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SAMPLING WITH UNEQUAL PROBABILITIES TO ESTIMATE CUBIC-FOOT VOLUME GROWTH FOR IMMATURE DOUGLAS-FIR ON PERMANENT SAMPLE PLOTS

by

John Frederick Bell

A dissertation submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
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ABSTRACT

SAMPLING WITH UNEQUAL PROBABILITIES TO ESTIMATE CUBIC-FOOT VOLUME GROWTH FOR IMMATURE DOUGLAS-FIR ON PERMANENT SAMPLE PLOTS

by
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The objectives of this study were to investigate and analyze the comparative merits of three different unequal probability sampling methods (Hartley-PPS, Three-P and SALT) when used in combination with the Barr and Stroud optical dendrometer for estimating cubic-foot volume growth on long-term research study plots.

The combination of unequal probability sampling and the optical dendrometer makes it possible to eliminate tree-volume tables along with their inherent errors and express the results on an area basis.

The populations for the study consisted of three young-growth populations containing a total of 190 Douglas-fir trees and one mature population of 158 Douglas-fir trees. Repeated computer drawn samples were used to analyze the three sampling methods. Since the

cubic-foot volume of each tree had been determined with the aid of the optical dendrometer, sampling results were checked against known parameters.

The Three-P sampling method was examined in depth. The bias of the adjusted estimate was found to be negligible for the populations studied. Tarif volume table values, various powers of diameter, and local volume table values were used to determine which estimate (x_i) of the tree characteristic of interest (y_i) gives the smallest variance. The repeated computer drawn samples indicated that tarif table values and diameter squared are effective in reducing the coefficient of variation.

A new finite population correction factor is recommended which appears to be more appropriate for unequal probability sampling methods.

The results of a field trial of the Hartley-PPS and Three-P sampling methods are comparable to the results of the repeated computer-drawn samples.

An analysis of repeated dendrometer measurements of individual trees over time suggests that upperstem diameter changes can be determined after only two growing seasons, and that there should be little difficulty in measuring cubic-foot volume growth on permanent research plots at four to five-year intervals.

The estimated cubic-foot volume growth for a given plot was determined as the difference between the estimated volume at the beginning of the growth period and the estimated volume at the end of the growth period. An analysis was made of data for sample estimates from 50 repeated computer samples each for Three-P and Hartley-PPS sampling representing four measurement times. The sample was selected at the time of the initial measurement. At subsequent measurement times the data were updated to reflect the changes in the population between measurement periods. The estimated total volumes varied little from the actual total volumes. The simulated expected estimated relative variance of the estimated total volume was highest at the initial measurement time and lowest after repeated thinnings. This anomaly appears to be explained by the fact that the population variance of the y_i/x_i ratios became smaller as the stand was thinned.

For the populations studied any one of the three sampling methods gave satisfactory results. Hartley-PPS and Three-P are sampling without replacement; therefore, they were selected in preference to SALT sampling. The Hartley-PPS sampling method is recommended for estimating cubic-foot volume on long-term research plots since sample size is fixed and all the trees are identified for relocation.

Although the populations used in this study were small, the results presumably are applicable, in general, for determining cubic-foot volume growth on research plots in all young-growth Douglas-fir stands.

ACKNOWLEDGEMENTS

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INTRODUCTION

Greater emphasis on the management of second growth timber in the Douglas-fir region has pointed up a need for more comprehensive measurement of the growth of individual trees and stands.

Improved methods, employing some type of unequal probability sampling, are being used increasingly by foresters for measuring forest stand volumes. One such method is Bitterlich's variable plot sampling which has received worldwide attention and is being used in various forms. If diameter is the measurement of greatest interest, horizontal line sampling is the most efficient[1/] sampling scheme. In line sampling some type of angle gauge is used and the chances of a tree being sampled are directly proportional to diameter. If basal area is the item of interest, the most efficient sampling procedure is horizontal point sampling. In the latter method the chances of a tree being sampled are directly proportional to diameter squared or basal area.

Another technique using sampling with unequal probability is probability proportional to size (PPS) sampling. In this method a list is made of the estimated values of the tree characteristic of interest for each tree in the population. A given observation in a

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[1/] Sampling scheme which minimizes the variance.

sample is then selected with probability proportional to that value. The process requires that each tree in the sample area be identified so that it can be relocated for careful measurement if it is selected to be sampled. Three-P is an unequal probability sampling scheme which eliminates the necessity of making an advanced list of estimated values. The technique allows the sampler to determine whether or not to sample as he visits each tree. A recent development in sampling with unequal probabilities is a method called selection at list time (SALT) which also allows the sampler to determine whether or not to sample as he visits each tree.

As the sampling scheme becomes more and more efficient it provides the opportunity to measure fewer and fewer trees provided those trees are measured more accurately Recently, instruments have been developed that make it possible to accurately measure upperstem diameters and lengths to the extent that tree-volume tables as such are eliminated. The Barr and Stroud optical dendrometer is an instrument with this capability.

Problem

The commercial forest land of the Douglas-fir region of Oregon and Washington comprises 25, 455, 000 acres. As old growth is replaced by second growth there is increased emphasis on intensive management of these new forests. It is imperative to have improved

research data to facilitate wise management. As a result, large numbers of permanent sample research plots have been established in the region. For example, the School of Forestry, Oregon State University, has over seventy acres of permanent sample research plots.

Present methods which enumerate each tree and employ some type of tree volume table are time consuming and only as accurate as the volume table used.

Repeated measurements of a permanent research plot over a period of time show the amount of growth made by the trees on the plot. Cubic-foot volume generally is considered to be the best unit of growth measurement for research purposes. In the past, standard volume tables have been adjusted to local conditions through the use of felled-tree measurements. However, it is difficult to remove completely the original bias of the standard volume tables since felled trees are seldom truly representative of the standing trees which are to be measured. Even if the volume tables were accurate, they fail to detect changes in the upperstem form. Another disadvantage of the felled-tree approach is that it is expensive and eliminates the trees from further study.

An improvement over the felled-tree approach is a local volume table developed by measuring representative trees with a high-performance optical dendrometer. It is difficult to arbitrarily select a

truly representative group of trees to be observed with the optical dendrometer. An alternative is to observe each tree in the study area with the dendrometer. However, the procedure is cumbersome and expensive.

Preliminary work done by the School of Forestry, Oregon State University, using the optical dendrometer to measure plots that have been thinned over a 10 to 15-year period as compared to unthinned plots indicate that there are upperstem form differences between thinned and control plots.

It becomes apparent that conventional methods of estimating volumes by diameter, height, and indirect form estimates are not sufficiently accurate to measure upperstem form changes. There is a need to eliminate tree volume tables and their inherent errors.

Unequal probability sampling is an efficacious method adaptable to this problem. The optical dendrometer makes it possible to determine upperstem change by repeated measurements over a period of time and these changes in upperstem volume can be expressed on a stand basis with the aid of unequal probability sampling.

Scope and Objectives

This study was an investigation and analysis of the comparative merits of three different types of unequal probability sampling, PPS, SALT (selection at list time) and Three-P, combined with an advanced

tree-measuring device for determining cubic-foot volume growth on long-term research study plots.

The cubic-foot volume growth for a given plot was determined as the difference between the estimated volume at the beginning of the growth period and the estimated volume at the end of the growth period. The estimated cubic-foot volume for the plot at each measurement time is obtained by one of the unequal probability sampling methods. The Barr and Stroud optical dendrometer is used to measure the cubic-foot volume of the same individual sample trees at each measurement time.

Computer drawn samples from known populations of Douglas-fir were used to analyze the three proposed sampling methods. The populations for this study consisted of three young-growth populations containing a total of 190 trees and one mature population of 158 trees. The dendrometer volumes of all trees were known. Therefore, sampling results were checked against known parameters. The results of this study are applicable for these populations.

The objectives of this study were:

1. To develop a computer program for drawing repeated PPS, SALT, and Three-P samples from the study populations.

2. To determine which estimate (x_i) of the tree characteristic of interest (y_i) gives the smallest variance for a Three-P sample using the study populations. Tarif volume table values, various powers

of diameter, and local volume table values were used.

3. To select the sampling method which minimizes the sample variance.

4. To analyze the bias of the adjusted Three-P estimator for immature Douglas-fir.

5. To estimate sample size for desired standard error of the mean ratio (y_i/x_i) in percent at given confidence levels over the range of coefficients of variation expected from procedures recommended by this study.

6. To analyze the problem of missing sample trees at subsequent measurement periods.

7. To analyze remeasurement of individual trees with the optical dendrometer and determine the minimum period over which definitive measurements can be made.

CHAPTER I

LITERATURE REVIEW

In addition to reviewing the proposed sampling methods and the accompanied use of the Barr and Stroud optical dendrometer the literature was examined to determine the extent of field experience with Three-P sampling. The ability to estimate upperstem diameters inside bark for the dendrometer-determined upperstem outside bark diameters was also studied.

The literature further suggests the inadequacy of tree-volume tables for determining volume growth on permanent research plots, and the potential usefulness of tarif volume table values as the x-values.

PPS Sampling

Introduction

Probability proportional to size (PPS) sampling was introduced by Hansen and Hurwitz (1949). Sample selection with variable probabilities was an important development in sampling theory since the errors of estimates depend only on the choice of selection probabilities and not on the population variance.

The general procedures of unequal probability sampling are illustrated with a hypothetical population of 500 trees. This population of trees could be characterized in many different ways such as basal area, cubic-foot volume, board-foot volume or value. For sake of simplicity, cubic-foot volume will be used in the discussions that follow.

Let y_i represent the measured cubic-foot volume of the i^{th} tree. If a cruiser were able to spend the time, each of the 500 trees could be accurately measured and the sum of the individual volumes would be the total volume of the population. In symbol form: $\sum_{i=1}^{500} y_i$ = total volume of the population.

A suitable technique is to make an ocular estimate (x_i) of the tree characteristic in which the cruiser is interested (in this case cubic-foot volume) for each tree in the population (N trees). A sample of n trees is then carefully measured to estimate the total volume in the population. The sample of n trees is obtained from the list of ocular volumes. The usual procedure is to total the individual estimated volumes ($\sum_{i=1}^{N} x_i$ or T_x) and then go back through the population obtaining a sample of size n with probabilities proportional to the estimated volume of the individual trees.

Table 1 is a hypothetical population of five trees. In actual practice, the y_i values are known only for the n trees.

TABLE 1

HYPOTHETICAL POPULATION OF FIVE TREES

Population of trees	Ocular Volume (x_i)	Measured Volume (y_i)	Dummy Frame
1	24	20	1 - 24
2	11	10	25 - 35
3	13	10	36 - 48
4	42	40	49 - 90
5	20	25	91 -110
	110	105	

It is noted that tree number one has an ocular volume of 24. Thus, tree number one has a probability of 24/110, tree number two a probability of 11/110, et cetera. The last column in Table 1 gives some insight into variable probability sampling. As indicated, each tree has a probability proportional to its predicted volume (x_i). Tree number one is arbitrarily assigned units one to 24, tree number two units 25 to 35, et cetera. Thus, the sample is drawn in such a way that there are 24 chances of getting dummy frame one to 24, 11 chances of getting dummy frame 25 to 35. It is obvious that the larger the characteristic of interest (volume in this case), the greater is its probability of being sampled.

PPS Sampling Procedures

In PPS sampling a sample of n elements is selected from a population of N elements in such a way that the probability of the i^{th} element being in the sample is proportional to its size.

The sampling rule for PPS sampling is as follows:

1. Make an ordered list (in order visited) of the estimated volume (x_i) for each of the N trees.
2. Select n random integers from the interval $[1, T_x]$

where $T_x = \sum_{i=1}^{N} x_i$.

3. Determine the volume (y_i) of the j^{th} tree if the random integer is greater than $\sum_{i=1}^{j-1} x_i$ but less than or equal to $\sum_{i=1}^{j} x_i$.

Since it is possible for more than one random integer to fall within a given interval, sampling is with replacement.

Madow (1949) is credited by Hartley (1966) with developing a systematic method for selecting a PPS sample of n elements (with a random start) from a population of N elements so that the probability of the i^{th} element being in the sample is proportional to its size. Hartley (1966) has further altered the procedures so that the x-values are listed in order of their size.

Table 2 illustrates how a sample of size three is selected using Hartley's method for a population containing the estimated values (x_i) 6, 3, 2, 4, 6 7, and 5.

TABLE 2

SELECTING A SAMPLE OF SIZE THREE
(Hartley Method)

i	1	2	3	4	5	6	7
y_i			3		7		8
x_i	2	3	4	5	6	6	7
Σx_i	2	5	9	14	20	26	33
x_i selected			*		*		*
r+(t-1)I			6		17		28

Where sampling interval (I) = $\frac{T_x}{n}$ such that the inequality $I \geq$ maximum x_i holds, r is a random integer in the interval [1, I] and t is an array of integers (1, 2, . . . , n).

Let n = 3 then I = 33/3 = 11.

Suppose r is 6, then the x_i elements are selected as follows:

r + (t-1) I = 6 + (1-1) 11 = 6 for t = 1 and $5 < 6 \leq 9$, therefore, unit i = 3 is selected and the x-value is 4. Its carefully measured y-value is 3.

$r + (t-1) I = 6 + (2-1) 11 = 17$ for $t = 2$ and $14 < 17 \leq 20$, therefore, unit $i = 5$ is selected and the x-value is 6 with a y-value of 7.

$r + (t-1) I = 6 + (3-1) 11 = 28$ for $t = 3$ and $26 < 28 \leq 33$, therefore, unit $i = 7$ is selected and the x-value is 7 with a y-value of 8.

Table 3 lists the x-values for the 11 different possible samples of size three for the illustrative population. It shows that even for this small population the larger the characteristic of interest the greater is its probability of being sampled.

TABLE 3

THE ELEVEN DIFFERENT POSSIBLE SAMPLES OF SIZE THREE FOR THE ILLUSTRATIVE POPULATION

Σx_i	Sample no.										
	1	2	3	4	5	6	7	8	9	10	11
	x values										
2	2	2									
5			3	3	3						
9						4	4	4	4		
14	5	5	5		5					5	5
20				6		6	6	6	6		
26	6	6	6	6						6	6
33					7	7	7	7	7	7	7

The estimated total volume (T_y) and its variance are calculated as follows:

$$\hat{T}_y = \frac{T_x}{n} \sum_{i=1}^{n} y_i/x_i$$

$$\hat{V}(\hat{T}_y) \frac{T_x^2}{n(n-1)} \left[\sum_{i=1}^{n} (y_i/x_t)^2 - \frac{\left(\sum_{i=1}^{n} y_i/x_i \right)^2}{n} \right]. \quad \underline{1}/$$

Hartley (1966) compared the ordered systematic PPS sampling with a random start (without replacement) to PPS sampling with replacement and to systematic PPS sampling with random order (without replacement). He found that the ordered systematic PPS sampling method had greater precision than the other two. This latter method is hereafter referred to as Hartley-PPS sampling.

Three-P Sampling

Three-P sampling as developed by Grosenbaugh (1964a, 1965) is a combination of simple sampling and unequal probability sampling (sometimes called variable probability sampling). As in simple

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1/ See page 61 for a discussion of a recommended finite population correction factor in unequal probability sampling.

sampling, the sampler goes through a population one unit at a time. As each unit is visited, the decision is made whether or not to include that unit in the sample. These decisions are made in such a way that they are completely independent of each other, i. e., the decision to include one does not affect the probability of including another.

Three-P sampling does not require a list of estimated volumes[1/] before sampling can begin, but it does require an estimate of the total volume, the volume of the largest tree and the expected number to be sampled. The technique allows a decision to be made as soon as a tree is visited whether or not that tree is sampled, i. e., whether or not the tree is a measured tree. Thus each tree in the cruise area is briefly visited, its volume estimated and the decision made whether or not to sample it. The big advantage of this technique is that the cruise area is visited only once during the sampling process.

The sampling rule for Three-P sampling is as follows:

1. Visit, in any order, each of the N trees in the population and estimate its volume, x_i (any whole number in the interval [1, K] where K is largest x_i used in the sampling scheme). All trees with x-values larger than K are considered as belonging to a separate population (see pages 54-55).

1/ In the Three-P system any characteristic of interest may be used, such as board-foot volume, cubic-foot volume or dollar value.

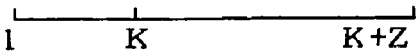
2. Compare each x_i as it is observed with a random number, r_i, selected from the range $1 \leq r_i \leq (K+Z)$ or from a list of computer generated random numbers--Grosenbaugh (1965).

3. Measure the observed tree if $x_i \geq r_i$. The measured volume is symbolized as y_i.

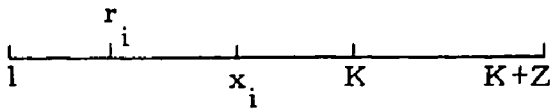
Figure 1 illustrates the sample selection process. From this figure it is obvious that the probability of a given tree being measured is $x_i/(K+Z)$.

Three things are needed in order to generate Grosenbaugh's computer program of random numbers--the value of K, n_e (expected sample size) the estimated $\sum_{i=1}^{N} x_i$. In Three-P sampling, sample size (n) is variable (see page 54) and the sampling is without replacement. However, there is no discussion of a finite population correction factor in the literature. The expected sample size (n_e) is the expected number of observations in a Three-P sample whereas n is the actual sample size. Grosenbaugh (1964a) points out that $n_e = \hat{T}_x/(K+Z)$ or $K+Z = \hat{T}_x/n_e$ where $\hat{T}_x$ is an estimate of T_x before sampling commences. The value of K+Z is easily obtained from this formula assuming values for n_e and $\hat{T}_x$. If $n_e = 1$, the probability (p_i) of selecting and measuring tree number five from the population in Table 1 is

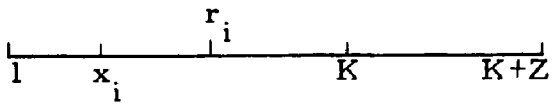
The probability that a given tree will be measured.

(a) 

(a) represents whole numbers between one and K+Z. Any integer (r_i) drawn at random that is larger than K but equal to or less than K+Z automatically rejects any x_i and in Grosenbaugh's random number computer program, it is designated -0. Z is defined as the number of unconditional chances to exclude an individual value from a Three-P sample.

(b) 

r_i = any whole number drawn at random. Since x_i is greater than r_i, this tree would be measured.

(c) 

Since x_i is less than r_i, this tree would not be measured.

$20/(K+Z)1$. From this $K+Z = \frac{110}{1}$ or 110 and $p_i = .182$. If $n_e = 2$, $K+Z$ would then equal 55 and the probability of selecting and measuring tree number five would be $20/(55)2$ or .182. If $n_e = 2$, the probability of selecting and measuring tree number four would be .382. This reiterates that the larger the x_i, the greater is its chance of being a measured tree. See Appendix A for a discussion of the theory of Three-P sampling.

The estimated total volume ($\hat{T}_y$) is calculated as follows:

$\hat{T}_y = (\hat{T}_x/n_e) \sum_{i=1}^{n} y_i/x_i$ which Grosenbaugh calls 3PFIRST (1965) since $K+Z = \hat{T}_x/n_e$. Schrueder, et al., (1968) calls it the unadjusted estimated total volume. A new $(K+Z)'$ could be calculated when the sampling is completed and T_x and n are known. Since $(K+Z)' = T_x/n$, the total volume of the population is estimated by the formula $(T_x/n) \sum_{i=1}^{n} y_i/x_i$ which Grosenbaugh calls 3PSEVENTH (1965) and Schrueder, et al., (1968) refer to it as the adjusted estimated total volume ($\hat{T}_{y_a}$). Grosenbaugh (1965) notes that "3PSEVENTH estimates may involve a small bias that completely disappears as N/ENZSN approaches unity or as variance of y_i/x_i approaches zero." ENZSN is the expected number of of non-zero samples.

The estimated variance of the adjusted estimated total ($\hat{V}(\hat{T}_{y_a})$) is calculated as follows:

$$\hat{V}(\hat{T}_{y_a}) = \frac{T_x^2}{n(n-1)} \left[\sum_{i=1}^{n} (y_i/x_i)^2 - \left(\sum_{i=1}^{n} (y_i/x_i) \right)^2 \right].$$

SALT Sampling

Introduction

SALT (selection at list time) sampling is similar to PPS sampling, except as the name implies, an advanced list of values is unnecessary. Thus, as in Three-P sampling the trees do not have to be referenced for future location and the sample area is visited only once. SALT sampling is sampling with replacement. Norick (1968) suggests that SALT sampling has the best features of PPS and Three-P sampling.

SALT Sampling Procedures

In SALT sampling, all that is needed is an estimate of the total volume (or any other parameter of interest) and a properly prepared list of numbers (integers). Each tree in the area to be sampled is briefly visited, its volume estimated and the decision made whether or not to sample it.

It is necessary to determine the minimum sample size desired for the population to be sampled. Table 4 developed by Norick (1968) gives minimum sample sizes for several coefficients of variation of total volume and two k values for five probability levels (γ) where k is the desired percent the estimate will be of the true population total.

In Table 4, V is the true population total volume and v is the estimated total volume, and according to Norick (1968) γ is the expected value of the probability that the estimate will be at least this accurate. CV is the coefficient of variation of the characteristic being sampled.

TABLE 4

MINIMUM SAMPLE SIZE WHICH

YIELDS $P\{\frac{|v-V|}{V} \leq k\} = \gamma$. [1/]

CV	γ	0.5		0.75		0.9		0.95		0.99	
	k	.1	.25	.1	.25	.1	.25	.1	.25	.1	.25
0.05		*	*	2	*	3	2	3	2	5	3
0.10		*	*	3	*	5	2	6	3	11	3
0.15		2	*	5	2	8	2	11	4	19	6
0.20		3	*	7	2	13	4	18	5	31	8
0.25		4	*	10	3	19	5	27	6	46	11
0.30		5	2	14	3	27	6	37	8	64	14
0.35		7	2	18	4	35	8	50	10	86	17

*A sample size of one gives the appropriate accuracy, however, no estimate of variance can be computed for sample sizes less than two.

Table 5 also developed by Norick (1968) gives the number of random integers to generate (R) for a given minimum sample size (M) and probability level from the interval [1, Z] where Z is two times the guessed total volume (G). In Table 5, n is the actual sample size. Z is the upper limit of the interval from which the random

[1/] Norick (1968)

integers are drawn. It is made equal to two times the estimated total volume of the area to be sampled whenever that estimate is reasonably accurate otherwise Z is made five to ten times greater than the estimated total volume. There is a risk of running out of integers before the sampling is completed if Z is too small.

Once R and Z have been determined, the randon integers can be selected. As the randon integer (r_i) is drawn, the corresponding number on a list of integers from one to Z is starred (*).

The sampling rule for SALT sampling is as follows:

1. Visit one of the N trees in the population and estimate its volume (x_i)
2. Start at the beginning of the list of integers in the interval $[I, Z]$ and count the number of integers (t_i), if any, that are starred which are less than or equal to x_i.
3. Measure the observed tree if t_i is greater than zero.
4. Estimate the volume (x_i) of the next tree and add that volume to the previous Σx_i.
5. Count the number of integers (t_i), if any, that are starred which are greater than the previous Σx_i, but less than or equal to the present Σx_i.
6. Measure the observed tree if t_i is greater than zero.
7. Repeat steps four through six for the remaining N-2 trees.

The estimated total volume $(\hat{T}_y)$ and its variance $\hat{V}(\hat{T}_y)$ are calculated as follows:

TABLE 5

HOW MANY RANDOM NUMBERS TO GENERATE IN THE INTERVAL (0, Z] for Z = 2G and P {n≥M} = γ.[1/]

M	γ 0.50	0.75	0.90	0.95	0.99
2	3	5	7	8	11
3	5	7	9	11	14
4	7	10	12	13	17
5	9	12	13	16	19
6	11	14	17	18	22
7	13	16	19	21	24
8	15	18	21	23	28
10	19	21	24	28	33
12	23	27	31	33	38
14	27	31	35	37	42
16	31	35	39	42	47
18	35	40	44	47	53
20	39	44	49	51	57
25	49	55	59	63	69
30	59	65	71	75	81
35	69	75	81	85	93
40	79	87	93	97	104
50	99	107	114	118	126
60	119	128	135	140	149
70	139	149	156	161	171
80	156	169	177	183	193
90	179	190	198	204	214

[1/] Norick (1968)

The estimated total volume ($\hat{T}_y$) and its variance $\hat{V}(\hat{T}_y)$ are calculated as follows:

$$\hat{T}_y = \frac{T_x}{n} \sum_{i=1}^{n} t_i \, y_i/x_i$$

$$\hat{V}(\hat{T}_y) = \frac{T_x^2}{n(n-1)} \left[\sum_{i=1}^{n} t_i (y_i/x_i)^2 - \frac{\left(\sum_{i=1}^{n} t_i y_i/x_i\right)^2}{n}\right].$$

Barr and Stroud Dendrometer

Three models of the Barr and Stroud Optical Dendrometer (FP9, FP12 and FP15) have been used in the United States. The FP12 model is described by Barr and Stroud (n. d.) as

> . . . a small specialised rangefinder incorporating an inclinometer. It is designed to measure the diameters of tree trunks at selected heights, for the purpose of assessing the volume, growth and value of timber in standing trees.

The optical dendrometer is an eight-inch base coincident type rangefinder employing 5.5x magnification. The dendrometer and tripod weigh approximately 15 pounds. Three readings, true coincidence (T Grad), false coincidence (F Grad) and sine value are recorded at each observation point of the upperstem of the tree being measured. The FP15 model incorporates refinements to the FP12 model that make it more convenient to use in the field.

The Bureau of Land Management experience with the optical dendrometer is that

> The instrument can be used practically in almost any of the stand conditions encountered in western Oregon which, of course, does make it possible to determine volumes based on the form of the trees which are of concern (Hartman, 1966).

Barr and Stroud (n. d.) claim an "uncertainty of observation" [1/] of 0. 1 inch for diameters between 1. 5 and 10 inches and 1 percent on diameters between 10 and 200 inches. Accuracy in height is listed as 1. 5 percent at vertical angles greater than 10 degrees. These accuracies pertain to readings by a "good observer under average practical conditions. " Values for diameters stated by the manufacturer are assumed to pertain to measurements on a cylinder.

There is limited reference in available literature on the accuracy of the Barr and Stroud optical dendrometer when used to measure upperstem diameter and length under field conditions. Grosenbaugh (1963) and Bruce (1966) have reported on results obtained when sighting with the optical dendrometer at points on the bole near the ground. Grosenbaugh's data related to caliper measurements were converted to basal area and indicated a coefficient of variation of basal area of 3. 9 percent. The study was conducted on trees with small diameter differences using a Type FP9 dendrometer, hand-held or crutch-supported. Grosenbaugh also measured eight trees for upperstem diameters and lengths. Because a sectional ladder was used to climb

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[1/] The term "uncertainty of observation" is not defined by the manufacturer.

the trees he was unable to make measurements with calipers on the upperstems. Five trees with one 16-foot log, two trees with two-logs and one three-log tree produced results from which inference can be drawn that there is one chance in three that a length measurement by dendrometer will differ from a taped estimate by more than 1.9 percent.

In Bruce's (1966) study, eight trees, from 15 to 26 inches in diameter, were measured at distances from 66 to 144 feet. A single instrument arrangement was used for each tree, and a small tag was put on the front of the tree to indicate the point of measurement. Each of five observers made two measurements on each tree. Coefficient of variation of caliper measurements was 0.42 percent, and the difference between the means of 80 measurements with dendrometer and 40 with calipers was 0.07 inch.

Bell and Groman (1968) reported on an intensive study of the accuracy of the optical dendrometer under field conditions. Precision was not a criterion of their investigation. The study was conducted on the 500-acre Forest Management Research Area at Black Rock near Falls City, Oregon, which is operated by the School of Forestry, Oregon State University. The area supports a stand of 55-year old Douglas-fir which is similar in topography and brush cover to many young-growth stands in western Oregon and Washington.

In the study, 12 Douglas-fir trees, free from abnormalities and visible defects, were chosen for dendrometer observations. The most satisfactory line of sight (LOS) for the maximum number of suitable observations on each tree was determined. A 25-foot collapsible leveling rod is an essential part of the School's optical dendrometer program. The rod eliminates sighting at the base of the tree to determine total height and reduces the amount of brushing necessary for the lower portion of the stem. Observations at approximate heights of 10, 17 and 25 feet were read directly on the rod to the nearest one-tenth foot. Four additional sighting marks, at variable heights, were made on the stem of the tree above 25 feet by means of bullet holes. One of these was at the crown base and there was a minimum of one observation within the crown.

The three observation points on the leveling rod and the four bullet marks above the rod gave a total of seven elevations at which diameter measurements were available on each tree. Six length references were possible, inasmuch as the elevation at the top of the first 16-foot log was recorded on the data card directly from the reading on the rod and was not determined by the sine of the vertical angle.

Observations were taken on the individual trees by an observer-recorder team that had gained experience measuring over 300 trees during the previous summer. One criterion for locating the instrument for each tree was satisfactory visibility of the observation

points from a point where the uppermost bullet mark was near the maximum vertical angle (+45 degrees) of the instrument. This point was designated Location A. On seven trees where sighting conditions warranted the instrument was moved as far away from the tree as possible along the LOS while still maintaining visibility of the marked points. This point was designated Location B.

The location of the sighting point of the upper or lower side of the bullet mark was recorded to facilitate measurement after felling. A subjective rating of one to ten depending on increasing order of visibility difficulty was also listed for each dendrometer observation. Subsequently, Bell and Groman (1968) eliminated three sightings of rating nine for Location A and three sightings of rating nine for Location B from their study because these observations would not normally be taken in the field.

After the trees were felled, diameters were measured to the nearest 0.1 inch with a caliper at all observation points and recorded. The caliper was continually checked to insure that the arms were parallel. Lengths were measured with a metallic tape and all measurements were rechecked.

Table 6 presents data showing diameter accuracy at Location A. Table 7 is a comparison of diameter accuracy between Location A and Location B and Table 8 is a comparison of measured volumes and dendrometer-determined volumes. The volumetric information

TABLE 6
DIAMETER ACCURACY AT LOCATION A [1/]

Diameter Criteria	Number of observations	Average diameter		Diam. mean difference	Chi-square test of accuracy	
		Calipers	Dendrometer		(E)	(P)
		Inches	Inches	Inches	Inches	Percent
Diameters under 10 in.	35	7.83	7.89	+0.06	±0.242	± 3.25
Diameters over 10 in.	46	13.14	13.18	+0.04	±0.270	± 2.12
Diameters on lower half of stem	48	12.60	12.64	+0.04	±0.281	± 2.36
Diameters upper half of stem	33	8.31	8.35	+0.04	±0.222	± 3.07
All observations	81	10.85	10.89	+0.04	±0.270	± 2.80

[1/] Bell and Groman (1968)

TABLE 7

COMPARISON OF DIAMETER ACCURACY BETWEEN LOCATION A AND LOCATION B [1/]

Measurement method	No. of observations	Horiz. dist. dendrometer to tree	Avg. Diameter dia-meter	mean difference	Chi-square test of accuracy (E)	(P)
		Feet	Inches	Inches	Inches	Percent
Caliper	42		11.04			
Location A	42	77.37	11.08	+0.04	±0.261	±2.48
Location B	42	115.10	11.08	+0.04	±0.298	±3.01

[1/] Bell and Groman (1968)

for the 12 trees was obtained from a modification of Grosenbaugh's (1964b) computer program.

TABLE 8
COMPARATIVE VOLUME MEASUREMENTS FROM GROUND LEVEL TO OBSERVATION SEVEN (IN CUBIC FEET OUTSIDE BARK). 1/

Tree number	Measured 2/ volume	Dendrometer 3/ volume	Volume difference	Volume difference
	Cu. ft.	Cu. ft.	Cu. ft.	Percent
1	62.4	62.5	+0.1	+0.16
2	72.3	72.4	+0.1	+0.13
3	31.4	31.7	+0.3	+0.96
4	99.1	100.1	+1.0	+1.01
5	64.7	65.0	-0.3	+0.46
6	38.6	38.2	-0.4	-1.04
7	24.8	25.0	+0.2	+0.81
8	87.1	86.8	-0.3	-0.34
9	63.8	63.6	-0.2	-0.31
10	78.3	80.8	+2.2	+2.80
11	35.9	35.3	-0.6	-1.67
12	32.3	32.5	+0.2	+0.62
Totals	691.0	693.9		
Average	57.6	57.8	+0.2	+0.35

1/ Bell and Groman (1968)

2/ Cubic-foot volume outside bark determined by caliper and tape measure.

3/ Cubic-foot volume outside bark determined by dendrometer measurements of diameter and length.

The accuracy tests in Table 6 and 7 are based on the two-tailed chi-square test described by Freese (1960), and indicate the maximum anticipated error at the 0.05 probability level.

The accuracy test values for diameters over ten inches in Table 6 are calculated as follows:

$$\chi^2 = \sum_{i=1}^{n} (x_i - \mu_i)^2/\sigma^2$$

where χ^2 = chi-square value for n degrees of freedom

x_i = the value of the i^{th} observational unit as estimated by the dendrometer

μ_i = the "true" value of the i^{th} observational unit as measured by the caliper

$\sigma^2 = E^2/(1.96)^2$
where E = the number of units the estimate is within the true value unless a 1-in-Z chance has occurred, and 1.96 is the value of the standard normal deviate corresponding to a two-tailed probability of 0.05.

The chi-square value for 46 degrees of freedom is 62.83; therefore, E is the only unknown in the above equation. In solving for E, E becomes the maximum anticipated error.

$$\sum_{i=1}^{n} (x_i-\mu_i)^2 = 1.19 \text{ (for diameters over 10 in. Table 6)}$$

and $62.83 = 1.19/(E^2/1.96^2)$

$E^2 = 4.571504/62.83$

$E = \pm .270.$

The maximum anticpated error can be expressed as a percentage as follows:

$E_i = P\mu_i/100$ where P = the percent the estimate is within the true value unless a 1-in-Z chance has occurred.

Substituting E_i into the chi-square formula,

$$\chi^2 = \sum_{i=1}^{n} \left[\left((x_i - \mu_i)^2 (196)^2 \right) / P^2 \mu_i^2 \right] = \left((196)^2 / P^2 \right) \sum_{i\,1}^{n} \left[x_i - \mu_i)^2 / \mu_i^2 \right]$$

$$62.83 = \left((196)^2 / P^2 \right) (0.0073452)$$

$$P^2 = 4.491058$$

$$P = \pm 2.12.$$

Field Experience with Three-P Sampling

Johnson, et al., (1967) compared Three-P cruising with standard Forest Service procedures for cruising old-growth ponderosa pine for timber sale purposes. The sale area contained 1,848 old-growth ponderosa pine trees. For the Three-P sample, 35 trees were measured with the Barr and Stroud optical dendrometer while the volumes for the 88 trees in the standard cruise were determined using 16-foot form class tables (Girard and Bruce n. d.).

The adjusted estimated total volume for the Three-P sample was 1,498 thousand board feet and for the standard cruise 1,860 thousand board feet. The total volume based on log scale after the trees were felled was 1,508 thousand board feet. The standard error of the estimated total volume was 59 thousand board feet, or 3.9 percent, as compared to 161 thousand board feet, or 8.7 percent, for the standard cruise. According to Johnson's calculations, 336 sample trees would have to be measured to reduce the standard error of the

estimated total for the standard cruise to 3.9 percent.

> One possible reason why the standard cruise estimate of total sale volume (1,860 thousand board feet) was so high in the Pringle Falls test is that the average form class 0.84 was used for all sample trees, whereas evidence from the 35 trees measured with the dendrometer indicates that average form class may have been only about 0.80 (Johnson, et al., 1967).

In addition, the average log height of the sample trees for the Three-P sample was 4.18 logs compared to 4.58 logs for the standard cruise sample. If the standard cruise sample were based on an average log height of 4.18 logs and a 0.80 form class, the estimated total volume would be 1,550 thousand board feet or only 2.8 percent above the log scale volume. This indicates that an accurate total volume estimate could have been obtained if the correct log height and form class had been used. The authors suggest that determining correct values for form class and log heights might be as costly as the Three-P sample.

Hartman (1966) reports that the Bureau of Land Management (Department of Interior) started investigating Three-P sampling which incorporated the Barr and Stroud optical dendrometer to determine timber sale area volumes in 1963. Since that time, much timber has been sold in western Oregon where the volumes were obtained using the above described method.

Estimating Upperstem Diameters Inside Bark

Since optical dendrometers measure upperstem diameters outside bark, there is a need to convert these diameters to diameters inside bark. Johnson (1966) has developed the following regression equation for determining upperstem bark factors (B_{us}) which is the ratio of diameter inside bark to diameter outside bark:

$$B_{us} = [6931 - 2.5A + 10.6DBH - 311(L/H)^2 + 1343(L/H)(dob/DBH) + 2326B_{LS}]\ 10^{-4}$$

where

L = length up the stem in feet to the point of measurement

A = tree age

DBH = diameter at breast height outside bark

dob = diameter outside bark at point of measurement on stem

H = total tree height

B_{LS} = bark factor at stump.

B_{us} times dob equals diameter inside bark (dib) at point of measurement. Johnson (1966) used 540 trees which ranged in age from 13 to 143 years and from 2 to 40 inches DBH. He considers the data to be reasonably representative of young-growth Douglas-fir in western Oregon and western Washington. Stump bark factor was

used only because bark factor at DBH data were not available.

Khan (1966) has developed regression equations for directly determining diameter inside bark (dib). He used data from 237 felled Douglas-fir trees in the Black Rock Forest Management Research Area, Polk County, Oregon and 302 of the trees that were used in developing the "Comprehensive Tree-Volume Tarif [1/] Tables." Diameters inside and outside bark were measured at 16-foot intervals up the tree. Khan used DBH, H, L times dob and BR as independent variables and dib as the dependent variable for each set of data. Since age was available for the Washington data, it was used as an additional independent variable for that set of data. BR is the bark ratio at breast height (dib/dob). He used all the squares and cross products of the basic independent variables.

The regression analysis produced the following equations:

Black Rock

(1) $dib = 0.928712 dob + 0.0416447$

(2) $dib = 2.01893\ BR + 0.909116 dob - 0.383417(10^{-4})\ L^2 + 0.849269(10^{-3})\ L\ (dob) - 1.76845$

Washington

(3) $dib = 0.934242 dob - 0.01916$

(4) $dib = 4.34159\ BR + 0.92833\ dob + 0.24567(10^{-3})\ L(dob) - 3.97289.$

[1/] See page 34 for definition of tarif.

The coefficient of determination (R^2) was 0.99 for all equations. Table 9 compares actual dib with computed dib for the four equations using tree measurements from three independent locations in addition to the Black Rock and Washington data.

Volume Tables

Local volume tables are normally used for measuring permanent research plots. Staebler (1954) suggests that when growth periods are five years or longer a local volume table should be prepared for each measurement. Since the local tables are based on samples, some of the apparent difference in volume between measurements is likely due to the inherent sampling error of the volume tables. Turnbull, et al., (1965) claim that the "Comprehensive Tree-Volume Tarif Tables" overcome this problem. "The term 'tarif' is of Arabic origin and simply means tabulated information" (Turnbull and Hoyer, 1965). European foresters have used the term "tarif" to represent local volume tables for many years. The tarif number is the total cubic-foot volume from the stump to a four-inch top for a tree of 1.0 square foot of basal area at DBH. They have further developed the work by Hummel (1955).

> In the strict sense, a local volume table is a volume/DBH trend for a stand derived from felled sample tree data for that specific stand. In a more general use of the term, a "local volume table" may be derived by smoothing a height/DBH(H/DBH) trend based on sample tree height measurements;

TABLE 9

SOME RESULTS OF THE FOUR EQUATIONS FOR ESTIMATING UPPERSTEM DIAMETER INSIDE BARK.

Diameter inside bark at the top of 16-foot log in inches	Locality	WILARK				N. SANTIAM			BLACK ROCK				STATE OF WASHINGTON					
	D.B.H.	5.2	10.2	15.0	20.1	10.2	15.1	21.2	5.0	9.9	14.9	20.8	4.7	10.0	15.1	20.1	25.0	30.0
1st Log	Actual	4.3	8.6	12.3	17.4	9.0	12.2	17.1	3.8	8.2	12.5	15.6	4.1	8.2	12.0	17.0	19.6	25.2
	Eq: (1)	4.3	8.7	12.2	17.2	8.9	12.3	17.0	3.8	8.2	12.4	15.6	-	-	-	-	-	-
	Eq: (2)	4.3	8.7	12.2	17.2	8.9	12.3	17.0	-	-	-	-	4.0	8.0	12.0	16.9	19.5	25.2
	Eq: (3)	4.2	8.7	12.2	17.1	8.8	12.4	17.2	3.9	8.2	12.4	15.6	-	-	-	-	-	-
	Eq: (4)	4.2	8.8	12.2	17.2	8.8	12.5	17.3	-	-	-	-	4.3	7.9	12.0	16.9	19.5	25.3
2nd Log	Actual	3.3	7.7	11.0	16.6	8.1	11.2	16.1	2.4	6.3	11.0	13.5		7.0	10.6	15.9	17.1	22.5
	Eq: (1)	3.3	7.8	10.9	16.4	8.0	11.1	16.0	2.5	6.3	11.0	13.5		-	-	-	-	-
	Eq: (2)	3.3	7.8	10.9	16.4	8.0	11.0	15.9	-	-	-	-		6.9	10.5	15.8	17.0	22.4
	Eq: (3)	3.3	7.8	10.8	16.3	8.0	11.1	16.0	2.6	6.3	11.0	13.5		-	-	-	-	-
	Eq: (4)	3.3	7.8	10.9	16.3	8.0	11.1	16.1	-	-	-	-		6.9	10.4	15.8	17.0	22.3
3rd Log	Actual	2.0	6.4	9.5	14.6	6.3	8.8	13.9		4.0	8.9	11.6		5.1	8.4	14.5	15.5	20.7
	Eq: (1)	2.0	6.4	9.4	14.6	6.3	8.9	13.9		3.9	8.9	11.6		-	-	-	-	-
	Eq: (2)	2.0	6.3	9.3	14.5	6.3	8.7	13.7		-	-	-		5.0	8.4	14.4	15.4	20.6
	Eq: (3)	2.0	6.3	9.2	14.4	6.3	8.9	13.7		3.9	8.8	11.6		-	-	-	-	-
	Eq: (4)	2.0	6.3	9.2	14.4	6.3	8.9	13.8		-	-	-		5.0	8.4	14.4	15.3	20.5
4th Log	Actual		4.7	7.1	12.4	4.0	5.7	11.1			5.7	8.1		1.8	3.9	12.5	11.8	18.5
	Eq: (1)		4.6	7.0	12.5	4.0	5.8	11.2			5.7	8.2		-	-	-	-	-
	Eq: (2)		4.6	7.0	12.4	4.0	5.6	11.0			-	-		1.8	4.0	12.5	11.7	18.5
	Eq: (3)		4.6	6.9	12.2	4.0	5.7	11.0			5.7	8.1		-	-	-	-	-
	Eq: (4)		4.6	7.0	12.2	4.0	5.7	11.0			-	-		1.9	4.0	12.5	11.7	18.3
5th Log	Actual		2.4	4.2	9.6			7.4			2.3	4.3			1.0	10.6	7.7	15.7
	Eq: (1)		2.4	4.1	9.7			7.4			2.3	4.3			-	-	-	-
	Eq: (2)		2.4	4.2	9.6			7.3			-	-			1.1	10.5	7.7	15.7
	Eq: (3)		2.5	4.2	9.4			7.3			2.4	4.3			-	-	-	-
	Eq: (4)		2.4	4.2	9.4			7.3			-	-			1.1	10.5	7.7	15.5

estimated average tree volume for each DBH class midpoint and smoothed height is then obtained from a H/DBH (form or nonform) standard volume table (Turnbull and Hoyer, 1965).

Four unique features of the "Comprehensive Tree-Volume Tarif Tables" are:

1. The proper tarif table to be used is determined from access tables.
2. Volume is expressed in several units of measure and to several merchantable top limits.
3. Volume/basal area ratios have been developed for each volume expression.
4. Growth multipliers are provided for each volume listing for volume growth estimation from increment core measurements.

The tarif tables are based on detailed measurements of 343 felled young-growth Douglas-fir trees that had an age range of 20 to 80 years. The trees were selected from 23 even-aged stands representative of the Douglas-fir region in western Washington. The authors indicate that subsequent field tests suggest the tables can be used successfully in conifers and hardwoods in many areas of the world. The diameters (DBH) of the felled trees ranged from two to 30 inches. Diameters were measured at length increments of 16.3 feet to a four-inch top. Total height and heights to four-inch, six-inch and eight-inch dib were determined.

An investigation by Turnbull and Hoyer (1965) of the basic curve forms of the various expressions of volume indicated that cubic-foot volume to a four-inch top would provide the basis for all the other expressions of volume. For cubic-foot volume to a four-inch top (CV4) there was no curvilinearity of trend (volume/basal area), the volume/basal area line was equally satisfactory for all locations and the horizontal (basal area) intercept was constant. The tarif number is merely an index of the slope of the line to which it refers, since all CV4/basal area lines have a common horizontal intercept (0.087 sq. ft.). This is illustrated in Figure 2.

The equation for the CV4/basal-area line is

$$CV4 = a + bB$$

where a = intercept on vertical axis

b = regression coefficient

B = basal-area square feet

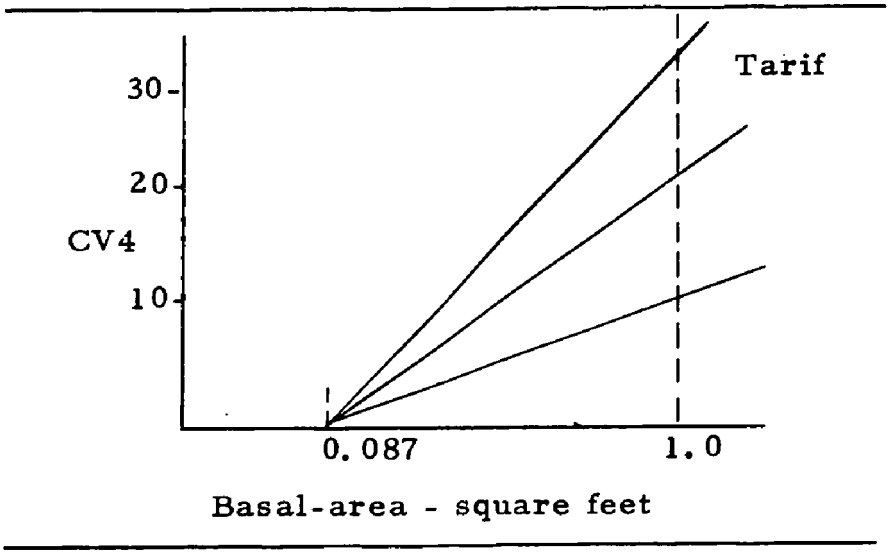
$a = -b(0.087)$ when $B = 0.087$

then $CV4 = -b(0.087) + bB$

or $CV4 = b(B - 0.087)$

Figure 2

Volume/BA lines and tarif number.



Tarif number (T) is defined as being equal to CV4 for a tree of 1.0 square feet of basal-area.

Therefore, $T = b(1.0-0.087)$

and $b = T/0.913$

hence $CV4 = (T/0.913)(B-0.087)$. This provides a convenient means of calculating CV4 when tarif is known or for calculating T when CV4 is known, i.e., $T = CV4(0.913)/B-0.087)$.

Only the tarif number varies for a tree of a given diameter in the CV4 equation. Thus, values from any convenient tarif tables can be used as the x-value in Three-P sampling (see page 53).

If CV4 can be accurately determined, then the latter formula above can be used for determining the proper tarif number (Table) to use. The correct tarif number is the mean T for at least 20 trees representative of the DBH range for the area to be sampled. Otherwise, the Access Tables (Turnbull, et al., 1963; Turnbull and Hoyer, 1965) are used to find the correct tarif number. Again, at least 20 representative trees are selected for measurement. The total height to the nearest foot and DBH to the nearest one-tenth inch is determined for each tree. The mean T is calculated from the T values obtained from the Height-DBH Access Tables.

The main advantages of the tarif tables are:

1. Volume estimating procedures are simplified since the tables are actually a series of pre-constructed local volume tables that have found application for conifer and hardwood stands in many parts of the world.
2. Tree volume is listed in several units of measure, and for differing minimum top diameter limits in each tarif table.
3. Standardized forms of curves are contained within the tarif system. This is important when estimating growth on remeasured permanent plots by substracting initial volume from remeasured volume. Apparent differences in volume could be the result of error derived from using different methods of curve fitting at each remeasurement.

This type of error is uniformly controlled within the tarif system (Turnbull, et al., 1963).

The tarif tables list a maximum DBH of 30 inches. King and Turnbull [1/] are in the process of making tarif tables with DBH extended to 48 inches.

However, the tarif volume tables do not provide a means of assessing upperstem form changes that occur between measurement periods.

[1/] James King, Weyerhaeuser Research Center, Centralia, Washington, and Kenneth J. Turnbull, College of Forest Resources, University of Washington, Seattle, Washington.

CHAPTER II

STUDY POPULATIONS

Three young-growth Douglas-fir populations and a mature Douglas-fir population were used to study the Hartley-PPS, Three-P and SALT sampling methods. The three young-growth Douglas-fir populations were selected from the Black Rock Forest Management Research Area. The research area is located on the George T. Gerlinger Experimental Forest which is approximately 25 miles air line northwest of Corvallis and three miles west of Falls City, Oregon. The land is owned by the Forestry Department of the State of Oregon, and has been made available to the School of Forestry, Oregon State University, for long-term research projects. The experimental forest is 500 acres in size and supports a 55 year-old stand of Douglas-fir. The site class ranges from II to IV with most of the area in site class III. The stocking varies from medium to fully stocked. Topography varies from gentle to very steep with an elevation range of 700 to 2000 feet. The research area is typical of thousands of acres of the Douglas-fir region in Oregon and Washington. Several methods, intensities, and frequencies of intermediate harvests are being applied to observe tree and stand responses.

The three young-growth populations represent unthinned, lightly thinned and heavily thinned stands. The unthinned population containing 52 trees, the lightly thinned population (62 trees) and the heavily thinned population (76 trees) were selected from several different permanent sample research plots. The fourth population (158 trees) is composed of the trees measured with the Barr and Stroud optical dendrometer in a Bureau of Land Management timber sale area. This latter population of mature Douglas-fir trees is of secondary importance to the study. It was added to study the function of the recommended finite population correction factor in a more variable population. Since all trees in each population have been measured with the optical dendrometer, the y-value is known for each tree. Also, because tarif table values and diameter squared are used as x-values, K and T_x are known for each population. Population parameters are shown in Table 10.

TABLE 10

PARAMETERS FOR THE POPULATIONS USED IN THE STUDY.

Population	x-value	r^2 [1]	σ_x^2	C_x%	σ_y^2	C_y%
52 [2]	Tarif	.948	990.8	51.2	1,041.6	50.3
52	$D^{2:0}$	.948	15,739.0	48.1		
62 [3]	Tarif	.966	1,573.2	62.9	1,842.2	65.1
62	$D^{2.0}$	.966	24,990.6	59.2		
76 [4]	Tarif	.942	1,958.3	55.3	2,434.9	53.9
76	$D^{2.0}$	.996	31,179.0	52.8		
190 [5]	Tarif	.948	1,628.2	58.1	2,010.4	59.2
190	$D^{2.0}$	.971	25,884.0	55.0		
158 [6]	$D^{2.0}$	.833	2,631,150.0	67.2	319,100.0	76.4

[1] r^2 is the coefficient of determination for the x and y-values in each population.

[2] Unthinned young growth.

[3] Lightly thinned young growth.

[4] Heavily thinned young growth.

[5] The young-growth populations combined.

[6] Unthinned old growth.

CHAPTER III

COMPUTER PROGRAM

Personnel and facilities of the Oregon State University Computer Center have been utilized to develop computer programs for drawing repeated samples from each test population for Hartley-PPS, Three-P and SALT sampling methods. The computer center has a CDC 3300 computer with time-sharing capabilities. Considerable preliminary calculations were made on a remote console.

The cubic-foot volume for each tree in the young-growth populations was determined by adapting Grosenbaugh's (1964b) program to the Oregon State University computer. The cubic-foot volume for each tree in the mature population was obtained from a Bureau of Land Management computer listing.

General Description

The computer program accepts data in the form of N ordered pairs (y_i, x_i) where y_i represents the cubic-foot volume of the i^{th} tree and x_i is the estimated volume of the i^{th} tree.

The program sees the above data as a population to be sampled. Any one of three unequal probability sampling methods (Hartley-PPS, SALT and Three-P) may be chosen. The user elects the number of

trials (samples to be selected) for each run. Thus, there are M trials in one run for a particular selection method from a given population.

Input

Files of the input data were created through the use of the remote console. A file was created for each population. In addition to the N ordered pairs (y_i, x_i), the first line of each file contained N, T_x, K, n_e and file identification.

Output

By changing the definition of some of the variables within the computer program, the output is common to the three sampling methods studied.

For each run the following is output:

1. The expected number of non-zero random integers which is nrz $(\frac{K}{K+Z})$ where nrz is the total number of random integers generated.
2. The actual number of non-zero random integers.
3. The expected sum of the random integers which is equal to nr(K+1)/2 where nr = number of non-zero random integers generated.
4. The actual sum of the random integers.

5. For Three-P sampling, the actual sample size for each sample and the mean of the M trials.

6. The total volume (T_y) for the population, $\sum_{i=1}^{N} y_i$ from the input data.

7. The estimated total volume ($\hat{T}_y$) for each sample (trial) and the mean and the variance for these M totals. For Three-P sampling, $\hat{T}_y$ is $\hat{T}_{y_a}$, the adjusted estimated total volume. The unadjusted estimated total volume ($\hat{T}_{y_u}$) is also calculated for each sample and the mean and variance for these M totals, $\hat{T}_{y_u} = \frac{\hat{T}_x}{n_e} \sum_{i=1}^{n} y_i/x_i$.

8. The absolute error in percent ($E_A\%$) for each sample and the mean for the M samples, $E_A\% = \left(\frac{\hat{T}_y - T_y}{T_y}\right) 100.$

9. The total of the estimated volumes for each tree in the population (T_x), $\sum_{i=1}^{N} x_i$ from the input data.

10. The total of the estimated volumes for each tree in the sample (t_x) where $t_x = \sum_{i=1}^{n} x_i$.

11. The ratio of t_x/T_x for each sample and the mean ratio for the M samples.

12. The square root of the finite population correction factor for each sample, $\sqrt{1 - t_x/T_x}$.

13. The estimated variance of the estimated total ($\hat{V}(\hat{T}_y)$) divided by T_y^2 for each sample and the mean for the M trials where

$$\hat{V}(\hat{T}_y)=\left[\sum_{i=1}^{n}\left(\frac{T_x}{n}\, y_i/x_i\right)^2 - \left(\sum_{i=1}^{n}\frac{T_x}{n}\, y_i/x_i\right)^2\right]\left[(1-t_x/T_x)\left(\frac{n}{n-1}\right)\right].$$

14. The estimated relative variance of the estimated total for each sample and the mean for the M trials which is the estimated variance of the estimated total ($\hat{V}(\hat{T}_y)$) divided by $\hat{T}_y^2$.

15. The standard error of mean R in percent ($S_{\bar{R}}\%$) for each sample and the mean for the M samples.

$$S_R\% = \left(\frac{\sqrt{\text{estimated relative variance of } \hat{T}_y}}{}\right)100 .$$

Sampling Methods

Random numbers used in drawing the repeated samples were produced by a mixed congruential random number generator program (see Appendix C for the program for Three-P sampling). A comparison of actual and expected values for the random numbers developed by the random number generator program for the Three-P samples is found in Table 1 in Appendix C.

The procedures for selecting samples are as follows:

1. Hartley-PPS. In this method n elements from a population of N elements (which are ordered according to increasing

x-values) are selected systematically with a random start. Let $X=T_x$ then the first step in selecting a sample of size n is to calculate the sampling interval (I) where $I = X/n$ and $I \geq$ maximum x_i holds. Next a random integer (r_i) is generated where $X_{j-1} < r_i + (t-1)I \leq X_j$, such that $1 \leq r_i \leq I$, $t = 1, 2, \cdots, n$ and x_j is the j^{th} partial sum $\sum_{i=1}^{j} x_i$. The j^{th} tree is sampled. The sampling process selects n trees without replacement.

2. Three-P. For each tree in the population, generate a random integer (r_i) (see Appendix C for program) such that $1 \leq r_i \leq K+Z$. If $r_i > K$, set $r_i = 0$. If $r_i \neq 0$ and $r_i \leq x_i$, the tree is sampled. In this method sample size is variable.

3. SALT. A computer sample in this study is selected as follows:

a. Define $T_1 = x_1$, and $T_i = T_{i-1} + x_i$ for $i > 1$.

b. Compute T_i for i = 1 to N.

c. Do the following for k = 1 to n:

(1) Generate a random integer r_k such that

$1 \leq r_i \leq T_x$. T_x corresponds to Norick's (1968) Z.

(2) Determine the interval such that $T_{i-1} < r_k \leq T_i$

(3) Sample the i^{th} tree.

SALT sampling is sampling with replacement. Since T_x is known in this study, sample size is n. However, when T_x is not

known, which is the case under normal field conditions, the estimate of T_x ($\hat{T}_x$) must be made large enough to avoid running out of numbers before the sampling is completed. See page 20 for procedure for selecting a field sample. Thus, under field conditions sample size is variable. In general, as $\hat{T}_x$ varies from T_x, the variation in n increases.

CHAPTER IV

THE THREE-P SAMPLING METHOD

Choice of x-values in Three-P Sampling

The estimated variance of the adjusted estimated total for a Three-P sample, $\hat{V}(\hat{T}_{y_a})$, is calculated from the variation of a carefully measured characteristic (y_i) divided by an estimate of the same characteristic (x_i) about the mean ratio (y_i/x_i).

$$\hat{V}(\hat{T}_{y_a}) = \left(T_x^2/n^2\right)\left[\sum_{i=1}^{n} (R_i-\bar{R})^2\right] n/n-1$$

where $y_i/x_i = R_i$.

Thus, it is readily seen that if R_i were a constant, the estimated variance in a Three-P sample would be equal to zero. Therefore, as the ratio y_i/x_i approaches a constant the estimated variance approaches zero. It is further noted that the constant can be any value. Various powers of diameter, local volume table values and tarif volume table values were chosen as the x-value in an effort to minimize the estimated variance of the adjusted estimated total.

Each of the 13 runs in Table 11 contains 30 repeated Three-P samples with an expected sample size of 20 in which n is the actual sample size, $\bar{S}_{\bar{R}}\%$ is the mean standard error of the mean R in

percent and $\bar{E}_A\%$ is the mean absolute error in percent for a given run.

$S_{\bar{R}}\%$ and $E_A\%$ are calculated as follows:

$$S_{\bar{R}}\% = \left(\sqrt{\frac{\sum_{i=1}^{n} R_i^2 - \left(\frac{\sum_{i=1}^{n} R_i}{n}\right)^2}{n(n-1)}} \Big/ \bar{R} \right) 100$$

which is the standard error of the mean R in percent.

$\bar{S}_{\bar{R}}\%$ is the mean $S_{\bar{R}}\%$ for the M repeated computer trials.

$$E_A\% = \frac{\left(\hat{T}_{y_a} - T_y\right)}{T_y} (100)$$

which is the absolute error in percent for a given trial.

$\bar{E}_A\%$ is the mean $E_A\%$ for the M trials.

$\left(\frac{S_{\bar{R}}\%}{100}\right)^2$ is the estimated relative variance of the adjusted estimated total in Table 12.

Table 11 data suggest that some power of diameter in the vicinity of diameter squared and tarif table values are the most satisfactory quick estimate of x_i. Other preliminary runs indicated that as the power of diameter decreased below 1.8 or increased above 2.3 the standard error of the mean and the absolute error increased. Diameter squared and tarif table values were used in subsequent work because of their more general application and ease of use. Honer (1965) in developing cubic-foot volume equations using a step-wise regression program found that "For each species the program chose D^2H as the first variable which accounted for a minimum of 97 per-

TABLE 11

COMPARISON OF VARIOUS x-VALUES.[1/]

x-value	n	$\bar{S}_{\bar{R}}$%	$\bar{E}_A$%
76 Tree Population			
$D^{1.8}$	21.03	2.72	1.73
$D^{1.9}$	20.03	2.40	1.78
$D^{2.0}$	19.26	2.37	1.48
$D^{2.1}$	21.53	2.37	1.32
$D^{2.2}$	21.23	2.39	1.44
$D^{2.3}$	20.63	2.64	1.18
B.201[2/]	20.83	1.94	1.20
B.R. Table[3/]	21.16	2.80	1.17
Tarif	18.53	2.48	1.25
52 Tree Population			
$D^{1.8}$	20.56	2.61	1.88
$D^{2.0}$	20.66	2.43	1.76
$D^{2.2}$	20.16	2.43	1.51
Tarif	19.56	2.40	1.52

[1/] These comparisons were made before the finite population correction factor was introduced into the study.

[2/] Technical Bulletin 201, Table 12.

[3/] Black Rock local volume table.

cent of the total variation, . . . " Therefore, it is not surprising that diameter squared is an excellent estimator of x_i. Any local volume table appropriate to the population being considered could be used for obtaining the x-values. However, since tarif tables are really local volume tables and they have wide application, they were used in the computer simulation runs.

Full advantage can be taken of the wide application of tarif tables since any tarif number can be used. An examination of the Turnbull and Hoyer (1965) formula for cubic-foot volume to a four-inch top (CV4) shows why this is true. CV4 = (T/0.913) (B-0.087) where T is the tarif number and B is the basal area for a given diameter. The only thing that varies for a tree of a given diameter is the tarif number. Two sets of y_i/x_i ratios based on the same dendrometer-determined values (y_i) but having quick estimates (x_i) obtained from two different tarif numbers (tables) would have the same relative variances. Thus, any convenient tarif number can be used as x-values.

Grosenbaugh (1965) in discussing the coefficient of variation for volume for a Three-P sample suggests that cruisers ". . . should have no difficulty in achieving a 20 to 30 percent coefficient, and talented assessors have been able to achieve 15 percent or less." The range of the coefficient of variation was from 2.4 to 13.0 percent for the 600 computer Three-P samples using diameter squared and tarif table values as the quick estimates and dendrometer-

determined cubic-foot volume as the y_i values for the three young-growth populations. This shows that tarif table values or diameter squared as the x_i estimate is effective in reducing the coefficient of variation for a Three-P sample.

Variability of Sample Size for a Three-P Sample

The expected sample size (n_e) is calculated from the following formula:

$$n_e = \frac{\hat{T}_x}{K+Z}$$

where $\hat{T}_x$ is the estimated sum of $\sum_{i=1}^{N} x_i$ or T_x. On the average, one sample tree will be selected each time the sum of the x-values is equal to K+Z.

The expected sample size (n_e) of a Three-P sample will vary because n_e is a random variable. The expected sample size will also vary due to the size of K, the estimated total volume ($\hat{T}_x$) of the area to be sampled and an improperly determined list of random numbers. If K is made larger than the largest actual x_i, there is no effect on sample size since those random numbers reject all observations from the sample. If K is made smaller than the largest actual x_i, n will be smaller than n_e. Grosenbaugh (1965) suggests that all trees with estimates (x_i) larger than K be measured. The volume of these latter trees is added to the estimated total volume for the sampled portion of the population. Grosenbaugh (1965) states that $\hat{T}_x$ should

be within 20 to 30 percent of the actual total (T_x). The reason for this is that if $\hat{T}_x$ is 20 percent larger than T_x, n_e will on the average be 20 percent larger. Likewise, if $\hat{T}_x$ is 20 percent smaller than T_x, n_e will on the average be 20 percent smaller.

The Bias of the Three-P Estimate

Grosenbaugh (1965) states that the adjusted Three-P estimate ". . . may involve a small bias that completely disappears as N/ENZSN approaches unity or as variance of YI/KPI approaches zero." ENZSN is the expected number of nonzero samples, YI is y_i, and KPI is x_i. Grosenbaugh (1965) also notes that for the adjusted Three-P estimate to be unbiased n times K (maximum x_i) must be less than T_x. Sharpnack (1965) found in extensive testing with computer simulation techniques of a population containing 3,439 trees that the bias of the adjusted estimate was negligible. Schreuder, et al., (1968) suggest that ". . . the bias of the adjusted estimate is of unknown magnitude, but may not be large for most practical applications." They used repeated computer trials (simulation) to assess the bias of the adjusted estimate and the relationship between the variance of the unadjusted estimate and the variance of the adjusted estimate. Schreuder, et al., (1968) used two populations of sawtimber trees (predominantly mature Douglas-fir) which came from a larger population of trees completely enumerated by the Bureau of Land Management for timber

sale purposes. Since these populations are composed of old growth, this writer deemed it necessary to analyze the bias of the adjusted estimate for young-growth Douglas-fir. In addition, Schreuder, et al., (1968) did not use a finite population correction factor in their study.

Table 12 presents the results for 15 different computer runs each containing 100 repeated Three-P samples. Three populations of young-growth Douglas-fir and one population of mature Douglas-fir were used. Tarif cubic-foot volumes and diameter squared were used as the x-values for all populations except population four. This population had diameters which ranged up to 76 inches. A sample size of 20 was used for the young-growth populations while sample sizes of 20, 35 and 50 were used for the mature Douglas-fir population and for the combined young-growth populations. The average sample size for the 100 repeated computer trials is symbolized by $\bar{n}$.

The remaining statistics in Table 12 are calculated as follows:

$$T_x = \sum_{i=1}^{N} x_i \quad \text{and} \quad t_x = \sum_{i=1}^{n} x_i$$

where N and n are respectively population size and sample size.

B_e[1/] $= E_e(\hat{T}_{y_a}) - T_y$ = estimated (simulated bias where T_y is the known population total, $E_e(\hat{T}_{y_a})$ is the mean of the estimated expected (E_e) adjusted estimated total volume for each of the M computer trials, this quantity constituting a simulated value of $E(\hat{T}_{y_a})$.

1/ Hereafter, the term bias means estimated bias.

TABLE 12

STATISTICS FOR 100 REPEATED COMPUTER THREE-P SAMPLES FOR FOUR DIFFERENT POPULATIONS.

Population	x	n_e	B_e/T_y	$B_e/S_e(\hat{T}_{ya})$	$\frac{E_e[\hat{V}(\hat{T}_{ya})]}{T_y^2}$	$\frac{E_e[\hat{V}(\hat{T}_{ya})]}{\hat{T}_{ya}^2}$	$\frac{V_e(\hat{T}_{ya})}{T_y^2}$	$\frac{S_e(\hat{T}_{ya})}{T_y}$	$\frac{V_e(\hat{T}_{yu})}{V_e(\hat{T}_{ya})}$	$\frac{E_e[\hat{V}(\hat{T}_{ya})]}{V_e(\hat{T}_{ya})}$
1	Tarif	20	+.00161	+.1004	.000316	.000316	.000255	.01598	121.8	1.239
1	$D^{2.0}$	20	+.00106	+.0496	.000318	.000319	.000456	.02135	68.4	0.697
2	Tarif	20	+.00266	+.1381	.000314	.000316	.000369	.01930	74.0	0.851
2	$D^{2.0}$	20	-.00137	-.0590	.000416	.000436	.000542	.02328	49.7	0.768
3	Tarif	20	-.00129	-.0794	.000382	.000372	.000256	.01628	136.8	1.442
3	$D^{2.0}$	20	+.00028	+.0295	.000107	.000112	.000091	.00955	336.4	1.176
1, 2, 3	Tarif	20	+.00384	+.1582	.000595	.000591	.000588	.02424	62.5	1.012
1, 2, 3	Tarif	35	-.00010	-.0059	.000296	.000295	.000311	.01765	61.2	0.952
1, 2, 3	Tarif	50	+.00146	+.1150	.000177	.000176	.000161	.01270	71.5	1.099
1, 2, 3	$D^{2.0}$	20	-.00045	-.0173	.000661	.000680	.000687	.02620	72.5	0.962
1, 2, 3	$D^{2.0}$	35	+.00066	+.0344	.000325	.000331	.000371	.01926	67.9	0.876
1; 2, 3	$D^{2.0}$	50	+.00200	+.1509	.000190	.000194	.000177	.01329	67.3	1.073
4	$D^{2.0}$	20	-.00242	-.0520	.001895	.002016	.002234	.04727	16.7	0.848
4	$D^{2.0}$	35	+.00396	+.1240	.000920	.000975	.001020	.03194	18.1	0.902
4	$D^{2.0}$	50	-.00027	-.0111	.000494	.000521	.000575	.02399	13.9	0.859

$$V_e(\hat{T}_{y_a}) = \frac{\sum_{j=1}^{M} \hat{T}_{y_{aj}}^2 - \left(\sum_{j=1}^{M} \hat{T}_{y_{aj}}\right)^2 / M}{M-1}$$

is the simulated variance of the adjusted estimated total where $\hat{T}_{y_{aj}}$ is the adjusted estimated total volume for the j^{th} sample and M is the number of repeated computer trials (samples).

Then, let

$$S_e = \sqrt{\hat{V}_e(\hat{T}_{y_a})}.$$

Similarly

$$V_e(\hat{T}_{y_u}) = \frac{\sum_{j=1}^{M} T_{y_{uj}}^2 - \left(\sum_{j=u}^{M} T_{uj}\right)^2 / M}{M-1}$$

is the simulated variance of the unadjusted estimated total.

Now, let

$$E_e[\hat{V}(\hat{T}_{y_a})] = \frac{\sum_{j=1}^{M} \hat{V}_j(\hat{T}_{y_a})}{M}$$

be the simulated expected estimated variance of the adjusted estimated total volume, where $\hat{V}_j$ is the estimated variance of the adjusted estimated total volume for the j^{th} sample, and

$$\hat{V}_j(\hat{T}_{y_a}) = \left[\sum_{i=1}^{n_j} \left(\frac{\sum x_i}{n_j} y_i/x_i\right)^2 - \left[\sum_{i=1}^{n_j} \left(\frac{\sum x_i}{n_j} y_i/x_i\right)\right]^2\right]$$

$$\left[\left(\frac{T_x - t_x}{T_x}\right)^{1/} \left(\frac{n_j}{n_j - 1}\right)\right].$$

1/ See page 61.

A comparison of the bias of the adjusted estimator to the known population total volume indicates that the bias for the 100 repeated computer samples is rather uniformly small. The range of the bias is from -0.242 percent to +0.396 percent of the true total volume. The ratio B/T_y (converted to percent) for the two populations reported by Schreuder et al., (1968) ranged from -1.27 percent to 2.12 percent which is considerably larger than those reported above. The range of the bias from six computer runs for the three young-growth populations is from 0.028 percent to 0.266 percent of the true total volume. These populations are typical of those that would be sampled for research purposes.

Since the $S_e(\hat{T}_{y_a})$ is calculated from the 100 trials, $B_e/S_e(\hat{T}_{y_a})$ times ten is a t-value. The largest t-value for the 15 computer runs is 1.582 with 99 degrees of freedom. Thus, the bias of the adjusted estimator is not proved significant at the 0.001 probability level. Nine of the computer runs had positive bias. There appears to be a slight tendency for the bias to be positive. The size of the bias of the adjusted estimator seems particularly inconsequential when the $V_e(\hat{T}_{y_u})/V_e(\hat{T}_{y_a})$ ratio is examined. This ratio varied from 13.9 to 336.4 for the 15 computer runs. The three lowest ratios were for the three runs of the mature Douglas-fir population. Schreuder, et al., (1968) defines $V_e(\hat{T}_{y_u})$ as V_u and his V_a is $V_e(\hat{T}_{y_a})$. For the two populations he reported, the range of V_u/V_a is from 1.73 to 12.17

which is vastly different from the 13.9 to 336.4 reported above.

Note the uniformity of the simulated expected estimated relative variance of the adjusted estimated total volume, $E_e[\hat{V}(\hat{T}_{y_a})/\hat{T}^2_{y_a}]$, and the simulated expected estimated variance relative to the known population total, $E_e[\hat{V}(\hat{T}_{y_a})]/T^2_y$. A further comparison of the relative simulated variance with the two previously mentioned relative variances shows how similar the expectation of the adjusted estimated variances are to the simulated variance. The mean difference between $E_e[\hat{V}(\hat{T}_{y_a})]/T^2_y$ and $V_e(\hat{T}_{y_a})/T^2_y$ for the 15 computer runs is 0.000046 which indicates that the relative bias of the variance estimator is negligible.

The Finite Population Correction Factor

Schrueder, et al., (1968 symbolizes the simulated variance of the adjusted estimated total, $V_e(T_{y_a})$, as V_a and the simulated expected estimated variance of the adjusted estimated total, $E_e[\hat{V}(\hat{T}_{y_a})]$, as v_a. They note there is "...tendency for v_a/V_a to increase as n_e/N increases." They suggested the finite population correction factor, (N-n)/N, would be a partial remedy for this tendency. During personal discussion with Dr. W. Scott Overton[1/] $(T_x-t_x)/T_x$) was suggested as the finite population correction factor. This latter factor

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[1/] Professor and biometrician Forest Research Laboratory, School of Forestry and Statistics Department, Oregon State University.

appears to be more realistic in unequal probability sampling since the inclusion probability of the i^{th} tree is proportional to the predicted x-value. The new finite population correction factor was also used for the Hartley-PPS samples. See Table 14 for a comparison of n_e/N with t_x/T_x.

The v_a/V_a values from the Schreuder, et al. (1968) study ranged from 1.00 to 1.59 and the n_e/N values ranged from 0.03 to 0.20. The comparable figures from this study are .697 to 1.442 (see Table 12) and 0.105 to 0.385 (see Table 13). Thus even though the sampling fraction n_e/N is considerably larger than theirs, the $E_e[\hat{V}(\hat{T}_{y_a})]/V_e(\hat{T}_{y_a})$ is not as large, and it is uniform for sample sizes 20, 35 and 50 from the combined young-growth populations and the mature population. Thus, the tendency of $E_e[\hat{V}(\hat{T}_{y_a})]/V_e(\hat{T}_{y_a})$ to increase as n_e/N increases is controlled by the new finite population correction factor based on the results of the test populations. A further indication of this is the mean difference for $(1-[E_e[\hat{V}(\hat{T}_{y_a})]/V_e(\hat{T}_{y_a})])$ for the 15 computer runs is 0.016 27. The above discussion indicates that the fpc, (T_x-t_x/T_x), is functioning as it should.

CHAPTER V

RESULTS

Results of Computer Runs

Hartley-PPS Samples

Table 13 presents statistics for sample size, mean standard error of mean (percent) and mean absolute error (percent) for the 15 computer runs for 100 repeated Hartley-PPS samples. In Hartley-PPS sampling n is fixed. The sampling ratio (n/N) varied from 0.105 for the combined young-growth populations to 0.385 for population one. A comparison of t_x/T_x to n/N for the 15 computer runs shows that this ratio varied from 1.229 to 1.453 which is nearly identical to the comparable figures for the Three-P and SALT samples. As in the case of the Three-P and SALT samples, there is only slight variation in the standard error of mean (percent) when tarif is used for the x-values and when diameter squared is used for the x-values except for population three.

Three-P Samples

The computer run of population three using diameter squared as the x-values had exceptionally low values for bias and relative

TABLE 13

STATISTICS FOR SAMPLE SIZE AND STANDARD ERROR OF MEAN (PERCENT) FOR 100 REPEATED COMPUTER HARTLEY-PPS SAMPLES.

Population	x	n	n/N	t_x/T_x	$\frac{t_x/T_x}{n/N}$	$\bar{S}_{\bar{R}}\%$	$\bar{E}_A\%$
1	Tarif	20	.385	.481	1.249	1.72	1.25
1	$D^{2.0}$	20	.385	.473	1.229	1.70	1.27
2	Tarif	20	.322	.449	1.394	1.76	1.38
2	$D^{2.0}$	20	.322	.434	1.348	2.09	0.65
3	Tarif	20	.263	.342	1.300	1.84	1.61
3	$D^{2.0}$	20	.263	.333	1.266	0.97	0.57
1, 2, 3	Tarif	20	.105	.141	1.343	2.36	1.46
1, 2, 3	Tarif	35	.184	.246	1.337	1.66	1.24
1, 2, 3	Tarif	50	.263	.352	1.338	1.27	1.36
1, 2, 3	$D^{2.0}$	20	.105	.137	1.305	2.59	1.63
1, 2, 3	$D^{2.0}$	35	.184	.240	1.304	1.83	0.80
1, 2, 3	$D^{2.0}$	50	.263	.342	1.300	1.41	1.13
4	$D^{2.0}$	20	.126	.194	1.540	4.30	2.50
4	$D^{2.0}$	35	.222	.321	1.446	3.05	1.98
4	$D^{2.0}$	50	.316	.459	1.453	2.21	2.44

variances and a very high value (336.4) for the ratio of the simulated variance of the unadjusted estimator to the simulated variance of the adjusted estimator. All of the young-growth populations (including the combined population) had slightly lower relative variances when tarif was used for the x-values rather than diameter squared as the x-values, except for the above mentioned population three. Table 14 shows the statistics for sample size for the 100 repeated computer Three-P samples. The variation of the mean sample size and expected sample size was smallest for the three young-growth populations. These populations had the largest sampling fraction. As expected, there was a considerable range for the values of n, yet mean n for each computer run was little different than the expected sample size (n_e).

The standard deviation of sample size for the M computer trials is represented by S_n and the coefficient of variation by C_n. A comparison of t_x/T_x to n/N for the 15 computer runs shows that this ratio varied from 1.234 to 1.453. This indicates the gain in the size of sampling fraction by using unequal probability sampling.

Table 15 is a comparison of mean standard error of the mean (percent) with mean absolute error (percent) for the 15 computer runs for 100 repeated Three-P samples. As expected the mean standard error of the mean (in percent) for the ratio y_i/x_i for the 100 repeated computer samples in each case was larger than the mean absolute

TABLE 14

STATISTICS FOR SAMPLE SIZE FOR 100 REPEATED COMPUTER THREE-P SAMPLES.

Population	x	n_e	$\bar{n}$	Range of n	S_n	C_n	n_e/N	t_x/T_x	$\frac{t_x/T_x}{n/N}$
1	Tarif	20	20.05	12-27	3.49	.174	.385	.487	1.262
1	$D^{2.0}$	20	20.02	13-29	3.58	.179	.385	.475	1.234
2	Tarif	20	20.21	12-28	3.26	.161	.322	.456	1.399
2	$D^{2.0}$	20	20.24	13-30	3.32	.164	.322	.440	1.350
3	Tarif	20	20.02	11-31	3.77	.188	.263	.339	1.289
3	$D^{2.0}$	20	19.67	11-30	3.53	.179	.263	.329	1.270
1, 2, 3	Tarif	20	19.82	9-31	3.84	.194	.105	.137	1.317
1, 2, 3	Tarif	35	33.85	19-51	4.69	.139	.184	.237	1.331
1, 2, 3	Tarif	50	48.56	38-64	5.41	.111	.263	.343	1.340
1, 2, 3	$D^{2.0}$	20	20.41	11-34	4.50	.220	.105	.137	1.280
1, 2, 3	$D^{2.0}$	35	34.86	23-49	5.38	.154	.184	.239	1.299
1, 2, 3	$D^{2.0}$	50	50.54	38-68	5.47	.108	.263	.349	1.312
4	$D^{2.0}$	20	19.62	11-33	3.81	.194	.126	.177	1.427
4	$D^{2.0}$	35	33.92	24-45	4.80	.142	.222	.309	1.437
4	$D^{2.0}$	50	48.88	37-60	4.42	.090	.316	.449	1.453

TABLE 15

COMPARISON OF MEAN STANDARD ERROR OF THE MEAN (PERCENT) WITH MEAN ABSOLUTE ERROR (PERCENT) FOR THREE-P SAMPLING.

Population	x	n_e	$\bar{n}$	t_x/T_x	$\bar{S}_{\bar{R}}\%$	$\bar{E}_A\%$
1	Tarif	20	20.05	1.262	1.73	1.26
1	$D^{2.0}$	20	20.02	1.234	1.76	1.72
2	Tarif	20	20.01	1.399	1.74	1.56
2	$D^{2.0}$	20	20.24	1.350	2.05	1.84
3	Tarif	20	20.02	1.289	1.89	1.30
3	$D^{2.0}$	20	19.67	1.270	1.00	0.77
1, 2, 3	Tarif	20	19.82	1.317	2.39	1.92
1, 2, 3	Tarif	35	33.85	1.331	1.70	1.38
1, 2, 3	Tarif	50	48.56	1.340	1.32	1.03
1, 2, 3	$D^{2.0}$	20	20.41	1.280	2.57	2.13
1, 2, 3	$D^{2.0}$	35	34.86	1.299	1.80	1.51
1, 2, 3	$D^{2.0}$	50	50.54	1.312	1.38	1.06
4	$D^{2.0}$	20	19.62	1.427	4.39	3.81
4	$D^{2.0}$	35	33.92	1.437	3.09	2.62
4	$D^{2.0}$	50	48.88	1.453	2.26	1.97
Mean					2.07	1.73

error (in percent) for the 15 computer runs. Populations one and two and populations one, two, and three combined had slightly smaller mean standard error of mean and mean absolute errors when tarif was used for the x-values instead of diameter squared.

SALT Samples

Statistics for sample size, mean standard error of mean (percent) and mean absolute error (percent) for the 15 computer runs for 100 repeated SALT samples are shown in Table 16. Normally sample size is variable in SALT sampling, but in this study T_x was known so the selection rule gave a fixed sample size. Thus, sample sizes for the Hartley-PPS and SALT samples were identical.

Evaluation of Unequal Probability Sampling Methods

In the Hartley-PPS sampling method n elements from a population of N elements which are listed in order of their size are systematically selected with a random start. Thus, it is sampling with a fixed sample size and without replacement.

A comparison of the mean standard error of the mean in percent ($\bar{S}_{\bar{R}}$ %) for the 15 computer runs of the 100 repeated samples for Hartley-PPS, Three-P and SALT from Tables 14, 15 and 16 shows only slight variations among the three sampling methods. This indicates that for the populations studied any one of the three sampling

TABLE 16

STATISTICS FOR SAMPLE SIZE AND STANDARD ERROR OF MEAN (PERCENT) 100 REPEATED COMPUTER SALT SAMPLES.

Population	x	n	n/N	t_x/T_x	$\frac{t_x/T_x}{n/N}$	$\bar{S}_{\bar{R}}\%$	$\bar{E}_A\%$
1	Tarif	20	.385	.478	1.242	1.72	1.99
1	$D^{2.0}$	20	.385	.474	1.231	1.67	2.03
2	Tarif	20	.322	.450	1.398	1.71	1.96
2	$D^{2.0}$	20	.322	.428	1.329	2.07	2.15
3	Tarif	20	.263	.342	1.300	1.80	1.95
3	$D^{2.0}$	20	.263	.336	1.278	0.93	0.81
1, 2, 3	Tarif	20	.105	.141	1.343	2.36	2.03
1, 2, 3	Tarif	35	.184	.246	1.337	1.64	1.68
1, 2, 3	Tarif	50	.263	.349	1.327	1.29	1.32
1, 2, 3	$D^{2.0}$	20	.105	.137	1.305	2.51	1.97
1, 2, 3	$D^{2.0}$	35	.184	.238	1.293	1.77	1.68
1, 2, 3	$D^{2.0}$	50	.263	.343	1.304	1.41	1.51
4	$D^{2.0}$	20	.127	.182	1.433	4.28	4.02
4	$D^{2.0}$	35	.222	.320	1.441	2.95	2.59
4	$D^{2.0}$	50	.316	.460	1.456	2.19	2.55

methods would give satisfactory results if a list of estimated volumes is available.

It is recommended that the Hartley-PPS sampling method be used whenever a complete list of the x-values is available since the sample size is fixed. This has application for sampling permanent research plots since each tree is identified. Three-P sampling is recommended whenever it is not desirable to identify the individual trees. Three-P sampling allows a decision to be made, as soon as a tree is visited, as to whether or not that tree is sampled, i.e., whether or not to measure the tree. Therefore, in Three-P sampling each tree is visited only once and it is not necessary to identify each tree for future reference. SALT sampling was developed because of the concern for the bias of Three-P sampling. However, the Three-P bias appears to be negligible based on the results of this study. A further advantage of Three-P sampling is that it is sampling without replacement. In addition to those given immediately above, there are no other particular advantages or disadvantages of one method over another in their application.

Sample Size Estimation

Sample size can be determined from Figures 3, 4, 5, 6, and 7 for the two recommended sampling methods, Hartley-PPS and Three-P.

Until the sampler gains experience with populations of his interest, it is suggested that a coefficient of variation of 10 percent be used for Hartley-PPS samples and a coefficient of variation of 12 percent for Three-P samples if tarif tables or diameter squared are used as the x-values. The higher coefficient of variation for Three-P sampling is recommended because sample size is variable. This would tend to assure an adequate minimum sample size. These suggestions are based on the results of this study where cubic-foot volume was being estimated. Sample size n, or expected sample size n_e, are read directly from the graphs.

Sample sizes for Figures 3, 4, 5, 6 and 7 were determined from the following formula which can be found in Dilworth (1969):

$$n = \frac{Nt^2C^2}{NA^2 + t^2C^2}$$

where

n = estimated sample size

N = number in population

A = desired accuracy in percent

C = coefficient of variation of sample units in percent

t = tabular value of Student's "t" .

This formula is derived from the formula

FIGURE 3

Sample size estimation (based on desired accuracy of 1-percent and 95-percent probability).

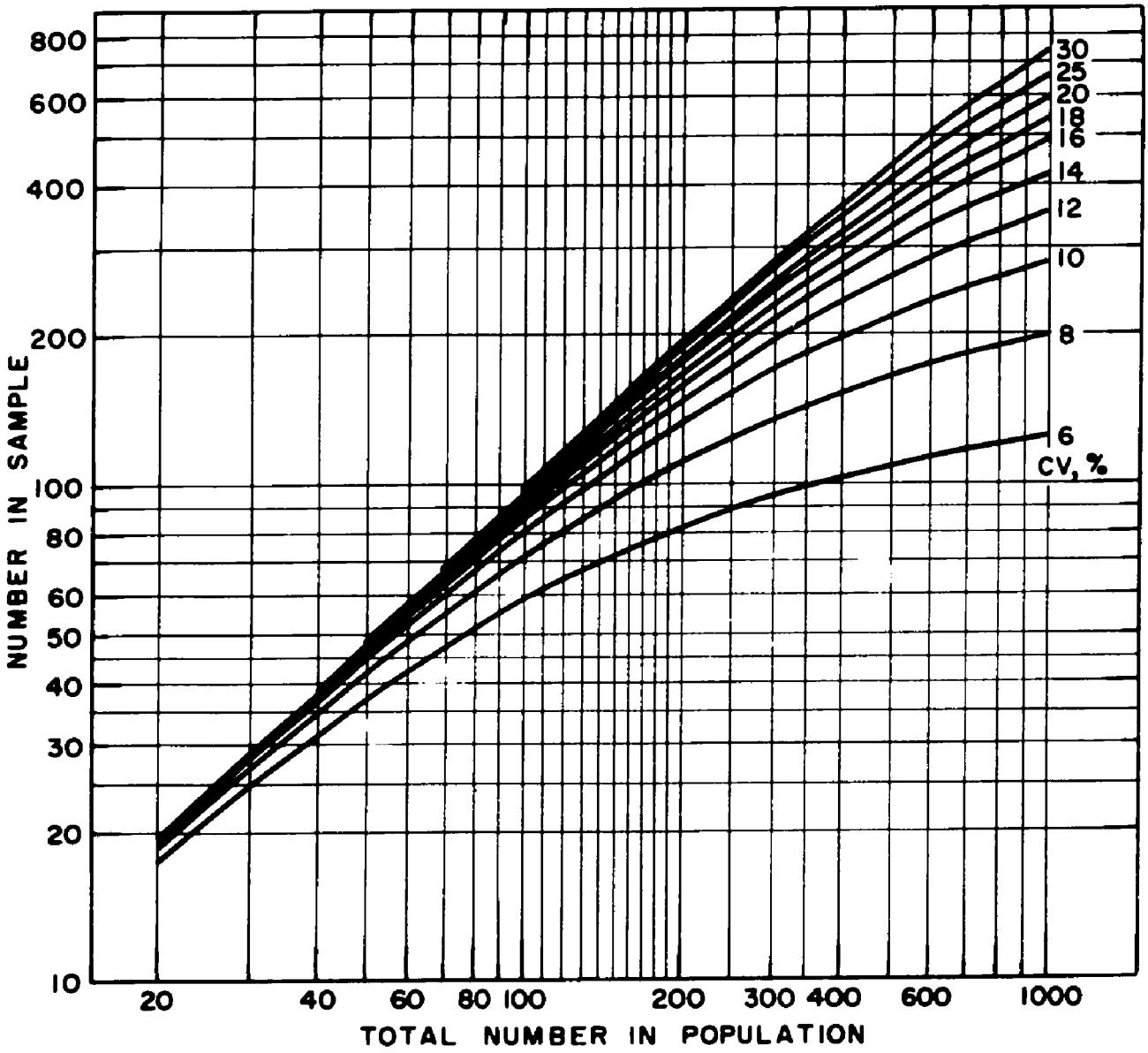


FIGURE 4

Sample size estimation (based on desired accuracy of 2-percent and 95-percent probability).

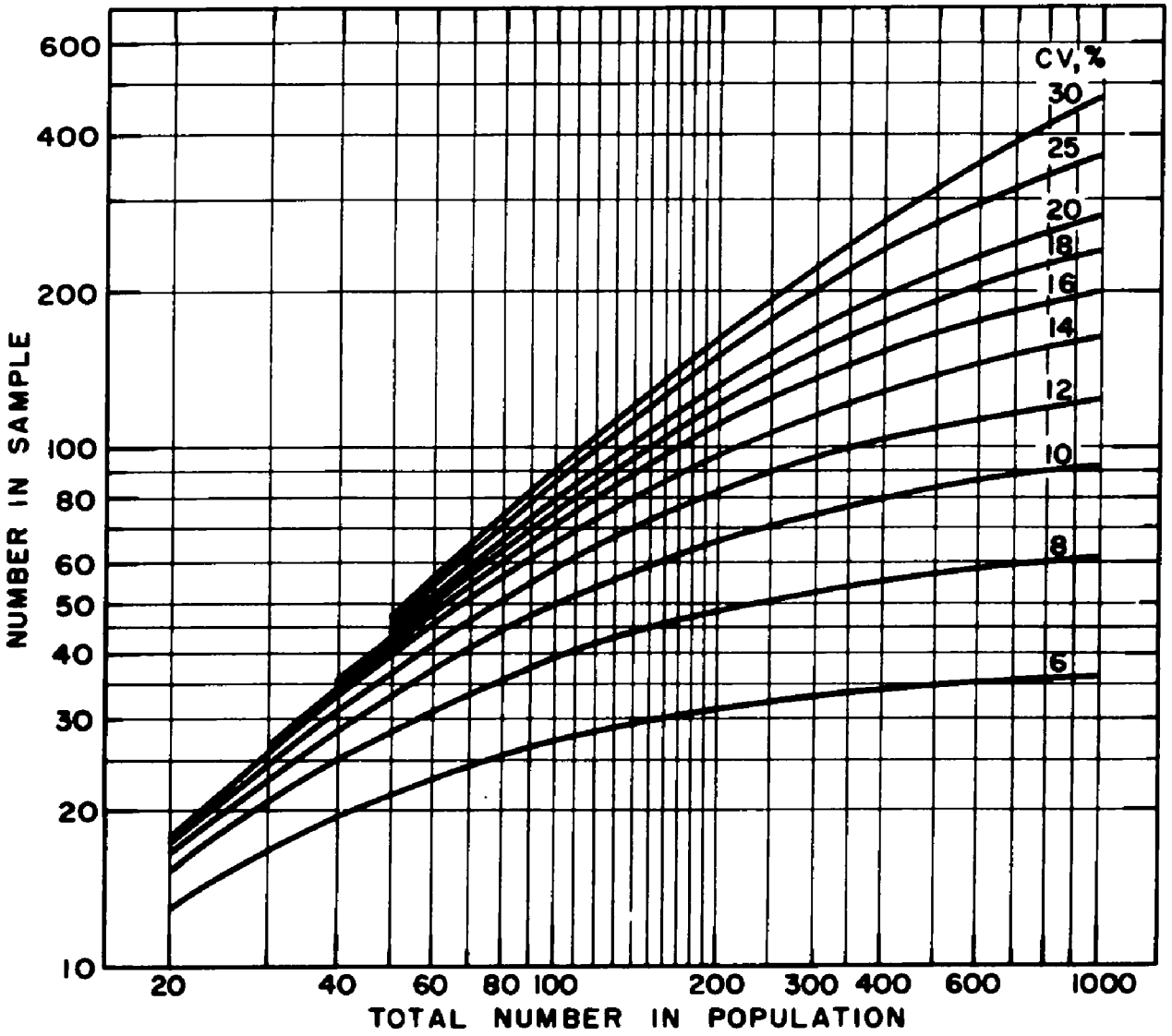


FIGURE 5

Sample size estimation (based on desired accuracy of 3-percent and 95-percent probability).

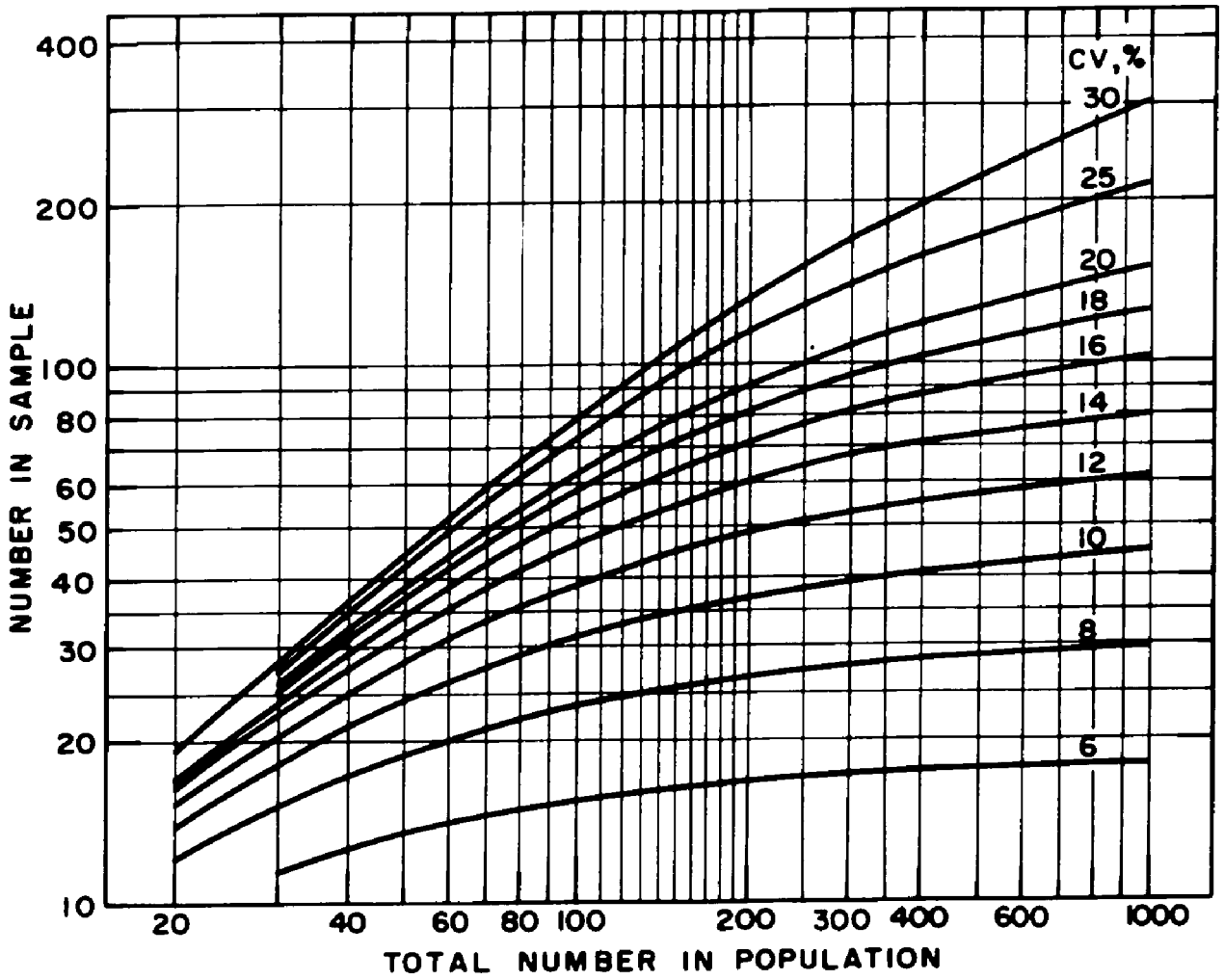


FIGURE 6

Sample size estimation (based on desired accuracy of 4-percent and 95-percent probability).

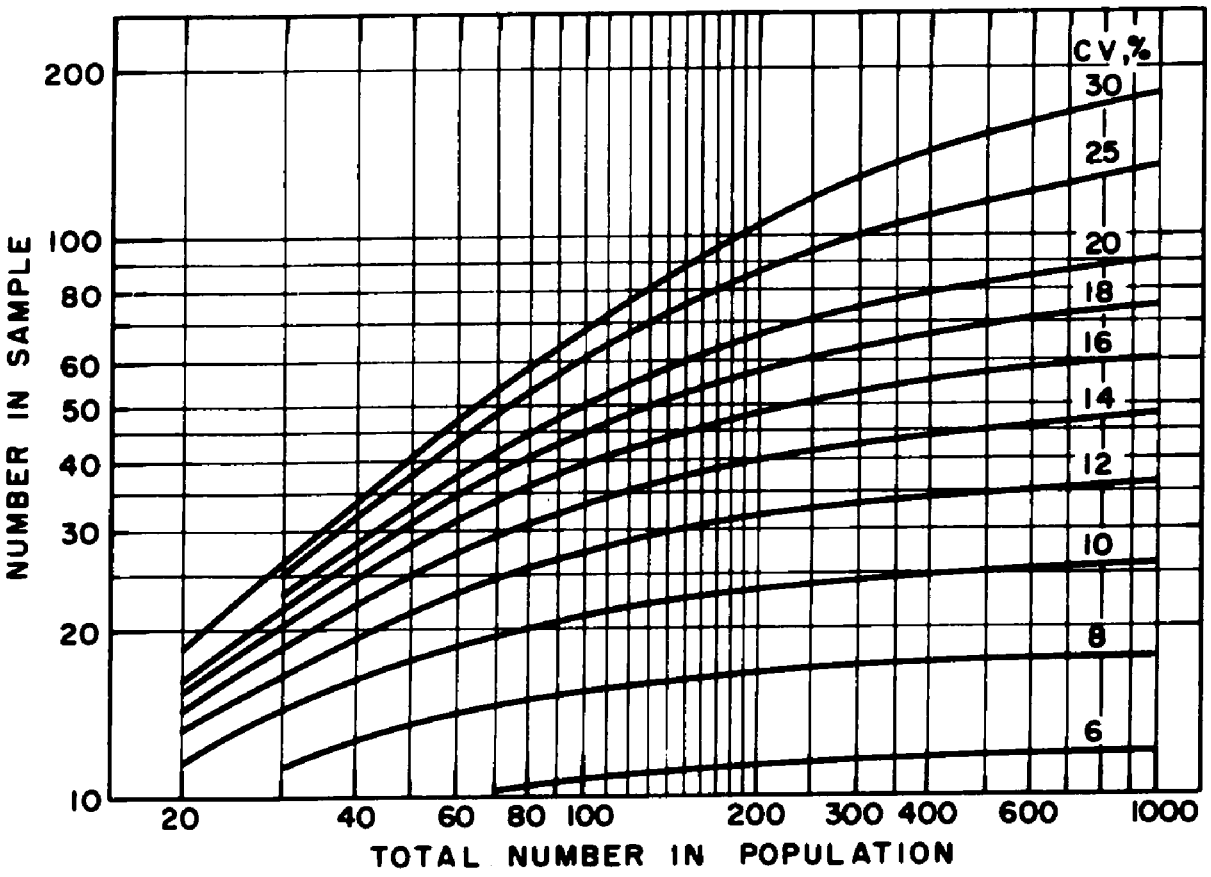
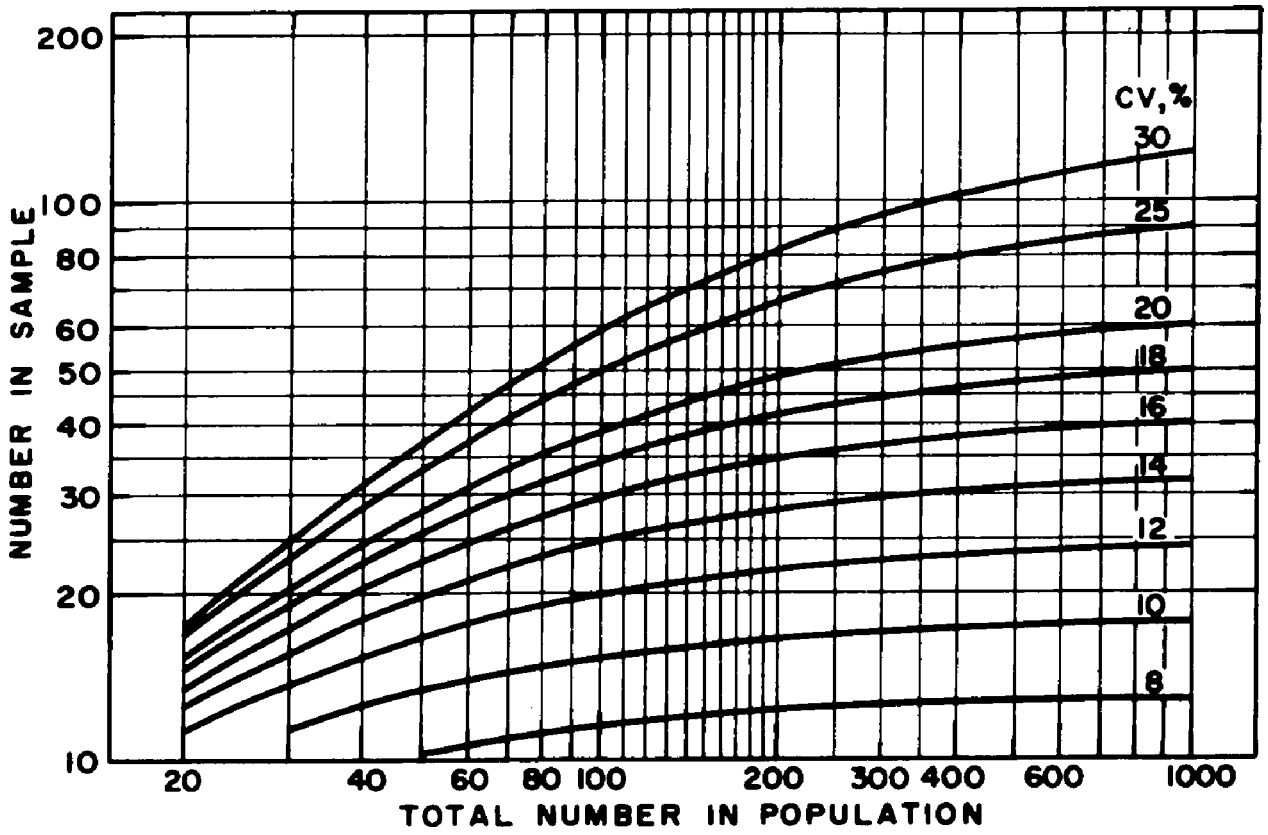


FIGURE 7

Sample size estimation (based on desired accuracy of 5-percent and 95-percent probability).



$$n = \frac{C^2 t^2}{A^2} (1 - n/N)$$

where $(1 - n/N)$ is the traditional finite population correction factor. The use of this finite population correction factor overestimates the estimated sample size compared to the finite population correction factor $(1 - t_x/T_x)$ recommended in this study. It is not meaningful to incorporate the recommended finite population correction factor in figures 3 through 7. The use of the traditional finite population correction factor is acceptable since the function of the charts is to provide reasonable guidelines for selecting sample sizes in advance of sampling. As the sampler gains experience, he will be able to compensate for the differences between the correction factors. The ratio $(t_x/T_x)/(n/N)$ for the 1200 computer Hartley-PPS samples ranged from 1.229 to 1.394 (Table 13). The same ratio for the 1200 Three-P samples ranged from 1.234 to 1.339 (Table 14).

Results of Field Trial

A field trial was made of the Three-P and Hartley-PPS sampling methods on a one-acre plot in the Black Rock Forest Management Research Area. The plot had been thinned several times and contained 146 trees. An estimated sample size of 40 was selected for the Three-P sample. The results of the field trial are found in Table 17

Ten trees were common to the two samples.

TABLE 17

RESULTS OF FIELD TRIAL

Sample Method	n	T_{y_a}	$\frac{\hat{V}(\hat{T}_{y_a})}{\hat{T}^2_{y_a}}$	$S_{\bar{R}}$ %	C	n/N	t_x/T_x	$\frac{t_x/T_x}{n/N}$
Hartley-PPS	40	5,973	.000139	1.18	9.9	.27	.41	1.48
Three-P	33	5,908	.000207	1.44	10.2	.23	.32	1.43

A comparison of each item in Table 17 with the corresponding items in Tables 12, 13 and 14 shows that the field trial results are well within the range suggested by the computer trials. These results substantiate the feasibility of estimating cubic-foot volume on permanent sample plots with either the Hartley-PPS or the Three-P sampling methods. The magnitude of the standard error of the mean ratio in percent ($S_{\bar{R}}$%) particularly indicates the potential of sampling permanent research plots. The results further indicate the desirability of using the Hartley-PPS sampling method for research plots where all the trees are identified since sample size is variable in the Three-P sampling method.

CHAPTER VI

ANALYSIS OF REPEATED MEASUREMENTS

Field Measurements of Individual Trees

Studies have been initiated at the School of Forestry, Oregon State University, by Groman and Berg (1970) to assess upperstem form changes in standing individual trees over a period of time and to determine the cause of upperstem form changes. This is made possible through the use of the Barr and Stroud optical dendrometer.

At the conclusion of the 1967 growing season, 190 trees on the Black Rock Forest Management Research Area were measured with the Barr and Stroud optical dendrometer. The dendrometer observation points were permanently marked and referenced so that these trees could be subsequently remeasured from exactly the same point. In establishing the points, care was taken, in so far as was possible, to be sure that the measurement points on the trees will be visible at the time of remeasurement.

The initial measurement procedures are as follows:

1. Locate dendrometer observation point from which entire tree is visible.

2. Measure diameter at breast height with calipers on

the line of sight from the dendrometer to the tree. The points on the tree where the caliper arms touch are marked with paint. This avoids observation difficulties due to low brush. To assist in relocating the tree, a vertical line is painted on the tree trunk facing the dendrometer observation point.

3. Determine double bark thickness for diameter at breast height.

4. Place a 25-foot telescoping leveling rod against the tree so that the 4.5 foot mark is at diameter breast height. The rod is graduated in feet and tenths. The use of the rod eliminates the need of determining the horizontal line of sight. It also allows greater flexibility in choosing dendrometer observation points since these points are possible where the horizontal line of sight is below the ground level of the tree.

5. Observe the tree at the 16.5 foot mark on the leveling rod with the optical dendrometer and record the true coincidence reading (T Grad), false coincidence reading (F Grad) and sine value. An additional five to eight points are measured on the tree including one at the crown base and a minimum of one within the crown when the trunk is visible. These additional observation point readings are referenced to the readings at the 16.5 foot point by the computer program.

The remeasurement procedures are as follows:

1. Relocate original dendrometer observation point and place the optical dendrometer over this point.

2. Measure diameter at breast height with caliper arms touching paint marks.

3. Determine double bark thickness for diameter at breast height.

4. Place a 25-foot telescoping leveling rod against the tree so that the 4.5 foot mark is at diameter breast height.

5. Set the original sine value for the 16.5 foot point on the inclinometer head with the level bubble centered, then raise or lower the dendrometer unit until the horizontal line viewed through the eyepiece bisects the 16.5 foot mark on the leveling rod. This places the dendrometer at the same height as during the original sighting. Record the T Grad and F Grad. Unless there are errors in the original observations, the T Grad will be essentially the same at the time of remeasurement since the T Grad reading determines distance. Diameter change is obtained through change in the F Grad reading.

6. Remeasure the remaining original observation points on the tree by setting the original sine value on the inclinometer head for each observation point then rotate the dendrometer

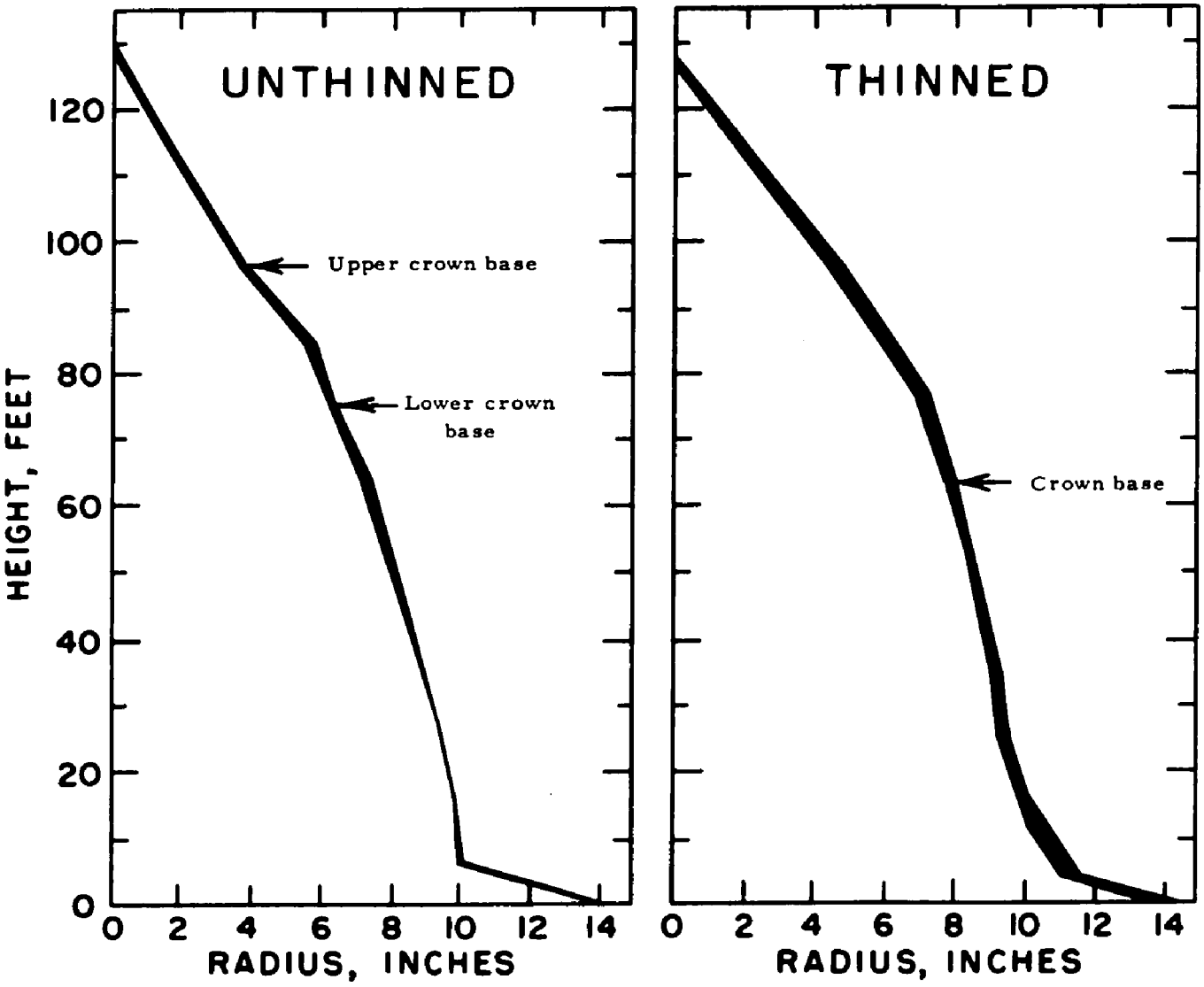
unit until the level bubble is centered. Record the T Grad and F Grad readings.

The remeasurement of the individual trees as outlined above makes it possible to assess upperstem form changes in addition to determining volume growth for the measurement period. Remeasurements indicate that upperstem diameter changes can be determined after only two growing seasons for trees on thinned plots or fast growing trees on unthinned plots. These measurements further indicate that there should be little difficulty in measuring cubic-foot volume growth on permanent research plots at four to five-year intervals.

Figures 8, 9 and 10 (Groman and Berg, 1970) illustrate upperstem form change comparisons for two growing seasons for thinned and unthinned Douglas-fir trees. In Figure 8, the increment pattern is compared for two dominant trees of similar height both of which were 22.1 inches diameter at breast height in 1967. Note the uniform and accelerated growth of the thinned dominant as contrasted with the unthinned dominant. Figure 9 shows a similar comparison for two dominant trees that were 19.6 inches diameter at breast height in 1967. Figure 10 compares two codominant trees.

FIGURE 8

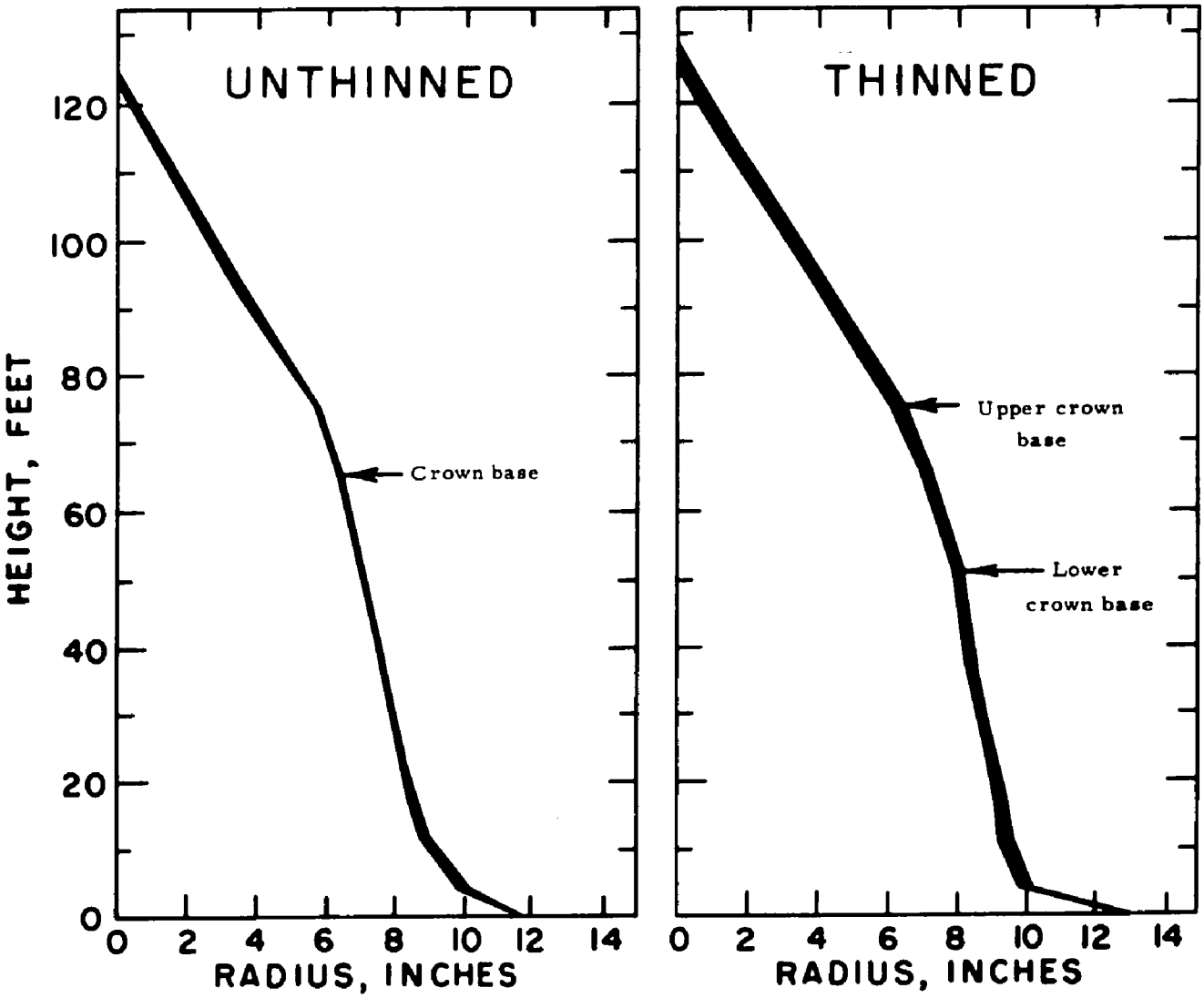
Increment pattern of two dominant Douglas-fir trees--thinned versus unthinned (DBH 22.1 inches). 1/



1/ Groman and Berg (1970).

FIGURE 9

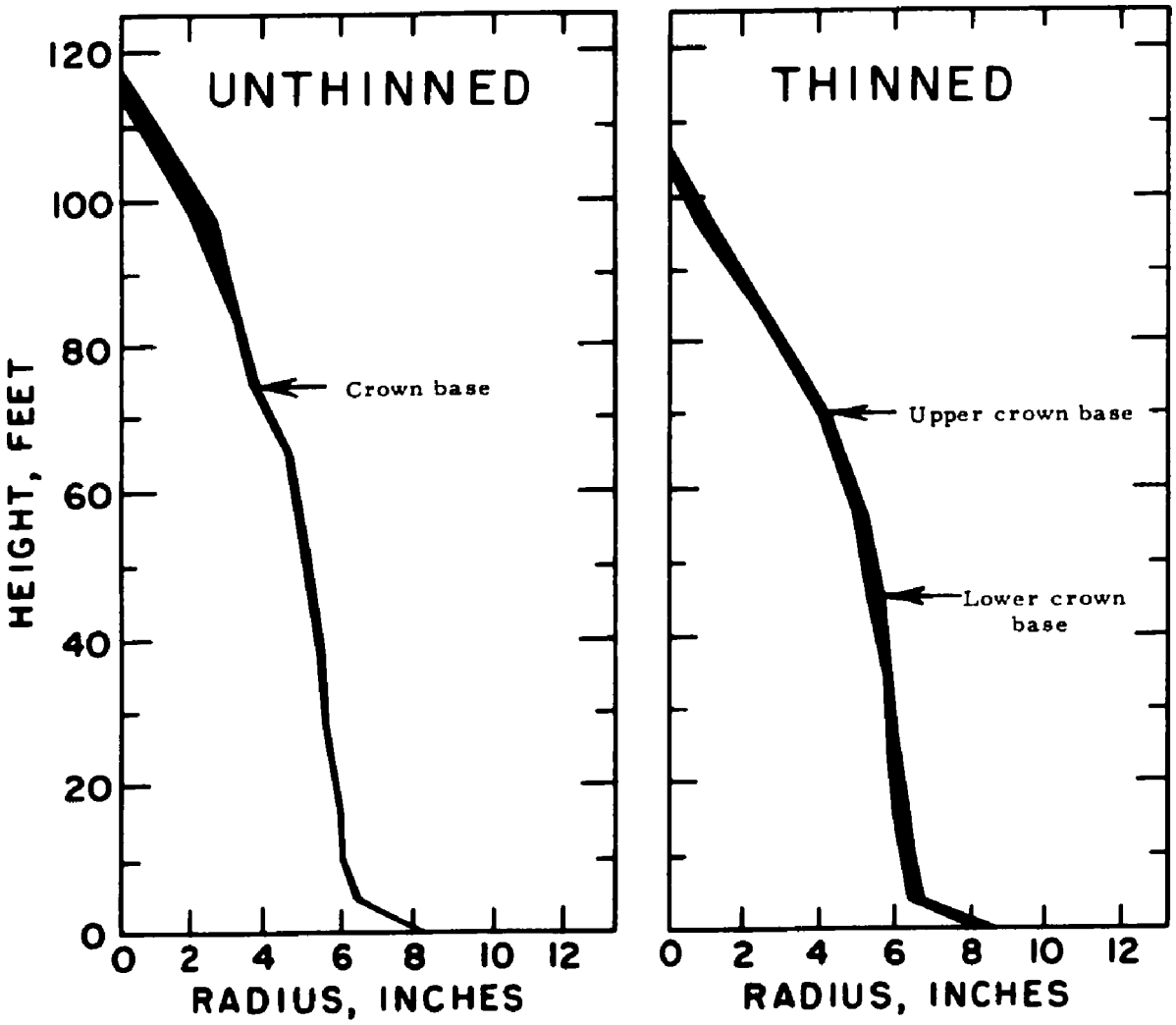
Increment pattern of two dominant Douglas-fir trees- thinned versus unthinned (DBH 19.6 inches). 1/



1/ Groman and Berg (1970)

FIGURE 10

Increment pattern of two codominant Douglas-fir trees--thinned versus unthinned. [1/]



[1/] Groman and Berg (1970).

Sample Estimates

Determination of Cubic-Foot Volume Growth

The cubic-foot volume growth for a given plot is determined as the difference between the estimated volume at the beginning of the growth period and the estimated volume at the end of the growth period.

The estimator of total volume at time one is:

$$\hat{T}_{y_{a1}} = T_{x1}/n_1 \sum_{i=1}^{n_1} y_{i1}/x_{i1}$$

and the estimator of total volume at time two is:

$$\hat{T}_{y_{a2}} = T_{x2}/n_2 \sum_{i=1}^{n_2} y_{i2}/x_{i2}$$

where subscripts 1 and 2 refer to times one and two respectively and y_i is the dendrometer-determined cubic-foot volume of the i^{th} tree.

The estimated cubic-foot volume growth for a given period is $\hat{T}_{y_{a2}} - \hat{T}_{y_{a1}}$ or G.

Table 18 presents data for sample estimates for 50 repeated computer Three-P samples over time. Table 19 presents data for sample estimates for 50 repeated computer Hartley-PPS samples

over time. The data are from a one-acre plot in the Black Rock Forest Management Research Area. The plot was established in 1953 and thinnings were made in 1957, 1963 and 1965. Tarif table volumes were used for the x-values and Black Rock local volume table volumes were used for the y-values. The total number of trees was reduced from 454 in 1953 to 184 in 1965. The sample was selected at the time of the initial measurement. At subsequent measurement times, T_x and the sample y_i/x_i ratios were updated to reflect the trees removed during the measurement period and the changes in those x and y-values. The mean estimated adjusted total volume for each measurement time varies little from the actual total volume. A comparison of the mean estimated cubic-foot volume growth (G) for each measurement period by sampling method for the 50 repeated computer trials shows that there is little difference between the Hartley-PPS and Three-P growth estimates. For the Three-P samples, the mean sample size ($\bar{n}$) varies from 60.96 to 31.82 while t_x/T_x varies only from 0.258 to 0.269. The simulated expected estimated relative variance of the adjusted estimated total volume was highest in 1953 and lowest in 1965. In general, the simulated relative variance of the estimated cubic-foot volume growth, $V_e(\bar{G})/\bar{G}^2$, for the 50 repeated computer trials are somewhat larger than the simulated relative variances of the adjusted estimated total volume. For the Hartley-PPS samples, the mean sample size varies from 60.0 to

TABLE 18

SAMPLE ESTIMATES FOR 50 REPEATED COMPUTER THREE-P SAMPLES OVER TIME

	1953	(57-53)	1957	(63-57)	1963	(65-63)	1965
T_x	6695.2		5842.3		6745.8		6166.9
T_y	6042.4		5376.7		6708.3		6299.9
$\hat{T}_{y_a}$ [1/]	6022.5		5383.2		6733.7		6318.6
$\bar{G}$		-639.3		1350.5		-415.1	
t_x/T_x [1/]	0.259		0.258		0.263		0.269
$\frac{E_e[\hat{V}(\hat{T}_{y_a})]}{\hat{T}^2_{y_a}}$	0.000309		0.000163		0.000179		0.000154
$\frac{V_e(\hat{T}_{y_a})}{\hat{T}^2_{y_a}}$	0.000360		0.000144		0.000173		0.000173
$\frac{V_e(\bar{G})}{\bar{G}^2}$		0.000454		0.000176		0.000358	
N	454		259		218		184
$\bar{n}$	60.96		42.80		37.46		31.82
$\sigma^2_{y_i/x_i}$	0.173		0.104		0.082		0.074

[1/] Mean for the 50 computer trials

TABLE 19

SAMPLE ESTIMATES FOR 50 REPEATED COMPUTER HARTLEY-PPS SAMPLES OVER TIME

	1953	(57-53)	1957	(63-57)	1963	(65-63)	1965
T_x	6695.2		5842.3		6745.8		6166.9
T_y	6042.4		5376.7		6708.3		6299.9
$\hat{T}_y$ [1/]	6030.2		5383.0		6727.0		6319.3
$\bar{G}$		-647.2		1,344.0		-407.7	
t_x/T_x [1/]	0.257		0.253		0.259		0.261
$\frac{E_e[\hat{V}(\hat{T}_y)]}{\hat{T}_y^2}$	0.000292		0.000170		0.000133		0.000169
$\frac{V_e(\hat{T}_y)}{\hat{T}_y^2}$	0.000078		0.000252		0.000237		0.000244
$\frac{V_e(\bar{G})}{\bar{G}^2}$		0.000324		0.000051		0.000205	
N	454		259	218		184	
$\bar{n}$	60.00		41.62		36.68		30.98
$\sigma^2_{y_i/x_i}$	0.173		0.104		0.082		0.074

[1/] Mean for the 50 computer trials.

30.98 while t_x/T_x varies only from 0.253 to 0.261.

The simulated relative variances of the estimated cubic-foot volume growth for the periods 1957-1963 and 1963-1965 are less than the simulated relative variances of the estimated total volume for 1957, 1963 and 1965 for the 50 repeated computer trials. The simulated expected estimated relative variance of the estimated total volume was highest in 1953 and lowest in 1963. The anomaly that the lowest relative variance for the two sampling methods occurred after repeated thinnings appears to be explained by the trend in population variance of the y_i/x_i ratios ($\sigma^2_{y_i/x_i}$) which varies from 0.173 to 0.074 for 1953 to 1965, respectively. This implies that as the stand was thinned the population of y_i/x_i ratios became more uniform. The similarity of the population variances for 1963 and 1965 suggests the reasonableness of the results that the relative variance for Three-P sampling was lowest in 1965 while the relative variance for Hartley-PPS sampling was lowest in 1963.

It is recommended that a new sample be selected should additional thinnings reduce the sample size to the point where the estimated variances are unacceptable.

Three-P Sampling Over Time for Nonresearch Areas

If the same x-values could be used at subsequent measurement times in Three-P sampling, the only trees which need referencing

are those in the sample. To ascertain the feasibility of this procedure a comparison was made of the y_i/x_i ratios when y-values are for a given measurement date and the x-values are for different measurement dates. This method has application for estimating stand volume changes in managed stands on a nonresearch basis.

The normal procedure would be to obtain the x-values for all trees and the y-values for the sample trees at the time of the initial measurement. New y-values are obtained at each remeasurement time. The initial total estimated volume is adjusted to reflect trees removed between measurements. Since only one set of y-values (1967) was available, x-values for different measurement times were used. For two of the young-growth populations x-values were available for 1957 and 1962 in addition to 1967.

Table 20 compares the y_i/x_i ratios over time where y_i is for a given measurement date and the x-values are for previous measurement periods. There is little difference between the actual total volume and the mean estimated adjusted total volume even though the x-values are from different measurement times. The magnitude of the simulated expected estimated variances in Table 20 is similar to that of those in Table 12.

TABLE 20

COMPARISON OF y_i/x_i RATIOS OVER TIME FOR 100 REPEATED COMPUTER THREE-P SAMPLES

Population	y_i	Tarif x_i	Tarif x_i	Tarif x_i	$E_e \frac{\hat{V}(\hat{T}_{ya})}{\hat{T}_{ya}^2}$	n_e	T_y	$\hat{T}_{ya}$ [1]
	1967	1967	1962	1957				
76	x	x			0.000372	20	6955.2	6946.2
76	x		x		0.000552	20	6955.2	6962.3
76	x			x	0.000363	20	6955.2	6962.6
62	x	x			0.000316	20	4086.9	4097.8
62	x		x		0.000335	20	4086.9	4091.8
62	x			x	0.000388	20	4086.9	4085.3

[1] Mean for the 100 computer trials.

CHAPTER VII

SUMMARY AND CONCLUSIONS

Summary

The objectives of this study were to investigate and analyze the comparative merits of three different unequal probability sampling methods (Hartley-PPS, Three-P and SALT) combined with the Barr and Stroud optical dendrometer for estimating cubic-foot volume growth on long-term research study plots.

A combination of unequal probability sampling and the Barr and Stroud optical dendrometer makes it possible to sample, rather than completely enumerate, permanent research plots.

The volume of standing trees can be directly measured with the aid of the Barr and Stroud optical dendrometer, which eliminates tree-volume tables, as such, along with accompanying errors. Repeated measurements of individual trees with the optical dendrometer provides the ability to assess upperstem form changes.

The unequal probability sampling methods studied have excellent sample selection procedures since the errors of estimates depend only on the choice of selection probabilities and not on the population variance. Repeated computer samples were drawn from

known populations in order to compare Hartley-PPS, Three-P and SALT sampling methods. For the populations studied any one of the three sampling methods gives satisfactory results if a list of estimated volumes is available. Hartley-PPS and Three-P are sampling without replacement; therefore, they were selected in preference to SALT sampling. The Hartley-PPS sampling method is recommended whenever a complete list of x-values is available since the sample size is fixed. Three-P sampling is recommended whenever it is a disadvantage to obtain a list of x-values in advance of the sampling.

Various powers of diameter, local volume table values and tarif table values were chosen as the x-value in an effort to minimize the estimated variance of the adjusted estimated total volume in Three-P sampling. The results from repeated computer samples showed that tarif table values and diameter squared were effective in reducing the coefficient of variation. The range of the coefficient of variation was from 2.4 to 13.0 percent, using tarif table values or diameter squared as the x-values and dendrometer determined cubic-foot volume as the y-values.

The expected sample (n_e) of a Three-P sample varies since n_e is a random variable. The expected sample size also varies due to the estimated total volume $(\hat{T}_x)$ of the area to be sampled and an improperly determined list of random numbers.

The magnitude of the bias of the adjusted estimate in Three-P sampling was examined. The results of 1500 repeated computer drawn samples showed that the bias of the adjusted estimate is negligible for the Douglas-fir populations studied.

In Hartley-PPS and Three-P sampling the sampling fraction is the ratio of the sample sum of the quick estimates and the population total of the quick estimates. Therefore, $(T_x-t_x)/T_x$ was recommended as a more appropriate finite population correction factor for these sampling methods.

Graphs were developed from which estimated sample sizes for Hartley-PPS and Three-P sampling methods can be read directly.

A field trial was made of the Three-P and Hartley-PPS sampling methods, the results of which were comparable to the repeated computer trials. An analysis of repeated dendrometer measurements of individual trees over time suggested that upperstem diameter changes can be determined after only two growing seasons for trees on thinned plots or fast growing trees on unthinned plots. These measurements indicate that there should be little difficulty in measuring cubic-foot volume growth on permanent research plots at four to five-year intervals.

The estimated cubic-volume growth for a given plot was determined as the difference between the estimated volume at the beginning of the growth period and the estimated volume at the end of

the growth period. An analysis was made of data for sample estimates from 50 repeated computer samples each for Three-P and Hartley-PPS sampling representing four measurement times. The estimated total volumes varied little from the actual total volumes. The simulated expected estimated relative variance of the estimated total volume was highest at the initial measurement time and lowest after repeated thinnings. This anomaly appears to be explained by the fact that the population variance of the y_i/x_i ratios became smaller as the stand was thinned.

Conclusions

This study has shown that it is highly efficient to estimate cubic-foot volume growth on long-term research plots by using a combination of repeated unequal probability sampling and repeated upperstem diameter and length measurements made with the Barr and Stroud optical dendrometer. The Hartley-PPS sampling method is recommended for long-term research plots since sample size is fixed and all the trees are identified for relocation. Thus, sampling lists are readily available. The use of the Barr and Stroud optical dendrometer eliminates tree-volume tables and their inherent errors. In addition, the optical dendrometer makes it possible to measure upperstem form changes over relatively short periods of time.

Although the populations used in this study were small, the results appear to be applicable, in general, for determining cubic-foot volume growth on research plots in all young-growth Douglas-fir stands.

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APPENDIX A

Theory of Three-P Sampling

The probability that any particular tree would be selected randomly from a population of N trees is $1/N$. The probability that a selected tree is measured is $x_i/(K + Z)$. Therefore, the probability (p_i) that a tree is selected and measured is $(\frac{1}{N})\,(\frac{x_i}{K+Z}) = x_i/N(K+Z) = p_i$. In this case, n could assume only the values of 0 or 1 where

$$\frac{y_i}{p_i} = \frac{y_i}{x_i/N(K+Z)} = \frac{y_i N(K+Z)}{x_i}$$

If $n = 0$, then $\frac{y_i}{p_i} = \frac{0}{p_i} = 0$

If $n = 1$, then $\frac{y_i}{p_i}$ is determined by the value of y_i.

$$E(y_i/x_i) = \sum_{i=1}^{N} [\frac{y_i}{p_i}] = \sum_{i=1}^{N} p_i (\frac{y_i}{p_i}) + (1-p_i) \cdot 0 = \sum_{i=1}^{N} y_i \quad \text{or} \quad T_y.$$

Thus $\frac{y_i}{p_i}$ is an unbiased estimate of T_y for the sample of size 0 or 1. Each $\frac{y_i}{p_i}$ is an unbiased estimate of T_y thus, $\sum_{i=1}^{N} \frac{y_i}{p_i}$ has the expected value of $N\ T_y$ and $\frac{1}{N} \sum_{i=1}^{N} \frac{y_i}{p_i}$ has the expected value of T_y and

$$\frac{1}{N}\sum_{i=1}^{N}\frac{y_i}{p_i} = \frac{1}{N}\sum_{i=1}^{N}\frac{y_i}{x_i/N(K+Z)} = \frac{1}{N}\sum_{i=1}^{N}\frac{N(K+Z)y_i}{x_i} = (K+Z)\sum_{i=1}^{N}\frac{y_i}{x_i}.$$

Likewise for a sample of size n, $\sum_{i=1}^{N}\frac{y_i}{p_i}$ has the expected value of $n\sum_{i=1}^{N} y_i$ and $1/n\sum_{i=1}^{N}\frac{y_i}{p_i}$ has the expected value of $\sum_{i=1}^{N} y_i$. (There are N-n values of $y_i/p_i = 0$). Therefore, $\sum_{i=1}^{N} y_i$ is estimated by $(K+Z)\sum_{i=1}^{N}\frac{y_i}{x_i}$ if n is not zero, otherwise = 0 if n = 0.

APPENDIX B

Theory of SALT Sampling

If R numbers are selected at random from the uniform distribution (0, Z), the probability that the number falls in a given interval (a, b] is $\frac{b-a}{Z}$ for $0 < a \leq b \leq Z$. The probability at each selection that the number falls into an interval j, is (j = 1, ···, L)

$$\frac{a_j - a_{j-1}}{\sum_{k=1}^{L} (a_k - a_{k-1})} = \frac{a_j - a_{j-1}}{a_L - a_o} = \frac{x_j}{Z} .$$

The number of random numbers which fall into the j^{th} interval is designated t_j. Using the properties of the Multinomial distribution, the expected value of t_j, its variance and the covariance of t_j and $t_{j'}$ can be determined.

$$E(t_j) = R\frac{x_j}{Z}$$

$$V(t_j) = R\left(\frac{x_j}{Z}\right)\left(1 - \frac{x_j}{Z}\right)$$

$$COV(t_j\, t_{j'}) = -R\left(\frac{x_j}{Z}\right)\left(\frac{x_{j'}}{Z}\right) \quad \text{for } j \neq j'$$

where x_j is the length of interval j. Sample size n is $\sum_{i=1}^{N} t_i$ which is also the numbers in the interval $(O, \sum_{i=1}^{N} x_i]$. The distribution

of n is Binomial (R, π) where $\pi = \frac{\sum_{i=1}^{N} x_i}{Z}$. The expected value of n and its variance are as follows:

$$E(n) = R\pi$$

$$V(n) = R\pi(1-\pi).$$

The conditional joint distribution, given n, of the number of random numbers falling in the intervals is Multinomial when

$$p_j = \frac{x_j}{\sum_{i=1}^{N} x_i}.$$

These probabilities are analogous to the probabilities in PPS sampling. Thus, the unbiased SALT estimate of T_y is $\frac{1}{n}\sum_{j=1}^{N} y_j/p_j$, and its variance $[V(\hat{T}_y)]$ is

$$E(1/n)\frac{\sum_{i}^{N} y_i^2}{p_i} - T_y^2).$$

APPENDIX C

Random Number Generator Program

The random numbers used in drawing the repeated samples were produced by a mixed congruential random number generator program. A comparison of actual and expected values for the random numbers developed by the random number generator program for the Three-P samples is found in Appendix C, Table 1.

The random number generator program for Three-P sampling is as follows:

```
      SUBROUTINE RANDOM (MAXR, KZ, NOPT, NRAND, INIT)
      IF (NOPT .NE. O) GO TO 100
      NOPT = 1
      CALL CLOCK (N)
      N = MOD(N, 6000)
      INIT = N
100   N = AND(2051*N + 1220519, 37777777B)
      NRAND = (N/8388608.)*KZ + 1
      IF (NRAND .GT. MAXR) NRAND = 0
      RETURN
      END
```

Input:

MAXR = K which is the largest x_i in the sampling scheme.

KZ = K + Z. Z is the number of unconditional chances to exclude an individual value from a Three-P sample.

NOPT = O call clock(computer) for a starting value - used for first call only.

NOPT = 1 use previous N for starting value.

Output:

NRAND = the random number.

INIT = the value obtained from clock (mod 6000).

Method:

Let N be either INIT or a previous random number, then

N(new) = AND(2051 * N(old) + 1220519, 37777777B) where 37777777B assures that N will be a positive number, and we have $0 \leq N \leq 8388607$. [1/]

NRAND = (N(new)/8388608.)* KZ + 1. If NRAND > K, NRAND is set equal to 0. Otherwise, we have $1 \leq$ NRAND $\leq K + Z$.

[1/] The largest positive value that can be represented as an integer on the Oregon State University computer.

TABLE 1

COMPARISON OF ACTUAL AND EXPECTED VALUES FROM THE RANDOM NUMBER GENERATOR PROGRAM FOR THE THREE-P SAMPLES.

(Trees)	Expected sample size	X	Number non-zero digits	Expected number non-zero digits	Sum of random numbers	Expected sum of random numbers
52	20	Tarif Vol.	4,609	4,582.50	326,524	327,239
52	20	$D^{2.0}$	4,470	4,417.70	1,293,163	1,289,595
62	20	Tarif Vol.	5,942	5,913.85	554,503	555,577
62	20	$D^{2.0}$	5,652	5,660.87	2,121,795	2,139,282
76	20	Tarif Vol.	6,199	6,200.00	763,679	771,775
76	20	$D^{2.0}$	5,995	6,004.72	3,017,693	3,015,485
190	20	Tarif Vol.	7,130	7,150.23	888,719	887,685
190	35	Tarif Vol.	12,426	12,531.91	1,561,374	1,547,037
190	50	Tarif Vol.	17,830	17,848.48	2,234,332	2,219,835
190	20	$D^{2.0}$	6,897	6,871.18	3,456,358	3,469,191
190	35	$D^{2.0}$	12,066	12,032.14	6,050,989	6,069,198
190	50	$D^{2.0}$	17,239	17,187.22	*	*

* Inadequate field size allowed for in computer program.

APPENDIX D

Relative Variances for Hartley-PPS and SALT Samples

The relative variances for 100 repeated computer Hartley-PPS samples for the four study populations are given in Appendix D Table 2, and Table 3 presents the same information for the SALT samples.

TABLE 2

RELATIVE VARIANCES FOR 100 REPEATED COMPUTER HARTLEY-PPS SAMPLES FOR FOUR DIFFERENT POPULATIONS.

Population	x	n	t_x / T_x	$\frac{E_e[\hat{V}(\hat{T}_y)]}{T_y^2}$	$\frac{E_e[\hat{V}(\hat{T}_y)]}{\hat{T}_y^2}$	$\frac{V_e(\hat{T}_y)}{T_y^2}$	$\frac{S_e(\hat{T}_y)}{T_y}$
1	Tarif	20	.481	.000303	.000298	.000203	.01423
1	$D^{2.0}$	20	.473	.000289	.000291	.000245	.01565
2	Tarif	20	.449	.000309	.000312	.000295	.01717
2	$D^{2.0}$	20	.434	.000425	.000442	.000070	.00835
3	Tarif	20	.342	.000347	.000339	.000391	.01972
3	$D^{2.0}$	20	.333	.000093	.000096	.000057	.00756
1, 2, 3	Tarif	20	.141	.000570	.000566	.000325	.01803
1, 2, 3	Tarif	35	.246	.000278	.000277	.000223	.01494
1, 2, 3	Tarif	50	.352	.000164	.000163	.000250	.01581
1, 2, 3	$D^{2.0}$	20	.137	.000669	.000691	.000371	.01925
1, 2, 3	$D^{2.0}$	35	.240	.000328	.000336	.000098	.00989
1, 2, 3	$D^{2.0}$	50	.342	.000195	.000200	.000179	.01338
4	$D^{2.0}$	20	.194	.001823	.001931	.001104	.03323
4	$D^{2.0}$	35	.321	.000886	.000934	.000585	.02418
4	$D^{2.0}$	50	.459	.000470	.000494	.000803	.02834

TABLE 3

RELATIVE VARIANCES FOR 100 REPEATED COMPUTER SALT SAMPLES FOR FOUR DIFFERENT POPULATIONS

Population	x	n	t_x/T_x	$\frac{E_e[\hat{V}(\hat{T}_y)]}{T_y^2}$	$\frac{E_e[\hat{V}(\hat{T}_y)]}{\hat{T}_y^2}$	$\frac{V_e(\hat{T}_y)}{T_y^2}$	$\frac{S_e(\hat{T}_y)}{T_y}$
1	Tarif	20	.478	.000308	.000303	.000592	.02434
1	$D^{2.0}$	20	.474	.000283	.000286	.000641	.02531
2	Tarif	20	.450	.000300	.000302	.000640	.02530
2	$D^{2.0}$	20	.428	.000424	.000446	.000769	.02774
3	Tarif	20	.342	.000348	.000337	.000627	.02504
3	$D^{2.0}$	20	.336	.000087	.000091	.000113	.01065
1, 2, 3	Tarif	20	.141	.000569	.000568	.000634	.02518
1, 2, 3	Tarif	35	.246	.000271	.000270	.000434	.02083
1, 2, 3	Tarif	50	.349	.000168	.000167	.000304	.01744
1, 2, 3	$D^{2.0}$	20	.137	.000631	.000648	.000589	.02426
1, 2, 3	$D^{2.0}$	35	.238	.000311	.000319	.000426	.02063
1, 2, 3	$D^{2.0}$	50	.343	.000194	.000199	.000330	.01817
4	$D^{2.0}$	20	.182	.001773	.001891	.002346	.04844
4	$D^{2.0}$	35	.320	.000840	.000887	.001039	.03223
4	$D^{2.0}$	50	.460	.000462	.000486	.000120	.03170

APPENDIX E

Glossary of Symbols

Symbol		Page
B_e	Estimated (simulated) bias	56
C	The coefficient of variation of sample units except for Table 19 where C is the coefficient of variation for population units	77
$E_A\%$	Absolute error in percent	46
E_e	Estimated (simulated) expected value	56
$E_e(\hat{T}_{y_a})$	The simulated expected adjusted estimated total volume for the repeated computer Three-P samples	56
G	Estimated cubic-foot volume growth	85
K	The largest x-value in a population to be sampled by the Three-P sampling method	14
M	The number of computer trials (samples)	46
N	The total number of trees in the population	8
n	Sample size	10
n_e	The expected sample size in Three-P sampling	15
r_i	The i^{th} random integer	15
S_e	Estimated (simulated) standard deviation	58
$S_{\bar{R}}\%$	The standard error of mean ratio ($R_i=y_i/x_i$) in percent	47
T_x	The population total of x-values	8

Gloassary of Symbols (continued)

Symbol		Page
$\hat{T}_x$	The estimated population total of x-values	15
t_x	The sample total of x-values.	46
T_y	The population total of y-values	46
$\hat{T}_y$	The estimated population total of y-values	13
$\hat{T}_{y_a}$	The adjusted estimated total volume in Three-P sampling	17
$\hat{T}_{y_u}$	The unadjusted estimated total volume in Three-P sampling	46
V_e	Estimated (simulated) variance	58
$\hat{V}(\hat{T}_y)$	The estimated variance of the estimated total	13
x_i	Estimate of the tree characteristic of interest of the i^{th} tree.	5
y_i	The tree characteristic of interest of the i^{th} tree	5
Z	The number of unconditional chances to exclude an individual value from a Three-P sample	16