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THE EFFECTS OF TWO INSTRUCTIONAL APPROACHES ON THE LEARNING OF ADDITION AND SUBTRACTION CONCEPTS IN GRADE ONE

by
Arthur Frank Coxford, Jr.

A dissertation submitted in partial fulfillment
of the requirements for the degree of
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1965

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CHAPTER I

THE PROBLEM

Learning mathematical concepts is the sine qua non of mathematics education at all levels of mathematical instruction. For some time mathematics courses in colleges have emphasized learning fundamental concepts, and recently scholarly reports (88, 98, 99) have emphasized teaching fundamental mathematical concepts at the secondary school and at the elementary school levels. One of the major objectives of this emphasis is to foster a continuous growth of mathematical concepts from the beginning to the end of formal mathematical education.

Among the major mathematical concepts which comprise the elementary school curriculum, the operation of subtraction has been one of the most difficult for young children to use and to explain. The evidence indicates that subtraction as taught is not easily generalized to include all appropriate instances (46) and that computations made in solving subtraction examples are more difficult to explain than those made in connection with the other three fundamental operations (13).

One of the difficulties in generalizing subtraction could be that the original introduction of the concept may itself lack the generality needed later. Inhelder (19) has suggested that young children should be taught general mathematical concepts in order to facilitate learning varied applications of those concepts. She suggested also that general concepts could be understood by young children when the concept was studied through manipulative materials. Ausubel (5) agreed with Inhelder's general thesis of providing a conceptual basis for specific later learnings but suggested that the time spent in doing it might be prohibitive at the primary level.

Another factor that could affect the difficulty encountered with subtraction is the time the learner symbolizes the operation.

Hendrix (56) suggested that immediate symbolization of a concept could interfere with its formulation as measured by utilization. She found that children who did not symbolize their generalizations were more able to apply the concepts in unfamiliar situations than were the children who made an immediate symbolization of the concept. These results were corroborated by Haslerud and Meyers (53).

The problem for this study grew from the current emphasis on mathematical concepts, the fact that children encounter difficulty with subtraction as taught, the suggestion that the generality of the concept taught could facilitate later learning, and the suggestion that the time a concept is symbolized could affect attainment of that concept. The problem for this study is summarized with the following questions.

How does an instructional approach which presents subtraction as a more general concept than is true in traditional instruction affect the arithmetic achievement of first grade pupils?

What are the effects of immediate versus delayed symbolization of addition and subtraction concepts relative to the instructional approaches?

Two approaches to subtraction were taught in grade one for a full school year, and the time of symbolization was varied also. The control or traditional approach to subtraction in the study was the Take-Away approach (TA). Removal of a subset from a more inclusive set was the concrete manipulation used to introduce subtraction in the control approach, and the relationship between addition and subtraction was not explicitly employed.

The experimental treatment consisted of a more general approach named the Part-Part-Whole approach (PPW). This approach to addition and subtraction was more general in character than the TA method because it emphasized the relationship between addition and subtraction explicitly and because the applications of subtraction were logically more easily related to the operation of subtraction. This approach was suggested by the work reported by Piaget (37). It was based on the fact that, when a finite set is partitioned into two subsets,

the number of members in the two subsets or parts is the same as the number of members in the set before partitioning When any two of these quantities are known, the third is uniquely determined. For example, when the two parts are known, the number of the whole set is found by adding the numbers of the parts; and when the whole set and a part are known, the other part is found by subtraction. Subtraction was introduced as the operation which asked that the other part be found when the whole and one part were given.

There was some question of whether or not general number concepts could be taught to young children because of the distinctive way in which young children form concepts. Results reported by Suppes (107) suggested that children tended to be very specific in their learning and therefore would tend not to relate the concepts of addition and subtraction Also young children have relatively few concepts (113). Gagne and Paradise (43) pointed to the importance of prior learned concepts in learning new concepts. However, Inhelder's contention (19) strongly suggested that general concepts could be effectively taught to young children, and the great emphasis on concept learning at all levels of mathematical instruction implied there was a need for experimentation to determine whether learning general concepts could be accomplished and with what results.

The specific questions this study sought to answer with respect to the approach and the time of symbolization variables are as follows:

1. How do the instructional approaches and/or the times of symbolization affect the first grade child's
   a. achievement in addition and subtraction computation,
   b. capacity to solve subtraction applications,
   c. capacity to solve unfamiliar subtraction examples,
   d. achievement in general problem solving, and
   e. total achievement in arithmetic?
2. How is achievement affected by sex?
3. How do the instructional approaches and/or the times of symbolization affect the first grade child's attainment of number concepts?
4. Which approach and/or time of symbolization procedures are conducive to positive feelings about arithmetic?

The remainder of the report of this study is divided into four chapters. In Chapter Two there is a summary of literature related to the problem. Chapter Three contains the detailed description of the experimental design, the variables, and the methods of data collection and statistical analysis. The analysis of the data collected is done in Chapter Four. Chapter Five concludes with the summary and conclusions drawn from the data analysis and the observations made in the classrooms.

CHAPTER II

RELATED LITERATURE

This chapter contains a summary of literature which is related to the problem of this study. The literature is discussed in two sections. The first includes the literature directly related to subtraction. The second contains literature pertinent to the problem of concept learning.

Subtraction

Significant research dealing with subtraction is not abundant in the accumulated literature of the past fifty years. Most studies have dealt with the subtraction algorithm preferred by students or with the algorithm that brings best results as measured by rate and accuracy (118). Three subtraction algorithms have been researched extensively: decomposition, equal additions, and the additive or Austrian method.

The three algorithms are used when, in the n^{th} place of the minuend and subtrahend, the digit b_n in the subtrahend names a number greater than a_n, the number named in the minuend. In the use of the decomposition algorithm the portion of the minuend in question, $a_{n+1}(10^n) + a_n(10^{n-1})$, is rewritten as $(a_{n+1} - 1)(10^n) + (10 + a_n)(10^{n-1})$, and the subtractions $(10 + a_n) - b_n$ and $(a_{n+1} - 1) - b_{n+1}$ are performed. In the notation which an elementary school pupil would encounter, the decomposition algorithm proceeds as follows:

```
 52        12 - 7 = 5, write 5;
-37        5 becomes 4;
           4 - 3 = 1, write 1.  Answer: 15
```

The equal additions algorithm has a slightly different format. The portion of the minuend $a_{n+1}(10^n) + a_n(10^{n-1})$ is added to 10^n yielding $a_{n+1}(10^n) + (10 + a_n)(10^{n-1})$. Also 10^n is added to the part

of the subtrahend $b_{n+1}(10^n) + b_n(10^{n-1})$ yielding $(1 + b_{n+1})(10^n) + b_n(10^{n-1})$. The subtractions $(10 + a_n) - b_n$ and $a_{n+1} - (1 + b_{n+1})$ are performed. In the usual notation the same problem given above would be done as follows employing the equal additions method:

52 — 12 - 7 = 5, write 5;
-37 — 3 becomes 4;
5 - 4 = 1, write 1. Answer: 15

In the additive or Austrian algorithm the first two steps in the solution are the same as in equal additions. Thereafter the process becomes one of addition, namely, what must be added to b_n to yield $10 + a_n$, and what must be added to $1 + b_{n+1}$ to yield a_{n+1}. Using the additive algorithm the problem above would be solved as follows:

52 — 7 + 5 = 12, write 5;
-37 — 3 becomes 4;
4 + 1 = 5, write 1. Answer: 15

Studies dealing with the subtraction algorithm began with Ballard (7) in England. The algorithms compared were equal additions and decomposition using a sample of 9,176 boys and 9,502 girls with an age range from eight through fourteen years. Ballard's results indicated that the equal additions children of both sexes and at all age levels performed with greater accuracy and speed. At the eight year level the equal additions group was approximately forty per cent faster in solving subtraction problems. This advantage steadily decreased as the age of the children increased until there was a ten per cent advantage for the equal additions group at age fourteen.

Mead and Sears (84) worked with two second grade groups. Each group was instructed for four months with one group using equal additions and the other group using decomposition. The differences in achievement scores were small, but they favored the decomposition algorithm. The authors also noted that the equal additions children tended to add in subtraction examples instead of subtracting more often than the decomposition children.

In England McClelland (82) studied 143 thirteen year olds, sixty-three of whom had learned subtraction using equal additions and eighty of whom had learned subtraction using decomposition. He compared

the groups with respect to accuracy, speed, and adaptability to new conditions. In all three categories the equal additions group were superior, being twenty per cent faster and making a lower per cent of errors, the error rates being 6.7% for equal additions and 9.7% for decomposition children. McClelland had the children practice on their method intensively for ten weeks. A test was given regularly every week, and at the end of the ten weeks the decomposition children surpassed the equal additions children in speed while the accuracy of both groups remained nearly constant.

Buckingham (21) compared grade school children in seven instructional centers. Each center had an additive group and a decomposition group which began learning the subtraction facts when the teacher decided they were ready. The author tested the children before they began the study of subtraction and retested when the work on subtraction had been completed for each group. Buckingham reported that in six of the seven centers the gains of the decomposition group surpassed the gains of the additive group. It was reported that none of the gains was statistically significant. In other research Taylor (111) had results favoring the decomposition algorithm over the additive, Osburn (90) the additive algorithm, and Johnson (63, 64) the additive and equal additions algorithms over decomposition.

It appears, then, that the results on the subtraction algorithm are very inconclusive and are, perhaps, of questionable reliability (100). Further, the major criteria for choosing one method over another in most studies were the rate and accuracy of the children. As Brownell and Moser (18) pointed out, these factors are important, but it is questionable to use them as the sole basis of decision. Other important criteria suggested were transfer value, ease of initial learning, applicability, and naturalness.

In 1942 Brownell and Moser (18) undertook a study of the relative merits of the decomposition and the equal additions algorithms. Three experimental centers of twelve, twelve, and seventeen third grade classes were used in the experiment. In each center four different treatments for teaching "subtraction with borrowing" were applied. The four treatments were obtained by the incorporation of a new variable dimension, the method of instruction, in the study of the subtraction

algorithm. The two levels of the instruction variable were "meaningful" and "mechanical" instruction in subtraction. In this context "meaningful" meant "through rationalization" and "mechanical" meant "rule and drill." The four experimental groups in each center were: (1) Decomposition-Mechanical (DM), (2) Decomposition-Rational (DR), (3) Equal Additions-Mechanical (EAM), and (4) Equal Additions-Rational (EAR).

To evaluate the treatments Brownell and Moser used six basic criteria. These criteria were measures of (a) accuracy, (b) rate, (c) transfer, (d) smoothness of performance, (e) understanding, and (f) retention. In all there were sixteen measures of achievement since the criteria a and b were measured immediately, in connection with criterion f and in connection with criterion c, and criteria d and e were measured initially and six weeks later.

The four methods of instruction were taught for a period of three school weeks in each center. Following this teaching period the children were tested by group tests and by interview techniques, given a six week period filled with only minimal maintenance work on borrowing, and retested for retention purposes.

As in earlier studies the results were not clearly in favor of any one approach. In comparing DR and DM, Brownell and Moser reported in favor of DR in most evaluative areas. In comparing DR and EAR, the experimenters found in favor of DR but not as unequivocally as in the DR and DM comparison. In the DR and EAM comparisons, two of the three centers were clearly in favor of DR, while the third center had seven areas favoring EAM and nine favoring DR. The discrepancy in the results of the last center was attributed to the fact that the children in that center had had much meaningful instruction in grades one and two and, thus, rationalized the mechanical process on their own. The remainder of the comparisons usually favored the rational group over the mechanical.

Brownell (14) summarized the results by saying that he would favor teaching borrowing by decomposition. He qualified this recommendation with, "This is true, however, only (a) when D [decomposition] is taught rationally and (b) when understanding and ability to transfer are regarded as important learning outcomes." (14:170)

The work of Brownell and Moser effectively ended the experimental

investigation of the subtraction algorithm. Researchers subsequently focused their attention on the problem of determining procedures which would facilitate learning of the decomposition algorithm and began to investigate the use of subtraction in applications.

The basis of the latter research was the three types of applications which can be solved using subtraction. The types of applications are commonly known as (a) the take-away, (b) the missing addend, and (c) the comparison cases of subtraction. An example of each type follows.

(a) Tommy collected sixteen eggs. He fell down and cracked seven of them. How many of Tommy's eggs were not cracked?

(b) Mary wanted to buy an ice cream cone which cost twelve cents. She has eight cents. How much more money does she need?

(c) Joseph had five pencils and seven erasers. How many more erasers than pencils did Joseph have?

Each of the types of practical problems above lends itself to solution using subtraction since, in each case, what is required is the number, not of either named set, but of a part of one of the sets or a subset. In the example for a, the part of the set desired is the part that is not cracked; for b, the part that is desired is the money she does not have; and, for c, the part that is required is the subset which is not used when the pencils and the erasers are matched in a one-to-one fashion.

An objective of mathematics instruction in the primary grades is to instruct the children in the use of subtraction in the three types of applications. This is not an easy task since primary children do not, in general, think of each of the practical problems a, b, and c as situations calling for subtraction. Instruction must, therefore, give careful consideration to the child's ways of thinking about the applications, as well as to the analysis of the mathematical process to be understood (46).

A well conceived study in the area of children's thinking in the activity of solving subtraction applications was done by Gibb (46, 47, 48). Gibb's (46) study was exploratory in nature. She was attempting to determine whether or not the three subtraction applications elicited the same or different responses from second grade children who had

been taught subtraction by means of taking one set from a second set.

Gibb interviewed the sample of children individually concerning each of the three applications. Three levels of abstraction were used in the problem materials to determine if the level of abstractness had an effect on the child's thinking. The first level of abstraction was the concrete--small toys and other objects were used in the presentation of the problems. The second level was semi-concrete--geometric figure's were pictured on cards to aid in the problem presentation. The third level of abstraction was abstract--the printed story was presented and read to the child in the problem presentation.

In the sample of second grade children, Gibb (48) found, in general, that the children did not think the same way in the three applications. In particular, the take-away problems were the easiest, were done in less time, were most readily associated with subtraction as taught, and were most accurately done. Gibb reported, "When taught the process of subtraction as a take-away process, it is much easier to use it in solving problems which are directly solved by taking a group away." (48:9)

The missing addend examples were solved more commonly as applications of addition. The comparative examples were most difficult to comprehend, were most often incorrectly answered, and were solved by addition and subtraction in nearly equal numbers. Finally, regarding the material used in the problems, the best results were obtained with the semi-concrete materials, followed by the concrete materials, and then the abstract, written description.

Gibb's results indicate that when the approach to subtraction was via take-away, children did not readily form the connection between subtraction and the missing addend and comparison applications. However, the take-away applications were well handled when taught subtraction via take-away. The better results with semi-concrete materials motivated the use of such materials in the Concept Attainment Test used in the present study. (See Appendix A.)

In a study of more limited scope, Schell and Burns (104) studied twenty-three second grade children to determine which of the three subtraction applications were most successfully worked and which

type was deemed most difficult by the children. Another aspect of the study was the determination of the degree of understanding the children had concerning each application.

Schell and Burns reported that the order of difficulty (difficult to easy) was comparison, missing addend, and take-away but that the differences for the total group, for the boys, and for the girls were not statistically significant. Both boys and girls deemed the take-away application the easiest but differed in the ranking of the hardest; the boys ranked comparison the hardest while the girls put missing addend applications first in difficulty. Finally, the boys and girls showed a lack of understanding of the comparison and missing addend problems. The authors suggested that further work should seek means of improving the child's understanding of the missing addend and comparison applications.

Addition and subtraction are inverse operations. The learning of one would seem to imply a facility with the other. Why, then, asked Eifermann and Etzion (33), did adults find subtraction more difficult than addition? The authors hypothesized that the way addition and subtraction were usually taught favored facility with addition over subtraction. Subtraction was less repeatedly and thoroughly trained. To test this hypothesis, a system isomorphic to the numbers one through ten was constructed, and adult subjects were trained in the composition of the system. It was reported that although addition was easier than subtraction in the isomorphic system, it was not significantly so. However, when these subjects solved the same addition and subtraction problems in the usual notational system, addition was significantly easier than subtraction when measured by time per correct response. Some of the subjects did make the shift from the experimental system to the standard notational system. However, the authors suggested that the same type of procedure could be tried with children and called for a method of teaching addition and subtraction which favored neither operation.

The results of Eifermann and Etzion supply some evidence indicating that instructional procedures in addition and subtraction may be biased in favor of addition. A similar analysis of instruction in the three types of subtraction applications indicates that the

take-away applications may be preferentially trained since subtraction is usually taught by means of concrete manipulation of taking away and "meaning" is supplied to the operation by appealing to the take-away application verbal problem. Luchins (79) has suggested that preferential instruction in one method produces mechanization which causes the learner to ignore differences in problems.

Recently Brownell (13) reported a study of the effects of instructional procedures on conceptual maturity in the four fundamental processes of arithmetic. The study was carried out in Great Britain in two parts: one in Scotland and the other in England. The sample included 1,406 seven and one-half to eight and one-half year old children who were completing the third grade. The three instructional procedures used in the three years prior to the testing program were the Cuisinaire, the Dienes, and the Conventional systems. Briefly, the Cuisinaire system used small multicolored sticks and based instruction on the idea of ratio; the Dienes system used specially designed blocks (e.g., longs, flats) and the basic approach was through child discovery; the Conventional program used various manipulative materials and was typically similar to many of the programs in the United States of a few years ago.

The method of testing was the individual interview in which the child was given eight number combinations (two of each process) orally, four pencil and paper computations (one for each process), twelve verbal problems (three for each process), and finally the eight original number combinations again for which the child was asked to "think out loud" while solving them. The criteria included rate, accuracy, level of performance, and the ability to explain procedures used.

Of the myriad of results reported only two will be included here. The first concerns the verbal problems in subtraction for the comparison of the Cuisinaire and Conventional systems in Scotland. Brownell reported, "For both groups, explanations of solutions were much more difficult for subtraction than for addition problems, especially for subtraction problem 5 which requires a comparison . . ." (13:80) It appears that neither approach solved the problem of the difficulty arising from the different types of subtraction applications.

The second result concerns the thought processes which the English children showed when "thinking out loud" while doing the eight number combinations. The comparison involved the three instructional approaches, and Brownell reported that "In ways of thinking answers for the number combinations the Con. [Conventional] subjects revealed a considerably higher degree of conceptual maturity than did the Cus. [Cuisinaire] and the Dien. [Dienes] subjects . . ." (13:192)

Brownell's conclusion was derived from the fact that when the per cents of meaningful habituation responses (i.e., the children could demonstrate the method used to arrive at a solution after they had replied promptly to the example) for the four processes were considered as a whole, the Conventional children attained a much higher value.

However, when the per cents for each process were compared, the Conventional children exceeded the Cuisinaire and Dienes children greatly for only three of the processes, the notable exception being subtraction. For subtraction the per cents of meaningful habituation responses for the three approaches (Cui., Dien., and Con.) respectively were four, two and five. Although the Conventional program surpassed the others for three processes, the three approaches were equally ineffective for subtraction.

Concept Learning

Undoubtedly Jean Piaget is the most prolific researcher in the area of concept development. The concept of number (92) was extensively studied, and the manner in which children reached a complete understanding of number was fully reported. The Swiss psychologist's emphasis in these investigations has not included the usual number learning topics of the classroom which have been reported extensively elsewhere (32, 58, 65, 86, 87, 96, 102). The major thesis of Piaget's work on number was that for number to be understood, certain concepts of conservation had to be attained. For example, three of the concepts of conservation were the conservation of discontinuous quantity, the conservation of one-to-one correspondence, and the conservation of the whole upon subdivision.

For each subconcept of number Piaget reported a series of typical child reactions arranged in broad age ranges or stages. These stages

came into being because, at similar ages, the reactions observed for each subconcept were very similar. The author reported that, on the average, each subconcept of number was attained by the children at the age of six to seven years. The conclusions were that all the subconcepts of number were attained at approximately the same age and that each child proceeded through qualitatively distinct stages of intellectual development for each subconcept. As Beard (8) and Ausubel (5) pointed out, the important result of Piaget's work was the articulation of a sequence of discernible stages of development for number concepts and not the age ranges which may vary from culture to culture.

Piaget (94, 78) is also of the view that thought is dependent upon actions. Inhelder said, ". . . the development of knowledge seems to be the result of a process of elaboration that is based essentially on the activity of the child." (62:20) In Piaget's "sensorimotor" period the beginnings of intelligent behavior are seen in the actions of the child. In the second or "pre-operational" period the child begins to internalize his actions, but they are not yet mentally reversible. The third period of "concrete operations" sees the child attain the ability to reverse his actions--actions such as combining, ordering, and setting up correspondences come to be done mentally and can be reversed mentally. Ultimately, the child reaches the final period of "formal operations" which is characterized by hypothetical reasoning. Thus it is reported that the child moves from actions, to internalized actions, to internalized, reversible actions, to hypothetical thinking (94). Flavell summarized well: "According to Piaget, actions performed by the subject constitute the substance or raw material of all intellectual and perceptual adaptation. . . . The overt, slow paced actions of the neonate eventually get transformed into lightning-quick, highly organized systems of internal operations." (37:82)

Flavell (37) reported a recent study (4) from the Geneva school. The experiment consisted of showing the child two rows of buttons which corresponded. One of the sets was then divided into two or more subsets, and the child was asked if the sets were still equivalent and on what basis. The responses were categorized on the basis of whether the child conserved number, needed to count to do so, or did not conserve

number. A highly intriguing aspect of the results was that the children seemed to show "progressive arithmetization" in their understanding of number. For smaller numbers some children could handle the conservation consistently, but for greater numbers these same children could not handle it with the same consistency. Flavell remarked, "Thus, curious as it sounds, it may make sense to think of a child as being in stage X for, say, numbers 1-15, in stage X - 1 for numbers 15-30, and in stage X - 2 for numbers greater than 30." (37:359-360)

Hood (59) reported that instruction had little effect on child behavior when dealing with Piaget's number concepts. Following an explanation of the necessary equivalence of two sets between which a one-to-one correspondence had been constructed, Hood reported that the children made no progress when retested. It was suggested that for slightly older children instruction could be profitable; results confirming Hood's suggestion were reported by Coxford (28). Inhelder (19), also, stated that it was possible to instruct young children in general number concepts. She pointed out that, "Basic notions in these fields [science and mathematics] are perfectly accessible to children of seven to ten years of age, provided they are . . . studied through materials that the child can handle himself." (19:43)

There are others who question the feasibility of teaching more general concepts to young children. In discussing Inhelder's views, Ausubel (5:124) pointed out that ". . . confirmatory empirical evidence is still unavailable. . . . No general answer can be given to this problem. The decision in each case must be based on the findings of particularized research." The experimental approach to addition and subtraction in this study was designed to be a first attempt to provide some information on the use of general procedures.

While Piaget has made a complex analysis of the development of the concept of number in children, others have studied the child's categorizations in simpler situations. Reichard, Schneider and Rapaport (97) suggested three methods which children used to form concepts: concretistic--formation based on non-essential incidental features, functional description--formation based on function or use, and abstract conceptual definition. For example, given a pear and an apple, a concretistic classification would be "they both have

stems", and a functional categorization would be "they are both edible." In a sorting test based on color and form Reichard et al. reported children below five sorted on only one attribute and could not shift to the other. By age seven or eight the two group sorting was predominant, and above eight seventy-five per cent could make the shift.

In studying young children and their ability to form concepts on different levels of abstractness, Welch and Long (116) reported that the children found it easier to learn a concept when the level of abstractness was constant than when two levels were attempted simultaneously. For example, the children found it easier to learn the names of two noncongruent triangles and a rectangle than to learn the names of the two triangles and the class name for triangles. In this case concept learning was equivalent to name learning. In a second report Long and Welch (76) indicated that for eight to eleven year olds the ability to use the learned concepts in reasoning exercises decreased as the level of abstractness increased.

In a series of experiments carried out at Stanford University, Suppes and Ginsberg (109) taught first grade children concepts involving sets. The children were to learn when two sets were identical, ordered, and equivalent. The children first learned one of the concepts, and then a second one was learned with various orders for the two learning tasks. It was reported that equivalence was the most difficult concept for the children, but more importantly, perhaps, in no case did the learning of one concept facilitate the learning of the second. Suppes and Ginsberg concluded that the young child's learning tended to be very specific.

In other studies reported by Suppes (107) it was found that a gradual withdrawal of a discriminative cue led to greater learning of the test concept than when there was no such cue or when the cue was not withdrawn. The authors concluded that incidental learning was not an efficient method for young children. In a second study it was reported that children who made an immediate overt correction response performed significantly better than those who made no such correction response. Attention was called to the fact that this result was contrary to results for adults (24) but agreed with results for lower organisms (60).

Kendler and Kendler (73) reported a study of reversal and nonreversal shifts in kindergarten children. The children who had learned the first discrimination fastest initially learned the reversal faster than the nonreversal shift; the slow initial learners learned the nonreversal shift faster than the reversal shift. The former result is consistent with adult learning results in the same type investigation (69); the latter result is consistent with that of animal learning, for ". . . a reversal shift in a simple discrimination situation with animals produces negative transfer effects." (68:173) These results point to the specificity of child learning which may hamper the learning of a relationship between two concepts; i.e., addition and subtraction.

In the area of concept attainment of mathematical ideas Gagné (39, 40) and Gagné and Paradise (43) suggested a distinction between "productive learning" and "reproductive learning." In the latter ". . . performance is usually measured on the specific task that is practiced during learning," while in the former ". . . measures of terminal performance are designed to demonstrate mastery of the defined class of tasks." (43:1)

Studying productive learning of mathematics Gagné and Paradise carried out several experiments using programmed learning materials as the vehicle of instruction for such varied topics as simple algebraic equations, deriving formulas, and adding integers. The results suggested that when the learner had ample time to complete the program, the end performance became independent of initial ability scores and highly dependent upon the specific subordinate knowledge the children had acquired. Gagné concluded, "Our results imply that there are many, many specific sets of 'readinesses to learn.' If these are present, learning is at least highly probable. If they are absent, learning is impossible. So, if we wish to find out how learning takes place, we must address ourselves to these specific readinesses, . . ." (40:626) Kendler and Vineberg (71) obtained results consistent with Gagné.

The results reported by Gagné and Paradise suggest that the conservation of the whole upon subdivision (37) may be a "specific readiness" for attaining the concept of subtraction.

On the adult level of concept learning Bruner, Goodnow, and

Austin (20) made a clear distinction between concept formation and concept attainment. The former is the process of forming meaningful classes for objects or behaviors while the latter ". . . refers to the process of finding predictive defining attributes that distinguish exemplars from nonexemplars of the class one seeks to discriminate." (20:23)

Concept attainment is characterized by a series of decisions for Bruner et al. Some of these decisions are whether or not to try to learn the concept, what method to use, what to do if a hypothesis proves false, and what are the consequences of changing the hypothesis completely or in part. "If you will, then," summarized the authors, "the steps involved in attaining a concept are successive decisions, earlier ones of which affect the degrees of freedom possible for later decisions." (20:54) .

Bruner et al. also enumerated five conditions affecting concept attainment: the definition of the task, the nature of validation, the nature of the instances encountered, the consequences of specific categorizations, and the nature of imposed restrictions. Further, Hunt (61) suggested that it is not always justifiable to assume that a person sees the stimuli in the same way as the investigator.

Bruner et al. suggested that increasing the memory load in concept attainment tasks resulted in fewer solutions than when the memory load was reduced. Kates and Yudin (66) also found that increasing memory load resulted in an increased time for the adult subjects to attain a concept. Bourne (10) confirmed these results.

In another study Bourne (9) reported that increasing the complexity of the task by increasing the number of irrelevant attributes led to greater error rates during the attainment of a concept. Fredrick (38) found that increasing the complexity of the stimulus array by increasing the number of relevant attributes resulted in decreased ". . . efficiency of concept attainment in terms of a time-to-criterion measure." (38:4) Fredrick also reported that increased memory load had the same effect.

There is evidence that the complexity of a concept attainment task increases the time needed to attain a concept. Two variables affecting task complexity are memory load and relevant-irrelevant

attributes. In one approach to subtraction in this study the concept which the child had to learn in order to do subtraction was more complex by either of these criteria. The stimulus array was more complex because there were two subsets visible, and the memory load was increased since the child had to distinguish between the part already known a d the part desired by remembering how many members were in the given part.

Summary

The research in subtraction indicated that the decomposition algorithm was most effective when taught rationally, that young children did not use subtraction in all cases where applicable with equal frequency, that three different instructional systems were equally ineffective in fostering conceptual maturity in using subtraction and that difficulties in subtraction could be the result of preferential addition training. The research in concept learning suggested that number concepts develop in qualitatively distinct stages, that concepts grow out of actions,and that for children there may be a "progressive arithmetization" of certain number concepts. Also it was reported that appropriate instruction in number concepts advanced young children's knowledge, and it was suggested that general number concepts could be taught and be beneficial to children as a basis of further learning. Studies dealing with learning of mathematical concepts by children indicated that acquired subordinate knowledge increased the probability of learning terminal concepts, that child learning tended to be quite specific, that incidental learning was not an efficient method for young children, and that there was a discrepancy between the results for child learning and adult learning of concepts. On the adult level evidence was cited which indicated that task complexity decreased efficiency in attaining concepts.

CHAPTER III

EXPERIMENTAL DESIGN AND IMPLEMENTATION

This chapter is divided into five sections. In the first section, Rationale, the manipulated variables are fully discussed and the differences illustrated. The second section, Design, deals with the topics of population and the independent and dependent variables, while in section three, Data and Instrumentation, the instruments used are described. The procedures used in the classrooms are discussed in section four, Instructional Procedures, and section five, Statistical Methods, contains a brief description of the procedures used in the analysis of data.

Rationale

The major variable defining the instructional programs used in this study was the approach to subtraction. The two levels of the approach variable were the Part-Part-Whole (PPW) approach and the Take-Away (TA) approach. Each approach involved a different manipulation of concrete materials and a different outcome of the manipulation. For the TA approach the task of the child was the removal of a given subset from a specified set and the determination of the number of the subset which remained. The task involved in the TA approach reduced the chance of confusion on the part of the child to a minimum, for the only thing left at the end of the process was the required subset.

The PPW approach arrived at the same end result, but the manipulative process did not specifically pick out the required subset for the child. For a given set and subset the task involved in the PPW approach was the partitioning of the given set so that one of the two subsets obtained was the given subset. The other subset obtained in the partitioning was the required subset. The results of the manipulation were more complex because there were two subsets remaining in the perceptual field of the child. The subset to be counted is not obvious to the learner as in the case of the TA approach. The learner

must remember which subset was given, determine the other subset, and count that subset.

The results of the two manipulations described above are shown in Figure 1 along with the initial sets. The pictures in the second row represent what the child would see upon manipulating five counters in the solution of the example 5 - 2 = _____ in the TA and PPW approaches respectively.

	TA	PPW
Given set	0 0 0 0 0	0 0 0 0 0
After manipulation	0 0 0	0 0 0 0 0

Fig. 1.--Given set and results of manipulation in solving the exercise 5 - 2 = _____ by the TA and PPW approach respectively.

Motivation for basing instruction in subtraction on the PPW manipulations rather than the more traditional TA approach had two aspects. In the first place motivation came from the fact that addition and subtraction are inverse operations. In addition two subsets or parts are known and the number of the union of these subsets is desired. Subtraction, on the other hand, requires the determination of a part when the union or whole set and one part are known. Thus in addition and subtraction the same three quantities are involved, the distinction being only in which quantity is to be found from the known quantities. Whenever a part is required, subtraction is employed; whenever the whole is required, addition is used.

Instruction in the PPW classes emphasized the connection between subtraction and addition perceptually as well as verbally. For addition the children were instructed to show the two named parts and find how many in the whole set. More specifically the children were instructed to show the whole set whose number satisfied an addition sentence in its partitioned form, that is, so the parts of the whole were obvious. The result of this manipulation of concrete materials while working addition examples was identical to that obtained while working related subtraction examples. (See Figure 2.)

The language used was designed to reinforce the connection between addition and subtraction also. For addition the term "part" was used for the subsets designated by the addends and the sum was the

number of the "whole set." In the same fashion, the "whole set" was used to designate the set for the minuend and "part" and "other part" were employed for the sets named by the subtrahend and the answer in a subtraction problem respectively. Moreover, sets of three numbers such as $\{1, 5, 6\}$ were given on occasion, and the children were asked to determine the addition and subtraction sentences which could be written using only these numbers.

	PPW Addition	PPW Subtraction
Results of Manipulations	00 000	00 000

Fig. 2.--Results of manipulating concrete materials in solving the examples 2 + 3 = _____ and 5 - 2 = _____ in PPW classes.

In the case of the TA approach addition and subtraction were not so closely related in the instructional procedures. The manipulations involved in subtraction and addition did not result in the same arrangement of the objects. The manipulations for subtraction have already been reported. For addition the manipulations resulted in two sets being produced by the child. The pupil produced these two sets in response to being told to show two given sets and to determine "how many there were altogether" in the two sets; whereas in subtraction the learner was asked to "take-away" a given set and to determine "how many were left." The results of both manipulations are depicted in Figure 3.

	TA Addition	TA Subtraction
Results of Manipulations	0 0 0 0 0	0 0 0

Fig. 3.--Results of manipulating concrete materials in solving the examples 2 + 3 = _____ and 5 - 2 = _____ in TA classes.

Very few attempts were made to connect the operations of addition and subtraction or to connect the concrete manipulations. However, on occasion, sets of numbers such as $\{1, 5, 6\}$ were given to these classes also, and the children were asked to write as many addition and subtraction sentences as possible using only the given numbers.

Summarizing then, in the PPW classes the language and the manipulations made to solve addition and subtraction examples were very similar in both addition and subtraction. Attempts to point out the connection between addition and subtraction were the rule. In the TA classes the language and the manipulations made to solve addition and subtraction examples were not similar. It was the exception when the connection between addition and subtraction was emphasized.

The second motivation for using the more complex PPW manipulation to instruct in addition and subtraction arose from the difficulty children had in applying subtraction to appropriate practical problems (13, 46, 104). The traditional TA approach to subtraction, by its very nature, emphasizes and provides a great deal of practice with the take-away application of subtraction. Every time a child is asked to "take-away" and find "how many are left," practice in the take-away application is occurring. Whether, when presented with a different type of application of subtraction, the traditional TA instruction interferes with the child's ability to use subtraction in the new instance is not known. There is some evidence indicating that such a possibility exists (79). However, the questions were: "Is there an approach which would minimize possible negative transfer or Einstellung effects?" "And, if there is, might not the PPW approach be such a method?"

The PPW approach to subtraction was designed to be a less biased method of instruction with respect to the three types of subtraction applications. In the first place, the language used in the classroom was noncommital. The PPW children were not asked "how many are left?" or "how many remain?", but "how many are there in the other part?", where the other part was the part that was not named in the subtraction sentence. Also in the concrete examples used to introduce subtraction, the emphasis was placed on the idea of partitioning a set rather than on removal of a set as was done in the TA classes. Stories made up to help the children attach meaning to the symbolism also stressed the concept of partitioning a set in the PPW classes.

While the PPW approach was designed not to favor any typical application, at the same time it was designed to be general enough (and specific enough) to be used in all types of subtraction

applications. The basis of this applicability was the fact that subtraction, as taught in PPW classes, was based on the action of finding a part of a set. And every subtraction application asks the same question: "What is the other part?" In the three typical applications, take-away, missing addend, and comparison, the parts desired are "the part left," "the part needed," and "the part which is not used in a one-to-one correspondence" respectively. If the child could be brought to see the unifying "part" idea of all subtraction applications, then the majority of the difficulties arising in the applications of subtraction could conceivably be diminished.

The second variable that was manipulated in this study was the time of symbolizing addition and subtraction statements. The two levels of the time of symbolization variable were early and late symbolization. Early symbolization was characterized by concurrent introduction of addition or subtraction concepts and the symbolic statements representing these concepts. For example, when addition was introduced in an early symbolization class, the statement a = b + c was also introduced at the same time.

Late symbolization was characterized by introducing the symbolic statements for addition or subtraction concepts six weeks after the concepts had been introduced. In the PPW-late symbolization classes the words "split" and "is" replaced the symbols "-" and "=" for six weeks. In the TA-late symbolization class the words "take-away" and "is" were used for "-" and "=" for six weeks. In all the late symbolization classes the words "and" and "is" replaced "+" and "=" for the six weeks following initial introduction of addition.

Design

Six first grade classes participated in the study during the 1964-65 school year. Two of these classes were in each of three elementary schools in Ypsilanti, Michigan. Previous achievement data supplied by the Ypsilanti school system indicated that there were great ability differences between the three schools. Hence, four classes in two of the schools were designated the higher ability classes and were chosen to be participants in the four experimental treatments. The remaining two classes were designated the lower ability classes

and were exposed to two of the experimental treatments. The code name, as defined by the treatment applied, of each class is given in Figure 4 along with the number of students in each class at the conclusion of the experimentation.

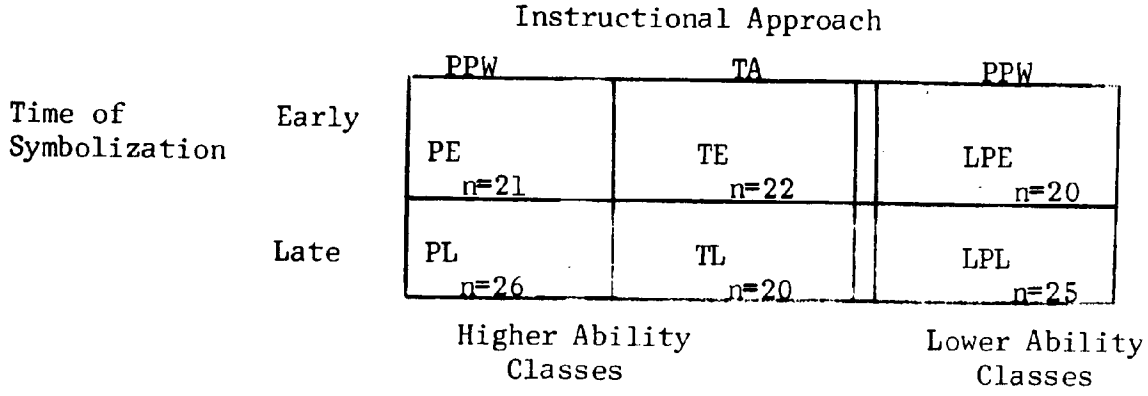


Fig. 4.--Code name, treatment applied and number of subjects in each class.

The classes not only were dissimilar in ability but they also had great differences in cultural backgrounds. Classes LPE and LPL were drawn from a low socio-economic, predominately Negro section of Ypsilanti. The children came from homes of unskilled laborers and domestics and had had few of the advantages of travel, family recreation, or individual artistic endeavors such as dancing or music lessons.

On the other hand, classes PE and TE were drawn from an upper middle class neighborhood. Many of the children had had the experiences not evident for the LPL and LPE children. In addition, the parents were very conscious of the importance of educational experiences and provided books and recordings for the children. The classes PL and TL were drawn from an area which combined features of the other two neighborhoods. Some of the children came from middle class homes and others from the lower socio-economic area of classes LPE and LPL. The number of children from each background was approximately the same. The three schools were quite representative of the population differences in the city of Ypsilanti, Michigan.

This sample of children was chosen because of the heterogeneous characteristics of the city's population, because of the willingness of the school system to experiment, and because there was little contamination of the subjects by previous extensive experimental work or

in-service training. The classes were chosen by the school system from those interested in participating in experimentation.

Variables

The independent manipulated variables were the instructional approach to addition and subtraction and the time when symbolism appropriate to addition and subtraction was introduced. Each independent variable had two levels. (See Figure 4.) The two approaches used were the Part-Part-Whole (PPW) and the Take-Away (TA) approach. The symbolization times were immediate (E), or at the time the operation of addition or subtraction was introduced, and late (L), or delayed six weeks from the times addition and subtraction were introduced.

The use of the two levels of each independent variable in the four higher ability classes permitted comparisons of the effects of the levels of each manipulated variable and, in addition, provided information concerning the relative effectiveness of one level of a variable over the two levels of the other. The two levels of the time of symbolization variable used in the lower ability classes allowed comparisons of the effects of the time variable when the PPW approach was used with less able subjects.

The instructional programs based on the independent variables for the six classes were designed and taught by the author in order to minimize the effects of the teacher variable. The demands of each treatment were adhered to faithfully, and each class received thirty minutes of instruction each school day for the thirty weeks of the experiment. As an aid to instruction each child in each class received a two-wire bead frame of forty beads with alternating sets of five red and five white beads. These manipulative instruments were in the possession of the children at all times and were used frequently to illustrate number concepts. The bead frames were the basic manipulative aids, but other aids such as bottle tops, pocket charts, a flannel board, felt materials, and demonstration counting frames were used when appropriate.

The dependent variables were (1) arithmetic achievement, (2) number concepts attainment, and (3) attitudes toward mathematics. In particular, (1) included achievement in subtraction computation,

addition computation, subtraction transfer, subtraction applications, problem solving, and arithmetic generally. The number concepts of (2) included different forms of counting skills, construction, conservation and use of a one-to-one correspondence and verbally given and pictorially represented subtraction applications.

Data and Instrumentation

Concept Attainment Test--Preparation

The Concept Attainment Test (CAT) was devised by the author to measure the extent to which first grade children could respond consistently to verbal and pictorial stimuli involving number concepts. (See Appendix A.) The development of the CAT was motivated by three factors. The first motive was very specific, namely, an instrument was needed to obtain initial measures of mathematical knowledge of the children in the study. Past achievement records of the students in the three schools indicated that there would be great ability differences between the schools. Hence an initial measure was needed for all children which could be used as the basis of statistical adjustment in the final analysis of data.

The second motive was also quite specific. An instrument was desired which would specifically measure the children's attainment of the PPW relationship. The CAT was designed in part to gain knowledge in this area. Also it was administered before and after instruction, and thus the CAT would provide some indication of the effects of instruction on the attainment of the PPW relationship.

The third motive was more general. If the CAT proved to correlate highly with the first grade children's mathematical achievement, it would be a helpful addition to the testing instruments available for use by the teacher in the first grade. The reasons are twofold; (1) the method of administration of the CAT, the individual interview, allows the teacher to become more intimately aware of the children's thinking and reactions in mathematical situations, and (2) the items included in the CAT are not only the more common counting items, but also there are items dealing with more fundamental concepts of number. The combination of these two types of items could facilitate a deeper understanding of the children's problems and strengths.

The CAT was developed during the nine months prior to the initiation of this study. During the first part of this period tentative item selection, wording, and order of presentation were adopted. With the aid of the staff and the administration of the John D. Pierce School at Northern Michigan University, Marquette, Michigan, the author interviewed thirty-five kindergarten and first grade children using the tentative CAT. Changes in item order and item wording were made at this time. Also the scoring procedure was refined and the average time of administration determined.

During August and early September 1964, final adjustments were made. The major revision was the addition of an item dealing with the three types of applications of subtraction (Item 11). Also two additional forms of the CAT were prepared. Finally the CAT was administered to ten beginning first grade children enrolled in The University Elementary School of The University of Michigan, Ann Arbor, Michigan. The results indicated that the procedure was satisfactory and the time of administration was reasonable for first graders (approximately twenty minutes).

Concept Attainment Test--Content, Materials, and Procedures

The content of the CAT can be conveniently put into three classifications. The first part which includes items one through five is essentially an exercise in different forms of counting; the second part which includes items six through ten is composed of four items dealing with one-to-one correspondence and one item concerning the order relationship between a set and a proper subset, that is, the number of a proper subset of a finite set is less than the number of the set; the last part (item eleven) is devoted to exercises involving the three types of applications of subtraction. These three classifications will be discussed for Form A of the Concept Attainment Test only, since the other forms (Form B and Form C) are essentially the same as Form A in general content.

The first part includes items one through five. Item one is concerned with rote counting and has two parts, 1a and 1b. In 1a the child is asked to count to ten. In 1b the task is counting backwards from fifteen. However, if the child fails in this, he is given

an opportunity to count backwards from ten and then from five. The task of item two is rational counting. The child is asked to count twenty-four green discs arranged in four rows of six each on an eleven by fourteen inch card.

In item three the child is asked to reproduce a set of an appropriate number of objects. He is given twenty red chips and asked to return sixteen of them. Item four consists of two parts, 4a and 4b. In each part the child is asked to identify a set of objects after being told how many members are in the desired set. In 4a three sets of flowers are depicted on an eleven by fourteen inch card, and in 4b three sets of balls are pictured on another eleven by fourteen inch card. Item five is also composed of two parts, 5a and 5b. In each the task is to determine which of two sets of lollypops has more members. In 5a and 5b the two sets of lollypops are depicted on eleven by fourteen inch cards. The task in 5a is the easier of the two since the pictures are of six and seven lollypops. The pictures for 5b contain sets of thirteen and twelve lollypops and are arranged in a random order.

The second part of the CAT includes items six through ten which involve number concepts suggested by Piaget. The first two items deal with one-to-one correspondence, item six requiring the formation of a one-to-one correspondence and item seven requiring the child to conserve the one-to-one correspondence he constructed in item six. For item six the materials are sixteen red chips and ten white chips. The child is given the set of sixteen red chips, and the interviewer puts the ten white chips in a slightly spread-out row before the child and asks the child to produce the same number of red chips.

The manner in which the child does this task is carefully observed. If the child does not make an obvious one-to-one correspondence as a first response, the interviewer asks him to place his chips so that it can be seen that there is the same number of red and white chips. When the one-to-one correspondence is not constructed by the child following the latter suggestion, the interviewer sets it up and asks the child for affirmation of the fact that both sets contain the same number of chips.

As soon as a one-to-one correspondence is constructed during the administration of item six, the interviewer moves directly to

item seven. The interviewer spreads out the row of red chips to a row about twice as long as the row of white ones and asks the child if the two rows contain the same number of chips. If the response is negative, the interviewer asks the child which row has more chips in it.

Item eight is a task which can be accomplished by using one-to-one correspondence or counting. The child is presented with two piles of red chips--one containing fourteen chips and the other containing eight. He is asked to use all the chips to form two new piles which are equal in number. The child is then given time to work on the problem while the interviewer observes the child's method of attack and correctness of solution.

The next item, item nine, deals with the relationship between a finite set and a proper subset. The materials are eighteen red and two white chips. These chips are arranged in two adjacent rows of ten each with nine red and one white chip in each row. The two rows are placed in front of the child, and the child is questioned concerning the red and white chips to make sure that the child sees that the classification of chips includes the white and the red chips. Five questions are then asked to test the child's understanding of the relationship: the part is less than the whole. Each of these questions is worded in a slightly different manner, but all ask the same basic question. The wording of the questions was varied because the preliminary work with children suggested that the wording affected the per cent of correct responses.

Item ten deals with the conservation of the whole upon subdivision. There are two different approaches to the idea of conservation of the whole within item ten. Each of these is composed of two tasks. In the first approach (10a and 10b) a one-to-one correspondence between a row of nine red and nine white chips is constructed by the interviewer. Then one row of the chips is partitioned, and the child is asked if there is the same number of chips in each row. When the question is completed, the two rows are covered with a seven by twenty-two inch card. After the response of the child, the task is repeated with a different subdivision.

The second approach of item ten (10c and 10d) is similar to the first except in this case only one row of eleven chips is used. The

row is placed before the child and is counted by the child. When the child has counted the chips, the interviewer partitions the set and asks the child if the number of the partitioned set is the same as the number of the set he counted. The row is covered at the conclusion of the question. Upon receiving a response the procedure is repeated with an alternative subdivision. (For the complete details concerning wording and exact procedures, the reader is referred to Appendix A.)

The third part of the Concept Attainment Test consists of item eleven. There are three questions which make up item eleven. The first (11a) is a take-away subtraction application. A story is told to the child in conjunction with the presentation of a picture of five apples on an eleven by fourteen inch card. The child is asked to indicate how many apples are left. The second (11b) and the third (11c) questions are a missing addend and a comparison application of subtraction respectively. The same type presentation is used in 11b and 11c as is used in 11a. The scores assigned to each item of the CAT are summarized in Table 1.

Administration of Concept Attainment Test--Interviewers

As indicated earlier the Concept Attainment Test is administered by means of an individual interview. The CAT was administered twice during the study. The first time was the week of September 21, 1964 and the second the week of May 12, 1965. For each administration of the CAT there were six interviewers in addition to the author. These interviewers were trained in the techniques and objectives of the CAT. In all instances the interviewers had studied the procedures, and they were required to observe an interview performed by the author before they were allowed to work independently. Even then, the author was present to answer any question which arose during or after the interview.

The training of the interviewers appeared adequate, but to be statistically sure that the results were independent of the interviewers a Chi-square analysis of independence of classification was performed for each item of the September administration. The classification tested was Interviewer--Author vs. Others. The other dimension was the correctness of response--Correct vs. Incorrect. Twenty Chi-square analyses were performed, and only one was statistically significant

TABLE 1

SCORE GIVEN FOR A CORRECT RESPONSE TO EACH ITEM OF CONCEPT ATTAINMENT TEST

Item	Score
1a	30
1b (15-1)	30
(10-1)	20
(5-1)	10
2	30
3	30
4a	30
4b	30
5a	30
5b	30
Subtotal 1-5	240
6	30
7	30
8	30
9a	6
9b	6
9c	6
9d	6
9e	6
10a	15
10b	15
10c	15
10d	15
Subtotal 6-10	180
11a	30
11b	30
11c	30
Subtotal 11	90
Total 1-11	510

at the five per cent level of confidence. In that one significant case the results of item 11a were dependent on the interviewer. The author obtained more favorable results than the other interviewers taken as a group. However, since there were twenty analyses and the confidence level was set at five per cent, the <u>one</u> significant result is not very important, for it is very likely that <u>one</u> of the twenty

values computed would exceed the critical value. Hence, there is evidence that the results obtained using the CAT were independent of the interviewer when the interviewers were the author and the others taken as a group.

Concept Attainment Test--Reliability

There are three forms of the CAT--Forms A, B, and C. It was desired to determine whether the three Forms were reliable or whether the results were item specific. That is, were the responses due to the way the items were constructed or were the responses to the items independent of the Form of the CAT used?

The procedure used to determine the reliability of the Forms A, B, and C of the CAT is fairly straightforward. First a table of random numbers was consulted and four one digit columns were chosen on four different pages. In reading down the columns A was written each time a one, two, or three was read; B was written for four, five, or six; C was written for seven, eight, or nine; and nothing was written when a zero was encountered. This procedure was continued until there were forty-five A's, B's, and C's written for each column read. Hence there were four columns of A's, B's, and C's, each in random order. This order of A's, B's, and C's was transferred to four sets of forty-five score sheets to be used in the first administration of the Concept Attainment Test. During the week of the interviews the Form of the CAT used with each child was determined by the random order of the A's, B's, and C's written in red on the score sheets in the possession of the interviewer. This technique effectively eliminated the need for any choice of Form to use with a particular child--the Form used was the one indicated by the next score sheet to be used by the interviewer.

When the week of interviewing was completed the total score for each child was put in one of three groups depending upon the Form used. A one-way analysis of variance was performed on the three sets of scores. The results are summarized in Table 2. Since the value of F is less than the critical value of 3.07, there is evidence supporting the contention that the three Forms of the CAT are reliable.

TABLE 2

ANALYSIS OF VARIANCE OF THE INDIVIDUAL SCORES FOR THE THREE FORMS OF THE CONCEPT ATTAINMENT TEST. INITIAL ADMINISTRATION.

Source of Variation	df	SS	MS	F
Between Groups	2	7,178	3,589.0	0.42
Within Groups	141	1,200,744	8,515.9	
Total	143	1,207,922		
$F_{.95}(2,143) = 3.07$				

Achievement Measures

Three instruments were used to measure the mathematical achievement of the pupils. The Stanford Achievement Test, Primary I Battery, Form W was used to assess the pupil's achievement in addition, subtraction, and problem solving. The battery is designed for use from the middle of grade one to the middle of grade two. Test Six, Arithmetic, is made up of three sections: Part A, Measures; Part B, Problem Solving; and Part C, Number Concepts. The Stanford Test was administered by the author on May 18-20, 1965, following the instructions in the test manual, but only Part B and Part C were used in the analysis of the data.

Part B, Problem Solving, consists of eighteen problems including examples of all four of the fundamental processes of arithmetic. Part C, Number Concepts, is made up of thirty-two items. There are ten addition sentences to solve, ten subtraction sentences to solve, and twelve miscellaneous excercises involving such ideas as counting, order, numeral writing, et cetera. For purposes of analysis Part C was divided into the addition, subtraction, and miscellaneous sections.

Reliability data is available only for Test Six, Arithmetic. For this test the split-half reliability coefficient corrected by the Spearman-Brown Prophecy Formula was 0.95 with a standard error of measurement of 1.0 grade scores.

The author-constructed Subtraction Applications and Transfer Test (SATT) was used to test the pupil's ability to solve subtraction applications and unfamiliar subtraction sentences. (See Appendix B.) The applications section of the test consists of six written

subtraction applications and two written addition applications. In the construction of this section three known subtraction facts were randomly assigned to the three types of applications of subtraction. Thus there were three applications involving known subtraction facts. These three problems and one addition application using a known addition fact were randomly assigned an order for presentation during administration. The same procedure was used in the construction of four problems using addition and subtraction facts which were not studied. In scoring one point was given for each correct answer to a subtraction problem, and the addition examples were not scored.

The transfer section of the Subtraction Applications and Transfer Test (SATT) was constructed in a very similar fashion. Seven subtraction sentences and one addition sentence which had not been studied by the children in the study were randomly assigned an order for presentation during administration. Then an addition and a subtraction fact which were well known were placed at the beginning of the randomly ordered list of unknown sentences. Thus the transfer section of the SATT consists of seven test problems and three filler problems. Only the seven test subtraction problems were scored, and one point was given for each correct response.

The SATT was administered in a group situation to each class during a forty-five minute period on May 10 and May 11, 1965. The children were provided with a test booklet and a piece of scratch paper. The author read each application problem once and allowed the children approximately one minute to work on each of the first four applications (known facts) and two minutes for each application involving the unknown facts. The children were allowed to work on the transfer section for approximately twenty minutes. Throughout the SATT administration the children were encouraged to use the scratch paper to help them solve the examples.

Intelligence, Attitude Measures

The Lorge-Thorndike Intelligence Test, Primary Battery, Form A was used to measure the mental ability of the children. The test was administered by the regular classroom teachers in October. In four classes the administration date was October 22, 1964; in one class the

date was October 23, 1964; and in one class the date was October 27, 1964. The measure used in this study was the IQ derived from the raw scores obtained on the Lorge-Thorndike.

The instrument used to measure the attitudes of the first grade children was the Arithmetic Attitude Scale developed by Harshman (52). The method of administration and scoring followed that described by Harshman (52:29). The Arithmetic Attitude Scale was administered three times to each class in the study. The dates of administration were November 3, 1964, February 3, 1965, and April 28, 1965. The mean attitude score for each class was derived from the average of the three scores for each child.

Instructional Procedures

The six first grade classes which were instructed by the author using the two approaches to subtraction and the two times of symbolizing addition and subtraction concepts were PPW-Early (PE), PPW-Late (PL), TA-Early (TE), TA-Late (TL), low ability PPW-Early (LPE), and low ability PPW-Late (LPL). The instructional procedures used with three groups made up of PE and PL, TE and TL, and LPE and LPL, denoted P, T, and LP respectively, will be discussed concurrently in order to facilitate making comparisons.

Differences within the groups P, T, and LP were almost entirely language and notational in nature and can be discussed apart from the other procedural aspects. The early symbolization PPW and TA classes (PE, LPE, and TE) used the symbols "+", "=", "-", the words "addition," "subtraction," and the phrases "addition sentence" and "subtraction sentence" from the time either addition or subtraction was introduced. Also the addition and subtraction sentences were always written in the form "A + B = C" and "C - B = A."

The late symbolization classes (PL, TL, and LPL) used the inclusive phrase "number sentence" and words for the mathematical symbols "+", "=", and "-" for six weeks before appropriate symbolism was introduced for addition and subtraction and before the words "addition," "subtraction," and the phrases "addition sentence" and "subtraction sentence" were used. The words used in all these classes in place of "+" and "=" were "and" and "is." The addition sentence

was written "A and B is C." The phrase used for "-" in the class TL was "take away." The subtraction sentence was written "C take away B is A." The classes PL and LPL used the word "split" for the minus sign. The subtraction sentence was written "C split B is A."

Weeks One through Eight

Group LP

The major emphasis during these weeks was on introducing the numbers one to five to the classes. In order to do this several concepts were needed. The first of these was the concept of a set. The idea of a set was developed through the use of concrete materials and by using the term "set" in familiar settings. For example, the children's first contacts with the term "set" were in connection with a set of dishes, a set of chairs, a set of golf clubs, and a set of glasses. From the familiar beginnings the idea of a set being any collection of objects was developed.

In addition to set other concepts relating to sets were studied. The concepts of subset and member of a set were introduced as a part of a set and any one of the objects making up a set respectively. For a sample worksheet on sets, see Worksheet 1, Appendix C.

The idea of a one-to-one correspondence between two sets was also necessary for the development of the concept of number. One-to-one correspondence was introduced through the idea of matching. For example, two sets of felt objects were put on a flannel board by the children. Then children were asked to choose a member of each set and connect them with a piece of yarn. If this process resulted in each member of each set being joined to one and only one member of the other set, the two sets were said to match.

After some experiences with matching sets, additional experience was provided with the idea of conservation of a one-to-one correspondence. For example, the members of one of two sets which had been matched were moved so that the one-to-one correspondence could no longer be seen easily. The children were asked if they thought the two sets still matched. One of those who thought not was asked to find out for the rest of the class. (See Worksheets 2 and 3, Appendix C.)

One-to-one correspondence was used to introduce the idea of number. Several sets of, say, two objects were put on the flannel board. These sets were matched by the children. It was suggested that they all matched one another. The sets were said to have the same number of members because they all matched.

A slightly different approach was used to introduce five. Sets of one, two, three, and four members were put on a flannel board in order. The teacher put a set of five members on the board also. The children were asked if this new set matched any of the other previously defined sets. When it was discovered that the new set did not match, it was decided that a new name for the number of members in that set was needed.

Writing the numerals one through five was also practiced along with recognition of sets of one through five members. Writing and recognizing numerals was practiced the entire year in all classes and will not be mentioned again. The ideas of more and greater were introduced using the idea of one-to-one correspondence. (See Worksheet 4, Appendix C.)

Groups P and T

These groups studied everything noted for Group LP, and, in addition, they studied fewer and less. (See Worksheet 5, Appendix C.) Moreover these groups began addition readiness work. The objective was to see how many different ways the children could show a set of five members, for example. A piece of manila paper which was creased in the middle and five counters were given to each child. The children were asked to show, for example, three counters on one side of the crease and two on the other. Questions were asked about how many counters were on the paper and how many were on each side of the crease. The procedure was repeated several times, and then the children were asked to find as many different ways of showing five as they could. The different ways were recorded on the chalkboard by the children, and each way the children found was discussed by the other children. When a way was shown on the chalkboard that had already been found, the repetition was left for the children to discover and erase.

The same procedure was employed for three and four also.

Variations were used in which the children wrote their names by their drawings. Finally the same procedure was used in worksheet exercises. (See Worksheet 6, Appendix C.)

Weeks Nine through Fourteen

Group LP

The instruction in this period centered around numeral formation, the numbers one through six, order concepts, and partitioning a set into two parts. In addition to the ideas of more and greater introduced in the first eight weeks, the concepts of less and fewer were also introduced. Generally, these order ideas were associated with the number sequence. A game was devised which was used as the vehicle of instruction. The game was "I'm Thinking of a Number." In this game the teacher gave clues about the numbers one through six. These clues involved the ideas of greater and less. For example, a typical clue in the game was: "I'm thinking of the numbers between three and six" or "I'm thinking of the number that comes just before two" or "I'm thinking of a number that is one less than five." Study also included individual work on order and the meaning of more, fewer, greater, and less. (See Worksheets 7 and 8, Appendix C.)

The second major item of instruction was set partitioning. For this purpose each child was provided with a two-wire bead frame. (The children in all the classes were given bead frames to use in connection with many of the number concepts studied.) Each bead frame had forty beads with alternating sets of five red and white beads. The children were asked to show, for example, a set of six beads on the bottom row of the bead frame. Then the children were asked to "split" the whole set into two parts, say, two and four, show an unpartitioned whole set of six on the top row of the bead frame, and were asked whether there were as many beads in the whole set on the top row as in the whole set on the bottom row. The objective was to have the children see matching sets in two different patterns and to see some of the ways whole sets could be made up of parts.

This approach was varied often. At times pictures were drawn on the chalkboard of the whole set and the partitioned set. At other times pictures were drawn on specially prepared worksheets. (See

Worksheet 9, Appendix C.) Often when the pictures were drawn on the chalkboard, someone would be asked to match the two whole sets using colored chalk. The children were repeatedly asked how many in the parts and how many in the whole set and if there was the same number in both whole sets.

Group P

During this period the children studied the sequences of numbers one through nine, the ideas of greater, less, more, and fewer, and partitioning sets. The procedures were like those described for Group LP except for the fact that the numbers seven, eight and nine were studied also.

A different type worksheet was also used in connection with set partitioning for the sake of variety. (See Worksheet 10, Appendix C.) This worksheet was used in connection with the numbers eight and nine to reduce the amount of time devoted to drawing pictures. The unpartitioned whole set was pictured in the box at the top of an item while the partitioned set was shown in the lower box. One of the parts in the lower box was encircled. The children were asked to count each whole set and part of a set pictured and to write the numeral naming the number of each in a space provided. Then questions were asked about the equivalence of the sets in the top and bottom rows, and, finally, the sets were matched.

Group T

The major emphases of instruction included order, numbers one through nine, more, fewer, greater, and less, and seeing different ways of representing sets of five through nine. The instructional procedures for the first three items were the same as in Groups LP and P.

Addition readiness was studied using the bead frames. The children, for example, were asked to show a set of three on the top row, a set of four on the bottom row, and tell how many altogether in the sets. This procedure was used to show all the different combinations which made six, seven, eight, and nine. Also a worksheet (Worksheet 11, Appendix C) was used to record the different combinations found. Choosing seven as the number studied, a picture of the top row of the

bead frames was drawn in the top box of an item on the worksheet. The number, say, four, was recorded in the box to the right. The same procedure was used for the set of three on the bottom row of the bead frame. All the beads were counted and the number was recorded in the box to the far right.

When the procedure was completed for all the possible combinations which made seven, the teacher asked what was similar in all the pictures drawn. For variation the same procedure was used with the pictures drawn on the chalkboard and a student was chosen to fill in the numerals. The procedure was very effective.

Weeks Fifteen through Twenty-one

Group LP

The major areas of instruction were the numbers one through ten, order, and partitioning sets, addition sentences, and missing addend sentences. The procedures used to teach the numbers one through ten, the order of the number sequence, and partitioning sets of the newly studied numbers were similar to those stated previously.

The majority of the time in this period was devoted to learning the operation of addition and to gaining skill in solving addition sentences and missing addend sentences. The addition sentence was introduced in response to the need of a way to record what was known and what was to be found when the children were asked to find the number of a whole set from two given subsets or parts. That is, in order to be sure that, after the number of the whole set was found, the number of each part was not forgotten, the addition sentence was written so that the number of each part was clearly noted.

The addition sentence was closely related to the manipulation of concrete materials (e.g., bead frames). When a pair of subsets was given for the children to show, the addition sentence was written, and the addends were shown to name the numbers of the subsets while the sum was shown to be the number of the whole set. A good deal of time was spent in group instruction in which the children pointed out the aspects of a set arrangement on the flannel board or chalkboard which

were named by the addends and the sum of an addition sentence.

The language used was invariably that of the PPW. The subsets shown by the children were the parts, and the parts together were the whole set. The children were asked, for example, to show parts of two and three on their bead frames and asked to determine the number in the whole set. The procedure was then retraced with the addition sentence which described the parts and the whole set being written. (See Worksheets 12, 13, Appendix C.)

All verbal problems which were used to supply more meaning to the addition sentence were couched in the language of parts and wholes and had the form of looking for the whole set. For example, "Mary had a set of glasses. One part of this set had three red glasses in it and the other part had two blue ones. How many glasses were in Mary's set of glasses?"

The ideas presented above were used throughout the period in connection with addition. In an attempt to force the children to read number sentences more carefully and to discriminate the parts of the sentences addition sentences were reversed. That is, a sentence such as $2 + 3 =$ _____ was written _____ $= 2 + 3$. The children had to read and discriminate the "=" and "+" in order to complete the sentence correctly.

Near the middle of the period missing addend sentences ($2 +$ _____ $= 5$) were introduced. The reason for studying the missing addend sentence was to provide practice in finding a part when the whole set and another part were given, i.e., subtraction. Everything done in connection with the missing addend sentence was done in the language of parts and whole sets, but two orders of presentation were used, and these led to different manipulations of the bead frames.

The first method used asked the children to find the other part, given <u>one part</u> and the whole set. For example, a part of two was shown on the bead frames. Then the children were asked to find the other part if the whole set was, say, five. The problem was solved in two ways. Enough more beads were shown to make the required whole set, or another distinct whole set was shown and then partitioned so that the known part and the required part were visible. Each of these manipulations was performed in the classes, but the majority of the time a second

approach was used.

In the second approach the order of problem presentation led to a different manipulation on the bead frames. First the number of the whole set was stated, say, five, and five beads were shown on the bead frames. Then the number of one part, say, two was given and the children were asked to find the other part. The manipulation of concrete materials consisted of showing the whole set and partitioning it in an appropriate manner to show the part of two and find the desired number of the other part.

Regardless of the manipulation and order of presentation used, in every case the manipulation was related to the missing addend sentence. It was emphasized that the sentence called for the discovery of a part of a set, and that the solution was the number of the other part. (See Worksheet 14, Appendix C.)

Group P

The same topics studied and procedures used in Group LP were pursued in this group. The differences were mainly that Group P began the study of addition a week earlier, began the missing addend sentences two weeks earlier, and studied the number facts with sums of six, whereas Group LP studied only number facts through sums of five.

Group T

The major topics studied were the numbers one through ten and order, addition, and subtraction. The work on the numbers one through ten and the number sequence was similar to that described earlier. The major points of interest are those connected with addition and subtraction instruction.

As in Groups LP and L the addition sentence arose from a need to record the sets which when combined made a larger set. Here also the addition sentence was closely related to manipulations on the bead frame. The main differences in the instruction of Groups LP and L and that of Group T were in the language used and the visual pattern resulting from manipulation. The differences in the visual stimuli are exemplified in Worksheets 12, 13, 15, and 16, Appendix C, and have been mentioned earlier.

The language differences were exemplified in the way the children were asked to set up an addition problem. The children were asked to show, for example, a set of two and a set of three on their bead frames. The final task required the children to find how many altogether by counting the two sets which were shown on the bead frames. The number of members in each set and in the two together were related to the numerals in an addition sentence by a process of questioning and suggesting. (See Worksheets 15 and 16, Appendix C.)

The approach to subtraction, which was introduced one week after addition and at the same time that Group P was studying missing addend sentences, was the more traditional procedure of removing a set from another set. For example, a set of five was shown on the bead frames, and two were "taken away" or "pushed back." The children counted the set which was left. The initial set, the set removed, and the set left were related to the minuend, subtrahend, and answer of the subtraction sentence. The children engaged in many exercises involving addition and subtraction during this period. The take away application was used occasionally to impart more meaning to the operation of subtraction. Drill work was done on addition and subtraction. (See Worksheet 17 and 18, Appendix C.)

During the last one and a half weeks of this period the number facts with sums of six and seven were introduced. The procedure was somewhat unusual. The children were asked to find all the addition sentences with sums of six. These were found by using the bead frames and were written on the chalkboard by the children. The sentences were checked by the other children to determine correctness; and, if correct, the contributor's name was written beside his sentence and the rest of the class wrote the sentence on their papers. Errors or repetitions were not mentioned by the teacher, but rather the children were asked to find and correct all errors. The same procedure was used in all classes to find the number facts with sums greater than five.

Weeks Twenty-two through Thirty

Group LP

The major emphases of instruction concerned addition, subtraction, base, place value, the number sequence, and the applications of

subtraction. During the first week of this period the symbolism "+" and "=" was introduced in the late symbolization class as a short way of writing "and" and "is." The topics addition and number sequence need little further comment since procedures outlined previously were continued. However, it should be noted that the number facts studied by Group LP did not include facts beyond sums of six, and the number sequence was not seriously studied beyond twenty-five.

The operation of subtraction and the subtraction sentence were introduced during the second week of this period. Although the subtraction sentence had not been introduced previously, a great deal of experience with the manipulations involved in solving subtraction problems was obtained in solving missing addend sentences. The subtraction sentence was introduced as a different way of writing a missing addend sentence. For example, 5 - 2 = _____ and 2 + _____ = 5 were shown to be different ways to write the same statement.

The method suggested for solving a subtraction sentence was set partitioning. For example, the children were directed to show a whole set of five on their bead frames, to show a part of two, and to count the other part of the partitioned set. To facilitate the manipulation the children were asked to "split" the whole set in order to show the known and unknown parts. The removal of the known part from the total visual pattern was not allowed, and, hence the two parts were visible at all times. (See Worksheet 19, Appendix C.)

The concept of base was developed in a rather unique manner by using the bead frames. For example, the children were asked to find all the sets of five on the forty-bead bead frame. The number of sets of five on the bead frame were counted, and it was seen that there were eight fives. This procedure was repeated with other numbers, for example, four, six, seven, eight, and ten.

The idea of "ones" was introduced when extra beads were found on the bead frame which were not used to make the sets of six and seven. For example, there were five sevens and five ones on the bead frame. The idea of how many sets of ten in a given set was exclusively studied after the introductory work on various sets. Group and individual work was undertaken to determine the number of tens and ones in a given set. (See Worksheet 20, Appendix C.)

The number of tens and ones in a set was related to the numeration system by suggesting the similarities between statements such as one ten and three ones and thirteen. The first numeral in a two digit numeral indicated the number of tens and the second numeral the number of ones. (See Worksheet 21, Appendix C.)

Work on the three applications of subtraction occupied ten class periods during the last five weeks. Missing addend and comparison applications accounted for eight class periods while the take away application was studied in the other two. These periods were not consecutive but, rather, were distributed throughout the last five weeks.

The first application studied was the missing addend problem. Two complementary approaches were used in the instruction. The first work was oral. The teacher stated the problem, drew a picture on the chalkboard, and asked the children what the story asked for--a part of a set or a whole set. When it was decided that a part of a set was required, the children were asked what kind of a number sentence should be used and what that sentence was. The main idea to be attained was that the problem asked for the number of a part of a set. This point was emphasized repeatedly. When the correct sentence was suggested, the facts of the problem were related to the subtraction sentence.

A very similar approach was used as a variation. The work was done with worksheets. The children read the problems to the class and suggested solutions after the questioning procedure stated above. The correct subtraction sentence was completed by every child in the space provided. (See Worksheet 22, Appendix C.)

The second application studied was the comparison case of subtraction. In this instance the overall procedure was identical to that suggested for missing addend problems. The part that the students were seeking was the part which was not matched after a correspondence had been set up. It was emphasized that the desired part could be obtained by partitioning the set with the greater number of members and, for this reason, the appropriate operation was subtraction. (See Worksheet 23, Appendix C.)

The take away application was the last one studied. The general procedures were like those for the other two applications. In this

case the part desired was the part which remained after another part was removed. Again subtraction was used because a part of a set was desired. (See Worksheet 24, Appendix C.)

Group P

The topics studied and procedures used in Group LP were also studied and used in this group. However, the number facts through sums of nine, place value for numerals through ninety-nine, and the applications of subtraction including the number facts through those with sums of nine were studied. (See Worksheet 24, Appendix C.) Otherwise the procedures were the same as those explained above, but learning was accomplished at a more rapid rate.

Group T

The topics studied in this group were addition and subtraction through the sums of nine, the number sequence through ninety-nine, base and place value through ninety-nine, and the applications of subtraction. Addition and subtraction were symbolically represented during the first two weeks of this period for the late symbolization class (TL). The approach was simply to suggest that "+", "-", and "=" were short ways of writing "and," "take away," and "equals" in the number sentences.

The only procedures which differed from those in Groups LP and P were those associated with the applications of subtraction. The difference here was especially the manner in which the applications were related to subtraction.

The first application of subtraction which was studied was the missing addend problem. Two related approaches of instruction were employed. The beginning method was oral. A verbal problem was stated, for example, "Billy wanted to buy a five cent candy bar. He had only two cents. How much more money did Billy need?" An associated picture was drawn on the chalkboard and solutions were requested from the children. It was suggested that the solution could be obtained by subtraction if two pennies were taken away from the cost of the candy bar. When it was found that subtraction worked, the number sentence was obtained from the children and written on the chalkboard. The sentence

was related to the picture and the verbal problem.

The second approach used worksheets. (See Worksheet 22, Appendix C.) Identical methods were used, but all the subtraction sentences were completed by each child in the spaces provided.

The second application studied was the comparison case of subtraction. The same general procedures were used in this case as in the missing addend application, but the children were asked to match the two sets and to take the matched part of the larger set away (erase it) to determine how many more were in one set than in the other. (See Worksheet 23, Appendix C.)

The take away application suited the take away method of subtraction, and no explanation was needed for the children to see that subtraction could be used to aid in the solution of the problems. (See Worksheet 24, Appendix C.)

Statistical Methods

The 135 subjects in the study at its conclusion were alphabetized by classes and coded by the assignment of the numerals 1 through 135 in order to facilitate statistical procedures and to insure that the names of the children would remain confidential.

It was indicated earlier that the six classes exhibited great individual differences. Therefore direct comparisons of means scores on the criterion measures was not appropriate. Two measures of individual differences were obtained through the administration of the Concept Attainment Test (CAT) and the Lorge-Thorndike Intelligence Test. These measures were used to control the individual differences between the classes statistically in an analysis of covariance of the criterion scores. This method was used for the analysis of sex differences in the achievement of the four higher ability classes and for the analysis of the achievement scores of the two lower ability classes.

The method of analysis of covariance (one-way) uses linear regression and analysis of variance techniques. Linear regression is used to predict the criterion means on the basis of the initial measures which reflect the individual differences between classes. The sum of squares (SS) of the residuals (errors in prediction) are used to calculate two independent estimates of the population variance. The

first variance estimate is the mean square (MS) for between groups. It is calculated from the formula (117)

$$MS(Groups) = \frac{SS(Group)}{df(Group)} , \qquad (1)$$

where df(Group) is one less than the number of groups. The second variance estimate is the mean square (MS) for within groups and is calculated from the formula (117)

$$MS(Within) = \frac{SS(Within)}{df(Within)} , \qquad (2)$$

where df(Within) = (N - 2) - df(Group), and N is the number of subjects in the total sample.

A test of significance is made by obtaining a value of F from the formula (117)

$$F = \frac{MS(Group)}{MS(Within)} . \qquad (3)$$

If the obtained value of F is greater than a prechosen value of F, the difference in means due to the manipulated variable is statistically reliable at a chosen level of confidence.

When the difference in means is statistically reliable, the adjusted group criterion means may be calculated from the formula (117)

$$\text{Adj. } \bar{Y}_g = \bar{Y}_g - b(\bar{X}_g - \bar{X}_t), \qquad (4)$$

where $\bar{Y}_g$ is a criterion mean of a group, $\bar{X}_g$ is an initial variable mean of a group, $\bar{X}_t$ is an initial variable mean of the sample, and b is the regression coefficient of the within groups regression line.

A second type analysis of covariance (two-way) was used in the analysis of the higher ability group achievement data. The analysis is similar to one-way analysis of covariance, but in one analysis three tests of significance are provided. The differences in the criterion mean scores due to the approach, the time of symbolization, and the interaction of the manipulated variables are simultaneously tested for statistical reliability. (For a discussion of interaction, see [74],

pp. 223-226, 235-238.) When the differences are reliable, the adjusted criterion mean scores may be calculated using formula four.

The t statistic for dependent samples was used to test the difference between mean scores on the first and last administration of the Arithmetic Attitude Scale for each class. The t statistic is defined by the equation (2)

$$t = \frac{\bar{d}}{s_d\sqrt{N}} \qquad df = N - 1, \qquad (5)$$

where $\bar{d}$ is the mean of the differences between the final and the initial scores, s_d is the standard deviation of the differences, and N is the number of members in the sample.

The t statistic for independent samples was used to test the difference between means of the average arithmetic attitude scores of various groups and the difference between Concept Attainment Test mean gain scores for LPE and LPL. The t statistic is defined by the equation (2)

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}\sqrt{\frac{n_1 + n_2}{n_1 n_2}}}, \quad df = n_1 + n_2 - 2 \qquad (6)$$

where $\bar{X}_1$ and $\bar{X}_2$ are the sample means, s_1^2 and s_2^2 are the sample variances and n_1 and n_2 are the number of sample points in each sample.

In the analysis of sex differences for the four higher ability classes a one-way analysis of covariance was employed. When the obtained F-ratio was significant, a post hoc analysis of differences in means was initiated. The method used was the Newman-Kuels method. This method provides a more conservative test of the significance of the difference between means than would be true if the t statistic were used in a planned program of testing means. The statistic is defined by the equation (119)

$$q = \frac{\bar{T}_i - \bar{T}_j}{\sqrt{\text{within MS}/\tilde{n}}}, \qquad df = (r,k.) \qquad (7)$$

where $\tilde{n} = \dfrac{4}{1/n_1 + 1/n_2 + 1/n_3 + 1/n_4}$

and $\bar{T}_i$ and $\bar{T}_j$ are the means of two groups, n_p ($p = 1,\ldots,4$) are the numbers of members in the groups, k is the degrees of freedom of the within mean square (within MS) and r is the number of steps between $\bar{T}_j$ and $\bar{T}_i$.

CHAPTER IV

ANALYSIS OF DATA

The content of Chapter IV is divided into six sections. In the first three sections the achievement data collected at the termination of the experiment is analyzed for the higher and lower ability groups and for the higher ability group reclassified by sex. The fourth and fifth sections contain the analysis of the concept attainment gain scores for the higher and lower ability groups respectively. The analysis of the attitude scores for all the groups is reported in the last section.

Higher Ability Group Achievement Data Analyzed

There were four classes in the higher ability category. Two of these classes were taught using the PPW approach and two employed the TA approach. One each of these pairs had early exposure to symbolized addition and subtraction concepts, and the two others had late symbolization. The four classes as defined by the treatment employed were PPW-Early (PE), TA-Early (TE), PPW-Late (PL), and TA-Late (TL).

The achievement data for these classes were analyzed using the method of two-way analysis of covariance. This method was chosen because it adjusted the means of the raw scores statistically on the basis of initial independent measures. Statistical adjustment was necessary because the ability differences between the classes were too large to ignore. Also the method of two-way analysis of covariance provided a more sensitive test of differences due to the manipulated variables.

Before the analysis of covariance could be conveniently performed, the number of students in each class had to be equalized. The equalization was accomplished by randomly deleting one student from PE, two students from TE, and six students from PL. These deletions brought the number of students in each class to twenty.

The initial measures used in the analysis of covariance were

(A) 1-5 Concept Pretest, (B) 6-10 Concept Pretest, (C) Total Concept Pretest, and (D) Lorge-Thorndike Intelligence Test. The means and standard deviations of the initial measures for each class are summarized in Table 3.

TABLE 3

MEANS AND STANDARD DEVIATIONS OF COVARIATES A, B, C, AND D FOR THE FOUR EQUALIZED CLASSES PE, TE, PL, AND TL

Covariate		Class			
		PE	TE	PL	TL
A	Mean	168.50	161.00	169.50	154.00
	SD	38.70	48.00	59.34	38.17
B	Mean	105.75	86.25	96.00	71.55
	SD	33.89	38.57	39.02	35.13
C	Mean	317.75	281.75	304.50	261.55
	SD	72.50	100.94	105.61	83.07
D	Mean	116.95	118.10	96.80	101.10
	SD	11.02	14.84	11.13	12.32

The criterion measures in the analysis of the effects of the two manipulated variables were the Subtraction Application Test (SA), the Test of Subtraction Applications excluding the take-away examples (NTA), the Subtraction Transfer Test (ST), the Addition Computation Test (AC), the Subtraction Computation Test (SC), the Problem Solving Test (PS), and the Stanford Achievement Test (SAT). The means and standard deviations of the criterion measures for each class are summarized in Table 4.

There are two trends evident in Table 4. The first is that the means for the class PE are consistently the greatest of the four classes. The means for classes TE and TL fluctuate between being the second and the third, while the means for PL are consistently the smallest. The second trend is that for the criteria AC and SC the mean scores for PE and PL are very nearly equal. On the other hand, for TE and TL the mean for SC is greater than the mean for AC. The reason appears to be inherent in the test items. The items in the addition computation test were, on the average, more difficult than the subtraction items. This was especially true for the TA classes. There were six addition

TABLE 4

MEANS AND STANDARD DEVIATIONS OF END OF YEAR TESTS SA, NTA, ST, AC, SC, PS, AND SAT FOR THE FOUR EQUALIZED CLASSES PE, TE, PL, AND TL

Criterion		Class PE	TE	PL	TL
SA (Sub.App.)	Mean	4.60	3.85	3.40	3.65
	SD	0.75	2.01	2.11	1.63
NTA (No TA.App.)	Mean	4.70	3.75	3.30	3.45
	SD	0.92	1.96	2.27	1.85
ST (Sub.Tran.)	Mean	4.65	4.65	3.35	3.90
	SD	2.52	2.91	3.13	2.40
AC (Add.Comp.)	Mean	6.45	4.75	4.15	4.45
	SD	2.26	2.40	3.01	3.05
SC (Sub.Comp.)	Mean	6.25	6.10	4.10	5.55
	SD	2.83	3.68	3.66	3.50
PS (Prob.Sol.)	Mean	12.60	10.65	7.85	9.70
	SD	2.62	4.39	4.03	4.67
SAT (Stan.Ach.)	Mean	30.45	26.60	20.20	24.90
	SD	7.84	11.74	11.95	12.95

items which involved ideas not studied during the school year. Three of these were addition sentences with three addends, and three were missing addend sentences. Neither of these topics were studied in the TA classes. Addition sentences with three addends were not studied in the PPW classes either, but the missing addend sentences were. The great number of addition problems involving topics not studied could explain the relatively small AC mean score when compared with the SC mean score for TE and TL.

The results of the two-way analysis of covariance of the criterion measures are summarized in Tables 5, 7, 9 and 11. The seven analyses of the criterion measures performed using a single covariate are combined in one table. For example, the seven analyses performed using Covariate A are recorded in Table 5. The adjusted means of the criterion measures for which there were statistically significant differences are reported in Tables 6, 8, 10, and 12.

The complete interpretation of the differences which were significant will not be explicitly stated in each case. However, an example of the method used to interpret a significant F-ratio is given for criterion AT in Table 5. Other interpretations were made following the same reasoning procedures.

The two-way analysis of covariance of the criterion measure Addition Computation (AC) using the Covariate A resulted in two F-ratios which were significant at the five per cent level of confidence. The adjusted means for AC are recorded in Table 6. The symbols P and T in Table 6 refer to the two levels of the approach variable; E and L refer to the early and late symbolization of the operations of addition and subtraction. Row means are the averages of the pairs of cell means in each row; column means are the averages of the pairs of cell means in each column; and M_t is the grand mean of the four cell means.

The F-ratio of 5.44 ($p < .05$) for the criterion AC for the source of variation Time directs attention to the row means for AC in Table 6. The mean of the early symbolization group (E) was significantly greater than the mean of the late symbolization group (L). To repeat the last statement in complete form: When the Arithmetic Computation Test (AC) means of the subgroups E and L were adjusted for individual differences in the ability of beginning first grade pupils to apply various counting skills as measured by 1-5 Concept Pretest, the difference between the mean achievement of early symbolization pupils and the mean achievement of those who had had late symbolization of addition and subtraction concepts was statistically significant at the five per cent level of confidence.

The F-ratio of 4.68 ($p < .05$) for criterion measure AC for the source of variation Interaction directs attention to the four cell means of AC in Table 6. To facilitate interpretation let it be said that interaction is the lack of uniformity of cell means when compared with the grand mean (M_t) of the sample. In the present case the lack of uniformity in the cell means is seen to exist in the two cells in the PPW column while the cell means in the TA column are relatively uniform when compared with the grand mean ($M_t = 4.9500$). Hence the significant F-ratio for Interaction would be interpreted to mean that,

when the AC means for the subgroups PE, TE, PL, and TL were adjusted for individual differences in the ability of beginning first grade pupils to apply various counting skills as measured by 1-5 Concept Pretest, the most effective combination of the manipulated variables was P and E while the least effectual was P and L. Little can be said about E and L for the TA approach.

TABLE 5

ANALYSIS OF COVARIANCE OF INDIVIDUAL SCORES FOR SA, NTA, ST, AC, SC, PS, AND SAT FOR CLASSES PE, TE, PL, and TL. COVARIATE: 1-5 CONCEPT PRETEST

Criterion	Source of Variation	df	Adj. SS	M.S.	F
SA (Sub.App.)	Approach	1	0.10	0.10	0.04
	Time	1	8.53	8.53	3.50
	Interaction	1	6.30	6.30	2.59
	Within	75	182.70	2.44	
NTA (No TA.App.)	Approach	1	0.85	0.85	0.30
	Time	1	12.78	12.78	4.55*
	Interaction	1	7.59	7.59	2.70
	Within	75	210.67	2.81	
ST (Sub.Tran.)	Approach	1	6.44	6.44	1.04
	Time	1	17.94	17.94	2.88
	Interaction	1	2.86	2.86	0.46
	Within	75	466.19	6.22	
AC (Add.Comp.)	Approach	1	2.42	2.42	0.45
	Time	1	29.20	29.20	5.44*
	Interaction	1	25.10	25.10	4.68*
	Within	75	402.47	5.37	
SC (Sub.Comp.)	Approach	1	22.28	22.28	2.46
	Time	1	30.82	30.82	3.40
	Interaction	1	17.79	17.79	1.96
	Within	75	680.31	9.07	
PS (Prob.Sol.)	Approach	1	4.47	4.47	0.39
	Time	1	147.01	147.01	12.66**
	Interaction	1	86.62	86.62	7.46**
	Within	75	870.41	11.61	
SAT (Stan.Ach.)	Approach	1	85.89	85.89	1.04
	Time	1	613.39	613.39	6.92*
	Interaction	1	470.26	470.26	5.69*
	Within	75	619.58	82.61	

*$F_{.95}(1,75) = 3.97$ **$F_{.99}(1,75) = 6.99$

TABLE 6

ADJUSTED CELL, ROW, COLUMN, AND GRAND MEANS FOR NTA, AC, PS, AND SAT
COVARIATE: 1-5 CONCEPT PRETEST

Criterion	Time	Approach P	Approach T	Row Means
NTA (No TA.App.)	E	4.6124	3.7875	4.2000
	L	3.1957	3.6043	3.4000
	Column Means	3.9041	3.6959	3.8000 (M_t)
AC (Add.Comp.)	E	6.2906	4.8183	5.5545
	L	3.9603	4.7308	4.3455
	Column Means	5.1254	4.7746	4.9500 (M_t)
PS (Prob.Sol.)	E	12.3596	10.7530	11.5563
	L	7.5638	10.1235	8.8437
	Column Means	9.9617	10.4383	10.2000 (M_t)
SAT (Stan.Ach.)	E	29.6905	26.9255	28.3080
	L	19.2958	26.2381	22.7670
	Column Means	24.4932	26.5818	25.5375 (M_t)

The remaining significant F-ratios in Table 5 and adjusted means in Table 6 lend themselves to the same type interpretation as that which was made for the analysis of criterion AC. For NTA, PS, and SAT the E group had significantly greater adjusted means, and the significance of the Interaction for criteria PS and SAT implied that P and E was the most effective combination of manipulated variables while P and L was the least effective. A different, but equivalent, manner of expressing the meaning of the significant F-ratio for Interaction for criterion SAT is that, when the choice of time of symbolization is fixed, at E for example, the approach P would be chosen, while if L is the fixed

TABLE 7

ANALYSIS OF COVARIANCE OF INDIVIDUAL SCORES FOR SA, NTA, ST, AC, SC, PS, AND SAT FOR CLASSES PE, TE, PL, AND TL. COVARIATE: 6-10 CONCEPT PRETEST

Criterion	Source of Variation	df	Adj. SS	M.S.	F
SA (Sub.App.)	Approach	1	0.78	0.78	0.33
	Time	1	3.86	3.86	1.62
	Interaction	1	6.08	6.08	2.55
	Within	75	178.39	2.38	
NTA (No TA.App.)	Approach	1	1.83	1.83	0.92
	Time	1	3.96	3.96	2.00
	Interaction	1	7.95	7.95	4.02*
	Within	75	148.28	1.98	
ST (Sub.Tran.)	Approach	1	16.54	16.54	2.58
	Time	1	8.18	8.18	1.28
	Interaction	1	2.46	2.46	0.38
	Within	75	480.46	6.41	
AC (Add.Comp.)	Approach	1	0.13	0.13	0.02
	Time	1	14.52	14.52	2.56
	Interaction	1	23.66	23.66	4.16*
	Within	75	426.11	5.68	
SC (Sub.Comp.)	Approach	1	49.03	49.03	5.33*
	Time	1	12.45	12.45	1.35
	Interaction	1	16.59	16.59	1.80
	Within	75	689.32	9.19	
PS (Prob.Sol.)	Approach	1	24.95	24.95	2.07
	Time	1	91.69	91.69	7.60**
	Interaction	1	82.91	82.91	6.87*
	Within	75	905.15	12.07	
SAT (Stan.Ach.)	Approach	1	326.33	326.33	3.70
	Time	1	289.65	289.65	3.28
	Interaction	1	441.84	441.84	5.00*
	Within	75	6622.77	88.30	

*$F_{.95}(1,75) = 3.97$ **$F_{.99}(1,75) = 6.99$

time, then the approach T would be the best choice. On the other hand, when the choice of approach is fixed at P, early symbolization (E) would be the best choice; while if it is fixed at T, either E or L could be chosen with nearly equal confidence.

Only the analysis of the criterion measure SC in Table 7 needs comment since the remainder of the significant F-ratios lend themselves

TABLE 8

ADJUSTED CELL, ROW, COLUMN, AND GRAND MEANS FOR NTA, AC, SC, PS, AND SAT
COVARIATE: 6-10 CONCEPT PRETEST

Criterion	Time	Approach P	Approach T	Row Means
NTA (No TA.App.)	E	4.1830	3.8686	4.0258
	L	3.1008	4.0477	3.5742
	Column Mean	3.6419	3.9581	3.8000 (M_t)
AC (Add.Comp.)	E	5.8847	4.8796	5.3822
	L	3.9322	5.1035	4.5178
	Column Mean	4.9084	4.9916	4.9500 (M_t)
SC (Sub.Comp.)	E	5.5370	6.2635	5.9002
	L	3.8252	6.3743	5.0998
	Column Mean	4.6811	6.3189	5.5000 (M_t)
PS (Prob.Sol.)	E	11.7204	10.8517	11.2861
	L	7.5111	10.7168	9.1139
	Column Mean	9.6158	10.7842	10.2000 (M_t)
SAT (Stan.Ach.)	E	27.7066	27.2291	27.4679
	L	19.1429	28.0714	23.6071
	Column Mean	23.4248	27.6502	25.5375 (M_t)

to interpretations similar to those stated above. The F-ratio of 5.33 ($p < .05$) for the criterion measure SC for the source of variation Approach directs attention to the two adjusted column means for SC in Table 8. The mean for group T was greater than that for group P. Hence, when individual differences in groups P and T were controlled by means of scores on 6-10 Concept Pretest, the difference between the

TABLE 9

ANALYSIS OF COVARIANCE OF INDIVIDUAL SCORES FOR SA, NTA, ST, AC, SC, PS, AND SAT FOR CLASSES PE, TE, PL, AND TL. COVARIATE: TOTAL CONCEPT PRETEST

Criterion	Source of Variation	df	Adj. SS	M.S.	F
SA (Sub.App.)	Approach	1	0.35	0.35	0.16
	Time	1	5.71	5.71	2.64
	Interaction	1	5.70	5.70	2.64
	Within	75	162.01	2.16	
NTA (No TA.App.)	Approach	1	0.02	0.02	0.01
	Time	1	8.81	8.81	3.66
	Interaction	1	6.91	6.91	2.87
	Within	75	180.61	2.41	
ST (Sub.Tran.)	Approach	1	14.64	14.64	2.55
	Time	1	11.76	11.76	2.05
	Interaction	1	2.15	2.15	0.38
	Within	75	429.77	5.73	
AC (Add.Comp.)	Approach	1	0.00	0.00	0.00
	Time	1	19.86	19.86	4.23*
	Interaction	1	22.56	22.56	4.80*
	Within	75	352.57	4.70	
SC (Sub.Comp.)	Approach	1	42.73	42.73	5.31*
	Time	1	19.48	19.48	2.42
	Interaction	1	15.29	15.29	1.90
	Within	75	603.28	8.04	
PS (Prob.Sol.)	Approach	1	20.17	20.17	2.03
	Time	1	113.54	113.54	11.43**
	Interaction	1	79.57	79.57	8.01**
	Within	75	744.98	9.93	
SAT (Stan.Ach.)	Approach	1	272.72	272.72	4.02*
	Time	1	410.93	410.93	6.06*
	Interaction	1	417.64	417.64	6.16*
	Within	75	5083.59	67.78	

*$F_{.95}(1,75) = 3.97$ **$F_{.99}(1,75) = 6.99$

group T mean achievement for subtraction computation and that of group P was statistically significant at the five per cent level of confidence.

The results reported in Tables 9 and 10 require little additional comment since they are very similar to those interpreted in Tables 7

TABLE 10

ADJUSTED CELL, ROW, COLUMN, AND GRAND MEANS FOR AC, SC, PS, AND SAT. COVARIATE: TOTAL CONCEPT PRETEST

Criterion	Time	Approach P	Approach T	Row Mean
AC (Add.Comp.)	E	5.9785	4.9224	5.4504
	L	3.9155	4.9837	4.4496
	Column Mean	4.9470	4.9530	4.9500 (M_t)
SC (Sub.Comp.)	E	5.6844	6.3068	5.9956
	L	3.8187	6.1902	5.0044
	Column Mean	4.7515	6.2485	5.5000 (M_t)
PS (Prob.Sol.)	E	11.8797	10.9133	11.3965
	L	7.4917	10.5152	9.0035
	Column Mean	9.6857	10.7143	10.2000 (M_t)
SAT (Stan.Ach.)	E	28.2081	27.4196	27.8138
	L	19.0849	27.4374	23.2612
	Column Mean	23.6465	27.4285	25.5375 (M_t)

and 8. However, the reduction of significant F-ratios for the analyses summarized in Table 11 deserves comment. The sole F-ratio which was great enough to surpass the critical value of F was associated with Interaction for the criterion measure PS. The adjusted cell means reported in Table 12 for PS exhibit a reversal of trends apparent in the adjusted means for PS in Tables 6, 8, and 10. The most effective combination of variables remained P and E, but the least effective combination was T and E, rather than P and L. Considered differently, and perhaps more clearly, if the approach T was chosen as a method, then late symbolization L was more effective than early symbolization E.

TABLE 11

ANALYSIS OF COVARIANCE OF INDIVIDUAL SCORES FOR SA, NTA, ST, AC, SC, PS AND SAT FOR CLASSES PE, TE, PL, and TL. COVARIATE: LORGE-THORNDIKE IQ

Criterion	Source of Variation	df	Adj. SS	M.S.	F
SA (Sub.App.)	Approach	1	3.47	3.47	1.47
	Time	1	2.59	2.59	1.09
	Interaction	1	3.22	3.22	1.36
	Within	75	177.49	2.37	
NTA (No TA.App.)	Approach	1	6.72	6.72	2.47
	Time	1	2.00	2.00	0.74
	Interaction	1	3.93	3.93	1.44
	Within	75	204.09	2.72	
ST (Sub.Tran.)	Approach	1	0.00	0.00	0.00
	Time	1	10.29	10.29	1.71
	Interaction	1	0.25	0.25	0.04
	Within	75	450.96	6.01	
AC (Add.Comp.)	Approach	1	19.35	19.35	3.42
	Time	1	5.76	5.76	1.02
	Interaction	1	13.80	13.80	2.44
	Within	75	423.47	5.65	
SC (Sub.Comp.)	Approach	1	2.30	2.30	0.23
	Time	1	7.16	7.16	0.72
	Interaction	1	7.70	7.70	0.77
	Within	75	745.84	9.94	
PS (Prob.Sol.)	Approach	1	4.09	4.09	0.32
	Time	1	0.10	0.10	0.01
	Interaction	1	55.28	55.28	4.31*
	Within	75	961.26	12.82	
SAT (Stan.Ach.)	Approach	1	15.19	15.19	0.16
	Time	1	105.87	105.87	1.13
	Interaction	1	247.11	247.11	2.64
	Within	75	7009.65	93.46	

*$F_{.95}(1,75) = 3.97$

However, the adjusted means reported in Tables 6, 8, and 10 indicated that there was little difference in E and L when combined with the approach T, and what difference there was tended to indicate that the combination of T and E was more effectual than that of T and L.

The reversal of trends mentioned above was due to the relatively low correlation coefficients for covariate D (IQ) and Covariates A,

TABLE 12

ADJUSTED CELL, ROW, COLUMN, AND GRAND MEANS FOR PS COVARIATE: LORGE-THORNDIKE INTELLIGENCE

Criterion	Time	Approach		Row Mean
		P	T	
PS (Prob.Sol.)	E	11.3057	9.1848	10.2452
	L	9.5492	10.7604	10.1548
	Column Mean	10.4274	9.9726	10.2000

B, and C (Concept Pretest) which were 0.243, 0.292, and 0.309 respectively and the relatively high correlations between PS and covariates A, B, C, and D which were 0.466, 0.488, 0.556, and 0.502 respectively for the sample. Hence relatively weakly related measures A, B, C, and D each were quite positively related to the criterion measure PS. In general the same statement is true for the remainder of the criteria.

Overall, the results of the statistical tests performed indicated that in no case were there reliable differences in the achievement of the four classes for the criterion subtraction applications (SA) and transfer (ST) and, thus, no inferences can be made concerning the effectiveness of either the approaches or the times of symbolization. However, the composite trend of the data on the criterion subtraction transfer suggested that the TA approach was slightly more effective than the PPW approach. There were no pronounced trends in the data on the criterion subtraction applications (SA); but when adjusted for intelligence differences, the PPW approach appeared more effective.

However, the analyses of the criterion subtraction applications excluding take-away applications (NTA) showed a statistically reliable difference favoring the early symbolization group and a reliable Interaction which suggested that the least effective combination of variables was P and L. Even though there were no reliable differences attributable to the approaches, the general trend of the data suggested that the PPW approach was slightly more effective than the TA approach. This was especially true when the means were adjusted for individual

differences in intelligence.

The differences in the means of the criteria addition computation, problem solving, and Stanford Achievement yielded statistically reliable evidence which suggested that early symbolization was more effective than late symbolization of addition and subtraction concepts. The positive evidence was tempered by the results obtained when the covariate D (IQ) was used in the analyses. For D there were no reliable differences in achievement for the three criterion measures.

The three criteria arithmetic computation, problem solving, and Stanford Achievement also showed statistically reliable interaction effects when the covariates were A, B, and C. In each case the combination of the approach variable P and the time of symbolization variable L was the least effective treatment. In all but one case (SAT-Covariate B) the most effectual treatment was the combination of the variables P and E. In that one contrary case there was little choice between the combinations P and E, T and E, and T and L. Again these results were tempered by the absence of reliable differences for AC and SAT when covariate D was employed to control individual differences. For the criterion problem solving (PS) a statistically reliable interaction was found in which the combination P and E was the most effective treatment, but the least effective treatment was T and E, not P and L as in the analyses based on the other covariates.

The analyses of covariance of the criterion subtraction computation showed two statistically reliable differences. In each case the TA approach (T) was the more effective method of instruction. Even though the other two tests were negative, the general trend of the data suggested that the TA approach was the most effective method for teaching subtraction computation.

Lower Ability Group Achievement Data Analyzed

The two lower ability classes in the study were instructed by the PPW method. One of these had early introduction of symbolism (LPE), and the other late introduction of symbolization (LPL). The classes LPE and LPL did not study as much material as the other classes in the study. For this reason the data collected in these classes were analyzed separately. The method of one-way analysis of covariance was

used to analyze the data in order to make adjustments in the data for individual differences reflected in four initial measures.

The means, effects, standard deviations, and N's for the initial measures used as covariates are tabulated in Table 13. These covariates are (A) 1-5 Concept Pretest, (B) 6-10 Concept Pretest, (C) Total Concept Pretest, and (D) Lorge-Thorndike Intelligence Test.

TABLE 13

MEANS, EFFECTS, STANDARD DEVIATIONS AND N'S OF COVARIATES A, B, C, AND D FOR CLASSES LPE AND LPL

Covariate	Class	Mean	Effect	SD	N
A	LPE	110.00	-16.00	53.48	20
	LPL	138.80	12.80	37.02	25
B	LPE	55.05	- 9.55	34.91	20
	LPL	72.24	7.64	38.40	25
C	LPE	190.55	-23.78	77.92	20
	LPL	233.36	19.03	65.84	25
D	LPE	95.95	4.08	10.17	20
	LPL	88.60	- 3.27	13.12	25

There was one significant difference in the pairs of means in Table 13. That significant difference occurred when the means of covariate A were tested using one-way analysis of variance, the difference being significant at the five per cent level of confidence. The significant difference indicates that the LPL children performed better on the counting part of the Concept Attainment Test during the first administration than did the LPE children. The LPL children brought greater counting skills with them when they entered the first grade than did the LPE children.

The means of the three covariates A, B, and C display a definite pattern. In each case the mean for LPL was greater. However, covariate D reversed the pattern, with the mean for LPE being the greater. The correlation coefficients for D and A-C also suggested that there was not a very close relationship between the scores on the Concept Attainment Test and the Lorge-Thorndike measure of intelligence. The correlation coefficients for A and D, B and D, and C and D were

respectively .327, .087, and .294.

The criterion measures were scores on the Subtraction Application Test (SA), the Subtraction Transfer Test (ST), the Addition Computation Test (AC), the Subtraction Computation Test (SC), the Problem Solving Test (PS), and the Stanford Achievement Test (SAT). The means, effects, and standard deviations for each of these measures are summarized in Table 14.

TABLE 14

MEANS, EFFECTS, AND STANDARD DEVIATIONS OF END OF YEAR TESTS SA, ST, AC, SC, PS, AND SAT FOR CLASSES LPE AND LPL

Criterion	Class	Mean	Effect	SD
SA (Sub.App.)	LPE	2.35	-0.14	1.53
	LPL	2.60	0.11	1.72
ST (Sub.Tran.)	LPE	1.45	-0.42	2.20
	LPL	2.20	0.33	2.55
AC (Add.Comp.)	LPE	1.85	-0.28	1.98
	LPL	2.36	0.23	2.64
SC (Sub.Comp.)	LPE	1.30	0.06	1.85
	LPL	1.20	-0.04	1.47
PS (Prob.Sol.)	LPE	6.90	0.17	3.51
	LPL	6.60	-0.13	3.71
SA (Stan.Ach.)	LPE	13.25	-0.04	7.48
	LPL	13.32	0.03	7.98

The means recorded in Table 14 indicate that the two classes had mean scores for each of the criterion measures which were nearly equal. In four cases LPL had the greater mean score, and in the remaining two cases the LPE mean was greater. In no case was the difference between means very great, the greatest difference being 0.75 for the criterion ST.

The results of the analyses of covariance of the criterion measures are summarized in Tables 15, 16, 17, and 18. In each table is reported the analysis of covariance of each criterion measure for one of the covariates. The tests of significance summarized in Tables 15, 16, and 17 indicate that, when initial differences were

TABLE 15

ANALYSIS OF COVARIANCE OF INDIVIDUAL SCORES FOR SA, ST, AC, SC, PS, AND SAT FOR CLASSES LPE AND LPL. COVARIATE: 1-5 CONCEPT PRETEST

Criterion	Source of Variation	df	Adj. SS	M.S.	F
SA (Sub.App.)	Between Groups	1	0.66	0.66	0.30
	Within Groups	42	92.28	2.20	
	Total	43	92.94		
	r = .482				
ST (Sub.Tran.)	Between Groups	1	0.08	0.08	0.02
	Within Groups	42	181.82	4.33	
	Total	43	181.90		
	r = .560				
AC (Add.Comp.)	Between Groups	1	0.26	0.26	0.05
	Within Groups	42	202.67	4.83	
	Total	43	202.93		
	r = .453				
SC (Sub.Comp.)	Between Groups	1	0.11	0.11	0.04
	Within Groups	42	122.20	2.91	
	Total	43	122.31		
	r = -.005				
PS (Prob.Sol.)	Between Groups	1	20.27	20.27	1.88
	Within Groups	42	452.17	10.77	
	Total	43	472.44		
	r = .448				
SAT (Stan.Ach.)	Between Groups	1	48.03	48.03	0.94
	Within Groups	42	2150.32	51.20	
	Total	43	2198.34		
	r = .434				

$F_{.95}(1,42) = 4.10$

controlled by using the Concept Pretest scores, there were no statistically significant differences in the criterion measures of achievement. This implies that the manipulated variable, time of symbolization of addition and subtraction concepts, produced few noticeable effects in the terminal achievement measures when individual differences were controlled by

TABLE 16

ANALYSIS OF COVARIANCE OF INDIVIDUAL SCORES FOR SA, ST, AC, SC, PS, AND SAT FOR CLASSES LPE AND LPL. COVARIATE: 6-10 CONCEPT PRETEST

Criterion	Source of Variation	df	Adj. SS	M.S.	F
SA (Sub.App.)	Between Groups	1	0.21	0.21	0.08
	Within Groups	42	118.17	2.81	
	Total	43	118.38		
	r = .154				
ST (Sub.Trans.)	Between Groups	1	1.96	1.96	0.35
	Within Groups	42	237.90	5.66	
	Total	43	239.86		
	r = .309				
AC (Add.Comp.)	Between Groups	1	0.58	0.58	0.10
	Within Groups	42	236.47	5.63	
	Total	43	337.05		
	r = .267				
SC (Sub.Comp.)	Between Groups	1	0.57	0.57	0.20
	Within Groups	42	118.55	2.82	
	Total	43	119.13		
	r = .161				
PS (Prob.Sol.)	Between Groups	1	3.68	3.68	0.27
	Within Groups	42	572.26	13.63	
	Total	43	575.94		
	r = .395				
SAT (Stan.Ach.)	Between Groups	1	10.14	10.14	0.17
	Within Groups	42	2480.44	59.06	
	Total	43	2490.58		
	r = .284				

$F_{.95}(1,42) = 4.10$

the Concept Pretest measures.

The results of the analyses of covariance summarized in Table 18 show two F-ratios significant at the five per cent level of confidence. These occurred for the comparisons of the criterion measures of Subtraction Applications and Subtraction Transfer. In both cases the

TABLE 17

ANALYSIS OF COVARIANCE OF INDIVIDUAL SCORES FOR SA, ST, AC, SC, PS, AND SAT FOR CLASSES LPE AND LPL. COVARIATE: TOTAL CONCEPT PRETEST

Criterion	Source of Variation	df	Adj. SS	M.S.	F
SA (Sub.App.)	Between Groups	1	0.33	0.33	0.14
	Within Groups	42	97.44	2.32	
	Total	43	97.77		
	r = .440				
ST (Sub.Tran.)	Between Groups	1	0.01	0.01	0.00
	Within Groups	42	183.07	4.36	
	Total	43	183.07		
	r = .557				
AC (Add.Comp.)	Between Groups	1	0.00	0.00	0.00
	Within Groups	42	219.37	5.22	
	Total	43	219.37		
	r = .375				
SC (Sub.Comp.)	Between Groups	1	0.62	0.62	0.22
	Within Groups	42	119.50	2.85	
	Total	43	120.12		
	r = .134				
PS (Prob.Sol.)	Between Groups	1	15.24	15.24	1.32
	Within Groups	42	483.24	11.51	
	Total	43	498.48		
	r = .395				
SAT (Stan.Ach.)	Between Groups	1	41.02	41.02	0.79
	Within Groups	42	2169.37	51.65	
	Total	43	2210.39		
	r = .429				

$F_{.95}(1,42) = 4.10$

adjusted means (Table 19) were greater for the late symbolization class than for the early symbolization class. Hence, when the criterion means of LPL and LPE were adjusted for individual differences in intelligence, the differences between the mean achievement in subtraction applications and subtraction transfer of LPL and LPE were so large

TABLE 18

ANALYSIS OF COVARIANCE OF INDIVIDUAL SCORES FOR SA, ST, AC, SC, PS, AND SAT FOR CLASSES LPE AND LPL. COVARIATE: LORGE-THORNDIKE INTELLIGENCE

Criterion	Source of Variation	df	Adj. SS	M.S.	F
SA (Sub.App.)	Between Groups	1	7.93	7.93	4.55*
	Within Groups	42	73.18	1.74	
	Total	43	81.11		
	r = .575				
ST (Sub.Tran.)	Between Groups	1	21.58	21.48	4.53*
	Within Groups	42	199.88	4.76	
	Total	43	221.46		
	r = .406				
AC (Add.Comp.)	Between Groups	1	12.81	12.81	2.59
	Within Groups	42	207.98	4.95	
	Total	43	220.79		
	r = .367				
SC (Sub.Comp.)	Between Groups	1	0.00	0.00	0.00
	Within Groups	42	121.33	2.89	
	Total	43	121.34		
	r = .089				
PS (Prob.Sol.)	Between Groups	1	13.78	13.78	1.72
	Within Groups	42	336.68	8.02	
	Total	43	350.46		
	r = .638				
SAT (Stan.Ach.)	Between Groups	1	76.95	76.95	1.76
	Within Groups	42	1860.38	44.29	
	Total	43	1937.33		
	r = .534				

*$F_{.95}(1,42) = 4.10$

that they could be attributed to the manipulated variable, i.e., the time of symbolization.

The correlation coefficients reported in Tables 15, 16, 17, and 18 for the criterion Subtraction Computation and the covariates A-D show a definite trend when compared with the correlation coefficients

TABLE 19

MEANS AND MEANS ADJUSTED FOR INDIVIDUAL DIFFERENCES IN INTELLIGENCE OF THE CRITERION MEASURES SA AND ST FOR CLASSES LPL AND LPE

Criterion	Class	Mean	Adj. Mean
SA (Sub.App.)	LPE	2.35	2.00
	LPL	2.60	2.88
ST (Sub.Trans.)	LPE	1.45	1.06
	LPL	2.20	2.51

for the other criteria. In each case the former correlation coefficients were very near zero. For the majority of the other criterion measures the correlation coefficients were much greater. The near zero correlation coefficients for SC are partially explained by the observation that these children did not have a firm grasp of the manipulations or the symbolism associated with subtraction as taught in the PPW approach. Difficulties always arose when the children were asked to work quickly and without a manipulative aid--both of which were true of the Subtraction Computation Test. Hence the achievement scores were uniformly low yielding a greater probability that the correlation would be low.

In summary, the only significant effect of the manipulated variable occurred in the criterion measures of Subtraction Applications and Transfer. When individual intelligence differences were controlled, the late symbolization class (LPL) had a greater adjusted mean score. This may be interpreted to mean that, through the use of words which conveyed the idea of partitioning a set, later symbolization was a positive factor which facilitated learning to use set partitioning as a means of solving unfamiliar subtraction examples and subtraction applications. The same effect was not observed for the criterion Subtraction Computation since the children did not have sufficient time to work out the answers. Four minutes were provided for ten subtraction sentences to be answered while approximately forty minutes were available for the Subtraction Applications and Transfer Test in which eighteen problems were to be done.

Sex Differences Analyzed

The analysis of sex differences was performed only for the higher ability classes, that is, PPW-Early (PE), PPW-Late (PL), TA-Early (TE), and TA-Lake (TL). A similar analysis was not performed for the lower ability PPW classes, low ability PPW-Early (LPE) and low ability PPW-Late (LPL), since the boys outnumbered the girls two to one, the N's being thirty and fifteen. Also the boys and girls in the lower ability PPW classes were not pooled with those of PE and PL because the topics studied by the LPE and LPL children did not include all that was studied in the other classes. The inclusion of LPE and LPL boys and girls would bias the results in favor of the TA classes.

Sex differences were analyzed using one-way analysis of covariance with four groups, namely PPW-Boys (PM), PPW-Girls (PF), TA-Boys (TM), and TA-Girls (TF). The four covariates used were the scores on (A) 1-5 Concept Pretest, (B) 6-10 Concept Pretest, (C) Total Concept Pretest, and (D) Lorge-Thorndike Intelligence Test. The means, standard deviations, effects, and N's for covariates A-D for the four groups are summarized in Table 20.

Analyses of variance of each of the covariates in Table 20 showed no statistically significant differences in the means. However, analysis of variance will show non-significant results in some cases where there actually is a significant difference between two means in a set of three or more. For covariate B, the mean of PF and the mean of TM have a difference of 28.63. This difference was tested using the conservative Newman-Kuels method and found to be significant at the five per cent level of confidence.

The large value of the mean of PF for covariate B is all the more striking when the means of covariate D (IQ) are ordered. For D, the mean of PF is the smallest. This reversal of order for PF means for covariates B and D is a partial explanation of the relatively small correlation coefficient (.292) for these measures.

The criterion measures in the analysis of sex differences were the scores on the Subtraction Applications Test (SA), the Subtraction Transfer Test (ST), the Addition Computation Test (AC), the Subtraction Computation Test (SC), the Problem Solving Test (PS), and the Stanford

TABLE 20

MEANS, EFFECTS, STANDARD DEVIATIONS, AND N'S OF COVARIATES A-D FOR THE FOUR GROUPS: PM, PF, TM, TF

Covariate	Group	Mean	Effect	SD	N
A	PM	176.96	12.35	49.29	23
	PF	165.42	0.81	43.11	24
	TM	152.17	-12.43	43.03	23
	TF	163.68	- 0.92	39.23	19
B	PM	93.26	3.40	36.22	23
	PF	103.63	13.76	32.54	24
	TM	75.00	-14.87	35.00	23
	TF	86.37	- 3.50	36.75	19
C	PM	314.57	20.66	90.67	23
	PF	310.29	16.38	76.04	24
	TM	261.09	-32.82	85.75	23
	TF	287.95	- 5.96	90.20	19
D	PM	107.35	- 0.66	14.79	23
	PF	104.96	- 3.05	14.25	24
	TM	107.00	- 1.01	17.59	23
	TF	113.89	5.88	13.40	19

Achievement Test (SAT). The means, effects, and standard deviations for each of these measures are given in Table 21.

The results of the analyses of covariance of the criterion measures are summarized in Tables 22, 23, 24, and 25. In each table is reported the analysis of covariance of each criterion measure for one of the covariates. For covariate A (1-5 Concept Pretest) in Table 22 there were no significant F-ratios for any of the six criterion measures. The same was true in Table 25 for covariate D (Lorge-Thorndike Intelligence). The only statistically significant F-ratios obtained were for the criterion measure SC in Tables 23 and 24 and for the criterion SAT in Table 23. Since there were four groups analyzed in each of these cases, it is not possible to say which means are significantly different without further tests. For this purpose the Newman-Kuels post hoc test of differences between means was employed. The results of the Newman-Kuels analysis are reported in Table 26. It will be noted that only the difference between one pair of means was found to be significantly different from zero using the Newman-Kuels method. The

TABLE 21

MEANS, EFFECTS, AND STANDARD DEVIATIONS OF END OF YEAR TESTS SA, ST, AC, SC, PS, AND SAT FOR GROUPS PM, PF, TM, TF

Criterion	Group	Mean	Effect	SD
SA (Sub.App.)	PM	3.57	-0.19	1.50
	PF	4.00	0.25	1.87
	TM	3.78	0.03	1.74
	TF	3.63	-0.12	1.78
ST (Sub.Tran.)	PM	3.57	-0.38	2.70
	PF	3.75	-0.19	3.06
	TM	4.09	0.14	2.57
	TF	4.47	0.53	2.72
AC (Add.Comp.)	PM	5.52	0.83	2.68
	PF	4.25	-0.45	3.02
	TM	4.30	-0.39	2.77
	TF	4.74	0.04	2.49
SC (Sub.Comp.)	PM	4.83	-0.47	3.55
	PF	4.71	-0.58	3.22
	TM	5.39	0.10	3.55
	TF	6.47	1.18	3.22
PS (Prob.Sol.)	PM	11.04	1.03	3.96
	PF	9.04	-0.97	3.63
	TM	9.39	-0.62	4.42
	TF	10.74	0.73	4.35
SAT (Stan.Ach.)	PM	26.35	1.60	11.28
	PF	22.17	-2.59	10.19
	TM	24.17	-0.58	12.52
	TF	26.79	2.04	11.02

reason for this is that the Newman-Kuels method is not as sensitive as a comparable t-test of means. However, in a post hoc analysis of means a t-test is not appropriate since all the means are obtained from one sample, and hence the confidence interval must be constructed to account for this. (See 54:483-489.)

TABLE 22

ANALYSIS OF COVARIANCE OF INDIVIDUAL SOCRES FOR SA, ST, AC, SC, PS, AND SAT FOR GROUPS PM, PF, TM, TF
COVARIATE: 1-5 CONCEPT PRETEST

Criterion	Source of Variation	df	Adj. SS	M.S.	F
SA (Sub.App.)	Between Groups	3	5.52	1.84	0.67
	Within Groups	84	229.83	2.74	
	Total	87	235.34		
	r = .352				
ST (Sub.Tran.)	Between Groups	3	21.71	7.24	1.05
	Within Groups	84	579.17	6.89	
	Total	87	600.89		
	r = .368				
AC (Add.Comp.)	Between Groups	3	10.62	3.54	0.55
	Within Groups	84	544.68	6.48	
	Total	87	555.30		
	r = .458				
SC (Sub.Comp.)	Between Groups	3	60.94	20.31	2.09
	Within Groups	84	814.65	9.70	
	Total	87	875.59		
	r = .421				
PS (Prob.Sol.)	Between Groups	3	41.29	13.76	0.98
	Within Groups	84	1174.72	13.98	
	Total	87	1216.02		
	r = .466				
SAT (Stan.Ach.)	Between Groups	3	291.95	97.32	1.01
	Within Groups	84	8088.71	96.29	
	Total	87	8380.66		
	r = .529				

$F_{.95}(3,84) = 2.73$

TABLE 23

ANALYSIS OF COVARIANCE OF INDIVIDUAL SCORES FOR SA, ST, AC, SC, PS, AND SAT FOR GROUPS PM, PF, TM, TF
COVARIATE: 6-10 CONCEPT PRETEST

Criterion	Source of Variation	df	Adj. SS	M.S.	F
SA (Sub.App.)	Between Groups	3	3.89	1.30	0.49
	Within Groups	84	221.86	2.64	
	Total	87	225.76		
	r = .399				
ST (Sub.Tran.)	Between Groups	3	28.52	9.51	1.37
	Within Groups	84	581.16	6.92	
	Total	87	609.68		
	r = .350				
AC (Add.Comp.)	Between Groups	3	33.57	11.19	1.78
	Within Groups	84	526.73	6.27	
	Total	87	560.30		
	r = .450				
SC (Sub.Comp.)	Between Groups	3	86.89	28.96	3.06*
	Within Groups	84	795.88	9.47	
	Total	87	882.78		
	r = .413				
PS (Prob.Sol.)	Between Groups	3	103.28	34.43	2.54
	Within Groups	84	1137.90	13.55	
	Total	87	1241.18		
	r = .448				
SAT (Stan.Ach.)	Between Groups	3	803.03	267.68	2.83*
	Within Groups	84	7953.73	94.69	
	Total	87	8756.77		
	r = .498				

*$F_{.95}(3,84) = 2.73$

TABLE 24

ANALYSIS OF COVARIANCE OF INDIVIDUAL SCORES FOR SA, ST, AC, SC, PS, AND SAT FOR GROUPS PM, PF, TM, TF
COVARIATE: 1-11 CONCEPT PRETEST

Criterion	Source of Variation	df	Adj. SS	M.S.	F
SA (Sub.App.)	Between Groups	3	6.10	2.03	0.82
	Within Groups	84	207.53	2.47	
	Total	87	213.63		
	r = .452				
ST (Sub.Tran.)	Between Groups	3	30.98	10.33	1.63
	Within Groups	84	533.61	6.35	
	Total	87	564.58		
	r = .433				
AC (Add.Comp.)	Between Groups	3	18.95	6.32	1.11
	Within Groups	84	478.69	5.70	
	Total	87	497.64		
	r = .540				
SC (Sub.Comp.)	Between Groups	3	83.96	27.99	3.25*
	Within Groups	84	724.36	8.62	
	Total	87	808.33		
	r = .490				
PS (Prob.Sol.)	Between Groups	3	69.31	23.10	1.93
	Within Groups	84	1003.22	11.94	
	Total	87	1072.53		
	r = .556				
SAT (Stan.Ach.)	Between Groups	3	594.07	198.02	2.49
	Within Groups	84	6680.20	79.53	
	Total	87	7274.27		
	r = .613				

*$F_{.95}(3,84) = 2.73$

TABLE 25

ANALYSIS OF COVARIANCE OF INDIVIDUAL SCORES FOR SA, ST, AC, SC, PS, AND SAT FOR GROUPS PM, PF, TM, TF
COVARIATE: I.Q.

Criterion	Source of Variation	df	Adj. SS	M.S.	F
SA (Sub.App.)	Between Groups	3	7.89	2.63	1.05
	Within Groups	84	209.99	2.50	
	Total	87	217.88		
	r = .434				
ST (Sub.Tran.)	Between Groups	3	3.73	1.24	0.20
	Within Groups	84	530.34	6.31	
	Total	87	534.07		
	r = .481				
AC (Add.Comp.)	Between Groups	3	24.93	8.31	1.33
	Within Groups	84	525.57	6.26	
	Total	87	550.50		
	r = .466				
SC (Sub.Comp.)	Between Groups	3	13.95	4.65	0.46
	Within Groups	84	846.52	10.08	
	Total	87	860.46		
	r = .438				
PS (Prob.Sol.)	Between Groups	3	41.87	13.96	1.05
	Within Groups	84	1119.12	13.32	
	Total	87	1161.00		
	r = .502				
SAT (Stan.Ach.)	Between Groups	3	127.76	42.59	0.43
	Within Groups	84	8237.33	98.06	
	Total	87	8365.09		
	r = .531				

$F_{.95}(3,84) = 2.73$

TABLE 26

NEWMAN-KUELS post hoc TEST OF ADJUSTED MEANS FOR SC AND SAT (COVARIATE B) AND FOR SC (COVARIATE C)

Covariate	Criterion	Group	Mean	Adj. Mean	q
B	SC (Sub.Comp.)	PM	4.83	4.67	*3.86 (TF vs PF)
		PF	4.71	4.08*	
		TM	5.39	6.07	
		TF	6.47	6.63*	
B	SAT (Stan.Ach.)	PM	26.35	25.75	3.71 (TF vs PF)
		PF	22.17	19.75	
		TM	24.17	26.79	
		TF	26.79	27.40	
C	SC (Sub.Comp.)	PM	4.83	4.38	3.58 (TF vs PF)
		PF	4.71	4.36	
		TM	5.39	6.10	
		TF	6.47	6.60	

*$q_{.95}(4,84) = 3.73$

From the analyses reported in Tables 22, 23, 24, 25, and 26 it appears quite clear that there were, generally, few sex differences associated with the two levels of the approach variable. The only point of contention with the general conclusion arises from the criterion measure of subtraction computation (SC). For this criterion measure the TA-Girls had a significantly greater adjusted mean than the PPW-Girls when individual differences, as reflected in covariate B, were controlled statistically. This significant difference is inexplicable by the author, especially when the other criterion measure of subtraction (ST) showed no significant differences.

Higher Ability Group Concept Attainment Gain Scores Analyzed

The four higher ability classes as defined by the manipulated variables were PPW-Early (PE), TA-Early (TE), PPW-Late (PL), and TA-Late (TL). The gain scores obtained from the two administrations of the Concept Attainment Test (CAT) were tested for significant differences using this method of two-way analysis of covariance. To facilitate the analysis the classes were reduced to twenty students by random procedures. The initial measures used to control the individual differences between

the classes were the Total Concept Pretest (C) and the Lorge-Thorndike Intelligence Test (D). The means and standard deviations of the covariates are summarized in Table 27.

TABLE 27

MEANS AND STANDARD DEVIATIONS OF COVARIATES C AND D FOR THE FOUR EQUALIZED CLASSES PE, TE, PL, AND TL

Covariate		Class			
		PE	TE	PL	TL
C	Mean	317.75	281.75	304.50	261.55
	SD	72.50	100.94	105.61	83.07
D	Mean	116.95	118.10	96.80	101.10
	SD	11.02	14.84	11.13	12.32

The gain scores were derived from the scores on the CAT by subtraction: CAT Terminal Score - CAT Initial Score. The gain scores for item 1-5, 6-8, 10, 6-10, 11, and 1-11 (total) were computed in this manner. The means and standard deviations of the gain scores for each class are summarized in Table 28. There is one clear trend evident in Table 28. The mean gains for PL were consistently smaller than was true for any other class. This trend follows most others found concerning data on these four classes. In nearly every instance PL had the smallest scores of the four classes.

The analyses of covariance for each of the six gain scores are reported in Tables 29 and 31. The tests of significance using covariate C are summarized in Table 29 and those employing covariate D are recorded in Table 31. The adjusted mean gain scores of the four classes for each test of significance which indicated a statistically reliable difference are tabulated in Tables 30 and 32.

There are two reliable interactions reported in Table 29. Both were significant beyond the one per cent level of confidence, and the adjusted means reported in Table 30 reveal the same fact for both measures. In each case the most effective combination of variables was P and E while the least effective combination was P and L. The results of the tests reported in Table 31 reinforce the differences revealed in Table 29.

TABLE 28

MEANS AND STANDARD DEVIATIONS OF CAT GAIN SCORES FOR ITEMS 1-5, 6-8, 10, 6-10, 11, AND 1-11 FOR FOUR EQUALIZED CLASSES PE, TE, PL, AND TL

Item		Class			
		PE	TE	PL	TL
1-5	Mean	54.00	43.00	29.00	62.00
	SD	32.67	40.28	47.12	46.86
6-8	Mean	13.50	13.50	1.50	25.50
	SD	32.97	24.77	20.59	32.68
10	Mean	9.75	6.75	1.50	0.00
	SD	22.45	29.79	37.03	25.75
6-10	Mean	29.55	22.05	4.20	30.60
	SD	43.13	45.19	37.02	36.54
11	Mean	37.50	31.50	25.50	33.00
	SD	29.00	28.34	24.38	27.36
1-11	Mean	121.05	96.55	58.70	125.60
	SD	74.63	87.98	68.75	77.81

Perhaps more interesting than the reliable differences found in Table 29 and 31 are those tests of significance which were not statistically significant. This is true of item ten (conservation of the whole upon subdivision) in particular. Greater mean gains in the score on item ten would be expected to occur for approach P than for approach T. Since they did not occur, there is evidence suggesting that the approach used had little effect on learning the concept of the conservation of the whole upon subdivision. However, the unadjusted mean gain scores for item ten and those adjusted by use of covariates C and D suggest that the children instructed by the PPW method tended to make the greater gains.

Overall the results suggest that the PPW approach had little significant effect upon learning the concepts measured by items six through ten of the CAT. The only Approach effect that was statistically reliable was found in the difference in the mean gains scores for 6-8 (Table 31). In this case the TA approach was superior to the PPW. However, all classes had very similar experiences with one-to-one

TABLE 29

ANALYSIS OF COVARIANCE OF CAT GAIN SCORES FOR ITEMS 1-5, 6-8, 10, 6-10, 11, AND 1-11 FOR CLASSES PE, TE, PL, and TL
COVARIATE: TOTAL CONCEPT PRETEST

Item	Source of Variation	df	Adj. SS	M.S.	F
1-5	Approach	1	7.32	7.32	0.01
	Time	1	1244.58	1244.58	1.15
	Interaction	1	8797.45	8797.45	8.26**
	Within	75	79850.00	1064.67	
6-8	Approach	1	1295.45	1295.45	1.77
	Time	1	50.24	50.24	0.07
	Interaction	1	2722.40	2722.40	3.72
	Within	75	54894.00	731.92	
10	Approach	1	120.66	120.66	0.14
	Time	1	1148.91	1148.91	1.32
	Interaction	1	10.56	10.56	0.01
	Within	75	65063.70	867.52	
6-10	Approach	1	355.82	355.82	0.23
	Time	1	2216.51	2216.51	1.45
	Interaction	1	5441.88	5441.88	3.56
	Within	75	114792.00	1530.56	
11	Approach	1	188.07	188.07	0.28
	Time	1	943.47	943.47	1.40
	Interaction	1	820.81	820.81	1.22
	Within	75	50562.00	674.16	
1-11	Approach	1	5.94	5.94	0.00
	Time	1	12785.80	12785.80	3.38
	Interaction	1	38499.80	38499.80	10.18**
	Within	75	283713.00	3782.83	

$^{**}F_{.99}(1,75) = 6.99$

correspondence, the topic of items six through ten, and it is difficult to conclude the approach used was the major factor.

The major reliable differences were found in the tests of interaction. These effects were due, not to the great magnitude of the adjusted mean score for a combination of variables, but rather to the diminutive adjusted mean score for class PL. Generally there were not great differences in three of the adjusted mean scores, but the three

TABLE 30

ADJUSTED CELL, ROW, COLUMN, AND GRAND MEANS FOR CAT ITEMS 1-5 AND 1-11
COVARIATE: TOTAL CONCEPT PRETEST

Item	Time	Approach		Row Mean
		P	T	
1-5	E	61.7600	40.1631	50.9616
	L	32.8598	53.2171	43.0384
	Column Mean	47.3099	46.6901	47.0000 (M_t)
1-11	E	134.8343	91.5108	113.1725
	L	65.5562	109.9987	87.7775
	Column Mean	100.1952	100.7548	100.4750 (M_t)

invariably exceeded the fourth substantially. However, when there was a distinct superior combination of approach and time of symbolization variables, that combination was P and E.

TABLE 31

ANALYSIS OF COVARIANCE OF CAT GAIN SCORES FOR ITEMS 1-5, 6-8, 10, 6-10, 11 AND 1-11 FOR CLASSES PE, TE, PL, AND TL
COVARIATE: LORGE-THORNDIKE PRETEST

Item	Source of Variation	df	Adj. SS	M.S.	F
1-5	Approach	1	2995.32	2995.32	1.70
	Time	1	1802.46	1802.46	1.02
	Interaction	1	10316.80	10316.80	5.85*
	Within	75	132268.00	1763.58	
6-8	Approach	1	3756.33	3756.33	5.07*
	Time	1	1876.88	1876.88	2.53
	Interaction	1	3384.02	3384.02	4.57*
	Within	75	55593.30	741.24	
10	Approach	1	258.35	258.35	0.31
	Time	1	41.55	41.55	0.05
	Interaction	1	0.03	0.03	0.00
	Within	75	62142.20	828.56	
6-10	Approach	1	1871.91	1871.91	1.12
	Time	1	1347.89	1347.89	0.81
	Interaction	1	5833.81	5833.81	3.49
	Within	75	125413.00	1672.17	
11	Approach	1	10.43	10.43	0.01
	Time	1	326.32	326.32	0.43
	Interaction	1	903.79	903.79	1.19
	Within	75	56744.10	756.59	
1-11	Approach	1	10246.40	10246.40	1.69
	Time	1	9454.32	9454.32	1.56
	Interaction	1	43269.90	43269.90	7.15**
	Within	75	453800.00	6050.66	

*$F_{.95}(1,75) = 3.97$ **$F_{.95}(1,75) = 6.99$

TABLE 32

ADJUSTED CELL, ROW, COLUMN, AND GRAND MEAN FOR CAT ITEMS 1-5, 6-8 1-11
COVARIATE: LORGE-THORNDIKE INTELLIGENCE TEST

Item	Time	Approach		Row Mean
		P	T	
1-5	E	58.2048	47.7598	52.9823
	L	23.4801	58.5553	41.0177
	Column Mean	40.8424	53.1576	47.0000 (M_t)
6-8	E	19.2266	19.9825	19.6046
	L	- 6.0177	20.8068	7.3954
	Column Mean	6.6044	20.3956	13.5000 (M_t)
1-11	E	126.0931	102.2588	114.1759
	L	52.0796	121.4686	86.7741
	Column Mean	89.0863	111.8637	100.4750 (M_t)

Lower Ability Group Concept Attainment Gain Scores Analyzed

Classes LPE and LPL were instructed using the PPW approach to subtraction and had nearly identical instruction in number concepts including counting, one-to-one correspondence, and the conservation of the whole upon subdivision. Also the instruction in the use of subtraction in practical situations was identical except for the symbolism used in expressing the subtraction sentences; LPE used the symbols "-" and "=" exclusively while LPL used "split" and "is" for six weeks prior to the introduction of the symbols "-" and "=".

The Concept Attainment Test was administered to the classes twice during the year. The mean scores for the two administrations are summarized in Table 33.

The obvious trend expressed in Table 33 is that LPL consistently scored greater mean scores for all the enumerated items except for item eleven of the first administration. A second slightly disconcerting fact evident from Table 33 is that both LPE and LPL decreased in mean score for item ten--the item which was expressly constructed to determine the extent to which the children grasped the concept of conservation of the whole upon subdivision. The differences between the second and first administrations for LPE and for LPL were tested with the t-test and in both cases found to be not significant ($t = -0.592$ for LPE; $t = -1.069$ for LPL).

Even though the differences for LPE and for LPL for item ten were not statistically significant, the fact that the mean scores decreased remains unexplained. There are two possible explanations. First, it could have been that the children did not learn anything about the concept of conservation of the whole upon subdivision during the year, and the resulting differences were purely random fluctuations from one testing to the next. If this is true, basing subtraction instruction on PPW was probably futile.

Second, it may have been true that at the outset the children actually knew less than the score for item ten indicated. This could have been due to the combination of the wording of the item (a "yes" is correct), and the observed tendency of these children to respond affirmatively to any question for which the answer is doubtful. If this is the case, the item and/or procedure must be reformulated. In

TABLE 33

MEAN SCORES FOR TWO ADMINISTRATIONS OF CONCEPT ATTAINMENT TEST FOR LPE AND LPL

Class	N	Items	Mean Pretest	Mean Post Test
LPE	20	1-5	110.00	176.00
LPL	25		138.80	177.20
LPE	20	6-8	34.50	57.00
LPL	25		43.20	69.60
LPE	20	10	14.25	10.50
LPL	25		20.40	14.40
LPE	20	6-10	55.05	80.70
LPL	25		72.24	94.80
LPE	20	11	25.50	46.50
LPL	25		22.32	50.40
LPE	20	1-11	190.55	303.20
LPL	25		233.36	322.40

either case the data available are not sufficient to resolve the issue.

The analyses of the mean gains for LPE and LPL are summarized in Table 34. Since there was no prior information to indicate either class would make greater gains, the two-tailed t-test was used for which the critical values of t are +2.021 or -2.021 for forty-three degrees of freedom. In each case the difference in mean gains was calculated by subtracting the LPL score from the LPE score. Therefore a positive t-ratio implies the gains of LPE were greater than those of LPL and conversely.

The summary in Table 34 indicates quite clearly that there were few differences between the two classes. The one significant difference occurred in the gains on the first five items in the Concept Attainment Test. These items deal predominately with the ability to count and may be answered correctly using only the number sequence. In LPE there were several children who had practically no knowledge of the number sequence at the beginning of the school year. During the year they gained some facility in counting, and this progress is reflected

TABLE 34

T-TEST FOR DIFFERENCES BETWEEN MEANS BASED ON INDIVIDUAL GAIN SCORES FOR CONCEPT ATTAINMENT TEST FOR LPE AND LPL

Class	N	Items	Mean Gain	SD	t
LPE	20	1-5	66.00	43.94	2.188*
LPL	25		38.40	40.48	
LPE	20	6-8	22.50	33.54	-0.354
LPL	25		26.40	39.04	
LPE	20	10	- 3.75	28.32	0.266
LPL	25		- 6.00	28.06	
LPE	20	6-10	25.65	39.15	0.217
LPL	25		22.56	53.06	
LPE	20	11	21.00	29.36	-0.646
LPL	25		28.04	37.67	
LPE	20	1-11	112.65	56.64	1.027
LPL	25		89.04	89.36	

*$t_{.95}(43) = 2.021$

in the mean gain score for items one through five.

Attitude Data Analyzed

The attitude data were analyzed in two ways: (a) the mean scores from the first administration of the Arithmetic Attitude Scale were compared with the mean scores of the third administration for each class using the two-tailed t-test and (b) the average scores on the three administrations for each subject were used to obtain group means which were compared using the two-tailed t-test as follows: (1) P vs. T, (2) E vs. L, (3) PE vs. TE, (4) PL vs. TL, (5) LPE vs. LPL, and (6) Boys vs. Girls (in the four unequalized classes). P and T refer to the two levels of the approach, that is, PPW and TA respectively. E and L refer to the two levels of the time of symbolization, that is, early and late respectively. Symbols like PE refer to the intersection of the time of symbolization and the approach, that is, to one class, while LPE and LPL refer to the two lower ability classes, namely, the lower ability PPW early and late symbolization classes respectively.

The results of the first analysis are reported in Table 35.

TABLE 35

T-TESTS FOR DIFFERENCE BETWEEN MEANS ON FIRST AND LAST ADMINISTRATIONS OF ARITHMETIC ATTITUDE SCALE FOR ALL CLASSES

Class	N	Difference (d) (Last - First)	$\bar{d}$	SD_d	t
PE	21	10	0.4762	1.067	2.046
TE	22	6	0.2727	1.188	1.078
PL	26	- 5	-0.1923	0.979	-1.000
TL	20	5	0.2500	1.032	1.084
LPE	20	- 5	-0.2500	0.803	-1.392
LPL	25	- 2	-0.0800	1.044	-0.383

$t_{.95}(20) = 2.086$; $t_{.05}(20) = -2.086$

There were no significant t-ratios for any of the differences obtained. Three classes gained in mean attitude score while three decreased. In the four higher ability classes there were three which increased in mean attitude score, the only one decreasing was PL. The two lower ability classes decreased in mean score.

The results of the second analysis are reported in Table 36. The first entry in Table 36 shows the comparison for the PPW group and the TA group (P and T). The mean score was slightly higher for the TA group but not significantly so. The following entry summarizes the comparison for the early symbolization and late symbolization groups (E and L). The late symbolization group mean is slightly higher, but again the obtained t-ratio is not significant.

The next three entries in Table 36 summarize the comparisons between classes. In the first case (PE vs. TE), with constant early symbolization, the TA class mean score was slightly greater; in the second (PL vs. TL), with constant late symbolization, the same is true. The third entry shows the comparison of the mean attitude scores for the two low ability PPW classes. The early symbolization class (LPE) had a slightly greater mean score. In no case was the difference statistically significant.

The last entry in Table 36 summarizes the comparison of the mean attitude scores for boys and girls in the four higher ability classes.

TABLE 36

T-TESTS FOR DIFFERENCE BETWEEN MEANS DERIVED FROM AVERAGED INDIVIDUAL SCORES FOR ARITHMETIC ATTITUDE SCALE FOR SIX GROUPS

Group	N	$\bar{X}$	SD	t
P	47	2.3475	0.5427	-0.588
T	42	2.4127	0.7403	
			$t_{.05}(87) = -2.000$	
E	43	2.3643	0.5030	-0.250
L	46	2.3913	0.5179	
			$t_{.05}(87) = -2.000$	
PE	21	2.3492	0.4998	-0.191
TE	22	2.3788	0.5174	
			$t_{.05}(41) = -2.021$	
PL	26	2.3462	0.5849	-0.549
TL	20	2.4500	0.4227	
			$t_{.05}(44) = -2.020$	
LPE	20	2.4833	0.5668	1.357
LPL	25	2.2533	0.5634	
			$t_{.95}(43) = 2.020$	
Boys	46	2.3840	0.5351	0.111
Girls	43	2.3720	0.4763	
			$t_{.95}(87) = 2.000$	

The boys obtained a mean score only 0.012 units greater than the girls. The obtained t-ratio was far from significant.

The greatest gain in attitude score was obtained by the PPW class PE, but, in general, the TA classes obtained greater mean attitude scores. Also the late symbolization group obtained a greater mean score than the early symbolization group, but in the case of the two lower ability classes the converse was true. There were no sex differences for mean attitude in the four higher ability classes.

CHAPTER V

SUMMARY AND CONCLUSIONS

In Chapter One four questions were asked concerning the effects of the two manipulated variables, approach to subtraction and the time of symbolization. These questions are not equally applicable to the lower and the higher ability groups since only the time of symbolization variable was manipulated in the lower ability group while both variables were manipulated in the higher ability group. Therefore, the questions are restated for each group throughout this chapter. However, the restatement of each question is in the form of a null hypothesis for which evidence is cited for acceptance or rejection.

Lower Ability Group

The lower ability group consisted of two classes. Both were taught using the PPW approach while the time of symbolizing addition and subtraction concepts was varied.

Hypothesis 1. For the two lower ability PPW classes, LPE (lower ability PPW-Early) and LPL (lower ability PPW-Late), the time of symbolization of the concepts of addition and subtraction has no effect on the achievement in

a. subtraction applications, as measured by the Subtraction Applications Test (SA),
b. subtraction transfer, as measured by the Subtraction Transfer Test (ST),
c. addition computation, as measured by the Stanford Achievement Test (AC),
d. subtraction computation, as measured by the Stanford Achievement Test (SC),
e. problem solving, as measured by the Stanford Achievement Test (PS), or
f. total arithmetic achievement, as measured by the Stanford Achievement Test (SAT).

Sub-hypotheses 1c-1f can be accepted without hesitation since there were no statistically reliable differences between the means for the classes LPE and LPL. In all cases the mean scores and the mean scores after being adjusted for individual differences were very similar.

Conclusion 1. There were no statistically reliable differences in achievement for the lower ability classes LPE and LPL in subtraction computation, addition computation, problem solving or arithmetic achievement as measured by the Stanford Achievement Test.

Data for sub-hypotheses 1a and 1b are ambiguous. Hendrix (56) and Haslerud and Meyers (53) reported that children who were not required to symbolize concepts immediately were better able to apply the concepts in unfamiliar instances. The results of the present study for the lower ability group tend to corroborate these results. The late symbolization class (LPL) tended to have greater mean scores for both the criterion measures SA and ST. The differences in means for both ST and SA were statistically reliable when individual differences were controlled by the initial measure Lorge-Thorndike Intelligence Test.

However, there were no statistically reliable differences when individual differences were controlled by scores on the initial administration of the CAT (Concept Attainment Test). Even though the results were not unequivocal, the evidence suggests:

Conclusion 2. When the PPW instructional approach was employed, the lower ability late symbolization class (LPL) tended to have a greater mean achievement in subtraction applications and subtraction transfer, as measured by the Subtraction Applications and Transfer Test, than did the lower ability early symbolization class (LPE).

Although later findings will support this conclusion, the ambiguity of the results for the criterion measures SA and ST suggest the following recommendation for further research:

Recommendation 1. For young children of lower ability the effects of immediate and delayed symbolization of mathematical concepts on the applicability and transfer of those concepts to familiar and unfamiliar instances should be investigated further.

Hypothesis 2. When the PPW instructional approach is employed, the time of symbolization of the concepts of addition and subtraction has no effect on the gain in number concepts as measured by the Concept Attainment Test (CAT) of the two lower ability classes LPE and LPL.

The first five items of the CAT were constructed to measure the child's counting ability. The mean gain score on these five items for LPE was significantly greater than that for LPL. However, since the two classes received nearly identical instruction in counting, it is difficult to attribute this difference to the time when addition and subtraction concepts were symbolized. It appears more likely that the difference was due to a sampling error. The class LPL had a significantly greater mean score on items one through five for the first administration of the CAT. On the second administration of the CAT, the class means were approximately the same. Hence it would seem reasonable to suggest that LPE was in some way initially deprived of experiences in counting, and during the school year the counting deficiency was overcome.

Perhaps more interesting are the gain scores for item ten (conservation of the whole upon subdivision), items six through ten (one-to-one correspondence tasks), and item eleven (subtraction applications). For item ten both classes had negative mean gain scores, that is, they had lower scores on the post test than on the pretest. Even though the children engaged in many activities motivated by this concept, the children did not exhibit any evidence of learning on the CAT. This result is contrary to a result reported by Coxford (28) for the effects of instruction on another number concept. There were differences not only in the concept studied but also the size of the groups instructed. These differences may have affected the results.

The results are further confounded by the fact that both LPE and LPL made positive gains on items six through ten which were each based on the work of Piaget (92). Hence, even though there are not unequivocal results, the evidence suggests:

Conclusion 3. The lower ability classes generally made positive gains on the sets of items of the Concept Attainment Test patterned after the examples of Piaget as well as on the items involving counting skills.

However, it must be pointed out that the gains cannot logically be attributed to the time of symbolization variable. Nor can they be attributed to the PPW approach since there was no opportunity to compare these gains with those of another class which had used a different instructional approach. For these reasons the following recommendation is made:

Recommendation 2. For young children of lower ability specific methods of instruction in the number concepts suggested by Piaget should be devised and tested in controlled experimentation.

The results for item eleven (subtraction applications) lend further support to Conclusion 2. Although the mean gain score for LPL was not significantly greater than that for LPE, it was greater and supports the trend suggested in Conclusion 2. In this case the time of symbolization variable appears to be a relevant factor, especially in view of the fact that LPL tended to have a greater mean score for the criterion measure SA (Subtraction Application Test).

Hypothesis 3. When the PPW instructional approach is employed, the time of symbolization of the concepts of addition and subtraction has no effect on the attitude toward arithmetic, as measured by the Arithmetic Attitude Scale, of the two lower ability classes LPE and LPL.

Although there were no reliable differences in mean attitude scores for LPE and LPL, it is significant to note that the mean scores

on the last administration of the Arithmetic Attitude Scale were lower than on the first administration for both LPE and LPL. The mean score for LPE decreased the most. But, on the other hand, the mean attitude score for the three administrations of the Arithmetic Attitude Scale was greater for LPE than for LPL.

With the confounding of the results for the arithmetic attitudes of the lower ability classes LPE and LPL, the only possible conclusion is:

Conclusion 4. There were no statistically reliable differences between the means for the Arithmetic Attitude Scale for the lower ability classes LPE and LPL.

In summary, there were few differences in mean achievement and arithmetic attitude between the classes LPE and LPL which could be attributed to the time of symbolization variable. The notable exception was the tendency for the late symbolization class (LPL) to have consistently greater mean scores for subtraction applications and subtraction transfer measures. It was recommended that experimentation be pursued which would provide clarifying evidence on the effects of the time of symbolization variable and which would yield information on the relationship between instruction and learning in teaching the number concepts suggested by Piaget to lower ability children.

Higher Ability Group

The higher ability group consisted of four classes. Both levels of the approach variable and the time of symbolization variable were employed in this group yielding four experimental treatments. The four classes as defined by treatment were the PPW-Early (PE), the TA-Early (TE), the PPW-Late (PL), and the TA-Late (TL). The first hypothesis for the higher ability group is as follows:

Hypothesis 4. For the four higher ability classes PE, TE, PL, and TL, the instructional approach and the time of symbolization of subtraction and addition concepts have no effect on the achievement in

a. subtraction applications, as measured by the Subtraction Applications Test (SA),
b. subtraction applications other than the take away type, as measured by the Subtraction Applications Test and the Concept Attainment Test (NTA),
c. subtraction transfer, as measured by the Subtraction Transfer Test (ST),
d. subtraction computation, as measured by the Stanford Achievement Test (SC),
e. addition computation, as measured by the Stanford Achievement Test (AC),
f. problem solving, as measured by the Stanford Achievement Test (PS), or
g. total arithmetic achievement, as measured by the Stanford Achievement Test (SAT).

The effects of the instructional approaches were most clearly evident for the criterion measure of subtraction computation (SC). In this case the TA approach was the most effective when individual differences were controlled by scores on the Concept Attainment Test (CAT) and the Lorge-Thorndike Intelligence Test. The consistent superiority of the TA approach in subtraction computation attains more importance when it is noted that for no other criterion measure did the TA group surpass the PPW group when the means were adjusted for individual differences in measured intelligence. The typical pattern was that the TA group had the higher adjusted criterion mean score when individual differences were controlled using the scores of the CAT; and when individual differences were controlled for intelligence, the PPW group had the higher mean score.

Hence there is positive evidence suggesting that the PPW approach to subtraction was not as effective as the TA approach for teaching subtraction computation skills. Also the general trend of the adjusted means for the criterion measure of subtraction transfer, which was predominately computational in nature, supported this result. Because of the general nature of the PPW approach and the complexity of the manipulations associated with subtraction for this approach, these results

were not too surprising. In view of the evidence the following conclusion seems reasonable:

Conclusion 5. The TA instructional approach was more effective than the PPW instructional approach for teaching subtraction computational skills in familiar and unfamiliar instances, as measured by the Stanford Achievement Test and the Subtraction Transfer Test.

The remainder of the data did not unequivocally support either the TA or the PPW approach, and no generalizations can be made with great confidence. However, the trends evident in the adjusted means of the measures of subtraction applications (SA and NTA) deserve comment because of Inhelder's (19) suggestion that early introduction of general mathematical concepts could facilitate the applicability of those concepts.

For both criterion measures of subtraction applications the PPW group had the greatest mean scores before adjustments for individual differences. In the case of the measure SA, which included take-away applications, the PPW group retained its superiority when the means were adjusted for individual differences in counting skills and intelligence, and relinquished that superiority for the other two initial measures. In no case was the difference between the mean scores very large. Therefore, when take-away applications were included, there was little difference in subtraction applications achievement between the PPW group and the TA group.

For the subtraction applications measure which excluded the take-away examples (NTA) the situation was similar to that for SA. But for NTA when the TA group had the greater mean score, that mean was only slightly greater than that of the PPW group. Moreover, when the means were adjusted for intelligence and counting skill differences, the PPW group had the greater mean, and the difference between the means for the PPW group and the TA group was quite large relative to the difference between the means when the TA group had the greater mean.

The relatively larger differences in favor of the PPW group suggest that there could be a positive relationship between the

generality of the introduction of the concept of subtraction and the applicability of that concept. Therefore, the following recommendation is made:

Recommendation 3: The relationship between the applicability of the concept of subtraction and the generality of the introduction thereof should be a topic for further research.

Considering the two levels of the time of symbolization variable, the general pattern of the adjusted means for the criterion measures was that the early symbolization group (E) had greater means than the late symbolization group (L) when the controlling variable was initial concept of number attainment and that L had the greater adjusted means when the controlling variable was intelligence. This pattern held for all criterion measures except PS (problem solving). In the case of problem solving, E had the greatest adjusted mean scores for all controlling variables, and the differences were highly reliable in three cases. Therefore, the following conclusion is made:

Conclusion 6. The early symbolization group had significantly greater problem solving achievement, as measured by the Stanford Achievement Test, than did the late symbolization group.

For the remainder of the criterion measures, even though there were reliable differences favoring E, it is difficult to conclude that early symbolization was more effective than late symbolization. This is true because in each case not only was the same reliable difference not found when the means were adjusted for intelligence differences, but also in every case L had the greater mean after adjustment for intelligence differences rather than E. About all that can be said is that when differences in initial number concepts were controlled, E tended to have greater mean scores for all the achievement measures used in this study but that the results were reversed (except for PS) when intelligence differences were controlled.

The effects of combinations of the approach variable and the time of symbolization variable were nearly as ambiguous as the effects

of the time of symbolization variable alone. However, the criterion measures of arithmetic computation, problem solving, and total arithmetic achievement showed reliable differences when individual differences in number concepts were controlled. In each case the combination of variables PPW and early symbolization was the most effective. Also when individual differences in intelligence were controlled, the most effectual combination of variables was PPW and E for two of the three measures. Hence the evidence suggests:

Conclusion 7. **The most effectual combination of variables for achievement in addition computation, problem solving, and total arithmetic achievement, as measured by the Stanford Achievement Test, was the PPW approach and early symbolization of addition and subtraction concepts.**

There were no reliable differences for measures of subtraction computation and subtraction transfer. **However, added support was given to Conclusion 5 since in every case one of the combinations of variables TA and E or TA and L was the most effective.**

Further support is also given to Recommendation 3 by the results for the criterion measures of subtraction applications SA and NTA. Generally the combinations of variables PPW and E was the most effective, and in the only case where it was not, the combination PPW and L was. Hence there was a strong indication that the generality of the instruction approach to subtraction facilitated achievement in applications of subtraction.

Hypothesis 5. For the PPW-Boys (PM), PPW-Girls (PF), TA-Boys (TM), and TA-Girls (TF), the instructional approach to addition and subtraction has no effect on achievement in

a. subtraction applications, as measured by the Subtraction Applications Test (SA),

b. subtraction transfer, as measured by the Subtraction Transfer Test (ST),

c. addition computation, as measured by the Stanford Achievement Test (AC),

d. subtraction computation, as measured by the Stanford Achievement Test (SC),

e. problem solving, as measured by the Stanford Achievement Test (PS), or

f. total arithmetic achievement, as measured by the Stanford Achievement Test.

The only reliable evidence for acceptance of Hypothesis 5 occurred in the comparison of TF and PF for the criterion measure of subtraction computation (SC). This one reliable difference is not sufficient to reject the hypothesis. Hence the following conclusion is suggested:

Conclusion 8. There was little evidence that the instructional approaches to addition and subtraction affected males and females of the four higher ability classes differentially.

Hypothesis 6. The instructional approach and the time of symbolization of addition and subtraction concepts have no effect on the gain in number concepts, as measured by the Concept Attainment Test (CAT), of the four higher ability classes PE, TE, PL, and TL.

The effects of the instructional approaches on the gain scores of the CAT were inconclusive. The only reliable difference occurred in the comparison of the gains on item six through eight of the CAT (one-to-one correspondence and the conservation thereof). In this case the TA group had the greater mean gain. Although not statistically reliable the TA group also had the greatest mean gain score on items six through ten and one through eleven (Total).

Item ten (conservation of the whole upon subdivision) was the only item tested for which the PPW group had consistently greater mean gain scores, but these were not statistically reliable. The PPW instructional approach provided many experiences with the topic of item ten, and it appears that this instruction had some effect on the gain score. For the same item the low ability group made no gains. This suggests that learning was affected by the ability level of the children. The absence of instruction in the topic of item ten in the

TA group was reflected in the zero gain score for the item for class TL.

Since the only major differences in instruction in the TA classes and the PPW classes centered around the ideas involved in item ten, it was not surprising that the gains were not consistently in favor of one approach. Therefore, the evidence suggests:

Conclusion 9. There were few differences in the mean gain scores on the Concept Attainment Test which could be reliably attributed to either of the instructional approaches.

Nearly all the differences between the early symbolization group (E) and the late symbolization group (L) in the mean gain scores on the CAT were favorable to E. However, it is difficult to attribute all these differences to the time of symbolization variable since many procedures in both groups were identical. There is one exception to this generalization. The exception concerns item eleven (subtraction applications).

The early symbolization group had the greatest mean gain scores for item eleven. This fact tends to contradict the suggestion of Hendrix (56) that immediate symbolization of a concept tended to decrease its applicability. Furthermore, the superiority of the early symbolization group tends to contradict the results reported for the lower ability group in the present study. Hence more support is given to Recommendation 1 made for the lower ability group which suggested further research on the effects of the time of symbolization of concepts.

The combination of variables which was most effectual in producing high mean gain scores on the Concept Attainment Test was not completely clear. For the five reliable interactions found, the combination PPW and E was most effective in three, and the combination TA and L was most effective in the other two. The only consistent and reliable interaction was found for the mean total gain score of the CAT. In this case PPW and E and TA and L were the two most effective combinations of variables. Since the remainder of the comparisons had approximately the same order of effectiveness, it is appropriate to suggest:

Conclusion 10. In general the most effective combination of variables in fostering gains on the Concept Attainment Test were (a) the PPW approach and early symbolization and (b) the TA approach and late symbolization.

Hypothesis 7. The instructional approach and the time of symbolizing addition and subtraction concepts have no effect on the attitudes toward arithmetic, as measured by the Arithmetic Attitude Scale, of the four higher ability classes PE, TE, PL, and TL.

In the analysis of the difference between the scores on the last and first administrations of the Arithmetic Attitude Scale for each class there were no statistically significant results. However, it is interesting to note that PE had the greatest gain in attitude score while PL was the only class which had a decrease in mean score. These results seem to reflect the achievement of the two classes since generally PE had the greatest achievement scores and PL the least.

The comparisons of mean attitude scores for different subgroups, too, resulted in no reliable differences. Two major trends were evident, however. The TA group had a greater mean score than the PPW group, and the late symbolization group had a greater mean score than did the early symbolization group. Furthermore, there were only slight differences between the girls and the boys in the four classes. Generally, then, the evidence suggests:

Conclusion 11. There were no reliable differences in attitude toward arithmetic, as measured by the Arithmetic Attitude Scale, attributable to the instructional approaches or the times of symbolization in the four higher ability classes.

Overview

Some aspects of this study stood out more than others. The first of these was the contradictory evidence concerning the time of symbolizing addition and subtraction concepts. The results reported for the lower ability classes were probably the most reliable. This is

suggested for two reasons. Observations made throughout the year in the lower ability late symbolization class and in the TA late symbolization class suggested that the delaying symbolization facilitated immediate learning. Secondly, in the higher ability group intelligence differences between the late and early symbolization groups were particularly great, and although the early symbolization group generally performed better, the adjustment for intelligence differences reversed the trend and showed the late symbolization group with the better performance. However, it seems that the effects of symbolization should be studied further because of immediate difficulties with the use of mathematical symbolization and because more symbolism is entering elementary school mathematics curricula.

A second prominent aspect of this study was the relative difficulty of teaching subtraction using the Part-Part-Whole approach. As stated earlier the procedure used was to study the missing addend sentence and then use this to introduce the subtraction sentence. The manipulations made on the bead frames in both cases resulted in two subsets being shown by the children. The task was to determine the number of the missing part. From reactions observed in the four PPW classrooms it can be declared that all the classes encountered difficulty when asked to name the missing part. Moreover, only about one class and a half of another ever overcame this difficulty at the time of the close of the study. Both of these classes were in the higher ability group. The two lower ability classes never progressed far beyond the basic task of finding the missing part. Only a few children could read a subtraction sentence and proceed to the solution without prompting.

The difficulties encountered with the PPW approach were not totally unexpected. The research reported by Bourne (9), Fredrick (38), and Kates and Yudin (66) suggested that such would be the case. These difficulties, it appears, were reflected in the decreases in mean arithmetic attitude in three of the PPW classes and in the superiority of the two TA classes in the measure of subtraction computation. However, in the midst of this negativism it should be emphasized that one PPW class overcame the difficulties and performed well on the Subtraction Computation Test and had an increase in mean arithmetic attitude.

The general uneasiness associated with the PPW approach for three classes may have been due to the more general nature of the instruction in subtraction. The leisurely pace at which learning proceeded, and many times the seeming lack of much comprehension, may be inherent in the PPW approach for children of average ability. In the TA classes, however, the learning pace seemed to be more rapid for subtraction skills. Perhaps, as the evidence suggests in a general way, the later concepts associated with subtraction will be more easily mastered because of the foundation provided by the PPW approach. In any case the immediate results were not outstandingly positive in three of the four PPW classes.

One wonders, then, if there was any factor which would recommend the PPW approach. Such a factor was suggested above. That factor was achievement in subtraction applications. Even though it could not be concluded that the PPW classes in the higher ability group were reliably more proficient in solving subtraction applications, it could be suggested that the results pointed in that direction. This trend was more pronounced when the take-away applications were deleted from the criterion measure than when they were included. Furthermore, when the mean scores were adjusted for intelligence differences, the superiority of the PPW classes was accentuated.

It should be noted that it could not be concluded that the PPW classes used subtraction to solve subtraction applications, but the suggestion was only that the PPW approach appeared to facilitate such solutions. There was further evidence that the PPW approach permitted the children to see the similarity of the three typical applications of subtraction. Observations made in the PPW classrooms generally suggested that these children were more prone to suggest a subtraction sentence when asked for a number sentence which was appropriate for a subtraction application than was true for the TA children.

There may be another factor to explore in connection with the TA and PPW approaches. While the PPW approach was very successful with children of quite high measured intelligence, it was less successful for children of lower ability. The case of the TA approach was somewhat reversed. The children with high ability were equaled and often surpassed in many achievement areas by children who, on the average,

were much less intelligent. Therefore, perhaps a particular instructional approach suits a specific kind of child better than another. In the present study the PPW approach appeared to suit the more intelligent children but not the children of lower ability. In a like manner the TA approach appeared to suit the average ability children better than the children of higher ability. This was borne out by the fact that the PPW-Early and the TA-Late classes were first and second in nearly all criterion measures.

What has been suggested above can be summarized as follows. The PPW approach did not lead to immediate subtraction computational skills, but it appeared to facilitate solutions of subtraction applications. Whether or not the computational skills will improve and, as Inhelder (19) suggested, the generality of the introductory work will produce a foundation upon which later learning can easily be built, are questions which must be answered by future experimentation. In the present study there were only suggestions that such could be the case.

APPENDIX A

CONCEPT ATTAINMENT TEST, FORMS A, B, AND C
CONCEPT ATTAINMENT SCORE SHEET

Concept Attainment Test Form A

Begin the interview by asking the child's name and recording the child's name, the child's school, the date, and the interviewer's name on the Concept Attainment Score Sheet.

1. ROTE COUNTING.

   a. "WILL YOU COUNT TO TEN FOR ME?"

   b. "STARTING WITH 15, WILL YOU COUNT BACKWARDS TO 1?"

If the child cannot do the backwards counting in part b, state the same question replacing 15 by 10. If still it is impossible for the child, replace 15 by 5. If still impossible, mark the question incorrect (I).

If the child begins with some number, such as 15, but makes mistakes in counting backwards, note the longest correct sequence which he recites and mark it on the Score Sheet.

2. RATIONAL COUNTING (1 picture of 24 green discs. Picture is marked #2).

   The interviewer places the picture on the table before the child and says:

   "WILL YOU COUNT THESE FOR ME?"

If the child is incorrect in his counting, mark the question incorrect and note (if possible) the last correctly used numeral in the space provided in the Score Sheet. Otherwise, just mark correct (C).

3. REPRODUCING A SET OF THE APPROPRIATE NUMBER OF CHIPS. (20 red chips)

   The interviewer spreads the 20 red chips before the child and says:

   "PLEASE GIVE ME 16 OF THE CHIPS."

If the child does not give the interviewer the correct number of chips, mark the number given in the space provided.

4. IDENTIFICATION OF THE NUMBER OF MEMBERS OF A SET. (2 pictures of flowers and red balls marked 4a and 4b).

Show the pictures one at a time to the child and say:

a. "POINT TO THE PICTURE OF 8 FLOWERS" (5, 8, 3 flowers).

b. "POINT TO THE PICTURE OF 13 BALLS" (13, 12, 15 balls).

If incorrect, note the number of objects pointed to in the space provided in the Score Sheet.

5. COMPARISON OF THE NUMBER OF MEMBERS IN TWO SETS. (2 pictures of lollypops marked 5a and 5b).

Show the pictures one at a time to the child and ask:

a. "WHICH PICTURE HAS MORE LOLLYPOPS IN IT?" (6 & 7 lollypops).

b. "WHICH PICTURE HAS MORE LOLLYPOPS IN IT?" (13 & 12 lollypops).

Whether the child is correct or incorrect in his answer, note whether he counts in the determination and circle HE COUNTED on the Score Sheet.

6. ONE-TO-ONE CORRESPONDENCE (16 red and 10 white chips).

The interviewer spreads 16 red chips before the child. Between the interviewer and the red chips, the interviewer places a row of 10 white chips so that there is a gap of about 1/2" between the white chips. The interviewer points to the row of white chips and says:

a. "THESE ARE MY CHIPS. TAKE SOME OF YOURS (pointing to the red ones) SO YOU HAVE THE SAME NUMBER AS I DO." (Push the red chips he does not use to one side.)

INTERVIEWER

White chips 0 0 0 0 0 0 0 0 0 0

Red chips 0 0 0 0 0 0 0 0 0 0

CHILD

If the child does not arrange the chips as in the diagram above in response to part 6a, say:

b. "PLACE YOUR CHIPS SO WE CAN SEE THAT YOU HAVE THE SAME NUMBER."

If the child still does not arrange the chips in the manner of

the diagram, note the way he has arranged them and record it under 6b of the Score Sheet. Then go on to part 6c. If at any time he does arrange the chips as in the diagram, go immediately to 7.

c. If b is incorrect, match the 10 white chips with the 10 red chips as in the diagram and say:
"IS THERE THE SAME NUMBER OF RED AND WHITE CHIPS?"

7. CONSERVATION OF ONE-TO-ONE CORRESPONDENCE (10 red and 10 white chips).

Use the same two rows of chips as arranged in #6. The interviewer then spreads the red chips into a row about twice as long as the row of white ones. This is done completely within the view of the child. The interviewer asks:
"ARE THERE AS MANY RED CHIPS AS WHITE CHIPS?"
If the child is incorrect in his answer to this question, ask:
"WHICH ROW HAS MORE CHIPS IN IT?"

8. ADDITIVE COMPARISON OF SETS (22 chips, red in color).

Place two piles (not stacked) of red chips, one of 14 and one of 8 chips, before the child and say:
"HERE ARE TWO PILES OF CHIPS. PUT THEM INTO TWO NEW PILES, SO EACH PILE HAS THE SAME NUMBER OF CHIPS IN IT."

On the Score Sheet note how the child carries out this task, whether he does it correctly or not.

9. CLASSIFICATION--PART LESS THAN THE WHOLE (18 red and 2 white chips).

The interviewer puts the 20 chips in two rows of ten each before the child--9 red and 1 white chip in each row with the colors matching and the white chips on the end. The interviewer then says several times:
"POINT TO A CHIP FOR ME." (No answer is recorded.)
If the child does not point to a white chip during this time, the interviewer points to several chips (including white ones) and asks:

"IS THIS A CHIP?" (No answer is recorded--this is only to have the child affirm that the white chips are chips.) Then the interviewer says:

"I'M GOING TO ASK YOU SOME QUESTIONS ABOUT THESE CHIPS."

a. "ARE THERE MORE RED CHIPS OR MORE CHIPS THERE?" (Pointing to the array of chips in general with the hand or finger about 4" away from the closest point of the chips.)

b. "ARE THERE MORE CHIPS OR MORE RED CHIPS THERE?" (Same pointing instructions as in a)

c. "ARE THERE MORE WHITE CHIPS OR MORE CHIPS THERE?" (Same pointing instructions as in a)

d. "IF WE PILE UP ALL THE RED CHIPS AND THEN PILE UP ALL THE CHIPS, WHICH PILE WOULD BE TALLER?"

e. "ARE THERE MORE RED CHIPS OR MORE CHIPS ALTOGETHER?" (Same pointing instructions as in a)

10. PART-PART-WHOLE (11 red chips and 9 white chips).

The interviewer puts 9 red chips in a row opposite the child and says:

a. "HERE ARE SOME CHIPS. COUNT THEM."

The interviewer puts 9 white chips in a one-to-one correspondence with the red ones as in the diagram below and says:

"HERE IS THE SAME NUMBER OF WHITE CHIPS (wait for child verification of this if need be). WATCH NOW."

The interviewer separates the row of red chips so that it has 6 chips in one part and 3 in the other (move 3 to your left as diagram shows) and says:

"IS THERE THE SAME NUMBER OF RED CHIPS (pointing to the complete row with a continuous hand motion from one end to the other; finger about 4" away) AND WHITE CHIPS?" (Repeat the same pointing movement as with the red chips.)

As the interviewer finishes asking this question, he covers the rows with a piece of cardboard marked 10.

INTERVIEWER

White chips	0 0 0 0 0 0 0 0 0 0		0 0 0 0 0 0 0 0 0 0
Red chips	0 0 0 0 0 0 0 0 0 0	to	0 0 0 0 0 0 0 0 0 0

CHILD

There are several possible child reactions to look for:

I. He may answer that the rows are not the same in number. In this case ask:

"ARE THERE MORE RED CHIPS OR MORE WHITE CHIPS?"

Record the result in 10a-A and go on to 10b.

II. He may answer immediately that the two rows are the same in number. In this case, circle DO NONE OF THESE and C in 10a-B and go on to 10b.

III. He may decline to say whether the rows have the same number or not and want to see the rows again. In this case uncover the rows, note whether he is correct or not, and put reactions in 10a-B. If he says nothing, ask:

"HOW COULD YOU FIND OUT IF THEY ARE THE SAME?" (No answer is recorded.)

The child will usually want to see the chips or want to count them. In any case uncover the rows and note the reactions in 10a-B.

b. The chips are uncovered if need be and rearranged in a one-to-one correspondence. The interviewer says:
"IS THERE THE SAME NUMBER OF RED CHIPS AND WHITE CHIPS?"
(No answer is recorded, but get a "yes".)
The interviewer then separates the row of red chips into parts of 5 (moved to the right) and 4 and covers the rows as he asks:
"IS THERE THE SAME NUMBER OF RED CHIPS (pointing instructions the same as in 10a) AND WHITE CHIPS?" (Same pointing instructions)

Note the same three responses as in 10a on the Score Sheet.

c. The interviewer removes all of the white chips and adds 2 red chips to the remaining row of red chips and says:
"COUNT THE CHIPS!" (No answer is recorded.)

The interviewer then separates the red chips into sets of 4 (move 4 to the left) and 7 chips and asks: "IS THERE THE SAME NUMBER OF CHIPS NOW (pointing to the two sets with a continuous finger movement from one end to the other, finger about 4" away) AS THERE WAS IN THE ROW YOU JUST COUNTED?" (As the interviewer completes this question, he covers the chips with the cardboard marked 10.)

There are several possible child reactions to look for:

I. He may answer that the two rows of chips do not have the same number of chips as the one row had. In this case ask:

"ARE THERE MORE CHIPS NOW?"

Record the result in 10c-A and go to 10d.

II. He may answer immediately that the two parts have the same number of chips as the one row of chips and, in this case, circle C and DO NEITHER in 10c-B and go on to 10d.

III. He may decline to say whether the rows have the same number or not and want to see the rows again. In this case, uncover the rows, note whether he is correct or not, and put reactions in 10c-B. If he says nothing at all, ask:

"HOW COULD YOU FIND OUT IF THEY ARE THE SAME?" (No answer is recorded.)

The child will usually want to see the chips or want to count them. In any case, uncover the chips and note the reactions in 10c-B of the Score Sheet.

d. The interviewer uncovers the chips if need be and puts them back in one row and says:

"COUNT THE CHIPS!" (No answer is recorded.)

The interviewer separates the 11 chips into two parts of 10 and 1 (moved to the right) and pointing to the two parts asks: "IS THERE THE SAME NUMBER OF CHIPS NOW (follow pointing instructions in 10c) AS THERE WAS IN THE ROW YOU JUST COUNTED?"

The interviewer covers the two parts and notes the 3 reactions as in part 10c.

*11. SUBTRACTION (3 pictures marked 11a, 11b, and 11c).

a. The interviewer shows the child a picture of 5 apples and says:
"JOHNNY BROUGHT 5 APPLES TO SCHOOL (pointing to the 5 apples with finger about 4" away). HE GAVE 2 OF THEM AWAY (pointing in the general direction of 2 apples with the finger no closer than 4".) HOW MANY DID HE HAVE LEFT?"

b. The interviewer shows the picture of the three bottle caps to the child and says:
"MARTY IS SAVING BOTTLE CAPS. HE HAS 3 NOW (pointing to the 3 bottle caps with finger about 4" away). HE WANTS TO GET 7 ALTOGETHER. HOW MANY MORE DOES HE NEED?"

c. The interviewer shows the child the picture of 9 balloons and 4 balloons and says:
"MARY HAS 9 BALLOONS. BETTY HAS 4 BALLOONS (pointing to the two sets of balloons with finger about 4" away). WHO HAS MORE BALLOONS? (No answer is recorded). HOW MANY MORE?" (Record answer).

*If you can judge, from watching the student, record the way he does each problem.

Minimum Instructions
Concept Attainment Test

Form A

1. a. 'WILL YOU COUNT TO TEN FOR ME?"
   b. "STARTING WITH 15, WILL YOU COUNT BACKWARDS TO 1?"
   (Try 10 and 5 if impossible at 15 for child)
2. (Picture--24 green circles) "WILL YOU COUNT THESE FOR ME?"
3. (20 red chips) "PLEASE GIVE ME 16 OF THE CHIPS."
4. a. (5, 8, 3 flowers) "POINT TO THE PICTURE OF 8 FLOWERS."
   b. (13, 12, 15 balls) "POINT TO THE PICTURE OF 13 BALLS."
5. a. (6, 7 lollypops) "WHICH PICTURE HAS MORE LOLLYPOPS IN IT?"
   b. (13, 12 lollypops) "WHICH PICTURE HAS MORE LOLLYPOPS IN IT?"
6. a. (16 red, 10 white chips) 10 white chips in a row about 1/2" gap. "THESE ARE MY CHIPS. TAKE SOME OF YOURS (pointing to the red ones) SO YOU HAVE THE SAME NUMBER AS I DO."
   b. If no obvious 1-1 correspondence, say: "PLACE YOUR CHIPS SO WE CAN SEE THAT YOU HAVE THE SAME NUMBER."
   c. If still incorrect, match the chips and say: "IS THERE THE SAME NUMBER OF RED AND WHITE CHIPS?"
7. (10 red, 10 white chips) Spread the red chips to twice length of white row. "ARE THERE AS MANY RED CHIPS AS WHITE CHIPS?" If incorrect, say: "WHICH ROW HAS MORE CHIPS IN IT?"
8. (22 red chips in groups of 14 and 8) "HERE ARE TWO PILES OF CHIPS. PUT THEM INTO TWO NEW PILES, SO EACH PILE HAS THE SAME NUMBER OF CHIPS IN IT." (Note counting or correspondence to do it.)
9. (18 red, 2 white chips) Two rows of 10 each with colors matching. "POINT TO A CHIP FOR ME." "IS THIS A CHIP?" "I'M GOING TO ASK YOU SOME QUESTIONS ABOUT THESE CHIPS."
   a. "ARE THERE MORE RED CHIPS OR MORE CHIPS THERE?" (Point to chips in general with finger about 4" away.)
   b. "ARE THERE MORE CHIPS OR MORE RED CHIPS THERE?" (Same pointing)
   c. "ARE THERE MORE WHITE CHIPS OR MORE CHIPS THERE?" (Same pointing)
   d. "IF WE PILE UP ALL THE RED CHIPS AND THEN PILE UP ALL THE CHIPS, WHICH PILE WOULD BE TALLER?" (Same pointing)

e. "ARE THERE MORE RED CHIPS OR MORE CHIPS ALTOGETHER?" (Same pointing)

10. (11 red, 9 white chips) 9 red chips in a row--matched later by 9 white.

a. "HERE ARE SOME CHIPS. COUNT THEM." "HERE IS THE SAME NUMBER OF WHITE CHIPS. WATCH NOW." (6 and 3 separation of the red chips) "IS THERE THE SAME NUMBER OF RED CHIPS AND WHITE CHIPS?" (cover) If incorrect, "ARE THERE MORE RED CHIPS OR MORE WHITE CHIPS?" If no answer, "HOW COULD YOU FIND OUT IF THEY ARE THE SAME?"

b. Rearrange in 1-1 correspondence: "IS THERE THE SAME NUMBER OF RED AND WHITE CHIPS?" (Separate red to 4 and 5.) "IS THERE THE SAME NUMBER OF RED CHIPS AND WHITE CHIPS?" (cover)

c. Remove white chips, add 2 red. "COUNT THE CHIPS!" (Separation of 4 and 7) "IS THERE THE SAME NUMBER OF CHIPS NOW AS THERE WAS IN THE ROW YOU JUST COUNTED?" (cover) If incorrect: "ARE THERE MORE CHIPS NOW?" If no answer: "HOW COULD YOU FIND OUT IF THEY ARE THE SAME?"

d. Put in one row again: "COUNT THE CHIPS!" (Separation of 10 and 1) "IS THERE THE SAME NUMBER OF CHIPS NOW AS THERE WAS IN THE ROW YOU JUST COUNTED?" (cover)

*11. a. Picture 11a. "JOHNNY BROUGHT 5 APPLES TO SCHOOL HE GAVE 2 OF THEM AWAY. HOW MANY DID HE HAVE LEFT?"

b. Picture 11b. "MARTY IS SAVING BOTTLE CAPS. HE HAS 3 NOW. HE WANTS TO GET 7 ALTOGETHER. HOW MANY MORE DOES HE NEED?"

c. Picture 11c. "MARY HAS 9 BALLOONS. BETTY HAS 4 BALLOONS. WHO HAS MORE BALLOONS? HOW MANY MORE?"

*If you can judge, from watching the student, record the way he does each problem.

Concept Attainment Test Form B

Begin the interview by asking the child's name and recording the child's name, the child's school, the date, and the interviewer's name on the Concept Attainment Score Sheet.

1. ROTE COUNTING.

   a. "WILL YOU COUNT TO TEN FOR ME?"

   b. "STARTING WITH 15, WILL YOU COUNT BACKWARDS TO 1?"

If the child cannot do the backwards counting in part b, state the same question replacing 15 by 10. If still it is impossible for the child, replace 15 by 5. If still impossible, mark the question incorrect (I).

If the child begins with some number, such as 15, but makes mistakes in counting backwards, note the longest correct sequence which he recites and mark it on the Score Sheet.

2. RATIONAL COUNTING. (1 picture of 24 blue squares. Picture is marked #2).

   The interviewer places the picture on the table before the child and says:

   "WILL YOU COUNT THESE FOR ME?"

If the child is incorrect in his counting, mark the question incorrect and note (if possible) the last correctly used numeral in the space provided in the Score Sheet. Otherwise, just mark correct (C).

3. REPRODUCING A SET OF THE APPROPRIATE NUMBER OF CHIPS. (20 blue chips)

   The interviewer spreads the 20 blue chips before the child and says:

   "PLEASE GIVE ME 17 OF THE CHIPS."

If the child does not give the interviewer the correct number of chips, mark the number given in the space provided.

4. IDENTIFICATION OF THE NUMBER OF MEMBERS OF A SET. (2 pictures of grapes and green squares marked 4a and 4b.)

   Show the pictures one at a time to the child and say:

   a. "POINT TO THE PICTURE OF 9 GRAPES (9, 4, 6 grapes).

   b. "POINT TO THE PICTURE OF 12 BOXES" (14, 11, 12 squares).

If incorrect, note the number of objects pointed to in the space provided in the Score Sheet.

5. COMPARISON OF THE NUMBER OF MEMBERS IN TWO SETS. (2 pictures of balloons marked 5a and 5b).

Show the pictures one at a time to the child and ask:

a. "WHICH PICTURE HAS MORE BALLOONS IN IT?" (8 & 7 balloons).

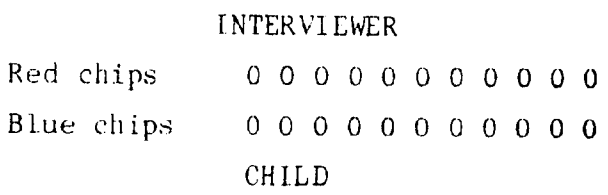
b. "WHICH PICTURE HAS MORE BALLOONS IN IT?" (13 & 14 balloons).

Whether the child is correct or incorrect in his answer, note whether he counts in the determination and circle HE COUNTED on the Score Sheet if he did.

6. ONE-TO-ONE CORRESPONDENCE (17 blue and 11 red chips).

The interviewer spreads 17 blue chips before the child. Between the interviewer and the blue chips, the interviewer places a row of 11 red chips so that there is a gap of about 1/2" between the red chips. The interviewer points to the row of red chips and says:

a. "THESE ARE MY CHIPS. TAKE SOME OF YOURS (pointing to the blue ones) SO YOU HAVE THE SAME NUMBER AS I DO." (Push the blue chips he does not use to one side.)



If the child does not arrange the chips as in the diagram above in response to part 6a, say:

b. "PLACE YOUR CHIPS SO WE CAN SEE THAT YOU HAVE THE SAME NUMBER."

If the child still does not arrange the chips in the manner of the diagram, note the way he has arranged them and record it under 6b of the Score Sheet. Then go on to part 6c. If at any time he does arrange the chips as in the diagram, go immediately to 7.

c. If b is incorrect, match the 11 red chips with the 11 blue chips as in the diagram and say:
"IS THERE THE SAME NUMBER OF RED AND BLUE CHIPS?"

7. CONSERVATION OF ONE-TO-ONE CORRESPONDENCE (11 blue and 11 red chips).

Use the same two rows of chips as arranged in #6. The interviewer then spreads the blue chips into a row about twice as long as the row of red ones. This is done completely within the view of the child. The interviewer asks:
"ARE THERE AS MANY BLUE CHIPS AS RED CHIPS?"
If the child is incorrect in his answer to this question, ask:
"WHICH ROW HAS MORE CHIPS IN IT?"

8. ADDITIVE COMPARISON OF SETS. (22 blue chips).

Place two piles (not stacked) of blue chips, one of 15 and one of 7 chips, before the child and say:
"HERE ARE TWO PILES OF CHIPS. PUT THEM INTO TWO NEW PILES, SO EACH PILE HAS THE SAME NUMBER OF CHIPS IN IT."

On the Score Sheet note how the child carries out this task, whether he does it correctly or not.

9. CLASSIFICATION--PART LESS THAN THE WHOLE (14 blue and 4 red chips).

The interviewer puts the 18 chips in two rows of 9 each before the child--7 blue and 2 red chips in each row with the colors matching and the red chips on the end. The interviewer then says several times:
"POINT TO A CHIP FOR ME." (No answer is recorded.)
If the child does not point to a red chip during this time, the interviewer points to several chips (including red ones) and asks:
"IS THIS A CHIP?" (No answer is recorded--this is only to have the child affirm that the red chips are chips.)

Then the interviewer says:

"I'M GOING TO ASK YOU SOME QUESTIONS ABOUT THESE CHIPS."

a. "ARE THERE MORE BLUE CHIPS OR MORE CHIPS THERE?" (Pointing to the array of chips in general with the hand or finger about 4" away from the closest point of the chips.)

b. "ARE THERE MORE CHIPS OR MORE BLUE CHIPS THERE?" (Same pointing instructions as in a)

c. "ARE THERE MORE RED CHIPS OR MORE CHIPS THERE?" (Same pointing instructions as in a)

d. "IF WE PILE UP ALL THE BLUE CHIPS AND THEN PILE UP ALL THE CHIPS, WHICH PILE WOULD BE TALLER?"

e. "ARE THERE MORE BLUE CHIPS OR MORE CHIPS ALTOGETHER?" (Same pointing instructions as in a)

10. PART-PART-WHOLE (11 blue and 8 red chips).

The interviewer puts 8 blue chips in a row opposite the child and says:

a. "HERE ARE SOME CHIPS. COUNT THEM."

The interviewer puts 8 red chips in a one-to-one correspondence with the blue ones as in the diagram below and says:

"HERE IS THE SAME NUMBER OF RED CHIPS (wait for child verification of this if need be.) WATCH NOW."

The interviewer separates the row of blue chips so that it has 6 chips in one part and 2 in the other (move 2 to your left as diagram shows) and says:

"IS THERE THE SAME NUMBER OF BLUE CHIPS (pointing to the complete row with a continuous hand motion from one end to the other; finger about 4" away) AND RED CHIPS?" (Repeat the same pointing movement as with the blue chips.)

As the interviewer finishes asking this question, he covers the rows with a piece of cardboard marked #10.

INTERVIEWER

Red chips	0 0 0 0 0 0 0 0		0 0 0 0 0 0 0 0
Blue chips	0 0 0 0 0 0 0 0	to	0 0 0 0 0 0 0 0

CHILD

There are several possible child reactions to look for:

I. He may answer that the rows are not the same in number. In this case ask:

"ARE THERE MORE BLUE CHIPS OR MORE RED CHIPS?"

Record the result in 10a-A and go on to 10b.

II. He may answer immediately that the two rows are the same in number. In this case circle DO NONE OF THESE and C in 10a-B and go on to 10b.

III. He may decline to say whether the rows have the same number of not and want to see the rows again. In this case uncover the rows, note whether he is correct or not, and put reactions in 10a-B. If he says nothing, ask:

"HOW COULD YOU FIND OUT IF THEY ARE THE SAME?" (No answer is recorded.)

The child will usually want to see the chips or want to count them. In any case uncover the rows and note the reactions in 10a-B.

b. The chips are uncovered if need be and rearranged in a one-to-one correspondence. The interviewer says:
"IS THERE THE SAME NUMBER OF BLUE CHIPS AND RED CHIPS?"
(No answer is recorded, but get a "yes".)
The interviewer then separates the row of blue chips into parts of 3 (move 3 to right) and 5 and covers the rows when he has asked:
"IS THERE THE SAME NUMBER OF BLUE CHIPS (pointing instructions the same as in 10a) AND RED CHIPS?" (Same pointing instructions)

Note the same three responses as in 10a on the Score Sheet.

c. The interviewer removes all of the red chips and adds 3 blue chips to the remaining row of blue chips and says:
"COUNT THE CHIPS!" (No answer is recorded.)

The interviewer then separates the blue chips into sets of 5 (move 5 to left) and 6 chips and asks:
"IS THERE THE SAME NUMBER OF CHIPS NOW (pointing to the two sets with a continuous finger movement from one end to the other, finger about 4" away) AS THERE WAS IN THE ROW YOU JUST COUNTED?" (As the interviewer completes this question, he covers the chips with the cardboard marked #10).

There are several possible reactions to look for:

I. He may answer that the two rows of chips do not have the same number of chips as the one row had. In this case ask:
"ARE THERE MORE CHIPS NOW?"
Record the result in 10c-A and go on to 10d.

II. He may answer immediately that the two parts have the same number of chips as the one row of chips and, in this case, circle C and DO NEITHER in 10c-B and go on to 10d.

III. He may decline to say whether the rows have the same number or not and want to see the rows again. In this case, uncover the rows, note whether he is correct or not, and put reactions in 10c-B. If he says nothing at all, ask:
"HOW COULD YOU FIND OUT IF THEY ARE THE SAME?" (No answer is recorded.)
The child will usually want to see the chips or want to count them. In any case, uncover the chips and note the reactions in 10c-B on the Score Sheet.

d. The interviewer uncovers the chips if need be and puts them back in one row and says:
"COUNT THE CHIPS!" (No answer is recorded.)
The interviewer separates the 11 chips into parts of 9 and 2 (2 moved to the right) and pointing to the two parts asks: "IS THERE THE SAME NUMBER OF CHIPS NOW (follow pointing instructions in 10c) AS THERE WAS IN THE ROW YOU JUST COUNTED?"

The interviewer covers the two parts and notes the 3 reactions as in part 10c.

*11. SUBTRACTION (3 pictures marked 11a, 11b, and 11c).

a. The interviewer shows the child a picture of 6 lemons and says:

"JOHNNY'S MOTHER BOUGHT 6 LEMONS (pointing to the 6 lemons with finger about 4" away). SHE USED 2 OF THEM IN A CAKE (pointing in the general direction of 2 lemons with the finger no closer than 4".) HOW MANY DOES SHE HAVE LEFT?"

b. The interviewer shows the picture of 4 rocks to the child and says:

"JOE IS SAVING ROCKS. HE HAS 4 NOW (pointing to the 4 rocks with finger about 4" away). HE WANTS TO GET 8 ALTOGETHER. HOW MANY MORE DOES HE NEED?"

c. The interviewer shows the child the picture of 9 balls and 5 balls and says:

"SUSIE HAS 9 BALLS. ALICE HAS 5 BALLS. (Pointing to the two sets of red balls.) WHO HAS MORE BALLS? (No answer recorded.) HOW MANY MORE?" (Record answer.)

*If you can judge, from watching the student, record the way he does each problem.

Minimum Instructions
Concept Attainment Test Form B

1. a. "WILL YOU COUNT TO TEN FOR ME?"
   b. "STARTING WITH 15, WILL YOU COUNT BACKWARDS TO 1?" (Try 10 and 5 if impossible for child at 15)
2. (Picture--24 blue squares) "WILL YOU COUNT THESE FOR ME?"
3. (20 blue chips) "PLEASE GIVE ME 17 OF THE CHIPS."
4. a. (9, 4, 6 grapes) "POINT TO THE PICTURE OF 9 GRAPES."
   b. (14, 11, 12 boxes) "POINT TO THE PICTURE OF 12 BOXES."
5. a. (8, 7 balloons) "WHICH PICTURE HAS MORE BALLOONS IN IT?"
   b. (13, 14 balloons) "WHICH PICTURE HAS MORE BALLOONS IN IT?"
6. a. (17 blue, 11 red chips) 11 red in a row about 1/2" gap. "THESE ARE MY CHIPS. TAKE SOME OF YOURS (pointing to the blue ones) SO YOU HAVE THE SAME NUMBER AS I DO."
   b. If no obvious 1-1 correspondence say: "PLACE YOUR CHIPS SO WE CAN SEE THAT YOU HAVE THE SAME NUMBER."
   c. If still_incorrect, match the chips and say: "IS THERE THE SAME NUMBER OF RED AND BLUE CHIPS?"
7. (11 blue and 11 red chips) Spread blue to twice length of red row. "ARE THERE AS MANY BLUE CHIPS AS RED CHIPS?" If incorrect, say: "WHICH ROW HAS MORE CHIPS IN IT?"
8. (22 blue chips in groups of 15 and 7) "HERE ARE TWO PILES OF CHIPS. PUT THEM INTO TWO NEW PILES, SO EACH PILE HAS THE SAME NUMBER OF CHIPS IN IT." (Note counting or correspondence to do it.)
9. (14 blue, 4 red chips) Two rows of 9 each with colors matching. "POINT TO A CHIP FOR ME." "IS THIS A CHIP?" "I'M GOING TO ASK YOU SOME QUESTIONS ABOUT THESE CHIPS."
   a. "ARE THERE MORE BLUE CHIPS OR MORE CHIPS THERE?" (Point to chips in general with finger about 4" away.)
   b. "ARE THERE MORE CHIPS OR MORE BLUE CHIPS THERE?" (Same pointing)
   c. "ARE THERE MORE RED CHIPS OR MORE CHIPS THERE?" (Same pointing)
   d. "IF WE PILE UP ALL THE BLUE CHIPS AND THEN PILE UP ALL THE CHIPS, WHICH PILE WOULD BE TALLER?" (Same pointing)

e. "ARE THERE MORE BLUE CHIPS OR MORE CHIPS ALTOGETHER?" (Same pointing)

10. (11 blue and 8 red chips) 8 blue in a row--matched later by 8 red.

a. "HERE ARE SOME CHIPS. COUNT THEM." "HERE IS THE SAME NUMBER OF RED CHIPS. WATCH NOW." (6 and 2 separation of blue chips) "IS THERE THE SAME NUMBER OF BLUE CHIPS AND RED CHIPS?" (cover) If incorrect, "ARE THERE MORE BLUE CHIPS OR MORE RED CHIPS?" If no answer, "HOW COULD YOU FIND OUT IF THEY ARE THE SAME?"

b. Rearrange in 1-1 correspondence: "IS THERE THE SAME NUMBER OF BLUE CHIPS AND RED CHIPS?" (Separate to 3 and 5) "IS THERE THE SAME NUMBER OF BLUE CHIPS AND RED CHIPS?" (cover)

c. Remove all red, add 3 blue. "COUNT THE CHIPS!" (5 and 6 separation) "IS THERE THE SAME NUMBER OF CHIPS NOW AS THERE WAS IN THE ROW YOU JUST COUNTED?" (cover) If incorrect, "ARE THERE MORE CHIPS NOW?" If no answer: "HOW COULD YOU FIND OUT IF THEY ARE THE SAME?"

d. Make in one row again: "COUNT THE CHIPS!" 9 and 2 separation: "IS THERE THE SAME NUMBER OF CHIPS NOW AS THERE WAS IN THE ROW YOU JUST COUNTED?" (cover)

*11. a. Picture 11a. "JOHNNY'S MOTHER BOUGHT 6 LEMONS. SHE USED 2 OF THEM IN A CAKE. HOW MANY DOES SHE HAVE LEFT?"

b. Picture 11b. "JOE IS SAVING ROCKS. HE HAS 4 NOW. HE WANTS TO GET 8 ALTOGETHER. HOW MANY MORE DOES HE NEED?"

c. Picture 11c. "SUSIE HAS 9 BALLS. ALICE HAS 5 BALLS. WHO HAS MORE BALLS? HOW MANY MORE?"

*If you can judge, from watching the student, record the way he does each problem.

Concept Attainment Test Form C

Begin the interview by asking the child's name and recording the child's name, the child's school, the date, and the interviewer's name on the Concept Attainment Score Sheet.

1. ROTE COUNTING.

   a. "WILL YOU COUNT TO TEN FOR ME?"

   b. "STARTING WITH 15, WILL YOU COUNT BACKWARDS TO 1?"

If the child cannot do the backwards counting in part b, state the same question replacing 15 by 10. If still it is impossible for the child, replace 15 by 5. If still impossible, mark the question incorrect (I).

If the child begins with some number, such as 15, but makes mistakes in counting backwards, note the longest correct sequence which he recites and mark it on the Score Sheet.

2. RATIONAL COUNTING. (1 picture of 24 brown hexagons. Picture is marked #2).

   The interviewer places the picture on the table before the child and says:

   "WILL YOU COUNT THESE FOR ME?"

If the child is incorrect in his counting, mark the question incorrect and note (if possible) the last correctly used numeral in the space provided in the Score Sheet. Otherwise, just mark correct (C).

3. REPRODUCING A SET OF THE APPROPRIATE NUMBER OF CHIPS (20 white chips).

   The interviewer spreads the 20 white chips before the child and says:

   "PLEASE GIVE ME 15 OF THE CHIPS."

If the child does not give the interviewer the correct number of chips, mark the number given in the space provided.

4. IDENTIFICATION OF THE NUMBER OF MEMBERS OF A SET (2 pictures of cherries and yellow buttons marked #4a and #4b).

   Show the pictures one at a time to the child and say:

   a. "POINT TO THE PICTURE OF 7 CHERRIES." (4, 7, 2 cherries).

   b. "POINT TO THE PICTURE OF 14 BUTTONS." (13, 14, 16 buttons).

If incorrect, note the number of objects pointed to in the space provided in the Score Sheet.

5. COMPARISON OF THE NUMBER OF MEMBERS IN TWO SETS (2 pictures of flags marked #5a and #5b).

Show the pictures one at a time to the child and ask:

a. "WHICH PICTURE HAS MORE FLAGS IN IT?" (5 & 6 flags)

b. "WHICH PICTURE HAS MORE FLAGS IN IT?" (12 & 11 flags)

Whether the child is correct or incorrect in his answer, note whether he counts in the determination and circle HE COUNTED on the Score Sheet if he did.

6. ONE-TO-ONE CORRESPONDENCE (15 white and 9 blue chips).

The interviewer spreads 15 white chips before the child. Between the interviewer and the white chips, the interviewer places a row of 9 blue chips so that there is a gap of about 1/2" between the blue chips. The interviewer points to the row of blue chips and says:

a. "THESE ARE MY CHIPS. TAKE SOME OF YOURS (pointing to the white ones) SO YOU HAVE THE SAME NUMBER AS I DO." (Push the white chips he does not use to one side.)

INTERVIEWER

Blue chips 0 0 0 0 0 0 0 0 0

White chips 0 0 0 0 0 0 0 0 0

CHILD

If the child does not arrange the chips as in the diagram above, in response to part 6a, say:

b. "PLACE YOUR CHIPS SO WE CAN SEE THAT YOU HAVE THE SAME NUMBER."

If the child still does not arrange the chips in the manner of the diagram, note the way he has arranged them and record it under 6b of the Score Sheet. Then go on to part 6c. If at any time he does arrange the chips as in the diagram, go immediately to 7.

c. If b is incorrect, match the 9 blue chips with the 9 white chips as in the diagram and say:

"IS THERE THE SAME NUMBER OF WHITE AND BLUE CHIPS?"

7. CONSERVATION OF ONE-TO-ONE CORRESPONDENCE (9 white and 9 blue chips).

Use the same two rows of chips as arranged in #6. The interviewer then spreads the white chips into a row about twice as long as the row of blue ones. This is done completely within the view of the child. The interviewer asks:

"ARE THERE AS MANY WHITE CHIPS AS BLUE CHIPS?"

If the child is incorrect in his answer to this question, ask:

"WHICH ROW HAS MORE CHIPS IN IT?"

8. ADDITIVE COMPARISON OF SETS (20 white chips).

Place two piles (not stacked) of white chips, one of 13 and one of 7 chips, before the child and say:

"HERE ARE TWO PILES OF CHIPS. PUT THEM INTO TWO NEW PILES, SO EACH PILE HAS THE SAME NUMBER OF CHIPS IN IT."

On the Score Sheet note how the child carries out this task, whether he does it correctly or not.

9. CLASSIFICATION--PART LESS THAN THE WHOLE (14 white, 6 blue chips).

The interviewer puts the 20 chips in two rows of 10 each before the child--7 white and 3 blue chips in each row with the colors matching and the blue chips on the end. The interviewer then says several times:

"POINT TO A CHIP FOR ME." (No answer is recorded.)

If the child does not point to a blue chip during this time, the interviewer points to several chips (including blue ones) and asks:

"IS THIS A CHIP?" (No answer is recorded--this is only to have the child affirm that the blue chips are chips.)

Then the interviewer says:

"I'M GOING TO ASK YOU SOME QUESTIONS ABOUT THESE CHIPS."

a. "ARE THERE MORE WHITE CHIPS OR MORE CHIPS THERE?" (Pointing to the array of chips in general with the hand or finger about 4" away from closest point of the chips.)

b. "ARE THERE MORE CHIPS OR MORE WHITE CHIPS THERE?" (Same pointing instructions as in a)

c. "ARE THERE MORE BLUE CHIPS OR MORE CHIPS THERE?" (Same pointing instructions as in a)

d. "IF WE PILE UP ALL THE WHITE CHIPS AND THEN PILE UP ALL THE CHIPS, WHICH PILE WOULD BE TALLER?"

e. "ARE THERE MORE WHITE CHIPS OR MORE CHIPS ALTOGETHER?" (Same pointing instructions as in a)

10. PART-PART-WHOLE (12 white chips and 10 blue chips).

The interviewer puts 10 white chips in a row opposite the child and says:

a. "HERE ARE SOME CHIPS. COUNT THEM."

The interviewer puts 10 blue chips in a 1-1 correspondence with the white ones as in the diagram below and says:

"HERE IS THE SAME NUMBER OF BLUE CHIPS (wait for child verification of this if need be). WATCH NOW."

The interviewer separates the row of white chips so that it has 7 chips in one part and 3 in the other (move 3 to your left as diagram shows) and says:

"IS THERE THE SAME NUMBER OF WHITE CHIPS (pointing to the complete row with a continuous hand motion from one end to the other; finger about 4" away) AND BLUE CHIPS?" (Repeat the same pointing movement as with the white chips.)

As the interviewer finishes asking this question, he covers the rows of chips with a piece of cardboard marked #10.

INTERVIEWER

Blue chips 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

White chips 0 0 0 0 0 0 0 0 0 0 to 0 0 0 0 0 0 0 0 0 0

CHILD

There are several possible child reactions to look for:

I. He may answer that the rows are not the same in number. In this case ask:

"ARE THERE MORE WHITE CHIPS OR MORE BLUE CHIPS?"

Record the result in 10a-A and go on to 10b.

II. He may answer immediately that the two rows are the same in number. In this case, circle DO NONE OF THESE and C in 10a-B and go on to 10b.

III. He may decline to say whether the rows have the same number or not and want to see the rows again. In this case, uncover the rows, note whether he is correct or not, and put reactions in 10a-B. If he says nothing, ask:

"HOW COULD YOU FIND OUT IF THEY ARE THE SAME?" (No answer is recorded).

The child will usually want to see the chips or want to count them. In any case, uncover the rows and note the reactions in 10a-B.

b. The chips are uncovered if need be and rearranged in a 1-1 correspondence. The interviewer says:

"IS THERE THE SAME NUMBER OF WHITE CHIPS AND BLUE CHIPS?" (No answer is recorded, but get a "yes".)

The interviewer then separates the row of white chips into parts of 6 (move 6 to right) and 4 and covers the rows as he completes:

"IS THERE THE SAME NUMBER OF WHITE CHIPS (pointing instructions the same as in 10a) AND BLUE CHIPS?" (Same pointing instructions)

Note the same 3 responses as in 10a on the Score Sheet.

c. The interviewer removes all of the blue chips and adds 2 white chips to the remaining row of white chips and says:

"COUNT THE CHIPS!" (No answer is recorded.)

The interviewer then separates the white chips into sets of 4 (move 4 to left) and 8 chips and asks:

"IS THERE THE SAME NUMBER OF CHIPS NOW (pointing to the

two sets with a continuous finger movement from one end to the other, finger about 4" away) AS THERE WAS IN THE ROW YOU JUST COUNTED?" (As the interviewer completes this question, he covers the chips with the cardboard marked #10).

There are several possible child reactions to look for:

I. He may answer that the two rows of chips do not have the same number of chips as the one row had. In this case ask:

"ARE THERE MORE CHIPS NOW?"

Record the result in 10c-A and go to 10d.

II. He may answer immediately that the two parts have the same number of chips as the one row of chips and, in this case, circle C and DO NEITHER in 10c-B and go on to 10d.

III. He may decline to say whether the rows have the same number or not and want to see the rows again. In this case, uncover the rows, note whether he is correct or not, and put reactions in 10c-B. If he says nothing at all, ask:

"HOW COULD YOU FIND OUT IF THEY ARE THE SAME?" (No answer is recorded.)

The child will usually want to see the chips or want to count them. In any case, uncover the chips and note the reactions in 10c-B of the Score Sheet.

d. The interviewer uncovers the chips if need be and puts them back in one row and says:

"COUNT THE CHIPS!" (No answer is recorded.)

The interviewer separates the 12 chips into two sets of 11 and 1 (1 moved to the right) and pointing to the two parts asks: "IS THERE THE SAME NUMBER OF CHIPS NOW (follow pointing instructions in 10c) AS THERE WAS IN THE ROW YOU JUST COUNTED?"

The interviewer covers the two sets and notes the three reactions as in part 10c.

*11. SUBTRACTION (3 pictures marked #11a, #11b, and #11c).

a. The interviewer shows the child the picture of 5 oranges and says:

"MARY BROUGHT 5 ORANGES TO SCHOOL (pointing to the 5 oranges with finger about 4" away). SHE GAVE 3 OF THEM TO HER FRIENDS (pointing in the general direction of 3 oranges with finger no closer than 4"). HOW MANY DID SHE HAVE LEFT?"

b. The interviewer shows the picture of 4 stamps to the child and says:
"JIMMY IS SAVING STAMPS. HE HAS 4 NOW (pointing to the 4 stamps with finger about 4" away). HE WANTS TO GET 7 ALTOGETHER. HOW MANY MORE DOES HE NEED?"

c. The interviewer shows the child the picture of 8 frying pans and 5 frying pans and says:
"JUDY HAS 8 TOY FRYING PANS. KATHY HAS 5. (Pointing to the two sets of blue frying pans). WHO HAS MORE TOY FRYING PANS? (No answer is recorded.) HOW MANY MORE?" (Record answer.)

*If you can judge, from watching the student, record the way he does each problem.

Minimum Instructions
Concept Attainment Test

Form C

1. a. "WILL YOU COUNT TO TEN FOR ME?"
   b. "STARTING WITH 15, WILL YOU COUNT BACKWARDS TO 1?" (Try 10 and 5 if impossible at 15 for child)
2. (Picture--24 brown hexagons) "WILL YOU COUNT THESE FOR ME?"
3. (20 white chips) "PLEASE GIVE ME 15 OF THE CHIPS."
4. a. (4, 7, 2 cherries) "POINT TO THE PICTURE OF 7 CHERRIES."
   b. (13, 14, 16 buttons) "POINT TO THE PICTURE OF 14 BUTTONS."
5. a. (5, 6 flags) "WHICH PICTURE HAS MORE FLAGS IN IT?"
   b. (12, 11 flags) "WHICH PICTURE HAS MORE FLAGS IN IT?"
6. a. (15 white, 9 blue chips) 9 blue in a row about 1/2" gap. "THESE ARE MY CHIPS. TAKE SOME OF YOURS (pointing to the white ones) SO YOU HAVE THE SAME NUMBER AS I DO."
   b. If no obvious 1-1 correspondence, say: "PLACE YOUR CHIPS SO WE CAN SEE THAT YOU HAVE THE SAME NUMBER."
   c. If still incorrect, match the chips and say: "IS THERE THE SAME NUMBER OF WHITE AND BLUE CHIPS?"
7. (9 white, 9 blue chips) Spread the white chips to twice length of blue row. "ARE THERE AS MANY WHITE CHIPS AS BLUE CHIPS?" If incorrect, say: "WHICH ROW HAS MORE CHIPS IN IT?"
8. (20 white chips in groups of 13 and 7) "HERE ARE TWO PILES OF CHIPS. PUT THEM INTO TWO NEW PILES, SO EACH PILE HAS THE SAME NUMBER OF CHIPS IN IT." (Note counting or correspondence to do it.)
9. (14 white, 6 blue chips) Two rows of 10 each with colors matching. "POINT TO A CHIP FOR ME." "IS THIS A CHIP?" "I'M GOING TO ASK YOU SOME QUESTIONS ABOUT THESE CHIPS."
   a. "ARE THERE MORE WHITE CHIPS OR MORE CHIPS THERE?" (Point to chips in general with finger about 4" away.)
   b. "ARE THERE MORE CHIPS OR MORE WHITE CHIPS THERE?" (Same pointing)
   c. "ARE THERE MORE BLUE CHIPS OR MORE CHIPS THERE?" (Same pointing)
   d. "IF WE PILE UP ALL THE WHITE CHIPS AND THEN PILE UP ALL THE CHIPS, WHICH PILE WOULD BE TALLER?" (Same pointing)

e. "ARE THERE MORE WHITE CHIPS OR MORE CHIPS ALTOGETHER?" (Same pointing)

10. (12 white, 10 blue chips) 10 white in a row--matched later by 10 blue.

a. "HERE ARE SOME CHIPS. COUNT THEM." "HERE IS THE SAME NUMBER OF BLUE CHIPS. WATCH NOW." (7 and 3 separation of white chips) "IS THERE THE SAME NUMBER OF WHITE CHIPS AND BLUE CHIPS?" (Cover) If incorrect, "ARE THERE MORE WHITE CHIPS OR MORE BLUE CHIPS?" If no answer, "HOW COULD YOU FIND OUT IF THEY ARE THE SAME?"

b. Rearrange in 1-1 correspondence: "IS THERE THE SAME NUMBER OF WHITE CHIPS AND BLUE CHIPS?" (Separate white to 6 and 4) "IS THERE THE SAME NUMBER OF WHITE CHIPS AND BLUE CHIPS?" (cover)

c. Remove all blue, add 2 white chips. "COUNT THE CHIPS!" (4 and 8 separation) "IS THERE THE SAME NUMBER OF CHIPS NOW AS THERE WAS IN THE ROW YOU JUST COUNTED?" (cover) If incorrect: "ARE THERE MORE CHIPS NOW?" If no answer: "HOW COULD YOU FIND OUT IF THEY ARE THE SAME?"

d. Put in one row again: "COUNT THE CHIPS!" (11 and 1 separation): "IS THERE THE SAME NUMBER OF CHIPS NOW AS THERE WAS IN THE ROW YOU JUST COUNTED?" (cover)

*11. a. Picture 11a. "MARY BROUGHT 5 ORANGES TO SCHOOL. SHE GAVE 3 OF THEM TO HER FRIENDS. HOW MANY DID SHE HAVE LEFT?"

b. Picture 11b. JIMMY IS SAVING STAMPS. HE HAS 4 NOW. HE WANTS TO GET 7 ALTOGETHER. HOW MANY MORE DOES HE NEED?"

c. Picture 11c. "JUDY HAS 8 TOY FRYING PANS. KATHY HAS 5. WHO HAS MORE TOY FRYING PANS? HOW MANY MORE?"

*If you can judge, from watching the student, record the way he does each problem.

SCORE SHEET: CONCEPT ATTAINMENT. Circle the correct response.

1. a. C I Counted to _____
   b. 15-1 10-1 5-1 I

2. C I Correct to _____

3. C I Child gave _____

4. a. C I Pointed to _____
   b. C I Pointed to _____

5. a. C I HE COUNTED
   b. C I HE COUNTED

6. a. CHILD COUNTS C I
   MAKES CORRESPONDENCE C I
   b. If I, child makes
   LONGER ROW
   SAME LENGTH ROW
   SHORTER ROW
   PILE
   c. C I

7. C I
   If I, child pointed to
   LONG SHORT ROW.

8. C I
   Done by COUNTING
   CORRESPONDENCE
   BOTH

9. a. C I
   b. C I
   c. C I
   d. C I
   e. C I

10. a. A. If I, more in SEPARATED
    CLOSED
    B. C I Child had to
    SEE THE ROWS
    COUNT
    MAKE CORRESPONDENCE
    DO NONE OF THESE

    b. A. If I, more in SEPARATED
    CLOSED
    B. C I Child had to
    SEE THE ROWS
    COUNT
    MAKE CORRESPONDENCE
    DO NONE OF THESE

    c. A. If I, more in SEPARATED
    CLOSED
    B. C I Child had to
    SEE THE ROWS
    COUNT
    DO NEITHER

    d. A. If I, more in SEPARATED
    CLOSED
    B. C I Child had to
    SEE THE ROWS
    COUNT
    DO NEITHER

11. a. C I
    b. C I
    c. C I

NAME_____SCHOOL_____
INTERVIEWER_____DATE_____
FORM A B C
INTERVIEW NUMBER _____

APPENDIX B

SUBTRACTION APPLICATION AND TRANSFER TEST

(Note: The original test was produced on a primer typewriter with large letters.)

NAME:

1. TED HAD 6 PENCILS.
   HE LOST 2 OF THEM.
   HOW MANY PENCILS
   DID HE HAVE THEN?

2. JUDY HAD 4 COOKIES.
   HER MOTHER GAVE
   HER 1 MORE.
   HOW MANY COOKIES
   DID SHE HAVE THEN?

3. PETER WANTS TO BUY
   A WHISTLE WHICH
   COSTS 7 PENNIES.
   HE HAS 4 PENNIES NOW.
   HOW MANY MORE
   DOES HE NEED?

4. ALICE HAS 5 DOLLS.
   SHE HAS 3 DOLL CHAIRS.
   HOW MANY MORE DOLLS
   THAN CHAIRS HAS SHE?

5. BEA HAD 11 GLASSES.
SHE HAD 6 STRAWS.
HOW MANY MORE
GLASSES THAN STRAWS
DID SHE HAVE?



6. JANE HAD 7 MARBLES.
HER BROTHER GAVE
HER 12 MORE.
HOW MANY MARBLES
DID SHE HAVE THEN?



7. BUD HAD 14 PENNIES.
HE SPENT 8 OF THEM
FOR AN ICE CREAM CONE.
HOW MANY PENNIES
DID HE HAVE THEN?



8. SALLY WANTS TO BUY
A DOLL HAT WHICH
COSTS 16 PENNIES.
SHE HAS 9 PENNIES
NOW. HOW MANY
MORE DOES SHE NEED?

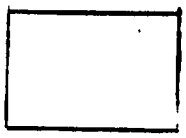


1. 3 - 1 = 

2. 3 + 2 = 

3. 15 - 4 = 

4. 14 - 5 = 

5. 17 - 9 = 

6. 8 + 15 = 

7. 33 - 25 = 

8. 13 - 8 = 

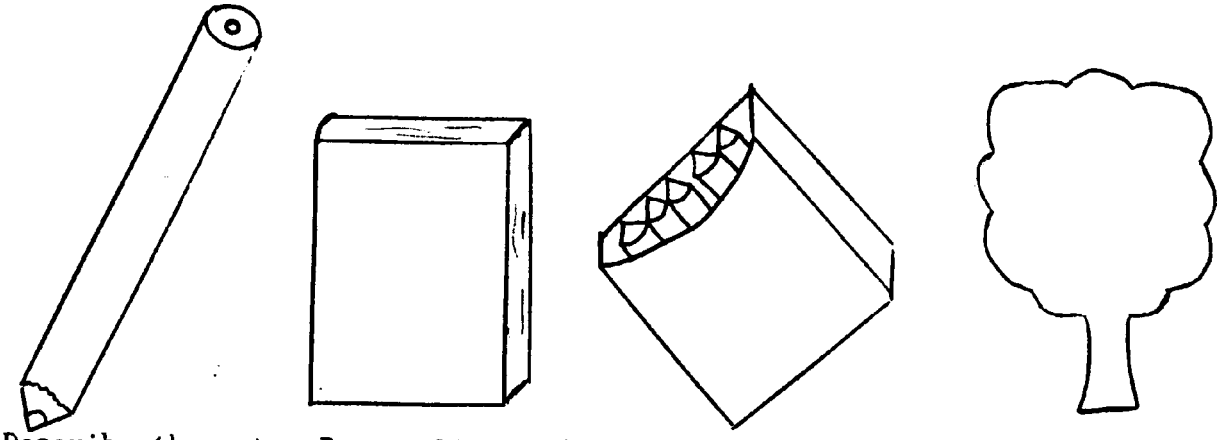
9. 29 - 22 = 

10. 94 - 38 = 

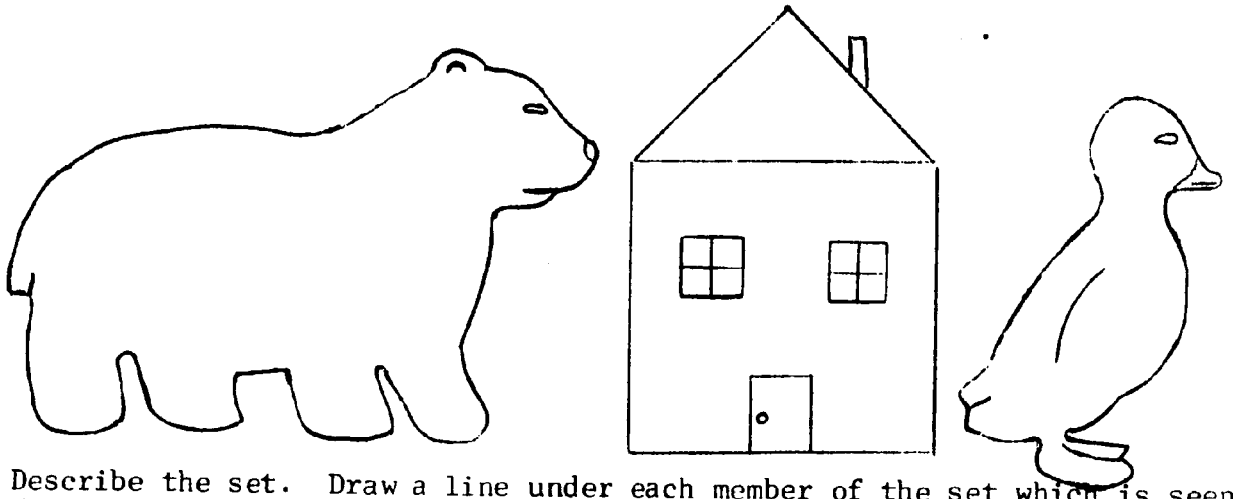
APPENDIX C

SAMPLE WORKSHEETS USED IN THE STUDY

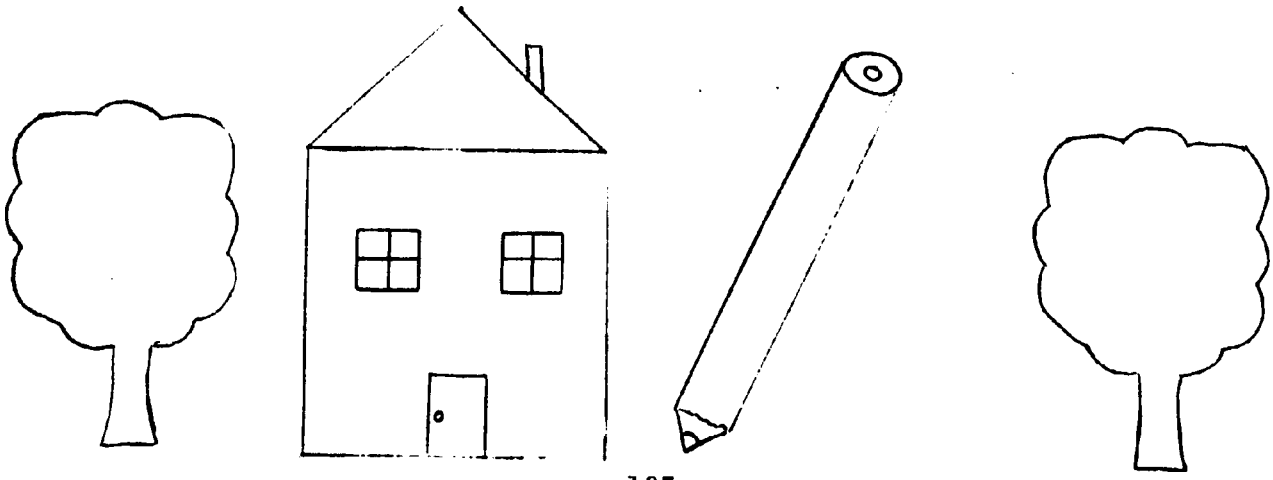
Describe the set. Draw a line under the members of the set of things you use in school.



Describe the set. Draw a line under the members of the set which are animals.

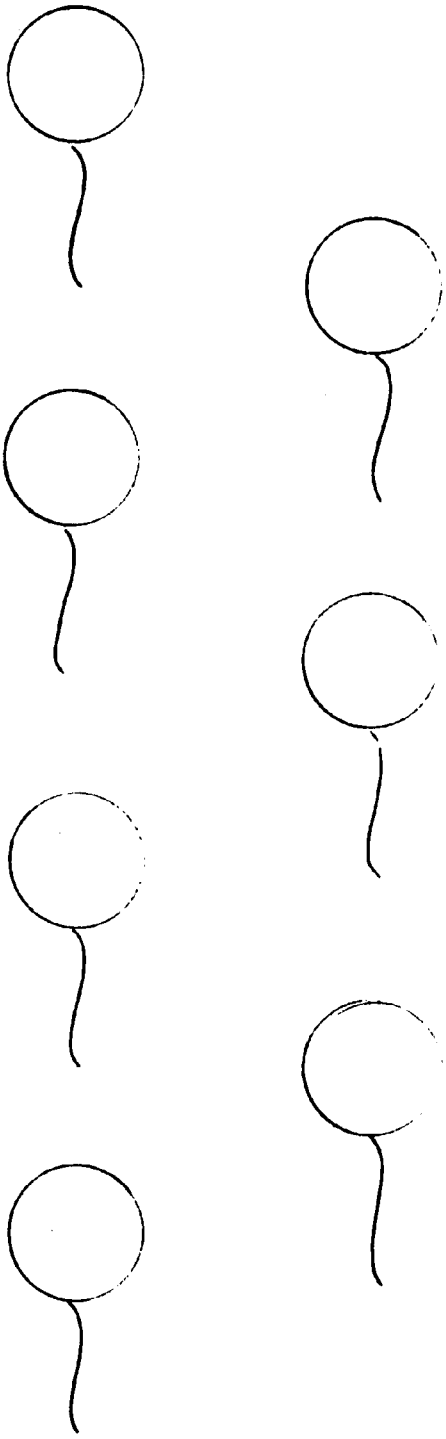


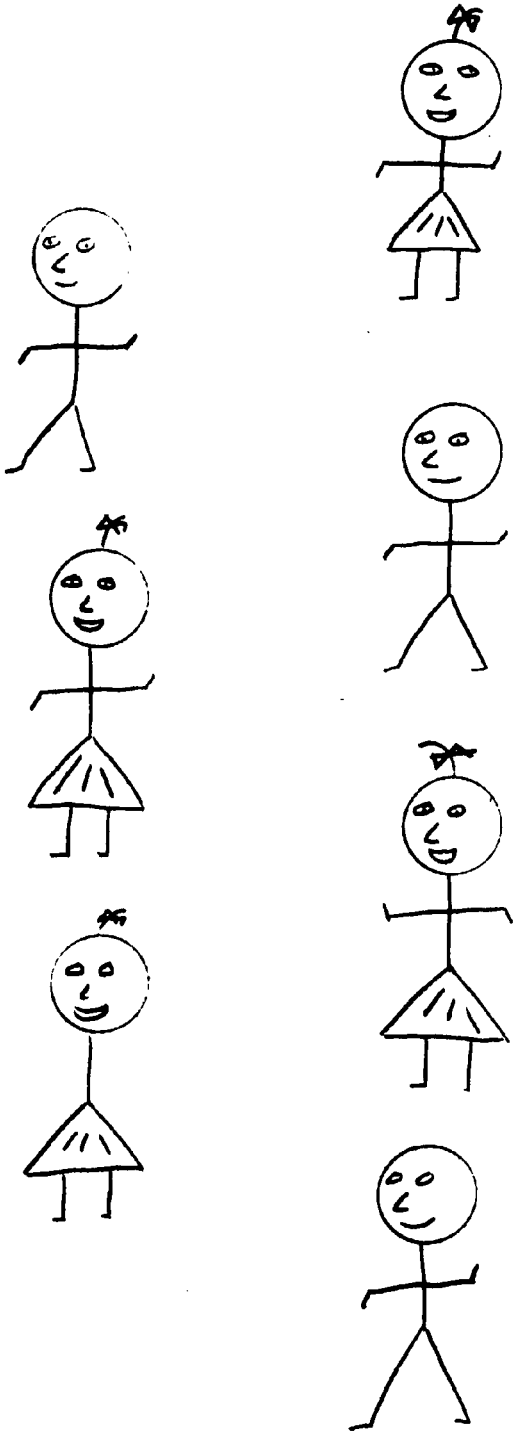
Describe the set. Draw a line under each member of the set which is seen on the way to school. Color all the pictures.



Worksheet 2

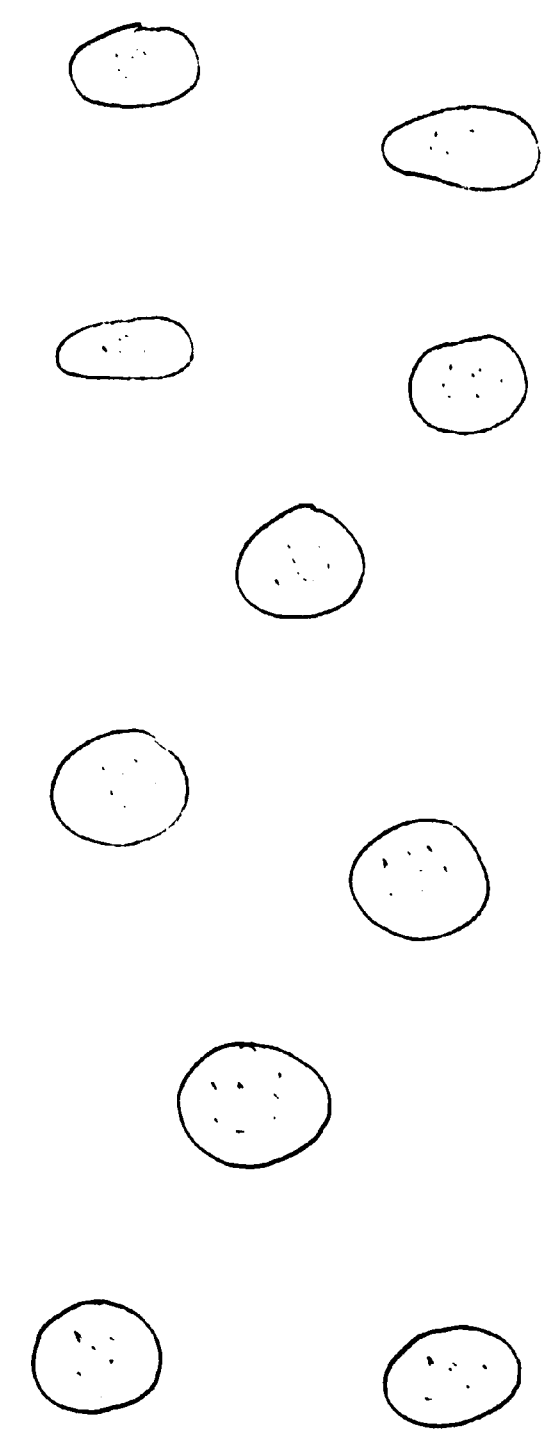
Is there a balloon for each child? Match the sets to see. Color the pictures.

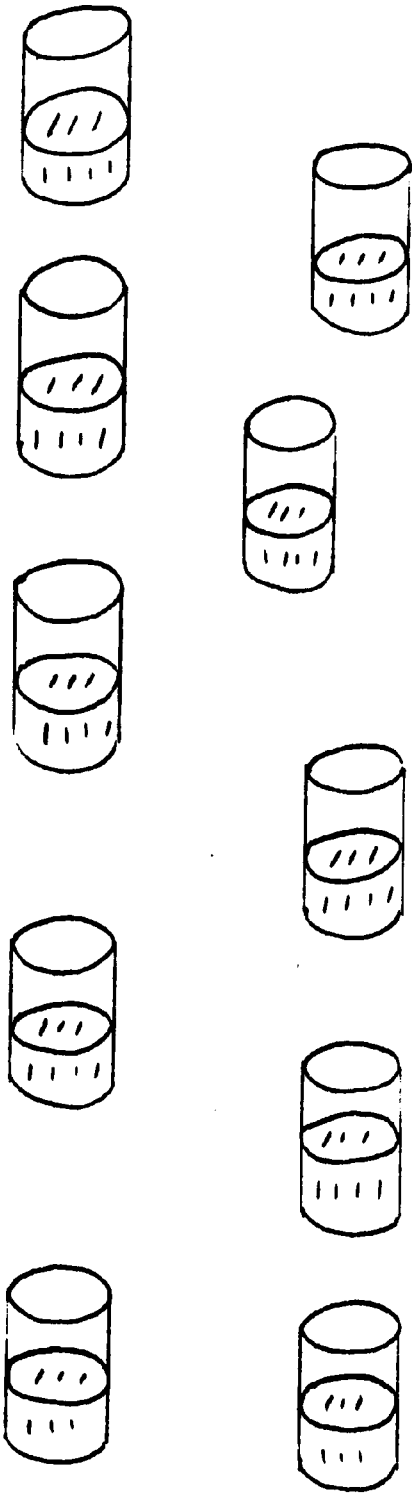




Worksheet 3

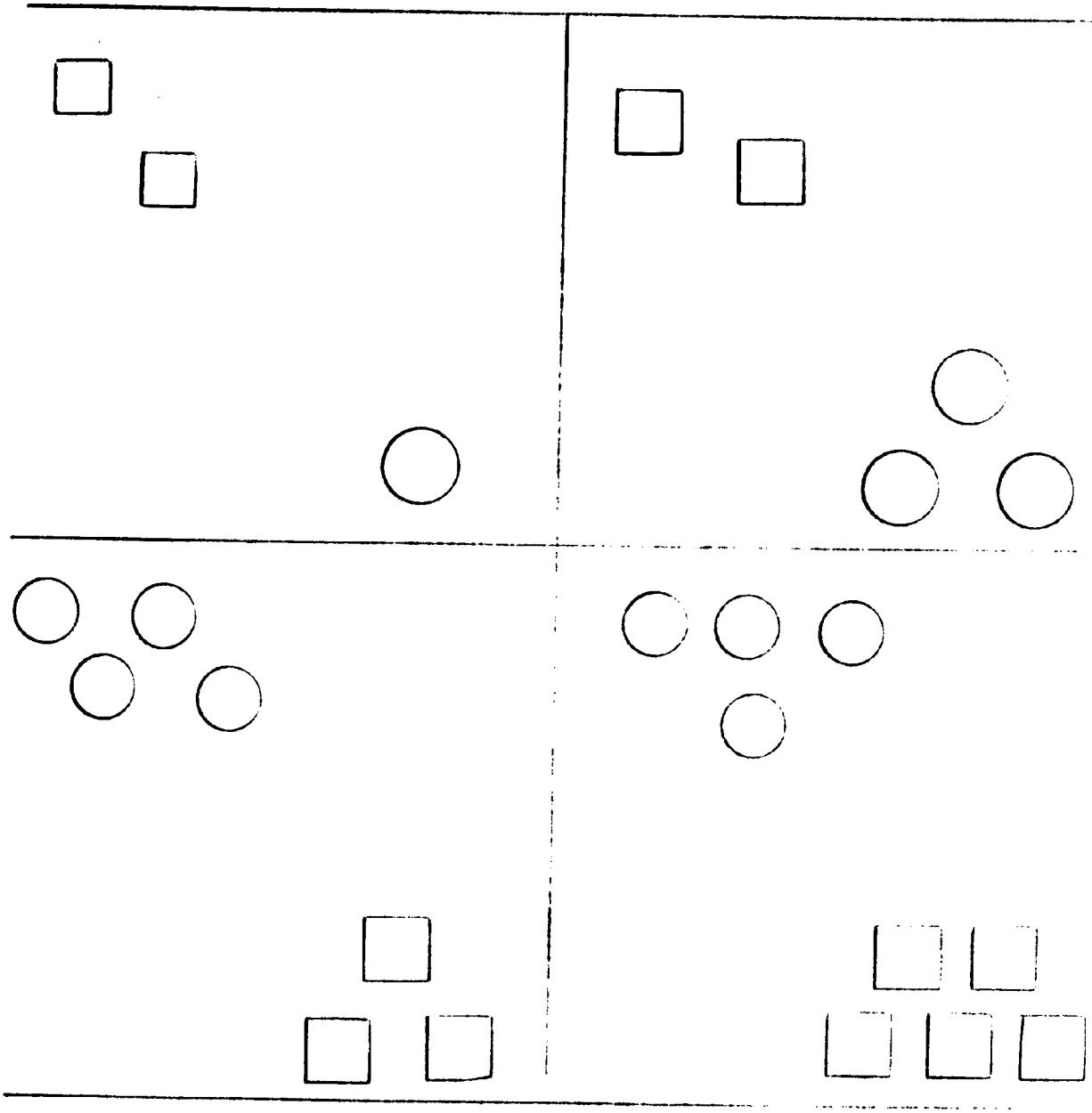
Is there a cookie for each glass of milk? Match the sets to see.
Color the pictures.





Worksheet 4

Circle the set with more members.



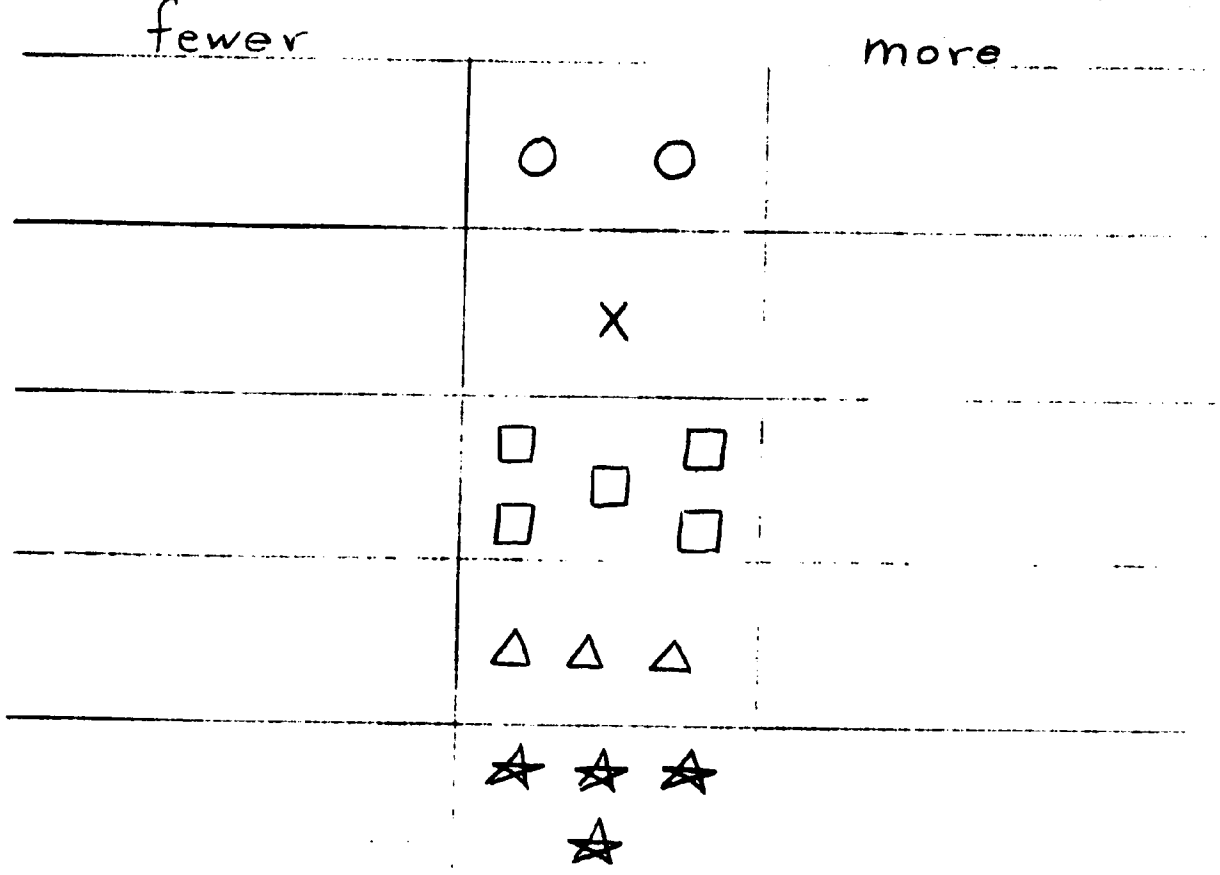
Circle the name of the greater number.

2	1	2	3
4	3	4	5

Worksheet 5

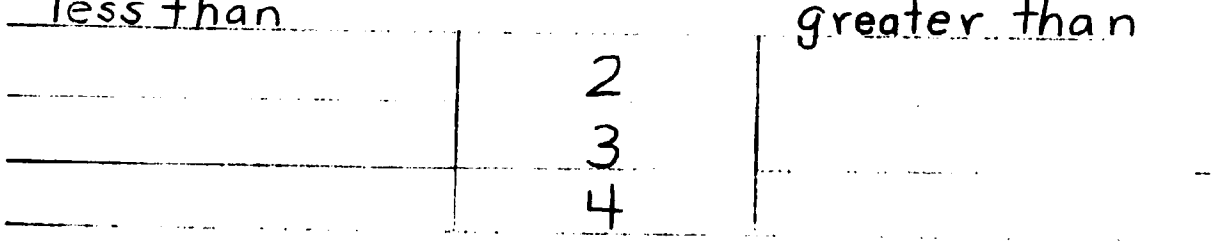
Draw a picture of a set with fewer members than are in the set in the box in the middle.

Draw a picture of a set with more members than are in the set in the box in the middle.



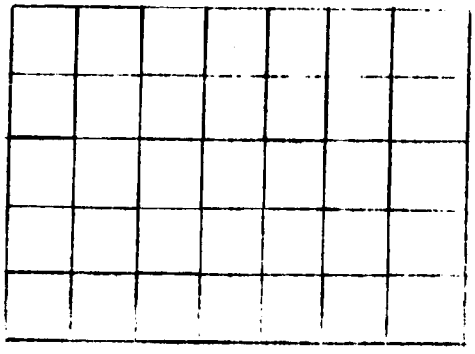
Write a numeral which names a number less than the one in the middle box.

Write a numeral which names a number greater than the one in the middle box.



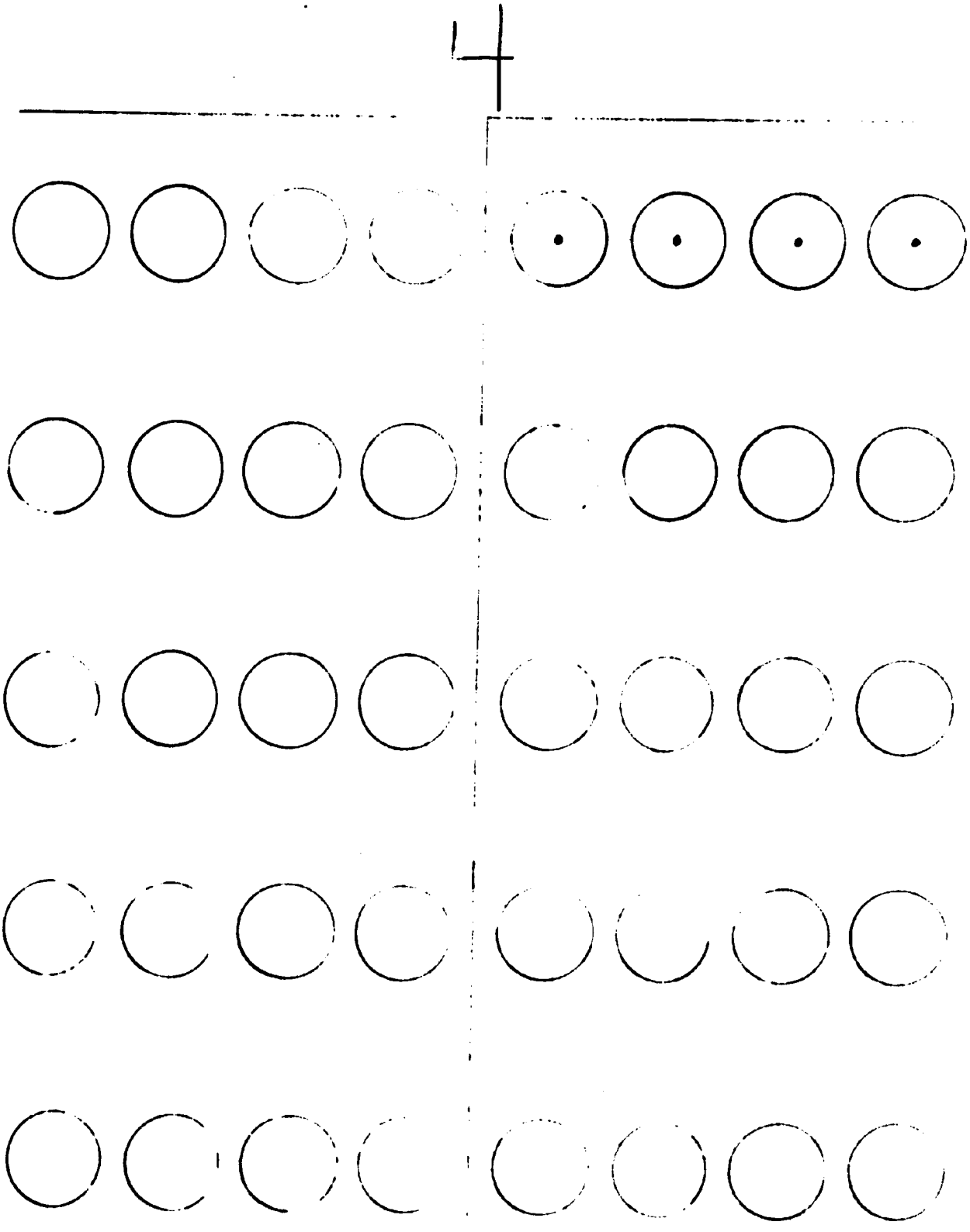
In each row color the correct number of boxes.

Color 1 □ black
Color 2 □'s red
Color 3 □'s green
Color 4 □'s yellow
Color 5 □'s blue



Worksheet 6

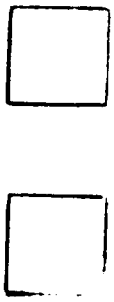
Supply the children with a strip of paper to keep the place. Have them suggest different ways of seeing four. When one is suggested, have the children put the correct number of dots in the circles. The first row is done to show 0 and 4. When all the ways are found, let the children color all the circles with dots in them.

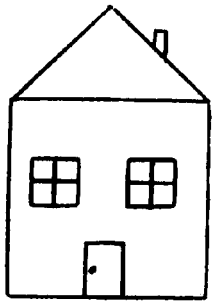


Worksheet 7

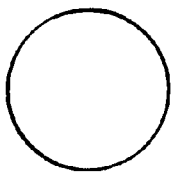
In each row write the names of all the numbers you can think of which are greater than the number in the first box.

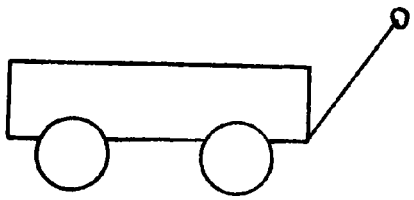
1	2	3	4	5
1				
2				
3				



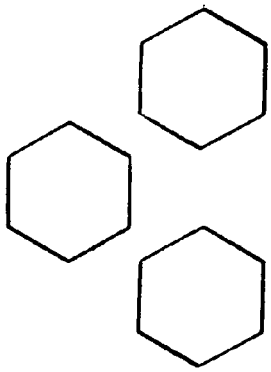


Draw a set of houses that has more members than the set of squares.





Draw a set of wagons which has more members than the set of circles.



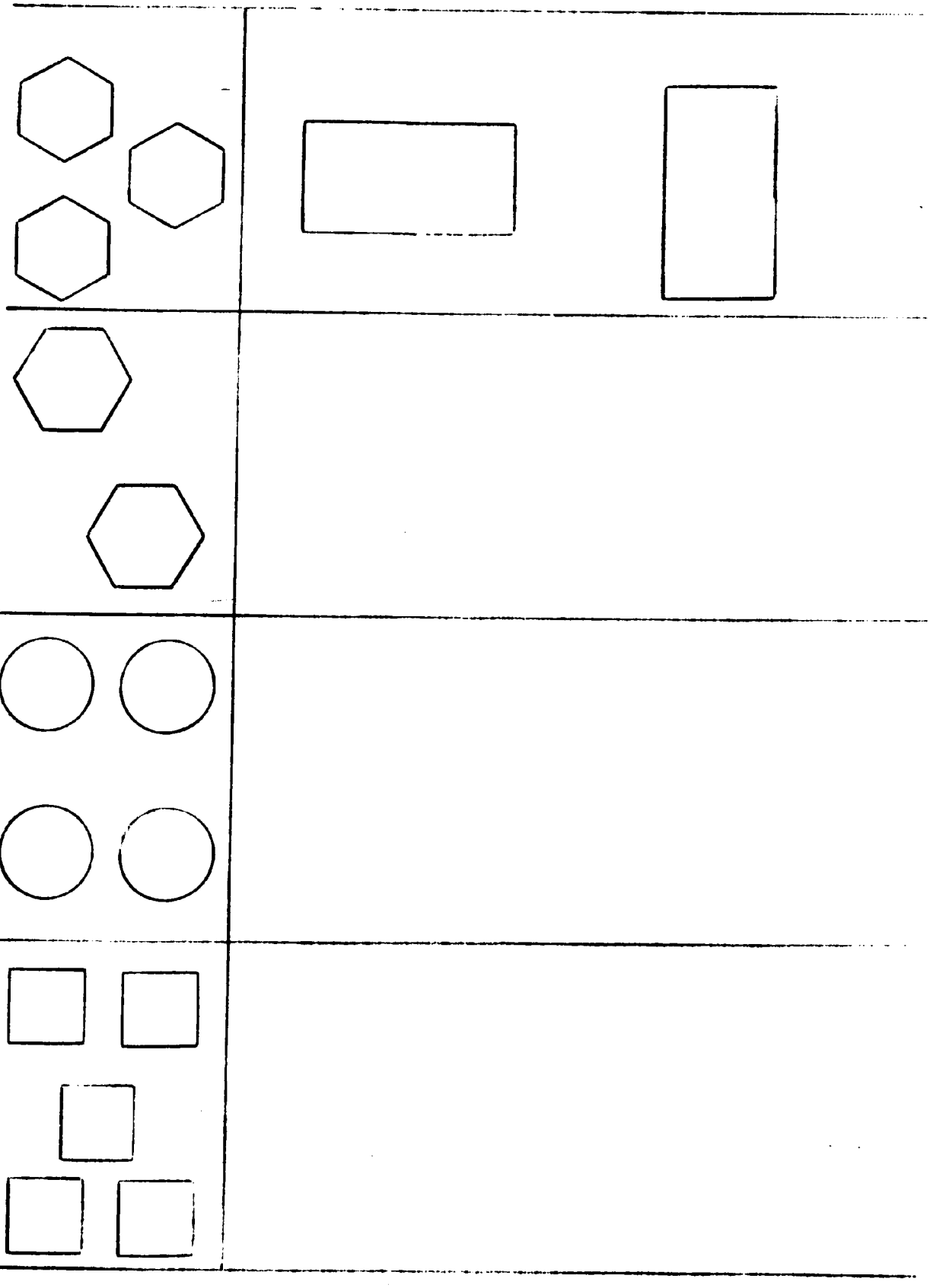


Draw a set of trees which has more members than the set of hexagons.

Color all the pictures.

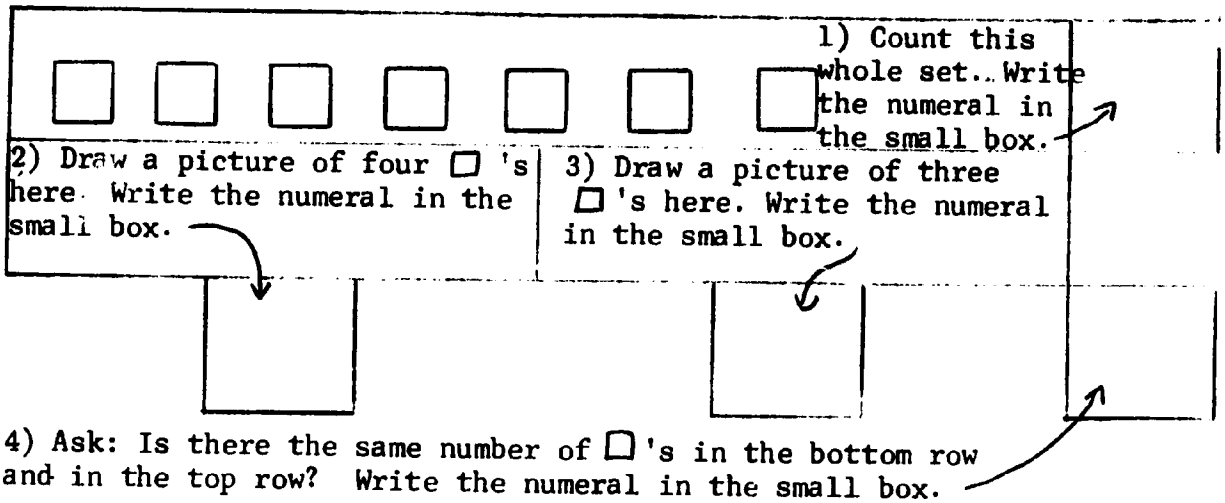
Worksheet 8

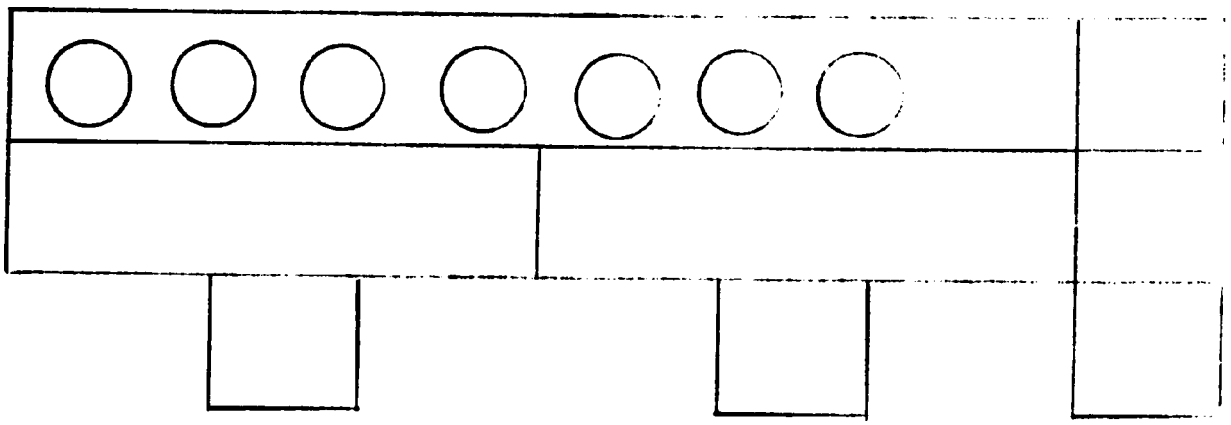
In each box draw a set with fewer members than are in the given set. The first one is done. The set of two boxes has fewer members than the set of three hexagons. Suggest what should be drawn, for example, balloons, cars, cookies, etc.

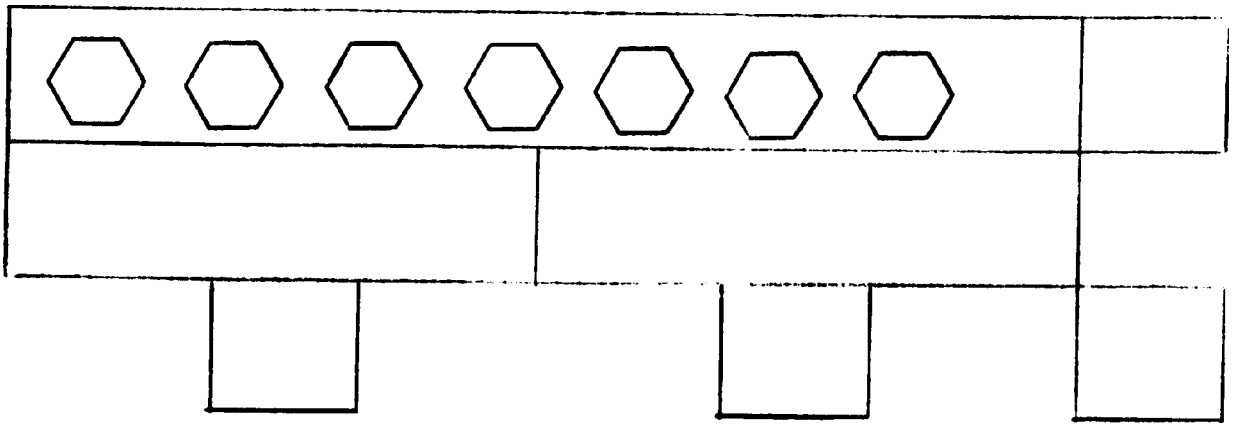


Worksheet 9

The teacher draws a figure like the first one shown and then carries out the procedure described below for each figure.

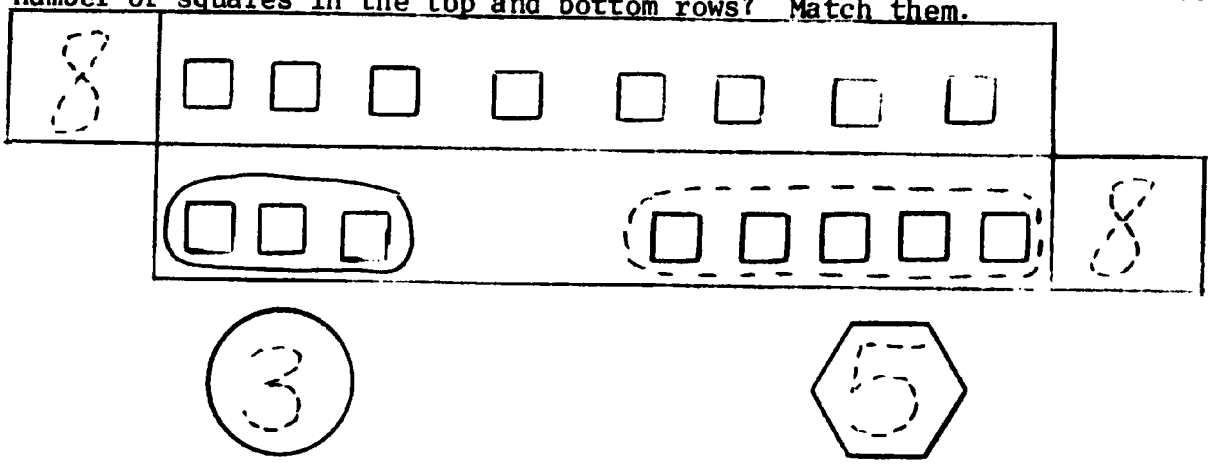


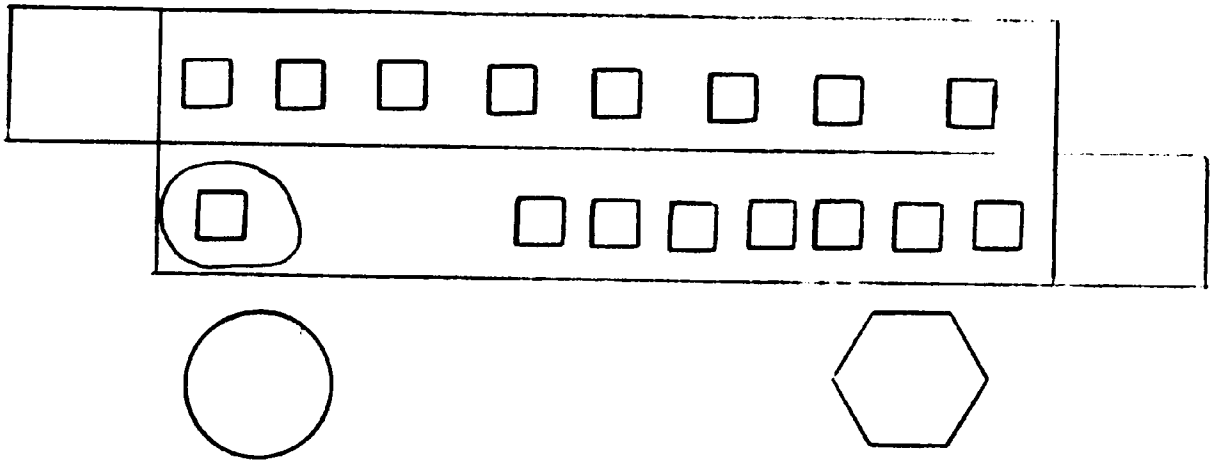


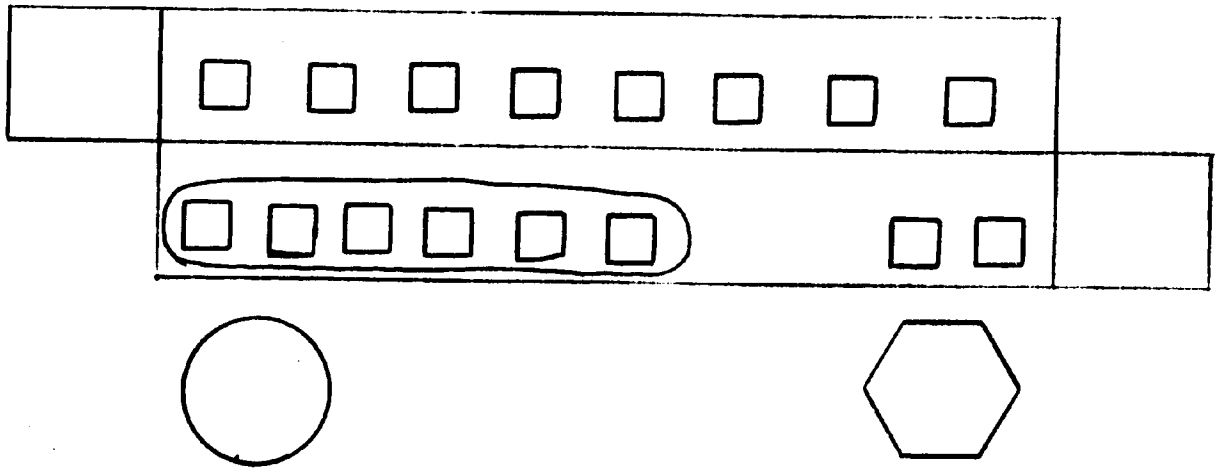


Worksheet 10

Count the set in the top row. Write the numeral in the box. How many squares in the encircled part in the second row? Write the numeral in the circle. Put a ring around the other part in the second row, count it, and write the numeral in the hexagon. How many members in the whole set in the bottom row? Write the numeral in the box. Is there the same number of squares in the top and bottom rows? Match them.



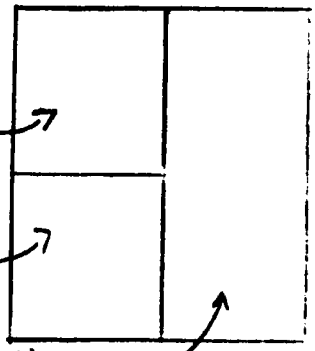




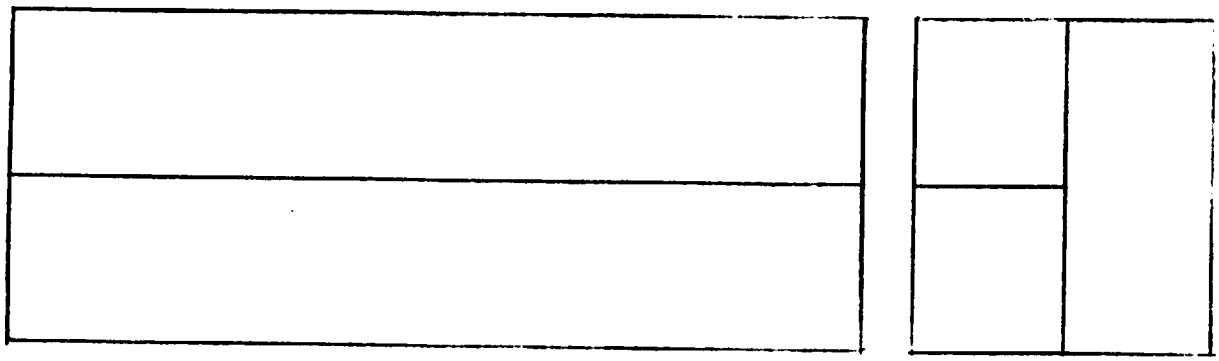
Worksheet 11

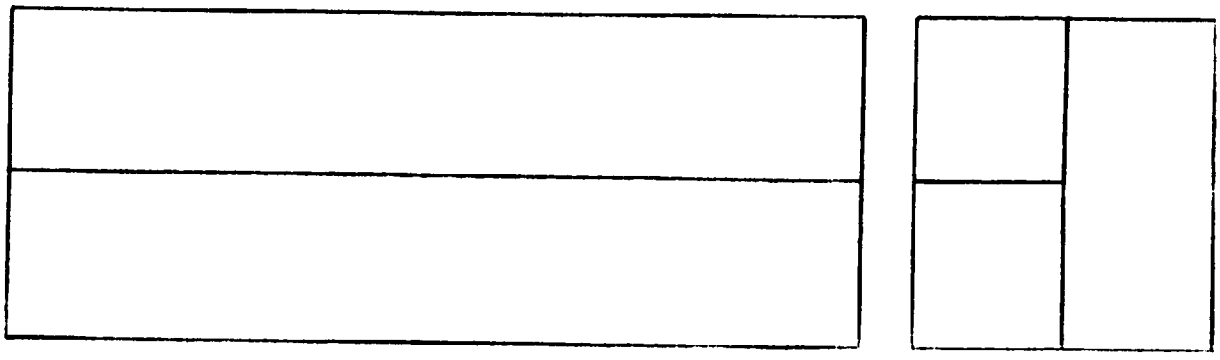
1) Ask the children to show 4 on the top row of their bead frames. Let them draw a picture of the top row here and write the numeral in the small box.

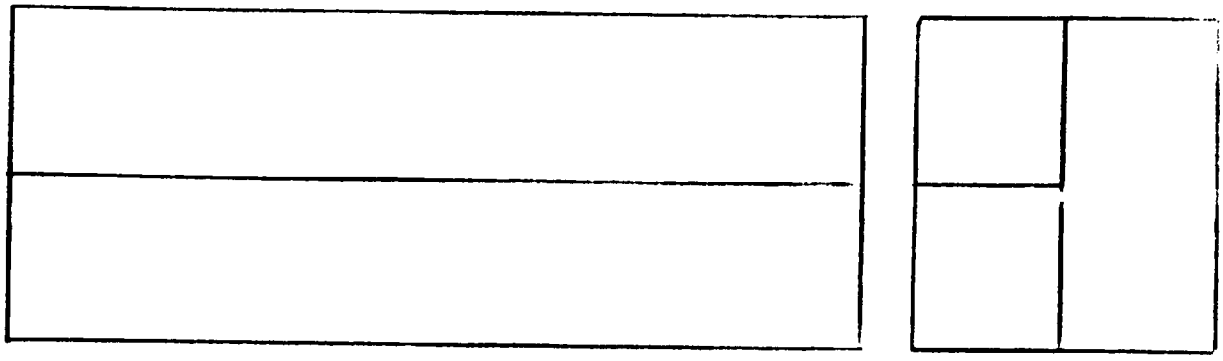
2) Ask the children to show a set of 3 on the bottom row of their bead frames. Draw a picture of that row here and write the numeral in the small box.



3) Have the children see how many in the two sets together and write the numeral in the big box.

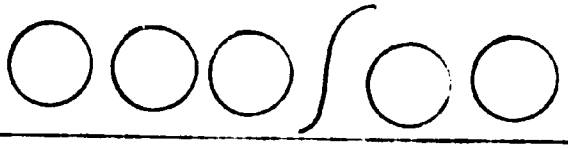
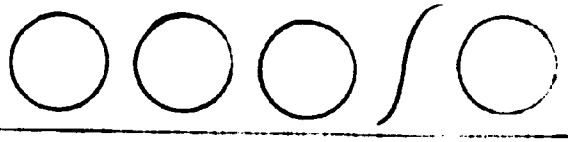






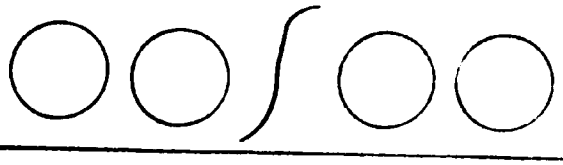
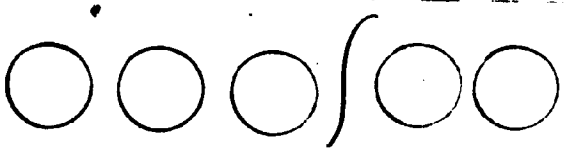
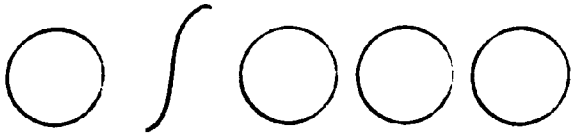
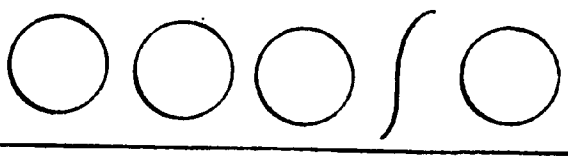
Worksheet 12

The teacher asks the children about the number in each subset and to find the numeral in the number sentence which tells about it. In the cases below the sentence must be completed. The teacher asks how many in the whole set and a picture is drawn in the space provided. The number sentence is read and completed.

Draw pictures of whole set here.	
	2 + 2 = ☐
	3 + 2 = ☐
	1 + 3 = ☐
	3 + 1 = ☐
	2 = ☐

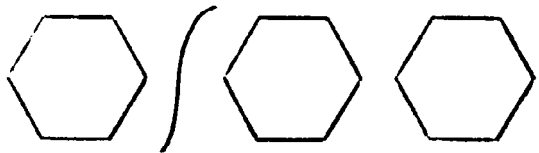
Worksheet 13

Same directions as in Worksheet 12.

Draw pictures of whole set here.	
	2 and 2 is ☐
	3 and 2 is ☐
	1 and 3 is ☐
	3 and 1 is ☐
	2 is ☐

Worksheet 14

Ask the children to color the pictures as a story is told about each one. Then ask them to read the number sentence and fill the box with the numeral for the missing part.

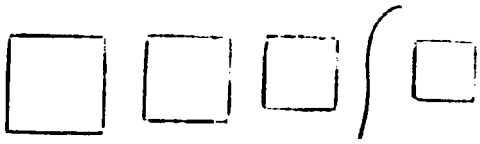


Example: Mary had a set of three beads. Two of them were green. The rest were yellow. How many were yellow?

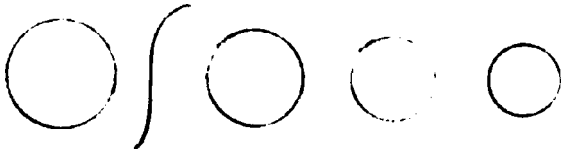
3 is ☐ and 2



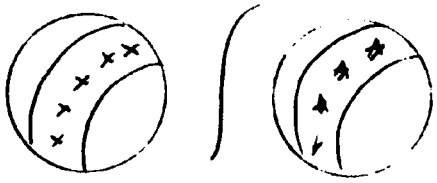
3 is ☐ and 1



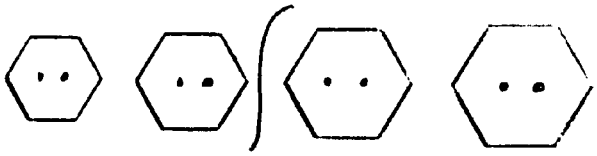
4 is ☐ and 1



4 is ☐ and 3



2 is ☐ and 1



4 is ☐ and 2

Read the number sentence and fill in the numeral for the missing part.

3 is ☐ and 1

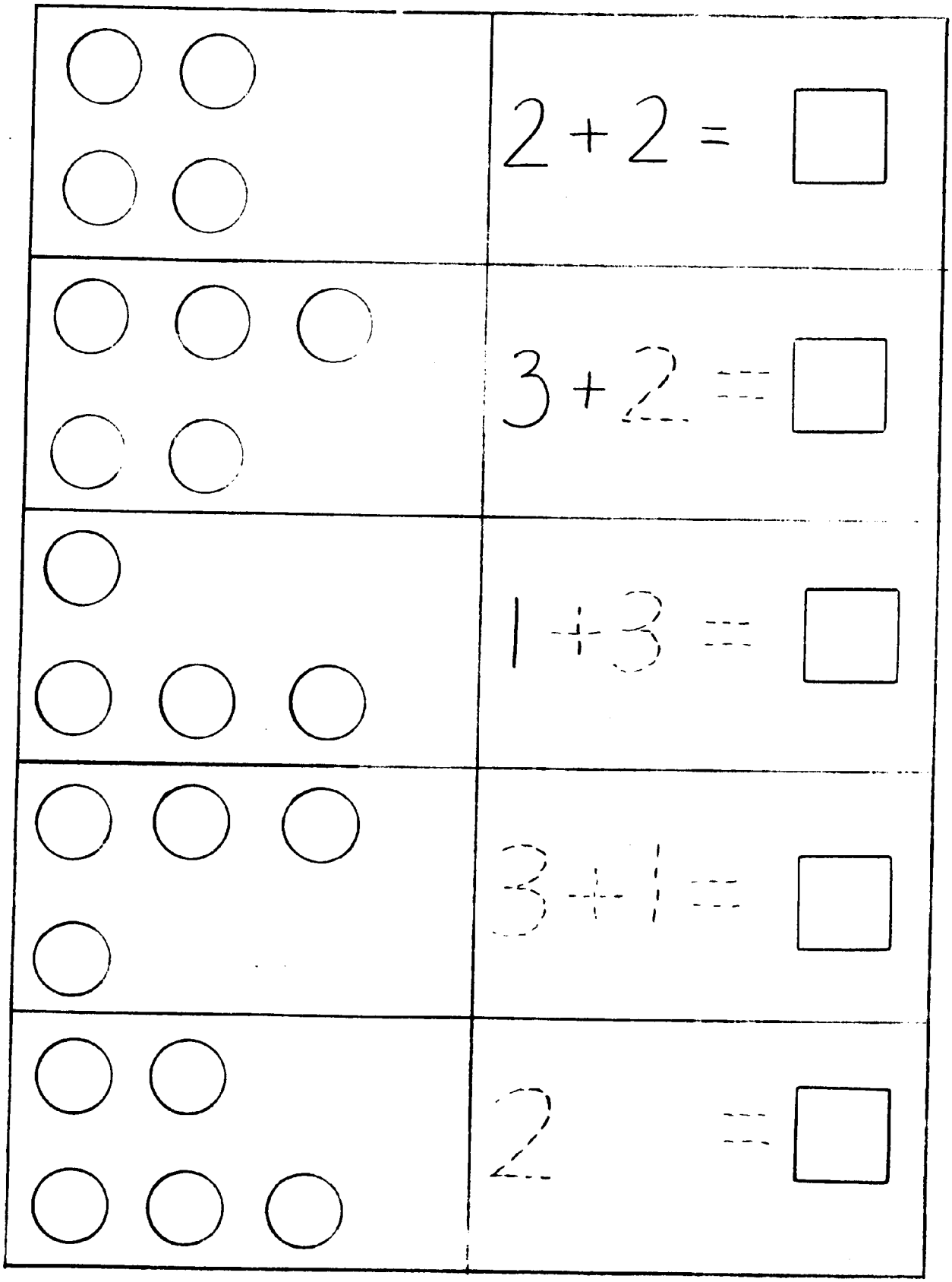
4 is ☐ and 3

4 is ☐ and 1

3 is ☐ and 2

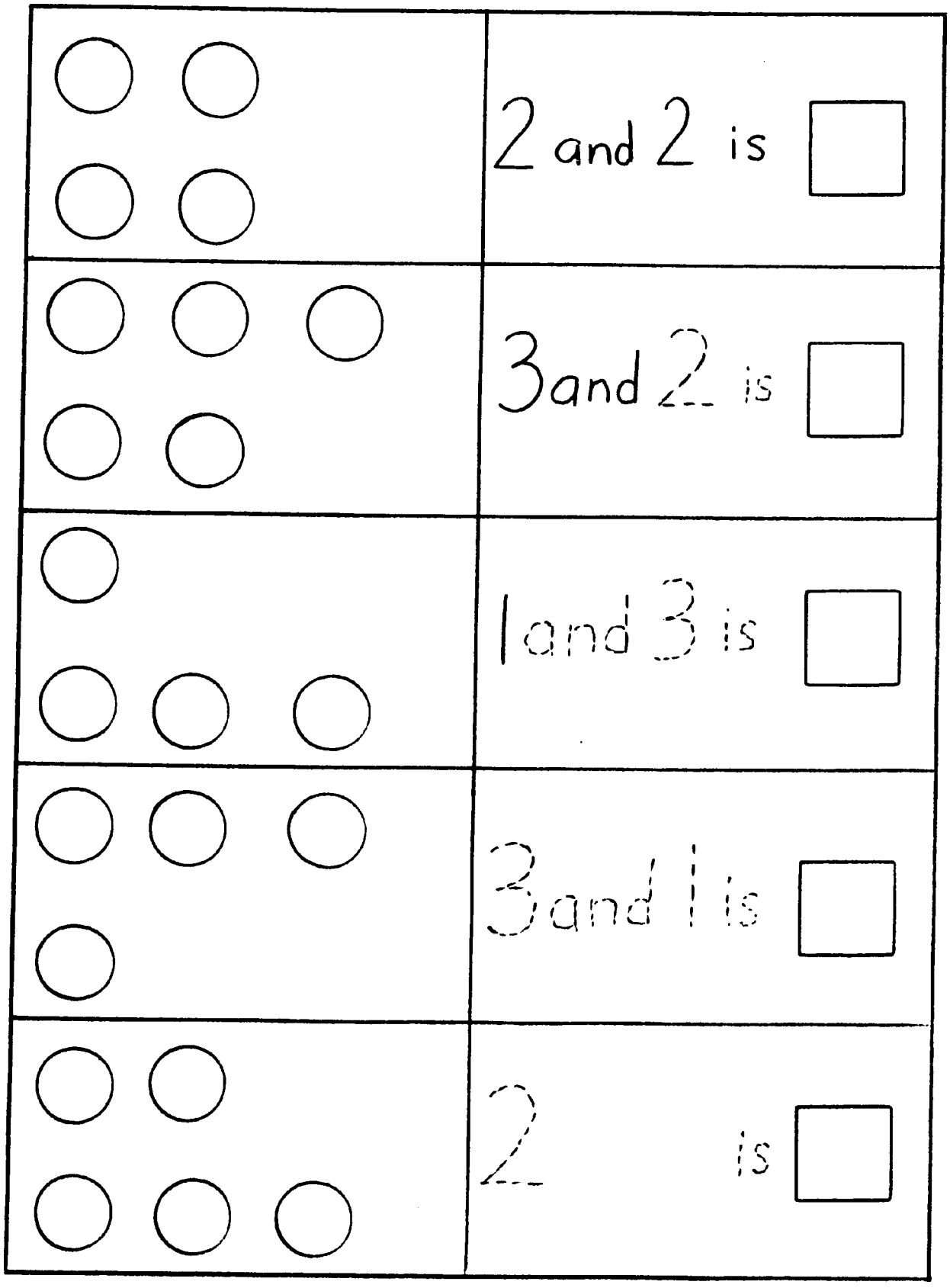
Worksheet 15

The teacher asks how many in the first set, the second set, and altogether. These numbers are found in the number sentence (or written in later cases). The number sentence is read and completed.



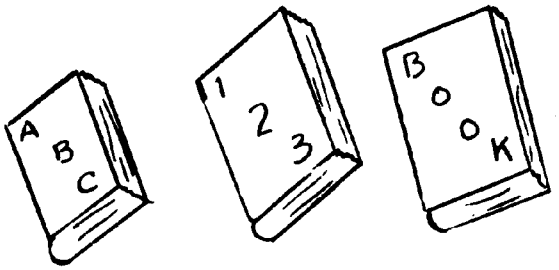
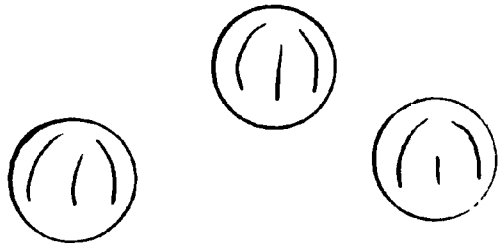
Worksheet 16

Same directions as in Worksheet 15.



Worksheet 17

Find the answer to each number sentence.

	
3 take away 2 is ☐	3 take away 1 is ☐
2 take away 1 is ☐	3 take away 1 is ☐
3 take away 0 is ☐	2 take away 1 is ☐
2 take away 2 is ☐	3 take away 3 is ☐
3 take away 2 is ☐	3 take away 2 is ☐
2 take away 0 is ☐	3 take away 1 is ☐
3 take away 1 is ☐	2 take away 1 is ☐

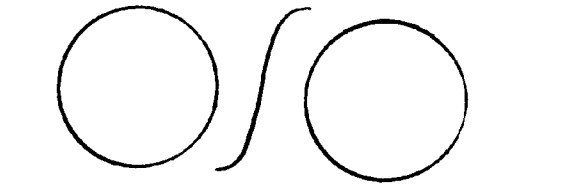
Worksheet 18

Find the answer to each addition sentence.

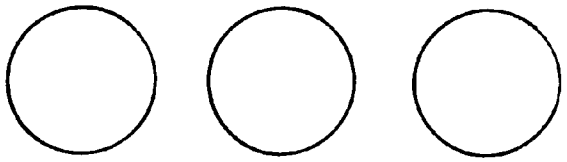
$4 + 3 = \square$	$\square = 5 + 2$
$7 + 0 = \square$	$\square = 2 + 4$
$2 + 5 = \square$	$\square = 1 + 6$
$2 + 4 = \square$	$\square = 4 + 3$
$6 + 1 = \square$	$\square = 0 + 7$
$3 + 4 = \square$	$\square = 3 + 3$
$2 + 3 = \square$	$\square = 2 + 5$
$1 + 6 = \square$	$\square = 6 + 1$
$5 + 2 = \square$	$\square = 2 + 2$
$0 + 7 = \square$	$\square = 3 + 4$

Worksheet 19

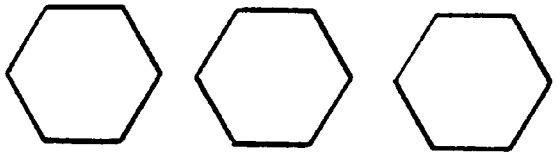
Find the answer to each subtraction sentence. Split the whole set so the parts can be seen.



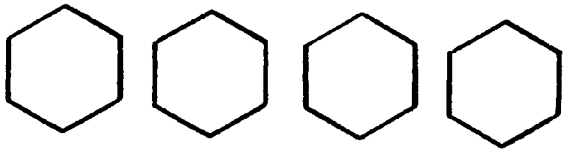
2 − 1 = ☐



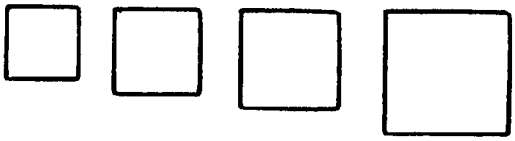
3 − 1 = ☐



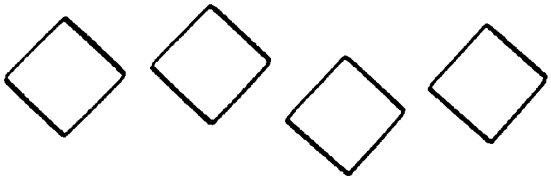
3 − 2 = ☐



4 − 1 = ☐



4 − 2 = ☐



4 − 3 = ☐

Find the answer to each subtraction sentence.

3 − 1 = ☐

4 − 1 = ☐

2 − 1 = ☐

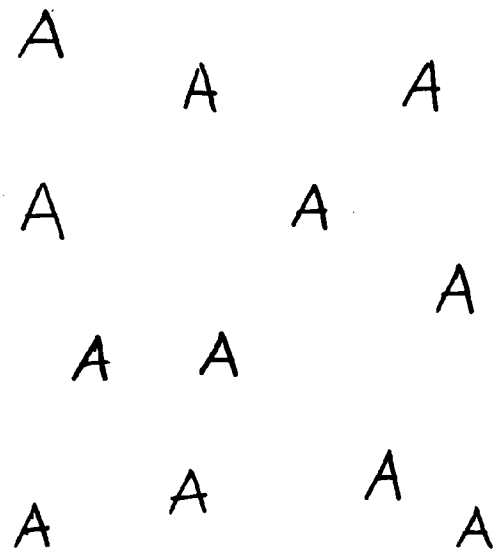
4 − 2 = ☐

3 − 2 = ☐

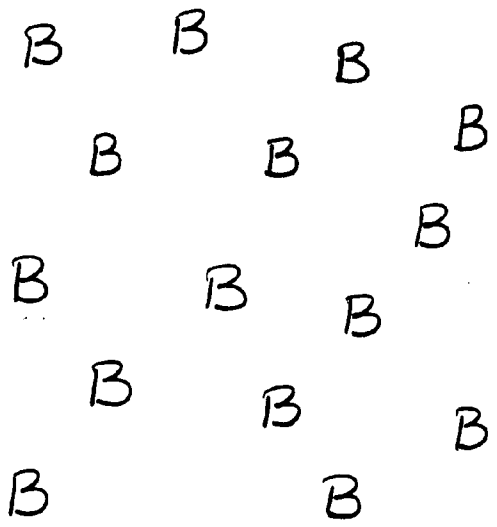
4 − 3 = ☐

Worksheet 20

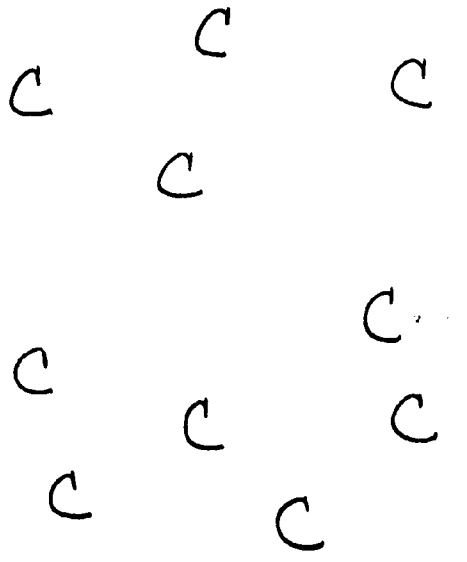
Circle each set of ten. Complete the statement: _____ tens _____ ones in each box.



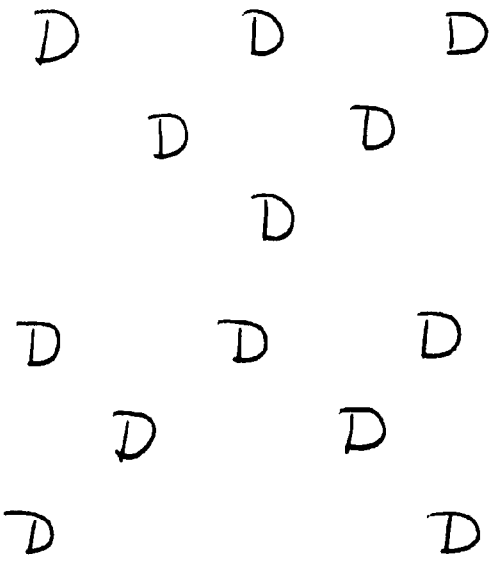
___ tens ___ ones



___ tens ___ ones



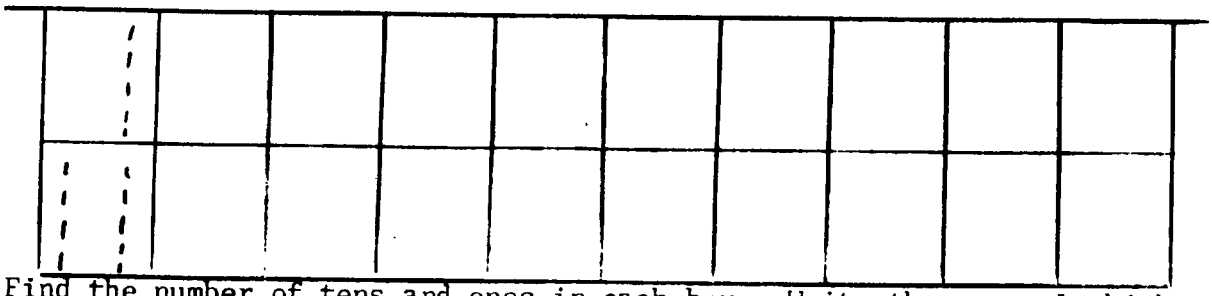
___ tens ___ ones



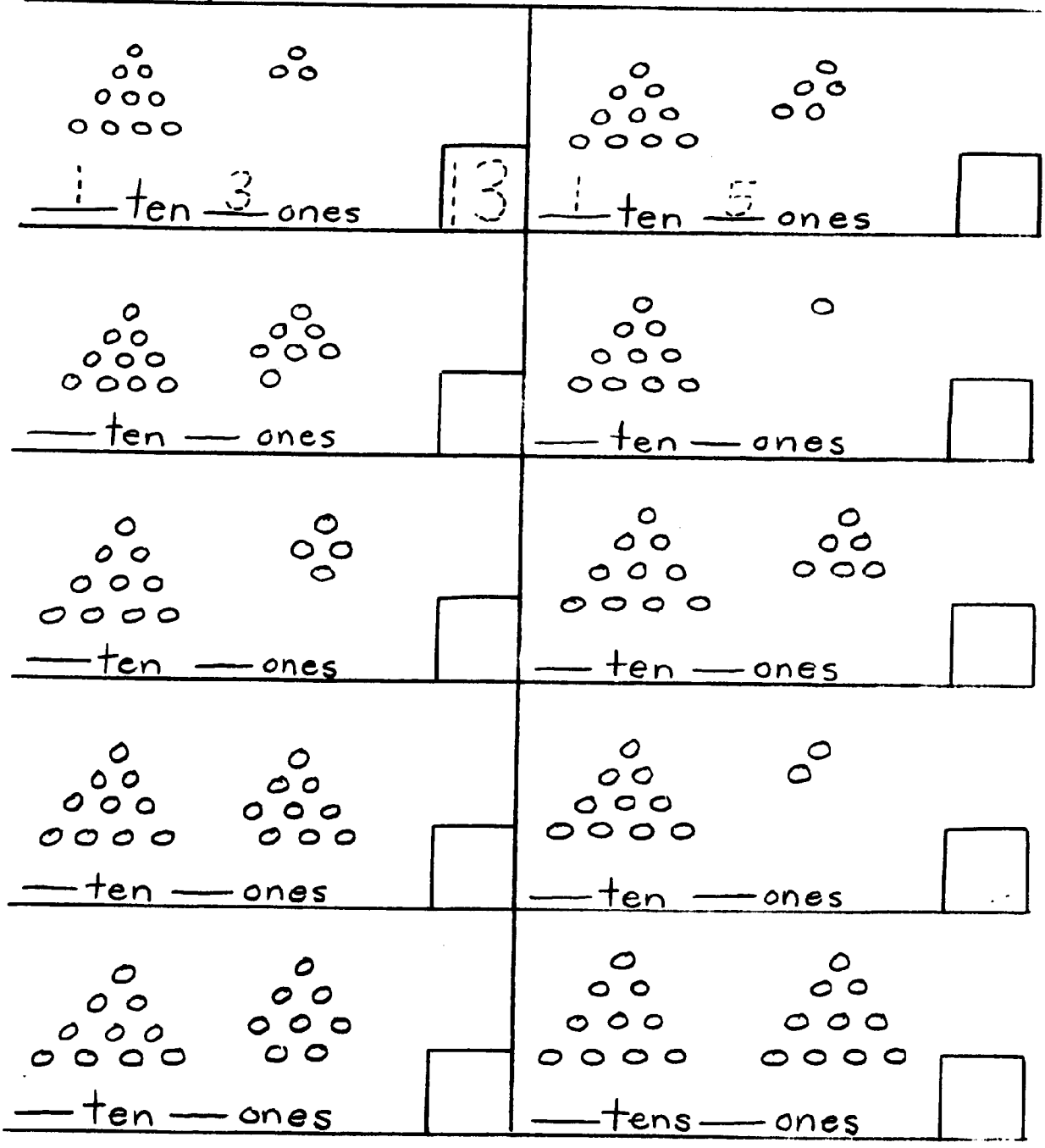
___ tens ___ ones

Worksheet 21

Write the numerals 1 through 20.

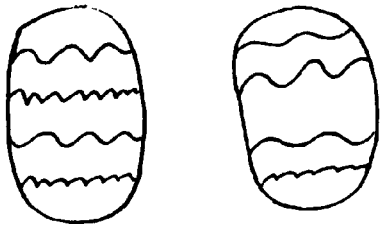


Find the number of tens and ones in each box. Write the numeral which tells how many.



Worksheet 22

Children read parts of the story. Discuss the story and fill in the blanks with appropriate numbers suggested by the children. Complete the number sentence and relate to the story.

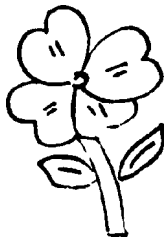


HERE ARE JOE'S EGGS. HOW MANY EGGS? _____

IF JOE WANTS 3, HE NEEDS _____ MORE. 3 SPLIT 2 IS _____.

IF JOE WANTS 5, HE NEEDS _____ MORE. 5 SPLIT 2 IS _____.

IF JOE WANTS 6, HE NEEDS _____ MORE. 6 SPLIT 2 IS _____.

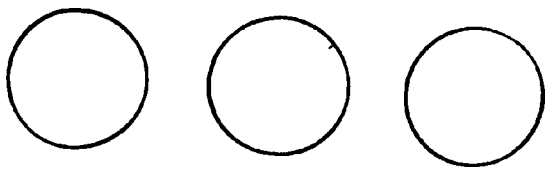


HERE IS MARY'S FLOWER. HOW MANY FLOWERS? _____

IF MARY WANTS 5, SHE NEEDS _____ MORE. 5 SPLIT 1 IS _____.

IF MARY WANTS 3, SHE NEEDS _____ MORE. 3 SPLIT 1 IS _____.

IF MARY WANTS 6, SHE NEEDS _____ MORE. 6 SPLIT 1 IS _____.



HERE ARE DICK'S MARBLES. HOW MANY MARBLES? _____

IF DICK WANTS 6, HE NEEDS _____ MORE. 6 SPLIT 3 IS _____.

IF DICK WANTS 4, HE NEEDS _____ MORE. 4 SPLIT 3 IS _____.

IF DICK WANTS 3, HE NEEDS _____ MORE. 3 SPLIT 3 IS _____.

Worksheet 23

The teacher draws a picture for each story on the chalkboard. The story is read by the children. Individuals match the sets and the subtraction sentence is suggested and written by the children.

1.

ANN HAS 5 DOLLS.

JANE HAS 3 DOLLS.

ANN HAS _____ DOLLS MORE THAN JANE.

5

2.

PETER HAS 6 PENNIES.

MARY HAS 5 PENNIES.

PETER HAS _____ PENNIES MORE THAN MARY.

3.

TOM HAS 5 APPLES.

JIM HAS 1 APPLE.

TOM HAS _____ APPLES MORE THAN JIM.

4.

JOE'S DOG ATE 7 BONES.

TED'S DOG ATE 4 BONES.

JOE'S DOG ATE _____ BONES MORE THAN TED'S DOG.

5.

EARL IS 6 YEARS OLD.

SALLY IS 4 YEARS OLD.

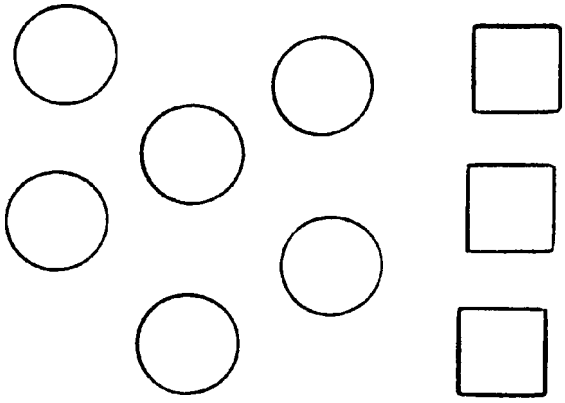
EARL IS _____ YEARS OLDER THAN SALLY.

6.

JUDY HAS 5 KITTENS.

PEGGY HAS 2 KITTENS.

JUDY HAS _____ KITTENS MORE THAN PEGGY.



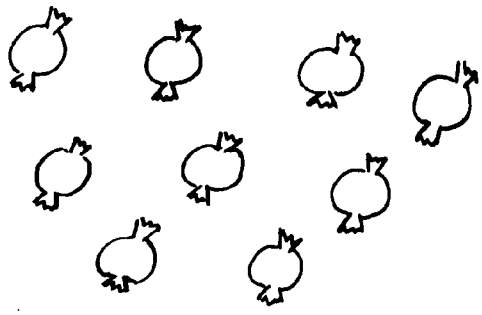
MATCH THE TWO SETS.

HOW MANY MORE ◯ 'S THAN □ 'S ARE THERE?

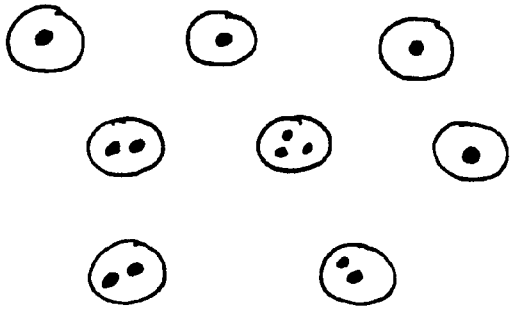
Worksheet 24

Ask individual children to read the stories and to suggest appropriate subtraction sentences. All children write the sentences in the spaces provided.

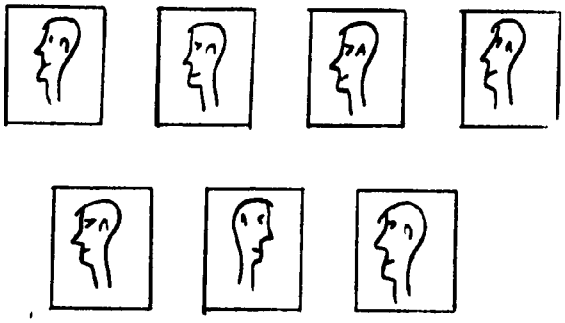
BILLY HAD 9 STICKS OF GUM.
HE GAVE FRANK 4 OF THEM.
HOW MANY DID HE HAVE LEFT?



BARBARA HAD 8 COOKIES.
SHE AND AMY ATE 4 OF THEM.
HOW MANY DID SHE HAVE THEN?



TIM HAD 7 STAMPS.
HE USED 1 ON A LETTER.
HOW MANY DID HE HAVE THEN?



CYNTHIA HAD 9 PENNIES.
SHE SPENT 7 ON CANDY.
HOW MANY DID SHE HAVE LEFT?

TOM HAD 7 JELLY BEANS.
HE ATE 6 OF THEM.
HOW MANY DID HE HAVE THEN?

APPENDIX D

CONCEPT ATTAINMENT PRE- AND POST- TEST SCORE, INTELLIGENCE SCORE, SUBTRACTION APPLICATION SCORE, SUBTRACTION TRANSFER SCORE, ADDITION COMPUTATION SCORE, PROBLEM SOLVING SCORE, SUBTRACTION COMPUTATION SCORE, STANFORD ACHIEVEMENT SCORE AND MEAN ARITHMETIC ATTITUDE SCORE FOR INDIVIDUAL STUDENTS

TABLE 37

CONCEPT ATTAINMENT PRE- AND POST- TEST SCORES FOR PUPILS IN SIX CLASSES PE, TE, PL, TL, LPE, LPL

Student Number	Concept Attainment Test					
	Pre-Test 1-5	Pre-Test 6-10	Pre-Test Total	Post Test 1-5	Post Test 6-10	Post Test Total
PE						
1	130	96	256	230	150	410
2	90	72	192	220	180	460
3	200	66	266	240	66	396
4	200	57	347	240	72	402
5	200	141	401	240	120	450
6	230	87	347	240	72	402
7	140	54	224	210	90	360
8	170	114	344	240	180	510
9	170	132	362	240	168	498
10*	230	72	362	240	159	459
11	180	87	357	210	96	396
12	180	90	300	200	174	464
13	220	168	448	240	162	462
14	180	102	312	200	72	362
15	160	153	373	230	168	488
16	100	72	172	200	168	458
17	150	141	381	200	165	455
18	130	96	256	210	162	462
19	180	108	318	210	180	480
20	140	132	302	210	87	387
21	220	147	397	240	174	474

* The * denotes the randomly selected pupils who were omitted from the two-way analysis of covariance for classes PE, TE, PL, and TL.

TABLE 37--Continued

Student Number	Concept Attainment Test					
	Pre-Test 1-5	Pre-Test 6-10	Pre-Test Total	Post Test 1-5	Post Test 6-10	Post Test Total
TE						
22	170	78	308	210	102	402
23	150	87	267	210	66	276
24	120	87	237	200	102	362
25	190	108	328	230	174	464
26	120	84	204	220	150	430
27	170	66	266	210	108	378
28	120	72	192	230	150	470
29	150	42	222	210	36	276
30	190	174	454	200	72	362
31	140	51	191	180	18	258
32	200	102	332	240	132	462
33	190	78	268	200	108	338
34	100	81	181	140	93	263
35	210	99	399	210	180	480
36	90	81	201	200	126	416
37	90	12	132	140	87	317
38*	180	108	318	200	66	296
39*	130	102	292	200	162	422
40	220	78	328	200	72	332
41	240	120	420	240	174	504
42	120	51	201	200	42	302
43	240	174	504	210	174	474
PL						
44	130	96	316	200	150	440
45	230	153	413	200	138	368
46	100	51	151	230	42	302
47	220	126	376	240	120	450
48	90	51	171	60	57	177
49	120	72	222	180	30	240
50	210	102	372	240	111	441
51	240	138	468	240	129	459
52*	160	96	286	160	132	322
53	220	132	382	230	120	440
54	220	72	352	190	162	442
55*	200	102	362	240	117	447
56*	200	120	380	200	36	296
57	240	141	441	240	96	426

* The * denotes the randomly selected pupils who were omitted from the two-way analysis of covariance for classes PE, TE, PL, and TL.

TABLE 37--Continued

Student Number	Concept Attainment Test					
	Pre-Test 1-5	Pre-Test 6-10	Pre-Test Total	Post Test 1-5	Post Test 6-10	Post Test Total
58	170	102	302	240	102	432
59	230	72	332	240	138	408
60	180	57	267	150	36	216
61	120	108	258	230	162	452
62	90	87	207	180	78	348
63*	150	78	288	220	108	388
64	90	36	126	130	33	193
65	170	132	332	200	102	392
66*	150	42	252	170	90	350
67*	190	87	307	190	138	358
68	230	156	446	230	126	416
69	90	36	156	120	72	222
TL						
70	120	90	210	240	153	483
71	160	96	286	210	66	336
72	180	72	252	190	72	292
73	150	30	210	230	105	365
74	150	102	282	170	72	302
75	170	72	272	240	123	453
76	90	36	186	240	102	402
77	150	72	222	190	84	334
78	160	30	220	230	96	416
79	180	126	396	180	174	444
80	150	87	327	200	99	389
81	170	72	272	240	90	420
82	140	78	278	240	72	402
83	210	153	423	210	117	417
84	150	30	210	230	96	386
85	190	72	292	220	111	421
86	150	30	180	210	81	351
87	120	27	177	180	48	258
88	60	51	111	230	102	362
89	230	105	425	240	180	510
LPE						
90	170	72	242	170	114	344
91	60	0	60	120	36	186
92	30	90	150	120	84	234

* The * denotes the randomly selected pupils who were omitted from the two-way analysis of covariance for classes PE, TE, PL, and TL.

TABLE 37--Continued

Student Number	Concept Attainment Test					
	Pre-Test 1-5	Pre-Test 6-10	Pre-Test Total	Post Test 1-5	Post Test 6-10	Post Test Total
93	30	42	132	90	42	192
94	150	96	276	200	153	383
95	120	90	240	200	78	308
96	60	42	102	200	42	272
97	60	30	120	150	81	261
98	90	48	138	150	78	318
99	180	36	276	230	96	356
100	120	60	210	180	42	282
101	230	141	401	210	81	381
102	90	6	96	150	42	222
103	80	27	137	200	102	392
104	90	81	201	190	108	328
105	60	90	180	180	72	282
106	120	36	186	190	126	346
107	180	66	276	190	87	337
108	160	6	196	160	96	316
109	120	42	192	240	54	324
LPL						
110	90	90	210	130	87	277
111	120	12	132	150	90	270
112	120	51	201	160	87	277
113	190	42	292	240	117	447
114	150	111	291	170	117	317
115	150	42	192	170	72	332
116	200	102	302	170	96	296
117	180	153	363	210	114	414
118	120	30	150	240	90	420
119	160	102	262	170	54	314
120	210	126	366	220	153	433
121	120	132	282	210	78	348
122	150	90	270	170	153	353
123	60	111	261	90	66	186
124	120	93	213	160	36	226
125	120	12	132	180	117	327
126	120	72	192	160	75	265
127	120	90	240	150	42	192
128	180	45	285	120	72	252
129	150	33	213	230	96	356
130	120	78	198	230	66	356
131	90	21	141	190	156	436
132	90	60	150	90	108	228
133	170	36	206	210	90	360
134	170	72	302	210	138	378

TABLE 38

INTELLIGENCE SCORE, SUBTRACTION APPLICATION SCORE, SUBTRACTION TRANSFER SCORE, ADDITION COMPUTATION SCORE, PROBLEM SOLVING SCORE, SUBTRACTION COMPUTATION SCORE, STANFORD ACHIEVEMENT SCORE, AND MEAN ARITHMETIC ATTITUDE SCORE FOR EACH PUPIL IN CLASSES PE, TE, PL, TL, LPE, LPL

Student Number	Intelligence Score	Subtraction Application Score	Subtraction Transfer Score	Addition Computation Score
PE				
1	124	4	2	4
2	97	5	2	4
3	103	4	4	8
4	113	4	1	9
5	112	4	3	5
6	132	4	7	7
7	108	3	2	6
8	93	5	6	10
9	130	5	7	10
10*	111	3	7	7
11	132	5	5	7
12	127	6	6	8
13	119	4	7	9
14	123	5	6	3
15	107	5	10	7
16	122	4	6	3
17	118	6	7	5
18	115	5	2	7
19	119	5	3	3
20	125	4	1	7
21	120	5	6	7
TE				
22	115	6	8	5
23	127	5	3	5
24	118	1	1	4
25	100	4	7	7
26	139	3	6	3
27	136	6	7	6
28	112	5	6	6
29	95	0	0	1
30	122	6	8	3
31	119	3	2	5
32	119	5	6	3
33	110	2	5	5

* The * denotes the randomly selected pupils who were omitted from the two-way analysis of covariance for classes PE, TE, PL, and TL.

TABLE 38--Continued

Student Number	Problem Solving Score	Subtraction Computation Score	Stanford Achievement Score	Mean Arithmetic Attitude Score
PE				
1	13	3	24	3.000
2	5	3	12	2.333
3	14	8	37	3.000
4	12	3	27	1.000
5	15	6	34	2.333
6	14	10	37	2.000
7	14	5	32	2.333
8	16	8	41	2.667
9	11	9	38	2.667
10*	14	8	34	2.000
11	15	8	39	2.333
12	12	5	28	2.667
13	13	10	37	2.333
14	10	1	16	2.000
15	13	8	33	2.000
16	9	9	23	3.000
17	12	9	29	3.000
18	12	4	27	2.333
19	11	6	24	1.667
20	15	8	37	2.667
21	16	2	34	2.000
TE				
22	15	9	35	3.000
23	6	1	16	2.000
24	8	0	14	1.667
25	8	10	28	3.000
26	7	8	22	2.333
27	17	9	42	2.333
28	12	9	33	2.000
29	6	2	11	2.333
30	15	8	32	2.000
31	10	4	23	3.000
32	12	8	33	1.667
33	10	10	27	3.000

* The * denotes the randomly selected pupils who were omitted from the two-way analysis of covariance for classes PE, TE, PL, and TL.

TABLE 38--Continued

Student Number	Intelligence Score	Subtraction Application Score	Subtraction Transfer Score	Addition Computation Score
34	93	3	3	6
35	134	6	5	8
36	104	3	0	3
37	102	3	2	1
38*	100	2	1	2
39*	141	4	7	3
40	130	4	6	6
41	146	6	8	10
42	114	0	1	1
43	127	6	9	7
PL				
44	119	6	7	6
45	90	3	0	6
46	96	6	4	2
47	106	5	8	7
48	82	1	0	1
49	95	2	4	3
50	107	4	5	7
51	107	6	6	8
52*	95	3	1	3
53	103	5	6	7
54	89	6	7	3
55*	91	3	2	1
56*	101	1	0	2
57	103	2	1	9
58	114	6	8	6
59	93	1	0	1
60	86	0	1	0
61	92	2	0	3
62	87	1	2	0
63*	101	0	0	0
64	102	2	0	4
65	86	4	1	2
66*	87	5	2	3
67*	127	3	0	1
68	103	5	7	8
69	76	1	0	0
TL				
70	117	2	1	4
71	82	3	1	3

* The * denotes the randomly selected pupils who were omitted from the two-way analysis of covariance for classes PE, TE, PL, and TL.

TABLE 38--Continued

Student Number	Problem Solving Score	Subtraction Computation Score	Stanford Achievement Score	Mean Arithmetic Attitude Score
34	6	4	20	3.000
35	17	8	40	2.000
36	13	4	21	3.000
37	6	0	8	2.667
38*	4	8	14	1.667
39*	9	6	21	2.333
40	8	7	27	3.000
41	18	10	49	2.667
42	4	1	9	1.667
43	15	10	42	2.000
PL				
44	8	7	26	2.333
45	5	0	14	3.000
46	4	0	9	2.667
47	17	10	42	2.333
48	3	1	6	2.333
49	4	0	10	1.000
50	9	8	32	2.000
51	14	7	36	2.000
52*	6	1	13	1.333
53	13	9	32	2.667
54	10	6	24	3.000
55*	10	3	21	2.667
56*	9	0	15	3.000
57	10	8	32	3.000
58	10	8	31	1.667
59	5	2	12	2.000
60	3	2	8	2.333
61	7	0	14	2.000
62	6	1	9	3.000
63*	7	2	11	1.667
64	6	2	12	2.667
65	6	1	12	2.667
66*	10	1	16	3.000
67*	6	2	15	1.333
68	13	8	36	2.667
69	4	2	7	2.667
TL				
70	4	4	17	2.667
71	10	7	27	2.333

* The * denotes the randomly selected pupils who were omitted from the two-way analysis of covariance for classes PE, TE, PL, and TL.

TABLE 38-- Continued

Student Number	Intelligence Score	Subtraction Application Score	Subtraction Transfer Score	Addition Computation Score
72	94	3	1	1
73	97	3	6	1
74	105	4	4	3
75	96	6	4	5
76	100	4	7	6
77	94	4	5	2
78	102	4	7	7
79	127	6	7	10
80	93	3	1	6
81	124	5	6	10
82	115	4	4	8
83	93	6	5	7
84	99	3	4	4
85	110	2	2	2
86	88	5	2	2
87	93	1	1	0
88	86	0	2	1
89	107	5	8	7
LPE				
90	107	3	0	4
91	80	1	1	0
92	87	2	0	2
93	82	2	0	0
94	105	5	6	2
95	100	1	0	1
96	103	3	0	1
97	96	2	2	0
98	86	4	1	2
99	104	3	1	1
100	96	0	1	1
101	99	4	7	5
102	73	0	0	1
103	102	4	6	8
104	91	0	0	0
105	108	1	0	0
106	114	2	0	1
107	92	5	3	2
108	100	3	1	4
109	94	2	0	2
LPL				
110	73	0	0	0
111	98	2	0	4

TABLE 38--Continued

Student Number	Problem Solving Score	Subtraction Computation Score	Stanford Achievement Score	Mean Arithmetic Attitude Score
72	8	4	15	2.000
73	3	1	6	3.000
74	6	6	19	2.333
75	15	9	39	2.333
76	13	8	33	2.667
77	5	4	14	3.000
78	11	9	34	2.000
79	17	10	48	1.667
80	8	9	27	2.667
81	17	9	44	3.000
82	15	7	38	2.667
83	16	8	40	1.667
84	7	3	17	2.667
85	12	0	16	2.667
86	6	0	12	2.333
87	3	1	7	2.333
88	7	2	12	2.000
89	11	10	33	3.000
LPE				
90	9	1	19	3.000
91	3	1	6	2.000
92	7	0	10	2.667
93	0	1	4	2.667
94	11	1	18	2.000
95	5	0	10	3.000
96	10	1	16	3.000
97	4	0	6	3.000
98	5	1	9	2.000
99	6	2	11	2.333
100	4	1	9	2.000
101	12	1	23	2.333
102	4	1	7	1.000
103	14	9	38	2.667
104	6	1	8	3.000
105	7	1	12	2.000
106	7	0	13	3.000
107	13	1	18	3.000
108	6	1	13	1.667
109	5	2	15	2.333
LPL				
110	7	0	9	1.000
111	6	1	12	2.000

TABLE 38--Continued

Student Number	Intelligence Score	Subtraction Application Score	Subtraction Transfer Score	Addition Computation Score
112	65	0	0	0
113	119	6	9	1
114	89	1	0	1
115	88	3	2	4
116	93	4	6	7
117	109	6	6	10
118	75	3	4	2
119	96	3	3	2
120	92	3	6	7
121	86	3	4	2
122	86	2	0	0
123	83	3	1	1
124	81	1	0	1
125	71	2	1	0
126	90	1	0	0
127	82	2	2	0
128	74	1	0	1
129	99	4	3	3
130	88	2	0	6
131	91	3	0	2
132	73	0	0	0
133	114	4	3	4
134	100	6	5	1

TABLE 38--Continued

Student Number	Problem Solving Score	Subtraction Computation Score	Stanford Achievement Score	Mean Arithmetic Attitude Score
112	3	2	8	3.000
113	16	0	19	2.333
114	4	1	8	1.667
115	7	1	17	2.667
116	6	0	19	2.333
117	15	6	42	2.333
118	9	0	16	1.667
119	5	1	12	2.333
120	8	1	20	2.667
121	7	4	17	3.000
122	5	0	8	2.333
123	5	3	11	3.000
124	2	2	8	2.333
125	3	0	4	2.667
126	5	2	9	2.667
127	0	0	0	1.333
128	6	3	13	1.333
129	8	1	15	1.667
130	6	1	18	2.333
131	6	0	9	2.000
132	3	0	3	2.667
133	13	0	20	3.000
134	10	1	16	2.000

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