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BLOCK RATE PRICING AND THE RESIDENTIAL
DEMAND FOR ELECTRICITY

by

Gail Ruth Blattenberger

A dissertation submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
(Economics)
in The University of Michigan
1977

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CHAPTER I

INTRODUCTION

1.1 Objectives and Rationale

The energy crisis has resulted in an increased interest in energy research of all kinds. Within the broad area of energy research, the topics of energy demand in general and electricity demand in particular have not been slighted. An accurate evaluation of energy demand is necessary in order to assess shortages in capacity and raw materials. Thus, research into energy demand has deservedly been emphasized.

The results of the last five or six years of research into the topic of residential electricity demand indicate that residential demand for electricity is price elastic in the long run.¹ If true, this has significant policy implications. It means that a given percentage price increase will result in an even greater percentage decrease in demand through either conservation or substitution of alternate fuels.

A problem exists in interpreting the estimated price effects discussed above. Electric utilities charge for

electricity according to a rate schedule rather than a single price. Representing a rate schedule by a single price variable may result in omitted variables bias which would magnify the estimated price effect.

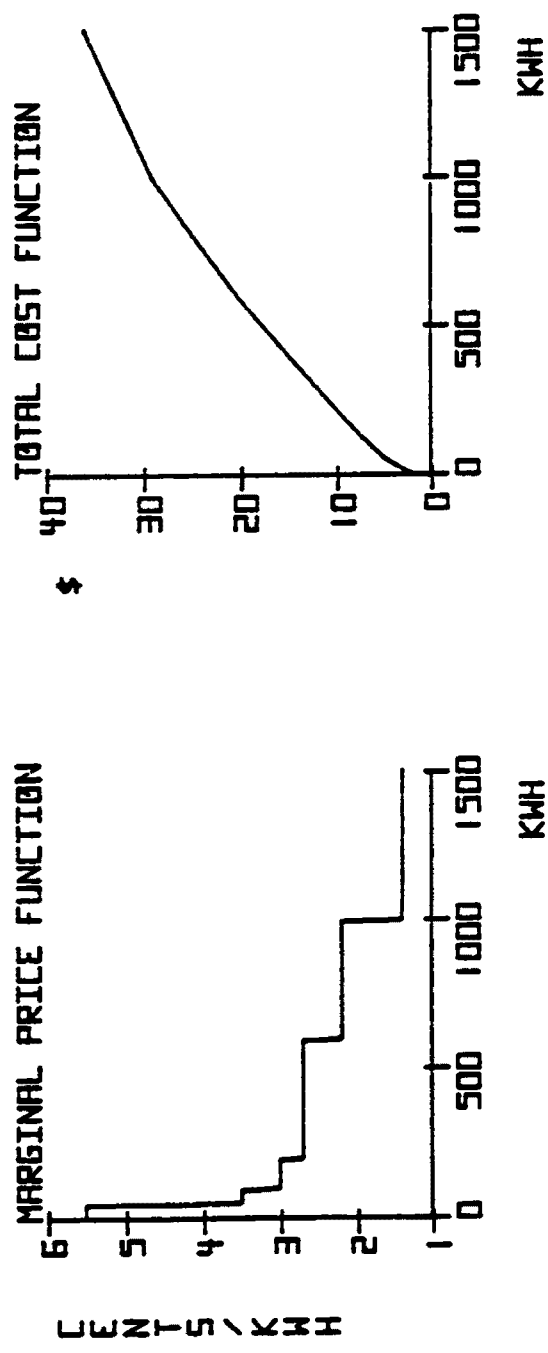
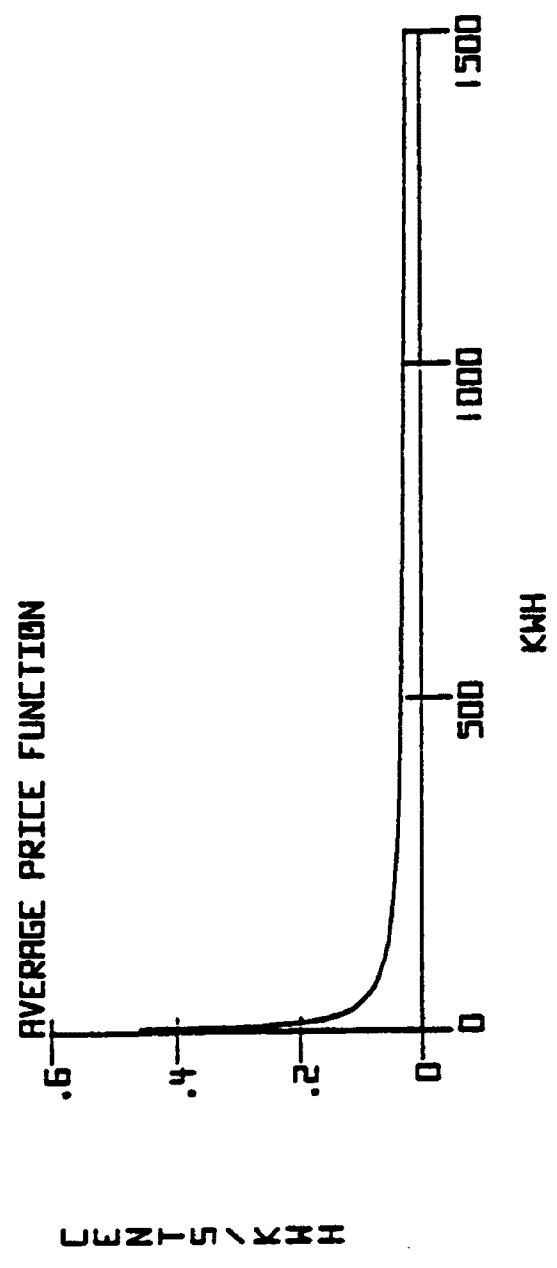
The present study is a thorough investigation into the relationship between residential electricity demand and electricity rate schedules. An illustrative example of a residential rate schedule for an electric utility is as follows.

Fixed Charge:	\$2.00
Block Rates:	
5.5¢/kwh.	first 50 kwh.
3.5¢/kwh.	next 50 kwh.
3.0¢/kwh.	next 100 kwh.
2.7¢/kwh.	next 400 kwh.
2.2¢/kwh.	next 400 kwh.
1.4¢/kwh.	all additional kwh.

This is the rate schedule charged by the Interstate Power Company in Minnesota in 1973.² The total cost function, the marginal price function, and the average price function for this rate schedule are graphed in Figure 1.1. The rate schedule above will be referred to as a block schedule since a constant marginal price is in effect over a "block" of quantities.

The price elasticities that are obtained in this study are more conclusive than those of previous studies

Figure 1.1
An Illustration of an Electricity Rate Schedule



for a number of reasons. First, the presence of rate schedules is explicitly recognized in the demand functions that are estimated. Second, actual rate schedules faced by consumers are the basis for the rate variables employed in the estimation.

The presence of rate schedules for electricity complicates the definition and determination of price elasticities and the modeling of demand. A search of both theoretical and empirical literature failed to yield a adequate solution to this problem. Therefore, the first part of this thesis contains a rigorous investigation into the implications of rate schedules for demand theory. The methods that are used subsequently to define rate variables are based on the results of this investigation. Although they are applied here to the special case of electricity demand, these results are applicable to a wide variety of situations.

In the second part of the thesis, the theoretical results are tested empirically. This part of the project consisted of the design and estimation of an econometric model. An objective of the modeling effort was to determine how to incorporate rate schedules into econometric models of demand.

The price data sets employed in previous electricity demand studies consist of either approximations to rate schedules or average prices. Since these data sets were not adequate for a comprehensive investigation of price

effects, it was necessary to compile an entirely new data set for the purpose of this study. This was a major effort in and of itself.

The original stated objective of this study was an investigation of demand determinants for residential electricity customers. However, the culmination of this research was not just an econometric model. The theoretical discussion of rate schedules, although it was initiated and motivated by the econometric model, is independent of the specific problem and generally applicable to other commodities and services which are priced according to a block rate schedule. The econometric modeling of rate schedules is also somewhat general, although it was designed specifically with reference to rate schedules of the kind employed in residential electricity sales in the United States. The compilation of a rate schedule data set is another by-product of this study.

1.2 An Outline of This Study

The current interest in energy topics has resulted in a large volume of empirical articles on electricity demand. In Chapter II the empirical literature on electricity demand is reviewed. This chapter serves as an introduction to the empirical problems encountered in estimating electricity demand, and to the state of the art in handling these issues. In the process, the issues

which are central to this investigation are identified and the general approach is introduced.

A theoretical discussion of rate schedules is presented in Chapter III. Although results of Chapter III are drawn upon later, this chapter is independent of the rest of the thesis. This chapter includes a review of the theoretical literature which has been written relevant to this issue, a presentation of the framework of assumptions, definitions and notation employed in the analysis, and a discussion of the theoretical results which are derived. Mathematical proofs are relegated to appendices.

Chapter IV addresses the problems associated with the derivation of an econometric model of electricity demand. There are two basic parts to this analysis. The first deals with the modeling of rate schedules for incorporation into an econometric model. The second is the analysis of the design of the econometric model itself. Where possible, the theoretical results of Chapter III are incorporated into the empirical model of Chapter IV.

Chapter V addresses the estimation of the model. In it the estimation techniques are discussed and the empirical results are analyzed. Finally, a summary of the results and conclusions of this investigation is given in Chapter VI.

Footnotes

¹A discussion of a representative sample of recent results in this area can be found in L. D. Taylor, "The Demand for Electricity: A Survey," The Bell Journal of Economics and Management Science 6 (Spring 1975): 74-110.

²Federal Power Commission, The National Electric Rate Book, Minnesota, 1973.

CHAPTER II

RESIDENTIAL DEMAND FOR ELECTRICITY: A REVIEW OF THE LITERATURE

2.1 Introduction

Before proceeding with a theoretical and empirical analysis of residential demand for electricity, a look at the experience and insight of past investigations is appropriate. This chapter reviews the empirical literature on electricity demand.

Electricity demand has been a subject of much current interest and there is a substantial body of literature in this area. Recently a quite comprehensive survey of the literature has been published.¹ This survey reviews the models of each major study individually and gives a summary and comparison of their results. Clearly it would be repetitive to duplicate such an effort in this chapter. Consequently, I will take a different approach to this subject.

The purpose of this chapter is twofold. First, it serves as an introduction to the issues which must be dealt with in estimating electricity demand. Second, it presents the methods applied in the past for dealing with these issues. In order to best attain these objectives

the discussion in this chapter is organized by topics. A separate and complete discussion of each of the studies considered is not given. The reader is referred to Taylor's survey for such a discussion. The emphasis of this chapter is on providing a comprehensive discussion of the nature and handling of specific topics.

Only a few selected studies will be considered in this discussion in line with the stated purposes of this chapter. The studies which have been selected are Houthakker (1951), Fisher and Kaysen (1962), Wilson (1971), Halvorsen (1972, 1973), Anderson (1972, 1973), Houthakker, Verleger and Sheehan (1973), Lyman (1973), and Mount, Chapman and Tyrrell (1972).² Other studies are excluded because they are in some way not comparable to this investigation, or do not furnish any additional insights into the topics of discussion.³ To reiterate, this is not meant to be a comprehensive review. Rather, it is intended to present the current state of the art in empirical research. Each of the studies selected will be compared on the basis of how they approach the topics which are discussed.

In the last section of this chapter, several as yet unpublished papers are considered. Since they were not taken into consideration in the design of this study, these papers are discussed separately.

2.2 Ex post and Ex Ante Price

Since it is the main thrust of this study, the first topic I will address is the determination of the electricity price variable. Because of the presence of a rate schedule there is no one single price facing consumers which can easily be incorporated into an empirical demand function. Furthermore, it is not possible to include the entire rate schedule in a demand equation. Therefore, some proxy for the rate schedule must be found.

Although many different techniques have been employed for this purpose, they can be generally grouped into two categories which shall be referred to as ex post and ex ante techniques.⁴ In selecting a price or prices to represent a rate schedule, it is natural to look at the typical prices actually paid. The problem with such approaches is that these prices are functionally dependent on the quantity demanded. Since the price variable can be determined only after the sales level is established, such techniques will be referred to as ex post techniques. Techniques which specify the price variable independently of the sales level are called ex ante techniques. With the latter techniques, prices may be specified prior to knowledge of the sales level. The basic problem involved in employing ex ante approaches is that they frequently do not yield the prices most relevant to the demand decision.

The most common ex post price employed is average price. This is calculated by dividing the total revenue collected by electric utilities from residential consumers by the total quantity sold. This technique is employed by Fisher and Kaysen, Mount, Chapman and Tyrrell, Halvorsen, and Lyman. Although this is the most common technique employed, it is actually least representative of the rate schedule, since for a given rate schedule there is only one sales level such that the average price is paid.

Another ex post technique is the use of a marginal price in the presence of a multi-block rate schedule. This technique is somewhat more general than the one above, since a given marginal price is applicable to all sales levels within that given block. Since price is not constant, this technique does not give any indication of overall cost.

In Houthakker's article on demand for electricity in Britain, he employs a marginal price. However, since electricity tariffs in Britain at that time consisted of a fixed charge combined with a constant marginal price, marginal price does not vary with quantity consumed. Therefore, in this instance the use of marginal price is classified as an ex ante technique. Houthakker tried to incorporate the fixed charge into the demand equation also, but this proved impossible because it was too highly correlated with income.

The three additional studies being considered here also used ex ante techniques. In each of these techniques, prices were taken from information in Typical Electric Bills.⁵ In Wilson, the average price was taken for a set consumption level, 500 kwh. In Houthakker, Verleger, and Sheehan, approximate average marginal prices were calculated for blocks designated as 100-250 kwh., 250-500 kwh., and 100-500 kwh. Anderson employs the incremental charge from 500-1000 kwh.

Halverson (1972, 1973) also does some experiments using Typical Electric Bills. In his 1972 study, he uses both ex post and ex ante price variables calculated from Typical Electric Bills in equations compared with his results using average price. In the 1973 study, he contrasts the results of specifying his structural equation for average price as a function of a set of typical bills with the results of specifying it as a cost function.

There are two problems with using Typical Electric Bills. First, it results in approximate rather than actual marginal prices, and second, the particular quantity selected, say 500 kwh., might not be the one appropriate to the demand decision.

In this study, one of the basic approaches to formulating rate variables is an ex post technique. The other is an ex ante technique. Both of them represent departures from techniques previously employed. One approach

is to represent a typical block of the rate schedule exactly. It is an ex post technique, but it is more accurate in representing cost than a straight marginal price, and more general than an average price. The other approach is to employ parameters defining an approximation to the entire rate schedule. This is an ex ante technique. By approximating the entire rate schedule, it is more general than the ex ante techniques discussed above.

All of the rate variables employed in this study are calculated from a newly compiled data set consisting of actual rate schedules for every utility in the country of significant size (service area of at least 2,500 in population). This has substantially increased the scope of potential methods of defining rate variables available to be used in this empirical investigation. In previous studies, the rate variables employed were to some extent a function of the unavailability of the data.

2.3 Simultaneity

Whenever two or more endogenous variables are jointly determined by a system of simultaneous equations, problems can exist in accurately estimating and identifying the parameters of these equations. It may be necessary to resort to simultaneous equations techniques to resolve these problems. In the case of electricity demand, there are two potential sources of simultaneous equations problems.

It is a basic principle of economics that prices and sales levels are jointly determined by demand and supply functions. Thus, one question to be examined is whether standard demand and supply considerations necessitate the use of simultaneous equations techniques in the estimation of electricity demand.

Another question concerning simultaneity arises from the presence of a rate schedule. With a rate schedule the price paid by individuals is itself a function of quantity demanded. The question then arises whether there are simultaneous equations problems peculiar to the presence of a rate schedule.

There seems to have been much confusion on these subjects in other discussions. Both of these questions have been muddled together, and statements have been made in reference to these questions without regard to whether the viewpoint is that of individual or average demand. Therefore, I will try to examine the nature of these questions explicitly, dealing first with standard problems of estimating demand and secondly with problems created by rate schedules.

The first question to be examined is whether a simultaneous equations model is necessitated by standard supply and demand considerations. A simple, standard supply and demand system may be written in the following way:

$$q = D(p, y)$$

$$p = S(q, c) \quad .$$

The quantity demanded, q , is a function of price, p , and income, y ; the supply price, p , is a function of quantity, q , and the cost of inputs to the production process, c . Income and cost are exogenous to this system, and p and q are simultaneously determined. Two situations can obviate the need for simultaneous equations estimation techniques to identify the demand function. If there are no shifts in demand, all observations lie along the same demand curve. This is the case if income is constant and

$$q = D(p) \quad .$$

In this case, the estimated function will be the demand function. However, this does not seem to be a reasonable assumption in general. Secondly, if supply is infinitely elastic then,

$$p = S(c)$$

and price is completely determined by exogenous factors. In this case the system of equations is recursive.

How are these standard demand and supply considerations stated in the presence of a rate schedule? It is shown in Chapter III that the quantity demanded is a function of the entire rate schedule. This can be written

$$q = D(R, y)$$

where R is a function defining the price at any consumption level. For the present, this is simply assumed and left to be rigorously established later. The question corresponding to the standard supply and demand situation is, "are quantity, q , and the function, R , jointly determined or is R based entirely on exogenous factors?" A simultaneous equations problem exists if the function R is modified in response to changes in the actual level of demand

$$R = S(q, c) \quad .$$

It does not exist if the function R is based solely on exogenous factors, e ,

$$R = S(e) \quad .$$

On this question it seems there is less reason to employ simultaneous equations than in the case of privately produced commodities. In the case of private goods, prices are set by the producers themselves, at will. In the case of electricity, prices (or rate schedules) are set jointly by the rate commissions and the utilities. This moves the determination of the rate schedule further from the market. Objections to viewing demand and the rate schedule as being simultaneously determined, thus, arise on two accounts: the lack of correspondence between actual firm costs and the rate schedule, and delays in the rate making process.

As in most industries, the cost of producing electricity is related to the amount produced. If rate schedules are designed to reflect the costs of production, including those related to the expected level of demand, then a simultaneous equations problem exists. To the extent that the cost of electricity is related to electric energy demand, it is related to the total electric energy demand of all customers. A change in the level of total demand will this affect the average cost of electricity, and presumably, the average overall price. However, not only are costs differentially allocated to different classes of customers, but they are also allocated differentially to large and small customers within a given class. The relationship among the average level of demand, production costs, and the setting of rate schedules is at best indeterminant.

It should be noted that although one objective in rate making is to cover costs, there are many other explicitly stated objectives in setting rates. In addition, electricity is a joint commodity and there is no one unquestionable procedure for allocating costs among consumers. However, the position generally taken by economists in the public utility field is that electric rates could be brought more in line with the actual costs of providing electric service.⁶

To the extent that the rate schedule is determined independent of the average consumption level,

$$R = S(e) \quad .$$

Second, although the price paid by individual customers is a function of their level of demand, electric utilities are required by law to meet whatever demand exists under the current rate schedule. There is no constraint on the total demand subject to that rate schedule beyond the capacity of the utility. Even then, the total capacity of an individual system is expandable by purchases from neighboring utilities. Thus, supply is infinitely elastic, at least in the short run, where short run is defined by the length of time it takes to get a new rate schedule approved by the rate commission. The lags involved in this process seem to obviate the need for a simultaneous equations model on this account. Under this interpretation,

$$R = S(q_1, c) \quad .$$

Joskow has pointed out that regulatory reviews are initiated by requests for nominal price increases. During periods of falling or constant prices, it may be assumed that utilities are behaving monopolistically and that prices are set to maximize profits.⁷ Therefore the above argument may not be valid during a period of decreasing or constant prices such as the period 1961-1966. This can be tested empirically. The above assumption seems reasonable in the face of rising prices.

Even if the premise is accepted that the rate schedule, R , can be considered exogenous from the viewpoint of consumer demand the issue of simultaneity is not removed. The question remains as to whether the rate

schedule causes simultaneity problems in and of itself. This question will be examined with respect to both individual and average demand. Without loss of generality, R , in this discussion is the average price rate schedule defining the average price of any quantity given by the rate schedule,

$$ap = R(q) \quad .$$

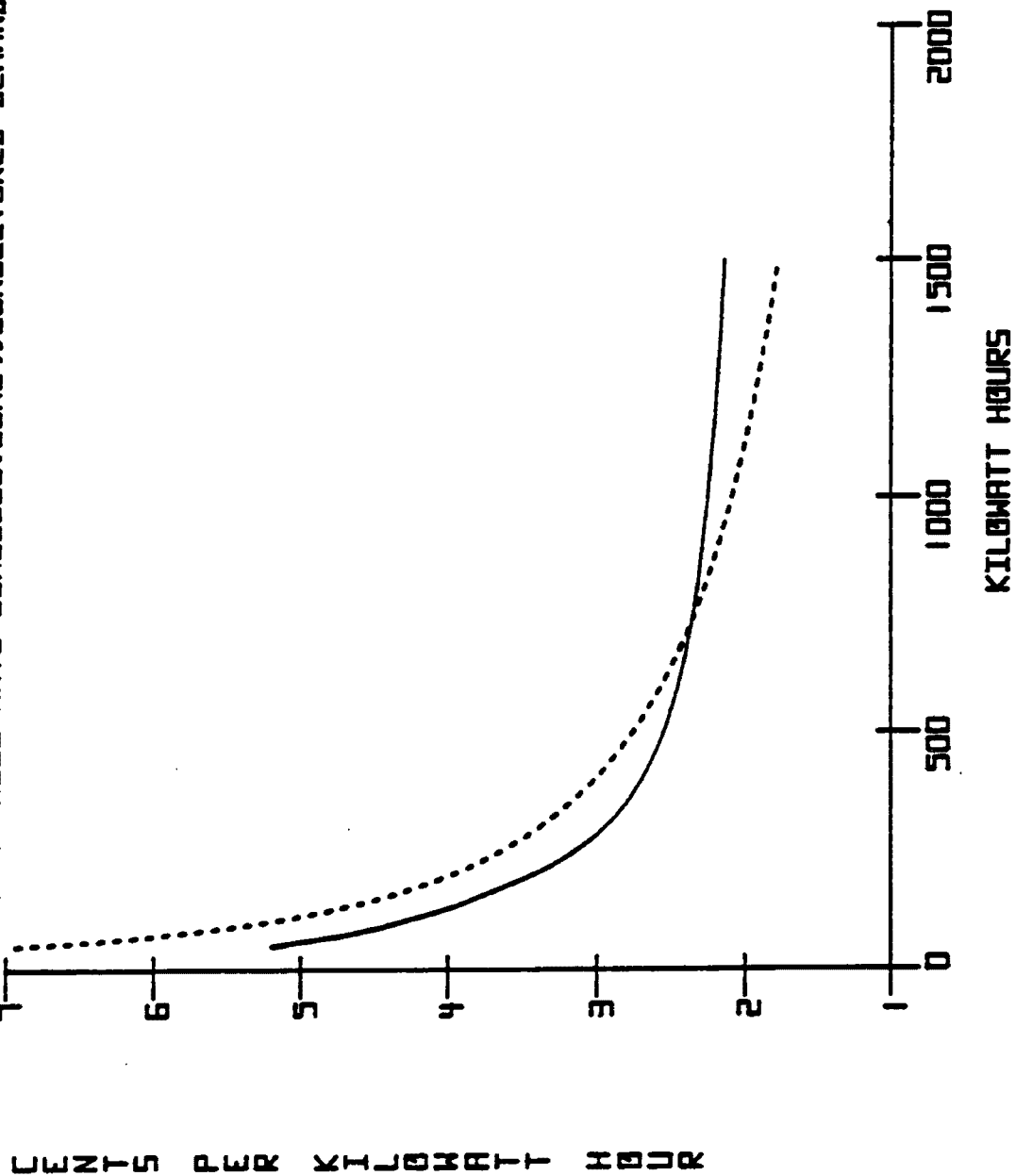
From the point of view of an individual consumer, a rate schedule is something like a supply function. It defines cost as a function of quantity purchased. Thus, one is tempted to illustrate individual demand and price as being jointly determined by an average price rate schedule and an individual demand function in terms of average price. This interpretation is illustrated in Figure 2.1. The average price rate schedule, R , is simply the average price defined by the rate schedule at any given quantity. This interpretation is expressed mathematically by the simultaneous equations system,

$$q = D(ap, y)$$

$$ap = R(q) \quad .$$

The interpretation presented above is misleading, however, because the demand function in the figure is a misrepresentation and does not exist. Individual demand is a function of the entire rate schedule and not a single

Figure 2.1
INTERPRETATION OF ELECTRICITY DEMAND FOR AN INDIVIDUAL
AVERAGE PRICE RATE SCHEDULE (LINE), CONJECTURED DEMAND



average price. Furthermore, an assumption is made in the derivation of demand functions that the supply function is flat. Conceptually demand defines the amount a consumer would purchase if faced with alternate supply functions, each of which has infinite elasticity. This is certainly not the situation illustrated in Figure 2.1.

The actual situation is that the average price rate schedule represents the locus of possible price-quantity combinations available to the individual. Given this rate schedule, the consumer selects one point from this set. (There may be several points in this set to which the consumer is indifferent, but only one will actually be selected.) Demand as a function of the rate schedule, R , can thus be represented as a point, r . To reiterate, this is different from the standard demand situation in that a flat rate is not offered, thus it is irrelevant to consider how much an individual would demand at a given flat rate. The actual situation can be expressed mathematically,

$$q = D(R, y)$$

$$ap = R(q) \quad .$$

If the function R is exogenous, this system of equations is recursive.

The discussion so far has employed an average price rate schedule. Similar points can be made with respect

to marginal price. If average price is defined by the function, R , then the total cost of q units of electricity is $q \cdot R(q)$, and the marginal price, mp , is

$$mp = R(q) + q \cdot R'(q) \quad .$$

The system,

$$q = D(R, y)$$

$$mp = R(q) + q \cdot R'(q)$$

is also recursive. Both quantity and marginal price are dependent on the rate schedule; demand is not a function of the actual level of marginal price.

Average demand is simply an aggregation of individual demand functions. Therefore, the same arguments given above apply to average demand. It is not correct to draw average demand as a function of average price. It is also a misrepresentation to have the levels of average price and average demand be determined jointly by an average demand function and the average price rate schedule. Average demand is a function of the entire rate schedule. Average price is defined by the rate schedule and the distribution of demand. Given N consumers, this can be written mathematically,

$$\bar{q} = \frac{1}{N} \sum_{i=1}^N q_i = \frac{1}{N} \sum_{i=1}^N D_i(R, y_i)$$

$$\overline{ap} = \frac{\sum_{i=1}^N q_i \cdot R(q_i)}{N} \quad .$$

There is no simultaneous equations problem created by the rate schedule in the actual determination of demand.

The presence of a rate schedule, however, does present problems in the modeling of demand. It is difficult to specify a rate schedule in an empirical demand equation. In fact, unless a rate schedule is very simple it is not possible to derive an analytical expression for a demand function. This point will be elaborated in Chapter III. Consequently, researchers have frequently found it necessary to resort to the use of ex post prices. The results of some of these investigations are estimated demand functions such as the one portrayed in Figure 2.1. In spite of the fact that the actual relationship is recursive, the use of average price as a proxy for the rate schedule in the demand equation does create simultaneous equations problems in the estimation of demand.

By definition, any ex post price, ep , is a function of quantity demanded,

$$ep = f(q) \quad .$$

Employing an ex post price as a proxy for R in the demand function,

$$q = D(ep, y)$$

creates a simultaneous equations problem.

Some ex post prices, however, are worse than others. The average price level is a bad proxy for R , since the function

$$ap = R(q)$$

is one-to-one. The simultaneity problems involved in employing an ex post marginal price are not as severe as those illustrated in Figure 2.1, since the relationship between marginal price and quantity is not one-to-one. In fact, mp is constant over a range of quantities.

The severity of a simultaneous equations problem when an ex post marginal price is used has been tested by Lacy and Street in some simulation experiments.⁸ They find that a severe simultaneous equations problem exists when an average price variable is employed. This problem is absent from experiments using a marginal price variable. The simultaneity problem in the experiments using average price is lessened when two-stage least squares is used.

Although one of the techniques employed in this study is an ex post technique, it does not employ an ex post average price as a price variable. Instead, ex post marginal price is used. Not only is the relationship between ex post marginal price and quantity not one-to-one, but marginal prices are aggregated to the state level further reducing the relationship between ex post marginal

price and quantity, which can be attributed to a rate schedule.

Only one study of those being considered here actually employs a simultaneous equations model, Halvorsen. In his analysis of the situation, Halvorsen implies that simultaneous equations problems are created by the presence of a rate schedule. However, the problem he actually deals with in his model is that of standard supply and demand considerations. The supply price function is defined in terms of quantity demanded and the cost of inputs to the production process. The fact that there was no significant difference between his demand functions obtained with ordinary least squares and those with two-stage least squares lends credence to the view that simultaneous equations are not necessitated by ordinary supply and demand considerations.

In one experiment in his study (1973), Halvorsen defines supply price as a function of quantity and a set of typical bills from Typical Electric Bills. This is analogous to estimating a rate schedule and including that in the demand function. This approach is more in line with my interpretation of the simultaneous equations problem, and it is gratifying to see that Halvorsen obtained much more significant results with this approach. In this case, however, it does not seem that a simultaneous equations estimating procedure is warranted because the schedule is well-defined and not stochastic.

Although in the rest of the studies single equation models are estimated, Lyman includes some variables which he considers to be supply side factors. The equations estimated by the other models include demand determinants only.

Fisher and Kaysen acknowledge the problems involved in employing an ex post average price. They justify by-passing this issue on a number of grounds. They argue that the relationship between average price and quantity attributable to the rate schedule is reduced by the use of real prices, first differences and aggregation over rate schedules and individuals. A further justification given is that the relationship between average price and quantity in the rate schedule is not exactly logarithmic. In actuality, this relationship seems fairly close to being logarithmic, negating this last assertion. The other factors may reduce the correlation, but they do not seem to obviate the issue.

In this study, the problems involved in the use of ex post techniques in the presence of a rate schedule are, at least, lessened by the use of a fairly general approach employing marginal prices. The ex ante techniques used avoid these problems altogether.

2.4 Aggregation and Estimation

In this section I will discuss the level of regional aggregation and aggregation over time selected in these studies. Since the estimation technique employed is frequently determined by the type of data set that is used, this issue will also be discussed.

The level of regional aggregation varies among the studies being discussed. Clearly, information is lost when greater degrees of aggregation are used and this can lead to biases in the results. Therefore, it is desirable to have the data as disaggregated as possible. On the other hand, accurate information on economic variables is frequently unavailable on a highly disaggregated level. Inaccuracies in data can also cause biases in the results. Consequently, some middle ground has to be selected. In the studies considered here, five employ data aggregated to the state level, one employs firm data of utilities, and two use towns or SMSA's.

Lyman employs firm data taken from FPC publications. He runs into problems, however, with the availability of economic data applicable to utility districts, and must, therefore, construct proxy variables using state and city data. Another problem that exists in using utility data is that the regions under a given utility's jurisdiction are not fixed over time, and may include non-contiguous

and heterogeneous areas. Lyman may have avoided this problem through a judicious selection of utilities.

By employing towns, and cities or SMSA's respectively, Houthakker, and Wilson avoid the problem of variation in the size and location of their regions. They also select regions for which additional statistical information is available. Both Houthakker and Wilson note, however, the tendency of this technique to eliminate private utilities from consideration.

The main advantage of using state data, as did the remaining studies, is the increased availability of data.⁹ In particular, much better income data is available by state than at a more disaggregated level. For this reason, state data will be employed in this study.

In all the studies discussed here, annual data were employed. Very little work on the estimation of seasonal demand has been done. Although Houthakker addresses this issue, there is room for much more work on this topic. In this study, there was not time to explore this problem. Daily variation in demand is another subject of much current interest which has been neglected.

A variety of types of data sets have been used in investigating electricity demand. Cross-section data, time series data and pooled cross-section-time-series data have been used. Wilson, Houthakker, Anderson (1973) and Halvorsen (1973) all use straight cross-section data and

except for Halvorsen, employ ordinary least squares. In the case of Houthakker and Anderson, they were able to select particular years for which singularly good data were available. Fisher and Kaysen estimate separate time series equations for each state. They also employ ordinary least squares. Halvorsen uses two-stage least squares to estimate his simultaneous equation model.

The remaining articles all use some form of pooling of cross-section and time series data. Houthakker, Verleger, and Sheehan are the only ones to explicitly allow for regional variation in the structure of the error term. They employ a variance components approach derived by Balestra and Nerlove.¹⁰ This is the same approach which will be employed in this study.

Mount, Chapman, and Tyrrell acknowledge the problems created by pooling time series and cross-sections. They accommodate these by including regional dummy variables. This yields asymptotically unbiased and consistent, although inefficient, estimators. Mount, Chapman and Tyrrell also attempt to deal with the problems created by the inclusion of a lagged dependent variable by means of an instrumental variables technique. Neither Lyman (1973) nor Halvorsen (1972) use techniques designed to deal with pooled cross-section and time series data. However, Lyman limits his pooling of data to within regions and may therefore avoid this problem.¹¹

Other statistical problems are also addressed in some of the empirical techniques. Halvorsen employs two-stage least squares to address simultaneous equations problems. The nature of these problems, and Halvorsen's handling of them, have been discussed. In an innovative approach, Lyman uses a nonlinear estimation technique of the type considered by Box and Cox.¹² This allows for a more general functional form which reduces to standard linear models in special cases.

2.5 The Measurement of Short-Run and Long-Run Elasticities

Consumption of electricity is a function of the existing stock of electric appliances. This stock of appliances represents durable goods of a fairly long life expectancy. Therefore, the adjustment of electricity to various economic variables can conceptually be divided into two stages, the short run, in which stock remains fixed, and the long run, in which all the effects of stock adjustments are included.

There are several methods of incorporating this distinction into a model. The most direct is through the use of a stock variable in the electricity demand equation. Long-run effects may then be determined by estimating stock equations. An alternative to this when stock data are unavailable is the use of a lagged dependent variable

in the electricity demand equation as a proxy for stock effects. This imposes additional constraints on the model, but is sometimes the only available option. Finally, as an alternative to either of these methods, straight time series models are generally thought to pick up short-run effects whereas cross-section models under the assumption that the market is an equilibrium are thought to pick up long-run effects.

Although Fisher, and Kaysen and Anderson estimate separate stock equations, only Houthakker explicitly includes a stock variable in his estimated model. Houthakker does not state whether he considers his coefficient to measure short- or long-run effects. However, the presence of a stock variable indicates that they are short run. Fisher and Kaysen include a stock variable in their theoretical model, but by means of some additional assumptions a simpler estimated model is derived where the stock effect is included in the constant term. By these assumptions they measure short-run effects in their demand equation and long-run effects from the stock equations. Anderson uses a cross-section model with census data. He measures long-run effects in both his stock equations and his electricity demand equations.

Both Houthakker, Verleger and Sheehan, and Mount, Chapman and Tyrrell use lagged dependent variables in their models. Consequently, short- and long-run effects

are differentiated in their demand equations. This is the general approach which will be followed in our model.

Wilson, Lyman and Halvorsen all employ static models. Since Wilson uses a cross-section of cities and Halvorsen (1973) uses a cross-section of states, the results of both these studies can be thought of as long-run effects. Both Lyman and Halvorsen (1972) pool their data. Therefore, it is indeterminate whether to interpret their results as long- or short-term effects.

A summary of elasticity results is given in Table 2.1. The last two studies listed are recent studies which are discussed in Section 2.7. The past results reported in this table indicate a price elasticity for electricity that is somewhat larger than one in absolute value. There is more variation in the income elasticities reported than in the price elasticities. The income elasticities obtained are positive with the exception of Lyman, but there is no indication as to whether income elasticity is larger or smaller than one.

TABLE 2.1

SUMMARY OF ECONOMETRIC ESTIMATES OF PRICE AND INCOME ELASTICITIES

Author	Type of Price	Price Elasticity		Income Elasticity	
		Short Run	Long Run	Short Run	Long Run
Houthakker	e.a.	-0.89	n.e.	1.16	n.e.
Fisher and Kaysen	e.p.	-0.15	0	0.10	small
Wilson	e.a.	n.e.	-2.0	n.e.	0
Halvorsen (1972)	e.p.	(-1.138)		(0.6113)	
Halvorsen (1973)	e.p.	n.e.	-1.044	n.e.	0.676
Anderson (1972)	e.a.	n.e.	-0.91	n.e.	1.13
Anderson (1973)	e.a.	n.e.	-1.12	n.e.	0.80
Lyman	e.p.	(-0.90)		(-0.20)	
Houthakker, Verleger and Sheehan	e.a.	-0.089	-1.02	0.14	1.64
Mount, Chapman and Tyrrell	e.p.	-0.14	-1.20	0.02	0.20
Acton <u>et al.</u>					
cross-section	e.p.	(-0.70)		(0.40)	
cross-section & time series	e.p.	(-0.34)		(0.41)	
Lacy and Street	e.p.	(-0.45)		(1.87)	

Note: n.e. - not estimated

e.a. - ex antee.p. - ex post

Source: This table has been adapted from Table 4 of Taylor, "Electricity Demand," p. 101.

1
3
1

2.6 Model Formulation

Many of the pertinent aspects of the models considered in this chapter have already been discussed. One issue that has not been mentioned so far is the functional form of estimated models. In this section I will examine the functional forms of the models for three of the studies considered. These three studies are selected because they all employ state data. Halvorsen is not included in this group because his is a simultaneous equations model. Anderson is excluded because he employs only census year data. Although the other models considered are not presented in detail, any particularly relevant aspects of them will be noted.

In Fisher and Kaysen's short-run model, the demand for electricity is defined by the utilization rate of the existing stock of appliances. This can be written,

$$x_t = \sum_{i=1}^I u_{it} k_{it} \quad (2.1)$$

where x_t is the level of electricity demand in period t , k_{it} is the appliance stock of the i^{th} appliance in period t , and u_{it} is the utilization rate. Fisher and Kaysen then proceed by assuming

$$u_{it} = B_i P_t^{\alpha_i} Y_t^{\beta_i}$$

where P_t is the average price of electricity, and Y_t is

the average income level at time t . By defining

$$C_i = B_i \bar{P}^{\alpha_i} \bar{Y}_t^{\beta_i}$$

where $\bar{P}$ and $\bar{Y}$ are averages over the entire estimation period (2.1) can be rewritten

$$x_t = \sum_{i=1}^I C_i (P_t/\bar{P})^{\alpha_i} (Y_t/\bar{Y})^{\beta_i} k_{it} \quad (2.2)$$

The rationalization for this is that C_i can be interpreted as the normal utilization rate. However, $\bar{P}$, $\bar{Y}$, and C_i are subsequently eliminated from the form of the equation specified for estimation. If C_i , α_i , and β_i are assumed to be constant over appliances, equation (2.2) can be rewritten

$$x_t = C (P_t/\bar{P})^{\alpha} (Y_t/\bar{Y})^{\beta} \sum k_{it} \quad .$$

This equation can then be estimated in the form

$$\ln x_t - \ln \sum k_{it} = \ln C^* + \alpha \ln P_t + \beta \ln Y_t \quad (2.3)$$

where $C^* = C \bar{P}^{-\alpha} \bar{Y}^{-\beta} = B$. At this point, however, Fisher and Kaysen reformulated their short-run model without the incorporation of any appliance stock variable because they felt their data were unreliable. They did this by assuming

$$\sum_{i=1}^I k_{it} = \left(\sum_{i=1}^I k_{i0} \right)^{\delta \cdot t} \quad .$$

Substituting this in (2.3) and taking first difference yields:

$$\Delta \ln x_t = \delta + \alpha \Delta \ln P_t + \beta \Delta \ln Y_t \quad . \quad (2.4)$$

This is the model actually estimated.

Although very poor results were achieved with this model, in terms of significance of the coefficients, some interesting results were found applying analysis of covariance to regional groupings.

In their long-term stock equations, Fisher and Kaysen reject a stock adjustment model in favor a 'disease model.' They theorize that with regard to most electrical appliances, people either have them or they don't. Incremental purchases are a function of the number of potential customers (marriages, electrification), prices, and income. In addition, they say it is possible that incremental purchases are affected by the current stock of appliances (demonstration affect - keeping up with the Jones'). The model is double-logarithmic in form and explains the percent change in stocks of different appliances. Since stock equations are not estimated in this study, they will not be presented here.

Houthakker, Verleger and Sheehan employ a Koyck-lag model for electricity demand. This is derived by specifying a flow adjustment process in discrete time. In contrast, in the Houthakker-Taylor flow adjustment model

the adjustment process is specified in continuous time.¹³ Houthakker, Verleger, and Sheehan assume that electricity consumption x_t adjusts to some desired level x_t^* by the following discrete process

$$x_t/x_{t-1} = (x_t^*/x_{t-1})^\theta .$$

$0 < \theta < 1$ since adjustment to the desired level is slowed by the role of appliance stocks in demand. The desired level of demand is formulated as follows:

$$x_t^* = \alpha P_t^\gamma Y_t^\beta .$$

In this case, P_t is an average marginal price defined from Typical Electric Bills. The estimating equation derived from the model defined above is as follows:

$$\ln x_t = (1-\theta) \ln x_{t-1} + \theta \ln \alpha + \gamma\theta \ln p_t + \beta\theta \ln y_t . \quad (2.5)$$

The short-run price elasticity derived from (2.5) is $\gamma\theta$, the long-run elasticity is γ .

Mount, Chapman and Tyrrell estimate several versions of their model. I will discuss the version of their variable elasticity model for which they present results. The form of this model is a variation of the one employed by Houthakker, Verleger, and Sheehan, although it is not derived explicitly from an adjustment process. (A form identical to Houthakker, Verleger, and Sheehan's is also estimated.) The variable elasticity model is formulated

as follows:

$$x_t = Ax_{t-1}^\alpha \prod_{i=1}^I V_{it}^{\beta_i} e^{\sum_{i=1}^I \gamma_i / V_{it}} \quad (2.6)$$

where the V_{it} is independent variable i at time t and A , α , β_i , and γ_i are parameters to be estimated. The estimated form of this model is:

$$\ln x_t = \ln A + \alpha \ln x_{t-1} + \sum_{i=1}^I \beta_i \ln V_{it} + \sum_{i=1}^I \gamma_i / V_{it} \quad (2.7)$$

The short-run elasticity of V_{it} is, thus, $(\beta_i - \gamma_i / V_{it})$ and the long-run elasticity is $(\beta_i - \gamma_i / V_{it}) / (1 - \alpha)$.

This model theorizes that there is no reason to assume elasticities are constant at different levels of the variables. Mount, Chapman and Tyrrell obtain significant variation in the income elasticity (significance γ).

Income elasticity for average income is inversely related to the average level of income. Lyman addresses this issue more comprehensively and explicitly allows for the distribution of income. He finds that a change in the distribution of income shifting income from rich to poor (with no change in the average level) also increases electricity consumption.

All the other models except Lyman's are linear or log-linear in functional form. Lyman uses a general form of a nonlinear model which reduces to linear and log-linear forms as special cases.

Additional variables besides price and income are included in a number of models. The price of gas is one factor which should be relevant in the long run, although statistically significant results were not common.¹⁴ Similarly, the price of appliances might be a factor in long-run demand. Good data on this factor are unavailable. Variables for which significant results were obtained are climactic variables, degree of urbanization and number of customers. These will be discussed separately below.

Both Halvorsen, and Mount, Chapman and Tyrrell predict total demand and include number of customers as an independent variable. All of the other studies estimate per-customer demand. Neither Halvorsen nor Mount, Chapman and Tyrrell find customer elasticities significantly different from 1. (The values range from .95 to 1.00). This reinforces the rationale for estimating per-customer demand. It should be noted that Lyman does find some significant results for the inclusion of number of customers in a per capita demand equation. He justifies this on the grounds that customers serve as a proxy for the influences of migration and scale factors. However, his is a utility model and not a state model.

A variety of climactic variables were investigated by Halvorsen, Lyman, Anderson, and Mount, Chapman and Tyrrell. Significant effects were measured by Halvorsen and Lyman.

A final feature included in several models (Halvorsen, Anderson, Lyman) were measures of the degree of urbanization and household sizes. These were thought to be indicators of electricity demand because they might influence the type of appliance used and utilization patterns for appliances.

Although this is not a comprehensive discussion of the modeling efforts in electricity demand, the significant features of the problem have been touched on. The results of the estimating equations explicitly presented here will be compared with the results obtained subsequently which employ a variety of rate schedule representations. A discussion of these results is given in Chapter V.

2.7 Some Recent Research

Several as yet unpublished studies of residential demand for electricity have recently been brought to my attention. Both of these studies employ marginal prices taken from actual rate schedules, and therefore, have some bearing on my research. The studies are Lacy and Street (1975), and Acton, Mitchell and Mowill (1975).¹⁵ These studies have other similarities in that they both employ monthly data for small geographic areas. In Lacy and Street, the area represented is the service area of Alabama Power Company. In Acton, Mitchell and Mowill, the

areas represented are meter read books (approximately 260 customers) of Los Angeles Department of Water and Power, and rate districts for Southern California Edison.

The price and income elasticities obtained in these two studies are given in Table 2.1. In neither case can these elasticities be conclusively labeled short- or long-run elasticities. Acton, Mitchell and Mowill estimate the utilization rate of the stock of electric appliances. Thus, the elasticities they estimate are theoretically short run. The size of the estimated elasticities indicates that some long-run effects are estimated also. Lacy and Street estimate a static time series model. On this basis their estimates would normally be considered short run, but they argue that their elasticities represent an intermediate time span. These elasticities also seem large for short-run estimates.

One reason for the price elasticities being so large could be the time unit of these models. There is a fair amount of seasonal variation in demand among the seasons. This variation is sufficiently large to vary the consumption block of a rate schedule even when the rate schedule does not change. Some sort of seasonal adjustment to the rate variables seems in order.¹⁶ Clearly, no seasonal adjustment is needed in the present study because annual variables are used.

Footnotes

¹L. D. Taylor, "The Demand for Electricity: A Survey," The Bell Journal of Economics and Management Science 6 (Spring 1975): 74-110.

²H. S. Houthakker, "Some Calculations of Electricity Consumption in Great Britain," Journal of the Royal Statistical Society, ser. A, 114 (1951): 351-71; F. M. Fisher and C. A. Kaysen, A Study in Econometrics: The Demand for Electricity in the United States, Contributions to Economic Analysis, XXVII (Amsterdam: North Holland Publishing Co., 1962); J. W. Wilson, "Residential Demand for Electricity," Quarterly Review of Economics and Business 11 (Spring 1971): 7-22; Robert Halvorsen, "Residential Electricity: Demand and Supply," presented at the Sierra Club conference in Jackson, Vermont, January 1972, and "Demand for Electric Power in the United States," Institute of Economic Research, Discussion Paper No. 73-12 (Seattle, University of Washington, December 1973); Kent P. Anderson, "Residential Demand for Electricity: Econometric Estimates for California and the United States," R-905-NSF (Santa Monica: The Rand Corporation, January 1972), and "Residential Energy Use: An Econometric Analysis," R-2197-NSF (Santa Monica: The Rand Corporation, October, 1973); H. S. Houthakker, P. K. Verleger, and D. P. Sheehan, "Dynamic Demand Analysis for Gasoline and Residential Electricity" (Lexington: Data Resources, Inc., 1973); R. A. Lyman, "Price Elasticities in the Electric Power Industry" (Tucson: University of Arizona, 1973); T. D. Mount, L. D. Chapman, and T. J. Tyrrell, "Electricity Demand in the United States: An Econometric Analysis," ORNL-NSF-49 (Oak Ridge National Laboratories, 1973).

³For example, T. E. Cargill and R. A. Meyer, "Estimating Demand for Electricity by Time of Day," Applied Economics 3 (1971): 233-46, look at daily variation in demand. Industrial demand is investigated by R. E. Baxter and R. Rees, "Analysis of the Industrial Demand for Electricity," Economic Journal 78 (June 1968): 277-98, and K. P. Anderson, "Toward Econometric Estimation of Industrial Energy Demand: An Experimental Application to the Primary Metals Industry," R-719-NSF (Santa Monica: The Rand Corporation, 1971); H. S. Houthakker and L. D. Taylor, Consumer Demand in the United States, 2nd ed. (Cambridge: Harvard University Press, 1970) employ national rather than regional data. Paul F. Levy, "The Residential Demand for Electricity in New England," MIT-EL 73-017 (Cambridge: Massachusetts Institute of

Technology, November 1973) duplicates Halvorsen's approach with utility data and does not add any additional insight relevant to this discussion.

⁴This terminology was first employed by Taylor in "The Demand for Electricity," p. 78.

⁵Federal Power Commission, Typical Electric Bills (Washington, D.C.: Government Printing Office, 1950-1976).

⁶See, for example, Barbara Epstein, "A Proposal to Modernize Electricity Rates," Public Utilities Fortnightly, April, 1963; William G. Shepherd, "Marginal Cost Pricing in American Utilities," Southern Economic Journal, July, 1966; Irwin M. Stelzer, "Marketing, Environment, and Rates," Public Utilities Fortnightly, February, 1973; Haskell Wald, "The Theory of Marginal Cost Pricing and Utility Rates," Public Utilities Fortnightly, June, 1967, and "Recent Proposals for Redesigning Utility Rates," Public Utilities Fortnightly, September, 1973.

⁷Paul L. Joskow, "Inflation and Environmental Concern: Structural Change in the Process of Public Utility Price Regulation," Journal of Law and Economics 17 (1974): 291-327.

⁸A. W. Lacy and D. R. Street, "A Single Firm Analysis of the Residential Demand for Electricity," Department of Economics, Auburn University, 1975 (mimeographed).

⁹Demand for electricity is estimated by Houthakker and Taylor in Consumer Demand in the United States with national data.

¹⁰P. Balestra and M. Nerlove, "Pooling Cross Section and Time Series Data in the Estimation of a Dynamic Model," Econometrica 34 (July 1966): 585-612.

¹¹Since neither Lyman's nor Halvorsen's models include lagged dependent variables, their estimation procedures yield unbiased if inefficient results.

¹²G. E. P. Box and D. R. Cox, "An Analysis of Transformations," Journal of the Royal Statistical Society, ser. B., 26 (1964): 211-43.

¹³The Houthakker-Taylor model is discussed in Chapter IV.

¹⁴Wilson finds a significant demand elasticity for gas of the correct sign (positive) as does Halvorsen.

¹⁵A. W. Lacy and D. R. Street, "A Single Firm Analysis of the Residential Demand for Electricity," Department of Economics, Auburn University, 1975 (mimeographed), and J. P. Acton, B. M. Mitchell, and R. S. Mowill, "Residential Demand for Electricity in Los Angeles: An Econometric Study of Disaggregated Data," The Rand Corporation (R-1899-NSF), December 1975 (preliminary draft).

¹⁶This is not done in Lacy and Street. I have not been able to obtain a copy of Acton, Mitchell and Mowill (it will be published in 1977) and I do not know if their rate variables are seasonally adjusted.

CHAPTER III

DEMAND THEORY IN THE PRESENCE OF BLOCK RATE PRICING

3.1 Introduction

In the structuring of an econometric model it is important to incorporate as much as possible of what economic theory has to offer. In this chapter I examine from a theoretical point of view the demand functions of consumers facing decreasing block prices. The framework of analysis is that of standard demand theory with the exception that rate schedules are applied to one commodity. Emphasis is therefore placed on the measurement of price effects in the presence of rate schedules. The representation of rate structures in the model is then based on this discussion.

In the econometric literature on electricity demand it is acknowledged that the presence of a rate schedule might have some significance for the measurement of price effects. However, this acknowledgment has been limited to a discussion of whether it is more appropriate to use marginal or average rates in the demand function. Which marginal or average rates to use, since both vary, is seldom discussed.

In order to determine how to characterize appropriately a rate schedule for use in an econometric model one must look at how quantity discounts affect demand. I show that a better characterization of the rate schedule than can be achieved with a single price variable is needed. The solution to the consumer choice problem where price depends on quantity consumed is determined by the entire market offer curve and not any single market price. Since demand depends on the entire market offer curve, the use of average price or marginal price alone biases the estimated price effects in an empirical investigation. The determination of the rate schedule characterization should thus be based on a reexamination of demand theory.

Although the ultimate objective of this study is the estimation of an empirical model, this chapter is strictly theoretical. The discussion here is general and applies to the demand for any commodity with decreasing block rates even though our particular interest is electricity demand. The incorporation of this information into an econometric model is discussed later and included with the development of the model itself. The theoretical analysis, however, concentrates on issues which have some econometric import.

There are a number of theoretical issues which are investigated with respect to demand functions of consumers facing decreasing block rates. An issue of primary

importance is the derivation and definition of demand curves for such consumers. I look at the shape of these curves, examine their properties and compare them to the properties of demand functions derived in classical demand theory. In particular, I look at the issues of continuity and single-valuedness. Other authors have examined cases where problems with continuity and single-valuedness arise from variation in the preference axioms assumed, but not from the non-convexity of the budget set.¹

A theoretical result of demand theory is that the demand function of a consumer maximizing a utility function over n goods subject to a given level of income satisfies the Slutsky symmetry conditions. Such demand functions are called theoretically plausible. None of the econometric studies of electricity demand to date deals with demand functions which are theoretically plausible, even though quite a few studies using theoretically plausible demand functions have been done in recent years.² The second issue I examine is the notion of theoretical plausibility with respect to demand functions of consumers facing decreasing block rates.

The presentation of the above-mentioned issues is organized in the following way. In Section 3.2 I discuss generally the problems created by decreasing block prices and the work that has been done in this area to date. I demonstrate the bias that is created by using a single

price variable in econometric studies. In the third section, I establish a notation and the assumptions necessary for determining a demand function when there are decreasing block rates, and define this demand function. In the fourth section, I look at the above-mentioned issues of continuity, and theoretical plausibility from the viewpoint of individual consumer demand. In the fifth section, I reassess the same issues, but then from the viewpoint of an average demand function for a group of consumers. Finally, in the sixth I summarize the theoretical findings of this chapter.

3.2 The Implications of Decreasing Block Rates in Demand Theory: A Discussion of the Issues Involved and the Current State of Theory

The approach taken to demand theory throughout this chapter will follow the lines of traditional demand theory. It is assumed, to begin with, that a consumer selecting a market basket of n goods maximizes a utility function over these goods subject to his level of income. Alternative assumptions have been discussed in the literature. One such assumption is "that the consumer is throughout under the assumption that the price paid for the last unit applies also to all additional quantities purchased, or even if he knows that the price is going to be lower at some stage, he never knows where this stage is until he has actually reached it."³ The problems and misconceptions

resulting from this assumption have been clearly stated by Gabor (1955), and we can dispense with this alternative. Another assertion is that consumers are totally unaware of the prices they pay for electricity. This assertion is equivalent to electricity being totally price inelastic. If this is the case, it certainly denigrates all the effort which goes into the construction of complicated rate structures. If everyone were charged the average price paid, no one would alter his demand and total revenue collected would remain the same. I feel there is no more reason to drop the utility maximization assumption here than throughout the rest of demand theory.

Before proceeding to examine rigorously the problems block rate structures create for demand theory in the following sections, I will illustrate these problems through graphical examples. For the purpose of these examples, I shall consider price schedules of the form:

$$\begin{array}{ll} p_0: & x \leq k_0 \quad \text{fixed charge} \\ p_1: & k_0 < x \leq k_1 \\ p_2: & k_1 < x \leq k_2 \quad \text{incremental prices} \\ p_3: & k_2 \leq x \end{array}$$

The choice illustrated in the graphical examples will be between expenditures on electricity, x , and another composite good, q . The cost of electricity is defined by the above schedule, and the price index for q is defined as 1.

As is common in electricity rate schedules, some consumption is included in with the fixed charge p_0 . Income is denoted by y .

The first figure, Figure 3.1, illustrates the general consumer choice problem for a consumer facing declining block rates. The relative marginal price of x with respect to q for consumption levels of x less than k_0 is zero. For consumption levels between k_0 and k_1 it is p_1 ; for consumption levels between k_1 and k_2 it is p_2 ; and for consumption levels greater than k_2 it is p_3 . $B(1, p_0, p_1, p_2, p_3, y)$ is the budget set. It is the set of all non-negative commodity bundles affordable for individuals with income y , and the specified prices. Given the iso-utility contour defined by U , the utility maximizing commodity bundle for the consumer illustrated in the diagram is the point $(\hat{x}, \hat{q})$. The marginal price of $\hat{x}$ is p_2 . Rates p_0 and p_1 will be referred to as intramarginal prices as they are reflected in the cost of $\hat{x}$, but are not marginal. p_3 is termed an extramarginal price since it does not affect the cost of $\hat{x}$.

In the case where there are decreasing block rates, that is,

$$p_1 > p_2 > p_3 \quad ,$$

the budget set, $B(1, p_0, p_1, p_2, p_3, y)$, is not convex. It is this feature of declining block rates that differs from classical demand theory, and prevents some of its

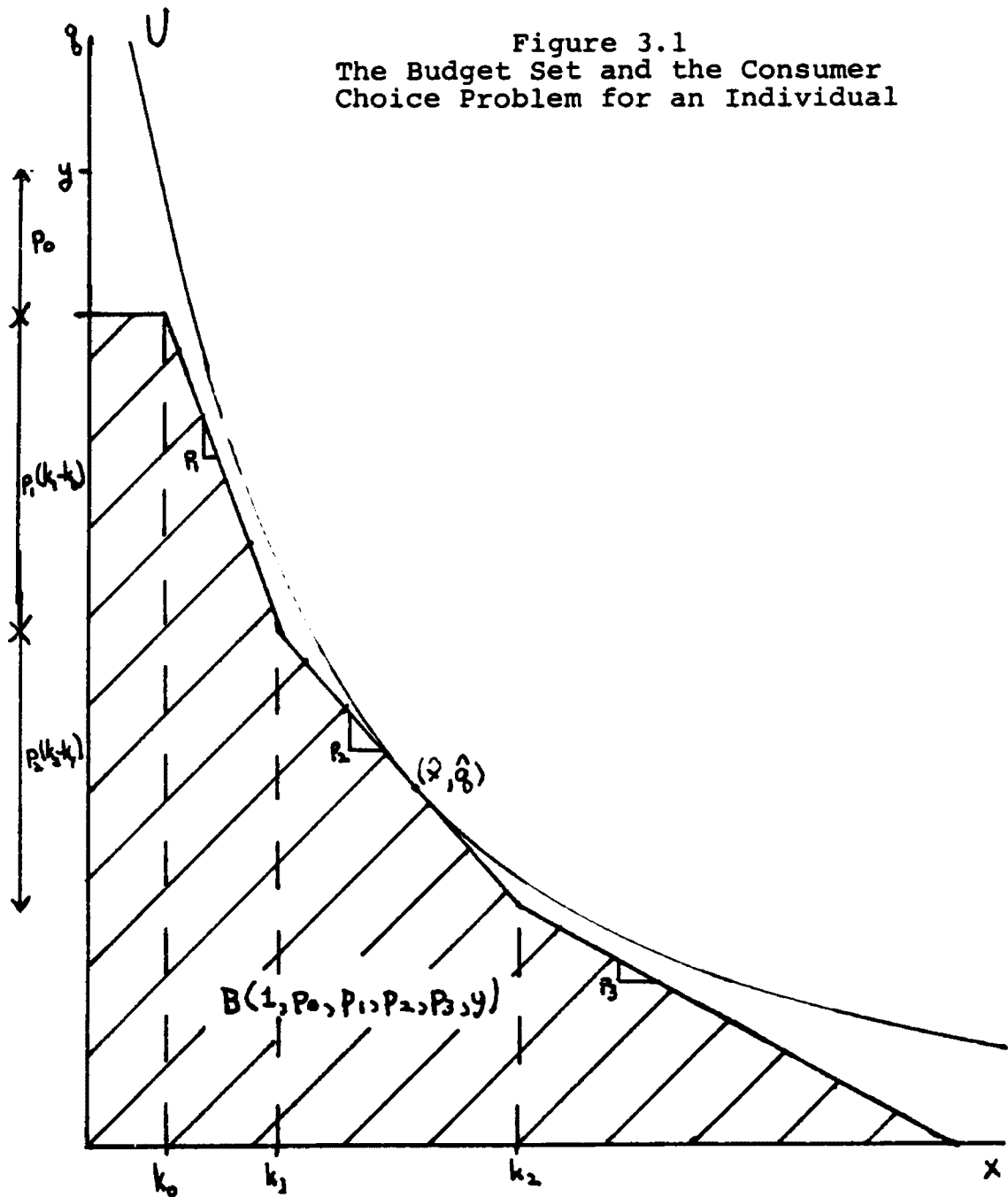


Figure 3.1
The Budget Set and the Consumer
Choice Problem for an Individual

conclusions from applying. Nevertheless, it should be noted that the budget set is compact if y is finite, all the block prices are non-negative, and the last block price and the price of q are strictly positive. These are restrictions I will make. The budget set is also independent of the particular utility function of the consumer.

Figures 3.2-3.4 illustrate the biases in estimated price effects that might result from the use of average or marginal price as an index of the rate schedule in an econometric model. These examples show that changes in the quantity demanded could be misattributed to price changes. On the other hand, a rate schedule change may not alter demand or may leave marginal and average rates unchanged.

Figure 3.2 illustrates a case where income is solely responsible for a shift in an individual's demand, but because of declining block rates this income shift also results in a change in both average and marginal prices. The average prices of the equilibrium quantities of demand for x at income levels y_1 and y_3 respectively are a and b . As income decreases from y_1 to y_3 , marginal price increases from p_3 to p_2 and average price from a to b . There is no change in the rate schedule in this example.

In Figure 3.3 a price schedule change alters quantity demand but not marginal or average price. In this example, marginal price remains at p_3 and average price is defined by the line ya in either case, yet this quantity change was

Figure 3.2
The Effect of a Change in Income on
Individual Demand

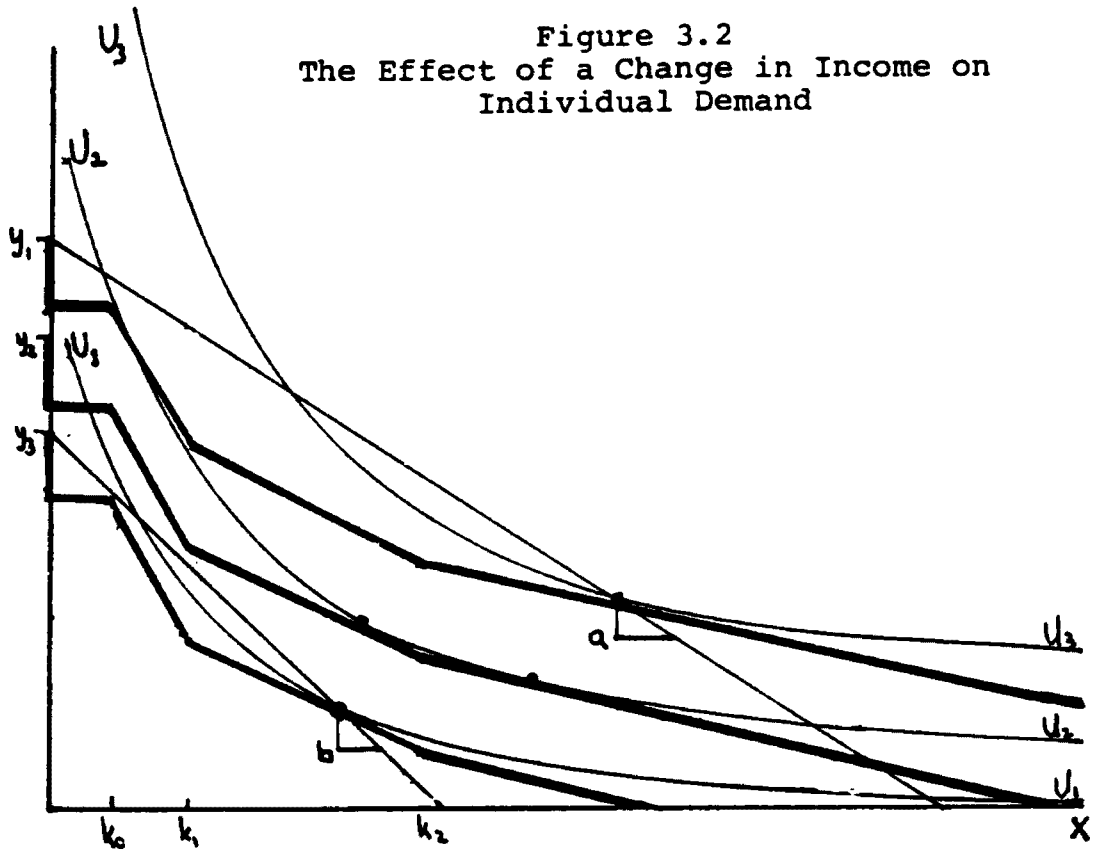
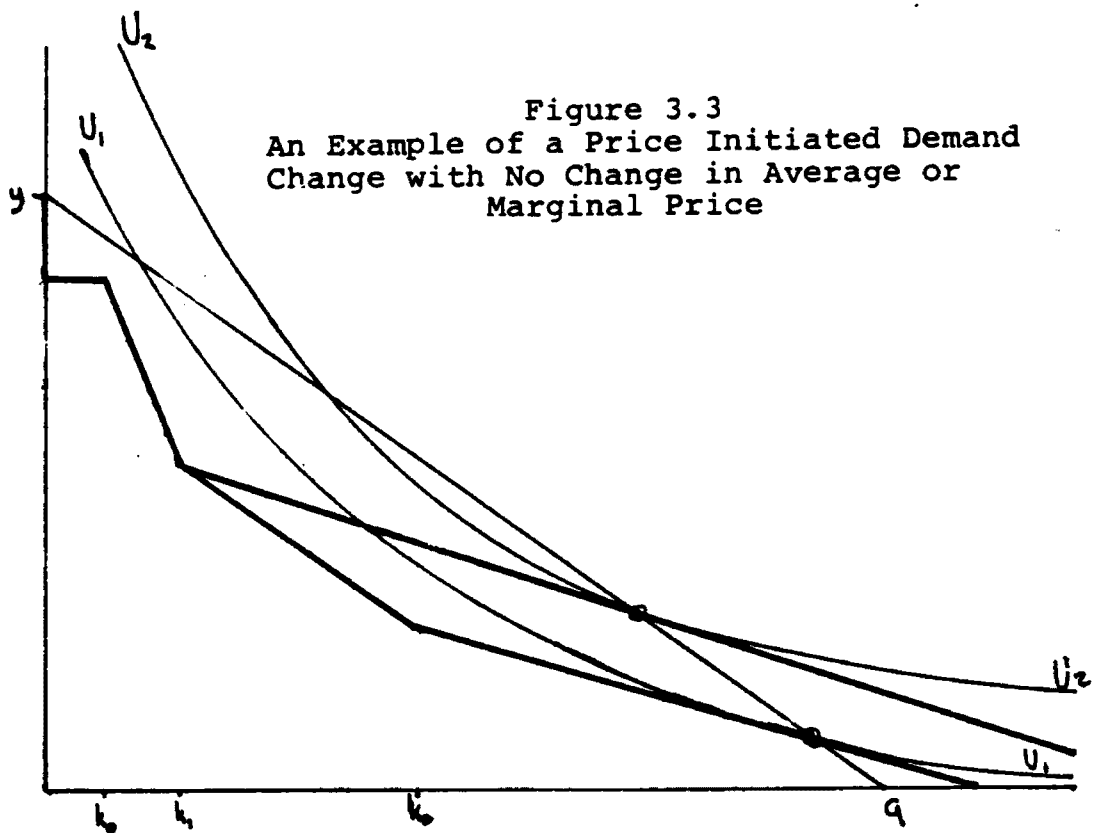


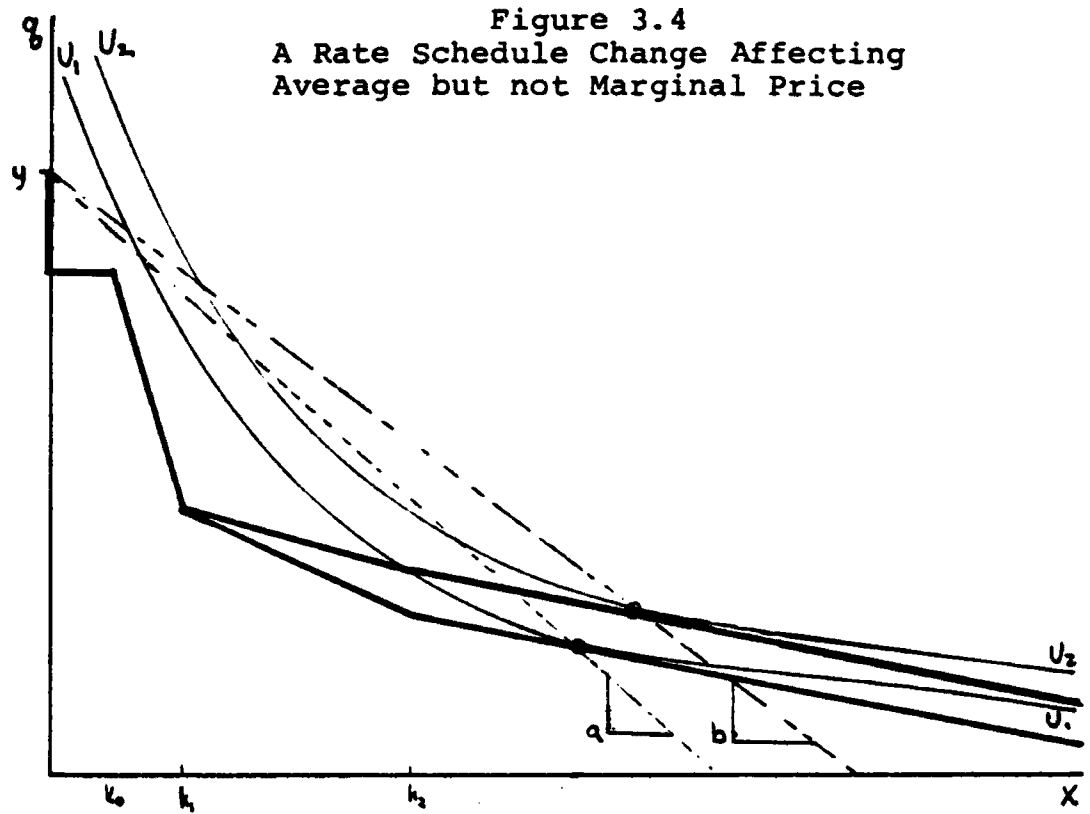
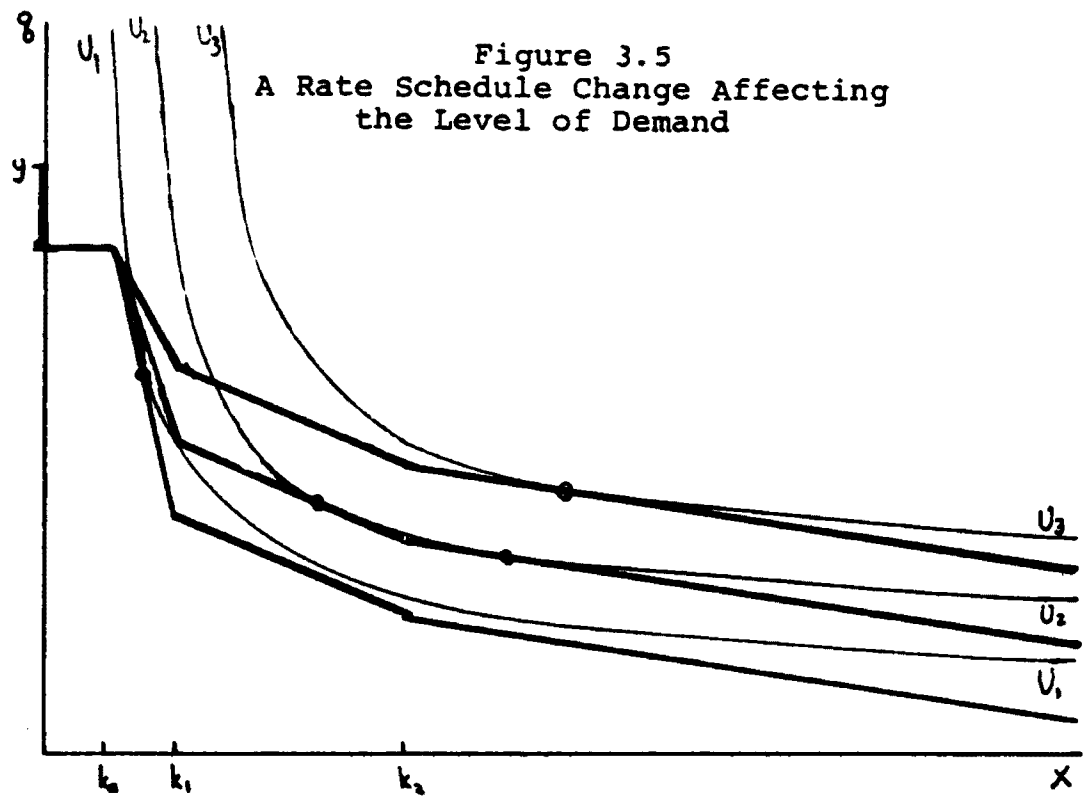
Figure 3.3
An Example of a Price Initiated Demand
Change with No Change in Average or
Marginal Price



caused by a price shift. Note that, although quantity falls, this example does not illustrate a Giffen good.

In Figure 3.4, a rate schedule change leaves marginal price unaltered while it induces a shift in both quantity demanded and average price. Average price shifts from a to b while the consumption level remains on the same facet of the budget constraint.

In each of these examples, the use of average price alone or marginal price alone might misrepresent the effect of price on demand. Average price is a declining function of quantity consumed. A given average price specifies only one point on the rate schedule. Thus, the average price variable, employed in many studies and calculated as total expenditure divided by total sales, is defined by the quantity it is then used to predict. A negative relationship between quantity and average price would exist with no price changes at all, simply reflecting the presence of declining block prices. Similar remarks could be made about marginal price. However, a given marginal price is in effect for a range of quantities not just a single point. Even so, the marginal price is also determined ex post as a function of quantity demanded, and further, marginal price does not entirely specify the cost situation facing a consumer even at a single level of demand.



Decreasing block prices also create problems with respect to the specification of a demand function. Figure 3.5 illustrates a shift from one facet of the budget constraint to another resulting from a price change. This example illustrates that the calculation of demand curves in all goods are not simple analytic problems. Discontinuities exist at the points where jumps are made from one facet to another, and at these prices the relationship between price and consumption will not be single-valued. This is true for all goods in the commodity bundle, not just electricity. Figure 3.2 illustrates that the same points can be made with respect to Engel curves.

The theoretical problems illustrated above have been understood since the articles of Buchanan (1953) and Gabor (1955). Until recently, no effort has been made to accommodate these problems in the econometric literature.⁴ This undoubtedly is partially because these authors were writing from a strictly theoretical viewpoint and did not attempt to determine any means of accommodating these problems econometrically. In addition, in most of the theoretical work that has been done on quantity discounts, the general approach has been to emphasize, sometimes exclusively, the producer's viewpoint.

Buchanan places more emphasis than anyone else on an attempt to characterize a consumer's consumption decision while facing monopolistic quantity discounts. However, rather than analyzing this situation with respect to block

prices, he employs as the budget constraint an arbitrary "opportunity curve which declines in slope as more units are purchased."⁵ In this situation neither a finite nor even a countable number of prices hold, and as Buchanan states, "No price-consumption function or demand function can be derived."⁶ With fixed block demarcations and a finite number of blocks, such a demand function can be derived. This is the situation which exists in practice.

Buchanan's results, of course, are equally valid for declining block prices. Buchanan was perhaps the first to recognize that quantity discounts are devices for extracting consumer surplus. He also recognized that quantity demanded is a function of the entire market offer in the case of quantity discounts and marginal price is an inadequate characterization of a rate schedule for determination of the level of demand. The generality of Buchanan's approach, however, prevents him from determining a demand relationship which allows income and the rate schedule to vary independently. He also does not allow for consumers to have varying preferences and income levels.

Gabor limits his discussion to block tariffs, but he does not attempt to determine any kind of functional demand relationship. He is concerned with other issues. He dispenses with the misconception that "unless each subsequent (small) unit is charged at a different price, it is impossible to deprive the consumer of the whole of his surplus, which is just as serious a fallacy as its corollary that

the corners of the price blocks must not extend beyond the point at which the consumer would be in equilibrium,"⁷ given the consumer is a utility maximizer. He proves the equivalence of a multi-block tariff and a fixed charge coupled with a flat rate per unit for a single consumer. He also discusses the efficiency of block tariffs from a producer's point of view. He does not consider the possibility of consumers with a distribution of preferences and incomes.

Another excellent paper on two-part tariffs is by Oi (1970). This paper views the problem solely from the producer's point of view. The most significant addition of Oi is that he does recognize the importance of variation in income levels and preferences among consumers when there are multiple block tariffs. This affects optimal pricing strategy as well as the nature of the demand function. His discussion of this from a producer's viewpoint complements my work on the demand side. Pricing strategy for setting block rates is discussed in Appendix 1.

As mentioned previously, there has been no work done in demand theory which departs from the assumption of a convex budget set with single prices for each commodity. Multivalued and discontinuous demand functions have appeared elsewhere, however, these were generated by relaxation of various standard preference axioms.

3.3 Notation, Assumptions, and the Definition of the Demand Function

In standard demand theory it is assumed that individual consumers face fixed prices or effectively horizontal supply curves. With block prices, price varies with quantity consumed and quantity consumed is determined by the entire market offer. The notion of a demand function must be expanded to accommodate this situation. In order to achieve a precise and rigorous definition, the premises on which the definition is based must be explicitly stated. In this section, I establish the notation and state the assumptions upon which the definition and the further analysis of demand functions are based. The definition of demand is specified. I also look at ways of dealing with the concept of continuity with respect to a possible multi-valued relationship.

It has been illustrated that the demand relationship for an individual facing block prices is not necessarily single-valued. The term function, however, will be used in its looser context to include a multi-valued mapping.⁸ This is equivalent to Debreu's term demand "correspondence."

A certain amount of notation is necessary to specify the situation facing the consumer. A vector of commodities, a vector of prices, the block demarcations for rate schedules, a cost function, and a budget set must all be specified.

There are assumed to be n commodities in the consumer's choice set. Ω is the set in n -space of all conceivable commodity bundles. It is usually possible to define units so that Ω is the non-negative quadrant of R^n . The vector $x = (x_1, x_2, \dots, x_n)$ represents a given commodity bundle where x_i is the amount of the i^{th} commodity. Subscripts are used to represent components of vectors and superscripts indicate particular vectors. In addition, small letters will generally indicate scalars and vectors, and capital letters will indicate sets. Throughout the discussion single prices prevail for the first $n-1$ commodities and the n^{th} is priced according to a block rate schedule.

The price vector, p , is $n + m - 1$ dimensional given m block prices applying to the n^{th} good.

$$p = (s, r) \quad ,$$

where s is the $n-1$ dimensional vector of single prices applying to $x_1, \dots, x_{n-1}$ and r is a vector of m block prices applying to x_n . The block prices apply to quantities purchased within sequential blocks. The sequential demarcations between the blocks where these prices apply are given by a vector k ,

$$k = (0, k_1, k_2, \dots, k_{m-1}, \infty) \quad .$$

For m blocks in the rate schedule k has dimension $m + 1$. r_1 applies to the first k_1 units of x_n , r_2 applies to units greater than k_1 but less than k_2 , and r_m applies to

all units purchased beyond k_{m-1} . It will be useful at times to refer to the sets K_i where

$$K_i = \{x \in \Omega \mid k_{i-1} \leq x_n \leq k_i\} \quad \text{for } i=1, \dots, m.$$

K_i defines the set of commodity bundles for which r_i is the marginal price of the n^{th} commodity. The set of commodity bundles for which r_i is an intramarginal price for the n^{th} commodity is called K_i^+ .

$$K_i^+ = \{x \in \Omega \mid x \geq k_i\} \quad \text{for } i=1, \dots, m-1; K_m^+ = \phi.$$

Similarly, the set of all commodity bundles for which i^{th} block is beyond the marginal block and thus, r_i is ineffective is called K_i^- . In this case, r_i is an extramarginal price.

$$K_i^- = \{x \in \Omega \mid x \leq k_{i-1}\} \quad \text{for } i=2, \dots, m; K_1^- = \phi.$$

Given these definitions,

$$K_i^+ = \bigcup_{j=i+1}^m K_j, \quad K_i^- = \bigcup_{j=1}^{i-1} K_j, \quad \text{and } \Omega = K_1^- \cup K_1 \cup K_1^+.$$

The set of admissible prices is called Π . The single price vector, s , is restricted to be positive. The block price vector, r , is restricted to be non-negative with the exception of its m^{th} component which must be strictly positive. These restrictions on r and s insure the compactness of the budget set given a finite income. The set of admissible prices for the i^{th} component of p is called Π_i ,

and the set of admissible rate schedules Π_r .

When only single prices apply, the cost of a commodity bundle can be concisely expressed as $p \cdot x$. The dimensionality of p and x as defined above are inconsistent. To circumvent this problem in expressing the cost of a commodity bundle, the $n + m - 1$ dimensional vector $g(x)$ is defined as follows:

$$(3.1) \quad g_i(x) = x_i \quad i = 1, \dots, n-1$$

$$g_n(x) = \begin{cases} x_n & \text{if } x \in K_1 \\ k_1 & \text{if } x \notin K_1 \end{cases}$$

$$g_{n-1+i}(x) = \begin{cases} 0 & \text{if } x \in K_i^- \\ x_n - k_{i-1} & \text{if } x \in K_i \\ k_i - k_{i-1} & \text{if } x \in K_i^+ \end{cases} \quad \text{for } 1 < i < m$$

$$g_{n-1+m}(x) = \begin{cases} 0 & \text{if } x \notin K_m \\ x_n - k_{m-1} & \text{if } x \in K_m \end{cases} .$$

The cost of a commodity bundle, x , can now be written $p \cdot g(x)$. A fixed charge or a minimum charge can be closely approximated in this framework by means of a redefinition of the term g_n . $g(x)$ is a continuous function of x .

An individual's choice set is constrained by his income, y , the price vector p , and block demarcation k to be the set of affordable commodity bundles under the given

conditions. The budget set is thus defined,

$$(3.2) \quad B(p, k, y) = \{x | x \in \Omega, \text{ and } p \cdot g(x) \leq y\} \quad .$$

If this set is to be compact and p is contained in the set Π , it is necessary that y be finite. For this reason y is constrained to be in the set Y ,

$$Y = \{y | 0 \leq y < \infty\} \quad .$$

The omission of any of the arguments in the expression $B(p, k, y)$ indicates that they remain constant. That vectors k and s do not vary is indicated by the expression $B(r, y)$. Similarly, $B(r_j)$ indicates that the only price variation is in r_j . The relevant characteristics of the budget set are:

- B1. $B(p, k, y)$ is compact where $p \in \Pi$, and $y \in Y$.
- B2. If $r_1 \leq r_2 \leq \dots \leq r_m$, then $B(p, k, y)$ is convex.
- B3. $B(p') \supset B(p'')$ if $p'' \geq p'$.⁹
- B4. $B(ap', ay) = B(p', y)$ where a is a positive scalar.
- B5. $B(r_j') \cap K_j^- = B(r_j) \cap K_j^-, \forall r_j \geq 0$.
- B6. $B(r_j', y) \cap K_j^+ = B(r_j'', y - (r_j'' - r_j')(k_j - k_{j-1})) \cap K_j^+, \forall r_j; r_j'' \geq 0$.

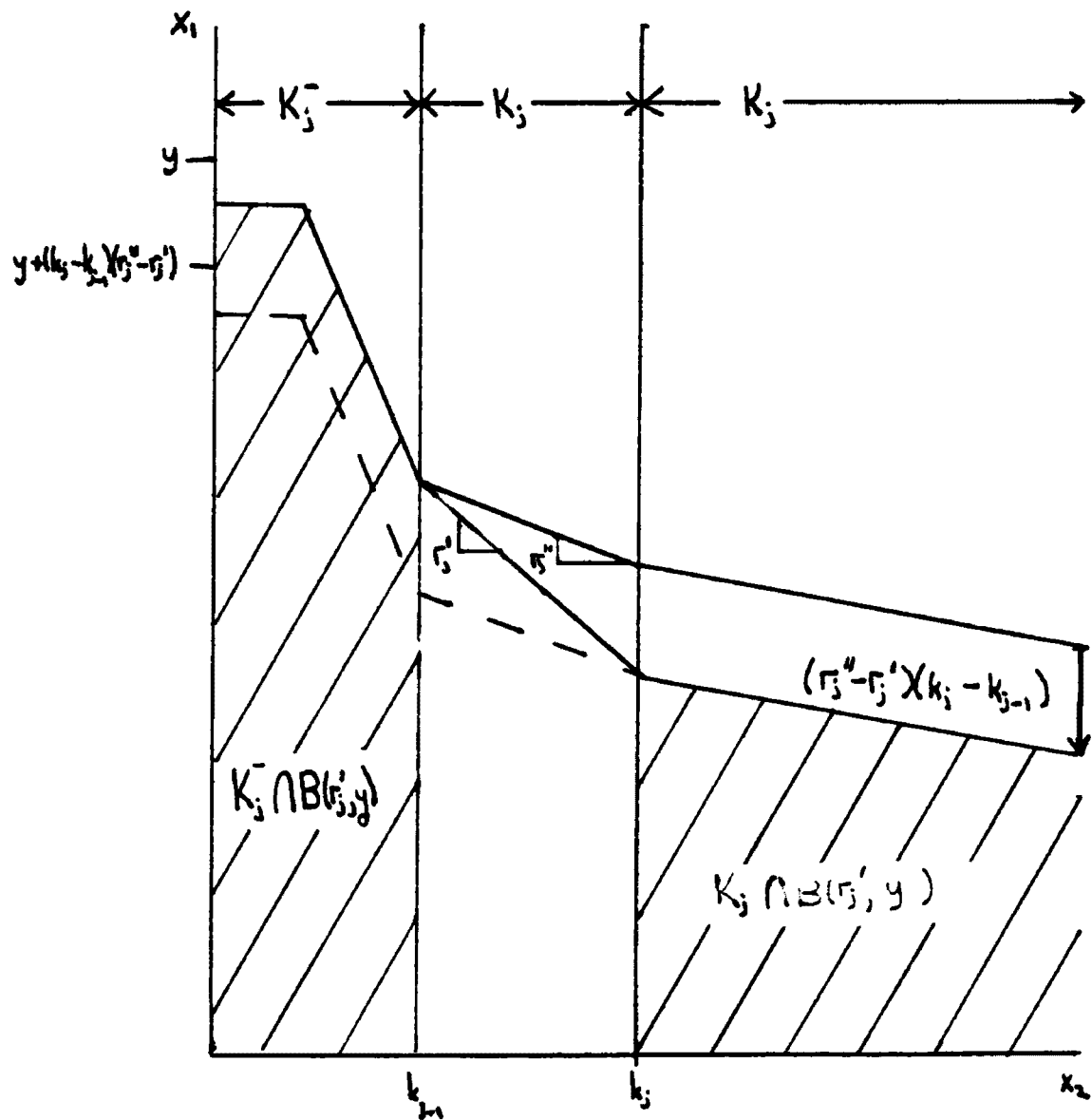
Characteristics B1 and B2 have been discussed earlier. Characteristics B3 and B4 are evident extensions of similar

statements about budget sets when single prices are in effect. B5 and B6 are illustrated in Figure 3.6. These characteristics have important implications for demand functions. B6 implies that a rate decrease in an intra-marginal block is equivalent to an income effect of a specified size. B5 implies that a rate increase in an extra-marginal block has no effect on demand whatsoever.

Individual demand functions will be derived from a standard set of axioms concerning individual preference. The axioms selected are sufficient for the continuity of ordinary demand functions with single prices for each commodity. They are also sufficient for the existence of a continuous utility function $u(x)$.¹⁰ The preference relation, P , is assumed to obey the following axioms:

- P1. Irreflexivity: For any $x \in \Omega$ we have $x \bar{P} x$.
- P2. Transitivity: For any $x, y, z \in \Omega$, the relations xPy and yPz imply xPz .
- P3. Convexity: For any $x, y \in \Omega$ such that $x \neq y$ and $x \bar{P} y$ we have $[(1 - \lambda)x + \lambda y] P x$, λ $0 < \lambda < 1$.
- P4. Monotonicity: For any $x, y \in \Omega$ such that $x \geq y$, xPy .
- P5. Continuity: For any $x^0 \in \Omega$ the set $\{x: x \in \Omega, x^0 Px\}$ is an open set in Ω .
- P6. Continuity: For any $x^0 \in \Omega$ the set $\{x: x \in \Omega, xPx^0\}$ is an open set in Ω .

Figure 3.6
An Illustration of Budget Characteristics B5 and B6



The derivation of a demand function is based upon the following theorem.

Theorem: If preference axioms P1 - P5 hold, then for any price vector, p , and income, y , there exists a commodity bundle $x^0 \in B(p, y)$ such that $x P x^0, \forall x \in B(p, y)$.

This is a restatement of Uzawa's theorem 5 without convexity of the budget set.¹¹ This theorem states that there is a point in any compact set which is at least as good as any other point in the set given the preference axioms. The proof which is based on the compactness of the budget set is identical to his; convexity would yield a unique x^0 .

The demand function of a consumer is designed to specify the relation of price and income vectors to a consumer's consumption choices. Here this is potentially a multivalued relationship. Accordingly, I define an individual's demand function

$$(3.3) \quad D(p, y) = \{x: x \in B(p, y), \forall z \in B(p, y), z \bar{P} x\} .$$

D is a set-valued function which maps into the commodity space, Ω , and designates all the options which might be selected under the given conditions. The demand for the i^{th} good, the i^{th} components of the vectors in $D(p, y)$, will be denoted $D_i(p, y)$. The function D satisfies the following conditions:

- D1. For any price vector p and income y , $D(p, y)$ defines a non-empty set of commodity bundles in Ω . The function is single-valued, if $B(p, y)$ is convex.
- D2. Any positive commodity bundle is chosen for a suitable price vector.
- D3. $\forall x$ such that $x \in D(p, y)$, $p \cdot g(x) = y$.
- D4. If the dimension of k is $m+1$, $D(p, y)$ contains at most m elements. That is, for any i , $i=1, \dots, m$, D contains at most one element such that $x \in K_i$.

Demand characteristics D1-D4 are extensions of similar characteristics of single-valued demand functions. D1 is an application of Uzawa's theorem which is stated above. D3 results from the continuity of the function $p \cdot g(x)$, and the monotonicity of preferences, P4. D4 is an extension of the preference axiom on convexity, P3. Finally, D2 is a restatement of a characteristic of single-valued demand. Single prices are a special case of block prices where all the block prices have the same value. The existence of a single price vector and income for any commodity bundle to be chosen is sufficient for D2.

Since D is rationalized by a transitive preference function, D also satisfies Richter's Congruence Axiom which reduces to Houthakker's strong axiom of revealed preference when demand is single-valued.¹²

Standard definitions of continuity apply to single-valued functions. Some expansion of this idea is necessary

with multivalued functions. In dealing with $D(p, y)$ the following definition of continuity will be used:

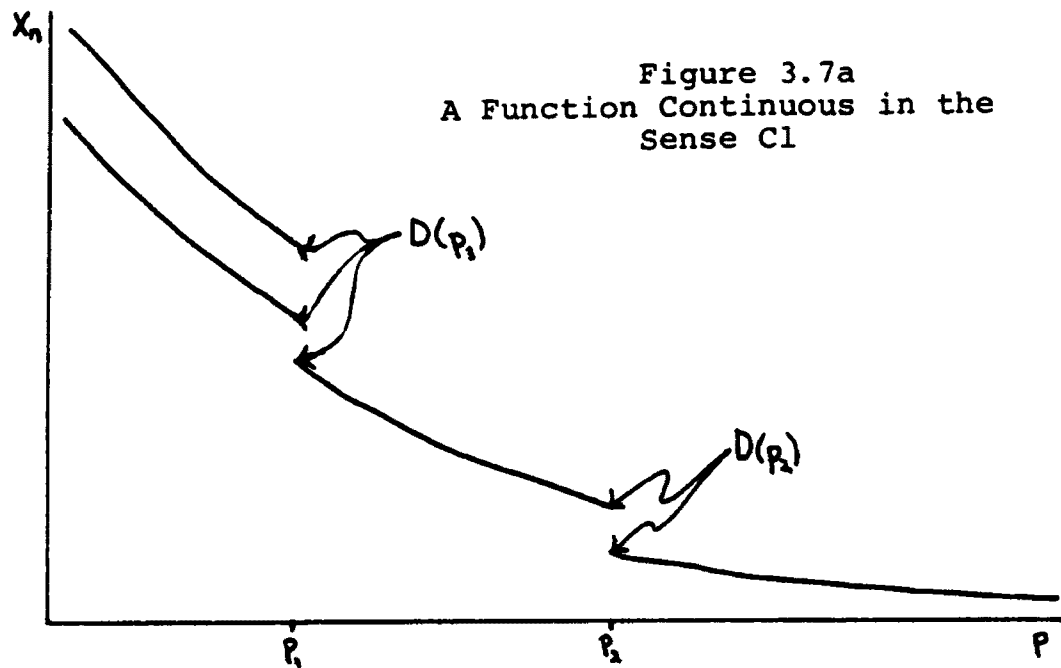
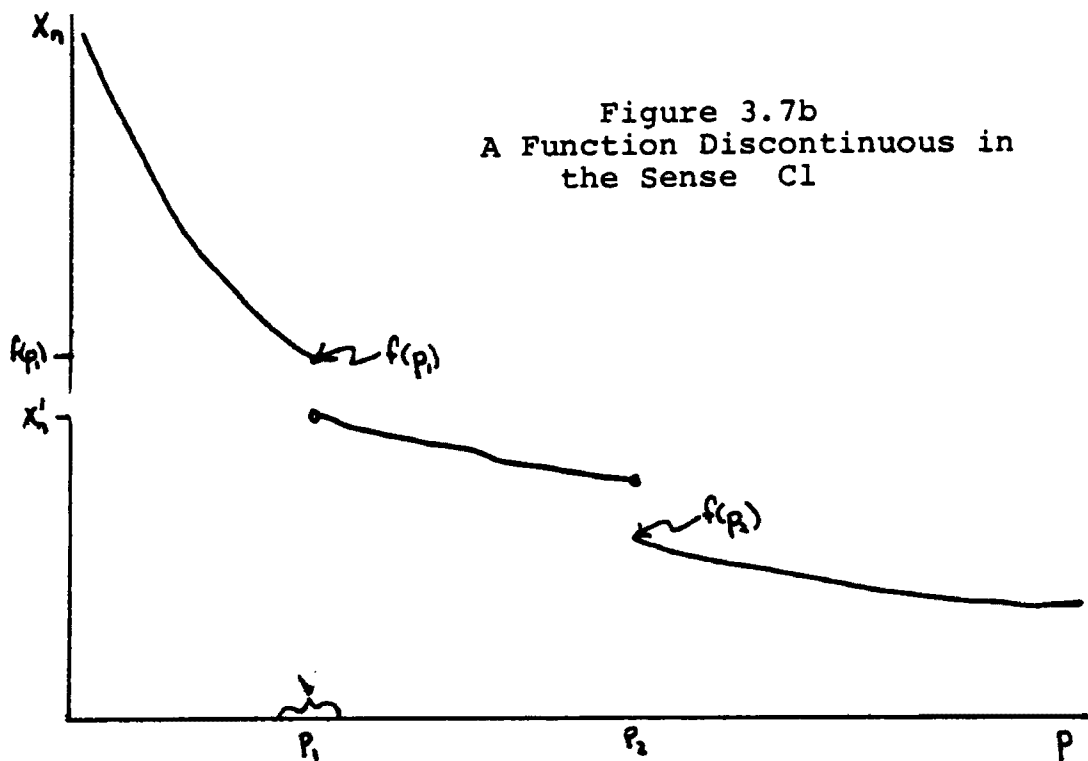
C1. A set valued mapping $D(p)$ is called continuous at p if for any open neighborhood W of the image set $D(p)$, there is an open neighborhood V of p such that $D(V) \subset W$ where $D(V) = \bigcup_{z \in V} D(z)$.¹³

This is a straightforward expansion of single-valued continuity and reduces to the ordinary definition when $D(p)$ is single valued. The image set of a function is continuous at a given point in the domain of the function if a neighborhood of that point can be selected so that elements in the image sets of the neighboring points are arbitrarily close to elements of the image set of the point in question. Figure 3.7a illustrates a function for which C1 holds; Figure 3.7b illustrates a function for which C1 does not hold. It is interesting to note where C1 breaks down in Figure 3.7b. At the point p_1 in Figure 3.7b, $x_n \notin f(p_1)$. However, for any open neighborhood V of p_1 , $x'_n \in f(p)$ for some $p \in V$. This contradicts the requirements of the definition.

A characteristic of set-valued mappings related to C1 is the following:

C2. A set valued mapping is closed at p , if $\lim p^V = p$, $\lim x^V = x$, and $x^V \in D(p^V)$ imply $x \in D(p)$.¹⁴

Figure 3.7
An Illustration of Continuity Definition C1



If the image set $D(p)$ is closed $C1$ implies $C2$. Note that the graph in Figure 3.7a is closed whereas the graph in Figure 3.7b is not. If the image set $D(p)$ is compact $C2$ implies $C1$. Since for any p , $D(p)$ contains a finite number of points by $D4$, the image set $D(p)$ is closed. The compactness of $D(p)$ in the neighborhood of any p , follows from the compactness of $B(p)$. Therefore $C1$ and $C2$ will be employed equivalently.

A function $d(p)$, defined as a weighted average of the elements of $D(p)$, shall also be employed. This function is single-valued by definition. In dealing with this function, a standard definition of continuity shall be used.

$C3$. $d(p)$ is continuous at p^* if $\forall \epsilon > 0$ there exists a $\delta > 0$ such that for all p where $||p^* - p|| < \delta$, then $||d(p^*) - d(p)|| < \epsilon$.

$||x||$ is the norm of the vector x . The particular norm employed in this chapter is $||x|| = \sum |x_i|$, but alternative norms can also be used.¹⁵ Note that $C1$ is identical to $C3$ if D is single-valued. However, since a norm applies to a vector and not a set of vectors, $C3$ is not equivalent to $C1$.

The use of d to circumvent the multivalued aspect of D does not pose serious restrictions because demand is de facto single-valued. That is, at any point in time only one commodity bundle is selected by any individual even though there may be other equally desirable commodity

bundles in the budget set. Since any one of the commodity bundles in $D(p, y)$ is equally desirable, it is just a matter of chance which one is selected. This is the justification for the use of $d(p, y)$. It will not itself represent a feasible choice for the individual if there is more than one point in $D(p, y)$ and all weights are positive, since P manifests strict convexity (P.4). It does represent in some sense the expected value of the individual's demand. This idea is also stated by Richter:

The set $h(B)$ may be thought of as the set of most preferred alternatives from the budget set B . However, since a single selection will often have to be made from the alternatives in B we may consider that the selection is to be made in two stages: First, the set $h(B)$ is selected from B , and second, an element is selected from $h(B)$ by some random device.¹⁶

The definitions and notation specified in this section apply throughout the rest of the chapter, unless explicitly stated otherwise.

3.4 Demand Functions for an Individual

3.4A Continuity of Demand Functions for an Individual

The continuity of the demand function $D(r)$ and the expected demand function $d(r)$ with respect to variation in the rate schedule is a subject of interest. In this section, I show that problems with the discontinuity of d and the multivaluedness of D occur over a small set and examine the nature of this set.

Initially, the issue of continuity will be examined within the framework of assumptions and definitions given in the last section. Within this framework it is established that the set-valued function D is continuous in the sense of C1. In addition, it is shown that the set of admissible rate schedules, Π_r , can be partitioned into m open sets over each of which D is single-valued and continuous, and a set for which D is multivalued. The set for which D is multivalued is shown to have Lebesgue measure zero. This last set is equivalent to the set on which d is discontinuous. Further statements about the discontinuities of d are only made with further assumptions.

Given certain additional assumptions, the continuity of d at any point in Π_r , the set of admissible rate schedules, is examined in directions parallel to the axes of R^m . Under these assumptions $d(r_j)$ is found to have at most $m - j + 1$ points of discontinuity. Furthermore, the m open sets over which $D(r)$ is single-valued are also shown to be connected. This result will be employed in the next section.

All of the assertions made above will be stated explicitly in the form of lemmas within this section. Rigorous mathematical proofs of these lemmas are given in Appendix 2. In the discussion in this section an attempt will be made to present a general idea of the proofs of the lemmas where relevant and to explain their key features and interrelationships.

In proving all the lemmas stated in this section, I employ the definitions of the demand function and the budget set (equations 3.2 and 3.3), the preference axioms (P1 - P6), and the resultant budget and demand characteristics which are deduced from these assumptions. A few of these characteristics play particularly important roles in proving the assertions made above. These roles will be discussed briefly before proceeding to an explicit discussion of the lemmas.

First of all, it should be clear that the preference axioms P5 and P6 are important. In the investigation of the continuity of demand it seemed appropriate to insure the continuity of demand when there are only single prices. This is effected through P5 and P6. These same characteristics result in the continuity of $D(r)$ in the sense C1.

The budget characteristics B5 and B6 are peculiar to block pricing. They also play an important role throughout the analysis. By B5 a change in an extramarginal block price has no effect on demand; by B6 a change in an intramarginal block price is equivalent to an income change. B5 and B6 are stated rigorously in the last section and illustrated in Figure 3.6.

Through B5 it is shown that the subset of Π_r over which $D(r)$ is multivalued has Lebesgue measure zero. This is done by showing that, at any point in this set, the set is zero dimensional in some direction. This point will be elaborated later.

B6 indicates that the nature of income effects are important. A rate change in an intramarginal block will be equated to an income change. B6 will therefore be used in conjunction with additional assumptions about the nature of income effects on demand. These assumptions will be specified at a later point. With these additional assumptions $d(r_j)$ is shown to have at most $m - j + 1$ points of discontinuity.

Finally, demand characteristic D4 limits the number of points in the set $D(r)$ for any $r \in \Pi_r$. This characteristic of $D(r)$ is useful in establishing the points of discontinuity of $d(r)$.

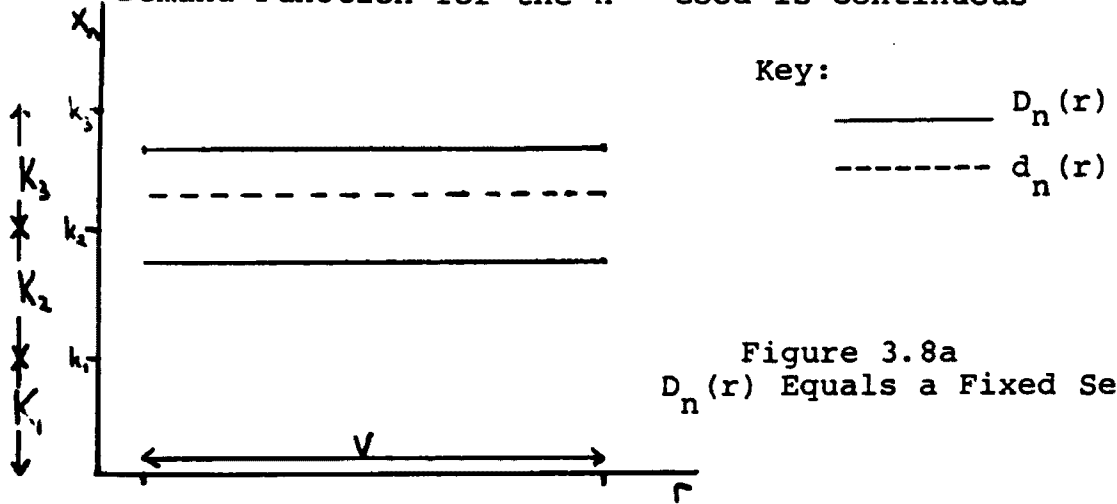
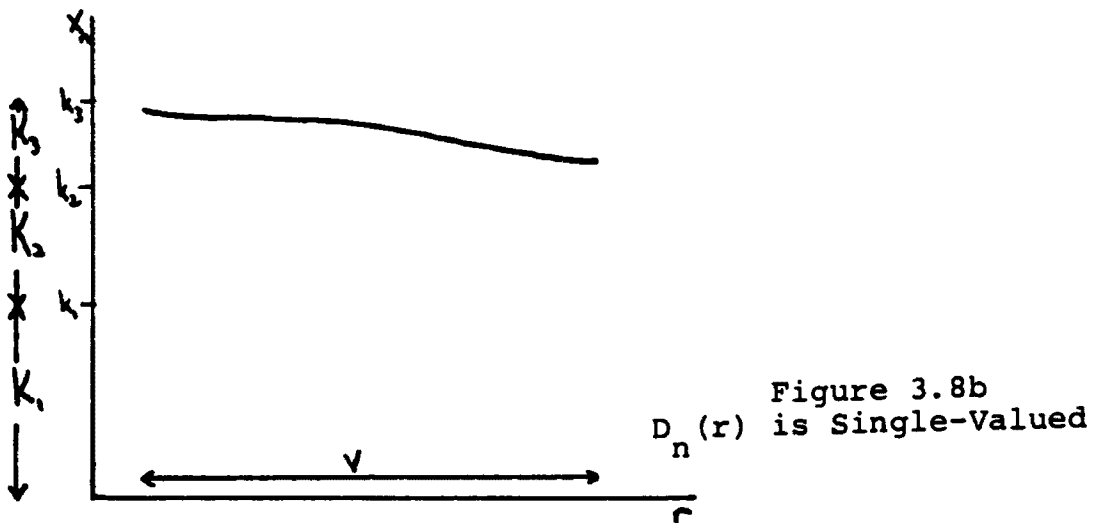
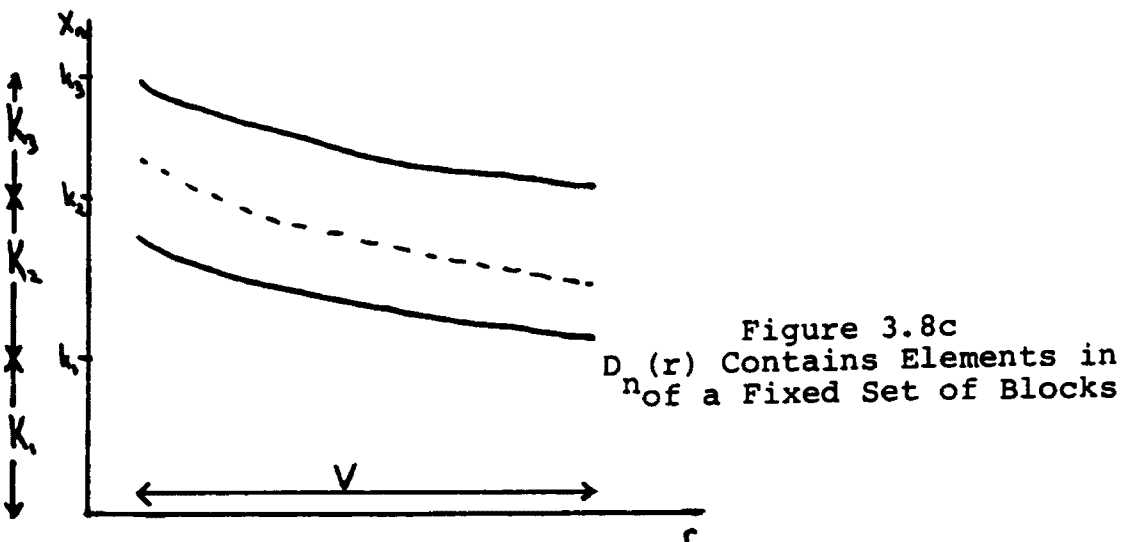
I will now proceed to present the results of this section explicitly. All of these results are derived from the ideas mentioned above. Rigorous proofs are included in Appendix 2.

The first lemma is a trivial application of C1.

Lemma 1. If a set-valued function, $D(r)$, is constant over a domain V , then it is continuous in the sense C1 on V , and $d(r)$ is constant and continuous on V also.¹⁷

When $D(r)$ is constant over a region it means that $D(r)$ equals some fixed set A for all r in the region. Of course by D4, A can contain at most one element of each set K_i . A graph of a function $D_n(r)$ on a set V for which $D(r)$ is constant is shown in Figure 3.8a.

Figure 3.8
Examples of Sets on which the Individual's Average
Demand Function for the n^{th} Good is Continuous



Given the equivalence of C1 and C2 in the stated framework of assumptions, it is possible to obtain results from the continuity of $D(R)$ by establishing C2. In establishing C2 it is necessary to examine the limit points of $D(r)$. This is done in Lemma 2.

Lemma 2. Given $\bar{r} \in \Pi_r$. If for all $\epsilon > 0$, there exists an $r \in \Pi_r$ such that $||r - \bar{r}|| < \epsilon$, and $x \in K_i$ for some $x \in D(r)$, then there exists an $\bar{x} \in D(\bar{r})$ such that $\bar{x} \in K_i$ and $\bar{x} = \lim_{r \rightarrow \bar{r}} (D(r) \cap K_i)$.

This lemma examines the limit points of $D(r) \cap K_i$. Of course, for any $r \in \Pi_r$, $D(r) \cap K_i$ contains at most one point by D4. An outline of the proof of this lemma follows. It is shown that the sequence of points $\{D(r) \cap K_i\}_{r \rightarrow \bar{r}}$ is contained in a compact set by B1 and B3. Therefore, this sequence contains a limit point $\bar{x}$. The continuity axioms, P5 and P6, are used to show $\bar{x} \in D(\bar{r})$. $\bar{x} \in K_i$, since K_i is closed. D4 insures that there can be only one such limit point, since $D(\bar{r}) \cap K_i$ contains at most one point.

A number of corollaries are the immediate result of this lemma:

Corollary 1: $D(r)$ is continuous in the sense C1 over the domain Π_r .

Corollary 2: If $D(r) \subset K_i$ for all r in the set V , then $D(r)$ is single-valued and continuous in

the sense C3 on V , and for any r' on the boundary of V , there exists an $x \in D(r')$ such that $x \in K_i$. $d(r)$ is also continuous in the sense C3 on V .

Corollary 3: If for some set $V \subset \Pi_r$, for all $r \in V$, $D(r) = \{f^i(r) | f^i(r) \in K_i, \text{ for all } i \in I\}$, then d is continuous on V in the sense C3 and for each point r' on the boundary of V there exists an $x \in D(r')$ such that $x \in K_i$.

The first corollary is one of the final objectives which I asserted would be proven in this section. It results from the equivalence of C1 and C2. There are only a finite number of sets K_i , and $\bigcup_{i=1}^m K_i = \Omega$. Therefore, any sequence $\{x^v\} \subset \Omega$, contains a subsequence in K_i for some i . Given any sequences $r^v \rightarrow \bar{r}$, and $x^v \in D(r)$ such that $x^v \rightarrow \bar{x}$, there is a subsequence of x^v in some K_i . Applying Lemma 2 to this subsequence shows that $\bar{x} \in D(\bar{r})$ establishing C2.

The next two corollaries deal with the nature of $D(r)$ on particular subsets of Π_r . The particular subsets selected are sets over which $d(r)$ is continuous in the sense C3 and this is established. As was indicated in the graphical examples in Section 2 of this chapter, $d(r)$ is discontinuous at the points where demand jumps from one block to another. In Corollary 2 demand is constrained to

fall within a single block. This is illustrated in Figure 3.8b for the function $D_n(r)$. In Corollary 3, demand is constrained to contain elements in each of a fixed set of blocks. This is illustrated in Figure 3.8c for the function $D_n(r)$. No jumps exist from one block to another on these specified sets and $d(r)$ is continuous in the ordinary sense. These corollaries are employed later in defining the family of sets over which $d(r)$ is continuous.

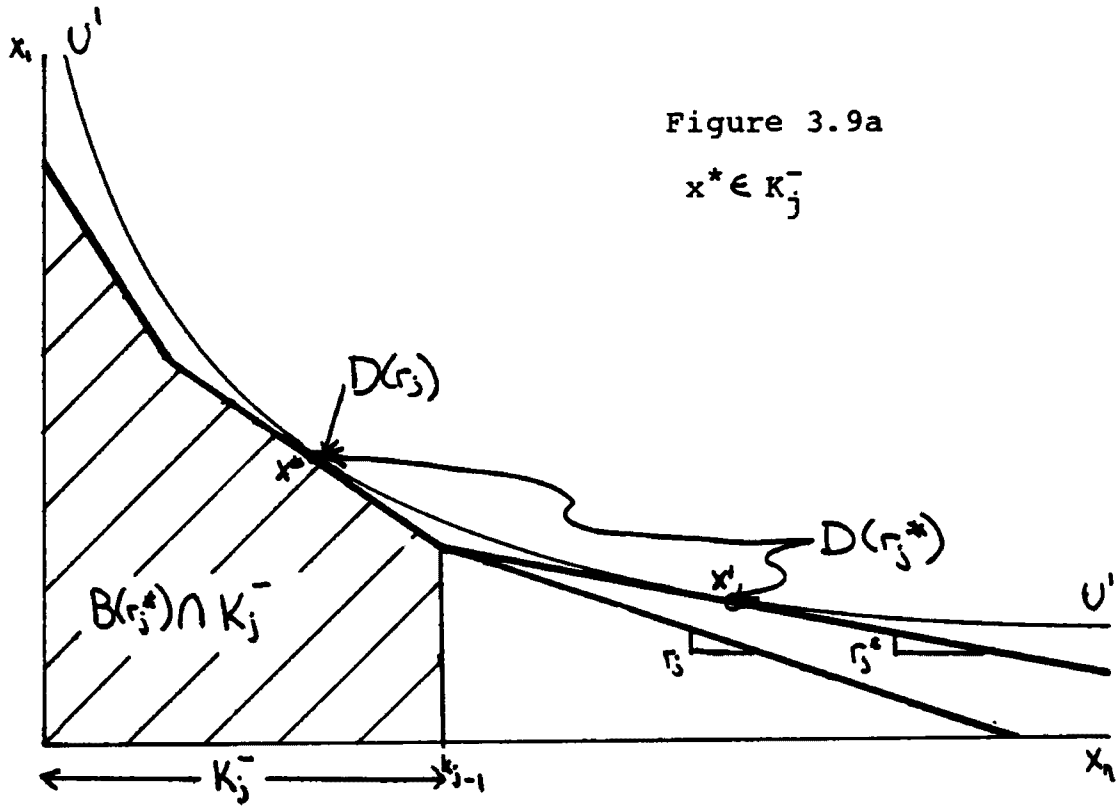
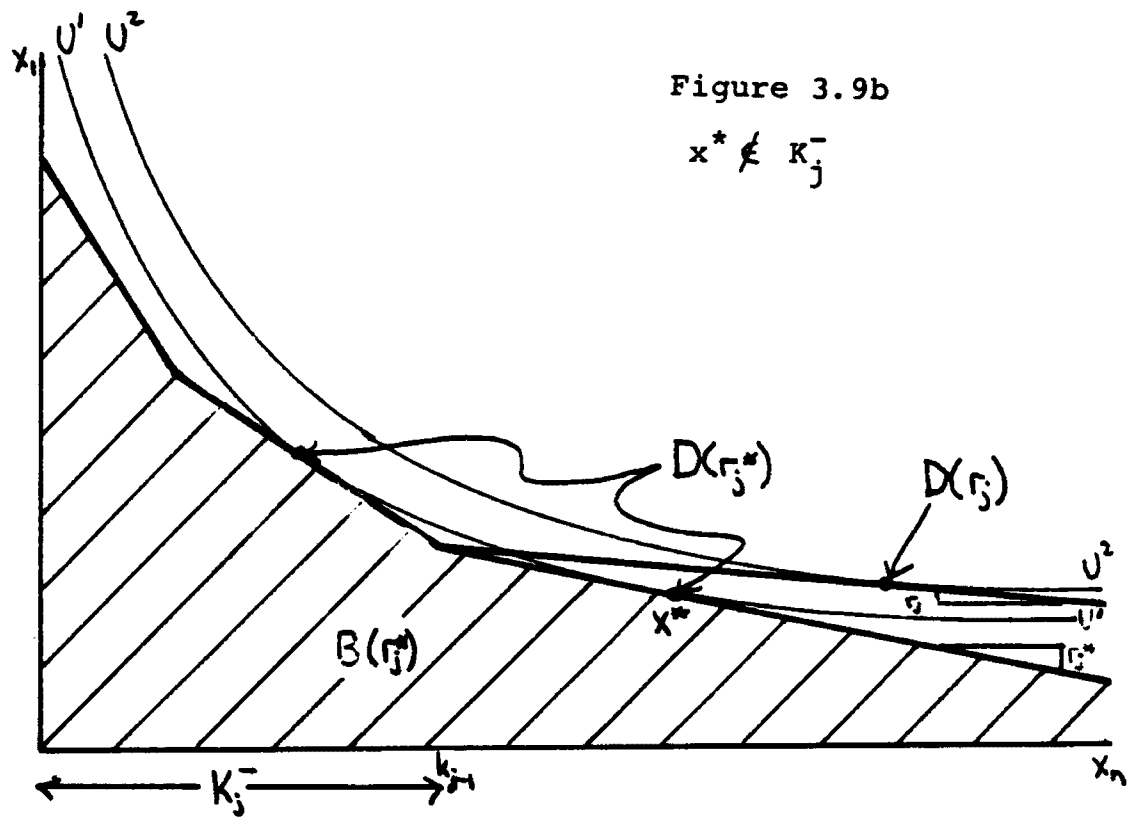
The implications of budget characteristics B5, have been alluded to earlier. It will be used to show that the points at which $D(r)$ is multivalued are boundary points to sets on which $D(r)$ is single-valued and that the set on which it is multivalued is of measure zero. It is later used to limit the number of points of discontinuity of $d(r_j)$. In each of these cases the implications of B5 are applied through the use of the following lemma which is based on B5. In this lemma all block prices except one remain constant.

Lemma 3. Given $x^* \in D(r_j^*)$. If $x^* \in K_j^-$, then
 $D(r_j) \subset K_j^-$ for all $r_j > r_j^*$. If $x^* \notin K_j^-$,
then $D(r_j) \cap K_j^- = \emptyset$ for all $r_j < r_j^*$.

Lemma 3 is illustrated in Figures 3.9a and 3.9b.

In Figure 3.9a there exists an x^* in $D(r_j^*)$ such that $x^* \in K_j^-$. As the j^{th} block price increases in this figure the budget set shrinks, but, by B5, the part of the budget

Figure 3.9
An Illustration of Lemma 3



set contained in K_j^- remains unaltered. Any optimal elements of the larger budget set, $B(r_j^*)$, contained in K_j^- , such as x^* , are also optimal elements of the smaller budget set, $B(r_j)$. Any optimal elements of the larger budget set not in K_j^- , such as x' , are not contained in the smaller budget set. Thus, $D(r_j) \subset K_j^-$, for all $r_j > r_j^*$.

In Figure 3.8b there exists an x^* in $D(r_j^*)$ such that $x^* \notin K_j^-$. As the j^{th} block price decreases in this figure, the budget set gets larger, but again by B5 the part of the budget set in K_j^- remains unaltered. For any element in the budget set $B(r_j^*)$ not in K_j^- , such as x^* , there are elements in the larger budget set, $B(r_j)$, which are greater than x^* , and thus strictly preferred to it. Therefore, for all $x \in D(r_j)$, xPx^* . Since the part of the budget set in K_j^- remains unaltered, there are no elements in this subset which are strictly preferred to the optimal elements of the budget set, $B(r_j^*)$. Thus, $D(r_j) \cap K_j^- = \emptyset$ for $r_j < r_j^*$.

The preliminaries have now been established to prove Lemma 4.

Lemma 4. It is possible to subdivide the set Π_r into m closed overlapping sets, H_i , $i = 1, \dots, m$ such that $D(r)$ is single-valued and continuous on the interior of each of these sets and multivalued on the boundaries.

This is one of the main assertions of the opening paragraphs of this section. Its proof consists of constructing the sets, H_i , and demonstrating that these sets satisfy the requirements of the lemma. The sets, H_i , are defined in the following way:

$$H_i = \{r \in \Pi_r \mid \exists x \in D(r), x \in K_i\} \quad i = 1, \dots, m.$$

Thus, H_i contains all the rate schedules, r , such that the consumption of electricity might fall within the i^{th} block. If $D(r)$ is multivalued, r is a member of more than one set H_i and the sets, H_i , overlap. For all $r \in \Pi_r$, $r \in H_i$ for some i since $D(r) \subset \Omega$ and $\bigcup_{i=1}^m K_i = \Omega$.

In proving Lemma 4, it is first shown that the sets H_i are closed and that they cover Π_r . It is then shown that $D(r)$ is multivalued if and only if r is in the boundary of some set H_i . This is established through the application of Lemma 3 to a comprehensive list of cases. Since $D(r)$ is single-valued on the interior of each set H_i , $D(r) \subset K_i$ on the interior of each H_i , and both $D(r)$ and $d(r)$ are continuous on the interior of each set H_i by Corollary 2.

Of the general results asserted in the beginning of the section which were obtained without the use of any additional assumptions, it remains only to show that the set over which $D(r)$ is multivalued has Lebesgue measure zero. This is done in Lemma 5. The sets, H_i^o , in this

lemma are the interior of the sets H_i just defined.

Lemma 5. The set $\Pi_r - \bigcup_{i=1}^m H_i^\circ$ has Lebesgue measure zero.

According to the preceding lemma, the set $\Pi_r - \bigcup_{i=1}^m H_i^\circ$ contains the set of points over which $D(r)$ is multivalued, and this set is equal to the union of the boundary points of each of the sets H_i . To prove Lemma 5, it is sufficient to show that the boundary of each of the sets H_i has measure zero. This is done by showing that this boundary is of zero dimension at each point in some direction. Thus, when its volume is integrated up it has measure zero. This is analogous to showing that the volume of space occupied by the surface of a sphere in three space has zero measure.

Corollary 4. The set of points over which $d(r)$ is discontinuous has Lebesgue measure zero.

This is an obvious extension of Lemma 5. Since $d(r)$ is continuous on H_i° for all $i = 1, \dots, m$ then the set over which $d(r)$ is discontinuous is also equal to $\Pi_r - \bigcup_{i=1}^m H_i^\circ$.

In the earlier part of this section, I alluded to the fact that further results could be obtained with additional assumptions. B6 equates price changes and income changes in certain situations. Thus, assumptions on the nature of income effects give insight into these situations.

In demand theory, the nature of income effects is specified through the concept of income elasticity. However, the standard concept of income elasticity is specific to single-valued demand functions. It is not easily expandable to set-valued functions. Initially, in examining the nature of income effects, I attempted to reduce the standard, single-valued demand concept of income elasticity to restrictions on the preference relation. Thus, zero income elasticity would imply vertically parallel indifference curves. Restrictions on the preference relation could be easily applied to the demand function as I have defined it. This is the most general way of approaching the problem of income elasticity for a set-valued function. However, the restrictions on the preference relation so defined proved too complicated to specify clearly on an n -dimensional commodity space. Consequently, I have re-defined strict zero income elasticity and strict positive income elasticity in terms of the demand function $D(r)$. The term strict is employed because, although these definitions imply their counterparts if demand is single-valued the converse is not true. Zero and positive income elasticity are defined through the elasticity assumptions E1 and E2 which follow.

- E1. A function has strict zero income elasticity with respect to the n^{th} good if $D_n(y^0) = D_n(y')$ for $y^0 \neq y'$.

E2. A demand function has strict positive income elasticity with respect to the n^{th} good if $\min\{D_n(y^0)\} > \max\{D_n(y')\}$ for $y^0 > y'$.

Note that if demand is single-valued these definitions reduce to the standard notions of zero and positive income elasticity. For single-valued demand E2 requires that

$$D_n(y^0) > D_n(y') \text{ for } y^0 > y'.$$

Figures 3.10a and 3.10b illustrate examples of Engel curves for functions having strict zero or positive income elasticity. In Figure 3.10a, zero income elasticity is illustrated. Although $D(y)$ is multivalued, it is constant for all y . Figure 3.9b illustrates strict positive income elasticity. In this case, demand is strictly increasing. Thus, if $x_n^* \in D_n(y^*)$ then for all $y > y^*$ and $x_n \in D_n(y)$, $x_n > x_n^*$.

These assumptions have the following implications for $d(r_j)$:

Lemma 6. If E1 holds everywhere, then $d(r_j)$ has at most two points of discontinuity.

Lemma 7. If E2 holds everywhere, then $d(r_j)$ has at most $m - j + 1$ points of discontinuity.

The central part of the proofs of these lemmas is the establishment of a partition of the set Π_{r_j} . Two points, r_j^0 and r_j^* are defined which partition Π_{r_j} into at most

Figure 3.10
An Illustration of the Income
Elasticity Constraints

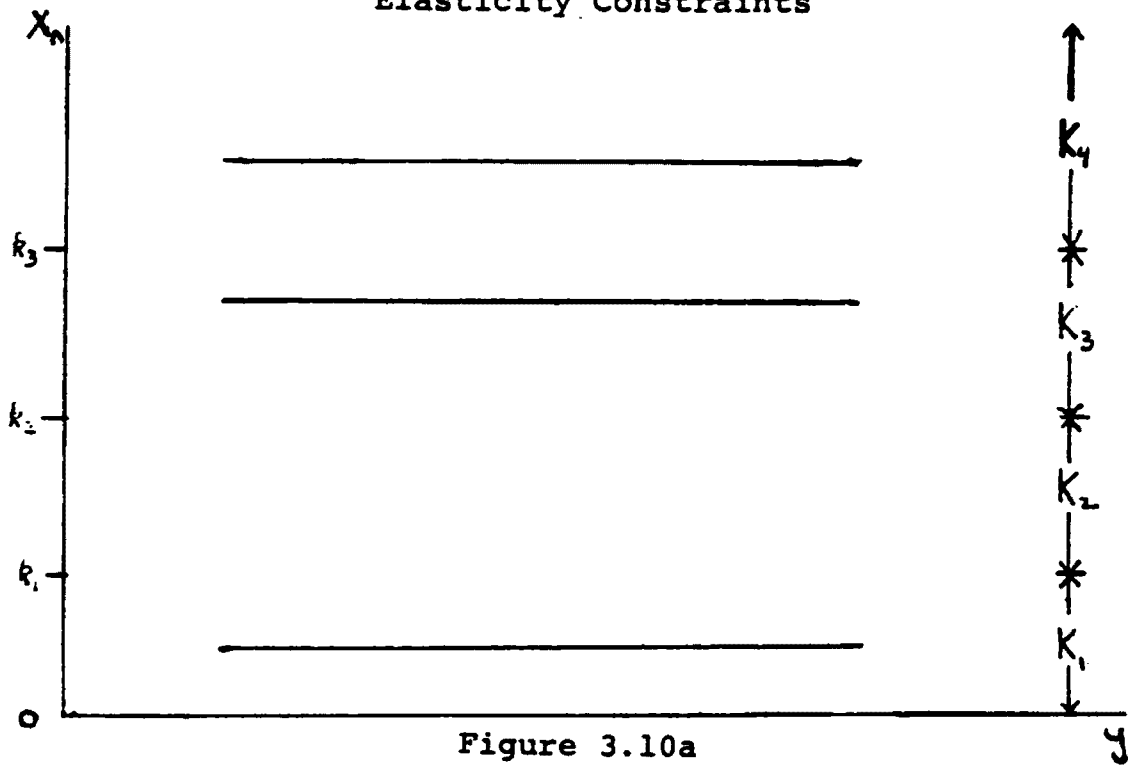


Figure 3.10a
An Illustration of E1

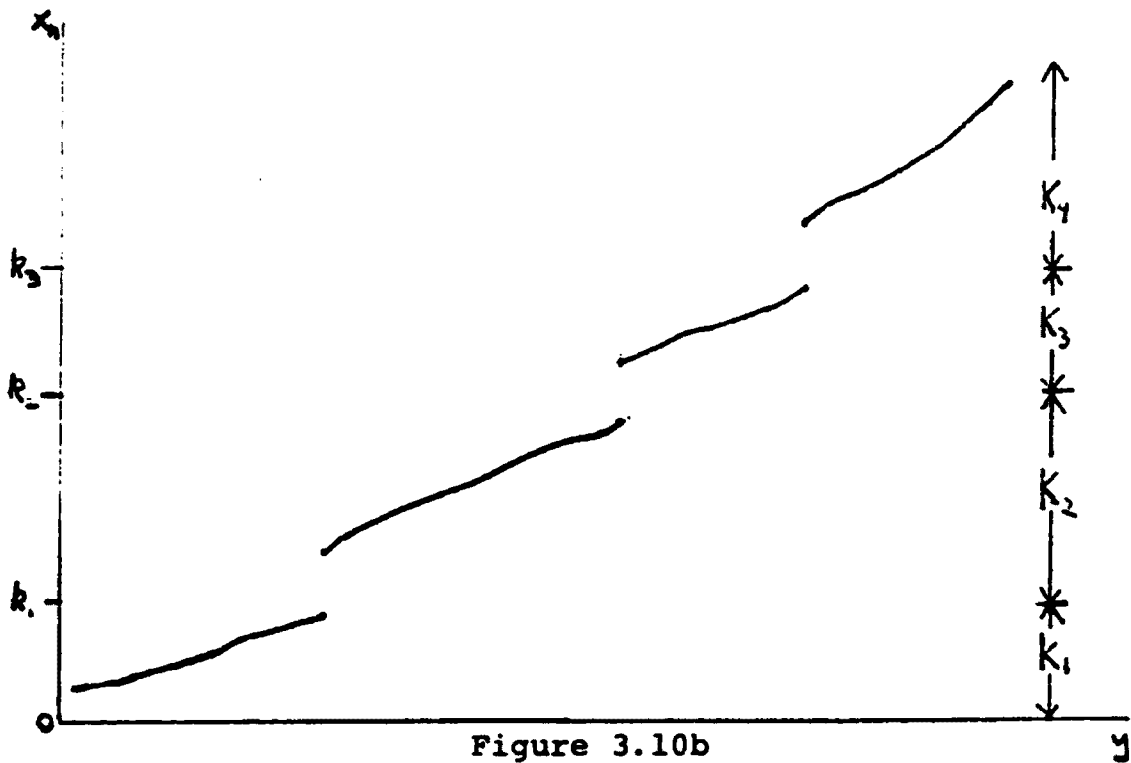


Figure 3.10b
An Illustration of E2

three non-empty subsets so that $D(r_j) \subset K_j^+$ for all $r_j < r_j^0$, $D(r_j) \subset K_j$ for all $r_j^0 < r_j < r_j^*$ and $D(r_j) \subset K_j^-$ for all $r_j^* < r_j$.

Given this partition, the continuity of $d(r_j)$ can be examined on the three regions $r_j > r_j^*$, $r_j > r_j^0$, and $r_j^0 > r_j > r_j^*$. For $r_j > r_j^*$, $D(r_j) \subset K_j^-$ and $D(r_j)$ is a fixed set. Therefore, $d(r_j)$ is continuous by Lemma 1. If $r_j^* > r_j > r_j^0$, then $D(r_j) \subset K_j$ and $D(r_j)$ is single-valued. Thus $d(r_j)$ is continuous by Corollary 2. If $r_j^0 > r_j$, then $D(r_j) \subset K_j^+$ and the continuity of $d(r_j)$ depends on assumptions E1 and E2.

If E1 holds it can be shown that $d(r_j)$ is continuous for $r_j^0 < r_j$. Since $r_j^0 < r_j$, a change in r_j is equivalent to an income change. By E1 $D_n(r_j)$ is fixed, and is constrained to contain elements in each of a fixed set of blocks. Therefore, $d(r_j)$ is continuous by Corollary 3 for $r_j < r_j^0$. This establishes Lemma 6 because $d(r_j)$ can be discontinuous at most at the points r_j^* and r_j^0 .

If E2 holds it can be shown that $d(r_j)$ is discontinuous at most at $m - j - 1$ points for $r_j < r_j^0$. According to E2, since a change in r_j is equivalent to an income change, $\max\{D_n(r_j')\} < \min\{D(r_j'')\}$ for $r_j', r_j'' < r_j^0$ and $r_j'' < r_j'$. Therefore, $D(r_j)$ is only multivalued at the points where demand shifts from one block to another and as price falls these shifts always occur from a lower to a higher block. Since there are only $m - j$ blocks in K_j^+ , there can be at most $m - j - 1$ points. With the additional discontinuity

points r_j^* and r_j^0 , there are most $m - j + 1$ points of discontinuity.

Figures 3.11a and 3.11b, the case for $d(r_m)$ is illustrated. The point r_m^0 is set equal to zero since $K_m^+ = \emptyset$. Zero is not in the set Π_{r_m} and there is at most one point of discontinuity of $d(r_m)$ at r_m^* . Since the set K_m^+ does not exist, assumptions E1 and E2 and budget characteristic B6 are irrelevant in this case.

In Figures 3.12 and 3.13, the case for $d(r_1)$ is illustrated under the assumptions E1 and E2 respectively. Since $K_1^- = \emptyset$, $r_1^* = \infty$. In Figure 3.12, $d(r_1)$ has at most one point of discontinuity even though as illustrated, $D(r_j)$ is multivalued for all $r_1 < r_1^0$. In Figure 3.13, $d(r_1)$ has at most $m - 1$ points of discontinuity as demand shifts from a lower to a higher block when prices fall. Note that $\max\{D(r_1')\} < \min\{D(r_1'')\}$ for $r_1' < r_1''$ and $r_1'' < r_1^0$.

Lemmas 6 and 7 are employed in showing that the sets H_i^0 are arcwise connected. This is stated in the following lemma:

Lemma 8. Each of the sets H_i^0 is arcwise connected, given either E1 or E2.

A set is called arcwise connected if for any two points in the set there exists a continuous mapping $f:[0,1]$ into the set such that $f(0)$ is the first point and $f(1)$ is the second. The proof of this lemma consists of constructing

Figure 3.12
An Illustration of Lemma 6 for
Variation in r_1

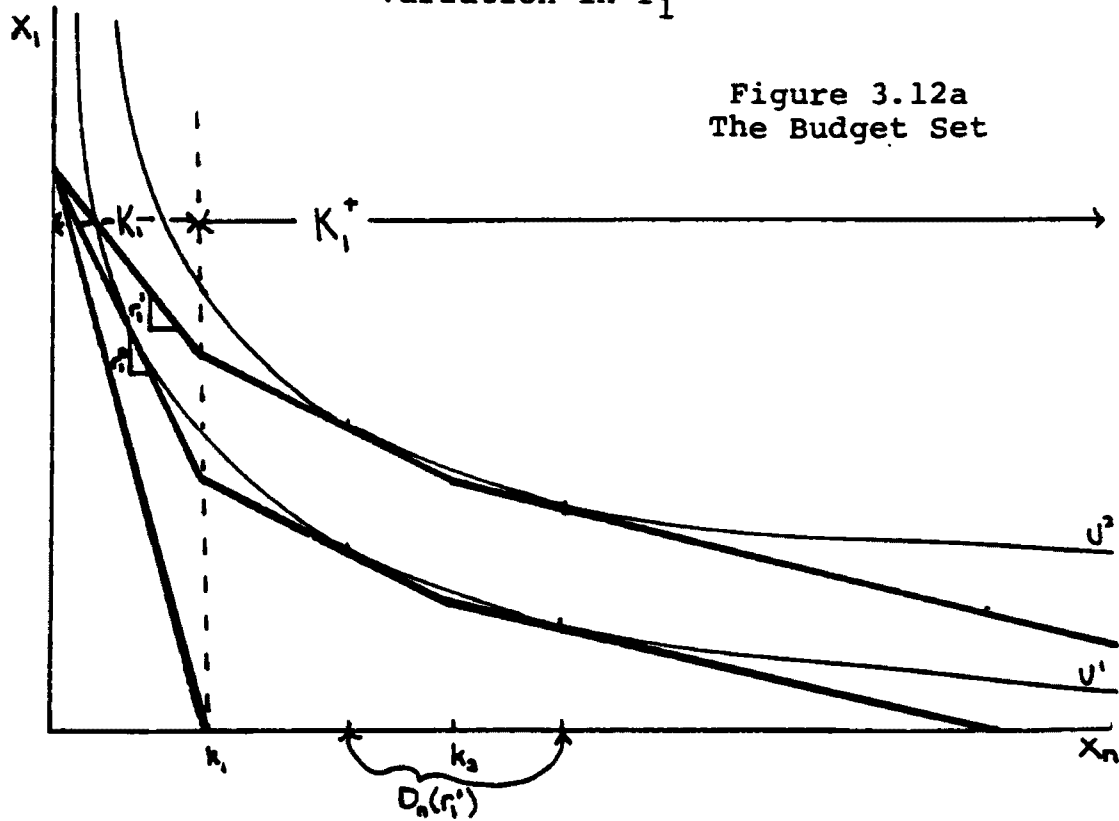
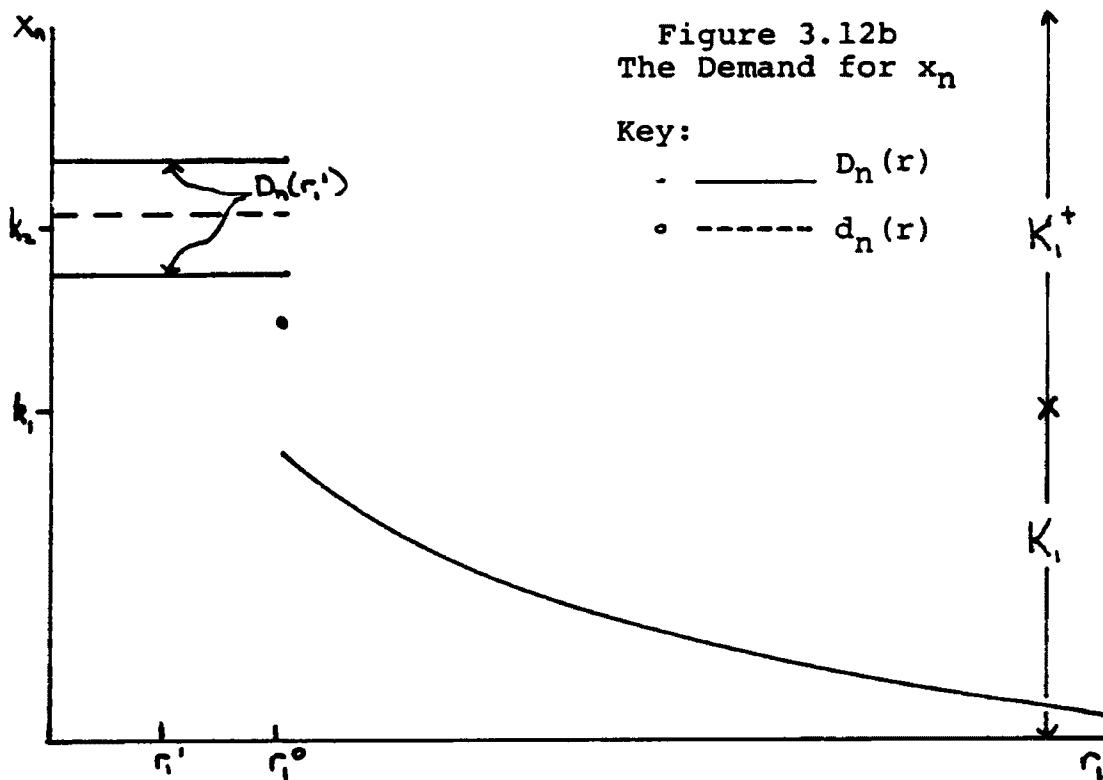
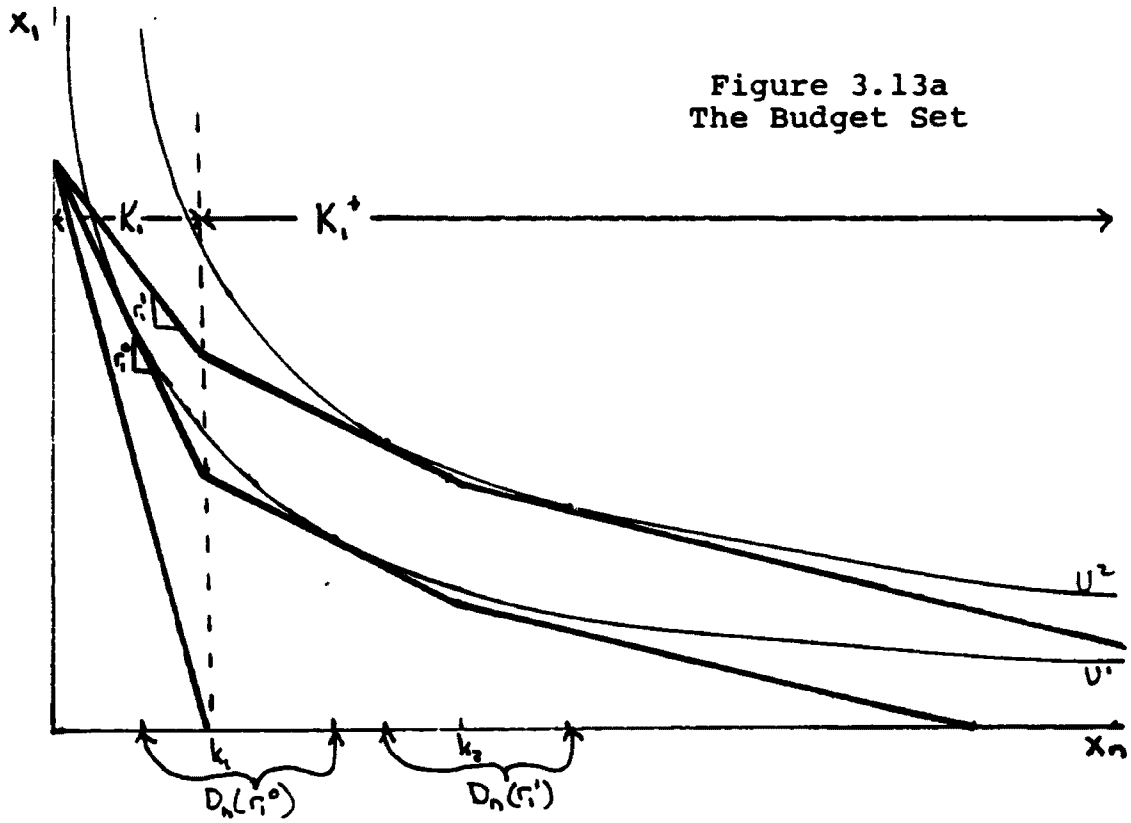
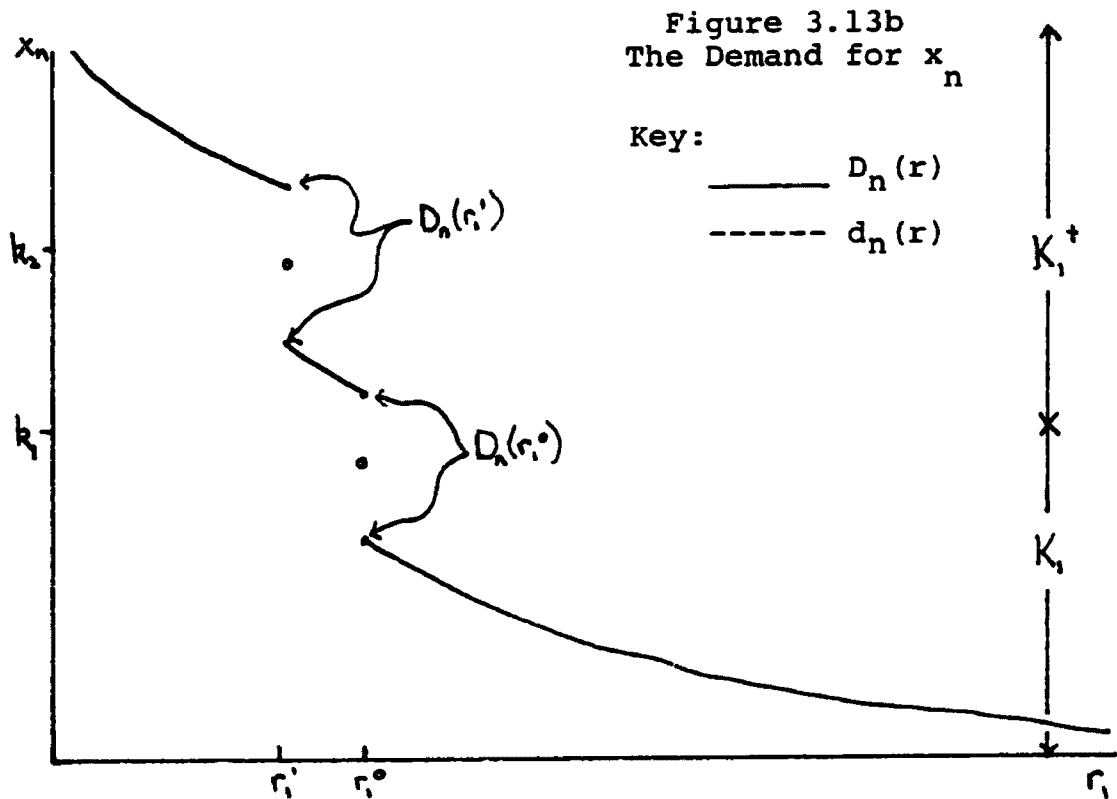


Figure 3.13
An Illustration of Lemma 7 for
Variation in r_1



such a function. This lemma will be used to justify certain assumptions which will be made in the second part of this section. These assumptions form the basis for several lemmas relating to theoretical plausibility of demand systems.

3.4B Theoretical Plausibility of the Demand Function of an Individual

Demand theory puts a number of constraints on the demand functions of preference maximizing consumers. Important among these constraints is the fact that the Slutsky substitution matrix must be symmetric and negative-semidefinite if the demand function is derived from a preference relation obeying specified axioms. Under certain conditions the symmetry and negative semi-definiteness of the Slutsky matrix are also sufficient for the existence of a utility function which is uniquely maximized on the budget set by the commodity bundle selected by the demand function. These conditions are called integrability conditions since the derivation of the resultant utility function involves the integration of a set of demand functions. If the Slutsky substitution matrix for a complete set of estimated demand equations is symmetric and negative semi-definite, the system is said to be theoretically plausible. The system is, then, consistent with the results of demand theory.

The empirical work done in this study does not include the estimation of theoretically plausible demand functions. This is because, first of all, a complete set of demand functions is not estimated. Second, since average demand functions are estimated and not individual demand, the applicability of the Slutsky conditions is no longer valid. This second point is examined and validated in the next section concerning average demand functions. In spite of the fact that theoretically plausible demand functions are estimated, it is still of theoretical interest to examine whether the Slutsky conditions are retained when a block rate schedule is applied to one good. In addition, these theoretical investigations do have import for some econometric problems including some that are encountered in this study.

In this section, I examine the extension of the results of demand theory concerning the Slutsky substitution matrix to demand systems in which a block rate schedule is applied to one good. I also discuss the empirical applicability of the results. The lemmas discussed in this section are based almost wholly on the work of Hurwicz and Uzawa.¹⁸ The adaptations of their proofs to this investigation are given in Appendix 3.

Some further assumptions and definitions are needed to facilitate this discussion. The Slutsky substitution matrix must be defined and the integrability conditions mentioned above must be specified. Among these conditions

are some restrictions on the demand function which apply to subsets of its domain. The domain of applicability of these restrictions is also defined.

The demand function in this part of Section 3.4 is not assumed to be rationalized by preference axioms unless so stated. This is because an explicit objective of one of the lemmas in this section is to prove the existence of a utility function which rationalizes a generally specified demand function. Nevertheless, the demand function is required to possess characteristics which are consistent with a demand function rationalized by preference axioms P1 - P6. These characteristics are among those established in Section 3.4A.

The demand function, $D(p, y)$, employed in this section is a set-valued mapping of a price vector, p , and income, y , into the set Ω . The restrictions on p and y are as specified earlier; $p \in \Pi$ and $y \in Y$. Therefore, D satisfies D1. D2, D3, and D4 are also satisfied. D can only contain one element in each block range of the rate schedule and the budget equation, $p \cdot g(D(p, y)) = y$, holds. In addition, for any $x \in \Omega$ there is some price income combination such that $x = D(p, y)$. Additional conditions imposed on the demand function apply to subsets of the domain of $D(p, y)$.

The subset χ_ℓ , of the domain of D is defined,

$$\chi_\ell = \{(p, y) \mid p \in \Pi, y \in Y, \text{ and for some } x \in D(p, y), \\ x \in K_\ell\}.$$

The set χ_ℓ is similar to the set H_ℓ defined in Section 3.4A. The set H_ℓ , however, is contained in Π_r and the only variation in demand determinants occurs in the rate schedule. Within χ_ℓ , variation occurs in the single price vector, s , and in income, y , as well as in the rate schedule. It can be proven that $D(p, y)$ is single-valued and continuous on the interior of χ_ℓ and that the set $(\Pi \times Y) - \bigcup_{\ell=1}^m \chi_\ell^O$ has Lebesgue measure zero if preference axioms P1 - P6 are assumed.¹⁹ These preference axioms are not assumed, but it will be required that $D(p, y)$ is single-valued and continuous on χ_ℓ^O for all ℓ and that $(\Pi \times Y) - \bigcup_{\ell=1}^m \chi_\ell^O$ has Lebesgue measure zero. In the second lemma in this section, it is also required, analogously to Lemma 8, that χ_ℓ^O is a connected set.²⁰

The restrictions discussed above and some additional ones are summarized in the following integrability conditions.²¹

- I1. $D(p, y)$ is single-valued and continuous on χ_ℓ^O for $\ell = 1, \dots, m$.
- I2. $\Pi \times Y - \bigcup_{\ell=1}^m \chi_\ell^O$ has Lebesgue measure zero.
- I3. The budget equation is satisfied, $p \cdot g(D(p, y)) = y$.
- I4. $D_j(p, y)$ has a differential $D_j^i(p, y)$ on χ_ℓ^O for all $i = 1, \dots, n+m-1$.
- I5. D_j^Y is bounded where D_j^Y is the partial of D_j with respect to y in the sense that, for any a', a'' with $0 < a' < a''$, there exists a positive $M_{a', a''}$, such

that $|D_j^Y(p, y)| \leq M_{a', a''}$ for all $(p, y) \in \chi_\ell$ for which $a' < p < a''$.

I6. χ_ℓ^0 is a connected set for $\ell = 1, \dots, m$.

The presence of block rates presents problems with respect to the dimensionality of the Slutsky substitution matrix. There are $n + m - 1$ prices and n commodities. Again, use is made of the function g (defined in 3.1). The terms of the Slutsky substitution matrix are redefined employing g .

$$(3.4) \quad S_{ij} = \frac{\partial g_i(x)}{\partial p_j} + g_j(x) \frac{\partial g_i(x)}{\partial y}$$

where $x = D(p, y)$. This reduces to the normal Slutsky matrix for single prices since $g(x) = x$ if all commodities have single prices.

The conditions required for a preference maximizing demand function to imply that the Slutsky substitution matrix is symmetric and negative semi-definite are that the demand functions be single-valued and differentiable, and that the budget condition is met. If these requirements are met almost everywhere then the Slutsky matrix is symmetric and negative semi-definite almost everywhere. This is stated explicitly:

Lemma 9. Let P be an irreflexive (P1), transitive (P2) relation defined on Ω such that the function $\bar{P}$ is a connected relation on Ω .

Let the function $D(p, y)$ mapping $\Pi \times Y$ into Ω maximize "satisfaction" represented by P subject to $p \cdot g(x) = y$. If $D(p, x)$ satisfies I1 - I4, then the Slutsky substitution matrix is symmetric and negative-semidefinite almost everywhere.²²

The existence of the Slutsky matrix and its symmetry and negative-semidefiniteness are local conditions. It suffices to show that these conditions persist at each point $(p^0, y^0) \in \chi_\ell^0$ for any $\ell = 1, \dots, m$. In proving this lemma, first an income compensation function $\mu(p)$ is defined which is concave and continuous on a subset of Π containing p^0 , and it is shown that $\mu(p^0) = y^0$. This function determines the income level necessary to maintain a given "satisfaction" level if prices are set at p . It is then shown that the derivative $\mu^i(p)$ exists,

$$\mu^i(p) = g_i[D(p, \mu(p))]$$

and that

$$\mu^{ij}(p^0) = S_{ij}(p^0, y^0) \quad .$$

The symmetry of $[S_{ij}]$ follows from the fact that $\mu^{ji} = \mu^{ij}$. The negative semi-definiteness of $[S_{ij}]$ from the fact that μ is concave. This lemma is proved in Appendix 3.

The Slutsky matrix in the case where single prices apply for all goods is a familiar entity. Therefore, the

results of the application of the above lemma to the sub-matrix consisting of the first $n - 1$ rows and columns of $[S_{ij}]$ do not come as a surprise. It is of some interest to us to look at the last m rows and columns of $[S_{ij}]$, and inquire if the above lemma has any particular implications for block rate schedules.

It is an explicit result of the symmetry of the Slutsky matrix that an intramarginal rate change is equivalent to an income change and that an extramarginal rate change has no effect on demand at all. This is consistent with the implications of B5 and B6 discussed earlier.

Consider the Slutsky substitution matrix, $[S_{ij}]$, for $D(p, y): \chi_\ell^0 \rightarrow K$. Within the domain χ_ℓ^0 , p_ℓ is the marginal block price for x_n . Note that p_j , $j=n, \dots, \ell-1$ are intramarginal block prices and p_j , $j=\ell+1, \dots, n+m-1$ are extramarginal block prices.

$$g_j(x) = k_j - k_{j-1} \text{ for } j=n, \dots, \ell-1$$

and

$$g_j(x) = 0 \text{ for } j=\ell+1, \dots, n+m-1.$$

Since g_j is constant for these values of j , $\frac{\partial g_j(x)}{\partial p_i} = 0$, and $\frac{\partial g_j(x)}{\partial y} = 0$. Therefore,

$$(3.5) \quad S_{ji} = 0, \text{ for } \forall i \text{ and for } j=n, \dots, \ell-1 \text{ or } j=\ell+1, \dots, n+m-1.$$

By the symmetry of S , the same is true for S_{ij} .

Suppose p_j is an intramarginal price. If $1 \leq i \leq n-1$, then

$$(3.6) \quad S_{ij} = \frac{\partial x_i}{\partial p_j} + (k_j - k_{j-1}) \frac{\partial x_i}{\partial y} = 0$$

since $g_i(x) = x_i$ for $1 \leq i \leq n-1$. On the other hand, if $i = n$, $g_i(x) = x_n - k_\ell$ and $\frac{\partial g_i(x)}{\partial p_i} = \partial x_n / \partial p_j$. Thus,

$$(3.7) \quad S_{\ell j} = \frac{\partial x_n}{\partial p_j} + (k_j - k_{j-1}) \frac{\partial x_n}{\partial y} = 0.$$

Equations (3.6) and (3.7) can be rewritten

$$(3.8) \quad \frac{\partial x_i}{\partial p_j} = -(k_j - k_{j-1}) \frac{\partial x_i}{\partial y} \text{ for } \forall i \text{ and } j=n, \dots, \ell-1.$$

Therefore, an intramarginal price change is equated to an income change of a specified size as implied by B6.

If p_j is an extramarginal price, (3.5) implies

$$S_{ij} = \frac{\partial g_i(x)}{\partial p_j} + 0 \cdot \frac{\partial g_i(x)}{\partial y} = 0, \text{ for } j=\ell+1, \dots, n+m-1$$

since $g_j(x) = 0$. Therefore

$$(3.9) \quad \frac{\partial x_i}{\partial p_j} = 0, \text{ for } \forall i \text{ and for } j=\ell+1, \dots, n+m-1.$$

This is the implication attributed to B5.

Equations (3.8) and (3.9) can be employed in defining a simplification of the function $D(p, y)$ on the domain χ_ℓ^0 . This simplification will be employed later in the specification of rate schedule variables for econometric estimation. Within the domain χ_ℓ^0 , the function $D(p, y)$ can be rewritten $D(s, r_j, y-t)$ where s is the vector of

single prices, $(p_1, \dots, p_{n_{J-1}})$, r_J is the marginal block rate, $J = l-n-1$, and $t = \sum_{j=1}^J r_j (k_j - k_{j-1})$. This incorporates the fact that rate changes for r_j are ineffective for $j = J + 1, \dots, m$ and equivalent to income effects for $j = 1, \dots, J-1$. Equations (3.8) and (3.9) imply

$$(3.10) \quad D_i^t = -D_i^y \quad \text{or} \quad \frac{\partial x_i}{\partial t} = - \frac{\partial x_i}{\partial y}$$

where D_i^t is the partial derivative of D_i with respect to t .

The symmetry and negative semi-definiteness of the Slutsky substitution matrix are necessary conditions for a single-valued, differentiable demand function to maximize preferences subject to a budget constraint. This has been established. The question, thus, arises if circumstances exist when these conditions on the Slutsky matrix are also sufficient for the existence of a utility function maximized on the budget set by the demand function.

The determination of sufficient conditions for utility maximization is more difficult than the determination of necessary conditions. In this investigation the requirements for the derivation of a utility function from a demand function are examined. In a very particular case, requirements are established such that a utility function can be derived on a subset of the commodity space, K_ℓ . Generally applicable conditions such as those established in Lemma 9 are not obtained. However, the

possibility of more generally applicable conditions existing is discussed.

The investigation of sufficient conditions mirrors that of Hurwicz and Uzawa. For reference and comparison, a brief outline of their proof follows below. The problems in adapting their proof to the demand function are then discussed.

First, Hurwicz and Uzawa prove the mathematical integrability of the system of differential equations $\partial \mu / \partial p_i = D(p, y)$. (Single prices apply to all goods and demand is single-valued.) An existence theorem establishes the necessary condition for the mathematical integrability of this system. These conditions include the symmetry of $[S_{ij}]$. Thus, there is a unique function $\mu(p; p^0, y^0)$ such that

$$\mu(p; p^0, y^0) = y^0$$

and

$$\mu^i(p; p^0, y^0) = D(p, \mu(p; p^0, y^0))$$

This function is the income compensation function employed earlier.

It is then shown that the ordering implied by this function is independent of (p^0, y^0) and that $\mu(p; p', y')$ is invariant for any (p', y') such that $D(p', y') = x'$. The utility function is then defined

$$U(x) = \mu(p^*; p, y)$$

where $x = D(p, y)$ and p^* is constant. The maximization of U over the budget set by the demand function results from the negative semi-definiteness of S .

Some difficulties exist in extending this proof to demand functions with multiple block pricing, when the income elasticity of D_n is positive. When the income elasticity is zero

$$(3.11) \quad \chi_\ell = A \times Y \quad ,$$

where A is a set of prices, $A \subset \Pi$, and Y is the set of non-negative incomes. When income elasticity is positive the prices vectors contained in χ_ℓ are not independent of what the income level is. The statement of the existence theorem employed in this proof requires a domain for D of the form (3.11). In addition, the definition of the utility function requires the existence of a p^* such that $(p^*, y) \in \chi^\ell$ for all y . These difficulties do not exist when the income elasticity of D_n is zero. It is probably true that these issues can be circumvented, but this has not been investigated further.

If demand exhibits zero income elasticity, then under block rate pricing $\chi^\ell = A \times Y$. An existence theorem can be established to show that the system of differential equations

$$\partial \mu / \partial p_i = g_i(D(p, \mu(p; p^0, y^0)))$$

$$\mu(p^0; p^0, y^0) = y^0$$

is uniquely integrable on the set $A \times Y$ under conditions I1, I3, I4, I5, I6 and the symmetry of $[S_{ij}]$. The ordering implied by the function μ is independent of (p^0, y^0) on $A \times Y$. However, it cannot be shown that $\mu(p; p', y')$ is invariant for any (p', y') such that $D(p', y') = x'$. This is only true on a convex subset of $A \times Y$.

This discussion indicates the feasibility of deriving a utility function from a demand function on a subset of K^ℓ . Required conditions for the derivation of a utility function are stated in the lemma below. This lemma applies to a demand function, $D(p, y): A \times Y \rightarrow K_\ell$, exhibiting zero income elasticity with respect to x_n .

Lemma 10. Let A_1 be a convex subset of A and $B \subset K_\ell$ be the image set of $A_1 \times Y$ under D . If conditions I1 and I3-I6 hold and the Slutsky substitution matrix is symmetric and negative semi-definite, then there exists a utility function U such that $D(p, y)$ uniquely maximizes U on B subject to $g(x) \cdot p \leq y$.²³

It is possible that a more general treatment of this subject exists which might be directly applicable in this case. In a discussion of some mathematical work of Hartman, Hurwicz discusses a more generalized form of integrability in which the symmetry and negative semi-definiteness of the Slutsky matrix are only required to

hold almost everywhere.²⁴ Here the demand functions must be Lipschitzian in all their arguments. He states

...much rougher situations are also covered. In particular, even discontinuity phenomena may be present. This is important because it may open a way for handling situations in which demand relations are not single-valued.²⁵

In light of Lemmas 4 and 5, this approach might be applicable for the function D over the entire set of prices and incomes.

A considerable amount has been learned about the demand functions of preference maximizing individuals in Sections 3.4A and 3.4B. It has been demonstrated that demand functions derived from a standard set of preference axioms are single-valued and continuous almost everywhere. If demand is also continuously differentiable on the set of points for which it is single-valued and continuous, then the Slutsky conditions hold almost everywhere. The requirement that Slutsky conditions hold almost everywhere is also sufficient for utility maximization on a subset of the budget set under additional conditions. This section establishes the extent to which the properties of individual's demand functions are retained when consumers face block rate schedules.

Some of the results of this section are relevant to the demand functions which are estimated in this study, even though the functions estimated are average demand functions. The properties of average demand functions are examined in Section 3.5. In examining aggregate behavior

it is relevant to understand individual behavior. The continuity of average demand curves to be proven in the next section will be examined in light of the results of this section. In addition, the intramarginal and extra-marginal price effects defined in this section (equations 3.8, 3.9, and 3.10) will be employed in assertions about price effects for average demand functions.

An additional potential application of the results of this section should be mentioned. There are a number of utilities which are now engaged in peak load pricing experiments, and which are gathering data on individual energy consumption patterns under different experimental rates. It might, thus, be possible to gather data on a group of individuals, all contained in the set χ_ℓ and to estimate the demand functions for these individuals. In this case, the results of this section concerning both continuity and the Slutsky conditions would be very relevant.

3.5 Average and Aggregate Demand Functions

3.5A The Continuity of Average Demand Functions

The discussion up to now has been restricted to the individual consumer choice problem. We will be estimating some sort of average or aggregate demand functions, not individual relationships. Therefore, it is important to determine if the properties discussed so far apply to average demand. Individual's demand functions have been

shown to be multi-valued and have discontinuities. There is no way to estimate a priori where such discontinuities exist since demand is de facto single-valued. The empirical techniques available are restricted to single-valued, continuous functions. Therefore, the demand function we estimate will be single-valued and continuous. It is, then, important to determine the significance of this restriction on the average demand curve.

A standard assumption made in aggregating individual demand functions is that the preferences and incomes of the consumers are identical. Thus, one representative consumer is adequate to determine the properties of average demand. This assumption is not meant to be realistic but it simplifies the problem. Since demand is summed up across consumers and relative prices remain the same across consumers in the standard consumer choice problem, the continuity of demand is not affected by this assumption even though other characteristics of demand are. However, given block pricing, the distribution of preferences and income becomes more significant. If income and preference relations are identical over all individuals, the discontinuities in demand always occur at the same price configurations. Then, the average demand curve is also discontinuous at these points. It is more realistic, however, to assume that preferences and incomes vary across individuals. In this section, the conditions under which the average demand function approaches continuity as the number

of customers gets large will be examined.

For this discussion the average demand function will be defined

$$(3.12) \quad \bar{d}_N(p) = \frac{\sum_{j=1}^N d^j(p, \alpha^j, y^j)}{N}$$

where d^j is the average demand function of individual j , y^j specifies the individual's income, and α^j the individual's preference relation P_{α^j} . The problem of the single-valuedness of demand is circumvented by the use of the demand function, d . To insure the boundedness of d , income is constrained to be the set Y' ,

$$Y' = \{y \in Y \mid 0 \leq y < W\}$$

where W is some very large number. N specifies the number of customers.

In investigating $\bar{d}_N$ it will be necessary to consider a distribution of preferences and incomes across customers. For this purpose, we define the probability space for α and y to be $(L, B, \mu \times \nu)$. L is the set of values in R^2 over which α and y may range, μ and ν are probability measures on α and y respectively, and B is the set of all $\mu \times \nu$ measurable sets in L .²⁶

A sufficient condition for $\bar{d}_N$ to approach continuity in the probability limit at p^* with respect to a given component of the price vector, p_i^* , is that the probability that $d(p_i, \alpha, y)$ is discontinuous at p_i^* for $(\alpha, y) \in L$ is zero.

This condition is also necessary if $d(p_i, \alpha, y)$ is monotonic in p_i . The function $\text{plim } \bar{d}_N$ is everywhere continuous with respect to p_i if it is continuous at each $p_i \in \Pi_i$. The set of (α, y) for which $d(p_i, \alpha, y)$ is discontinuous at p_i^* is precisely written,

$$(3.13) \quad A(p_i^*) = \{(\alpha, y) \in L : \exists \xi(\alpha, y) > 0 \text{ such that}$$

$$\forall \delta > 0, \exists p_i \in \Pi_i, \text{ where } |p_i^* - p_i| < \delta, \\ \text{and } ||d(p_i^*, \alpha, y) - d(p_i, \alpha, y)|| > \xi(\alpha, y) \}.$$

Given this definition, the point continuity condition is established in the following theorem.

Theorem 1: Given $p_i^* \in \Pi_i$. If $\mu \times \nu(A(p_i^*)) = 0$, then $\forall \varepsilon > 0, \exists \delta > 0$ such that for $p_i \in \Pi_i$ where $|p_i^* - p_i| < \delta$,

$$(3.14) \quad ||\text{Plim}_{N \rightarrow \infty} \bar{d}_N(p_i^*) - \text{Plim}_{N \rightarrow \infty} \bar{d}_N(p_i)|| < \varepsilon.$$

If $d(p_i', \alpha, y) \geq d(p_i'', \alpha, y)$ for all $p_i' \leq p_i''$ or for all $p_i' \geq p_i''$, then $\mu \times \nu(A(p_i^*)) = 0$ is a necessary condition for (3.14) as well as a sufficient condition.

This theorem states that the function $\text{Plim } \bar{d}_N(p_i)$ is continuous at p_i if $\mu \times \nu(A(p_i)) = 0$. $\mu \times \nu(A(p_i)) = 0$ means that the probability is zero that $d(p_i)$ is discontinuous at i for any randomly selected individual. If

demand is monotonic in p_i , then the continuity of the function $\text{plim } \bar{d}_N(p_i)$ implies $\mu \times v(A(p_i)) = 0$.

The proof of this theorem is given in Appendix 4. I will discuss the applicability of the theorem, and give an illustrative example in the rest of this section.

The theorem as stated deals with the continuity of $\text{plim } \bar{d}_N$ as a function of one price p_i . If the condition $\mu \times v(A(p_i)) = 0$ is fulfilled for all i at all $p \in \Pi$, then the theorem holds for all prices and $\text{plim } \bar{d}_N$ is continuous at every point in its domain in directions parallel to the coordinate axes. While it is possible to construct a function that is continuous at every point in these directions, but which nevertheless has a point of discontinuity, such counter examples are unusual. If, in addition, it is known that for any $p^* \in \Pi$ there is a neighborhood of p^* for which $\text{plim } \bar{d}_N$ is Lipschitz continuous in the axial directions, then $\text{plim } \bar{d}_N(p)$ is everywhere continuous in this neighborhood with respect to p .

Although this Lipschitz condition is difficult to visualize, it can be used to establish a more intuitive criterion. The Lipschitz condition is met at any positive price p^* , if the price elasticities of $\text{plim } \bar{d}_N$ with respect to p_i are bounded for all i on some neighborhood of p^* . Although there is some contention over whether or not price elasticities are greater or less than one, there is no contention over whether they are finite. The assumption

of bounded price elasticities, sufficient for the continuity of $\text{plim } \bar{d}_N$, is therefore justified.

The above assertions are stated explicitly in the following lemma and its corollary:

Lemma 11. If $\text{plim } \bar{d}_N(p_i)$ is Lipschitz continuous with respect to p_i for all i on a neighborhood of p^* then $\text{plim } d_N(p)$ is continuous with respect to p at p^* .

Corollary 5. If $p^* > 0$ and the elasticity of $\text{plim } d_N(p)$ with respect to p_i is bounded on a neighborhood of p^* for all i , then $\text{plim } \bar{d}_N(p)$ is continuous at p^* with respect to p .

There remains the question of the generality of the condition $\mu \times v(A) = 0$. As of yet a complete set of conditions that define the distribution of preference relations and incomes which qualify has not been established. It is clear that $\mu \times v(\alpha, y)$ must equal zero for all $(\alpha, y) \in L$ but this condition is generally not sufficient. However, some observations can be made which are indicative of the generality of the condition; in addition, an example of a commonly employed class of utility functions that satisfies the condition under certain circumstances will be given.

In Section 3.4, it was established that the set of prices at which an individual's expected demand function is discontinuous is a set of measure zero. This is true at any finite income, for any preference relation satisfying P1 - P6. This does not insure that $\mu \times \nu(A) = 0$. In fact, a counter-example has already been given in the case where all individuals have identical incomes and preferences. However, if it was generally true that individual's demand functions contradicted Lemmas 4 and 5, and $m(\Pi - \bigcup_{i=1}^m H_i^0) \neq 0$, then it would also be likely that $\mu \times \nu(A) \neq 0$.

An example of a suitable class of preference relations for two commodities is the class of linearly homogeneous utility functions, $U = x_1^\alpha x_2^{1-\alpha}$ where $0 < \alpha < 1$. In this case the condition $\mu \times \nu(A) = 0$ reduces to a simple restriction on the distributions of α and y . The condition

$$\mu(\alpha|y) = 0 \text{ or } \nu(y|\alpha) = 0 \text{ for all } (\alpha, y) \in L,$$

is sufficient for $\mu \times \nu(A) = 0$ in this case. Any continuous distributions for $\mu(\alpha|y)$ and $\nu(y|\alpha)$ are sufficient for this purpose.

This class of utility functions exhibits positive income elasticity in both goods. Therefore, $d(p_i, \alpha, y)$ is monotonic in p_i for all i . The above theorem states that $\mu \times \nu(A) = 0$ is necessary as well as sufficient for the continuity of $\text{plim}_{N \rightarrow \infty} \bar{d}_N$ in these circumstances.

The comments made above about this example are established through the application of the following lemma:

Lemma 12. Let $D(p, \alpha, y)$ maximize the utility function, $U = x_1^\alpha x_2^{1-\alpha}$, subject to the budget constraint $p \cdot g(x) \leq y$. If $D(p, \alpha, y)$ is multivalued at p such that $\bar{x} \in K_j$ and $\tilde{x} \in K_i$ for $\bar{x}, \tilde{x} \in D(p, y, \alpha)$, then

$$\alpha = 1 - \ln\left(\frac{y-a_i}{y-a_j}\right) / \ln(r_i/r_j)$$

where

$$p = (s_1, r_1, \dots, r_m)$$

and

$$a_i = \sum_{t=1}^{i-1} (k_t - k_{t-1}) r_t \quad .$$

This lemma is proven in Appendix 4. Through this lemma it can be shown that $\mu \times \nu(A) = 0$ if $\mu(\alpha|y) = 0$ or $\nu(y|\alpha) = 0$ for all $(\alpha, y) \in L$.

To establish that $\mu \times \nu(A) = 0$, consider the set A . If $(\alpha, y) \in A$, then $d(p, \alpha, y)$ is discontinuous at p , and $D(p, \alpha, y)$ is multivalued at p . The question can then be asked at what values of (α, y) is D multivalued. This set of points contains the set A .

If D is multivalued at p , then the utility function must be tangent to at least two facets of the budget set. Given any two facets, Lemma 12 defines the set of (α, y) such that D contains elements in these two blocks. For the

i^{th} and j^{th} blocks this set is defined

$$T(i,j) = \{(\alpha, y) \mid (\alpha, y) \in L \text{ and } \alpha =$$

$$1 - \ln\left(\frac{y-a_i}{y-a_j}\right) / \ln(r_i/r_j)\}$$

Since $A = \bigcup_{i \neq j} T(i,j)$, the measure of A is constrained by the measure of $T(i,j)$, $0 \leq \mu \times \nu(A) \leq \sum_{i \neq j} \mu \times \nu(T(i,j))$. Therefore, if $\mu \times \nu(T(i,j)) = 0$ for all $i \neq j$, then

$\mu \times \nu(A) = 0$. It can be shown that if $\mu(\alpha|y) = 0$, or $\nu(y|\alpha) = 0$ for all $(\alpha, y) \in L$, then $\mu \times \nu(T(i,j)) = 0$.

Define T_y and T_α to be the crosssections of $T(i,j)$ given a fixed y or α respectively.

$$\begin{aligned} \mu \times \nu(T(i,j)) &= \int_y \mu(T_y|y) d\nu \\ &= \int_\alpha \nu(T_\alpha|\alpha) d\mu \end{aligned}$$

By Lemma 12 the sets T_y and T_α each contain at most one point. Therefore,

$$\mu(T_y|y) = 0 \quad \text{if} \quad \mu(\alpha|y) = 0 \quad ,$$

$$\nu(T_\alpha|\alpha) = 0 \quad \text{if} \quad \nu(y|\alpha) = 0 \quad ,$$

and

$$\mu \times \nu(T(i,j)) = 0 \quad ,$$

if $\mu(\alpha|y) = 0$ or $\nu(y|\alpha) = 0$. In the special case of the class of utility functions, $U = x_1^\alpha x_2^{1-\alpha}$ the condition $\mu \times \nu(A) = 0$ is required by the restriction on the distributions of α and y , $\mu(\alpha|y) = 0$ or $\nu(y|\alpha) = 0$, $V(\alpha, y) \in L$.

In spite of the discontinuities and multi-valuedness of individual demand functions, it has been shown that these properties are not retained by an average demand function in the probability limit. Thus, the assumption of single-valuedness and continuity imposed on empirically estimated functions does not seem to be a severe constraint.

3.5B Theoretical Plausibility and Average Demand

It would be remiss for us, having discussed substitution effects with respect to individual demand, not to mention in our discussion of average demand, the recent work that has been done on the extension of these substitution properties to aggregate demand.²⁷ In this work it is shown that any arbitrary excess demand function which is continuous and which satisfies Walras' Law can be written as the aggregation of n individual demand functions where n is the number of commodities, each of these demand functions satisfying all the properties of preference maximization. Inversely, this means that in a market of n or more preference maximizing consumers, the aggregate demand function does not necessarily satisfy the conditions implied by preference maximization for individuals.²⁸

This has significant implications for empirical applications. In estimating demand functions where the unit of observation represents a large population, this implies that there is no reason to require the concavity properties

of the demand function to hold. Since single price demand functions are a special case of block price demand functions where all the block prices are identical, this result extends to the case of block rate schedules.

Some useful information is still available from the aggregation process. In Section 3.4 it was shown (3.8 and 3.9) that

$$\frac{\partial x_i}{\partial r_j} = -(k_j - k_{j-1}) \frac{\partial x_i}{\partial y}$$

for intramarginal block rates, r_j , and that

$$\frac{\partial x_i}{\partial r_j} = 0$$

for extramarginal block rates, r_j . In addition, for $t = \sum_{j=1}^{J-1} (k_j - k_{j-1})r_m$, and r_J the marginal block rate, it was shown that (3.10)

$$\frac{\partial x_i}{\partial t} = - \frac{\partial x_i}{\partial y} .$$

The results have considerable impact if they are still valid for the aggregate demand function $\bar{d}_N$.

Consider the case where the rate schedule r applies to the n^{th} commodity, and the average demand level is

$$\bar{x} = \sum_{\ell=1}^N 1/N x^{\ell} .$$

What can be said about the value of $\partial \bar{x}_i / \partial r_j$? Arrange the customers according to increasing levels of consumption

of x_n .

$$\begin{aligned} x_n^\ell &< k_{j-1} & \ell &\leq N_1 \\ k_{j-1} &< x_n^\ell &< k_j & N_1 < \ell \leq N_2 \\ k_j &< x_n^\ell & & N_2 < \ell \leq N \end{aligned}$$

From (3.8) and (3.9) it can be seen that

$$\begin{aligned} (3.15) \quad \frac{\partial x_i}{\partial r_j} &= \sum_{\ell=1}^N 1/N \frac{\partial x_i^\ell}{\partial r_j} \\ &= 1/N \left\{ 0 + \sum_{\ell=N_1+1}^{N_2} \frac{\partial x_i^\ell}{\partial r} - (k_j - k_{j-1}) \sum_{\ell=N_2+1}^N \frac{\partial x_i^\ell}{\partial y} \right\}. \end{aligned}$$

For the first N_1 consumers r_j is an extramarginal block price, for the next $N_2 - N_1$ consumers r_j is the marginal block price, and for the remaining consumers, r_j is an intramarginal block price.

It is clear from (3.15) that (3.8) and (3.9) do not extend to the aggregate demand function in general. Further insight into (3.15) can be gained by consideration of alternative assumptions on individual price and income effects and the distribution of consumers across rate blocks. In econometric estimation it is frequently assumed that price and income effects are constant across customers. A similar assumption will be made in this case.

$$\frac{\partial x_i^\ell}{\partial y^\ell} = \frac{\partial \bar{x}_i}{\partial \bar{y}} \text{ for all } \ell$$

and

$$\frac{\partial x_i^\ell}{\partial r_j} = \frac{\partial x_i}{\partial r_j} \text{ for all } N_1 < \ell \leq N_2 \quad .$$

With these assumptions incorporated, (3.15) becomes

$$(3.16) \quad \frac{\partial \bar{x}_i}{\partial r_j} = \frac{N_2 - N_1}{N} \frac{\partial x_i}{\partial r_j} - \frac{N - N_2}{N} (k_j - k_{j-1}) \frac{\partial \bar{x}_i}{\partial y} \quad .$$

In light of varying additional assumptions, (3.16) shall be examined.

If either customers are evenly distributed across blocks, with equal number of customers consuming within each block, or if

$$0 > \frac{\partial x_i}{\partial r_j} > -(k_j - k_{j-1}) \frac{\partial \bar{x}_i}{\partial y} \quad ,$$

then $\partial \bar{x}_i / \partial r_j$ increases [approaches zero from below] as $j \rightarrow m$. This is because as j goes to m there will be fewer customers from whom r_j is an intramarginal rate and more customers for whom r_j is an extramarginal rate. This is true even if the marginal price effect, $\partial x_i / \partial r_j$, is constant across blocks.

In the opposite situation where there exists a median block containing a disproportionately large number of customers, and

$$(3.17) \quad \frac{\partial x_i}{\partial r_j} < -(k_j - k_{j-1}) \frac{\partial \bar{x}_i}{\partial y} \quad ,$$

then it is possible that $\partial \bar{x} / \partial r_j$ decreases as j approaches the median block and increases (approaches zero from below)

thereafter. This is a feature of the aggregate demand function which can be tested empirically. It is undoubtedly true that customers are concentrated in a large medium block on the rate schedule. If there is a minimum value for $\partial \bar{x} / \partial r_j$ at some interior block, then it is likely that this is the median block and that (3.17) holds.

In special situations it is possible that (3.10) can hold in the aggregate. This is not always true. The relative sizes of $\partial \bar{x}_i / \partial t$ and $\partial \bar{x}_i / \partial \bar{y}$ are empirically measurable, and the size of $\partial \bar{x} / \partial r_j$ can have implications for this comparison. If for individuals $N_1 + 1$ to N_2 ,

$$\frac{\partial x_i}{\partial r_j} = - \frac{N_2}{N_2 - N_1} (k_j - k_{j-1}) \frac{\partial \bar{x}_i}{\partial \bar{y}} \text{ for all } j < J$$

then by (3.16)

$$\frac{\partial \bar{x}_i}{\partial r_j} = -(k_j - k_{j-1}) \frac{\partial \bar{x}_i}{\partial \bar{y}} \text{ for all } j < J$$

and

$$\frac{\partial \bar{x}_i}{\partial t} = - \frac{\partial \bar{x}_i}{\partial \bar{y}}$$

where

$$t = \sum_{j=1}^{J-1} (k_j - k_{j-1}) r_j \quad .$$

(3.8) and (3.10) hold for the aggregate demand function, $\bar{d}_N$, in this particular situation. On the other hand, by (3.16)

$$\frac{\partial \bar{x}_i}{\partial r_j} \leq -(k_j - k_{j-1}) \frac{\partial \bar{x}_i}{\partial y}$$

if, for individuals $N_1 + 1$ to N_2 .

$$\frac{\partial x_i}{\partial r_j} \leq -\frac{N_2}{N_2 - N_1} (k_j - k_{j-1}) \frac{\partial \bar{x}_i}{\partial y} \quad .$$

Furthermore, if

$$(3.18) \quad \frac{\partial \bar{x}_i}{\partial r_j} \leq -(k_j - k_{j-1}) \frac{\partial \bar{x}_i}{\partial y} \text{ for all } j < J, \text{ then}$$

$$\frac{\partial \bar{x}_i}{\partial t} \leq -\frac{\partial x_i}{\partial y}$$

where t is as defined above. Since $\partial \bar{x}/\partial r_j$, $\partial \bar{x}/\partial y$ and $(k_j - k_{j-1})$ are all empirically measurable, it is possible to determine if these values are consistent with one another.

The above discussion establishes a number of different configurations of price and income effects which are consistent with the results of Section 3.4B for individual consumers. It should be emphasized that all of these configurations are based on assumptions requiring constant income and marginal price effects for all consumers. If these assumptions are dropped, configurations contradicting those discussed above could also be consistent. Clearly, there is not much that can be firmly established with regard to the aggregate demand function. However, the relative sizes of the empirically measured price and income effects will be examined in light of the discussion

in this section as one plausible framework of examination.

The model that will be estimated is an average demand function. Therefore, an attempt will be made to specify the model in conformity with the results of this section. The continuity of the average demand function in the probability limit is achieved by allowing N , the size of the population, to increase. The necessity of the Slutsky conditions is removed for the same reason. The unit of observation is the state and demand is assumed to be continuous. The Slutsky conditions are not assumed to hold although, since a complete set of demand equations is not estimated in this study, this last conclusion is not relevant. It will be assumed that the Slutsky conditions do hold for individual demand in the manner discussed in Section 3.4B. In addition, it is assumed that the aggregate demand function is consistent with individual demand functions in the ways discussed in this section, but it is recognized that many configurations of price effects can be consistent.

3.6 Conclusion

The purpose of this chapter has been to investigate with some rigor the properties of demand functions of consumers facing block rate schedules. In the organization of this chapter, it seemed useful to separate the discussion of average demand and individual demand. This was

done because the properties of average demand are affected by aggregation, and, thus, differ from those of individual demand.

Individual demand functions are continuous and single-valued almost everywhere, but they are multivalued on a non-empty set. In addition, the Slutsky conditions are necessary for utility maximization everywhere the demand function is single-valued. In some circumstances the Slutsky conditions are also sufficient for utility maximization.

The same properties are not exhibited by average demand functions as are exhibited by individual demand functions. Average demand functions are everywhere continuous in the probability limit given certain assumptions concerning the distribution of preferences and incomes across consumers. However, the Slutsky conditions are no longer a requirement for utility maximization at any point on the demand function.

Footnotes

¹See the work of Sonnenschein, Chipman, Hurwicz, and Richter in Preferences, Utility and Demand, Chipman et al. (ed.) (New York: Harcourt Brace Jovanovich, Inc., 1971), Chapters 2, 3, 9, 10, 11.

²Lester D. Taylor, "The Demand for Electricity: A Summary," The Bell Journal of Economics (Spring 1975): 74. For examples, see Parks (1969), Houthakker and Taylor (1970), Philips (1972), Taylor and Wieserbs (1972), Brown and Heien (1972), and Christenson, Jorgensen and Lau (1973).

³Andre Gabor, "A Note on Block Tariffs," Review of Economic Studies (1955), 34.

⁴Taylor, "The Demand for Electricity," 74-110.

⁵James Buchanan, "The Theory of Monopolistic Quantity Discounts," The Review of Economic Studies 20 (1952-1953): 200.

⁶Ibid., p. 201.

⁷Andre Gabor, "A Note on Block Tariffs," p. 33.

⁸Leonid Hurwicz, and Hirofumi Uzawa, "On the Intregrability of Demand Functions," Preference Utility and Demand, Chipman et al. (eds.), p. 115.

⁹ $p'' > p'$, where p'' and p' are vectors, means $p''_i \geq p'_i \quad \forall i$ and $p''_j > p'_j$ for some j .

¹⁰Hirofumi Uzawa, "Preference and Rational Choice in the Theory of Consumption," Preference, Utility and Demand, p. 13.

¹¹Ibid., p. 23.

¹²Marcel Richter, "Rational Choice," Preference, Utility and Demand, p. 37.

¹³Hukukane Nikaido, Convex Structures and Economic Theory (New York: Academic Press, 1968), p. 65.

¹⁴Ibid.

¹⁵It is required that the norm satisfy the following criteria:

- 1) $\|x\| \geq 0$ and $\|x\| = 0$ iff $x = 0$.
- 2) $\|ax\| = |a| \cdot \|x\|$.
- 3) $\|x+y\| \leq \|x\| + \|y\|$.

The same topological structure can be obtained from the norm above as for the Euclidean norm or the norm defined $\|x\| = \max_i \{ |x_i| \}$.

¹⁶Marcel Richter, "Rational Choice," p. 31.

¹⁷As stated in Section 3, continuity in the sense C3 is implicit in any discussion of the continuity of $d(r)$. Of course, since $d(r)$ is single-valued, C1 is equivalent to C3 and distinguishing these notions of continuity is irrelevant.

¹⁸Leonid Hurwicz and Hirofumi Uzawa, "On the Integrability of Demand Functions," Preference, Utility and Demand, pp. 114-49.

¹⁹The proof of this statement is identical to the proofs of Lemmas 4 and 5.

²⁰Lemma 8 is also extendable to X^L , but the proof is not identical to that of Lemma 3 and would require some assumptions on the income effects on the single priced commodities $x_1, \dots, x_{n-1}$.

²¹Hurwicz and Uzawa, p. 117.

²²Ibid., p. 19.

²³Ibid., p. 124.

²⁴Leonid Hurwicz, "On the Problem of Integrability of Demand Functions," Preference, Utility and Demand, p. 182.

²⁵Ibid.

²⁶ μ and ν are probability measures. Therefore, the notation $\mu \times \nu(A)$ indicates the probability that the random variable (α, y) is contained in the set A . $\mu \times \nu(A)$

is defined for any set $A \subset L$ such that A is an element of B . The notation $\mu(\alpha|y)$ indicates the probability of given y and $\nu(y|\alpha)$ indicates the probability of y given α .

²⁷Hugo Sonnenschein, "Do Walras' Identity and Continuity Characterize the Class of Community Excess Demand Functions?" Journal of Economic Theory 6 (1973): 345-54; Rolf Mantel, "On the Characterization of Aggregate Excess Demand," Journal of Mathematical Economics, 1 (1974): 15-21; Daniel McFadden, Andrew Mas-Colell, and Rolf Mantel, "A Characterization of Community Excess Demand Functions," Journal of Economic Theory 9 (1974): 361-74.

²⁸Kelvin Lancaster, "The Theory of Household Behavior; Some Foundations," Annals of Economic and Social Measurement 41 (1975):9.

CHAPTER IV

MODELLING THE DEMAND FOR ELECTRICITY

4.1 Introduction

In Chapter III the theoretical implications of decreasing block rate schedules were discussed in considerable detail. However, little attempt was made to deal with these implications on an empirical level. In this chapter, I address the problem of integrating the results of the last chapter into an empirically estimable model.

This chapter is organized around two basic topics. The first topic is the representation of rate schedules in empirical models. This is discussed in Section 4.2. The second topic is the design of the model itself. This is discussed in Sections 4.3 - 4.5. In Section 4.3 the role of appliance stocks is considered; in Section 4.4 the flow adjustment model which will be estimated is derived; and in Section 4.5 alternate models to which the flow adjustment model will be compared are presented. Some concluding remarks are made in Section 4.6.

4.2 The Characterization of the Rate Schedule

In the theoretical discussion of the last chapter, average demand was represented as a function of an entire

rate schedule,

$$(4.1) \quad \bar{x} = \bar{d}(s, r) \quad .$$

In practical terms, it will not be feasible to include the entire rate schedule in an empirical model. Even at the individual level the consumer choice problem does not admit to a simple analytic solution. This presents problems in model specification. It is easy enough to write a general function for average demand as in (4.1); it is harder to write a precise expression for $\bar{d}$. In addition, because the blocks are not of a uniform size or number as the rate schedules vary, including all of the block rates in the model would entail enlarging the number of blocks used or a departure from representing the entire rate schedule. With a large number of block rates included in the model, there would be difficulties in determining and interpreting the roles of each of these rates and, of course, they would be highly interdependent. It is, thus, necessary to find some means of characterizing a rate schedule for the purpose of incorporating it into an econometric model.

Several techniques for representing a rate schedule in a model will be employed in this study. In selecting these techniques, there were trades-off between how adequately the technique characterized the actual rate schedule and how easily the characterization could be incorporated into an empirically testable model. Several

criteria were used in devising these techniques. How adequately the techniques approximate the rate schedules was important. The procedures selected either represent the rate schedule exactly over an interval of quantities, or approximately but independent of quantity. At the same time these procedures employ a minimum number of variables. In each case, the two variables selected are chosen so as to simplify the specification of the model, eliminate duplication of the price effects among several variables, and facilitate the interpretation of the coefficients. The two techniques selected possess each of these qualities to varying degrees.

The first technique derives from the fact that as long as an individual is consuming within a certain block, $(p, y) \in \chi^i$ in the notation of Chapter III, the rate schedule the individual is facing is completely characterized by two variables, the marginal price, r_i , and the expenditure on electricity up to the i^{th} block, t_i ,

$$(4.2) \quad t_i = \sum_{j=1}^{i-1} r_j (k_j - k_{j-1}) \quad .$$

For an individual the coefficient on r_i measures the effect of a change in marginal price. The sign of this coefficient would be negative if the commodity is not a Giffen good as shown in Chapter III. In addition, according to equation (3.10),

$$dx_n/dt_i = -dx_n/dy$$

providing an empirically testable hypothesis on the side and the sign of the coefficient of t for individual demand.

Ideally, information would be known about the distribution of consumers across the blocks of a rate schedule. Suppose there are N_i consumers within the i^{th} block of the rate schedule,

$$N = \sum_{i=1}^m N_i .$$

The average demand function could then be specified by including an average marginal price variable,

$$\bar{r} = \sum_{i=1}^m \frac{N_i r_i}{N} ,$$

and a variable indicating the average individual expenditure on the intramarginal blocks,

$$\bar{t} = \sum_{i=1}^m \frac{N_i t_i}{N} .$$

If the size of the income effect, dx_n/dy , is uniform across consumers,

$$(4.3) \quad \partial \bar{x}_n / \partial \bar{t} \sim - \partial \bar{x}_n / \partial \bar{y} .$$

As long as the consumption blocks of individuals do not shift, this technique results in a very accurate representation of the rate schedule. It is computationally simple. The variables are easy to interpret economically, and (4.3) yields an empirically testable hypothesis. These are the main virtues of this technique.

Unfortunately, the consumption blocks of individuals do shift. There is substantial seasonal variation in individual's consumption levels, and only average monthly consumption levels over a year are employed in this study. This means that the values of the rate schedule variables are a function of the particular distribution of consumption levels in effect at the time. These variables are then to be used in estimating average consumption. A technique which was derived only from the rate schedule without reference to consumption would be advantageous especially to a model that is to be used in forecasting. This feature is lacking in the model being discussed.

In addition, several modifications to this technique are necessary. The first modification results from the fact that information is not generally available on the distribution of customers across blocks in the rate schedule; the second is due to the fact that the number and size of rate blocks vary across utilities.

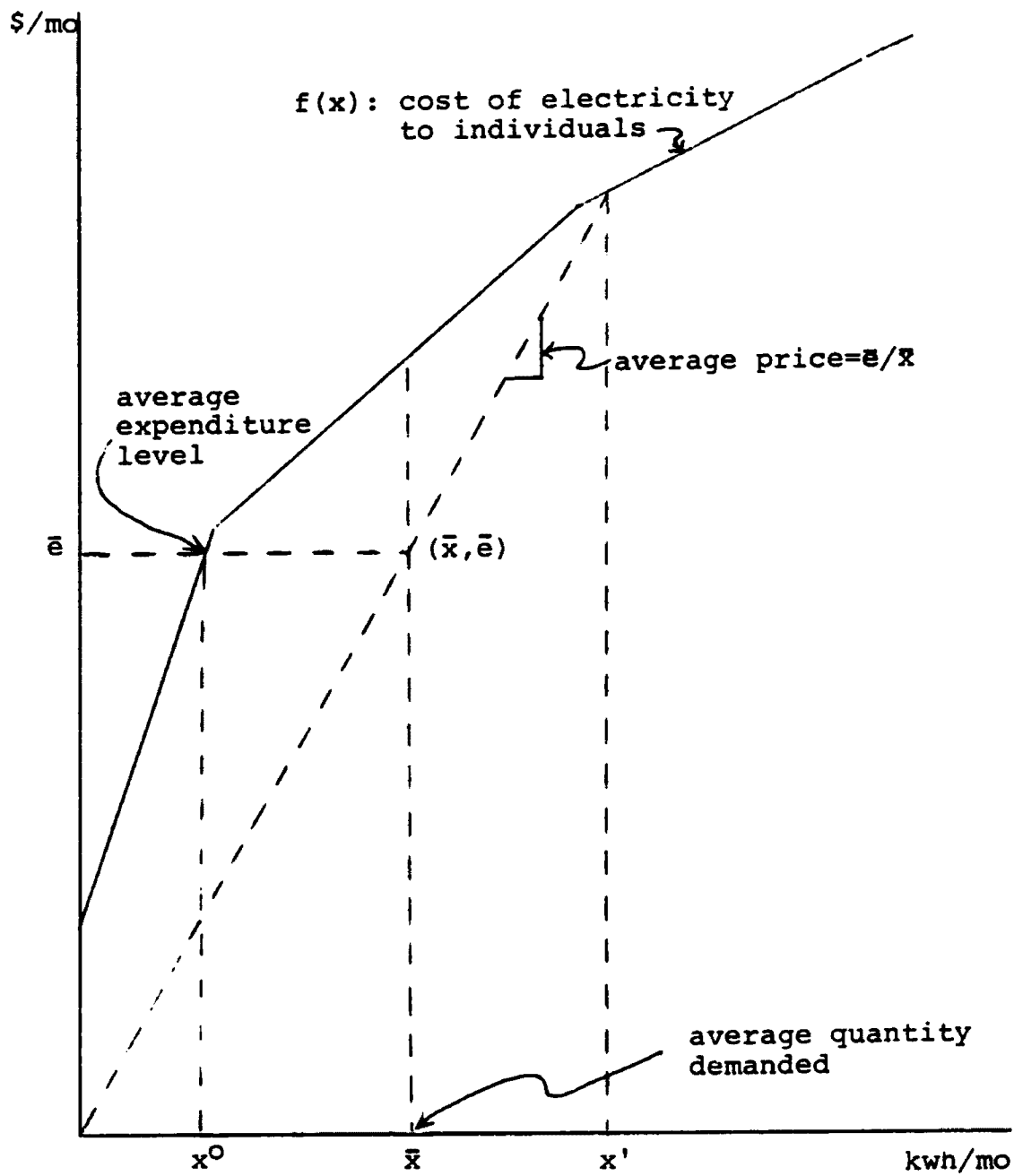
Because the distribution of consumption levels is unavailable, this technique must be modified so that the rate schedule is characterized by the values t_i and r_i defined for a particular "typical" block in the rate schedule. Because of this, it shall be referred to as the typical block technique.

By typical block it seems best to employ the modal consumption block, but again there is a data problem.

Information is only available on the mean consumption level and the mean expenditure level of customer. Since the distribution of consumption levels is skewed to the right, the modal consumption level is less than the mean. The question now becomes how to best approximate the modal level with the available information.

The graph in Figure 4.1 shows individual expenditure on electricity, $f(x)$, as a function of quantity, x , given decreasing block rates. It also shows the average consumption level, $\bar{x}$, and the average expenditure level, $\bar{e}$. The function $f(x)$ and the quantities $\bar{x}$ and $\bar{e}$ are known. Since there are decreasing block rates $f(x)$ is concave. The concavity of f means that $f(\bar{x}) > \bar{e}$, if consumption levels of individuals vary among block levels. x^0 indicates the consumption level at which the cost of electricity is equal to the average expenditure level; $f(x^0) = \bar{e}$. Since $f(x)$ is monotonically increasing and $f(x^0) = \bar{e} < f(\bar{x})$, $x^0 < \bar{x}$. x' indicates the point at which the average price of electricity per kwh is equal to the average price actually paid; $f(x')/x' = \bar{e}/\bar{x}$. Since $f(\bar{x}) > \bar{e}$, $f(\bar{x})/\bar{x} > \bar{e}/\bar{x}$. Since f is concave and $f(0) > 0$, $f(x)/x$ is monotonically decreasing. Therefore, $x' > x$, and $x^0 < \bar{x} < x'$. Since demand is skewed to the right, the modal consumption level is less than $\bar{x}$. Therefore, it is best approximated by x^0 or $\bar{x}$ and not by x' . The values for r_i and t_i will be determined in the typical block method by using each of the levels x^0 , x' and $\bar{x}$ as

Figure 4.1
Methods of Selecting the Modal Block



approximations of the mode of x . It is assumed that the best results will be obtained with either x^0 or $\bar{x}$. This can be tested empirically.

A comparison of the values of $\partial x / \partial r_i$ will be made using average expenditure (x^0), average consumption ($\bar{x}$), and average price (x') to define the typical block and the corresponding marginal price, r_i . Under the assumptions of 3.5B, if $\partial x / \partial r_i$ is small (in absolute value) relative to $\partial x / \partial y$ for individual demand, then $\partial \bar{x} / \partial r_i$ will decrease (in absolute value) as the block level increases. If $\partial x / \partial r_i$ is large (in absolute value) relative to $\partial x / \partial y$ (see equation (3.17)), then $\partial \bar{x} / \partial r_i$ can peak at the modal block. This is an observable phenomenon.

Since only one block is used to represent the rate schedule in this case instead of a weighted average of all the blocks, equation (4.3) is no longer valid. However, if $\partial \bar{x} / \partial r_i$ is large with respect to $\partial \bar{x} / \partial y$ in absolute value, this is consistent with

$$(4.4) \quad \partial \bar{x} / \partial t < -\partial \bar{x} / \partial y$$

under the assumptions of Section 3.5B (see equation (3.18)). This again is an observable phenomenon. Although there is no definitive relationship between $\partial \bar{x} / \partial t$ and $\partial \bar{x} / \partial y$ in the case of average demand, we can look for a consistency among the values $\partial \bar{x} / \partial t$, $\partial \bar{x} / \partial r$, and $\partial \bar{x} / \partial y$.

The rate variables t_i and r_i represent the rate schedule exactly for consumption levels in the interval $k_{i-1} \leq x < k_i$. Thus, although this is still an ex post technique, it is a better representation of the rate schedule than a simple average price which specifies one point on the rate schedule, or a marginal price which gives no indication of the total cost at all. As mentioned in Chapter II, some ex post simultaneity exists between x and the rate variables r_i and t_i since x is employed in evaluating these variables. This simultaneity will be the strongest for $\bar{x}$. However, since r_i and t_i are valid for all x in the i^{th} block, this relationship is not one-to-one. For this reason it is not considered to be critical.

A further modification of this technique is necessary due to the fact that the vector of block demarcations, k , varies. Therefore, the value of t_i might measure the expenditure on electricity up to different points, k_{i-1} , even at the same mean consumption level. Thus, this technique would not differentiate between the two cost curves, $f_1(x)$ and $f_2(x)$, in Figure 4.2a; it would differentiate between the cost curves $f_1(x)$, $f_2(x)$ in Figure 4.2b.

To standardize the effect of the block demarcations, t_i is modified so that it represents the expenditure on electricity up to some point c . This cost is defined so that the total cost of $\bar{x}$ (or x^0 , x') units of electricity

Figure 4.2
Problems in the Typical Block
Characterization of a Rate Schedule

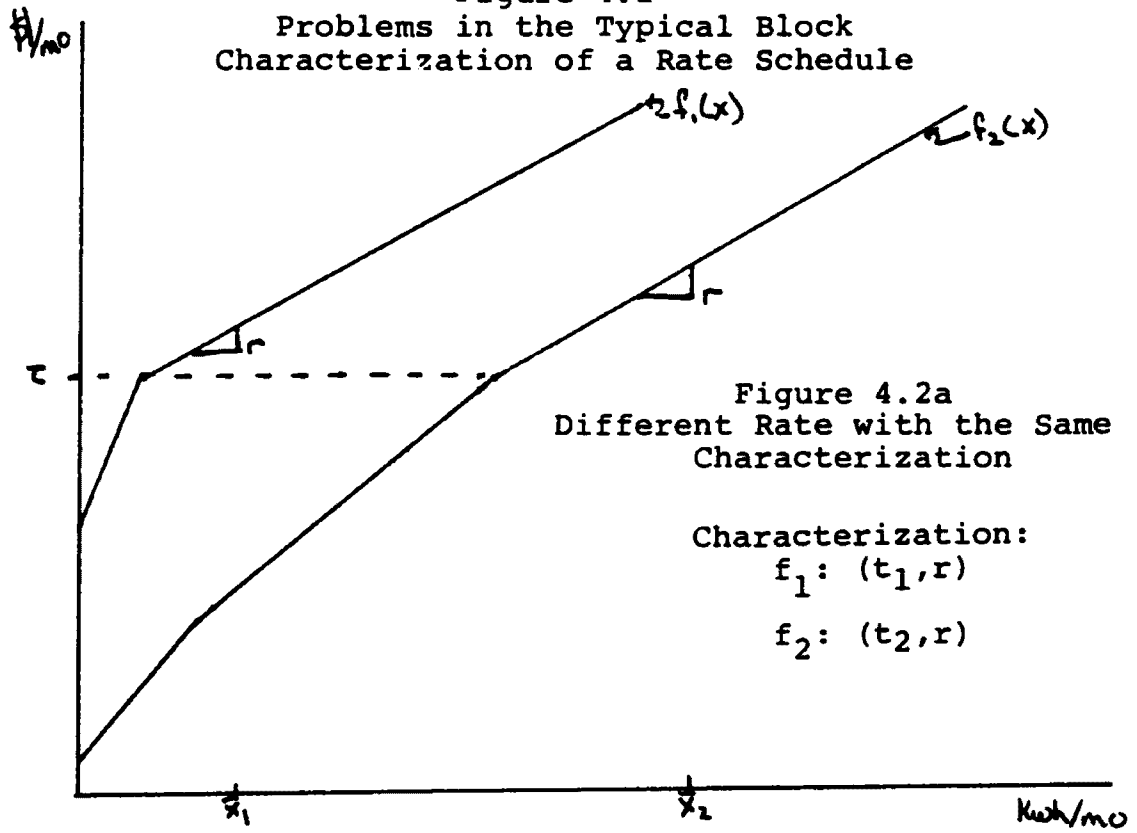


Figure 4.2a
Different Rate with the Same
Characterization

Characterization:

$$f_1: (t_1, r)$$

$$f_2: (t_2, r)$$

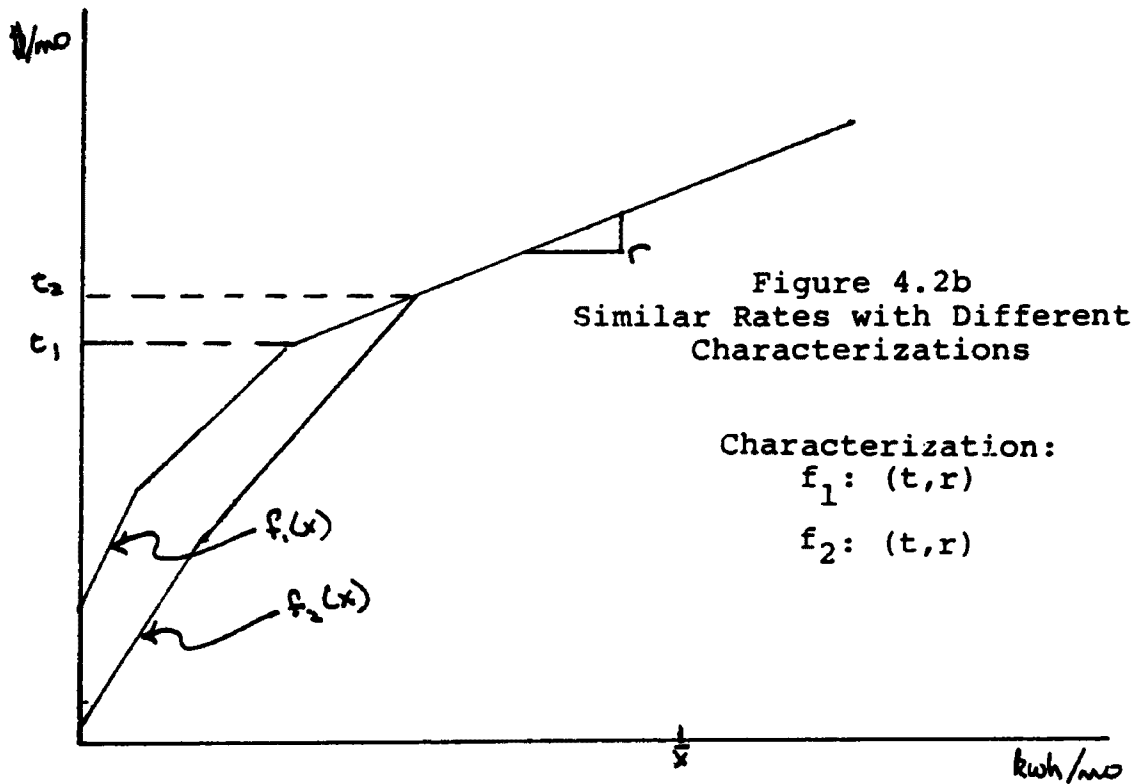


Figure 4.2b
Similar Rates with Different
Characterizations

Characterization:

$$f_1: (t, r)$$

$$f_2: (t, r)$$

is unaltered if a marginal price, r , applies for units greater than c . Thus, the modified value of t_i , t , is written

$$(4.5) \quad t = t_i - (k_{i-1} - c)r_i$$

where $k_{i-1} < \bar{x}$ (or x^0, x') $< k_i$ and $t_i = f(k_{i-1})$. If $(k_{i-1} - c)$ is small, then

$$d\bar{x}/dt \sim d\bar{x}/dt_i \quad .$$

Two values of c are initially selected in this investigation. In some experiments c is set at a single point, zero. In others it is set at

$$c = \frac{\sum_j N_j k_{i-1}}{\sum_j N_j} \quad ,$$

the average value of k_{i-1} for all districts, where N_j is the number of customers in district j . This value of t varies in different years. The results of the empirical investigation led to an alternate specification of t .

To reiterate, the advantages of this technique are that it produces variables that are computationally simple, easily incorporated into a model and yield easily interpretable results. The disadvantages are that these variables are dependent on the quantity variable and only accurately represent the rate schedule over an interval of quantities.

The second technique has qualities that complement the first. It approximates the rate schedule fairly well over a broad range of quantities, and is independent of the level of consumption; the incorporation of this technique into a model presents some difficulty, however, since the most intuitive way of incorporating this rate representation into a model requires some form of nonlinear estimation technique. The coefficients estimated by this technique are less easily interpreted than those in the first technique.

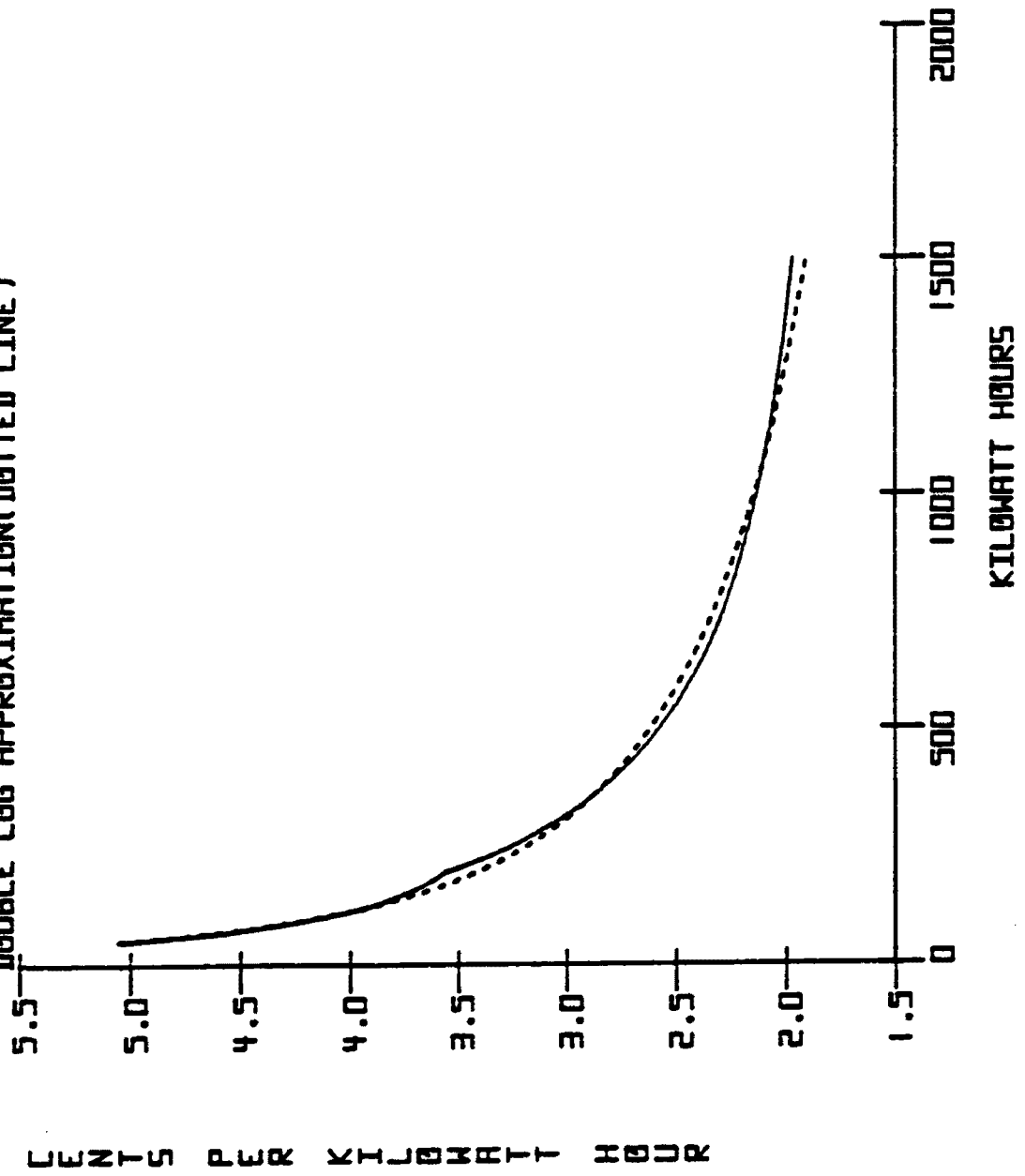
The average price paid for electricity is a function of quantity consumed and this function is specified by the information given in the rate schedule. If $f(x)$ is the cost of x units of electricity, the average price function is $f(x)/x$. In the second technique, this average price function will be approximated by a double logarithmic function. This approximation is written

$$f(x)/x \sim p_1 x^{p_2} .$$

This will be referred to as the average price technique.

Figure 4.3 illustrates an average price function and its double logarithmic approximation. A very close approximation is achieved over a wide range of quantities as is seen in this figure. Thus, the consumption decision facing a consumer can be specified quite accurately by the variables p_1 and p_2 .

Figure 4.3
AVERAGE PRICE OF ELECTRICITY - ACTUAL (SOLID LINE)
DOUBLE LOG APPROXIMATION (DOTTED LINE)



These variables will be calculated prior to model estimation from the actual rate schedules. They will then be included in the specification of the model. The variable, P_1 , can be interpreted as an indicator of the level of prices; the variable, P_2 , can be viewed as a measure of the rate at which prices fall with increased purchase levels. An increase in the level of prices can be thought of as an increase in the entire rate vector, r . Then, according to Chapter III,

$$(4.6) \quad \partial \bar{x} / \partial P_1 < 0 \quad .$$

An increase in the rate at which prices fall can be thought of as an increase in intramarginal prices and a decrease in extramarginal prices. Then, again according to Chapter III,

$$(4.7) \quad \partial \bar{x} / \partial P_2 < 0 \quad .$$

Neither of these relations can be thought to correspond precisely to a marginal price effect.

In order to measure the effect of marginal price on demand, note that, if the average price of electricity is $P_1 \times P_2$, then the total cost is $P_1 \times P_2 + 1$. The marginal price of electricity, MP , is then

$$MP = P_1 \cdot (P_2 + 1) \cdot x^{P_2} \quad .$$

The elasticity of demand with respect to marginal price, using this expression for marginal price, is measured

$$(4.8) \quad \left(\frac{\partial x}{\partial MP}\right) \left(\frac{MP}{x}\right) = [(\eta_{P1})^{-1} + (1 + P2)^{-1} \left(\frac{\eta_{P2}}{P2}\right)^{-1} \\ + (P2) + \left(\frac{\eta_{P2}}{P2}\right)^{-1} \ln x]^{-1}$$

where η_{P1} and η_{P2} are the elasticities of x with respect to $P1$ and $P2$ respectively. This can be evaluated at the average values of x , $P1$ and $P2$ and compared to the elasticity of x with respect to r obtained in the first technique.

The calculation of the values of $P1$ and $P2$ are entirely independent of the actual average demand level, $\bar{x}$ (or x_0 , x'). Therefore, a model incorporating this technique of representing the rate schedule should be ceteris paribus better suited for forecasting. This technique does not give a precise representation of the rate schedule. Nevertheless, it does yield a good approximation. The main difficulties lie in the interpretation of the coefficients and in the estimation technique. One model employing these rate variables requires some form of nonlinear estimation technique. This makes estimation of the model more difficult. A nonlinear estimation technique is available for this purpose which yields consistent although inefficient estimates of the desired coefficients.

4.3 The Role of Appliance Stocks

The demand for electricity is a derived demand. Electricity serves no purpose except when employed in conjunction with electricity-burning appliances. Thus, the stock of appliances is clearly an important determinant of the demand for electricity.

Furthermore, while electricity is used up instantaneously as it is consumed, appliances are a capital stock of some durability. Therefore, although it is possible to adjust the demand for electricity quickly to changes in demand determinants, there can be substantial lags in the adjustment of the stock of appliances. It will, thus, be important to differentiate between short- and long-run elasticities. Short-run elasticities will measure the demand adjustments excluding the effects of capital stock adjustments. Long-run elasticities will include the effects of capital stock adjustments.

Optimally, this differentiation in elasticities could be handled by following the general approach of Fisher and Kaysen, and estimating separately the demand for electricity given a stock of appliances and the investment decision in appliances. This procedure would allow short- and long-run elasticities to vary independently. It could also allow for utilization rates to vary across appliances.

This study is a part of a larger effort, just being completed, which includes appliances stocks directly in the demand equations.¹ However, the scope of this study will be limited to simpler models. There are a number of reasons for this.

The primary reason is that I have attempted to undertake a thorough examination of rate schedule representation, and this has proved to be a quite extensive study in itself. An investigation of the demand for appliances and the incorporation of appliance stock is also a major study in itself. It has not been done to date with any degree of adequacy.² Although Fisher and Kaysen attempted to deal with these issues, they were hampered by data inadequacies. They did not, in fact, include any stock variable in their electricity demand equation.

In addition, the completion of the compilation of a new appliance stock data set was delayed and it has only just been completed.

4.4 The Flow Adjustment Model

The first model I will employ in estimating electricity demand is the Houthakker-Taylor flow adjustment model. It is called the HT model in the discussion of the results. Although it does not include an appliance stock variable, it is a dynamic model and does differentiate between short- and long-run effects. This model has not previously

been applied to electricity demand estimated at the regional level.

In this model it is assumed that consumption adjusts to the desired level by the following process:

$$(4.9) \quad \dot{x} = \theta (x - \hat{x})$$

where $\hat{x}$ is the desired level of consumption and θ is an adjustment parameter. It is further assumed that the desired level of consumption is a function of exogenous variables and can be expressed,

$$(4.10) \quad \hat{x} = a_0 + a_1 y + a_2 r + a_3 t + a_4 z + \varepsilon$$

Here y is income, r and t are rate schedule parameters from the typical block technique, and z represents any other exogenous variable.

The model as defined is a continuous dynamic model. However, observed values of most variables represent discrete intervals of time. The variable $\dot{x}$ is unobservable. In order to express a finite approximation to this dynamic model, the within-period changes in consumption are examined.

$$\Delta^* x = x(\tau_0 + \tau) - x(\tau_0)$$

defines the within-period changes in consumption. The observable variables in this model represent average values in each period. The difference between the average

values in two periods can be approximated

$$(4.11) \quad \bar{x}_{\tau_0+\tau} - \bar{x}_{\tau} = 1/2(\Delta^*x_{\tau_0+\tau} + \Delta^*x_{\tau_0}) \quad ,^3$$

where $\bar{x}_{\tau_0}$ is the average consumption in the period

$$\bar{x}_{\tau_0} = 1/\tau \int_{\tau_0}^{\tau_0+\tau} x(\tau) \quad .$$

Although all variables will represent average values, the bar will be omitted in the notation from here on.

Equation (4.11) allows a finite approximation to be made to the continuous model. From (4.9) and (4.11)

$$x - x_{-1} = \theta/2(\hat{x} - x + \hat{x}_{-1} - x_{-1}) \quad .$$

Therefore,

$$x = \left(\frac{1-\theta/2}{1+\theta/2}\right)x_{-1} + \frac{\theta/2}{1+\theta/2} (\hat{x} + \hat{x}_{-1}) \quad .$$

Incorporating (4.10) into this expression yields

$$(4.12) \quad x = \left(\frac{1-\theta/2}{1+\theta/2}\right)x_{-1} + \left(\frac{\theta/2}{1+\theta/2}\right)\{2a_0 + a_1(y+y_{-1}) \\ + a_2(r+r_{-1}) + a_3(\tau+\tau_{-1}) + a_4(z+z_{-1})\} + \eta \quad .$$

This equation can then be estimated in the form,

$$x = b_0 + b_1x_{-1} + b_2(y+y_{-1}) + b_3(r+r_{-1}) \\ + b_4(t+t_{-1}) + b_5(z+z_{-1}) + \eta \quad .$$

All the parameters in (4.12) are identified in the estimated equation:

$$\begin{aligned}
 \theta &= \frac{2(1 - b_1)}{1 + b_1} \\
 a_0 &= \frac{b_0}{1 - b_1} \\
 (4.13) \quad a_1 &= \frac{2b_2}{1 - b_1} \\
 a_2 &= \frac{2b_3}{1 - b_1} \\
 a_3 &= \frac{2b_4}{1 - b_1} \\
 a_4 &= \frac{2b_5}{1 - b_1} .
 \end{aligned}$$

The reasoning behind this model is equally valid when it is written in its logarithmic form. For this form the basic equations become

$$(4.14) \quad \dot{x}/x = \ln(\hat{x}/x)^\theta$$

and

$$(4.15) \quad \hat{x} = e^{a_0} y^{a_1} r^{a_2} t^{a_3} z^{a_4} e^\varepsilon .$$

The estimating equation derived from this form of the model is

$$\begin{aligned}
 (4.16) \quad \ln x &= b_0 + b_1 \ln x_{-1} + b_2 (\ln y + \ln y_{-1}) \\
 &\quad + b_3 (\ln r + \ln r_{-1}) + b_4 (\ln t + \ln t_{-1}) \\
 &\quad + b_5 (\ln z + \ln z_{-1}) + \eta .
 \end{aligned}$$

The relations between the parameters $\theta, a_0, \dots, a_4$ and the estimated coefficients remain as given in (4.13).

Deriving elasticities from the logarithmic form of this model is easy since the coefficients themselves are elasticities. A discrete approximation to the short-run elasticity of x with respect to r can be calculated from equation (4.16) as

$$(4.17) \quad \eta_r = 2b_3 \quad .$$

This includes the total direct effect of r (or r_{-1}) on x and omits the indirect effect of the lagged dependent variable. In the long run, x approaches $\hat{x}$. The long-run elasticity of x with respect to r as calculated from equation (4.15) is

$$(4.18) \quad \begin{aligned} \eta_r \text{ (long run)} &= a_2 \\ &= 2b_3/1 - b_1 \quad . \end{aligned}$$

The discrete approximation to this long-run elasticity as calculated from the estimating equation (4.16) is also $2b_3/(1-b_3)$. The elasticities of x with respect to the other variables are calculated in a similar manner.

In the linear form of this model the expressions derived in (4.17) and (4.18) are derivatives instead of elasticities. In this model elasticities will be estimated at the mean values of the sample. The short-run elasticity of x with respect to r is

$$(4.19) \quad \eta_r = 2b_3 \bar{r}/\bar{x}$$

where $\bar{r}$ and $\bar{x}$ are sample means of the variables r and x .

The long-run elasticity is

$$\eta_r \text{ (long run)} = (2b_3/(1 - b_1)) \bar{r}/\bar{x} \quad .$$

Both of these forms of the model will be tested employing the typical block technique of representing the rate schedule.

The second technique of representing a rate schedule is with a double logarithmic approximation to the average price function. With this technique the logarithmic form of the model is used. Two different ways of incorporating this representation of the rate schedule into the model are employed.

In the first method, the variables defining the average price function are incorporated into the desired consumption equation replacing the variables r and t .

(4.15) is then expressed

$$(4.20) \quad \hat{x} = e^{a_0} y^{a_1} P_1^{a_2} e^{a_3 P_1} z^{a_4} e^\epsilon \quad .$$

The variable P_2 is negative and enters as an exponent. This does not alter the derivation of the model. The estimating equation is identical to (4.16) with $\ln P_1$ and P_2 replacing $\ln r$ and $\ln t$ respectively. The relation between the estimated coefficients and the model parameters is the same as before (see equation (4.13)). The short-

run elasticity of x with respect to P_1 is $\eta_{P_1} = 2b_3$ and with respect to P_2 it is $\eta_{P_2} = 2b_4 \cdot \bar{P}_2$. The marginal price elasticity can be obtained by incorporating this information into (4.8).

In the second method of incorporating this representation of the rate schedule into the model, the average price function is included in the desired consumption equation in its estimated form. Equation (4.13) is then expressed

$$(4.21) \quad x = e^{a_0} y^{a_1} (P_1 x^{a_2} P_2^{a_2} z^{a_3}) e^{\eta}.$$

Therefore, (4.12) becomes

$$(4.22) \quad \ln x = \left(1 + \frac{\theta}{2} - \frac{\theta a_2 P_2}{2}\right)^{-1} \left\{ \theta \ln a_0 + (1 - \theta/2) \ln x_{-1} \right. \\ \left. + \frac{\theta a_1}{2} (\ln y + \ln y_{-1}) + \frac{\theta a_2}{2} (\ln P_1 + \ln P_{1-1} \right. \\ \left. + \ln x_{-1}) + \frac{\theta a_3}{2} (\ln z + \ln z_{-1}) + \eta \right\}.$$

The estimating equation for this model can be written

$$(4.23) \quad \ln x = \frac{1}{(b_5) - b_0 P_2} \{ b_0 + b_1 \ln x_{-1} + b_2 (\ln y + \ln y_{-1}) \\ + b_3 (\ln P_1 + \ln P_{1-1} + P_{2-1} \ln x_{-1}) \\ + b_4 (\ln z + \ln z_{-1}) \}.$$

This equation clearly requires some form of nonlinear estimating technique. In addition, unless it is required that

$$(4.24) \quad b_5 = 2 - b_1 \quad ,$$

and

$$b_6 = b_3 \quad ,$$

the parameters of the model will be over identified by this equation. Given (4.24),

$$(4.25) \quad \theta = 2(1 - b_1) \quad ,$$

$$a_0 = b_0/2(1 - b_1) \quad ,$$

$$a_1 = b_2/(1 - b_1) \quad ,$$

$$a_2 = b_3/(1 - b_1) \quad ,$$

$$a_3 = b_4/(1 - b_1) \quad .$$

Even though the logarithmic form of the model is employed, because of the nonlinearity of the model in this case, the elasticities are not constant. They will be evaluated at the sample means. A discrete approximation to the elasticity of x with respect to P_2 in the short run is

$$(4.26) \quad \eta_{P_2} = \frac{b_3 + b_6}{b_5 - b_3 P_2} \cdot P_2 \cdot \ln x$$

$$= \frac{2b_3 P_2 \ln x}{2 - b_1 - b_3 P_2}$$

given (4.24). The short-run elasticity of demand with respect to P_1 is

$$\begin{aligned}
 (4.27) \quad \eta_{P2} &= \frac{2b_3}{b_5 - b_6 P2} \\
 &= \frac{2b_3}{2 - b_1 - b_3 P2} \quad .
 \end{aligned}$$

These short-run elasticities are calculated by including all the direct effect of the specified variable and by omitting any effect due to the lagged dependent variable. Given

$$c = \frac{b_1 + b_3 P2}{2 - b_1 - b_3 P2} \quad ,$$

the long-run elasticities of demand with respect to these variables can be written

$$\eta \text{ (long run)} = \frac{\eta \text{ (short run)}}{1 - c} \quad .$$

The expressions for η_{P1} and η_{P2} can be employed in calculating the marginal price elasticity (equation (4.8)).

These two versions of the logarithmic model will be tested for the average price representation of the rate schedule.

A major virtue of a model such as the one discussed in this section is that it is dynamic. It also eliminates the need for the inclusion of a stock variable for which data are frequently unavailable. This presents some problems which should be stated explicitly. In all the forms of the model presented in this section, long-run elasticities are proportional to short-run elasticities. The

model also implies that the utilization rates of all types of appliances are the same or that the composition of the appliance stock is constant even though the utilization of different types of appliances may vary. Neither of these assumptions is very realistic. They are unavoidable without including a separate appliance stock variable. Nevertheless, this model should be adequate for testing our rate schedule hypotheses. It is also at least no worse than other models that have been employed in similar studies.

4.5 Other Models

Several other models will be tested for comparison purposes. All of them have been previously employed in estimating electricity demand on pooled state data. They are all similar to the flow adjustment model described in the last section in a number of respects. They are all dynamic, and they all avoid the use of an appliance stock variable. In addition, they are all expanded to include the same set of demand determinants. This set of variables includes a rate schedule representation, an income variable, the price of natural gas, and indices of weather. Since all of these models are discussed in Chapter II, I will only discuss the derivation of the models to the extent they differ from the earlier description. Since the rate representations I employ differ from those in the

earlier implementations of these models, they shall be specified in each case.

The first of these models is the Fisher-Kaysen model which is described in Chapter II. As stated in that discussion, it assumes a constant growth rate of the appliance stock. The long-run elasticity is, thus, effectively infinite. The formulation of this model is only applicable in its logarithmic form. Consequently, only this formulation will be used. The general form of this model is stated in equation (2.4). This formulation is expanded to include the variables r and t of the typical block representation instead of the variable p . It also includes the additional variables listed above. The average price representation of a rate schedule is not employed in this model. In the discussion of the empirical results, this model is labeled the FK model.

The second comparison model is the one employed by Houthakker, Verleger, and Sheehan. It is also described in Chapter II. It can be interpreted as either a flow adjustment model or a Koyck-lag model. The flow adjustment interpretation is given in Chapter II. This differs from the model of the last section in that a discrete adjustment process is used. In the Koyck-lag interpretation, the basic equation is formulated as follows,

$$x = a_0 + a_1 \sum_{i=0}^{\infty} \Lambda^i y_{-i} + a_2 \sum_{i=0}^{\infty} \Lambda^i r_{-i} \quad .$$

Subtracting Λx_{-1} from x yields

$$x = (1 - \Lambda)a_0 + \Lambda x_{-1} + a_1 y + a_2 r \quad .$$

This model in its logarithmic form is identical to the one estimated by Houthakker, Verleger, and Sheehan (equation (2.5)). The interpretation I will give to this model is that of a flow adjustment model and it will be referred to as the HVS model.

The flow adjustment model with a discrete adjustment process is estimated in both its linear and logarithmic forms. In its logarithmic form it will be estimated using both the typical block and the average price technique of representing a rate schedule. Since a variety of forms of this model will be estimated, I will describe each one explicitly.

The incorporation of the typical block method of representing a rate schedule into this model is straightforward. Both the linear and logarithmic forms are used. The desired consumption equations are the same as were used in the last section for the typical block technique (equations (4.10) and (4.15)). Only the adjustment process is different. The adjustment process for the logarithmic form of the model was stated in Chapter II. This process is

$$x/x_{-1} = (\hat{x}/x_{-1})^\theta \quad .$$

The linear form of this adjustment process is

$$x - x_{-1} = \theta(\hat{x} - x_{-1}) \quad .$$

The estimating equations for the linear and logarithmic forms of this model respectively are as follows:

$$(4.28) \quad x = b_0 + b_1 x_{-1} + b_2 y + b_3 r + b_4 t + b_5 z \quad .$$

$$(4.29) \quad \ln x = b_0 + b_1 \ln x_{-1} + b_2 \ln y + b_3 \ln r \\ + b_4 \ln t + b_5 \ln z \quad .$$

The relationship between the estimated coefficients and the model parameters is the same for both these equations. This relationship is,

$$(4.30) \quad \begin{aligned} \theta &= 1 - b_1 \\ a_0 &= b_0 / 1 - b_1 \\ a_1 &= b_2 / 1 - b_1 \\ a_2 &= b_3 / 1 - b_1 \\ a_3 &= b_4 / 1 - b_1 \\ a_4 &= b_5 / 1 - b_1 \quad . \end{aligned}$$

The only difference between the logarithmic form of this model and the one used by Houthakker, Verleger, and Sheehan is the rate schedule representation.

The average price technique of representing a rate schedule is only incorporated into the logarithmic form of the model. Two ways of including this rate schedule representation in the desired demand equation are discussed

in the last section (equations (4.20) and (4.21)). Both of these are used in conjunction with the logarithmic adjustment process to define an estimatable model. The estimating equation derived from the first expression for the desired demand equation (4.20) is

$$(4.31) \quad \ln x = b_0 + b_1 \ln x_{-1} + b_2 \ln y + b_3 \ln P_1 \\ + b_4 P_2 + b_5 \ln z \quad .$$

For this form of the model, the relationship between the estimated coefficient and the model parameters is the same as given in (4.30). The estimating equation derived from the second desired demand equation (4.21) is

$$(4.32) \quad \ln x = (1 - b_4 P_2)^{-1} \{ b_0 + b_1 \ln x_{-1} + b_2 \ln y \\ + b_3 \ln P_1 + b_5 \ln z \}$$

This model requires a nonlinear estimation routine. In addition, unless

$$(4.33) \quad b_4 = b_3 \quad ,$$

the model coefficients are overidentified. Given (4.33)

$$(4.34) \quad \theta = 1 - b_1 \\ a_0 = b_0 / (1 - b_1) \\ a_1 = b_2 / (1 - b_1) = b_4 / (1 - b_1) \\ a_2 = b_3 / (1 - b_1) \\ a_3 = b_5 / (1 - b_1) \quad .$$

The final model estimated to which our results will be compared is the variable elasticity model of Mount, Chapman, and Tyrrell, abbreviated MCT in the discussion of the empirical results. As stated in Chapter II, this model can be viewed as a flow adjustment model with a discrete adjustment process. In this interpretation, the logarithmic adjustment process is employed. The desired demand equation necessary to yield an estimating equation of their form is as follows:

$$\ln \hat{x} = \frac{1}{\theta} \ln A + \sum_{i=1}^I \left(\frac{\beta_i}{\theta} \ln V_i + \frac{r_i}{\theta} V_i^{-1} \right) .$$

The estimated form of this model is given in equation (2.7). The model is estimated only with the typical block method of representing the rate schedule. In this case, the variables t and r are included as independent variables, V_i .

Having estimated a variety of models for comparison purposes, we must establish some sort of procedure for evaluating and comparing these results. Clearly, the significance of the estimated coefficients and the fit of the equations is important. In addition, the reasonableness of the results and the extent to which they conform to our prior notions will be used in evaluating the models. An estimated price elasticity of -10 would, for example, be entirely unreasonable, and could lead us to reject a model on that account. The reasonableness of the

model itself is also relevant. Because of its assumption of a constant growth rate in the appliance stock, the Fisher-Kaysen model seems somewhat shakey. This model is included in the set of models to be estimated because the Fisher-Kaysen study is the best known and most comprehensive study of electricity demand to date. Our evaluation of this model can be tested through an assessment of its empirical results. Finally, to quote Anderson,

It is to be emphasized, however, that any particular choice [of functional form] must be somewhat arbitrary, since a well estimated equation is likely to be no more than a good approximation of a more complicated real-world relationship, and good approximation usually come in assorted shapes and sizes.⁴

It is to be expected, therefore, that if any faith is to be placed in the results of our efforts, some level of consistency exists among the results of the various models.

4.6 Conclusion

In this chapter the problems associated with trying to represent a rate schedule in an empirical model are discussed and two techniques are selected. The first technique is the "typical block" technique. The several different variations of this technique which are discussed will be tested. The second technique is the average price technique. The main virtue of the second technique is that the price variables are constructed completely independently of any sales information.

In addition, the models in which these techniques will be tested are presented. The problems in modeling electricity demand which lead to the selection of these models are explained, and several alternative models in which these techniques can be tried are suggested. Some of the models have been previously applied to data sets consisting of pooled samples of state data. These models will be useful for the purpose of comparing our results to those of previous studies.

Footnotes

¹See Lester Taylor and Gail Blattenberger, "The Residential Demand for Energy," Draft Report, July 1976.

²Excluding the study cited above, which is preliminary and unpublished.

³H. S. Houthakker and L. D. Taylor, Consumer Demand in the United States (Cambridge: Harvard University Press, 1970), p. 14-15.

⁴Kent P. Anderson, Residential Demand for Electricity: Econometric Estimates for California and the United States (Santa Monica: The Rand Corporation, January 1972), p. 7.

CHAPTER V

ESTIMATION OF THE MODEL

5.1 Introduction

In this chapter, I discuss the actual estimation of the electricity demand models which were specified in Chapter IV. I test the hypotheses specified in Chapter IV with regard to the characterization of rate schedules, and report the results of these tests. Also, I evaluate the models and the techniques of representing rate schedules within models according to the criteria which have been established.

The first topic of discussion in this chapter is the choice of estimation technique. This is discussed in Section 5.2. Although this technique is not novel, its use implies the existence of certain conditions and, thus, some justification is relevant.

The next three sections all concern the empirical results obtained employing the typical block technique to represent a rate schedule. Section 5.3 compares the results of different models, all using the same version of the typical block technique and evaluates these model specifications. Section 5.4 reports on the effect of variation in the typical block technique itself.

Finally, Section 5.5 evaluates Typical Electric Bill data by employing it in models using the typical block method and contrasting these results with those previously obtained.

Section 5.6 presents the results of using a double-log approximation to represent the actual average price rate schedule. Only two models and two methods of incorporating this rate representation into a model are used. Therefore, this topic is covered within one section.

Section 5.7 summarizes the conclusions of this chapter.

Throughout this chapter a different set of variable names is employed than in earlier chapters. This is done for two reasons. The multi-letter variable names in this chapter are abbreviations for the variable definitions. Thus, their meanings are readily understood in the estimated equations presented in this chapter. These names were not employed in earlier sections because they would have been unwieldy in mathematical derivations. Secondly, this switch in variable names distinguishes the actual data series employed in estimation from the theoretical constructs of earlier chapters. The variable names given to the actual data series and their definitions are summarized in Table 5.1. A more detailed description of the data calculation and data sources is given in Appendix 5.

TABLE 5.1

VARIABLE DEFINITIONS

SAL	Average monthly electricity consumption per customer	kwh/mo
MP	Marginal price of electricity	cents/kwh
FC	Implicit fixed charge for electricity consumption	\$/mo
YPC	Income per capita (average monthly)	\$/mo
PNG	Average price of natural gas to residential customers	\$/10 ⁶ BTU
HDD	Heating degree days	
CDD	Cooling degree days	
MP100T250 MP100T500 MP250T500	Average marginal prices as calculated from <u>Typical Electric Bills</u> (ex. MP100T250 is the average marginal price between 100 and 250 kwh.)	cents/kwh
FC100T250 FC100T500 FC250T500	Implicit fixed charge as calculated from <u>Typical Electric Bills</u> , associated with the average marginal price for the corresponding block	\$/mo
P1 P2	Approximation parameters of the fitted average price rate schedule	

5.2 The Choice of Estimation Technique

The data employed in this study are a pooled sample of time series and cross section observations. There are a variety of estimation techniques which have been applied to pooled data. The technique applied in this case is an error components technique designed by Balestra and

Nerlove.¹ My selection of this technique is briefly discussed in this section.

As a point of departure for this discussion and for the purpose of contrast later, I will begin with a simple linear model. This model is autoregressive since the models to be estimated are autoregressive. The simple linear autoregressive model commonly used in econometrics is stated,

$$(5.1) \quad y_{it} = \alpha + \beta x_{it} + \gamma y_{it} + \epsilon_{it}$$

where if

$$(5.2) \quad \epsilon' = (\epsilon_{11}, \epsilon_{12}, \dots, \epsilon_{1T}, \epsilon_{21}, \dots, \epsilon_{NT}),$$

$$\text{then } E(\epsilon\epsilon') = \sigma_{\epsilon}^2 I_{NT}.$$

In this model the dependent variable, y , is explained by x and lagged values of y as well as unknown or unmeasurable factors aggregated into a constant term, α , and an error term, ϵ . There are N regions and T periods in this model. Although individual unknown factors may fluctuate widely, it is assumed that there are many of these effects which cancel out such that the aggregate variation is small. There is no differentiation of any kind among the behavior of different regions in this model. Under the assumptions given above, ordinary least squares (OLS) is the appropriate technique to estimate the parameters.

In a pooled, cross-section-time-series model, the existence of unknown or unmeasured factors which are essentially constant over time becomes a possibility.² Climate is an example of such a factor. Although heating and cooling degree days are included in this study, all aspects of climate are not captured by these terms.

If time-invariant, regional factors are included in (5.1) this model becomes,

$$(5.3) \quad y_{it} = \alpha + \beta x_{it} + \gamma y_{it-1} + \varepsilon_{it} + \eta_i$$

Equation (5.3) can be estimated either under the assumption that η_i is a fixed constant, or the assumption that η_i is a regional error term. Which of these assumptions is made can have substantial effect on the estimated values of the parameters.

If it is assumed that η_i is a fixed constant, the correct method of approaching the model in (5.3) is to include a separate regional dummy variable for each region and apply least squares to obtain an estimate of $(\hat{\alpha} + \hat{\eta}_i)$ for each i . This technique is referred to as least squares with dummy variables (LSDV). Intuitively, this is inefficient because N degrees of freedom are used in estimating the values of each η_i . Since the values of η_i in most cases lack any economic interpretation, economists are not generally concerned with these values. The η_i are estimated because economists want their estimators of β and γ to have good properties. Finally, this

estimation technique requires that the distribution of the error term be the same in all regions.

An alternative method of approach to (5.3) is to assume that η_i is a random variable independent of ε such that,

$$(5.4) \quad E(\eta\eta') = \sigma_{\eta}^2 I_N$$

where η is the N-dimensional vector,

$$\eta' = (\eta_1, \dots, \eta_N) \quad .$$

In the simple linear model, the error term, ε_{it} , represents unmeasured or unmeasurable factors that vary over individuals and time. It is assumed that the aggregate effect of these factors is random. The η_i represent time-invariant, unmeasured or unmeasurable factors that vary by region. Off-hand there does not seem to be any reason to treat η differently from ε . Furthermore, in this approach it is only necessary to estimate the standard errors of ε and η rather than each individual η_i .

With the interpretation of η as a regional component of the error term, the model described in (5.3) becomes,

$$(5.5) \quad y_{it} = \alpha + \beta x_{it} + \gamma y_{it-1} + u_{it}$$

where the error term, u_{it} , is defined,

$$u_{it} = \varepsilon_{it} + \eta_i \quad .$$

The covariance matrix of this error term is defined,

$$E(uu') = \sigma^2 \Omega.$$

Given assumptions (5.2) and (5.4) regarding the structure of the error term, the covariance matrix is explicitly described by the conditions,

$$\sigma^2 = \sigma_{\varepsilon}^2 + \sigma_{\eta}^2 ,$$

$$(5.6) \quad \Omega = \begin{vmatrix} A & & & 0 \\ & \cdot & & \\ & & \cdot & \\ & & & \cdot & \\ 0 & & & & A \end{vmatrix}$$

$$A = \begin{vmatrix} 1 & \rho & \dots & \rho \\ \rho & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \rho & \dots & \rho & \cdot \end{vmatrix} \quad \text{and}$$

$$\rho = \sigma_{\eta}^2 / \sigma^2 .$$

Generalized least squares (GLS) is the best estimator of the parameters in equation (5.5) under the assumed error structure.³ In GLS a matrix T is determined such that

$$(5.7) \quad T'T = \Omega^{-1} \quad \text{and} \quad T\Omega T' = I .$$

The variables in (5.5) are transformed by the matrix T so that equation (5.5) becomes

$$(5.8) \quad Ty = \alpha T + Tx\beta + Ty_{-1}\gamma + Tu .$$

By (5.7)

$$(5.9) \quad E(Tuu'T') = \sigma^2 T\Omega T' = \sigma^2 I$$

and the transformed equation can be estimated by OLS. In this case, the matrix T is a function of ρ . If $\rho = 0$, $\Omega = I_{NT}$ and GLS reduces to OLS. If $\rho = 1$, the transformation, T , transforms the variables into deviations from regional means and GLS reduces to LSDV. Therefore, this model is in some sense a general model which subsumes both OLS and LSDV as polar cases. On this basis the approach of viewing the regional factors as a component of the error term seems superior to the other interpretations discussed.

Under the assumed error structure it has been noted that GLS is the "best" estimator and that GLS requires knowledge of ρ . A priori knowledge of ρ is unavailable in applied problems and some kind of feasible estimator must be employed.

One criterion for evaluating alternate feasible estimators is their asymptotic properties. Unless $\rho = 0$, it is known that the OLS estimator is both asymptotically biased and inconsistent. OLS may be ruled out of consideration on this basis. On the other hand, Amemiya shows that, for the model described above, if both N and T are allowed to approach infinity in any way, maximum likelihood estimators (ML), LSDV, and feasible GLS employing a consistent estimate of ρ are asymptotically

equivalent. All three of these estimators are consistent and asymptotically efficient.⁴ It is not possible to differentiate between these estimators in this way.

Small sample properties must be examined to determine the most generally preferable estimator in this case. Nerlove has performed Monte Carlo studies on a variety of estimators for precisely the model being discussed.⁵ Nerlove compares the results of different estimators on 6000 samples using a variety of combinations of parameter values for β , γ , and ρ . The estimation techniques were GLS given ρ for comparison purposes, OLS, LSDV, instrumental variables (IV), ML, and two two-round techniques applying GLS with a consistently estimated ρ from the first round. One of these estimators obtains $\hat{\rho}$ from the LSDV regression (2RDV), the other from IV (2RIV).

The results of Nerlove's experiments were that over a wide range of parameter values, 2RDV performs best both on the basis of bias and on the basis of mean squared error. OLS severely biases β down and γ up. LSDV severely biases β up and γ down. The bias for 2RDV is small but in the same direction as LSDV. The adequacy of the IV estimators is a function of the value of β . The ML estimator is erratic. This seems to be because the likelihood function is extremely flat in the direction of ρ , and boundary solution at $\rho = 0$ are frequently produced even when the true value of ρ is substantially above zero.

These results are fairly conclusive evidence for applying the 2RDV technique to models possessing the error structure assumed in this discussion.

Several questions remain to be dealt with regarding the choice of 2RDV as the estimation technique in this instance. First, how does this estimator behave when some of the assumptions of the model are violated and second, are the assumptions of the model violated in this particular case? Some evidence on the first question is available from Monte Carlo studies done by Johnson and Lyon.⁶ This evidence is discussed briefly below. Subsequently, some tests of the assumptions of this model are reported and their results examined.

The Monte Carlo experiments of Johnson and Lyon are less complete than Nerlove's since they examine only 50 data samples and one set of parameter values for each model. However, they examine several model specifications with differing error structures. One of these is a model like the one discussed above expanded to allow β to be a normally distributed random variable. Thus, the performance of 2RDV can be evaluated for a looser error structure than the one assumed thus far.

Johnson and Lyon test eleven different estimators on their data sets. Among these estimators 2RDV has the lowest bias and mean squared error for β , and is third for γ , performing better than Swamy's random coefficient

estimator. This evidence strengthens the argument for applying a 2RDV estimator to a pooled cross-section time series sample for a simple autoregressive model. Monte Carlo experiments have not been done for other violations in the assumed error structure of the model.

The theoretical and experimental evidence on pooling time series and cross sections has been presented, if only in summary form. It is relevant now to look at the sample at hand. A number of questions are examined with regard to the validity of the specification of the error term in the models estimated on this data set. First, the existence of a regional component of the error term is tested. Second, the inclusion of an additional time component to the error term is discussed. Third, the possibility of variation in coefficient values by region is examined. Finally, the presence of serial correlation in the error term is investigated.

All the theoretical results presented in the section assume that there is indeed a regional component to the error term. A test of this assumption can be made by analysis of covariance comparing the sum of squared residuals of an OLS regression and an LSDV regression. According to the hypothesis that there is no regional component to the error term,

$$(5.10) \quad \frac{(SSR_{OLS} - SSR_{LSDV})/N-1}{SSR_{LSDV}/NT-N-K}$$

is distributed as an F with $N-1$ and $NT-N-K$ degrees of freedom where K is the number of regressors other than the regional dummies.⁷ The test is made for all but the non-linear forms of the model. The results are presented in Table 5.2. In all the tests H_0 is rejected at the 1 percent significance level.

Although these results clearly indicate the presence of some kind of regional factor they do not say anything about the nature of η . In particular, the assumption that,

$$E(\eta_i \eta_j) = 0$$

seems rather strong given the arbitrariness of political boundaries. To the extent that η represents climatological factors, this assumption is strong. The assumption is weakened somewhat by the fact that the regional areas represented are fairly large. To the extent that η includes political factors, the assumption is further weakened. Analysis of variance could be performed on the estimated values of η_i to test if the values of η are correlated among neighboring states. This has not been done at this time.

TABLE 5.2
TESTS OF THE ERROR STRUCTURE OF THE ESTIMATED MODELS

Test	Regional Dummy	Time Dummy	Variable Coefficient	Durbin Watson Average Value	No. of States in Critical Region
Type	F ⁴⁷ ₇₆₁	F ¹⁷ ₇₄₄	F ⁵⁵ ₇₀₆		
Critical Value	1.59(1%)	2.02(1%)	1.54(1%)		
Model:					
Typical block logarithmic					
HT	2.91	6.97	.22	2.02	1
MCT	4.06	6.38	.55	2.01	1
HVS	3.83	6.49	.56	1.99	0
FK	3.37	7.33	.33	2.04	0
Linear					
HT	2.46	8.64	1.04	1.80	2
HVS	3.37	8.83	1.80	1.63	4
Average price method					
HT	3.12	6.25	.22	2.08	0
HVS	3.99	5.89	.52	2.04	0

The estimation procedure allows for a regional component to the error term, but disregards the possibility of a time component. Such a time component could be treated identically to the regional component. The existence of such a factor can be tested by a similar test to the one above. In this test the sum of squared residuals are compared from two LSDV regressions, one of which, SSR_N , includes only N regional constants, and the other of which, SSR_T , includes $T-1$ time dummies also. Under the hypothesis H_0 , that there is no time component to the error term but there is a regional component,

$$\frac{SSR_N - SSR_T / T-1}{SSR_T / NT - N - K - T}$$

is distributed as an F with $T-1$ and $NT-N-K-T$ degrees of freedom. Again, the null hypothesis is rejected at the 1 percent level in all cases.

A caveat is in order about including too many dummy variables. Although dummy variables may explain fully the variation in the dependent variable, this explanation is likely to be devoid of any interpretation and not worthwhile. In addition, if these dummy variables are colinear with some of the economic variables they may detract from the significance of the economic effects. In

this case, the time coefficients might represent the average national consumption by year and the regional dummy the average deviation of any state from the national average. Although the fit may be good in this case, any causal interpretation is weak. On this basis, a time dummy is excluded from the analysis.

Although there is some evidence in Johnson and Lyon that 2RDV performs adequately in the presence of random coefficients, a further assumption of the estimation technique is that the variable coefficients are constant across regions. This assumption has not been thoroughly tested at this time. Some interesting work might be done on this issue in the future. Only one hypothesis in this area has been tested at this time. The sample was divided into two parts, one part consisting of those states in which at least 10 percent of the customers have all-electric homes, the second part consisting of the remaining states. The combined sum of squared residuals from LSDV regressions on each of these samples was compared to the sum of squared residuals from an LSDV regression on the entire sample. The results of this F-test are reported in Table 5.2. These results indicate no substantial difference between the two samples.

The use of a feasible GLS technique depends on the assumption of a scalar covariance matrix for the transformed error term as given in equation (5.9). The derivation of the transformation matrix in turn depends on

the assumptions of scalar covariance matrices for the error components as given in (5.2) and (5.4). The assumption of independence of the regional component of the error term has been discussed. Autocorrelation in either the non-regional component of the error term, ϵ , or the entire transformed error term, Tu , would cause the estimation in an autoregressive model to be both biased and inconsistent.

To test for autocorrelation in the transformed error term a Durbin-Watson statistic could be calculated for each state using the estimate of the transformed error term, Tu . This is not done in the regression package employed here. A test is made for autocorrelation in the non-regional component of the error term. Given the estimated coefficients, η_i is estimated for each region,

$$(5.11) \quad \hat{\eta}_i = \bar{y}_i - \bar{\alpha} - \beta \bar{x}_i - \gamma \bar{y}_{i-1} \quad .$$

Given η_i , ϵ_{it} is estimated for each observation in the sample,

$$\hat{\epsilon}_{it} = y_{it} - \alpha - \beta x_{it} - \gamma y_{it-1} - \hat{\eta}_i \quad .$$

A Durbin-Watson statistic is then calculated for each state for the error component ϵ_{it} . The results of this test are presented in Table 5.2 in summary form. The two statistics reported are the average value of the Durbin-Watson over all states for each model, and the number of states for which the Durbin-Watson falls into

the critical region. Since there are only 17 observations for each state, the indeterminant region for the Durbin-Watson is rather large and most states fall within this region.

There are many more interesting tests which could be performed on this data set with regard to the error structure. The evidence at this time indicates that the assumed error structure of 2RDV is not unreasonable for the models estimated. It is also the best available technique to deal with the data.

5.3 A Comparision of Models Using the Typical Block Technique

The typical block method of representing a rate schedule was empirically tested in a variety of ways. Experiments were conducted on the model specification, on the required data base, and on the definition of the typical block technique itself. For the sake of clarity, these topics are addressed in separate sections. This section will discuss the comparative merits of various model specifications in fitting the actual data. For the purpose of this discussion, the marginal block is defined by the average consumption level and the implicit fixed charge is defined at zero consumption units.

The discussion of models in this section makes it possible to dispense with model specifications which are

inappropriate for one reason or another. The first model discussed is the Fisher-Kaysen model. This model is discussed in some detail because of its prominence in the literature, but it is rejected for a variety of reasons. Second, the two linear models are discussed. These are rejected on the basis of unreasonable coefficient values. Finally the three forms of the log-linear, flow-adjustment models are presented. These models yield highly acceptable results and the remaining analysis concentrates on these three models. Although this model evaluation considers only one variation of the rate schedule characterization, it is representative of the other results obtained.

The models in this section are evaluated on a number of criteria. Clearly, the overall fit of the model is relevant, as is the significance of the coefficients. In addition, the signs of the coefficients and their magnitudes are evaluated on reasonableness and agreement with theoretically determined results.

Income effects and electricity price effects have been discussed in detail in Chapters III and IV. Electricity price effects are assumed to be negative and income effects positive. No a priori judgment has been made on whether these coefficients are larger or smaller than one. However, very large elasticities are viewed with suspicion. In addition, the relative values of $dSAL/dYPC$, $dSAL/dMP$, and $dSAL/dFM$ are investigated for consistency in light of the results of Section 3.5B.

The remaining explanatory variables in the model are the price of natural gas, two weather variables, and in some cases the lagged dependent variable. The signs of the coefficients on these variables have not been discussed. A priori one would expect the price of a substitute good to have a positive effect on electricity demand. Extreme weather, either hot or cold, is also expected to have a positive effect on electricity demand. The coefficient of the lagged dependent variable is a function of the adjustment parameter θ (for example, see equation (4.12)). By definition, this coefficient should be positive and less than one if demand effects are to be stable. Furthermore, if this coefficient is too close to one, demand effects damp out very gradually and unreasonable long-run elasticities result.

The estimation equations for all the models considered in this section are given in Table 5.3. The results of the estimation are reported in Table 5.4 for the logarithmic forms of these models and in Table 5.5 for the linear forms of the models. The same set of coefficient names is given for each equation. It should be remembered that although these coefficient values are comparable, they are not the same.

The most noticeable feature of these results at first glance is the extremely poor fit of the FK model. This is not so much because of the low R^2 , which is to be expected

TABLE 5.3

ESTIMATING EQUATIONS FOR THE TYPICAL BLOCK METHOD

Logarithmic Models

$$\begin{aligned} \text{HT: } \ln \text{ SAL} = & b_0 + b_1 \ln \text{SAL}_{-1} + b_2 (\ln \text{YPC} + \ln \text{YPC}_{-1}) \\ & + b_3 (\ln \text{MP} + \ln \text{MP}_{-1}) + b_4 (\ln \text{FC} + \ln \text{FC}_{-1}) \\ & + b_5 (\ln \text{PNG} + \ln \text{PNG}_{-1}) + b_6 (\ln \text{HDD} + \ln \text{HDD}_{-1}) \\ & + b_7 (\ln \text{CDD} + \ln \text{CDD}_{-1}) \end{aligned}$$

$$\begin{aligned} \text{HVS: } \ln \text{ SAL} = & b_0 + b_1 \ln \text{SAL}_{-1} + b_2 \ln \text{YPC} + b_3 \ln \text{MP} \\ & + b_4 \ln \text{FC} + b_5 \ln \text{PNG} + b_6 \ln \text{HDD} + b_7 \ln \text{CDD} \end{aligned}$$

$$\begin{aligned} \text{MCT: } \ln \text{ SAL} = & b_0 + b_1 \ln \text{SAL}_{-1} + b_2 \ln \text{YPC} + b_3 \ln \text{MP} \\ & + b_4 \ln \text{FC} + b_5 \ln \text{PNG} + b_6 \ln \text{HDD} + b_7 \ln \text{CDD} \\ & + b_8 \cdot 1/\text{MP} \end{aligned}$$

$$\begin{aligned} \text{FK: } \ln \text{ SAL} - \ln \text{SAL}_{-1} = & b_0 + b_2 (\ln \text{YPC} - \ln \text{YPC}_{-1}) + b_3 (\ln \text{MP} - \ln \text{MP}_{-1}) \\ & + b_4 (\ln \text{FC} - \ln \text{FC}_{-1}) + b_5 (\ln \text{PNG} - \ln \text{PNG}_{-1}) \\ & + b_6 (\ln \text{HDD} - \ln \text{HDD}_{-1}) + b_7 (\ln \text{CDD} - \ln \text{CDD}_{-1}) \end{aligned}$$

Linear Models

$$\begin{aligned} \text{HT: } \text{SAL} = & b_0 + b_1 \text{SAL}_{-1} + b_2 (\text{YPC} + \text{YPC}_{-1}) + b_3 (\text{MP} + \text{MP}_{-1}) \\ & + b_4 (\text{FC} + \text{FC}_{-1}) + b_5 (\text{PNG} + \text{PNG}_{-1}) + b_6 (\text{HDD} + \text{HDD}_{-1}) \\ & + b_7 (\text{CDD} + \text{CDD}_{-1}) \end{aligned}$$

$$\begin{aligned} \text{HVS: } \text{SAL} = & b_0 + b_1 \text{SAL}_{-1} + b_2 \text{YPC} + b_3 \text{MP} + b_4 \text{FC} + b_5 \text{PNG} \\ & + b_6 \text{HDD} + b_7 \text{CDD} \end{aligned}$$

TABLE 5.4
RESULTS OF THE ESTIMATION OF THE LOGARITHMIC MODELS
(t-statistics are given in parentheses)

Coefficient	Model			
	HT	MCT	HVS	FK
b ₀ Constant Term	-.00840 (-.15)	-.0629 (-1.32)	-.102 (-2.21)	.0288 (21.65)
b ₁ Lagged Sales	.912 ⁺ *** (77.83)	.903 ⁺ *** (79.61)	.911 ⁺ *** (81.64)	
b ₂ Income	.0482 ^{***} (4.64)	.0949 ^{***} (4.79)	.0936 ^{***} (4.72)	.0174 (.49)
b ₃ Marginal Price	-.0367 ^{***} (-4.57)	-.149 ^{***} (-5.45)	-.072 ^{***} (-4.91)	-.0627 ^{**} (-2.68)
b ₄ Fixed Charge	-.00759 [*] (-1.79)	-.0166 [*] (-2.14)	-.1022 (-1.59)	-.00478 (-.38)
b ₅ Natural Gas Price	.00111 (.208)	.00326 (.30)	-.000384 (-.035)	-.0768 ^{***} (-3.11)
b ₆ Heating Days	.00702 (.82)	.0662 ^{***} (4.80)	.0658 ^{***} (4.78)	.0766 ^{***} (6.10)
b ₇ Cooling Days	.00690 (1.51)	.0312 ^{***} (4.79)	.0321 ^{***} (4.91)	.0211 ^{***} (4.66)
b ₈ Reciprocal of Marginal Price		-.135 ^{***} (3.33)		
R ²	.990	.991	.991	.087

⁺Significantly less than one at the .01 level.
^{*}.001 level of significance.
^{***}Significant at the .001 level.
^{*}Significant at the .05 level.

TABLE 5.5
RESULTS OF ESTIMATION OF THE LINEAR MODELS
(t-statistics are given in parentheses)

Coefficient	Model	
	HT	HVS
b ₀ Constant Term	8.49 (1.37)	-14.0 (-2.67)
b ₁ Lagged Sales	.981 ⁺ (105.11)	.988 ⁺⁺ (115.45)
b ₂ Income	.0652 ^{***} (3.64)	.111 ^{***} (3.64)
b ₃ Marginal Price	-5.34 ^{***} (4.26)	-10.9 ^{***} (-4.66)
b ₄ Fixed Charge	-.207 ^{***} (-.46)	-.0433 ^{***} (-.05)
b ₅ Natural Gas Price	.000229 (.23)	.00139 (.72)
b ₆ Heating Days	-.00612 (.474)	.979 ^{***} (5.23)
b ₇ Cooling Days	-.00813 (-.26)	.319 ^{***} (6.90)
R ²	.989	.989

⁺Not significantly less than one at the .01 significance level.

⁺⁺Not significantly less than one at the .1 significance level.

^{***}Significant at the .001 level.

in first differences, as it is from the signs and significances of the coefficients. The most important predictor of the growth rate of demand is a constant. In addition to the constant, the most significant predictor of demand is weather. The price of natural gas is also very significant, but it has the wrong sign. Neither income nor the implicit fixed charge is at all significant.

One problem with this model is that electricity prices varied more cross-sectionally than over time in the sample period. This information is suppressed in first differences and might have some import in explaining regional differences in the growth in demand.

The major reason Fisher and Kaysen employ 1st differences is that it enables them to dispense with the stock of appliances under the assumption that appliances grow at a uniform rate. No explanation is given as to why the stock of appliances is growing at a constant rate or how this rate is determined. Implicit in the assumption of a constant growth rate of the appliance stock is an effectively infinite elasticity of demand with respect to the utilization rate or any determinant of the utilization rate in the long run. Thus, a change in marginal price results in a once and for all change in the utilization rate. However, because the appliance stock is growing, this has an undiminished effect on the growth in demand. In a more realistic model, it would seem

reasonable for marginal price to have an initial impact through the utilization rate and a long-run impact on the stock of appliances itself. Predictions based on a model claiming that the stock of appliances will continue to grow at some constant rate are suspect.

Fisher and Kaysen give several additional justifications for the use of first differences. One of these is that first differences eliminates any trend effect from the dependent variable and, thus, yields more "honest" results. It seems more reasonable to the author to estimate such a relationship if it exists than to assume it to be of a very specific form. Secondly, first differences are employed to reduce multicollinearity. However, multicollinearity does not seem to be a serious problem in this data set. A correlation matrix for the variables is given in Table 5.6. A potential problem with multicollinearity among independent variables only arises in the case of HDD and CDD. However, since weather relationships are fairly stable, this does not present problems in forecasting. A third additional justification for employing first differences is that this eliminates some of the simultaneity problems caused by the use of average price. This problem has been eliminated from the present analysis by the choice of price variable.

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TABLE 5.6
A CORRELATION MATRIX FOR THE LOGARITHMIC REGRESSION VARIABLES¹

	SAL	SAL ₋₁	YPC	FC	MP	PNG	HDD	CDD
SAL	1.0000							
SAL ₋₁	0.9969	1.0000						
YPC	0.3552	0.3616	1.0000					
FC	0.2962	0.2965	-0.1170	1.0000				
MP	-0.8318	-0.8300	-0.1474	-0.3563	1.0000			
PNG	-0.0960	-0.0915	0.0447	0.0299	0.2389	1.0000		
HDD	-0.1783	-0.1596	0.2385	-0.1788	0.2481	0.0238	1.0000	
CDD	0.0627	0.0403	-0.2643	0.1928	-0.0862	-0.2779	-0.7718	1.0000

¹All variables are in their logarithmic forms.

In spite of the criticisms made above, it should be noted that the fit obtained with this version of the FK model and this data set is substantially better than that obtained by Fisher and Kaysen. Fisher and Kaysen employed average price to represent the rate schedule. In addition, they omitted the price of natural gas and the weather variable from the model.

The constant term reported in the results in Table 5.4 is the constant term from the transformed regression equation (5.8). The average value of the constant term for each state including the regional component of the error term is .058. (See equation (5.11) for calculation of the regional component of the error term.) This value is comparable to the average constant term estimated by Fisher and Kaysen of .1111. In Fisher and Kaysen's results the constant term was the only variable generally significant for all states.

In contrast to Fisher and Kaysen's results, a significant price effect was obtained for the marginal price variable. Fisher and Kaysen obtained price effects significant at the 5 percent level for only 15 out of 47 states. The short-run marginal price elasticity obtained in this version of the FK model is consistent with those of the other models estimated.

The linear models are eliminated from contention with only a cursory examination of their fit. The problem in

the linear models lies in the size of the coefficients of the lagged dependent variable. The value of the coefficient in each case is too close to one and this results in unreasonably large long-run elasticities. The price and income elasticities for both of these models is given in Table 5.7. The unrealistic values for the long-run elasticities make it impossible to consider seriously the fit of these models.

TABLE 5.7
PRICE AND INCOME ELASTICITIES FOR THE
LINEAR MODELS

Variable	Model			
	HT		HVS	
	Short Run	Long Run	Short Run	Long Run
YPC	0.0825	4.342	0.696	5.800
MP	0.0580	3.053	0.0597	4.978
FC	0.00133	0.0705	0.0002	0.0169

Finally, I shall address the remaining three logarithmic models. All these models are variations on flow adjustment models with discrete or continuous adjustment processes. Of these models, MCT with variable elasticity for marginal price only perhaps fits best overall. However, these results are not conclusive and the features

of all these models are similar. Therefore, they will be discussed together.

The first things to be examined in these models are the electricity price and income effects. The signs and significance levels of these coefficients are extremely good. The only exception is the fixed charge, FC, and even that is significant at the 5 percent and 7 percent levels in the HT and HVS models respectively, and at the 2 percent level in the MCT model. In addition, the magnitudes of the short-run and long elasticities are consistent with the theoretical conditions derived in Chapter III. These results are presented in Table 5.8.

The long-run elasticities are dependent on the coefficient of the lagged dependent variable. In each case, the coefficient of this variable is significantly less than one,⁸ which damps the effect of variable shifts on the level of demand in subsequent periods. This stability in demand supports our confidence in the values of the long-run elasticities. In addition, there is consistency among elasticity estimates obtained in each of the separate models. This also lends credence to their values.

Although the point estimate for the long-run elasticity for the marginal price variable is less than one in absolute value in each of these models, it should be noted that none of these estimates are significantly less than one. This is tested by the following procedure as applied to the HVS model. The long-run marginal price

TABLE 5.8
ELASTICITIES AND DERIVATIVES FOR PRICE AND INCOME
IN THE LOGARITHMIC MODELS

	Model		
	HT	MCT	HVS
<u>Elasticities</u>			
YPC			
Short run	0.0962	0.0949	0.0935
Long run	1.093	0.978	1.050
MP			
Short run	-0.0734	-0.0925	-0.0721
Long run	-0.8340* (1.09)	-0.9530* (0.13)	-0.8101* (1.27)
FC			
Short run	-0.0152	-0.0166	-0.0123
Long run	-0.1727	-0.1711	-0.1382
<u>Derivatives</u>			
dSAL/dYPC	0.157	0.150	0.149
dSAL/dMP	-1351.48	-1703.16	-1327.54
dSAL/dFC	-1.542	-1.68	-1.248

* Not significantly less than one at the 10 percent level of significance. The value in parentheses is the t-statistic from this test.

elasticity for the HVS model is calculated:

$$\eta_{MP} = b_3/1-b_1 \quad .$$

If

$$b_3/1-b_1 > -1 \quad , \quad \text{then}$$

$$b_3-b_1+1 > 0 \quad .$$

The hypothesis,

$$H_1: \quad b_3-b_1+1 > 0$$

can then be compared to the null hypothesis,

$$H_0: \quad b_3-b_1+1 = 0 \quad ,$$

in a one-tailed t-test. The null hypothesis in this case is equivalent to a long-run elasticity of one. This hypothesis is not rejected for each of the models under consideration.

It is not only the sign and the sizes of these elasticities that is important but their relative values as well. It was established that,

$$(5.12) \quad dx_i/dt = dx_i/dy$$

for individual consumers. This does not translate into

$$(5.13) \quad dSAL/dFC = -dSAL/dYPC$$

for a number of reasons which have been discussed. To reiterate, first the data employed here is aggregated over an entire state. Second, FC is not the average

expenditure up to the marginal block for all individuals. It represents only one block on each rate schedule, and all of a utility's customers are not within that one block. Third, even for individuals, (5.12) is only valid for consumption within the marginal block and individual's consumption blocks change. Finally, since block demarcations are not consistent from one rate schedule to another and consumption blocks change, FC is defined as the expenditure up to some fixed point. For these reasons it is not expected that (5.13) will hold in the estimated models.

On the other hand, if the income effect is assumed to be relatively constant across consumers, it was shown in Chapter III that if,

$$(5.14) \quad d\bar{x}_i/d\bar{r}_j \gtrless (k_j - k_{j-1})d\bar{x}_i/d\bar{y} \text{ for all } j < J, \text{ then} \\ d\bar{x}_i/d\bar{t} \gtrless -d\bar{x}_i/d\bar{y} \quad .$$

This hypothesis can be tested with these models.

The derivatives $dSAL/dYPC$, $dSAL/dMP$, and $dSAL/dFC$ are given in Table 5.8 for the models being discussed. They are evaluated at the mean variable levels. The derivative with respect to marginal price has been scaled up to make the units comformable. MP was measured in cents whereas both YPC and FC are measured in dollars. A reasonable upper bound for $(k_j - k_{j-1})$ is 200 kwh. Clearly,

$$dSAL/dMP < -200 \ dSAL/dYPC$$

for the mean consumption block in all the models. This is consistent with the fact that,

$$dSAL/dFC < - \ dSAL/dYPC$$

in all the models. Although there would be plausible justifications for this result not holding, it is reassuring to find the results consistent with our a priori beliefs.

In examining the signs and significance of the coefficients, it is apparent that the coefficient on the price of natural gas is at best insignificant with the right sign. At worst (in the FK model) it is significant with the wrong sign. The problem with the natural gas price can be attributed to a number of causes. Two of these stand out. First, the natural gas price included in this model is an average price. Since natural gas customers face rate schedules, natural gas prices really should be dealt with in the same way as the electricity rates in this study. The necessary data compilation was too major an undertaking to deal with at this time. Secondly, the availability of natural gas is not uniform across states. This fact has not been dealt with at all in these models. For these reasons, it is not surprising that weak results were obtained for natural gas.

Mixed results were obtained for the weather variables. The results are much stronger for the models with a discrete adjustment process than for the models in which a continuous adjustment process was employed. The approximation used in the continuous adjustment model to convert all the variables into discrete units is the following:

$$x_t - x_{t-1} = \frac{1}{2}(\Delta^* x_t + \Delta^* x_{t-1}) \quad .$$

This approximation is better the shorter the time period and the less fluctuation in x . The time period in this model is a year and, clearly, within a year there is more fluctuation in weather than in prices or income. The HT model would probably work better if a shorter time unit were employed. In both the HVS and MCT models, both HDD and CDD are highly significant and of the right sign.

In conclusion, all the log-linear, flow-adjustment models fit the data extremely well. The coefficients of these models are in accord with predetermined beliefs as to their relative and absolute values. In addition, the results of each of these models are consistent with the others. Long-run price elasticities less than one in absolute value were obtained in contrast to the results in Table 2.1. However, these values were not significantly less than one.

5.4 Variations on the Typical Block Technique

The typical block technique, as defined in Section 4.2, provides a general framework for characterizing a rate schedule in an econometric model. A variety of definitions of actual rate variables is consistent with this general framework. Some variations in the definition of the typical block variables are explored in this section.

The first topic of investigation in this section is the choice of the typical block itself. All the definitions of the typical block employed are proxies for the modal block of the rate schedule. The derivation of these definitions was discussed in detail in Chapter IV.

Three definitions of the typical block are employed in this investigation. The typical block is defined by the average consumption level, the average expenditure level, and the average price level. The average price level provides the highest estimate of the modal block; the average expenditure level the lowest estimate of the modal block. The average consumption definition was employed in the last section. A priori the modal block is best approximated by either the block defined by the average expenditure level or the block defined by the average consumption level. The results of fitting the HVS model to data employing the three definitions of the modal block are reported in Table 5.9. These results are representative of results obtained for other models.

TABLE 5.9
A COMPARISON OF ALTERNATIVE METHODS OF SELECTING
THE TYPICAL BLOCK
(Logarithmic HVS Model)

Coefficient	Method of Selecting the Typical Block		
	Average Expenditure	Average Consumption	Average Price
b_0 Constant	-0.100 (-2.31)	-0.102 (-2.21)	-0.177 (-2.61)
b_1 Lagged Sales	0.916 (82.29)	0.911 (81.64)	0.912 (86.58)
b_2 Income	0.0970 (4.86)	0.0936 (4.72)	0.0942 (4.85)
b_3 Marginal Price	-0.0472 (-3.67)	-0.0721 (-4.91)	-0.0652 (-5.01)
b_4 Fixed Charge	-0.00223 (-0.29)	-0.0122 (-1.59)	-0.00319 (-0.47)
b_5 Natural Gas Price	-0.0102 (-0.94)	-0.000385 (-0.03)	-0.0011 (-0.10)
b_6 Heating Days	0.0666 (4.80)	0.0658 (4.78)	0.0692 (5.04)
b_7 Cooling Days	0.0339 (5.18)	0.0321 (4.91)	0.0322 (4.97)
R^2	0.991	0.991	0.991
Elasticities			
YPC			
Short run	0.0970	0.0935	0.0942
Long run	1.155	1.050	1.070
MP			
Short run	-0.0472	-0.072	-0.0652
Long run	-0.562	-0.810	-0.741
Derivatives			
$dSAL/dYPC$	0.153	0.149	0.149
$dSAL/dMP$	-869.07	-1327.54	-1200.49
t^*	(3.47)	(4.73)	(4.86)

* The t-statistic tests the hypothesis $dSAL/dMP = -200 dSAL/dYPC$ as compared to the alternative hypothesis $dSAL/dMP < -200 dSAL/dYPC$.

Varying the definition of the typical block in this way only affects the coefficients of the electricity rate variables. The coefficient of FC is most significant for the average consumption definition, but of marginal significance in any case. The coefficient of MP is highly significant for all the equations but peaks in absolute value for the average consumption definition.

It is of interest to consider several explanations for the pattern of coefficient values for the marginal price variable. Since marginal price is not fixed for all customers it is not surprising that the marginal price effect should vary depending on which marginal price is chosen. Two further explanations are consistent with the particular pattern of price effects obtained in these equations.

In Section 3.5B, it was argued that if there is a modal consumption block and if $dSAL/dMP$ is sufficiently larger than $dSAL/dYPC$ in absolute value (3.16), then $dSAL/dMP$ will peak in the modal consumption block. There undoubtedly exists a modal consumption block, and $dSAL/dMP$ is significantly less than $-dSAL/dYPC$ in all the versions of the definitions for the typical block (See Table 5.9 for test values.) This indicates that the average consumption level best approximates the modal block.

On the other hand, it might be argued that a greater degree of ex post simultaneity exists between the dependent

variable and the marginal price variable when it is defined by the average consumption level than when the other determinations of the typical block are used. Some simultaneity is present with any ex post price including all those presented here. This simultaneity has been minimized through the use of marginal price instead of average price.

Undoubtedly both of these arguments have some validity with regard to the empirical data. To the extent that simultaneity exists, the price effect is biased upward (in absolute value). Since a greater degree of simultaneity exists for the average consumption definition of the typical block, the bias is the larger in this case. However, the pattern of price effects observed empirically is also consistent with the mathematical derivations of Section 3.5B. On this basis the average consumption definition of the typical block is selected as the best approximation to the typical block.

The second topic of investigation in this section is the definition of the implicit fixed charge. The fixed charge variable is defined by equation (4.5). It defines an expenditure for a given consumption level such that, if a constant marginal price is charged for all subsequent units, the cost in the marginal block is accurately represented. This variable is designed to represent intra-marginal cost.

The intra-marginal cost employed in the mathematical results of Chapter III was the expenditure up to the marginal block demarcation. This was rejected as the intra-marginal cost variable in the econometric model because the marginal block demarcation varies. Thus, this cost variable would be more indicative of the variation in the block demarcation than of variation in actual cost. For this reason a fixed point was chosen as the marginal block demarcation.

In all the results presented so far, the fixed charge is defined at zero consumption units. This is the first part of Gabor's two-part tariff equivalent to the typical block of a multi-block rate schedule. Given the definition of this variable (equation (4.5)), it clearly incorporates much of the marginal price effect. This should bias the elasticity of sales with respect to FC towards zero.

The bias involved in defining FC at zero consumption units should be reduced by selecting a consumption level closer to the marginal block demarcation. In the second definition, FC was defined at the average level of the marginal block demarcation among states within each year. In this way the effect of marginal price changes on FC was limited as much as possible, while some but not all of the variation in the marginal block demarcation was removed. This experiment did not work and its results are not reported. In this case, the variation in average

block demarcation over time biases the coefficient of FC and positive coefficient values are obtained.

Based on the above results, a final experiment on the definition of the FC was designed. In this case, FC was defined for a fixed consumption level as in the other results reported. However, the consumption level chosen was the average level of the marginal block demarcation for the entire sample, 226.13 kwh. The results of this experiment are presented in Tables 5.10 and 5.11 for the logarithmic flow-adjustment models.

Several differences can be noted between these results and the results of defining FC at zero consumption units. The most important effect of this experiment is that FC is significant at the .01 level with the new definition while the significances of the other variables are not appreciably altered. In addition, the natural gas price coefficient is at least of the right sign if not significant in these equations. The relative values of the price and income derivatives also remain consistent with a priori results as discussed in the earlier set of equations.

TABLE 5.10

ESTIMATION RESULTS DEFINING THE FIXED CHARGE AS THE
EXPENDITURE UP TO 226 kwh
(t-statistics in parentheses)

Coefficient	Model		
	HT	MCT	HVS
b ₀ Constant Term	0.0161 (0.29)	-0.0587 (-1.22)	-0.0824 (-1.74)
b ₁ Lagged Sales	0.908 ⁺ (76.57)	0.902 ⁺ (79.06)	0.908 ⁺ (80.55)
b ₂ Income	0.0471 (4.63)	0.0965 (4.92)	0.0918 (4.71)
b ₃ Marginal Price	-0.0295 (-4.51)	-0.122 (-4.99)	-0.0609 (-5.00)
b ₄ Fixed Charge	-0.0229 (-2.71)	-0.0342 (-2.24)	-0.0367 (-2.39)
b ₅ Natural Gas Price	0.00242 (0.45)	0.00433 (0.40)	0.00203 (0.19)
b ₆ Heating Days	0.00669 (0.79)	0.0656 (4.75)	0.646 (4.71)
b ₇ Cooling Days	0.00692 (1.53)	0.0311 (4.77)	0.0317 (4.87)
b ₈ Reciprocal of Mar- ginal Price		-0.116 (-2.89)	
R ²	0.991	0.991	0.991

⁺Significantly less than one at the .001 significance level.

TABLE 5.11
PRICE AND INCOME ELASTICITIES WHEN THE FIXED
CHARGE IS DEFINED AT 226.12 kwh

	Model		
	HT	MCT	HVS
YPC			
Short run	0.0942	0.0965	0.0918
Long run	1.024	0.985	1.018
MP			
Short run	-0.0590	-0.0735	-0.0609
Long run	-0.641*	-0.746	-0.675*
t	(-2.81)	(-0.64)	(-2.64)
FC			
Short run	-0.0458	-0.0342	-0.0367
Long run	-0.498	-0.349	-0.407

* Significantly less than one at the .01 level of significance. The t-statistic for this test is shown in parentheses.

One further observation on the results of this definition of FC is in order. In two of the models estimated, the long-run marginal price elasticity is significantly less than one. No a priori belief as to the value of this elasticity was specified and this result does not condition the acceptance of the FC definition. It is nevertheless an important result. As Table 2.1 illustrates, long-run price elasticities less than one are obtained infrequently. In light of this present study,

it appears that past results on price elasticities have been caused by misspecification of the rate variables and omitted variables bias.

Two final experiments were conducted on the typical block method and they shall be briefly discussed. Each of these experiments extends the typical block method to a more generalized framework.

One experiment conducted includes three blocks in the models instead of one typical block. A marginal price and a fixed charge variable is included for each of these blocks. Not surprisingly the significance of individual price effects is reduced and wrong signs are obtained in some cases. This is attributable to multicollinearity among the rate variables used.

A second experiment eliminates the fixed charge variable altogether and substitutes a variable defined as the cost of 1500 kwh. in its place. This variable reflects any overall change in a rate schedule. Although the coefficient of this variable is significant and of the right sign, its value is hard to interpret. In addition, it includes marginal rate changes as part of its measured effect. Consequently, the marginal price effects obtained in this experiment are reduced.

In conclusion, the selection of the best representation of a rate schedule within the framework of the typical block method is based on the experiments in this section. In accordance with these empirical results, the

typical block is defined by the average consumption level and the fixed charge is defined at 226 kwh. These conclusions are appropriate for the particular data set investigated and are not general conclusions.

5.5 Typical Electric Bill Data

Summary information about rate schedules is contained in a Federal Power Commission publication called Typical Electric Bills. From this publication, the average marginal rates between fixed points on a rate schedule can be calculated. This data is more readily tabulated than actual rate data. Therefore, it is useful to investigate whether or not rate data is adequately represented in econometric models by Typical Electric Bills (TEB) data. This is the topic of this section.

In Typical Electric Bills, the FPC publishes the cost of an electric bill of set sizes in communities across the country. The sizes of these bills are 100 kwh., 250 kwh., and 500 kwh. In more recent years, 750 kwh. and 1000 kwh. bills have also been published. DRI's energy databank contains series which are weighted averages of these bills within each state for selected communities. Some ad hoc adjustments were made in these series to eliminate incongruous results.

From the DRI series average marginal price variables were calculated for each state between 100 and 250 kwh.,

between 100 and 500 kwh., and between 250 and 500 kwh. These series are called MP100T250, MP100T500, and MP250T500 respectively. Correspondingly, fixed charge variables FC100T250, FC100T500, and FC250T500 are calculated. These series are available from 1960 to 1972.

The three logarithmic flow adjustment models are estimated for the period 1961 to 1972 employing the TEB variables. These results are compared to the results of the same models estimated over the same period using actual rate variables. The estimation results for the HVS model are given in Table 5.12. This model is selected because the best results are achieved with this model.⁹

During the period 1961 to 1972, the average value of the marginal block demarcation for the entire sample is 255.12. In light of the results of Section 5.4, MP is defined for the average consumption level and FC is defined at 255.12 kwh. For comparability in the TEB representation, the fixed charge variable is defined as the typical bill for the lower bill used in calculating marginal rates.

An average value for the marginal block demarcation of 255.12 kwh. indicates that the TEB variable most indicative of the marginal price ought to be MP250T500. However, in the calculation of TEB data, the FPC employs hot water heating rates when they are available for the 250 to 500 kwh. block. There is no reason to assume that

hot water heating is the marginal electricity usage of consumers in the 250 to 500 kwh. block.

The actual correlation between MP and the TEB marginal price variables is given by the following correlation matrix:

	<u>MP</u>	<u>MP100T250</u>	<u>MP100T500</u>	<u>MP250T500</u>
MP	1.0000			
MP100T250	0.7495	1.0000		
MP100T500	0.6340	0.8444	1.0000	
MP250T500	0.3975	0.5279	0.9008	1.0000

On the basis of this empirical evidence, it appears that MP250T500 is biased by the use of hot water heating rates. MP100T250 is the best proxy for MP on this empirical evidence.

Among the models estimated with TEB data, the closest approximation to the model with actual rate data is the model employing the 100 to 500 kwh approximation. In general, the TEB data biases the income effect down slightly and the marginal price effect down to a greater degree. In the 250 to 500 kwh. approximation, the marginal price effect is substantially underestimated. The fixed charge effect is substantially overestimated in each case. The use of TEB data has little or no effect on the other coefficients of the model. The short- and long-run elasticities of the price and income effects are given in Table 5.12.

TABLE 5.12

A COMPARISON OF TEB DATA WITH ACTUAL RATE DATA

(Logarithmic HVS model - t-statistics
in parentheses)

Coefficient	NERB*	100T250 ⁺	100T500 ⁺	250T500 ⁺
b ₀ Constant	-0.0951 (-1.20)	-0.0739 (-0.94)	-0.0871 (-1.11)	-0.0371 (-0.422)
b ₁ Lagged Sales	0.850 (45.05)	0.865 (49.01)	0.861 (48.51)	0.862 (48.14)
b ₂ Income	0.136 (4.63)	0.114 (3.39)	0.121 (4.17)	0.117 (3.94)
b ₃ Marginal Price	-0.0843 (-5.22)	-0.0531 (-2.65)	-0.0662 (-3.22)	-0.0258 (-1.87)
b ₄ Fixed Charge	-0.0523 (-2.56)	-0.131 (-4.45)	-0.119 (-3.95)	-0.157 (-4.36)
b ₅ Natural Gas Price	0.00619 (0.354)	0.0106 (0.61)	0.0101 (0.58)	0.0100 (0.57)
b ₆ Heating Days	0.0807 (4.12)	0.0790 (4.65)	0.0780 (3.98)	0.0775 (3.95)
b ₇ Cooling Days	0.0390 (4.71)	0.0385 (4.65)	0.0391 (4.74)	0.0393 (4.74)
R ²	0.981	0.981	0.981	0.981
<u>Elasticities</u>				
YPC				
Short run	0.136	0.114	0.121	0.117
Long run	0.937	0.846	0.874	0.843
MP				
Short run	-0.0844	-0.0531	-0.0662	-0.0258
Long run	-0.578	-0.384	-0.476	-0.186
FC				
Short run	-0.0523	-0.131	-0.119	-0.157
Long run	-0.361	-0.975	-0.853	-1.136

* Rate variables calculated from the National Electric Rate Book.

⁺ Rate variables calculated from Typical Electric Bills.

A comparison of the results using TEB data and the actual rate data was also made defining the fixed charge variable at zero consumption units. In contrast to the results of Table 5.12, the closest representation of the actual rate data was achieved with the 250 to 500 kwh. block representation. Both the 100 to 250 and the 100 to 500 block representations overestimate the marginal price effect. The fixed charge effect is still overestimated substantially in this case for all the TEB representations.

Although the coefficients of MP in the models employing actual rate data are relatively stable for the two definitions of FC, the marginal price coefficients for the TEB representations are not. This may be attributed to multicollinearity between the TEB marginal price variable and the combined effect of the income variable and the TEB fixed charge variable.

No further experiments have been done with TEB data at this time. The current evidence indicates that TEB data can be used as a proxy for actual rate data, but that this should be done with caution. Certainly there is room for further research in this area. One possibility for such research is analysis of covariance to determine the relationship between TEB variables and actual rate variables among regions. In addition, since the FPC is about to institute new procedures for calculating TEB data, it is not certain that the relationship

between the TEB data and actual rate data observed in this study will continue to apply in the future.

5.6 The Average Price Technique

An alternative technique to the typical block method of representing a rate schedule was presented in Chapter IV. This is an ex ante technique. In this technique the entire rate schedule is approximated and the parameters defining the approximation to the entire rate schedule are included in the model.

Summary statistics on the adequacy of the double-log approximation to the average price rate schedule are presented in Table 5.13 for each state. These statistics are the average value within the state for the R^2 of the fixed rate approximation to a set of points on the actual rate schedule, and the t-statistics of the parameters defining the fitted equation. (See Appendix 5 for the method of calculating these parameters.) Clearly, a double-log function approximates an average price rate schedule very well.

Two different methods were employed for incorporating the parameters of the rate approximation in the desired demand equation. One of these (4.20) simply replaces the typical block variables with variables consisting of the values of the approximation parameters. The other incorporates the average price function in its fitted form

TABLE 5.13
SUMMARY STATISTICS ON THE ADEQUACY OF THE AVERAGE
PRICE RATE SCHEDULE APPROXIMATION

t-Statistic (P2)	t-Statistic (P1)	R-Squared	State
-175.62564	218.96901	0.99043	Alabama
-172.20035	256.00966	0.98986	Arizona
-335.23361	465.97408	0.99010	Arkansas
-135.00033	175.32067	0.98336	California
-100.76024	163.29219	0.96822	Colorado
-120.32338	192.83699	0.96366	Connecticut
-118.23747	179.47412	0.97363	Delaware
-109.65398	154.36712	0.97637	Florida
- 96.14357	119.05958	0.96004	Georgia
-119.70337	166.74701	0.97897	Idaho
- 97.37735	152.75907	0.96499	Illinois
-106.46419	153.24662	0.96757	Indiana
-138.19062	220.47185	0.97971	Iowa
- 81.02389	123.58179	0.95744	Kansas
-126.33542	178.53401	0.97803	Kentucky
-222.00262	325.38742	0.99166	Louisiana
-121.96110	180.55816	0.95528	Maine
-207.75652	288.74246	0.99045	Maryland
- 70.84249	135.72112	0.94074	Massachusetts
-183.86914	274.55046	0.97674	Michigan
-125.56637	186.88315	0.97219	Minnesota
-483.38395	621.95465	0.99815	Mississippi
-123.56932	188.45738	0.97398	Missouri
- 99.91884	129,43042	0.96816	Montana
- 86.55937	122.19233	0.96078	Nebraska
-275.77254	381.80282	0.99611	Nevada
- 61.50617	100.92854	0.92560	New Hampshire

Table 5.13 (Continued)

t-Statistic (P2)	t-Statistic (P1)	R-Squared	State
-107.37755	175.24793	0.97038	New Jersey
-128.23252	182.97605	0.97788	New Mexico
-109.22349	166.97995	0.97285	New York
-109.44957	156.46381	0.97218	North Carolina
-101.14457	158.05967	0.96423	North Dakota
-122.64492	187.49657	0.97775	Ohio
-179.76497	286.58553	0.96726	Oklahoma
-106.43100	127.53298	0.97479	Oregon
- 93.57892	138.41458	0.96607	Pennsylvania
-111.21989	192.62596	0.96226	Rhode Island
-135.15959	184.78886	0.97582	South Carolina
-133.84520	203.55614	0.97453	South Dakota
-239.91311	266.49482	0.99339	Tennessee
-123.44116	177.98953	0.97841	Texas
-116.16164	163.11646	0.97898	Utah
- 77.10051	125.90625	0.94727	Vermont
-261.54066	349.65763	0.98699	Virginia
-143.58612	162.65750	0.98589	Washington
-145.95517	197.75257	0.98275	West Virginia
- 88.17398	141.46747	0.95995	Wisconsin
- 65.51084	106.25188	0.93432	Wyoming
-141.55078	202.27656	0.97242	United States

into the desired demand equation (4.21). Each of these desired demand equations was associated with the discrete and continuous adjustment processes of the HT and HVS models resulting in four different estimating equations.

The inclusion of the fitted average price function in the desired demand equations results in a non-linear estimating equation; direct inclusion of the approximation parameters results in an estimating equation log-linear in form. The estimating equations for all models are given in Table 5.14.

TABLE 5.14
ESTIMATING EQUATIONS FOR THE AVERAGE PRICE METHOD

Log-linear Models:

$$\begin{aligned} \text{HT: } \ln \text{SAL} = & b_0 + b_1 \ln \text{SAL}_{-1} + b_2 (\ln \text{YPC} + \ln \text{YPC}_{-1}) \\ & + b_3 (\ln \text{P2} + \ln \text{P2}_{-1}) + b_4 (\ln \text{P1} + \ln \text{P1}_{-1}) \\ & + b_5 (\ln \text{PNG} + \ln \text{PNG}_{-1}) + b_6 (\ln \text{HDD} + \ln \text{HDD}_{-1}) \\ & + b_7 (\ln \text{CDD} + \ln \text{CDD}_{-1}) \end{aligned}$$

$$\begin{aligned} \text{HVS: } \ln \text{SAL} = & b_0 + b_1 \ln \text{SAL}_{-1} + b_2 \ln \text{YPC} + b_3 \ln \text{P2} \\ & + b_4 \ln \text{P1} + b_5 \ln \text{PNG} + b_6 \ln \text{HDD} + b_7 \ln \text{CDD} \end{aligned}$$

Non-linear Models:

$$\begin{aligned} \text{HT: } \ln \text{SAL} = & (2 - b_1 - b_3 \text{P2})^{-1} \{ b_0 + b_1 \ln \text{SAL}_{-1} \\ & + b_2 (\ln \text{YPC} + \ln \text{YPC}_{-1}) + b_3 (\ln \text{P1} + \ln \text{P1}_{-1} \\ & + \text{P2} \cdot \ln \text{SAL}_{-1}) + b_5 (\ln \text{PNG} + \ln \text{PNG}_{-1}) \\ & + b_6 (\ln \text{HDD} + \ln \text{HDD}_{-1}) + b_7 (\ln \text{CDD} + \ln \text{CDD}_{-1}) \} \end{aligned}$$

$$\begin{aligned} \text{HVS: } \ln \text{SAL} = & (1 - b_3 \text{P2})^{-1} (b_0 + b_1 \ln \text{SAL}_{-1} + b_2 \ln \text{YPC} \\ & + b_3 \ln \text{P1} + b_5 \ln \text{PNG} + b_6 \ln \text{HDD} + b_7 \ln \text{CDD}) \end{aligned}$$

The estimation results for the log-linear estimating equations are given in Table 5.15. An extremely good fit was obtained with these models. All the coefficients are of the right sign and highly significant except the price of natural gas which is at least of the right sign. As in the typical block method, the weather variables are less significant when a continuous adjustment process is used. The fit of the model using the discrete adjustment process (HVS) is perhaps the best obtained in this study. The smallest t-statistic with the exception of natural gas is 4.42. The elasticities obtained with this model are fairly consistent with those obtained in the typical block method corroborating those results. The marginal price elasticity is calculated according to equation (4.8). It should be noted that since ex ante price variables are used in these models, any simultaneity problem due to the use of rate schedules is removed.

In contrast to Table 5.15, the results for the non-linear estimating equations as presented in Table 5.16 are very poor. Few of the coefficients are significant and many are of the wrong sign. In addition, the elasticity with respect to the lagged dependent variable is quite close to one dominating the exogenous effects in the long-run elasticities. For these reasons this model specification is eliminated.

TABLE 5.15
ESTIMATION RESULTS OF THE LOG-LINEAR MODELS
(t-statistics in parentheses)

Coefficient	Adjustment Process	
	HT	HVS
b_0 Constant Term	-0.110 (2.15)	-0.192 (-4.38)
b_1 Lagged Sales	0.939 (98.01)	0.939 (98.98)
b_2 Income	0.0581 (5.79)	0.110 (5.71)
b_3 (P2)	-0.326 (-5.13)	-0.577 (-4.90)
b_4 (P1)	-0.0458 (-4.75)	-0.0778 (-4.42)
b_5 Natural Gas Price	0.00202 (0.37)	0.000347 (0.03)
b_6 Heating Days	0.0110 (1.29)	0.0709 (5.16)
b_7 Cooling Days	0.00977 (2.16)	0.0356 (5.48)
R^2	0.991	0.991
<u>Elasticity</u>		
YPC		
Short run	0.116	0.110
Long run	1.901	1.744
MP		
Short run	0.0557	0.0473
Long run	0.911	0.766

TABLE 5.16
ESTIMATION RESULTS OF THE NON-LINEAR MODELS
(t-statistics in parentheses)

Coefficient	Adjustment Process	
	HT	HVS
b_0 Constant Term	-0.490 (0.86)	-0.243 (-0.44)
b_1 Lagged Sales	0.993 (453.19)	0.987 (230.39)
b_2 Income	0.00737 (1.88)	0.0139 (1.81)
b_3 Electricity Price	-0.00215 (-0.44)	-0.00563 (0.61)
b_5 Natural Gas Price	-0.00126 (-0.59)	-0.000435 (-0.10)
b_6 Heating Days	-0.00487 (-1.59)	0.00192 (0.34)
b_7 Cooling Days	0.00408 (2.02)	0.0153 (4.15)
R^2	0.993	0.992
<u>Elasticities</u>		
SAL ₋₁	0.986	0.973
YPC		
Short run	0.00728	0.0138
Long run	0.520	0.512
MP		
Short run	-0.00276	-0.00273
Long run	-0.209	-0.197

The non-linear equations were estimated with a package employing the Newton method. The variables in the equations were defined in terms of deviations from state means. This is the only compensation that was made for a regional component to the error term. Another accommodation to a regional error component might result in more efficient estimation, but the results obtained here are poor enough that this effort seems unwarranted.

5.7 Conclusion

Two different techniques of representing rate schedules in econometric models are employed in this chapter. The empirical results support the use of both the techniques.

The appropriateness of each of these techniques is judged on the basis of the fit of regression equations employing them. For each of the rate schedule representations a good fit is obtained for some model specification.

The fit of each regression equation is evaluated on the significance of the coefficients, and the conformity of the signs and the sizes of the coefficients with a priori beliefs. For each of the rate representation methods equations were estimated in which all the coefficients are highly significant and of the right sign except the coefficient of the price of natural gas.

In these equations the relative sizes of the price and income effects are in agreement with theoretically established a priori results. The absolute magnitudes of the coefficients are also reasonable. Finally, the elasticities obtained in the models using the different rate representations are consistent with each other, further corroborating the results. The empirical results seem to indicate a long-run price elasticity of about -0.75.

Footnotes

¹P. Balestra and M. Nerlove, "Pooling Cross Section and Time Series Data in the Estimation of a Dynamic Model: The Demand for Natural Gas," Econometrica (July, 1966): 585-612.

²Although regional factors seem more intuitively plausible in the general case, a time varying factor which is constant across regions is also possible. Discussion of this possibility is deferred until later.

³Since the model is autoregressive, GLS is not unbiased. However, it is consistent and asymptotically efficient.

⁴T. Amemiya, A Note on the Estimation of Balestra-Nerlove Models, Institute for Mathematical Studies in the Social Sciences, Technical Report No. 4 (Palo Alto: Stanford University, 1967).

⁵M. Nerlove, "Further Evidence on the Estimation of Dynamic Economic Relations from a Time Series of Cross Sections," Econometrica 39 (March, 1971):359-82.

⁶K. H. Johnson and H. L. Lyon, "Experimental Evidence on Combining Cross-Section and Time Series Information," Review of Economics and Statistics 55 (November, 1973): 465-74.

⁷The test is made employing an LSDV model and not a 2RDV model since the numerator and denominator of (5.10) must be independent. Under the assumption, H_0 , that there is no regional component to the error term,

$$SSR_{OLS} = \hat{e}'\hat{e}_{OLS} = u'M_1u, \text{ and}$$

$$SSR_{LSDV} = \hat{e}'\hat{e}_{LSDV} = u'M_2u \text{ where } E(uu') = \sigma^2 I.$$

Since $M_2(M_1 - M_2) = 0$, the numerator and denominator of the expressions in (5.10) are distributed as independent χ^2 , and (5.10) is distributed as an F under H_0 . In 2RDV the variables are transformed and the orthogonality between M_2 and $M_0 - M_2$ is not maintained.

⁸At the .001 level of significance.

CHAPTER VI

CONCLUDING REMARKS

Throughout this investigation the results and conclusions have been evaluated individually on their own merit. In this chapter, I will try to tie these results together into a cohesive unit and evaluate the results on the basis of their contribution to the work as a whole. To this purpose I will review the stated objectives of this study and its major conclusions. In addition, in order to achieve some perspective on this evaluation I will reiterate the basic assumptions upon which this investigation is founded. How realistic these assumptions are and how critical they are to the conclusions are important to the overall relevance of the study. The assumptions and omissions of this investigation are also useful in determining directions for future research.

A number of separate but interrelated objectives were specified in the introduction to this study. The first of these was a comprehensive investigation of the implications of block rate schedules in demand theory. The second objective was the determination of a method of representing rate schedules in empirical models. The final objective was the estimation of an empirical model of

electricity demand. The methods of representing rate schedules determined in fulfillment of the second objective are derived from the theoretical results obtained in carrying out the first objective. The econometric model of electricity demand is an application of the rate schedule representation. Some of the criteria on which the econometric models are evaluated are also drawn from the theoretical discussion.

The theoretical investigation is divided into an examination of individual demand and average demand. Although the results obtained with regard to individual demand are not specifically relevant to the empirical models estimated in this study, they form a framework for the theoretical discussion of average demand. In addition, they are relevant to models that might be estimated in the future with individual customer data. Data of this kind is being collected in some current pricing experiments.

The theoretical results obtained regarding average demand are directly relevant to the empirical models estimated. In the examination of average demand conditions are determined which are sufficient for the average demand function to be a continuous function of any given block rate. Under additional assumptions, conditions are established regarding the relative sizes of $\bar{dx}/dy$ and $\bar{dx}/dt$ as a function of the relative sizes of $\bar{dx}/dy$ and

$d\bar{x}/dr$. The same assumptions have implications for the relative sizes of $d\bar{x}/dr_i$ for different block rates.

Two methods of incorporating a rate schedule into an empirical model are examined in this study. The typical block method falls directly out of the theoretical results concerning average demand. The theoretical results mentioned above show explicitly the sufficient conditions for the continuity of average demand as a function of the typical block variables. These results also provide testible hypotheses on the relative sizes of the coefficients of the typical block and income variables.

The average price technique compliments the typical block technique in that it is an ex ante technique. Explicit testible hypotheses of the kind derived for the typical block technique are not obtained. However, the results can be tested for consistency with the results obtained with the other method.

Both methods of representing rate schedules in empirical models were tested in econometric models of electricity demand. The evaluation of these models concentrated on an assessment of the methods of dealing with rate schedules. Both of the rate representations work very well in this empirical application. The rate variables are highly significant and in agreement with the hypotheses specified above. In addition, the results of the two methods are consistent with each other.

The above discussion indicates that this investigation has been a highly successful undertaking. A brief examination of the issues which have either been omitted in this analysis or avoided through qualifying assumptions should provide a clearer perspective on this evaluation. Simultaneously, a consideration of these omissions presents an opportunity to make suggestions for extension of this research in the future.

The most important assumptions to the theoretical aspects of this study are that electricity demand responds to the standard set of preference axioms and that customers are indeed aware of their budget sets. Alternative hypotheses were eliminated in the theoretical discussion and have not been investigated empirically in this study. Such an investigation might be possible with individual customer data. In general, the hypotheses made in this study do seem to be supported by the data at hand. Furthermore, block rates occur more frequently than just in electricity pricing so that an analysis of block rate pricing within the framework of traditional demand theory has wide applicability.

A large number of assumptions are required for empirical models. One of the issues raised in Chapter II was that of simultaneity. Problems of simultaneity resulting from the presence of a rate schedule have been adequately handled. Problems of simultaneity resulting

from normal supply and demand considerations have been assumed away on the basis of regulatory constraints. Jaskow has argued that while this assumption may be reasonable during periods of rising prices, it is not realistic while prices are constant or falling.¹

I assume that problems of simultaneity resulting from supply and demand considerations can be ignored. If this assumption is critical to the conclusions, the results of this study could be criticized on these grounds. Therefore, a test of this assumption was designed.² Constant or falling nominal prices prevailed nationally during the period 1961 through 1966. Separate price variables were defined in each of the models for this period. An F-test was then calculated to determine if significantly different price effects were measured during periods of falling and rising prices. Separate income and weather variables for the two periods were not defined since these are demand determinants only. This test indicates that significantly different price effects are not obtained at the 5% level in any of the models except the FK model and the conclusions of this study are not based on that model in any case. Therefore, the assumption employed regarding supply effects does not seem critical.

Another issue introduced in Chapter II is the problem of measuring short- and long-run elasticities. I assumed that these could be adequately measured with flow adjustment models circumventing the necessity of appliance stock data. Models have also been estimated using appliance stock data gathered by DRI and including the same rate representations used in this study.³ Although impressive results have been obtained with this data, they are consistent with the results obtained in this study. Therefore, this assumption seems to be reasonable.

Oil prices were omitted entirely from the models estimated in this study, and gas prices were calculated as ex post average prices. The rationality behind this is simply the unavailability of other data. There is no way of assessing the criticalness of this omission at this time. This issue has been deferred for future research.⁴

The most critical omissions to the econometric model have been reviewed. Only ad hoc models have been estimated in this study, and many extensions and modifications to this effort are possible. However, the empirical results obtained are certainly within the realm of reason and consistent with a priori beliefs. Acceptance of the empirical results is justified on these grounds.

The remaining assumptions employed in this study are those pertaining to the error structure of the model. These have been discussed in some detail in Chapter V.

The tests of these assumptions were not exhaustive and some further experiments were suggested in that discussion. The tests performed do indicate that the assumed error structure is quite reasonable.

There is one remaining extension to this model which I feel deserves mention. This model has not been used or tested as a forecasting model. I feel that a complete evaluation of the ideas introduced in this study would require testing them in a forecasting model. This also has been left for future research.

My intent in reviewing the assumptions and omissions of this study is not to downplay the results and conclusions which I obtained. My purpose has been to give some perspective to the relevance of the conclusions and to indicate their relation to past and future research. It is expected that any well-researched investigation will provide many new research topics. I feel that this review of the basic assumptions upon which this study is founded indicates that it rests on fairly solid ground.

Footnotes

¹Paul L. Joskow, "Inflation and Environmental Concern: Structural Change in the Process of Public Utility Price Regulation," Journal of Law and Economics 17 (1974): 291-327.

²Performing a test such as the one described here was suggested by Professor Edward Mitchell.

³Lester D. Taylor, Gail R. Blattenberger, and Phillip K. Verleger, The Residential Demand for Energy (Palo Alto: Electrical Power Research Institute, 1977), Chapter 6.

⁴Lee Phillips is undertaking such an effort at this time.

APPENDICES

APPENDIX 1

OPTIMAL PRICING STRATEGY FOR BLOCK RATE PRICING

The investigation of demand functions of consumers facing decreasing block rates raises questions about the nature of block rate pricing itself. How should block rate schedules be determined, and when are they applicable? When is it functional to have multiple blocks, and how should the block rates be optimally set?

The potential functions served by block rate pricing will be discussed briefly in this section. Profit maximization and cost allocation are among these potential uses. Certainly many other criteria are employed by rate commissions in setting rate schedules. There is a substantial body of utility rate literature on how a "fair" rate of return ought to be calculated and how capital should be valued. These are some of the things that go into the actual rate-making process. However, the viewpoint of this discussion is to look at what the potential advantages are of multiple block rates to an electric utility. We will not discuss the actual rate-making process and rate regulation.

Optimal pricing strategies and their advantages vary depending upon whether or not consumers are identical in their tastes and incomes. If all individuals are identical

or if prices are set individually, then there is no purpose to having more than two block prices, or equivalently a fixed customer charge and a constant unit price. On the other hand, if prices are set uniformly across a group of consumers with varying tastes and incomes, then the number of blocks necessary to maximize profit is a function of the number of customers. Furthermore, in this situation it is possible to achieve load improvements at peaks by means of multiple block prices.

If rate schedules are to be effective in any meaningful way it must be the case that large quantities cannot be bought up at a cheap discount price and stored at a reasonable cost for the following periods. There must also be barriers that prevent the resale of the commodity, or else consumers would join together to avoid paying the high initial block rates. These conditions are necessary for block rate schedules to be applicable; they generally hold in the case of electricity.

The optimal pricing strategy for profit maximization and the optimal number of blocks is affected by the number of customers for whom the rate schedule is to apply. First we will examine the case for one consumer.

The maximum amount of profit can be extracted from a single consumer by means of a fixed charge and a single unit price. This was shown by Gabor (1955) and his argument is illustrated in Figures A1.1a and A1.1b. Figure A1.1a shows an indifference relation U between electricity,

Figure A1.1
Extracting Maximum Profit from
An Individual Consumer with a Block
Rate Schedule

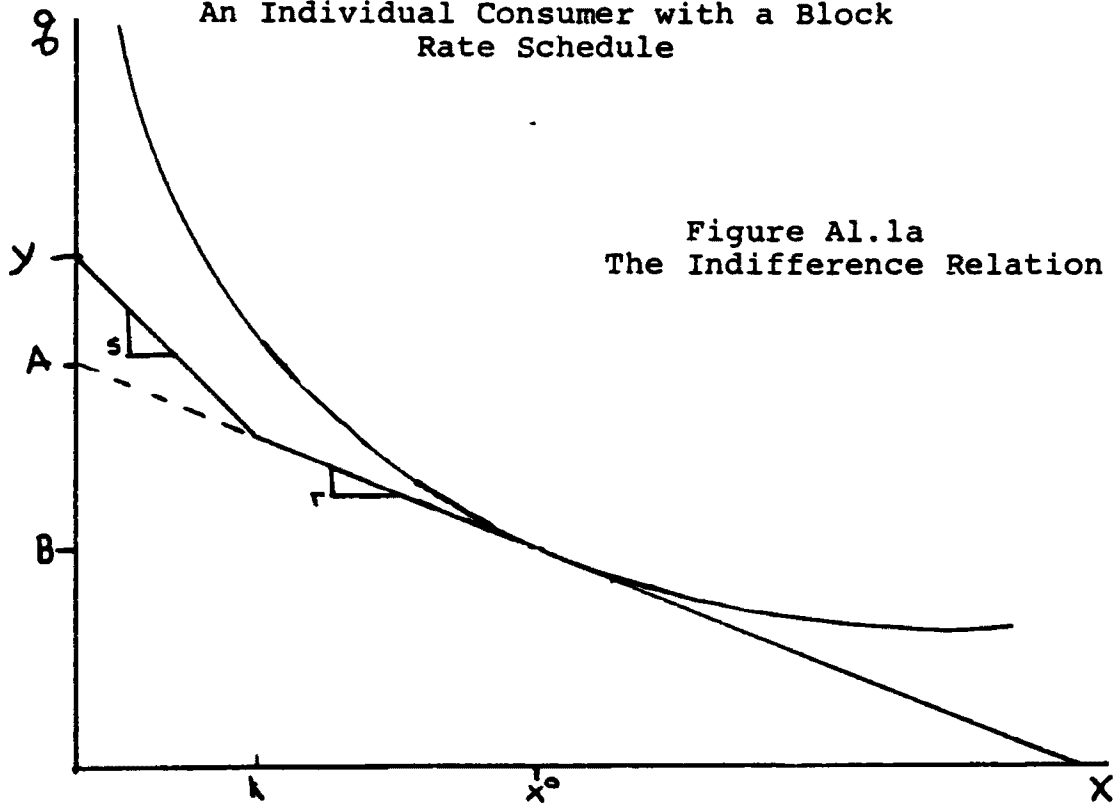


Figure A1.1a
The Indifference Relation

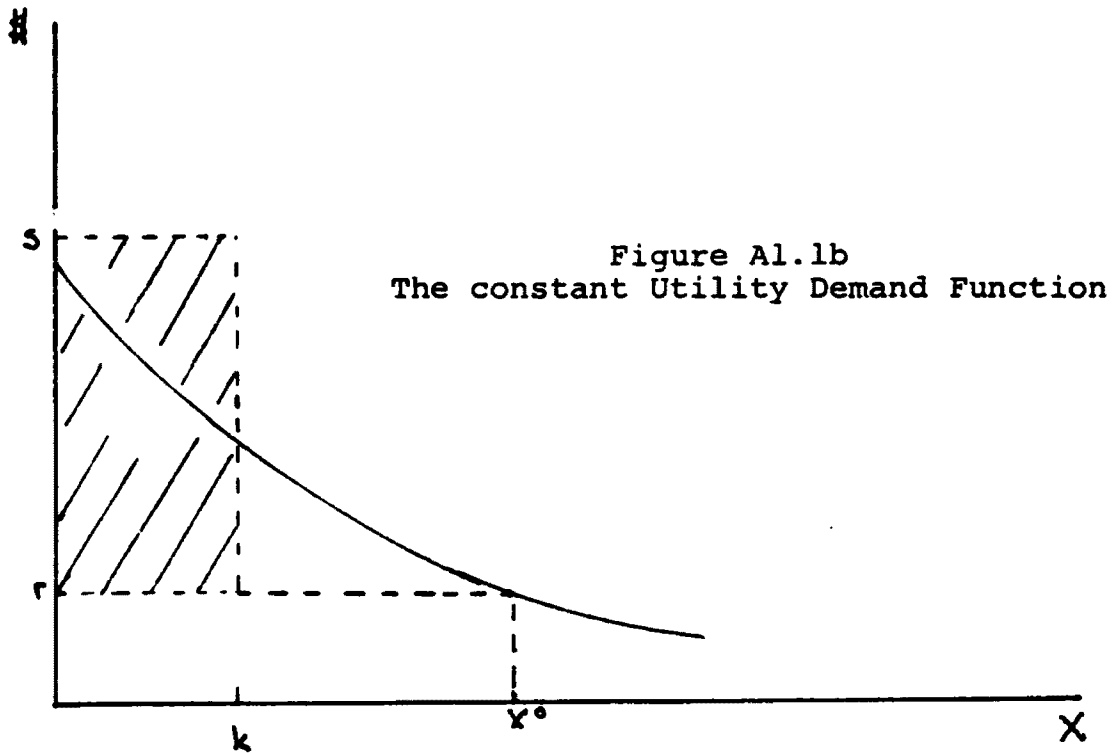


Figure A1.1b
The constant Utility Demand Function

x , and a composite good, q , for an individual with income y . Figure A1.1b shows the corresponding constant utility demand curve, D . At quantity x^0 the maximum amount of revenue that can be extracted from a consumer is YB . This can be done by means of a fixed charge YA combined with a unit price, r , or correspondingly by charging a price s for the first K units and r thereafter. No additional profit can be gained by using more block prices. A suitable revenue-maximizing two block tariff can be designed for any consumption level x^0 . The profit maximizing point for the firm occurs when r is equal to the marginal cost of producing x and s (or YA) is set at the maximum level that still retains the customer in the market.

If individuals are identical all consumption takes place in one block. Thus, any multiple block rate schedule is equivalent to some two-part tariff. In this case it seems more reasonable to employ only two rates because it is operationally simpler and there is nothing to be gained by the additional blocks. In some localities it has been legally required that some electricity be included in with the fixed charge. Such a restriction might require two block rates in addition to the fixed charge for profit maximization. However, multiple block rate schedules only become operationally different from two-part tariffs when consumption is taking place within

different blocks. This requires that there be individuals with different preferences or incomes in the market.

The derivation of an optimal two-part tariff to be applied uniformly across customers was derived by Oi (1969). His argument for the two person case is illustrated graphically in Figure A1.2. The constant-utility demand curves of the two individuals are given by D^1 and D^2 . Marginal cost is set at C . If the unit price is set equal to C , the maximum fixed charge that can be set and still retain both customers in the market is represented by the area of the triangle ABC . The maximum revenue is then gotten from the first customer but not the second. Oi shows that by raising the unit price to P^1 , the total profit can be increased even though the fixed charge must then be reduced to AEP^1 . This can equivalently be written as a two-block rate schedule. For example, a price, P^2 , could be charged for the first x^0 units, and a price P^1 thereafter; $(P^2 - P^1)x^0 = AEP^1$. Under this price situation consumer 1 demands x^1 units of electricity and consumer 2, x^2 units.

A similar procedure can be carried out for any desired number of consumers to be retained in the market. Then, further optimization can be carried out over the number of consumers. This procedure yields the optimal two-part tariff or two-block rate schedule.

Figure A1.2
An Optimal Uniform Two-Part Tariff

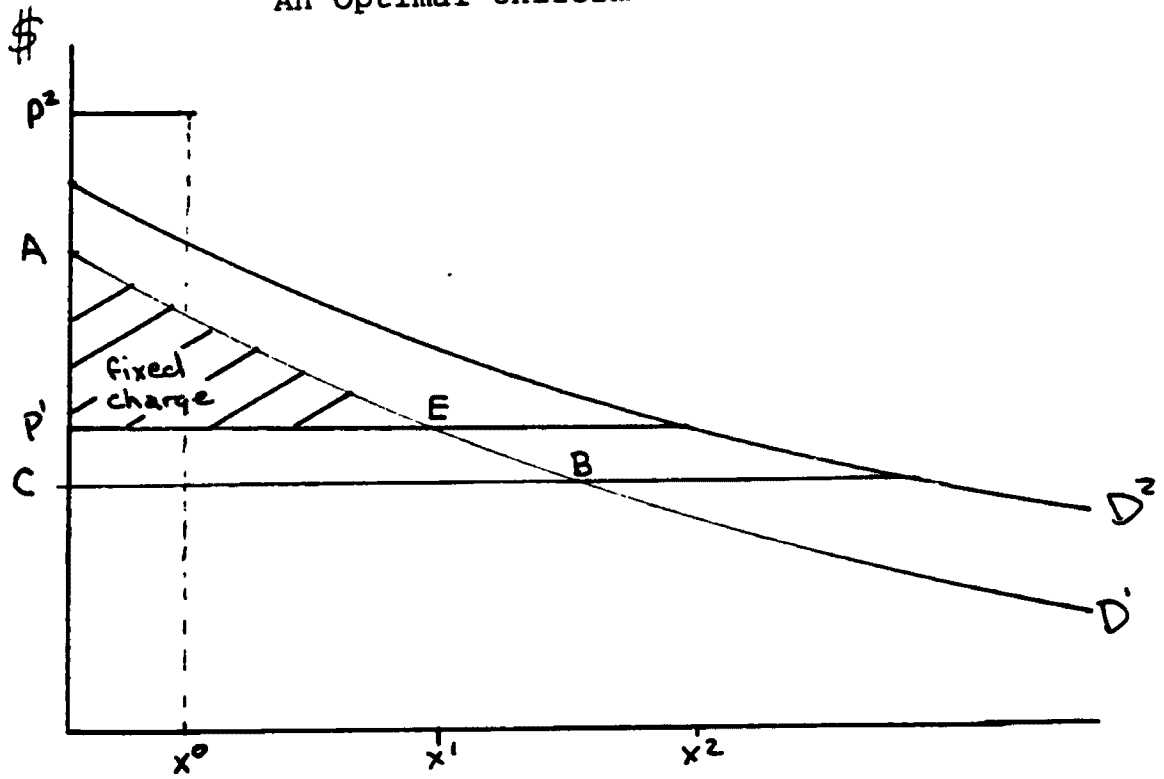
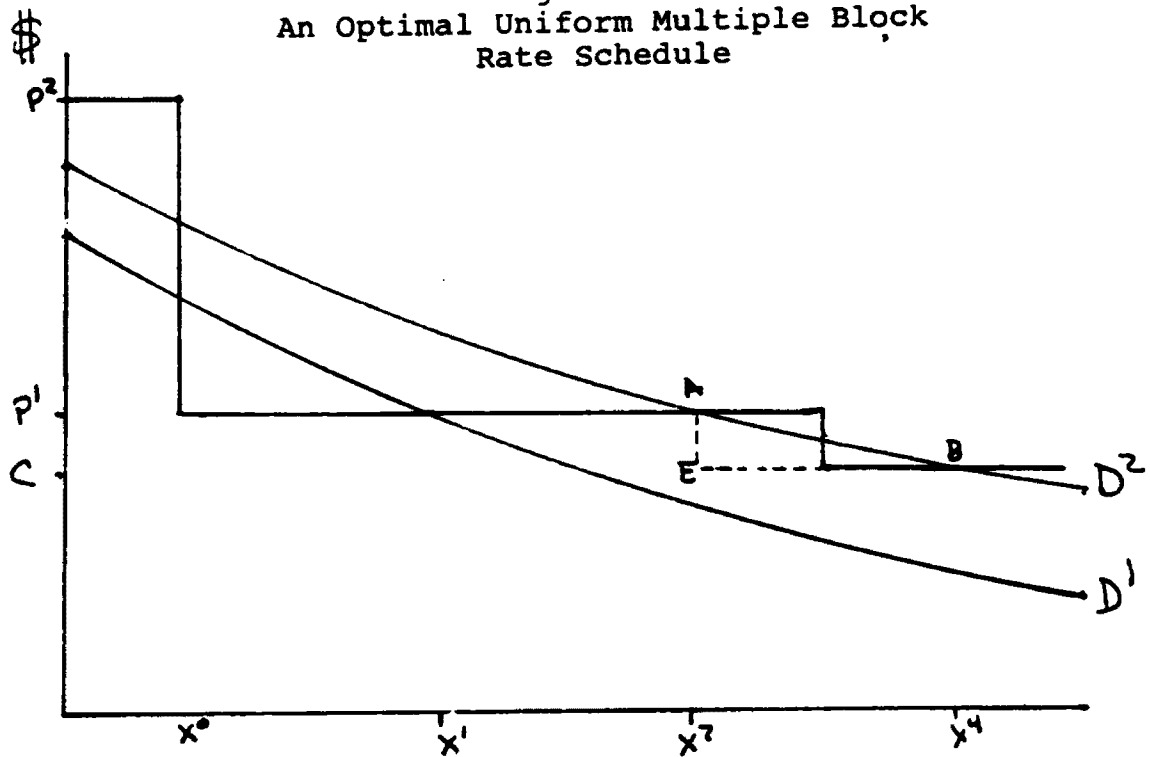


Figure A1.3
An Optimal Uniform Multiple Block Rate Schedule



A two-block rate schedule will not necessarily yield the globally maximum profit in a market with many consumers. Additional profit can sometimes be achieved through the use of additional block rates.

In Figure A1.3, the demand functions of Figure A1.2 are duplicated, but with a three-block rate schedule. As in Figure 1, price p^2 is charged for the first x^0 units. However, the price p^1 applies only up to x^3 units, and c thereafter. The consumption level of consumer 1 remains at x^1 with no change in expenditure. Consumer 2 increases his consumption level to x^4 , and profit increases by an amount equal to the area of the triangle AEB. Including more than three blocks does not raise the optimum profit level in this case.

No additional blocks are necessary for profit maximization in some cases. Under certain conditions Oi shows that the optimal two-part tariff yields a marginal price lower than marginal cost. In this case there is no advantage to the third block.

A corollary to Gabor's and Oi's work can be inferred from this analysis. In a market of N consumers, no more than $N + 1$ blocks are necessary in the rate schedule for the firm to achieve a globally optimum profit, and when a globally optimum profit is achieved, the last block price is less than or equal to marginal cost. If there are more than $N + 1$ blocks, then there must be at least one block other than the first which contains the

consumption level of no consumer. This block could then be eliminated without altering the cost of electricity to any consumer, if the block rates are decreasing. Further, if the last block is greater than marginal cost, then as illustrated in Figure A1.3, profit can be increased by the addition of a block price equal to marginal cost.

Multiple block rate schedules are frequently justified on cost grounds. We will see to what extent this is reasonable. There are three basic kinds of costs facing an electricity utility: fixed costs, cost proportionate to the amount of energy consumed, and costs related to power (capacity) requirements at different times of day. This last cost results from the difficulty in storing electricity. A fixed charge or high initial block rate can enable a firm in a decreasing cost industry to meet its costs while still setting marginal price equal to marginal cost. This still does not explain the presence of as many as 10 block rates in a rate schedule. However costs are assessed, it is clear that it should not matter whether one person demands ten kilowatts during a certain hour or ten people demand one kilowatt during the same hour. If, however, customers' consumption patterns over time are related to their level of consumption, then multiple block rates may be related to time of day pricing. This is the only cost justification for multiple block rates.

Under the particular condition that there is a one-to-one relationship between consumption levels of individuals and their consumption patterns over time, multiple block rates are equivalent to peak load pricing. Of course, this condition is never met exactly. There are situations, nevertheless, such as when large consumers have all-electric homes and, thus, a large base load demand, where some load factor improvement might be effected through block rate schedules.

This discussion is not intended as a recommendation for using multiple block rate schedules instead of peak load pricing. This is a very indirect method, and there is no guarantee that the distribution of customers is such that peaking costs would be allocated appropriately. Although block rate pricing is frequently justified on the grounds discussed here, there have been no empirical tests of the validity of this justification. Such empirical tests would not be possible without the very meters with which peak load pricing is made possible. Given the availability of reasonably priced time-of-day meters, peak load pricing is the appropriate technique.

One further point can be made about the potential advantages of multiple block rates from a utility's point of view. For a period of time sales growth has been a clearly stated goal of utilities. Block rate schedules are devices for sales expansions. In addition, utilities are constrained in their profit maximization as to their

profit rate. This can lead to over-expansion of capital, in order to raise the profit ceiling, and necessitates higher sales levels than in the unconstrained case.¹ The expanded sales resulting from block rate schedules can also influence profit in this way.

¹Averch-Johnson effect.

APPENDIX 2

THE CONTINUITY OF INDIVIDUAL'S DEMAND FUNCTIONS

Throughout the discussion in Section 3.4A, statements are made about the continuity of $D(r)$ and $d(r)$. Rigorous proofs of these statements are contained in this appendix. The lemmas and their corollaries apply to an n dimensional commodity space.

Lemma 1. If a set-valued function, $D(r)$, is constant over a region V , then it is continuous in the sense C1 on V , and $d(r)$ is constant and continuous on V .

Proof:

This is a trivial application of C1.

Assume $D(r) = A$, where A is a fixed set for all r within a domain V . There are a finite number of elements in A by D4. Consider any open set W such that $W \supset A$. For any $r^* \in V$, and any open neighborhood V^* of r^* where $V^* \subset V$,

$$D(V^*) = \bigcup_{r \in V^*} D(r) = A.$$

Therefore $D(V^*) \subset W$, and $D(r)$ is continuous at r^* in the sense C1.

Since $d(r)$ is a weighted average of the elements in $D(r) = A$, then $d(r) = \sum w_i a_i$ for $a_i \in A$. $\sum w_i a_i$ is a fixed scalar. Clearly, $d(r)$ is continuous on V . Q.E.D.

Lemma 2. Given $\bar{r} \in \Pi_r$. If for all $\epsilon > 0$ there exists an $r \in \Pi_r$ such that $||r - \bar{r}|| < \epsilon$ and $x \in K_i$ for some $x \in D(r)$, then there exists an $\bar{x} \in D(\bar{r})$ such that $\bar{x} \in K_i$ and $\bar{x} = \lim_{r \rightarrow \bar{r}} (D(r) \cap K_i)$.

Proof:

For all $N > 0$ select r^N such that $||r^N - \bar{r}|| < \epsilon/N$ and $x^N \in K_i$ for some $x^N \in D(r^N)$. $\lim_{N \rightarrow \infty} r^N = \bar{r}$.

First it is shown that the sequence of points $\{D(r) \cap K_i\}$ is contained in a compact set. Set $r^* = \inf \{r_i^N\}$. Since r_i^N is bounded below such an inf exists. $D(r^N)$ is contained in the compact set $B(r^*)$ by B1 and B3.

Since $\{D(r) \cap K_i\}$ is contained in a compact set it has a convergent subsequence. Set $\{r^{\ell}\}$ equal to a subsequence of $\{r^N\}$ such that $\{D(r^{\ell}) \cap K_i\}$ converges. Set $\bar{x} = \lim_{r \rightarrow \bar{r}} (D(r^{\ell}) \cap K_i)$.

Next it is shown that $\bar{x} \in D(\bar{r})$. Consider any point x' in $D(\bar{r})$. There exists a sequence $\{x^v\}$ which converges to x' such that x^v is in the interior of $B(\bar{r})$ for all v . Since x^v is in the interior of $B(\bar{r})$, there exists an $L > 0$, such that x^v is in the interior of $B(r^{\ell})$ for all $\ell > L$. Therefore, for any x^v there exists an L such that

$$x^l P x^v \text{ for all } l > L$$

where $x^1 = D(r^1) \cap K_i$. By P5, $x^v \overline{P} X$. Since $x^v \overline{P} X$ for any v , then by P6

$$x' \overline{P} X \quad .$$

Therefore $\bar{x} \in D(\bar{r})$.

Finally, $\bar{x}$ is the only limit point of the sequence $\{D(r^N) \cap K_i\}$. $\bar{x} \in K_i$ since $\{x^l\} \in K_i$ and K_i is closed. If there is another limit point to this sequence, then this second limit point is also in $D(\bar{r})$ and in K_i . This is impossible by D4. Q.E.D.

Corollary 1. $D(r)$ is continuous in the sense C1 over the domain Π_r .

Proof:

There are only a finite number of sets K_i , and $\bigcup_{i=1}^m K_i = \Omega$. Therefore, any sequence $x^v \in \Omega$ contains an infinite subsequence in some K_i . Given any sequences $r^v \rightarrow \bar{r}$ and $x^v \in D(r^v)$ such that $x^v \rightarrow \bar{x}$, there is an infinite subsequence of x^v in some K_i . Applying Lemma 2 to this subsequence proves that $\bar{x} \in D(\bar{r})$ and establishes C2. C2 is equivalent to C1 since $D(\bar{r})$ is compact. Q.E.D.

Corollary 2. If $D(r) \subset K_i$ for all r in the set V , then $D(r)$ is single-valued and continuous in the sense C3 on V , and for any r' on the boundary of V , there exists an $x \in D(r')$ such

that $x \in K_i$. $d(r)$ is also continuous in the sense C3 on V .

Proof:

$D(r)$ is single-valued for all $r \in V$ by D4. Therefore, $D(r) = d(r)$. C1 is equivalent to C3 for single valued functions, and C1 is established from Corollary 1.

By lemma 2, for any sequence $\{r^v\}$ such that $\lim_{v \rightarrow \infty} r^v = r'$, $r^v \in V$, and $\bar{x} = \lim_{v \rightarrow \infty} D(r^v)$, then $\bar{x} \in D(r')$. Q.E.D.

Corollary 3. If for some set $V \in \Pi_r$, all for $r \in V$,
 $D(r) = \{f^i(r) \mid f^i(r) \in K_i \text{ for } \forall i \in I\}$ $d(r)$ is continuous on V in the sense C3 and for each point r' on the boundary of V there exists an $x \in D(r')$ such that $x \in K_i$ for all $i \in I$.

Proof:

Set $D(r) = \{f^i(r) \mid f^i(r) \in K_i \text{ for all } i \in I\}$ for all r in some set V . Since $D(r)$ contains at most one point in each set K_i by D4, the functions $f^i(r)$ are single-valued and continuous in the sense C3. $d(r) = \sum w_i f^i(r)$ where w_i are constant. Therefore $d(r)$ is continuous in the sense C3. $f^i(r') \in D(r')$ by Lemma 2. Q.E.D.

In the next lemma all block prices except one remain constant.

Lemma 3. Given $x^* \in D(r_j^*)$. If $x^* \in K_j^-$, then $D(r_j) \subset K_j^-$
for all $r_j > r_j^*$. If $x^* \notin K_j^-$, then $D(r_j) \cap K_j^- = \emptyset$
for all $r_j < r_j^*$.

Proof:

Suppose $x^* \in D(r_j^*)$ and $x^* \in K_j^-$. For all $r_j > r_j^*$, $B(r_j) \subset B(r_j^*)$ by B3. By B5, $x^* \in B(r_j)$ for all r_j . Therefore, $x^* \in D(r_j)$, for all $r_j > r_j^*$. If $x \in D(r_j)$ such that $x \neq x^*$, then $x^* \bar{P}x$. $D(r_j) \subset B(r_j) \subset B(r_j^*)$ and since $x^* \bar{P}x$, $x \in D(r_j^*)$. However, if $x \notin K_j^-$ and $x \in D(r_j)$, then $x \notin B(r_j)$. Thus, $x \in K_j^-$ and $D(r_j) \subset K_j^-$.

Suppose $x^* \in D(r_j^*)$ and $x^* \notin K_j^-$. For any $r_j' < r_j^*$, x^* is in the interior of the set $B(r_j')$. Therefore, there exists an $x' \in B(r_j')$ such that $x' P x^*$. For all $x \in B(r_j')$ such that $x \in K_j^-$, by B5 $x \in B(r_j^*)$. Therefore, $x \bar{P} x^*$ and $x' P x$. Thus, $x \notin D(r_j')$ and $D(r_j') \cap K_j^- = \emptyset$. Q.E.D.

Lemma 4. It is possible to divide the set Π_r into m closed overlapping sets H_i , $i = 1, \dots, m$ such that $D(r)$ is single-valued and continuous on the interior of these sets and multivalued on the boundaries.

Proof

The sets H_i , $i = 1, \dots, m$ are defined in the following manner:

$$H_i = \{r \in \Pi_r \mid x \in D(r), \quad x \in K_i\} \quad .$$

Clearly, $\bigcup H_i = \Pi_r$ since for all $r \in \Pi_r$, r belongs to one of the sets H_i . The sets, H_i , are closed, since the function D is closed and the image sets are closed.

First it will be shown that $D(r)$ is single-valued on the interior, H_i^0 , of H_i . This will be done by showing that $D(r)$ is multivalued if and only if r is on the boundary of H_i .

Suppose $r^* \in H_i$ and $D(r)$ is multivalued at r^* . Since $r^* \in H_i$, there exists an $x^* \in D(r^*)$ such that $x^* \in K_i$. Since $D(r)$ is multivalued at r^* , there exists an $x' \in D(r^*)$ such that $x' \neq x^*$ and $x' \in K_j$ for some $j \neq i$. Either $j > i$ or $j < i$. First suppose $j > i$ and $x' \notin K_j^-$. Consider $r'(a)$ such that

$$\begin{aligned} r'_k &= r_k^* & \text{where } k \neq j, & \text{ and} \\ r'_j &= ar_j^* & \text{where } 0 < a < 1 & . \end{aligned}$$

By Lemma 3 $D(r'(a)) \cap K_j^- = \emptyset$. Therefore, for all such values of a , $r'(a) \notin H_i$ and, thus r^* is on the boundary of H_i . Secondly, suppose $j > i$ and $x' \in K_j^-$. Since $x' \in K_j^-$ and $x' \in K_j$, then $x' \in K_{j-1}$ and $x' \notin K_{j-1}^-$. Since $x' \in K_{j-1}$ $x^* \notin K_{j-1}$ by D4, and since $i < j$, $i < j-1$. Consider $r'(b)$ such that

$$\begin{aligned} r'_k &= r_k^* & \text{where } k \neq j-1, & \text{ and} \\ r'_{j-1} &= br_{j-1}^* & \text{where } 0 < b < 1 & . \end{aligned}$$

By Lemma 3, $D(r'(b)) \cap K_{j-1}^- = \emptyset$. Therefore, for all such b , $r'(b) \notin H_i$ and, thus r^* is on the boundary of H_i .

Finally, suppose $i > j$. Consider $r'(c)$ such that

$$r'_k = r_k^* \quad \text{where } k \neq j+1, \quad \text{and}$$

$$r'_{j+1} = c r_{j+1}^* \quad \text{where } c > 1.$$

By Lemma 3, $D(r'(c)) \subset K_{j+1}^-$. Therefore, for all such values of c , $r'(c) \notin H_i$, and, thus, r^* is on the boundary of H_i . If $D(r)$ is multivalued at r^* , r^* is on the boundary of H_i , and thus $D(r)$ is single-valued on the interior of H_i .

If r^* is on the boundary of H_i , then for all $\sigma > 0$, there exists an $r' \notin H_i$ such that $\|r^* - r'\| < \sigma$. Take a sequence of σ_k going to zero and an associated set of r^k . Without loss of generality $r^k \in H_j$ for some $j \neq i$. Clearly $\lim_{\sigma \rightarrow 0} r^k = r^*$ and since H_j is closed $r^* \in H_j$. Thus, $D(r)$ is multivalued at r^* .

If $D(r)$ is single-valued at r^* , then it is single-valued on some neighborhood of r^* , since r^* is in the interior of some H_i . $D(r)$ is continuous at r^* in the sense C3 by corollary 2. Q.E.D.

Lemma 5. The set $\Pi_r - \bigcup_{i=1}^m H_i^0$ has Lebesgue measure zero.

Proof:

Set $Z = \Pi_r - \bigcup_{i=1}^m H_i^0$. This set is Lebesgue measurable since the H_i^0 are open sets and are Lebesgue measurable, and Π_r is Lebesgue measurable.

Define the boundary of H_i to be $\hat{H}_i = H_i - H_i^0$. Since $\Pi = \bigcup_{i=1}^m H_i$, $Z = \bigcup_{i=1}^m \hat{H}_i$. Therefore,

$$m(Z) \leq \sum_{i=1}^m m(\hat{H}_i) .$$

If $m(\hat{H}_i) = 0$ for all i , then $m(Z) = 0$. It will be shown next that $m(\hat{H}_i) = 0$ for any $\hat{H}_i$.

For any arbitrary $\hat{H}_i$ define the sets $E^1, \dots, E^{i-1}$ to be

$$E^j = \{r \in H_i \mid k_{j-1} \leq \min\{x_n \in D_n(r)\} < k_j\} .$$

The sets, E^j , $j = 1, \dots, i-1$ are contained in the intersection,

$$E^j \cap \hat{H}_i \cap \hat{H}_j \quad j = 1, \dots, i-1 .$$

The set E^i is defined,

$$E^i = \{r \in H_i \mid k_{i-1} \leq \min\{x_n \in D_n(r)\} \leq k_i\} .$$

$$E^i \cap \hat{H}_i \cap \left(\bigcup_{j=i+1}^m \hat{H}_j \right) .$$

By definition, $\forall r \in \hat{H}_i$, $x \in D(r)$ such that $x_n < k_i$. Therefore

$$m(\hat{H}_i) = \sum_{j=1}^i m(E^j) .$$

If $m(E^j) = 0$ for all j , then $m(\hat{H}_i) = 0$.

Either E^j is empty or it is nonempty. If E^j is empty $m(E^j) = 0$. If it is nonempty, look at the cross-section of E^j in the $j+1^{\text{st}}$ direction, E_{j+1}^j . Given ν as a one-dimensional Lebesgue measure in the direction of the

$j+1^{\text{st}}$ coordinate axes and μ an $m-1$ dimensional measure in the space defined by the other $m-1$ coordinate axes (Π_r is an m dimensional space), then

$$m(E^j) = \int v(E_{j+1}^j) d\mu \quad .^1$$

Thus, if $v(E_{j+1}^j) = 0$ at all points in E^j then $m(E^j) = 0$.

Finally, it is shown that $v(E_{j+1}^j) = 0$ at any arbitrary point in E^j . Suppose $r^* \in E^j$ and $j < i$. Consider $r'(a)$ such that

$$r'_k = r^*_k \quad \text{for } k \neq j+1$$

$$r'_{j+1} = ar^*_{j+1} \quad \text{where } a > 0 \quad .$$

$r'(1) = r^*$, and it will be shown that $r'(a) \notin E^j$ for $a \neq 1$.

Since $r^* \in H_j$, $\exists x \in D(r^*)$ such that $x \in K_{j+1}^-$. By Lemma 3, $D(r'_j(a)) \cap K_{j-1}^- \neq \emptyset \quad \forall a$ such that $a > 1$. Therefore, $r'(a) \notin H_i$ and $r'(a) \notin E^j$. Also, since $i > j$, there exists an $x \in D(r^*)$ such that $x \notin K_{j+1}^-$. Therefore, by Lemma 3, $D(r'(a)) \cap K_{j+1}^- = \emptyset$ for all a such that $0 < a < 1$. This means that $r'(a) \notin E^j$ for $0 < a < 1$. Therefore, $r'(a) \notin E^j$ for $a \neq 1$.

$$E_j^{j+1} = \{r^*\}, \quad \text{and}$$

$$v(E_j^{j+1}) = 0$$

Similar arguments hold if $j = i$. Since $r^* \in H_i$, $x \in D(r^*)$ such that $x \in K_{i+1}^-$. By Lemma 3, $D(r'(a)) \cap K_{i+1}^- \neq \emptyset \quad \forall a$

¹H. Royden, Real Analysis, 2nd ed. (New York: MacMillan, 1968), p. 266.

such that $a > 1$. However, by definition of E^i ,
 $D(r'(a)) \cap K_{i+1}^- \cap H_i$. Therefore $r'(a) \in H_i^0 \forall a > 1$, and
 $r'(a) \notin \hat{H}_i$. $\forall a > 1$, $r'(a) \notin E^i$. Also, since $r^* \in \hat{H}_i$, $D(r)$
is multivalued at r^* . Since $r^* \in E^i$, $\exists x \in D(r^*)$ such that
 $x \notin H_i$ and $x \in H_j$ where $j < i$. Therefore $\exists x \in D(r^*)$ such that
 $x \notin H_i$ and $x \in H_j$ where $j > i$. This $x \notin K_{i+1}^-$. By Lemma 3,
 $D(r'(a)) \cap K_{i+1}^- = \emptyset$, $\forall a$, $0 < a < 1$. Thus $r'(a) \notin H_i$ for
 $0 < a < 1$ and $r'(a) \notin E^i$. Again this shows $r'(a) \notin E_i \forall a \neq 1$.

$$E_i^{i+1} = \{r^*\} \quad , \quad \text{and}$$

$$v(E_i^{i+1}) = 0 \quad . \quad \text{Q.E.D.}$$

Corollary 4. The set of points over which $d(r)$ is discontinuous has Lebesgue measure zero.

Proof:

$D(r)$ is single-valued and continuous at all r such that $r \in H_i^0$ for some $i = 1, \dots, m$. By Corollary 2 $d(r)$ is also continuous. The set of points over which $d(r)$ is discontinuous is contained in $\Pi_r = \bigcup_{i=1}^m H_i^0$. This set has Lebesgue measure zero by Lemma 5. Q.E.D.

Lemma 6. If E1 holds everywhere, then $d(r_j)$ has at most two points of discontinuity.

Lemma 7. If E2 holds everywhere, then $d(r_j)$ has at most $m-j+1$ points of discontinuity.

Proof:

The proofs of these lemmas follow the same general outline. Therefore they shall be proven together.

The central part of the proof of these lemmas is the establishment of a partition of the set Π_{r_j} . Two points r_j^0 and r_j^* are defined which partition Π_{r_j} into at most three nonempty subsets so that $D(r_j) \subseteq K_j^+$ for all $r_j < r_j^0$, $D(r_j) \subseteq K_j$ for all $r_j^0 < r_j < r_j^*$ and $D(r_j) \subseteq K_j^-$ for all $r_j^* < r_j$.

Once this partition is established the continuity of $d(r_j)$ can be examined on the subsets defined by this partition. Price changes affect demand differently on each of these subsets according to budget conditions B5 and B6. For $r_j > r_j^*$, $D(r_j) \subseteq K_j^-$. K_j^- is a fixed set by E5 so $D(r_j)$ is also a fixed set on this region, and $d(r_j)$ is continuous by lemma 1. If $r_j^* > r_j > r_j^0$, then $D(r_j) \subseteq K_j$ and $D(r_j)$ is single-valued by D4. Therefore $d(r_j)$ is continuous by corollary 2. If $r_j^0 > r_j$ then $D(r_j) \subseteq K_j^+$ and any price change in r_j is equivalent to an income change by B6. Thus, the continuity of $d(r_j)$ on this set depends on E1 or E2.

If E1 holds, $d(r_j)$ is continuous for $r_j^0 > r_j$. This is true since $D_n(r_j)$ is fixed for $r_j < r_j^0$ by E1 and it is constrained to contain elements in each of a fixed set of blocks. $d(r_j)$ is then continuous on this region by Corollary 3. Lemma 6 is thus established since $d(r_j)$ can be discontinuous at most at the points r_j^* and r_j^0 .

If E2 holds, then for $r_j^1, r_j^0, r_j^* < r_j^0, r_j^0 < r_j^1$,

$$(A2.1) \quad \max\{D_n(r_j^1)\} < \min\{D_n(r_j^0)\} \quad .$$

$d(r_j)$ is discontinuous only where $D(r_j)$ is multivalued. But since $D_n(r_j)$ is monotonic (A2.1) demand shifts to a higher block only as price falls and $D(r_j)$ is multivalued at most at $m-j-1$ points in this region. Including the discontinuity points r_j^0 and r_j^* , there are at most $m-j+1$ points of discontinuity.

It remains to establish the required partition of Π_{r_j} . In establishing this partition it should be remembered that $K_m^+ = \emptyset$ and $K_1^- = \emptyset$. Therefore $D(r_m) \cap K_m^+ = \emptyset$ for all r_m and $D(r_1) \cap K_1^- = \emptyset$ for all r_1 . This means that $r_m^* = \infty$ and $r_1^0 = 0$.

Given K_j^- exists ($j \neq 1$) and given an r_j^1 such that $x \in K_j^-$ for some $x \in D(r_j^1)$, by Lemma 3 $D(r_j) \subset K_j^-$ for all $r_j > r_j^1$. Define r_j^*

$$r_j^* = \inf \{r_j^1 \mid x \in K_j^- \text{ for some } x \in D(r_j^1)\}$$

r_j^* exists since $\{r_j^1\}$ is nonempty and bounded below by zero. By definition for $r_j < r_j^*$,

$$D(r_j) \cap K_j^- = \emptyset \quad .$$

On the other hand for $r_j > r_j^*$

$$D(r_j) \subset K_j^-$$

by lemma 3. If there is no r_j^1 such that $x \in K_j^-$ for some

$x \in D(r_j')$, then

$$D(r_j) \cap K_j^- = \emptyset$$

for all r_j . Then set $r_j^* = \infty$.

Given K_j^+ exists ($j \neq m$) and given r_j' such that $x \in K_j^+$ for some $x \in D(r_j', y)$ where y is a fixed income level. By B6 for all $r_j < r_j'$ $B(r_j, y) \cap K_j^+$ is equivalent to $B(r_j', y') \cap K_j^+$ for some $y' > y$ and $B(r_j', y') \supset B(r_j, y)$. Consider $D(r_j', y')$. Since $x \in D(r_j', y)$ such that $x \in K_j^+$, by either E1 or E2 $x \in D(r_j', y)$ such that $x \in K_j^+$. Further, since

$$B(r_j, y) \cap K_j^+ = B(r_j', y') \cap K_j^+$$

then

$$D(r_j, y) \cap K_j^+ = D(r_j', y') \cap K_j^+.$$

Since $B(r_j, y) \subset B(r_j', y')$ and $D(r_j, y) \cap D(r_j', y') \neq \emptyset$,

$$D(r_j, y) \subset D(r_j', y').$$

However, if $x \in D(r_j', y')$ and $x \notin K_j^+$, then $x \notin B(r_j, y)$.

Therefore

$$D(r_j, y) = D(r_j', y') \cap K_j^+$$

and $D(r_j, y) \subset K_j^+$ for all $r_j < r_j'$.

Define r_j^0 ,

$$r_j^0 = \sup\{r_j' | x \in D(r_j', y), x \in K_j^+\}.$$

If $r_j^0 = \infty$, then by definition, $D(r_j, y) \cap K_j^+ = \emptyset$ for all $r_j > r_j^0$, and by the argument above $D(r_j, y) \subset K_j^+$ for all $r_j < r_j^0$. If no r_j' exists such that $x \in D(r_j', y)$ and

$x \in K_j^+$, then $D(r_j', y) \cap K_j^+ = \emptyset$, $\forall r_j' > 0$ and $r_j^0 = 0$.

Clearly $r_j^0 \leq r_j^*$. Suppose $r_j^0 \neq r_j^*$. For any r_j , $r_j^0 < r_j < r_j^*$, $D(r_j) \cap K_j^+ = \emptyset$ since $r_j > r_j^0$ and $D(r_j) \cap K_j^- = \emptyset$ since $r_j < r_j^*$. Therefore

$$D(r_j) \cap K_j = \emptyset \quad \text{for} \quad r_j^0 < r_j < r_j^* .$$

If $r_j^0 = r_j^*$, $\{r_j | r_j^0 < r_j < r_j^*\} = \emptyset$. Q.E.D.

Lemma 8. Each of the sets H_i^0 is arcwise connected.

Proof:

A set is called arcwise connected if for any two points in the set, there exists a continuous mapping, $f: [0,1]$, into the set such that $f(0)$ is the first point and $f(1)$ is the second.

This proof is straight forward in the case of H_m^0 . Suppose r and $\bar{r}$ are in H_m^0 . Then define,

$$r_j' = \min\{\tilde{r}_j, \bar{r}_j\} \quad j = 1, \dots, m-1 .$$

and define the two points $\bar{r}'$ and $\tilde{r}'$

$$\bar{r}' = (r_1', \dots, r_{m-1}', \bar{r}_m) , \quad \text{and}$$

$$\tilde{r}' = (r_1', \dots, r_{m-1}', \tilde{r}_m) .$$

By E1 or E2, $\forall r, \bar{r} \geq r \geq \tilde{r}'$

$$r \in H_i^0 .$$

Thus $\bar{r}$ is arcwise connected to $\tilde{r}'$. The same reasoning

follows for $\bar{r}$ and $\bar{r}'$. By lemma 5 and lemma 6, since $\bar{r}' \in H_i^0$ and $\tilde{r}' \in H_i^0$, then $r \in H_i^0 \forall \bar{r}' \geq r \geq \tilde{r}^0$ where without loss of generality $\bar{r}_m \geq \tilde{r}_m$.

It is therefore possible to define a continuous function $f(x): [0,1] \rightarrow H_m^0$ such that $f(0) = \tilde{r}$ and $f(1) = \bar{r}$. One such function is:

$$f(x) = \begin{cases} (1-3x)\tilde{r} + 3x\tilde{r}' & 0 \leq x \leq 1/3 \\ (2-3x)r' + (3x-1)\bar{r}' & 1/3 \leq x \leq 2/3 \\ (3-3x)\bar{r} + (3x-2)\tilde{r} & 2/3 \leq x \leq 1 \end{cases} .$$

A similar argument follows for $i < m$. Suppose $\bar{r}$ and $\tilde{r} \in H_i^0$ where $i < m$. Define

$$\begin{aligned} r'_j &= \max\{\tilde{r}_j, \bar{r}_j\} \quad j = i+1, \dots, m \\ r'_j &= \min\{\tilde{r}_j, \bar{r}_j\} \quad j = 1, \dots, i-1 \end{aligned} .$$

Then define the points

$$\begin{aligned} \bar{r}' &= (\bar{r}_1, \dots, \bar{r}_i, ar'_{i+1}, \dots, ar'_m) \quad , \quad \text{and} \\ \tilde{r}' &= (\tilde{r}_1, \dots, \tilde{r}_i, ar'_{i+1}, \dots, ar'_m) \quad , \end{aligned}$$

where $a > 1$. An increase in an extramarginal price does not affect demand. Therefore for all $a > 0$ and $\bar{r} < r < \bar{r}'$, $r \in H_i^0$. The same argument holds for $\tilde{r}'$ and $\tilde{r}$.

Define

$$\begin{aligned} \bar{r}'' &= (r'_1, \dots, r'_{i-1}, \bar{r}_i, ar'_{i+1}, \dots, ar'_m) \quad , \quad \text{and} \\ \tilde{r}'' &= (r'_1, \dots, r'_{i-1}, \tilde{r}_i, ar'_{i+1}, \dots, ar'_m) \quad . \end{aligned}$$

Intramarginal price decreases are equivalent to income increases and by either E1 or E2

$$D(\bar{r}'') \subset K_i \cup K_i^+ \quad \text{and}$$

$$D(\tilde{r}'') \subset K_i \cup K_i^+ .$$

Price increases in blocks $i+1, \dots, m$ reduce the demand $D_n(r)$ and for sufficiently large a

$$D(\bar{r}'') \subset K_i \quad \text{and}$$

$$D(\tilde{r}'') \subset K_i .$$

Therefore, for all r , $\bar{r}' \leq r \leq \bar{r}''$, $r \in H_i^0$ by E1 or E2; likewise for all r , $\tilde{r}' \leq r \leq \tilde{r}''$. By lemma 5 or lemma 6, for all r , $\tilde{r}'' \leq r \leq \bar{r}''$ where without loss of generality. $\tilde{r}_i < \bar{r}_i$, $r \in H_i^0$.

Again it is possible to construct a continuous function $f(x): [0,1] \rightarrow H_i^0$ such that $f(0) = r$ and $f(1) = \bar{r}$. One such function is:

$$f(x) = \begin{cases} (1-5x)\tilde{r} + 5x\tilde{r}' & 0 \leq x \leq 1/5 \\ (2-5x)\tilde{r}'' + (5x-1)\tilde{r}'' & 1/5 \leq x \leq 2/5 \\ (3-5x)\tilde{r}'' + (5x-2)\bar{r}'' & 2/5 \leq x \leq 3/5 \\ (4-5x)\bar{r}'' + (5x-3)\bar{r}' & 3/5 \leq x \leq 4/5 \\ (5-5x)\bar{r}' + (5x-4)\bar{r} & 4/5 \leq x \leq 1 \end{cases}$$

Q.E.D.

APPENDIX 3

THEORETICAL PLAUSIBILITY OF DEMAND FUNCTIONS FOR AN INDIVIDUAL

This appendix contains the mathematical validation of statements made in Section 3.4B. Section 3.4B concerns the theoretical implications of the Slutsky substitution matrix in demand theory. For the purpose of this investigation, the Slutsky matrix has been redefined (equation (3.4)) so as to accommodate the presence of a rate schedule for one good. The first lemma in this section shows that the symmetry and negative semi-definiteness of the Slutsky matrix are necessary conditions for a preference maximizing demand function.

Lemma 9. Let P be an irreflexive (P1), transitive (P2) relation defined on Ω such that the function $\bar{P}$ is a connected relation on Ω . Let the function $D(p,y)$ mapping $\Pi \times Y$ into Ω maximize "satisfaction" as represented by P subject to $p \cdot g(x) \leq y$. If $D(p,x)$ satisfies I1-I4, then the Slutsky substitution matrix is symmetric and negative semi-definite almost everywhere.

Proof:

Since the set $(\Pi \times Y - \bigcup_{\ell=1}^m \chi_{\ell}^0)$ has Lebesgue measure zero (I2), it is sufficient to show that the Slutsky substitution matrix is symmetric and negative semi-definite at any arbitrary point $(p^0, y^0) \in \chi_{\ell}^0$, for $\ell = 1, \dots, m$. Therefore, the symmetry and negative semi-definiteness of the Slutsky matrix are considered at an arbitrary point (p^0, y^0) which is contained in χ_{ℓ}^0 for some ℓ .

Set

$$x^0 = D(p^0, y^0) \quad .$$

Since $(p^0, y^0) \in \chi_{\ell}$, $x^0 \in K_{\ell}$.

The first part of this proof consists of defining an income compensation function μ , and specifying a neighborhood of p^0 on which this function is defined. Since χ_{ℓ}^0 is an open set, there exists an open, convex neighborhood, A' , of p^0 such that

$$(p, y^0) \in \chi^0 \quad \text{for all } p \in A' \quad .$$

The income compensation function is defined on the set A' . An additional set R is employed in the definition of this function. R is the set of commodity bundles in K_{ℓ} at least as good as x^0 .

$$R = \{x | x \in K_{\ell} \quad \text{and} \quad x^0 \bar{P} x\} \quad .$$

Given the set R , the income compensation function is defined on A' as follows:

$$(A3.1) \quad \mu(p) = \inf_{x \in R} p \cdot g(x) \quad .$$

Both R and A' are composed of non-negative vectors. For any $x \in R$ and $p \in A'$, $p \cdot g(x) \geq 0$. Therefore, the inf of $p \cdot g(x)$ is bounded below over the set R . This establishes that the function $\mu(p)$ is well-defined. Furthermore, for any $x \in R$, the function,

$$f_x(p) = p \cdot g(x) \quad ,$$

defined on the convex set A' is a linear function. Therefore, $\mu(p) = \inf_{x \in R} f_x(p)$ is concave [Fenchel, p. 60, proposition 7]. Since $\mu(p)$ is concave, it is continuous on A' [Fenchel, p. 75, proposition 23].

At this point a subset of A' containing p^0 can be established such that

$$(p, \mu(p)) \in \chi_\ell^0 \quad ,$$

for all p in this subset. First, it is shown that,

$$(A3.2) \quad (p^0, \mu(p^0)) \in \chi_\ell^0 \quad , \quad \text{since}$$

$$\mu(p^0) = y^0 \quad .$$

By the definition of $\mu(p)$,

$$\mu(p^0) \leq y^0$$

since $x^0 \in R$ and $y^0 = p^0 \cdot g(x^0)$. Suppose $\mu(p^0) < y^0$.

Then by the definition of μ there exists an $x' \in R$, $x' \neq x^0$ such that,

$$p^0 \cdot g(x') < y^0 \quad .$$

Since $x' \in R$, $x^0 \bar{P} x'$. Thus, $x' \in D(p^0, y^0)$ contradicting I1.

Since $(p^0, \mu(p^0))$ is contained in the open set X_ℓ^0 , and since $\mu(p)$ is continuous, there exists an open neighborhood, A'' , of p^0 such that $A'' \subset A'$ and for all $p \in A''$

$$D(p, \mu(p)) \in K_\ell \quad .$$

Next the function $Z(p)$ is defined on A'' .

$$(A3.3) \quad Z(p) = D(p, \mu(p)) \quad .$$

$Z(p)$ is continuous since both $\mu(p)$ and $D(p, y)$ are continuous. By I3,

$$(A3.4) \quad p \cdot g(Z(p)) = \mu(p) \quad , \quad \forall p \in A'' \quad .$$

From this point the proof follows that of Hurwicz and Uzawa verbatim except for a few notational changes to meet this particular case.

First, it is shown that for any $p, p' \in A''$,

$$(A3.5) \quad p \cdot g(Z(p)) \leq p' \cdot g(Z(p)) \quad .$$

By the definition of the function $\mu(p)$, for all $p' \in A''$ there exists a sequence $\{x^i\}$ such that,

$$x^0 \bar{P} x^i \quad \forall_i \quad , \quad \text{and}$$

$$\lim_{i \rightarrow \infty} p' \cdot g(x^i) = \inf_{x \in R} p' \cdot g(x) \quad .$$

For any $\varepsilon > 0$, there exists an i_ε such that

$$p' \cdot g(x^i) < \mu(p') + \varepsilon, \quad \forall i > i_\varepsilon.$$

Consider now

$$x^\varepsilon = D(p', \mu(p') + \varepsilon).$$

For small enough ε , $x^\varepsilon \in K_\ell$. x^ε maximizes satisfaction subject to,

$$p' \cdot g(x) \leq \mu(p') + \varepsilon.$$

Therefore,

$$p' \cdot g(x^i) < \mu(p') + \varepsilon \quad \text{yields}$$

$$x^\varepsilon P x^i \quad \forall i \geq i_\varepsilon.$$

Since $D(p, y)$ is continuous in y , $x^\varepsilon \rightarrow Z(p)$ as $\varepsilon \rightarrow 0$.

To establish (A3.5), consider first the case $Z(p) \in R$. $Z(p)$ minimizes $p \cdot g(x)$ over the set R by (A3.4). Since $x^\varepsilon \in R$, this implies

$$g(Z(p)) \cdot p \leq g(x^\varepsilon) \cdot p. \quad \text{Therefore,}$$

$$g(Z(p)) \cdot p \leq g(Z(p')) \cdot p,$$

since $\lim_{\varepsilon \rightarrow 0} x^\varepsilon = Z(p')$, establishing (A3.5) in this case.

On the other hand, consider the case $Z(p) \notin R$. Since $Z(p) \notin R$,

$$x^0 P Z(p).$$

Since $x^0 \bar{P} x^i$, and $x^\varepsilon P x^i$,

$$x^\epsilon \cdot p \cdot Z(p) \quad .$$

If $p \cdot g(Z(p)) > p \cdot g(Z(p'))$, then for small enough ϵ

$$p \cdot g(Z(p)) > p \cdot g(x^\epsilon) \quad .$$

But then $Z(p)$ does not maximize satisfaction subject to $p \cdot g(x) < \mu(p)$ and,

$$Z(p) \neq D(p, \mu(p)) \quad ,$$

contradicting A3.3. This establishes the statement (A3.5).

Next it is shown that for every $j = 1, \dots, n + m = 1$ the partial derivative

$$(A3.6) \quad \partial \mu / \partial p_j = g_j(Z(p)) \quad .$$

Consider two price vectors p and $p + \Delta p$ contained in A'' and the corresponding $Z(p)$ and $Z(p + \Delta p)$. By A3.5

$$p \cdot g(Z(p)) \leq p \cdot g(Z(p + \Delta p)) \quad .$$

By defining

$$\Delta g(Z(p)) = g(Z(p + \Delta p)) - g(Z(p)) \quad ,$$

this becomes

$$(A3.7) \quad p \cdot g(Z(p)) \leq p \cdot g(Z(p)) + p \cdot \Delta g(Z(p)) \quad , \quad \text{and}$$

$$p \cdot \Delta g(Z(p)) \geq 0 \quad .$$

If $\Delta \mu(p)$ is defined similarly, then

$$\begin{aligned}\Delta\mu(p) &= \mu(p + \Delta p) - \mu(p) \\ &= (p + \Delta p) \cdot [g(Z(p)) + \Delta g(Z(p))] - p \cdot g(Z(p)) \\ &= p \cdot \Delta g(Z(p)) + \Delta p \cdot g(Z(p)) + \Delta p \cdot \Delta g(Z(p)) \quad .\end{aligned}$$

Then by A3.7,

$$(A3.8) \quad \Delta\mu(p) \geq \Delta p \cdot g(Z(p)) + \Delta p \cdot \Delta g(Z(p)) \quad .$$

Considering the special case,

$$\begin{aligned}\Delta p &= (0, \dots, 0, \Delta p_j, 0, \dots, 0), \text{ where} \\ \Delta p_j &> 0, \\ \frac{\Delta\mu(p)}{\Delta p_j} &> g_j(Z(p)) + \Delta g_j(Z(p)) \quad .\end{aligned}$$

Since $\mu(p)$ is concave it possesses both left- and right-hand derivatives [Fenchel, pp. 71, 79 ff.]. Letting $\Delta p_j \rightarrow 0$,

$$(A3.9) \quad \lim_{\Delta p_j \rightarrow 0^+} \frac{\Delta\mu(p)}{\Delta p_j} \geq g_j(Z(p)) \quad ,$$

from (A3.8). (Note g is continuous and $\Delta g_i \rightarrow 0$ as $\Delta p_j \rightarrow 0$.)

Similarly in the special case

$$\Delta p = (-, \dots, 0, \Delta p_j, 0, \dots, 0) \quad \text{where } \Delta p_j < 0,$$

it can be shown from (A3.8) that

$$(A3.10) \quad \lim_{\Delta p_j \rightarrow 0^-} \frac{\Delta\mu}{\Delta p_j} \leq g_j(Z(p)) \quad .$$

Combining (A3.9) and (A3.10) yields,

$$\lim_{\Delta p_j \rightarrow 0^-} \frac{\Delta \mu}{\Delta p_j} \leq g_i(Z(p)) \leq \lim_{\Delta p_j \rightarrow 0^+} \frac{\Delta \mu}{\Delta p_j} .$$

However, since $\mu(p)$ is a concave function of p ,

$$\lim_{\Delta p_j \rightarrow 0^-} \frac{\Delta \mu}{\Delta p_j} > \lim_{\Delta p_j \rightarrow 0^+} \frac{\Delta \mu}{\Delta p_j} ,$$

establishing A3.7.

The function $g(\cdot)$ is either constant on A'' ($i = n, \dots, l-1, l+1, n+m-1$), the identity function ($i=1, \dots, n-1$), or the identity function minus a constant ($i=l$). Therefore, g is continuous and differentiable on A'' . $Z_i(p)$ is continuous and differentiable on A'' by I4. Consequently,

$$\mu^j(p) = \partial \xi(p) / \partial p_j = g_j(Z(p)) ,$$

is continuous and differentiable on A'' .

Differentiating $g_j(Z(p))$ with respect to p_i yields,

$$\begin{aligned} \text{(A3.11)} \quad g_j^i(Z(p)) &= \frac{\partial g_j(Z(p))}{\partial p_i} \\ &= g_j^i[D(p, \mu(p))] + g_i^Y[D(p, \mu(p))] \mu^i(p) \\ &= g_j^i[D(p, \mu(p))] + g_j^Y[D(p, \mu(p))] \cdot g_i \\ &\quad [D(p), \mu(p)] \\ &= S_{ji}(p, \mu(p)) . \end{aligned}$$

This implies that

$$\mu_{ji}(p) = \frac{\partial^2 \mu(p)}{\partial p_j \partial p_i} = S_{ji}(p, \mu(p)) \quad .$$

By A3.2

$$\mu_{ji}(p^0) = S_{ji}(p^0, y^0) \quad .$$

Since μ is twice differentiable, Young's Theorem may be applied to get

$$\mu_{ji}(p^0) = \mu_{ij}(p^0) \quad ,$$

establishing the symmetry of $S(p^0, y^0)$. The negative semi-definiteness of the matrix, $[\mu_{ji}(p^0)]$ results from the fact that μ is a twice differentiable concave function [Fenchel, pp. 87-88]. Q.E.D.

In establishing Lemma 10, the following existence theorem is employed.

Existence Theorem. Let the functions $f_i(p, y)$, $i = 1, \dots, n$ all be defined on the set $A \times Y$ where $p \in A$, a connected set, and $y \in Y$,

$$Y = \{y | 0 \leq y < \infty\} \quad .$$

If the following conditions hold:

- a) for each $i = 1, \dots, n$, $f_i(p, y)$ possesses a differential at every point of $A \times Y$.
- b) for each $i = 1, \dots, n$, the partial derivative $f_i^y(p, y)$ is bounded, in the sense that there exists

a finite number $K_{a', a''}$ such that

$$|f_i^Y(p, y)| < K_{a', a''}$$

for all (p, y) in $A \times Y$ such that $a' < p < a''$.

c) the symmetry condition holds

$$\begin{aligned} f_i^j(p, y) + f_i^Y(p, y) \cdot f_j(p, y) &= f_j^i(p, y) \\ &+ f_j^Y(p, y) \cdot f_i(p, y) \end{aligned}$$

then the system of equations

$$\frac{\partial \mu}{\partial p_i} = f_i(p, y)$$

where $\mu(p^*) = y^*$, is uniquely solvable on A for each $(p^*, y^*) \in A \times Y$.

Proof:

Under the given conditions it is established that the system is solvable in the neighborhood of p^* , and two solutions must agree on any convex neighborhood of p^* , by Lemma 2 of Berger and Meyer.¹

The solution is extended to the connected set A by pasting together local solutions in the same manner that it is done in the main theorem of Berger and Meyer.² Q.E.D.

This existence theorem plays a key role in Lemma 10. This lemma applies to a demand function exhibiting zero income elasticity with respect to x_n . Therefore, the set χ_ℓ equals $A \times Y$, $A \subset \Pi$.

Lemma 10. Let A_1 be a convex subset of A and $B \subset K$ be the image set of $A_1 \times Y$ under the demand function, $D(p, y)$. If conditions I1 and I3-I6 hold and the Slutsky substitution matrix is symmetric and negative semi-definite, then there exists a utility function U such that $D(p, y)$ uniquely maximizes U on B subject to $g(x) \cdot p \leq y$.

Proof:

Consider the system of differential equations
 $\mu(p^0) = y^0,$

$$(A3.12) \quad \partial \mu / \partial p_i = g(D_i(p, y)) \quad .$$

By the existence theorem above, since $g(D)$ satisfies E1, I1, I4, I5, and I6, and the Slutsky matrix is symmetric, the system (A3.12) is uniquely integrable on the set A where

$$A = \{p \in \Pi \mid (p, y) \in \chi_\ell, \forall y \in Y\}$$

and $A \times Y = \chi_\ell$. Therefore, for any $(p^0, y^0) \in A \times Y$ there exists a unique function $\mu(p; p^0, y^0)$ such that

$$\mu(p^0; p^0, y^0) = y^0 \quad , \quad \text{and}$$

$$\mu^i(p; p^0, y^0) = g(D_i(p, y)) \quad .$$

For every $p \in A$, $\mu(p; p^0, y^0)$ is continuous with respect to (p^0, y^0) .

The function, μ , is used to define the utility function U . A price income combination (p', y') and its corresponding demand level $x' = D(p', y')$ is preferred to (p'', y'') and $x'' = D(p'', y'')$ if for some p^*

$$\mu(p^*; p', y') > \mu(p^*; p'', y'') \quad .$$

Before the utility function can be defined, however, it must be shown that the ordering implied by μ is independent of the base price p^* . Following Hurwicz and Uzawa, this is done in several steps.

First, let (p', y') and $(p'', y'') \in A \times Y$. If

$$\mu(p^0; p', y') = \mu(p^0; p'', y'') \quad \text{for}$$

some $p^0 \in A$, then

$$\mu(p; p', y') = \mu(p; p'', y'') \quad \text{for}$$

all $p \in A$. This is implied by the uniqueness of the solution to (A13.12).

Second, if

$$(A3.13) \quad \mu(p^0; p', y') < \mu(p^0; p'', y'') \quad \text{for some } p^0 \in A,$$

then

$$\mu(p; p', y') \geq \mu(p; p'', y'') \quad \text{for all } p \in A.$$

This follows from Lemma 2 of Hurwicz and Uzawa. It requires the connectedness of A . Suppose $p^0, p^1 \in A$ such that (A3.13) holds for p^0 but

$$\mu(p^1; p', y') \geq \mu(p^1; p'', y'') \quad .$$

Since A is arc wise connected, there exists a continuous function, $f(t)$, such that

$$\begin{aligned} f(0) &= p^0, \quad f(1) = p^1, \quad \text{and} \\ f(t) &\in A \quad \forall 0 \leq t \leq 1 \quad . \end{aligned}$$

Let $p^t = f(t)$, and let

$$\phi(t) = \mu(p^t; p', y') - \mu(p^t; p'', y'') \quad .$$

Since μ is continuous and f is continuous, ϕ is continuous. $\phi(0) < 0$ and $\phi(1) > 0$. Therefore, $\phi(t) = 0$ for some t , $0 < t < 1$, and $\mu(p^t; p', y') = \mu(p^t; p'', y'')$. By the argument given above, this implies $\phi(t) = 0$ for all t contradicting (A3.13).

The utility function $U(x)$ is now defined in terms of some base price p^* ,

$$U_{p^*}(x) = \mu(p^*; p, y) \text{ where } x = D(p, y) \quad .$$

It has been shown that the ordering implied by this function is independent of the choice of p^* . It has yet to be shown that $U_{p^*}(x)$ is uniquely defined. That is,

$$U_{p^*}(x) = \mu(p^*; p, y) \quad \forall (p, y) \text{ such that } x = D(p, y) \quad .$$

It also has to be shown that $D(p, y)$ maximizes $U(x)$ for all $x \in K_\ell$ such that $p \cdot g(x) \leq y$. In this lemma these

properties are only established for a convex subset of A , A_1 , and a subset of K_ℓ , B , where B is the image set of $A_1 \times Y$ under D .

Hurwicz and Uzawa establish these properties in a series of lemmas.³ Given $(p^0, y^0), (p', y') \in A' \times Y$ such that

$$x^0 = D(p^0, y^0) \neq x' = D(p', y')$$

their Lemma 4 shows that if

$$y' \geq u(p'; p^0, y^0) \quad , \quad \text{then}$$

$$p^0 \cdot g(x') > p^0 \cdot g(x^0) \quad ,$$

and their Lemma 5 shows that if

$$p^0 \cdot g(x^0) \geq p^0 \cdot g(x') \quad , \quad \text{then}$$

$$p' \cdot g(x^0) > p' \cdot g(x') \quad .$$

Lemma 6 in Hurwicz and Uzawa shows that the set $\{(p, y) \in A \times Y: D(p, y) = x\}$ is a convex set and Lemma 7 shows that $u(p; p^0, y^0) = u(p; p', y')$ for all p if $D(p^0, y^0) = D(p', y')$. This establishes that the utility function is uniquely defined. Lemma 8 establishes that the demand function maximizes this utility function on the set of $x \in B$ such that $g(x) \cdot p < y$. All of these lemmas cited in Hurwicz and Uzawa, Lemmas 4-8, require the negative semi-definiteness of the Slutsky matrix. This constraint has a direct role in the proof of Lemma 4 upon which the

remaining lemmas are based. Since the proof of Hurwicz and Uzawa remain applicable under the particular set of constraints in effect in this case, their proof are not repeated here. Q.E.D.

¹Melvin S. Berger and Norman G. Meyer, "Utility Functions for Finite Dimensional Commodity Spaces," in Preferences, Utility, and Demand, ed. Chipman et al. (New York: Harcourt, Brace, Johanovich, Inc., 1971), p. 151.

²Ibid., p. 158.

³Leonid Hurwicz and Hirofumi Uzawa, "On the Integrability of Demand Functions," in Preference, Utility, and Demand, pp. 126-30.

APPENDIX 4

THE CONTINUITY OF AVERAGE DEMAND FUNCTIONS

In Chapter 3, the set $A(p_i^*)$ was defined.

$$A(p_i^*) = \{(\alpha, y) \in L : \xi(\alpha, y) > 0 \text{ such that } \forall \delta > 0, p_i \in \Pi_i,$$

where $|p_i^* - p_i| < \delta$, and $||d(p_i^*, \alpha, y) - d(p_i, \alpha, y)|| > \xi(\alpha, y)\}$.

Given this definition, the point continuity condition is established in the following theorem.

Theorem 1. Given $p_i^* \in \Pi_i$. If $\mu_{xy}(A(p_i^*)) = 0$, then

$\forall \epsilon > 0, \delta > 0$ such that for $p_i \in \Pi_i$ where

$$|p_i^* - p_i| < \delta ,$$

$$(A4.1) \quad ||\lim_{N \rightarrow \infty} \bar{d}_N(p_i^*) - \lim_{N \rightarrow \infty} d_N(p_i)|| < \epsilon .$$

If $d(p_i', \alpha, y) \geq d(p_i'', \alpha, y)$ for all $p_i' \leq p_i''$ or for all $p_i' \geq p_i''$, then $\mu_{xy}(A(p_i^*)) = 0$ is a necessary condition for (A4.1) as well as a sufficient condition.

Proof:

Part 1

From earlier discussion (B1) it is known that the set $B(p_i, y)$ is compact for all $p_i \in \Pi_i, y \in Y'$. Consequently,

for some $\delta_1 > 0$, there exists an $\hat{x}$, such that for all $x \in B(p_i, Y)$, for all $|p_i^* - p_i| < \delta_1$, and for all $y \in Y'$, $\hat{x} > x$. Then since the budget set is independent of preferences, $d(p_i, y, \alpha) < \hat{x}$ for all $(y, \alpha) \in L$ and $|p_i^* - p_i| < \delta_1$. Since $d(p_i, y, \alpha) \geq 0$, the function d is bounded, as is also the covariance matrix $\text{cov}(d(p_i, y, \alpha)) \leq \hat{x}\hat{x}'$.

The quantity we are interested in is

$$\begin{aligned} p \lim_{N \rightarrow \infty} d_N(p_i^*) - p \lim_{N \rightarrow \infty} \bar{d}_N(p_i) || &= || p \lim_{N \rightarrow \infty} \sum_{j=1}^N \frac{1}{N} d(p_i^*, \alpha^j, y^j) \\ &- p \lim_{N \rightarrow \infty} \sum_{j=1}^N d(p_i, \alpha^j, y^j) || , \end{aligned}$$

and we wish to show that for any $\epsilon > 0$ this is less than ϵ , when $|p_i^* - p_i| < \delta$ for some δ . To do this, we use the strong law of large numbers.

Consider then an independent sequence (α^j, y^j) . Let

$$M(p_i) = E(d(p_i, \alpha, y)) .$$

Since d is bounded, M is finite. Next, let

$$f^j(p_i) = d(p_i^*, \alpha^j, y^j) - M(p_i) .$$

Then $E(f^j) = 0$. Also, since $d(p_i, \alpha^j, y^j)$ are independent and identically distributed, $\sum_{j=1}^{\infty} \frac{1}{j^2} \text{cov}(d(p_i, \alpha^j, y^j))$ is finite for all $p_i \in \Pi_i$, such that $|p_i^* - p_i| < \delta_1$. Therefore, by Theorem D of Halmos (1950, p. 204), the limit of

$(\frac{1}{N} \sum_{j=1}^N f^j)$ as $N \rightarrow \infty$ converges to zero almost everywhere.

Consequently, since

$$p \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{j=1}^N f^j(p_i) = 0 = E(f^j)$$

then

$$p \lim_{N \rightarrow \infty} \left\{ \sum_{j=1}^N \frac{1}{N} d(p_i^*, \alpha^j, y^j) \right\} = M(p_i^*) = \int_L d(p_i^*, \alpha, y) d\mu_{\alpha\nu} ,$$

and

$$p \lim_{N \rightarrow \infty} \left\{ \sum_{j=1}^N \frac{1}{N} d(p_i, \alpha^j, y^j) \right\} = M(p_i) = \int_L d(p_i, \alpha, y) d\mu_{\alpha\nu} .$$

Therefore,

$$\begin{aligned} & \left\| p \lim_{N \rightarrow \infty} \sum_{j=1}^N (d(p_i^*, \alpha^j, y^j) - d(p_i, \alpha^j, y^j)) - p \lim_{N \rightarrow \infty} \sum_{j=1}^N d(p_i, \alpha^j, y^j) \right\| \\ &= \left\| \int_L \{d(p_i^*, \alpha, y) - d(p_i, \alpha, y)\} \right\| . \end{aligned}$$

However,

$$\begin{aligned} (A4.2) \quad & \left\| \int_L \{d(p_i^*, \alpha, y) - d(p_i, \alpha, y)\} d\mu_{\alpha\nu} \right\| \\ & \leq \int_L \|d(p_i^*, \alpha, y) - d(p_i, \alpha, y)\| d\mu_{\alpha\nu} . \end{aligned}$$

If $\bar{f}(\Pi_i, \alpha)$ is monotonic, this last expression is an equality. Furthermore,

$$\begin{aligned} (A4.3) \quad & \int_L \|d(p_i^*, \alpha, y) - d(p_i, \alpha, y)\| d\mu_{\alpha\nu} \\ &= \int_{L-A} \|d(p_i^*, \alpha, y) - d(p_i, \alpha, y)\| d\mu_{\alpha\nu} \\ &+ \int_A \|d(p_i^*, \alpha, y) - d(p_i, \alpha, y)\| d\mu_{\alpha\nu} . \end{aligned}$$

By hypothesis d is continuous at p_i^* for $(\alpha, y) \in L-A$. Therefore, $\forall \epsilon > 0$, $\exists \delta_2 > 0$, $\delta_2 < \delta_1$ such that for $\forall (\alpha, y) \in L-A$, $||d(p_i^*, \alpha, y) - d(p_i, \alpha, y)|| < \epsilon$ for all p_i for which $|p_i - p_i^*| < \delta_2$. For such p_i ,

$$\int_{L-A} ||d(p_i^*, \alpha, y) - d(p_i, \alpha, y)|| d\mu_{xv} < \epsilon \int_{L-A} d\mu_{xv} = \epsilon \cdot \mu_{xv}(L-A) .$$

In addition, since $\delta_2 < \delta_1$, for such p_i

$$||d(p_i^*, \alpha, y) - d(p_i, \alpha, y)|| < ||\hat{x}|| ,$$

so that

$$\int_A ||d(p_i^*, \alpha, y) - d(p_i, \alpha, y)|| d\mu_{xv} < ||\hat{x}|| \mu_{xv}(A) .$$

If $\mu_{xv}(A) = 0$, then $\mu_{xv}(L-A) = 1$, and, for $p_i \in \Pi_i$ such that $|p_i^* - p_i| < \delta_2$. From (A4.3)

$$\int_L ||d(p_i^*, \alpha, y) - d(p_i, \alpha, y)|| d\mu_{xv} < \epsilon .$$

Therefore,

$$p \lim_{N \rightarrow \infty} ||\bar{d}_N(p_i^*) - \bar{d}(p_i)|| < \epsilon .$$

Part 2

If $d(p_i, \alpha, y)$ is monotonic in p_i , then from (A4.2) and (A4.3)

$$\begin{aligned} & || \int_L (d(p_i^*, \alpha, y) - d(p_i, \alpha, y)) d\mu_{xv} || \\ &= \int_{L-A} ||d(p_i^*, \alpha, y) - d(p_i, \alpha, y)|| d\mu_{xv} \\ &+ \int_A ||d(p_i^*, \alpha, y) - d(p_i, \alpha, y)|| d\mu_{xv} . \end{aligned}$$

By hypothesis, $d(p_i^*, \alpha, y)$ is continuous at p_i^* for all $(\alpha, y) \in L-A$. However, since d is monotonic in p_i , then $\forall (\alpha, y) \in A, \exists \xi(\alpha, y) > 0$ such that

$$||d(p_i^*, \alpha, y) - d(p_i, \alpha, y)|| > \xi(\alpha, y)$$

for either $\forall p_i > p_i^*$ or $\forall p_i < p_i^*$.

Define

$$A_1(p_i^*) = \{(\alpha, y) \in A(p_i^*) : ||d(p_i^*, \alpha, y) - d(p_i, \alpha, y)|| > \xi(\alpha, y) \forall p_i > p_i^*\}$$

$$A_2(p_i^*) = A(p_i^*) - A_1(p_i^*)$$

Suppose $\mu(A) > 0$. Then either $\mu(A_1) > 0$ or $\mu(A_2) > 0$.

Therefore, either

$$\eta_1 = \int_{A_1} \xi(\alpha, y) d\mu_{xv} > 0$$

or

$$\eta_2 = \int_{A_2} \xi(\alpha, y) d\mu_{xv} > 0$$

since $\xi(\alpha, y)$ is strictly positive on A . Let $\eta = \max(\eta_1, \eta_2)$.

Then $\forall \delta > 0, p_i \in \Pi_i$ for which $|p_i^* - p_i| < \delta$, such that

$$\int_A ||d(p_i^*, \alpha, y) - d(p_i, \alpha, y)|| d\mu_{xv} > \eta > 0$$

Consequently $\forall \delta > 0, p_i \in \Pi_i, |p_i^* - p_i| < \delta$ for which

$$p \lim_{N \rightarrow \infty} d_N(p_i^*) = p \lim_{N \rightarrow \infty} \bar{d}_N(p_i) > \eta,$$

contradicting (A4.1).

Hence, when d is monotonic in p_i , the continuity of $p \lim_{N \rightarrow \infty} \bar{d}_N$ requires $\mu \times \nu(A) = 0$. QED.

Continuity is extended to the entire domain of prices by means of Lemma 11.

Lemma 11. If $p \lim_{N \rightarrow \infty} \bar{d}_N(p_i)$ is Lipschitz continuous with respect to p_i for all i on a neighborhood of p^* , then $p \lim_{N \rightarrow \infty} \bar{d}_N$ is continuous with respect to p at p^* .

Proof:

If there exists a neighborhood of p^* , such that for all p in this neighborhood, $p \lim_{N \rightarrow \infty} \bar{d}_N(p_i)$ is Lipschitz continuous with respect to p_i for all i , then $\forall i$, there exists a δ_i, M_i such that if $|p_i' - p_i''| < \delta_i$, and $p_j' = p_j''$ for $j \neq i$,

$$(A4.4) \quad ||p \lim_{N \rightarrow \infty} \bar{d}_N(p_i') - p \lim_{N \rightarrow \infty} \bar{d}_N(p_i'')|| > M_i |p_i' - p_i''| \quad .$$

To show $p \lim_{N \rightarrow \infty} \bar{d}_N$ is continuous at p^* , $\forall \epsilon > 0$ consider any p' in the described neighborhood such that

$$||p^* - p'|| < \delta$$

where

$$(A4.5) \quad \delta = \min\{\delta_i, \frac{\epsilon}{(n+m-1) \cdot M_i} \mid i = 1, \dots, n+m-1\} \quad .$$

Consider the sequence of points $p^0, p^0, \dots, p^{n+m-1}$, such that

$$p^0 = p^* \quad \text{and}$$

$$p^i = p^{i-1} + (p_i' - p_i^*) \cdot i$$

where i is the unit vector with its i^{th} component equal to 1,

$$p' = p^{n+m-1}.$$

By the triangle inequality,

$$\begin{aligned} ||p \lim_{N \rightarrow \infty} d_N(p^*) - p \lim_{N \rightarrow \infty} d_N(p')|| &\leq \sum_{i=1}^{n+m-1} ||p \lim_{N \rightarrow \infty} d_N(p^{i-1}) \\ &\quad - p \lim_{N \rightarrow \infty} d_N(p^i)||. \end{aligned}$$

From (A4.4) and (A4.5)

$$||p \lim_{N \rightarrow \infty} d_N(p^{i-1}) - p \lim_{N \rightarrow \infty} d_N(p^i)|| < M_i \cdot \delta \leq \frac{\epsilon}{n+m-1}.$$

Therefore,

$$\forall p', ||p^* - p'|| < \delta, ||p \lim_{N \rightarrow \infty} d_N(p^*) - p \lim_{N \rightarrow \infty} d_N(p')|| > \epsilon. \text{ QED.}$$

Corollary 5. If $p^* > 0$ and the elasticity of $p \lim \bar{d}_N(p)$ with respect to p_i is bounded on a neighborhood of p^* for all i , then $p \lim \bar{d}_N(p)$ is continuous at p^* with respect to p .

Proof:

Set $q(p) = p \lim_{N \rightarrow \infty} \bar{d}_N(p)$. Suppose

$$\left| \frac{dq_j}{dp_i} \cdot \frac{p_i}{q_j} \right| < N_i \quad \forall i, \forall j \quad \text{on a given neighborhood of } p^*.$$

Then, if $p_i \neq 0$, $\left| \frac{dq_j}{dp_i} \right| < N_i \frac{q_j}{p_i}$ since $p_i, q_j \geq 0$. Set

$M_i = N_i \cdot \frac{\max \{q_j\}}{p_i}$. For any p_i' , p_i'' in the given neighborhood where $p_k' = p_k''$ for $k \neq i$,

$$||p \lim_{N \rightarrow \infty} d_N(p_i') - p \lim_{N \rightarrow \infty} d_N(p_i'')|| > M_i |p_i' - p_i''| \quad .$$

$p \lim_{N \rightarrow \infty} d_N(p)$ is continuous at p^* by Lemma 11. QED.

Lemma 12 examines a utility function for which the set $A(p_i)$ is readily defined.

Lemma 12. If $D(p, \alpha, y)$ is multivalued at p so that $\tilde{x} \in K_j$ and $\bar{x} \in K_i$ for $\bar{x}, \tilde{x} \in D(p, \alpha, y)$, and if $D(p, \alpha, y)$ maximizes the utility function $U(x) = x_1 x_2^{1-\alpha}$ subject to the budget constraint $p \cdot q(x) \leq y$ then

$$\alpha = 1 - \ln \left(\frac{y - a_i}{y - a_j} \right) / \ln \left(\frac{r_j}{r_i} \right)$$

where r_i is the i^{th} block price for x_2 , and

$$a_i = \sum_{t=1}^{i-1} r_t (k_t - k_{t-1}) \quad .$$

Proof:

Since U is monotonic, $\bar{x}$ and $\tilde{x}$ are on the budget constraint,

$$(A4.6) \quad y - a_i - r_i \bar{x}_2 - s_1 \bar{x}_1 = 0, \quad \text{and}$$

$$(A4.7) \quad y - a_j - r_i \tilde{x}_2 - s_1 \tilde{x}_1 = 0 \quad .$$

Since $(\bar{x}_1, \bar{x}_2)$ maximizes U subject to (A4.6), then

$$(A4.8) \quad \alpha \bar{x}_1^{\alpha-1} \bar{x}_2^{1-\alpha} - \Lambda_1 s_1 = 0$$

$$(1-\alpha) \bar{x}_1^\alpha \bar{x}_2^{-\alpha} - \Lambda_1 r_i = 0 \quad .$$

From (A4.6) and (A4.8),

$$(A4.9) \quad \frac{\alpha}{1-\alpha} \frac{\bar{x}_2}{\bar{x}_1} = \frac{s_1}{r_i} \quad , \quad \text{and}$$

$$(A4.10) \quad \bar{x}_2 = (1-\alpha)(y-a_i)/r_i \quad .$$

Similarly, if $(\tilde{x}_1, \tilde{x}_2)$ maximizes U subject to (A4.7), then

$$(A4.11) \quad \frac{\alpha}{1-\alpha} \frac{\tilde{x}_2}{\tilde{x}_1} = \frac{s_1}{r_j} \quad , \quad \text{and}$$

$$(A4.12) \quad \tilde{x}_2 = (1-\alpha)(y-a_j)/r_j \quad .$$

If $(\bar{x}_1, \bar{x}_2)$ and $(\tilde{x}_1, \tilde{x}_2)$ are both in $D(p, y)$ then $U(\bar{x}) = U(\tilde{x})$ and $\tilde{x}_1^\alpha \tilde{x}_2^{1-\alpha} = \bar{x}_1^\alpha \bar{x}_2^{1-\alpha}$. Then

$$(A4.13) \quad (\tilde{x}_1/\bar{x}_1)^\alpha = (\bar{x}_2/\tilde{x}_2)^{1-\alpha} \quad .$$

From (A4.9) and (A4.11)

$$(A4.14) \quad \tilde{x}_1/\bar{x}_1 = \tilde{x}_2 r_j / \bar{x}_2 r_i$$

and from (A4.13) and (A4.14)

$$(A4.15) \quad (r_j/r_i)^\alpha = (\bar{x}_2/\tilde{x}_2) \quad .$$

Incorporating (A4.10) and (A4.12) into (A4.15) yields

$$(A4.16) \quad (r_i/r_j)^{1-\alpha} = (y-a_i)/(y-a_j) \quad .$$

Solving (A4.16) for

$$\alpha = 1 - \ln \left(\frac{y-a_i}{y-a_j} \right) \div \left(\ln \frac{r_i}{r_j} \right) \quad . \quad QED.$$

APPENDIX 5

DATA APPENDIX

The specific variables selected for use in this study and their sources will be given in this appendix. Some of the variables employed are taken directly from established sources, and need no further explanation than their source and the reasoning behind their selection. On the other hand, some of the data were compiled for the first time for the purpose of this study and a more detailed definition of this data is necessary. The discussion of variables in this appendix will be organized around the nature of the compilation process and the sources of the data.

All of the price and quantity data for electricity were compiled for the first time for the purpose of this study. These variables will be discussed in Section A5.1. In addition to explicitly defining the variables, the compilation process and the methods of aggregation will be discussed.

Exclusive of the electricity price and quantity data, all the data employed in the electricity demand equations were taken from DRI consumer data banks. The exact sources of these variables and the reasoning behind their selection will be given in Section A5.2.

A5.1 Electricity Price and Quantity Data

The price and quantity data for electricity are interrelated by the aggregation process. In both methods of characterizing rate schedules, the rate schedules operative in each state were weighted by number of customers. In one method, information was also employed on the typical block level. Raw information on revenue, sales level, and number of customers by company are contained in one data set; raw rate schedules in another. Because the quantity data was to be used in conjunction with rate data in our models, companies were included in the aggregated state data only if information was available on the rate schedule, the number of customers and the consumption level.

The sources for the quantity data set were Statistics on Privately-Owned Electric Utilities in the United States¹ and Statistics of Publicly-Owned Electric Utilities in the United States.² In some cases this was augmented by information in Form 1, Table 414. Form 1 is a document filled out annually by large private utilities as required by the Federal Power Commission. It was particularly useful in

¹Statistics of Privately-Owned Electric Utilities in the United States--classes A and B, Federal Power Commission, U.S. Government Printing Office, Washington, D.C., 1955-1972.

²Statistics of Publicly-Owned Electric Utilities in the United States--classes A and B, Federal Power Commission, U.S. Government Printing Office, Washington, D.C., 1955-1972.

cases where there were multiple rate districts for a utility each having different rate schedules and, thus, each having to be considered separately. These reports give data by company (or rate district) on the number of residential customers, the sales level, and the amount of revenue collected. All of these reports are produced annually and give annual data. The public utility data, however, are reported on fiscal rather than calendar years. Approximate calendar year data was computed from the data by fiscal year.

The aggregation process for the quantity data was simply a matter of adding up the total number of customers for the companies included, and dividing this into total sales. There are, however, differences between the average sales data employed in this investigation and that employed in other studies. This is due to the spotty reporting of company data and to the exclusion of companies for which no rate data were available. Because of these factors the average sales data in this report differ slightly from the average sales data employed in other studies. In order to evaluate this difference a comparison was made between our average monthly sales variable SAL, and the variable employed in the DRI study of residential demand for electricity.³ This study employed published state figures.⁴ For 768 observations (the DRI data are

³Houthakker, Verleger, and Sheehan (1974).

⁴Statistical Yearbook, Edison Electric Institute, 1957-1972.

available back to 1957 only) the correlation between the two series is .983.

The rate data were processed in a number of stages prior to the aggregation process. The raw rate data were gotten from the National Electric Rate Book.⁵ In this publication rate schedules are given by rate district for every utility in the country. The rate book is published almost every year for every state, and gives the date at which the currently listed rate became effective. Given this information annual rates were calculated by interpolation of all the rates effective within a year. These rates were weighted by the number of days within the year that they were in effect. Spot rates for fuel adjustments are also given in the rate book. Annual fuel adjustments were calculated, where they were in effect for residential customers, by linearization between the spot rates. The average monthly fuel adjustment for the year was then added to the average monthly rate schedule.

The first method of approximating the rate schedule was by the typical block. In this technique the marginal price, MP, and an equivalent fixed charge, FC, to maintain the cost of the mean consumption level were calculated:

$$FC = \text{COST}(\text{SAL}) - \text{MP} * \text{SAL} \quad .$$

⁵National Electric Rate Book - rate schedules for electric service in communities of 2,500 population or more, residential, commercial and industrial services, Federal Power Commission, U.S. Government Printing Office, Washington, D.C., 1950-1972.

Although the distribution of customers by consumption levels was not available, the distribution of customers across utilities was, and this was taken advantage of. The price variables, FC and MP, were calculated for each company and then aggregated across the state weighting by number of customers. Several variations on the method of selecting the typical block were employed. Besides using the mean consumption level, the mean expenditure level and the mean price were used as explained in the text of this report. However, the aggregation procedure and the method of defining the variables remain the same regardless.

The second method of characterizing the rate schedule was by a double-logarithmic approximation to the entire average price schedule. It is not dependent on any particular consumption level. Therefore, the distribution of consumption levels is not important. In this technique a state average rate schedule was calculated for each year weighting each rate schedule in the state by number of customers. A double-logarithmic approximation was made to the average price function implied by this rate schedule by picking points off this function (at 5 Kwh intervals from 50 to 1500 Kwh) and employing the standard linear regression technique. The resulting price variables are called P1 and P2.

Data Obtained from DRI Consumer Databanks

Before proceeding to discuss other variables in the model, it should be noted that all price and income variables were deflated by the implicit price deflator for personal consumption expenditures. This series is from the DRI central databank.⁶ The same deflator was used for all states, since there was no personal consumption deflator available on a state-by-state basis. The deflator from the revised national income accounts was used, and prices and income are measured in 1972 dollars.

The income variable employed in this study was income per capita, YPC. This is the ratio of personal income⁷ to the total resident population in each state.⁸ These variables were taken from the DRI central and regional databanks respectively. It should be noted that the income variable is income per capita, not income per customer. The ratio of total residents to total customers is probably fairly stable across states, whereas the ratio of total to reported customers is not. Since personal income is measured for the whole state and total sales only for reporting utilities, the per capita income

⁶The source for this series is the U.S. Department of Commerce, Bureau of Economic Analysis, National Income and Product Accounts of the United States, Table 8.

⁷U.S. Department of Commerce, Bureau of Economic Analysis.

⁸U.S. Department of Commerce, Bureau of the Census. Current Population Reports, Series P-25. Estimates are for July 1.

variable is more comparable to the customer sales variable than income per reported customer would be.

The price variable included for natural gas was average price. To be consistent with the approach taken in this study, a similar sort of characterization of natural gas rate schedules should be employed. However, such an effort was not feasible within the constraints of this study. The average price of natural gas, PNG, is a ratio of total gas utility revenues to total sales within each state.⁹ These variables came from the DRI energy databank.

Two weather variables, heating degree days, HDD, and cooling degree days, WDDC, were also included in the model. These also came from the DRI energy data bank. They are composite indices of the demand for heating and the demand for cooling. The heating degree days variable was calculated in the following manner. The difference between sixty-five degrees and the local temperature, if it was less than sixty-five, was taken at set intervals within each day in weather stations across a state. These numbers were then multiplied by some measure of fuel demand and weighted across the state by population. This is not an ideal measure of the effects of weather on electricity demand. It is the best currently available. Cooling degree days were calculated in a similar manner.

⁹The source for these series was Gas Facts, American Gas Association, 1955-1972.

¹⁰Source, National Oceanic and Atmospheric Admin.

Finally, for comparison purposes, some experiments were done using prices derived from Typical Electric Bills rather than rate schedules. DRI has compiled a weighted average of information in typical electric bills for a sample of utilities in each state.¹¹ This includes the typical bill for 100, 250, and 500 kilowatt hours. From this information the average marginal price between 100 and 250 kwh, MP100T250, could be calculated. Similarly, MP100T500, and MP250T500 were calculated. For each of these marginal prices there is an implicit fixed charge needed to maintain the same typical bills as were employed in calculating the marginal price. For MP100T250 this implicit fixed charge, FC100T250, was defined:

$$FC100T250 = PELCRES100 - 100*MP100T250 \quad ,$$

where PELCRES100 was the average expenditure on 100 Kwh given in the DRI Energy Databank. These variables were used in experiments to determine how critical it is to derive the price variables from actual rate schedules.

¹¹Typical Electric Bills, U.S. Federal Power Commission.

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