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A simple memristive jerk system

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Abstract: In this paper, a simple memristive chaotic jerk system with one variable to represent the internal state is found. The proposed equilibria-free memristive system yields hidden chaotic oscillation in a narrow parameter space. A circuit modeling the jerk system is also constructed and shows agreement with the predicted oscillation. To our knowledge, the new memristive jerk system is one of the algebraic simplest memristive chaotic systems.

1. Introduction

Memristors have attracted great interest in engineering for their potential applications [1-5]. They have been used as a dynamic element to generate chaotic signals. The presence of a memristor usually leads to a 4-D system [6-10], although only a 3-D system is required for chaos. Several 3-D memristive chaotic systems have been reported [11-15], but the memristor is implemented with a complicated operational-amplifier-based equivalent device.

Jerk systems are a simple type of dynamical system that can generate chaos [16-20]. Such a compact structure is composed of a couple of integral operation units in series. Some jerk systems are chaotic when they either contain a nonlinearity from quadratic terms [16-18], an exponential function [19] or a cubic term [20]. Other jerk systems that produce chaos based on the memristor are all 4-D [21, 22]. The novelty of this work is that compared with the prior works in the following we aim to construct a totally 3-D jerk memristive circuit. The most important challenge in this work is that how to introduce a suitable memristor to break the existing oscillation in a 2-D structure and finally to bring chaos.

Furthermore, many memristive systems exhibit chaotic oscillation combined with other states because of the integral effect from the memristor [6-10, 21-25]. Unlike these memristive jerk systems, a second-order jerk structure with a memristor is explored in this paper, which is both simple and robust for giving chaos. In Sec. 2, the model is given and analyzed with basic dynamical analysis. In Sec. 3, the circuit is built for proving the theoretical analysis. Finally a short conclusion is made to summarize the whole work.

2. System model

A simple chaotic jerk oscillator containing a memristor as one of the state variables was found by an exhaustive computer search based on Euler method. Suppose there is a 2-D jerk structure $\dot{y} = z, \dot{z} = f(y, z)$, and introduce a flux-controlled memductance $W(x)$ in it, the following system is found for chaos producing:

$$\begin{cases} \dot{x} = y, \\ \dot{y} = z, \\ \dot{z} = -z - az^2 - W(x)y + b. \end{cases} \tag{1}$$

where a flux-controlled memductance $W(x) = 1.3x^2\text{-}1$ is introduced in the z-dot equation. Here the variables y and z are the external system variables, and x is the internal variable in memristor and indicates the magnetic flux. When $a = 0.239$ and $b = 1$, system (1) produces chaos with Lyapunov exponents (0.0529, 0, -1.0529) after a time of t = 2e7 and a corresponding Kaplan-Yorke dimension of D_{KY} = 2.0502 for initial condition (0, -2, -2), as shown in Fig. 1, whose basins of attraction (in the z = 0 plane) is shown in Fig. 2.

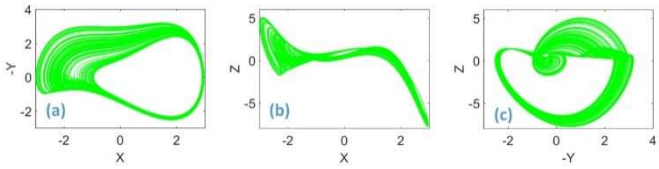


Fig. 1 *Strange attractor projections of system (1) with a = 0.239, b = 1 and initial condition (x_0, y_0, z_0) = (0, -2, -2)* (a) *x-(-y) plane,* (b) *x-z plane,* (c) *(-y)-z plane.*

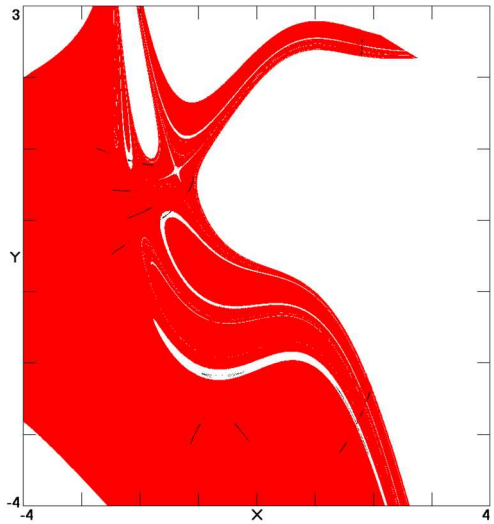


Fig. 2 *Basins of attraction of system (1) in the plane of z = 0*

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This system is fairly delicate with chaos only in a narrow range of the parameter space, as indicated in Fig. 3. The internal variable x comes from the integration of the system state variable y. System (1) is asymmetric with a speed of volume contraction determined by a derivative proposed by Lie,

$$\nabla V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} = -1 - 2az\ .$$

And also system (1) has no equilibria [26-30], and therefore the attractor is hidden [31-35].

More interestingly, unlike other memristive systems [8-10], system (1) outputs relatively stable oscillations except when switching between chaos and sliding initial values which agree with the plots of the dynamical region and basins of attraction. To further verify this, the solution based on offset boosting under a fixed initial condition is used for diagnosing multistability [39-40]. Taking the offset boosting d in the dimension $y \to y + d$, it is shown that when the offset d varies in [-5, 5], except the long transient process system (1) remains chaotic unless it is dragged sliding with the initial condition, as shown in Fig. 4.

The embedded memristor is defined as

$$\begin{cases} \dot{x} = y, \\ W(x) = 1.3x^2 - 1, \\ i_M = W(x)y. \end{cases} \tag{2}$$

The flux-dependent memductance is related to the internal variable x, which is of quadratic degree [36-38] and is determined by the system variable y,

$$W(x) = 1.3x^2 - 1 = 1.3(\int_{-\infty}^{t} y ds)^2 - 1 = 1.3(\int_{0}^{t} y ds)^2 - 1 + W_0 \tag{3}$$

where $W_0 = 1.3((\int_{-\infty}^{t} y ds)^2 - (\int_{0}^{t} y\, \mathrm{d}s)^2)$. Element memductance and the corresponding theoretical loop of pinched hysteresis are plotted in Fig. 5.

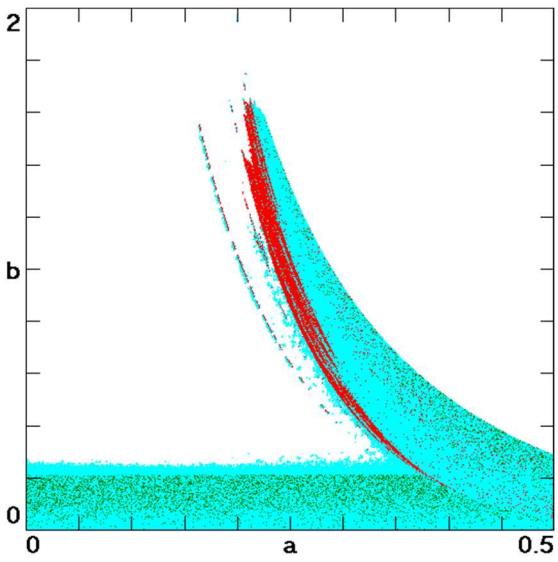


Fig. 3 *Dynamical regions in the parameter space of a, b, red means chaos, and cyan means limit cycle*

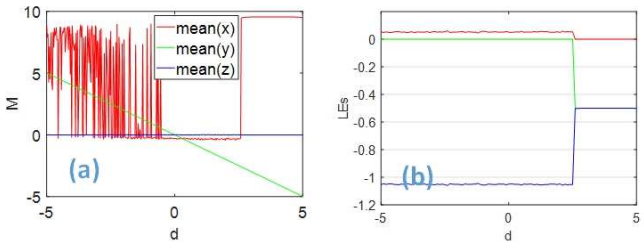


Fig. 4 *Dynamical behaviors in system (1) with a = 0.239, b = 1, and initial condition (x_0, y_0, z_0) = (0, -2, -2), when the offset parameter d varies within [-5, 5] a.* average value of the variables,
b. Lyapunov exponents.

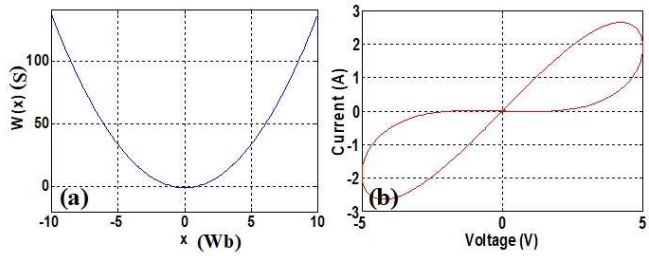


Fig. 5 *The memductance and pinched hysteresis loop*

3. Jerk circuit implementation

For obtaining an analog circuit to realize system (1), it is a relatively easy way to introduce a memristor in the operational amplifier-based integration circuit. Firstly we construct a 2-D Jerk main structure. Secondly, the equivalent circuit is designed for the applied memristor resorting to another amplifier-based integration element. The basic principle is based on the characteristics of virtual break and virtual short of an operational amplifier. An analog circuit based on Eq. (1) is designed as shown in Fig. 6, and according to the Kirchhoff law, the circuit equations can be written as follows,

$$\begin{cases} \dot{y} = \dfrac{1}{R_3 C_2} z, \\ \dot{z} = -\dfrac{z}{R_1 C_1} - \dfrac{z^2}{R_2 C_1} - \dfrac{W(x)y}{C_1} + \dfrac{V_b}{R_0 C_1}. \end{cases} \tag{4}$$

where the memristor $W(x)$ is equivalent to the circuit simulator as shown in Fig. 7,

$$\begin{cases} i_M = W(x)y, \\ W(x) = \dfrac{1}{R_b} x^2 - \dfrac{1}{R_c}, \\ \dot{x} = \dfrac{1}{R_a C_a} y. \end{cases} \tag{5}$$

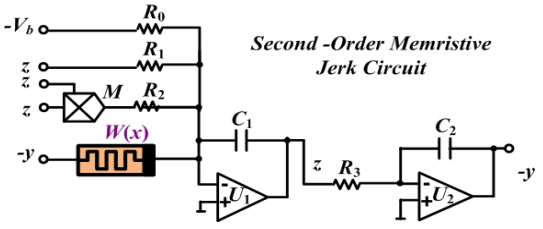


Fig. 6 *Circuit schematic of the memristive jerk oscillator*

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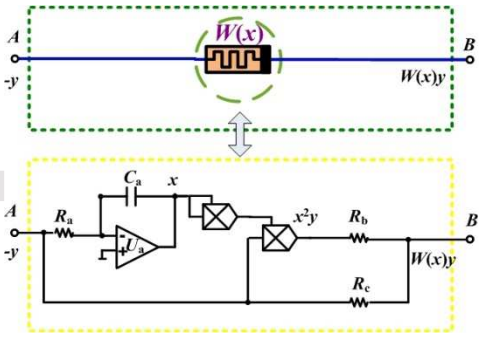


Fig. 7 *Equivalent element of the flux-dependent memristor*

In this process, a 2-D jerk structure provides a simple structure for the proposed system. The circuit structure contains two parts based on the integration including addition, and subtraction of the system variables y and z according to Eq. (1). Here the analog multiplier AD633/AD is applied to realize the nonlinear product operation while the operational amplifier OPA404/BB combined with its peripheral circuit is used to construct the addition, inversion, and integration. The memductance $W(x) = 1.3x^2 - 1$ is realized through the circuit parameters C_a = 100nF, R_a=10kΩ, R_b = 7.69kΩand R_c =10kΩ. The system parameters a = 0.239, b = 1 are realized through the circuit parameters $R_1 = R_3$ = 10kΩ, R_2 = 41.8kΩ, R_0 = 30kΩ, V_b = 3V. The capacitor $C_1 = C_2$ = 100 nF are selected for establishing a robust phase trajectory which only rescales the time of the oscillation. The equations used in the circuit implementation contain different constants with those from System (1) owing to amplitude and time rescaling, which is normal in circuit implementation. Figure 8 displays a plot of the experimental constraints of memductance and inherent relation of pinched hysteresis, while Fig. 9 displays the phase portraits observed in the oscilloscope.

The experimental constraints of memductance, inherent pinched hysteresis effect, and the experimental phase portraits agree well with the numerical simulation proving the system dynamics and the effectiveness of hardware circuit. As a main element, the equivalent memristor poses a great effect on the performance of the jerk system, which is dominantly determined by two analog multipliers and one operational amplifier. These three components define the memristor applied in this work, and therefore some other physical memristor models (e.g. HP memristor) cannot guarantee chaos in the 3-D jerk structure.

4. Conclusions

By introducing a memristor into a second-order jerk structure, chaotic oscillation is found in a 3-D jerk system. The proposed simple memristive jerk system has only six terms while without any equilibria, one of which is quadratic. Circuit experiments show the same oscillation agreeing with the numerical simulation. When the flux-controlled memductance is revised as $W(x) = 1.3|x| - 1$, minor parameter adjustment (a = 0.432, b = 1) can still recover chaos with Lyapunov exponents (0.0328, 0, -1.0332) and a corresponding attractor dimension of D_{KY} = 2.0321, which simplifies the circuit realization. Compared to other memristive systems [41, 42], this system is also unique for its robust chaotic oscillation although it is hidden [43, 44]. This feature is attractive for its application in chaos-based communication or image encryption. Future work on this circuit can investigate introducing other memristors such as the HP memristor into this proposed jerk structure for chaos.

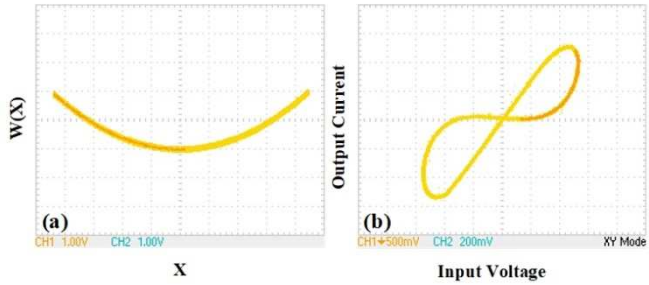


Fig. 8 *Experimental value of memductance and inherent constraints of pinched hysteresis*

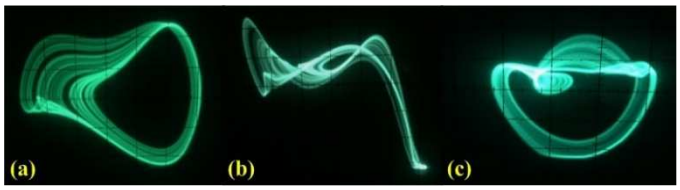


Fig. 9 *Strange attractor projections of system (6) shown in oscilloscope (a) x - (-y) (b) x - z (c) (-y) – z* (0.5V/div)

5. Acknowledgments

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