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DILUTION REFRIGERATION AND VORTEX RINGS
IN He^3 - He^4 MIXTURES

by

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PART I

INTRODUCTION TO $\text{He}^3\text{-He}^4$ MIXTURES AND DILUTION REFRIGERATION

CHAPTER I. INTRODUCTION

The traditional method for producing temperatures in the millikelvin range has been the adiabatic demagnetization of a paramagnetic salt. Although magnetic cooling has become a highly refined technique since it was first suggested in 1926 by Debye and Giauque, it is not a continuous process and, by its very nature, cannot extract large amounts of heat at the lowest temperatures. A low temperature requires a salt with a small lattice entropy which, in turn, implies a small specific heat and a poor refrigerating capability. This might well be expected in view of the Third Law of Thermodynamics and much important ultra-low temperature work has been done using magnetic cooling. These limitations, however, have prohibited many important experiments that were not consistent with the allowable heat load of a few tens of ergs/min. Clearly a new refrigeration process was needed.

Early in the 1950's, H. London suggested a cooling process¹ using the entropy of mixing He^3 in He^4 . Below 1 K, the entropy of the He^4 is negligible compared to the entropy of a small amount of He^3 dissolved in the superfluid. The resulting solution behaves much like a gas of He^3 atoms. London pointed out that an adiabatic expansion of this "gas" might produce cooling; however, his suggestion was not explored at that time because of the lack of sufficient quantities of He^3 .

During the late 1950's and early 1960's, mixtures of He^3 and He^4 began to generate much excitement in the world of low temperatures. In 1956 Walters and Fairbank discovered that mixtures of He^3 and He^4 underwent a phase separation² below 0.8 K. Phase separation promised an even

more powerful refrigeration technique than London's adiabatic expansion of He^3 --a process similar to the evaporation of a liquid. 1962 brought the discovery of the finite solubility of He^3 in He^4 at absolute zero by Edwards and Daunt.³ A year later--in a paper on the osmotic pressure of dilute solutions--London, Clark and Mendoza proposed several models for a refrigerator that utilized the dilution of pure He^3 into a He^3 - He^4 mixture.⁴ This "dilution refrigerator" was a continuous process and promised to reach temperatures of a few millikelvin under many times the typical heat load of magnetic cooling. Four years later, two groups--one in Russia,⁵ and the other at Oxford⁶--reported the successful operation of dilution refrigerators built along the lines of the model that was suggested by H. Hall. An earlier attempt by a group at Leiden produced a refrigerator that would not cool below 0.2 K.⁷

Encouraged by these successes and armed with some design data generously shared by J. C. Wheatley, a program in ultra-low temperature physics was started at The University of Michigan (Michigan) in 1967 by Dr. S. C. Whitmore, and S. R. Ryan. This thesis is a description of that ultra-low temperature program and the dilution refrigerator designed and constructed at Michigan. An experimental measurement of the frictional drag on vortex rings in dilute mixtures will also be discussed.

Ions and Vortex Rings in Liquid Helium

Charged particles have long been used as microscopic probes of the superfluid nature of liquid helium. Charges interact strongly with the liquid to produce ions of very interesting structure; these ions have been

the subject of intensive investigation over the past decade. At very low temperatures, the friction on the ions in $\text{He}^3\text{-He}^4$ mixtures is dominated by the He^3 present, and the ions can be used to examine the properties of London's gas of He^3 atoms.

In a regime where the friction is small, the ions can be accelerated in an electric field until they reach a critical velocity and create a macroscopic flow pattern in the liquid helium similar to a smoke ring. This flow field is just the vortex ring of classical hydrodynamics with one important distinction--the circulation⁸ of the vortex ring is quantized in units of h/m_4 . These vortex rings with attached ions, discovered in liquid helium by Rayfield and Reif,⁹ have interesting dynamics, as their velocity is inversely proportional to their energy. In dilute mixtures, a measurement of the energy and velocity of the rings in an electric field will provide information on the He^3 -vortex interaction and can test the understanding of dilute $\text{He}^3\text{-He}^4$ mixtures.

Outline of the Thesis

The remaining two chapters of Part I are intended to give the reader an introduction to $\text{He}^3\text{-He}^4$ mixtures and the principles of dilution refrigeration. The material discussed in these two chapters is presented in a heuristic manner and the reader interested in rigor is referred to the many excellent review articles on the subject listed in Appendix D.

Part II is a description of the Michigan dilution refrigerator. It is written to provide an operating manual for future use as well as supply new design data to supplement the existing literature. Because a program

in ultra-low temperature physics has many pitfalls, the false steps, as well as the successes of the program at Michigan, will be discussed for the benefit of those who, like us, do not have the heritage of adiabatic demagnetization on which to draw. The last chapter of Part II is a compendium of miscellaneous techniques, discussion of materials and suggestions that have proved useful in ultra-low temperature work.

It is intended that anyone familiar enough with dilution refrigeration to utilize Part II of the thesis may safely omit the reading of Part I. A useful--although not exhaustive--bibliography of the literature on dilution refrigeration and $\text{He}^3\text{-He}^4$ mixtures is given in Appendix D. This list was collected during the design and construction of the refrigerator.

Part III of the thesis reports a measurement of the frictional drag on vortex rings in dilute solutions. The measurements were made by the stopping potential technique and were taken at several temperatures between 60 and 200 mK. Four different mixtures between 6 and 80 parts per million of He^3 were examined.

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CHAPTER II. MIXTURES OF He^3 IN He^4

Properties of He^3 and He^4

Below 2 K the thermodynamic and hydrodynamic properties of liquid helium are strongly dependent on the quantum statistics of the helium atom. Helium ^{4} is a boson and undergoes a superfluid transition at 2.17 K similar to the Bose-Einstein condensation of an ideal Bose gas. A consequence of this transition or "condensation in phase space" is that liquid helium ^{4} may be described in terms of a two-fluid model, consisting of a superfluid component carrying no entropy or viscosity, and a normal fluid composed of a gas of thermal excitations or quasiparticles. These quasiparticles, which are responsible for the entropy and viscosity of the normal fluid, are called phonons or rotons depending on the region of the dispersion curve from which they come. Below 0.8 K the density of rotons is negligible, and only the phonons need be considered when characterizing the normal fluid.

He^3 , on the other hand, has an odd number of nucleons, is a fermion, and does not undergo a superfluid transition--at least at currently attainable temperatures. Consequently, a simple description, such as the two-fluid model, does not exist for liquid helium ^{3} , although much success has been achieved in developing an adequate Fermi liquid theory.

Because He^3 is distinguishable in a quantum mechanical sense from He^4 , it cannot be described by the same symmetric ground state wave function that characterizes the bulk superfluid. The He^3 , therefore, in a dilute mixture does not partake in the superfluid motion, but contributes to the normal fluid. Since He^3 and He^4 are isotopes and chemically identical, the forces between two He^3 atoms, two He^4 atoms, and a He^3 and He^4 atom are all

equal. In a dilute solution, where the He³ atoms are separated by many He⁴'s, He⁴ atoms screen the interaction between two He³ impurities, and the He³ atoms do not interact strongly with each other. The superfluid component, having no viscosity, exerts no drag on the He³ atoms, and the He³ impurities behave much like an ideal gas in a "mechanical vacuum" of superfluid He⁴.

Landau and Pomeranchuk¹ have written the energy spectrum for this gas of He³ atoms as

$$E = E_0 + p^2/2m^* ,$$

where E_0 is the binding energy of a helium 3 atom to the bulk liquid and m^* is an effective mass (experimentally, 2.3 times the mass of the bare He³ atom).² E_0 is akin to a latent heat and is the energy necessary to remove a single He³ impurity atom from pure He⁴ at absolute zero. m^* is the sum of the bare He³ mass plus a contribution from a flow field in the superfluid, created by the motion of the He³, which takes the He⁴ from in front of the impurity and fills in the void left behind it. Accordingly, one should not think of a bare He³ atom in superfluid He⁴ but, rather, a potential well of depth E_0 containing a quasiparticle composed of the bare impurity dressed with a dipole flow field. It is this gas of quasiparticles which behaves as an ideal gas--in fact, an ideal Fermi-Dirac gas since the quasiparticle is a fermion. With this picture in mind, it would be more accurate to describe the superfluid component as an "ether" since the He⁴ background affects the energy and the dynamics of the He³.

A He³ atom, having $3/4$ the mass of a He⁴, has a larger zero point energy than a He⁴ atom, and, consequently, each He³ occupies a slightly larger volume than a He⁴. The interaction between two He³ quasiparticles arises from the fact that the dipole flow field of one quasiparticle sees a reduced density at the site of another, slightly changing the effective mass of the quasiparticle. This interaction is small, and the properties of a dilute solution may be calculated to high accuracy using the model of an ideal Fermi-Dirac gas with an effective mass that is concentration dependent. The effective mass m^* increases from $2.35 m_3$ in the limit of zero He³ concentration to $2.5 m_3$ for a ten percent mixture.

So far, the phonon contribution to the properties of dilute mixtures has been neglected. At 0.5 K the phonon number density is equal to the number density of the He³ atoms in a 3 percent mixture. However, the energy and momentum of the average phonon are so small in comparison to that of a He³ quasiparticle that the He³ dominates both the thermodynamic and hydrodynamic properties of mixtures. (For the same reason, rotons dominate the normal fluid properties above 1 K even though the phonon density is many times the roton number density.) At 0.5 K phonons contribute only 0.3 percent of the entropy in a 3 percent mixture. Since the phonon number density is proportional to T^3 , at lower temperatures the He⁴ contribution to the normal fluid is truly negligible. The picture of a dissipation-free mechanical ether of He⁴ supporting a gas of He³ quasiparticles is valid below 0.5 K. (Phonons do remain important in some processes, such as thermal conductivity and thermal boundary resistance, down to very low temperatures.)

Phase Separation

While the model of an ideal Fermi-Dirac gas is valid for low concentrations, it is obvious that as the concentration of He^3 is increased this description must break down; the mixture will cease to be superfluid when there is enough He^3 present to destroy the long range order. At suitably high concentrations, He^3 - He^4 mixtures undergo a phase transition analogous to the condensation of a real gas. Starting with a 70 percent mixture, the He^3 atoms begin to condense at 0.8 K forming a non-superfluid, He^3 -rich, phase which floats on top of a superfluid phase, rich in He^4 . As the temperature is lowered, separation continues until by 60 mK the top phase is virtually pure He^3 . (Fig. 1) Prior to 1961, the superfluid phase was assumed to purify in the same manner, on the basis of entropy of mixing. The fact that this assumption was erroneous inhibited much early interest in mixtures. In 1961, detailed measurements of the specific heat of dilute mixtures by Edwards and Daunt indicated that even at absolute zero some 6 percent He^3 can be dissolved in He^4 ; the full physical significance of mixtures then became apparent.³ Dilute solutions of He^3 in He^4 present a unique opportunity to study the properties of an ideal Fermi-Dirac gas from conditions of complete quantum-mechanical degeneracy (6 percent mixtures near absolute zero) to the non-degenerate Boltzmann regime (very low concentrations above 200 mK). This 6 percent limiting solubility is of crucial importance for the operation of a refrigeration process utilizing He^3 - He^4 mixtures.

The finite solubility at absolute zero does not violate the Third Law because a Fermi-Dirac gas becomes completely degenerate at that

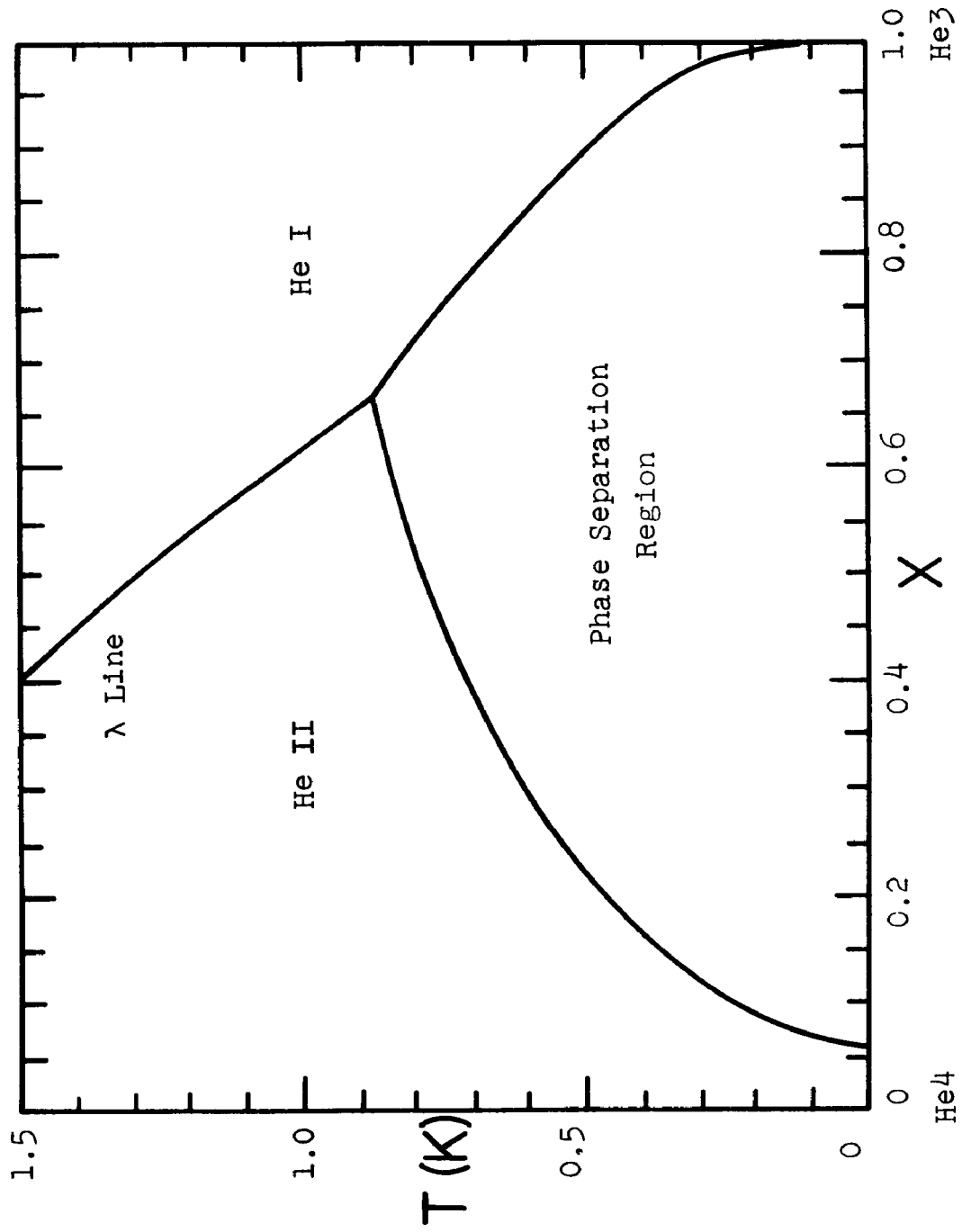


FIG. 1 PHASE SEPARATION DIAGRAM FOR He³-He⁴ MIXTURES (AFTER RADEBAUGH).

temperature. To understand the equilibrium behavior of a phase-separated mixture at absolute zero, the chemical potential of the He³ should be examined. At absolute zero, the chemical potential has a particularly simply interpretation--it is the amount of energy needed to "evaporate" a He³ atom from the liquid. (Zero energy being the state in which the He³ atom is infinitely far from the liquid and at rest.) If L_0 is the latent heat of pure He³ at 0 K, and E_0 is the energy necessary to remove a single He³ impurity from pure liquid He⁴ at 0 K, then there will be a finite solubility only if E_0 is more negative than L_0 . Experimentally, $-L_0$ is 2.5 K and $-E_0$ is 2.8 K. If the dissolved He³ quasiparticles were a Maxwell-Boltzmann gas, E_0 more negative than L_0 would imply an infinite solubility at absolute zero; however, the quasiparticles are fermions, and only two He³ atoms (corresponding to the two spin states) are allowed per energy state. Thus the translational kinetic energy levels of the ideal gas are filled with He³ quasiparticles until the Fermi level of the dilute solution equals the Fermi level of the pure liquid He³. This occurs when

$$L_0 = E_0 + p_f^2/2m^* = E_0 + kT_f, \quad ,$$

where T_f is the Fermi temperature of the mixture, p_f is the momentum at the Fermi surface, and k is Boltzmann's constant. The Fermi temperature is related to the concentration by the equation

$$kT_f = p_f^2/2m^* = \frac{\hbar^2}{2m^*} \left[\frac{3\pi^2}{v} \right]^{2/3} x^{2/3}, \quad ,$$

where v is the average volume per atom and $x = n_3 / (n_3 + n_4)$ is the concentration.

Osmotic Pressure

An osmotic pressure is developed across a semipermeable membrane separating a dilute mixture from pure liquid helium ⁴. The membrane--in this case a "superleak"--is permeable to the superfluid but prevents the passage of normal fluid. Consider a U tube with pure He⁴ on one side, separated by a superleak from a dilute mixture in the other U tube arm. The superfluid will flow through the superleak, establishing a hydrostatic pressure head, until the chemical potential of the He⁴ on both sides is equal

$$\mu_0(T, P = 0, x = 0) = \mu_M(T, P_{\text{osm}}, x) .$$

Here μ_0 is the chemical potential of pure He⁴ at zero pressure and μ_M is the chemical potential of the He⁴ component of the mixture at concentration x and osmotic pressure P_{osm} (which is equal to the hydrostatic pressure head in the U tube at equilibrium). Using the Gibbs-Duhem relationship for a two-component mixture, it can be shown that van't Hoff's equation is obeyed for sufficiently dilute and non-degenerate mixtures

$$vP_{\text{osm}} = kxT ,$$

where x is the concentration, T is temperature, k is Boltzmann's constant and v is the volume per atom of He⁴. This has the same form as the ideal gas law.

In preparation for Chapter III, where equilibrium conditions do not prevail and there is a steady flow of quasiparticles, let us consider a closed tube filled with a dilute mixture in the presence of a temperature gradient. The concentration at any point along the tube is given by van't Hoff's equation. If the tube is in mechanical equilibrium, the osmotic pressure is constant, and the product $xT = vP_{\text{osm}}/k$ is a constant along the tube. If the solute is in steady flow through the solvent, then van't Hoff's equation is no longer valid unless the solvent happens to have a rather exceptional property--like superfluidity. For low flow velocity, the equation of motion of the superfluid is

$$\vec{\nabla}_{\mu_4} = -m_4 (\partial \vec{v}_s / \partial t) ,$$

where m_4 is the mass of a He^4 atom, $\vec{v}_s$ is the velocity of the superfluid and $\vec{\nabla}_{\mu_4}$ is the gradient of the chemical potential of the He^4 . Under conditions of steady flow, $(\partial \vec{v}_s / \partial t)$ and $\vec{\nabla}_{\mu_4}$ are equal to zero. If the equilibrium concentration is disturbed, the superfluid will accelerate, until the gradient of the chemical potential of the superfluid is again zero, and xT is a constant. This development is valid as long as the mixture is not very degenerate. As the temperature decreases, x increases, approaching the phase separation curve, as shown in Fig. 2. Below 100 mK, the concentration tends asymptotically to 6.4 percent.

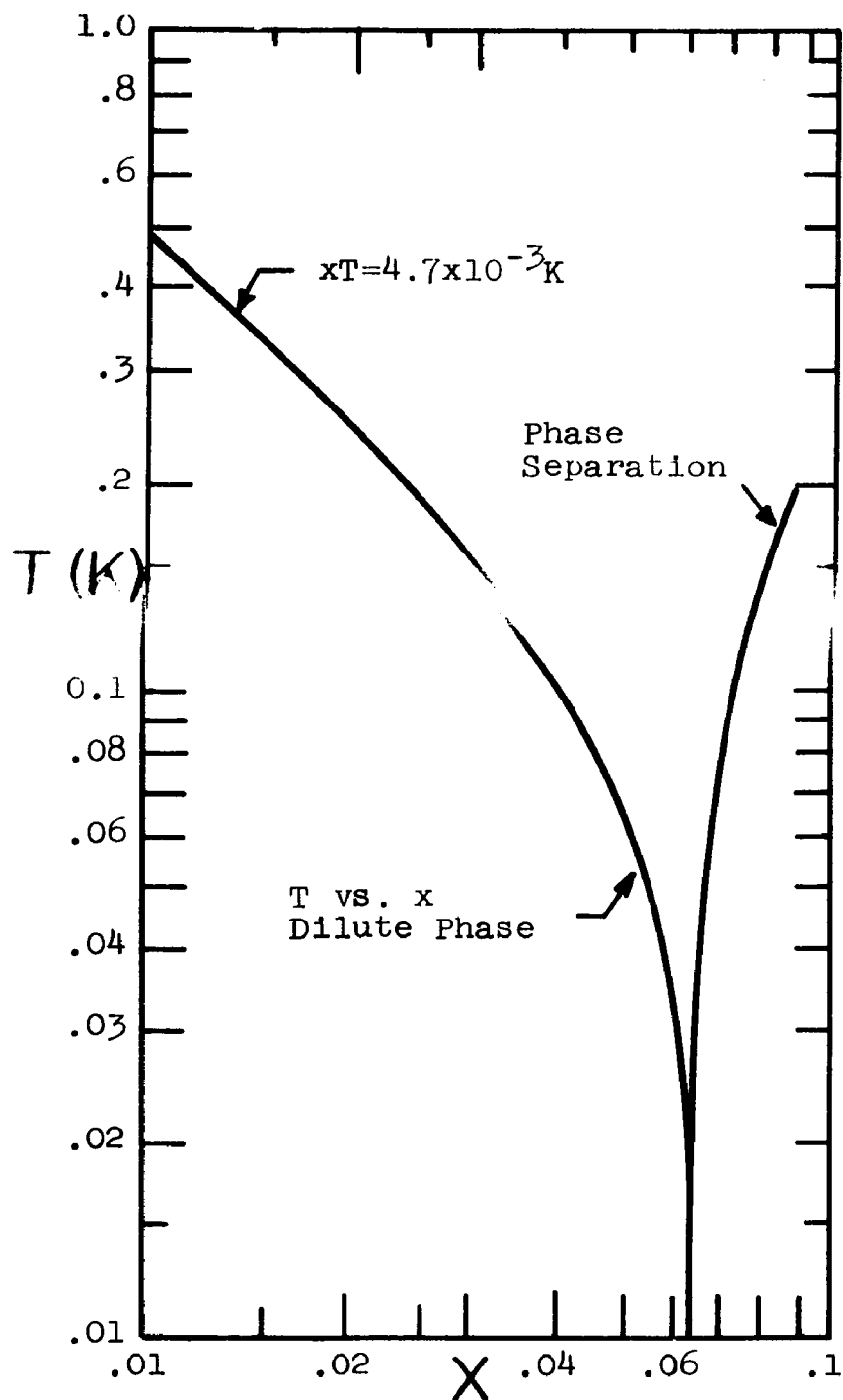


FIG. 2 DETAIL OF THE PHASE SEPARATION CURVE BELOW 200 mK AND T VS. x FOR THE DILUTE PHASE IN A TUBE UNDER A TEMPERATURE GRADIENT. THE LOWER END OF THE TUBE IS AT 0 K AND 6.4% (AFTER WHEATLEY).

Useful Properties of Mixtures

As was mentioned earlier, the thermodynamic properties of dilute mixtures can be calculated quite accurately from ideal Fermi-Dirac gas theory if the effective mass m^* is allowed to vary slightly with concentration. At temperatures well above the Fermi temperature of a mixture, its properties are those of an ideal Boltzmann gas. The specific heat per atom is $3k/2$, for example. Near the Fermi temperature, numerical methods must be employed to calculate the thermodynamic properties of a Fermi-Dirac gas. When the temperature is much less than T_F , the results again become relatively simple. Near absolute zero, the specific heat and entropy are proportional to T , and the pressure approaches a constant 12 Torr (for a 6.4 percent mixture). This non-vanishing pressure is the osmotic pressure (not the external vapor pressure above the mixture) and is a consequence of the Pauli principle, which prevents the He³ quasi-particles from condensing into the state of zero kinetic energy. The viscosity of a dilute mixture is proportional to $1/T^2$, becoming molasses-like near absolute zero, while the thermal conductivity varies as $1/T$. At higher temperatures the thermal conductivity again increases, with an increase in phonon density, until there are sufficient thermal excitations present to propagate second sound.

Finally, the problem of thermal transport across a liquid-solid interface should be mentioned. A thermal boundary resistance, which varies approximately as $1/T^3$, develops at very low temperatures and makes heat transfer difficult. Known as the Kapitza resistance, it is thought to arise from a mismatch in the acoustic impedance for phonons between the solid

and liquid helium. The Kapitza resistance is not unique to dilute mixtures, although the problem is complicated by the fact that heat is transferred between solid and liquid in the form of thermal phonons, while the heat content of a mixture resides in the He³ quasiparticles.⁴ At very low temperatures, the phonon density is so low that He³-phonon collisions become infrequent. To overcome these problems, good thermal contact requires a large volume of dilute mixture, to insure that He³ quasiparticles and the phonons are in equilibrium, and a very large surface area for phonon transfer between the solid and the liquid.

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CHAPTER III. PRINCIPLES OF DILUTION REFRIGERATION

Refrigerator Principles

The standard technique for producing temperatures in the vicinity of 1 K is the evaporation of liquid helium. As an atom evaporates across a liquid-gas interface, it extracts an amount of energy L , the latent heat per atom, which is finite even at absolute zero. The rate $\dot{n}$ at which atoms can be made to evaporate has the same exponential dependence as the vapor pressure: $\exp(-L/kT)$. The refrigerating power of an evaporating liquid is $\dot{n}L$, and goes to zero exponentially with temperature. The latent heat of He^4 is 7.2 K and, by a temperature of 0.9 K, the vapor pressure above the liquid is too small to provide much useful cooling, even with the largest vacuum pump. Liquid He^3 , with a latent heat of 2.7 K, will refrigerate to 0.3 K. What is needed is a substance with a finite vapor pressure at absolute zero--an impossibility unless one remembers that an ideal Fermi-Dirac gas exerts a finite pressure (the osmotic pressure) at all temperatures. If He^3 can be persuaded to "evaporate" into He^4 , it will form an ideal Fermi-Dirac gas, and the "evaporation" can provide cooling. It is now the latent heat, rather than the vapor pressure, that is zero at absolute zero.

Consider a two phase He^3 - He^4 mixture near absolute zero, with virtually pure liquid He^3 floating on top of a 6 percent mixture of He^3 in He^4 , close to the solubility limit. A He^3 atom can then dissolve from the concentrated liquid He^3 into the ideal gas of quasiparticles. Because the entropy per He^3 atom is increased in the dilute phase, heat must be absorbed (in an isothermal process). The Fermi temperature of concentrated

liquid He³ is 1.62 K; below a few tenths of a degree, pure He³ is highly degenerate, with a very low entropy. The only disorder comes from the region within kT of the Fermi surface. The He³ in the dilute solution, on the other hand, has a much larger entropy per atom--a consequence of the fact that it is an ideal gas and is less ordered than a Fermi liquid. Below 100 mK the entropy per atom of the pure liquid He³ is 5 times smaller than that of a 6.4 percent mixture.

If the dilution occurs at a concentration lower than the limiting solubility, the process is irreversible, and heating will occur if a He³ atom dissolves. For this reason, a dilution refrigerator must operate along the phase separation curve at the limit of solubility. Operating along the phase separation curve is equivalent to dilution at constant μ , the chemical potential of either the He³ or He⁴. Below 60 mK, this implies a constant 6.4 percent concentration of He³.

The heat that must be added to a dilute solution when n_3 atoms of He³ dilute across the phase boundary in an isothermal reversible process at constant μ_4 is

$$Q = n_3 T \left(S_d[T, x(T)] - S_c(T) \right)$$

and

$$S_d[T, x(T)] = S_{fd}[T, x(T)] + S_4(T) [1 - x(T)]/x(T) .$$

Here $S_d[T, x(T)]$ is the average entropy per atom in the dilute phase and S_4 , S_{fd} and S_c are the average entropy per atom of He⁴, the ideal Fermi-Dirac gas of He³ quasiparticles, and the concentrated He³ phase respectively.¹

Because of the "etherial" nature of the He^4 , it is permissible to write the total entropy of the solution as the sum of the quasiparticle entropy and the entropy of the He^4 . $x(T)$ is the equilibrium concentration as a function of temperature (Fig. 2). Since S_4 and S_c are much less than S_{fd} , which is proportional to the temperature, the refrigerating power of the dilution process is proportional to T^2 . Clearly, "evaporation" of He^3 atoms will stop once the limiting solubility has been reached, and no further cooling will occur unless some means can be found for "pumping" the He^3 atoms out of the dilute phase. Here the large osmotic pressure and the ideal gas properties of the mixture come to the rescue.

The "pumping system" used to remove He^3 atoms from the dilute solution is shown in Fig. 3. The "mixing chamber," which contains the dilute solution and phase boundary, is connected to a "still" by a tube filled with dilute solution. The still contains a gas-liquid interface at high temperature. In spite of the temperature gradient, the osmotic pressure is constant along the tube, implying that the product xT is constant in the tube. (Actually Fig. 2 shows that xT is a constant above 50 mK and x approaches a concentration of 6.4 percent below 50 mK.) He^3 atoms can be preferentially evaporated from the dilute solution in the still by applying heat.

Consider, for the moment, the effect of a pulse of heat applied to the still. The concentration of He^3 at the gas-liquid interface is lowered. He^4 will superflow around each He^3 as the quasiparticles move toward the still under a concentration gradient. The product xT is readjusted along the tube, and the concentration of He^3 at the mixing chamber phase

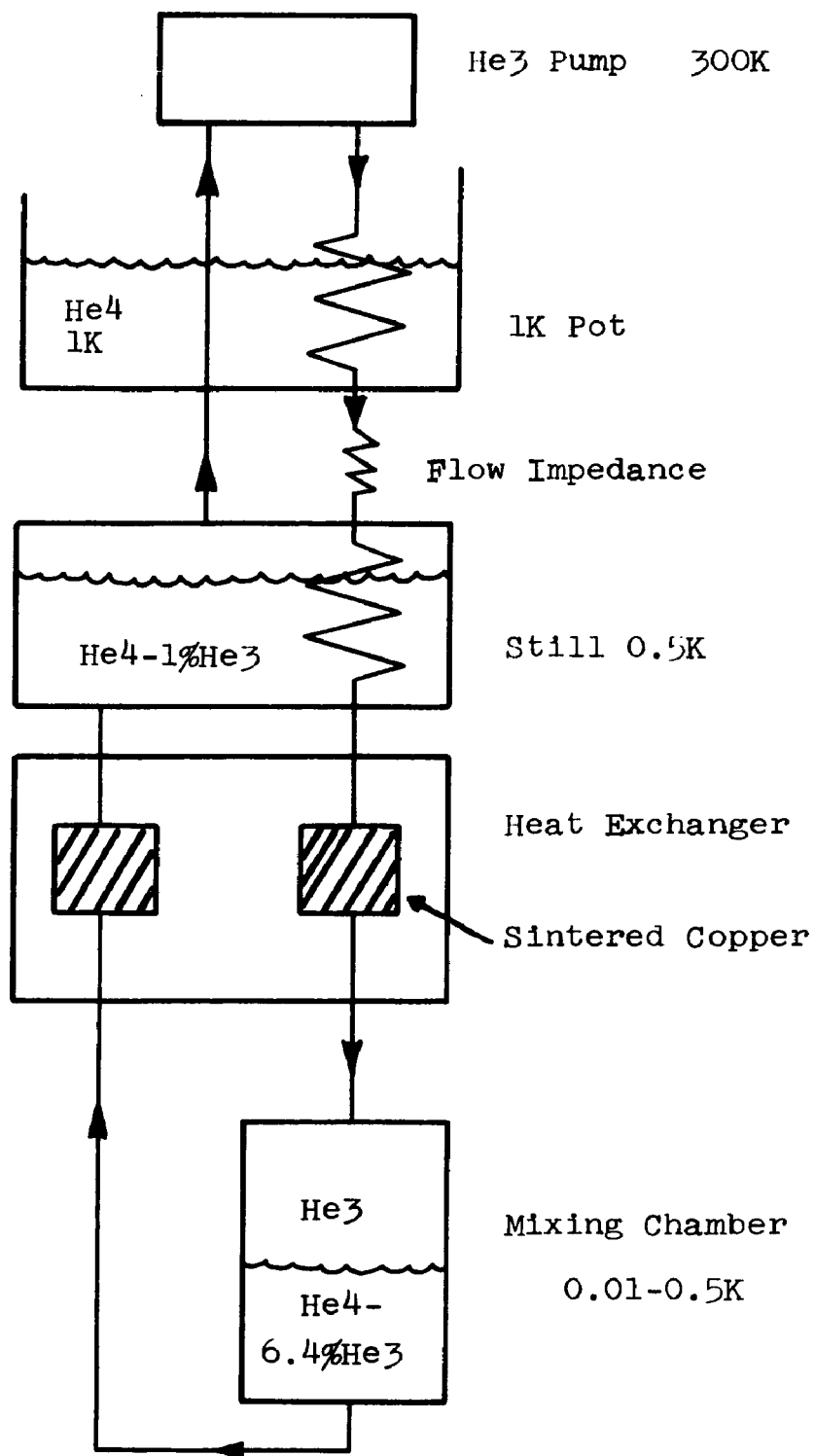


FIG. 3 SCHEMATIC DIAGRAM OF A DILUTION REFRIGERATOR.

boundary is decreased. He³ atoms are then able to dilute from the concentrated phase again, and x returns to its initial value at each point along the tube. In actual operation, heat is applied continuously, and the dilution process occurs at constant μ_4 , since for every He³ atom that is evaporated at the still, another He³ dilutes across the phase boundary in the mixing chamber. The column of He³ quasiparticles in the dilute solution connecting the two phase boundaries is in steady flow, carrying He³ away from the mixing chamber up to the still.

If, for example, the mixing chamber is at a temperature of 10 mK, then the concentration at the phase boundary is 6.4 percent while the still, kept at a temperature near 0.5 K by applying power, has a 1 percent concentration. He³ is preferentially evaporated from the still at 0.5 K, since the partial pressure of He³ in the vapor above the still is approximately 1000 times the partial pressure of the He⁴ above the mixture. The evaporated He³ is removed by a room temperature pump, compressed, condensed, and returned via heat exchangers to the virtually pure He³ in the top of the mixing chamber.

It is possible to operate the dilution refrigerator in a transient mode in which the He³ is not returned. The dilution process then no longer need cool the warm He³ returning to the mixing chamber, and lower temperatures can be reached for a short period of time. A numerical evaluation of the quantity $[S_d(T, x(T)) - S_c(T)]$ gives the refrigeration in transient operation to be

$$\dot{Q} = 10 \dot{n}_3 kT^2 = (10^4 \text{ erg/sec/K}^2) T^2 ,$$

where He3 is being removed from the mixing chamber at a rate $\dot{n}_3$ of 10^{-5} mole/sec. This is a useful equation for both single shot and continuous operation, because in practice the efficiency of the heat exchangers, and therefore the temperature of the returning He3 , is not known a priori.

In a continuous mode, when the returning He3 must be cooled, the refrigeration available is:

$$\dot{Q} + \dot{n}_3 [u_{3c}(T_h) - u_{3c}(T)] = \dot{n}_3 T [S_d(T, x(T)) - S_c(T)] ,$$

where $\dot{Q}$ is the refrigeration power, $\dot{n}_3$ is the He3 circulation rate, u_{3c} is the internal energy of the concentrated He3 , T_h is the temperature of the last heat exchanger, and T is the temperature of the mixing chamber.

In typical refrigerators, the He3 circulation rate is between 10^{-5} and 10^{-4} mole/sec--a number determined by the speed of available room temperature pumps and practical dimensions for components in the dilution stage. Performance data, from several different refrigerators operating in this range of $\dot{n}_3$, indicate a minimum temperature of 10 mK can be reliably obtained in a continuous mode of operation, while temperatures on the order of 5 mK may be reached by a single shot device. The dilution refrigerator described in Part II produces 5 erg/sec usable refrigeration at 40 mK and 1,000 erg/sec at 100 mK.

The intrinsic limitation of the dilution process appears to be viscous heating in the dilute solution. Calculations by J. C. Wheatley indicate that viscous heating in the cold dilute mixtures, at a circulation rate of 10^{-5} mole/sec, limits the refrigerator to temperatures of the order

of 5 mK or higher.² The viscous heating may be reduced by increasing the diameter of the tube that goes to the still; however, improvements in viscous heating with larger tubes will be balanced by the increased thermal conductivity down the tube. (In addition, the dilute mixture in the tube to the still is convectively unstable since the lower concentration mixture in the still is heavier than the 6.4 percent solution in the mixing chamber. The tube size must be small enough to prevent the dilute solution from turning over.)

The dilution refrigerator is a powerful method for obtaining temperatures of the order of 10 mK while extracting relatively large heat loads. Ultra-low temperatures in the sub-millikelvin range require other methods such as compressional cooling of He^3 , or nuclear cooling--both of which rely on the dilution refrigerator to provide a heat sink at 10 mK for successful operation.

Refrigerator Design

The remainder of this chapter will outline the basic elements of a dilution refrigerator. The mixing chamber, still, and four or more heat exchangers comprise the dilution stage, and all require special attention to maximize heat transfer in light of the Kapitza resistance. The mixing chamber is constructed of copper and has a large quantity of copper powder sintered into the bottom half of the interior, to afford good thermal contact to the mixture. The concentrated He^3 is returned at the top of the mixing chamber, and the dilute solution is drawn off at the bottom. The experiment is usually appended to the outside of the mixing chamber.

The cold He^3 quasiparticles flow through a series of heat exchangers, which cool the warm He^3 returning to the mixing chamber. The heat exchangers, currently in vogue in many laboratories, are copper blocks with two unconnected holes packed with copper powder--one for the dilute mixture and the other for concentrated He^3 . The powder provides the large surface area and is sintered to obtain good thermal contact between the powder grains and to the block itself. Heat exchanger design must maximize phonon transfer and He^3 -phonon interaction while minimizing viscous heating.

The still is responsible for fractionating the He^3 from the dilute solutions and for precooling the hot returning, concentrated He^3 , and provides a convenient 0.5 K thermal anchor. Because power is applied to the still under normal operating conditions, a large heat load on the still is tolerable. Any He^4 which is circulated along with the He^3 has an undesirable effect on the refrigerator performance. If circulated in significant quantities, the He^4 will phase separate in the concentrated return line, releasing a heat of separation. This heat of separation must then be removed in the heat exchangers. Once He^3 is evaporated from the still, it is pumped at room temperature, compressed, precooled by a 4 K bath, and finally condensed in a He^4 bath known as the 1 K pot.

Since the dilution process is carried out at very low external pressure, the returning He^3 must flow through a flow-limiting impedance, in order to raise the pressure in the condensing line above the vapor pressure of He^3 at 1 K. Otherwise the returning gas would condense at the still, depositing a large latent heat. Concentrated He^3 then returns to the top of the mixing chamber via the still and heat exchangers to repeat the cycle.

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2. W. R. Abel and J. C. Wheatley, Phys. Letters 27A, 599 (1968).

PART II

CONSTRUCTION AND OPERATION OF A DILUTION REFRIGERATOR

CHAPTER IV. THE MICHIGAN DILUTION REFRIGERATOR

Introduction

The Michigan dilution refrigerator ran successfully for the first time in April of 1968, attaining a temperature of 10 mK, as measured by a magnetic thermometer. This early version (hereafter known as Mark I) had many problems related to its mechanical reliability and stability of operation. Attempts to operate the Mark I refrigerator a second time were frustrated by a series of leaks that defied repair.¹ The following two years were spent in rebuilding the dilution stage, designing experiments to be carried out on the refrigerator, and acquiring a background in ultra-low temperature techniques. The Mark I refrigerator is described in Appendix A and will be mentioned when pertinent to the discussion.

The refrigerator has subsequently undergone several stages of modification, with the current edition labeled Mark II for convenience. This chapter describes the construction of the Mark II refrigerator. Refrigerator operation and thermometry will be discussed in the next chapter. Desirable features, as well as possible modifications, will be mentioned when appropriate in these two chapters. An annotated check list for refrigerator operation is presented in Appendix B.

A program in ultra-low temperature physics requires much more than an operating refrigerator. The success of an experimental program demands the development of a set of techniques and materials to cope with the problems of the millikelvin temperature range. These problems generally fall into two categories--thermometry and thermal contact and insulation.

The time-honored technique for thermometry in the millikelvin region is the measurement of the susceptibility of a paramagnetic salt which obeys Curie's law. This method places a great constraint on the materials used for cryostat construction. Problems connected with temperature measurement have continued to plague the ultra-low temperature program at Michigan to this date. The discussion of temperature in the next chapter will outline progress that has been made in thermometry. Appendix C describes a new ac Hartshorn mutual inductance bridge designed and built for the dilution refrigerator at Michigan.

It should be mentioned that many of the observations on refrigerator performance are based on an educated guess. Preparing for each experimental run and operating the refrigerator require a major effort. This has been particularly true for the Michigan refrigerator since the program has been carried out singly since 1969. Consequently, many modifications were made prior to each experimental run, and a systematic investigation of each change was often not possible.

The Dilution Stage

The dilution stage--comprising the mixing chamber, heat exchangers, and still--is a modular unit connected by an indium O-ring at the still to the He³ pumping line. The dilution-stage components are designed to maximize heat transfer at low temperatures, and make use of sintered copper powder to provide a very large surface area.

The mixing chamber (Fig. 4) is machined from OFHC copper and has approximately 6 g of copper powder sintered into the bottom half of the

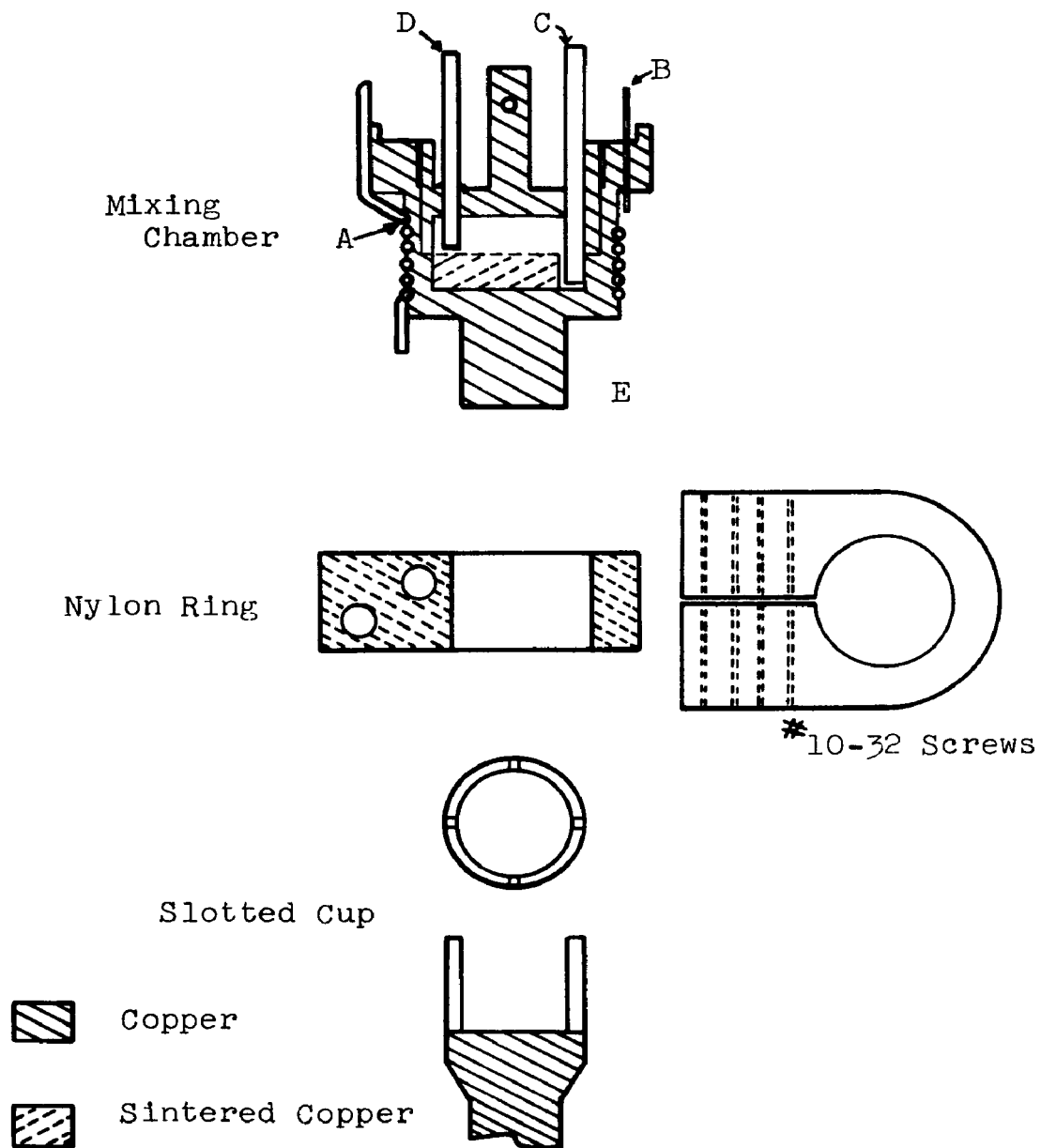


FIG. 4 THE DILUTION REFRIGERATOR MIXING CHAMBER, THERMAL CONTACT CUP, AND NYLON CRIMP RING. A--CELL PRECOOL LINE, B--THERMAL ANCHOR FOR ELECTRICAL LEADS, C--TUBE CONNECTING DILUTE PHASE, D--CONCENTRATED He3 RETURN LINE, E--THERMAL CONTACT POST.

interior. The amount of He³ used in the refrigerator must be adjusted to keep the phase boundary out of the sintered copper, to prevent excessive viscous heating. The copper powder is No. 325 mesh (44 micron average size) and is packed in layers and compressed at 300 atmospheres with an arbor press. The sintering process is described in Chapter VI. The total liquid volume of the mixing chamber is 2 cm³. Thermal contact to the experimental cell is made through a gold-plated copper post (1/2 inch diameter x 1/2 inch long) machined on the outside of the mixing chamber.² A slotted copper cup fits over the post and a nylon crimp ring clamps the cup to the post. Differential contraction upon cooling applies pressure to the joint. A solid ring, designed to be a push fit at room temperature, proved very difficult to assemble. In our present design, the nylon ring is split and is tightened at room temperature using two No. 10-32 nylon screws. The fill line for the experimental cell is cooled in a 75 cm coil of copper tubing 0.020 inch o.d. by 0.008 inch i.d., hard-soldered into grooves cut on the outside of the mixing chamber. This method provides adequate thermal anchoring even at the lowest temperatures, since there is no mass flow in the tube once the cell is filled. The coil presents a high impedance to the flow of gas in the fill line and effectively shunts the heat leak in the fill line to the mixing chamber.

Electrical leads, including a coaxial lead for an electrometer, are thermally anchored to the mixing chamber. These leads are soldered to short lengths of No. 16 copper wire, which are epoxied in holes drilled in the rim of the mixing chamber. A portion of the outer conductor of the coaxial line is removed, and the center conductor is epoxied to the

mixing chamber. Two carbon resistance thermometers are also mounted on the mixing chamber. A 100 ohm Speer resistor is epoxied into a hole in the rim, while a 220 ohm Speer, used in the Mark I refrigerator, is epoxied on the side. Table I summarizes data for the mixing chamber and other refrigerator components.

The heat exchangers for the Mark II refrigerator (Fig. 5) are made from solid blocks of OFHC copper. Two blind holes, cut with an end mill, are packed with four or six layers of presintered copper felt forced into the holes to provide thermal contact between the liquid and the copper block.³ The felt pads are not sintered to the heat exchanger, but are cut slightly oversized and packed into the heat exchanger on an arbor press. (This procedure may not provide adequate thermal contact between the pad and copper body.) A fine mesh, copper screen is placed in the bottom of each heat exchanger to encourage a uniform flow of the liquid through the packed copper felt. The heat exchange sections are closed with copper caps soldered into place. Holes are drilled in the cap and body of the heat exchangers for bushings to connect the inlet and outlet tubes. The heat exchangers are mounted to a central nylon support rod with brass brackets. A 220 ohm Speer resistor is epoxied in a hole in each exchanger body. A copper block, coated with Apeizon N grease, can be fastened to the side of each exchanger to anchor the fill line and coax. The Mark II refrigerator has 6 heat exchangers; the top four exchangers each have four felt pads compressed into each side. Each section in the dilute side has 3.4 g of copper felt and 0.42 cm³ liquid volume, with a packing fraction (volume of copper/total volume) of 48 percent. The concentrated side has

Table 1

Refrigerator Design Data

I. Mixing Chamber

Total volume	2.7 cm ³
liquid	2.0 cm ³
copper powder	0.7 cm ³ (lower half interior)
Post for heat exchange	1/2" dia. x 1/2" long
Fill line precool coil	75 cm long x 0.008" i.d. Cu
volume of coil	2.4 x 10 ⁻² cm ³
impedance of coil	2.0 x 10 ¹⁰ cm ⁻³
Heater--0.003" dia. wire	200 ohms (Evanohm)
Powder used in sintering	325 mesh 99.9% pure Cu

II. Heat Exchangers

Two largest exchangers

Dilute side	1.9 cm ³
liquid	1.1 cm ³
copper felt	0.8 cm ³ --7.2 g
impedance factor	0.35 x 10 ⁸ cm ⁻³ (approx)
Concentrated side	0.3 cm ³
liquid	0.17 cm ³
copper felt	0.13 cm ³ --1.1 g
impedance factor	1.5 x 10 ⁸ cm ⁻³ (approx)

Four small exchangers

Dilute side	0.8 cm ³
liquid	0.42 cm ³
copper felt	0.38 cm ³ --3.4 g
impedance factor	0.3--1.0 x 10 ⁸ cm ⁻³
Concentrated side	0.2 cm ³
liquid	0.1 cm ³
copper felt	0.095 cm ³ --0.85 g
impedance factor	1.5--3.0 x 10 ⁸ cm ⁻³

Table I (cont.)

Connections	
Dilute side--3 cm of 1/16" Cu-Ni tubing between each exchanger, approx. 25 cm total with 0.4 cm ³ estimated volume	
Total volume of dilute side--3.9 cm ³	
Concentrated side--5 cm of 1/64" Cu-Ni tubing between each exchanger, total length of concentrated side (including pot)--1 meter	
volume of tubing 0.02 cm ³	
Total volume of concentrated side--0.75 cm ³	
III. Still	
Total internal volume	10.7 cm ³
liquid	8.0 cm ³
copper powder	2.7 cm ³ (lower half interior)
Precool coils	
(He3 return and cell fill line each)	
	75 cm long x 0.008" i.d. Cu
volume of coil	2.4 x 10 ⁻² cm ³
impedance factor	2.0 x 10 ⁸ cm ³
Glass orifice	0.052" dia (fire-polished)
IV. Flow limiting impedance	7.0 x 10 ¹⁰ cm ⁻³
V. Liquid to fill dilution stage	
(assuming dilute solution occupies half of the total volume of both still and mixing chamber)	
Dilute side	9.9 cm ³
Concentrated side	1.8 cm ³
Liquid charge	1.3 NTP liters He3
	8.7 NTP liters He4
Room temperature gas	0.2 NTP liters He3
VI. Volume of storage tanks	6.05 liters
VII. Volume of 1 K pot	0.35 liters (36 hour hold time)

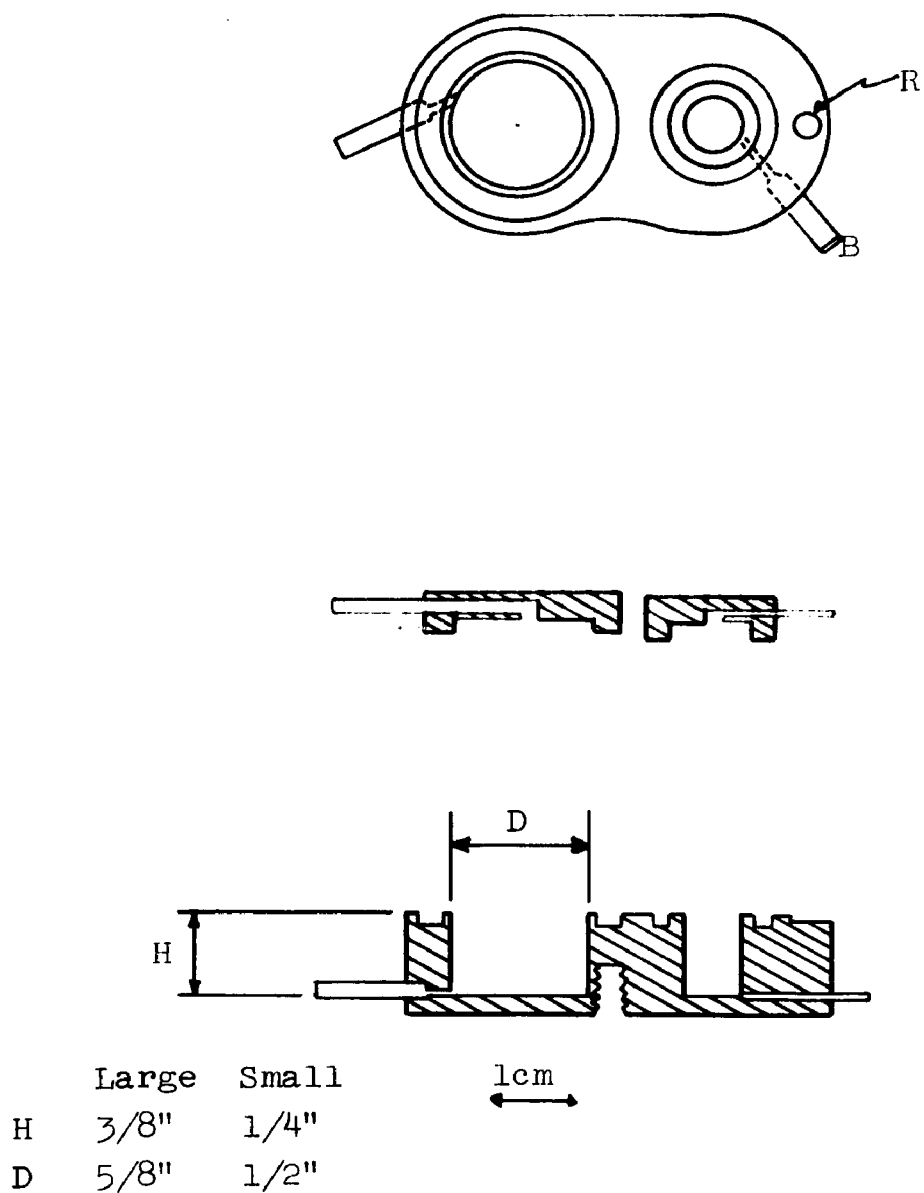


FIG. 5 THE DILUTION REFRIGERATOR HEAT EXHCANGERS BEFORE THE SINTERED COPPER IS ADDED. DIMENSIONS ARE GIVEN FOR THE LARGE AND SMALL EXCHANGERS. R--HOLE FOR RESISTOR, B AND C--BUSHINGS FOR CONNECTING TUBES.

1/4 the volume of the dilute heat exchanger and has the same packing fraction. The two coldest heat exchangers have larger volumes, to compensate for the increased Kapitza resistance at low temperatures. The dilute side, on each of the two coldest exchangers, contains 7.2 g of copper felt and has a 1.1 cm^3 liquid volume. The concentrated side has approximately 1/6 the volume of the dilute side, with the same packing fraction as before. The tubing that connects the dilute phase on the mixing chamber to the still is 1/16 inch o.d., Cu-Ni, with 3 cm of tubing between adjacent heat exchangers. The concentrated side is connected using 1/64 inch Cu-Ni tubing--5 cm between heat exchangers. The flow impedance of each side is in the range of 10^8 cm^{-3} .

The still (Fig. 6) is machined from an OFHC copper block and has approximately 24 g of copper powder (No. 325 mesh), sintered in the bottom half of the interior. The still has a liquid volume of 8 cm^3 . Two sets of precooling coils--75 cm of 0.008 inch i.d. copper tubing each--are hard-soldered into threads cut on the outside of the still.

One coil cools the returning concentrated He3, while the other thermally anchors the experimental fill line. (The flow impedance of these lines is $2 \times 10^{10} \text{ cm}^{-3}$.) A 225 ohm Evahohm resistance wire is varnished to the outside of the still with GE-7031 and serves as the still heater. A copper disc, coated with Apeizon N grease, is clamped to the bottom of the still, to provide a thermal anchor, and support the dilution stage. Short lengths of No. 16 copper wire are epoxied into holes in the plate for thermal grounding of the lead.

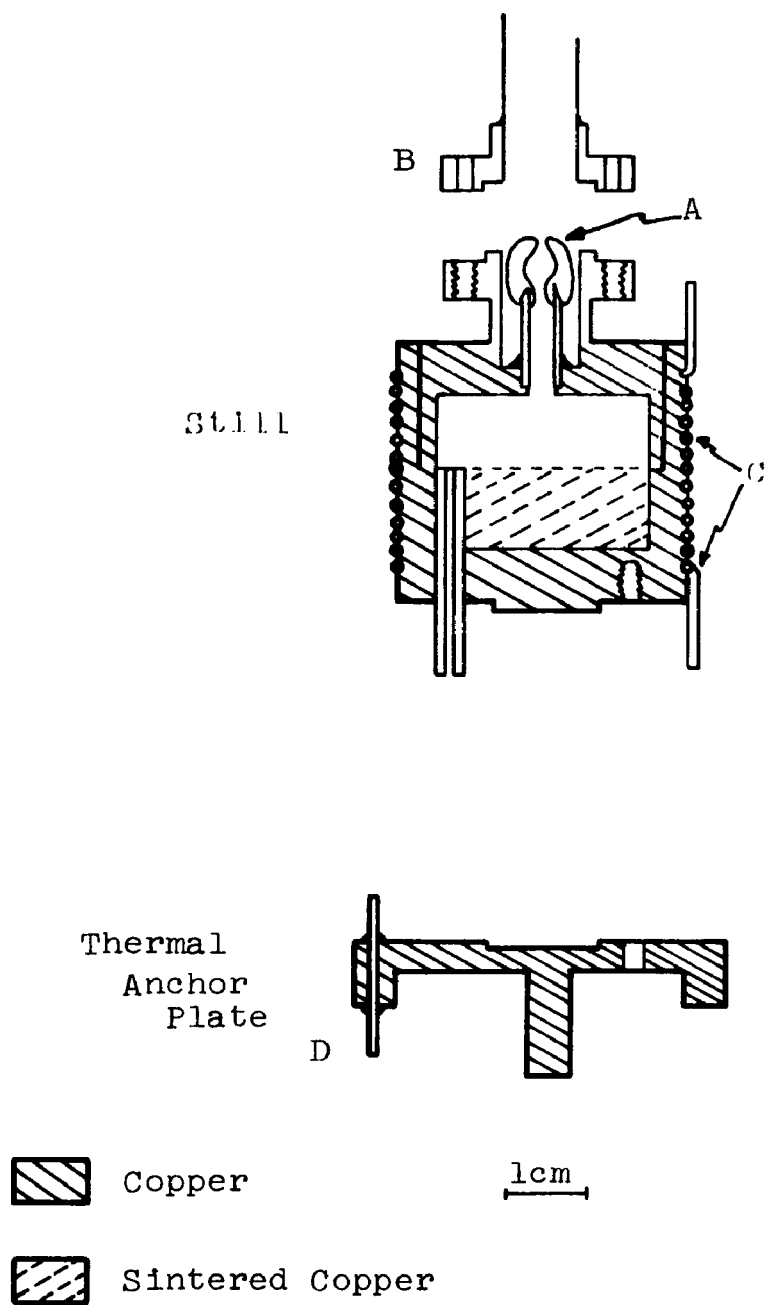


FIG. 6 DILUTION REFRIGERATOR STILL. A--GLASS PUMPING ORIFICE, B--BRASS FLANGE FOR INDIUM O-RING, C--PRECOOL COIL (ONLY ONE SHOWN), D--THERMAL ANCHOR FOR ELECTRICAL LEADS.

Any He⁴ circulating along with the He³ has a deleterious effect on the performance of the refrigerator. The superfluid film flow up the pumping tube is the dominant factor in determining He⁴ circulation. (The equilibrium concentration of He⁴ in the vapor above the still is only 0.1 percent at 0.5 K.) Mass transport in the film flow is determined by the minimum periphery that the film must traverse. An orifice is used to limit film flow. Because it is the length of the perimeter on a microscopic scale that is important, the edge of the orifice must be extremely smooth. A 0.050 inch diameter orifice was made by a glassblower in the end of a glass-copper "Housekeeper" seal and carefully fire-polished for maximum smoothness. The copper portion of the seal was then soft-soldered into the top of the still, with the glass orifice extending inside the 3/8 inch Cu-Ni pumping line. The orifice was very successful at first, limiting the He⁴ circulation to less than 5 percent. In subsequent runs this performance deteriorated, perhaps as a result of pump oil deposited on the clean glass surface. The still of the Mark I refrigerator contained a baffle and heater attached to a short 1/8 inch pumping tube designed to evaporate the He⁴ film before it reached the orifice. While the baffle proved ineffective, the 1/8 inch tube allowed the still to operate at 0.35 K, and the low ultimate temperature of the Mark I refrigerator may have been due, in part, to this low still temperature. About 20 percent He⁴ was circulated with this design.

To condense the returning He³ in the pot at 0.9 K, the pressure in the He³ return line must be greater than 5 Torr. Since this pressure would otherwise be determined by the vapor pressure in the still (zero

for all practical purposes) plus any viscous pressure drop in the heat exchangers, a flow-limiting impedance must be placed between the pot and the still to raise the pressure above 5 Torr. The viscous pressure drop Δp across an impedance can be written in the form

$$\Delta p = \dot{m} z \nu ,$$

where $\dot{m}$ is the mass flow rate, ν is the kinematic viscosity, and z is a geometric factor, independent of the material (equal to $8L/\pi a^4$ for a uniform tube of length L and radius a). The required impedance z can be calculated from the desired circulation rate and measured experimentally at room temperature. Several different types of flow-limiting impedance were tried; the most reliable was made from a 1/32 inch Cu-Ni tube, packed with copper powder, and sintered. Construction details are given in Chapter VI. An impedance of $7 \times 10^{11} \text{ cm}^{-3}$ is used in the Mark II refrigerator.

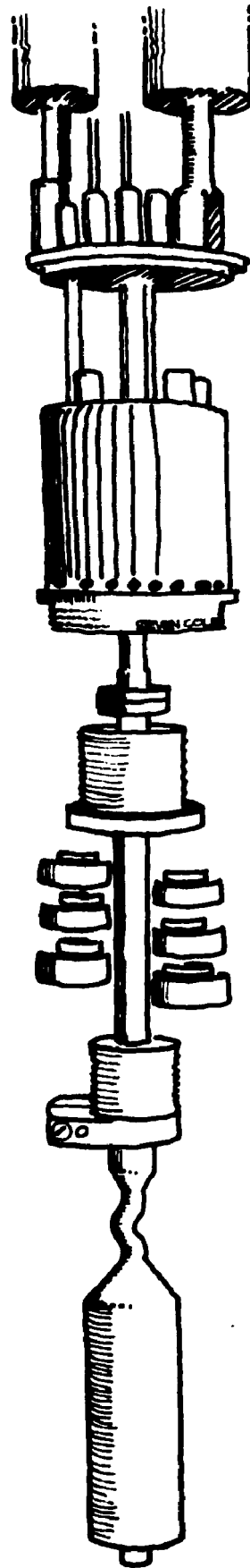
The impedance is located between the pot and still, and its temperature is not well defined. If the temperature is too high, the liquid in the line between the still and the impedance can phase separate and form a gas bubble. However, the precooling coil on the still has a flow impedance of $2 \times 10^{10} \text{ cm}^{-3}$, and approximately 1/6 of the total condensing pressure appears across the precooling line on the still. The vapor pressure of He3 at the still temperature is approximately 0.1 Torr, and the viscous pressure drop in the remainder of the dilution stage is sufficient to prevent the formation of a gas bubble below the still.

The dilution stage is supported by a 1/2 inch nylon rod, clamped to the bottom of the still thermal-anchor plate. The heat exchangers are mounted to the rod with small brass clamps; the mixing chamber is fastened to the lower end of the rod. The Mark I refrigerator used a graphite rod for thermal isolation. This rod was replaced with nylon after breaking several times in assembly. Nylon, however, does not provide a sufficiently rigid support, and produced a severe vibration problem. A complicated mechanical anchor was built to clamp the mixing chamber to the radiation shield (see below). It consists of two stages of nylon rods connected in series. The junction between the two stages is thermally anchored to the still to reduce the heat leak. An expansion joint, made with banana plugs, allows for differential contraction between the nylon support rod and the copper radiation shield.

The total liquid volume in the dilution stage is 11 cm^3 when the still is half-filled with liquid. If the phase boundary between the concentrated He^3 and the dilute solution is half-way between the top of the mixing chamber and the sintered copper plug, the volume of liquid He^3 needed to fill the dilution stage is 1.85 cm^3 while approximately 9.3 cm^3 of He^4 is needed. The volume of the mixing chamber must be large enough to retain the dilution phase boundary, for reasonable variations in the amount of condensed He^3 . A large still volume makes operation of the refrigerator insensitive to the total amount of liquid used.

Cryostat

Figure 7 shows the lower end of the dilution refrigerator. (The vacuum can and radiation shield are not shown.) The 1 K pot, made of copper, contains a 350 cm^3 bath of liquid He^4 to condense the returning He^3 and provide a thermal anchor at 1 K. The bath is pumped through a $3/8$ inch Cu-Ni tube with a $1/8$ inch brass orifice to limit film flow. The He^3 condensing line, and experimental cell fill line--both 80 cm of $1/64$ inch Cu-Ni tubing--are coiled inside the pot. The condensing lines, as well as a coax for the experimental cell, enter and leave the pot through brass "castles" (Fig. 14). The use of castles allows a line to be soldered into place without reheating other solder connecting to the pot. The pot provides mechanical support and thermal anchor for the $3/8$ inch Cu-Ni still pumping line. The bottom of the pot is threaded to accept a copper radiation shield, which encloses the dilution stage. The tail section of the radiation shield is made of coil foil⁴ to reduce eddy-current heating from the susceptibility coils, which are positioned on the narrow tail section of the vacuum can that covers the experiment and tail of the radiation shield. (Except in calorimetry experiments, the radiation shield is of doubtful value, as the radiation heat leak is only a few erg/sec.) The manganin electrical leads are varnished to the outside of the pot for thermal anchoring. The pot is filled through a valve on the "light trap" (see below).⁵



DILUTION
CRYOSTAT

FIG. 7

Vacuum Can and Light Trap

The dilution stage, experiment, and 1 K pot are enclosed in a vacuum can for thermal isolation. The can is made entirely of brass, and reduces to 2 inch o.d. at the level of the experimental cell, to accept a coil set for susceptibility thermometry. The vacuum can is sealed with an indium O-ring to a flange--the "light trap"--through which all connections to the outside world are made. The pumping lines that come through the light trap from room temperature are offset in this flange, to stop 300 K black-body radiation inside the pumping lines. The electrical leads, a coax, and various fill lines enter the vacuum can through castles on the light trap.

Pumping Head, Lines, and Dewar

Four large, thin-walled, stainless steel lines connect the light trap to the top flange of the cryostat (the Pumping Head). They are: (1) a 2.0 inch x 0.020 inch wall He3 circulation line, (2) a 1.5 inch 1 K pot line, (3) a 3/4 inch isolation vacuum line, (4) a spare 1.5 inch line to be used in the future for a second stage dilution refrigerator. These diameters were chosen to prevent the pumping lines from limiting the speed of the diffusion pumps. Since the conductance of a tube increases at lower temperatures, the diameter of each line at the cold end is larger than necessary. The large pumping lines are 52 cm long and reduce to 5/8 inch (or 1/2 inch) lines 9 cm above the light trap.

To utilize the cooling power of the evaporating helium in the Dewar, five copper baffles are soldered 5 cm apart on the pumping lines. A 77 K precool plate is soldered to the lines just below the pumping head. Liquid nitrogen can be circulated through a 1/4 inch tube on this plate.⁶

The He3 return line (1/8 inch Cu-Ni tube) is thermally anchored to the 4 K bath with a coil of copper tubing (0.080 inch o.d. x 75 cm long) wrapped around the light trap. Similarly, the experimental fill line, 1/32 inch Cu-Ni tubing, is anchored at 4 K by a 50 cm coil of 0.060 inch o.d. copper tubing before entering the vacuum can through a castle. Care was taken in the experimental fill line to avoid a local low spot in which He3 could collect. Both lines had sintered copper filters, located part way down the Dewar, to trap impurities before they could enter the narrow capillaries in the vacuum can. (The internal volume of the filter in the cell fill line was 0.06 cm³.) The construction of lead tubes and electrometer coax is described in Chapter VI. The pumping head provides connections to the external plumbing systems, and has the usual provisions for electrical leads, transferring liquid He, cooling the main helium bath, etc. Valves, on the pumping head, seal each cryostat line to retain vacuum when the refrigerator is removed from the Dewar.

The liquid helium Dewar (glass) is 4 feet deep and 5 inch i.d., reducing to 4 inch i.d. below the light trap; it requires two transfers per day during a run. The volume of liquid helium above the light trap is marginal for 12 hours of continuous He3 circulation; a 5 foot Dewar would be more suitable. Both the helium and nitrogen Dewars went

"soft" after three years of operation, and vacuum lines were added to each Dewar.

The Dewar stand is mounted on poured concrete blocks set in the sand beneath the floor of the room, to reduce building vibrations. Vacuum lines from the mechanical pumps are anchored to the wall of the room, through vibration eliminators. Additional vibration isolators decouple the Dewar from the wall. The vibration isolation proved inadequate; the amplitude of low frequency vibrations on the Dewar stand was several times that of the floor or ceiling in the room.

Gas Handling System

The external gas handling system (Figs. 8 and 9) is divided into three sections: a He³ circulation system, a He⁴ pumping system for the 1 K pot, and a house vacuum and leak detector. Booster diffusion pumps for the He³ and He⁴ systems are mounted on the Dewar stand close to the pumping head.⁷ Backstreaming of the diffusion pump oil (Narcoil 10) is so low that the 30 cm of 7 cm i.d. room temperature pumping line serves as an adequate "cold trap." The booster pumps are capable of a throughput of 5×10^{-5} mole/sec of He³. The He³ booster pump is backed by a 5cfm mechanical pump,⁸ sealed to prevent the loss of He³.

The He³-He⁴ operating mixture is stored in two 6.05 liter tanks (tanks Nos. 1 and 2 in Fig. 8), the original He³ bottle (tank No. 3), and the exhaust side of the sealed mechanical pump. He³ is returned to the Dewar through a 77 K cold trap filled with molecular sieve. A

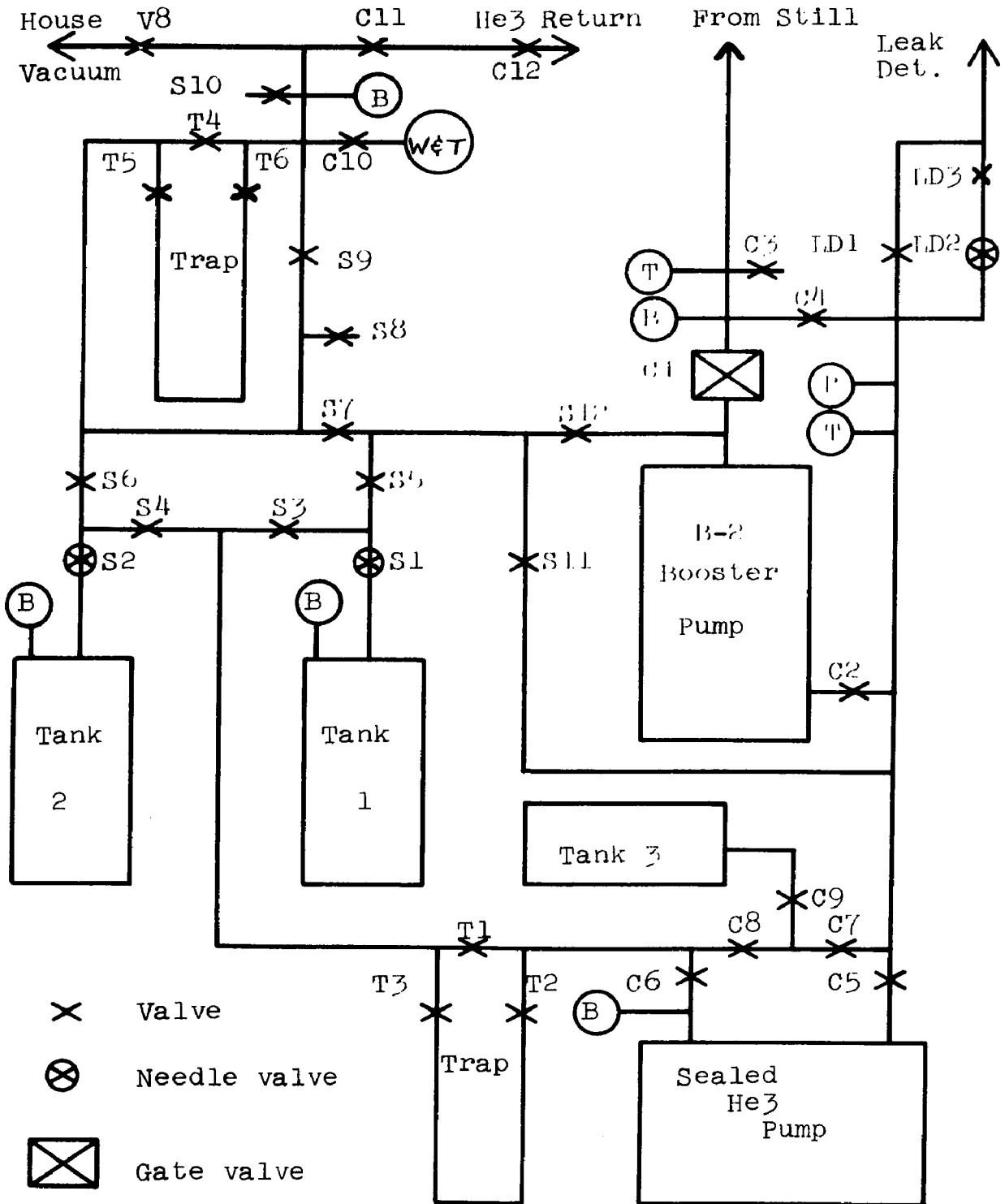


FIG. 8 He3 CIRCULATION AND STORAGE SYSTEM. B--BOURDON GAUGE, T--THERMO-
COUPLE GAUGE, P--PIRANI GAUGE, WT--100 TORR PRECISION BOURDON,
WALLACE AND TIERNAN FA 160.

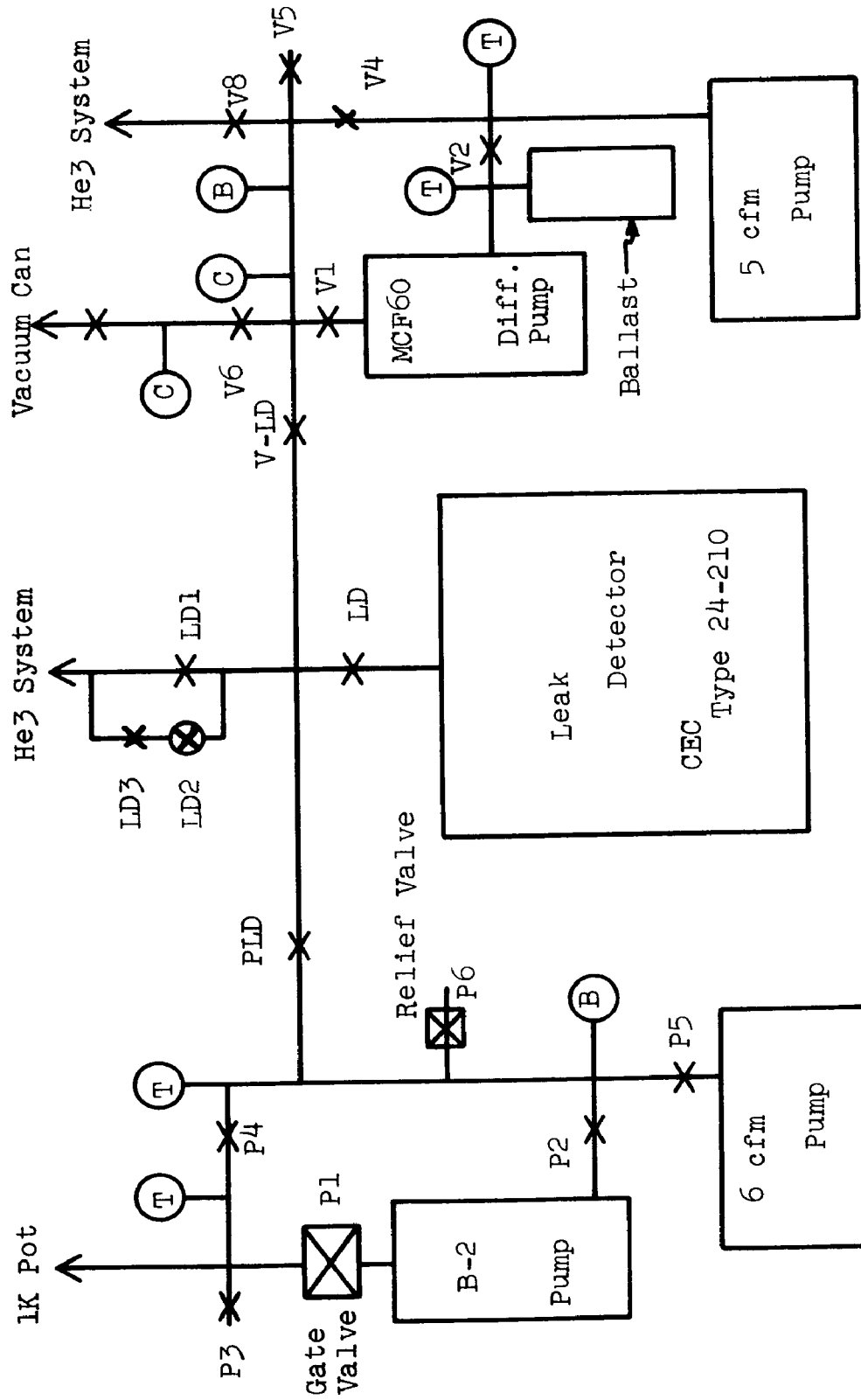


FIG. 9 He4 AND HOUSE VACUUM SYSTEMS. B--BOURDON GAUGE, T--THERMOUCOUPLE GAUGE, C--COLD CATHODE (PHILLIPS) GAUGE.

second cold trap removes oil vapor from the exhaust of the sealed mechanical pump. The house vacuum is provided by a 5 cfm mechanical pump and 60 liter/sec diffusion pump. Smaller capacity pumps proved inadequate. The function of this system is to provide the isolation vacuum in the vacuum can, although it is also used for numerous house-keeping tasks. A leak detector, permanently connected to the refrigerator, is used to check each system at 300 K, 77 K, and 4 K, and to measure the He3-He4 ratio in the circulation system. The leak detector is left on the isolation vacuum during a run, to indicate a malfunction. A precision Bourdon gauge measures the condensing pressure, and is useful in determining when the refrigerator has reached steady-state operation. Inexpensive Bourdon gauges and thermocouple gauges measure pressure at appropriate locations. Unfortunately, neither style gauge offers sufficient sensitivity for the quantitative measurements needed in refrigerator operation. The circulation rate is measured by a Pirani vacuum gauge,⁹ at the intake of the sealed He3 pump. (The pressure times the pump displacement gives the throughput or circulation rate.) While the Pirani gauge has excellent sensitivity and proved extremely valuable in general refrigerator operation, the control circuit did not have the long-term stability needed to compare circulation rates from different runs.

REFERENCES AND NOTES

1. The leaks were traced to hydrogen embrittlement in OFHC copper and the use of a Cd-Bi solder, with a very low superconducting transition, but poor mechanical strength.

See: W. Steyert, Jr., Sci. Instr. 38, 964 (1967)
2. R. I. Boughton, N. R. Brubaker, and R. J. Sarwinski, Rev. Sci. Instr. 38, 1177 (1967).
3. Presintered copper felt, initial density 20 percent, FM1006, Huyck Metals, Inc., Milford, Conn.
4. A. C. Anderson, G. L. Salinger, and J. C. Wheatley, Rev. Sci. Instr. 32, 1110 (1961).
5. The valve is of rather complicated design, and could easily be replaced with a valve made from the body and stem of a common "Hoke" needle valve. Such valves are now in common use at Michigan. The valve does not need to be superleak tight.
6. While the precool plate reduced the heat load on the bath by a third, the cost of liquid nitrogen used is comparable to the savings in helium. Space limitations in the Dewar prohibited a liquid nitrogen pot of reasonable volume on the pumping line.
7. NRC Equipment Corp., Newton, Mass., B-2 booster diffusion pump, Type 123 with a speed of 80 liters/sec at 10^{-3} Torr and 15 liters/sec at 0.1 Torr. These excellent pumps are no longer manufactured.
8. Welch Scientific Co., Model 1402 KBG.
9. CEC, Autovac Gauge, Type 3294B.

CHAPTER V. OPERATION OF THE DILUTION REFRIGERATOR

The Mark II dilution refrigerator is capable of producing temperatures of the order of 50 mK indefinitely. While the ultimate temperature of Mark II has never approached the performance of the Mark I refrigerator, none of the instabilities or long-term temperature drifts inherent in Mark I, or several of the intermediate editions, plague the Mark II refrigerator. Steady-state operation is easily achieved with Mark II, allowing the operator to turn his attention to an experimental problem.

Appendix B presents a detailed outline of the operating procedure for the refrigerator. The first part of the outline covers steps to be taken at room temperature and during precool. While this part may seem unnecessarily detailed, preparations for a run are lengthy and a step omitted at room temperature might easily add several days to a run, if discovered at 4 K.

Leak checking the various systems at each stage of operation is essential. Hydrogen and helium 3 should be used as exchange gas in the liquid nitrogen precool and helium transfer. This procedure will materially reduce the time to evacuate the vacuum can; in addition, a small background of either will not interfere with leak checking. The vacuum can, pot, and both the He3 pumping and return lines are initially connected to the leak detector. Maximum sensitivity is achieved with the booster diffusion pumps on and the mechanical pumps and house vacuum

off. As leak checking proceeds, each line is valved off from the leak detector and helium gas is added, to test for leaks to the remaining systems. It is also desirable to check the value of the flow-limiting impedance in the He3 return line to insure that it has not plugged during precool. This is easily accomplished, as part of the leak checking routine, by admitting a known pressure of He4 in the return line between valves C-11 and C-12. C-12 is opened and the gas on the still side of the flow impedance is pumped into the He3 fore line with the booster diffusion pump. The rate at which the pressure increases on the Pirani gauge indicates the size of the impedance. After this check, the diffusion pump is valved off, and valve C-4 is opened, to allow adequate He4 in the still line for leak checking. He4 cleanup time in each system is long, and the suggested order for leak checking is important.

Leak checking from the experimental cell to the vacuum can is made difficult by the large impedance of the cell fill line. The problem is usually complicated by the fact that the paramagnetic salt in the cell will lose its water of hydration if evacuated at room temperature. A leak from the cell may be found by monitoring the background count on the leak detector when the He4 pressure in the cell is doubled. A similar procedure can be used to insure that the cell fill line is not plugged. The pressure in the cell is increased to 2 atm, and the fill line valve vented (to 1 atm) and quickly shut. The pressure at the Bourdon gauge on the fill line will drop and then gradually rise to half its original value, if the volume of the cell is 10 cm³.

Efficient liquid helium transfers require utilization of the cold He gas to cool the large mass of metal in the Dewar. The transfer

tube fits into an extension, mounted on the cryostat, which carries the liquid helium past the light trap to the bottom of the Dewar. A small funnel, which fits into the transfer bung, aids in mating the transfer tube with the extension. This extension can reduce the amount of liquid helium used by a factor of two in the initial transfer. It is also helpful in subsequent transfers. The 1 K pot is filled during liquid helium transfer, for maximum efficiency. A mechanical pump on the pot will aid the filling process and fill the pot with helium below the lambda point. A manometer on the pumping line indicates when the pot is full, as the pressure above the pot increases abruptly when the liquid level passes the 1/8 inch orifice.

Refrigerator Operation

The operating mixture for the refrigerator is condensed from the storage tanks in the gas handling system. If the storage procedure outlined in Appendix B has been followed, the bulk of the He3 is stored in tank No. 1. This material is condensed first, through the flow limiting impedance; then the remaining gas, stored in tanks No. 2 and 3 and the sealed He3 pump, is condensed. When all tanks are empty, the pot booster diffusion pump is started, and the refrigerator charge cools to 0.9 K. The mixture is then circulated with the sealed He3 pump. At a pressure of 1 Torr, the He3 booster pump can be turned on. In this mode, the still operates as a He3 refrigerator and begins to cool. It is important to have enough liquid He3 in the still to provide the initial refrigeration to phase-separate the mixture. (In the dilution mode, nearly all of the He3 is on the return side of the dilution stage.)

The amount of He3 needed is determined by the requirement that the phase boundary between the dilute mixture and concentrated He3 be in the mixing chamber. The volume of the mixing chamber should be large enough to retain the phase boundary for reasonable variations in the amount of He3 in the dilution stage. These variations occur with changes in the condensing pressure, or simply because the precise volume of the concentrated side is not known. The mixing chamber is large enough to accommodate a ± 20 percent change in the total amount of He3. The amount of gas contained in the external gas handling system, under operating conditions, is of the order of 0.2 NTP liters of He3. This number, together with data from Table 1, indicates that 1.5 NTP liters of He3 is required to position the phase boundary properly in the mixing chamber. (The refrigerator was originally charged with 2.5 STP liters of He3.) Approximately 7.9 NTP liters of He4 is needed, although the large volume of the still provides for a large margin of error. Unfortunately, it is not possible to check the position of the phase boundary visually, and adjustments in the amount of He3 used must be deduced from the operating characteristics of the refrigerator. Because too much or too little He3 places the phase boundary in the same place--the lowest heat exchanger--it is not immediately clear whether one should add or subtract He3 from the system when the refrigerator is operating poorly.

When the still reaches 0.4 K, the still heater can be turned on, and adjusted to the desired circulation rate (2×10^{-4} watts in the still gives a circulation rate of 10^{-5} mole/sec). The heat exchangers will successively cool, as the phase boundary passes through them--propagating

a temperature minimum down toward the mixing chamber. The temperature of the mixing chamber may remain relatively constant and high for an unseemly time, as heat is extracted from the experimental liquid in the cell, and the heat load from the mechanical support is reduced by lowering the thermal conductivity of the support system. Eventually the mixing chamber will cool, when the phase boundary has reached the mixing chamber.

Operating Characteristics

Several hours after the still heater is turned on, the refrigerator reaches its ultimate temperature; it will continue to operate at this temperature for hours, with little attention. A circulation rate of $2-3 \times 10^{-5}$ mole/sec will generally produce the lowest temperature; however, the still power should be varied to confirm this. The time to reach equilibrium, after a change in heat load or circulation rate, depends on the condensing pressure (nominally 70 Torr), which determines the amount of material in the room temperature lines. A 25 percent change in still power requires approximately 40 minutes for equilibrium to be re-established. This is not intolerable, although it could be improved by using a smaller flow impedance. The Mark I refrigerator, with a condensing pressure of one atmosphere, required several hours to reach equilibrium.

The ultimate temperature of the Mark II refrigerator (as measured by the resistor calibrated in the first Mark I refrigerator run) is a rather disappointing 45 mK, compared to 10 mK for the Mark I. While the

resistance thermometers used in the Mark II refrigerator have not been calibrated, enough is known to indicate that Mark II has a problem in this respect. In steady-state operation, the lowest heat exchanger appears to be colder than the mixing chamber. This, coupled with the high ultimate temperature, might indicate:

1. An exceptionally large heat leak to the mixing chamber--this has been investigated and is unlikely.
2. The phase boundary is in the lowest heat exchanger--also unlikely.
3. The extrapolation of the temperature vs. resistance curve of the resistor on the lowest heat exchanger is wrong.
4. Bad thermal contact between the presintered copper felt and the body of the heat exchanger.
5. The sintered copper plug has broken away from the body of the mixing chamber.

Items 4 and 5 are the most likely candidates. These possibilities are substantiated by the fact that the ultimate temperature rose slightly with each succeeding run, as would be case if the bond between the sintered copper and the body deteriorated upon thermal cycling. Clearly, the thermometry must be settled to assess the magnitude of the problem.

The refrigerator operates without much direct operator intervention; however, certain parameters should be monitored often, such as the condensing pressure, pressure in the He3 and He4 fore lines and the vacuum can, and the temperature of the mixing chamber. The liquid helium level in the Dewar should not be allowed to drop below the light trap, while He3 is circulating; in fact, both the liquid helium and liquid

nitrogen levels should be kept high, to prevent impurities frozen at the top of the He3 return line (or cell fill line) from re-evaporating and freezing further down the line, as the liquid level drops. This is a problem during a long run, since the effect is cumulative, and a plug can be formed. A plug in the cell fill line forms a potentially disastrous helium bomb.

Overnight Operation

Once the operating mixture has been condensed, leaving the refrigerator unattended for any extended period presents a problem. The most convenient mode in which to leave the refrigerator is with the mixture circulating under still power. The refrigerator remains cold, and there is no delay in start-up. This mode is hazardous, since any malfunction (e.g., the 1 K pot emptying) will dump the entire refrigerator charge into the small volume of the external gas handling system. Also possible, although not as convenient, is the mode in which the still power and booster pumps are turned off, but circulation is continued with the mechanical pump at a rate determined by the residual heat leak. This mode is subject to the same result in the event of a major malfunction; however, the time constant of the system is longer and the heat load on the pot and bath is greatly reduced. The phase boundary is preserved in the mixing chamber, and start-up requires approximately 90 minutes. Safest, although least desirable, is the mode in which circulation is completely stopped. The phase boundary is not preserved and start-up takes much

longer. Modifications detailed in the last section of this chapter would enable safe, unattended operation under still power.

At the end of a refrigerator run, it is convenient to store the bulk of the He3 in one tank. The return valve C-12 is shut and the He3, fractionated by the still, is stored in tank No. 1. The still power can be increased to speed the removal of He3, and the 1 K pot should be valved off. This procedure will remove most of the He3 in 30 minutes. The He3 booster pump is then by-passed, and He4 exchange gas is added to the vacuum can. The remaining liquid (mostly He4) in the He3 system is stored in tank No. 2. When the He3 system is emptied, tank No. 2 is shut and the gas in the plumbing lines to tank No. 1 and No. 2 is pumped into tank No. 3. This tank and the exhaust valve on the sealed pump are shut, and the gas in the line to tank No. 3 is pumped into the exhaust side of the pump. The 1 K pot is then emptied. Since the impedance of the fill line is large, approximately five hours are required to empty the cell. This process should be started during the storage procedure. The Dewar must be kept at 4 K during this period. After the cell is emptied, it is possible to speed the return to room temperature, with exchange gas in the nitrogen Dewar vacuum space. The house vacuum is used to remove any frozen material in the pot, cell and He3 system. One cautionary note: Do not remove the cryostat from the Dewar until it is warm!

Thermometry

The susceptibility of cerium magnesium nitrate (CMN) was the thermometric property used to measure temperature in the Mark I refrigerator. CMN is a paramagnetic salt, known to obey Curie's law above a temperature of 6 mK. The salt was powdered and packed into a right circular cylinder (with diameter equal to height) in the mixing chamber. The susceptibility of the CMN was measured with an ac Hartshorn bridge, and a mutual-inductance coil set mounted on the vacuum can. Appendix C describes both in detail.

A 220 ohm Speer carbon resistor in the mixing chamber was calibrated against the CMN in the first run.¹ The insulation was ground off the carbon body of this resistor for better thermal contact to the experimental liquid, and the tinning was removed from the copper leads, to improve the magnetic properties of the resistor. Additional Speer resistors on the heat exchangers, still, and pot are used to monitor refrigerator performance. Carbon resistors must be calibrated against a known thermometer--such as CMN--since their resistance is not known a priori as a function of temperature. The Speer resistor calibrated in the mixing chamber during the first run has been used as a thermometer for later work. Subsequent attempts at susceptibility thermometry have been frustrated by magnetically bad materials in the experimental ion cell.

Some time after the initial CMN calibration, it was discovered that the CMN sample² was probably adulterated with cerium magnesium acetate. For this reason there is some uncertainty in the original thermometry, although the sample did obey Curie's law above 1 K. Figure 10 shows the

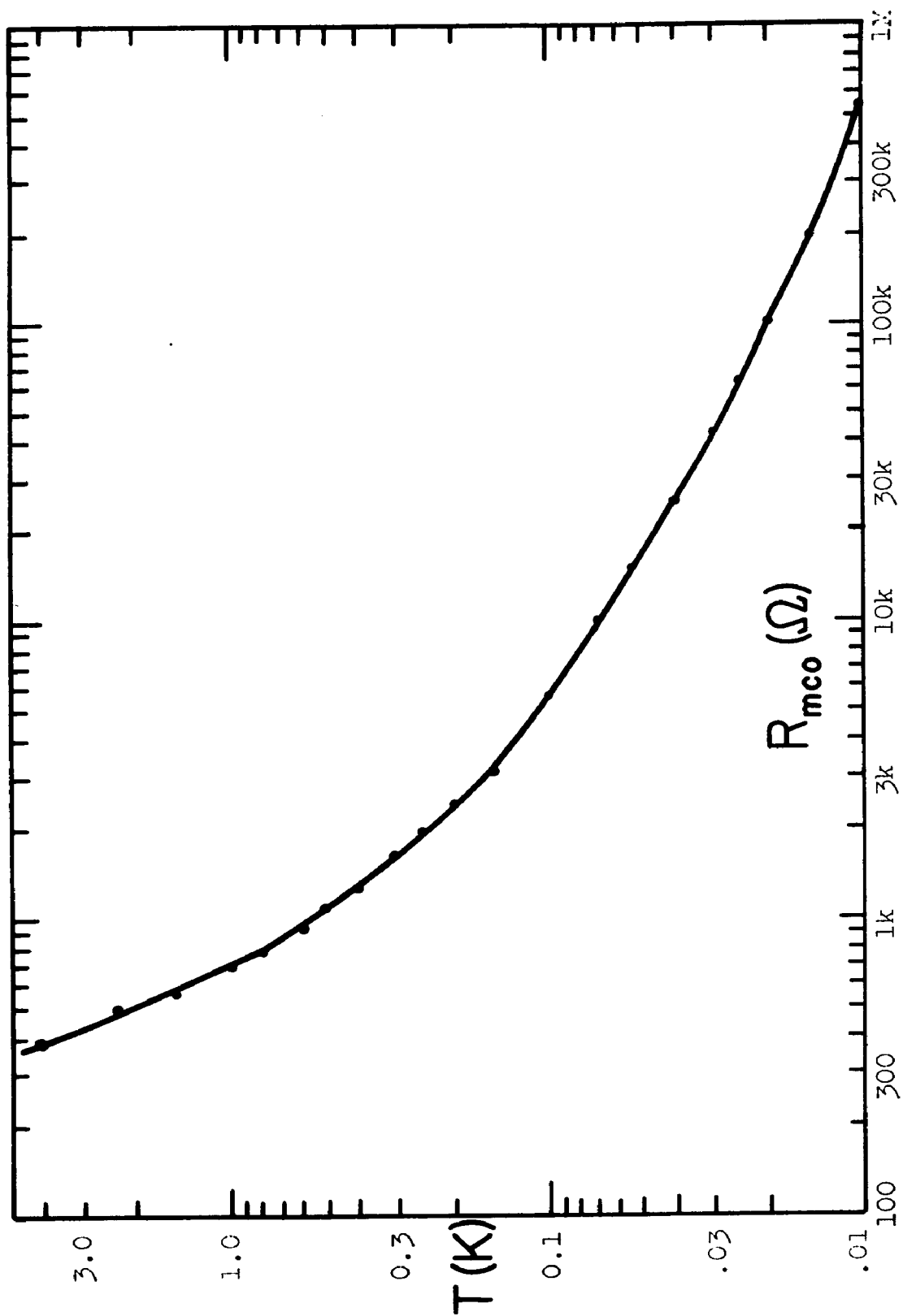


FIG. 10 RESISTANCE VS. MAGNETIC TEMPERATURE FOR THE ORIGINAL MIXING CHAMBER RESISTOR, R_{mco} .

mixing chamber resistor as a function of T^* , the magnetic temperature measured by the original salt pill. For the rest of this discussion, it will be assumed that T^* is equal to T , the absolute temperature.

Table 2 gives the values of the other refrigerator resistors at several temperatures as well as the resistance of the connecting leads. The last column in Table 2 shows a typical value of each resistor when the refrigerator is operating at 50 mK. While poor reproducibility upon thermal cycling precludes their use as absolute thermometers, carbon resistors are exceedingly useful for monitoring the refrigerator. These resistors should be replaced with a set calibrated against CMN.

Because of the large Kapitza resistance between a solid and liquid at very low temperatures, it is desirable to place the CMN thermometer directly in the experimental liquid. The coil set for measuring the susceptibility of the CMN is mounted on the tail-section of the vacuum can over the salt pill and experimental cell. This necessitates an experimental cell which is compatible with magnetic thermometry--a requirement that has proved difficult to fulfill. All materials used must have a much smaller magnetic moment than CMN. Since an ac susceptibility bridge is used, attention must be given to reducing eddy currents in the cell. Thermal contact to the cell is made through a brush of 10,000 copper wires (No. 44), which are parallel to the measuring field. Metal must be kept to a minimum, and all conductors are slit in a plane perpendicular to the magnetic field. The materials used in the cell were selected on the basis of susceptibility measurements made above 1 K.

Table 2

Refrigerator Resistors^a

Name and Location	Nominal Resistance	R _{300 K}		R _{91 K}		R _{4.2 K}		R _{1.23 K}		R _{0c}	
		Total	Leads	Total	Leads	Total	Leads	Total	Leads	Total	Leads
1 K Pot	470 Speer	571.1	83.7	636.3		1097.	80	1959.		2410	
Still	220 "	363.3	112.0	392.8		576.4	107	882.1		1335	
Heat Ex. 1	220 "	349.3	121.2	379.0		547.1	116	827.6		2540	
Heat Ex. 2	220 "	348.2	121.4	376.2		538.6	116	805.4		5370	
Heat Ex. 3	220 "	352.7	122.8	383.0		558.3	118	854.1		13.9 k	
Heat Ex. 4	220 "	316.1	89.1	346.2		519.9	75	808.2		21.5 k	
Heat Ex. 5	220 "	354.5	124.3	383.8		546.4	119	814.0		10.9 k	
Heat Ex. 6	220 "	349.3	124.6	380.1		552.0	119.5	841.6		36.0 k	
Mixing chamber Old (MCO)	220 "	317.2	75.1	350.9		520.0	61	793.0		17.4 k	
Mixing chamber New (MCN)	100 "	139.4	39.0	148.0		201.5	16	279.5		3222	
Cell	100 "	144.8	37.5	155.5		205.2	15	276.8		1770	
R ₁ (cell) _b	Germanium			4.2		27.35		49.05		6500	
R ₂ (cell) _b	Germanium			4.7		37.85		74.50		1240	

a. Resistance in ohms (except k--kilohms) and includes lead resistance; b. Germanium resistors are Cryocal CR-30, measured in a four-terminal network; c. R₀ is measured with the refrigerator operating and the mixing chamber is at 50 mK.

Even with this care, the first run with the experimental ion cell produced a large, spurious, magnetic-field-dependent signal at 2 K. Investigation showed that one of the tungsten grids used in the ion cell had a field-dependent magnetic moment when the plane of the grid was perpendicular to the measuring field. (Initial tests had been made with the plane parallel to the measuring field.) The offending grid was removed and the others carefully tested above 1 K before the cell was reassembled. The same problem occurred again, this time at a temperature of 0.2 K. Cell design made it difficult to replace the grids with another style; instead, two germanium resistance thermometers were added to the cell, in place of the CMN.³ Once calibrated, germanium resistors are reliable thermometers that are reproducible upon thermal cycling. The calibration of the germanium thermometers against CMN has been left for a later date. Figure 11 is a graph of resistance vs. temperature for the two germanium resistors, the 100 ohm Speer resistor in the experimental cell, and the original 220 ohm Speer resistor calibrated in the first run. Above 1 K temperature is measured using He⁴ vapor pressure thermometry. Below 1 K the resistance is plotted against T*, the magnetic temperature (Fig. 10). The dashed line indicates the data prior to a correction for the Kapitza resistance of the thermal contact brush. Figure 11 has been used to assign temperatures for the vortex ring experiment of Part III.

The carbon resistors were measured using an ac Wheatstone bridge, with a lock-in amplifier for a null detector. The bridge dissipates approximately 10^{-12} to 10^{-13} watt in the resistors, producing no

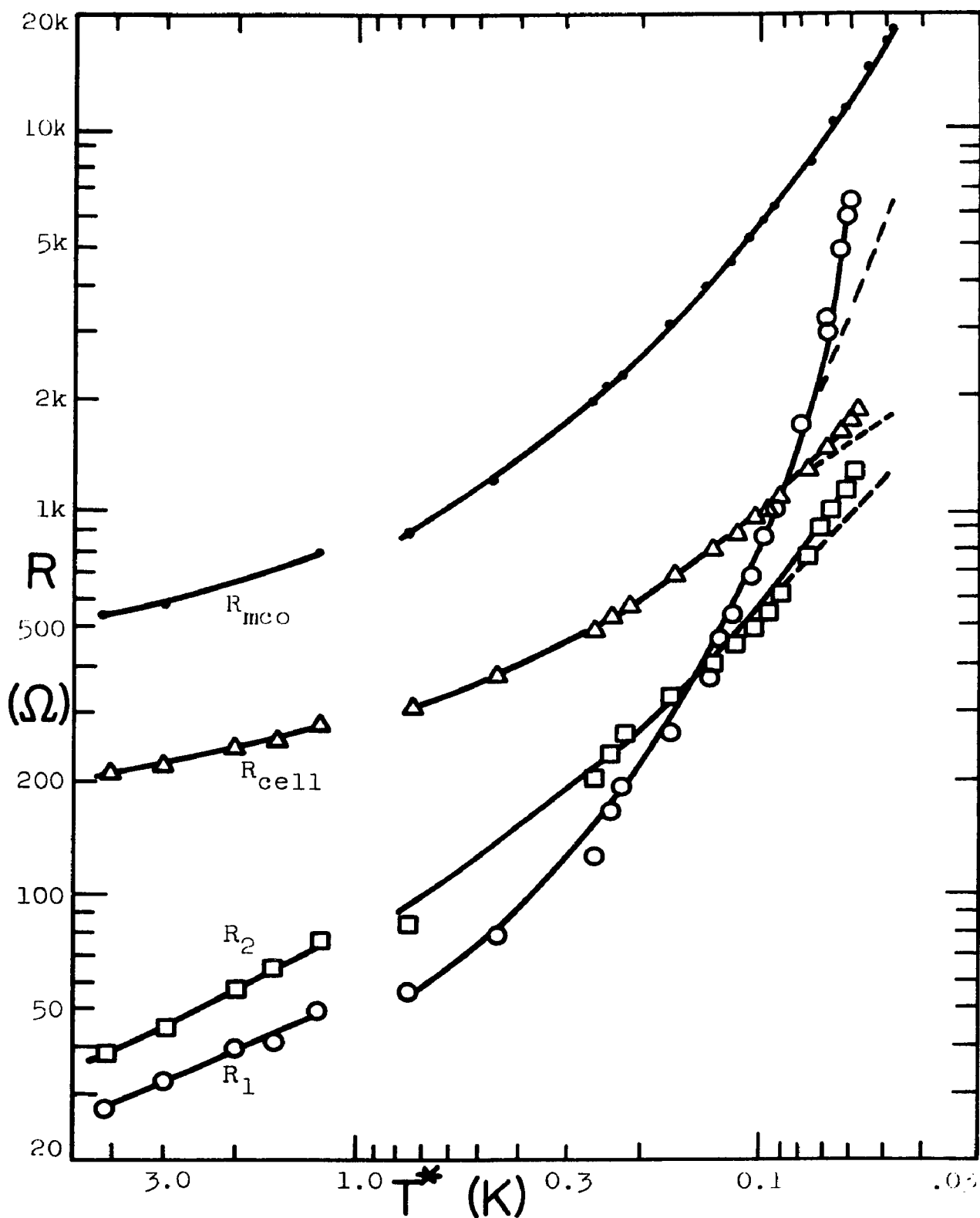


FIG. 11 RESISTANCE VS. MAGNETIC TEMPERATURE FOR THE EXPERIMENTAL CELL RESISTORS. ABOVE 1 K, TEMPERATURE IS MEASURED USING He^4 VAPOR PRESSURE; BELOW 1 K, FIG. 10 IS USED. RESISTORS IDENTIFIED IN TABLE 2.

measurable heating effects in the resistors epoxied to the outside of the mixing chamber.

The germanium resistors were measured with a constant-current source and microvolt digital dc voltmeter, in a four-terminal configuration. R_1 is sealed in a metal case and contains He^4 exchange gas, while R_2 is mounted in a perforated case exposed to the experimental liquid. Measurements indicate that the maximum allowable power to the sealed resistor is approximately 10^{-12} watt at 70 mk, 10^{-11} at 100 mK, and 10^{-10} at 200 mk. A factor of 3 to 5 more power is permissible in the unsealed resistor. A typical refrigerator performance curve is shown in Fig. 12.

Future Modifications

After thermometry problems have been laid to rest, the high ultimate temperature should be improved. The prime culprit appears to be the copper felt in the heat exchangers. These should be replaced with exchangers that do not rely on a push-fit for thermal contact.⁴ The sintered copper plug in the mixing chamber should also be inspected. The present support system is needlessly complicated, although not actually a problem. If new heat exchangers are built, the present system should be replaced with a three-rod support which is essentially the inverse of the present arrangement. The heat exchangers, positioned coaxially with the still and mixing chamber, would be supported by three nylon rods, placed 120 degrees apart on the circumference of the exchangers. The rods should be staggered at alternate levels to thermally anchor the nylon at each exchanger.

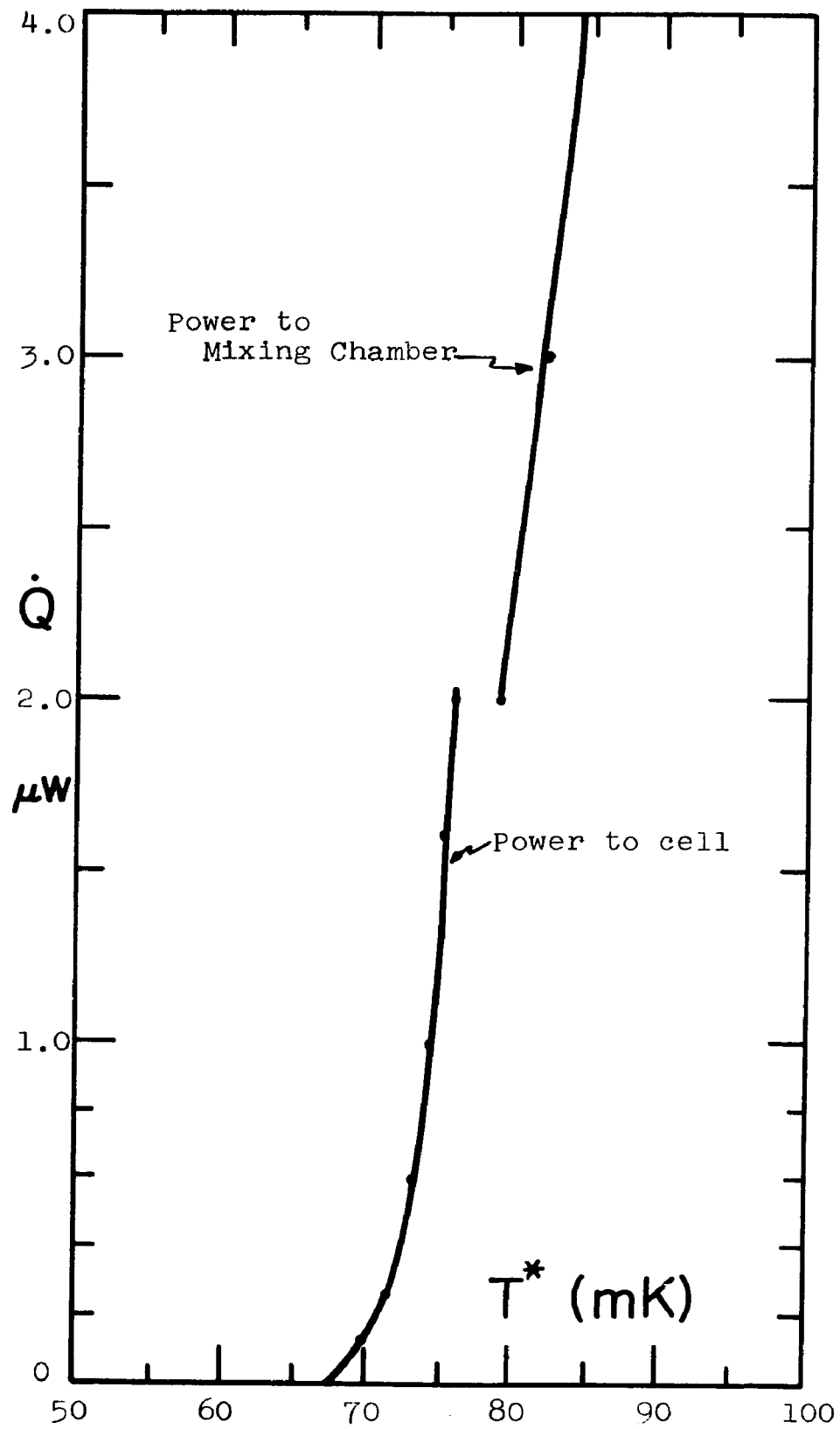


FIG. 12 PERFORMANCE CURVE FOR THE DILUTION REFRIGERATOR.

Some improvements in the gas handling system are also desirable. The multiplicity of valves and gauges have proved valuable and, if anything, more should be added. For instance, the time required to reduce the He⁴ background in the house vacuum can be annoying during leak checking. An independent means of introducing clean He⁴ gas into the various systems would be convenient. A large diameter pumping line connecting the He³ and He⁴ systems would reduce clean-up time in the circulation system. A general upgrading of pressure gauges--both thermocouple and Bourdon--is desirable, especially on the gas storage tanks. An integrating gas-flow meter on the 1 K pot would indicate the amount of liquid remaining in the pot--an important consideration for over-night operation. The cold traps in the He³ circulation system should be enlarged. Presently the "hold" time is only 8 hours.

A series of safety features would allow continuous, unattended operation. Currently, a malfunction--such as a power failure or loss of isolation vacuum--would result in the loss of the entire He³-He⁴ mixture. A 15 liter tank, evacuated and connected to the back side of the sealed He³ pump through a relief valve, would alleviate this problem. High pressure electrical interlocks on the diffusion pumps, and a high-vacuum solenoid valve, to maintain the isolation vacuum in the event of a power failure, might be considered. A high-low pressure alarm on the He³ condensing line, pot, Dewar vacuum and house vacuum would enable the operator to devote full attention to an experiment.⁵

The Dewar support system is easily the most unsatisfactory feature of the refrigerator. To work on the refrigerator, the cryostat (weighing

nearly 100 lb) had to be removed from the Dewar with a winch and carried by hand to a rack mounted on an I-beam. Because all plumbing and electrical connections are disconnected for this operation, leak checking and system tests are hampered. A more satisfactory arrangement (for future refrigerators) would allow the Dewars to be lowered into the floor leaving the cryostat connected. An improved vibration isolation system could be included at the same time. Any new refrigerator should include a liquid nitrogen pot to precool the pumping lines and a larger (metal) Dewar.

REFERENCES AND NOTES

1. Speer Carbon Co., Bradford, Pa., Grade 1002.

See: W. C. Black, Jr., W. R. Roach and J. C. Wheatley, Rev. Sci. Instr. 35, 587 (1964).
2. Alfa Inorganics, Inc., Beverly, Mass.
3. CryoCal Inc., Riviera Beach, Florida. The resistors were loaned through the courtesy of Mr. T. Herder. R_1 is model CR30, S/N 1247, and is sealed with He4 exchange gas. R_2 is model CR30, S/N 1698 and is in a perforated can.
4. R. Soulens of the N.B.S. has designed some exchangers in which a presintered copper plug is heli-arc welded into the copper heat exchanger body for good thermal contact. The plugs are cut from presintered stock, type C and F, manufactured by the Pall Corp., Glen Cove, L.I., New York.
5. A simple pressure alarm using an I.C. voltage comparator and an N.R.C. type 501 thermocouple gauge has been designed by George Schofield for the Michigan He3 cryostat.

CHAPTER VI. USEFUL MATERIALS AND TECHNIQUES

Materials

The materials for an ultra-low temperature program must be carefully chosen, with attention paid to problems of thermal contact and magnetic properties. The bibliography lists many excellent papers on materials--notably those co-authored by J. C. Wheatley. While the materials mentioned in this section are covered, to a large extent, in this literature, we will provide some comments regarding their care and feeding.

Copper-nickel tubing soft-solders easily, and is invaluable in cryostat construction. An apparatus such as a dilution refrigerator has many soldered connections; unless care is taken, solder flux on the first joint will corrode the apparatus before the last joint is finished. Solf-solder flux can be neutralized with a solution of trisodium phosphate (1 oz/gal of water). After soldering, copper alloys may be cleaned and polished with an ammonium persulfate solution (250 g/liter of water). These solutions are less hazardous than the conventional acid brite-dip, but the work should be rinsed well afterwards. Ordinary lead-tin soft solder is used for joints that cannot be hard soldered. An attempt to use a low-superconducting-transition, Cd-Bi alloy as a solder was disastrous; the alloy lacked mechanical strength, and a multitude of leaks appeared upon thermal cycling.¹ Castles are used to make solder connections to flanges with more than one solder joint, and

are highly recommended (Fig. 14). A convenient tool for soldering to castles is a 300 watt soldering iron with a removable copper tip machined to match the cylindrical surface of the castle.

The sintered copper powder used for thermal contact is No. 325 mesh (44 micron average particle size) copper, compressed to 300 atmospheres on an arbor press. The plugs were sintered at 625 °C for one hour in a hydrogen atmosphere.² For best adhesion, the copper body, in which the powder is packed, should be sandblasted. It is important to use OFHC (oxygen-free-high-conductivity) copper for the body to avoid hydrogen embrittlement, and leaks after sintering. It is possible to sinter without a hydrogen atmosphere by burying the work in lamp black, although copper oxidizes slightly in this process.

Graphite is often used as a structural support, because of its rigidity and low thermal conductivity below 0.5 K. Graphite, however, is easily broken and has large surface area which readily absorbs He exchange gas (subsequently time consuming to remove). Nylon does not suffer from either of these problems but does not provide a very rigid support. Nylon can be joined to nylon with a glue made from nylon shavings dissolved in 88 percent phenol and 12 percent water. Joints made in this fashion are leak tight at room temperature, and stronger than any of the commercial nylon epoxies.

Epibond 100A is a cast, machinable, non-magnetic epoxy which is often used in ultra-low temperature work.³ Gas is evolved in the curing process and some effort is required to produce bubble-free castings. Bar stock is cast in a cylindrical mold coated with a teflon spray. The mold

is packed with powdered Epibond and heated in an oven to 120 °C. The mold is alternately evacuated and pressurized during the first 1/2 hour of curing to remove gas, and then pressurized to 50 psi to keep any gas subsequently evolved in small bubbles. The epoxy is fully cured in 24 hours at 120 °C. Cured Epibond 100A softens at 50 °C, and a machined part will lose its dimensional stability above that temperature. The finished product is extremely resistant to thermal shock.

Parts machined from Epibond 100A may be joined with Stycast 2850 FT (24LV catalyst)--an all-purpose, low temperature epoxy with exceptional adhesive strength. Because Stycast is slightly magnetic, Epibond 121 is often used for joining Epibond parts. Epibond 121 shatters easily upon thermal cycling and must be used sparingly. The surface of the Epibond 100A joint is wet with 121; excess epoxy is blotted with an absorbent tissue, leaving a very thin film. The joint must be clamped to produce a good bond. Wire lead seals in 100A epoxy can be made with 121. Holes, slightly larger than the wire diameter (e.g., 0.001 inch), are drilled, and the wire is coated with Epibond 121. The wire is then pulled through the hole several times to insure that the inside of the hole is coated with an epoxy film. Excess epoxy should be wiped off.

An epoxy that is useful for making low temperature electrical contacts is CA-10--an epoxy matrix filled with silver powder. The electrical conductivity of CA-10 increases a factor of two upon cooling to liquid helium temperatures, and its susceptibility is small and constant below 4 K. The mechanical strength of the epoxy is only moderate.

If magnetic thermometry is anticipated, the materials used in construction must be selected with care. Table 3 presents a list of

Table 3

Magnetic Properties of Materials

This table gives a qualitative indication of the magnetic properties of cryostat materials. Data are taken at 4 K and 1 K, and the magnetic moment is compared to 1 g of CMN. The quantity of each material tested is comparable to the amount that might be used in construction. KEY: Excellent--use in any quantity; Good--use in moderation; Poor--use sparingly; Bad--avoid; T.I.--temperature independent over the range tested; S.U.--source unknown.

<u>Material</u>	<u>Magnetic Moment</u>	<u>Comment</u>
Masking tape	Excellent	Several brands tested
Teflon tubing-red	Excellent	G.C. Electronics
Glass fiber filter (single disk)	Good	Reeve Angel 934AH Clifton, N.J.
Fritted glass disk	Good	Similar to Corning No. 31000
Glass sleeving	Poor	Ben-Har ST, Bentley Harris, Conshohocken, Pa.
Bakelite-linen	Excellent	S.U., coil forms
Nylon	Good	S.U., small quantities do not use for coil forms or sample holders
Varnish-GE 7031	Excellent	General Electric Co.
CA-10, Silver conducting epoxy	Good, T.I.	Techkit Adhesives Darmest, N. J.
Silver print	Good	G. S. Electronics
Speer resistor	Poor	Speer Carbon Co. Bradford, Pa.
Speer resistor-tinning removed from leads	Good	Speer Carbon Co.
Cupron wire (constantan)	Bad	Wilber Driver Co. Newark, N. J.

(cont.)

Table 3 (cont.)

<u>Material</u>	<u>Magnetic Moment</u>	<u>Comment</u>
Evanohm wire (0.003" dia)	Good, T.I.	Wilber Driver Co. (100 ohm quantities)
Cu-Ni tubing	Poor	Superior Tube Co. Collegeville, Pa.
Phosphor Bronze	Good	S.U., wire and shim
Superinsulation	Bad	Aluminized mylar N.R.C. Equipment Co. (Norton Corp.)
Brass	Poor, T.I.	S.U., small quantities such as screws

materials, measured over a period of several years. A qualitative judgment of the susceptibility is given, since many of the measurements were made on a "go-no-go" basis. In each case the amount of material measured is comparable to the quantity that might be used in cryostat construction; the moment is compared to several grams of CMN. This list, and other published data on cryostat materials, should be used as a guide in material selection. Each lot of material should be checked prior to use, and the assembled apparatus should be tested in the millikelvin range before a paramagnetic salt is added.

Flow-Limiting Impedance

The flow impedance is made by packing layers of copper powder in a 1/32 inch Cu-Ni tube, and sintering the assembly in a hydrogen atmosphere at 625 °C. A 2 inch length of tubing is mounted on a vertical 1/4 inch peg, of 0.025 inch o.d. steel wire, set in a brass block. A small amount of copper powder is added to the tube, and compressed by a "tamper" made from a short length of wire, with a 50 g mass on one end. The tamper is lifted approximately 1 cm and dropped several times, compacting the powder. A 2 cm long plug of copper powder is built up, using many layers of powder. Although it is possible to adjust the impedance upward after sintering, by drawing the tube through a die, it is more convenient to pack several dozen tubes, varying the number of layers, and select the proper impedance later. Impedances between 10^{10} and 10^{13} cm^{-3} were made using up to 40 layers of copper powder compacted an average of 10 strokes per layer. Similar impedances constructed with lead powder were unsatisfactory.

Electrical Connections

The first electrometer lead for the ion experiment was a rigid coax which proved extremely sensitive to pressure and temperature fluctuations. The coax had a rigid stainless steel jacket, teflon insulation and copper-weld center conductor.⁴ Relative motion between the insulation and outer jacket, caused by changes in pressure or temperature, generated a triboelectric charge in the teflon insulator. A new coax (Fig. 13) was designed, and was particularly successful in minimizing these effects. The minimum resolvable current, with this coax, is 2×10^{-15} A (with a 3 sec time constant), virtually independent of the helium or nitrogen level in the Dewar.

The teflon jacket was stripped from a special miniature coaxial cable. This cable is similar to Microdot cable No. 250-3919, with these exceptions: (1) both the center conductor and braided shield are copper-weld wire, (2) the Microdot "Mininoise" graphite coating is applied between the shield and teflon insulation. The graphite reduces triboelectric charge by a factor of 100. The coax was then pulled into a long 1/16 inch o.d. Cu-Ni tube for support. (The braided shield was left on the coax, and provided a tight fit.) The 1/16 inch tube was then inserted into a 1/4 inch o.d. Cu-Ni tube, and centered with a series of Styrofoam spacers. The top end of the coax is terminated in a hermetic BNC connector, sealed by an O-ring to a brass bushing soldered on the 1/4 inch tube. The bushing extends through a castle on the pumping head. The lower end of the 1/4 inch tube enters the vacuum can through a castle on the light trap. The tube and center conductor are sealed at the lower ends

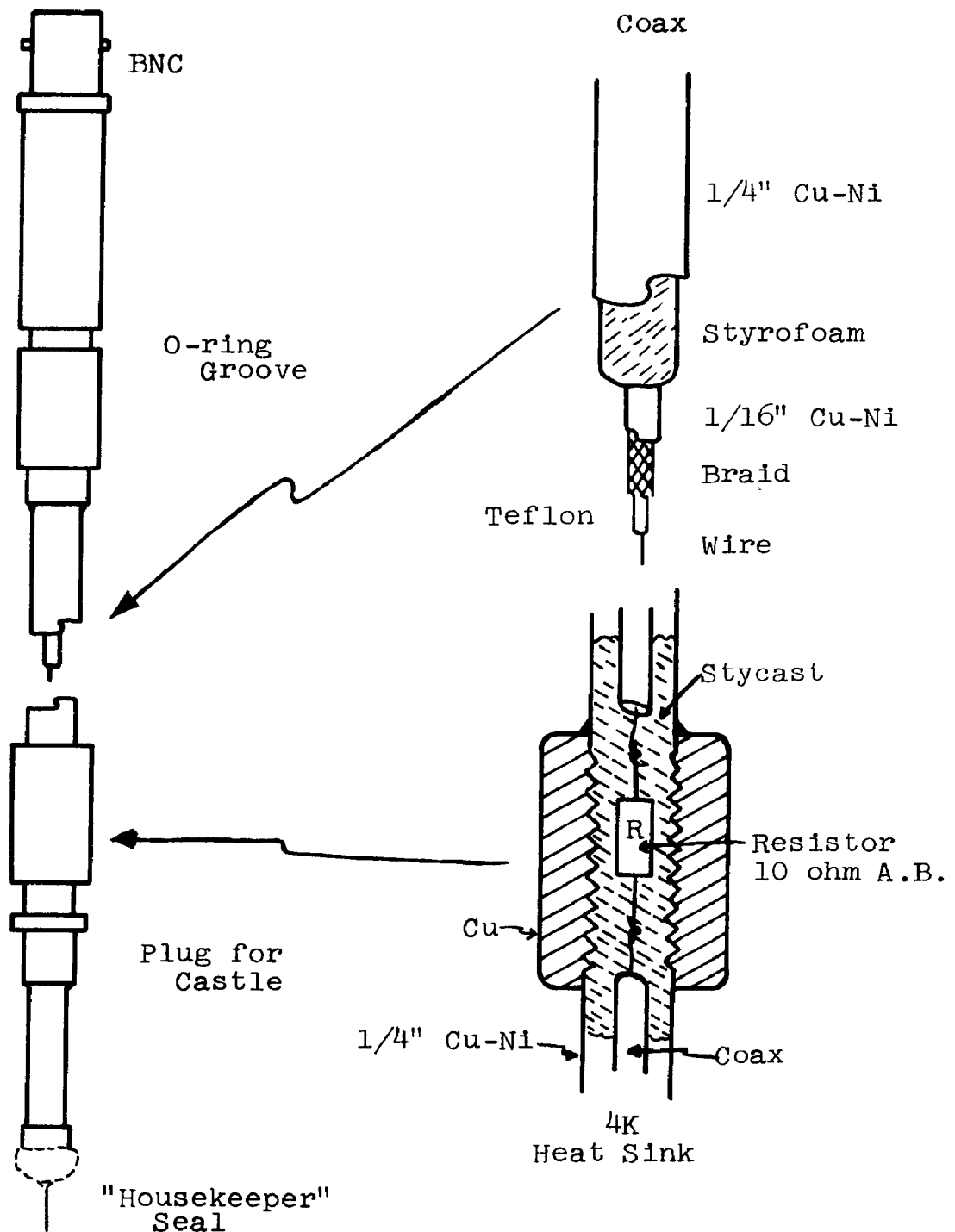


FIG. 13 COAXIAL LEAD FOR THE ELECTROMETER SHOWING DETAILS OF THE CONSTRUCTION AND THE 4 K HEAT SINK. THE RESISTOR IS A 10 OHM TYPE BB ALLEN BRADLEY.

with a Stycast-copper "Housekeeper" seal.⁵ The center conductor is anchored to the 4 K bath just above the light trap, with the resistor-Stycast-copper arrangement shown in the detail in Fig. 13. The coax is anchored to the 4 K bath at one end, and room temperature at the other. The 1/4 inch tube, and Styrofoam spacers, insulate the coax, keeping the temperature gradient independent of bath level. The 4 K bath cryopumps the coax.

The coaxial line, from the light trap to the experimental cell inside the vacuum can, does not need such elaborate precautions, since it is all below 4 K. This section of the coaxial line is a rigid micro-miniature coax--0.020 inch o.d., with a 0.0045 inch center conductor.⁶ Approximately 30 cm of coax are coiled inside the pot, for thermal anchoring. Below the pot, the coax line has a stainless steel center conductor, instead of copper-weld wire, to reduce the heat leak to the mixing chamber and the experiment. The center conductor is anchored to the still, with a resistor-Stycast-copper combination (see Fig. 13); the outer conductor is soldered to every other heat exchanger. Cooling the pot has a minimal effect on the coax. The current is measured at the pumping head with an operational amplifier electrometer.⁷ The electrometer is connected directly to the hermetic BNC connector on top of the coax.

Electrical leads from the pumping head enter the vacuum can through lead tubes shown in Fig. 14. The leads are sealed inside the vacuum can in conventional Stycast-copper "Housekeeper" seals. The top end of each lead tube is fitted with a nylon cap, sealed with the

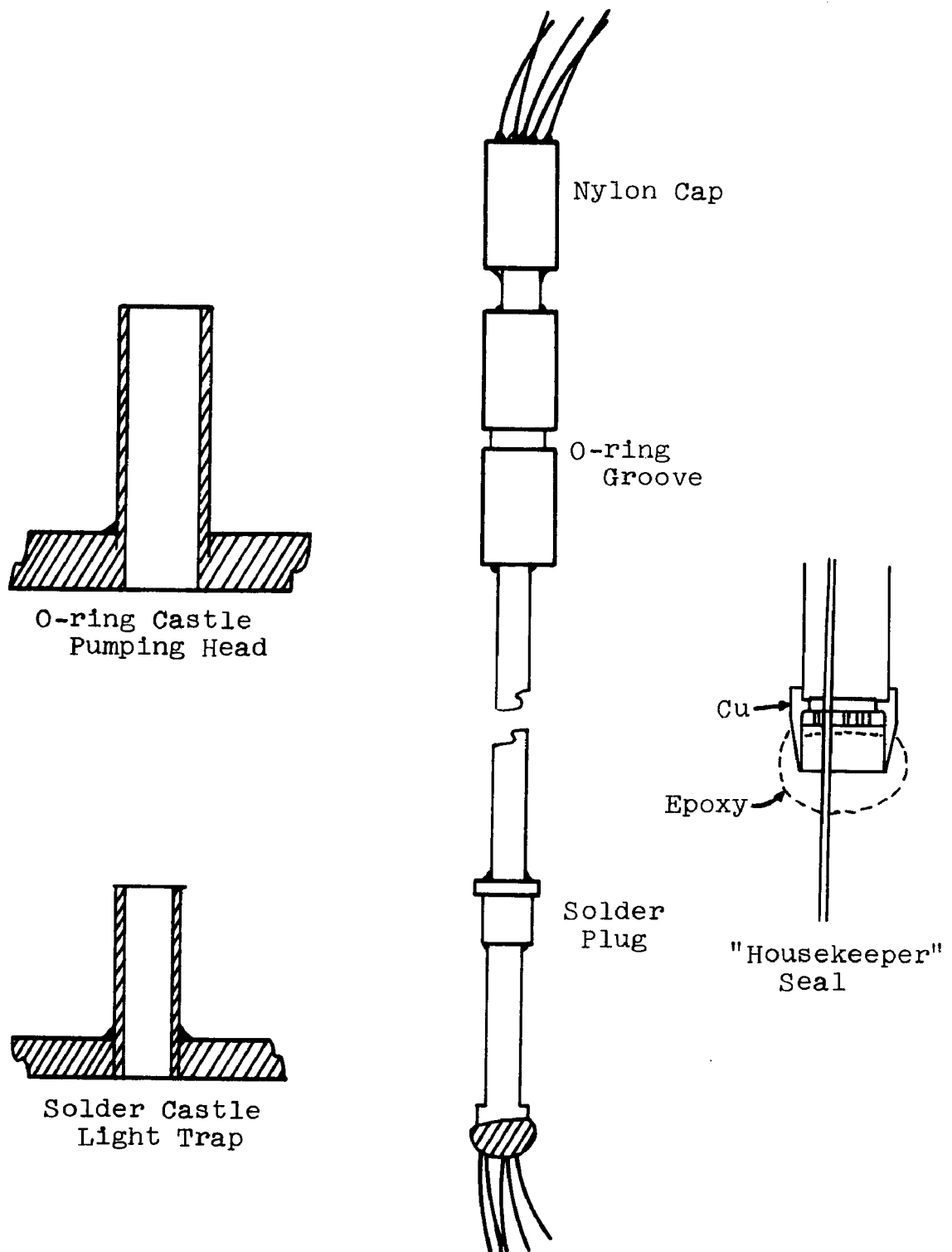


FIG. 14 CASTLES AND ELECTRICAL LEAD TUBE.

phenol-nylon glue mentioned earlier.⁸ Manganin wire (0.005 inch dia.) is used for the electrical leads, because of its low thermal conductivity and thermal emf against copper. A miniature 10 pin connector that works well at cryogenic temperatures can be made from two teflon-insulated integrated circuit sockets.⁹

Level Detector

A resonant LC circuit and back-diode oscillator is used to indicate the level of liquid helium in the experimental cell. The dielectric constant of the liquid helium in the capacitor changes the capacitance and, therefore, the frequency of the resonant circuit. The capacitor is made from two strips of Invar tape, separated by a nylon mesh, and rolled on a teflon mandrel, like a conventional paper capacitor. This assembly fits into a teflon-insulated brass can, and is connected to the experimental cell through a 1/64 inch Cu-Ni tube. The back diode and LC circuit are mounted on the mixing chamber. The oscillator is connected to the room-temperature bias supply and amplifier by a single coaxial line. Figure 15 shows the schematic of the tunnel diode circuit, bias supply, and amplifier.

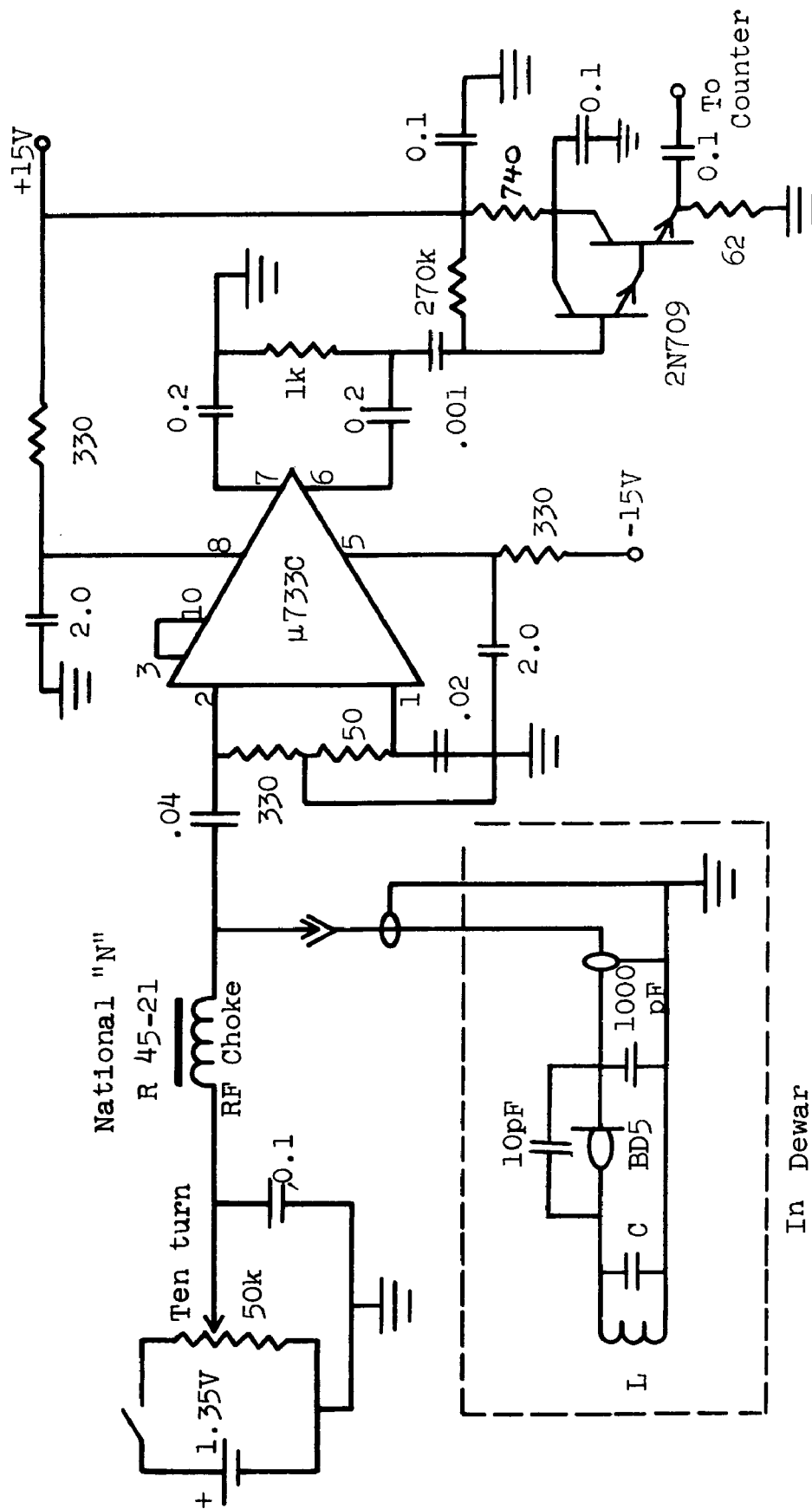


FIG. 15 BACK-DIODE LIQUID HELIUM LEVEL DETECTOR. RESISTANCE VALUES IN OHMS, AND CAPACITANCE IN MICROFARADS UNLESS SPECIFIED. BACK-DIODE--GENERAL ELECTRIC CO., VIDEO AMPLIFIER--FAIRCHILD SEMICONDUCTOR.

REFERENCES AND NOTES

1. W. Steyert, Jr., Rev. Sci. Instr. 38, 964 (1967).
2. In lieu of the sintering process, presintered copper felt (FM 1006, Huyck Metals, Inc., Milford, Conn.), or a porous plug of copper (Type C or F, Pall Corp., Glen Cove, L.I., New York) can be used.
3. The Epibond epoxies are manufactured by Furane Plastics, Inc., Fairfield, N. J., while Stycast is a product of Emerson and Cuming, Inc., Canton, Mass. CA-10 is a silver conducting epoxy made by Techkits, Demarest, N. J. Another conducting epoxy, filled with copper, is Ablebond 163-4 from Ablestik Adhesive Co., Gardena, Calif. Since the magnetic properties have not been tested, this epoxy has never been used in the ultra-low temperature program.
4. UT-85-SS, Uniform Tube Inc., Collegeville, Pa.
5. A. C. Anderson, Rev. Sci. Instr. 39, 605 (1968).
6. UT-20-SS, Uniform Tubes, Inc., Collegeville, Pa.
7. Analog Devices Inc., Model 310J, and a 10^{12} ohm Victoreen Hi-Meg feedback resistor.
8. In an attempt to prevent arcing at high voltages, two of the lead tubes were filled with transformer oil. After a period of time, the oil reacted with the Stycast seal, to produce spurious conduction paths to ground (approximately 10^8 ohms). This is only a problem at room temperature.
9. Augat 8058-1G34, and 8058-1G31.

PART III
VORTEX RINGS IN He^3 - He^4 MIXTURES

CHAPTER VII. FRICTIONAL DRAG ON POSITIVE VORTEX RINGS IN He³-He⁴ MIXTURES

Introduction

Vortex rings are created in liquid helium when ions reach a critical velocity in a regime of low friction. First investigated by Rayfield and Reif¹ in 1963, the vortex rings² obey the same classical hydrodynamic equations as the smoke rings that fascinated Helmholtz and Kelvin in the mid-19th century. The quantum nature of the superfluid requires that the circulation κ of the superfluid around the core of the ring be quantized in units of h/m_4 .

$$\kappa = \oint \mathbf{v} \cdot d\mathbf{l} = N h/m_4 \cong 10^{-3} \text{ N cm}^2/\text{sec} .$$

The ion creates a macroscopic flow pattern quantized in units of Plank's constant divided by the mass of the helium 4 atom--quantities usually reserved for microscopic phenomena. Only one unit of circulation ($N = 1$) has ever been observed.

The equations³ for the kinetic energy, drift velocity, and momentum⁴ of a vortex ring with a hollow core⁵ are:

$$E = (\rho \kappa^2 R/2) [\ln(8R/A) - 2] \quad (1a)$$

$$v = (\kappa/4\pi R) [\ln(8R/A) - 1/2] \quad (1b)$$

$$P = \rho \kappa \pi R^2 , \quad (1c)$$

where R is the radius of the vortex ring, A is the radius of the core, and ρ is the density of the fluid. Experimentally, A is $1.28 \text{ \AA}$ in pure He^4 at 0.27 K . The ion is trapped on this core, and provides a "handle" enabling work done on the ion by an electric field to be transformed into hydrodynamic energy of the ring. The quantity $[\ln(8R/A) - 1/2]$ is conventionally abbreviated by $\chi(E)$. The radius and drift velocity of the ring as a function of energy and $\chi(E)$ are shown in Fig. 16.

In the presence of scatterers, such as phonons, rotons, or He^3 quasiparticles, the ring experiences a frictional force which is proportional to the circumference and drift velocity of the ring. Since the velocity is (approximately) inversely proportional to the radius of the ring, the friction is only logarithmically dependent on the size or energy of the ring. That is,

$$F = (2\alpha'/\kappa)(2\pi R)v = \alpha'\chi(E) ,$$

where α' is the friction coefficient. Unless the power dissipated in friction is supplied by an electric field, the ring loses energy, and its velocity increases, until it quite literally meets a "sudden death."

Vortex Rings in an Electric Field

In this section, we will examine the energy of a vortex ring in an electric field when friction is present. For simplicity, the problem will be considered in one dimension: (velocity and field are positive in the plus z direction). The differential equation for the energy of a vortex ring is

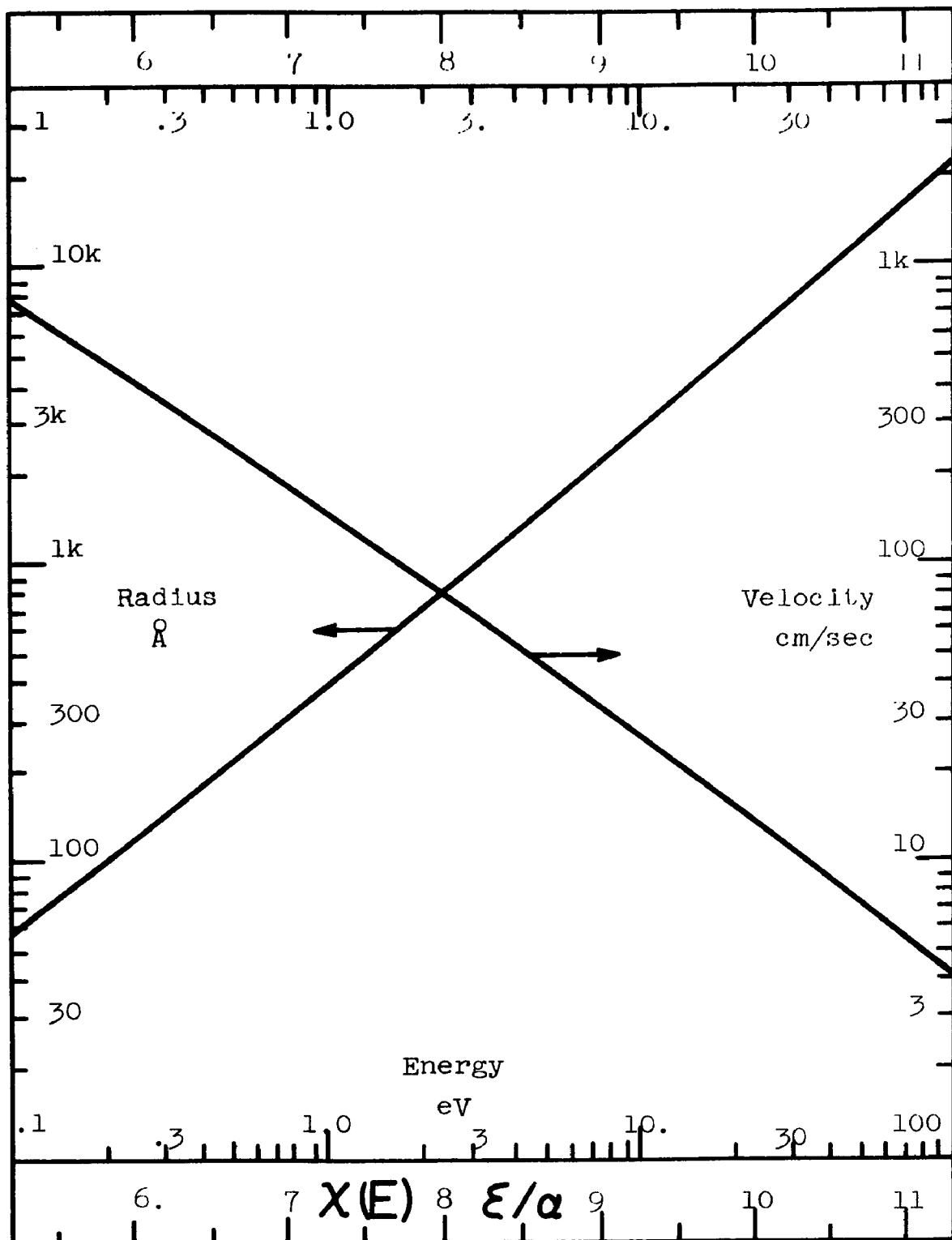


FIG. 16 THE RADIUS AND VELOCITY OF A VORTEX RING AS A FUNCTION OF ENERGY E , AND $\chi(E)$. ALSO SHOWN IS $E/\alpha = \chi(E_\infty)$, WHERE E_∞ IS THE ASYMPTOTIC ENERGY OF A VORTEX RING IN AN ELECTRIC FIELD WITH FRICTION α .

$$(dE/dz) = F_{\text{total}} = e\mathcal{E} - \alpha'X(E) \quad .$$

(It will be convenient to measure E and $\alpha = \alpha'/e$ in units of eV and eV/cm, respectively.) Since $X(E)$ is explicitly a function of the ring radius, it is convenient to write the differential equation in terms of the radius, by using (1a).

$$(dR/dz) = \frac{\mathcal{E} - \alpha[\ln(8R/A) - 1/2]}{(\rho\kappa^2/2)(\ln(8R/A) - 1)} \quad . \quad (2)$$

The ring will asymptotically approach an equilibrium radius R_∞ in the electric field, given by the relation

$$R_\infty = (A/8) \exp(\mathcal{E}/\alpha + 1/2) \quad .$$

The friction coefficient α is thus a scale factor for the electric field:

$$\mathcal{E}/\alpha = [\ln(8R/A) - 1/2] = X(E_\infty) \quad ,$$

where E_∞ is the asymptotic energy of the ring. The solution of equation (2) can be written in terms of the logarithmic integral. After traveling a distance z in the electric field, the ring has a radius R_f given by the root of the equation $F(R_f) = 0$, where

$$F(R_f) \equiv R_f - R_i + \frac{2\alpha z}{\rho\kappa^2} + (\mathcal{E}/\alpha - 1/2) \int_{R_i}^{R_f} \frac{dR}{\ln(R/R_\infty)} \quad . \quad (3)$$

Here i and f refer to the initial and final radii of the ring. This equation is valid for all values of $\mathcal{E}$.⁶

Friction Due to He³ Quasiparticles

Below 0.4 K, where the number density of phonons and rotons is small (Fig. 17), a few parts per million of He³ dominates the friction. If the gas of He³ quasiparticles is non-degenerate, classical kinetic theory gives the He³ contribution to the friction coefficient¹ to be

$$\alpha_3 = (3/8) k n_3 \sigma_3 (2\pi m^* kT)^{1/2},$$

where k is Boltzmann's constant, n_3 is the quasiparticle number density, and σ_3 is the effective cross section for He³ vortex ring scattering. σ_3 has dimensions of a length.

Recent theoretical calculations^{7,8} predict α_3 to be temperature independent, and that σ_3 should vary as $T^{-1/2}$. Measurements on the drift velocity of vortex rings in an electric field, for $x > 100$ ppm, are in general agreement with this prediction.^{7,9}

This work will describe measurements of the friction coefficient of positively charged vortex rings in He³-He⁴ mixtures, with $x < 100$ ppm, at temperatures between 60 and 200 mK. The measurements are made by a "stopping potential" technique.¹

A beam of vortex rings of known energy is admitted to a long "drift region," in which the friction is approximately balanced by an electric field. The energy of the rings is measured again at the end of this drift space, and the loss due to friction is equal to the change in ring energy, minus the accelerating voltage across the drift region. The energy of the rings is determined by applying a retarding field to the rings and measuring the reverse bias necessary to cut off the

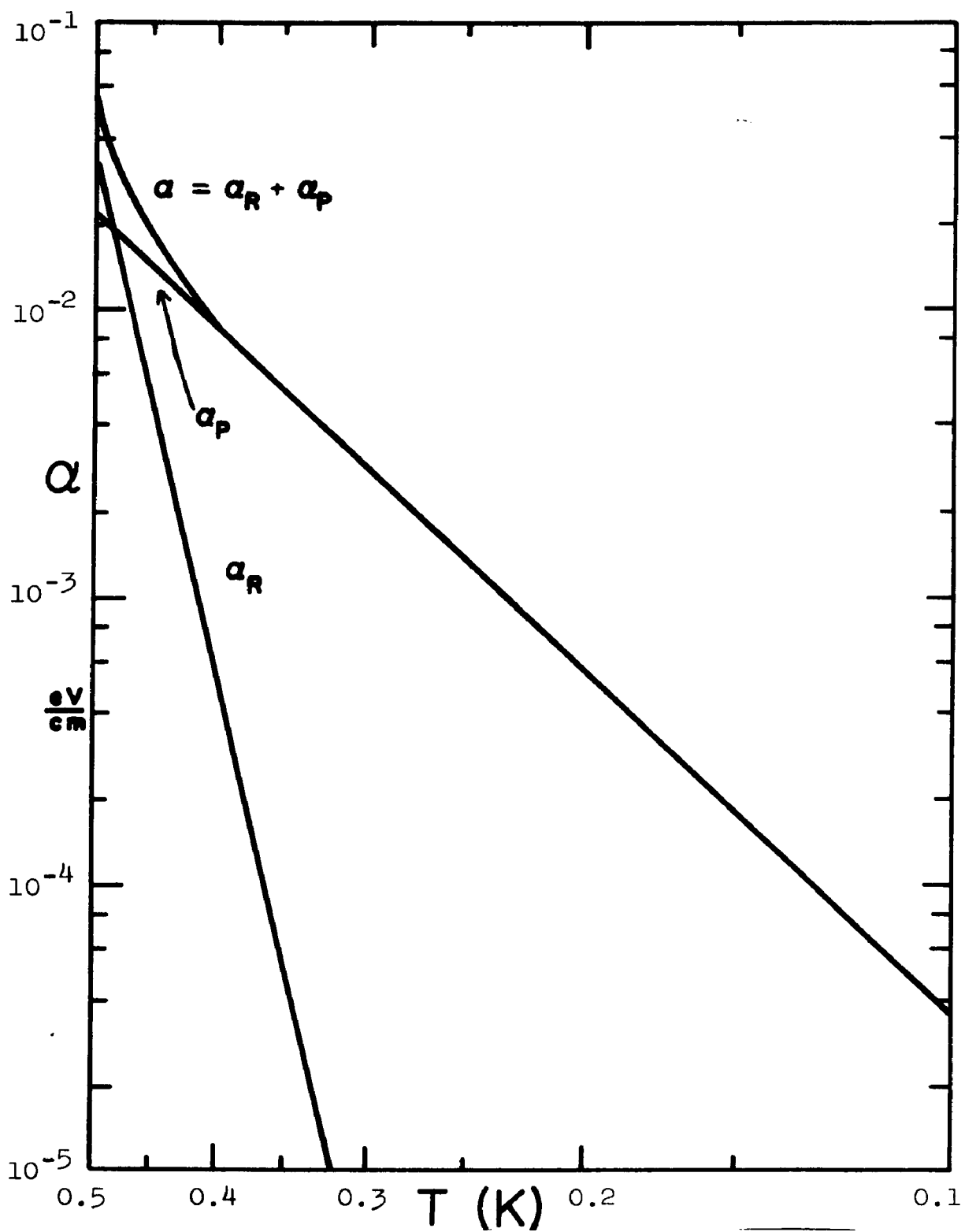


FIG. 17 CONTRIBUTION TO THE FRICTION COEFFICIENT FROM ROTONS (α_r) AND PHONONS (α_p). DATA EXTRAPOLATED FROM REF. 1.

current. This technique is particularly suited to the regime in which the friction is small, and the distance a ring must travel to reach equilibrium can be long.

Ion Cell

The cell (Fig. 18) used in the friction coefficient studies was originally designed as a double-gate ion velocity spectrometer,¹⁰ to measure ion mobilities in He³-He⁴ mixtures below 100 mK. A slight digression on the operation of a velocity spectrometer will serve to identify the various elements in the cell.

A radioactive source mounted on grid A ionizes the liquid helium, creating charged particles. Ions of the appropriate sign are extracted from the region in front of the source by an electric field between grids A and B. Grids B and C operate as a gate for the ion beam. These grids, known collectively as Gate I, are normally reverse-biased to cut off the current extracted from the source. This gate is periodically "opened" by applying a forward bias between grids B and C with a square wave generator, allowing a pulse of charge to enter the drift region. The ions then drift in an electric field to grid D, where they encounter a second gate formed by grids D and E. These grids, which constitute Gate II, operate in the same manner as Gate I, and, in typical operation, are pulsed synchronously with Gate I. If the pulse of ions arrives at Gate II just as it opens, the ions will pass through grids D, E, and F, and be collected. Grid F reduces the capacitive coupling between the pulsed gate and the collector. The collector is mounted on a guard ring, at ground potential, to prevent spurious currents from being collected.

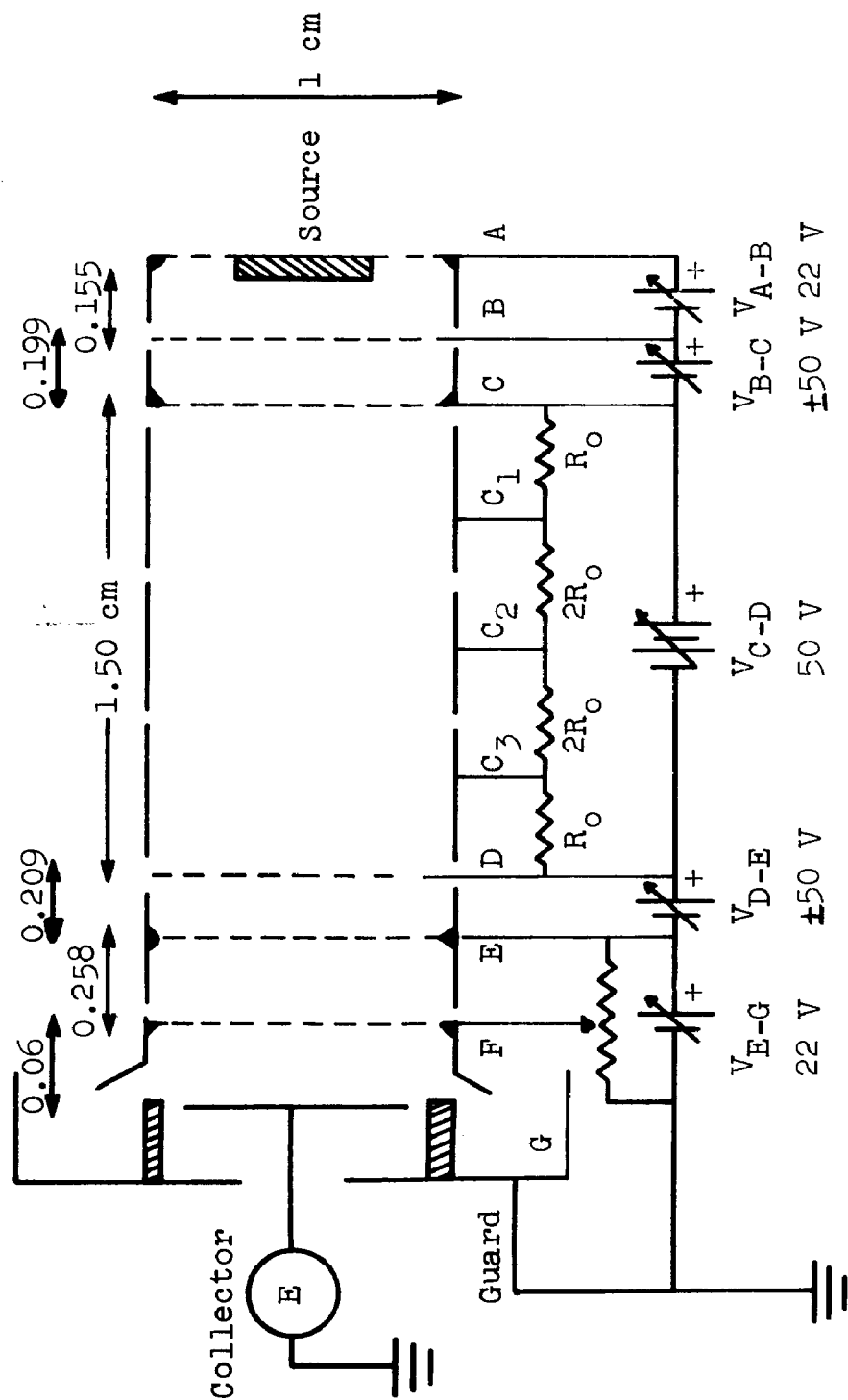


FIG. 18 SCHEMATIC DIAGRAM OF THE ION CELL AND BATTERY SUPPLY.

The average current in the cell is measured with a dc electrometer, and is plotted as a function of gate frequency. The time required for the ions to cross the drift space--as indicated by a maximum in the collected current--is an integral multiple of the square-wave period.

The radioactive source determines the magnitude of the ion current and is the major heat load on the refrigerator. The ultimate temperature of the refrigerator must be balanced against the minimum detectable current when selecting a radioactive source. Tritium is the most effective source--in terms of collected current per erg of heatload; the tritium source used in the cell is nominally 15mCi and produces a current of 0.2 pA at the collector with a 5 erg/sec heat load.

The magnitude of the current involved dictates that a dc method be used, although in principle the same information can be obtained using Gate I alone and measuring the arrival time of the charge pulse at the collector with a wideband current amplifier.

Although intended for ion mobility work, the cell is readily adaptable to vortex-ring studies. The energy of the vortex rings at each end of the drift region is determined by applying a retarding field in each gate and measuring the reverse bias necessary to stop the vortex rings from reaching the collector. Since the method just described does not insure that the rings have reached terminal velocity in the drift region, a computer program which evaluates equation (3) in each region is required to calculate the friction coefficient from the Gate I and Gate II cutoff characteristics and the drift field.

In principle, the energy of the rings, when they enter the drift region, is known from the potential difference between grids A and C

(corrected for the energy loss due to friction in that region). Unfortunately, when ring studies were attempted, it was discovered that the surface of the radioactive source was partially oxidized and a layer of electric charge accumulated on the source. Consequently, the potential difference through which the rings were accelerated was indeterminate. Since the energy of the rings can be measured at Gate I, the charging of the source causes no problem as long as the surface charge remains constant, and relatively uniform during measurements. It was found that the charging rate depended on the sign of the current extracted from the source, a positive current producing a smaller charging rate and reaching equilibrium faster. (Presumably, beta emission from the source partially cancels the charging effect of a positive vortex ring current.) All the measurements were made using positively charged vortex rings.

Cell Construction

Details of the experimental cell construction are shown in Figs. 19 and 20. The structural members of the cell are machined from Epibond 100A to be compatible with magnetic thermometry. A top, which covers the actual electrode assembly, is sealed to a wire brush, that provides thermal contact between the experimental liquid and the refrigerator. The brush¹¹ consists of 10,000 No. 44 copper wires hard-soldered at the top to the slotted cup described earlier. The internal volume of the cell was kept small to reduce the amount of He³ needed

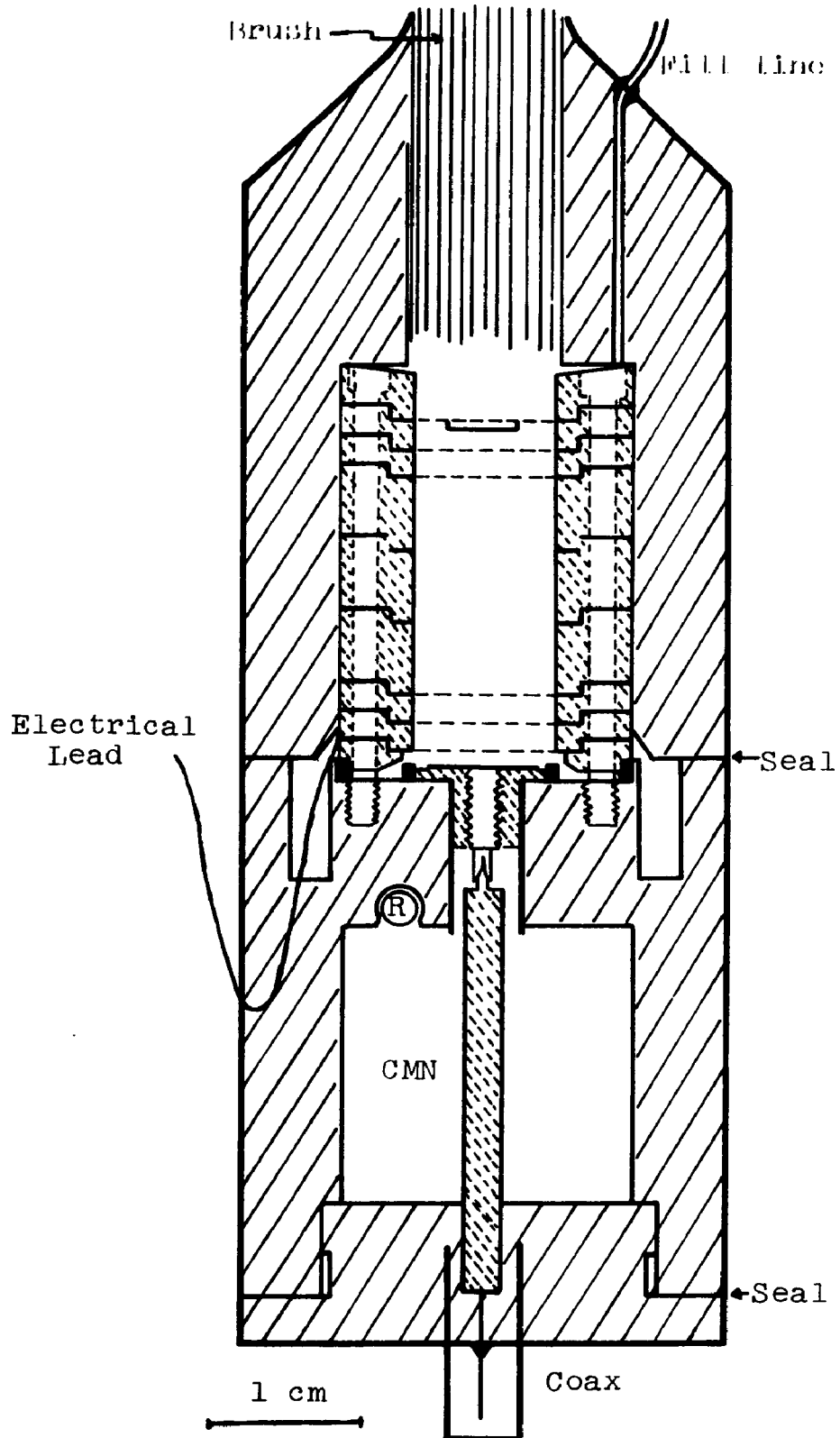


FIG. 19 VIEW OF THE ION CELL. R IS A 100 OHM SPEER RESISTOR.

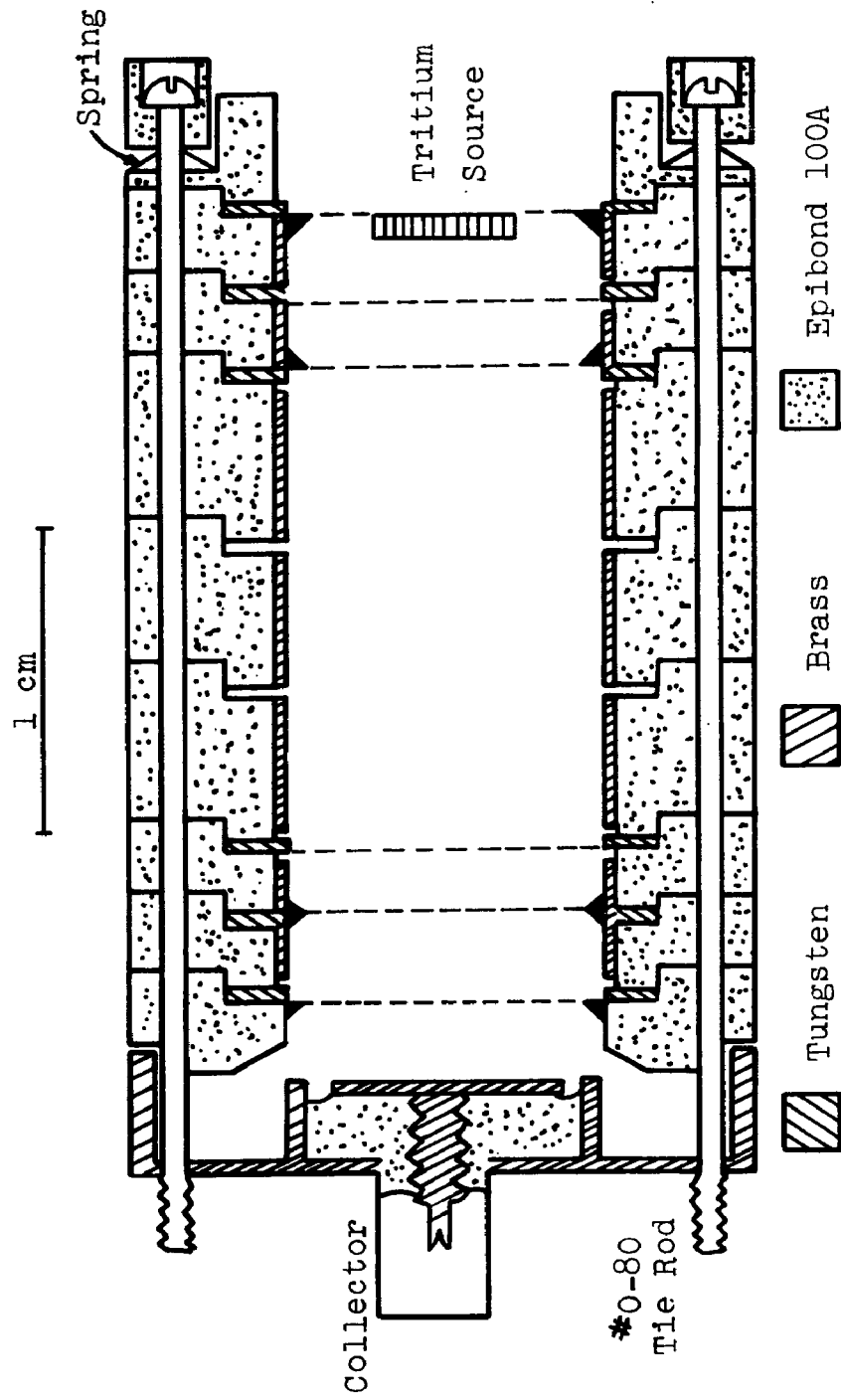


FIG. 20 DETAILS OF THE ELECTRODE ASSEMBLY.

to fill the cell--a major consideration for 6 percent mobility studies, but not important at the 100 ppm level.

The body of the cell, to which the ion electrode assembly is attached, contains a cavity for the CMN thermometer. The electrometer lead is brought out through the center of the CMN and consists of a phosphor bronze wire inside a rod of Epibond 100A shielded with CA-10. An Epibond 100A plug caps the CMN space. The plug and the cell top with the thermal contact brush are sealed to the body of the cell with Epibond 121. (These seals could be cut apart with a thin, circular saw mounted in a milling machine.) To insure thermal contact, the CMN space is connected to the rest of the cell by a series of holes around the guard ring.

The germanium resistors (which replaced the CMN for thermometry, when magnetic problems developed) are mounted on the removable end plug, inside the CMN cavity. A 100 ohm Speer carbon resistor and an Evanohm wire heater are mounted on the top of the CMN cavity. The 0.005 inch dia. phosphor bronze electrical leads are brought out through the cell wall and sealed with Epibond 100A. The electrode assembly mounts on the collector guard ring; it is held together by No. 0-80 brass tie rods, threaded into the cell body. The gold-plated collector is hard-soldered to a brass screw threaded into an Epibond 100A insulator cast into the guard ring.

The grids¹² are gold-plated tungsten mesh, spot-welded to a tungsten annulus, with a transmission coefficient of 85 percent. These grids exhibited strange magnetic properties, and should not be used if magnetic thermometry is anticipated. The grids are held in place with

epoxy spacers machined from Epibond 100A cast on a thin-walled brass sleeve to provide a conducting wall for the cell. The brass sleeve, guard ring, and annulus on the grids are slit to reduce eddy currents. The electrical leads are fastened to the brass sleeves and tungsten grids with CA-10 epoxy. The radioactive source is a tritiated-titanium film, vapor deposited on a copper foil substrate. The copper foil is epoxied to grid A with CA-10; it is placed just below the thermal-contact brush to avoid a temperature gradient in the cell.

To allow for differential contraction between the epoxy spacers and the brass tie rod, a leaf spring, cut from phosphor-bronze shim stock, is placed under the tie-rod retaining ring. The cell is filled through a 1/64 inch o.d. Cu-Ni tube sealed to the cell with Stycast. A second 1/64 inch Cu-Ni tube, not shown in Fig. 19, connects the cell to the level detector. The capacitor of the level detector is placed just below the mixing chamber.

Electronics

The battery supplies, used in the vortex ring studies, (Fig. 18) are part of the double-gate velocity spectrometer electronics. Meters and reversing switches on each battery, and a master switch to reverse the polarity of the cell, are not shown in Fig. 18. Ten-turn potentiometers on the batteries for Gate I and Gate II provide the resolution necessary for determining the cutoff characteristics. The potential difference between each pair of grids can be measured with digital voltmeter with 3 1/2 digit resolution.

The vortex ring current is measured with an operational amplifier electrometer mounted on the pumping head. The electrometer lead from the mixing chamber to the pumping head is described in Chapter VI. Below the mixing chamber, the electrometer lead is a teflon-insulated copper coax, to avoid magnetic problems. The copper, outer conductor of the coax is slit longitudinally to reduce eddy currents. The slit is varnished, and shielded with CA-10.

Experimental Procedure

To check the operation of the cell, pure He⁴ was condensed from a 10 liter storage tank, through a liquid nitrogen cold trap. The amount of He⁴ condensed from the storage tank was measured with a precision Bourdon gauge.¹³ This gauge, together with an accurately measured 1 liter flask, had been used previously to calibrate the volume of the storage tank and connecting lines to the cell. (These volumes are known to an accuracy of 0.5 percent.)

The mixtures were made by adding He³ to the cell fill line after the He⁴ had been condensed, with the refrigerator at 200 mK. It is impractical to condense each mixture separately, since it requires six hours to empty the cell at 4 K. To make mixtures in the ppm range, He³ is admitted to the Bourdon gauge and the pressure (approx. 100 Torr) accurately measured. The He³ in the gauge is then expanded into the connecting lines several times to reduce the pressure in the gauge by a known amount. When the appropriate quantity of He³ is left, the

connecting lines are evacuated, and the He3 remaining in the gauge is condensed into the cell. Approximately 14 hours was allowed to condense the He3 and establish equilibrium.

With the appropriate mixture condensed, the source voltage was turned on several hours prior to the actual measurements to allow the surface charge layer to reach equilibrium. The drift field was set to (approximately) compensate for the frictional loss, and the electric fields in the remaining regions were adjusted to maximize the current. The stopping potential in Gate I was then measured by plotting the vortex-ring current at the collector as a function of the voltage between grids A and C. At cutoff, this voltage minus V_s , the (undetermined) voltage on the source, is equal to the energy lost by the rings through friction between A and C. After the Gate I cutoff characteristic was plotted, the electric field in Gate I was adjusted to admit vortex rings of the desired energy to the drift region, and the Gate II stopping potential was measured. The collector current was plotted as a function of the voltage between grid A and E. Once again, the voltage at which the current vanishes (minus V_s) is equal to the energy lost by the rings in traversing the distance between A and E. Both cutoff characteristics were taken with the same value of drift field.

To measure the dependence of the friction coefficient on the energy of the ring, a set of Gate II characteristics was taken by changing the energy of the vortex rings in Gate I (with the drift field held constant). The Gate I cutoff characteristic was checked periodically, to insure that the source potential had not changed during the course of the Gate II measurements. The entire procedure was then repeated for different values of drift field and temperature.

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2. For a general review of ions and vortex rings see:

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G. Gamota, Supplement au Journal de Physique, Fasc. 10, C3-39, 1970.

F. Reif, Quantum Fluids, Proceedings of the Batsheva Seminar, Edited by Nathan Wised, (Gordon and Breach, New York, 1970).
3. See, for example, A. B. Bassett, Treatise on Hydrodynamics, (Dover Publ. Inc., New York, 1961) Vol. II, Chapter XIV.
4. More accurately, the impulse; see: C. C. Lin, Liquid Helium, Proceedings of the International School of Physics, "Enrico Fermi," Course 21, Edited by G. Careri, (Academic Press, New York, 1963), p. 98.
5. There are several possible models for the core of the vortex ring. The constants (2 and $1/2$) after the logarithmic term in equations (1a) and (1b) correspond to a hollow core. A core filled with fluid, rotating as a solid body, corresponds to constants of ($7/4$ and $1/4$).
6. Equation (3) is valid as long as $\ln(8R/A) > 1$. This is no restriction since (1a), (1b), and (1c) are valid only for $R \gg A$.
7. M. Kuchnir, J. B. Ketterson, and Pat Roach, Phys. Rev. Letters 26, 879 (1971).
8. W. Titus, Phys. Rev. A 2, 206 (1970).
9. M. Kuchnir, J. B. Ketterson, and P. R. Roach, to be published.
10. The double gate ion velocity spectrometer was originally used by A. M. Tyndall and C. F. Powell for ion studies in gases, and was adapted for liquid helium work by F. Reif and L. Meyer. (Phys. Rev. 119, 1164 (1960)).
11. The brush is similar in construction to ones used at the University of Illinois. See M. Kuchnir, Ph.D. Thesis, (University of Illinois, Urbana, Illinois, 1966), p. 80.

12. The grids are made by Eimac Corp. for 2C39A lighthouse tube.
13. Wallace and Tiernan FA-145.

CHAPTER VIII. ANALYSIS OF THE FRICTION COEFFICIENT

Introduction

The energy lost by the vortex rings in the drift field is given by the difference between the stopping potentials measured at each gate (neglecting loss in gates). The friction coefficient is equal to this loss divided by the average value of $\chi(E)$ in the drift region. Since $\chi(E)$ is not measured directly in this experiment, a computer program, which solves equations (3) of Chapter VII in each region of the cell, is used to calculate the friction coefficient.

Stopping Potential

Examples of the Gate I and Gate II cutoff characteristics taken at four different He3 concentrations, are shown in Fig. 21. (The curves have been drawn smoothly through the point-by-point plot of the I vs. V gate characteristics.) With the exception of the Gate II curves at 80 ppm, the cutoff curves approach the abscissa with zero slope, making the exact stopping potential subject to some uncertainty. Several factors contribute to the formation of this "toe" on the cutoff curve. As illustrated in A of Fig. 22, the cutoff characteristic is a step function if all the rings created at the source have the same energy. Experimentally, a monoenergetic beam of rings is hard to realize.

The tritium source emits beta particles, with an 18 keV end-point energy, that have a range of 0.1 mm in the liquid helium. Since ions can be created anywhere along this path, the vortex rings are "born"

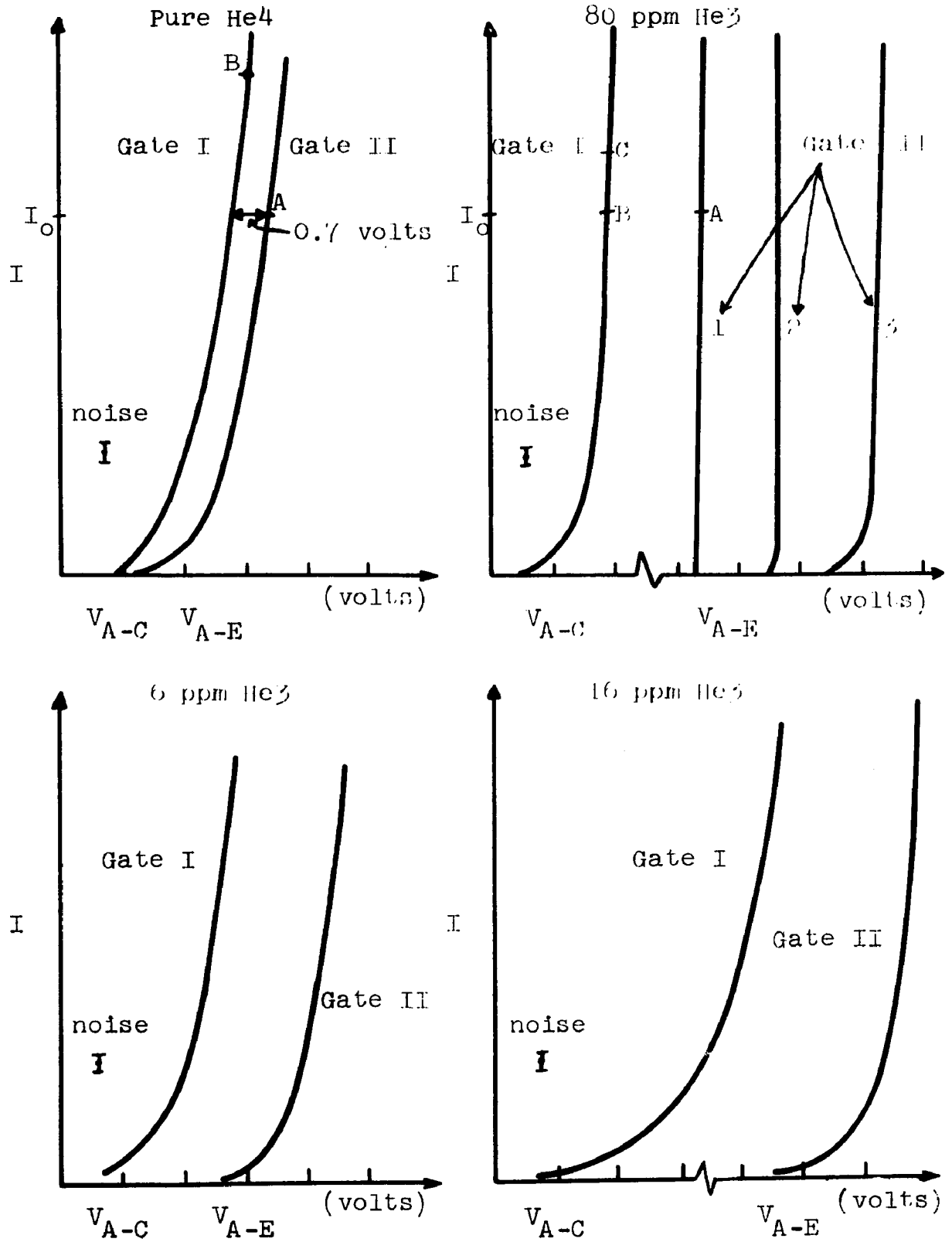


FIG. 21 GATE I AND GATE II CUTOFF CHARACTERISTICS. COLLECTOR CURRENT IS PLOTTED AGAINST THE VOLTAGE BETWEEN THE SOURCE AND GRID AT THE FAR END OF THE GATE.

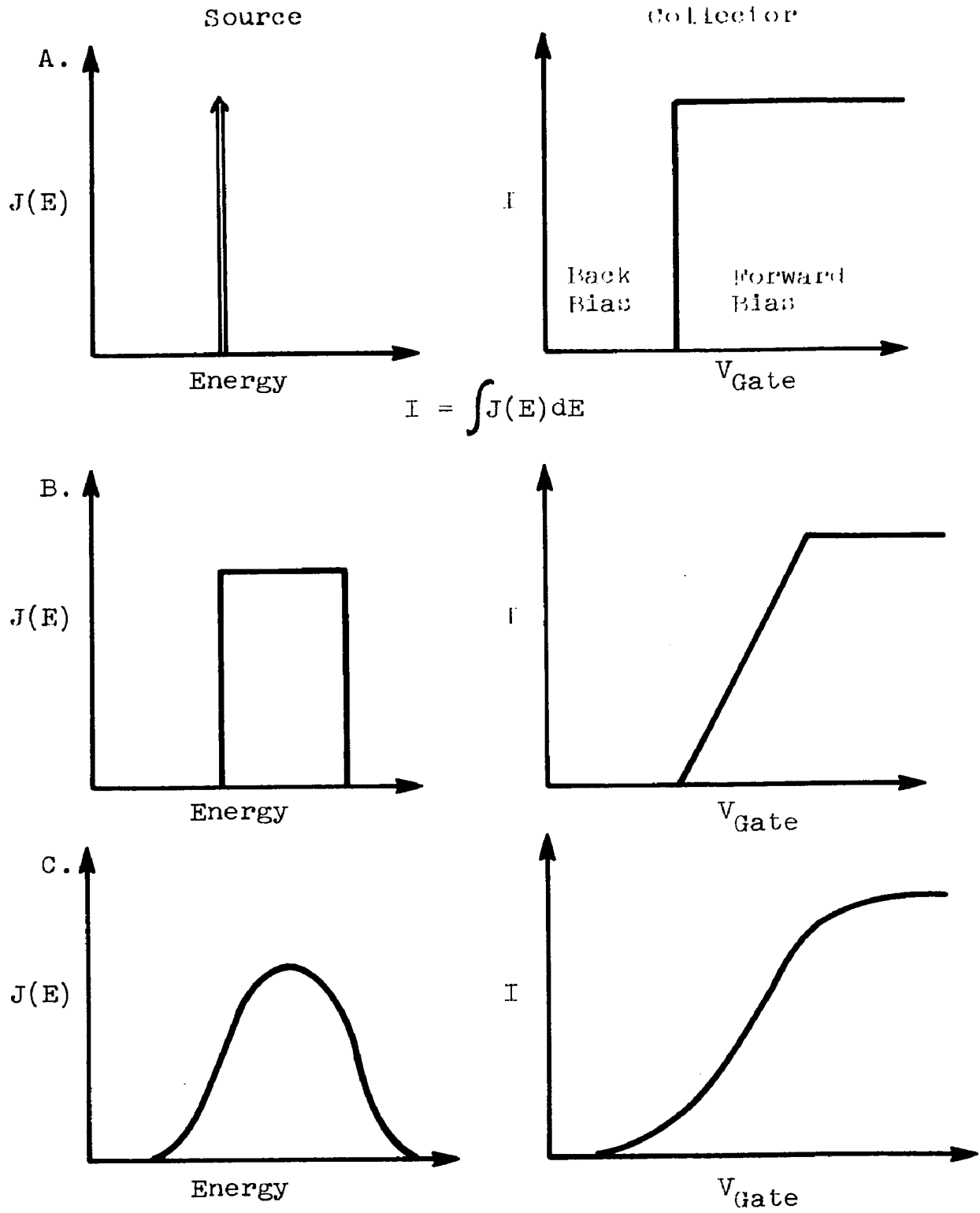


FIG. 22 IDEALIZED CUTOFF CHARACTERISTICS FOR VARIOUS VORTEX RING ENERGY DISTRIBUTIONS.

at different places in the electric field. This effect gives rise to a 5 percent spread in the energy of the rings. Source charging can add to this variation in energy. If the layer of charge is not uniform, the potential will vary across the source, and the rings will not be accelerated by a uniform field. The shape of the cutoff curves for two possible ring energy profiles is shown in B and C of Fig. 22.

In addition to effects at the source, any ring with a transverse component of velocity will contribute to the toe of the cutoff curve. A ring entering the gate region at an angle ω to the axis of the cell will require a smaller stopping field in the gate. The stopping potential¹ for this ring is reduced by a factor of $(1 - \omega^2)^{1/2}$. This effect is accentuated by the fact that any ring, whose trajectory is not parallel to the axis of the cell, must travel slightly farther, losing more energy than an axial ring. From these considerations, it can be seen that the voltage at which the current vanishes is complicated, and not a particularly good measure of ring energy.

Fortunately, the intercept is not needed to calculate the friction coefficient. Each point on the stopping potential curve corresponds to the "death" of a ring with a certain energy. All that is necessary is to choose a point on the Gate II characteristic and find the place on the Gate I curve that corresponds to a ring of the same energy. The friction coefficient can be calculated from any such pair of points. In fact, the friction coefficient can be measured as a function of ring energy using the entire cutoff curve.

In pure He4 (Fig. 21), the shape of the two curves is different in the toe region. Both curves appear to intersect the voltage axis

at the same point--as they should in the absence of friction. (The curves in pure He⁴ were taken at 60 mK.) Since the slope of the "steep" portion is very similar for the two curves, it is likely that the Gate II curve is attenuated relative to Gate I; a ring that "died" at point A in Gate II dies at point B in Gate I.

The 80 ppm curves indicate that the addition of friction improves the shape of the cutoff curves. These curves have a steeper slope and, under certain conditions, Gate II loses its toe. The 80 ppm, Gate II characteristics also illustrate the measurement of α as a function of $\chi(E)$. The set of Gate II curves was generated by increasing the energy of the rings at Gate I by one volt between each curve. (Curve 3 corresponds to the most energetic rings.)

Two different methods have been used to analyze the cutoff curves and select the stopping potential. The first method was an attempt to find an empirical equation that would fit both the Gate I and Gate II curves for a given concentration. A non-linear least squares fit,² of the cutoff data was made to the general equation³

$$I = C(V - V_c)^n, \quad V > V_c$$

$$I = 0, \quad V < V_c.$$

V_c is the cutoff voltage, and the quantity of interest. C , n , and V_c are the free parameters that can be simultaneously varied. When the exponent n is allowed to vary as a free parameter, the best fit to the data is obtained with a value of $n \geq 4$, and a V_c several volts lower than any

reasonable extrapolation of the toe. Clearly, this value of V_c is not the desired cutoff voltage. A two (free) parameter equation, in which the exponent n is fixed, produces more reasonable cutoff voltage. When $n = 2$, V_c is close to the voltage that would be extrapolated from the steep portion of the curve with a ruler, while the cutoff from $n = 3.5$ corresponds to an extrapolation of the toe of the curve.

At 6 ppm, the difference between the Gate I and Gate II cutoff voltage, $[V_c(II) - V_c(I)]$, is (approx.) constant, and independent of the n used in the two parameter fit (for n between 2 and 3.5). Clearly, the $n = 2$ cutoff corresponds to one energy ring, while the cutoff from $n = 3.5$ is associated with a ring of higher energy. The friction coefficient calculated from these cutoff voltages depends only slightly on n .

The second method used to select the cutoff is illustrated on the 80 ppm curves in Fig. 21. A current I_o , well above the toe of the curve, is chosen arbitrarily, and the stopping potential is taken to be the voltage at I_o (points A and B). The curves in pure He^4 indicate that the correct Gate I cutoff is slightly higher than B, perhaps at point C. This error will be small as long as the Gate I curve has a steep slope at I_o . The 0.7 volt shift, between Gate I and Gate II in pure He^4 , provides an estimate of the uncertainty in the Gate I cutoff. This method of selecting the cutoff will be abbreviated as $V(I_o)$ in the future.

At high concentrations, the friction coefficient is independent of the method used to select the cutoff, as will be seen later. At low concentrations, the difference between the two stopping potentials is

independent of the exponent n used in the least squares fit and is always less than the difference between the two curves at I_0 . The pure He^4 data indicate that the two-parameter fit is more appropriate at low concentrations.

Computer Calculation of the Friction Coefficient

The friction coefficient is calculated by a computer program, which solves equation (3), $F(R_f) = 0$ in each region of the cell. Using an initial estimate of the friction, the program takes the ring through the cell, a grid at a time, and calculates the distance required to stop the ring in the retarding field of Gate II. The difference between this distance and the length of Gate II is used to make a better estimate of the friction coefficient. The process is repeated until the ring is stopped at grid E.

The program has three subroutines. The first evaluates the logarithmic integral. A second subroutine calculates the distance required to stop the ring in the retarding field of Gate I and Gate II. (A ring is considered to "die" and be "born" at a radius of $10 \text{ \AA}$, or an energy of 10 mV . Thus both the initial and final radius of the ring are known for this subroutine. (The friction coefficient is expected to be very insensitive to the actual value of the terminal radius.) The third subroutine calculates the final radius of the ring, in each of the remaining regions, by solving for the root of $F(R_f) = 0$. The root is easily found using Newton's method,

$$R_f^{n+1} = R_f^n - F(R_f^n)/F'(R_f^n) .$$

Here R_f^{n+1} and R_f^n are successive approximations to the final radius, R_f , and $F'(R_f)$ is the derivative of $F(R_f)$. The subroutine terminates when $(R_f^{n+1} - R_f^n)$ is less than 5 Å.

The program first calculates the radius of the ring at grid B, using estimates of the source potential and friction coefficient. Next, the program calculates the distance required to stop the ring in the Gate I retarding field. If this distance is not equal to the length of Gate I, the initial value of source potential was wrong. The difference between the stopping distance and the length of Gate I is used to estimate a better value for the source potential. The process is repeated until the ring dies at grid C.

With the source potential determined, for this initial estimate of α , the ring is then started through the entire cell. The radius of the ring is calculated at the end of each region using the third subroutine. When the ring reaches Gate II (grid D), the program again calculates the distance required to stop the ring. If this distance is not equal to the length of the Gate II, the estimate of the friction coefficient is incorrect. α is then changed by several percent and the ring is run through the entire cell again to measure $dz/d\alpha$. A new value of the friction coefficient is calculated using this derivative. The source potential is re-determined for the new α and the program repeated until the ring dies at grid E. Usually two or three iterations are sufficient. It should be noted that the radius and model (e.g., hollow or solid core) of the core are input parameters. Any change in the core with temperature, energy, or concentration will appear as a variation in α .

Results

The results of the computer analysis of the 80 ppm data are shown in Fig. 23. Since the friction is expected to be proportional to the concentration, the ordinate is α/x , where x is the He3 concentration in parts per million. The abscissa of Graph A (Fig. 23) is $\bar{\chi}$, the average value of $\chi(E)$ in the drift region. $\bar{\chi}$ is calculated by the computer program from the equation

$$E_D - E_C = V_D - V_C - \alpha Z_{DC} \bar{\chi} ,$$

where E and V are the energy of the ring and the voltage at grids C and D, and Z_{CD} is the length of the drift region. Data are shown for three different temperatures--indicated by dots, crosses, and stars. The value of the electric field in the drift region is denoted by a circle or triangle around the mark that specifies temperature. Since it is not $\mathcal{E}$, but $\mathcal{E}/\alpha$ that enters the differential equation, $\chi(E_\infty)$, equal to $\mathcal{E}/\alpha$, is used to designate the electric field (where E_∞ is the asymptotic energy of the ring).

Figure 23 indicates that α systematically increases with an increase in the energy of the ring ($\bar{\chi}$) and the electric field in the drift region. This variation is not expected. (Since the variation in α may be an artifact, the electric field is designated by χ_0 equal to $\mathcal{E}/\alpha_0$, where $\alpha_0 = 0.030x$.) An examination of α measured at the same value of $\bar{\chi}$ and χ_0 , but different temperatures, reveals that the friction coefficient changes by 5 percent between 60 mK and 100 mK, and is independent of

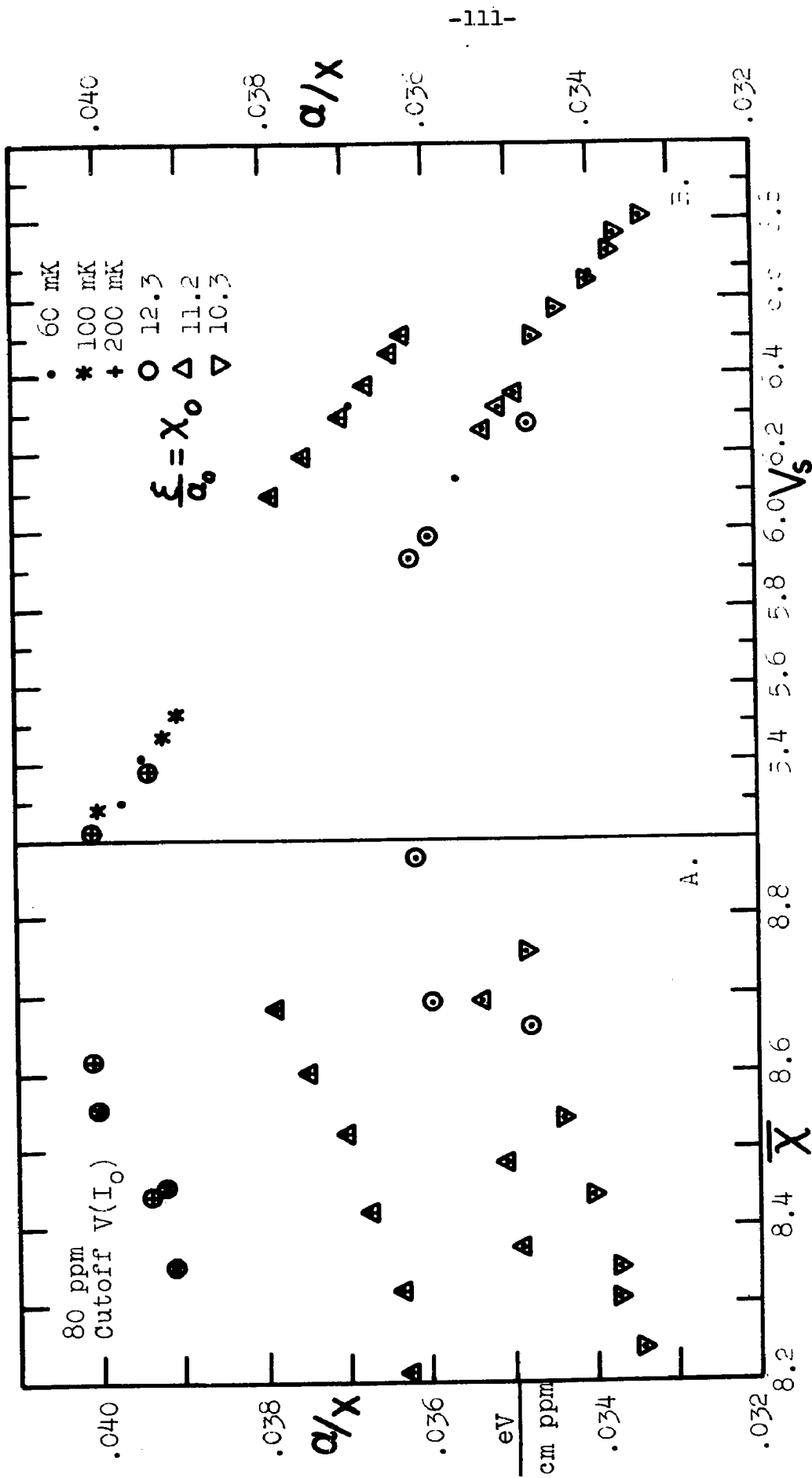


FIG. 23 α/x AT 80 ppm VS. V_s , THE SOURCE POTENTIAL, AND $\bar{X}$, THE AVERAGE VALUE OF $[\ln(8R/\lambda) - 1/2]$ IN THE DRIFT REGION.

temperature above 100 mK. The $V(I_0)$ method is used to select the cutoff voltages for Fig. 23.

The source potential calculated by the computer program is a function of α (Graph B, Fig. 23). Although it would seem that the potential of the source V_s should be a constant, V_s is calculated from the energy of the rings and reflects the energy profile of the vortex beam. It is conceivable that the variation of the source potential with friction is an artifact of the computer program. Any pair of values of V_s and α , that lie on the α vs. V_s curve in Fig. 23 might cause the program to terminate before the best value of α is found. To test this possibility, each set of measurements at the same T and χ_0 was recalculated using as initial conditions, a single pair of values for V_s and α from the α vs. V_s curve. If the program were indeed terminating too soon, a new value of the friction coefficient would be found. This was not the case, and α and V_s were unchanged.

It is reasonable to inquire whether the uncertainty in cutoff is responsible for the systematic variation of the friction coefficient with $\bar{\chi}$ and χ_0 . Figure 24 shows the friction coefficient calculated using different methods to select the cutoff. Graph A contains all values of α calculated using stopping potentials from the least squares program. Graph B shows the values calculated with the $V(I_0)$ method. The bar through each point illustrates the effect of a ± 0.7 volt shift in the Gate I characteristic--0.7 volt being the difference between the two gate characteristics at the current I_0 in pure He⁴. (The points in B fall on top of the curves in Graph A, and are plotted separately for clarity.) The

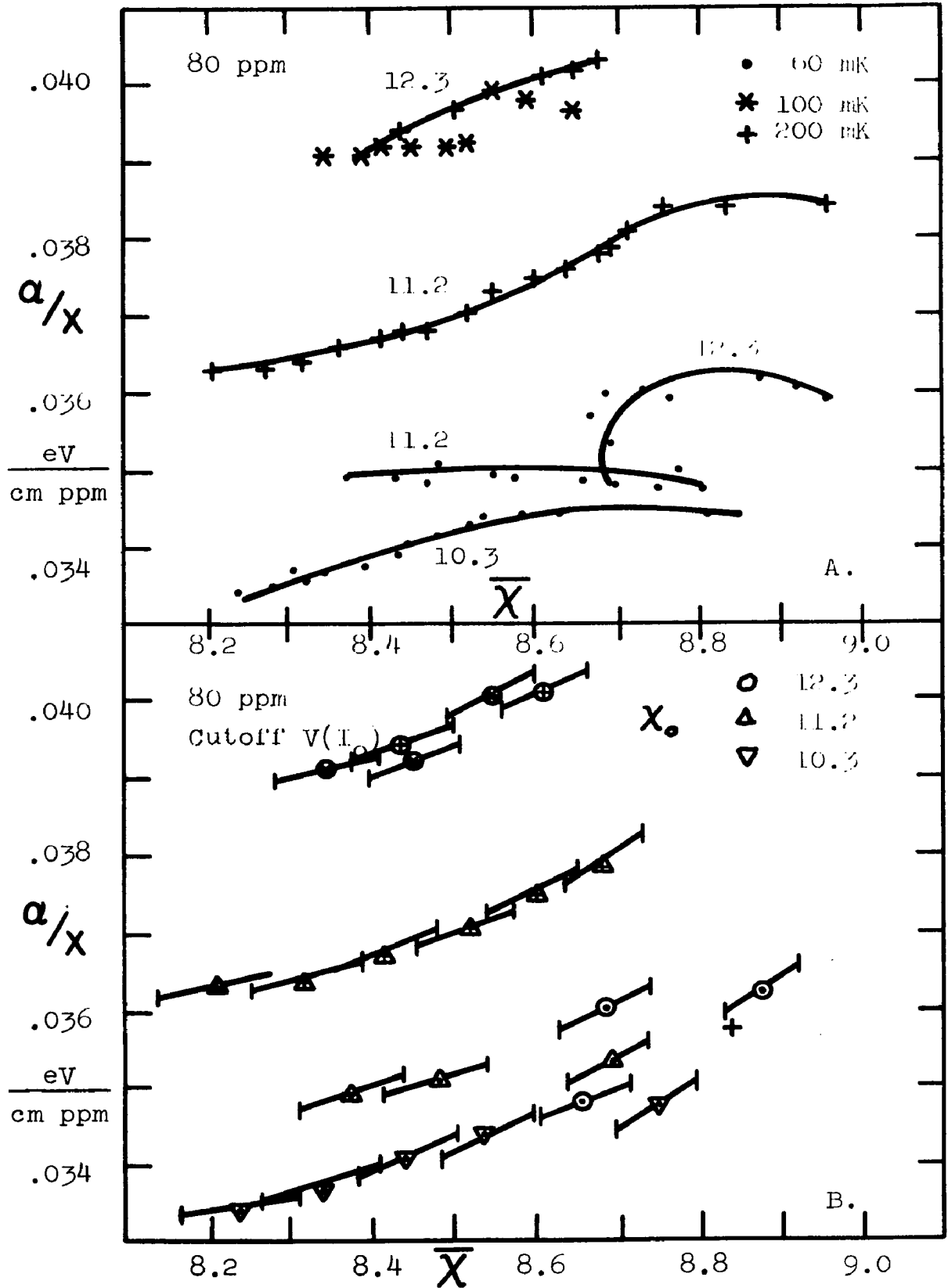


FIG. 24 THE FRICTION COEFFICIENT CALCULATED USING DIFFERENT METHODS FOR SELECTING THE CUTOFF VOLTAGE.

systematic variation of α with $\bar{\chi}$ and χ_0 is obviously not caused by the uncertainty in the cutoff.

The data have been analyzed using the parameters for a vortex ring with a hollow core of 1.28 Å--the pure He4 value measured at 0.27 K. It is quite possible, however, that the core radius does not remain constant, but increases at lower temperatures as He3 is condensed on the core.⁴ Figure 25 illustrates a variation of the core parameters. Only one value of χ_0 and temperature is shown, since the net effect is just a shift in the origin. If the core of the vortex ring is filled with fluid, rotating as a solid body, the constants after the logarithmic terms in Eqs. (1a) and (1b) should be 7/4 and 1/4, instead of 2 and 1/2. The change in α with a solid core is small, and, more important, the dependence of α on energy and field is unchanged. Increasing the vortex core increases the friction coefficient and shifts the curves to smaller values of $\bar{\chi}$. The effect of a 6 percent (1 mm) increase in the length of the drift region is also illustrated in Fig. 25.

The analysis of the 40 ppm data corroborates the observations at 80 ppm, although the variation of α with $\bar{\chi}$ is not as marked (Fig. 26). More important, α/χ is the same for the 40 and 80 ppm mixtures. Data taken in the 16 and 6 ppm mixtures are shown in Figs. 27 and 28. Stopping potentials for these last two concentrations were calculated using the two parameter least-squares program. Each pair of cutoff curves is plotted twice in these graphs, once for $n = 2$ and again for $n = 3.5$. The point labeled "a" is calculated using the $V(I_0)$ cutoff; the line through "a" indicates a ± 0.7 volt variation in Gate I. The data at the two lower concentrations have rather large uncertainties.

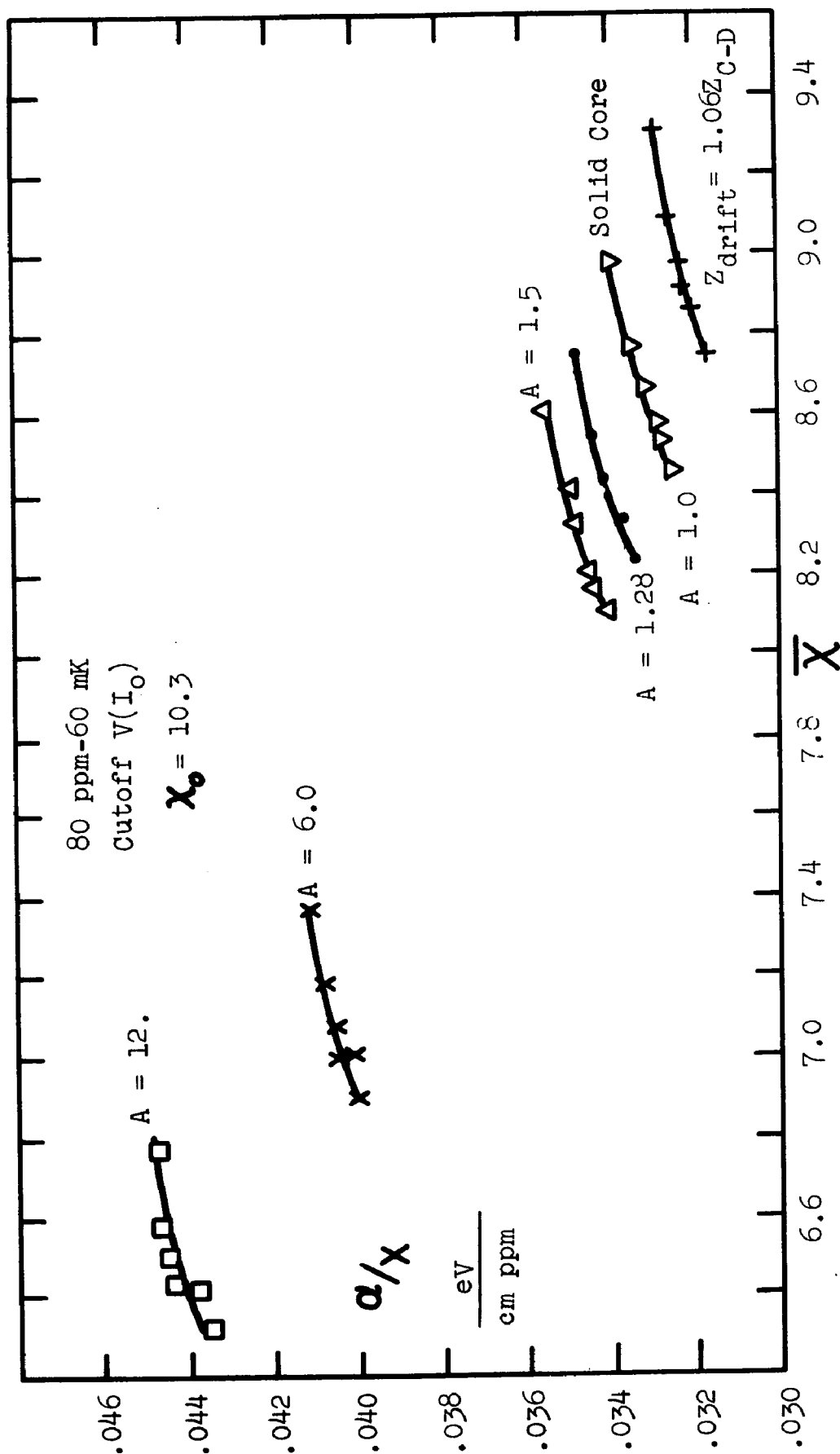


FIG. 25 α/x vs. $\bar{X}$ FOR VARIOUS CORE PARAMETERS. A HOLLOW CORE HAS BEEN ASSUMED EXCEPT WHERE INDICATED. ALSO ILLUSTRATED IS THE EFFECT OF AN INCREASE IN THE DRIFT REGION.

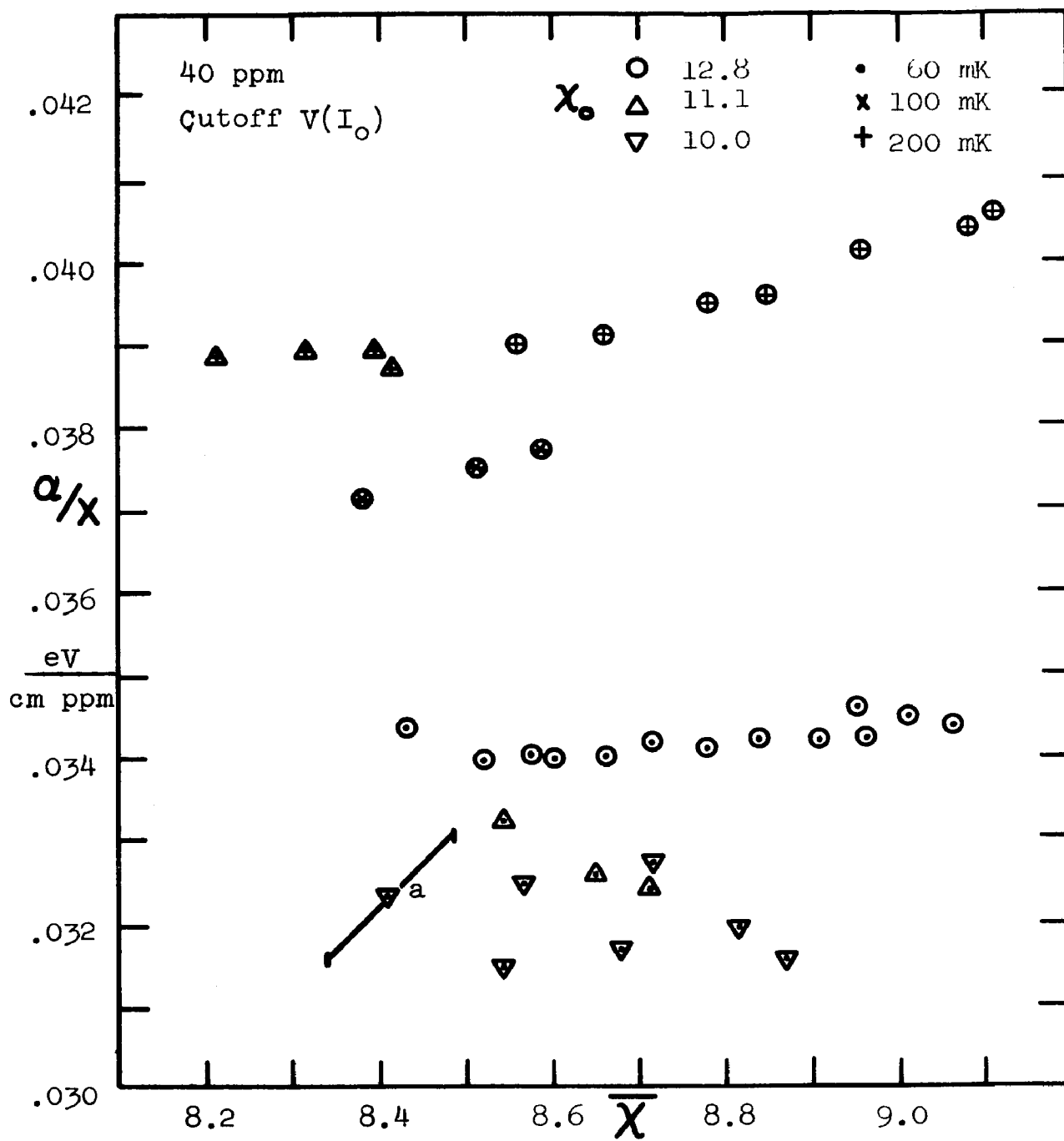


FIG. 26 α/x AT 40 ppm VS. $\bar{\chi}$. "a" SHOWS THE EFFECT OF A ± 0.7 VOLT SHIFT IN THE GATE I CUTOFF VOLTAGE.

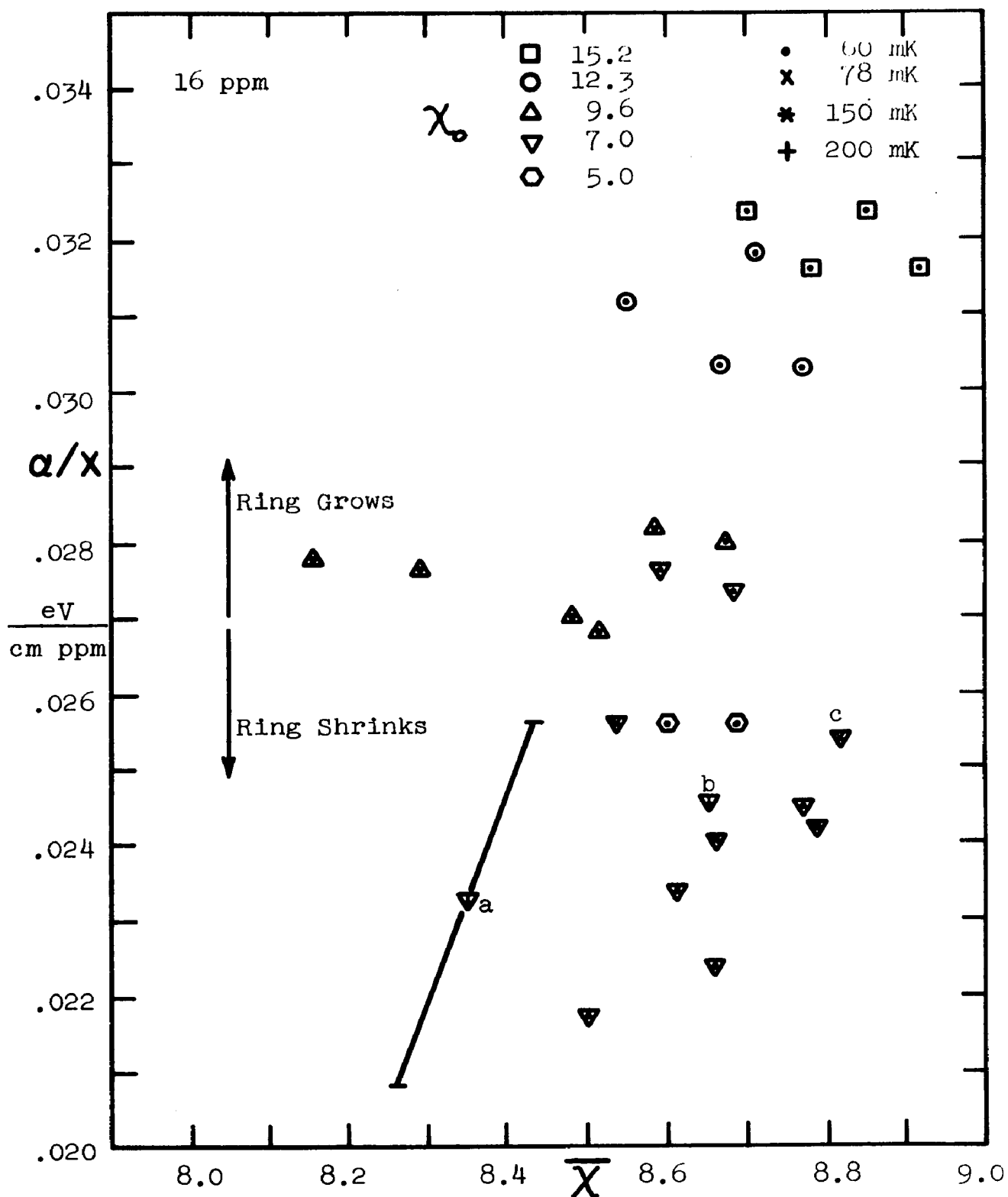


FIG. 27 α/x AT 16 ppm VS. $\bar{\chi}$. "a" IS CALCULATED USING THE $V(I_o)$ METHOD, WHILE "b" ($n = 2$) AND "c" ($n = 3.5$) ARE CALCULATED FROM THE LEAST SQUARES FIT PROGRAM.

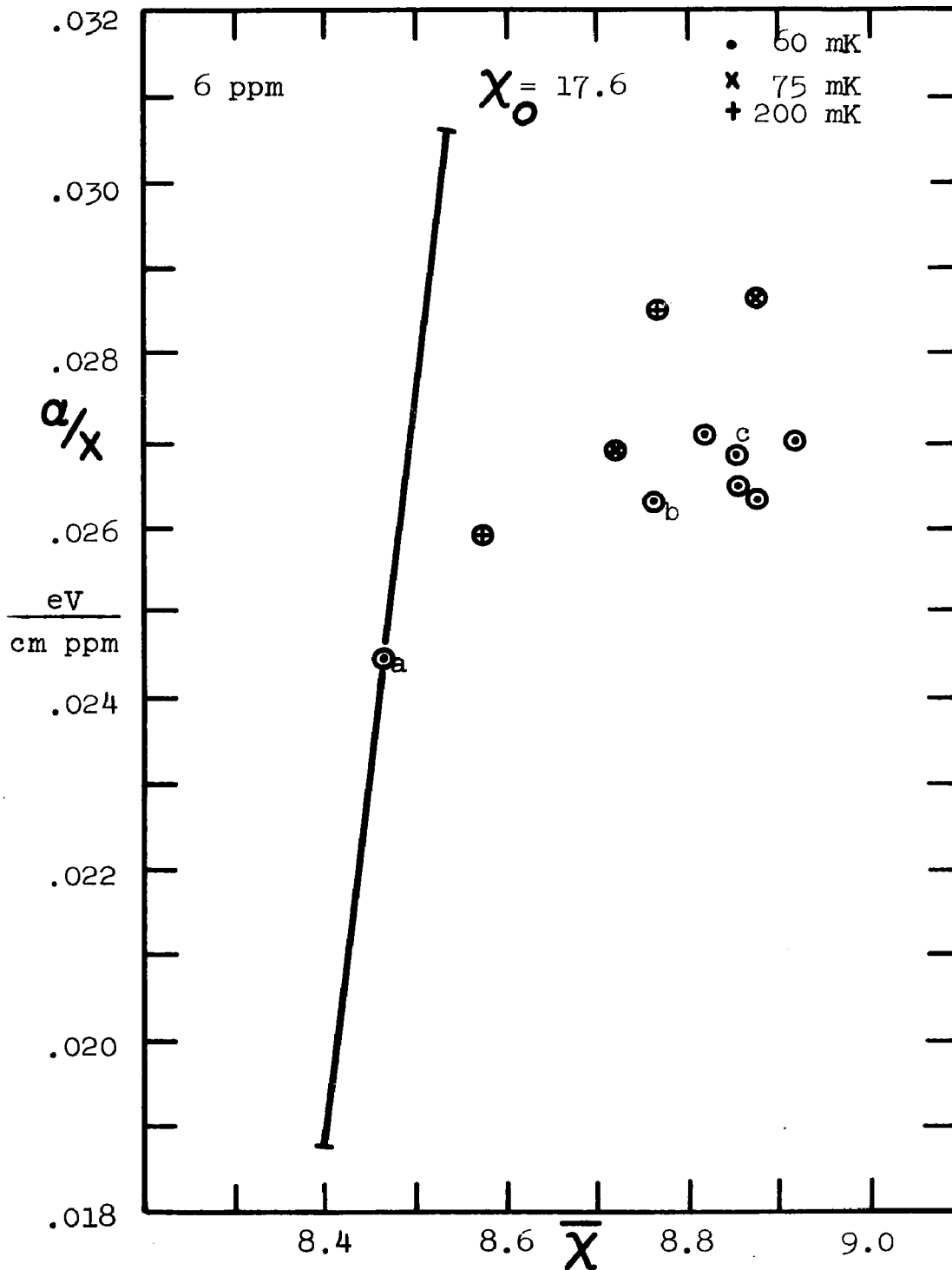


FIG. 28 α/x AT 6 ppm VS. $\bar{X}$. "a" IS CALCULATED USING THE $V(I_0)$ METHOD, WHILE "b" ($n = 2$) AND "c" ($n = 3.5$) ARE CALCULATED FROM THE LEAST SQUARES FIT PROGRAM.

The measurements at 80 ppm provide the most accurate determination of α , because the friction is large and the fractional uncertainty in the cutoff voltage is small. It is difficult to assign an absolute error to the determination of α because of its systematic variation with energy and field, which is not understood. The error (at 80 and 40 ppm) due to the ambiguity of the stopping potential is much smaller than these systematic variations. A measure of the relative error for each mixture is indicated by the $V(I_0)$ cutoff method. Figures 24, 26, 27 and 28 (point "a") show the change in α with a ± 0.7 volt shift in the Gate I characteristics. This estimate of the error is not proportional to x , since the line that connects the values of α calculated from $(V_c \pm 0.7)$ tilts at high friction to lie along the α vs. $\bar{X}$ curve. The error at 40 ppm is five times the error at 80 ppm, while this estimate indicates that the 16 and 6 ppm measurements are 16 and 40 times as uncertain as the 80 ppm data.

The value of α/x for the 16 and 6 ppm mixtures is approximately 30 percent lower than at 80 or 40 ppm. This may be an artifact of the relatively large error at the lower concentrations. This large uncertainty may also be responsible for the fact that no systematic variation of α with $\bar{X}$ and T can be seen at 16 and 6 ppm.

The dependence of the friction coefficient on temperature (at 40 and 80 ppm), although small, is undoubtedly not an artifact. It is entirely consistent with an increase in the core radius below 100 mK --Figure 25 indicates a core radius of 3 or 4 Å below 100 mK. On the basis of these measurements, there is no way to distinguish

between a decrease in α and an increase in the core radius as temperature is decreased.

There are several possible reasons for the observed dependence of α on $\bar{\chi}$ and χ_0 . It is known that the drag on the positive ion trapped on the ring will contribute to the total friction.⁵ This effect will make the calculated value of α size-dependent. However, the ion occupies less than 0.5 percent of the circumference of the rings shown in Fig. 23. No difference between the He3 cross section for an ion and a ring can possibly account for the 6 percent change in the friction coefficient that is seen in Fig. 23. It is also unlikely that the core radius is a function of energy and field.

The small internal volume of the cell dictates a very unfavorable aspect ratio (the ratio of cell diameter to length) for vortex ring measurements. A ring that does not travel along the central axis of the cell experiences appreciable transverse electric fields. Since the dynamics of the ring depend on energy, field, and friction, it is quite possible that the dependence on $\bar{\chi}$ and χ_0 is the result of a systematic lengthening of the path that the ring travels in the drift region (see Fig. 25). Operationally, a measurement of the friction coefficient in pure He4, at temperatures high enough to provide the same friction as the 80 ppm measurements, would distinguish between systematic errors due to cell design and effects attributable to the He3. Unfortunately, the only pure He4 measurements in the present apparatus were made at 60 mK.

Summary

The friction coefficient α for positively charged vortex rings has been measured at 60, 100, and 200 mK in dilute He³-He⁴ mixtures at concentrations x of 6, 16, 40, and 80 ppm of He³. A stopping potential method was used. The measurements are analyzed using the classical hydrodynamic equations for a vortex ring with a hollow core of 1.28 Å. The effect of changing the core parameters has been investigated. The friction coefficient calculated from these measurements depends systematically on the energy, electric field, and temperature. The 80 and 40 ppm mixtures indicate that the friction is proportional to the concentration, while α/x decreases for the two lowest concentrations. The large error associated with low friction may be responsible for this apparent decrease.

At 80 and 40 ppm, α/x is 0.037 eV/(cm ppm) for temperatures between 100 mK and 200 mK, decreasing by 5 percent at a temperature of 60 mK. This decrease is consistent with a constant α and an increase of the core radius to 3 Å. The dependence of α on electric field and ring energy is not understood, and limits the accuracy of the measurements to 20 percent at 80 ppm. The error in α due to uncertainties in the stopping potential is a factor of ten smaller. At 16 and 6 ppm, α/x is 0.028 eV/(cm ppm). The scatter in the data obscures any dependence of α on temperature and energy.

Previously, measurements of the friction coefficient⁶ have been made at 28 and 106 ppm. Rayfield and Reif⁷ report the value α/x of 0.05 at 28 ppm and 280 mK. Kuchnir, Ketterson and Roach⁸ have made

measurements in 10^6 ppm mixtures at temperatures of 19, 85, 370 and 550 mK. They report an α/x of 0.07, 0.064, 0.069, and 0.067 eV/(cm ppm) at these temperatures, with core radii of 15, 11, 4.5, and 2.8 Å, respectively.

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APPENDIXES

APPENDIX A. THE MARK I DILUTION REFRIGERATOR

This appendix was presented as a paper at the Conference on Low Temperature Physics (LT 11) held at St. Andrews in 1968. It was entitled "Operation of a Dilution Refrigerator" and is printed in the Proceedings of the Eleventh International Conference on Low Temperature Physics, edited by J. F. Allen, (University of St. Andrews Press, St. Andrews, Scotland, 1968) p. 661. The figures in this appendix are taken from slides shown at the conference.

We have recently constructed a ^3He - ^4He dilution refrigerator of medium capacity which reached 10 mK in continuous operation. The refrigerator is based on the design proposed by Hall^{1,2}, in which ^3He is circulated through a standing column of ^4He , and is similar to refrigerator II described by Wheatley, Vilches, and Abel.³ The design is quite flexible; some of the construction details reported here can readily be modified.

Circulation of the ^3He is provided by a 2 in. NRC B-2 oil booster diffusion pump backed by a sealed Welch 1402 mechanical pump. The maximum circulation rate of this system is about 7×10^{-5} moles sec^{-1} . However, in practice the circulation rate was limited to about 2×10^{-5} moles sec^{-1} because at this rate a misdesigned flow impedance in the condensing line raised the condensing pressure to 1 atm.

The "still" or "evaporator" was pumped through a tube 0.30 cm in diameter, and contained a heated diaphragm baffle designed to reduce ^4He film flow into the tube. This design was not particularly successful,

since the circulating vapor contained about 20% ^4He . Nevertheless, it should be noted that the large pumping tube allowed the still to operate at 0.4 K, well below the phase separation temperature of the returning mixture. Thus the heat generated by phase separation was transferred to the still rather than to the heat exchanger. During construction of the still it was found that Cd-Bi eutectic solder, which is recommended for its high thermal conductivity,⁴ is mechanically unreliable under thermal cycling.

The refrigerator uses four heat exchangers between the still and mixing chamber, each one containing two copper powder sponges sintered into holes in a single OFHC copper block.⁵ The sponges were made from 325 mesh 99.99% pure copper powder compressed in thin layers to 300 atm, then sintered at 625°C for 1/2 hour in a hydrogen atmosphere. When this procedure was used the sponges adhered to the copper block. The sponges carrying the concentrated phase of the mixture contained 0.7 g of powder; those carrying the dilute phase contained 3 g. The Cu-Ni connecting tubes were each 5 cm long, with .025 cm i.d. on the concentrated side and 0.14 cm i.d. on the dilute side.

The mixing chamber was machined from Epibond 100A, and for thermometry contained a 220 ohm Speer resistor and 1 g of powdered cerium magnesium nitrate in the form of a right circular cylinder with diameter equal to height. The mixing chamber was sealed with Stycast 2850 FT, which we found more reliable than Epibond 121.

The susceptibility of the CMN was measured using phase sensitive detection and an ac Hartshorn inductance bridge⁶ adapted to operational amplifiers. The bridge proved quite sensitive and simple to construct and was relatively inexpensive.

The refrigerator operated with 2.5 STP liters of ^3He and 7 STP liters of ^4He . In order to start the refrigerator we first condensed in all the required gas and then began circulating with pumps and still power. The refrigerator cooled steadily to 20 mK in less than 2 hours after turning on the still power. There was no indication of convective instability.

The lowest temperature reached in continuous operation was 10 mK. For a circulation rate of 1.8×10^{-5} moles sec^{-1} as measured by the applied still power, the refrigerator extracted 3 μW at 30 mK, 25 μW at 50 mK, and 100 μW at 150 mK. These are magnetic temperatures of the CMN cylinder and are quoted here without correction. With no applied heat load typical operating temperatures were 450 mK at the still, 140, 60, 30, and 25 mK for the four heat exchangers, and 11 mK for the mixing chamber.

We are indebted to J. C. Wheatley for hospitality extended to one of us (S. C. W.) and for continuing to inform us of his results. We also wish to acknowledge the support given this work by the U.S. Atomic Energy Commission.

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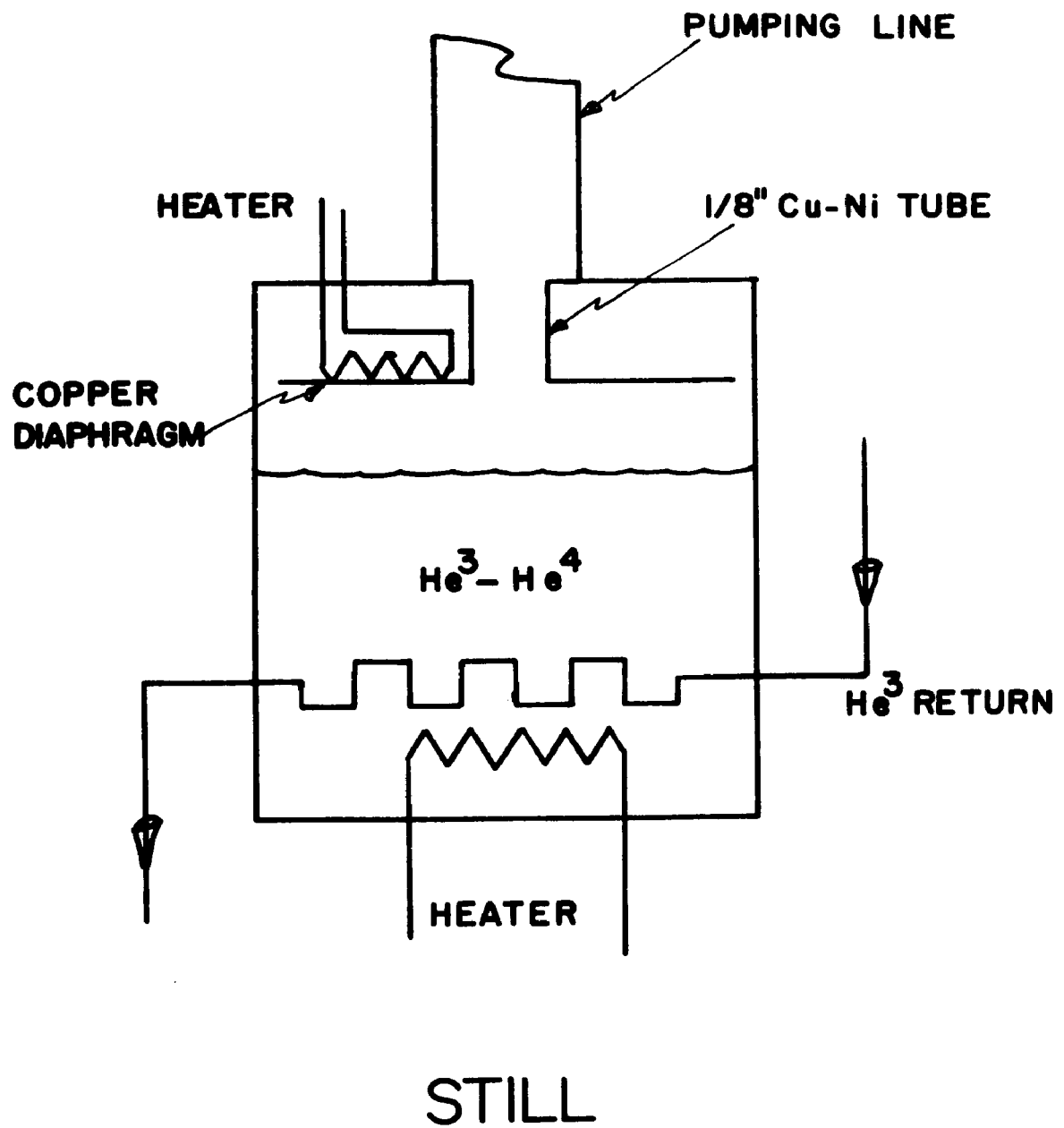


FIG. A-1 THE MARK I STILL.

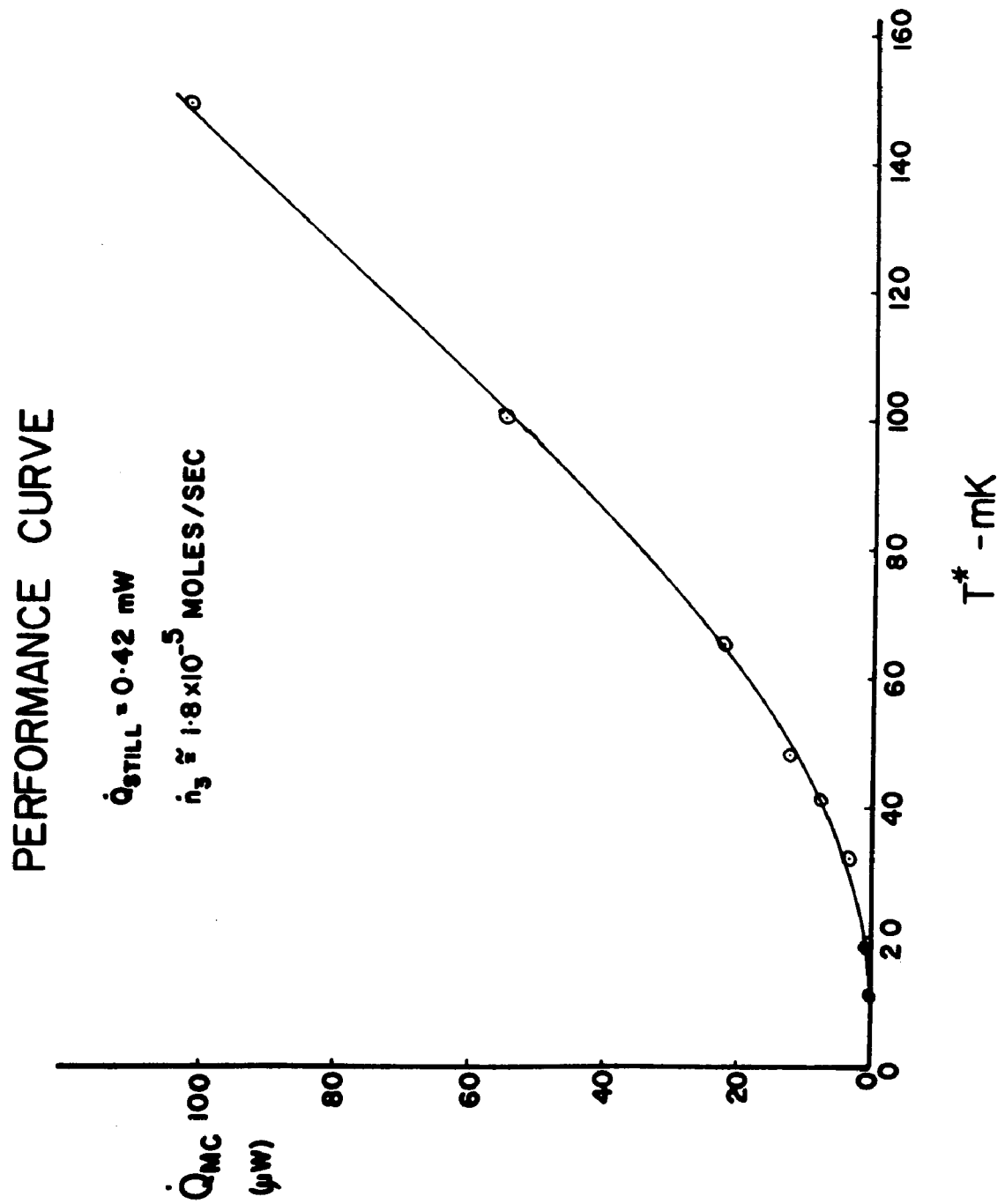


FIG. A-2 PERFORMANCE CURVE .

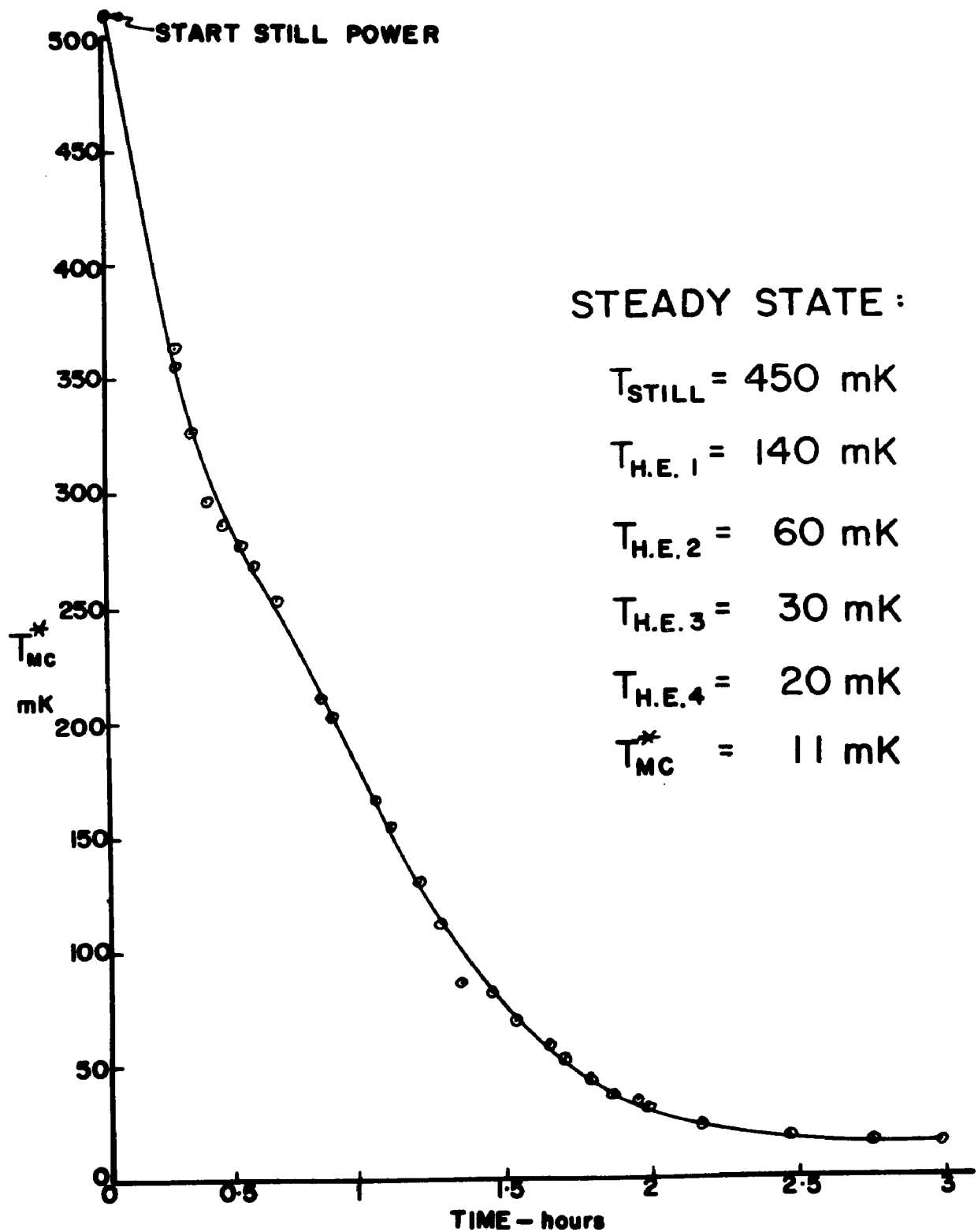


FIG. A-3 TEMPERATURE VS. TIME.

APPENDIX B. DILUTION REFRIGERATOR CHECK LIST

I. Cryostat out of Dewar

1. Check electrical leads
 - a. Resistors
 - b. Heaters
 - c. Experimental leads and experiment
 - d. Level detector
 - e. Coax and electrometer
2. Visually inspect refrigerator
3. Install vibration eliminator on mixing chamber
4. Install radiation shield
 - a. Fasten vibration eliminator
 - b. Visually inspect for stray thermal contact between electrical leads and radiation shield
5. Install indium O-ring and vacuum can
6. Connect susceptibility coil set and check continuity; check bath regulator, thermometer, and heater
7. Install transfer tube extension
8. Install lead counterbalance on pumping head
9. Flush and backfill He Dewar with N_2 gas
10. Start diffusion pump on LN_2 Dewar vacuum space
11. Adjust Pirani gauge--zero and full scale
12. Clean O-rings for pumping head connections

II. Cryostat in Dewar at Room Temperature

13. Lower cryostat into Dewar with winch
 - a. Bolt pumping head into place on Dewar stand
 - b. Remove lead counterbalance and install Dewar pumping line
14. Connect and tighten the following:
 - a. 2 gate valves; He³, He⁴ pumping lines
 - b. 1 Cajon coupling; cell experimental line
 - c. 4 stainless steel flanges; He³ fore-line, He⁴ fore-line, vacuum can, He³ return line
15. Evacuate:
 - a. Vacuum can
 - b. Dewar
 - c. Experimental cell fill line
 - d. He³ return line to Dewar
 - e. B-2 pumps (and outgas B-2's)
 - f. Turn on cooling fan on He⁴ mechanical pump housing
16. Repeat steps Nos. 1 and 6 and
 - a. Zero inductance bridge
 - b. Measure heaters and resistors
 - c. Check bath regulator
17. Check insulation on He Dewar vacuum connections and turn on blower to prevent icing
18. Pump out cryostat lines:
 - a. 1 K pot
 - b. He³ circulation and return line
 - c. Experimental cell--only if no CMN is being used

19. Leak checking (L.C.):

- Notes:
1. Keep cold trap on leak detector filled whenever diffusion pump is on
 2. Leak check in proper order since He clean-up time in each system is long
 3. Connect both ends of He³ system to leak detector (C-1 and C-12) response time though flow impedance is long
 4. Use B-2 pumps on He³ system and 1 K pot to improve sensitivity
 5. Valve off house vacuum and He⁴ mechanical pump, if possible, to improve sensitivity
- a. L.C. external plumbing and connections to the pumping head (including the B-2 pump drain plugs)
 - b. L.C. Dewar to all systems (except cell)
 - c. L.C. cell to all systems--procedure depends on whether or not CMN is in the cell
 - d. L.C. 1 K pot to vacuum can and He³ system

20. Flow impedance:

- a. Seal off He³ system, close C-12, put 1 atm clean He⁴ between C-11 and C-12
- b. With B-2 on, C-1 open, C-5 shut--open C-12 and measure time for Pirani gauge to reach full scale (typically 100 sec)
- c. Shut C-1 and C-2, open C-4 and leak check He³ system to vacuum can

21. L.C. Dewar and pumping head (if necessary)
22. Connect thermocouples, ice bath, and D.V.M.--if monitor of precool is desired
23. Put exchange gas in Dewar and vacuum can
 - a. N_2 gas in Dewar--15 inch vac. precools to 20 inch vac.
(0 inch vac. on Bourdon gauge equals 1 atm)
 - b. H_2 gas in vacuum can--0.5 Torr (makes L.C. at 77 K easy)
24. Pump on He3 system and 1 K pot

III. Liquid Nitrogen Temperatures

25. Transfer LN_2
26. Monitor thermocouple, or thermistor on mixing chamber, for 0 C if CMN is in cell (Note: one lead of thermistor is grounded, other lead is connected to the octal socket on pumping head. Resistance of the thermistor triples between room temperature and 0 C.)
27. Cell flush (only if CMN is in cell):
 - a. Approximately 45 minutes after LN_2 transfer, cell is cooled to 0 C; start pump and flush cycle
 - b. Pump cell for approximately 10 minutes
 - c. Pressurize cell with clean He^4 gas for 3 minutes
 - d. Repeat 10 times
28. Precool overnight or equivalent--check resistors or thermocouple for adequate precool--will precool to 90 K without LN_2 precool plate.
29. Check electrical leads:
 - a. Repeat No. 16
 - b. Test experiment

30. Evacuate vacuum can (H_2 exchange gas makes this easy)
31. Leak check--77 K (see notes for No. 19):
 - a. L.C. cell to all systems by evacuating He^4 from cell--repressurize if any doubt
 - b. L.C. Dewar to all systems
 - c. L.C. 1 K pot to He^3 system and vacuum can
2. Check flow impedance, and leak check He^3 to vacuum can--follow procedure for No. 20 (1/2 atm He^4 at C-12 gives 100 sec to full scale on Pirani)
33. Pump all external gas lines including cold trap
34. Pump He^4 out of all systems:
 - a. 1 K pot
 - b. He^3 --still and return
 - c. cell

Note: Use B-2 pumps and house vacuum-monitor He^4 background with leak detector--pump at least two hours

35. Put He^3 exchange gas in vacuum can--0.1 Torr adequate (Use He^3 bottle on experimental gas handling system.)

IV. Liquid Helium Transfer

36. Flush Dewar with He gas
37. a. Insert 1/2 inch Cu-Ni funnel for transfer tube in Dewar bung to insure that transfer tube mates with extension in Dewar
- b. Turn on heater tape on pumping head to prevent frost (60 volts on Variac)

38. Transfer:

- a. Transfer from front of Dewar with transfer cart slightly to right of Dewar (need extra-long transfer tube)
 - b. Start transfer at 7 inch H_2O pressure, begin to collect LHe in 1/2 hr., then increase pressure to 15 inch or 20 inch H_2O
 - c. Open large pumping valve on Dewar to reduce plume
 - d. Make sure insulated section of transfer tube extends into storage volume of helium storage Dewar for efficient transfer
 - e. Initial transfer--9 to 11 liters is good. (Have used up to 17 liters when transfer tube does not fit into extension.)
 - f. Subsequent transfers 4-7 liters depending on conditions
 - g. 1 K pot often filled on transfer--most economical--start when liquid is above 1 K pot valve, LHe transfer pressure is reduced to 5 inch--7 inch H_2O (i.e., just barely transferring)--Filling takes 1/2 hour, plus--resume normal transfer pressure after pot is filled (see No. 44 below).
 - h. Keep LN_2 level high in Dewar--above 95 cm.
39. Repeat lead and experiment check--see No. 29.
40. Pump bath down to make temperature calibration on CMN, if necessary.
41. Evacuate vacuum can--3 hours plus--check progress on leak detector, L.C. is possible during this period if $He3$ is used as exchange gas.
42. Leak check:
- a. Bath vacuum can, $He3$ system, and 1 K pot
 - b. Cell to vacuum can, $He3$ and 1 K pot
 - c. Pot to vacuum can, $He3$

43. Check flow impedance and leak check He3 system. Repeat No. 20 using 100 Torr clean He4 gas
44. Pot filling procedure
- Start with liquid level 10 cm above 1 K pot valve
 - Put oil manometer on pot (at P-3), pump on pot with mechanical pump, B-2 valved off
 - Open fill valve 7 or 8 turns--pressure in oil manometer will rise to (30-50 cm). Filling takes approximately 1/2 hour.
 - Pot full when liquid level rises above 1/8 inch orifice--pressure on manometer rises drastically--shut valve.
 - Continue pumping with mechanical pump, turn on B-2 pump at 1 cm oil.
 - Pot lasts 36 hr. with 12 hr of He3 circulation--less if refrigerator charge or experimental liquid is condensed

V. Refrigerator Operation

45. Pump external gas handling system and fill cold trap--needs refilling every 8 hours
46. a. Begin to condense material in tank No. 1 via cold trap and C-12. Regulate condensation rate with needle valve S-1.
- b. B-2 may be used on 1 K pot if low starting temperature desired--slightly more economical in pot hold time if it is not
- c. Condensation may also proceed via the 2 inch pumping line to speed process up--must regulate condensation rate more carefully
- d. Shut C-12 and expand gas in back side of sealed He3 pump into tank No. 1, pump contents of tank No. 3 into tank No. 1

- e. Condense material again from tank No. 1
 - f. Condense material from tank No. 2
 - g. Turn on 1 K pot booster pump and condense until equilibrium vapor pressure is reached
 - h. Close C-12, use sealed He3 pump to empty tank No. 1, tank No. 2, tank No. 3 and miscellaneous lines--store gas in back side of pump, then condense via C-12
 - i. Keep liquid level high in LN₂ and LHe Dewar
47. Condense experimental liquid--most economical to condense with 1 K pot booster pump off and refrigerator at 1 K
48. After 1 K pot reaches 0.9 K, circulate mixture with sealed He3 pump, turn on He3 booster pump at 0.4 Torr--system operates as a He3 refrigerator
49. Turn on still heater (nominally 1.8 mA) when B-2 pump at 0.1 Torr
50. Store desired amount of mixture (mostly He3) in tank No. 1, (shut C-12, open S-1 to store)
51. Operation:
- a. Monitor temperature of heat exchangers--watch phase boundary propagate toward mixing chamber--indicated by temperature minimum
 - b. Monitor He3/He4 ratio with leak detector

VI. Overnight procedure:

52. Before leaving unattended
- a. Fill LN₂ Dewar to 100 cm
 - b. Fill LHe Dewar to 95 cm
 - c. Fill LHe Dewar every 10 hours when circulating, do not let level drop below light trap--fill every 12 hours when not circulating He3

- d. Fill pot when filling Dewar
 - e. Fill leak detector cold trap, He3 circulation cold trap
 - f. Leave house vacuum on vacuum can
53. Various modes for leaving refrigerator unattended:
- a. Still power on--system refrigerating--no start-up
 - b. Shut off still power, both B-2 pumps--circulate with sealed He3 pump--system warms to 200 mK--starts easily
 - c. Shut off: (1) still power, (2) both B-2 pumps, (3) sealed He3 pump--long start-up--does not retain phase boundary in mixing chamber

VII. Shut-down procedure:

54. Pump on experimental cell (empties in 5 hr)
55. Store He3 gas in tank No. 1. Shut C-12
- a. Valve off 1 K pot and warm to 4 K
 - b. Increase still power--remove all He3 in 30 minutes--shut tank No. 1 when He3 is removed (check with leak detector)--remove gas between C-12 and flow impedance
56. a. Store remaining gas in tank No. 2, turn off He3 B-2 pump, continue pumping with sealed He3 mechanical pump
- b. Put exchange gas in vacuum can,--refrigerator empties rapidly--shut tank No. 2 when all liquid removed from refrigerator
 - c. Pump gas remaining in lines into tank No. 3, shut tank No. 3
 - d. Pump gas from line to tank No. 3 into exhaust side of sealed mechanical pump, valve off sealed pump

57. Empty 1 K pot with mechanical pump
 - a. Clean 1 K pot, He³ system, and cell with house vacuum
 - b. Keep liquid in both Dewars until cell is empty
 - c. Add exchange gas to LN₂ Dewar if speedy warm-up desired
 - d. Put bubbler on LHe Dewar
 - e. DO NOT TAKE CRYOSTAT OUT OF DEWAR UNTIL WARM
48. Quantity of gas stored in each tank
 - a. Tank No. 1 3 inch vac (mostly He³)
 - b. Tank No. 2 10 inch vac (mostly He⁴)
 - c. Tank No. 3 12 inch vac (mostly He⁴)
 - d. Sealed mechanical pump--6.5 inch vac (mostly He⁴)
59. Liquid He⁴ use rate--approximately 14 liter per day with 2 transfers per day (190 liters in 13 days)

APPENDIX C. A MUTUAL INDUCTANCE BRIDGE FOR LOW TEMPERATURE THERMOMETRY

Introduction

A property useful for low temperature thermometry is the susceptibility of a paramagnetic salt, which obeys Curie's law at the temperatures of interest. Of the several techniques which are used to measure susceptibility, one of the most popular is an adaptation of the ac Hartshorn bridge¹ in which a paramagnetic salt is placed in the core of a transformer and the variation of susceptibility with temperature is measured by a change in the mutual inductance.

A disadvantage of the ac Hartshorn bridge is that it requires a precise variable mutual inductance which is both expensive and limited in range. It is possible, however, to overcome this drawback by simulating a variable mutual inductance electronically with an amplifier and a fixed inductance. A bridge of this design was first reported by Jastram, Pillinger, and Daunt² and is available commercially.³ This circuit relies on the stability of the transconductance of a vacuum tube⁴ and requires periodic gain adjustment. In addition, the range of the bridge is limited by the restriction that the circuit must operate in a linear portion of the tube's characteristics. With the advent of solid-state operational amplifiers, it is now possible to design a bridge whose accuracy is limited only by passive elements, independent of the gain of any active device.⁵

Basic Bridge Design

The basic Hartshorn bridge circuit is shown in Fig. 1. The measuring coil is in the cryostat and consists of two identical secondaries and a coaxial primary. The secondaries are separated along the axis of the primary coil and connected in opposition. In principle, the net voltage induced across the two secondaries is zero until a paramagnetic sample is inserted into one of them. The induced emf is then opposed by the voltage across the secondary of the variable mutual inductance M and variable resistance R . When the bridge is balanced, so that the current in the secondary is zero, M is proportional to the real part of the susceptibility of the sample while R is proportional to the power loss. The resistive component is not relevant to temperature measurement and is balanced only to produce a null. The balance point of the bridge is independent of the current through the primary circuit, although the sensitivity is not.

Figure 2 shows the circuit for the simulated, variable mutual inductance. If the gain A of the operational amplifier is high (A of 10^5 is typical), then $e_o/A = (e_+ - e_-)$ is very small, and we have

$$i_p f R_o = i_o R ,$$

giving

$$V_s = f R_o (M/R) (di_p/dt) .$$

As f varies from zero to one, the effect is to simulate a variable mutual inductance. Provided that the gain and common mode rejection ratio of the operational amplifier are high, the linearity and stability

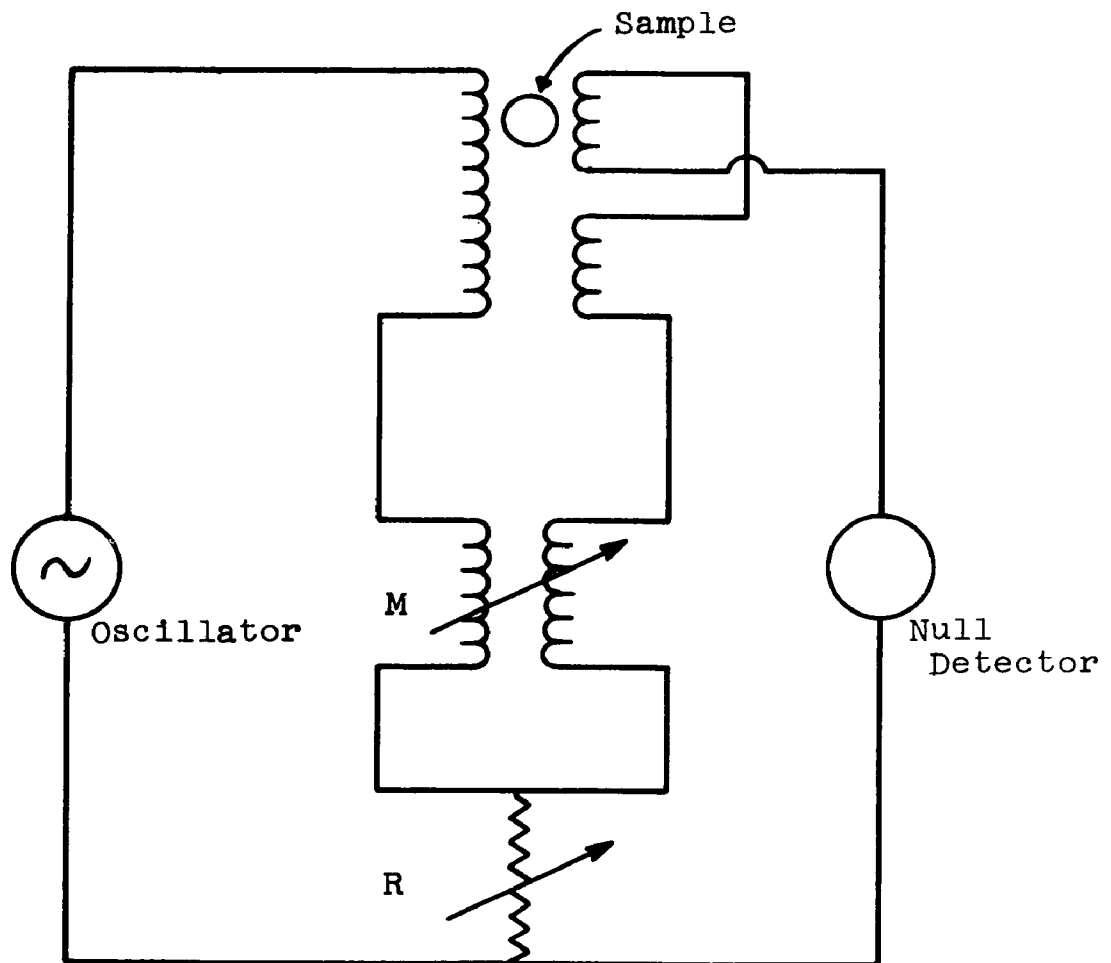


FIG. C-1 THE ac HARTSHORN MUTUAL INDUCTANCE BRIDGE. M IS A VARIABLE MUTUAL INDUCTANCE AND R IS A VARIABLE RESISTOR.

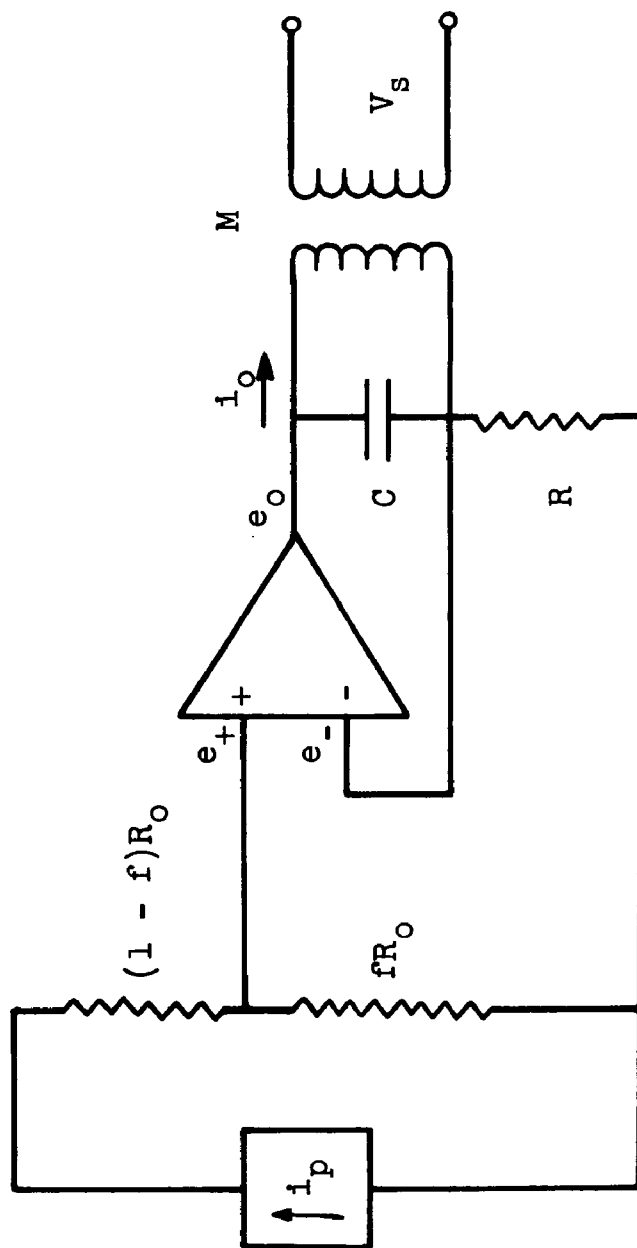


FIG. C-2 THE SIMULATED, VARIABLE MUTUAL INDUCTANCE CIRCUIT.

of the circuit are determined by resistors R and R_0 and transformer M . The resistance R_0 is a precision voltage divider⁶ and R is a metal film resistor which can be changed by a decade switch to provide an additional factor of 10^3 in the range of the mutual inductance. Capacitance C has a negligible susceptance and stabilizes the amplifier against high frequency oscillations.

In practice, it is hard to wind a measuring coil so that the two secondaries exactly cancel; asymmetries in the cryostat introduce a residual imbalance independent of the sample. For these reasons, it is convenient to have two identical, mutual-inductance circuits (Fig. 3). One is used to balance the bridge at 4.2 K and is locked at that temperature. The other mutual inductance then measures the change in the sample susceptibility below 4.2 K.

Details of Bridge Design

In previous bridge designs,⁷ as well as earlier models of this bridge,⁵ there have been a number of problems caused by capacitive coupling between the primary and secondary circuits, and the capacitance between secondary and ground. Shifts in the bridge balance with changes in lead resistance, inconsistent readings between ranges, and changes in balance when reversing switches are used, can all be traced to capacitive coupling.⁸ For this reason, the fixed mutual inductances and the measuring coil in the Dewar should have electrostatic shields between primary and secondary.

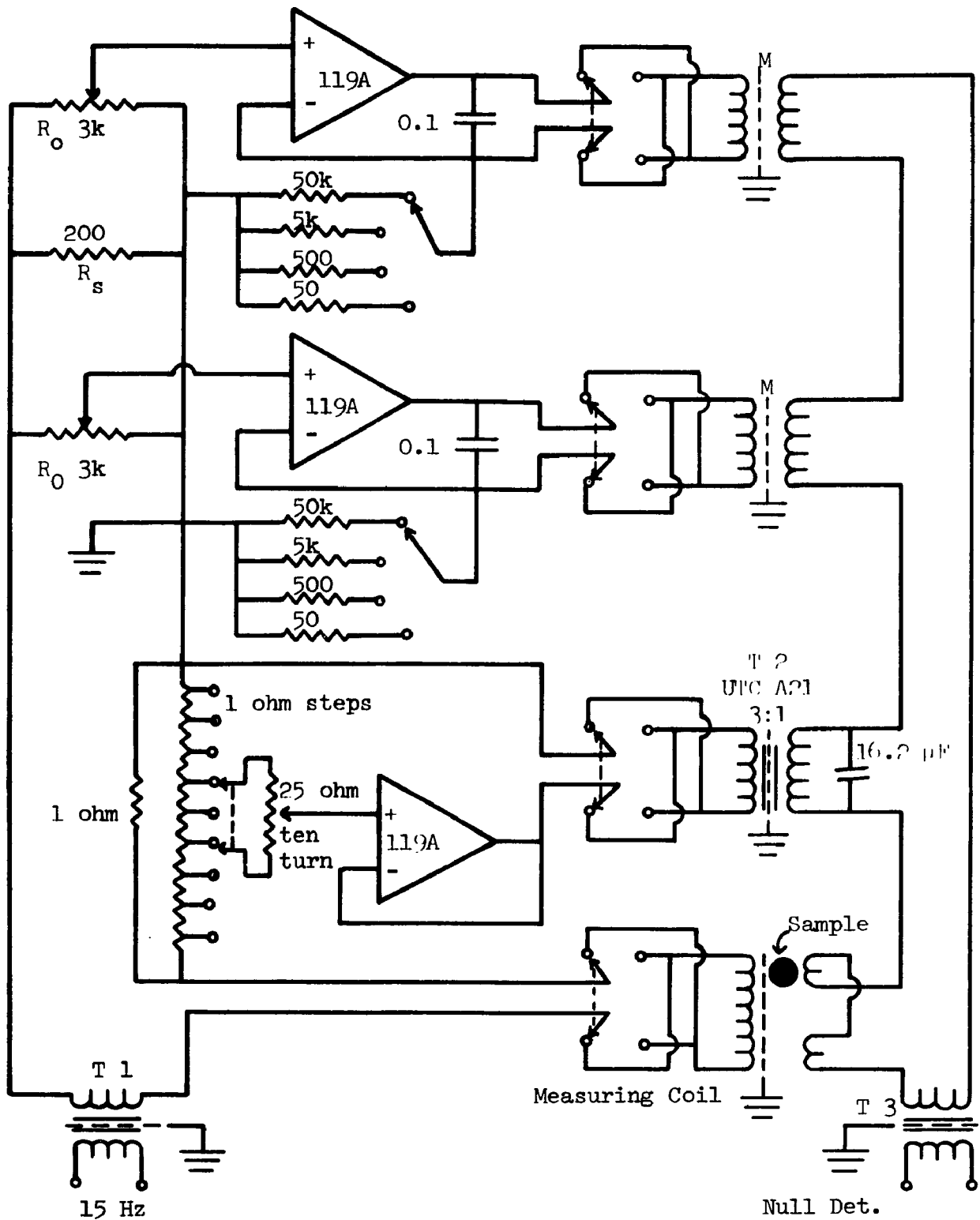


FIG. C-3 THE MUTUAL INDUCTANCE BRIDGE. OPERATIONAL AMPLIFIERS ANALOG DEVICES.

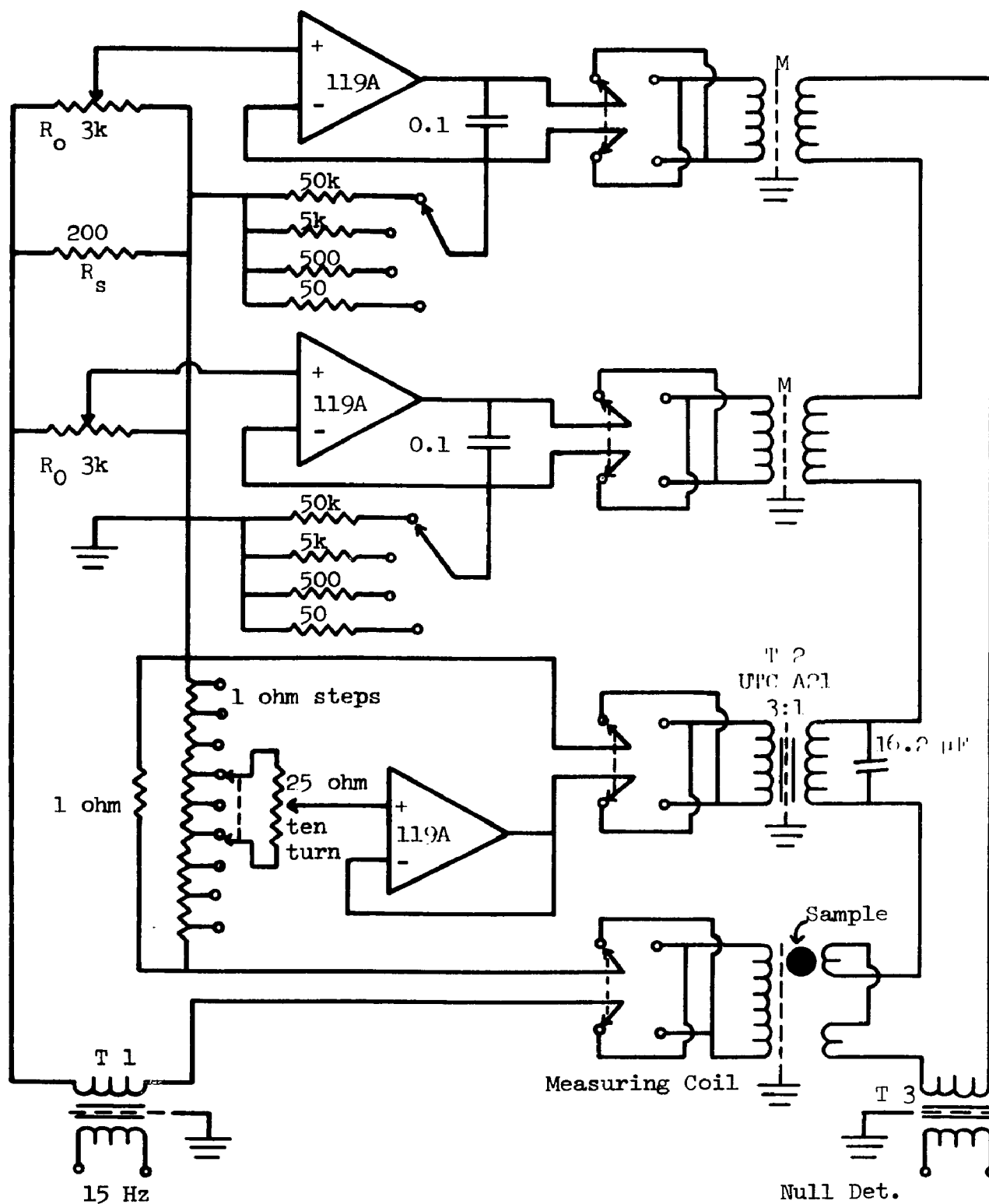


FIG. C-3 THE MUTUAL INDUCTANCE BRIDGE. OPERATIONAL AMPLIFIERS ANALOG DEVICES.

The capacitance to ground, introduced by the addition of the electrostatic shield, will also give rise to current loops in the secondary circuit which do not include the null detector. Consequently, even at balance, there may be an appreciable current through the measuring coil, and the voltage across the coil will depend on its resistance. As a result, the balance point of the bridge will shift when the coil is cooled. If the secondary-to-shield capacitance is less than 1000 pF in each coil, the shift in balance from room temperature to 4.2 K is less than 1 percent.

Reversing switches should be placed in the primary rather than secondary of each coil set, so that the sign of any residual magnetic pickup in the secondary will not change when the reversing switch is used. The mutual inductance coils are astatically wound⁹ to reduce pickup from external fields and have air cores for stability. The measuring coil is patterned after a design of Abel, Anderson, and Wheatley⁷ with the addition of an electrostatic shield. However, the bridge is sufficiently versatile to operate with a wide variety of measuring coil designs. Pertinent coil data are summarized in Table I and Fig. 4.

The resistive balance circuit shown in Fig. 5 eliminates a direct connection between primary and secondary circuits and allows the secondary to float with respect to ground. If the inductive reactance ωL of the primary of transformer T_2 is much larger than the output impedance of the operational amplifier circuit, then the voltage across the secondary T_2 is in phase with the current in the primary circuit. A capacitance C is used to resonate the transformer and reduce the phase

Table I

BRIDGE COILS

Mutual Inductance Coils

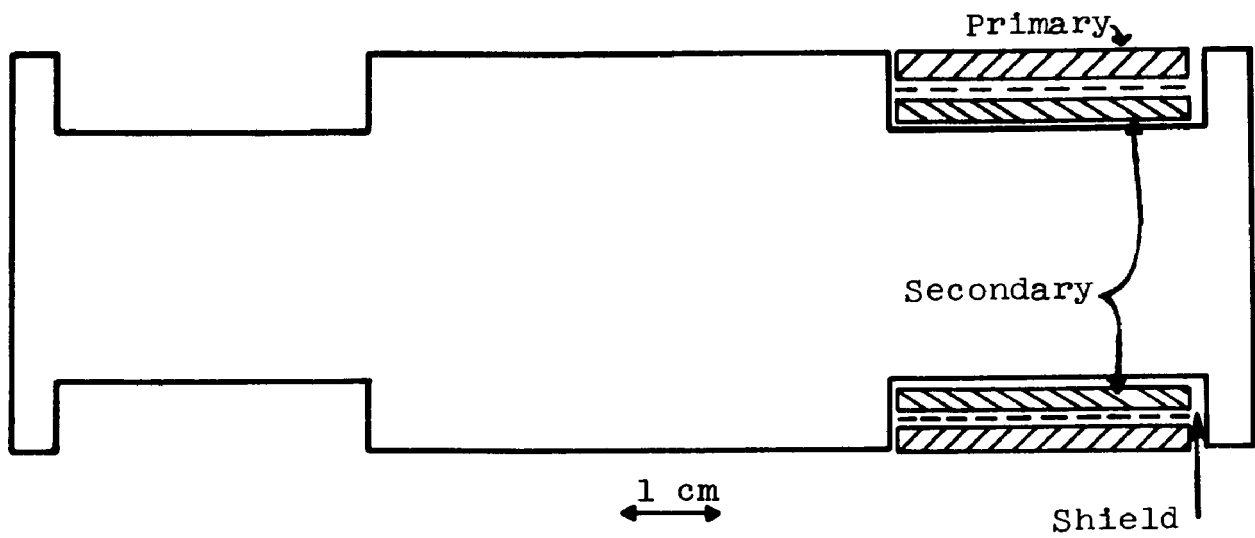
(Astatically wound--two primaries and two secondaries)

Mutual inductance	43 mH
Turns (each primary)	1000
Turns (each secondary)	1000
Capacitance (primary to secondary)	14 pF
Capacitance (shield to secondary)	260 pF
Wire size (copper)	28

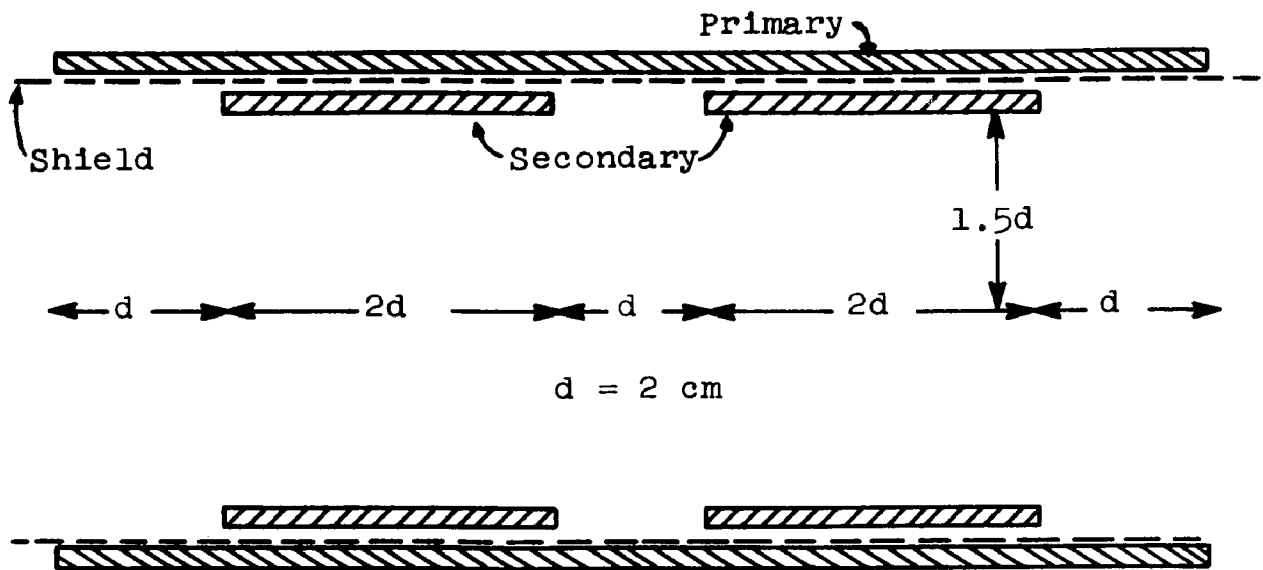
Measuring Coil

(One primary and two secondaries)

Mutual inductance (primary and one secondary)	1 H
Turns (primary)	5,600 (400 turns/cm)
Turns (secondary)	8,000 (2,000 turns/cm)
Capacitance (primary to secondary)	1 pF
Capacitance (shield to secondary)	870 pF
Wire size (copper)	40
Magnetic field in primary	0.5 gauss/mA
(dM/dT)	5 H/K/(g of CMN)
(T = 1 K)	1,000 $\frac{\text{bridge units}}{\text{K (g of CMN)}}$



COIL FORM--MUTUAL INDUCTANCE M



MEASURING COIL

FIG. C-4 BRIDGE COILS.

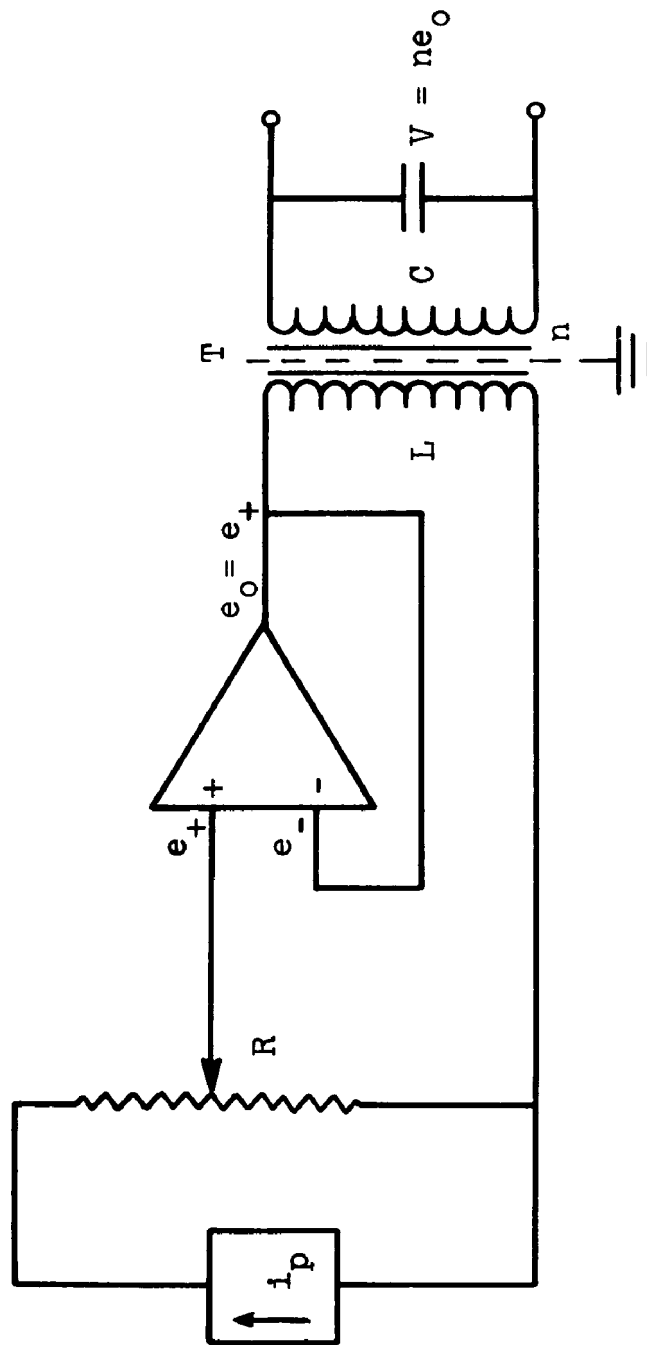


FIG. C-5 RESISTIVE BALANCE CIRCUIT.

shift to less than 0.001 rad. (Since the inductance of T_2 depends on the operating level in the transformer, C must be selected to provide zero phase shift at the very low levels encountered in bridge operation.) The balance circuits of Fig. 2 and 3 use either current gain or a step-down transformer to reduce the noise from the bridge operational amplifiers below that of the detector circuit.¹⁰

Null Detector

Null is detected with a Lissajous figure on an oscilloscope. The horizontal axis of the scope is driven by the 15 Hz bridge supply (Fig. 6) via a phase shift network which is adjusted to bring the horizontal component of the display into phase with the inductive component of the signal. The 15 Hz is produced by a scale-of-four divider from the 60 Hz line to keep any line-related pickup stationary on the Lissajous figure. (Another phase shift network allows adjustment of the phase of the 15 Hz supply relative to the pickup.) As a result, the bridge balance using the scope is limited by random noise, not line-related pickup. A phase-sensitive detector can also be used as a null detector and has the advantage that the inductive component can easily be nulled in the presence of a relatively large resistive imbalance. Because the 15 Hz supply is phase-locked to the line, the lock-in has a high rejection of all even harmonics of 15 Hz including the 60 Hz line.

The preamplifier (Fig. 7) is designed to have low noise at 15 Hz and provide a high gain, narrow band output to the scope and a wide band

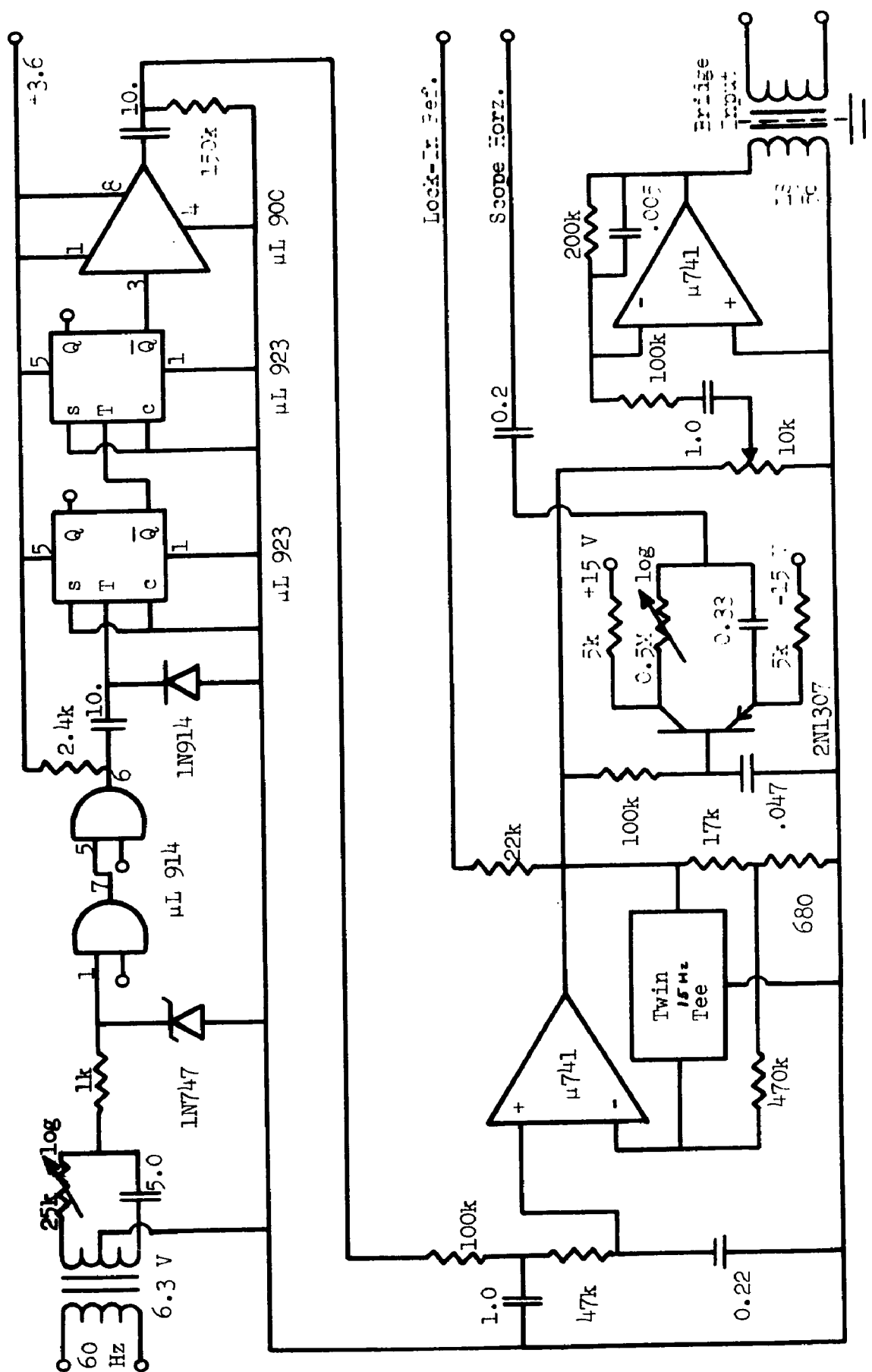


FIG. C-6 THE 15 Hz BRIDGE SUPPLY. I.C.'s---FAIRCHILD SEMICONDUCTOR.

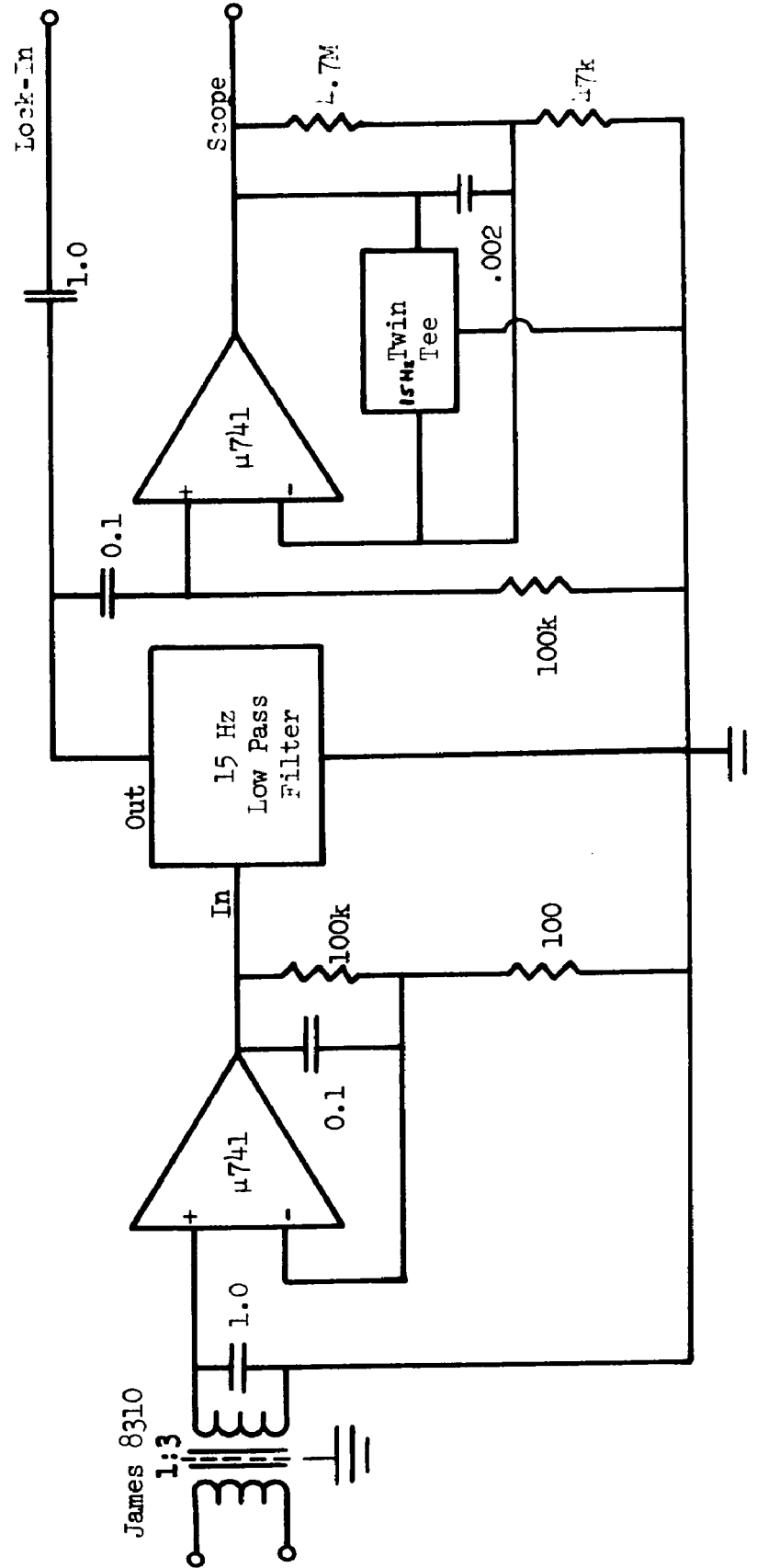


FIG. C-7 NULL DETECTOR PREAMPLIFIER.

output to the lock-in amplifier. It incorporates a 4-pole active filter.¹¹ The gain of the narrow band amplifier is 4×10^4 and the noise is less than 20 nV rms in a 3 Hz bandwidth.¹² The wide band amplifier has a gain of 1.5×10^3 . The output transformer T_3 provides an additional voltage gain of 3 and should be a good quality, low-level input transformer with an electrostatic shield and low capacitance between the primary and the shield. A capacitor is used to resonate the secondary of T_3 .

Details of Construction

Care must be taken to isolate the input transformer T_1 from the mutual inductances and output transformer. The 15 Hz generator and power supplies are mounted in a chassis, physically separated from the bridge proper. Any stray magnetic coupling in the bridge will not affect its accuracy as long as the coupling is linear in the primary current. However, the input transformer T_1 operates at a high level, and its leakage flux is non-linear in i_p . If care is taken, the mutual inductance coils, the resistive balance transformer T_2 , and the output transformer T_3 can all be mounted on the same chassis. The components should be mounted one at a time and located to minimize coupling to other components. T_2 is mounted in a mu-metal shield. Separate power supplies are desirable for the 15 Hz generator, preamplifier, and bridge operational amplifiers. All bridge resistors are metal film or wire-wound for temperature stability. All ground connections are made at one point on the chassis.

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9. Astatic coils are wound with two coaxial sets of primaries and secondaries each contributing half of the required mutual inductance. The two sets are mounted on their common axis and separated by approximately a coil diameter. The secondaries are connected to cancel any voltage induced by a uniform stray field common to them both, while the primaries are connected to give a net mutual inductance.
10. Current state-of-the-art integrated circuits have approximately 10 times the noise of the better discrete op amps and are therefore avoided in low-level portions of the circuit.
11. We have used an Analog Devices model 9083 tunable low pass filter with a 4 pole 1/2 db Tchebyscheff response. The Tchebyscheff response was chosen to provide maximum rejection (relative to 15 Hz) of 1800 rpm motor interference, third harmonic distortion from the input transformer T₁, and the 60 Hz line. A narrow band amplifier with an equivalent rejection of 15 Hz harmonics would require a very high Q and therefore a poor transient response.

12. See J. Levine, Rev. Sci. Inst. 41, 1358 (1970) for a description of a narrow band amplifier with logarithmic compression and good transient response. We are also indebted to J. Levine for pointing out that parametric amplifiers (such an Analog Devices Model 301) have a factor of ten lower noise at 15 Hz than conventional operational amplifiers.

APPENDIX D. SUGGESTED PAPERS FOR DILUTION REFRIGERATION

Articles are grouped according to subject and listed in reverse chronological order. Starred papers were found especially useful.

I. Properties of He³-He⁴ Mixtures

- * 1. J. C. Wheatley, "Experimental Properties of Pure He³ and Dilute Solutions of He³ in Superfluid He⁴ at Very Low Temperatures--Application to Dilution Refrigeration," Prog. in Low Temperature Phys. 6, Edited by C. J. Gorter (North Holland Publ. Co., Amsterdam, 1970), p. 77.
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- 9. W. R. Abel, R. T. Johnson and J. C. Wheatley, "Thermal Conductivity of Pure He³ and of Dilute Solutions of He³ in He⁴ at Low Temperatures," Phys. Rev. Letters 18, 737 (1967).
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15. S. G. Sydoriak and T. R. Roberts, "Vapor Pressures of He³-He⁴ Mixtures," Phys. Rev. 118, 901 (1960).
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II. Properties of He³

17. J. C. Wheatley, "Experimental Properties of Liquid He³ near Absolute Zero," Phys. Rev. 165, 304 (1968).
18. W. R. Abel, A. C. Anderson, W. C. Black and J. C. Wheatley, "Low Temperature Heat Capacity of Liquid He³," Phys. Rev. 147, 191 (1966).
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