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BASES FOR PREFERENCES AMONG THREE-OUTCOME BETS.

University of Michigan, Ph.D., 1962
Psychology, experimental

University Microfilms, Inc., Ann Arbor, Michigan

BASES FOR PREFERENCES AMONG THREE-OUTCOME BETS

by
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A dissertation submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy in the
University of Michigan
1962

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ACKNOWLEDGEMENTS

This research was supported in part by the United States Air Force under Contract AF 49(638)-769, monitored by the Air Force Office of Scientific Research of the Air Research and Development Command.

I am especially grateful to my chairman, Associate Professor Ward Edwards, for his unceasing guidance and encouragement of my work.

My thanks, also, to Alice Kugelman and Phyllis Spiegel for their clerical and administrative assistance.

Finally, my gratitude to my husband Edward, for his invaluable patience and help.

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INTRODUCTION

Research on decision making has focused in recent years on a model called the subjectively expected utility maximization model (SEU model). This model states that choices among bets can be predicted from a function similar to expected value, but employing subjective rather than objective values for both money and probability (see Edwards, 1955, 1961).

Support for the SEU model has come from research exploring the subjective money measure, called utility (e.g., Mosteller & Nogee, 1951), or the subjective probability measure (e.g., Preston & Baratta, 1948; Shuford, 1959), or both measures at once (Edwards, 1955; Davidson, Suppes & Siegel, 1957). Many of these experiments, however, are based on limited sets of stimuli, and fail to test whether the curves obtained for subjective probability and utility will predict choices in other situations, among quite different bets.

Recent criticism of the SEU model has come from Coombs and Pruitt (1960), who feel that the SEU model is undesirable because of its relative invulnerability to experimental refutation. Utility and subjective probability curves may be found to fit any reasonably orderly set of data. Accordingly, they argue against the SEU model not by collecting evidence which refutes it, but by proposing an alternative set of variables which might be used as the bases for a new model.

Viewing a bet as a probability-density distribution, Coombs and Pruitt point out that the value, or Expected Value (EV) of the bet is the first of a series of moment functions which uniquely describe the bet. Other moment functions are Variance (V), Skewness (Sk), and Kurtosis (K). The equations for these functions are shown in Table 1. Since EV has long been recognized as an important, though not sufficient, determiner of choices among bets, and since several authors (Allais, 1953; Edwards, 1954b; Royden, Suppes & Walsh, 1959) have proposed that V, too, is important, Coombs and Pruitt proposed the moment functions as a possible set of variables useful in predicting choices among bets.

A decision between the moment functions and the SEU model, however, is hampered by a lack of systematic, atheoretical information about the effects of a number of bet variables upon Ss' choices. This experiment is designed to explore the relevance and relative importance of the following bet parameters:

EV: There is no question that the value of a bet is a primary determiner of Ss' responses. Indeed, so sure are we that Ss attend to EV that failure to find a substantial EV effect would invalidate the study.

V: Coombs and Pruitt (1960) found that Ss exhibit systematic preference orderings among bets varying in V.

Sk: Skewness may reflect a facet of the risk of a bet, but its intuitive meaning as applied to bets is not clear.

K: No research has ever included K as an independent variable, nor is there any apparent intuitive meaning to it as a bet variable. It is included merely to complete the commonly accepted group of

Table 1

Formulas for EV, Variance and Skewness

Name	Moment Notation	Definitional Formula	A Computational Formula for Two-outcome bets	A Computational Formula for n-outcome bets
EV	μ_1	$\sum_i p_i \$_i$	$pA + qB$	$\sum_i p_i \$_i$
Variance	$\mu_{2,c}$	$\sum_i p_i(\$_i - \mu_i)^2$	$pq(A-B)^2$	$\sum_{i>j} p_i p_j(\$_i - \$_j)^2$
Skewness	$\frac{\mu_{3,c}}{[\mu_{2,c}]^{3/2}}$	$\frac{\sum_i p_i(\$_i - \mu_i)^3}{[\mu_{2,c}]^{3/2}}$	$\frac{1 - 2p}{[pq]^{1/2}}$	not known
Kurtosis	$\frac{\mu_{4,c}}{[\mu_{2,c}]^2}$	$\frac{\sum_i p_i(\$_i - \mu_2)^4}{[\mu_{2,c}]^2}$	$\frac{1}{pq} - 3$	$\frac{\sum_{i>j} p_i p_j(\$_i - \$_j)^4}{\left[\sum_{i>j} p_i p_j(\$_i - \$_j)^2\right]^2} - 3$

moment functions.

Probability: Probability preferences have been demonstrated by Edwards (1953) and by Coombs and Pruitt (1960). These studies establish that Ss exhibit systematic and consistent preferences for certain probabilities of winning, while avoiding others.

Outcome Sum: Outcome sum is simply the algebraic sum of all the outcomes of a bet. This variable is explored because of the possibility that Ss may not pay as much attention to probabilities as they should. If Ss totally disregarded probabilities, attending only to the money outcomes, the sum of the outcomes would correlate highly with Ss' responses.

Pattern: Pattern refers to the combination of wins and losses within a single bet. Subjects might react differently to a bet in which all outcomes are positive (winning) than to a bet in which one outcome is a loss, even if the bets were equated for EV. It was hypothesized that Ss would prefer bets with more winning outcomes or with fewer losing outcomes.

COMPUTATIONS OF BETS

In order to evaluate properly the effectiveness of the moment function variables, it is important that they be mutually independent in the experimental design. This has not been true in previous research. Edwards (1953), in establishing probability preferences, completely confounded this variable with V. Coombs and Pruitt (1960), avoiding this difficulty, confounded probability with Sk. Such confounding is inevitable when bets having only two outcomes are used, as may be seen from the formulae for these variables for a two-outcome bet, shown in Table 1. This study uses three-outcome bets in order to permit independent variation of probabilities, EV, V, and Sk. The problems resulting from confounded variables, inevitable when using two-outcome bets, are thus avoided for these variables, which constitute the major independent variables of this study.

When two-outcome bets are used, a bet (or pair of bets) is uniquely determined by specifying the probabilities, EV, and V. The set of equations used to determine the bet's two outcomes may be directly solved by quadratic methods, and, except when the probabilities both equal 1/2, provide two solutions. These two bets differ in the sign, but not the amount, of Sk.

When three-outcome bets are used, the mathematical situation becomes more complicated. There exist an infinite set of bets all having the same probabilities, EV, and V. Additional specification of Sk uses up the degrees of freedom. Unfortunately, the addition of the equation for Sk creates a system of equations for which there is

no general solution, so that approximation methods are necessary to find the bets so specified.[1] Nor is there any assurance, as there was in the two-outcome case, that a solution exists for any particular values of the independent variables. While values for the probabilities, EV, and V may be chosen freely, there are constraints on the choice of Sk, constraints determined by choice of probabilities. The probabilities in this study are limited to units of eighths. These units provide an opportunity to study the existence of probability preferences, and were used by Edwards (1953) and others.

When probabilities in units of eighths are used, no bets exist when Sk is outside the interval ±2.26. Within this range, from zero to four bets exist having the same probabilities, EV, V and Sk. Table 2 shows the maximum number of different bets which exist for each of several Sk and probability combinations, when EV and V are held constant.

Table 2

Number of Different Bets for

Levels of Skewness and Probability

	Skewness				
Probability	-1.8	-.9	0	+.9	+1.8
1/8, 1/8, 6/8	2	1	1	1	2
1/8, 2/8, 5/8	2	4	2	4	2
1/8, 3/8, 4/8	2	2	2	2	2
2/8, 2/8, 4/8	0	2	1	2	0
2/8, 3/8, 3/8	0	1	3	1	0

[1] I am grateful to Dr. L. J. Savage for suggesting the graphic method of solution used to obtain the bets in this study. The method is described in Appendix I.

METHOD

Levels of the independent variables. The stimuli chosen for use in the experiment were 182 three-outcome bets and seven two-outcome bets. The three-outcome bets were drawn from the following levels of the independent variables.

Probability: All five possible probability triples based on the unit of an eighth were used. The number of different bets for each level is shown in parentheses: 1/8, 1/8, 6/8 (16); 1/8, 2/8, 5/8 (96); 1/8, 3/8, 4/8 (28); 2/8, 2/8, 4/8 (18) and 2/8, 3/8, 3/8 (24).

EV, three levels: -83¢ (39), zero (104) and +83¢ (39).

V, four levels: .25 (107), 1.00 (25), 2.25 (25) and 4.00 (25). These levels were chosen so that the outcomes of bets from successive V levels differed by a constant amount, and so that the largest single win or loss would be about $5.00.

Sk, five levels: -1.8(16), -.9 (42), 0 (66), +.9 (42) and +1.8 (16). The maximum and minimum Sk possible with the probabilities here used are ±2.26. Lesser values were chosen so that no two outcomes of a single bet would be nearly equal. In the bets used, the smallest difference between two outcomes in a single bet was 23¢.

Other variables represented in the experiment were:

Kurtosis: This variable ranged 1.3 to 5.2. K remains constant with changes in EV and V, and with changes in the sign of Sk.

Outcome Sum: This is the simple arithmetic sum of a bet's outcomes, disregarding probability. It varied from +$4.24 to -$4.24.

Pattern: This variable refers to the signs of the outcomes, win (+), lose (-), or win nothing (0). For three-outcome bets, there are seven possible patterns, shown in Table 3, along with the number

Table 3

Pattern of Bets

Pattern:	+++	++0	++-	+0-	+--	0--	---
Number of Bets:	17	7	61	12	61	7	17

of each represented in the experiment. Bet pattern is partially determined by the EV of the bet. In the present study, bets with positive EV showed patterns +++, ++0 and ++-; zero EV bets showed patterns ++-, +0- and +--, and negative EV bets showed patterns +--, 0-- and ---.

Number of bets. The number of stimuli chosen for use in the experiment was determined by the following considerations:

When probability and Sk are varied, while EV and V are held constant, 41 different bets may be calculated (see Table 2). A linear transformation on any one of these bets may be made to produce a new bet having the same probabilities and Sk, but new EV and V values. Thus when EV and V are varied, a total design of 492 bets (41 x 3EV x 4V is produced. When five replications of each bet are given, this design provides a total of 2460 stimuli. Presenting this many stimuli to each S would be too lengthy a process; one could not expect even approximate stability of preference over the time needed for five replications. Accordingly, a smaller subset of the total design was chosen for the experiment, consisting of 182 different bets. This

subset was obtained by holding constant every possible pair of variables in turn from the four: probability, EV, V and Sk, and varying fully the other two. A few additional bets were included which represent rare patterns. Appendix III shows the bets used in the experiment.

The principal loss which results from using the subset of 182 bets instead of the total design of 492 bets is that this subset does not permit the study of second- and third-order interactions among the four major independent variables.

Seven two-outcome bets, with EV = 0 and V = .25, were also included. They had Sk values ranging from -2.26 to +2.26.

The total number of bets in the bidding experiment was 945 (182 + 7; 5 replications).

The design of this study is peculiarly incomplete. This is due primarily to the variable number of bets which can be computed for given values of probability and Sk. This incompleteness is inherent in the mathematics of moment functions and cannot be avoided. A further, systematic incompleteness has been introduced by reducing the number of three-outcome bets used from 492 to 182.

The bidding technique. Because of the large number of bets chosen for use in this experiment, the method of paired-comparisons often used in such gambling-preference studies is impractical, requiring too many choices for even a single replication. In addition, it was felt that Ss must actually play many of the bets during the experiment, in order to maintain motivation for several hours. It was hoped that the beneficial effects of increased interest in the experiment would more than offset possible changes in strategy due to unusually large wins or losses during the course of the game.

Accordingly, a single stimulus, real gambling, method of presentation was devised.

For each bet, S was asked to state the largest amount of money he would be willing to pay E in order to play the bet. If S did not like the bet well enough to pay for it, he was asked to state the smallest amount of money E had to pay him before S would play the bet. The S was told that this amount of money, his bid for the bet, would be compared with bids made by other Ss in former experiments, and that his bid would be accepted and the bet played only if it fell in the upper quarter of previous bids for the bet. This competitive bidding was a fiction used to motivate Ss to give careful responses; there was no such comparison group. The E actually accepted a random third of Ss' bids, so that S did not receive any consistent feedback useful to him in developing a strategy of play.

However, E never accepted a bid which was below a certain minimum level; if a bid scheduled for acceptance were below the minimum, the next acceptable bid thereafter was accepted instead. The minimum was 75¢ below EV, or, when the largest possible loss in the bet was less than $1.00 below EV, the minimum level was:

$$EV - .75\,(EV - \text{Largest Loss}).$$

This minimum bid was used to avoid accepting bids so low that S would become suspicious of the competitive bidding fiction.

When a bid was declared acceptable by E, the bid amount was exchanged and the bet was played. When the bid was refused, the bet was not played. Instructions to S are given in Appendix II.

Previous work with the bidding method (Slovic, Lichtenstein & Edwards, 1961) indicated that it provides powerful ratio scale data

which, when reduced to order relationships, is similar to data obtained by paired comparisons. Its greater economy over paired-comparison methods and its attractiveness to Ss further argue for its use.

Subjects. The Ss were twelve male volunteers, undergraduates from the University of Michigan. Some had familiarity with gambling (for example, horse racing) while others had never gambled before. None had had previous experience in comparable research.

Sessions. The Ss were run singly in seven 2-hour sessions spread over a 2-week period. Five of the sessions were devoted to the experimental bets; in each session Ss bid on 189 bets, a single complete replication. The bets were presented in a different random order for each session; all Ss saw the bets in the same sequence.

Two sessions, the fourth and last, were adjustment sessions. In these sessions, Ss' winnings and losses were carefully controlled to ensure that over the course of the seven sessions, all Ss won an average amount of money in the range $1.50 to $2.50 per hour of experimental time. Including an adjustment session in the middle as well as at the end of the experiment helped to keep Ss' winnings or losses within a reasonable range.

In the adjustment sessions Ss made paired-comparison choices between pairs of bets. They were told that E decided whether or not to play a bet by consulting a random arrangement of letters, "P" for play and "N" for not play. Actually, E made up the sequence of plays and not plays as the experiment progressed, so that Ss who needed to win money played more positive EV bets than negative EV bets, while Ss who had previously won excessive amounts played more losing bets. The proportion of bets played varied from session to session.

Bets for the Adjustment Sessions. For the two paired-comparison money adjustment sessions, a different but related set of bets was used. Seventy-eight pairs of three-outcome bets were prepared. Fifteen of these represented a complete pairing of six bets with EV = +$1.00, V = .36 and Sk = -.9. The bets differed in probability. Fifteen more were the same bets with all signs of outcomes reversed, so that EV = -$1.00, V = .36 and Sk = +.9. The remaining 48 stimuli were pairs in which the EV of the two bets differed by 14¢ to 21¢. Half of these pairs had EVs ranging from +$1.00 to +$1.37, V = .36, S = -.9; the others had EVs ranging from -$1.00 to -$1.37, V = .36 and Sk = +.9. These pairs were chosen to test whether differences in pattern would induce Ss to prefer a bet with lower EV. Thus for example, a bet with pattern ++0 and EV = +$1.00 was paired with a bet with pattern ++-, EV = +$1.16.

The 78 pairs of bets were presented twice to each S in each of the two adjustment sessions, so that four replications were available for analysis. The bets used are shown in Appendix III.

Financial exchanges. All Ss were told at the beginning of the experiment that they would be gambling for real money, and that while the game was biased in their favor, there was a possibility that they could lose some of their own money.

Financial exchanges during a session were made with poker chips. The Ss were advanced $30 in chips at the beginning of each session and required to return the same amount at the end of the session. In order to emphasize the reality of the gambling situation, financial settlements were made at the end of every session. The Ss were paid in cash, or, when their winnings were unusually large (over $10), were

given IOU's signed by E. When Ss lost they were asked to pay E their losses. An IOU signed by S was accepted whenever, as often happened, S said he had no money. These IOU's however, were always paid up at the beginning of the next session.

Bet display. The bets were presented to Ss on 4 x 6 in. cards; one bet per card for the bidding sessions, two bets per card for the paired-comparison sessions. The bets were shown as pie diagrams, with the amounts of money written inside the sectors of the pie, e.g., "win $1.00," "lose $.50," or "win nothing." The probabilities were nowhere written down, but shown only by the size of the pie sectors. Figure 1 shows a sample bet.

The bets were played by spinning a balanced aluminum pointer placed over a plastic disc eight inches in diameter. Five discs

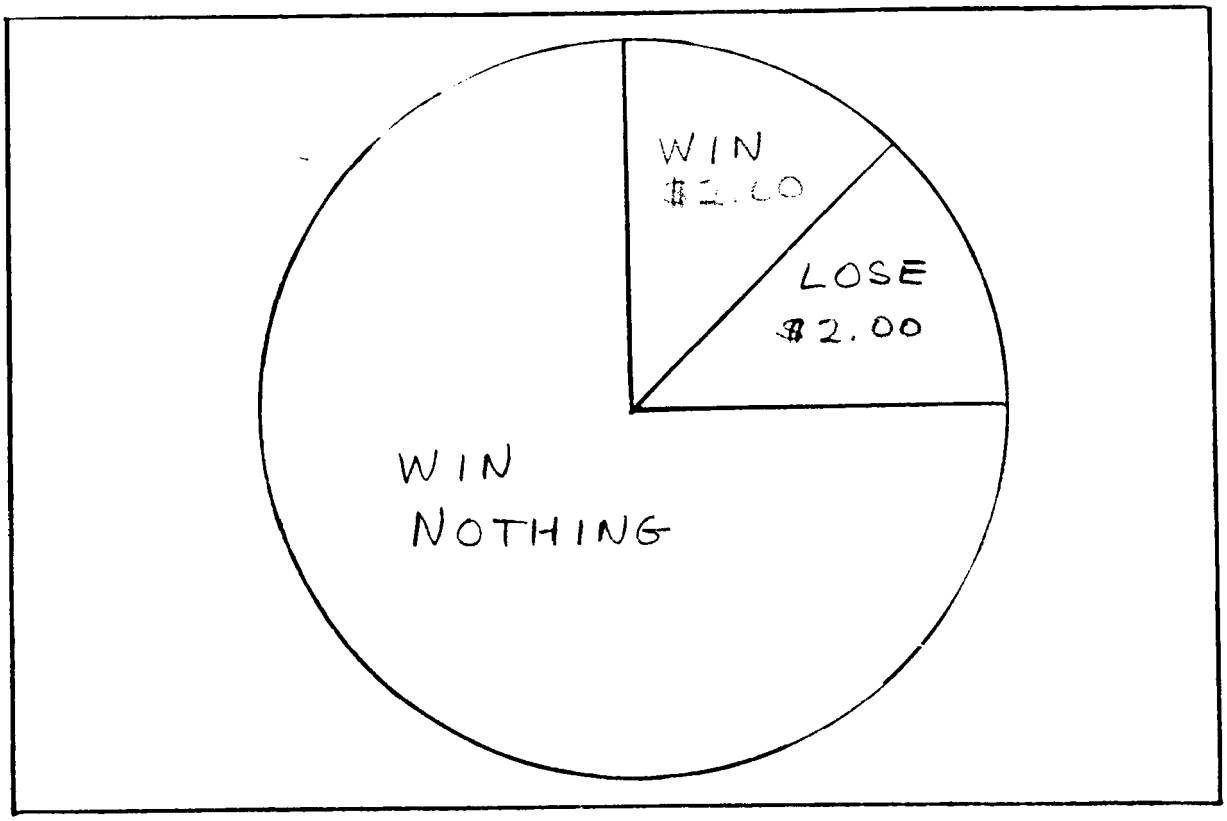


Fig. 1. Sample Bet as Shown to Ss.

corresponding to the five probability combinations for three-outcome bets, and four discs with two-outcome probabilities, were prepared by painting the backs in colors red, blue, and green, in pie-slice sectors. These colors corresponded to the colors with which the outcomes were written on the bet cards.

Before starting the first session, the discs were shown to S and the probabilities were explained, both in fractional and percentage terms.

Data analysis. Analysis of the bidding data was based on the amount of money each S bid for each replication of each bet. Because of the incompleteness of design, when means are computed across all the bets used in the study, certain levels of the independent variables are over-represented. For example, Table 4 shows the number of three-outcome bets broken down by EV and V.

Table 4

Number of Bets

Expected Value x Variance

	Variance				
Expected Value	.25	1.00	2.25	4.00	Total
+83¢	33	2	2	2	39
zero	41	21	21	21	104
-83¢	33	2	2	2	39
Total	107	25	25	25	182

In order to avoid drawing false conclusions from data in which several variables are unevenly represented, five small subsets of the data, each factorially complete, and each representing the variation of two independent variables while two are held constant, were subjected to analyses of variance. Thus any conclusion drawn from the total data may be checked in two or three narrower but purer settings. Each analysis of variance was of the form A x B x Subjects (Lindquist, 1953, p. 237). The subsets and their analyses are given in Appendix IV.

Analysis of the data also included a detailed look at the effects of real gambling on the betting responses. This was done to check whether Ss change their strategy because of large wins or losses in the course of the experiment. For this purpose the following variables were defined and studied:

Status: The amount of money S has won or lost during a given session (e.g., a given replication), at the time he faces a given bet.

Total Status: The amount of money S has won or lost over the entire experiment, at the time he faces a given bet.

Local Success: The sum over the three most recently played bets of the amount won or lost minus the amount bid.

Delay: The number of bets S has bid on since last he played a bet.

The mean amount bid for different levels of each of these variables was computed for each S separately. The results were also broken down by EV and V to check for possible interactions with these variables.

RESULTS

All Ss made clear by their comments during the experiment that they accepted the competitive bidding fiction and believed in the game; they were trying very hard to play the game wisely and to win money. When Ss were winning, they were happy, cheerful and "chatty." When Ss were losing they became silent, moody, and often rude, picking arguments over trivia.

Subjects bid carefully and with moderate consistency. The average standard deviation of the five bids a single S made to a single bet in five replications was 14¢, for all Ss and all bets. The most consistent S had a per-bet average standard deviation of 5¢; for the most variable S this figure was 23¢.

Financial transactions. Subjects won money in 46 of the 60 bidding sessions. The average outcome for all bidding sessions was a win of $5.71. The largest win for a single bidding session was $24.30; the largest loss was $11.57.

The largest cumulative win was $57.44; one S had won this much after 13 hours of play, before his final adjustment session. The largest cumulative loss was $23.21; one S had lost this much after three sessions.

Expected Value. All Ss were sensitive to the EV of the bets. Over the entire experiment, the average bid was 12¢ below EV. Table 5 shows the mean bid minus EV for the three different EV levels, along with the highest and lowest means among individual Ss. One S often bid above EV; 68 per cent of his bids averaged over replications were

Table 5

Effect of EV:

Mean Amount Bid

EV	Mean for 12 Ss	Highest	Lowest
-83¢	-$.91	-$.72	-$1.06
zero	-$.11	+$.20	-$ .32
+83¢	+$.67	+$.83	+$ .56

above EV. For the other 11 Ss this figure was 16 per cent.

Variance. The Ss were also sensitive to changes in V. The mean amounts bid for the four levels of V are shown in Table 6, for the entire data and for the three subsets subjected to analysis of variance (see Appendix IV).

The Ss bid more for the lower V bets than for the higher V bets. Thirty-six sets of data, obtained by analysing separately each

Table 6

Mean Amounts Bid for

Four Variance Levels

V	All three-outcome bets	Subset A: EV x V	Subset D: V x Sk	Subset E: V x Pr
.25	-$.07	-$.10	-$.05	-$.04
1.00	-$.12	-$.13	-$.09	-$.08
2.25	-$.17	-$.19	-$.15	-$.15
4.00	-$.25	-$.25	-$.22	-$.21

of 12 Ss and each of three EV levels, were examined for V preferences. Of these, 21 sets showed a simple ordering from low V to high (10 sets with zero EV, six with positive EV, and five with negative EV). Of the other 15 sets, 11 had no clear preference order but differed from the order shown in Table 6 by a few cents, while only four sets had clearly different orders. Only one S showed a marked preference for high V bets.

The analyses of variance made on subsets of the data, confirming the V effect, showed V highly significant, as was the V x S interaction.

An even more pronounced result with respect to V was an increase in the variability of amount bid as V increased. Table 7 shows the standard deviation of amount bid taken across all bets for each V and each EV level, computed for each S individually and averaged across Ss.

Table 7

Standard Deviations of Amount Bid

Classified by EV and V

EV \ V:	.25	1.00	2.25	4.00
-83¢	.17	.21	.32	.44
zero	.12	.21	.30	.39
+83¢	.15	.19	.21	.35

Further inspection of the data indicates that this increase was due primarily to an increase in the variability of amount bid for a single bet across replications, rather than an increase in variability

among the several bets of a single V level. Standard deviations as large as $.40 to $.80 for a single bet across replications for a single S were not uncommon at high V levels.

Skewness. Skewness had little or no effect on Ss' bidding. Inspection of the data revealed that no single S had a clear preference ordering among the five Sk levels, nor was there any small effect cumulative across Ss. Table 8 shows the mean amount bid for different levels of Sk for all the data, and for two factorially complete subsets of the data. No large or consistent preference ordering is seen.

Table 8

Mean Amounts Bid

for Five Skewness Levels

Sk	All three-outcome bets	Subset B: EV x Sk	Subset D: V x Sk
-1.8	-$.06	-$.08	-$.07
- .9	-$.09	-$.07	-$.13
zero	-$.13	-$.10	-$.13
+ .9	-$.12	-$.11	-$.15
+1.8	-$.12	-$.09	-$.15

The analyses of variance further indicated no significant first order interactions among the variables EV, V and Sk.

Kurtosis. Correlations between K and amount bid averaged over replications were computed for each S and for each EV level separately. These correlations ranged from -.32 to +.30, with a mean of +.02. More detailed analysis checking for possible curvilinear relationships, and separate analyses for each V level, indicated no relationship between K and amount bid.

Probability. There was no evidence for probability preferences for any single S, nor for all Ss cumulatively, either in the entire data or in the two relevant factorially complete subsets. Probability did not interact with Ss, nor with the other independent variables, EV, V and Sk. Table 9 shows the amounts bid for different probability levels, averaged over Ss, for all the data and for two subsets of the data.

Table 9

Mean Amounts Bid

for Five Probability Levels

Probability	All three-outcome bets	Subset C: EV x Pr	Subset E: V x Pr
1/8, 1/8, 6/8	-$.09	-$.07	-$.13
1/8, 2/8, 5/8	-$.13	-$.10	-$.13
1/8, 3/8, 4/8	-$.09	-$.09	-$.13
2/8, 2/8, 4/8	-$.08	-$.09	-$.08
2/8, 3/8, 3/8	-$.10	-$.08	-$.12

Pattern. The Ss showed no differences in amount bid attributable to pattern for negative EV and positive EV bets. For the zero EV bets, there was a faint trend in the expected direction. The mean amount bid for bets with pattern ++-, was -5¢; for bets with pattern +0-, -9¢; and for bets with pattern +--, -17¢. This trend was neither large nor consistent across Ss: only 7 Ss bid most for pattern ++-, and for only 7 Ss was the difference between their most preferred and least preferred patterns larger than their own per-bet average standard deviation.

Outcome Sum. Outcome sum was correlated with amount bid averaged over replications, for each EV level and each S separately. The average correlation was +.11; the range was -.39 to +.62. Four Ss might be said to show the effect of this variable; correlations broken down by EV for these four Ss are shown in Table 10. Further analyses indicated no curvilinear relationship and no interaction with V.

Table 10

Correlations between Outcome Sum and Mean Amount Bid

Subject Number	EV: -83¢	zero	+83¢
1	-.30*	-.29**	-.39**
4	-.03	+.62**	+.14
8	+.40**	+.52**	+.28*
10	+.38**	+.51**	+.36*

*significant at .05 level
**significant at .01 level

Bets with equal independent variables. The existence of multiple bets with constant probability, EV, V and Sk provides an excellent opportunity to test for bet preferences unrelated to the moment functions. These sets of bets were examined to see if differences in amount bid among the bets were large or interpretable. The design includes 12 such sets of four bets each and six sets of three bets each.

Figure 2 shows the amount bid averaged across Ss and replications for four of these sets of bets. All these bets have EV = 0, Sk = -.9, and probabilities 1/8, 2/8, 5/8. The V differs between sets but is the same within each set. For the set with V = .25, the bets are

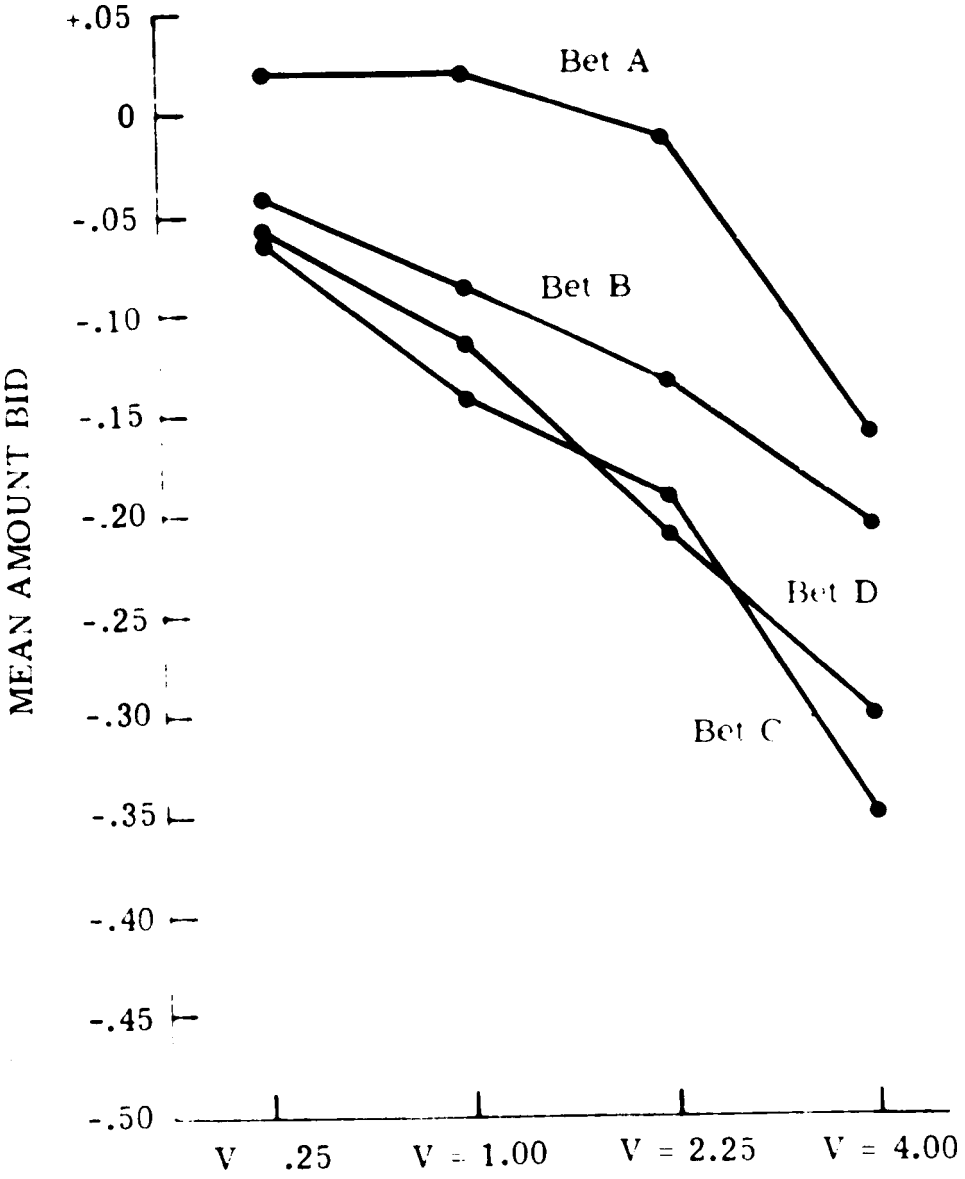


Fig. 2. Mean Amount Bid for sets of bets with equal independent variables. For all bets, probability = 1/8, 2/8, 5/8; EV = 0; Sk = -.9.

as follows:

A: 1/8 to win $ .61; 2/8 to lose $ .83; 5/8 to win $ .21.

B: 1/8 to lose $ .12; 2/8 to lose $ .83; 5/8 to win $ .35.

C: 1/8 to lose $1.11; 2/8 to win $ .62; 5/8 to lose $ .03.

D: 1/8 to lose $1.00; 2/8 to lose $ .41; 5/8 to win $ .36.

The other bets in Fig. 2 are constant multiples of these bets. For example, bet D in the set $\underline{V}$ = 4.00 is:

D: 1/8 to lose $4.00; 2/8 to lose $1.66; 5/8 to win $1.46.

Figure 2 shows that for all $\underline{V}$ levels, bet A was preferred over bet B, and bet B preferred over bets C and D. The downward trend of each curve across $\underline{V}$ indicates the effect of $\underline{V}$.

Two other sets of bets, not shown in Fig. 2, but related to these bets, had $\underline{EV}$ = ±83¢, $\underline{V}$ = .25; $\underline{Sk}$ = -.9, and probabilities 1/8, 2/8, 5/8. They showed the same trend as seen in Fig. 2 .

Figure 3 shows four comparable sets of bets with $\underline{EV}$ = 0, $\underline{Sk}$ = +.9, and probabilities 1/8, 2/8, 5/8. These bets are mirror images of those shown in Fig. 2; that is, all wins are changed to losses, all losses to wins. Thus for Fig. 3, the A bet for $\underline{V}$ = .25 reads:

A: 1/8 to lose $ .61; 2/8 to win $ .83; 5/8 to lose $ .21.

Figure 3 shows consistent preference orderings in the reverse direction from the mirror-image bets of Fig. 2. Two additional sets of bets, not shown in Fig. 2, had $\underline{EV}$ = ±83¢, $\underline{V}$ = .25, $\underline{Sk}$ = ±.9, and probabilities 1/8, 2/8, 5/8, and showed the same trend as seen in Fig. 3.

Figure 4 shows four sets of three bets each. All bets have $\underline{EV}$ = 0, $\underline{Sk}$ = 0, and probabilities 2/8, 3/8, 3/8. Again, $\underline{V}$ is a constant

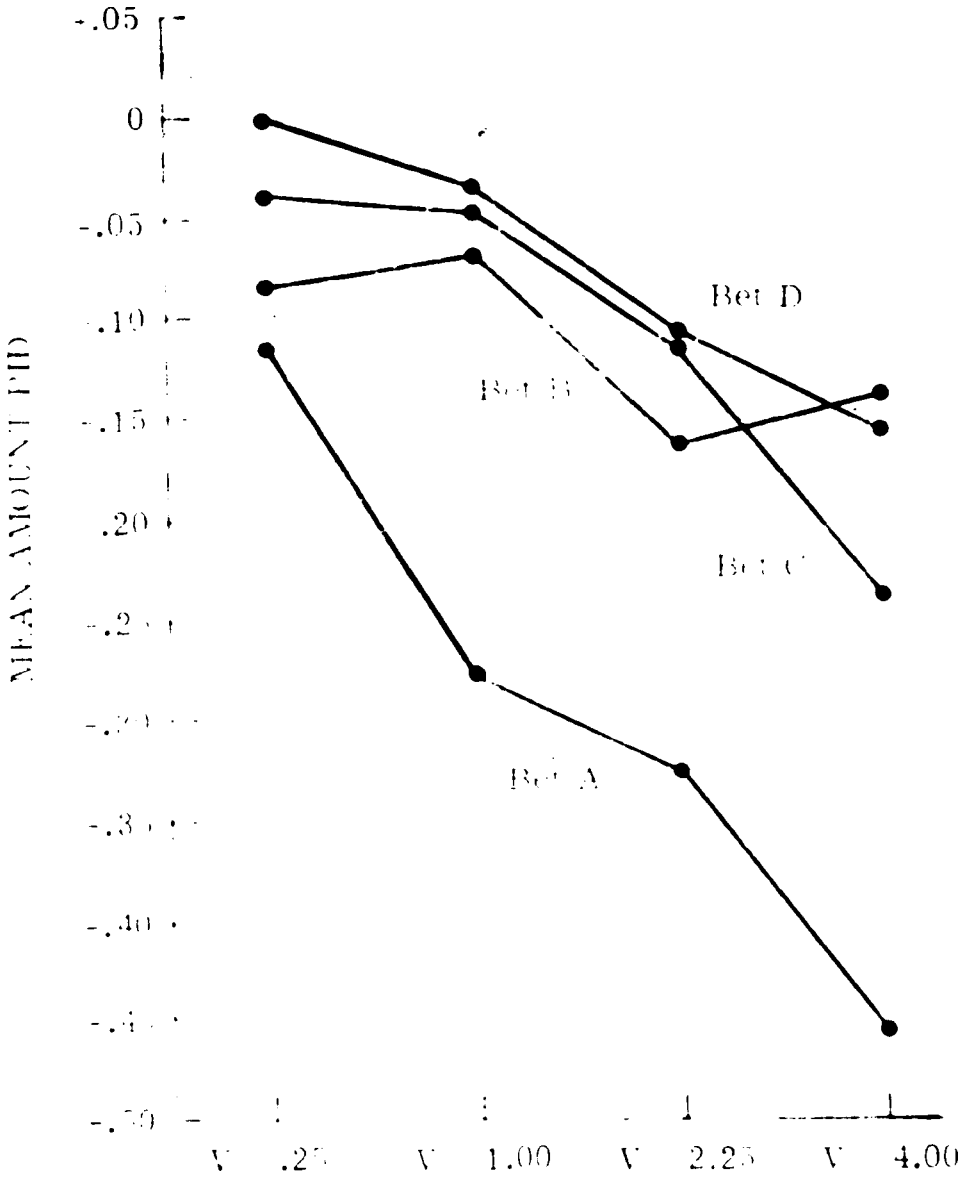


Fig. 3. Mean Amount Bid for sets of bets with equal independent variables. For all bets, probability = 1/8, 2/8, 5/8; EV = 0; Sk = +.9.

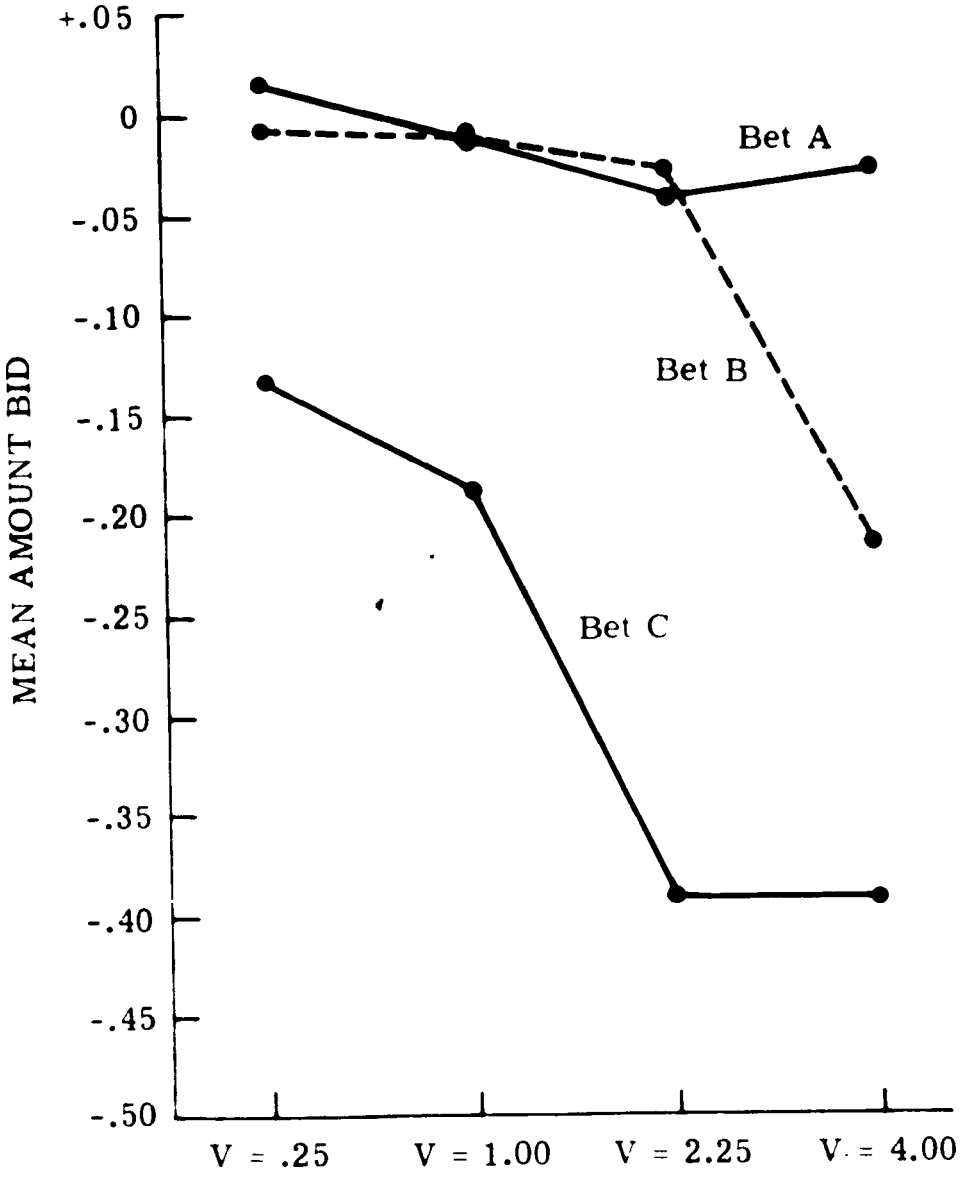


Fig. 4. Mean Amount Bid for sets of bets with equal independent variables. For all bets, Probability = 2/8, 2/8, 3/8; EV = 0, Sk = 0.

within a set but differs between sets. For the set with V = .25, the bets are:

A: 2/8 to win $ .67; 3/8 to win $ .14; 3/8 to lose $ .59.

B: 2/8 to win nothing; 3/8 to win $ .58; 3/8 to lose $ .58.

C: 2/8 to lose $ .67; 3/8 to lose $ .14; 3/8 to win $ .59.

The other sets of bets are constant multiples of these.

The Ss clearly preferred bet A over bet C, with bet B intermediate. Two additional sets, with EV = ±83¢, V = .25, Sk = 0 and probabilities 2/8, 3/8, 3/8, omitted from Fig. 4, showed the same trend as seen in Fig. 4.

In summary, the average difference in amount bid between the most preferred and the least preferred bets for all 18 sets of bets was 18¢. The results are not attributable to differences in Kurtosis nor Outcome Sum.

The consistent orderings of bets shown in Figs. 2, 3, and 4 may be explained by supposing that Ss attended primarily to the amounts of money associated with the smallest probability. Note that in Figs. 2 and 3, the bets A and B were sharply differentiated, despite the similarity of their more probable outcomes, the 2/8 and 5/8 outcomes. In contrast, there was no consistent preference shown between bets C and D, where the least probable outcomes were similar. The only major exception to this interpretation is the relative overbidding for bet B in Fig. 3, V = 4.00. Six of the 12 Ss showed this reversal of trend.

This importance of the amount of money associated with the least likely outcome, which is so clearly seen in the sets of three and four bets with equal independent variables, may also be seen in the instances of pairs of bets with equal independent variables. The

design includes 46 such pairs. Of these 18 pairs differ sharply in the least likely amount, while 28 have equal or nearly equal least likely amounts.

For the 18 pairs that differ, Table 11 shows that 15 had differences in mean amount bid in the predicted direction. The mean difference in amount bid for all 18 pairs was 13¢.

For the 28 pairs not differing in least likely amount, the prediction was made that Ss would base their preferences on the amount of the next least likely outcome (i.e., the outcome of the middle probability). Table 11 shows that this hypothesis accounted for 75 per cent of the results, but that the mean difference in amount bid was small.

This experiment included 45 zero EV mirror-image pairs of bets (i.e., bets identical except for a reversal of outcome signs). Of these, 12 have identical independent variables and thus are included in the results already reported. The other 33 mirror-image pairs differ in sign of Sk. The prediction that Ss will prefer the bet which has a winning, rather than a losing, least likely outcome is borne out in these pairs, but not as strongly as with the previously reported groups of bets. Table 11 summarizes these results.

Gambling Status variables. Of the several variables reflecting the effect of real gambling, only Total Status revealed any differences, for only three Ss. Data showing the effects for these Ss are given in Table 12. Total Status refers to the total amount of money S had won or lost over the several sessions up to a given point. Note that the three Ss reacted differently to their status. In the case of S 12, he bid most when he was winning; S 7 bid most when he was losing,

Table 11

Pairs of Bets with Equal Independent Variables:

Frequency of Differences in Mean Amount Bid

	Total N	No. with diff. in predicted direction	(%)	No. with no differences	No. with reversed differences	Mean difference
Bets with equal independent vars., differing in least likely amount	18	15	(83%)	1	2	13¢
Bets with equal independent vars., same in least likely amount	28	21	(75%)	2	5	4¢
Mirror Image Bets with equal independent variables	12[a]	12	(100%)	0	0	22¢
Mirror Image Bets differing only in Sk	33	20	(61%)	0	13	3¢

[a]Eight of these pairs are included in the 18 pairs in the first row of this table; the other 4 are included in Fig. 3.

Table 12

Total Status Data for Three Ss

Showing Mean Amount Bid

Total Status	S 11[a]	S12[b]	S 7[c]
+$20 or more	-$.08	+$.04	-$.07
+$10 to +$20	- .05	+ .08	- .01
+$5 to +$10	+ .01	- .07	+ .04
+$1.50 to +$5	+ .09	- .14	+ .05
-$1.50 to +$1.50	+ .07		+ .12
-$5 to -$1.50	+ .01	- .32	+ .13
-$10 to -$5	+ .02		
-$10 or worse	.00		

[a] incomplete data based on 520 responses.

[b] incomplete data based on 745 responses.

[c] incomplete data based on 480 responses.

and S 11 bid most when he was about even.

The other variables relating to real gambling, Status, Local Success, and Delay, showed no relationship to amount bid.

Two-outcome bets. Data from the seven two-outcome bets were examined for evidence of preferences among the bets, but little preference was found. Only six Ss showed a difference in amount bid of 15¢ or more between the most and least preferred bets. These six Ss had no common pattern in their orderings of the seven bets.

Adjustment Sessions. While the adjustment sessions were designed primarily to control Ss' winnings, some information is pro-

vided by the data. For the pairs of bets in which the EVs of the two bets were unequal, Ss chose the bet with the higher EV 86 per cent of the time, indicating satisfactory sensitivity to these rather complicated three-outcome gambles.

The adjustment sessions also included two sets of six bets with equal EV paired exhaustively within each set. The bets in each set varied only in probabilities; thus probability preferences were expected from the data. No probability preferences were found, however. Ss split their choices, choosing one bet twice and the other bet twice in four replications, 18 per cent of the time. This introduced so many tied choices that preference orderings inferred from the data are not clear.

Although firm conclusions can not be drawn concerning precise ordering preferences among the bets, seven Ss showed preferences for the three bets patterned ++- over those patterned ++0 in the positive EV pairs, and preferred bets patterned 0-- over those patterned +-- in the negative EV pairs. This result cannot be interpreted as probability preference, since two of the six bets had the same probabilities yet were widely separated in preference. Nor is this result consistent with the finding in the bidding sessions that the least likely amount determines Ss' behavior. The patterning preferences seen here are not found in the main study.

Subject differences. Finally, a global view of each S's data for the entire experiment was taken to see if there were any striking differences between Ss. Two Ss were notable.

S 2 often bid above EV for bets, and thus lost much money in the bidding sessions. His unprofitable play, however, was not highly

erratic; his mean standard deviation across replications was below the group average. He was the only S who preferred high V bets.

S 6 carefully attended to the EV of bets and fairly consistently bid 3¢ below EV. His data are striking for their evenness, showing no effects attributable to any variable other than EV included in this study.

The other 10 Ss, although varying slightly in different aspects of their performance, seem well represented by the group data.

DISCUSSION

The moment functions. The primary purpose of this study was to examine the relevance and relative importance of a number of bet variables in determining Ss' responses to risky bets. In particular, the set of moment functions, EV, V, Sk and K, were proposed as a possible alternative to the SEU model of decision making. If this alternative were fruitful, we would expect to find experimental effects attributable to the functions EV, V and Sk. In addition, we would expect that within the sets of bets for which these functions are constant, Ss would show no systematic preferences, or, that if preferences were shown, that these preferences be related to the fourth moment function, K.

The results of the present study may be evaluated in the light of these expectations.

First, there was a sizeable effect attributable to EV. Indeed, failure to find such an effect would invalidate the study, so sure are we that Ss ought to, and do, attend to the value of bets when gambling.

Variance also had an effect on Ss' behavior. Ten of the 12 Ss showed preference for low V bets. In a sense, this result replicates the results of Coombs and Pruitt (1960), who also found V preferences. But while Coombs and Pruitt's Ss showed wide variation in their most preferred V, most Ss in the present study preferred low V bets. This difference may be due to the difference in motivating conditions in the two studies. In the Coombs and Pruitt study, Ss did not play any bets during the experiment. They knew that their choices could not

affect the amount of money they received for participation in the experiment. In contrast, Ss in this study played one-third of the bets for real money, and believed that their decisions were the major determinant of how much money they would win or lose. The fears of losing money under such conditions may account for the avoidance of the riskier, high V bets. However, evidence by Edwards (1954a) and Slovic et al. (1961) in slightly different settings indicates that playing for real money or worthless chips makes little or no difference in Ss' preferences; the issue remains undecided.

In the present study, neither Sk nor K was related to Ss' behavior. An even more serious obstacle to the moment function approach was the finding of large and consistent differences in bidding among bets whose first three moment functions are equal. These differences were unrelated to the fourth moment function, K. These findings lead to the rejection of the moment functions as a set of variables sufficient to build a model. However, the first two moment functions, EV and V, are shown to be of sufficient importance that any future theory must account for them.

The least likely amount. The unexpected finding of consistent preferences among bets of equal EV, V, Sk and probabilities is of interest. Apparently Ss lay undue emphasis on the amount of money associated with the smallest probability, bidding more for a bet when this amount is positive than when it is negative. This effect of the least likely amount is quite distinct from the effect of V. To illustrate this distinction, consider the following bets:

A: 1/8 to win $ .12; 2/8 to win $ .83; 5/8 to lose $ .35.

B: 1/8 to win $1.00; 2/8 to win $ .41; 5/8 to lose $ .36.

These two bets have equal EV, V and Sk. Ss bid more for bet B, the bet with the largest least likely amount. In contrast are the following bets:

A: 1/8 to win $ .12; 2/8 to win $ .83; 5/8 to lose $ .35.

B: 1/8 to win $ .50; 2/8 to win $3.50; 5/8 to lose $1.42.

These two bets have equal EV and Sk but differ in V. Ss bid more for bet A, the bet with the smaller V, even though bet B has the larger least likely amount.

This example illustrates a general finding of the study, that Ss were sensitive to the overall money range of the bets, as reflected in bet V, and, at the same time, to the relative configuration of the separate parts of the bet, as reflected in the least likely amount. In reporting the latter result, it would be more accurate to say that Ss are over-sensitive to the amount of the least likely outcome relative to the other amounts of the bet.

Probability preferences. Another important result was the surprising finding that Ss showed no probability preferences. The lack of probability preferences among the few two-outcome bets is in sharp contrast with previous findings. This failure can not be attributed to a weakness in the bidding technique because Slovic et al. (1961), using the bidding technique, did find probability preferences. It may be that some contextual effect was operating, whereby these few bets, involving relatively small amounts of money, and seemingly more simple to understand, did not command Ss' attention and thus were not discriminated.

The lack of preferences among different probability triples may not be so easily discounted. Instead, the results of former

studies may be reinterpreted in the light of the present results. As discussed eariler, there is, in two-outcome bets, a confusion among independent variables. Thus what Edwards (1953) called probability preferences, Coombs and Pruitt (1960) chose, with equal accuracy, to call Skewness preferences. One can proceed to even further renaming. For two-outcome bets with zero EV, the bet labeled "1/8 probability" has 1/8 probability of winning money and 7/8 chance of losing money, and is a mirror image of the bet labeled "7/8 probability." Thus when S prefers the 1/8 bet over the 7/8 bet, this means that he prefers the bet in which the least likely outcome is positive rather than negative. Accordingly, probability preference results may be attributed to the effect of the least likely amount, so that these former findings may be seen as consistent with the findings of the present study. Further research is required, however, to tell whether this proposed relabeling is any improvement over the labels "probability preference" or "Skewness preference."

The SEU model. The three most important results already discussed, the effect of V, the effect of the least likely outcome, and the lack of probability preferences, may be brought together cogently within the framework of the SEU model. The preference for low V bets indicates that the utility curve for money is not symmetric in its extreme ranges; that is, that large losses appear larger than large wins. The effect of the least likely outcome suggests that Ss overestimate small probabilities; in other words, that the subjective probability curve is not linear with objective probability. Such a subjective probability curve has been found before by several experimenters (e.g., Preston & Baratta, 1948).

Subjective probability may be used as a unifying concept to reconcile the results of this study with previous research. A non-linear subjective probability function applied to two-outcome bets results in clear probability preferences. The complex, non-monotonic nature of three-outcome bets, however, obscures probability preferences; here subjective probability is seen in the importance of the least likely outcome.

Thus this study, which was originally designed to explore alternatives to the SEU model, provides data which the SEU model fits best. Only the failure to find two-outcome probability preferences is inconsistent with both previous results and the expectations of the SEU model.

Other variables. This study showed slight effects attributable to the variables outcome sum and pattern. Both these faint effects are probably due to the confounding of these variables with the amount of the least likely outcome.

The amount of the least likely outcome is often the largest amount of the bet; thus it tends to dominate the outcome sum. This would explain the low positive correlations found between outcome sum and amount bid.

Similar considerations apply to pattern. There is a tendency for bets with ++- pattern to have a positive least like amount, while +-- bets most often have a negative least likely amount. Thus, assuming that the truly influential variable is the least likely amount, a small, muffled effect from pattern might be expected.

An unexplained result of this study is the finding in the paired comparison sessions that ++- bets are preferred over ++0 bets,

and that 0-- bets are preferred over +--. This finding is nowhere reflected in the main study. While Ss were subjected to unusual conditions in these sessions (Ss won or lost large amounts of money in a short time with little or no apparent control over their fate), it is difficult to see how these conditions could account for the deviant results.

Real gambling. It should be comforting to anyone contemplating research with real gambling that the effects of wide variation in wins and losses are minor for most Ss. Actually playing the bets during the course of the experiment, a device which is necessary to the competitive bidding fiction used in this study, kept Ss alert throughout the long experimental sessions. Indeed, it is possible that Ss were too strongly motivated in the present study--not only alert but so worried about the possibility of losing their own money that they all became pessimistic conservatives. In contrast with previous studies in which wide ranges of S behavior have been observed, 10 of the 12 Ss in this study consistently avoided high V bets and bets in which there was a small possibility of large loss.

Future research. The most important new contribution of this study is the apparent importance of the least likely amount in determining Ss' responses, and it is this result which most urgently needs to be investigated further. Three-outcome bets can be well employed for future research on the least likely amount. Using three-outcome bets, the specification of probabilities, EV and V still leaves room for an additional independent variable. Since Sk was shown to have little or no effect on Ss' responses, the least likely amount may be chosen instead as the fourth main variable determining the bets. This

amount is not completely independent of the other variables. The bounds on A, the least likely amount of money, are determined by the relationship $(q + r)\ \underline{V} \geq p\ (A - \underline{EV})^2$. This means, for example, that if the probabilities chosen are 1/8, 2/8, 5/8, $\underline{EV} = 0$, and $\underline{V} = 1.00$, then the least likely amount must lie within the range $-\$2.64 \leq A \leq +\$2.64$. For every permissible choice of A, there exist two solutions for the other amounts of money, B and C.[2] Thus one can find two bets equal in probabilities, $\underline{EV}$, $\underline{V}$, and least likely amount. This could easily provide a set of stimuli to test the importance of the least likely amount and to explore further its relation to $\underline{V}$.

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[2]Exception: When $(q + r)\ \underline{V} = p(A - \underline{EV})^2$, there is only one solution for B and C. In general,

$$C = \frac{-r(pA - \underline{EV}) \pm \sqrt{qr(q + r)\ \underline{V} - pqr(A - \underline{EV})^2}}{r(q + r)}$$

$$B = \frac{\underline{EV} - pA - rC}{q} .$$

SUMMARY

Evaluation of the subjectively expected utility maximization (SEU) model of decision making is hampered by a lack of systematic, atheoretical information about the relevance and relative importance of a number of bet variables. This study was designed to remedy that lack. The most important variables of the study were the moment functions which uniquely describe a bet and which have been suggested as an alternative to the SEU model. These variables are Expected Value (EV), Variance (V), Skewness (Sk), and Kurtosis (K). Other variables in the study were probability, pattern (the relative number of wins and losses in a bet), and outcome sum (the algebraic sum of the outcomes of a bet).

Three-outcome rather than two-outcome bets were chosen for stimuli, enabling the independent assignment of probabilities, EV, V, and Sk. These variables have been confounded in previous research. The existence of more than one bet (up to four in some cases) with equal probabilities, EV, V and Sk permitted a direct assessment of the adequacy of these variables in predicting Ss' responses. The stimuli were 182 bets representing several levels of the independent variables.

A single-stimulus bidding technique providing ratio scale data was employed for data collection. The Ss were asked to state how much they would be willing to pay E to play the bet. For unattractive bets, S stated the smallest amount E would have to pay him before he would play the bet. A fictitious competitive bidding scheme encouraged Ss

to bid as high as possible. One third of the bets were played for real money as the experiment progressed.

Twelve male student volunteers were run in five sessions, each a single replication of the bets, and in two additional paired-comparison adjustment sessions, in which Ss' winnings were carefully controlled.

Data analysis was based on the mean amount bid for each bet over five replications. Analyses of variance performed on five factorially complete subsets of the data were used as statistical checks on conclusions drawn from the entire data.

Results indicated that Ss were well motivated and reasonably consistent in their bids. The Ss bid an average of 12¢ below EV. They preferred low V bets, bidding 18¢ more for low than high V bets. They showed no preferences for Sk nor for probabilities. The variables K, pattern, and outcome sum had little or no effect on Ss' responses.

Analysis of the sets of bets constant in probabilities, EV, V, and Sk revealed that Ss showed marked preferences which were consistent over changes in V. The Ss bid more for such bets in which the amount assiciated with the least likely outcome was largest.

In view of this variation in response among bets with constant moment functions, and the failure to find effects due to Sk or K, the moment functions were rejected as an adequate set of variables with which to build a model.

The failure to find three-outcome probability preferences is in contrast with earlier studies using two-outcome bets. This difference may be due to the more complex, non-monotonic nature of probability triples. The effect of the least likely amount, essentially a prob-

ability effect, is seen as parallel to the earlier probability findings.

The SEU model seems well able to handle the results of this study. The preference for low V bets suggests a utility curve which is non-symmetric in its extremes, with large losses overvalued. The effect of the least likely amount indicates a subjective probability function with low probabilities overvalued.

Appendix I

Computation Method for Three-Outcome Bets

This appendix gives a method for finding three-outcome bets which have specified values of probabilities, Expected Value (EV), Variance (V), and Skewness (Sk). A three-outcome bet is any bet of the form:

p to win \$A; q to win \$B; r to win \$C; $p + q + r = 1$.

For three-outcome bets,

$$EV = pA + qB + rC$$

$$V = p(A - EV)^2 + q(B - EV)^2 + r(C - EV)^2$$

$$Sk = \left[p(A - EV)^3 + q(B - EV)^3 + r(C - EV)^3\right] / V^{3/2}$$

Computational Method. The method in brief is this: fix p, q, r, A and B, and graph the relationship between C and Sk, as C varies between A and B. Read from this graph the needed C value for a given Sk value. Then linear transformations on A, B and C will produce a bet having the needed values of EV and V without changing the value of Sk. Details of each of these steps follow.

Computational Steps. 1. Fix p, q and r. For a given probability triple, all different permutations of the three probabilities must be considered separately. When units of eighths are used, just five different probability triples are possible: 1/8, 1/8, 6/8; 1/8, 2/8, 5/8; 1/8, 3/8, 4/8; 2/8, 2/8, 4/8; and 2/8, 3/8, 3/8. For each triple in which two elements are equal, there are three different permutations. For example, 1/8, 1/8, 6/8

may be arranged:

$$p = 1/8,\ q = 1/8,\ r = 6/8$$

$$\text{or } p = 1/8,\ q = 6/8,\ r = 1/8$$

$$\text{or } p = 6/8,\ q = 1/8,\ r = 1/8.$$

For the triples in which no elements are equal, as in the triple 1/8, 2/8, 5/8, there are six different permutations. Thus to encompass all possible bets using units of eighths, 21 different cases must be considered. These 21 cases are listed in Table 13.

2. Fix A and B. These may be chosen arbitrarily. For the curves presented here, A = -1.00 and B = +1.00. These values make step 3 slightly simpler than would otherwise be the case.

3. Graph the relationship between C and Sk, as C varies between A and B. This must be done separately for each case. Choose several C values between A and B. For the curves here shown, C took on the 21 values from -1.0 to +1.0 in steps of tenths. Fewer points would probably suffice to define the curve if the computations must be done by hand. For each case, that is, for each permutation of each probability triple, the following equations were computed for each C value, resulting in corresponding values for Sk.

$\mu_1 = q - p + rC$ (this is EV, when A = -1.00 and B = +1.00)

$\mu_2 = q + p + rC^2$ (this is the second moment about zero)

$\mu_3 = q - p + rC^3$ (this is the third moment about zero)

$\mu_{2,c} = \mu_2 - \mu_1^2$ (this is V, or second central moment)

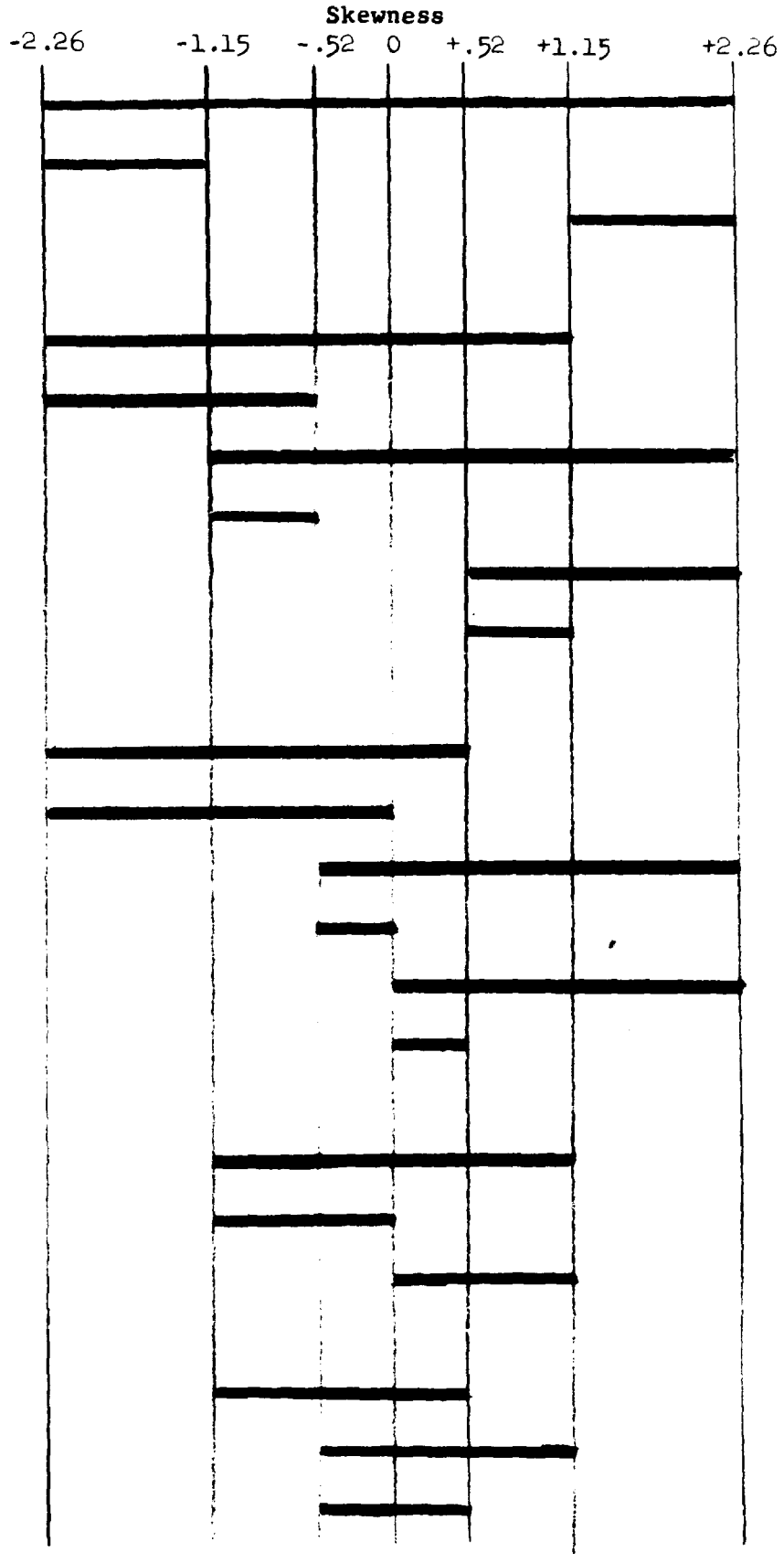
$\mu_{3,c} = \mu_3 - 3\mu_1\mu_2 + 2\mu_1^3$ (this is the third central moment)

$Sk = \mu_{3,c}[\mu_{2,c}]^{-3/2}$

Table 13

Range of Skewness for
Each Probability Permutation

Case No.	Probabilities p	q	r	Figure No.
1	.125	.125	.750	5
2	.125	.750	.125	9
3	.750	.125	.125	10
4	.125	.250	.625	5
5	.125	.625	.250	9
6	.250	.125	.625	5
7	.250	.625	.125	11
8	.625	.125	.250	10
9	.625	.250	.125	12
10	.125	.375	.500	6
11	.125	.500	.375	6
12	.375	.125	.500	7
13	.375	.500	.125	13
14	.500	.125	.375	7
15	.500	.375	.125	13
16	.250	.250	.500	8
17	.250	.500	.250	11
18	.500	.250	.250	12
19	.250	.375	.375	8
20	.375	.250	.375	8
21	.375	.375	.250	13



The curves shown in Figures 5 through 13 were made by computing and plotting a series of (Sk, C) points using these equations and connecting them smoothly with a French Curve, for each probability case.

Range of Skewness. I found it helpful to chart the range of Sk for each probability case. This is shown in Table 13. The end points of these ranges may be found easily by setting C equal to A and to B in turn, thus reducing the three-outcome case to a two-outcome case (mathematically, the bet 1/8 to lose \$1.00, 2/8 to lose \$1.00, and 5/8 to win \$1.00 is the same as the bet 3/8 to lose \$1.00 and 5/8 to win \$1.00). The Sk of these end-points, these two-outcome bets, may be found by using the equation $\frac{1-2(p+r)}{[q(p+r)]^{1/2}}$ when C = A, or the equation $\frac{1-2p}{[p(q+r)]^{1/2}}$ when C = B.

Table 13 may be used to tell how many different bets exist which have the same probability triple and the same Sk, V, and EV. For example, four different bets may be found, each of which has probabilities 1/8, 2/8, and 5/8, any Sk value in the open interval $-1.15 < Sk < -.52$ or the open interval $+.52 < Sk < +1.15$, and any given EV and V values. It may first appear from Table 13 that there are six different bets with probabilities 1/8, 3/8, and 4/8, and Sk = 0, but notice that four of these six (cases 11, 13, 14 and 15) are end points at Sk = 0, and thus all four turn out to be the same two-outcome bet. Only cases 10 and 12 give genuine three-outcome bets.

It is easy to compute, without worrying about permutations of probabilities or exact numbers of bets, the range of Sk within

which at least one bet may be found. For any probability triple, call the smallest probability p, and the sum of the other two q. Then the Sk range is $\pm \frac{1-2p}{(pq)^{1/2}}$. It is always symmetric around zero. Thus to obtain a bet with Sk as large as 10, the smallest probability must be slightly smaller than 1/100.

Note: If probabilities in units of eighths are to be used, the preceding three computational steps need not be repeated. To compute a bet, start with step 4.

Final Computations. 4. Suppose a bet is wanted with the following values: Probabilities = (p, q, r)

Expected Value = M

Variance = V and

Skewness = S.

(a) Find the graph or graphs with the wanted p, q and r which include the wanted value S. The graphs are indexed in Table 13. For each applicable graph do the following:

(b) Read off the C value corresponding to the given S. Note the particular permutation of probabilities used in the graph (shown in the upper right-hand corner), and follow that assignment of values for p, q and r in the equations following. (By finding this C value, we have found a bet with Skewness = S, to wit, p to lose $1.00, q to win $1.00 and r to win C. This bet, however, does not have the Expected Value and Variance needed. In step (c) the actual Expected Value, M', and Variance, V', of this bet are computed. Steps (d), (e) and (f) make a linear transformation on the amounts of money re-

sulting in a bet with Expected Value = M, and Variance = V, as desired. Skewness is invariant under linear transformations).

(c) Compute $M' = q - p + rC$

$$M_2' = p + q + rC^2$$

$$V' = M_2' - M'^2$$

(d) Find a value t such that $t^2V' = V$, or $t = (V/V')^{1/2}$

(e) Find a value s such that $tM' + s = M$, or $s = M - tM'$

(f) Then the desired bet has values:

p to win $\$(-t + s)$

q to win $\$(+t + s)$

r to win $\$(Ct + s)$.

Example. Suppose we wish to find a bet with these characteristics: $(p, q, r) = (2/8, 3/8, 3/8)$

Expected Value = +1.00

Variance = 2.00

Skewness = +1.00

Table 13 shows that there exists only one such bet, case 20, graphed in Figure 8. Figure 8 shows that when Skewness = +1.00, C = -.61. For this case, p = .375, q = .250 and r = .375. Then:

$M' = -.35375$

$M_2' = +.7642375$

$V' = .6391$

$t = +1.77$

$s = +1.63$

Thus the only three-outcome bet using probabilities (2/8, 3/8, 3/8)

and having Expected Value = $1.00; Variance = 2.00 and Skewness = +1.00 is:

2/8 to win $3.40

3/8 to lose $ .14

3/8 to win $ .55.

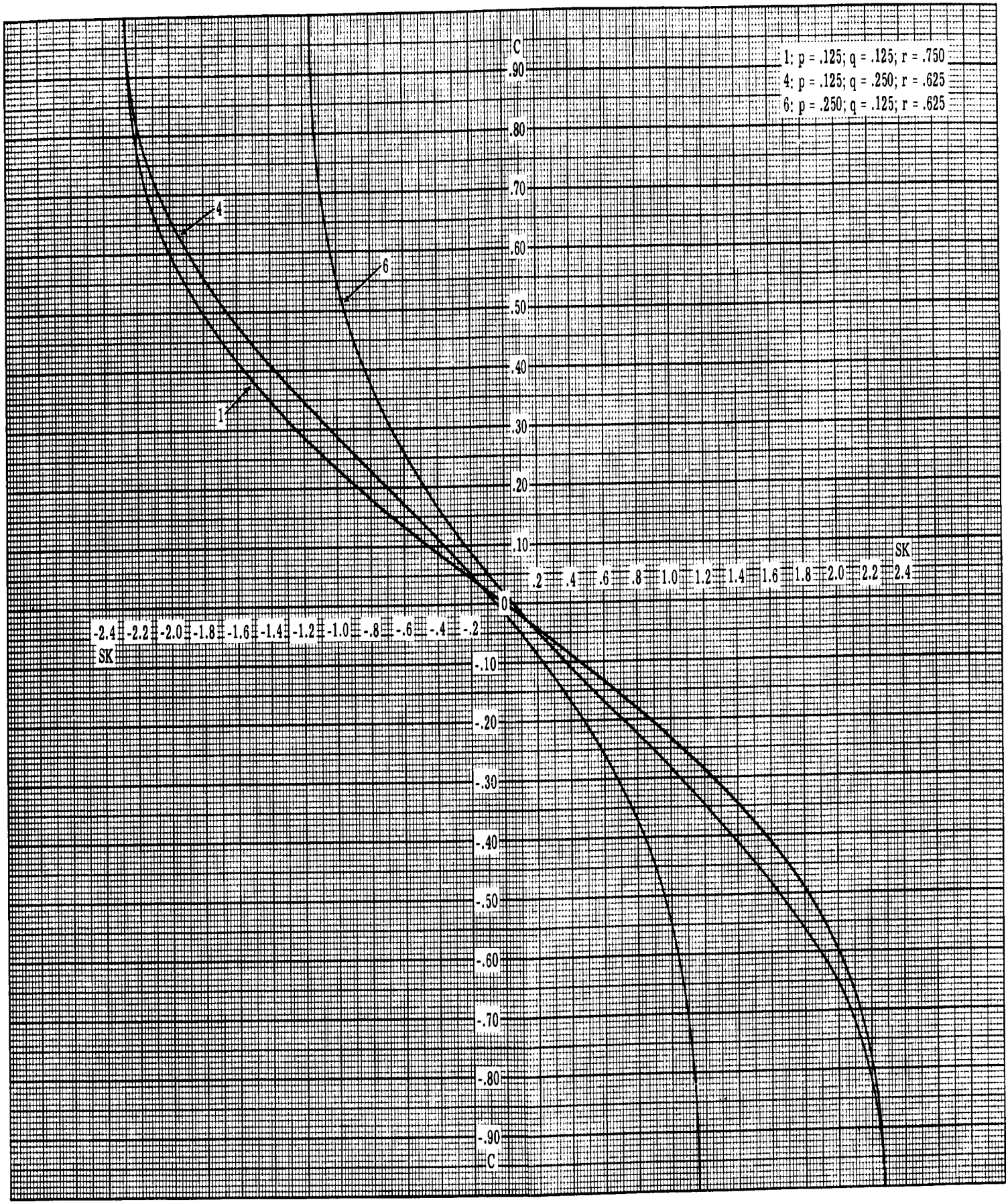


Fig. 5. Nomographs 1, 4 and 6

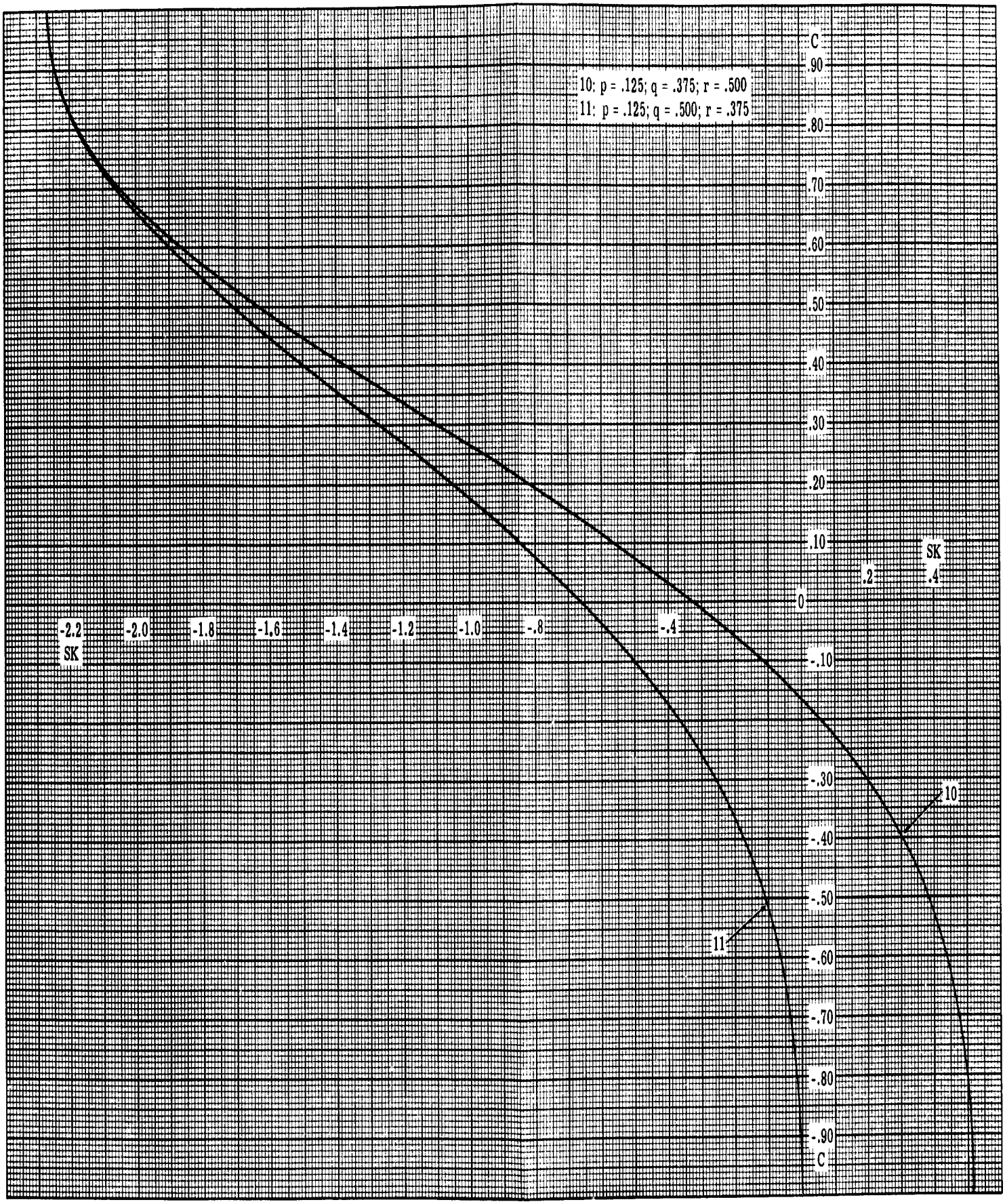


Fig. 6. Nomographs 10 and 11

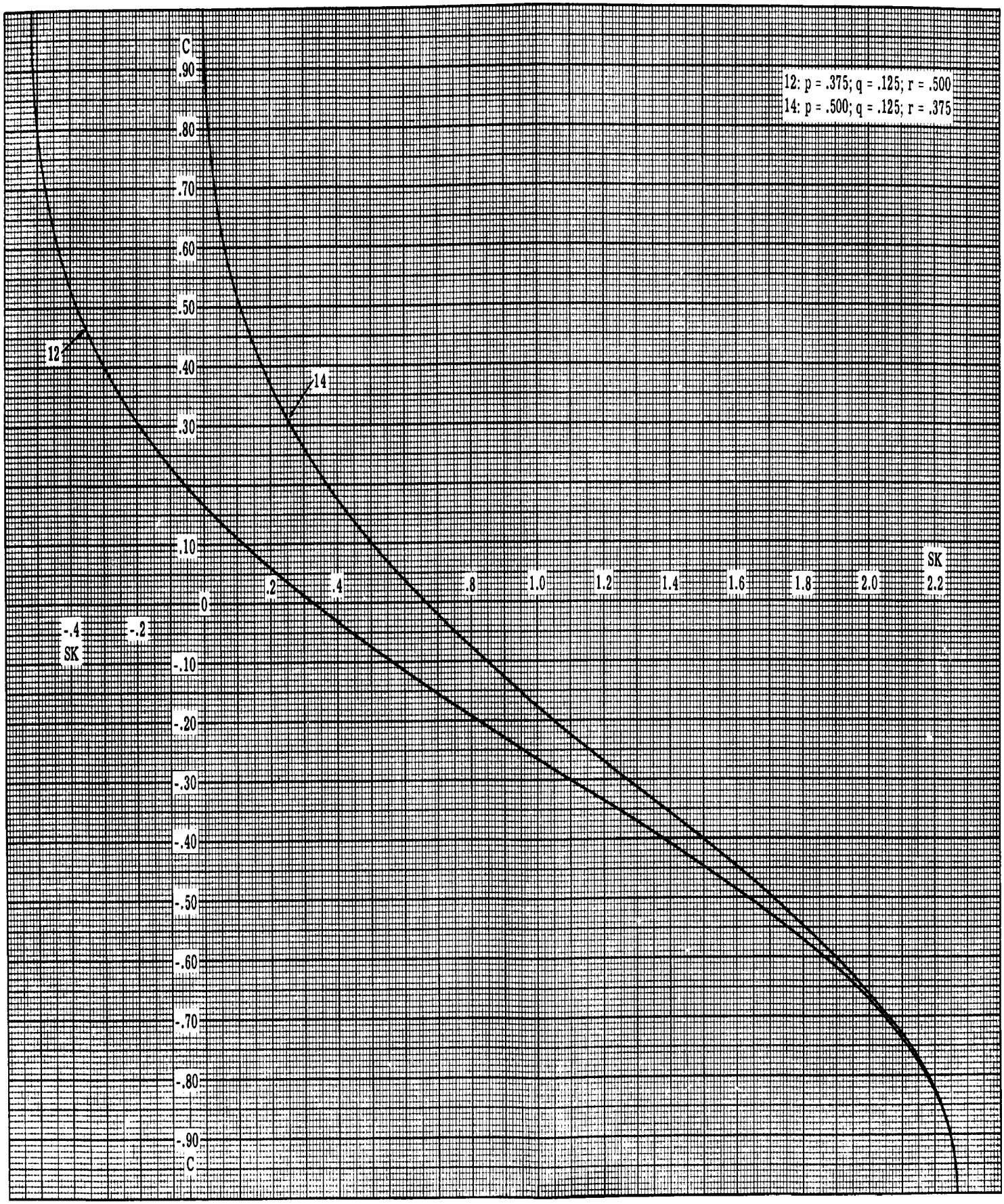


Fig. 7. Nomographs 12 and 14

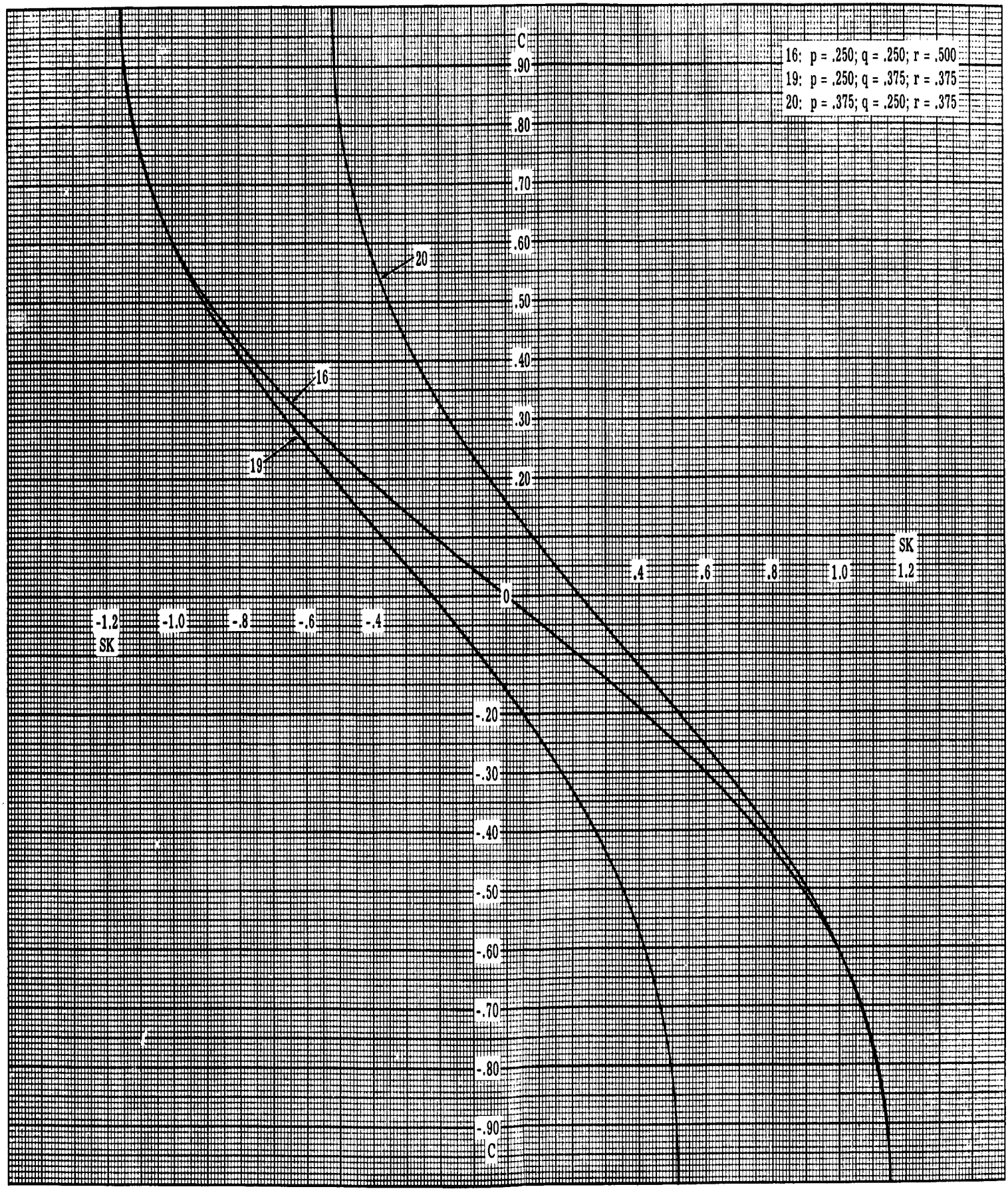


Fig. 8. Nomographs 16, 19 and 20

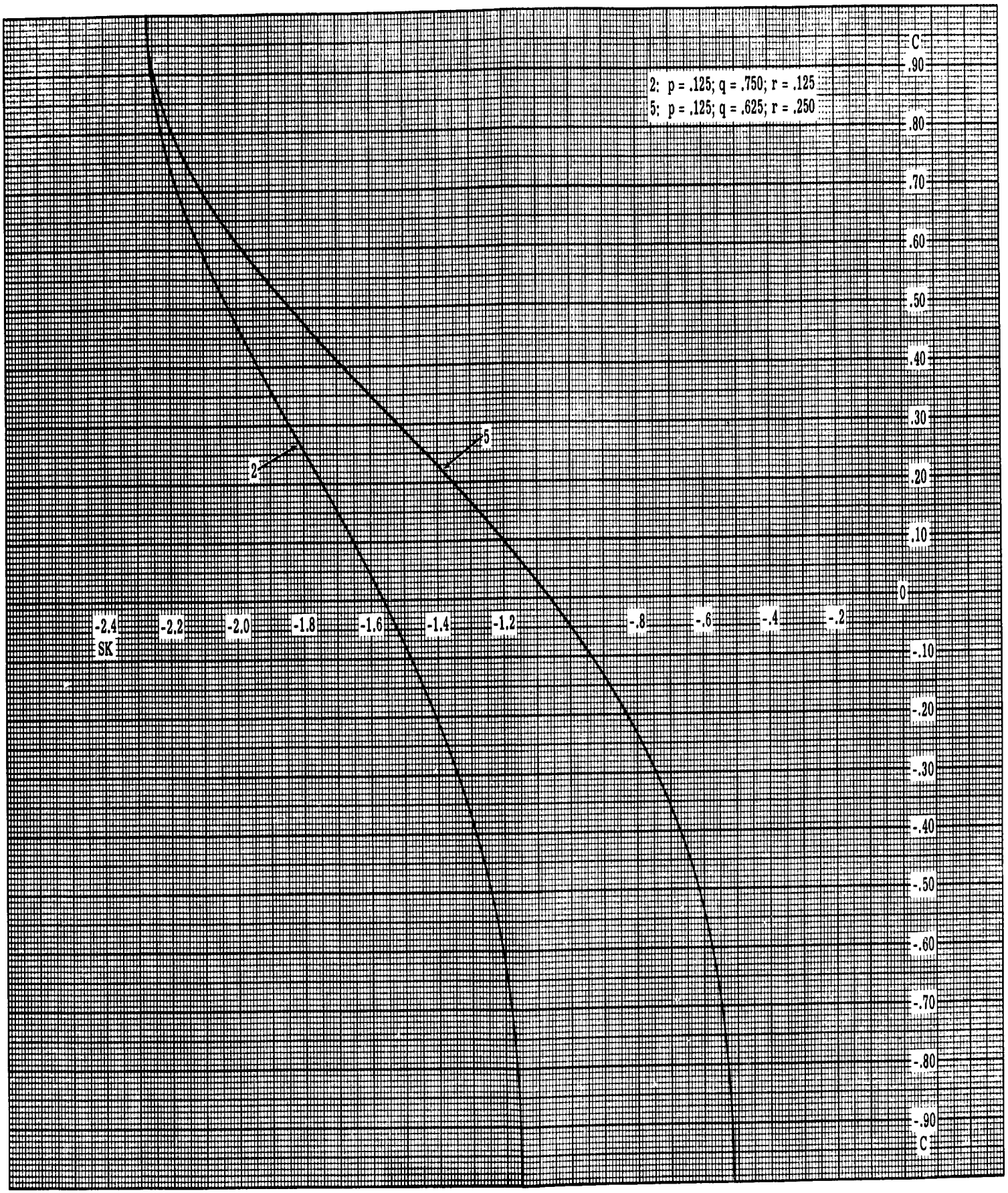


Fig. 9. Nomographs 2 and 5

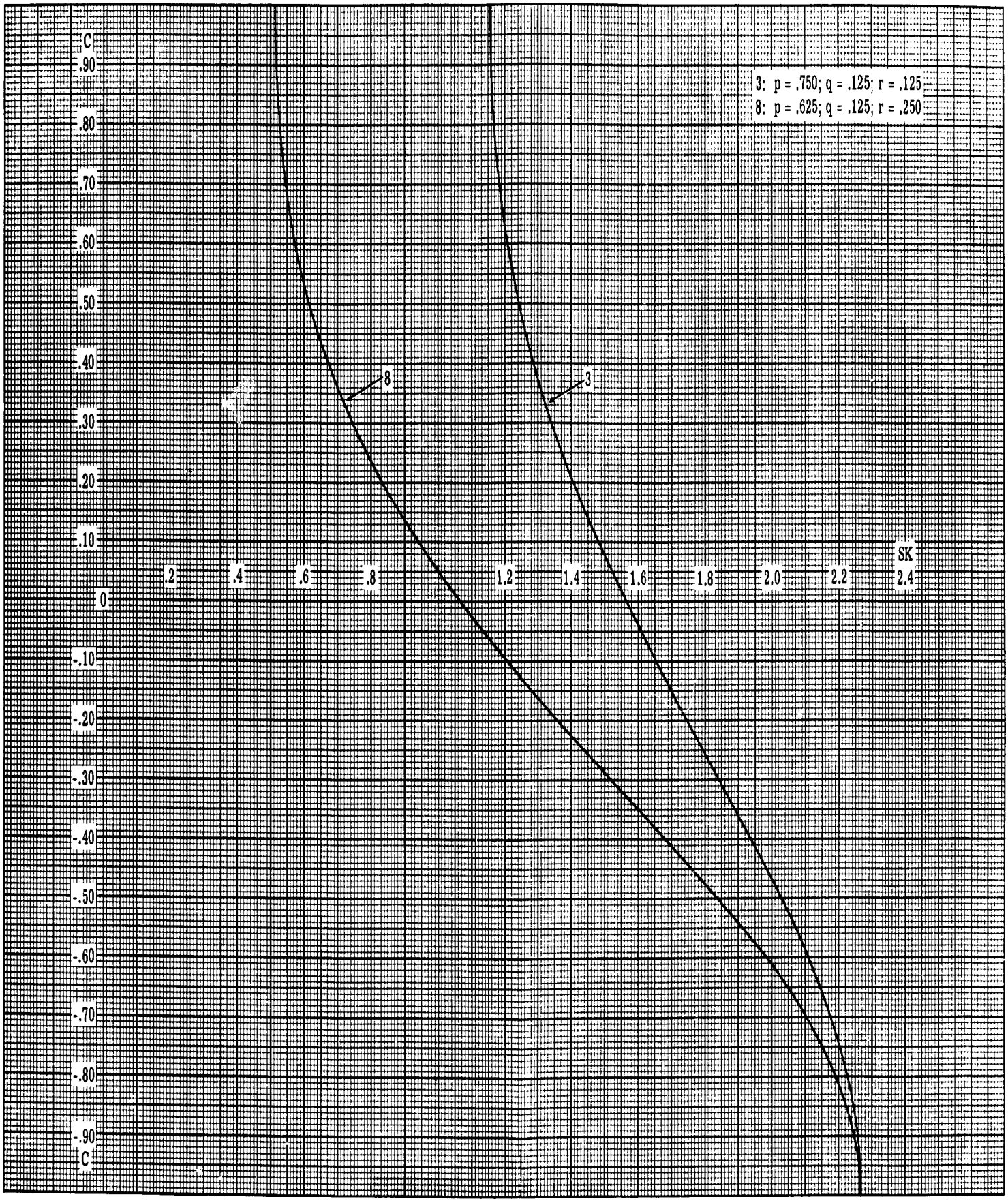


Fig. 10. Nomographs 3 and 8

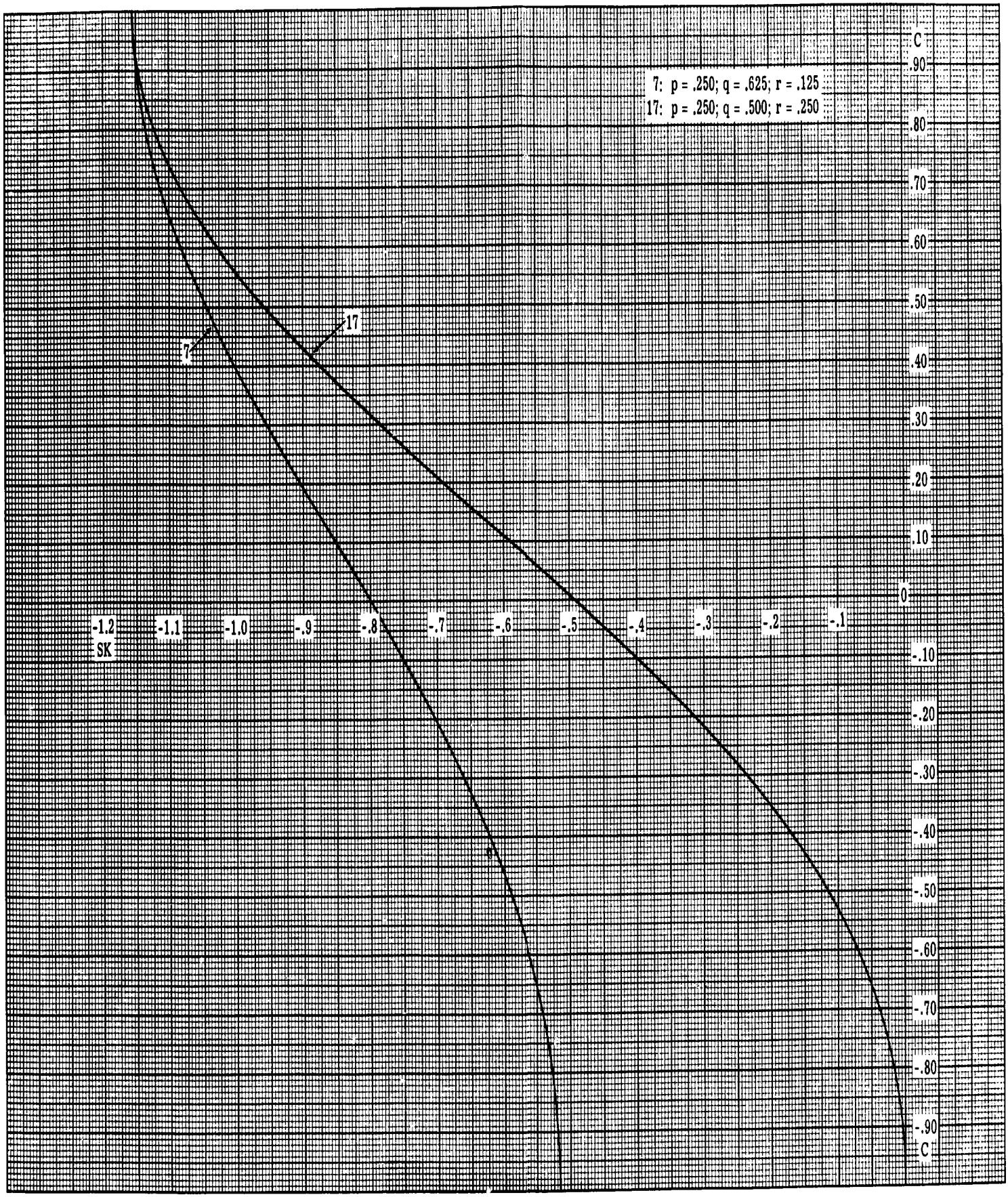


Fig. 11. Nomographs 7 and 17

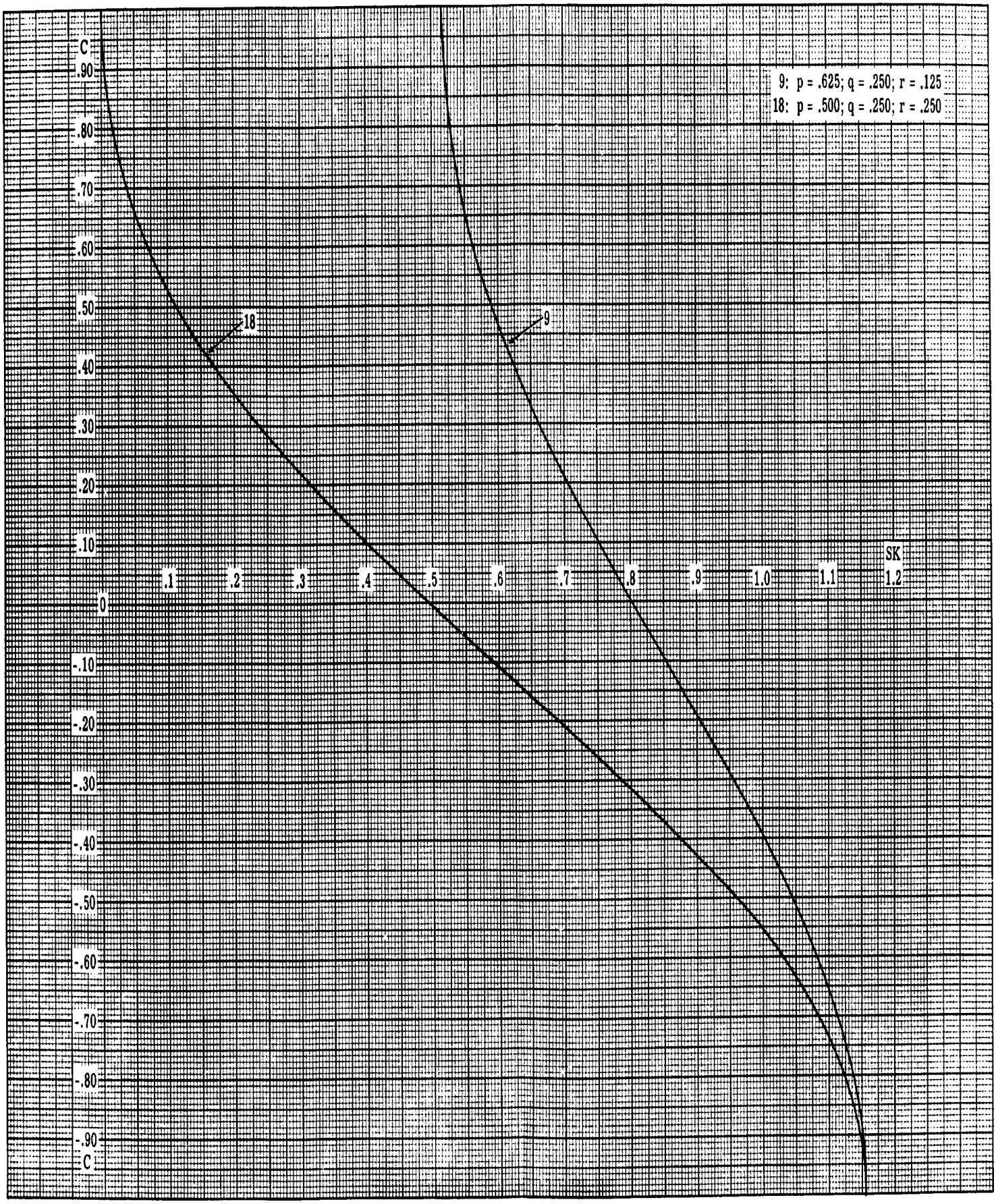


Fig. 12. Nomographs 9 and 18

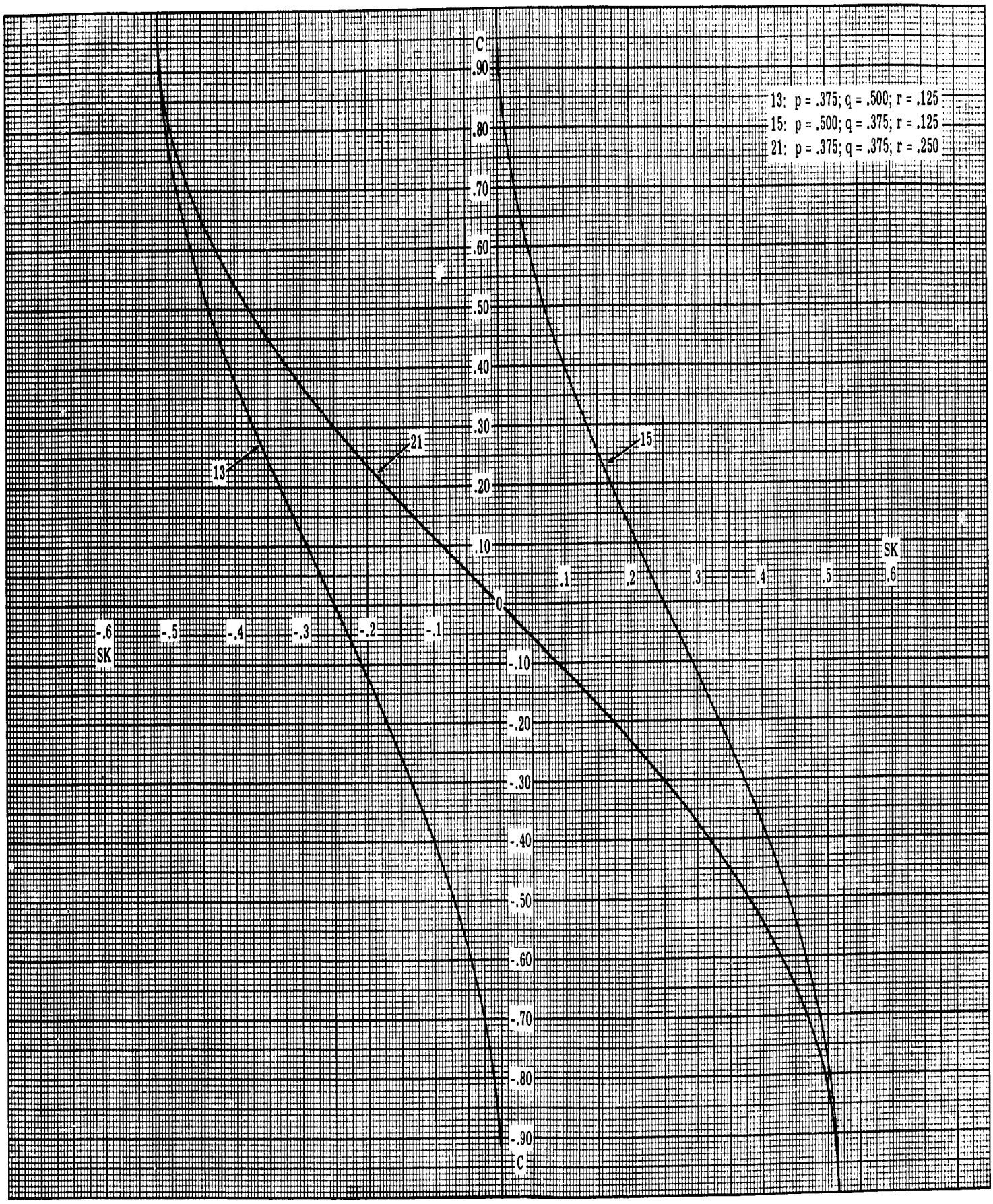


Fig. 13. Nomographs 13, 15 and 21

Appendix II

Instructions to Subjects

Instructions for the first bidding session. I am interested in studying decision making under risk. The decisions I will ask you to make are about bets or gambles. I am going to play a gambling game with you, and you are going to be asked to gamble with your own money. Playing this game, you should expect to win money. Subjects in this experiment usually have ended up winning money. However, you should understand before starting the experiment that there is a small chance that you could lose some of your own money. Balanced against this is the chance that you will win more money than the average. How well you do will depend partly on how wisely you play the game, and partly on pure luck. I do not expect you to lose money, for the chances of this are very slight, but it is only fair to warn you of this possibility before the experiment starts. I would like to get your assurance now that if you start the experiment, you will finish it. Is this okay with you?

Here are chips worth $30. [Chips are counted with the S] At the end of the session you must return the $30 in chips to me. What is left represents your winnings, and I will pay you that amount in cash. If you do not have $30 in chips to return to me, you must make up the difference out of your own pocket. Is that clear?

The bets look like this [show example]. The bets will be played by spinning this pointer [show]. If the pointer lands in a

sector which is labeled "win" on the card, I pay you chips. If it lands in a sector which is labeled "lose" on the card, you pay me chips. Notice that I have written "win" or "lose" on the card in the same color as this plastic pie, so that there will be no confusion. (You aren't color blind, are you?) Almost all the bets you will see have three sectors. A few bets have only two sectors, like this [show]. The smallest sector is just 1/8 of the pie, or 12 and a half per cent. The pointer will land in such a sector 12.5% of the time. All the larger sectors are multiples of eighths. [Show each pie and enumerate probabilities.]

When we play a bet, I will spin the pointer. If the pointer lands on the line between two colors, I will always spin it again. I, not you, decide whether or not it is on the line.

Now, for each bet you see, I want you to tell me how much money you would be willing to pay me for the privilege of playing the bet with me. This amount of money you would pay me before we played the bet, and I would keep it whether or not you won the bet. If you win the bet, I would pay you the amount you won, so your net winnings would be the amount you won minus the amount you paid me to play the bet. If you lost, you would pay me the losing amount, so your net loss would be the amount you paid me to play the bet plus the amount you lose when the bet was played.

Let's taken an example: Suppose you bid 50¢ for this bet. I'm not saying that's the best bid; I'm just using it as an example. You would pay me 50¢ and then we would play the bet, by spinning the pointer. If it landed here [point to blue sector], I would pay you

$1.60, so you would be ahead $1.10, the difference between what you paid me, and what I paid you. If the pointer landed here [point to red sector], I would pay you 80¢, and you would have a net profit of 30¢. If it landed here [point to green sector], you would have to pay me $1.20 in addition to what you already gave me, so you would have a net loss of $1.70.

You may find some bets that seem so poor to you that you wouldn't play them unless you were paid to. In that case, I want you to tell me how much I would have to pay you before you'd play the bet. This would be money I would pay you, before we ran the bet. For example, you'd be foolish to offer me money to play this bet. Here, your task would be to decide the least amount of money I'd have to pay you before you played the bet. The money I would pay you, you would keep no matter how the bet came out. Suppose your bid on this bet is that I must pay you $1.25 before you'd play the bet. Again, take this just as an example, not necessarily right. Then I'd pay you the amount, and we'd spin the pointer. If it landed here [point to red sector], you'd be ahead $1.65, the $1.25 I paid you plus the 40¢ you'd won. If it landed here [point to green sector], you'd be ahead by 5¢, and if it landed here [point to blue sector], you'd lose a net of 75¢.

I cannot accept every bid you make. I will only accept your bid when you offer to pay me enough money on the favorable bets or ask me for a sufficiently small amount of money on the unfavorable bets. Here's how I decide that: I ran an experiment like this before, and collected bids from a number of people. In order for me to accept

your bid for the bet, it must be in the upper quarter of the bids I collected before. In other words, you are really bidding against my former subjects for the bet. This means that you must bid just as high as you can. Of course, if you bid too high for the bet, you will get to play every bet, but in the long run you will lose money. If you bid too low for the bet, I will refuse your bid, and you will never get to play any bets. Your task is to decide just what each bet is worth to you, and bid that amount. When I accept your bid, we will exchange the appropriate amount of money and play the bet. When I refuse your bid, we will not play the bet, but simply go on to the next bet.

Quite frankly, the Ss I ran before were pretty inconsistent in their responses. You will find that sometimes I, using their responses to guide me, will refuse a bid you think is pretty good, or accept one you think is not so good. This can be confusing for you, particularly if you spend your time trying to figure out that other group. I advise you that you don't pay too much attention to this variability. In order to win as much money as possible, concentrate on deciding just how much money you think the bet is worth. If you do this you will have the best chance of doing well, regardless of how my other Ss bid.

Do you have any questions? Would you like to go over the examples again?

Instructions for the first adjustment session. Today we will play a different game. The same type of bets will be used, and the bets will be decided in the same way, by spinning the pointer. But

this time you will see two bets at a time, like this [show example]. There will be no bidding. Your task is to decide which of the two bets you would rather play. You must always pick one of the two bets, even if you don't like either bet, or if you like both bets.

We will not play all the bets. Which bets will be played will be decided randomly. When you look at a pair of bets, tell me which bet you prefer. Then I will tell you whether we will play that bet. Your choice between the two bets will not affect whether the bet is played, but I won't tell you whether we'll play the bet until after you make your choice. I will decide whether to play the bet by following this [wave page] random arrangement of P's for "play", and N's, for "not play."

Make your choices carefully, for that will increase your chance of winning money.

The game is an even game. I will bias it $1.50 in your favor by giving you $31.50 in chips, and asking you to return $30 at the end of the session.

Before you start, please shuffle the cards. Then as you look at a card, read to me the number in the right hand corner. Then tell me if you prefer the left bet or the right bet.

Any questions?

[If S asks how many bets will be played, say "I can't tell you".]

Appendix III

Stimuli Used in the Experiment

Bets used in the bidding sessions. All the bets listed in Table 14 have EV = 0 and V = .25. Probability (p) indicates probability of winning a given amount when the sign is positive, and of losing when the sign is negative. Checks in the appropriate columns show which of these bets also appeared with different EV and V values. To compute the unlisted bets, make the following linear transformations on the money outcomes: to change EV, add or subtract 83¢ to each outcome; to change V, multiply each outcome by 2 for V = 1.00, by 3 for V = 2.25, and by 4 for V = 4.00.

Table 14

Bets Used in the Bidding Sessions

Bets				Related Bets				
				EV=+83¢	EV=-83¢	EV=0	EV=0	EV=0
Sk	p=1/8	p=1/8	p=6/8	V=.25	V=.25	V=1.00	V=2.25	V=4.00
-1.8	-$1.26	+$ .58	+$ .11					
-1.8	-1.24	- .30	+ .26					
- .9	+ .83	-1.14	+ .05	x	x			
0	+1.00	-1.00	0	x	x	x	x	x
+ .9	- .83	+1.14	- .05	x	x			
+1.8	+1.26	- .58	- .11					
+1.8	+1.24	+ .30	- .26					
	p=1/8	p=2/8	p=5/8					
-1.8	-$1.25	-$ .10	+$ .29	x	x	x	x	x
-1.8	-1.25	+ .45	+ .07	x	x	x	x	x
- .9	+ .61	- .83	+ .21	x	x	x	x	x
- .9	-1.11	+ .62	- .03	x	x	x	x	x
- .9	-1.00	- .41	+ .36	x	x	x	x	x
- .9	- .12	- .83	+ .35	x	x	x	x	x
0	- .93	+ .73	- .11[a]	x	x	x	x	x
0	+ .93	- .73	+ .11[a]	x	x	x	x	x
+ .9	- .61	+ .83	- .21	x	x	x	x	x
+ .9	+1.11	- .62	+ .03	x	x	x	x	x
+ .9	+1.00	+ .41	- .36	x	x	x	x	x
+ .9	+ .12	+ .83	- .35	x	x	x	x	x

[a]These bets also took the forms: EV = +83¢, V = 1.00; EV = +83¢, V = 2.25; EV = +83¢, V = 4.00; EV = -83¢, V = 1.00; EV = -83¢, V = 2.25, and EV = -83¢, V = 4.00.

Table 14--Continued

Bets				Related Bets				
				EV=+83¢	EV=-83¢	EV=0	EV=0	EV=0
Sk	p=1/8	p=2/8	p=5/8	V=.25	V=.25	V=1.00	V=2.25	V=4.00
+1.8	+$1.25	+$ .10	-$ .29	x	x	x	x	x
+1.8	+1.25	- .45	- .07	x	x	x	x	x
	p=1/8	p=3/8	p=4/8					
-1.8	-1.25	- .03	+ .33					
-1.8	-1.25	+ .38	+ .03					
- .9	-1.08	+ .51	- .11	x	x			
- .9	-1.06	- .22	+ .43	x	x			
0	- .83	+ .60	- .24	x	x	x	x	x
0	+ .83	- .60	+ .24	x	x	x	x	x
+ .9	+1.08	- .51	+ .11	x	x			
+ .9	+1.06	+ .22	- .43	x	x			
+1.8	+1.25	+ .03	- .33					
+1.8	+1.25	- .38	- .03					
	p=2/8	p=2/8	p=4/8					
- .9	-$ .83	+$ .50	+$ .17	x	x			
- .9	- .83	+ .04	+ .39	x	x			
0	+ .71	- .71	0	x	x	x	x	x
+ .9	+ .83	- .50	- .17	x	x			
+ .9	+ .83	- .04	- .39	x	x			
	p=2/8	p=3/8	p=3/8					
- .9	-$ .83	+$ .43	+$ .12	x	x			
0	- .67	+ .59	- .14	x	x	x	x	x

Table 14--Continued

Bets				Related Bets				
				EV=+83¢	EV=-83¢	EV=0	EV=0	EV=0
Sk	p=2/8	p=3/8	p=3/8	V=.25	V=.25	V=1.00	V=2.25	V=4.00
0	+$ .67	-$ .59	+$ .14	x	x	x	x	x
0	0	+ .58	- .58	x	x	x	x	x
+ .9	+ .83	- .43	- .12	x	x			
	p=1/8	p=7/8						
+2.26	+$1.33	-$ .19						
-2.26	-1.33	+ .19						
	p=2/8	p=6/8						
+1.15	+$ .87	-$ .29						
-1.15	- .87	+ .29						
	p=3/8	p=5/8						
+.52	+$ .65	-$ .39						
-.52	- .65	+ .39						
	p=4/8	p=4/8						
0	+$ .50	-$ .50						

Bets used in the paired-comparison adjustment sessions.

All the bets listed in Table 15 have V = .36 and Sk = -.9. Fifteen additional bets were used which had all signs of outcomes reversed. These reversed bets had negative EV's, and all had V = .36 and Sk = +.9.

Table 15

Bets Used in the Paired-Comparison Adjustment Sessions

Bet Number	EV	Bet
1a	$1.00	1/8 to win $2.00; 1/8 to lose $..37; 6/8 to win $1.06
1b	1.18	1/8 to win $2.18; 1/8 to lose $ .19; 6/8 to win $1.25
1c	1.37	1/8 to win $2.37; 1/8 to win nothing; 6/8 to win $1.34.
2a	1.00	1/8 to lose $ .33; 2/8 to win $1.74; 5/8 to win $ .96
2b	1.16	1/8 to lose $ .17; 2/8 to win $1.90; 5/8 to win $1.12
2c	1.33	1/8 to win nothing; 2/8 to win $2.07; 5/8 to win $1.29
3a	1.00	1/8 to win $ .86; 2/8 to win nothing; 5 /8 to win $1.42
3b	1.16	1/8 to win $1.02; 2/8 to win $ .16; 5/8 to win $1.58
4a	1.00	1/8 to lose $ .30; 3/8 to win $1.61; 4/8 to win $ .88
4b	1.15	1/8 to lose $ .15; 3/8 to win $1.76; 4/8 to win $1.03
4c	1.30	1/8 to win nothing; 3/8 to win $1.91; 4/8 to win $1.18
5a	1.00	2/8 to win nothing; 2/8 to win $1.60; 4/8 to win $1.20
5b	1.16	2/8 to win $ .16; 2/8 to win $1.76; 4/8 to win $1.36
6a	1.00	2/8 to win nothing; 3/8 to win $1.52; 3/8 to win $1.14
6b	1.16	2/8 to win $ .16; 3/8 to win $1.68; 3/8 to win $1.30

The bets were paired as follows:

1b - 3a	2b - 6a	4c - 5b	2a - 4b	1a - 5a	3a - 4a
2b - 3a	4b - 6a	1c - 6b	1b - 4a	1a - 6a	3a - 5a
3a - 4b	1c - 3b	2c - 6b	2b - 4a	2a - 3a	3a - 6a
1b - 5a	2c - 3b	4c - 6b	1a - 2a	2a - 4a	4a - 5a
2b - 5a	3b - 4c	1a - 2b	1a - 3a	2a - 5a	4a - 6a
4b - 5a	1c - 5b	1a - 4b	1a - 4a	2a - 6a	5a - 6a
1b - 6a	2c - 5b	1b - 2a			

Appendix IV

Results of Analyses of Variance Performed on Five Subsets of the Data

Note: Certain aspects of these analyses make interpretation of F-ratios difficult. Each cell contains the five bids made by a single S to a single bet across replications, for several bets. Thus a typical cell has 15 entries, the responses to three different bets. In addition to this complication, the data shown in Table 7 of the text indicate considerable heterogeneity of variance within some parts of the data. For these reasons, the following summaries do not include F-ratios, because such ratios would be tenuous and misleading.

The unusually large Mean Squares for some main effects, however, indicate that conclusions can be drawn confidently from these analyses without precise estimation of F-ratios and their probability values.

(A) **EV** x **V** for all bets with probability = 1/8, 2/8, 5/8 and **Sk** = 0.

Source of Variation	d.f.	Mean Square
EV	2	309.34
V	3	1.673
Subjects	11	2.249
EV x **V**	6	.100
EV x S	22	.300
V x S	33	.236
EV x **V** x S	66	.055
Within Cells	1296	.085
Total	1439	

(B) **EV** x **Sk** for all bets with probabilities = 1/8, 2/8, 5/8; **V** = 0.25.

Source of Variation	d.f.	Mean Square
EV	2	526.410
Sk	4	.123
Subjects	11	.955
EV x **Sk**	8	.078
EV x S	22	.234
Sk x S	44	.064
EV x **Sk** x S	88	.019
Within Cells	2340	.023
Total	2519	

(C) EV x Probability for all bets with V = .25 and Sk = 0.

Source of Variation	d.f.	Mean Square
EV	2	337.950
Probability	4	.043
Subjects	11	.661
EV x Pr	8	.135
EV x S	22	.175
Pr x S	44	.029
EV x Pr x S	88	.028
Within Cells	1440	.025
Total	1619	

(D) V x Sk for all bets with EV = 0 and probabilities = 1/8, 2/8, 5/8.

Source of Variation	d.f.	Mean Square
V	3	4.473
Sk	4	.598
Subjects	11	6.797
V x Sk	12	.155
V x S	33	.532
Sk x S	44	.528
V x Sk x S	132	.077
Within Cells	3120	.081
Total	3359	

(E) V x Probability for all bets with EV = 0 and Sk = 0.

Source of Variation	d.f.	Mean Square
V	3	2.997
Probability	4	.153
Subjects	11	6.648
V x Pr	12	.032
V x S	33	.614
Pr x S	44	.285
V x Pr x S	132	.042
Within Cells	1920	.085
Total	2159	

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The existence of several bets all having the same probabilities, EV, V, and Sk is a powerful feature of the experimental design. It permits a direct assessment of the value of the moment functions as a useful set of predictive variables. If the variation of S's response within such a set is as large as the variation between such sets, the moment functions will be judged inadequate, unless Kurtosis, the next such function, is related to the variations in response.