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THE EFFECTS OF VARIATIONS IN INSTRUCTION AND FINAL UNIT EVALUATION PROCEDURES ON COMMUNITY COLLEGE BEGINNING ALGEBRA CLASSES

by

Paul William Merritt

A dissertation submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
(Education)
in The University of Michigan
1972

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CHAPTER I

STATEMENT OF THE PROBLEM

A major problem facing community college mathematics instructors in beginning level classes is that of designing meaningful instruction and evaluation procedures for students with wide differences in background and ability. Related to this problem is the large number of students that either fail or drop out of these classes. Chapter I presents a discussion of this problem, a rationale for its study, and a list of the hypotheses investigated.

One of the fastest growing institutions for higher education today is the two-year junior or community college. According to Richardson, nearly two million students are presently enrolled in the more than one thousand institutions of this type. (1968, p.iii) Reynolds reports an estimate that 50 new two year colleges will be created each year in the foreseeable future. (1969, p.vi) Support for the likelihood of a continuing growth is given by the Carnegie Commission on Higher Education. The Commission estimates that, by 1980, 3.7 to 4.4 million students will be enrolled in two-year colleges. (1970, p. 34)

Part of the reason for the continued growth of the community college has been their widespread acceptance of an open-door admissions policy. Schenz (1963) found that 91% of the junior colleges responding to his survey indicated that the door was "wide open" for all high school graduates and "open" for all persons eighteen years of age and over who could profit from the instruction.

While such an admissions policy has served to implement the goal of universal higher education, it has also pointed out the need for innovation in instruction due to the emergence of a student population characterized by wide ranges of ability and aptitude. Medsker reports that although the average academic aptitude level of students entering two-year colleges is somewhat below that of those who enter four-year colleges, "there are a wide range of abilities among two-year college students, and many of them are superior in ability to many students in four-year institutions." (1960, p. 30) In contrast with the latter portion of this statement are the findings of Schenz (1963) and Cross (1968). Schenz (1963) noted that many junior colleges were observing that an increasingly larger proportion of their full time student body were students with low ability. Cross found that "approximately one-third of the students who enter junior college have not taken a secondary course of study that would permit them to enter a four-year college." (1968, p. 50)

Although Schenz (1963) found community college administrators to be aware of the need for instructional innovation, there is little research to indicate that this need is being met. Johnson attests to this fact for junior college teaching in general. "There is a paucity of evidence regarding the success of new developments in junior college teaching. Evaluation in instruction is largely a missing entity in the junior colleges of our nation, . . ." (1969, p. 95) Roueche, after briefly summarizing the state of the two-year college in regard to remedial education, arrives at a similar conclusion for instruction of the low achieving student.

> In the last decade there has been evidence of a growing concern in junior colleges with the remedial student. Rapidly increasing enrollments in recent years have served to emphasize the problem. Increasing numbers of full-time students in the community colleges are low achieving students. Millions of dollars are expended annually on programs for these students. Indeed, programs and instructors for remedial students presently constitute a major institutional endeavor by the community colleges. Yet, with very few exceptions, little research has been implemented to evaluate the effectiveness of these programs and instructors. (1968, p. 41)

Closely related to the need to design and evaluate research in instruction at the two-year college level is the problem of student dropouts and failures. Kipps polled 165 junior college basic arithmetic teachers and found that of the 12,468 students who had enrolled and remained at least four weeks, less than half were likely to earn a grade of "C" or higher. (1966, p. 40) Grasser studied the beginning algebra courses of four Michigan community colleges and reported an average dropout rate over 25%. Beyond this he reported an average failure rate for the four community colleges of 20%. (1969, p. 4) Aligning himself with those who have termed the community college door "revolving" as well as "open" for many students, Roueche has called the attrition rate "one of the most pressing problems of the community junior college." (1968, p. 2)

It was within the context of the problems and needs identified above that this study was designed. The major areas of emphasis for the study were classroom organization for instruction and final unit evaluation.

One of the classroom organization for instruction procedures commonly used to provide for individual differences is that of grouping. Although the subject of grouping is fraught with controversy, recent studies have attempted, through carefully formulated design, to meet the major criticisms of: 1) lack of a combination of criteria in formulating

groups, 2) inflexibility in the grouping process, 3) failure to provide for varying methods and materials, and 4) tending to cause a breakdown in whole class unity.

Bierden (1968) and Mortlock (1969) utilized a combination of whole class instruction and flexible intra-class grouping based on achievement of the objectives at the junior and senior high school level, respectively. McBride (1970) used a similar approach at the college level in a content course for elementary school teachers. The design of each of the studies was an attempt to meet the major objections listed above. Each of the studies recommended further modification and experimentation with the classroom scheme at a variety of levels. The results of all the studies indicate potential benefits from the use of such a scheme with beginning algebra students at the community college level.

The statement of instructional objectives has also proven beneficial as a means of facilitating the teaching-learning process. Bierden (1968) and Mortlock (1969) found objectives useful both to students and to the teacher. For the teacher they provided a basis for the selection of content, the design of instruction, and the assessment of achievement. For the student they furthered communication with the teacher by defining exactly what was to be learned. Johnson sees the use of instructional objectives as important for the junior college level as well.

> To date, the definition of instructional objectives as a basis for improving instruction can by no means be identified as a trend in the junior colleges of our nation. But such definition is emerging as an important emphasis at some junior colleges, and this emergence may well point to a dynamic focus for the future. (1969, p. 320)

There also appears to be a need for research at the junior college level in the area of evaluation and in particular with classroom testing.

According to Altshuler, "For the junior college student, the part of the course which is of vital concern is also the part about which very little research has been done - the uses, administration, and evaluation of the classroom test." (1967, p. 1) Perry (1971) allowed students the opportunity to retake a final unit test as many times as they desired in a community college elementary algebra course. Reporting favorable results, he found that "an average student took only one or two times to complete a test satisfactorily, . . ." (1971, p. 44)

The major purpose of this study was to determine the effects of variations in classroom management and instruction and final unit evaluation procedures on mathematics achievement, attitudes toward mathematics, test anxiety, and the dropout and failure rate in community college beginning algebra classes. The treatments included:

1. Whole class instruction with one final unit test.
2. Whole class instruction with a final unit test-retest option.
3. A combination of whole class instruction and differentiated small group instruction with one final unit test.
4. A combination of whole class instruction and differentiated small group instruction with a final unit test-retest option.

The hypotheses tested were the following:

H(1): There are no significant differences in the changes in mathematics achievement, attitude toward mathematics, or test anxiety for the total population of beginning algebra students.

H(2): There are no significant differences in mean achievement between the treatment groups of community college beginning algebra students as measured by post-CMAT scores.

H(3): There are no significant differences in mean achievement between the treatment groups of community college beginning algebra students as measured by students' final unit test averages.

H(4): There are no significant differences in mean mathematics achievement for Low and High subgroups of grouping and whole class treatment groups as measured by:

a. Pre- and post-test forms of the CMAT.
b. Final unit test averages.

H(5): There are no significant differences in mean basic level objective achievement for Low and High subgroups of grouping and whole class treatment groups on:

a. Best test I
b. Best test II
c. Best test III
d. Best test IV
e. Best test V
f. Final exam

H(6): There are no significant differences in mean attitude toward mathematics between the treatment groups as measured by the MAO.

H(7): There are no significant differences in mean mathematics test anxiety between the treatment groups as measured by the TAQ.

H(8): There are no significant differences between the treatment groups in the percentage of students who drop out of the beginning algebra course.

H(9): There are no significant differences between the treatment groups in the percentage of students who receive a grade of "C" or better (success) in the beginning algebra course.

The subsequent chapters give a review of the literature, the design of the study, an analysis of the data, and a summary and discussion of the findings.

CHAPTER II

REVIEW OF THE LITERATURE

The purpose of this study was to determine the effects of variations in classroom organization for instruction and final unit evaluation procedures on community college beginning algebra classes. The major areas of emphasis were the use of instructional objectives stated in behavioral terms (behavioral objectives) and sample tests, flexible within-class grouping with differentiated instruction at three levels of difficulty, and a test-retest evaluation procedure.

Chapter I provided motivation for this research by briefly describing the present state of the community college in the areas of student population, instructional innovation and research, and student dropout and failure rate. The first chapter also outlined the means by which this study would attempt to meet the problems identified in those areas.

This chapter presents a further review of the literature in each of the major areas of emphasis: instructional objectives, within-class grouping, and two-test evaluation. The aim is to provide perspective for the design of this study in relation to the problem under investigation.

Behavioral Objectives

Mortlock (1969) has extensively reviewed the literature on behavioral objectives in a study with students in eleventh grade mathematics.

As this has been done with particular reference to the significance of the literature for a study designed such as this one, it is intended here to simply summarize the relevant information and provide for a brief extension with reference to the community college level.

Simply defined, a behavioral objective is a statement which describes what the learner will be able to do after completing some unit of instruction. Its major purpose is to provide direct and unambiguous communication between teacher and student regarding the requirements for successful completion of the instructional unit.

Preparation of Behavioral Objectives

One of the most popular and widely accepted guides for the preparation of behavioral objectives in education is that given by Mager (1962). According to Mager, there are three components which characterize an effective behavioral objective. These are:

1. The objective specifies the kind of terminal behavior that will be accepted as evidence that the learner has achieved the objective.
2. The objective defines the desired behavior further by describing the important conditions under which the behavior will be expected to occur.
3. The objective specifies the criteria of acceptable performance by describing how well the learner must perform to be considered acceptable. (1962, p. 12)

Although the above characteristics serve to clarify the intended objective, it is important to note that, for Mager, it may not be necessary to include all three characteristics in each objective. Rather, "the object is to write objectives that communicate." (1962, p. 12)

The criteria of an effective instructional objective for Gagne (1965), whose instructional theory is based on hierarchies of learning and mastery of prerequisite tasks, are very similar to those of Mager. Emphasizing the need for instructional objectives to be descriptions of performance centering on "what" the student will be able to do after learning, it is necessary that they be complete, unambiguous, and internally consistent. "And above all, such objectives should be reliable; two readers should have no disagreement about the kind of performance expected of the learner." (1965, p. 33)

The essential components of behavioral objectives identified by Mager and Gagne are also cited and enlarged upon by Miles and Robinson.

1. Conditions: a description of the class of stimuli to which the student is to respond (e.g., the type of questions, tasks or problems, and the form in which they will be presented; the relevant conditions under which the student will be expected to perform - materials or equipment which will be available, environmental conditions which may affect the performance, special physical or psychological demands which may exist).
2. Behavior: a statement containing an action or behavioral verb which connotates or denotes the behavior the student is to perform (e.g., identify, write, describe, solve, classify) and a general reference to the product of the student's behavior (e.g., an essay, a diagram, a three-dimensional model).

3. Criteria: a description of the success criteria by which the student's behavior is to be judged acceptable or unacceptable (e.g., correctly applies three principles; identifies eight out of ten; solves the problem; the idea must be different from any in the textbook, discussed in class or produced by other students). (1971, p. 39)

An important feature of preparing instructional objectives which, although not explicitly stated, is suggested by the sources above is that of providing sample test items as a means of further clarifying the stated objectives. Tyler (1951) points this out in suggesting that the learner can demonstrate mastery, only if the teacher writes items that measure attainment of a particular objective. Popham (1970) also acknowledges this fact with reference to both the pre-college and junior college levels.

> A factor which has not been perceived by all advocates of precise objectives is that such objectives may be necessary, but by themselves they are far from sufficient. Too often even a behaviorally stated objective may be used as a window dressing for instruction as usual . . . .
>
> Few school districts or junior colleges have made this logical jump from the development of objectives to the necessity of developing test items. (1970, pp. 44-45)

The presentation of test items complements the statement of instructional objectives and provides for a direct communication of course objectives.

Rationale For Stating Behavioral Objectives

A number of writers have set forth a detailed rationale for the statement of behavioral objectives as an aid in instructional and curriculum design. Most notable among these are Tyler (1964), Mager (1962), Gagne (1965), and Glaser (1967). Mortlock (1969) has summarized their positions in three basic reasons for the use of behavioral objectives.

1. Behavioral objectives clearly communicate the terminal behaviors to be developed and guide the selection of content and of instructional procedures. (p. 18)

2. Behavioral objectives facilitate the evaluation of the extent to which students individually and collectively are achieving the objectives of the instructional program. (p. 21)

3. Behavioral objectives provide advantages to the student by providing goals, the means to assess his progress, and the means for organizing his learning. (p. 29)

For Glaser (1967), educational attainment can be immediately improved through the teacher's involvement in the analysis and specification of educational objectives and learning outcomes. The following statement by him offers support for the first reason given above:

> The definition of instructional objectives instructs the curriculum designer and the teacher how to proceed. Vague specification leave the teacher with little concrete information about what to provide the student to attain this performance. Recognition of the details of the competence to be acquired permits the teacher to guide some students to reach this level and to quickly react to the students who surpass it. (1967, p. 2)

Tyler (1969) sees behavioral objectives as the most critical criteria for guiding all of the remaining activities of the curriculum developer. In terms of evaluation, there is a complementary relationship existing between the behavioral objectives defined and the process of evaluation.

> The process of evaluation begins with the objectives of the educational program. . . . If they have not yet been clearly defined, it is absolutely essential that they be defined in order to make an evaluation since unless there is some clear conception of the sort of behavior implied by the objectives, one has no way of telling what kind of behavior to look for in the students in order to see to what degree these objectives are being realized. This means that the process of evaluation may force persons who

> have not previously clarified their objectives to a further process of clarification. Definition of objectives, then, is an important step in evaluation. (1969, pp. 110-111)

Mager (1962) points out clearly and simply the positive aspects of behaviorally stated objectives for students.

> An additional advantage of clearly defined objectives is that the student is provided the means to evaluate his own progress at any place along the route of instruction and is able to organize his efforts into relevant activities. With clear objectives in view, the student knows which activities on his part are relevant to his success, and it is no longer necessary for him to "psych out" the instructor. (1962, p. 4)

Support for the statement and utilization of behavioral objectives for the college and junior colleges is also in abundance. Popham (1969), writing on the concept of a bank of behavioral objectives for the junior college as a means of clarifying and standardizing course objectives, lists three advantages of behaviorally stated objectives as (1) improved quality of objectives, (2) more relevant instruction, and (3) effective evaluation. Over a period of 18 months, Johnson (1969) made a survey of the innovations in junior college instruction. After covering 22 states and more than four of every ten of United States junior colleges, Johnson pointed to the distribution of instructional objectives as "the single most important contribution which a college and an instructor can make to the learning of many students." (1969, p. 94) "The definition of specific instructional objectives for developing the curriculum and improving instruction emerged as a thematic emphasis in the survey of innovations in junior college instruction." (1969, p. 94)

The American Institutes for Research stressed the importance of objectives for the college level in a curriculum development proposal written by them. Although not based on formal research, the following statement was included:

> The importance to the success of an educational venture of clearly defined objectives has been emphasized by various investigators of educational problems, including Tyler. . . , Dressel. . . , Bloom. . . , and his collaborators. . . , among others. At the level of college instruction, for example, teachers who worked at the process of specifying objectives are reported to have achieved great clarification in the content and method of course presentation, resulting in improved student achievement. When such objectives are made clear to the student, his interest and motivation provide a basis for realistic appraisals of student achievement and progress. (1965, p. 3)

The continuing importance of the procedure of defining educational objectives to improve instruction in junior colleges is seen in Cohen's paradigmatic picture of a community college as it might look in the year 1979. In such a college, for Cohen, "student achievement of specific learning objectives is the focus - the acknowledged, sanctioned overriding purpose of the institution." (1969, pp. 5-6)

While the preceding arguments strongly support the statement of educational objectives in behavioral terms, the process as yet has not gained universal acceptance. Many, for example, feel that the clear and specific statement of educational objectives can serve to hinder as well as promote the goals of instruction and curriculum development. Eisner has summarized the major objections in pointing out what he views as four "limitations to theory in curriculum regarding the functions educational objectives are to perform." (1967, p. 253) The four points are:

1. The dynamic and complex process of instruction yields outcomes far too numerous to be specified in behavioral and content terms in advance.
2. The theory concerning educational objectives has not taken into account the particular relationship that holds between the subject matter being taught and the degree to which educational objectives can be predicted and specified.

3. Curriculum theory which views educational objectives as standards by which to measure educational achievement overlooks those modes of achievement incapable of measurement.
4. Educational objectives need not precede the selection and organization of content in curriculum development. (1967, pp. 254-258)

In reference to point one, while Eisner feels that it is possible to identify the outcomes of instruction in a general way, he sees the nature of the instructional process and classroom interaction as producing numerous and complex results which are only in part predictable. Ebel (1967) lends support to this view in pointing out the difficulty involved with specificity in objectives.

> But if objectives are stated in specific detail, they tend to become very numerous. If one tried to state all of them explicitly in advance, he could easily spend all his time writing objectives and have none left for actual teaching. (1967, p. 261)

In response to this limitation of behavioral objectives, both Hastings (1967) and Payne (1967) agree that while it is not possible to specify all the outcomes of instruction, neither is it necessary. The attempt should be, rather, for teachers to be selective in the process of listing the desired outcomes of instruction which can be identified. The list can then be used by the teacher as a basis for making instructional decisions both in and out of the classroom on how best to achieve the desired outcomes. While the initial list of objectives is gradually clarified and possibly enlarged, the instruction is given direction in the process.

Eisner's second point is based on the feeling that it is relatively simple to specify terminal student behaviors in areas such as science, mathematics, and the languages while somewhat more difficult, if not impossible, in other subject areas such as the arts. Although differing in his classification of science, support for this view comes from Silberman (1970). For Silberman, teaching by behavioral objectives is a prescription for training and not education. In reference to Eisner's second point, he quotes Cronbach who writes, "the person who wants us to 'specify the acts the learner is to perform as a result of instruction' is deluding himself when he talks of history, science or other nonskill subjects." (1970, p. 202)

The fact that much work in writing behavioral objectives has been done in the area of mathematics may in part be a reason for Eisner's second point. However, as Hastings points out, it may also be due to the view of mathematics held by the untrained observer.

> The relative simplicity is illusory, and the illusion is held only by those who have not closely examined the new curriculum developments in mathematics and science. Surely the science and mathematics projects supported by the National Science Foundation are much more complex in intent and in outcome then Eisner suggests. (1967, p. 269)

In his third point, Eisner expressed his feeling that not all outcomes of curriculum and instruction are amenable to measurement. In this category are placed such outcomes as the ability to produce an aesthetic object or an impressive narrative. Difficulty arises for Eisner since to evaluate outcomes such as these a qualitative judgment is required rather than a clearly defined standard. Ebel points out in response, however, that all measurement requires the exercise of some sort of judgment, and if evaluation is to take place some sort of criteria for

for excellence must be identified. He adds that what may be necessary for the encouragement of artistic freedom, if this is a concern, is that we settle for somewhat imprecise measurements of artistic excellence.

Eisner's fourth point deals with the function of educational objectives in curriculum construction. Although he agrees that the statement of objectives is seemingly the rational approach to curriculum development, it may not be the most psychologically efficient. Relating that it is possible for teachers to first identify potentially rewarding activities from which objectives can then be derived, he quotes MacDonald (1965) in support of his view.

> This view would state that our objectives are only known to us in an complete sense after the completion of our act of instruction. No matter what we thought we were attempting to do, we can only know what we wanted to accomplish after the fact. . . .
>
> In the final analysis, it could be argued, the teacher in actuality asks a fundamentally different question from "What am I trying to accomplish?" The teacher asks "What am I going to do?" and out of the doing comes accomplishment. (1965, pp. 613-614)

In response to this point, Hastings (1967) agrees that if there are curriculum developers who try to state objectives and then develop instruction without modifying, adding, or deleting objectives, they should be subjected to Eisner's admonition. Payne (1967) offers a similar response and quotes Tyler (1951) and Herrick (1950) to support her feeling that both of these men, as well, share a position similar to that of Eisner.

Two statements will serve to summarize the preceding discussion on the statement of behavioral objectives. The first comes from Glaser, a proponent of such a procedure, with reference to science education.

> The fear of many educators that the detailed specification of objectives forces us to work with only simple behaviors which can be forced into measurable and observable terms is, indeed, an incorrect notion. The situation rather is, that if we do not attempt to specify the complex processes we want to see in the student, then we are in danger of omitting them and following the path of least effort toward teaching more easily observable and trivial behavior. . . . The challenge, then, is to find ways to recognize this kind of behavior in a student (and the recognition of its presence or absence is a kind of measurement) and to derive instructional situations which allow these objectives to be attained . . . At the very least, in the time we have available as teachers, objectives provide standards of competence which most students can reach and which many will surpass. (1967, p. 2)

The second statement, by Forbes (1971), is also that of a proponent of the statement of behavioral objectives. It is taken, however, from an article in which he writes in opposition to the "cause" of behavioral objectives. Expressing a current outlook and concern of educators, it is a fitting conclusion for the foregoing discussion.

> We have entered an "age of accountability" in education in which tests results are used to evaluate not only the performance of students, but also, by inference, the performance of teachers and schools. Such tests measure the achievement of behavioral objectives. If this is a bad process, as opponents believe, they must organize their arguments, prove their points, and stop the process. If it is a good process, as proponents believe, they must determine if it is being carried out properly and if not, see that it is improved. (1971, p. 747)

In view of the preceding discussion the design of the present study was affected as follows:

1. The behavioral objectives utilized were written in accordance with the characteristics of Mager and Miles and Robinson.
2. Sample tests for each unit were included with the list of behavioral objectives in an attempt to:
   a. reduce the length of the stated objectives.
   b. provide for a more direct communication of desired terminal behaviors.

c. write items that measure the attainment of the desired learning outcomes as suggested by Tyler and Popham.

d. minimize the ambiguity as to what the student was required to do.

3. Selection of the course content and instructional procedures was made, to the extent possible, so as to facilitate student achievement of the objectives.

4. Evaluation instruments were patterned after sample tests and graded according to previously stated procedures and minimum performance levels.

Studies Involving Behavioral Objectives

Any attempt to secure empirical evidence that suggests that students of teachers who use behavioral objectives achieve better than those who do not seems difficult due to the experimental design required of such as undertaking. For Miles and Robinson, there is no such evidence available ". . . and it does not appear that there can be." (1971, p. 44) Their rationale for such a statement is supported by the comments of Raths (1967).

> Raths pointed out that any study which hopes to compare the two treatments - one with pre-specified objectives and one with no pre-specified objectives - is biased in favor of the treatment using pre-specified objectives, because the criterion test usually reflects the specific objectives. This is true when there is a common criterion test to assess both treatments, and when the test is based upon the specific objectives in the one treatment. The use of a dual criteria, one for each treatment, leaves no basis for comparison. (1971, p. 44)

The writers do contend, however, that it is possible to answer questions concerning the benefits for students in achieving stated objectives as well as the influence of objectives specified prior to instruction on student attitudes.

A detailed review of eleven studies which provide answers to questions such as those above is given by Mortlock (1969). Following the

review, he concludes that the studies lend support to the three claims made as the rationale for using behavioral objectives in the instructional process. The reader is directed to Mortlock (1969) for further detail on this review.

Three studies which further support the reasons given earlier for the use of behavioral objectives are those of Bierden (1968), Mortlock (1969), and McBride (1970). Each of the three studies utilized behaviorially stated objectives as a part of the design and attempted to evaluate their effectiveness by means of student questionnaires.

Bierden (1969) explored provisions for individual differences in seventh grade mathematics classes through grouping and behavioral objectives. Although time consuming to prepare, he found that the behavioral objectives offered advantages for both the students and the teacher.

> The objectives guided the selection of appropriate conditions of learning, facilitated communication between the teacher and students of exactly what the students were to learn, and provided a basis for assessing and differentiating student achievement. (1969, p. 197A)

Mortlock (1969), in a study concurrent with that of Bierden, explored similar provisions for individual differences in eleventh grade mathematics. Although reaction was least favorable from students of very high general ability and mathematics achievement, Mortlock concluded:

> Student reactions to the use of behavioral objectives . . . were strongly favorable. . . . With the exception of improving achievement, the objectives lived up to claims made for them with respect to both the teacher and students. (1969, p. 242)

McBride (1971) based flexible grouping and differentiated instruction procedures on achievement of behavioral objectives in a mathematics course for prospective elementary school teachers. Finding behavioral objectives helpful in the areas of communication and assessment McBride

concluded: "The behavioral objectives provided a means of adequately communicating what was to be learned and contributed to more effective evaluation." (1971, p. 4041A)

Within-Class Grouping

The method of with-in class (intra-class) grouping as a means of classroom organization for instruction has been the subject of much educational literature in years past. Included therein are the comprehensive studies of Weaver (1966) and Brewer (1966) on within-class grouping in elementary school mathematics classes. Extensive reviews of the literature in general and as it relates to mathematics education are also available by Ebeid (1964) and Perkins (1967). The study of Bierden (1968) is of particular importance due to its in-depth review of the literature as it relates specifically to the design of the present study. Since these reviews are readily accessible to the interested reader, the intent of this section will be to present a summary of the relevant information.

Shane (1960) has defined within-class grouping as "part-time ability grouping" which is designed to permit the teacher to work with pupils of roughly comparable ability. (1960, p. 316) The definition of within-class grouping used for the purpose of this study will coincide with that given by Shane.

Rationale For Within-Class Grouping

Any attempt to build a rationale for grouping within the classroom must by definition be two pronged. It must present a case for potential educational benefits from the grouping of students, and indicate why this procedure should be carried out by individual teachers in a single classroom.

A first major reason for the use of grouping as a classroom organization for instruction is the wide range of individual differences present in the normal classroom situation. Brewer (1966) points out that early evidence of this fact can be seen in the common practice of grouping in the elementary grades for reading instruction. While Bierden found that grouping in mathematics has also been most frequently used in the elementary grades he added that "the longitudinal growth studies of Olson (1959) have shown that the range of abilities in any school subject increases year by year as children move through school." (1968, p. 9)

While the presence of a wide range of individual differences in the normal classroom setting is seemingly common knowledge for teachers in general, nowhere, as evidenced by the findings of Medsker (1960), Schenz (1963), and Cross (1968) reported in Chapter I, is the fact more apparent than in the community college. Further evidence is seen in unpublished data, reported by Cross, from the School to College: Opportunities for Postsecondary Education (SCOPE) study. Testing 35,000 high school seniors with the School and College Ability Test II, the study placed students in the top, middle, or lowest third of the sample according to scores they obtained. Results indicated that whereas students entering four-year colleges tended to cluster in the top third and non-college students in the lowest third, the junior college group had a "substantial number of students at all three levels." (1968, p. 13) Reynolds has stressed the importance of this situation in pointing out that in junior colleges where there is a wide range of ability among students, there is a strong probability that the same characteristic will prevail in individual classes. (1965, p. 47)

If it is true, as Tyler and Brownell point out, that "no matter what difficulty they make for school men, individual differences are real, inevitable, ineradicable, desirable, and, indeed, essential," then they must be creatively dealt with by educators. (1962, p. 316) The use of within-class grouping seems a potential means of securing this end and has been suggested as such by Jones and Pingry in the following:

> If each child is seen as completely unique, it follows that instruction must be completely individualized; if all children are viewed as alike, teaching should be entirely a group process. Neither position is tenable, and all good teaching accepts a compromise somewhere between these extremes. (1960, p. 122)

A second reason for the use of grouping in the classroom is that it provides for more efficient use of instruction time through rapid identification of student ability levels and prescription of appropriate instructional procedures. The usual attempt to provide instruction for varying ability levels with the class as a whole is only too familiar to the average teacher and has been pointed out by Inlow.

> In a typical classroom situation, where pupils differ widely in ability and in other significant learning traits, the instructor who "teaches" the whole group at any one time can never relate equally to all. If he reaches the bright, he excludes the slow; if he appeals to the average, he slights both the slow and the bright, although for contrasting reasons. (1966, pp. 173-74)

The tragedy of a procedure such as this is not only that it is always of limited value for a certain segment of the class, but that the teacher may proceed unaware of the ability level make up of his class until the results of the first test are in.

Perkins has expressed the feeling that a situation such as this can be alleviated through the procedure of grouping.

> I believe very strongly that the first step in creating a perception of individual differences and a belief on the teacher's part that she could and should try to make provisions for them,

> is to get the teacher to group the children in some way. The very fact that there is more than one group to plan for assures that there will be some planning. Since part of it will be for a group which has some difficulties the teacher is forced to do more diagnosis and planning. (1967, p. 47)

The decision to utilize grouping as an instructional procedure also provides for an immediate awareness of the various levels of ability and achievement in the classroom due to the fact that the grouping is most often based on the results of some initial diagnostic test instrument.

If the teacher is aware of the relative strengths and weaknesses of the individual groups he is better able to adapt his methods of instruction accordingly. For Reeve (1956) the result is efficient use of class time.

> The teacher knows fairly well at the outset which of the groups is the stronger, and he can therefore adapt his methods of instruction to each section as a whole. This enables each section to make as much progress as possible and helps the slower ones to go at a pace that will insure learning that is worthwhile. This makes for efficiency in instruction and time. (1956, p. 76)

A corollary to the above procedure according to Reeve is that the number of failures is reduced "because the students who might otherwise fail are given individual attention earlier in the course and their chances of passing the course are materially increased." (1956, p. 77)

A third major reason for the use of grouping in instruction is that it allows for increased utilization of the interaction between pupils as an aid in instruction. Probably the most common method of instruction in mathematics classes at the senior high and college levels is that of the whole-class lecture. But while the lecture provides benefits for students which a textbook is unable to provide, it fails to provide adequately for individual differences when given in the whole class setting. As DeVault and Kriewall (1970) pointed out "We recognize that the book

is not a teacher but too often we fail to recognize that the lecturer is also not a teacher." (1970, p. 420) Individualization proceeds, for DeVault and Kriewall, rather, through discussion and the opening of both teacher-pupil and pupil-pupil communication lines. While the communication most commonly present in the classroom is of the teacher-pupil variety "We have long known that much learning takes place when peers communicate among themselves." (1970, p. 422)

> Experienced teachers report times when their own efforts to explain something to a pupil have been eclipsed by another pupil who interjected briefly with an effectiveness to be envied by any master teacher. . . . A kind of empathy and understanding exists in such a situation which is difficult for an outsider to emulate. And the teacher is, for the most part, an outsider. (1970, p. 422)

Three major reasons, then, have been pointed out for the use of grouping as a classroom organization for instruction. These are that (1) there are a wide range of individual differences in the normal classroom setting which grouping may help to alleviate, (2) grouping provides for more efficient use of instruction time through early diagnosis of achievement levels and prescription of differentiated instruction, and (3) grouping allows for increased utilization of the interaction between pupils for instructional purposes.

While the grouping discussed above could be carried out through a physical separation of classes and assignment of groups to separate classrooms, there is some basis for the use of grouping within a single classroom. Probably a major reasons for grouping within the normal classroom setting is the ease with which it can be carried out by individual teachers. Rather than introducing the administrative problems of student schedule changes and teacher assignments to various ability levels, the within-class organization allows for grouping, regrouping, or no grouping at all as the

individual classroom teacher sees fit for maximum instructional effectiveness.

A second reason for grouping within the classroom is that it maintains classroom unity and the interaction of students of all ability levels. While it may be that instruction is more efficiently and effectively carried out through grouping based on achievement, there are additional portions of the total classroom experience that seem appropriate for use with the class as a whole. Among these are the introduction and motivation for a unit of instruction, the need to deal with a subject matter problem common to the whole class, and the provision for a final summary and review of a given instructional unit. Utilization of a whole-class approach for these and other activities may serve to lessen the feelings of inferiority and superiority which often result when the total classroom experience is based on groups formed by achievement or ability levels.

A third and probably the most important reason for grouping within the classroom is the success that such a procedure has demonstrated at a variety of different grade levels in past studies and the indications that a similar procedure will work at the community college level. Evidence to this effect will be reviewed later in this section following a discussion of the criteria necessary for successful within-class organization for instruction.

Criteria For Successful Within-Class Grouping

Various lists of criteria, including those of Lerch (1961) and Brewer (1966), have been established as a basis for successful intra-class grouping as an organization for instruction. Bierden (1968), using these and other sources, identifies five criteria as being important for successful within-class organization for instruction. These are as follows:

1. Some combination of subjective and objective criteria should be used as a basis for establishing groups.
2. Grouping procedures should maintain flexibility that will allow each pupil to work with the group which will most appropriately serve his needs, interests and abilities.
3. Grouping should be accompanied by proper selection of content, methods, and materials.
4. Intra-class grouping should not break up the unit of the total class group.
5. Grouping should minimize loss of self-respect on the part of the "slow" pupils. (1968, pp. 17-20)

The use of subjective as well as objective criteria for establishing groups within the classroom is necessary to insure that each student is placed in a situation where optimum learning can occur. Lerch (1961) pointed out, in reference to within-class grouping for arithmetic instruction, that neither a completely objective nor a completely subjective approach to the placement of students in groups is by itself effective. While a purely objective approach may work to the detriment of the student who is unable to demonstrate his achievement or ability on a given test, a purely subjective approach may cause student placement on the basis of personality or behavior. What is needed, rather, are "subjective judgments based on objective data." (1961, p. 406)

Further support for this feeling at the secondary level comes from Hoover and Hoover who see student as well as teacher input necessary for the most beneficial placement of students into groups.

> It should be emphasized. . . that actual achievement and the wishes of the student probably provide the best basis for subgrouping. . . . In every case, the instructor recommends but the student selects his group. (1968, p. 209)

A second criterion for the successful use of within-class grouping as an organization for instruction is that the grouping procedure maintain flexibility. One means of accomplishing this is by providing individual group changes when necessary and periodic regrouping of the entire class. As any form of classroom organization for instruction has as its goal the optimization of student learning, the within-class grouping procedure should provide for changes in group membership by individuals whenever teacher and student alike feel such a change would enhance learning. The most expeditious time for change in group membership, however, appears to be at the inception of a new unit of instruction.

Lerch sees the need for an ongoing assessment of student abilities and understandings and suggests it be done by gathering data from "pretests, inventories, or diagnostic tests which pertain specifically to the next topic to be studied." (1961, p. 406) Hoover and Hoover express a similar feeling in outlining a specific plan for within-class grouping.

> Based upon pretest scores for the first unit, three initial subgroups can be established. Then on the basis of actual achievement in the unit, subsequent subgroups can be formulated (Some consideration might also be given to subsequent unit pretest scores). This process is repeated for each unit of the course. If instructional processes have been adjusted appropriately, the subgroups will soon closely parallel differentiated ability levels. (1968, p. 209)

Another criterion on which a large number of sources agree is that grouping, if it is to have a positive effect on pupil achievement, must be accompanied by appropriate differentiated instruction for each of the groups formed. Evidence to this effect is reported by Jones and Pingry (1960), Ekstrom (1961), and Goldberg and Passow (1966). Support for

this point is also present in the Research Division of the National Education Association's report on ability grouping. Summarizing the findings and conclusions from 50 research studies published since 1960 it found that "ability grouping tends to succeed when there is modification of materials, objectives, curriculum and teaching methods." (1968, p. 75) The report also stated that despite the considerable disagreement surrounding the matter of ability grouping, one of the three major areas of agreement was that objectives, materials, curriculum, and methods should change to meet the needs of the various groups.

It seems important to note as well that providing the modifications described above requires a somewhat greater commitment on the part of the teacher than would normally be the case. Brewer (1966), for example, found that one of the six chief reasons given for not grouping pupils for arithmetic instruction was that grouping required too much teacher time for planning and correction work. He also found that it was necessary for teachers to have access to a wide variety of materials.

One of the criticisms of the use of grouping within the classroom is that classroom unity is disrupted when various groups are studying separate topics which may or may not be related. In such a situation it is somewhat difficult to maintain a sense of classroom unity due to the limited number of occasions that arise for whole class activities. Lerch (1961) suggests that this is remedied if groups are studying the same topic or area but at different levels of difficulty or abstraction. "Under such a plan it would be more possible to bring the entire class together for review sessions or for discussions of particular ideas within a topic." (1961, p. 405)

The need for whole class unity is also reported by Ivie, Gunn, and Holladay (1957). They found that teachers participating in an experiment on within-class grouping for arithmetic instruction recommended the use of some activities designed to pull the group together as a whole. Bierden's fourth criterion, then, for successful organization for instruction through within-class grouping is that it maintain the unity of the total class.

A final criterion identified by Bierden is that grouping be done in such a way that the loss of self respect of the less able student is minimized. While the stigma of being constantly assigned to a low group is pointed out by critics of grouping as being a major defect, Lerch (1961) points out that students who perform poorly in academic work would have some feelings of inferiority regardless of the classroom organization. On the other hand, the negative impact of the grouping procedure would seem to be lessened when grouping that is flexible and whole class activities are combined. Brewer recommends that "Teachers should be encouraged to explain the idea of individual differences to their pupils so that pupils will not feel that they are being stigmatized by not always being in the top group. (1966, p. 313)

A procedure which, in the writer's experience, is more appropriate to the community college level is that given by McMeen (1959). McMeen points out that the loss of self respect is lessened when the student is brought to a point where he can take pride in accomplishing goals which are appropriate for him.

Based on the preceding discussion, the design of the within-class grouping organization for instruction of the present study was characterized as follows:

1. Group membership was based on diagnostic test scores, teacher recommendation and student choice.
2. Grouping was flexible and regrouping occurred at the inception of each of the units.
3. Instruction was differentiated for each of the three groups. A major aspect of the differentiation was the use of exercise sheets written at three levels of difficulty.
4. Classroom unity was maintained through a period of whole class lecture for the first third of each unit, a whole class review for unit tests, and whole class administration of unit tests.

Experiments And Studies Using Within-Class Grouping

Despite the large number of studies involving the use of grouping, the inconclusiveness of the research findings is apparent. For Ekstrom this is due to "the wide variety of experimental conditions, methods, and purposes," and the fact that "few studies have much in common in content, method, or type of subjects used." (1961, p. 222) Goldberg, Passow, and Justman point out that "the essential weakness in many of the studies is that they simply have been poorly designed as experiments." (1966, p. 17) Passow has listed the problems involved with attempting to equate and synthesize the research findings in the area of ability grouping as follows:

1. The studies vary considerably in scope of aim and purpose.
2. The studies differ in the number of students, the number of groups, and the size of the classes involved.

3. The studies differ in their duration - ranging from a semester or less to a year or more.
4. The studies differ in the adequacy of the selection bases and the means of matching experimental and control groups.
5. The studies differ in the "treatment" - i.e.; the differentiation of curricula and methods of teaching.
6. The studies differ in the deployment of teachers in various groups.
7. The studies differ in the instruments and techniques used in evaluating changes in students.
8. The studies have generally failed to assess the effects of grouping on teacher and administrators. (1962, pp. 285-87)

While the above list refers to inter-class grouping it is equally as appropriate for intra-class grouping.

Bierden (1968) pointed out that most of the research reported on intra-class grouping in mathematics was done at the elementary school level. For this reason, much of the review to follow will be concerned with the elementary school level. In addition to the intra-class literature cited, studies involving inter-class ability grouping will be included as seem appropriate. Particular emphasis will be given to the studies of Bierden (1968), Mortlock (1969), and McBride (1970) due to their similarity of design with the present study and their results at a variety of different grade levels.

A series of studies representative of those whose results support the use of within-class grouping are those of Spence (1959), Smith (1960),

Dewar (1962), and Smith (1970). Each of the studies utilized intra-class grouping of pupils for instruction of arithmetic, the first three at the elementary level and the fourth at the middle or junior high school level.

The study of Spence (1959) involved the establishment of experimental and control groups with the control classes taught on a whole class basis. The experimental classes were partitioned into three subgroups designated A, B, and C with subgroups A and C being high and low achievers respectively. Although a variety of means were used for subgroup assignment, emphasis was placed upon available objective data such as intelligence and standardized arithmetic achievement tests. A variety of teaching methods was used in an attempt to vary instruction. The major variation was that fewer concrete materials were used with the upper two subgroups. The objectives of the study included an assessment of the effects of grouping on arithmetic achievement, attitudes of parents, pupils and teachers.

The results showed the arithmetic achievement test scores of pupils in the experimental classes were higher than those in the control classes, although not significantly so. In addition, the attitudes of parents, pupils, and teachers were very favorable toward the classroom use of subgroups for instruction. While careful planning on the part of the teacher and a variety of teaching materials were necessary, Spence concluded that "subgrouping within the classroom for instruction in arithmetic is a practical and successful method of classroom organization to provide differentiated instruction for the various ability levels." (1959, p. 1682)

Smith (1960) established experimental and control classes in grades two through five using pupil pairs matched on the basis of achievement

and intelligence tests. Subgroups were formed in a manner similar to that of Spence and experimental classes spent at least three quarters of the daily arithmetic period working in subgroups. Control classes were taught using a whole class procedure.

The results indicated that significantly higher gains were achieved by the experimental classes in arithmetic average and computation for grades two, four, and five, and in problem solving for grade two. The achievement of subgroup A was significantly higher than that of the matched control group in grade five on computation and arithmetic average. The achievement of subgroup B and C was significantly higher for grade four in computation, problem solving, and arithmetic average, and in computation and arithmetic average respectively. The grade three control classes achieved significantly higher gains than did subgroup B in problem solving. On the basis of these results Smith concluded that "intra-class grouping is a procedure which may be used to improve arithmetic achievement in grades two through five of the elementary school" and that "the effects of intra-class grouping appear to be about the same for the three intra-class groups used in this study." (1960, p. 564)

Dewar (1962) studied intra-class grouping in sixth grade arithmetic classes for the specific purpose of determining whether arithmetic achievement under this organization was significantly better than with a traditional whole class approach. Establishing three subgroups on the basis of achievement test results, teacher opinion, and school records for three levels of ability, instruction was differentiated through a variety of textbook materials and a curriculum outline prepared for each teacher.

The results showed that the low and high subgroups of the experimental classes gained significantly over their control class counterparts

with no significant difference between the middle subgroups. The teachers felt that the grouping and differentiated instruction were definitely valuable for their pupils, but felt themselves limited by the available time and the difficulty of planning for each group. A large majority of the pupils favored grouping "because they were able to learn more, received more individual attention from the teacher, and studied at a level at which they were capable of working. (1962, p. 2247)

Smith (1970) studied within-class grouping in seventh and eighth grade mathematics classes using a combination of independent study and flexible small study groups formed on the basis of achievement. The experiment involved only seventh grade classes the first year and was extended to include eighth grade classes the following year. First year results showed a mean gain of 1.2 grade levels on the Stanford Achievement Test and a high degree of increase in the interest of students in mathematics. Second year results showed all but four students completing the usual five book basic eighth grade series with substantial numbers of students progressing beyond that point through supplementary materials on sets, preparation for algebra, and algebra. The Stanford Test results for the second year were also "very gratifying." "The eighth grade pupils achieved an overall average of 1.2 grade levels with one class attaining 1.5 grade levels. The seventh grade achieved an average of 1.6 grade levels and in some areas went to 1.8." (1970, p. 8)

Moench (1962) utilized a somewhat similar experimental design in an attempt to individualize practice in fourth grade arithmetic classes. Specifically, the study attempted to provide practice for pupils at a variety of difficulty levels through a more independent use of textbooks. "The overall plan was that the teacher would follow class-as-a-whole

organization except during work study time practice periods at which time terminal sub-grouping at three levels would be operative." (1962, p. 323)

The achievement results of Moench are interesting in that they did not surface following the school year study, but only after the summer vacation period when the students returned to school. Although finding no achievement advantage between experimental and control groups at the close of the school year, there were appreciable differences one year after the study had begun.

> At this time experimental groups did excel more than the controls both in reasoning and computation. This held for the low and superior performers with something of a "tie" effect appearing in the middle-performance groups. (1962, pp. 326-27)

It was also concluded by Moench that an important and necessary aspect of the grouping procedure was that of flexibility.

> . . . complete flexibility is one of the advantages of this type of grouping. The subgroups can be set aside and taken up again at any time without interruption to the ongoing program. Also, complete mobility is possible from topic to topic. (1962, pp. 324-25)

Some of the findings of studies involving inter-class ability grouping are also appropriate for inclusion in a review of this sort in relation to the design of the present study. Platz (1965) found that ability grouping for below average students was more effective than ungrouped classroom organization for academic achievement on 9th grade science examinations. Bailey (1968) studied the effectiveness of ability grouping on success in first year algebra and indicated that the mere grouping of students without consideration for teacher differences, and adaptation of course content, methods, and procedures would not assure significantly higher first year algebra achievement.

Willcutt (1969) ability grouped junior high mathematics students by content topics on a flexible plan. Although finding no significant differences in achievement, he did report a general positive attitude change toward mathematics for the grouped students and especially for those with very negative thoughts at the beginning of the year. Finding a low degree of stability in group membership of students from topic to topic and a lack of different curricula for each level, Willcutt recommended that "A flexible grouping procedure, similar to the one used in this research project, but one which included definite curricula for the different grouping levels, would be a further attempt to produce positive academic results." (1969, p. 156)

Some evidence indicative of the fact that an intra-class grouping approach to instruction may be no better than the traditional whole class method is seen in the studies of Campbell (1965) and Wallen and Vowles (1960). Campbell (1965) compared the effectiveness of the two methods with seventh grade arithmetic classes. Although subgrouping at three levels of achievement with varying instruction was utilized, there were no significant differences in the arithmetic achievement between the whole-class and within-class grouping methods. Whole class method students, in fact, made greater gains in achievement than did their experimental counterparts. Wallen and Vowles in a similar study at the sixth grade level also found no significant differences between grouping and nongrouping procedures.

The studies of Bierden (1968), Mortlock (1969), and McBride (1970) are of particular importance for the present study due to their similarity of design, results, and recommendations for further research at a variety of different grade levels. Each of the studies included the use of

behavioral objectives and flexible grouping at three levels of ability accompanied by differentiated instruction. The management procedure for each topic covered in the course included initial instruction of the whole class at an intermediate level of difficulty, a test for the achievement of the objectives at this level, assignment of the students to one of the three groups, small group and individual study in groups, and a final unit test.

Bierden (1968) studied seventh grade mathematics classes of the University School, The University of Michigan during the 1967-68 school year. Reporting significant gains ($P < .01$) in computational skills and knowledge of mathematical concepts, he failed to find significant differences between the experimental students and four seventh grade comparison groups with respect to knowledge of mathematical concepts. The mean gain in attitudes toward mathematics, however, was significantly higher ($P < .01$) for the experimental students than for each of the four comparison groups.

In a concurrent study, Mortlock (1969) studied the effectiveness of flexible within-class grouping as a means of providing for individual differences in eleventh grade mathematics classes at the University High School of The University of Michigan. The control group was formed by comparable classes from the preceding year. The findings included no significant difference in achievement between experimental and control groups. There was also a positive gain in pre-post attitudes toward mathematics, although not significantly so.

The findings of both Bierden and Mortlock indicated significant flexibility in the group membership to warrant a need for flexible grouping. Each of the studies also found a reduction in mathematics test anxiety, with Mortlock's result statistically significant ($P < .01$).

Student reactions to the use of objectives and to the classroom management plan were "favorable" for Bierden and "strongly favorable" for Mortlock.

The study of McBride (1970), which utilized flexible grouping and differentiated instruction based on achievement of behavioral objectives in a mathematics course for prospective elementary teachers, was somewhat different from that of the former two studies in that the small group instruction following the whole class lecture was carried out in separate classrooms. Using as a comparable control group a representative elementary education population studied by Withnell in 1967, McBride reported significantly higher scores for the experimental group on a test of mathematics achievement and a sub-test of basic concepts relative to elementary school teaching ($P < .001$). Additional findings included significant changes in group membership ($P < .001$) indicating a need for flexible grouping, a significant reduction in test anxiety ($P < .001$), and a positive, but not significant, gain in attitude toward mathematics. Student reactions to the classroom management plan were "extremely favorable" with all students summarizing their attitudes as either "highly favorable" or "favorable."

Based on their success with the classroom management procedure, Bierden and Mortlock recommended that this technique or some measure of it be investigated at other grade levels and in other mathematics courses. McBride, noting the least effective instruction to be that in the small group intermediate section, suggested further modification at this point and subsequent investigation.

Two-Test Evaluation

The community college in the United States has been described as the only educational institution that can truly be considered an American social invention. (Gleazer, p. 3) The two year college is probably referred to in this way due to its ideals of universal post high school education through an open door admissions policy, and equal educational opportunity through remedial education and superior instruction. But with the continuing increase in enrollment the community college has been hard pressed to meet the challenge of its espoused ideals.

In reference to the challenge of the open door, Roueche, Baker, and Brownell (1971) note that "few community colleges . . . have lived up to their bright promise. The 'open door' is too often a glib admissions statement rather than a true concept of accountable reality." (1971, p. 11) Concerning instruction in the community college, Roueche and Boggs state that "The claim of 'superior instruction' does not appear to be based upon any research - institutional or otherwise." (1968, p. 53) Although Dare (1970), sampling 120 institutions from the 1968 American Association of Junior College's Directory, found that the conceived purpose of remedial programs for most colleges was that of salvage, second chance, and redirection, the statements of Cohen (1963), Schenz (1963), and Bossone (1966) support the feeling of Roueche, Baker, and Brownell that "Attrition rates at community colleges generally are alarming." (1971, p. 11)

Aiken (1968), in a report comparing the junior college dropout or withdrawee with the junior college continuer, indicates that a part of the problem described above may be due to the way in which evaluation takes place in the junior college. In a comprehensive comparison, Aiken

found the groups alike in vocational goals upon entrance, educational and occupational levels of parents, and demographic characteristics that would contribute to the prediction of withdrawal. While he did find a significant difference in the grade point average of the two groups, he cites a report by Marks (1967) that the academic ability of the two groups is the same. The conjecture posed by Aiken is: "If the withdrawees are like the continuers, it may be possible that because of our grading and evaluation practices that we are forcing good ability students out of college." (1968, p. 9)

Other sources have echoed Aiken's feeling concerning community college evaluation procedures and have seen the need for more research on the classroom test. Seibel conducted a survey of testing practices and problems in junior colleges under the auspices of the Educational Testing Service and concluded that "One third of the colleges felt they needed to do more research with tests." (1966, p. 60) Roueche, in an article on institutional research, points out that "Considering its importance, there is need for more information on the proper function of the classroom test in the junior college." (1967, p. 23) Altshuler, in a discussion on classroom testing practices in the junior college, arrives at a similar conclusion.

> With tests an important part of the evaluation of both teacher and student and with grades so vital an evidence of success, there seems to be a clearcut need for more interest in the construction of good tests and for better reporting of test practices in junior college classrooms. (1967, p. 3)

It was on the basis of this need and the writer's experiences with evaluation in junior college mathematics classes that the two-test evaluation procedure of the present study was formulated. The important aspects of this procedure were the following:

1. Students were given the opportunity to retake a parallel form of any final unit test within one week of the day on which the graded tests were returned.
2. During the one week period various opportunities for instruction on the unfulfilled objectives were available to the student.
3. If a student took advantage of the test-retest option, the higher of the two scores was his final unit test score.

The rationale for such a procedure, in part, was to give an actual second chance for success to those students interested, and to provide motivation for all students to raise their final unit test averages.

Studies Involving Two-Test Evaluation

There appear to be very few studies in the literature which report the use of a two-test approach to final unit evaluation. McBride (1970) utilized a two-test form of grading in the study discussed earlier on flexible grouping in a mathematics content course for prospective elementary school teachers. Following a period of five or six days of whole class lecture the students were given the first of the two tests. Grouped on the basis of these scores the students then studied in groups with an appropriate level of instruction and materials. Following the group study the students were retested and their grades determined by weighting the second test three times the first test. McBride concluded that "The strongly favorable student reactions to the 'two-test' form of grading indicates that this procedure should receive more consideration in mathematics teaching and evaluation." (1971, p. 4041A)

Komaridis (1971) studied the effects of four testing systems on schedules of study, test performance, and retention of materials in an introductory educational psychology course. The four testing systems involved were:

1. tests scheduled by teacher and administered only once to students,
2. tests scheduled by teacher and administered a second time if the minimum mastery level was not reached on the first trial,
3. tests scheduled by students and administered only once to students, and
4. tests scheduled by students and administered a second time if the minimum mastery level was not reached on the first trial. (1971, p. 790A)

Although the study found that the testing system did not significantly affect final test performance, "it was found that the retesting condition resulted in significantly higher retention test scores than the scores obtained under the one-trial testing method." (1971, p. 790A) The results also suggested that students with low grade point averages were able to profit most from the retesting method. Students with low grade point averages in testing system three seemed to retain significantly less of what they had learned than students under other testing systems.

Mitzel (1970) studied the effects of the variable "prior experience with test-type problems" on minority group test performance, with the effects estimated using test-retest correlational analysis. A sample of 151 Negro second-grade children were randomly assigned to

three treatment groups. The first two received instruction on nonverbal and verbal test problems, respectively, while the third was administered a placebo treatment. Each of the three groups received four test trials on a verbal and nonverbal test over a period of six months. Included in the test-retest correlation coefficient findings were indications that "experiences with test related material effected systematic changes in the test retest correlations of the group studied," and "major changes in group performance occurred between the first and second testing." (1970, p. 611A)

> Results of the investigation suggest that when subjects are tested on material on which it cannot be assumed they have had equivalent prior exposure, first test scores are less stable and less typical of later performance than second testing. In such situations, retesting would seem to be appropriate before decisions based on test results are made. (1970, p. 611A)

Perry (1971) studied the use of an unlimited retest option in a junior college elementary algebra course. Noting that while it is necessary for most junior colleges to include a course of this type, Perry states that the elementary algebra course is one of the most difficult to teach.

> However, it is one of the most difficult courses in the mathematics curriculum to teach because of the diversity of student interest and preparation and because primarily it is not a transfer course, but a remedial one. Thus students have hesitated to take the course even though their mathematical background indicated a need for the course. (1971, p. 40)

Under the retest option of Perry, if the student did not receive an acceptable score he could retake an alternate form of the test in the following week. This procedure was continued until the desired score was achieved. The unit test score received by the student was the highest score obtained on the tests taken for a given unit. Although Perry notes the need for a large number of parallel forms of a given unit test, it is

important for the present study that "An average student took only one or two times to complete a test satisfactorily . . . ." (1971, p. 44)

The results showed no significant differences in achievement between experimental and control group classes, but did indicate a significant difference ($P < .05$) in attitude toward mathematics. An interesting comparison reported was that arrived at by assuming that the score of the first test taken by experimental group students would have been their score in a traditional classroom situation. On a four point system, the mean of the experimental group students using the first test was 2.42 as opposed to 3.21 for the best unit test for the same group. Commenting on the latter mean as almost a full grade higher, Perry expresses the feeling that this "is a truer picture of the knowledge that the student has gained from the course." (1971, p. 45)

Summary

The review of the literature on behavioral objectives points out three characteristics of an effective objective. For Mager, these are that the objective should specify the terminal behavior, describe the conditions under which the behavior will occur, and specify the criteria of acceptable performance. Mager stresses, however, that the major concern is to write objectives which communicate. One way of furthering this communication, as seen by Popham and Tyler, is to provide sample test items for each of the stated objectives.

Three basic reasons for the use of behavioral objectives were presented from a summary by Mortlock. These were: communication of terminal behaviors and guidance in the selection of content and instructional procedures; facilitation in the evaluation of the extent of achievement of

objectives by students; advantages to the student of providing goals, assessing his progress, and organizing his learning.

The definition of within-class grouping given by Shane of "part time ability grouping" was accepted for the purposes of the present study. A rationale for the use of such a procedure was that it (1) is a practical and easily controlled method of providing for individual differences, (2) provides for more efficient use of instruction time, and (3) maintains classroom unity as well as allows for increased utilization of the interaction between pupils as an aid in instruction. Five criteria identified by Bierden as important for successful within-class organization for instruction were the use of objective and subjective criteria for establishing groups, group flexibility, differentiation of content, methods and materials for each of the groups, maintenance of total class unity, and minimization of the loss of self-respect for slower students.

The need for research in evaluation and the use of the classroom test at the community college level was supported by Aiken, Seibel, Roueche, and Altshuler. A potential means of partially realizing this objective and of allowing the community college to better meet the needs of all of its students was presented as the use of a two-test procedure for final unit evaluation.

CHAPTER III

DESIGN OF THE STUDY

The purpose of this study was to determine the effects of variations in classroom organization for instruction and final unit evaluation procedures on mathematical achievement, attitudes toward mathematics, mathematics test anxiety, and the success and dropout rate in community college beginning algebra classes. The treatments were:

1. Whole class instruction with one final unit test. (W-1)
2. Whole class instruction with a final unit test-retest option. (W-2)
3. A combination of whole class instruction and differentiated small group instruction with one final unit test. (G-1)
4. A combination of whole class instruction and differentiated small group instruction with a final unit test-retest option. (G-2)

Each treatment incorporated the following features: 1) a list of instructional objectives covering the material taught for each of the five units of the course, 2) a sample test for each unit of the course, 3) exercise sheets at three levels of difficulty for each topic of the course.

In chapter II a review of the literature pertaining to instructional objectives, flexible grouping and the test-retest procedure for unit evaluation was presented. This chapter describes the way in which these variations were utilized in the design of the study. A rationale for the various aspects of the design, where appropriate, will be based on reference

to the literature. Additionally, this chapter includes a description of the experimental population, the selection of classes, the classroom controls, the preparation of materials, the four classroom treatments, and the instrumentation used.

Experimental Population

The study was conducted at Highland Park Community College, Highland Park, Michigan, during the fall semester of the 1971-72 school year. Although the college is located in Highland Park, a majority of the students enrolled lived in the city of Detroit which completely encircles Highland Park. The college is a predominantly black, inner city, two year institution which offers a wide variety of two year terminal programs as well as courses for students planning to transfer to four year colleges and universities.

The initial experimental population consisted of 220 beginning algebra students enrolled in 8 of the college's "Mathematics 100" classes. Four of the classes were scheduled during the day school sessions of the college. The four classes were taught by one instructor and met for three 50 minute periods a week between the hours of 8:00 a.m. and 3:00 p.m. The remaining four classes were classified as late afternoon and evening classes. These were taught by the second instructor in the study and met for two 75 minute periods a week between the hours of 3:00 p.m. and 8:45 p.m. Additional information comparing the characteristics of day and night students is reported in Chapter V.

Selection of Classes

Mathematics 100 is a one semester beginning algebra course with content coverage largely that of a first year high school algebra course. The

topics of study for the five unit course are:

Unit I - Sets, variables and open sentences, rational numbers, mathematical properties

Unit II - Signed numbers, inverses, absolute value, removing parentheses

Unit III - Algebraic expressions, equations, formulas, word problems, solution and graphing of inequalities

Unit IV - Polynomials, exponent laws, factoring, polynomial equations

Unit V - Rational algebraic expressions, complex fractions, rational equations

A nontransferable course, Mathematics 100 was designed for those with deficient mathematical backgrounds who must either take a higher level mathematics course in order to complete their chosen area of study, or satisfy a prerequisite requirement for another course.

As the opportunity to place students randomly into various treatment groups was not available, classes were chosen in advance of registration in such a way that the two experiment instructors could each meet four Mathematics 100 classes. The four classroom treatments were then randomly assigned to the four classes of each instructor.

Classroom Controls

Initial guidelines were laid down for each of the instructors participating in the experiment in terms of methods and procedures for each of the four treatments. In addition, the writer and instructors met weekly for purposes of careful in-service training, discussion, and distribution of materials. Periodic classroom visits by the writer were also made in

an attempt to insure, as far as possible, the intended definition of the treatments.

Classrooms were also controlled in terms of content coverage, total time allotments for instruction, and materials used. All instructors were expected to cover the material designated for inclusion in the course. No controls were placed on the amount of time spent on individual units, but guidelines for unit lengths were distributed and discussed.

One aspect of classroom activities that was not constant when comparing whole class instruction classes with grouping instruction classes was the presence of an extra day of instruction in the former and an extra day of testing in the latter. This occurred due to the use of the diagnostic test for the purpose of defining group membership in the grouping approach and the absence of such an instrument in the whole class approach. Beyond the attempt to minimize the effect of the differences in instruction time by shortening the diagnostic test from an hour to 20-30 minutes, the situation must be included as part of the definition of the treatments.

Preparation of Materials

The instructional materials used for the course, in addition to the textbook, were written by the writer during the spring and summer semesters of the 1970-1971 school year. They were used during the summer semester by the writer in a Mathematics 100 course as a means of modifying the materials, where necessary, based on student feedback. These materials included: 1) a list of instructional objectives, 2) a unit outline, and 3) a sample test.

The use of stated objectives has met with success at a variety of different grade levels and has gained wide support from educators and teacher training institutions. In summary from Chapter II, instructional

objectives provide goals for students by communicating exactly what is to be learned, guide the teacher in the selection of content and appropriate instructional procedures, and facilitate evaluation of the extent to which students are achieving the objectives of the educational program.

The selection of the objectives was based on the content definition and purpose of the course as communicated by the college mathematics department, and in accordance with the textbook for the course, Introductory Algebra For College Students by Johnson, Lendsey, and Slesnick (1967).

For each list of instructional objectives those considered necessary for successful completion of the course were labeled as "basic" and starred for recognition purposes. In addition to being emphasized for all students, regardless of treatment, the basic level objectives were those which received instructional emphasis for group 1 students in the flexible grouping treatments.

The procedure commonly used to clarify instructional objectives for students is to either key each objective to a set of textbook exercises or include a series of examples following each of the objectives. In an attempt to further organize and clarify the content to be learned, the writer, in place of this procedure, included a sample test with the instructional objective list for each unit. Similar in format and content to the final test, the sample test was designed to communicate to the student not only what he was required to know, but the way in which it would be asked, as well. Thus the only task left for the student was to demonstrate achievement of the objectives.

The unit outline given to each student at the beginning of the unit included a list of topics for discussion for each class meeting of the unit and the text book pages on which the topic was covered. It also listed

a set of suggested homework exercises in the textbook for each topic.

A unit outline, sample test, and list of instructional objectives for one of the five units of the course are included in Appendix A.

In addition to the above package, exercise sheets at three levels of difficulty for each topic of the course were written by the writer and utilized as a supplement to the textbook. The materials were prepared at three levels to correspond with the three level grouping design of the G-1 and G-2 treatments. The first level exercise sheets were limited to basic level objective content and were characterized by detailed explanations of problem types, planned problem sets, and judicious amounts of repetition. The second level exercise sheets covered all of the objectives of the unit and were characterized by a brief presentation of the content with problem set emphasis on a wide variety of problem types. The third level exercise sheets contained exercises reviewing the basic level objectives, a brief presentation with problem sets for the remaining objectives, and horizontal enrichment material.

Sample exercise sheets for each of the difficulty levels are included in Appendix B.

Classroom Treatments

W-1

The whole class treatment was designed as a modified traditional approach to classroom organization for instruction. It is here defined as an approach in which a majority of the within class instruction time is carried out by the teacher with the class as a whole. Classroom time was divided so that approximately 75% was devoted to informal lecture and other developmental activities with 25% devoted to classroom discussion of

such things as homework and example problems. The informal lecture was presented at a level appropriate for a majority of the class members as perceived by the instructor.

The evaluation procedure utilized for the W-1 treatment was that commonly used in the traditional classroom. This procedure provides a measure of achievement on the basis of a test given at the conclusion of the instruction period of each unit. The one-test procedure offers no alternative for the student who receives a failing grade other than to "make it up" on the next unit test.

Some of the modifications of the traditional whole class approach used in this treatment have already been detailed. Each of the students received the package of materials containing instructional objectives, a sample test, and a topical outline for each unit of the course. In addition students were able to choose one of the first two levels of exercise sheets for each topic of the course. Those students desiring both levels of exercise sheets were accommodated where the supply of materials allowed.

In summary, the modified traditional whole class approach to instruction was characterized by:

1. Informal lecture and heuristic questioning by the teacher in the development of concepts at an average level of sophistication.
2. Class discussion, flowing through the teacher, of sample problems solved by the teacher or presented by the teacher and solved by the students at their desks.
3. A list of instructional objectives, a sample test, and a topical outline for each unit.
4. Use of textbook problem sets, and supplementary exercise

sheets for planned practice and homework assignments at one of two levels of difficulty.

5. Teacher response in whole class setting to questions on homework assignments.
6. Teacher led summary and review of units.

W-2

The W-2 treatment was designed exactly the same in terms of instructional procedures and materials as the W-1 treatment. It differed in its use of a 2-test approach to final unit evaluation.

The 2-test procedure offered to any student who desired the opportunity to retake a parallel form of the unit test within a prescribed number of class days of the termination of the unit. The period within which a test could be retaken was one week from the date on which the graded tests were returned. During this time the student was encouraged to take advantage of various options for preparing himself for the second testing. These included individual study, opportunities for conference with the teacher during office hours, utilization of the mathematics department tutoring program, or study with fellow classmates. As class time was not provided for retests, the student was required to make an appointment with his instructor to take the test.

The purpose of the test-retest option was to provide a "second chance" for securing a passing unit test score and to motivate all students to improve their final unit test scores. For the student who elected to take advantage of the test-retest option, the higher of the two scores was used as his final unit test score.

Bierden (1968) and Mortlock (1969) utilized a two-test approach to final unit evaluation and found student attitudes favorable toward it.

In addition, Bierden found a reduction and Mortlock a significant reduction in mathematics test anxiety. McBride (1970) found student reactions to a two-test form of grading to be very favorable. Perry (1971) utilized an unlimited retest procedure as part of a community college experiment in elementary algebra. While reporting no significant differences in mathematics achievement, there was a significant positive change in attitude.

G-1

The classroom organization for instruction used in the G-1 and G-2 treatments was patterned with only a few modifications after the design utilized by Bierden (1968), Mortlock (1969), and McBride (1970). The organization involved a combination of whole class and flexible small group instruction with the group instruction differentiated for varying levels of achievement.

The Mathematics 100 course was divided into five units. As a planned sequence of activities was repeated for each of the five units, a detailed description of these activities will be discussed below in the order in which they occurred in the classroom. A flowchart of the classroom procedure is given in Illustration 1. The flowchart sections are keyed to the description by Arabic numerals.

1. Each unit of the course was begun with a period of whole class instruction for purposes of introduction, motivation, and informal lecture on the basic ideas of the unit. Approximately 1/3 of the time allotted for the unit was devoted to this form of instruction. During the whole class instruction period an attempt was made to focus mainly on basic

concepts and skills rather than on all topics of the unit. The purpose of this portion of the unit was to provide the basis for a sound conceptual framework from which all of the students in the class could proceed as well as to provide a period of meaningful review for students who had, in part, already attained many of the basic ideas of the unit. Assignments given during the whole class instruction period were basic in nature and reflected the ideas covered in the informal lectures presented by the teacher.

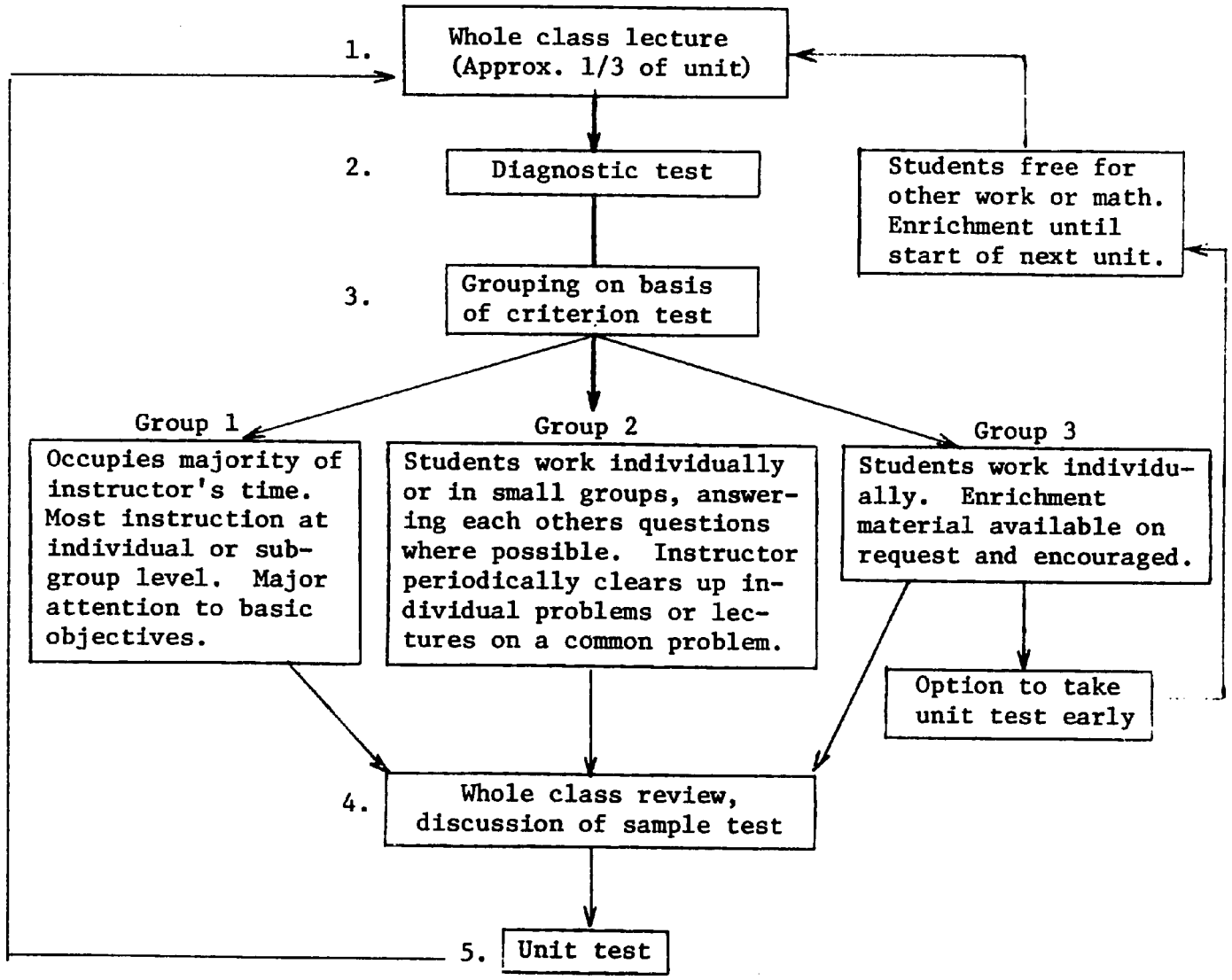
At the beginning of the whole class lecture portion of the first unit each student was given a topical outline, sample test, and list of instructional objectives for the unit. Thereafter, for the remaining four units, these materials were distributed immediately following the test on the previous unit.

In addition to fulfilling the purpose defined above, the whole class lecture portion of the instruction seemed important for a number of other reasons:

1. It provided for a continuing whole class identity for all of the students, one of the five criteria identified by Bierden (1968) for the effective use of intra-class grouping.
2. It provided a period of instruction in which all students were able to benefit from the questions and discussion of their classmates.
3. It allowed students the opportunity of a well organized overview of the unit followed by a chance to determine their group level anew each unit by demonstrating the level of their achievement of the objectives.

ILLUSTRATION I

CLASSROOM PROCEDURE FLOWCHART FOR COMBINATION
WHOLE-CLASS, FLEXIBLE SMALL GROUP INSTRUCTION



4. It provided for a meaningful change of pace to the general classroom routine.

2. Following the whole class instruction period, a test sampling all topic areas of the unit and similar to the final unit test was administered for diagnostic purposes. The diagnostic test scores were not used in assessing final student achievement for the unit, but rather to identify student strengths and weaknesses and to provide guidance for partitioning the class into three small subgroups. Students were placed in various groups on the basis of their diagnostic test scores as well as on the basis of the instructors assessment of their level of attainment. The approximate diagnostic test score percentages defining the groups were as follows:

Group 3 85% - 100%

Group 2 60% - 85%

Group 1 59% or less

3. On the class meeting immediately following the administration of the diagnostic test, students received their diagnostic tests back with an indicated group assignment on it. Each of the three groups was then assigned to a portion of the classroom where study and instruction was carried on in groups for approximately the remaining 2/3 of the allotted instruction time for the unit. The differentiation in instruction for each group in this period was sought through variations in content expectation, learning time, and teaching methods and materials.

The major objective in providing small group instruction was to free students to work at a rate and with a level of materials more commensurate with their own ability so that each student could be assured, as far as possible, success in learning the desired basic concepts and skills. Since strong motivation for the proposed study arose from the

problem of how best to meet the needs of the low achieving student in the whole class setting, instructional modifications for group 1 will be discussed first.

Group 1 Instruction

A first important aspect of group 1 instruction was that it was, in general, limited to those basic concepts and skills judged essential for successful completion of the course. Thus, expectations for content mastery were limited to the starred objectives on the list of instructional objectives for the unit. A major reason for this limitation was to provide for the slower learning rate which is characteristic of this level of student. Final unit tests were constructed in such a way that achievement of the basic objectives would insure a passing test score, so that the limited content emphasis would not work as a detriment to the group 1 member. If any group 1 individual appeared, in the instructor's judgment, to have achieved the basic objectives prior to the end of the group instruction period, he was encouraged to begin work on the remaining objectives.

Instruction in group 1 was centered around a maximum amount of student-teacher interaction and planned homework and practice assignments. Approximately half of the teacher's within-class instruction time was devoted to work with the students of this group. During this time the teacher concentrated on sequential demonstrations of various problem types and on eliciting student discussion concerning them. The instructor attempted to involve the students and to provide for continuing feedback as to their level of proficiency. During periods of concept and skill development, a special attempt was made to explain problems in a step by step fashion.

In summary, group 1 instruction was characterized by:

1. Emphasis on student involvement in the learning process through a maximum amount of student-teacher interaction.
2. Close supervision of student worked sample problems with guidance for diagnosis of errors.
3. Initial presentation of concepts in concrete form with slow movement to a more abstract level.
4. An abundance of repetition.
5. Planned practice and study assignments to provide for characteristics 3 and 4 above.
6. Step by step explanations of difficult skill and concept developments.
7. Elimination of non-essential ideas from the first line of instruction.

A brief description of instructor activities for a given class period may give some indication as to how the classroom activities were managed to provide time for the procedures discussed above. A typical class period began with a few minutes devoted to making assignments for groups 2 and 3 with distribution of planned exercise sheets and an indication to group 2 members that a certain topic would be discussed later in the period. The next 20-30 minutes was spent with group 1, introducing a given topic and laying the groundwork for meaningful practice either from the textbook or supplementary exercise sheets. The teacher then returned to group 2 and carried on an informal lecture period for 10-15 minutes on a difficult topic or one not clearly discussed in the textbook. The final minutes of the period were used to make assignments to each of the groups as well as to outline activities for the following

period or periods. On another day the time allotments might have been varied so that planning for group 3 members could be accomplished while still allowing for group 1 instruction time.

Group 2 Instruction

Group 2 procedures differed from those of group 1 in that a larger portion of students' time was devoted to independent study. The instructor in this setting functioned primarily as a resource person for individual questions and for periodic lecture to the whole group. The lecture served to present areas that had proven difficult for students in the past or that a majority of the group members appeared to be having difficulty with as evidenced by their individual questions. Students were encouraged to work with fellow group members for purposes of giving and receiving help when the instructor was not available. Instruction was characterized by less repetition than in group 1, although emphasis on skills was maintained and fostered through the planned homework and practice exercises.

Content expectations for group 2 members ranged from a minimum of the starred instructional objectives for the unit to encouragement for achievement of all the objectives.

Group 3 Instruction

The instruction in group 3 was similar to that of group 2 with the instructor acting as a resource person and offering encouragement for members to discuss ideas and seek help from each other. Students in this group had, in a sense, already achieved most of the objectives for the unit, as evidenced by their diagnostic test scores. However, in the instructors' experience, many times there was still a need to solidify concepts and skills. Thus, group 3 members concentrated not only on

unachieved objectives and horizontal enrichment material but on problems sampling all of the objectives for the unit. The sampling attempted to provide for a wider variety of problem types and levels of difficulty than it was possible to include in a diagnostic test instrument. Content expectations for group 3 members included achievement of all unit objectives.

Open to members of group 3 was the opportunity to complete the final unit test before the designated testing day. This was done only upon consultation with the instructor. Upon successful completion of the unit previous to the designated testing day, the student was free to terminate his attendance until the beginning of the next unit or to work on some related mathematical topic which was of interest to him.

4. At the conclusion of the unit, the three groups were reassembled for a whole class review. The review was usually centered around the sample unit test.

5. The final unit test was administered on the class day immediately following the whole class review.

6. On the next regularly scheduled class meeting, the class met as a whole to receive and discuss, where necessary, their graded unit tests. The next unit was then begun where the above procedure was repeated.

The final unit evaluation procedure for the G-1 treatment was the same as that utilized in the W-1 treatment. It provided a measure of achievement on the basis of one test given at the conclusion of the instruction period of each unit.

G-2

The G-2 treatment was designed, in terms of instructional procedures and materials, exactly the same as the G-1 treatment. It also

utilized the two-test procedure for final unit test evaluation as outlined in the W-2 treatment discussion.

Instrumentation

Pre- and Post-Test Instruments

Three pre- and post-test instruments were utilized for the purpose of this study. These included instruments to measure mathematical achievement, attitudes toward mathematics, and mathematics test anxiety. The pre-test forms were given during the first two class meetings of the semester while the post-test forms were administered during the final two days of the semester previous to the final exam.

Mathematical achievement was measured using the Cooperative Mathematics Test, Algebra I (1962), forms A and B. Parallel forms B and A of the Test were used as pre- and post-test instruments respectively. A publication of the Educational Testing Service, each of the forms is a 40 minute test of algebra achievement with coverage equal roughly to one year of high school algebra. For the purposes of this study, this test will be referred to as the CMAT (Cooperative Mathematics Achievement Test).

Student attitude toward mathematics was measured using the Aiken and Dreger Opinionnaire (1961). A copy of this instrument is included in Appendix D. The 20 item instrument is scored using Likert's method of summated ratings. Under this method, the respondent indicates whether he strongly agrees, agrees, is undecided, disagrees, or strongly disagrees with each of the 20 items expressing positive or negative attitudes towards mathematics. The response alternatives for positive items are weighted from 4 (strongly agree) to 0 (strongly disagree) with the weightings reversed for negative items. A student's score is the sum of the weighted

alternatives he has chosen. Possible scores on the instrument range from 0 to 80 with a high score reflecting a positive attitude toward mathematics. This test will be referred to as MAO (Mathematics Attitude Opinionnaire).

The instrument used to assess student test anxiety was the Questionnaire on Attitude Toward Mathematics Testing, a modification of the Test Anxiety Questionnaire developed by Mandler and Sarason (1958). A copy of this questionnaire is included in Appendix D. To complete the questionnaire, the respondent recorded his feeling for each of the 15 items by placing an X along a continuum defined by brief phrases at its mid- and end-points. For scoring purposes, each of the 15 continua is split into 10 divisions which are weighted from 1 (low anxiety) to 10 (high anxiety). A student's score, which may range from 15 to 150, is the sum of his 15 responses with a high score indicating a high test anxiety. This test will be referred to as TAQ (Test Anxiety Questionnaire).

Opinionnaire

This instrument was prepared by the writer and contained statements relating to identification of the material to be learned, a student's most beneficial learning resource, a student's perceived attitude change toward mathematics, and the thing that most influenced this change. For all but the last question the student was instructed to choose one of the listed responses or to specify one of his own. The final question was an open-ended one. The opinionnaire was given once, in December, following the conclusion of instruction for the course and previous to the final exam. A copy of the opinionnaire is found in Appendix D.

Treatment Verification Form

The purpose of the treatment verification form was to obtain student verification that each of the treatments had been carried out by the instructors as intended by the writer. Verification was sought in the areas of grouping, testing, materials, and handouts. The questions posed on these areas were open-ended in nature and designed to allow the student to respond as he desired. Also included on the verification form were three questions designed to investigate student opinions concerning the class. A copy of this instrument is included in Appendix D.

CHAPTER IV

ANALYSIS OF THE DATA

The purpose of this chapter is to present an analysis of the data collected in this study. The chapter is divided into sections dealing with total group comparisons, mathematical achievement, attitudes toward mathematics, test anxiety, success and dropout rate, and student reactions.

The initial experimental population consisted of 220 beginning algebra students at Highland Park Community College, Highland Park, Michigan. The experiment was undertaken for the duration of the Fall, 1971, semester. For all but the section dealing with success and dropout levels, the analysis concerns itself with the 161 students who finished the course and for whom the data are complete.

For the reader interested in examining raw scores, actual pre- and post-test means and standard deviations are given in Tables 1-3 for (1) the total population, (2) each of the four treatment groups, and (3) each of the four treatment groups by instructor.

The variable and treatment abbreviations that will be used throughout this chapter are as follows:

1. CMAT: Cooperative Mathematics Achievement Test, Algebra I. A measure of algebra achievement, this test has a maximum score of 40.
2. MAO: Mathematics Attitude Opinionnaire. This instrument has a maximum score of 80, with a high score indicating favorable attitude.

3. TAQ: Questionnaire on Attitude Toward Mathematics Testing. A measure of test anxiety, this instrument has a maximum score of 150, with a high score indicating a high test anxiety.
4. FUTA: Final Unit Test Average. A student's final test average for the five unit tests, with the higher of the two unit test scores used for test-retest option students.
5. W-1: Whole class instruction with one final unit test.
6. W-2: Whole class instruction with a final unit test-retest option.
7. G-1: Combination whole class and differentiated small group instruction with one final unit test.
8. G-2: Combination whole class and differentiated small group instruction with a final unit test-retest option.

TABLE 1

MATHEMATICAL ACHIEVEMENT, ATTITUDES TOWARD MATHEMATICS, AND TEST ANXIETY, PRE-TEST AND POST-TEST SCORES, ACTUAL MEANS ($\bar{X}$), STANDARD DEVIATIONS (SD), AND MEAN GAIN SCORES ($\bar{d}$), FOR THE TOTAL GROUP

VARIABLE		N	POSSIBLE SCORE	$\bar{X}$	SD	$\bar{d}$
CMAT	Pre-	161	40	10.13	4.53	6.78
	Post-	161	40	16.91	5.12	
MAO	Pre-	161	80	44.13	17.89	4.72
	Post-	161	80	48.85	16.68	
TAQ	Pre-	161	150	77.94	19.90	-7.02
	Post-	161	150	70.92	22.54	

TABLE 2

MATHEMATICAL ACHIEVEMENT, ATTITUDES TOWARDS MATHEMATICS, AND TEST ANXIETY, PRE-TEST AND POST-TEST SCORES, ACTUAL MEANS ($\bar{X}$), AND STANDARD DEVIATIONS (SD), FOR EACH OF THE SUBGROUPS

		W-1 N = 40		G-1 N = 42		W-2 N = 39		G-2 N = 40	
VARIABLE		$\bar{X}$	SD	$\bar{X}$	SD	$\bar{X}$	SD	$\bar{X}$	SD
CMAT	Pre-	10.33	4.85	11.17	4.04	9.85	4.83	9.13	4.29
	Post-	17.28	4.72	16.52	4.50	17.05	6.03	16.83	5.33
MAO	Pre-	46.58	19.13	41.64	16.51	43.59	17.54	44.83	18.64
	Post-	51.78	17.53	46.79	15.00	47.41	17.47	49.50	16.85
TAQ	Pre-	78.78	22.55	79.64	17.73	76.80	19.63	76.43	20.08
	Post-	72.73	20.60	70.24	26.11	70.80	23.59	69.95	19.92

TABLE 3

MATHEMATICAL ACHIEVEMENT, ATTITUDES TOWARD MATHEMATICS, AND TEST ANXIETY, PRE-TEST AND POST-TEST SCORES, ACTUAL MEANS ($\bar{X}$), AND STANDARD DEVIATIONS (SD), FOR FINAL UNIT TEST AVERAGES, BY INSTRUCTOR, FOR EACH OF THE SUBGROUPS

I_1		W-1 N = 20		G-1 N = 26		W-2 N = 21		G-2 N = 20	
VARIABLE		$\bar{X}$	SD	$\bar{X}$	SD	$\bar{X}$	SD	$\bar{X}$	SD
CMAT	Pre-	9.90	5.09	11.58	4.34	10.76	4.25	9.35	3.69
	Post-	17.65	4.63	16.92	4.27	18.71	5.27	17.15	5.09
MAO	Pre-	49.00	18.65	42.12	17.27	44.52	17.86	45.80	15.41
	Post-	55.95	16.39	48.58	13.78	50.95	14.56	51.10	13.59
TAQ	Pre-	80.15	20.05	76.77	17.39	77.38	18.41	72.65	16.29
	Post-	71.65	22.58	64.42	22.44	71.81	20.90	70.40	17.74
FUTA		70.4	14.0	72.8	17.9	74.5	14.2	73.1	17.0

I_2		W-1 N = 20		G-1 N = 16		W-2 N = 18		G-2 N = 20	
VARIABLE		$\bar{X}$	SD	$\bar{X}$	SD	$\bar{X}$	SD	$\bar{X}$	SD
CMAT	Pre-	10.75	4.69	10.50	3.52	8.78	5.35	8.90	4.91
	Post-	16.90	4.91	15.88	4.91	15.11	6.42	16.50	5.67
MAO	Pre-	44.15	19.77	40.88	15.72	42.50	17.60	43.85	21.77
	Post-	47.60	18.04	43.88	16.85	43.28	19.99	47.90	19.82
TAQ	Pre-	77.40	25.25	84.31	17.81	76.11	21.48	80.20	23.08
	Post-	73.80	18.94	79.69	29.51	69.61	26.96	69.50	22.34
FUTA		67.8	20.2	63.5	24.0	71.9	20.4	78.0	12.5

Total Population Comparisons

H(1): There are no significant differences in the changes in mathematics achievement, attitude toward mathematics, or test anxiety for the total population of beginning algebra students.

The measurements used to describe the changes were the pre-and post-test scores on the CMAT, MAO, and TAQ. The hypothesis was tested by t-tests for the significance of the difference of the mean change scores from zero. The results for hypothesis H(1) are shown in Table 4.

TABLE 4

CMAT, MAO, AND TAQ PRE- AND POST-TEST SCORES, MEAN CHANGE SCORES ($\bar{d}$), STANDARD DEVIATIONS (SD), AND t-RATIOS (t), FOR THE TOTAL GROUP

Source		CMAT	MAO	TAQ
Total Class	$\bar{d}$	6.78	4.72	-7.02
N = 61	SD	4.44	12.73	22.03
	t	19.38*	4.70*	4.04*

*Indicates significance at the .001 level.

The hypothesis of no significant changes in mathematics achievement, attitude toward mathematics, or test anxiety is rejected ($P < .001$). The data show significant changes in each of the three categories.

The significant change in mathematics achievement is consistent with the results of similar studies including that of Grasser (1969). The significant positive change in mathematics attitude and significant reduction in test anxiety are especially noteworthy. Few experimenters have reported significant gains in both test anxiety and attitudes. That this result occurred with a population of predominantly black, inner city, community colleges beginning algebra students offers encouragement to the writer for future related experimentation.

Mathematical Achievement

H(2): There are no significant differences in mean achievement between the treatment groups of community college beginning algebra students as measured by post-CMAT scores.

This hypothesis was tested using a three-way analysis of covariance. The three variables of classification were instructors, classroom organization for instruction, and final unit evaluation procedures. Two instructors took part in the study. One was classified by the Community College as a day instructor and the second as a late afternoon and evening instructor. Two methods of classroom organization for instruction were utilized. One used a modified traditional approach with the class operating as a whole and with instruction carried out through a lecture-discussion combination. The second classroom organization utilized a combination whole class-flexible grouping approach for each of the five units of the course. Instruction was varied for each of three group levels. The two forms of final unit evaluation procedures were one final unit test, and the option to retake a final unit test within one week of the date the tests were returned to the students. In the latter procedure the student received the higher of the two test scores.

The data pertinent to this hypothesis are shown in Table 5. The null hypothesis of no significant differences in mathematics achievement as measured by pre- and post-scores of the CMAT is accepted. The data show no significant differences in mathematics achievement between the four treatment groups.

An area of concern at the onset of the experiment was the possible occurrence of interaction between instructors, classroom organization for

instruction (Groups), and final unit evaluation procedures (Tests). If statistically significant interaction had been found, it would have caused difficulty in drawing definite conclusions regarding the variables being tested. The data of Table 5 show no significant interaction between instructors, groups, or tests.

TABLE 5

ANOCOVAR FOR MATHEMATICS ACHIEVEMENT
USING CMAT POST-SCORES

Source	df	MS	F
Instructors (I)	1	44.83	2.59
Tests (T)	1	28.65	1.65
Groups (G)	1	10.09	0.58
I x T	1	2.35	0.14
I x G	1	21.07	1.22
G x T	1	28.38	1.64
I x G x T	1	2.13	0.12
Error	152	17.34	

Table 6 shows adjusted mean post-CMAT scores for each of the four treatment groups.

TABLE 6

ADJUSTED MEAN POST-CMAT SCORES FOR
THE FOUR TREATMENT GROUPS

	Whole Class	Grouped	
1 Test	17.14	15.79	16.47
2 Tests	17.15	17.50	17.33
	17.15	16.65	16.90

Although the result is not significant, the adjusted post-CMAT scores of Table 6 show the mean achievement of the whole class treatments to be greater than that of the flexibly grouped classes. Table 6 also shows a nonsignificant difference between final unit evaluation procedures in favor of the 2-test procedure.

The treatment that produced the highest mean post-CMAT score was that involving the flexible grouping organization for instruction with the test-retest option evaluation procedure.

H(3): There are no significant differences in mean achievement between the four treatment groups of community college beginning algebra students as measured by students' final unit test averages.

This hypothesis was tested using a three-way analysis of covariance. The covariate for the analysis was a student's score on Form A of the CMAT, Algebra I. The data important for this analysis are given in Table 7.

TABLE 7

ANOCOVAR FOR MATHEMATICS ACHIEVEMENT
USING FINAL UNIT TEST AVERAGES

Source	df	MS	F
Instructors (I)	1	88.02	0.32
Tests (T)	1	2114.48	7.61**
Groups (G)	1	16.12	0.06
I x T	1	721.76	2.60
I x G	1	4.74	0.02
G x T	1	250.09	0.90
I x G x T	1	230.16	0.83
Error	152	277.79	

*Indicates significance at the .05 level.
**Indicates significance at the .01 level.

For this analysis, as for that of H(2), there are no significant interactions between instructors, groups, or tests.

The hypothesis of no significant differences in mathematics achievement between the four treatment groups as measured by the students' final unit test averages is rejected ($P < .01$). There is a significant difference in final unit test averages between groups taking one final unit test and those with the option to retake any given unit test a second time. The direction of this difference is seen in Table 8.

The adjusted mean final unit test average for the retest option (2 tests) classes is significantly higher than that of the classes taking final tests in the traditional manner of one per unit. This result is consistent with the larger average score for the 2-test classes reported in Table 6 where mathematics achievement was measured using the standardized CMAT.

TABLE 8

ADJUSTED MEAN FINAL UNIT TEST AVERAGES
FOR THE FOUR TREATMENT GROUPS

	Whole Class	Grouped	
1 Test	68.9	67.0	68.0
2 Tests	73.7	76.9	75.3
	71.3	72.0	71.7

It is interesting to note in considering the four treatment means of Table 8 that regardless of the classroom organization for instruction, the two test classes produced higher adjusted mean final unit test averages. The whole class organization shows a difference of 4.8, while that of the flexibly grouped classes is 9.9.

Important results from the analysis of hypotheses H(2) and H(3) may be briefly summarized as follows:

1. There was a significant difference ($P < .01$) in mean mathematics achievement as measured by students' final unit test averages between 1-test and 2-test treatments in favor of the 2-test procedure.
2. There was a nonsignificant difference in mean mathematics achievement as measured by the CMAT between 1-test and 2-test treatments in favor of the 2-test procedure.
3. Regardless of whether achievement is measured by the CMAT or the students' final unit test averages, the treatment producing the highest adjusted mean score involved the flexible grouping organization for instruction and the test-retest option procedure for final unit test evaluation. (G-2)

Subgroup Comparisons

A major goal in the design of this study was to identify instruction and evaluation procedures appropriate for the wide range of individual differences present in a community college beginning algebra course. Of special concern in this respect were those students who demonstrated low achievement in algebra. Ideally, the intent was to define and evaluate a classroom management plan which would provide maximum benefits for the low achieving students but which would maintain and enhance the achievement of the remaining students of the class.

In an attempt to evaluate the effectiveness of the whole-class and grouping treatments for students with varying levels of achievement, comparable subgroups of low and high achieving students from each of these treatments were identified. Grouping treatment students were used as the basis for this identification since the achievement profiles obtained for them through the five unit criterion tests were considered to be representative of their actual achievement levels.

A student was initially labeled low achieving (Low) if he had been in group 1 for four out of five units of the course and high achieving (High) if he had been in either groups 2 or 3 for four out of five of the unit regroupings. Means and standard deviations on the CMAT pre-test were then figured for each of the two subgroups. A maximum CMAT score for the Low subgroup and a minimum CMAT score for the High subgroup were then set using the CMAT pre-test computations. The resulting criteria for Low and High subgroup membership were as follows:

1. Low achieving subgroup
    a) Group 1 member for 4 out of the 5 units
    b) CMAT pre-score ≤ 9
2. High achieving subgroup
    a) Group 2 or 3 member for 4 out of the 5 units
    b) CMAT pre-score ≥ 10

Once the Low and High subgroups were identified in the G-1 and G-2 treatment groups, the number of students having various levels of CMAT pre-scores for each subgroup was tallied. Comparable students in treatment groups W-1 and W-2 were then selected by randomly choosing the same number of students at each of the CMAT pre-score levels.

The hypothesis used to compare the mathematics achievement of the Low and High subgroups was the following:

H(4): There are no significant differences in mean mathematics achievement among Low and High subgroups of grouping and whole class treatment groups as measured by:

a. Pre- and post-test forms of the CMAT.
b. Final unit test averages.

This hypothesis was tested using a one-way analysis of covariance to initially check for nonchance variations among means. Significant F-statistics were investigated further through pairwise t-tests to compare differences in means between appropriate combinations of subgroups. Since the latter comparisons were the result of an a priori hypothesis, there was no need to compute a more "robust" t through a method such as that of Scheffe (McNemar, p. 323).

The analysis of covariance results for Low and High subgroups of grouping and whole class treatment groups are shown in Table 9.

TABLE 9

ANOCOVAR ON MATHEMATICS ACHIEVEMENT
FOR LOW AND HIGH SUBGROUPS

Variable	Source	df	SS	MS	F
CMAT	Equality of Adjusted cell means	3	132.53	44.18	2.83*
	Error	87	1359.5	15.63	
FUTA	Equality of Adjusted cell means	3	1954.8	651.59	2.72*
	Error	87	20844.	239.58	

*Indicates significance at the .05 level.

The hypothesis of no significant differences in mathematics achievement among Low and High subgroups of the grouping and whole class treatments is rejected ($P < .05$). Although the significance levels are somewhat marginal ($P < .043$ and $P < .049$ respectively), the F-statistics indicate nonchance differences in subgroup means for mathematics achievement as measured either by the CMAT or final unit test averages.

Table 10 shows adjusted post-CMAT scores and selected t-tests for the desired Low-High subgroup comparisons.

TABLE 10

ADJUSTED POST-CMAT MEANS, MEAN DIFFERENCES ($\bar{d}$), AND SELECTED PAIRWISE t-TESTS (t) FOR LOW AND HIGH SUBGROUPS

	Low	High
Grouped N = 22,24	1 15.0	2 19.4
Whole Class N = 22,24	3 17.5	4 19.4

Pairwise Comparisons	$\bar{d}$	t
1 2	-4.37	-2.97**
1 3	-2.48	-1.64
2 4	-0.09	-0.06
3 4	-1.97	-1.33
Interaction	-2.40	-1.45

*Indicates significance at the .05 level.
**Indicates significance at the .01 level.

The pairwise comparisons reported in Table 10 include only one which is statistically significant. The High group in the grouping treatment had a significantly greater mean score on the post-CMAT than did the Low group.

The interaction effect reported in Table 10 is defined, for subgroups S_1, S_2, S_3, and S_4, by $(S_1 - S_2) - (S_3 - S_4)$. If the interaction were significantly different from zero, then one would conclude that the treatments differentially affected High and Low students

within the grouped and whole class treatments. As reported in Table 10, the interaction is not significant for mathematics achievement as measured by the CMAT.

In Table 11 are reported the adjusted final unit test averages and t-statistics for subgroup comparisons. The High subgroup had a significantly greater mean score on FUTA than did the Low subgroup in the Grouping treatment. This result is consistent with that reported in Table 10. Thus for the achievement measures used in this study, High students in the grouping treatment, on the average, had significantly greater achievement than did their low classmates.

TABLE 11

ADJUSTED MEAN FINAL UNIT TEST AVERAGES, MEAN DIFFERENCES ($\bar{d}$), AND SELECTED PAIRWISE t-TESTS (t) FOR LOW AND HIGH SUBGROUPS

	Low	High
Grouped N = 22,24	1 64.3	2 83.3
Whole Class N = 22,24	3 69.9	4 77.9

Pairwise Comparisons	$\bar{d}$	t
1 2	-18.97	-3.29**
1 3	- 5.57	-0.94
2 4	5.37	0.96
3 4	- 8.03	-1.39
Interaction	-10.94	-1.69

*Indicates significance at the .05 level.
**Indicates significance at the .01 level.

Although the interaction of High and Low subgroups and classroom organization was statistically significant for neither measure of achievement, definite trends are apparent in Tables 10 and 11. First, the Low scoring subgroup in the grouping treatment achieved less than did the corresponding group in the whole class treatment. This trend is somewhat discouraging since the grouping treatment was designed to accommodate the less able student. Apparently these students fair better in a more traditionally organized class.

The second trend is that the High subgroup in the grouping treatment did no worse, and for FUTA did better, than the comparable subgroup in the whole class treatment. This is encouraging and suggests a differential effect of treatment on High and Low subgroups even though the interaction is not statistically significant. A natural question which arises is: Can one maintain the greater achievement for High students and simultaneously improve the achievement of Low students by varying the classroom organization in some different manner?

An important consideration in designing the flexible grouping organization for instruction was that of improving achievement of basic algebra skills for low achieving students. Through homogeneous grouping and appropriate instructional materials for middle and upper level subgroups, the instructor was to be freed to give more individual help to low achieving students and to place additional emphasis on those objectives designated as "basic" and necessary for successful completion of the course. It was hoped that this would provide for increased basic level objective achievement for low achieving students without affecting the achievement of high achieving students.

The hypothesis was the following:

H(5): There are no significant differences in mean basic level objective achievement for Low and High subgroups of grouping and whole class treatment on:

a. Best Test I
b. Best Test II
c. Best Test III
d. Best Test IV
e. Best Test V
f. Final Exam

Hypothesis H(5) was tested using a one-way analysis of covariance, followed by selected t-tests were significant F-statistics occurred. The covariate for the analysis was the CMAT pre-test.

The analysis of covariance results for comparing basic level achievement on best unit tests I-V and the final exam for Low and High subgroups are shown in Table 12. The results indicate nonchance variations in subgroup means for unit II ($P < .05$), unit III ($P < .01$), and the final exam ($P < .05$).

TABLE 12

ANOCOVAR OF BASIC LEVEL OBJECTIVES ON BEST TESTS I-V AND FINAL EXAM FOR LOW AND HIGH SUBGROUPS OF GROUPING AND WHOLE CLASS TREATMENTS

Variable	Source	df	SS	MS	F
Best Test I Basic	Equality of adjusted cell means	3	201.88	67.29	1.25
	Error	87	4668.4	53.66	
Best Test II Basic	Equality of adjusted cell means	3	1204.4	401.46	3.36*
	Error	87	10395.	119.49	
Best Test III Basic	Equality of adjusted cell means	3	2987.7	995.90	4.08**
	Error	87	21229.	244.01	
Best Test IV Basic	Equality of adjusted cell means	3	2128.8	709.61	2.47
	Error	87	24946.	286.74	
Best Test V Basic	Equality of adjusted cell means	3	756.48	242.16	0.50
	Error	87	43715.	502.47	
Final Exam Basic	Equality of adjusted cell means	3	2343.7	781.22	3.84*
	Error	87	17698.	203.43	

*Indicates significance at the .05 level.
**Indicates significance at the .01 level.

Table 13 gives the mathematical coverage of each of the five units of the course and the number of points on each of the five unit tests given for problems testing basic level objectives.

TABLE 13

MATHEMATICAL COVERAGE, BASIC OBJECTIVE AND TOTAL POINT VALUES FOR UNITS I-V AND THE FINAL EXAM

Unit	Mathematical Coverage	Unit Test Point Total	Basic Obj. Point Total
I	Sets, variables and open sentences, rational numbers, mathematical properties	102	69
II	Signed numbers, inverses, absolute value, removing parentheses	100	70
III	Algebraic expressions, equations, formulas, word problems, solution and graphing of inequalities	100	75
IV	Polynomials, exponent laws, factoring, polynomial equations	100	76
V	Rational algebraic expressions, complex fractions, rational equations	100	85
Final Exam	Comprehensive	100	79

Table 14 shows adjusted basic level objective means for best tests I-V and for the final exam. For those portions of Table 12 producing significant F-statistics, Table 14 shows selected pairwise t-tests between Low and High subgroups, and a t-test for the interaction effect defined on page 79.

The best test basic level objective means for tests I-V and the final exam given in Table 14 all reflect a similar pattern in terms of individual subtable results. Each of the six subtables of Table 14 shows the mean basic level objective achievement for low achieving students in

TABLE 14

ADJUSTED BASIC LEVEL OBJECTIVE MEANS FOR BESTS TESTS I-V AND THE FINAL EXAM, MEAN DIFFERENCES ($\bar{d}$), AND SELECTED PAIRWISE t-TESTS (t) FOR LOW AND HIGH SUBGROUPS

Best Test I Basic	Low	High
Grouped	1 60.3	2 64.5
Whole Class	3 62.6	4 61.2

Best Test II Basic	Low	High
Grouped	1 47.0	2 61.9
Whole Class	3 52.3	4 58.6

Pairwise Comparisons		$\bar{d}$	t
1	2	-14.89	-3.66**
1	3	- 5.23	-1.25
2	4	3.31	0.84
3	4	- 6.35	-1.55
Interaction		- 8.54	-1.87

Best Test III Basic	Low	High
Grouped	1 43.2	2 65.3
Whole Class	3 54.5	4 62.6

Pairwise Comparisons		$\bar{d}$	5
1	2	-22.08	-3.80**
1	3	-11.26	-1.88
2	4	2.75	0.49
3	4	- 8.07	-1.38
Interaction		-14.00	-2.15*

Best Test IV Basic	Low	High
Grouped	1 45.2	2 64.6
Whole Class	3 47.7	4 58.7

*Indicates significance at the .05 level.
**Indicates significance at the .01 level.

TABLE 14 (Continued)

Best Test

V Basic	Low	High
	1	2
Grouped	54.6	66.0
Whole Class	3 55.4	4 62.9

Final Exam

Basic	Low	High
	1	2
Grouped	43.0	63.7
Whole Class	3 50.6	4 59.2

Pairwise

Comparisons	$\bar{d}$	t
1 2	-20.71	-3.91**
1 3	- 7.55	-1.37
2 4	4.53	0.88
3 4	- 8.64	-1.62
Interaction	-12.07	-2.03*

*Indicates significance at the .05 level.
**Indicates significance at the .01 level.

whole class treatments slightly higher than that of low achieving students in the flexible grouping treatments. In none of the six subtables is the difference significant and, thus, free from the possibility of chance occurrence. The results seem to indicate, however, that a whole class approach to classroom organization for instruction provides for a somewhat greater achievement of basic level objectives in beginning algebra than does a flexible grouping approach.

There is also a slightly higher mean achievement of basic level objectives in each of the six subtables for the High subgroup of the grouping treatments than for the High subgroup of the whole class treatments. Although none of the mean differences is significant, the results again seem to indicate some slight benefits in mean basic level achievement for high achieving students when using a flexible grouping organization for

instruction. These trends support those noted in connection with the achievement data.

Subtables 2, 3, and 6, which display basic objective achievement for units II, III, and the final exam each show a significant difference between Low and High subgroups of the grouping treatments ($P < .01$). A corresponding difference for Low and High subgroups is not present for whole class treatments. Significant interaction effects are reported for two measures. This supports the contention that the treatments had a differential effect on High and Low subgroups.

The following is a brief summary of findings from the analysis of hypotheses H(4) and H(5):

1. There was a significant difference ($P < .01$) in mean mathematics achievement as measured by the CMAT and final unit test averages between Low and High subgroups for the grouping treatment.
2. The mathematics achievement of the Low achieving subgroup of the whole class treatment was consistently greater, but not significantly, than that of the Low achieving subgroup of the grouping treatment as measured by adjusted mean post-CMAT scores and final unit test averages.
3. The mathematics achievement of the High achieving subgroup of the grouping treatment was not inferior and tended to be superior to that of the comparable subgroup of the whole class treatment.

4. There was a significant difference ($P < .01$) in mean basic level objective achievement between Low achieving and High achieving subgroups of the grouping treatment for best tests II, III, and the final exam.
5. There is a tendency for the grouping and the whole class treatments to affect the High and Low subgroup differently.

Attitudes Toward Mathematics

Student attitude toward mathematics was measured using the Aiken and Dreger Opinionnaire (1961). A copy of this instrument is included in Appendix D. The 20 item instrument is scored using Likert's method of summated ratings. With the Opinionnaire, under this method, the respondent indicates whether he strongly agrees, agrees, is undecided, disagrees, or strongly disagrees with each of the 20 statements expressing positive or negative attitudes toward mathematics. The student's score is the sum of the weights (0, 1, 2, 3, and 4) which have been assigned to the responses he has made. Possible scores on the instrument range from 0 to 80 with a high score reflecting a positive attitude toward mathematics.

The hypothesis tested was:

H(6): There are no significant differences in mean attitude toward mathematics between the treatment groups as measured by the MAO.

This hypothesis was tested using a three-way analysis of covariance. The covariate was the students' pre-score on the MAO. Data pertinent to the analysis of this hypothesis are shown in Table 15.

TABLE 15

ANOCOVAR FOR ATTITUDES TOWARD MATHEMATICS USING THE MAO

Source	df	MS	F
Instructors (I)	1	728.36	5.59*
Tests (T)	1	24.22	0.19
Groups (G)	1	4.12	0.03
I x T	1	1.44	0.01
I x G	1	79.23	0.61
G x T	1	131.59	1.01
I x G x T	1	25.16	0.19
Error	152	130.20	

*Indicates significance at the .05 level.

The hypothesis of no significant differences in mean attitude toward mathematics between the treatment groups is accepted. Adjusted mean post-MAO scores are shown in Table 16.

TABLE 16

ADJUSTED MEAN POST-MAO SCORES FOR THE FOUR TREATMENT GROUPS

	Whole Class	Grouped	
1 Test	50.2	48.0	49.1
2 Tests	47.6	49.1	48.4
	48.9	48.6	48.8

Table 15 does indicate, however, a significant difference ($P < .05$) in adjusted post-MAO means between the classes of Instructor 1 and those of Instructor 2. The adjusted post-MAO mean of Instructor 1 was 50.9 as compared with 46.6 for Instructor 2. This would seem to indicate that Instructor 1 was more effective in bringing about positive attitude change than Instructor 2. This is supported by Table 17 in which actual pre- and post-MAO means for each of the two instructors are reported.

TABLE 17

ACTUAL PRE- AND POST-MAO MEANS ($\bar{X}$), STANDARD DEVIATIONS (SD), AND t-RATIOS (t), BY INSTRUCTOR

	Source	N	X	SD	t
I_1	Pre-MAO	87	45.1	17.22	4.37**
	Post-MAO	87	51.4	14.56	
I_2	Pre-MAO	74	43.0	18.70	2.11*
	Post-MAO	74	45.8	18.51	

*Indicates significance at the .05 level.
**Indicates significance at the .01 level.

Both Instructor 1 ($P < .01$) and Instructor 2 ($P < .05$) were successful in bringing about a significant positive mean attitude toward mathematics gain. Thus, even though I_1 students had greater attitude means than I_2 students, both sets of students significantly improved in attitude. This result is encouraging and important, in the writer's opinion, due to the seemingly large numbers of Mathematics 100 students that enter the course with unpleasant experiences in mathematics courses and noticeably negative feelings toward the subject matter.

Test Anxiety

The instrument used to assess student anxiety was the Questionnaire on Attitude Toward Mathematics Testing, a modification of the Test Anxiety Questionnaire developed by Mandler and Sarason (1958). A copy of this questionnaire is included in Appendix D. To complete the questionnaire, the respondent recorded his feeling for each of the 15 items by placing an X along a continuum defined by brief phrases at its mid- and endpoints. For scoring purposes, each of the 15 continua is split into 10 divisions which are weighted from 1 to 10. A student's score, which may range from 15 to 150, is the sum of his 15 responses with a high score indicating a high test anxiety.

The hypothesis tested was:

H(7): There are no significant differences in mean mathematics test anxiety between the treatment groups as measured by the TAQ.

This hypothesis was tested using a three-way analysis of covariance. The covariate was the student's pre-TAQ score. The data pertinent to the analysis of this hypothesis are given in Table 18.

The results of Table 18 show no significant difference in mean test anxiety as measured by the TAQ either between 1-test and 2-test treatments (Tests), or whole class and the combination whole class-flexible grouping treatments (Groups). Since there is, in addition, no Group-Test interaction, the hypothesis is accepted.

TABLE 18

ANOCOVAR FOR TEST ANXIETY
USING THE TAQ

Source	df	MS	F
Instructors (I)	1	178.50	0.44
Tests (T)	1	7.98	0.02
Groups (G)	1	47.33	0.11
I x T	1	1121.66	2.76
I x G	1	47.19	0.12
G x T	1	9.95	0.02
I x G x T	1	301.78	0.74
Error	152	406.13	

Unlike the results of the attitudes toward mathematics analysis, there was no significant difference due to instructors on mathematics test anxiety. Table 18 also indicates that there were no significant interactions between Instructors, Tests, or Groups.

Success and Dropout Rate

Student success and dropout rate were tested using the chi square statistic. The dropout and success frequencies for each of the treatment groups were compared using an expected frequency.

For the purposes of this study, the terms "success" and "dropout" are defined as follows:

SUCCESS - A student who has completed all requirements of the course (5 unit tests and the final exam) and who received a grade of C or better.

DROPOUT - A student who has failed to finish the course by not completing all of the requirements.

The term "dropout" as used here is synonymous with "nonfinisher" and includes those students who dropout early in the course, gradually fade out after the midsemester, or merely choose not to take the final exam.

The hypothesis tested for dropout rate was the following:

H(8): There are no significant differences between the treatment groups in the percentage of students who drop out of the beginning algebra course.

TABLE 19

STUDENT DROPOUT FREQUENCIES, PERCENTAGES, AND CHI SQUARE (X^2), BY TREATMENT

Treatment	Dropouts Frequency	%	Finishers Frequency	%	
W-1	17	29.8	40	70.2	
W-2	13	25.0	39	75.0	
G-1	13	23.6	42	76.4	
G-2	16	28.6	40	71.4	
Totals	59		161		220
df = 3		$X^2 = 0.722$			

As seen in Table 19, 59 of the initial 220 students or 26.8% were classified as dropouts. The largest percentage of dropouts (29.8%) resulted for the traditional whole class treatment with one final unit test. The treatment with the smallest dropout percentage (23.6%) was the flexible grouping treatment with one final unit test. The data show the percentage of dropouts, however, to be statistically unaffected by any of the four treatments. Hypothesis H(8) is, therefore, accepted.

The hypothesis tested for student success rate was the following:

H(9): There are no significant differences between the treatment groups in the percentage of students who receive a grade of C or better (Success) in the beginning algebra course.

The important data for this hypothesis are given in Table 20 which shows frequencies and percentages of successful and nonsuccessful students for each of the four treatment groups and the chi square statistic for significance.

TABLE 20

STUDENT SUCCESS FREQUENCIES, PERCENTAGES, AND CHI SQUARE (X^2) BY TREATMENT

Treatment	Success Frequency	%	Nonsuccess Frequency	%	
W-1	26	45.6	31	54.4	
W-2	28	53.8	24	46.2	
G-1	27	49.1	28	50.9	
G-2	30	53.6	26	46.4	
Totals	111		109		220
df = 3		$X^2 = 1.032$			

Hypothesis H(9) is accepted. The chi square statistic of Table 20 shows no significant differences between the four treatment groups in the percentage of students who receive a grade of C or better. One hundred eleven of the 220 initially enrolling students or 50.5% were successful. The treatment yielding the smallest percentage of successful students (45.6%) was the traditional whole class approach with one final unit test. The 2-test treatment offered the highest percentage of successful students with an average success rate of 53.7%.

An interesting result is obtained when considering the success rate of only those students who finish or complete all requirements of the course. Of the 161 students finishing the course, 111 earned a grade of C or better. Thus, over two-thirds of the students (68.9%) that finished the course were successful.

Student Reactions

Student reactions to various aspects of the classroom treatments were obtained through two instruments administered previous to the final exam at the conclusion of the semester. The instruments, both of which were prepared by the writer, consisted of an Opinionnaire and a Treatment Verification Form. A copy of each of these can be found in Appendix D.

Opinionnaire

The purpose of the brief four question Opinionnaire was to identify those things which students saw as most beneficial for identifying and learning the required content of the course. It was designed to provide feedback on how students' perceived the change in their attitudes toward mathematics as a result of the course and what they saw as foremost in bringing about this change.

The purpose of the Treatment Verification Form was to provide a verification by students of the treatment procedures defined by the writer for each of the four treatment groups. It also provided an opportunity for student response to open-ended questions regarding their opinions, likes, and dislikes of the way their class was run.

Some preliminary conjectures of the writer concerning instructional objectives and student use of the current textbook adoption were the basis for including the first two questions on the Opinionnaire. It was the writer's opinion that instructional objectives, often so meticuously formulated by teachers, are most beneficial for just that group while students more readily identify what is to be learned from sample test items. Motivation for writing the supplementary exercise sheets was in part due to the writer's feeling that the students could not read the textbook with understanding and thus were unable to use it as a learning resource.

Table 21 shows the results of the Opinionnaire for each of the four treatment groups.

Question 1 of the Opinionnaire shows this writer's preliminary hypothesis concerning student use of instructional objectives to be somewhat inaccurate. The results for question 1 in Table 21 indicate a larger percentage of students consulted the instructional objectives than the sample test items to determine what was to be learned for all but the W-1 group. Combined percentages show that at least 87% of the students in each of the four treatment groups consulted either the instructional objectives or the sample test to determine exactly what was to be learned. The textbook appears to have been consulted for this purpose only to a limited extent with percentages ranging from a low of 2% for the W-1 treatment to a high of 6% for the G-1 treatment. An average of 7% of the students for the four treatment groups specified some other source than one of those listed. The sources listed for students choosing this response were usually a combination of two or more of the remaining three responses.

TABLE 21

STUDENT OPINIONNAIRE FREQUENCIES (f) AND PERCENTAGES FOR EACH OF THE FOUR TREATMENT GROUPS

1. To find out exactly what I was expected to learn for each unit, I looked at:

	W-1		W-2		G-1		G-2	
	f	%	f	%	f	%	f	%
a) The instructional objectives	17	44	18	55	23	49	17	46
b) The sample test items	19	49	11	33	18	38	15	41
c) The textbook	1	2	1	3	3	6	2	5
d) Other (please specify)	2	5	3	9	3	6	3	8
	39		33		47		37	

2. In learning the mathematics in this course, the thing that helped me the most was:

a) The exercise sheets	14	34	6	18	32	67	24	67
b) The classroom organization	1	2	1	3	0	0	2	6
c) The textbook	0	0	0	0	0	0	0	0
d) The teacher's lecture	26	63	27	79	16	33	9	25
e) Other (please specify)	0	0	0	0	0	0	1	3
	41		34		48		36	

3. During this semester I think my overall attitude towards mathematics:

a) Has become more favorable	27	66	26	70	27	55	30	79
b) Has remained the same	11	27	9	24	16	33	7	18
c) Has become less favorable	2	5	1	3	3	6	0	0
d) Don't know	1	2	1	3	3	6	1	3
	41		37		49		38	

TABLE 21 (Continued)

4. What <u>one thing</u> has influenced your change in attitude?

	W-1		W-2		G-1		G-2	
	f	%	f	%	f	%	f	%
Teacher	12	67	13	50	8	30	8	36
Success	4	22	9	35	6	23	9	41
Exercise sheets	2	11	1	4	3	12	2	9
Lectures	0	0	1	4	0	0	0	0
Classroom organization	0	0	0	0	5	19	3	14
Subject	0	0	1	4	4	15	0	0
Objectives	0	0	1	4	0	0	0	0
	18		26		26		22	

The responses for question 2 as reported in Table 21 are consistent with the design of each of the four treatments. Whole class treatments (W-1 and W-2) utilized the teacher and the exercise sheets as primary and secondary resources, respectively, for learning. Grouping treatments (G-1 and G-2) reversed this result, utilizing the exercise sheets as the primary resource for learning.

It is interesting to note that not one student in any of the four treatment groups chose the textbook as a help in learning the mathematics in the course. This would seem to support the writer's feeling concerning the ability of students to read with understanding the textbook currently being used. As this feeling is openly shared by the two instructors in the study, it is also very possible that the use of the exercise sheets was more strongly emphasized than that of the textbook.

The response to question 3 of the Opinionnaire indicates that at least 55 percent of the responding students in each of the four treatment groups perceived their attitudes toward mathematics as having become more favorable as a result of their experiences during the semester. The perceived attitude change is somewhat more prevalent in the 2-test classes than the 1-test classes with the average percentages being 75% and 61% respectively. The general perceived attitude change is consistent with the results of the measured attitude toward mathematics change as given in Table 4.

Question 4 of the Opinionnaire produced a variety of responses which were categorized into one of seven areas as seen in Table 21. The frequency totals indicate that only about 57% of the students finishing the course responded to this question. If these responses can be considered, however, as representative of the total sample a few observations might be made.

The role of the teacher as a catalyst in attitude change, for example, seems extremely important for the whole class approach to instruction. When asked to identify the one thing which most influenced their change in attitude, an average of 59% of the students in the whole class treatments named the teacher as opposed to an average of 33% for the students of the grouping treatments. This result is important in that it does indicate that teachers can have an effect on student attitude. If the major responsibility for positive attitude change rests with the classroom teacher, however, this result may cause some concern for teachers who are unable to bring about such a change.

The results of question 4 also suggest that the students in the grouping treatments attributed their attitude change to a more varied set of reasons. An average of 17% of the grouping treatment students, for example, indicated that the classroom organization had changed their attitude as opposed to none of the whole class treatment students. A larger percentage of grouping than whole class treatment students also named the subject matter as influencing their change in attitude. It may be that a grouping approach to organization for instruction provides for a situation in which a larger variety of classroom experiences positively affect attitude.

A representative sampling of student responses given to question 4 of the Opinionnaire are given below. These are included by treatment, without editing, not only to present student opinion but to reflect the type and spontaneity of the students under analysis.

The question asked was:

"What <u>one</u> <u>thing</u> has influenced your change in attitude?"

<u>W-1</u>

- Good Grades!
- I seem to have beat a lot of my old problems from high school.
- The one thing that influenced me was that I didn't think I was able to do the work. But now since I can it changes my attitude toward this class. And I might even takes mathematics up to gemtory or triagmentry.
- The way my instructor taught his classes. He made his lectures clear, to the point and if there were any problems I didn't understand I could tell him about it and he would help me until I understood.

W-2

- My understanding and test scores.

- My attitude is the same. I never liked math.

- Good instructor made learning surprisingly easy. Never thought I could learn math. Always my worst subject. Thrilled that I could master it with such ease.

G-1

- I got more understanding this semester.

- I have never liked math. This is not the first time taking math 100 and I still don't.

- I've always liked math, but the method of learning with the exercise sheets simplified the work and made it more enjoyable (it was fun) rather than a boring, complicated task.

- The way the class was organized. The way Mr. _____ handed the class in the three different groups.

- I had a very patient instructor.

- The grades that I received on tests.

G-2

- The system of teaching.

- The teacher.

- The exercise sheets that show you how to work problems. The sheet explained better than the textbook. Because the textbook was for college, who already expect you to know so much.

- I was able to take a test which showed me actually what I needed to study more of. (Note: The reference here is to the criterion test.)

- I found out that if I study I could really get good marks. The hand out sheets helped to change my attitude.

- I think the freedom of working alone and not having to listen to a lecture every class period.

Treatment Verification Form

The initial four questions on the Treatment Verification Form were posed to verify each of the treatments in the areas of grouping, testing, materials, and handouts. These questions were analyzed with the specific treatment in mind by computing frequencies on whether the response was judged to verify or not verify the treatment under consideration. On the basis of this analysis each of the eight treatment groups were judged to have been verified for classroom organization, procedures, and materials.

The remaining three questions of the Treatment Verification Form were concerned with student opinions about their class. The results of these questions are shown in Table 22.

The results of question 5 as seen in Table 22 indicate, as in question 4 of the Opinionnaire, that only a portion of the students offered responses. Approximately 50% of the students finishing the course responded to the question and all but a few students responded concerning the things they liked about the class. The frequencies reported in question 5, therefore, refer to the positive responses of students.

The response pattern for question 5 was similar in some respects to that of question 4 of the Opinionnaire. The percentage of students naming the teacher as the thing they liked about the class was more than twice as large for the whole class treatments as for the grouping treatments. A slightly larger average percentage of students reported liking the handouts in the grouping than the whole class treatments.

TABLE 22

TREATMENT VERIFICATION FORM FREQUENCIES (f) AND PERCENTAGES FOR EACH OF THE FOUR TREATMENT GROUPS

5. Opinion About Class. How did you like the way this class was run? What did you like about the class? What didn't you like about the class?*

	W-1		W-2		G-1		G-2	
	f	%	f	%	f	%	f	%
Teacher	18	82	11	73	9	53	5	19
Retest	0	0	4	27	0	0	6	23
Handouts	4	18	0	0	2	12	3	12
Grouping	0	0	0	0	6	35	12	46
	22		15		17		26	

6. Would you take another math class that was run the way yours was run?

Yes	40	98	34	94	42	98	38	95
No	1	2	1	3	1	2	2	5
Maybe	0	0	1	3	0	0	0	0
	41		36		43		40	

7. Please explain why you answered as you did in question 6 above.

Teacher	17	74	6	35	9	50	5	19
Retest	0	0	5	29	0	0	5	19
Handouts	2	9	0	0	1	6	2	8
Grouping	0	0	0	0	7	39	10	39
Success	4	17	6	35	1	6	4	16
	23		17		18		26	

*All but a few students responded concerning the things they liked about the class. The frequencies reported, therefore, refer to the positive responses of students.

It is interesting to note that the two experimental portions of the treatment definitions were singled out as having wide appeal. An average of 40% of the grouping treatment students mentioned the division of the class into groups as the thing they liked about the class. An average of 25% of the retest option treatment students indicated the chance to retake a final unit test as the thing that most appealed to them.

The percentages for question 6 were very similar for each of the four treatment groups. Approximately 96% of all students finishing the course indicated satisfaction with the way their course was run and that they would take another mathematics class run using similar procedures.

Question 7 of the Treatment Verification Form was an attempt to again secure feedback on what students most liked about their individual classes. With again only approximately 50% of the students that finished the course responding, a larger percentage of whole class than grouping treatment students named the teacher as responsible for their desire to take another mathematics class with procedures similar to those used this semester. The experimental 2-test and grouping procedures were again identified by students at approximately the same rate as having had a positive effect on their classroom experience. In a pattern similar to that of question 4 on the Opinionnaire, the percentage of students indicating their own success as the reason for agreeing to take another mathematics class run in a way similar to their own was more than twice as large for the 2-test classes than for those taking final unit tests once.

A sample of student responses to the questions of Table 22 for each of the four treatments is given without editing below. The question

for which the response is given appears in parentheses immediately preceding the response.

W-1

- (5) I liked the class but the teaching method was to fast for me, also we cover to much before we have a test.

- (7) The way this class was run I didn't get a headache trying to figure out what the teacher was talking about nor did I get confused doing the problems.

- (5) The class was runned excellent, I had no trouble understanding anything. What I liked about the class was that handouts were given each day we covered new material, and the teacher explained everything excellent.

- (5) The class was ran fairly. The teacher didn't assign nothing he couldn't explain.

- (5) It was together.

- (7) When ever you have sample test along with an outline of a unit, its easier for you to understand what is expected of you, and also what to expect on the test.

W-2

- (5) The thing I like most of all was the way the work was presented, and the opportunity to take a make-up chance.

- (7) The reason I would take another class like this is because of the retake test.

- (7) Easy to understand - patient instructor, chance to take all test over. Beautiful.

G-1

- (7) Because I like the way my instructor explained everything.

- (5) I think that Mr. _____ ran the class just perfectly. I liked the way it was divided into group and the criterion test and the handouts. You had to walk from the building to the portable in the cold.

- (7) I feel that in the grouping each student was able to work as he thought best and not get bored going over something he may already know.

- (7) Because it gave each student a chance to have individual instruction if needed.

- (7) Because you can learn more when you have a teacher that will take time to teach.

- (5) This was the first time a math class was really fun. I not only enjoyed it, but I was doing quit well and never brought or seen a textbook.

- (7) Yes because the way the class was run, it really was benefical to those who were not good.

- (7) I didn't like getting up at 6 to make it here at 8 oclock.

G-2

- (5) I approve very much of how the class was run. I think the division into groups is a good idea. It gives students a chance to work alone and gives the instructor more time to give other students his personal attention.

- (5) I appreciated the way the class was conducted, I really like the grouping arrangement.

- (7) Always when taking a test there is a certain amount of fear of failure but knowing that we could take a retest made one feel more relaxed.

- (5) I enjoyed working in groups with other students and helping and being helped by them. When a problem arose we couldn't solve, the instructor was always able to assist.

- (7) Because the person which was slow had more help when the fast one so they could keep up.

- (5) Like the class because you can learn at your own speed.

- (7) The class was run good to me, because I understand the handout sheets.

- (5) It was run ok. Needed more class discussion, working more problems on the board.

- (7) I like to be able to make up my test.

A brief summary of the results of the analysis of student reactions is as follows:

1. The one source most consulted by students (average of 49%) to determine what they were expected to learn was the list of instructional objectives.
2. At least 87% of the students in each of the four treatment groups consulted either the instructional objectives or the sample test items to determine what they were expected to learn.
3. The most beneficial learning resource for the combined whole class treatments was the teacher with the exercise sheets serving as the secondary resource.
4. The most beneficial learning resource for the combined grouping treatments was the exercise sheets with the teacher serving as the secondary resource.
5. Positive attitude change as perceived by students is dependent on the teacher for a majority of the responding whole class students while dependent for the responding grouping treatment students on a wider variety of classroom related experiences.
6. Each of the four treatments were verified by students in the areas of classroom organization, testing, materials, and handouts.

7. In response to the question "What did you like about the class?" a majority of the responding students for all but the G-2 treatment named the teacher. The G-2 treatment responses were more evenly distributed over the indicated responses.
8. An overwhelming number (96%) of the students finishing the course indicated they would take another mathematics course that was run as theirs had been. The reason for this decision in the W-1 treatment was the teacher (74%) with the remaining three treatments indicating reasons distributed between student success, grouping, retest option, teacher, and course handouts.

CHAPTER V

SUMMARY AND DISCUSSION OF THE DATA

This chapter contains a summary and discussion of the major findings of this study. It begins with a brief restatement of the problem and concludes with some recommendations for further research.

Restatement of the Problem

The purpose of this study was to determine the effects of variations in classroom organization for instruction and final unit evaluation procedures on mathematical achievement, attitudes toward mathematics, test anxiety, and the success and dropout rate in community college beginning algebra classes. The treatments were:

1. Whole class instruction with one final unit test. (W-1)
2. Whole class instruction with a final unit test-retest option. (W-2)
3. A combination of whole class instruction and differentiated small group instruction with one final unit test. (G-1)
4. A combination of whole class instruction and differentiated small group instruction with a final unit test-retest option. (G-2)

The initial experimental population consisted of 220 beginning algebra students a Highland Park Community College, Highland Park, Michigan. For all but the section dealing with the success and dropout rate, the

analysis was concerned with the 161 students who finished the course and for whom the data were complete. The experiment was undertaken for the duration of the Fall, 1971, semester.

All of the students were provided with an outline, a set of instructional objectives, and a sample test prepared by the author for each of the five units of the course. Exercise sheets at three levels of difficulty for each topic of the course were also written by the author and keyed to the textbook. Whole class instruction students were allowed to select the exercise sheet of their choice from the first two levels of difficulty for each of the topics. Flexible grouping instruction students were given a specific exercise sheet depending on their group assignments. Although a few of the latter group of students utilized the most difficult exercise sheets for Unit I, the grouping and exercise sheet assignments for Units II-V were essentially at the first two levels of difficulty.

The whole class treatments utilized a lecture-discussion approach to instruction. All within-class instruction was carried out by the teacher with the class as a whole. Classroom time was divided so that approximately 75% was devoted to informal lecture with 25% devoted to classroom discussion of such things as homework, example problems, and sample test items.

Classroom organization for instruction for the flexible grouping treatments followed a planned sequence of activities for each of the five units of the course. The sequence of activities for each unit was the following:

1. Each student was given an outline, a list of instructional objectives, and a sample test for the unit.

2. Lectures, based on basic level objectives, were given to the entire class with only a limited amount of classroom discussion allowed. (Approximate time - 1/3 of the unit or 2 - 3 days)
3. A short diagnostic test sampling all topic areas of the unit and similar to the final unit test was administered to the entire class.
4. Students were partitioned into three groups on the basis of their diagnostic test scores and the teacher's assessment of their level of attainment.
5. Differentiated instruction was provided for the group as follows: (Approximate time - 2/3 of the unit or 4 - 5 days)

   a. Group 1

      Content - limited to basic skills and concepts necessary for successful completion of the course.

      Instruction - individual or small group help from the instructor.

      Exercise sheets - a gradual development of content with emphasis on practice.

   b. Group 2

      Content - all of the objectives.

      Instruction - students work individually or in small groups on their own with the instructor acting only as a resource person.

      Exercise sheets - a brief and direct presentation of content with emphasis on practice with a wide variety of problem types.

c. Group 3

Content - all of the objectives and enrichment.

Instruction - same as group 2.

Exercise sheets - a brief review of topics and horizontal enrichment.

6. The entire class was reassembled for a review of the unit centered around the sample test.

7. The final unit test was administered to the entire class.

The two procedures for assessing final unit achievement were a one final unit test approach and a final unit test-retest option. The former procedure is recognized as the conventional method of classroom evaluation. The test-retest option offered the student the opportunity to retake a parallel form of the unit test within one week of the date on which the graded tests were returned. A student who took advantage of the retest option received the higher of the two unit test scores.

The intended purpose of the test-retest option and the flexible grouping variables was to present a setting where the individual needs of low and high achieving beginning algebra students in a single classroom could be more efficiently and effectively dealt with. The flexible grouping organization for instruction was designed to free the instructor for more in-depth work with low achieving students while allowing high achieving students to learn at their own rate rather than that of the majority of the class. The test-retest option within one week of the date the graded tests were returned was designed to provide low achieving students in particular with an extended period of time to utilize the various instructional resources available to them and to offer a second chance to demonstrate achievement of the unit objectives.

Findings and Discussion

The following measurements were obtained during the study for students finishing the beginning algebra course.

1. CMAT: Pre- and post-scores on the Cooperative Mathematics Achievement Test, Algebra I.
2. MAO: Pre- and post-scores on the Mathematics Attitude Opinionnaire.
3. TAQ: Pre- and post-scores on the Questionnaire on Attitudes Toward Mathematics Testing.
4. FUTA: Final Unit Test Averages based on the five unit test scores, with the higher of the two unit test scores chosen for test-retest option students.

The paragraphs which follow present the hypotheses tested in this study, the procedures used in the analysis, a list and discussion of the findings, and the conclusions drawn.

H(1): There are no significant differences in the changes in mathematics achievement, attitude toward mathematics, or test anxiety for the total population of beginning algebra students.

Procedures:

The measurements used to describe the changes were the pre- and post-scores on the CMAT, MAO, and TAQ. The hypothesis was tested by t-tests for the significance of the difference of the mean change scores from zero.

Findings and Discussion:

1. Statistically significant gains ($P < .001$) were made in both mathematics achievement and attitudes toward

mathematics with mean gain scores of +6.78 and +4.72 respectively.

2. A statistically significant reduction ($P < .001$) in test anxiety occurred with a mean gain score of -7.02.

Conclusion 1: The total population made significant gains in mathematics achievement, attitudes toward mathematics, and a significant reduction in test anxiety.

H(2): There are no significant differences in mean achievement between the treatment groups of community college beginning algebra students as measured by the post-CMAT.

H(3): There are no significant differences in mean achievement between the treatment groups of community college beginning algebra students as measured by students' final unit test averages.

H(4): There are no significant differences in mean mathematics achievement among Low and High subgroups of grouping and whole class treatment groups as measured by:

a. Pre- and post-test forms of the CMAT.
b. Final unit test averages.

H(5): There are no significant differences in mean basic level objective achievement for Low and High subgroups of grouping and whole class treatment groups on:

a. Best Test I
b. Best Test II
c. Best Test III
d. Best Test IV
e. Best Test V
f. Final Exam

Procedures:

Hypotheses H(2) and H(3) were tested using a three-way analysis of covariance. The three variables of classification were instructors, classroom organization for instruction, and final unit evaluation procedures. The covariate was the students' pre-score on the CMAT.

Hypotheses H(4) and H(5) were tested using a one-way analysis of covariance to initially check for nonchance variations among means. Significant F-statistics were investigated further through pairwise t-tests to compare differences in means between selected combinations of subgroups. The covariate was the CMAT pre-test.

Findings and Discussion:

3. There were no significant differences in mathematics achievement between the treatment groups as measured by the post-CMAT.
4. There was a significant difference ($P < .01$) in mean mathematics achievement as measured by FUTA between 1-test and 2-test treatments in favor of the 2-test procedure.
5. There was a significant difference ($P < .01$) in mean mathematics achievement as measured by the CMAT and FUTA between Low and High subgroups for the grouping treatment.
6. There was a significant difference ($P < .01$) in mean basic level objective achievement between Low and High subgroups of the grouping treatment for best unit tests II, III, and the final exam.

7. The mathematics achievement of the Low subgroup of the whole class treatment was consistently greater, but not significantly, than that of the Low subgroup of the grouping treatment as measured by the CMAT and FUTA.
8. The mathematics achievement of the High subgroup of the grouping treatment was not inferior and tended to be superior to that of the High subgroup of the whole class treatment.
9. The highest mean mathematics achievement as measured by the CMAT and FUTA occurred for the G-2 treatment group.

The 2-test final unit evaluation procedure proved beneficial for increasing mathematics achievement and raising the final course grades of a sizeable number of students. The mean mathematics achievement of the 2-test treatment was greater than that of the 1-test treatment as measured by the CMAT and significantly greater ($P < .01$) as measured by FUTA. Twenty-two of the 79 students finishing in retest classes or approximately 28% were able to raise their grades as a result of the 2-test procedure. Nine of the 22 students were able to raise their grades either from W to D (2 students) or from D to C (7 students).

It seems important to note as well that of the 79 students finishing in the four 2-test classes, 63 or approximately 80% took advantage of the retest option for at least one of the five units. None of the students taking retests did so for all five of the units and only 5% took a retest four of the five units. Seventy-three percent of the students taking retests did so on only one or two of the units. Thus it appears, for a majority of the participating students, that the retest was not used consistently and as a crutch but only occasionally when circumstances

prevented them from demonstrating achievement of the objectives on the first testing.

> Conclusion 2: The difference in mathematics achievement between 1-test and 2-test treatments supports the use of a 2-test final unit evaluation procedure for increasing mathematics achievement.

The mathematics achievement findings suggest definite trends in relation to the differential effect of whole class and grouping treatments on subgroups of Low and High achieving students. A first trend was for the Low subgroup of the grouping treatment to achieve lower than the Low subgroup of the whole class treatment. This was apparent, although not significantly so, whether mathematics achievement was measured using post-CMAT scores or FUTA. A similar result occurred consistently for basic level objective achievement on best unit tests I-V and the final exam. In each instance the mean score of the Low achieving subgroup of the grouping treatment was lower than that of the corresponding subgroup of the whole class treatment. Since the grouping treatment was designed to accommodate the less able student, this trend is somewhat discouraging. It appears that the needs of the Low achieving student are better met in a more traditionally organized class.

A possible explanation for this trend, in the writer's opinion, may be found in the grouping treatment design and the level at which the whole class treatment lecture and discussion was carried on. With approximately half of any given algebra class made up of low achieving students, classroom instruction was directed, by necessity, toward this level with little if any special attention given to those students not having difficulty. Although the grouping treatment instructors spent

approximately 70% of the time allotted to group study with the Low achieving students, only two-thirds of the class time in the grouping treatment was devoted to study in groups. The whole class instruction in the initial third of the unit appears to have been of questionable worth for Low achieving students. This feeling is supported by instructor comments and a diagnostic test average for Low achieving students of only 25. Thus, given this contention, only approximately 50% of the total classroom instruction time was spent working specifically with the less able student. This, combined with the extra day of instruction for the whole class treatment due to treatment design, seems to indicate a considerably greater amount of instruction time for the Low achieving student in the whole class setting.

The low achieving student, in addition, may have been able to benefit from the questions and comments of his more able classmates. The opportunity to listen while other class members attempted to verbalize their understandings and difficulties would have been especially helpful for the student who had difficulty understanding his instructor.

The reduced content coverage and basic objective emphasis did not result in greater basic level objective achievement for the Low subgroup of the grouping treatment as compared with the Low subgroup of the whole class treatment. A possible explanation for this result, in the writer's opinion, may have been the nature of the beginning algebra course and the time factor discussed above. The hierarchical content development of the beginning algebra course demands that the student continually use the basic concepts of the material already studied. Thus there would have been a natural reinforcement of basic skills in both the whole class and grouping treatments in the course of normal instruction. Although

basic objectives were emphasized in group 1 work, the reduced instruction time for Low achieving students in the grouping treatment reduced, as well, the amount of basic skills reinforcement.

A second trend in the mathematics achievement findings was for the High subgroup in the grouping treatment to achieve as well, if not better, than the High subgroup of the whole class treatment. While the High subgroups had equal mathematics achievement as measured by post-CMAT scores, the achievement of the High subgroup of the grouping treatment was greater than that of the comparable subgroup of the whole class treatment when measured using FUTA. As in the case of the Low subgroups, this trend was again supported consistently by the basic level objective achievement on best unit tests I-V and the final exam. Although not significantly so, the mean score of the High subgroup of the grouping treatment in each instance was greater than that of the High subgroup of the whole class treatment. While this result is obviously encouraging, its effect is somewhat reduced due to the accompanying trend.

Some additional findings further support the trends noted. First, there was a significant difference ($P < .01$) in mathematics achievement as measured by post-CMAT scores and FUTA between Low and High subgroups of the grouping treatment. There was also a significant difference ($P < .01$) in basic level objective achievement between Low and High subgroups of the grouping treatment for best unit tests II, III, and the final exam. In none of these instances was there a corresponding difference for Low and High subgroups of the whole class treatment. In addition, there was a significant interaction effect ($P < .05$) in basic level objective achievement for best unit test III and the final exam. These results serve to strengthen the contention that the treatments had a

differential effect on High and Low subgroups.

Conclusion 3: There were definite, although nonsignificant, trends in mathematics achievement in relation to the effect of whole class and grouping treatments on Low and High subgroups as follows:

a. The Low subgroup of the whole class treatment had greater achievement than the Low subgroup of the grouping treatment.

b. The High subgroup of the grouping treatment had greater achievement than the High subgroup of the whole class treatment.

H(6): There were no significant differences in mean attitude toward mathematics between the treatment groups as measured by the MAO.

Procedures:

This hypothesis was tested using a three-way analysis of covariance. The covariate was the students' pre-score on the MAO.

Findings and Discussion:

10. There were no significant differences in mean attitude toward mathematics between the treatment groups as measured by the MAO.

11. There was a significant difference ($P < .05$) on adjusted post-MAO means between the classes of Instructor 1 and those of Instructor 2.

There was a significant positive mean attitude toward mathematics gain for the classes of both Instructor 1 ($P < .01$) and Instructor 2 ($P < .05$). Thus, even though there was a significant differences ($P < .05$) between instructors in mean attitude toward mathematics, both sets of students

significantly improved in attitude. This result is encouraging and important, in the writer's opinion, in view of the seemingly large numbers of mathematics 100 students that openly express anxiety and negative attitudes toward mathematics.

H(7): There are no significant differences in mean test anxiety between the treatment groups as measured by TAQ.

Procedures:

This hypothesis was tested using a three-way analysis of covariance. The covariate was the students' pre-TAQ score.

Findings and Discussion:

12. There were no significant differences in mean test anxiety between the treatment groups as measured by TAQ.

H(8): There are no significant differences between the treatment groups in the percentage of students who drop out of the beginning algebra course.

H(9): There are no significant differences between the treatment groups in the percentage of students who receive a grade of C or better (success) in the beginning algebra course.

Procedures:

Student success and dropout rate were tested using the chi square statistic. The dropout and success frequencies for each of the treatment groups were compared using an expected frequency.

Findings and Discussion:

13. There were no significant differences between the treatment groups in the percentage of students who:

a. Dropped out of the beginning algebra course.

b. Received a grade of C or better (success in the beginning algebra class.

The findings show the dropout and success rate to be essentially the same for each treatment. Although no firm conclusions can therefore be drawn concerning the dropout and success rate and the variables investigated in this study, certain nonsignificant results seem worthy of mention. The treatment with the largest percentage of dropouts (29.8%) and the smallest percentage of successful students (45.6%) was the traditional W-1 treatment. The treatment with the smallest dropout percentage (23.6%) was the G-1 treatment. With an average success rate of 53.7%, the 2-test treatment had the highest percentage of successful students. Thus, for this study, it appears that the dropout and success rate are most favorable when the traditional classroom organization for instruction and evaluation procedure are modified in some way.

Question 1: What were the reactions of the students toward various features of the classroom treatments?

Procedures:

Student reactions to various aspects of the classroom treatments were obtained through two instruments administered previous to the final exam at the conclusion of the semester. For both of the instruments, student responses frequencies were tallied for each of the questions and percentages were computed. No statistical analysis was done and all findings reported were gained through inspection.

Findings and Discussion:

14. An overwhelming majority of the students in each of the treatments consulted either the instructional objectives or the sample test to determine exactly what they were expected to learn.
15. The most beneficial learning resource for the whole class treatment was the teacher with the exercise sheets serving as the secondary resource.
16. The most beneficial learning resource for the grouping treatment was the exercise sheets with the teacher serving as the secondary resource.
17. In addition to the teacher, the grouping and 2-test procedures were singled out by students as the thing they most liked about the class.
18. A majority of the students in each of the treatments perceived their attitude toward mathematics as having become more favorable as a result of their experiences during the semester.

Both the instructional objectives and the sample test were utilized by students to determine exactly what they were expected to learn. The one source most consulted by students (average - 49%) was the list of instructional objectives. Combined percentages showed that a least 87% of the students in each of the treatment groups consulted either the instructional objectives or the sample test. In addition to their usefulness to students, the writer found them to be beneficial in writing exercise sheets and preparing tests.

Conclusion 4: The instructional objectives and sample unit tests were an important part of each of the treatments. The students found them effective in communicating exactly what they were expected to learn for each unit.

The exercise sheets were a valuable learning resource along with the teacher for each of the treatment groups. Consistent with the design of the treatments, the grouping treatment students were more dependent on the exercise sheets and less dependent on the teacher than the whole class treatment students. The fact that the textbook was not named by students as a learning resource is, in the writer's opinion, probably due to two reasons. First, the textbook was difficult to read and is poorly organized for use in a beginning algebra course. Second, the instructors did not emphasize the use of the textbook as much as would normally have been the case due to an awareness of these shortcomings.

While the teacher was clearly an important factor for each treatment in all aspects of classroom procedure from the students' point of view, the grouping and 2-test procedures were also singled out as having wide student appeal. In response to the question "What did you like about the class?" a majority of the responding students in all but the G-2 treatment named the teacher. In response to the same question, 40% of the grouping treatment students mentioned the division of the class into groups as the thing they liked about the class. Twenty-five percent of the 2-test treatment students indicated the chance to take a final unit test a second time as the thing they liked most about the way their class was run.

Conclusion 5: The favorable student reaction to the grouping and 2-test procedures suggests the use of these approaches as aids in positively affecting student attitude.

The findings indicate positive attitude change toward mathematics and positive attitudes toward the Mathematics 100 course for a majority of students in each of the treatment groups. With the result somewhat more prevalent in 2-test than 1-test classes, at least 55% of the responding students in each of the treatments perceived their attitudes toward mathematics as having become more favorable as a result of their experiences during the semester. More than 90% of the students finishing the course in each treatment indicated they would take another mathematics course that was run as theirs had been. These results are consistent with the significant positive attitude change found for the total population as measured by the MAO.

Recommendations for Further Research

1. The results of this study indicate that instructional objectives and sample tests are useful to students in determining what they are expected to learn. Since both of the above were distributed at the beginning of each unit, it is not clear whether one is more beneficial to students than the other or if either a list of instructional objectives or a sample unit test could be used apart from the other. Research might be done to clarify these issues.

2. The test retest procedure for final unit evaluation was shown to produce significantly larger mean mathematics achievement as measured by students final unit test averages than the traditional 1-test final unit evaluation approach. In addition, 28% of the finishing 2-test

treatment students were able to raise their grade as a result of the second test chance. It seems important that additional research be carried out with a test-retest approach to evaluation and that it be done at a variety of grade levels. An interesting extension of this writer's research might involve the inclusion of a retention test administered after a given period of time following the conclusion of the course. Such an inclusion might shed some light on the permanency of the achievement gains made by 2-test treatment students, and on their retention as compared with that of students taking a single test.

3. There appears to be a need for research involving a modification of the combination whole class-flexible grouping organization for instruction as defined in this study. This study has shown grouping beneficial for high achieving students in terms of mathematics achievement and for positively effecting the attitudes of students. Grouping also allows for greater awareness of the ability level make-up of a class in the opinion of the instructors of this study. It has seemingly failed, however, to provide for increased mathematics achievement for low achieving students, as was originally intended. The question to be answered at this point is: Is it possible to maintain the greater achievement for high achieving students and simultaneously improve the achievement of low achieving students by varying the classroom organization in some different manner?

A possible beginning point for modification might be the initial lecture portion of the grouping treatment. While consuming approximately one-third of each unit of the course, it seemed, in the writer's opinion, to be neither an efficient nor an effective time of instruction for low achieving students. An initial diagnostic pre-test, at a somewhat

simpler level, might serve as well for identifying previous student preparation in the various unit topics.

4. This study has further demonstrated for the writer the need for an in depth study of the characteristics of successful and nonsuccessful students in a course of this sort. There seems to be a need for a complete student profile, not only in areas such as age, sex, hours of work per week, and previous mathematics preparation, but in the more difficult to measure areas of motivation and long range personal and professional goals and aims. If such a profile can be used to identify potential nonsuccessful students previous to the beginning of the course, it may be possible to prevent such an outcome as the course unfolds through counseling and supplementary instruction.

APPENDIX A

UNIT TOPICAL OUTLINE, INSTRUCTIONAL OBJECTIVES, AND SAMPLE TEST

Mathematics 100 - Unit III

TOPICAL OUTLINE

	Topic for Discussion	Pages in Text	Related Exercises
1.	Algebraic Expressions	55-57	p. 57 Ext 1,5,7,9,11, 12,14,17-33 odds.
2.	Introduction to Equations, Solutions	73-77	pp. 75-76 ExD 1a,c,d, 5.
3.	Equations	73-77	pp. 76-77 Ex 1-17,22, 26,44,46,49,51,54,55.
4.	Equations: Formulas, Word Problems	80-88	p. 82 ExD 1a,b,c,Ex 2-4, p. 83 Ex 7, pp. 86-87 ExD 1-7, pp. 87-88 Ex 1-3,5,7,8.
5.	Inequalities: Solutions	101-104	p. 104 ExD 1-7, p. 105 Ex 5,9-13.
6.	Inequalities: Solutions and Graphing	105,109-110	p. 110 ExD 2,5,6,8, 10a,b,11, p. 111 Ex 1-8.
7.	TEST III		

Mathematics 100 - Unit III

OBJECTIVES

At the conclusion of Unit III the student will be able to:

*1. Simplify algebraic expressions by removing grouping symbols and combining like terms. (1)+

*2. Solve and check simple linear equations in one variable. (2,a-e)

3. Solve and check linear equations in one variable with fractional coefficients. (2,f-g)

*4. Evaluate formulas for a specific variable by substituting known values of the remaining variables into the equation. (3)

5. Solve word problems involving "age" or "mixture" situations. (5)

*6. Solve and graph linear inequalities in one unknown. (4)

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*Starred objectives are those considered minimum for successful completion of the unit.

+Numbers following each objective refer to corresponding sample test items.

SAMPLE TEST ITEMS - UNIT III

1. Simplify the following algebraic expressions by removing grouping symbols and combining like terms:

   a) $2(x - 3) + 4x =$

   b) $2(a + 7) + 3(a - 5) =$

   c) $2(x - 5) - (3 + 4x) =$

   d) $2[x + 3(x - 4)] =$

SAMPLE TEST ITEMS - UNIT III (continued)

2. Solve and check the following equations:

   a) $x - 7 = 15$

   b) $\frac{2}{3}x = 12$

   c) $2y + 3 = 11$

   d) $3x + 7 = x - 13$

   e) $2x - 4 = 8x + 8$

   f) $\frac{3}{5}x + 3 = 12$

   g) $\frac{2x - 3}{4} - 1 = \frac{x - 5}{3}$

3. Solve each of the following formulas for the variable indicated:

   a) $V = \frac{1}{3}Bh$ Find h, if V = 8, B = 12

   b) $C = \frac{5}{9}(F - 32)$ Find C, if F = 77

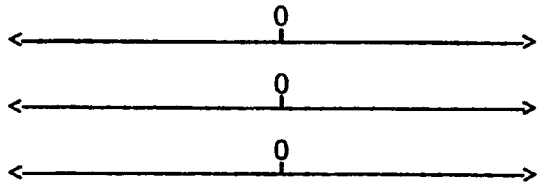
   c) $P = 2e + 2w$ Find e, if P = 26, w = 10

4. Solve the following inequalities and draw a graph of each of their solution sets on the number line provided:

   a) $x > -4$

   b) $3x + 1 < 7$

   c) $-5x + 1 < 2x - 13$



5. Solve the following word problems:

   a) Sue is two years older than Mary. Eleven years ago Sue was twice as old as Mary. How old is each of the girls now?

   b) A clerk sells one dozen more $4.00 blouses than $5.00 blouses. If the total amount she took in for the sale of the blouses was $318.00, how many of each did she sell?

APPENDIX B

INSTRUCTIONAL MATERIALS

Mathematics 100 - Unit III

CRITERION TEST

1. Simplify the following algebraic expressions by removing the grouping symbols and combining like terms:

   a) $4(x - y) + 7x$

   b) $3(x - 5) - 2(2x + 3)$

2. Solve and check the following equations:

   a) $x + 3 = 11$

   b) $\frac{4}{5}x = 12$

   c) $6x + 7 = 2x - 5$

   d) $\frac{a}{4} - 2 = \frac{4a + 7}{3}$

3. Solve each of the following formulas for the variables indicated:

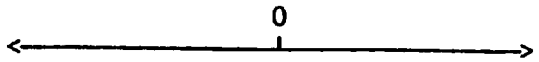
   a) $A = \frac{1}{2}bh$ Find A, if b = 16, h = 3.

   b) $C = 2L + 2W$ Find C, if L = 5, W = 7.

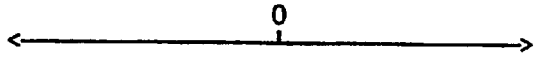
   c) $F = \frac{9}{5}C + 32$ Find C if F = 50.

4. Solve each of the following inequalities and graph their solution sets on the number lines provided.

   a) $x < -5$

   b) $4x + 9 > -7$

5. Solve the following word problem:

   a) Thomas is 6 years older than James. In three years the sum of their ages will be 44. Find their ages now.

Mathematics 100 - Unit III Exercises

Algebraic Expressions, Equations

GROUP 1

The related exercises for this portion of Unit III are:

p. 57 Ex 1,5,7,9,11,12,14,17-33 odds. pp. 75-76 ExD 1a,c,d,5. pp. 76-77 Ex 1-17,22,26,44, 46,49.

These should be completed in addition to the exercises below.

A necessary skill in algebra is that of simplifying algebraic expressions by removing the grouping symbols and combining like terms. Recall that "like terms" are terms which have the exact same letter parts. Thus, $5x$, x, $-2x$, and $\frac{2}{3}x$ are like terms since their letter part is x. $8y$, $-3y$, $\frac{1}{2}y$, and y are also like terms since their letter part is y. $5xy$, $7x$, and $3y$ are NOT like terms since each has a different letter part. Study the examples below before proceeding to the exercises.

(a) $2(x - 4) + 7 = 2x - 8 + 7 = 2x - 1$

(b) $3(x - y) - 4y = 3x - 3y - 4y = 3x - 7y$

(c) $5(x + y) - 2(x - y) = 5x + 5y - 2x + 2y$
$= 3x + 7y$

1. Simplify the following algebraic expressions by removing grouping symbols and combining like terms:

a) $3x + 2y - 8 + 5y - x =$

b) $2(x - 3) + 9 =$

c) $5x + 4(x + 7) =$

d) $3x - 7y + 2(x + y) =$

e) $4(2x + y) + 3(x - y) =$

f) $5x - (3x + 4y) =$

g) $6y + 7 - 2(y + 8) =$

h) $4(2x - 3y) - 2(x + y) =$

An equation is a statement that two algebraic expressions are equal. In solving equations the important thing to remember is that this equality between sides must be preserved. Thus, whatever is done to one side of an equation must be done to the opposite side of the equation as well.

The primary aim in solving any equation is to isolate all variables on one side of the equation and all numbers on the opposite side of the equation. In general, solving an equation is like untying a knot in that we always do the opposite of what has been done to form the equation.

If a number has been added to one side of an equation and it is desirable to remove it from that side, we subtract the number from BOTH sides of the equation. If a number has been subtracted from one side of an equation and it is desirable to remove it from that side, we add the number to BOTH sides of the equation. For example,

$$x - 3 = 4$$
$$x - 3 + 3 = 4 + 3$$
$$x = 7$$

We wish to remove the number 3 from the left side of the equation. It has been subtracted from the variable so we reduce it to zero by adding 3 to both sides.

$$y + 5 = 9$$
$$y + 5 - 5 = 9 - 5$$
$$y = 4$$

We wish to remove the number 5 from the left side of the equation. It has been added to the variable so we reduce it to zero by subtracting 5 from both sides.

2. Solve each of the following by adding or subtracting the same number on BOTH sides of the equation. Check your answer when finished by substitution.

a) $x - 4 = 8$

b) $y + 2 = 7$

c) $x - 5 = -7$

d) $y + 4 = -2$

e) $4 = x - 2$

f) $8 = x + 7$

g) $7 = 5 + x$

h) $x = 7 = -3 + 9$

i) $4 + x = 7 - 9$

j $5 + x = 2$

The equation $2x = 10$ states that 2 times a number x is 10. Following our general procedure of doing the opposite of what has been done to the variable, we can solve the equation by dividing both sides by 2 since division is the opposite of multiplication.

$2x = 10$

$\frac{2x}{2} = \frac{10}{2}$ (divide by 2)

$x = 5$

Thus to find the value of x when x has a whole number as a coefficient, divide both sides of the equation by the numerical coefficient of x. For example,

a) $3x = 12$

$\frac{3x}{3} = \frac{12}{3}$

$x = 4$

b) $5x = 4$

$\frac{5x}{5} = \frac{4}{5}$

$x = \frac{4}{5}$

c) $-2x = 7$

$\frac{-2x}{-2} = \frac{7}{-2}$

$x = \frac{-7}{2}$

If the coefficient of x is a fraction such as in the equation $\frac{3}{4}x = 12$, we multiply both sides of the equation by the multiplicative inverse of $\frac{3}{4}$ or $\frac{4}{3}$ since this will give us 1x or x. Notice we are still doing the opposite since to divide by $\frac{3}{4}$ is to multiply by its reciprocal $\frac{4}{3}$. For example,

a) $\frac{2}{3}x = 4$

$\frac{3}{2}(\frac{2}{3}x) = \frac{3}{2}(4)$

$x = 6$

b) $\frac{4}{9}y = 16$

$\frac{9}{4}(\frac{4}{9}y) = \frac{9}{4}(16)$

$y = 36$

c) $\frac{-3}{5}x = 9$

$\frac{-5}{3}(\frac{-3}{5}x) = \frac{-5}{3}(9)$

$x = -15$

3. Solve the following equations:

a) $5x = 10$

b) $\frac{2}{3}x - 16$

c) $-4x = 12$

d) $-6x = -18$

e) $\frac{5}{7}y = 30$

f) $\frac{-6}{7}y = 42$

g) $\frac{1}{3}z = 5$

h) $-12x = 24$

i) $3x = -7$

j) $-7y = 25$

Many times solving an equation requires more than one operation. Study the examples below before proceeding further.

a) $3x + 2 = 8$

$3x + 2 - 2 = 8 - 2$ (Subtract 2)

$3x = 6$ (Combine like terms)

$\frac{3x}{3} = \frac{6}{3}$ (Divide 3)

$x = 2$

CHECK:

$3(2) + 2 \stackrel{?}{=} 8$

$6 + 2 = 8$

$8 = 8$

b) $2x - 5 = x + 7$

$2x - 5 + 5 = x + 7 + 5$ (Add 5)

$2x = x + 12$ (Combine terms)

$2x - x = x + 12 - x$ (Subtract x)

$x = 12$ (Combine like terms)

CHECK:

$2(12) - 5 \stackrel{?}{=} 12 + 7$

$24 - 5 = 19$

$19 = 19$

c) $2x - 2 = 5x + 7$

$2x - 2 + 2 = 5x + 7 + 2$ (Add 2)

$2x = 5x + 9$ (Combine like terms)

$2x - 5x = 5x + 9 - 5x$ (Subtract 5x)

$-3x = 9$ (Combine like terms)

$\frac{-3x}{-3} = \frac{9}{-3}$ (Divide -3)

$x = -3$

CHECK:

$2(-3) - 2 \stackrel{?}{=} 5(-3) + 7$

$-6 - 2 = -15 + 7$

$-8 = -8$

Notice that we always wait until the next to the last step before dividing both sides by any number.

4. Solve the following equations and check your answers by substitution.

a) $3x - 15$

b) $x - 1 = 8$

c) $2x + 3 = 11$

d) $3x + 4 = -14$

e) $4x + 7 = 3x - 2$

f) $2x - 5 = x - 3$

g) $4x + 6 = 3x + 7$

h) $11y + 2 = 3 + y$

i) $3a + 7 = a + 21$

j) $3x = 5x + 15 - 17$

k) $5x + 7 = 6x + 8$

l) $-10 - x = 12$

Exercise Key

1. a) $2x + 7y - 8$ b) $2x + 3$ c) $9x + 28$ d) $5x - 5y$

   e) $11x + y$ f) $2x - 4y$ g) $4y - 9$ h) $6x - 14y$

2. a) $x = 12$ b) $y = 5$ c) $x = -2$ d) $y = -6$

   e) $x = 6$ f) $x = 1$ g) $x = 2$ h) $x = -1$

   i) $x = -6$ j) $x = -3$

3. a) $a = 2$ b) $x = 24$ c) $x = -3$ d) $x = 3$

   e) $\frac{5}{7}y = 30$ f) $y = -49$ g) $z = 15$ h) $x = -2$

   $\frac{7}{5}(\frac{5}{7}y) = \frac{7}{5}(30)$ i) $x = \frac{-7}{3}$ j) $y = \frac{-25}{7}$

4. a) $x = 5$ b) $x = 9$ c) $x = 4$ d) $x = -6$

e) $x = -9$ f) $x = 2$ g) $x = 1$ h) $y = \frac{1}{10}$

i) $3a + 7 = a + 21$

$3a + 7 - 7 = a + 21 - 7$

$3a = a + 14$

$3a - a = a + 14 - a$

$2a = 14$

$\frac{2a}{2} = \frac{14}{2}$

$a = 7$

j) $3x = 5x + 15 - 17$

$3x = 5x - 2$

$3x - 5x = 5x - 2 - 5x$

$-2x = -2$

$\frac{-2x}{-2} = \frac{-2}{-2}$

$x = 1$

k) $5x + 7 = 6x + 8$

$5x + 7 - 7 = 6x + 8 - 7$

$5x = 6x + 1$

$5x - 6x = 6x + 1 - 6x$

$-x = 1$

$(-1)(-x) = (-1)(1)$

$x = -1$

Notice that we multiplied both sides by -1 to obtain the positive value of the variable.

l) $x = -22$

Mathematics 100 - Unit III Exercises

Algebraic Expressions, Equations

GROUP 2

The related exercises for this portion of Unit III are:

p. 57 Ex 1,5,7,9,11,12,14,17-33 odds. pp. 75-76 ExD 1a,c,d,5. pp. 76-77 Ex 1-17,22,26,44, 46,49.

These should be completed in addition to the exercises below.

Many times in algebra it is necessary to simplify algebraic expressions by removing parentheses and combining like terms. Recall that like terms are terms which have the exact same letter parts. For example, in the algebraic expression

$$3xy + 4x + 7 - 5y - 3x + 9$$

there are 6 terms and the only like terms to be combined are 4x and 3x, and 7 and 9. Thus the simplified form for the above expression is $3xy + 4x + 7 - 5y - 3x + 9 = 3xy + x - 5y + 16$.

1. Simplify the following algebraic expressions by removing grouping symbols and combining like terms:

   a) $3x + 5 + 7x - 2 =$

   b) $4(x+2) - 5 =$

   c) $4x + 3(x - 4) + 10 =$

   d) $3(x + y) - 5x + y =$

   e) $4(x - 3) + 2(x - 5) =$

   f) $3x - (x + 4) =$

   g) $2(x + 3) - 2(2x - 6) =$

   h) $3(2x + 4y - 7) + 2(x - 3) =$

An equation may be solved by adding, subtracting, multiplying, or dividing BOTH sides of the equation by the SAME number. In solving any equation our primary aim is to isolate all variables on one side of the equation and all numbers on the other side of the equation. In general, solving an equation is like untying a knot in that we always do the opposite of what has been done to form the equation. Below are some examples of solved equations. Study them carefully before proceeding to the exercises.

a) $x + 7 = 15$

$x + 7 - 7 = 15 - 7$ (Since 7 is added to x, we substract 7)

$x = 8$ (Combine like terms)

CHECK:

$8 + 7 \stackrel{?}{=} 15$

$15 = 15$

b) $2x - 5 = 17$

$2x - 5 + 5 = 17 + 5$ (Since 5 was subtracted from 2x, we add 5 to both sides)

$2x = 22$ (Combine like terms)

$\frac{2x}{2} = \frac{22}{2}$ (Divide by 2)

$x = 11$

CHECK:

$2(11) - 5 \stackrel{?}{=} 17$

$22 - 5 = 17$

$17 = 17$

c) $3x + 2 = 5x - 4$

$3x + 2 - 2 = 5x - 4 - 2$ (Subtract 2)

$3x = 5x - 6$ (Combine like terms)

$3x - 5x = 5x - 6 - 5x$ (Subtract 5x)

$-2x = -6$

$\frac{-2x}{-2} = \frac{-6}{-2}$ (Divide -2)

$x = 3$

CHECK:

$$3x + 2 \stackrel{?}{=} 5x - 4$$
$$3(3) + 2 = 5(3) - 4$$
$$9 + 2 = 15 - 4$$
$$11 = 11$$

d) $\frac{3}{4}x = 27$

$\frac{4}{3}(\frac{3}{4}x) = \frac{4}{3}(27)$ (Multiply by the reciprocal of $\frac{3}{4}$)

$x = 36$

CHECK:

$\frac{3}{4}(36) \stackrel{?}{=} 27$

$27 = 27$

2. Solve and check the following equations:

a) $x + 5 = 11$
b) $x - 4 = 9$
c) $x + 7 = -3$
d) $4x = 12$
e) $5x + 7 = 12$
f) $3x - 2 = -2$
g) $2x = x + 8$
h) $3x = x - 5$
i) $2x - 4 = x + 6$
j) $5x + 3 = 2x + 9$
k) $\frac{2}{3}x = 10$
l) $\frac{3}{4}x - 7 = 8$
m) $x - 4 = 3x + 8$
n) $3x + 9 = 27 + 6x$
o) $4x + 7 = x - 9$

Many times it is necessary to solve equations which have fractions as some of their terms. To solve an equation of this sort we will first perform an operation to eliminate the fractions from the equation and then solve the resulting equation as those in exercise 2 above.

Consider the equation $\frac{x}{2} - 4 = \frac{x}{3}$. To clear the equation of all fractions it is necessary to multiply every term of the equation by the <u>least common denominator</u> (l.c.d.) of the fractions present in the equation. Since the l.c.d. for the denominators 2 and 3 is 6 we multiply every term by 6.

$$6\left(\frac{x}{2}\right) - 6(4) = 6\left(\frac{x}{3}\right)$$

$$3x - 24 = 2x$$

$$3x - 24 + 24 = 2x + 24 \qquad \text{(Add 24)}$$

$$3x = 2x + 24 \qquad \text{(Combine like terms)}$$

$$3x - 2x = 2x + 24 - 2x \qquad \text{(Subtract 2x)}$$

$$x = 24 \qquad \text{(Combine like terms)}$$

CHECK:

$$\frac{24}{2} - 4 \stackrel{?}{=} \frac{24}{3}$$

$$12 - 4 = 8$$

$$8 = 8$$

A second example is the following:

$$\frac{2x}{7} + 1 = \frac{5x}{14} - 2$$

$$14\left(\frac{2x}{7}\right) + 14(1) = 14\left(\frac{5x}{14}\right) - 14(2) \qquad \text{(Multiply by the l.c.d. 14)}$$

$$4x + 14 = 5x - 28$$

$$4x + 14 - 14 = 5x - 28 - 14 \qquad \text{(Subtract 14)}$$

$$4x = 5x - 42 \qquad \text{(Combine like terms)}$$

$$4x - 5x = 5x - 42 - 5x \qquad \text{(Subtract 5x)}$$

$$-x = -42$$

$$-1(-x) = -1(-42) \qquad \text{(Multiply by -1 to obtain the positive value of the variable)}$$

$$x = 42$$

3. Solve the following equations by first clearing each equation of fractions using the least common denominator.

a) $\frac{x}{2} = 5 + \frac{x}{3}$

b) $\frac{2x}{3} - 5 = \frac{x}{4}$

c) $\frac{1}{3}x - \frac{3}{2} = \frac{1}{6}$

d) $\frac{2x}{3} - \frac{2}{5} = \frac{2x}{5}$

e) $\frac{3y}{4} - \frac{y}{2} = -2$

f) $\frac{5}{6}y + \frac{1}{3}y = \frac{1}{2}$

4. Solve the following equations:

a) $3(2x - 1) = 5$

b) $2(2x - 3) = 3(x + 4)$

c) $2(x + 2) - 3(2x - 1) = 5$

d) $\frac{1}{3}(9x - 6) = -10$

e) $\frac{1}{2}(4x - 2) = 7$

f) $\frac{y + 2}{3} = \frac{2y - 7}{5}$

Exercise Key

1. a) $10x + 3$

b) $4(x+2) - 5 = 4x + 8 - 5 = 4x + 3$

c) $7x - 2$

d) $-2x + 4y$

e) $6x - 22$

f) $3x - (x + 4) = 3x - x - 4 = 2x - 4$

g) $2(x + 3) - 2(2x - 6) = 2x + 6 - 4x + 12 = -2x + 18$

h) $8x + 12y - 27$

2. a) $x = 6$

b) $x = 13$

c)
$$x + 7 = -3$$
$$x + 7 - 7 = -3 - 7 \quad \text{(Subtract 7)}$$
$$x = -10$$

d) $x = 3$

e)
$$5x + 7 = 12$$
$$5x + 7 - 7 = 12 - 7 \quad \text{(Subtract 7)}$$
$$5x = 5 \quad \text{(Combine like terms)}$$
$$\frac{5x}{5} = \frac{5}{5} \quad \text{(Divide 5)}$$
$$x = 1$$

f)
$$3x - 2 = -2$$
$$3x - 2 + 2 = -2 + 2 \quad \text{(Add 2)}$$
$$3x = 0 \quad \text{(Combine like terms)}$$
$$\frac{3x}{3} = \frac{0}{3} \quad \text{(Divide 3)}$$
$$x = 0$$

g) $x = 8$

h) $3x = x - 5$

$3x - x = x - 5 - x$ (Subtract x)

$2x = -5$

$\frac{2x}{2} = \frac{-5}{2}$ (Divide 2)

$x = \frac{-5}{2}$

i) $2x - 4 = x + 6$

$2x - 4 + 4 = x + 6 + 4$ (Add 4)

$2x = x + 10$

$2x - x = x + 10 - x$ (Subtract x)

$x = 10$

j) $x = 2$

k) $\frac{2}{3}x = 10$

$\frac{3}{2}(\frac{2}{3}x) = \frac{3}{2}(10)$ (Multiply by $\frac{3}{2}$ the reciprocal of $\frac{2}{3}$)

$x = 15$

l) $\frac{3}{3}x - 7 = 8$

$\frac{3}{4}x - 7 + 7 = 8 + 7$ (Add 7)

$\frac{3}{4}x = 15$

$\frac{4}{3}(\frac{3}{4}x) = \frac{4}{3}(15)$ (Multiply by $\frac{4}{3}$)

m) $x = -6$

n) $x = -6$

o) $x = \frac{-16}{3}$

3. a) The l.c.d. is 6; $6(\frac{x}{2}) = 6(5) + 6(\frac{x}{3})$

$3x = 30 + 2x$

$3x - 2x = 30 + 2x - 2x$ (Subtract 2x)

$x = 30$

b) The l.c.d. is 12; $x = 12$

c) The l.c.d. is 6; $x = 5$

d) The l.c.d. is 15; $x = \frac{3}{2}$

e) The l.c.d. is 4; $y = -8$

f) The l.c.d. is 6; $6(\frac{5}{6}y) + 6(\frac{1}{3}y) = 6(\frac{1}{2})$

$5y + 2y = 3$

$7y = 3$

$\frac{7y}{7} = \frac{3}{7}$ (Divide 7)

$y = \frac{3}{7}$

4. a) $x = \frac{4}{3}$

b) $x = 18$

c)
$$\begin{aligned} 2(x + 2) - 3(2x - 1) &= 5 \\ 2x + 4 - 6x + 3 &= 5 \\ -4x + 7 &= 5 && \text{(Combine terms)} \\ -4x + 7 - 7 &= 5 - 7 && \text{(Subtract 7)} \\ -4x &= -2 \\ \frac{-4x}{-4} &= \frac{-2}{-4} && \text{(Divide -4)} \\ x &= \frac{1}{2} \end{aligned}$$

d)
$$\begin{aligned} \tfrac{1}{3}(9x - 6) &= -10 \\ 3x - 2 &= -10 \\ 3x - 2 + 2 &= -10 + 2 \\ 3x &= -8 \\ x &= \frac{-8}{3} && \text{(Divide 3)} \end{aligned}$$

e) This could be solved as d), another way is the following:

$$\begin{aligned} 2[\tfrac{1}{2}(4x - 2)] &= 2(7) && \text{(Multiply by l.c.d. 2)} \\ 1\ 4x - 2) &= 14 \\ 4x - 2 &= 14 \\ 4x - 2 + 2 &= 14 + 2 && \text{(Add 2)} \\ 4x &= 16 && \text{(Combine like terms)} \\ x &= 4 && \text{(Divide by 4)} \end{aligned}$$

f)
$$\begin{aligned} 5(y + 2) &= 3(2y - 7) \\ 5y + 10 &= 6y - 21 \\ 5y + 10 - 10 &= 6y - 21 - 10 \\ 5y &= 6y - 31 \\ 5y - 6y &= 6y - 31 - 66 \\ -y &= -31 \\ (-1)(-y) &= (-1)(-31) \\ y &= 31 \end{aligned}$$

Mathematics 100 - Unit III Exercises

Algebraic Expressions, Equations

GROUP 3

The related exercises for this portion of Unit III are:

p. 57 Ex 1,5,7,9,11,12,14,17-33 odds. pp. 75-76 ExD 1a,c,d,5. pp. 76-77 Ex 1-17,22,26,44,46, 49,51,52-57. p. 82 ExD 1a,b,c, Ex 2-4. p. 83 Ex 7.

These should be completed in addition to the exercises below.

To simplify an algebraic expression is to remove all grouping symbols and combine like terms. Recall that <u>like terms</u> are those having the exact same letter parts.

1. Simplify the following algebraic expressions:

   a) $5x + 7y + 4(x - 3) - 2y$
   b) $3(x + y) - 2(x - y)$
   c) $4[3x - 4(x + y)]$
   d) $2(x + y) - 3(x - y + 2)$
   e) $3[(x - y) + 2(x + y)]$
   f) $-2[-3(x + 7) + 4]$

2. Solve and check the following equations:

   a) $x + 4 = -2$
   b) $3x + 4 = 7$
   c) $-x = 7$
   d) $5x + 2 = 3x$
   e) $4x + 7 = 5x - 4$
   f) $-5x = 30$
   g) $\frac{4}{5}y = 12$
   h) $\frac{-3}{4}x = -9$

   h) $\frac{-3}{4}x = -9$

   i) $\frac{-3}{4}x - 7 = 8$
   j) $15 - (y - 6) = 6 - (3y + 5)$
   k) $3(x + 5) + 4(x + 5) = 21$
   l) $\frac{-2}{3}k + 1 = -5$
   m) $6 - 8x = 4 - 4x$
   n) $9 - 2y = y + 30$

Many times it is necessary to solve equations involving fractions. When solving equations of this sort, it is advantageous to first clear the equation of all fractions. This may be done by multiplying <u>every term</u> of the equation by the least common denominator (l.c.d.) of the fractions present in the equation. The following examples demonstrate this technique:

a) $\frac{x}{2} - 4 = \frac{x}{3}$ (The l.c.d. is 6)

$$6(\tfrac{x}{2}) - 6(4) = 6(\tfrac{x}{3})$$
$$3x - 24 = 2x$$
$$3x - 24 + 24 = 2x + 24$$
$$3x = 2x + 24$$
$$3x - 2x = 2x + 24 - 2x$$
$$x = 24$$

b) $\frac{2x - 3}{4} = 1 + \frac{x - 5}{3}$ (The l.c.d. is 12)

$$12(\tfrac{2x - 3}{4}) = 12(1) + 12(\tfrac{x - 5}{3})$$
$$3(2x - 3) = 12 + 4(x - 5)$$
$$6x - 9 = 12 + 4x - 20$$
$$6x - 9 + 9 = 12 + 4x - 20 + 9$$
$$6x = 4x + 1$$
$$6x - 4x = 4x + 1 - 4x$$
$$2x = 1$$
$$x = \frac{1}{2}$$

3. Solve each of the following equations:

a) $\frac{x}{2} = 5 + \frac{x}{3}$

b) $\frac{2x}{3} - 5 = \frac{x}{4}$

c) $\frac{1}{3}x - \frac{3}{2} = \frac{1}{6}$

d) $\frac{2x}{3} - \frac{2}{5} = \frac{2x}{5}$

e) $\frac{1}{3}(9x + 6) = -10$

f) $\frac{1}{2}(4x - 2) = 7$

g) $\frac{2x - 1}{4} - \frac{x}{5} = \frac{x + 1}{2}$

h) $\frac{3x}{5} - \frac{x + 1}{2} = 4$

i) $\frac{4x + 1}{2} - \frac{3x - 1}{3} = 1 - \frac{x + 5}{4}$

Note: Other fractional equations of this sort are found on page 77 of your textbook, Exercises 52 - 57.

Formulas

One of the most widespread applications of mathematics to other areas of study is the use of equations for stating formulas. In general, a formula is a statement of the relationship between a series of different or similar things.

A common formula used in business, for example, relates interest (I), the principal (P, the amount borrowed or saved), the rate (R, percent), and the time (T, in years). The formula is I = PRT. The interest on a loan of 500 dollars, with a rate of 7 percent for 2 years, then, is computed as follows.

$$\begin{aligned} I &= PRT \\ &= (\$500)(.07)(2) \\ &= (\$35)(2) \\ &= \$70 \end{aligned}$$

A formula used for finding the Perimeter (distance around) of a rectangular figure is P = 2L + 2W where L = Length and W = Width. Thus, if the Perimeter is 54 and the width is 9, the length is found as follows:

$$\begin{aligned} P &= 2L + 2W \qquad && P = 54,\ W = 9 \\ 54 &= 2L + 2(9) \\ 54 &= 2L + 18 \\ 54 - 18 &= 2L + 18 - 18 \\ 36 &= 2L \\ 18 &= L \end{aligned}$$

A formula is useful when all but one of the variables involved is known.

4. Solve each of the following formulas for the variables indicated:

a) $C = \frac{5}{9}(F - 32)$ Find C, if $F = 41^{\circ}$

b) $P = 2L + 2W$ Find L, if P = 148, W = 22

c)	$V = \frac{1}{3}Bh$	Find V, if B = 27, h = 6
d)	$A = LW$	Find A, if L = 13, W = 25
e)	$d = rt$	Find r, if d = 60, t = 2
f)	$F = \frac{9}{5}C + 32$	Find F, if C = 80
g)	$I = PRT$	Find I, if P = 200, r = .09, t = 1 year
h)	$I = PRT$	Find I, if P = 1,000, r = .05, t = 3 years

When working with formulas it is sometimes helpful to solve the formula for one of the variables present. Suppose, for example, we are working with the formula,

$$\text{distance} = (\text{rate})(\text{time})$$
$$d = rt$$

If it will be necessary to evaluate the formula for t a large number of times it is advantageous to first solve the formula for t. This can be cone as follows:

$$d = rt$$
$$\frac{d}{r} = \frac{rt}{r}$$
$$\frac{d}{r} = t$$

Similarly, if working with the formula P = 2L + 2W and we have ten or twenty values for P and L from which we must find W, it is helpful to first solve the equation for W. Thus, we have

$$P = 2L + 2W$$
$$P - 2L = 2L + 2W - 2L$$
$$P - 2L = 2W$$
$$\frac{P - 2L}{2} = \frac{2W}{2}$$
$$\frac{P - 2L}{2} = W$$

Notice the procedures for solving formulas for a variable are the same as those for solving equations. We isolate the desired variable on one side of the equation and all numbers and other letters on the other side of the equation.

5. Solve the following formulas for the variable indicated.

a)	$d = rt$	For r
b)	$P = 2L + 2W$	For L
c)	$L = 2\pi r + 3s$	For r
d)	$A = LW$	For W
e)	$F = \frac{9}{5}C = 32$	For C
f)	$I = PRT$	For R
g)	$V = \frac{1}{3}Bh$	For h
h)	$I = \frac{C}{D^2}$	For C

Exercise Key

1. a) $9x + 5y - 12$

b) $x + 5y$

c) $4[3x - 4(x + y)]$
$= 4[3x - 4x - 4y]$
$= 4[-x - 4y]$
$= -4x - 16y$

d) $-x + 5y - 6$

e) $9x + 3y$

f) $6x + 34$

2. a) $x = -6$

b) $x = 1$

c) $x = -7$

d) $x = -1$

e) $x = 11$

f) $x = -6$

g) $y = 15$

h) $x = 12$

i) $x = -20$

j) $y = -10$

k) $x = -2$

l) $k = 9$

m) $x = \frac{1}{2}$

n) $y = -7$

3. a) l.c.d. = 6, $x = 30$

b) l.c.d. = 12, $x = 12$

c) l.c.d. = 6, $x = 5$

d) l.c.d. = 15, $x = \frac{3}{2}$

e) l.c.d. = 3, $x = -4$

f) l.c.d. = 2, $x = 4$

g) l.c.d. = 20, $x = \frac{-15}{4}$

h) l.c.d. = 10

$$10(\tfrac{3x}{5}) - 10(\tfrac{x+1}{2}) = 10(4)$$
$$2(3x) - 5(x+1) = 40$$
$$6x - 5x - 5 = 40$$
$$x = 45$$

i) l.c.d. = 12

$$12(\tfrac{4x+1}{2}) - 12(\tfrac{3x-1}{3}) = 12(1) - 12(\tfrac{x+5}{4})$$
$$6(4x+1) - 4(3x-1) = 12 - 3(x+5)$$
$$24x + 6 - 12x + 4 = 12 - 3x - 15$$
$$12x + 10 = -3x - 3$$
$$x = \frac{-13}{15}$$

4. a) $C = 5$

b) $L = 52$

c) $V = 54$

d) $A = 325$

e) $r = 30$

f) $F = 176$

g) $I = \$18$

h) $I = \$150$

5. a) $r = \frac{d}{t}$

b) $L = \frac{P - 2W}{2\pi}$

c) $r = \frac{L - 3s}{2}$

d) $W = \frac{A}{L}$

e) $F = \frac{9}{5}C + 32$

$$F - 32 = \frac{9}{5}C$$
$$\frac{5}{9}(F - 32) = \frac{5}{9}(\frac{9}{5}C)$$

f) $R = \frac{I}{PT}$

g) $V = \frac{1}{3}Bh$

$$3V = 3(\tfrac{1}{3}Bh)$$
$$3V = Bh$$
$$\frac{3V}{B} = h$$

h) $I = \frac{C}{D^2}$ (l.c.d. = D^2)

$$D^2(I) = D^2(\frac{C}{D^2})$$
$$D^2I = C$$

APPENDIX C

CRITERION TEST, UNIT TEST, AND RETEST

Class Hour _____

Name _____

Math 100 - Unit III CRITERION TEST

1. Simplify the following algebraic expressions by removing the grouping symbols and combining like terms:

   a) $4(x - y) + 7x$ b) $3(x - 5) - 2(2x + 3)$

2. Solve and check the following equations:

   a) $x + 3 = 11$ b) $\frac{4}{5}x = 12$

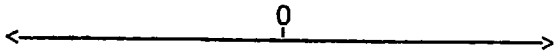
   c) $6x + 7 = 2x - 5$ d) $\frac{a}{4} - 2 = \frac{4a + 7}{3}$

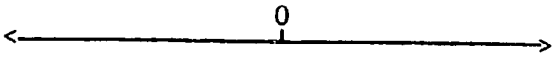
3. Solve each of the following formulas for the variable indicated:

   a) $A = \frac{1}{2}Bh$ Find A, if B = 16, h = 3

   b) $F = \frac{9}{5}C + 32$ Find C, if F = 50

4. Solve each of the following inequalities and graph their solution sets on the number lines provided:

   a) $x < -5$ 

   b) $4x + 9 \geq -7$ 

5. Solve the following word problem:

   Thomas is 6 years older than James. In three years the sum of their ages will be 44. Find their ages now.

Class Hour _____

Name _____

Math 100 TEST III

1. Simplify the following algebraic expressions by removing grouping symbols and combining like terms.

   a) $3x + 7y - x - 2xy - 3y$ _____

   b) $5(y - 6) - 3y + 17$ _____

   c) $4(x - y) + 2(x + 2y)$ _____

   d) $3y - 2(2x - 3y)$ _____

2. Solve each of the following equations and check any three of the five equations. (Points will be given both for solutions and the three checks)

   a) $x + 7 = 13$ b) $2x + 3 = -5$ c) $\frac{4}{7}y = 28$

   d) $8x - 7 = 3x + 8$ e) $\frac{3x}{5} - 4 = \frac{x + 1}{2}$

3. Solve each of the following formulas for the variable indicated:

   a) $P = 2L + 2W$ Find L, if $P = 52$, $W = 10$ $P =$ _____

Class Hour _____

Name _____

Math 100 TEST III p. 2

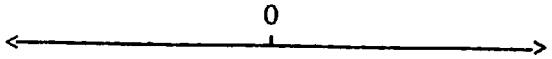
3. (Continued) Solve each of the following formulas:

b) $C = \frac{5}{9}(F - 32)$ Find C, if F = 86 C = _____

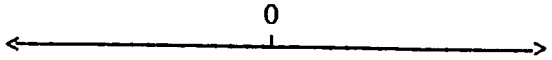
c) $F = \frac{9}{5}C + 32$ Find F, if C = 85 F = _____

4. Solve each of the following inequalities and graph their solution sets on the number line provided.

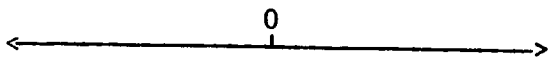
a) $-3x < 6$



b) $3x + 7 \geq 10$



c) $\frac{-5}{4}y > 20$



5. Solve the following word problems: (Please show all work - points will be given both for the answer and the set up of the problem)

a) John is four years older than Rose. The sum of their ages is 40. Find their ages.

Rose _____

John _____

b) Two kinds of candy sell for 35 cents and 75 cents a pound. A clerk wishes to make a 20 pound mixture to sell for 49 cents a pound. How many pounds of each type candy should be used.

35 cent type _____

75 cent type _____

Class Hour _____

Name _____

Math 100 RETEST III

1. Simplify the following algebraic expressions by removing grouping symbols and combining like terms.

   a) $5y - 7x + 2x - 3y - 4xy$ _____

   b) $4(3y - 5) - 4y + 16$ _____

   c) $3(x + y) + 4\ (x - 3y)$ _____

   d) $2y - 3(3x - 2y)$ _____

2. Solve each of the following equations and check any three of the five equations. (Points will be given for both solutions and checks).

   a) $X + 9 = 5$ b) $3x + 4 = {}^{-}5$ c) $\frac{3}{8}x = 24$

   d) $7x - 8 = 4x + 7$ e) $\frac{2}{3}x - 6 = \frac{x + 1}{2}$

3. Solve each of the following formulas for the variable indicated:

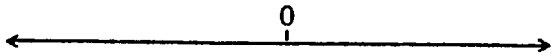
   a) $P = 2L + 2W$ Find W, if P = 56, L = 21 W = _____

   b) $C = \frac{5}{9}(F - 32)$ Find C, if F = 104 C = _____

   c) $F = \frac{9}{5}C + 32$ Find F, if C = 65 F = _____

4. Solve each of the following inequalities and graph their solution sets on the number line provided.

   a) ${}^{-}2x < 8$

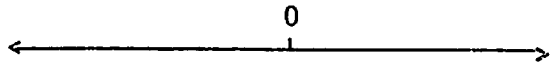


Class Hour _____

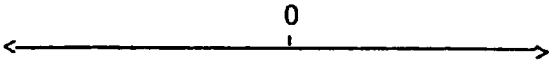
Name _____

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b) $4x + 5 \geq 13$



c) $\frac{6}{5}y > 2$



5. Solve the following word problems: (Please show all work - Points will be given both for the answer and the set up of the problem).

a) Paul is 3 years older than John. The sum of their age is 59. Find their ages.

Paul _____

John _____

b) Two kinds of candy sell for 45 cents and 75 cents a pound. A clerk wishes to make a 30 pound mixture to sell for 55 cents a pound. How many pounds of each type candy should be used?

45 cent type _____

75 cent type _____

APPENDIX D

QUESTIONNAIRES AND OPINIONNAIRES

Name _____

Date _____

Class Hour _____

OPINIONNAIRE

Directions: Please write your name in the upper right hand corner and fill in the date and class hour of your course. Each of the statements on this opinionnaire expresses a feeling which a particular person has toward mathematics. You are to express on a five-point scale, the extent of agreement between the feeling expressed in each statement and your own personal feeling. The five points are: Strongly Disagree (SD), Disagree (D), Undecided (U), Agree (A), Strongly Agree (SA). You are to circle the letter which best indicates how closely you agree or disagree with the feeling expressed to each statement as it concerns you.

1.	I do not like mathematics. I am always under a terrible strain in a math class.	SD D U A SA
2.	I do not like mathematics, and it scares me to have to take it.	SD D U A SA
3.	Mathematics is very interesting to me. I enjoy math courses.	SD D U A SA
4.	Mathematics is fascinating and fun.	SD D U A SA
5.	Mathematics makes me feel secure, and at the same time it is stimulating.	SD D U A SA
6.	I do not like mathematics. My mind goes blank and I am unable to think clearly when working math.	SD D U A SA
7.	I feel a sense of insecurity when attempting mathematics.	SD D U A SA
8.	Mathematics makes me feel uncomfortable, restless, irritable, and impatient.	SD D U A SA
9.	The feeling that I have toward mathematics is a good feeling.	SD D U A SA
10.	Mathematics makes me feel as though I'm lost in a jungle of numbers and can't find my way out.	SD D U A SA

Name _____

Class Hour _____

Page 2

OPINIONNAIRE

11.	Mathematics is something which I enjoy a great deal.	SD	D	U	A	SA
12.	When I hear the word math, I have a feeling of dislike.	SD	D	U	A	SA
13.	I approach math with a feeling of hesitation - hesitation resulting from a fear of not being able to do math.	SD	D	U	A	SA
14.	I really like mathematics.	SD	D	U	A	SA
15.	Mathematics is a course in school which I have always liked and enjoyed studying.	SD	D	U	A	SA
16.	I don't like mathematics. It makes me nervous to even think about having to do a math problem.	SD	D	U	A	SA
17.	I have never liked math, and it is my most dreaded subject.	SD	D	U	A	SA
18.	I love mathematics. I am happier in a math class than in any other class.	SD	D	U	A	SA
19.	I feel at ease in mathematics, and I like it very much.	SD	D	U	A	SA
20.	I feel a definite positive reaction to mathematics; it's enjoyable.	SD	D	U	A	SA

QUESTIONNAIRE ON ATTITUDES TO MATHEMATICS TESTING

Name _____ Date _____

This questionnaire is designed to give you an opportunity to indicate how and what you feel in regard to mathematics tests.

One of the main reasons for construction of this questionnaire is that very little is known about people's feelings towards taking various kinds of tests. We can assume that people differ in the degree to which they are affected by taking a test. What we are particularly interested in here is how widely people differ in their opinions of and reactions to testing situations.

The value of this questionnaire will in large part depend on how frank you are in stating your opinions, feelings and attitudes. Your answers will be kept confidential.

For each question there is a line or scale on the ends of which are statements of opposing feelings or attitudes. In the middle of the line you will find either the word "midpoint" or a phrase, both of which are intended to reflect a feeling or attitude which is in-between the statements of opposing feelings described above. You are required to put a mark (X) on that point of the line you think best indicates the strength of your feeling or attitude about the particular question. The midpoint is only for your guidance. Do not hesitate to put the mark (X) on any point of the line as long as that mark reflects the strength of your feeling or attitude.

If you have any questions at this time please ask them.

THERE ARE NO CATCH QUESTIONS IN THIS QUESTIONNAIRE. PLEASE READ EACH QUESTION AND EACH SCALE VERY CAREFULLY. THERE IS NO TIME LIMIT.

THE MIDPOINT IS ONLY FOR YOUR GUIDANCE. DO NOT HESITATE TO PUT THE MARK (X) ON ANY POINT OF THE LINE AS LONG AS THAT MARK REFLECTS THE STRENGTH OF YOUR FEELING OR ATTITUDE.

The following questions relate to your attitude and experience with mathematics tests. By a mathematics test we mean a test such as one which is given at the end of a topic or unit and which counts towards your final grade. Please try to remember how you usually reacted toward these tests and how you felt while taking them.

1. How valuable do you think mathematics tests are in determining a person's ability?

Very valuable	Valuable in some respects and valueless in others	Valueness

2. Do you think that mathematics tests should be used more widely than at present to grade students?

Should be used less widely	Should be used	Should be used more widely

3. Would you be willing to stake your grade in a math course on the outcome of one mathematics test which has previously predicted success in a highly reliable fashion?

Very willing	Uncertain	Not willing

4. If you know that you are going to take a math test, how do you feel beforehand?

Feel very unconfident	Midpoint	Feel very confident

5. After you have taken a math test, how confident do you feel that you have done your best?

Feel very unconfident	Midpoint	Feel very confident

THE MIDPOINT IS ONLY FOR YOUR GUIDANCE. DO NOT HESITATE TO PUT A MARK (X) ON ANY POINT ON THE LINE AS LONG AS THAT MARK REFLECTS THE STRENGTH OF YOUR FEELING OR ATTITUDE.

6. When you are taking a mathematics test, to what extent do your emotional feelings interfere with or lower your performance?

Do not interfere at all	Midpoint	Interfere a great deal

7. Before taking a mathematics test, to what extent are you aware of an "uneasy feeling"?

Am very much aware of it	Midpoint	Am not aware of it at all

8. While taking a mathematics test, to what extent do you experience an accelerated heart-beat?

Heart-beat does not accelerate at all	Midpoint	Heart-beat noticeably accelerated

9. Before taking a mathematics test, to what extent do you experience an accelerated heart-beat?

Heart-beat does not accelerate at all	Midpoint	Heart-beat noticeably accelerated

10. While taking a mathematics test, to what extent do you worry?

Worry a lot	Midpoint	Worry not at all

11. Before taking a mathematics test, to what extent do you worry?

Worry a lot	Midpoint	Worry not at all

12. While taking a mathematics test, to what extent do you perspire?

Perspire not at all	Midpoint	Perspire a lot

13. Before taking a mathematics test, to what extent do you perspire?

Perspire not at all	Midpoint	Perspire a lot

14. In comparison with other students, how often do you think of ways of avoiding a mathematics test?

Less often than other students	Midpoint	More often than other students

15. Do your emotional feelings interfere with your performance on a mathematics test more than on tests of similar importance in most other subjects?

Interfere more on a mathematics test	Midpoint	Interfere less on a mathematics test

Name _____

Class Hour _____

OPINIONNARE

Read each of the following questions carefully and answer only after you have considered all of the responses possible.

1. To find out exactly what I was expected to learn for each unit, I looked at: (circle one)

   a) The instructional objectives

   b) The sample test items

   c) The textbook

   d) Other (please specify) _____

2. In learning the mathematics in the course, the thing that helped me the most was: (choose one)

   a) The exercise sheets

   b) The classroom organization

   c) The textbook

   d) The teacher's lecture

   e) Other (please specify) _____

3. During this semester I think my overall attitude towards mathematics: (circle one)

   a) Has become more favorable

   b) Has remained the same

   c) Has become less favorable

   d) Don't know

4. What one thing has influenced your change in attitude?

Name _____

Class Hour _____

TREATMENT VERIFICATION FORM

The purpose of this questionnaire is to find out from you how your class was run this semester. In filling out the form it is important to keep in mind how your class was usually run during each of the five units of the course. Your cooperation in responding to each of the questions in an open and honest way will be greatly appreciated.

1. Grouping How was your class grouped for instruction? Did your teacher usually teach the class as a single group? Or did he teach the whole class part of the time, then divide the class into smaller groups and work with each of these groups?

2. Testing How was unit testing done in your class? Did you have only one chance to take a test on a unit? Or could you take a unit test a second time if you wanted?

3. Materials What kinds of instructional materials were made available to you for each of the units of the course? Was the textbook the only source you used? Were exercise sheets made available for each topic? Did you have a choice of which exercise sheets you would use to study with?

4. Handouts What kinds of handouts were given at the beginning of each unit of the course? What was included in the handout given at the beginning of each unit?

5. Opinion About Class How did you like the way this class was run? What did you like about the class? What didn't you like about the class?

6. Would you take another math class that was run the way yours was run?

YES NO

7. Please explain why you answered as you did in question 6 above. (Use other side if necessary).

BIBLIOGRAPHY

Aiken, James. A Comparison of Junior College Withdrawees. California: University of California, Los Angeles, Clearinghouse For Junior College Information (ED 023 389), August, 1968.

Aiken, L.R., and Dreger, Ralph M. "The Effect of Attitude on Performance in Mathematics." The Journal of Educational Psychology, 52: 19-24, February, 1961.

Altshuler, Thelma C. "Classroom Testing Practices in the Junior College." Junior College Research Review, Vol. 2, No. 1, September, 1967.

American Institutes for Research and Quincy Public Schools. Development and Evaluation of an Experimental Curriculum for the New Quincy (Mass.) Vocational Technical School, Pittsburgh, Pennsylvania: American Institutes for Research, September, 1965.

Bailey, Herman P. "A Study of The Effectiveness of Ability Grouping on Success in First Year Algebra." Dissertation Abstracts, 28: 3061A, February, 1968.

Bierden, James E. "Provisions for Individual Differences in Seventh Grade Mathematics Based on Grouping and Behavioral Objectives: An Exploratory Study." Unpublished Doctoral Dissertation, The University of Michigan, 1968.

_____. "Provisions for Individual Differences in Seventh Grade Mathematics Based on Grouping and Behavioral Objectives: An Exploratory Study." Dissertation Abstracts, 30: 196A-197A, July, 1969.

Bossone, Richard M. Remedial English Instruction in California Public Junior Colleges; An Analysis and Evaluation of Current Practices. California: California State Department of Education, September, 1966.

Brewer, Emery. "A Survey of Arithmetic Intra-Class Grouping Practices." The Arithmetic Teacher, 13: 310-314, April, 1966.

Campbell, Azzie Leedy. "A Comparison of The Effectiveness of Two Methods of Class Organization For The Teaching of Arithmetic in Junior High Schools." Dissertation Abstracts, 26: 813-814, August, 1965.

Carnegie Commission of Higher Education. The Open-Door Colleges. New York: McGraw-Hill Book Company, 1970.

Cohen, Arthur M. Dateline '79: Heretical Concepts for the Community College. California: Glencoe Press, 1969.

Cross, K. Patricia. The Junior College Student: A Research Description. Educational Testing Service in Cooperation With Center for Research and Development in Higher Education, University of California, Berkeley, and the American Association of Junior Colleges, 1968.

Dare, James L. "A Study of Remedial Programs in Public Two-Year Colleges." Dissertation Abstracts, 31: 3896A, February, 1971.

DeVault, M. Vere, and Kriewall, Thomas E. "Differentiation of Mathematics Instruction." The Sixty-ninth Yearbook of the National Society for the Study of Education, Part I, Chicago: University of Chicago Press, 1970.

Dewar, John A. "An Experiment in Intra-Class Grouping for Arithmetic Instruction in the Sixth Grade." Dissertation Abstracts, 22: 2247, January, 1962.

Ebeid, William T. "An Experimental Study of the Scheduled Classroom Use of Student Self Selected Materials in Teaching Junior High School Mathematics." Unpublished Doctoral Dissertation, The University of Michigan, 1964.

Ebel, Robert L. "Some Comments by Robert L. Ebel." School Review, 75: 261-266, Autumn, 1967.

Eisner, Elliot W. "Educational Objectives Help or Hindrance?" School Review, 75: 250-260, Autumn, 1967.

Ekstrom, Ruth B. "Experimental Studies of Homogeneous Grouping: A Critical Review." School Review, 69: 216-226, Summer, 1961.

Forbes, Jack E. "The Utility of Behavioral Objectives: A Source of Dangers and Difficulties." The Mathematics Teacher, 64: 687, 744-747, December, 1971.

Gagne, Robert M. "The Analysis of Instructional Objectives For The Design of Instruction." Teaching Machines and Programmed Learning. Ed. Robert Glaser. Washington, D.C.: National Educational Association, 1965.

Glaser, Robert. "An Individualized System." Science Education News, American Association for the Advancement of Science, Miscellaneous Publication No. 67-7: 1-3, June, 1967.

Gleazer, Edmund J., Jr., Ed. American Junior Colleges, Sixth Edition. Washington, D.C.: American Council on Education, 1963.

Goldberg, Miriam L., and Passow, A. Harry. "The Effects of Ability Grouping." Grouping in the Elementary School. Ed. Anne Morgenstein, New York: Pitman Publishing Corporation, 1966.

Goldberg, Miriam L.; Passow, A. Harry; and Justman, Joseph. The Effects of Ability Grouping. New York: Teachers College Press, 1966.

Hastings, J. Thomas. "Some Comments by J. Thomas Hastings." School Review, 75: 267-271, Autumn, 1967.

Herrick, Virgil E. "The Concept of Curriculum Design." Toward Improved Curriculum Theory, Ed. Virgil E. Herrick and Ralph W. Tyler, Supplementary Educational Monographs, No. 7, Chicago: University of Chicago Press, 1950.

Hoover, Kenneth H., and Hoover, Helen M. "A Plan for Grouping in the Secondary Classroom." Education, 88: 208-212, February, 1968.

Inlow, Gail M. The Emergent in Curriculum. New York: John Wiley & Sons, Inc., 1966.

Johnson, B. Lamar. Islands of Innovation Expanding: Changes in the Community College. California: Glencoe Press, 1969.

_____. "Toward Change and Improvement in Junior College Instruction." The Improvement of Junior College Instruction. Ed. B. Lamar Johnson. Occasional Report No. 15, Junior College Leadership Program, Graduate School of Education, University of California, Los Angeles, March, 1970.

Jones, R. Stewart, and Pingry, Robert E. "Individual Differences." Instruction in Arithmetic: Twenty-fifth Yearbook. Washington, D.C.: National Council of Teachers of Mathematics, 1960.

Kipps, Carol. "Arithmetic in College." Junior College Journal. 37: 38-44, December, 1966.

Komaridis, George V. "The Effects of Four Testing Systems on Schedules of Study, Test Performance, and Retention of Materials." Dissertation Abstracts, 32: 790A, August, 1971.

Lerch, Harold H. "Intra-Class Grouping for Arithmetic Instruction: Critique and Criteria." The Arithmetic Teacher, 8: 404-407, December, 1961.

MacDonald, James B. "Myths About Instruction." Educational Leadership, 22: 613-614, May, 1965.

McBride, Ralph B. "Flexible Grouping Based on Achievement of Behavioral Objectives in a Mathematics Course For Prospective Elementary Teachers." Unpublished Doctoral Dissertation, The University of Michigan, 1970.

_____. "Flexible Grouping Based on Achievement of Behavioral Objectives in a Mathematics Course For Prospective Elementary Teachers." Dissertation Abstracts, 31: 4040A-4041A, February, 1971.

McMeen, George H. "Differentiating Arithmetic Instruction for Various Levels of Achievement." The Arithmetic Teacher, 6: 113-120, April, 1959.

McNemar, Quinn. Psychological Statistics. New York: John Wiley & Sons, Inc., 1969.

Mager, Robert F. Preparing Instructional Objectives. Palo Alto: Fearon Publishers, 1962.

Mandler, George, and Cowen, Judith E. "Test Anxiety Questionnaires." Journal of Consulting Psychology, 22: 228-229, June, 1958.

Marks, E. "Student Perceptions of College Persistence, and their Intellective Personality and Performance Correlates." Journal of Educational Psychology, 58: 210-221, August, 1967.

Medsker, Leland L. The Junior College: Progress and Prospect. New York: McGraw-Hill Book Company, Inc., 1960.

Miles, David T., and Robinson, Robert E. "Behavioral Objectives: An Even Closer Look." Educational Technology, June, 1971, pp. 39-44.

Mitzel, Margaret A. "An Experimental Study of the Effect of Specific Experiences on Test-Retest Correlations." Dissertation Abstract, 31: 611A, August, 1970.

Moench, Laurel. "Individualized Practice in Arithmetic - A Pilot Study." The Arithmetic Teacher, 9: 321-329, January, 1962.

Mortlock, Roland S. "Provisions for Individual Differences in Eleventh Grade Mathematics Using Flexible Grouping Based on Achievement of Behavioral Objectives: An Exploratory Study." Unpublished Doctoral Dissertation, The University of Michigan, 1969.

National Education Association, Research Division. "Ability Grouping." National Education Association Research Bulletin, 46: 74-76, October, 1968.

Olson, Willard C. Child Development. Boston: D.C. Heath and Company, 1959.

Passow, A. Harry. "The Maze of the Research on Ability Grouping." Educational Forum, 26: 281-288, March, 1962.

Payne, Arlene. "Some Comments by Arlene Payne." School Review, 75: 272-277, Autumn, 1967.

Perkins, Ruth M. "Ways of Providing for Individual Differences in Elementary Mathematics." Unpublished Doctoral Dissertation, The University of Michigan, 1967.

Perry, Donald. "An Experiment in Teaching Elementary Algebra." The Two Year College Journal, 2: 40-46, Fall, 1971.

Platz, Eugene F. "The Effectiveness of Ability Grouping in General Science Classes." Dissertation Abstracts, 26: 1459, September, 1965.

Popham, W. James. "A Bank of Instructional Objectives For The Junior College." The Improvement of Junior College Instruction. Ed. B. Lamar Johnson. Occasional Report No. 15, Junior College Leadership Program, Graduate School of Education, University of California, Los Angeles, March, 1970.

Raths, James. "Another Look at Behavioral Objectives." CAREL Report, University of Maryland, July, 1967.

Reeve, William D. "The Problem of Varying Abilities Among Students in Mathematics." The Mathematic Teacher. 49: 70-78, February, 1956.

Reynolds, James W. The Comprehensive Junior College Curriculum. California: McCutchan Publishing Corporation, 1969.

_____. The Junior College. New York: The Center for Applied Research in Education, Inc., 1965.

Richardson, Richard D., Jr. Student's Guide To The Two Year College. New Jersey: Prentice Hall, Inc., 1968.

Roueche, John E. "Gaps and Overlaps in Institutional Research." Junior College Journal, 38: 20-23, November, 1967.

_____. Salvage, Redirection, or Custody? Remedial Education in the Community Junior College. Washington, D.C.: American Association of Junior Colleges, 1968.

_____. and Boggs, John R. Junior College Institutional Research: The State of The Art, Washington, D.C.: American Association of Junior Colleges, 1968.

_____; Baker, George A., III; and Bronwell, Richard L. Accountability and the Community College: Directions for the 70's. Washington, D.C.: American Association of Junior Colleges, January, 1971.

Schenz, Robert F. "An Investigation of Junior College Courses and Curricula For Students With Low Ability." Unpublished Doctoral Dissertation, Los Angeles, California: University of California, School of Education, 1963.

Seibel, Dean W. Testing Practices and Problems in Junior Colleges - A Survey, New Jersey: Educational Testing Service, Field Studies Report, September, 1966.

Shane, Harold G. "Grouping in the Elementary School." Phi Delta Kappan, 41: 313-319, April, 1960.

Silberman, C.S. Crisis in the Classroom: The Remaking of American Education. New York: Random House, 1970.

Smith, Jerry E. "How To Succeed in Mathematics By Really Trying." School and Community, 56: 6-7, January, 1970.

Smith, William M. "The Effects of Intra-Class Ability Grouping on Arithmetic Achievement in Grades Two Through Five." Dissertation Abstracts, 21: 563-564, September, 1960.

Spence, Eugene S. "Intra-Class Grouping of Pupils for Instruction in Arithmetic in the Intermediate Grades of Elementary School." Dissertation Abstracts, 19: 1682, January, 1959.

Tyler, Ralph W. Basic Principles of Curriculum and Instruction. Chicago: University of Chicago Press, 1951.

_____. Basic Principles of Curriculum and Instruction. Chicago: University of Chicago Press, 1969.

_____. "Some Persistent Questions on the Defining of Objectives." Defining Educational Objectives. Ed. C.M. Lindvall, Pittsburgh: University of Pittsburgh Press, 1964.

_____. and Brownell, William A. "Facts and Issues: A Concluding Statement." Individualizing Instruction. Ed. Nelson B. Henry. Chicago: The University of Chicago Press, 1962.

Wallen, Norman E., and Vowles, Robert O. "The Effect of Intra-Class Grouping on Arithmetic Achievement in the Sixth Grade." Journal of Educational Psychology, 51: 159-163, March, 1960.

Weaver, J. Fred. "Differential Instruction and School-Class Organization for Mathematical Learning Within the Elementary Grades." The Arithmetic Teacher, 13: 495-506, October, 1966.

Willcutt, Robert E. "Ability Grouping by Content Topics in Junior High School Mathematics." The Journal of Educational Research, 63: 152-156, December, 1969.

Withnell, Melvin C. "A Comparison of the Mathematical Understandings of Prospective Elementary Teachers in Colleges Having Different Mathematics Requirements." Unpublished Doctoral Dissertation, The University of Michigan, 1967.