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PAULSON, James Marvin. THE ANALYSIS OF MULTIPLE AND CONTINUOUS FOLDED PLATE STRUCTURES.

University of Michigan, Ph. D., 1958
Engineering, civil

Negative microfilm received from University.

THE ANALYSIS OF MULTIPLE AND CONTINUOUS FOLDED PLATE STRUCTURES

by

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A dissertation submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy in the
University of Michigan

1958

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This paper presents the theoretical analysis and the results of an experimental investigation of multiple folded plates, continuous folded plates and continuous multiple folded plate structures. Folded plates can be used economically to span large areas with minimum interior obstruction. In such cases interior frames or additional interior plates may be provided to limit displacements and to provide a reasonable distribution of stress in the plate system. The solutions presented are concerned with folded plate structures made indeterminate through the use of multiple folded plate construction and through the use of interior frames not rigid in their plane.

The analysis of a two bay multiple folded plate is undertaken treating the additional interior plate as a redundant member. The interaction of the simple folded plate and the redundant plate is assumed to be provided through a distributed edge pressure and a distributed edge shear force acting at the joint common to the simple folded plate and the redundant plate. The simple folded plate is analyzed for the effect of the applied load and unit values of the redundant forces; the redundant member is analyzed for the effect of unit values of the redundants. Equations are written expressing the consistency of the displacements and of the edge stresses of the simple folded plate and the redundant plate at their common edge. The solution of these equations yields the intensity of the redundant edge forces.

Symmetrical and unsymmetrical loadings are considered in this analysis.

A single bay folded plate continuous over a frame not rigid in its plane is analyzed treating the interior frame as a redundant member. The interaction of the folded plate and the frame is assumed to be provided through a system of resultant forces acting in the middle plane of the plates and on the cross sections adjacent to the interior frame. After analyzing the simple folded plate for the effect of the applied load and unit values of the redundants, the frame is analyzed for displacements induced by unit values of the redundants. Equations written to express the consistency of the displacements of the folded plate and the frame are solved for the redundants.

The third part of the analytical investigation consists of the analysis of a folded plate structure combining the problems of both the multiple and the continuous systems. In this case the various force systems or the plate moments produced by them are represented by trigonometric series. The final solution is obtained considering the first six terms of the series.

The results of an experimental investigation, conducted by the writer on small scale aluminum models, are presented. Satisfactory agreement between the theoretical and experimental results is demonstrated for each of the three structural systems investigated.

The agreement between the analytical and the experimental results indicates that the accepted methods of analysis for simple folded plate structures can be extended through the use of the principles of superposition and consistency of displacement and stress to the solution of more complex problems in folded plate analysis.

ACKNOWLEDGMENT

The writer wishes to express his appreciation to Professor L. C. Maugh for the patient and understanding counsel given during the writing of this dissertation.

The writer is indebted to Professors B. G. Johnston, L. M. Legatski, P. M. Naghdi, and E. H. Rothe for giving generously of their time in serving on the writer's Doctoral Committee and to Associate Professor R. B. Harris who kindly agreed to serve in Professor Johnston's absence.

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CHAPTER I

INTRODUCTION

1. Synopsis

The folded plate is one of several structural systems developed during the past three decades as the result of the increase in interest in reinforced concrete construction and the growing need to enclose maximum space with minimum structural material. The increase in use of this type of construction in the industrial field has led to problems in long span and continuous construction. Recent recognition of the versatility of the folded plate by architects has led to problems concerned with the analysis of a wide variety of plate arrangements and framing conditions.

This thesis presents the results of analytical and experimental investigations of a multiple folded plate, a continuous folded plate and a multiple continuous folded plate. The fundamental principles and procedures of analysis developed originally for the analysis of simple folded plates are presented in Chapter II. These fundamentals are utilized in the remaining chapters to aid in the analysis of more complex structures.

Certain applications of folded plate construction require the use of multiple folded plates or folded plates which are continuous over interior frames not rigid in their plane or combinations of these systems. Chapters III, IV, and V consider these problems in turn,

presenting methods for analysis and experimental results obtained by the writer from tests made on small scale models.

2. Historical Background

According to Born[1*], the first development of a method for the analysis of structural elements acting as a folded plate system was made by Schwizer[2] in 1920. An independent development of a rudimentary method of analysis was made by Ehlers[3], whose first analysis considered the plates joined at their edges by hinges capable of resisting longitudinal sliding motion. However, in 1930, both Ehlers[4] and Craemer[5] published methods of analysis comparable to the fundamental solution presented in Chapter II.

In 1932 Gruber[6] presented a method that considered the distortion of the cross section as well as the flexural continuity at the edges. The solution is obtained through a system of simultaneous differential equations of fourth order written in terms of the transverse moments where these moments and the remaining functions are expressed as trigonometric series. Grüning[7] also published a solution in 1932 which was similar to Gruber's, except that the functions were represented by geometric series.

Following the work presented by Gruber[6], a great deal of interest developed in this field, particularly in Germany and Switzerland, so that a number of specialized problems were solved during the period from 1935 to 1952. The problems of stiffening diaphragms, edge beams in the form of frames, and multiple edge beams were treated by Gruber[8] in 1935. The solutions represent applications of his

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*All numbers refer to the bibliography.

basic solution presented in 1932[6].

Perhaps the most important recent advances in the development of practical methods for the analysis of simple folded plate systems were provided by Winter and Pei[9] and Gaafar[10]. Winter and Pei[9] developed a distribution method for the analysis of simple folded plate systems, while Gaafar[10] developed a means of including the effect of cross section distortion in this solution. The two contributions are reviewed in Chapter II, since they are drawn on in the remaining chapters.

In 1950 Huang[11] developed a method for designing diaphragms for folded plate systems, and in 1951 Ali[12] presented a method for analyzing folded plates continuous across rigid interior supports. Both of these dissertations utilized the methods of Winter and Pei[9] and Gaafar[10]. Gruber[13] published a solution for folded plate systems continuous over rigid supports in 1952. In this he made brief reference to the effect of introducing a flexible support.

Another contribution to the analysis of the simple folded plate was made by Craemer[14] in 1953 in which he provided a solution to the problem undertaken by Gruber[6] in 1932. The solution is provided through a system of simultaneous differential equations of second order. Craemer also developed a parameter which may be used as a guide in anticipating the nature of the structural response of the simple folded plate.

In 1954 Ashdown[15] presented a solution for the simple folded plate developed through the use of a system of simultaneous algebraic equations, which include the effect of relative edge displacement. The solution provided is comparable to that given by the distribution

method of Winter and Pei[9] and Gaafar[10].

In 1954 a major contribution to the literature on folded plate analysis appeared in the form of a book by Born[1]. The fundamental and the more rigorious theories were reviewed as well as many specialized topics. Among the special problems included were the statically indeterminate problems of multiple and continuous construction. Multiple folded plates were analyzed by introducing the "four shear equation" and by considering the edge force at the multiple joint as a redundant. Reference was made to Gruber's[8] solution for folded plates continuous over interior supports. A case was considered in which the folded plate was continuous across isolated interior columns, the solution being provided by setting the displacement of the folded plate equal to zero at the point at which the folded plate joins the column.

Many other publications have appeared in this field; the ones referred to above have a distinct relation to the matter discussed in this dissertation.

CHAPTER II

FUNDAMENTAL RELATIONSHIPS AND PROCEDURES

1. Notations and Symbols

The following notations and symbols will be used throughout the following chapters.

x = coordinate measured from the end frame parallel to the longitudinal axis of the roof.

$1,2,3---m$ = subscripts used to denote plates or frame elements.

$a,b,c---j$ = subscripts used to denote longitudinal plate edges or frame joints. Plate 1 has edges a and b.

L = span of the structure measured in the direction parallel to the longitudinal axes of the plates.

h_1 = dimension of plate 1 measured in its middle plane perpendicular to the longitudinal axis of the plate.

t_1 = thickness of plate 1.

A_1 = cross sectional area of plate 1 = $h_1 \times t_1$.

S_1 = section modulus of plate 1 = $t_1 h_1^2 \div 6$.

I_T = moment of inertia of a strip of unit width = $1 \times t_1^3 \div 12$.

δ_1 = component of displacement of plate 1 parallel to the middle plane of plate 1 and perpendicular to the longitudinal axis of plate 1.

δ_{12} = displacement δ_1 caused by the application of a unit force to plate 2 in the plane of plate 2.

δ_{1a} = component of displacement of edge a in the direction of δ_1.

δ_{1b} = component of displacement of edge b in the direction of δ_1. $\delta_{1a} = \delta_{1b} = \delta_1$.

Δ_{1a} = component of displacement of edge a perpendicular to the plane of plate 1.

Δ_{1b} = component of displacement of edge b perpendicular to plate 1.

Δ_1 = total relative displacement of edges a and b perpendicular to the plane of plate 1.

$\Delta_{1a} + \Delta_{1b} = \Delta_1$.

W = total load acting on the structure.

V_{1a} = shearing force per unit length acting on edge a of plate 1 in the direction perpendicular to the plane of plate 1.

Q_{1a} = equilibrant of V_{1a}.

P_{1a} = concentrated edge force acting normal to edge a in the plane of plate 1.

p_{1b} = distributed edge force applied to plate 1 at edge b acting in the plane of plate 1.

m_{1a} = bending moment per unit length in plate 1 at edge a acting in planes perpendicular to the longitudinal axis of plate 1, hereafter called slab bending moment.

M_1 = bending moment in plate 1 acting in the middle plane of the plate and on the cross section of the plate, hereafter called plate bending moment.

T_a = shearing force per unit length at joint a acting on planes perpendicular to the middle plane of the plates framing into joint a and acting in a direction parallel to the longitudinal axes of the plates.

N_a = total edge shear acting on edge a of a plate at a section x distance from the end frame = $\int_o^x T_a dx$.

F_1 = resultant applied to plate 1 acting in the middle plane of the plate on its cross section and directed perpendicular to the longitudinal axis of the plate.

f_{1a} = normal stress in plate 1 at edge a and acting parallel to the x axis. Hereafter f will be referred to as edge stress.

Other symbols and notations used in the following chapters are defined where they first appear.

2. Assumptions

The recently devised methods of analysis for folded plates are based on certain assumptions concerning the nature of the materials used, the geometry of the structure, and the force systems acting in the structures' components. Most of the assumptions have been shown to be reasonable as the result of recently performed investigations.

1. The material is homogenious, isotropic and elastic.

2. Plane sections in individual plates before bending remain plane after bending.
3. The structure is made up of prismatic members of depth h and thickness t. The depth of each plate, h, is sufficiently small compared to the span, L, so that slab bending moments in planes perpendicular to the middle plane of the plate and parallel to the longitudinal axis of the plate can be considered small. The chickness, t, is small as compared with the depth h.
4. Loads applied to the surface of a plate are distributed uniformly over the entire plate.
5. Assumptions 3 and 4 make it reasonable to assume that slab moments, m, can be computed by analyzing a unit transverse strip of the plate system and that torsional moments in the plates can be considered small and shear deformation neglected.
6. The plates are joined at their edges to form a monolithic plate system.
7. The end frames are assumed to be rigid in their plane and are assumed to be entirely flexible perpendicular to their plane.

The dimension ratios referred to in Assumption 3 have not been defined experimentally or analytically. Recommendations for the minimum value of $\frac{L}{h}$ have varied from two to four. It is presumed that the ratio $\frac{h}{t}$ will be approximately thirty; practical conditions will usually insure that it not exceed this value, while requirements of economy will usually require that it will not be much less.

3. Development of Basic Relations

The fundamental theory for the analysis of a folded plate is based on the additional assumption that all edges displace the same amount. With this assumption and the preceding ones it is deemed reasonable to consider the action of the structure broken down into two phases for analysis purposes. The transverse action and the longitudinal action are considered separately but related through the edge force, p.

The first part of the procedure is concerned with the transverse action of the plate system involving the determination of the forces m, V, and p. After resolving the applied loads into components parallel to and perpendicular to the plate surface to which they are applied, a unit transverse strip is cut from the plate system and analyzed for moments, m, induced by the components of the loads acting perpendicular to the plates. This is accomplished through the use of a method such as moment distribution, considering the strip as a beam continuous over a series of non-yielding, knife edge supports. These supports exist at the edges at which two plates join. If the angle of inclination of a plate relative to its adjacent plate is not small, it is reasonable to assume that each plate framing into the joint will provide a reaction component in its plane at the joint. This is possible in view of the large stiffness of the plate in its plane as

compared to its stiffness in planes perpendicular to its plane. The shear forces, V, developed at the joints are computed from the equilibrium conditions for each member in the strip. The equilibrants, Q, of the shear forces are resolved into components in the plane of the plates framing into the joint as shown in Fig. 2.1. These components are the edge forces, p, defined in Section 1. In this way the applied loads are resolved into force systems producing transverse bending in the plates and subsequently into forces which are assumed to act in the middle plane of the plates.

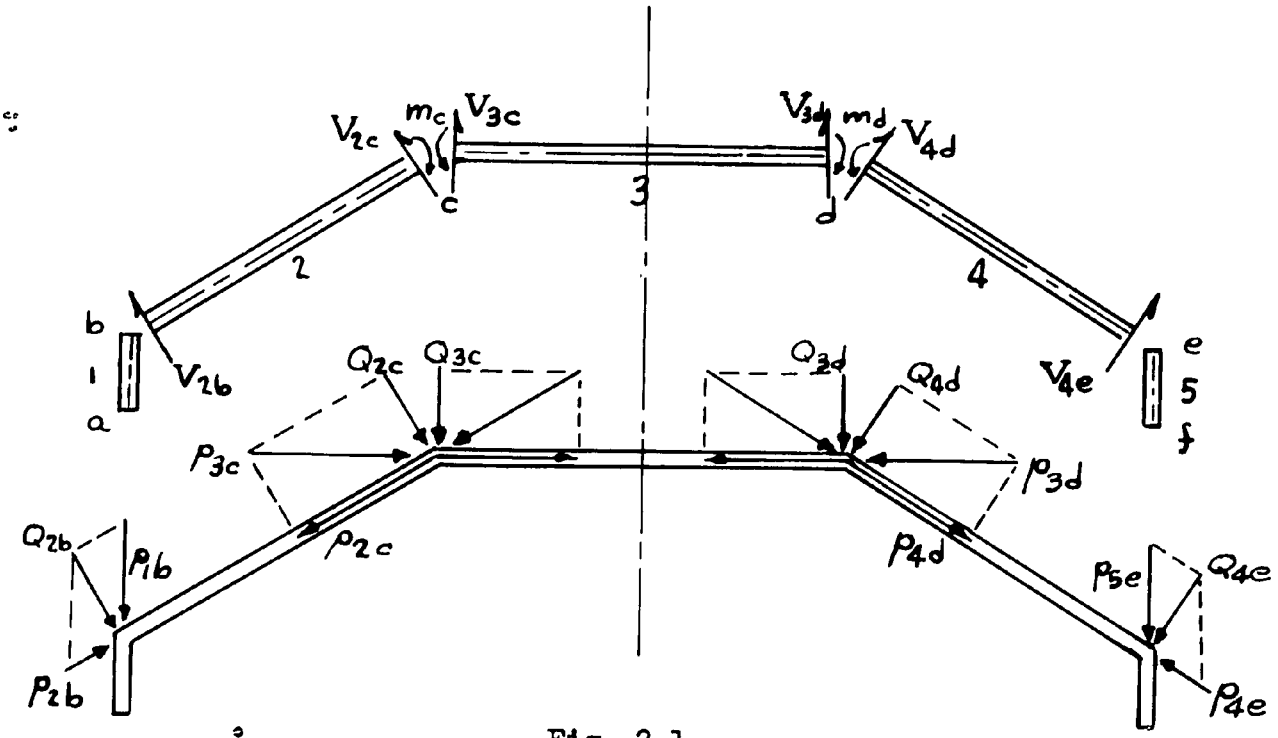


Fig. 2.1

After summing the load and edge force components acting in the plane of each plate, the edge stresses are computed using the ordinary flexure formula. The edge stresses computed in this manner will be referred to as free edge stresses. The computations for free edge stresses are based on the assumption that the plates act in an isolated manner. Since this produces edge stresses in obvious disagreement with the actual monolithic response of the structure, it is necessary to apply a system of forces to the plates to bring about the agreement between edge stresses, f_j, of plates common to edge j. Edge shear forces, T_j, are introduced to provide this continuity of edge stress. For a plate system such as that shown in Fig. 2.2, shear forces, T, would be applied at all edges, j, at which two plates join. Plates m+1 and m-2 exist but are not shown.

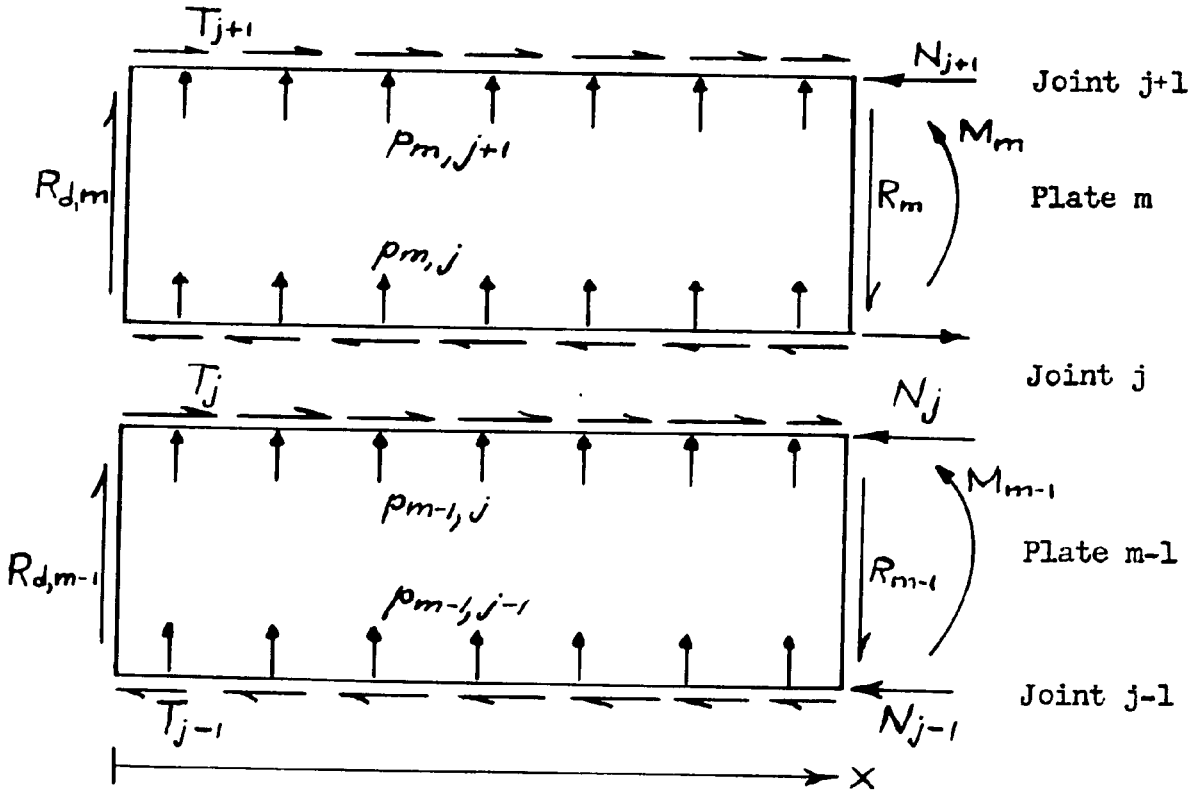


Fig. 2.2

Since N_j is the total edge shear acting on edge j at the section shown, the total edge stress at edge j of plate m can be written in terms of N_j, N_{j+1}, and M_m. Assuming that the stresses on the plates' cross section caused by N_j and N_{j+1} can be computed as a direct stress of constant magnitude and a flexural stress computed using the ordinary flexure formula, the following expressions can be written for the stress at edges j and j+1 of plate m. The assumed positive directions for N and M are shown in Fig. 2.2; tension stress is positive.

$$f_{m,j} = \frac{M_m}{S_m} + \frac{2}{A_m}(2N_j + N_{j+1}) \quad \ldots\ldots\ldots\ldots\ldots \quad 2.1$$

$$f_{m,j+1} = -\frac{M_m}{S_m} - \frac{2}{A_m}(2N_{j+1} + N_j) \quad \ldots\ldots\ldots\ldots\ldots \quad 2.2$$

Two such equations can be written for each plate, providing 2m equations for edge stress. The condition that the edge stress at a joint common to two plates be a single value can be met by equating the expressions for stress for the plates joining at this edge. This provides j-2 equations to solve for the j-2 unknown values of N. The expression obtained by equating the edge stresses at edge j for plates m-1 and m appears as follows.

$$-\frac{M_{m-1}}{S_{m-1}} - \frac{2}{A_{m-1}}(2N_j + N_{j-1}) = \frac{M_m}{S_m} + \frac{2}{A_m}(2N_j + N_{j+1}) \quad \ldots \quad 2.3$$

The values of N are the only unknowns in the equations.

The transverse as well as the longitudinal action having been considered, one finds that all of the forces assumed to be acting on the plate cross section or on the unit strip have been defined. It then remains to evaluate the magnitude of these forces on other

sections and on other strips. Since it was assumed that the load is uniformly distributed and since the edges are assumed not to displace relative to each other at any section of the structure, it is apparent that the analysis made for the strip is typical of all other strips in the structure. The moments, m, the shears, V, and the forces, p, are thus known throughout the structure. The shear force, R, and the moment, M, are functions of p and x only so that these forces are known at all sections. If the expressions

$$M_m = p_m f(x)$$

were substituted into equations 2.3 and the equations were then solved for N_j the following functions would be obtained.

$$N_j = R_m f(x)$$

Also, since

$$M_m = \int_o^x R_m \, dx$$

and

$$N_j = \int_o^x T_j \, dx,$$

it is known that T_j varies in the same way as F_m. Thus the fundamental solution for the forces at any section can be made once equations 2.3 have been solved.

As suggested by Winter and Pei[9], equation 2.3 can be differentiated yielding equation 2.4. The resulting j-2 equations can be solved simultaneously for the unknowns T_j. This may be done rather than solving equations 2.3 for N_j.

$$-\frac{R_{m-1}}{S_{m-1}} - \frac{2}{A_{m-1}} (2T_j + T_{j-1}) = \frac{R_m}{S_m} + \frac{2}{A_m} (2T_j + T_{j+1}) \quad \ldots \quad 2.4$$

4. Distribution Methods

Winter and Pei[9] presented a distribution method for determining N based on the similarity between equation 2.3 and the three moment equation. Equation 2.3 can be rewritten in the form shown in equation 2.5.

$$\frac{N_{j-1}\,h_{m-1}}{S_{m-1}} + 2N_j\left[\frac{h_{m-1}}{S_{m-1}} + \frac{h_m}{S_m}\right] + \frac{N_{j+1}\,h_m}{S_m} = -3\left[\frac{M_{m-1}}{S_{m-1}} + \frac{M_m}{S_m}\right] \quad \ldots \quad 2.5$$

The three moment equation for two adjacent spans in a beam system continuous across non-yielding, knife edge supports and loaded symmetrically on each span is as follows.

$$M_{j-1}\frac{L_{m-1}}{I_{m-1}} + 2M_j\left[\frac{L_{m-1}}{I_{m-1}} \quad \frac{L_m}{I_m}\right] + M_{j+1}\frac{L_m}{I_m} = -3\left[\frac{(-M^F_{m-1})\,L_{m-1}}{I_{m-1}} + \frac{(-M^F_m)\,L}{I_m}\right]$$

The values $-M^F_{m-1}$ and $-M^F_m$ are fixed end moments at support j for spans m-1 and m respectively. The form of the equations is the same, so that the analogous "fixed end" quantities become $-\frac{M_{m-1}}{h_{m-1}}$ and $-\frac{M_m}{h_m}$ and the analogous "stiffness factors" become $\frac{S_{m-1}}{h_{m-1}}$ and $\frac{S_m}{h_m}$.

Noting that

$$\frac{S_{m-1}}{h_{m-1}} = \frac{A_{m-1}}{6}$$

and

$$\frac{S_m}{h_m} = \frac{A_m}{6}$$

the distribution factors at joint j are

$$r_{m-1,j} = \frac{A_{m-1}}{A_{m-1}+A_m}$$

$$r_{m,j} = \frac{A_m}{A_{m-1}+A_m}$$

The carry over factor is a positive 0.50.

Similarity also exists between equations 2.1 and 2.2 and the slope deflection equations for a prismatic member derived using the sign convention shown in Fig. 2.3. This implies that a method for balancing free edge stresses is feasible.

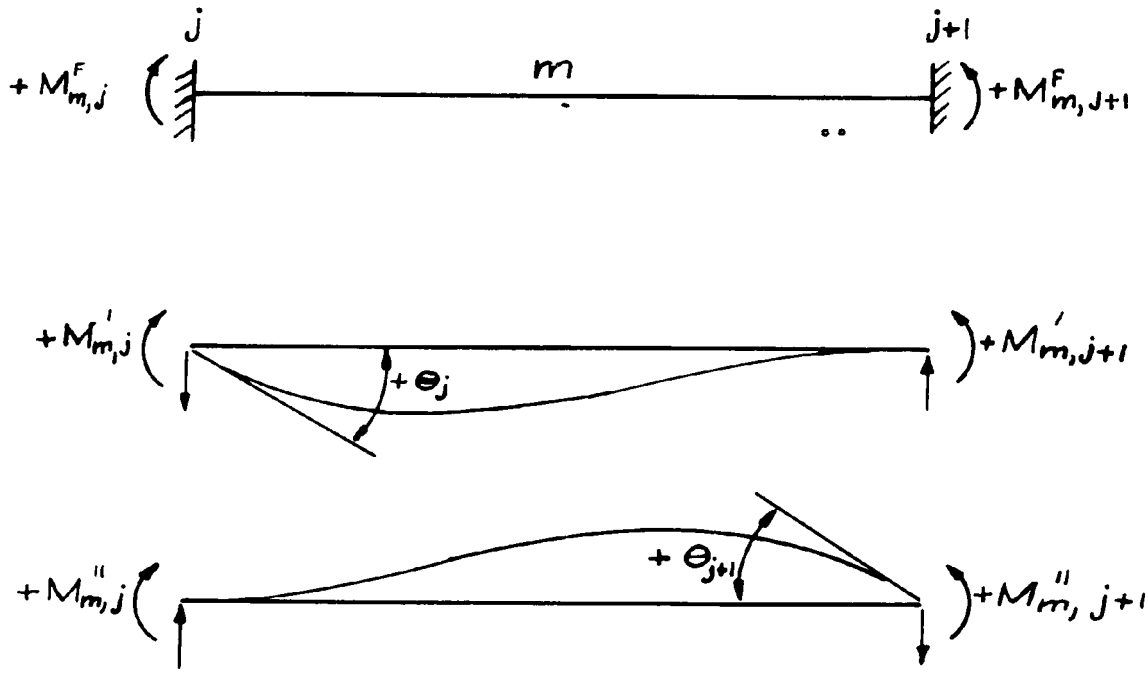


Fig. 2.3

The following expressions can be written from Fig. 2.3.

$$M_j = M^F_{m,j} + M'_{m,j} + M''_{m,j}$$

$$M_{j+1} = M^F_{m,j+1} + M'_{m,j+1} + M''_{m,j+1}$$

$$M'_{m,j} = 4k_m\theta_j$$

$$M'_{m,j+1} = -2k_m\theta_j$$

$$M''_{m,j+1} = -4k_m\theta_{j+1}$$

$$M'''_{m,j} = +2k_m\theta_{j+1}$$

$$M_j = M^F_{m,j} + 4k_m\,\theta_j + 2k_m\,\theta_{j+1}$$

$$M_{j+1} = M^F_{m,j+1} - 2k_m\,\theta - 4k_m\,\theta_{j+1}$$

The similarity between equations 2.1 and 2.2 and the preceding expressions requires that the "fixed end" quantities be $\frac{M_m}{S_m}$ at edge j and $-\frac{M_m}{S_m}$ at edge j+1. The "stiffness factors" are

$$C_1 k = \frac{4}{A_m} \text{ at edge } j$$

and

$$C_3 k = \frac{4}{A_m} \text{ at edge } j+1$$

Completing the derivation for the moment distribution properties of the beams, m-1 and m, one finds the correction moments applied to joint j are defined as

$$M'_{m-1,j} = -\frac{k_{m-1}}{k_{m-1} + k_m} (M^F_{m-1,j} - M^F_{m,j})$$

$$M'_{m,j} = \frac{k_m}{k_{m-1} + k_m} (M^F_{m-1,j} - M^F_{m,j})$$

Thus the distribution factors for joint j are

$$r_{m-1,j} = -\frac{\frac{4}{A_{m-1}}}{\frac{4}{A_{m-1}} + \frac{4}{A_m}} = -\frac{A_m}{A_{m-1} + A_m}$$

$$r_{m,j} = \frac{A_{m-1}}{A_{m-1} + A_m}.$$

The carry over factors are given by the basic equations

$$f_{j-1} = -\frac{N_j}{A_{m-1}} + \frac{\frac{h}{2} N_j}{S_{m-1}} = +2 \frac{N_j}{A_{m-1}}$$

$$f_j = -\frac{N_j}{A_{m-1}} - \frac{\frac{h}{2} N_j}{S_{m-1}} = -\frac{4N_j}{A_{m-1}}$$

Thus the carry over factors are -0.50.

The stress distribution method will be used as a basic tool for analysis in the following chapters.

5. Relative Edge Displacement

The development of the basic relationships presented in Sections 3 and 4 is based upon the assumption that the cross section of the folded plate is displaced without being distorted by relative edge displacement. This assumption makes it possible to carry out the analysis of the transverse action in which the edge loads, p, are defined as functions of the applied load, w. The introduction of a displacement, Δ, between two adjacent supports of the unit strip, as shown in Fig. 2.4, produces additional moments, m, and shears, V, at the supports of the strip. The equilibrants, Q, of the shear forces are resolved into components in the plane of the plates common to the joint at which the shears are computed. The relative edge displacement thus produces a change in the previously computed values of m, V, and p.

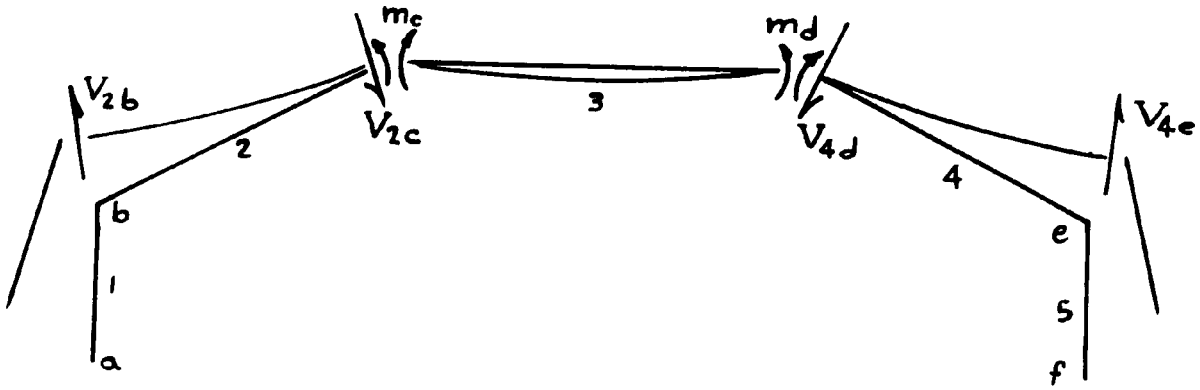


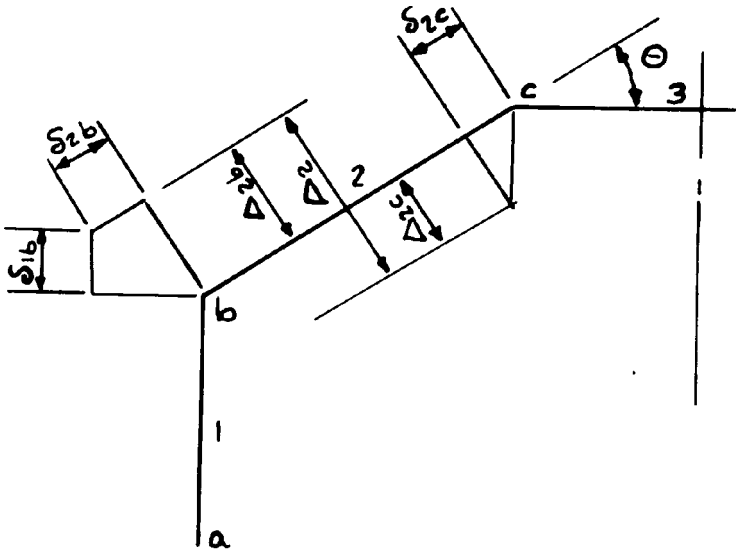
Fig. 2.4**

Gaafar[10] developed a method for considering the effect of relative edge displacement to be used in conjunction with the basic procedure discussed in Section 3. The revised procedure is as follows.

STEP 1. To the edge loads, p, dependent upon the applied loads and computed using the fundamental procedure discussed in Section 3, are added edge loads dependent upon relative edge displacements, Δ. Adjacent edges in the unit strip are displaced relative to each other; as many Δ displacements are imposed in turn as there are plates in which slab moments were considered in the elementary analysis. Fixed end moments are computed for the member of the displaced strip whose relative edge displacement is being computed. After balancing the fixed end moments using the moment distribution procedure, the shear at the supports is computed. The equilibrants of these shear forces are then resolved to provide a set of p forces that are functions of the imposed displacement, Δ. This process is repeated for each of the possible Δ values.

STEP 2. The free edge stresses, produced in each plate by the edge loads just obtained, are computed as in the fundamental procedure. In this case the free edge stresses are computed as functions of the displacements, Δ. Each of these free edge stress systems is balanced using the stress distribution method discussed in Section 4. The displacement of each plate in its plane is computed from the balanced edge stresses. The δ displacement of each plate can be expressed as a function of the applied loads and of the imposed displacements. The variation of Δ with x must be known or assumed for this computation.

STEP 3. The geometrical relations between the displacements δ and Δ are written from an assumed deformed position of the cross section of the folded plate such as that shown in Fig. 2.5.



$$\Delta_{2c} = \delta_{2c} \cot\theta$$

$$\Delta_{2b} = \delta_{1b}\sec\theta + \delta_{2b}\tan\theta$$

$$\Delta_2 = \Delta_{2b} + \Delta_{2c}$$

Fig. 2.5

There is only one system of Δ values that will satisfy the geometrical relations and the equilibrium conditions. Substituting the expressions for δ into the geometrical relations provides as many independent equations as there are values of Δ. These equations are solved simultaneously for the Δ displacements. This solution is obtained at some particular value of x, usually at midspan, so that the solution provides for the conditions at one section only.

STEP 4. The total edge stresses, moments, shears, and displacements can now be computed as the sum of the values obtained from the fundamental solution and those induced by the relative edge displacements, Δ. Since the magnitude of each of these functions has been computed in terms of the Δ displacements, the effect of the displacement is given by multiplying the function by the magnitude of Δ. The importance of the effect of relative edge displacement will be noted in the analytical investigations presented in the following chapters.

6. Continuous Folded Plate Structures

Folded plate structures continuous over rigid interior supports were treated by Ali[12] and it was determined that a reasonable solution to the problem can be provided by varying the procedure recommended in Gaafar's[10] investigation of simple folded plates. In Ali's[12] solution the interior support is assumed to be rigid in its plane and entirely flexible in planes perpendicular to its plane. Thus, if in the free edge computations one considers the plates as isolated beams loaded

with the p forces and acting as beams continuous across knife edge supports, it is possible to account for the continuity in a rather convenient manner. To facilitate making the correction for relative edge displacement, the free edge stresses are computed at the mid span sections. The stresses are balanced in the manner recommended in Section 3 and the δ displacements computed as recommended as in Section 4. The resulting δ values are once again functions of the applied load and the imposed Δ displacements. The geometrical relationships for the deformed mid span cross section are written and the δ expressions substituted as discussed in Section 4. The resulting equations are solved for the unknown Δ values and the final magnitudes of the edge stresses, moments, shears, and displacements computed.

This approach to the analysis of continuous folded plate systems will be utilized as a step in the solution to the problem considered in Chapter IV.

7. Sign Convention

No particular sign convention is necessary in the application of these fundamental procedures, but consistency in the use of an assumed convention is mandatory. In the remaining chapters the convention for signs of forces will be that shown in Fig. 2.6 which presents the assumed positive quantities for the slab and plate force systems. The δ displacements are assumed positive when they develop in the sense of increasing plate designation.

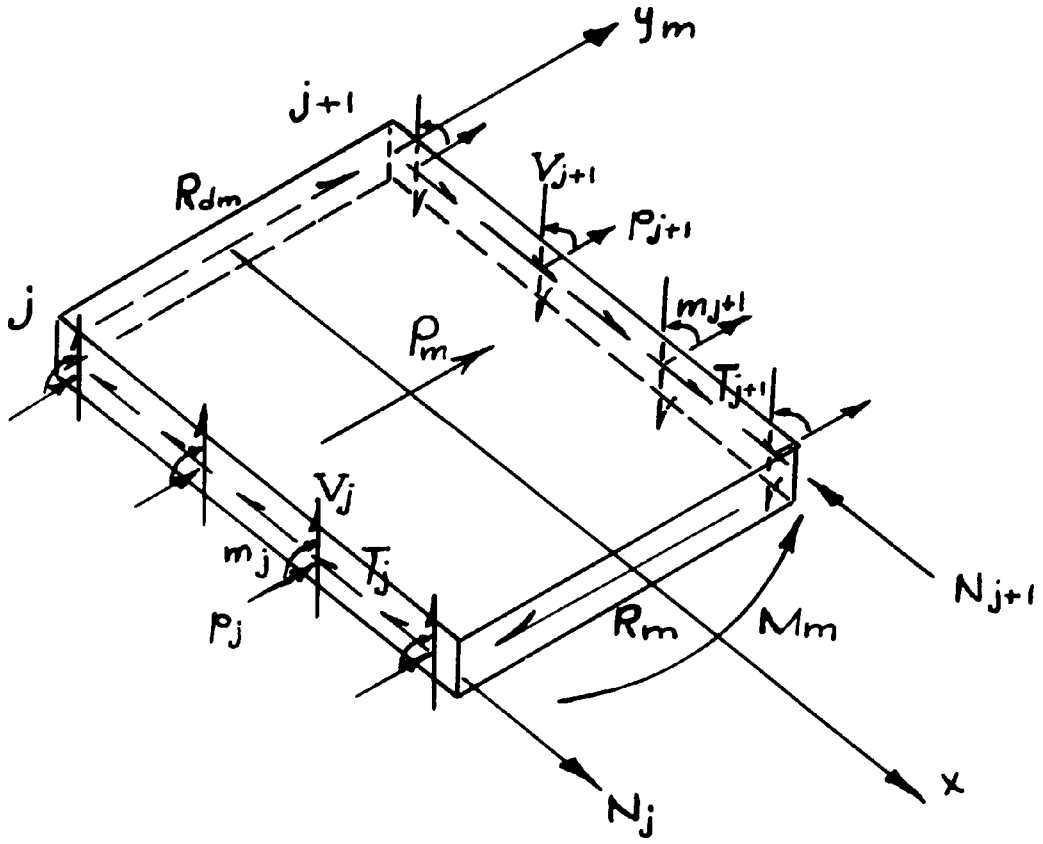


Fig. 2.6

CHAPTER III

THE INVESTIGATION OF A MULTIPLE FOLDED PLATE STRUCTURE

1. Nature of the Problem

The method of analysis of folded plate structures, as improved by Winter and Pei[9] and Gaafar[10], is applicable to structures comprised of plates arranged in such a manner that no more than two plates join at a single edge. As defined in Chapter II, there are j equations available to solve for the longitudinal stresses at j edges.

The introduction of a third plate framing into a single longitudinal joint creates a condition for which the previously discussed methods are inadequate. In the case of an open folded plate structure, made up of j-2 joints at which two plates frame, there are j-2 unknown values of edge shear, N, and there are j-2 equations available, written in terms of M. The plate moments, M, are functions of the edge loads, p, and can be defined from statics. If a third plate is added at one of the joints, the edge loads, p, cannot be determined from statics alone. The distribution of p and N at this joint depends upon the flexural interaction of the plates framing at the joint and thus upon the joint displacement.

Since a simple folded plate, one having two plates only framing at any joint, can be analyzed using the equations of statics,

it is reasonable to think of the multiple folded plate as a statically indeterminate structure. This latter structure is redundant to the degree defined by that number of forces considered necessary to establish the monolithic interaction of the added plate with the statically determinate system.

In structures of civil engineering application, it is reasonable to assume that the largest number of plates framing at a joint will be three. The functional requirements will usually be such that this added member will be of limited depth so that edge forces, p and T only, will be considered as acting at the junction of the redundant member with the statically determinate system of plates. The structure thus defined becomes 2r times statically indeterminate, where there are r joints at which three plates frame.

The solution to the problem can be obtained by using the principle of consistent deformations and stresses and the principle of superposition. The analysis may be carried out in the following manner.

STEP 1. Reduce the structure to a simple folded plate structure. Utilizing the method of Winter and Pei[9] and Gaafar[10], analyze the simple folded plate for the stresses and displacements induced by the applied loads.

STEP 2. Analyze the simple folded plate for the stresses and displacements caused by the application of unit values of the redundants.

STEP 3. Analyze the redundant member for stresses and displacements caused by the application of unit values of the redundants.

STEP 4. Since the displacement and the stress at the common edge must be the same for the simple folded plate as it is for the redundant member, two equations are written describing this consistency at each of the r joints. These equations are then solved for the redundants.

STEP 5. The final stresses, displacements, forces, and moments are determined by summing the quantities obtained from the analysis of the simple folded plate, Step 1, and those values induced by the redundants.

As stated previously the edge load and the edge shear are chosen as the redundants. The edge load, p, is assumed to be sinusoidally distributed with amplitude p_o. Since N is the resultant of the distributed edge shear, T, the shear redundant can be expressed as the amplitude, T_o, of the distributed shear force. The edge shear, T, is assumed to vary as a cosine function with amplitude T_o. Thus the first term of a sine and cosine series is recommended for the solution for p_o and T_o; this is consistent with the treatment of the edge load and shear by previous writers and is sufficient for this particular problem. Using the first term of a sine series to represent p and the first term of a cosine series to represent T permits consistency equations for displacement and stress to be written for one section only, a condition similar to that met in approximating the effect of relative edge displacement. A more general approach is presented in

Chapter V. The break down of the redundant structure and the redundant force systems involved in the analysis is shown in Fig. 3.1.

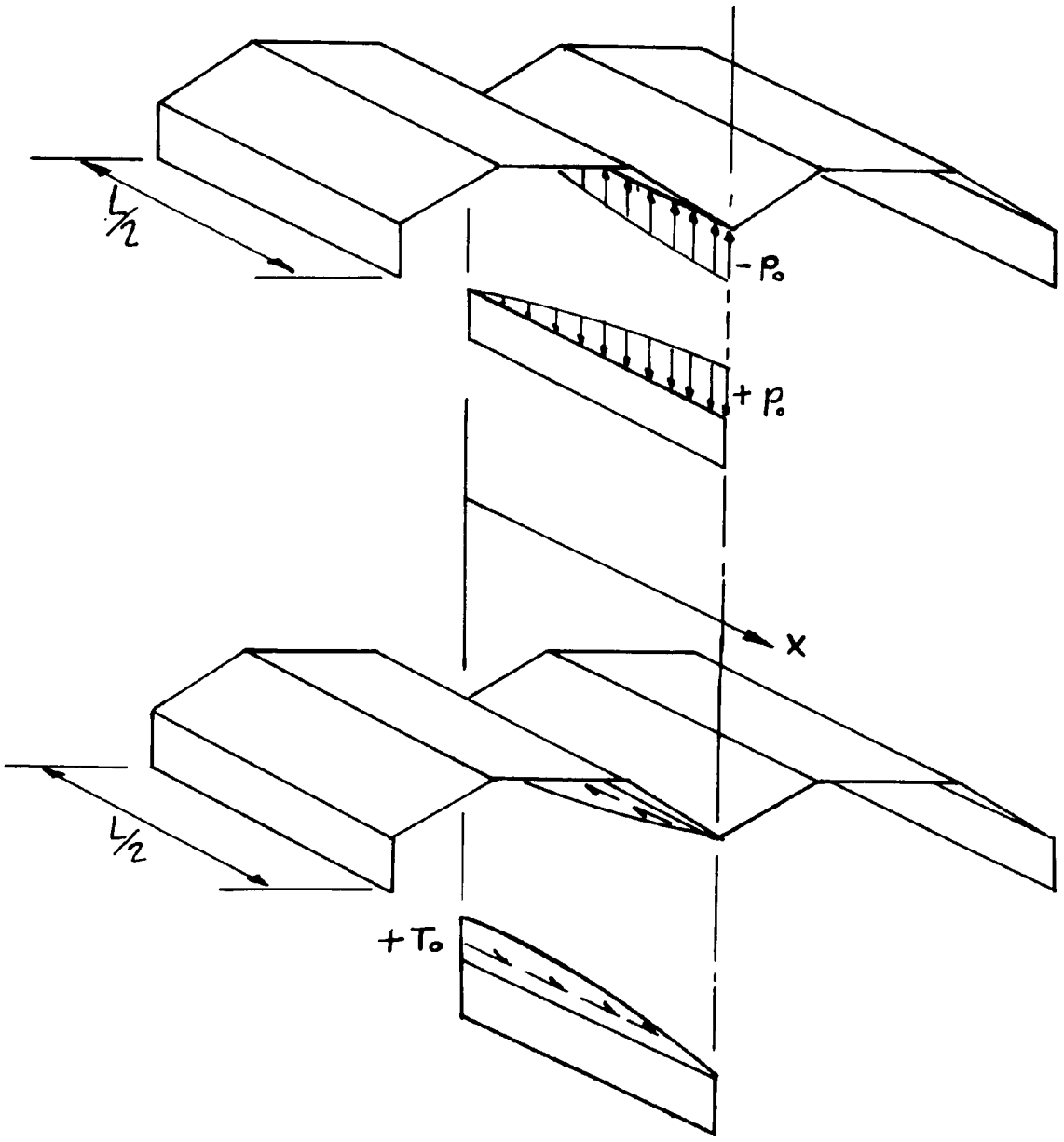


Fig. 3.1

The proposed method will be applied to the analysis of a small scale model of a multiple folded plate roof. The description of the tests and the test results supporting the analysis will be discussed at the end of this chapter.

2. Example 3.1

The folded plate roof shown in Fig. 3.2 is loaded with eight equal load concentrations of 73 pounds. The loads are applied at the third points of the ridge edges.

STEP 1. The interior edge beam is cut from the roof isolating the simple folded plate 1-2-3-4-6-7-8-9. This statically determinate roof is analyzed for the effects of the eight concentrated loads.

The 73 pound loads are resolved into components in the plane of the plates common to the joint at which the forces are applied as shown in Fig. 3.3. The free edge moment at mid span of plates 2,4,6, and 8 is computed for the isolated simply supported plates.

$$M_2' = - M_4' = M_6' = - M_8' = - P_{2c} \frac{L}{3} = \frac{136 \times 35}{3} = + 1{,}590 \text{ in. lb.}$$

Free edge stresses are computed from this moment using the simple flexure formula.

$$f_{2b} = - f_{2c} = \frac{M_2'}{S_2} = \frac{1{,}590}{0.265} = 6{,}000 \text{ lb. per sq. in.}$$

The free edge stresses are now balanced using the procedure discussed in Chapter II. In this case the structure and the load are symmetrical about the plane of plate 5, so that the edge shear, N_e, will be zero.

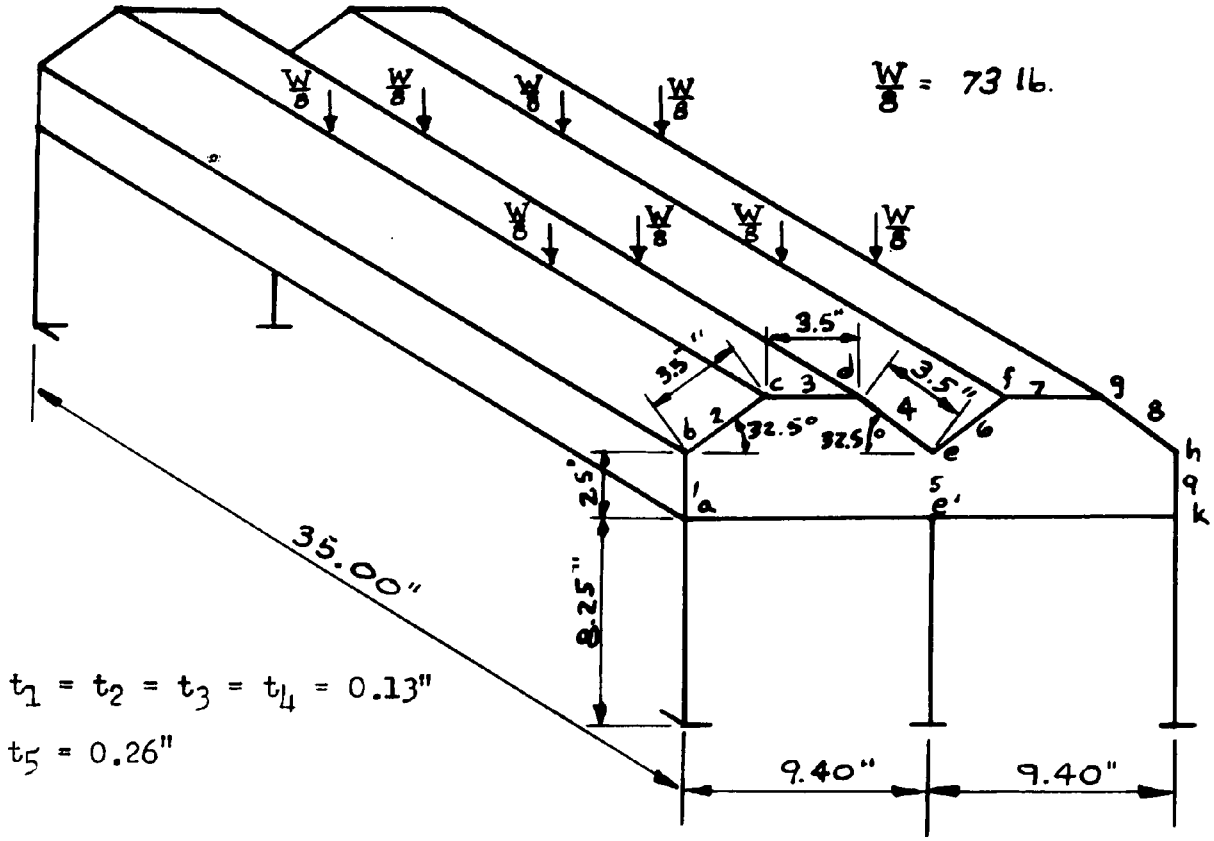
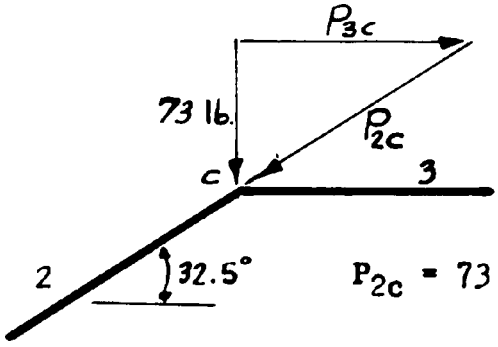


Fig. 3.2



P_{2c} = 73 cosec 32.5° = 136 lb.

Fig. 3.3

Consequently no correction stresses will be applied to joint ● but there will be stresses "carried over" to the joint equally from joints d and f. Only one half of the cross section is considered in the distribution. The distribution and carry over factors are computed from the information presented in Chapter II.

$$r_{1b} = -\frac{A_2}{A_1+A_2} = -\frac{0.13x3.5}{0.13x2.5 + 0.13x3.5} = -\frac{0.455}{0.325+0.455} = -0.58$$

$$r_{2b} = \frac{A_1}{A_1+A_2} = \frac{0.325}{0.780} = 0.42$$

$$r_{2c} = -r_{3c} = -0.50$$

$$r_{3d} = -r_{4d} = -0.50$$

Carry over factor = -0.50

Joint	a		b		c		d		e
r		-0.58	0.42	-0.50	0.50	-0.50	0.50		
Free Edge Stress			+6000	-6000			-6000	+6000	
Operation									
1		+3480	-2520	+3000	-3000	-3000	+3000		
2	-1740		-1500	+1260	+1500	+1500		-1500	
1		- 870	+ 630	+ 120	- 120	- 750	+ 750		
2	+ 435		- 60	- 315	+ 375	+ 60		- 375	
1		- 35	+ 25	+ 345	- 345	- 30	+ 30		
2	+ 18		- 173	- 13	+ 15	+ 173		- 15	
1		- 101	+ 72	+ 14	- 14	- 86	+ 87		
2	+ 50		- 7	- 36	+ 42	+ 7		- 42	
1		- 4	+ 3	+ 39	- 39	- 4	+ 3		
2	+ 2		- 20	- 2	+ 2	+ 20			
1		- 12	+ 8	+ 2	- 2	- 10	+ 10		
2	+ 6		- 1	- 4	+ 5	- 1		- 5	
1			+ 1	+ 5	- 4	+ 1			
Total	-1229	+2458	+2458	-1585	-1585	-2120	-2120	+4063	

Operation 1 Balancing

Operation 2 Carry over

Edge	Balanced Edge Stress
a	-1229
b	+2458
c	-1585
d	-2120
e	+4063

The proportions of the structure are such that the effect of relative edge displacement cannot be neglected. Assuming the folded plate cross section to be deformed as shown in Fig. 3.4, the effect of the relative displacement of the pairs of edges bc, cd, and de can be solved for in terms of Δ_2, Δ_3, and Δ_4. Each of the relative displacements, Δ, is assumed to be induced separately requiring three additional analyses of the folded plate.

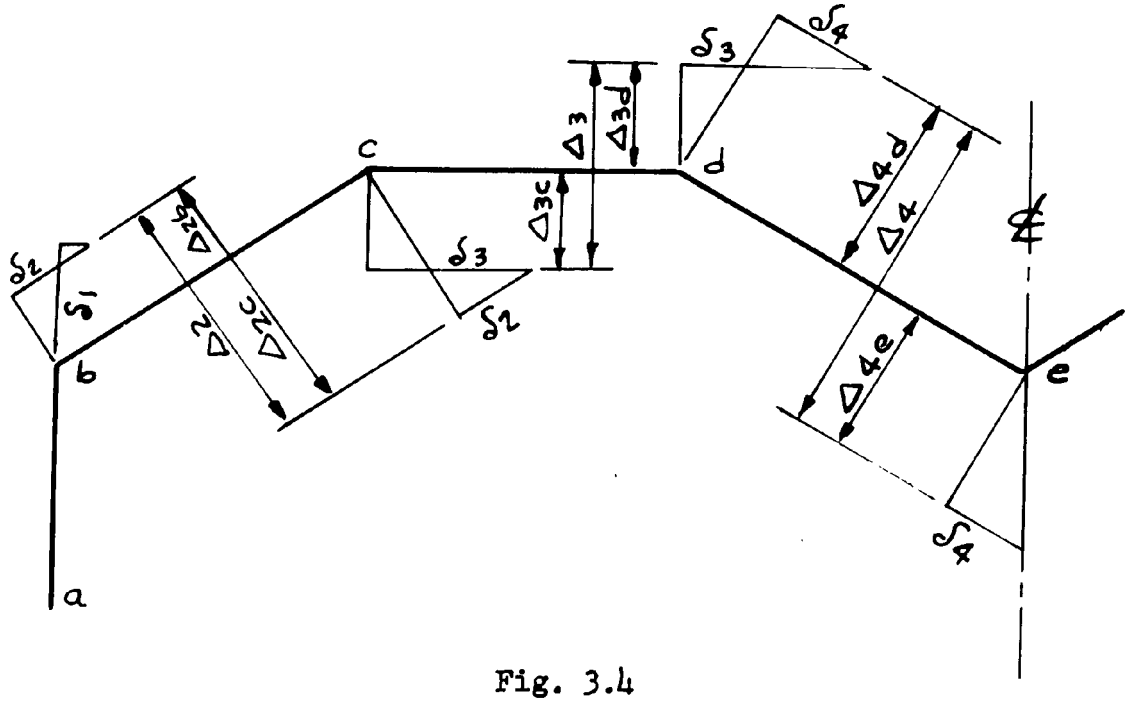


Fig. 3.4

A transverse strip of unit width of the folded plate is analyzed as a beam continuous over knife edge supports at joints b, c, d. The strip is assumed fixed at e from symmetry; plate 1 is assumed to act as a cantilever. Joint b is displaced relative to c through the distance Δ_2, joint c being fixed against rotation. The fixed end moment at c is

$$m_{2c}^F = -\frac{3EI_T}{h_2^2}\,\Delta_2 = -3\alpha_2 \text{ in. lb.}$$

$$\alpha_2 = \frac{EI_T}{h_2^2}\,\Delta_2 = \frac{10.5 \times 183}{3.5^2}\,\Delta_2 = 156.5\Delta_2 \text{ in. lb.}$$

$$I_T = \frac{1 \times 0.13^3}{12} = 183 \times 10^{-6} \text{ in.}^4$$

The fixed end moment, m_{2c}^F, is distributed using the usual moment distribution procedure. Following this, the shears V_{mj} are computed as illustrated in the following computation.

Joint	b		c		d	e	
		0.43	0.57	0.50	0.50		
Moment Distribution		$-3.00\alpha_2$					
		+1.29	-1.71	+0.86			
			-0.21	-0.43	-0.43		
		+0.09	+0.12	+0.06			
			-0.02	-0.03	-0.03		
		+0.01	+0.01				
Moments m		$-1.61\alpha_2$	$+1.61\alpha_2$	$+0.46\alpha_2$	$-0.46\alpha_2$	$-0.23\alpha_2$	
Shears V	$0.460\alpha_2$	$0.460\alpha_2$	$0.591\alpha_2$	$0.591\alpha_2$	$0.197\alpha_2$	$0.197\alpha_2$	

The equilibrants, Q_{mj}, of the shear forces are resolved into components in the plane of the plates as illustrated in Fig. 3.5.

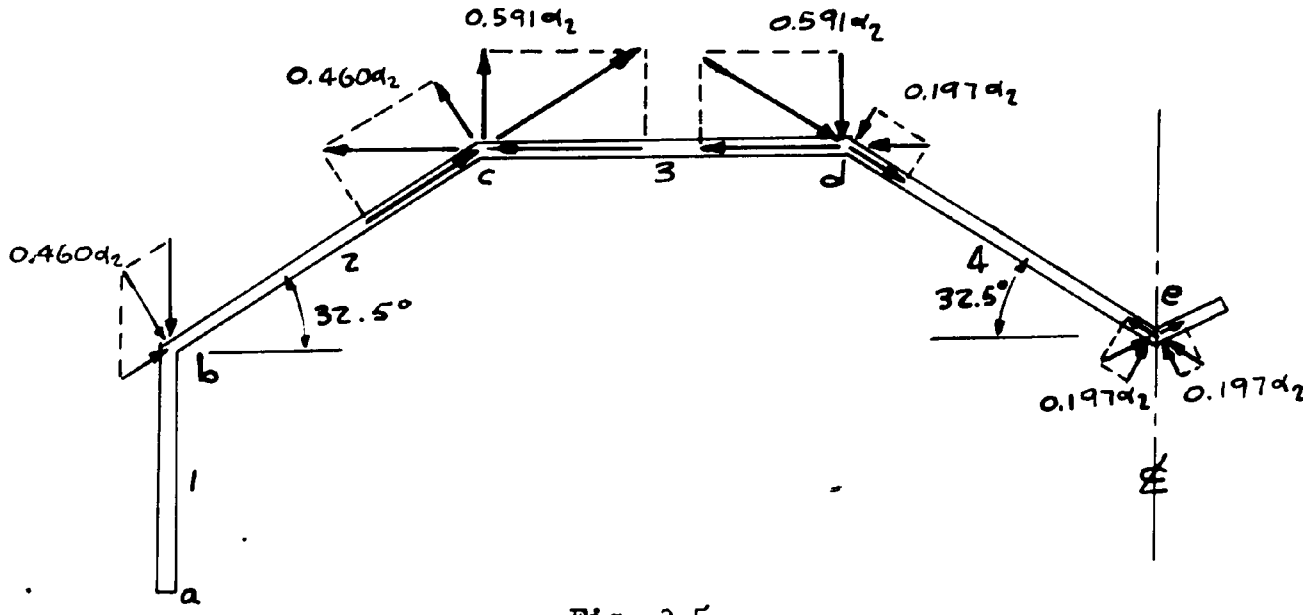


Fig. 3.5

The edge forces p_{mj} are computed as follows.

$$p_{1b} = -0.460\alpha_2 \sec 32.5^\circ = -0.546\alpha_2$$

$$p_2 = p_{2b} + p_{2c} = 0.460\alpha_2 \tan 32.5^\circ + 0.460\alpha_2 \cot 32.5^\circ + 0.591\alpha_2 \operatorname{cosec} 32.5^\circ = 2.119\alpha_2$$

$$p_3 = p_{3c} + p_{3d} = -0.460\alpha_2 \operatorname{cosec} 32.5^\circ - 0.197\alpha_2 \operatorname{cosec} 32.5^\circ - 2 \times 0.591\alpha_2 \operatorname{cosec} 32.5^\circ = -3.085\alpha_2$$

$$p_4 = p_{4d} + p_{4e} = 0.591\alpha_2 \operatorname{cosec} 32.5^\circ + 0.197\alpha_2 \cot 32.5^\circ - 0.197\alpha_2 (\sec 25^\circ + \tan 25^\circ) = 1.102\alpha_2$$

It is apparent that p varies in the same manner as Δ. Consequently the free edge moments and stresses produced by these p forces will vary similarly. If Δ is assumed to vary sinusoidally, the free edge moment at midspan becomes

$$M_m' = -\frac{p_m L^2}{\pi^2} = -\frac{35^2}{\pi^2} p_m = -124 p_m.$$

The free edge stresses are

$$f_{1a} = -f_{1b} = +124 \times 0.546 \alpha_2 = +502\alpha_2$$

$$f_{2b} = -f_{2c} = -124 \times 2.119 \alpha_2 = -990\alpha_2$$

$$f_{3c} = -f_{3d} = +124 \times 3.085 \alpha_2 = +1{,}447\alpha_2$$

$$f_{4d} = -f_{4e} = -124 \times 1.102 \alpha_2 = -515\alpha_2.$$

The free edge stresses are balanced yielding the total edge stresses induced by Δ_2.

a		b		c		d	e
	- 0.58	0.42	- 0.50	0.50	- 0.50	0.50	
$+502\alpha_2$	$-502\alpha_2$	$-990\alpha_2$	$+990\alpha_2$	$+1447\alpha_2$	$-1447\alpha_2$	$-515\alpha_2$	$+515\alpha_2$
	-283	+205	+ 228	- 229	+ 466	-466	
+142		-114	- 103	- 233	+ 114		+233
	- 66	+ 48	- 65	+ 65	- 57	+ 57	
+ 33		+ 33	- 24	+ 28	- 32		- 28
	+ 19	- 14	+ 26	- 26	+ 16	- 16	
- 10		- 13	+ 7	- 8	+ 13		+ 8
	- 8	+ 5	- 7	+ 8	- 6	+ 7	- 3
+ 4		+ 3	- 2	+ 3	- 4		
- 1	+ 2	- 1	+ 3	- 2	+ 2	- 2	+ 1
$+670\alpha_2$	$-838\alpha_2$	$-838\alpha_2$	$+1053\alpha_2$	$+1053\alpha_2$	$-935\alpha_2$	$-935\alpha_2$	$+726\alpha_2$

The effects of relative edge displacements Δ_3 and Δ_4 are computed in a similar manner, the final edge stresses for the three corrective computations are as follows.

Balanced Edge Stresses

Edge	Δ_2	Δ_3	Δ_4
a	$+ 670\alpha_2$	$+1058\alpha_3$	$+ 617\alpha_4$
b	$- 838\alpha_2$	$-1465\alpha_3$	$-1017\alpha_4$
c	$+1053\alpha_2$	$+2101\alpha_3$	$+1843\alpha_4$
d	$- 935\alpha_2$	$-1993\alpha_3$	$-1872\alpha_4$
e	$+ 726\alpha_2$	$-1544\alpha_3$	$+1368\alpha_4$

The displacement of each plate in its plane is computed as the sum of the effects of the applied loads and relative edge displacement. The δ displacements at midspan are computed using the following expressions.

Concentrated loads at the one third points:

$$\delta = \frac{23}{648}\frac{PL^3}{EI} = \frac{23}{216}\left[\frac{PL}{3}\right]\frac{L^2}{EI} = \frac{23}{216}\frac{f_o L^2}{Eh}$$

$$\text{where } f_o = \frac{PL}{3}\frac{h}{I}$$

Sinusoidally distributed loads:

$$\delta = \frac{L^4}{\pi^4 EI} = \frac{f_o L^2}{\pi^2 Eh}$$

$$\text{where } f_o = \frac{\pi^2}{L^2}\frac{h}{I}$$

Substituting the computed edge stresses the following displacements are computed.

$$\delta_1 = + 18.31\ (10)^{-3} - 1.116\ \Delta_2 - 1.867\Delta_3 - 1.209\Delta_4$$

$$\delta_2 = - 14.34\ (10)^{-3} + 0.9994\Delta_2 + 1.880\Delta_3 + 1.512\Delta_4$$

$$\delta_3 = - 1.907(10)^{-3} - 1.051\ \Delta_2 - 2.164\Delta_3 - 1.963\Delta_4$$

$$\delta_4 = + 21.93\ (10)^{-3} + 0.8773\Delta_2 + 1.869\Delta_3 + 1.712\Delta_4$$

The relationships between the and the Δ displacements are written from the geometrical requirements portrayed in Fig. 3.4.

$$\Delta_4 = \Delta_{4e} + \Delta_{4d} = 1.862\,\delta_3 - 1.572\,\delta_4 + 1.572\,\delta_4$$
$$= 1.862\,\delta_3$$

$$\Delta_3 = \Delta_{3c} + \Delta_{3d} = -1.862\,\delta_2 + 3.144\,\delta_3 - 1.862\,\delta_4$$

$$\Delta_2 = \Delta_{2b} + \Delta_{2c} = 1.186\,\delta_1 - 2.209\,\delta_2 + 1.862\,\delta_3$$

Substituting the δ displacement expressions into the above relationships yields three equations containing the relative edge displacements as unknowns.

$$- 49.84(10)^{-3} + 6.489\Delta_2 + 10.396\Delta_3 + 8.429\Delta_4 = 0$$

$$+ 20.13(10)^{-3} + 6.799\Delta_2 + 14.785\Delta_3 + 12.175\Delta_4 = 0$$

$$+ 3.55(10)^{-3} + 1.957\Delta_2 + 4.029\Delta_3 + 4.655\Delta_4 = 0$$

The solution of these equations provides the following values for Δ and α.

$\Delta_2 = + 37.33\ (10)^{-3}$ in. $\qquad \alpha_2 = + 5.842$ lb. in.

$\Delta_3 = - 17.32\ (10)^{-3}$ in. $\qquad \alpha_3 = - 2.711$ lb. in.

$\Delta_4 = - 1.465(10)^{-3}$ in. $\qquad \alpha_4 = - 0.229$ lb. in.

The corrected edge stresses, computed for the simple folded plate, are the sum of the stresses computed neglecting relative edge displacement and those induced by the relative edge displacements.

Edge	Stresses Neglecting Relative Edge Displacement	Relative Edge Displacement Δ_2	Δ_3	Δ_4	Total lb. per sq. in.
a	- 1,229	+ 3,914	- 2,868	- 141	- 324
b	+ 2,458	- 4,896	+ 3,972	+ 233	+ 1,767
c	- 1,584	+ 6,152	- 5,696	- 422	- 1,550
d	- 2,119	- 5,462	+ 5,403	+ 429	- 1,749
e	+ 4,059	+ 4,235	- 4,186	- 313	+ 3,795

The vertical displacement at edge e is computed from the expression written for δ_4.

$$\delta_4 = + 21.93(10)^{-3} + 0.8773\Delta_2 + 1.869\Delta_3 + 1.712\Delta_4$$

$$= + 19.80(10)^{-3} \text{ in.}$$

$$\delta_{5e} = \delta_4 \text{ cosec } 32.5^\circ = 36.87(10)^{-3} \text{ in. (downward).}$$

STEP 2. The statically determinate form of the roof is now analyzed for the edge stresses and displacements developed upon the application of unit values of the redundants.

The p force will be considered first. As described earlier, this force is assumed to vary sinusoidally, the amplitude, p_o, being set equal to unity in this step of the analysis. This distributed force is applied to edge e and acts in the plane of plate 5 as shown in Fig. 3.1. The distributed edge load is resolved into components acting in the planes of plates 4 and 6.

$$p_{6e} = p_{4e} = - \frac{\text{cosec } \theta}{2} \sin \frac{\pi x}{L} = - 0.9311 \sin \frac{\pi x}{L}$$

The free edge stresses developed in plate 4 at the midspan section are

$$f_{4d} = -f_{4e} = \frac{0.9311\ L^2}{S_4 \pi^2} = \frac{0.9311 \times 35^2}{0.265\pi^2} = 435.7 \text{ lb. per sq. in.}$$

The free edge stresses are then distributed in the usual manner; the results appear in the following tabulation.

Edge	Balanced Edge Stress lb. per sq. in.
a	+ 8
b	- 17
c	+ 55
d	- 203
e	+ 320

The analysis made in Step 1 for the effect of relative edge displacement may be used here. The δ displacement relationships may be written directly from those of Step 1 by simply recomputing the constant term which in this step is a function of the applied edge load,

$$p_e = \sin \frac{\pi x}{L} .$$

$$\delta_1 = + \quad 118.3(10)^{-6} - 1.116\ \Delta_2 - 1.867\Delta_3 - 1.209\Delta_4$$

$$\delta_2 = - \quad 243.1(10)^{-6} + 0.9994\Delta_2 + 1.880\Delta_3 + 1.512\Delta_4$$

$$\delta_3 = + \quad 871.3(10)^{-6} - 1.051\ \Delta_2 - 2.164\Delta_3 - 1.963\Delta_4$$

$$\delta_4 = - 1{,}766.2(10)^{-6} + 0.8773\Delta_2 + 1.869\Delta_3 + 1.712\Delta_4$$

Substituting these expressions into the geometrical relationships between δ displacements and the Δ displacements yields the following equations which are solved for Δ_2, Δ_3, and Δ_4.

$$- 2.2996(10)^{-3} + 6.489\Delta_2 + 10.396\Delta_3 + 8.429\Delta_4 = 0$$

$$- 6.4808(10)^{-3} + 6.799\Delta_2 + 14.785\Delta_3 + 12.175\Delta_4 = 0$$

$$- 1.6223(10)^{-3} + 1.957\Delta_2 + 4.029\Delta_3 + 4.655\Delta_4 = 0$$

$\Delta_2 = - 1.322 \quad (10)^{-3}$ in. $\qquad \alpha_2 = - 0.2068$ lb. in.

$\Delta_3 = + 1.050 \quad (10)^{-3}$ in. $\qquad \alpha_3 = + 0.1643$ lb. in.

$\Delta_4 = - 0.00476(10)^{-3}$ in. $\qquad \alpha_4 = - 0.0008$ lb. in.

The corrected edge stresses at the midspan section are computed as the sum of the stresses obtained neglecting relative edge displacements and those induced by Δ_2, Δ_3, and Δ_4.

Edge	Neglecting Relative Edge Displacement	Relative Edge Displacement Δ_2	Δ_3	Δ_4	Total lb. per sq. in.
a	- 8	- 139	+ 174	---	+ 27
b	+ 17	+ 173	- 241	+ 1	- 50
c	- 55	- 218	+ 345	- 1	+ 71
d	+ 203	+ 193	- 327	+ 1	+ 70
e	- 320	- 150	+ 254	- 1	- 217

The displacement of edge e in the plane of plate 5 is computed as

$$\delta_{5e} = \delta_4 \operatorname{cosec} 32.5^\circ = 1.862 \left[- 1.7662(10)^{-3} + 0.8773\Delta_2 + 1.869\Delta_3 + 1.712\Delta_4 \right] = - 1.8089(10)^{-3} \text{ in. (upward)}$$

Fig. 3.1 shows the simple folded plate roof loaded with an edge shear force applied with an assumed cosine distribution of unit amplitude. The shear force N_e is the resultant edge shear at any section.

$$N_e = \int_o^x T_e dx = \int_o^x T_o \cos \frac{\pi x}{L} \, dx = T_o \frac{L}{\pi} \sin \frac{\pi x}{L}$$

at midspan,

$$N_e = \frac{L}{\pi} T_o$$

for

$$T_o = 1$$

$$N_o = \frac{L}{\pi}$$

From the symmetry of the structure and the load about plate 5 it follows that equal edge shears will be developed at edge e of plates 4 and 6.

At midspan this force system produces free edge stresses in plates 4 and 6 given by the following computation.

$$f_{4d} = f_{6f} = \frac{- 2N_o}{2A_4} = - \frac{2L}{2\pi A_4} = - \frac{35.0}{0.455\pi} = - 24.48 \text{ lb. per sq. in.}$$

$$f_{4e} = f_{6e} = + \frac{4N_o}{2A_4} = \frac{4L}{2\pi A_4} = + 48.96 \text{ lb. per sq. in.}$$

The free edge stresses are distributed in the usual manner; the balanced edge stresses are shown in the following tabulation.

Edge	Balanced Edge Stresses
a	+ 0.5
b	- 1.0
c	+ 3.1
d	- 11.4
e	+ 42.5

Here again the analysis for relative edge displacement made in Step 1 may be utilized, the constant term being computed from the balanced edge stresses tabulated above.

$$\delta_1 = - 7.09(10)^{-6} - 1.116\,\Delta_2 - 1.867\Delta_3 - 1.209\Delta_4$$

$$\delta_2 = + 13.85(10)^{-6} + 0.9994\Delta_2 + 1.880\Delta_3 + 1.512\Delta_4$$

$$\delta_3 = - 48.97(10)^{-6} - 1.051\,\Delta_2 - 2.164\Delta_3 - 1.963\Delta_4$$

$$\delta_4 = + 182.02(10)^{-6} + 0.8773\Delta_2 + 1.869\Delta_3 + 1.712\Delta_4$$

The substitution of the above displacements into the geometrical relationships written from the deformed cross section shown in Fig. 3.4 yields the following three equations which are solved for Δ_2, Δ_3, and Δ_4.

$$+ 130.18(10)^{-6} + 6.489\Delta_2 + 10.396\Delta_3 + 8.429\Delta_4 = 0$$

$$+ 518.67(10)^{-6} + 6.799\Delta_2 + 14.785\Delta_3 + 12.175\Delta_4 = 0$$

$$+ 91.18(10)^{-6} + 1.957\Delta_2 + 4.029\Delta_3 + 4.655\Delta_4 = 0$$

$\Delta_2 = + 139.31(10)^{-6}$ in. $\qquad \alpha_2 = + 21.80\ (10)^{-3}$ lb. in.

$\Delta_3 = - 121.10(10)^{-6}$ in. $\qquad \alpha_3 = - 18.95\ (10)^{-3}$ lb. in.

$\Delta_4 = + 26.67(10)^{-6}$ in. $\qquad \alpha_4 = + 4.174(10)^{-3}$ lb. in.

The edge stresses induced in the simple folded plate by applying the unit edge shear load are obtained as the sum of the stresses computed neglecting relative edge displacement and those computed as functions of Δ_2, Δ_3, and Δ_4.

Edge	Neglecting Relative Edge Displacement	Relative Edge Displacement Δ_2	Δ_3	Δ_4	Total lb. per sq. in.
a	+ 0.5	+ 14.6	- 20.0	+ 2.6	- 2.3
b	- 1.0	- 18.3	+ 27.8	- 4.2	+ 4.3
c	+ 3.1	+ 23.0	- 39.8	+ 7.7	- 6.0
d	- 11.4	- 20.4	+ 37.8	- 7.8	- 1.8
e	+ 42.5	+ 15.8	- 29.3	+ 5.7	+ 34.7

The midspan displacement of edge e, measured in the plane of plate 5, is computed as follows.

$$\delta_{5e} = \delta_4 \operatorname{cosec} 32.5^\circ = 1.862 \left[+ 182.0(10)^{-6} + 0.8773\Delta_2 + 1.869\Delta_3 + 1.712\Delta_4 \right] = 230.2(10)^{-6} \text{in.} \quad \text{(downward)}.$$

STEP 3. The analysis of plate 5 for stresses and displacements at midspan developed upon the application of unit values of the redundants p_e and T_e is presented in the following computations.

Stresses and displacement caused by the edge load, p_e, of unit amplitude are computed at the midspan section.

$$M = \frac{L^2}{\pi^2}$$

$$f_{5e} = -\frac{(35.0)^2}{0.270\pi^2} = -459.3 \text{ lb. per sq. in.}$$

$$\delta_{5e} = \frac{L^2}{\pi^2}\frac{f_o}{Eh} = \frac{(35.0)^2 \times 918.6}{\pi^2 \times 10.5(10)^6 \times 2.5} = 4.344\ (10)^{-3} \text{ in. (downward)}$$

Stresses and displacements caused by the edge shear, T_e, of unit amplitude are computed at midspan.

$$f_{5e} = -4\frac{L}{\pi A} = -\frac{4 \times 35.0}{0.650\pi} = -68.56 \text{ lb. per sq. in.}$$

$$\delta_{5e} = \frac{L^2}{\pi^2}\frac{f_o}{Eh} = \frac{(35.0)^2 \times 102.84}{\pi^2 \times 10.5(10)^6 \times 2.5} = 486.33\ (10)^{-6} \text{ in. (downward)}$$

STEP 4. The assumed positive values of p_e and T_e were used in Steps 2 and 3. Assuming positive displacement as downward at joint e and tension stress as positive, the following equations can be formed expressing the consistency of displacement and stress at $x = \frac{L}{2}$ of the simple folded plate and the additional plate.

$$+36.87(10)^{-3} - 1.809(10)^{-3}p_o + 230.2(10)^{-6}T_o = +4.344(10)^{-3}p_o + 486.3(10)^{-6}T_o$$

$$+3,795 - 217p_o + 34.7T_o = -459.3p_o - 68.56T_o$$

The solution of these equations yields amplitudes, p_o and T_o, of the redundant functions.

$$p_o = 8.33 \text{ lb. per in.}$$

$$T_o = 56.3 \text{ lb. per in.}$$

STEP 5. The solution for the final edge stresses is obtained by summing the edge stresses in the simple folded plate caused by the applied load and those developed by the redundants. The final edge stresses at midspan are presented in the following tabulation.

Edge	Simple Folded Plate	p_e	T_e	Total lb. per sq. in.
a	- 324	+ 225	+ 129	+ 30
b	+ 1,767	- 417	- 242	+ 1,108
c	- 1,550	+ 591	+ 338	- 621
d	- 1,749	+ 583	+ 101	- 1,065
e	+ 3,795	- 1,808	- 1,953	+ 34
e'		+ 3,826	- 1,930	+ 1,896

The final stresses at sections 3.89 in. and 7.78 in. from the end support are presented at the end of the chapter as are the final displacements and slab moments.

3. Example 3.2

The analysis of the structure investigated in Example 3.1, but with one bay only loaded, proceeds in a manner similar to that just illustrated. The fact that symmetry of load about plate 5 does not exist in this case lengthens Steps 1 and 5 of the analysis but has only a small effect on the other parts of the solution. Since the solution is quite similar to that of Example 3.1 results of the various phases of the analysis will be presented.

STEP 1. In this case the structure is the same as in Example 3.1 but each of the concentrated loads is 75 pounds, the loads being applied to the left bay only. The simple folded plate is analyzed for the effect of these loads.

The 75 pound edge concentrations are resolved into components in the plane of plates common to the joint of application. The magnitude of the component P_{2c} and of P_{4d} is 140 pounds. The free edge stresses developed by these concentrated forces are balanced providing the edge stresses shown in the following tabulation.

Edge	Balanced Edge Stress
a	- 1286
b	+ 2575
c	- 1780
d	- 1617
e	+ 2086
f	- 560
g	+ 153
h	- 51
k	+ 25

The solution for the effect of relative edge displacement lengthens the problem since there are six Δ values to be considered. Assuming the roof cross section to be distorted as shown in Fig. 3.6, the effect of each of the Δ displacements is considered in turn. The results of the analyses for stress induced by Δ_2, Δ_3, Δ_4, Δ_6, Δ_7, and Δ_8 are presented in the following tabulation.

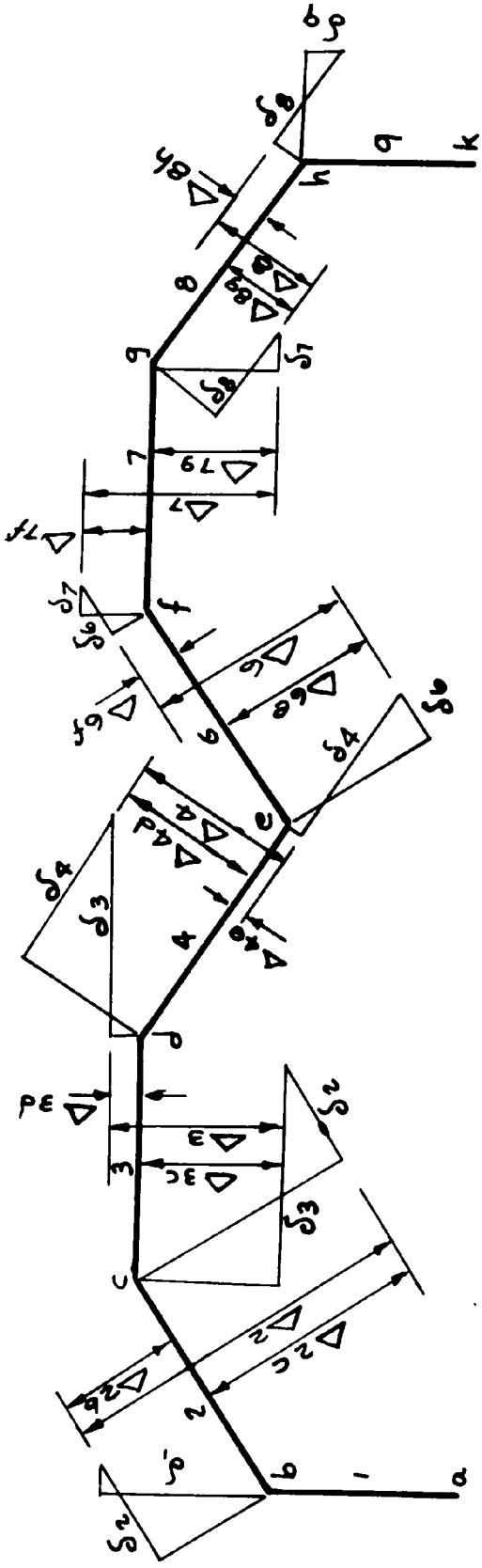


Fig. 3.6

Balanced Edge Stresses

Edge	Δ_2	Δ_3	Δ_4	Δ_6	Δ_7	Δ_8
a	$+661\alpha_2$	$+1011\alpha_3$	$+470\alpha_4$	$+140\alpha_6$	$+44\alpha_7$	$+10\alpha_8$
b	$-821\alpha_2$	$-1388\alpha_3$	$-767\alpha_4$	$-234\alpha_6$	$-75\alpha_7$	$-19\alpha_8$
c	$+1005\alpha_2$	$+1933\alpha_3$	$+1328\alpha_4$	$+583\alpha_6$	$+161\alpha_7$	$+47\alpha_8$
d	$-832\alpha_2$	$-1748\alpha_3$	$-1326\alpha_4$	$-565\alpha_6$	$-240\alpha_7$	$-104\alpha_8$
e	$+363\alpha_2$	$+770\alpha_3$	$+742\alpha_4$	$+742\alpha_6$	$+770\alpha_7$	$+363\alpha_8$
f	$-104\alpha_2$	$-240\alpha_3$	$-565\alpha_4$	$-1326\alpha_6$	$-1748\alpha_7$	$-832\alpha_8$
g	$+47\alpha_2$	$+161\alpha_3$	$+583\alpha_4$	$+1328\alpha_6$	$+1933\alpha_7$	$+1005\alpha_8$
h	$-19\alpha_2$	$-75\alpha_3$	$-234\alpha_4$	$-767\alpha_6$	$-1388\alpha_7$	$-821\alpha_8$
k	$+10\alpha_2$	$+44\alpha_3$	$+140\alpha_4$	$+470\alpha_6$	$+1011\alpha_7$	$+661\alpha_8$

Displacements, δ, are now computed and are expressed in terms of the applied load and the unknown relative edge displacements. The coefficients for the δ equations are tabulated below.

	Load	Δ_2	Δ_3	Δ_4	Δ_6	Δ_7	Δ_8
δ_1	$+19.19\ (10)^{-3}$	-1.097	-1.775	-0.9154	-0.2768	-0.0881	-0.0215
δ_2	$-15.46\ (10)^{-3}$	+0.9651	+1.755	+1.107	+0.4318	+0.1247	+0.0349
δ_3	$+\ 0.580\ (10)^{-3}$	-0.9709	-1.945	-1.403	-0.6067	-0.2119	-0.0798
δ_4	$+13.15\ (10)^{-3}$	+0.6316	+1.331	+1.093	+0.6908	+0.5338	+0.2468
δ_6	$-\ 9.392\ (10)^{-3}$	-0.2468	-0.5338	-0.6908	-1.093	-1.3308	-0.6316
δ_7	$+\ 2.530\ (10)^{-3}$	+0.0798	+0.2119	+0.6067	+1.403	+1.945	+0.9709
δ_8	$-\ 0.7233(10)^{-3}$	-0.0349	-0.1247	-0.4318	-1.107	-1.755	-0.9651
δ_9	$+\ 0.3780(10)^{-3}$	+0.0215	+0.0881	+0.2768	+0.9154	+1.775	+1.097

The relationships between the δ and the Δ displacements are derived from the geometry of the distorted cross section. Positive displacements have been assumed as those developed in the direction of increasing edge numbers; the distorted cross section shown in Fig. 3.6 has been drawn showing positive displacements facilitating the writing of the following expressions.

$$\Delta_2 = +1.186\delta_1 \quad -2.209\delta_2 \quad +1.862\delta_3$$

$$\Delta_3 = -1.862\delta_2 \quad +3.144\delta_3 \quad -1.862\delta_4$$

$$\Delta_4 = +1.862\delta_3 \quad -1.106\delta_4 \quad -1.103\delta_6$$

$$\Delta_6 = +1.103\delta_4 \quad +1.106\delta_6 \quad -1.862\delta_7$$

$$\Delta_7 = +1.862\delta_6 \quad -3.144\delta_7 \quad +1.862\delta_8$$

$$\Delta_8 = -1.862\delta_7 \quad +2.209\delta_8 \quad -1.186\delta_9$$

Substituting the computed δ values into the above geometrical relationships yields six equations which are solved simultaneously for the Δ displacements.

$$+6.240\,\Delta_2 + 9.604\,\Delta_3 + 6.143\,\Delta_4 + 2.4118\Delta_6 + 0.7746\Delta_7 + 0.2512\Delta_8 - 58.38(10)^{-3} = 0$$

$$+6.026\,\Delta_2 + 12.861\,\Delta_3 + 8.507\,\Delta_4 + 3.998\,\Delta_6 + 1.8923\Delta_7 + 0.7754\Delta_8 - 6.12(10)^{-3} = 0$$

$$+2.234\,\Delta_2 + 4.505\,\Delta_3 + 4.059\,\Delta_4 + 0.6881\Delta_6 - 0.4829\Delta_7 - 0.2751\Delta_8 + 3.10(10)^{-3} = 0$$

$$-0.2751\Delta_2 - 0.4829\Delta_3 + 0.6881\Delta_4 + 4.059\,\Delta_6 + 4.505\,\Delta_7 + 2.234\,\Delta_8 + 0.60(10)^{-3} = 0$$

$$+0.7754\Delta_2 + 1.8923\Delta_3 + 3.9978\Delta_4 + 8.5072\Delta_6 + 12.861\,\Delta_7 + 6.026\,\Delta_8 + 26.79(10)^{-3} = 0$$

$$+0.2512\Delta_2 + 0.7746\Delta_3 + 2.4118\Delta_4 + 6.143\,\Delta_6 + 9.604\,\Delta_7 + 6.241\,\Delta_8 + 6.76(10)^{-3} = 0$$

The solution of these equations provides the following Δ values.

$\Delta_2 = + 30.677(10)^{-3}$ in. $\quad \alpha_2 = + 4.801$ lb. in.

$\Delta_3 = - 9.734(10)^{-3}$ in. $\quad \alpha_3 = - 1.523$ lb. in.

$\Delta_4 = - 8.441(10)^{-3}$ in. $\quad \alpha_4 = - 1.321$ lb. in.

$\Delta_6 = + 6.879(10)^{-3}$ in. $\quad \alpha_6 = + 1.077$ lb. in.

$\Delta_7 = - 8.110(10)^{-3}$ in. $\quad \alpha_7 = - 1.269$ lb. in.

$\Delta_8 = + 7.867(10)^{-3}$ in. $\quad \alpha_8 = + 1.231$ lb. in.

The final edge stresses are obtained as the sum of the stresses computed neglecting relative edge displacement and those caused by relative edge displacement. Stresses are in lb. per sq. in.

Edge	Neglecting Displacement	Relative Edge Displacement Δ_2	Δ_3	Δ_4	Δ_6	Δ_7	Δ_8	Total
a	-1286	+3173	-1540	- 621	+ 151	- 56	+ 12	- 167
b	+2575	-3942	+2114	+1013	- 252	+ 95	- 23	+1580
c	-1780	+4825	-2944	-1754	+ 628	- 204	+ 58	-1171
d	-1617	-3994	+2662	+1752	- 609	+ 305	- 128	-1629
e	+2086	+1743	-1173	- 980	+ 799	- 977	+ 447	+1945
f	- 560	- 499	+ 366	+ 746	-1428	+2218	-1024	- 187
g	+ 153	+ 226	- 245	- 770	+1430	-2453	+1237	- 422
h	- 51	- 91	+ 114	+ 309	- 826	+1761	-1011	+ 205
k	+ 25	+ 48	- 67	- 185	+ 506	-1283	+ 814	- 142

The vertical displacement of edge e at midspan is computed as a function of δ_4 and δ_6.

$$\delta_5 = \delta_4 \left[\cot 2\theta \cos\theta + \sin\theta\right] - 6 \frac{\cos\theta}{\sin 2\theta}$$

$$\delta_5 = 0.01886 \text{ in. (downward)}$$

STEP 2 and STEP 3. Both of these steps are identical with the corresponding steps in Example 3.1 and the results of that example will be used in setting up the equations written in Step 4 of the solution.

STEP 4. The equations written for the solution of the redundants in Example 3.1 are applicable here except for the constant terms which are functions of the applied loading.

$$+ 18.86(10)^{-3} - 6.153(10)^{-3}p_o - 0.2561(10)^{-3}T_o = 0$$

$$+ 1945 + 242.3p_o + 103.3T_o = 0$$

The solution of these equations yields the redundant amplitudes p_o and T_o.

$$p_o = 4.28 \text{ lb. per in.}$$

$$T_o = 28.9 \text{ lb. per in.}$$

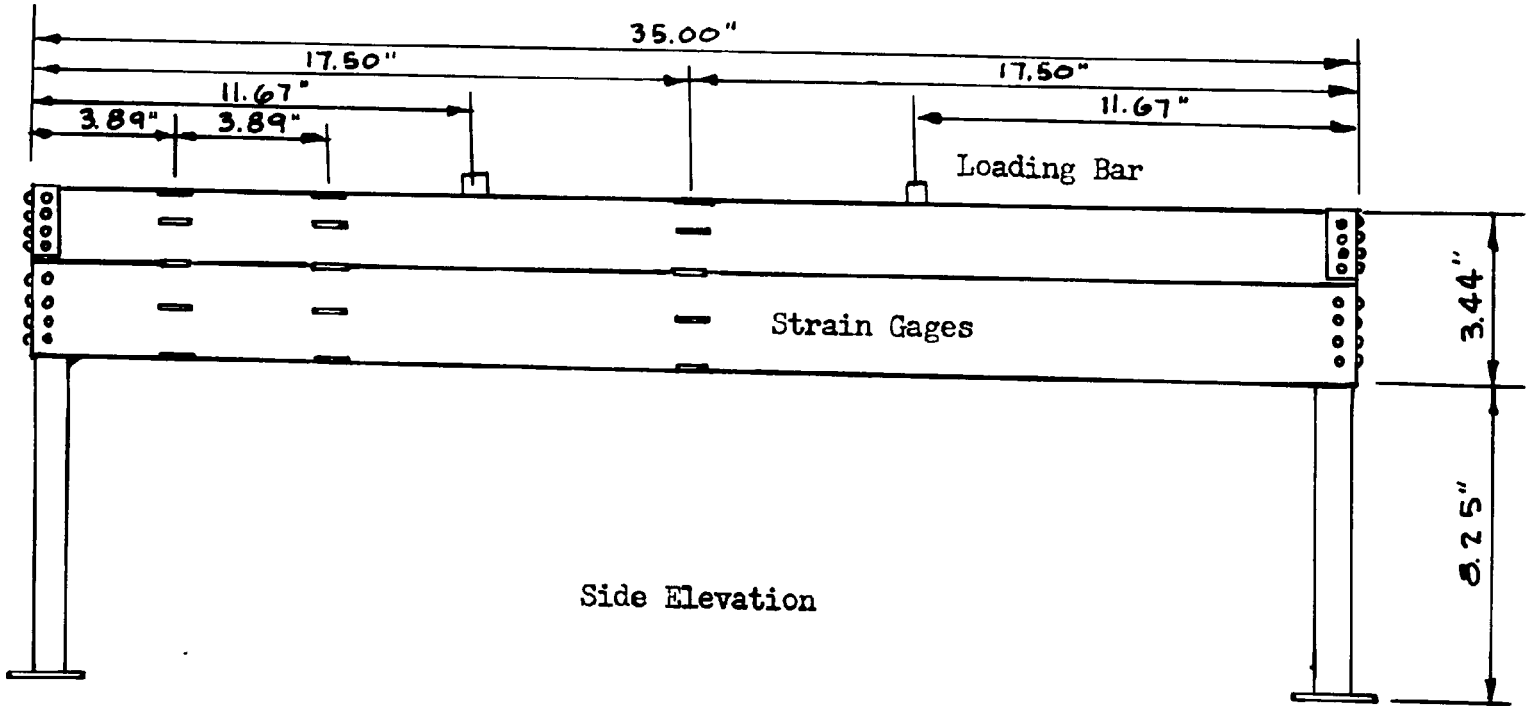
The final edge stresses at midspan are given as the sum of those computed for the simple folded plate and those induced by the redundants.

Edge	Simple Folded Plate	p_e	T_e	Total lb. per sq. in.
a	- 167	+ 114	+ 68	+ 15
b	+1580	- 216	- 127	+1237
c	-1171	+ 300	+ 176	- 695
d	-1629	+ 295	+ 53	-1281
e	+1945	- 915	-1015	+ 15
f	- 187	+ 296	+ 53	+ 162
g	- 422	+ 300	+ 176	+ 54
h	+ 205	- 210	- 126	- 131
k	- 142	+ 114	+ 67	+ 39
e'		+1940	-1003	+ 937

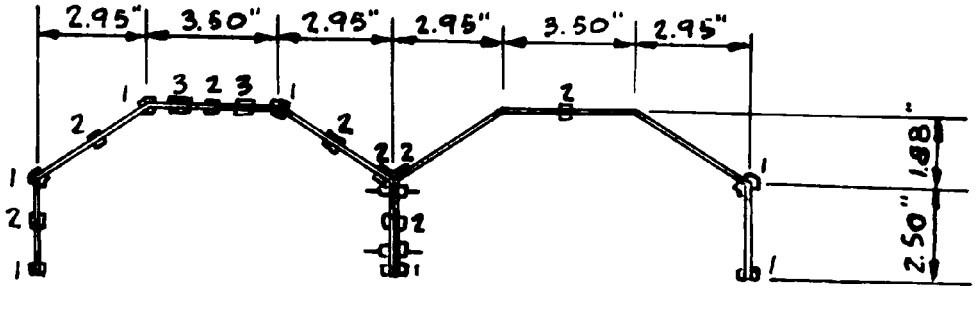
The final displacements and transverse moments will be presented at the end of the chapter along with the values obtained experimentally.

4. Experimental Investigation

The experimental investigation consisted of two phases. The first phase involved the testing of a symmetrically loaded model corresponding to the analysis made in Example 3.1; the second phase was concerned with the testing of the same model loaded unsymmetrically as considered in Example 3.2. The model employed was fabricated from two single bay, simple folded plates of the type investigated by Gaafar[10], Huang[11], and Ali[12]. The two units were bolted securely together at the common interior edge plate to simulate the monolithic construction assumed in the preceding analyses. The dimensions of the model are those used in Example 3.1 and 3.2 and shown in Fig. 3.7.



Side Elevation



Gage Types

1 - A 12

2 - A 1

3 - A 7

Cross Section

Fig. 3.7

The roof was fabricated from 0.125 inch thick aluminum sheets of 2024-T3 grade. End frames were bolted to the roof to provide end supports sufficiently rigid to prevent distortion of the roof cross section at the end supports. After bolting the model to a heavy, reinforced table, ten 0.0001 inch dial indicators were mounted along each edge plate, a total of thirty indicators being used. Five dial indicators at each plate were oriented to indicate vertical displacement and the other five, horizontal displacement. Fig. 3.8 shows the unloaded model ready for testing.

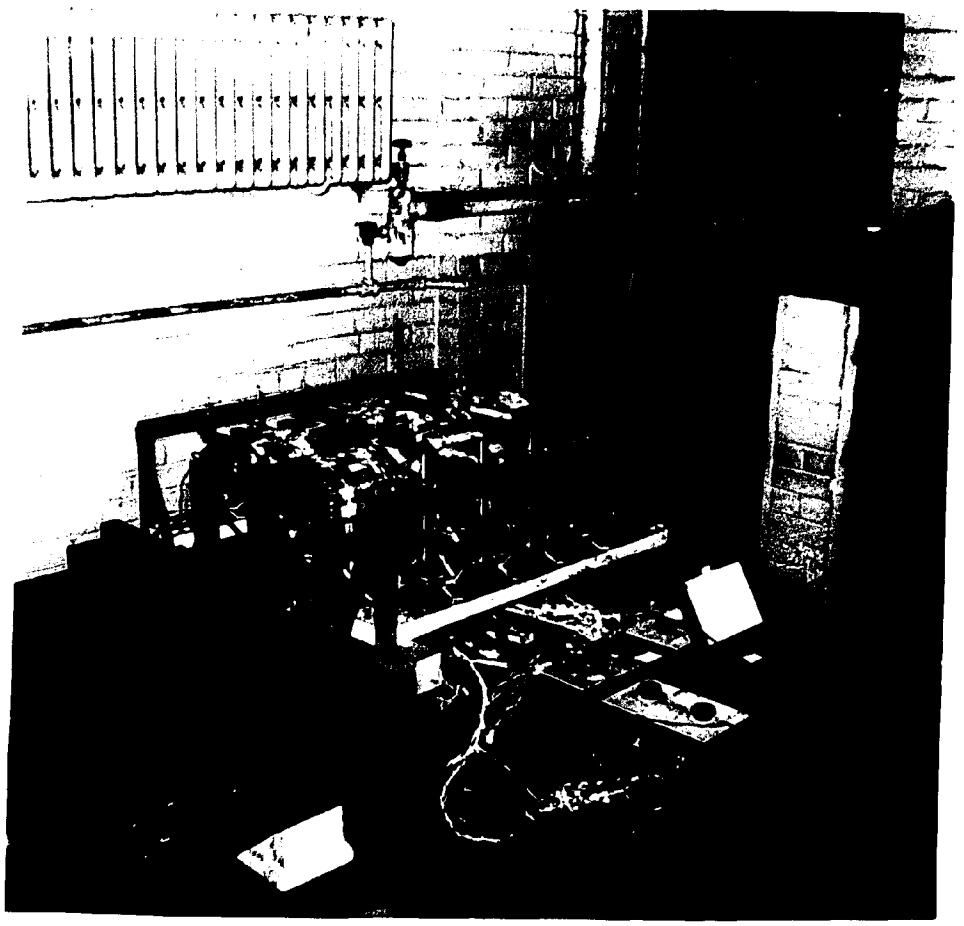


Fig. 3.8

SR-4 strain gages were attached to both the inner and outer surfaces of the roof. The gages oriented to indicate longitudinal strain were placed at each of the edges and at the mid-depth of each plate at a particular section. Gages were applied at three sections, the 1/9, 2/9, and midspan sections. The gages on the outer surface were connected in series with the corresponding gages on the inner surface to indicate average longitudinal strains.

Transversely oriented gages were attached to both surfaces at the above mentioned sections to provide information concerning transverse strains. The transverse gages were wired independently of one another since they were intended to provide information concerning slab bending. Gage types A 1 and A 12 were used for measuring longitudinal strains. A 7 gages were employed for measuring the transverse strains.

The model was loaded by placing fifty pound steel ingots on loading bars resting on the ridge edges at the one-third points of the span. The load was applied in two hundred pound increments in the investigation wherein both spans were loaded and one hundred pound increments in the testing of the unsymmetrically loaded model. The total load placed on the model with both spans loaded was 584.0 pounds, while 300.0 pounds was used in loading the model unsymmetrically. Strain readings as well as dial gage readings were recorded after the application of each load increment.

5. Summary and Comparison of Experimental and Analytical Results

The results of the experimental and analytical investigations are presented in graphical form in Diagrams 3.1 through 3.7. Diagrams 3.1 through 3.3 present the results of the studies made of the symmetrically loaded structure and Diagrams 3.4 through 3.7 present the results of the investigation of the unsymmetrically loaded structure.

Diagram 3.1 shows the distribution of longitudinal stress at midspan and at the 2/9 and 1/9 section. The plot is the vertical projection of the distribution. Diagram 3.2 presents the transverse bending stresses at midspan. Transverse strains were measured at the quarter points of plates 3 and 7 at this section. The horizontal and vertical displacements of the edges of plates 1, 5, and 9 are presented in Diagram 3.3.

Diagrams 3.4 through 3.7 present data obtained from the investigation of the unsymmetrically loaded folded plate. Diagram 3.4 shows the distribution of longitudinal stress at the midspan, 2/9 and 1/9 sections. The vertical projections of the stress distributions for the loaded span are shown. Diagram 3.5 shows the plot of longitudinal stresses in the unloaded span. Diagram 3.6 presents the transverse stresses and Diagram 3.7 the edge displacements.

Step 1 of the preceding analysis considered the computations for stresses, moments and displacements for the simple folded plate. The "free edge" phase of this analysis considered the plates adjacent to the loaded edges as carrying the full magnitude of the components of the applied load, with no aid from those plates framing one joint away from the loaded edges. If the transfer of a portion of the

applied load to plates one plate away from the loaded joint is considered, a considerable improvement in computed response results. Ali[12] recommended a method for estimating the magnitude of the load induced edge force to be considered for plates away from the loaded joint. The influence of the redistribution of the load is shown in Diagrams 3.1 through 3.7. Much more favorable agreement between computed and measured values results from the use of the correction.

The stiffening influence of the end diaphragms is indicated in Diagrams 3.1 and 3.4. The longitudinal stress distribution at the 1/9 section becomes quite similar for the interior and exterior plates.

The redistribution of the load as recommended by Ali has a noticeable effect on transverse bending stress as well as displacement. In each case an improvement of the calculated results is provided.

The analytical results are in good agreement with the experimental results in the unsymmetrically loaded case. The improvement in the agreement resulting from considering plates 1 and 6 to aid in carrying the applied loads is particularly evident in the longitudinal stress distribution and the displacement of plate 1.

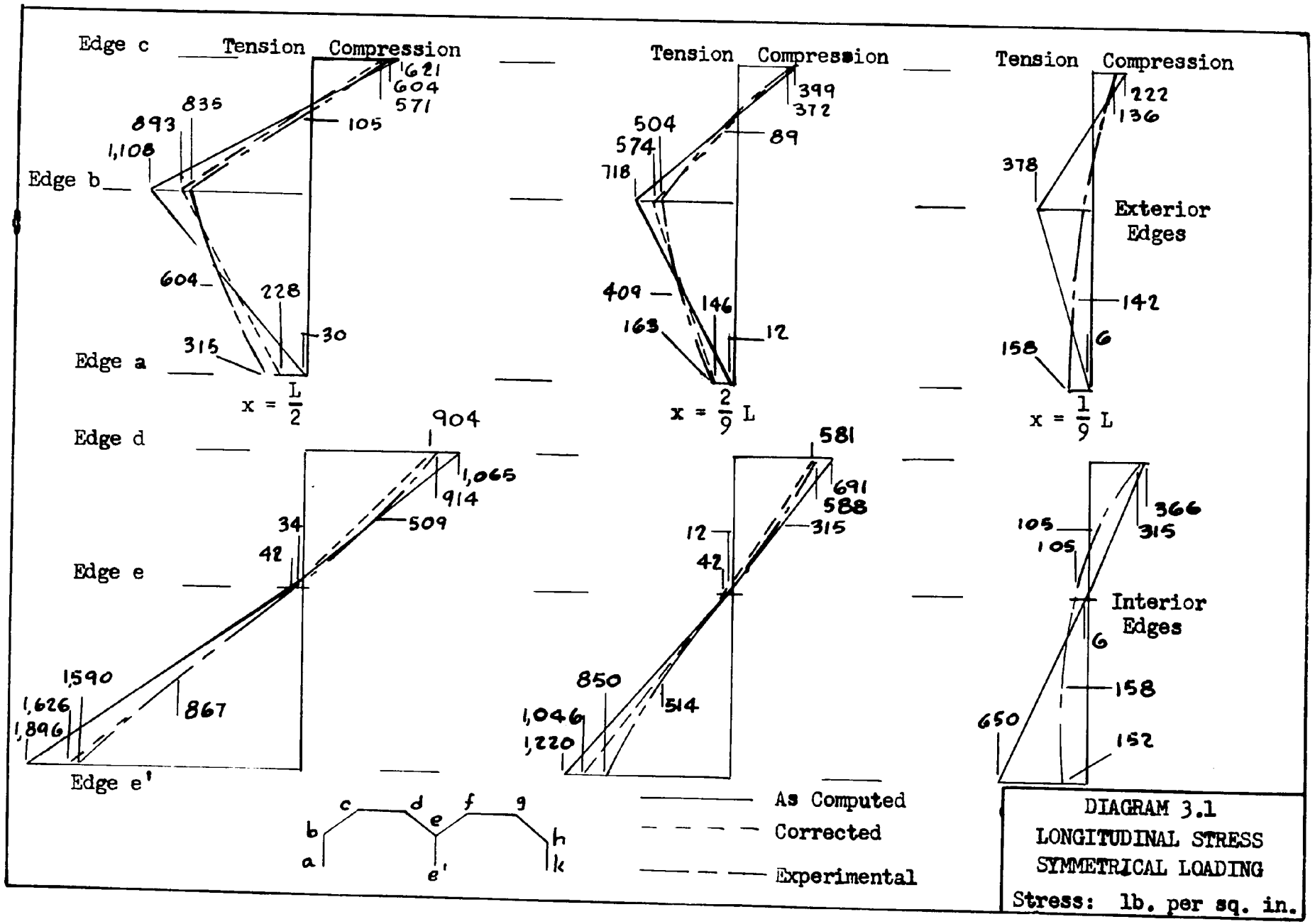
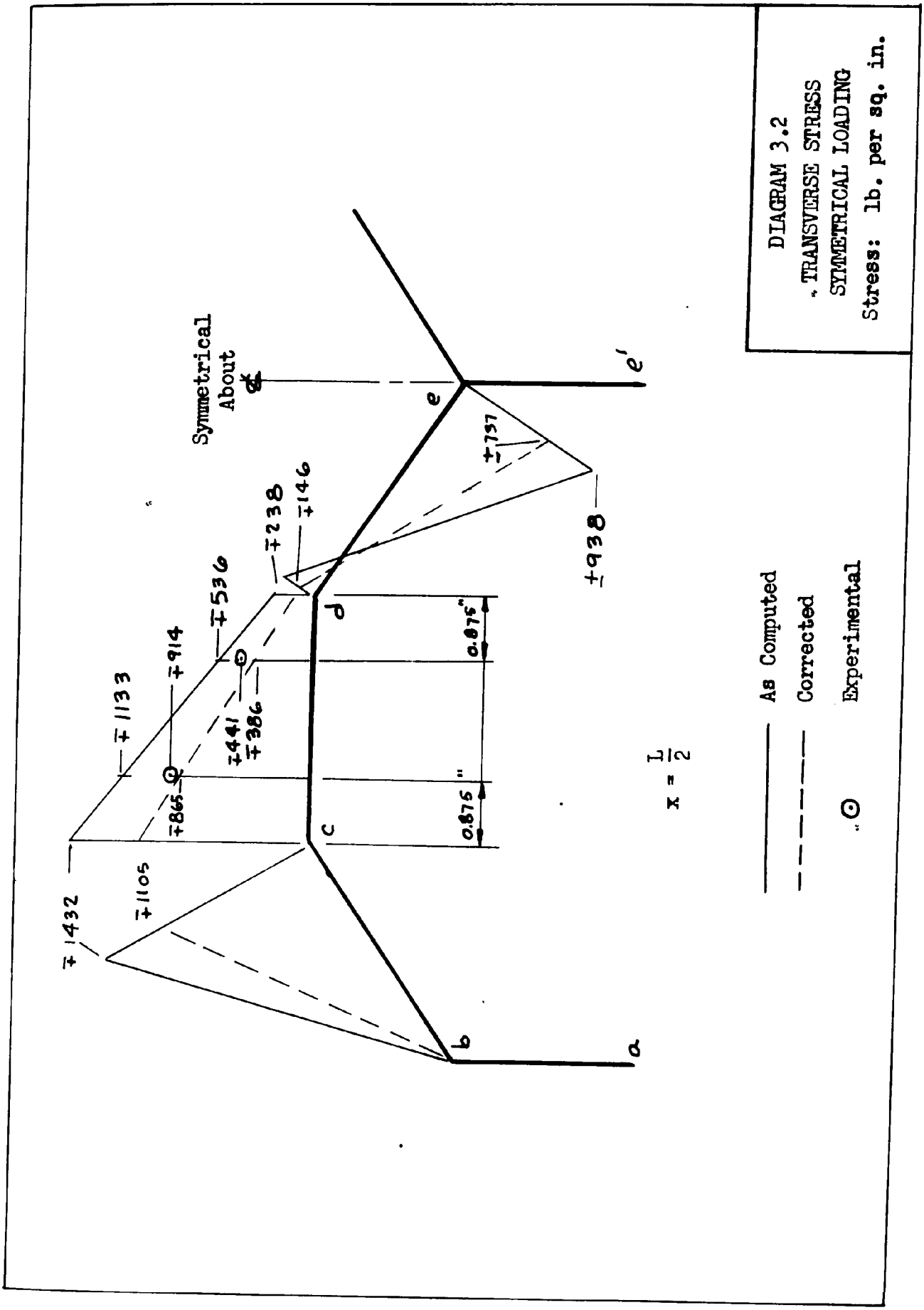
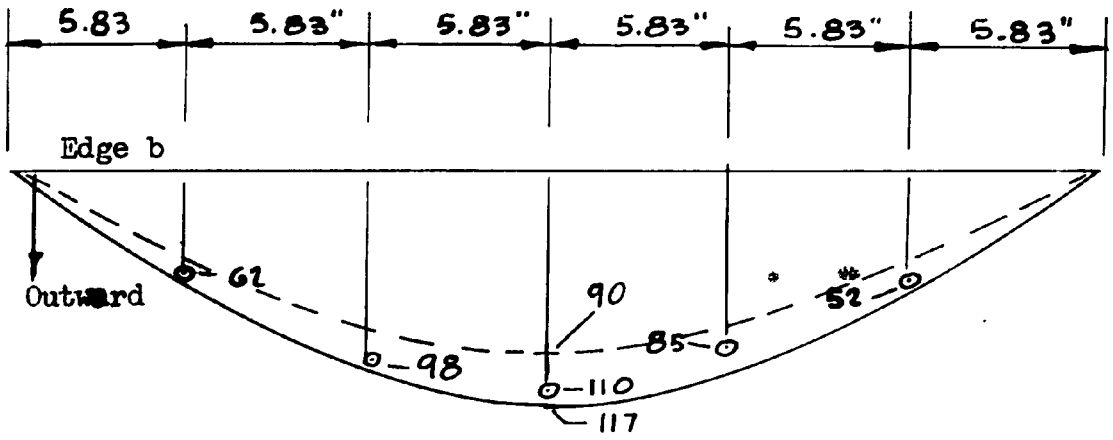


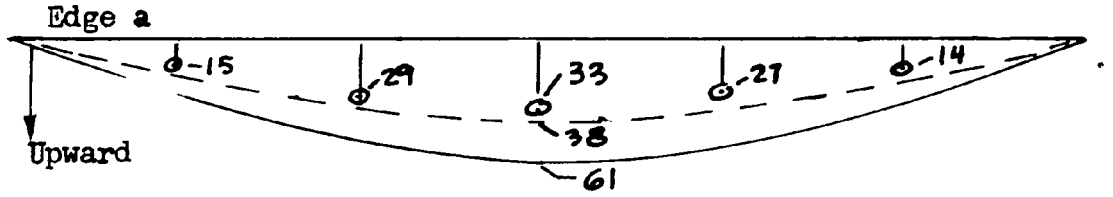
DIAGRAM 3.1
LONGITUDINAL STRESS
SYMMETRICAL LOADING
Stress: lb. per sq. in.

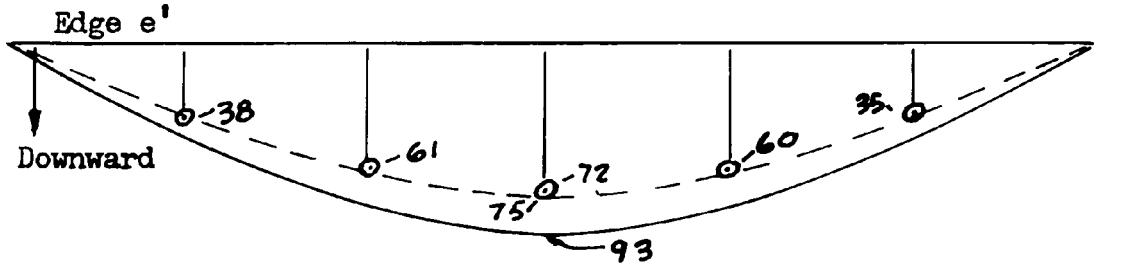

DIAGRAM 3.2
TRANSVERSE STRESS
SYMMETRICAL LOADING
Stress: lb. per sq. in.
As Computed
Corrected
Experimental
x = L/2
Symmetrical About
±1432
±1105
±1133
±866
±914
±441
±386
±536
±238
±146
±737
±938
0.875"
0.875"
a
b
c
d
e
e'



Edge e' (No Measureable Displacement)

HORIZONTAL DISPLACEMENT





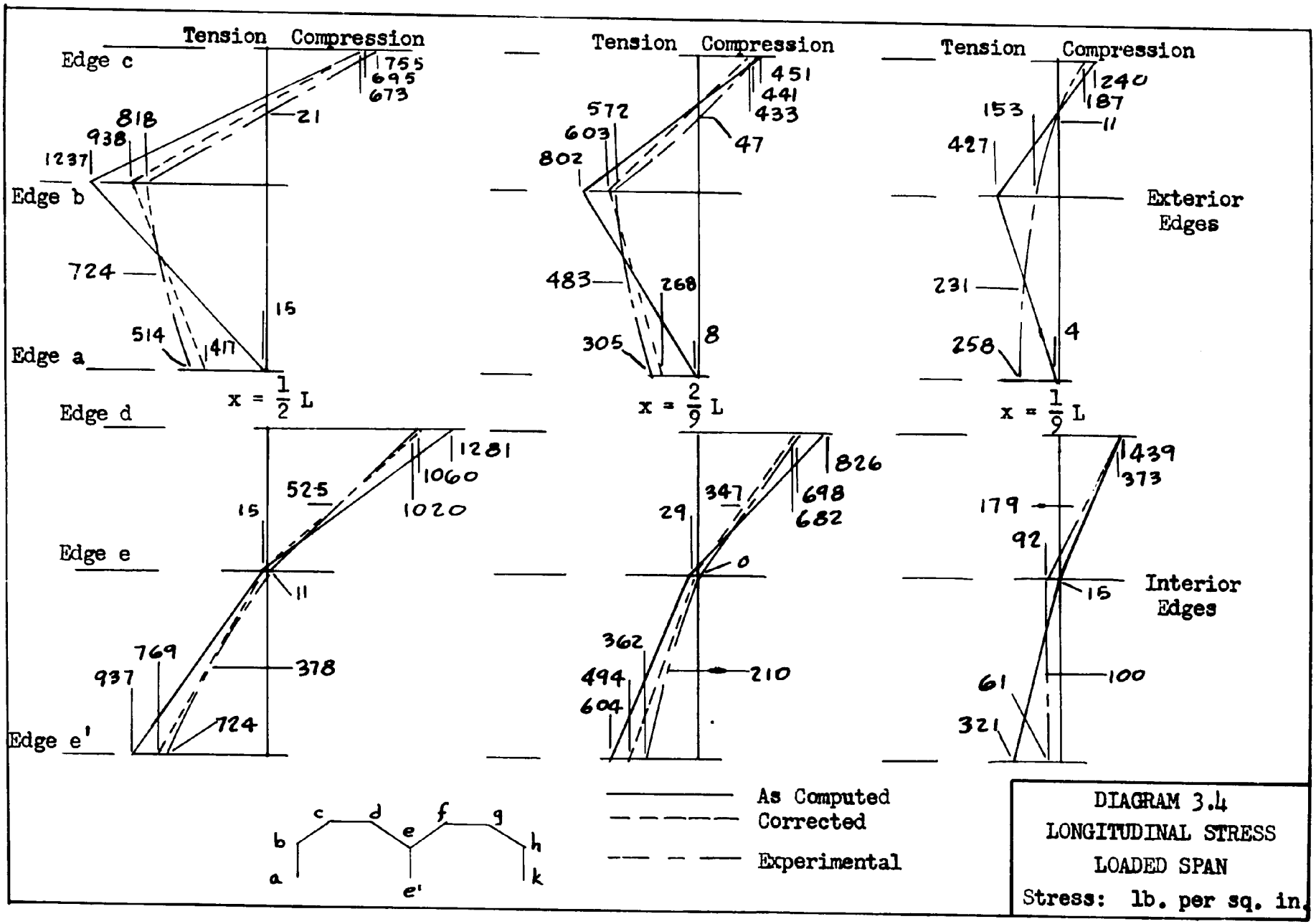
VERTICAL DISPLACEMENT

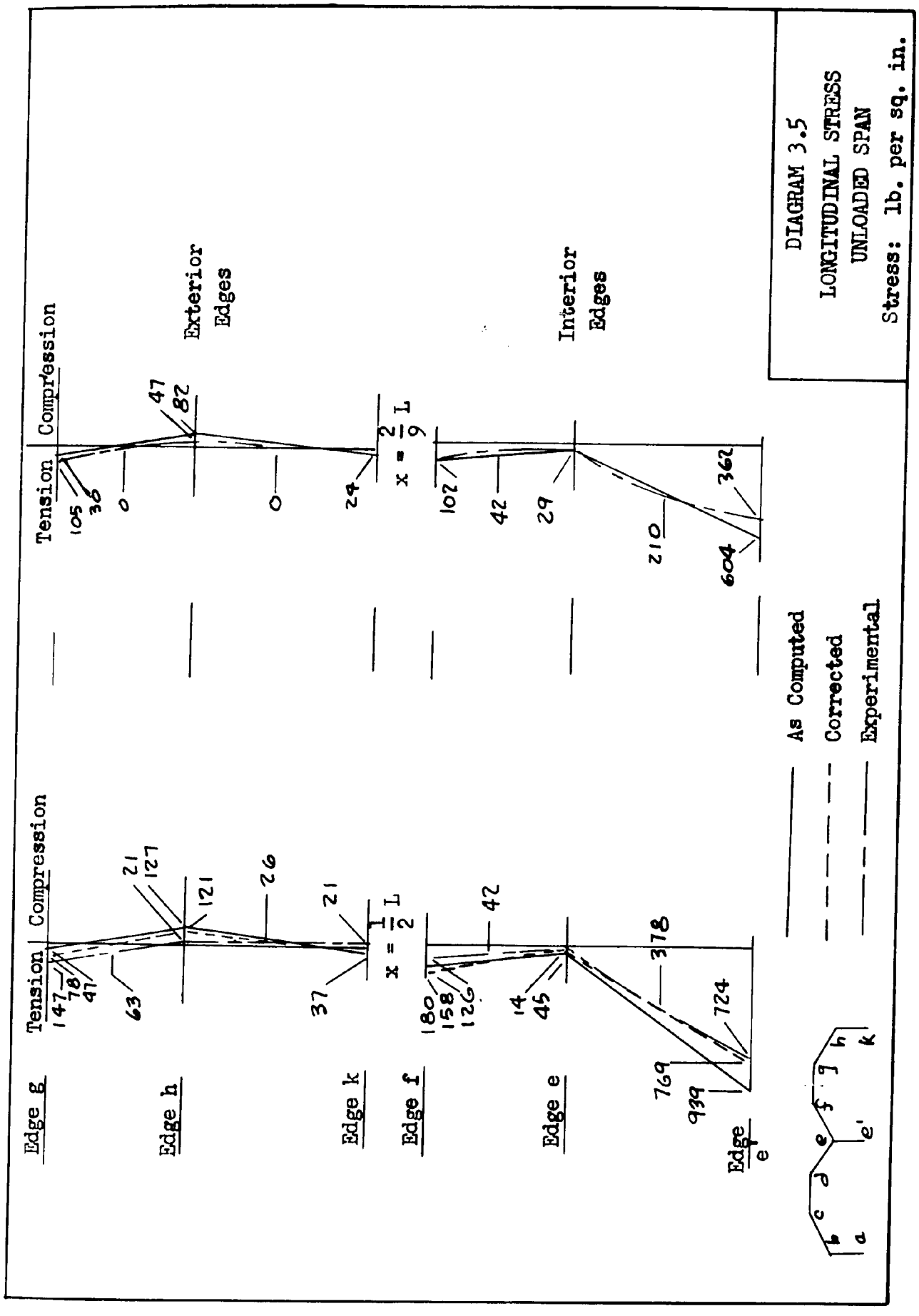
——— As Computed

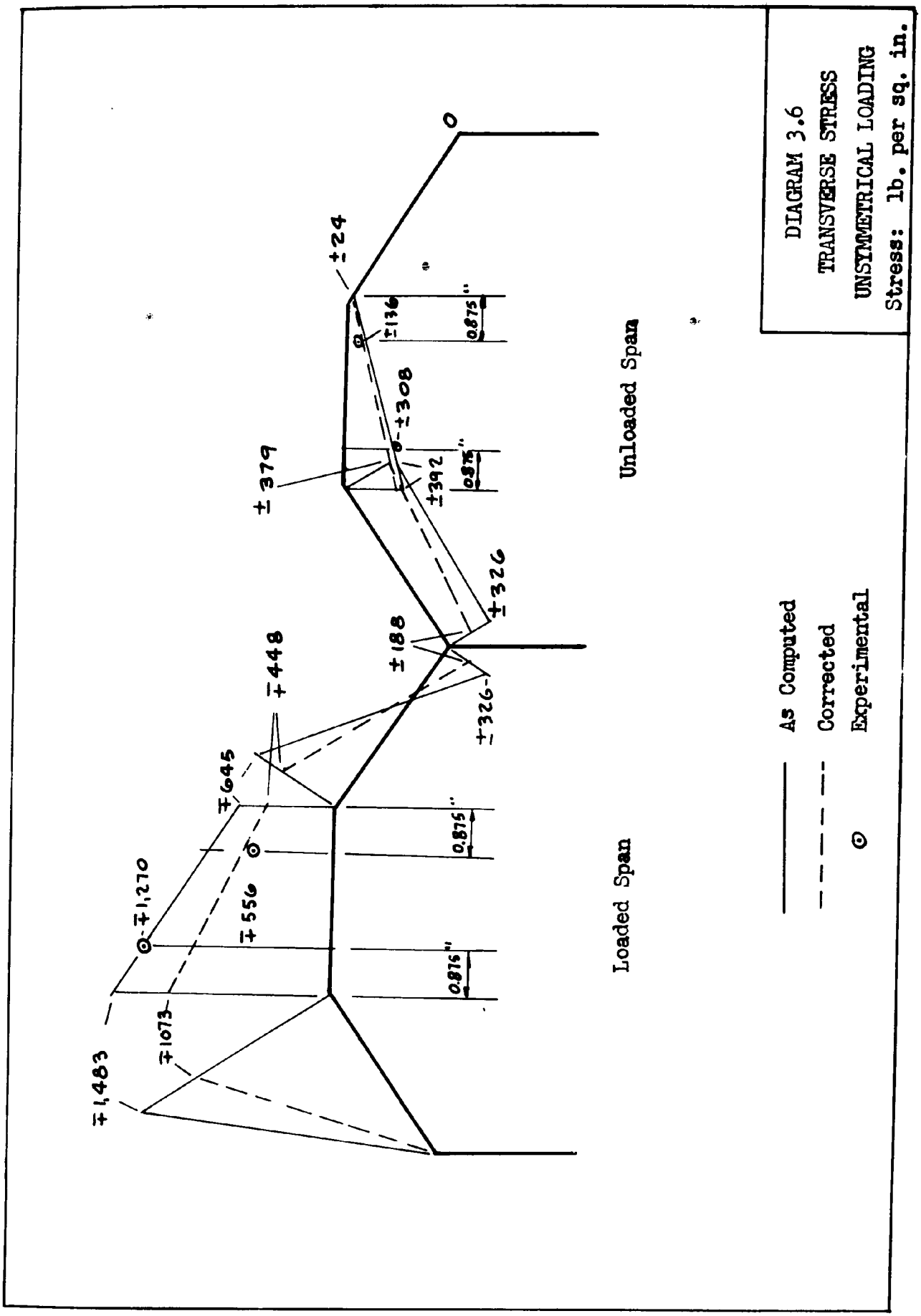
— — — — — Corrected

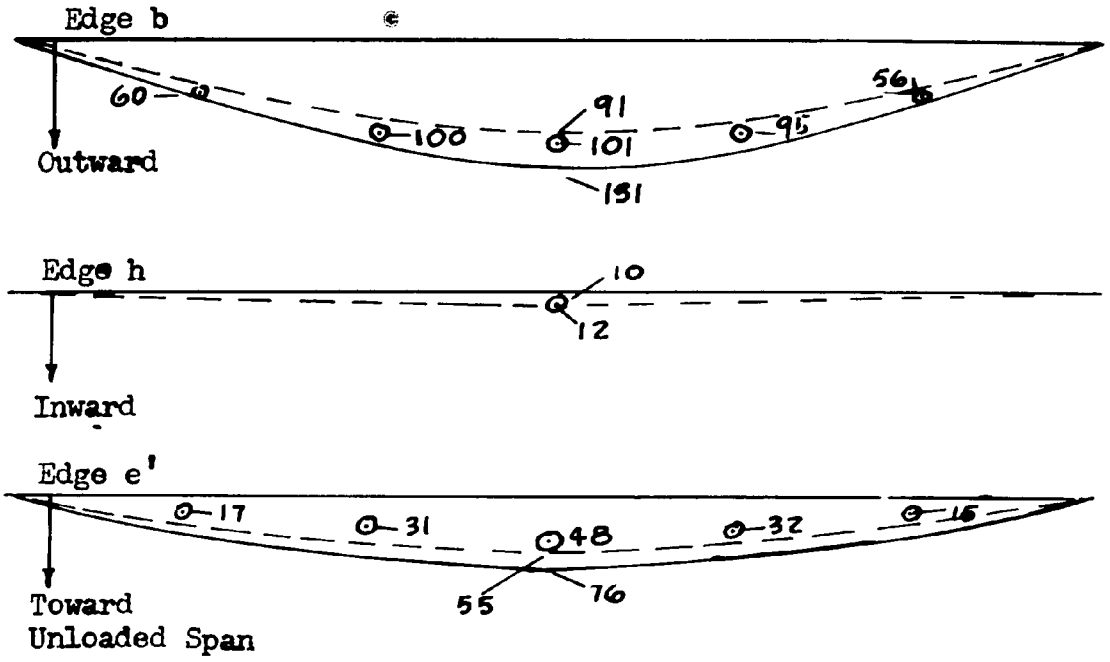
⊙ Experimental

DIAGRAM 3.3
EDGE DISPLACEMENT
SYMMETRICAL LOADING
Displacements: in.$(10)^{-4}$

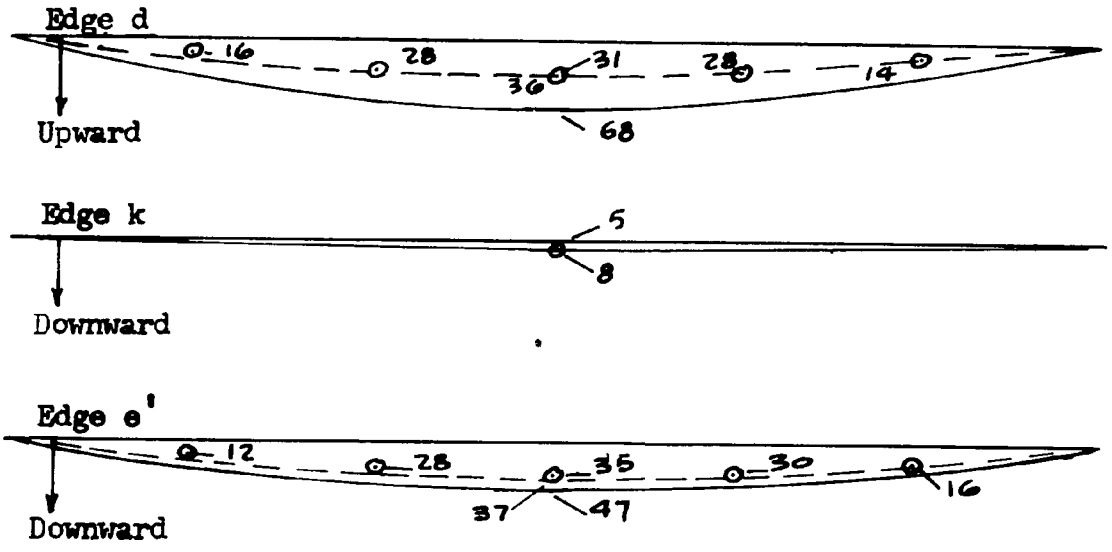

Tension Compression
Edge c
755
695
673
818
938
1237
21
Edge b
724
15
514
417
Edge a
x = 1/2 L
Tension Compression
451
441
433
572
603
802
47
483
268
305
8
x = 2/9 L
Tension Compression
240
187
153
11
427
Exterior Edges
231
258
4
x = 1/9 L
Edge d
1281
1060
525
1020
15
Edge e
11
769
937
378
724
Edge e'
826
347
698
682
29
0
362
494
604
210
439
373
179
92
Interior Edges
16
61
100
321
As Computed
Corrected
Experimental
a
b
c
d
e
e'
f
g
h
k
DIAGRAM 3.4
LONGITUDINAL STRESS
LOADED SPAN
Stress: lb. per sq. in.


DIAGRAM 3.5
LONGITUDINAL STRESS
UNLOADED SPAN
Stress: lb. per sq. in.
Tension
Compression
Exterior Edges
Interior Edges
Edge g
Edge h
Edge k
Edge f
Edge e
Edge e'
x = 2/9 L
x = 1/2 L
As Computed
Corrected
Experimental
a
b
c
d
e
f
g
h
k
e'


DIAGRAM 3.6
TRANSVERSE STRESS
UNSYMMETRICAL LOADING
Stress: lb. per sq. in.
As Computed
Corrected
Experimental
Loaded Span
Unloaded Span
0
±24
±136
0.875"
±308
±379
±392
0.875"
±326
±188
±326
±448
±645
0.875"
±1,270
±556
0.875"
±1073
±1,483



HORIZONTAL DISPLACEMENT



VERTICAL DISPLACEMENT

As Computed

Corrected

⊙ Experimental

DIAGRAM 3.7
EDGE DISPLACEMENT
UNSYMMETRICAL LOADING
Displacement: in.$(10)^{-4}$

CHAPTER IV

THE INVESTIGATION OF A FOLDED PLATE STRUCTURE CONTINUOUS OVER A FRAME NOT RIGID IN ITS PLANE

1. Introduction

Frequently the design of folded plate roofs can be improved through the use of multiple and continuous construction. The effect of both of these types of construction is to reduce the stresses, moments, and displacements in the folded plate. Interior frames providing deep, rigid diaphragms improve the structural action of the plate system by reducing edge stresses, slab moments and deformations. Frequently, however, it is not desirable to construct interior frames containing deep diaphragms because of their interference in the utilization of the space enclosed by the roof. In a case where the diaphragm is omitted from the construction, the dimensions of the frame are no longer so large and the frame so stiff that the frame displacements can be neglected in the analysis of the structure.

Ali[12] has presented a method of analysis for folded plate structures continuous over rigid interior supports. The method is similar to that presented by Gaafar[10] for the analysis of simple folded plate roofs, except that Ali considered the individual plates as continuous over non-yielding supports in the "free edge" phase of the analysis.

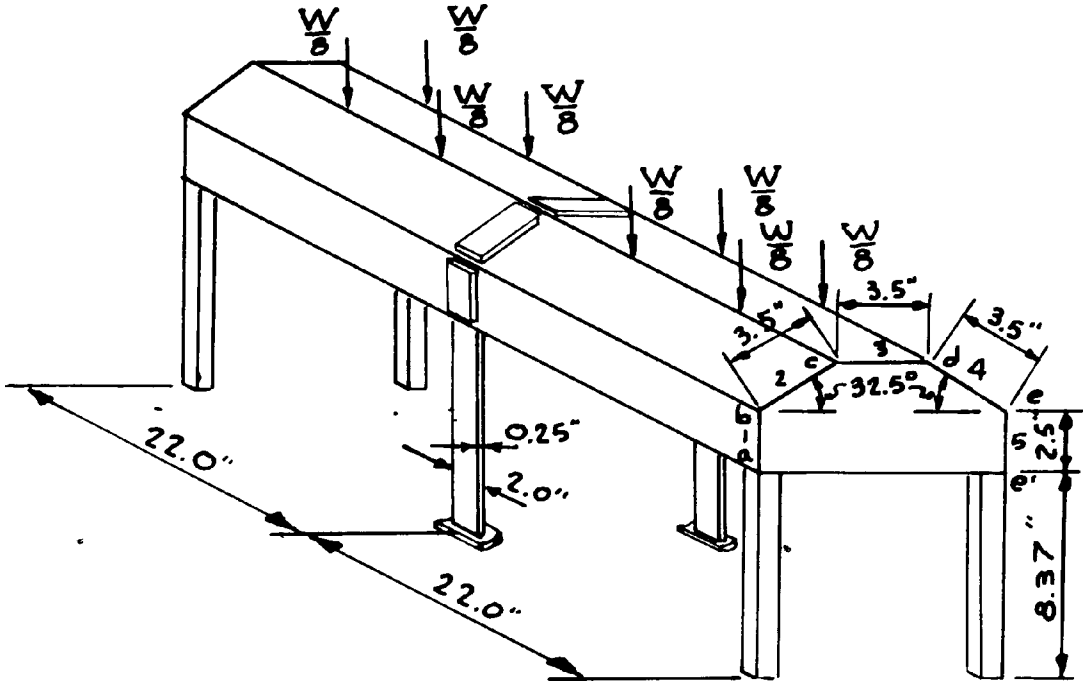

W/8
3.5"
3.5"
3.5"
32.5°
22.0"
22.0"
0.25"
2.0"
8.37"
2.5"

Fig. 4.1

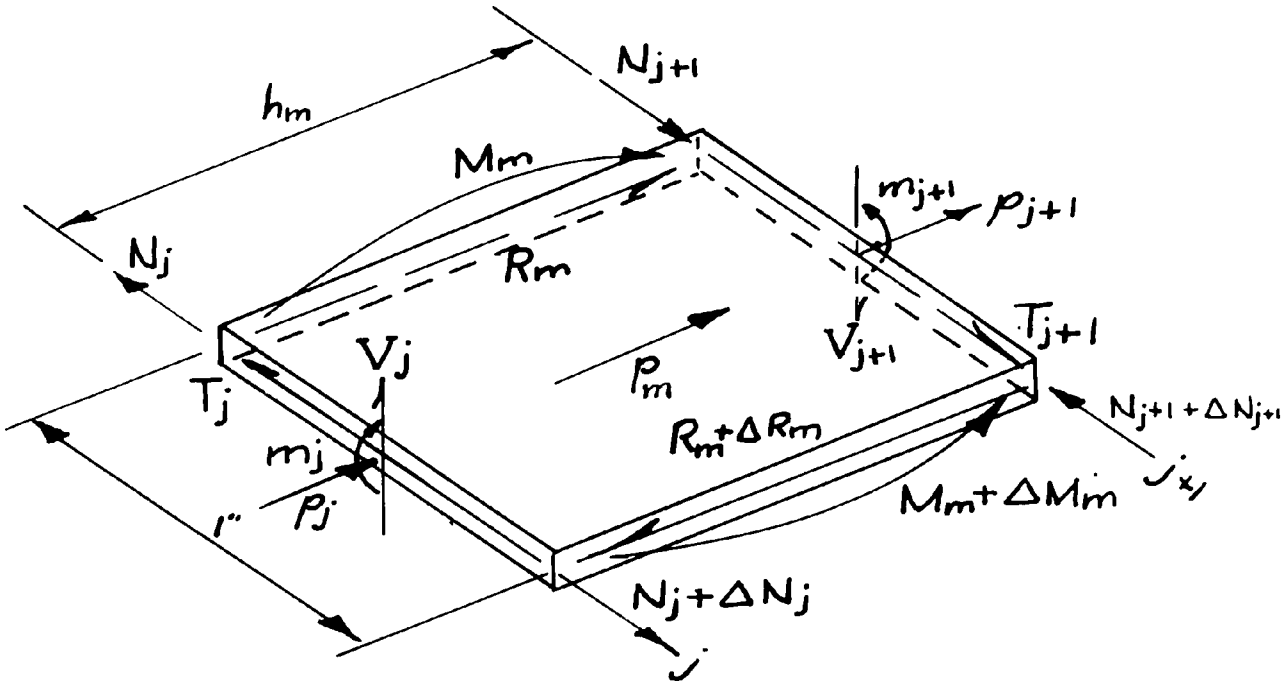

h_m
N_{j+1}
M_m
m_{j+1}
p_{j+1}
N_j
R_m
T_{j+1}
V_j
V_{j+1}
T_j
p_m
$N_{j+1}+\Delta N_{j+1}$
$R_m+\Delta R_m$
m_j
$M_m+\Delta M_m$
p_j
1"
$N_j+\Delta N_j$
j
x

Fig. 4.2

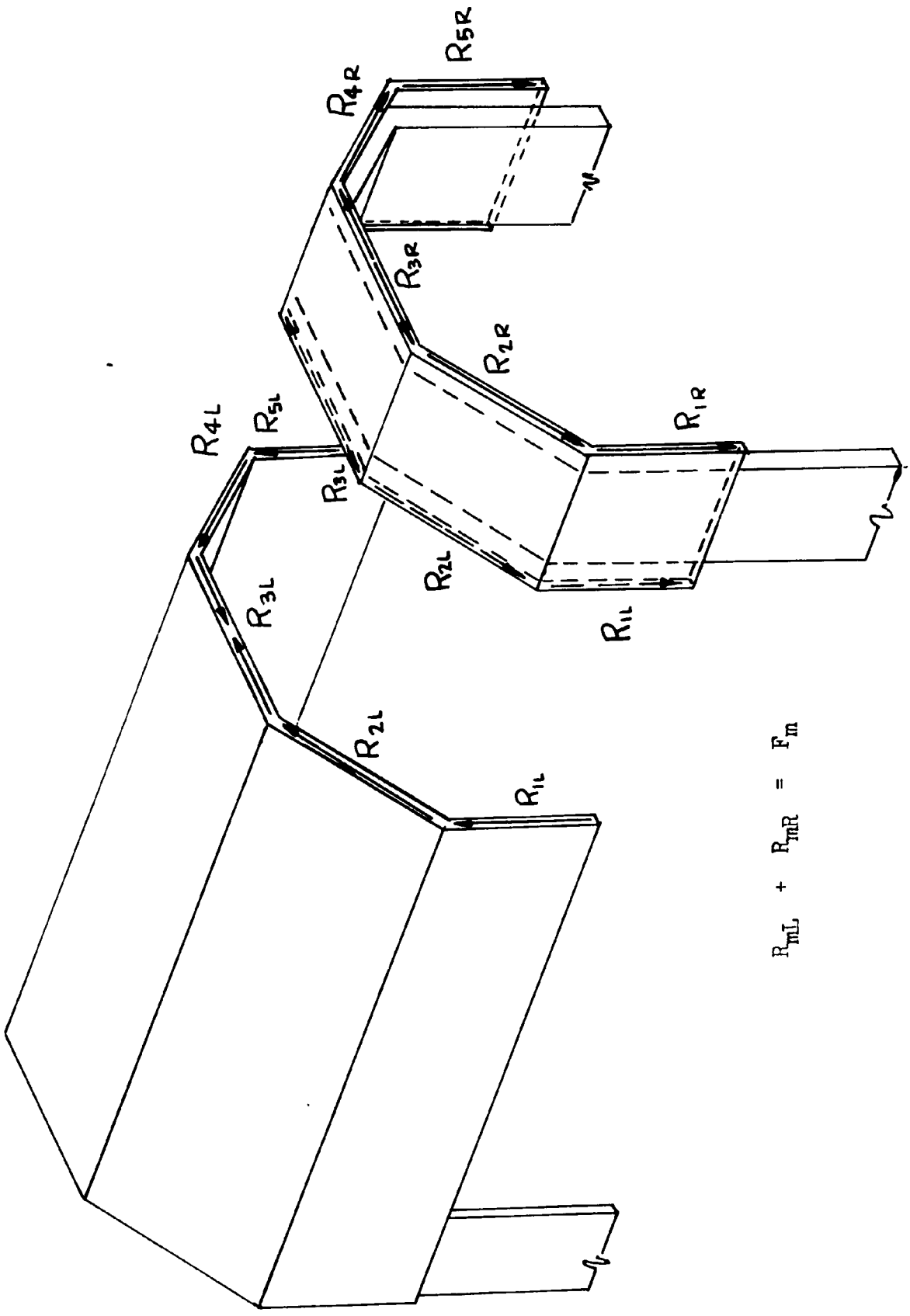
This chapter presents the results of an investigation of a folded plate continuous over a frame not rigid in its plane. The structure to be analyzed is presented in Fig. 4.1. A section of unit length of one of the plates is shown in Fig. 4.2; the applied and internal forces are shown in their assumed positive sense. Consistent with the assumptions listed in Chapter II, all other internal forces are considered small and are neglected.

In the following analysis it will be assumed that the approximate methods of analysis as improved by Winter and Pei[9], Gaafar[10], and Ali[12] may be utilized. It is further assumed that the interaction of the plates with the frame can be provided through resultants, F_m, which act in the middle plane of each plate on the cross section adjacent to the frame as shown in Fig. 4.3. The principles of superposition and consistent deformation are assumed as being applicable.

The change in the magnitude of the resultants, F_m, is computed as the frame undergoes a change in stiffness from the rigid condition assumed by Ali[12] to the more flexible condition provided in this investigation. The resultant, F_m, acting on the roof is the sum of the R_m forces acting on plate cross sections adjacent to the frame. The shear forces, R_m, can be computed from the equilibrium equations for the portions of plate on either side of the frame, considering each portion as a free body. R_m is a function of p_j and M_m only.

2. General Procedure

The steps in the analysis can be enumerated as follows.



$$R_{mL} + R_{mR} = F_m$$

Fig. 4.3

STEP 1. Analyze the structure for the effects of the applied loads, assuming the interior support to be non-yielding. Compute the resultants, F_m, for this rigid support condition; the resultants will be designated F_{om}.

STEP 2. Remove the frame and analyze the simple folded plate for the effect of unit values of F_m.

The unit forces applied in Step 2 represent unit values of the correction resultant, F'_m. The sum of F'_m and F_{om} computed for plate m is equal to the final resultant F_m. The computations in Steps 1 and 2 include corrections for relative edge displacement.

STEP 3. Analyze the frame for the effect of unit forces applied in the middle plane of the plates which act as effective frame flanges. These forces are unit values of the final resultants F_m acting on the frame.

STEP 4. Write the equations expressing the equality of the δ_m displacements of the folded plate and the frame. One equation can be written for each plate in the system. The equations are independent and sufficient to provide for the solution for the unknown resultants, F_m.

STEP 5. The final values for stress, moment, and displacement for the folded plate are calculated as the sum of the quantities obtained from the solution in Step 1 and the values from Step 2 multiplied by the appropriate values of F'_m.

The application of this procedure will be demonstrated by an example in the remaining portion of this chapter. The results of the analysis will be compared with the results of tests performed to verify the foregoing recommendations.

3. Example 4.1

The structure shown in Fig. 4.1 is a model of a single bay, two span folded plate roof continuous over an interior frame not rigid in its plane, the frame being placed at the midspan section of the folded plate. The load consists of eight 50 pound concentrations applied to the top edges at the one third points of the spans.

STEP 1. Assuming the interior support to be rigid, the structure is analyzed for the effect of the applied load.

After resolving the loads into components in the plane of the plates joining at the loaded edges, Ali's[12] recommendation for the transfer of a portion of the applied load to adjacent plates is employed. The following values for the plate loads, P, are computed.

$$P_1 = -P_5 = -6.6 \text{ lb.}$$

$$P_2 = -P_4 = -81.0 \text{ lb.}$$

The free edge bending moments caused by the above P forces are computed assuming the interior support to be non-yielding and assuming the loaded plates to act as isolated two span members loaded at the third points. The edge stresses are calculated at the sections 0.4375 L, 0.500 L, 0.8635 L, and at the interior support.

Section	Plate	Free Edge Stress lb. per sq. in.
0.4375 L	1	± 201
	2	± 1256
0.500 L	1	± 179
	2	± 1120
0.8635 L	1	∓ 163
	2	∓ 1015
L	1	∓ 358
	2	∓ 2240

The free edge stresses are distributed in the usual manner producing the following balanced edge stresses.

Section	Edge	Balanced Edge Stresses lb. per sq. in.
0.4375 L	a	- 123
	b	+ 446
	c	- 340
0.500 L	a	- 110
	b	+ 399
	c	- 304
0.8635 L	a	+ 100
	b	- 362
	c	+ 275
L	a	+ 220
	b	- 798
	c	+ 608

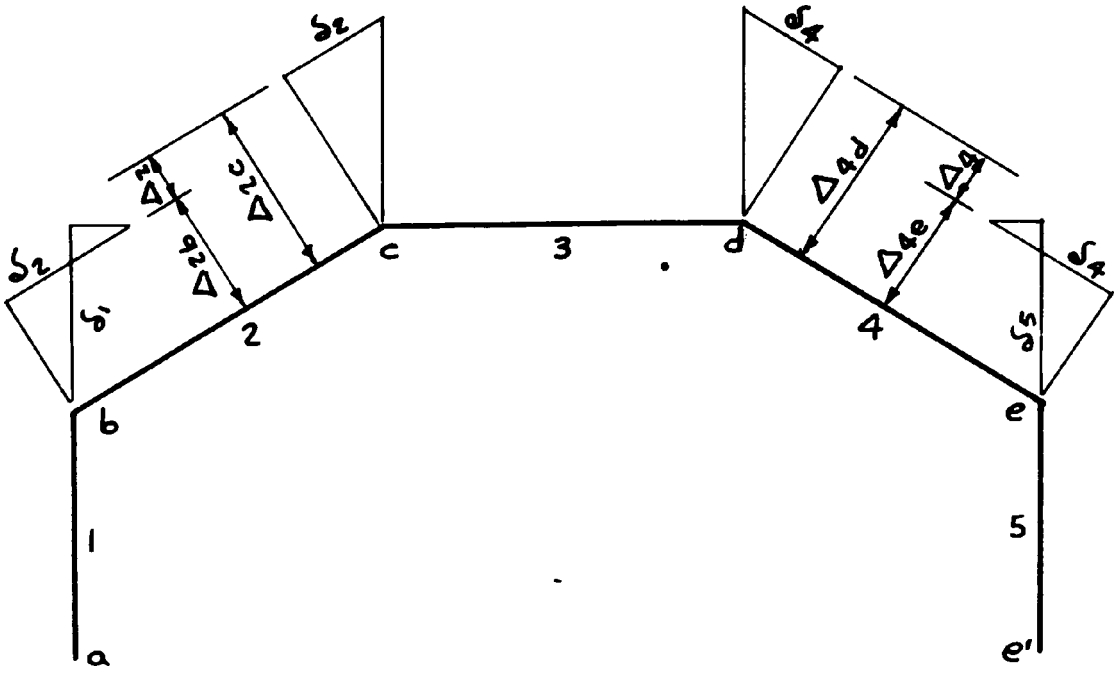


Fig. 4.4

The proportions of the structure are such that the effect of relative edge displacement must be considered. The relative edge displacement, Δ, shown in Fig. 4.4, is assumed to vary sinusoidally with x. The computations for the free edge stresses in this phase of the analysis are based once again on the assumption that the plates span continuously across a non-yielding interior support. The load on the plates is an edge force, p, which varies in the same way that Δ varies. The free edge stresses are computed at the several sections and are presented in the following tabulation.

Section	Plate	Free Edge Stress
0.4375 L	1	$\pm$ 82.5α
	2	$\mp$ 78.9α
0.500 L	1	$\pm$ 76.6α
	2	$\mp$ 73.3α
0.8635 L	1	$\mp$ 59.6α
	2	$\pm$ 57.0α
L	1	$\mp$ 140 α
	2	$\pm$ 134 α

Distributing the free edge stresses yields the balanced edge stresses tabulated below.

Section	Edge	Balanced Edge Stress
0.4375 L	a	+ 73 α
	b	- 64 α
	c	+ 29 α
0.500 L	a	+ 68 α
	b	- 59 α
	c	+ 27 α
0.8635 L	a	- 53 α
	b	+ 46 α
	c	- 21 α
L	a	- 124 α
	b	+ 109 α
	c	- 49 α

The δ displacements at 0.500 L are computed as the sum of the values taken from the calculations neglecting relative edge displacement and those induced by relative edge displacement.

$$\delta_1 = - \frac{0.088 \times (22)^2}{10.5\ (10)^6 \times 2.5} (-110 - 399) - \frac{0.080 \times (22)^2 \alpha}{10.5\ (10)^6 \times 2.5} (68+59)$$

$$= 825\ (10)^{-6} - 29{,}300\ (10)^{-6}\ \Delta$$

$$\delta_2 = -\frac{0.088 \times (22)^2}{10.5\ (10)^6 \times 3.5}(399 + 304) - \frac{0.080 \times (22)^2 \alpha}{10.5\ (10)^6 \times 3.5}(-59 - 27)$$

$$= -815\ (10)^{-6} + 14{,}200\ (10)^{-6}\ \Delta$$

where $\Delta = \frac{EI_T}{h^2}\alpha = \frac{(10.5)\ (10)^6\ (183)\ (10)^{-6}}{(3.5)^2} = 156.5\alpha$

The displacements δ and Δ are related through the geometry of the distorted cross section shown in Fig. 4.4.

$$\Delta = \frac{\delta_1}{\cos\theta} - \delta_2\ (\tan\theta + \cot\theta)$$

$$\Delta = 1.186\,\delta_1 - 2.207\,\delta_2$$

Substituting the expressions for δ_1 and δ_2 into the above expression for the relative edge displacement provides an equation which can be solved for Δ.

$$\Delta = 2.60\ (10)^{-3}\ \text{in.}$$

$$\alpha = 156.5\Delta = 0.407\ \text{in. lb.}$$

The corrected edge stresses are obtained as the sum of the stresses computed neglecting relative edge displacement and those induced by edge displacement.

Section	Edge	Neglecting Relative Edge Displacement	Relative Edge Displacement	Total lb. per sq. in.
0.4375 L	a	- 123	+ 30	- 93
	b	+ 446	- 26	+ 420
	c	- 340	+ 12	- 328
0.500 L	a	- 110	+ 28	- 82
	b	+ 399	- 24	+ 375
	c	- 304	+ 11	- 293
0.8635 L	a	+ 100	- 22	+ 78
	b	- 362	+ 19	- 343
	c	+ 275	- 9	+ 266
L	a	+ 220	- 50	+ 170
	b	- 798	+ 44	- 754
	c	+ 608	- 20	+ 588

Having determined the total edge stresses, the basic equations 2.1 and 2.2, derived in Chapter II, are applied to determine the longitudinal shear resultants N_j and the plate moments M_m at the section adjacent to the interior frame.

Plate 1: $+ 170 = \frac{M_1}{0.1354} + \frac{2}{0.325}(0 + N_b)$

$- 754 = - \frac{M_1}{0.1354} - \frac{2}{0.325}(2N_b + 0)$

from which $N_b = + 95.0$ lb.

$M_1 = - 56.1$ in. lb.

Plate 2: $- 754 = \frac{M_2}{0.266} + \frac{2}{0.455}(2 \times 95.0 + N_c)$

$+ 588 = - \frac{M_2}{0.266} - \frac{2}{0.455}(2N_c + 95.0)$

from which $N_c = + 133$ lb.

$M_2 = - 578$ in. lb.

$M_3 = 0$ from symmetry.

The plate moment, M_m, is a function of the edge loads, p and P, and the support conditions only. P is the applied plate load and p is the edge force resulting from relative edge displacement. In the preceding analysis M_m and N_j were determined; the edge loads p_j are known from the computations for relative edge displacement. Thus, each of the plates may be isolated and the resultants, F_{om}, computed using the equations of statics. The free body sketches are shown in Fig. 4.5.

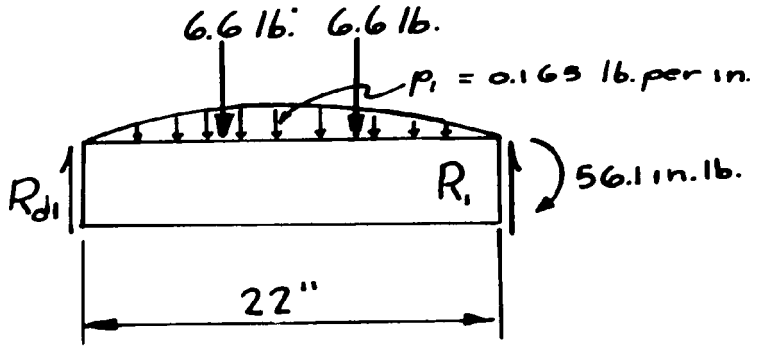


Plate 1

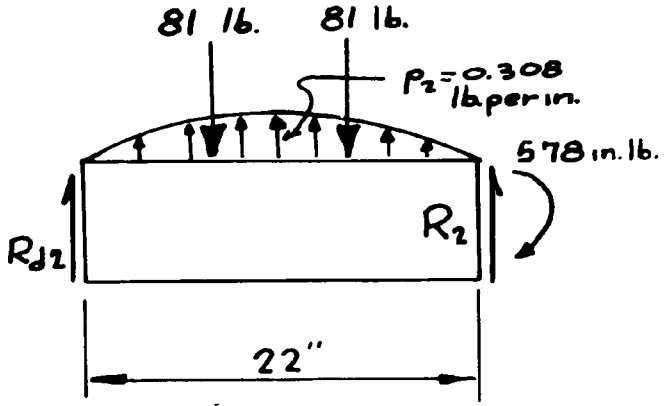


Plate 2

Fig. 4.5

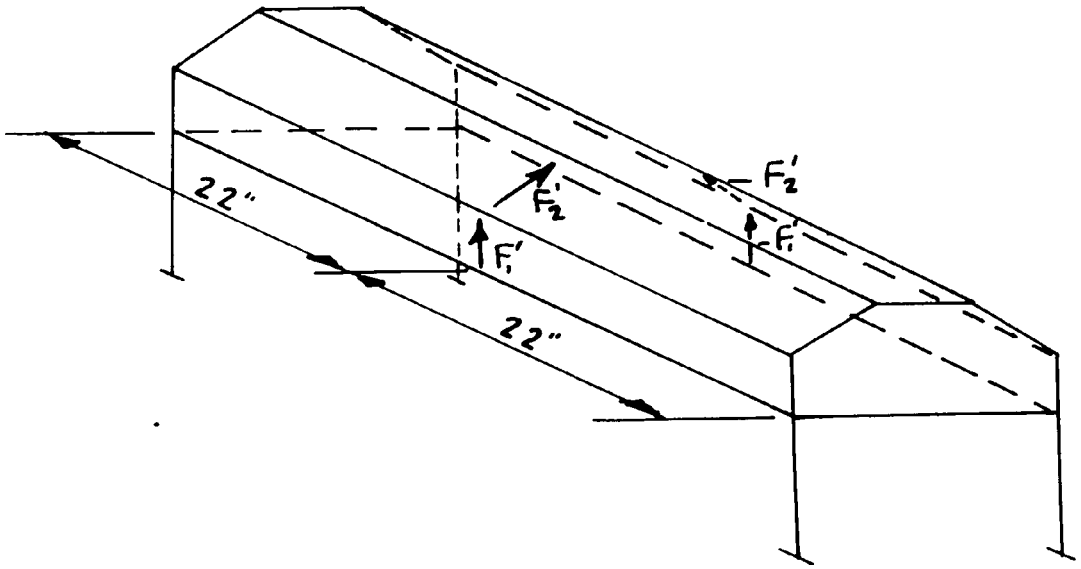


Fig. 4.6

$$F_{01} = 2\left[\frac{11(13.2 + 2.31) + 56.1}{22}\right] = + 20.6 \text{ lb.}$$

$$F_{02} = 2\left[\frac{11(162.0 - 4.3) + 578}{22}\right] = +210.2 \text{ lb.}$$

The positive sign indicates the sense of the resultants to be that shown in Fig. 4.5 for the end shears R_m.

STEP 2. The analysis of the simply supported roof for the effect of the correction resultants proceeds in a manner similar to that demonstrated in Chapter II for the analysis of simple folded plates. The forces are applied in the middle plane of the plates at the section at which the frame joins the roof, as is shown in Fig. 4.6.

The free edge stresses at the midspan section are computed as follows.

$$f_{1a} = -f_{1b} = -\frac{11.00\ F_1'}{0.1354} = -81.24F_1'$$

$$f_{2b} = -f_{2c} = -\frac{11.00\ F_2'}{0.2660} = -41.35F_2'$$

After distributing the free edge stresses, the following balanced edge stresses are obtained. All stresses are lb. per sq. in.

Section	Edge	$F_1' = 1$	$F_2' = 1$
L	a	- 55.92	+ 8.55
°	b	+ 30.59	- 17.09
	c	- 6.12	+ 11.89

The balanced edge stresses at sections x = 0.4375 L and 0.8635 L are as follows.

Section	Edge	Balanced Edge Stresses $F_1' = 1$	$F_2' = 1$
0.4375 L	a	- 24.47	+ 3.74
	b	+ 13.38	- 7.48
	c	- 2.68	+ 5.20
0.8635 L	a	- 48.29	+ 7.38
	b	+ 26.41	- 14.76
	c	- 5.28	+ 10.27

The effect of relative edge displacement is considered, producing the following balanced edge stresses in terms of $\propto$.

Section	Edge	Balanced Edge Stresses
0.4375 L	a	+ 316 $\propto$
	b	- 276 $\propto$
	c	+ 123 $\propto$
0.8635 L	a	+ 487 $\propto$
	b	- 424 $\propto$
	c	+ 189 $\propto$
L	a	+ 498 $\propto$
	b	- 434 $\propto$
	c	+ 193 $\propto$

Displacements δ_1 and δ_2 at x = L are computed as the sum of the displacement neglecting the effect of relative edge displacement and the displacement component δ caused by relative edge displacement.

$$\delta_1 = + 531.7(10)^{-6}F_1' - 157.6(10)^{-6}F_2' - 1.090\Delta$$

$$\delta_2 = - 161.0(10)^{-6}F_1' + 127.2(10)^{-6}F_2' + 0.5238\Delta$$

Substituting the expressions for $_1$ and $_2$ into the geometrical relation between Δ and $_1$ and $_2$, an equation results which can be solved for Δ.

$$\Delta = 1.186\delta_1 - 2.209\delta_2$$

$$\Delta = +986.4(10)^{-6}F_1' - 467.6(10)^{-6}F_2' - 2.449\Delta$$

$$\Delta = 286.0(10)^{-6}F_1' - 135.6(10)^{-6}F_2'$$

$$\alpha = 44,759(10)^{-6}F_1' - 21,221(10)^{-6}F_2'$$

The values of δ_1 and δ_2 are now computed. In the resulting expressions, the coefficients are displacements produced by unit values of F_1' and F_2'.

$$\delta_1 = 220.0(10)^{-6}F_1' - 9.8(10)^{-6}F_2'$$

$$\delta_2 = -11.4(10)^{-6}F_1' + 56.2(10)^{-6}F_2'$$

from which

$$\delta_{11}' = +220.0(10)^{-6} \text{ in.}$$

$$\delta_{12}' = - 9.8(10)^{-6} \text{ in.}$$

$$\delta_{22}' = + 56.2(10)^{-6} \text{ in.}$$

$$\delta_{21}' = - 11.4(10)^{-6} \text{ in.}$$

The first subscript identifies the displacement, the second subscript identifies the force causing it.

Final edge stresses are obtained as the sum of the stresses calculated neglecting relative edge displacement and those induced by relative edge displacement. All stresses are lb. per sq. in.

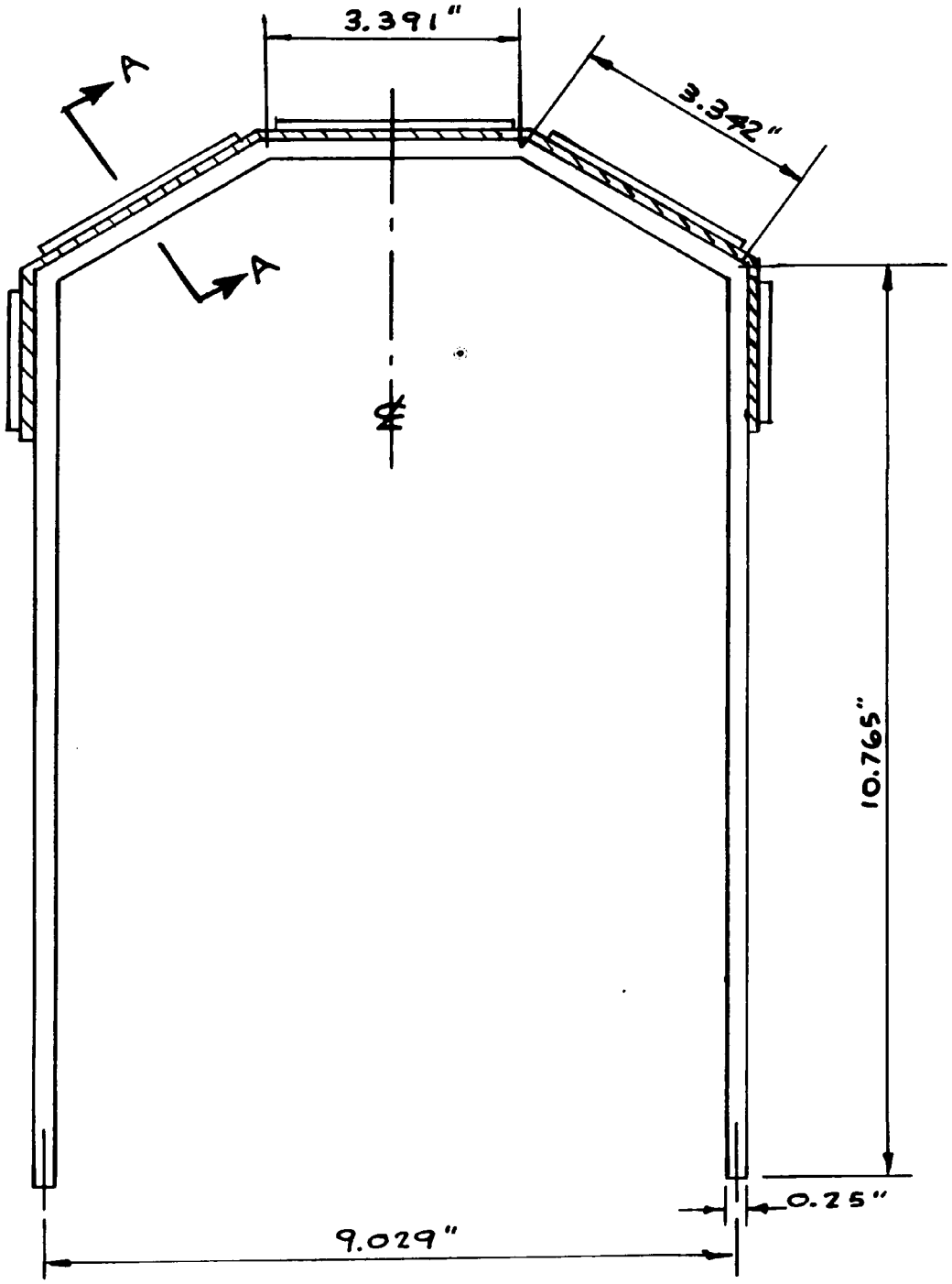
Section	Edge	$F_1' = 1$ lb.			$F_2' = 1$ lb.		
		Case I	Case II	Total	Case I	Case II	Total
0.4375 L	a	-24.47	+14.14	-10.33	+ 3.74	- 6.71	-2.97
	b	+13.38	-12.35	+ 1.03	- 7.48	+ 5.86	-1.62
	c	- 2.68	+ 5.51	+ 2.83	+ 5.20	- 2.61	+2.59
0.8635 L	a	-48.29	+21.80	-26.49	+ 7.38	-10.33	-2.95
	b	+26.41	-18.98	+ 7.43	-14.76	+ 9.00	-5.76
	c	- 5.28	+ 8.46	+ 3.18	+10.27	- 4.01	+6.26
L	a	-55.92	+22.31	-33.61	+ 8.55	-10.58	-2.03
	b	+30.59	-19.44	+11.15	-17.09	+ 9.22	-7.87
	c	- 6.12	+ 8.65	+ 2.53	+11.89	- 4.10	+7.79

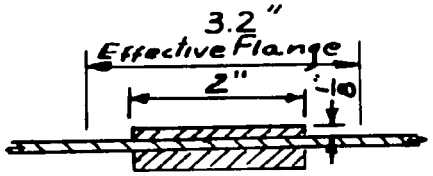
Case I. Neglecting relative edge displacement

Case II. The effect of relative edge displacement

STEP 3. Fig. 4.7 shows the interior frame and its dimensions. The frame is analyzed for the displacements produced by the resultants, F_m.

The supports for the frame are hinged and the frame is symmetrical about its centerline. The splice bars aid in the attachment of the roof to the frame and consequently increase the stiffness of the frame. In the analysis of the frame it is assumed that the portion of the splice bar between the outermost bolts connecting it to the frame acts monolithically with the frame.





Section A-A

Fig. 4.7

The plates provide an effective flange for the frame, the total effective width being assumed as 3.2 inches. This agrees closely with the value 0.363h + b recommended by Timoshenko[16]. The conditions upon which this recommendation is based are:

(1) the flanged beam is simply supported.

(2) the expanse of the plates forming the flange is large as compared to the beam span.

(3) the moment diagram for the beam varies sinusoidally.

In this case the response of the frame is not particularly sensitive to the choice of flange width, so that, even though the conditions at hand do not correspond closely with the above limitations, the 3.2 inch dimension will be used. The effective cross section is shown in Fig. 4.7.

The analysis of the frame loaded symmetrically with unit values of F_1 and F_2 is presented in the Appendix. This analysis yields the following δ displacements.

$$\delta_{11} = - 0.5981(10)^{-6}\text{in.}$$

$$\delta_{12} = + 1.8770(10)^{-6}\text{in.}$$

$$\delta_{22} = -26.180\ (10)^{-6}\text{in.}$$

$$\delta_{21} = + 2.9071(10)^{-6}\text{in.}$$

STEP 4. Equations are written equating the total displacement of the frame, produced by the final value of the resultant, F_m, to the displacements experienced by the folded plate in changing from a rigid support condition to a non rigid support condition

at the interior frame. Since the structure and the loading are symmetrical about a vertical plane containing the longitudinal axis of plate 3, two independent equations can be written to solve for F_1 and F_2.

$$F_1 \delta_{11} + F_2 \delta_{12} = F_1' \delta_{11}' + F_2' \delta_{12}'$$

$$F_1 \delta_{21} + F_2 \delta_{22} = F_1 \delta_{21}' + F_2' \delta_{22}'$$

The final resultants F_1 and F_2 are related to the rigid support resultants and to the correction resultants.

$$F_1 = F_{01} + F_1'$$

$$F_2 = F_{02} + F_2'$$

Substituting F_1' and F_2' from these expressions into the consistency equations yields the necessary equations which can be solved for F_1 and F_2.

$$(\delta_{11}' - \delta_{11})F_1 + (\delta_{12}' - \delta_{12})F_2 = \delta_{11}'F_{01} + \delta_{12}'F_{02}$$

$$(\delta_{21}' - \delta_{21})F_1 + (\delta_{22}' - \delta_{22})F_2 = \delta_{21}'F_{01} + \delta_{22}'F_{02}$$

Substituting the numerical values of F_{01} and F_{02} from Step 1 and the values of δ' and δ from Steps 2 and 3 respectively, provides the following two equations with two unknowns.

$$(220.0 + 0.6)F_1 + (-9.8 - 1.9)F_2 = 220.0 \times 20.6 - 9.8 \times 210.2$$

$$(-11.4 - 2.9)F_1 + (56.2 + 26.2)F_2 = -11.4 \times 20.6 + 56.2 \times 210.2$$

Solving these equations yields the following value of F_1 and F_2.

$$F_1 = + 18.84 \text{ lb.}$$

$$F_2 = +143.8 \text{ lb.}$$

$$F_1' = F_1 - F_{01} = 18.8 - 20.6^{\circ} = - 1.8 \text{ lb.}$$

$$F_2' = F_2 - F_{02} = 143.8 - 210.2 = -66.4 \text{ lb.}$$

STEP 5. The final values for the stresses, displacements and moments are computed as the sum of the values taken from Step 1, wherein, a non-yielding support was assumed, and the values resulting from Step 3 multiplied by F_1' or F_2' as required. The final edge stresses are tabulated below. All stresses are lb. per sq. in.

Final Edge Stresses

Section	Edge	Case I	Case II	Case III	Total
0.4375 L	a	- 93	+ 19	+ 197	+ 123
	b	+ 420	- 2	+ 108	+ 526
	c	- 328	- 5	- 172	- 505
0.8635 L	a	+ 78	+ 48	+ 196	+ 322
	b	- 343	- 13	+ 382	+ 26
	c	+ 266	- 6	- 416	- 156
L	a	+ 170	+ 60	+ 135	+ 365
	b	- 754	- 20	+ 523	- 251
	c	+ 588	- 5	- 517	+ 66

Case I: Stresses in the folded plate with a non-yielding interior support.

Case II: Stresses in the folded plate obtained by multiplying the stresses computed in Step 2 for a unit value of F_1' by the actual value of F_1' solved for in Step 4.

Case III: Stresses in the folded plate obtained by multiplying the stresses computed in Step 2 for a unit value of F_2' by the actual value of F_2' solved for in Step 4.

The final values for slab moments and displacements are computed in a similar manner, summing the quantities obtained for a rigid support with those produced by the correction resultants F_1' and F_2'.

Slab Moments (lb. in.)

Section	Edge	Case I	Case II	Case III	Total
0.4375 L	a	0	0	0	0
	b	0	0	0	0
	c	+ 0.479	- 0.062	+ 1.074	+ 1.491
0.8635 L	a	0	0	0	0
	b	0	0	0	0
	c	+ 0.203	- 0.095	+ 1.651	+ 1.759

δ Displacements (in. x 10^4)

Section	Plate	Case I	Case II	Case III	Total
0.500 L	1	+ 8.49	- 2.80	+ 4.60	+ 10.29
	2	- 7.78	+ 0.14	- 26.38	- 34.02
L	1	0	+ 3.96	- 6.51	- 2.55
	2	0	+ 0.21	- 37.32	- 37.11

4. Experimental Investigation

The structural model analyzed in the preceding section was built and then tested for the loading specified in Section 3. The roof was fabricated from a 0.125 inch thick aluminum sheet of 2024-T3 grade. The interior frame was bent from a 2" x 1/4" bar of 6061-T4 grade aluminum. The continuity of the roof was provided through the use of splice bars placed on the outer surface of the roof and bolted to the roof and the frame. Base plates for the interior frame were rounded to provide as closely as possible a hinged end condition. The unloaded model is shown in Fig. 4.8.

SR-4 strain gages were applied to the model at sections 0.4375 L and 0.8635 L from the end frame. The gages were placed at each edge and at the mid-depth of each plate element and oriented to indicate strain in the longitudinal direction. Additional gages were attached to plate 3 of the model and oriented to indicate transverse strain. All gages were placed in pairs, one on the outer surface and one on the inner surface of the plates. The longitudinal gages were wired in series to indicate the mean longitudinal strain at the point of application of the gage. The transverse gages were wired independently of one another to provide information concerning transverse bending.

The model was set up on a heavy concrete slab and securely attached to the slab through plugs which had been inserted in the concrete. Displacements were measured with 0.0001 inch dial gages placed to indicate the horizontal displacement at edge b and the vertical displacement at edge a.

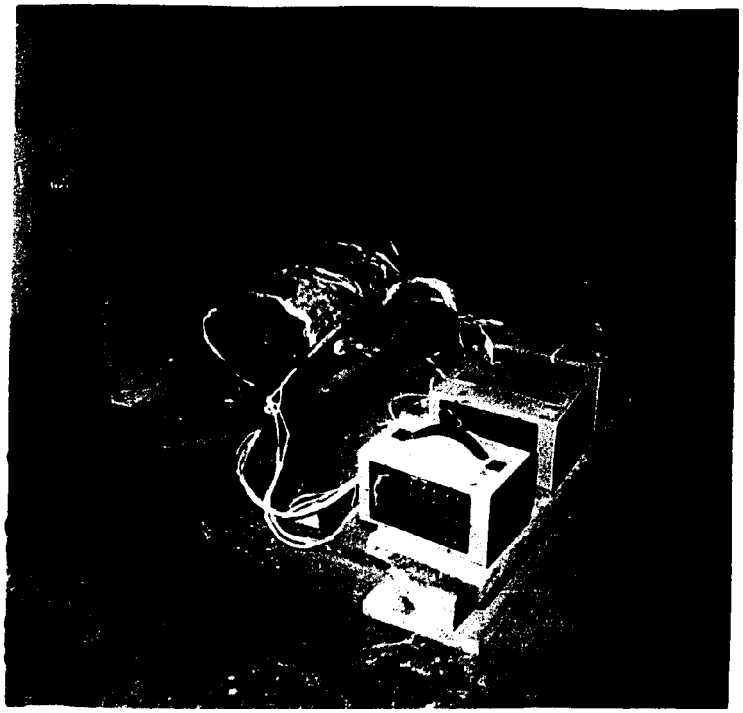


Fig. 4.8

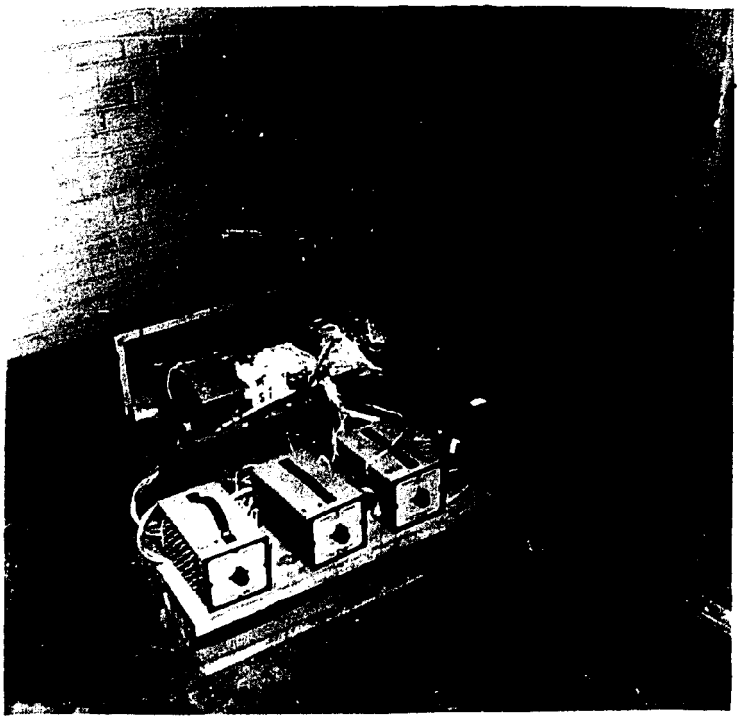


Fig. 4.9

The model was loaded with fifty pound steel ingots placed on loading bars which rested on the ridge edges at the one-third points of the spans. Load increments were one hundred pounds each. Strain readings and displacements were recorded after the application of each load increment, readings being taken during both the loading and unloading of the model. The partially loaded model is shown in Fig. 4.9.

5. Comparison of Analytical and Experimental Results

The results of the experimental work carried out in the investigation of the roof compare favorably with the results obtained analytically. The mean longitudinal stresses corresponding to the computed edge stresses are plotted in Diagram 4.1; good agreement is displayed. For purposes of comparison Diagram 4.1 presents the edge stresses computed for an assumed non-yielding interior support. Considerable error, with complete reversal of computed stress from the experimental results, is shown in the region of the interior support.

Good agreement is provided between the measured and computed transverse stresses at the section x = 0.8635 L. The agreement at the 0.4375 L section is not particularly good. In this case the transverse gages were placed 2.29 inches from the load point and the local distortion of the model undoubtedly influenced the results. Fair agreement between the computed and measured displacements is shown.

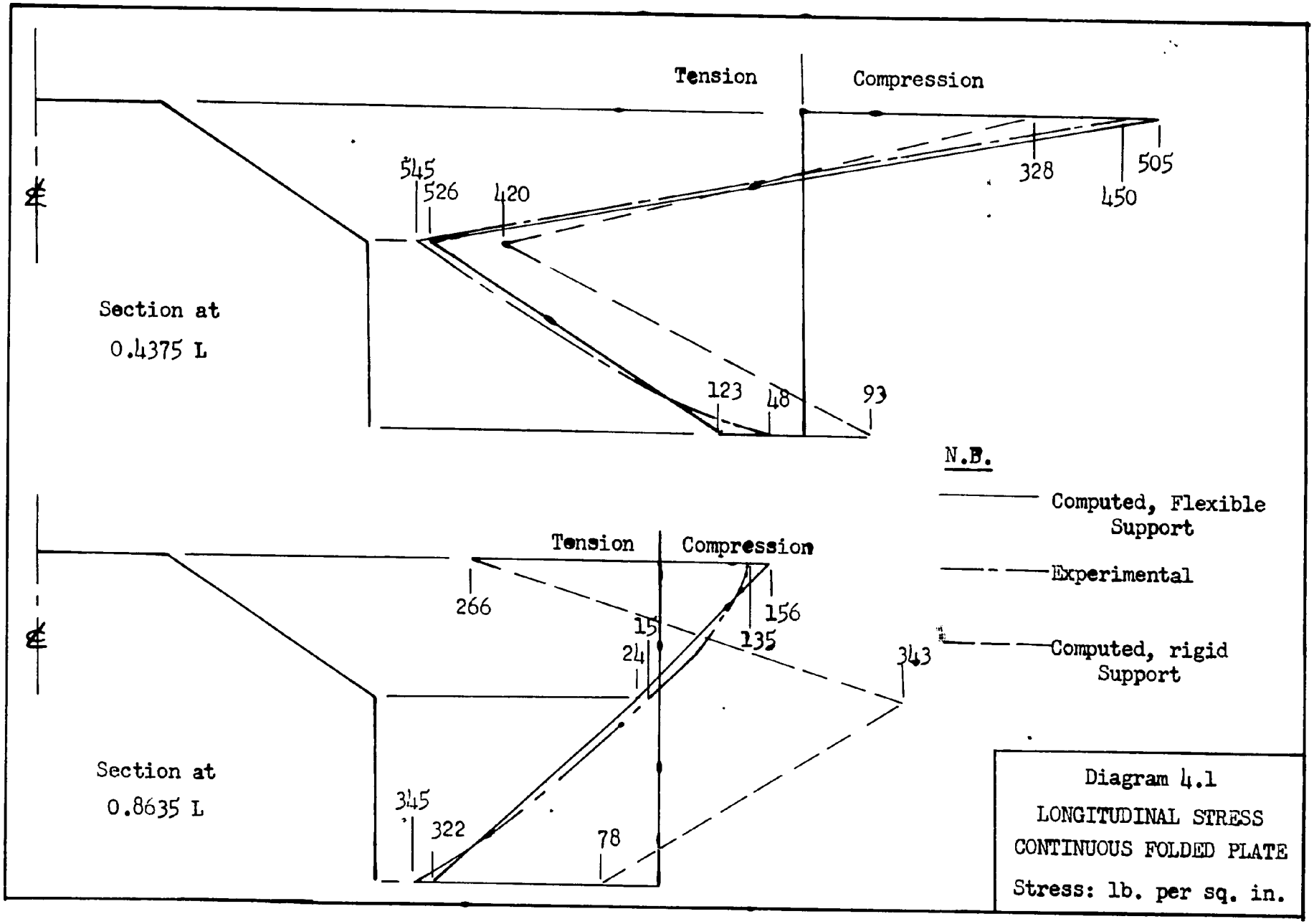


Diagram 4.1

LONGITUDINAL STRESS

CONTINUOUS FOLDED PLATE

Stress: lb. per sq. in.

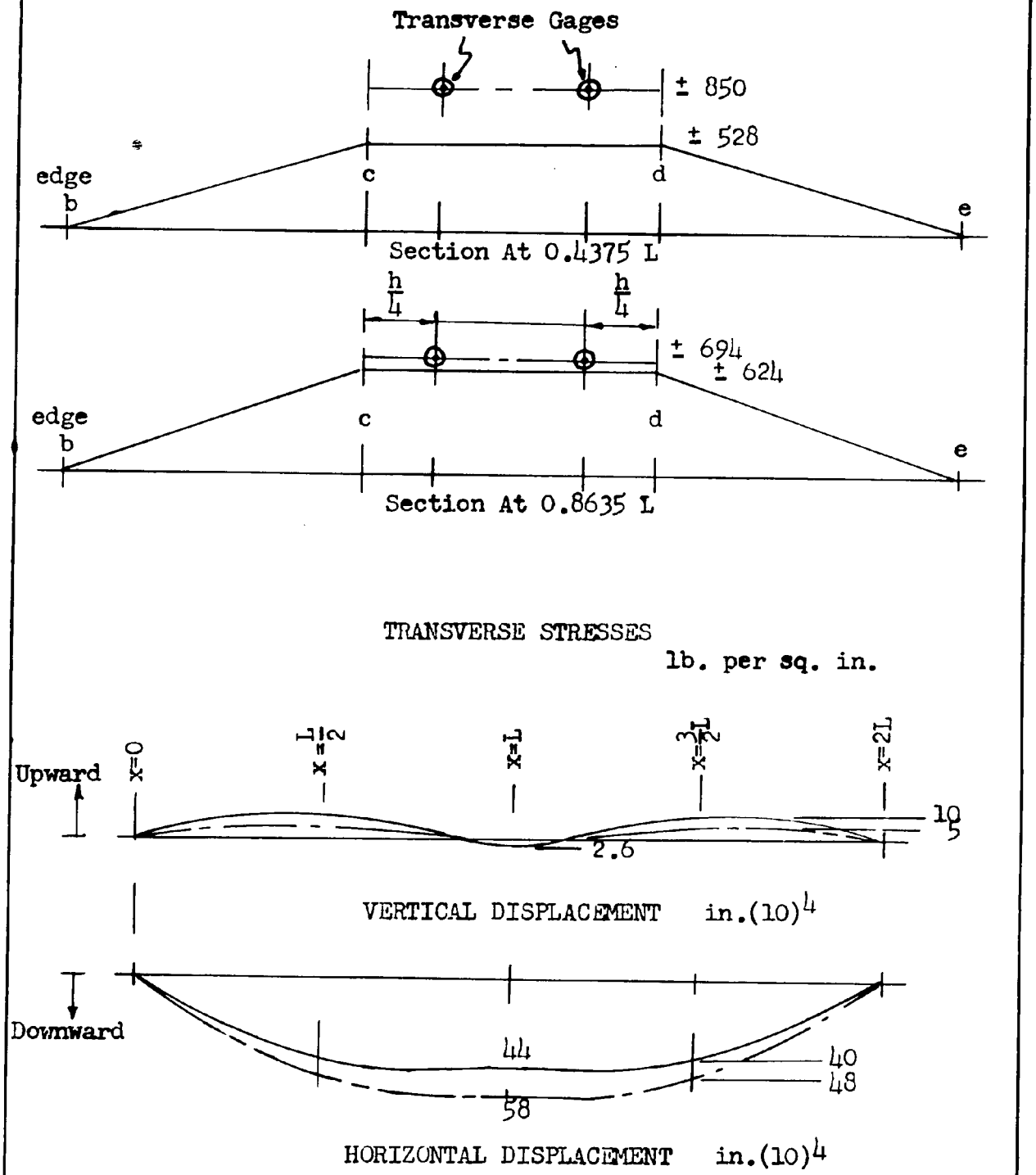


Diagram 4.2

TRANSVERSE STRESSES and EDGE DISPLACEMENTS

CHAPTER V

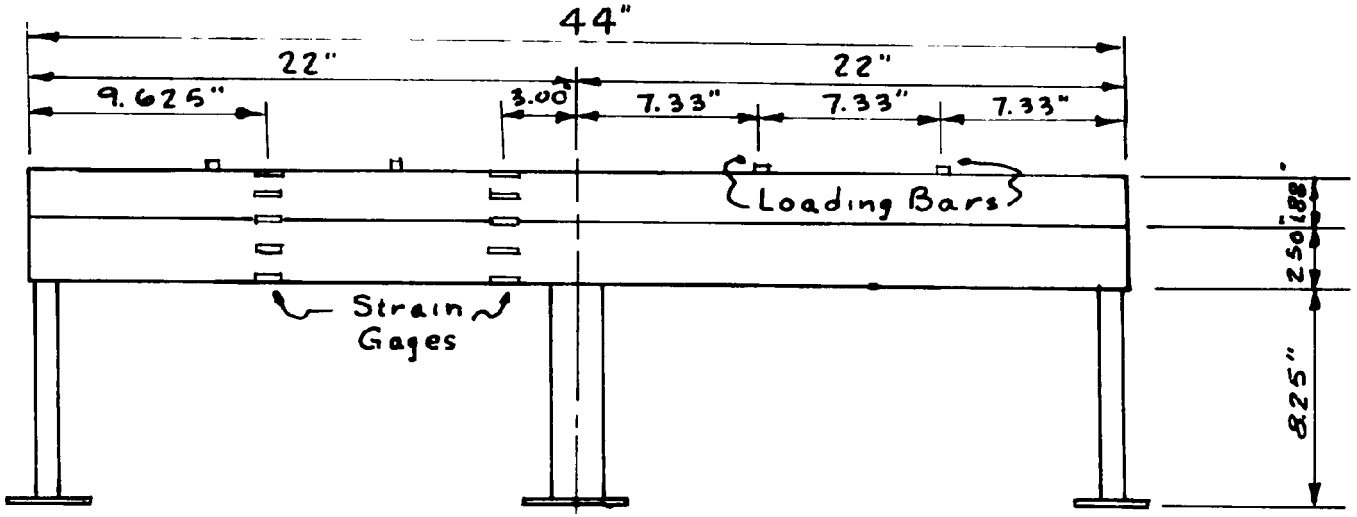
THE INVESTIGATION OF A MULTIPLE FOLDED PLATE STRUCTURE CONTINUOUS OVER A FRAME NOT RIGID IN ITS PLANE

1. The Nature of the Problem

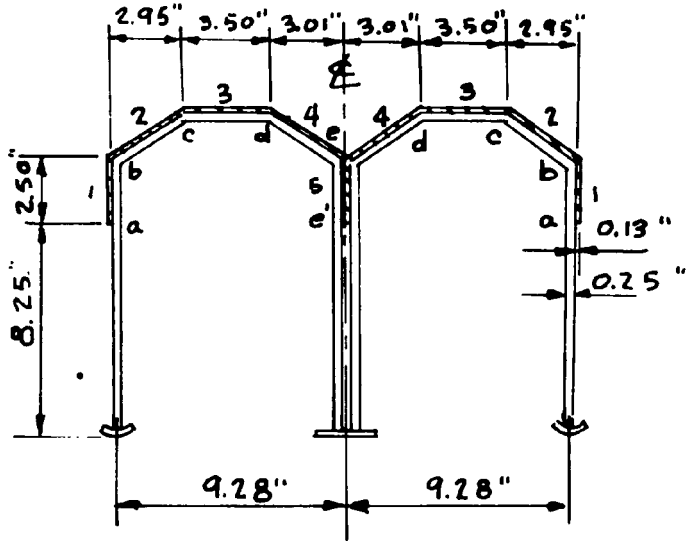
The investigation of a structure which combines the problems considered in Chapters III and IV is now presented. The feasibility of solving the problem utilizing the principles of superposition and consistent deformation is investigated.

The structure to be analyzed is shown in Fig. 5.1. It consists of a folded plate supported by end diaphragms which are rigid in their plane and by an interior frame not rigid in its plane. At the middle longitudinal joint multiple folded plate construction is employed, providing a third plate at this joint.

As demonstrated in Chapters III and IV, the use of multiple folded plate construction and interior frames introduces systems of redundant forces in the basic folded plate. The influence of the additional interior plate on the response of the roof to load will again be computed assuming that only a distributed edge force of intensity p_e and a distributed edge shear of intensity T_e act along the edge common to the folded plate and to the additional plate. The interaction of the roof and the frame will be computed assuming that all



Side Elevation



Cross Section

Fig. 5.1

forces acting at the section common to the roof and the frame are negligible except the resultants defined in Chapter IV and referred to as F_m.

The redundants F_m are assumed to be uniformly distributed along the m segments of the roof and the frame and are assumed to act in the middle plane of the plates. The nature of the distribution of the force systems, p_e and T_e, is unknown and complicated by the presence of the frame so that they will be represented by the trogonometric series

$$p_e = \sum_{n=0}^{\infty} a_n \sin \frac{n\pi x}{L}$$

$$T_e = \sum_{n=0}^{\infty} b_n \cos \frac{n\pi x}{L}$$

The origin for x is taken at the left end diaphragm and x is measured parallel to the longitudinal axis of the roof. The free edge moments, which are the basic functions computed in the process of determining the effects of the loads and the redundants, F_m, are also represented by trigonometric series. Consequently, all of the computed displacements, forces and stresses caused by these force systems appear in series form.

The solution is provided by the superposition of these various force systems and the writing of equations providing for the consistency of the strains and deflections at the edge common to the folded plate and to the additional interior plate and for the consistency of the displacements of the roof and the frame at their common section. The solution presented in this chapter differs from the solution presented in Chapter IV in the matter of treating the interaction of the roof and the frame. In the previous chapter the change in the resultants, F_m, was computed as the frame changed from a rigid condition to a

non rigid condition. In this solution the rigid condition is not employed.

2. Description of the Solution

The steps in the solution can be enumerated as follows.

STEP 1. The roof is first considered as a simple folded plate. Thus, the additional interior plate is removed as is the interior frame. The forces necessary to provide for the continuity between the folded plate and the additional plate and the frame are assumed. In this case they will be assumed as

a. The edge force systems

$$p_e = \sum_{n=1}^{\infty} a_n \sin \frac{n\pi x}{L}$$

$$T_e = \sum_{n=1}^{\infty} b_n \cos \frac{n\pi x}{L}$$

b. The resultant force system

$$F_1,\ F_2,\ F_3,\ F_4,\ \text{and}\ F_5.$$

The free edge moments produced in the simple folded plate by the resultants, F_m, are represented by the series

$$M'_m = F_m \sum_{n=1}^{\infty} c'_{mn} \sin \frac{n\pi x}{L} .$$

The free edge moments produced by the loads are also represented by a series

$$M'_m = \sum_{n=1}^{\infty} \phi'_n \sin \frac{n\pi x}{L} .$$

The simple folded plate is now analyzed for its response to the applied load. In this analysis the correction for relative edge displacement is made. An analysis is also made of the response of the simple folded plate to unit values of the redundants; this includes the correction for relative edge displacement. Upon completion of the analysis of the simple folded plate for each of these force systems, the various displacements, forces and stresses will appear in series form for each of the force systems considered.

STEP 2. The additional interior plate is analyzed for stresses and displacements produced by the application of the edge load, p_e, and the edge shear, T_e. An analysis of the interior frame for the δ displacements produced by unit values of the resultants, F_1, F_2, F_3, F_4, and F_5, is made at this time.

STEP 3. Equations are written formulating the consistent strain and deflection conditions at the edge common to the folded plate and the additional interior plate for the applied load and for each of the force systems, F_1, F_2, F_3, F_4, and F_5. Solving each of these systems of simultaneous equations, the edge forces, p_e and T_e, are defined for each of the above load systems.

The displacements, δ_m, are computed at the section common to the roof and to the frame for the applied load and for unit values of each of the force systems F_m. For this computation the roof is considered as a simply supported, multiple folded plate. The equations

expressing the consistency of the δ displacements of the roof and the frame at their common section are written. The resulting equations are then solved simultaneously for the values of F_1, F_2, F_3, F_4, and F_5.

STEP 4. Final values of the plate moments, M_m, and edge shears, N_j, are computed as the sum of the effects of the applied loads and the redundants. The edge stresses are computed from these final expressions for M_m and N_j. The slab moments and the final δ displacements are computed in a similar manner.

The above procedure will be illustrated with an example.

3. Example 5.1

The structure to be analyzed is a small scale aluminum model having the proportions and dimensions shown in Fig. 5.1. The structure is loaded with a total load of 800 pounds distributed as 16 equal concentrations as shown in Fig. 5.2. The analysis will provide a solution for the final edge stresses and the slab moments at the section x = 9.625 in., the final edge stresses at the section x = 19.00 in., and the final displacement of plate 1 at sections x = 11.00 in. and x = 22.00 in.

STEP 1. The redundants are defined as p_e, T_e, F_1, F_2, F_3, F_4, and F_5. After analyzing the simply supported folded plate for the effect of the applied load, the folded plate will be analyzed for unit values of each of these redundants in turn.

Having removed the redundant members, the simple folded plate is analyzed for the effect of the applied loads. The concentrated loads

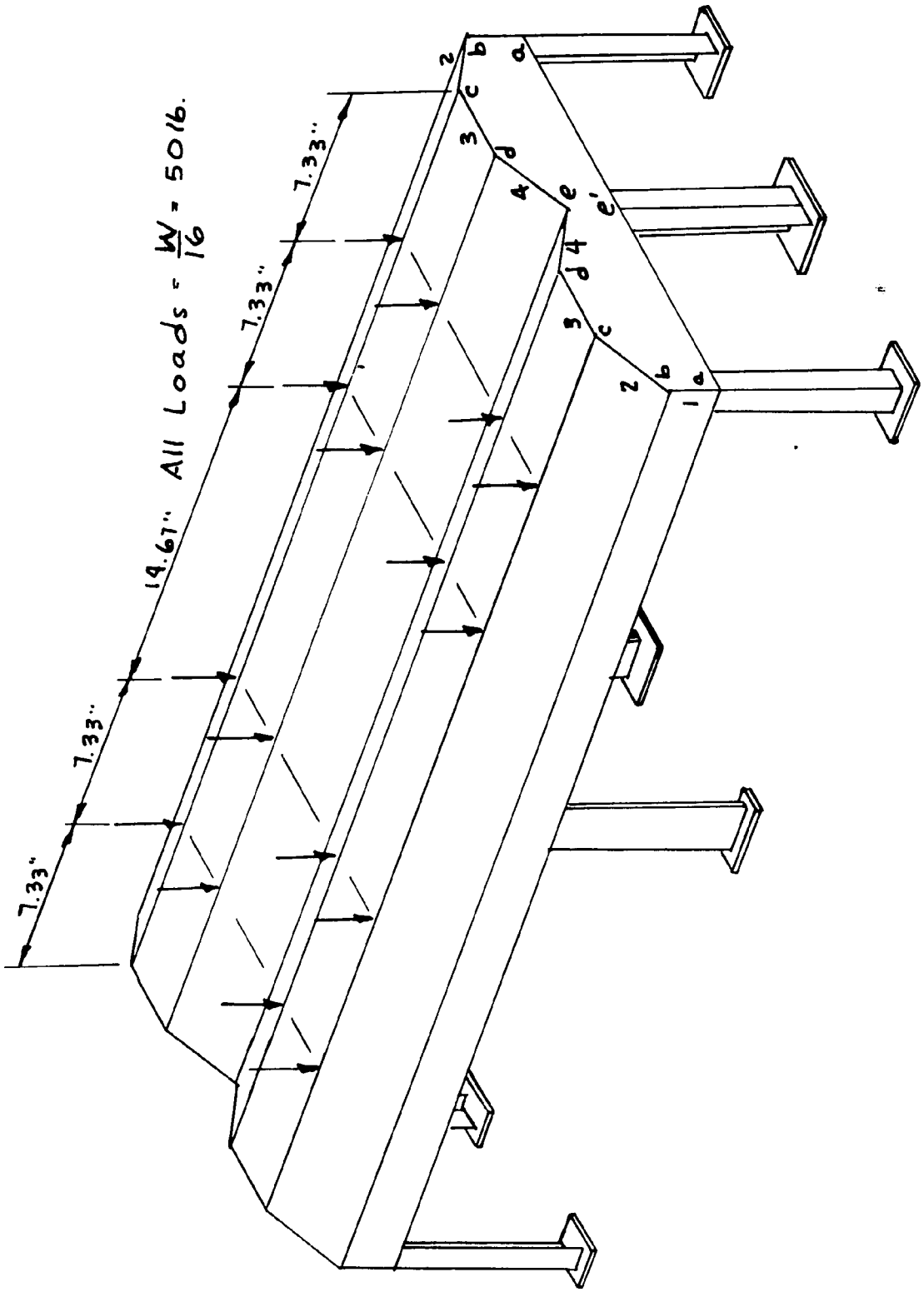


Fig. 5.2

are resolved into components parallel to the plates that join at the edge to which the load is applied.

$$P_2 = -P_4 = -(50.00)(1.865) = -93.25 \text{ lb.}$$

The free edge moments for plates 2 and 4 can be expressed as

$$M_2' = -M_4' = -\frac{2P_2L}{\pi^2}\sum_{n=1}^{\infty}\frac{\phi_n}{n^2}\sin\frac{n\pi x}{L}$$

where $$\phi_n = \sin\frac{n\pi}{6} + \sin\frac{n\pi}{3} + \sin\frac{2n\pi}{3} + \sin\frac{5n\pi}{6}$$

Using the first six terms of the series, the moment M_2' can be written as:

$$M_2' = +2,271.5\sin\frac{\pi x}{L} + 184.75\sin\frac{3\pi x}{L} - 24.345\sin\frac{5\pi x}{L}$$
$$+12.422\sin\frac{7\pi x}{L} - 20.529\sin\frac{9\pi x}{L} - 18.773\sin\frac{11\pi x}{L}$$

The solution for the edge shears, N_j, can be obtained from the three shear equations.

$$N_a\frac{h_1}{S_1} + 2N_b\left[\frac{h_1}{S_1} + \frac{h_2}{S_2}\right] + N_c\frac{h_2}{S_2} = -3\left[\frac{M_1'}{S_1} + \frac{M_2'}{S_2}\right]$$

$$N_b\frac{h_2}{S_2} + 2N_c\left[\frac{h_2}{S_2} + \frac{h_3}{S_3}\right] + N_d\frac{h_3}{S_3} = -3\left[\frac{M_2'}{S_2} + \frac{M_3'}{S_3}\right]$$

$$N_c\frac{h_3}{S_3} + 2N_d\left[\frac{h_3}{S_3} + \frac{h_4}{S_4}\right] + N_e\frac{h_4}{S_4} = -3\left[\frac{M_3'}{S_3} + \frac{M_4'}{S_4}\right]$$

In the above equations,

$N_a = 0$, since it is a free edge, and

$N_e = 0$, from symmetry.

Solving these equations for N_b, N_c, and N_d yields the following expressions for the unknown edge shears in terms of the free edge moments, M_m'.

$$N_b = -0.37093M_1' - 0.13841M_2' + 0.037796M_3' - 0.012599M_4'$$

$$N_c = +0.099010M_1' - 0.19183M_2' - 0.18167M_3' + 0.060556M_4'$$

$$N_d = -0.024752M_1' + 0.047957M_2' - 0.16886M_3' - 0.22942M_4'$$

For the applied load,

$$M_1' = M_3' = 0 \quad \text{and} \quad M_2' = -M_4'$$

Substituting the series for M_2' and M_4' into the expressions for N_b, N_c, and N_d yields

$$N_b = -285.78 \sin\frac{\pi x}{L} - 23.243 \sin\frac{3\pi x}{L} + 3.0629 \sin\frac{5\pi x}{L}$$
$$-1.5628 \sin\frac{7\pi x}{L} - 2.5828 \sin\frac{9\pi x}{L} - 2.3618 \sin\frac{11\pi x}{L} \ldots$$

$$N_c = -573.29 \sin\frac{\pi x}{L} - 46.629 \sin\frac{3\pi x}{L} + 6.1443 \sin\frac{5\pi x}{L}$$
$$-3.1351 \sin\frac{7\pi x}{L} + 5.1813 \sin\frac{9\pi x}{L} + 4.7381 \sin\frac{11\pi x}{L} \ldots$$

$$N_d = +630.06 \sin\frac{\pi x}{L} + 51.245 \sin\frac{3\pi x}{L} - 6.7527 \sin\frac{5\pi x}{L}$$
$$+3.4455 \sin\frac{7\pi x}{L} - 5.6944 \sin\frac{9\pi x}{L} - 5.2073 \sin\frac{11\pi x}{L} \ldots$$

The displacements in the plane of the plates can be computed as

$$\delta_m = \frac{1}{EI_m} \int_0^x \int_0^x \left[M_m' + \frac{h_m}{2} (N_j + N_{j+1}) \right] dx\, dx$$

$$\delta_1 = \frac{h_1}{2EI_1} \int_0^x \int_0^x N_b\, dx\, dx$$

$$(10)^6 \delta_1 = 39{,}426 \sin\frac{\pi x}{L} + 356.29 \sin\frac{3\pi x}{L} - 16.902 \sin\frac{5\pi x}{L}$$
$$+4.4000 \sin\frac{7\pi x}{L} - 4.3990 \sin\frac{9\pi x}{L} - 2.6928 \sin\frac{11\pi x}{L} \ldots$$

$$\delta_2 = \frac{1}{EI_2} \int_0^x \int_0^x \left[M_2' + \frac{h_2}{2} \left(N_b + N_c \right) \right] dx\, dx$$

$$(10)^6 \delta_2 = -30,894 \sin \frac{\pi x}{L} - 281.99 \sin \frac{3\pi x}{L} + 13.244 \sin \frac{5\pi x}{L}$$
$$- 3.4478 \sin \frac{7\pi x}{L} + 3.4470 \sin \frac{9\pi x}{L} + 2.1101 \sin \frac{11\pi x}{L} \ldots$$

$$\delta_3 = \frac{h_3}{2EI_3} \int_0^x \int_0^x \left(N_b + N_c \right) dx\, dx$$

$$(10)^6 \delta_3 = -3,995.9 \sin \frac{\pi x}{L} - 36.101 \sin \frac{3\pi x}{L} + 1.7129 \sin \frac{5\pi x}{L}$$
$$- 0.44588 \sin \frac{7\pi x}{L} + 0.44588 \sin \frac{9\pi x}{L} + 0.27294 \sin \frac{11\pi x}{L} \ldots$$

$$\delta_4 = \frac{1}{EI_4} \int_0^x \int_0^x \left[M_4' + \frac{h_4 N_d}{2} \right] dx\, dx$$

$$(10)^6 \delta_4 = +47,014 \sin \frac{\pi x}{L} + 424.87 \sin \frac{3\pi x}{L} - 20.156 \sin \frac{5\pi x}{L}$$
$$+ 5.2469 \sin \frac{7\pi x}{L} - 5.2456 \sin \frac{9\pi x}{L} - 3.2110 \sin \frac{11\pi x}{L} \ldots$$

The relative edge displacements between edges b and c, between edges c and d and between edges d and e are designated Δ_2, Δ_3, and Δ_4, respectively. It is assumed that these relative edge displacements can be approximated by the following series.

$$\Delta_2 = \sum_{n=1}^{\infty} b_n' \sin \frac{n \pi x}{L}$$

$$\Delta_3 = \sum_{n=1}^{\infty} b_n'' \sin \frac{n \pi x}{L}$$

$$\Delta_4 = \sum_{n=1}^{\infty} b_n''' \sin \frac{n \pi x}{L}$$

The solution for the slab moments induced by relative edge displacement is approached in the same manner as in the previous chapters. The cross section of the roof is assumed to be deformed as shown in Fig. 3.4. If the joints have not been permitted to rotate, the fixed end moments for a strip of unit width of the roof may be expressed as functions of the relative edge displacements.

$$m_2^F = -\frac{3EI_T}{h_2^2}\,\Delta_2 = -\frac{3EI_T}{h_2^2}\sum_{n=1}^{\infty} b_n' \sin\frac{n\pi x}{L}$$

$$m_3^F = \frac{6EI_T}{h_3^2}\,\Delta_3 = \frac{6EI_T}{h_3^2}\sum_{n=1}^{\infty} b_n'' \sin\frac{n\pi x}{L}$$

$$m_4^F = -\frac{6EI_T}{h_4^2}\,\Delta_4 = -\frac{6EI_T}{h_4^2}\sum_{n=1}^{\infty} b_n''' \sin\frac{n\pi x}{L}$$

Distributing these fixed end moments and solving for the associated plate edge loads for each of the relative displacements Δ_2, Δ_3, and Δ_4 yields the following:

For Δ_2 :

$$p_1 = -\,85.64\sum_{n=1}^{\infty} b_n' \sin\frac{n\pi x}{L}$$

$$p_2 = +\,332.38\sum_{n=1}^{\infty} b_n' \sin\frac{n\pi x}{L}$$

$$p_3 = -\,483.90\sum_{n=1}^{\infty} b_n' \sin\frac{n\pi x}{L}$$

$$p_4 = +\,172.86\sum_{n=1}^{\infty} b_n' \sin\frac{n\pi x}{L}$$

$$\text{For } \Delta_3: \quad p_1 = -\ 111.05 \sum_{n=1}^{\infty} b_n'' \sin \frac{n\pi x}{L}$$

$$p_2 = +\ 574.10 \sum_{n=1}^{\infty} b_n'' \sin \frac{n\pi x}{L}$$

$$p_3 = -1{,}083.25 \sum_{n=1}^{\infty} b_n'' \sin \frac{n\pi x}{L}$$

$$p_4 = +\ 367.20 \sum_{n=1}^{\infty} b_n'' \sin \frac{n\pi x}{L}$$

$$\text{For } \Delta_4: \quad p_1 = -\ 36.70 \sum_{n=1}^{\infty} b_n''' \sin \frac{n\pi x}{L}$$

$$p_2 = +\ 357.63 \sum_{n=1}^{\infty} b_n''' \sin \frac{n\pi x}{L}$$

$$p_3 = -1{,}144.90 \sum_{n=1}^{\infty} b_n''' \sin \frac{n\pi x}{L}$$

$$p_4 = +\ 289.40 \sum_{n=1}^{\infty} b_n''' \sin \frac{n\pi x}{L}$$

The free edge moments are computed from the edge loads for each of the relative edge displacement systems.

$$M_m' = \int_0^x \int_0^x p_m \, dx \, dx$$

The edge shears are computed utilizing the expressions obtained for N_j on page 96.

$$\text{For } \Delta_2: \quad M_1' = +\ 16{,}800 \sum_{n=1}^{\infty} \frac{b_n'}{n^2} \sin \frac{n\pi x}{L}$$

$$M_2' = -\ 65{,}200 \sum_{n=1}^{\infty} \frac{b_n'}{n^2} \sin \frac{n\pi x}{L}$$

$$M_3' = + 94,922 \sum_{n=1}^{\infty} \frac{b_n'}{n^2} \sin \frac{n \pi x}{L}$$

$$M_4' = - 33,907 \sum_{n=1}^{\infty} \frac{b_n'}{n^2} \sin \frac{n \pi x}{L}$$

$$N_b = + 6,807.6 \sum_{n=1}^{\infty} \frac{b_n'}{n^2} \sin \frac{n \pi x}{L}$$

$$N_c = - 5,127.1 \sum_{n=1}^{\infty} \frac{b_n'}{n^2} \sin \frac{n \pi x}{L}$$

$$N_d = - 11,792.2 \sum_{n=1}^{\infty} \frac{b_n'}{n^2} \sin \frac{n \pi x}{L}$$

For Δ_3 : $M_1' = + 21,784 \sum_{n=1}^{\infty} \frac{b_n''}{n^2} \sin \frac{n \pi x}{L}$

$$M_2' = - 112,615 \sum_{n=1}^{\infty} \frac{b_n''}{n^2} \sin \frac{n \pi x}{L}$$

$$M_3' = + 212,491 \sum_{n=1}^{\infty} \frac{b_n''}{n^2} \sin \frac{n \pi x}{L}$$

$$M_4' = - 72,030 \sum_{n=1}^{\infty} \frac{b_n''}{n^2} \sin \frac{n \pi x}{L}$$

$$N_b = +16,445.5 \sum_{n=1}^{\infty} \frac{b_n''}{n^2} \sin \frac{n \pi x}{L}$$

$$N_c = -19,205.3 \sum_{n=1}^{\infty} \frac{b_n''}{n^2} \sin \frac{n \pi x}{L}$$

$$N_d = -25,296 \sum_{n=1}^{\infty} \frac{b_n''}{n^2} \sin \frac{n \pi x}{L}$$

For Δ_4 :

$$M_1' = +\ 7{,}199.9 \sum_{n=1}^{\infty} \frac{b_n'''}{n^2} \sin \frac{n \pi x}{L}$$

$$M_2' = -\ 70{,}153 \sum_{n=1}^{\infty} \frac{b_n'''}{n^2} \sin \frac{n \pi x}{L}$$

$$M_3' = +\ 224{,}583 \sum_{n=1}^{\infty} \frac{b_n'''}{n^2} \sin \frac{n \pi x}{L}$$

$$M_4' = -\ 56{,}769 \sum_{n=1}^{\infty} \frac{b_n'''}{n^2} \sin \frac{n \pi x}{L}$$

$$N_b = +\ 16{,}242.7 \sum_{n=1}^{\infty} \frac{b_n'''}{n^2} \sin \frac{n \pi x}{L}$$

$$N_c = -\ 30{,}067.4 \sum_{n=1}^{\infty} \frac{b_n'''}{n^2} \sin \frac{n \pi x}{L}$$

$$N_d = -\ 28{,}441.7 \sum_{n=1}^{\infty} \frac{b_n'''}{n^2} \sin \frac{n \pi x}{L}$$

$N_e = 0$, from symmetry

The δ displacements are computed from the above values for M_m' and N_j using the same procedure as that presented on page 97 so that the δ displacements induced by relative edge displacements are as follows.

$$\delta_1 = -2.7934 \sum_{n=1}^{\infty} \frac{b_n'}{n^4} \sin \frac{n\pi x}{L} - 4.6732 \sum_{n=1}^{\infty} \frac{b_n''}{n^4} \sin \frac{n\pi x}{L} - 3.0355 \sum_{n=1}^{\infty} \frac{b_n'''}{n^4} \sin \frac{n\pi x}{L}$$

$$\delta_2 = +2.5041 \sum_{n=1}^{\infty} \frac{b_n'}{n^4} \sin \frac{n\pi x}{L} + 4.7238 \sum_{n=1}^{\infty} \frac{b_n''}{n^4} \sin \frac{n\pi x}{L} + 3.7947 \sum_{n=1}^{\infty} \frac{b_n'''}{n^4} \sin \frac{n\pi x}{L}$$

$$\delta_3 = -2.6270 \sum_{n=1}^{\infty} \frac{b_n'}{n^4} \sin \frac{n\pi x}{L} - 5.4143 \sum_{n=1}^{\infty} \frac{b_n''}{n^4} \sin \frac{n\pi x}{L} - 4.9147 \sum_{n=1}^{\infty} \frac{b_n'''}{n^4} \sin \frac{n\pi x}{L}$$

$$\delta_4 = +2.1938 \sum_{n=1}^{\infty} \frac{b_n'}{n^4} \sin \frac{n\pi x}{L} + 4.6776 \sum_{n=1}^{\infty} \frac{b_n''}{n^4} \sin \frac{n\pi x}{L} + 4.2852 \sum_{n=1}^{\infty} \frac{b_n'''}{n^4} \sin \frac{n\pi x}{L}$$

The geometrical relationships between the δ_m and the Δ_m displacements are derived from Fig. 3.4 and are the same as those used in Chapter III.

$$\Delta_2 = + 1.186\,\delta_1 - 2.209\,\delta_2 + 1.862\,\delta_3$$

$$\Delta_3 = - 1.862\,\delta_2 + 3.144\,\delta_3 - 1.862\,\delta_4$$

$$\Delta_4 = + 1.862\,\delta_3$$

Substituting the expressions for δ_m and Δ_m into the geometrical relationships yields the following equations.

$$\sum_{n=1}^{\infty}\left[1 + \frac{13.7361}{n^4}\right] b_n' \sin\frac{n\pi x}{L} + 26.0587 \sum_{n=1}^{\infty}\frac{b_n''}{n^4}\sin\frac{n\pi x}{L} + 21.1338 \sum_{n=1}^{\infty}\frac{b_n'''}{n^4}\sin\frac{n\pi x}{L} - \sum_{n=1}^{\infty} A_{n2}\sin\frac{n\pi x}{L} = 0$$

$$17.0068 \sum_{n=1}^{\infty}\frac{b_n'}{n^4}\sin\frac{n\pi x}{L} + \sum_{n=1}^{\infty}\left[1 + \frac{34.5280}{n^4}\right] b_n''\sin\frac{n\pi x}{L} + 30.4965 \sum_{n=1}^{\infty}\frac{b_n'''}{n^4}\sin\frac{n\pi x}{L} - \sum_{n=1}^{\infty} A_{n3}\sin\frac{n\pi x}{L} = 0$$

$$4.8915 \sum_{n=1}^{\infty}\frac{b_n'}{n^4}\sin\frac{n\pi x}{L} + 10.0814 \sum_{n=1}^{\infty}\frac{b_n''}{n^4}\sin\frac{n\pi x}{L} + \sum_{n=1}^{\infty}\left[1 + \frac{9.1512}{n^4}\right]\frac{b_n'''}{n^4}\sin\frac{n\pi x}{L} - \sum_{n=1}^{\infty} A_{n4}\sin\frac{n\pi x}{L} = 0$$

where $\sum_{n=1}^{\infty} A_{n2}\sin\frac{n\pi x}{L}$, $\sum_{n=1}^{\infty} A_{n3}\sin\frac{n\pi x}{L}$, and $\sum_{n=1}^{\infty} A_{n4}\sin\frac{n\pi x}{L}$ represent the series that would result from the substitution of the δ_m expressions for the load into the geometrical relationships. The resulting system of simultaneous equations is solved for b', b", and b"' for each value of n used to evaluate the series. Six sets of b', b", and b"' values are thus obtained in terms of the series constants A_{n2}, A_{n3}, and A_{n4}.

n = 1

$$b_1' = + 0.43179\ A_{12} - 0.41769\ A_{13} + 0.35588\ A_{14}$$
$$b_1'' = - 0.19045\ A_{12} + 0.37503\ A_{13} - 0.73018\ A_{14}$$
$$b_1''' = - 0.018926\ A_{12} - 0.17118\ A_{13} + 0.65219\ A_{14}$$

n = 3

$$b_3' = + 0.89657\ A_{32} - 0.18947\ A_{33} - 0.14608\ A_{34}$$
$$b_3'' = - 0.12277\ A_{32} + 0.74840\ A_{33} - 0.22439\ A_{34}$$
$$b_3''' = - 0.034918\ A_{32} - 0.073410\ A_{33} + 0.93151\ A_{34}$$

n = 5

$$b_5' = + 0.97962\ A_{52} - 0.038235\ A_{53} - 0.030809\ A_{54}$$
$$b_5'' = - 0.025043\ A_{52} + 0.94932\ A_{53} - 0.044820\ A_{54}$$
$$b_5''' = - 0.0071583\ A_{52} - 0.014797\ A_{53} + 0.98655\ A_{54}$$

n = 7

$$b_7' = + 0.99419\ A_{72} - 0.010603\ A_{73} - 0.0085851\ A_{74}$$
$$b_7'' = - 0.00697\!46\ A_{72} + 0.98596\ A_{73} - 0.012416\ A_{74}$$
$$b_7''' = + 0.000029021\ A_{72} - 0.0041026\ A_{73} + 0.99627\ A_{74}$$

n = 9

$$b_9' = + 0.99789\ A_{92} + 0.0039378\ A_{93} - 0.0031916\ A_{94}$$
$$b_9'' = + 0.0025879\ A_{92} - 0.99479\ A_{93} - 0.0046091\ A_{94}$$
$$b_9''' = - 0.00074690\ A_{92} + 0.0015235\ A_{93} + 0.99862\ A_{94}$$

n = 11

$$b_{11}' = + 0.99906\ A_{11\,2} - 0.0017729\ A_{11\,3} - 0.0014375\ A_{11\,4}$$
$$b_{11}'' = - 0.0011623\ A_{11\,2} + 0.99765\ A_{11\,3} - 0.0020751\ A_{11\,4}$$
$$b_{11}''' = - 0.00033278\ A_{11\,2} - 0.00068593\ A_{11\,3} + 0.999377\ A_{11\,4}$$

The series constants for δ_m, computed for the applied load, are substituted into the geometrical relationships to yield the following values for A_{n2}, A_{n3}, and A_{n4}.

n = 1 :

$A_{12} = +107,564(10)^{-6}$; $A_{13} = -42,666(10)^{-6}$; $A_{14} = -7,440.4(10)^{-6}$

n = 3 :

$A_{32} = +978.26(10)^{-6}$; $A_{33} = -379.54(10)^{-6}$; $A_{34} = +67.220(10)^{-6}$

n = 5:

$A_{52} = -46.113(10)^{-6}$; $A_{53} = +18.255(10)^{-6}$; $A_{54} = +3.1894(10)^{-6}$

n = 7:

$A_{72} = +12.0044(10)^{-6}$; $A_{73} = -4.7517(10)^{-6}$; $A_{74} = -0.83023(10)^{-6}$

n = 9 :

$A_{92} = -12.0014(10)^{-6}$; $A_{93} = +4.7508(10)^{-6}$; $A_{94} = +0.83023(10)^{-6}$

n = 11:

$A_{11\,2} = -7.3467(10)^{-6}$; $A_{11\,3} = +2.9080(10)^{-6}$; $A_{11\,4} = +0.50821(10)^{-6}$

Substituting these coefficients into the expressions for b_n', b_n'', and b_n''', yields the following values for the coefficients b_n', b_n'', and b_n'''.

n	$(10)^6 b_n'$	$(10)^6 b_n''$	$(10)^6 b_n'''$
1	+61,618	-31,054	+415.2
3	+958.81	-389.07	- 68.913
5	- 45.969	+ 18.342	+ 3.2065
7	+ 11.9917	- 4.7584	- 0.80729
9	- 11.9600	- 4.7609	+ 0.84528
11	- 7.3457	+ 2.9086	+ 0.50834

The coefficients b_n', b_n'', and b_n''' are now substituted into the previously defined series for δ_m, page 103, yielding the corrections for relative edge displacement. These corrections are added to the δ displacements computed for the applied load neglecting relative edge displacement. The final values for the δ displacements of the folded plate loaded with the 800 pound load are as follows.

$$(10)^6\delta_1 = +\ 11{,}170 \sin\frac{\pi x}{L} + 348.25 \sin\frac{3\pi x}{L} - 16.850 \sin\frac{5\pi x}{L}$$
$$+\ 4.3963 \sin\frac{7\pi x}{L} - 4.3909 \sin\frac{9\pi x}{L} - 2.6924 \sin\frac{11\pi x}{L} \cdots$$

$$(10)^6\delta_2 = -\ 21{,}700 \sin\frac{\pi x}{L} - 278.27 \sin\frac{3\pi x}{L} + 13.211 \sin\frac{5\pi x}{L}$$
$$-\ 3.4458 \sin\frac{7\pi x}{L} + 3.4395 \sin\frac{9\pi x}{L} + 2.1098 \sin\frac{11\pi x}{L} \cdots$$

$$(10)^6\delta_3 = +\ 230.00 \sin\frac{\pi x}{L} - 37.009 \sin\frac{3\pi x}{L} + 1.7220 \sin\frac{5\pi x}{L}$$
$$-\ 0.44662 \sin\frac{7\pi x}{L} + 0.45397 \sin\frac{9\pi x}{L} + 0.27301 \sin\frac{11\pi x}{L} \cdots$$

$$(10)^6\delta_4 = +\ 38{,}800 \sin\frac{\pi x}{L} + 424.72 \sin\frac{3\pi x}{L} - 20.158 \sin\frac{5\pi x}{L}$$
$$+\ 5.2472 \sin\frac{7\pi x}{L} - 5.2524 \sin\frac{9\pi x}{L} - 3.2110 \sin\frac{11\pi x}{L} \cdots$$

The displacement of edge e measured in the plane of the additional interior plate is obtained as

$$\delta_5 = \delta_4 \frac{1}{\sin\theta} = 1.862\,\delta_4.$$

$$\delta_5 = +\ 72{,}210 \sin\frac{\pi x}{L} + 790.47 \sin\frac{3\pi x}{L} - 37.517 \sin\frac{5\pi x}{L}$$
$$+\ 9.7659 \sin\frac{7\pi x}{L} - 9.7755 \sin\frac{9\pi x}{L} - 5.9762 \sin\frac{11\pi x}{L} \cdots$$

The substitution of the values of b', b'', and b''' into the series for the plate moments and the edge shears developed on pages 101 and 102 provides the relative edge displacement corrections for the plate moments and edge shears. Adding the corrections to the series for M_m' and N_j computed neglecting relative edge displacements yields the following values for M_m and N_j for the simple folded plate.

$$M_1 = +\ 361.7 \sin\frac{\pi x}{L} + 0.7929 \sin\frac{3\pi x}{L} - 0.013987 \sin\frac{5\pi x}{L}$$
$$+\ 0.0018775 \sin\frac{7\pi x}{L} - 0.0036858 \sin\frac{9\pi x}{L} - 0.0004661 \sin\frac{11\pi x}{L} \cdots$$

$$M_2 = + 1,722.0 \sin \frac{\pi x}{L} + 183.22 \sin \frac{3\pi x}{L} - 24.317 \sin \frac{5\pi x}{L}$$
$$+ 12.4178 \sin \frac{7\pi x}{L} - 20.5136 \sin \frac{9\pi x}{L} - 18.772 \sin \frac{11\pi x}{L} \cdots$$

$$M_3 = - 656.55 \sin \frac{\pi x}{L} - 0.7936 \sin \frac{3\pi x}{L} + 0.01017 \sin \frac{5\pi x}{L}$$
$$- 0.00111 \sin \frac{7\pi x}{L} - 0.02417 \sin \frac{9\pi x}{L} + 0.000298 \sin \frac{11\pi x}{L} \cdots$$

$$M_4 = - 2,147.6 \sin \frac{\pi x}{L} - 184.82 \sin \frac{3\pi x}{L} + 24.347 \sin \frac{5\pi x}{L}$$
$$- 12.4221 \sin \frac{7\pi x}{L} + 20.5377 \sin \frac{9\pi x}{L} + 18.773 \sin \frac{11\pi x}{L} \cdots$$

$$N_b = - 370.27 \sin \frac{\pi x}{L} - 23.353 \sin \frac{3\pi x}{L} + 3.0646 \sin \frac{5\pi x}{L}$$
$$- 1.5630 \sin \frac{7\pi x}{L} + 2.5810 \sin \frac{9\pi x}{L} + 2.3619 \sin \frac{11\pi x}{L} \cdots$$

$$N_c = - 305.29 \sin \frac{\pi x}{L} - 46.115 \sin \frac{3\pi x}{L} + 6.1357 \sin \frac{5\pi x}{L}$$
$$- 3.1340 \sin \frac{7\pi x}{L} + 5.1829 \sin \frac{9\pi x}{L} + 4.7378 \sin \frac{11\pi x}{L} \cdots$$

$$N_d = + 677.18 \sin \frac{\pi x}{L} + 51.301 \sin \frac{3\pi x}{L} - 6.7532 \sin \frac{5\pi x}{L}$$
$$+ 3.4456 \sin \frac{7\pi x}{L} - 5.6915 \sin \frac{9\pi x}{L} - 5.2073 \sin \frac{11\pi x}{L} \cdots$$

Edge stresses are computed from the corrected plate moments and edge shears. The edge stress, f_e, is computed as follows.

$$f_e = - \frac{M_4}{S_4} - \frac{2}{A_4} (N_d + 2N_e) = - 3.7594 M_4 - 4.3956 N_d$$

$$f_e = + 5,114.9 \sin \frac{\pi x}{L} + 470.85 \sin \frac{3\pi x}{L} - 62.048 \sin \frac{5\pi x}{L}$$
$$+ 31.658 \sin \frac{7\pi x}{L} - 52.362 \sin \frac{9\pi x}{L} - 47.842 \sin \frac{11\pi x}{L} \cdots$$

The analysis of the simply supported folded plate system loaded with unit values of the redundants proceeds in a manner similar to that just described for the load. Since these solutions are similar to the

solution just described, it will be sufficient to state the series used to represent either the force system or its free edge moment and then tabulate the final results of the computations for displacements, plate moments, edge shears, and edge stresses. In these analyses the solution for relative edge displacement is completed using the expressions for b_n', b_n'', b_n''' presented on page 104. Particular values for $\sum_{n=1}^{\infty} A_{n2}$, $\sum_{n=1}^{\infty} A_{n3}$, and $\sum_{n=1}^{\infty} A_{n4}$ are computed for each loading case and substituted into the expressions for b_n', b_n'', and b_n'''.

Solution for a unit value of F_1:

The resultants F_1 are applied in the middle plane of plates 1 at $x = \frac{L}{2}$. The free edge moment for plate 1 can be represented by the first six terms of the following series.

$$M_1' = -\frac{2L}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\sin\frac{n\pi}{2}\sin\frac{n\pi x}{L} = -8.9163\sum_{n=1}^{\infty}\frac{1}{n^2}\sin\frac{n\pi}{2}\sin\frac{n\pi x}{L}$$

$$M_1' = -8.9163\sin\frac{\pi x}{L} + 0.99069\sin\frac{3\pi x}{L} - 0.35665\sin\frac{5\pi x}{L}$$
$$+ 0.18197\sin\frac{7\pi x}{L} - 0.11008\sin\frac{9\pi x}{L} + 0.073688\sin\frac{11\pi x}{L} \cdots$$

$$M_2' = M_3' = M_4' = 0$$

The series constants for the corrected δ displacements, the plate moments, the edge shears, and the edge stress f_e are presented in the following tabulation.

n	1	3	5	7	9	11
$(10)^6\delta_1$	+251.31	-5.9885	+0.83125	-0.21888	+0.080423	-0.036030
$(10)^6\delta_2$	- 20.75	+1.6152	-0.26036	+0.070155	-0.025993	+0.011625
$(10)^6\delta_3$	- 25.99	-0.04360	+0.060585	-0.018399	+0.0070880	-0.0031554
$(10)^6\delta_4$	+ 22.91	-0.25759	-0.012974	+0.0056096	-0.0023567	-0.0010375
$(10)^6\delta_5$	+ 42.64	-0.47942	-0.024147	+0.0104403	-0.0043862	+0.0019310
$(10)^3M_1$	- 6174.4	+964.11	-354.89	+181.72	-110.07	+73.672
$(10)^3M_2$	- 6184.0	+111.866	- 7.8309	+ 1.10465	- 0.04159	+ 0.074614
$(10)^3M_3$	- 1102.0	-178.34	+ 13.311	- 1.8932	+ 0.03592	- 0.12801
$(10)^3M_4$	-10754.0	+ 62.345	- 4.5406	+ 0.64371	- 0.012568	- 0.043515
$(10)^3N_b$	+3117.9	-380.63	+133.28	-67.639	+40.834	-27.336
$(10)^3N_c$	+ 710.0	+110.17	- 36.328	+18.164	-10.897	+ 7.3057
$(10)^3N_d$	+ 289.3	- 2.688	+ 7.2026	- 4.2730	+ 2.7191	- 1.8083
$(10)^3f_e$	+2780.2	-223.08	- 14.552	+16.357	-11.905	+ 7.7846

Solution for a unit value of F_2:

The free edge moment produced in plates 2 upon the application of unit values of the resultant F_2 can be represented by the following series.

$$M_2' = -8.9163 \sin\frac{\pi x}{L} + 0.99069 \sin\frac{3\pi x}{L} - 0.35665 \sin\frac{5\pi x}{L} + 0.18197 \sin\frac{7\pi x}{L} - 0.11008 \sin\frac{9\pi x}{L} + 0.073688 \sin\frac{11\pi x}{L} \ldots$$

$$M_1' = M_3' = M_4' = 0$$

The corrected series constants for the δ displacements, the plate moments, the edge shears, and the edge stress f_e are presented in the following tabulation.

n	1	3	5	7	9	11
$(10)^6\delta_1$	-20.51	+1.6106	-0.25974	+0.069999	-0.025960	+0.011605
$(10)^6\delta_2$	+40.72	-1.3865	+0.22957	-0.062137	+0.023081	-0.010314
$(10)^6\delta_3$	+ 5.45	+0.5726	-0.130285	+0.036580	-0.013779	+0.0061393
$(10)^6\delta_4$	-44.32	+0.09162	+0.036013	-0.011658	+0.0046018	-0.0020317
$(10)^6\delta_5$	-82.487	+0.17052	+0.067026	-0.021697	+0.0085647	-0.0037813
$(10)^3M_1$	-1162.1	+ 22.321	- 1.5685	+ 0.22117	+ 0.000769	+ 0.014955
$(10)^3M_2$	-5499.4	+885.93	-349.14	+180.91	-110.10	+73.616
$(10)^3M_3$	-2584.4	+189.738	- 13.880	+ 1.9681	+ 0.04986	+ 0.13312
$(10)^3M_4$	+1252.3	- 63.185	+ 4.5905	- 0.65017	- 0.020740	- 0.04398
$(10)^3N_b$	+1078.7	-122.93	+ 48.323	- 25.038	+ 15.241	-10.188
$(10)^3N_c$	+1485.2	-206.03	+ 69.620	- 35.078	+ 21.112	-14.148
$(10)^3N_d$	- 85.87	+ 24.391	- 15.414	+ 8.4871	- 5.2839	+ 3.5177
$(10)^3f_e$	-4340.8	+130.85	+ 50.458	- 34.856	+ 23.304	-15.296

Solution for a unit value of F_3:

The free edge moment developed in plates 3 upon the application of the unit values of the resultants, F_3, can be represented by the following series.

$$M_3' = -8.9163 \sin\frac{\pi x}{L} + 0.99069 \sin\frac{3\pi x}{L} - 0.35665 \sin\frac{5\pi x}{L} + 0.18197 \sin\frac{7\pi x}{L} - 0.11008 \sin\frac{9\pi x}{L} + 0.073688 \sin\frac{11\pi x}{L} \cdots$$

$$M_1' = M_2' = M_4' = 0$$

The corrected series coefficients for the δ displacements, the plate moments, the edge shears and the edge stress f_e are presented in the following tabulation.

n	1	3	5	7	9	11
$(10)^6\delta_1$	-26.021	-0.04216	+0.060405	-0.018353	+0.0071213	-0.0031478
$(10)^6\delta_2$	+ 5.458	+0.5723	-0.13023	+0.036577	-0.013796	+0.0061390
$(10)^6\delta_3$	+11.527	-1.0855	+0.20547	-0.056566	+0.021167	-0.0094361
$(10)^6\delta_4$	+ 6.295	+0.7697	-0.15552	+0.043124	-0.016187	+0.0072105
$(10)^6\delta_5$	+11.716	+1.4325	-0.28945	+0.080261	-0.030127	+0.013420
$(10)^3M_1$	+ 107.71	- 22.151	+ 1.6353	- 0.23185	- 0.012917	- 0.015703
$(10)^3M_2$	-1728.4	+113.89	- 8.3155	+ 1.1778	+ 0.071689	+ 0.079719
$(10)^3M_3$	-3071.8	+765.50	-340.38	+179.68	-110.20	+73.532
$(10)^3M_4$	-15282.2	+ 72.626	- 5.2694	+ 0.74594	+ 0.047602	+ 0.050471
$(10)^3N_b$	+ 102.43	+ 20.471	- 12.254	+ 6.7042	- 4.1708	+ 2.7733
$(10)^3N_c$	+ 807.60	-158.72	+ 63.276	- 32.843	+ 20.007	-13.373
$(10)^3N_d$	+ 783.7	-139.91	+ 58.247	- 30.448	+ 18.601	-12.425
$(10)^3f_e$	+2313.1	+341.36	-236.18	+131.03	- 81.942	+54.425

Solution for a unit value of F_4:

The free edge moment developed in plates 4 upon the application of unit values of the resultants F_4 can be represented by the following series.

$$M_4' = -8.9163 \sin\frac{\pi x}{L} + 0.99069 \sin\frac{3\pi x}{L} - 0.35665 \sin\frac{5\pi x}{L} + 0.18197 \sin\frac{7\pi x}{L} - 0.11008 \sin\frac{9\pi x}{L} + 0.073688 \sin\frac{11\pi x}{L} \cdots$$

$$M_1' = M_2' = M_3' = 0$$

The series coefficients for the corrected δ displacements, plate moments, edge shears, and edge stress f_e are presented in the following tabulation.

n	1	3	5	7	9	11
$(10)^6\delta_1$	+ 23.240	-0.25848	-0.012903	+0.0055950	-0.0024150	+0.0010349
$(10)^6\delta_2$	- 44.490	+0.09231	+0.036108	-0.011655	+0.0046392	-0.0020316
$(10)^6\delta_3$	+ 6.330	+0.7694	-0.15569	+0.043127	-0.016212	+0.0072110
$(10)^6\delta_4$	+107.66	-2.1844	+0.33130	-0.088523	+0.032765	-0.014636
$(10)^6\delta_5$	+200.37	-4.0655	+0.61660	-0.16476	+0.060981	-0.027240
$(10)^3M_1$	+ 257.60	+ 18.152	- 1.3636	+ 0.19367	+ 0.02051	+ 0.01313
$(10)^3M_2$	+1259.1	- 96.862	+ 7.0977	- 1.006	- 0.1053	- 0.06809
$(10)^3M_3$	-5161.3	+194.57	- 14.029	+ 1.984	+ 0.1794	+ 0.1343
$(10)^3M_4$	-7178.5	+927.64	-352.15	+181.33	-110.15	+73.645
$(10)^3N_b$	- 374.46	+ 2.340	+ 3.4291	- 2.1421	+ 1.4015	- 0.9182
$(10)^3N_c$	+ 286.90	+ 41.206	- 20.269	+ 10.832	- 6.6805	+ 4.4497
$(10)^3N_d$	+2572.5	-250.77	+ 83.520	- 41.988	+ 25.235	-16.922
$(10)^3f_e$	+15738.	-2392.8	+959.68	-498.64	+304.09	-203.09

The simply supported roof is next analyzed for the effect of an edge pressure, p_e, distributed along edge e of the simple folded plate and acting in the plane of plate 5. The edge force applied to the folded plate is represented by the first six terms of the following series.

$$p_e = -\sum_{n=1}^{\infty} a_n \sin \frac{n \pi x}{L}$$

The edge force is resolved into components acting in the middle plane of plates 4 which have joint e as a common joint. The free edge moment developed in plates 4 can be expressed as follows.

$$M_4' = 0.93110 \frac{L^2}{\pi^2} \sum_{n=1}^{\infty} \frac{a_n}{n^2} \sin \frac{n \pi x}{L}$$

$$M_4' = + 182.645a_1 \sin \frac{\pi x}{L} + 20.294a_3 \sin \frac{3\pi x}{L} + 7.3058a_5 \sin \frac{5\pi x}{L}$$
$$+ 3.7274a_7 \sin \frac{7\pi x}{L} + 2.2549a_9 \sin \frac{9\pi x}{L} + 1.5095a_{11} \sin \frac{11\pi x}{L} \cdots$$

$$M_1' = M_2' = M_3' = 0$$

Having defined the free edge moment, the remainder of the solution for this loading condition proceeds in the same manner as that described for the applied load. The final series coefficients are presented in the following tabulation.

a_n	a_1	a_3	a_5	a_7	a_9	a_{11}
$(10)^6 \delta_1$	- 475.44	- 5.2947	+ 0.26433	+0.11461	+0.049468	+0.021200
$(10)^6 \delta_2$	+ 910.70	+ 1.8912	- 0.73732	-0.23874	-0.095035	-0.041617
$(10)^6 \delta_3$	- 128.81	+15.761	+ 3.1856	+0.88336	+0.33208	+0.14771
$(10)^6 \delta_4$	-2205.99	-44.745	- 6.7867	-1.8133	-0.67120	-0.29984
$(10)^6 \delta_5$	-4105.70	-83.277	-12.631	-3.3749	-1.2492	-0.55806
M_1	- 5.2782	+ 0.37183	+0.027932	+0.0039670	-0.0004202	+0.0002689
M_2	- 25.780	- 1.9842	-0.14540	-0.020598	+0.002158	-0.001395
M_3	+105.68	+ 3.9855	+0.28738	+0.040645	-0.0037385	+0.002750
M_4	+147.06	+19.003	+7.2124	+3.7142	+2.2563	+1.5086

a_n	a_1	a_3	a_5	a_7	a_9	a_{11}
N_b	+ 7.6679	+0.047930	-0.070245	-0.043879	-0.028708	-0.018809
N_c	- 5.8720	+0.84410	+0.41521	+0.22188	+0.13684	+0.091149
N_d	-52.687	-5.1369	-1.7108	-0.86007	-0.51690	-0.34664
f_e	-322.49	-49.018	-19.654	-10.2135	-6.2290	-4.1603

The edge shear force, T_e, is applied along the joint e which is common to the simple folded plate and the additional plate, the edge shear is represented by the first six terms of the following series.

$$T_e = \sum_{n=1}^{\infty} b_n \cos \frac{n \pi x}{L}$$

$$N_e = \int_0^x T_e dx = \frac{L}{\pi} \sum_{n=1}^{\infty} \frac{b_n}{n} \sin \frac{n \pi x}{L} = 14.0056 \sum_{n=1}^{\infty} \frac{b_n}{n} \sin \frac{n \pi x}{L}$$

N_e is the resultant shear at any value of x acting on the edge e of the additional interior plate. From the equilibrium condition and from symmetry at joint e, the shear resultant acting on the edge of one of the plates, 4, will be $\frac{N_e}{2}$. The following six terms of the series for $\frac{N_e}{2}$ will be used to represent the shear resultant applied to edge e of plate 4.

$$\frac{N_e}{2} = + 7.0028 b_1 \sin \frac{\pi x}{L} + 2.3343 b_3 \sin \frac{3\pi x}{L} + 1.4006 b_5 \sin \frac{5\pi x}{L}$$
$$+ 1.00040 b_7 \sin \frac{7\pi x}{L} + 0.77809 b_9 \sin \frac{9\pi x}{L} + 0.63662 b_{11} \sin \frac{11\pi x}{L} \cdots$$

With the edge force specified, the remaining edge shears are computed using the equations presented on page 96, wherein, the free edge plate moments are zero, but the edge shear resultant, N_e, of the equation is $\frac{N_e}{2}$ expressed in the above series. The resulting series coefficients for the edge shears N_b, N_c, and N_d are as follows.

b_n	b_1	b_3	b_5	b_7	b_9	b_{11}
N_b	-0.1029346	-0.034312	-0.020587	-0.014705	-0.011437	-0.0093577
N_c	+0.49476	+0.16492	+0.098955	+0.070680	+0.054974	+0.044978
N_d	-1.8751	-0.62503	-0.37502	-0.26787	-0.20834	-0.17046

The remainder of the solution involving the calculations for the δ displacements and the correction for relative edge displacement is the same as for the applied load. The corrected constants for the δ displacements, the plate moments, the edge shears, and the edge stress f_e are presented in the following tabulation.

b_n	b_1	b_3	b_5	b_7	b_9	b_{11}
$(10)^6\delta_1$	- 41.507	- 1.09479	+0.04233	+0.034190	+0.020210	+0.010358
$(10)^6\delta_2$	+ 64.010	+ 0.64317	-0.14794	-0.073044	-0.038558	-0.020404
$(10)^6\delta_3$	- 33.866	+ 1.6698	+0.69340	+0.27480	+0.13410	+0.072628
$(10)^6\delta_4$	-230.98	-11.7007	-2.8152	-1.04493	-0.49582	-0.27085
$(10)^6\delta_5$	-429.89	-21.777	-5.2395	-1.9448	-0.92282	-0.50411

o o

b_n	b_1	b_3	b_5	b_7	b_9	b_{11}
M_1	-0.26077	+0.065758	+0.0081998	+0.0016295	-0.0002804	+0.00017353
M_2	-2.0681	-0.35039	-0.042646	-0.0084537	+0.0014441	-0.000899
M_3	+5.2185	+0.69177	+0.083356	+0.016508	-0.0025652	+0.001756
M_4	-2.5502	-0.22788	-0.027371	-0.0098614	+0.0009216	-0.00057619
N_b	+0.50939	+0.018811	-0.014231	-0.013447	-0.011642	-0.009224
N_c	-0.23683	+0.099170	+0.091147	+0.069135	+0.055190	+0.044823
N_d	-2.2639	-0.70799	-0.38507	-0.26986	-0.2080	-0.17067
f_e	-42.003	-16.5507	-10.5173	-7.5713	-5.9293	-4.843

The calculation for the effect of a resultant F_5 must be delayed until the conditions are written providing for the continuity of the simple folded plate and plate 5.

STEP 2. In this step the analysis of the redundant members is undertaken. The additional interior plate will be considered first.

The edge load, p_e, applied to edge e and in the plane of the additional interior plate is represented by the first six terms of the following series.

$$p_e = \sum_{n=1}^{\infty} a_n \sin \frac{n \pi x}{L}$$

The plate moment produced by this loading on the simply supported plate is

$$M_5 = \int_0^x \int_0^x p_e dx dx = - \frac{L^2}{\pi^2} \sum_{n=1}^{\infty} \frac{a_n}{n^2} \sin \frac{n \pi x}{L} .$$

The first six terms of this series are used to represent the moment, M_5.

$$M_5 = -196.16a_1 \sin\frac{\pi x}{L} - 21.796a_3 \sin\frac{3\pi x}{L} - 7.8464a_5 \sin\frac{5\pi x}{L}$$
$$- 4.0033a_7 \sin\frac{7\pi x}{L} - 2.4217a_9 \sin\frac{9\pi x}{L} - 1.6212a_{11} \sin\frac{11\pi x}{L} \cdots$$

$$f_e = \frac{M_5}{S_5} = \frac{M_5}{0.27084}$$

$$f_e = -724.38a_1 \sin\frac{\pi x}{L} - 80.488a_3 \sin\frac{3\pi x}{L} - 28.975a_5 \sin\frac{5\pi x}{L}$$
$$- 14.783a_7 \sin\frac{7\pi x}{L} - 8.9430a_9 \sin\frac{9\pi x}{L} - 5.9866a_{11} \sin\frac{11\pi x}{L} \cdots$$

$$\delta_5 = \frac{1}{EI_5}\int_0^x\int_0^x M_5 dxdx = \frac{L^4}{\pi^4 EI_5}\sum_{n=1}^{\infty}\frac{a_n}{n^4}\sin\frac{n\pi x}{L}$$

$$(10)^6\delta_5 = +10.825a_1 \sin\frac{\pi x}{L} + 133.64a_3 \sin\frac{3\pi x}{L} + 17.320a_5 \sin\frac{5\pi x}{L}$$
$$+ 4.5085a_7 \sin\frac{7\pi x}{L} + 1.6499a_9 \sin\frac{9\pi x}{L} + 0.73936a_{11} \sin\frac{11\pi x}{L} \cdots$$

The edge shear distributed along edge e of plate 5 is represented by the first six terms of the following series.

$$T_e = \sum_{n=1}^{\infty} b_n \cos\frac{n\pi x}{L}$$

The resultant edge shear force is

$$N_e = \int_0^x T_e dx = \frac{L}{\pi}\sum_{n=1}^{\infty}\frac{b_n}{n}\sin\frac{n\pi x}{L} .$$

$$N_e = +14.006b_1 \sin\frac{\pi x}{L} + 4.6687b_3 \sin\frac{3\pi x}{L} + 2.8012b_5 \sin\frac{5\pi x}{L}$$
$$+ 2.0009b_7 \sin\frac{7\pi x}{L} + 1.5562b_9 \sin\frac{9\pi x}{L} + 1.2733b_{11} \sin\frac{11\pi x}{L} \cdots$$

$$f_e = + \frac{4N_e}{A_5}$$

$$f_e = + 86.190b_1 \sin \frac{\pi x}{L} + 28.730b_3 \sin \frac{3\pi x}{L} + 17.238b_5 \sin \frac{5\pi x}{L} \cdots\cdots$$
$$+ 12.312b_7 \sin \frac{7\pi x}{L} - 9.5767b_9 \sin \frac{9\pi x}{L} + 7.8355b_{11} \sin \frac{11\pi x}{L} \cdots$$

$$\delta_5 = \frac{h_5}{2EI_5} \int_0^x \int_0^x N_e dx dx = - \frac{L^3 h_5}{2\pi^3 EI_5} \sum_{n=1}^{\infty} \frac{b_n}{n^3} \sin \frac{n\pi x}{L}$$

$$(10)\ {}^6\delta_5 = - 966.14b_1 \sin \frac{\pi x}{L} - 35.781b_3 \sin \frac{3\pi x}{L} - 7.7286b_5 \sin \frac{5\pi x}{L}$$
$$- 2.8166b_7 \sin \frac{7\pi x}{L} - 1.3252b_9 \sin \frac{9\pi x}{L} - 0.72586b_{11} \sin \frac{11\pi x}{L} \cdots$$

Fig. 5.3 presents elevation and cross section views of the interior frame which is to be analyzed to determine the δ_m displacements produced by unit values of the resultants, F_m. The F_m forces are assumed to act in the middle plane of the folded plate elements and are assumed to be uniformly distributed along each segment of the frame. The frame is symmetrical about the center line of the middle column and the distribution of the F_m forces is also symmetrical, consequently only one half of the frame is considered in the analysis. Half of the frame is divided into thirty-eight Δ_S divisions.

The folded plate system participates in the frame action. The increase in effective section of the frame is approximated in the same manner as that discussed in Chapter IV. The effective section for the frame from section 16 through 38 is shown in Fig. 5.3.

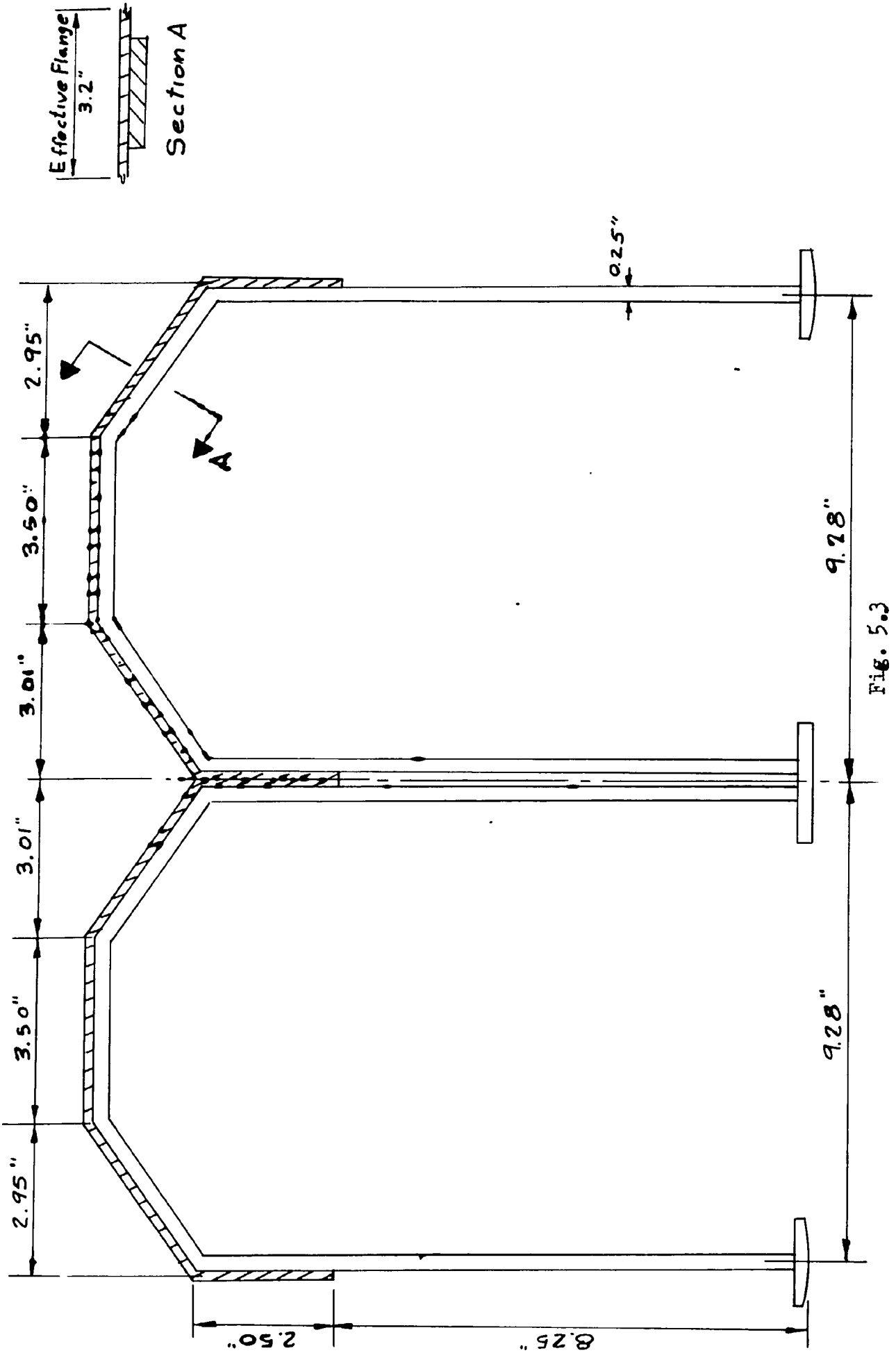

Effective Flange
3.2"
Section A
.25"
2.95"
3.50"
3.01"
3.01"
3.50"
2.95"
9.28"
9.28"
2.50"
8.25"
A

Fig. 5.3

The analysis of the frame is presented in the Appendix. The displacements of the frame, produced by the force systems F_1, F_2, F_3, F_4, and F_5, are as follows.

$$(10)^6\delta_1 = + 0.3865F_1 - 0.9829F_2 - 0.411F_3 - 0.02472F_4$$

$$(10)^6\delta_2 = + 2.564\ F_1 - 43.00\ F_2 - 23.179F_3 - 1.3949\ F_4$$

$$(10)^6\delta_3 = + 0.959\ F_1 - 22.45\ F_2 - 14.231F_3 - 0.8502\ F_4$$

$$\delta_4 = 0$$

$$\delta_5 = 0$$

The displacements were computed along the controidal axis of the frame segments, thus yielding the value of $\delta_4 = 0$. Computing the displacement at the centroid of the frame rather than in the middle plane of the plate segments produces a maximum error of 5% in the coefficients of the final equations written for the solution for F_1, F_2, F_3, F_4, and F_5.

STEP 3. In this step, equations are written providing for the continuity of the simple folded plate and the additional plate and the interior frame. These equations will be solved yielding the redundant forces.

Two equations are written providing for the continuity of the simple folded plate and the additional plate. One equates the displacement of edge e of the simple folded plate to the displacement of plate 5, the displacements being measured in the plane of plate 5.

The second equates the stresses at edge e of the simple folded plate and plate 5.

$$\delta_{on} + \delta_{pn}^{R} + \delta_{Tn}^{R} = \delta_{pn}^{B} + \delta_{Tn}^{B}$$

$$f_{on} + f_{pn}^{R} + f_{Tn}^{R} = f_{pn}^{B} + f_{Tn}^{B}$$

δ_{on} and f_{on} are respectively the displacement at edge e, $(10)^{6}\delta_{5}$, and the edge stress, f_e, of the simple folded plate produced by either the applied load or the redundants F_1, F_2, F_3, F_4, F_5. The superscripts R and B in the remaining expressions refer respectively to the simple folded plate and to the additional plate. The subscripts p and T refer to the edge forces p and T. The displacements δ_{pn}^{R} and δ_{pn}^{B} and the stresses f_{pn}^{R} and f_{pn}^{B} are in series form with series coefficients a_n. The displacements δ_{Tn}^{R} and δ_{Tn}^{B} and the stresses f_{Tn}^{R} and f_{Tn}^{B} are written in series form with series coefficients b_n. Six sets of this system of simultaneous equations are written in this case, one for each value of n considered in this solution. Solving these sets of equations simultaneously yields values of a_n and b_n in terms of δ_{on} and f_{on}.

The series constants for the edge loadings p and T are tabulated on pages 113 to 118. The systems of equations to be solved are as follows.

$n = 1$: δ_{o1} -4,105.70 a_1 -429.89 b_1 = +10,825 a_1 -966.14 b_1

f_{o1} - 322.49 a_1 - 42.003 b_1 = - 724.38 a_1 + 86.190 b_1

$n = 3$: δ_{o3} - 83.277 a_3 - 21.777 b_3 = + 133.64 a_3 - 35.781 b_3

f_{o3} - 49.018 a_3 - 16.550 b_3 = - 80.488 a_3 + 28.730 b_3

$n = 5$: δ_{o5} - 12.631 a_5 - 5.2397 b_5 = + 17.32 a_5 - 7.7286 b_5

f_{o5} - 19.654 a_5 - 10.5137 b_5 = - 28.975 a_5 + 17.238 b_5

$n = 7$: δ_{o7} - 3.3749 a_7 - 1.9448 b_7 = + 4.5085 a_7 - 2.8166 b_7

f_{o7} - 10.2135 a_7 - 7.5713 b_7 = - 14.783 a_7 + 12.312 b_7

$n = 9$: δ_{o9} - 1.2492 a_9 - 0.92282b_9 = + 1.6499 a_9 - 1.3252 b_9

f_{o9} - 6.2290 a_9 - 5.9293 b_9 = - 8.9430 a_9 + 9.5767 b_9

$n = 11$: δ_{o11} - 0.55806a_{11} - 0.50411b_{11} = + 0.73936a_{11} - 0.72586b_{11}

f_{o11} - 4.1603 a_{11} - 4.8443 b_{11} = - 5.9866 a_{11} + 7.8355 b_{11}

The solutions for a_n and b_n are presented in the following tabulation.

$(10)^6 a_1$ = + 315.71 f_{o1} + 75.473 δ_{o1}

$(10)^6 b_1$ = + 8,790.5 f_{o1} + 236.61 δ_{o1}

$(10)^6 a_3$ = + 1,492.7 f_{o3} + 4,826.6 δ_{o3}

$(10)^6 b_3$ = + 23,122 f_{o3} + 3,354.6 δ_{o3}

$(10)^6 a_5$ = + 3,080.3 f_{o5} + 34,347 δ_{o5}

$(10)^6 b_5$ = + 37,068 f_{o5} + 11,536 δ_{o5}

$$(10)^6 a_7 = + 5,707.0\ f_{o7} + 130,160\ \delta_{o7}$$

$$(10)^6 b_7 = + 51,605\ f_{o7} + 29,912\ \delta_{o7}$$

$$(10)^6 a_9 = + 9,174.3\ f_{o9} + 353,520\ \delta_{o9}$$

$$(10)^6 b_9 = + 66,097\ f_{o9} + 61,876\ \delta_{o9}$$

$$(10)^6 a_{11} = + 13,820\ f_{o11} + 790,210\ \delta_{o11}$$

$$(10)^6 b_{11} = + 80,856\ f_{o11} + 113,810\ \delta_{o11}$$

Substituting the values for δ_{on} and f_{on} produced by the applied load and the unit values of the force systems, F_m, yields the following series constants for the edge force, p_e, and edge shear, T_e, for the simply supported, multiple folded plate.

n / Applied Load	1	3	5	7	9	11
a_n	+ 7.0647	+ 4.51812	-1.47973	+1.45180	-3.93621	-5.3837
b_n	+62.049	+13.5387	-2.7328	+1.9258	-4.0659	-4.5485
F_1						
$(10)^6 a_n$	+ 4,095.83	-2,646.96	-874.202	+1,452.258	-1,659.83	+1,633.48
$(10)^6 b_n$	+34,528	-6,766.4	-817.97	+1,156.39	-1,058.28	+ 849.23
F_2						
$(10)^6 a_n$	- 7,595.97	+1,018.35	+2,457.57	-3,023.00	+3,241.59	-3,199.41
$(10)^6 b_n$	-57,675	+3,597.5	+2,643.59	-2,447.74	+2,070.27	-1,667.1
F_3						
$(10)^6 a_n$	+ 1,614.51	+ 7,423.65	-10,669.25	+11,194.56	-11,402.26	+11,356.77
$(10)^6 b_n$	+23,105	+12,698.4	-12,093.8	+ 9,162.56	- 7,280.2	+ 5,928
F_4						
$(10)^6 a_n$	+ 20,091.1	-23,194.2	+24,134.5	-24,291	+24,348	-24,332
$(10)^6 b_n$	+185,755	-68,964	+42,686	-30,660	+23,872	-19,521

At this time the analysis of the multiple folded plate could be made for a unit value of the redundant, F_5, since the conditions are now available for considering the continuity of the simple folded plate and plate 5. It is not necessary to present this solution, however, since in this case it is apparent that the forces, displacements, and stresses produced in the multiple folded plate by a unit value of the force F_5 would be proportional to the forces, displacements and stresses produced by the force system made up of unit values of F_4. The unit value of F_5 can be resolved into components in the plane of plates 4 which are common to plate 5 at edge e. The magnitude of these components of $F_5 = 1$ are:

$$\frac{1}{2 \sin \theta} = 0.9311$$

Thus, the application of a unit value of F_5 will produce forces, displacements and stresses in the multiple folded plate which are 0.9311 as great as those produced by unit values of F_4 as computed on page 112. The a_n and b_n coefficients for a unit value of F_5 would not be 0.9311 of those presented on page 123 resulting from the application of unit values of F_4, however.

To determine the magnitude of the redundant resultants F_1, F_2, F_3, F_4, and F_5, equations representing the consistency of the δ displacements of the roof and the frame will be written. To facilitate writing these equations the displacements of the roof at the section common to the frame are computed for the applied load, the load systems F_m, and the edge loads p_e and T_e. The computations are made

for the six values of n and summed to give the resultant displacements of the multiple folded plate at the frame. Details of the computations are presented in the Appendix; the results of the computations are as follows.

$$\delta_1 = +4{,}902.7 + 255.063F_1 - 16.478F_2 - 27.570F_3 + 6.032(F_4 + 0.9311F_5)$$

$$\delta_2 = -11{,}012.8 - 16.784F_1 + 31.813F_2 + 7.641F_3 - 14.286(F_4 + 0.9311F_5)$$

$$\delta_3 = -2{,}844.97 - 27.510F_1 + 7.613F_2 + 11.714F_3 - 2.908(F_4 + 0.9311F_5)$$

$$\delta_4 = +8{,}815.6 + 5.942F_1 - 14.226F_2 - 2.962F_3 + 20.836(F_4 + 0.9311F_5)$$

$$\delta_5 = \frac{1}{\sin\theta}\,\delta_4$$

In the preceding equations $F_4 + 0.9311F_5$ represents the resultant F_4 combined with the component of F_5 resolved in the plane of plate 4. This combined resultant will be represented by F_4^*.

Equating the displacements computed for the multiple folded plate to those of the frame, presented on page 120, five equations result, the last two of which describe the consistency of the displacements of the multiple folded plate and the frame in the plane of plates 4 and 5. Since the displacements of the frame were computed along the centroidal axis of the frame, δ_4 and δ_5 for the frame are zero. Thus the last two equations are identical. The first four equations can be solved for F_1, F_2, F_3, and F_4^*, however. After equating the frame and folded plate displacements, the equations are as follows.

$$+254.676F_1 - 15.495F_2 - 27.159F_3 + 6.057F_4^* + 4,902.7 = 0$$

$$- 19.348F_1 + 74.813F_2 + 30.820F_3 - 12.891F_4^* - 11,012.8 = 0$$

$$- 28.469F_1 + 30.063F_2 + 25.945F_3 - 2.058F_4^* - 2,844.97 = 0$$

$$+ 5.942F_1 - 14.226F_2 - 2.962F_3 + 20.836F_4^* + 8,815.6 = 0$$

Solving these equations simultaneously yields the following values for F_1, F_2, F_3, and F_4^*.

$$F_1 = - 9.72 \text{ lb.}$$

$$F_2 = +103.53 \text{ lb.}$$

$$F_3 = - 49.27 \text{ lb.}$$

$$F_4^* = -356.63 \text{ lb.}$$

STEP 4. The final values of the plate moments and edge shears are computed at sections 9.625 inches and 19.00 inches from the end support. These final moments and edge shears are used to compute the final edge stresses.

The final plate moments and edge shears are functions of the applied load and the redundant force systems F_1, F_2, F_3, F_4^*, p_e and T_e. Computing the plate moments and edge shears for each of the force systems yields the following final summations at sections x = 9.625 in. and x = 19.00 in. Sample computations for the final plate moments caused by the load are presented in the Appendix.

Final Plate Moments	Final Edge Shears
x = 9.625 in.	
M_1 = +130.85 in. lb.	N_b = -136.63 lb.
M_2 = +782.66 in. lb.	N_c = -230.29 lb.
M_3 = +569.27 in. lb.	N_d = - 84.922 lb.
M_4 = + 73.212 in. lb.	N_e = + 46.684 lb.
M_5 = -106.995 in. lb.	
x = 19.00 in.	
M_1 = +212.06 in. lb.	N_b = -140.269 lb.
M_2 = +638.70 in. lb.	N_c = -230.436 lb.
M_3 = +902.75 in. lb.	N_d = -259.965 lb.
M_4 = +764.24 in. lb.	N_e = -317.57 lb.
M_5 = +436.93 in. lb.	

The final edge stresses at these two sections are computed from the final plate moments and edge shears using the basic equations 2.1 and 2.2.

Final Edge Stresses (lb. per sq. in.)

Edge	x = 9.625 in.	x = 19.00 in.
a	+ 125	+ 703
b	+ 716	+ 160
c	- 324	+ 235
d	- 386	- 103
e	- 108	- 341
e'	+ 251	- 636

The final values of the displacements δ_1 and δ_2 are computed at sections x = 11.00 inches and x = 22.00 inches.

x = 11.00"

$$\delta_1 = + \ 810(10)^{-6} \text{ in.}$$

$$\delta_2 = -2,700(10)^{-6} \text{ in.}$$

x = 22.00"

$$\delta_1 = 0$$

$$\delta_2 = -2,837(10)^{-6} \text{ in.}$$

The final slab moments are derived entirely from the relative edge displacement computations for each loading case. The final values for these moments at the section x = 9.625 in. are as follows.

$$m_a = m_b = 0$$

$$m_c = +1.8024 \text{ in. lb.}$$

$$m_d = -0.4805 \text{ in. lb.}$$

$$m_e = +0.4038 \text{ in. lb.}$$

4. Experimental Investigation

The structural model that was analyzed in the preceding section was fabricated and tested to check the results of the analysis. The dimensions of the model are shown in Fig. 5.1. Aluminum alloys 2014 and 6061 were used for the folded plate and the frame respectively.

After fabricating the model, SR-4 strain gages, type A 7, were attached to both the top and bottom surfaces of the folded plate at sections 9.625 in. and 19.00 in. from the end diaphragm as shown in Fig. 5.1. All of the gages were placed in pairs at a given location on the folded plate, one on the upper surface and one on the bottom surface. Those gages oriented to measure strain in the longitudinal direction were wired so that the gages in a pair were in series. Gages oriented to measure strain in the direction normal to the longitudinal axis of the roof were placed in pairs but wired independently.

The model was set up for testing on a heavy concrete slab, care being taken to have all column bases bearing solidly on the supporting slab. The strain gage leads were connected to Baldwin twenty channel switching units. Leads from these units along with compensating gage leads were connected to a Baldwin portable strain indicator, type MA. Horizontal and vertical displacements of plate 1 were measured with Federal dial gages sensitive to 0.0001 in. A photograph of the model ready for testing is shown in Fig. 5.4.

The 800 pound load was distributed to the model through 16 load points and was applied in 200 pound increments. The model supporting the first two load increments is shown in Fig. 5.5. Strain and displacement readings were taken before loading and after the application of each load increment.

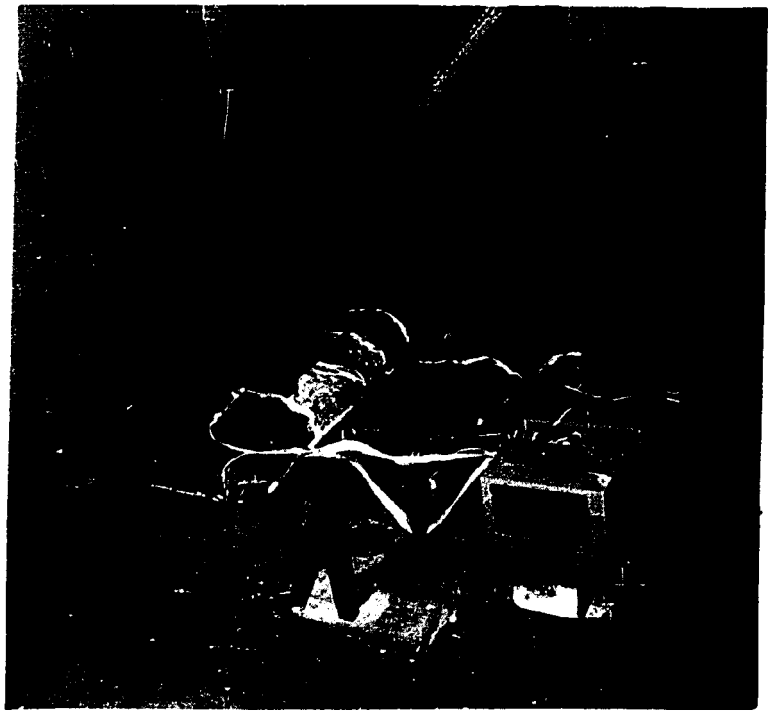


Fig. 5.4

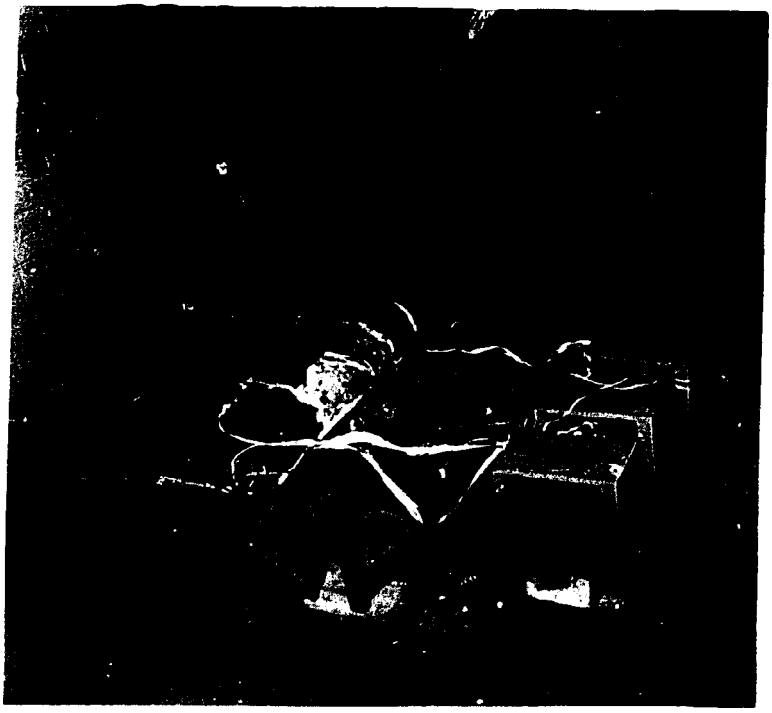


Fig. 5.5

Normal stresses, f_x, acting in the middle plane of the plates were computed from the measured strains in the manner discussed in Appendix I. These stresses correspond to the edge stresses computed in the preceding section. Normal stresses, f_{ym}, acting in a direction normal to the edge stresses and at the surface of the plates were computed from the measured strains using the expressions appearing in Appendix I.

5. Comparison of Analytical and Experimental Results

Graphical presentations of the results of the analysis and experimental work appear in Diagrams 5.1, 5.2, and 5.3 in which the longitudinal stresses, the transverse stresses, and the edge displacements are plotted. General agreement between the computed and the measured values was obtained.

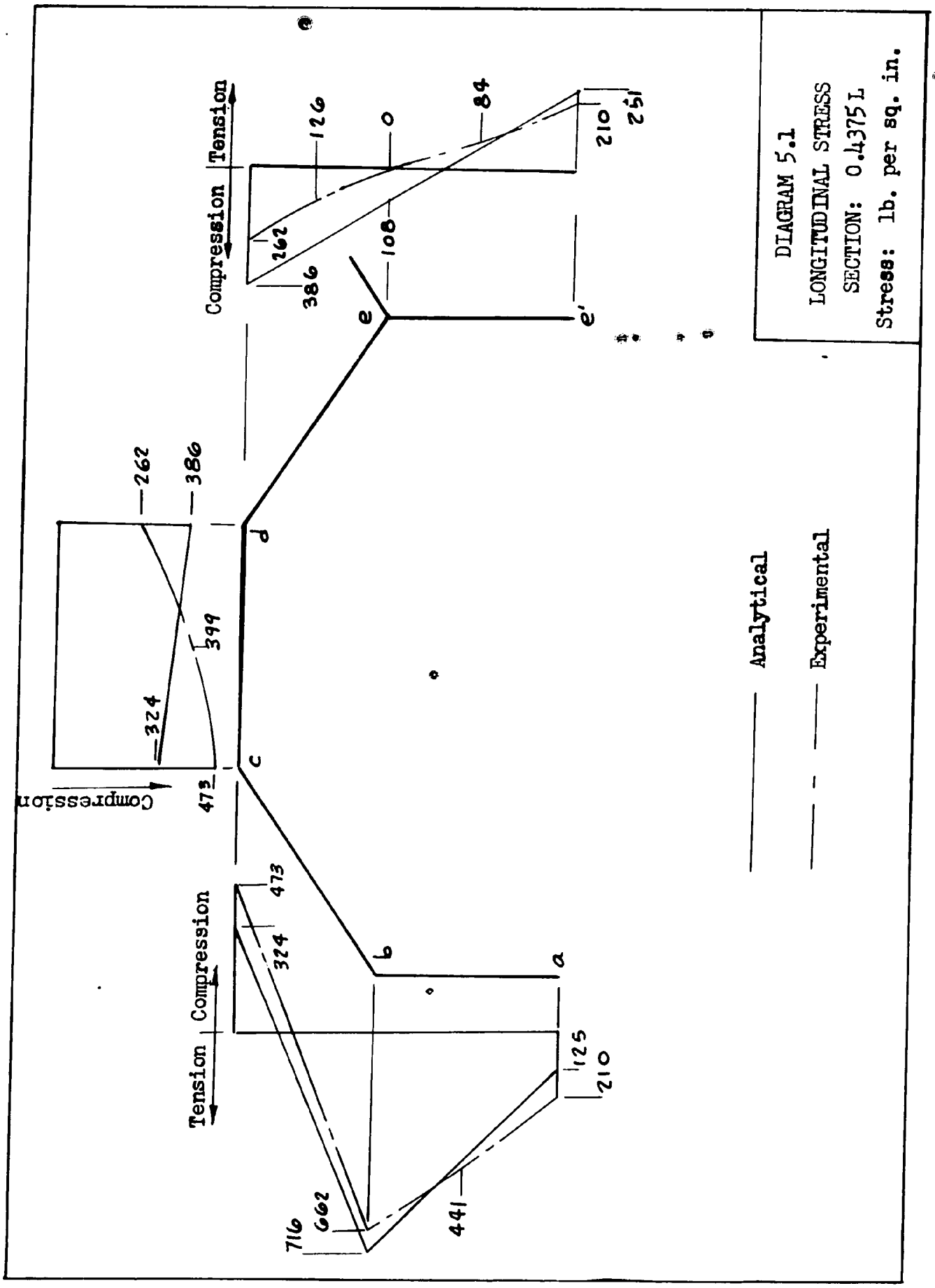
Good agreement for longitudinal stresses, is shown for section x = 9.625 in. except in the region of plate 3. At section x = 19.00 in. the analytical results agree fairly well with the experimental results except in the region of plate 5.

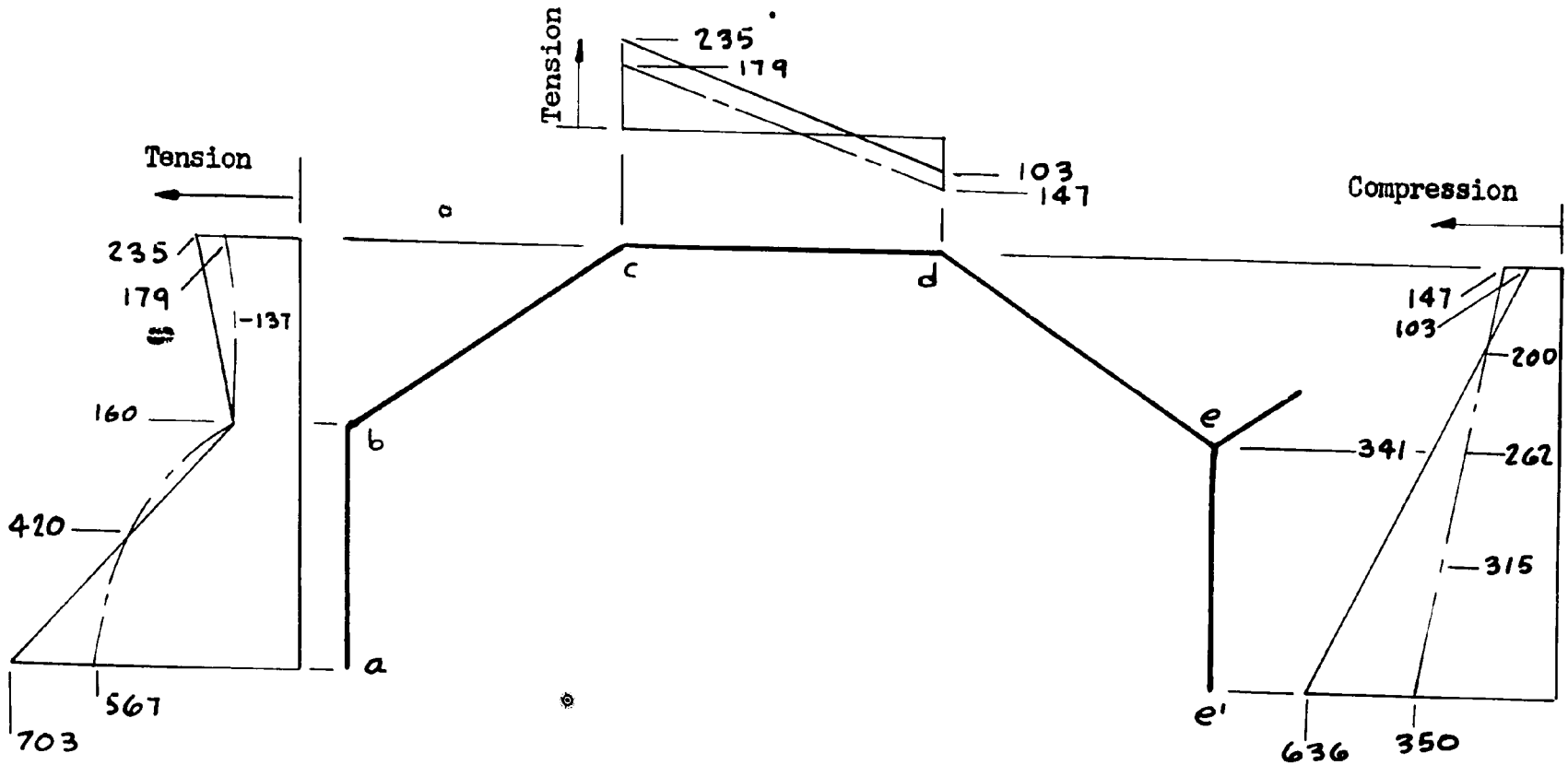
The transverse stresses computed from the measured transverse strains are in excess of the analytical values. In this investigation, as in the investigation presented in Chapter IV, the indicated transverse strains may have been influenced considerably by the closeness of the gages to the load points. The gages were 2.29 in. from the point of load application whereas the distance between these points, in the investigation presented in Chapter III, was nearly four inches. During the experimental investigation the transverse stresses

were particularly sensitive to the applied load. The computed displacements are in fair to good agreement with the measured quantities.

The correction recommended by Ali[12] for transfer of the applied load to plates adjacent to the loaded plates would undoubtedly have improved the agreement between the computed and measured results particularly for longitudinal stress in the region of plates 1 and 5.

In this case the computations were particularly lengthy having considered six values of n in the solution. The final results indicate that three terms at the most would have been sufficient. The resulting agreement between analytical and experimental work does indicate that the methods of superposition and consistent deformation can be used to provide acceptable results in the analysis of these complex forms of folded plate construction.

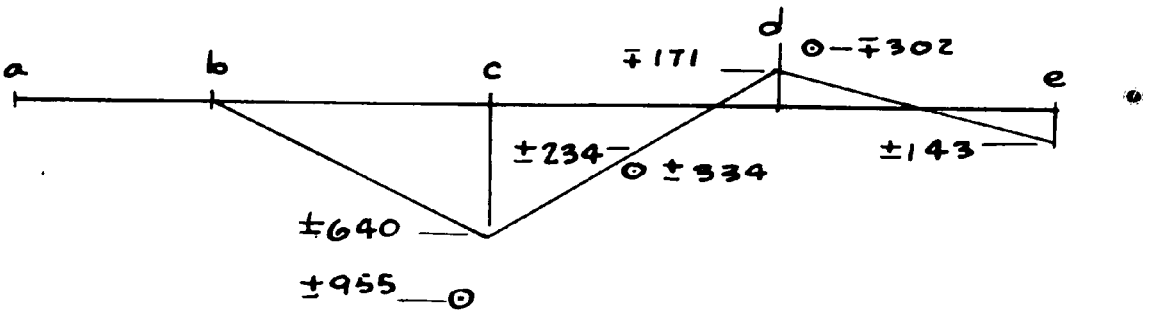

DIAGRAM 5.1
LONGITUDINAL STRESS
SECTION: 0.4375 L
Stress: lb. per sq. in.
Analytical
Experimental
Compression
Tension
Tension
Compression
Compression
126
0
84
210
251
162
386
108
e
e'
262
386
d
399
324
c
473
473
324
b
a
125
210
716
662
441



Analytical

Experimental

DIAGRAM 5.2
LONGITUDINAL STRESS
SECTION: 0.9635 L
Stress: lb. per sq. in.



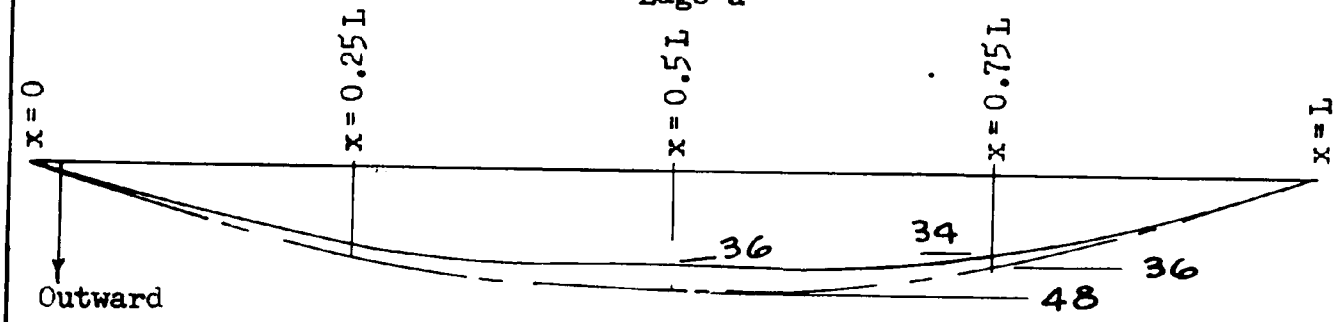
Transverse Stress (lb. per sq. in.)

x = 9.625"



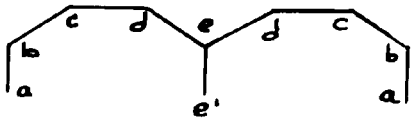
Vertical Displacement in.$(10)^4$

Edge a



Horizontal Displacement in.$(10)^4$

Edge b



——— Analytical

——— ⊙ ——— Experimental

DIAGRAM 5.3
TRANSVERSE STRESS
EDGE DISPLACEMENT

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APPENDIX I

1. Experimental Data

The experimental results presented in Chapters III, IV, and V provide an indication of the total strain measured at the various gages but do not indicate the variation of strain with each load increment. Diagrams I and II present typical load-strain curves for gages attached to the model investigated in Chapter V. The curves are typical of those obtained for the investigations presented in Chapters III and IV.

The data was taken after loading and unloading the model several times which represented an effort to place the model in a state of uniform response. The models were all redundant structures and the preworking was an attempt to remove any unplanned slippage or settlement resulting from small discrepancies in fabrication and set up.

2. Stress Strain Relations

The SR-4 strain gages were oriented and wired to indicate longitudinal strains in the middle plane of the plates and transverse strains at the surface of the plates. The measured strains are related to the computed edge stresses and slab bending stresses through the following relationships.

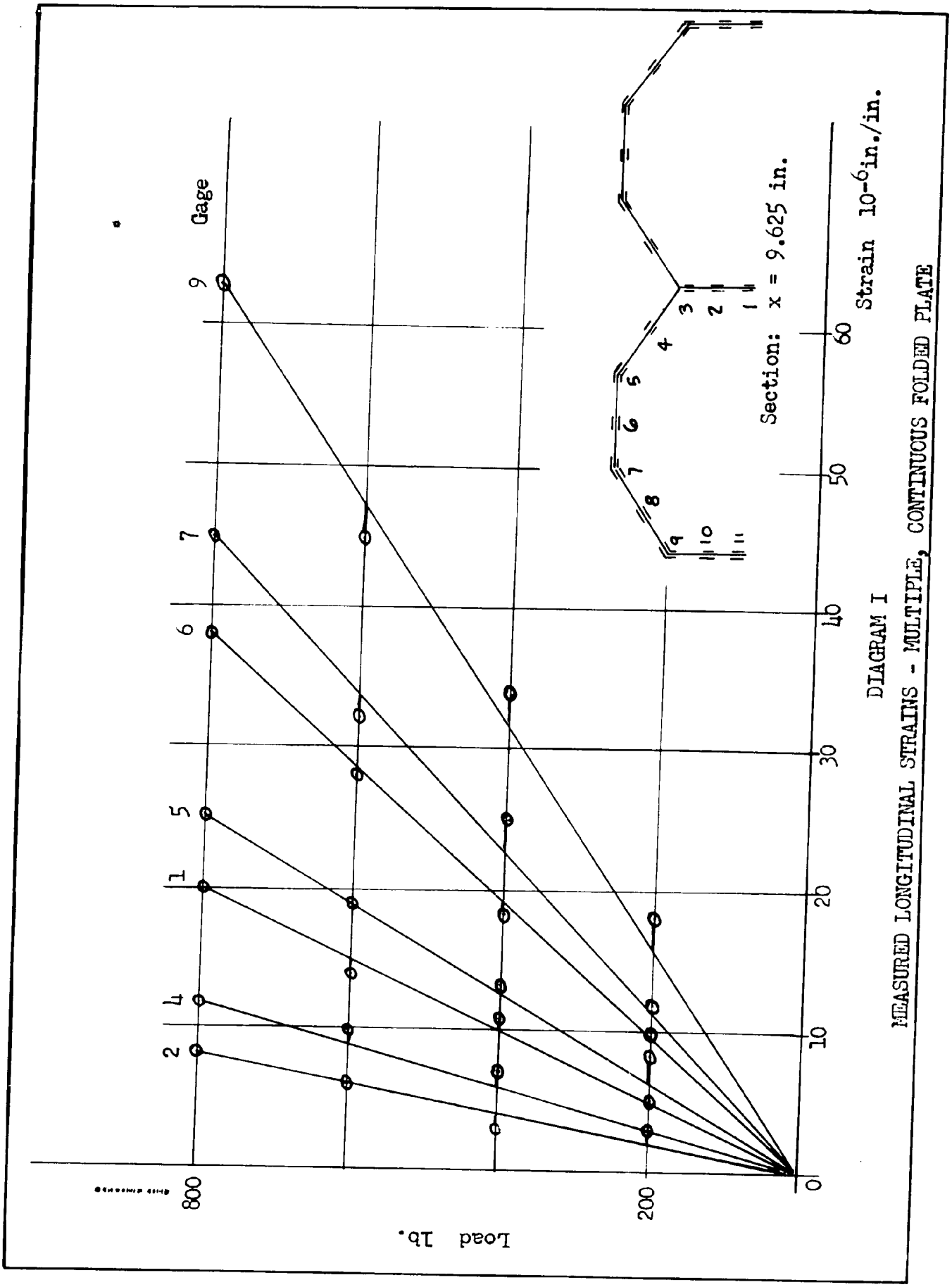


DIAGRAM I

MEASURED LONGITUDINAL STRAINS - MULTIPLE, CONTINUOUS FOLDED PLATE

DIAGRAM II

MEASURED LONGITUDINAL STRAINS - MULTIPLE, CONTINUOUS FOLDED PLATE

$$f_x = \frac{E}{1-\mu^2} (\epsilon_x + \mu \epsilon_y)$$

$$f_y = \frac{E}{1-\mu^2} (\epsilon_y + \mu \epsilon_x)$$

Placing the longitudinally oriented gages in pairs and wiring the gages in a pair in series eliminated the effect of slab bending. It did not, however, eliminate the influence of the forces acting in the middle plane of the plates. These forces exist primarily as the result of the edge loads, p, which, in the case of these investigations, are due primarily to relative edge displacement. However, the most severe edge force occurs as p_e in the investigation of Chapter III. This force is from five to ten times as large as the other edge forces in the folded plate. The stress in the middle plane of the plate produced by this edge force is

$$\frac{(8.33)\ (0.9311)}{0.13} = 59.6 \text{ lb. per sq. in.}$$

The influence of this stress on the measured strain ϵ_x would be:

$$\epsilon_x = -\frac{1}{E} (f_x - 0.33 \times 59.6)$$

$$= \frac{1}{E} (f_x - 19.8)$$

The effect is small and can be neglected when comparing measured and computed edge stresses. Thus the following relationship is used to indicate longitudinal stress from the measured strains.

$$f_x = \frac{\epsilon_{x \text{ ave.}}}{E}$$

In this expression, $\epsilon_{x \text{ ave.}}$ is the indicated longitudinal strain.

The strains indicated by the transversely oriented, independently wired gages include the influence of the stress f_x and the transverse stress in the middle plane of the plate. The stress f_x at the surface of the plate is closely approximated by $\frac{\epsilon_{x\ ave.}}{E}$. For the region in which the transverse strains were measured, the transverse stress in the middle plane of the plate is small compared with those computed from p_e, above. Thus the slab bending stresses were computed from the indicated transverse strains using the following relationship.

$$f_y = \frac{E}{1-\mu^2} (\epsilon_y + \mu \epsilon_{x\ ave.})$$

APPENDIX II

1. Analysis of the Single Bay Frame

The analysis of the frame shown in Fig. 4.7 is presented in this section. The frame and the F_m forces, with which the frame is loaded, are symmetrical about the center line of the frame. Hinged supports are simulated in the model, so that the frame is considered as a symmetrical two hinged frame. Symmetry simplifies the analysis of the frame whose redundant will be taken as the horizontal component of the reaction. The frame is divided into fifty-six equal divisions of 0.569 in. measured along the line through the centroids of the frame cross sections. From symmetry, the summations made over these divisions need to be made over only one half of the total, or twenty-eight divisions.

Three different moments of inertia are computed for the analysis, since the splice bars are assumed to be effective between their outermost bolts. The three moments of inertia which follow are respectively for the 2" x 1/4" bar section, the bar with a 3.2" flange and the bar with a flange and a splice bar added to the section.

$$I_O = 2{,}604(10)^{-6} \text{ in.}^4$$

$$I_F = 10{,}937(10)^{-6} \text{ in.}^4$$

$$I_S = 21{,}538(10)^{-6} \text{ in.}^4$$

$$\frac{I_F}{I_O} = 4.200 \quad ; \quad \frac{I_S}{I_O} = 8.271$$

The total strain energy due to bending stored in the frame upon the application of the F_m forces is

$$U = 2 \sum_{0}^{28} \frac{M^2}{2EI} \Delta s$$

where

$$M = M' + H_y$$

The statically determinate moment is denoted as M'. From Castigliano's Second Theorem and from the support conditions

$$\Delta_H = \frac{\partial U}{\partial H} = 0 = \sum_{0}^{28} \frac{M'y\Delta s}{EI} + H \sum_{0}^{28} \frac{y^2 \Delta s}{EI} .$$

Solving this equation for H yields

$$H = - \frac{\sum_{0}^{28} \frac{M'y\Delta s}{EI}}{\sum_{0}^{28} \frac{y^2 \Delta s}{EI}} = - \frac{\sum_{0}^{28} \frac{M'y}{I/I_0}}{\sum_{0}^{28} \frac{y^2}{I/I_0}} .$$

Substituting these summations yields the following value for H.

$$H = - \frac{-4.2759F_1 + 15.8345F_2}{632.081}$$

$$= + 67.648(10)^{-4}F_1 - 250.514(10)^{-4}F_2$$

The magnitude of V is obtained from the equilibrium conditions.

$$V = F_1 + 0.5373F_2$$

The moment at the center of each Δs division is now computed. The quantity $\frac{M\Delta s}{EI}$ represents the total rotation occurring in the length Δs and is considered as a concentrated rotation developed at the center of the Δs division. Since the cross section at the center line of the

frame does not rotate, the total rotation of the hinge at the support is given as follows.

$$\alpha_a = \sum_0^{28} \frac{M\Delta s}{EI} = +4.422(10)^{-6}F_1 - 19.274(10)^{-6}F_2$$

The displacement of any point in the frame can be computed as the difference between the displacement produced by the rotation α_a and the displacement induced by the individual Δs rotations developed between the support and the point whose displacement is being computed.

$$\delta_y = X_m \alpha_a - \sum_{n=o}^{n=m} (X_m - X_n)\, \Delta\alpha_n$$

$$\delta_x = Y_m \alpha_a - \sum_{n=o}^{n=m} (Y_m - Y_n)\, \Delta\alpha_n$$

In the above equations, m is the point whose displacement is to be computed and n is any Δs division between the support and m. The displacement δ_2 is obtained from the geometry of the frame and the computed displacements δ_x and δ_y.

$$\delta_2 = \delta_x \cos\theta - \delta_y \sin\theta$$

The δ_1 displacements for points 16 through 19 are the δ_y displacements for these points.

Sample calculations are shown in Tables I and II. The resulting δ displacements required for the analysis of the folded plate are as follows.

$$\delta_{11} = -0.5981(10)^{-6}\text{in.}$$

$$\delta_{12} = +1.877\,(10)^{-6}\text{in.}$$

$$\delta_{22} = -26.180\,(10)^{-6}\text{in.}$$

$$\delta_{21} = +2.9071(10)^{-6}\text{in.}$$

TABLE I

ANALYSIS OF SINGLE BAY FRAME

Δs	x	y	$\frac{y^2}{I/I_0}$	M'		$\frac{M'y}{I/I_0}$	
				F_1	F_2	F_1	F_2
1	0	+ 0.2845	+ 0.081	0	0	0	0
2		+ 0.8535	+ 0.729				
3		+ 1.4225	+ 2.024				
4		+ 1.9915	+ 3.966				
5		+ 2.5605	+ 6.556				
6		+ 3.1295	+ 9.794				
7		+ 3.6985	+ 13.679				
8		+ 4.2675	+ 18.212				
9		+ 4.8365	+ 23.392				
10		+ 5.4055	+ 29.219				
11		+ 5.9745	+ 35.695				
12		+ 6.5435	+ 42.817				
13		+ 7.1125	+ 50.588				
14		+ 7.6815	+ 59.005				
15		+ 8.2505	+ 68.071				
16	-0.1332	+ 8.8195	+ 9.404	-0.1492	-0.1332	-0.1591	- 0.1420
17	-0.1332	+ 9.3885	+ 10.657	-0.1615	-0.1332	-0.1834	- 0.1512
18	-0.1332	+ 9.9575	+ 11.988	-0.1739	-0.1332	-0.2094	- 0.1603
19	+0.0833	+10.5265	+ 26.383	-0.1875	-0.0833	-0.4699	- 0.2087
20	+0.1567	+10.9639	+ 28.621	-0.1875	+0.0656	-0.4894	+ 0.1713
21	+0.6098	+11.3117	+ 15.470	-0.1875	+0.3091	-0.2564	+ 0.4227
22	+1.0897	+11.6174	+ 16.318	-0.1875	+0.5581	-0.2634	+ 0.7839
23	+1.5696	+11.9231	+ 17.188	-0.1875	+0.8071	-0.2703	+ 1.1635
24	+2.0495	+12.2288	+ 18.080	-0.1875	+1.0562	-0.2772	+ 1.5616
25	+2.5562	+12.4924	+ 37.157	-0.1875	+1.2692	-0.5577	+ 3.7751
26	+3.0909	+12.6445	+ 38.067	-0.1875	+1.3969	-0.5645	+ 4.2056
27	+3.6599	+12.6866	+ 19.460	-0.1875	+1.4385	-0.2876	+ 2.2065
28	+4.2289	+12.6866	+ 19.460	-0.1875	+1.4385	-0.2876	+ 2.2065
Σ			+632.081			-4.2759	+15.8345

TABLE II

ANALYSIS OF SINGLE BAY FRAME

Δs	$\frac{M}{I/I_0}$		$(10)^6 \delta_x$		$(10)^6 \delta_y$	
	F_1	F_2	F_1	F_2	F_1	F_2
1	+0.00192	-0.0071	+ 1.2865	- 5.483		
2	+0.00577	-0.0214	+ 3.8369	-16.367		
3	+0.00962	-0.0356	+ 6.3190	-27.001		
4	+0.01347	-0.0499	+ 8.6871	-37.204		
5	+0.01732	-0.0641	+10.8959	-46.820		
6	+0.02127	-0.0784	+12.8995	-55.679		
7	+0.02502	-0.0927	+14.6513	-63.612		
8	+0.02887	-0.1069	+16.1067	-70.444		
9	+0.03272	-0.1213	+17.2205	-76.011		
10	+0.03657	-0.1354	+17.9468	-80.141		
11	+0.04042	-0.1497	+18.2400	-82.668		
12	+0.04427	-0.1639	+18.0545	-83.421		
13	+0.04812	-0.1782	+17.3049	-82.235		
14	+0.05196	-0.1924	+16.0655	-78.939		
15	+0.05581	-0.2067	+14.1708	-73.364		
16	-0.01082	-0.0428	+11.6153	-65.341	+0.5981	- 1.877
17	-0.01185	-0.0445	+ 9.1878	-56.811	+0.5981	- 1.877
18	-0.01289	-0.0463	+ 6.9008	-47.755	+0.5981	- 1.877
19	-0.02769	-0.0826	+ 4.7663	-38.151	+0.4110	- 1.034
20	-0.02698	-0.0498	+ 3.3777	-30.018	-0.3511	+ 3.429
21	-0.01342	+0.0031	+ 2.4687	-23.189	-1.5352	+12.326
22	-0.01318	+0.0323	+ 1.7551	-17.206	-2.6556	+21.717
23	-0.01291	+0.0615	+ 1.1254	-11.431	-3.6440	+30.786
24	-0.01267	+0.0907	+ 0.5777	- 6.045	-4.5037	+39.243
25	-0.02452	+0.2277	+ 0.1750	- 1.900	-5.2778	+47.212
26	-0.02429	+0.2572	+ 0.0204	- 0.227	-5.8220	+53.090
27	-0.01230	+0.1355	0	0	-6.1134	+56.299
28	-0.01230	+0.1355	0	0	-6.2590	+57.904
Σ	+0.21731	-0.9262				

2. Analysis of the Two Bay Frame

The frame shown in Fig. 5.3 is symmetrical about its centerline and has hinged supports. The applied forces F_m are symmetrically disposed about the centerline of the frame so that the analysis can be made assuming the frame to be fixed at the top of the interior column. Splice bars are not present in this case so that only two moments of inertia are involved.

$$I_O = 2,604(10)^{-6} \text{in.}^4$$

$$I_F = 10,937(10)^{-6} \text{in.}^4$$

The portion of the frame from the exterior support to the top of the interior column is divided into thirty seven equal divisions 0.569 in. long and one division 0.198 in. long.

The analysis proceeds in the same manner as that presented in the preceding section, except, in this case, there are two redundants. The vertical and horizontal reaction components at the exterior support are chosen as the redundants. The total strain energy due to bending absorbed by the portion of the frame from the exterior support to the top of the interior column is:

$$U = \sum_{0}^{38} \frac{M^2 \Delta s}{2EI}$$

$$M = M' - Hy + Vx$$

The support conditions at the point of application of H and V are such that there is no horizontal or vertical displacement at that point. This permits the following two equations to be written.

$$\Delta_H = \frac{\partial U}{\partial H} = 0 = -V \sum_0^{38} xy \frac{\Delta s}{EI} + H \sum_0^{38} y^2 \frac{\Delta s}{EI} - \sum_0^{38} M'y \frac{\Delta s}{EI}$$

$$\Delta_V = \frac{\partial U}{\partial V} = 0 = V \sum_0^{38} x^2 \frac{\Delta s}{EI} - H \sum_0^{38} xy \frac{\Delta s}{EI} + \sum_0^{38} M'x \frac{\Delta s}{EI}$$

The statically determinate moments, M', are computed assuming the frame to act as a cantilever supported at the top of the interior column and loaded with the resultants F_1, F_2, F_3, and F_4. The resultants are applied in the sense opposite to that shown as positive in the analysis of the folded plate.

The summations over the Δs divisions 0 through 38 are obtained from Table III and Table IV and substituted into the above equations. Solving the equations for H and V yields the following:

$$H = -0.0064974 8F_1 + 0.0232971F_2 + 0.0105323F_3 + 0.000633064F_4$$

$$V = +1.017428F_1 + 0.447285F_2 + 0.129160F_3 + 0.00741825F_4$$

Following the computation for the moment at the center of the Δs divisions, the displacements are computed as shown in Table V. The procedure is the same as that demonstrated in the preceding section, except that in this case there are four F quantities.

The final values for the δ displacements are as follows:

$$(10)^6 \delta_1 = +0.3865F_1 - 0.9829F_2 - 0.4115F_3 - 0.02472F_4$$

$$(10)^6 \delta_2 = +2.564F_1 - 43.00F_2 - 23.179F_3 - 1.3949F_4$$

$$(10)^6 \delta_3 = +0.959F_1 - 22.45F_2 - 14.231F_3 - 0.8502F_4$$

$$\delta_4 = 0$$

TABLE III

ANALYSIS OF TWO BAY FRAME

Δs	x	y	xy	x^2	y^2
1	0	+ 0.2845	0	0	+ 0.0809
2		+ 0.8535			+ 0.7285
3		+ 1.4225			+ 2.0235
4		+ 1.9915			+ 3.9661
5		+ 2.5605			+ 6.5562
6		+ 3.1295			+ 9.7938
7		+ 3.6985			+ 13.6789
8		+ 4.2675			+ 18.2116
9		+ 4.8365			+ 23.3917
10		+ 5.4055			+ 29.2194
11		+ 5.9745			+ 35.6946
12		+ 6.5435			+ 42.8174
13		+ 7.1125			+ 50.5877
14		+ 7.6815			+ 59.0054
15		+ 8.2505			+ 68.0708
16	-0.0833	+ 8.8195	- 0.7349	+ 0.0069	+ 77.7836
17	-0.0833	+ 9.3885	- 0.7821	+ 0.0069	+ 88.1439
18	-0.0833	+ 9.9575	- 0.8295	+ 0.0069	+ 99.1518
19	-0.0833	+10.5265	- 0.8769	+ 0.0069	+110.8072
20	+0.1567	+10.9639	+ 1.7180	+ 0.0246	+120.2071
21	+0.6366	+11.2696	+ 7.1742	+ 0.4053	+127.0039
22	+1.1165	+11.5753	+12.9238	+ 1.2466	+133.9876
23	+1.5964	+11.8810	+18.9668	+ 2.5485	+141.1582
24	+2.0763	+12.1867	+25.3032	+ 4.3110	+148.5157
25	+2.5562	+12.4924	+31.9331	+ 6.5342	+156.0601
26	+3.0909	+12.6445	+39.0829	+ 9.5537	+159.8834
27	+3.6599	+12.6445	+46.2776	+13.3949	+159.8834
28	+4.2289	+12.6445	+53.4723	+17.8836	+159.8834
29	+4.7979	+12.6445	+60.6670	+23.0198	+159.8834
30	+5.3669	+12.6445	+67.8618	+28.8036	+159.8834
31	+5.9359	+12.6445	+75.0565	+35.2349	+159.8834
32	+6.4706	+12.4924	+80.8333	+41.8687	+156.0601
33	+6.9505	+12.1867	+84.7037	+48.3095	+148.5157
34	+7.4304	+11.8810	+88.2806	+55.2108	+141.1582
35	+7.9103	+11.5753	+91.5641	+62.5728	+133.9876
36	+8.3902	+11.2696	+94.5542	+70.3955	+127.0039
37	+8.8701	+10.9639	+97.2509	+78.6787	+120.2071
38	+9.1934	+10.7579	+98.9017	+84.5186	+115.7324

$$\sum_{o}^{38} \frac{x^2 \Delta s}{I} = + 27{,}541 \text{ in.}^{-1}$$

$$\sum_{o}^{38} y^2 \frac{\Delta s}{I} = + 237{,}093 \text{ in.}^{-1}$$

$$\sum_{o}^{38} xy \frac{\Delta s}{I} = + 52{,}481 \text{ in.}^{-1}$$

TABLE IV
ANALYSIS OF TWO BAY FRAME

Δs	$(10)^3$ M'x				$(10)^3$ M'y			
	F_1	F_2	F_3	F_4	F_1	F_2	F_3	F_4
1	0	0	0	0	0	0	0	0
2								
3								
4								
5								
6								
7								
8								
9								
10								
11								
12								
13								
14								
15								
16	+ 2				- 270			
17	+ 4				- 511			
18	+ 6				- 778			
19	+ 8				- 1,097			
20	- 54	- 2			- 3,774	- 204		
21	- 524	- 23			- 9,287	- 401		
22	- 1,456	- 59			- 15,094	- 608		
23	- 2,848	- 110			- 21,194	- 825		
24	- 4,700	- 179			- 27,588	- 1,053		
25	- 7,013	- 266			- 34,275	- 1,302		
26	-10,133	- 815	- 56.3		- 41,454	- 3,336	- 230.3	
27	-14,081	- 2,085	- 128.6		- 48,648	- 7,202	- 444.6	
28	-18,676	- 3,701	- 220.3		- 55,843	-11,068	- 658.8	
29	-23,919	- 5,666	- 331.2		- 63,038	-14,933	- 872.9	
30	-29,810	- 7,979	- 461.4		- 70,233	-18,799	- 1,087.2	
31	-36,348	-10,640	- 618.5		- 77,427	-22,665	- 1,317.6	
32	-43,082	-14,299	- 1,663.6	-120.42	- 83,176	-27,607	- 3,211.8	- 232.48
33	-49,613	-18,944	- 3,911.7	-247.11	- 86,989	-33,216	- 6,858.7	- 433.27
34	-56,604	-24,084	- 6,453.3	-390.07	- 90,508	-38,510	-10,318.6	- 632.70
35	-64,056	-29,719	- 9,288.3	-549.20	- 93,734	-43,488	-13,591.7	- 803.65
36	-71,969	-35,849	-12,417.5	-724.67	- 96,667	-48,152	-16,679.0	- 973.37
37	-80,342	-42,473	-15,839.3	-924.26	- 99,306	-52,500	-19,578.2	-1,142.44
38	-86,242	-47,215	-18,309.6	-957.95	-100,919	-55,250	-21,425.4	-1,120.97
37								
$\sum_1$	-515,207	-196,897	-51,390.2	-2,955.73	-1,020,893	-325,867	-74,849.3	-4,208.91

TABLE V

ANALYSIS OF TWO BAY FRAME

Δs	$E\delta x$				$E\delta y$			
	F_1	F_2	F_3	F_4	F_1	F_2	F_3	F_4
1	+ 12.65	- 59.1	- 27.86	- 1.675	0	0	0	0
2	+ 37.73	-176.6	- 83.20	- 5.002				
3	+ 62.12	-291.6	-137.42	- 8.262				
4	+ 85.36	-402.4	-189.77	-11.410				
5	+107.00	-507.5	-239.53	-14.402				
6	+126.55	-605.2	-285.93	-17.190				
7	+143.59	-693.7	-328.23	-19.734				
8	+157.64	-771.6	-365.70	-21.986				
9	+168.24	-837.2	-397.56	-23.903				
10	+174.93	-888.7	-423.10	-25.438				
11	+177.25	-924.5	-441.55	-26.549				
12	+174.75	-943.1	-452.20	-27.189				
13	+166.97	-942.8	-454.26	-27.314				
14	+153.44	-921.7	-447.01	-26.879				
15	+133.69	-878.5	-429.71	-25.840				
16	+107.29	-811.4	-401.61	-24.151	+ 3.8652	- 9.829	- 4.115	- 0.2472
17	+ 82.61	-737.0	-370.43	-22.279	+ 3.8652	- 9.829	- 4.115	- 0.2472
18	+ 60.24	-655.1	-336.00	-20.212	+ 3.8652	- 9.829	- 4.115	- 0.2472
19	+ 40.80	-565.3	-298.16	-17.941	+ 3.8652	- 9.829	- 4.115	- 0.2472
20	+ 28.58	-489.8	-266.30	-16.030	- 2.8390	+ 31.607	+ 13.366	+ 0.8018
21	+ 21.85	-433.7	-242.52	-14.601	-13.4102	+119.551	+ 50.699	+ 3.0436
22	+ 16.76	-377.5	-218.16	-13.135	-21.4051	+207.829	+ 88.942	+ 5.3456
23	+ 13.14	-324.1	-194.16	-11.683	-27.0825	+291.684	+126.628	+ 7.6238
24	+ 10.84	-276.5	-171.44	-10.301	-30.7004	+366.355	+162.291	+ 9.7941
25	+ 9.68	-237.8	-150.94	- 9.041	-32.5175	+427.084	+194.463	-11.7724
26	+ 9.59	-224.5	-142.31	- 8.502	-32.8235	+473.937	+224.784	+13.6692
27	+ 9.59	-224.5	-142.31	- 8.502	-31.6253	+499.399	+249.714	+15.2459
28	+ 9.59	-224.5	-142.31	- 8.502	-29.1970	+501.985	+265.634	+16.2558
29	+ 9.59	-224.5	-142.31	- 8.502	-25.8319	+483.208	+270.870	+16.5740
30	+ 9.59	-224.5	-142.31	- 8.502	-21.8239	+444.584	+263.748	+16.0756
31	+ 9.59	-224.5	-142.31	- 8.502	-17.4662	+387.629	+242.593	+14.6356
32	+ 8.41	-204.8	-132.47	- 7.832	-13.3185	+318.309	+207.989	+12.2802
33	+ 6.15	-158.8	-105.58	- 6.143	- 9.7568	+246.274	+165.770	+ 9.6298
34	+ 4.07	-111.4	- 75.40	- 4.323	- 6.5150	+171.759	+118.393	+ 6.7724
35	+ 2.32	- 67.1	- 45.76	- 2.581	- 3.7524	+102.103	+ 71.863	+ 4.0372
36	+ 0.97	- 30.4	- 20.48	- 1.125	- 1.6282	+ 44.643	+ 32.185	+ 1.7533
37	+ 0.12	- 6.3	- 3.40	- 0.168	- 0.3017	+ 6.718	+ 5.365	+ 0.2500
38	0	- 1.6	- 0.02	- 0.009	- 0.0527	0	+ 0.004	0
38.5	0	0	0	0	0	0	0	0

3. Tabulations Accompanying Example 5.1

Sample detailed computations which are part of the solution of Example 5.1 are presented in the following tables. Table VI presents tabulated computations for the δ displacements of the simple folded plate loaded with the 800 lb. test load.

The δ displacements obtained neglecting relative edge displacement are tabulated in each row headed with the symbol a. The numbers shown are the series constants computed on pages 97 and 98. The influence of edge displacements is tabulated in rows headed by the symbols b', b'', and b'''.

The series which represent the contribution to the δ displacements by the relative edge displacements appear on page 102 and are written in terms of b'_n, b''_n, and b'''_n which are expressed in terms of the series constants A_{nm} as presented on page 104. The values of b'_n, b''_n, and b'''_n are substituted into the series for δ_m. Table VI presents the calculations which includes the summation of the δ displacements computed neglecting edge displacement and those caused by edge displacement. This summation yields the final displacements δ_m for each value of n considered. For example, the computation for the contribution to δ_1 by Δ_2 for the first term in the series would involve the following product:

$$\frac{-2.7934 \times 61,618 \times (10)^{-6}}{(1)^4} = -0.17212$$

TABLE VI

DISPLACEMENT OF SIMPLE FOLDED PLATE DUE TO LOAD

n	1	3	5	7	9	11
$(10)^6\delta_1$						
a	+ 39,426	+356.29	-16.902	+4.4000	-4.3990	-2.6928
b'	-172,120	- 33.07	+ 0.205	-0.0140	+0.0051	+0.0014
b''	+145,120	+ 22.45	- 0.137	+0.0093	+0.0034	-0.0009
b'''	- 1,260	+ 2.58	- 0.016	+0.0010	-0.0004	-0.0001
$(10)^6\delta_1$	+ 11,170	+348.25	-16.850	+4.3963	-4.3909	-2.6924
$(10)^6\delta_2$						
a	- 30,894	-281.99	+13.244	-3.4478	+3.4470	+2.1101
b'	+154,300	+ 29.64	- 0.187	+0.0127	-0.0046	-0.0013
b''	-146,690	- 22.69	+ 0.139	-0.0094	-0.0034	+0.0009
b'''	+ 1,576	- 3.23	+ 0.019	-0.0013	+0.0005	+0.0001
$(10)^6\delta_2$	- 21,700	-278.27	+13.211	-3.4458	+3.4395	+2.1098
$(10)^6\delta_3$						
a	- 3,996	- 36.101	+ 1.7129	-0.44588	+0.44588	+0.27294
b'	-161,870	- 31.096	+ 0.1932	-0.01312	+0.00479	+0.00132
b''	+168,140	+ 26.007	- 0.1589	+0.01073	+0.00393	-0.00108
b'''	- 2,041	+ 4.181	- 0.2521	+0.00165	-0.00063	-0.00017
$(10)^6\delta_3$	+ 230	- 37.009	+ 1.7220	-0.44662	+0.45397	+0.27301
$(10)^6\delta_4$						
a	+ 47,014	+424.87	-20.156	+5.2469	-5.2456	-3.2110
b'	+135,180	+ 25.97	- 0.161	+0.0110	-0.0040	-0.0011
b''	-145,260	- 22.47	+ 0.137	-0.0093	-0.0034	+0.0009
b'''	+ 1,779	- 3.65	+ 0.022	-0.0014	+0.0006	+0.0002
$(10)^6\delta_4$	+ 38,800	+424.72	-20.159	+5.2472	-5.2524	-3.2110
$(10)^6\delta_5$	+ 72,210	+790.47	-37.517	+9.7659	-9.7755	-5.9762

Table VII presents the tabulated computations for the displacement of the multiple folded plate at the midspan section; the load producing the displacement is the 800 lb. test load. The rows in the tabulation headed by the symbol L contains the series constants for the δ displacement of the simple folded plate. The rows headed by p and T represent the terms contributed by the redundants p_e and T_e. The series for the δ displacements produced by p_e and T_e are presented on pages 113 and 115. The series are written in terms of constants a_n and b_n for the redundants p_e and T_e respectively which are presented on page 123. Substituting the constants a_n and b_n into the series coefficients presented on pages 113 and 115 permits the calculation of the δ contribution of p_e and T_e.

The computation for the displacement δ_1 for n = 1 is as follows:

Load: + 11,170$(10)^{-6}$, coefficient for the first term of the series, page 106.

p_e : a_1 = + 7.0647, page 123.

(+7.0647) (-475.44) $(10)^{-6}$ = - 3,359, page 113.

T_e : b_1 = + 62,049, page 123.

(+62.049) (-41.507) $(10)^{-6}$ = - 2,575, page 115.

The first term for the series representing δ_1 at $x = \frac{L}{2}$ becomes:

$$+ (+11{,}170 - 3{,}359 - 2{,}575)\,(10)^{-6} \sin\frac{\pi}{2} = + 5{,}236(10)^{-6}\text{in.}$$

TABLE VII

DISPLACEMENT OF MULTIPLE FOLDED PLATE AT $x = \frac{L}{2}$ DUE TO LOAD

n	1	3	5	7	9	11	Total
$\sin \frac{n\pi}{2}$	+1	-1	+1	-1	+1	-1	
$(10)^6 \delta_1$ L	+11,170	+348.25	-16.850	+4.3963	-4.3909	-2.6924	
p	- 3,359	- 23.92	- 0.391	+0.1664	-0.1947	-0.1141	
T	- 2,575	- 14.82	- 0.116	+0.0658	-0.0822	-0.0471	
Σ	+ 5,236	+309.51	-17.357	+4.6285	-4.6678	-2.8536	
$(10)^6 \delta$	+ 5,236	-309.51	-17.357	-4.6285	-4.6678	+2.8536	+ 4,902.7
$(10)^6 \delta_2$ L	-21,700	-278.27	+13.211	-3.4458	+3.4395	+2.1098	
p	+ 6,434	+ 8.54	+ 1.090	-0.3466	+0.3741	+0.2241	
T	+ 3,972	+ 8.71	+ 0.404	-0.1407	+0.1568	+0.0928	
Σ	-11,294	-261.02	+14.705	-3.9331	+3.9704	+2.4267	
$(10)^6 \delta$	-11,294	+261.02	+14.705	+3.9331	+3.9704	-2.4267	-11,012.8
$(10)^6 \delta_3$ L	+ 230.00	- 37.009	+ 1.7220	-0.44662	+0.45397	+0.27301	
p	- 910.00	+ 71.210	- 4.7138	+1.28246	-1.30714	-0.79523	
T	- 2,101.35	+ 22.607	- 1.8949	+0.52921	-0.54523	-0.33035	
Σ	- 2,781.35	+ 56.808	- 4.8867	+1.36505	-1.39840	-0.85257	
$(10)^6 \delta$	- 2,781.35	- 56.808	- 4.8867	-1.36505	-1.39840	+0.85257	- 2,844.97
$(10)^6 \delta_4$ L	+38,800	+424.72	-20.158	+5.2472	-5.2524	-3.2110	
p	-15,585	-202.16	+10.042	-2.6325	+2.6420	+1.6142	
T	-14,332	-158.41	+ 7.693	-2.0123	+2.0159	+1.2320	
Σ	+ 8,883	+ 64.15	- 2.423	+0.6024	-0.5945	-0.3648	
$(10)^6 \delta$	+ 8,883	- 64.15	- 2.423	-0.6024	-0.5945	+0.3648	+ 8,815.6
$(10)^6 \delta_5$ L	+72,210	+790.47	-37.517	+9.7659	-9.7755	-5.9762	
p	-29,006	-376.26	+18.690	-4.8997	+4.9171	+3.0044	
T	-26,674	-294.83	+14.319	-3.7453	+3.7521	+2.2929	
Σ	+16,530	+119.38	- 4.508	+1.1209	-1.1063	-0.6789	
$(10)^6 \delta$	+16,530	-119.38	- 4.508	-1.1209	-1.1063	+0.6789	[illegible]

Table VIII presents the tabulated computations for the plate moments in the multiple folded plate at $x = 0.2188$ L due to the 800 lb. test load. The nature of the computations is almost identical with those presented in Table VII, except in this instance the computations are for moment rather than displacement and the value of x is 0.2188 L rather than $\frac{L}{2}$.

The computation for the first term of the series for the plate moment, M_1, at $x = 0.2188$ L due to load is accomplished as follows:

Load: select the series constant + 361.7
from the first term of the series
for M_1, page 106.

P_e : $a_1 = + 7.0647$, page 123.
$(+7.0647)\ (-5.2782) = -37.3$, page 113.

T_e : $b_1 = + 62.049$, page 123.
$(+62.049)\ (-0.26077) = -16.2$, page 116.

The first term in the series representing the plate moment, M_1, at $x = 0.2188$ L due to load becomes:

$$(+361.7 - 37.3 - 16.2) \sin 0.2188\pi = + 195.52 \text{ in. lb.}$$

The moment contributed by the redundants, F_m, is also computed, yielding the total moments tabulated on page 127.

TABLE VIII

FINAL PLATE MOMENTS AT x = 0.2188 L DUE TO LOAD

	n	1	3	5	7	9	11	Σ
M_1	M_1^p	+ 361.7	+ 0.7929	- 0.013987	+ 0.0018775	- 0.0036858	- 0.0004661	
	p	- 37.3	+ 1.6800	- 0.041332	+ 0.0057593	+ 0.0016540	- 0.0014476	
	T	- 16.2	+ 0.8903	- 0.022408	+ 0.0031381	+ 0.0011402	- 0.0007893	
	Σ	+ 308.2	+ 3.3632	- 0.077727	+ 0.0107749	- 0.00089160	- 0.0027030	
	M	+ 195.52	+ 2.9661	+ 0.022563	- 0.010723	+ 0.00008739	- 0.0025866	+ 198.495
M_2	M_2^p	+1,722.0	+183.22	-24.317	+12.4178	-20.5136	-18.772	
	p	- 182.1	- 8.96	+ 0.215	- 0.0299	- 0.0085	+ 0.008	
	T	- 128.3	- 4.74	+ 0.117	- 0.0163	- 0.0059	+ 0.004	
	Σ	+1,411.6	+169.52	-23.985	+12.3716	-20.5280	-18.760	
	M	+ 895.51	+149.50	+ 6.9624	-12.312	+ 2.0119	-17.952	+1,023.720
M_3	M_3^p	- 656.6	- 0.7936	+ 0.01017	- 0.00111	- 0.02417	+ 0.000298	
	p	+ 746.6	+ 18.0070	- 0.42524	+ 0.05901	+ 0.01472	- 0.014806	
	T	+ 323.8	+ 9.3657	- 0.22780	+ 0.03179	+ 0.01043	- 0.007985	
	Σ	+ 413.8	+ 26.5791	- 0.64287	+ 0.08969	+ 0.00098	- 0.022493	
	M	+ 262.51	+ 23.4406	+ 0.18661	- 0.08926	- 0.000096	- 0.021524	+ 286.027
M_4	M_4^p	-2,147.6	-184.82	+24.347	-12.4221	+20.5377	+18.773	
	p	+1,038.9	+ 85.86	-10.672	+ 5.3923	- 8.8813	- 8.122	
	T	- 158.2	- 3.09	+ 0.075	- 0.0190	- 0.0037	+ 0.003	
	Σ	-1,266.9	-102.05	+13.750	- 7.0488	+11.6527	+10.654	
	M	- 803.72	- 90.000	- 3.9914	+ 7.0148	- 1.1421	+10.195	- 881.643
M_5	M_5^p							
	p	-1,385.8	- 98.477	+11.6106	- 5.8120	+ 9.5323	+ 8.7281	
	T							
	Σ							
	M	- 879.14	- 86.8488	- 3.3703	+ 5.7840	- 0.93426	+ 8.3523	- 956.157