Options in Construction Decisions Under Uncertainty

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Options

Definition: An *option* is a decision alternative that permits or may permit a future decision following revelation of some information.

Key elements:

- An option allows the decision maker (or his agents) to acquire new information
- On the basis of this new information, the decision maker (or his agents) will be able to exercise their own judgment and select the alternatives they prefer in the future decisions created by the option.
    - The decision maker is not tied down to specific decision alternatives or choices ahead of time.
    - Future decisions will be made based on information available then (in the future).
    - Future decisions may include future decision alternatives that were not even known or technologically possible when the option was created.

Finance: The above definition includes the various types of financial options, such as

- [Call | put] options: [buy | sell] securities at predetermined price
- [European | American] options: [on | before] a predetermined date

Options, however, also encompass many personal and professional decision situations, such as the ones below.

Testing: A classic example of an option is the decision alternative to perform testing, obtain the results of the test, and then decide how to act given the new information provided by the test.

Contracting: For example, when a contractor decides to bid a job it is embarking on an option. If its bid is successful, the contractor will be able to choose whether certain work will be done with its own forces or subcontracted. For work to be subcontracted, the contractor will be able to choose the subcontractors (subject to the approval of the owner or A/E) and how much to pay.

R&D: Similarly, companies that invest in research and development with the expectation of finding a profitable product or a cost-saving solution are engaged in an option because numerous decisions will have to be made throughout the development and commercialization process as information is revealed.

Lotto: Purchasing a lottery ticket is also an option because if the lottery prize is won it creates several decisions, such as how much to keep, give away, invest for retirement, spend now, etc.

Decision Synthesis: The generalized concept of an option is a fundamental notion in decision analysis and an important aid to

- Decision synthesis: creating or devising appropriate decision alternatives that create or enable subsequent or future decisions after new information is revealed.

Contractual Options

A **contractual** option is an agreement between two parties that allows future decision alternatives (for one or both parties) that otherwise might not exist.

Financial options tend to be of a contractual nature where one party is typically in the role of the buyer and the other party in the role of the seller.

An example of a contractual option in construction is *a letter of intent*. It is a mini contract signed by both the owner and the contractor that allows construction to start without having to sign the agreement for the construction of the entire project. This is typically done to allow construction to begin while waiting for all financial arrangements (construction loans) and even permits to be secured by the owner. A letter of intent must specify clearly the included scope of work and the obligations of the owner and contractor. It must also make provisions in case the agreement for the entire project is executed (signed) or not. It allows the two parties to continue working together or to withdraw amicably if things do not turn out as intended. I.e., it allows for important decisions to be made later based on additional information.

Non-Contractual Options

Non-contractual options exist when the decision maker recognizes, or creates, the possibility to decide later after new information is revealed. The decision to be made in the future may have the same set of choices as the decision to be made now, or it may have a modified set of alternatives. Non-contractual options may even involve the acquisition of contractual options after the revelation of new information.

The typical example of a non-contractual option is in-house research and development. In this case, there is no contract that obliges the company to make a choice or to act in a certain way in the future. Furthermore, research can sometimes be so unpredictable that the future decision possibilities it creates could not have been foreseen ahead of time.

A construction example is provided by the Athens Metro. The city of Athens, Greece, needed to build a new subway system in preparation for the 2004 Olympics. To this end, the city decided to perform first an extensive subsurface exploration program and investigate possible routes and depths for the tunnels and stations to avoid

- Unfavorable ground conditions, and
- Interfering with or damaging existing or even undiscovered archeological sites.

When new information from the exploration program was collected, the city exercised the option to create a series of contractual options with designers, construction managers, contractors, etc. to do the work. In this case,

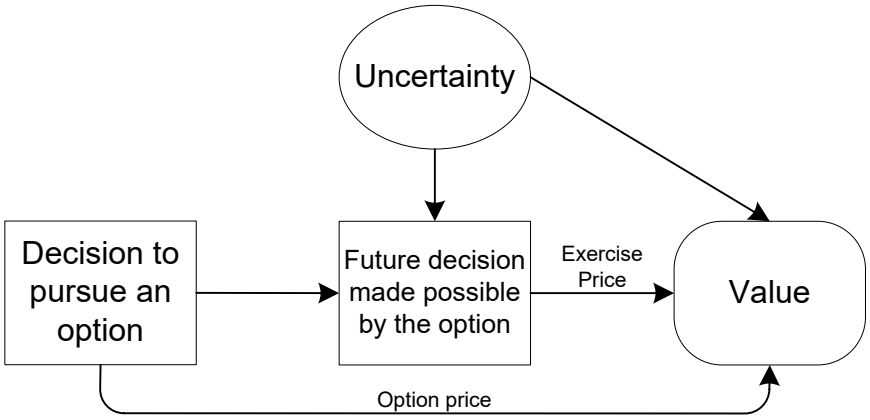
- Exploration program was a non-contractual option to collect information, and
- Create appropriate contractual options for the design and construction that followed.

Option Price, Exercise Price, and Option Value

A decision is defined as *an irrevocable allocation of resources*. Options, too, are decision alternatives and as such are seldom free. Typically, the decision to create an option will require some type of payment or commitment of resources. The money one needs to pay to create or acquire an option is called the ***option price***.

A generic influence diagram for the decision to pursue an option is shown below. In this influence diagram the option price is represented by the arrow from the first decision to the value node.

After a decision maker pays the option price and obtains an option, he usually collects additional information (see the arrow from the uncertainty node in the influence diagram) before selecting the appropriate alternative for the decision made possible by the option (the second decision node in the influence diagram). Each alternative of this second decision may also require some payment. This payment is called the ***exercise price*** for that specific alternative.



Influence Diagram for Option Analysis

To compute *the **value** of the option we must **know** the **exercise prices*** for **all** future decision alternatives:

> The *value* of an option is that special option price (i.e., the price for acquiring the option) that makes the decision maker indifferent between obtaining the option or not. I.e., the *value of an option* is its *PIBP*.

In general, the *value* of an option *increases* as the level of initial risk increases and as the number of possible future alternatives created by the option increases. In general, the value of an option *decreases* as the exercise price increases and as the number of possible future alternatives allowed by the option decreases.

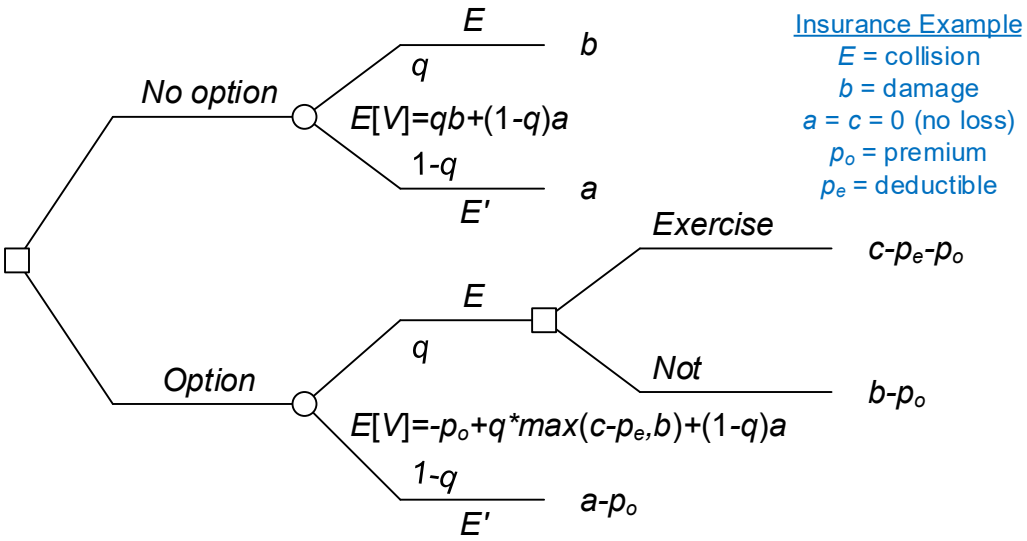
Insurance as Option

Insurance is a classic example of an *option* with a *well-defined* option price and exercise price that is purchased by most companies (including contractors) and individuals. The *option price* is the ***premium*** paid to buy the insurance policy. Having an insurance policy allows a person to decide whether to file a claim when a loss occurs in the future. If a claim is filed, the *exercise price* is the ***deductible***. As an example, consider the collision portion of a car insurance policy that has a *deductible* of $500. For the car owner, the $500 *deductible* is the *exercise price*. If a collision results in a $800 damage, and the car owner decides to file a claim, then he pays the first $500 of the damage and the policy pays the remaining $300. If the damage is less than $500 then the car owner pays for the repair (the option is not exercised).

Decision Tree for an Option

An option provides the decision maker with the opportunity to adjust future decisions in the light of new information or experience. The *value* of the option stems from the *likelihood* that such future adjustment may be possible and the *benefit* that would result by adjusting. These concepts should be recognized as central to decision analysis because they deal directly with inference based on new evidence, decision alternatives that allow future flexibility, and the deliberate inclusion and evaluation of testing and experimentation as part of such choices.

The following is a simple decision tree that corresponds to the influence diagram for an option:



Without the option, the decision maker faces a chance node with two outcomes. Outcome E will occur with probability q and result in a monetary payoff b. Outcome E' will occur with probability $1-q$ and result in payoff a. For example, E may represent an automobile accident, in which case b may be the losses due to the accident and E' may be the condition of no accident in which case the payoff, a, is the current situation with no accident.

The decision maker can also buy an option at an option price of p_o. In this case, with probability q the exercise condition E will still arise and with probability $1-q$ it will not. (There is no reason to believe that buying an option will increase or decrease the probability of E occurring). If the

condition E does not occur, then the resulting payoff is $a\text{-}p_o$, i.e., the outcome without the option minus the option price. However, if the condition E does occur, it will lead to a second decision: whether to exercise the option or not.

If the choice is made *not* to exercise the option, then the payoff will be $b\text{-}p_o$, which is the payoff for condition E minus the option price. On the other hand, if the option is exercised, then the payoff will be $c\text{-}p_e\text{-}p_o$, where p_e is the exercise price and c is the outcome due to the usage of the option. For example, c may be the result of some monetary amount added to b, or any other payment agreed upon when purchasing the option.

When to Exercise an Option

An option should be exercised if the certain equivalent of exercising the option is greater than that of not exercising it at the time of that decision (i.e., when condition E actually occurs):

$$c\text{-}p_e\text{-}p_o > b\text{-}p_o$$

So, the option should be exercised if:

$$c\text{-}b > p_e$$

Note that if the exercise price is zero, and if $c>b$, then the option should always be exercised.

In the case of insurance, we often have c=\$0 and b<\$0 (a loss), which means that the option should be exercised if the loss (as a positive number) is greater than the deductible

$$-b > p_e$$

Value of a Free Option (p_o=\$0)

For a risk-neutral decision maker, the value of a free option is found by comparing the value of the deal with the free option (which should be high) to the value of the deal without the free option (which should be low).

The expected value of the no-option alternative is given in the decision tree as:

$$qb+(1\text{-}q)a$$

The expected value of an option that would be exercised if necessary, $c\text{-}b > p_e$, is:

$$q(c\text{-}p_e\text{-}p_o)+(1\text{-}q)(a\text{-}p_o)$$

If the option is free, $p_o = 0$, and if it would be exercised if necessary, $c\text{-}b > p_e$, then its value is

$$q(c\text{-}p_e)+(1\text{-}q)a$$

Thus, the value of an option with zero option price for a risk-neutral decision maker is

$$q(c\text{-}p_e)+(1\text{-}q)a - qb\text{-}(1\text{-}q)a =$$

$$q(c\text{-}b\text{-}p_e)$$

Maximum Option Price

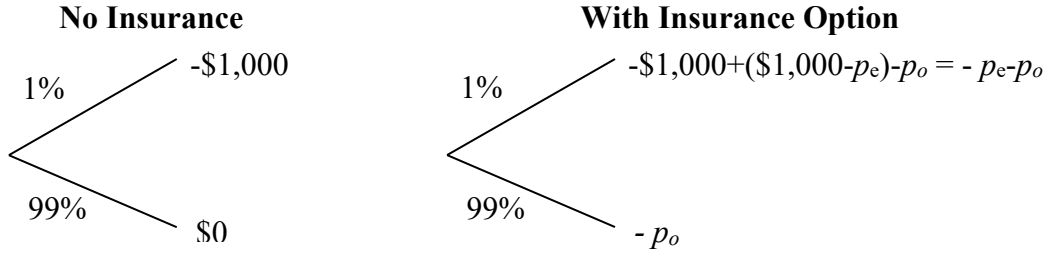
The value of the option, if it is free, is $q(c\text{-}b\text{-}p_e)$. If it is not free, then the option should be purchased if its cost (i.e., its market price p_o) is less its value. *I.e.*, an option should be purchased when its price is less than its value:

$$q(c\text{-}b\text{-}p_e) > p_o$$

The calculation of the value of an option for a more general utility curve requires trial and error. In that case, different values for the option price p_o must be tried until the certain equivalent of *the deal with the option* is equal to the certain equivalent of *the deal without the option*. The *value of the option* is that special price p_o that makes the two certain equivalents equal. It is the most one would pay to acquire the option.

Insurance Example

We shall now investigate an insurance example where there is a 1% chance of a \$1,000 loss and a 99% chance of zero loss. The decision maker can buy insurance at a price p_o that, if the loss occurs, will reimburse him for the \$1,000 loss minus the exercise price p_e (the deductible).



For a risk-neutral decision maker, the case of no insurance has a negative expected value. Thus, a risk-neutral person would pay up to \$10 to transfer (sell) this undesirable lottery to someone else:

$$-\$1{,}000 * 1\% + \$0 * 99\% = -\$10.$$

Let us also consider a risk-averse decision maker with risk tolerance =\$1000 and an exponential utility curve:

$$u(x) = 1 - \exp(-x/1000)$$

For the risk-averse decision maker the case with no insurance gives:

$$u(\$0) = 0,\ u(-\$1000) = -1.72,\ E[u(x)] = -1.72*1\% = -0.0172,\ CE[X] = -\$17.04$$

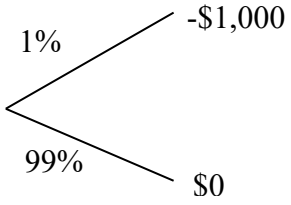
As expected, the risk-averse person is willing to pay more to avoid this lottery, \$17.04.

The certain equivalents -\$10 and -\$17.04 are the personal indifferent selling prices (PISP) for these two persons. Let us now investigate the option to buy insurance as one possible way to avoid the potential losses of this lottery, in part ($p_e>0$), or in full ($p_e=0$).

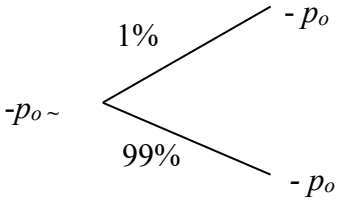
Value of Option with Zero Exercise Price (Zero Deductible)

Let us consider first the case where the insurance option can be exercised for free, p_e=0. In this case, both branches of the lottery with insurance lead to the same dollar outcome, $-p_o$.

No Insurance Option



With Insurance Option and p_e=0



If the person does not acquire insurance, then he *owns* an undesirable lottery with a *negative* certain equivalent calculated above, -$10 if risk-neutral and -$17.04 if risk tolerance is $1000. This person can *sell* this *undesirable* lottery to an insurance company for a *negative* selling price, $-p_o$. (In practical terms, he can obtain an insurance policy and pay the premium $- p_o$ for it.) The *most* he would be willing to pay as insurance premium, $- p_o$, would be his negative PISP, i.e., the negative CE[X] of the undesirable lottery.

Thus, for a delta-person, the *value of the option* is $0 (value with *free* insurance) minus the *negative* personal indifferent selling price PISP (i.e., the CE[X]) for the undesirable lottery without insurance. (The subtraction of a negative CE[X] makes the value of the option positive.)

To summarize, for a risk neutral person the *value* of the insurance option with a zero-exercise price is $10. For a person with risk tolerance of $1,000 the *value* of the option is $17.03.

For a risk neutral person, the same result can be obtained using the algebraic formula above:

$$q(c\text{-}b\text{-}p_e) > p_o$$

For this example, the values of the relevant variables are:

$$q = 1\%, \quad a = \$0,\ b = -\$1{,}000,\ \ c = \$0$$

Thus, the value of the option to a risk-neutral person is:

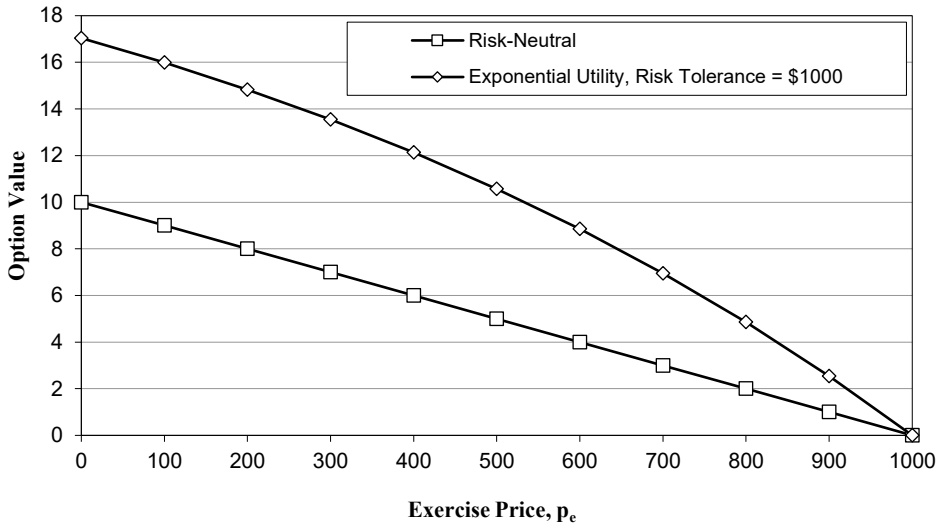
$$q(c\text{-}b\text{-}p_e) = 1\%[\$0\text{-}(-\$1{,}000)\text{-}p_e] = 1\%[\$1{,}000\ \text{-}p_e]$$

For a *zero*-exercise price p_e, the value of the option to a risk-neutral person is $10, as above.

Value of Option with Non-Zero Exercise Price (Non-Zero Deductible)

When the exercise price is positive, p_e>0, we would expect that *the value of the insurance option decreases as the exercise price increases*. In the limit, when the exercise price becomes very high and reaches p_e=\$1000, there will be no monetary difference between exercising and not exercising the option. In both cases, if the event leading to the \$1000 loss does occur, the cash outflow will be \$1000 (either as a loss without an insurance claim, or as reimbursed insurance claim but with a \$1000 deductible). It is clear that when p_e=\$1000 the value of the option drops to *zero* for both a risk-neutral and for a risk-averse person.

These common-sense observations can be verified easily through direct calculations as shown in the figure below. As expected, when p_e=\$0 the value of the insurance option starts out at \$10 for a risk-neutral person and \$17.03 for a risk-averse person on the left of the graph and declines to a value of zero when p_e=\$1,000 on the right of the graph.



Effect of Exercise Price on Option Value

To calculate the *value* of the insurance option at an exercise price p_e for a *delta-person* we compute the certain equivalent of the deal with a free insurance option, i.e., when p_o = \$0, (which in algebraic terms should be high), from which we subtract the certain equivalent value of the deal without the free option (which should be low).

As an example, let us consider an exercise price p_e = \$500. For a risk-neutral person the value of the option is:

$$q(c\text{-}b\text{-}p_e) = 1\%[\$1{,}000 - p_e] = \$5$$

For a risk-averse delta-person whose risk tolerance is \$1000, the case with insurance gives:

$$u(\$0) = 0; \quad u(-p_e) = u(-\$500) = -0.65; \quad E[u(x)]=1\%(-0.65) = -0.0065; \quad CE[X] = -\$6.45$$

Hence the value of the option is:

$$CE[X \mid \text{Option}] - CE[X \mid \text{No Option}] = -\$6.45 - (-\$17.04) = \$10.57$$

Consequences of not Recognizing Options

Sequential decisions are sometimes misrepresented as simple decisions leading to decision errors and diminished value. We shall illustrate this using a simple example. Suppose that a risk-neutral person is offered the opportunity to participate in an investment determined by three trials that result in "Success(S) or Failure (F)". Observing Successes makes future Successes more likely; observing Failures makes future Failures more likely. The probability of Success on the first trial is 1/2. As successive trials are observed, the probability of Success on the next trial will be

$$P[S \mid n,s] = (s+1)/(n+2)$$

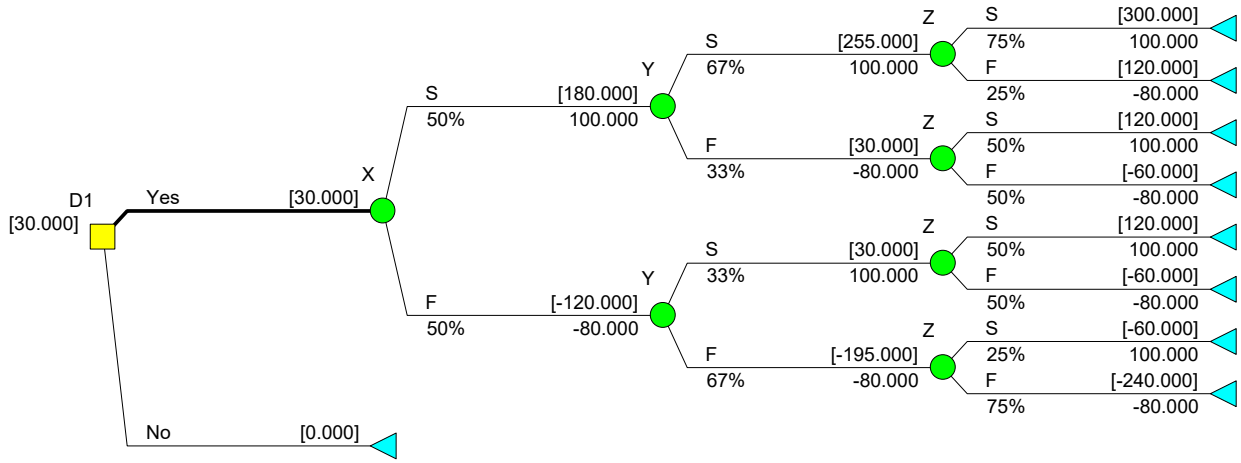
Where s is the number of Successes already observed in the previous n trials. For example, if the first trial is a Success, then the probability of Success on the second trial will be 2/3. However, if the first trial is a Failure, then the probability of Success on the second trial will be 1/3. The pattern continues: if the first two trials have been Successes, then the probability of Success on the third trial will be 3/4, etc.

Note that the probability of Success on the second trial if we do not know the result of the first trial is 1/2*2/3+ 1/2*1/3 = 1/2. Likewise, the probability of Success on the third trial if we do not know the result of previous trials is also 1/2.

The investment pays $100 for a Success and costs $80 for a Failure.

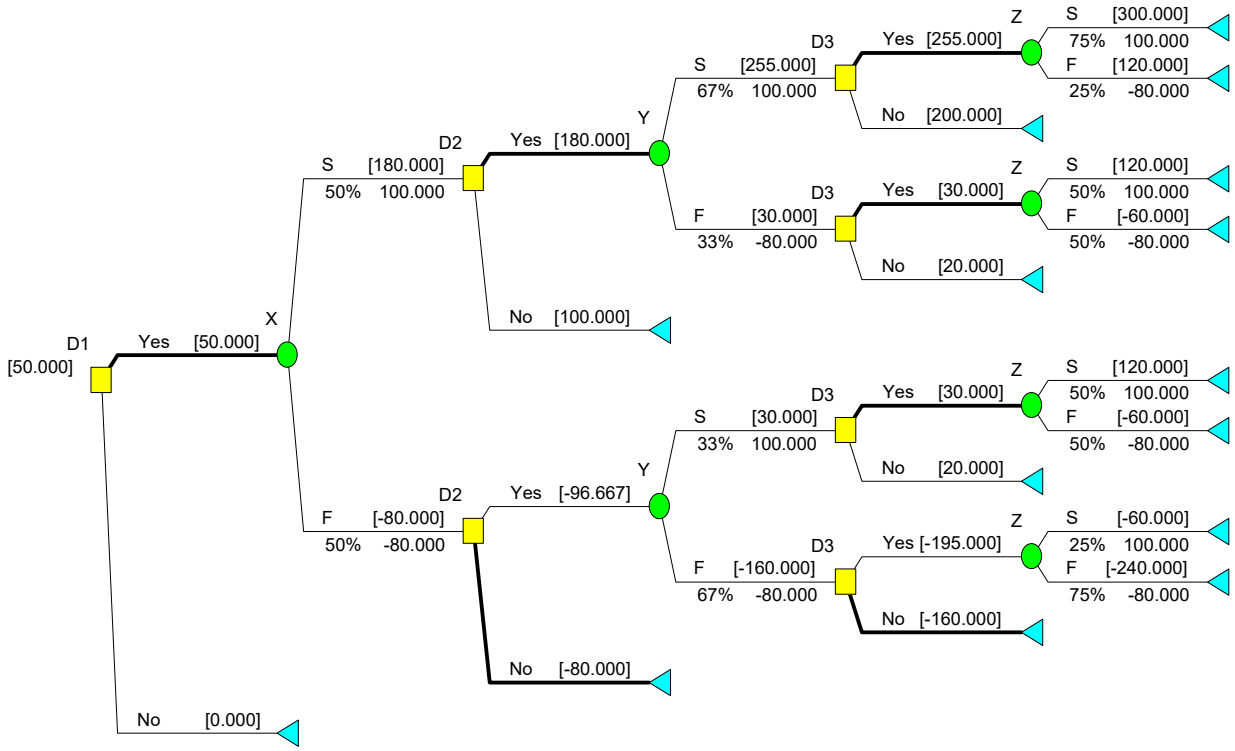
This investment opportunity could be viewed as a *simple* decision that needs to be made *now* with only two alternatives: (a) invest in all three trials, or (b) do not invest in all three trials. Since, a priori there is a 0.5 chance of Success on each of the three trials, the expected value of each trial is $10. Thus, the alternative to invest in all three trials has an expected value of $30.

The same expected value of $30 can also be obtained by considering all possible sequential outcomes as shown in the tree below:



The Value of the Option to Stop

Suppose now that the investor is offered an option to stop further participation in the trials at any point he chooses. The decision tree for evaluating this option is shown below and has an expected value of $50. Thus, the value of the option to stop is significant: $50-$30 =$20.



Clearly, if the option to stop does exist but is not recognized, the decision policy to go ahead and commit to invest in all three trials is not optimal. It will lead to the reduced expected value of $30 instead of the potential $50 if the option to stop is recognized.

If the first trial, *X*, is a *Success*, the decision maker should participate in a second trial, *Y*, and irrespective of the outcome, he should continue with a third trial, *Z*, too. But, if the first trial, *X*, is a *Failure*, then the decision maker should stop; and this potential stop increases value by $20.

The Value of the Option to Skip

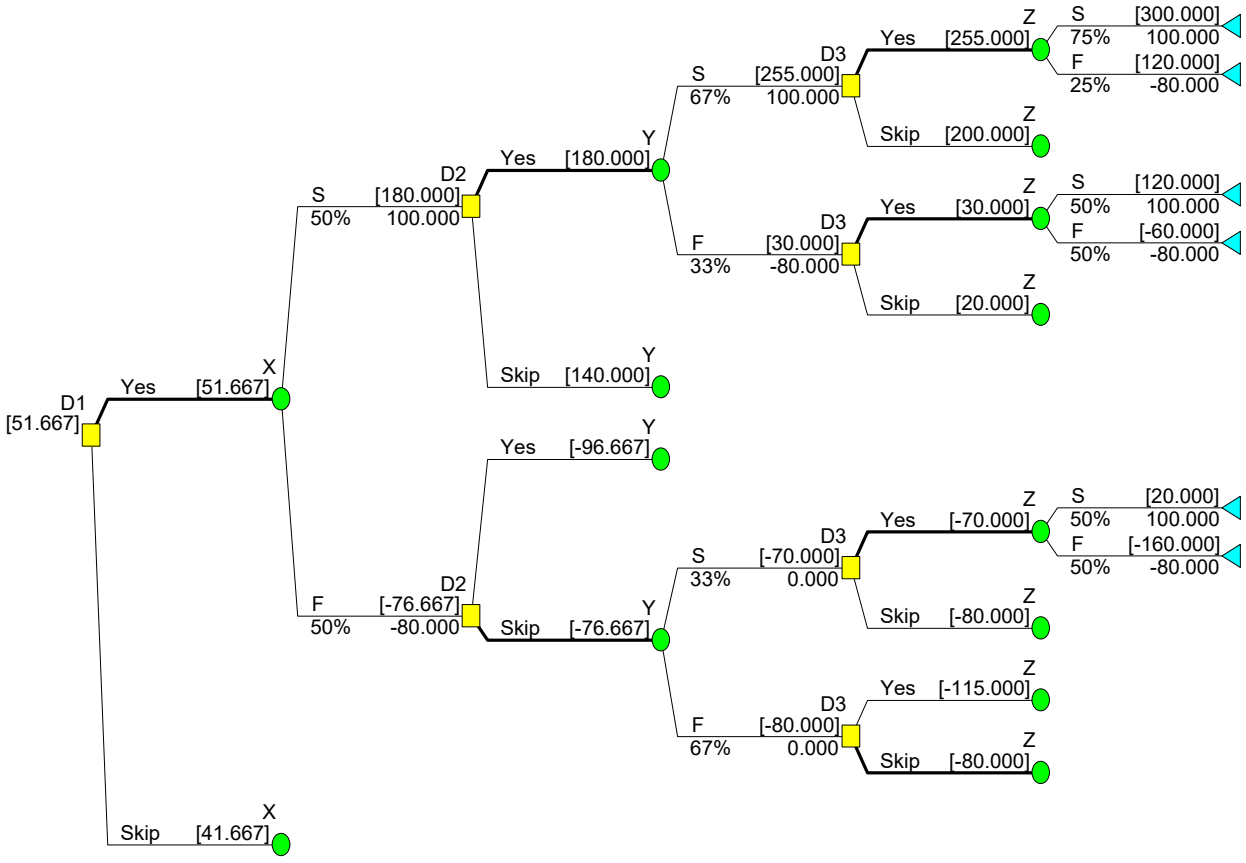
The option to stop does not have to be an "all or nothing" deal. That is, the investor does not have to *continue* or *stop* participating altogether.

Instead, he may choose to *pass*, i.e., to *skip* participating in any given trial with the possibility to participate in future trials. The decision tree for the option to *skip* is shown below. It has an expected value of $51.67 (>$50) and is a just a little more valuable than the option to *stop*.

In this case, if the first trial, *X*, results in *Failure*, then the decision maker should *skip* the second trial, *Y*, and simply observe its outcome. If the second trial, *Y*, proves to be a *Success*, then the decision maker should participate in the third trial, *Z*. This additional possibility to participate in the third trial, *Z*, even if the first trial, *X*, was a *Failure*, increases the value of the option by:

$$E[Z|F_x,S_y]\ P[S_y|F_x]\ P[F_x] = \$10\ (1/3)\ (1/2) = \$1.67$$

And this possibility increases the value of the option from $20 to $21.67.



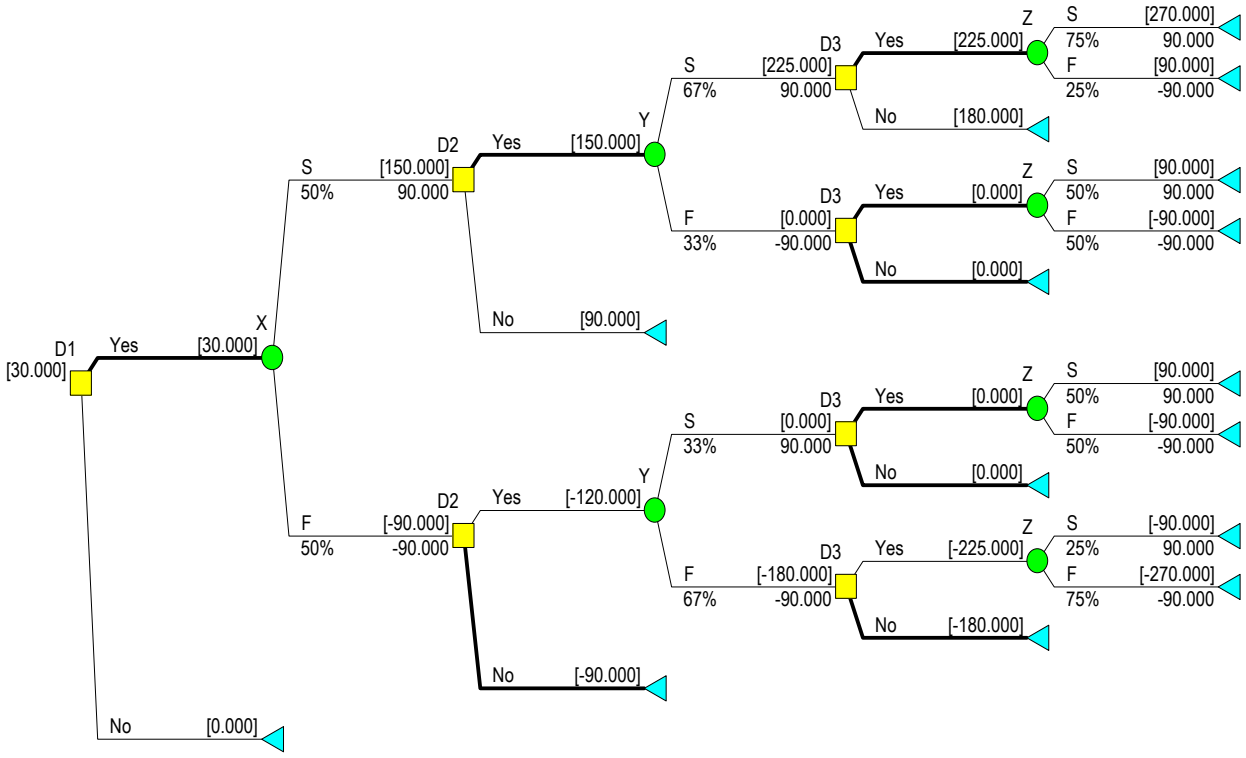
The Option to Stop gives Value to a Series of Investments without Value

Let us now consider the exact same situation where the payoffs of the investments for *Success* and *Failure* are each reduced by $10 to $90 and -$90.

The expected value of each investment taken separately is now $0.

Thus, without an option, this series of three investments would have zero value to a risk-neutral person.

With the option to stop at any time, however, the situation changes. As shown in the decision tree below, the expected value of this series of investments with the option to stop at anytime is $30. Thus, the value of the option to stop anytime is $30-$0 = $30.



The Option to *Skip* and the Option to *Stop* give the Same Value

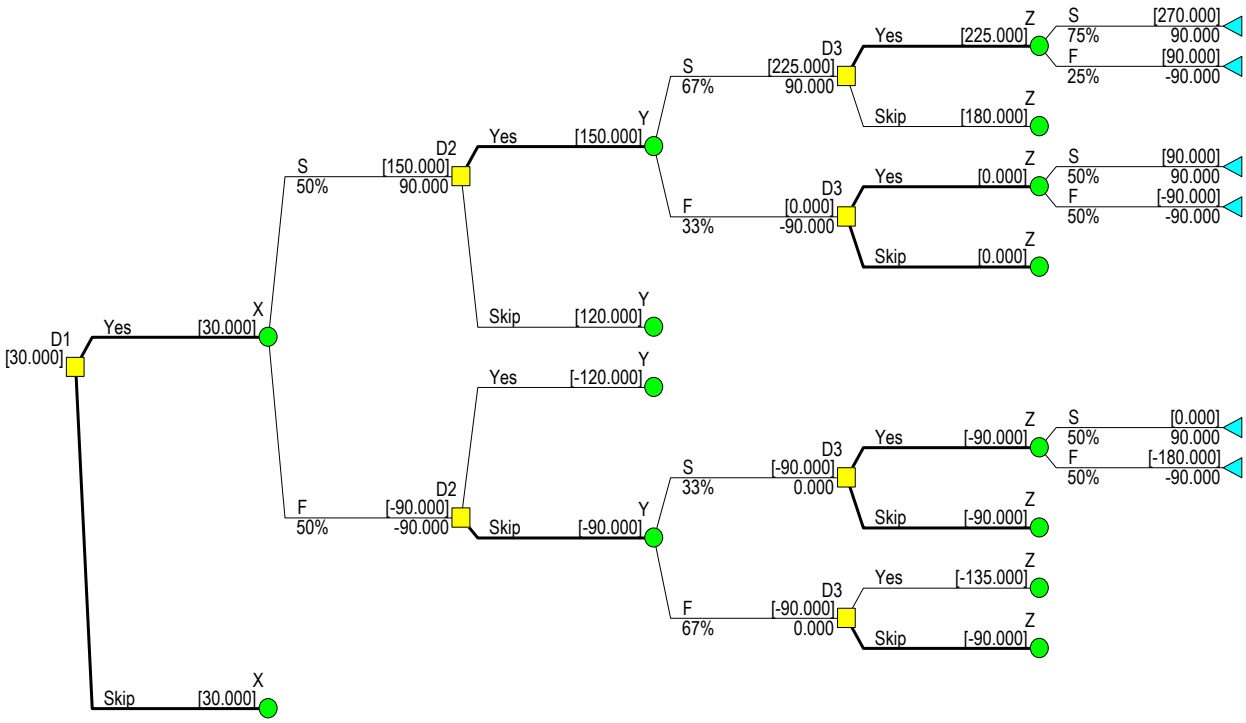
The decision tree for the option to skip trials when the payoffs of the investments for *Success* and *Failure* are each reduced by \$10 to \$90 and -\$90 is shown below.

The expected value of this tree is again \$30, the same as for the tree without the skip option. Thus, the option to skip trials does not have any more value than the option to stop anytime. Both lead to the same decision policy (compare the branches highlighted with heavy lines).

The reason is that because each investment has an expected value of \$0, the alternative to continue at decision $\{D_3|S_y,F_x\}$ leads to an expected value of

$$E[Z|F_x,S_y] = \$0$$

This is the same as the outcome of the alternative to stop at decision $\{D_3|S_y,F_x\}$. Thus, the option to continue does not lead to additional value.



Experimentation and Information Gathering as Options

A decision alternative to perform experimentation (or information gathering) is an option that allows making future decisions after the outcomes of selected uncertainties are revealed. For example, conducting a concrete slump test allows making the decision to accept or reject a batch of concrete after the outcome of the slump test is known. Similarly, the destructive testing of concrete cylinders provides experimental information by which to evaluate the performance of concrete from the same batch that will be used in other parts of a structure and then make decisions.

In general, any decision alternative to obtain either *perfect* information or *sampled* information is an *option* because it results in the revelation of new information on which subsequent decisions can be based. Hence, the calculation of the *expected value of perfect information* (EVPI) and the *expected value of sampled information* (EVSI) are closely related to the evaluations of *options*:

EVPI and EVSI is the *value of an option* if it had a *zero-exercise price*.

It represents the most money that the decision maker would be willing to spend to buy this option (the information) if using the information subsequently was free (zero exercise price). Notice that the concept of an *option* allows us to expand the concept of *value of information* because each exercise price leads to an associated option value. Thus, If the exercise price is very high, then the value of information may be reduced to zero.

The value of information is a function of its exercise price.

An example of testing that may be associated with an exercise price is the performance of multiple sequential tests. In this case, performing each successive test may be associated with a *variable cost* that represents the *exercise price* of the option to observe one more sample. In this case, the *price* of the option may be the *fixed cost* that must be paid up front to initiate sequential testing. We will investigate these concepts below.

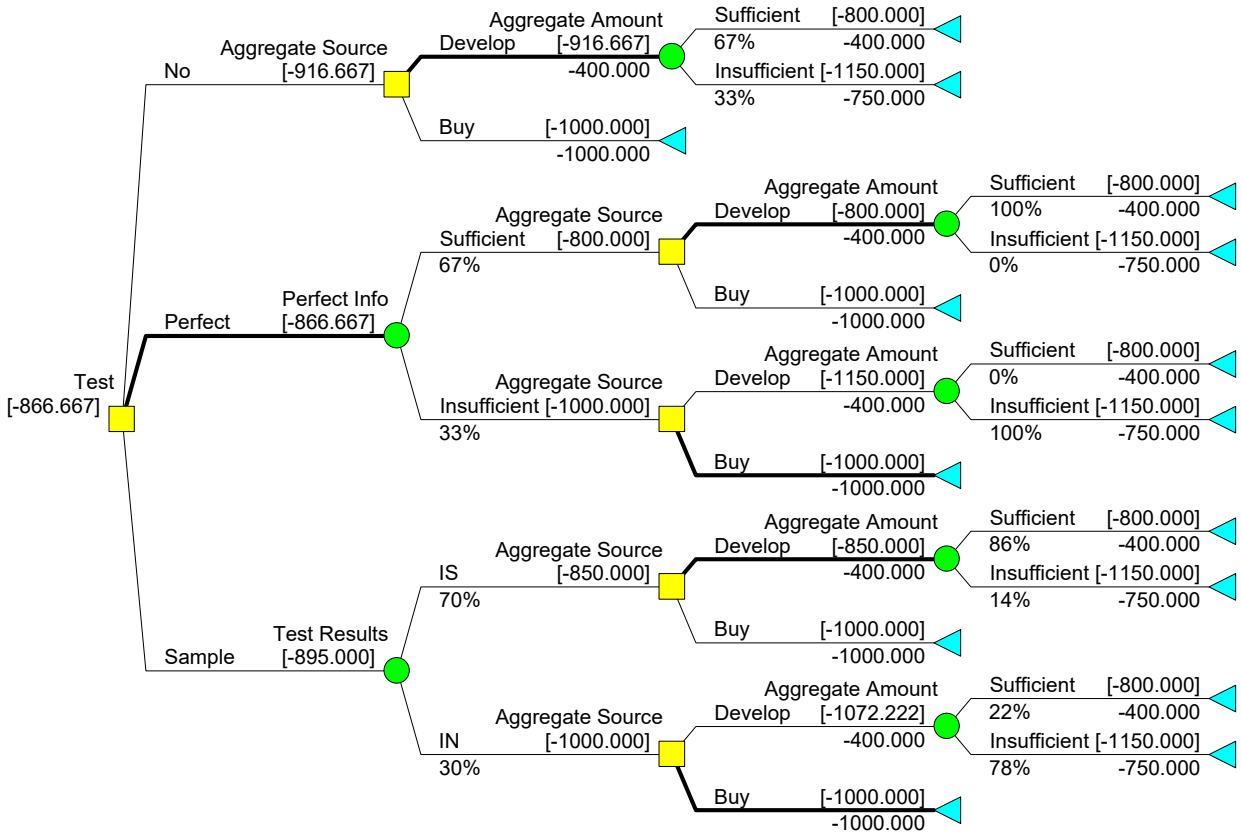
The introduction to the concepts of *perfect* and *sampled information* assumes that the *same* decision alternatives are available to the decision maker irrespective of whether new information is obtained or not. This situation is called *the case of complete flexibility*. We can now qualify that the observation that EVPI or EVSI could never be negative is based on two implicit assumptions: (a) *complete flexibility* and (b) *zero exercise price* to use the acquired information.

It should be made clear that experiments with *limited flexibility*, such as the breaking of concrete cylinders, are also *options*. They represent those cases where obtaining information limits the set of decision alternatives that can be made after testing. Typical examples are all types of destructive testing where the item that was tested can no longer be used, but the information collected can be used for similar items.

However, if the new decision situation following perfect or sampled information *eliminates* one or more of the alternatives in the original decision and does not add any new alternatives, then, the value of new information given this *limited flexibility* could be *negative*, even with a zero-exercise price. Conversely, the value of information (an option) with a zero-exercise price and no change in alternatives, that is, with complete flexibility, cannot be negative.

Sequential Information Options: Sampled Information followed by Perfect Information

Let us consider again the *source-of-aggregate* decision problem presented earlier. The decision tree for this problem appears below and shows all three alternatives of "*no testing*", "*obtain perfect information*", and "*perform one test at the site*".



This decision tree shows that the expected cost without any testing is -$917k. The expected cost with one test drops to -$895k, giving an EVSI of -$895k+917k=$22k. Perfect information leads to an expected cost of -$867k, giving an EVPI of -$867k+917k=$50k.

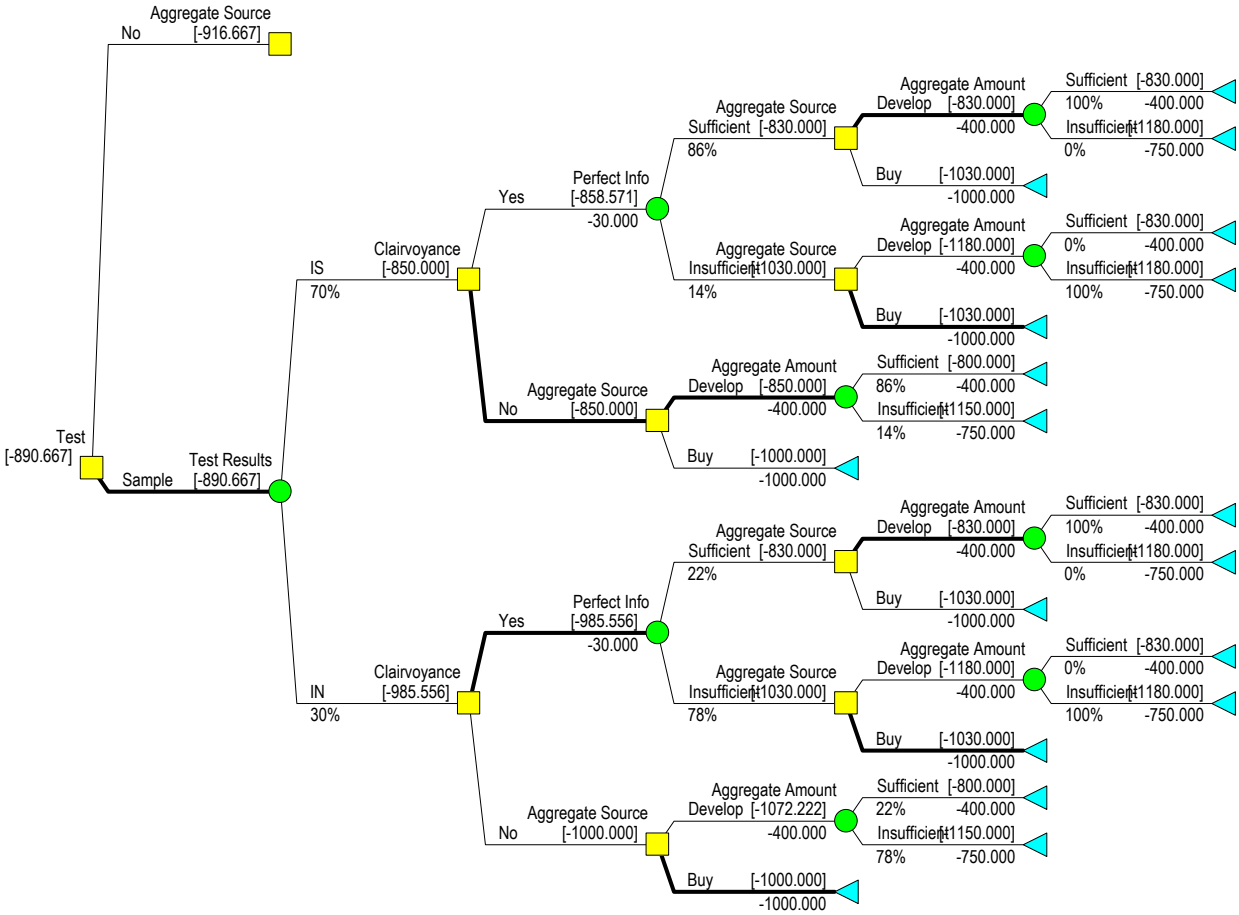
Suppose that the alternative to obtain the results of a single test will be provided for free but the actual outcome has not yet been observed. What is the EVPI about the aggregate amount at the site? Without the test, the EVPI = $50k. With the one test available for free the EVPI about the actual aggregate amount is reduced to EVPI = -$867k+895k=$28k. Thus, the decision maker should be willing to pay up to $28k *now* for perfect information to be obtained after the results of the single test are known. (Notice that even though the outcome of the test will be *known* before perfect information is obtained, the reduced EVPI still exceeds the original EVSI of the test: $28k > $22k. This is an indication that the test being employed is not particularly accurate.)

If a clairvoyant charges $30k to provide perfect information, then clearly the decision maker will choose not to buy ($30k>$28k). Instead it is best to observe the outcome of the single test and make a decision based on that alone: if IS then "Develop"; if IN then "Buy".

Sequential Option Structure Makes Buying Future Information Possible

Let us now introduce a variation that will allow the clairvoyant to sell his services to the decision maker for $30k or even more. In this case, the decision maker does not have to commit to buy perfect information ahead of time. Instead he can buy an option for $1k that will allow him to decide whether to buy clairvoyance *after* he observes the outcome of the single test. If he decides to obtain perfect information then, it will cost him $30k. Is this an acceptable deal?

The decision tree for this option is shown below.

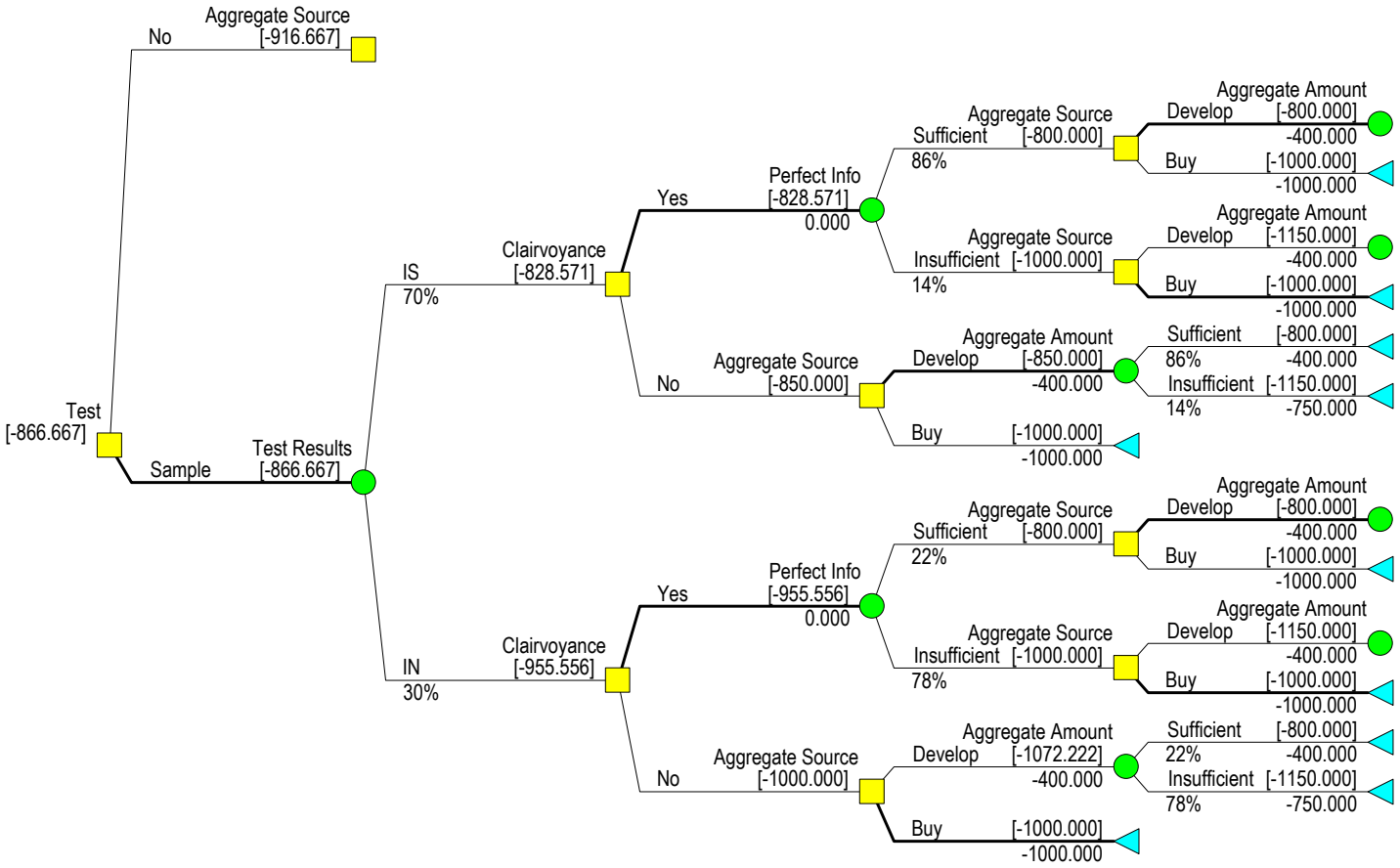


The expected cost with this option is -$891k and the value of the option is $-891k+895k= $4k. Thus, the offer to buy this option for $1k is acceptable (<$4k).

Notice that after the option to obtain clairvoyance is purchased, the test will indicate sufficient aggregate (IS) with probability 70%, in which case perfect information (clairvoyance) should not be purchased, and the aggregate pit should be developed. With probability 30% the test will indicate not-sufficient aggregate (IN) in which case perfect information (clairvoyance) should be purchased at a cost (exercise price) of $30k. If this clairvoyance reveals that there will be sufficient aggregate, then the aggregate pit should be developed; otherwise, the pit should not be developed, and all aggregate should be purchased from other sources.

Effect of Exercise Price on Sequential Option Value

We will now investigate the general case and examine the effect of the exercise price on (a) the expected cost with *the option to test followed by clairvoyance* and (b) the corresponding option value. To illustrate the process and to make calculations easy, we assume at first that both the option price and the exercise price are *zero* as shown in the decision tree below.



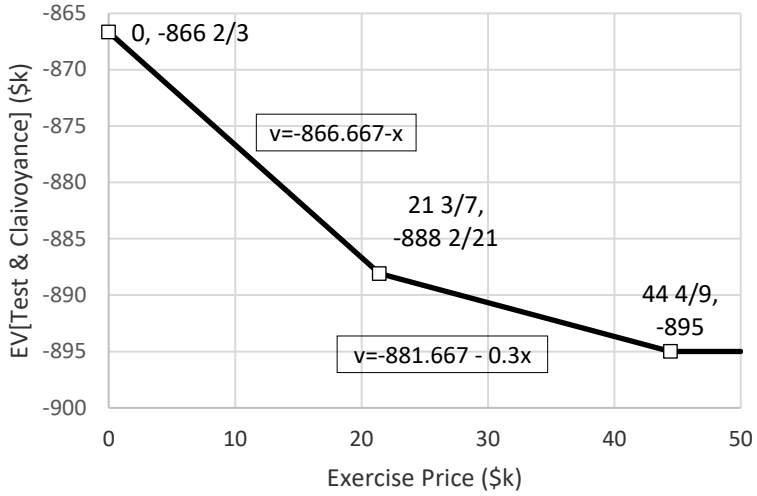
The decision tree shows that if the test indicates sufficient aggregate (IS), the expected cost values with and without clairvoyance are -$828.571k and -$850k respectively. In this case, clairvoyance should be purchased if its exercise price does not exceed -$828.571k+850k = $21.429k (in fractional form $21 3/7k).

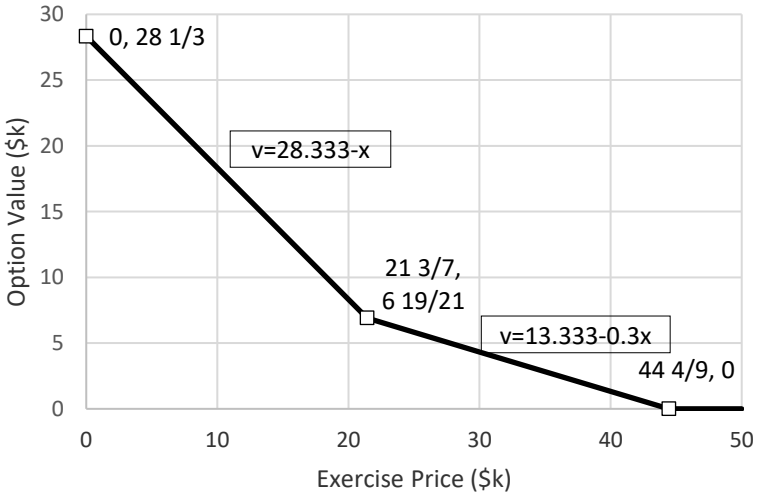
The decision tree also shows that if the test indicates insufficient aggregate (IN), the expected cost values with and without clairvoyance are -$955.556k and -$1000k respectively. In this case, clairvoyance should be purchased if its exercise price does not exceed -$955.556k+1000k = $44.444k (i.e., in fractional form $44 4/9k).

Taken together, these observations indicate that if the exercise price is less than $21 3/7k, clairvoyance should be purchased following either of the test observation (IS, IN). If the exercise price is between $21 3/7k and $44 4/9k then clairvoyance should be purchased only if the test

observation is IN. If the exercise price exceeds \$44 4/9k then clairvoyance has no value and should not be purchased.

The two graphs below show the effect of the *exercise price* on (a) the *expected cost* with the option to pay for clairvoyance after the test outcome is known and (b) the *value* of the option. As expected, the value of the option declines as the exercise price increases.

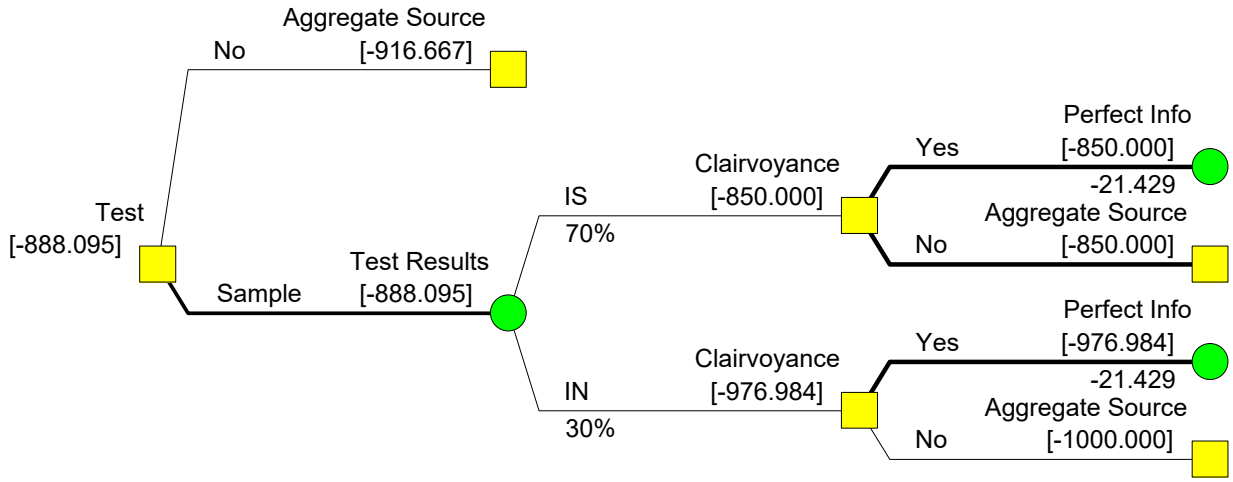




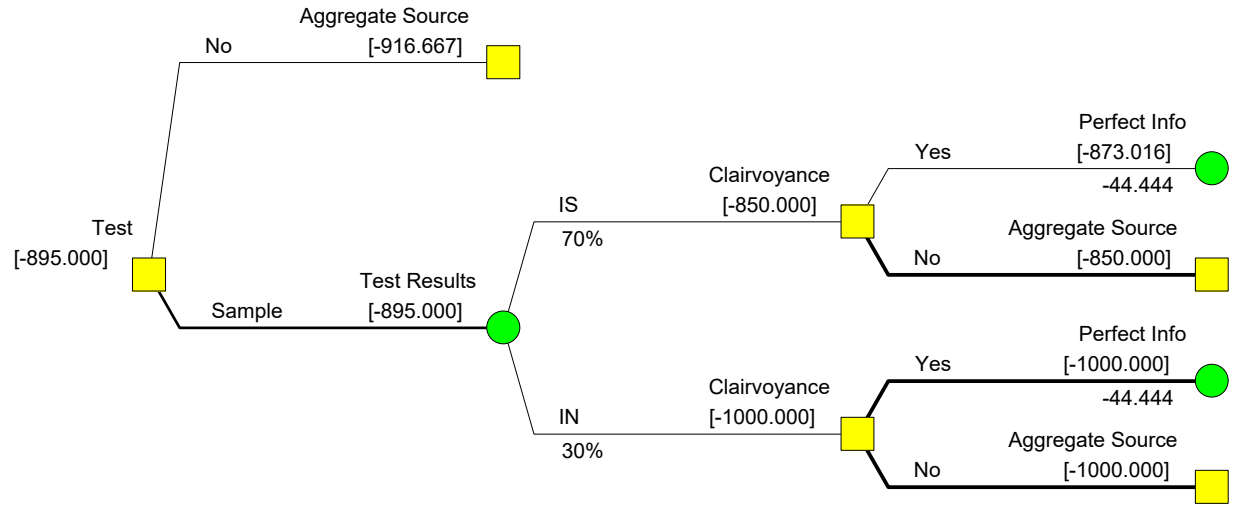
When the exercise price is zero, the value of the option to obtain clairvoyance after the test outcome is known, is -866.667k+895k = \$28.333k. For \$1 increase in the exercise price, the value of the option declines by \$1, too, because, when the exercise price is less than \$21 3/7k, clairvoyance should be purchased following either of the test observation (IS, IN).

When the exercise price is between \$21 3/7k and \$44 4/9k, however, clairvoyance should be purchased only if the test observation is IN which occurs with probability 0.3. Thus, for each \$1 increase in the exercise price, the value of the option declines only by \$0.3. These observations are reflected in the slopes of the linear segments in the above graphs.

The tree below verifies the above observations for the pivotal exercise price of $21 3/7k. If the test indicates sufficient aggregate (IS) then the decision maker would be indifferent between obtaining and not obtaining clairvoyance. If the test indicates insufficient aggregate (IN) then the decision maker should pay the $21 3/7k exercise price and obtain clairvoyance.



The tree below verifies the above observations for the other pivotal exercise price of $44 4/9k. If the test indicates sufficient aggregate (IS) then the decision maker should not obtain clairvoyance. If the test indicates insufficient aggregate (IN) then the decision maker would be indifferent between obtaining and not obtaining clairvoyance at an exercise price of $44 4/9k. Notice that for this exercise price the expected cost is -$895k which is the same as the expected cost of the single test without the option of clairvoyance.



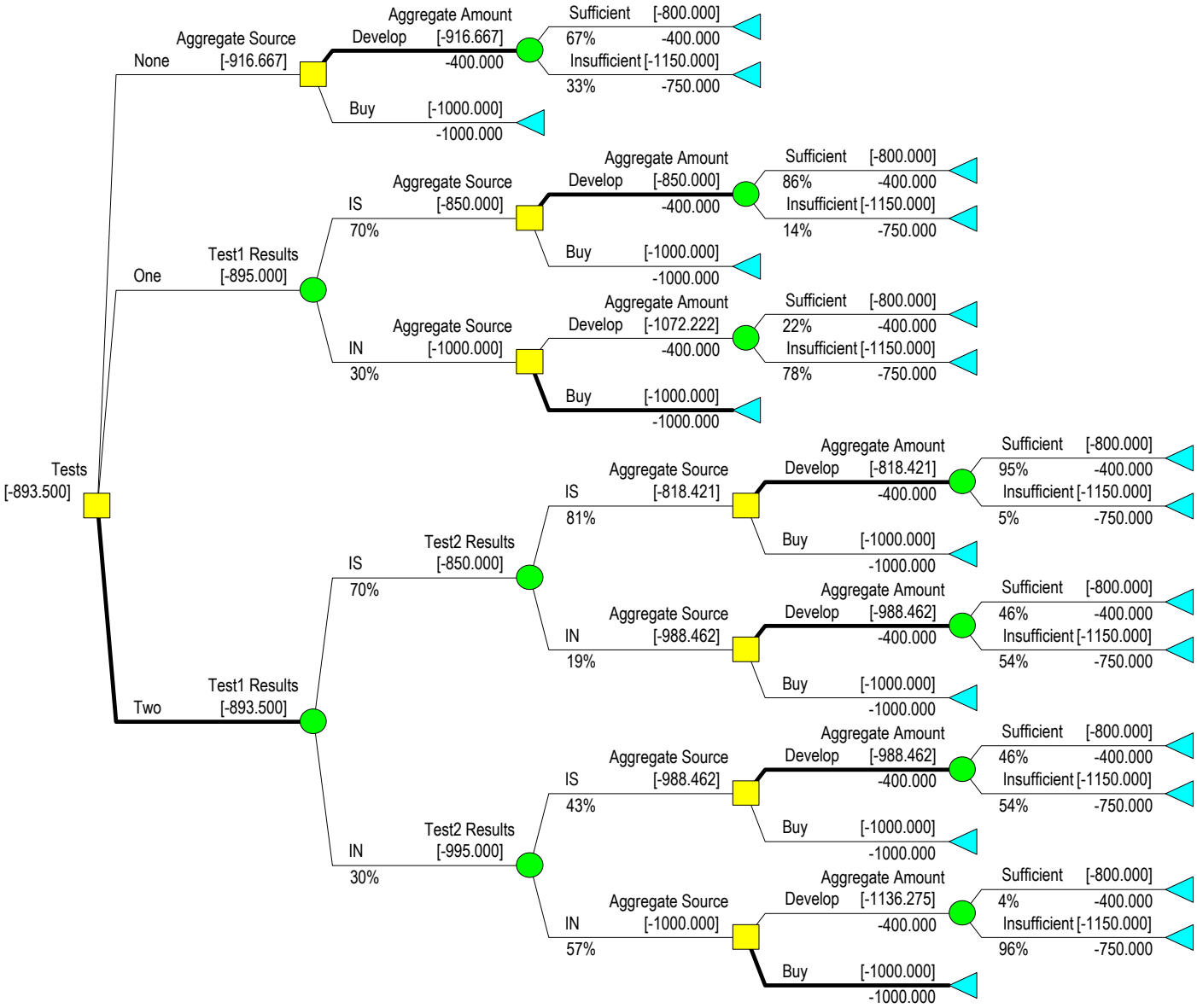
In summary, when the exercise price is zero, the decision maker would be willing to pay up to a maximum of $28 1/3k ahead of time, for the option to obtain clairvoyance after observing the outcome of the first test. As the exercise price increases to $21 3/7k, the maximum price for the option declines linearly to $6 19/21k. As the exercise price increases from $21 3/7k to $44 4/9k

the decision maker would be willing to buy the option only when the test observation indicates insufficient aggregate. The option value declines linearly from $6 19/21k when the exercise is $21 3/7 k to zero when the exercise price is $44 4/9k (which is quite large).

Sequential Sampled (Imperfect) Information Options

We now revisit the situation where the first test at the aggregate pit can be followed by a second test with the same reliability. Because of large spatial variability at the site, we assume that when the true state of nature at the aggregate pit is known, the outcome of the first test is independent of the outcome of the second test. Note that there are cases where this is may not be a valid assumption. For a given true state of nature, the observations of the two tests may be positively or even perfectly correlated. In such cases, the value of the second test would be diminished because it adds little to no information to that provided by observing the first test.

The decision tree below corresponds to the case where all test decisions must be made before any test results are known. The alternative "no test" has an expected cost of -$916.667, "one test" has an expected cost of -$895, and "two tests" has an expected cost of -$893.5.

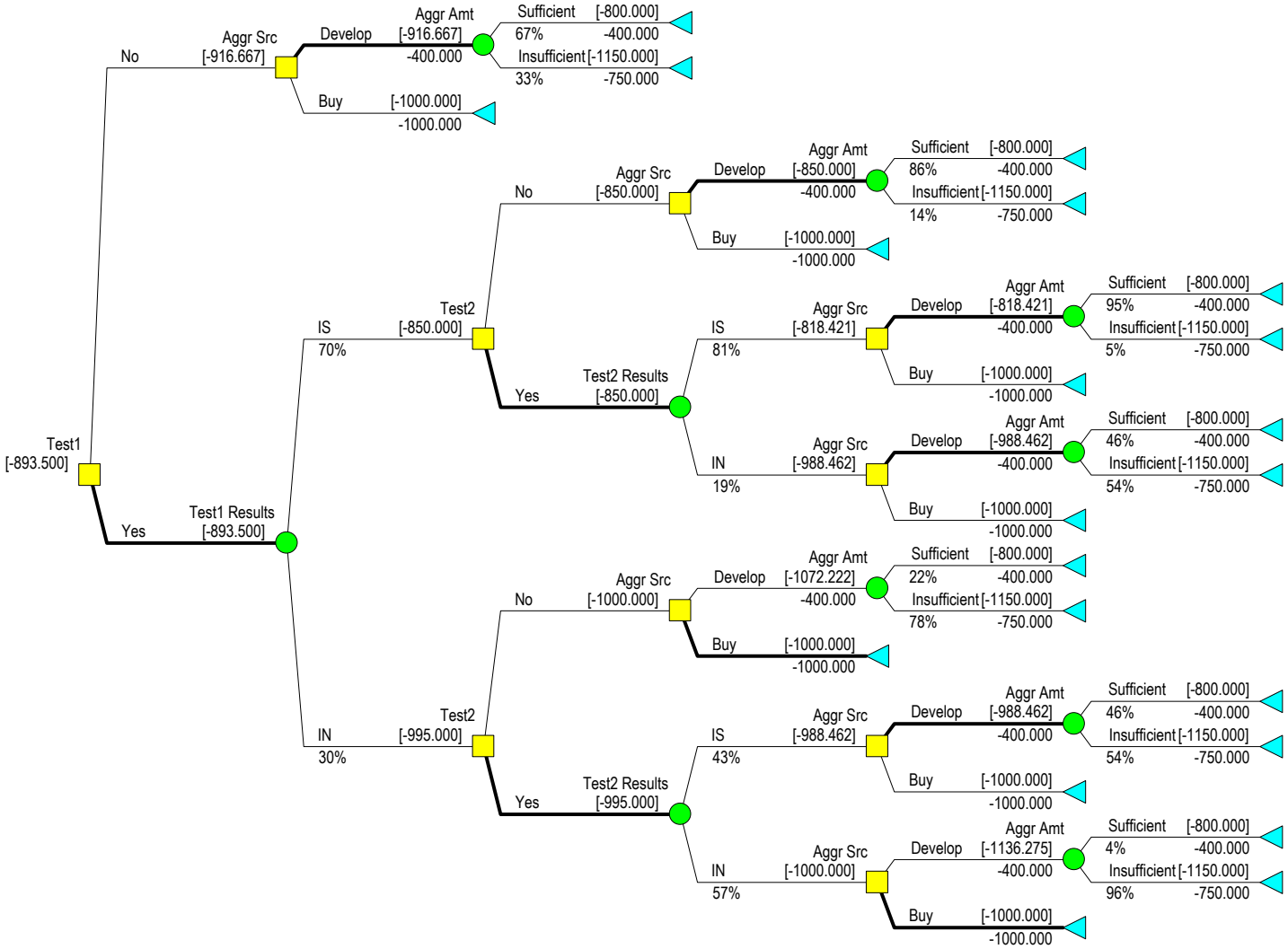


The EVSI of a single test is -$895k+916.667k= $21.667k. The EVSI of two tests (as a package) is -$893.5k+916.667k= $23.167k.

Suppose that the alternative to obtain the results of a single test is provided for free but the actual outcome has not yet been observed. In this case the expected value of sampled information for the second test is EVSI = -$893.5k+895k= $1.5k.

Clearly, the second test has little value. Part of the reason is the fact that when the first test indicates IS the optimal decision does not depend on the outcome of the second test; it is always the same: "Develop". Thus, if the result of the first test is IS then the second test has no value. Only when the first test results in IN does the result of the second test change the optimal decision: If IS then "Develop"; otherwise, if IN then "Buy".

Let us know consider the *option* where the decision to perform the second test is made sequentially, *after* the result of the first test is known. To allow easy calculations, the decision tree below assumes a *zero* option price and a *zero* exercise price for the second test.

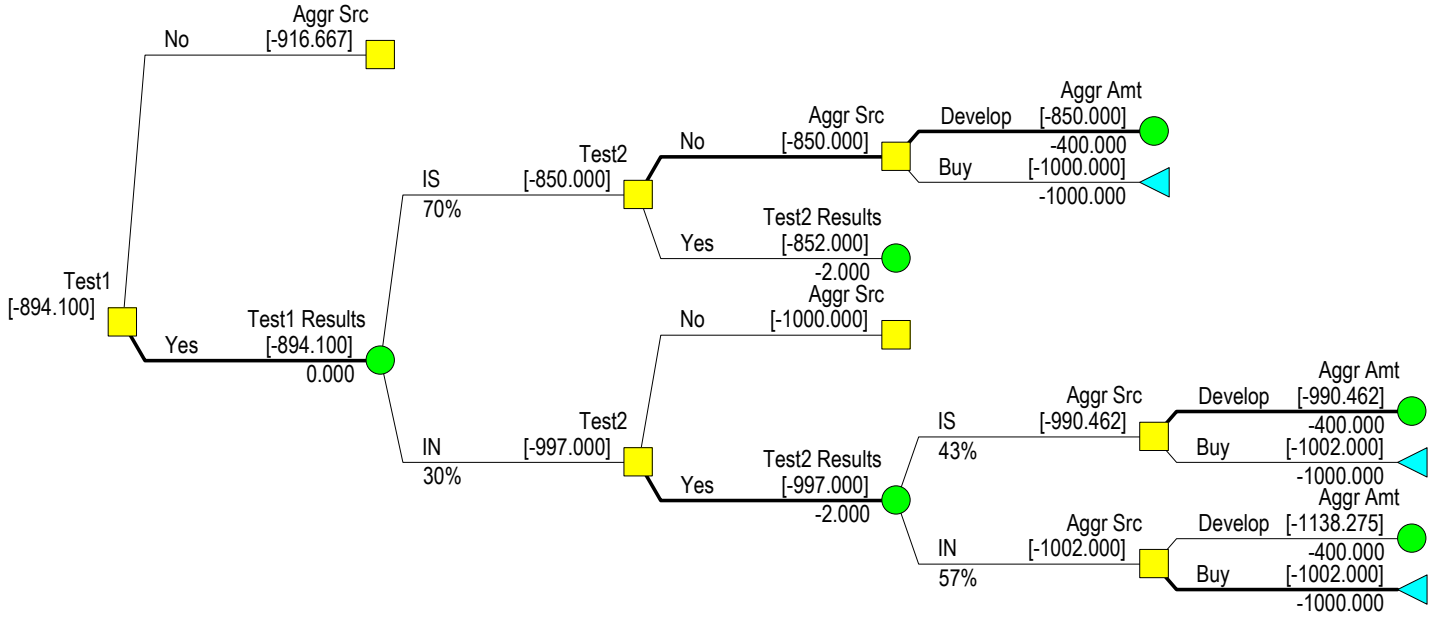


If the first test indicates IS, then both branches of the decision about the second test lead to an expected cost of -$850k. In this case, the second test should not be performed at any exercise price (any cost). On the other hand, if the first test indicates IN, then performing the second test has an expected cost of -$995k whereas not performing the second test has an expected cost of -$1000k. Thus, the second test can be performed up to an exercise price of -$995k+1000k = $5k.

Effect of Exercise Price on the Value of Sequential Imperfect Test Option

Let us assume that the cost of the second test is \$2k. If the decision maker was presented with an offer to perform a second test at that price before the outcome of the first test was known, he would decline because its cost (\$2k) exceeds its EVSI (\$1.5k).

On the other hand, if the second test was presented to the decision maker as an option with an exercise price of \$2k, then the situation would be that shown in the decision tree below:

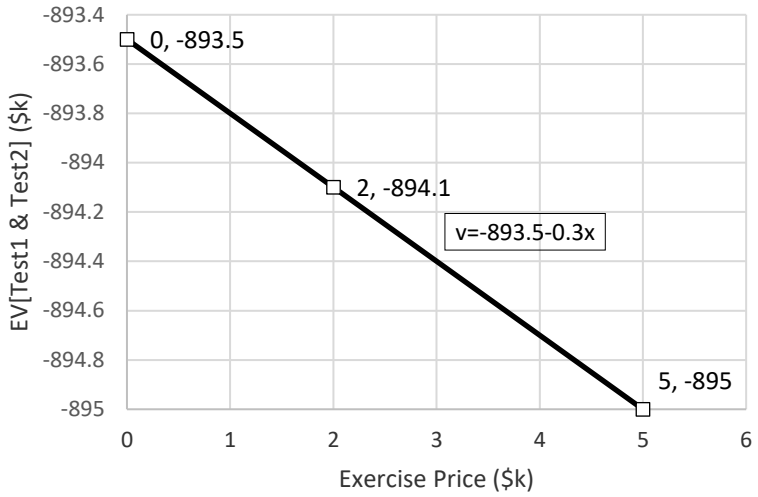


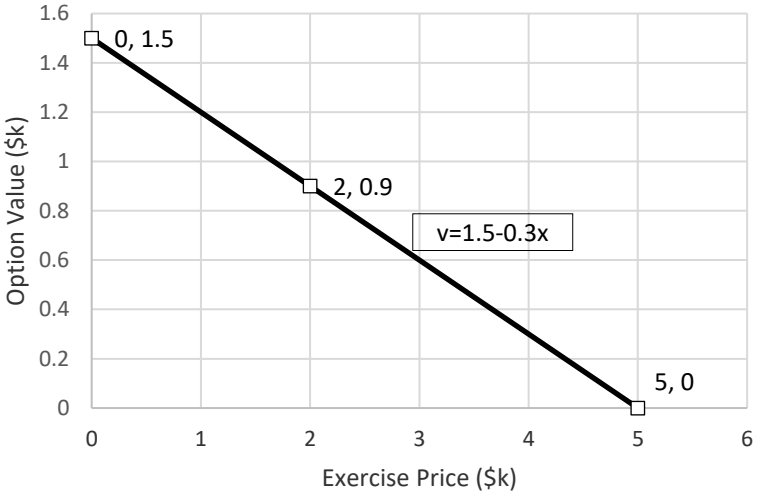
The expected cost of the option with an exercise price of \$2k is -\$894.1. Thus, the corresponding option value is -\$894.1+\$895 = \$0.9k. This means that the decision maker would be willing to pay up to \$0.9k ahead of time to have the option to buy the second test for \$2k after the outcome of the first test is observed. Of course, this option will be exercised, and the \$2k paid, only if the first test indicates IN. Otherwise, if the first test indicates IS, the option will not be exercised and the only cost to the decision maker would be the initial option price.

The two graphs below show the linear relationship between the *exercise price* and (a) the *expected cost* of the option to buy the second test after the outcome of the first test is known and (b) the *value* of the option.

As expected, the value of the option, v, declines linearly as the exercise price p_e increases:

$$v_o = 1.5 - 0.3\ p_e$$





When rearranged, this equation also states the "obvious": that the option value, v_o, plus the *expected* exercise price, $0.3p_e$, equals the EVSI of the second test ($1.5k):

$$v_o + 0.3p_e = 1.5 = \text{EVSI}$$

The seller of the test also has a choice to make. (A) Should he charge an option price of $1.5k with a zero-exercise price (a sure thing for himself)? (B) Should he offer the option for free and then charge an exercise price of $5k (a pure gamble for himself)?

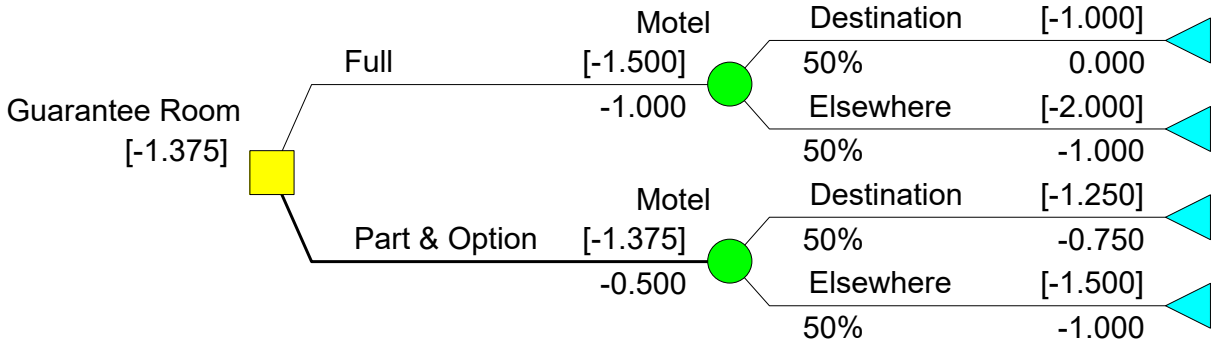
The above equation argues that it makes no difference. Either way, the seller expects to make $1.5k. However, this requires at least two strong assumptions: (a) the seller knows exactly what the decision maker knows, and (b) the seller is also risk-neutral. If (a) is not true, then choice (A) is risky because a slight overestimate of the maximum option value ($1.5k) might cause the decision maker to reject the option altogether and the seller cannot make any money. On the other hand, the certain equivalent for the gamble in (B) may be much less than $1.5k for a risk-averse seller. Thus, the problem for the seller requires its own in-depth investigation.

Creating Options

Every discussion on generating alternatives for a decision should include the question, "What options could we and should we create?" Options result from recognizing or creating new decision opportunities in anticipation of the total or partial resolution of uncertainty.

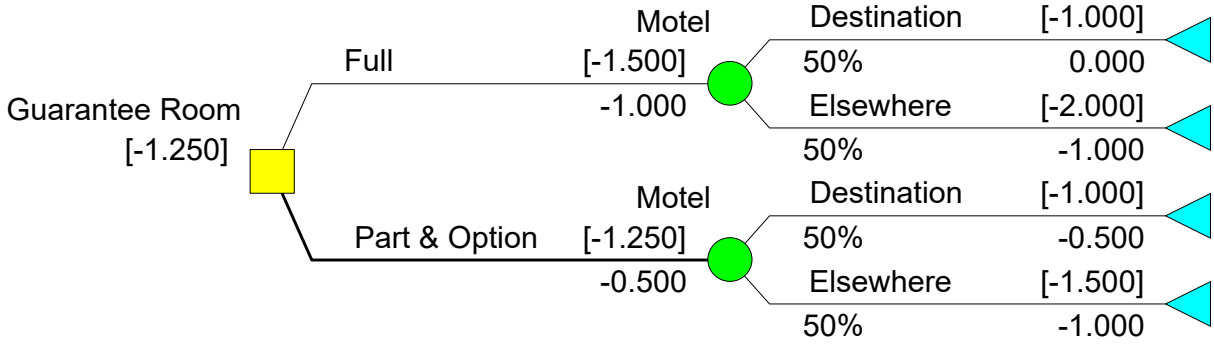
Suppose that you are traveling by car and you are concerned whether you will find a motel room at your desired destination. Given the highway's bad traffic conditions, you assign only a 50% chance of reaching your destination. You call a motel and find that you can guarantee a room if you prepay for it now by credit card. Your alternatives are to prepay the room or not.

Suppose that you make a counteroffer to prepay the innkeeper only half the cost of the room (the option price), and but also offer to pay an additional 3/4 of the cost of the room if you arrive (the exercise price). If you are risk-neutral, your evaluation of the cost of the option is 1/2 + (1/2)(3/4), or 7/8 the cost of the room, which is less than the cost of the guarantee.



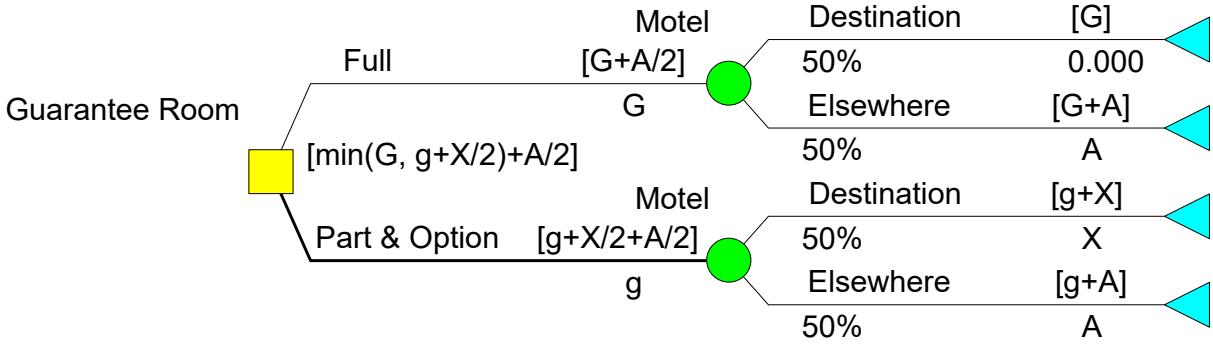
If the innkeeper is also risk-neutral and assigns the same 50% chance of your appearing, then the value to the innkeeper is also 7/8. The option will be a good deal to an innkeeper who assigns less than a 7/8 probability to renting the room to someone else.

You can adjust the option to fit other situations. For example, if the exercise price is reduced to 50% of the room cost, the option cost is 1/2 + (1/2)(1/2)=75%. Selling this option will be desirable to the innkeeper if there is less than 75% chance of renting the room otherwise.



To create a more general model let us define the following variables:

G = full guarantee = per diem cost room at destination
g = partial guarantee & option = prepaid partial cost of room at destination
X = exercise price = remaining cost of room at destination (if reached)
A = cost of alternate lodgings elsewhere



The option value in $ is that value of g, such that G+A/2 = g + X/2 +A/2. I.e., option value is:

$$g = G - X/2$$

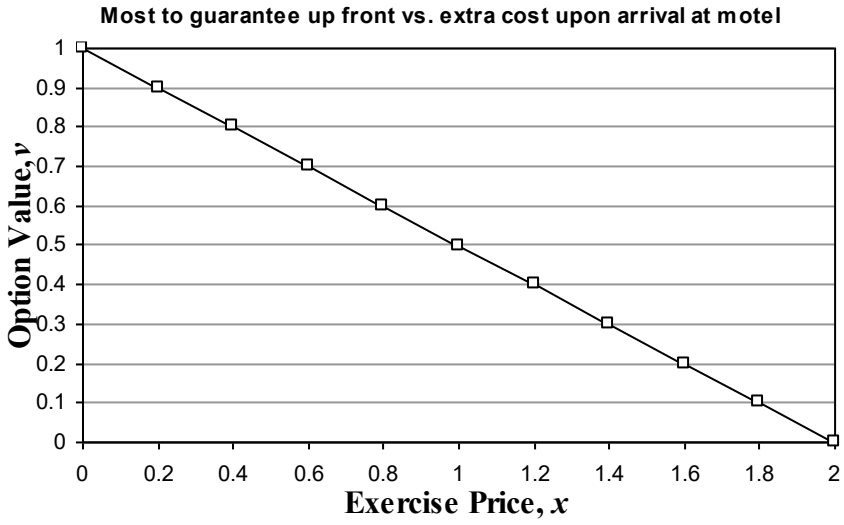
Notice that the option value does not depend on A, the per diem cost of alternate lodgings.

Let v=g/G and x=X/G (i.e., v and x are % of G). Then, the proportional option value is:

$$v = 1 - x/2$$

Case 1: x = 2 (i.e., pay twice G if arrive at destination); then v=0 (no guarantee up front).
Case 2: x = 0 (i.e., pay nothing extra at destination); then v=1 (full guarantee up front).
Case 3: x = 3/4 (i.e., pay 3/4G if arrive at destination); then v=5/8 (guarantee 5/8G up front).
Case 4: x = 1/2 (i.e., pay 1/2G if arrive at destination); then v=3/4 (guarantee 3/4G up front).

(Notice that v represents the maximum that the traveler would be willing to pay up front.)



Use in Developing Strategies

Options should have an important place in selecting business *strategies*. Possible *strategies* are defined by first identifying every decision required to define the strategy and then specifying the alternatives for that decision. Often, only *few* strategies are defined and investigated further: Those that appear to combine the most promising *coherent* selections of alternatives for each decision. The rest of the decision combinations are discarded without analysis as inferior.

As an example, consider a cosmetics company in the process of developing possible strategies for a new perfume. In this case, the decision "*Type of Product*" might have alternatives like "*mass-market*", "*medium-priced*", and "*exclusive*, *very-high priced*". The decision "*Distribution Channel*" might have alternatives like "*top-level department stores*" and "*discount chains*". A strategy that combines the "*exclusive, very-high priced*" product alternative and the "*discount chain*" distribution alternative would not appear to be *coherent* and would not be investigated. Clearly, the notion of *strategy construction* (i.e., the identification of coherent combinations of decision alternatives) is useful in that it reduces the dimensionality of decision problems to a manageable size (by pruning the non-coherent alternative combinations).

Sometimes, however, strategies present all decision alternatives as a package and not as a series of sequential decisions that could be made with additional information. I.e., the "framing of the choices" for each decision as "alternatives" may lead to insufficient consideration of strategies that contain options and hence *sequential* decisions.

One can evoke options by asking first whether we would be in a better position to make this decision after the revelation of one or more distinctions about which we are uncertain. If so, we should then ask what options we could create to give us the decision opportunity we seek. Then, we would evaluate whether they make sense through option analysis.

In the context of the perfume example, the company may buy an "option on manufacturing facilities" rather than committing to them *before* the results of a "market survey" are available.

Summary of Key Ideas

- An option is an alternative that permits or may permit the decision maker to exercise free will and make a future decision following revelation of information.
- An option has a value, an option price, and an exercise price.
- To calculate the value of an option we must know the exercise price for each possible alternative.
- If the option price is zero and the exercise price is also zero, then the value of an option is like the value of clairvoyance and must be non-negative.
- Acquiring information is an option even there is no clear future decision to make.
- The option to wait and make a decision later often adds value.
- Decision synthesis and the development of well-structured options can enable deals (make them profitable) that otherwise might appear to have no value.
- Well-structured two-party options can lead to win-win situations.

- There are many good reasons to generate options in management and our daily lives.