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AN EMPIRICAL EXAMINATION AND ANALYSIS OF THE DIVERSIFICATION REQUIREMENTS OF THE COMMON STOCK INVESTOR

by
Edward Harrington Jennings

A dissertation submitted in partial fulfillment
of the requirements for the degree of
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Doctoral Committee:

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ABSTRACT

AN EMPIRICAL EXAMINATION AND ANALYSIS OF THE DIVERSIFICATION REQUIREMENTS OF THE COMMON STOCK INVESTOR

by

Edward Harrington Jennings

Chairman: Douglas A. Hayes

The objective of this work is to develop empirically based generalizations regarding the portfolio size requirements, in terms of numbers of securities, of the long-term common stock investor. The analysis employes three important propositions which include: (1) that ex ante portfolio size decisions are closely related to the probabilities associated with achieving various target rates of return; (2) that ex post portfolio performance judgments are related to the actual average market return; and (3) that diversification is not a "free good" (i.e., gain opportunities foregone are considered to be a cost of risk protection).

The probabilities of achieving the various target rates of return were calculated from the monetary performance of all common stocks listed on the NYSE at any time during the period 1955-65, and simple random sampling was used as the technique of security selection. No assumptions were made regarding returns necessitating the use of the Cornish and Fisher series in order to obtain the required probabilities.

The results of the study suggest that a portfolio which contained approximately 12 securities would have been optimum for most investors with an objective of long-term growth. Several aspects of the analysis support this conclusion.

In the first place, a 12 security portfolio was found to produce an equal "trade-off" between risk exposure and gain opportunities as indicated by the fact that the addition of the twelfth security affected a reduction in the probability of a loss which was approximately equal to the corresponding reduction in the probability of a gain.

Secondly, variations in the definitions of gains and losses did not alter the conclusion implying that 12 securities would have been optimum for a wide range of risk-aversion schedules. Even when the objective was to minimize risk, it was found that a portfolio size of 12 was appropriate since at that size the probability of loss was negligible and changed little with larger portfolios. However, if the objective was to maximize gain opportunities, a portfolio size of 12 would have been excessive, but in this case some diversification was needed since the probability of an exceptional gain was found to increase with portfolio additions for "small" portfolio sizes.

Third, the analysis indicated that the length of the investor's holding period was important only when contemplating time horizons of seven years or less. Thus, it was concluded

that a portfolio size of 12 was applicable provided the holding period was at least eight years.

Finally, it was found that variations in the method of security selection did not involve significantly different relationships. In particular, inter-industry diversification and dollar averaging were associated with approximately the same portfolio size requirements as was the case for simple random selection. However, the sampling procedures used to represent inter-industry diversification made no allowance for correlations between industry categories. Thus, a conclusion that the technique is without value was not warranted -- only that a random selection of industries is not likely to benefit the investor.

When consideration was given to the investment objective of principal protection, the data indicated that a portfolio size of approximately 12 would also have been optimum. In the case of the current income objective, it was apparent that approximately 40 securities were necessary to assure real long-term income stability. To achieve income stability in the short-run, prohibitively large portfolios would have been required. It was also suggested that the probabilities associated with long-term dividend increases were not so small as to prohibit a portfolio size of 12 when the investment objectives of long-term growth, principal protection, and current income were combined.

ACKNOWLEDGEMENTS

Encouragement, guidance, and support for this study came from many sources. Foremost of these sources has been Professor D.A. Hayes, the Chairman of my dissertation committee. His ability to focus upon the most significant aspects of the many problems that arise in this type of work was invaluable. In addition, the dissertation has been an important learning experience instead of merely another Ph.D. requirement as a direct result of the penetrating and thought-provoking suggestions offered by Professor Hayes. His time and efforts are greatly appreciated.

A special note of thanks is also extended to other members of my committee. Professor D.A. Jones was extremely helpful with regard to the mathematical aspects of the study. Professors D.J. Brophy, D.F. Reinmuth, and W.J. Wrobleski also offered suggestions and provided encouragement which were helpful in the development and completion of the study.

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INTRODUCTION

In the parlance of finance, risk and uncertainty are defined with respect to the degree of knowledge about future events which, if occur, will cause the initiator of an activity to absorb an economic or utilitarian loss. It is said that one is exposed to risk when all future alternatives are known and the degree of knowledge is such that objective probabilities can be assigned to each possible event. Uncertainty becomes important when future alternatives are known but objective probabilities cannot be assigned without an increase in one's knowledge of the future.[1] Consequently, a given course of action is rarely totally void of risk and uncertainty. Risk exists by definition, and the elimination of uncertainty will often require either clairvoyance or a marginal cost in excess of the marginal revenue of the proposed action.

As a result of this condition, man has devised an infinite variety of behavior patterns designed not to eliminate risk or uncertainty, but instead to reduce the consequences

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[1] H. Igor Ansoff, Corporate Strategy: An Analytic Approach to Business Policy for Growth and Expansion (New York: McGraw-Hill Co., 1965), pp. 118-121.

of the occurance of an undesirable event. For example, one may carry an umbrella on a cloudy day; most of us carry auto insurance; the wheat farmer buys wheat futures; and the manufacturer makes less than a full commitment to a proposed innovation. However, while each technique may effectively accomplish the objective of reducing the impact of an unfavorable event, their use also reduces the impact of a favorable event. Therefore, loss protection is not a free good, and the extent to which such techniques are used will be a function of the trade-off between risk aversion and the opportunity for gain which is foregone.

In the area of investment in common stocks one such risk protection device that is widely recommended is diversification which may be defined as simply constructing a common stock portfolio composed of a group of equity instruments. The underlying assumption is that securities can be found where their returns exhibit a "poor" correlation. Otherwise, diversification could not protect the investor from the risk and uncertainty inherent in the purchase of common stocks.[1] However, while it is widely held that diversification is analogous to effective portfolio management, the definition

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[1]Harry M. Markowitz, Portfolio Selection: Efficient Diversification of Investments (New York: John Wiley & Son, 1959), p. 5.

of the word group, or the number of securities required, is at best ambiguous. For example, Sauvain suggests that 30-40 randomly selected stocks will provide adequate protection;[1] Hayes recommends 5-15 with the limit based on the number of securities which can be followed;[2] Eiteman, Dice, and Eiteman consider 30 common stocks to be excessive and 10 adequate for the small investor.[3]

Thus it seems clear that the extent to which one should diversify his common stock holdings is as yet undefined for fairly obvious reasons. In the first place, man is basically a risk averter but some are more capable of absorbing loss than others, either for economic or psychological reasons as was originally recognized by Bernoulli over 200 years ago.[4] Yet, to date, no definitive evidence exists regarding the protection provided by adding securities to one's portfolio. Secondly, security analysts' have no objective method of

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[1]Harry Sauvain, Investment Management (3rd ed. revised, Englewood Cliffs, N.J.: Prentice-Hall, 1967), p. 402.

[2]Douglas A. Hayes, Investments: Analysis and Management (2nd ed. New York: The Macmillan Co., 1966), pp. 446-8.

[3]Wilford J. Eiteman, Charles A. Dice, and David K. Eiteman, The Stock Market (4th ed., New York: McGraw-Hill Co., 1966), p. 531.

[4]Daniel Bernoulli, "Exposition of a New Theory on The Measurement of Risk," The Theory of Business Finance: A Book of Readings, ed. Stephen H. Archer and Charles A. D'Ambrosio (New York: Macmillan Co., 1967), pp. 22-35.

determining the trade-off between diversification and its opposite, concentration. Third, it is simply not possible to prepare a definition of the word loss which is universally applicable because economic and personal objectives vary from investor to investor, as does the buy-sell holding period, and the general ability to absorb a certain type of loss. Fourth, investment analysis remains an "art" such that the amount of uncertainty contained in a particular investment decision varies widely from individual to individual and security to security. Finally, there exists many variations to the basic theory of diversification where common stock selection may be random, inter-industry, intra-industry, by date of purchase, by various analytical techniques, and by geographic location.

As a result, one must conclude that the general theory of diversification is well formulated but that its application is heuristic at best due to a general lack of objective risk-return criteria, based upon empirical research, which an individual investor can use to determine the number of securities which should be included in his common stock portfolio.

In addition, several specific areas within the investment's discipline require knowledge of the number of securities which should be included within a given portfolio.

The Markowitz approach to portfolio selection requires an initial decision regarding the number of securities to be analyzed,[1] but of greater importance is the fact that the investment community is as yet unable to evaluate the potential contribution of his technique. That is, it is possible that the underlying probabilities of gain or loss from a given portfolio are such that the marginal cost of application of the Markowitz model is greater than its expected marginal return. Thus, if adequate diversification can be achieved with, say, five securities, it would seem that the cost of determining millions of covariances would be prohibitive.

It is also significant to recognize that one of the inherent limitations upon the portfolio manager is his inability to evaluate an infinite number of securities, but at this time one cannot readily determine the economic consequences of limiting the number so analyzed. In a similar vein, it is alleged that one of the primary functions of the mutual fund industry is to provide adequate diversification without the associated analytical problems. Again there are no objective measures of the value of this function. Finally, it is generally accepted that the risks of common stock

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[1]Harry Markowitz, "Portfolio Selection," The Journal of Finance, Vol. VII (March, 1952), 77-91.

investment are related to the investor's time horizon yet the specifics of the relationship have not been clearly defined.

As a result of these needs, this study is designed to create meaningful generalizations with regard to the number of common stock issues which should be included in a portfolio composed entirely of common stocks. The emphasis is placed upon the needs of the investor who follows the tenants of the common stock theory.

CHAPTER I

INVESTMENT IN COMMON STOCKS: THEORY AND HISTORICAL PERFORMANCE

Introductory Definitions

Investment

As all elementary students of economics quickly learn, one invests only when the activity results in an ex post addition to the community's stock of capital regardless of the ex ante intention. The farmer invests when he builds a new barn; General Motors invests when it constructs new dies; and the government invests when it builds a new highway. On the other hand, the individual purchasing shares in a Savings and Loan Association does not invest, nor does the corporate financial officer who buys treasury bills--they are merely parties to a financial transaction which results in a transfer of wealth but is devoid of capital creation.

Consequently, it is technically incorrect to refer to the purchase of the existing equity shares of a corporate enterprise as investment. Such a transaction is merely financial with no direct effect upon society's stock of capital.

However, while this subtle distinction is esthetically pleasing to those so inclined, the financial community and society in general have preferred to apply the term to the purchase of any income producing quantum without regard to whether or not the transaction actually increased the stock of capital or merely transferred some portion of it. It is this convention which is adopted here.

One should further recognize that several authors have seen fit to distinguish between common stock investment and speculation or gambling[1] on the basis of any one of the following characteristics: (1) the amount of risk and uncertainty to which the principles of a transaction are exposed; (2) the expected time period which will elapse between purchase and sale; (3) whether or not the expected return is to be derived from "intrinsic value" or merely a deviation from "intrinsic value;" (4) the particular financial needs and objectives of the parties to the transaction. Again, the purpose of the study will not be well served by insistence upon any or all of these modifications despite their obvious importance to particular situations. Thus, any purchase of common stock is considered an investment regardless of intent or status of the buyer.

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[1]See, e.g., Lawrence Chamberlain and William Wren Hay, Investment and Speculation: Studies of Modern Movements and Basic Principles (New York: Henry Holt and Company, 1931), pp. 8-11.

Income

It is also necessary to define the income an investment in common stock is designed to produce. That is, income could be measured in terms of personal utility, social utility, some monetary return, or institutional prestige to name a few. Furthermore, upon selection of the "proper" unit, it becomes necessary to state quite explicitly the minimum number of such units which represent accomplishment of the investor's objectives. Several alternatives are available where the selection of a particular quantity is dependent upon the specific objectives, the investment alternatives available, the investor's aversion for or love of risk and uncertainty, and the time period under consideration.

As before, an acceptable resolution of these factors lies well beyond the scope of the study thereby necessitating the employment of arbitrary definitions. Regarding the unit of measure, it will be assumed that investors purchase common stock in order to receive a monetary return knowing full well that many other motives prevail. The second aspect of this problem, however, is not so easily disposed of since it is conceptually impossible to generalize monetary objectives in any meaningful sense. Moreover, even if generalizations were possible, it would be necessary to compare returns to those available from other investment opportunities which is

also impossible in a normative sense. Consequently, the study adopts several alternative definitions of objective accomplishment ranging from a constant value of principal and income to achievement of the "average" portfolio return for the investment media in question, namely common stocks. Specific definitions of a gain will be detailed where appropriate.[1]

It becomes necessary, then, to briefly examine the common stock theory in terms of its basic logic, the historic development, the problems of application, and the resulting implications applicable to the investor. Upon completion of this discussion, the theory's predictability will be detailed from the accumulation of empirical work that has occurred to date. The basic purpose of the discussion will be to set the stage for a logical development of appropriate diversification criteria.

The Common Stock Theory

As originally conceived

In the early 1920's, Edgar Smith proposed a theory of common stock investment which, at that time, was revolutionary

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[1]The general problems of measuring investment gain or profitability have recently been outlined quite clearly in: James H. Lorie, "Preliminary Comments: NABAC Study on Measuring Investment Performance of Pension Funds," Financial Analysts Journal, XXIV (March-April, 1968), 139-43.

in every respect. As originally proposed, he suggested that:

> (1) Over a period of years, the principal value of a well diversified holding of the common stocks of representative corporations, in essential industries, tends to increase in accordance with the operation of compound interest.
> (2) Such stock holdings may be relied upon over a term of years to pay an average income return on such increasing values of something more than the average current rate on commercial paper.[1]

Smith based the theory upon the results of his empirical work concerning the returns achieved from equity investment, but nevertheless the underlying logic of the hypothesis is difficult to fault under certain minimum conditions. That is, given a portfolio which is statistically representative of the corporate sector, the investor will receive some current income with certainty provided the corporate population operates at a profit, and follows a policy of distributing some portion of those profits to the stockholders. In addition, profits will grow at some positive rate so long as the overall economy grows, and the corporate sector maintains its proportionate share of national income.[2]

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[1]Edgar Lawrence Smith, Common Stocks as Long Term Investments (New York: The Macmillan Company, 1924), p. 79.

[2]These conditions were not explicitly stated by Smith until early in 1930 when their importance became far more visible than may have been the case earlier in the century. "The Cult of The Common Stock - Now," Commerce and Finance, XIX (February 19, 1930), 396.

Such conditions, while far from certain, do not for practical purposes cast doubt upon the above statements. In the first place, while predictions based on history are often dangerous, it is instructive to note that these conditions have held throughout the history of the industrial United States despite wars, depressions, changing political philosophies, increasing taxes, and so on. But of greater importance is the rather simple fact that if these conditions do not hold in the future, then it can be confidently predicted that most private investments will be unprofitable. As a consequence, to conclude that the common stock theory is incorrect is to imply that one should not invest at all.[1]

Application of the theory

Applying this basic hypothesis, the casual observer might conclude that common stocks are riskless assets with equity profits open to all who will bother to phone their broker. Interestingly enough this may have been the original interpretation as evidenced by the rather unusual stock market activity which followed publication of Smith's book. While the late 1920's cannot be blamed on Smith, the period does present interesting examples of the problems to be anticipated in application of the theory of common stocks to

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[1]Laurence H. Sloan, Everyman and His Common Stocks: A Study of Long Term Investment Policy (New York: McGraw-Hill Book Company, 1931), p. 305.

individual portfolios.

Initially it is recognized that the theory is confined to long-term commitments with long-term defined such that a sale will be made only when new information indicates a change in the initial analysis, a permanent industry or economic change becomes apparent, or when the investor realizes that a mistake was made in his initial analysis.[1] In essence, the investor must remain committed regardless of temporary or cyclical fluctuations which may occur.[2] The implication is that the common stock investor who might be required to liquidate his portfolio for reasons other than those mentioned, as well as the investor who is psychologically unequipped to accept "paper losses" has not chosen a riskless investment media.

This leads to a second problem which by most standards is far more critical than the first. Here it is recognized that a given corporation may pay lucrative dividends and have a favorable growth pattern, yet purchase of the common stock would be unwise due to the phenomenon of price.[3] Very simply, the arithmetic of monetary returns is such that a

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[1] Ibid., pp. 299-300.

[2] Smith, Common Stocks as Long Term Investments, pp. 12-13.

[3] Chelcie C. Bosland, The Common Stock Theory of Investment (New York: The Roland Press Company, 1937), pp. 9-10.

ceteris paribus increase in a security's price reduces the relative return to be realized from that point forward. The result is that the firm's internal rate of return is generally different from that received by the stockholders. In effect, the relationship between the price per share and the book value per share must be the same on the purchase and sale dates in order for the profits of the corporation to be transferred directly to the owners.

In a similar vein, the measure of the owner's return cannot be stated merely in a positive or negative context, but instead must be related to some alternative investment opportunity. In Smith's study, reference is made to the yield on commercial paper as the rough comparison for the current yield on common stocks, and as will be shown later, other empirical studies make similar comparisons to high-grade bonds, changes in the value of the dollar, real estate returns, and real growth of the United States economy. The specific criterion used, however, is not relevant at this point. The relevant factor is that such a comparison must be made.

It is also important to note that the common stock theory will operate only so long as the investment community does not believe it! That is, if the theory is generally accepted, stock prices will be bid up to a point where any yield advantage is eliminated which in turn implies that the

monetary return on common stocks relative to other alternatives will be zero. In essence, general acceptance of the theory will invalidate it due to the phenomenon of price.[1]

These difficulties lead to a fourth which, in reality, is a manifestation of the first three. Here it is recognized that the timing of an investor's purchase and sale will significantly influence the return achieved, and while the importance of this factor declines as the holding period lengthens, it is not eliminated.[2] Witness, for example, the difference between the average performance of all New York Stock Exchange (NYSE) securities where the average annual portfolio return from 1929 to 1962 was 8.1 percent, compared to a return of 15.4 percent from 1932 to 1962, and 17.5 percent from 1949 to 1962.[3] Each return is positive and covers a "long" period of time yet the variation is wide, and while it is true that 1929 and 1932 were extreme cases, the fact remains that a long-run commitment cannot eliminate the timing problem, only reduce it.

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[1]Wilford J. Eiteman, Charles A. Dice, and David K. Eiteman, The Stock Market (4th ed.; New York: The McGraw-Hill Book Company, 1966), p. 370.

[2]Eugene M. Lerner, "Rate of Return on Common Stocks," The Financial Analysts Journal, XVI (September-October, 1960), 48-49.

[3]Lawrence Fisher and James H. Lorie, "Rates of Return on Investments in Common Stocks: The Year-By-Year Record, 1926-65," The Journal of Business of The University of Chicago, XXXX (July, 1968), 6.

Fluctuations in the value of the dollar

As indicated, the theory alleges that the investor will not only achieve a positive return from a purchase of common stocks, but in addition, the claim is made that the return will be sufficient to protect principal and income from the potential problems of purchasing power erosion. Again, the basic logic is irrefutable in two significant respects. As originally conceived, the value of a given common stock was tied to the asset value of the firm exclusive of "water." Thus, a general price increase would have the equivalent effect upon the firm's assets thereby increasing the price of the common stock, and in turn preserving the purchasing power of the investor's principal.[1]

The modern view is that common stock value is a reflection of future earning power, but here again equities should provide an inflation hedge. Revenues are current and as a result will immediately reflect price level changes, yet expenses are historic in the sense that they are incurred prior to receipt of revenues. Therefore, expenses must lag price level changes creating an increase in real profits. In addition, even the "all-variable cost" firm will maintain a constant real level of profits provided the cost-revenue

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[1]Benjamin Graham, "Long-Term Outlook for Equities," The Commercial and Financial Chronicle, CLXXVI (November 27, 1952), 4.

ratio remains constant which should occur in a general price increase. In essence, real values will at least remain constant, and thus preserve the common stock investor's purchasing power.[1]

As with other aspects of the theory, certain problems arise in application of the inflation-hedge portion. In the first place, the current revenue-historic cost relationship is, in large part, a function of the average life of the assets of the firm, and thus the organization with short lived assets will "benefit" less from inflation. Secondly, such important factors as government regulations, elasticity of demand, "sticky" prices, and cost sensitivity operate to make the specific results of inflation widely varied from firm to firm. Third, the problem of price (of the security) also enters the picture in terms of the significance assigned to inflation by the investment community. Fourth, while deflation today appears unlikely, its potential effects must not go unrecognized particularly when comparing equity yields to those available from fixed-income securities. In summary, the specific relationship is not general but depends upon the circumstances surrounding a given security. However, the phenomenon is general in the sense that no real gain can

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[1]Stephan H. Archer, "Common Stocks as an Inflation-Hedge," The Financial Analysts Journal, XVI (September-October, 1960), 41-45.

accrue to all shareholders when it is assumed that the corporate sector will maintain its share of national income during an inflationary period.

Relationship to corporate bonds

In addition to purchasing power protection, the theory holds that the return on common stocks must exceed the return available to the owners of corporate bonds. Here again the logic is basic. Assuming that corporations will utilize debt only so long as their after-tax internal rate of return exceeds their after-tax cost of debt, it follows that the return to equity must exceed interest payments to the bondholders. In essence, a bond is "safe" only so long as the corporation is profitable, but if the corporation is profitable, the equity will be just as "safe" and more profitable.[1]

However, one must again separate the internal affairs of the corporation from the market forces acting upon its securities. For example, high-grade bond yields today exceed 6.5 percent with common stocks priced such that the current yield is approximately 3.0 percent. As a result, equity prices and dividends taken together must increase at an annual rate in excess of 3.5 per cent in order for the common stock

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[1] Smith, Common Stocks as Long Term Investments, pp. 15-16.

investor to recoup the bond opportunity foregone.[1] It is true that such a growth rate may not be difficult to achieve, but the common stock theory claims superiority, and to this extent the current situation may leave little margin for error. Other examples could be used, but the point is that common stock superiority over corporate bonds is a function of both the internal operations of the firm and the prevailing relationships within the capital markets.

Summary

To summarize the discussion to this point, it is clear that one is on sound theoretical ground when concluding that common stocks can be relied upon to produce a monetary return which exceeds the return available on alternative fixed-income securities, and is sufficient to compensate the investor for upward movements in the general price level, thereby allowing the shareholder to participate in the economic growth of the corporate sector. However, application of the theory must be qualified as follows: (1) the commitment is long-run; (2) the investor is psychologically equipped to accept market fluctuations; (3) the purchase and sale prices are "proper;" (4) the investment community has

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[1]Tax considerations may lower this required growth rate slightly provided the capital gain portion of the bond yield is less than the equity capital gains. Such a situation can be generally expected except when coupon rates are "low."

failed to completely discount the predictions of the theory. In addition, it is quite clear that implementation requires "adequate" diversification, but since this particular problem is directly related to the purpose of the study, a detailed description has been postponed to a more appropriate point in the discussion.

Under these conditions, it would seem reasonable to investigate the predictability of the theory in terms of ex post returns on common stock investment. As a result, the discussion to follow is concerned with the empirical work which has been conducted regarding the historic behavior of common stock returns, and is designed to reflect upon the validity of the theory, the importance of the problems encountered in its application, and whether or not a specific quantitative measure of expected value is appropriate.

Empirical Evidence

Initial research

In the investments discipline, like any experimental science, the researcher can never expect to prove (or disprove) an hypothesis regardless of the sophistication of his techniques. He must adopt the more modest goal of providing evidence such that the student can accept or reject an hypothesis based on the weight of evidence rather than a complete inability to discover conceptual errors in the

research.[1] As a consequence, the synopsis to follow is more concerned with relative comparisons between studies than the specifics of each researcher's methodology and the conceptual problems that may have arisen.

With this in mind, the work of Edgar Smith was neither the first[2] nor the most comprehensive as judged by modern standards, and yet in terms of influence it was probably the most significant. His basic approach was to compare the total income and principal appreciation actually obtained from a $10,000 investment in common stocks with the income received from a riskless investment in corporate bonds. Twelve separate tests were made utilizing various techniques of selecting common stocks and various holding periods ranging in length from seventeen to twenty-two years between 1866 and 1922. The basic philosophy was to favor bonds whenever conceptual doubts arose.[3]

The results were revolutionary in that every test except one confirmed, by a substantial margin, the financial wisdom of common stock investment relative to bond investment.

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[1]Eugene F. Fama, "Random Walks in Stock Market Prices," Financial Analysts Journal, XXI (September-October, 1965), 57.

[2]How to Invest When Prices are Rising, cited by Gilbert Harold. "A Reconsideration of the Common Stock Theory," The Journal of Business of The University of Chicago, VII (January, 1934), 43.

[3]The details of Smith's methodology are well summarized in Ibid., pp. 43-45.

Moreover, the single exception gave only a slight advantage to bonds and relied upon a period highly favorable to fixed-income securities where interest rates and prices were falling.[1] In essence, the significant point was not that bonds did better, but that stocks performed as well as they did under the circumstances of the particular test. The explanation was that equity prices will, over the long-run, parallel real economic growth thereby requiring a quite substantial price level deflation in order to offset this inherent advantage.[2]

From these results, Smith anticipated his critics by addressing himself to the question of stability and again found support for his hypothesis. While no specific measures of stability were introduced, he found that approximately 95 percent of the time the market value of the several equity portfolios examined exceeded the initial purchase value within four years. In no case would the investor have been required to wait longer than fifteen years in order to realize a capital gain.[3] Furthermore, current income from the equity portfolios equalled or exceeded bond income in all but approximately 10 percent of the annual increments studied.

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[1]Smith, Common Stocks as Long Term Investments, p. 48.

[2]Ezra Solomon, "Economic Growth and Common Stock Value," The Journal of Business, XXVIII (July, 1955), 217.

[3]Smith, Common Stocks as Long Term Investment, p. 82.

Thus while some instability existed, its effect was minor relative to the aggregate advantages of common stock.

Following Smith's work but preceeding the 1930's, several authoritative studies appeared, and while each differed in detail, their results were in complete conformity with those predicted by the common stock theory.[1] Under the weight of such consistent evidence the theory appeared confirmed, but with the benefit of hindsight their contemporary relevance must be questioned both with regard to their historic distance and from their inability to take cognizance of the early 1930's. Therefore, it is appropriate to examine those studies which closed in the early 1930's, and then proceed to an analysis of the more modern works.

Influence of the 1930's

From the point of view of earlier work, perhaps the most significant study concerned with the 1930's was simply an extension of Smith's work to December, 1932, when the DJIA was a mere twenty points above the depression low of

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[1]The more significant of these studies include: Irving Fisher, "When are Gilt-Edge Bonds Safe?," The Magazine of Wall Street, XXXV (April 25, 1925), 1096-98ff; James Roy Jackson, "Common and Preferred Stocks as Investments," The Journal of Business of The University of Chicago, I (October, 1928), 397-416; Dwight C. Rose, A Scientific Approach to Investment Management (New York: Harper and Brothers Company, 1928); and Kenneth S. VanStrum, Investing in Purchasing Power (New York: Barron's, 1926).

41.[1] In this study Harold updated three of the six 1901-1922 tests made by Smith with the primary objective of testing the theory under the worst possible conditions. Again, the results significantly favored common stocks with respect to income, and although two of the three tests favored bonds with respect to principal, the difference was slight. Furthermore, the equity principal was compared to a default-free bond portfolio which was not even remotely realistic at the end of 1932. However, speculation with regard to this point is unnecessary since Rodkey's study[2] confirmed the theory under similar adverse conditions with respect to both income and principal.[3] Here the period examined was 1908-1932 with actual common stock performance compared to the actual performance of the bonds and preferred stocks issued by the same firms included in the common stock portfolio.

Unfortunately, unlike the pre-depression studies, all the evidence did not favor the common stock theory. In a comprehensive study by Kellogg and Kilbourne, covering the period 1922-1933, it was found that stocks were definitely

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[1]Harold, "A Reconsideration of the Common Stock Theory."

[2]Robert G. Rodkey, Preferred Stocks as Long-Term Investments, Michigan Business Studies, Vol. IV, No. 3 (Ann Arbor: The University of Michigan, 1932).

[3]As did J.L. Amberg, "Testing the Theory of 'Stocks for Long Term Investment' in a Bear Market," The Magazine of Wall Street, XLVII (February 7, 1931), 503-05ff.

inferior to bonds.[1] Moreover, the study differed significantly from previous work in that the "sample" was not a sample at all, but instead included all stocks and bonds listed on the NYSE at both the beginning and ending dates of the study period. However, despite this coverage, the research had several conceptual faults such that its conclusions were highly suspect. In the first place, current income was ignored. Secondly, the comprehensiveness was questionable in view of the fact that those securities which failed to survive the entire period were excluded. Finally, while eleven years is a "long time," it had been demonstrated earlier by Smith that on rare occasions one may require as much as fifteen years to realize the benefits of the theory.[2] As a result, the study, at best, lends credence to the holding period and timing qualifications but does not destroy the general theory.

In a similar vein, it is equally important to examine the theory under adverse buying conditions--namely a 1929 purchase. Here again, the theory survives quite well although the long-run requirement is underscored since the

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[1]H. Dudley Kellogg and Radcliffe E. Kilbourne, "Bonds Generally More Satisfactory than Stocks as Long-Term Investment," The Annalist, XLII (February 9, 1934), 259-60ff.

[2]See supra., p. 22.

investor would have had to wait until 1946 before his average annual return became positive. Moreover, it was not until 1955 that the common stock return began to exceed a similar bond return, and 1959 marked the beginning of a higher common stock return than that which could have been achieved from an investment in preferred issues.[1]

However, despite the fact that the theory was eventually vindicated, a thirty year holding period is not very reassuring. As a consequence, it is interesting to introduce some portfolio "management" into the analysis. Thus, the investor who followed a dollar-averaging plan, beginning in 1929 using the DJIA as his portfolio, would have achieved an annual return of 13.8 percent by 1955[2] compared to 6.9 percent without "management."[3] It is ture, of course, that this period was "ideal" for a dollar-averaging plan, but nevertheless, the differences are striking.

Contemporary studies

With one major exception, little was done following the early 1930's until the post-war years when several sig-

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[1]John P. Herzog, "Investor Experience In Corporate Securities: A New Technique for Measurement," The Journal of Finance, XIX (March, 1964), 52-53.

[2]Paul L. Howell, "Common Stocks and Pension Fund Investing," Harvard Business Review, XXXVI (November-December, 1958), 94.

[3]Fisher and Lorie, "Rates of Return on Investments in Common Stock: The Year-By-Year Record, 1926-65," 6.

nificant studies emerged.[1] The research in general provided further verification of the theory, but while comparisons between alternative investments continued, the emphasis turned to determination of an expected annual return, the mathematical "beauty" of the yield equations, the performance of mutual funds, and the question of random walk. This being the case, a detailed examination of each of the modern studies is beyond the scope of the study, but as will become apparent later, it is appropriate to inquire after the question of an expected annual return.

Cowles Commission

From the studies available, it is clear that the

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[1]Wilford J. Eiteman and Dean S. Eiteman, Common Stock Values and Yields, 1950-61: A Study of Common Stocks as Long-Term Investments with Particular Emphasis on Investment Yields, Michigan Business Reports, No. 40 (Ann Arbor: The University of Michigan, 1962); Wilford J. Eiteman and Frank P. Smith, Common Stock Values and Yields: A Study of Common Stocks as Long-Term Investments with Particular Emphasis on Investment Yields, Michigan Business Studies, Vol. XI, No. 3 (Ann Arbor: The University of Michigan, 1953); William C. Greenough, A New Approach to Retirement Income (New York: Teachers Insurance and Annuity Association of America, 1951); Donald L. Kemmerer, "For Long-Term Investment: Stocks or Bonds," The Commercial and Financial Chronicle, CLXXIII (February 1, 1951), 1ff; Lerner, "Rate of Return on Common Stocks;" James J. O'Leary, Record of Life Insurance Investments (New York: Life Insurance Association of America, 1957 and 1964); R. Minturn Sedwick, "A New Pension Plan," Harvard Business Review, XXXI (January-February, 1953), 70-82; U.S. Congress, House, A Study of Mutual Funds, by Irwin Friend, et al, H.R. 2274, 87th Cong., 2nd sess., (Washington, D.C.: Government Printing Office, 1962); and Paul F. Wendt and Sui N. Wong, "Investment Performance: Common Stocks Versus Apartment Houses," The Journal of Finance, XX (December, 1965), 633-46.

Cowles Commission Report[1] coupled with the updating task performed by Molodovsky[2] provides the most comprehensive estimate of an expected value. Briefly, the Cowles study constructed an index covering the period 1871-1937 including virtually all stocks listed on the NYSE at any time during the period. Each security was weighted by its total market value with the index adjusted monthly to reflect weighting revisions. The market price selected was the average of the high-low for the month and taxes and commissions were ignored. As is well known, the indexes were capable of being "spliced" to the contemporary Standard & Poor indexes--a fact which enabled Molodovsky to update the study to include the entire period 1871-1959.

Under these conditions, the "index investor" received an average annual return of 7.3 percent for the entire eighty-nine year period assuming that all dividends were reinvested which breaks down to 4.9 percent current income and 2.4 percent principal growth.[3] These data can be compared with the original Cowles report which found an annual return of 6.8 percent through 1937. The striking similarity between

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[1]Alfred Cowles, 3rd and Associates, Common Stock Indexes: 1871-1937 (Bloomington, Indiana: Principia Press, 1938).

[2]Nicholas Molodovsky, "Stock Values and Stock Prices," The Financial Analysts Journal, XVI (May-June, 1960), 9-32ff.

[3]Ibid., p. 81.

the two returns serves to remind the reader of the powerful influence of a long period of time in that the close of 1937 was hardly a "good" sale date, but judged by subsequent market performance, it was an extremely good purchase date. To this extent, it would seem that a return of roughly 7.5 percent can be used as an estimate of the expected value of the common stock theory, particularly when it is recognized that over the entire period earnings, dividends, and common stock prices grew at approximately the same annual rate.[1] In addition, the estimate is conservative in view of the fact that the general price level advanced at an average annual rate of slightly more than one percent[2] during the period--a rate which is probably below today's expectations. However, this influence may be balanced by the lower current yields existing today.

Fisher and Lorie

Nevertheless, any modern examination of the common stock theory would be incomplete without a brief foray into the recently published Fisher and Lorie studies.[3] Again, the

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[1] Ibid.

[2] U.S. Department of Commerce, Bureau of the Census, Long Term Economic Growth: 1860-1965 (Washington, D.C.; Government Printing Office, 1966), pp. 200-03.

[3] L[awrence] Fisher and J[ames] H. Lorie, "Rates of Return on Investments In Common Stocks," The Journal of Business of the University of Chicago, XXXVII (January, 1964), 1-21, and Fisher and Lorie, "Rates of Return on Investments In Common Stock: The Year-by-Year Record, 1926-65,"

findings merely add weight to the evidence already available in terms of varifying the common stock theory, yet their study is significant with respect to answering many of the criticisms which have been directed at previous studies.

In the first place, the data included every common stock listed on the NYSE during the period 1926-1965, and while the Cowles Commission study was equally inclusive to 1937, its "spliced" index is the Standard & Poor's Composite Index. Secondly, Fisher and Lorie used actual monthly closing prices rather than some hypothetical average price. Third, taxes and commissions were included. Fourth, the portfolio plan was to invest an equal amount of funds in each security rather than in amounts proportionate to the total market value. This technique makes their results less representative of the total market, yet at the same time the conclusions derived are more applicable to individual portfolios. Fifth, the calculations were made in a fashion which enables one to compare the differences between a portfolio which reinvests dividends and one that does not. Finally, the results are reported for several periods enabling the student to examine yield differences which result from "proper" or "improper" timing, and in another study, Fisher examines the results achieved when all possible purchase-sale combinations during the 1926-1960 period are included

in the caluclations.[1]

As indicated, the Fisher and Lorie results are in general agreement with those studies published earlier providing additional empirical support for the common stock theory. However, due to the publicity given the findings, some comments are necessary regarding the aggregate results.[2] Excluding taxes and commissions and reinvesting dividends, the study found an average annual return of 9.3 percent[3] for the 1926-1965 period which is well above the 7.3 percent return determined by the Cowles Commission and updated by Molodovsky. The difference, however, is not difficult to determine and serves to emphasize the importance of the price phenomenon.

During the 1926-1965 period, earnings per share increased by approximately 420 percent while common stock prices grew more than 680 percent. In contrast, the Cowles-

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[1]Lawrence Fisher, "Outcomes for 'Random' Investments In Common Stocks Listed on The New York Stock Exchange," The Journal of Business of The University of Chicago, XXXVIII (April, 1965), 149-61.

[2]Leo Barnes, "What Difference Does KNOWLEDGE make to Investors?," Financial Analysts Journal, XXI (September-October, 1965), 60-61.

[3]It is often assumed that the 9.8 percent figure obtained in Fisher's second study, "Outcomes for 'Random' Investments in Common Stocks Listed on The New York Stock Exchange," can be used as an expected value. Clearly this is incorrect since the figure is a median rather than a mean, and comes from a population which includes all possible holding period combinations from one month to thirty-five years.

Molodovsky period showed a parallel movement of these variables. Adjusting the Fisher and Lorie data for the differences, one finds complete agreement between the two--a fact which underscores the danger of assuming a direct relationship between the fortunes of the firm and those of the stockholders.

Turning to the unique aspects of the study, certain items deserve mention. The effect of commissions turns out to be a function of time, as expected, with the relative difference becoming insignificant after a period of approximately three years. Taxes cannot be generalized, but for the "moderate" income investor the reduction is less than 1.0 percent per year which is again in general agreement with expectations due to the tax treatment of capital gains, dividend exclusions, and compounding effects. On the other hand, it is important to note that the tax rates employed were historic rather than current.

Regarding the question of the investor's use of current dividends, the study shows little difference between the returns achieved by the "reinvestor" and those received by the "spender." Furthermore, where differences exist the holding period is relatively short, and no positive or negative pattern is discernible. The explanation lies in the fact that while the "reinvestor" will terminate his investment with more monetary wealth, he will also have a higher quantity of funds invested at any given time. Therefore, the existence of differences will be a function of whether

or not price growth is higher or lower after receipt of the dividend. As has been demonstrated, "long-run" growth rates are relatively constant. Consequently, if the predictions of the common stock theory are applicable only to the "long-run," the return achieved by the investor who reinvests dividends will not be significantly different than the return realized assuming dividends are taken as current income.

To the extent that the Fisher and Lorie results are quite similar to previous studies, it would appear that the equal weighting technique compared to weighting by total market value makes very little difference which is contrary to theoretical opinion.[1] However, the results must be considered inconclusive since the portfolios are not continuously adjusted as the theory requires. Moreover, in another article Fisher found that monthly adjustments to maintain equal dollar representation of each security produced an annual return of 12.6 percent for the period 1926-1960 compared to 9.0 percent without adjustment.[2]

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[1]Henry A. Latané, "Investment Criteria: A Three Asset Portfolio Model," The Review of Economics and Statistics, XLV (November, 1963), 427-30.

[2]Lawrence Fisher, "Some New Stock Market Indexes," The Journal of Business of The University of Chicago, XXXIX, (January, 1966; Part II), 197.

Summary

From the accumulation of evidence, it would appear that the assumptions and logic of the common stock theory are valid, and that the model generated can be utilized as a reliable predictor of reality. In terms of a specific return expectation, the investor can conservatively anticipate approximately 7.5 percent per year. However, the empirical work has also shown that risk and uncertainty remain essential ingredients of equity investment and are related to the length of the investor's time horizon, the strength of the relationship between corporate performance and security performance, the elements of price and timing, the general acceptance of the theory, the psychological importance of income and principal stability, and the personal objectives of the investor.

In addition, the theory is applicable only to the common stock "population," and, as such, its tenets cannot be expected to hold without the use of the concept of diversification. Therefore, the chapter to follow is concerned with the theory of diversification, and the various decision criteria which can be used to determine when the common stock theory becomes applicable to a given portfolio.

CHAPTER II

THE CONCEPT OF DIVERSIFICATION

To this point, the theory of common stock investment has been established, but it has also been recognized that implementation of an investment program designed to take advantage of the theory requires use of the principles of diversification. Thus, the discussion to follow is concerned with the general theory of diversification, the problems of application, and the various techniques which can be used to diversify a given portfolio. The objective of the inquiry will be to establish the scope of the study as well as providing certain basic definitions to be employed in attempting to solve the problem at hand--namely, how many securities are required to achieve "adequate" diversification.

General Theory of Diversification

The problem

Despite the general tenets of the common stock theory, its application to an individual portfolio involves recognition and action upon various conditions which require the investor to be something more than merely a "market buyer." In the first place, investors have different monetary

objectives which include, either singularly or in combination, current income, long-term growth, purchasing power protection, or stable income and principal. Secondly, the acceptability of risk will vary depending upon one's economic and psychological ability to incur a loss. Third, individual issues possess various characteristics of risk, return, and form of income production with the particular combination widely varied between securities. Fourth, the problem of timing must be dealth with not only in terms of when to buy and sell, but also with regard to what period of time shall elapse between the purchase and sale dates. Finally, there are several different risk categories with the importance of each a function of the investor's objectives, his degree of risk aversion, and the timing strategies utilized.[1]

Diversification defined

As a result of these difficulties, it has been widely recognized that an individual portfolio should consist of more than one investment unit, or to use the time honored phrase, the investor should not "put all his eggs in one basket."[2] He should diversify, which implies that diversification enables the investor to balance the undesirable performance of some elements of his portfolio with the desirable

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[1]For a more complete discussion of these problems see Hayes, Investments: Analysis and Management, Ch. 27.

[2]Sauvain, Investment Management, p. 401.

performance of other elements such that the aggregate return is consistent with his particular monetary objectives. However, this concept does not imply that diversification will eliminate risk or even reduce it--only that the consequences of an event, whether favorable or unfavorable, will be abridged. Furthermore, diversification cannot eliminate or reduce the absolute consequences of a given event--only the proportionate effect.

On the other hand, it should not be assumed that diversification is concerned with risk to the exclusion of the monetary rewards of investment. Such a conclusion would be an over-simplification since any economic transaction is a "trade-off" between risk and return. Thus, the modern concept holds that the objective of diversification is to achieve the maximum return consistent with a given level of risk.[1] As a result, it is recognized that any particular investment media or unit is incapable of satisfying both of these fundamental constraints providing further justification for diversification. The study, however, will be confined to the media of common stocks thereby excluding diversification by "type" of security.

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[1]Ibid., p. 406.

The Markowitz approach

The origin of these concepts is lost to history,[1] but the modern formulation can be credited to Markowitz.[2] Here, the concept was formalized mathematically in recognition of the fact that the power of diversification is a function of the degree to which the behavior patterns of the future returns of individual securities are correlated. That is, if the behavior of the future returns of each potential issue exhibit a perfect positive correlation, diversification becomes an irrational investment strategy since the most efficient "portfolio" contains only one issue. Namely, the issue with the highest expected return. Otherwise, the investor's performance will be less than optimum due to the fact that the return on all other issues must be less than the performance of the issue with the highest expected return regardless of the size of that return. As a consequence, the power or efficiency of diversification is a function of the degree to which future returns are not positively correlated.[3]

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[1]Bruce A. Valentine, "Shakespeare Revisited," Financial Analysts Journal, XXI (May-June, 1965), 91.

[2]Markowitz, "Portfolio Selection;" and Markowitz, Portfolio Selection: Efficient Diversification of Investments.

[3]For a more thorough and contemporary discussion of this concept see: Basil J. Moore, An Introduction to The Theory of Finance: Assetholder Behavior Under Uncertainty (New York: The Free Press, 1968), pp. 42-46.

Based upon this relationship, Markowitz developed an implementation technique which measured risk as the semivariance[1] of the future returns of all combinations, or portfolios, which could be generated from a given population of securities. Using this measure, certain portfolios will be efficient in the sense that for a given level of risk (semivariance) the various combinations of securities with that risk will have different future returns with the efficient portfolio having the highest expected return. Correspondingly, for a given return there will be portfolios with different levels of risk where the efficient portfolio has the lowest risk or semivariance. As a result, any given population will have an efficient set of portfolios with selection of a particular element of the set related to the especial investor's risk aversion function which by definition is individualistic. Therefore, the model has maximized the return within the investor's risk constraints by producing a solution which is unique to each investor with a unique risk aversion schedule.[2]

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[1] If $S_{\bar{r}}$ is the semivariance around the mean return $\bar{r}$ of a portfolio containing n securities with i used as a subscript to denote the i^{th} security then:

$$S_{\bar{r}} = \frac{1}{n} \sum_{i=1}^{n} (r_i - \bar{r})^2 \quad \text{where } r_i < \bar{r}$$

[2] A more complete review of the mathematical model can be found in: Moore, An Introduction to The Theory of Finance: Assetholder Behavior Under Uncertainty, pp. 44-50; Sauvain, Investment Management, pp. 405-07; and an interesting variation where the measure of risk is a lower confidence limit which is defined as K standard deviations from the mean is

In essence, the mathematical solution is consistent with the intuitive solution of extensive diversification when risk is to be avoided, but the avoidance of risk also reduces the investor's ability to achieve a return which is significantly above the market average. Thus when risk becomes less important, the model and intuition advise the investor in the direction of the opposite of diversification, or concentration, in order that high returns will not be diluted by low or mediocre returns. For example, it is conceivable that the model could generate an efficient "portfolio" containing only one security.

In this context, the basic model is difficult to fault as a formal description of the theory of diversification, but utilization by the investment community is a different matter. To begin with, while the variance of annual returns has been used frequently to measure risk,[1] it will be shown below that this approach is conceptually sound only if the investor may

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introduced in William J. Baumol, "An Expected Gain-Confidence Limit Criterion for Portfolio Selection," Management Science, X (October, 1963), 174-82; and William J. Baumol, "Mathematical Analysis of Portfolio Selection: Principles and Application," Financial Analysts Journal, XXII (September-October, 1966), 95-99.

[1]See, e.g., John Lintner, "Security Prices, Risk and Maximal Gains from Diversification," The Journal of Finance, XX (December, 1965), 587-615; William F. Sharpe, "Risk Aversion in the Stock Market," The Journal of Finance, XX (September, 1963), 416-22; and Jack L. Treynor, "How to Rate Management of Investment Funds," Harvard Business Review, XLIII (January-February, 1965), 63-75.

be required to sell at any future point in time which is inconsistent with the basic tenets of the common stock theory. Secondly, the physical problems of model construction are enormous in recognition of the fact that the required covariance terms increase by the number of securities in the original population with the addition of one security to the population. Third, the model makes no allowance for costs which may be incurred with the addition of securities to a given portfolio. Finally, as with the common stock theory, the Markowitz approach is relevant only so long as it is not generally applied by the investment community. That is, given broad acceptance, the securities which possess desirable covariance characteristics will be bid up such that their advantage is quickly eliminated.[1]

Basic principles

As a result, the more traditional principles of diversification remain an important ingredient of portfolio management and include: (1) diversification by investment media in recognition of the fact that no individual security or group of securities can satisfy each of an investor's objectives; (2) diversification by place in order to reduce the unfavorable consequences of local difficulties; (3) di-

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[1]Irwin Friend and Douglas Vickers, "Portfolio Selection and Investment Performance," The Journal of Finance, XX (September, 1965), 395.

versification by industry under the assumption that inter-industry correlations will be less than intra-industry correlations; (4) diversification within a given industry in order to avoid the consequences of the inevitable failures within even the most profitable industries; (5) diversification by time to protect against and profit from unpredictable price movements.

However, for purposes of this study, diversification by type of investment media is excluded from the analysis since the concern is with the common stock portion of the investor's portfolio. In addition, geographic diversification is excluded in view of the national characteristics of the majority of issues included in the data to be used. Moreover, assignment of an issue to a particular area would require fundamental security analysis which is also excluded from the study as is technical analysis. Finally, intra-industry diversification is not examined in view of the fact that such an analysis would also require the use of security analysis.

Implementation Techniques

Despite these exclusions, it is clear that several alternative diversification techniques are available for analysis. As a consequence, the discussion to follow is concerned with the various diversification techniques which result from the basic principles mentioned above. Those

techniques which fall outside the scope of the study will be excluded from the discussion.

Random diversification

Perhaps the most straight-forward method of diversification is simply to include a sufficient number of securities in the portfolio such that the portfolio performance parallels that of the market. The investor employs sampling theory by selecting a random sample of issues from a known population, say the NYSE, under the assumption that a sample of sufficient size will be "representative" of the population. The problems with such an approach, however, are considerable. To begin with, there is the question of how many issues are "sufficient," and of equal importance is the problem of establishing a range of portfolio returns which can be considered as an acceptable approximation of the population. Secondly, the individual investor's objectives are ignored, e.g., the "current income" investor may select from a different population than would the "growth" investor. Third, the approach, by definition, prohibits the achievement of a return which exceeds the "average;" thus ignoring the basic concept of a trade-off between risk and return. In effect, even random selection does not eliminate the cost of adding issues. Finally, the market fluctuates quite substantially in the short-run, and thus the investor who is unable to

tolerate these movements, for whatever reason, will find a "representative" sample to be inappropriate.

Despite these problems, this study employes random diversification in a direct attempt to shed light on the problem of sample size, as well as to establish a basis for determining the value of other methods of diversification. No allowance is made for the possibility that different investor objectives will command selection from different populations. On the other hand, objectives are considered through variations in the methods of calculating a rate of return and the definitions of "loss" utilized. The length of the expected holding period is also considered.

Inter-industry diversification

Many variations to random selection have been suggested and frequently applied. Perhaps the most popular recommendation is to employ inter-industry diversification which has as its objective the avoidance of a portfolio which contains a concentration of firms operating in the same industry or industries which exhibit a high degree of positive correlation. In theory this approach is more acceptable than simple random selection, but the problem of industry dependency is formidable since the required relationships are not only physically difficult to determine, but also require future orientation which may or may not be related to the historical data available. Furthermore, the problems of

investor objectives, quantification of risk and uncertainty, cost of security additions, and short-run fluctuations remain. Nevertheless, the technique is used in the study employing random selection of industries and random selection of individual issues within each industry selected.

Diversification of risk category

In response to certain of these problems, the investor can also diversify by the various risk categories which include long-term secular risk, cyclical risk, interest rate risk, monetary risk, and valuation risk. Here, a firm with high cyclical fluctuations might be balanced against a stable or counter-cyclical firm. Correspondingly, the growth stock might be coupled with the firm which is in the mature stage of its life cycle. As before, the approach is designed to reflect the independency concept, but since application is heavily dependent upon fundamental security analysis, the technique is excluded from the study.

Time diversification

In addition to these techniques, the investment community also recognizes the problem of security prices and attempts to provide a remedy through the use of any one of several time diversification procedures. The basic assumption is that prices will fluctuate about some norm or warranted level, but that the investor is unable to consistently

predict such fluctuations. In dollar-averaging, the objective is to invest a fixed quantity of funds at predetermined intervals such that the average cost per share is less than the average price. Formula plans vary the amount of money invested in a particular type of security as a function of the size and timing of deviations from the market norm.[1]

Unfortunately, these techniques are reliable only under certain conditions. In the case of dollar-averaging, market fluctuations which are either "small" or of limited duration will reduce the predicted advantage over the portfolio constructed at one point in time as will long-term economic growth. Regarding formula plans, all the variations assume that the range of market movements can be predicted, and that some normal price level exists. Both approaches require the investor to rigidly maintain the plan regardless of current information, but by doing so the investor may have a tendency to ignore the dynamics of his investment program. For purposes of the study, time diversification will be confined to an examination of dollar-averaging since formula plans require diversification by type of issue in addition to market predictions. As before, security selection is randomized.

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[1]For a detailed examination of the many variations of time diversification see: Sherman F. Feyler, Income Growth with Security: The Formula-Plan Solution (New York: The Macmillan Company, 1958); and Jerome B. Cohen and Edward D. Zinbarg, Investment Analysis and Portfolio Management (Homewood, Illinois: Richard D. Irwin, Inc., 1967), Ch. 15.

Scope of The Study

Each of these techniques, of course, cannot be considered in isolation just as the addition of a security to one's portfolio cannot be divorced from his aggregate investment policy. For example, time diversification could be combined with selection by industry, and the industries could be chosen with reference to their particular risk characteristics. However, each technique or combination has in common the assumption that the returns of individual issues are not perfectly correlated. The implication is that risk is reduced as securities are added to the portfolio as is the opportunity for achieving a gain which is substantially above the market average.[1] In recognition of this point, the term portfolio size is used to refer to the number of issues within a given portfolio rather than the total monetary quantity invested.

Nevertheless, the risk reducing properties of a "large" portfolio, while necessary, are not sufficient to determine the size requirements of a particular portfolio. Instead, the investor must also consider several independent variables which can be summarized as follows: (1) his unique set of monetary objectives; (2) the expected time duration of the investment and/or the likelihood of a forced sale; (3) the

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[1]Hayes, Investments: Analysis and Management, p. 447.

method or combination of methods of diversification to be used; (4) his ability or inability to reduce uncertainty and partial ignorance through security analysis; (5) his unique risk aversion function.

Loss toleration level

From this listing, the direction of the analysis becomes clear, but first it is necessary to define the term loss. As suggested earlier, it is incorrect to imply that an investment has been profitable simply if it produced a positive monetary return. Instead, the return must be compared to some return which is capable of being achieved within the particular investment media and is sufficient to cover the economic cost of other investment alternatives. In addition, it was noted that this comparative return falls within a wide range not being subject to normative judgments.

As a result, the term loss, or loss toleration level, is applied to any return which is less than a reasonable expectation of the future performance of common stocks in general or is insufficient to meet the investor's objectives which will be defined in terms of his next best investment alternative as specified below. For example, if the average market return is expected to be 7.5 percent, then a portfolio return of, say, 6.0 percent would be considered a loss when compared to the market average. Correspondingly, if the investor's next best alternative is, say, 5.0 percent, then the

same 6.0 percent return could be considered a gain when used in the context of an opportunity cost. Since any specific return lacks theoretical justification, various "loss points" are employed in the analysis.

Portfolio objectives

The variable of portfolio objectives is to be handled in the following manner. For the investor interested in long-term growth, the return is measured by the relative difference between the initial investment and the closing monetary wealth assuming the reinvestment of dividends with the difference compared to various loss toleration levels.

Regarding current income, it is assumed that whatever current yield is available is sufficient to satisfy the income objective since the study is limited to the common stock portion of an individual's portfolio. That is, given that common stocks have been selected as the vehicle to obtain current income, it follows that the absolute size of the yield is irrelevant. As a consequence, the emphasis is placed upon the change in dividend income from period to period, and the risk that such a change will be insufficient to maintain the real and monetary value of the beginning income.

Correspondingly, the objective of principal maintenance is measured by the compound interest return exclusive of

dividends. The risks are analyzed relative to the portfolio requirements of the long-term growth investor using similar loss points.

As noted, no allowance is made for the possibility that different objectives may dictate different populations to draw upon. Further, the fact that objectives are often combined is not considered directly.

The dimension of time

With respect to the investor's time horizon, holding period lengths ranging from one year to eleven years in one year increments are examined.[1] As will be discussed below, the data employed cover the period December 31, 1954 to December 31, 1965, but it should be noted that these dates represent holding period boundaries (buy-sell dates) only in the case of an eleven year holding period. For shorter periods, there could be an extremely large number of buy-sell dates, but in the Fisher convention[2] the potential dates are

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[1]It is true that many of these holding periods will be too short to be relevant to the common stock theory. However, short is difficult to define quantitatively, and since the marginal effort necessary to examine many holding periods instead of one is small, this approach would seem warranted. Furthermore, the data obtained may provide an empirical definition of the short-run or at least a clearer view of the risk of forced sales.

[2]Fisher, "Outcomes for 'Random' Investments in Common Stocks Listed on The New York Stock Exchange," 150.

limited to the last business day of every third month.[1] Thus, for example, given a holding period of five years, a purchase can be made on 12/31/54, 3/31/55, 6/31/55, ... 12/31/60 and sales made on 12/31/59, 3/31/60, 6/31/60, ... 12/31/65 producing a total of 25 different five year holding periods. This approach is used under the assumption that the combination of holding periods is more representative than would be some unique period.[2] Moreover, this approach is not subject to the criticism of the Fisher study that was previously noted since different holding period lengths will not be combined.[3]

Techniques of diversification

Regarding the particular method of diversification, three specific techniques are examined and include: (1) ran-

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[1]Fisher, of course, used the last business day of each month in "Outcomes for 'Random' Investments in Common Stocks Listed on The New York Stock Exchange." For purposes of this study, it is felt that the marginal precision obtained from such detail is insignificant relative to the additional computer time required.

[2]This judgment is not based on the fact that different holding periods will produce different average returns since the average return is to be removed from the data employed. However, the study is concerned with the behavior of the variance of individual returns. As will be discussed there is good reason to believe that, relative to the mean, the variance should be roughly constant from period to period. Nevertheless, an average of several periods of the same length is more appropriate than some unique sample period due to inevitable sampling variations. Thus combining equal length period lengths is preferred.

[3]See supra., p. 31, n. 3.

dom diversification; (2) inter-industry diversification with the selection of the industries and issues within an industry randomized; (3) time diversification which will be represented by dollar-averaging where the purchase increment is six months with selection randomized on the initial purchase date. In each method, an equal probability is assigned to the selection of each individual issue included in the population, and no allowance is made for variations in the market price or shares outstanding. The effect will be the equivalent of investing $1 in each security in a given portfolio. However, no adjustment is made for the passage of time such that upon liquidation, there will be a larger representation of those issues which have achieved a return in excess of the population average.

Risk aversion

With regard to the important problem of the individual investor's risk aversion schedule, even the most casual observer of the literature will quickly recognize that a detailed examination of this subject would take the reader far afield, and much of the material would be only indirectly relevant to the topic at hand. As a result, the chapter to follow will outline the research methodology and decision criteria which are used to form conclusions. Incorporated into the discussion will be an examination of the character-

istics of risk aversion and the techniques available to measure risk which are pertinent to the analysis.

CHAPTER III

THE RESEARCH METHODOLOGY

The theory of common stocks has now been discussed and verified with the empirical evidence currently available. In addition, the concepts of diversification have been reviewed with particular emphasis upon the dimensions of the present analysis. It becomes appropriate, then, to describe the research methodology which is used in order to form conclusions regarding the portfolio size requirements of the long-term common stock investor. As a result, the subsequent discussion takes the form of a conceptual description of the various research techniques enlisted followed by a development of the means with which the concepts are quantified.

An Over-view of The Methodology

An example

For purposes of clarity, it will be instructive to develop the concepts employed around a rather straightforward diversification problem. The problem assumes that the objective is to determine the number of common stock issues which should be included in the common stock portion of the portfolio of an investor who: (1) expects to maintain his

investment for a period of two years, (2) will measure his return by the compound rate of return method assuming annual compounding and reinvestment of dividends, and (3) will confine his method of diversification to simple random selection from a known population of common stocks.

The basic procedure

The initial step of the research is to determine the properties of the frequency distribution of the total monetary wealth derived, over the two-year period, from an investment of $1 in each security contained in the population. For example, if a given security were priced at $10 at the beginning of the period; paid no dividends throughout; and sold for $12.10 at the end of the period, it would occupy the point 1.21 (= 12.10/10) on the frequency distribution which is the equivalent of an average annual return of 10.0 percent compounded annually.[1]

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[1]Using standard compounding techniques this equivalency is derived as follows:

$$r = (V_T/V_0)^{1/T} - 1.0$$

where:

r = the average annual rate of return
V_0 = the value of the initial investment
V_T = the value of the ending investment
T = the number of years in the investment period

Before proceeding, it is necessary to comment on the fact that the total wealth quantity is used instead of the more conventional annual return.[1] It is well known that when one is involved with holding periods which are greater than one year in length, the average of the annual rates of return from each issue is an inappropriate measure of the monetary performance of a given portfolio. For example, if a portfolio contained one share of each of two stocks with the following performance characteristics, the average annual rate of return on the portfolio would appear to have been plus 25.0 percent given a two-year holding period.

	Stock A	Stock B
Purchase Price	$10	$10
Sale Price	$40	$ 2.50
Dividends Received	0	0
Average annual rate of return	100%	-50%
Wealth per dollar invested	4.0	.25

However, if $20 (the initial investment) is compounded at 25.0 percent per year for two years, the portfolio would have had a terminal value of $31.25[2] instead of $42.50

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[1]As implied from the example, this measure is applied with regard to the objective of long-term growth. Consequently, other objectives will dictate different measures of performance which will be described in detail at a later point in order to avoid confusion over the basic methodology.

[2]Which is arrived at as follows:

$$20.0(1.25)^2 = 31.25$$

(40.0 + 2.5) as actually occurred. The error lies in the fact that the different rates of return created unequal dollar representation of each security in the portfolio as time passed even though the initial representation was equal. If the total wealth derived from a one dollar investment in each security is used, the problem evaporates such that the performance of the portfolio is accurately reflected.[1] Thus, the measure of performance in the example portfolio would be 2.125 (= [4.0 + .25]/2) which is the equivalent of an average annual return of approximately 45.7 percent. Since the concern is with all possible combinations of individual issues, the wealth ratio must be used to represent portfolio performance. However, in order to avoid confusion in the discussion to follow, the more conventional annual return is referred to as the measure of portfolio performance, but it should be kept in mind that a given annual return must be converted to an equivalent wealth ratio for purposes of statistical analysis.

Supplied with the frequency distribution of the returns available from common stocks, the investor can quickly determine the risk associated with each potential, randomly

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[1] A more detailed discussion of the problems can be found in: Robert M. Soldofsky and Roger Biderman, "Yield-Risk Measurements of The Performance of Common Stocks," Journal of Financial and Quantitative Analysis, III (March, 1969), 59-74.

selected, portfolio size. Specifically, he can determine the probability that a given portfolio, or sample size, will produce a return which is less than his particular loss toleration level, and select his portfolio size accordingly.

The use of probability

Still, selecting a portfolio size "accordingly" requires considerable comment in terms of: (1) the mathematical determinates of the probabilities required; (2) whether or not probability is an accurate quantification of risk; and (3) how a probability measure can be used to select the appropriate portfolio size. Initially, the question of the usefulness of the measure of probability is considered, reserving for a later point the examination of the first two questions.

The average market return

Under the investment conditions set forth in the example, it can be said that the investor will expect to receive a return on his portfolio which is equal to the arithmetic mean, or average, of the returns available within the common stock population used to select his portfolio.[1] In

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[1] As will be shown, there is more than one "mean" or average return that the investor must examine, but for the moment a unique value is assumed which is the simple average of all returns (wealth ratios) that actually occurred during the holding period.

other words, regardless of the size of the portfolio, the expected value will be the mean of the population. Furthermore, if the investor's objective is to minimize the risk that his actual performance will deviate from the expected value, his only recourse is to construct a portfolio containing all issues in the population. Anything less creates an opportunity for the actual return to be different than the expectation.

This solution, however, is untenable in several respects. In the first place, it is simply impractical for obvious reasons, but of greater importance is the fact that complete assurance that a portfolio will perform, in an absolute sense, as expected is not an achievable objective of common stock investment. That is, regardless of the sophistication of an investor's methods of measuring risk, partial ignorance and uncertainty remain to make possible an average outcome which is significantly different than that which was expected. For example, twenty years ago few investors would have predicted the present level of interest rates, and had one been so bold, it is possible that he would have also predicted a general decline in equity prices which may accompany an interest rate increase. In effect, the population average is subject to uncertainty. Therefore, purchase of the population does not eliminate the potential for unfavorable events.

In addition, it is recognized that complete elimination of risk, to the exclusion of uncertainty, is undesirable from the point of view of the capital markets taken as a whole. If the investment community demanded the elimination of risk, it is reasonable to assume that the rate of return advantage on common stocks would be eliminated to the consternation of many investors, not to mention the potential distortion of resource allocation.

As a result, the portfolio size necessary to provide assurance that the portfolio return will approximate the average market return, can be defined as one in which there is a small probability that the portfolio return will not deviate significantly from the actual average market return. Such a definition must, of course, rely upon an essentially arbitrary assessment of small and significant, and to this problem judgments are based upon the nature of the distribution of returns. To be specific, it would be desirable to unambiguously state that the portfolio size required to achieve the market average should contain, say, 75 securities since at that level the probability is, say, 0.95 that the portfolio return will be equal to a return which is within, say, plus or minus 100 basis points of the market average. Thus, small is 5 percent and significant is 100 basis points. On the other hand, if small is expanded to, say, 10 percent and/or significant to, say, 150 basis points, the "required"

portfolio size may decline quite rapidly to, say, 25 securities resulting in a condition where the portfolio size must be tripled in order to achieve the slight increase in risk protection.

In essence, the nature of the data may require a trade-off to be made between a small probability, a significant deviation from the average market return, and the portfolio size. If so, it will be necessary to examine the changes in the probability of a significant deviation from the market average that result from portfolio additions in order to establish a portfolio size designed for the investor concerned solely with a portfolio return which approximates the average market return.[1]

The problem of concentration

However, even if unambiguous criteria could be established, this approach must be considered incomplete since it implies that the average market return is the only element of the portfolio decision. It is well known that every diversification plan contains the basic assumption that the addition of securities to a portfolio involves not only greater risk protection but also greater investment costs[2] which

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[1]This approach implies that portfolio additions involve a cost which, as will be shown, appears to be a reasonable assumption.

[2]See e.g., John L. Evans and Stephen H. Archer, "Diversification and The Reduction of Dispersion: An Empirical Analysis," The Journal of Finance, XXIII (December, 1968), 762.

generally include: (1) brokerage fees; (2) analytical costs; (3) management costs; and (4) the opportunity for an exceptional portfolio gain which is assumed to decline as securities are added to the portfolio. As has been shown, brokerage fees are insignificant when considering a long-term commitment and thus are ignored in the analysis. Furthermore, management and analytical costs will not be given explicit treatment in view of the random selection techniques employed in each method of diversification to be examined.

While each of these costs may have importance in specific instances, by far the cost with the most general significance is the opportunity for an exceptional gain which is foregone with the addition of securities to a portfolio. As has been previously discussed, this "trade-off" between the risk of loss and the opportunity to achieve an exceptional gain implies greater concentration and less diversification as the consequences of risk become less important to the investor.

It has been further noted, that several portfolio loss points are used each of which is defined within the context of the average market return. Correspondingly, an exceptional portfolio gain is defined as a return which is significantly above the average market return where significant is determined within the context of the nature of the data. Specifically, an exceptional gain is defined within the

feasible set of gains available within the common stock population utilized. If the investor finds that such exceptional returns are "insufficient," his only alternative is to turn to another investment media.

Based on these definitions, probability theory indicates that, in general, the addition of securities to a portfolio will reduce the probability that the portfolio return will be less than some loss toleration level, but at the same time the probability of an exceptional gain will also decline. With respect to investor objectives, the former behavior is desirable and the latter undesirable. As a consequence, the amount of reduction in each becomes the significant factor such that as long as the probability of a loss is declining more rapidly than is the probability of an exceptional gain, diversification remains a desirable strategy. When the changes are approximately equal, the investor is indifferent to further additions to his portfolio. When the change in the probability of an exceptional gain exceeds the change in the probability of a loss, the investor will prefer to avoid adding securities. The added risk protection is achieved at the expense of a larger decline in the opportunity for an exceptional gain. Thus, the appropriate portfolio size will exist where the change in the probability of loss is approximately equal to the change in the probability of an exceptional gain with the addition of the last security.

Investor utility

In view of the above comments, it is possible that several objections could be raised to the proposed methodology, each of which has theoretical validity but in actual fact will not significantly alter the final results. In the first place, it is quite correct to note that a basic assumption underlying the approach appears to be that the investor is a risk-neutral: i.e., the magnitude of the utility of a loss is assumed to equal the magnitude of the utility of an exceptional gain. Such an approach would conflict with the contemporary Bayesian approach to business decisions where probability is only one dimension of the decision-making process with personal utility an equally important ingredient.[1]

The conflict, however, will prevent only completely definitive conclusions from being made. That is, the assumption of neutrality toward risk prevails only when the loss and gain points are equidistant from the average. For example, if a loss point is defined as being less than 80 percent of the mean and an exceptional gain as being greater

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[1]The most comprehensive source in this field as applied to business decisions is: Robert Schlaifer, Probability and Statistics for Business Decisions (New York: The McGraw-Hill Book Company, 1959). For a more condensed review of the subject see: Jack Hirshleifer, "The Bayesian Approach to Statistical Decision, An Exposition," The Journal of Business of The University of Chicago, XXXV (October, 1961), 471-89.

than 120 percent of the mean, only the risk-neutral will consider an approximately equal trade-off of probability changes to be a satisfactory criterion.

If, on the other hand, the loss toleration level is reduced and the exceptional gain point held constant, the equal trade-off of probability changes becomes appropriate for the individual willing to accept greater risk exposure. Thus, the study analyzes the relationship between the changes in the probability of a loss and gain affected by security additions for various loss and exceptional gain definitions. In addition, the absolute probability of loss for various loss toleration levels is to be examined in order to give consideration to the investor with a high degree of risk-aversion.

Probability as a measure of risk

A second objection to the methodology which might be raised lies in the fact that the measure of risk selected is the "probability of loss." At first glance, this framework would appear to conflict with much of the contemporary thinking in the field of finance where risk is generally quantified with some measure of the dispersion of the annual returns about the average return of the individual issue. Such a conclusion, while partially correct, will not detract from the usefulness of the analysis.

In the first place, the dispersion of annual returns is a relevant measure of risk only when the investor may be required to liquidate all, or some portion, of his portfolio within a short-run and unpredictable period of time. As a consequence, the investor who adheres to the tenets of the common stock theory is immuned to this eventuality, and thus will not be concerned with short-run fluctuations in his "paper" return. Furthermore, a significant number of investors face the almost certain prospect of never being required to liquidate even a small portion of their current portfolio. That is, many institutional investors, such as pension funds, are in a position where cash inflows exceed outflows with every assurance that such a condition will exist for a period long enough to be considered infinite.[1] In addition, many individual investors can be classified as "inert investors" who "are disinclined to engage in market trading"[2] implying that this group will rarely, if ever, be motivated to sell regardless of their knowledge of the common stock theory.

In addition to these conceptual difficulties, it should be noted that a measure of dispersion is an acceptable

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[1]See, e.g., Lorie, "NABAC Study on Measuring Investment Performance of Pension Funds," 141.

[2]Eiteman, Dice, and Eiteman, The Stock Market, pp. 523-24.

quantification of risk only when the distribution is approximately normal. Otherwise dispersion may indicate very little about the properties of the distribution. As will be shown below, normality is indeed a poor assumption when dealing with the returns available from common stocks. Furthermore, even if normality could be assumed, the behavior of dispersion becomes significant only when it is related to a particular point on the distribution. As a result, dispersion cannot be considered to be a "complete" measure of risk.

In essence, probability of loss as a measure of risk is merely a manifestation of dispersion without the associated mathematical and conceptual problems. In addition, it is more useful in terms of examining the effects of different holding period lengths, and as such, it is more relevant to the study in view of the objective of applicability to the long-term investor who follows the tenets of the common stock theory.

Summary

In summary, the analysis breaks down into two basic steps. The first step consists of determining the portfolio size required in order to provide the investor with considerable assurance that his portfolio return will approximate the average market return. The second step of the analysis consists of determining a portfolio size where the last security

added produces a decline in the probability of loss which is approximately equal to the decline in the probability of an exceptional gain. The analysis will employ various definitions of a loss and an exceptional gain with consideration given to the absolute level of the probability of loss.

The Relevance of The Common Stock Theory

To this point in the discussion, the implication has been that the investor to which the conclusions of the study are directed is one who follows the basic tenets of the common stock theory. The observable difference between such an individual (or institution) and "other" investors is the length of the investment holding period. Thus, in order for one to achieve that which is predicted by the common stock theory, the investment time horizon must be "long-run." Unfortunately, an unambiguous definition of long-run is not available. Consequently, the study gives explicit treatment to the effects upon the required portfolio size that result from variations in the length of the investor's holding period.

The methodology employed to achieve this end is quite straightforward and consists simply of a comparison of the portfolio size differences created from variations in the investor's time horizon with all other variables held constant. The objective is to determine a minimum holding period where

a lengthening of the time horizon does not significantly alter the portfolio size requirements. In other words, if the common stock theory has empirical validity, there should exist some minimum time period which has sufficient length such that short-run or temporary market conditions become insignificant relative to the long-run forces prevailing in the economy. At this point, the risks and gain opportunities should behave in a fashion that will enable one to conclude that a longer investment life is a relatively insignificant variable.

The Relevance of History

The actual market performance

From the description of the methodology, it becomes clear that the characteristics of the frequency distributions to be developed are critical to the analysis. Therefore, unless the study is to be of historical interest only, it is not sufficient to merely compile data from some representative period, determine the mathematical properties of the distribution of that data, and proceed with the tests as outlined above. Instead, one must be reasonably certain that the data employed are representative of the future.

In particular, the actual average market return can create a wholly misleading comparison for the long-term investor. For example, during the ten-year period January 1,

1956 to January 1, 1966 the average annual return on common stocks listed on the New York Stock Exchange was 12.5 percent[1] which is considerably above the long-term market expectation. As was noted, the arithmetic explanation of such deviations lies largely in the behavior of price-earnings ratios during the specific period. In the example period, price-earnings ratios increased at approximately 3.4 percent per year[2] implying that the investment community was engaged in a re-evaluation of either their predictions of the future earnings of the corporate sector, the risks associated with such earnings, or both. The result of these judgments was a rate of return which substantially exceeded the actual performance of the corporate sector and correspondingly the long-term market expectation or market average.

The significance of this condition is that during the period suggested the average annual rate of return on common stocks would have compared quite favorably to the returns available on alternative investments. That is, if it is assumed, for the moment, that the loss toleration level is

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[1]Assuming annual compounding, reinvestment of dividends, and equal funds invested initially in each security. See: Fisher and Lorie, "Rates of Return on Investments in Common Stock: The Year-By-Year Record, 1926-1965," 6.

[2]Based on the published price-earnings ratios of the Standard & Poor's Composite Index. See: Standard & Poor's Corporation, Standard & Poor's Trade and Securities Statistics: Security Price Index (1966 ed.; Orange, Conn.: Standard & Poor's, 1967), pp. 120-21.

equal to an average annual return of 6.0 percent, the results of a comparison between this point and the actual average return of 12.5 percent would be quite different than a similar comparison with the long-term expectation of approximately 7.5 percent. Therefore, since the structural increases in the price-earnings ratios which created the "excessive" return cannot continue indefinitely, a comparison with the actual average may create wholly misleading conclusions. In effect, the investor following the common stock theory looks only to the long-term performance of the corporate sector for the determinates of his monetary return, and therefore cannot consider market adjustments to the values generated by the corporate sector.

As a consequence, it is assumed that the long-term common stock investor will assess the risks of his decisions in terms of the expected performance of the corporate sector. The result is that the data employed must come from a period where price-earnings ratios remained roughly constant. However, the data also must be reasonably "fresh," and unfortunately the recent history of equity performance contains substantial upward adjustments in price-earnings ratios. Furthermore, even if a satisfactory period could be found, its applicability may be questionable in view of the fact that such a period is so difficult to find. In other words, while the long-term performance on common stocks will directly

reflect the economics of the corporate sector, any finite period will contain some elements of re-evaluation due to the inherent dynamics of investor judgments.

The condition leads to a second assumption where it is recognized that no amount of common stock diversification can protect the investor from the fact that a finite time horizon will produce an average market return which is different than the long-term expected return. If the long-term expected return is 7.5 percent per year, the actual time period selected will produce an average return which is not equal to 7.5 percent. This element of risk and uncertainty exists regardless of the number of securities in a given portfolio, and as a consequence, common stock diversification becomes a meaningless strategy in the context of market fluctuations. This fact, of course, is why the common stock theory insists upon a long-term holding period such that the fluctuations noted become insignificant relative to the total. It is also a partial justification for diversification by type of security.

Method of comparison

In relation to the study, then, it is assumed that variations in the market average which result from the necessity of a finite time horizon will be ignored by the long-term investor. Specifically, comparisons are made to the

actual market average regardless of its value. However, it has been noted that such an approach can create misleading conclusions if the loss points are compared to the actual results of a given period rather than the long-term market average. The dilemma leads to a third fundamental assumption which underlies the results of the analysis. It is assumed that the probability of a sample mean exceeding some multiple of the actual population mean is constant with respect to changes in the population mean for each particular holding period length. Thus, given a holding period length of, say, two years, the assumption allows one to make the following statement. If the actual population mean is equal to, say 10.0 percent per year, the probability of a sample mean exceeding, say, 8.0 percent per year (80 percent of the mean) will be the same as the probability of a sample mean exceeding 16.0 percent per year if the actual population mean were equal to 20.0 percent per year. In the subsequent discussion, this assumption will be referred to as the change of scale assumption.

The analytical procedure which results from this assumption can be explained by means of an example which employs the following market information.

Actual Average Annual Rate of Return	10.0 percent
Long-Term Average Annual Rate of Return	7.5 percent

If the comparison is to be made with a return which is, say, 5.6 percent per year, it can be said that such a return, or loss point, is 75 percent of the long-term average annual return available from common stocks (= 5.6/7.5). In terms of the actual return of 10.0 percent which was recorded during the given period, the equivalent loss point would be a return of 7.5 percent (= 10.0[.75]). Thus, for purposes of analysis, a return of 5.6 percent relative to the long-term expectation would be defined as being 75 percent of the actual average market return.

The change of scale assumption allows one to conclude that the probabilities associated with this point are equal to the probabilities associated with a return of 5.6 percent relative to the long-term common stock expectation of 7.5 percent. The result is an analytical procedure which avoids the problems created by structural changes in price-earnings ratios, but at the same time allows data to be drawn from a period which contains such changes.

The change of scale assumption

Consequently, the importance of the change of scale assumption is clear and requires considerable justification.[1]

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[1]Despite the fact that the assumption is generally consistent with much of the contemporary efforts in the field of finance. That is, the Markowitz model has little practical value unless historical variances can be applied to the future, and Baumol agrees that they can in "Mathematical Analysis of Portfolio Selection: Principles and Application,"

To begin with, a stable relationship is intuitively appealing resting on the proposition that the monetary rewards (annual rate of return) associated with a given level of risk will be roughly constant relative to other levels of risk within the equity markets even if the rewards from common stocks in general should change.[1] However, it will be quickly recognized that a theoretical justification of this proposition goes far beyond the scope of the analysis involving such concepts as the risk-free rate of return, an efficient market, true investment value, and the dynamics of funds flows, to name a few. Fortunately, such a discussion is unnecessary.

In the first place, it will be recalled that the proposed methodology is directed at the tails of the distributions of sample means. As a result, even "substantial" errors in the measure of the variance and all higher moments of the distributions are unlikely to significantly alter the final conclusions. For example, the variance statistic may be in error by, say, 25 percent, but at a calculated probability of .01 the "real" probability would be .019, or .001,

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99. In terms of risk measurements, it has been observed (see supra., p. , n.) that many authors utilize the historic variance, and again their work will be of little consequence unless history approximates the future with reference to the behavior of the variance and the resulting probability implications.

[1]See, e.g., J. Fred Weston and Eugene F. Bringham, Managerial Finance (2nd ed.; New York: Holt, Rinehart and Winston, 1966), pp. 304-06.

depending upon the direction of the error and assuming all higher moments to be zero. At a calculated probability of .10 the "real" probability would be .127 or .050. In view of the massive amounts of data to be utilized in the study, an error of this size probably overstates the case considerably, and thus "proof" of the assumption would contribute little to the credibility of the conclusions.

Nevertheless, it is conceivable, although unlikely, that "errors" of this size and larger do, in fact, occur. But here the results of the analysis provide confidence in the validity of the assumption. That is, the statement of the change of scale assumption implied that the relationship was not expected to hold with variations in the length of the holding period. Otherwise, one would be forced to conclude that the predictions of the common stock theory were applicable in the short term as well as in the long-term. The probability of a loss would be the same for a one-year period as for a ten-year period. However, it has been recognized that some minimum holding period length should exist such that longer periods do not significantly alter the probabilities associated with a given annual return.

Thus, it is possible that a minimum holding exists, but due to a lack of realism in the change of scale assumption, the analysis may fail to reveal it. To be specific, the effects of compounding will produce substantially

different average wealth ratios for different holding period lengths which will, in turn, produce significantly different probabilities for long-term holding periods if the assumption is incorrect. Fortunately, the analysis demonstrates that such problems do not occur.

As a final comment, the magnitude of the problem should be placed in perspective. That is, the necessity of making the assumption lies in the fact that any finite time horizon will produce a return which is different than the long-term expectation. However, the size, and consequently the significance, of such differences is largely a decreasing function of the holding period length. For example, during the post World War II period the average annual return on New York Stock Exchange common stocks had a range of 71.2 percentage points when the holding period was one year, but only 6.8 percentage points when a ten-year period was examined.[1] As a consequence, the study requires only that the change of scale assumption is a reasonable approximation of reality within a rather narrow range of possible market averages. If it breaks down for short-run periods, and wider ranges, the conclusions to be suggested would not be altered.

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[1]Fisher and Lorie, "Rates of Return on Investments in Common Stock: The Year-By-Year Record, 1926-65," 6-7.

Summary

In summary, comparisons are related to the market average regardless of the actual value achieved during a given period in order to generate conclusions which have future applicability. However, calculations will be made based upon the actual performance of the market during the period; the justification of which lies in the change of scale assumption. It should be emphasized that this approach enables one to make statements which are related only to the behavior of the actual market average thereby prohibiting any conclusions regarding the effects of variations in the market mean. As has been shown, this variable is related to the decision area of which investment instrument to purchase, and should not effect the diversification requirements within a given media. Moreover, the "long-term" nature of the study implies that the problem is relatively insignificant.

In addition, it is far more reasonable to interpret a loss or a gain in terms of the actual market average. For example, if the actual average return during the next ten years is, say, 20 percent per year, few investors would consider a return of, say, 7.0 percent to be a gain even though such a return exceeds other contemporary investment alternatives. In essence, the average market return is used as

a bench mark and variations in the bench mark are considered to be a risk of common stocks in general rather than as a risk which can be affected through common stock diversification.

With this summary, the general outline of the methodology has been completed and the philosophy and techniques of data utilization established. The task remaining is to describe certain specifics which have been omitted for purposes of clarity. In particular, it is necessary to determine: (1) the quantitative representation of each of the three categories of portfolio objectives, (2) the specifics of the sampling techniques which represent the three methods of diversification to be tested, and (3) the mathematical techniques to be employed in order to determine the desired probabilities.

Portfolio Objectives

Long-term growth

It has already been noted that the objective of long-term growth is measured by the wealth ratio or the total monetary value to which $1 will accumulate over the appropriate holding period assuming the reinvestment of dividends.

Principal protection

The measure to be used to represent the objective of principal protection is the total value of the investment on

the closing date of the particular holding period relative to an initial investment of $1 with dividends ignored. This measure is equivalent to that portion of the average annual rate of return which eminates from the growth of the original principal, or the rate of return exclusive of dividends. As noted earlier, the objective of principal protection is to be analyzed relative to the objective of long-term growth. In addition, the risks associated with the growth of principal are given specific consideration.

Current income

It has been established that the analysis of the current income objective will concentrate upon the change in current income rather than the absolute level of income. As a result, the relevant measure of performance is simply the anticipated average annual growth of dividends. Unfortunately, several conceptual problems are encountered when attempting to apply this measure.

However, prior to undertaking a discussion of the conceptual problems, the precise meaning of a change in current income must be established. In particular, the concern is with the growth of dividends per dollar invested and not changes in the current yield. That is, current income as measured in dollars received from a common stock investment is wholly dependent upon the dividend policy of the firm, and consequently, will be unaffected by changes in the

market price of the security. For example, given the following information on a hypothetical security,

Purchase price per share	$10.00
Price per share - one year later	13.75
Dividend per share - first year	0.50
Dividend per share - second year	0.55
Current Yield - first year	5.0%
Current Yield - second year	4.0%

it is clear that the investor's current income has increased by 10 percent, but use of the current yield would indicate a decline. It is true, of course, that such a decline is relevant to annual performance comparisons with other investment media, but in order to implement such a comparison the investor must be allowed to make portfolio changes based entirely upon market price behavior which is not in keeping with the common stock theory. Furthermore, the objective of current income is, by definition, concerned with <u>income</u> rather than yield. Specifically, an "inadequate" current yield on common stocks may cause the investor to select some alternative investment media which can satisfy his income needs. As a result, the change in dividend income, as opposed to the current yield, is used to represent the objective of current income. In the above example, the change would be 10.0 percent.

The initial conceptual problem encountered is the establishment of a definition of the beginning dividend

income. Here it is assumed that the beginning dividend, or dividend at time of purchase, was the annual dividend which accumulated during the year immediately preceding the purchase date. For example, if the purchase date were December 31, 1964, then the beginning dividend would be the income payments which the shareholder would have received between December 31, 1963 and December 31, 1964. The problem with such an approach is that the investor does not receive this dividend since payment by the particular firm was history on the purchase date. On the other hand, because the investor does not practice security analysis and cannot employ hindsight, the most recent dividend payment is the only logical approach. Moreover, the risk measure will be represented by subsequent changes in this dividend thereby fully accounting for the future.

A much more serious conceptual difficulty is encountered with regard to the year-to-year variability of the annual growth rate. That is, if the average annual growth rate, over several years, of the dividends of an individual issue were, say, 10 percent, and the investor was attempting to protect his income against an annual inflation rate of, say, 5 percent, then one might conclude that protection was achieved. On the other hand, since the investor's objective is <u>current</u> income, the analysis must be related to the change which occurs in <u>each</u> year of the holding period in addition

to the average change. For example, if the beginning dividend paid by a particular firm were $1.00, and the dividends in the first and second years after purchase were $.90 and $1.21, respectively, then the average growth rate would be 10 percent per year. However, one could hardly conclude that such a performance was completely satisfactory since the $.90 in the first year after purchase was clearly inadequate.

As a result, the study will examine the probability that the dividends received during the first year of any given holding period will fall below the appropriate loss point in addition to the probability of an inadequate average annual growth rate over the entire holding period.

The loss points will be defined as a zero change such that the monetary value of the beginning income is maintained, and an annual increase which is sufficient to protect the purchasing power of the investor's beginning income. It should be re-emphasized that the comparisons will be made to the beginning dividend income rather than to each preceding year. In the example, this implies that the $1.21 received in the second year is compared to the beginning dividend of $1 rather than the $.90 received in the first year. Finally, only those holding periods which correspond to calendar years are used rather than all possible holding periods in order to maintain consistency with the generally accepted methods of determining annual income.

Continuing with the conceptual problems, it is recognized that the use of the annual growth rate as the random variable contains an "averaging" problem similar to that which was encountered when attempting to use annual rates of return. In addition, meaningful growth rates for individual issues can be established only when the beginning dividend is greater than zero.[1] For both of these reasons, the absolute change in income must be used rather than the relative change, but in order to maintain an equivalency with growth rates, the absolute changes must be measured as the absolute income change per dollar of initial investment. That is, in the above example the income per dollar of investment would be $.05 (= .50/10.0) in the first year and $.055 (= .55/10.0) in the second, producing a change of $.005 per $1 invested. It is the change per $1 invested which is used as the random variable.

Techniques of Diversification

With the establishment of these several measures, it becomes appropriate to proceed to the task of determining the specific security selection techniques which are used to

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[1]It is possible, of course, to reverse the calculations such that a 100 percent growth rate would be generated during the first year in which dividends became positive. However, this procedure is equally unsatisfactory when it is recalled that it is possible for a dividend to become zero at some later date in which case the growth rate would also be undefined.

represent the various methods of diversification described earlier. These include: random selection; time diversification; and inter-industry diversification. Random selection has already been examined, and thus the following discussion concentrates upon time and inter-industry diversification.

Time diversification

It has been noted that dollar-averaging is to be used to represent time diversification with purchases made in six month increments. The measure of return is the total monetary value accumulated during the holding period relative to the initial investment where dividends are reinvested when received. The objectives of current income and principal protection are ignored, and the minimum holding period length to be tested is two years. The actual selection of securities is by simple random selection where the basic assumption is that the same issues are acquired on each purchase date, rather than the alternatives of either random selection on each purchase date, or an accumulation of issues during the over-all holding period.

Inter-industry diversification

With regard to the technique of inter-industry diversification, the random variable employed is the total monetary value accumulated, during the holding period, relative to the initial investment where dividends are reinvested

when received. As with time diversification, the objectives of current income and principal protection are ignored. The method of security selection consists of a random selection of an industry followed by a random selection of one issue within each of the industries chosen. No industry is to be represented more than once unless the portfolio reaches a size where the industrial categories are exhausted.

Summary

The remaining descriptive tasks involve an examination of the characteristics of the data and a development of the mathematical techniques which must be used to determine the probabilities necessary for implementation of the research methodology. However, prior to embarking upon such a discussion, it is appropriate to summarize the methodology as presented to this point.

It has been suggested that the study will attempt to determine a portfolio size which provides the investor with considerable assurance that his portfolio return will approximate the average market return. From this point, the "trade-off" between the risk of loss and the opportunity to achieve an exceptional gain will be analyzed using the change in the probability of gains and loss with security additions as the determining criteria. Included in the basic approach will be an examination of the time period necessary to achieve the investment advantages which adhere to the investor who follows the tenets of the common stock theory.

In order to make the conclusions relevant to the future, the analysis will center upon comparisons related to the average market return from common stocks thereby eliminating the opportunity for errors which may result from the inclusion of temporary market adjustments. In addition, various investor objectives will be analyzed including long-term growth, principal protection, and current income. Finally, the analysis will give attention to certain techniques of diversification in order to determine whether or not the specific methods can be used to improve upon the results of simple random selection. The methods are inter-industry diversification and dollar-averaging which is used to represent time diversification.

CHAPTER IV

THE MATHEMATICAL TECHNIQUES OF THE ANALYSIS

Introduction

As indicated, it now becomes necessary to enter into a discussion of the mathematical tools and concepts which were applied to the data in order to arrive at the probabilities required by the research methodology. It will be shown that there exist three fundamental problems which must be given attention in order to achieve the objectives of the study. These include: (1) the shape or characteristics of the distributions and the resulting effects upon the probability calculations; (2) the fact that the various measures of performance of the individual issues contained in the population are not independent of each other; and (3) the mathematical properties of the various sampling techniques which are used to represent the several methods of diversification to be tested. However, prior to examining these questions directly, it is appropriate to review certain specifics in order to assure clarity and set the stage for a more detailed examination of the problems mentioned.

To begin with, the familiar approach to determining probabilities is reviewed. As has been shown, the random

variable used to represent the objective of long-term growth is the wealth ratio generated from an investment in an individual common stock contained in an, as yet, undefined population. The variance of the random variable is defined as:[1]

$$V(X_i) = E(X_i - EX_i)^2 \qquad (1)$$

where:

X_i = the random variable or the i^{th} value of all possible wealth ratio values
EX_i = the arithmetic mean of all possible values of X_i
$V(X_i)$ = the variance of the random variable X_i

If, then, a simple random sampling plan is adopted as the method of diversification it follows that the variance of the sample means, or portfolio returns, is defined as:[2]

$$V(\bar{x}) = \frac{1}{n}[V(X_i)] \qquad (2)$$

where:

n = the sample size or portfolio size.
$V(\bar{x})$ = the variance of the sample means or portfolio returns

Based on these definitions and assuming that the random variable describes a normal distribution, the elementary method of determining the probabilities associated with a

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[1]Taro Yamane, Elementary Sampling Theory (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1966), p. 19.

[2]Ibid., 33.

given point on the distribution, or a given wealth ratio, can be described as follows: the distribution is "standardized" such that the wealth ratio under consideration can be described in terms of occupying a point which is a specific number of standard deviations to the left or right of the mean. With this information it is generally sufficient to enlist the aid of a Normal Area Table and simply read off the appropriate probabilities.

Before moving on, it is worth noting that the relationship in equation (2) is the foundation of all diversification models where it is assumed that as the portfolio size (n) increases, the variance of the portfolio means declines thereby providing increased risk protection while at the same time reducing the opportunity for an exceptional gain.

In addition, it should be recalled that security selection for diversification is generally without replacement. Otherwise, a given security might be purchased more than once. The relationship as set forth in equation (2) assumes that sampling is with replacement. Nevertheless, the problem is a minor one since the actual portfolio sizes, or sample sizes, to be considered are very small relative to the total population (a maximum of approximately 3.0 percent). As a result, the use of the hypergeometric correction will not significantly alter the probabilities and subsequent conclusions.

The Shape of The Distributions

The above review assumed that the random variable described a normal distribution. Unfortunately, such an assumption is very far from reality as the results of the analysis clearly demonstrate. Moreover, the Central Limit Theorem does not make the assumption of normality any more acceptable for purposes of the study. The theorem recognizes that the distribution of sample means will be asymptotically normal as the sample size (n) approaches infinity assuming mild regularity conditions of the parent population.[1] The great value of the theorem is that in most applications the sample size is large enough to enable one to employ the normal distribution as a good approximation of the actual distribution of sample means. In terms of the objectives of the study, the sample sizes to be considered relative to the characteristics of the population are not large enough to make the normal approximation an acceptable alternative. Furthermore, considerable emphasis is given to the change in probability which results from the addition of securities to a portfolio, and as a consequence, the significance of these changes would be largely hidden if the assumption of normality were retained.

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[1]Taro Yamane, Mathematics for Economists: An Elementary Survey (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1962), pp. 420-23.

Therefore, the required probabilities are calculated from the Cornish and Fisher series[1] which theoretically allows consideration to be given to all definable moments of a distribution thereby eliminating the problems of "non-normality." However, a close examination of the data reveals that only the first two terms of the series are relevant to the objectives of the study. Specifically, the calculated probabilities, using the first four terms of the series, show very little deviation from those calculated with only the first two terms. Experiments with the terms beyond the fourth were not conducted and are assumed to be negligible. In essence, the distributions exhibit substantial skewness which is considered in the second term, and while kurtosis is not absent,[2] its effect upon the calculated probabilities is negligible.

The first two terms of the series are as follows:

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[1]The original development of this series can be found in: E.A. Cornish and R.A. Fisher, "Moments and Cumulants in The Specification of Distributions," in Contributions To Mathematical Statistics, ed. by Walter A. Shewhart (New York: John Wiley & Sons, 1950), Ch. 30. A more modern examination can be found in: Maurice G. Kendall and Alan Stuart, The Advanced Theory of Statistics, Vol. I: Distribution Theory (2nd ed.: New York: Hafner Publishing Co., 1963), pp. 163-67. An example of an application can be found in Newton L. Bowers, Jr., "An Approximation To The Distribution of Annunity Costs," Society of Actuaries: Transactions, XIX (March, 1968), 295-309.

[2]Specifically, the distributions appear to exhibit large kurtosis where there is a relatively high concentration of items near the center and in the tails with a relatively low concentration of items in between.

$$X_\varepsilon = Z_\varepsilon - \frac{\gamma_1}{6}(Z_\varepsilon^2 - 1) \qquad (3)$$

where:

Z_ε = a value of some random variable, Z such that:

$$P[Z > Z_\varepsilon] = \varepsilon$$

X_ε = a value of some random variable, X which is distributed normally such that:

$$P[X > X_\varepsilon] = \varepsilon$$

γ_1 = the ratio of the third moment around the mean to the standard deviation cubed of Z.

In order to clarify the usage of the series as it is applied to the objectives of the study, an example is presented below which employs the following table of hypothetical information.

Actual Average Wealth Ratio	= E(X)	= 1.2100
Standard Deviation of Wealth Ratios	= σ	= .4000
Third Moment of Wealth Ratios	= K_3	= .0768
Wealth Ratio of Loss Point	= L	= 1.1700

With a loss toleration level (L) of 1.17, it can be said that such a point is 0.10 standard deviations to the left of the mean. Algebraically this is:

$$Z = [L - EX]\frac{1}{\sigma} \qquad (4)$$

$$= [1.1700 - 1.2100]\frac{1}{.4}$$

$$= -.10$$

where:

Z = the number of standard deviations the point L is distant from the mean.

L = the appropriate loss criterion.

which allows one to conclude, with the aid of a Normal Area Table, that the probability of achieving a wealth ratio which is less than 1.17 (the assumed loss point) for a sample of unity is approximately 0.46 when the wealth ratios are distributed normally. Thus, equation (4) can be used to represent Z_ε in the Cornish and Fisher series where $\varepsilon = .46$. Applying equation (3) to the example data results in:

$$X_\varepsilon = -.10 - \frac{.0768/(.4)^3}{6}[(.10)^2 - 1]$$

$$= -.10 - .20(-.99)$$

$$= .098$$

which is used in the same fashion as Z under the assumption of normality. Therefore, using a Normal Area Table, it would be concluded that the probability of achieving a wealth ratio which is less than 1.17 is 0.54 when the portfolio contains one security. This is, as opposed to a probability of 0.46 under the assumption of normality. For portfolios, or samples,

containing more than one security, equation (5) is presented without comment:

$$X_{\varepsilon,n} = Z_{\varepsilon}(n)^{1/2} - \frac{\gamma_1}{6}[(Z_{\varepsilon})^2(n)^{1/2} - 1/n^{1/2}] \quad (5)$$

where:

n = the sample, or portfolio, size

Element Dependence Within The Random Variable

It becomes appropriate, then, to turn to the second mathematical problem mentioned which eminates from the fact that the performances of the various issues within the market population are not independent of each other. For example, if two issues are perfectly correlated, the selection of one implies that the selection of the second will produce a predictable performance measure rather than any one of all possible performance values which the second issue could assume given independence. As a consequence, the variance of the sample means may be different than that which equation (2) suggests.

In the context of the contemporary literature in the field of investments, this problem is generally attacked in two steps.[1] That is, the behavior of the performance

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[1]See, e.g., William F. Sharpe, "A Simplified Model for Portfolio Analysis," Management Science, IX (January, 1963), 277-93; Evans and Archer, "Diversification and The Reduction of Dispersion: An Empirical Analysis," 761-2; and Jack L. Treynor, William W. Priest, Jr., Lawrence Fisher, and Catherine A. Higgins, "Using Portfolio Composition to Estimate Risk," Financial Analysts Journal, XXIV (September-October, 1968), 93-100.

measure for a given issue is divided into the following categories: (1) the correlation between the performance of the particular issue and the average performance of the market population; and (2) the correlation between the individual issue and all other issues exclusive of the average performance.

With regard to the first category, it has been suggested that the proposed methodology is designed to make comparisons to the average market return regardless of the value it may assume. The implication is that the interest is with deviations from any mean value rather than from some particular mean value. As a consequence, it is possible to adjust the random variable by simply subtracting the actual market average from the particular measure of performance of each issue. Algebraically this is:

$$Z_i = X_i - EX$$

where:

X_i = the wealth ratio of the i^{th} security
EX = the arithmetic mean of the wealth ratios of all securities contained in the market population
Z_i = the wealth ratio of the i^{th} security exclusive of the market mean

in the case of the objective of long-term growth. The net effect of this adjustment is to produce a distribution which has an invariant mean equal to zero. This, in turn, implies

that the covariance between any issue and the population mean must also equal zero thereby eliminating the problems of the first category of dependence.

Unfortunately the second category of the problem is not so easily disposed of. However, here again, the proposed methodology generates a satisfactory solution. That is, the discussion to follow will demonstrate that it is possible to determine the average variance of the portfolio returns in the same fashion as would be the case if all covariances were equal to zero provided the market average is always equal to zero.

Thus, it is known that the distribution of the wealth ratios of all individual securities has the following characteristics:[1]

$$\sum_{i=1}^{N} Z_i = 0 \qquad (6)$$

and

$$\text{Cov}\ (Z_i Z_j) \neq 0$$

where:

Z_i = the wealth ratio of the i^{th} security exclusive of the market mean.

N = the number of securities contained in the population

$\text{Cov}\ (Z_i Z_j)$ = the covariance between the wealth ratios exclusive of the market mean of the i^{th} and j^{th} securities

[1] As before, the discussion concentrates upon the performance measure used to represent the objective of long-term growth where it is assumed that any resulting conclusions are applicable to other performance measures.

In addition, it is known that the variance of the sum of any i, j combination of securities is:[1]

$$V(Z_i + Z_j) = V(Z_i) + V(Z_j) + 2\ \text{Cov}\ (Z_i\ Z_j) \qquad (7)$$

where:

$V(Z_i)$ = the variance of the wealth ratio of the i^{th} security exclusive of the market mean

Correspondingly, the variance of the mean of any i, j combination of securities is:

$$V\left(\frac{Z_i + Z_j}{2}\right) = \frac{1}{4}[V(Z_i) + V(Z_j) + 2\ \text{Cov}\ (Z_i\ Z_j) \qquad (8)$$

Given these relationships, Table 4.1 is constructed which shows the characteristics of a specific sampling plan from a "market" containing three securities. Each "portfolio" contains two securities selected at random with replacement.

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[1] Yamane, Elementary Sampling Theory, p. 32.

TABLE 4.1

EXAMPLE SAMPLING PLAN

Portfolio	Portfolio Components	Portfolio Mean	Portfolio Variance
1	Z_1, Z_1	$E(Z_1)$	$V(Z_1)$
2	Z_1, Z_2	$1/2(EZ_1 + EZ_2)$	$1/4[V(Z_1) + V(Z_2) + 2\ Cov\ (Z_1Z_2)]$
3	Z_1, Z_3	$1/2(EZ_1 + EZ_3)$	$1/4[V(Z_1) + V(Z_3) + 2\ Cov\ (Z_1Z_3)]$
4	Z_2, Z_1	$1/2(EZ_2 + EZ_1)$	$1/4[V(Z_2) + V(Z_1) + 2\ Cov\ (Z_2Z_1)]$
5	Z_2, Z_2	$E(Z_2)$	$V(Z_2)$
6	Z_2, Z_3	$1/2(EZ_2 + EZ_3)$	$1/4[V(Z_2) + V(Z_3) + 2\ Cov\ (Z_2Z_3)]$
7	Z_3, Z_1	$1/2(EZ_3 + EZ_1)$	$1/4[V(Z_3) + V(Z_1) + 2\ Cov\ (Z_3Z_1)]$
8	Z_3, Z_2	$1/2(EZ_3 + EZ_2)$	$1/4[V(Z_3) + V(Z_2) + 2\ Cov\ (Z_3Z_2)]$
9	Z_3, Z_3	$E(Z_3)$	$V(Z_3)$

The preliminaries dispensed with, it will be recalled that the specific problem at hand is one of deriving the average variance of all possible portfolio, or sample means. To this end, it is noted that:

V_n = the variance of all possible portfolio means of size $\underline{n}$

and in terms of Table 4.1

$n = 2$ = the number of securities contained in each portfolio

Furthermore, it should be recalled that the sampling plan, or diversification technique, is simple random sampling with replacement. Consequently, it is proper to recognize that the sample means of each portfolio are independent of the sample means of each other portfolio, and as a result V_n will be the expected value of the variances of all possible portfolios.

If, for the moment, attention is directed at the portfolios labeled 2 and 3 in Table 4.1, it is seen that these portfolios are unique in the sense that they contain different securities <u>and</u> have a common first draw: i.e., the security labeled Z_1 was the first security drawn in both portfolios. Thus, if it is possible to show that:

$$\text{Cov}(Z_1Z_2) + \text{Cov}(Z_1Z_3) = -V(Z_1) \tag{9}$$

it can be concluded that the sum of the variances of portfolios 2 and 3 is equal to:

$$1/4[V(Z_1) + V(Z_2) + 2\ Cov(Z_1Z_2)]$$

$$+ \quad 1/4[V(Z_1) + V(Z_3) + 2\ Cov(Z_1Z_3)]$$

$$= \quad 1/4[V(Z_2) + V(Z_3)]$$

Moreover, if equation (9) can be generalized as follows:

$$\sum_{\substack{j=1 \\ j\neq i}}^{N} Cov(Z_i, Z_j) = -V(Z_i) \tag{10}$$

when:

$$\sum_{i=1}^{N} Z_i = 0$$

The sum of all portfolio variances would reduce to:

$$V(Z_1) + 1/4[V(Z_2) + V(Z_3)] + 1/4[V(Z_1) + V(Z_3)]$$

$$+ V(Z_2) + 1/4[V(Z_1) + V(Z_2)] + V(Z_3)$$

$$= 3/2[V(Z_1) + V(Z_2) + V(Z_3)]$$

and since there are a total of 9 possible portfolios, the average variance of the portfolios would be:

$$V_2 = 1/6[V(Z_1) + V(Z_2) + V(Z_3)]$$

which is the formula for the variance of sample means given a distribution containing three independent random variables and a sample size of two.

Therefore, the fact that the covariances are not equal to zero is of no consequence provided that the sum of all covariances associated with a given issue is the negative of the variance of that issue. Thus, attention is directed to equation (10) which is:

$$\sum_{\substack{j=1 \\ j\neq i}}^{N} \mathrm{Cov}(Z_i, Z_j) = -V(Z_i) \qquad (10)$$

when:

$$\sum_{i=1}^{N} Z_i = 0$$

With regard to the left side of equation (10) it is known that:[1]

$$\sum_{\substack{j=1 \\ j\neq i}}^{N} \mathrm{Cov}(Z_i, Z_j) = \mathrm{Cov}\left(Z_i, \sum_{i=1}^{N} Z_i - Z_k\right) \qquad (11)$$

and since

$$\sum_{i=1}^{N} Z_i = 0$$

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[1]Henry Scheffe, The Analysis of Variance (New York: John Wiley & Sons, 1959), p. 8.

equation (11) becomes

$$\sum_{\substack{j=1 \\ j\neq i}}^{N} \mathrm{Cov}(Z_i, Z_j) = \mathrm{Cov}(Z_i, -Z_i)$$

$$= -\mathrm{Cov}(Z_i, Z_i)$$

$$= -V(Z_i)$$

which completes the proof of equation (10).

As a result, it is concluded that the average variance of the sample means is unaffected by the value of the covariances when the population mean is invariate and equal to zero.[1] Therefore, the problems associated with the second category of dependence are eliminated given the solution to the first category.

Techniques of Diversification

Dollar-averaging

The remaining task, then, is to examine the mathematical properties of the various sampling techniques which are used to represent the various methods of diversification to be tested. As has been discussed, simple random sampling

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[1] While the behavior of the covariances is generalized to any N security population, the use of this relationship has been demonstrated only in the special case of N = 3 and n = 2. However, equation (10) indicates that the sum of all portfolio variances will always produce a cancellation of the covariances, and thus it is assumed that the conclusions mentioned are general.

is the primary technique employed, but in addition, the effects of dollar-averaging and inter-industry diversification are analyzed. In dollar-averaging, it is assumed that the initial selection of securities is a random selection with equal quantities of funds invested in each issue. Subsequent purchases involve the same securities as were selected initially.

As a consequence, it is appropriate to utilize a simple random sampling plan provided that the measure of performance, or random variable, is reflective of the results of all possible purchases. In order to satisfy this constraint, the random variable is calculated as indicated by the following example. If the concern is with the performance of a given issue during a two-year holding period beginning on January 1, 1963 which had the following price and dividend activity, the dollar-averaging investor's total wealth at the end of the period would be $5.04 if $1 were invested in the particular issue every six months.

Date	Dividends/Share	Price/Share
1/1/63	0	10
7/1/63	0	10
1/1/64	0	11
7/1/64	0	11
1/1/65	0	13.2

Specifically, the initial investment of $1 (on 1/1/63) would accumulate to $1.32 by the end of the two-year period.

The second purchase (on 7/1/63) would also accumulate to $1.32, but this accumulation would occur over 18 months instead of two years. The third purchase (on 1/1/64) would accumulate to $1.20 (= 13.2/11.0) as would the fourth (on 7/7/64) for a total accumulated wealth of $5.04. It is this figure which is used to measure the investment performance of each issue included in the market population. However, in accordance with the previous discussion, the random variable is this figure exclusive of the market mean.

Inter-industry diversification

It has been suggested that inter-industry diversification is to be represented by a two-step process of security selection. Specifically, an industry is selected with simple random sampling followed by a simple random selection of one security contained in the given industry. In terms of the statistical techniques required, this method of diversification can be represented by two-stage cluster sampling where the cluster sizes (industries) are allowed to vary, and selection of clusters is without replacement in order to prevent a multiple representation of one industry.[1]

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[1]For detailed description of the general concepts of cluster sampling see: Kendall and Stuart, The Advanced Theory of Statistics, Vol. III: Design and Analysis, and Time-Series, pp. 187-95; Leslie Kish, Survey Sampling (New York: John Wiley & Sons, 1965), Chs. 5 and 6; and Yamane, Elementary Sampling Theory, Ch. 8.

However, under these constraints, the standard techniques of determining the properties of the distribution of sample means do not apply. In most such sampling plans the objective is to determine the nature of the population. Thus, the concern is with determining an unbiased estimator of the moments of the population such that the effects of cluster size variations are removed. With regard to the present analysis, the objective is reversed such that the distribution of the actual sample means must be obtained given the characteristics of the population. In general, this distribution will be different than the distribution of the unbiased estimator. As a result, it is necessary to employ the following equations which are designed to yield the first three moments of the distribution of sample means of size n. Given these, probabilities are determined as indicated in the previous discussion.[1] If:

$\mu_{K,n}$ = the K^{th} moment about the mean of the distribution of sample means of size n

$\mu_{K,i}$ = the K^{th} moment about the mean of the distribution of wealth ratios for the i^{th} industry.

I = the number of industrial categories contained in the population.

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[1]The author is indebted to The Statistical Research Laboratory, The Rackham Graduate School, The University of Michigan, Ann Arbor, Michigan for assistance in developing these equations.

then the mean of the sample means is:

$$\mu_{1,n} = \frac{1}{I} \sum_{i=1}^{I} \mu_{1,i} \tag{12}$$

and the variance of the sample means is:

$$\mu_{2,n} = \frac{1}{In} \sum_{i=1}^{I} \mu_{2,i} + \frac{n-1}{n}\left[\frac{1}{I(I-1)}\right]\left[2 \sum_{i=1}^{I} \sum_{j>i} (\mu_{1,i})(\mu_{1,j})\right]$$

$$- (\mu_{1,n})^2 \tag{13}$$

Finally, the third moment around the mean of the sample means is:

$$\mu_{3,n} = \frac{1}{I(n)^2} \sum_{i=1}^{I} \mu_{3,i} + \frac{n-1}{n^2}\left[\frac{1}{I(I-1)}\right]\left[3 \sum_{i=1}^{I} \sum_{j>i} (\mu_{2,i})(\mu_{i,j})\right]$$

$$+ \frac{n-1}{n^2}\left[\frac{1}{I(I-1)}\right]\left[3 \sum_{i=1}^{I} \sum_{j>i} (\mu_{1,i})(\mu_{2,j})\right]$$

$$-3[(\mu_{1,n})(\mu_{2,n} + (\mu_{1,n})^2] + 2[(\mu_{1,n})^3] \tag{14}$$

While a detailed presentation of the development of these equations is unnecessary in the context of the study, certain properties deserve emphasis. In the first place, it should be recognized that the within cluster (industry) sample size is a constant and is equal to unity. The result is that equations (13) and (14) are applicable only so long as one is concerned with portfolio sizes which do not exceed

the number of industrial categories in the market population. Fortunately, this limitation does not impair the ability to make conclusions.

Secondly, the equations contain the assumption that the performance of a given industry or cluster is independent of the performance of all other industries. Furthermore, it is recognized from equation (12) that exclusion of the average market performance will not necessarily produce a random variable which has a mean equal to zero, since the number of firms in each industry is not constant. Moreover, it would be incorrect to adjust for such differences since their existence may result in a decided advantage (or disadvantage) to the technique of inter-industry diversification even if the higher moments are not significantly different from those obtained through simple random sampling. Therefore, it is necessary to use the suggested performance measure exclusive of the market average, the mean of which may not equal zero. As a result, the covariances will not "fall-out" such that it is necessary to assume that all covariances are equal to zero in the special case of inter-industry diversification. However, the data indicate that this assumption will not significantly alter the general conclusions of the analysis.

Thus, the description of the research methodology and the necessary mathematical tools is complete which leads to

the final descriptive task of describing the source and nature of the data employed. It is with this subject that the following chapter is concerned.

CHAPTER V

THE SOURCE AND NATURE OF THE DATA EMPLOYED

The description of the research methodology in Chapters III and IV suggests that massive quantities of data are required to obtain reasonably good approximations of the characteristics of the distribution of the returns available from common stock investment. This chapter is concerned with analyzing the important features of the data with particular emphasis upon how the conclusions of the study will be affected by the nature of these data. In addition, the parameters of the time period are established and the reasons for the selection of the particular period are discussed.

The Basic Common Stock Data

With regard to the nature of the data employed, it is noted that the data source was the Center for Research in Security Prices, Monthly NYSE Price-Relative Tape (hereinafter referred to as the CRSP Tape) which was originally developed in 1963 by the Center for Research in Security Prices, Graduate School of Business, University of Chicago.

As a result, a complete and detailed presentation of the various definitions, collection techniques, adjustments to reflect capital changes, and verification procedures used to compile the CRSP Tape are omitted in view of the widespread circulation and acceptance of the data.[1]

The market population

Nevertheless, certain specifics are of particular importance to the present study. In the first place, the CRSP Tape includes price and dividend information on all common stocks listed on the New York Stock Exchange at any time during the period 1926 to 1965.[2] Consequently, the common stock population is confined to only those issues listed on the New York Stock Exchange (hereinafter referred to as NYSE). In addition, it should be recognized that information on an individual issue is available only during the time period in which the security was listed on the NYSE which immediately raises the problem of delisting.[3]

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[1]This information can be found in: Fisher and Lorie, "Rates of Return on Investments in Common Stocks," pp. 9-12; Fisher, "Outcomes for 'Random' Investments in Common Stocks Listed on The New York Stock Exchange," pp. 154-61. Additional detail can be made available upon request from Standard Statistics Company, 345 Hudson Street, New York, New York.

[2]A common stock is defined as any issue with residual rights to dividends. The problem of voting rights is ignored.

[3]Delisting occurs when a security ceases to be traded on the NYSE for whatever reason including absorption of all outstanding shares through a merger.

Specifically, if the data are to be reflective of the performance of the entire NYSE, it is necessary to include all issues listed at the beginning of a particular period regardless of whether or not a given security was listed throughout the entire period. In other words, hindsight is not allowed, and thus delisted securities cannot be ignored.

As a consequence, a given holding period includes all common stocks which are listed on the beginning date of the particular period.[1] If a security was delisted at some date prior to the close of the holding period, it is assumed that the issue was sold at the closing price of the last complete month of listing. From that date to the close of the particular holding period, the proceeds of the sale are assumed to earn the average market return as actually occured during the remaining months.[2] In effect, a delisted issue is sold and the proceeds spread over the entire market at the date of delisting. It is true, of course, that such an approach is not a realistic investment strategy, but since such issues generally perform poorly prior to delisting, the

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[1]Any issue which obtained listed status after the beginning date of the holding period is excluded. Such an issue was not an element in the population at the time of purchase despite its later inclusion.

[2]For computer programming convenience, this average return is represented by Fisher's market indexes. See Fisher, "Some New Stock-Market Indexes," pp. 191-225.

"errors" should be small.[1] That is, the techniques of compounding create a condition where the "real" return achieved after delisting will have a reduced effect upon the total return provided the return prior to delisting was "low." In essence, the return after delisting receives less weight due to the lower base from which it begins.

Continuing with the problems of defining the population, it will be recalled that the methodology related to the objective of current income requires knowledge of the dividend paid by each issue during the twelve months immediately preceding the purchase date. Therefore, it is assumed that in the case of the current income objective, an issue is not eligible for purchase unless it has been listed for at least one year prior to the purchase date under consideration.

Fortunately this assumption does not create a major limitation. In the first place, the annual number of issues added to the NYSE during any given year of the study period did not exceed 5 percent of the total number of listed issues. Thus the exclusion of such issues should not significantly alter the results. In addition, the distribution of dividend changes was analyzed assuming that the beginning annual dividend was the dividend paid during the first year

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[1]The performance of a random sample of 28 delisted securities showed an average annual return during the three years prior to delisting of approximately 6.0 percent or approximately 45 percent of the average market return.

of the holding period which makes possible the inclusion of the recently listed issues. The results obtained from these distributions were virtually identical to the results obtained using the suggested methodology. As a consequence, it is concluded that the exclusion of such issues would not alter the results of the analysis.

Finally, it should be noted that it was not necessary to adjust the data for various capital adjustments in order to determine the required rates of return.[1] The source material provided the necessary adjustments in a format which enabled the user to calculate rates of return for any given period without an examination of capital changes. It is assumed that the adjustments have been made correctly on the CRSP Tape.

Industry categories

The data is in a form suitable to conduct the inter-industry diversification tests suggested. Each listed security is categorized according to the three-digit Standard Industrial Classification (SIC) assigned by the Securities and Exchange Commission.[2] Moreover, the format of the

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[1]Capital adjustments to raw stock market data generally include such items as stock dividends, stock splits, subscription rights, reorganizations, and liquidating dividends.

[2]A complete description of each classification can be found in: U.S. Bureau of The Budget Office of Statistical Standards, Standard Industrial Classification Manual, 1967 (Washington, D.C.: Government Printing Office, 1967), pp. 3-321.

data permits a determination of the appropriate classification during each month a particular security was listed which makes possible the classification of issues as they would have been at the time of purchase. Since hindsight cannot be employed, this was the classification used rather than the current classification.

On the other hand, use of the three-digit classification creates certain problems which deserve mention. In the first place, the fundamental objective of inter-industry diversification is to avoid constructing a portfolio which contains a concentration of firms operating in the same industry or in industries which are expected to exhibit similar performance characteristics. Use of the three-digit classification is unlikely to accomplish this objective. For example, the classification implies that electric utility companies, gas utility companies, and combination gas and electric utility companies are members of three separate industries which, of course, they are. Nevertheless, the performance of the common stocks of these firms should exhibit a high degree of positive correlation. As a result, industry classifications, for purposes of the study, were confined to the Major, or two-digit, SIC categories.

In addition, the utilization of the three-digit categories would have created a condition where any potential advantages of inter-industry diversification were likely to

have been eliminated. With few exceptions, each three-digit classification contains a very small number of firms listed on the NYSE (generally less than five) with many containing only one company. As a consequence, several of the "clusters," or industries, would not have been clusters at all but individual securities. Thus, the cluster sampling plan would have come quite close to a simple random sampling plan thereby preventing the investor from gaining any portfolio advantage from the technique of inter-industry diversification.

Moreover, an industry which contained a small number of issues lacked a sufficient number of data points to create confidence that the sample distributions represented a good approximation of the "real" distributions of the returns within a given industry. This problem, however, was not relieved entirely by the broader two-digit classification. As a result, all major two-digit classifications which contained three companies or less were excluded from the analysis. The result was an exclusion of ten industry classifications containing a total of twenty firms.[1]

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[1]It should be noted that Major Groups 11 (Anthracite Coal Mining) and 12 (Bituminous Coal Mining) are combined into one industrial category in accordance with general practices as are Major Groups 70 through 73 combined into a single General Service Industry. See, e.g., Fisher, "Some New Stock-Market Indexes," pp. 209-17.

In summary, all securities listed on the NYSE were categorized by their SIC two-digit classification according to the assignments made by the Securities and Exchange Commission.[1] Those major groups which contained three firms or less were excluded. Furthermore, no attempt was made to determine the accuracy of these classifications, nor was consideration given to potential improvements in the diversification strategy which may have been available through the establishment of specific relationships between the various industry categories. Such an examination would have involved security analysis which has been excluded from the scope of the study.

The calculations

The required calculations were conducted on the computer facilities operated by the Computation Center, The University of Chicago. In addition, only the calculated probabilities are reported in the analysis since the great bulk of the programming required was related to the format of the data rather than to the calculation of the various moments and resulting probabilities. Should the reader wish to duplicate the computations, the programs used can be made available by the author. The program validating procedures are also available upon request.

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[1]Should the reader require a listing of the firms contained in each industry, the information can be made available. However, since the data are privately owned by The Standard Statistics Company such a request must be made through that firm. Prices are subject to negotiation.

The Period of Study

As was suggested earlier,[1] the period used to obtain data was the eleven year period beginning on December 31, 1954 and ending on December 31, 1965. Thus, the immediate question is the logic behind the selection of this particular period as opposed to some other period. As a detailed historical review of the behavior of stock prices on the NYSE is beyond the scope of the analysis, only the broad features of their behavior as they relate directly to the objectives of the study are considered. The emphasis is upon the future applicability of a given period.[2]

The pre-World War II period

Because of major structural changes in, and growth of the economy it is reasonable to assume that the performance of the NYSE prior to the close of World War II is

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[1] See supra, p. 49.

[2] Unless otherwise noted, the following discussion is taken from: Benjamin Graham, David L. Dodd, and Sidney Cottle, Security Analysis: Principles and Technique (4th ed.: New York: McGraw-Hill Book Company, 1962), Ch. 31; Benjamin Graham, The Intelligent Investor (3rd ed.; New York: Harper & Row, 1965), Ch. 3; Paul D. Harbrecht, Pension Funds and Economic Power (New York: The Twentieth Century Fund, 1959), Chs. 1 and 2; Nicholas Molodovsky, "The Many Aspects of Yields," Financial Analysts Journal, XVIII (March-April, 1962), 49-62ff; John M. Birmingham, "The Quest for Performance," Financial Analysts Journal, XXII (September-October, 1966), 93-94; and Henry C. Wallich, "What Does the Random Walk Hypothesis Mean to Security Analysis?" Financial Analysts Journal, XXIV (March-April, 1968), 159-62.

irrelevant to the contemporary market. The inclusion of the years 1925-45 would undoubtedly produce different conclusions.

The basic reason for concentrating on the period subsequent to World War II is the federal government's contemporary pursuit of a vigorous national income policy. While it cannot be assumed that macro-economic instability has been eliminated, it is reasonable to assume that the magnitude of fluctuations has been moderated such that the pre-war era would not be representative of the future. In essence, the probabilities are high that economic fluctuations and trends prior to World War II are not likely to be similar to contemporary and future economic conditions. Thus it appears reasonable to exclude from the analysis the behavior of the common stocks listed on the NYSE as was exhibited during the years prior to World War II.

The 1945-1954 period

It also can be argued that the period 1945-54 is equally suspect in terms of its contemporary applicability for several reasons. In the first place, acceptance of the argument that the events prior to World War II should be excluded from the analysis implies that the aftermath of such events should be excluded. Specifically, the late 1940's were dominated by the general belief that history would be

repeated with a major depression occuring sometime after the war. The result was that common stocks sold at exceptionally low prices relative to earnings and dividends and alternative investments under the basic assumption that the risks of equity investment were too great for most investors. Furthermore, during the middle of 1949 a correction began which resulted in an average price increase of more than 350 percent through the middle of 1955. The effect was a doubling of price-earnings ratios from about 6 times to approximately 12 times earnings.[1] In essence, the stock market behavior during the period 1945-54 could still be traced to the depression years creating a strong argument for exclusion.

Secondly, the period probably marked the beginning of the "institutional" investor. While mutual funds, pension funds, trust funds, and the like certainly existed prior to the late 1940's, their influence could hardly be considered significant not only in terms of volume but in terms of performance orientation as well. Therefore, it would be wrong to include a period when such an important market force was just emerging. However, it should be noted that the present influence from this segment of the capital markets may also represent an inaccurate reflection of the future.

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[1]Based upon the performance of the Standard & Poor's Composite Index. See: Standard & Poor's Trade and Securities Statistics: Security Price Index Record, 1966 ed. pp. 119-21.

Third, it is important to note that the early 1950's marked the beginning of the active use of monetary policy as a stabilization device with the advent of the now famous "accord." As a result, the nations' money and capital markets were once again faced with the possibility of cyclical interest rate fluctuations. However, it was not until several years after the "accord" that the Federal Reserve's stabilization policies began to have a significant impact. Thus the capital markets of the early 1950's were not yet operating under the influence of an active monetary policy.

Finally, it is interesting to reflect upon the relationship between bond yields and common stock prices that existed prior to about 1955. That is, until quite recently current yields on common stocks were expected to exceed yields on corporate bonds. When the reverse occured, one could confidently predict a corrective decline in stock prices. Unfortunately, since about 1955 such predictions have been wrong and for good reason. Specifically, if the corporate sector experiences very little real and monetary growth (as was the case in the 1930's), the current yield on common stocks represents the primary ingredient of the total return from that media, and thus, can be compared to the returns available from alternative investments. However, the existence of real long-term growth plus inflation reduces the importance of the current yield as a comparative device.

With the modern emphasis on economic growth, the historic behavior of current yields relative to bond yields is indicative of the fact that the years prior to about 1955 are not representative of the expectations existing in the contemporary economic environment.

Summary

In summary, it seems apparent that the periods prior to approximately 1955 are not likely to be applicable to the future behavior of common stock returns. As a result, the conclusions of the study are based upon the price and dividend performance of NYSE common stocks during the period December 31, 1954 to December 31, 1965.

Perhaps the strongest argument against the future applicability of this period lies in the fact that the increases in price-earnings ratios which began in the middle of 1949 continued past 1955 reaching a peak in early 1962.[1] The result was an average annual return of 13.1 percent[2] during the 1955-1965 period which is substantially above the long-term empirical record. It is for this reason that the methodology of the study is designed to assess the risks and returns of common stock portfolios in relation to the average market returns. In that way, changes in price-

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[1] Ibid.

[2] Fisher and Loris, "Rates of Return on Investments In Common Stock: The Year-By-Year Record, 1926-65," 6.

earnings ratios should not bias the results. Thus, the actual average return during these years should not alter the conclusions of the study.

The descriptive portion of the study is now complete. Therefore, it is appropriate to turn to the task of reporting and analyzing the results which is the subject matter of the following two chapters.

CHAPTER VI

THE PORTFOLIO SIZE REQUIREMENTS FOR THE LONG-TERM INVESTOR

Introduction

With the description of the data complete, it becomes appropriate to describe and analyze the results of the study. In view of the multiplicity of the results, the following discussion will be confined to an examination of the optimum diversification strategy of the common stock investor, during the 1955-65 period, who: (1) had an investment objective of long-term growth, (2) used simple random sampling as his method of diversification, and (3) had a time horizon of ten years. In the subsequent chapter, other portfolio conditions will be established in order to determine the effects of different investor objectives, variations in the length of the holding period, and alternative diversification techniques.

The Average Market Return

The probabilities

With regard to the probabilities associated with the achievement of a portfolio return which approximated the

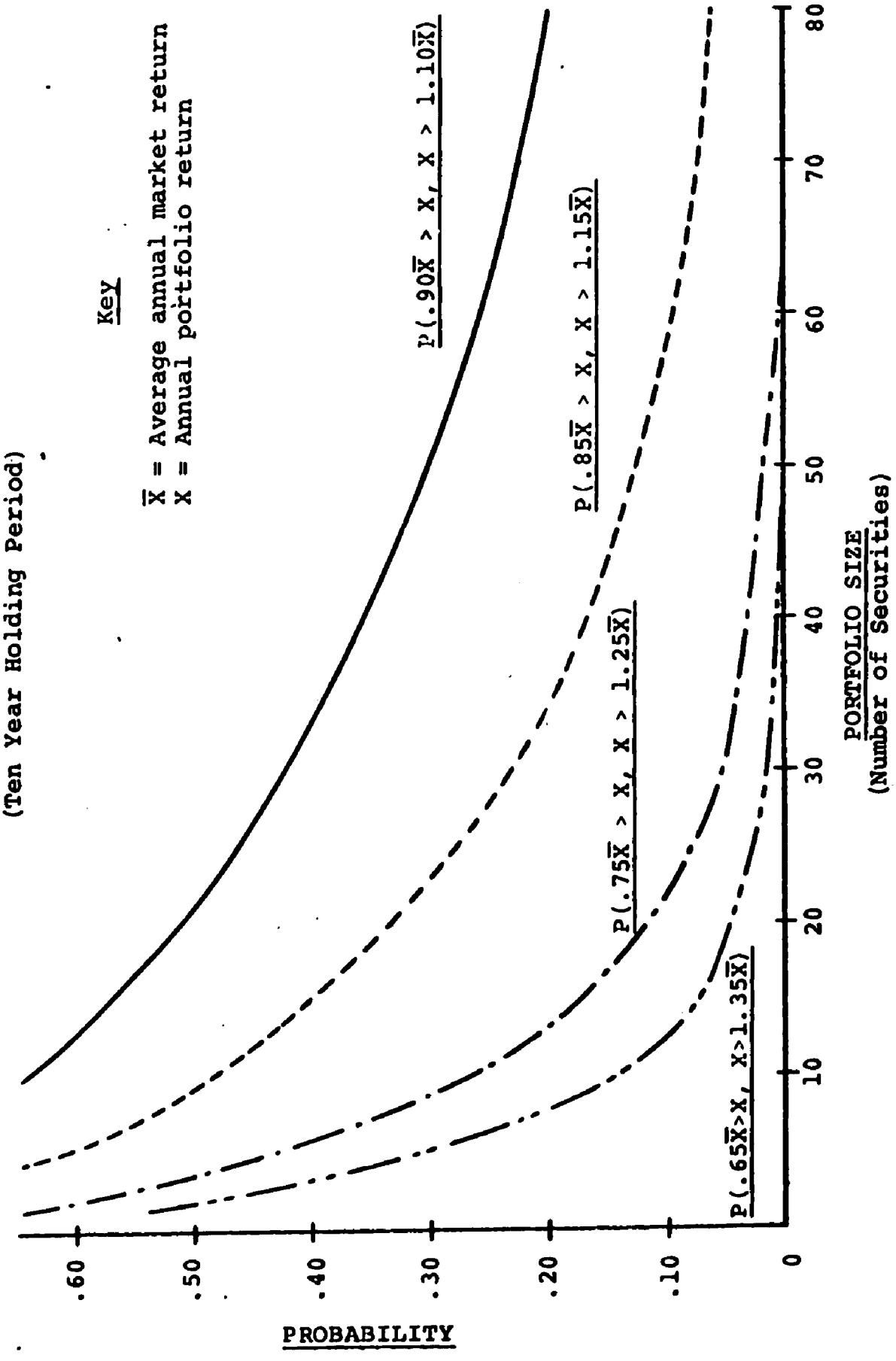
average market return, Chart 6.1 is presented for analysis. The horizontal axis of the chart depicts the portfolio size in numbers of securities, while the vertical axis is the probability that the portfolio return did not approximate the average market return. Each curve represents a unique range of deviations from the market average.

To illustrate, if a portfolio contained 20 randomly selected securities with equal funds placed in each, there was a probability of approximately 0.12 that the portfolio return would have been less than 75 percent of the market average or greater than 125 percent of the market average. If the range of acceptable deviations had been 90 and 110 percent of the market average, the probability that the actual portfolio return would have fallen outside this range is read from Chart 6.1 as approximately 0.50 given a portfolio size of 20 securities.

The initial objective of the analysis is to determine a portfolio size which had a "small" probability that the actual portfolio return would not have closely approximated the average market return. Thus, it is immediately apparent from Chart 6.1 that the investor faced a dilemma. In order to have obtained a small probability (less than 0.05) of not approximating the market average, he would have had to choose a large portfolio size unless his definition of "approximate" was wider than 65 and 135 percent of the market average. If

CHART 6.1

PROBABILITY OF A FAILURE TO ACHIEVE AN APPROXIMATION OF THE AVERAGE MARKET RETURN
(Ten Year Holding Period)



he insisted upon a more narrow definition of "approximate," he had to be content with either a large (greater than 0.10) probability that his actual portfolio return would not "approximate" the market average or a large portfolio size. In essence, it appears that very large portfolio sizes were required in order to satisfy the criterion of a small probability of a close approximation of the market average unless the terms "small" and "close" approximation are rather loosely defined.

The problem can be demonstrated more forcefully with the use of Table 6.1 which presents the various definitions of a "small" probability of a "close" approximation of the market average. The table emphasizes that the investor was required to make a choice between: (1) a high probability of a deviation from the market average; (2) a wide range of acceptable deviations, or close approximations; or (3) a large portfolio size. He could not have obtained "small" definitions of each. The implication is that while an individual investor may have been capable of making such a choice, it is inappropriate to make generalizations regarding a specific portfolio size applicable to all investors on the basis of these data. In effect, a trade-off between a small probability and a close approximation of the market average was required.

TABLE 6.1

PORTFOLIO SIZES AND PROBABILITIES ASSOCIATED WITH ACHIEVING AN APPROXIMATION OF THE AVERAGE MARKET RETURN (Ten Year Holding Period)

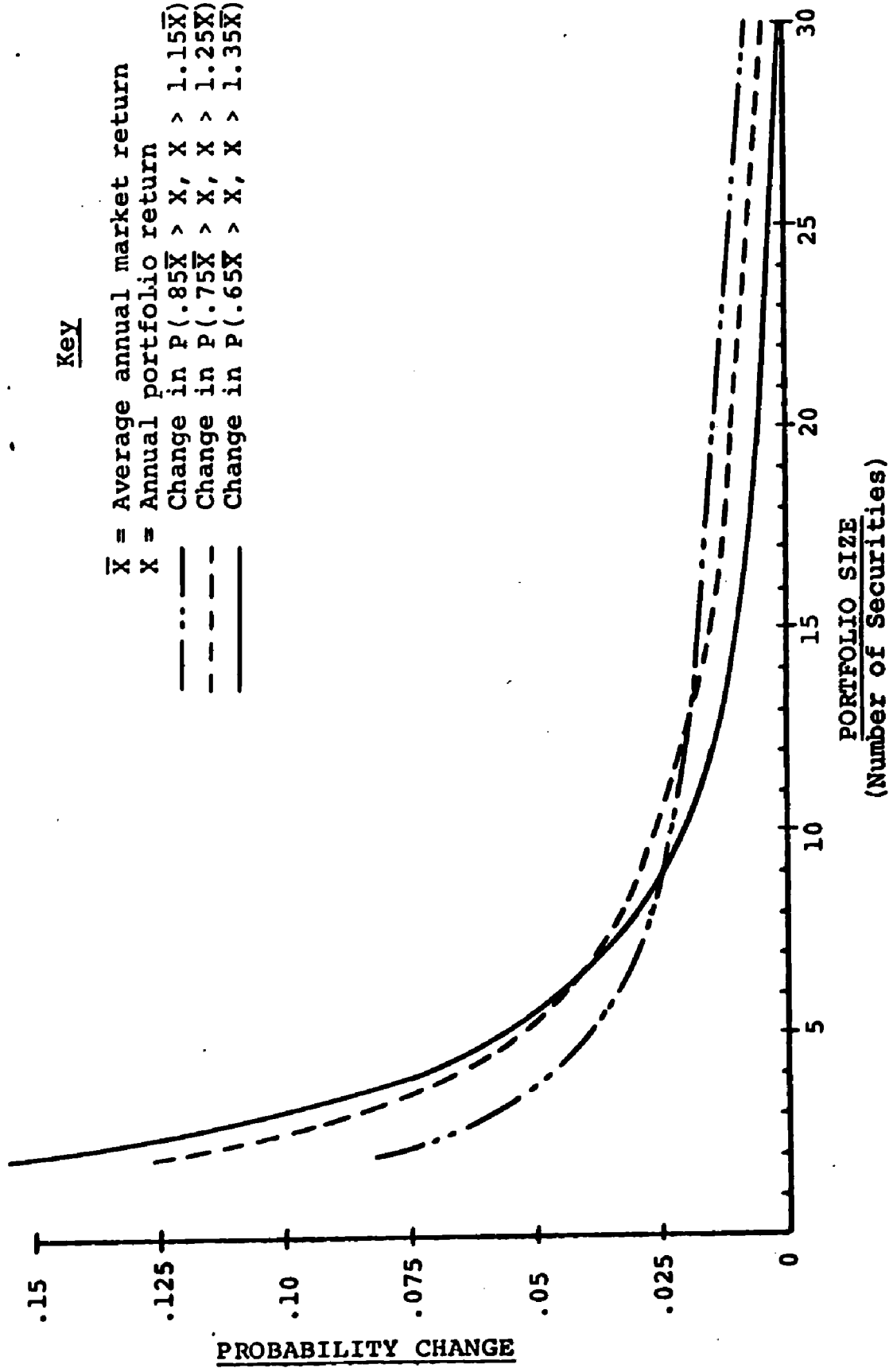
Probability of A Failure of the Portfolio Return to Fall Within The Indicated Range	Portfolio Size (Number of Securities)				
	Range of Portfolio Returns as a Percent of The Average Market Return				
	90% to 110%	85% to 115%	80% to 120%	75% to 125%	65% to 135%
0.05	178	79	48	32	19
0.10	125	56	33	22	13
0.15	95	44	25	17	10
0.20	76	35	20	13	8
0.25	62	28	16	11	6
0.30	50	21	13	9	5

The effects of security additions

Consequently, Chart 6.2 is introduced which has a horizontal axis depicting the portfolio size. The vertical axis describes the change in the probability of a failure to achieve an approximation of the market average, which results from the addition of one security to the indicated portfolio size. Each curve reflects a unique definition of an approximation of the market average. For example, the addition of the fourth security reduced the probability that the portfolio return would have been less than 85 percent of the market average or greater than 115 percent of the market

CHART 6.2

PORTFOLIO ADDITIONS: CHANGE IN PROBABILITY OF A FAILURE TO ACHIEVE THE AVERAGE MARKET RETURN
(Ten Year Holding Period)



average by approximately 0.05. When the range of deviations is expanded to 75 and 125 percent, the reduction is read as approximately 0.07.

An examination of Chart 6.2 reveals two significant relationships. In the first place, the change in the probabilities generated by security additions was, for all practical purposes, unaltered by variations in the range of deviations from the market average. That is, unless the investor was contemplating portfolio sizes of approximately 4 securities or less, differences in the range of acceptable fluctuations had no discernable effect upon the reduction in probabilities created through security additions.[1] The implication is that while the range of acceptable deviations altered the absolute level of probability, it did not affect the risk-reducing properties of security additions.

Secondly, and of greater importance, is the fact that the change in the probabilities which resulted from security additions became negligible for the portfolio sizes greater than approximately 16 to 18 securities. The result is that further reductions of portfolio risk required large increases in the size of the portfolio in order for the risk

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[1]This relationship is to be expected from the mathematical derivation of the distrib ution of sample means where all higher moments decline as a function of the sample size. Thus, the significant variable is the sample size.

reduction to be significant. Since the objective of diversification is to reduce risk, security additions to portfolio sizes of 16 to 18 did not accomplish the objective in a practical sense. Furthermore, while the costs of adding securities have not been given explicit treatment,[1] the underlying assumption is that such costs exist. Therefore, if the risk-reducing properties of further security additions were negligible, portfolio sizes larger than 16 to 18 would have been uneconomical regardless of the absolute level of the probability of a failure of the portfolio return to approximate the market average.[2]

Summary

To summarize this initial aspect of the analysis, it is apparent that very large portfolios were necessary in order to satisfy the requirement of a "small" probability of a failure of the portfolio return to "closely" approximate the average market return. With small portfolio sizes, a trade-off between the size of the probability and the definition of a close approximation of the market average was required. Nevertheless, it was equally clear that security

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[1]Except, of course, the opportunities for an exceptional gain which are foregone. This cost is examined below in the section entitled Diversification vs. Concentration.

[2]This approach and line of thinking is quite similar to that used in Evans and Archer, "Diversification and The Reduction of Dispersion: An Empirical Analysis," p. 762.

additions beyond a portfolio size of approximately 16 to 18 did not substantially reduce the probability of obtaining an approximation of the market average. Since portfolio additions were assumed to have involved a cost, larger portfolio sizes were not likely to produce an economic gain.

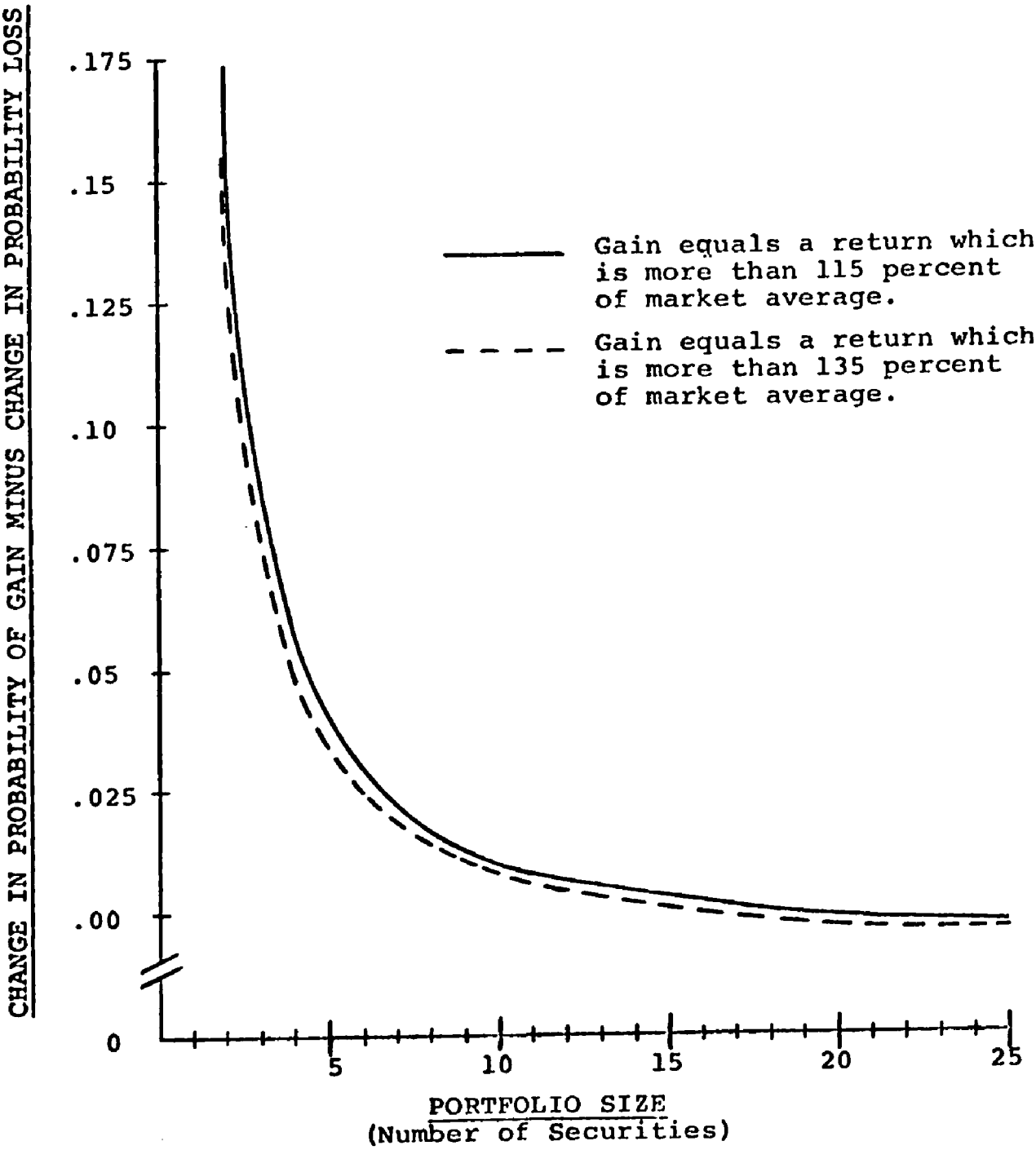
However, the conclusion has two fundamental limitations. In the first place, the absolute risk-averter in common stocks is more concerned with the "downside" risks of his portfolio. In other words, he is more concerned with the probability that his portfolio return will be less than, say, 75 percent of the market average than with the probability of a portfolio return which is, say, less than 75 percent and greater than 125 percent of the market average. Secondly, the investor with less risk-aversion may attempt to achieve a trade-off between risk exposure and the opportunity for an exceptional gain.

Diversification vs. Concentration

In order to provide insight into the second limitation, Chart 6.3 is introduced. The horizontal axis represents the portfolio size in numbers of securities. The vertical axis measures the change in the probability of an exceptional gain minus the change in the probability of a loss (defined as a return that is less than 75 percent of the market average) which resulted from the addition of one

CHART 6.3

PORTFOLIO ADDITIONS: CHANGE IN LOSS vs. GAIN PROBABILITIES
(Ten Year Holding Period)



Note: Loss equals a return which is less than 75 percent of market average.

security to the indicated portfolio size during the 1955-65 period. Thus, the positive portion of each curve represents the area where portfolio additions created a larger reduction in the probability of a loss than the reduction in the probability of an exceptional gain. The zero value represents the equal trade-off point. The negative area represents those portfolio sizes where the trade-off was unfavorable: i.e., the probability of an exceptional gain declined more rapidly than the probability of a loss.

Each curve represents a unique exceptional gain defined as a percentage of the average market return. For example, the addition of the sixth security produced a reduction in the probability of loss which was approximately 0.025 greater than the reduction in the probability that the portfolio return was more than 135 percent of the market average.

An examination of Chart 6.3 initially reveals that variations in the definition of an exceptional gain appear to have had little, if any, effect upon the trade-off between gains and losses.[1] The implication is that within the constraint of the gains which were available from the common

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[1]As with the change in the probability of not achieving an approximation of the market average, this relationship is to be expected since the extent of the changes were governed primarily by the size of the higher moments relative to the sample size.

stock population during the 1955-65 period, variations in individual gain definitions did not significantly alter the differences between the changes in the probability of losses and gains that accompanied security additions.

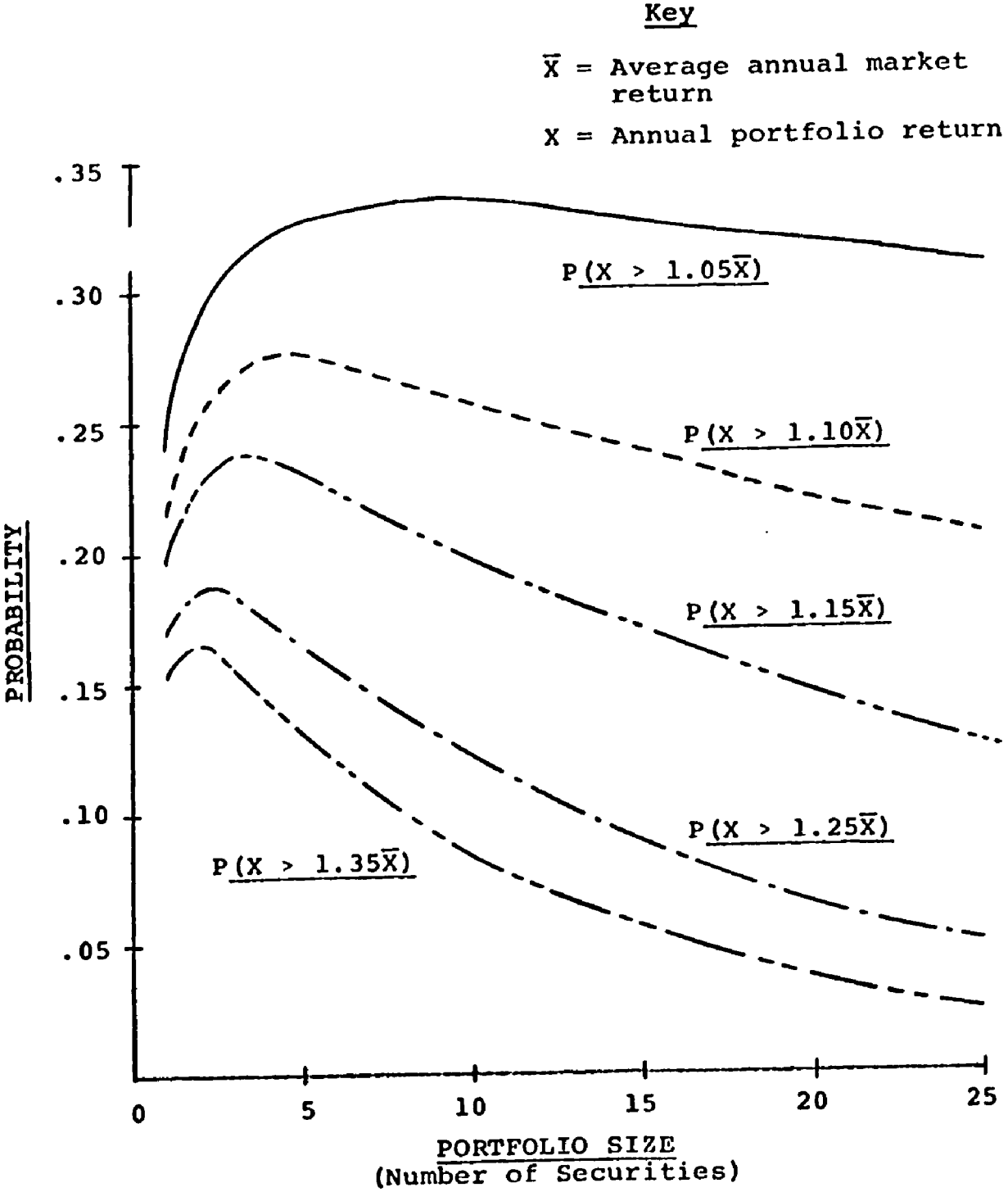
As a result, Chart 6.3 indicates that, in general, a portfolio size of 12 was sufficient to obtain an equal trade-off between loss and gain. Specifically, larger portfolios involved an equal or greater reduction in the probability of an exceptional gain than the reduction in the probability of a loss. It is true that portfolios larger than 12 continued to exhibit a positive trade-off, but the actual difference was so small that it can be considered zero in a practical sense. In essence, absolute equality was achieved at approximately 20 securities, but realistically one can conclude that 12 securities satisfied the trade-off criterion.

An Exceptional Gain

Reinforcement of this conclusion is obtained from several aspects of the data. In the first place, the probability of an exceptional gain behaved in an interesting manner which is demonstrated in Chart 6.4. The horizontal axis of the chart is the portfolio size while the vertical axis describes the probability of an exceptional gain. Each curve represents a unique definition of an exceptional gain. For example, a portfolio which contained 10 securities had

CHART 6.4

PROBABILITY OF AN EXCEPTIONAL GAIN
(Ten Year Holding Period)



a 0.20 probability that the portfolio return would have been more than 115 percent of the market average.

The chart reveals that the classic predictions of diversification theory are not entirely correct. In particular, it is generally assumed that security additions not only reduce risk but also reduce the opportunity to achieve an exceptional gain. However, Chart 6.4 illustrates that to a limited extent the investor could have actually increased the probability of an exceptional gain through the addition of securities to his portfolio. For example, the probability of achieving a portfolio return which was more than 105 percent of the market average increased with security additions up to a portfolio size of 9 securities. At this point the expected decline began. If the definition of an exceptional gain is expanded to a return which is greater than 110 percent of the market average, the probability of an exceptional gain increased up to a portfolio size of 5 securities.

Thus, as the criteria for an exceptional gain was widened in terms of a target departure from the market average, a fewer number of securities was required to achieve the optimum probability of achieving such gains, but in all cases presented,[1] some diversification was required in order

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[1]It should be noted that "very large" exceptional gain criteria are not associated with probability increases with security additions.

to achieve the maximum probability of obtaining an exceptional gain. The implication is that some diversification would have been appropriate even for the investor willing to accept a large amount of risk in order to maximize his gain opportunities. In addition, the result provides an explanation of the very favorable differences between changes in the probabilities of gains and losses up to approximately 12 securities and the negligible change differences found to have existed for larger portfolios.[1]

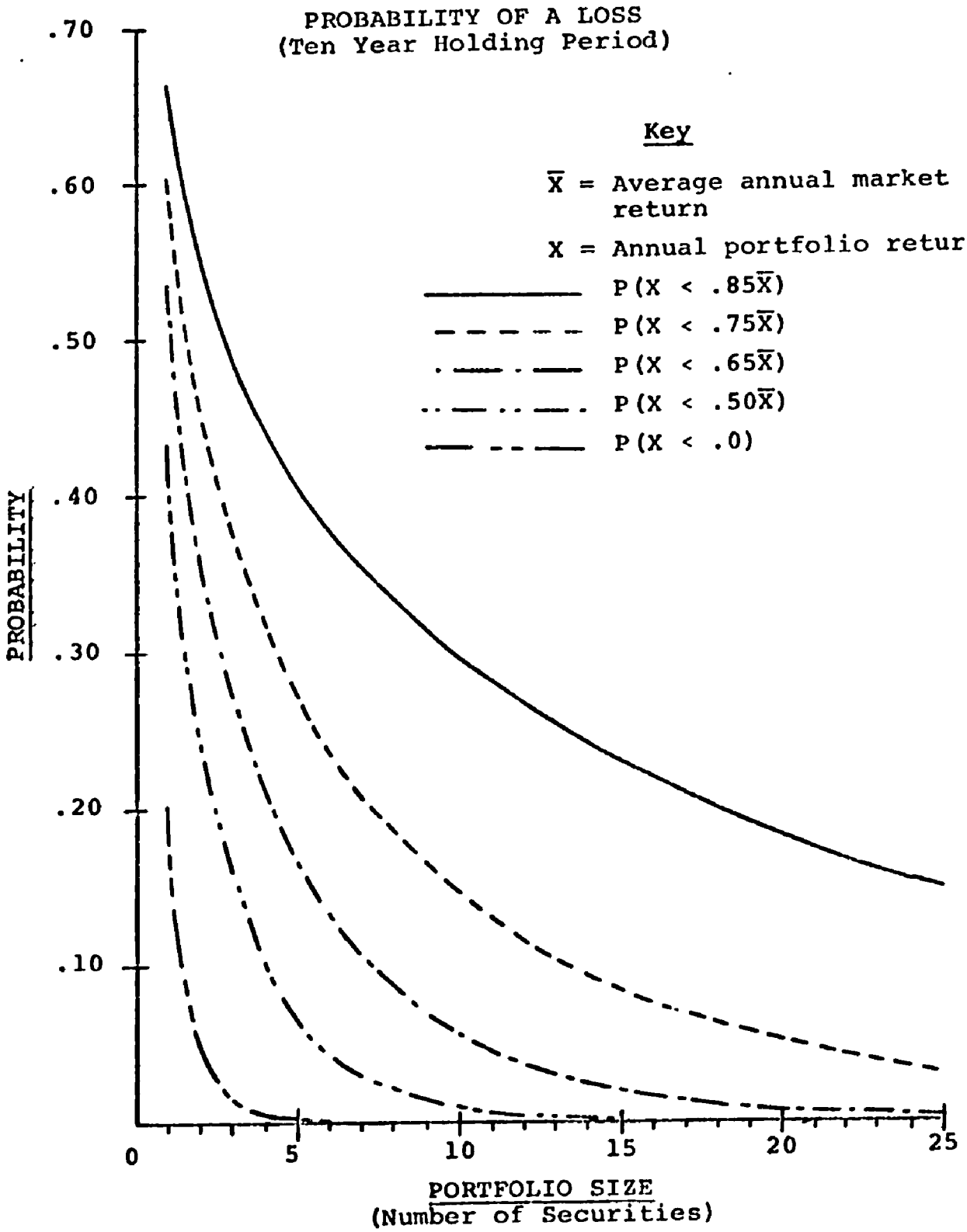
The Absolute Level of Risk

It is also important to recognize that a portfolio which contained 12 securities selected at random with equal funds placed in each provided the investor with considerable risk protection. This condition is demonstrated with Chart 6.5 which depicts the portfolio size along the horizontal axis. The vertical axis is the probability of loss. Each curve represents a unique loss definition. To illustrate, when there were 4 securities in the portfolio there was a 0.10 probability that the actual return would have been 50 percent or less than the average market return.

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[1]The fact that security additions created an increasing probability of an exceptional gain is related to the skewness of the distribution of wealth ratios. The theoretical explanation of the relationship is provided in Appendix A, which is presented at the conclusion of this chapter.

CHART 6.5


PROBABILITY OF A LOSS
(Ten Year Holding Period)
Key
$\bar{X}$ = Average annual market return
X = Annual portfolio return
P(X < .85$\bar{X}$)
P(X < .75$\bar{X}$)
P(X < .65$\bar{X}$)
P(X < .50$\bar{X}$)
P(X < .0)
PROBABILITY
.70
.60
.50
.40
.30
.20
.10
0
5
10
15
20
25
PORTFOLIO SIZE
(Number of Securities)

The striking feature of Chart 6.5 is the magnitude of the probabilities.[1] When the portfolio contained 12 securities, the probability of a zero return was virtually nonexistent as was the probability of a return which is less than 50 percent of the market average. The loss probabilities became measurable only when the loss point was expanded to a return which is less than 65 percent of the market average. Even here the probability of loss was less than 0.04 with 12 securities. Regarding a loss definition which is less than 75 percent of the average market return, the probability of loss is read as approximately 0.12 when the portfolio contained 12 securities. Thus, it seems apparent that this portfolio size not only produced an equal trade-off between risk exposure and exceptional gain opportunities, but also provided the investor with a low absolute level of risk.

The implication is that 12 securities is applicable to a wide range of risk-aversion schedules. That is, as the investor's aversion for risk increases, it is assumed that he will desire a lower absolute level of risk. Since a portfolio size of 12 involved a small level of risk, it would seem that increases in the investor's aversion for risk would not generate a requirement of a portfolio size greater than 12 securities.

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[1]The probabilities presented here are much lower than those relating to the achievement of an approximation of the market average since the concern is with the left-hand tail of the distribution rather than both tails.

Other Loss Definitions

It has been noted[1] that generalizations also require an examination of other loss definitions. Specifically, an investor less concerned with risk may be inclined to increase his loss exposure when examining the trade-off between risk and gain. As a consequence, Chart 6.6 is introduced for analysis. The chart is similar to Chart 6.3 in that it reflects the difference between the change in the probability of an exceptional gain and the change in the probability of a loss for each portfolio addition. The difference is in the definition of a loss. The higher curve defines a loss as a return which is less than 30 percent of the market average, and the lower curve defines a loss as a portfolio return which is less than zero. Both curves define an exceptional gain as a return which is more than 115 percent of the market average.

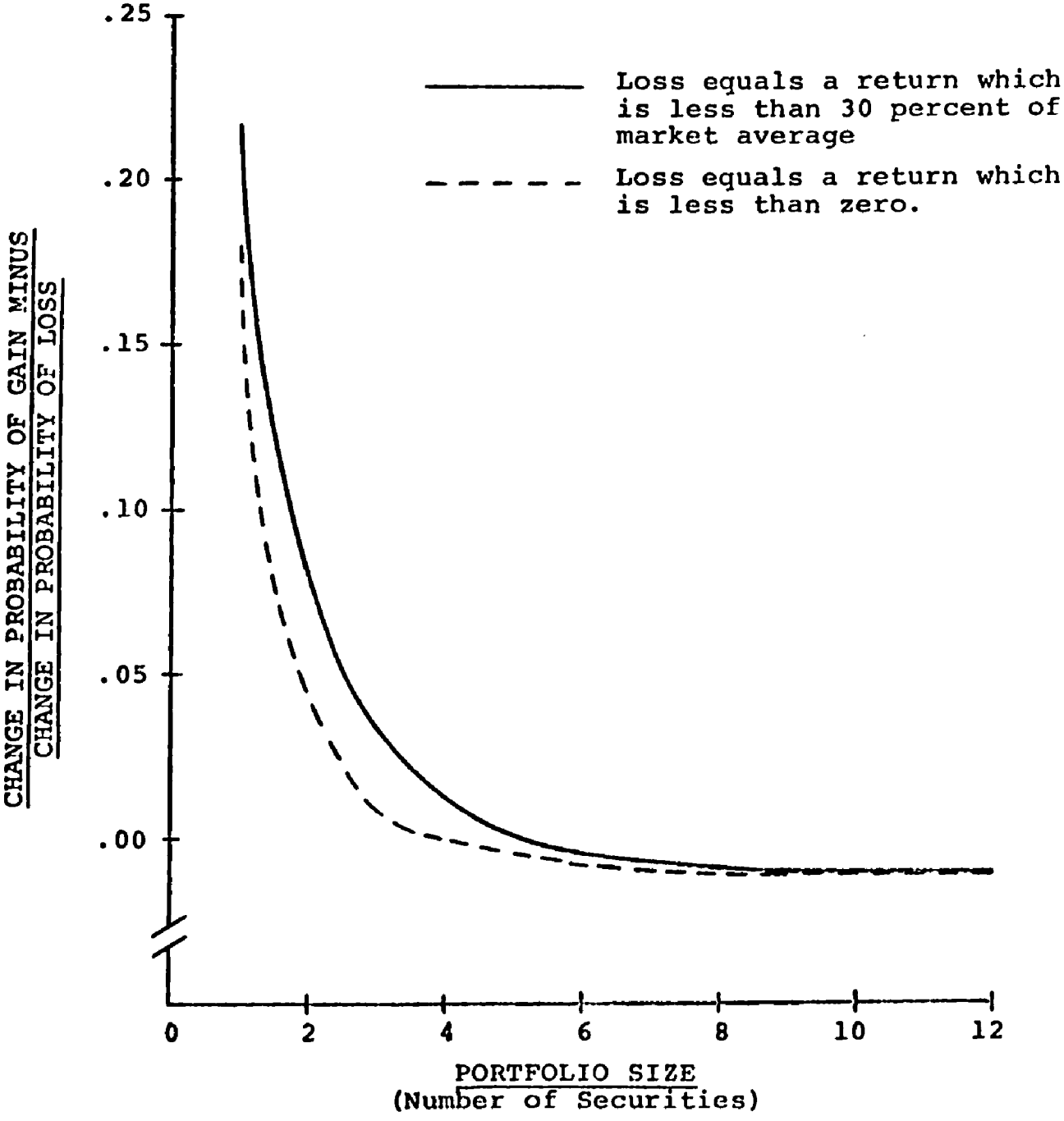
The curves indicate an equal trade-off point at a portfolio size of approximately 6 securities which is consistent with the hypothesis that less aversion for risk implies a lower portfolio size requirement. At first glance, then, it would appear that the investor would have had to select a portfolio size between a range of 6 and 12 securities where the choice was a function of his particular attitude toward risk.

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[1] See, supra, pp. 65-66.

CHART 6.6

PORTFOLIO ADDITIONS: CHANGE IN LOSS vs. GAIN PROBABILITIES (Ten Year Holding Period)



Note: Gain equals a return which is more than 15 percent of market average.

However, prior to making such a conclusion, it should be recognized that lower loss definitions are appropriate only under certain conditions. It was suggested[1] that less aversion for risk implies a willingness to accept a greater loss exposure provided the potential gains are sufficiently rewarding. Thus, in order to accept the lower portfolio size which was associated with these loss points, it must be shown that the gain opportunities were substantially improved.

As is apparent from Chart 6.4, this requirement did not occur. If the definition of an exceptional gain is relatively close to the market average, a reduction of the portfolio size from 12 to 6 actually produced a decline in the probability of an exceptional gain. If the definition of an exceptional gain is increased, the probability of an exceptional gain was higher with a portfolio size of 6 than with a size of 12, yet the difference was small. For example, the probability of realizing a portfolio return which was greater than 135 percent of the market average was approximately 0.07 with 12 securities. When the portfolio size was reduced to 6, the probability of a similar return increased to 0.12.[2]

In essence, the lower portfolio size which resulted

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[1] See, supra, pp. 65-66.

[2] See, Chart 6.4, p. 136.

from a willingness to accept greater risk exposure did not significantly increase the investor's opportunities to achieve an exceptional gain. Therefore, the result suggests some doubt that even investors concerned primarily with gain opportunities would have selected a portfolio size less than 12 or so securities, assuming simple random selection. It should be recognized, however, that the conclusion is not applicable to the extreme of zero, or very little, risk aversion since any reduction in the probability of an exceptional gain is unacceptable given a complete absence of risk aversion.

Summary

To sum up this section of the analysis, it is suggested that, during the 1955-65 period, optimum diversification would have been obtained with a portfolio size of approximately 12 securities under the investment conditions of a ten year holding period, simple random selection, and an objective of long-term growth. At that size the difference between the change in the probability of a loss and the change in the probability of an exceptional gain was approximately zero. A loss was defined as a return which is less than 75 percent of the market average. Variations in the definition of an exceptional gain did not alter the conclusion.

Furthermore, the evidence indicated that 12 securities was applicable to a wide range of risk-aversion schedules. If the investor fell into the risk-neutral category he would have selected this portfolio size based upon the equal trade-off criterion. If the investor placed the primary emphasis upon risk, he would have been content with 12 securities since the probability of loss was low. Finally, the investor who was willing to accept greater risk exposure would have chosen this portfolio size since smaller portfolios did not generate a significant increase in the opportunity for an exceptional gain. It was recognized that the conclusion was not applicable to the investor willing to accept a very large amount of risk. However, even here the evidence suggested that a small amount of diversification would have been appropriate.

It should be emphasized that the conclusion is limited in its applicability by the fact that no consideration was given to variations in the investor's holding period, different investment objectives, or other diversification strategies. Therefore, it is necessary to examine the effects of each of these variables which is the subject matter of the following chapter.

Other Research

However, before proceeding, it is interesting to compare the initial conclusion to the results of other diversification studies. In particular, Evans and Archer conclude that:

> The results also raise doubts concerning the economic justification of increasing portfolio sizes beyond 10 or so securities.[1]

The similarity of this conclusion to the results presented here suggests a highly stable relationship in view of the substantial differences in methodology. That is, Evans and Archer obtained their data from the 1958-67 period using 470 of the securities contained in the 1958 Standard & Poor's Composite Index. The measure of dispersion was the standard deviation of the semi-annual returns of each security around the 1958-67 average annual return of the particular security. Skewness was considered by using the natural log of the returns. The decision criterion employed was the change in the standard deviation of the portfolio return created by security additions. Finally, the portfolio standard deviations were obtained by simulating the random selection of one portfolio for each portfolio size.

In another study Eiteman and Eiteman suggest that:

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[1]Evans and Archer, "Diversification and The Reduction of Dispersion: An Empirical Analysis," p. 767.

> ...almost any combination consisting of nine or more issues and picked at random is certain to approach the typical portfolio return.[1]

Again, the similarity of the conclusions is striking in view of the substantial differences in approach. Eiteman and Eiteman utilized data from the 1950-1961 period, and in an earlier study Eiteman and Smith used the 1937 to 1950 period.[2] In each analysis only 91 securities were used to estimate the population. What is more important, the 91 issues were selected on the basis of trading volume, and thus cannot be considered as a random sample of the population. In addition, dollar-averaging was employed, and no adjustments were made to reflect differences between the actual market average and the long-term expected return. Finally, selection was without replacement, and no specific considerations were given to the characteristics of the distribution of returns.

However, there is a possible inconsistency between the results of this examination and the conclusions made by Eiteman and Eiteman. Specifically, Eiteman and Eiteman imply that 9 securities were sufficient in order to assure

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[1] Eiteman and Eiteman, Common Stock Values and Yields, 1950-61: A Study of Common Stocks as Long-Term Investments With Particular Emphasis on Investment Yields, p. 40.

[2] Eiteman and Smith, Common Stock Values and Yields: A Study of Common Stocks as Long-Term Investments With Particular Emphasis on Investment Yields.

a close approximation of the market average. In contrast, it was suggested here that while a portfolio size of 12 was adequate, it is also suggested that "very large" portfolios were required in order to obtain assurance that the portfolio return would have approximated the market average.[1]

The inconsistency is illusory. In particular, Eiteman and Eiteman based their conclusions upon the actual results of 9 separate portfolios ranging in size from 47 securities to 9 securities.[2] Of these portfolios, the worst performance was an average annual return of approximately 8.0 percent[3] which is 56 percent of the average market return of 14.2 percent utilized in their study. The next worst portfolio produced an annual return of approximately 9.7 percent which is 68 percent of the market average. The best performance was an annual return of approximately 19.0 percent which is 134 percent of the market average.

In effect, 3 of the 9 portfolio returns fell outside the range which has upper and lower boundaries of 125 and

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[1] See, supra, p. 130.

[2] Eiteman and Eiteman, Common Stock Values and Yields, 1950-61: A Study of Common Stocks as Long-Term Investments With Particular Emphasis on Investment Yields, p. 17.

[3] In the Eiteman and Eiteman study, the measure of performance was quite similar to the wealth ratios used here. For purposes of convenience and clarity, the present author has converted these values to their equivalent annual returns. For the actual values used by Eiteman and Eiteman see, Ibid.

75 percent of the market average, respectively. In the context of this study, reference to Table 6.1[1] indicates that when a portfolio contained 9 securities, there was a 0.30 probability that the portfolio return would have fallen outside the range of 75 and 125 percent of the market average. Therefore, it would appear that inconsistencies do not in fact exist. The difference lies in the fact that Eiteman and Eiteman based their conclusions upon a wider range of acceptable deviations around the market average.

As a consequence, it seems apparent that the results of this study are generally consistent with other research into the problem of common stock portfolio size requirements despite significant methodological differences. Nevertheless, it remains necessary to examine the effects of variations in the investor's objectives, other diversification strategies, and reduced holding period lengths. It is these factors with which the following chapter will be concerned.

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[1]See, supra, p. 128.

APPENDIX A

AN INCREASING PROBABILITY OF AN EXCEPTIONAL GAIN

In the preceding discussion it was shown that, under certain conditions, the probability of an exceptional gain increased with portfolio additions which is contrary to the general assumptions of diversification theory. The cause of the increase lies in the fact that the distribution of wealth ratios was skewed to the right. To understand the relationship between skewness and the behavior of the exceptional gain probabilities, attention is directed to Figure 1.

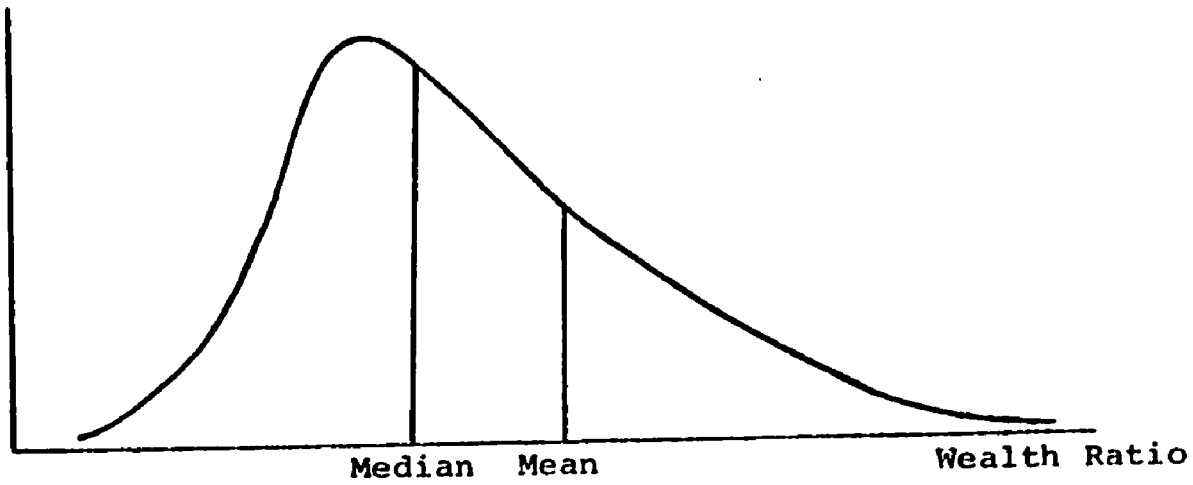


Figure 1 -- A probability distribution which is skewed to the right.

The essential feature of Figure 1 is that the median is less than the mean, or average. If the distribution were normal, the median would equal the mean. Since the area to the right of the median is always 0.50, it follows that the area to the right of the mean must be less than 0.50 when the distribution is skewed to the right. Thus, if one item is selected at random, the probability that its value will exceed the mean will be less than 0.50 when the distribution is skewed to the right.

However, the Central Limit Theorum suggests that the distribution of sample means will be asymptotically normal as the sample size approaches infinity assuming mild regularity conditions of the parent population. Therefore, the median of the distribution of sample means will approach the population mean as the sample size is increased. In other words, the area to the right of the mean will approach 0.50 as the sample size is increased.

Consequently, when the parent distribution is skewed to the right, the probability that the value of a given sample mean will exceed the population mean will increase as the sample size is increased. In the context of common stock investment, the implication is that the investor can increase the probability of his portfolio return exceeding the market average simply by adding securities to his portfolio.

A similar situation is encountered with regard to the behavior of the probability of an exceptional gain or a return which exceeds some value which is greater than the market average. The difference is that the behavior of the probabilities is not general. That is, whether or not the probability of an exceptional gain will actually increase with portfolio additions, or decline more slowly than would be expected under a normality assumption, will depend upon the following variables: (1) the extent to which the parent distribution is skewed to the right (increases in exceptional gain probabilities are more likely with extensive skewness); (2) the location of the exceptional gain value (higher values are less likely to be associated with increases in the probability of an exceptional gain as securities are added); and (3) the size of the portfolio prior to the addition of the next security (probability increases can be anticipated with "small" portfolios). The actual behavior of the probability of an exceptional gain, then, is an empirical question which depends upon the interaction of these three variables.

CHAPTER VII

THE EFFECTS OF ADDITIONAL VARIABLES UPON THE PORTFOLIO SIZE REQUIREMENTS OF THE LONG-TERM INVESTOR

Introduction

In the preceding chapter, the portfolio size requirements were suggested for the investor whose objective was long-term growth, utilized simple random sampling as his method of diversification, and had a ten year time horizon. Therefore, in order to make generalizations, the following discussion is directed at the effects of each of these variables. Specifically, reduced holding period lengths will be examined in order to determine a minimum time horizon which is applicable to the long-term investor. Secondly, the discussion will consider the investment objectives of principal protection and current income. Finally, the effects of time and inter-industry diversification will be examined.

The Holding Period Length

It has been suggested that a minimum time horizon for the long-term investor can be defined as one in which longer holding period lengths do not significantly alter the port-

folio size requirement.[1] Since the portfolio size established thus far relied upon the trade-off between changes in the probability of a loss and changes in the probability of an exceptional gain as well as the absolute level of the probability of loss, it is these criteria which will be used to establish a minimum holding period length.

Diversification vs. concentration

Turning to the trade-off between loss and gain, Chart 7.1 is introduced to aid the analysis. The horizontal axis depicts the length of the holding period in years. The vertical axis reflects the portfolio size where further security additions involved a reduction in the probability of an exceptional gain which was approximately equal to the reduction in the probability of loss. A loss is defined as any portfolio return which was less than 75 percent of the market average. An exceptional gain is undefined in view of the negligible effects that definitional variations had upon earlier results.[2]

An examination of Chart 7.1 reveals that holding periods of eight years or longer had virtually no effect

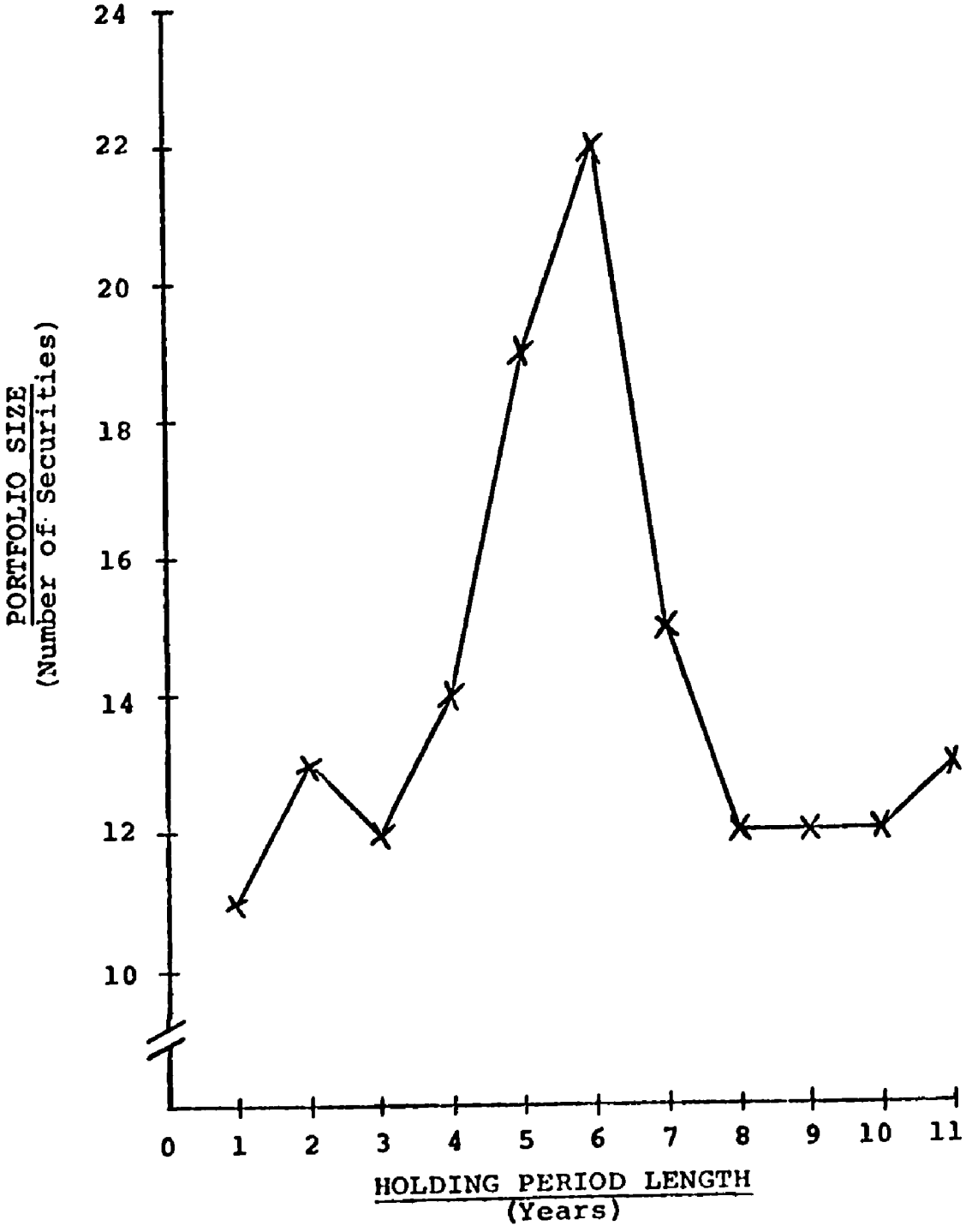
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[1] See supra, pp. 69-70.

[2] That is, the point of equal trade-off for a ten year holding period did not change when the definition of an exceptional gain varied either up or down. See supra, pp. 134-136. The same condition is encountered with other holding period lengths.

CHART 7.1

CHANGE IN LOSS vs. GAIN PROBABILITIES:
NUMBER OF SECURITIES EQUATING
SUCH PROBABILITIES



Note: Loss equals a return which is less than 75 percent of market average. Gain undefined.

upon the portfolio size which equated the changes in the probability of a loss with the changes in the probability of an exceptional gain. Specifically, holding periods of eight, nine, ten, and eleven years each produced an equal trade-off at a portfolio size of approximately 12 securities. In addition, it is important to note the range of portfolio sizes that exist for the holding period lengths presented. The largest portfolio size contained 22 securities and the smallest contained 11 securities. Moreover, if the holding periods of four through seven years are excluded, the range becomes 11 and 13 securities.

The implication would seem to be that the long-term investor required a minimum time horizon of eight years since longer periods did not alter the portfolio size which equated the change in the probability of loss with the change in the probability of an exceptional gain. On the other hand, it is also apparent that very short periods did not substantially alter the size requirements. In other words, from Chart 7.1 it would appear that a portfolio size of approximately 12 securities was applicable to both the short-term and the long-term investor, but that the intermediate-term investor required a somewhat larger number of securities.

The effects of skewness

The reason for this result is contained in the change in the characteristics of the distribution of wealth ratios as the length of the holding period is varied. It was demonstrated in Appendix A of Chapter VI that the more a distribution is skewed to the right, the greater is the likelihood that the probability of an exceptional gain will increase with security additions. At the very least, the rate of decline of the probability of an exceptional gain will be smaller with increased skewness to the right.[1] Therefore, larger portfolio sizes should be associated with larger right skewness when the portfolio size is based upon the difference between changes in the probability of losses and gains as securities are added to the portfolio.

The actual existence of this condition, during the 1955-65 period, is demonstrated in Chart 7.2 which has a horizontal axis depicting the length of the holding period in years. The left vertical axis describes a general measure of skewness[2] where a zero value would indicate a symmetrical distribution.[3] The right vertical axis is the

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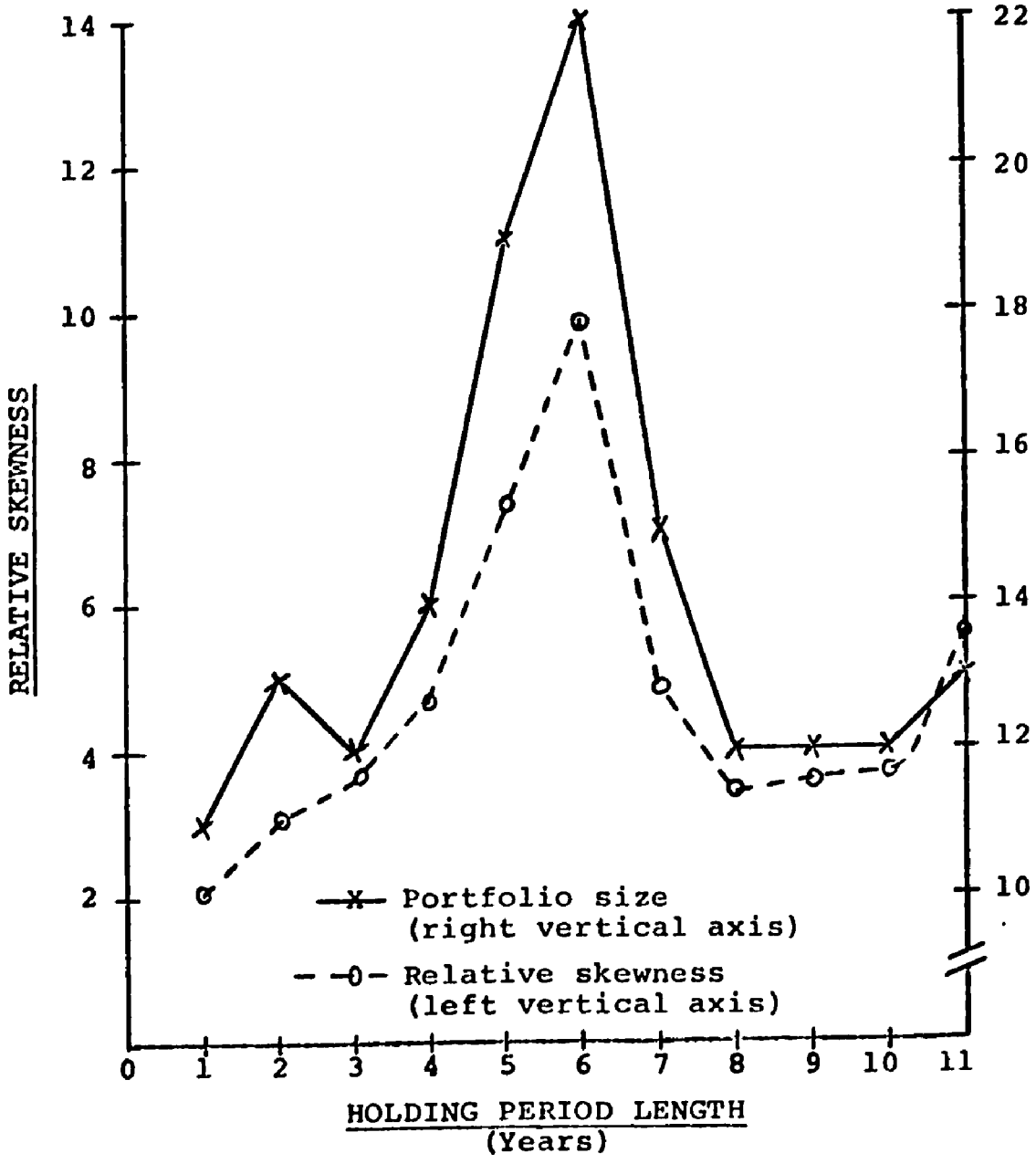
[1] See supra, pp. 150-152.

[2] Which is γ_1, in equation (3) of Chapter IV (see supra, p. 93) and γ_1, in Kendall and Stuart, The Advanced Theory of Statistics, Vol. 1, p. 85.

[3] Although it should be recalled that a non-symmetrical distribution could produce a zero value. A negative value would indicate left skewness.

CHART 7.2

COMPARISON OF DISTRIBUTION SKEWNESS TO NUMBER OF SECURITIES EQUATING CHANGES IN LOSS AND GAIN PROBABILITIES



Note: Loss equals a return which is less than 75 percent of market average. Gain undefined.

number of securities in the portfolio, and is included in order that the portfolio size curve of Chart 7.1 can be superimposed upon the skewness curve.

The striking feature of Chart 7.2 is the close relationship between the extent of positive skewness and the portfolio size which equates the probability changes. While the relationship is not exact, the curves indicate that the dominate variable affecting the difference between the changes in the probability of gains and losses with security additions was the extent of skewness exhibited by the distributions.

A determination of the underlying financial conditions which led to the substantial increases in skewness for holding period lengths of four to seven years goes beyond the scope of the analysis. However, the results suggest an interesting area of further research. Namely, what financial and/or economic factors are responsible for the behavior of skewness. In particular, why did the intermediate holding period lengths of four to seven years exhibit unique qualities, and why was the extent of skewness otherwise roughly constant with time?[1]

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[1]It should be noted that in terms of absolute values, skewness cannot be considered constant. However, in terms of the affect upon the portfolio size and the actual probabilities, the results would seem to indicate that "constant" can be defined rather loosely.

The absolute probability of loss

It has been recognized that in addition to the trade-off between a loss and a gain, the investor will wish to examine the absolute level of risk. It was further noted that a portfolio size of 12 not only produced an equal change in the probability of loss and the probability of an exceptional gain, but also provided the investor with a low absolute level of risk when the holding period was ten years. Therefore, in order to determine the significance of time, both criteria must be examined with respect to the holding period length.

That the absolute level of risk changed substantially with time is illustrated in Table 7.1. Thus, for example, the probability that the actual portfolio return would have been less than 75 percent of the market average was approximately 0.11 when the holding period was eleven years and the portfolio contained 12 securities. When the holding period was one year the corresponding loss probability was approximately 0.37.

Table 7.1, then, reveals quite clearly the significance of time where longer holding periods reduced the absolute level of risk quite markedly as the common stock theory predicts. In terms of establishing a minimum holding period for the long-term investor, the probability of loss data indicate that a holding period of about seven years was required. That is, at a holding period length of six years

TABLE 7.1

PROBABILITY OF LOSS FOR A PORTFOLIO OF TWELVE RANDOMLY SELECTED SECURITIES (By Holding Period Length)

Holding Period Length (Years)	Probability of Loss		
	Loss Defined as a Portfolio Return Which Is Below The Indicated Percent of The Average Market Return		
	75%	65%	50%
1	.37	.31	.22
2	.35	.26	.16
3	.32	.22	.12
4	.31	.20	.09
5	.30	.18	.06
6	.29	.15	.03
7	.19	.09	.02
8	.16	.07	.01
9	.15	.06	.01
10	.12	.04	.01
11	.11	.03	*

*Less than .005

or less, the probability of loss was large in an absolute sense, and at seven years there was a substantial decline in the probability of loss. For example, the probability of a return which is less than 75 percent of the market average was reduced from 0.29 to 0.19 when the holding period was lengthened from six to seven years.[1]

Beyond seven years the changes were less dramatic. In particular, the probability of loss was small and changed little with time. For example, the probability of a portfolio return which is less than 75 percent of the market average was 0.19 with a seven year holding period. When the holding period was extended to eleven years, the corresponding probability of loss declined to only 0.11. This condition, coupled with the fact that the portfolio size which equated changes in the probability of loss with changes in the probability of an exceptional gain was not altered by an extension of the holding period beyond eight years, suggests that the long-term investor required a holding period of at least eight years in order to choose a portfolio size of 12 securities selected at random with equal funds placed in each.

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[1]Arithmetically, this probability change is closely related to the substantial change in skewness which was associated with a lengthening of the holding period from six to seven years. (See supra, Chart 7.2, p. 158.)

Summary

In summary, it seems apparent that a portfolio size of 12 securities was appropriate for the long-term investor during the 1955-65 period provided the time horizon was at least eight years. The conclusion is based upon the following characteristics of an eight year holding period: (1) a small absolute level of risk, (2) small changes in the absolute level of risk with longer holding period lengths, and (3) a point of equal trade-off between probability changes which resulted from security additions that did not change with longer holding period lengths.

Shorter holding periods appeared to have produced substantially different characteristics. In particular, time horizons of six years or less were associated with a high level of risk that could have been considerably reduced with a longer holding period. Furthermore, holding period lengths of four to seven years required a larger portfolio size in order to satisfy the criterion of an equal trade-off between the changes in the probability of loss and the changes in the probability of an exceptional gain.

It should be recalled, however, that the conclusion is reached without an examination of holding periods longer than eleven years. Moreover, no attention has been given to the possible fluctuations in the "long-term" market average under the assumption that common stock diversifi-

cation is not designed to protect against these risks.

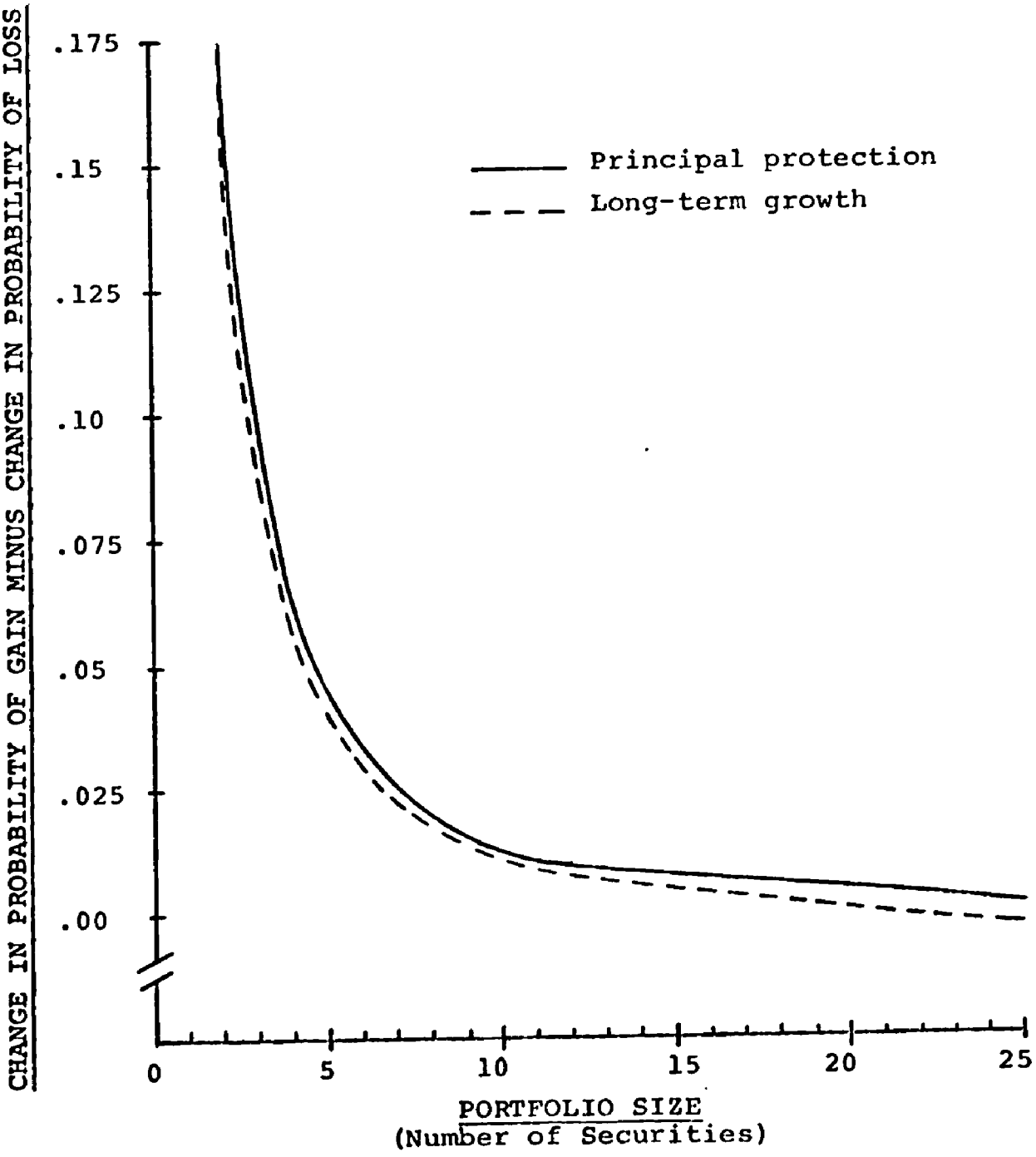
Principal Protection

As might be expected, the objective of principal protection involved the same portfolio size requirement as when the objective was long-term growth during the 1955-65 period. In terms of the differences between the changes in the probability of loss and changes in the probability of an exceptional gain, the indicated size was 12 securities. To illustrate, Chart 7.3 is presented which, as before, plots the difference between changes in the probability of a loss and changes in the probability of an exceptional gain. A loss is defined as any return which is less than 75 percent of the market average. An exceptional gain is any return which is more than 115 percent of the market average. The holding period was ten years.

For comparative purposes, the corresponding long-term growth curve is also presented on Chart 7.3 which enables one to clearly recognize the indistinguishable difference between the requirements of the two objectives. The similarity is to be expected since the only difference between the two measures of performance was the treatment of dividends. The objective of principal protection excluded dividends while the long-term growth objective assumed that dividends were reinvested. In either case, it has been sug-

CHART 7.3

PORTFOLIO ADDITIONS: CHANGE IN LOSS vs. GAIN PROBABILITIES (Ten Year Holding Period)



Note: Loss and gain equal returns which are less than and more than 75 and 115 percent of market average, respectively.

gested that the dominate variable will be the growth of principal.[1]

With regard to the absolute probability of loss, or the risk that the investor's original principal will be impaired, a portfolio size of 12 again provided a negligible probability of loss. The situation is demonstrated by Chart 7.4 which relates the probability of a loss for a portfolio composed of 12 securities to the length of the holding period. The upper curve defines a loss as any return (exclusive of dividends) which is less than 50 percent of the market average. The middle curve defines a loss as any return which was not sufficient to maintain the purchasing power of the investor's original principal.[2] The lower curve represents a zero return such that the monetary value of the original principal would have been unimpaired.

The chart suggests that unless the time horizon was "short-run," the absolute level of risk can be considered to have been insignificant when the portfolio contained 12 securities. For example, a holding period length of eight years was associated with a probability of a purchasing

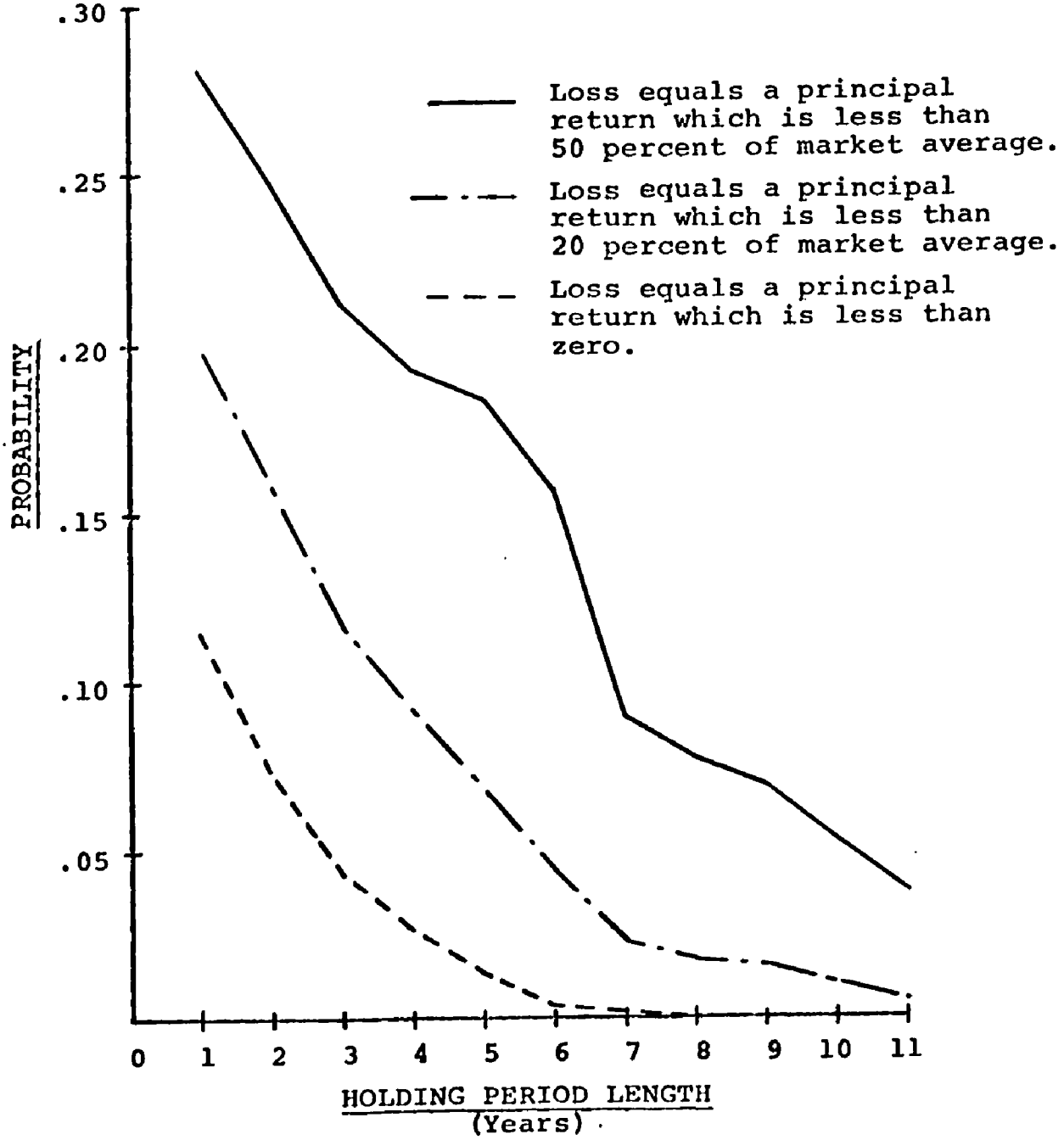
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[1] See supra, pp. 32-33.

[2] During the 1955-65 period, the Consumer Price Index advanced at an average annual rate of 1.55 percent. Thus, a growth of principal greater than 1.55 percent per year was considered sufficient to maintain purchasing power and was equal to a growth rate which was 20 percent of average market growth rate.

CHART 7.4

PROBABILITY OF A PRINCIPAL LOSS
FOR A PORTFOLIO SIZE OF 12
(Different Holding Periods)



power loss of approximately 0.02. The probability of a monetary loss was, for all practical purposes, zero for holding periods longer than five years.

As a result, it seems clear that the investor interested in principal protection would have had the same portfolio size requirements as the long-term growth investor. Specifically, principal protection was satisfied with a portfolio which contained 12 securities selected at random provided the holding period length was eight years or more.[1] As indicated, the explanation of the similarity lies in the fact that the measures of performance for each objective were dominated by the variable of price appreciation.

Current Income

The objective of current income, however, presents an entirely different situation. In this case, the analysis is designed to evaluate the stability of annual income rather than the absolute level of current income. The purpose is to determine the portfolio size required in order to provide the investor with reasonable assurance that his annual income will be stable. Stability is defined as: (1) an increase in annual income of sufficient size to maintain

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[1]The complete analysis of the effects of holding period length variations is omitted since the results virtually duplicate those obtained when the objective was long-term growth.

the purchasing power of the beginning income, and (2) for comparative purposes, an annual income of sufficient size to maintain the monetary value of the beginning income or a zero increase.

The analysis begins with an examination of year-to-year increases where it is assumed that a decline in the first year of the holding period below the original income would have violated the income objective even if the average annual increase in dividends during the entire holding period was satisfactory.[1] The second step of the analysis is related to the probabilities associated with the average annual dividend increases which occured over the entire holding period. In each aspect of the analysis, the probability of an income decline and the probability of a failure of the dividend income to increase sufficiently to maintain purchasing power are the primary decision criteria.

First year income increases

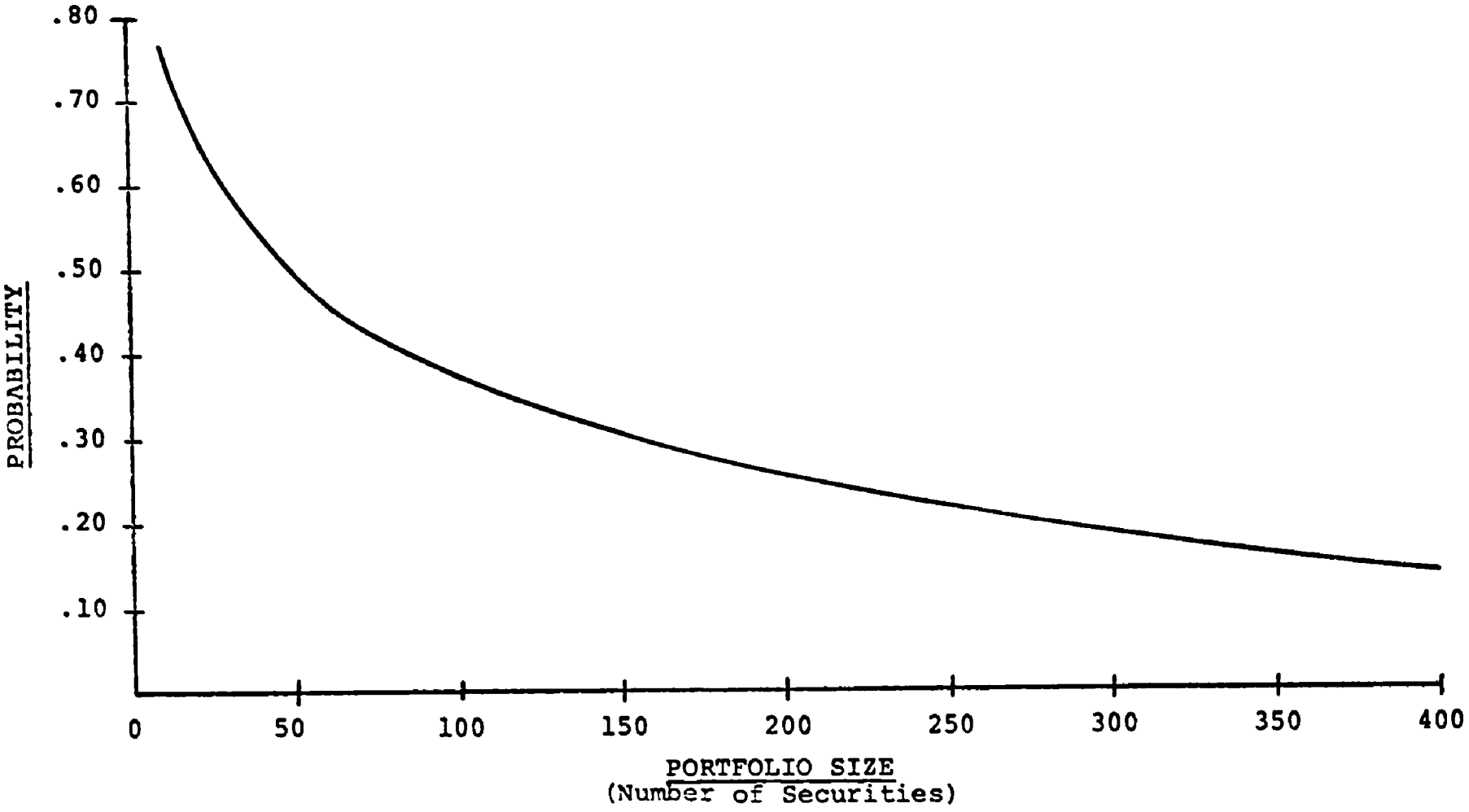
In order to examine the potential for an income decline during the first year of the holding period, Chart 7.5 is presented. The horizontal axis is the portfolio size, and the vertical axis is the probability of a decline in the absolute value of the beginning level of dividend income

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[1] See supra, pp. 80-84.

CHART 7.5

PROBABILITY OF A DECLINE IN DIVIDEND INCOME
(One Year Holding Period)



during the first year of the holding period.[1]

The most important feature of the chart is the extremely high level of probabilities. For example, when a portfolio contained 50 securities, the probability of an income decline is read as approximately 0.50 during the first year of the holding period. In other words, approximately one-half of the portfolios experienced a decline in current income during the first year of the holding period when the portfolio size was 50. When the portfolio size was 100, the probability of an income decline remained substantial at approximately 0.35.[2]

Without attempting to place a specific value judgment upon these probabilities, it would seem that reasonable assurance of a stable current income could have been obtained during the 1955-65 period only with very large portfolio sizes when securities were selected at random and equal funds placed in each security.

The explanation of this result is three-fold and includes: (1) the pattern of annual income changes that

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[1]The purchasing power definition of income stability is omitted at this point for purposes of clarity.

[2]The substantial right skewness that can be inferred from the behavior of the probabilities presented in Chart 7.5 is attributable to approximately 4 percent of the NYSE listings which had an income increase greater than 50 times the average income increase. Exclusion of these issues from the calculations lowered the probabilities, but the reduction was not sufficient to alter the conclusions suggested below.

existed during the period 1955-65; (2) the relative magnitude of the dividend changes of individual issues, and (3) the fact that equal weight was given to the selection of individual issues.

It is possible that the pattern of annual income changes as was exhibited during the 1955-65 period overstates the risks associated with current income. As is shown in Table 7.2 the average year-to-year increase in dividend income during the period examined was 5.35 percent, or .0017 per $1 invested, based on the behavior of the securities contained in the Standard & Poor's Composite Index.[1]

However, Table 7.2 also indicates an average income decline in 1958 and a small average increase during 1957 and 1961. Since the comparison for the first year of the current income investor's holding period centers upon annual changes, it is possible that the average income changes of 1957, 1958, and 1961 overstate the risks of an income decline during the first year of future investment holding periods. In effect, a more stable future pattern of year-to-year income changes

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[1]It is unlikely that the 5.35 percent average annual growth of dividends can explain the magnitude of the probabilities. That is, if a 10 percent growth is assumed, the probability of an income decline was 0.41 with a portfolio size of 50 using the same higher moments as were used in the original calculations. This is as compared to a probability of 0.50 using the actual rate of change.

TABLE 7.2

STANDARD & POOR'S COMPOSITE INDEX

Year	Year-end Price Index	Dividends per share	Dividend Growth from Previous Year (%)	Average Income per $1 Invested[1] ($)	Average Income Change from Previous Year[2] ($)
1955	45.45	1.64	6.49	.0361	.0028
1956	46.67	1.74	6.10	.0373	.0022
1957	39.99	1.79	2.87	.0448	.0011
1958	55.21	1.75	-2.29	.0317	-.0010
1959	59.89	1.83	4.57	.0306	.0014
1960	58.11	1.95	6.56	.0336	.0020
1961	71.55	2.02	3.59	.0282	.0012
1962	63.10	2.13	5.45	.0338	.0015
1963	75.02	2.28	7.04	.0304	.0024
1964	84.75	2.50	9.65	.0295	.0029
1965	92.43	2.72	8.80	.0294	.0026
1966	80.33	2.87	5.54	.0357	.0016
1967	96.47	2.92	1.74	.0303	.0006
1968	103.86	3.07	5.14	.0296	.0016

[1]Dividends per share divided by year-end price index.

[2]Dividend growth multiplied by previous year's average income per $1 invested.

Source: Standard and Poor's Trade and Securities Statistics: Security Price Index Record (1968 Edition).

would imply less risk than is indicated by the actual behavior of dividend income during the 1955-65 period.

A second explanation of the magnitude of the probabilities lies in the potential for the dispersion of dividend changes to be quite wide. To illustrate, it is assumed that an investor purchased a portfolio of common stocks on December 31, 1965. Included in the portfolio were the securities of American Motors which at that time were selling to yield 6.94 percent[1] thereby producing a beginning income of $.0694 per $1 invested.

During 1966, the dividend income from American Motors was reduced to zero creating an income change of minus $.0694 per $1 invested in the securities of that firm. This is as compared to an average change during 1966 of .0016 per $1 invested as reported in Table 7.2. In relative terms, then, the income change of the securities of American Motors was approximately 43 times as large as the average market income change [= $^{.0694}/_{.0016}$] of 1966. In other words, the investor's portfolio would have had to contain 43 additional securities which achieved the average income increase of .0016 per $1 invested in order to have avoided an income

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[1]Based on the 1965 dividend of $.625 per share and a closing market price of $9 per share.

decline during the first year of the holding period.[1]

An objection to this example may be raised on the grounds that the current income investor would not have selected American Motors as an income stock. Indeed he probably would not have, but such a rejection requires the use of security analysis which is not possible under the assumption of random selection. As a matter of fact, the value of security analysis is brought into sharp focus by the results.

As further evidence, the probability of loss was calculated utilizing data from the securities included in the Dow-Jones Industrial Average (DJIA). The difference in probabilities is astonishing. For example, the probability of an income decline during the first year of the holding period was 0.76 when the portfolio size was 10 and securities were selected at random from the entire NYSE.[2] The corresponding probability when the random selection was from the DJIA was approximately 0.10.[3] Thus, if one accepts

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[1]It should not be inferred from this example that the distributions of income changes were skewed to the left. As with the distributions of wealth ratios, the income changes exhibited right skewness. The example of American Motors is given only to illustrate the potential for a large dispersion of income changes.

[2]See, Chart 7.5, p. 170.

[3]Based upon the dividend performance of the securities included in the DJIA during the years 1963 to 1964.

the premise that the inclusion of a security in the DJIA was based upon security analysis as opposed to random selection, it seems apparent that security analysis has considerable value from the point of view of the diversification requirements of the current income investor.

A third explanation of the magnitude of the probabilities presented in Chart 7.5 lies in the weights assigned to the selection of each security. For purposes of the study, it was assumed that there was an equal probability of selecting each security listed on the NYSE, and that equal funds were invested in each security selected.

It is suggested that the dividend pattern of the smaller firm is generally less stable than the dividend pattern of the larger enterprise. If the premise is valid, the probability of an income decline will be greater when the size of the firm is not considered in either the selection process or the quantity of funds placed in each security selected. In essence, random selection by some measure of size, such as total market value, is likely to reduce the income decline probabilities presented in Chart 7.5.

In summary, it seems apparent that under the investment conditions used in the analysis, the probabilities associated with changes in dividend income were of such a magnitude that prohibitively large portfolio sizes would have been required in order to have provided the investor

with a reasonable assurance of a stable dividend income during the first year of the holding period.

A ten year holding period

However, while instability in the short-run was apparent, the benefits of a long-term holding period were equally as apparent. As evidence, Chart 7.6 is presented which has a horizontal axis depicting the portfolio size. The vertical axis is the probability of an increase in dividend income over the ten year period, 1955-65. The upper curve defines an income increase in absolute values. The lower curve defines an increase in real values or a dividend increase which is greater than the increase required to maintain the purchasing power of the beginning income.[1]

In contrast to the first year probabilities, the striking feature of Chart 7.6 is the magnitude of the probabilities. That is, at a portfolio size of approximately 15 securities, the probability of an increase in the dollar value of dividend income was 0.95. With a portfolio size of 40 randomly selected securities, the probability of a real income increase was approximately 0.95,[2] and the

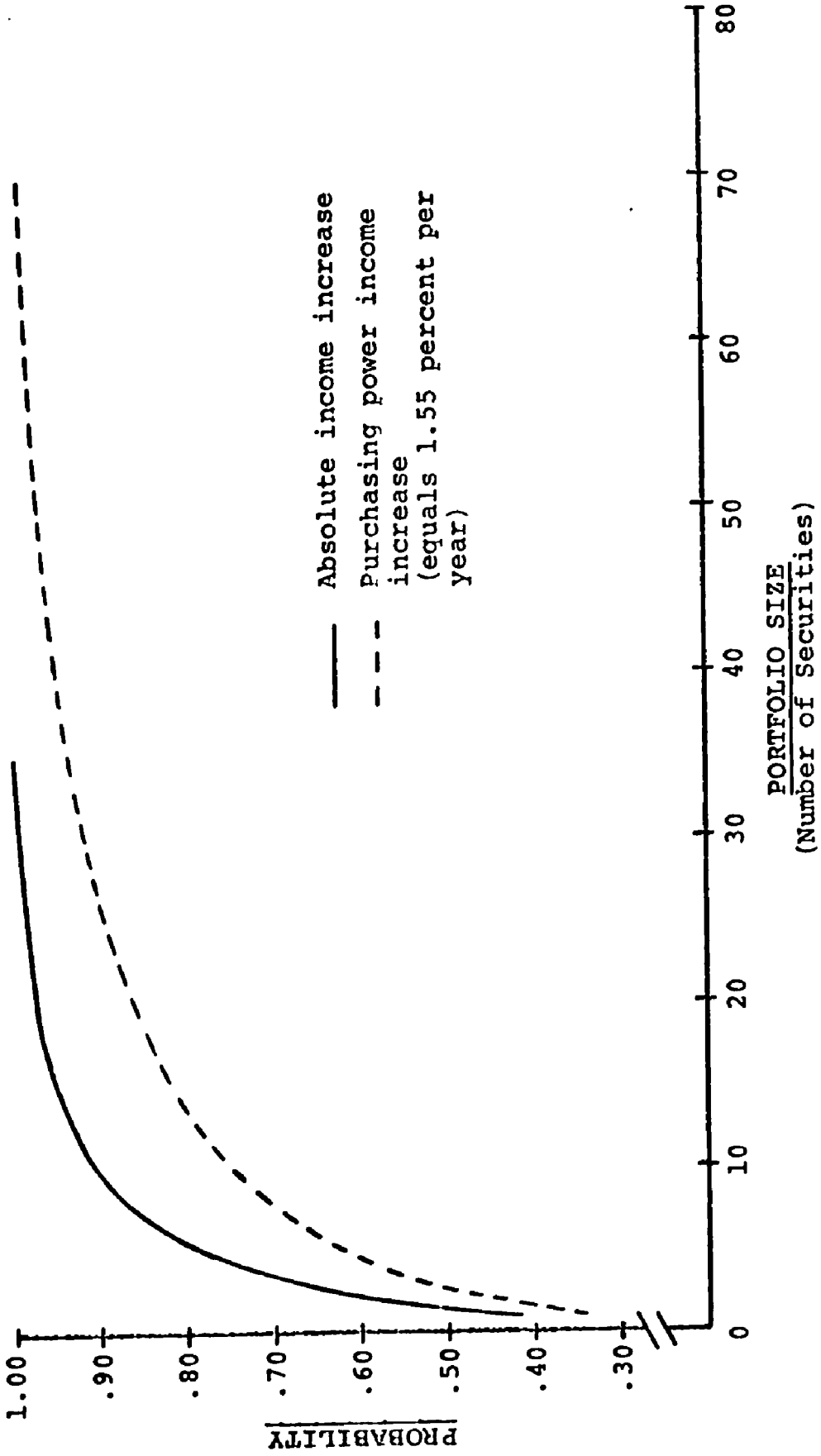
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[1] During the 1955-65 period, the Consumer Price Index advanced at an average annual rate of 1.55 percent. Thus, a dividend increase greater than 1.55 percent per year was considered sufficient to maintain purchasing power.

[2] In addition, it is noteworthy that portfolio additions beyond 40 securities provided an increase in the probability of a real dividend increase which was less than 0.001 per security added.

CHART 7.6

PROBABILITY OF AN INCREASE IN DIVIDEND INCOME
(Ten Year Holding Period)



potential for a dollar increase was virtually assured with 40 securities.

Therefore, it is reasonable to conclude that a portfolio size of approximately 40 securities would have been sufficient to protect the purchasing power of the investor's dividend income over the ten year period with virtually no risk of a decline in the absolute value of the beginning income. This is as compared to a high-grade fixed income alternative where the risk of an absolute income decline is also negligible, but excludes the opportunity to achieve purchasing power protection.

In essence, the long-term growth of dividends exhibited during the period[1] was sufficient to overcome the short-run instability and the actual rate of inflation provided that the portfolio contained approximately 40 randomly selected securities and the holding period was ten years.

It is also interesting to recognize the relationship between the long-term portfolio size requirements of the current income investor and the size requirements for other investment objectives. It was suggested that a portfolio size of 12 would have been sufficient when the objective was long-term growth and/or principal protection. The data

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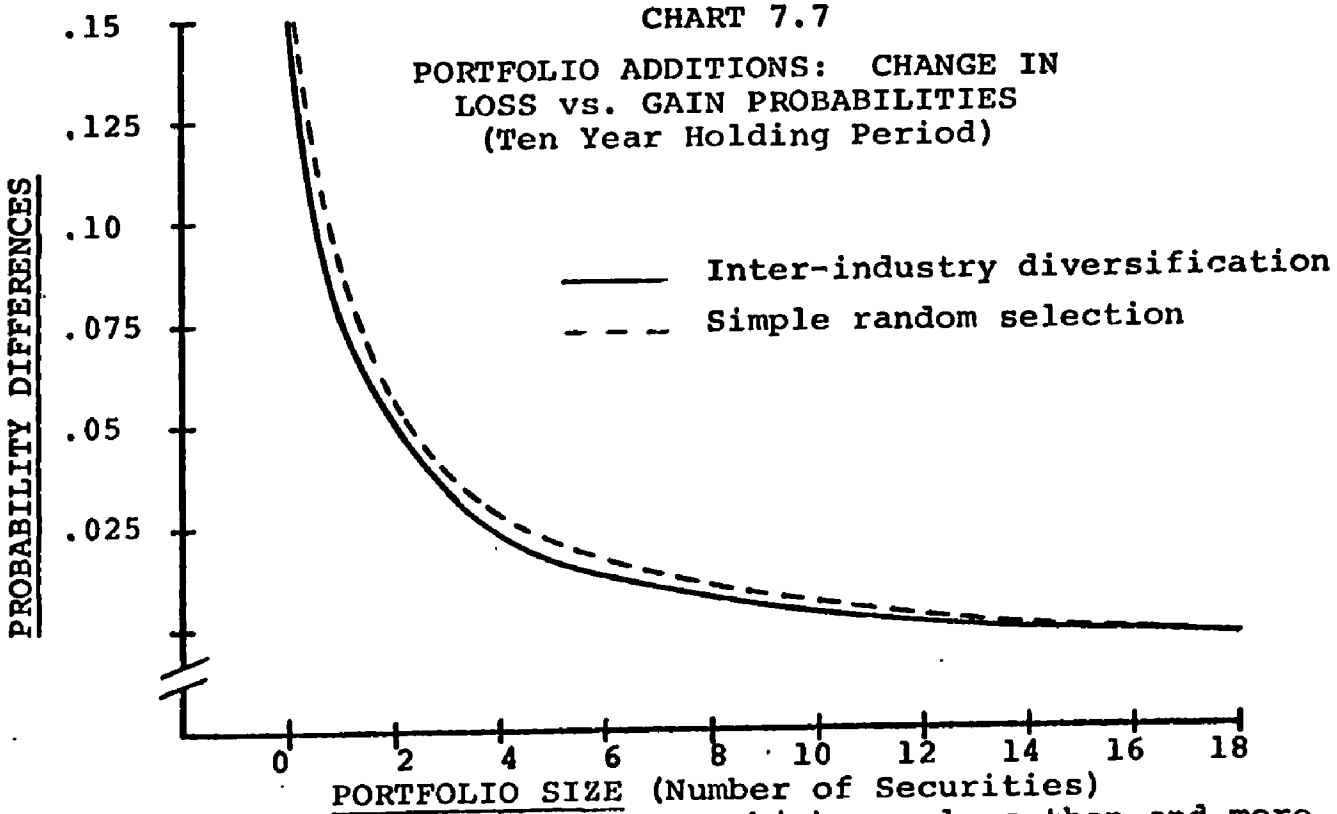
[1]The average annual growth rate of dividends per share was 5.35 percent during the 1955-65 period. (See supra, Table 7.2, p. 173.

of Chart 7.6 show that at a portfolio size of 12, the probability of an increase in the dollar value of the beginning income was 0.[illegible] which would appear to be reasonable assurance of long-term income stability in terms of current dollars.

In this connection, it is appropriate to suggest that a portfolio size of 12 would have been sufficient when the objectives of growth, principal protection, and income were combined. With 12 securities, the investor would have: (1) obtained an equal trade-off between long-term risk exposure and gain opportunities; (2) been virtually assured of principal recoverability in both real and monetary terms, as was shown in Chart 7.4; and (3) had reasonable assurance of an increase in the absolute value of the beginning income. The only disadvantage was a risk that the dividend income would not have increased sufficiently to compensate for inflation. However, Chart 7.6 indicates that the probability of at least a purchasing power income increase was 0.78 with 12 securities which, relative to the advantages of this portfolio size, does not appear unreasonably low. Furthermore, a large number of security additions would have been required to have avoided this disadvantage. For example, Chart 7.6 indicates that a portfolio size of 26 would have been necessary to obtain a 0.90 probability of a purchasing power income increase.

CHART 7.7

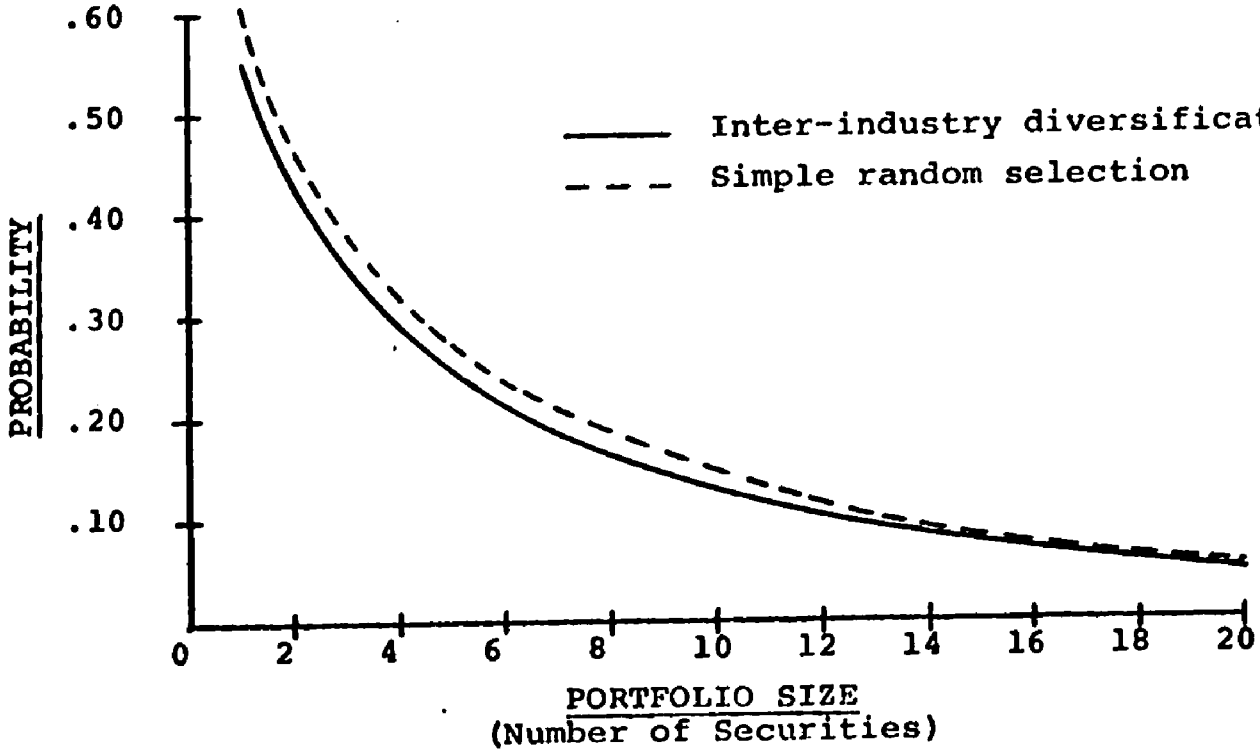
PORTFOLIO ADDITIONS: CHANGE IN LOSS vs. GAIN PROBABILITIES (Ten Year Holding Period)



Note: Loss and gain equal returns which are less than and more than 75 and 115 percent of market average, respectively.

CHART 7.8

PROBABILITY OF LOSS (Ten Year Holding Period)



Note: Loss equals a return which is less than 75 percent of market average.

Summary

In summary, the data suggest that a prohibitively large portfolio size was required to provide the current income investor with assurance that his dividend income would have been stable during the first year of the holding period. On the other hand, a ten year time horizon provided virtual assurance of both real and monetary income growth when the portfolio contained approximately 40 securities. Finally, the probabilities associated with long-term increases in dividend income appeared sufficiently large to justify a portfolio size of 12 when the objectives of long-term growth, principal protection, and income were combined.

Inter-Industry Diversification

As previously suggested, it is also important to compare the value of other methods of diversification in relation to simple random sampling. To accomplish this objective, attention is directed to Chart 7.7 which describes the portfolio size along the horizontal axis. The vertical axis is the difference between the changes in the probability of loss affected by portfolio additions. For comparative purposes, two curves are presented. One plots the changes which were associated with simple random sampling, while the other assumes selection by industry, or inter-industry diversification. Each utilize the objective of

long-term growth and a ten year holding period.[1]

An examination of Chart 7.7 indicates that the portfolio size which equated changes in the probability of a loss and an exceptional gain was unaffected by the use of inter-industry diversification. When the portfolio contained 12 securities, the change in the probability of loss was approximately equal to the change in the probability of an exceptional gain for each diversification technique. Furthermore, the absolute probability of loss did not change with the use of inter-industry diversification as evidence by Chart 7.8. The horizontal axis of this chart is the portfolio size and the vertical axis is the probability of loss which is defined as a return which is less than 75 percent of the market average.

As a result, it seems clear that the sampling technique used to represent inter-industry diversification had little value relative to simple random sampling both from the point of view of the trade-off between loss and gain, and the absolute level of risk during the 1955-65 period. Moreover, it seems equally clear that such a condition is to be expected.

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[1]In addition, a loss is defined as a return which was 75 percent or less than the market average. An exceptional gain is 115 percent of the market average. The simple random sampling curve is the same curve as was presented on Chart 6.3. See supra, p. 134.

In the first place, it is obvious that a company's principal line of business is not the only variable which relates to the performance of its securities. For example, technological advances have made Xerox a very different company than other office equipment firms. McLouth Steel and Armco Steel are both members of the steel industry yet the former sells more than two-thirds of its output to the auto industry while the latter has a highly diversified group of customers. Merck & Co. sells more than one-third of its drug and chemical output in the international markets creating significant geographic differences between it and other drug firms. Management skills probably account for the superior performance of Hewlett-Packard relative to its electronics industry competitors. National Lead and Sherman-Williams are both categorized as paint companies yet the former is diversified to the extent that 60 percent of sales are in metals and metal products with 40 percent in paint. The latter confines its activities almost entirely to paint and related products.

However, even if industry categories are more descriptive than these examples suggest, it is also important to recognize the dynamic nature of industry classifications. The fortunate investor who, in the late 1950's, purchased the shares of Textron and Houdaille Industries may have considered himself to be diversified by industry. Textron was producing textile products and Houdaille, auto parts. Today

both firms are widely diversified with Textron having eliminated textiles from its product line, and Houdaille selling only one-fourth of its output within the auto parts industry.

Finally, and perhaps most important, the sampling technique used to represent inter-industry diversification is a random selection of an industry followed by a random selection of a security within that industry. Thus, it is possible for a portfolio to contain, say, McLouth Steel and Ford Motor Company representing steel and autos, respectively. The result would have been a portfolio of two closely related firms and in this example, the relationship would be two-fold. In the first instance, the auto industry, in general, purchases approximately 20 percent of the output of the steel industry. Secondly, the auto industry purchases approximately two-thirds of the output of McLouth Steel.

As with the current income objective, an individual investor would probably not have accepted these selections given a desire for inter-industry diversification. Nevertheless, random selection requires that he must. A rejection implies the utilization of security analysis. Consequently, it is reasonable to conclude that the classification of securities by industry does not necessarily imply the reduction of risk when no allowance is made for the fact that the performance of any one industry is not independent[1] of the

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[1]It will be recalled that the suggested correlation is exclusive of the average market return.

performance of all other industries.[1]

This condition coupled with the fact that industry categories did not necessarily isolate the most significant variables affecting the performance of the securities of a given industry, as well as the dynamic properties of such classifications suggests that the ineffectiveness of the sampling technique used to represent inter-industry diversification was to be expected. Thus, rejection of the principles of inter-industry diversification is not warranted despite the fact that the probabilities presented showed no advantage for the diversification strategy. As with the current income objective, the result appears to indicate that even the most basic security analysis techniques have value.

Time Diversification

In addition to inter-industry diversification, the relative value of dollar-averaging is also examined. It will be recalled that this approach to common stock investment is designed to lower the average price per share and achieves its greatest effectiveness when the market fluctuates about some constant level. With positive growth its effectiveness

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[1]This statement does not indicate that the probabilities presented are incorrect since consideration of the total population cancels the effects of inter-industry correlations provided the expected return is zero. See supra, pp. 108-109.

is reduced.[1] This potential benefit, however, is ignored in the analysis to follow since the concern is with the long-term investor who anticipates positive market growth in the future.

Nevertheless, it is appropriate to examine the technique since many individual investors and institutions make periodic contributions to their portfolio. Furthermore, it is to be expected that the portfolio size requirements will be smaller than in a simple buy and hold strategy. That is, exclusive of the average market return, the performance of an individual issue will not be constant over time. By making periodic purchases over the long-term, the investor's risk will be reduced since it is less likely that his entire investment in a given issue will be purchased at a high point in the price activity of a particular issue. At the same time, the opportunity for an exceptional gain will be lower for the opposite reason. The result should be a lower absolute probability of loss as well as a lower portfolio size which equates the changes in the probability of a loss and a

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[1]It should be noted that advantages may accure to the technique when market growth is positive provided the fluctuations are large. However, it is unlikely that the necessary fluctuations will be large enough. For example, if it is assumed that the market will grow at an average annual rate of 5 percent and that the range of fluctuations will not exceed 25 percent of the trend value, the strategy of buy and hold is generally superior to dollar-averaging in terms of total wealth generated assuming a holding period of at least four years.

gain. The implication is that the portfolio size requirement will be lower than the 12 securities suggested for a strategy of buy and hold with simple random selection.

On the other hand, the reduction should be small. Specifically, it has been noted that price fluctuations are of minor importance to the long-term investor since he will not buy or sell on such movements. Of greater importance is the fact that the long-term trend will overpower poor timing on the initial purchase and final sale.[1]

The relationship between dollar-averaging and a buy and hold strategy exhibited during the 1955-65 period is illustrated in Charts 7.9 and 7.10. Chart 7.9 reflects the difference between changes in the probability of a loss and an exceptional gain for each diversification technique. Chart 7.10 is the probability of loss for each strategy where a loss is defined as a return which is less than 75 percent of the market average. Both charts represent a ten year holding period and utilize the long-term growth objective to reflect the buy and hold strategy.

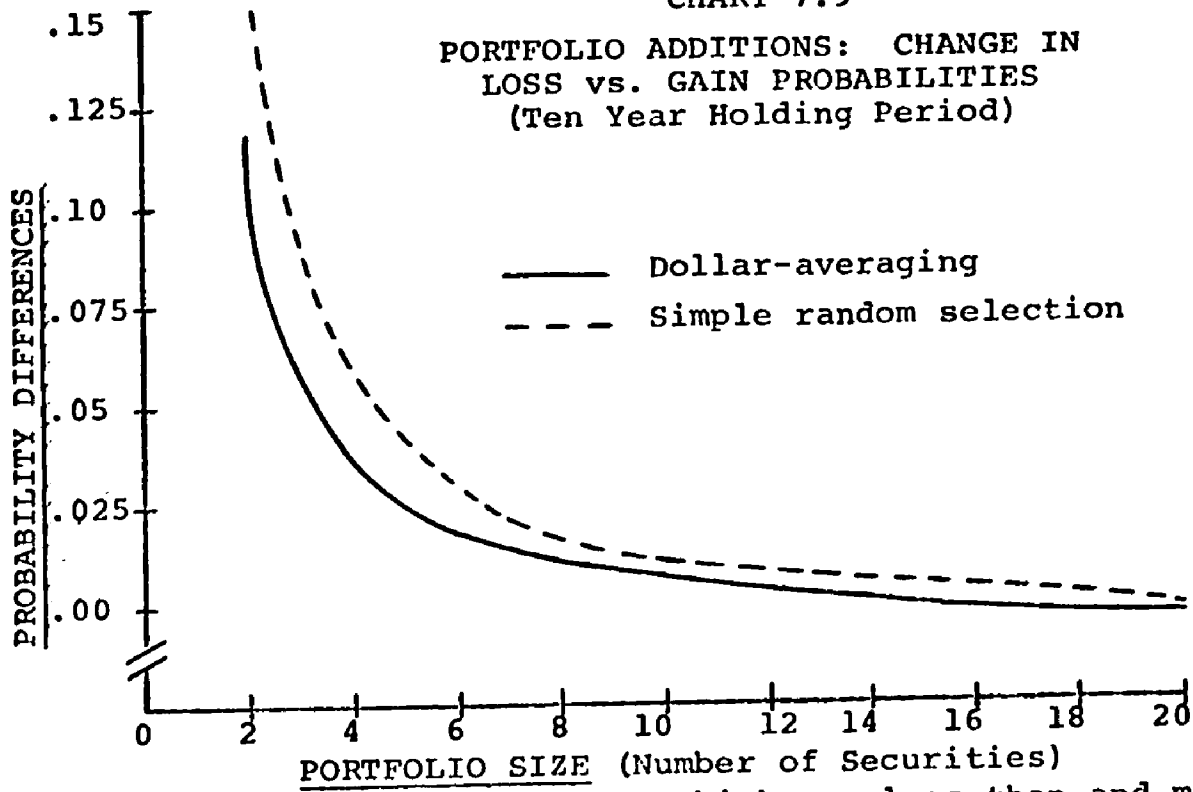
Thus, it appears that the investor who employed dollar-averaging required a slightly smaller portfolio size, or approximately 10 securities as opposed to 12 when the strategy was buy and hold. However, for practical purposes,

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[1]See supra, pp. 20-28.

CHART 7.9

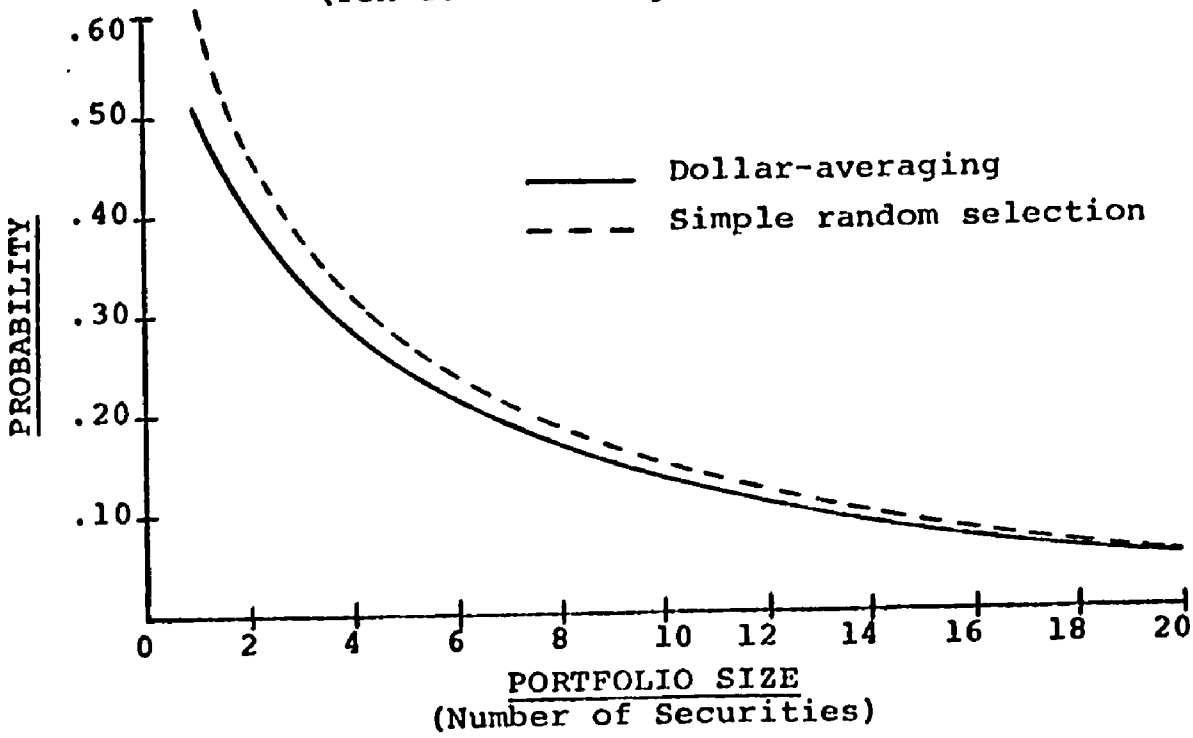
PORTFOLIO ADDITIONS: CHANGE IN LOSS vs. GAIN PROBABILITIES (Ten Year Holding Period)



Note: Loss and gain equal returns which are less than and more than 75 and 115 percent of market average, respectively.

CHART 7.10

PROBABILITY OF LOSS (Ten Year Holding Period)



Note: Loss equals a return which is less than 75 percent of market average.

it seems reasonable to conclude that the portfolio requirements did not change materially with the use of dollar-averaging.

CHAPTER VIII

SUMMARY AND CONCLUSIONS

Summary and Methodology

The objective of the study has been to develop empirically based generalizations regarding the portfolio size requirements, in terms of numbers of securities, of the long-term common stock investor. The data employed to accomplish this objective came from the period 1955-65 and included the monetary performance of all common stocks listed on the NYSE at some time during that period.

It was assumed that the long-term common stock investor makes ex ante judgments regarding the efficiency of his particular diversification strategy in terms of the probability of achieving a particular set of monetary returns which are acceptable in the context of the particular investment objectives, the risk-aversion schedule, and other investment alternatives which may have been available. Therefore, the primary decision criteria used to establish an appropriate portfolio size centered upon the probability that a given portfolio size would have produced a "loss" or a "gain."

It was further assumed that the investor who purchases common stocks for the long-term looks only to the performance of the corporate sector for the source of his return, and that the anticipation of structural changes in the market for common stocks (such as price-earnings ratio increases) will not affect the diversification requirements of the common stock portion of his portfolio. Therefore, the methodology made comparisons of portfolio returns relative to the average market return regardless of the actual returns on the universe of stocks achieved during the particular time period. Consequently, no consideration was given to potential variations in the average market return. In essence, the study did not include a judgment regarding the investment value of common stocks in general--only the diversification requirements of the investor who considered common stocks an appropriate investment media.

In order to implement this approach, it was necessary to define the various target returns used to represent gains and losses in terms relative to the average market return. For example, a frequently used loss point was defined as a return which is 75 percent of the average return from common stocks. This is as opposed to a particular rate of return value. The results of the analysis suggested that substantial variations in the relative definitions of the target returns would not have altered the conclusions.

In order to make the conclusions of the study applicable to as broad a group of long-term investors as possible, several specific portfolio decision criteria were examined. These included: (1) the probabilities associated with achieving a portfolio return which approximated the average market return; (2) the changes in the average market return probabilities which accompanied security additions; (3) the difference between the changes in the probability of gains and losses which resulted from portfolio additions; and (4) the magnitude of the probability of a loss. The emphasis was placed on the fact that diversification is not a "free good." That is, the risk protection that diversification is designed to provide is accomplished at the expense of a reduced opportunity to achieve an exceptional portfolio gain. Thus, it was assumed that most investors will attempt to "trade-off" risk exposure and gain opportunities when making the portfolio size decision.

An initial examination of the data revealed that the distribution of common stock returns, during the 1955-65 period, exhibited considerable right skewness such that an assumption of normality, in order to calculate the required probabilities, would have been inappropriate. As a consequence, probability calculations were based upon the first two terms of the Cornish and Fisher series which allowed consideration to be given to the effects skewness can have

upon the probabilities associated with a given rate of return value. In addition, the probabilities presented assumed a random selection of securities where each security had an equal chance of being selected,[1] and upon selection each security received an equal quantity of funds. No allowance was made for variations in the total monetary value of a given portfolio.

Summary of Conclusions

The objective of long-term growth

The results of the study indicate that a portfolio of approximately 12 randomly selected common stocks would have been appropriate for most investors who had an objective of long-term growth as opposed to principal protection or current income. Several aspects of the analysis support this conclusion.

In the first place, it was found that 12 securities produced an approximately equal trade-off between gains and losses. A reduction in the portfolio size to less than 12 involved an increase in the investor's risk exposure without an equal or greater increase in the opportunity to have achieved an exceptional portfolio gain. With more than 12 securities, the gain opportunities foregone by security

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[1]Except in the case of inter-industry diversification which is discussed below. (See infra, p. 200.

additions were not compensated for by larger reductions in risk exposure. In other words, the addition of the twelvth security created a reduction in the probability of loss which was approximately equal to the reduction in the probability of an exceptional gain.

Secondly, the data indicated that the conclusion would not have been altered by variations in the definitions of gains and losses. When the definition of an exceptional gain was varied, the point of equal trade-off remained at approximately 12 securities. It was noted, however, that if the investor's objective was to maximize his gain opportunities, a portfolio size of 12 would have been excessive. On the other hand, the analysis indicated that even in this case some diversification would have been appropriate. The reason was that, contrary to the general assumptions of diversification theory, the probability of an exceptional gain increased with portfolio additions for "small" portfolio sizes.

When the definition of a loss was varied, the portfolio size which equated gain and loss probability changes also varied, yet the condition did not alter the 12 security portfolio size recommendation. It was assumed that an investor willing to accept a greater degree of risk exposure (a lower definition of loss) would do so only if his willingness would have been compensated for by a significant

increase in the opportunity to achieve an exceptional gain. It was demonstrated that such a requirement did not occur in any significant fashion when the portfolio size of 12 was reduced. The implication seemed to be that the investor willing to accept considerable risk exposure would have chosen a portfolio size of approximately 12 randomly selected securities.

Third, it was found that the long-term growth investor who placed the primary emphasis upon risk would have selected a portfolio size of approximately 12 securities. The conclusion was based on the fact that the level of risk, or the probability of loss, was small. For example, the probability of realizing a return which is less than 75 percent of the market average was only 0.12 with a portfolio size of 12 and a ten year holding period.

In essence, the accumulation of evidence suggested that, during the 1955-65 period, a portfolio size of 12 randomly selected securities with equal funds placed in each would have been appropriate for a wide range of long-term investors. The implication was that, in general, the long-term common stock investor will optimize his diversification strategy with a portfolio size of approximately 12 securities.

The average market return

The data also suggested that when the investment objective was to obtain a close approximation of the average market return, a portfolio size greater than 16 to 18 securities would have been uneconomical. That is, security additions beyond 16 to 18 produced very small increases per security added in the probability of obtaining a close approximation of the market average. If it is assumed that security additions involved a cost (over and above opportunity costs), it is unlikely that larger portfolios could have been justified economically when attempting to achieve the market average.

The length of the holding period

Throughout the study the emphasis was placed upon the long-term investor. Thus, it was necessary to obtain an empirical definition of "long-term." In this connection, it was found that a portfolio size of 12 securities would have been appropriate provided the holding period was at least eight years. The conclusion was based on the fact that 12 randomly selected securities satisfied the several decision criteria for all holding periods of eight years and longer. With shorter holding period lengths, it was doubtful that a portfolio size of 12 would have represented an optimum diversification strategy for most investors with an objective of long-term growth.

The objective of principal protection

It should not be inferred, however, that the conclusions are applicable only to the investment objective of long-term growth. In particular, it was found that when the objective was principal protection, a portfolio size of 12 and a minimum holding period of eight years provided the investor with virtual assurance that the real and monetary value of his initial investment would not have been violated. In addition, a portfolio size of 12 was optimum in the sense that the difference between changes in the probability of gains and losses of principal was approximately zero with 12 securities.

The objective of current income

A somewhat different situation arose when attention was directed at the objective of dividend income. In this case, the analysis evaluated the probabilities associated with an increase in income sufficient to have maintained purchasing power. It was concluded that a portfolio of approximately 40 securities was required, during the 1955-65 period, in order to assure the investor that the purchasing power of his beginning income would have been maintained.

The proviso of long-term was found to be quite important. Specifically, prohibitively large portfolios were required to assure income stability during the first year of

a given holding period. In effect, the dispersion of short-term dividend changes was such that a reasonable amount of random diversification could not have been expected to "average out" the risks and gain opportunities of corporate dividends. Nevertheless, the long-term growth of dividends during the study period was sufficient to negate the effects of this short-term instability when the portfolio size was 40.

In addition, it was suggested that the probabilities associated with long-term income increases were not so low that the investor would have required a portfolio size greater than 12 when the investment objectives of long-term growth, principal protection, and current income were combined. With 12 securities the long-term probabilities were 0.93 and 0.78 of at least a monetary and a real income increase, respectively. Relative to the several advantages of a 12 security portfolio, it is doubtful that these income probabilities would have been considered unacceptable by the investor with a combination of investment objectives.

Inter-industry diversification

While the major portion of the analysis assumed simple random sampling as the method of security selection, the effects of inter-industry diversification also received attention. The sampling plan used to represent this

diversification technique consisted of a random selection of an industry without replacement followed by a random selection of an individual issue within the industry chosen.

The results of the evaluation showed no discernible improvement over the technique of simple random selection. The explanation of the similarity between the two approaches appeared to lie in the fact that the analysis gave no consideration to the correlations that existed between the performance of the securities of one industry and the performance of the securities of all other industries. Therefore, the conclusion that inter-industry diversification is inefficient was not warranted--only that a random selection of industries is not sufficient to improve upon simple random selection.

Time diversification

In addition to inter-industry diversification, the technique of dollar-averaging was examined. Again, relative to simple random sampling, the dollar-averaging method did not produce a significant improvement although a slight reduction in the various probability measures was recorded. The result was to be expected since the objective of the technique is to profit from market fluctuations when the long-term growth trend is zero. Nevertheless, the results imply that a portfolio size of approximately 12 securities

would have been appropriate when the investment plan required periodic contributions to the portfolio.

Security analysis

As a final note, it was found that very basic security analysis could have served the investor by substantially reducing the risk exposure of a given portfolio. In the case of the current income investor, the data indicated that exclusion of those issues where a dividend reduction was obviously imminent and consideration of the size of each firm in the population would have substantially increased the probabilities associated with income growth. In addition, it was suggested that an examination of only the most obvious industry correlations could have reduced the investor's risk exposure, and, in turn, his portfolio size requirements.

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