Supplementary Appendix for

"Distributed Lag Interaction Models with Two Pollutants"

Yin-Hsiu Chen[1], **Bhramar Mukherjee**[1], **Veronica Berrocal**[1]

[1]Department of Biostatistics, University of Michigan

A.1 Iterative Algorithm for Tukey's Distributed Lag Model (TDLM)

Define Vec$(\cdot)$ as an operator that transforms a $q \times r$ matrix $\boldsymbol{A}$ to a column vector $(a_{11}, \cdots, a_{1r}, \cdots, a_{qr})^T$ of length qr and Vec$^{(-1)}(\cdot)$ be the reversed operator. Let $Q(\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \eta)$ be the likelihood function of TDLM evaluated at $\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \eta$, namely

$$Q(\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \eta) = \sum_{t=1}^{T} [y_t [\boldsymbol{W}_{1t}^\top \boldsymbol{\theta}_1 + \boldsymbol{W}_{2t}^\top \boldsymbol{\theta}_2 + \boldsymbol{W}_{It}^\top \boldsymbol{\theta}_I]^\top - e^{\boldsymbol{W}_{1t}^\top \boldsymbol{\theta}_1 + \boldsymbol{W}_{2t}^\top \boldsymbol{\theta}_2 + \boldsymbol{W}_{It}^\top \boldsymbol{\theta}_I} - \log(y_t!)]$$

The iterative algorithm for fitting a Tukey's distributed lag model proceeds as follows:
Step 1: Initialize $\hat{\boldsymbol{\theta}}_1^{(0)}$, $\hat{\boldsymbol{\theta}}_2^{(0)}$, and $\hat{\eta}^{(0)}$

Step 2: Given $\hat{\boldsymbol{\theta}}_2^{(m-1)}$ and $\hat{\eta}^{(m-1)}$, update $\hat{\boldsymbol{\theta}}_1^{(m)}$.

Let $W_{0t}^* = \boldsymbol{W}_{2t}^\top \hat{\boldsymbol{\theta}}_2^{(m-1)}$ and $\boldsymbol{W}_{1t}^* = \boldsymbol{W}_{1t} + \hat{\eta}^{(m-1)} \text{Vec}^{-1}(\boldsymbol{W}_{It}) \hat{\boldsymbol{\theta}}_2^{(m-1)}$.

$$\hat{\boldsymbol{\theta}}_1^{(m)} = \underset{\boldsymbol{\theta}_1}{\text{argmin}} \sum_{t=1}^{T} [y_t [W_{0t}^* + \boldsymbol{W}_{1t}^{*\top} \boldsymbol{\theta}_1] - e^{W_{0t} + \boldsymbol{W}_{1t}^{*\top} \boldsymbol{\theta}_1} - \log(y_t!)]$$

Step 3: Given $\hat{\boldsymbol{\theta}}_1^{(m)}$ and $\hat{\eta}^{(m-1)}$, update $\hat{\boldsymbol{\theta}}_2^{(m)}$.

Let $W_{0t}^* = \boldsymbol{W}_{1t}^\top \hat{\boldsymbol{\theta}}_1^{(m)}$ and $\boldsymbol{W}_{2t}^* = \boldsymbol{W}_{2t} + \hat{\eta}^{(m-1)} \text{Vec}^{-1}(\boldsymbol{W}_{It}^T) \hat{\boldsymbol{\theta}}_1^{(m)}$.

$$\hat{\boldsymbol{\theta}}_2^{(m)} = \underset{\boldsymbol{\theta}_2}{\text{argmin}} \sum_{t=1}^{T} [y_t [W_{0t}^* + \boldsymbol{W}_{2t}^{*\top} \boldsymbol{\theta}_1] - e^{W_{0t} + \boldsymbol{W}_{2t}^{*\top} \boldsymbol{\theta}_2} - \log(y_t!)]$$

Step 4: Given $\hat{\boldsymbol{\theta}}_1^{(m)}$ and $\hat{\boldsymbol{\theta}}_2^{(m)}$, update $\hat{\eta}^{(m)}$.

Let $W_{0t}^* = \boldsymbol{W}_{1t}^\top \hat{\boldsymbol{\theta}}_1^{(m)} + \boldsymbol{W}_{2t}^\top \hat{\boldsymbol{\theta}}_2^{(m)}$ and $W_{It}^* = \boldsymbol{W}_{It}^\top (\hat{\boldsymbol{\theta}}_1^{(m)} \otimes \hat{\boldsymbol{\theta}}_2^{(m)})$.

$$\hat{\eta}^{(m)} = \underset{\eta}{\text{argmin}} \sum_{t=1}^{T} [y_t [W_{0t}^* + \eta W_{It}^*] - e^{W_{0t}^* + \eta W_{It}^*} - \log(y_t!)]$$

Step 5: Compute relative increase in likelihood

$$\Delta^{(m)} = \frac{Q(\hat{\boldsymbol{\theta}}_1^{(m)}, \hat{\boldsymbol{\theta}}_2^{(m)}, \hat{\eta}^{(m)})}{Q(\hat{\boldsymbol{\theta}}_1^{(m-1)}, \hat{\boldsymbol{\theta}}_2^{(m-1)}, \hat{\eta}^{(m-1)})} - 1$$

Stop the algorithm if $\Delta^{(m)}$ is smaller than the pre-specified value. Otherwise, repeat steps 2-4 until convergence.

A.2 Full Conditional Distribution of Bayesian Tukey's Distributed Lag Model (BTDLM)

With constraints $\boldsymbol{\beta}_1 = \boldsymbol{C}_1\boldsymbol{\theta}_1$, $\boldsymbol{\beta}_2 = \boldsymbol{C}_2\boldsymbol{\theta}_2$, and $\boldsymbol{\gamma} = \boldsymbol{\eta}\odot(\boldsymbol{\beta}_1\otimes\boldsymbol{\beta}_2)$, we have $f(\boldsymbol{Y}|\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \boldsymbol{\eta}) =$

$\exp\{\boldsymbol{Y}^T[\boldsymbol{X}_1\boldsymbol{C}_1\boldsymbol{\theta}_1+\boldsymbol{X}_2\boldsymbol{C}_2\boldsymbol{\theta}_2+\boldsymbol{X}_I(\boldsymbol{\eta}\odot((\boldsymbol{C}_1\otimes\boldsymbol{C}_2)(\boldsymbol{\theta}_1\otimes\boldsymbol{\theta}_2)))]-e^{\boldsymbol{X}_1\boldsymbol{C}_1\boldsymbol{\theta}_1+\boldsymbol{X}_2\boldsymbol{C}_2\boldsymbol{\theta}_2+\boldsymbol{X}_I(\boldsymbol{\eta}\odot((\boldsymbol{C}_1\otimes\boldsymbol{C}_2)(\boldsymbol{\theta}_1\otimes\boldsymbol{\theta}_2)))}\}$

The full conditional distributions of $\boldsymbol{\theta}_1$, $\boldsymbol{\theta}_2$, $\boldsymbol{\eta}$, σ^2 and ω are

$$f(\boldsymbol{\theta}_1|\boldsymbol{\theta}_2, \boldsymbol{\eta}, \sigma^2, \omega, \boldsymbol{Y}) \propto f(\boldsymbol{Y}|\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \boldsymbol{\eta}) \cdot \exp(-\frac{\boldsymbol{\theta}_1^T\boldsymbol{\theta}_1}{2\cdot 100^2})$$

$$f(\boldsymbol{\theta}_2|\boldsymbol{\theta}_1, \boldsymbol{\eta}, \sigma^2, \omega, \boldsymbol{Y}) \propto f(\boldsymbol{Y}|\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \boldsymbol{\eta}) \cdot \exp(-\frac{\boldsymbol{\theta}_2^T\boldsymbol{\theta}_2}{2\cdot 100^2})$$

$$f(\boldsymbol{\eta}|\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \sigma^2, \omega, \boldsymbol{Y}) \propto f(\boldsymbol{Y}|\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \boldsymbol{\eta}) \cdot \exp(-\frac{\boldsymbol{\eta}^T\boldsymbol{\Sigma}(\omega)\boldsymbol{\eta}}{2\sigma^2})$$

$$f(\sigma^2|\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \boldsymbol{\eta}, \omega, \boldsymbol{Y}) \sim IG(a + (L_1+1)(L_2+1)/2, b + \boldsymbol{\eta}^T\boldsymbol{\Sigma}^{-1}(\omega)\boldsymbol{\eta}/2)$$

$$p(\omega|\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \boldsymbol{\eta}, \sigma^2, \boldsymbol{Y}) = \frac{|\sigma^2\boldsymbol{\Sigma}(\omega)|^{-1/2}\exp(-\frac{1}{2\sigma^2}\boldsymbol{\eta}^T\boldsymbol{\Sigma}^{-1}(\omega)\boldsymbol{\eta})}{\sum_{\omega^*}|\sigma^2\boldsymbol{\Sigma}(\omega^*)|^{-1/2}\exp(-\frac{1}{2\sigma^2}\boldsymbol{\eta}^T\boldsymbol{\Sigma}^{-1}(\omega^*)\boldsymbol{\eta})}$$

A.3 Full Conditional Distribution of Bayesian Constrained Distributed Lag Model (BCDLM)

With constraints $\boldsymbol{\beta}_1 = \boldsymbol{C}_1\boldsymbol{\theta}_1 + \boldsymbol{C}_1^{\mathsf{c}}\boldsymbol{\theta}_1^{\mathsf{c}}$, $\boldsymbol{\beta}_2 = \boldsymbol{C}_2\boldsymbol{\theta}_2 + \boldsymbol{C}_2^{\mathsf{c}}\boldsymbol{\theta}_2^{\mathsf{c}}$, and $\boldsymbol{\gamma} = \boldsymbol{C}_I\boldsymbol{\theta}_I + \boldsymbol{C}_I^{\mathsf{c}}\boldsymbol{\theta}_I^{\mathsf{c}}$, we have $f(\boldsymbol{Y}|\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \boldsymbol{\theta}_I, \boldsymbol{\theta}_1^{\mathsf{c}}, \boldsymbol{\theta}_2^{\mathsf{c}}, \boldsymbol{\theta}_I^{\mathsf{c}})$
$= \exp\{\boldsymbol{Y}^T[\boldsymbol{X}_1\boldsymbol{C}_1\boldsymbol{\theta}_1 + \boldsymbol{X}_2\boldsymbol{C}_2\boldsymbol{\theta}_2 + \boldsymbol{X}_I\boldsymbol{C}_I\boldsymbol{\theta}_I + \boldsymbol{X}_1\boldsymbol{C}_1^{\mathsf{c}}\boldsymbol{\theta}_1^{\mathsf{c}} + \boldsymbol{X}_2\boldsymbol{C}_2^{\mathsf{c}}\boldsymbol{\theta}_2^{\mathsf{c}} + \boldsymbol{X}_I\boldsymbol{C}_I^{\mathsf{c}}\boldsymbol{\theta}_I^{\mathsf{c}}]$
$-e^{\boldsymbol{X}_1\boldsymbol{C}_1\boldsymbol{\theta}_1+\boldsymbol{X}_2\boldsymbol{C}_2\boldsymbol{\theta}_2+\boldsymbol{X}_I\boldsymbol{C}_I\boldsymbol{\theta}_I+\boldsymbol{X}_1\boldsymbol{C}_1^{\mathsf{c}}\boldsymbol{\theta}_1^{\mathsf{c}}+\boldsymbol{X}_2\boldsymbol{C}_2^{\mathsf{c}}\boldsymbol{\theta}_2^{\mathsf{c}}+\boldsymbol{X}_I\boldsymbol{C}_I^{\mathsf{c}}\boldsymbol{\theta}_I^{\mathsf{c}}}$. Without loss of generality, we only present the conditional distribution of $\boldsymbol{\theta}_1$, $\boldsymbol{C}_1^{\mathsf{c}}$, and σ_1^2. The conditional distributions of other parameters can be constructed by symmetry.

$$f(\boldsymbol{\theta}_1|\boldsymbol{\theta}_2, \boldsymbol{\theta}_I, \boldsymbol{\theta}_1^{\mathsf{c}}, \boldsymbol{\theta}_2^{\mathsf{c}}, \boldsymbol{\theta}_I^{\mathsf{c}}, \sigma_1^2, \sigma_2^2, \sigma_I^2, \boldsymbol{Y}) \propto f(\boldsymbol{Y}|\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \boldsymbol{\theta}_I, \boldsymbol{\theta}_1^{\mathsf{c}}, \boldsymbol{\theta}_2^{\mathsf{c}}, \boldsymbol{\theta}_I^{\mathsf{c}}) \cdot \exp(-\frac{\boldsymbol{\theta}_1^\top\boldsymbol{\theta}_1}{2\cdot 100^2})$$

$$f(\boldsymbol{\theta}_1^{\mathsf{c}}|\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \boldsymbol{\theta}_I, \boldsymbol{\theta}_2^{\mathsf{c}}, \boldsymbol{\theta}_I^{\mathsf{c}}, \sigma_1^2, \sigma_2^2, \sigma_I^2, \boldsymbol{Y}) \propto f(\boldsymbol{Y}|\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \boldsymbol{\theta}_I, \boldsymbol{\theta}_1^{\mathsf{c}}, \boldsymbol{\theta}_2^{\mathsf{c}}, \boldsymbol{\theta}_I^{\mathsf{c}}) \cdot \exp(-\frac{\boldsymbol{\theta}_1^{\mathsf{c}^\top}\boldsymbol{\theta}_1^{\mathsf{c}}}{2\sigma_1^2})$$

$$f(\sigma_1^2|\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \boldsymbol{\theta}_I, \boldsymbol{\theta}_1^{\mathsf{c}}, \boldsymbol{\theta}_2^{\mathsf{c}}, \boldsymbol{\theta}_I^{\mathsf{c}}, \sigma_2^2, \sigma_I^2, \boldsymbol{Y}) \sim IG(a_0 + (L_1 - p_1 + 1)/2, b_0 + \boldsymbol{\theta}_1^{\mathsf{c}^\top}\boldsymbol{\theta}_1^{\mathsf{c}}/2)$$

A.4 Dual Representation of Bayesian Constrained Distributed Lag Model (BCDLM)

The dual representation of BCDLM in a frequentist framework is to minimize

$$-\sum_{t=1}^{T}\{y_t[\boldsymbol{X}_{1t}^{\top}(\boldsymbol{C}_1\boldsymbol{\theta}_1+\boldsymbol{C}_1^{\mathsf{c}}\boldsymbol{\theta}_1^{\mathsf{c}})+\boldsymbol{X}_{2t}^{\top}(\boldsymbol{C}_2\boldsymbol{\theta}_2+\boldsymbol{C}_2^{\mathsf{c}}\boldsymbol{\theta}_2^{\mathsf{c}})+\boldsymbol{X}_{It}^{\top}(\boldsymbol{C}_I\boldsymbol{\theta}_I+\boldsymbol{C}_I^{\mathsf{c}}\boldsymbol{\theta}_I^{\mathsf{c}})]$$
$$-e^{\boldsymbol{X}_{1t}^{\top}(\boldsymbol{C}_1\boldsymbol{\theta}_1+\boldsymbol{C}_1^{\mathsf{c}}\boldsymbol{\theta}_1^{\mathsf{c}})+\boldsymbol{X}_{2t}^{\top}(\boldsymbol{C}_2\boldsymbol{\theta}_2+\boldsymbol{C}_2^{\mathsf{c}}\boldsymbol{\theta}_2^{\mathsf{c}})+\boldsymbol{X}_{It}^{\top}(\boldsymbol{C}_I\boldsymbol{\theta}_I+\boldsymbol{C}_I^{\mathsf{c}}\boldsymbol{\theta}_I^{\mathsf{c}})}-\log(y_t!)\}$$
$$+\lambda_1\boldsymbol{\theta}_1^{\mathsf{c}\top}\boldsymbol{\theta}_1^{\mathsf{c}}+\lambda_2\boldsymbol{\theta}_2^{\mathsf{c}\top}\boldsymbol{\theta}_2^{\mathsf{c}}+\lambda_I\boldsymbol{\theta}_I^{\mathsf{c}\top}\boldsymbol{\theta}_I^{\mathsf{c}}$$

where λ_1, λ_2, and λ_I are tuning parameters that control the amount of shrinkage. When λ_1, λ_2, $\lambda_I \to 0$, the DL coefficients estimates approach $\hat{\boldsymbol{\Psi}}_{UDLM}$ as full-rank transformation of regression coefficients and that does not change the model fit of the unpenalized likelihood. When λ_1, λ_2, $\lambda_I \to \infty$, the DL coefficients estimates approach $\hat{\boldsymbol{\Psi}}_{BiDLM}$ as $\boldsymbol{\theta}_1^{\mathsf{c}}$, $\boldsymbol{\theta}_2^{\mathsf{c}}$, and $\boldsymbol{\theta}_I^{\mathsf{c}}$ are all shrunk to zero.

The asymptotic mean squared error (MSE) of $\hat{\boldsymbol{\Psi}}_{BCDLM}$ can be decomposed into the sum of squared bias

$$\boldsymbol{\Psi}^{\top}(\boldsymbol{W}^{\top}\hat{\boldsymbol{\Omega}}\boldsymbol{W}+\boldsymbol{S})^{-2}\boldsymbol{\Psi}$$

and variance

$$\sum_{j=1}^{p_1}\frac{1}{k_j}+\sum_{j=p_1+1}^{L_1+1}\frac{k_j}{(k_j+\lambda_1)^2}+\sum_{j=L_1+2}^{d_1}\frac{1}{k_j}+\sum_{j=d_1+1}^{d_2}\frac{k_j}{(k_j+\lambda_2)^2}+\sum_{j=d_2+1}^{d_3}\frac{1}{k_j}+\sum_{j=d_3+1}^{d_4}\frac{k_j}{(k_j+\lambda_I)^2}$$

where $\boldsymbol{W}=\left[\boldsymbol{X}_1\boldsymbol{C}_1|\boldsymbol{X}_1\boldsymbol{C}_1^{\mathsf{c}}|\boldsymbol{X}_2\boldsymbol{C}_2|\boldsymbol{X}_2\boldsymbol{C}_2^{\mathsf{c}}|\boldsymbol{X}_I\boldsymbol{C}_I|\boldsymbol{X}_I\boldsymbol{C}_I^{\mathsf{c}}\right]$, $\hat{\boldsymbol{\Omega}}$ is a diagonal matrix with the mean value of $\boldsymbol{Y}$ along the diagonal, $\boldsymbol{S}=\text{diag}[\boldsymbol{0}_{p_1},\lambda_1\boldsymbol{1}_{L_1+1},\boldsymbol{0}_{p_2},\lambda_2\boldsymbol{1}_{L_2+1},\boldsymbol{0}_{p_1p_2},\lambda_I\boldsymbol{I}_{(L_1+1)(L_2+1)}]$, k_j is the j^{th} eigenvalue of $\boldsymbol{W}^{\top}\hat{\boldsymbol{\Omega}}\boldsymbol{W}$, and $d_1=L_1+p_2+1$, $d_2=L_1+L_2+2$, $d_3=L_1+L_2+p_1p_2+2$, and $d_4=(L_1+1)+(L_2+1)+(L_1+1)(L_2+1)$, respectively. Through some algebra, it can be shown that the squared bias is a monotonically increasing function and the variance is a monotonically decreasing function of the tuning parameters. Along with the convexity of the asymptotic MSE, there always exist λ_1, λ_2, $\lambda_I > 0$ that achieve greater bias-variance tradeoff.

A.5 Computational Times

Table 1: Average computational times for applying five two-pollutant distributed lag models on an Intel i7-2600 CPU with a single 3.4GHz core in one simulation scenario with time series length $T = 1000$, maximum number of lags for the first pollutant $L_1 = 9$, and maximum number of lags for the second pollutant $L_2 = 9$ based on 1000 simulated datasets.

Methods	Time
UDLM	0.07 seconds
BiDLM	0.01 seconds
TDLM	0.33 seconds
BTDLM[1]	8.9 mins
BCDLM[1]	8.8 mins

[1] Gibbs sampler is based on 20000 burn-ins and 10000 posterior draws with thinning interval equal to 10

Table 2: Computational times for applying five two-pollutant distributed lag models on an Intel i7-2600 CPU with a single 3.4GHz core to the National Morbidity and Mortality Air Pollution Study (NMMAPS) to estimate the lagged effects of air particulate matter with aerodynamic diameter less than 10 micrometers (PM_{10}) and ozone (O_3) concentration on mortality in Chicago, Illinois from 1987 to 2000.

Methods	Time
UDLM	9.2 seconds
BiDLM	3.2 seconds
TDLM[1]	190 mins
BTDLM[2]	79 mins
BCDLM[2]	82 mins

[1] Standard error estimate is based on 1000 bootstrap samples

[2] Gibbs sampler is based on 20000 burn-ins and 10000 posterior draws with thinning interval equal to 10

A.6 Simulation Settings

Main effect coefficients $\boldsymbol{\beta}_1$ and $\boldsymbol{\beta}_2$:

- (a) Cubic:

$$\beta_{ij} = 0.0014[j(j-7)(j-9)+8] \text{ for } i = 1, 2 \text{ and } j = 0, \cdots, 9$$

- (b) Function with departure from cubic:

$$\boldsymbol{\beta}_1 = \boldsymbol{\beta}_2 = (0.25, 0.2, 0, -0.1, -0.15, 0, 0.2, 0.1, 0, -0.05)^\top$$

Interaction coefficients $\boldsymbol{\gamma}$:

- (1) No interaction: $\gamma_{ij} = 0$ for $i = 0, \cdots, L_1$ and $j = 0, \cdots, L_2$
- (2) Tukey's style interaction: interaction effects are scaled product of their corresponding main effects - $\boldsymbol{\gamma} \propto (\boldsymbol{\beta}_1 \otimes \boldsymbol{\beta}_2)$
- (3) Kronecker product interaction: basis functions for interaction effects are tensor product of main-effect basis functions - $\boldsymbol{\gamma} \propto (\boldsymbol{C}_1 \otimes \boldsymbol{C}_2)\boldsymbol{\theta}_I$
- (4) Sparse interaction: only the interaction terms between exposure 1 at lag 1-2 and exposure 2 at lag 1-2 are nonzero

$$\begin{cases} \gamma_{ij} = 0.7, & i = 1, 2 \text{ and } j = 1, 2 \\ \gamma_{ij} = 0, & \text{otherwise} \end{cases}$$

- (5) Unstructured interaction:

$$v = (0.2, 0.6, 1, 0.5, 0, 0.1, 0.7, 0.9, 0.2, 0.4)^\top$$
$$\boldsymbol{\gamma} = v \otimes v$$

A.7 Summary Statistics of NMMAPS

The mean concentrations (standard deviations in parentheses) of PM_{10} and O_3 are 37.06 (19.25) $\mu g/m^3$ and 19.14 (10.20) ppb, respectively. The three quartiles of PM_{10} are 24.29 $\mu g/m^3$, 34.25 $\mu g/m^3$, and 45.78 $\mu g/m^3$ and the three quartiles of O_3 are 13.51 ppb, 20.53 ppb, and 27.92 ppb, respectively. We observed minimal skewness in PM_{10} and O_3 measurements and no extreme values needed further investigation. The partial autocorrelation function (PACF) plots indicate that the autocorrelation PM_{10} and O_3 have autocorrelation 0.42 and 0.74, respectively, at lag 1. O_3 time series displays strong correlation and slower decay than PM_{10}. The correlation on both time series suggests that some smoothing on the DL coefficients is needed. The average daily non-accidental mortality count is 38.47 with standard deviation equal to 15.89.

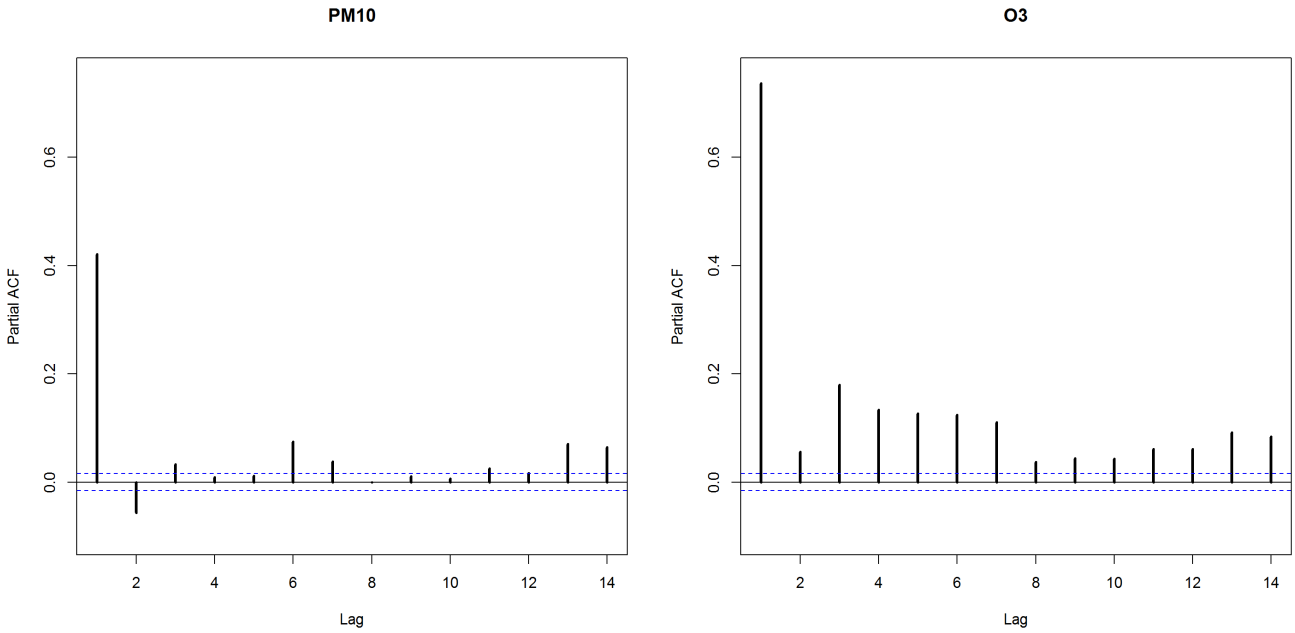


Figure 1: Partial autocorrelation function (PACF) plots for daily measurements of PM_{10} (left) and O_3 (right) in Chicago, Illinois from 1987 to 2000 based on the National Morbidity, Mortality, and Air Pollution Study (NMMAPS) data.

A.8 NMMAPS Analysis

Table 3: Estimated marginal lag effects (% change in mortality count) and standard deviations (in parenthesis) associated with a 10 $\mu g/m^3$ increase in PM_{10} on mortality where O_3 is kept at average winter levels and average summer levels under five estimation methods. Estimates refer to mortality in Chicago, Illinois from 1987 to 2000 and have been derived using the National Morbidity, Mortality, and Air Pollution Study (NMMAPS) data.

	UDLM		CDLM		TDLM		BTDLM		BCDLM	
	Winter	Summer	Winter	Summer	Winter	Summer	Winter	Summer	Winter	Summer
Lag 0	-0.22	-0.31	-0.10	-0.19	0.03	-0.01	-0.19	-0.21	0.00	0.00
	(0.14)	(0.15)	(0.11)	(0.11)	(0.06)	(0.04)	(0.07)	(0.08)	(0.04)	(0.03)
Lag 1	-0.17	-0.18	-0.09	0.01	-0.08	0.02	-0.17	-0.16	-0.14	-0.08
	(0.14)	(0.14)	(0.06)	(0.06)	(0.04)	(0.06)	(0.07)	(0.07)	(0.06)	(0.06)
Lag 2	-0.07	-0.03	-0.08	0.11	-0.12	0.03	-0.02	0.09	-0.06	0.10
	(0.14)	(0.14)	(0.05)	(0.06)	(0.06)	(0.08)	(0.06)	(0.07)	(0.06)	(0.08)
Lag 3	0.16	0.53	-0.07	0.14	-0.13	0.03	0.05	0.23	0.12	0.36
	(0.14)	(0.14)	(0.05)	(0.05)	(0.06)	(0.08)	(0.06)	(0.06)	(0.05)	(0.08)
Lag 4	-0.01	0.17	-0.06	0.12	-0.11	0.03	-0.03	0.15	-0.02	0.14
	(0.13)	(0.14)	(0.05)	(0.05)	(0.05)	(0.06)	(0.06)	(0.08)	(0.04)	(0.06)
Lag 5	-0.05	0.18	-0.05	0.07	-0.08	0.02	-0.13	0.01	-0.14	-0.06
	(0.13)	(0.14)	(0.04)	(0.05)	(0.04)	(0.04)	(0.04)	(0.06)	(0.05)	(0.06)
Lag 6	-0.18	-0.35	-0.05	0.01	-0.05	0.01	-0.16	-0.09	-0.11	-0.05
	(0.13)	(0.14)	(0.04)	(0.05)	(0.04)	(0.03)	(0.06)	(0.08)	(0.05)	(0.07)
Lag 7	-0.10	0.16	-0.05	-0.04	-0.03	0.01	-0.05	-0.03	-0.06	-0.01
	(0.13)	(0.14)	(0.04)	(0.05)	(0.04)	(0.02)	(0.05)	(0.05)	(0.03)	(0.04)
Lag 8	-0.03	-0.09	-0.06	-0.07	-0.03	0.01	0.05	0.02	-0.06	-0.02
	(0.14)	(0.14)	(0.04)	(0.04)	(0.04)	(0.03)	(0.06)	(0.08)	(0.03)	(0.03)
Lag 9	0.16	-0.03	-0.07	-0.09	-0.03	0.01	0.02	-0.02	-0.08	-0.06
	(0.14)	(0.14)	(0.04)	(0.04)	(0.04)	(0.03)	(0.05)	(0.06)	(0.04)	(0.04)
Lag 10	0.00	0.05	-0.09	-0.09	-0.05	0.01	-0.07	-0.06	-0.07	-0.04
	(0.14)	(0.14)	(0.04)	(0.05)	(0.04)	(0.03)	(0.06)	(0.07)	(0.04)	(0.04)
Lag 11	-0.09	-0.23	-0.10	-0.07	-0.06	0.02	-0.17	-0.07	-0.09	-0.05
	(0.14)	(0.14)	(0.05)	(0.05)	(0.04)	(0.03)	(0.06)	(0.08)	(0.04)	(0.05)
Lag 12	-0.35	0.18	-0.10	-0.04	-0.07	0.02	-0.17	-0.08	-0.06	-0.01
	(0.14)	(0.14)	(0.05)	(0.05)	(0.04)	(0.03)	(0.06)	(0.07)	(0.04)	(0.05)
Lag 13	-0.02	-0.46	-0.09	-0.02	-0.05	0.01	-0.08	-0.07	-0.06	-0.02
	(0.13)	(0.14)	(0.05)	(0.05)	(0.04)	(0.03)	(0.06)	(0.08)	(0.03)	(0.04)
Lag 14	0.06	0.33	-0.05	-0.02	0.03	-0.01	-0.02	-0.04	0.01	0.01
	(0.13)	(0.13)	(0.09)	(0.09)	(0.07)	(0.06)	(0.05)	(0.06)	(0.07)	(0.05)

Table 4: Estimated marginal lag effects (% change in mortality count) and standard deviations (in parenthesis) associated with a 10 ppb increase in O_3 on mortality where PM_{10} is kept at average winter levels and average summer levels under five estimation methods. Estimates refer to mortality in Chicago, Illinois from 1987 to 2000 and have been derived using the National Morbidity, Mortality, and Air Pollution Study (NMMAPS) data.

	UDLM		CDLM		TDLM		BTDLM		BCDLM	
	Winter	Summer	Winter	Summer	Winter	Summer	Winter	Summer	Winter	Summer
Lag 0	0.49	0.22	0.36	0.30	-0.04	0.16	0.05	0.12	0.11	0.36
	(0.34)	(0.33)	(0.25)	(0.23)	(0.17)	(0.23)	(0.14)	(0.15)	(0.10)	(0.13)
Lag 1	-0.48	0.06	0.06	0.33	-0.10	0.33	0.13	0.35	-0.09	0.22
	(0.35)	(0.34)	(0.13)	(0.12)	(0.17)	(0.20)	(0.13)	(0.12)	(0.14)	(0.15)
Lag 2	0.68	1.09	-0.11	0.28	-0.11	0.35	0.14	0.56	0.13	0.50
	(0.35)	(0.34)	(0.13)	(0.12)	(0.17)	(0.19)	(0.14)	(0.15)	(0.14)	(0.20)
Lag 3	-0.54	-0.70	-0.16	0.18	-0.09	0.28	-0.07	0.32	-0.11	0.14
	(0.35)	(0.34)	(0.12)	(0.11)	(0.15)	(0.16)	(0.14)	(0.15)	(0.15)	(0.16)
Lag 4	-0.24	0.46	-0.16	0.06	-0.06	0.16	-0.34	-0.19	-0.10	0.05
	(0.34)	(0.34)	(0.10)	(0.10)	(0.10)	(0.11)	(0.16)	(0.17)	(0.08)	(0.10)
Lag 5	0.00	-0.65	-0.09	-0.02	-0.02	0.05	-0.21	-0.20	-0.13	-0.08
	(0.35)	(0.35)	(0.10)	(0.09)	(0.05)	(0.06)	(0.14)	(0.14)	(0.10)	(0.08)
Lag 6	-0.44	0.21	-0.02	-0.07	0.02	-0.04	0.14	0.12	0.01	0.00
	(0.35)	(0.35)	(0.10)	(0.09)	(0.04)	(0.05)	(0.17)	(0.17)	(0.04)	(0.02)
Lag 7	0.56	0.53	0.06	-0.07	0.04	-0.09	0.13	0.02	0.00	-0.07
	(0.35)	(0.35)	(0.10)	(0.10)	(0.07)	(0.06)	(0.13)	(0.14)	(0.08)	(0.07)
Lag 8	-0.26	-0.42	0.11	-0.03	0.06	-0.10	0.01	-0.19	0.04	-0.05
	(0.35)	(0.36)	(0.10)	(0.10)	(0.08)	(0.07)	(0.18)	(0.19)	(0.05)	(0.06)
Lag 9	0.14	-0.06	0.13	0.00	0.05	-0.09	0.12	-0.07	0.05	-0.04
	(0.35)	(0.36)	(0.09)	(0.09)	(0.08)	(0.06)	(0.15)	(0.16)	(0.07)	(0.07)
Lag 10	0.76	0.17	0.11	0.04	0.04	-0.06	0.25	0.13	0.10	0.03
	(0.35)	(0.36)	(0.10)	(0.10)	(0.06)	(0.05)	(0.18)	(0.21)	(0.07)	(0.06)
Lag 11	-0.49	0.11	0.05	0.05	0.01	-0.02	0.15	0.14	0.09	0.06
	(0.35)	(0.35)	(0.12)	(0.11)	(0.06)	(0.05)	(0.17)	(0.15)	(0.08)	(0.06)
Lag 12	0.32	0.34	-0.05	0.01	-0.01	0.01	-0.09	-0.02	0.02	0.01
	(0.35)	(0.35)	(0.12)	(0.12)	(0.07)	(0.06)	(0.16)	(0.14)	(0.06)	(0.05)
Lag 13	-1.06	-1.09	-0.18	-0.09	-0.01	0.03	-0.35	-0.25	-0.03	-0.01
	(0.34)	(0.35)	(0.12)	(0.11)	(0.08)	(0.07)	(0.14)	(0.16)	(0.07)	(0.05)
Lag 14	0.27	0.23	-0.32	-0.28	0.00	-0.01	-0.23	-0.18	-0.05	-0.03
	(0.31)	(0.31)	(0.23)	(0.22)	(0.11)	(0.12)	(0.11)	(0.12)	(0.08)	(0.07)