Ch. 10 - TIME-COST TRADEOFF

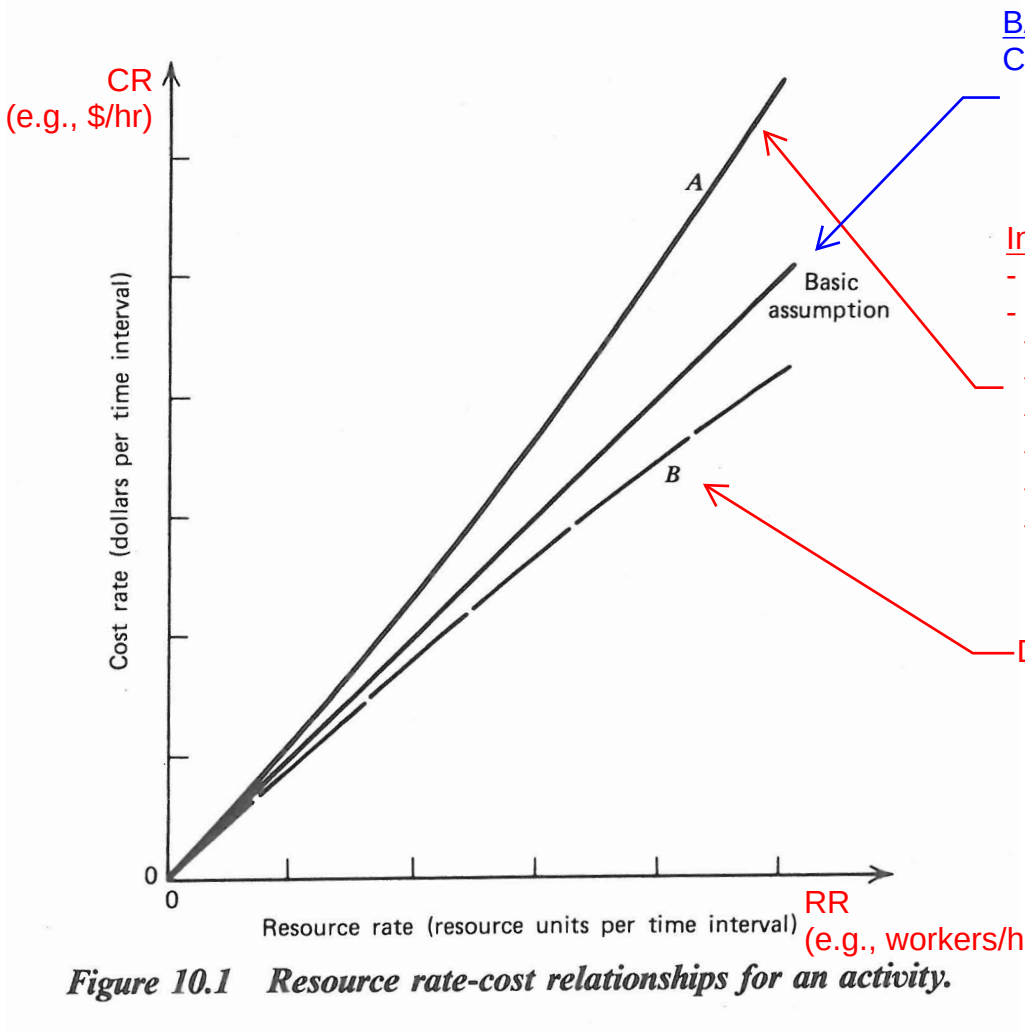


Figure 10.1 Resource rate-cost relationships for an activity.

BASIC ASSUMPTION (Ideal conditions):
Cost Rate / Resource Rate =
= $/hr / Workers/hr
= $ / worker
= Constant slope

In practice:
- Least costly resources applied first.
- Successive resources tend to cost more due to:
  * Supervision
  * Union Rules
  * Buying or leasing new/additional equipment
  * Using material suppliers with higher prices
  * Increased OH
  * etc.

Does not occur in practice

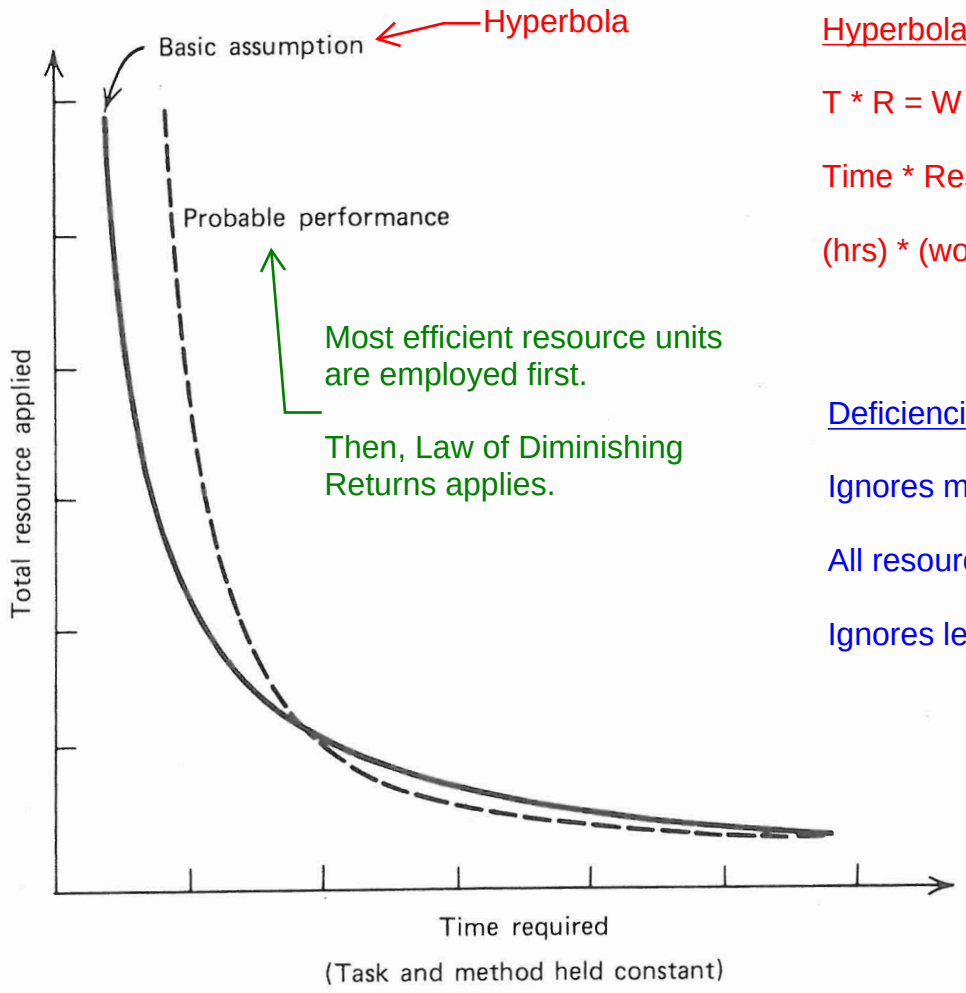


Figure 10.2 Resource-time relationships for an activity.

Hyperbola (basic assumption):

T * R = W

Time * Resources = Work content

(hrs) * (workers) = (manhours)

Deficiencies of Basic Assumption :

Ignores max & min crew sizes

All resource units assumed equally efficient

Ignores learning-curve effects for repetitive tasks

EXAMPLE

TABLE 10.1

Time (hrs)	Laborers	Hourly Rate	Remarks
56.0	1	$10	
22.5	2	20	
18.7	3	32	Third man has a longevity rate of $12 per hour
17.5	4	44	Second $12 per hour man added
16.0	5	59	Forman added at $15 per hour

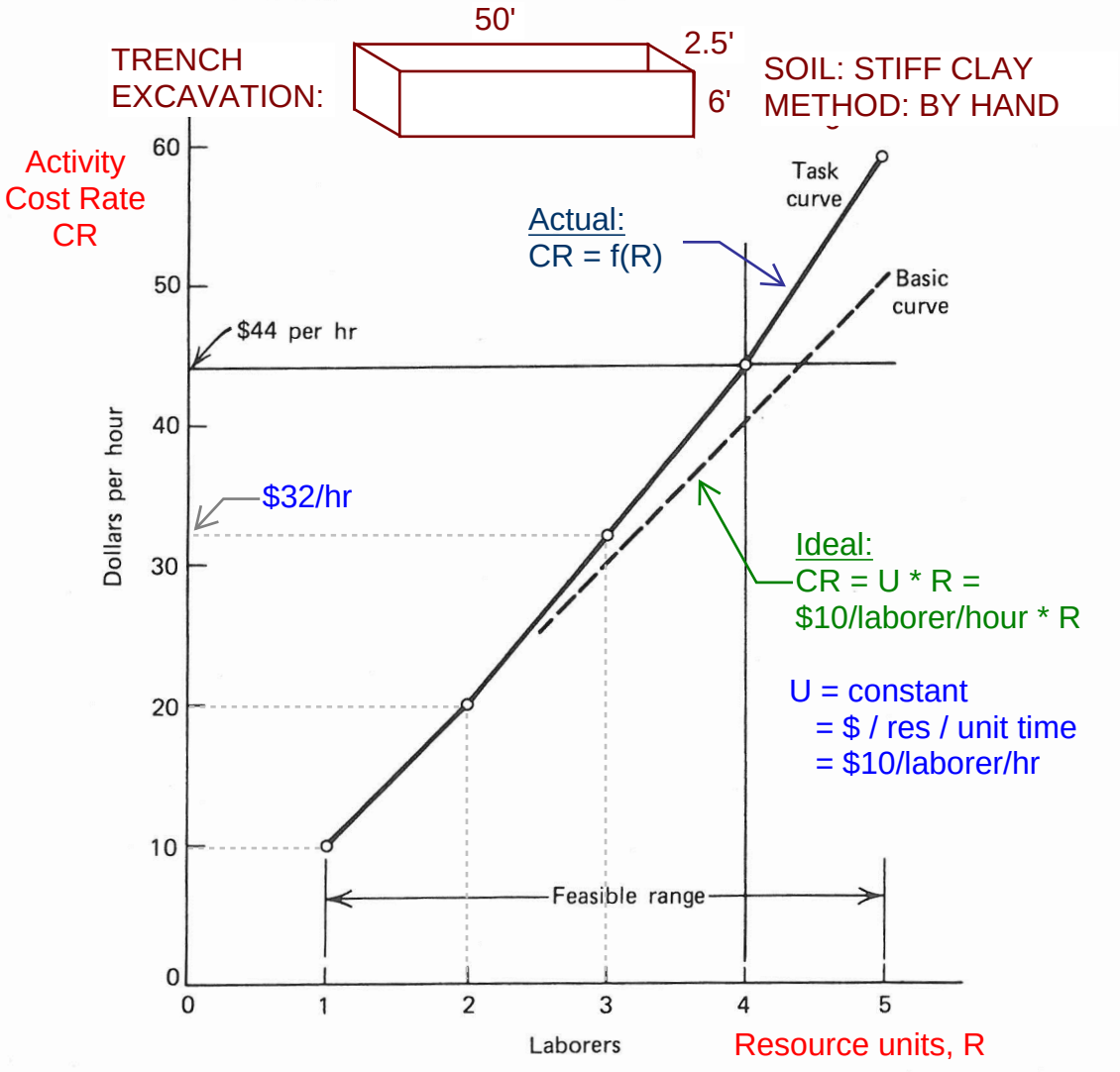


Figure 10.3 Resource-cost curve for a trenching activity.

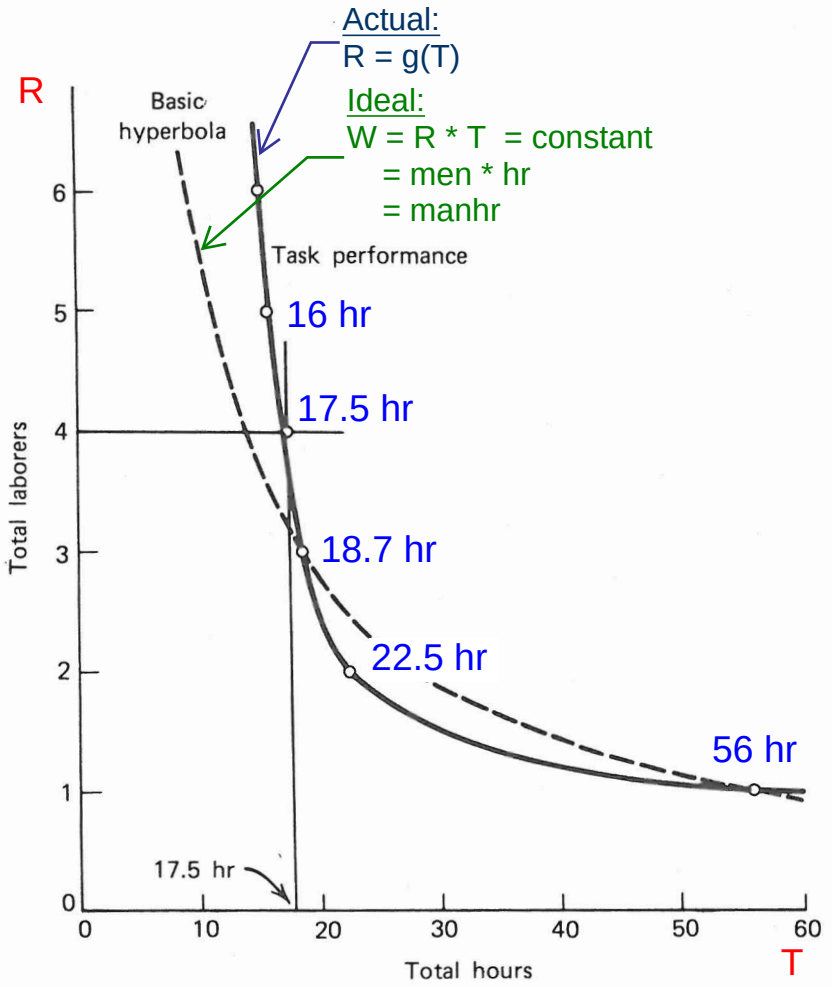


Figure 10.4 Time-resource curve for a trenching activity.

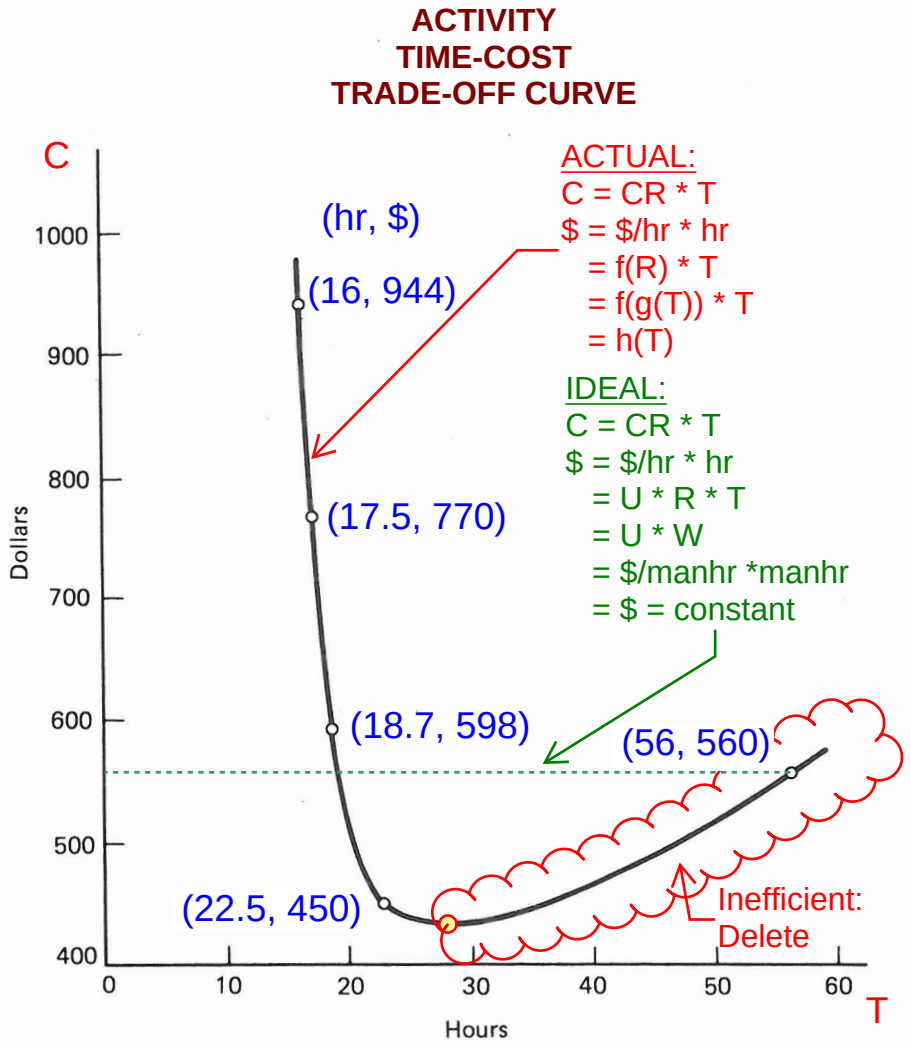


Figure 10.5 Time-cost curve for a trenching activity.

Slope: S

$$S = \frac{CC - CN}{TN - TC} = \frac{\Delta C}{\Delta T}$$

To make analysis by hand easier
use absolute value, S > 0.

= CRASH point
= min TIME

Points above the Time-Cost curve are feasible

E = Estimated
(if E is on T-C curve, then it is efficient)

= NORMAL point
= min COST

Assumptions
Time-Cost curve exists
Time-Cost is linear
- Method does not change
- No extreme events
- Curve is continuous (not discrete)

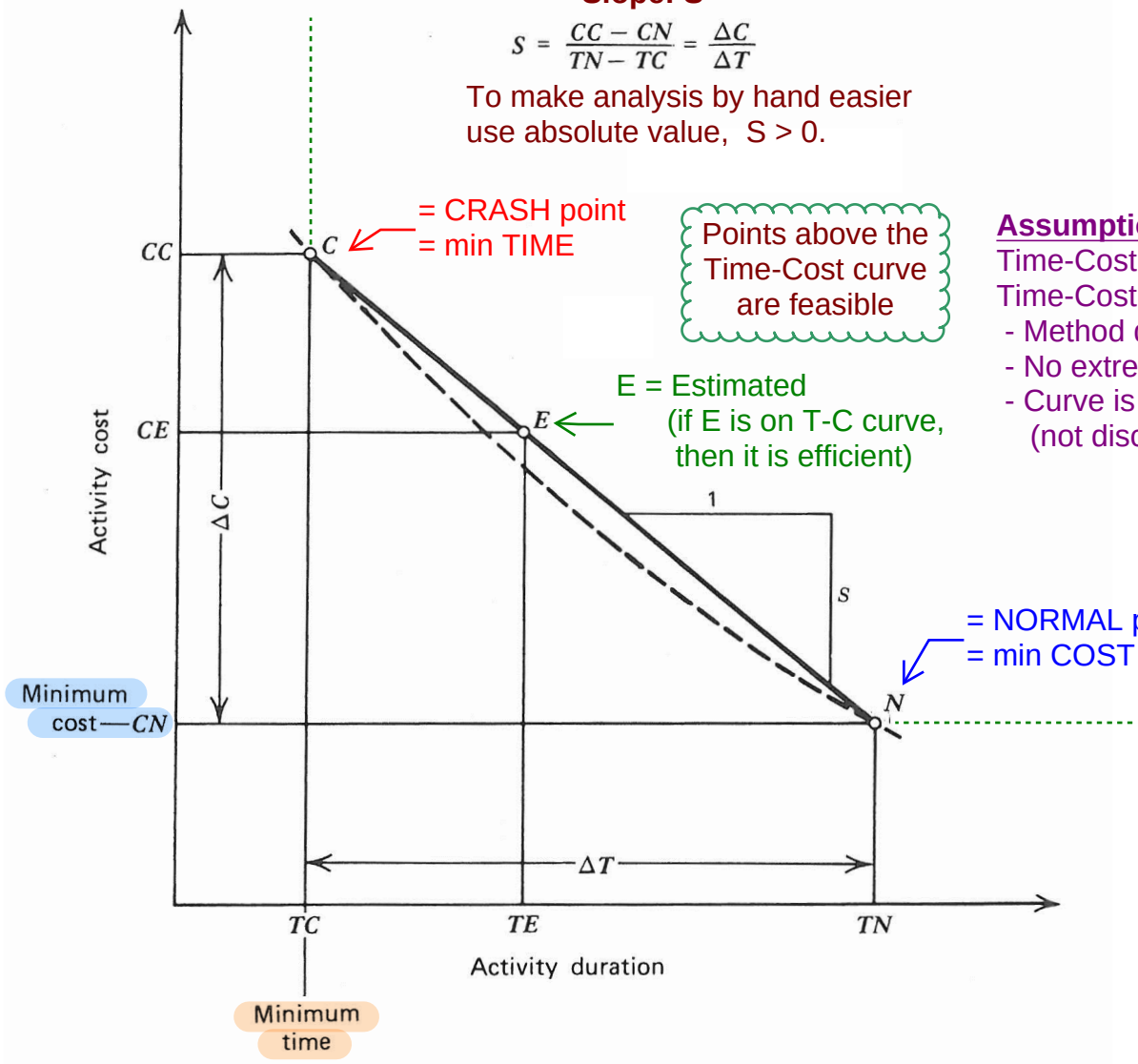


Figure 10.6 Activity time-cost relationships.

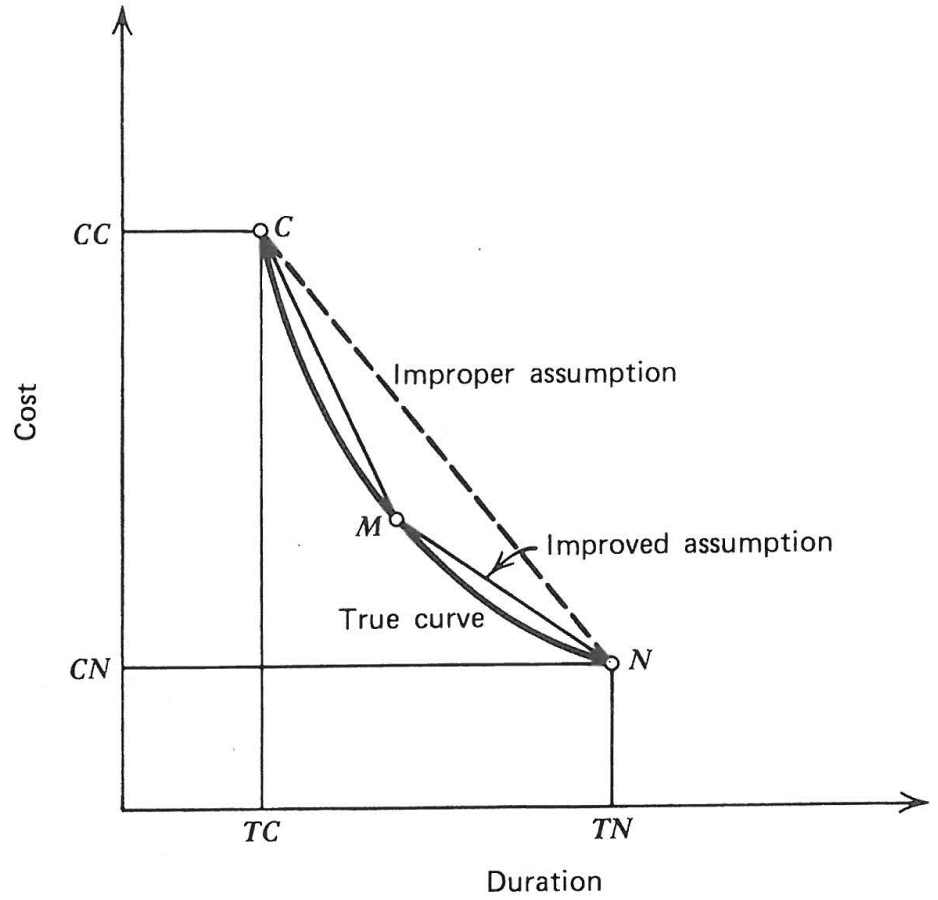


Figure 10.7 Creation of a piecewise linear time-cost curve.

How to Split a Convex Piecewise Linear Time-Cost Curve

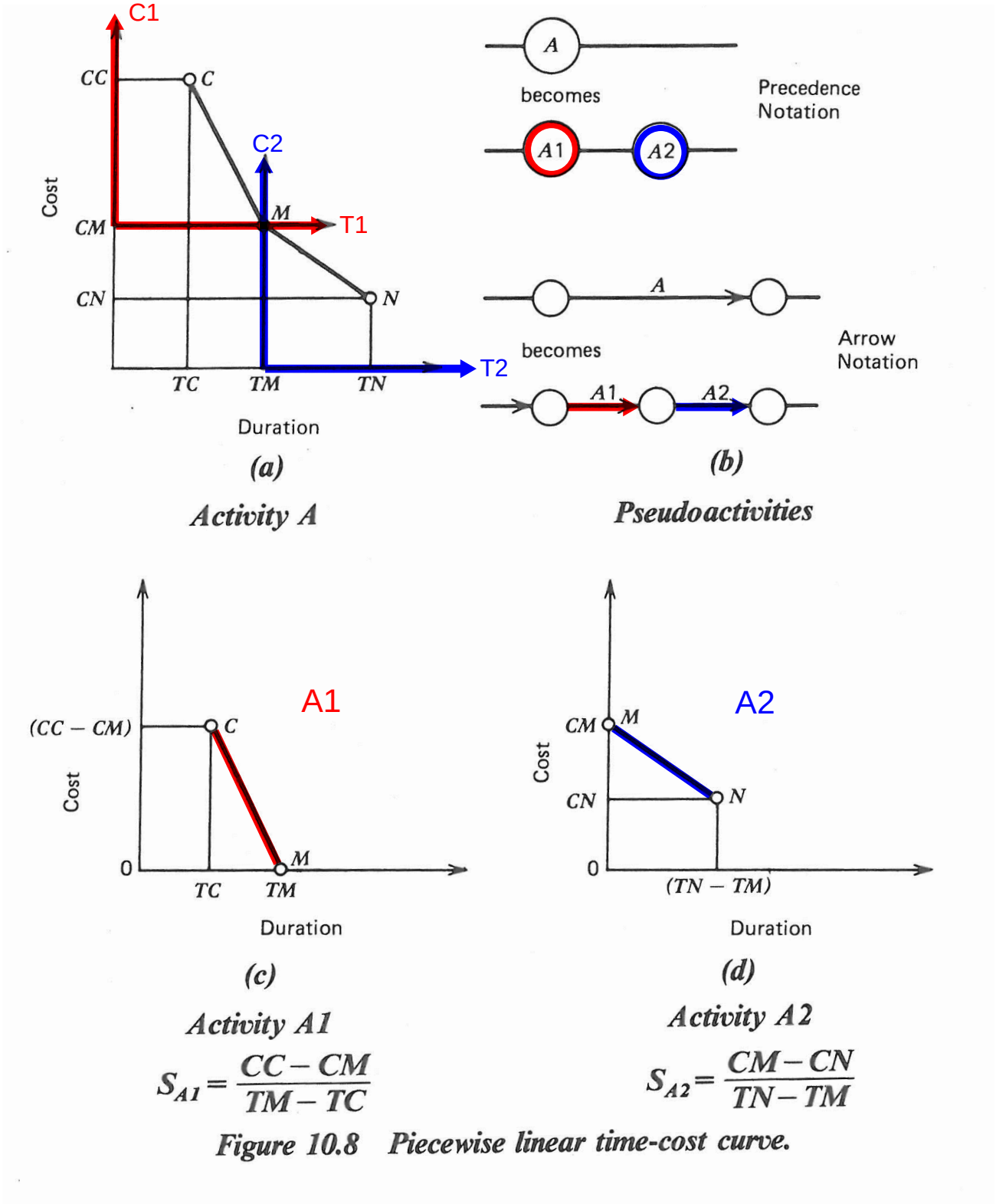


Figure 10.8 Piecewise linear time-cost curve.

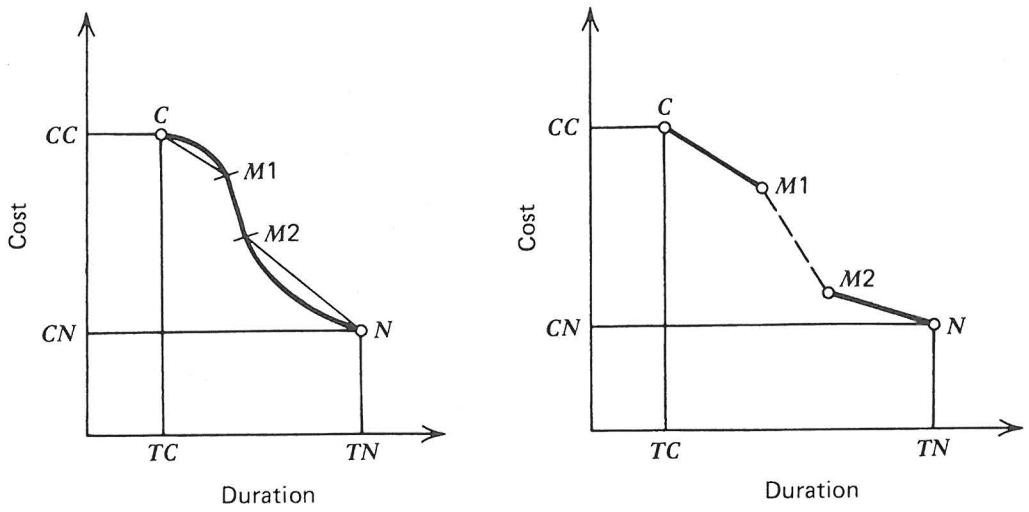


Figure 10.9 Varying time-cost curves.

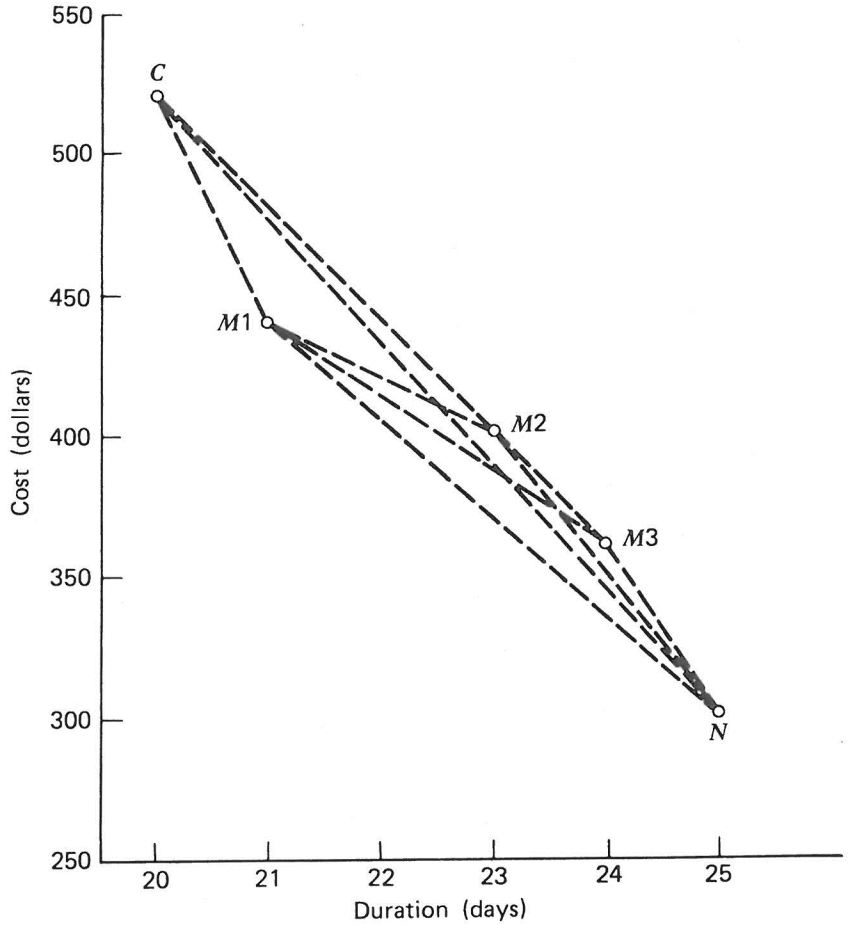


Figure 10.10 Activity with discrete time-cost values.

			To				
	Points	Days	C	M1	M2	M3	N
			20	21	23	24	25
FROM	C	20	0	−80	−40	−40	−44
	M1	21	80	0	−20	−26.7	−35
	M2	23	40	20	0	−40	−50
	M3	24	40	26.7	40	0	−60
	N	25	44	35	50	60	0

Figure 10.11 Cost slope matrix—activity with discrete time-cost values.

TABLE 10.3

Duration Times and Costs for an Activity with Discrete Time-Cost Values

Point	Duration Time	Cost
C	20 days	$520
M1	21	440
M2	23	400
M3	24	360
N	25	300

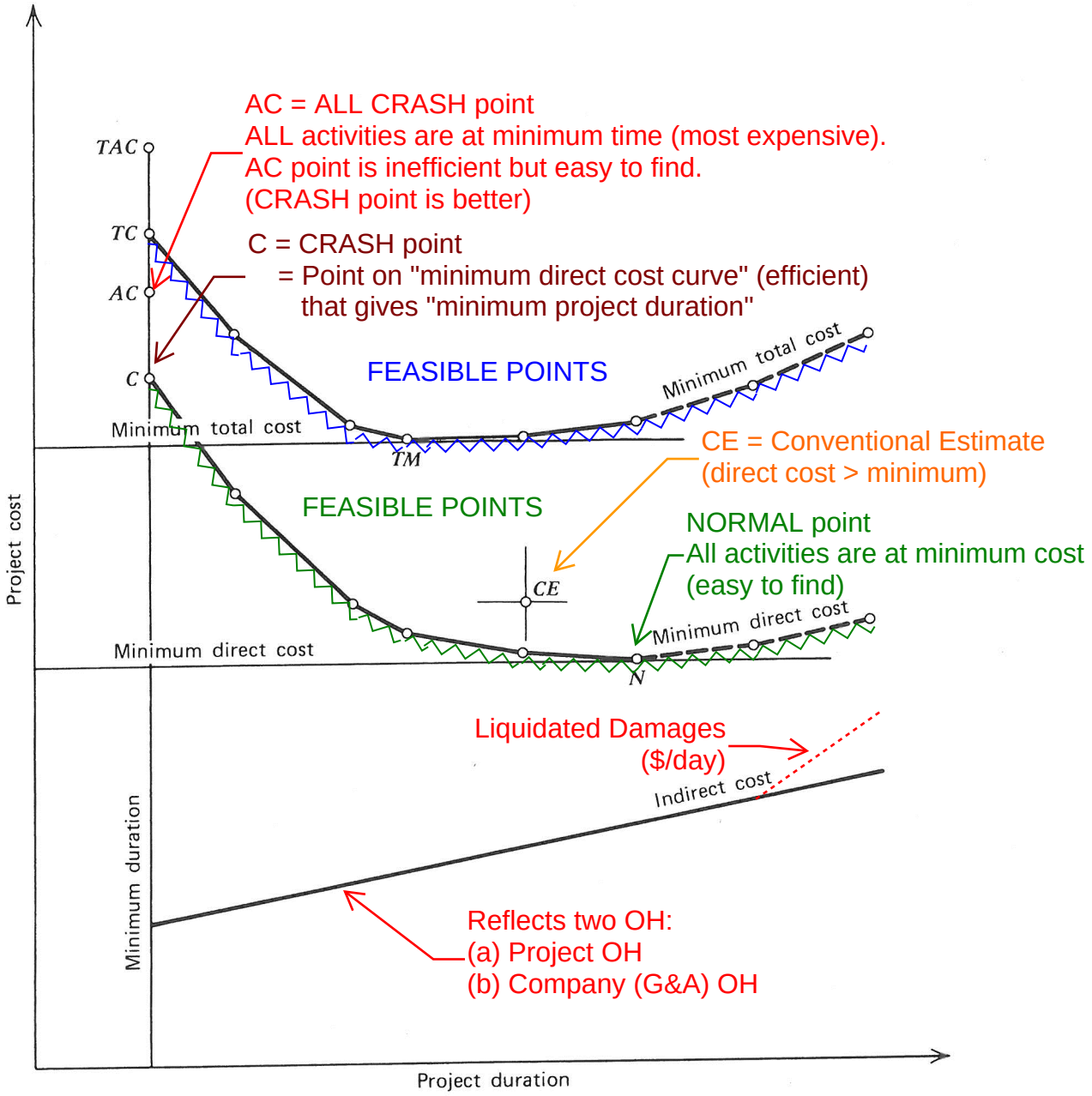
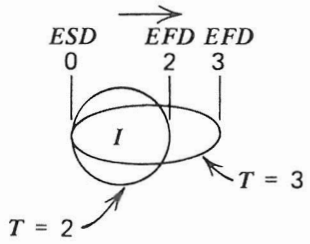


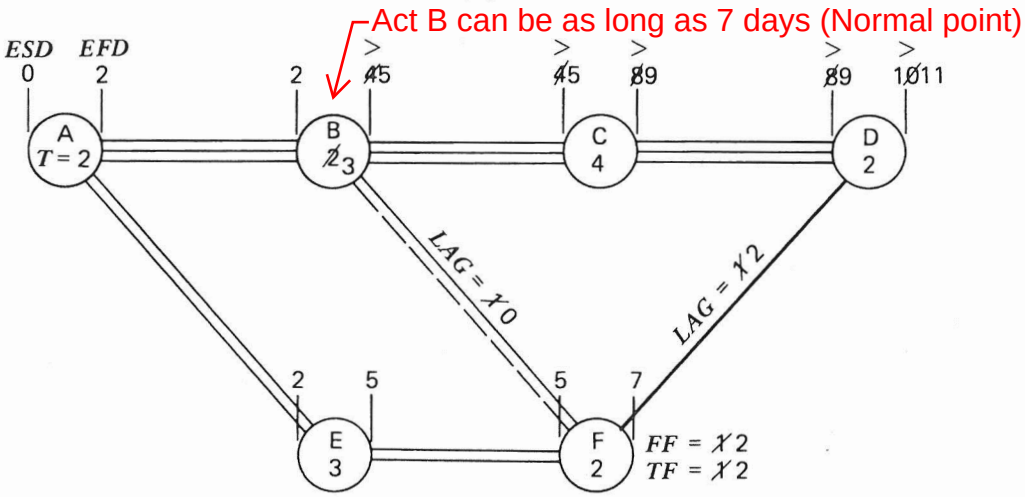
Figure 10.12 Project time-cost relationships.

Effects on Network from Lengthening/Shortening Activity Durations

Assumptions:
* Use ESD (Early Start)
* No breaks in Activities



(a)



(b)

Activity B, lengthened one day

Links							
I Act	*J Act*	*I EFD*	*J ESD*	*LAG*	*I EFD*	*J ESD*	*Rev. LAG*
A	B	2	2	0			
A	E	2	2	0			
B	C	4	4	0	≫	>	
B	F	4	5	1	≫		0
E	F	5	5	0			
C	D	8	8	0	>	>	
F	D	7	8	1		>	2

Network limit

(c)

Figure 10.13 Network limit concepts.

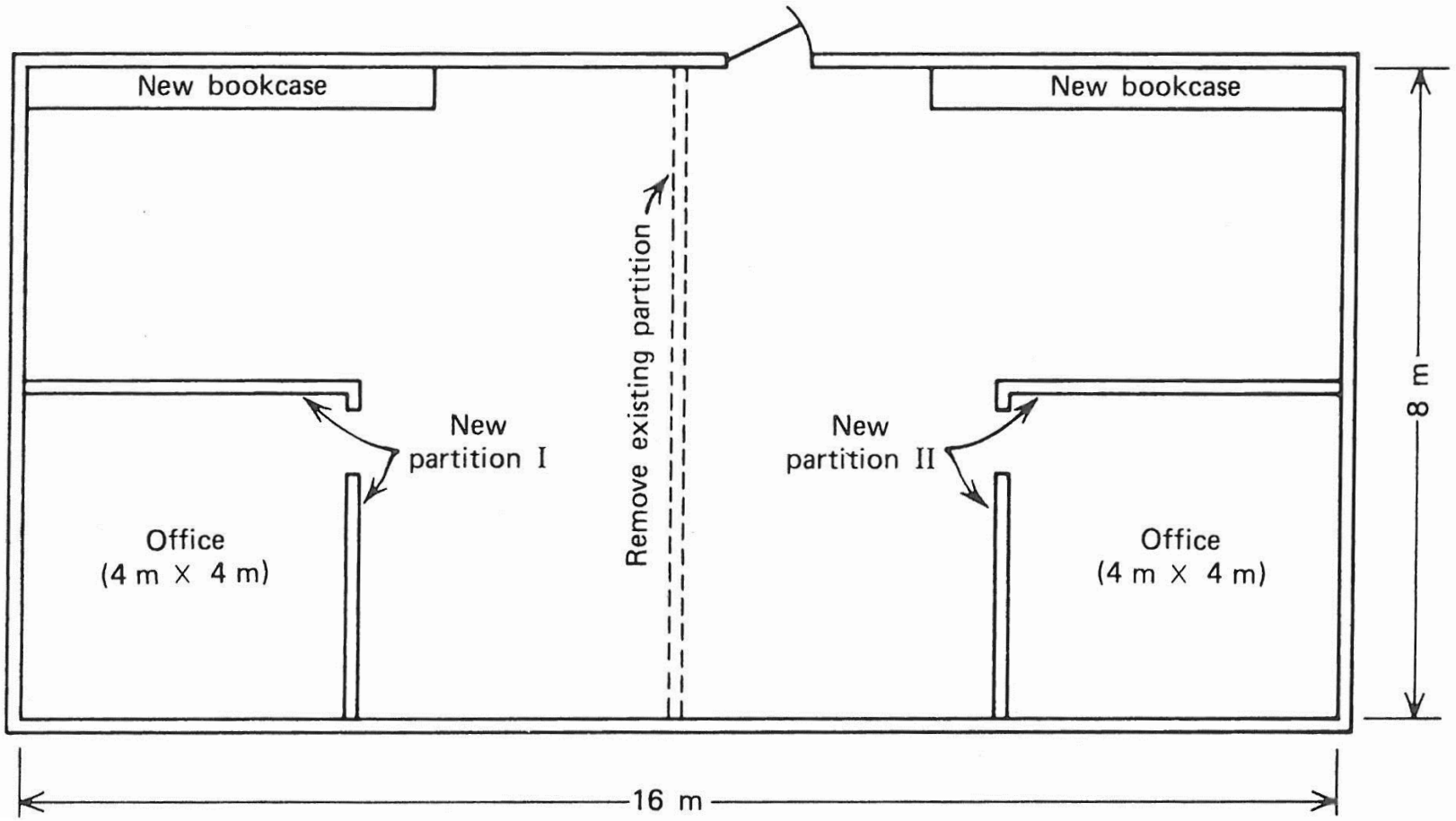


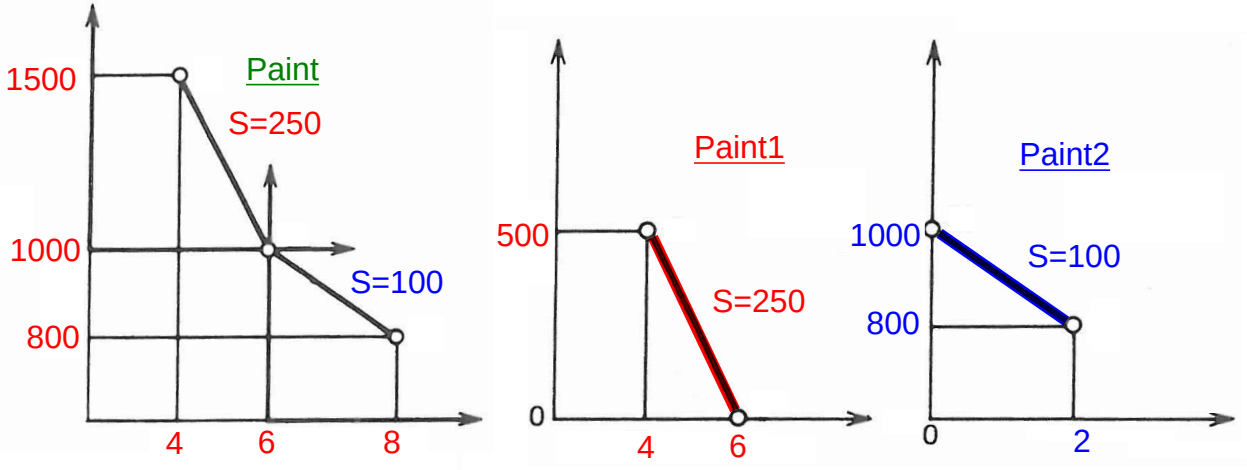
Figure 10.14 Plan for remodeling purchasing department.

TABLE 10.4

Times and Costs—Remodeling Purchasing Department

Activity	Act. No.	Normal		Conventional		Crash		ΔT	Cost
		TN	*CN*	*TCE*	*CCE*	*TC*	*CC*	*(TN-TC)*	Slope
Clear rooms	10	5	$ 500	5	$ 500	5	$ 500	—	—
Remove partition	20	3	900	2	1050	1	1200	2	$150
Rough mechanical and electrical	30	7	3250	5	3850	4	4150	3	300
Sand floors	32	5	1000	3	1300	3	1300	2	150
Partition I	40	6	1400	5	1850	3	2750	3	450
Partition II	42	5	1100	4	1500	3	1900	2	400
New bookcases	44	7	1500	4	2400	4	2400	3	300
Finish mechanical and electrical	50	10	4200	8	5400	5	7200	5	600
Paint	52	8	800	6	1000	4	1500	4	Variable (See Matrix)
Carpet	60	2	1100	2	1100	1	1300	1	200
Furnish	70	3	1300	3	1300	2	1500	1	200
		36*	$17,050	30*	$21,250	21*	$25,700		

** From CPM diagram.*



All Activities have "Normal durations"
Hence, Project also has its "Normal duration" = 36 days

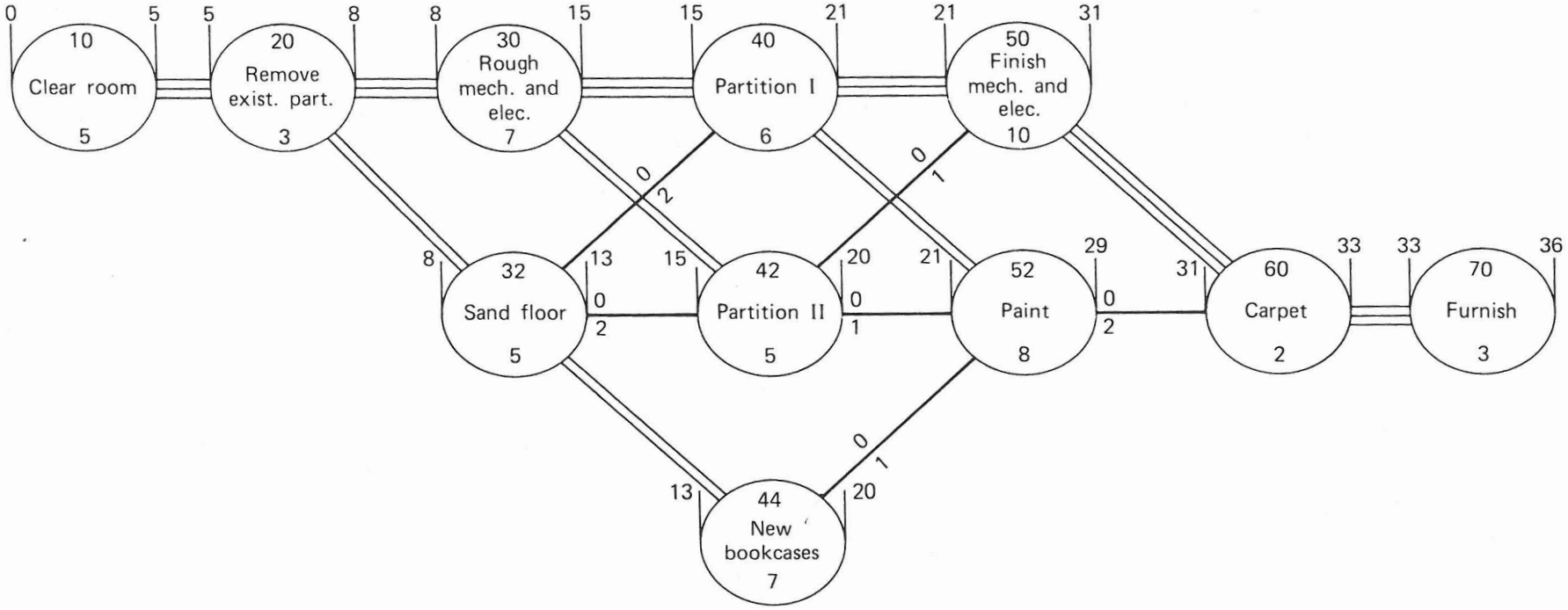


Figure 10.15 Precedence diagram—remodeling purchasing department.

TIME-COST TRADEOFF TABLES

SUMMARY TABLE

CY	Combination 1	2	3	ΔT 1	2	3	NL	Days Changed	$ Slope	$ Change	Project Cost	Project Duration
0												
1												
2												
3												
4												
5												
6												
7												
8												
9												
10												
11												
12												

NETWORK LIMIT DETERMINATION TABLE

CYCLE	1			2			3			4			5			6			7			8			9			10			11			12		
ACTS Changed																																				
Network Limit																																				
Days Changed																																				
LINK I	J	Lag	I	J	LAG	I	J	LAG	I	J	LAG	I	J	LAG	I	J	LAG	I	J	LAG	I	J	LAG	I	J	LAG	I	J	LAG	I	J	LAG	I	J	LAG

ACTIVITY SELECTION AND TALLY TABLE

COMBINATION SELECTION	ACT	$ SLOPE	CP CY	FIN CY	T	ΔT	REVISED T/ΔT 1	2	3	4	5	6	7	8	9	10	11	12

TALLY OF CRITICAL PATHS AT END OF CYCLE

STEP	ACTIVITIES																																					
1																																						
2																																						
3																																						
4																																						
5																																						
6																																						
7																																						

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From CE to Minimum Direct Cost Curve

1. Lengthen non-critical activities
   (with max cost slope)
2. Lengthen critical activities
   (with Max cost slope)
3. Shorten critical activities
   (with Min cost slope)
4. Repeat (2) & (3) until
   (Max cost slope) = (Min cost slope)

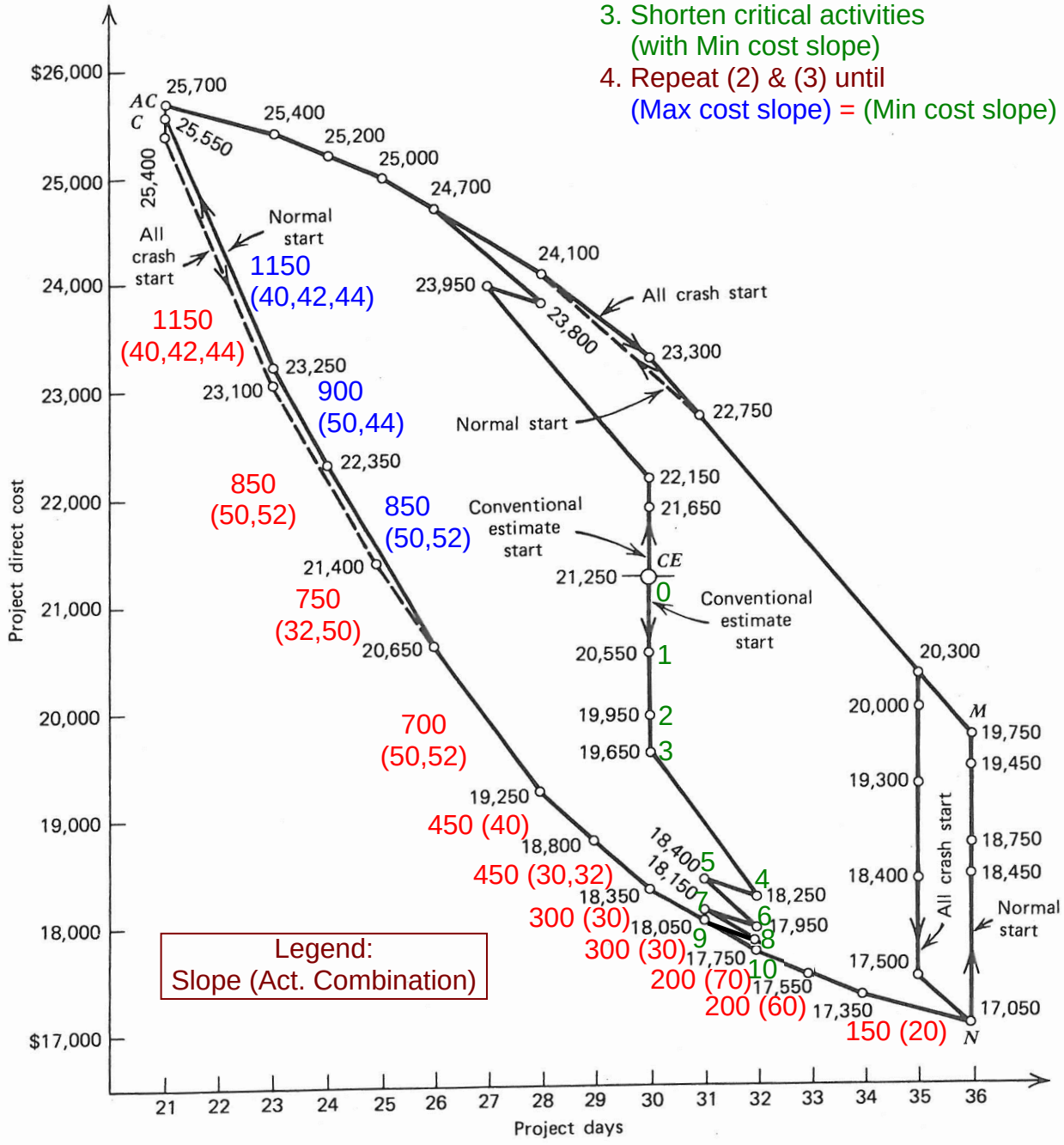

Figure 10.18 Project time-cost curves—remodeling purchasing department.

TIME-COST TRADEOFF TABLES
Normal to Crash Solution

SUMMARY TABLE

CY	Combination 1	2	3	ΔT 1	2	3	NL	Days Changed	$ Slope	$ Change	Project Cost	Project Duration
0											$17,050	36
1	20			2			N	2	150	300	$17,350	34
2	60			1			N	1	200	200	$17,550	33
3	70			1			N	1	200	200	$17,750	32
4	30			3			1	1	300	300	$18,050	31
5	30			2			1	1	300	300	$18,350	30
6	30	32		1	2		N	1	450	450	$18,800	29
7	40			3			1	1	450	450	$19,250	28
8	50	52		5	2		N	2	700	1400	$20,650	26
9	50	52		3	2		N	2	850	1700	$22,350	24
10	50	44		1	3		2	1	900	900	$23,250	23
11	40	42	44	2	2	2	N	2	1150	2300	$25,550	21
12	32			1			1	1	150	150	$25,700	21

NETWORK LIMIT DETERMINATION TABLE

CYCLE	1	2	3	4	5	6	7	8	9	10	11	12
ACTS Changed	20	60	70	30	30	30,32	40	50,52	50,52	44,50	40,42,44	32
Network Limit	None	None	None	1	1	None	1	None	None	2	None	1
Days Changed	2	1	1	1	1	1	1	2	2	1	2	1

LINK I	J	Lag	1 I	1 J	1 LAG	2 I	2 J	2 LAG	3 I	3 J	3 LAG	4 I	4 J	4 LAG	5 I	5 J	5 LAG	6 I	6 J	6 LAG	7 I	7 J	7 LAG	8 I	8 J	8 LAG	9 I	9 J	9 LAG	10 I	10 J	10 LAG	11 I	11 J	11 LAG	12 I	12 J	12 LAG
10	20	0																																				
20	30	0	«	<																																	x		
20	32	0	«	<																																			
30	40	0	<	<								«	<		«	<		«	<																		x		
30	42	0	<	<								«	<		«	<		«	<																		x		
32	40	2	<	<								<	<	1		<	0	«	<																		«	x	1
32	42	2	<	<								<	<	1		<	0	«	<																		«	x	1
32	44	0	<	<														«	<																		«	<	
40	50	0	<	<								<	<		<	<		<	<		«	<														«	<		x	x	
40	52	0	<	<								<	<		<	x	1	<	<		«	2									<	1	«	<		x	<	0
42	50	1	<	<								<	<		<	<		<	<			<	0											«	<		x	x	
42	52	1	<	<								<	<		<	x	2	<	<												<	1	«	<		x	<	0
44	52	1	<	<									<	0		x		<	<											«	<		«	<		<	<	
50	60	0	<	<								<	<		<	<		<	<		<	<		«	<		«	<		«	<		<	<		x	x	
52	60	2	<	<								<	<		x	<	1	<	<			<	0	«	<		«	<		<	<		<	<		<	x	1
60	70	0	<	<		«	<					<	<		<	<		<	<		<	<		<	<		<	<		<	<		<	<		x	x	

ACTIVITY SELECTION AND TALLY TABLE

COMBINATION SELECTION

12	11	10	9				8				7		6		5	4	3	2	1	ACT	$ SLOPE	CP CY	FIN CY	T	ΔT	REVISED T/ΔT 1	2	3	4	5	6	7	8	9	10	11	12
																				10	-	0	0	5	-												
				x		x		x												52	Var	7	9	8	4								6/2	4/0			
																			X	20	150	0	1	3	2	1/0											
X														X						32	150	5	12	5	2						4/1						3/0
																		X		60	200	0	2	2	1		1/0										
																	X			70	200	0	3	3	1			2/0										
														X	X	X				30	300	0	6	7	3				6/2	5/1	4/0						
	X	x	X	x		x		x												44	300	7	11	7	3									6/2	4/0		
	X	x		x	x			x	x											42	400	7	11	5	2										3/0		
	X	x		x	x			x	x			X		x						40	450	0	11	6	3							5/2			3/0		
			X			x	X			x	X		x							50	600	0	10	10	5								8/3	6/1	5/0		
150	**1,150**	1,150 **900**	1,150 1,100 900 **850**				1,150 950 900 **700**				600 **450**		600 450 **450**		**300**	**300**	**200**	**200**	**150**																			

"Activity Combinations" should NOT include "more than one activity" from the SAME critical path (it is inefficient to crash 2 acts when 1 is enough).

TALLY OF CRITICAL PATHS AT END OF CYCLE

STEP	ACTIVITIES	0	1	2	3	4	5	6	7	8	9	10	11	12
1	10	~~10~~	~~10~~	~~10~~	~~10~~	~~10~~	~~10~~ ~~10~~	~~10~~ ~~10~~	~~10~~ ~~10~~ ~~10~~ ~~10~~ ~~10~~	~~10~~ ~~10~~ ~~10~~ ~~10~~ ~~10~~	~~10~~ ~~10~~ ~~10~~ ~~10~~ ~~10~~	~~10~~ ~~10~~ ~~10~~ ~~10~~ ~~10~~	~~10~~ ~~10~~ ~~10~~ ~~10~~ ~~10~~	~~10~~ ~~10~~
2	20	20	~~20~~	~~20~~	~~20~~	~~20~~	~~20~~ ~~20~~	~~20~~ ~~20~~	~~20~~ ~~20~~ ~~20~~ ~~20~~ ~~20~~	~~20~~ ~~20~~ ~~20~~ ~~20~~ ~~20~~	~~20~~ ~~20~~ ~~20~~ ~~20~~ ~~20~~	~~20~~ ~~20~~ ~~20~~ ~~20~~ ~~20~~	~~20~~ ~~20~~ ~~20~~ ~~20~~ ~~20~~	~~20~~ ~~20~~
3	30,32	30	30	30	30	30	30 32	~~30~~ 32	~~30~~ 32 32 ~~30~~ 32	~~30~~ 32 32 ~~30~~ 32	~~30~~ 32 32 ~~30~~ 32	~~30~~ 32 32 ~~30~~ 32	~~30~~ 32 32 ~~30~~ 32	~~30~~ ~~30~~
4	40,42,44	40	40	40	40	40	40 40	40 40	40 40 42 42 44	40 40 42 42 44	40 40 42 42 44	40 40 42 42 44	~~40~~ ~~40~~ ~~42~~ ~~42~~ ~~44~~	~~40~~ ~~42~~
5	50,52	50	50	50	50	50	50 50	50 50	50 50 50 50 52	50 50 50 50 52	50 50 50 50 ~~52~~	~~50~~ ~~50~~ ~~50~~ ~~50~~ ~~52~~	~~50~~ ~~50~~ ~~50~~ ~~50~~ ~~52~~	~~50~~ ~~50~~
6	60	60	60	~~60~~	~~60~~	~~60~~	~~60~~ ~~60~~	~~60~~ ~~60~~	~~60~~ ~~60~~ ~~60~~ ~~60~~ ~~60~~	~~60~~ ~~60~~ ~~60~~ ~~60~~ ~~60~~	~~60~~ ~~60~~ ~~60~~ ~~60~~ ~~60~~	~~60~~ ~~60~~ ~~60~~ ~~60~~ ~~60~~	~~60~~ ~~60~~ ~~60~~ ~~60~~ ~~60~~	~~60~~ ~~60~~
7	70	70	70	70	~~70~~	~~70~~	~~70~~ ~~70~~	~~70~~ ~~70~~	~~70~~ ~~70~~ ~~70~~ ~~70~~ ~~70~~	~~70~~ ~~70~~ ~~70~~ ~~70~~ ~~70~~	~~70~~ ~~70~~ ~~70~~ ~~70~~ ~~70~~	~~70~~ ~~70~~ ~~70~~ ~~70~~ ~~70~~	~~70~~ ~~70~~ ~~70~~ ~~70~~ ~~70~~	~~70~~ ~~70~~

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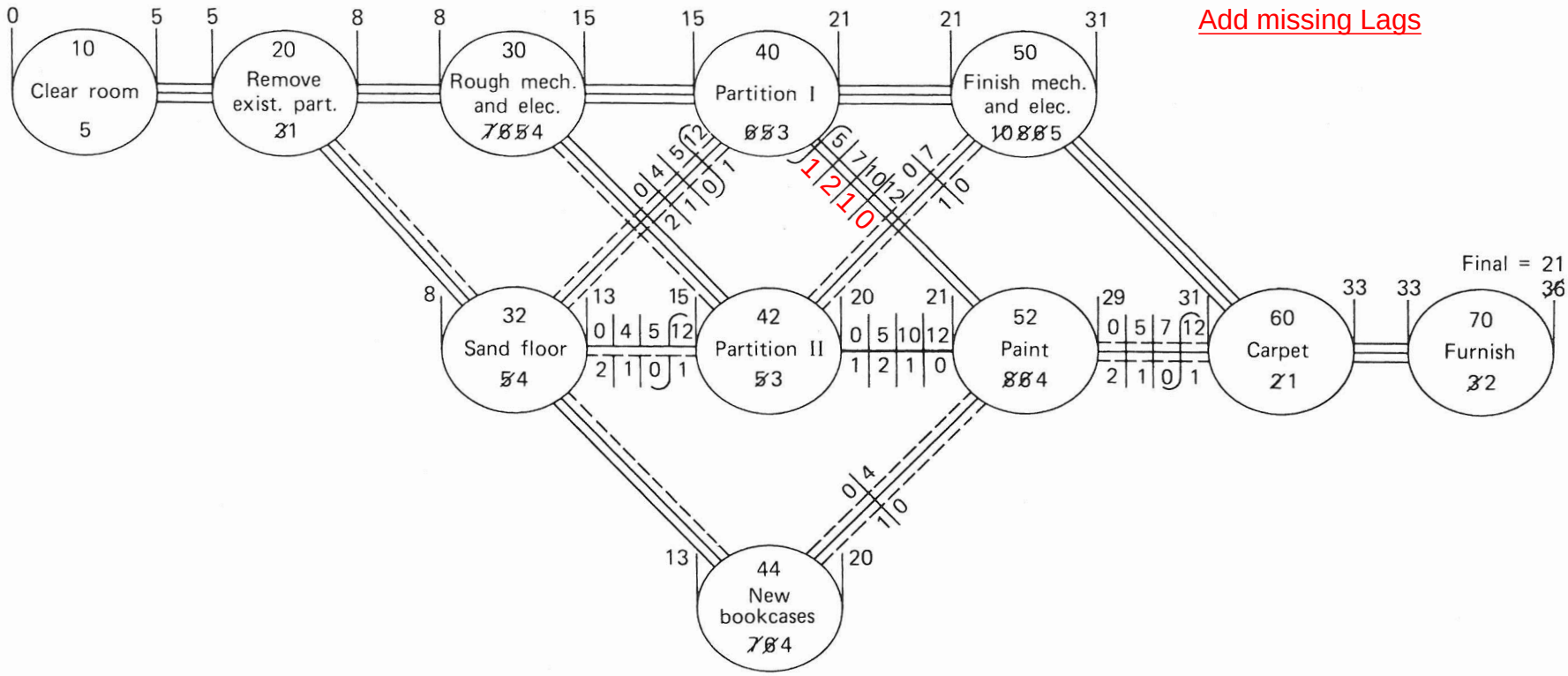


Figure 10.17 Normal start solution—remodeling purchasing department.

TIME-COST TRADEOFF TABLES
All Crash to Normal Solution

SUMMARY TABLE

CY	Combination 1	2	3	ΔT 1	2	3	NL	Days Changed	$ Slope	$ Change	Project Cost	Project Duration
0											$25,700	21
1	44			3			1	1	300	300	$25,400	21
2	40	42	44	3	2	2	N	2	1150	2300	$23,100	23
3	50	52		5	2		N	2	850	1700	$21,400	25
4	32	50		2	3		1	1	750	750	$20,650	26
5	50	52		2	2		N	2	700	1400	$19,250	28
6	40			1			2	1	450	450	$18,800	29
7	30	32		3	1		N	1	450	450	$18,350	30
8	30			2			1	1	300	300	$18,050	31
9	30			1			N	1	300	300	$17,750	32
10	70			1			N	1	200	200	$17,550	33
11	60			1			N	1	200	200	$17,350	34
12	20			2			N	2	150	300	$17,050	36

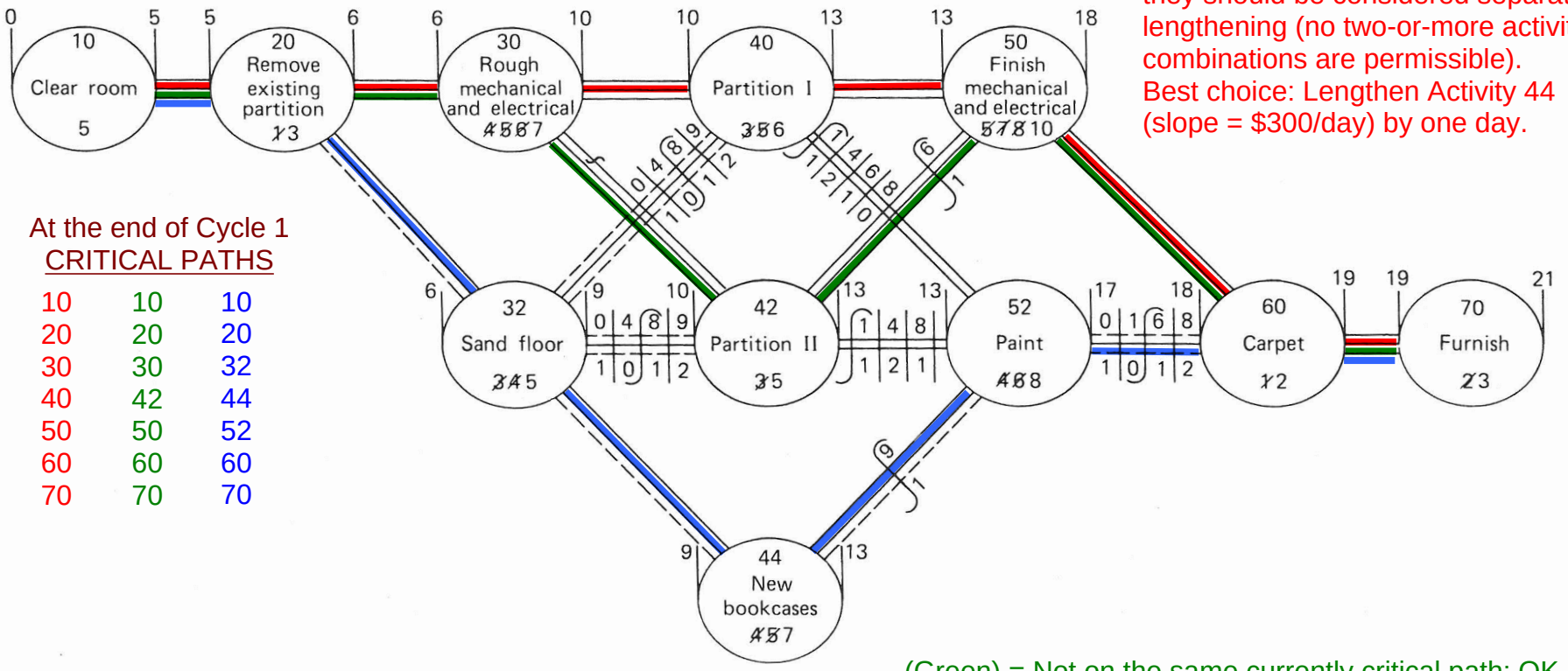
NETWORK LIMIT DETERMINATION TABLE

CYCLE			1			2			3			4			5			6			7			8			9			10			11			12		
ACTS Changed			44			40,42,44			50,52			32,50			50,52			40			30,32			30			30			70			60			20		
Network Limit			1			None			None			1			None			2			None			1			None			None			None			None		
Days Changed			1			2			2			1			2			1			1			1			1			1			1			2		
LINK I	J	Lag	I	J	LAG	I	J	LAG	I	J	LAG	I	J	LAG	I	J	LAG	I	J	LAG	I	J	LAG	I	J	LAG	I	J	LAG	I	J	LAG	I	J	LAG	I	J	LAG
10	20	0																																				
20	30	0																																	»	>		
20	32	0																																	»	>		
30	40	0																			»	>		»	>		»	>							>	>	
30	42	0																			»	>		»	>		»	>							>	>	
32	40	1										»		0							»	>			>	1		>	2						>	>	
32	42	1										»		0							»	>			>	1		>	2						>	>	
32	44	0										»	>								»	>													>	>	
40	50	0				»	>											»	>		>	>		>	>		>	>						>	>	
40	52	0		>	1	»	>						>	2				»		1	>	>		>		0	>	>						>	>	
42	50	0				»	>												>	1	>	>		>	>		>	>						>	>	
42	52	0		>	1	»	>						>	2							>	>		>		1	>	>						>	>	
44	52	0	»	>		»	>					>	>								>	>						>	1					>	>	
50	60	0				>	>		»	>		»	>		»	>		>	>		>	>		>	>		>	>					>	>	
52	60	1	>		0	>	>		»	>		>	>		»	>			>	1	>	>			>	2	>	>					>	>	
60	70	0				>	>		>	>		>	>		>	>		>	>		>	>		>	>		>	>			»	>		>	>	

ACTIVITY SELECTION AND TALLY TABLE

| COMBINATION SELECTION | | | | | | | | | | | | | | | | | | |

In Cycle 0, the three non-critical Activities, 32, 44, 52, are on the same branch of the early spanning tree. Hence, they share the 1-day lag from 52-60 and they should be considered separately for lengthening (no two-or-more activities combinations are permissible).
Best choice: Lengthen Activity 44 (slope = $300/day) by one day.



At the end of Cycle 1
CRITICAL PATHS

10	10	10
20	20	20
30	30	32
40	42	44
50	50	52
60	60	60
70	70	70

(Green) = Not on the same currently critical path: OK

Figure 10.19 All crash start solution—remodeling purchasing department.

Cycle From-->To	In Table 10.10						Should have been in Table 10.10 (but did not fit; book page too narrow)					
	30	30	40	50	50	50	20	30	40	40	60	70
	32	44	42	44	52	32		32	42	42		
			44						32	52		
1-->2	450	600	1150	900	850	750	150	550	1000	1100	200	200
	30		40	40	50	50	20	30			60	70
	32		52	32	52	32		32				
2-->3	450		~~700~~	600	~~850~~	750	150	550			200	200
3-->4	450		550	600	700	750						

TIME-COST TRADEOFF TABLES
Conventional Estimate Solution

SUMMARY TABLE

CY	L/S	Combination 1	2	3	ΔT 1	2	3	NL	Days Changed	$ Slope	$ Change	Project Cost	Project Duration
0												$21,250	30
1	L	42	44		1	3		1	1	700	700	$20,550	30
2	L	44			2			2	2	300	600	$19,950	30
3	L	32			2			2	2	150	300	$19,650	30
4	L	50	52		2	2		N	2	700	1400	$18,250	32
5	S	20			1			N	1	150	150	$18,400	31
6	L	40			1			2	1	450	450	$17,950	32
7	S	60			1			N	1	200	200	$18,150	31
8	L	30			2			1	1	300	300	$17,850	32
9	S	70			1			N	1	200	200	$18,050	31
10	L	30			1			N	1	300	300	$17,750	32
11													
12													

NETWORK LIMIT DETERMINATION TABLE

CYCLE			1L			2L			3L			4L			5S			6L			7S			8L			9S			10L								
ACTS Changed			42,44			44			32			50,52			20			40			60			30			70			30								
Network Limit			1			2			2			None			None			2			None			1			None			2								
Days Changed			1			2			2			2			1			1			1			1			1			1								
LINK I	J	Lag	I	J	LAG	I	J	LAG	I	J	LAG	I	J	LAG	I	J	LAG	I	J	LAG	I	J	LAG	I	J	LAG	I	J	LAG	I	J	LAG	I	J	LAG	I	J	LAG
10	20	0																																				
20	30	0													«	<																						
20	32	0													«	<																						
30	40	0													<	<								»	>				»	>								
30	42	0													<	<								»	>				»	>								
32	40	2							»		0				<	<								>		1			>		2							
32	42	2							»		0				<	<								>		1			>		2							
32	44	0							»	>					<	<																						
40	50	0													<	<		»	>					>	>				>	>								
40	52	0								>	2				<	<		»		1				>		0			>	>								
42	50	1	»		0										<	<			>	1				>	>				>	>								
42	52	1	»		0					>	2				<	<								>		1			>	>								
44	52	3	»		2	»		0	>	>					<	<														>	1						
50	60	0											»	>	<	<		>	>					>	>				>	>								
52	60	2							>		0	»	>		<	<			>	1				>		2			>	>								
60	70	0										>	>		<	<		>	>		«	<		>	>				>	>								

ACTIVITY SELECTION AND TALLY TABLE

COMBINATION SELECTION 10	9	8	7	6	5	4	3	2	1	ACT	$ SLOPE	CP CY	FIN CY S	FIN CY L	ΔT S	ΔT L	REVISED ΔT$_S$/ΔT$_L$ 1	2	3	4	5	6	7	8	9	10	11	12
										10	-	0	0	0	0	0												
						X x x				52	Var	3		4	2	2				4/0								
					X	x				20	150	0	5		1	1					0/2							
							X			32	150	~~3~~	~~0~~	3	0	2			2/0									
			X		x					60	200	0	7	~~0~~	1	0							0/1					
	X				x					70	200	0		~~0~~	1	0								0/1				
X		X				x x				30	300	0			1	2								2/1		3/0		
								X	X	44	300	~~3~~	~~0~~	2	0	3	1/2	3/0									
									X	42	400	~~1~~		1	1	1	2/0											
				X		x x				40	450	0		6	2	1						3/0						
						x x				50	600	0		4	3	2				5/0								
300	**200**	**300**	**200**	**450**	200 200 **150**	**700** 600 550 450 400 300 150	**150**	**300**	**700**																			

TALLY OF CRITICAL PATHS AT END OF CYCLE

STEP	ACTIVITIES	0	1L		2L		3L					4L					5S					6L		7S		8L	9S	10L
1	10	~~10~~	~~10~~	~~10~~	~~10~~	~~10~~	~~10~~	~~10~~	~~10~~	~~10~~	~~10~~	~~10~~	~~10~~	~~10~~	~~10~~	~~10~~	~~10~~	~~10~~	~~10~~	~~10~~	~~10~~	~~10~~	~~10~~	~~10~~	~~10~~	~~10~~	~~10~~	~~10~~	~~10~~
2	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	~~20~~	~~20~~	20	20	~~20~~	20	~~20~~
3	30,32	30	30	30	30	30	30	30	~~32~~	~~32~~	~~32~~	30	30	32	32	32	30	30	~~32~~	~~32~~	~~32~~	30	32	30	~~32~~	30	30	30
4	40,42,44	40	40	~~42~~	40	~~42~~	40	~~42~~	40	~~42~~	~~44~~	40	42	40	42	44	40	~~42~~	40	~~42~~	~~44~~	40	40	~~40~~	~~40~~	40	~~40~~	40
5	50,52	50	50	50	50	50	50	50	50	50	52	50	50	50	50	52	~~50~~	~~50~~	~~50~~	~~50~~	~~52~~	50	50	~~50~~	~~50~~	50	~~50~~	50
6	60	~~60~~	~~60~~	~~60~~	~~60~~	~~60~~	~~60~~	~~60~~	~~60~~	~~60~~	~~60~~	60	60	60	60	60	~~60~~	~~60~~	~~60~~	~~60~~	~~60~~	60	60	60	60	~~60~~	60	~~60~~
7	70	~~70~~	~~70~~	~~70~~	~~70~~	~~70~~	~~70~~	~~70~~	~~70~~	~~70~~	~~70~~	70	70	70	70	70	~~70~~	~~70~~	~~70~~	~~70~~	~~70~~	70	70	~~70~~	~~70~~	70	70	~~70~~

P.G. Ioannou, 11/6/2014

In Cycle 0, the four non-critical Activities, 32, 44, 42, and 52, are on three different branches of the early spanning tree (in blue).
Lengthening Act 52 (which has the least slope of 100) takes away the Lag(52,60)=2 which can be used to lengthen Act 32 in Cycle 3 at a slope of 300 (when Act 30 and Act 52 are on the same branch of the early spanning tree 10-20-32-44-52). So 52 should not be lengthened before 44 and 32.
This same observation applies to Cycles 1, 2, and 3.
Hence, the combinations from which to choose in Cycle 1 are:
32 & 42: Slope = 150+400 = $550/d
44 & 42: Slope = 300+400 = $700/d (Best)

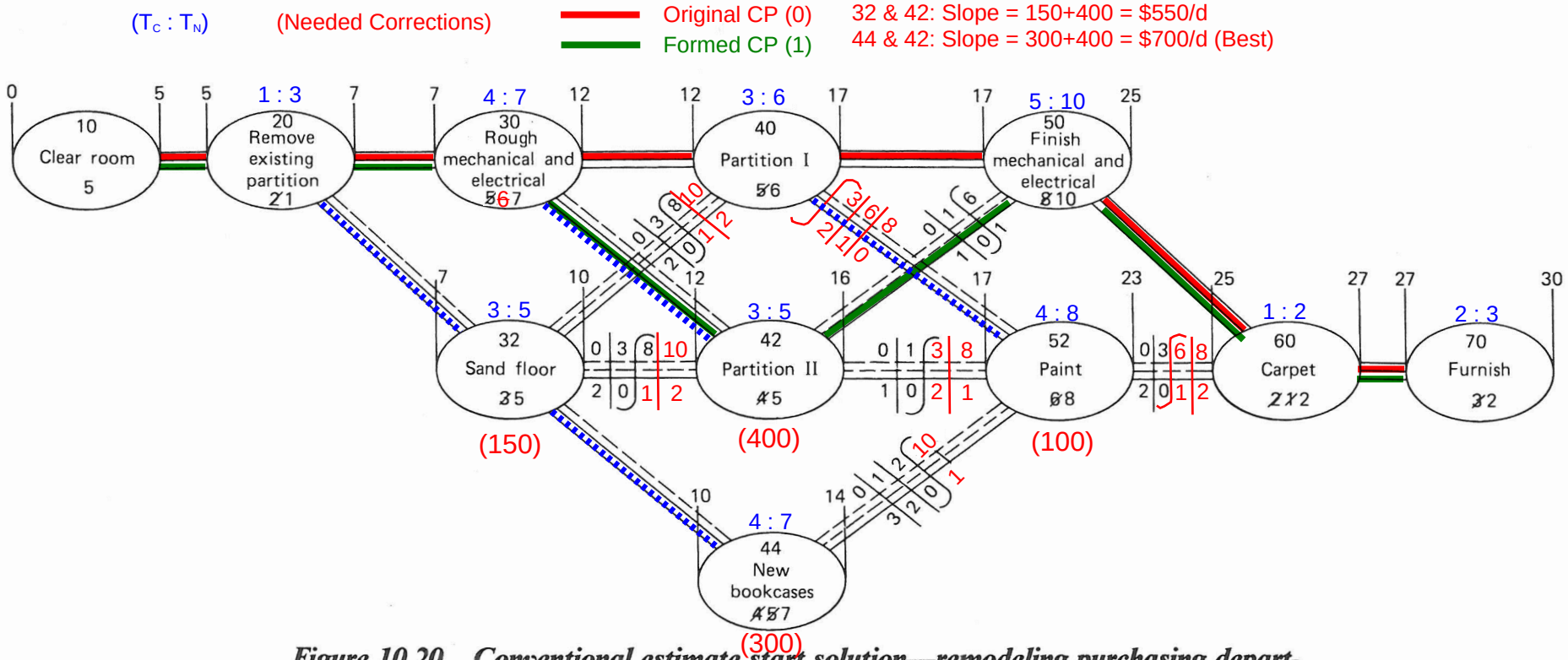


Figure 10.20 Conventional estimate start solution—remodeling purchasing department.

From CE to Minimum Direct Cost Curve

1. Lengthen non-critical activities
   (with max cost slope)
2. Lengthen critical activities
   (with Max cost slope)
3. Shorten critical activities
   (with Min cost slope)
4. Repeat (2) & (3) until
   (Max cost slope) = (Min cost slope)

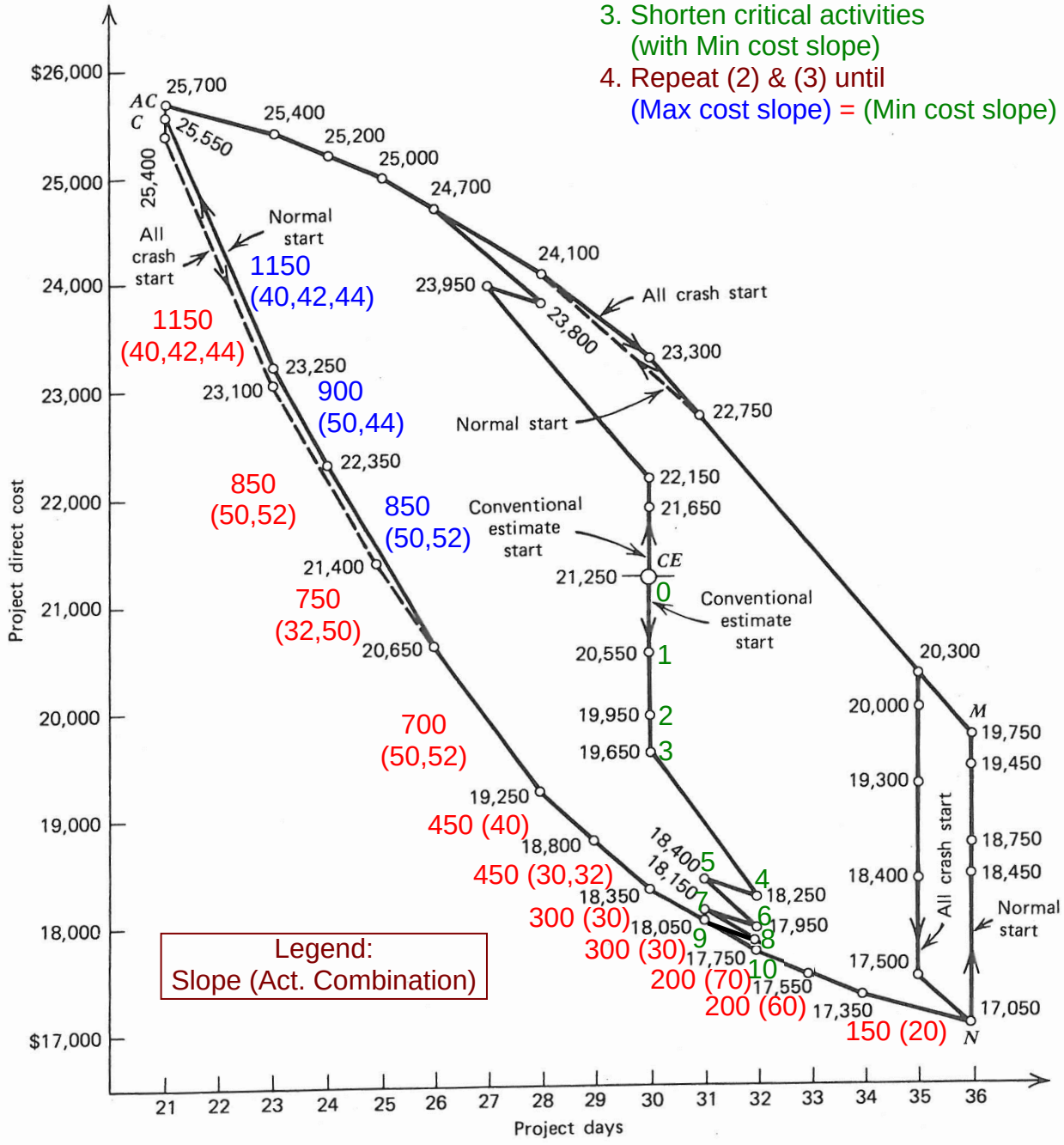


Figure 10.18 Project time-cost curves—remodeling purchasing department.

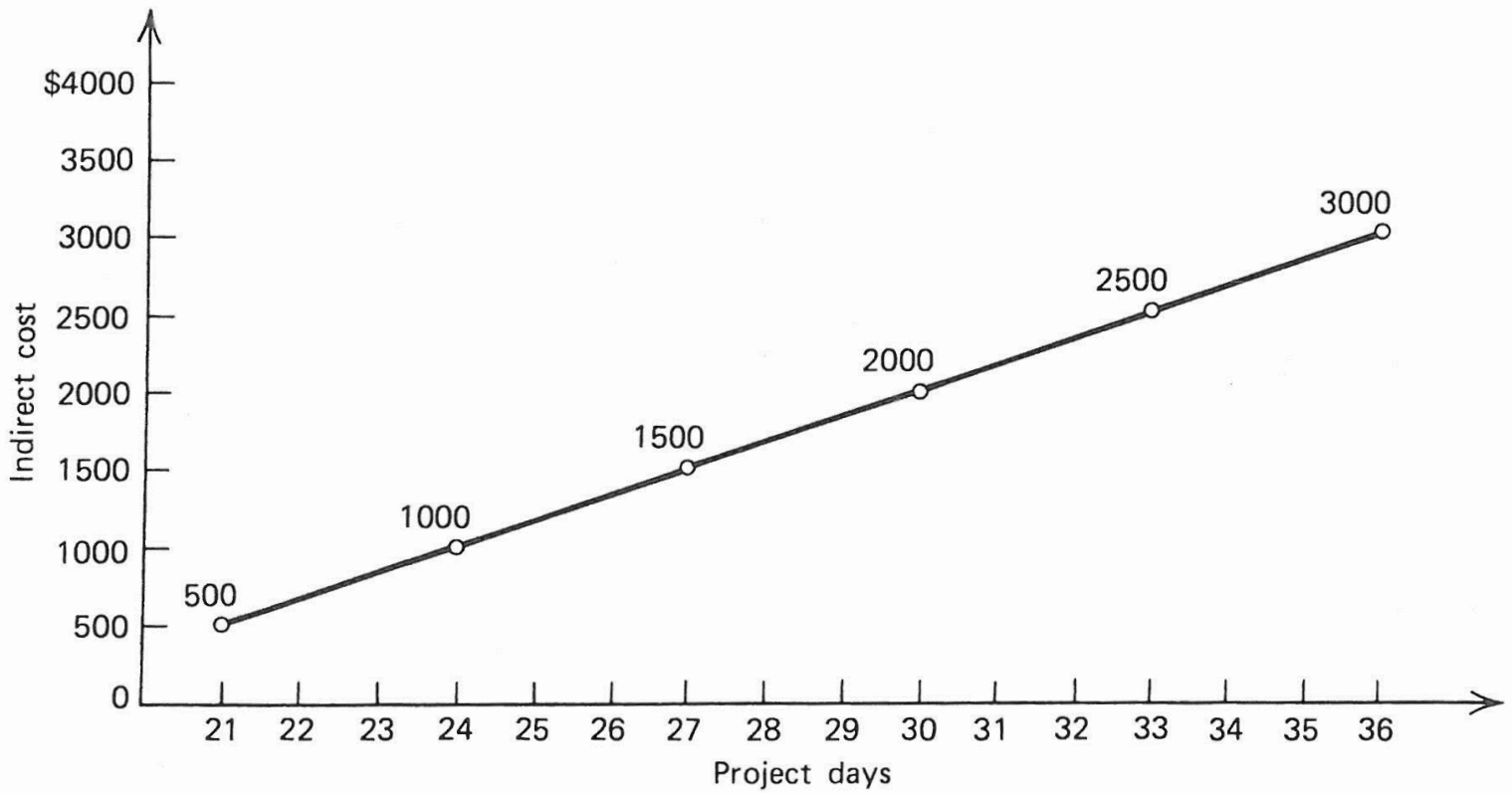


Figure 10.21 Project indirect costs—remodeling purchasing department.

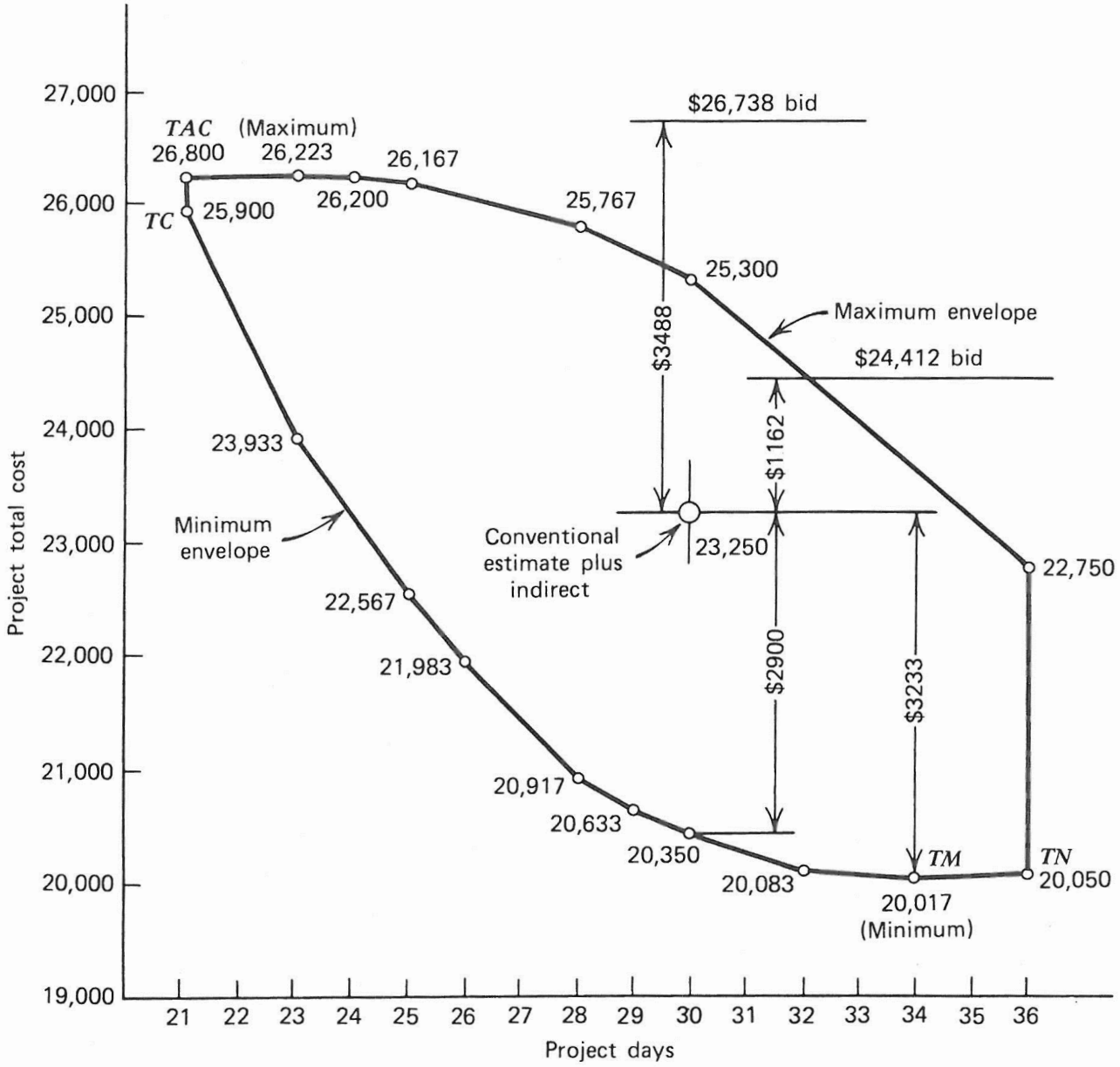
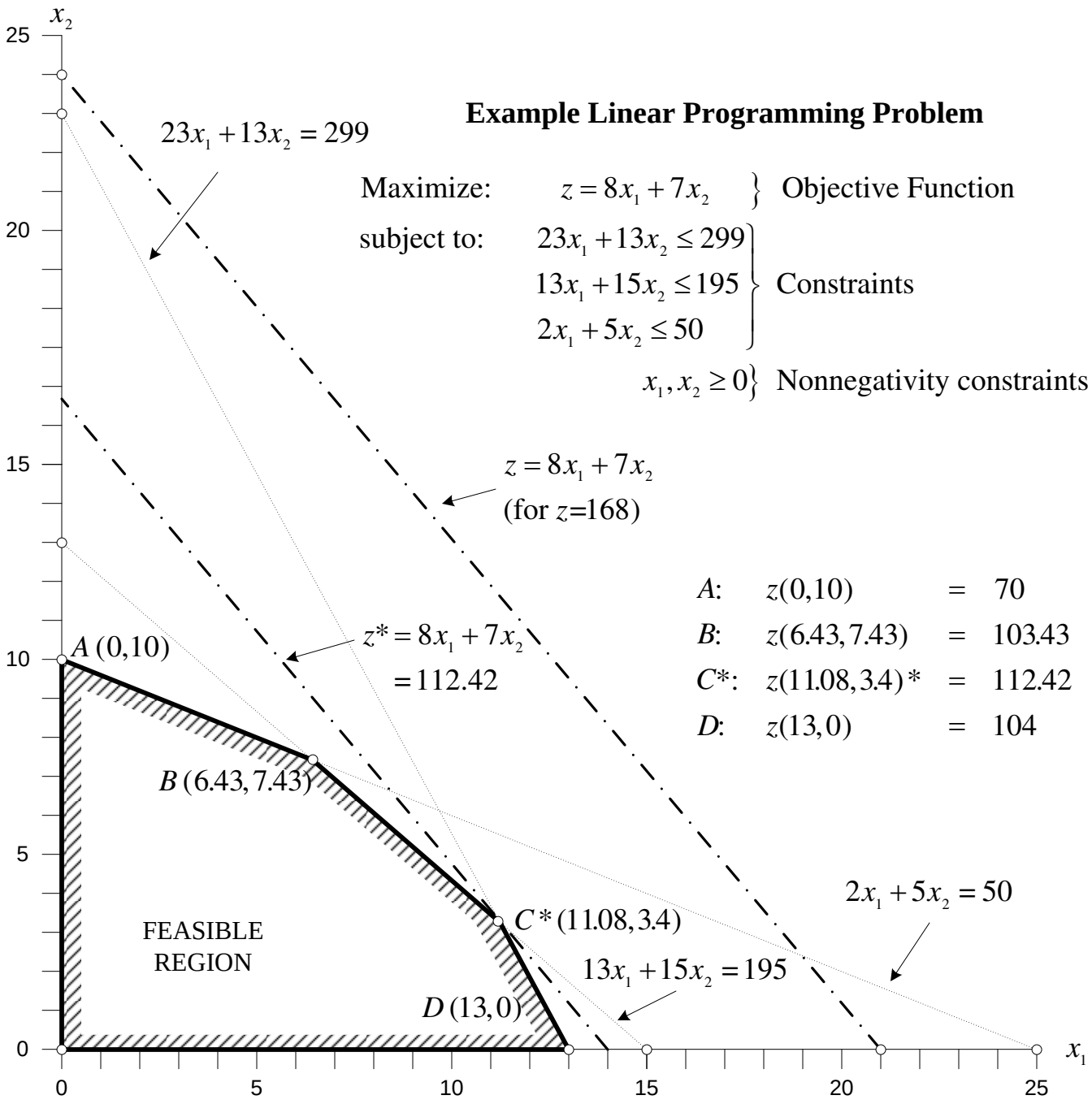


Figure 10.22 Project total cost curves—remodeling purchasing department.



Example Linear Programming Problem

Maximize: $z = 8x_1 + 7x_2$ } Objective Function

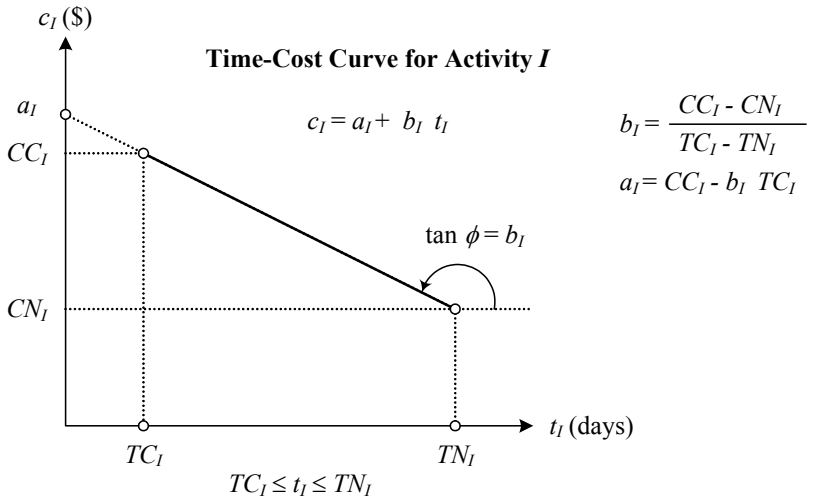
subject to:
$23x_1 + 13x_2 \le 299$
$13x_1 + 15x_2 \le 195$ } Constraints
$2x_1 + 5x_2 \le 50$

$x_1, x_2 \ge 0$ } Nonnegativity constraints

A:	$z(0,10)$	=	70
B:	$z(6.43, 7.43)$	=	103.43
*C**:	$z(11.08, 3.4)^*$	=	112.42
D:	$z(13, 0)$	=	104

The set of points (x_1, x_2) that satisfy all constraints is called the *feasible region*.
The *optimal solution* $z^* = (x_1, x_2)^*$ always occurs at a corner point of the feasible region.
Corner points are defined as the *intersection* of two or more constraints.
Constraints satisfied as *equalities* at the optimal solution are called *binding constraints*.
The remaining constraints are *non-binding*, i.e., they have *slack*.

TIME-COST TRADEOFF — LP SOLUTION
PRECEDENCE NETWORK (ACTIVITY-ON-NODE)



Linear Programming Formulation of the Time-Cost Tradeoff Problem Using Precedence Network Format

$$\min z = \sum_I c_I = \sum_I a_I + \sum_I b_I t_I$$

s.t.

$$ESD_I + t_I - ESD_J \le 0 \qquad \forall \text{ link } I, J$$

$$t_I \ge TC_I \qquad \forall I$$

$$t_I \le TN_I \qquad \forall I$$

$$ESD_f + t_f \le T$$

$$ESD_S = 0$$

$$ESD_I, t_I \ge 0$$

Notes:
z = Direct project cost
s = Start activity
f = Finish activity
$ESD_f + t_f$ = Project Duration
T = Selected project duration for this solution. Must be selected to be feasible, *e.g.*, $T = \sum_I t_I$.

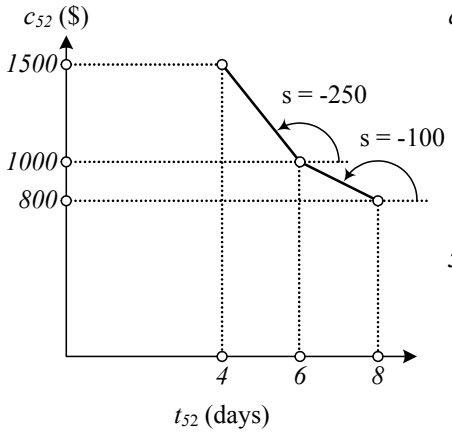
Time-Cost Coefficients

Use this worksheet to compute the a_I and b_I coefficients for the LP solution. The table below illustrates the approach using the textbook example.

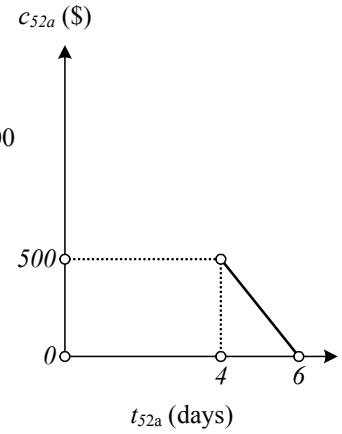
Formulas:	a= Cn-Tn*b	b= (Cn-Cc)/(Tn-Tc)

	Normal		Crash		Cost = a + b*Time	
Act No	Time Tn	Cost Cn	Time Tc	Cost Cc	a	b
10	5	500	5	500	500	**0**
20	3	900	1	1200	1350	**-150**
30	7	3250	4	4150	5350	**-300**
32	5	1000	3	1300	1750	**-150**
40	6	1400	3	2750	4100	**-450**
42	5	1100	3	1900	3100	**-400**
44	7	1500	4	2400	3600	**-300**
50	10	4200	5	7200	10200	**-600**
52a	6	0	4	500	1500	**-250**
52b	2	800	0	1000	1000	**-100**
60	2	1100	1	1300	1500	**-200**
70	3	1300	2	1500	1900	**-200**
	Normal Cost	17050	All Crash Cost	25700	**35850** = Sum of Fixed Costs	

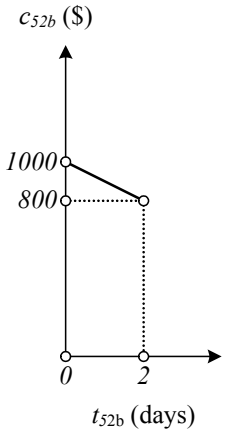
Time-Cost Curve for Activity 52



Time-Cost for 52a

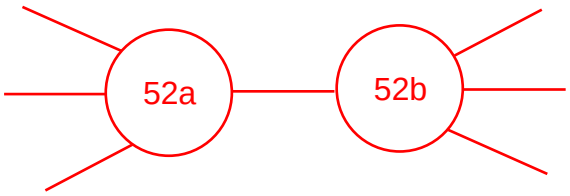


Time-Cost for 52b



LINDO Input File.(Text-only file)

```
      ! Time-Cost Tradeoff. Activity-On-Node formulation.
      ! This is the example in Prof. Harris's book on Time-Cost Tradeoff.
      ! Minimize the objective function: i.e., Minimize Direct Project Cost
1     MIN   - 150 T20 - 300 T30 - 150 T32 - 450 T40 - 400 T42 - 300 T44
            - 600 T50 - 250 T52a - 200 T60 - 100 T52b - 200 T70
            + 35850 FIXED
      ! Subject to the following constraints:
      ST
      ! ESDi + Ti <= ESDj
2                T10 + ESD10 - ESD20  <=   0
                 T20 + ESD20 - ESD30  <=   0
4                T20 + ESD20 - ESD32  <=   0
                 T30 + ESD30 - ESD40  <=   0
6                T30 + ESD30 - ESD42  <=   0
                 T32 + ESD32 - ESD40  <=   0
8                T32 + ESD32 - ESD42  <=   0
                 T32 + ESD32 - ESD44  <=   0
10               T40 + ESD40 - ESD50  <=   0
                 T40 + ESD40 - ESD52a <=   0
12               T42 + ESD42 - ESD50  <=   0
                 T42 + ESD42 - ESD52a <=   0
14               T44 + ESD44 - ESD52a <=   0
                 T50 + ESD50 - ESD60  <=   0
16               T52a+ ESD52a- ESD52b <=   0
                 T52b+ ESD52b- ESD60  <=   0
18               T60 + ESD60 - ESD70  <=   0
      ! Tc <= T <= Tn
                 T10  >=   5
20               T10  <=   5
                 T20  >=   1
22               T20  <=   3
                 T30  >=   4
24               T30  <=   7
                 T32  >=   3
26               T32  <=   5
                 T40  >=   3
28               T40  <=   6
                 T42  >=   3
30               T42  <=   5
                 T44  >=   4
32               T44  <=   7
                 T50  >=   5
34               T50  <=   10
                 T52a >=   4
36               T52a <=   6
                 T60  >=   1
38               T60  <=   2
                 T52b >=   0
40               T52b <=   2
                 T70  >=   2
42               T70  <=   3
      ! Project ESD = 0
                  ESD10 =  0
      ! Project EFD <= Target (The problem parameter; must be feasible)
44                T70 + ESD70 <=  36
      ! The fixed cost coefficient is 1
                  FIXED = 1
      END
      LEAVE
```



To direct LINDO to take its input from a file (such as the one shown in the previous page) type the command:

```
TAKE filename
```

The last line in the input file (*i.e.*, `LEAVE`) directs LINDO to stop reading its input from the file and to return input control to the keyboard.

To direct LINDO to solve the Linear Program type the command:

```
GO
```

This produces the solution to the Minimum Direct Cost problem that gives a project duration that is less than or equal to the specified *T*.

To plot the Minimum Direct Cost graph it is necessary to change *T* and repeat the solution procedure for all possible values of project duration.

The easiest way to accomplish this is to perform *parametric analysis* on *T*. That is, to change the value of *T* from its current value to some other level and to observe the effect of this change on the optimal value of the objective function (i.e., on the minimum direct project cost).

To perform parametric analysis, first decline the offer to perform *range analysis* (this question is part of the output produced by the `GO` command). Then issue the `PARA` command (*i.e.*, parametric analysis). In response to the "`ROW`" prompt type the "*number*" of the constraint that limits project duration to T (the objective function is `ROW 1`, the first constraint is `ROW 2`, etc.). Respond to the "`NEW RHS VAL=`" prompt with the value 0 (zero). This asks for solutions all the way down to a zero duration project! (Obviously, LINDO stops at the Crash point).

The points for constructing the Minimum Direct Cost graph are given by the two columns labeled "`RHS VAL`" (*i.e.*, Time in days) and "`OBJ VAL`" (*i.e.*, Minimum Direct Project Cost).

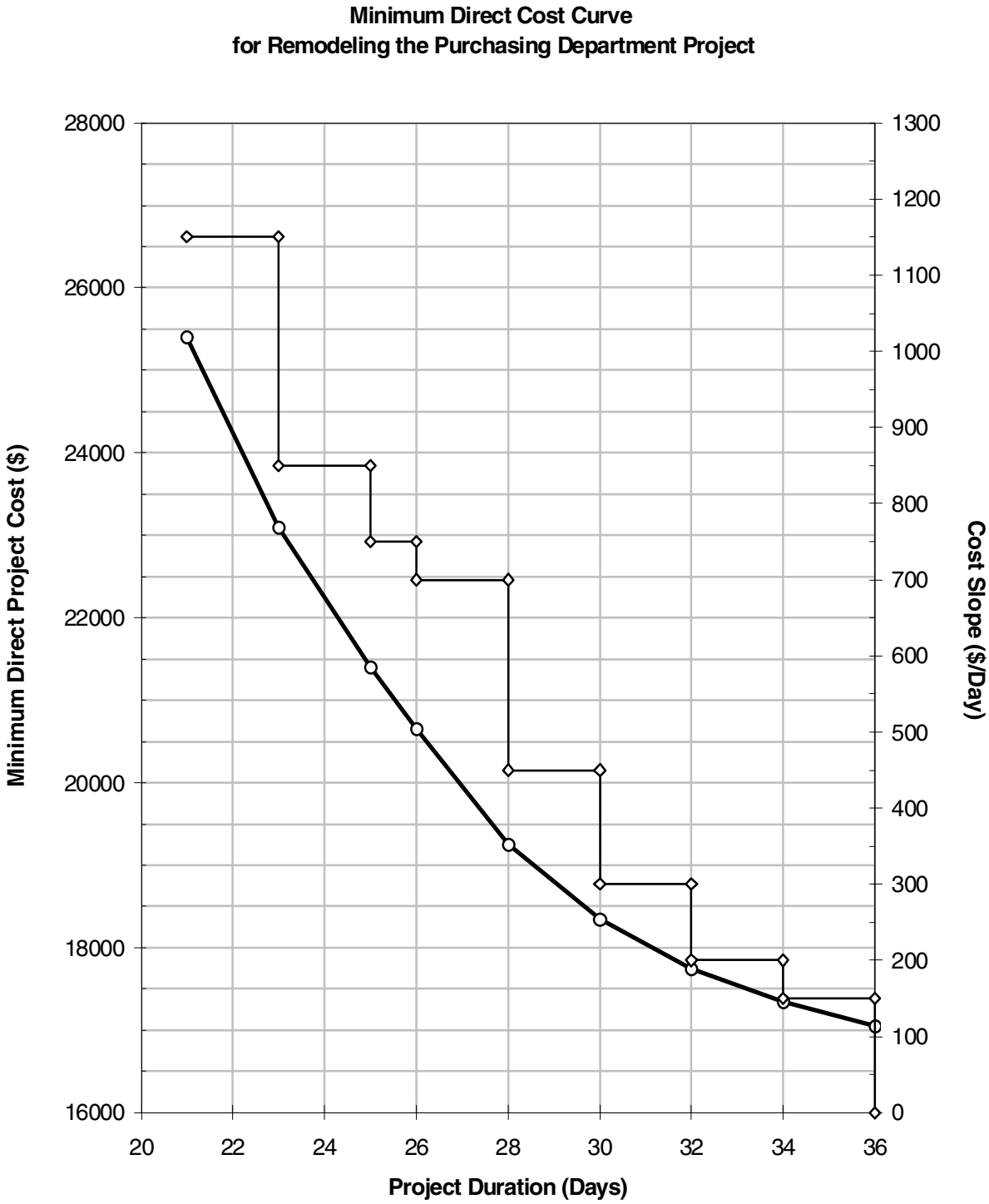
```
DO RANGE(SENSITIVITY) ANALYSIS?
? n
: para
ROW:
44
NEW RHS VAL=
0
```

When SLK=0 then constraint becomes binding

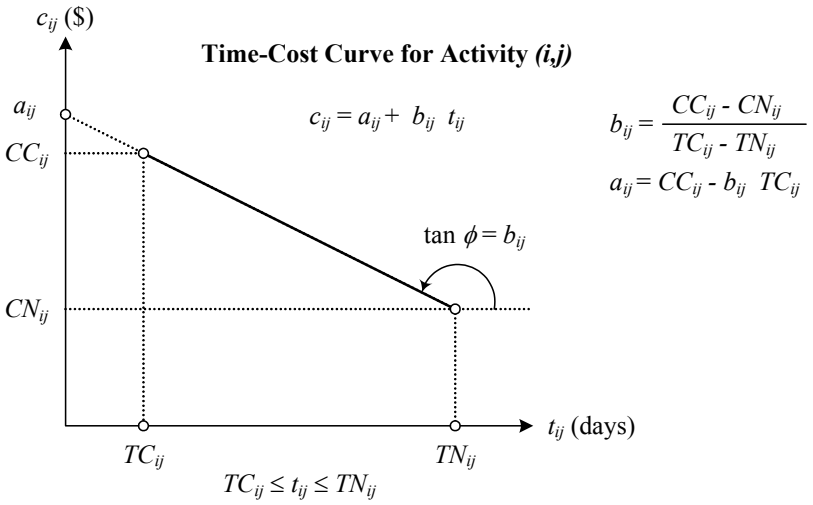
~~10~~	~~10~~	~~10~~	~~10~~	~~10~~						
~~20~~	~~20~~	~~20~~	~~20~~	~~20~~						
~~30~~	32	32	~~30~~	32						
40	40	42	42	44						
50	50	50	50	52						
~~60~~	~~60~~	~~60~~	~~60~~	~~60~~						
~~70~~	~~70~~	~~70~~	~~70~~	~~70~~						

Critical Paths @ end of cycle 8. For cycle 9, Fondahl chooses (50,52) @ a slope = 850. Yet, (32,50) @ slope = 750 is better, even though 32 & 50 belong to same Critical Paths.

VAR OUT		VAR IN		PIVOT ROW		RHS VAL		BY LP ACT	BY HAND ACT	DUAL VARIABLE	OBJ VAL
				Cycle		T (days)				dC/dT	Cost ($)
SLK	44	SLK	22	0	44	36				0	17050
SLK	21	SLK	38	1	21	34		20	20	150	17350
SLK	37	SLK	42	2	37	33		60	60	200	17550
SLK	41	SLK	24	3	41	32		70	70	200	17750
SLK	4	SLK	26	4,5	4	30		30	30	300	18350
SLK	23	SLK	28	6	23	29		32	30,32	450	18800
SLK	12	SLK	34	7	12	28		40	40	450	19250
SLK	16	SLK	40		16	28		50		600	19250
SLK	39	SLK	7	8	39	26		52b	50,52	700	20650
SLK	25	SLK	36	9?	25	25		32&50	32,50	750	21400
SLK	33	SLK	30		33	23		52a	50,52	850	23100
SLK	35	SLK	32		35	23		42		1100	23100
SLK	27	ART			27	21		44	40,42,44	1150	25400


Minimum Direct Cost Curve
for Remodeling the Purchasing Department Project
Minimum Direct Project Cost ($)
Cost Slope ($/Day)
Project Duration (Days)
28000
26000
24000
22000
20000
18000
16000
1300
1200
1100
1000
900
800
700
600
500
400
300
200
100
0
20
22
24
26
28
30
32
34
36

TIME-COST TRADEOFF — LP SOLUTION
ACTIVITY-ON-ARROW NETWORK



Linear Programming Formulation of the Time-Cost Tradeoff Problem Using Activity-On-Arrow Network Format

$$\min z = \sum_i \sum_j c_{ij} = \sum_i \sum_j a_{ij} + \sum_i \sum_j b_{ij} t_{ij}$$

$$s.t.$$

$$TE_i + t_{ij} - TE_j \le 0 \qquad \forall (i,j)$$

$$t_{ij} \ge TC_{ij} \qquad \forall (i,j)$$

$$t_{ij} \le TN_{ij} \qquad \forall (i,j)$$

$$TE_S = 0$$

$$TE_f \le T$$

$$TE_i, t_{ij} \ge 0$$

Notes:

T = Selected project duration for the solution. Must be selected to be feasible, *e.g.*, $T = \sum_i \sum_j t_{ij}$

s = Start event

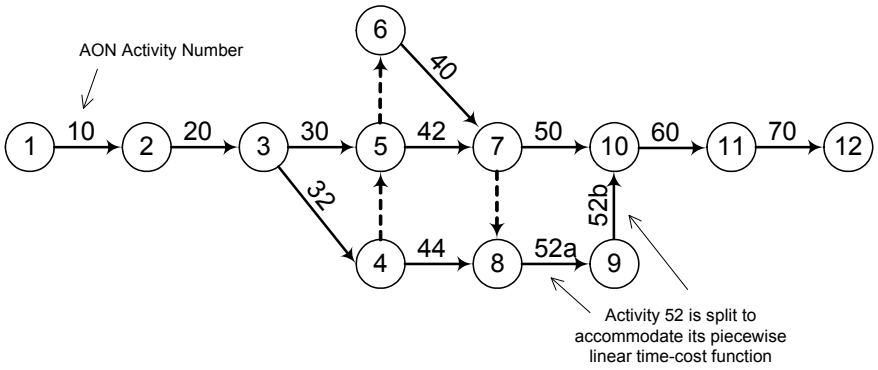
f = Finish event

To direct LINDO to take its input from a file (such as the one shown below) type the command:

TAKE *filename*

```
! Time-Cost Tradeoff. Activity-On-Arrow formulation.
! This is the example used in Harris's book on time cost tradeoff.
! Objective Function: Minimize the Direct Project Cost
MIN   - 150 T23 - 300 T35 - 150 T34 - 450 T67 - 400 T57 - 300 T48
      - 600 T710 - 250 T89 - 200 T1011 - 100 T910 - 200 T1112
      + 35850 FIXED
! Subject to the following constraints:
ST
! TEi + Tij <= TEj
           T12   + TE1  - TE2  <= 0
           T23   + TE2  - TE3  <= 0
           T34   + TE3  - TE4  <= 0
           T35   + TE3  - TE5  <= 0
                   TE4  - TE5  <= 0
                   TE5  - TE6  <= 0
           T67   + TE6  - TE7  <= 0
           T57   + TE5  - TE7  <= 0
           T48   + TE4  - TE8  <= 0
                   TE7  - TE8  <= 0
           T710  + TE7  - TE10 <= 0
           T89   + TE8  - TE9  <= 0
           T910  + TE9  - TE10 <= 0
           T1011 + TE10 - TE11 <= 0
           T1112 + TE11 - TE12 <= 0
! TCij <= Tij <= TNij
           T12   >= 5
           T12   <= 5
           T23   >= 1
           T23   <= 3
           T35   >= 4
           T35   <= 7
           T34   >= 3
           T34   <= 5
           T67   >= 3
           T67   <= 6
           T57   >= 3
           T57   <= 5
           T48   >= 4
           T48   <= 7
           T710  >= 5
           T710  <= 10
           T89   >= 4
           T89   <= 6
           T1011 >= 1
           T1011 <= 2
           T910  >= 0
           T910  <= 2
           T1112 >= 2
           T1112 <= 3
! Project ESD = 0
           TE1   =  0
! Project EFD <= Target (This is the problem parameter; must be feasible)
           TE12  <= 100
! The fixed cost coefficient is 1
           FIXED = 1
END
LEAVE
```

The Network of Fig. 10.15 in Activity-On-Arrow Format




```
DO RANGE(SENSITIVITY) ANALYSIS?
? n
: PARA
ROW:
42
NEW RHS VAL=
0
```

VAR OUT		VAR IN		PIVOT ROW	RHS VAL	DUAL VARIABLE	OBJ VAL
SLK	42	SLK	20	42	36.0000	.000000	17050.0
SLK	19	SLK	36	19	34.0000	150.000	17350.0
SLK	35	SLK	40	35	33.0000	200.000	17550.0
SLK	39	SLK	22	39	32.0000	200.000	17750.0
SLK	10	SLK	11	10	31.0000	300.000	18050.0
SLK	6	SLK	24	6	30.0000	300.000	18350.0
SLK	21	SLK	26	21	29.0000	450.000	18800.0
SLK	9	SLK	32	9	28.0000	450.000	19250.0
SLK	14	SLK	38	14	28.0000	600.000	19250.0
SLK	37	SLK	6	37	26.0000	700.000	20650.0
SLK	23	SLK	34	23	25.0000	750.000	21400.0
SLK	31	SLK	28	31	23.0000	850.000	23100.0
SLK	33	SLK	30	33	23.0000	1100.00	23100.0
SLK	25	ART		25	21.0000	1150.00	25400.0

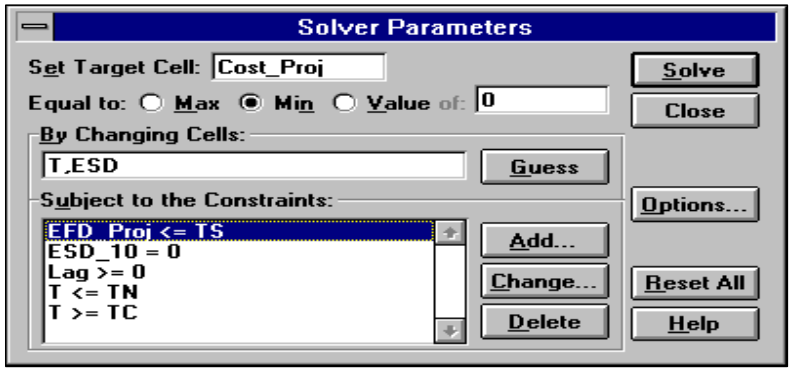
Time-Cost Tradeoff Problem Using Excel and its Solver (Version 5.0)

P.G. Ioannou
11/22/1994

Activity Durations T, and Early Start Dates ESD Are the decision variables

	Activity No	Normal Time T_N	Normal Cost C_N	Crash Time T_C	Crash Cost C_C	Cost = f + v*Time v = $(C_N-C_C)/(T_N-T_C)$	f = C_N-v*T_N	T	C	ESD	EFD	
_10	10	5	500	5	500	0	500	5	500	0	5	
_20	20	3	900	1	1,200	- 150	1,350	1	1200	5	6	
_30	30	7	3,250	4	4,150	- 300	5,350	4	4150	6	10	
_32	32	5	1,000	3	1,300	- 150	1,750	4	1150	6	10	
_40	40	6	1,400	3	2,750	- 450	4,100	5	1850	10	15	
_42	42	5	1,100	3	1,900	- 400	3,100	5	1100	10	15	
_44	44	7	1,500	4	2,400	- 300	3,600	7	1500	10	17	
_50	50	10	4,200	5	7,200	- 600	10,200	10	4200	15	25	
_52.1	52.1	6	0	4	500	- 250	1,500	6	0	17	23	
_52.2	52.2	2	800	0	1,000	- 100	1,000	2	800	23	25	
_60	60	2	1,100	1	1,300	- 200	1,500	1	1300	25	26	
_70	70	3	1,300	2	1,500	- 200	1,900	2	1500	26	28	=EFD_Proj
		SC_N =	17,050	SC_C =	25,700	Sf =	35,850	SC =	19,250	TS =	28	

Link		Lag	
10	20	0	= ESD _20-ESD _10-T _10
20	30	0	= ESD _30-ESD _20-T _20
20	32	0	= ESD _32-ESD _20-T _20
30	40	0	= ESD _40-ESD _30-T _30
30	42	0	= ESD _42-ESD _30-T _30
32	40	0	= ESD _40-ESD _32-T _32
32	42	0	= ESD _42-ESD _32-T _32
32	44	0	= ESD _44-ESD _32-T _32
40	50	0	= ESD _50-ESD _40-T _40
40	52.1	2	= ESD _52.1-ESD _40-T _40
42	50	0	= ESD _50-ESD _42-T _42
42	52.1	2	= ESD _52.1-ESD _42-T _42
44	52.1	0	= ESD _52.1-ESD _44-T _44
50	60	0	= ESD _60-ESD _50-T _50
52.1	52.2	0	= (ESD _52.2)-ESD _52.1-T _52.1
52.2	60	0	= (ESD _60)-(ESD _52.2)-(T _52.2)
60	70	0	= ESD _70-ESD _60-T _60



Project Cost: Cost_Proj is the objective function to minimize

TS is the desired project T. EFD_Proj must be <= TS. Change TS and run Solver again for more T/C points

These lags are constrained to be >= zero