STRATEGIC AUDITING WITH INCOMPLETE AND ASYMMETRIC INFORMATION

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Technical Report 96-6

March 1996

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* Author's research partially supported by the grant CCR - 9321550 from the National Science Foundation

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ABSTRACT

This paper examines the value of prior information to an internal auditor in a strategic setting. The strategic setting is an incomplete information game (developed in Choi et al, 1996) in which an auditee issues a financial report on the auditee's performance, and the auditor must determine whether or not to perform testing on the report. Incomplete information is modeled as follows. There are two auditor types, experienced and inexperienced, and the auditee is not certain which auditor is the opponent. Similarly, there are two auditee types, fraudulent (low integrity) and not fraudulent, and the auditor does not know which is the opponent. The strategic setting is also characterized as having asymmetric information – the auditor has prior information about the auditee, but the auditee has no prior information about the auditor. The analysis of the incomplete information game with asymmetric information provides new insights into strategic auditing.

To study the role and usefulness of prior information, we develop three equilibria and derive and interpret optimal strategies for each. The equilibria represent control environments of different strengths – a weak, moderate and a strong control environment. The equilibrium strategies show that in a weak control environment, the auditor is more likely to choose a testing strategy, and the auditee is more likely to choose to commit fraud. In contrast, in the strong control environment, the auditor is more likely to choose a no-audit strategy, and the auditee is more likely to choose no-fraud. In the intermediate control environment, the strategies fall in between these extremes.

In a weak or moderate control environment, the role of prior information is consistent with auditing concepts and practices. With prior information indicating fraud, the auditor performs more testing. Also, with an increase in the accuracy of the prior information, the auditor substitutes the improved accuracy of the prior information for the subsequent tests,

and thus performs less testing.

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1 Introduction

Auditing is now commonly viewed as a strategic process in which the auditor and manager (hereafter, "auditee") observe and respond to each other's actions in determining the outcomes of the audit and the auditee's actions. The process thus presumes conflicting interests between auditor and auditee, which is consistent with auditing concepts and professional standards for both internal and external auditors.[1] In this paper the nature of the conflict is operationalized as that of an auditee who prepares a financial report upon which the auditee's compensation is based. Since the auditee has an incentive to report fraudulently, the auditor's role is to investigate the accuracy of the report. The paper investigates the case of an internal auditor, because of the importance of fraud detection in internal auditing:

> "... internal auditors should be alert to the possibility of intentional wrongdoing, errors and omissions ... the possibility of material irregularities or noncompliance should be considered whenever the internal auditor undertakes an internal audit assignment." (Standards for the Professional Practice of Internal Auditing, Section 280, "Due Care.")

Moreover, Shibano (1990) has shown that the strategic formulation is particularly important to describe the auditor-auditee relationship when potential fraud is involved. In contrast, the single-person, or decision theoretic approach is more appropriate when only simple error, and no fraud, is involved. This is because no play of opponents with conflicting interests is involved for simple errors. In contrast, the strategic approach is a particularly apt way to view the case of potential fraud, where conflicting interests are involved. In this paper we consider the strategic, game-theoretic approach to modeling the internal auditor and auditee in a case of potential fraud by the auditee.

A common representation of the strategic process is the game of complete information (Fellingham and Newman, 1985; Anderson and Young 1988; Fellingham et al, 1989; Newman

[1] As in the prior work in this area, we presume a non-cooperative game setting.

and Noel, 1989; Shibano, 1990; Patterson, 1993; Hansen, 1993; Morton, 1993). A complete information game is one in which both the auditor and auditee have common knowledge of (a) all the players (the auditor and auditee), (b) all the actions available to all the players, and (c) all the potential outcomes to all the players from all available actions.

1.1 Incomplete Information Games

The assumption of complete information is not representative of the actual conflicting relationship between the auditor and auditee in internal auditing. It is more likely that the auditor will be uncertain of the integrity of the auditee (and therefore the potential for fraud) and the auditee will be uncertain about the nature of the auditor due to changes in management and operations, and changes in audit staff. These changes are even more likely if the internal auditor allocates internal audit resources randomly to achieve the benefits of an unexpected audit.[2]

For these reasons, Choi et al (1996) developed an incomplete information game for the strategic audit setting. The findings were that the optimal strategies for the internal auditor are likely to be quite different from the strategies in complete information games. First, the optimal strategies are equally likely to be pure and randomized strategies in the incomplete information game, in contrast to complete information games in which the optimal strategy is always a randomized strategy. Second, when the optimal strategy is a randomized strategy, the auditor's actions are exaggerated, to compensate for the auditee's lack of complete information about the auditor. Thus, the formulation of the incomplete information game provides new insights about strategic auditing. While the Choi et al (1996) paper examined the effect of moving from complete to incomplete information, the objective of this paper is to examine the effect of adding information asymmetry to the game with incomplete information.

[2]Anderson and Young (1988, p 29) observe that complete information games "may not be representative of certain audits, e.g., initial audits."

1.2 Information Asymmetry

Information asymmetry is operationalized in the game by allowing the auditor access to prior information about the integrity of the auditee, while the auditee has no prior information about the type of auditor. This condition is descriptive of the actual audit context, in which auditor independence is maintained for compliance with professional standards, and thus the auditee has little or no information about the timing, nature, or extent of auditing procedures. In contrast, it is common for the auditor to seek out prior information about the auditee in the form of inquiry and analyses, to detect errors and frauds, and to properly design the timing, nature and extent of audit tests.

The auditor's access to prior information can be viewed as a Bayesian characterization of the audit process, as described by Johnstone (1995) and Edwards (1995), and as modeled by Kinney (1975) and others. These authors present strong arguments for the relevance and practicality of taking the Bayesian view in auditing. In the Bayesian view, the auditor incorporates prior information as additional evidence to supplement the information obtained from audit tests. The incomplete information game developed in this paper takes the same view. The auditor uses prior information to determine the extent of subsequent audit tests (in this case, whether or not additional audit tests are to be performed).

There are broadly two types of prior information available to the internal auditor – risk indicators, as summarized in control environment documentation and questionnaires, and analytical procedures. Both types of prior information are commonly viewed in internal audit practice as effective tools for detecting fraud (Sawyer, 1981; Jacobson, 1990). Moreover, both internal and external audit research and professional guidance support the use of these procedures for the detection of fraud. Statement on Internal Auditing Standards Number 8 (Analytical Auditing Procedures) states: "Analytical auditing procedures are useful in identifying, among other things, potential errors, irregularities, or illegal acts." And, the Auditing Standards Board of the American Institute of CPAs is currently developing new guidance regarding the detection of fraud, which includes extensive reference to specific risk indicators the auditor should consider in the audit. Also, research by Coglitore and Berryman (1988) and Blocher (1992) show the ways that analytical procedures can be effective in detecting fraud. In summary, both conceptually (based on the Bayesian model for

incorporating prior information) and practically (based on current guidance and practical experience) prior information is an appropriate means to address the detection of fraud.

In the following section we develop the incomplete information game with asymmetric information, including the cost and payoff functions. The next section develops three equilibria for the game which are distinguished by the strength of the control environment. The final section includes a discussion and interpretation of the findings from the three equilibria.

2 Game with Incomplete and Asymmetric Information

The game setting is the same as that described in Choi et al (1996). The auditees are division managers in a large firm with many divisions. The auditees are guaranteed a base salary and a cash bonus based on a financial report prepared by the auditee. The auditee thus has the incentive, and we presume also the opportunity to misstate the division's financial results. The auditee has two pure strategies: no fraud, or fraud of a material amount. Also, there are two types of auditees: one with integrity (auditee number one), and one lacking integrity (auditee two). Later, we develop a condition (Inequality One) such that the penalty for material fraud is sufficiently high for auditee one that this auditee never has the incentive to commit fraud; only auditee two, who has a lower penalty for fraud, has an incentive to commit fraud. [3] The payoffs to auditees, conditional on the actions of the auditor, are shown in Exhibit 1.

[3]Inequality One is used to restrict auditee one to the no-fraud action. It reduces the number of potential equilibria in the game to a limited number of useful and interesting equilibria, as explained more fully in the next section.

Exhibit 1

The cost matrix of type i auditor

	No fraud	Material fraud
No testing	0	M
100% testing	S_i	S_i

The payoff matrix of type j auditee

	No fraud	Material fraud
No testing	0	F
100% testing	0	$-P_jF$

where M = Loss to company due to undetected material fraud

S_i = Cost of "100% testing" to type i auditor

F = Incremental bonus to auditee due to material fraud

P_j =Penalty for material fraud to type j auditee when detected

The internal audit function in the firm investigates auditees for potential fraud. Individual internal auditors are assigned randomly to auditees. The auditor has two action choices: no testing, or 100% testing which will perfectly detect fraud. [4] Also, there are two types of auditors: auditor one is experienced while auditor two is not. Higher experience in this case means higher cost of testing. It is assumed that auditor one, being more experienced, has higher compensation and therefore a higher testing cost than auditor two, even after taking into account the presumed greater testing efficiency of the more experienced auditor. Later we develop the condition (Inequality Two) such that the cost of testing for auditor one is sufficiently high that the type one auditor always does less testing than the type two auditor. [5] The costs of the auditors, conditional on the auditee's actions, are shown in Exhibit 1.

[4]We assume there are no random errors, only fraud.

[5]Inequality Two is used to reduce the number of potential equilibria in the game to a limited number of useful and interesting equilibria, as explained more fully in the next section."

2.1 Prior Information

Information asymmetry is incorporated in the game through the auditor's prior information about the auditee, the auditee having no prior information. Further, we assume that only the type one auditor is able to effectively utilize the prior information, that is, the effective use of prior information requires experience. [6] This assumption is made to simplify the game, and to focus on the most useful game equilibria.

Prior information is modeled as follows. The type one auditor receives a signal (the prior information) about the type of auditee. The signal reports either θ_1 (the auditee is type one) or θ_2 (the auditee is type two). The reliability of the signal is ϕ, where $\phi > .5$. Thus, if the signal is θ_1, the probability is ϕ that the auditee is a type one auditee; and if the signal is θ_2, the probability is ϕ that the auditee is type two.

2.2 Expected Auditor Cost Functions and Auditee Payoff Functions

We assume that the probability of either type of auditor is .5, and the probability of either type of auditee is also .5, that is, apart from the signal and its accuracy, both types or auditors and auditees exist in equal proportions. [7] Given the accuracy of the signal and the proportions of auditors, the probability function of the type one auditee is as follows. The probability that the type one auditee has a type one auditor opponent and the auditor uses a strategy contingent on θ_1 is $.5\phi$. Also, the probability that the type one auditee has a type one auditor opponent and the auditor uses a strategy contingent on θ_2 is $.5(1-\phi)$. The probability that the opponent is a type two auditor is .5 The probability distribution

[6] The assumption that only the experienced auditors can effectively utilize analytic procedure and interpret risk indicators is supported in both research and practice. Johnson et al (1992) show and explain why an experienced auditor detects the fraud in an experimental case, while the inexperienced auditor fails to do so. Also, Hirst et al (1995) report that auditors consider experience to be a critical factor in determining the extent to which analytic procedures are performed.

[7] In Choi et al (1995), the proportion of auditors was assumed to be 50%, as assumed here. For greater generality, the the proportion of type one auditees was assumed to be π and the proportion of type two auditees was assumed to be $1-\pi$. For simplicity and clarity, both types of auditors and auditees are assumed to be equally likely in this paper.

of the type two auditee can be derived similarly. From these probability functions and the costs and payoffs in Exhibit 1, we derived the expected cost functions for the auditors and expected payoff functions for the auditees, which are presented in Exhibit 2. The game is solved by having each auditor minimize expected cost, and each auditee maximize expected payoff.

Exhibit 2: Notation

ϕ represents signal accuracy, and $\phi > 0.5$.

$X_{1\theta_1}$ represents probability of "100% testing" by type 1 auditor contingent on θ_1.

$X_{1\theta_2}$ represents probability of "100% testing" by type 1 auditor contingent on θ_2.

X_2 represents probability of "100% testing" by type 2 auditor.

Y_j represents probability of "material fraud" by type j auditee $j \in \{1, 2\}$.

The cost function to type 1 auditor with θ_1:
$\phi[X_{1\theta_1}S_1 + \{(1 - X_{1\theta_1})Y_1M\}] + (1 - \phi)[X_{1\theta_1}S_1 + \{(1 - X_{1\theta_1})Y_2M\}]$.

The cost function to type 1 auditor with θ_2:
$(1 - \phi)[X_{1\theta_2}S_1 + \{(1 - X_{1\theta_2})Y_1M\}] + \phi[X_{1\theta_2}S_1 + \{(1 - X_{1\theta_2})Y_2M\}]$.

The cost function to type 2 auditor:
$X_2S_2 + \{(1 - X_2)M\}(1/2\sum_{j=1}^{2} Y_j)$.

The payoff function to type 1 auditee:
$0.5\phi\{1 - (1 + P_1)X_{1\theta_1}\}Y_1F + 0.5(1 - \phi)\{1 - (1 + P_1)X_{1\theta_2}\}Y_1F + 0.5\{1 - (1 + P_1)X_2\}Y_1F$.

The payoff function to type 2 auditee:
$0.5(1 - \phi)\{1 - (1 + P_2)X_{1\theta_1}\}Y_2F + 0.5\phi\{1 - (1 + P_2)X_{1\theta_2}\}Y_2F + 0.5\{1 - (1 + P_2)X_2\}Y_2F$.

3 Solution of the Game: Three Equilibria

This section solves the complete information game with asymmetric information for each of three equilibria. Each of the three equilibria are characterized by successively stronger control environments, that is, larger values for P_2, the penalty for fraud by the type two auditee. In deriving the equilibria, three assumptions are made. First, we assume that $S_1 > S_2$. This assumption requires that the testing cost of the experienced auditor, auditor one, be greater than for auditor two. This assumption is made in order to eliminate equilibria which are less useful and interesting, that is, equilibria similar to existing equilibria. [8]

The second assumption is that one of the auditees, auditee one, never has an incentive to commit fraud. This assumption is equivalent to Inequality One which requires that the expected payoff from material fraud for the type one auditee is always less than zero. The terms, $X^*_{1\theta_1}$, $X^*_{1\theta_2}$, and X^*_2 , in Inequality One represent the equilibrium probabilities of 100% testing by type one auditor with θ_1, by type one auditor with θ_2, and by the type two auditor, respectively.

$$[1 - (1 + P_1)\{\phi X^*_{1\theta_1} + (1 - \phi)X^*_{1\theta_2} + X^*_2\}/2] < 0. \tag{1}$$

The purpose for the second assumption is to focus on the more likely equilibria in the game by eliminating unlikely or unreasonable equilibria.

The third assumption limits the value of ϕ such that, even with θ_2, the expected incremental cost to auditor one of no testing is less than the incremental cost of 100% testing. That is, the type one auditor never tests to a greater extent than the type two auditor. This assumption is equivalent to Inequality Two, where Y^*_1 and Y^*_2 represent the equilibrium probabilities of material fraud by the type one and type two auditee, respectively.

$$(\phi - 0.5)(Y^*_2 - Y^*_1)M < (S_1 - S_2). \tag{2}$$

As for the above assumptions, the purpose of the third assumption is to restrict the number of equilibria by focusing on the more likely equilibria. [9]

[8] If $S_1 < S_2$, the role of each type of auditor is reversed. The equilibrium strategy of a type one auditor becomes the equilibrium strategy of the type two auditor, and vise versa.

[9] Equilibrium one exists without Inequality Two. Equilibria Two and Three require Inequality Two.

Equilibrium One ($S_1 > S_2$, $0 < S_1 \leq (1-\phi)/M$, [10] $0 < P_2 < (1-\phi)/(1+\phi)$) [11]

The unique Nash strategies for this equilibrium are shown in Exhibit 3. The appendix presents a proof. This equilibrium represents a relatively weak control environment, the weakest of the three equilibria. The penalty for material fraud is the lowest and the probability of material fraud is the highest of the three equilibria. Exhibit 4 illustrates how the three equilibria differ as to auditee type two's penalty (P_2) for material fraud (since by Inequality One auditee one never commits fraud, we focus our attention on the penalty for and behavior of auditee two). Because the control environment is weak, auditor type two who must rely on audit tests (no prior information) adopts the pure strategy of 100% testing. Auditor one, when presented with the unfavorable signal θ_2, also adopts the pure strategy of 100% testing. But when auditor one receives the favorable signal θ_1, this auditor moves to a randomized strategy which includes no testing. Overall, the testing strategies of the two auditor types reflects the weak control environment. Additionally, auditor one

However, Inequality Two does not change the result that more accurate information leads to less testing. See footnote 13 in the Appendix.

[10] The condition, $0 < S_1 \leq (1-\phi)M$, results from Inequality One. When Inequality One holds, $Y_1^* = 0$, and $Y_2^* = S_1/[(1-\phi)M]$, as long as $0 < S_1 < (1-\phi)M$, $0 < Y_2^* \leq 1$. Therefore Inequality One is equivalent to $0 < S_1 \leq (1-\phi)M$. That is, Inequality One is equivalent to assuming that testing is not costly.

If Inequality One does not hold, $Y_1^* = (1/\phi)[S_1/M - (1-\phi)]$, and $Y_2^* = 1$. As long as $M(1-\phi) < S_1 \leq 1$, $0 < Y_1^* \leq 1$. Thus, assuming that Inequality One does not hold is equivalent to assuming that testing is costly.

[11] For expositional ease, let's assume that the equilibrium strategy of type one auditor with θ_1 is no testing; however, the equilibrium strategy of type one auditor with θ_2 and the type two auditor is 100% testing. The probability that the type two auditor gets away with material fraud is (1) the probability of having the type one auditor as opponent (equal to $\frac{1}{2}$), which is multiplied by (2) the probability of θ_1 observed by auditor one $(1-\phi)$. The probability that the type two auditee gets caught for material fraud and pays the penalty (P_2F) is $\frac{1}{2} + \frac{1}{2}(\phi)$ - - equal to the sum of (1) the probability of auditor type two plus (2) the probability of auditor type one multiplied by the probability of θ_2. Thus, the expected payoff to the type two auditee from material fraud is $F(1-\phi)\frac{1}{2} - FP_2\frac{1}{2}(1-\phi)$. The expression is 0 when $P_2 = (1-\phi)/(1+\phi)$. In other words, the type two auditee is indifferent about no fraud versus material fraud. However, as P_2 decreases from $(1-\phi)/(1+\phi)$ (Equilibrium One), the type one auditor with θ_1 should increase the probability of 100% testing to make the auditee type two indifferent about no fraud versus fraud. As P_2 increases from $(1-\phi)/(1+\phi)$ (Equilibrium Two), the type two auditor with θ_2 can decrease the probability of 100% testing and still can make auditee type two indifferent about no fraud versus fraud.

responds in the expected manner to the prior information.

Equilibrium Two ($S_1 > S_2$, $0 < S_1 \leq \phi/M$, $(1-\phi)/(1+\phi) \leq P_2 < 1$)

The unique Nash strategies for this equilibrium are shown in Exhibit 3. The appendix presents a proof. This equilibrium represents a moderately strong control environment. The penalty for material fraud is between that of the other two equilibria (see Exhibit 4) and the probability of material fraud is between that of the other two equilibria. The penalty is such that auditee two clearly has an incentive to commit fraud, though not as high an incentive as for Equilibrium One. In response, auditor two continues to do 100% testing, while auditor one, because of the improvement in control, does less testing overall – under a favorable signal (θ_1) the type one auditor does no testing (in contrast to a randomized strategy in Equilibrium One), while under the unfavorable signal (θ_2), auditor one does less testing, using a randomized strategy instead of the pure 100% test strategy used in Equilibrium One.

Overall, the testing strategies of the two auditor types reflect the moderately strong control environment. Additionally, auditor one responds in the expected manner to the prior information.

Exhibit 3: Equilibrium Strategies

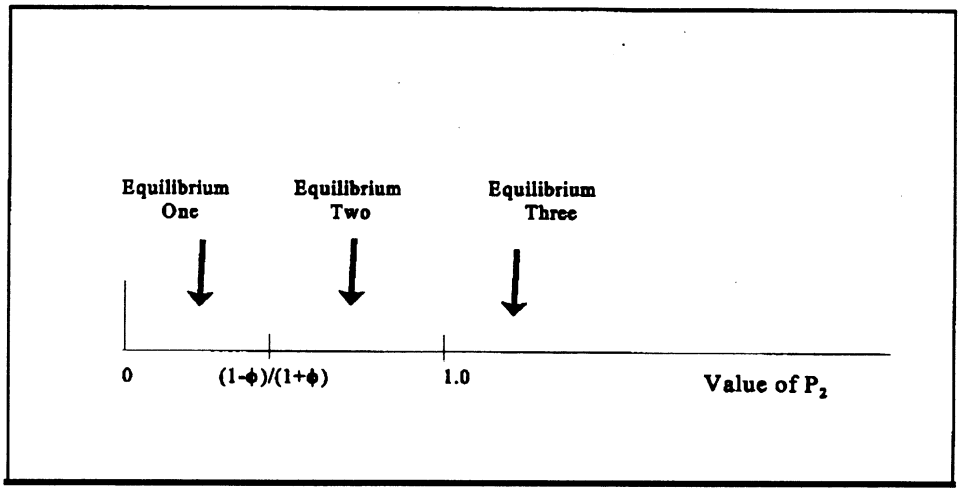
		Equilibrium One	Equilibrium Two	Equilibrium Three
Control Environment - - - >		Very weak; $0 \leq P_2 <$ $(1-\phi)/(1+\phi)$	Moderate: $(1-\phi)/(1+\phi) \leq P_2$ < 1	Strong: $P_2 \geq 1$
	Strategy			
Auditor 1 with $\theta 1$	No test	$1-[1/(1-\phi)]$ $[2/(1+P_2)-$ $(1+\phi)]$	1	1
	100% test	$[1/(1-\phi)]$ $[2/(1+P_2)-$ $(1+\phi)]$	0	0
Auditor 1 with $\theta 2$	No test	0	$1-\{1/\phi[2/(1+P_2)-$ $1]\}$	1
	100% test	1	$1/\phi[2/(1+P_2)-1]$	0
Auditor 2	No test	0	0	$(1-P_2)/(1+P_2)$
	100% test	1	1	$2/(1+P_2)$
Auditee 1	No fraud	1	1	1
	Fraud	0	0	0
Auditee 2	No fraud	$1-S_1/\{(1-\phi)/M\}$	$1-(S_1/\phi M)$	$1-(2S_2/M)$
	Fraud	$S_1/(1-\phi)M$	$S_1/\phi M$	$2S_2/M$

Equilibrium Three ($S_1 > S_2$, $0 < S_2 \leq .5M$, $P2 \leq 1.0$)

The unique Nash strategies for this equilibrium are shown in Exhibit 3. The appendix presents a proof. This equilibrium represents the strongest control environment of the three

equilibria. The penalty for material fraud is the greatest of the three equilibria (see Exhibit 4) and the probability of material fraud is the lowest. [12]

Exhibit 4: The relationship among the three equilibria



These results for Equilibrium Three are identical to those for the game with incomplete information but no information asymmetry (Choi et al, 1996). Thus, with the very strong control environment in Equilibrium Three ($P_2 \geq 1$), the auditor with the ability to use prior information acts the same as if there were no prior information. The prior information is dominated by the strength of the control environment. That is, in the presence of a very strong control environment, the auditor is not influenced further by the results of preliminary audit information such as the review of risk indicators or the use of analytical procedures. The assurance that the auditee's control system will prevent fraud dominates the evidence value of detection-oriented procedures. Reliance on prevention of fraud obviates the necessity to obtain prior information or to perform 100% testing to detect fraud.

4 Discussion

The incomplete information game with asymmetric information provides useful additional insights about strategic internal auditing. The insights are consistent with current professional guidance and with projected new directions in audit practice and guidance regarding the detection of management fraud. The prevention and detection of management fraud

[12] As required by Inequality Two.

continues to be a critical issue in auditing, and auditors continue to look for improved methods (American Institute of CPAs, 1996). In Equilibria One and Two, the auditor is more likely to apply 100% testing when the prior information signals potential for fraud, a low integrity auditee. And conversely, the auditor is more likely to apply no tests if the prior information is favorable. In both Equilibria One and Two, the auditor does more testing when the signal (θ_2) is unfavorable. As observed earlier, in equilibrium three the control environment is so strong that the experienced auditor chooses never to test, irrespective of the nature of the prior information (Choi, et al, 1996). This is consistent with the notion that a sufficiently strong control environment will provide sufficient assurance that fraud will be *prevented*, and there is therefore no need for the additional assurance provided by procedures (that is, the prior information or the 100% tests) to *detect* fraud. While current professional guidance emphasizes the importance of a strong control environment, and the need for the auditor to study and assess the control environment, it is not clear that the guidance indicates that a sufficiently strong control environment obviates the necessity of any audit tests to detect potential management fraud.

Looking again to Equilibria One and Two, in which auditor one utilizes prior information, we see that as the accuracy of the prior information (ϕ) increases, the auditor decreases the probability of 100% testing. That is, as the accuracy of prior information improves, the auditor substitutes the prior information for subsequent testing – the randomized strategies for auditor one in Equilibrium One (with θ_1) and Equilibrium Two (with θ_2) show a lower probability for 100% testing. This result is consistent with current professional guidance and practice for both internal and external auditors. As the precision of analytical procedures and the diagnosticity of risk indicators improve, the value of these procedures (and therefore of prior information) increases, and the auditor is able to obtain the desired level of assurance of no fraud with a lower extent of other tests.

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Appendix A

Equilibrium 1. If $S_1/M > S_2/M, 0 < S_1/M \leq (1-\phi)$, and $0.5(\phi+1) \leq 1/(1+P_2) \leq 1$, the unique Nash strategies are as follows:

		Strategy	Probability
Auditor	Type 1 with θ_1	"no testing"	$1-[1/(1-\phi)][2/(1+P_2)-(1+\phi)]$
		"100% testing"	$[1/(1-\phi)][2/(1+P_2)-(1+\phi)]$
	Type 1 with θ_2	"no testing"	0
		"100% testing"	1
	Type 2	"no testing"	0
		"100% testing"	1
Auditee	Type 1	"no fraud"	1
		"material fraud"	0
	Type 2	"no fraud"	$1-S_1/[(1-\phi)M]$
		"material fraud"	$S_1/[(1-\phi)M]$

The equilibrium strategies also satisfy

$$[1-(1+P_1)\{\phi X^*_{1\theta_1}+(1-\phi)X^*_{1\theta_2}+X^*_2\}/2] < 0 \tag{1}$$

where $X^*_{1\theta_1}, X^*_{1\theta_2}$, and X^*_2 represent the equilibrium probabilities of "100% testing" by type 1 auditor with θ_1 and θ_2 and type 2 auditor. The condition represented by Inequality (1) assures that the expected payoff from "material fraud" for the type 1 auditee is always less than 0.

Proof: The equilibrium strategies of the four players must meet the following conditions.

$[S_1 - M\{\phi Y^*_1+(1-\phi)Y^*_2\}][X_{1\theta_1}-X^*_{1\theta_1}] \geq 0$ for all $0 \leq X_{1\theta_1} \leq 1$

$[S_1 - M\{(1-\phi)Y^*_1+\phi Y^*_2\}][X_{1\theta_2}-X^*_{1\theta_2}] \geq 0$ for all $0 \leq X_{1\theta_2} \leq 1$

$[S_2 - M(Y^*_1+Y^*_2)/2][X_2-X^*_2] \geq 0$ for all $0 \leq X_2 \leq 1$

$[1-(1+P_1)\{\phi X^*_{1\theta_1}+(1-\phi)X^*_{1\theta_2}+X^*_2\}/2][Y^*_1-Y_1] \geq 0$ for all $0 \leq Y_1 \leq 1$

$[1-(1+P_2)\{(1-\phi)X^*_{1\theta_1}+\phi X^*_{1\theta_2}+X^*_2\}/2][Y^*_2-Y_2] \geq 0$ for all $0 \leq Y_2 \leq 1$

$\hat{Y}_1 = \phi Y^*_1+(1-\phi)Y^*_2$, $\hat{Y}_2 = (1-\phi)Y^*_1+\phi Y^*_2$, and $\hat{Y}_3 = 0.5(Y^*_1+Y^*_2)$

$$\hat{X}_1 = 0.5\{\phi X^*_{1\theta_1}+(1-\phi)X^*_{1\theta_2}+X^*_2\}$$

$$\hat{X}_2 = 0.5\{(1-\phi)X^*_{1\theta_1} + \phi X^*_{1\theta_2} + X^*_2\}.$$

Using the result of Proposition 1 of Choi and Saigal (1995a),

$\hat{Y}_1 = \phi Y^*_1 + (1-\phi)\hat{Y}_2 = S_1/M$. Therefore, $[S_1 - M\{\phi Y^*_1 + (1-\phi)Y^*_2\}] = 0$. Then, $[S_1 - M\{(1-\phi)Y^*_1 + \phi Y^*_2\}] < 0$ since $Y^*_2 > Y^*_1$. $Y^*_2 = Y^*_1$ or $Y^*_2 < Y^*_1$ cannot happen by a contradiction. $[S_2 - M(Y^*_1 + Y^*_2)/2] < 0$ since $Y^*_2 > Y^*_1$. Thus, $X^*_2 = 1$ and $X^*_{1\theta_1}$ is arbitrary. $X^*_{1\theta_2} = 1, \hat{X}_2 = 0.5\{(1-\phi)X^*_{1\theta_1} + \phi X^*_{1\theta_2} + X^*_2\} = 1/(1+P_2)$. Therefore, $[1-(1+P_2)\{(1-\phi)X^*_{1\theta_1} + \phi X^*_{1\theta_2} + X^*_2\}/2] = 0$. Because $[1-(1+P_1)\{\phi X^*_{1\theta_1} + (1-\phi)X^*_{1\theta_2} + X^*_2\}/2] < 0$ based upon Inequality (1). Thus, $Y^*_1 = 0$ and Y^*_2 is arbitrary. Since $[1-(1+P_2)\{(1-\phi)X^*_{1\theta_1} + \phi X^*_{1\theta_2} + X^*_2\}/2] = 0$ and $x^*_{1\theta_2} = 1$, $X^*_2 = 1, X^*_{1\theta_1} = 1/(1-\phi)[2/(1+P_2) - (1+\phi)]$. Since $\hat{Y}_2 = \phi Y^*_1 + (1-\phi)Y^*_2 = S_1/M$ and $Y^*_1 = 0, Y^*_2 = S_1/[(1-\phi)M]$. Since $0 < S_1/M \leq (1-\phi)$, and $0.5(\phi+1) \leq 1/(1+P_2) \leq 1, 0 \leq X^*_{1\theta_1} \leq 1$ and $0 \leq Y^*_2 \leq 1$.

Equilibrium 2. If $S_1/M > S_2/M, 0 < S_1/M \leq \phi$, and $0.5 \leq 1/(1+P_2) \leq 0.5(\phi+1)$, the unique Nash strategies are as follows:

		Strategy	Probability
Auditor	Type 1 with θ_1	"no testing"	1
		"100% testing"	0
	Type 1 with θ_2	"no testing"	$1 - 1/\phi[(1-P_2)/(1+P_2)]$
		"100% testing"	$1/\phi[(1-P_2)/(1+P_2)]$
	Type 2	"no testing"	0
		"100% testing"	1
Auditee	Type 1	"no fraud"	1
		"material fraud"	0
	Type 2	"no fraud"	$1 - S_1/(\phi M)$
		"material fraud"	$S_1/(\phi M)$

The equilibrium strategies also satisfy the property by Inequality (1).

$$(\phi - 0.5)(Y^*_2 - Y^*_1)M < (S_1 - S_2) \tag{2}$$

and Y^*_1 and Y^*_2 represent the equilibrium probabilities of "material fraud" by type 1 and

2 auditee respectively. The condition represented by Inequality (2) assures that the type 1 auditor never performs testing to a greater extent than does the type 2 auditor.

Proof: The equilibrium strategies of the four players must meet the following conditions.

$[S_1 - M\{\phi Y_1^* + (1-\phi)Y_2^*\}][X_{1\theta_1} - X_{1\theta_1}^*] \geq 0$ for all $0 \leq X_{1\theta_1} \leq 1$

$[S_1 - M\{(1-\phi)Y_1^* + \phi Y_2^*\}][X_{1\theta_2} - X_{1\theta_2}^*] \geq 0$ for all $0 \leq X_{1\theta_2} \leq 1$

$[S_2 - M(Y_1^* + Y_2^*)/2][X_2 - X_2^*] \geq 0$ for all $0 \leq X_2 \leq 1$

$[1 - (1+P_1)\{\phi X_{1\theta_1}^* + (1-\phi)X_{1\theta_2}^* + X_2^*\}/2][Y_1^* - Y_1] \geq 0$ for all $0 \leq Y_1 \leq 1$

$[1 - (1+P_2)\{(1-\phi)X_{1\theta_1}^* + \phi X_{1\theta_2}^* + X_2^*\}/2][Y_2^* - Y_2] \geq 0$ for all $0 \leq Y_2 \leq 1$

$\hat{Y}_1 = \phi Y_1^* + (1-\phi)Y_2^*$, $\hat{Y}_2 = (1-\phi)Y_1^* + \phi Y_2^*$, and $\hat{Y}_3 = 0.5(Y_1^* + Y_2^*)$

$$\hat{X}_1 = 0.5\{\phi X_{1\theta_1}^* + (1-\phi)X_{1\theta_2}^* + X_2^*\}$$

$$\hat{X}_2 = 0.5\{(1-\phi)X_{1\theta_1}^* + \phi X_{1\theta_2}^* + X_2^*\}.$$

Using the result of Proposition 1, Choi and Saigal (1955a),

$\hat{Y}_2 = (1-\phi)Y_1^* + \phi Y_2^* = S_1/M$. Therefore, $[S_1 - M\{(1-\phi)Y_1^* + \phi Y_2^*\}] = 0$. Then, $[S_1 - M\{\phi Y_1^* + (1-\phi)Y_2^*\}] > 0$ since $Y_2^* > Y_1^*$. $Y_2^* = Y_1^*$ or $Y_2^* < Y_1^*$ cannot happen by a contradiction. $[S_2 - M(Y_1^* + Y_2^*)/2] < 0$ based upon $Y_2^* > Y_1^*$ and Inequality (2).[13] Thus, $X_2^* = 1$ and $X_{1\theta_1}^* = 0$. $X_{1\theta_2}^*$ is arbitrary. $\hat{X}_2 = 0.5\{(1-\phi)X_{1\theta_1}^* + \phi X_{1\theta_2}^* + X_2^*\} = 1/(1+P_2)$. Therefore, $[1-(1+P_2)\{(1-\phi)X_{1\theta_1}^* + \phi X_{1\theta_2}^* + X_2^*\}/2] = 0$. Because $[1-(1+P_1)\{\phi X_{1\theta_1}^* + (1-\phi)X_{1\theta_2}^* + X_2^*\}/2] < 0$ based upon Inequality (1). Thus, $Y_1^* = 0$ and Y_2^* is arbitrary. Since $[1-(1+P_2)\{(1-\phi)X_{1\theta_1}^* + \phi X_{1\theta_2}^* + X_2^*\}/2] = 0$ and $x_{1\theta_1}^* = 0$, $X_2^* = 1, X_{1\theta_2}^* = 1/\phi[2/(1+P_2)-1]$. Since $\hat{Y}_2 = (1-\phi)Y_1^* + \phi Y_2^* = S_1/M$ and $Y_1^* = 0, Y_2^* = S_1/(\phi M)$. Since $0 < S_1/M \leq \phi$, and $0.5 \leq 1/(1+P_2) \leq 0.5(\phi+1), 0 \leq X_{1\theta_2}^* \leq 1$ and $0 \leq Y_2^* \leq 1$.

Equilibrium 3. If $S_1/M > S_2/M, 0 < S_1/M \leq 0.5$, and $1/(1+P_2) \leq 0.5$, the unique Nash strategies are as follows:

[13] If Inequality (2) does not hold, then $[S_2 - M(Y_1^* + Y_2^*)/2] > 0$. $X_2^* = 0$ and $X_{1\theta_1}^* = 0$. $X_{1\theta_2}^*$ is arbitrary. Since $\{(1-\phi)X_{1\theta_1}^* + \phi X_{1\theta_2}^* + X_2^*\}/2 = 1/(1+P_2), X_{1\theta_2}^* = 1/\phi[2/(1+P_2)]$. $Y_1^* = 0$ and $Y_2^* = S_1/(\phi M)$. Therefore, in Equilibrium 2, being able to better recognize "red flags" still is a substitute for actual testing.

		Strategy	Probability
Auditor	Type 1 with θ_1	"no testing"	1
		"100% testing"	0
	Type 1 with θ_2	"no testing"	1
		"100% testing"	0
	Type 2	"no testing"	$(1-P_2)/(1+P_2)$
		"100% testing"	$2/(1+P_2)$
Auditee	Type 1	"no fraud"	1
		"material fraud"	0
	Type 2	"no fraud"	$1-2S_2/M$
		"material fraud"	$2S_2/M$

The equilibrium strategies also satisfy the conditions represented by Inequalities (1) and (2).

Proof: The equilibrium strategies of the four players must meet the following conditions.

$[S_1 - M\{\phi Y_1^* + (1-\phi)Y_2^*\}][X_{1\theta_1} - X_{1\theta_1}^*] \geq 0$ for all $0 \leq X_{1\theta_1} \leq 1$

$[S_1 - M\{(1-\phi)Y_1^* + \phi Y_2^*\}][X_{1\theta_2} - X_{1\theta_2}^*] \geq 0$ for all $0 \leq X_{1\theta_2} \leq 1$

$[S_2 - M(Y_1^* + Y_2^*)/2][X_2 - X_2^*] \geq 0$ for all $0 \leq X_2 \leq 1$

$[1 - (1+P_1)\{\phi X_{1\theta_1}^* + (1-\phi)X_{1\theta_2}^* + X_2^*\}/2][Y_1^* - Y_1] \geq 0$ for all $0 \leq Y_1 \leq 1$

$[1 - (1+P_2)\{(1-\phi)X_{1\theta_1}^* + \phi X_{1\theta_2}^* + X_2^*\}/2][Y_2^* - Y_2] \geq 0$ for all $0 \leq Y_2 \leq 1$

$\hat{Y}_1 = \phi Y_1^* + (1-\phi)Y_2^*, \hat{Y}_2 = (1-\phi)Y_1^* + \phi Y_2^*$, and $\hat{Y}_3 = 0.5(Y_1^* + Y_2^*)$

$$\hat{X}_1 = 0.5\{\phi X_{1\theta_1}^* + (1-\phi)X_{1\theta_2}^* + X_2^*\}$$

$$\hat{X}_2 = 0.5\{(1-\phi)X_{1\theta_1}^* + \phi X_{1\theta_2}^* + X_2^*\}.$$

Using the result of Proposition 1, Choi and Saigal (1995a),

$\hat{Y}_3 = (Y_1^* + Y_2^*)/2 = S_2/M$. Therefore, $[S_2 - M(Y_1^* + Y_2^*)/2] = 0$. Then, $[S_1 - M\{\phi Y_1^* + (1-\phi)Y_2^*\} > 0$ since $\phi > 0.5$ and $Y_2^* > Y_1^*$. $Y_2^* = Y_1^*$ or $Y_2^* < Y_1^*$ cannot happen by a contradiction. $[S_1 - M\{(1-\phi)Y_1^* + \phi Y_2^*\}] > 0$ because of Inequality (2). Thus, X_2^* is arbitrary and $X_{1\theta_1}^* = 0$. $X_{1\theta_2}^* = 0, \hat{X}_2 = 0.5\{(1-\phi)X_{1\theta_1}^* + \phi X_{1\theta_2}^* + X_2^*\} = 1/(1+P_2)$. Therefore, $[1-(1+P_2)\{(1-\phi)X_{1\theta_1}^* + \phi X_{1\theta_2}^* + X_2^*\}/2] = 0$. Because $[1-(1+P_1)\{\phi X_{1\theta_1}^* + (1-$

$\phi)X^*_{1\theta_2} + X^*_2\}/2] < 0$ based upon Inequality (1).[14] Thus, $Y^*_1 = 0$ and Y^*_2 is arbitrary. Since $[1-(1+P_2)\{(1-\phi)X^*_{1\theta_1} + \phi X^*_{1\theta_2} + X^*_2\}/2] = 0$ and $X^*_{1\theta_1} = 0$ and $X^*_{1\theta_2} = 0, X^*_2 = 2/(1+P_2)$. Since $\hat{Y}_3 = (Y^*_1 + Y^*_2)/2 = S_2/M$ and $Y^*_1 = 0, Y^*_2 = 2S_2/M$. Since $0 < S_2/M \leq 0.5$, and $1/(1+P_2) \leq 0.5, 0 \leq X^*_2 \leq 1$ and $0 \leq Y^*_2 \leq 1$.

[14]When Inequality (1) holds, $Y^*_1 = 0, Y^*_2 = S_2/M$. As long as $0 < S_2/M \leq 0.5, 0 \leq Y^*_2 \leq 1$. Therefore, Inequality (1) is equivalent to $0 < S_2/M \leq 0.5$. That is, Inequality (1) is equivalent to assuming that testing is not costly. If Inequality (1) does not hold, $Y^*_1 = 2S_2/M - 1, Y^*_2 = 1$. As long as $0.5 \leq S_2/M \leq 1, 0 \leq Y^*_1 \leq 1$. Thus, assuming that Inequality (1) does not hold is equivalent to assuming that testing is costly.