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EFFECTS OF MICROSTRUCTURE, COMPOSITION AND STRENGTH ON THE NIL DUCTILITY TRANSITION TEMPERATURE OF A LOW ALLOY NI-CR-MO STEEL

by
Marcel Leon Salive

A dissertation submitted in partial fulfillment
of the requirements for the degree of
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ABSTRACT

EFFECTS OF MICROSTRUCTURE, COMPOSITION AND STRENGTH ON THE NIL DUCTILITY TRANSITION TEMPERATURE OF A LOW ALLOY NI-CR-MO STEEL

by
Marcel Leon Salive

Chairman: Maurice J. Sinnott

Steel from 22 heats of low-carbon Ni-Cr-Mo steel (ASTM A543-65) were heat treated to study the effects on the drop weight nil ductility transition (NDT) temperature of (1) commercial variation in composition and inclusion content, (2) variation in microstructure such as prior austenitic grain size and the relative amount of isothermally produced ferrite or bainite in a tempered martensitic matrix, and (3) the observed variation in strength obtained after a one hour 1150F temper followed by a water quench to prevent embrittlement while cooling from the tempering temperature.

Using a tempered 100 percent martensitic microstructure as a base line, the variables found to be most significant are linearly related to the measured Nil Ductility Transition (NDT) temperature; the range of the variable and its effect on the NDT temperature is:

VARIABLE	RANGE	EFFECT ON NDT
Producer	(Coded 1 or 2)	-21.9F/Unit
Prior Austenitic Grain Size	(3 to 9.5)	-19.2F/ASTM G.S. No.
Percent Bainite	(0 to 76%)	0.62F/Pct B
Percent Ferrite	(0 to 69%)	-0.23F/Pct F
Percent Pearlite	(0 to 32%)	5.4F/Pct P
Tensile Yield Strength	(61.5 to 124.7 Ksi)	0.68F/Ksi

Higher order terms and interaction terms did not significantly improve the relationship, nor did consideration of inclusion content of the steel. Other than hydrogen, no alloying or trace elements were found to correlate with the measured NDT temperature of this steel.

The difference between the tensile and compressive yield strength, sometimes called the strength differential (SD) effect was observed in this study to be at least 5% of the tensile yield strength. Data are cited to show that in the low-carbon Ni-Cr-Mo steels studied here, the S-D effect observed was a relatively constant percent of the tensile yield strength, and is markedly structure sensitive to prior austenitic grain size, microstructural constituents, tempering temperature, type and distribution of carbides formed during tempering, and tempering slightly above the lower critical (A_{c1}) temperature.

INTRODUCTION

At the present there is a gap in the body of general metallurgical knowledge concerning the mechanisms that control the notch toughness, as measured by the drop weight Nil Ductility Transition NDT* temperature, of the low carbon, medium alloy, quenched and tempered steels. Most prior studies have been limited to the process of characterizing specific classes of commercial steels by their expected NDT temperature frequency distribution range. No previous studies have been made of the interaction of chemical composition, mechanical properties and microstructure with the NDT temperature of any steel.

Studies of the notch toughness of higher alloy steels, such as those recently performed by U. S. Steel, have concerned themselves with the fracture mechanics or critical flaw size approach that is more appropriate for such higher strength, low toughness materials. Previous studies of the higher toughness steels, however, have always concerned themselves with the effect of variation in a single alloying element and usually only with its effect on strength or on some Charpy impact property. In addition, these previous studies have been concerned only with the effects on a normalized steel or on an oil quenched alloy steel of un-

*The NDT is the temperature of initial transition from plane strain to mixed-mode fracture and represents the temperature at which very small flaw sizes suffice for dynamic fracture initiation. The NDT temperature is generally not indexable to the sharply falling Charpy energy curve.

determined microstructure.

The normalized structural steels are gradually being replaced in critical, high toughness, low transition temperature applications by the quenched and tempered, medium alloy steels such as HY-80 and HY-100. Therefore, it is important that the mechanisms controlling the interaction of composition, mechanical properties and microstructure with NDT temperature be determined. This is so, particularly since the NDT temperature is one of the most important criteria for the selection of a steel for structural application under severe service conditions.

There has long been an interest in the structural use of high toughness, low transition temperature steels, and more recently with the higher strength, quenched and tempered steels such as the ASTM A543-65 (Class I and II), HY-80, HY-100, and HY-130 steels currently being used for the shells of pressure vessels and hulls of oceanographic vehicles. The steel used in this study is a quenched and tempered medium alloy steel having an 85,000 psi yield strength and the following nominal composition: 0.16% C, 0.36% Mn, 0.015% P, 0.015% S, 0.25% Si, 2.75% Ni, 1.50% Cr and 0.35% Mo. This steel is commonly classed as a low carbon nickel-chrome-molybdenum alloy steel.

LITERATURE SURVEY

The impact transition temperature is some arbitrary point in the temperature range over which the fracture characteristics of a metal with a body centered cubic lattice, and sometimes with a hexagonal close packed lattice, exhibit a marked loss in ductility as evidenced by (1) a large drop in the energy required to produce fracture, (2) a marked change in the fracture surface appearance from a dull, fibrous-looking transcrystalline shear fracture associated with high energy absorption to a bright, faceted cleavage or intercrystalline fracture associated with low energy absorption, and (3) a marked loss of ductility as evidenced by either a decrease in the amount of lateral contraction at the bottom of the notch used to initiate the crack, or a decrease in the amount of lateral expansion at the "hinge" point of the impact specimen opposite the notch. The temperature range over which this transition from ductile to brittle behavior occurs is sensitive to test conditions such as strain rate, notch acuity, and residual or applied stress levels, as well as the material's toughness at the time of testing; as a result the transition range is usually determined using a standardized impact specimen and test method to test the material at a series of temperatures. The "transition temperature" then is defined as that temperature in the transition range at which some specific condition occurs

such as:

1. The fracture appearance transition temperature, FATT, is the temperature at which the specimen is 50 or 100 percent fibrous and is called the 50% FATT or the 100% FATT, respectively.

2. The fracture temperature is the temperature at which the Charpy V-notch, C_V, impact specimen absorbs some specific amount of energy; a common energy value used to characterize mild steel ship plate is 15 ft-lb and this criterion would be referred to as the Charpy V-notch 15 ft-lb transition temperature.

3. The temperature at which the drop weight impact test specimen exhibits no ductility is the nil ductility transition, NDT, temperature, and is the temperature of initial transition from plane strain to mixed-mode fracture. The NDT temperature represents the temperature at which very small flaw sizes suffice for dynamic fracture initiation. The drop weight NDT temperature is not generally indexable to some specific point on the sharply falling Charpy energy curve.

An extensive body of literature exists on the effects of microstructure, composition, and strength on the impact transition temperature. The data are mainly concerned with the effects of alloy additions to plain carbon steels, or alloy steels having 0.30 percent or more carbon, whose microstructures are frequently not specified other than as resulting from a normalizing treatment, and these data

often ignore the austenitic grain size of the steel prior to cooling. These studies are seldom based on the drop weight nil ductility transition temperature.

GRAIN SIZE EFFECT

Changes in grain size have been reported to have a detrimental effect on the ductile to brittle fracture transition temperature of steels[1-32]. Some authors report the effect of change in the average grain diameter, others in terms of the change in ASTM micro-grain size. Some authors report grain size in terms of:

1. Ferritic grain size as measured on a polished and etched section;

2. Prior austenitic grain size, measured using the McQuaid-Ehn test method;

3. Prior austenitic grain size as measured using an etheral picric acid etch to outline the prior austenitic grain boundaries on a polished specimen;

4. Fracture grain size as determined by comparing a specimen broken at low temperature and exhibiting a crystalline fracture appearance to the fracture grain size of a set of specially prepared specimens used as a standard.

The various methods of reporting grain size are not independent. The ASTM micro-grain size is related to the average grain diameter as follows:

1. The ferritic grain size is highly dependent upon

the prior austenitic grain size and to a lesser degree upon other factors[2,4-7,13,17-19,22,24,26,28,30,31] such as:

a. cooling rate from the austenitizing temperature. The slower the cooling rate the coarser the resultant ferrite due to fewer nucleation sites and more time for these sites to grow.

b. temperature at which transformation from austenite to ferrite, or some other product, takes place. The higher the temperature the fewer nucleation sites and therefore larger grains.

c. the amount of ferrite grain growth that takes place after transformation while cooling, tempering or stress relieving. Longer times allow growth of some grains at the expense of smaller or less favorably oriented neighbors.

d. the quantity and dispersion of inclusions, oxides and nitrides that may inhibit ferritic as well as austenitic grain growth, and

e. deoxidation practice.

2. The fracture grain size of embrittled material is usually the same as the prior austenitic grain[11,12]. The strong dependence on prior austenitic grain size, all other things being equal, is due to the fact that the austenite to ferrite reaction nucleates at the prior austenite, gamma, grain boundary. The finer the prior austenitic grain the greater the grain boundary surface area and thus the more nucleation sites available for the gamma to ferrite trans-

formation. Slower cooling rates or longer times at temperature allow growth of existing grains either from existing gamma or at the expense of other smaller ferrite grains.

Most of the investigators who have reported on the effects of grain size have used the Charpy V-notch impact test to determine the fracture transition temperature[1,3,4,7-11,13-21,23,24,29-31]; only a few have used the Charpy keyhole specimen[2,6], the drop weight impact test specimen[21,24,32] or the crack or tear test specimen[6]. How the fracture transition temperature is defined has a marked effect on the magnitude of the shift in transition temperature caused by changes in grain size[26]. Some of the definitions of fracture transition temperature used in the literature are as follows:

1. For Charpy V-notch and keyhole specimens

   a. Level of impact energy absorbed was frequently used to determine the transition temperature[7,8,13,14,16,20,21,24]. The values used ranged from 8 to 40 ft-lbs, with perhaps the most common value being 15 ft-lbs. The 15 ft-lb level is commonly used for plain carbon steels becuase it is the level found by the National Bureau of Standards and the Ship Structures Committee to characterize the susceptibility of plain carbon steel ship plate to brittle fracture. The higher values are associated with quenched and tempered alloy steels.

   b. A specific amount of fibrous fracture appear-

ance on the broken surface of the impact specimen is sometimes used to define the fracture transition temperature[4,8,16,21,23,26,27]. This is usually referred to as the fibrous fracture appearance transition temperature, FATT, and 50 percent fibrous fracture is the most commonly used value.

c. An amount of lateral expansion or contraction in the deformed area of fracture is used occasionally to define the transition temperature[21].

d. The inflection point or sharp drop in the energy absorbed vs. temperature curve is used[3,20].

e. A mean energy level such as the mean energy absorbed at +100 and -196C has set the transition temperature[4], as has the temperature at which the Charpy energy absorbed is one-half the maximum energy absorbed at upper energy shelf[24], and as "...the middle of the temperature range over which an impact value of 40 ft-lbs was obtained".[2]

f. Sometimes just a visual comparison is made of energy absorption vs. test temperature curves for one material or condition as opposed to another[1].

2. For drop weight impact test specimens, the transition temperature is defined as the highest temperature at which the crack from the brittle crack starter weld will propagate across the surface of the specimen and around the edge on at least one side[21,24,32].

3. The crack arrest or tear test transition tempera-

ture is the temperature at which a running crack is halted or arrested by the material. In a specimen having a thermal gradient, the crack runs toward the warm side of the specimen. These tests are not used extensively and apparently are seldom used to evaluate the effect of grain size on the transition temperature.

Other parameters associated with the material, its treatment and microstructure have been reported to have an interaction with the grain size to change the magnitude of the shift in the impact transition temperature. These factors are: impurity level and composition changes[5,7,8,19,23]; commercial size heats as opposed to laboratory size heats or melts[9,31]; straining or cold deformation[5,12,26]; thermal treatments that cause a secondary effect to overshadow grain size effects, retention of austenite[3], temper embrittlement[10,15,20,23,26], and growth of grains[2,5,7,23]; and microstructure changes[1,3,14,17,20,22,26,27]. The effect of each on the apparent shift in transition temperature with grain size ($\Delta T/\Delta GS$) may either be positive or so negative in nature as to make grain size appear to have no effect.

As a result of these complications, each reported value of $\Delta T/\Delta GS$ must be evaluated in terms of the specimen used; the criterion defining transition temperature; the composition, including impurity levels; source of the material, lab vs. production melts; thermal treatment; straining; microstructure; and the range over which grain size was actually varied. If this is done, a few general obser-

vations may be made on the specific effect of changes in grain size on the impact transition temperature. For convenience, the following will always refer to grain size as the ASTM micro-grain size. The following equation represents the relationship between the number of grains per square inch (n) and ASTM micro-grain size (N): $n = 2^{(N-1)}$.

Ferritic Grain Size

The ferritic grain size is commonly used in the literature to report the effects of grain size on $\Delta T/\Delta GS$. There are three reasons for this: first, ferrite is the major room temperature equilibrium phase in the hypoeutectoid plain carbon steels used in most of the studies reported in the literature; second, ferrite with varying amounts of carbide is the primary room temperature microconstituent of quenched and tempered alloy steels; and third, the final ferritic grain size is readily measured using standard metallographic techniques.

Changes in the size of the ferrite subgrain size are reported to affect the impact transition temperature[18,31]. An increase in the sub-boundaries, a decrease in the ferritic subgrain size, raises not lowers, the Charpy V-notch 15 ft-lb transition temperature 20F/per ferrite grain size number. The ferrite subgrain boundaries examined with an electron microscope show a lower dislocation density than the boundaries associated with the true ferrite grain boundary[31]. While these authors were puzzled by this

effect, it must be pointed out that there is a possible explanation. Current dislocation theory says that the subgrain structure is caused by a preferential alignment of dislocations having like signs. As the number of dislocations increase, the subgrain size would be expected to decrease and the strain in the grain increase. The usual effect of straining is to raise the transition temperature for ductile to brittle fracture. In fact, temper embrittlement has even been attributed to strain developed across prior austenitic grain boundaries as a result of the contraction of the ferrite lattice when dissolved alloying elements are removed from solution in ferrite by a reaction with existing minor carbide phases[12]. Thus it is logical to expect that as the ferritic subgrain size gets smaller indicating increasing dislocation densities and higher strains in the lattice, the impact transition temperature would increase as opposed to the usual decrease in $\Delta T/\Delta GS$ associated with a finer true ferritic grain size.

References 2, 6, 7, 13, 17, 18, 24, 28 and 31 show that a decrease in the ferritic grain size of a plain carbon steel sufficient to increase the ASTM grain size number one unit will reduce the transition temperature 22F on the average, the effect ranging from an increase of 9F to a decrease of 39F. The 9F value, reference 31, is undoubtedly due to the normal variation in the measurement of transition temperature of ±10F reported by Gross[21] and to the small variation in grain size, less than one unit[31]. The -39F

value for $\Delta T/\Delta GS$ is difficult to rationalize since the author reports neither the type of impact specimen he used nor the criterion used for determining the transition temperature.

These $\Delta T/\Delta GS$ values seem to be independent of carbon content in these plain carbon steels. It is reported that of the factors: ferrite grain size, pearlite island size, pearlite spacing and amount of proeutectoid ferrite, as it influences pearlite carbon content, only the ferrite grain size showed any degree of correlation with the Charpy V-notch 15 ft-lb transition temperature[17]. The $\Delta T/\Delta GS$ value's dependence on test method is seen in reference 24 where the author reports a value of $-18F/\Delta GS$ based on the Charpy V-notch 15 ft-lb energy transition temperature and $-11F/\Delta GS$ based on drop weight NDT.

Austenitic Grain Size

Increases in the prior austenitic grain size have usually been associated with increasing impact transition temperatures[1,3,4,8-14,16,19,23,27]. In fact, as temper brittleness increases in martensitic-ferrite, the fracture paths become increasingly intergranular and follow the prior austenitic grain boundaries[11,12,29,36,37,41]. The relationship between the Charpy V-notch 10 ft-lb and 50% FATT transition temperatures and prior austenitic grain size was linear for hardened 0.84 and 0.95C steels[16].

It was reported for tests on SAE 1045, 2340 and 3140

having various microstructures and tempered to the same hardness, that increasing the prior austenitic grain size increased the impact transition temperature of air cooled and tempered (pearlitic) material ($\Delta T/\Delta \gamma GS$ from -25 to $\simeq$ -120F) but had the opposite effect (+5 to +20F) on quenched and tempered material[3]. The slight improvement in the quenched and tempered steels as austenitic grain size increased was attributed to the increase in hardenability as the grain size increased and to the resultant increase in the amount of martensite formed during quenching. The impact fracture transition temperature of the tempered martensitic structure was always better, i.e. lower, than that of the tempered pearlitic structures. Increasing gamma grain size also increased the notch sensitivity of the pearlitic microstructure[3]. In a later study it was reported for 0.80% C plain carbon steel that as the hardness was increased, pearlite spacing decreased, the impact transition temperature went up, more so for fine than for coarse prior austenitic grain size material; in fact, there was a minimum in the pearlite spacing (hardness) vs. impact transition temperature relation for coarse material whereas the finer prior gamma grain size material showed a continuous increase[14].

Prior austenitic grain size is also reported to have a controlling effect on the mean impact energy absorbed transition temperature of 0.12 C bainitic steels, but the effect is lessened as the strength increases with increased dis-

persion of carbides[27]. The prior austenitic grain diameter, which is directly related to either the rolling finishing temperature or the reheating temperature used during austenitizing, has been shown to strongly affect the average Charpy energy impact transition temperature of 70,000 psi tensile strength bainitic steel, with the transition temperature increasing in an S shaped relationship with increasing prior austenitic grain diameter[27]. The transition temperature of the bainitic steels reportedly ranged from -76F for an austenite grain diameter of 3.9×10^{-4} in. to 140F for a 22.3×10^{-4} in. diameter.

Grain refiners are frequently cited as and demonstrated to have a beneficial effect on lowering the impact transition temperature[4,5,7] and even materials which are not specific grain refiners, such as lead, but which react to inhibit grain growth, will show a beneficial effect, especially when added to normally coarse grained steels[8].

Petch is reported[30] to have shown that the Charpy V-notch 15 ft-lb transition temperature (T) of ferritic steels is linearly related to the natural logarithm of the square root of the reciprocal of the mean grain diameter (d) in the following way: $T = A + B\ell n(d^{-\frac{1}{2}})$. If one uses $n = 2^{(N-1)}$ where n is the number of grains per square inch and N is the ASTM grain size number, and notes the fact that n is proportional to d^2, one can rewrite the Petch relationship in terms of the ASTM grain size and show that impact transition temperature should be a direct linear function of N,

$T = A_1 + B_1(N)$.

An investigation of unhardened C1095 and C1080 plain carbon steels shows that a coarser prior austenitic grain size raises the impact transition temperature, regardless of the temperature at which the austenite is transformed to pearlite[8]. In addition, it was shown that while lower transformation temperatures raised the ultimate tensile strength, it also lowered the impact transition temperatures by refining the microstructure. The data below shows the observed range of change in Charpy V-notch, C_v, impact transition temperature ΔT per unit change in prior austenitic grain size, $\Delta T/\gamma GS$ and demonstrates the dependence of $\Delta T/\gamma GS$ on both the criteria used to measure ΔT and the steel composition.

STEEL	C_v (10 ft-lb.)	C_v (50% FATT)
C1095	46-35F/γGS	17-19F/γGS
C1080	27-29F/γGS	33-23F/γGS

Studies of the effects of various factors on the impact transition temperature of plain carbon steel ship plate have shown that the higher transition temperatures associated with thicker plates are more closely related to differences in the ferrite grain size resulting from rolling finishing temperature and cooling rate than to differences in chemical composition[13]. The finer the grain size, the higher the ASTM grain size number, the lower the impact transition temperature. The magnitude of the effect of grain size on $\Delta T/\Delta GS$ depends on the method used to measure grain size;

based on ferrite grain size the $\Delta T/\Delta GS$ for ship plate was $-14F/\Delta GS$, and based on fracture, which may partially reflect prior austenitic grain size[11,12,29,36,37,41], $\Delta T/\Delta GS$ was $-9°F/\Delta GS$[13].

In a study of the effects of impurity level on the way changes in prior austenitic grain size influenced Charpy V-notch 100% FATT transition temperature of a 0.3 C, 3.0 Ni, 0.75 Cr steel, it was observed that the relative effect of grain size was strongly influenced by both impurity level and thermal treatment[23].

	As Oil Quenched and Tempered at 1112F	OQ and T Plus 168 Hr at 842F
High purity	$-4.1F/\Delta\gamma GS$	$-5.9/\Delta\gamma GS$
Plus 0.023% P*	$-13.6F/\Delta\gamma GS$	$-19.8/\Delta\gamma GS$
Plus 0.074% Sn*	$-28.5F/\Delta\gamma GS$	$-42.3/\Delta\gamma GS$

These data show that the impact transition temperature of high purity alloys is affected by changes in gamma grain size; that tin is more detrimental than phosphorus at equal atomic percents; and that the detrimental effect of increasing prior austenitic grain size is greater after an embrittling treatment than after a simple tempering treatment.

Microstructure

The interaction of microstructure and grain size appears to be rather straightforward; the finer the microstructure for any given prior austenitic grain size, the

*Each about 0.04 atomic percent

lower will be the impact transition temperature. The finer the ferrite grains in ferritic steels[2,4-7,15,18,22,24,26,28,32], the finer the pearlite colonies and the finer the interlaminar ferrite-carbide spacing[1,3,14,16,17], the finer the bainitic-ferrite[20,22,27] and the finer the martensitic-ferrite[3,22,27], the lower will be the impact transition temperature. It should be remembered that the finer the prior austenitic grain size[2,6], the lower the rolling finishing temperature[13], the faster the cooling rate[2,6,28], and the lower the temperature of transformation from austenite[8,28], the finer will be the resultant final grain size.

Because of the increasingly acicular nature of the ferrite grain size as the microstructure is changed from polygonal ferrite to bainitic-ferrite and then to martensitic-ferrite, it becomes increasingly difficult to make a change in ferritic grain size and as a result, ferritic grain size can hardly be considered a variable[22]. As a result of the increasing acicular nature of the microstructure and the fracture path following the embrittled prior austenitic grain boundary[11,12,29,36,37,41], one finds the tendency is for researchers to report the effects of grain size on $\Delta T/\Delta GS$ in terms of prior austenitic grain size when dealing with tempered bainitic and martensitic steels[3,20,22,27]. Researchers also tend to report ferritic grain sizes when dealing with plain carbon steels where the structure is essentially ferrite or some mixture of ferrite

and pearlite.

For 0.02 percent carbon, plain carbon and alloy steels there is reported to be a slight increase in Charpy V-notch transition temperature due to the acicular shaped ferrite grains over what might otherwise be expected from simple changes in ferrite grain size alone[20]. This may be due to the fact that the identical grain sizes were developed by a combination of furnace and air cooling from different austenitizing temperatures, which means that prior gamma grain size differences or embrittlement during cooling may have caused the differences rather than simply the differences between polygonal and acicular ferrite grains.

For 0.32 percent carbon, plain carbon steel, it was found that of ferrite grain size, pearlite island size, pearlite spacing and amount of proeutectoid ferrite as it affects pearlite carbon content, only the ferrite grain size showed any degree of correlation with the Charpy V-notch 15 ft-lb transition temperature[17]. As the pearlite spacing is decreased in a 0.80 percent carbon, plain carbon steel, the hardness is increased and the Charpy V-notch 8 ft-lb transition temperature increased, more so for fine than for coarse prior austenitic grain size material[14]. In tests on SAE 1045, 2340, and 3140 having various microstructures tempered to the same hardness, the Charpy V-notch transition temperature of tempered martensite was always lower than that of the tempered pearlitic structures[3].

Fracture Path

The sudden decrease in impact energy shown by the truly ferritic, low carbon, little or no alloying steels is accompanied or brought about by a marked increase in the amount of cleavage fracture and by a marked reduction in the plastic deformation accompanying fracture[6,7,13,15,20]. The cleavage fracture of the ferrite grains initiates at the ferrite or ferrite-pearlite grain boundaries[20]. The properties of the ferrite rather than the pearlite control notch toughness in killed and semikilled steels[31]. The low temperature brittle fracture in these ferritic steels is primarily transcrystalline cleavage of the ferrite grains, though there has been reported an increased tendency to intergranular brittleness with increasingly coarse grains and as the oxygen content is raised above 0.003 percent[15,18,20].

In martensitic steels transgranular and intergranular fracture refer to austenitic grains present prior to quenching and not to the ferritic grain structure developed by tempering the martensite[11,12,23,29,36,37,41]. The relative amount of transgranular and intergranular fracture developed by the impact fracture of a tempered martensite depends on the following:

1. The relative temperature in the fracture transition region. The lowest temperatures tend to give more intergranular fracture[11].

2. The amount of temper embrittlement. Certain combi-

nations of temperatures and long holding times, will increase the tendency to intergranular fracture if the material is susceptible to temper embrittlement[12].

3. Blue brittleness, associated with a 500F tempering temperature. This tends to give an intergranular fracture and shows a brittle appearance over a considerable temperature range and which slowly disappears at higher temperatures without ever exhibiting an abrupt transition[11].

4. Non-embrittling tempering causes fracture to show a progression from transgranular fracture after tempering at 400°F, to intergranular fracture after a 500°F temper, and a return to the transgranular fracture mode after a 1200F temper. Tempers from 550F to 1200F exhibit brittle fractures composed of a mixture of intergranular and transgranular brittle partings[11].

5. High purity tempered martensite will exhibit only transgranular cleavage when it breaks in a brittle way whereas steels containing 0.04 atomic percent phosphorus or tin showed at least some intergranular fracture along the prior austenitic grain boundaries with the proportion increasing with grain size and with embrittling treatments[23]. Phosphorus containing steel, 0.023 wt% P in a 0.3 C, 3.0 Ni, 0.75 Cr steel, tended to show more intergranular fracture than the same steel doped with 0.074 wt% Sn.

Summarizing, the impact fracture path is characterized by the transgranular or intergranular mode relative to the ferritic grains in plain carbon steels having a micro-

structure consisting of ferrite or ferrite and pearlite, and to the prior austenitic grains in quenched and tempered martensite which shows some degree of temper embrittlement. The impact properties of bainitic steels are separated into two distinct types, with the high temperature bainitic structures behaving like ferrite-pearlite structures and low temperature bainitic structures behaving like martensitic structures; the impact properties of the low temperature bainite improve with decreasing prior austenitic grain sizes[27].

MICROSTRUCTURE

The general effect reported in the literature is for tempered bainitic steels to have impact properties intermediate to those of tempered martensitic and tempered pearlitic steels of the same hardness, with the tempered martensitic steel having the lowest impact transition temperature and the highest level of energy absorbed above the transition temperature[27,33,42]. The fine bainitic structure responds to tempering like a quenched steel; the transition temperature steadily decreases with increasing tempering temperature[27,35]. Composition of low carbon bainitic steels has little effect on the impact properties if similar microstructures having the same grain size and same hardness are compared[27]. In order to get adequate impact resistance in bainitic or martensitic steels, they must be tempered beyond the stage where maximum secondary

hardening takes place; secondary hardening due to the presence of other alloying elements, has a deleterious effect on impact resistance[22]. Even after tempering, a portion of upper bainite is more detrimental to the impact properties of an otherwise martensitic microstructure than is a portion of lower bainite[42].

COMPOSITION

The effects of compositional changes in tempered martensitic and lower-bainitic microstructures on impact properties are threefold: first, the basic impact fracture transition temperature is shifted due to the presence of trace impurity elements[23,36,37]; second, the susceptibility to embrittlement at low tempering temperatures, 500 to 800F, may be affected[10,37]; and third, there may be susceptibility to temper embrittlement during either long time tempering treatments or slow cooling instead of quenching from the tempering temperatures[9,10,22,23,34,36,37,41]. In addition, the composition changes may be detrimental to impact properties if they increase the strength markedly or if they tend to retard softening during tempering[27,35,37].

Tests on special high purity quenched and tempered steels have shown that certain minor impurity elements, some of which are invariably present in commercial steels and for which many are not routinely analyzed, P, Sb, As, Sn, and N_2, are primarily responsible for embrittlement[23,36,37]. The high purity steels show very low impact

transition temperatures and very little susceptibility to embrittlement as a result of usually detrimental tempering treatments. Susceptibility to low tempering temperature embrittlement is increased by Mn, P, Si, Cr, As, Sn, N_2, not affected much by Ni, Cr and Mo and seems to be inhibited by Al[10,37]. The most detrimental effects were observed for P and N_2[37].

Susceptibility to temper embrittlement due to either long tempering times in the region from 900 to 1100°F or slow cooling from the tempering temperature is increased by C[9,22], Mn[34,36,37], P[10,23,36,37,41], Si[36,37], Ni[34], Cr[34], As[36,37], Sb[36,37,39,41] and Sn[23,36,37,39], not affected much by N_2, Bi, Cu, Co, Ge, Ga, and Zr[36 37], and inhibited by Mo[10 34 36 39] and W[39]. There was no evidence in studies of a quenched and tempered Ni-Cr-Mo steel that there was any interaction between regular alloying additions and trace elements that exaggerated their total effect[39,41]. For some hardened steels, there is evidence from the impact transition temperature and the fracture appearance that some embrittlement occurred during the initial tempering at an elevated (1112 to 1202°F) temperature[37,41].

A study of a quenched and tempered Ni-Cr-Mo steel that was not deliberately given an embrittling treatment shows that P, Sb and Sn have a detrimental effect on impact transition temperature, As had little or no effect and Al additions were actually beneficial[39]. Phosphorus was the most damaging, and the effect of As, which had been shown

to be detrimental to deliberately embrittled steel, showed little effect in the not deliberately embrittled steel. The effect of Al shifted from detrimental in embrittled material to beneficial in non-embrittled material; the benefit of Al additions was attributed to a decrease in the prior gamma grain size. Other effects reported are that the optimum carbon content in quenched and tempered steels is about 0.38 percent and that this amount optimizes both NDT temperature and maximum Charpy V-notch shelf energy[40]. Sulphur is reported to have no effect on impact transition temperature but to lower the maximum Charpy shelf energy[10].

B, C, Mn, Ni and Cr additions can be detrimental to the impact properties of low-carbon bainitic steels if they reduce the austenite decomposition temperature and as a result either increase the strength excessively or produce a mixed microstructure of bainite and martensite[27]. Secondary hardening elements like Mo and V have a tendency to reduce impact properties of low-carbon bainites, in that they increase the hardness of the initial microstructure and they tend to precipitate carbides which retard softening during tempering[27].

STRENGTH

The effect of strength level on impact transition temperature is complicated by the dependency of strength on the interaction between composition, grain size, microstructure produced by the transformation of austenite under

a wide variety of conditions, and tempering treatment. Increases in strength produce corresponding increases in impact transition temperatures unless other parameters are adjusted to compensate for the increased strength level[27,35]. Conversely, a decrease in strength brought about by a long time temper is no guarantee that the impact transition temperature will improve, since for a wide variety of quenched and tempered steels a 100 hour stress relief will reduce the strength and at the same time increase the transition temperature; this is attributed to a susceptibility to temper embrittlement[38].

In bainitic steels, increases in strength brought about by refining the bainitic-ferrite microstructure will have an adverse effect on impact transition temperature unless compensation is made by refining the prior austenitic grain size also[27]. The dependence of the impact transition temperature of bainitic steels on prior austenitic grain size is attributed to the fact that the upper limit on the size of the bainitic-ferrite plates is the prior austenite grain diameter[27]. Tempering bainitic steels at a temperature above the temperature at which the bainite formed improves its impact transition temperature by lowering the strength[42].

THE STRENGTH DIFFERENTIAL EFFECT IN STEELS

Hirth and Cohen[43] have called attention to the strength-differential phenomenon in martensitic steels

wherein the strength of such steels are notably higher in compression than in tension. Recently Leslie and Sober observed that martensitic 4300 series steels are significantly stronger in uniaxial compression than in uniaxial tension[44]. Lessells has observed a similar effect for 1035 and 1045 steels[45]. Hirth and Cohen have presented a partial explanation for the strength-differential (S-D) effect in as-quenched martensitic steel based on the interstitial carbon atoms straining the body centered iron from a cubic to a tetragonal lattice. They attribute the S-D effect to the difference between the nonlinear stress-strain curves in tension and compression and demonstrate that their proposed nonlinear model can account for a 3 to 6 percent strength-differential.

The work of Hirth and Cohen was stimulated by Leslie and Sober's work on the effects of .12, .21, .34 and .41 percent carbon additions on the strength of as-hardened martensitic AISI 4300 series steels in the brine quenched condition. Figure 7 gives a plot of the S-D effect ($\Delta\sigma_{S-D}$) versus carbon content and a plot of the ratio of the magnitude of the S-D effect to the tensile yield strength (σ_{TYS}) for various carbon contents[43]. The solid line represents the experimental values and the hatched area gives the region in which the true strength differentials would lie, based on the 1/3 to 1/2 correction to $\Delta\sigma_{S-D}$ that Rauch and Leslie recommended be applied to this data.

It should be pointed out that Leslie and Sober heat treated finished specimens in evacuated quartz capsules which were crushed upon contact with the brine quench[44]. The specimens were reported not to have been distorted, although the higher carbon specimens increased in length slightly. They also reported that about one-third of the 4340 specimens had to be discarded since they were cracked when they were taken from the liquid nitrogen bath. This cracking is frequently observed for the higher carbon steel and is attributed to the lowering of the M_S temperature as the C content is raised. The lower M_S eliminates the possibility of self-tempering of the fresh martensite to relieve residual stresses in the brittle, non-ductile as-quenched martensite. It is generally believed that the lower carbon martensites are not only inherently more ductile, but that they self temper somewhat due to the higher temperature at which they form.

Leslie and Sober's compression tests were reported to have been made using a subpress without either a self-aligning swivel head or lubrication between the ends of the specimen and the anvils, as recommended by the ASTM. In addition, they used a deflectometer to measure strains. As is usually the case when using deflectometers, instead of an averaging compressometer with a fixed gauge length, the apparent modulus of elasticity of the steel specimen was $\leq 20 \times 10^6$ psi instead of the expected 30×10^6 psi; this frequently leads to mistakes when taking the 0.2% offset

yield strength, and may account for the need of a correction factor. Tension tests were made using a one inch gauge length extensometer. The uncorrected data shows that as the carbon content of the martensite is lowered, the S-D effect decreases. Other data is cited to show that the S-D effect: (a) is typically 10 percent[48]; (b) is reported to go to zero in as-quenched 0.025% C ferrite[49]; (c) persists beyond yielding out to about three percent strain; (d) is characteristically smaller for a bainitic structure than for martensitic structures; and (e) does not vanish until a relatively high tempering temperature (>1250°F) is reached[44].

In addition to the data on as-quenched 4300 steels, Leslie and Sober gave the following data on the effects of tempering for one hour on the S-D effect in 4330 steel:

Tempering Temperature	Strength, Ksi σ_{CYS}	σ_{TYS}	$\Delta\sigma_{S-D}$
72F	271	220	51
400F	237	210	27
800F	200	180	20
1200F	125	115	10

It is uncertain whether the 1/3 to 1/2 correction factor should be applied to these values, but it is most likely necessary since these data are from the same set of experiments. The hatched areas on Figure 7c represent the effect of applying this correction.

Hirth and Cohen offer and discuss six tentative explanations of the observed S-D effect, most of which they

reject. Briefly, their explanations and observations are as follows:

1. MICROCRACKS--rejected because while they may be more detrimental to σ_{TYS} than to σ_{CYS} in some cases, they are not detrimental for fine-grained fracture tough steels.

2. RESIDUAL STRESSES--caused by the volume expansion that accompanies the austenite to martensite transformation and by the thermal gradients caused during quenching, residual stresses are rejected as a source of the S-D effect since:

(a) they must be balanced by equal and opposite forces to maintain micro and macro equilibrium;

(b) the S-D effect persists out to about three percent strain where it should have been wiped out by that amount of plastic strain;

(c) it persists even after 25% reduction in thickness by rolling.

3. RETAINED AUSTENITE--austenite transforms more readily to martensite under tensile loading and the accompanying volume expansion could lead to premature yielding, but this theory must be rejected since:

(a) the Leslie-Sober specimens were held at liquid nitrogen temperature and there was essentially no retained austenite as verified by X-ray diffraction;

(b) the S-D effect is observed even after tempering at temperatures high enough to have transformed any remaining austenite, and

(c) it persists after 25% deformation, which is sufficient to transform any retained austenite to below detectable levels.

4. INTERNAL BAUSCHINGER EFFECT--this hypothesis, which is based on the residual stresses set up by the volume expansion accompanying the austenite to martensite transformation and the resultant plastic compression of the martensite plates or needles by the surrounding material, is rejected by Hirth and Cohen for two reasons:

(a) it requires selective orientation of the martensite relative to the axis of testing, and this is unlikely even within a single parent grain, since there are 24 different crystallographic orientations that the martensite can form along in the parent grain, which itself is usually randomly oriented.

(b) if there were some S-D effect in one of the principal directions of a hardened plate due to a Bauschinger effect, it should be reversed or appreciably lower in the transverse direction, and such is not the case.

5. VOLUME EXPANSION--is rejected as plausible but difficult to evaluate since it would be dependent upon whether the actual expansion accompanying plastic strain is sufficient to account for the S-D effect and whether it takes place during a rate controlling event. As an example, Hirth and Cohen indicate that the sign (±) of normal stress across the glide plane may affect the Peierls stress.

6. SOLUTE/DISLOCATION INTERACTION IN SOLID SOLUTION--this is the hypothesis selected by Hirth and Cohen for their model and based on the distortion caused to the iron lattice when the austenite to martensite transformation traps most of the available carbon atoms in solution. This has two possible effects:

(a) the Peierls stress for movement of dislocations across the glide plane is dependent on the sign of the normal stress, with the Peierls stress increasing as σ_n decreases and eventually becomes negative, and

(b) the tetragonal distortion of the iron lattice around the trapped carbon atom causes non-linear elastic strains which result in the plastic force-displacement relations not being the same in tension and compression, thus causing the S-D effect. Hirth and Cohen state that the Peierls stress is not controlling since the S-D effect is not as temperature dependent as would be expected if it did control; observed temperature dependence is related instead to obstacle spacing and dislocation density. The Hirth-Cohen model is based on the distortions caused by the interstitial carbon atom and determines the order of magnitude of the contribution of the non-linear lattice distortions to the S-D effect.

Hirth and Cohen derive equations based on this last theory to calculate that the nonlinear interaction energy

difference associated with the S-D effect is of the order of four percent of the carbon/dislocation interaction energy for various values of strain associated with a dislocation strain field and for an applied strain $\varepsilon_a = \pm 0.01$. They state that they expect the S-D effect to persist at a decreased level in tempered martensite and tempered bainite, since the interaction is then associated with carbon atmospheres adsorbed at dislocations. In addition, they expect linear dependence of flow stresses on both solute (carbon) concentration and the solute/dislocation energy for drag of Cottrell, Snoek or core atmospheres and for dislocation motion without drag effects in concentrated solid solutions or solutions exhibiting order. They state that the solid solution strengthening is expected to be proportional to the carbon/dislocation interaction energy as well as some power of the carbon concentration.

Additional data are available on the S-D effect in an aluminum killed lower carbon, 0.16 percent, quenched and tempered HY-80 steel of the following composition: 0.16 C, 0.30 Mn, 0.009 P, 0.017 S, 0.25 Si, 2.78 Ni, 1.52 Cr, 0.36 Mo, 0.058 Cu, balance Fe[46,47]. Figure 8 is taken from reference 46 and 47 and shows the effects of tempering temperature on the S-D effect in quenched and tempered martensite. This material was austenitized at 1640°F for 1/2 hour. All tempering was performed on as quenched martensite that had been held for one hour at -120°F in dry ice and acetone. Tempering was for one hour at the in-

dicated temperature and was followed by a water quench from the tempering temperature. The lightly dashed line on Figure 8a is for the same steel after austenitizing at 2000°F for one hour.

Figure 9 shows similar data for this material[46] after austenitizing at 2000°F for one hour, transferring to 1640°F for 5 minutes and then either directly brine quenching or isothermally treating to produce some bainite or ferrite and then water quenching. The material was given a low temperature treatment at -120°F for one hour immediately after quenching. Tempering was for one hour at the indicated temperature and was followed by a brine quench from the tempering temperature. All the data in Figures 8 and 9 were obtained using self-aligning test fixtures and averaging, one inch gauge length compressometers and extensometers.

EXPERIMENTAL PROCEDURES

SUMMARY

The general test program followed in this study can be summarized as follows. Test coupons were collected from commercially produced plates made by the two major producers of a low carbon, nickel-chrome molybdenum steel*. These coupons were selected to represent the broad range of chemical composition to which this steel is melted in commercial practice. These coupons were slabbed to convenient size, analyzed for chemical composition, heat treated and tested. All heat treating and testing was performed on 5/8 × 2 × 5 in. drop weight blanks heat treated so as to eliminate the effects of variation in hardenability, plate thickness, and mill heat treatment. Isothermal heat treatments were used to produce controlled microstructures, and a uniform tempering temperature of 1150F, slightly above the minimum value permitted by the specification, was used. The isothermal holding times and temperature were tentatively selected to produce approximately 25 percent ±10 percent of each non-martensitic product, bainite or ferrite, for an average composition. Microstructures produced by a given heat treat were measured for each coupon using micro-specimens taken from the broken drop weight test specimens; mechanical properties were measured on these same test specimens.

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*Commonly called HY-80 steel and made in accordance with ASTM Specification A543-65.

MATERIALS

Coupons were collected over a period of years from plates made during actual mill runs at the plants of the two major producers of this steel and represent only a small fraction of heats melted during this period. The coupons were obtained after final heat treatment from the end of the plate when the plate was cropped to ordered size. All coupons were cut from material adjacent to the ordered plate. The actual coupons selected were chosen by the investigator on the basis of the producer's chemical analysis to represent a fairly broad range of the chemical compositions used for this steel in commercial mill practice. A total of 27 coupons representing 22 heats were collected; 16 coupons were from one producer and 11 coupons from the other. Table 1 summarizes the heat numbers, mechanical properties and chemical compositions reported by the producers; in addition, the table lists the identification numbers assigned to each coupon. Coupons made by the largest producer have a number starting with an X and coupons made by the other producer have been assigned a number prefixed with a Y. To obtain the maximum use of each coupon and to make specimen location as uniform as possible for all specimens, specimens were taken from adjacent to the original plate surfaces, see Figure 1. In fact, to minimize machining costs, the original plate surface was made one of the 2 × 5 in. surfaces of the drop weight specimen blank. The specimen blanks were 5/8 × 2 × 5

in. with the 2 inch dimension in the direction of plate roll and the 5 inch dimension parallel to the plate surface and perpendicular to the direction of plate roll, see Figure 2. This is the size of specimen required for subsize drop weight Nil Ductility Transition (NDT) temperature tests in ASTM Method E208. Each specimen blank was coded on the end with the appropriate number, location, test direction, and specimen number. These specimen blocks were used for drop weight tests; after that the broken specimens were used to make mechanical property test specimens.

HEAT TREATING PROCEDURES

The 5/8 × 2 × 5 in. drop weight specimen blanks were prepared for heat treatment by (1) welding handling loops onto one of the 5/8 × 2 in. faces, (2) depositing a brittle crack starter bead on the 2 × 5 in. surface of the specimen that represented the original plate surface of the specimen using a 3/16 in. diameter Hardex N hardfacing electrode, (3) normalizing the specimens by heating to 1650F, holding for 1 hour and then air-cooling, and (4) sandblasting the specimen prior to final heat treatment.

The final heat treatment of the drop weight specimen blanks consisted of austenitizing in a neutral salt bath at 1640F for 1/2 hour and then either water quenching in a brine solution or quenching into an isothermal neutral salt bath, holding for a prescribed time and then brine quenching. After quenching, all specimens were immediately given

a low temperature (about -110F) treatment in a bath made of acetone and chunks of dry ice; all specimen blanks were given at least 1/2 hour in this bath to ensure that they came down to temperature and that all retained austenite was transformed to martensite by the treatment. All specimens were subsequently tempered in a circulating air furnace at 1150F for 1 hour at temperature and then water quenched in a brine solution (BQ). In a few cases, where sufficient material was available, a few sets of drop weight specimen blanks were austenitized at 2000F for one hour in a circulating air furnace, then transferred to the 1640F salt bath for five minutes, minimum, to standardize the temperature differential between the isothermal bath and the specimen prior to immersion. All furnace and bath temperatures are manually controlled using potentiometers to within ±5F. Specimens were heat treated by suspending them by their loop from a fixture that allowed them to be treated as a group. This fixture held the specimens 1½ inches apart and suspended them vertically in the salt baths to permit circulation of the salt.

Fully Quenched (Martensitic)

Specimen blanks that were fully quenched to form a fine prior austenitic grain size to ultimately give a tempered martensitic microstructure were treated as follows: 1640F - ½ hr - BQ: -110F - ½ hr; temper 1150F - 1 hr - BQ. Specimen blanks from coupons X-12, -13, -15 through -18 which were quenched from a coarse prior austenitic size were treated

as follows: 2000F - 1 hr - 1640F 5 min (minimum) - BQ; -110F - ½ hr; temper 1150F - 1 hr - BQ.

875F Isothermal Treatment (Semi-Bainitic)

Specimen blanks that were treated isothermally at 875F to ultimately give a microstructure consisting of a mixture of tempered bainite and tempered martensite were treated as follows: 1640F - ½ hr - transferred to 875F - held 152 (or 1600) sec-BQ; -110F - ½ hr; temper 1150F - 1 hr - BQ. Additional specimen blanks were available from coupons X-12, -13, -15, -16, -17, and -18 and treated as follows: 2000F - 1 hr - transferred to 1640F - held 5 min. (minimum) - transferred to 875F - held 1600 sec - BQ; -110F - ½ hr; 1150F - 1 hr - BQ.

1200F Isothermal Treatment (Semi-Ferritic)

Specimen blanks from all coupons were isothermally treated at 1200F to ultimately give a microstructure of tempered martensite and tempered ferrite. In several cases a little pearlite was found to have been formed during the isothermal treatment. These coupons were treated as follows: 1640F - ½ hr - transferred to 1200F - held for 3350 sec (or extra sets of specimen blocks from coupons X-1, -2, -12, -13, -15 through -18 and Y-2, -4, and -11 were also held for 8500 sec) -BQ; -110F - ½ hr; 1150F - 1 hr - BQ. There was insufficient material to give any specimens with a coarse austenitic grain size the 1200F isothermal treatment.

TESTING PROCEDURES

In general an attempt was made to adhere to standard ASTM testing procedures. In a few cases it was necessary to make slight deviations from standard practice to take into account the specific conditions of these tests; any such deviations will be indicated below.

Drop Weight NDT Tests

All drop weight tests were performed in accordance with ASTM Method E 208-66T, using the type P-3 specimen which is 5/8 × 2 × 5 in. The specimens were supported as a simple beam and subjected to an impact load of 259 ft-lb produced by dropping a 223 lb weight 1 ft 2 in.; this drop energy was found to consistently crack the crack starter weld bead and to cause the specimen to consistently contact the anvil stop.

As stated previously, the specimens were prepared with one, as rolled, 2 × 5 in. face. The Hardex-N crack starter bead was deposited in the center of this face prior to heat treatment of the specimen so that the heat-affected zone created by deposition of the weld bead would not interfere with the correct measurement of the NDT temperature of the base material. After final heat treatment, the crack starter weld beads were notched as required in ASTM E 208, using a thin abrasive disk. The combination of the isothermal treatments and the subsequent 1150F temper was not found to prevent proper functioning of the crack starter bead; upon impact, a good crack formed in the bead in every case.

A group of four or five, usually five, specimens were given a selected heat treatment and used to determine the NDT temperature. Since tests were conducted on a known material with a known thermal history, the NDT temperature was determined within 10F by three to five tests; the usual number being four. Prior to testing, each specimen was placed in a large insulated liquid, isopentane, low temperature bath with at least one in. of liquid all around the specimen. A thermocouple embedded in a dummy specimen was attached to an automatic recorder and used to control the bath temperature. The bath temperature was maintained for a period of at least 45 min. prior to test by manually stirring the bath and adding dry ice or liquid nitrogen as required. Gloves were used to handle the specimens by the heat treating loop welded to the end (5/8 × 2 in. face). Interpretation of the specimen fracture as to "Break" or "No Break" was as recommended in ASTM E 208. The only "No-Test" specimens encountered in these tests were with a few specimens that were cut a little thin, less than the recommended 0.62 ± 0.02 in. thickness. These specimens were not included in the measurement of the NDT temperature.

Tensile Tests

All tensile tests were performed in accordance with ASTM Standard E 8-65T, Tension Testing of Metallic Materials. A small size specimen, 0.250 in. diameter with a one in. gage length, proportioned to the standard Type 1, 0.5050 in. diameter tension test specimen was used. The specimens were

taken parallel to the original direction of plate roll, thus the overall length of the specimen was limited to 2 in. since this is the size of the P-3 drop weight specimen in that direction, see Figure 3. The 2 in. overall length was achieved by slightly reducing the length of the threaded ends of the specimen.

The specimens were tested in hydraulically loaded universal testing machines calibrated in accordance with ASTM Standard E 4-64, Verification of Testing Machines, using proving rings. The testing machines were found to indicate the correct load within ±0.5 percent in the loading ranges used in these tests. Self aligning, spherical bearing devices were used to ensure axial loading of the test specimens. Load:strain curves were recorded for all specimens using an averaging, one in. gage length, microformer strain indicator (extensometer) and an autographic recording device calibrated over the range of the expected yield. Specimens were tested at a speed of 0.002 in./in./min. or less, up to and slightly beyond the yield point. The 0.2 percent offset method was used to determine the yield strength. The values reported are the average of two specimens.

Compression Tests

All compression tests were performed in accordance with ASTM Standard E 9-61. The specimens tested were ½ in. diameter by 2 in. long cylinders made with special care to ensure that the ends were parallel to each other within 0.002 in. and perpendicular to the long axis of the specimen

within 0.005 in. The specimens were taken parallel to the original direction of plate roll; thus they were machined parallel to the 2 in. dimension of the drop weight specimen they were cut from, see Figure 3.

The specimens were tested using hydraulically loaded testing machines calibrated in accordance with ASTM Standard E 4-64, Verification of Testing Machines. The testing machines were verified using proving rings and found to indicate the correct load within ±0.5 percent in the loading ranges used in these tests.

All compression tests were run using a subpress with a spherical seated bearing block at the end of its shaft to ensure full contact with the end of the specimen and to ensure true axial loading. Hardened steel bearing blocks with smooth, ground, parallel faces were used in contact with the ends of specimen to prevent possible damage to the faces of the subpress. The ends of the specimen were greased with lubriplate to eliminate end restraints. Because of the shape of the specimen it was not necessary to use a jig to prevent lateral buckling. Load:strain curves were recorded for all specimens using an averaging, 1 in. gage length, microformer strain indicator (compressometer) and an autographic recording device calibrated over the range of the expected yield. Specimens were tested at a speed of 0.002 in./in./min. or less up to and slightly beyond the yield point. The 0.2 percent offset method was used to determine the yield strength. The values reported

are the average of two specimens.

Charpy Impact Tests

All Charpy V-notch impact tests were performed in accordance with ASTM Standard E 23-64 using a pendulum type of impact machine with 268 ft-lb capacity and a striking velocity of 16.85 ft/sec. Energy losses due to friction were negligible (less than 1.5 ft-lb) over the full range and therefore were disregarded. A Type A Charpy V-notch (simple-beam) impact test specimen 0.394 × 0.394 × 2.165 in. was used for these tests.

Most Charpy specimens were cut parallel to the 5 in. dimension of the drop weight test block and thus were transverse to the major direction of plate roll, see Figure 3. Transverse Charpy specimens were selected for this study so that if banding or nonmetallic inclusions were present in the test specimen, they would be oriented in such a way as to offer the least resistance to crack propagation and so as not to act as crack arresters and thus mask the properties being studied. However, a few coupons were large enough to provide extra material to make a few oversize 5/8 × 2¼ × 5½ in. specimen blanks, see Figure 4. These blanks were heat treated and used to make a set of longitudinal, 2.165 in. dimension parallel to the major direction of plate roll, Charpy V-notch impact specimens. Each specimen was identified before notching on both 0.394 × 0.394 end faces with the drop weight specimen number, test direction and specimen number. All Charpy specimens were

notched so that the notch was perpendicular to the plate surface through-the-thickness of the plate.

The Charpy V-notch specimens were machined to closer tolerances than recommended by the ASTM in order to minimize scatter in test results; the permissible variations used were as follows:

Adjacent sides---------- 90° ±30'
Cross-section dimension---------±0.001 in.
Length of specimen (L)----------±0.002 in.
Centering of the notch (L/2)----±0.001 in.
Angle of notch---------- 45° ±22½ min.
Radius of notch----------±0.005, -.002 in.
Dimensions to bottom of notch---±0.001 in.
Finish requirements----------32 microinches all over

Specimens were cooled to test temperature using a liquid bath in an insulated container. Specimens were removed from the bath and centered in the test machine using self-centering tongs similar to those shown in ASTM E 23; the tongs were chilled in the low temperature bath with the specimens for at least 15 min. and were immediately returned to the bath each time after they were used. Bath temperatures were controlled using a dummy specimen with an embedded thermocouple. The values of energy absorbed by the specimen reported herein are usually the average of two specimens broken at the indicated temperature; individual specimen results are plotted on the energy absorbed vs. test temperature curves.

METALLOGRAPHIC PROCEDURES

Metallographic specimens were cut from the undeformed areas of the drop weight test specimen blank and far enough from the crack starter bead to be sure that no alloying elements had diffused into the specimen. Longitudinal and transverse specimens were prepared and mounted in transoptic mounting resin. After the specimens were mounted, 1/8 in. of specimen surface was machined off in a lathe under coolant using a sharp cutting tool. The surface was progressively ground using 180-, 240-, and 600-grit silicon carbide papers. Final polishing was performed in two stages using 60-cycle automatic vibratory polishers. The semi-final polishing bowl was covered with bleached silk cloth and a slurry of Linde A, distilled water and aerosol wetting agent. The final polishing bowl was covered with Gamal cloth and a slurry of Linde B, distilled water, and aerosol. All specimens were lightly etched in 2 percent Nital and repolished several times during the final stages of polishing to ensure the removal of any smeared structure and correct development of the microstructure.

The inclusion content of the specimens was measured in both the longitudinal and transverse, through-the-thickness planes in accordance with ASTM Standard E 45-63, Determining the Inclusion Content of Steel. Microscopic Method B was used to determine the length of the longest inclusion, the number and average length of all inclusions over 0.005 in. long and the background rating including all inclusions less

than 0.005 in. long. The vibratory polishing technique was found to leave the inclusions intact and in the specimen.

The prior austenitic grain size was measured on fully-quenched specimens taken from the quenched and tempered fully martensitic specimens in accordance with ASTM Standard E 112-63, Estimating the Average Grain Size of Metal, using the intercept or Heyn procedure. The prior austenitic grain size was revealed by the use of the etching reagents described in Appendix A. Comparisons made on several different specimens from different melts showed that both etches indicated the same prior austenitic grain size.

The percent of isothermal transformation product resulting from the isothermal treatments was measured for each treatment. The microspecimens were each etched in three reagents. Between each of the reagents, the specimens were rinsed and dried. The triple etch consisted of (1) a saturated picric acid solution to develop the prior austenitic grain boundaries, (2) a one percent Nital solution to develop the structure in the areas of tempered martensite, and (3) a 20 percent solution of anhydrous sodium metabisulfite for staining (darkening) the tempered martensite areas. The last etch leaves the areas of the tempered bainite or ferrite white, in sharp contrast to the darker areas of stained tempered martensite. The details of these etching solutions and how they were used are given in Appendix B. The percent of transformation product was estimated by superimposing a grid on the specimen and

determining the percentage of the grid intersections that fell over each microconstituent. Because of the finely banded nature of the transformation product and the high magnification (500X/1000X) needed to resolve the structure, extreme care was taken to systematically traverse the specimen and read at least 10 areas so as to get a representative average value to report.

EXPERIMENTAL RESULTS

CHEMICAL COMPOSITION

The actual chemical compositions of the coupons tested in this study are listed in Table 2. These analyses were made by a commercial testing laboratory on material located adjacent to the original plate surface of the coupon. The material analyzed came from the top 5/8-in. of the plate but did not include any heat treating scale or decarburized plate surface which would have influenced the analysis.

Wet chemical analyses were performed for percent carbon (C), manganese (Mn), phosphorus (P), silicon (Si), nickel (Ni), chrome (Cr), molybdenum (Mo), copper (Cu), and acid soluble and total aluminum (Al).

Spectrographic analysis was performed to determine the percent present of the following trace elements: vanadium (V), lead (Pb), tin (Sn), magnesium (Mg), cobalt (Co) and titanium (Ti). No values are reported for Pb, Sn and Mg in Table 2 since they were not found to be present in detectable amounts. After several samples had been analyzed it was found that V and Co were present in negligible amounts and therefore, analysis for these elements was discontinued. The producers informed the author that the Co was a tramp element introduced by the nickel addition. The Co was left in with the nickel since it was too difficult to remove and since they found that it had essentially no effect on the hardenability or properties of the steel. The V was also

present only as a tramp element but it came from a variety of sources that went into the make up of a given heat of steel.

Gaseous analysis was performed for oxygen (O_2), hydrogen (H_2), and nitrogen (N_2). O_2 and N_2 are reported as a percent and H_2 as parts per million (ppm) in Table 2.

A review of Table 2 shows that in general the range of the alloying elements found in the samples selected for this study almost completely cover the full composition range permitted for nickel-chrome-molybdenum steels such as ASTM A543 and Hy-80. The only element that falls outside the combined range for the two steels is Mn; coupons Y-8 and Y-10 have 0.49 percent Mn. This is only 0.04 percent over the 0.45 percent (0.40 upper limit plus the allowance of 0.05 percent variation over the upper limit) maximum allowed for these steels.

METALLOGRAPHIC ANALYSIS

The results of the metallographic analysis of coupons used in this study are given in Tables 3, 4 and 5. Table 3 presents the results of the inclusion analysis of these coupons at the mid-thickness and adjacent to the surface of each coupon in both the longitudinal and transverse directions on planes perpendicular to the plate surface through the thickness of the plate. The reporting method recommended in ASTM E 45-63 is used to describe the inclusion content. The length of the longest inclusion at 100X is

reported in units equivalent to 0.005 in. on the specimen along with a superscript describing whether it is grouped (g), very disconnected (vd) or disconnected (d). The average length of all inclusions over one unit long and excluding the longest is reported with a superscript denoting the number of inclusions averaged. The background rating A, B, C or D corresponds to the sample photographs in E45 with A being a rather clean steel and D a rather dirty steel. Figures 5 and 6 are photomicrographs (100X) of representative samples and will give the reader a better understanding of the significance of the data reported in Table 3.

The prior austenitic micro-grain size of each coupon is reported in Table 4. The grain size is reported for the as-received coupon and for the coupons after heat treatment. This table shows that the coupons austenitized at 1640F had a slightly finer prior austenitic grain size than as-received coupons, and a slightly coarser grain size after the 2000F treatment.

The percent of isothermal transformation products produced by the various isothermal treatments are summarized for each coupon in Table 5. The large variation in percent transformation product resulting from a given heat treatment reflects the large range in chemical composition and hardenability of these coupons. The high hardenability, rich composition coupons had the least transformation take place for a given isothermal treatment. Photomicrographs of

typical areas of each coupon for each isothermal treatment are given in Appendix C. The dark areas are tempered martensite that has been darkened as described in Appendix B and the light areas are tempered isothermal products.

MECHANICAL PROPERTY TEST RESULTS

Table 6 summarizes the mechanical property test results for each heat treatment; the corresponding microstructures are also included for the reader's convenience. The table lists the longitudinal tensile and compressive properties, the NDT temperature and the transverse Charpy V-notch impact properties at the NDT temperature, -120F and +212F (representative of the maximum energy shelf). All values reported are the average of two specimens. Also longitudinal Charpy data are reported for the few cases where it was measured. Appendix D contains the full Charpy V-notch curves for most of the coupons.* The Charpy curves show the energy absorbed, the lateral expansion at the notch after fracture and the percent fibrous fracture appearance of the fracture surface.

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*A few coupons (X-4, -5, -6 and Y-5 and -6) only had a pair of Charpy specimens tested at each of the three indicated temperatures, NDT, -120F and +212F. This was not enough data to draw a meaningful Charpy data curve so they are not shown.

DISCUSSION

A comparison of Tables 1 and 2 shows that there are some significant differences between the producer's reported chemical composition of the heats and the actual composition of the coupons. The biggest differences are in C, Mn, P and S with the coupon values generally higher than the producer's values. These variations are slightly more than what might be expected from normal variation in analytical results and from variation caused by the differences in the position of the sample in the heat of steel, and point up the need to check the actual composition of coupons being evaluated in laboratory experiments rather than to rely on the producer's data. Table 2 shows a good compositional range for the usual alloying elements and a fair spread for most of the rest of them.

Table 3 shows the mid-thickness and surface inclusion content of the coupons. In general, the material adjacent to the surface is cleaner than at the mid-thickness of the coupon. Even though a good range of inclusion sizes is represented in these samples, it must be pointed out that no large continuous inclusions were found.

Table 4 compares the prior austenitic grain size of the as-received material at mid-thickness to the prior austenitic, gamma grain size that resulted from the austenitizing treatments used in this study. While the as-received gamma grain size was fairly coarse, ranging from ASTM 8 to 4, the treatments used in this study bracketed an

equally wide range in gamma grain size of ASTM 9.5 to 3.0.

The isothermal holding times referred to in Tables 5 and 6 are time in the bath, not actual holding times at bath temperature. Appendix F gives a comparison of the actual time-temperature history of 5/8 × 2 × 5 in. drop weight specimens when they are transferred from 1640F into either 875 or 1200F salt baths and compares it to the cooling curves predicted using the method of Sinnott and Shyne[52]. The mid-thickness of the drop weight specimen is within 20F of 875F in about 120 seconds, a considerable portion of the 152 second treatment at 875F but a reasonable fraction of the 1600 second treatment. The data indicate that the mid-thickness of the specimens is within 20F of 1200F in about 90 seconds, which is negligible compared to the 3350 and 8500 second holding times used at 1200F. The important matter concerning holding times is that they produced a reasonable range in the amount of microconstituents.

Table 5 reports the amount of bainite produced during the 875F isothermal treatments and the amount of ferrite and pearlite produced by the 1200F treatments. The balance of the austenite that was untransformed at the end of the isothermal treatment was completely transformed to martensite by the brine-water quench and -120F treatment. The amount of bainite produced by the 875F treatment ranged from 0 to 76 percent and varied with the hardenability of the particular steel and the isothermal holding time. As

would be expected, the steels with the highest hardenabilities transformed the least. A similar effect was observed for the 1200F isothermal treatment, the amount of ferrite ranging from 0 to 69 percent. Only four of the 1200F treatments produced measurable amounts of pearlite, ranging from 4 to 32 percent; these steels were low hardenability, low Ni and low Mn compositions. The fact that only four samples had pearlite in the microstructure must be considered during the subsequent discussion of the effects of microstructure on the NDT temperature. There are so few points that not much confidence should be placed on the estimated effect of pearlite as opposed to the other microconstituents. It must be pointed out that one normally would not find pearlite in the microstructure of this low-carbon, Ni-Cr-Mo alloy steel unless something were grossly wrong. Any discussion of the effect of pearlite is of academic rather than a practical interest.

Table 6 gives a summary of the microstructure, tempered-bainite, -ferrite, -pearlite, and -martensite, longitudinal mechanical properties, the drop weight NDT temperature, and transverse and longitudinal Charpy V-notch impact properties resulting from the various heat treatments. Because of the uniform 1150F temper, the strengths reported here are more characteristic of an HY-100 steel rather than an HY-80 steel. The variations in strength reflect the variety of chemistries studied and the different initial hardnesses, the slightly different responses to

tempering, and the different microstructures. The mechanical property data in Table 6, the chemistry data in Table 2, the inclusion content data in Table 3, the prior austenitic grain size data in Table 4 and the microconstituent data in Table 5 were combined to form the set which is the basis of this analysis. The result is that the 115 data points represent the 27 different coupons which constitute the sample used in this analysis.

INDEPENDENCE OF VARIABLES

When performing a regression analysis to determine the functional relationship between a dependent variable and more than one independent variable (in this case NDT temperature and such things as composition, microstructure and strength), it is obvious that the magnitude of each independent variable must be randomly distributed relative to the magnitude of each other independent variable; otherwise it will be difficult if not impossible to apportion a part of the observed changes in the dependent variable to one or another independent variable. In experiments where the investigator is melting his own material he can ensure independence between the independent variable he wishes to study by careful randomization in statistically designed experiments. In this case we have 27 coupons with a wide variation in chemical composition and mechanical properties. In this study to ensure independence, or at least to ensure that there was not a strong correlation between the vari-

ables considered, the data making up the sample (115 data sets) were run on a correlation matrix as described in Appendix E.

The results from this correlation matrix are summarized in Table 7, showing the cross-correlated variables with a correlation coefficient from 0.85 to 0.999. Table 7 gives a numbered list of the 83 variables considered and indicates which variables are related to another for three ranges of correlation. The identification number of the variable having the indicated level of correlation is listed next to each. For example V1-Sample Number* is strongly related (correlation from 0.900 to 0.949) to variables 10 and 15 (amount of copper and amount of cobalt). A look at Table 2 shows that this is indeed the case. Producer X has much less copper in his melts than producer Y, and cobalt was measured only on coupons from producer X.

Of the individual alloying elements, V8-Amt. Cr is slightly related to V15-Amt. Co, related to V50-V7×V9-Ni× Cr, the product of Amt. Ni times Amt. Cr, a variable representing an interaction effect, and highly related to V79-V8 ×V8-Cr×Cr, a second order effect in this variable itself. Table 7 shows that many of the interaction effects and higher order effects using a given variable are strongly correlated to that variable and may not be capable of being

*Sample number was derived by numbering the coupons from producer X in the one hundreds and those from producer Y as the two hundreds. For example, material from coupon X-3 was identified in the data set as 103 and material from coupon Y-11 as 211.

properly evaluated using these data. For example, the effect of total aluminum may not be separable from the effect of acid soluble Al in these data, V11 and V12. Similarly, ultimate tensile strength, V27, tensile yield strength, V28, and compressive yield strength, V31 are so strongly correlated as to be difficult to evaluate individually, and thus each must be regarded as just a different way of quantifying the "strength" of the sample.

Of interest in Table 7 is the failure of most of the Charpy V-notch properties at +210F, -120F and even at the NDT temperature itself to correlate with the NDT transition temperature of this steel. It is not surprising that Gross[21] was able to obtain only a ±50F correlation between Charpy V-notch and drop weight NDT transition temperatures for the variety of steels he was using, since the Charpy transition properties measured in the present study did not correlate very well with NDT temperature. Of the Charpy impact properties, only V36 -120F Pct Fiber has a correlation coefficient greater than 0.47, and it was 0.82. Gross did not try to correlate this with NDT. The present study was not able to relate Charpy and NDT transition temperatures even though this steel (a) was of a single grade, (b) had carefully controlled heat treatments, (c) had the drop weight specimen crack starter bead deposited before heat treatment to eliminate the effects of a heat affected zone under the brittle bead, and (d) that the Charpy specimens were taken from the drop weight specimen used to

measure NDT.

SINGULARLY SIGNIFICANT VARIABLES AFFECTING NDT

The correlation matrix can perform an important function for the data analyst by giving an indication of those independent variables that by themselves have a linear relationship with the dependent variable. The correlation matrix showed that only three variables have a fairly high linear correlation with NDT temperature; they are:

1. V20 - Prior Austenitic Grain Size, r = -0.78
2. V36 - Percent Fibrous Fracture at -120F, r = -0.82
3. V64 - Gamma Grain Size × TYS, r = -0.77

The next seven best variables have correlation values with NDT temperature ranging between 0.4 and 0.48; the rest of the variables were lower. These seven variables and their values were:

1. V33 - C_v Energy Absorbed at the NDT Temperature, r = 0.48
2. V21 - Percent Bainite, r = 0.47
3. V24 - Percent Martensite, r = -0.43
4. V4 - Amount of Phosphorous, r = 0.41
5. V37 - C_v Energy Absorbed at +210F, r = 0.41
6. V61 - $Ti \times N_2$, r = -0.41
7. V45 - $Cr \times N_2$, r = 0.40

The correlation of some of these factors with NDT temperature is understandable. V20 - Prior Gamma Grain Size

has been reported in the literature to be a significant variable affecting the transition temperature of tempered martensitic and bainitic steels[11,12,29,36,37,41,46], as has strength[27,35,38,42,46,47]. However, the correlation between NDT temperature and the interaction between austenitic grain size and tensile yield strength (TYS), V64, is more than likely due in this case to the dominating effect of changes in ASTM grain size number on a relatively constant 106 ±10 ksi tensile yield strength and, as shown later, will not prove to be statistically or practically significant in the multiple variable analysis. The Charpy V-notch fibrous fracture appearance at -120F, V36, has a strong correlation with NDT temperature. It is logical, since -120F is close enough to the NDT temperature of most of these samples for the percent fibrous fracture appearance to give a good indication of the change in impact ductility. It is interesting to note that the Charpy V-notch energy absorbed at -120F, V35, has only $r = -0.37$ in the correlation matrix, reflecting the fact that the producer's original mill rolling practice has introduced a large variability in the energy absorbed and markedly reduced its level of correlation with the NDT temperature.

The seven variables with correlation values between $|0.40$ and $0.48|$ are obviously not dominant factors affecting NDT temperature and require evaluation in terms of multiple regression to see if they still contribute significantly to the observed variation in NDT temperature when they are in-

cluded with a more dominant factor such as prior gamma grain size. As will be subsequently shown, the inclusion of a singularly dominant variable in a multiple regression analysis will eliminate almost all of the variables that show a markedly lower singular correlation.

RESULTS OF REGRESSION ANALYSIS ON NDT

Almost 100 computer analyses were made using the regression analysis program XIRE and the statistical methods described in Appendix E to evaluate the functional relationship between the NDT temperature, V32, and the other 82 variables listed in Table 7. In the initial computer analyses about 15 variables selected for their mutual independence were selected to determine their functional relationship to NDT temperature. The Charpy V-notch impact values were not included since they are just another aspect of the transition from ductile to brittle impact fracture characterized by the NDT temperature itself. One criterion for selection of significant independent variables was that the term coefficient, Bi in the computer output, consistently demonstrate that it was statistically significant, i.e., not likely to be zero in several regressions with a variety of other variables. As described in Appendix E, this requires that the term's regression coefficient divided by the standard deviation of the coefficient, T in the output tables, be greater than some value of "t", Student "t",

based on the degrees of freedom* (DF) for that particular run and a confidence level of at least 90%. Because of the large sample size (here 115 data sets) the usual value of acceptance was about $|T| > 1.70$ since small changes in a large value DF do not affect the value of "t" much. Another requirement for acceptance was that additional terms should significantly improve the functional relationship as evidenced by statistically significant improvements in the multiple correlation coefficient R and the standard deviation Sx.y, i.e., reduce the residual variance not accounted for by the terms in the regression. It was noted that if an additional term or terms significantly improved R it also significantly improved Sx.y; see examples 4 through 11 in Appendix E.

Composition and Producer

The results of many regression analyses can be summarized by saying that most of the alloying additions and alloy interaction terms failed to indicate a consistent and statistically significant relationship to NDT temperature. This is not surprising since the range, minimum to maximum, of each element present is a result of commercial melting practice specifically aimed at trying to stay within procurement specification limits** as opposed to an artificially large range as would be the case in laboratory melts

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* DF = Sample size minus number of terms in regression, including the dependent variable.
**HY-80 steel is melted to ASTM A543-65.

designed to explore the effect of a specific alloy addition.

The only element that displayed a measurable effect was hydrogen; Table 2 shows that it ranged from 0 to 12 ppm with a typical value of 0 to 2 ppm. As a result, even though hydrogen content is statistically significant, its total effect is small, about 2F/ppm H_2, resulting in a 0 to +24F change in NDT transition temperature.

The producer also turned out to be a statistically significant variable. The producer was coded as 1 for producer X and 2 for producer Y for the regression, and the results indicate that the NDT temperature of material made by producer Y is about 24F less than that made by producer X, everything else being equal. In the sample of 115 data sets, 76 represent material made by producer X and 39 represent material made by producer Y. The reason underlying this observation is not simple chance and may be due to either some compositional difference, such as amount of P or Cu which by themselves were not consistently significant, or some difference in mill practice. In any case, the summation of differences is great enough to make the producer a significant variable that must be included in the analysis.

Prior Austenitic Grain Size

As indicated in Table 4, the prior gamma grain size represented in this sample ranged from ASTM 9.5 to ASTM 3, and was varied by changes in austenitizing temperature and

holding time, in this study 1640F - ½ hr and 2000F - 1 hr, as well as by changes in composition from heat to heat. The regression analysis confirmed the importance of grain size reported in the literature and showed that adding a second order grain size term did not significantly improve the correlation.

It is interesting to note that these results indicate that Petch's linear relation between the 15 ft-lb Charpy V-notch transition temperature, T, and ferritic grain diameter, d, ($T = A_o + A_1 \ell nd^{-\frac{1}{2}}$) of ferritic steels has a linear corollary for martensitic steels and drop weight NDT temperature. As shown previously, Petch's relationship can be expressed as a linear function of ASTM grain size (N), $T = C + D(N)$. In terms of NDT temperature and prior austenitic grain size, it can be written $NDTT = B_o + B_1(\gamma GS)$; this is the form of the regression results given in Table 8. The correlation matrix and the regression analysis both indicate that this simple linear relationship will account for about 80 percent of the observed variability in drop weight nil ductility transition temperature.

An additional factor that must be considered when discussing prior austenitic grain size is that final gamma grain size prior to quenching reflects the interaction of a number of variables such as:

1. Austenitizing temperature. The higher the temperature above the upper critical, the coarser will be the gamma grain size with controlled grain size steels tending

to retain finer grain sizes until their grain coarsening temperature is exceeded.

2. Compositional variations will be represented in that if, as in the present case, the austenitizing time and temperature are fixed, composition changes will cause the difference between the upper critical temperature (A_{C_3}) and the austenitizing temperature to vary slightly. Composition changes that increase this difference will result in a coarser gamma grain size. Increasing C, Mn, Ni, Cu, N_2, B, Cb and Zn or decreasing Cr, Si, Mo, Ti, W, V, Sn, Al, P and Zr will tend to increase the prior austenitic grain size if the austenitizing treatment is held constant for a hypoeutectoid steel[53-59].

3. The longer the time at the austenitizing temperature, the coarser the gamma grain size, especially at temperatures above the grain coarsening temperature for controlled grain size steels.

4. Deoxidation practice or addition of grain refiners such as Al, Si, V, Ti, Mo, W and rare earth additions will affect or inhibit gamma grain growth.

Such compositional variations would account for the range in gamma grain size resulting from the 1640F - ½ hr austenitizing treatment; Table 4 indicates that gamma grain size ranged from 7 to 9.5 for this treatment. No attempt was made to pin point a specific compositional variation for this difference in grain size, but a quick review of the chemistry data in Table 2 indicates that the two heats

that resulted in grain size 7 (X-3 and Y-11) had the lowest nitrogen contents, 0.003 and 0.002 percent respectively versus an average value of 0.007 percent. Whether or not this is the underlying cause for the difference can not really be settled from these experiments. However, it has been reported in the literature that Al additions combine with nitrogen to form aluminum nitrides that inhibit grain growth[57].

Microstructure

The previous studies reviewed herein showed that the tempered martensitic structure generally had the lowest transition temperature and that bainitic structures are intermediate to those of the same steel treated to produce martensite or ferrite. Hall indicates that grain size has an effect on transition temperature of the same order as that of degree of hardening[56]. The data in Table 5 show that the steels tested had been treated to produce from 0 to 69 percent bainite with the balance martensite and from 0 to 67 percent ferrite with the balance martensite, except in four cases where there was some pearlite formed during the final part of the 1200F treatment. Up to 32 percent pearlite was found.

Table 9 gives the regression results for the effects of percent microconstituents on NDT temperature allowing for the effect of prior austenitic grain size, expressed as ASTM grain size number; 100 percent tempered martensite

is the implicit base line. Linear regression terms are used since higher order and interaction terms were not found to statistically improve the relationship. The results show that: (a) bainite raises the NDT transition temperature about one-half a degree per percent; (b) ferrite lowers the NDT transition temperature about three-fourths of a degree F per percent; (c) pearlite raises the NDT temperature about 5F per percent; and (d) refining the prior austenitic grain size lowers the NDT transition temperature about 20.4F per unit increase in ASTM grain size number. Thus, one sees that changes in prior austenitic grain size expressed in terms of ASTM grain size has an effect on the NDT temperature that is about an order of magnitude greater than the effect of percentage changes in microconstituents of the HY-80 steel studied here.

A question might arise as to whether there is a change in controlling effect on NDT temperature by the type of grain size, i.e., prior austenitic or ferritic, as the amount of isothermally produced ferrite increases. This question can be answered either by metallographic examination of the fracture path in the broken specimens or by a graphical analysis of the difference between experimental and calculated values of the NDT temperature as a function of the ferrite content. The latter method has been selected to answer this question since such a graph can be quickly prepared from the back calculation data in the regression analysis computer output, whereas the former would require

the laborious preparation of a series of metallographic specimens.

Figure 12 is a plot of the difference between the experimentally observed NDT temperature and the calculated value (DE = Y - YC) using the regression results of Table 10 for those specimens which contained a measurable amount of up to 69 percent ferrite. This shows that there is a slight tendency for the calculated value of the NDT temperature to be high, but this tendency is essentially confined to those very fine prior austenitic grain, 8.5 to 9.5, specimens containing only a trace of ferrite. Examining this figure shows that there does not appear to be a systematic difference between experimental and calculated value as the ferrite (and pearlite) is increased to large amounts, until as little as 8 percent martensite remains in a matrix of 69 percent ferrite and 23 percent pearlite. See Figures C1, C16 and C17 for the photomicrographs of the predominately ferrite and pearlite microstructures.

Judging by the results of regression analysis of these data, it is likely that prior austenitic grain size remains the dominant factor controlling NDT temperature of this steel, even as the amount of ferrite is increased to over 50 percent and significant amounts of pearlite are found. This is confirmed also by the relative magnitude of the term coefficients for prior austenitic grain size and percent ferrite in Table 10, which show that a change of one ASTM grain size number has about 100 times as much

effect in determining the NDT temperature as a change of one percent ferrite, -19F/ASTM G.S. Number versus -0.2F/ Percent Ferrite.

No attempt was made in the present study either to deliberately vary ferritic grain size or to deliberately vary the prior austenitic grain size before the 1200F transformation, even though the prior gamma grain size ranged from ASTM 7 to 9.5. While Geil, et al.[4] have reported some data on the relative effects of ferrite and prior gamma grain size on the Charpy transition temperature of 0.3 C steels; they came to the conclusion that ferritic grain size is more important. It must be pointed out that they only changed the gamma grain size from 50% ASTM 4 plus 50% ASTM 6-7 to ASTM 7-8 with few 4-5 in three heats of an unkilled steel. Neither of these experiments has varied the prior gamma grain size enough to say with certainty that either predominantly controls transition temperature in ferritic steels. A definitive set of experiments remains to be done on the relative effects of prior austenitic grain size and final ferritic grain size of ferritic steels.

Inclusion Content

Examination of Table 3 and Figures 5 and 6 shows that the inclusions observed and measured in these samples were small and uniformly dispersed in a way that should not have affected the measured NDT temperature. This was confirmed by the regression anslysis, which showed that the inclusion

content of these samples did not correlate with the observed variations in NDT temperature of the HY-80 steel studied.

Strength

Reference 46 shows that the strength level of HY-80 steel, varied by tempering at various temperatures for one hour and then quenching, has a strong correlation with the NDT temperature of the steel. The NDT temperature increases as the strength is increased. The tensile yield strengths measured for the 115 data sets that make up the sample being analyzed have an average value of 106 ksi and a standard deviation of 10.2 ksi. Over such a restricted range a linear approximation should adequately describe the effect of strength on NDT temperature even though a higher order function would be required for a wider range.

Table 10 shows the results of linear regression analysis for the effects of producer, gamma grain size, percent microconstituents (bainite, ferrite and pearlite) and tensile yield strength in psi. These results show for these data, after accounting for the other variables, that tensile yield strength contributes to determining the NDT temperature only in a minor way, about 0.68F per 1000 psi increase in tensile yield strength. This is slightly less than the 3 to 4F per 1000 psi shown graphically in Reference 46 for the range in tensile yield strength, TYS, from 100 to 120 ksi for coarse and fine grained material respectively. The reason for this difference in $\Delta T/\Delta TYS$ between the two

studies is that this study covers a slightly wider range in tensile yield strength, from about 86 to about 126 ksi, which includes the range from 80 to 100 ksi that Reference 46 shows has little or no effect on NDT temperature, as opposed to the 100 to 120 ksi range where tensile yield strength has about the maximum effect on NDT temperatures.

Analysis of other computer runs shows that higher order strength terms and terms expressing an interaction between austenitic grain size and either strength or micro-constituents do not significantly improve the correlation for these sets of data. Correlations based on ultimate tensile strength (UTS) show no statistical or practical improvement over those based on tensile yield strength.

The data in Reference 46 shows that strength level, as changed by variation in tempering temperature, has a larger effect on NDT temperature than is indicated in this study. This is due to the fact that in Reference 46 the strength was varied by tempering for one hour at temperatures ranging from 400 to 1330F to produce a tensile yield strength ranging from 161 to 81 ksi in the tempered martensitic structure alone; in the present study the tensile yield strength ranged from only 107 to 122 ksi in the 100 percent martensitic structure tempered at 1150F. Reference 46 also shows that the functional relationship between strength and NDT temperature cannot be represented over the full range of strength by a simple higher order polynomial and thus does not lend itself to regression; in fact, it would appear

from Reference 46 that even higher order interaction terms are required to represent the interaction between strength and microstructure. Such a study is not warranted here since the graphs shown in Reference 46 clearly demonstrate the complexity of this interaction.

Summary of Factors Controlling NDT Temperature

This study has shown that under carefully controlled conditions which eliminate the possibility of retained austenite and of temper embrittlement due to slow cooling after tempering, the dominant factor controlling the NDT temperature of low-carbon, Ni-Cr-Mo steel* is the prior austenitic grain size. The results in Table 10 show that prior austenitic grain size has from four to 100 times as much effect on NDT temperature as either the percent non-martensitic isothermal products, percent bainite, ferrite and pearlite, in a tempered martensitic matrix or the tensile yield strength. The significant variables and the relative effect they had on the NDT temperature of the Ni-Cr-Mo steel used in this study are:

VARIABLE	RANGE	EFFECT ON NDT
Producer	(Coded 1 or 2)	-21.9F/Unit
Prior Austenitic Grain Size	(3 to 9.5)	-19.2F/ASTM G.S. No.
Percent Bainite	(0 to 76%)	0.62F/Pct B
Percent Ferrite	(0 to 69%)	-0.23F/Pct F
Percent Pearlite	(0 to 32%)	5.4F/Pct P
Tensile Yield Strength	(61.5 to 124.7 Ksi)	0.68F/Ksi

*ASTM A 543-65

The standard deviation (Sx.y) of this correlation is 18.3F and the multiple corrleation coefficient (R) is 0.90 with an associated degrees of freedom of 103. Higher order terms and interaction terms did not significantly improve the correlation.

The value of the standard deviation for the correlation reported here, Sx.y = 18.3F is close to the theoretical limit of the best possible correlation based on the ±10F repeatability of the drop weight test determination of the NDT temperature and the repeatability of measurment of the supposedly fixed valued independent variables*. For example, a repeatability of ±½ in the ASTM prior gamma grain size measurements would indicate that the best possible value of Sx.y for the regression would be about ±15F. Because of the small contribution of the other variables to the NDT temperature's variation, small errors in their measurement would not contribute as much to the magnitude of the standard deviation for the regression as would an error in the measurement of the prior austenitic grain size. However, the effect of the errors associated with each of the independent variables plus the variation in the experimental determination of the NDT temperature indicates that the value of the standard deviation associated with the

*Regression analysis programs such as the one used to analyze these data assume the error is associated with the dependent variable and that the independent variables are exact values. While this is not the true case, it is a commonly used assumption. The more rigorous technique has computational and analytical disadvantages that are discussed by Gross in Reference 21.

final regression equation given in Table 10, Sx.y = 18.3F, is close to the theoretically best possible functional relationship between a set of variables and the NDT temperature; lower values can be attained but only by using a set of dummy variables that would have no true significance.

The producer who melted the steel and rolled out the plate is a significant variable when analyzing the variables affecting NDT temperature. This is attributed to basic differences in composition and mill practice that could not be quantified by themselves but are lumped together in this variable.

No chemical composition terms (other than amount of hydrogen) were found to consistently correlate with NDT temperatures, and the low level of hydrogen found in these tests, typically 0 to 2 ppm, results in little practical effect on NDT temperature, 2F per ppm H_2. The failure of other alloying and trace elements to significantly affect NDT temperature is attributed to the effect of these elements on the prior austenitic grain size which in itself is the dominant factor controlling NDT temperatures measured in these tests. In addition, the alloying and trace elements affect the microstructure and strength through their influence on quantity and type of transformation product produced by austenite decomposition, ferrite strength, temperability, secondary hardening and susceptibility to temper embrittlement, and through their effect on strength and microstructure exert an effect on the

secondary factors that determine NDT temperature.

The steel used in these studies was relatively clean and free from inclusions. Inclusion content was not found to be a significant variable affecting the NDT temperature.

The range of tensile yield strength produced by this study was rather limited by the decision to use a single (1150F) tempering treatment for all samples; the mean value was 106 ksi and ranged from 124.7 ksi for the highest strength tempered martensitic structure to 61.6 ksi for the lowest strength structure containing large amounts of tempered ferrite and pearlite and the least tempered martensite. Reference 46 indicates that the rather small effect of strength on NDT temperature (0.68F/ksi) is related to the limited range of strengths obtained in this study. Reference 46 indicates that there should be an interaction between strength level and microstructure in their effect on the NDT temperature; this was not observed to be the case in the present study, again probably due to the limited strength range.

The order of magnitude of the effect of prior gamma grain size on the NDT temperature found in this study, -19F/(γGS), is of the same order of magnitude as that reported in the literature for both ferritic, bainitic and martensitic steels, in terms of ferritic and prior austenitic grain sizes respectively. In plain carbon steels a unit increase in the ASTM ferritic grain size, $\Delta\alpha$GS, will reduce the transition temperature 22F on the average with the

change ranging from +9 to -39F[2,6,7,13,17,18,24,28,31]. Drop weight NDT test of a ferritic steel gave -11F/$\Delta\alpha$GS[24]. Pearlitic steels[8] (C1080 and C1095) and steels air cooled and tempered[3] (SAE 1045, 2340, and 3140) had $\Delta T/\Delta\gamma GS$ ranging from -17 to $\simeq$ -125F per unit change in prior austenitic grain size. In a study of the effect of impurity, P and Sn level, $\Delta T/\Delta\gamma GS$ ranged from -4 to -42F per unit change in prior austenitic grain size in a 0.3 C, 2.0 Ni, 0.75 Cr steel with the value increasing with impurity level and embrittling treatment[23].

The linear dependence of transition temperature of ferritic steels on ferrite grain size, $T = A_0 + A_1 \ell nd^{-\frac{1}{2}}$, reported by Petch[30], and of pearlitic, 0.84 and 0.95 C, steels on prior austenitic grain size reported by Gross and Stout[16] is in agreement with the linear dependence of NDT temperature on prior austenitic (ASTM) grain size observed for the HY-80 steel used in this study. Irvine and Pickering[27] report that the transition temperature of low-carbon bainitic steels increases in an S shaped relationship, third order, with increasing prior gamma grain diameter as varied by reheating to various austenitizing temperatures. Replotting their data on semi-log graph paper shows that the transition temperature of low-carbon bainitic steels also varies in an approximately linear manner with the logarithm of the grain diameter. The results of the present study indicate that a low-carbon tempered martensitic steel such as HY-80 also has a linear relation between

NDT temperature and prior austenitic grain size as expressed in ASTM grain size numbers. This relationship is not affected by appreciable amounts of isothermally produced bainite and ferrite in a structure tempered at 1150F.

EFFECT OF MICROSTRUCTURE ON STRENGTH DIFFERENTIAL EFFECT

The difference in uniaxial tensile and compressive yield strength in martensitic steels has been attributed to the difference in the non-linear effects in tension and compression caused by the carbon atoms trapped in the martensite. Hirth and Cohen's model of the non-linear stress: strain relation is based on a number of assumptions[43]. They assume the following:

1. The non-linear effects are due to carbon atoms trapped in the interstices of the iron lattice and are not due to the substitutional alloying effects.

2. The solid solution strengthening caused by the carbon is proportional to the carbon/dislocation interaction energy as well as some power of the carbon concentration. (It should be noted that Hirth and Cohen make no attempt to proportion this effect to carbon content in either their model or their sample calculation with 4340 steel. In fact, their only attempt to explain carbon concentration effects was to point out that the uncorrected $\Delta\sigma_{S-D}$ data of Leslie and Sober indicated that the S-D effect appears to increase linearly with carbon content.)

3. It is safe to neglect lattice forces and strains

of the Fe/C bond between the interstitial C atom and its two nearest neighbors.

4. The lattice strains existing in the eight nearest Fe/Fe pairs will represent the non-linear shift in the stress:strain relationship to within at least the correct order of magnitude.

5. The C atom causes a large dipole distortion which results in an 8% compressive strain in the bonds between the eight surrounding Fe/Fe atom pairs.

6. The nonlinear normal forces along the (111) direction are adequately estimated from Brenner's stress: strain curves for (111) iron whiskers[50].

7. It is safe to disregard the straining effects of substitutional alloy additions on the basic iron lattice structure, since they would only add to the effect.

8. The S-D effect is maximized in the as-quenched martensitic structure and will disappear or persist at a decreased level in tempered martensite and bainite, the persistance being attributed to C atmospheres adsorbed at dislocations.

Some of these assumptions (5 and 6) are based on experimental data and need not be questioned; some (3, 4 and 7) are necessary to the development of the model with a minimum of complicating factors. Others of these assumptions (1, 2 and 8) can be examined in light of available experimental data, and it is these latter assumptions to which we turn to evaluate Hirth and Cohen's model of the

S-D effect.

Leslie and Sober's data for as-quenched 4300 series steel martensite is shown as line A in Figure 7a, and the hatched area represents the same data with the recommended 1/3 to 1/2 correction factor[44]. Figure 7b gives a plot of the S-D effect as a percent of the uniaxial tensile yield strength, with the corrected values again represented by the hatched area.

The data are replotted in Figure 7b to see if the S-D effect is a constant percentage of the tensile yield strength since the tensile yield strength of the as-quenched martensite increases as the carbon is increased. Both parts of Figure 7 have been projected down to run through zero as is assumed by Hirth and Cohen. The hatched area of Figure 7a shows that the true S-D effect may range anywhere from about 5 to 15 percent for the 0.10 to 0.40 carbon range actually studied by Leslie and Sober.

For the 0.16% carbon content typical of HY-80 steel the S-D effect would be expected to be about 5 to 10 percent of the tensile yield strength of the as-quenched martensite, based on Figure 7b; the solid line in Figure 8a shows it to have been measured as 9 percent for an HY-80 steel austenitized at 1640F, water quenched and tempered at various temperatures[47].

In addition, Figure 8a shows that tempering does not reduce the S-D effect to zero as the tempering temperature is increased. Instead, Figure 8a shows that the S-D effect

increases to a maximum as the tempering temperature* is raised to about 600F corresponding to the precipitation of Fe_3C. It then decreases slightly as the temperature is raised to about 1150F, shows a slight inflection at about 1200F in the region where coherent alloy carbides start to form, and drops markedly from 10 to 5 percent as the temperature is raised closer to the lower critical temperature (A_{c_1})** and as the alloy carbides grow and lose coherency. Beyond 1300F, slightly below the A_{c_1}, there is a sudden and dramatic increase in the S-D effect from 5 to 28% as the tempering temperature passes into the critical temperature range where increasing proportions of austenite are formed and which tranform to fresh martensite upon quenching from the tempering temperature. This effect begins to reverse itself about 1400F, while the percent of austenite formed during tempering is still relatively small. This effect may be due either to the lowering of the carbon content of the ferrite matrix, accompanied by the change in shape and distribution of the hard particles in the soft matrix, or to the strain imposed on the ferrite lattice when the carbon enriched austenite expands as it transforms to martensite. It is most likely the former, since examination of the lightly dashed line of Figure 8a for a coarse grained material originally austenitized at 2000F shows that the same effect appears to occur but at the temperatures

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* One hour at temperature and then brine quenched from the tempering temperature.
**The A_{c_1} is about 1330F for this steel.

normally associated with secondary hardening due to the formation of alloy carbides instead of at temperatures slightly above the A_{c1}. It is unfortunate that the coarse grained steel was not tempered in the range from 1330 to 1400F to see if a second peak occurred in the S-D effect, as the A_{c1} was exceeded during tempering of the coarse grained material.

Comparison of the solid and dashed lines on Figure 8a shows three things: first, that the S-D effect definitely does not decrease to zero as the tempering temperature is raised, but that it follows a closed loop, see Figure 8b; second, that the prior austenitic grain size of the steel can have a marked effect on the magnitude of the S-D effect; and third, that either secondary hardening or tempering above the lower critical temperature can have a marked effect on the S-D effect.

Figure 9 shows the effects of microstructure on the S-D effect, Figure 9a as a function of tempering temperature and Figure 9b as a function of strength[46]. This material was austenitized at 2000F to produce a coarse grained austenitic structure prior to transformation. The open circles represent 100% martensite, the open triangles represent 3% bainite-97% martensite, the closed triangles 17% bainite-83% martensite, and the open squares 9% ferrite-91% martensite. These figures show the following for tempered steels with a mixed microstructure: first, the S-D effect persists at a significant level even after tempering;

second, it does not go to zero as the tempering temperature is raised; third, that disregarding the secondary hardening of the fully martensitic structure, the S-D effect is approximately the same for martensitic and semi-bainitic structures; fourth, a semi-ferritic structure tends to have a markedly larger S-D effect at low tempering temperatures, up to about 1000F, and fifth, that after an 1150F temper for one hour, the percent S-D effect is quite consistent for all microstructures, 8 to 12% of the tensile yield strength.

The increased S-D effect for the semi-ferritic structure may be attributed to either the preferential lowering of the uniaxial tensile yield strength by the weak ferrite or to the increased carbon content of the martensite due to the carbon enrichment of the remaining austenite during the formation of the ferrite from austenite. The latter is not likely to be the significant factor, since the carbon increase would not be significant with the formation of 9% ferrite raising the carbon content of the remaining austenite from its original 0.16% to less than 0.18 percent. Examining Figure 7b, it can be seen that this is not enough to account for the increase in S-D effect from 5 to 10% observed for the semi-ferritic structure in Figure 9a.

Figure 10 is a plot of the percent S-D effect reported in Table 6 for 27 different steel plates heat treated to have a fully martensitic structure and then tempered at 1150F for one hour. The circles represent data on plates austenitized at 1640F for ½ hour to give a fine prior aus-

tenitic grain size, and the squares represent plates heated at 2000F for one hour to give a coarse prior austenitic grain size. The open circles represent plate material from heats made by one producer and the closed circles* represent heats made by the other. Figure 10 shows plots of the percent S-D effect as a function of carbon content, prior austenitic grain size and ultimate tensile strength.

Figure 10 shows: (a) for the fine grained material where comparison can be made between producers, both producers' steel has the same magnitude S-D effect; (b) carbon content ranging from 0.14 to 0.22% had no effect on the magnitude of the S-D effect; (c) prior austenitic grain size appears to have little or no effect on the magnitude of the S-D effect, considering the range of observed values for a given grain size; and (d) there appears to be a slight correlation between the magnitude of the S-D effect and the ultimate tensile strength of the steel. As can be seen from Figure 10c, the solid line representing the relationship between ultimate tensile strength and percent S-D effect is given by the equation below and indicates that the S-D effect (Y)

$$Y = \tfrac{1}{4}X - 23\tfrac{3}{4} \qquad (1)$$

goes to zero at an ultimate strength (X) of 95 ksi; it should be kept in mind that this projection is only for a fine grained, fully quenched martensitic HY-80 steel

*None of these heats were austenitized at 2000F.

tempered for one hour at 1150F and that the usefulness of the equation should be limited to the range of the ultimate tensile strength data used to derive it, 117.5 to 138.8 ksi. Equation (1) does not hold for other tempering temperatures, as can be seen from Figures 8 and 9, and the data in Figure 10 does not agree with the hypothesis that the S-D effect goes to zero at zero strength as is represented by the dashed line in Figure 10c.

Figure 11 is a plot of selected data from Figure 10; the number next to each datum point plotted on Figure 11 refers to the plate identification number. Figure 11 gives only the data for the six plates which were tested after both the fine and coarse grain austenitizing treatments prior to quenching and tempering. Again, the S-D effect seems unrelated to carbon content, at least for the limited range shown, 0.16 to 0.22% C. There does appear to be a slight correlation of percent S-D effect to prior austenitic grain size and to ultimate strength. It should be kept in mind, however, that strength and grain size are not truly independent variables, since both yield strength and ultimate strength are a function of grain size[51], with the finer grained material expected and observed here to have higher strengths.

A comparison of the corrected data for 4300 steels in Figure 7 and for the 0.16% C HY-80 steel in the other figures shows that the S-D effect is almost a constant percentage of the tensile yield strength regardless of carbon

content and regardless of the tempering temperature, as long as some secondary effect does not intervene, such as secondary hardening or exceeding the A_{c1}. This tends to support the contention that the S-D effect is the result of either substitutional elements straining the lattice, since their effect would exist even after tempering, or that the sign (±) of the normal stress across the glide plane during plastic deformation has a small but finite effect on the Peierls stress. The data of Lessel's for 1035 and 1045 steel show a similar effect for plain carbon steels and thus supports the latter[45].

CONCLUSIONS

The following conclusions are based on the results of a study on the effects on the NDT temperature of: (1) commercial variation in composition and inclusion content, (2) variation in microstructure, prior austenitic grain size and microconstituents produced by quenching and by partial isothermal transformation at 1200 and 875F to produce ferrite and bainite prior to quenching, and (3) variation in strength level after an 1150F temper for one hour followed by a water quench to prevent embrittlement while cooling from the tempering temperature.

The steel used in these studies is a high toughness, low impact transition temperature, quenched and tempered Ni-Cr-Mo steel of the type made to ASTM Standard A 543-65. Sixteen coupons were obtained from one producer and eleven coupons from another. The average composition of the steels used in this study are given below:

0.18% C	0.10% Cu
0.36% Mn	0.032% Acid Soluble Al
0.016% P	0.040% Total Al
0.019% S	0.002% Ti
0.30% Si	0.014% O_2
2.75% Ni	0.007% N_2
1.51% Cr	1 ppm H_2
0.38% Mo	

FACTORS AFFECTING THE NDT TEMPERATURE

The results of this study show that the NDT temperature of the low-carbon Ni-Cr-Mo steel tested is primarily determined by a single variable and that the other significant

variables act in a secondary way. The conclusions drawn from this study are as follows:

1. Using a tempered 100 percent martensitic microstructure as the baseline, the significant variables acting to determine the NDT temperature measured in this study and their relative (from Table 10) effects are:

VARIABLE	RANGE	EFFECT ON NDT
Producer	(Coded 1 or 2)	-21.9F/Unit
Prior Austenitic Grain Size	(3 to 9.5)	-19.2F/ASTM G.S. No.
Percent Bainite	(0 to 76%)	0.62F/Pct B
Percent Ferrite	(0 to 69%)	-0.23F/Pct F
Percent Pearlite	(0 to 32%)	5.4F/Pct P
Tensile Yield Strength	(61.5 to 124.7 Ksi)	0.68F/Ksi

The effect of a unit change in prior austenitic grain size on NDT temperature is one and one-half to two orders of magnitude greater than the effect of either a percentage change in the amount of isothermally produced bainite and ferrite or a 1000 psi change in tensile yield strength.

$$\text{NDT Temp. (F)} = -77 - 22\,(\text{Producer Code}) - 19.2\,(\gamma GS) + 0.62\,(\%B) - 0.23(\%F) + 5.4(\%P) + 0.68(\text{TYS in ksi})$$

2. A linear function of the significant terms gives the best expression of the functional relationship between these variables and the NDT temperature since it was found that higher order and interaction terms did not significantly improve the correlation.

3. No chemical composition terms were found to consistently correlate with measured NDT temperatures in a

statistically significant manner, except for the amount of hydrogen in the steel, and the low level of hydrogen found in these steels, typically 0 to 2 ppm, results in little practical effect on NDT temperature, 2F per ppm H_2. The failure of alloying and trace elements to significantly affect NDT temperature is attributed primarily to the fact that these elements interact with: (a) the austenitizing treatment to control prior austenitic grain size; (b) the transformation product; (c) temperability and final strength of the steel, and (d) basic susceptibility to embrittlement during normal tempering, as well as the fact that ranges in the amount of each element were so restricted by the specifications and commercial melting practice as to make it impossible to detect their relatively weak effect on a sound statistical basis.

4. The inclusion content was not found to be a significant factor affecting the NDT temperatures measured in these studies.

5. The weak correlation between tensile yield strength and NDT temperatures observed in this study, as opposed to the strong effect reported in References 46 and 47 is attributed to the limited range of strengths produced by the 1150F temper used on all material in the present study.

6. The linear relationship between ASTM prior austenitic grain size and NDT temperature of the low-carbon Ni-Cr-Mo steel used in this study is similar to the linear dependence between: (a) prior austenitic grain diameter and

the transition temperature of low-carbon bainitic steels reported by Irvine and Pickering[27]; (b) prior austenitic grain size and the transition temperature of pearlitic 0.84 and 0.95 C steels reported by Gross and Stout[16], and (c) the logarithm of the ferritic grain diameter and the transition temperature of ferritic steels reported by Petch[30].

7. The results of these studies indicate that the steel producer is a statistically significant variable that can account for about 22F of the observed variation in NDT temperature, depending upon which producer made the steel; this is attributed to a combination of small compositional differences, such as amount of P or Cu which by themselves were not consistently significant, and to differences in mill practice. In any case, these results indicate that the producer is a significant variable that should be considered in any analysis of variables affecting the NDT temperature of low-carbon Ni-Cr-Mo steels.

FACTORS AFFECTING THE STRENGTH-DIFFERENTIAL EFFECT

The experimental data presented here and in References 46 and 47 show that the strength-differential (S-D) effect* is sensitive to microstructure and that it appears to be a function of the tensile yield strength rather than a function of the lattice strain caused by carbon atoms in the martensitic lattice as proposed by Hirth and Cohen in

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*The difference between tensile and compressive uniaxial yield strength. The percent S-D effect is ((CYS-TYS)/TYS) ×100, and $\Delta\sigma_{S-D}$ is (CYS-TYS).

Reference 43. The following conclusions may be drawn from these data:

1. The data presented here on low-carbon Ni-Cr-Mo steels show that for the as-quenched structure, the magnitude of the S-D effect is markedly influenced by the prior austenitic grain size, which should not be the case if it was solely due to non-linear strains induced in the body centered tetragonal lattice by interstitiial carbon atoms.

2. The percent S-D effect observed in these studies persists and is essentially undiminished by tempering below the lower critical (A_{c_1}).

3. The S-D effect observed in this study is about 8% (±4%) of the tensile yield strength, regardless of strength level and carbon content for the group of low-carbon Ni-Cr-Mo steels having about the same composition and given the same tempering treatment.

4. The existence of the S-D effect in plain carbon steels, such as 1035 and 1045 reported by Lessels[45], indicate that large percentages of substitutional atoms distorting the iron lattice may not be necessary to produce the S-D effect.

5. Evaluation of Figure 7 in light of the data in Figures 8-11 indicates that the projection $\Delta\sigma_{S-D}$ goes to zero as the carbon content goes to zero may be correct, but that this is attributed to the fact that $\Delta\sigma_{S-D}$ is effectively a constant percentage of the tensile yield strength which is also decreasing as the carbon content is lowered.

6. Based on the data reported in Reference 46, the S-D effect seems to be greatest for a mixed microstructure of ferrite and tempered martensite rather than a maximum for either a tempered fully martensitic or bainitic structure, and that this is probably an additive effect due in part to the more detrimental effect of small amounts of the weak ferrite in the stronger martensite when testing in tension as opposed to compression.

7. The relatively constant level of the S-D effect when expressed as a percent of the yield strength for various microstructures, as-quenched and as-quenched and tempered below the lower critical temperature, suggest that interstitial carbon atoms are not the sole cause and probably not the major cause of the S-D effect.

8. The data presented here and in References 46 and 47 show that the percent S-D effect is markedly structure sensitive to such things as (a) prior austenitic grain size, (b) microstructural constituents, (c) tempering temperature, (d) type and distribution of carbides formed during tempering and (e) tempering slightly above the lower critical (A_{C1}) temperature.

9. The results of the tests reported here on low-carbon Ni-Cr-Mo steel treated to produce various microstructures strongly suggest that the S-D effect has a minimum value of about 5% of the tensile yield strength of the material in residual stress free, relatively isotropic, homogeneous structures not subject to unusual phenomenon

such as preferring to twin to accomplish plastic deformation under compressive loading, and may range to much higher values due to its sensitivity to final microstructure.

10. The microstructural sensitivity of the percent S-D effect strongly indicates that the applied stress required to move a dislocation (the Peierls Stress) is being affected and is responsible for the S-D effect, and that, like frictional forces, it is sensitive to the sign of the resolved normal stresses across the glide plane, as well as to the apparent "roughness" of the glide plane.

REFERENCES

1. Meklowitz, J.,"The Effect of Temperature and Material Structure on the Fracture Properties of Medium-Carbon Steel," ASTM Proc., Vol. 49, 1949, pp. 602-617.

2. Hodge, J. M., Manning, R. D. and Reichold, H. M., "The Effect of Ferritic Grain Size on Notch Toughness," Metals Transactions, March 1949, pp. 233-240.

3. Sinclair, G. M. and Dolan, T. J., "Some Effects of Austenitic Grain Size and Metallurgical Structure on the Mechanical Properties of Steel," ASTM Proc., Vol. 50, 1950, pp. 587-616.

4. Geil, G. W., Carwile, N. L., and Digges, T. G., "Influence of Nitrogen on the Notch Toughness of Heat-Treated 0.3 Percent-Carbon Steels at Low Temperatures," J. Res. of National Bureau of Standards, Vol. 48, No. 3, March 1952, pp. 193-200.

5. Armstrong, T. N., Kahn, N. A., and Thielach, H., "Transition from Ductile to Brittle Behavior in Pressure Vessel Steels," Welding J., Vol. 31, August 1952, pp. 371s-380s.

6. Frazier, R. H., Boulger, F. W., and Lorig, C. H., "Influence of Heat Treatment on the Ductile-Brittle Transition Temperature of Semikilled Steel Plate," Trans. AIME, February 1955, pp. 323-329.

7. Barrett, C. S., "Metallurgy at Low Temperatures--Edward DeMille Campbell Memorial Lecture," ASM Trans., Vol. 49, 1957, pp. 53-117.

8. Cross, J. H. and Stout, R. D., "Effect of Microstructure on Notch Toughness, Part II," Welding J., March 1955, pp. 117s-122s.

9. Schwartzbart, H. and Sheehan, J. P., "How Carbon Content Affects Impact Properties," Iron Age, August 9, 1956, pp. 85-89.

10. Schwartzbart, H., and Sheehan, J. P., "How Boron, Grain Size Affects Impact Properties," Iron Age, August 16, 1956, pp. 102-106.

11. Schwartzbart, H. and Sheehan, J. P., "How Fracture Type, Specimen Size Affect Impact Properties," Iron Age, August 23, 1956, pp. 107-110.

12. Mikus, E. B. and Siebert, C. A., "A Study of the Role of Carbon in Temper Embrittlement," Trans. ASM, Vol. 50, 1958, pp. 682-704.

13. Williams, M. L., "Correlation of Metallurgical Properties and Service Performance of Steel Plates from Fractured Ships," Welding J., October 1958, pp. 445s-454s.

14. Gross, J. H. and Stout, R. D., "The Effect of Microstructure on Notch Toughness--Part I," Welding J., Vol. 30, October 1951, pp. 481s-485s.

15. Rees, W. P., Hopkins, B. E., and Tipler, H. R., "Tensile and Impact Properties of Iron and Some Iron Alloys of High Purity," J. Iron and Steel Institute, Vol. 169, October 1951, pp. 157-168.

16. Gross, J. H. and Stout, R. D., "The Effect of Microstructure on Notch Toughness--Part II," Welding J., Vol. 35, March 1955, pp. 117s-122s.

17. Gross, J. H. and Stout, R. D., "The Effect of Microstructure on Notch Toughness--Part III," Welding J., Vol. 36, February 1956, pp. 72s-76s.

18. Danko, J. C. and Stout, R. D., "The Effect of Subboundaries and Carbide Distribution on the Notch Toughness of an Ingot Iron," Trans. ASM, Vol. 49, 1957, pp. 189-203.

19. Weaver, A. P., "Impact Characteristics and Mechanical Properties of Leaded and Nonleaded C-1050 and C-1141 Steels," Trans. ASM, Vol. 49, 1957, pp. 464-481.

20. Kottcamp, E. H. and Stout, R. D., "Effect of Microstructure on Notch Toughness--Part IV," Welding J., Vol. 38, November 1959, pp. 435s-440s.

21. Gross, J. H., "Comparison of Charpy V-Notch and Drop-Weight Tests for Structural Steels," Welding J., Vol. 39, February 1960, pp. 59s-69s.

22. Irvine, K. J., "The Development of High-Strength Steels," J. Iron and Steel Institute, Vol. 200, October 1962, pp. 820-833.

23. Capus, J. M., "Austenite Grain Size and Temper Brittleness," J. Iron and Steel Institute, Vol. 200, November 1962, pp. 922-927.

24. Boulger, F. W. and Hansen, W. R., "The Effect of Metallurgical Variables in Ship-Plate Steels on the Transi-

tion Temperatures in the Drop-Weight and Charpy V-Notch Tests," Ship Structure Committee Report SSC-145, December 3, 1962, AD 294827.

25. Gulyayev, A. P., "Effect of Chromium and Nickel on Toughness of Steel," Metal Science and Heat Treatment of Metals, November-December, 1962, pp. 491-493.

26. Newhouse, D. L., "Relationships Between Charpy Impact Energy, Fracture Appearance and Test Temperature in Alloy Steels," Welding J., Vol. 42, March 1963, pp. 105s-111s.

27. Irvine, K. J. and Pickering, F. B., "The Impact Properties of Low Carbon Bainitic Steels," J. Iron and Steel Institute, Vol. 201, June 1963, pp. 518-531.

28. Irvine, K. J. and Pickering, F. B., "Low-Carbon Steels with Ferrite-Pearlite Structures," J. Iron and Steel Institute, Vol. 201, November 1963, pp. 944-959.

29. Spektor, Y. I., Sarrak, V. I. and Entin, R. I., "Brittle Failure and the Fine Structure of Steel," Phys. Metal. Metallogr., Vol. 18, n. 6, 1964, pp. 107-111.

30. Masubuchi, K., Monroe, R. E. and Martin, D. C., "Interpretative Report on Weld-Metal Toughness," Ship Structure Committee Report SCC-169, July 1965.

31. Jolley, W. and Kottcamp, E. H., "A Study of the Toughness Differences Between Aluminum-Killed and Semikilled Steels," Trans. ASM, Vol. 59, September 1966, pp. 439-456.

32. Roper, C. R. and Stout, R. D., "Weldability and Notch Toughness of Heat-Treated Carbon Steels," Welding J., September 1968, pp. 385s-392s.

33. Hollomon, J. H., Jaffe, L. D., McCarthy, D. E. and Norton, M. R., "The Effects of Microstructure on the Mechanical Properties of Steel," Trans. ASM, Vol. 38, 1947, pp. 807-844.

34. Taber, A. P., Thorlin, J. F. and Wallace, J. F., "Influence of Composition on Temper Brittleness in Alloy Steels," Trans. ASM, Vol. 42, 1950, pp. 1033-1055.

35. Irvine, K. J. and Pickering, F. B., "Low-Carbon Bainitic Steels," J. Iron and Steel Institute, December 1957, pp. 292-309.

36. Steven, W., Balajiva, K., "The Influence of Minor Elements on the Isothermal Embrittlement of Steels," J. Iron and Steel Institute, Vol. 193, October 1959, pp. 141-147.

37. Capus, J. M. and Mayer, G., "The Influence of Trace Elements on Embrittlement Phenomena in Low-Alloy Steels," Metallurgia, Vol. 62, October 1960, pp. 133-138.

38. Pense, A. W., Stout, R. D. and Kottcamp, Jr., E. H., "A Study of Subcritical Embrittlement in Pressure Vessel Steels," Welding J., Vol. 42, n. 12, December 1963, pp. 541s-546s.

39. Baron, H. G. and Turner, S., "Effects of Residual Elements on Brittleness in Hardened and Tempered Forgings of Nickel-Chromium-Molybdenum Steel," J. Iron and Steel Institute, Vol. 203, December 1965, pp. 1229-1236.

40. Lubahn, J. D. and Chu, H. P., "Optimum Carbon Content for Tempered Martensitic Steels," Trans. ASME, J. Basic Eng., March 1968, pp. 1-7.

41. Low, Jr., J. R., Goodman, S. R. and Smith, C. L., "Grain Boundary Segregation of Impurities in Metals and Intergranular Brittle Fracture," Carnegie-Mellon University Report No. 031-727-1, ONR Tech. Report No. 1, March 1969.

42. Hehemann, R. F., Luken, V. J., and Troiano, A. R., "The Influence of Bainite on Mechanical Properties," Trans. ASM, Vol. 49, 1957, pp. 409-426.

43. Hirth, J. P. and Cohen, M., "On the Strength-Differential Phenomenon in Hardened Steel," Met Soc of AIME and ASM Metallurgical Transactions, Vol. 1, No. 1, January 1970, pp. 1-8.

44. Leslie, W. C. and Sober, R. J., "The Strength of Ferrite and of Martensite as Functions of Composition, Temperature and Strain Rate," Trans. ASM, Vol. 60, No. 3, September 1967, pp. 459-484.

45. Lessels, J. M., Strength and Resistance of Metals, J. Wiley and Sons, New York, 1954, p. 36.

46. Willner, A. R. and Salive, M. L., "The Effects of Tempered Nonmartensitic Products on the Notch Toughness and Mechanical Properties of an HY-80 Steel," David Taylor Model Basin Report 1605, January 1965.

47. Willner, A. R. and Salive, M. L., "Effects of Tempering Above the Lower Critical Temperature A_{c_1} on the Properties of an HY-80 Steel," David Taylor Model Basin Report 2140, January 1966.

48. Kalish, D. and Cohen, M., "Anisotropy Properties in Martensite as Developed by Thermomechanical Treatments," Trans. ASM Qtrly, Vol. 62, No. 2, June 1969, pp. 353-361.

49. Speich, G. R. and Kek, A. S., Private communication with Hirth and Cohen, U.S. Research Center, Monroeville, Pennsylvania, May 1969.

50. Brenner, S. S., J. Applied Physics, Vol. 27, 1956, p. 1484.

51. Dieter, G. E., Jr., Mechanical Metallurgy, McGraw-Hill, 1961, p. 121 and pp. 188-189.

52. Sinnott, M. J. and Shyne, J. C., "An Investigation of the Quenching Characteristics of Salt Bath," Trans. ASM, Vol. 44, 1952, pp. 758-744.

53. Seitz, F., The Physics of Metals, McGraw-Hill, New York, 1943.

54. Crafts, W. and Lamont, J. L., Hardenability and Steel Selection, Sir Isaac Pitman and Sons, Ltd., London, 1949.

55. Hollomon, J. H. and Jaffe, L. D., Ferrous Metallurgical Design, J. Wiley and Sons, Inc., New York, 1947.

56. Hall, A. M., Nickel in Iron and Steel, J. Wiley and Sons, Inc., New York, 1953.

57. Case, S. L. and VanHorn, K. R., Aluminum in Iron and Steel, J. Wiley and Sons, Inc., New York, 1953.

58. Comstock, G. F., Titanium in Iron and Steel, J. Wiley and Sons, Inc., New York, 1955.

59. Grange, R. A., Shortsleeve, F. J., and Hilty, D. C., Binder, W. O., Motock, G. S., and Offenhauer, C. M., Boron, Calcium, Columbium and Zirconium in Iron and Steel, J. Wiley and Sons, Inc., New York, 1957.

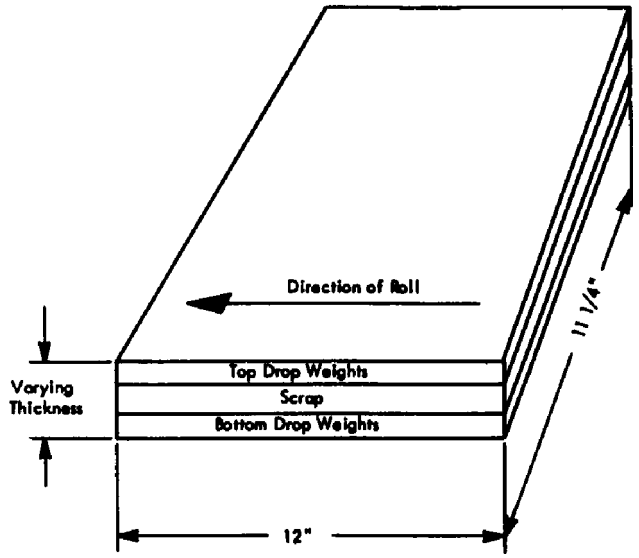
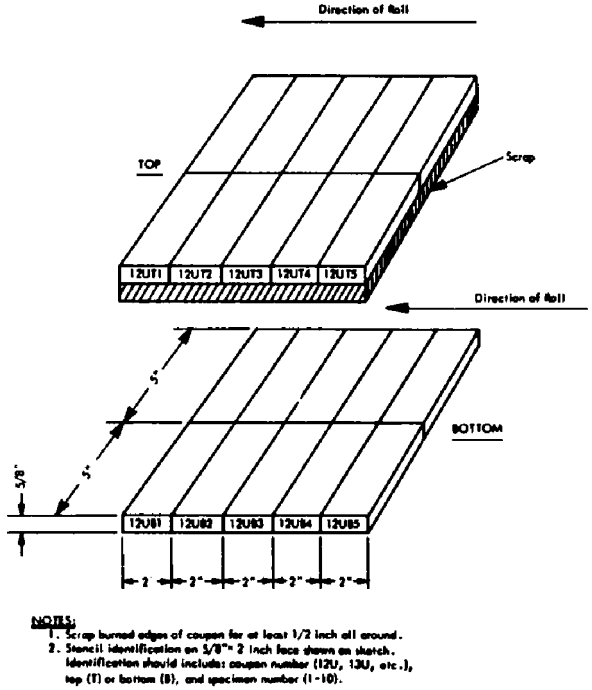


FIGURE 1 – LOCATION OF MATERIAL USED IN THIS STUDY



NOTES:
1. Scrap burned edges of coupon for at least 1/2 inch all around.
2. Stencil identification on 5/8"× 2 inch face shown on sketch. Identification should include: coupon number (12U, 13U, etc.), top (T) or bottom (B), and specimen number (1-10).

FIGURE 2 – LAYOUT OF DROP WEIGHT TEST SPECIMENS

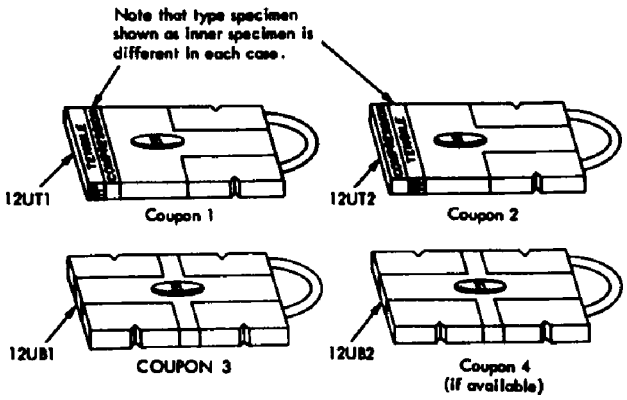


FIGURE 3A - LOCATION OF TEST SPECIMENS

NOTE:

1. Take the following specimens at the locations shown from each group of coupons:
   (A.) 2 longitudinal modified S-914 tensile specimens.
   (B.) 2 longitudinal S-12914 compression specimens.
   (C.) 10 transverse B-328-4 Charpy V-notch specimens. (See sheet No. 2 for additional instructions regarding the Charpy specimens.)
2. Identify each specimen with the following:
   (A.) Coupon No. (12UT4 for example)
   (B.) Direction (L or T)
   (C.) Specimen No. 1—4 (or 6 if necessary)
3. Identify tensile and Charpy specimens on both ends by stamping.
4. Identify compression specimens by scribing on the side only.
5. If any Charpy specimens have to come from under weld bead they should come from coupons 1 and 2.

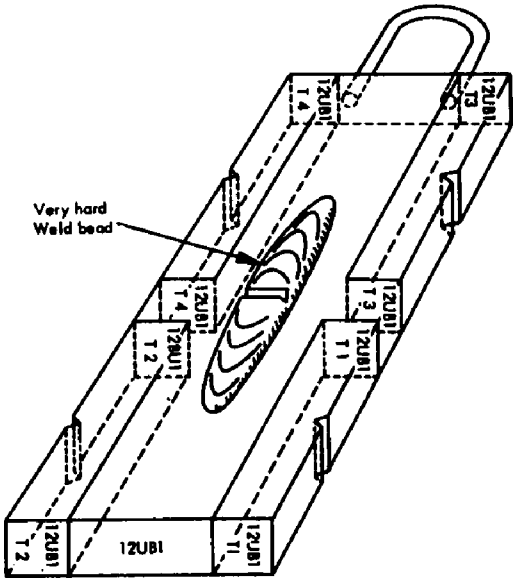


FIGURE 3B - DETAILS OF LAYOUT OF CHARPY V-NOTCH SPECIMENS

NOTES:

1. Transverse Charpy V-Notch Impact Specimens in accordance with NSRDC drawing B-328-4.
2. Note direction of notch and specimen identification numbers, see drawing B-328-4 for details.
3. When sawing specimens out of coupon stay away from the weld deposit; it is very hard.
4. Reidentify coupon after sawing specimens.

FIGURE 3 – LOCATION OF MECHANICAL PROPERTY AND TRANSVERSE CHARPY V–NOTCH IMPACT SPECIMENS MADE FROM TESTED DROP WEIGHT SPECIMENS.

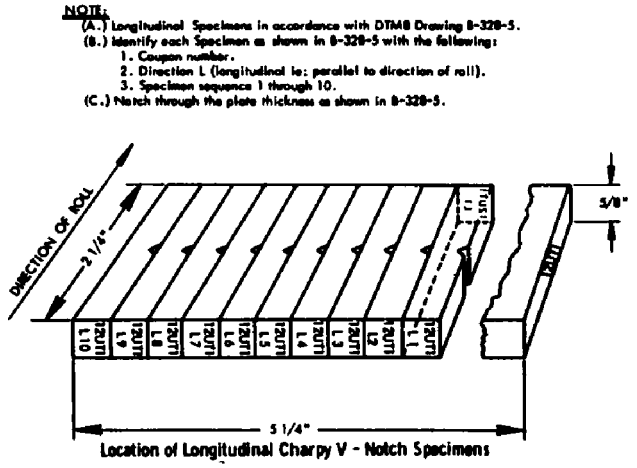


FIGURE 4 – LOCATION OF LONGITUNDINAL CHARPY V–NOTCH IMPACT SPECIMENS MADE FROM SPECIAL SPECIMEN BLOCKS

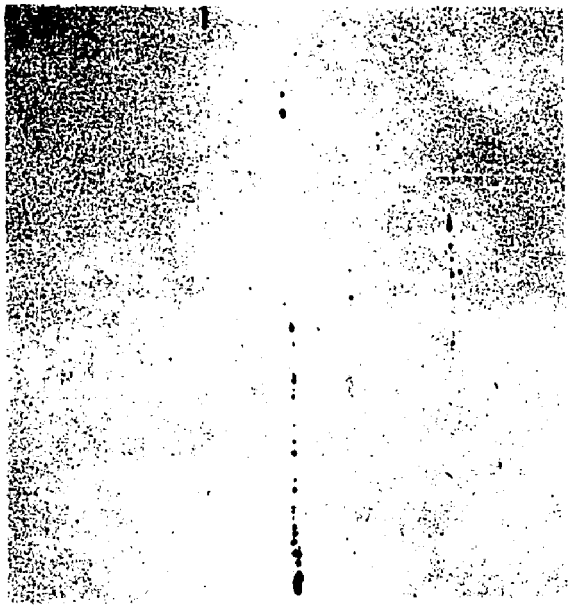


Figure 5a - Mid-Thickness, Longitudinal



Figure 5b - Mid-Thickness, Transverse



Figure 5c - Adjacent to Surface, Longitudinal

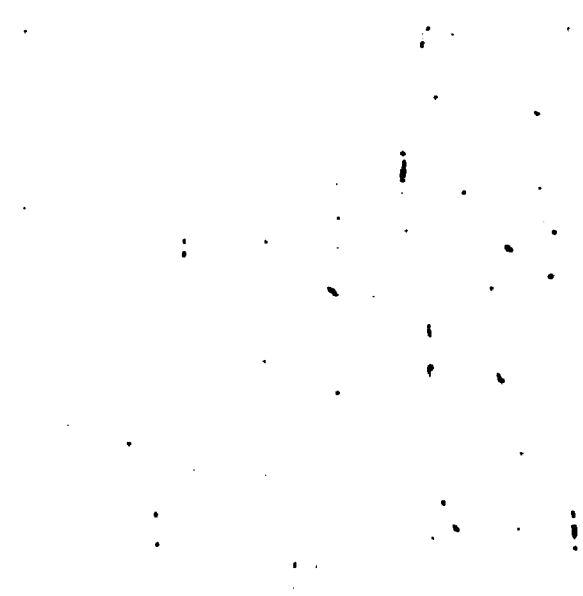


Figure 5d - Adjacent to Surface, Transverse

Figure 5 - Photomicrographs of the Inclusion Content of Coupon X-18

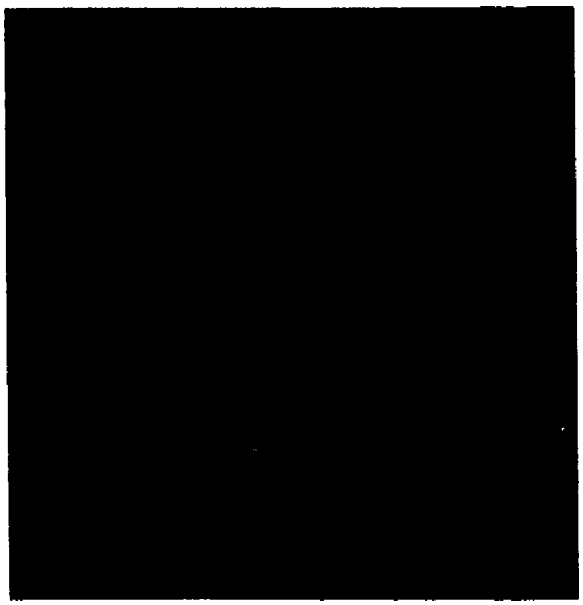


Figure 6a - Mid-Thickness, Longitudinal

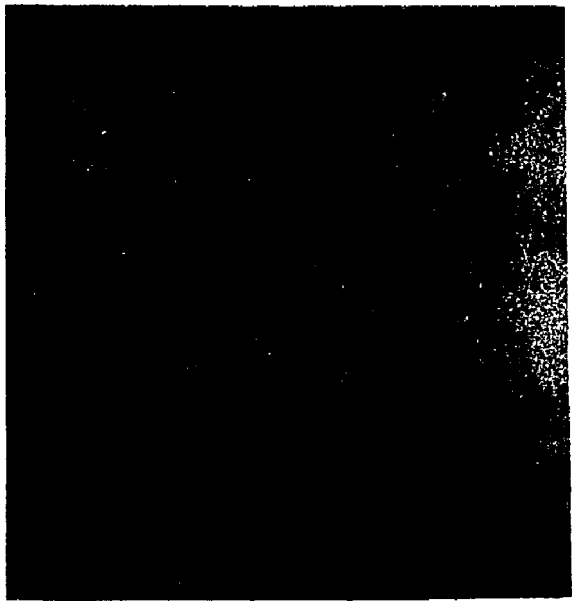


Figure 6b - Mid-Thickness Transverse

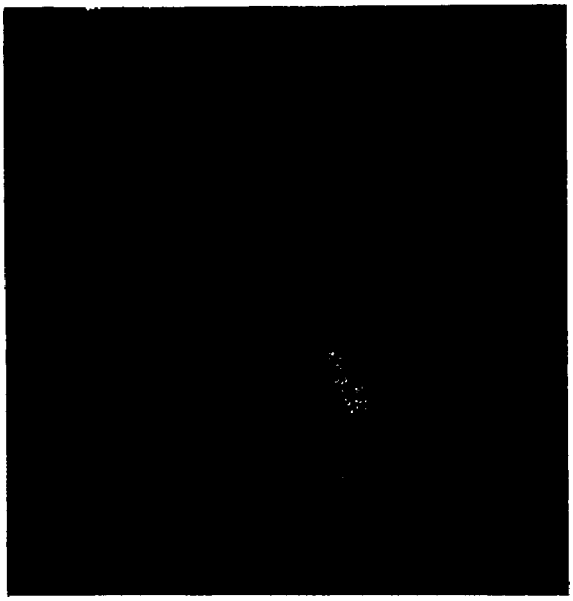


Figure 6c - Adjacent to Surface, Longitudinal

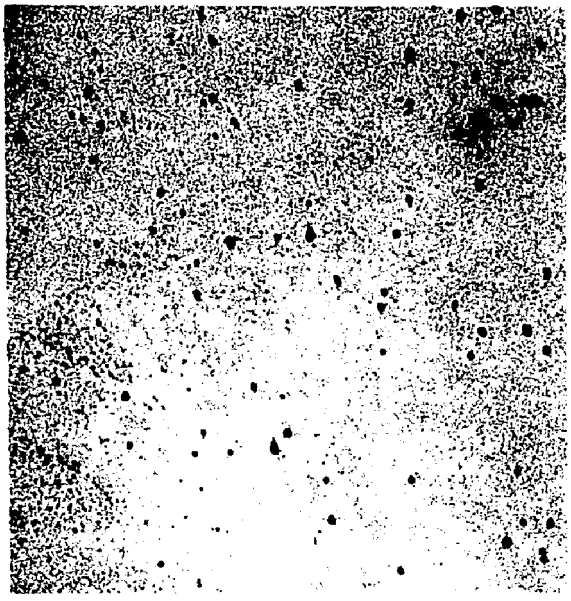


Figure 6d - Adjacent to Surface, Transverse

Figure 6 - Photomicrographs of the Inclusion Content of Coupon Y-1

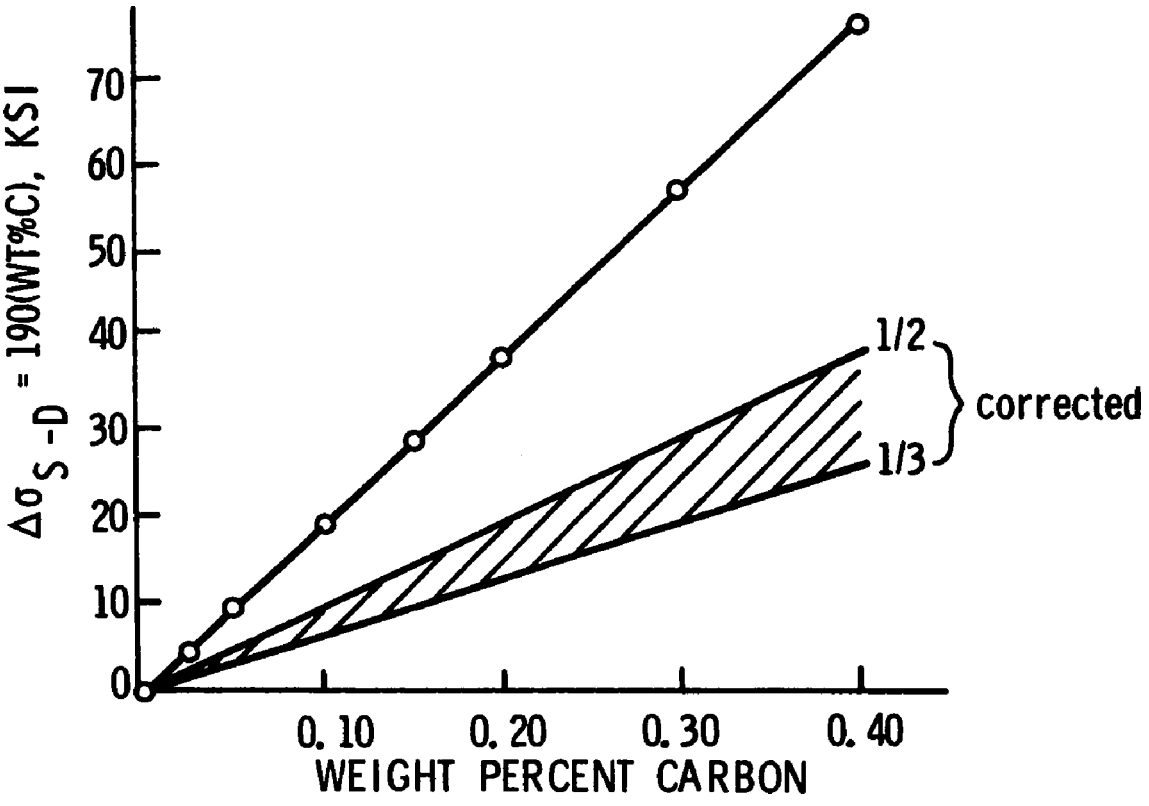


Figure 7a - Difference in Uniaxial Tensile and Compressive Yield Strength

Figure 7 - A Plot of the Strength Differential Effect Observed by Leslie and Sober as a Function of the Carbon Content of As-Quenched, Fully Martensitic 4300 Series Steels and a Quenched and Tempered 4330 Steel.

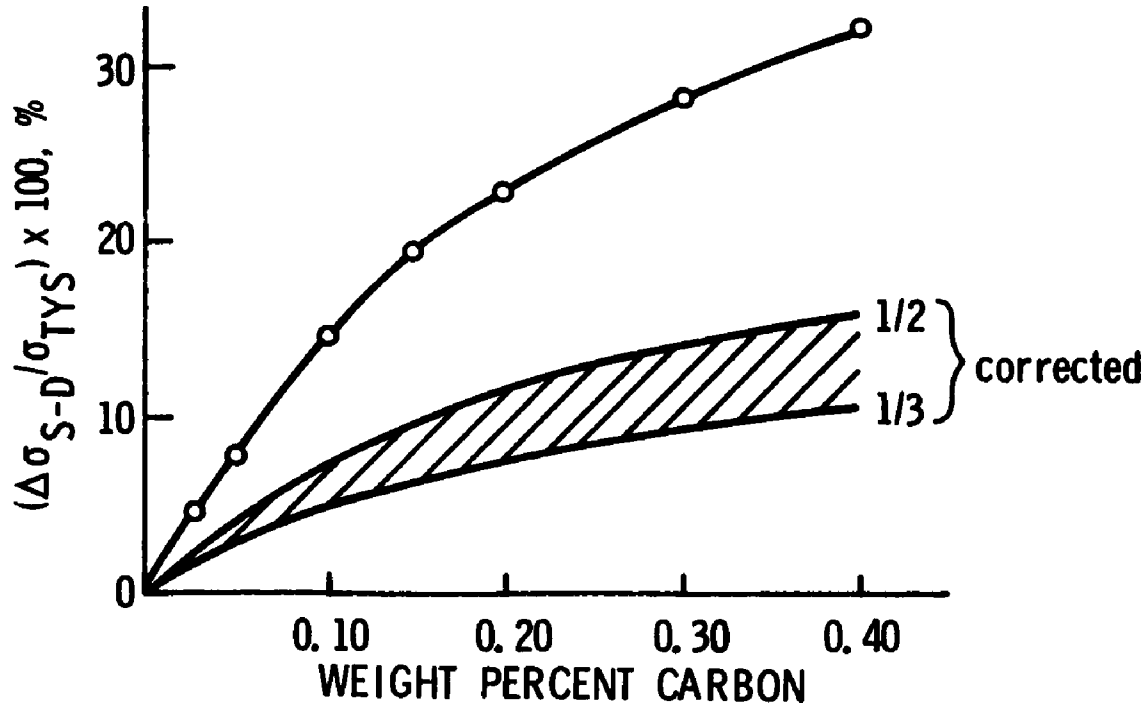


Figure 7b - Strength Differential as a Percent of Tensile Yield Strength

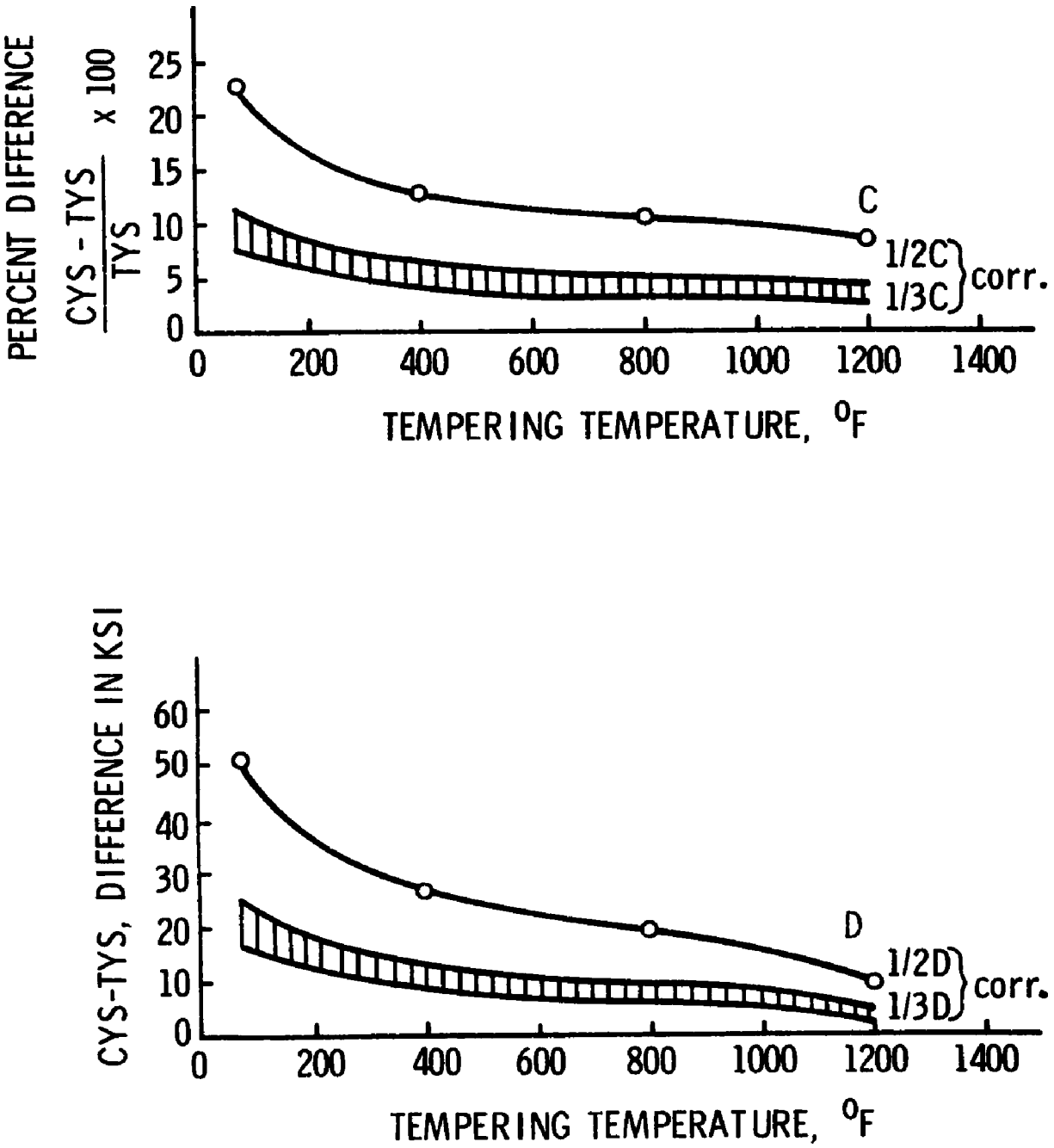


Figure 7c - S-D Effect in a Quenched and Tempered 4330 Steel.

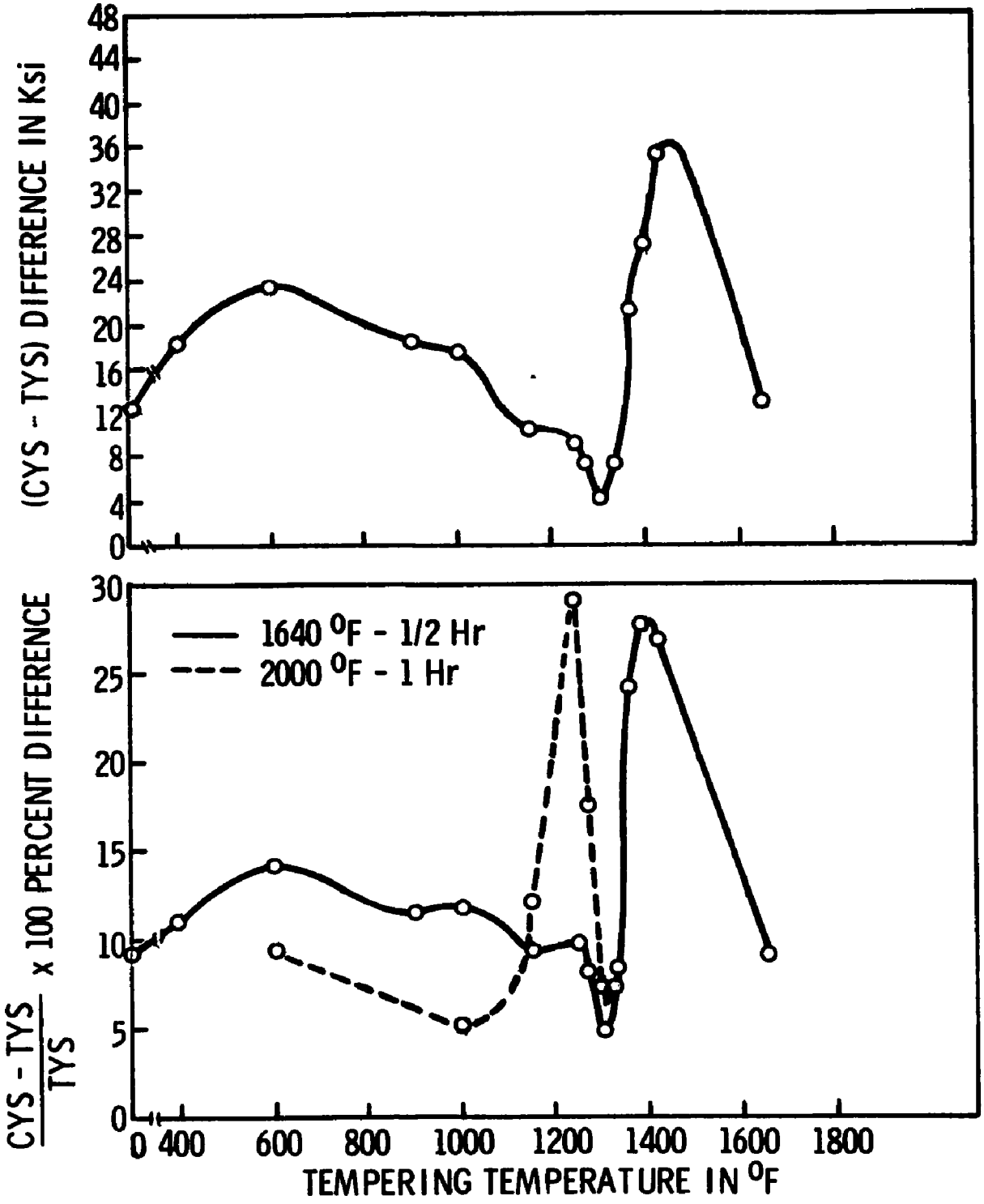


Figure 8a - Effects of Tempering Temperature on the S-D Effect.

Figure 8 - Effects of Tempering Temperature and Strength Level on the Relationship Between Tensile and Compressive Yield Strength of an HY-80 Steel Austenitized at 1640F for ½ Hour and Quenched to Produce a Fully Martensitic Structure.

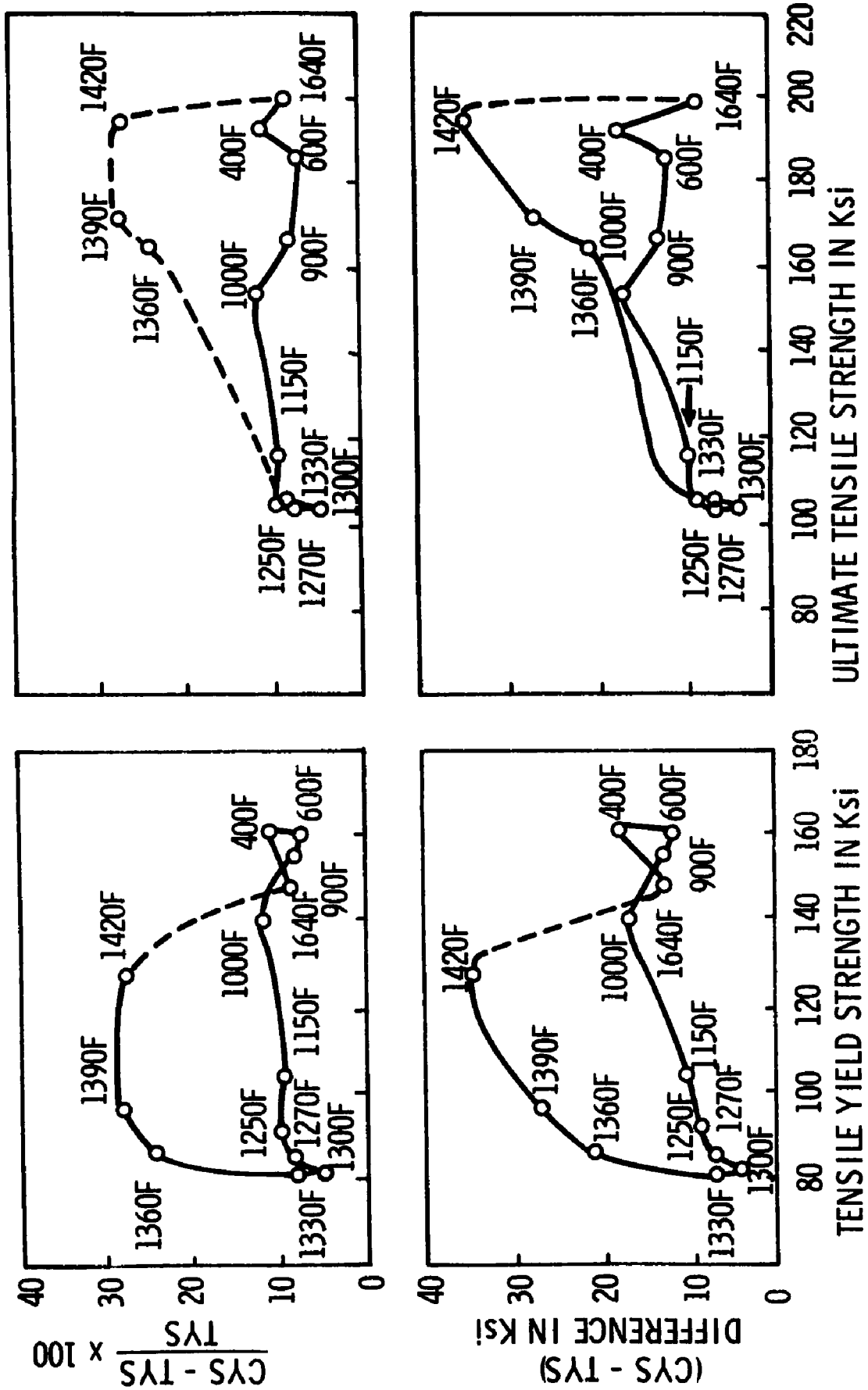


Figure 8b - Effects of Strength on the S-D Effect.

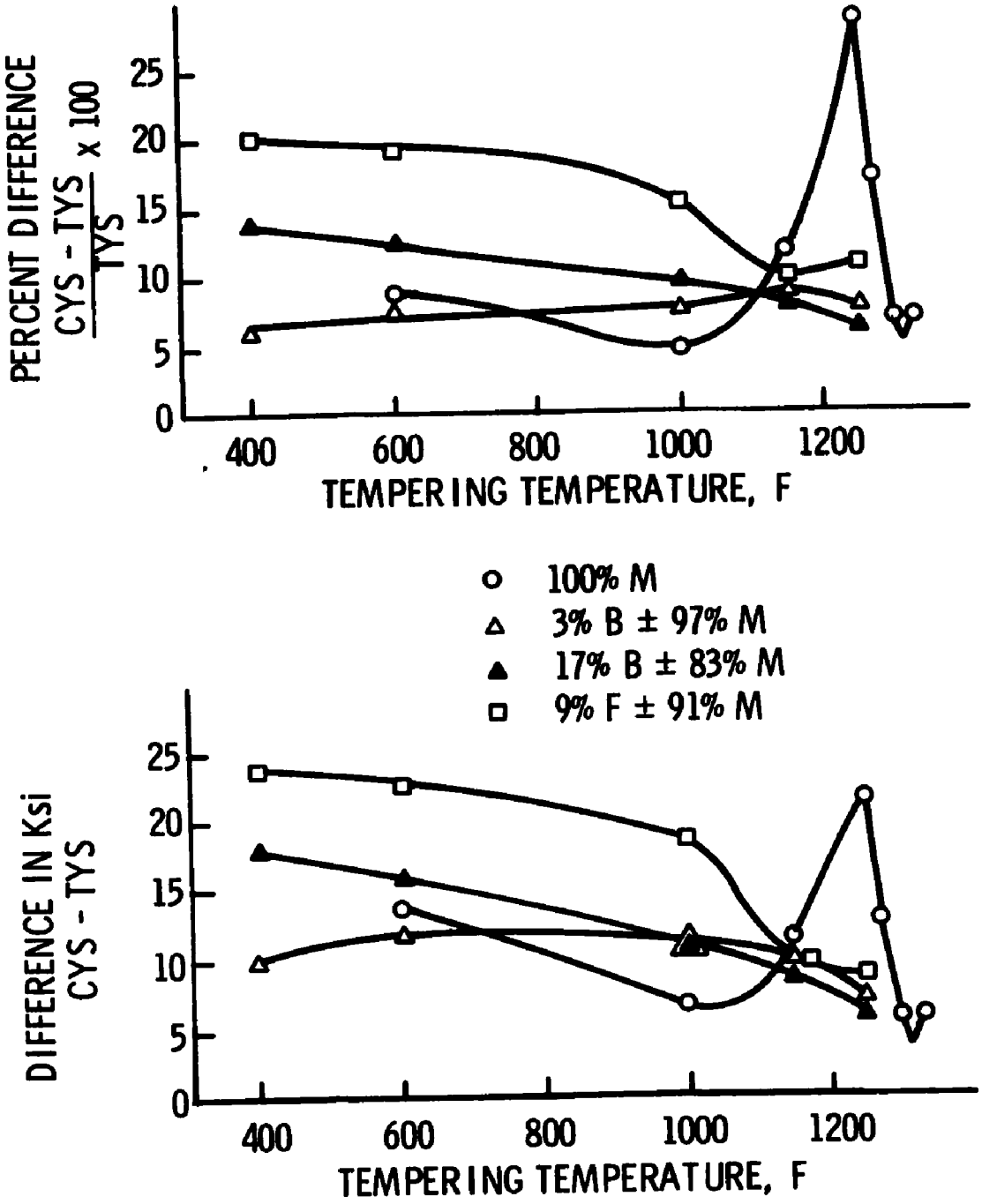


Figure 9a - Effects of Tempering Temperature and Microstructure on the S-D Effect.

Figure 9 - Effects of Tempering Temperature, Microstructure and Strength Level on the S-D Effect Observed in an HY-80 Steel Austenitized at 2000F for One Hour and then Treated to Produce Various Microstructures.

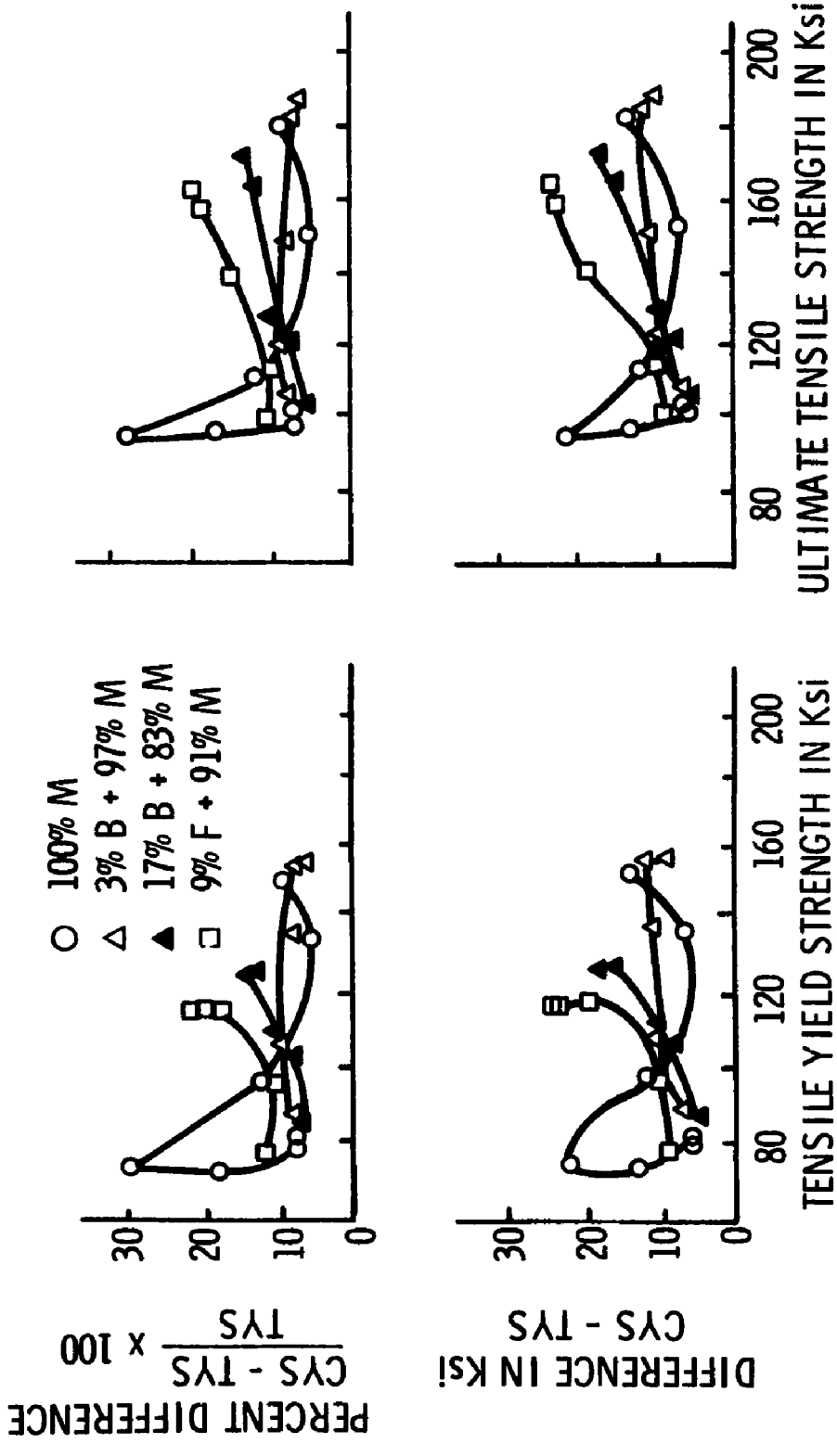


Figure 9b - Effects of Strength Level and Microstructure on the S-D Effect.

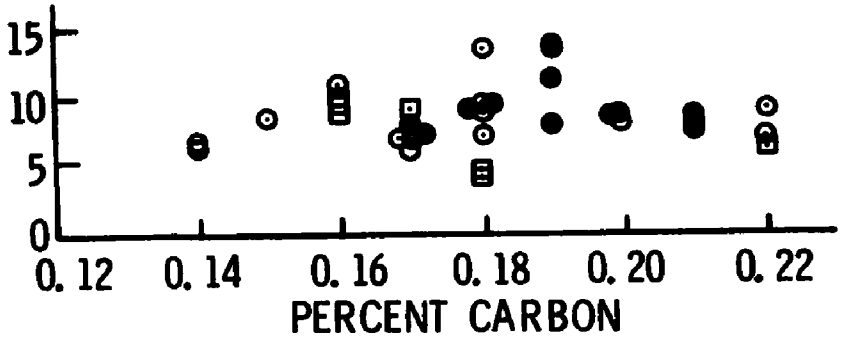


Figure 10a - Effect of Percent Carbon

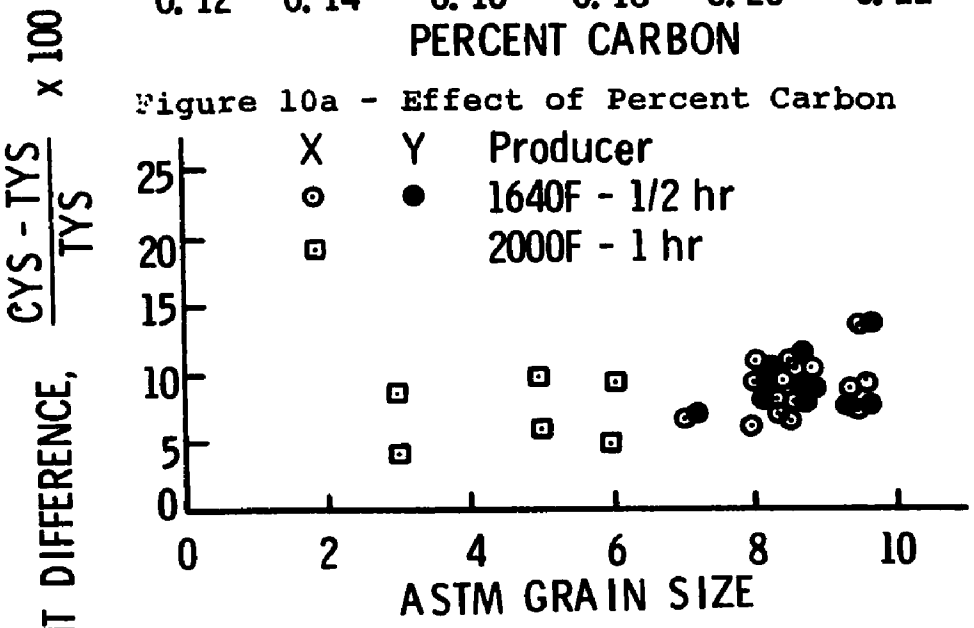


Figure 10b - Effect of Prior Austenitic Grain Size

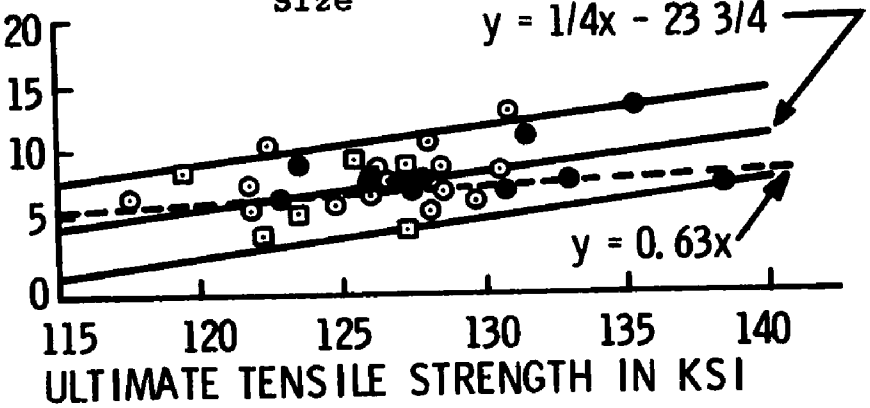


Figure 10c - Effect of the Ultimate Strength

Figure 10 - Effects of Percent Carbon, Prior Austenitic Grain Size and Ultimate Tensile Strength on the S-D Effect Observed in 27 HY-80 Steel Plates Quenched to a Fully Martensitic Microstructure and Tempered at 1150F for One Hour.

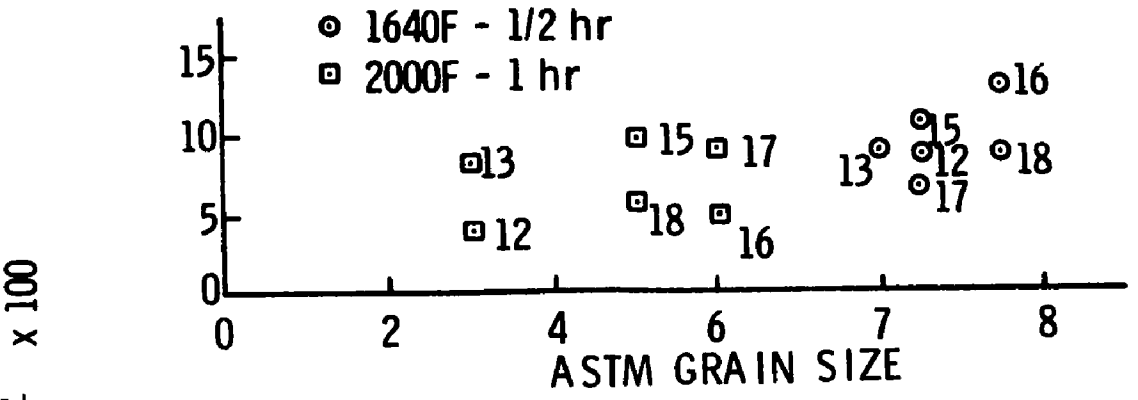


Figure 11a - Effect of Prior Austenitic Grain Size

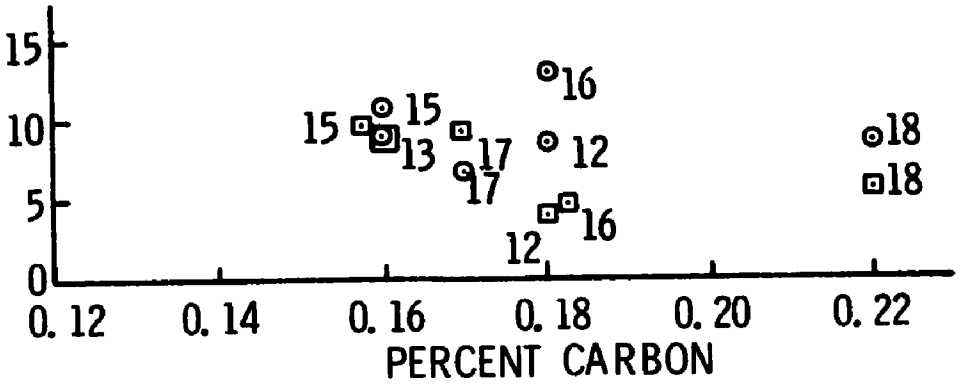


Figure 11b - Effect of Carbon Content

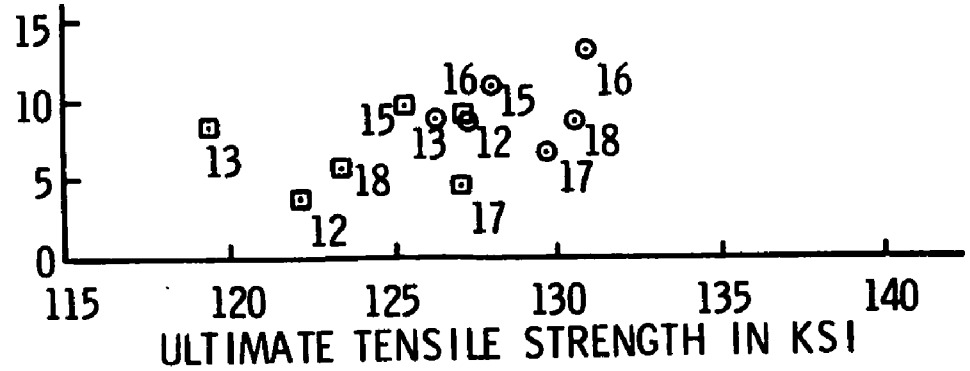


Figure 11c - Effect of Ultimate Tensile Strength

Figure 11 - Effect of Percent Carbon, Prior Austenitic Grain Size and Ultimate Tensile Strength on the S-D Effect in Six HY-80 Steel Plates After Austenitizing at 1640F for ½ Hour or 2000F for One Hour, Quenching to a Fully Martensitic Structure and Tempering at 1150F for One Hour.

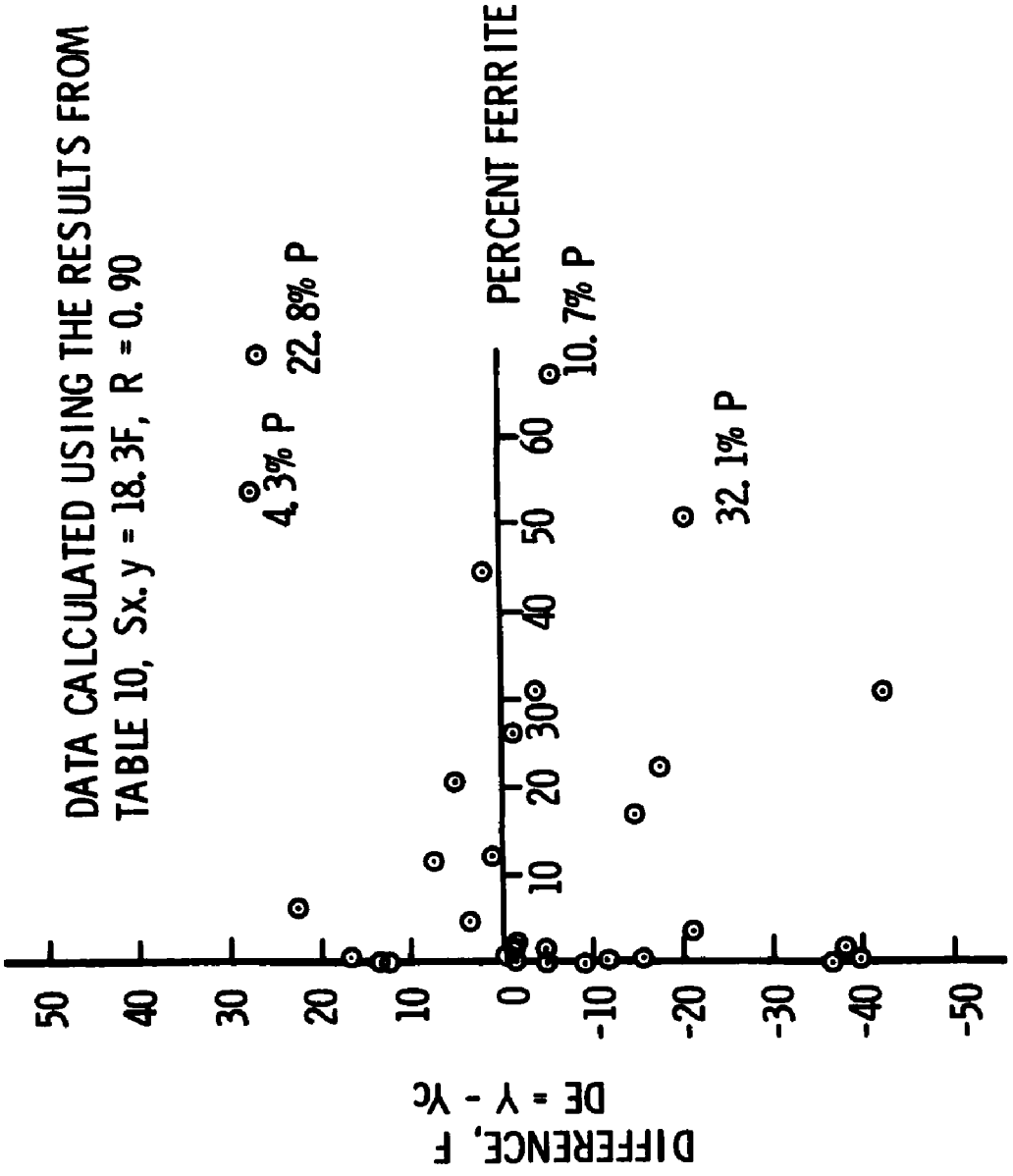


Figure 12 - A Plot of the Difference Between Calculated and Experimental NDT Temperature of Material Containing From A Trace to 69 Percent Ferrite Versus Ferrite Content.

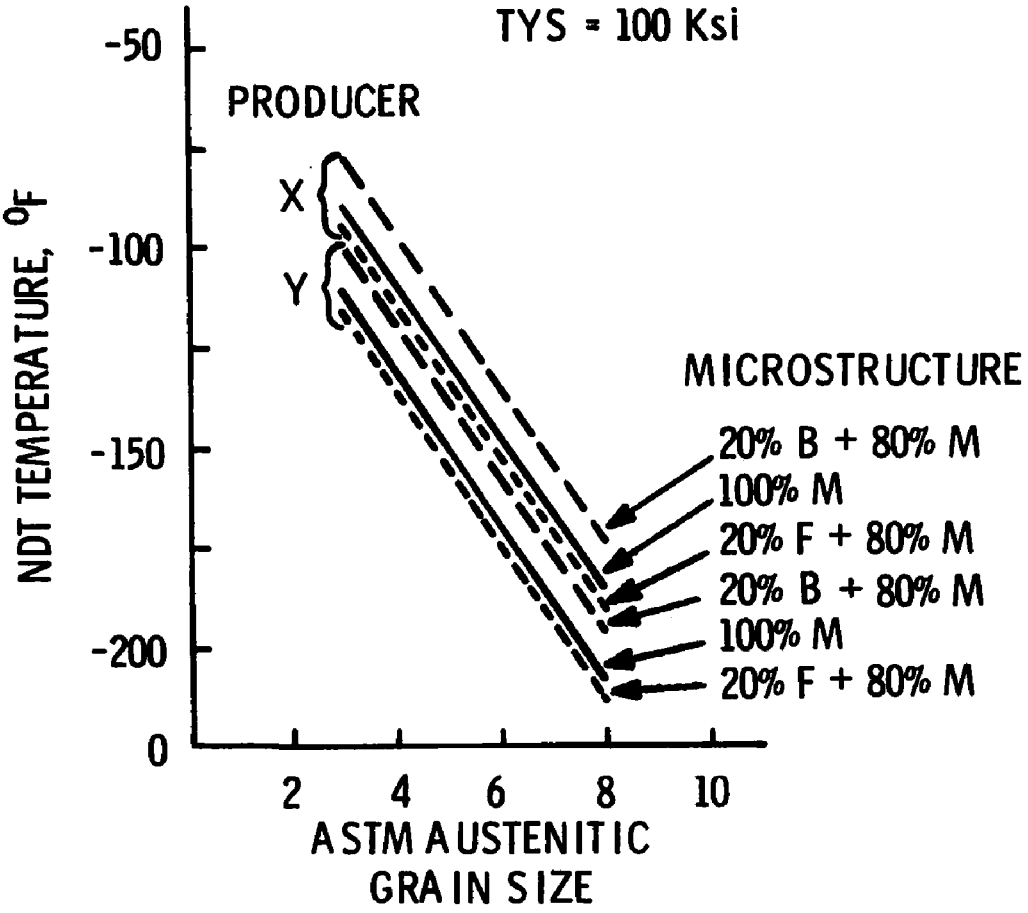


Figure 13a - Functional Relationship Between NDT Temperature and Prior Austenitic Grain Size in Terms of Microconstituents and Producer.

Figure 13 - Plots Showing the Relative Effect of Prior Austenitic Grain Size, Producer, Percent Non-martensitic Isothermal Transformation Product (Bainite, Ferrite and Pearlite), and Tensile Yield Strength on the NDT Temperature of a Low-Carbon Ni-Cr-Mo Alloy Steel Tempered at 1150F.

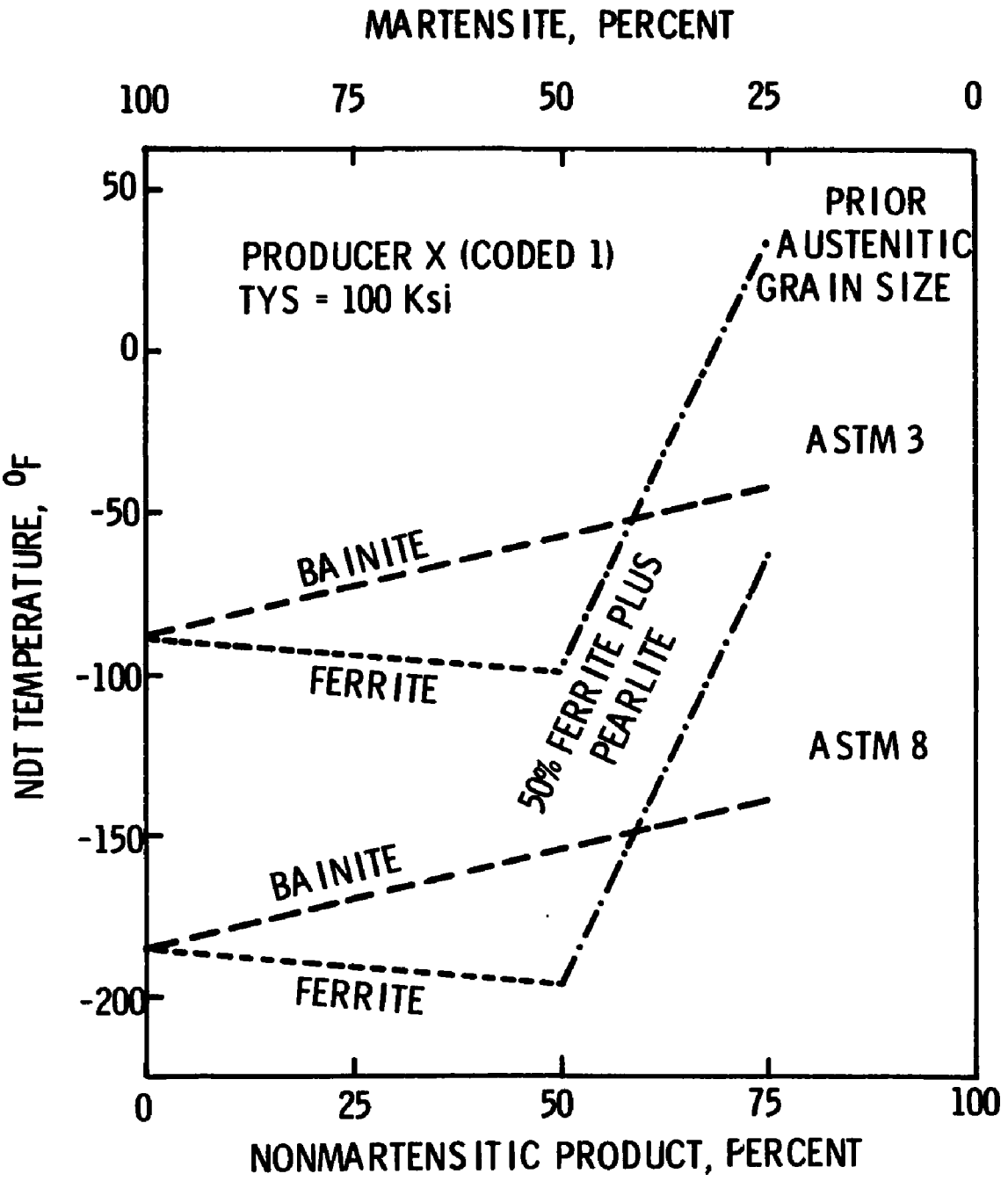


Figure 13b - Functional Relationship Between NDT Temperature and Microconstituents in Terms of Prior Austenitic Grain Size.

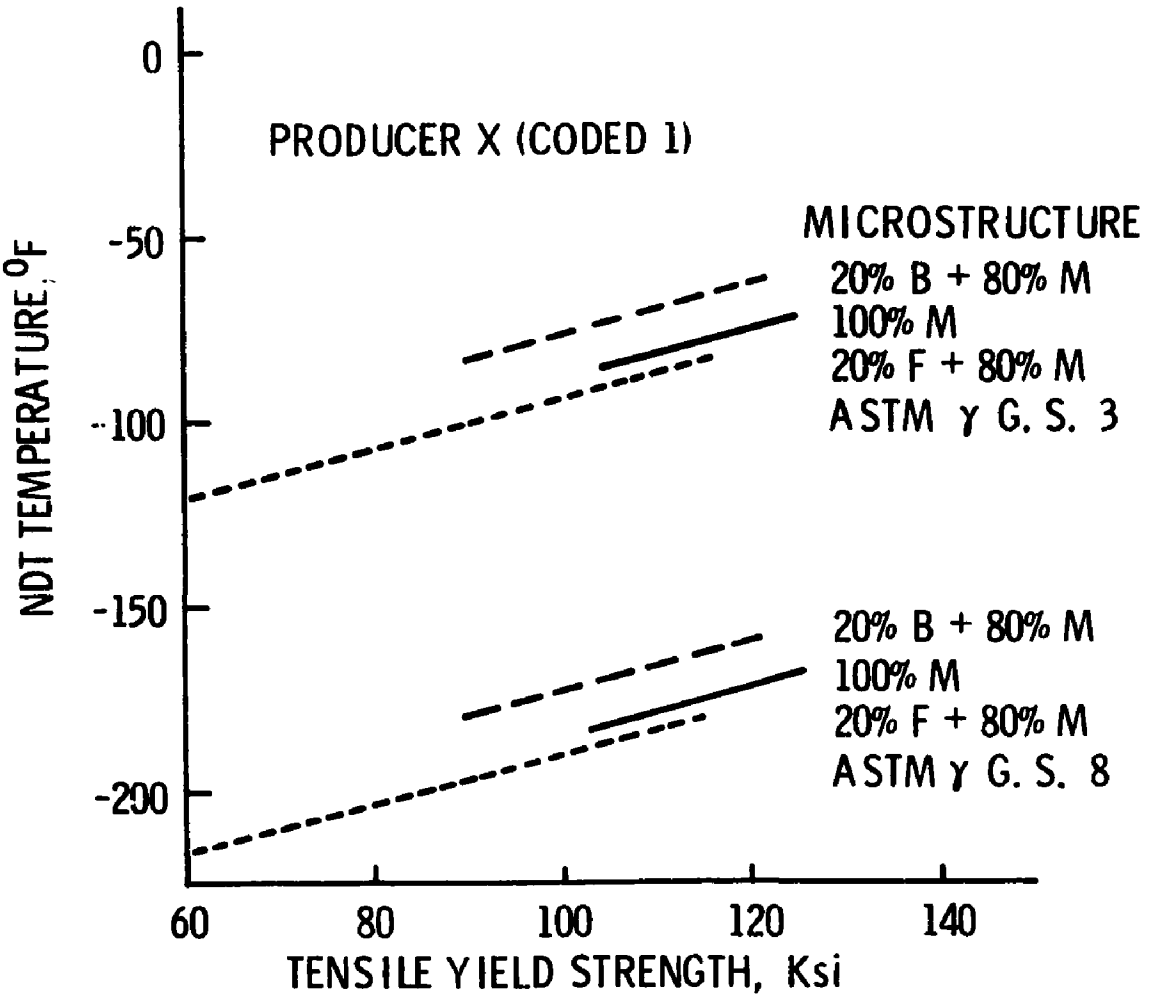


Figure 13c - Functional Relationship Between NDT Temperature and Tensile Yield Strength in Terms of Micro-constituents and Prior Austenitic Grain Size.

IDENT. NO.	HEAT NO.	LOC.	CHEMICAL COMPOSITION, PCT C	MN	P	S	CU	SI	NI	CR	MO	COUPON SIZE TXWXL, IN.	SPEC. DIR.	MECHANICAL PROPERTIES STRENGTH, KSI YIELD	TENSILE	ELONG. IN 2 IN., PCT	RED. IN AREA, PCT	LONGITUDINAL CHARPY V-NOTCH ENERGY, FT-LB
		L*	.16	.26	.011	.020		.23	2.60	1.35	.47							
X- 1	6S5399	F	.16	.25	.014	.020		.22	2.59	1.40	.44	2X21X90	TRANS	88.1	108.0	24.0	63.8	101-108-108
		R	.16	.28	.014	.019		.24	2.61	1.40	.36		LONG.	84.2	102.8	24.0	73.0	90- 89- 92
		L	.14	.30	.013	.016		.24	2.85	1.53	.41							
X- 2	72S686	F	.15	.30	.015	.017		.22	2.83	1.58	.38	6X18X126	TRANS	80.8	100.7	26.0	70.1	
		R	.15	.28	.020	.017		.18	2.80	1.50	.39		LONG.	80.7	100.8	24.0	71.9	96- 96-104
		L	.14	.30	.013	.016		.24	2.85	1.53	.41							
X- 3	72S686	F	.15	.30	.015	.017		.22	2.83	1.58	.38	1.62X18X118	TRANS	88.8	106.2	22.5	68.7	99- 97-97
		R	.15	.28	.020	.017		.18	2.80	1.50	.39		LONG.	86.3	102.0	25.0	75.0	110- 85- 85
		L	.16	.34	.016	.019		.28	2.84	1.66	.46							
X- 4	66S411	F	.19	.34	.016	.019		.27	2.85	1.72	.41	2X21X140	TRANS	93.7	113.7	23.0	57.2	84- 87- 86
		R	.18	.34	.027	.020		.24	2.85	1.74	.40		LONG.	96.3	114.5	25.0	68.8	82- 92-102
		L	.17	.30	.011	.018		.28	2.80	1.69	.45							
X- 5	66S357	F	.16	.32	.015	.019		.24	2.82	1.71	.42	2.13X17X133	TRANS	91.3	112.0	22.0	63.9	110-106-112
		R	.16	.30	.022	.018		.22	2.83	1.71	.42		LONG.	89.1	110.4	24.0	71.5	110-110-124
		L	.16	.32	.013	.015		.27	2.82	1.70	.46							
X- 6	65S799	F	.15	.30	.013	.015		.27	2.80	1.75	.45	3.75X18X76	TRANS	86.0	104.0	22.0	69.1	
		R	.14	.32	.012	.016		.25	2.85	1.75	.44		LONG.	80.1	101.2	22.0	74.3	124-130-120
		L	.15	.25	.013	.017		.23	2.76	1.52	.35							
X- 8	66U040	F	.15	.26	.015	.016		.24	2.76	1.50	.27	2X23X140	TRANS	92.0	107.6	24.0	69.6	113-117-114
		R	.14	.27	.021	.016		.18	2.74	1.38	.32		LONG.	87.5	105.7	24.0	63.9	121-125-113
		L	.14	.29	.013	.016		.28	2.85	1.78	.38							
X- 9	70U032	F	.16	.28	.017	.017		.26	2.88	1.76	.36	1.5X24X100	TRANS	84.0	108.2	23.0	64.0	120-132-133
		R	.14	.28	.012	.017		.25	2.81	1.77	.34		LONG.	80.1	102.1	24.0	75.2	127-117-124
		L	.16	.35	.018	.017		.27	3.00	1.62	.36							
X-10	65U178	F	.16	.35	.012	.014		.24	2.91	1.68	.42	1.75X42X130	TRANS	90.8	111.2	22.0	64.4	123-125-122
		R	.17	.38	.013	.015		.24	2.90	1.58	.36		LONG.	88.2	108.8	22.0	73.2	99- 91- 85
		L	.15	.26	.009	.011		.23	2.95	1.50	.50							
X-11	X51294	F	.16	.28	.009	.012		.23	2.90	1.50	.48	2.13X36X139	TRANS	83.1	102.0	26.0	66.6	85- 82- 89
		R	.17	.28	.010	.012		.23	2.89	1.49	.48		LONG.	80.1	104.9	26.0	71.6	99- 99- 94
		L	.16	.27	.012	.018		.27	3.00	1.58	.40							
X-12	68U369	F	.17	.29	.011	.018		.22	2.99	1.50	.38	1.5X28X96	TRANS	91.6	109.2	23.0	66.8	127-135-131
		R	.16	.29	.017	.017		.18	3.03	1.52	.37		LONG.	90.7	108.5	24.0	72.7	122-125-130
		L	.16	.27	.012	.018		.27	3.00	1.58	.40							
X-13	68U369	F	.17	.29	.011	.018		.22	2.99	1.50	.38	4.5X23X96	TRANS	86.3	110.5	24.0	68.5	
		R	.16	.29	.017	.017		.18	3.03	1.52	.37		LONG.	82.6	103.6	24.0	73.8	115-102-119

TABLE 1 – PRODUCERS' REPORTED CHEMICAL COMPOSITION AND MECHANICAL PROPERTIES

X-18	66U397	L	.19	.27	.010	.017		.25	2.90	1.75	.41	2X20X135	TRANS	85.7	110.2	24.0	59.7	91- 68- 97
		F	.19	.29	.011	.016		.27	2.87	1.73	.39		LONG.	85.6	108.6	22.0	68.9	102-100- 70
			.16	.25	.016	.017		.22	2.87	1.75	.40							
X-15	66U397	L	.19	.27	.010	.017		.25	2.90	1.75	.41	4.5X26X114	TRANS	92.8	110.8	20.0	67.9	94- 96- 97
		F	.19	.29	.011	.016		.27	2.87	1.73	.39		LONG.	89.7	109.1	24.0	62.9	
		R	.16	.25	.016	.017		.22	2.87	1.75	.40							
X-16	N51533	L	.16	.30	.014	.015		.22	2.92	1.63	.53	1.8X74X97	TRANS	91.1	109.3	22.0	63.5	95- 97- 97
		F	.16	.31	.013	.014		.21	2.89	1.65	.51		LONG.	89.8	108.9	24.0	68.3	
		R	.16	.33	.012	.015		.22	2.87	1.65	.52							
X-17	N51533	L	.16	.30	.014	.015		.22	2.92	1.63	.53	6X22X67	TRANS	91.2	108.5	25.0	68.3	86- 60- 65
		F	.16	.31	.013	.014		.21	2.89	1.65	.51		LONG.	88.2	106.8	26.0	71.4	
		R	.16	.33	.012	.015		.22	2.87	1.65	.52							
Y- 1	A2171-4A		.15	.22	.021	.024		.22	2.15	1.28	.27	.69X15X148	TRANS	87.4	104.5	26.0		90- 92- 97
													LONG.	90.5	106.5	26.0		
Y- 2	A2345-4			.26	.010	.017				1.28		2X15X161	TRANS	88.4	107.9	27.0		89- 96- 90
													LONG.	91.9	110.8	26.0		
Y- 3	A2472-1D		.14	.19	.010	.026		.22	2.11	1.10	.26	.75X15X103	TRANS	86.2	103.4	23.0		96- 92- 95
													LONG.	82.8	101.0	25.0		
Y- 4	A2480-4C		.17	.35	.011	.022		.28	2.77	1.55	.37	2.62X15X75	TRANS	84.8	106.9	25.0		64- 71- 73
													LONG.	86.6	107.4	26.0		
Y- 5	A2848-2		.17	.31	.010	.023	.19	.26	2.81	1.49	.39	2.25X15X155	TRANS	88.5	109.0	24.0		87- 88- 86
													LONG.	87.1	107.9	26.0		
Y- 6	20730-5		.17	.32	.010	.024	.17	.25	2.75	1.54	.42	2X15X121	TRANS	90.0	109.5	24.0		89- 90- 88
													LONG.	88.9	107.6	25.0		
Y- 7	19540-6		.16	.35	.010	.025	.16	.30	2.74	1.67	.40	1.5X15X96	TRANS	85.1	104.1	25.0		86- 90- 90
													LONG.	82.1	102.5	26.0		
Y- 8	19540-1		.16	.35	.010	.025	.16	.30	2.74	1.67	.40	2X15X161	TRANS	85.1	105.0	25.0		83- 81- 83
													LONG.	90.6	108.9	26.0		
Y- 9	20844-2		.16	.29	.010	.025	.15	.25	2.76	1.47	.43	2X8.5X109	TRANS	88.6	108.4	25.0		109-109-108
													LONG.	88.4	108.1	28.0		
Y-10	21222-2		.16	.34	.013	.023	.23	.27	2.80	1.53	.42	2.5X15X168	TRANS	85.1	105.0	25.0		78- 72- 75
													LONG.	90.6	108.9	26.0		
Y-11	22616-1		.15	.25	.010	.025	.17	.20	2.85	1.42	.23	3.62X15X158	TRANS	81.6	103.5	25.0		45- 55- 53
													LONG.	80.1	99.0	27.0		

* L IS LADLE, F IS FRONT AND R IS REAR

TABLE 1 – CONTINUED

CHEMISTRY DATA

SPEC. NO.	AMT. C	AMT. MN	AMT. P	AMT. S	AMT. SI	AMT. NI	AMT. CR	AMT. MO	AMT. CU	ACID SOL AL	TOTAL AL	AMT. TI	AMT. V	AMT. CO	AMT. O(2)	AMT. N(2)	AMT H(2) PPM	D(I) SOM	GRAIN SIZE
X- 1	0.15	.280	.015	.018	0.210	2.590	1.450	0.360	0.076	.0280	.04C0	.0040	.0010	.	.0110	.0050	1	5.78	8.50
X- 2	0.16	.380	.016	.021	0.30C	2.750	1.470	0.350	0.048	.0320	.C360	.0010	.0020	.	.0050	.0060	0	7.83	8.00
X- 3	0.14	.39C	.023	.020	0.270	2.760	1.470	0.350	0.049	.0230	.C260	.0010	.0020	.	.0150	.0030	1	7.95	7.00
X- 4	0.18	.360	.017	.018	0.260	2.930	1.800	0.310	0.075	.0240	.0420	.0050	.0010	.	.0490	.0080	7	9.33	9.50
X- 5	0.17	.300	.019	.017	0.230	2.780	1.740	0.320	0.044	.0230	.0470	.0050	.	.	.0480	.0060	3	8.96	8.50
X- 6	0.14	.290	.016	.011	0.210	2.710	1.730	0.290	0.040	.0230	.C330	.0040	.	.	.0200	.0070	1	5.71	8.00
X- 8	0.20	.370	.014	.023	0.320	2.750	1.400	0.300	0.047	.0410	.0460	.0010	.0020	.	.0150	.0090	1	8.41	8.50
X- 9	0.17	.420	.020	.015	0.300	2.810	1.620	0.310	0.048	.0320	.0360	.0010	.0030	.	.0060	.0080	0	8.85	8.50
X-10	0.22	.440	.015	.017	0.300	2.780	1.460	0.310	0.041	.0470	.0530	.0010	.0020	.	.0060	.0080	0	13.83	9.50
X-11	0.18	.39C	.013	.015	0.310	2.890	1.420	0.470	0.074	.0400	.0480	.0020	.0030	.	.0130	.0110	0	10.27	8.50
X-12	0.18	.360	.021	.018	0.360	2.930	1.450	0.350	0.069	.0260	.0300	.0020	.0050	.0210	.0130	.0050	0	7.62	8.50
X-13	0.16	.34C	.020	.017	0.320	2.850	1.420	0.350	0.054	.0360	.0410	.0010	.0040	.0190	.0110	.0060	0	7.58	8.00
X-18	0.22	.360	.017	.018	0.360	2.750	1.600	0.370	0.049	.0350	.0410	.0010	.0020	.0100	.0120	.0060	1	8.00	9.50
X-15	0.16	.37C	.019	.018	0.300	2.700	1.610	0.350	0.060	.0360	.0410	.0010	.0040	.0110	.0100	.0050	0	7.53	8.50
X-16	0.18	.4C0	.017	.018	0.300	2.830	1.550	0.500	0.088	.0580	.0650	.0020	.0060	.0100	.0100	.0070	0	12.10	9.50
X-17	0.17	.390	.019	.022	0.310	2.780	1.540	0.500	0.101	.0640	.071C	.0020	.0060	.0100	.0080	.0070	1	10.50	8.50

SPEC. NO.	AMT. C	AMT. MN	AMT. P	AMT. S	AMT. SI	AMT. NI	AMT. CR	AMT. MO	AMT. CU	ACID SOL AL	TOTAL AL	AMT. TI	AMT. V	AMT. CO	AMT. O(2)	AMT. N(2)	AMT H(2) PPM	D(I) SOM	GRAIN SIZE
Y- 1	0.21	.320	.013	.023	0.310	2.150	1.200	0.270	0.160	.0220	.0250	.0010	.0020	.	.0040	.0090	2	4.68	8.50
Y- 2	0.19	.260	.008	.017	0.260	2.690	1.560	0.420	0.170	.0060	.0280	.0040	.	.	.0090	.0090	1	7.17	9.50
Y- 3	0.18	.29C	.015	.021	0.320	2.100	1.100	0.270	0.170	.0320	.0360	.0010	.0010	.	.0160	.0090	1	3.87	8.50
Y- 4	0.21	.370	.010	.017	0.290	2.910	1.580	0.510	0.190	.0030	.0250	.0050	.	.	.0240	.0090	12	13.29	9.50
Y- 5	0.19	.32C	.007	.019	0.280	2.890	1.520	0.340	0.260	.0150	.0360	.0040	.	.	.0310	.0100	1	8.14	8.50
Y- 6	0.18	.34C	.011	.021	0.240	2.770	1.570	0.350	0.250	.0270	.C370	.0040	.	.	.0240	.0070	7	8.61	9.50
Y- 7	0.20	.380	.009	.025	0.300	2.750	1.670	0.550	0.190	.0270	.0470	.0050	.0010	.	.0250	.0090	1	13.47	8.50
Y- 8	0.17	.490	.015	.028	0.330	2.650	1.520	0.370	0.150	.0380	.0430	.0020	.0030	.	.0040	.0090	1	8.28	8.50
Y- 9	0.19	.420	.015	.018	0.300	2.730	1.400	0.400	0.150	.0320	.0360	.0010	.0020	.	.0040	.0070	0	10.02	8.50
Y-10	0.20	.490	.021	.018	0.330	2.700	1.450	0.380	0.190	.0380	.0420	.0110	.0020	.	.0110	.0070	2	9.69	8.50
Y-11	0.17	.380	.020	.022	0.290	2.790	1.400	0.390	0.160	.0100	.0130	.0010	.0030	.	.0080	.0020	0	9.89	7.00

TABLE 2 – ACTUAL CHEMICAL COMPOSITION OF COUPONS

INCLUSION RATING AT THE SURFACE AND MID-THICKNESS
(ASTM E45-63, METHOD A)

SPECIMEN NUMBER	MID-THICKNESS LONGITUDINAL INCLUSIONS LONGEST	MID-THICKNESS LONGITUDINAL INCLUSIONS OTHERS	MID-THICKNESS LONGITUDINAL BACKGROUND	MID-THICKNESS TRANSVERSE INCLUSIONS LONGEST	MID-THICKNESS TRANSVERSE INCLUSIONS OTHERS	MID-THICKNESS TRANSVERSE BACKGROUND	BOTTOM SURFACE LONGITUDINAL INCLUSIONS LONGEST	BOTTOM SURFACE LONGITUDINAL INCLUSIONS OTHERS	BOTTOM SURFACE LONGITUDINAL BACKGROUND	BOTTOM SURFACE TRANSVERSE INCLUSIONS LONGEST	BOTTOM SURFACE TRANSVERSE INCLUSIONS OTHERS	BOTTOM SURFACE TRANSVERSE BACKGROUND
X- 1	3.5[B]	1.2[2]	A	1.0[wd]	1.2[1]	A	3.0[wd]	1.6[3]	B	3.7[d]	1.1[4]	A
X- 2	3.5[wd]	.0	D	2.5[d]	1.0[1]	C	1.5[wd]	1.4[1]	A	2.0[d]	1.0[1]	B
X- 3	5.0[d]	2.1[5]	A	3.5[wd]	1.5[1]	A	3.5[B]	1.5[4]	C	5.7[wd]	1.7[1]	B
X- 4	3.0[d]	1.8[2]	A	1.5[d]	1.3[3]	C	1.9[wd]	1.2[3]	B	2.0[wd]	.0	B
X- 5	8.0[wd]	1.6[2]	A	1.3[wd]	.0	B	1.0[d]	.0	A	6.3[d]	3.6[2]	C
X- 6	6.1[B]	1.0[2]	A	6.0[B]	1.0[6]	B	2.8[d]	2.0[1]	A	1.5[wd]	1.0[1]	A
X- 8	3.5[wd]	2.1[7]	B	2.7[d]	1.5[4]	A	3.5[d]	1.7	D	4.2[wd]	1.5[3]	D
X- 9	2.5[B]	1.3[7]	B	3.5[B]	1.0[2]	D	3.7[B]	1.7[4]	C	.6[d]	.0	A
X-10	2.3[d]	1.7[6]	D	2.0[d]	1.5[3]	B	1.5[d]	1.1[5]	C	.7[wd]	.0	A
X-11	2.8[d]	1.4[13]	C	5.0[wd]	1.5[9]	C	1.6[d]	1.3[10]	D	1.3[d]	.0	B
X-12	2.5[wd]	1.6[13]	C	1.0[wd]	.0	A	4.3[wd]	1.6[21]	C	2.7[wd]	1.1	B
X-13	1.7[wd]	1.4[11]	B	1.3[d]	1.1[2]	A	1.1[d]	1.0[1]	D	2.2[B]	1.2[2]	A
X-14	4.3[wd]	2.0[24]	C	1.9[B]	1.0[3]	C	3.2[wd]	1.6[16]	C	1.2[d]	.0	A
X-15	2.1[d]	1.2[8]	B	1.5[B]	.0	B	2.7[wd]	1.3[6]	C	3.2[d]	1.4[1]	B
X-16	3.6[d]	2.1[63]	C	2.2[wd]	1.5[3]	B	6.0[d]	2.1[29]	C	2.6[B]	1.2[3]	D
X-17	4.3[B]	1.6[5]	B	1.2[B]	1.1[3]	A	2.1[d]	1.3[6]	A	.6[d]	.0	A
Y- 1	2.9[wd]	2.1[5]	C	7.4[d]	2.7[8]	D	3.0[wd]	.0	A	6.3[wd]	.0	D
Y- 2	1.2[d]	.0	A	4.2[d]	1.1[1]	A	1.9[d]	1.2[5]	A	2.6[B]	1.5[1]	A
Y- 3	4.1[wd]	1.5[7]	A	7.0[d]	.0	A	4.2[wd]	2.6[2]	A	1.5[d]	.0	A
Y- 4	2.4[wd]	1.2[2]	A	1.5[B]	1.3[1]	B	4.0[d]	1.0[1]	A	5.4[d]	1.3[3]	A
Y- 5	4.2[d]	1.3[1]	A	5.5[wd]	2.2[4]	C	4.0[wd]	1.3[4]	A	2.5[wd]	1.3[2]	C
Y- 6	2.5[d]	1.1[3]	B	3.2[B]	1.4[4]	C	2.0[wd]	1.2[4]	B	1.1[wd]	1.1[1]	B
Y- 7	3.0[wd]	1.6[13]	C	1.0[d]	.0	A	2.1[d]	1.2[1]	B	1.8	1.1[2]	A
Y- 8	2.0[d]	1.0[1]	B	1.2[d]	1.1[5]	A	1.9[d]	.0	A	2.0[d]	1.0[1]	D
Y- 9	2.1[d]	1.4[4]	A	1.2[d]	1.1[1]	C	5.0[B]	2.9[3]	A	2.0[wd]	.0	B
Y-10	3.0[B]	1.6[16]	C	3.0[d]	1.3[8]	B	1.5[d]	1.3[1]	C	3.1[d]	1.7[2]	A
Y-11	2.5[d]	1.7[4]	A	3.0[wd]	1.0[1]	A	2.5[d]	.0	B	.7[d]	.0	A

TABLE 3 – INCLUSION CONTENT

SPECIMEN NUMBER	AS RECEIVED MATERIAL MID-THICKNESS	AUSTENITIZING TREATMENT * 1640 F-1/2 HR-W.Q.	2000 F-1 HR-W.Q.
X- 1	6	8.5	
X- 2	6	8	
X- 3	5	7	
X- 4	6	9.5	
X- 5	4	8.5	
X- 6	7	8.	
X- 8	5	8.5	
X- 9	6	8.5	
X-10	4	9.5	
X-11	6	8.5	
X-12	8	8.5	3
X-13	6	8	3
X-18	5	9.5	5
X-15	7	8.5	5
X-16	5	9.5	6
X-17	6	8.5	6
Y- 1	7	8.5	
Y- 2	7	9.5	
Y- 3	7	8.5	
Y- 4	5	9.5	
Y- 5	6	8.5	
Y- 6	5	9.5	
Y- 7	5	8.5	
Y- 8	7	8.5	
Y- 9	5	8.5	
Y-10	7	8.5	
Y-11	4	7	

* AFTER AUSTENITIZING AND QUENCHING, -120 F 1 HR, TEMPERED AT 1150 F 1 HR THEN WATER QUENCHED.

TABLE 4 – PRIOR AUSTENITIC GRAIN SIZE

AUSTENITIZING							
TEMPERATURE	1640 F	1640 F	1640 F		1640 F		2000 F
TIME	1/2 HR	1/2 HR	1/2 HR		1/2 HR		1 HR
ISOTHERMAL							
TEMPERATURE	875 F	875 F	1200 F		1200 F		875 F
TIME	152 SEC	1600 SEC	3350 SEC		8500 SEC		1600 SEC
MICRO-CONSTITUENTS	BAINITE	BAINITE	FERRITE	PEARLITE	FERRITE	PEARLITE	BAINITE
COUPON NUMBER							
X- 1	29	54	31	0	67	11	
X- 2	14	54	TRACE	0	2	0	
X- 3	21		31	0			
X- 4	2		11	0			
X- 5	1		0	0			
X- 6	0		1	0			
X- 8	14		5	0			
X- 9	7		26	0			
X-10	1		TRACE	0			
X-11	0		0	0			
X-12	1	50	TRACE	0	1	0	63
X-13	14	60	TRACE	0	TRACE	0	67
X-14	TRACE	21	0	0	0	0	56
X-15	1	27	0	0	0	0	58
X-16	0	16	TRACE	0	TRACE	0	58
X-17	0	32	0	0	TRACE	0	30
Y- 1	53		50	32			
Y- 2	9	41	22	0	53	4	
Y- 3	76		69	23			
Y- 4	1	39	7	0	21	0	
Y- 5	3		17	0			
Y- 6	1		12	0			
Y- 7	1		3	0			
Y- 8	1		1	0			
Y- 9	6		2	0			
Y-10	4		1	0			
Y-11	27	63	3	0	44	0	

TABLE 5 – PERCENT MICROCONSTITUENTS PRODUCED BY VARIOUS ISOTHERMAL TREATMENTS

TABLE 6 – RESULTS OF MECHANICAL PROPERTY TESTS

TRANSVERSE

SPEC. NO.	PCT. BAINITE	PCT. FERRITE	PCT. PEARLITE	MARTEN-SITE	ISO-THERMAL HOLDING TEMP. (F)	ISO-THERMAL HOLDING TIME (SEC)	ULT. TEN. STH. (PSI)	TEN. YIELD STH. (PSI)	PCT. EL. IN 2 IN.	PCT. RED. IN AREA	COMP. YIELD STH. (PSI)	NDT TEMP (F)	CHARPY V-NOTCH IMPACT PROPERTIES NDT ENERGY (FT-LBS)	NDT PCT. FIBER	-120F ENERGY (FT-LBS)	-120F PCT. FIBER	+210F ENERGY (FT-LBS)	+210F PCT. FIBER
X- 1	0.	0.	0.	100.0	0	0	125907	113820	22.0	73.9	123334	-190	31.0	24.5	49.5	100.0	66.0	100.0
X- 2	0.	0.	0.	100.0	0	0	122392	107343	24.0	73.5	118860	-170	48.0	37.0	62.0	72.0	90.0	100.0
X- 3	0.	0.	0.	100.0	0	0	124850	114128	22.0	72.7	121686	-120	39.0	43.5	39.0	43.5	61.0	100.0
X- 4	0.	0.	0.	100.0	0	0	128526	113023	23.0	72.6	123578	-190	28.0	57.8	46.0	92.8	56.0	100.0
X- 5	0.	0.	0.	100.0	0	0	128115	113043	22.0	72.5	119830	-190	30.0	57.8	69.2	92.8	69.0	100.0
X- 6	0.	0.	0.	100.0	0	0	121792	107759	23.0	74.5	114383	-170	38.0	57.8	64.0	92.8	73.0	100.0
X- 8	0.	0.	0.	100.0	0	0	121756	109280	23.0	74.7	118068	-190	37.0	34.0	62.5	93.5	77.0	100.0
X- 9	0.	0.	0.	100.0	0	0	117448	104138	23.0	76.5	111430	-190	35.5	35.0	60.0	77.0	79.0	100.0
X-10	0.	0.	0.	100.0	0	0	125926	111361	23.0	74.2	119220	-190	44.0	59.0	68.5	100.0	74.0	100.0
X-11	0.	0.	0.	100.0	0	0	128527	118118	22.0	69.6	126668	-190	30.5	35.0	45.5	82.5	61.5	100.0
X-12	0.	0.	0.	100.0	0	0	127346	114571	22.5	74.7	124813	-180	42.5	41.5	58.0	85.5	66.5	100.0
X-13	0.	0.	0.	100.0	0	0	126275	112988	23.0	72.7	123228	-180	48.0	38.5	83.0	99.0	93.0	100.0
X-18	0.	0.	0.	100.0	0	0	130601	116000	23.0	72.8	126265	-190	44.0	71.0	60.5	100.0	62.5	100.0
X-15	0.	0.	0.	100.0	0	0	128074	112289	23.0	73.4	124506	-190	44.0	47.0	70.0	94.0	74.5	100.0
X-16	0.	0.	0.	100.0	0	0	130870	116883	23.0	74.5	132264	-210	40.0	36.5	56.0	100.0	62.0	100.0
X-17	0.	0.	0.	100.0	0	0	129743	116617	22.0	71.9	124533	-180	47.0	71.0	63.0	100.0	69.0	100.0
Y- 1	0.	0.	0.	100.0	0	0	125874	113686	21.0	70.2	122785	-210	34.0	39.0	50.5	86.5	68.5	100.0
Y- 2	0.	0.	0.	100.0	0	0	135325	122294	22.0	70.3	138652	-190	28.5	33.0	44.0	90.0	58.0	100.0
Y- 3	0.	0.	0.	100.0	0	0	123600	114600	22.0	72.0	125312	-230	32.5	70.5	53.5	97.0	61.0	100.0
Y- 4	0.	0.	0.	100.0	0	0	138777	124698	22.0	68.7	134012	-210	27.5	26.0	42.0	86.5	48.0	100.0
Y- 5	0.	0.	0.	100.0	0	0	131421	115871	22.0	69.1	129079	-230	27.0	57.8	46.0	92.8	52.0	100.0
Y- 6	0.	0.	0.	100.0	0	0	130814	115277	23.0	71.7	123819	-190	31.0	57.8	51.0	92.8	56.0	100.0
Y- 7	0.	0.	0.	100.0	0	0	133000	117150	21.0	66.5	126756	-220	27.0	31.0	41.0	99.0	42.5	100.0
Y- 8	0.	0.	0.	100.0	0	0	127600	112100	22.0	71.8	120685	-210	26.5	51.5	34.5	99.5	38.5	100.0
Y- 9	0.	0.	0.	100.0	0	0	126527	111611	22.0	74.2	120598	-190	35.5	35.5	55.0	100.0	62.5	100.0
Y-10	0.	0.	0.	100.0	0	0	128001	112000	23.0	71.1	121602	-230	26.5	12.0	54.0	100.0	58.5	100.0
Y-11	0.	0.	0.	100.0	0	0	122955	109532	22.5	71.3	117168	-150	35.0	56.0	40.5	72.5	64.0	100.0

LONGITUDINAL

SPEC. NO.	PCT. BAINITE	PCT. FERRITE	PCT. PEARLITE	MARTEN-SITE	ISO-THERMAL HOLDING TEMP. (F)	ISO-THERMAL HOLDING TIME (SEC)	ULT. TEN. STH. (PSI)	TEN. YIELD STH. (PSI)	PCT. EL. IN 2 IN.	PCT. RED. IN AREA	COMP. YIELD STH. (PSI)	NDT TEMP (F)	CHARPY V-NOTCH IMPACT PROPERTIES NDT ENERGY (FT-LBS)	NDT PCT. FIBER	-120F ENERGY (FT-LBS)	-120F PCT. FIBER	+210F ENERGY (FT-LBS)	+210F PCT. FIBER
X- 3	0.	0.	0.	100.0	0	0	124850	114128	22.0	72.7	121686	-120	102.0	100.0	102.0	100.0	103.5	100.0
X-12	0.	0.	0.	100.0	0	0	127346	114571	22.5	74.7	124813	-180	94.0	92.5	101.0	99.5	114.0	100.0
X-13	0.	0.	0.	100.0	0	0	126275	112988	23.0	72.7	123228	-180	83.0	91.5	100.0	100.0	103.0	100.0
X-18	0.	0.	0.	100.0	0	0	130601	116000	23.0	72.8	126265	-190	84.5	100.0	93.0	100.0	103.0	100.0
X-15	0.	0.	0.	100.0	0	0	128074	112289	23.0	73.4	124506	-190	64.0	75.5	87.0	100.0	100.0	100.0
X-16	0.	0.	0.	100.0	0	0	130870	116883	23.0	74.5	132264	-210	70.5	72.0	97.5	100.0	102.5	100.0
X-17	0.	0.	0.	100.0	0	0	129743	116617	22.0	71.9	124533	-180	56.0	56.0	75.0	98.0	87.0	100.0
Y- 1	0.	0.	0.	100.0	0	0	125874	113686	21.0	70.2	122785	-210	77.0	100.0	82.0	100.0	92.5	100.0
Y- 3	0.	0.	0.	100.0	0	0	123600	114600	22.0	72.0	125312	-230	67.5	49.5	104.5	100.0	107.0	100.0
Y- 7	0.	0.	0.	100.0	0	0	133000	117150	21.0	66.5	126756	-220	37.0	48.5	60.0	99.0	68.5	100.0

TABLE 6A – SPECIMENS AUSTENITIZED AT 1640F FOR ½ HOUR, WATER QUENCHED AND TEMPERED AT 1150F FOR ONE HOUR

TRANSVERSE

SPEC. NO.	PCT. BAINITE	PCT. FERRITE	PCT. PEARLITE	MARTEN-SITE	ISO-THERMAL HOLDING TEMP. (F)	ISO-THERMAL HOLDING TIME (SEC)	ULT. TEN. STH. (PSI)	TEN. YIELD STH. (PSI)	PCT. EL. IN 2 IN.	PCT. RED. IN AREA	COMP. YIELD STH. (PSI)	NDT TEMP (F)	CHARPY V-NOTCH IMPACT PROPERTIES NDT ENERGY (FT-LBS)	NDT PCT. FIBER	-120F ENERGY (FT-LBS)	-120F PCT. FIBER	+210F ENERGY (FT-LBS)	+210F PCT. FIBER
X- 1	29.0	0.	0.	71.0	875	152	120984	105422	24.0	75.3	111571	-210	30.0	16.5	42.0	69.0	66.0	100.0
X- 2	14.2	0.	0.	85.8	875	152	119382	103331	24.0	4.3	111926	-150	54.0	50.5	63.5	68.0	90.0	100.0
X- 3	21.2	0.	0.	78.8	875	152	118600	105099	22.0	73.1	111957	-110	38.0	23.5	36.0	19.0	68.0	100.0
X- 4	1.5	0.	0.	98.5	875	152	127008	109036	23.0	72.9	118620	-191	26.0	36.3	46.0	80.6	52.0	100.0
X- 5	0.6	0.	0.	99.4	875	152	127204	109175	23.0	72.9	119614	-191	30.0	36.3	55.0	80.6	68.0	100.0
X- 6	0.3	0.	0.	99.7	875	152	121291	105253	24.0	75.8	115366	-191	50.0	36.3	51.0	80.6	81.0	100.0
X- 8	13.5	0.	0.	86.5	875	152	116168	101897	23.0	73.9	110132	-190	37.5	34.5	64.0	81.0	81.0	100.0
X- 9	6.8	0.	0.	93.2	875	152	118165	104594	23.0	76.1	113073	-180	41.0	37.5	60.0	76.0	81.5	100.0
X-10	1.1	0.	0.	98.9	875	152	125459	116784	23.0	76.0	118691	-210	41.0	45.0	66.0	94.5	74.0	100.0
X-11	0.	0.	0.	100.0	875	152	126658	113550	23.0	72.0	124016	-190	36.0	41.0	47.1	75.0	65.0	100.0
X-12	1.1	0.	0.	98.9	875	152	120801	105399	23.0	75.1	115540	-170	49.0	43.5	67.0	86.0	78.5	100.0
X-13	14.1	0.	0.	85.9	875	152	118327	102789	23.0	73.7	109078	-170	52.0	50.0	70.5	81.5	95.5	100.0
X-18	0.7	0.	0.	99.3	875	152	124799	107999	23.0	74.7	120065	-220	32.5	26.0	54.0	86.5	69.0	100.0
X-15	0.7	0.	0.	99.3	875	152	123355	106389	23.0	73.1	115127	-210	33.0	34.0	66.0	89.0	79.0	100.0
X-16	0.	0.	0.	100.0	875	152	123755	109473	23.0	74.3	119487	-210	35.0	33.0	59.0	91.0	64.0	100.0
X-17	0.	0.	0.	100.0	875	152	123307	108168	23.0	72.9	120074	-190	39.0	43.0	55.0	88.0	65.5	100.0
Y- 1	53.0	0.	0.	47.0	875	152	114885	98902	23.0	70.9	106646	-170	33.0	57.0	44.5	96.5	65.0	100.0
Y- 2	9.0	0.	0.	91.0	875	152	130814	116780	23.0	72.1	125320	-240	25.5	18.5	52.5	87.0	60.0	100.0
Y- 3	76.0	0.	0.	24.0	875	152	107480	92724	24.0	73.1	103014	-170	36.0	34.5	52.5	68.0	69.0	100.0
Y- 4	0.7	0.	0.	99.3	875	152	134932	119382	22.0	69.9	132787	-210	28.0	41.0	45.0	100.0	54.0	100.0
Y- 5	2.2	0.	0.	97.8	875	152	131526	116466	22.0	71.3	129051	-210	32.0	36.3	48.0	80.6	72.0	100.0
Y- 6	0.9	0.	0.	99.1	875	152	126906	109350	23.0	73.3	124078	-210	28.0	36.3	52.0	80.6	56.0	100.0
Y- 7	1.4	0.	0.	98.6	875	152	132600	116201	21.0	67.9	123508	-220	23.5	29.0	39.5	98.5	43.0	100.0
Y- 8	1.1	0.	0.	98.9	875	152	128614	113709	22.5	71.1	124195	-220	22.0	38.0	36.0	97.5	38.5	100.0
Y- 9	6.3	0.	0.	93.7	875	152	125896	109761	22.5	74.3	121646	-210	31.0	33.0	48.0	84.5	61.5	100.0
Y-10	4.0	0.	0.	96.0	875	152	129155	114828	22.0	71.6	123200	-210	33.0	42.5	46.0	95.0	56.0	100.0
Y-11	27.0	0.	0.	73.0	875	152	119841	106182	22.0	71.9	112789	-150	30.5	22.5	33.0	41.0	56.5	100.0

LONGITUDINAL

SPEC. NO.	PCT. BAINITE	PCT. FERRITE	PCT. PEARLITE	MARTEN-SITE	ISO-THERMAL HOLDING TEMP. (F)	ISO-THERMAL HOLDING TIME (SEC)	ULT. TEN. STH. (PSI)	TEN. YIELD STH. (PSI)	PCT. EL. IN 2 IN.	PCT. RED. IN AREA	COMP. YIELD STH. (PSI)	NDT TEMP (F)	CHARPY V-NOTCH IMPACT PROPERTIES NDT ENERGY (FT-LBS)	NDT PCT. FIBER	-120F ENERGY (FT-LBS)	-120F PCT. FIBER	+210F ENERGY (FT-LBS)	+210F PCT. FIBER
X- 1	54.0	0.	0.	46.0	875	1600	112337	92530	24.0	73.4	104176	-130	96.0	79.0	101.0	84.0	115.0	100.0
X- 2	53.7	0.	0.	46.3	875	1600	109457	93914	25.0	74.2	104142	-170	78.0	64.0	106.5	94.5	118.5	100.0
Y- 2	40.8	0.	0.	59.2	875	1600	123818	107536	23.0	71.3	116268	-180	60.5	55.0	73.0	90.0	89.5	100.0
Y- 4	38.6	0.	0.	61.4	875	1600	124374	109128	23.0	70.5	118196	-190	29.5	54.0	79.0	100.0	91.0	100.0

TABLE 6B – SPECIMENS AUSTENITIZED AT 1640F FOR ½ HOUR, ISOTHERMALLY TREATED AT 875F FOR 152 SECONDS, WATER QUENCHED AND TEMPERED AT 1150F FOR ONE HOUR

SPEC. NO.	PCT. BAINITE	PCT. FERRITE	PCT. PEARLITE	MARTEN-SITE	ISO-THERMAL HOLDING TEMP. (F)	ISO-THERMAL HOLDING TIME (SEC)	ULT. TEN. STH. (PSI)	TEN. YIELD STH. (PSI)	PCT. EL. IN 2 IN.	PCT. RED. IN AREA	COMP. YIELD STH. (PSI)	NDT TEMP (F)	TRANSVERSE CHARPY V-NOTCH IMPACT PROPERTIES NDT ENERGY (FT-LBS)	NDT PCT. FIBER	-120F ENERGY (FT-LBS)	-120F PCT. FIBER	+210F ENERGY (FT-LBS)	+210F PCT. FIBER
X- 1	54.0	0.	0.	46.0	875	1600	112337	92530	24.0	73.4	104176	-130	44.0	37.0	45.5	40.0	77.0	100.0
X- 2	53.7	0.	0.	46.3	875	1600	109457	93914	25.0	74.2	104142	-170	61.0	32.5	78.0	93.0	102.0	100.0
X-12	49.4	0.	0.	50.6	875	1600	114795	98893	23.0	75.3	104176	-190	34.5	33.0	58.0	71.0	80.0	100.0
X-13	60.0	0.	0.	40.0	875	1600	115485	98102	23.0	73.2	105022	-190	36.0	31.0	67.0	67.0	95.0	100.0
X-18	20.7	0.	0.	79.3	875	1600	124573	108323	23.0	73.7	115765	-190	38.5	48.5	62.0	100.0	69.0	100.0
X-15	26.9	0.	0.	73.1	875	1600	123476	105394	22.0	72.5	113029	-190	42.0	49.0	59.5	85.5	81.5	100.0
X-16	16.2	0.	0.	83.8	875	1600	124000	110000	23.0	73.8	116458	-190	37.5	63.0	60.0	94.5	62.5	100.0
X-17	31.5	0.	0.	68.5	875	1600	121878	105794	22.0	73.3	112890	-170	38.0	42.0	56.5	79.5	77.0	100.0
Y- 2	40.8	0.	0.	59.2	875	1600	123818	107536	23.0	71.3	116268	-180	33.0	32.0	47.0	58.0	62.0	100.0
Y- 4	38.6	0.	0.	61.4	875	1600	124374	109128	23.0	70.5	118196	-190	32.5	32.5	55.0	93.0	63.5	100.0
Y-11	63.0	0.	0.	37.0	875	1600	112949	96912	23.0	71.1	101911	-150	30.5	25.0	35.5	34.0	59.0	100.0

TABLE 6C – SPECIMENS AUSTENITIZED AT 1640F FOR ½ HOUR, ISOTHERMALLY TREATED AT 875F FOR 1600 SECONDS, WATER QUENCHED AND TEMPERED AT 1150F FOR ONE HOUR

SPEC. NO.	PCT. BAINITE	PCT. FERRITE	PCT. PEARLITE	MARTEN-SITE	ISO-THERMAL HOLDING TEMP. (F)	ISO-THERMAL HOLDING TIME (SEC)	ULT. TEN. STH. (PSI)	TEN. YIELD STH. (PSI)	PCT. EL. IN 2 IN.	PCT. RED. IN AREA	COMP. YIELD STH. (PSI)	NDT TEMP (F)	TRANSVERSE CHARPY V-NOTCH IMPACT PROPERTIES NDT ENERGY (FT-LBS)	NDT PCT. FIBER	-120F ENERGY (FT-LBS)	-120F PCT. FIBER	+210F ENERGY (FT-LBS)	+210F PCT. FIBER
X- 1	0.	31.0	0.	69.0	1200	3350	108000	91000	23.0	74.0	110597	-250	23.0	18.0	54.0	76.0	72.0	100.0
X- 2	0.	0.1	0.	99.9	1200	3350	117581	103248	24.0	74.9	110570	-170	49.5	42.0	70.5	74.1	95.0	100.0
X- 3	0.	31.3	0.	68.7	1200	3350	104296	81169	24.0	71.0	88185	-190	26.0	23.0	44.5	75.5	63.5	100.0
X- 4	0.	11.4	0.	88.6	1200	3350	118690	98588	24.0	74.2	109708	-210	26.0	26.8	51.6	77.8	56.0	100.0
X- 5	0.	0.	0.	100.0	1200	3350	124199	105168	23.0	74.2	116642	-190	32.0	26.8	50.0	77.8	72.0	100.0
X- 6	0.	0.5	0.	99.5	1200	3350	113545	979700	25.0	77.3	104710	-170	38.0	26.8	61.0	77.8	82.0	100.0
X- 8	0.	5.0	0.	95.0	1200	3350	117050	103446	22.5	75.5	111654	-190	42.5	35.0	65.0	70.0	77.0	100.0
X- 9	0.	26.1	0.	73.9	1200	3350	108566	88396	24.0	76.3	96054	-210	30.5	26.5	59.0	77.5	76.0	100.0
X-10	0.	0.1	0.	99.9	1200	3350	121592	106070	24.0	75.5	113179	-210	40.0	40.0	68.0	96.0	78.0	100.0
X-11	0.	0.	0.	100.0	1200	3350	122954	108184	23.0	71.3	115221	-230	24.5	20.5	50.0	74.5	70.5	100.0
X-12	0.	0.5	0.	99.5	1200	3350	122986	108459	23.0	75.3	119321	-190	41.0	38.5	59.5	79.0	76.0	100.0
X-13	0.	0.1	0.	99.9	1200	3350	121315	106574	23.0	73.5	117327	-190	51.0	42.5	75.5	97.0	88.5	100.0
X-18	0.	0.	0.	100.0	1200	3350	128016	112414	23.0	74.5	110597	-210	34.0	32.0	53.0	87.5	72.5	100.0
X-15	0.	0.	0.	100.0	1200	3350	126620	110320	23.0	72.1	119382	-190	38.0	37.0	61.5	88.5	78.5	100.0
X-16	0.	0.2	0.	99.8	1200	3350	127218	113260	23.0	73.9	124414	-210	34.0	36.0	53.5	97.0	61.0	100.0
X-17	0.	0.	0.	100.0	1200	3350	127019	113360	22.0	71.9	120455	-190	34.0	30.5	53.0	72.0	72.0	100.0
Y- 1	0.	50.2	32.1	17.7	1200	3350	88823	65869	29.0	62.5	71587	-100	21.0	9.0	18.0	5.0	60.5	100.0
Y- 2	0.	22.3	0.	77.7	1200	3350	129000	113000	23.0	73.0	110597	-250	25.0	23.0	53.0	83.0	61.0	100.0
Y- 3	0.	69.0	22.8	8.1	1200	3350	82672	61554	31.0	69.1	65068	-110	27.0	9.0	24.0	7.5	85.5	100.0
Y- 4	0.	6.5	0.	93.5	1200	3350	124398	107343	24.0	72.5	116627	-210	29.0	33.0	46.0	100.0	58.0	100.0
Y- 5	0.	17.0	0.	83.0	1200	3350	123395	108347	23.0	70.5	115424	-230	30.0	26.8	52.0	77.8	55.0	100.0
Y- 6	0.	12.3	0.	87.7	1200	3350	147000	115000	23.0	69.0	110597	-230	24.0	26.8	56.0	77.8	64.0	100.0
Y- 7	0.	3.3	0.	96.7	1200	3350	130609	113161	21.0	67.3	124597	-230	21.5	11.5	41.5	100.0	42.0	100.0
Y- 8	0.	0.5	0.	99.5	1200	3350	124852	109544	23.0	71.6	117527	-250	18.0	23.5	41.5	100.0	39.0	100.0
Y- 9	0.	1.8	0.	98.2	1200	3350	123200	107800	23.0	74.8	117051	-250	29.0	13.5	58.0	100.0	61.5	100.0
Y-10	0.	0.7	0.	99.3	1200	3350	124303	103765	23.0	71.5	116863	-230	26.5	20.0	53.0	100.0	62.5	100.0
Y-11	0.	2.6	0.	97.4	1200	3350	116485	100900	23.0	71.8	109661	-190	28.5	16.0	40.0	50.5	57.0	100.0

TABLE 6D – SPECIMENS AUSTENITIZED AT 1640F FOR ½ HOUR, ISOTHERMALLY TREATED AT 1200F FOR 3350 SECONDS, WATER QUENCHED AND TEMPERED AT 1150F FOR ONE HOUR

SPEC. NO.	PCT. BAINITE	PCT. FERRITE	PCT. PEARLITE	MARTEN-SITE	ISO-THERMAL HOLDING TEMP. (F)	ISO-THERMAL HOLDING TIME (SEC)	ULT. TEN. STH. (PSI)	TEN. YIELD STH. (PSI)	PCT. EL. IN 2 IN.	PCT. RED. IN AREA	COMP. YIELD STH. (PSI)	NDT TEMP (F)	TRANSVERSE CHARPY V-NOTCH IMPACT PROPERTIES NDT ENERGY (FT-LBS)	NDT PCT. FIBER	-120F ENERGY (FT-LBS)	-120F PCT. FIBER	+210F ENERGY (FT-LBS)	+210F PCT. FIBER
X- 1	0.	67.0	10.7	22.2	1200	8500	94518	68993	28.0	67.1	75343	-180	13.0	14.5	35.0	48.5	69.0	100.0
X- 2	0.	1.4	0.	98.6	1200	8500	115339	100598	24.0	74.3	106871	-190	47.0	28.5	62.0	72.5	101.5	100.0
X-12	0.	1.1	0.	98.9	1200	8500	120400	106200	23.0	74.8	114385	-190	37.0	36.0	65.0	84.5	82.0	100.0
X-13	0.	0.1	0.	99.9	1200	8500	120041	104188	23.0	73.7	111387	-170	48.0	47.5	74.5	90.5	93.5	100.0
X-18	0.	0.	0.	100.0	1200	8500	125001	108417	22.5	73.3	116716	-230	27.5	20.0	61.0	97.5	68.0	100.0
X-15	0.	0.	0.	100.0	1200	8500	124378	107428	23.0	72.7	113091	-190	41.0	34.0	67.5	89.0	82.0	100.0
X-16	0.	0.4	0.	99.6	1200	8500	124400	108598	22.0	73.6	117777	-220	35.5	36.5	55.5	86.5	72.5	100.0
X-17	0.	0.3	0.	99.7	1200	8500	123753	108892	23.0	72.1	114733	-190	36.0	33.0	64.0	93.5	76.5	100.0
Y- 2	0.	53.2	4.3	42.5	1200	8500	106878	81354	26.0	65.2	88944	-210	19.0	16.0	32.0	56.1	56.0	100.0
Y- 4	0.	20.5	0.	79.5	1200	8500	127019	108176	24.0	67.0	118509	-230	15.0	25.0	44.5	100.0	53.0	100.0
Y-11	0.	44.3	0.	55.7	1200	8500	103984	79931	25.0	70.3	85639	-210	20.5	9.5	39.5	81.5	53.0	100.0

TABLE 6E – SPECIMENS AUSTENITIZED AT 1640F FOR ½ HOUR, ISOTHERMALLY TREATED AT 1200F FOR 8500 SECONDS, WATER QUENCHED AND TEMPERED AT 1150F FOR ONE HOUR

SPEC. NO.	PCT. BAINITE	PCT. FERRITE	PCT. PEARLITE	MARTEN-SITE	ISO-THERMAL HOLDING TEMP. (F)	ISO-THERMAL HOLDING TIME (SEC)	ULT. TEN. STH. (PSI)	TEN. YIELD STH. (PSI)	PCT. EL. IN 2 IN.	PCT. RED. IN AREA	COMP. YIELD STH. (PSI)	NDT TEMP (F)	TRANSVERSE CHARPY V-NOTCH IMPACT PROPERTIES NDT ENERGY (FT-LBS)	NDT PCT. FIBER	-120F ENERGY (FT-LBS)	-120F PCT. FIBER	+210F ENERGY (FT-LBS)	+210F PCT. FIBER
X-12	0.	0.	0.	100.0	0	0	122123	107608	21.5	72.2	111884	-110	36.0	29.0	35.5	25.5	77.0	100.0
X-13	0.	0.	0.	100.0	0	0	119341	104686	22.0	71.3	113546	-90	54.5	52.0	44.5	32.5	91.0	100.0
X-18	0.	0.	0.	100.0	0	0	123430	106780	22.0	72.3	113127	-100	60.5	93.5	50.0	65.0	69.0	100.0
X-15	0.	0.	0.	100.0	0	0	125423	108174	22.0	72.5	118627	-110	47.5	56.0	44.5	49.0	77.0	100.0
X-16	0.	0.	0.	100.0	0	0	127199	113600	21.5	71.6	118993	-110	36.0	31.0	33.5	25.5	66.0	100.0
X-17	0.	0.	0.	100.0	0	0	127146	112874	22.0	71.5	123237	-110	50.5	57.5	47.5	48.5	77.5	100.0

TABLE 6F – SPECIMENS AUSTENITIZED AT 2000F FOR ONE HOUR, TRANSFERRED TO 1640F FOR 5 MINUTES MINIMUM, WATER QUENCHED AND TEMPERED AT 1150F FOR ONE HOUR

SPEC. NO.	PCT. BANITE	PCT. FERRITE	PCT. PEARLITE	MARTEN-SITE	ISO-THERMAL HOLDING TEMP. (F)	ISO-THERMAL HOLDING TIME (SEC)	ULT. TEN. STH. (PSI)	TEN. YIELD STH. (PSI)	PCT. EL. IN 2 IN.	PCT. RED. IN AREA	COMP. YIELD STH. (PSI)	NDT TEMP (F)	TRANSVERSE CHARPY V-NOTCH IMPACT PROPERTIES NDT ENERGY (FT-LBS)	NDT PCT. FIBER	-120F ENERGY (FT-LBS)	-120F PCT. FIBER	+210F ENERGY (FT-LBS)	+210F PCT. FIBER
X-12	62.8	0.	0.	37.2	875	1600	116733	101494	22.0	71.3	107197	-80	35.5	18.0	29.5	9.5	75.5	100.0
X-13	66.9	0.	0.	33.1	875	1600	115547	100051	21.5	70.4	103241	-60	45.0	25.0	34.0	6.5	90.0	100.0
X-18	56.2	0.	0.	43.8	875	1600	121480	104797	22.0	70.7	108874	-80	36.5	29.0	28.5	14.0	78.5	100.0
X-15	58.4	0.	0.	41.6	875	1600	123000	106300	22.0	67.8	112219	-60	43.0	36.0	32.0	16.5	79.0	100.0
X-16	56.7	0.	0.	43.3	875	1600	124029	109772	22.0	71.1	116274	-110	34.0	35.5	32.5	30.0	64.0	100.0
X-17	29.9	0.	0.	70.1	875	1600	125000	111000	22.0	70.3	115943	-120	35.5	30.0	35.5	30.0	76.5	100.0

TABLE 6G – SPECIMENS AUSTENITIZED AT 2000F FOR ONE HOUR, TRANSFERRED TO 1640F FOR 5 MINUTES MINIMUM, ISO–THERMALLY TREATED AT 875F FOR 1600 SECONDS, WATER QUENCHED AND TEMPERED AT 1150F FOR ONE HOUR

VARIABLE IDENTIFICATION	RESULTS FROM CORRELATION MATRIX .850 TO .899	0.900 TO 0.949	0.950 TO 0.999
V1 - SAMPLE NO.		10,15	
V2 - AMT CARBON	70		78
V3 - AMT MN	82		81
V4 - AMT P	68		
V5 - AMT S			
V6 - AMT SI	69		
V7 - AMT NI			
V8 - AMT CR	15	50	79
V9 - AMT MO	72,83		51,80
V10- AMT CU		1	
V11- ACID SOLUBLE AL		12,48,57	73
V12- TOTAL AL	48,57,73	11	
V13- AMT TI			45,61
V14- AMT V	63		
V15- AMT CO	8,79,82	1	46
V16- AMT OXYGEN		54,60	52
V17- AMT NITROGEN		53	
V18- AMT HYDROGEN (PPM)			
V19- D(I)50M			
V20- ASTM MICRO GRAIN SIZE	64		
V21- PCT BAINITE		75	
V22- PCT FERRITE			76
V23- PCT PEARLITE			77
V24- PCT MARTENSITE			
V25- HOLDING TIME (SEC)			
V26- HOLDING TEMP (F)			
V27- UTS (PSI)		31	28
V28- TYS (PSI)			27,31,64
V29- PCT EL. IN 2 IN.			
V30- PCT RA			
V31- CYS (KSI)		27	28
V32- NDT TEMP (F)			
V33- NDT ENERGY(FT-LB)CHARPY			
V34- NDT PCT FIBER			
V35- -120F ENERGY (FT-LB)			
V36- -120F PCT FIBER			
V37- +210F ENERGY (FT-LB)			
V38- +210F PCT FIBER			
V39- LONGEST INCLUSION (L)			
V40- AV SIZE OTHER INCLUSIONS (L)			
V41- BACKGROUND RATING (L)			
V42- LONGEST INCLUSION (T)			
V43- AV SIZE OTHER INCLUSIONS (T)			
V44- BACKGROUND RATING (T)			

Table 7 - List of Variables and Correlation Coefficients Showing Highly Correlated Variables

Table 7 - Continued

VARIABLE IDENTIFICATION	RESULTS FROM CORRELATION MATRIX .850 TO .899	0.900 TO 0.949	0.950 TO 0.999
V45- V 2*V13 - C * TI			13,61
V46- V 2*V15 - C * AMT CO	82		15
V47- V 3*V 5 - MN * S			
V48- V 5*V11 - S * ACID SOL AL	12,57	11,73	
V49- V 6*V 8 - SI * CR			
V50- V 7*V 8 - NI * CR		8,79	
V51- V 7*V 9 - NI * MO	72,83		9,80
V52- V 8*V16 - CR * OXYGEN		54,60	16
V53- V 8*V17 - CR * NITROGEN		17	
V54- V 9*V16 - MO * OXYGEN		16,52,60	
V55- V 9*V17 - MO * NI			
V56- V11*V13 - ACID SOL AL * TI			
V57- V11*V17 - ACID SOL AL * NI	12,48,59	11,73	
V58- V11*V16 - ACID SOL AL * OXYGEN			
V59- V12*V17 - TOTAL AL * NITROGEN	57		
V60- V13*V16 - TI * OXYGEN		16,52,54	
V61- V13*V17 - TI * NITROGEN			13,45
V62- V14*V16 - V * OXYGEN			
V63- V14*V70 - V * NITROGEN	14		
V64- V20*V28 - GR SZ * TYS	20		28
V65- V20*V37 - GR SZ * +210F ENERGY			
V66- V 6*V11/V17 - SI * A S AL / N			
V67- V 2*V 3 - CARBON * MN			
V68- V 2*V 4 - CARBON * P	4		
V69- V 2*V 6 - CARBON * S	6		
V70- V 2*V 7 - CARBON * NI	2,71,78		
V71- V 2*V 8 - CARBON * CR	70		
V72- V 2*V 9 - CARBON * MO	9,51,80		
V73- V 2*V11 - CARBON * A S AL	12	48,57	11
V74- V39*V42 - INCL (L) * INCL (T)			
V75- V20*V21 - GR SZ * BAINITE		21	
V76- V20*V22 - GR SZ * FERRITE			22
V77- V20*V23 - GR SZ * PEARLITE			23
V78- V 2*V 2 - CARBON * CARBON	70		2
V79- V 8*V 8 - CR *CR	15	50	8
V80- V 8*V 9 - CR * MO	72		9,51
V81- V 3*V 3 - MN * MN	82		3
V82- V 3*V 8 - MN * CR	3,15,46,81		
V83- V 3*V 9 - MN * MO	9,51		

REGRESSION STATISTICS

							* * * * * * * TERM * * * * * *		
REGRESSION COEFFICIENTS			TERM NAMES			T	PRODUCT	MEANS	STD. DEVS.	RANGE
B	1	-0.2283212E 02	V20- GRAIN SIZE			-14.950	-132.0369263	8.1858406	1.4457368	5.66
Y			V32 NDT					-185.3362827	40.3826804	-4.59
		BO	1.5637894	R2,	0.6681601		DF 111			
		SX.Y	23.3672152	R	0.8174106		N 113.			

BO - CONSTANT CALCULATED FOR REGRESSION EQUATION
R2 - STANDARD ERROR OF ESTIMATE (R SQUARED)
DF - DEGREES OF FREEDOM
SX.Y - STANDARD DEVIATION OF A PREDICTED POINT (RESIDUAL UNACCOUNTED FOR AFTER REGRESSION)
R - REGRESSION COEFFICIENT
N - NUMBER OF DATA POINTS
T - COEFFICIENT DIVIDED BY THE STD DEV OF THE COEFFICIENT
(USE WITH STUDENT T DISTRIBUTION TABLE AND DEGREES OF FREEDOM TO DETERMINE
CONFIDENCE INTERVALS OF THE REGRESSION COEFFICIENTS)
PRODUCT - 4*(REGRESSION COEF)*(TERM STD DEV)
TERM RANGE - (TERM MEAN)/(TERM STD DEV)
* TERM RANGE IS LESS THAN 2.0, THE VALUE OF PRODUCT SHOULD BE
RECOMPUTED BEFORE BEING EVALUATED SINCE VALUES OF THIS VARIABLE ARE DISTRIBUTED IN A SKEW MANNER

$$NDT(F^\circ) = 1.56 - 22.8 \times (\text{ASTM } \gamma \text{ Grain Size})$$

Table 8 - Results of Linear Regression Relating NDT Temperature to Prior Austenitic Grain Size

REGRESSION STATISTICS

		REGRESSION COEFFICIENTS	TERM NAMES	T	PRODUCT	MEANS	STD. DEVS.	RANGE
				* * * * * * * TERM * * * * * * *				
B	1	-0.2044782E 02	V20- GRAIN SIZE	-14.296	-117.2553663	8.1913043	1.4335923	5.71
B	2	0.4640463E 00	V21- PCT BANITE	4.289	35.5681038	9.3226086	19.1619370	0.49*
B	3	-0.7432906E 00	V22- PCT FERRITE	-3.551	-38.1352696	4.1808695	12.8264999	0.33*
B	4	0.4872587E 01	V23- PCT PEARLITE	6.953	73.9413671	0.6078261	3.7937424	0.16*
Y			V32 NDT			-183.9391289	41.3986368	-4.44

BO	-20.6249943	R2,	0.7640074	DF	110
SX.Y	20.4734623	R	0.8740752	N	115.

BO - CONSTANT CALCULATED FOR REGRESSION EQUATION
R2 - STANDARD ERROR OF ESTIMATE (R SQUARED)
DF - DEGREES OF FREEDOM
SX.Y - STANDARD DEVIATION OF A PREDICTED POINT (RESIDUAL UNACCOUNTED FOR AFTER REGRESSION)
R - REGRESSION COEFFICIENT
N - NUMBER OF DATA POINTS
T - COEFFICIENT DIVIDED BY THE STD DEV OF THE COEFFICIENT
(USE WITH STUDENT T DISTRIBUTION TABLE AND DEGREES OF FREEDOM TO DETERMINE
CONFIDENCE INTERVALS OF THE REGRESSION COEFFICIENTS)
PRODUCT - 4*(REGRESSION COEF)*(TERM STD DEV)
TERM RANGE - (TERM MEAN)/(TERM STD DEV)
* TERM RANGE IS LESS THAN 2.0, THE VALUE OF PRODUCT SHOULD BE
RECOMPUTED BEFORE BEING EVALUATED SINCE VALUES OF THIS VARIABLE ARE DISTRIBUTED IN A SKEW MANNER

$$NDT,°F = -20.6-20.4(\text{ASTM } \gamma \text{ Grain Size})+0.464(\%\text{ Bainite})-0.743(\%\text{ Ferrite}) +4.87(\%\text{ Pearlite})$$

Table 9 - Results of Linear Regression Relating NDT Temperature to Prior Austenitic Grain Size, Percent Bainite, Percent Ferrite and Percent Pearlite

REGRESSION STATISTICS

		REGRESSION COEFFICIENTS	TERM NAMES	T	PRODUCT	MEANS	STD. DEVS.	RANGE
					* * * * * * * TERM * * * * * * *			
B	1	-0.2193804E 02	V1 - PRODUCER	-5.379	-41.7249303	1.33913C4	0.4754860	2.82
B	2	-0.1920520E 02	V20- GRAIN SIZE	-14.501	-110.1297035	8.1913C43	1.4335923	5.71
B	3	0.6156752E 00	V21- PCT BAINITE	5.347	47.1901188	5.3226C8E	19.161937C	0.49*
B	4	-0.2285341E 00	V22- PCT FERRITE	-0.796	-11.7251731	4.18C8695	12.8264999	0.33*
B	5	0.5355528E 01	V23- PCT PEARLITE	8.193	81.269973E	C.6C76261	3.7937424	0.16*
B	6	0.6771273E-03	V28- TYS (PSI)	1.875	27.7613032	106514.5126953	1C249.661377C	10.39
Y			V32- NDT TEMP			-183.9291289	41.3986368	-4.44

BO	-77.4089441	R2,	0.8138912	DF	108
SX.Y	18.3488998	R	0.9021592	N	115.

BO - CONSTANT CALCULATED FOR REGRESSION EQUATION
R2 - STANDARD ERROR OF ESTIMATE (R SQUARED)
DF - DEGREES OF FREEDOM
SX.Y - STANDARD DEVIATION OF A PREDICTED POINT (RESIDUAL UNACCOUNTED FOR AFTER REGRESSION)
R - REGRESSION COEFFICIENT
N - NUMBER OF DATA POINTS
T - COEFFICIENT DIVIDED BY THE STD DEV OF THE COEFFICIENT
(USE WITH STUDENT T DISTRIBUTION TABLE AND DEGREES OF FREEDOM TO DETERMINE
CONFIDENCE INTERVALS OF THE REGRESSION COEFFICIENTS)
PRODUCT - 4*(REGRESSION COEF)*(TERM STD DEV)
TERM RANGE - (TERM MEAN)/(TERM STD DEV)
* TERM RANGE IS LESS THAN 2.0, THE VALUE OF PRODUCT SHOULD BE
RECOMPUTED BEFORE BEING EVALUATED SINCE VALUES OF THIS VARIABLE ARE DISTRIBUTED IN A SKEW MANNER

$$NDT,°F = -77.4-21.9(\text{Producer, Coded 1 or 2})-19.2(\text{ASTM } \gamma \text{ Grain Size})+0.616(\%B) -0.229(\%F)+5.36(\%P)+0.677(\text{TYS,Ksi})$$

Table 10 - Results of Linear Regression Relating NDT Temperature to Producer, Prior Austenitic Grain Size, Percent Bainite, Percent Ferrite, Percent Pearlite and Tensile Yield Strength

APPENDIX A

REAGENTS USED TO DEVELOP PRIOR AUSTENITIC GRAIN BOUNDARIES

ETHERAL PICRIC ACID PRIOR AUSTENITIC GRAIN BOUNDARY OR TEMPER BRITTLENESS ECHANT* AND ETCHING TECHNIQUE

The etheral picric acid reagent consists of the following reagents:

Picric Acid (solid with 11% water)	50 grms
Purified Ethyl Ether	250 ml
Zephiran Chloride (12.8% solution)**	10 ml
Water	240 ml

The reagent is prepared and used as follows:

1. Dissolve the picric acid in ether (most of the picric acid will remain undissolved.) Then add 250 ml of zephiran chloride and water solution.

2. Shake thoroughly; keep in tightly stoppered bottle. This mixture separates into two distinct layers. The top layer is saturated solution of picric acid in ether with some zephiran chloride. Bottom layer is aqueous zephiran chloride - picric acid solution.

3. Let stand overnight. No further shaking is necessary or desirable.

4. Decant small portion of the top layer and dilute with 1/3 volume ether. (Prevents supersaturation with picric acid.) If picric acid crystals precipitate upon the specimen, a badly pitted, uneven etch results. After use,

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* Transactions of A.S.M., Vol. XXXIX, 1947, p. 109-138 inclusive.
**A brand of benzalkonium chloride manufactured by Winthrop-Stearns, New York, New York.

the etching solution is usually returned to the bottle. That is, the etchant may be reused and like Picral appears to improve with use.

5. Technique:

a. To develop prior austenitic grain boundaries, immerse without agitation for one to 15 minutes. The exact time depends on how long it takes to develop the network of the prior austenitic grain boundaries. Keep solution at 80 ±2F. Repeat, if too lightly etched, after washing. If removed for cursory examination, wash with alcohol before re-immersion because of a precipitated layer of picric acid crystals. This film is insoluble in water.

b. To determine the presence of embrittling grain boundary precipitates the specimen is taken after step (a) and repolished to remove most of the grain boundary network. Repolish for approximately one minute on final wheel. Note: This polishing will take six minutes if a vibratory polisher is used. This etch is removed but grain boundary pitting due to an embrittling constituent present in the prior austenitic grain boundaries will persist and provide a definite indication in an improperly heat treated or temper embrittled specimen; a properly treated specimen will only have an occasional pit remaining at the location of the prior austenitic grain boundaries, whereas a netowrk of pits is characteristic of a material suffering from temper embrittlement.

ALTERNATE GRAIN BOUNDARY ETCH

The picric acid and alcohol reagent consists of the following reagents:

Picric Acid (11% water)	Excess
Ethyl Alcohol (180 proof, undenatured)	100 ml
Sodium Tridecylbenzelene Sulfonate*	1 gm

The reagent is prepared as follows:

1. Prepare a saturated solution of picric acid in ethyl alcohol and add the tridecylbenzelene.

2. To etch, immerse for three to six seconds to develop network or prior austenitic grain boundaries.

3. Rinse in stream of hot water and then re-rinse immediately with 180 proof undenatured ethyl alcohol.

4. Repeat rinses in step (3) above.

5. Dry with hot air after second alcohol rinse.

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*One source is Chemithon Corporation, 5430 W. Marginal Way, Seattle, Washington.

Note: If this etch is used instead of the etheral picric acid etch, it should be used first, before the one percent Nital etch, when using the triple etching technique to identify isothermal transformation products. See Appendix B.

APPENDIX B

ETCHING TECHNIQUES FOR IDENTIFYING ISOTHERMAL PRODUCTS

REAGENTS

The first step is to prepare the following four solutions:

1. One Percent Nital

2. Saturated Picric Acid Etch. (See Appendix A for details describing how to prepare this etch or its alternate.)

(a) Before using the etheral picric acid etch described in enclosure (2), add two drops of saturated Zephiran Chloride (a brand of benzalkonium chloride manufactured by Winthrop-Stearns, N. Y. 18, N. Y., and available in drug stores) to each 50 ml.

(b) Appendix A gives an alternate alcohol base etch that will do an adequate job of revealing the prior austenitic grain boundaries. This alcohol base etch has the distinct advantage of ease of preparation and a much less objectionable odor.

3. Neutralizing Rinse. Add 10 ml concentrated ammonium hydroxide to 90 ml ethyl alcohol.

4. Twenty Percent Sodium Metabisulfite Solution.

(a) Add 20 grams of Fisher's sodium metabisulfite (Cat. No. S-243) to 80 ml of distilled water. Allow to go completely into solution. Add two drops of saturated aerosol solution; aerosol is a wetting agent.

(b) Filter to remove any particles before using.

(c) Solution must be made fresh every day since decomposition gradually takes place. Store in brown, glass-stoppered bottle.

The specimens evaluated using this technique have all been mounted in "Trans Optic" mounting powder. Both Lucite and the various bakelites were found unsatisfactory for mounting specimens: the ordinary metallographic Lucite because it quickly crazes when immersed in these solutions, and the bakelites because they are slightly soluble and will deposit a film on the specimen. The latter is readily apparent when a cotton swab that has been dipped in Nital is rubbed over the bakelite.

ETCHING TECHNIQUES

When ready to etch a specimen, a small beaker full of each etching solution is poured out. An amount sufficient to cover the specimen about 3/8 in. is put in a small, glass-covered beaker. The covered beakers containing the three etches, one percent Nital, saturated picral, and 20 percent sodium metabisulfite, are put in a constant temperature bath. The temperature of these etches is maintained at 80 degrees Fahrenheit plus or minus two degrees. For consistent results, the temperature of these solutions must be kept constant and within the indicated range.

The following etching procedure is now undertaken:

1. Remove micro-specimen from final polishing oper-

ation, wash with liquid green soap, rinse, and dry with ethyl alcohol and hot air. If the specimen has stood overnight, the final polishing operation should be repeated before etching.

2. Etch the specimen in the one percent Nital by immersion. Etch until the structure just begins to become visible, three to five seconds, then rinse and dry.

3. Etch the specimen in etheral picric acid with the zephiran chloride by immersion. Etch until grain boundaries are slightly delineated. Time may have to be adjusted to ensure good grain boundary attack. Gently shake specimen while immersed.

4. Neutralize at end of etching time with ten percent ammonium hydroxide in alcohol. Rinse with alcohol to remove all traces of nitric acid. Dry with ethyl alcohol and hot air. All picric acid must be neutralized before going on to the next etch.

5. Etch or stain with the 20 percent sodium metabisulfite and aerosol. Etching time depends on the temper given to the specimen; in any case etching with this solution is cumulative and may be repeated until the desired contrast is obtained. Start with about two seconds immersion. Once staining starts, care must be taken not to over etch since the process seems to accelerate once it has started.

6. Rinse with hot water (if necessary due to overetching with the metabisulfite, try wiping with a cotton

swab to improve contrast) and then dry with ethyl alcohol and hot air.

7. Specimen is handled with nickel tongs during the etching sequence.

8. The ethyl alcohol mentioned above is pure and undenatured ethyl alcohol from government stores.

9. The above etching procedure has been used for etching HY-80 steel and seems to work best on a tempered structure.

10. If higher nickel content steels, such as HY-140 are being examined, the best etching procedure is to use just the alcohol picric acid etch followed by the Nital etch. On this steel the metabisulfite just seems to cloud over and obscure the microstructure.

INTERPRETATION

When finally etched, the microstructure will exhibit the following characteristics:

1. Tempered martensite will appear grey due to the staining action of the sodium metabisulfite, and will exhibit typical martensitic substructure.

2. The isothermal products will be a bright metallic color since they are untouched by the stain.

(a) Proeutectoid ferrite will usually exhibit smooth, even grain boundaries and a flat, unbroken, blocky area.

(b) Bainites will exhibit ragged grain boundaries,

with this becoming more pronounced at the lower transformation temperatures. In general, it may also be said that the bainites will not exhibit the large flat blocky areas of the ferrite but will be smaller and peppered with darker areas of martensite or precipitated carbides; this becomes more pronounced as the transformation temperature is lowered.

3. A smeared structure or an oxidized surface will make proper evaluation of the results of this technique impossible.

APPENDIX C

PHOTOMICROGRAPHS

OF

MICROSTRUCTURES



29% Bainite-71% Martensite
875 F - 152 Sec



31% Ferrite-69% Martensite
1200F - 3350 Sec



54% Bainite
46% Martensite
875 F - 1600 Sec



67% Ferrite-11% Pearlite
22% Martensite
1200 F - 8500 Sec

Cropped End No. X - 1

Figure C1 - Microstructure (1000X) of Specimens from Cropped End No. X-1 Austenitized at 1640 F, Isothermally Treated at 875 F and 1200 F, and then Tempered at 1150 F



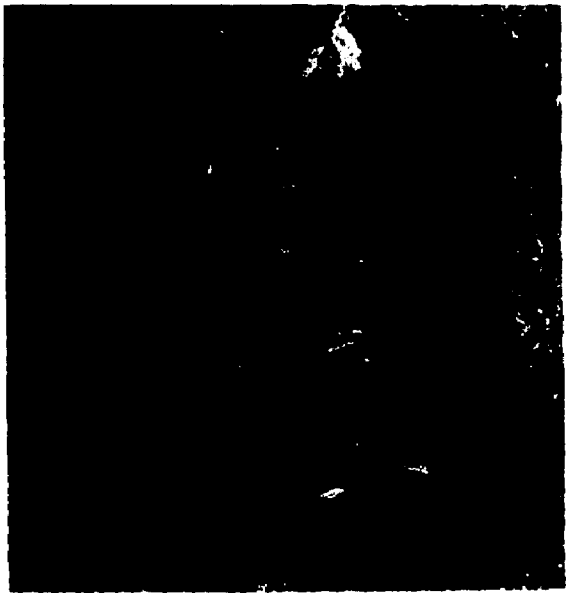
14% Bainite-86% Martensite
875 F - 152 Sec



Trace Ferrite-100% Martensite
1200 F - 3350 Sec



54% Bainite-46% Martensite
875 F - 1600 Sec



2% Ferrite-98% Martensite
1200 F - 8500 Sec

Cropped End No. X - 2

Figure C2 - Microstructure (1000X) of Specimens from Cropped End No. X-2 Austenitized at 1640 F, Isothermally Treated at 875 F and 1200 F, and then Tempered at 1150 F



21% Bainite-79% Martensite
875 F - 152 Sec

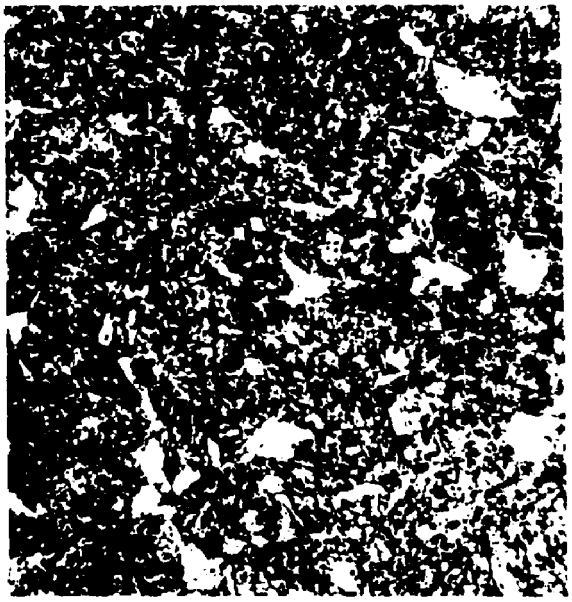


31% Ferrite-69% Martensite
1200 F - 3350 Sec

Cropped End No. X - 3



2% Bainite-98% Martensite
875 F - 152 Sec



11% Ferrite-89% Martensite
1200 F - 3350 Sec

Cropped End No. X - 4

Figure C3 - Microstructure (1000X) of Specimens from Cropped Ends X-3 and X-4 Austenitized at 1640 F, Isothermally Treated at 875 F and 1200 F, and then Tempered at 1150 F



1% Bainite-99% Martensite
875 F - 152 Sec



100% Martensite
1220 F - 3350 Sec

Cropped End No. X - 5



Trace Bainite-100% Martensite
875 F - 152 Sec



1% Ferrite-99% Martensite
1220 F - 3350 Sec

Cropped End No. X - 6

Figure C4 - Microstructure (1000X) of Specimens from Cropped Ends X-5 and X-6 Austenitized at 1640 F, Isothermally Treated at 875 F and 1200 F, and then Tempered at 1150 F



14% Bainite-86% Martensite
875 F - 152 Sec



5% Ferrite-95% Martensite
1200 F - 3350 Sec

Cropped End No. X - 8



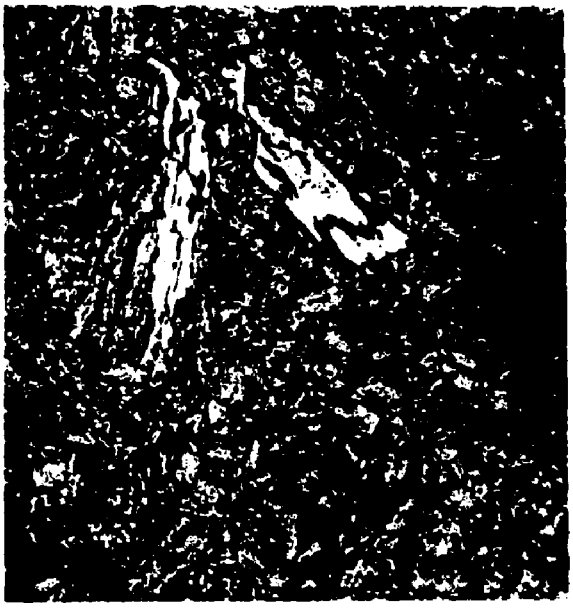
7% Bainite-93% Martensite
875 F - 152 Sec



26% Ferrite-74% Martensite
1200 F - 3350 Sec

Cropped End No. X - 9

Figure C5 - Microstructure (1000X) of Specimens from Cropped Ends X-8 and X-9 Austenitized at 1640 F, Isothermally Treated at 875 F and 1200 F, and then Tempered at 1150 F

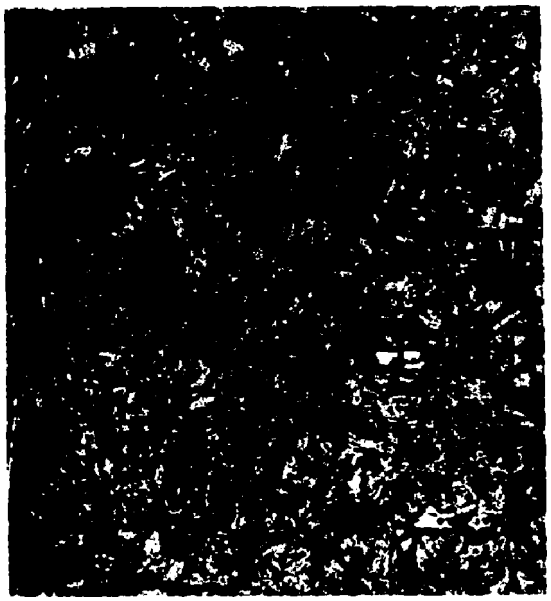


1% Bainite-99% Martensite
875 F - 152 Sec



Trace Ferrite-100% Martensite
1200 F - 3350 Sec

Cropped End No. X - 10



100% Martensite
875 F - 152 Sec



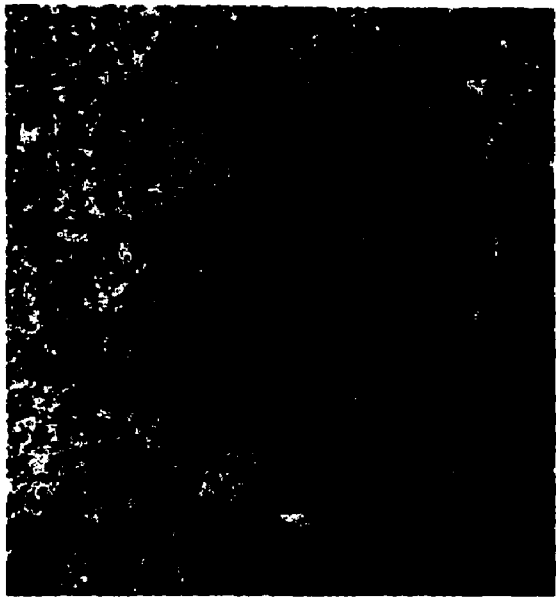
100% Martensite
1200 F - 3350 Sec

Cropped End No. X - 11

Figure C6 - Microstructure (1000X) of Specimens from Cropped Ends X-10 and X-11 Austenitized at 1640 F, Isothermally Treated at 875 F and 1200 F, and then Tempered at 1150 F



1% Bainite-99% Martensite
875 F - 152 Sec



Trace Ferrite-100% Martensite
1200 F - 3350 Sec



50% Bainite-50% Martensite
875 F - 1600 Sec



1% Ferrite-99% Martensite
1200 F - 8500 Sec

Cropped End No. X - 12

Figure C7 - Microstructure (1000X) of Specimens from Cropped End No. X-12 Austenitized at 1640 F, Isothermally Treated at 875 F and 1200 F, and then Tempered at 1150 F



50% Bainite-50% Martensite
875 F - 1600 Sec



63% Bainite-37% Martensite
875 F - 1600 Sec

Cropped End No. X - 12



60% Bainite-40% Martensite
875 F - 1600 Sec



67% Bainite-33% Martensite
875 F - 1600 Sec

Cropped End No. X - 13

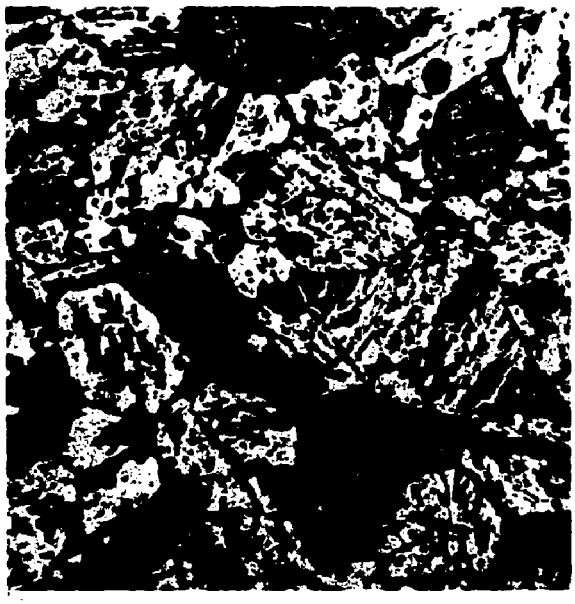
Figure C8 - Effects of Prior Austenitic Grain Size on the Microstructure (1000X) of Cropped Ends X-12 and X-13



14% Bainite-86% Martensite
875 F - 152 Sec



Trace Ferrite-100% Martensite
1200 F - 3350 Sec



60% Bainite-40% Martensite
875 F - 1600 Sec



Trace Ferrite-100% Martensite
1200 F - 8500 Sec

Cropped End No. X - 13

Figure C9 - Microstructure (1000X) of Specimens from Cropped End No. X-13 Austenitized at 1640 F, Isothermally Treated at 875 F and 1200 F, and then Tempered at 1150 F



21% Bainite-79% Martensite
875 F - 1600 Sec



56% Bainite-44% Martensite
875 F - 1600 Sec

Cropped End No. X - 18



27% Bainite-73% Martensite
875 F - 1600 Sec



58% Bainite-42% Martensite
875 F - 1600 Sec

Cropped End No. X - 15

Figure C10 - Effects of Prior Austenitic Grain Size on the Microstructure (1000X) of Cropped Ends X-15 and X-18



1% Bainite-99% Martensite
875 F - 152 Sec



100% Martensite
1200 F - 3350 Sec



27% Bainite-73% Martensite
875 F - 1600 Sec



100% Martensite
1200 F - 8500 Sec

Cropped End No. X - 15

Figure C11 - Microstructure (1000X) of Specimens from Cropped End No. X-15 Austenitized at 1640 F, Isothermally Treated at 875 F and 1200 F, and then Tempered at 1150 F



100% Martensite
875 F - 152 Sec



Trace Ferrite-100% Martensite
1200 F - 3350 Sec



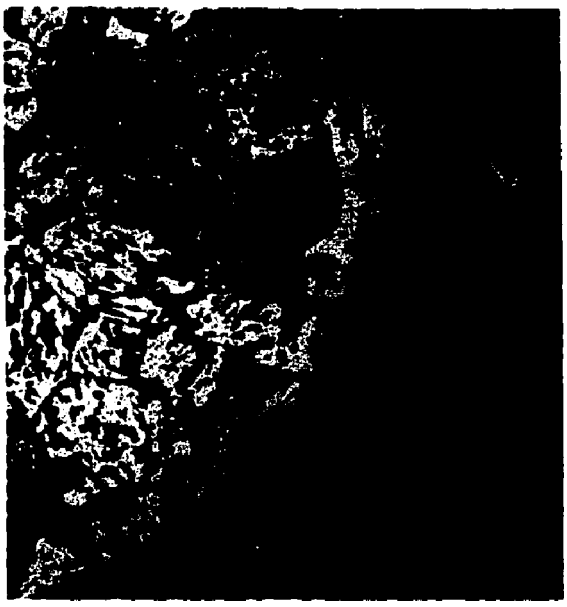
16% Bainite-84% Martensite
875 F - 1600 Sec



Trace Ferrite-100% Martensite
1200 F - 8500 Sec

Cropped End No. X - 16

Figure C12 - Microstructure (1000X) of Specimens from Cropped End No. X-16 Austenitized at 1640 F, Isothermally Treated at 875 F and 1200 F, and then Tempered at 1150 F



16% Bainite-84% Martensite
875 F - 1600 Sec



58% Bainite-42% Martensite
875 F - 1600 Sec

Cropped End No. X - 16



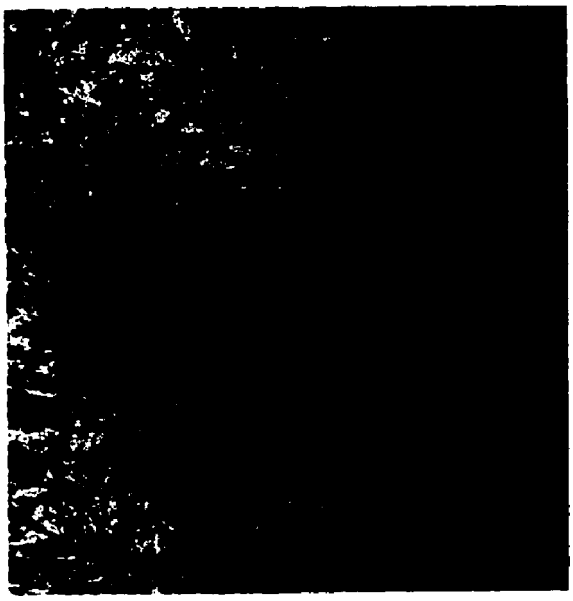
32% Bainite-68% Martensite
875 F - 1600 Sec



30% Bainite-70% Martensite
875 F - 1600 Sec

Cropped End No. X - 17

Figure C13 - Effects of Prior Austenitic Grain Size on the Microstructure (1000X) of Cropped Ends X-16 and X-17



100% Martensite
875 F - 152 Sec



100% Martensite
1200 F - 3350 Sec



32% Bainite-68% Martensite
875 F - 1600 Sec



Trace Ferrite-100% Martensite
1200 F - 8500 Sec

Cropped End No. X - 17

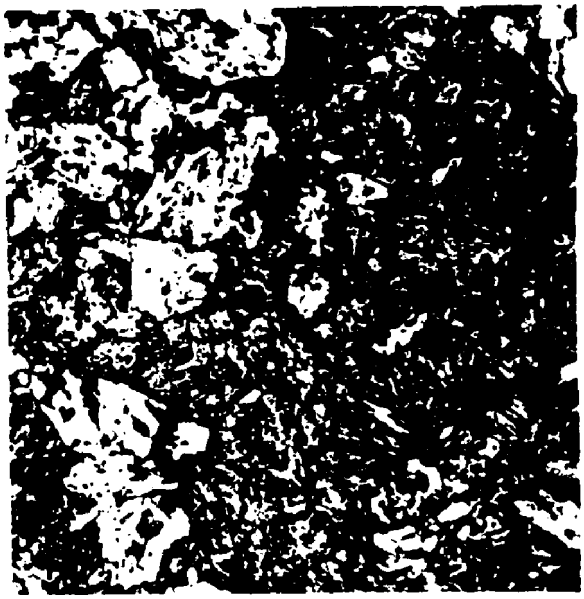
Figure C14 - Microstructure (1000X) of Specimens from Cropped End No. X-17 Austenitized at 1640 F, Isothermally Treated at 875 F and 1200 F, and then Tempered at 1150 F



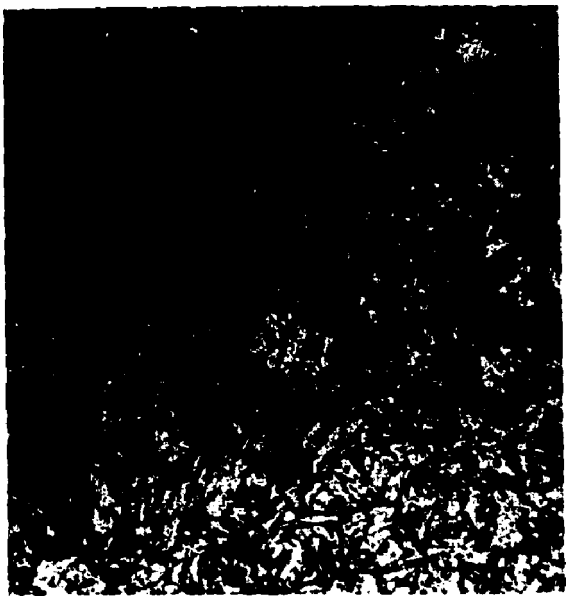
Trace Bainite-100% Martensite
875 F - 152 Sec



100% Martensite
1200 F - 3350 Sec



21% Bainite-79% Martensite
875 F - 1600 Sec



100% Martensite
1200 F - 8500 Sec

Cropped End No. X - 18

Figure C15 - Microstructure (1000X) of Specimens from Cropped End No. X-18 Austenitized at 1640 F, Isothermally Treated at 875 F and 1200 F, and then Tempered at 1150 F



53% Bainite-47% Martensite
875 F - 152 Sec



50% Ferrite-32% Pearlite
18% Martensite
1200 F - 3350 Sec

Cropped End No. Y - 1



76% Bainite
24% Martensite
875 F - 152 Sec



69% Ferrite-23% Pearlite
8% Martensite
1200 F - 3350 Sec

Cropped End No. Y - 3

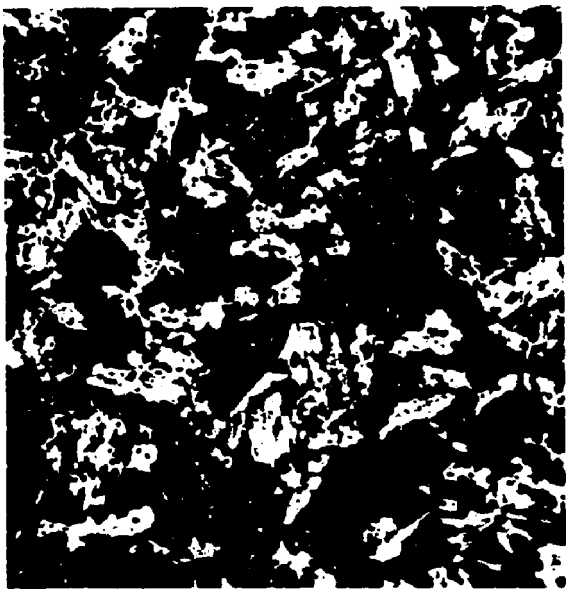
Figure C16 - Microstructure (1000X) of Specimens from Cropped Ends Y-1 and Y-3 Austenitized at 1640 F, Isothermally Treated at 875 F and 1200 F, and then Tempered at 1150 F



9% Bainite-91% Martensite
875 F - 152 Sec



22% Ferrite-78% Martensite
1200 F - 3350 Sec



41% Bainite
59% Martensite
875 F - 1600 Sec



53% Ferrite-4% Pearlite
43% Martensite
1200 F - 8500 Sec

Cropped End No. Y - 2

Figure C17 - Microstructure (1000X) of Specimens from Cropped End No. Y-2 Austenitized at 1640 F, Isothermally Treated at 875 F and 1200 F, and then Tempered at 1150 F



1% Bainite-99% Martensite
875 F - 152 Sec



7% Ferrite-93% Martensite
1200 F - 3350 Sec



39% Bainite-61% Martensite
875 F - 1600 Sec



21% Ferrite-79% Martensite
1200 F - 8500 Sec

Cropped End No. Y - 4

Figure C18 - Microstructure (1000X) of Specimens from Cropped End No. Y-4 Austenitized at 1640 F, Isothermally Treated at 875 F and 1200 F, and then Tempered at 1150 F

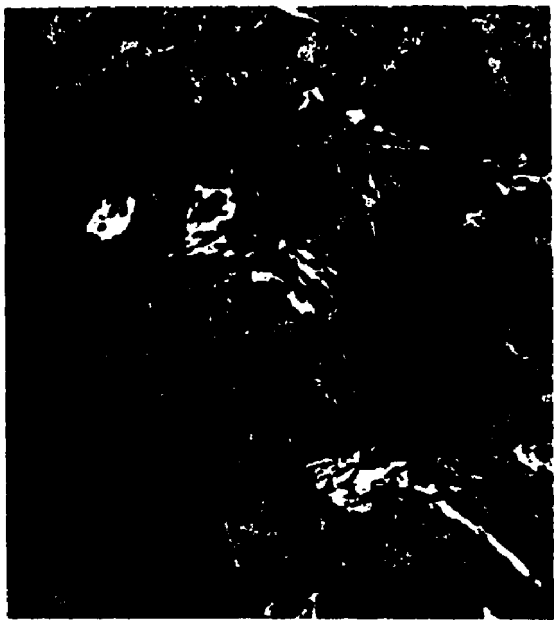


3% Bainite-97% Martensite
875 F - 152 Sec

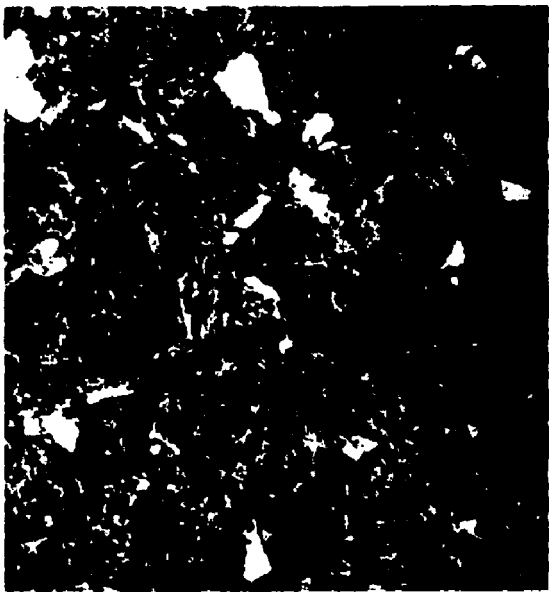


17% Ferrite-83% Martensite
1200 F - 3350 Sec

Cropped End No. Y - 5



1% Bainite-99% Martensite
875 F - 152 Sec



12% Ferrite-88% Martensite
1200 F - 3350 Sec

Cropped End No. Y - 6

Figure C19 - Microstructure (1000X) of Specimens from Cropped Ends Y-5 and Y-6 Austenitized at 1640 F, Isothermally Treated at 875 F and 1200 F, and then Tempered at 1150 F



1% Bainite-99% Martensite
875 F - 152 Sec



3% Ferrite-97% Martensite
1200 F - 3350 Sec

Cropped End No. Y - 7



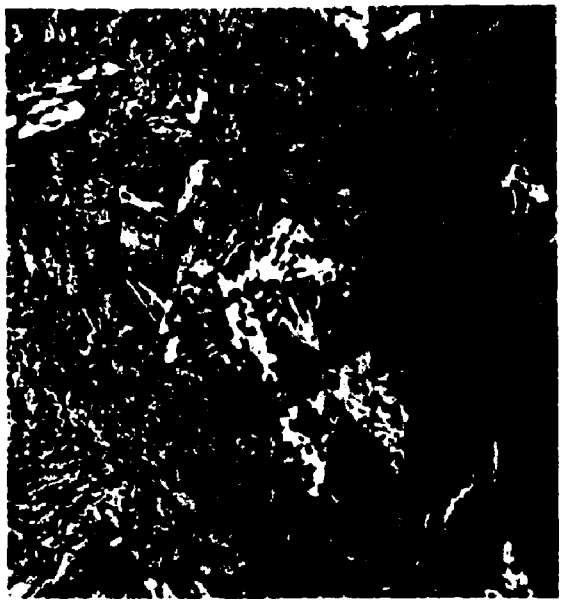
1% Bainite-99% Martensite
875 F - 152 Sec



1% Ferrite-99% Martensite
1200 F - 3350 Sec

Cropped End No. Y - 8

Figure C20 - Microstructure (1000X) of Specimens from Cropped Ends Y-7 and Y-8 Austenitized at 1640 F, Isothermally Treated at 875 F and 1200 F, and then Tempered at 1150 F



6% Bainite-94% Martensite
875 F - 152 Sec

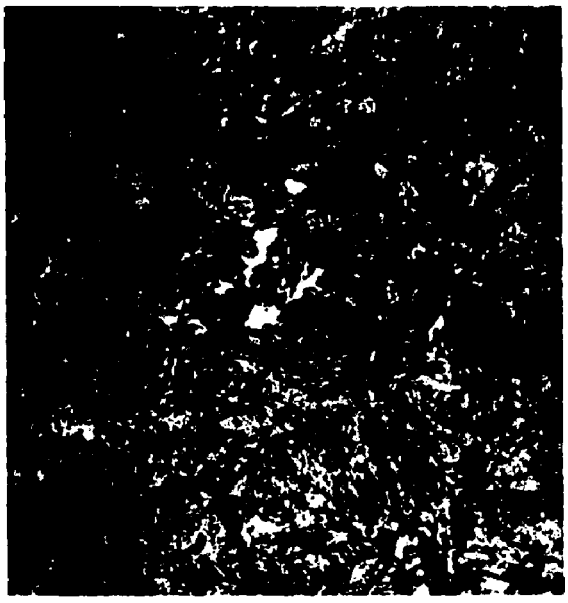


2% Ferrite-98% Martensite
1200 F - 3350 Sec

Cropped End No. Y - 9



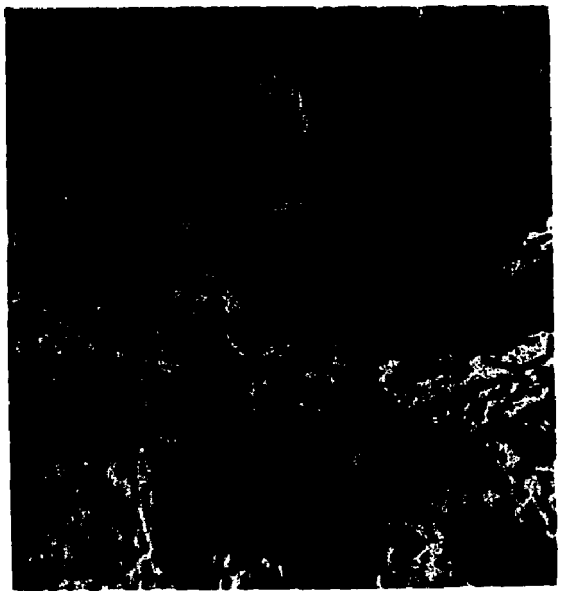
4% Bainite-96% Martensite
875 F - 152 Sec



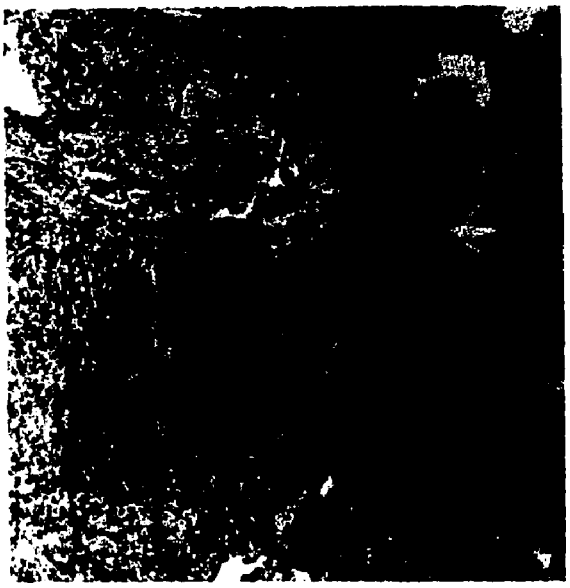
1% Ferrite-99% Martensite
1200 F - 3350 Sec

Cropped End No. Y - 10

Figure C21 - Microstructure (1000X) of Specimens from Cropped Ends Y-9 and Y-10 Austenitized at 1640 F, Isothermally Treated at 875 F and 1200 F, and then Tempered at 1150 F



27% Bainite-73% Martensite
875 F - 152 Sec



3% Ferrite-97% Martensite
1200 F - 3350 Sec



63 % Bainite-37% Martensite
875 F - 1600 Sec



44% Ferrite-56% Martensite
1200 F - 8500 Sec

Cropped End No. Y - 11

Figure C22 - Microstructure (1000X) of Specimens from Cropped End No. Y-11 Austenitized at 1640 F, Isothermally Treated at 875 F and 1200 F, and then Tempered at 1150 F

APPENDIX D

CHARPY V-NOTCH IMPACT

PROPERTY CURVES

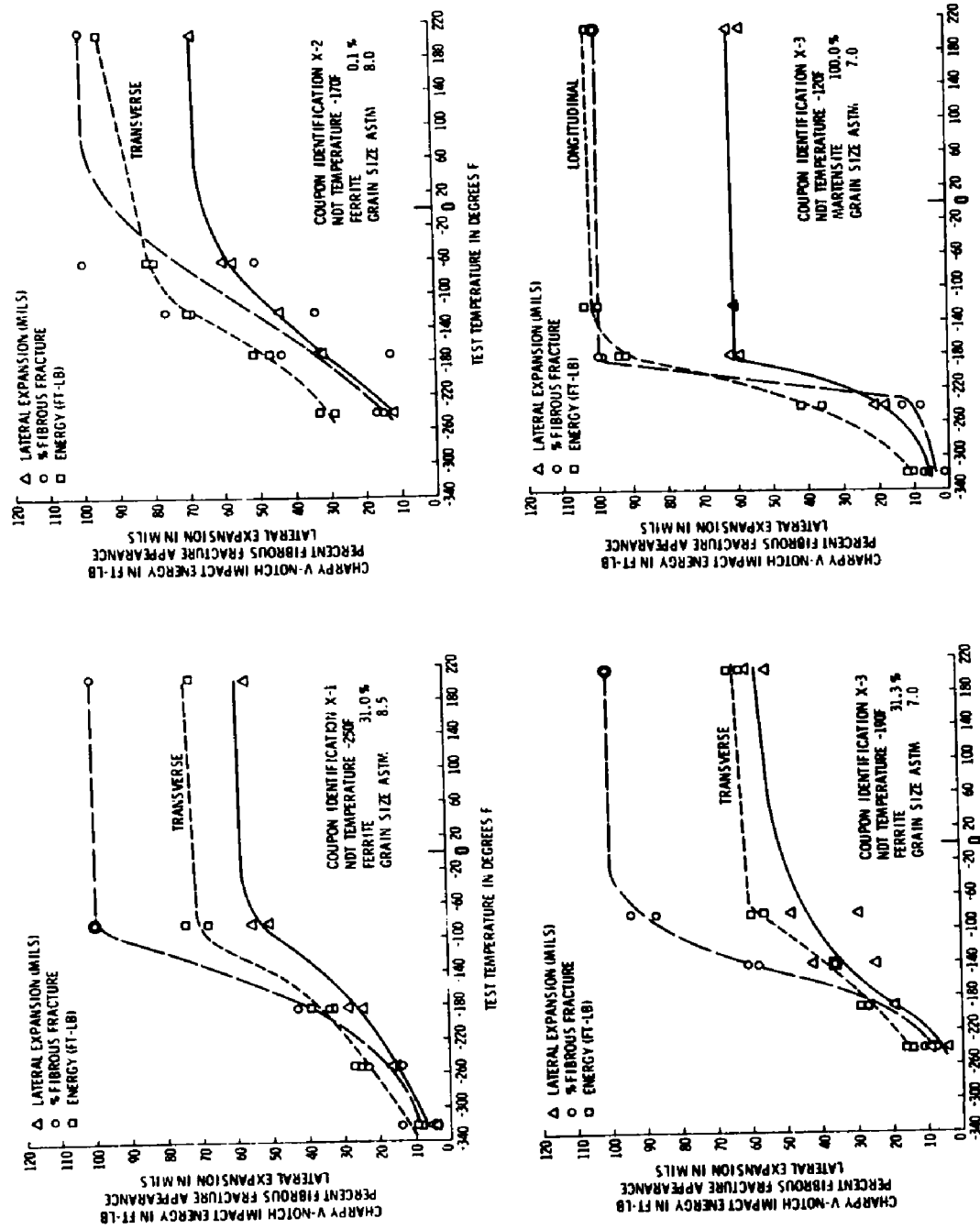


FIGURE D 1 – TRANSVERSE AND LONGITUDINAL CHARPY V–NOTCH IMPACT DATA FOR SPECIMENS AUSTENITIZED AT 1640F FOR ½ HOUR, WATER QUENCHED AND TEMPERED AT 1150F FOR ONE HOUR

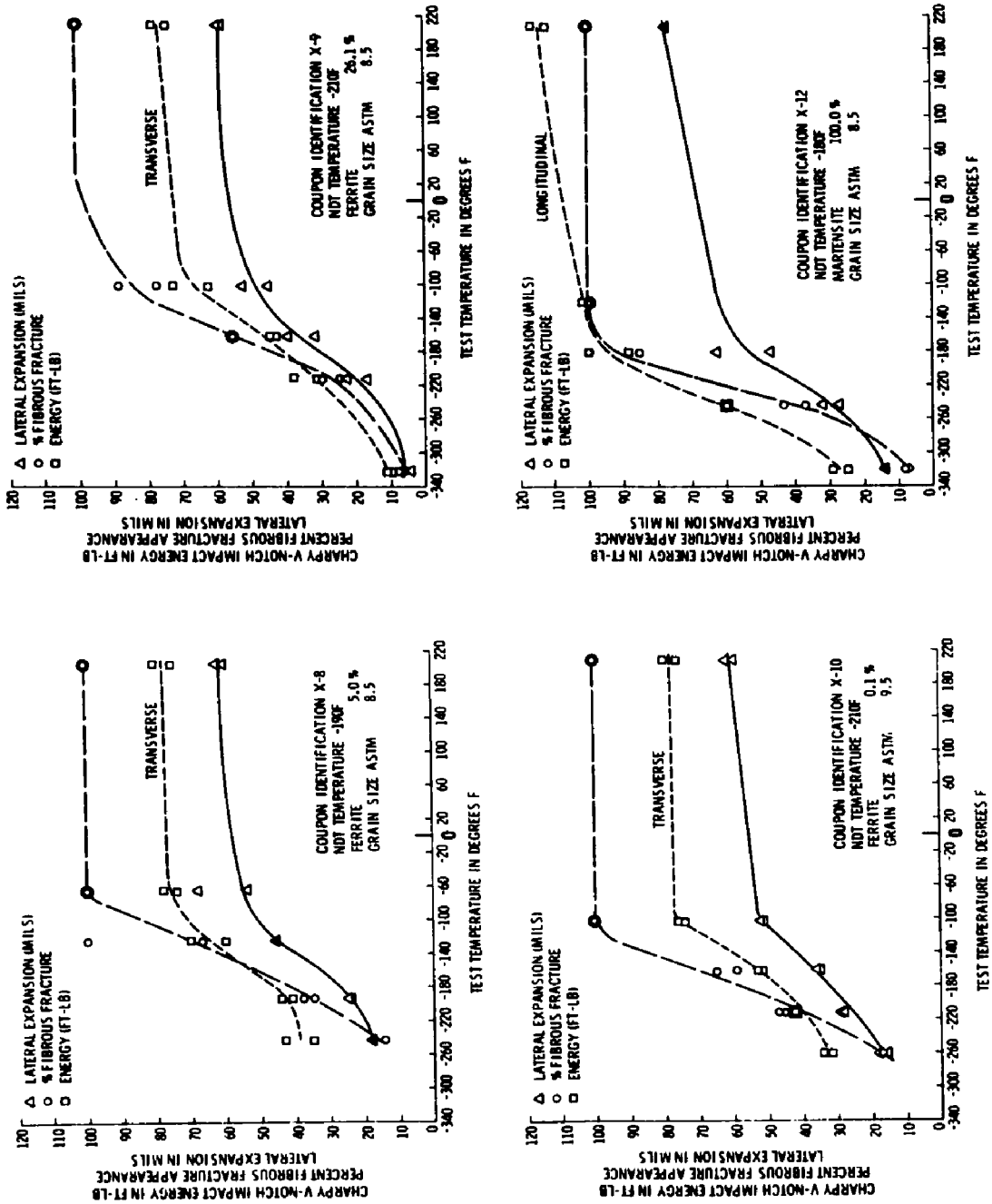


FIGURE D 1 – CONTINUED

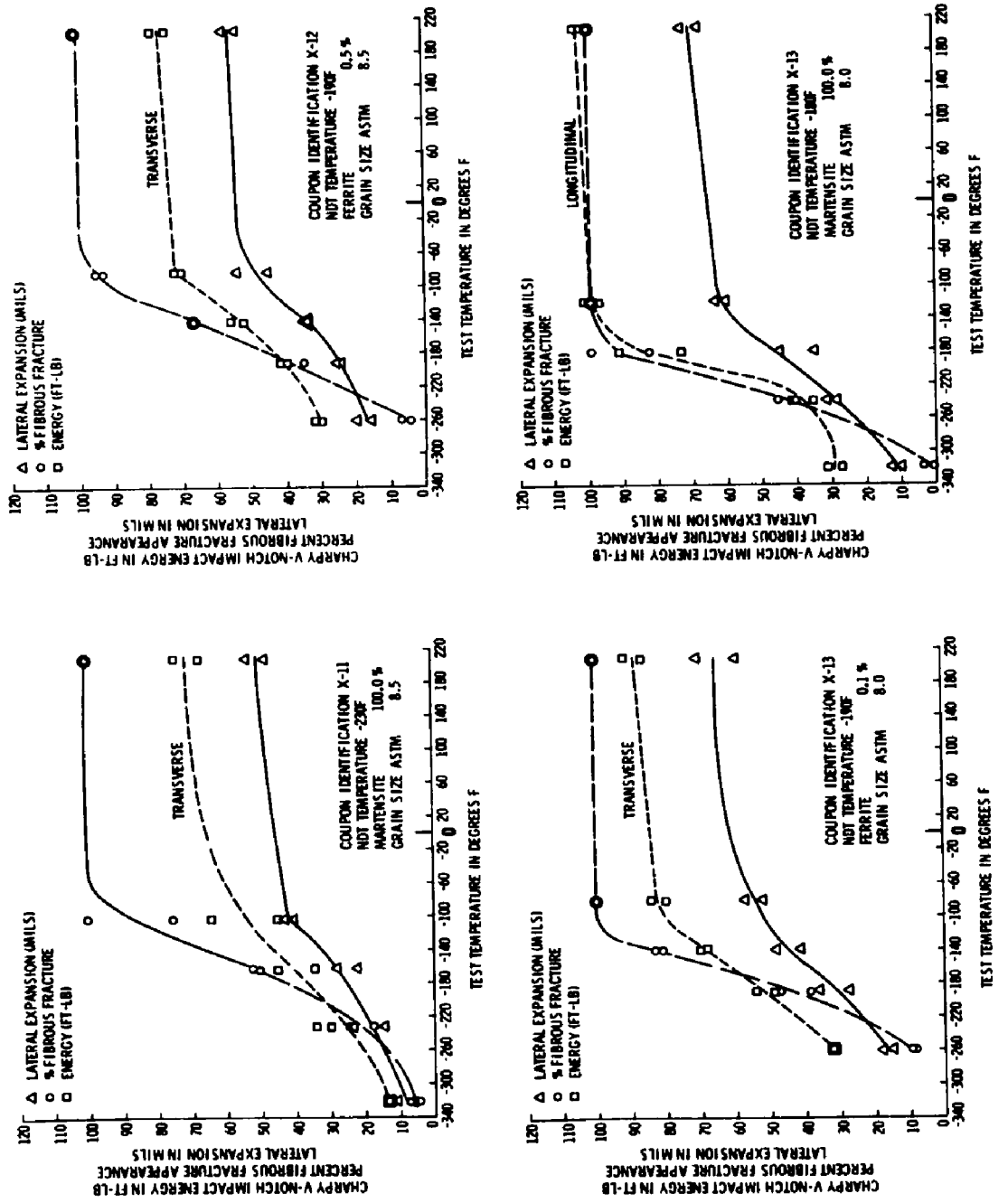


FIGURE D 1 – CONTINUED

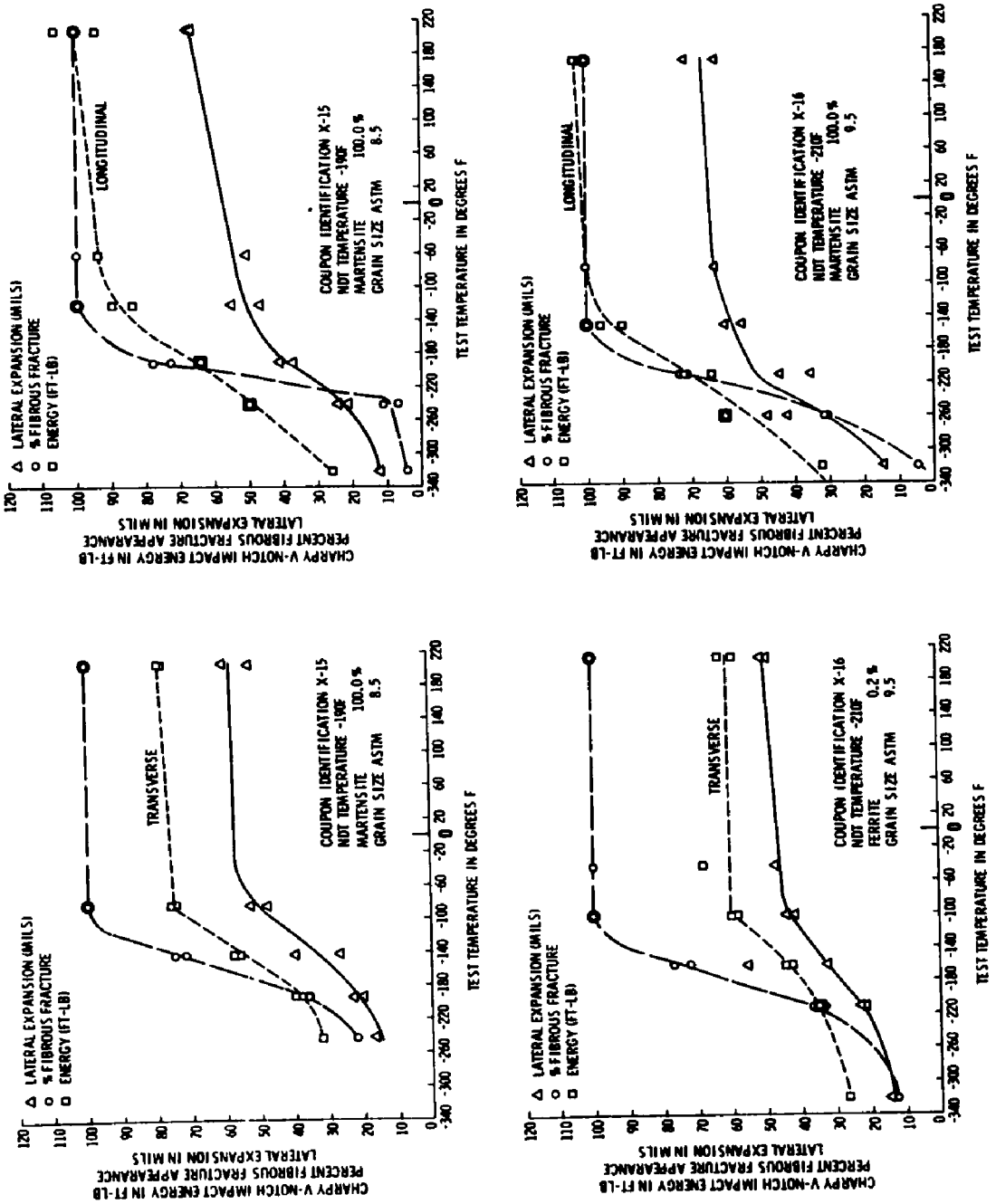


FIGURE D 1 – CONTINUED

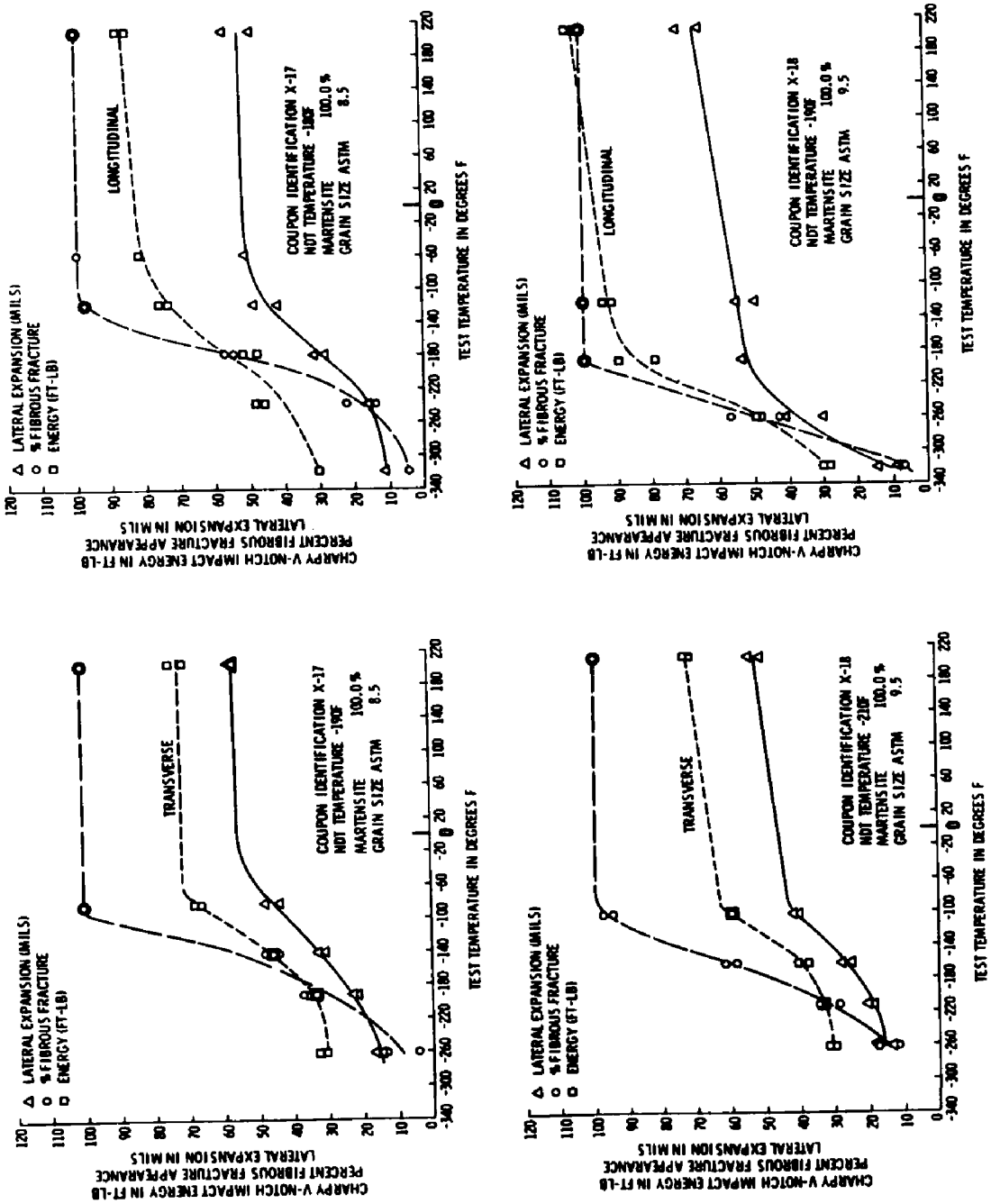


FIGURE D 1 – CONTINUED

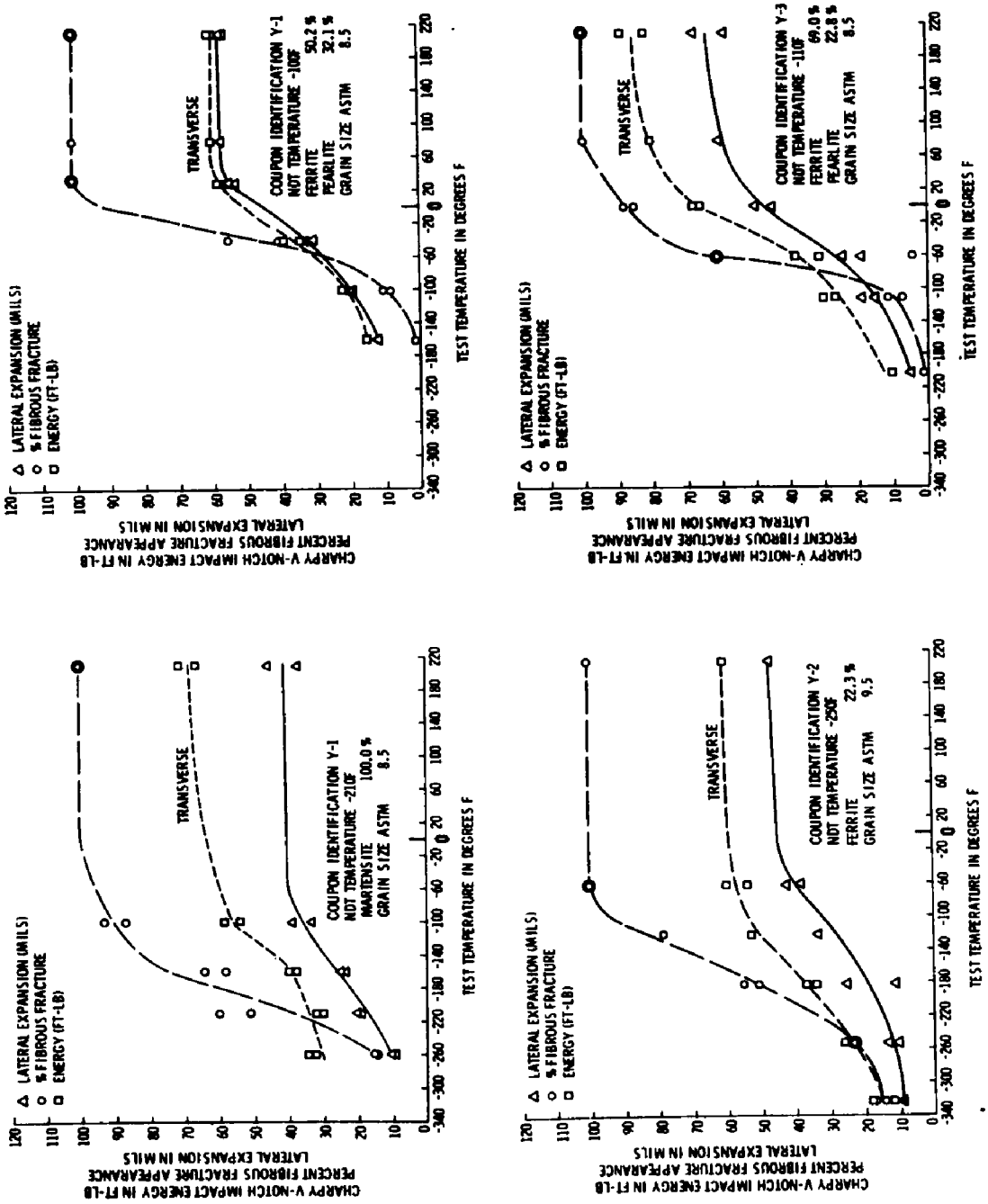


FIGURE D 1 – CONTINUED

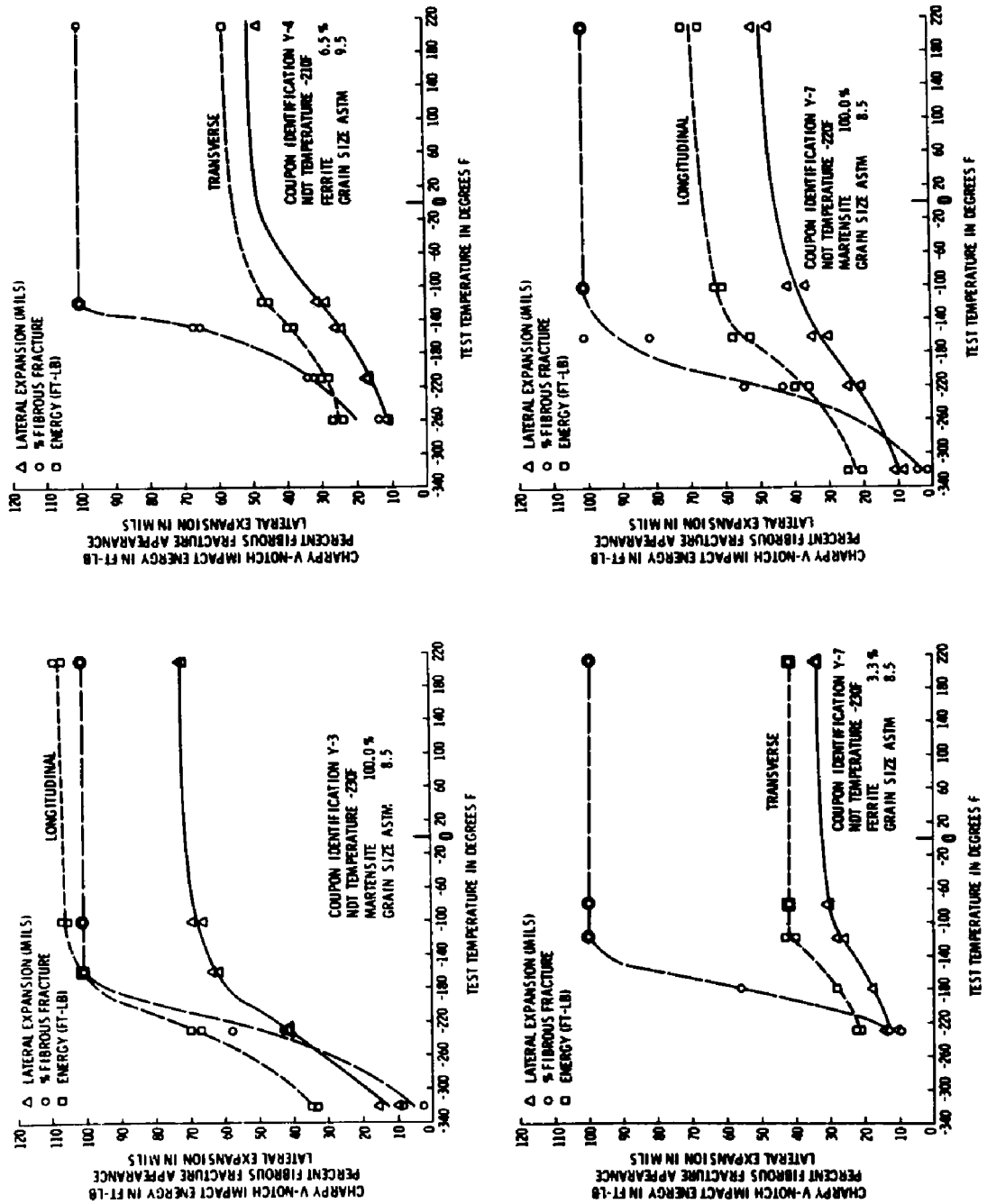


FIGURE D 1 – CONTINUED

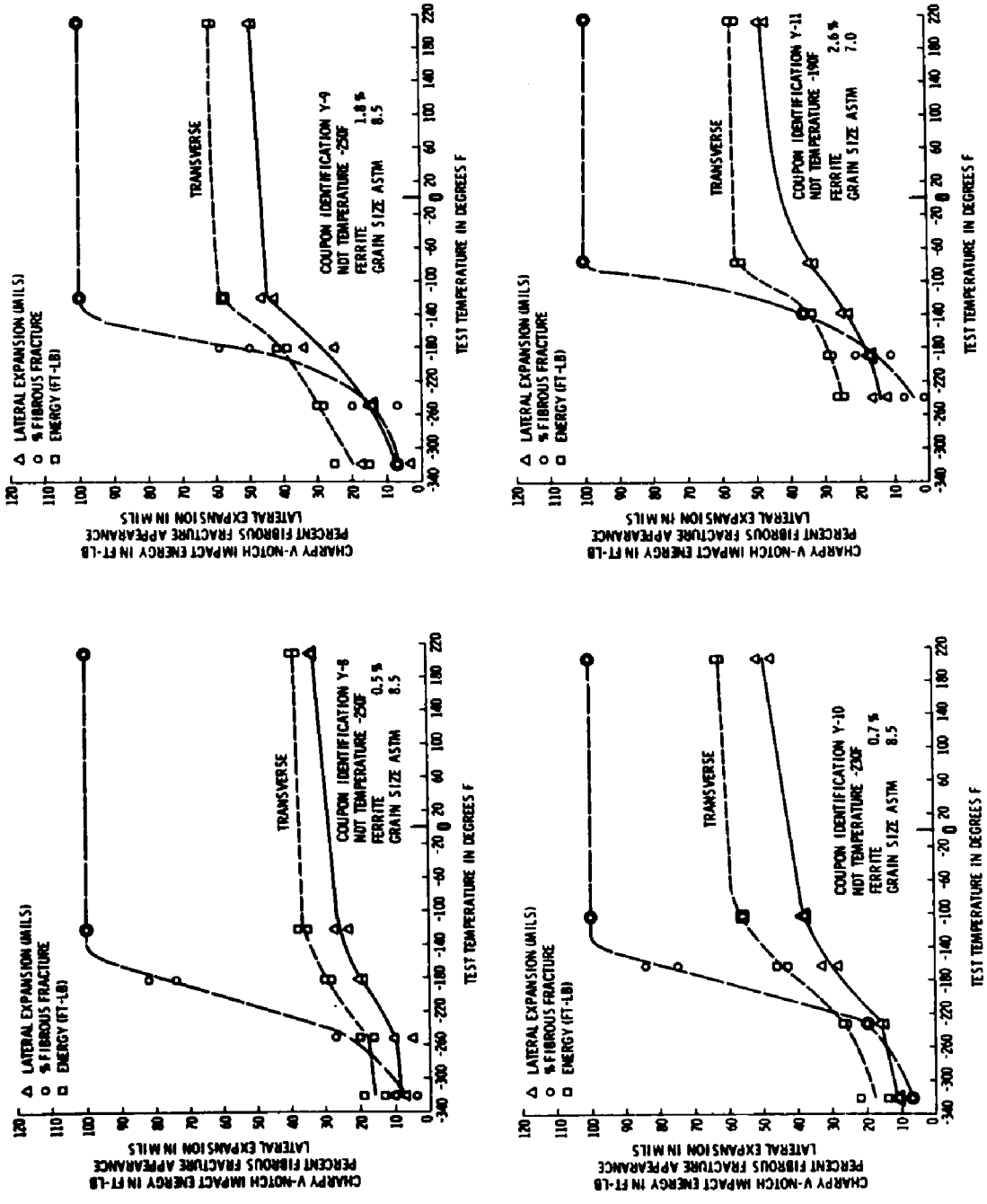


FIGURE D 1 – CONTINUED

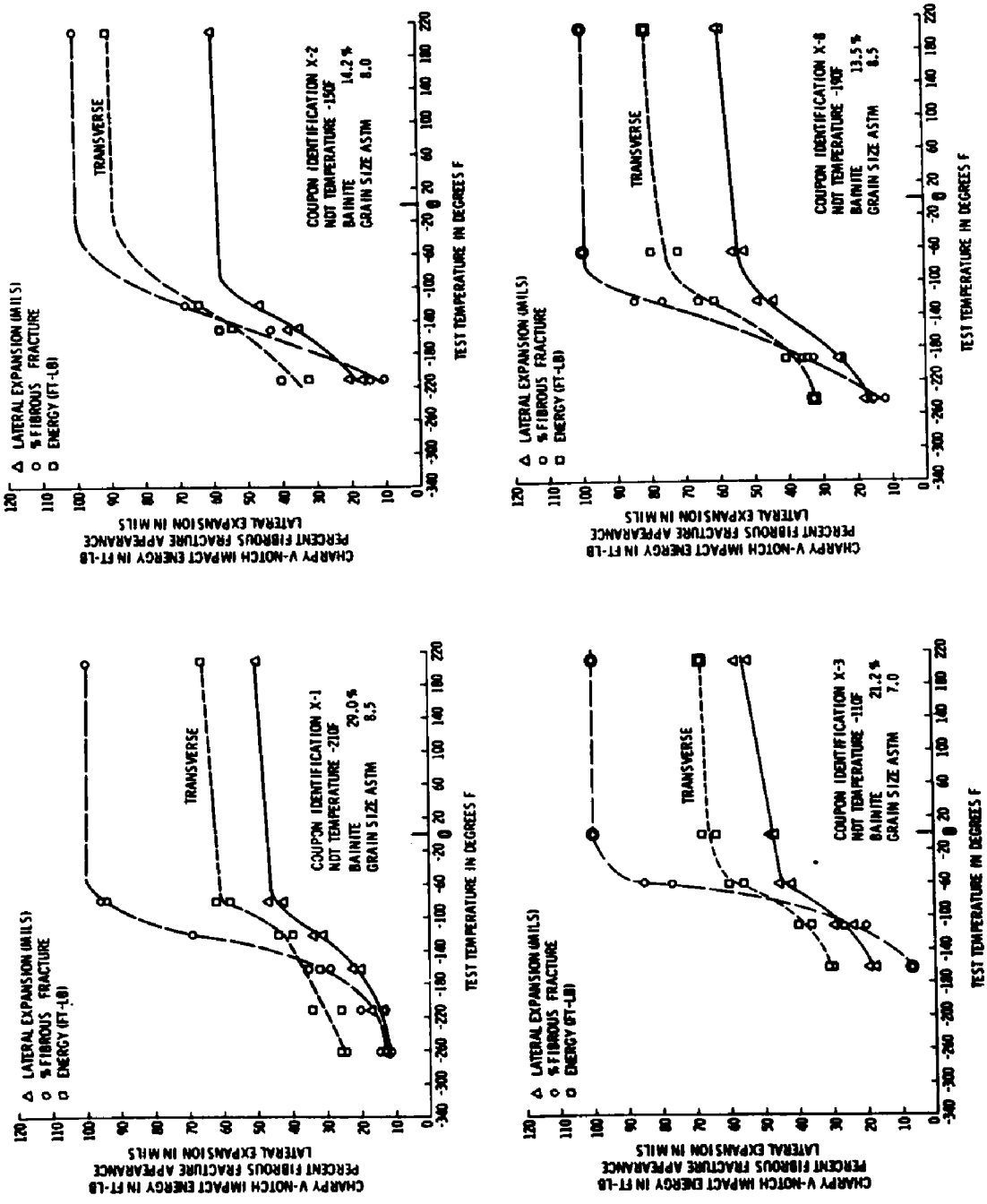


FIGURE D 2 -- TRANSVERSE CHARPY V-NOTCH IMPACT DATA FOR SPECIMENS AUSTENITIZED AT 1640F FOR ½ HOUR, ISOTHERMALLY TREATED AT 875F FOR 152 SECONDS, WATER QUENCHED AND TEMPERED AT 1150F FOR ONE HOUR

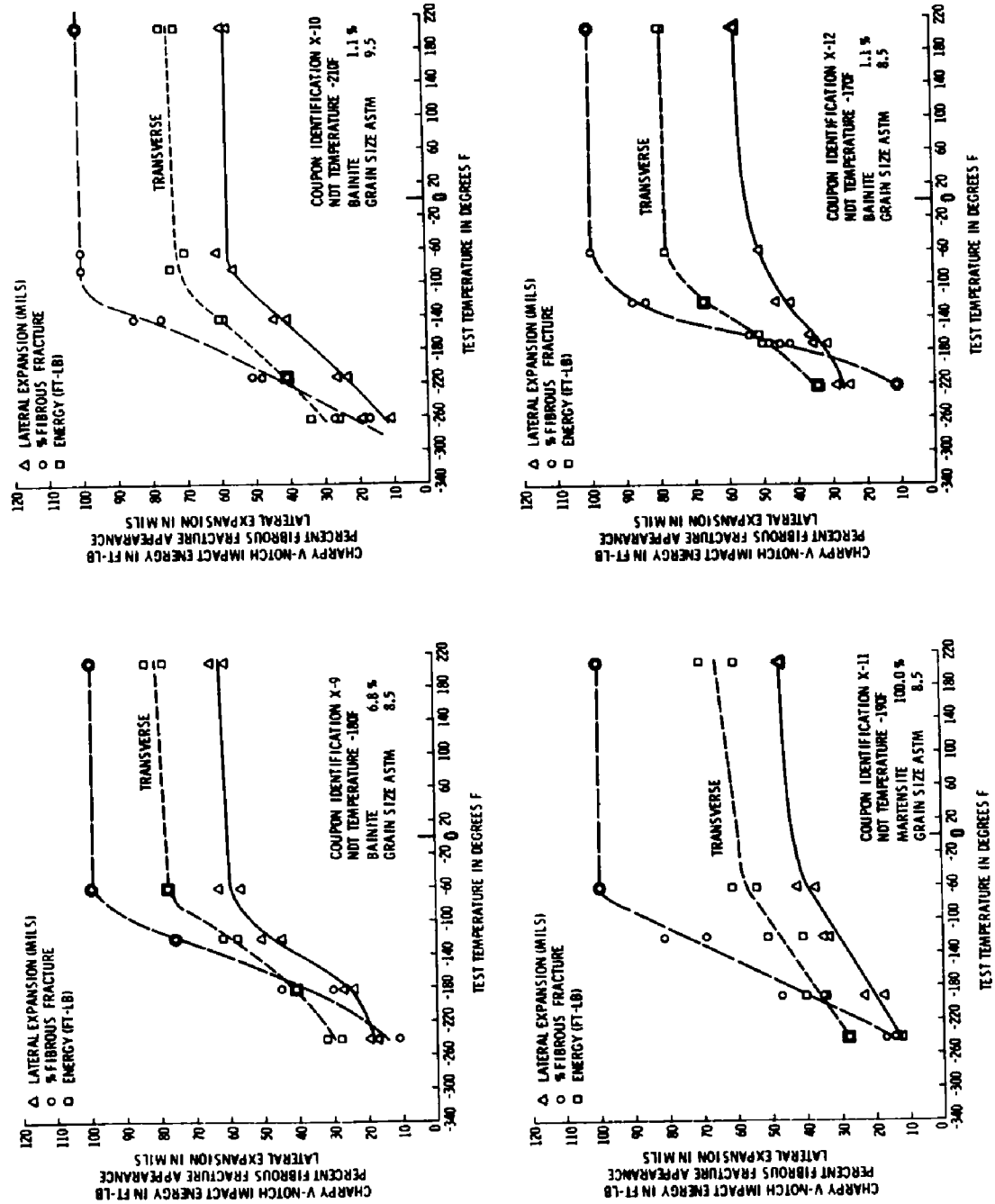


FIGURE D 2 – CONTINUED

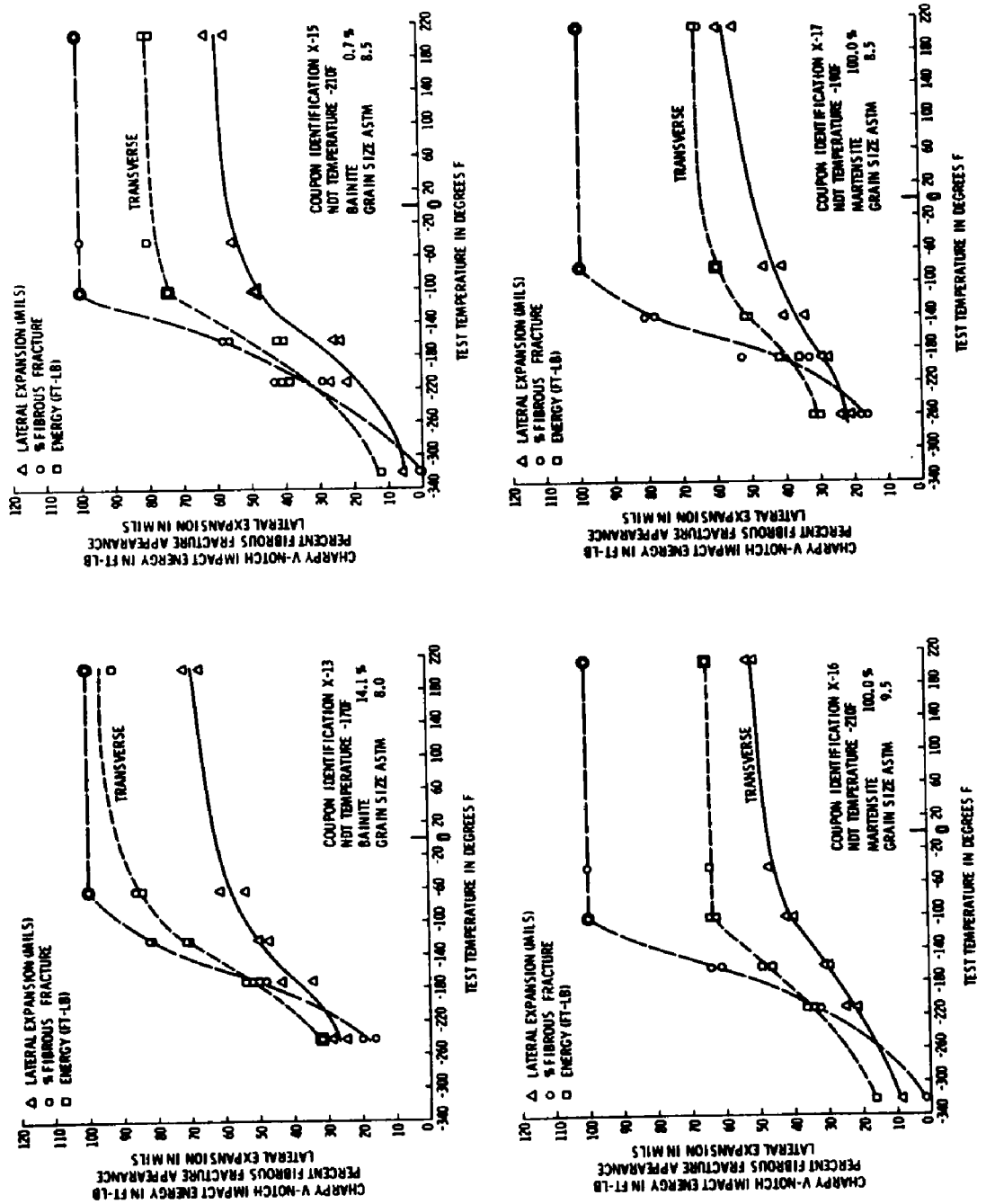


FIGURE D 2 – CONTINUED

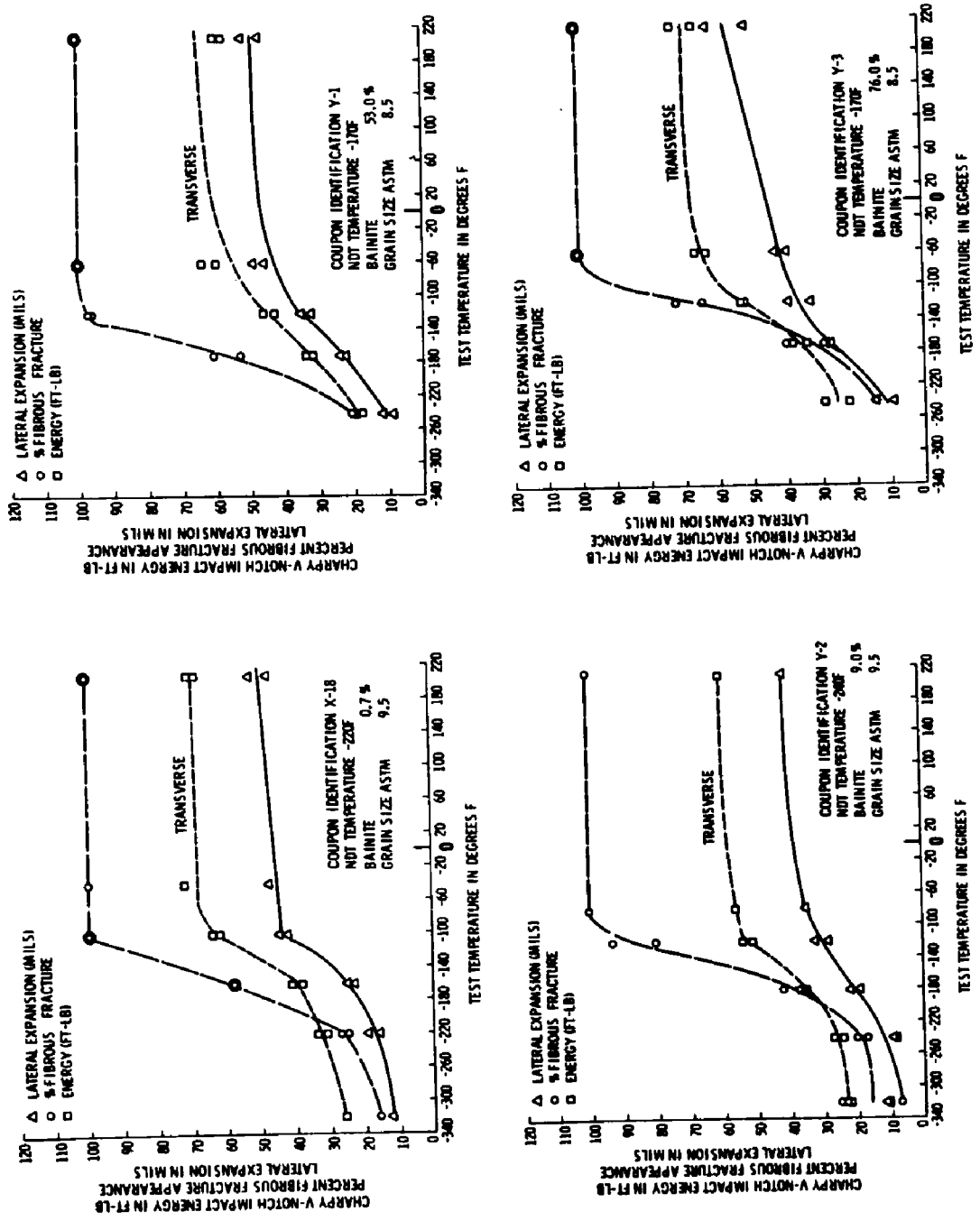


FIGURE D 2 – CONTINUED

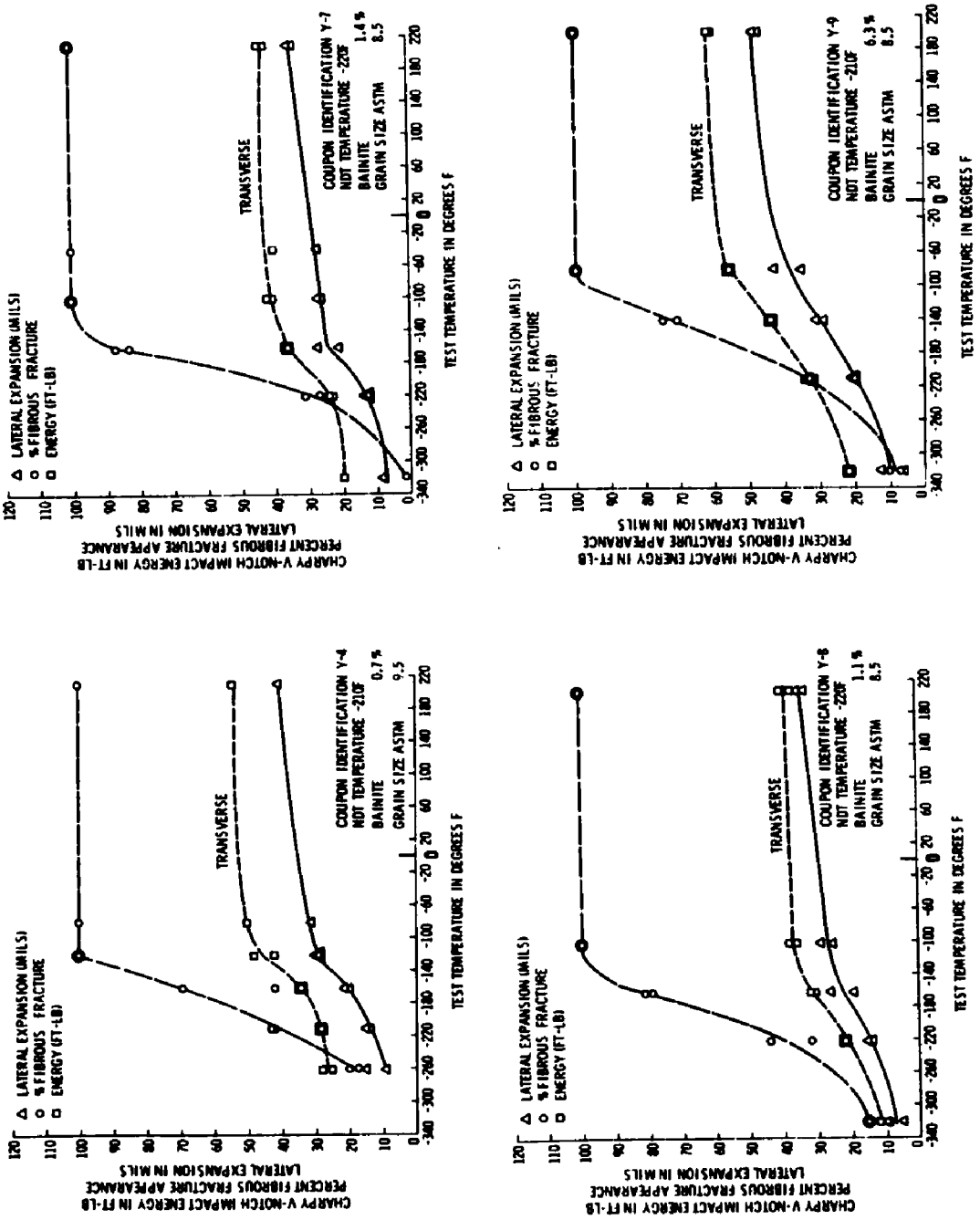


FIGURE D 2 – CONTINUED

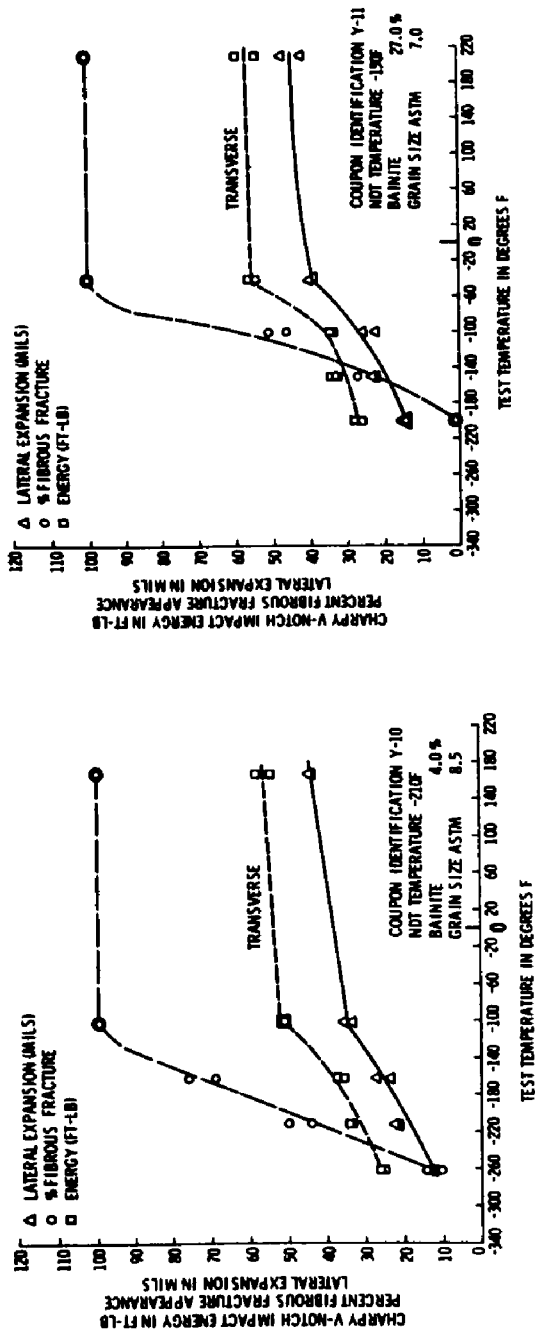


FIGURE D 2 – CONTINUED

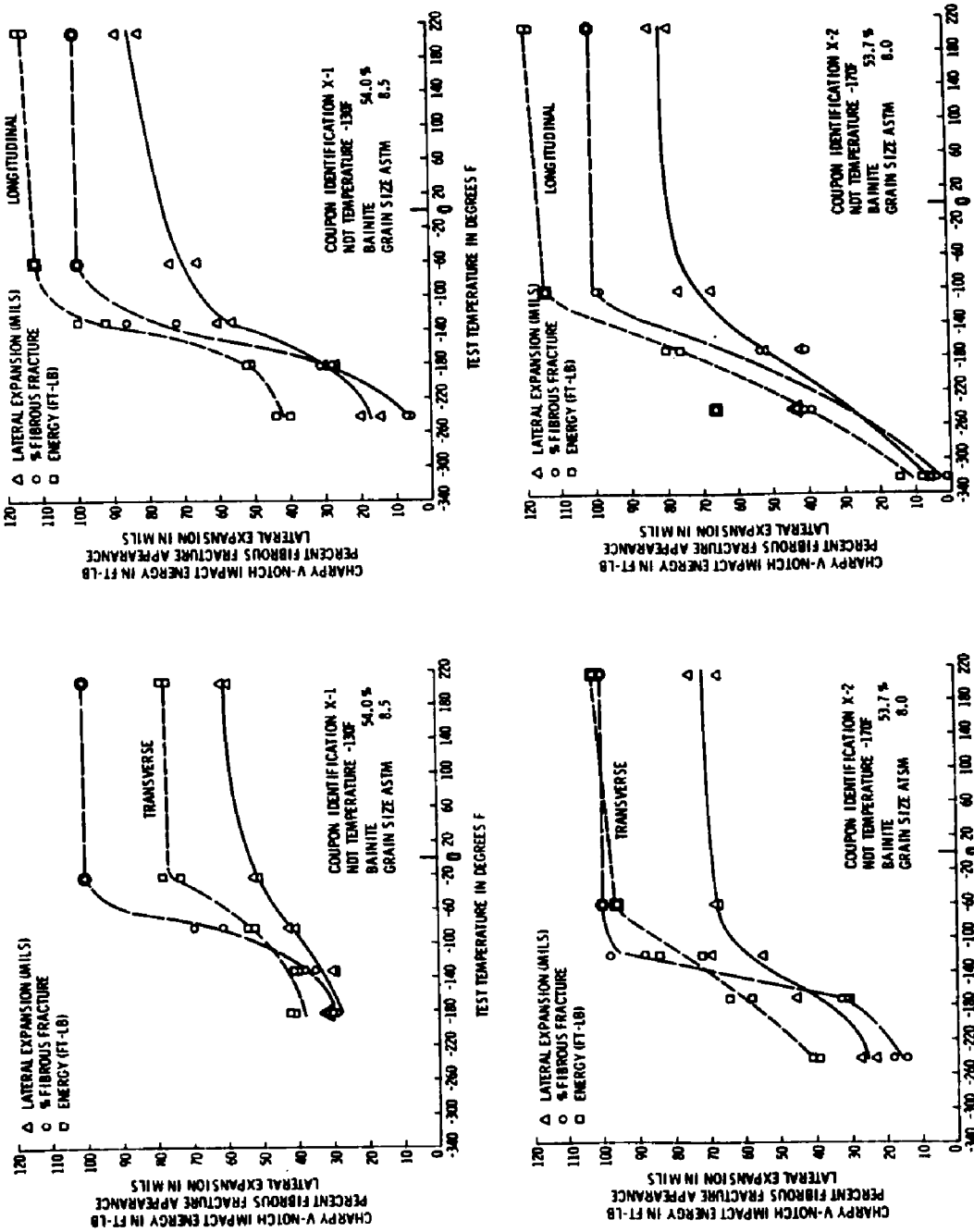


FIGURE D 3 – TRANSVERSE AND LONGITUDINAL CHARPY V–NOTCH IMPACT DATA FOR SPECIMENS AUSTENITIZED AT 1640F FOR ½ HOUR, ISOTHERMALLY TREATED AT 875F FOR 1600 SECONDS, WATER QUENCHED AND TEMPERED AT 1150F FOR ONE HOUR

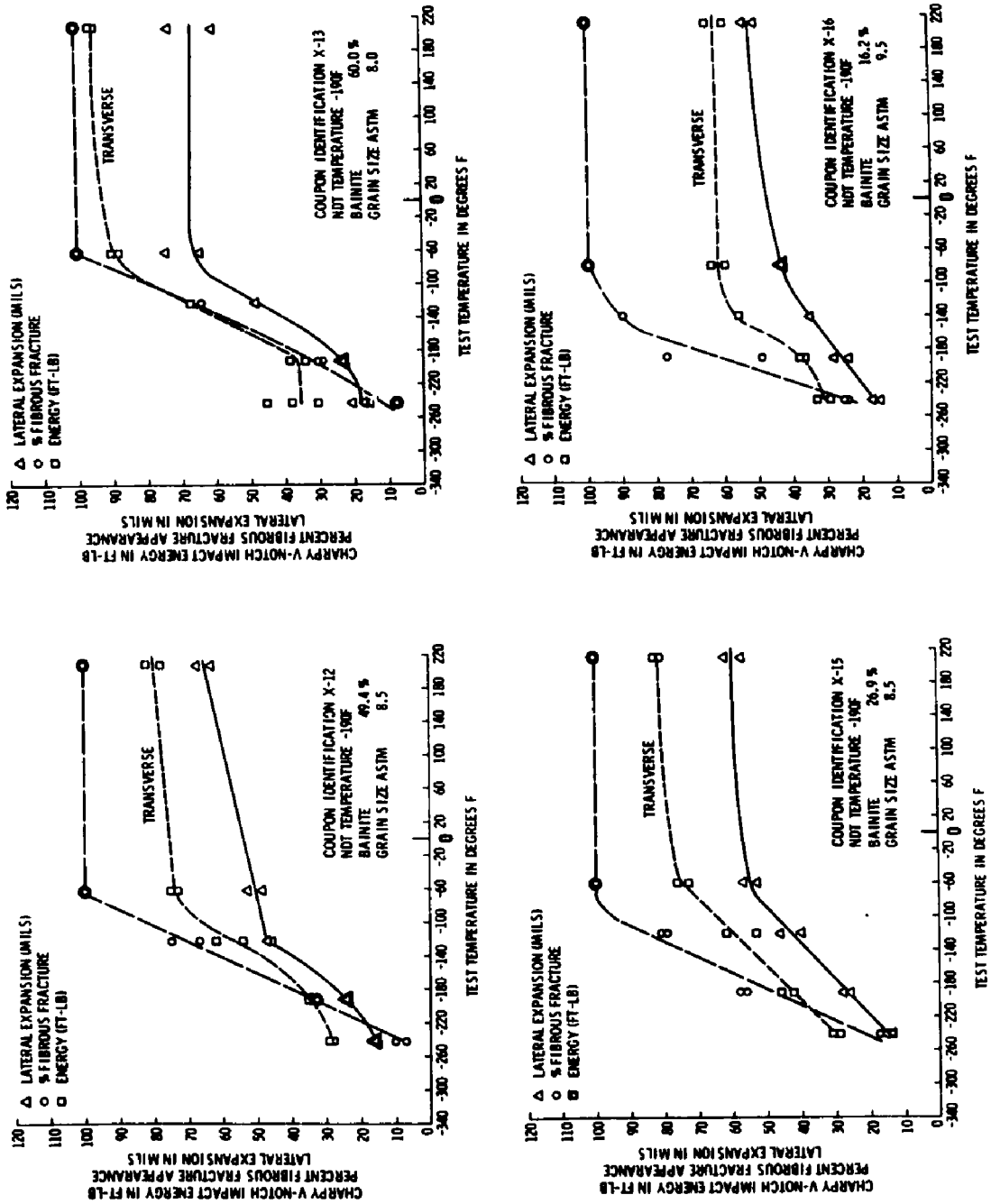


FIGURE D 3 – CONTINUED

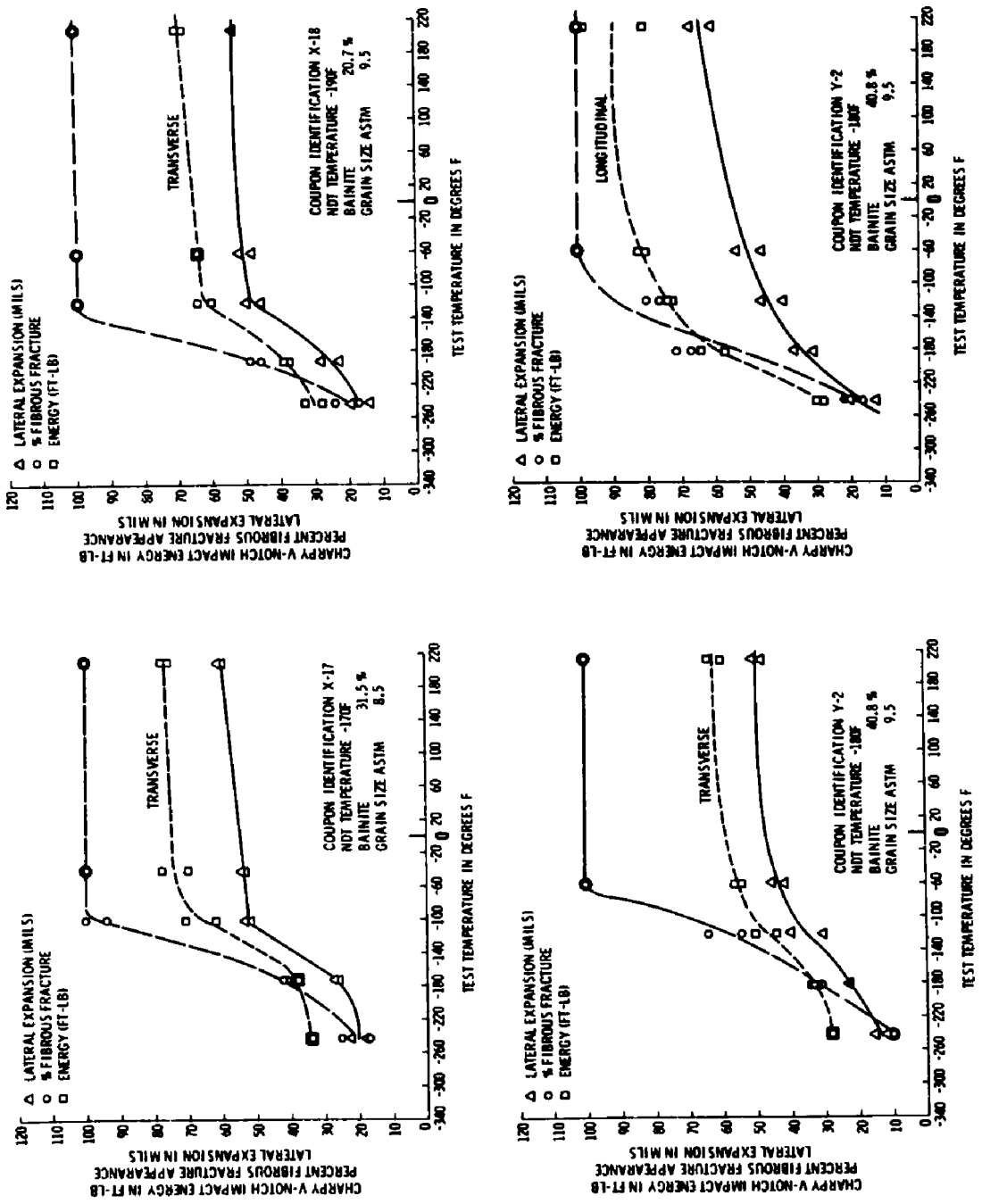
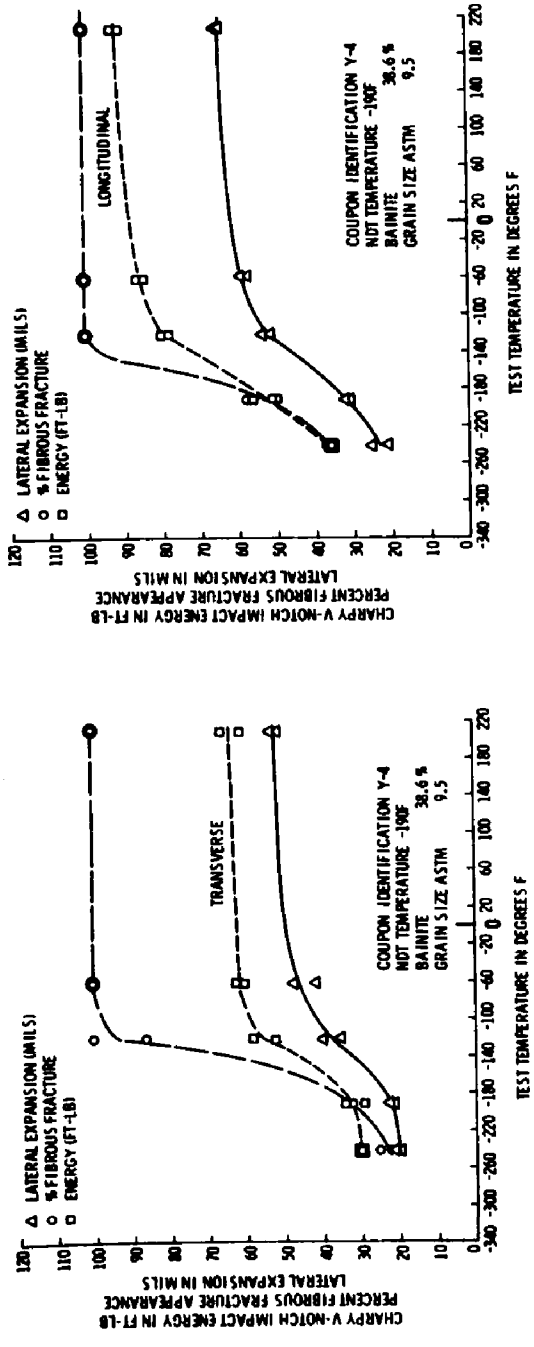


FIGURE D 3 – CONTINUED



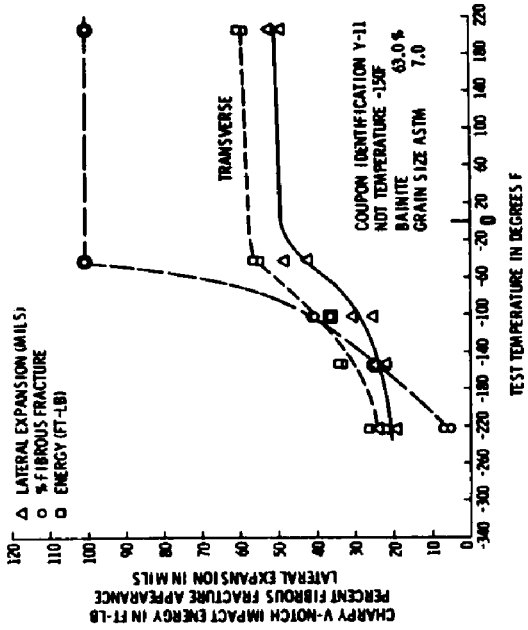


FIGURE D 3 – CONTINUED

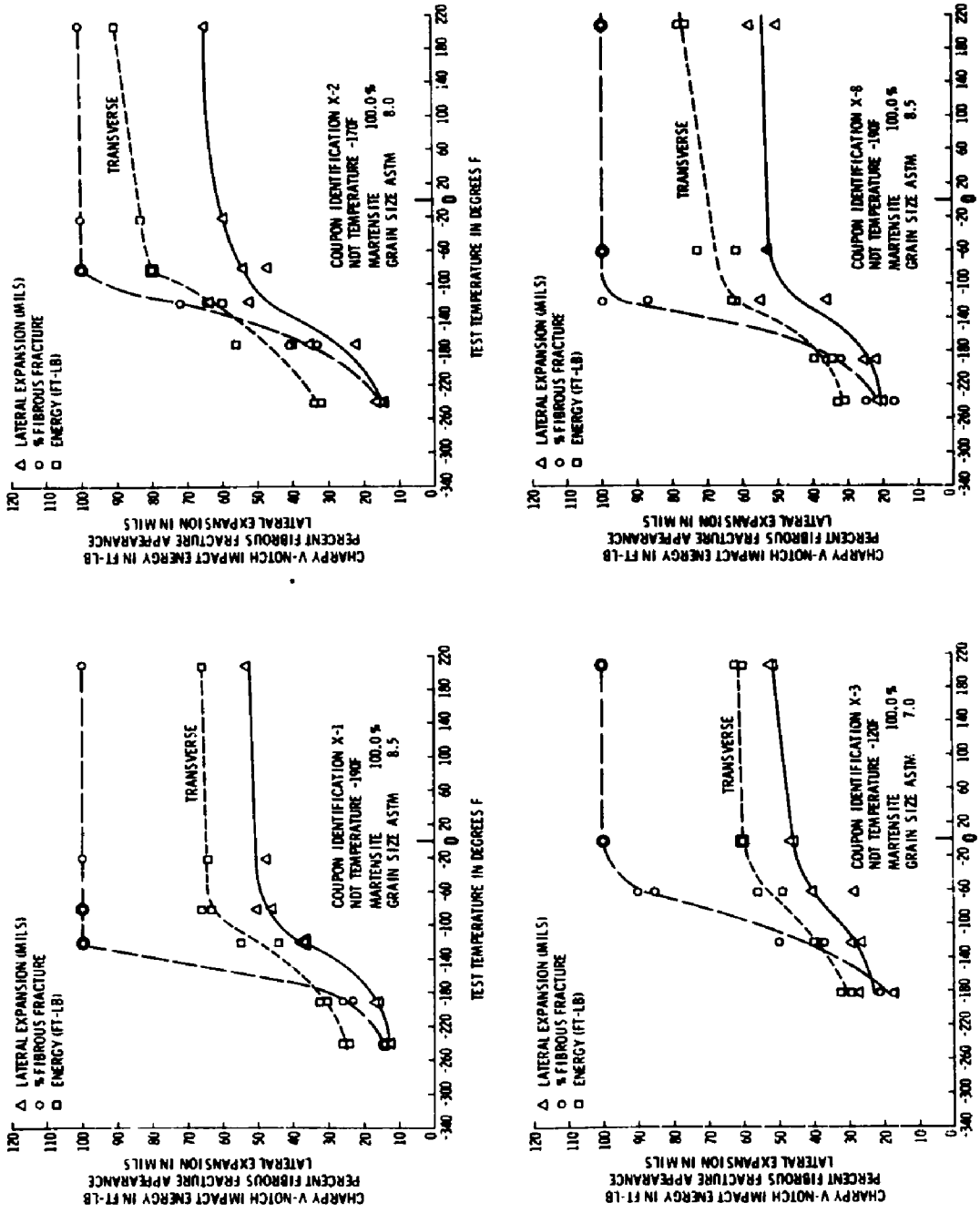


FIGURE D 4 – TRANSVERSE CHARPY V–NOTCH IMPACT DATA FOR SPECIMENS AUSTENITIZED AT 1640F FOR ½ HOUR, ISOTHERMALLY TREATED AT 1200F FOR 3350 SECONDS, WATER QUENCHED AND TEMPERED AT 1150F FOR ONE HOUR

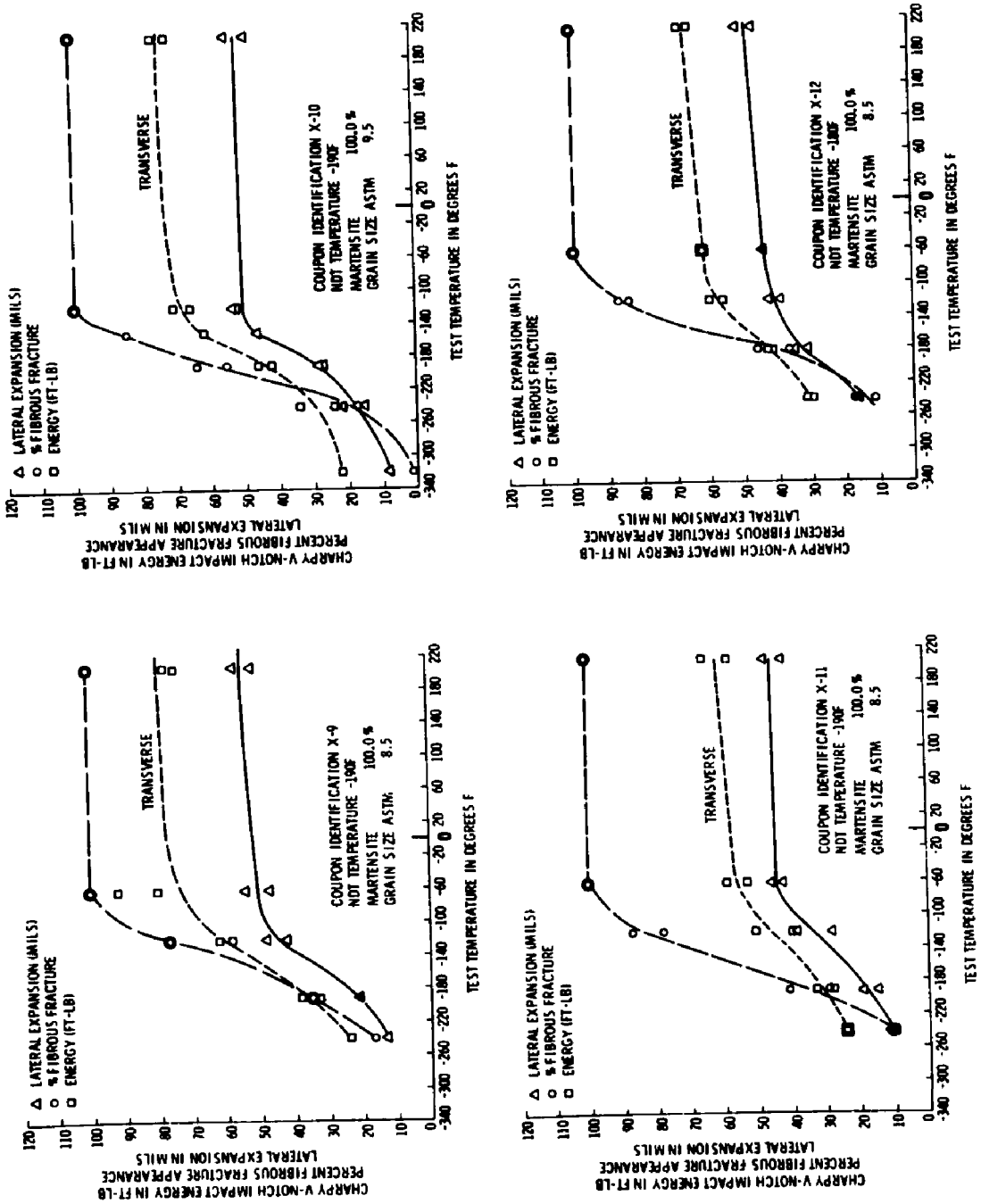


FIGURE D 4 – CONTINUED

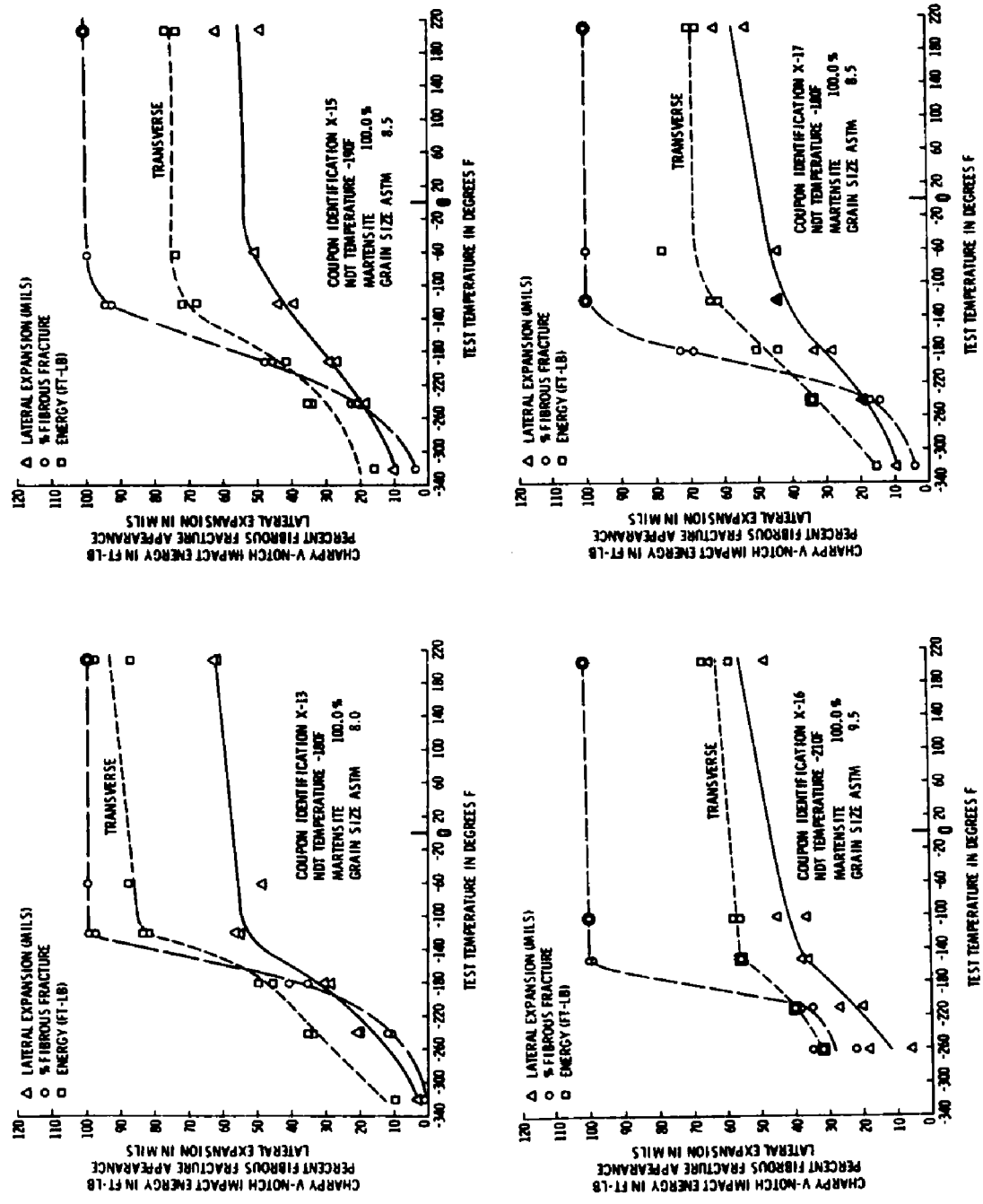


FIGURE D 4 – CONTINUED

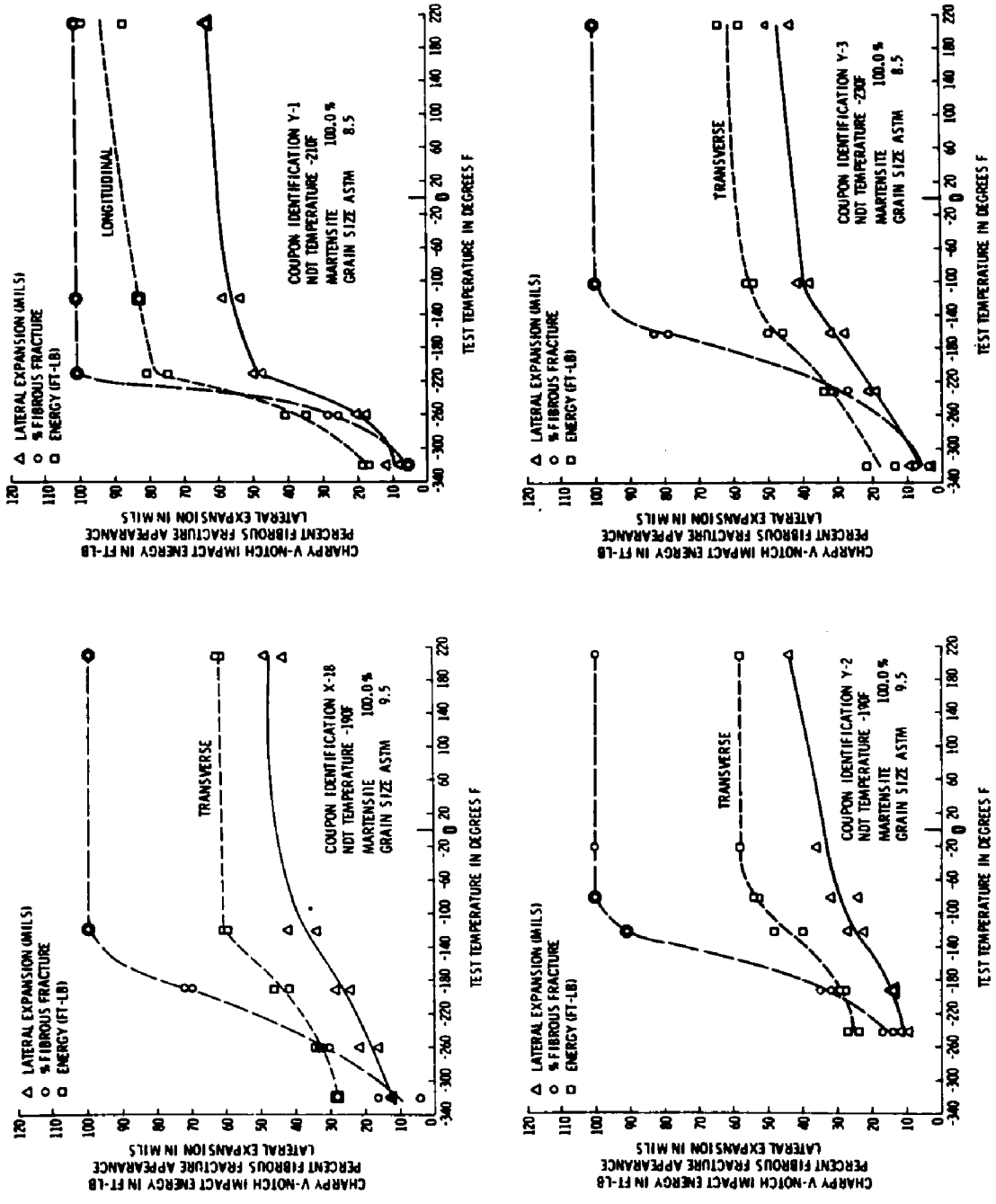


FIGURE D 4 – CONTINUED

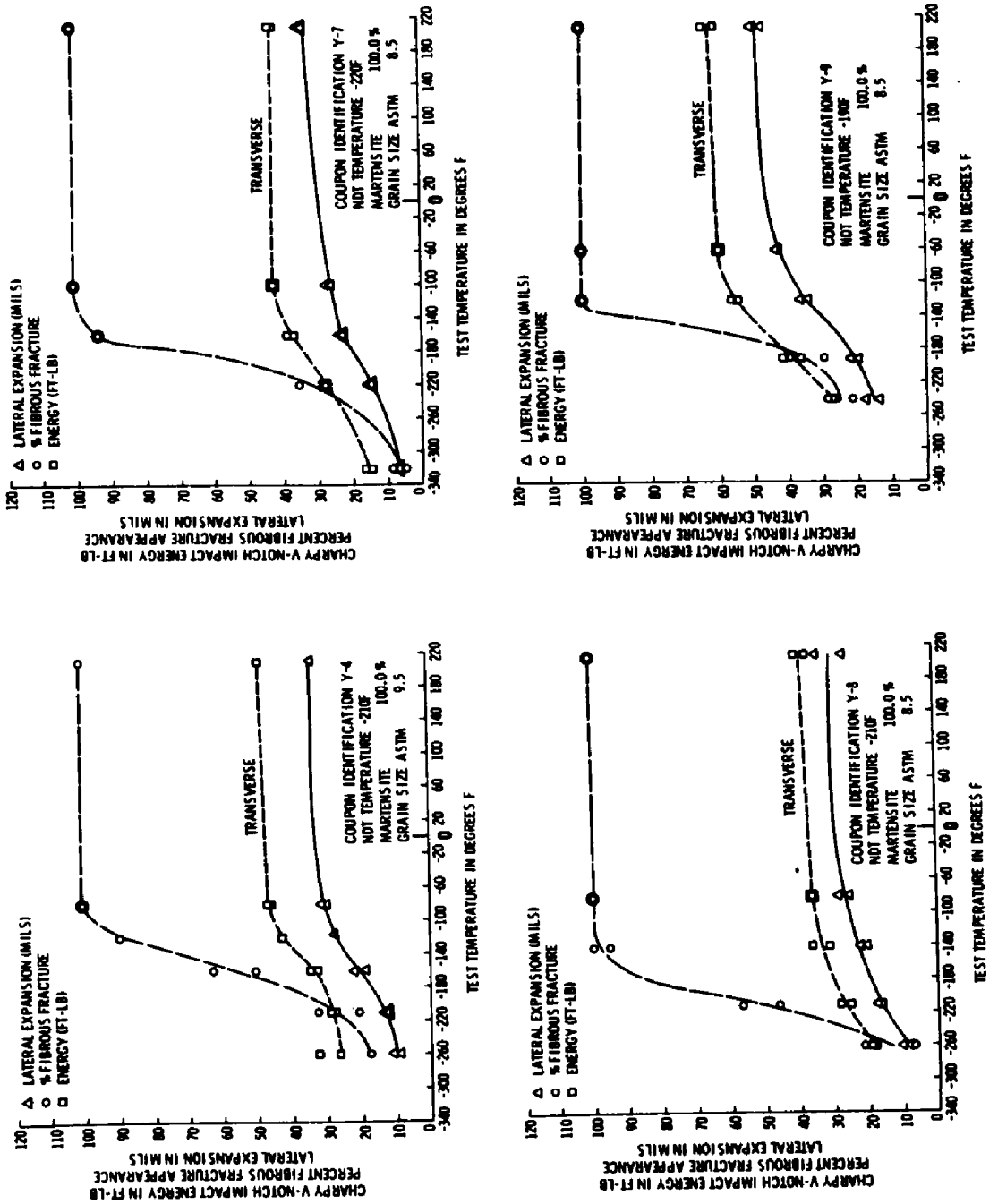


FIGURE D 4 – CONTINUED

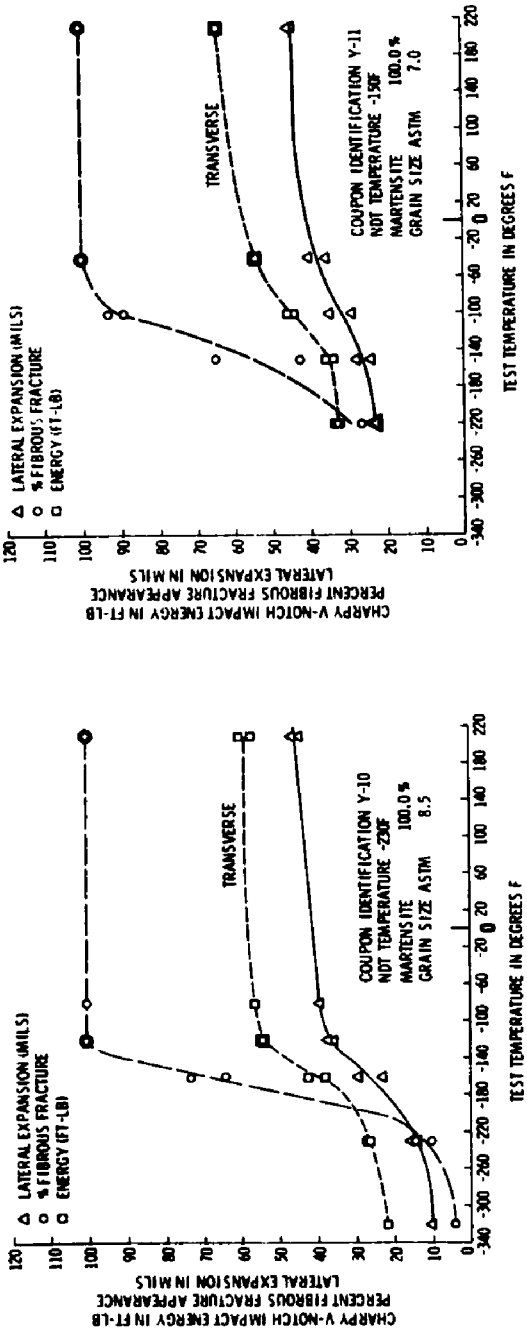


FIGURE D 4 – CONTINUED

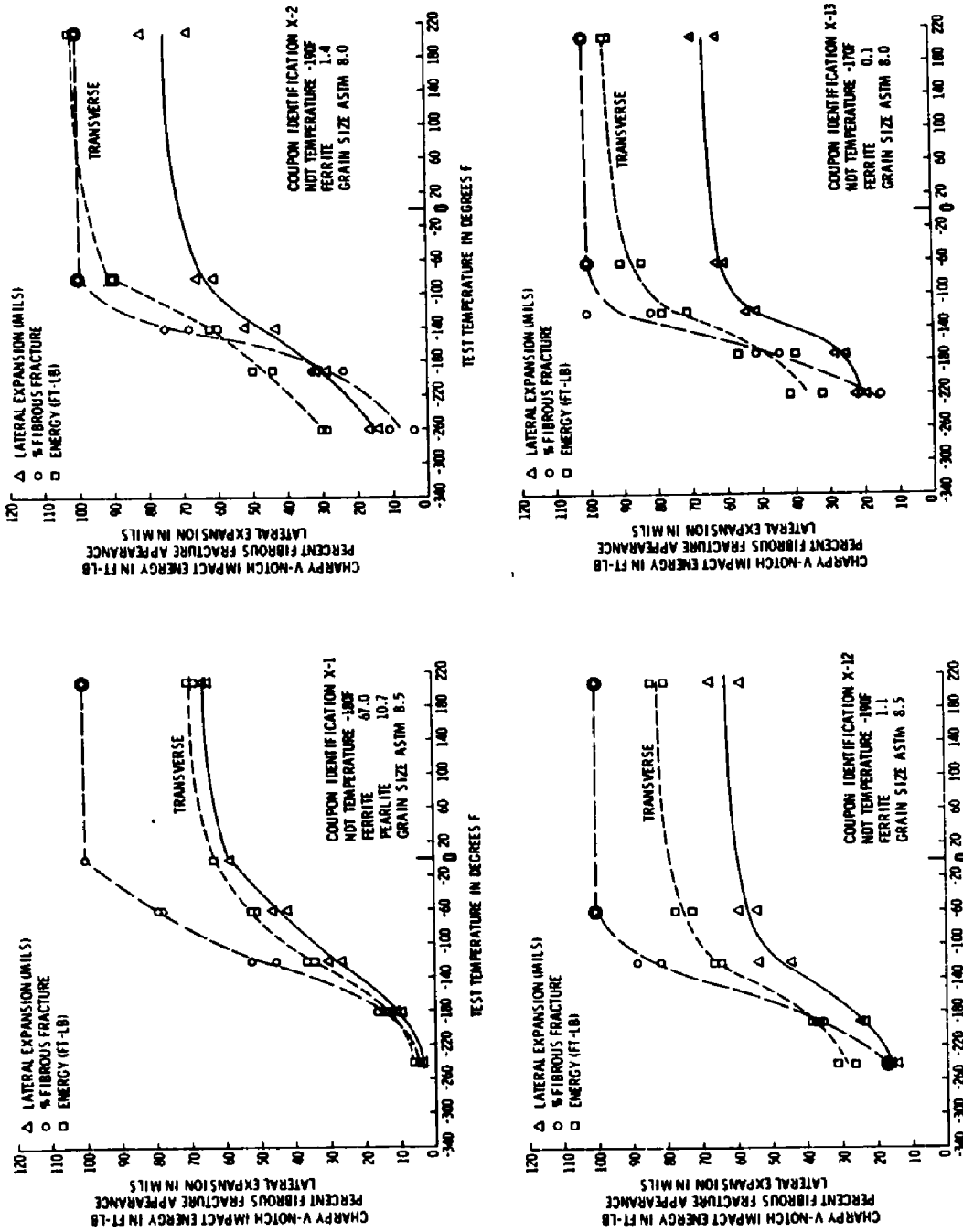


FIGURE D 5 – TRANSVERSE CHARPY V–NOTCH IMPACT DATA FOR SPECIMENS AUSTENITIZED AT 1640F FOR ½ HOUR, ISOTHERMALLY TREATED AT 1200F FOR 8500 SECONDS, WATER QUENCHED AND TEMPERED AT 1150F FOR ONE HOUR

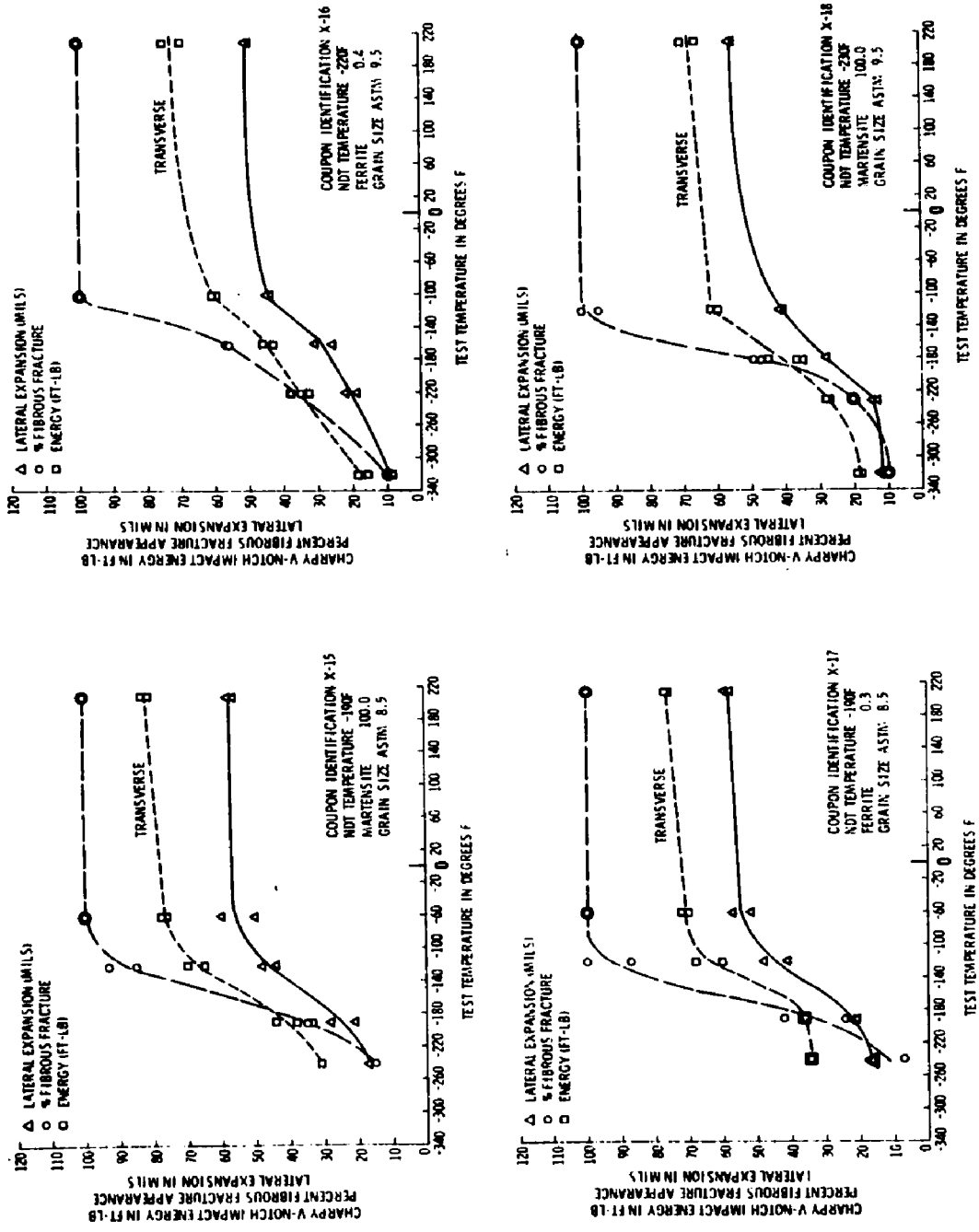


FIGURE D 5 – CONTINUED

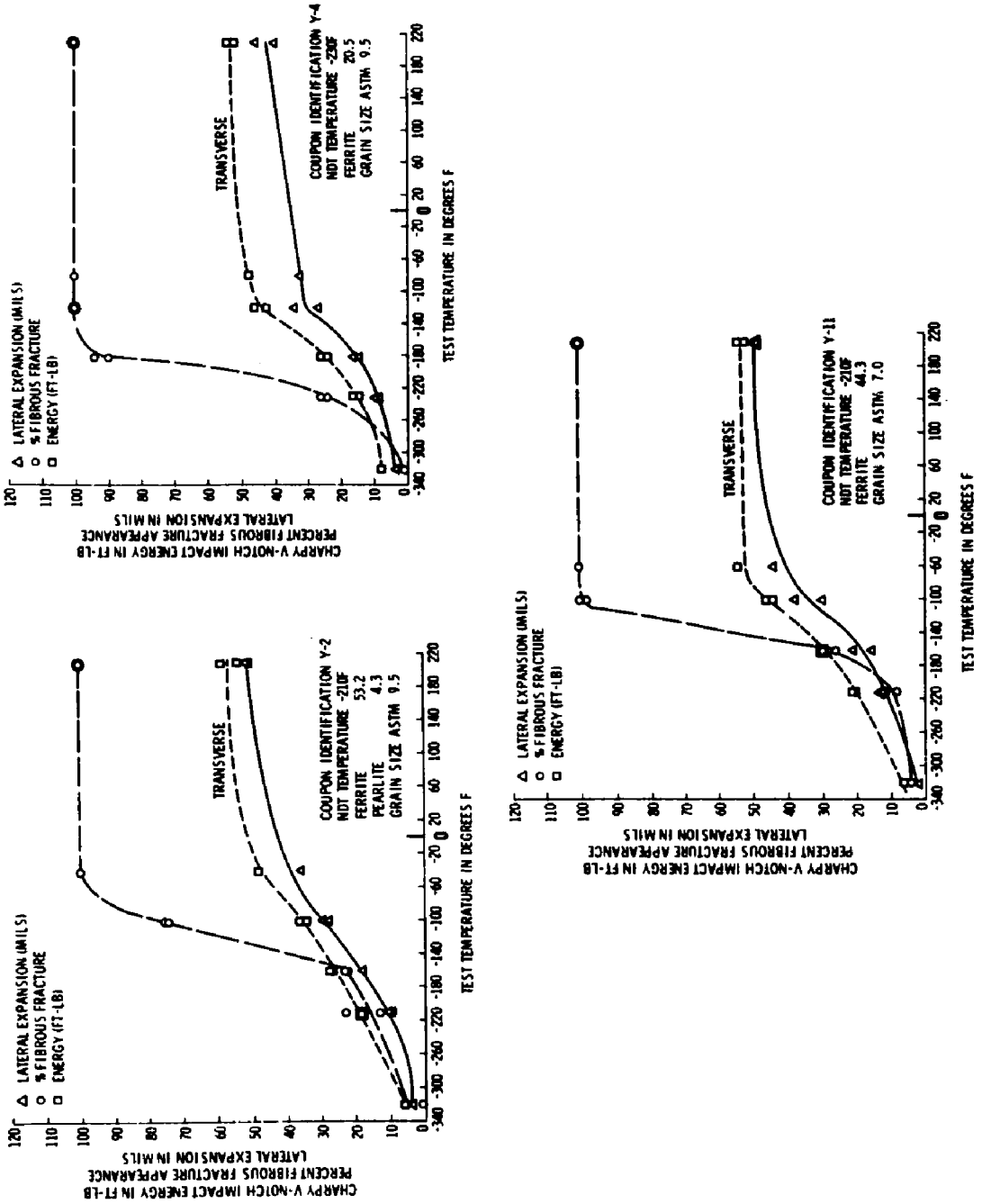


FIGURE D 5 – CONTINUED

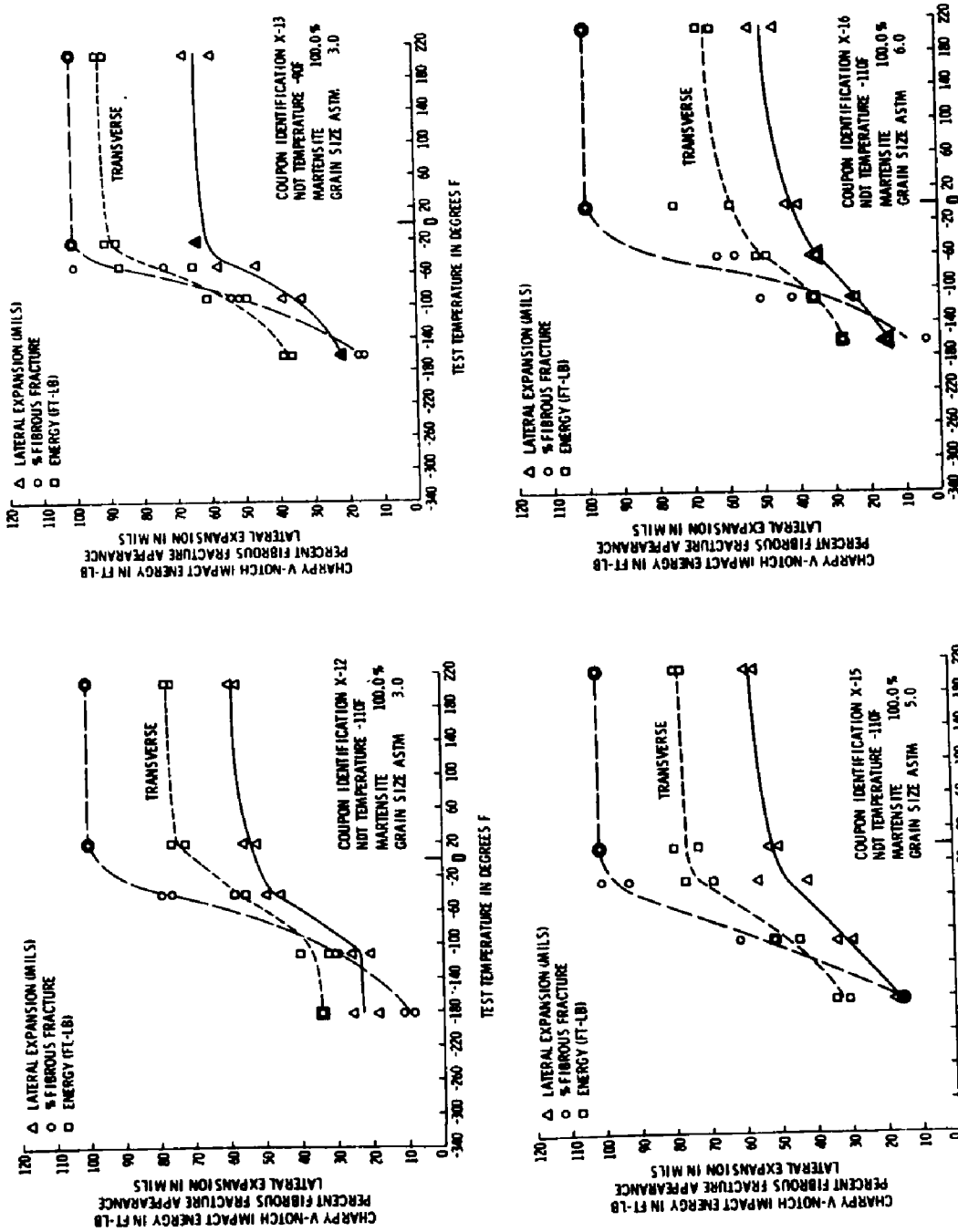


FIGURE D 6 – TRANSVERSE CHARPY V–NOTCH IMPACT DATA FOR SPECIMENS AUSTENITIZED AT 2000F FOR ONE HOUR, TRANSFERRED TO 1640F FOR 5 MINUTES MINIMUM, WATER QUENCHED AND TEMPERED AT 1100F FOR ONE HOUR

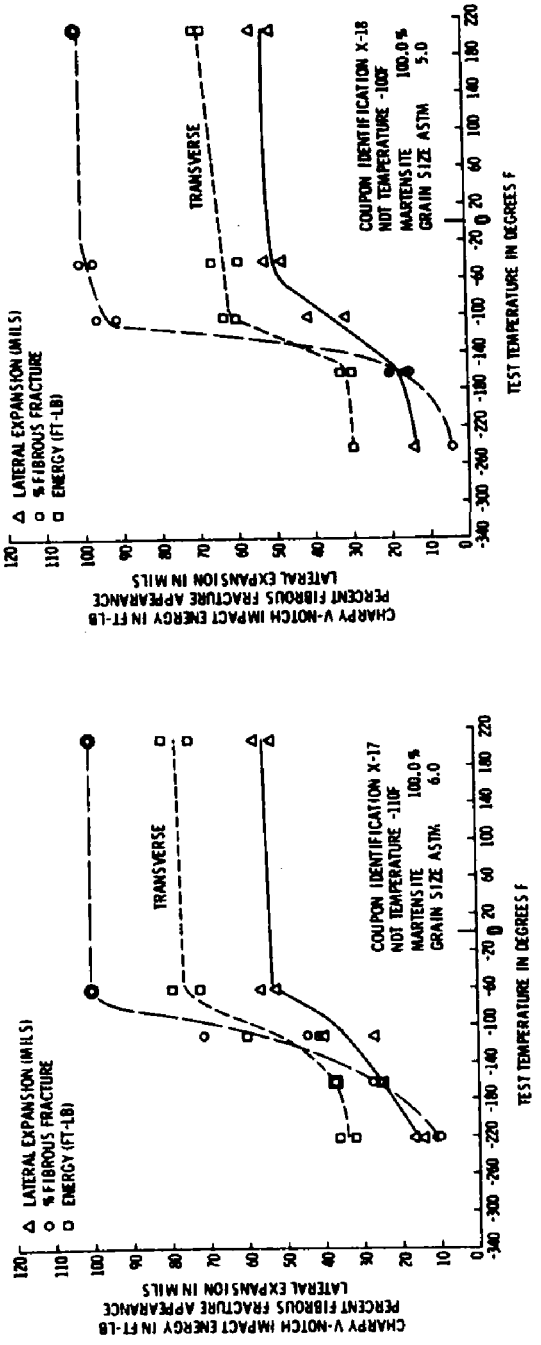


FIGURE D 6 – CONTINUED

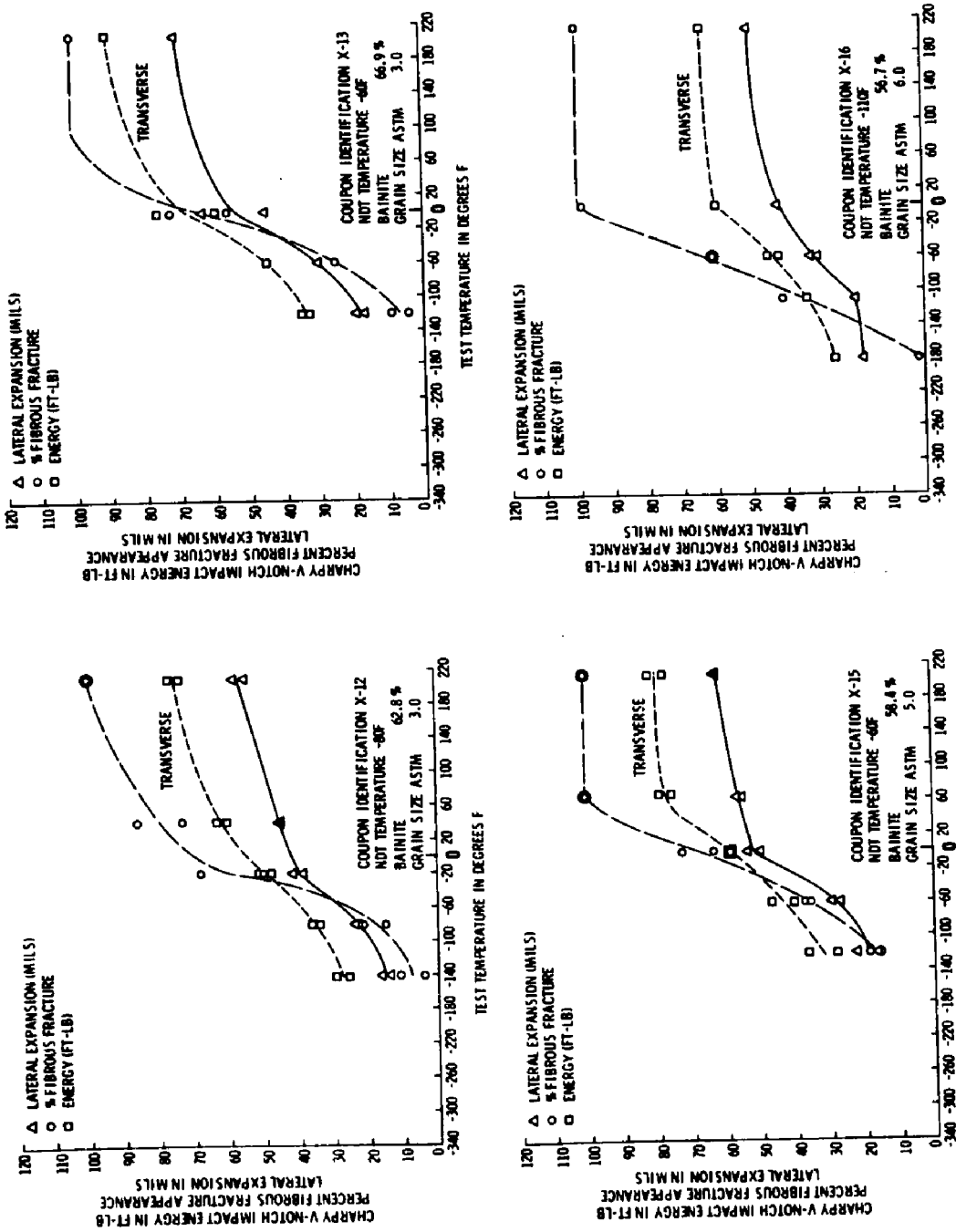


FIGURE D 7 – TRANSVERSE CHARPY V–NOTCH IMPACT DATA FOR SPECIMENS AUSTENITIZED AT 2000F FOR ONE HOUR, TRANSFERRED TO 1640F FOR 5 MINUTES MINIMUM, ISOTHERMALLY TREATED AT 875F FOR 1600 SECONDS, WATER QUENCHED AND TEMPERED AT 1150F FOR ONE HOUR

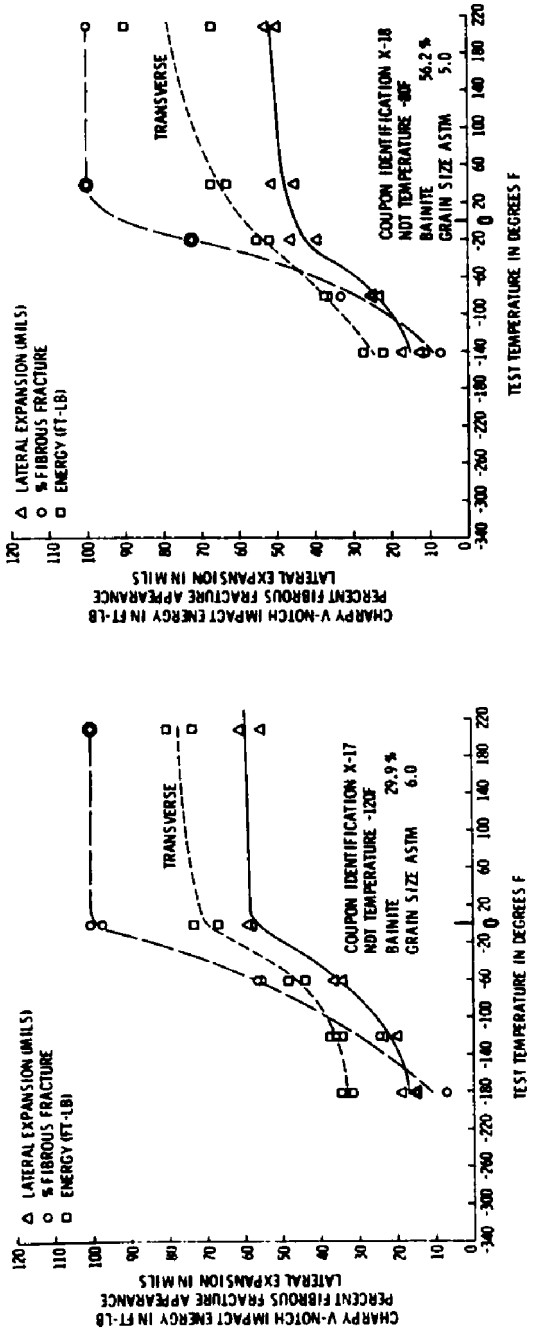


FIGURE D 7 – CONTINUED

APPENDIX E

DOCUMENTATION OF REGRESSION ANALYSIS PROGRAM

Documentation of XIRE - A Multi-variable Tape Regression Analysis Program with Rule of Thumb Guides for Interpretation of Results by Non-Statisticians.

INTRODUCTION

BACKGROUND

XIRE is a modified and expanded version of REGD, a multivariate tape regression program developed by Daniel Embody while at the Naval Ship Systems Command (NAVSHIPS).* It was written for the IBM 1620 in 1964 and rewritten for the IBM 7090 in 1967. REGD was intended primarily for analyzing a large quantity of test data to determine the most statistically significant functional relationship between variables. The functional relationships derived from regression are used in engineering applications to provide a mathematical expression of the relationship between two or more independent variables and one dependent variable, the effect that it is desired to control. Regression can provide information on the effect of variations in structural design parameters on the strength of a ship's structure, or on the effect of composition changes on the notch toughness of steel hull plate.

At the suggestion of the Head, Personnel Protections Concepts Branch, Department of Structural Mechanics at NSRDC, REGD was modified to permit the program to be used by an engineer with minimal knowledge of statistics and

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*Mr. Embody is now a mathematical statistician on the staff of the Biometrical Service, Agricultural Research Service, U.S. Department of Agriculture, Beltsville, Maryland. REGD is also available as USDA Program 1604 (see reference E1) for use on the IBM 360 Computer. References for Appendix E are listed on page 296.

programming. This involved writing a much more detailed explanation of how to use the program, expanding the output format, and preparing a guide explaining common causes of computational error dumps and suggesting and illustrating some of the simple statistical tests that can be applied to the output to prevent spurious correlations.

PURPOSE OF APPENDIX E

The purpose of this appendix is:

1. To document the details of the XIRE regression program.

2. To explain how to use the program.

3. To identify the various items of information in the output.

4. To explain uses of the regression equation itself.

5. To demonstrate how to calculate confidence intervals.

6. To show how to perform statistical tests of the results so as to avoid spurious correlations.

7. To give some rule-of-thumb guides on the practical significance of these tests and their implications.

8. To explain some of the common causes of computational dumps that frequently occur when using the program.

9. To provide a list of useful references that contain more details on items of specific interest to the user.

XIRE has the capability to develop a wide variety of relationships: (1) simple linear relationships between two

variables, (2) curvilinear relationships between two variables, (3) planar relationships between several variables, (4) curvilinear planar relationships among several variables, (5) interaction of the effects of several variables, and (6) evaluation of nonparametric effects such as differences between producers or materials.

The original documentation of REGD was used freely in assembling the information given in this report. Although XIRE is user-oriented and assumes that the user may not know much about statistics, it also assumes that the user is aware that unless he examines his data very carefully in some systematic way, he may end up with regression coefficients for variables that are not the most important ones. Some frequently used statistical tests are described in terms of how to ensure truly independent variables, how to ensure the significance of the individual regression coefficients (i.e., parameters), and how to evaluate the significance of different regressions.

GENERAL DESCRIPTION

XIRE uses matrix algebra methods to solve for polynomial and multivariable relationships by the method of least squares. The basis technique is fully documented in a variety of sources[E2-E6]. One of the best is Chapter 6 of Reference E6, wherein Natrella provides a step-by-step description of the technique in mathematical and matrix notation along with a sample calculation for each. Computer

Listing E1* gives a FORTRAN IV source statement listing of XIRE and associated memory and storage maps. Numerous comment cards have been put in the listing to identify various parts of the program.

The basic equation solved by XIRE has the form

$$Y = B_0 + B_1X_1 + B_2X_2 + \ldots B_{59}X_{59}$$

The X_1, X_2, ... X_{59} terms must consist of a variety of independent variables that in their direct form may be a cross product of terms or a transformation of one or a number of terms. Each X_i may be a complex term built up by a combination of several variables and constants, transformed and raised to an exponential power. The equation can be structured to fit theoretical or known relationships or to provide an empirical fit to several variables to give a parametric expression of the effect of the independent variables on some property, dependent variable. The equation can be used to determine the relative effect of several parameters or to provide a method of predicting the most likely value of some property or structure given a set of known conditions.

The program uses tape storage of an unlimited number of data points with up to 60 different input variables, one dependent and 59 independent. The program will search out data sets that are aberrant at a level read in on the

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*Computer Listings start on page 264.

control cards, discard them, and then recompute the regression calculations. Double precision routines, DOPE and DOPOP are elementary routines calling on AM DPAF* for this purpose, are used for the matrix calculations in order to minimize the possibility of creating an apparently singular matrix.

The use of tape for storage of data and matrix calculations allows the accommodation of an almost unlimited number of data sets. The actual number, which is in excess of several thousand data sets, is restricted only by the size of available scratch tapes. The user must be responsible for ensuring that the proper scratch tapes will be available for use.

The standard FORTRAN IV library subroutines ABS, SQRT, ALOG, SIN and ARCSIN are required in addition to the subroutines DOPOP, DPAF, and DATA, which are available at NSRDC. DOPOP and AM DPAF are double precision subroutines used to improve accuracy in the matrix manipulation. The program occupies 27401 octal places in the memory core of the IBM 7090.

A maximum of 60 variables may be accommodated, including the dependent term. The number of complete sets of data must be at least equal to the number of terms being correlated and preferably should exceed it. Data sets which have missing values of the independent variables are

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*Computer Listing E2 gives FORTRAN IV source statements listing and documentation of AM DPAF.

not used in the regression although they will be back calculated.

OPERATION

Data are read in the form of variables in an order specified on the variable arrangement card. Each data set is transferred from core to tape storage as it is read, and the process is repeated until all data are on tape. The Embody original format for input data was 6(1X, E11.5), but this can easily be modified to accommodate data which have already been punched with a different format, as is shown on Cards 1090 through 1300. Statements 15-18 inclusive are used to read data, with Statement 2002 as the format. The user must be extremely careful not to omit a statement number required for branching. In particular, the IF statement which tests for a trailer card indicating the end of the data deck must be compatible with any new FORMAT statements to be used.

After his data have been read in, the user of XIRE has the option of transforming them by use of transformation and transformation constant cards.* In the event that more than one transformation on a variable is desired, the variable arrangement card is used to place the data in

*An alternative method is to insert a routine in the program to manipulate the data prior to the statement labeled 19, Card 1330. The author has used this method to transform and sum a series of variables to produce a single variable in the final form required for regression. Reference E5 suggests several transformations used to normalize the distribution of an otherwise skewly distributed variable prior to regression.

several columns. The transformations of $\log_{10}$, exponential, SIN, ARCSIN, inverse, and linear multiplication are available by specifying the code of the transformation desired on the transformation card. Constants within the transformation are set by use of the transformation constant card. Note that a code of 00 is required for variables on the problem specification card. Each transformation other than 00 required a transformation constant card.

The transformed variables are combined into terms by use of the term card. Any number of terms (not to exceed 60) may be formed from only a limited number of variables. Likewise, a limited number of terms could be selected from a larger group, up to 100 as D(I) as dimensioned now, see card XIRE 0490, of available variables. For example, to set up the equation

$$V_5 = b_0 + b_1V_1 + b_2V_2V_3 + b_3V_2^4 + b_4V_1\log_{10}V_3$$

it is first reduced to the form

$$Y = b_0 + b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4$$

where the terms are of the form

$$X_1 = V_1$$

$$X_2 = V_2V_3$$

$$X_3 = V_2^4$$

$$X_4 = V_1\log_{10}V_3$$

$$Y = V_5$$

In order to define the above terms, term cards are used as described in the selection on Input Information. Note that the $\log_{10}$ in the fourth term is accounted for by the transformation card before its use in the term card.

The data are read and written on tape for storage. When the program encounters the trailer card, a calculation is performed to determine the regression coefficients using matrix manipulations to obtain a least squares solution.

Following the regression, back calculations are performed, and a comparison of the predicted term to the actual term is made. Should the difference exceed the product of the standard error of estimate and the value of ABT specified, the data set is tagged as aberrant and the remaining data sets are tested. The regression is then repeated, omitting the aberrant values identified.

The program prevents aberrant values from occurring during the second set of back calculations by doubling the value of ABT used in the first test.

INPUT INFORMATION

Each correlation requires a complete set of control cards followed by the data, as shown in Figure 1. Several correlations may be run on the same data set only if a duplicate data deck is placed behind each set of control cards. The eight control cards are described in the order of their use.

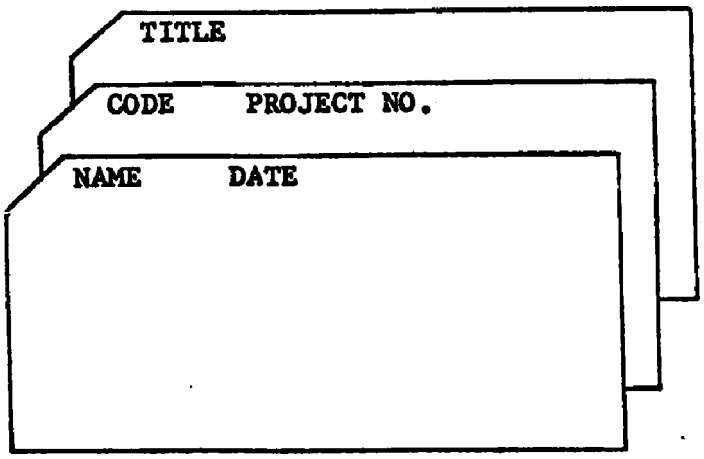
TITLE CARDS (Three Required)

These cards are read at the beginning of the program using format statement 2000 and are immediately printed out to identify the output:

Card 1: Columns 1-18 - Name of User
Columns 19-30 - Date

Card 2: Columns 1-18 - Navy Code
Columns 19-30 - Project Number

Card 3: Columns 1-30 - Title of Project



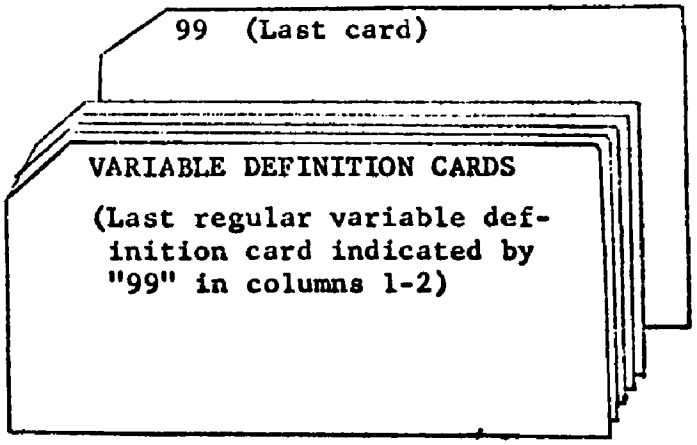
Example:

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
1	J	ϕ	H	N		D	ϕ	E		S	M	I	T	H					0	8	/	1	6	/	6	9				
2	C	ϕ	D	E		0	0	1																						
3	M	E	T	A	L	L	U	R	G	I	C	A	L			A	N	A	L	Y	S	I	S							

VARIABLE DEFINITION CARDS

The sole purpose of the variable definition cards is to provide verbal identification tables for the variables and terms being used in the regression. Any number of cards may be used, stating whatever information is desired in

Columns 3-36. The last card is indicated by a 99 in Columns 1 and 2. The author has found that the variables should be labeled by numbers in the sequence in which they are read and should match number for number the variables as they are stored into the Y array. This system is illustrated in the example below.



Example:

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
1			V	1		-		T	I	M	E		(	S	E	C	Φ	N	D	S	)											
2			V	2		-		D	I	S	T	A	N	C	E		(	F	E	E	T	)										
3			V	3		-		R	A	N	D	Φ	M		N	U	M	B	E	R												
4			T	H	E	R	E		A	R	E		N	Φ		T	R	A	N	S	F	Φ	R	M	A	T	I	Φ	N	S		
5			T	H	I	S		I	S		A		T	E	S	T		R	U	N		F	Φ	R		C	Φ	D	E		X	Y
6	9	9	S	E	E		F	Φ	L	D	E	R		2	9	9		F	Φ	R		M	Φ	R	E		I	N	F	Φ		

PROBLEM SPECIFICATION CARD

1. Number of variables on cards (I2, Columns 1, 2).
2. Number of transformed variables (I2, Columns 3, 4). Note that 00 is considered a transformation and must be included in the count of transformed variables.

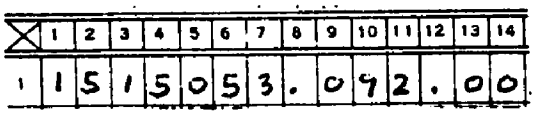
3. Number of terms to be generated (I2, Columns 5, 6); equals the number of independent variables to be used for this run plus one for the dependent variable.

4. Value of ABT (F4.2, Columns 7-10).
   ABT is an input constant used to detect aberrant values among the residuals. It should be selected from the Student "t" distribution tables on the basis of the degrees of freedom for the correlation. For very large sample sizes (degrees of freedom greater than 120), the "t" tables approach the Z values of the standardized normal distribution. In such a case, setting ABT equal to 3.09 would be expected to discard only one out of 1000 normally distributed values. Setting ABT equal to 2.58 would discard one out of 100 true values in addition to values that are really aberrant. The author recommends using the 3.09 value to scan the data for correct coding and the 2.58 value to flag data points that are not being correctly handled by the independent variables selected for regression. If a value of 9.00 is used, scanning for aberrant values is skipped.

5. Value of TCL (F4.2, Columns 11-14).
   TCL is the value of "t" used with the standard error of estimate to construct confidence limits on the predicted dependent term. It should be

selected from the "t" tables on the basis of degrees of freedom for the correlation and the desired confidence intervals. For a correlation involving just a few independent variables and having a great many data points, a good approximation of desired "t" values can be made using the value of the standardized normal distribution. Embody included this feature to provide information in the back calculations for constructing quality control charts.

Example of Problem Specification Card:



Number of Variables = 15

Number of Transformed Variables = 15

Number of Terms = 5

ABT = 3.09

TCL = 2.00

TERM ID CARDS

The term ID cards are used in the output to label regression coefficients in the output with information to make the results readily meaningful. One card is allowed for each term and must be in the same order as the term cards used later. The program continues after reading the proper number of cards as indicated in Columns 5 and 6 of the

problem specification card. Any alphanumeric characters may be used in card Columns 1-36 for identification of terms and should be expressive of the transformed form of the variable(s) actually used in the regression as opposed to the untransformed listing of variables required on the Variable Definition Cards above.

It is recommended that short titles be centered within the 36-column format in order to create an aesthetically pleasing output format. A shortcut is to duplicate the variable definition cards for whichever variables are going to be analyzed.

VARIABLE ARRANGEMENT CARD

Each of the variables entered on data cards is assigned an identification number from 1 to 60 by the variable arrangement card for later use in specifying transformations and defining terms. The format is as follows:

Variable 1 Variable 2 .... Variable 36	Columns 1, 2 Columns 3, 4 .... Columns 71, 72	Card 1
Variable 37 .... Variable 60	Columns 1, 2 .... Columns 47, 48	Card 2 (if required)

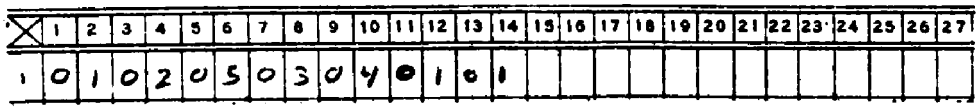
The program determines whether to read one or two cards from the number of variables specified on the problem specification card.

Example:

The five numbers following are read as data:

11.3 12.2 7.2 1.9 50.8

The variable arrangement card used is:



Then

Variable 1 = 11.3

Variable 2 = 12.2

Variable 3 = 1.9

Variable 4 = 50.8

Variable 5 = 7.2

Variable 6 = 11.3

Variable 7 = 11.3

TRANSFORMATION CARDS

Each variable must be transformed by one of the following forms:

Code	Transformation
00	Untransformed data
01	$C_1 * LOG_{10}(V + C_2)$
02	$(C_1 + V) ** C_2$
03	$C_1 * 10 ** (C_2 * V)$
04	$1.0/(V + C_1)$
05	ARCSIN (SQRT(V/100.0))
06	$SIN (C_1 * (V + C_2))$

If the desired transformation is not on this list, the user may easily add his own transformations between Cards XIRE 4100 and XIRE 4240. The computed GO TO (Card XIRE 4100) must also be modified to include these additions.

The appropriate code for the transformation should be placed on the transformation card in the following format:

Card 1	Transformation for Variable 1 Transformation for Variable 2 . . . . Transformation for Variable 36	Columns 1, 2 Columns 3, 4 . . . . Columns 71, 72
Card 2 (if req)	Transformation for Variable 37 Transformation for Variable 38 . . . . Transformation for Variable 60	Columns 1, 2 Columns 3, 4 . . . . Columns 47, 48

The reader is reminded that Transformation 01 requires that the sum in parentheses be a positive number since a zero or a negative sum will result in termination of the program by the computer's control program.

The transformation ARCSIN computes the transformation result in degrees. The transformation SIN requires the argument to be expressed in radians. The program determines whether one or two cards are to be read from the number of variables specified on the problem specification card. The constants C1 and C2 are discussed below.

TRANSFORMATION CONSTANT CARD

Some of the transformations listed above require the constants C1 and C2. These cards provide the means by which these constants are entered. Constants must be specified

even if they are to be set equal to one or zero in the equation. E-type floating point numbers are used for the two constants. The program reads transformation constant cards until the final card is indicated by 99 in Columns 31-32.

The format for the constant cards is:

Number		Format
Variable Requiring Constant	I2	Columns 2, 3
C1	E12.6	Columns 5-16
C2	E12.6	Columns 18-29
Trailer in Last Card Only	99	Columns 31, 32

As many constant cards may be entered as needed. If no constants are required, then only one card with dummy constants and the 99 is required. The example below shows the single card with dummy constants.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
	0	1		+	.	1	0	0	0	0	0	E	+	0	1		+	.	0	0	0	0	0	0	E	+	0	0		9	9		

TERM CARDS

Term cards are the means by which transformed variables are converted into terms of the regression equation. In general, all transformed variables may be raised to any exponent desired, and a maximum of three variables may then be multiplied together to form a term. In many cases, a single variable raised to the appropriate power will be used as a term in the regression equation.

For example, $X = V_i^q V_j^r V_k^s$ represents the general form-

ation of a term X, where V_i, V_j, and V_k are transformed variables and q, r, and s are desired exponents. This feature may be used to set up variables to evaluate interaction effects between pairs or even groups of variables.

The format for each term card is as follows:

First Variable	I2	Columns 3,4
Exponent First Variable	E9.5	Columns 6-14
Second Variable	I2	Columns 16,17
Exponent Second Variable	E9.5	Columns 19-27
Third Variable	I2	Columns 29,30
Exponent Third Variable	E9.5	Columns 32-40

Zeros or blanks are used for the second and third variables and their exponents when no variable is to be used in these positions. This effectively makes these last two terms one and leaves just the first variable for use in the regression. The dependent variable is identified as the last term card used. It will be the last card read before data are encountered. The total number of term cards must match the number of terms stated on the problem specification card and which included the dependent variable in that number. In addition, the total number of term cards must match the number of term ID cards. The examples below show a sample term card in E format and in F format, respectively.

	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40
1	06 +.100E+01 11 +.100E+01 17 -.050E+01
2	06 1.0 11 1.0 17 -0.5

*Because of the operational characteristics of the 7090 system, F format will work just as well here when coding the data.

DATA CARDS

General Format

Data are recorded in E-type floating point numbers at a rate of only six variables per card in the original Embody writeup. When the number of variables in an analysis exceeds six, additional cards are required. In the same manner, 10 cards are required to record a set of 60 variables. Card columns are as follows:

Variable	Format	Columns
1	E11.5	2-12
2	E11.5	14-24
3	E11.5	26-36
4	E11.5	38-48
5	E11.5	50-60
6	E11.4	62-72

Card Columns 73-80 are not used by the program; hence they may be used for identification codes or sequencing if desired.

The author found that as a practical matter, every problem will require the use of an especially devised coding format to suit its particular needs. That is, the user will normally want more than six variables recorded per card and as few cards as possible per data set; this will require special formating of the read statement for each problem. Users are strongly urged to record all possible variables during the tests and to code them all

up the first time; this will minimize subsequent handling of the data. Along this vein, it is important for potential users to note that up to 100 variables may be read in each time even though only 60 terms may be used in the regression; D(J) is dimensioned D(100) and Y(I) is dimensioned Y(60). Even more variables may be coded onto the cards if the user is willing to limit each run to reading in no more than 100 variables by the simple expedient of controlling the READ and FORMAT statements.

Missing Value Code

At times there are sets of independent variables where the dependent variable is missing or is judged to be aberrant and for which it is desired that the regression function be evaluated with confidence limits. To accomplish this evaluation, the independent variables are entered on the cards the same as for other sets of data. In the position of the dependent variable, the number .999999E + 38 is entered. The program senses this number on input and marks the set of data as aberrant. This aberrant set is not used in the correlation but is included during back calculations. This is very useful in generating tables of predicted points with confidence limits. It should be noted that the missing value code may not be used in the position of the first variable in the first card, name, Columns 2-12. Also, it may not be used in place of missing variables.

Running Time

In general, the running time seems to be roughly proportional to the number of data points and to the square root of the number of variables; five minutes is usually a good first guess. Table E1 is a compilation of actual running times for a dozen different runs on the 7090 computer.

TRAILER CARDS

The first trailer card stops the reading of data and causes the program to begin the regression calculations. It consists of a single card with the number 999 in Columns 1-3. The trailer card follows each data deck being run.

The program is terminated by an end file card; this consists of a card with 7 and 8 overpunched in Column 1 and END FILE punched in Columns 8-15. This card should follow the trailer card (999) of the last data deck.

Computer Listing E3 contains a sample input and results for a test case run in XIRE.

TABLE E1

SOME ACTUAL RUNNING TIMES FOR THE XIRE PROGRAM ON THE 7090 COMPUTER

Run No.	Program Deck	Number of Variables Read In	Printed Corrected Sums of Square and Products and Inverse Matrix		No. Data Sets Used As Input	No. of AB Values	No. of Terms Run	Running Time, Minutes
			Yes	No				
1	Binary	44	X		115	2	3	1.70
2	Source	44	X		115	0	14	2.49
3	Binary	44	X		115	0	33	7.07
4	Source	44	X		143	3	14	3.34**
5	Binary	44	X		143	2	15	4.15
6	Binary	44	X		143	2	17	3.98
7	Source	44	X		143	1	19	4.83**
8	Source***	46		X	176	0	4	1.94**
9	Source***	46		X	176	0	5	2.18**
10	Source***	58		X	186	0	6	4.56*
11	Source***	58		X	186	0	7	2.59
12	Source***	46		X	348	0	17	6.60

* Debugging Post Processor Printed

** Storage Map Printed

***Special Version of XIRE (XIMP) that Calculates and Transforms Data before Storing in the Y Array.

OUTPUT INFORMATION

IDENTIFICATION

The output is identified as to originator, Navy code, project number, date, and title of regression. A list of variables being used and other information desired by the user is permitted.

MATRICES

The matrix of corrected sums of squares and products of the terms being correlated and its inverse are printed. This is useful for comparing values in symmetrical positions about the diagonal connecting value (1,1) to value (u,u). If the symmetrical values are the same out to 7 or 8 significant figures, this will indicate that the calculation has proceeded with sufficient precision.

TABLE OF COEFFICIENTS

The regression coefficients for all the terms are printed in a table which lists the following:

1. The calculated coefficients B_1 through B_n.
2. The term name corresponding to each variable or transformed variable used in this particular run.
3. A value of T where T = (term coefficient)/(standard deviation of the term coefficient).
4. A value PRODUCT = 4*(TERM STD. DEV.)*(REG. COEFFICIENT).
5. The TERM MEAN is the mean of all values of the

indicated variable.

6. The TERM STANDARD DEVIATION is the deviation of the indicated variable.

7. The TERM RANGE is the TERM MEAN divided by TERM STANDARD DEVIATION. TERM RANGE provides an indicator of the range of the values of the specified variable and is an indicator that the values may not be normally distributed if its absolute value is less than 2.*

REGRESSION STATISTICS

The values printed are as follows:

B_0 - The constant term for the regression

Sx.y - The standard error of estimate for the selected regression

R - The regression coefficient

R2 - The regression coefficient squared

DF - The degrees of freedom of the system (DF = N - number of terms)

N - The number of data sets used in the regression (N = number of input data sets - number of aberrant values)

T AND PRODUCT

The value of T is used with the Student "t" distribution tables and the degrees of freedom to determine confi-

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*When the value of TERM RANGE is less than 2, it is flagged with an asterisk(*) to attract the attention of the user. If TERM RANGE is less than 2.0, the value of PRODUCT should be recomputed before being evaluated since the values of this variable are probably distributed in a skewed manner. In other words, if RANGE < 2, the value of PRODUCT calculated as the Bi term coefficient multiplied by four times the TERM STANDARD DEVIATION may be too large.

dence intervals of the term coefficients, i.e., to test the hypothesis that the coefficient is not zero. Capital T is the calculated value of t for each Bi and its variance; lower case t refers to the tabulated values of Student's t.

Values of T cannot be used as the sole criterion of the significance of a given independent variable. Values of T and the associated value of PRODUCT are much more informative. PRODUCT is an approximation of the range of the effect produced on the dependent variable when only the single given independent variable is allowed to range from its maximum to its minimum value, assuming that the value of the variable was normally distributed; see the footnote at the bottom of the previous page.

PRODUCT = 4*(TERM STD. DEV.)*(REG. COEFFICIENT)

Thus, large absolute values of PRODUCT indicate that the given independent variable contributes significantly to variations in the dependent variable. In contrast, a large absolute value of T is merely assurance that the true value of the regression coefficient is very unlikely to be zero. Thus we see that a large absolute value of T in association with a small value of PRODUCT is a signal that although there is definitely a correlation between the dependent and independent variables, varying the independent variable over its full range will have an insignificant effect on the dependent variable for all practical purposes. Various combinations of large and small values of T and PRODUCT are

interpreted in the tabulation given below.

Value of T	Value of Product	Interpretation
Small	Small	Very likely that true regression coefficient is zero.
Small	Large	Quite likely that regression coefficient is zero and that large value of product is of questionable engineering value.
Large	Small	Definite correlation but possibly of little or no engineering value.
Large	Large	Highly significant variable with great practical effect on dependent variable.

Another good indication of a truly significant variable is one whose absolute value of T is consistently greater than say 1.7* when several regressions are made with a variety of supposedly independent variables. Related pairs of variables will frequently have one with a T value of +2.691, for example, and the other with a value of -2.581. In addition, the value PRODUCT will have nearly identical values, frequently very large, with opposite signs. In such cases, the net effect is usually negligible and when one variable is run without the other, the resulting T value is so close to zero as to require discarding that variable since its regression coefficient term is not statistically different from zero.

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*If a highly correlated variable is run as V_1 and V_1^2, the T value of V_1 may drop and become misleadingly small because of obvious strong correlation between V_1 and V_1^2.

A useful rule of thumb for engineers using the program is that large values of PRODUCT or T, i.e., PRODUCT >> TERM MEAN, T>6.00, may indicate that these supposedly independent variables are really interrelated in some way. This interrelationship among independent variables may be due to a true dependence or, in many cases, to a poorly distributed data sample. Attempts to regress on a set of variables that contains a pair of highly correlated independent variables or several sets of correlated variables will frequently set up a singular matrix that dumps the program or results in such nearly identical values of sums of squares and products that even with double precision, the computer ends up trying to take the square root of zero or of a negative number, and the system will give an "error dump" for the particular run. For proper correlation the level of each independent variable in a run should be randomly distributed with respect to the level of every other independent variable.

STANDARD ERROR OF ESTIMATE Sx.y

The standard error of estimate Sx.y represents the residual of the dependent term unaccounted for by the regression and should be compared to the experimental error associated with the measurement of the dependent variable. Values of PRODUCT should be considered in light of the magnitude of Sx.y as an aid in judging the importance of a given variable to the analysis. In order to make relative comparisons of the significance of a coefficient between

separate regressions, the value of PRODUCT should be divided by the appropriate value of Sx.y. Sx.y can be used in connection with Snedecor's F ratio test[E2,E7] to test the ratio of the variances of two runs, assuming the values of dependent variables are normally distributed or transformed so that they were normal when the regression was performed, to determine whether the difference between the two values of Sx.y is statistically significant. These can be used either to tell whether adding a variable(s) has significantly improved the correlation or whether deleting a variable(s) to eliminate spurious terms has not significantly changed the usefulness of the correlation. A similar evaluation could be made using the regression coefficient R and the charts on pages 332 and 333 of Reference E7.

REGRESSION COEFFICIENT R

The regression coefficient R is a measure of the correctness of the correlation, with R = ±1 indicating a perfect correlation, absolute value of 1. In the absence of prior knowledge to indicate how good a correlation is attainable, Schrock[E8] has suggested a range for general use in evaluating the quality of the correlation in terms of the absolute value of R. As indicated by the following comparison, the author recommends higher quality standards for good scientific or engineering type relations than does Schrock.

Quality of Correlation	Schrock	Author
Excellent	1.00 to 0.75	1.00 to 0.85
Good	0.75 to 0.50	0.85 to 0.70
Fair	0.50 to 0.25	0.70 to 0.50
Doubtful	0.25 to 0.00	0.50 to 0.00

It has been the author's experience that significant variables rapidly raise correlations of engineering data to 0.85-0.90 and that thereafter many nonsense terms, i.e., no true significance, must be added to raise the value of R by even 0.03. Such spurious correlations can be avoided by applying Snedecor's F test to the ratio of $(Sx.y)_1^2/(Sx.y)_2^2$ in order to justify the corresponding decrease in the degrees of freedom. Schrock also points out that R^2, R2 in the output, is a measure of the portion of the variance of Y explained by the correlation. Statistical tests should be applied to the value of R using the degrees of freedom DF for this regression to ensure that a real relationship has been achieved as contrasted to a chance relationship; see Reference E2, E7, and E8.

DATA AND BACK CALCULATIONS

The ORIGINAL DATA*, the TRANSFORMED DATA*, and the terms formed are printed for each data set with the aberrant values identified. All aberrant values should be closely

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*These items help identify data sets when reviewing the back calculations; they are particularly useful when examining aberrant data.

examined for keypunching, coding and experimental errors, as well as for clues as to variables which should be used in the initial search for significant independent variables. In addition, a table with the following results is printed:

YC The calculated Y value using the regression coefficients and the values of the independent input data.

DE The difference between the calculated Y value and the actual value of the dependent variable given in the input data. This value, called the residual, should be correlated against each independent variable to ensure that it is random in nature. If it is not randomly distributed, it can give important clues as to deficiencies in the regression, such as failure to use a higher order term to account for periodic or systematic variation.

LL The lower confidence level.

UL The upper confidence level.

OYC The inverse of any transformation which may have been applied to the dependent term.

OLL The inverse of lower confidence level.

OUL The inverse of upper confidence level.

A value for each of the above confidence limits is printed for a predicted point, the mean line, and the control lines based on the value of TCL given on the problem specification cards.

Confidence limits are printed for the predicted point and the mean line. Confidence levels on the predicted point are necessarily larger than those of the mean line since the latter is based on more complete information than a single point. The confidence levels printed under the heading CONTROL are useful in quality control engineering. See Reference E8 for specific examples of industrial applications of this feature.

Aberrant values are flagged or indicated by having the program append a symbol AB at the end of the back calculations for each data set or case. The number of AB data sets can be calculated as

Number of AB values = (No. of input data points - N)

where N is the number of sets used in the regression. AB, TERM MEANS and TERM STAND DEV.. give the user checks of input data. During initial runs, data sets tagged AB frequently have a punching or coding error in one or more of the variables being run. The user's basic knowledge of expected values of TERM MEAN and knowledge of the range covered by each independent variable allow him to scan these columns and quickly check whether any fall outside their expected value.* When they do, this signals errors such as card positioning in the input data deck, incorrect

*The values of TERM MEANS and TERM STAND DEV should agree closely with the similar values given in the output form BMDO3D. This is a useful cross check on the output from these two programs.

formating either on the data cards or in the read statements, and possibly even minor coding errors. Card positioning errors in the data deck can be checked for by getting a listing and scanning it for out of position cards; end punched data decks speed this process up considerably. Quick scans of a listing of the input data for symmetrical repetition of visual patterns is a good check to be sure the data deck hasn't been shuffled during handling. Data flagged AB should also be examined in detail to see if there is some obvious reason for its having been tagged, such as failure to use a term in the correlation to account for a basic difference between this data and the rest.

USES OF THE RESULTS OF THE REGRESSION

The regression program XIRE can be used to develop a wide variety of relationships:

1. Simple linear relationships between two variables

   $Y = B_0 + B_1x$

2. Curvilinear relationships between two variables

   $Y = B_0 + B_1x + B_2x^2 + B_Mx^M$

3. Planar relationships between several variables

   $Y = B_0 + B_1x_1 + B_2x_2 + B_3x_3 + \ldots B_mx_m$

   This is even a good approximation of a curved surface if the segment area is limited by small ranges for each independent variable.

4. Curvilinear planar relationships between several variables for optimization or minimization

$$Y = B_0 + B_1x_1 + B_2x_1^2 + B_3x_2 + B_4x_2^2 + B_5x_2^3$$

$$+ B_6x_6^{-4} + \ldots B_n x_m^{\ell}$$

5. Interaction of the effects of several variables

$$Y = B_0 x_1^{B_1} X_2^{B_2} X_3^{B_3} \quad , \text{ etc.}$$

Taking the logarithm of both sides will arrange this in a form suitable for regression

$$\ell n\, Y = \ell n B_0 + B_1 \ell n x_1 + B_2 \ell n x_2 + B_3 \ell n x_3 + \ldots \text{ etc.}$$

Interactions between two variables such as the interaction of grain size and yield strength on NDT temperature, of pressure and grit size on wear, of stiffener spacing with hull thickness on the collapse pressure of deep submergence hulls and almost any other interaction imaginable can be evaluated by letting the value of X_i in the re-regression analysis be the product of the two variables of interest

$$X_i = V_k * V_l$$

These interaction effects are extremely difficult to evaluate without using special care in the design of the experiment; an applied statistician should be consulted early in the planning stages to help ensure that the experimental data will be acquired in a way that will permit a clearcut evaluation of interaction effects.

6. Evaluations of nonparametric effects such as the difference between producers or materials.

These equations can be used to determine: (1) the relative effect of variables, (2) whether the effect is linear or curvilinear, (3) whether there is some interaction between variables, (4) whether there is some optimum or minimum, (5) what variables are controlling, (6) what proportion of the observed variability of the dependent variable is associated with the combination independent variables, (7) what proportion of the observed variability of Y is not accounted for by the regression, (8) most likely value of the parameter expressed by the dependent variable when values are given for the independent variables, (9) the variability to be attached to the independent variable in order to establish quality control charts, and (10) what variables may require more complete examination as a result of screening of preliminary tests on a large number of variables.

To use this program wisely and with meaningful results will also require an understanding of the variability of the measurements of each of the independent variables as well as of the dependent variable so that the user will have some grasp of the effect variances in the supposedly fixed independent variables has on the apparent quality of the regression. Variance in the measurement of supposedly fixed valued independent variables will tend to increase the resultant value of Sx.y just as will decreasing precision in the measurement of the dependent variable. Natrella discusses the relationship between two variables when both are

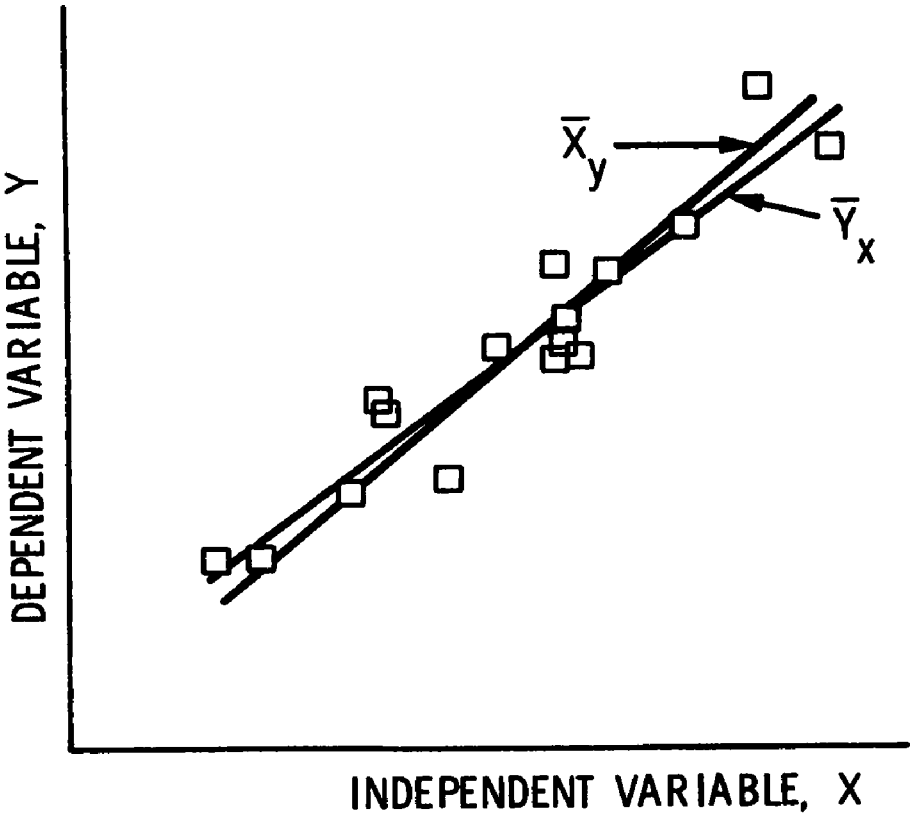
subject to errors of measurement on pp. 5-27 and 5-30 of Reference E6.

These empirically or theoretically derived equations can be used for design purposes and to graphically present the relationship between variables. Families of curves or three-dimensional plots can be prepared to visually present all or portions of these regression surfaces.

If one fits the equation Y = f(A,B,C,D), it will be a useful tool for predicting the most likely value of Y given a knowledge of A,B,C, and D. If one now wants to estimate a value of, say, C given a knowledge of Y, A, B, and D, solution of the previous equation for C will not give the most likely value of C but only a fair estimate of C unless Y = f(A,B,C,D) represents a known physical relationship rather than an empirically derived relation. To get an empirical equation to predict the most likely value of C given Y,A,B, and D, one should fit his data to such an equation. Examples of this observation are given in most statistics books for the simple case of a linear relation of X and Y. The expected value of Y given X ($\bar{Y}_x$) and the expected value of X given Y ($\bar{X}_y$) are shown on the following schematic.

It is readily apparent from this plot that if one intends to use a regression line or surface most effectively to empirically relate several known factors or parameters in order to estimate the value of some term, the relationship should be derived to express the term to be estimated

as a function of the known parameters.



MISSING DATA

When one has a missing observation for one of the independent variables and wants to salvage the data, a good rule of thumb is to insert the average value of that variable. This is one way of handling this with minimal effect on the regression and the estimated error. Whenever this is done, the degrees of freedom should be reduced by one for each estimate used. Cochran and Cox discuss another method for handling missing data on pages 81 and 82 of Reference E9. If this is crucial, professional opinion should be obtained.

INDEPENDENT VARIABLES

The user of regression analysis equations must be especially careful to ensure that the independent variables used for the regression are truly independent, i.e., that they do not contain one or more of the dependent variables as a component. Special care must be taken to ensure:

1. That the independent variables are mutually independent and not varied together in some systematic pattern. The producer may have increased Cr and Ni simultaneously to increase hardenability. If so, one cannot attribute or apportion an observed effect to one or the other, or separate their independent effects.

2. That the independent variables are measured with sufficient precision relative to the total range of the independent variables or to its size to be able to analyze its effect. An independent variable measured to ±0.10 and ranging from 0.40 to 0.50 in the total set of data will not be of much use. Similarly a variable ranging from 0 to 5 ppm and measured with an accuracy of ±5 ppm will not be particularly informative.

3. That the independent variables are not themselves dependent on or a reflection of the same controlling parameters. For example, the tensile yield strength, ultimate tensile strength, and compressive yield strength of quenched and tempered steels are so intimately related to the basic composition of the steel and its thermal treatment that they are directly related to one another. A second example,

chemical composition determines the upper critical temperature of a steel and susceptibility to grain coarsening. Austenitizing temperature, i.e., degrees above the upper critical, time at temperature and susceptibility to grain coarsening at a given temperature interact to determine the prior austenitic grain size. Thus, use of prior austenitic grain size as an independent variable may limit what else can be run as an independent variable in that regression analysis.

4. That the independent variables are not intimately interrelated with the dependent variable and thus influenced or controlled by the same dependent variable. If the dependent variable is the ultimate tensile strength of a quenched and tempered steel, obviously one could not run yield strength together with composition and thermal treatment as the independent variables and expect to get a significant correlation on the latter two variables. The direct interrelationship between UTS and YS (YS = .8 to .9 * UTS) would account for most of the variability and mask the contribution of the other variables. A less obvious case is when the dependent variable is the ductile-to-brittle transition temperature of a steel and the independent variables are the impact strength at some fixed low temperature in or slightly above the usual transition region and the other properties that control transition temperature. The movement of the transition temperature markedly affects the impact strength, making these variables so highly correlated

as to mask the effect of other properties.

5. That the independent variables are not related in the sense that A + B + C = constant, since if A and B are specified, C is automatically defined. For example, if the percent bainite, ferrite, and pearlite are recorded as variables, the percent martensite cannot also be run as an independent variable since the total of these variables is 100 percent.

6. That the independent variables are not indirectly specified by some nonparametric independent variable. For example, a coding for producer as a variable may eliminate degree of cross rolling as a variable if one producer cross rolls only 4 to 1 and the other cross rolls only 1 to 1. A system of dummy variables, using one independent variable for each and coding the input as 0 or 1 (or +1 and -1) to indicate its absence or presence, may eliminate such things as density and strength as independent variables if they are also specified by the material independent variables: for example, if Material 1 is an aluminum alloy, Material 2 is a titanium alloy, and Material 3 is a certain steel, specifying the material code specifies density and strength. Interactions between quantitative and qualitative factors may also be tested; this is why +1 and -1 are used. Note that when specifying material codes as independent variables, Item 5 above must be adhered to.

7. That the degrees of freedom for a specific independent variable is compatible with the number of degrees of

freedom for that variable. For example, if there are only four cases with Pearlite (V23) in the microstructure, then attempting to use V23, $(V23)^2$, V23 * V20, and V23 * V28 as independent variables leaves no degrees of freedom with regard to V23. Thus, the number of terms in which a specific independent variable appears should be less than the number of input data cases that contain a significant amount of that variable.

Items 1, 3, and 4 above are sometimes not readily apparent and may be overlooked; Items 2, 5, 6, and 7 are related to the experiment and the subsequent analysis and are readily controlled by the engineer. Overlooking any of these items can lead to spurious correlations or interfere with the regression analysis.

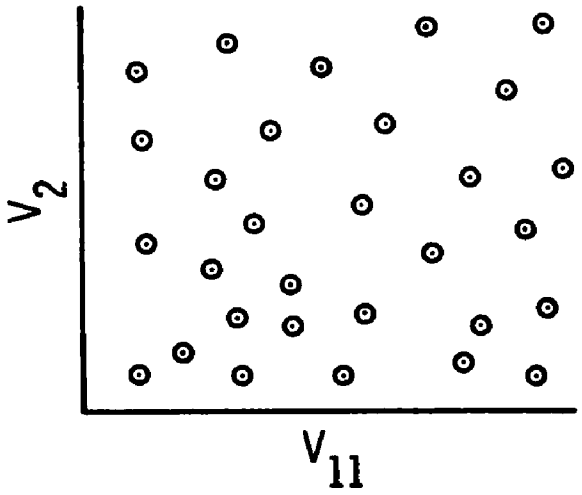
Items 1, 3, and 4 are sometimes difficult to watch for when analyzing data collected from production quality control records or from laboratory or engineering design experiments conducted over a period of time and not necessarily obtained in the formalized way recommended by the rigorous methods required by statisticians. Statistical design of experiments using randomized blocks, factorial experiments, lattice designs, and latin squares are found to some degree in most applied statistics books and more completely in Cochran and Cox[E9]; their text gives the research worker a formalized way (1) to ensure the independence of his variables, (2) to minimize the amount of experimental work required and to maximize the amount of

useful information to be gained, and (3) to ensure that the desired knowledge of the interaction of certain variables will be evaluated to the desired level of complexity. This method is rather complicated to set up and requires the assistance of a professional statistician.

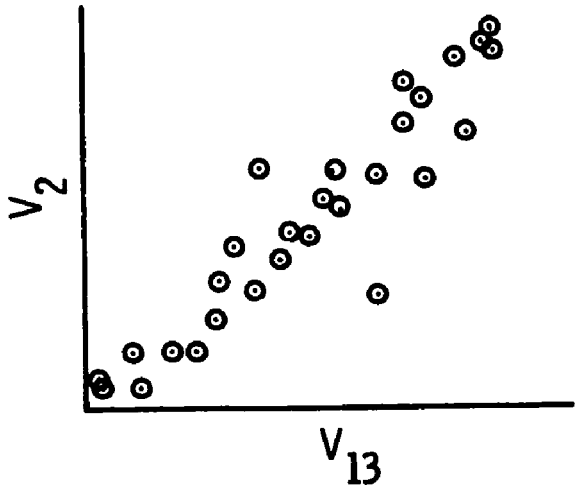
Assuming that data were not collected during a statistically designed experiment but rather are the result of a series of engineering studies to evaluate the effects of some parameter, the engineer will need some method of screening his proposed list of all possible variables, independent and dependent, to avoid spurious correlations introduced by the previously discussed factors. Such a tool is available in the form of a correlation coefficient matrix which is the result of a systematic comparison of each variable to every other variable. Such programs are usually available in the libraries of most computing centers. The program used by the author is BMDO3D-Correlation With Item Deletion[E10]. This version was prepared at the University of California as a part of a biomedical data analysis package which consists of a set of interrelated statistical analysis programs.

As indicated on page 221, a good rule of thumb for engineers using the program is that large values of PRODUCT or T, i.e., PRODUCT>>TERM MEAN, T>6.00, may be an indication that some of these supposedly independent variables are really related to each other in some way. For proper correlation, each independent variable in a run should be ran-

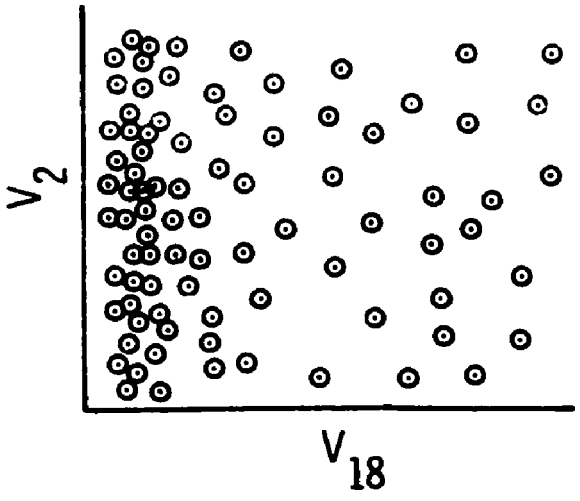
domly distributed with respect to every other independent variable. Several distributions of two supposedly independent variables V_{1k} and V_2 are illustrated below.



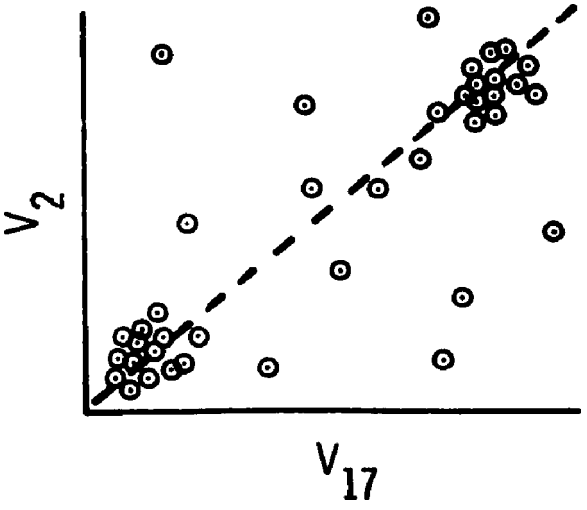
Random distribution suitable for proper multivariable regression. This is the ideal case and is frequently achieved by proper statistical design of the experiments during early planning stages.



Variables are linearly dependent; poor correlation will result. This is frequently seen in production data when the producer increases both variables simultaneously in order to compensate for some demand made by the finished product. For example, the producer may increase both the Ni and Cr alloy content of his steel in order to provide adequate hardenability when he is producing heavy or thicker plates. The effect of one variable cannot be separated from the effect of the other.



Data are weighted toward small values of V_{18}; poor correlation may result since any corrleation present in the lower data range will be imposed on the upper range.
Note: This situation might be corrected by judicious elimination of some data if there is enough, or by a suitable transformation; for example, a log transformation will separate out small values.



Date are grouped such that the linear dependence depicted by the dotted line will result. This dependence will yield a poor correlation since the effects of the two variables may not be separable.

The possibility of such unfortunate interrelations between variables should be checked by using Subroutine BMDO3D*, Reference E10, to run a preliminary check on the data using the matrix of correlation coefficients r. Values of r ranging from 0.8 to 0.9 indicate that the pair of independent variables should be examined and only used together with caution. Values greater than 0.9 indicate that the pair of variables should definitely not be used together as independent variables in a given run; they should be separately run, though running just one will usually give an adequate idea of how the other will correlate. Again this is where the user must depend on his knowledge of theory and not blindly follow these rule of thumb guides. A large value of r in the matrix column relating the dependent variable to some other variable is a good sign that this variable will account for a large portion of the observed variability in the dependent variable. This observation may give a good indication of some of the independent variables to use in the initial regression analyses.

One can also make use of BMDO3D to screen variables and determine if they are or seem to be mutually dependent, or whether they were varied in some systematic way so that

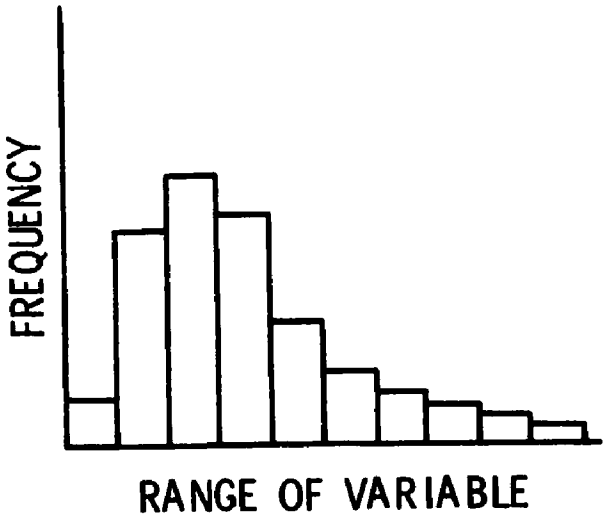
*Because elaborate documentation is available with this program, it will not be discussed in detail here other than to say it can be run with the same data cards as used for the XIRE and permits similar transformations to those described for this program. In addition, it has the valuable feature of permitting item deletion when the variable has a specific value, e.g., zero.

it is impossible to say that an observed effect on the dependent variable is due to one or the other of these related variables. The author has found that the correlation coefficient matrix printed out by BMDO3D quickly becomes too cumbersome to use as the number of variables is increased. To overcome this problem it is suggested that the program be modified to print out the list of variable titles, number them consecutively, sort out the correlation coefficients that indicate potential trouble into three groups 0.800 to 0.899, 0.900 to 0.949, and 0.950 to 0.999, and then to print after the variable name the number of each variable that falls in each group.

DEPENDENT VARIABLES

Another important factor that must be adhered to when performing least square regression analysis is that the dependent variable must be normally distributed about the regression line or about the regression surface and that the variance or standard deviation is the same everywhere along the line or about the surface. Most engineers are familiar with the technique of using such transformations as taking the logarithm or the reciprocal of a variable to linearize a relationship between variables. These and similar transformations are used to normalize the distribution of variables and to equalize the variance all along the line or plane of regression. In Chapter 20 of Reference E6, Natrella shows some frequently encountered non-normal dis-

tributions and shows how to normalize them using a variety of transformations: (1) taking the square root, (2) taking the natural log, (3) taking the log to the base 10, (4) using $\log_e\left(\frac{x}{1-x}\right) = 2\ \tanh^{-1}(2X-1)$, (5) the arcsin $\sqrt{X}$, and (6) $\frac{1}{2}\log_e((1+x)/(1-x)) = \tanh^{-1}X$. If there is enough replication of the test data, a relative frequency histogram as shown below can be plotted to indicate the distribution pattern and get an idea of a suitable transformation. A similar check of the distribution can be made using probability graph paper. After regression, the surface can be used with its standard deviation to construct confidence intervals that, upon inverse transformation,

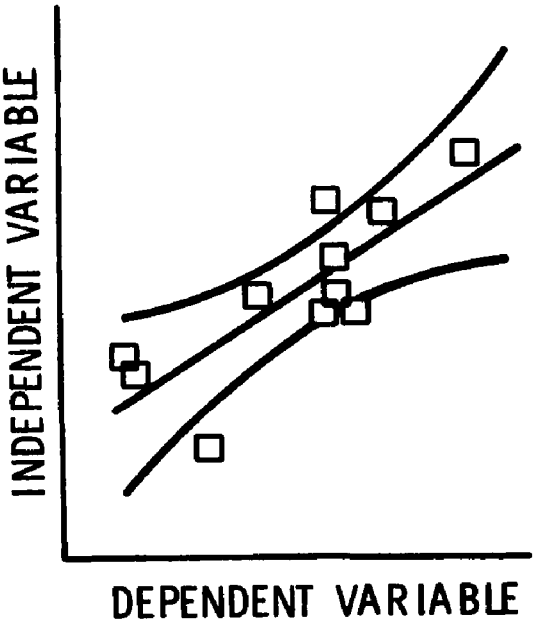


will reflect the previously observed characteristics of a non-normal distribution and/or a non-uniform standard deviation about the regression surface.

CONFIDENCE INTERVALS AND EXTRAPOLATION OF RESULTS

Confidence intervals are used to bracket the range of values that will contain a given proportion (50, 80, 95, or 99 percent) of the sample or population that will have values of the dependent variable that fall in this range. To compensate for uncertainty due to small sample size, the Student t values are larger than the Z values for equal confidence intervals and are usually used to construct these confidence intervals except in the case where (1) the sample size is very large (DF>120) or (2) where the relationship is for a physical phenomena having (a) a known form, (b) a normal distribution of the observed values of the dependent variable, and (c) a known standard deviation based on extensive prior tests; in this last case, the Z values from the standardized normal distribution are sometimes used to construct confidence intervals.

The statistician is usually cautious about using even the tail ends of regression lines, let alone extrapolating beyond the range of the test data used to derive the line--particularly in the case of empirical correlations. This reluctance is due to an uncertainty that the sample is representative, let alone that the line has the correct slope beyond the limits of the data. As known from calculus, a series of short straight lines will give a good approximation of a curved line, but each straight line is useful only along a very restricted portion of the curve. This



uncertainty is reflected in the statistical procedure which requires that the confidence interval expand as it approaches the limits of the input test data as shown in this sketch. Note that the confidence interval grows smaller as one moves in toward the center of the data.

Exactly the same effect holds for fitted surfaces resulting from multivariable regression. Ideally the confidence interval for a surface would vary to reflect the distribution and number of points for each variable. For example, if 115 data sets are run with several variables, one of which has values that vary from zero in only four instances, much certainty can be placed on the exact magnitude of the effect of this variable because of the small effective sample size used to establish it. Bear in mind that the T test of the coefficient for this variable can only tell whether the difference between this coefficient and

zero can be judged significantly, not how accurately it is defined by the sample.

Because of the large sample size that should be used when performing a regression analysis and because the high degrees of freedom, sample size minus the number of terms in regression, that hopefully remain after regression, it is expedient to adopt the less exact solution of using just the t value for the degrees of freedom and constructing parallel surfaces as the confidence intervals. As a result, one must be cautious when using confidence intervals near the limits of the regression surface. In addition, it means that the experiments designed to provide data for regression analysis should be laid out to ensure (1) that the area of maximum interest falls at or near the center of the range of each independent variable, (2) that the range of each variable is broad enough to encompass any expected deviations from this center of interest, as frequently happens because of changes in the factors entering into a design when one is using or trying to provide an empirical design equation, (3) that the distribution of the values of each variable is sufficient to allow determination of whether the effect of the variable is linear or curvilinear, (4) that there is sufficient replication, enough tests are run, to ensure that the observed effect is real and not a spurious relation caused by a combination of chance and a large scatter in the test results, and (5) that the variable is not varied in some systematic way relative to the variation of another

variable so that the effect of each can be separated and attributed to each in the correct proportion.

Satisfaction of all the above requirements is usually too costly in material, time, and money; consequently some compromise is made in the design of the experiment. When this is done with a series of experiments designed to provide an empirical equation relating the effects of various design, physical, or metallurgical parameters, the exact set of compromises and the actual range of the input data for each variable should be clearly set forth so that potential users will know the restrictions on the regression equation and thus be in a better position to judge whether or not it would be wise to try to extrapolate using the equation.

PREVENTION OF SPURIOUS CORRELATIONS

The first step to be taken to prevent spurious correlations is to ensure that the independent variables are independent of one another as was discussed in the preceeding sections of this appendix. Next one must test each term coefficient to ensure that the independent variable represented by that term has a valid functional effect on the dependent variable. Finally, one should test the standard deviation associated with the regression equation and the sample data to ensure that the regression provides a good representation of the data and to determine if adding or omitting a variable will significantly affect the usefulness

of the functional relationship.

TESTS OF THE TERM COEFFICIENTS, B_i

Confidence interval type tests are used to evaluate the term coefficients, B_i, in order to ensure that significance is not attributed to some variable term when its coefficient is likely to be zero or even when it is likely to have the opposite sign and thus an opposite effect on the dependent variable. The confidence interval is based on the Student "t" value for the degrees of freedom in the correlation. The use of confidence intervals for the B_i term coefficients is explained by Guttman and Wilks for linear and planar regression, pp. 249-256 of Reference E7, and by Natrella in Chapter 6, p. 16 of Reference E6*.

Examining the simple cases used by Guttman and Wilks to illustrate the construction of confidence intervals, CI, for the variable term coefficients and the constant term, one can see that going from an equation of the form $y = B_0 + B_1x_1$ to one of the form

$$y = B_0 + B_1x_1 + B_2x_2$$

the addition of a single term results in a tremendous increase in the complexity of the calculation of the standard deviation of the term coefficient B_i. The confidence interval scale factor can be effectively reduced by increasing

*On page 14, Natrella gives a test of the significance of a partial set of term coefficients.

the sample size and by working near the center of the mass of data used for the correlation.

The confidence interval for the ith variable's term coefficient is

$$B_i \pm (t_{DF;\alpha/2}) \frac{Sx.y_i}{\sqrt{\text{CI Scale Factor for ith term}}}$$

When evaluating the B_i values, it is important to know whether the confidence interval for B_i would include zero or to know at what point the sign of B_i, the term coefficient, changes from plus to minus. If there is a reasonable probability that B_i may be zero or even have the opposite sign, implying that the term has the opposite effect on the dependent variable, the variable term should be discarded. To facilitate this decision, XIRE prints out a T value next to the TERM NAME for each B_i term coefficient. A similar test could be applied to the constant of the regression equation B_0 even though XIRE is not presently set up to facilitate this.

The value of T is the B_i term coefficient divided by its standard deviation. This T has the sign and magnitude of the Student's $t_{DF;\alpha}$ value required to make zero one of the bounds of the confidence interval for the B_i term coefficient. Thus using the degrees of freedom, the DF for the correlation, one can determine the level of significance α for the required confidence interval and ascertain whether B_i is significantly different from zero. Values of $t_{DF;\alpha}$

are tabulated in the references cited in this report and certainly in almost all statistics books. Interpolation of the data in these tables is easily accomplished by making a plot of $t_{DF;\alpha}$ versus 1/DF which is almost linear.

An alternative method which eliminates the problem of interpolation is to use the equation for $t_{DF;\alpha}$ given by Fisher and Cornish, see page 216 of Reference E11, and have the Student's $t_{DF;\alpha}$ values printed in the computer output. This is the method currently being adopted by the author. An additional embellishment would be to have the program indicate the names of terms that are significantly different from zero by printing an asterisk before the term name.

Example 1:

Having screened the independent variables to eliminate pairs that were really not independent, 32 were selected and run versus NDT temperature (V32) as the dependent variable. A portion of the output section labeled REGRESSION STATISTICS is given in Table E2 along with the $t_{DF;\alpha/2}$ values for confidence intervals at the 90, 95, 98, and 99 percent levels of significance for the indicated degrees of freedom, DF = 82. In general, the more terms used, the higher the significance level that should be applied to each so that the cumulative confidence in the regression will not fall too low and introduce a significant probability that one of the terms is spurious or even might have an opposite effect; see pages 12-16 of Chapter 6, Reference E6.

REGRESSION COEFFICIENTS			TERM NAMES	* * * * * * * TERM * * * * * * *				
				I	PRODUCT	MEANS	STD. DEVS.	RANGE
B	1	-0.1880757E 04	V2 - AMT CARBON	-1.129	-161.5126095	0.1796783	0.0214691	8.37
B	2	0.5991357E 03	V3 - AMT MN	0.717	124.0108557	0.3645217	0.0517457	7.04
B	3	-0.5775777E 04	V4 - AMT P	-0.440	-93.0578060	0.0162174	0.0040279	4.03
B	4	-0.1106146E 04	V6 - AMT SI	-0.435	-170.6937561	0.2976522	0.0385785	7.72
B	5	-0.9320779E 03	V7 - AMT NI	-1.946	-360.4743462	2.7476521	0.1693710	16.22
B	6	-0.2852184E 04	V8 - AMT CR	-1.378	-1489.3882904	1.5104348	0.1305480	11.57
B	7	0.4543247E 06	V13- AMT TI	1.028	3692.6995239	0.0024783	0.0020320	1.22*
B	8	-0.5878877E 05	V16- AMT OXYGEN	-1.206	-2406.7886963	0.0139652	0.0102349	1.36*
B	9	-0.3725347E 02	V18- AMT HYDROGEN	-0.881	-410.3698120	1.4608696	2.7539030	0.53*
B	10	-0.2468059E 02	V20- GRAIN SIZE	-6.205	-141.5276279	8.1913043	1.4335923	5.71
B	11	0.5431619E 00	V21- PCT BANITE	4.193	41.6321335	9.3226086	19.1619370	0.49*
B	12	-0.4404706E 00	V22- PCT FERRITE	-1.203	-22.5987847	4.1808695	12.8264999	0.33*
B	13	0.5944824E 01	V23- PCT PEARLITE	8.089	90.2125311	0.6078261	3.7937424	0.16*
B	14	0.7566911E-03	V28- TYS (KSI)	1.541	31.0233083	106514.5126953	10249.6613770	10.39
B	15	-0.1639538E 03	V43- AU SIZE OTHER INCL.	-1.123	-499.1767998	0.8765217	0.7611545	1.15*
B	16	0.5232668E 02	V44- BACKGROUND RATING	1.425	219.3171577	1.8173913	1.0478266	1.73*
B	17	-0.9021310E 03	V46-V2*V15 - C * AMT CO	-1.346	-826.0175323	1.0525896	0.2294616	4.59
B	18	0.8574413E 03	V47-V3*V5 - MN * S	1.824	1740.2201843	4.1625608	0.5073875	8.20
B	19	0.7084524E 03	V48-V5*V11 - S * A S AL	0.447	171.5122013	0.4480470	0.0605235	7.40
B	20	-0.3468536E 05	V51-V7*V9 - NI * MO	-1.143	-172.5342140	0.0008692	0.0012436	0.70*
B	21	-0.4473722E 05	V53-V8*V17 - CR * N	-1.515	-551.8357849	0.0103505	0.0030838	3.36
B	22	0.7475624E 05	V54-V9*V16 - MO * O	1.141	1107.5983734	0.0052650	0.0036873	1.43*
B	23	0.7894164E 05	V55-V9*V17 - MO * NI	1.732	326.4350662	0.0026371	0.0010338	2.55
B	24	-0.1355793E 08	V56-V11*V13 - A S AL * TI	-1.024	-3773.1130066	0.0000703	0.0000696	1.01*
B	25	0.1068818E 07	V57-V11*V17 - A S AL * NI	0.613	523.5405502	0.0002174	0.0001225	1.78*
B	26	0.1925334E 07	V58-V11*V16 - A S AL * O	1.083	1929.0791626	0.0003909	0.0002505	1.56*
B	27	-0.1840561E 07	V59-V12*V17 - TOT. AL * N	-0.700	-895.0766907	0.0002826	0.0001216	2.32
B	28	0.1138517E 08	V61-V13*V17 - TI * N	0.978	740.4513321	0.0000180	0.0000163	1.11*
B	29	-0.1199995E 08	V62-V14*V16 - V * O	-1.033	-994.5758438	0.0000261	0.0000207	1.26*
B	30	0.2209580E 08	V63-V14*V17 -V * N	0.931	1164.0068054	0.0000162	0.0000132	1.23*
B	31	0.5225190E-01	V65-V20*V37 - GR.SZ.*210F ENG	1.098	27.1013036	560.2086945	129.6665974	4.32
B	32	0.4077350E 03	V66-V6*V11/V17-SI *A S AL/N	1.044	1190.7434845	1.4963829	0.7300964	2.05
Y			V32 NDT			-183.9391289	41.3986368	-4.44

BO	2595.5610657	R2.	0.8757823	DF	82
SX.Y	17.2037582	R	0.9358324	N	115.

CI	$t_{DF;\ \alpha/2}$
90	1.665
95	1.990
98	2.373
99	2.639

Table E2 - Regression Statistics Used in Example 1

Examination of Table 2 shows that only the three variables V20-GS, V21-B, and V23-P had |T| values greater than the 2.639 value of $t_{82;.005}$ required for the 99 percent confidence interval. Only three more, V7, V47, and V55 had |T| values greater than the 1.665 needed for the 90 percent confidence interval.

This indicates that although the correlation shown here is excellent, R2 shows that about 87 percent of the variability of the dependent variable has been explained, the correlation would most likely be equally good if at least 26 of the 32 independent variables had not been used. This is easily checked by repeating the regression using only the six terms with high |T| values or even using only the three terms with |T| values greater than 2.639 and seeing whether there is a significant decrease in R and Sx.y.

Example 2:

Table E3 gives a portion of the output section from XIRE that is labeled REGRESSION STATISTICS after running only V20-GS, V21-B, V22-F, and V23-P as independent variables and V32-NDT as the dependent variable. The Student's $t_{DF;\alpha/2}$ values for the 90 to 99 percent confidence intervals are given in Table E3 for the DF = 110. The degrees of freedom, DF, have increased from the 82 (given in Example 1) since we have eliminated 28 of the variables in this run.

All the T values are significant at the 99 percent

REGRESSION STATISTICS

		REGRESSION COEFFICIENTS	TERM NAMES	T	PRODUCT	* * * * * * * TERM * * * * * * * MEANS	STD. DEVS.	RANGE
B	1	-0.2044782E 02	V20- GRAIN SIZE	-14.296	-117.2553663	8.1913043	1.4335923	5.71
B	2	0.4640463E 00	V21- PCT BANITE	4.289	35.5681038	9.3226086	19.1619370	0.49*
B	3	-0.7432906E 00	V22- PCT FERRITE	-3.551	-38.1352696	4.1808695	12.8264999	0.33*
B	4	0.4872587E 01	V23- PCT PEARLITE	6.953	73.9413671	0.6078261	3.7937424	0.16*
Y			V32 NDT			-183.9391289	41.3986368	-4.44

B0	-20.6249943	R2,	0.7640074	DF	110
SX.Y	20.4734623	R	0.8740752	N	115.

B0 - CONSTANT CALCULATED FOR REGRESSION EQUATION
R2 - STANDARD ERROR OF ESTIMATE (R SQUARED)
DF - DEGREES OF FREEDOM
SX.Y - STANDARD DEVIATION OF A PREDICTED POINT (RESIDUAL UNACCOUNTED FOR AFTER REGRESSION)
R - REGRESSION COEFFICIENT
N - NUMBER OF DATA POINTS
T - COEFFICIENT DIVIDED BY THE STD DEV OF THE COEFFICIENT
(USE WITH STUDENT T DISTRIBUTION TABLE AND DEGREES OF FREEDOM TO DETERMINE
CONFIDENCE INTERVALS OF THE REGRESSION COEFFICIENTS)
PRODUCT - 4*(REGRESSION COEF)*(TERM STD DEV)
TERM RANGE - (TERM MEAN)/(TERM STD DEV)
* TERM RANGE IS LESS THAN 2.0, THE VALUE OF PRODUCT SHOULD BE
RECOMPUTED BEFORE BEING EVALUATED SINCE VALUES OF THIS VARIABLE ARE DISTRIBUTED IN A SKEW MANNER

CI	$t_{DF;\ \alpha/2}$
90	±1.660
95	±1.983
98	±2.362
99	±2.632

Table E3 - Regression Statistics Used in Example 2

level, and R2 = 0.764 indicates that they are still accounting for 76 percent of the variability on the dependent variable, V32. Sx.y has increased to only 20.5 from the 17.2 in Example 1 which employed 32 independent variables.

Example 3:

In addition to the variables run in Example 2, V1 is added as a variable named Producer, coded 1 for Producer X and 2 for Producer Y, and a variable named V28-TYS is added along with second and third order terms of V20-GS. The output from this run is given in Table E4 along with the Student's $t_{DF;\alpha}$ values for DF = 106.

For the range of V28-TYS in the input data, 106 ksi ± about 20 ksi, the regression coefficient for this term is quite likely to be zero or even have the opposite sign*. Other runs made adding 28 data points specifically developed to demonstrate the effect of the YS term provided a better estimate of its effect. The T values indicate that V22-F is significant at about the 97 percent level.

The T values for GS^2 and GS^3, -5.454 and +5.384, respectively, are very large, almost equal in magnitude but opposite in sign; this reflects the high correlation between these two terms. The sum of the PRODUCT values for GS, GS^2 and GS^3 is about equal to the -117 value given in Example 2 for GS alone.

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*Regression on a term that is a constant will frequently result in $B_i = 0$ and $T_i = 0$. This signifies that the regression has automatically eliminated a term that should not have been run in the first place.

REGRESSION STATISTICS

		REGRESSION COEFFICIENTS	TERM NAMES	T	PRODUCT	MEANS	STD. DEVS.	RANGE
			* * * * * * * TERM * * * * * * *					
B	1	-0.1979449E 02	V1 - PRODUCER	-5.382	-37.6480160	1.3391304	0.4754860	2.82
B	2	0.1810719E 03	V20- GRAIN SIZE	4.952	1038.3332214	8.1913043	1.4335923	5.71
B	3	-0.3197421E 02	V84- V20(GR SIZE)**2	-5.454	-2503.4501953	69.1347818	19.5739791	3.53
B	4	0.1591907E 01	V85- V20(GR SIZE)**3	5.384	1380.9633331	593.7782593	216.8725376	2.74
B	5	0.4887996E 00	V21- PCT BARITE	4.540	37.4654064	9.3226086	19.1619370	0.49*
B	6	-0.6204721E 00	V22- PCT FERRITE	-2.254	-31.8335403	4.1808695	12.8264999	0.33*
B	7	0.5433532E 01	V23- PCT PEARLITE	9.309	82.4536800	0.6078261	3.7937424	0.16*
B	8	0.1337441E-03	V28- TYS (PSI)	0.373	5.4833276	106514.5126953	10249.6613770	10.39
Y			V32- NDT TEMP			-183.9361289	41.3986368	-4.44

BO	-394.8674469	R2,	0.8549800	DF	106
SX.Y	16.3493137	R	0.9246513	N	115.

BO - CONSTANT CALCULATED FOR REGRESSION EQUATION
R2 - STANDARD ERROR OF ESTIMATE (R SQUARED)
DF - DEGREES OF FREEDOM
SX.Y - STANDARD DEVIATION OF A PREDICTED POINT (RESIDUAL UNACCOUNTED FOR AFTER REGRESSION)
R - REGRESSION COEFFICIENT
N - NUMBER OF DATA POINTS
T - COEFFICIENT DIVIDED BY THE STD DEV OF THE COEFFICIENT
(USE WITH STUDENT T DISTRIBUTION TABLE AND DEGREES OF FREEDOM TO DETERMINE
CONFIDENCE INTERVALS OF THE REGRESSION COEFFICIENTS)
PRODUCT - 4*(REGRESSION COEF)*(TERM STD DEV)
TERM RANGE - (TERM MEAN)/(TERM STD DEV)
* TERM RANGE IS LESS THAN 2.0, THE VALUE OF PRODUCT SHOULD BE
RECOMPUTED BEFORE BEING EVALUATED SINCE VALUES OF THIS VARIABLE ARE DISTRIBUTED IN A SKEW MANNER

CI	$t_{DF;\alpha/2}$
90	±1.661
95	±1.984
98	±2.363
99	±2.625

Table E4 - Regression Statistics Used in Example 3

TESTS OF STANDARD DEVIATION OF THE REGRESSION Sx.y

Tests may be made using the standard deviation of the regression to check whether it has been significantly changed from one regression to another. The usual check is to apply the Snedecor F ratio test to evaluate the null hypothesis that the ratio of the variances is one

$$H_0: \frac{(Sx.y)_1^2}{(Sx.y)_2^2} = 1$$

against the alternate hypothesis

$$H_1: \frac{(Sx.y)_1^2}{(Sx.y)_2^2} > 1 \quad .$$

The critical region to reject H_0 in favor of H_1 for this right-sided test is

$$\frac{(S_1)^2}{(S_2)^2} \geq F_{DF_1;\ DF_2;\alpha} \quad ,$$

where (1-alpha) is the confidence level of this test. A description of this test and an example is given in Reference E7 (pages 171-173) along with tables of F values (pages 320-329) and their nonsymmetrical distribution (pages 146 and 151). Note that this test is simplified by using the larger value of Sx.y in the numerator.

Example 4:

For regression on the variable GS versus NDT temperature as the dependent variable, $r_1 = 0.8174$, $Sx.y_1 = 23.367$, and $DF_1 = 111$; for regression on the four variables GS, B,

F, and P versus NDT temperature, $r_2 = 0.8741$, $Sx.y_2 = 20.473$, $DF_2 = 110$. Testing the hypothesis that the ratio of the variances is one at the 5 percent significance level*, we have $F_{111;110;.05} = 1.371$ and $\frac{(23.367)^2}{(20.473)^2} = 1.303$. Since $1.303 < 1.371$, one accepts the hypothesis that the variances are the same at the 5 percent level of significance. In this case, the 5 percent means that we have only 5 percent probability of rejecting H_0 (accept H_1) when H_0 is true (Guttman and Wilks discuss hypothesis testing on page 156 of Reference E7). Thus we see that adding only the variables B, F, and P has not reduced the observed variation for this regression at the 5 percent level of significance.

Example 5:

For regression on the four independent variables GS, B, F and P versus NDT temperature as the dependent variable, $r_1 = 0.8741$, $Sx.y_1 = 20.473F$, and $DF_1 = 110$; for regression on GS, $(GS)^2$, $(GS)^3$, B, F and P versus NDT temperature, $r_2 = 0.9103$, $Sx.y_2 = 17.367F$, and $DF_2 = 106$. Testing the null hypothesis that the ratio of the variances is one at the 5 percent significance level, we have $F_{110;106;.05} = 1.400$ and $\frac{(20.473)^2}{(17.367)^2} = 1.390$. Again we must accept the null hypothesis that the variances are the same for the two regressions at the 5 percent significance level. This implies that adding the second and third order GS terms did not reduce the observed variance, a statistically significant

*1-alpha is the 95 percent confidence level.

amount even though $Sx.y_2$ is 3F less than $Sx.y_1$.

Example 6:

For regression on GS versus NDT, one saw in Example 4 that $r_1 = 0.8174$, $Sx.y_1 = 23.367$, and $DF_1 = 111$; for regression on GS, GS^2, GS^3, B, P and F versus NDT, $r_2 = 0.9103$, $Sx.y_2 = 17.367$, and $DF_2 = 106$. Testing H_0: $(S_1^2/S_2^2) = 1$ against the alternative $S_1^2 > S_2^2$ at the 5 percent level, one has $F_{111;106;.05} = 1.399$ and $\frac{(23.367)^2}{(17.367)^2} = 1.810$. As a result, we must say that the ratio of the variances is not equal to one, reject H_0, and judge that the additional variables have reduced the variance for the second regression to a significant extent at the 5 percent level.

Example 7:

One regression gives $r_1 = 0.894$, $Sx.y_1 = 25.529$, and $DF_1 = 129$. With everything the same except for the addition of a variable to specify the producer of the material, another regression gives $r_2 = 0.9323$, $Sx.y_2 = 20.145$, and $DF_2 = 126$. Testing H_0: $S_1^2 = S_2^2$ versus the alternative H_1: $S_1^2 > S_1^2$ at the 5 percent level, one has $F_{129;136;.05} \approx 1.340$ and $\frac{(25.529)^2}{(20.145)^2} = 1.606$. H_0 must be rejected in favor of H_1, leading to the conclusion that adding a variable to specify the producer significantly improved the correlation by reducing the observed variance that remained unexplained after the second regression analysis.

TESTS OF THE REGRESSION CORRELATION COEFFICIENT r

Tests of the regression correlation coefficient r are accomplished in the usual way: a confidence interval, CI, is established for the true hypothesis that R or some other sample value r falls in the confidence interval. In these examples we will work with the 95 percent interval.

There are two ways to establish the desired confidence interval for R: (1) by use of the charts by F. N. David, given on pages 332 and 333 of Reference E7, or (2) by taking advantage during calculation of the fact that $\frac{1}{2}$ $\ell n \frac{1+r}{1-r}$ is normally distributed with a mean of $\frac{1}{2}$ $\ell n \frac{1+r}{1-r}$ and a standard deviation of $1/\sqrt{DF}$, where DF* is the degrees of freedom for the correlation. In either case, one can see that the confidence interval is dependent on the regression coefficient r, the sample size N, and the number of terms used in the regression. Both of these methods are described on pages 251-253 of Guttman and Wilks[E7]. Schrock[E8] indicates that DF should be at least 30 to get a good estimate of R, the true correlation coefficient.

The David charts are quite useful for a quick check and for values of r from about 0.85 to -0.85. The curves are too compressed to be read easily for $|r| > 0.85$, and it may be desirable to calculate the confidence interval using Equation [1] and inversely interpolating:

---

*DF = N - (number of terms), where N is the number of data points, and the number of terms is the number of independent variables plus one for the dependent variable in the regression analysis.

$$(\tfrac{1}{2}\ \ell n\ \frac{1+r}{1-r} \pm 1.96\ \frac{1}{\sqrt{DF}}) \qquad [1]$$

For convenience in subsequent discussion and calculations, let $g = \tfrac{1}{2}\ \ell n\ \frac{1+r}{1-r}$. Guttman and Wilks[E7] give tabulated values for g in terms of r; however, with modern desk top calculators, this function can easily be calculated directly.

Example 8:

Regression on GS versus NDT temperature gave $r_1 = 0.8174$ for $DF_1 = 111$ and regression on GS, B, F and P versus NDT temperature gave $r_2 = 0.8741$ and $DF_2 = 110$. To check whether the second run has significantly improved the correlation, one must establish the confidence interval for r_1 at the 95 percent significance level using r_1, DF_1, and Equation [1] and determine whether r falls outside those bounds.

$$(1.179 \pm 1.96\ \frac{1}{\sqrt{111}}) = (0.992 \text{ to } 1.366)\ .$$

Inversely interpolating to find the confidence interval, CI, for r_1 gives

$$CI = (0.7583 \text{ to } 0.8777)$$

Comparing the confidence interval for r_1 to the given value of r_2 shows that r_2 falls in this interval and that r_2 is not different than r_1 at the 95 percent level of significance although it would obviously not be the same at the 90

percent level*.

Example 9:

Regression on GS, B, F and P versus NDT temperature gave $r_1 = 0.8741$ for $DF_1 = 110$. Regression on GS, GS^2, GS^3, B, F and P versus NDT temperature gave $r_2 = 0.9103$ for $DF_2 = 106$ (two data sets were eliminated from the correlation by the aberrance value test which was set at T = 2.58 versus T = 3.09 for the first run). To check whether the second run significantly improved the correlation at the 95 percent level of significance, one must establish the CI for that run, using r_1 and DF_1 to see whether r_2 falls in that interval. Using Equation 1 we get:

$$(1.349 \pm 1.96 \frac{1}{\sqrt{110}}) = (1.162 \text{ to } 1.536)$$

Inversely interpolating to find the confidence interval, CI, for $r_1 = 0.87$ with $DF_1 = 110$ gives CI = (0.82 to 0.91) which indicates that r_2 falls right at the upper limit of the 95 percent confidence interval for r_1. This implies that using GS to the third power has not improved the correlation coefficient at the 95 percent significance level.

---

*It should be noted that the conclusions reached in Examples 8 to 11 using confidence interval tests of the regression coefficients, R values, are exactly the same conclusions reached using the Snedecor F ratio test and the standard deviations, Sx.y values, for the same regression analyses in Examples 4, 5, 6 and 7 respectively. Both methods indicate whether the addition or deletion of terms has significantly affected the usefulness of one regression analysis compared to another.

Example 10:

A comparison of r_2 from Example 9 to the CI for r_1 in Example 8 shows that GS, GS^2, and GS^2, B, F and P (r_2 = 0.9103, DF_2 = 106) have significantly improved the correlation of these factors with NDT over that for GS alone (r_1 = 0.8174, DF_1 = 111) since r_2 does not fall in the confidence interval for r_1, (0.7583 to 0.8777).

Example 11:

Regression on a set of variables gave r_1 = 0.932 for DF_1 = 126; deleting only the variable used to specify the producer of the material, regression gave, r_2 = 0.894 for DF_2 = 129. The 95 percent confidence interval, CI, is calculated as follows:

$$(1.674 \pm 1.06 \frac{1}{\sqrt{126}}) = (1.585 \text{ to } 1.849)$$

Inversely interpolating, CI = (0.9194, 0.9515). Since r_2 falls outside the confidence interval for r_1, we can conclude that the producer constitutes a significant variable at the 95 percent significance level and that this variable should be included in the regression.

CAUSES OF COMPUTATIONAL DUMPS

Possible causes of computational dumps are as follows:

1. Mixed up data deck. A listing of the input data will show breaks in the normal repetitive pattern when first setting it up; sequential end punching the final data package will help to keep it in order.

2. Mistakes in coding or punching the data. Even though the aberrance test and TERM MEANS will help to spot this trouble in the terms being run, 100 percent proof-reading of the data is a must.

3. Running as independent variables terms that are not independent. This sets up a singular matrix that usually causes an error dump labeled as being caused by trying to take the square root of a negative number or of zero; the solution is to sift the terms and their cross products using a correlation coefficient matrix such as in the BMO3D program described earlier. A single pair of highly correlated terms or several pairs of less highly correlated terms can cause this type of error dump.

4. Trying to take the logarithm of a term or its transformation when they are zero or negative. Obviously, one must be careful not to do this since it will cause an error dump; one can get around it by adding a constant that makes all the values of this term positive.

5. Running an inherently singular set of terms and variables. Possible solutions are to call the independent variable terms in a different order, to rearrange the sequence of the data sets, or to delete a few of either the variable terms or the data sets. Computational schemes are available to permit the user to check a matrix first and automatically shift columns or rows until the singularity is relieved.

6. Forgetting one of the many control cards. This often happens unless one develops the habit of using a pre-flight check list to check each and every one of the required control cards. When manipulating data decks, it is particularly important to carefully label the trailer card to avoid misplacing or forgetting it. Using a uniquely colored card for this helps reduce user mistakes.

Successful use of this program is entirely dependent on the user's constant attention to detail. It is also highly recommended that the program be run as often as possible so that the user becomes thoroughly imbued with the apparent vagaries in its operation. Successful regression and meaningful results can be accomplished only by a continual interaction between the project engineer whose data are being run, the applied statistician, and the programmer responsible for the day-to-day running of the program and data.

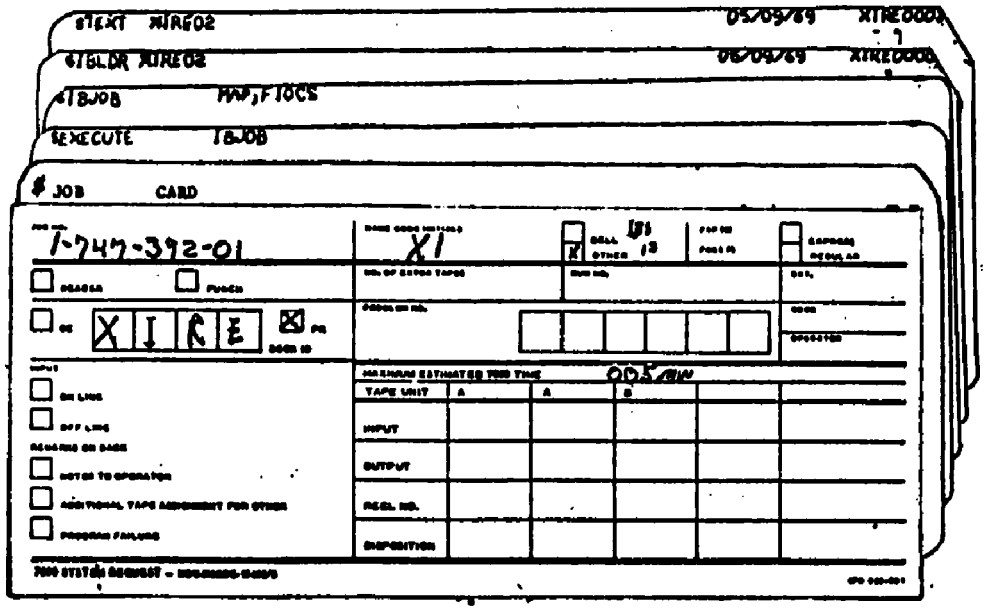

Figure 1a - For Running With a Binary Program Deck.

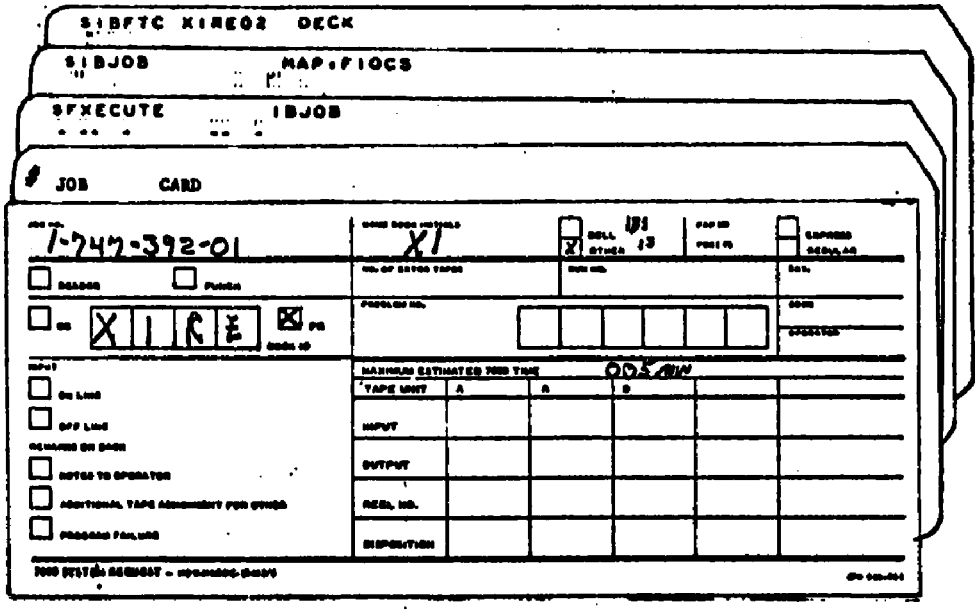

Figure 1b - For Running with FORTRAN IV Source Deck (Note, this includes a request for a binary program deck; if this is not desired, just omit the word DECK on the IBFTC card.)

Figure 1 - Request Form and Control Cards Required Before Program Deck to Run XIRE on 7090 at NSRDC. (While the above control card setup will work, a better set of control cards would consist of three additional cards placed immediately after the JOB CARD; they are (1) a $IBSYS card; (2) a $RESTORE card, and (3) a $DATE card.)

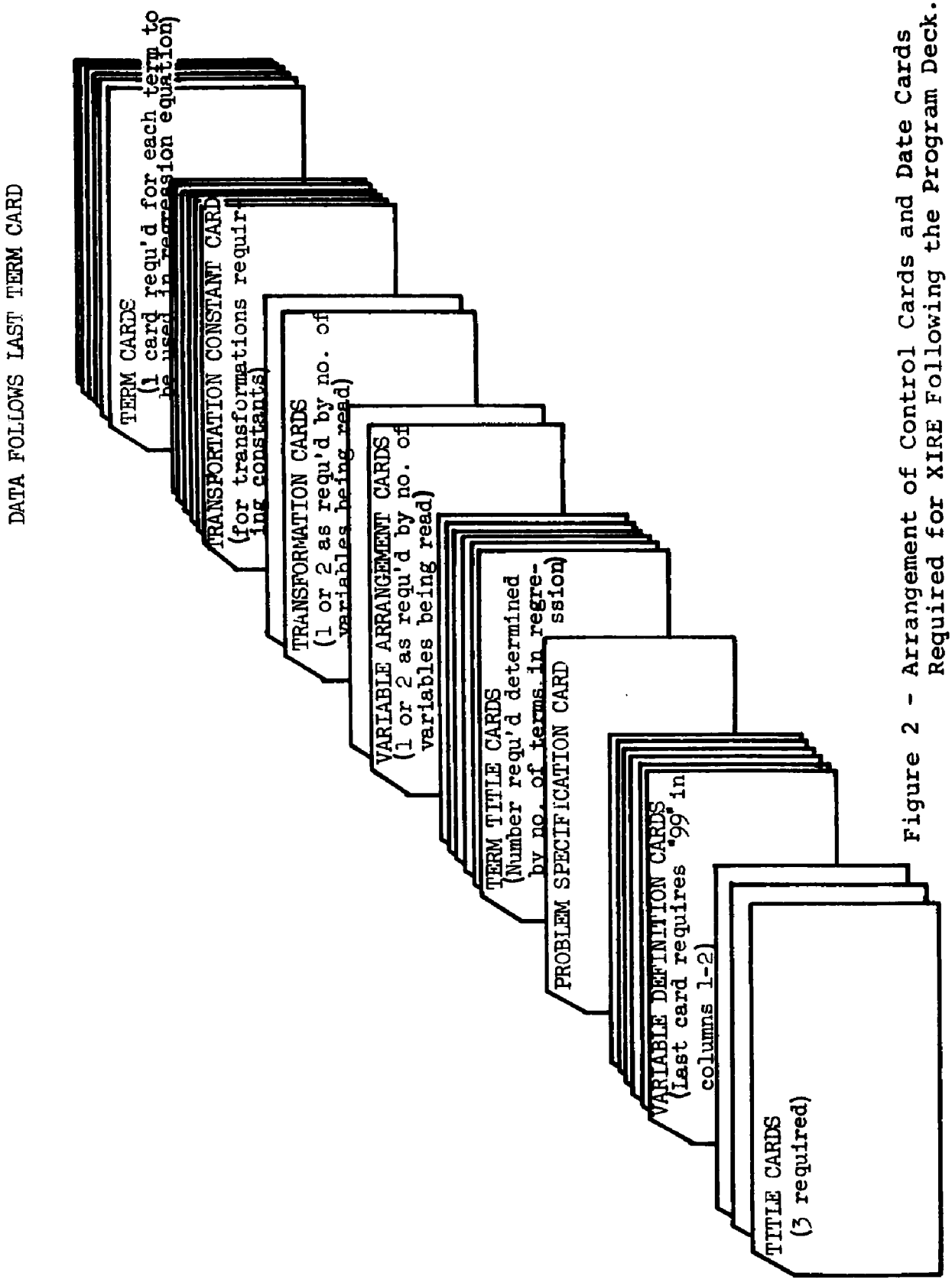

Figure 2 - Arrangement of Control Cards and Date Cards Required for XIRE Following the Program Deck.

COMPUTER LISTING E1

FORTRAN IV SOURCE STATEMENTS, LISTING OF PROGRAM XIRE, WITH STORAGE AND MEMORY MAPS FOR A 7090 COMPUTER

08/04/70

XIRE - EFN SOURCE STATEMENT - IFN(S) -

```
C      XIRE DECK 2 FORTRAN IV
C     M.L. SALIVE  EXT. 3237  NSRDC 09/30/69   TAPE REGRESSION
C
C*************************************************************************
C     THIS DECK SET UP TO READ 6 CARD FORMAT AND TO CACULATE TERM RANGE
C*************************************************************************
C
 1000 FORMAT (1H1,47X,22HDEPARTMENT OF THE NAVY)                          XIREC010
  998 FORMAT(1HO,37X,42HNAVAL SHIP RESEARCH AND DEVELOPMENT CENTER)
  999 FORMAT(1HO,25X,67HPROTECTION SYSTEMS CONCEPTS BRANCH (747) - STRUCXIREC020
     1TURAL MECHANICS LAB)                                                XIREC025
 1001 FORMAT (1HO,23X,32HTAPE REGRESSION 2 - 60 VARIABLES,17X,25HPROGRAMXIREC030
     1 NAME XIRE  DECK 2)
                                                                          XIREC050
 1002 FORMAT (1H1)                                                        XIREC060
 1003 FORMAT (1HO)                                                        XIREC070
 1004 FORMAT (1X)                                                         XIREC080
 1006 FORMAT (1HO, 8X,11HORIGINATOR , 3A6, 27X, 13HPJCT DATE     ,2A6)    XIREC090
 1007 FORMAT (19X,10HNAVY CODE ,3A6,19X,17HPROJECT NUMBER    ,2A6)        XIREC100
 1008 FORMAT (1HO,18X,6HTITLE , 5A6)                                      XIREC110
 1009 FORMAT (1HO,18X,75HVARIABLES ON CARDS  TRANSFORMED VARIABLES  TERMXIREC120
     1S  ABERRANCE T  CONFIDENCE T)                                       XIREC130
 1010 FORMAT (26X,I3,19X,I3,11X,I3,6X,F5.2,8X,F5.2)                       XIREC140
 1012 FORMAT (1HO,39X,38HCORRECTED SUMS OF SQUARES AND PRODUCTS)          XIREC150
C     PRINT SS, SP, AND INVERSE                                          XIREC160
 1013 FORMAT (7E16.8)                                                     XIREC170
 1014 FORMAT (1HO,51X,15HINVERSE  MATRIX)                                 XIREC180
 1016 FORMAT (1H1,48X,21HREGRESSION STATISTICS)
 1017 FORMAT(1HO,23HREGRESSION COEFFICIENTS,14X,10HTERM NAMES,23X,1HT,8X
     1,1CH   PRODUCT,6X,9H    MEANS,3X,14H    STD. DEVS.,3X,5HRANGE)
 1018 FORMAT(1X,2HB ,I3,3X,E14.7,5X,6A6,4X,F7.3,2X,F14.7,2(1X,F14.7),
     12X,F6.2)
 1040 FORMAT(100X,32H* * * * * * * TERM * * * * * * *   )
 1041 FORMAT(1X,2HB ,I3,3X,E14.7,5X,6A6,4X,F7.3,2X,F14.7,2(1X,F14.7),
     12X,F6.2,1H*)
 1019 FORMAT(1X,1HY,26X,6A6,27X,2(1X,F14.7),3X,F5.2)
 1042 FORMAT(1X,1HY,26X,6A6,27X,2(1X,F14.7),3X,F5.2,1H*)
 1020 FORMAT (1HO,18X,6HBO    ,F14.7,10X,4HR2,  ,F14.7,10X,4HDF   ,I4)    XIREC240
C     PRINT BO, R, DF, ETC                                               XIREC230
 1021 FORMAT (19X,6HSX.Y  ,F14.7,10X,4HR   ,F14.7,10X,4HN    ,F5.0)       XIREC250
 1022 FORMAT (1H1,38X,29HDATA  AND  BACK  CALCULATIONS)
C     PRINT DATA FORMATS                                                 XIREC270
 1023 FORMAT (1HO,8X,13HORIGINAL DATA)                                    XIREC280
 1024 FORMAT (9X,16HTRANSFORMED DATA)                                     XIREC290
 1025 FORMAT (9X,5HTERMS)                                                 XIREC300
 1026 FORMAT (10E13.5)                                                    XIREC310
 1027 FORMAT (24X,2HYC,11X,2HDE,11X,2HLL,11X,2HUL,11X,3HOYC,10X,3HOLL,   XIREC320
     110X,3HOUL)                                                          XIREC330
 1028 FORMAT (10X,4HLINE,5X,7(E13.5),2X,A2)                               XIREC340
 1029 FORMAT (10X,5HPOINT,4X,E13.5,13X,5(E13.5))                          XIREC350
 1129 FORMAT (10X,7HCONTROL,2X,E13.5,13X,5(E13.5))                        XIREC360
 1030 FORMAT(19X,7A6)                                                     XIREC370
 1031 FORMAT (22X,F3.1,14X,F3.1)                                          XIREC380
C     READ FORMATS                                                       XIREC390
 2000 FORMAT(11A6)                                                        XIREC400
 2001 FORMAT (3I2,2F4.2,I1,12I2)                                          XIREC410
```

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                                                                                08/04/70
          XIRE       - EFN   SOURCE STATEMENT  -  IFN(S)  -

 2002 FORMAT(6(1X,E11.5))                                                      XIREC420
 2003 FORMAT(I2,11A6)                                                          XIREC430
 2004 FORMAT (40I2)                                                            XIREC440
 2005 FORMAT (1X,I2,2(1X,E12.6),1X,I2)                                         XIREC450
 2006 FORMAT(1X,3(1X,I2,1X,E9.5),1X,I2)                                        XIREC460
C     BEGIN CALCULATIONS                                                      XIREC470
      DIMENSION Y(60),Z(80),    C(60,2),SS(60,62,2),X(60),S(60,2),            XIREC480
     1IT(60,3),TR(60,3),L1(6C),L2(6C),T(3),D(100)                             XIREC490
      DIMENSION TNAMES(60,6)
      DIMENSION CROSS(60)
      DIMENSION RANGE(60)
      DATA SA8/2HA8/,BL/2H  /                                                 XIREC500
      M1=60                                                                   XIREC510
      ID=0                                                                    XIREC511
    1 WRITE (6,1000)                                                          XIREC520    3
      WRITE(6,998)                                                                        4
      WRITE (6,999)                                                           XIREC530    5
      WRITE (6,1001)                                                          XIREC540    6
      READ(5,2000) (Z(J),J=1,11)                                              XIREC550    7
      WRITE(6,1006) (Z(J),J=1,5)                                              XIREC560   14
      READ(5,2000) (Z(J),J=1,11)                                              XIREC570   21
      WRITE(6,1007) (Z(J),J=1,5)                                              XIREC580   28
      READ(5,2000) (Z(J),J=1,11)                                              XIREC590   35
      WRITE(6,1008) (Z(J),J=1,5)                                              XIREC600   42
    3 READ(5,2003)N, (Z(J),J=1,5)                                             XIREC610   49
      WRITE(6,1030) (Z(J),J=1,5)                                              XIRE0620   57
      IF(N-99)3,4,4                                                           XIREC630
    4 READ (5,2001)NCD,NT,M1,ABT,TCL,IYC,IY2,IY3,IY4,IY5,IY6                  XIREC640   65
      WRITE(6,1009)                                                           XIREC650   75
      WRITE(6,1010)NCD,NT,M1,ABT,TCL                                          XIRE0660
C     READ STATEMENT ADDED TO READ TERM NAMES IMMEDIATELY AFTER PROBLEM
C     SPECIFICATION CARD.   THESE WILL BE PRINTED BESIDE REGRESSION COEFFICIENTS  76
      DO 3000 IIIII=1,M1                                                                  80
 3000 READ(5,3001) (TNAMES(IIIII,KKKKK),KKKKK=1,6)
 3001 FORMAT(6A6)                                                             XIRE0670
      IF (NCD-36)2,2,22                                                       XIRE0680   88
    2 READ(5,2004) (L2(J),J=1,36)                                             XIREC690   95
      READ(5,2004) (L1(J),J=1,36)                                             XIREC700
      GO TO 11                                                                XIREC710  103
   22 READ (5,2004) (L2(J),J=1,36)                                            XIREC720  110
      READ (5,2004) (L2(J),J=37,MT)                                           XIREC730  117
      READ (5,2004) (L1(J),J=1,36)                                            XIREC740  124
      READ (5,2004) (L1(J),J=37,MT)                                           XIREC750
   11 DO 6 I=1,MT                                                             XIREC760
      DO 6 J=1,2                                                              XIREC770
    6 C(I,J)=0.0                                                              XIRE0780
      DO 7 I=1,MT                                                             XIRE0790  144
      READ(5,2005)J,YC,QYC,N                                                  XIREC800
      C(J,1)=YC                                                               XIRE0810
      C(J,2)=QYC                                                              XIREC820
      IF(N-99)7,8,8                                                           XIRE0830
    7 CONTINUE                                                                XIREC840
    8 DO 9 I=1,M1                                                             XIRE0850  157
    9 READ(5,2006)IT(I,1),TR(I,1),IT(I,2),TR(I,2),IT(I,3),TR(I,3),N           XIRE0860
      M4=IT(M1,1)                                                             XIRE0870
      M=M1-1
```

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      XIRE       - LRN   SOURCE STATEMENT  -  IFN(S)  -

      M2=M1+1                                                           XIREC880
      M3=M1+2                                                           XIREC890
      CM1=M1                                                            XIREC900
      E1C=.43429448                                                     XIREC910
      REWIND 1                                                          XIREC920    172
      REWIND 2                                                          XIREC930    173
      AB=BL                                                             XIREC940
      NA6=-1                                                            XIREC950
      NA=0                                                              XIREC960
      IF(ABT-9.0)12,13,13                                               XIREC970
   12 NI=0
      GO TO 14                                                          XIREC990
   13 NI=4                                                              XIRE1000
   14 NP=2                                                              XIRE1010
      NB=0                                                              XIRE1020
      ZR=0.0                                                            XIRE1030
      N=0                                                               XIRE1040
      DO 10 I=1,NT                                                      XIRE1050
   10 L1(I)=L1(I)+1                                                     XIRE1060
      L14=L1(M4)                                                        XIRE1070
C     THESE CARDS REPLACE STATEMENTS 15 THRU 18 INCLUSIVE TO MODIFY     XIRE1090
C     FORMAT OF DATA INPUT FOR SPECIAL RUN.   NORMAL INPUT CARDS        XIRE1100
C     FOLLOW AS COMMENT CARDS.                                          XIRE1110
C                                                                       XIRE1120
   15 CONTINUE                                                          XIRE1130
   16 CONTINUE                                                          XIRE1140
C     FOR METALLOGRAPHIC DATA ONLY (TEMPORARY INPUT FORMAT)             XIRE1150
   17 READ(5,20002)(D(J),J=1,14)                                        XIRE1160    195
20002 FORMAT(F3.0,2X,13F5.4)                                            XIRE1170
      IF(D(1)-999.)18,27,27                                             XIRE1180
   18 READ(5,20003)(D(J),J=15,NCD)                                      XIRE1190    203
20003 FORMAT(5F5.4/F5.4/4F4.3,2(F4.0,1X),2(F6.0,1X),2(F4.2,1X),F6.0,1X,FXIRE1200
     14.0,1X,F5.3/5F10.5/6F10.1)
C                                                                       XIRE1220
C     ********************************************************************XIRE1230
C  15 IF(NCD-6)16,16,17                                                 XIRE1240
C  16 READ(5,2002) (D(J),J=1,NCD)                                       XIRE1250
C     IF(D(1)-.99999E38)19,27,27                                        XIRE1260
C  17 READ(5,2002)(D(J),J=1,6)                                          XIRE1270
C     IF(D(1)-.99999E38)18,27,27                                        XIRE1280
C  18 READ(5,2002)(D(J),J=7,NCD)                                        XIRE1290
C     ********************************************************************XIRE1300
C                                                                       XIRE1310
C     REARRANGE COLUMNS                                                 XIRE1320
   19 DO 25  K=1,NT                                                     XIRE1330
      L22=L2(K)                                                         XIRE1340
   25 Y(K)=D(L22)                                                       XIRE1350
      ID=(IFIX(Y(1)))/100                                               XIRE1352
      Y(1)=FLOAT(ID)                                                    XIRE1353
C     IF(Y(M4)-.99999E38) 120,121,121                                   XIRE1360
      IF(Y(M4)-999.)120,121,121                                         XIRE1361
  120 AB=BL                                                             XIRE1370
      GO TO 26                                                          XIRE1380
  121 AB=SAB                                                            XIRE1390
      Y(M4) =1.0                                                        XIRE1400
   26 WRITE (1) (Y(J),J=1,NT),AB                                        XIRE1410    227
```

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XIRE - EFN SOURCE STATEMENT - IFN(S) -

```
      N=N+1                                                          XIRE1420
C     INSERT TEMPORY PRINT HERE                                      XIRE1430
      GO TO 15                                                       XIRE1440
   27 ENC FILE 1                                                     XIRE1450    235
      REWIND 1                                                       XIRE1460
C     N SETS REARRANGED DATA ON TAPE 1                               XIRE1470    236
   30 CNC=0.0                                                        XIRE1480
      DO 31 I=1,M1                                                   XIRE1490
      DO 31 K=1,2                                                    XIRE1500
   31 S(I,K)=0.0                                                     XIRE1510
      DO 32 I=1,M1                                                   XIRE1520
      DO 32 J=1,M3                                                   XIRE1530
      DO 32 K=1,2                                                    XIRE1540
   32 SS(I,J,K)=0.0                                                  XIRE1550
   21 DO 101 I=1,M                                                   XIRE1560
    5 IF(NAB)33,33,34                                                XIRE1570
   33 READ (1) (X(J),J=1,NT),AB                                      XIRE1580    267
      GO TO 35                                                       XIRE1590
   34 READ (2) (X(J),J=1,NT),AB                                      XIRE1600    273
   35 IF(AB.EQ.SAB)GO TO 101                                         XIRE1610
   36 CNC=CNO+1.0                                                    XIRE1620
      GO TO 40                                                       XIRE1630
C     TEMPORARY PRINT TO TEST TERMS                                  XIRE1640
C     TERM VECTOR IN Z(J) ADD TO S, SS, +SP                          XIRE1650
C     DOUBLE PRECISION SUMS IN S(I,J) ARRAY                          XIRE1660
   95 DO 100 J=1,M1                                                  XIRE1670
      IP=1                                                           XIRE1680
      CALL DOPOP (S(J,1),S(J,2),Z(J),ZR,IP)                          XIRE1690    290
      DO 100 K=1,M1                                                  XIRE1700
      IP=3                                                           XIRE1710
      A1=Z(J)                                                        XIRE1720
      A2=ZR                                                          XIRE1730
      CALL DOPOP (A1,A2,Z(K),ZR,IP)                                  XIRE1740    297
      IP=1                                                           XIRE1750
      CALL DOPOP (SS(J,K,1),SS(J,K,2),A1,A2,IP)                      XIRE1760    302
  100 CONTINUE                                                       XIRE1770
  101 CONTINUE                                                       XIRE1780
      IF (NAB)102,102,103                                            XIRE1790
  102 END FILE 1                                                     XIRE1800    310
      REWIND1                                                        XIRE1810    311
      GO TO 104                                                      XIRE1820
  103 END FILE 2                                                     XIRE1830    313
      REWIND 2                                                       XIRE1840    314
  104 DO 105 J=1,M1                                                  XIRE1850
      DO 105 K=1,M1                                                  XIRE1860
      IP=3                                                           XIRE1870
      A1=S(J,1)                                                      XIRE1880
      A2=S(J,2)                                                      XIRE1890
      CALL DOPOP (A1,A2,S(K,1),S(K,2),IP)                            XIRE1900    326
      IP=4                                                           XIRE1910
      CALL DOPOP (A1,A2,CNO,ZR,IP)                                   XIRE1920    329
      IP=2                                                           XIRE1930
      CALL DOPOP (SS(J,K,1),SS(J,K,2),A1,A2,IP)                      XIRE1940    334
  105 CONTINUE                                                       XIRE1950
C     TRANSFER M1 COL. TO M3 COL                                     XIRE1960
      DO 108 J=1,M1                                                  XIRE1970
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XIRE - EFN SOURCE STATEMENT - IFN(S) -

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      DO 108 K=1,2                                                      XIRE1980
  108 SS(J,M3,K)=SS(J,M1,K)                                             XIRE1990
C     TEST FOR PRINT                                                    XIRE2000
      IF(N1)112,112,109                                                 XIRE2010
  109 WRITE(6,1012)                                                     XIRE2020    351
      DO 110 I=1,M1                                                     XIRE2030
      WRITE(6,1004)                                                     XIRE2040    354
  110 WRITE(6,1013)(SS(I,J,1),J=1,M1)                                   XIRE2050    355
  112 SV2=SS(M1,M1,1)                                                   XIRE2060
C     TERM STANDARD DEVIATIONS IN S(1,2)                                XIRE2070
      DO 113 I=1,M1                                                     XIRE2080
  113 S(I,2)=SQRT (SS(I,I,1)/(CNO-1.0))                                 XIRE2090
C                                                                       XIRE2100
C     INVERT MATRIX DOUBLE PRECISION                                    XIRE2110    369
      IF(M1-2)115,115,915                                               XIRE2120
  115 SS(1,2,1)=1.0/SS(1,1,1)                                           XIRE2130
      SS(1,1,1)=SS(1,2,1)*SS(1,M3,1)                                    XIRE2140
      GO TO 178                                                         XIRE2150
  915 DO 936 KIT=1,M                                                    XIRE2160
      DO 924 I=1,M1                                                     XIRE2170
      DO 924 K=1,2                                                      XIRE2180
  924 SS(I,M2,K)=0.0                                                    XIRE2190
      SS(1,M2,1)=1.0                                                    XIRE2200
      DO 926 J=1,M1                                                     XIRE2210
      IP=4                                                              XIRE2220
      A1=SS(1,J+1,1)                                                    XIRE2230
      A2=SS(1,J+1,2)                                                    XIRE2240
      B1=SS(1,1,1)                                                      XIRE2250
      B2=SS(1,1,2)                                                      XIRE2260
      CALL DOPOP (A1,A2,B1,B2,IP)                                       XIRE2270    395
      SS(M1,J,1)=A1                                                     XIRE2280
  926 SS(M1,J,2)=A2                                                     XIRE2290
      DO 928 I=1,M                                                      XIRE2300
      SS1=SS(I,1,1)                                                     XIRE2310
      SS2=SS(I,1,2)                                                     XIRE2320
      DO 928 J=1,M1                                                     XIRE2330
      IP=3                                                              XIRE2340
      A1=SS1                                                            XIRE2350
      A2=SS2                                                            XIRE2360
      B1=SS(M1,J,1)                                                     XIRE2370
      B2=SS(M1,J,2)                                                     XIRE2380
      CALL DOPOP (A1,A2,B1,B2,IP)                                       XIRE2390    41C
      B1=A1                                                             XIRE2400
      B2=A2                                                             XIRE2410
      A1=SS(I,J+1,1)                                                    XIRE2420
      A2=SS(I,J+1,2)                                                    XIRE2430
      IP=2                                                              XIRE2440
      CALL DOPOP (A1,A2,B1,B2,IP)                                       XIRE2450    415
      SS(I,J,1)=A1                                                      XIRE2460
  928 SS(I,J,2)=A2                                                      XIRE2470
      DO 936 I=1,M                                                      XIRE2480
      DO 936 J=1,M1                                                     XIRE2490
      DO 936 K=1,2                                                      XIRE2500
  936 SS(I,J,K)=SS(I+1,J,K)                                             XIRE2510
C     TEST FOR PRINT                                                    XIRE2520
  178 IF(N1)183,183,179                                                 XIRE2530
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  179 WRITE(6,1014)                                                    XIRE2540    438
      DO 182 I=1,M                                                     XIRE2550
      WRITE(6,1004)                                                    XIRE2560    441
      WRITE(6,1013)(SS(I,J,1),J=2,M1)                                  XIRE2570    442
  182 WRITE(6,1004)                                                    XIRE2580
C     CALCULATE TERM MEANS, R2, BO, TBS, DF                            XIRE2590
C     CALCULATE TERM STANDARD DEVIATIONS - STORE IN S(I,2)             XIRE2600    447
  183 S(M1,1)=S(M1,1)/CNO                                              XIRE2610
      XM1=M1                                                           XIRE2620
      SEQE=0.0                                                         XIRE2630
      SS(M1,1,1)=0.0                                                   XIRE2640
      DO 186 I=1,M                                                     XIRE2650
      S(I,1)=S(I,1)/CNO                                                XIRE2660
  186 SEQE=SEQE+SS(I,1,1)*SS(I,M3,1)                                   XIRE2670
      R2=SEQE/SY2                                                      XIRE2680
      R=SQRT (R2)                                                      XIRE2690    464
      SEQE=SQRT ((SY2-SEQE)/(CNO-XM1))                                 XIRE2700
C     TBS IN SS(I,M2,1), BO IN SS(M1,1,1)                              XIRE2710    465
      DO 187 I=1,M                                                     XIRE2720
      SS(I,M2,1)=SS(I,1,1)/(SEQE*SQRT (SS(I,I+1,1)))                   XIRE2730    470
  187 SS(M1,1,1)=SS(M1,1,1)+SS(I,1,1)*S(I,1)                           XIRE2740
      SS(M1,1,1)=S(M1,1)-SS(M1,1,1)                                    XIRE2750
      DF=CNO-XM1                                                       XIRE2760
C     TEST FOR PRINT                                                   XIRE2770
      DO 189 I=1,M
  188 CROSS(I) = SS(I,1,1)*S(I,2)*4.0
      IF(NI)201,201,189
C     CALCULATE TERM RANGE (TERM MEAN / TERM STD. DEV.)
  189 RANGE(I)=S(I,1)/S(I,2)
C     WRITE REGRESSION COEFFICIENTS AND CONSTANTS                      XIRE2790
  190 WRITE(6,1016)                                                    XIRE2800    497
      WRITE(6,1003)                                                                498
      WRITE(6,1040)                                                                499
      WRITE(6,1017)                                                    XIRE2810
C     PRINTOUT ADDED TO ORIGINAL PROGRAM TO LIST TERM NAME BESIDE
C     REGRESSION COEFFICIENT
C     TEST RANGE TO SEE IF LESS THAN 2.0 (THESE VALUES TO BE TAGGED *  )           500
      DO 192 I=1,M
      IF(RANGE(I).LT.2.0.AND.(-2.0.LT.RANGE(I))) GO TO 193
      WRITE(6,1018) I,SS(I,1,1),(TNAMES(I,I9),I9=1,6), SS(I,M2,1),
     1CROSS(I),S(I,1),S(I,2),RANGE(I)                                              508
      GO TO 192
  193 WRITE(6,1041)I,SS(I,1,1),(TNAMES(I,I9),I9=1,6),SS(I,M2,1),
     1CROSS(I),S(I,1),S(I,2),RANGE(I)                                              520
  192 CONTINUE
      RANGE(M1)=S(M1,1)/S(M1,2)
      IF(RANGE(M1).LT.2.0.AND.(-2.0.LT.RANGE(M1))) GO TO 195
      WRITE(6,1019) (TNAMES(M1,J9),J9=1,6),S(M1,1),S(M1,2),RANGE(M1)                541
      GO TO 196
  195 WRITE(6,1042) (TNAMES(M1,J9),J9=1,6),S(M1,1),S(M1,2),RANGE(M1)                552
  196 CONTINUE
      IDF=IFIX(DF)
      WRITE(6,1020)SS(M1,1,1),R2,IDF                                   XIRE2850    563
      WRITE(6,1021)SEQE,R,CNO                                          XIRE2860
C     EXPLANATIONS OF PRINTOUT NAMES IS PRINTED IMMEDIATELY AFTER THEIR USE        565
      WRITE(6,3002)                                                                566
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                                                                      08/04/70
      XIRE       - FTN   SOURCE STATEMENT  -  IFN(S)  -

3002 FORMAT(/50H 80 - CONSTANT CALCULATED FOR REGRESSION EQUATION   /
     161H R2 - STANDARD ERROR OF ESTIMATE (R SQUARED)               /
     261H DF - DEGREES OF FREEDOM                                   /
     363H SX.Y - STANDARD DEVIATION OF A PREDICTED POINT (RESIDUAL UNALC
     A37HOUNTED FOR AFTER REGRESSION)                               /
     461H R - REGRESSION COEFFICIENT                                /
     561H N - NUMBER OF DATA POINTS                                 /
     661H T - COEFFICIENT DIVIDED BY THE STD DEV OF THE COEFFICIENT /
     861H (USE WITH STUDENT  T  DISTRIBUTION TABLE AND DEGREES OF FREE ,
     C16HDOM TO DETERMINE  /
     D61H CONFIDENCE INTERVALS OF THE REGRESSION COEFFICIENTS)       /
     761H PRODUCT - 4*(REGRESSION COEF)*(TERM STD DEV)               /
     861H TERM RANGE - (TERM MEAN)/(TERM STD DEV)                   /
     962H * TERM RANGE IS LESS THAN 2.0, THE VALUE OF PRODUCT SHOULD BE/
     E61H     RECOMPUTED BEFORE BEING EVALUATED SINCE VALUES OF THIS V ,
     F61HARIABLE ARE DISTRIBUTED IN A SKEW MANNER                    )
      WRITE(6,1022)                                                  XIRE2970    567
      WRITE(6,3003)                                                              568
 3003 FORMAT(///
     161H ORIGINAL DATA - IN SAME ORDER AS VARIABLE DEFINITION LIST  /
     261H TRANSFORMED DATA - IN SAME ORDER AS ORIGINAL DATA          /
     356H TERMS - IN SAME ORDER AS REGRESSION COEFFICIENTS, WITH     ,
     461H DEPENDENT VARIABLE LAST                                   )
      WRITE(6,1004)                                                  XIRE2980    569
      NB=9                                                           XIRE2990
      DO 230 I=1,N                                                   XIRE3000
      IF(N-8)2201,2201,2202                                          XIRE3010
 2201 READ (1)(X(J),J=1,NT),AB                                       XIRE3020    578
      GO TO 2203                                                     XIRE3030
 2202 READ (2)(X(J),J=1,NT),AB                                       XIRE3040    582
 2203 IF(NT)203,203,202                                              XIRE3050
  202 IF(NP-4)1202,1201,1201                                         XIRE3060
 1201 NP=0                                                           XIRE3070
 1202 WRITE(6,1023)                                                  XIRE3080    592
      WRITE(6,1026)(X(J),J=1,NT)                                     XIRE3090    593
  203 GO TO 40                                                       XIRE3100
  204 IF(NT)206,206,205                                              XIRE3110
  205 WRITE(6,1024)                                                  XIRE3120
C     WRITE(6,1026)(Y(J),J=1,NT)                                     XIRE3130    601
  206 GO TO 60                                                       XIRE3140
  207 IF(NT)209,209,208                                              XIRE3150
  208 WRITE(6,1025)                                                  XIRE3160    605
      WRITE(6,1026)(Z(J),J=1,M1)                                     XIRE3170    606
  209 YC=0.0                                                         XIRE3180
      DO 210 J=1,M                                                   XIRE3190
  210 YC=YC+SS(J,1,1)*Z(J)                                           XIRE3200
      YC=YC+SS(M1,1,1)                                               XIRE3210
      CL=0.0                                                         XIRE3220
      DO 212 J=1,M                                                   XIRE3230
      DO 212 K=2,M1                                                  XIRE3240
  212 CL=CL+SS(J,K,1)*(Z(J)-S(J,1))*(Z(K-1)-S(K-1,1))                XIRE3250
      CL1=(1.0/CNO)*CL                                               XIRE3260
      CL=TCL*SEOE*SQRT(CL1)                                          XIRE3270    631
      AUCL=YC+CL                                                     XIRE3280
      ALCL=YC-CL                                                     XIRE3290
      CL1=TCL*SEOE*SQRT(1.0+CL1)                                     XIRE3300    637
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                                                                        C8/04/70
      XIRE       - EFN   SOURCE STATEMENT  -  IFN(S)  -

      BUCL=YC+CL1                                                       XIRE3210
      BLCL=YC-CL1                                                       XIRE3220
      CUCL=YC+(TCL*SEDE)                                                XIRE3230
      CLCL=YC-(TCL*SEDE)                                                XIRE3240
      DEV=Z(M1)-YC                                                      XIRE3250
      IF (AB.EQ.SAB)GO TO 213                                           XIRE3260
  211 DE=ABS(DEV/SEDE)                                                  XIRE3270
      IF (DE-ABT)213,215,215                                            XIRE3280
  215 AB=SAB                                                            XIRE3290
      NA=NA+1                                                           XIRE3300
  213 IF(NAB)314,314,315                                                XIRE3310
  314 WRITE(2) (X(L),L=1,NT),AB                                         XIRE3320    642
      GO TO 313                                                         XIRE3330
  315 WRITE(1) (X(L),L=1,NT),AB                                         XIRE3340    646
  313 IF (NI)230,230,316                                                XIRE3350
  316 GO TO (221,216,217,218,219,220,221),L14                           XIRE3360
  216 OYC=-C(M4,2)+(10.0**( YC /C(M4,1)))                               XIRE3370    658
      OALCL= -C(M4,2)+(10.0**(ALCL/C(M4,1)))                            XIRE3380    661
      OALCL = -C(M4,2)+(10.0**(ALCL/C(M4,1)))                           XIRE3390    664
      OBLCL=-C(M4,2)+(10.0)**(BLCL/C(M4,1))                             XIRE3400    667
      OBLCL=-C(M4,2)+(10.0)**(BUCL/C(M4,1))                             XIRE3410    67C
      OCLCL=-C(M4,2)+(10.0)**(CLCL/C(M4,8))                             XIRE3420    673
      OCUCL=-C(M4,2)+(10.0)**(CUCL/C(M4,1))                             XIRE3430    67E
      GO TO 225                                                         XIRE3440
  217 DE=C(M4,1)                                                        XIRE3450
      DO=1.0/C(M4,2)                                                    XIRE3460
      OYC=DE+(YC**DO)                                                   XIRE3470    682
      OALCL=DE+(ALCL**DO)                                               XIRE3480    683
      OBLCL=DE+(BLCL**DO)                                               XIRE3490    684
      OCLCL=DE+(CLCL**DO)                                               XIRE3500    685
      OAUCL=DE+(AUCL**DO)                                               XIRE3510    686
      OBUCL=DE+(BUCL**DO)                                               XIRE3520    687
      OCUCL=DE+(CUCL**CO)                                               XIRE3530    688
      GO TO 225                                                         XIRE3540
  218 DE=C(M4,2)*E10*ALOG(C(M4,1)*10.0)                                 XIRE3550    692
      DE=E10/DE                                                         XIRE3560
      OYC   =DE*ALOG(YC)                                                XIRE3570    694
      OALCL=DE*ALOG(ALCL)                                               XIRE3580    695
      OBLCL=DE*ALOG(BLCL)                                               XIRE3590    696
      OCLCL=DE*ALOG(CLCL)                                               XIRE3600    697
      OAUCL=DE*ALOG(AUCL)                                               XIRE3610    698
      OBUCL=DE*ALOG(BUCL)                                               XIRE3620    695
      OCLCL=DE*ALOG(CUCL)                                               XIRE3630    70C
      GO TO 225                                                         XIRE3640
  219 DO=-C(M4,1)                                                       XIRE3650
      OYC=(1.0/YC)+DO                                                   XIRE3660
      OALCL=(1.0/ALCL)+DO                                               XIRE3670
      OBLCL=(1.0/BLCL)+DO                                               XIRE3680
      OCLCL=(1.0/CLCL)+DO                                               XIRE3690
      OAUCL=(1.0/AUCL)+DO                                               XIRE3700
      OBUCL=(1.0/BUCL)+DO                                               XIRE3710
      OCLCL=(1.0/CUCL)+DO                                               XIRE3720
      GO TO 225                                                         XIRE3730
  220 DO=57.295780                                                      XIRE3740
      OYC=((SIN(YC/DO))**2)*100.0                                       XIRE3750    706
      OALCL=((SIN(ALCL/DO))**2)*100.0                                   XIRE3760    707
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      OBLCL=((SIN(BLCL/DD))**2)*100.0                                   XIRE3770   708
      OCLCL=((SIN(CLCL/DD))**2)*100.0                                   XIRE3780   709
      OALCL=((SIN(AUCL/DD))**2)*100.0                                   XIRE3790   710
      OBLCL=((SIN(BUCL/DD))**2)*100.0                                   XIRE3800   711
      OCUCL=((SIN(CUCL/DD))**2)*100.0                                   XIRE3810   712
      GO TO 225                                                         XIRE3820
  221 OYC=YC                                                            XIRE3830
      OALCL=ALCL                                                        XIRE3840
      OBLCL=BLCL                                                        XIRE3850
      OCLCL=CLCL                                                        XIRE3860
      OALCL=AUCL                                                        XIRE3870
      OBUCL=BUCL                                                        XIRE3880
      OCUCL=CUCL                                                        XIRE3890
  225 WRITE(6,1027)                                                     XIRE3900   715
      WRITE(6,1028)YC,DEV,ALCL,AUCL,OYC,CALCL,UAUCL,AB                  XIRE3910
C     WRITE(6,1029)YC,BLCL,BLCL,OYC,OBLCL,OBUCL                         XIRE3920
C     WRITE(6,1129)YC,CLCL,CLCL,OYC,OCLCL,OCUCL                         XIRE3930   718
  230 CONTINUE                                                          XIRE3940
      REWIND1                                                           XIRE3950   719
      REWIND2                                                           XIRE3960   720
      NB=0                                                              XIRE3970
      IF(NA)233,233,231                                                 XIRE3980
  231 NA=0                                                              XIRE3990
      NAB=NAB*(-1)                                                      XIRE4000
      ABT=1.3*ABT                                                       XIRE4010
      GO TO 30                                                          XIRE4020
  233 IF(NI)234,234,301                                                 XIRE4030
  234 NI=NI+1                                                           XIRE4040
      GO TO 30                                                          XIRE4050
  301 GO TO 1                                                           XIRE4060
C     TEMPORARY PRINT                                                   XIRE4070
   40 DO 55 J=1,NI                                                      XIRE4080
      LLL=LL(J)                                                         XIRE4090
      GO TO (42,43,44,45,46,47,51),LLL                                  XIRE4100
   42 Y(J)=X(J)                                                         XIRE4110
      GO TO 55                                                          XIRE4120
   43 Y(J)=C(J,1)*E10*ALOG(X(J)+C(J,2))                                 XIRE4130   745
      GO TO 55                                                          XIRE4140
   44 Y(J)=(X(J)+C(J,1))**C(J,2)                                        XIRE4150   753
      GO TO 55                                                          XIRE4160
   45 Y(J)=(C(J,1)*10.0)**(C(J,2)*X(J))                                 XIRE4170   760
      GO TO 55                                                          XIRE4180
   46 Y(J)=1.0/(X(J)+C(J,1))                                            XIRE4190
      GO TO 55                                                          XIRE4200
   47 Y(J)=57.29578*ARSIN (SQRT (X(J)/100.0))                           XIRE4210   770
      GO TO 55                                                          XIRE4220
   51 Y(J)=SIN (C(J,1)*(X(J)+C(J,2)))                                   XIRE4230   778
   55 CONTINUE                                                          XIRE4240
C     TRANSFORMED VARIABLES IN Y(J) ARRAY                               XIRE4250
C     TEMPORARY PRINT                                                   XIRE4260
      IF(NB)60,60,204                                                   XIRE4270
C     DEVELOP TERMS IN Z(J) ARRAY                                       XIRE4280
   60 DO 85 J=1,M1                                                      XIRE4290
      DO 80 K=1,3                                                       XIRE4300
      IF(IE(J,K))61,61,62                                               XIRE4310
   61 T(K)=1.0                                                          XIRE4320
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                                                                                08/04/70
          XIRE      - EFN   SOURCE STATEMENT  - IFN(S) -

      GO TO 80                                                                  XIRE4330
   62 K1=IT(J,K)                                                                XIRE4340
      IF(TR(J,K)-1.0)79,64,65                                                   XIRE4350
   64 T(K)=Y(K1)                                                                XIRE4360
      GO TO 80                                                                  XIRE4370
   65 IF(TR(J,K)-2.0)79,66,67                                                   XIRE4380
   66 T(K)=Y(K1)*Y(K1)                                                          XIRE4390
      GO TO 80                                                                  XIRE4400
   67 IF(TR(J,K)-3.0) 79,68,69                                                  XIRE4410
   68 T(K)=Y(K1)*Y(K1)*Y(K1)                                                    XIRE4420
      GO TO 80                                                                  XIRE4430
   69 IF(TR(J,K)-4.0)79,70,79                                                   XIRE4440
   70 T(K)=Y(K1)*Y(K1)*Y(K1)*Y(K1)                                              XIRE4450
      GO TO 80                                                                  XIRE4460
   79 T(K)=Y(K1)**TR(J,K)                                                       XIRE4470     833
   80 CONTINUE                                                                  XIRE4480
   85 Z(J)=T(1)*T(2)*T(3)                                                       XIRE4490
      IF(NB)20,20,207                                                           XIRE4500
   20 CONTINUE                                                                  XIRE4510
      GO TO 95                                                                  XIRE4520
      END                                                                       XIRE4530

ERROR MESSAGE NUMBER     1

ERROR MESSAGE NUMBER     2

ERROR MESSAGE NUMBER     3
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                                                                08/04/70
                                                  COMPILATION       XIRE

                                          DIAGNOSTIC   MESSAGES

1     SOURCE ERROR   127  LEVEL 1 - WARNING ONLY
TRANSFER INTO DO NEST TO IFN  283     FROM OUTSIDE ITS RANGE


2     SOURCE ERROR   127  LEVEL 1 - WARNING ONLY
TRANSFER INTO DO NEST TO IFN  599     FROM OUTSIDE ITS RANGE


3     SOURCE ERROR   127  LEVEL 1 - WARNING ONLY
TRANSFER INTO DO NEST TO IFN  603     FROM OUTSIDE ITS RANGE
```

XIRE

STORAGE MAP

MAIN PROGRAM

DIMENSIONED PROGRAM VARIABLES

SYMBOL	LOCATION	TYPE
Y	00001	R
SS	00405	R
IT	17311	I
L2	20155	I
TNAMES	20420	R

SYMBOL	LOCATION	TYPE
Z	00075	R
X	17025	R
TR	17575	R
T	20251	R
CROSS	21170	R

SYMBOL	LOCATION	TYPE
C	00215	R
S	17121	R
L1	20061	I
D	20254	R
RANGE	21264	R

UNDIMENSIONED PROGRAM VARIABLES

SYMBOL	LOCATION	TYPE
MT	21360	I
N	21363	I
M1	21366	I
IYC	21371	I
IY4	21374	I
IIIII	21377	I
YC	21402	R
M	21405	I
CM1	21410	R
BL	21413	R
NI	21416	I
ZR	21421	R
L22	21424	I
IP	21427	I
SY2	21432	R
B2	21435	R
XNL	21440	R
R	21443	R
J9	21446	I
CL1	21451	R
BUCL	21454	R
CLCL	21457	R
L	21462	I
OBLCL	21465	R
OCUCL	21470	R
K1	21473	I

SYMBOL	LOCATION	TYPE
ID	21361	I
NCD	21364	I
ABT	21367	R
IY2	21372	I
IY5	21375	I
KKKKK	21400	I
OYC	21403	R
M2	21406	I
E1C	21411	R
NAB	21414	I
NP	21417	I
L14	21422	I
SAB	21425	R
A1	21430	R
KIT	21433	I
SS1	21436	R
SEDE	21441	R
DF	21444	R
IOF	21447	I
AUCL	21452	R
BLCL	21455	R
DEV	21460	R
DALCL	21463	R
OBUCL	21466	R
DC	21471	R

SYMBOL	LOCATION	TYPE
J	21362	I
NT	21365	I
TCL	21370	R
IY3	21373	I
IY6	21376	I
I	21401	I
M4	21404	I
M3	21407	I
AB	21412	R
NA	21415	I
NB	21420	I
K	21423	I
CNL	21426	R
A2	21431	R
B1	21434	R
SS2	21437	R
R2	21442	R
I9	21445	I
CL	21450	R
ALCL	21453	R
CUCL	21456	R
DE	21461	R
DALCL	21464	R
OCLCL	21467	R
LL1	21472	I

ENTRY POINTS

.....	SECTION	2

SUBROUTINES CALLED

.FWRD.	SECTION	3
.FWRB.	SECTION	6
DOFOP	SECTION	9
ALCG	SECTION	12
.FXEN.	SECTION	15

.FRCD.	SECTION	4
.FEFT.	SECTION	7
SQRT	SECTION	10
SIN	SECTION	13
.UNOG.	SECTION	16

.FRWT.	SECTION	5
.FRLB.	SECTION	8
.XP3.	SECTION	11
ARSIN	SECTION	14
.FFIL.	SECTION	17

XIRE STORAGE MAP

.FCNV.	SECTION	18	.UN05.	SECTION	19	.FRTN.	SECTION	20
.UN01.	SECTION	21	.UN02.	SECTION	22	.FWLR.	SECTION	23
.FBLT.	SECTION	24	.FBCT.	SECTION	25	.FRLR.	SECTION	26
E.1	SECTION	27	E.2	SECTION	28	E.3	SECTION	29
E.4	SECTION	30	CC.1	SECTION	31	CC.2	SECTION	32
CC.3	SECTION	33	CC.4	SECTION	34	SYSLOC	SECTION	35

EFN IFN CORRESPONDENCE

EFN	IFN	LOCATION	EFN	IFN	LOCATION	EFN	IFN	LOCATION
1000	FORMAT	21533	998	FORMAT	21541	999	FORMAT	21553
1001	FORMAT	21571	1002	FORMAT	21606	1003	FORMAT	21607
1004	FORMAT	21610	1006	FORMAT	21611	1007	FORMAT	21623
1008	FORMAT	21634	1009	FORMAT	21640	1010	FORMAT	21657
1012	FORMAT	21666	1013	FORMAT	21677	1014	FORMAT	21701
1016	FORMAT	21706	1017	FORMAT	21714	1018	FORMAT	21741
1040	FORMAT	21755	1041	FORMAT	21765	1019	FORMAT	22001
1042	FORMAT	22010	1020	FORMAT	22020	1021	FORMAT	22032
1022	FORMAT	22043	1023	FORMAT	22052	1024	FORMAT	22057
1025	FORMAT	22063	1026	FORMAT	22065	1027	FORMAT	22067
1028	FORMAT	22103	1029	FORMAT	22110	1129	FORMAT	22116
1030	FORMAT	22125	1031	FORMAT	22127	2000	FORMAT	22133
2001	FORMAT	22134	2002	FORMAT	22140	2003	FORMAT	22143
2004	FORMAT	22145	2005	FORMAT	22146	2006	FORMAT	22153
1	3A	22553	3	49A	22735	4	65A	23001
3000	80A	23073	3001	FORMAT	22160	2	88A	23126
22	103A	23165	11	131A	23267	6	137A	23310
7	150A	23362	8	153A	23364	9	157A	23374
12	177A	23471	13	179A	23473	14	180A	23475
10	187A	23512	15	193A	23522	16	194A	23522
17	195A	23522	20002	FORMAT	22161	18	203A	23545
27	235A	23676	20003	FORMAT	22164	19	210A	23567
25	215A	23601	120	223A	23637	121	225A	23642
26	227A	23647	3C	237A	23706	31	243A	23730
32	255A	23773	21	261A	24005	101	306A	24223
5	265A	24031	33	267A	24034	34	273A	24055
35	278A	24075	36	281A	24101	40	732A	27316
95	283A	24105	100	303A	24214	102	310A	24230
103	313A	24241	104	315A	24251	105	335A	24362
108	344A	24422	112	362A	24513	109	351A	24435
110	355A	24464	113	367A	24531	115	374A	24593
915	377A	24565	178	436A	25103	936	428A	25065
924	385A	24654	926	398A	24720	928	418A	25024
183	450A	25172	179	438A	25106	182	447A	25162
186	459A	25230	187	473A	25323	185	491A	25361
188	486A	25351	201	570A	25751	190	497A	25366
192	531A	25554	193	520A	25511	195	552A	25627
196	562A	25662	3002	FORMAT	22203	3003	FORMAT	22462
230	717A	27256	2201	576A	26056	2202	582A	26077
2203	587A	26117	203	598A	26154	202	589A	26122
1202	592A	26127	1201	591A	26126	204	599A	26155
206	602A	26166	205	601A	26160	60	784A	27465
207	603A	26167	209	611A	26217	208	605A	26172
210	614A	26225	212	623A	26263	213	640A	26415
211	637A	26376	215	639A	26410	314	642A	26420

315	648A	26441	313	653A	26461	316	655A	26464
221	714A	27204	216	656A	26500	217	679A	26607
218	690A	26676	219	702A	27000	22C	705A	27046
225	715A	27222	233	727A	27306	231	723A	27274
234	729A	27311	301	731A	27315	55	780A	27460
42	738A	27344	43	742A	27347	44	749A	27365
45	756A	27402	46	763A	27417	47	768A	27426
51	774A	27446	85	838A	27637	8C	836A	27633
61	792A	27521	62	795A	27524	79	831A	27617
64	800A	27533	65	804A	27541	66	807A	27546
67	812A	27556	68	815A	27563	69	821A	27576
70	824A	276C1	20	843A	27651			

```
                    * MEMORY MAP *

SYSTEM                                  00000 THRU  02717
FILE BLOCK ORIGIN                       02720
FILES       1.      UNIT01
            2.      UNIT02
            3.      UNIT05
            4.      UNIT06
FILE LIST ORIGIN                        030CC
PRE-EXECUTION INITIALIZATION            03010
CALL ON OBJECT PROGRAM                  03041
OBJECT PROGRAM                          03046 THRU  50015

      DECK  ORIGIN      CONTROL SECTIONS  (/NAME/=NON 0 LENGTH, (LOC)=DELETED, *=NOT REFERENCED)

  1. XIRE    03046      ......  32747 *
  2. DPAF    32765      FAD     32765     FSB     33004     FMP     33024     FDV     33C44     TEST    33070 *
  3. DOPE    33134      DOPOP   33232
  4. .LXCON  33302      .LXSTR  33302 *   .LXSTP  33305     .LXOUT  33353     .LXERR  33362     .LXCAL  33365 *
                        .LXRTN  33365     IBEXIT  33365 *   .DBCLS  33547 *   .LXARG  33716     .LO     33741 *
                        .CLSE   33747     .LFBL   33750 *   .LUNB   33751     .DFOUT  33752
  5. .IODEF  33756      .DEFIN  33756     .ATTAC  33762 *   .CLOSE  33764     .OPEN   33766     .READ   33770
                        .WRITE  33772     .DSR    34002 *   .READR  34012     .RELES  34C14 *   .LAREA  34025
                        .LFBLK  34043     .LTSX   34046 *   .RLHLD  34C56 *   .AREA1  34C61     .LUNBL  34067
                        .ENTRY  34073     .GOA    34130     .GO     34134     .DERR   34150     .NOPXI  34151
                        .COMXI  34153     .EX34   34175
  6. .LXSL   34202      .LXSEL  34202     .LXSL1  34203     .LXTST  34206 *   .LXOVL  34246 *   .LXMOD  34310 *
                        .LXIND  34334 *   .LXDIS  34337 *   .LXFLG  34340 *   .LTCH   34341
  7. .FPTR   34347      .FFPT.  34347 *   OVFLOW  34377 *   .FPOUT  34412     .COUNT  34416 *   .FPARG  34425
  8. .ERAS.  34442      E.1     34442     E.2     34443     E.3     34444     E.4     34445
  9. .ACC.   34446      CC.1    34446     CC.2    34447     CC.3    34450     CC.4    34451
 10. XIT     34452      EXIT    34452     .EXIT.  34452 *
 11. FXEM    34453      ERRINT  34453 *   ERRTRM  34457 *   .FXEM.  34467     .FXOUT  34730     .FXARG  34736
                        .OPTN.  35121
 12. FOUT    35140      .FOUT.  35140
 13. FCNV    35501      .FCON.  35501     .FCNV.  35522     .ENDFS  35530     .CNVSW  35533     .FDX1   35537
                        .FDX2   35540     .DBC    35542     .DBC10  35700     .DBC20  35726     .DDSW   35744
                        .DDFIX  35750     .FIXSW  35751     .DDBC   36016     .DDRS1  36267     .DDRS2  36271 *
                        .D1     36274     .D2     36276     .FERR2  36363 *   .AMPT   36417     .DNPT   36434
                        .LNTP   36517     .AOUT   36566     .DOUT   36577     .LOUT   36626     .DFLT   36636
                        .FLT    37103     .FXFL1  37234     .FXD    37240     .FXFL2  37243     .FXFL3  37247
                        .INTG   37253     .TOPAC  37271     .WIDTH  37275     .FPACK  37302     .TEST   37303
                        .KOUNT  37336     .LIST   37341     .DONE   37347     .OUTBF  4CC40     .CHAR   40274
                        .FBDBF  4031C     .DDDFL  4C342     .DDFLG  40343     .WORD   40344     .MOD    4C345
                        .PEX    40346     .FEXP   4C347     .DIG    4C350     .DEXPN  40351
 14. FIOB    404C3      .FIOB.  40403     .FCNT   40506     .FBLT.  4C604     .FBDT.  40624     .FRLR.  40650
                        .FRLR. (40650)    .FWLR.  4C714     .FWLR. (4C714)    .FBIBF  40754     .FRITE  41046
 15. FIOS    41054      .FIOS.  41054     .FSEL.  41214     .FILR.  41220 *   .FKTB.  41227     .FRTD.  41234
                        .FILL.  41237     .FCLS   41241 *   .FOPN   41245 *   REOF    41251 *   .TOUT.  41414
                        .REED   41422 *   .BIN    41423 *   .FCT    41424     .FCKSZ  41426
 16. FIOH    41510      .FIOH.  41510     .FFIL.  42351     .FRTN.  42377
 17. FWRC    42567      .FWRD.  42567
 18. FWRB    42613      .FWRB.  42613
 19. FRDC    42637      .FRDD.  42637
 20. FRDB    42665      .FRDB.  42665
 21. UN01    42711      .UN01.  42711
```

22. UN02	42712	.UN02.	42712										
23. UN05	42713	.UN05.	42713										
24. UN06	42714	.UN06.	42714		.BUFSZ	42715							
25. FLOG	42720	ALOG10	42720 *		ALOG	42721							
26. FXPF	43124	EXP	43124										
27. FSCN	43245	COS	43245 *		SIN	43246							
28. FSQR	43441	SQRT	43441										
29. FXP?	43514	.XP3.	43514										
30. FEFT	43641	.FEFT.	43641										
31. FRNT	43741	.FRNT.	43741										
32. FASC	44046	ARCOS	44046 *		ARSIN	44047							
33. .IOCS	44177	.L(0)	44177		.MONSW	44217		.TEOR	44266		.DEFI.	44346		.JOINX	44412 *
		.CLOS.	44431		.ATTC.	44444		.SH1	44656 *		.SH9	44720 *		.OPEN.	44741
		.OP4	44767 *		.OP7	45020 *		.OP9.2	45034 *		.RLSE.	45106		.RER2.	45106
		.READ.	45107		.RER1.	45132		.WRIT.	45134		.WAIT1A	45324 *		.EOFEX	45405 *
		.FEEIT	45455		.GTIOX	45476		.RW7	45614 *		.RE7	46237 *		.ENDTR	46700
		.SEL59	46702 *		.BSR.	47321		.EOTOF	47446		.ETOF3	47454 *		.SWITC	47503
		.TCHEX	50C10		.BASIC	5C013 *									
34. .IOCSM	50016														

I/O BUFFERS 50016 THRU 76634

UNUSED CORE 76635 THRU 77014

COMPUTER LISTING E2

FORTRAN IV SOURCE STATEMENTS AND LISTING AND DOCUMENTATION FOR SUBROUTINE AM DPAF

FORTRAN SUBROUTINE

IDENTIFICATION

FORTRAN Double Precision Arithmetic AM DPAF

Sharon E. Good
David Taylor Model Basin
July 10, 1959

PURPOSE

To perform double precision add, subtract, multiply, divide, and compare two double precision numbers with a tolerance factor; to take advantage of floating point trap; to use Bell System error procedure.

RESTRICTIONS

To be used with BE SYS2 with corrected floating trap routine.

METHOD

The package was adapted from FORTRAN II routines written by A. Gartenhaus of GP and from SAP routines written by R. Johnson of CL. A double precision number is as the contents of the AC and MQ after a FAD instruction. The characteristic of the least significant part (MQ) is 27_{10} less than that of the most significant part (AC). A machine zero (36 binary zeros) may be used for the least significant part of the double precision word. The sign of the double precision word is the sign of the most significant part even though the sign of the least significant part may be different. The arithmetic routines are FORTRAN II subroutines.

Full accuracy is obtained only when the characteristic of the most significant half is greater than 27_{10}. If less, the accuracy will be reduced to 8 decimal digits rather than 16 and the least significant part is machine zero in the memory. A floating point trap routine is required which will set the proper register (AC and/or MQ) to zero when underflow occurs.

When two double precision floating point numbers are compared, absolute equality is seldom realized. The positive tolerance factor E allows a test for equality which is more realistic. To compare A and B, A-B is calculated and the result tested for sign and (most significant part |A-B|) -E is tested for sign. TEST is a FORTRAN II function.

USAGE

```
CALL FAD(AM,AL,BM,BL,CM,CL)
CALL FSB(AM,AL,BM,BL,CM,CL)
CALL FMP(AM,AL,BM,BL,CM,CL)
CALL FDY(AM,AL,BM,BL,CM,CL)
F=TEST(AM,AL,BM,BL,E)
IF (TEST(AM,AL,BM,BL,E))n1,n2,n3
```

where AM is the most significant part of A

AL is the least significant part of A

C = A operation B

F is 1. if A > B and |A-B|>E

0. if A = B or |A-B|<E

-1. if A < B and |A-B|>E

CODING INFORMATION

DFAF requires 109 storage cells plus 4 common. The binary deck consists of 7 cards. Accuracy is 16 significant figures except for arguments less than 10^{-30}.

Timing assuming minimum cycles:

FAD 1.104 m.s.

FSB 1.176 m.s.

FMP 1.656 m.s.

FDY 1.776 m.s.

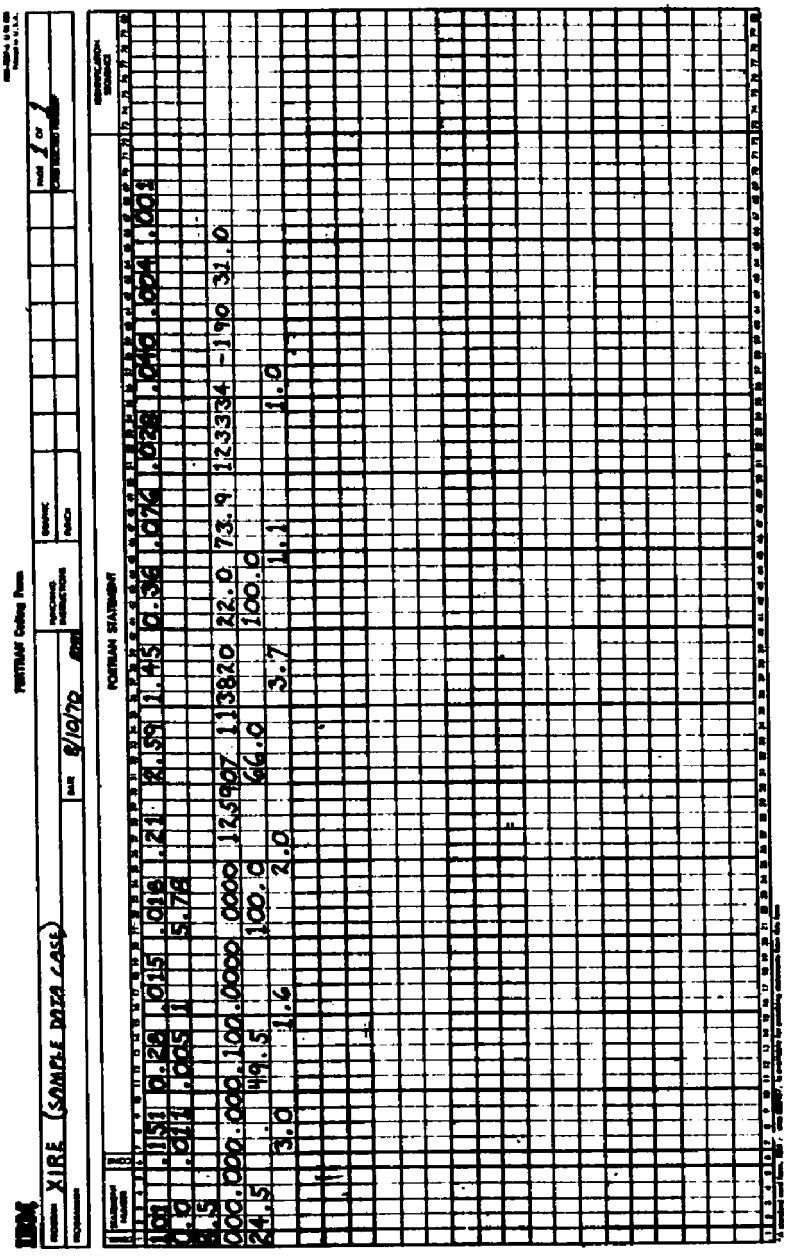
TEST 1.442 m.s.

```
$IBMAP DPAF    DECK
*      DOUBLE PRECISION  FORTRAN  PACKAGE   7090  BE  SYS3          DPAF
       ENTRY   FAD                                                 DPAF0002
       ENTRY   FSB                                                 DPAF0003
       ENTRY   FMP                                                 DPAF0004
       ENTRY   FDY                                                 DPAF0005
       ENTRY   TEST                                                DPAF0006
  FAD  SXA     END+2,2                                             DPAF0018
       TXI *+1,4,-2                                                DPAF0019
       TSX ALL,2                                                   DPAF0020
       CLA COMMON+2          B1                                    DPAF0021
  FADE FAD COMMON            A1                                    DPAF0022
       STO COMMON                                                  DPAF0023
       CLA COMMON+1          A2                                    DPAF0024
       STO COMMON+1                                                DPAF0025
       FAD COMMON+3          B2                                    DPAF0026
       FAD COMMON+1                                                DPAF0027
       FAD COMMON                                                  DPAF0028
  END  STO*    5,4                                                 DPAF0029
       STO*    6,4                                                 DPAF0030
       AXT     **,2                                                DPAF0031
       TRA 7,4                                                     DPAF0033
  FSB  SXA     END+2,2                                             DPAF0034
       TXI *+1,4,-2                                                DPAF0035
       TSX ALL,2                                                   DPAF0036
       CLS COMMON+3          CHANGE SIGN B AND ADD                 DPAF0037
       STO COMMON+3                                                DPAF0038
       CLS COMMON+2                                                DPAF0039
       TXL FADE,,**                                                DPAF0042
  ALL  CLA*    1,4           SECURE INPUTS AND OUTPUT              DPAF0043
       STO COMMON                                                  DPAF0046
       CLA*    2,4                                                 DPAF0047
       STO COMMON+1                                                DPAF0050
       CLA*    3,4                                                 DPAF0051
       STO COMMON+2                                                DPAF0054
       CLA*    4,4                                                 DPAF0059
       STO COMMON+3                                                DPAF0060
       TRA 1,2                                                     DPAF0062
  FMP  SXA     END+2,2                                             DPAF0061
       TXI *+1,4,-2                                                DPAF0063
       LDQ COMMON                                                  DPAF0064
       TSX ALL,2                                                   DPAF0065
       FMP COMMON+3          A1 TIMES B2                           DPAF0066
       STO COMMON+3                                                DPAF0067
       LDQ COMMON                                                  DPAF0068
       FMP COMMON+2          A1 TIMES B1                           OPAF0069
       STQ COMMON                                                  DPAF0070
       LDQ COMMON+1                                                DPAF0071
       STO COMMON+1                                                DPAF0072
       FMP COMMON+2          A2 TIMES B1                           DPAF0073
       FAD COMMON+3                                                DPAF0074
       FAD COMMON                                                  DPAF0075
       FAD COMMON+1                                                DPAF0076
       TRA END                                                     DPAF0077
  FDY  SXA     END+2,2                                             DPAF0078
       TXI *+1,4,-2                                                DPAF0079
       TSX ALL,2
       CLA COMMON            A1 OVER B1
       FDP COMMON+2
       STQ COMMON
```

```
        FAD  COMMON+1          REMAINDER A1 OVER B1 + A2                DPAF0080
        FDP  COMMON+2          DIVIDE BY B1                             DPAF0081
        STQ  COMMON+1                                                   DPAF0082
        CLA  COMMON+3                                                   DPAF0083
        ANA  FDYC              ISOLATE POWER                            DPAF0084
        FSB  COMMON+3                                                   DPAF0085
        FDP  COMMON+2          -B2 OVER B1                              DPAF0086
        FMP  COMMON            TIMES A1 OVER B1                         DPAF0087
        FAD  COMMON+1          ADD LESSER PARTS                         DPAF0088
        FAD  COMMON            ADD A1 OVER B1                           DPAF0089
        DCT                                                             DPAF0090
        TRA  ERR                                                        DPAF0092
        TRA  END                                                        DPAF0093
 FDYC   OCT  377000000000                                               DPAF0094
 TEST   LDQ  TESTC                                                      DPAFZ097
        TXI  **1,4,-2                                                   .
        STQ      END                                                    DPAF0098
        TRA      FSB                                                    DPAF0100
 TESTR  LDQ  TESTC+1                                                    DPAF100A
        STQ      END                                                    DPAF0
        TMI      TESTX+2                                                DPAF0101
        FSB*     5,4                                                    DPAF0102
        TMI  TESTX+4                                                    DPAF0105
        CLA      ERR-1         PLUS ONE                                 DPAF0106
 TESTX  LXA   END+2,2                                                   DPAFA106
        TRA  6,4                                                        DPAF0107
        FAD*     5,4                                                    DPAF0108
        TMI      TESTX+6                                                DPAF0109
        ZAC                    ZEERO                                    DPAF0110
        TRA  TESTX                                                      DPAF0111
        CLS      ERR-1         MINUS ONE                                DPAF0112
        TRA  TESTX                                                      DPAF0115
        DEC  1.                                                         DPAF0116
 ERR    SXA      ERR+4,4                                                DPAF
        TSX  .MWR,4                    MAP        PRINT BUT PROCEED    DPAF
        PZE  1          7/14/64
        PZE  FORM,,5
        AXT      **,4                                                   DPAF
        TRA  END
 FORM   BCI      5, DBL PREC DIV CKECK                                  DPAF0122
 TESTC  TRA  TESTR
        STO*    5,4
 COMMON BSS  4
        END                                                             DPAF0128
                                                                        104
```

COMPUTER LISTING E3

SAMPLE INPUT AND RESULTS FROM A TEST CASE RUN USING XIRE



Sample Coding of One Data Point Requiring Six Cards for XIRE

DEPARTMENT OF THE NAVY

NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER

PROTECTION SYSTEMS CONCEPTS BRANCH (747) - STRUCTURAL MECHANICS LAB

TAPE REGRESSION 2 - 60 VARIABLES PROGRAM NAME XIRE DECK 2

ORIGINATOR SALIVE
NAVY CODE CODE 747

PJCT DATE 01/23/70
PROJECT NUMBER 1-747-183-01

TITLE METALLOGRAPHIC REGRESSION
V1 - PRODUCER
V2 - AMT CARBON
V3 - AMT MN
V4 - AMT P
V5 - AMT S
V6 - AMT SI
V7 - AMT NI
V8 - AMT CR
V9 - AMT MO
V10- AMT CU
V11- ACID SOL AL
V12- TOTAL AL
V13- AMT TI
V14- AMT V
V15- AMT CO
V16- AMT OXYGEN
V17- AMT NITROGEN
V18- AMT HYDROGEN
V19- O(1)SCP
V20- GRAIN SIZE
V21- PCT BAINITE
V22- PCT FERRITE
V23- PCT PEARLITE
V24- PCT MARTENSITE
V25- HOLDING TIME (SEC)
V26- HOLDING TEMP (F)
V27- UTS(KSI)
V28- TYS (KSI)
V29- PCT EL. IN 2 IN.
V30- PCT RA
V31- CVS (KSI)
V32- NDT TEMP
V33- NDT ENERGY(FT-LBS)CHARPY
V34- NDT PCT FIBER
V35- -120F ENERGY
V36- -120F PCT FIBER
V37- +210F ENERGY
V38- +210F PCT FIBER
V39- LONG.(LONGEST INCL.)
V40- AU SIZE OTHER INCL.
V41- BALKGROUND RATING
V42- TRAN.(LONGEST INCL.)
V43- AU SIZE OTHER INCL.
V44- BACKGROUND RATING
V45-V2*V13 - C * TI
V46-V2*V15 - C * AMT CO
V47-V3*V5 - MN * S

Sample Output from XIRE

```
V48-V5*V11 - S * A S AL
V49-V6*V8 - SI * CR
V50-V7*V8 - NI * CR
V51-V7*V9 - NI * MO
V52-V8*V16 - CR * O
V53-V8*V17 - CR * N
V54-V9*V16 - MO * O
V55-V9*V17 - MC * NI
V56-V11*V13 - A S AL * TI
V57-V11*V17 - A S AL * NI
V58-V11*V16 - A S AL * O
V59-V12*V17 - TOT. AL * N
V60-V13*V16 - TI * O
V61-V13*V17 - TI * N
V62-V14*V16 - V * O
V63-V14*V17 -V * N
V64-V20*V28 - GR.SZ. * TYS
V65-V20*V37 - GR.SZ.*210F ENG
V66-V6*V11/V17-SI *A S AL/N
V67-V2*V3 - C * MN
V68-V2*V4 - C * P
V69-V2*V6- C * S
V70-V2*V7 - C * NI
V71-V2*V8 - C* CR
V72-V2*V9 - C * MC
V73-V2*V11 - C * A S AL
V74-V39*V42 - LONG. * TRAN.
V75-V20*V21 - GR.SZ.*BANITE
V76-V20*V22 - GR.SZ.*FERRITE
V77-V20*V23 - GR.SZ.*PEARLITE
V78-V2*V2 -CARBON *CARBON
V79-V8*V8 -   CR  *  CR
V80-V8*V9 -   CR  *  MC
V81-V3*V3 -   MN*  MN
V82-V3*V8 -   MN  *  CR
V83-V3*V9 -   MN  *  MC

VARIABLES ON CARDS  TRANSFORMED VARIABLES  TERMS  ABERRANCE T  CONFIDENCE T
        44                   44              11       2.58          2.00

                    CORRECTED SUMS OF SQUARES AND PRODUCTS

 0.28288732E 02  0.28158450E 02  0.35895951E 03  0.367C2825E 04 -0.23457C42E 02  0.93050704E 02  0.40002113E 02
 0.47166?17E 05  0.10743044E 11  0.17276357E 16 -0.10574859E 04

 0.28158450E 02  0.47631866E 03  0.61298741E 04  0.63112709E 05 -0.84369471E 03  0.45066197E 03  0.44910915E 02
 0.28183764E 06  0.53469794E 11  0.73493C29E 16 -0.1C838510E 05

 0.35895951E 03  0.61298741E 04  0.80187209E 05  0.83821339E 06 -0.11616434E 05  0.58665697E 04  0.50972165E 03
 0.42405292E 07  0.82957962E 12  0.11936834E 18 -0.13948348E 06

 0.36702825E 04  0.63112709E 05  0.83821339E 06  0.88859242E 07 -0.12657011E 06  0.60738347E 05  0.43862227E 04
 0.50720C30E 08  0.10136347E 14  0.15036C23E 19 -0.14312100E 07

-0.23457C42E 02 -0.84369471E 03 -0.11616434E 05 -0.12657011E 06  0.47437298E 05 -0.68493583E 04 -0.62274992E 03
-0.22184388E 07 -0.55819381E 12 -0.99414791E 17  0.44494383E 05

 0.93050704E 02  0.45066197E 03  0.58665697E 04  0.60738347E 05 -0.68493583E 04  0.25536992E 05  0.37518355E 04
-0.16430759E 08 -0.30623647E 13 -0.43725502E 18 -0.12289363E 05
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 0.40002113E 02  0.44910915E 02  0.50972165E 03  0.43862227E 04 -0.62274992E 03  0.37518355E 04  0.16488214E 04
-0.27878577E 07 -0.48873645E 12 -0.66010806E 17  0.37706598E 04

 0.47166317E 05  0.28183764E 06  0.42405292E 07  0.50720030E 08 -0.22184388E 07 -0.1643C755E 08 -0.27878577E 07
 0.23788641E 11  0.48099166E 16  0.74743132E 21  C.14108231E 08

 0.10743C44E 11  0.53469794E 11  0.82957982E 12  0.10136347E 14 -0.55819381E 12 -0.30623647E 13 -0.48873645E 12
 0.48099166E 16  0.986C5919E 21  0.15532929E 27  C.35269094E 13

 0.17276357E 16  0.73493029E 16  0.11936834E 18  C.15036023E 19 -0.99414791E 17 -0.43725502E 18 -0.66010806E 17
 0.74743132E 21  0.15532929E 27  0.24796935E 32  0.66108474E 18

-0.10574859E 04 -0.10838510E 05 -0.13948348E 06 -0.14312100E 07  0.44494383E 05 -0.12289363E C5  0.37706598E 04
 0.14108231E C8  0.35269094E 13  0.66108474E 18  0.41883223E 06
```

INVERSE MATRIX

```
 0.41648724E-01  0.92049698E-C3 -0.52408726E-03  0.31792595E-04 -0.12329775E-03 -0.28838410E-03 -0.25510914E-03
 0.20262343E-04 -0.20514245E-09  0.66543556E-15

 0.92049698E-C3  0.471C7500E 01 -0.74371820E 00  C.36735545E-01 -0.11365030E-02 -0.258CC063E-02  0.11381956E-01
 0.16287C90E-C3 -0.16264513E-08  0.52155383E-14

-0.52408726E-C3 -0.74371820E 00  0.11835629E 00 -0.58882433E-02  0.17648C13E-C3  0.40342145E-03 -0.19267718E-02
-0.26372C96E-04  0.26136891E-09 -0.83187151E-15

 0.31792595E-04  0.36735545E-01 -0.58882433E-02  0.29494695E-03 -0.84352820E-05 -0.208818C4E-C4  0.1C152039E-03
 0.13464436E-05 -0.13289100E-10  0.42097772E-16

-0.12329775E-03 -0.11365030E-02  0.17648C13E-03 -C.84352820E-05  C.26465274E-04  0.15007731E-C4 -0.48295339E-04
-0.92457439E-06  0.90197552E-11 -0.28382C71E-16

-0.28838410E-03 -0.258CC063E-02  0.40342145E-03 -C.2C881804E-04  0.15007731E-04  0.13754920E-03 -0.14011285E-04
 0.82323257E-06 -0.47264878E-11  0.73503C32E-17

-0.2551C914E-03  0.11381956E-01 -0.19267718E-02  C.10152039E-03 -0.48295339E-04 -C.14011285E-04  0.2344793CE-02
 0.23321986E-04 -0.21496288E-09  0.64913C37E-15

 0.20262343E-04  0.16287090E-03 -0.26372C96E-04  0.13464436E-05 -0.92457439E-06  0.82323257E-C6  0.23321986E-04
 0.42848614E-06 -0.40378622E-11  0.12446431E-16

-0.20514245E-09 -0.16264513E-08  0.26136891E-09 -C.13289100E-10  0.90197552E-11 -0.47264878E-11 -0.21496288E-09
-0.40378622E-11  0.38233283E-16 -0.11836115E-21

 0.66543556E-15  0.52155383E-14 -0.83187151E-15  0.42097772E-16 -0.28382C71E-16  0.735C3032E-17  0.64913C37E-15
 0.12446431E-16 -0.11836115E-21  0.36790285E-27
```

REGRESSION STATISTICS

	REGRESSION COEFFICIENTS		TERM NAMES	T	PRODUCT	MEANS	STD. DEVS.	RANGE
					• • • • • • • TERM • • • • • • •			
B	1	-0.2706778E 02	V1 - PRODUCER	-6.267	-48.4964385	1.2746479	0.4479166	2.85
B	2	0.1350784E 03	V20- GRAIN SIZE	2.940	993.0817871	7.9190140	1.8379735	4.31
B	3	-0.2463136E 02	V84- V20**2	-3.383	-2349.5857544	66.0651407	23.8475072	2.77
B	4	0.1210213E 01	V87- V20**3	3.329	1215.2446899	566.3309053	251.0393181	2.26
B	5	0.7207947E 00	V21- PCT BANITE	6.620	52.8837156	8.9091548	18.3421574	0.49*
B	6	0.6121776E 00	V22- PCT FERRITE	2.466	32.9543490	5.4140844	13.4578384	0.40*
B	7	0.7234394E 01	V23- PCT PEARLITE	7.059	98.9553795	0.4922535	3.4196154	0.14*
B	8	0.3355919E-01	V28- TYS (KSI)	2.422	1743.5995026	105775.8378906	12988.9864502	8.14
B	9	-0.3115599E-06	V85- V28**2	-2.381	-3295.6686401	719168.0000000	640314.0000000	4.29
B	10	0.1006726E-11	V86- V28**3	2.480	1688.7317657	615832.0000000	983778.0000000	2.95
Y			V32- NOT TEPP			-176.2183094	54.5017552	-3.23

BO	-1537.8538666	R2.	0.8598858	DF	131
SX.Y	21.1653504	R	0.9273003	N	142.

BO - CONSTANT CALCULATED FOR REGRESSION EQUATION
R2 - STANDARD ERROR OF ESTIMATE (R SQUARED)
DF - DEGREES OF FREEDOM
SX.Y - STANDARD DEVIATION OF A PREDICTED POINT (RESIDUAL UNACCOUNTED FOR AFTER REGRESSION)
R - REGRESSION COEFFICIENT
N - NUMBER OF DATA POINTS
T - COEFFICIENT DIVIDED BY THE STD DEV OF THE COEFFICIENT
(USE WITH STUDENT T DISTRIBUTION TABLE AND DEGREES OF FREEDOM TO DETERMINE
CONFIDENCE INTERVALS OF THE REGRESSION COEFFICIENTS)
PRODUCT - 4*(REGRESSION COEF)*(TERM STD DEV)
TERM RANGE - (TERM MEAN)/(TERM STD DEV)
* TERM RANGE IS LESS THAN 2.0, THE VALUE OF PRODUCT SHOULD BE
RECOMPUTED BEFORE BEING EVALUATED SINCE VALUES OF THIS VARIABLE ARE DISTRIBUTED IN A SKEW MANNER

DATA AND BACK CALCULATIONS

ORIGINAL DATA - IN SAME ORDER AS VARIABLE DEFINITION LIST
TRANSFORMED DATA - IN SAME ORDER AS ORIGINAL DATA
TERMS - IN SAME ORDER AS REGRESSION COEFFICIENTS, WITH DEPENDENT VARIABLE LAST

ORIGINAL DATA
0.10000E 01 0.15100E 00 0.20000E 00 C.15000E-01 0.18000E-01 0.21000E 0C 0.25900E 01 0.14500E 01 0.36000E 00 0.76000E-01
0.28000E-01 0.40000E-01 0.40C00E-02 C.1CC00E-02 0. 0.11CCCE-01 0.5C00CE-02 0.1C0000E 01 0.57800E 01 0.85000E 01
0. 0. 0. C.1CC00E 03 0. 0. 0.12591E 06 0.11382E 06 C.22000E 02 0.739C0E 02
0.12933E 06 -0.19000E 03 0.31CC0E 02 C.24500E 02 0.49500E 02 0.10C00E 03 0.660000E 02 0.1C0000E 03 0.30000E 01 0.16000E 01
0.20000E 01 0.37000E 01 0.11CC0E 01 C.1CC00E 01
TRANSFORMED DATA
TERMS
0.10000E 01 0.85000E 01 0.72250E 02 C.61412E 03 0. C. 0. 0.11382E 06 0.12955E 11 0.14745E 16
-0.19000E 03

	YC	DE	LL	UL	OYC	OLL	OUL
LINE	-0.18524E 03	-0.47565E 01	-0.19221E 03	-0.17827E C3	-0.18524E 03	-0.19221E 03	-0.17827E 03

ORIGINAL DATA
0.10000E 01 0.16C00E 00 0.38CC0E 00 C.16C00E-01 0.21CC0E-01 0.30CCCE 0C 0.275CCE 01 0.14700E 01 0.35000E 00 0.48000E-01
0.32000E-01 0.360C0E-01 0.1C000E-02 C.2C000E-02 0. C.5CCCCE-02 0.6CC0CE-02 0. 0.78300E 01 0.80000E C1
0. 0. 0. C.1C000E C3 0. C. 0.12239E 06 C.10734E 06 0.24000E 02 0.735C0E 02
0.11886E 06 -0.17000E 03 0.48000E 02 C.37000E 02 0.62000E 02 0.72C0CE 02 0.9C000E 02 0.1C000E 03 C.15000E 01 0.14000E 01
0.10000E 01 0.20C00E 01 0.1CCC0E 01 C.2CC00E 01
TRANSFORMED DATA
TERMS
0.10000E 01 0.8CCC0E 01 C.64CC0E 02 C.512C0E C3 0. 0. 0. 0.10734E 06 0.11523E 11 0.12369E 16
-0.17000E 03

	YC	DE	LL	UL	OYC	OLL	OUL
LINE	-0.18350E C3	0.13503E 02	-0.19011E 03	-0.17689E C3	-0.18350E 03	-0.19C11E 03	-0.17689E 03

ORIGINAL DATA
0.10000E 01 0.14C00E 00 0.39CC0E 00 C.23000E-01 0.20000E-01 C.27CCGE 0C 0.27600E 01 0.14700E 01 0.35000E 00 0.49000E-01
0.23000E-01 0.26C00E-01 0.1C000E-02 C.2C000E-02 0. 0.15C00E-01 0.30000E-02 0.1C000E 01 G.79500E 01 0.70000E 01
0. 0. 0. C.1CC00E C3 0. 0. 0.12485E 06 C.11413E 06 0.22000E 02 0.727C0E 02
0.12169E 06 -0.12000E 03 C.39C00E 02 C.43500E 02 0.39000E 02 0.4350CE 02 0.61000E 02 0.10000E C3 G.35000E 01 0.15000E 01
0.30C00E 01 0.57000E 01 0.17C00E C1 C.20000E C1
TRANSFORMED DATA
TERMS
0.10000E 01 0.7C000E 01 C.49CC0E 02 C.34300E C3 0. C. 0. 0.11413E 06 0.13025E 11 0.14865E 16
-0.12000E 03

	YC	DE	LL	UL	OYC	OLL	OUL
LINE	-0.14276E C3	0.22755E 02	-0.15221E 03	-0.13330E C3	-C.14276E 03	-0.15221E 03	-0.13330E 03

ORIGINAL DATA
0.10000E 01 0.18300E 00 0.36000E 00 C.17000E-01 0.18000E-01 0.26C0CE 0C 0.293CCE 01 0.18000E 01 G.31C00E 00 0.75000E-01
0.24000E-01 0.42000E-01 C.5CCC0E-02 C.1C000E-02 0. 0.49C00E-01 0.8C000E-02 G.70000E 01 0.93300E 01 0.95000E 01
0. 0. C. C.1CC00E 03 0. C. 0.12853E 06 0.11302E 06 0.23000E 02 0.72600E 02
0.12358E 06 -0.19CC0E 03 0.28C00E 02 C.57800E 02 0.46000E 02 0.92e00E 02 0.56000E 02 0.10000E 03 0.19000E 01 0.12000E 01
0.20000E 01 0.20C00E 01 0. C.20000E 01
TRANSFORMED DATA
TERMS
0.10000E 01 0.95000E 01 C.90250E 02 C.857376 03 0. 0. 0. 0.11302E 06 0.12774E 11 0.14438E 16
-0.19000E 03

	YC	DE	LL	UL	OYC	OLL	OUL

```
       TERMS
 0.10000E 01  0.35000E 01  0.12250E 02  0.42875E 02 -0.          -0.          -0.          0.75000E 05  0.56250E 10  0.42187E 15
-0.14500E 03
                     YC           DE           LL           UL          DYC          DLL          DUL
        LINE      -0.15287E 03  0.78665E 01 -0.17759E 03 -0.12814E 03 -0.15287E 03 -0.17759E 03 -0.12814E 03

       ORIGINAL DATA
 0.10000E 01  0.16000E 00  0.30000E 00  0.90000E-02  0.17000E-01  0.25000E 00  0.27800E 01  0.15200E 01  0.36000E 00  0.58000E-01
 0.38000E-01  0.42000E-01  0.10000E-02  0.10000E-02 -0.           0.80000E-02  0.90000E-02 -0.           0.52000E 01  0.35000E 01
 0.30000E 01 -0.          -0.           0.97000E 02  0.15200E 03  0.87500E 03  0.15100E 06  0.13900E 06  0.18000E 02  0.66000E 02
 0.15000E 06 -0.10000E 02  0.28000E 02  0.51000E 02  0.15000E 02  0.80000E 01  0.39000E 02  0.10000E 03  0.50000E 00  0.
 0.10000E 01  0.11000E 01  0.           0.30000E 01
       TRANSFORMED DATA
       TERMS
 0.10000E 01  0.35000E 01  0.12250E 02  0.42875E 02  0.30000E 01 -0.          -0.          0.13900E 06  0.19321E 11  0.26856E 16
-0.10000E 02
                     YC           DE           LL           UL          DYC          DLL          DUL
        LINE       0.89289E 01 -0.18929E 02 -0.12561E 02  0.30419E 02  0.89289E 01 -0.12561E 02  0.30419E 02

       ORIGINAL DATA
 0.10000E 01  0.16000E 00  0.30000E 00  0.90000E-02  0.17000E-01  0.25000E 00  0.27800E 01  0.15200E 01  0.36000E 00  0.58000E-01
 0.38000E-01  0.42000E-01  0.10000E-02  0.10000E-02 -0.           0.80000E-02  0.90000E-02 -0.           0.52000E 01  0.35000E 01
 0.30000E 01 -0.          -0.           0.97000E 02  0.15200E 03  0.87500E 03  0.12200E 06  0.10900E 06  0.22000E 02  0.73000E 02
 0.11900E 06 -0.11000E 03  0.42000E 02  0.82000E 02  0.40000E 02  0.78000E 02  0.54000E 02  0.10000E 03  0.50000E 00  0.
 0.10000E 01  0.11000E 01  0.           0.30000E 01
       TRANSFORMED DATA
       TERMS
 0.10000E 01  0.35000E 01  0.12250E 02  0.42875E 02  0.30000E 01 -0.          -0.          0.10900E 06  0.11881E 11  0.12950E 16
-0.11000E 03
                     YC           DE           LL           UL          DYC          DLL          DUL
        LINE      -0.79784E 02 -0.30216E 02 -0.90951E 02 -0.68617E 02 -0.79784E 02 -0.90951E 02 -0.68617E 02

       ORIGINAL DATA
 0.10000E 01  0.16000E 00  0.30000E 00  0.90000E-02  0.17000E-01  0.25000E 00  0.27800E 01  0.15200E 01  0.36000E 00  0.58000E-01
 0.38000E-01  0.42000E-01  0.10000E-02  0.10000E-02 -0.           0.80000E-02  0.90000E-02 -0.           0.52000E 01  0.35000E 01
 0.30000E 01 -0.          -0.           0.97000E 02  0.15200E 03  0.87500E 03  0.10900E 06  0.93000E 05  0.25000E 02  0.74900E 02
 0.97000E 05 -0.11000E 03  0.42000E 02  0.82000E 02  0.42000E 02  0.68000E 02  0.64000E 02  0.10000E 03  0.50000E 00  0.
 0.10000E 01  0.11000E 01  0.           0.30000E 01
       TRANSFORMED DATA
       TERMS
 0.10000E 01  0.35000E 01  0.12250E 02  0.42875E 02  0.30000E 01 -0.          -0.          0.93000E 05  0.86490E 10  0.80436E 15
-0.11000E 03
                     YC           DE           LL           UL          DYC          DLL          DUL
        LINE      -0.10374E 03 -0.62587E 01 -0.11654E 03 -0.90947E 02 -0.10374E 03 -0.11654E 03 -0.90947E 02

       ORIGINAL DATA
 0.10000E 01  0.16000E 00  0.30000E 00  0.90000E-02  0.17000E-01  0.25000E 00  0.27800E 01  0.15200E 01  0.36000E 00  0.58000E-01
 0.38000E-01  0.42000E-01  0.10000E-02  0.10000E-02 -0.           0.80000E-02  0.90000E-02 -0.           0.52000E 01  0.35000E 01
 0.17000E 02 -0.          -0.           0.83000E 02  0.16000E 04  0.87500E 03  0.13000E 06  0.11200E 06  0.21000E 02  0.67900E 02
 0.12300E 06  0.10000E 02  0.26000E 02  0.25000E 02  0.20000E 02  0.50000E 01  0.51000E 02  0.10000E 03  0.50000E 00  0.
 0.10000E 01  0.11000E 01  0.           0.30000E 01
       TRANSFORMED DATA
       TERMS
 0.10000E 01  0.35000E 01  0.12250E 02  0.42875E 02  0.17000E 02 -0.          -0.          0.11200E 06  0.12544E 11  0.14049E 16
 0.10000E 02
                     YC           DE           LL           UL          DYC          DLL          DUL
        LINE      -0.64941E 02  0.74941E 02 -0.78676E 02 -0.53206E 02 -0.64941E 02 -0.78676E 02 -0.53206E 02  AB

       ORIGINAL DATA
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       TERMS
 0.10000E 01  0.35000E 01  0.12250E 02  0.42875E 02 -0.          -0.          -0.          0.75000E 05  0.56250E 10  0.42187E 15
-0.14500E 03
                     YC           DE           LL           UL           OYC          OLL          OUL
        LINE      -0.15287E 03  0.78665E 01 -0.17759E 03 -0.12814E 03 -0.15287E 03 -0.17759E 03 -0.12814E 03

       ORIGINAL DATA
 0.10000E 01  0.16000E 00  0.30000E 00  0.90000E-02  0.17000E-01  0.25000E 00  0.27800E 01  0.15200E 01  0.36000E 00  0.58000E-01
 0.38000E-01  0.42000E-01  0.10000E-02  0.10000E-02 -0.           0.80000E-02  0.90000E-02 -0.           0.52000E 01  0.35000E 01
 0.30000E 01 -0.          -0.           0.97000E 02  0.15200E 03  0.87500E 03  0.15100E 06  0.13900E 06  0.18000E 02  0.66000E 02
 0.15000E 06 -0.10000E 02  0.28000E 02  0.51000E 02  0.15000E 02  0.80000E 01  0.39000E 02  0.10000E 03  0.50000E 00  0.
 0.10000E 01  0.11000E 01  0.           0.30000E 01
       TRANSFORMED DATA
       TERMS
 0.10000E 01  0.35000E 01  0.12250E 02  0.42875E 02  0.30000E 01 -0.          -0.          0.13900E 06  0.19321E 11  0.26856E 16
-0.10000E 02
                     YC           DE           LL           UL           OYC          OLL          OUL
        LINE       0.89289E 01 -0.18929E 02 -0.12561E 02  0.30419E 02  0.89289E 01 -0.12561E 02  0.30419E 02

       ORIGINAL DATA
 0.10000E 01  0.16000E 00  0.30000E 00  0.90000E-02  0.17000E-01  0.25000E 00  0.27800E 01  0.15200E 01  0.36000E 00  0.58000E-01
 0.38000E-01  0.42000E-01  0.10000E-02  0.10000E-02 -0.           0.80000E-02  0.90000E-02 -0.           0.52000E 01  0.35000E 01
 0.30000E 01 -0.          -0.           0.97000E 02  0.15200E 03  0.87500E 03  0.12200E 06  0.10900E 06  0.22000E 02  0.73000E 02
 0.11900E 06 -0.11000E 03  0.42000E 02  0.82000E 02  0.40000E 02  0.78000E 02  0.54000E 02  0.10000E 03  0.50000E 00  0.
 0.10000E 01  0.11000E 01  0.           0.30000E 01
       TRANSFORMED DATA
       TERMS
 0.10000E 01  0.35000E 01  0.12250E 02  0.42875E 02  0.30000E 01 -0.          -0.          0.10900E 06  0.11881E 11  0.12950E 16
-0.11000E 03
                     YC           DE           LL           UL           OYC          OLL          OUL
        LINE      -0.79784E 02 -0.30216E 02 -0.90951E 02 -0.68617E 02 -0.79784E 02 -0.90951E 02 -0.68617E 02

       ORIGINAL DATA
 0.10000E 01  0.16000E 00  0.30000E 00  0.90000E-02  0.17000E-01  0.25000E 00  0.27800E 01  0.15200E 01  0.36000E 00  0.58000E-01
 0.38000E-01  0.42000E-01  0.10000E-02  0.10000E-02 -0.           0.80000E-02  0.90000E-02 -0.           0.52000E 01  0.35000E 01
 0.30000E 01 -0.          -0.           0.97000E 02  0.15200E 03  0.87500E 03  0.10900E 06  0.93000E 05  0.25000E 02  0.74900E 02
 0.97000E 05 -0.11000E 03  0.42000E 02  0.82000E 02  0.42000E 02  0.68000E 02  0.64000E 02  0.10000E 03  0.50000E 00  0.
 0.10000E 01  0.11000E 01  0.           0.30000E 01
       TRANSFORMED DATA
       TERMS
 0.10000E 01  0.35000E 01  0.12250E 02  0.42875E 02  0.30000E 01 -0.          -0.          0.93000E 05  0.86490E 10  0.80436E 15
-0.11000E 03
                     YC           DE           LL           UL           OYC          OLL          OUL
        LINE      -0.10374E 03 -0.62587E 01 -0.11654E 03 -0.90947E 02 -0.10374E 03 -0.11654E 03 -0.90947E 02

       ORIGINAL DATA
 0.10000E 01  0.16000E 00  0.30000E 00  0.90000E-02  0.17000E-01  0.25000E 00  0.27800E 01  0.15200E 01  0.36000E 00  0.58000E-01
 0.38000E-01  0.42000E-01  0.10000E-02  0.10000E-02 -0.           0.80000E-02  0.90000E-02 -0.           0.52000E 01  0.35000E 01
 0.17000E 02 -0.          -0.           0.83000E 02  0.16000E 04  0.87500E 03  0.13000E 06  0.11200E 06  0.21000E 02  0.67900E 02
 0.12300E 06  0.10000E 02  0.26000E 02  0.25000E 02  0.20000E 02  0.50000E 01  0.51000E 02  0.10000E 03  0.50000E 00  0.
 0.10000E 01  0.11000E 01  0.           0.30000E 01
       TRANSFORMED DATA
       TERMS
 0.10000E 01  0.35000E 01  0.12250E 02  0.42875E 02  0.17000E 02 -0.          -0.          0.11200E 06  0.12544E 11  0.14049E 16
 0.10000E 02
                     YC           DE           LL           UL           OYC          OLL          OUL
        LINE      -0.64941E 02  0.74941E 02 -0.76676E 02 -0.53206E 02 -0.64941E 02 -0.76676E 02 -0.53206E 02  48

       ORIGINAL DATA
```

```
      TERMS
 0.10000E 01  0.35000E 01  0.12250E 02  0.42875E 02 -0.          -0.          -0.          0.75000E 05  0.56250E 10  0.42187E 15
-0.14500E 03
                     YC           DE           LL           UL           OYC          OLL          OUL
        LINE      -0.15287E 03  0.78665E 01 -0.17759E 03 -0.12814E 03 -0.15287E 03 -0.17759E 03 -0.12814E 03

      ORIGINAL DATA
 0.10000E 01  0.16000E 00  0.30000E 00  0.90000E-02  0.17000E-01  0.25000E 00  0.27800E 01  0.15200E 01  0.36000E 00  0.58000E-01
 0.38000E-01  0.42000E-01  0.10000E-02  0.10000E-02 -0.          0.80000E-02  0.90000E-02 -0.          0.52000E 01  0.35000E 01
 0.30000E 01 -0.          -0.          0.97000E 02  0.15200E 03  0.87500E 03  0.15100E 06  0.13900E 06  0.18000E 02  0.66000E 02
 0.15000E 06 -0.10000E 02  0.28000E 02  0.51000E 02  0.19000E 02  0.80000E 01  0.39000E 02  0.10000E 03  0.50000E 00  0.
 0.10000E 01  0.11000E 01  0.          0.30000E 01
      TRANSFORMED DATA
      TERMS
 0.10000E 01  0.35000E 01  0.12250E 02  0.42875E 02  0.30000E 01 -0.          -0.          0.13900E 06  0.19321E 11  0.26856E 16
-0.10000E 02
                     YC           DE           LL           UL           OYC          OLL          OUL
        LINE       0.89289E 01 -0.18929E 02 -0.12561E 02  0.30419E 02  0.89289E 01 -0.12561E 02  0.30419E 02

      ORIGINAL DATA
 0.10000E 01  0.16000E 00  0.30000E 00  0.90000E-02  0.17000E-01  0.25000E 00  0.27800E 01  0.15200E 01  0.36000E 00  0.58000E-01
 0.38000E-01  0.42000E-01  0.10000E-02  0.10000E-02 -0.          0.80000E-02  0.90000E-02 -0.          0.52000E 01  0.35000E 01
 0.30000E 01 -0.          -0.          0.97000E 02  0.15200E 03  0.87500E 03  0.12200E 06  0.10900E 06  0.22000E 02  0.73000E 02
 0.11900E 06 -0.11000E 03  0.42000E 02  0.82000E 02  0.40000E 02  0.78000E 02  0.54000E 02  0.10000E 03  0.50000E 00  0.
 0.10000E 01  0.11000E 01  0.          0.30000E 01
      TRANSFORMED DATA
      TERMS
 0.10000E 01  0.35000E 01  0.12250E 02  0.42875E 02  0.30000E 01 -0.          -0.          0.10900E 06  0.11881E 11  0.12950E 16
-0.11000E 03
                     YC           DE           LL           UL           OYC          OLL          OUL
        LINE      -0.79784E 02 -0.30216E 02 -0.90951E 02 -0.68617E 02 -0.79784E 02 -0.90951E 02 -0.68617E 02

      ORIGINAL DATA
 0.10000E 01  0.16000E 00  0.30000E 00  0.90000E-02  0.17000E-01  0.25000E 00  0.27800E 01  0.15200E 01  0.36000E 00  0.58000E-01
 0.38000E-01  0.42000E-01  0.10000E-02  0.10000E-02 -0.          0.80000E-02  0.90000E-02 -0.          0.52000E 01  0.35000E 01
 0.30000E 01 -0.          -0.          0.97000E 02  0.15200E 03  0.87500E 03  0.10900E 06  0.93000E 05  0.25000E 02  0.74900E 02
 0.97000E 05 -0.11000E 03  0.42000E 02  0.82000E 02  0.42000E 02  0.68000E 02  0.64000E 02  0.10000E 03  0.50000E 00  0.
 0.10000E 01  0.11000E 01  0.          0.30000E 01
      TRANSFORMED DATA
      TERMS
 0.10000E 01  0.35000E 01  0.12250E 02  0.42875E 02  0.30000E 01 -0.          -0.          0.93000E 05  0.86490E 10  0.80436E 15
-0.11000E 03
                     YC           DE           LL           UL           OYC          OLL          OUL
        LINE      -0.10374E 03 -0.62587E 01 -0.11654E 03 -0.90947E 02 -0.10374E 03 -0.11654E 03 -0.90947E 02

      ORIGINAL DATA
 0.10000E 01  0.16000E 00  0.30000E 00  0.90000E-02  0.17000E-01  0.25000E 00  0.27800E 01  0.15200E 01  0.36000E 00  0.58000E-01
 0.38000E-01  0.42000E-01  0.10000E-02  0.10000E-02 -0.          0.80000E-02  0.90000E-02 -0.          0.52000E 01  0.35000E 01
 0.17000E 02 -0.          -0.          0.83000E 02  0.16000E 04  0.87500E 03  0.13000E 06  0.11200E 06  0.21000E 02  0.67900E 02
 0.12300E 06  0.11000E 02  0.26000E 02  0.25000E 02  0.20000E 02  0.90000E 01  0.51000E 02  0.10000E 03  0.50000E 00  0.
 0.10000E 01  0.11000E 01  0.          0.30000E 01
      TRANSFORMED DATA
      TERMS
 0.10000E 01  0.35000E 01  0.12250E 02  0.42875E 02  0.17000E 02 -0.          -0.          0.11200E 06  0.12544E 11  0.14049E 16
 0.10000E 02
                     YC           DE           LL           UL           OYC          OLL          OUL
        LINE      -0.64941E 02  0.74941E 02 -0.76676E 02 -0.53206E 02 -0.64941E 02 -0.76676E 02 -0.53206E 02  AB

      ORIGINAL DATA
```

REFERENCES FOR APPENDIX E

E1. Penney, Leonie, "Multiple Regression Program Capable of Handling Missing Data and Scanning for Aberrant Data," U.S. Department of Agriculture, Data Systems Application Division, Program 1604 (December 1969).

E2. Snedecor, G. W. and Cochran, W. G., Statistical Methods, Fifth Edition, The Iowa State College Press, Ames, Iowa, 1956, pp. 122-159 and 413-471.

E3. Plackett, R. L., Principles of Regression Analysis, Oxford University Press, London, 1960, pp. 1-165.

E4. Deming, W. E., Statistical Adjustment of Data, John Wiley and Sons, Inc., New York, 1948, pp. 1-251.

E5. Neyman, J. and Scott, E. L., "Correction for Bias Introduced by a Transformation of Variables," Annals of Mathematical Statistics, Vol. 31, 1960, pp. 643-655.

E6. Natrella, M. G. et al., "Engineering Design Handbook, Experimental Statistics, Section 1, Basic Concepts and Analysis of Measurement Data," U.S. Army Materials Command, AMC Pamphlet AMCP 706-110, Chapter 6, (Polynomial and Multivariable Relationships, Analysis by the Method of Least Squares) pp. 6-4 through 6-16, July 1963.

E7. Guttman, I. and Wilks, S. S., Introductory Engineering Statistics, John Wiley and Sons, Inc., New York, 1965.

E8. Schrock, E. M., Quality Control and Statistical Methods, Second Edition, Reinhold Publishing Corp., New York, 1967.

E9. Cochran, W. G. and Cox, G. M., Experimental Designs, Second Edition, John Wiley and Sons, Inc., New York, 1957.

E10. "BMDO3D, Correlation with Item Deletion," Health Services Computing Facility, University of California at Los Angeles (13 November 1964).

E11. Fisher, R. A. and Cornish, E. A., "The Percentile Points of Distributions Having Known Cumulants," Technometrics, Vol. 1, No. 2, May 1960, pp. 209-225.

E12. Box, G. E. P., "Use and Abuse of Regression," Technometrics, Vol. 8, No. 4, November 1966, pp. 625-629.

E13. Mordecai, E., Methods of Correlation Analysis, Second Edition, John Wiley and Sons, Inc., New York, 1950.

APPENDIX F

TIME:TEMPERATURE COOLING CURVES FOR DROP WEIGHT SPECIMENS WHEN TRANSFERRED FROM 1640F FURNACE TO MOLTEN SALT BATHS AT EITHER 1200F OR 875F

INTRODUCTION

The actual time-temperature history was desired for the low-carbon Ni-Cr-Mo steel (5/8 × 2 × 5 in.) drop weight impact specimens used in these tests when they were transferred from a high temperature salt bath at 1640F to a salt bath at either 1200F or 875F. In order to determine this thermal history a specimen was instrumented with a thermocouple at its center as recommended by Sinnott and Shyne[52] and the time-temperature history of the specimen and the bath were recorded at least every five seconds on a strip chart recorder. The experimental cooling curve is compared to the curve calculated using the method of Sinnott and Shyne.

SPECIMEN PREPARATION

A 0.087 inch diameter hole was drilled from one end, 2½ inches deep into the center of the specimen. The hole was countersunk and tapped for ¼-28 threads to a depth of 3/8 inch. A 7 inch length of ¼ inch high pressure steel tubing was screwed tightly into the hole. An insulated 28-gauge Chromel-Alumel thermocouple was inserted in the tube and forced against the specimen at the bottom of the well. Another thermocouple was positioned in the low temperature salt bath adjacent to the specimen. Both thermocouples were attached to a multichannel recorder to alternately record the temperatures indicated by both thermocouples. The specimen was sand blasted between each run to

remove all scale and salt and return it to a representative condition.

HEAT TREATMENT

The specimen was suspended vertically in a high temperature salt bath and allowed to come to 1640F. It was held at 1640F ±15F for 30 minutes, minimum. The ±15F bath temperature tolerance was used here and for the low temperature baths because it represented the tolerance of the commercial temperature controller used in this test as opposed to the closer ±5F manual control used for the actual specimen heat treatments. From 4 to 5 inches of the thermocouple tubing was kept below the surface of the molten salt to minimize heat losses from the tube.

The specimen was transferred from the 1640F bath to the low temperature bath within one or two seconds. The specimen was quenched vertically into the center of the low temperature bath and mildly agitated near the thermocouple recording the bath temperature. Multiple runs were made to allow for slight variations in the bath temperatures between runs.

RESULTS

The results of the cooling rate studies are plotted in Figure F1 and are compared to the salt-bath cooling rate predicted using the instantaneous film coefficient method of Sinnott and Shyne with the following parameters:

Specimen Area	0.200 sq. ft.
Specimen Weight	1.771 lb.
Mean Specific Heat, C_p*	
1640 to 1200F	0.127 BTU/lb/F
1640 to 875F	0.128 BTU/lb/F

Table F1 is a listing of the computer program used to predict cooling rates.

The variable names used in this program and their significance are given below:

ITB — Temperature of low temperature bath, 875, 900 or 1200F.

ITS — Initial temperature of specimen, 1640F.

HF — Instantaneous film coefficient based on the difference between the average surface temperature of the specimen and the bath during the time interval THETA.

QTH — Average cooling rate in BTU/sec when cooling from MTS1 to MTS2.

THETA — Time to cool from a temperature MTS1 to MTS2, seconds.

SUMTH — Total elapsed time to cool from ITS to MTS2, seconds.

The data shown in Figure F1 indicates reasonable agreement between experimental and predicted cooling rates. The low bath temperature experiments were run with an initial bath temperature of about 900F rather than 875F because of

*Based on data from U.S. Steel Corporation

the characteristics of the commercial salt mix that was available in the salt bath when these tests were run.

The data shows that it takes about 125 seconds for the center of the specimen to cool from 1640F to 1210F and about 140 seconds to cool from 1640F to 885F when a drop weight specimen is transferred to a 1200F and an 875F bath respectively.

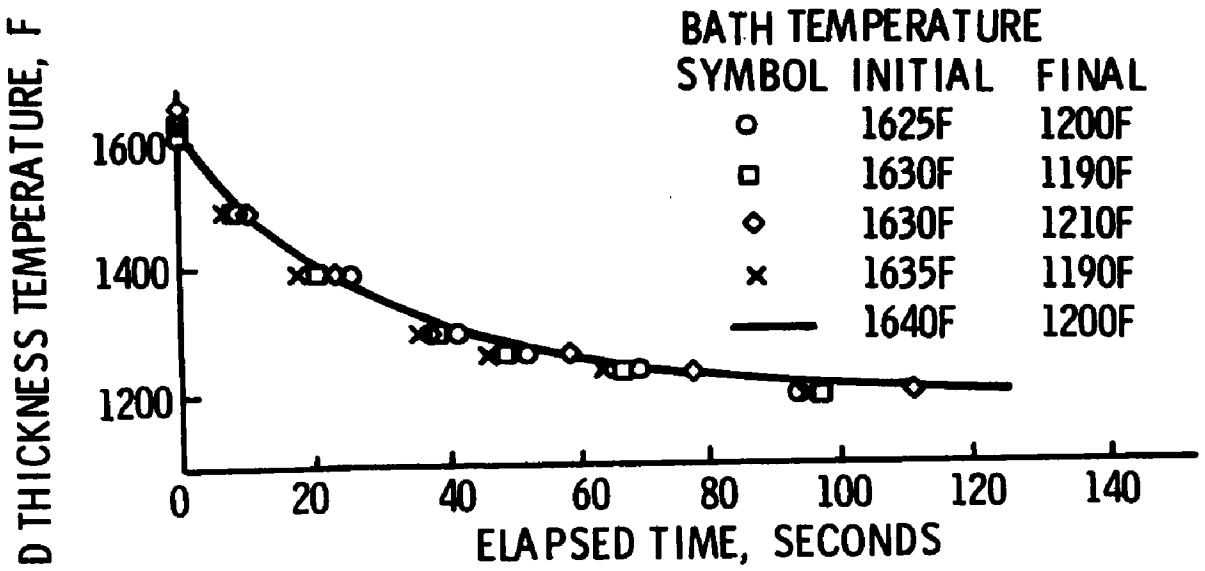

Figure F1a - Cooling Rate from 1640 to 1200F
(Average C_p = 0.127 BTU/lb/F)

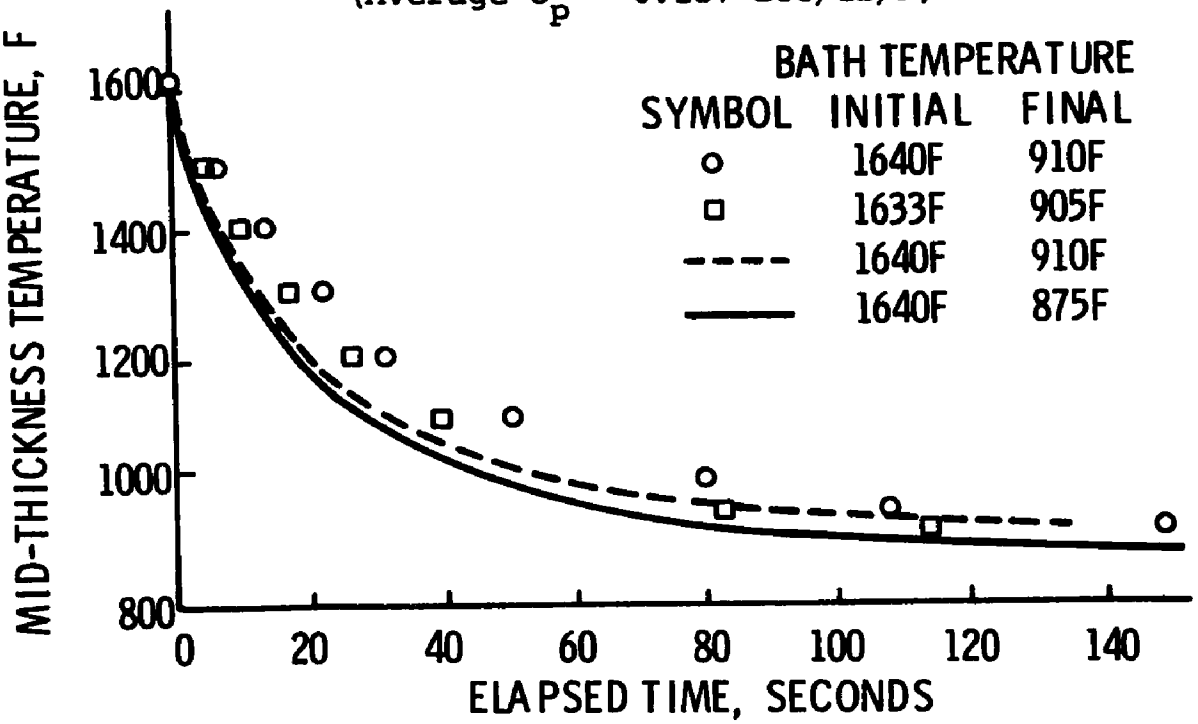

Figure F1b - Cooling Rate from 1640 to 875F
(Average C_p = 0.128 BTU/lb/F)

Figure F1 - Comparison of Actual Cooling Rates with Computed Cooling Rates for the Center of a 5/8 × 2 × 5 Inch Drop Weight Specimen.

```
BUB

100 CP=.128
110 ITB=875.0
120 SUMTH=0.0
130 A=.19965
140 W=1.7708
150 ITS=1640.0
160 PRINT,"          *******  AGITATED BY HAND  *******"
170 PRINT,^
180 PRINT,"  MTS2          HF           QTH          THETA        SUMTH"
190 DØ 20 M=1,80
200 MTS2=ITS-M*10
210 IF(ITB-MTS2)15,20,20
220 15 MTS1=ITS-(M-1)*10
230 HF=93.0*EXP(0.0015*((MTS1+MTS2)*0.5)-ITB))
240 QTH=HF*A*(MTS1-ITB)/3600.0
250 THETA=(W*CP*(MTS1-MTS2)/QTH)
260 SUMTH=SUMTH+THETA
270 PRINT,MTS2,HF,QTH,THETA,SUMTH
280 20 CONTINUE
290 END
```

Table F1 - Listing of a Computer Program Using the Instantaneous Film Coefficient Method of Sinnott and Shyne[52] to Predict the Instantaneous Cooling Rate and Time-temperature Cooling Curve for a Low-Carbon Ni-Cr-Mo Alloy Steel Specimen 5/8 × 2 × 5 Inches When Transferred from a Neutral Salt Bath at 1640F to a Neutral Salt Bath at 875F.