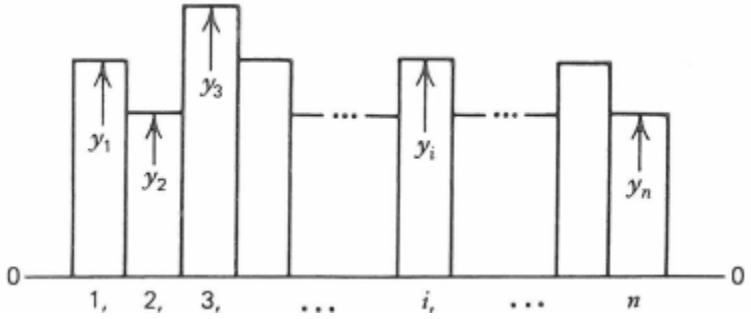
RESOUCE LEVELING: MINIMUM MOMENT METHOD

Which resource histogram (of given area) has minimum static moment?



$$\min M = \sum_{i=1}^{n} \frac{1}{2} y_i^2$$

$$\text{s.t.} \qquad \sum_{i=1}^{n} y_i = A$$

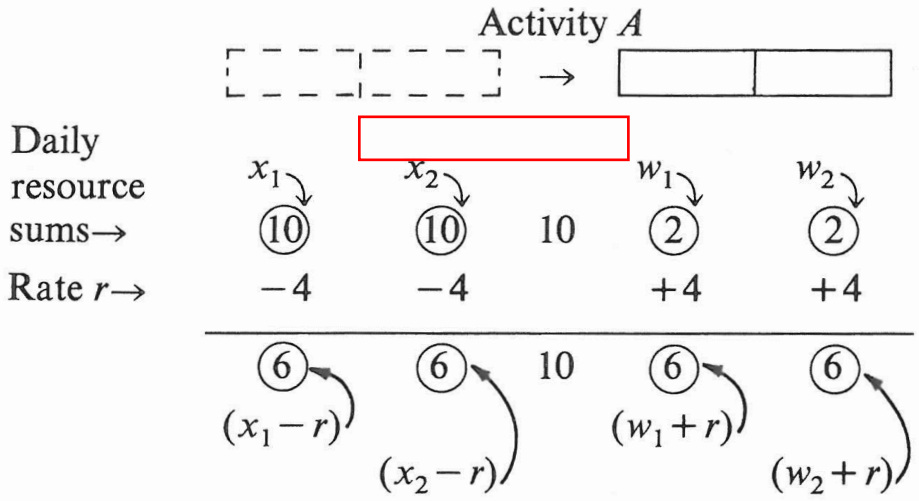
$$\mathcal{L}(y_i, \lambda) = \sum_{i=1}^{n} \frac{1}{2} y_i^2 - \lambda(\sum_{i=1}^{n} y_i - A)$$

$$\frac{\partial \mathcal{L}(y_i, \lambda)}{\partial y_i} = y_i - \lambda = 0 \qquad \Rightarrow \quad y_i = \lambda$$

$$\frac{\partial \mathcal{L}(y_i, \lambda)}{\partial \lambda} = -\sum_{i=1}^{n} y_i + A = 0 \quad \Rightarrow \quad \sum_{i=1}^{n} y_i = A$$

$$\Rightarrow \quad n\lambda = A \quad \Rightarrow \quad y_i = \lambda = \frac{A}{n}$$

Conclusion: A rectangular (smooth) resource histogram has minimum static moment.
Thus, an algorithm that minimizes static moment should lead to a smooth histogram.



Activity A

Duration, $T = 2$

Days to shift, $S = 3$

$m = \min(S, T)$

$= \min(3, 2)$

$= 2$

$$M_1 = 10^2 + 10^2 + 10^2 + 2^2 + 2^2 = 308$$

$$M_2 = 6^2 + 6^2 + 10^2 + 6^2 + 6^2 = 244$$

$$M_1 - M_2 = 308 - 244 = 64$$

$$\text{IF}(\text{Act}, S) = r\left(\sum_1^m x_i - \sum_1^m w_i - mr\right)$$

$$\text{IF}(A, 3) = 4[(10 + 10) - (2 + 2) - 2 \cdot 4]$$

$$\text{IF}(A, 3) = 4(20 - 4 - 8) = 32$$

Figure 11.9 Improvement factor relationships.

THE IMPROVEMENT FACTOR

An activity *ACT* requires *r* resources per day.
It will be shifted by *S* days to a later time.
The shift *S* can be in the range: $0 \le S \le FF$.

As a result of the shift *S*, the daily resource requirements for 2*m* days will change:
The daily resource requirements $x_1, x_2, ..., x_m$ for *m* days will be **decreased** by *r*.
The daily resource requirements $w_1, w_2, ..., w_m$ for *m* days will be **increased** by *r*.

If *S* is the shift and *T* is the duration of activity *ACT*, then $m = \min\{S,T\}$.

Moment of the resource histogram due to the daily resource requirements of the 2*m* days before the shift:

$$M_1 = \frac{1}{2}\sum_{i=1}^{m}(x_i^2 + w_i^2)$$

Moment of the resource histogram due to the resource requirements of the same 2*m* days after the shift:

$$M_2 = \frac{1}{2}\sum_{i=1}^{m}[(x_i - r)^2 + (w_i + r)^2]$$

Net reduction in the moment of the resource histogram caused by the shift *S*:

$$\begin{aligned} M_1 - M_2 &= \frac{1}{2}\sum_{i=1}^{m}[x_i^2 + w_i^2 - (x_i - r)^2 - (w_i + r)^2] \\ &= \frac{1}{2}\sum_{i=1}^{m}[r(2x_i - r) - r(2w_i + r)] \\ &= \frac{1}{2}r\sum_{i=1}^{m}(2x_i - 2w_i - 2r) \\ &= r(\sum_{i=1}^{m}x_i - \sum_{i=1}^{m}w_i - mr) \end{aligned}$$

This reduction in the resource histogram's static moment is called the Improvement Factor due to shifting activity *ACT* by a shift *S*:

$$IF(ACT, S) = r(\sum_{i=1}^{m}x_i - \sum_{i=1}^{m}w_i - mr)$$

Setup

A. Prepare an arrow or precedence network of the project and determine the sequence step number for each activity.
B. Prepare a link matrix and compute the *ESD* and *EFD* for each activity. Compute the lags for each link. Determine the total float for each activity and identify the critical path.
C. Select a resource to be leveled.
D. Determine the amount of the selected resource which is required per day for each network activity. Call this required resource rate, "*r*."
E. Prepare a bar chart showing the early start time for each activity and its duration. Plot critical activities first, followed by noncritical activities.
F. For each day of the project, total the resource rates at the foot of the bar chart to obtain the daily resource sums.

Procedure

The procedure begins with the activities on the last sequence step.

1. Examine the activities on the sequence step.
   (a) Every activity on the sequence step having zero free float is passed over.
   (b) Every activity on the sequence step having a zero resource rate is shifted to the limit of the activity free float to allow preceding activities to be shifted.
   (c) For each activity on the sequence step having a positive resource rate, determine the extent of its free float and compute improvement factors for all possible days that the activity can be shifted until the free float limit is reached.
2. Select the activity having the largest improvement factor determined in algorithm step 1(c).
   (a) If the largest improvement factor is negative, no shifting takes place. Go to the next sequence step and examine the activities as in algorithm step 1.
   (b) If there is a tie in the value of the largest improvement factor for several activities:
      (i) Select the activity with the greatest "*r*" value.
      (ii) If still tied, select the activity which creates the greatest free float for preceding activities.
      (iii) If still tied, select the activity with the latest start date.
      (iv) If still tied, select the first activity in the queue.
3. Shift the selected activity subject to the following:
   (a) If the largest improvement factor is zero or positive, shift the selected activity by subtracting the activity resource rate from each of the daily resource sums at the position being vacated and adding this same rate to each of the daily resource sums at the position being occupied.
   (b) If there is a tie in the value of the improvement factor at several of the possible activity positions, shift the activity the greatest number of time units.
4. If shifting has occurred in algorithm step 3, update the lags, *ESD* and *EFD*, in the link matrix.
5. Reexamine the activities on the sequence step and repeat algorithm steps 1 through 4 until all shifting is complete on this step.
6. Examine the next earlier sequence step and repeat algorithm steps 1 through 5. Continue in this manner until all activities have been considered and all possible shifting has taken place on every sequence step.
7. Repeat algorithm steps 1 through 6 until no further shifting takes place. This is the end of the forward cycle.
8. Beginning with the first sequence step, using back float instead of free float, and progressing to the next latest sequence step instead of to the next earliest sequence step, repeat algorithm steps 1 through 7 until all activities have been considered and shifted, where possible, to an earlier time position. This is the end of the backward cycle and completes the leveling.

The example chosen for discussion is the same one that was used in Section 11.4. The diagram may be either the one shown in Figure 11.3 or Figure 11.4. The activity durations and the resource rates are taken from Table 11.1.

Next Sequence Step

Same Sequence Step

1. For current Seq. Step:

Compute all possible IF(Act,S)

2. Select max IF(Act,S) to find Act and shift S.

3. Shift activity Act by shift S.

4. Update Link Matrix.

5. Repeat 1-4 for the same Sequence Step!

6. Goto next earlier Sequence Step!

7. Repeat Forward Pass 1-6!

8. Backward Pass Repeat 1-7 in reverse!

TABLE 11.2

Sequence Step Numbers Computed for an Arrow Network

*I**	*J*	ACT	Seq. Step Value	Start Step No.	End Step No.
1	2	A	1	1†	2
1	4	B	1	1	2
2	5	C	1	2	3
2	3	D	1	2	3
3	4	Dummy	0	3	3
3	5	F	1	3	4
4	5	E	1	3	4

Sequence step numbers

**Refer to Figure 11.3.*

†*Initial number assumed to be 1.*

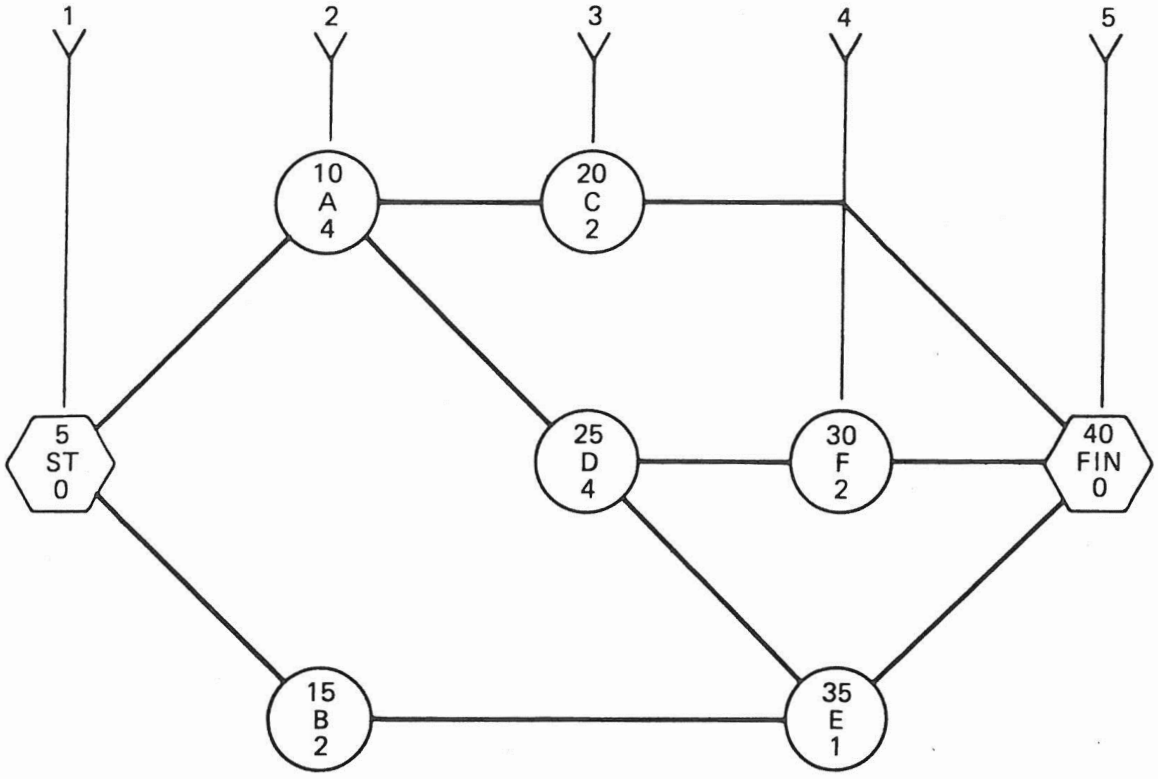


Figure 11.4 Precedence diagram for leveling example.

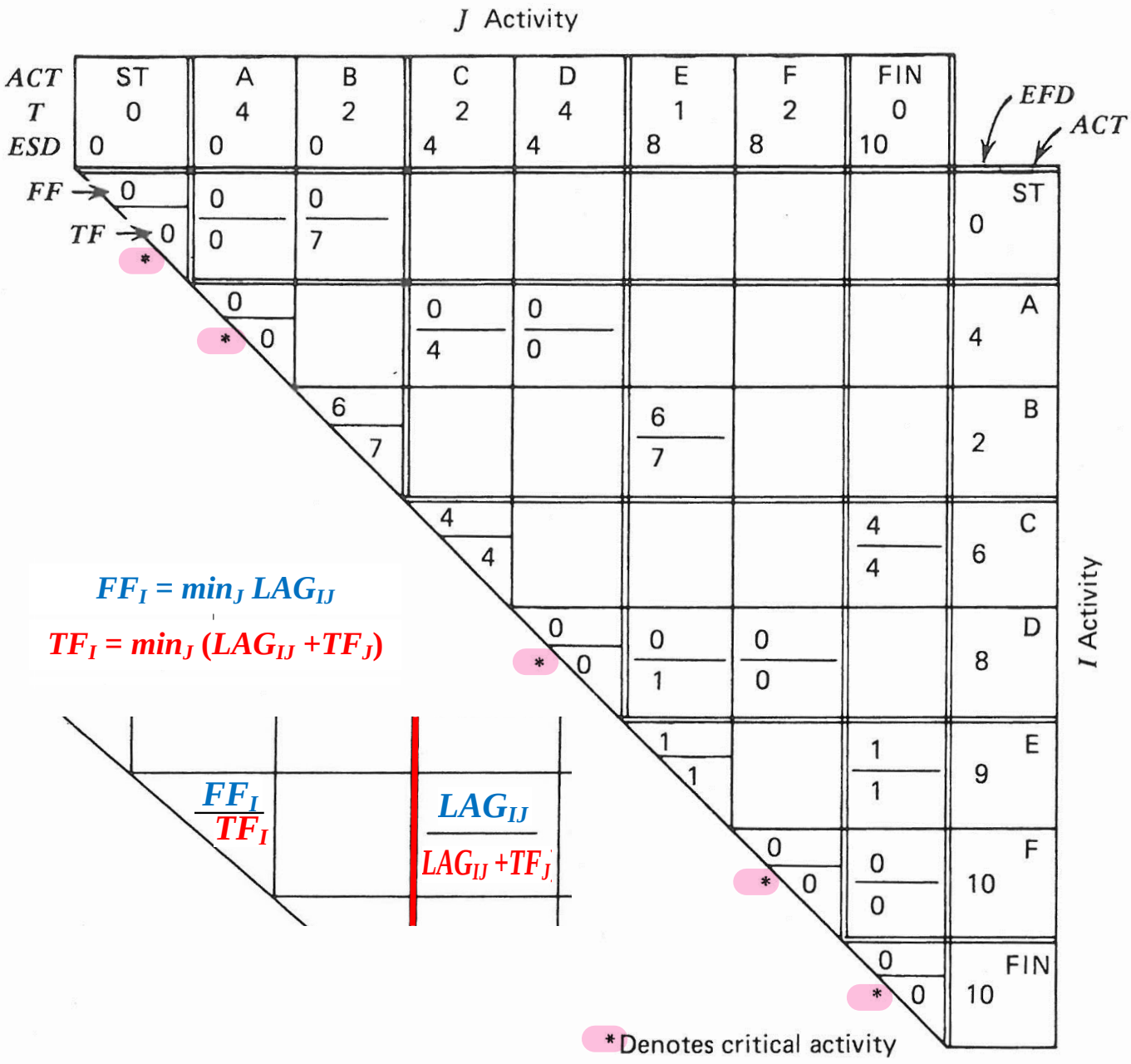
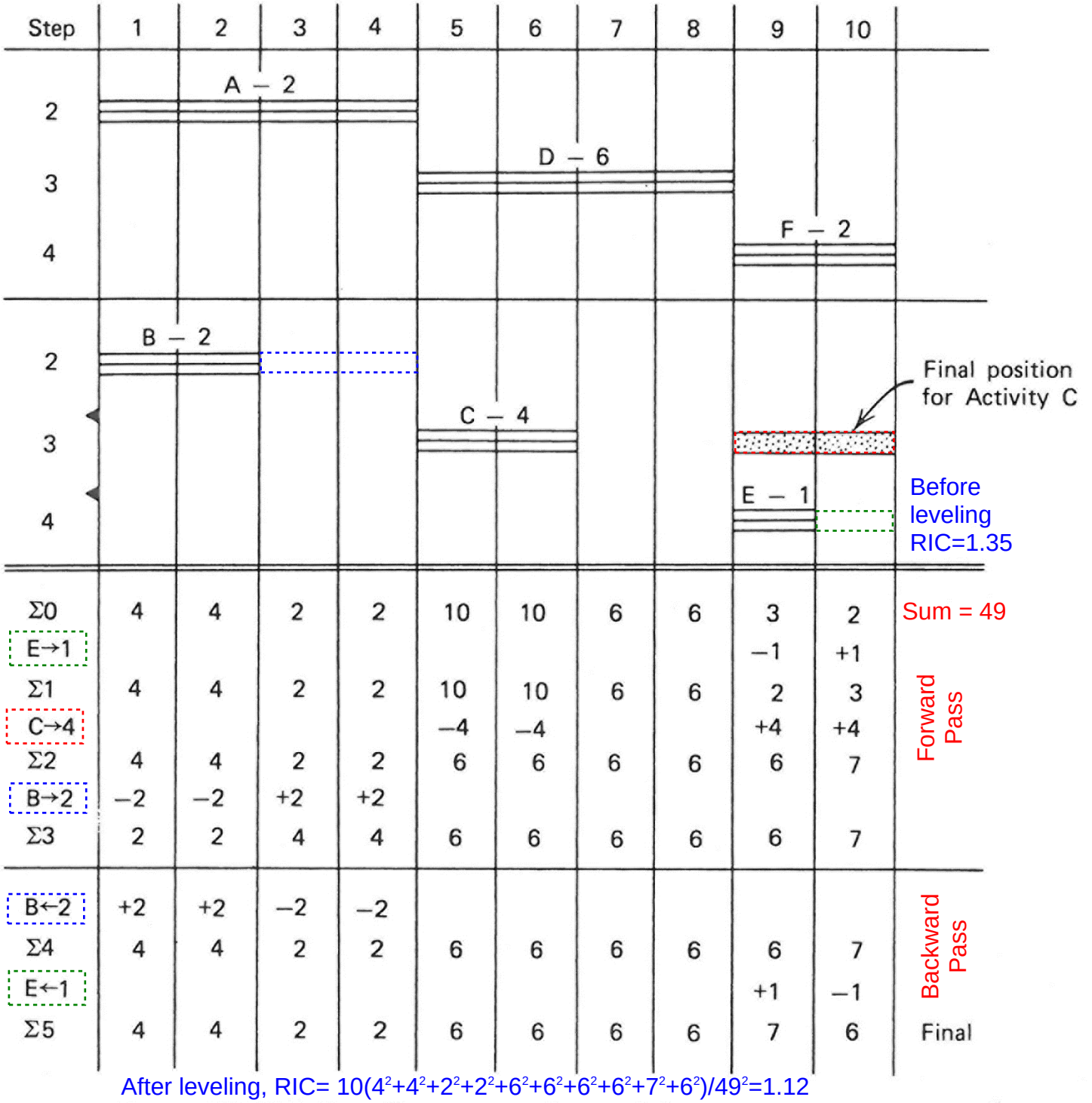


Figure 11.11 Link matrix for leveling example before leveling.

$$IF(Act,S) = r\left(\sum_{1}^{m} x_i - \sum_{1}^{m} w_i - mr\right)$$



After leveling, RIC= $10(4^2+4^2+2^2+2^2+6^2+6^2+6^2+6^2+7^2+6^2)/49^2=1.12$

Figure 11.12 Leveling example—minimum moment approach.

$$RIC = \frac{M_1'}{M_0'} = \frac{n\sum_{i=1}^{n} y_i^2}{\left(\sum_{i=1}^{n} y_i\right)^2} = \frac{n\sum_{i=1}^{n} y_i^2}{A^2} \geq 1$$

MINIMUM MOMENT METHOD

Forward Pass

S.S. 4

F: has FF=0, bypass (1a)

IF(E,1) = 1(3 – 2 - 1*1) = 0 (*)
S=1 (3a)
Update barchart and matrix

S.S. 3

D: has FF=0, bypass (1a)

IF(C,1) = 4[(10) – (6) - 4*1) = 0
IF(C,2) = 4[(10+10) – (6+6) - 4*2) = 0
IF(C,3) = 4[(10+10) – (6+2) - 4*2) = 16
IF(C,4) = 4[(10+10) – (2+3) - 4*2) = 28 (*)
S=4 (3a)
Update barchart and matrix

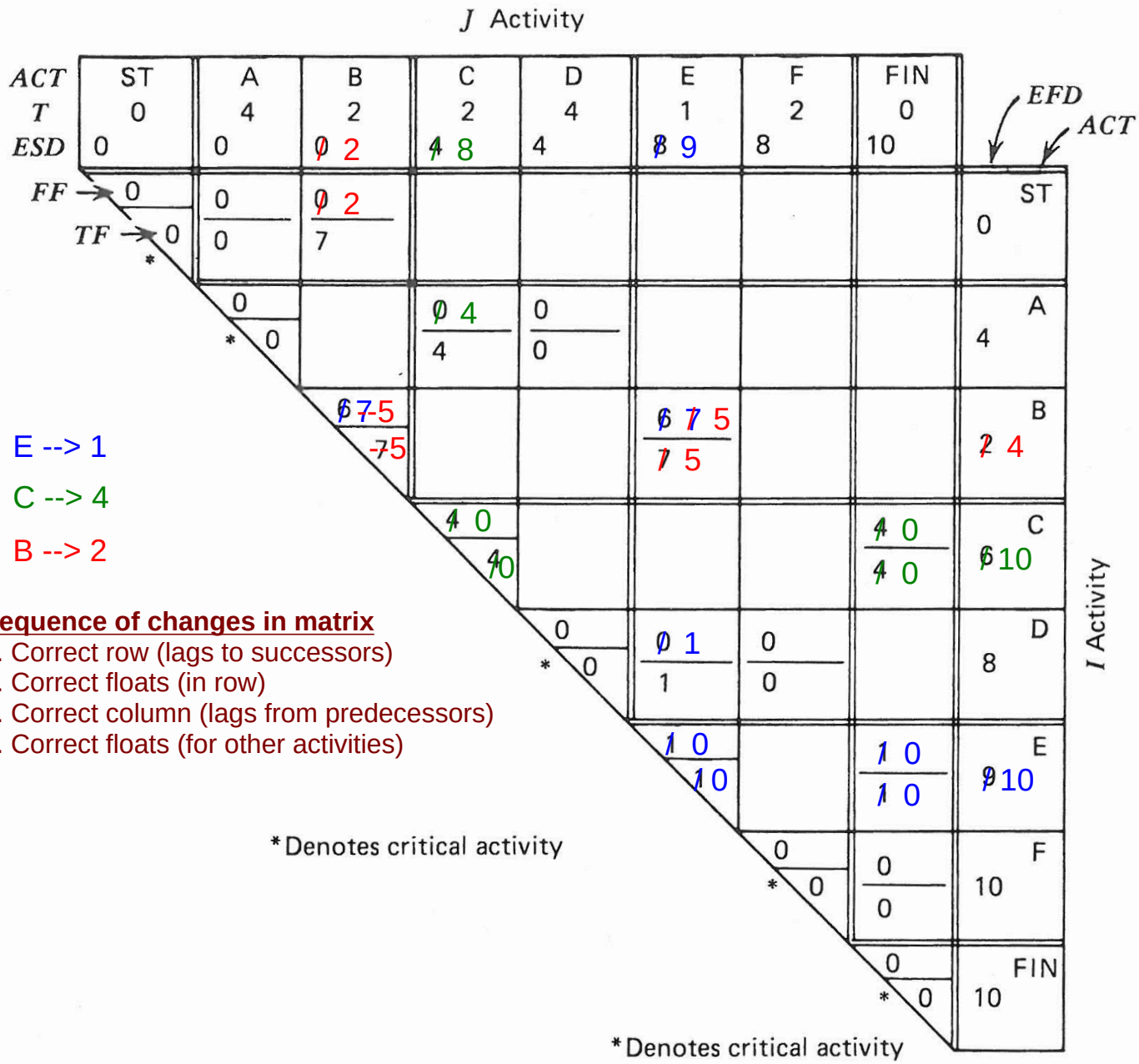
S.S. 2

A: has FF=0, bypass (1a)

IF(B,1) = 2[(4) – (2) - 1*2) = 0
IF(B,2) = 2[(4+4) – (2+2) - 2*2) = 0 (*)
IF(B,3) = 2[(4+4) – (2+6) - 2*2) = -8
IF(B,4) = 2[(4+4) – (6+6) - 2*2) = -16
…
IF(B,7) = 2[(4+4) – (6+6) - 2*2) = -16
S=2 (3a, 3b)
Update barchart and matrix

End of Forward Pass

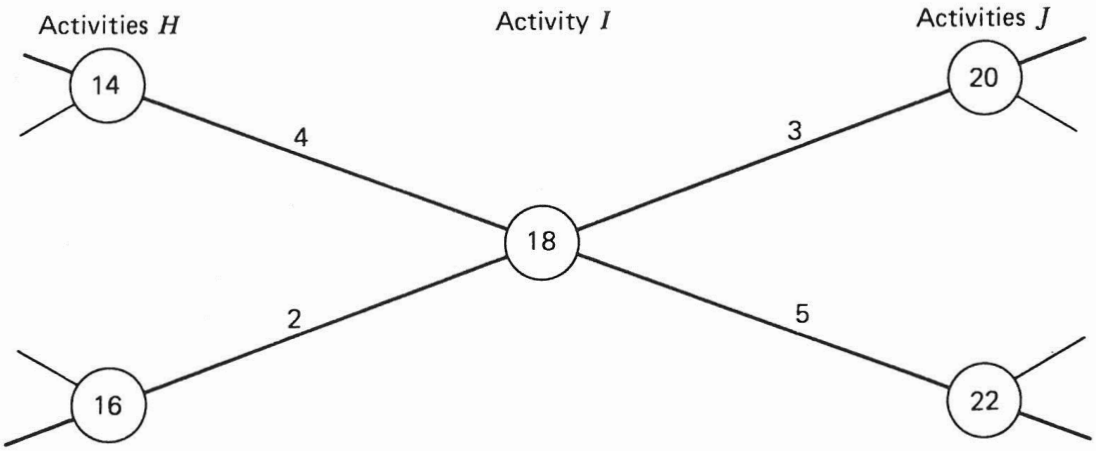
Forward Pass



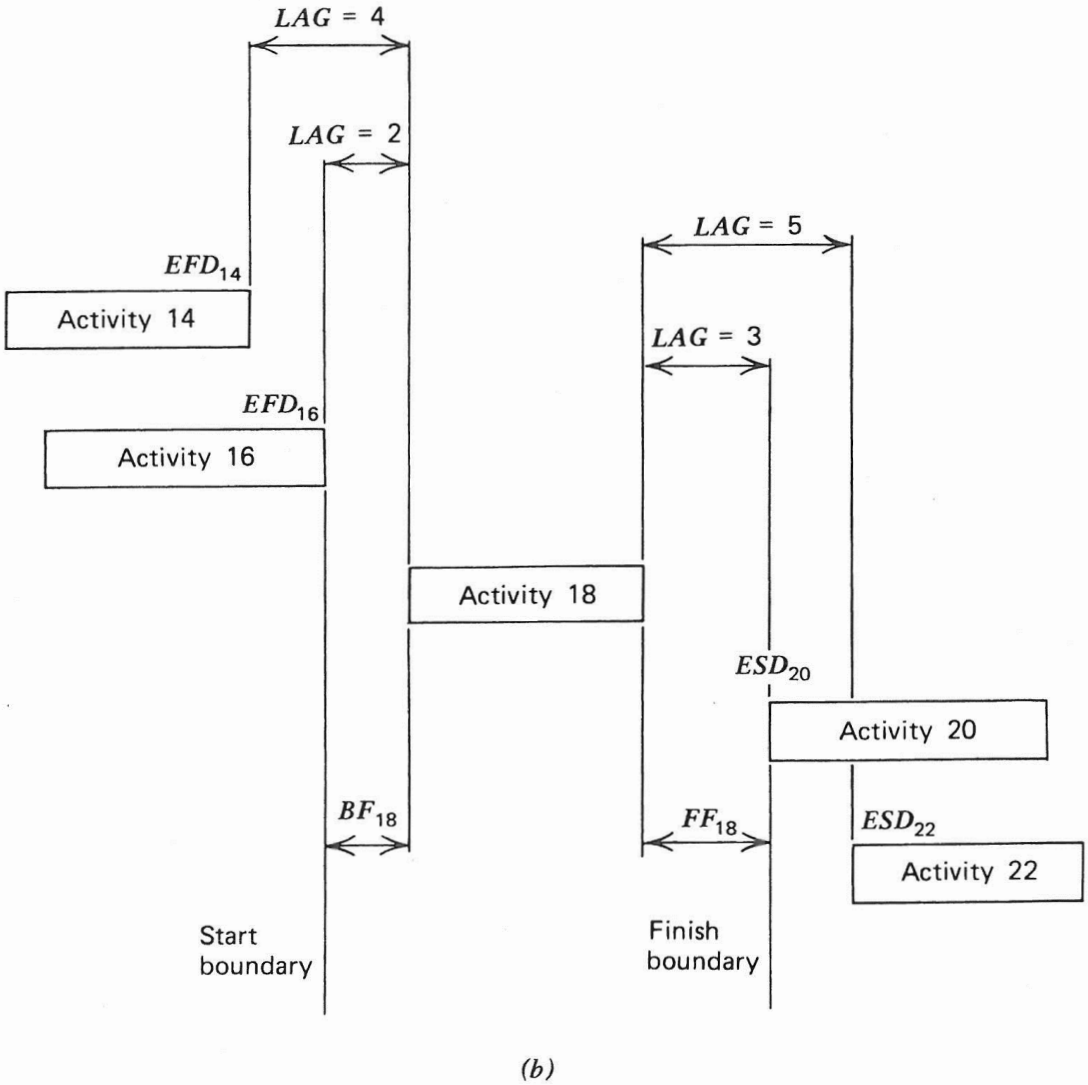
Sequence of changes in matrix

1. Correct row (lags to successors)
2. Correct floats (in row)
3. Correct column (lags from predecessors)
4. Correct floats (for other activities)

Figure 11.11 Link matrix for leveling example before leveling.



(a)



(b)

Figure 11.10 Free float and back float relationships.

MINIMUM MOMENT METHOD

Backward Pass

S.S. 1

ST: has BF=0, bypass (1a)

S.S. 2

A: has BF=0, bypass (1a)

IF(B,1) = 2[(4) – (2) - 1*2) = 0
IF(B,2) = 2[(4+4) – (2+2) - 2*2) = 0 (*)
S=2 (3a, 3b)
Update barchart and matrix

S.S. 3

D: has BF=0, bypass (1a)

IF(C,1) = 4[(7) – (6) - 4*1) = -12
IF(C,2) = 4[(7+6) – (6+6) - 4*2) = -28
IF(C,3) = 4[(7+6) – (6+6) - 4*2) = -28
IF(C,4) = 4[(7+6) – (2+6) - 4*2) = -28
Do Not Shift C.

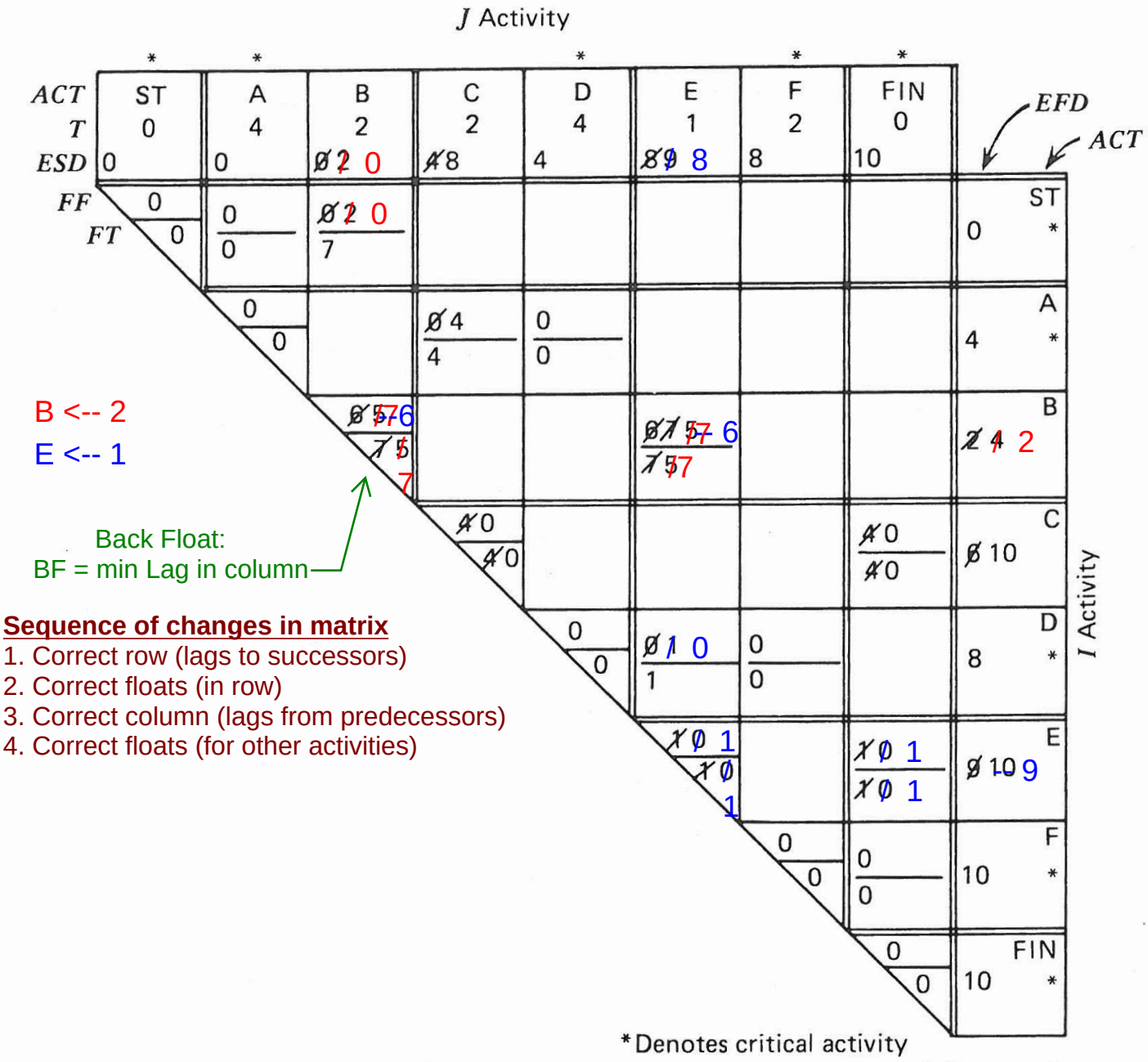
S.S. 4

F: has BF=0, bypass (1a)

IF(E,1) = 1(7 – 6 - 1*1) = 0 (*)
S=1 (3a)
Update barchart and matrix

End of Backward Pass
End of Algorithm

Backward Pass



Sequence of changes in matrix

1. Correct row (lags to successors)
2. Correct floats (in row)
3. Correct column (lags from predecessors)
4. Correct floats (for other activities)

Figure 11.13 Link matrix for leveling example of end of forward pass.

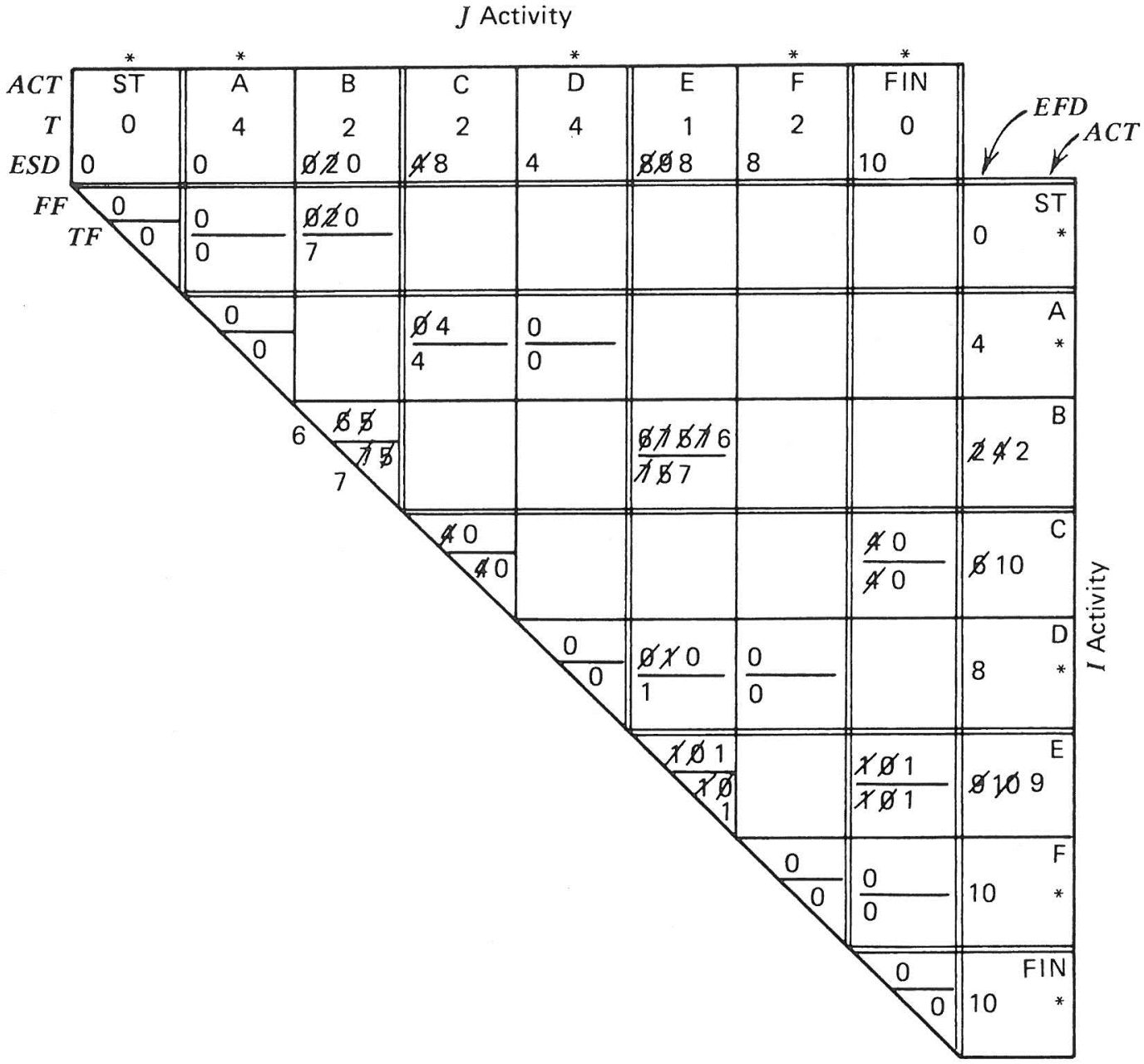


Figure 11.14 Link matrix for leveling example at end of backward pass.

Resource Improvement Coefficient, RIC

RIC measures "how good is the leveled histogram relative to the absolute best".

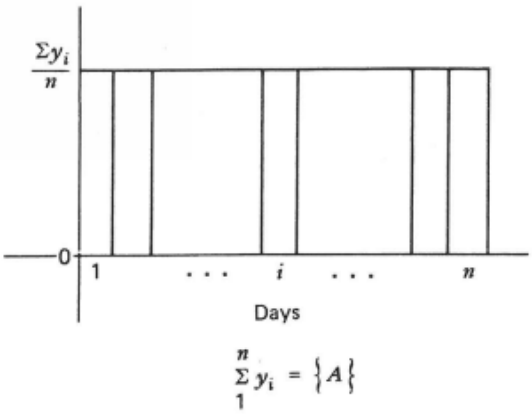


Figure 11.15 Ideal histogram.

(Static) Moment of ideal resource histogram:

$$M_0' = \frac{1}{2}hA = \frac{1}{2}\frac{A}{n}A = \frac{1}{2n}A^2 = \frac{1}{2n}\left(\sum_{i=1}^{n} y_i\right)^2$$

(Static) Moment of leveled resource histogram:

$$M_1' = \sum_{i=1}^{n}\frac{1}{2}y_i^2 = \frac{1}{2}\sum_{i=1}^{n} y_i^2$$

Cauchy–Schwarz inequality: $$n\sum_{i=1}^{n} y_i^2 \geq \left(\sum_{i=1}^{n} y_i\right)^2 \Rightarrow M_1' \geq M_0'$$

$$RIC = \frac{M_1'}{M_0'} = \frac{n\sum_{i=1}^{n} y_i^2}{\left(\sum_{i=1}^{n} y_i\right)^2} = \frac{n\sum_{i=1}^{n} y_i^2}{A^2} \geq 1$$

$$\sum\begin{bmatrix} y_1^2 & y_1^2 & \cdots & y_1^2 \\ y_2^2 & y_2^2 & \cdots & y_2^2 \\ \vdots & \vdots & \ddots & \vdots \\ y_n^2 & y_n^2 & \cdots & y_n^2 \end{bmatrix} \geq \sum\begin{bmatrix} y_1^2 & y_1y_2 & \cdots & y_1y_n \\ y_2y_1 & y_2^2 & \cdots & y_2y_n \\ \vdots & \vdots & \ddots & \vdots \\ y_ny_1 & y_ny_2 & \cdots & y_n^2 \end{bmatrix}$$

RIC and Statistics

Sample Average: $$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i = \frac{A}{n}$$

Sample Variance: $$S^2 = \frac{1}{n}\sum_{i=1}^{n}\left(y_i - \bar{y}\right)^2 = \frac{1}{n}\sum_{i=1}^{n} y_i^2 - \bar{y}^2$$

(Square of the) Coefficient of Variation (measures relative variability):

$$V^2 = \frac{S^2}{\bar{y}^2} = \frac{\frac{1}{n}\sum_{i=1}^{n} y_i^2 - \bar{y}^2}{\bar{y}^2} = \frac{\frac{1}{n}\sum_{i=1}^{n} y_i^2}{\left(A/n\right)^2} - 1 = \frac{n\sum_{i=1}^{n} y_i^2}{A^2} - 1 = RIC - 1$$

Thus, $RIC = V^2 + 1$ measures the relative variability at the top of the resource histogram when compared to the average height of the resource histogram.