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# **SLAB-BAND FLOOR CONSTRUCTION**

by

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## **Doctoral Committee:**

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## PREFACE

The prime object of this thesis is to investigate, both experimentally and theoretically, the structural behavior of slab-band floor construction. In this construction the floor slab is supported by wide shallow beams called slab-bands. Because of the advantages of this floor construction as well as its increasing popularity in newly-built buildings it is, therefore, rather important to present a standard design **method in order to provide a rational way of designing this kind of floor system.**

The building for tests and frame analysis has four stories and three bays in both directions. All the panels are square at the third floor and all the exterior and corner panels are nearly square at other floors because of dimensions of exterior and corner columns. This building was designed as light-load type so that the results obtained from this study can be used for designing buildings of this category.

A model made of Plexiglas was used for testing and its scale was one-fifteenth. It consisted of three floors and columns between **them and on one quarter of its middle floor, which was tested,** **151 SR-4 A-7 strain gages were installed.** The strains measured by gages due to different combinations of loading were converted into stresses by use of modulus of elasticity and Poisson's ratio of

Plexiglas which were determined by testing specimens cut from the same sheet of Plexiglas as the tested floor of the model. These model stresses were further converted into prototype stresses by means of scale of the model and moments of inertia of and unit load on the model and the prototype in order to compare with the stresses obtained from theoretical analysis.

In the theoretical frame analysis four bents were analyzed at each floor in order to get stresses in the slab, interior slab-band, exterior slab-band and wall beams. Assumptions which are suggested for the analysis of flat slab frame by ACI Building Code were used here to simplify the analysis of slab-band floor frame.

Comparisons between stresses in the prototype obtained from results of model tests by use of conversion formulas and stresses in the prototype calculated by approximate methods of frame analysis were made in order to show how close the results obtained from model tests and frame analysis agree. The comparisons showed that these stresses were very close in many instances at critical design sections of the members and a few revealed large differences at some not important places. Because the model was made exactly as the prototype in shape and the modulus of elasticity, Poisson's ratio and strains in the model were determined accurately therefore it is believed that the results from model tests gave more accurate stresses than those from frame analysis.

A specification for designing the slab-band floor is suggested in order to give a definite guide of designing it as well as to encourage engineers and architects to adopt this kind of floor construction. A comparative design of four kinds of floor constructions was also made in order to show that the slab-band floor saved in total cost of concrete and steel as compared to the one-way slab, two-way slab and flat slab

floors under the same architectural layout in addition to the savings in story height, plumbing, electric conduits, partition wall, labor and so forth.

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## CHAPTER 1

### INTRODUCTION

Reinforced concrete has long been used as one of the most important materials in construction. New ideas in reinforced concrete construction are being introduced and new types of reinforced concrete structures are being built. But so far as methods of designing these structures are concerned most of them, if not all, are designed according to theories and experience. As for theories, there are always limitations and ideal conditions accompanying them, and as for experience, it is expensive to get. In recent years structural laboratories have been established in universities and by private concerns and extensive researches in testing reinforced concrete slabs, beams, columns and so forth have been completed. It is the intention of the author to investigate, both theoretically and experimentally, the action of a building frame in which slab-band floor construction is used.

#### 1.1. Slab-Band Floor Construction

In slab-band floor construction the slab is supported by slab-bands which are wide and shallow beams as shown in figure 1.01. This

variation of floor system is something between conventional slab-beam-girder and flat-slab floor constructions. In slab-band construction the slab-bands are used only in one direction except that wall beams are used normal to the slab-bands at the outside as shown in figure 2.01.

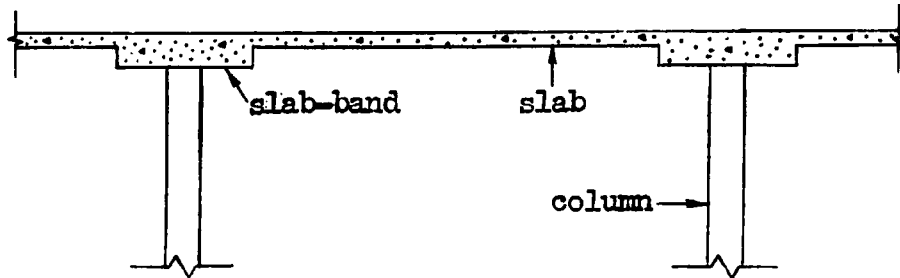


Figure 1.01.

#### Slab-Band Floor Construction

The slab-band construction might be likened to a flat slab system in which the drop panels are extended in one direction until a continuous band is formed. The slab-band has a double action. It performs as a part of the slab in the direction normal to its length and it acts as a wide beam spanning between columns. Concrete protection over steel reinforcement in a slab-band can probably be the same as for a slab without violating the intent of the ACI Building Code.

#### 1.2. History of Slab-Band Floor Construction

The following is the brief history of the development of slab-band floor construction.

It is believed that the first building in which the slab-band floor construction was used was built in 1927 in Seattle, Washington. Its name is Rhododendron Apartment Building and it has nine stories. The total width of the concrete floor of the building is 44'2" and the

floor slab is supported by two spandrel beams at the sides and by a wide slab-band at the middle. These supporting members are running in one direction only and there are no stiffening members in the other direction. The ratio of the thickness of the slab to that of the slab-band is 0.684 and the ratio of the width to the total depth of the slab-band is 15.58. The width of the slab-band is more than eight-tenths of the clear span of the slab. Figure 1.02 shows the arrangement and dimensions of this floor construction.

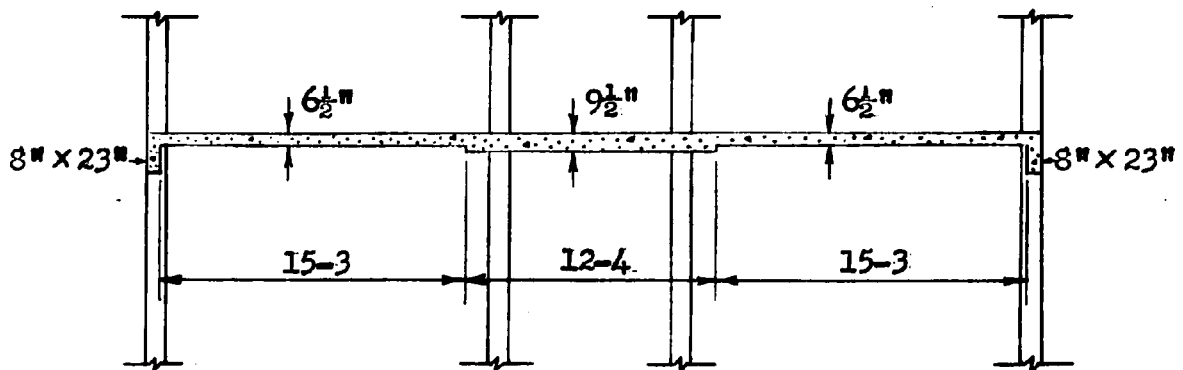


Figure 1.02.

Floor Section, Rhododendron Apartment Building  
Seattle, Washington.

The slab-band floor construction was again adopted in eight men's dormitories at the University of Wisconsin, Madison, Wisconsin. These dormitories were completed in 1939 and each of them has three stories and a basement. Each dormitory is 36'0" wide and 112'0" long. The arrangement of this floor system in the dormitories is not very different from that in the Rhododendron apartment building. The ratio of the thickness of the slab to that of the slab-band is 0.556 and the ratio of the width to the total depth of the slab-band is 7.5. The width of the slab-band is about four-tenths of the clear span of the slab. Figure 1.03 shows the cross-section of this floor system.

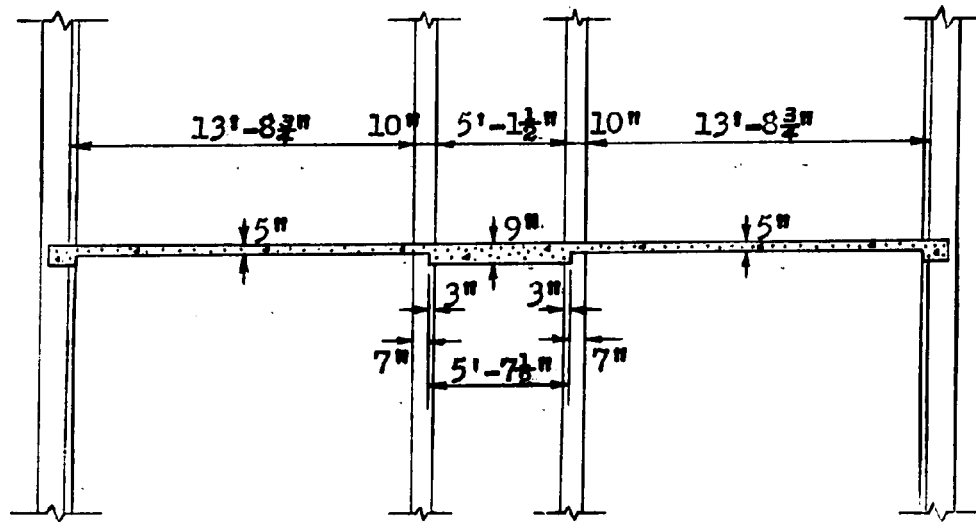


Figure 1.03.

Floor Section, Men's Dormitory,  
University of Wisconsin, Madison, Wisconsin.

In 1944 a design for the building used as residence and school for nurses at Bellevue hospital in New York was done. A transverse section of a typical floor for the nurses' living quarters is shown in figure 1.04. The slab of the floor is supported by two interior slab-bands and two exterior spandrel beams whose size is between conventional narrow, deep beam and slab-band. The ratio of the thickness of the slab to that of the slab-band is 0.264 and the ratio of the width to the total depth is 2.7 for the interior slab-band and 0.79 for the exterior spandrel beam. The width of the slab-band is 0.45 of the average clear spans of two adjacent slabs. It is interesting to note the functional reason for the adoption of these interior slab-bands on the floor. The corridor is directly under the center slab span and has a width of the clear span of the slab. On each side of the corridor the space under the interior slab-band is used for lavatory and closet and the adjoining space under the exterior slab is used for bedroom.

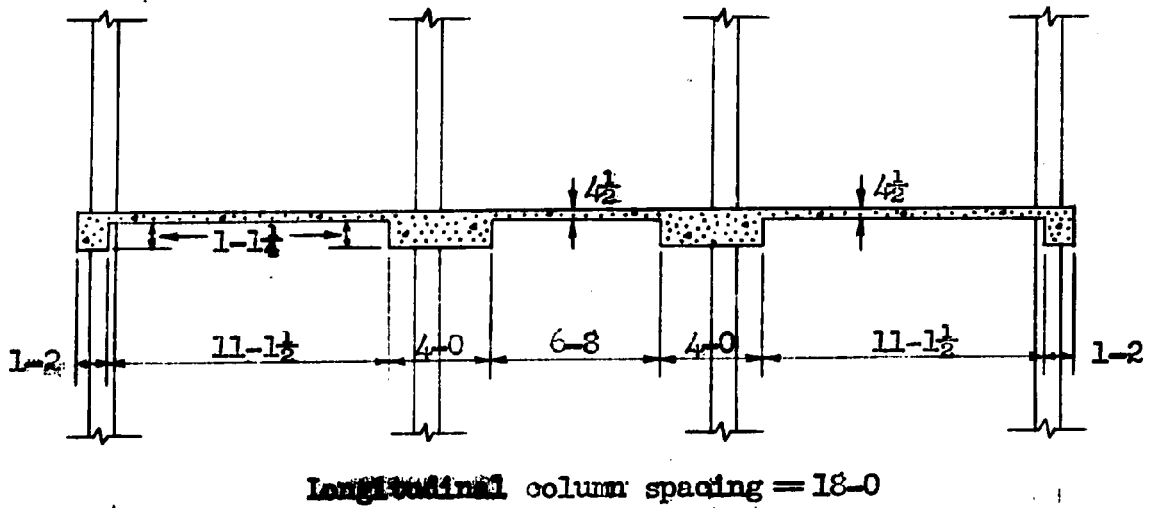


Figure 1.04.

Floor Section, Residence and School for Nurses,  
Bellevue Hospital, New York.

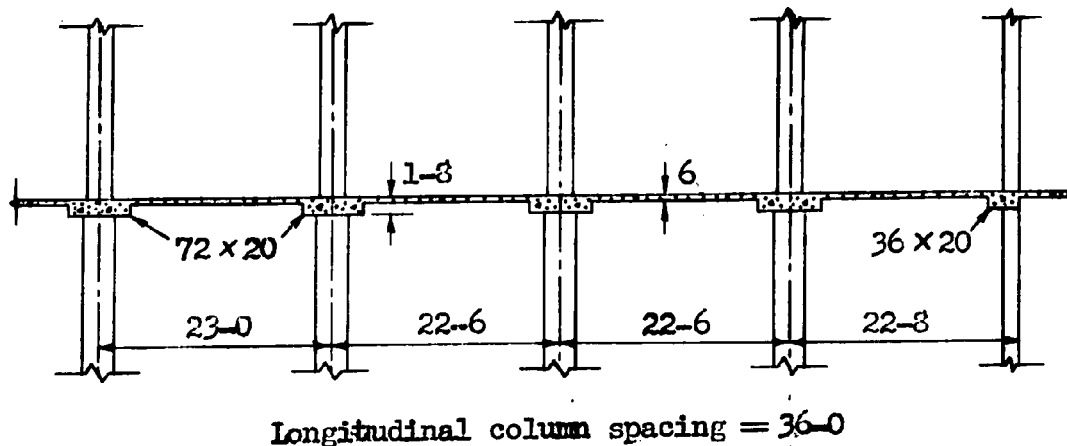


Figure 1.05.

Floor Section, The New Tripler General Hospital,  
Oahu Island, Hawaii.

During the years of 1945 and 1946 the New Tripler General hospital on Oahu Island, Hawaii was under construction. One of the structural features of the main hospital building is the adoption of slab-band floor construction. Figure 1.05 illustrates the partial

cross-section of this floor system. The ratio of the thickness of the slab to that of the slab-band is 0.3 and the ratio of the width to the total depth of the slab-band is 3.6. The width of the slab-band is 0.364 of the clear span of the slab.

In 1946 two apartment projects, the John Lovejoy Elliot houses and the Brownsville houses, were being built using slab-band floor construction in New York City under the auspices of the New York City Housing Authority. Figure 1.06 shows a section through a typical wing of the John Lovejoy Elliot houses. The ratio of the thickness of the slab to that of the slab-band is 0.377 and the ratio of the width to the total depth of the slab-band is 2.11. The width of the slab-band is 0.177 of the average clear spans of two adjacent slabs.

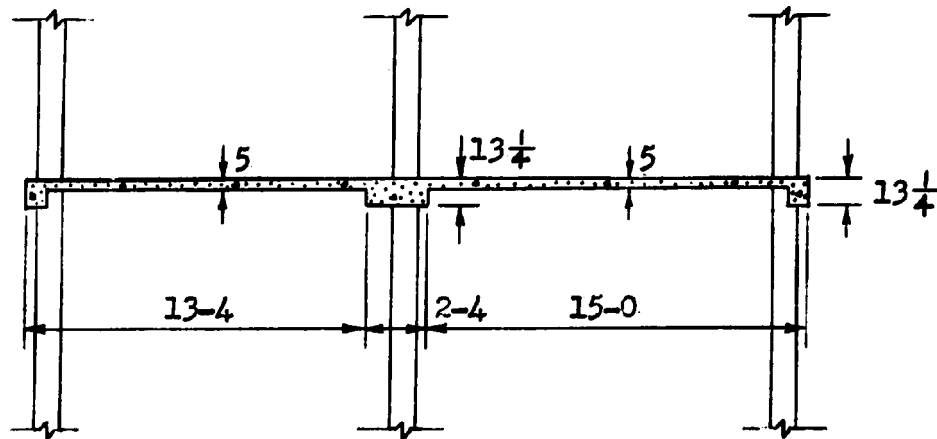


Figure 1.06.

Floor Section, John Lovejoy Elliot Houses,  
New York City.

Many other buildings including apartments, schools, industrial buildings have been built since then with the slab-band floor construction instead of the conventional type.

In order to show the different proportions of the slab-band floor construction in some of the existing buildings as described above the table 1.01 is prepared for comparison.

Table 1.01.

Range of Proportions of Slab-Band Construction

| Buildings                                               | $\frac{t_1}{t_2}$ | $\frac{b}{t_2}$ | $\frac{b}{L}$ |
|---------------------------------------------------------|-------------------|-----------------|---------------|
| Rhododendron Apartment Building<br>Seattle, Washington. | 0.684             | 15.58           | 0.809         |
| Men's Dormitories, Univ. of Wis.<br>Madison, Wisconsin. | 0.556             | 7.5             | 0.400         |
| Nurses' School in Bellevue<br>Hospital, New York.       | 0.264             | 2.7             | 0.450         |
| New Tripler General Hospital,<br>Oahu Island, Hawaii.   | 0.3               | 3.6             | 0.364         |
| John Lovejoy Elliot Houses,<br>New York City.           | 0.377             | 2.11            | 0.177         |
| Building proposed for study<br>in this thesis:          | 0.409             | 4.37            | 0.333         |

The symbols used in the table have the following meanings and are also indicated in figure 1.07.

$t_1$ : Thickness of the slab.

- $t_2$  : Thickness of the slab-band.  
 $b$  : Width of the slab-band.  
 $L$  : Clear span of the slab.

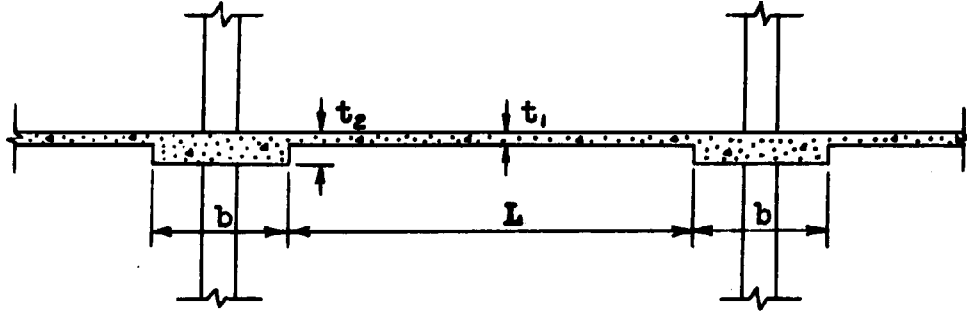


Figure 1.07.

### Floor Section of Slab-Band Construction

#### 1.3. Comparison with Other Types of Floor Construction

It can be said that one type of floor construction is superior to others in some respects but is inferior to others in other respects and therefore each type of floor construction has its own advantages and its own sphere of application. The slab-band floor construction is a new one and it is not widely known to the engineers. It is therefore necessary to demonstrate the advantages that this floor system offers in its application to certain kinds of buildings. The following is a comparison of different types of floor systems with the slab-band floor construction.

##### 1.3.1. Slab-Band and One-Way Slab Floor Constructions

It is supposed that both of these two floor constructions have the same architectural layout and that both the slab-bands of the former system and the beams of the latter one are in one direction only.

### A. Bending Moments.

In the design of slab for the slab-band floor construction the slab, the slab-bands and the columns are considered as one whole frame in the case of frame analysis and the slab and the slab-bands are considered as an entity in the case of the analysis of slab with haunched ends, but in the design of slab for the one-way slab construction the slab is conventionally considered to be separated from supporting beams. This is the fundamental difference in designing these two floor systems.

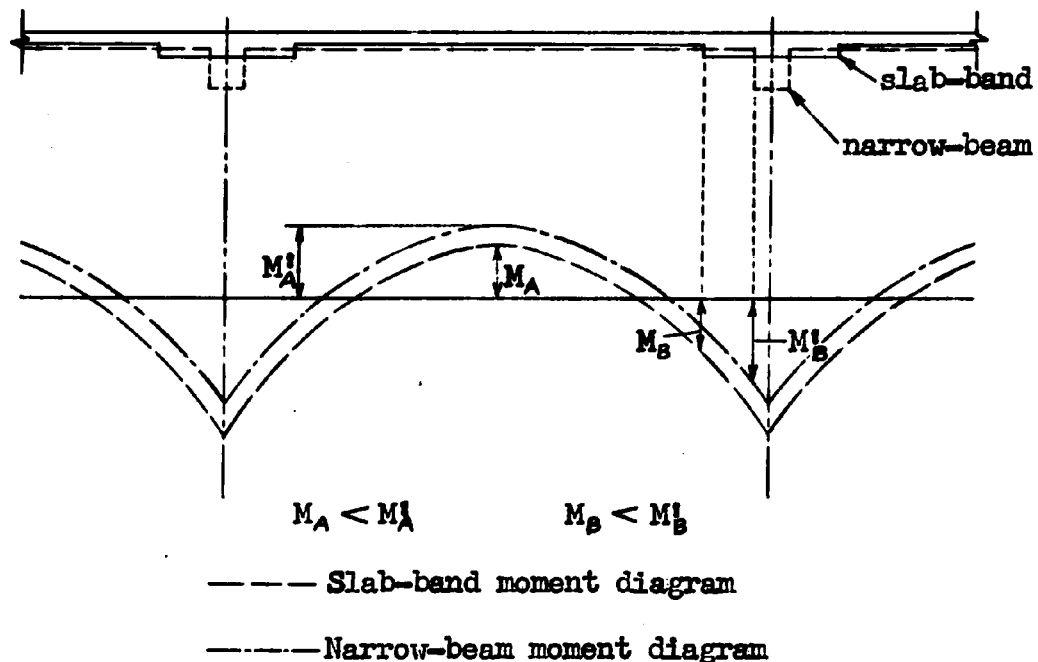


Figure 1.08.

### Bending Moment Curves for Slabs of Slab-Band Floor and One-Way Slab Floor

When the method of moment distribution is used to analyze the slab-band floor system both the fixed-end moments at centers of supports and the carry-over factors of the slab-band floor are larger than those of the one-way slab floor respectively. In other words the

negative bending moments along the centers of supports are larger and the positive bending moments are smaller for the slab-band floor than corresponding terms for the one-way slab floor. As a matter of fact the negative bending moments along the edges of the supports are critical in design. The slab-bands are much wider than the conventional beams and therefore the negative bending moments in the slab along the edges of the slab-bands are smaller than those in the slab along the edges of the beams in the corresponding panels. Figure 1.08 shows this fact.

#### B. Thickness of Slab

The slab is thinner for the slab-band floor than for the one-way slab floor because both the positive and negative bending moments for the former floor are smaller than those for the latter floor respectively. In practice the thickness of the slab is limited to 4" or  $4\frac{1}{2}$ " minimum and thus both floor systems will need the same slab thickness in some cases. In this case the difference of these two floors will be in steel.

#### C. Reinforcing Steel

Because of the difference of bending moments in these two floor systems less main reinforcing steel is required in the slab of the slab-band floor than in the slab of the one-way slab floor. The temperature and shrinkage reinforcement in the direction perpendicular to that of the main reinforcement is nearly the same for both floor systems.

#### D. Concrete Protection over Steel Reinforcement

The slab-bands are wide and therefore they need no more concrete over steel reinforcement for fire protection than the slab does except at their corners. Furthermore the unit shear as a measure

of diagonal tension is usually low for slab-bands and there is no need for more concrete over the main steel reinforcement because of the diameter of stirrups. As a result the total thickness of concrete protection needed over main steel reinforcement for slab-bands is three-quarters of an inch while that for the beam is one and a half inches plus diameter of stirrups.

#### E. Story Height

The height of a story consists of two elements, one of which is the total depth of the slab-band or deep beam and the other is the space between the bottom of the slab-band or the deep beam and the floor surface below. The second factor is the same for both floor systems and the first one is different because the slab-band is thinner than the deep beam by about one foot or so. It is thus seen that the adoption of slab-band floor instead of the one-way slab system can reduce the story height by about one foot.

#### F. Columns

The columns needed for the slab-band floor construction are about one foot shorter than those for the slab-beam floor construction and the amount of concrete, longitudinal reinforcing steel and ties saved in the columns is considerable.

#### G. Plumbing and Electrical Conduits

Length of both the plumbing and the electrical conduits will be saved because of the less story height for buildings using slab-band floor system. However, the slab-band construction is perhaps at a disadvantage in the matter of space in which to run services.

#### H. Partition Walls

The height of partition walls for the slab-band floor

construction is reduced because of the reduced height of stories. In addition, if the slab-band is located over corridor, lavatory, closet or any other small rooms the partition walls need only be extended up to the bottom of slab-band and thus material is further saved.

#### I. Formwork

The length of formwork for the columns and the height of supports for the slab formwork are reduced for the slab-band floor construction for the same reason as above. Furthermore the formwork for the slab-band floor system is easier to make.

#### J. Placing of Steel Reinforcement

It is easier to place the steel reinforcement in the shallow slab-bands than in the deep beams.

#### K. Pouring of Concrete

It is easier to pour concrete into shallow slab-bands than into deep beams and consequently concrete of higher quality with less water-cement ratio can be used.

#### L. Locations of Columns

The columns used to support slab-bands can be located any place laterally within the confines of the slab-bands but those used to support conventional narrow, deep beams have not much choice in this respect. In some cases the columns which are located beyond the center line of the slab-band need not to be increased in amount of steel or size of concrete section. In other cases the columns would have to be more heavily reinforced to care for the eccentricity introduced by shifting the columns off the slab-band center line but this is offset by the advantage of having some freedom in column locations.

#### M. Turn of Slab-Bands

Conventional beams are either straight or curved between two consecutive supporting columns but the slab-band can turn between two consecutive supporting columns in a way as shown in figure 1.09 because of architectural layout.

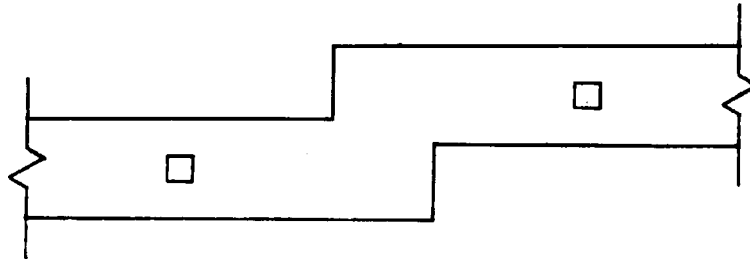


Figure 1.09.

Turn of Slab-Band

#### N. Brick Wall

The height of brick wall around the building for the slab-band construction is reduced because of the reduced height of the stories.

#### O. Appearance

When slab-bands are used over corridor, lavatory, closet and so on the other rooms will have an unbroken ceiling line which is desirable because of appearance. On the other hand in some large rooms the presence of wide, shallow slab-bands will break the monotony of a large flat ceiling area without intruding into the room as do the deep beams.

#### P. Relative Economy of Slab-Bands and Conventional Beams

The slab-bands alone, compared to the beams in conventional one-way slab construction, are not as economical of steel and concrete. However, this is offset by savings in the slab.

### 1.3.2. Slab-Band and Two-Way Slab Floor Construction

It is supposed that both of these two floor constructions have the same architectural layout and that the slab-bands of the former system are in one direction only and the beams of the latter one are in two directions perpendicular to each other.

#### A. Bending Moments

For a square two-way interior slab panel the moment coefficient  $c$  in the equation  $M = cwL^2$  is equal to 0.033 for negative bending moment along the edges and 0.025 for positive bending moment along the center line of the slab as specified by ACI Building Code section 709.

For an interior slab-band slab with the same column spacing the moment coefficient  $c$  in the equation  $M = cwL'^2$  is equal to 0.074 for negative bending moment along the edge of the slab-band or is equal to 0.075 for positive bending moment along the center line of the slab. The term  $L'$  in this equation is the clear span of the slab between edges of two adjacent slab-bands and is equal to three-fourths of the panel length. The coefficients 0.075 and 0.074 were determined by the present study and is given in section 7.6. The moment equation for the slab of the slab-band floor can be rewritten as thus:  $M = cwL'^2 = 0.075w(\frac{3}{4}L)^2 = 0.042wL^2$ .

The coefficients in the slab-band slab are larger than the corresponding ones in the two-way slab. This is due to the fact that the two-way slab has supporting beams on all sides and spans in two directions while the slab-band slab is mainly supported in one direction only.

#### B. Thickness of Slab

The slab-band floor requires a thicker slab on the basis

of bending moment. However the thickness of a two-way slab is limited by ACI Building Code to a minimum of the perimeter of the slab divided by 180 but not less than 4 inches. Therefore, at least for light floor loads the two types of construction could result in same slab thickness.

### C. Reinforcing Steel

Although the slab-band slab needs more steel in the direction of its span length, only temperature and shrinkage reinforcement is needed in the other direction. In the two-way slab, steel in both directions is main reinforcement. As a whole the two-way slab needs more steel than the slab-band slab does.

D. to O. Same as D. to O. in section 1.3.1. except the term "one-way slab" is replaced by the term "two-way slab".

### P. Relative Economy of Slab-Bands and Conventional Beams

The slab-bands alone, compared to the beam in conventional two-way slab construction, are not as economical of steel and concrete in some cases. However, this is offset by savings in the slab.

## 1.3.3. Slab-Band and Flat Slab Floor Constructions

It is supposed that both of these floor constructions have the same architectural layout and that the slab-bands of the former system are in one direction only.

### A. Bending Moments

The sum of the positive and the average negative bending moments at the critical design sections of a flat slab panel as specified by ACI Building Code is expressed by the following formula:

$$M_u = 0.09WL\left(1 - \frac{2c}{3L}\right)^2$$

in which: L = Span length of slab center to center of columns in the

direction in which bending is considered.

$c$  = Diameter of column capital at the under side of drop panel.

=  $0.225L$  as often used.

$W$  = Total dead and live load uniformly distributed over a single panel area.

=  $wL^2$  when the panel is square and  $w$  is the unit dead and live load.

This formula can be rewritten as follows:

$$M_o = 0.09wL^2 \times L \left(1 - \frac{2 \times 0.225L}{3L}\right)^2$$

$$= 0.065wL^3$$

The thickness of slab is controlled by the positive moment in the column strip which is  $0.20M_o$  for flat slab with drop panel, i. e.

$$M = 0.20 \times 0.065wL^3$$

$$= 0.013wL^3$$

In order to compare with the bending moment in the slab of the slab-band construction it is necessary to express this positive moment in the column strip in the unit of ft.-lbs per ft. width of slab. The width of the column strip is half of the panel width, i. e.  $\frac{1}{2}L$ , and only three-fourths of the width of the column strip shall be taken as the width of the section in computing compression due to bending according to ACI Building Code, i. e.  $\frac{3}{4} \times \frac{1}{2}L = 0.375L$ . The positive moment in the column strip is therefore

$$M = \frac{0.013wL^3}{0.375L} = 0.0347wL^2 \text{ \#/ft. of slab.} \quad (1.01)$$

In the case of slab-band construction the maximum bending moment in the slab of an interior panel as given in section 1.3.2.A is

$$M = 0.042wL^2 \quad (1.02)$$

Comparing the equations (1.01) and (1.02), the maximum bending moment in the slab of the flat slab construction is about 83% of that in the slab of the slab-band construction.

### B. Thickness of Slab

The slab for the flat slab construction is thinner than that for the slab-band construction because the former has smaller bending moment than the latter. However the ACI Building Code limits the minimum thickness of flat slab to  $\frac{L}{40}$  while the general practice uses  $\frac{L'}{32}$  as the minimum thickness for one-way slab. The clear span  $L'$  for the slab of the slab-band construction is three-fourths of the panel length and therefore the limitation of slab thickness for this construction is  $\frac{\frac{3}{4}L}{32} = \frac{L}{42.7}$  which is smaller than  $\frac{L}{40}$ .

### C. Reinforcing Steel

The steel used in both directions in the flat slab is main reinforcement while the slab of the slab-band construction needs main reinforcement in one direction only and temperature and shrinkage reinforcement in another.

### D. Partition Walls

The height of the partition walls is constant and is reduced for the slab-band construction if it is built below the slab-band.

### E. Relative Economy of Slab-Band and Drop Panel

The slab-band needs more concrete than the drop panel of the flat slab construction, and the former needs positive reinforcement while the latter does not. However, this is offset by saving in the slab.

## 1.4. Plan of Investigation, General Description

Both theoretical frame analysis and experimental model tests will be used in this project for the investigation of the slab-band

floor construction. Following is a general description of the principal steps followed in this investigation.

#### 1.4.1. Analytical Studies

Analytical studies included the following analyses.

A. The exterior slab-band together with columns framing from above and below was analysed as a bent.

B. The interior slab-band together with columns framing from above and below was likewise analysed as a bent.

C. Normal to the bents in A. and B. the bent made up of a strip of slab and slab-band one bay wide and the columns framing from above and below was analysed to determine the slab action. The horizontal members of this bent were considered as haunched beams.

D. The bent at the end of the building, composed of columns and wall beams normal to the slab-bands was analysed.

E. From the moments determined in A. to D. above the stresses in concrete and steel were calculated.

F. Similar buildings, using other type of floor construction, were partially designed in order to obtain comparisons.

A detailed description of the analysis of slab-band construction is given in chapter 8.

#### 1.4.2. Model Studies

The structural model which was tested is described in chapter 3 and the model tests are described in chapter 6. Briefly, the middle floor of the model was loaded with uniform full panel loads in various arrangements to obtain maximum strain conditions at critical sections in the floor system. Strains were measured by SR-4 strain gages.

The measured strains were converted to stresses in the model by the equations derived in section 7.1. These stresses were converted to stresses in the full scale building, assuming homogeneous material, by means of equation 9.05 in section 9.1. By means of equations 9.07 and 9.09 in section 9.1 the comparable stresses for a reinforced concrete structure were determined.

#### 1.4.3. Comparisons

Finally, in chapter 9, the stresses obtained by theoretical analysis are compared with corresponding stresses obtained by model tests. On the basis of these comparisons design recommendations have been prepared for slab-band construction.

## CHAPTER 2

### BUILDING FOR ANALYSIS AND TEST

#### 2.1. Type of Building

A building with moderate column spacing and designed for relatively light floors is suitable for a quite wide range of uses. Among them are offices, stores, schools, hospitals, hotels, and apartments etc. The general dimensions of the building chosen for this study place it in the class of buildings noted above. The general dimensions of the frame are shown in figure 2.01.

#### 2.2. Number of Stories

The number of stories of such a building of the type under consideration varies from a few floors to more than ten according to the particular use of that building and the architectural layout. As for a more complete frame analysis of a building, the result of which will be used as reference for practical design, four stories are the minimum number that should be used. In this way there is at least one floor which is the third one from either the top or the bottom of the building. For buildings with more than four stories their third and inner floors have the stress conditions not far from those as in

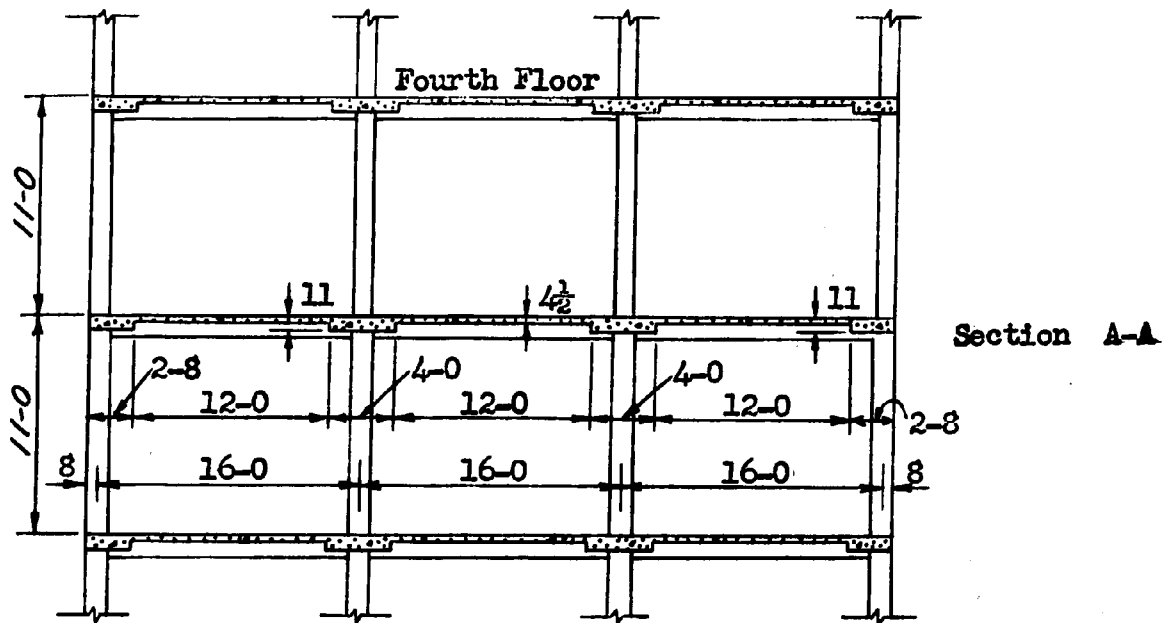
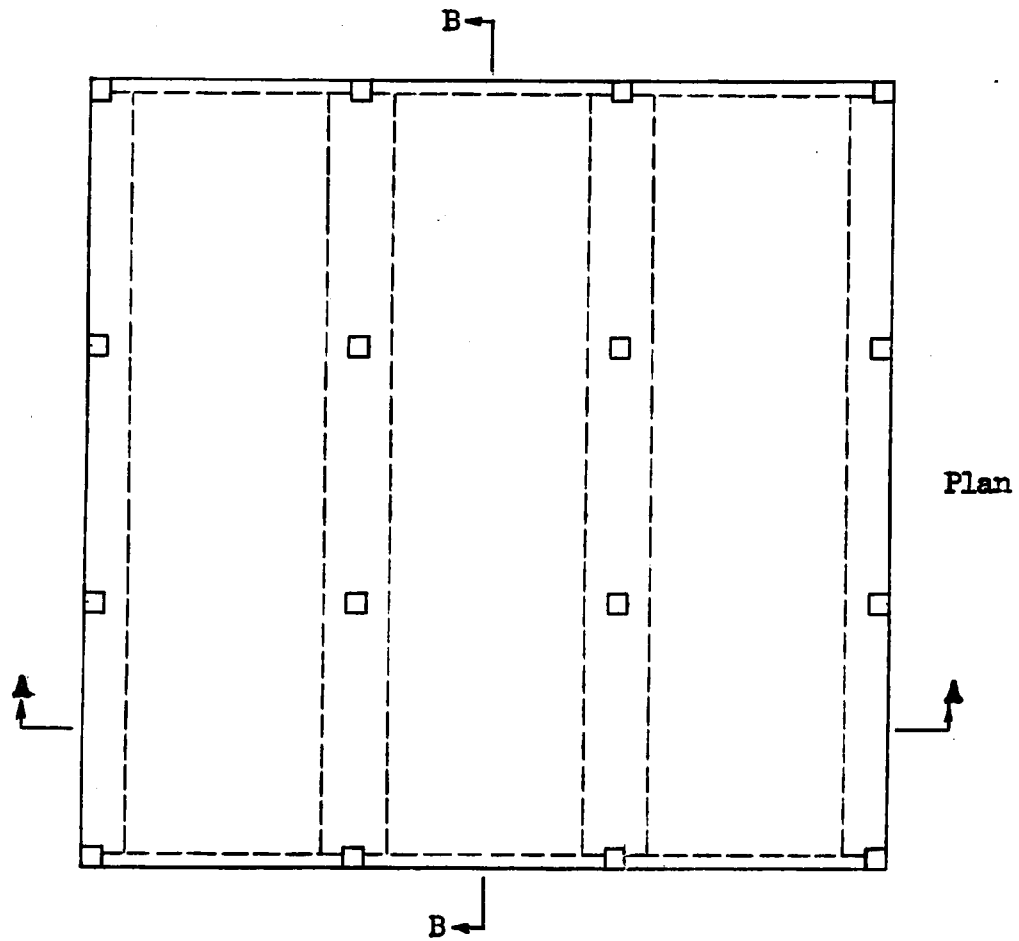


Figure 2.01

Building for Analysis and Test

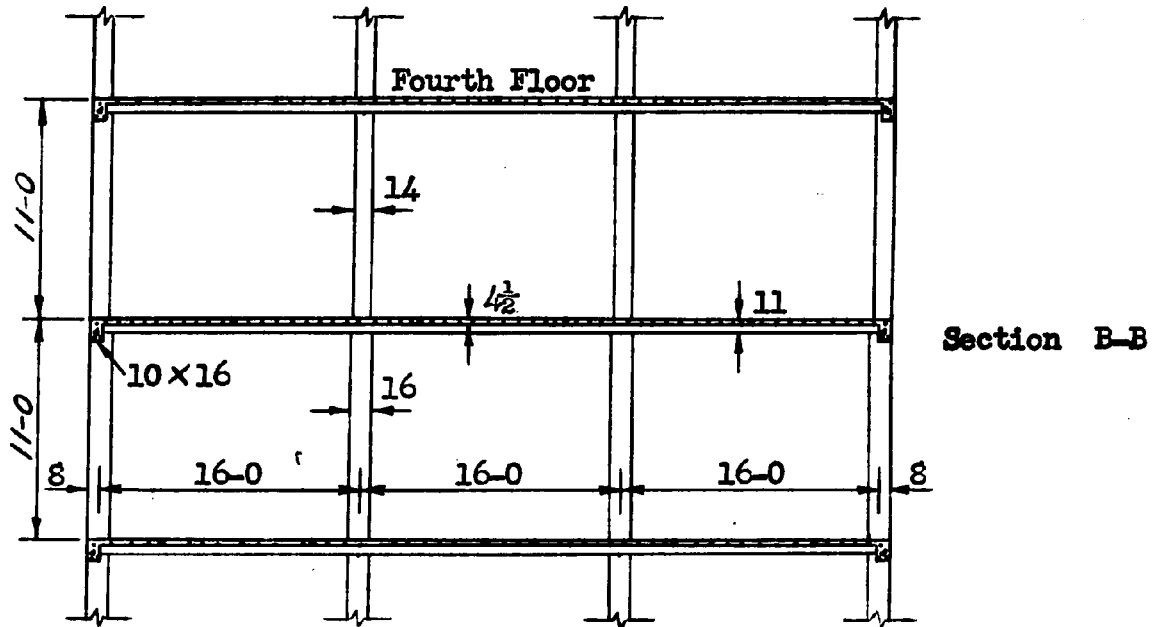


Figure 2.01. (Cont.)

## Building for Analysis and Test

the third floor of the four-story building.

The height of any one story between tops of floor slabs of any two consecutive floors is 11'0".

## 2.3. Number of Panels

There are three panels in each direction of this proposed building. Three panels are the minimum number for frame analysis and tests involving panels with all types of edge conditions. The panel length from center to center of supporting columns on the third floor is 16'0" in both directions. The dimensions of the columns directly under this floor is 16" x 16" and therefore the overall dimension of this building frame is 49'4" x 49'4". The length of exterior panels is longer than 16'0" for the floors above the one considered and shorter than 16'0" for those below the one considered because of the varying

size of columns.

#### 2.4. Slab-Bands

The slab-bands are used to support the slab and are to be supported by columns instead of the conventional narrow and deep beams or girders and they are running in one direction only. The width of the interior slab-band is assumed to be one-fourth of the center to center distance between slab-bands, i. e. 4'0", and that of the exterior slab-band to be one-eighth of the distance between slab-bands plus half the exterior or corner columns, i. e. 2'8". The total depth of the slab-band is determined by bending moment and shear requirements and in this case is 11 inches. The diagonal tension is usually low in slab-bands and stirrups are not needed. The concrete protection needed for slab-bands is only three quarters of an inch outside the main reinforcement instead of inch and half plus diameter of stirrups for conventional beams. This is a matter of the interpretation of the present ACI Building Code.

#### 2.5. Slab

The slab is principally a one-way slab because the slab-bands which support it are running in one direction only. As a matter of fact stresses are also developed in the slab in the direction perpendicular to that of the slab span because of loads, frame action, change of temperature, and shrinkage. The clear span of the slab is the panel length less half of the width of the slab-band on each side, i. e. 12'-0".

## 2.6. Spandrel Beams

There are two conventional continuous spandrel beams at two edges of the floor in the direction perpendicular to that of the slab-bands. These beams are used to support the slab and the spandrel wall along those edges of the building as well as to stiffen the whole floor in that direction.

## 2.7. Columns

Square tied columns will be used in this building. For the sake of appearance and simplicity in construction all the columns between two consecutive floors have the same dimension and only the steel reinforcement in them are different according to their respective bending moments and direct loads.

## 2.8. Loading

Although the buildings for offices, stores, schools, hospitals, hotels, and apartments belong to the light-load type, actually the live load used for designing these buildings is different in certain degree because of different codes by local governments, different function of buildings and different arrangement in a building. It seems to be desirable to have a fairly average intensity of live load for designing and investigating the building for this study in order that the results obtained from analyses and tests can be used for designing buildings of the light-load type. The live load intensity of 80 lbs. per sq. ft. has therefore been chosen for this purpose.

In addition to the weight of the concrete of the slab and

slab-and the other dead loads due to partition wall, wooden floor surface, and ceiling should also be taken into consideration. Twenty lbs. per sq. ft. is used for the equivalent partition wall, three for the wooden floor surface, and ten for the ceiling.

## CHAPTER 3

### THE MODEL

#### 3.1. Selection of Material for the Model

There are six important factors affecting the selection of material for the model and they are discussed as follows.

##### 3.1.1. Quality

The material should be homogeneous in quality and should be isotropic so that its physical properties will be the same at all spots and in all directions,

##### 3.1.2. Elasticity

The material should have a constant relationship between unit applied load and the induced change of strain within a certain limit so that not only the calculation of the testing results is simplified but also the initial stress which already exists before the application of testing loading because of some other reasons can be neglected.

##### 3.1.3. Connections

It is most ideal if the whole model is cast in one piece so that the joints between columns and beams will be rigid just as those

in the actual reinforced concrete building frame. The cost of such a model would be prohibitive. An alternative construction would be to make the floors and the columns of the model separately and then assemble them by means of welding, soldering, or cementing. It is essentially required that the physical properties of the material should not be changed due to the process of making connections and it is important as well that the joints thus made should be as strong as the material itself.

#### 3.1.4. Comparison of Moduli of Elasticity

Since the model will be rather small and have a limited space for loading the total amount of load that can be applied is limited. In order to build up reasonably large strains with a limited load and still stay within the elastic limit it is desirable that the modulus of elasticity for the material of the model be rather small.

#### 3.1.5. Workability

The ease of machining various materials to the required shapes and dimensions and of assembling the different parts varies with the material. The cost of machining is considerably greater than the material cost. The machining cost is affected not only by the cost of the actual machining but also by the ease with which the piece can be handled. Therefore weight is a consideration too.

#### 3.1.6. Cost of Materials

The cost of possible materials for a model of the kind required varies greatly. The cost of some of the materials is greatly affected by the size of pieces required. For example a single slab of brass

large enough for one floor and of the required thickness cost, in 1950, \$720. This compared with \$150 for steel and \$270 for Plexiglas. In addition some materials were very difficult to obtain so that finally the consideration of material cost was necessarily affected by availability.

### 3.1.7. Final Choice of Material

To summarize, the combination of properties most desirable in a material for this model are:

- A. Homogeneous and, if possible, isotropic.
- B. Elastic within a reasonable range.
- C. Good jointing properties.
- D. Low modulus of elasticity.
- E. Easily Handled and machined.
- F. Reasonable cost.

Four materials were considered: steel, aluminum, brass and Plexiglas. Plexiglas was finally chosen on the basis of the above properties.

## 3.2. Plexiglas

Plexiglas is one of the trade names for the methyl methacrylate plastics. The following is a summary of the history, manufacture, properties, and uses of the Plexiglas.

### 3.2.1. History

A brief history of the acrylic plastics, of which the methyl methacrylate is one, is outlined as follows.

In 1843 the ethyl ester of acrylic acid was prepared by Redtenbacher.

In 1872 Linnemann made the earliest observation of the tendency of polymerization of the acrylic acid.

In 1873 the acid and its methyl, ethyl, and allyl esters were prepared by Caspary and Tollens.

In 1877 a glassy polymer of ethyl methacrylate was obtained by Fittig.

In 1880 the transparent polymer of methyl acrylate was obtained by Kahlbaum.

In 1901 Rohm prepared the solid transparent polymers of acrylic acid derivatives.

In 1927 the Rohm and Haas Co. in Germany started to produce polymethyl acrylate.

In 1931 the Rohm and Haas Co. in the United States began manufacture of polymethyl acrylate under the trade names "Acryloid" and "Plexigum" for different applications.

From 1930 onwards the development of a methyl methacrylate resin was carried out in Great Britain by research by Hill and others for Imperial Chemical Industries, Ltd.

In 1936 rigid sheeting of methyl methacrylate resin was introduced by the Rohm and Haas Co. under the trade name "Plexiglas" and molding powder of the same resin was produced the next year under the same trade name.

In the same year "Lucite", the trade name of sheets, rods, tubes, and molding powder of the methyl methacrylate resin, began to be produced by E. I. Du Pont de Nemours and Company.

### 3.2.2. Manufacture

Methyl methacrylate is the polymerization product of esters of methacrylic acid and is at first prepared from acetone. Acetone is either obtained directly by fermentation of grain or is prepared from propylene which is in turn derived from petroleum or natural gas. Acetone is then treated with hydrocyanic acid to produce acetone cyanohydrin which reacts with sulfuric acid and methyl alcohol to produce monomeric ester of methacrylic acid. These methyl methacrylate monomers are at last polymerized by the action of heat, light, ozone, and oxygen. The properties of the finished product are largely controlled by the process of polymerization.

### 3.2.3. Physical Properties

The average values of physical properties of Plexiglas as given by the Rohm and Haas Company, the manufacturer of this material, are listed below.

#### A. Mechanical

|                                                        |         |
|--------------------------------------------------------|---------|
| Specific Gravity (25°C)                                | 1.18    |
| Tensile Strength (psi)                                 | 8,000   |
| Flexural Strength (psi)                                | 13,000  |
| Compressive Strength (psi)                             | 13,000  |
| Impact Resistance (ft. lbs.) ASTM Charpy Unnotched Bar | 3.5     |
| Modulus of Elasticity (psi)                            | 375,000 |
| Water Absorption (max. wt. gain, 24 hrs.) (%)          | 0.3     |

#### B. Electrical

|                                       |               |
|---------------------------------------|---------------|
| Dielectric Strength (short time test) | 500 volts/mil |
| Power Factor (60 cycles)              | 0.06          |
| Dielectric Constant (60 cycles)       | 3.9           |

## C. Thermal

|                                     |                                |
|-------------------------------------|--------------------------------|
| Resistance to Heat (°F. continuous) | 140-160                        |
| Coefficient of Thermal Expansion    | $5 \times 10^{-5}$ in./in./°F. |

## D. Optical

|                                |       |
|--------------------------------|-------|
| Total White Light Transmission | 92%   |
| Haze                           | 1%    |
| Refractive Index $N_D$         | 1.49  |
| Dispersion $N_F - N_C$         | 0.008 |

## E. Chemical

| Unaffected by:                        | Affected by:          |
|---------------------------------------|-----------------------|
| Automotive gasoline                   | Concentrated alcohols |
| Many chemicals, in solution           | Benzene               |
| Kerosene, hexane, white gasoline      | Acetone               |
| Mineral oils and greases              | Lacquer thinners      |
| Animal oils and greases               | Aviation gasoline     |
| Photographic solutions                | Carbon tetrachloride  |
| Battery solutions                     |                       |
| Mineral acids                         |                       |
| Strong alkalis                        |                       |
| Most liquid window-cleaning compounds |                       |

## F. Rate of Burning

Plexiglas is classified as "Slow Burning" under Federal Specifications LP-406A. Hot metals will not ignite it; if ignited by an open flame, it burns as slowly as medium density wood.

In addition to the properties listed above the following characteristics of Plexiglas are important too.

G. Plexiglas has excellent dimensional stability. It is featured by a marked resistance to cold flow or creep, and its impact

strength remains high at low temperatures.

H. It is resistant to age, weather, and sun.

I. It is light in weight. In comparison with other materials Plexiglas is less than half as heavy as glass; 43% as heavy as aluminum and 70% as heavy as magnesium.

J. It is brilliantly attractive in appearance.

K. It is as transparent as the finest optical glass. The purity of the material makes possible its manufacture in a highly effective and extensive range of transparent, translucent and opaque colors.

L. It has excellent workability. It can be sawed, machined and drilled like wood or soft metals, and can be formed, when heated to a pliable state, to almost any shape or curve.

M. The efficiency of Plexiglas as an insulator is not affected by age, weather or immersion in water.

N. In addition to its excellent transmission of all visible colors of the spectrum, Plexiglas transmits infra-red, ultra-violet and X-rays. Plexiglas is also noted for its ability to "pipe" light, even around curves and angles.

#### 3.2.4. Uses:

The most important application of methyl methacrylate plastics is transparent airplane enclosures. This is because of its excellent optical properties, its low density, its resistance to weathering and its retention of impact resistance at low temperatures.

The other applications of this material are automobile tail light and trim, watch crystals, machine windows and guards, transparent models, optical equipment, scientific instruments, edge-lighted pointers

and signs, edge-lighted instrument panels, curved dental, surgical and industrial instruments for piping cool light to inaccessible locations, furniture, display fixtures, better-grade novelty jewelry, compacts, brush backs and handles, dentures and teeth, and contact lenses.

In addition to its applications to various fields as mentioned above this kind of plastics is also used as structural and architectural models.

### 3.3. Description of the Model

A general description of the model is given below. Figure 3.01 shows this model. The perspective views, elevations, plans, sections, and dimensions of the model are shown in figures 3.02, 3.03, and 3.04.

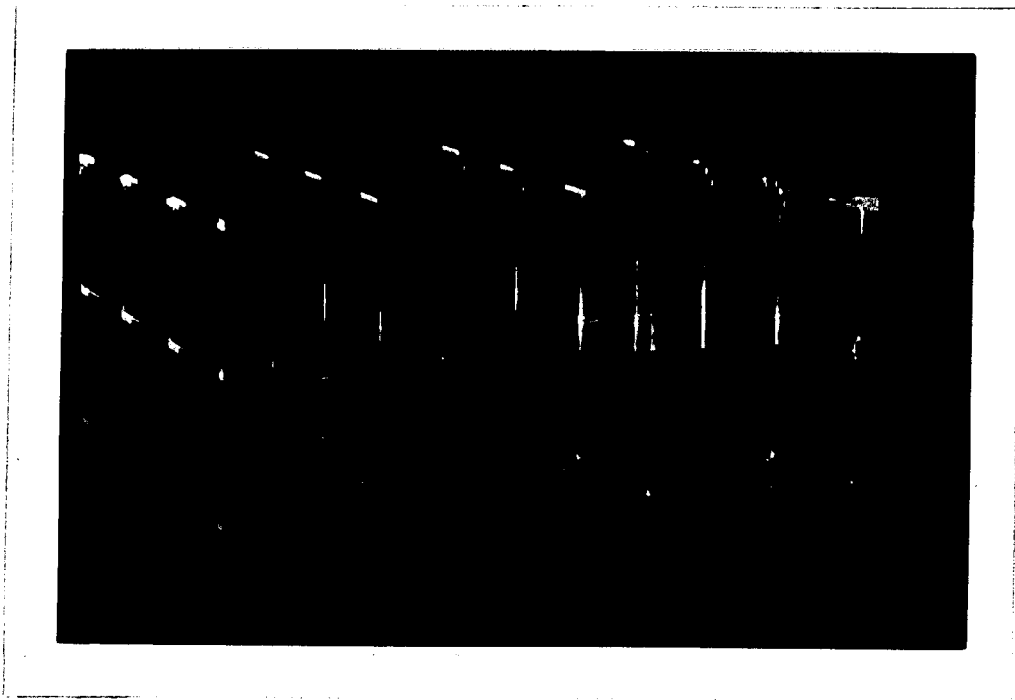
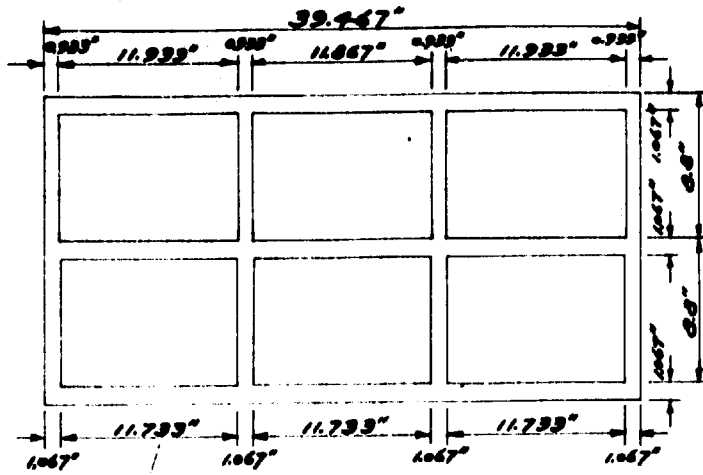
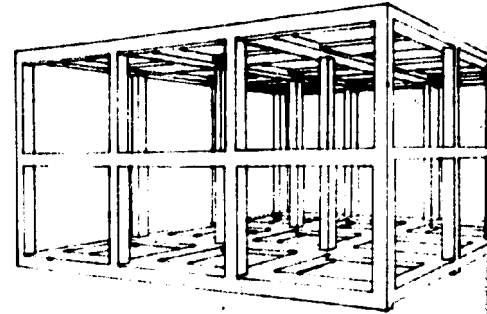


Figure 3.01.

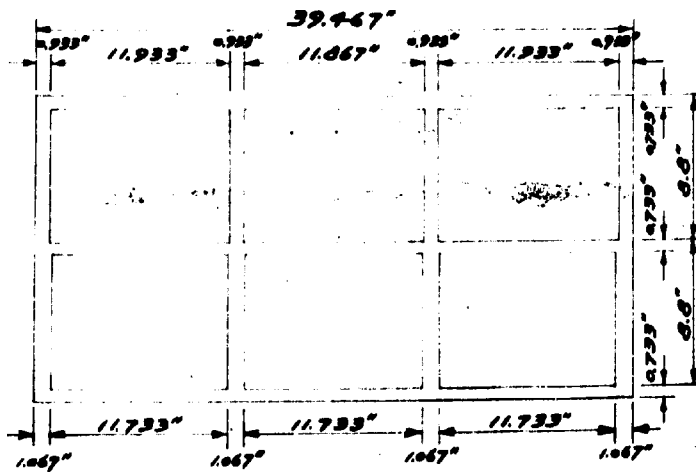
The Model



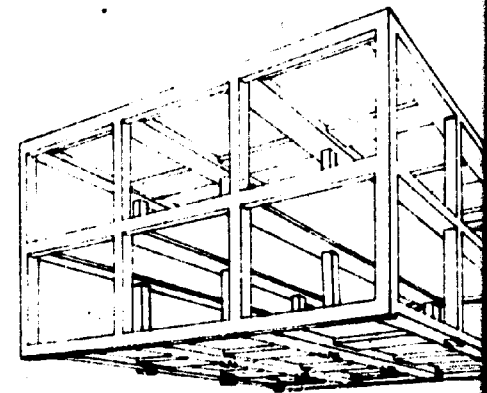
FRONT ELEVATION



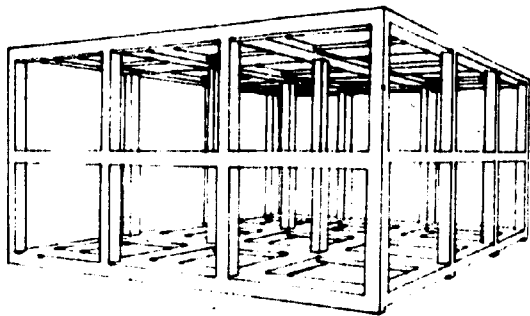
PERSPECTIVE VIEW FROM M



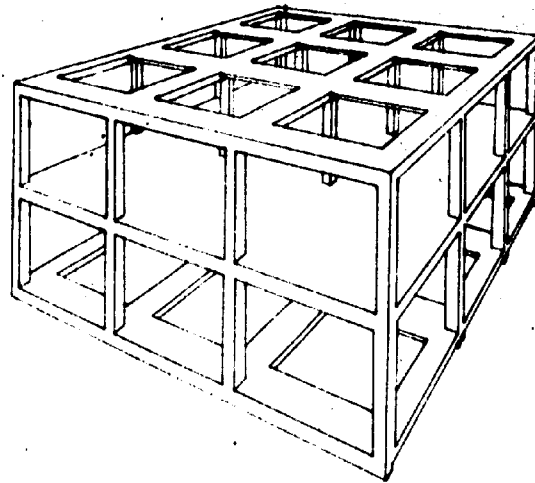
SIDE ELEVATION



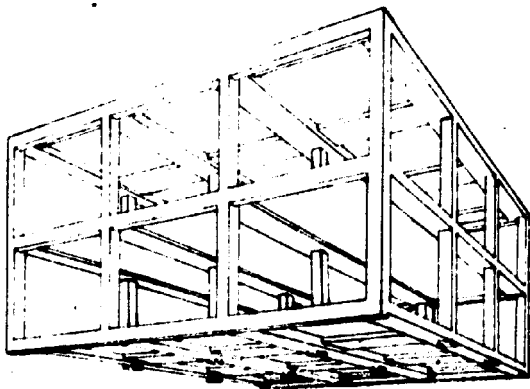
PERSPECTIVE VIEW FROM B



*PERSPECTIVE VIEW FROM MIDDLE*



*PERSPECTIVE VIEW FROM TOP*



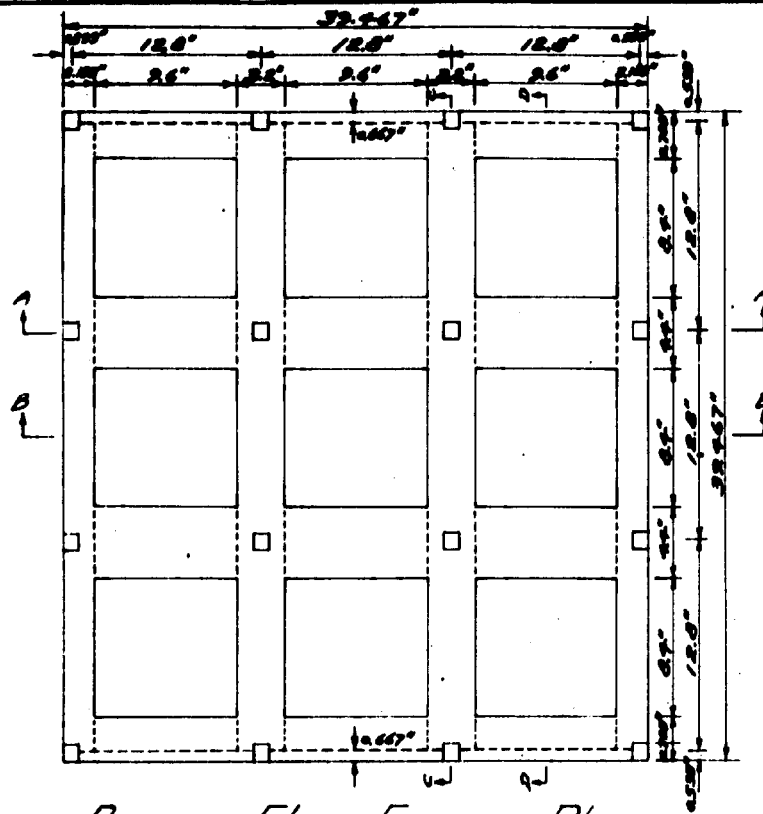
*PERSPECTIVE VIEW FROM BOTTOM*

**Figure 3.02.**

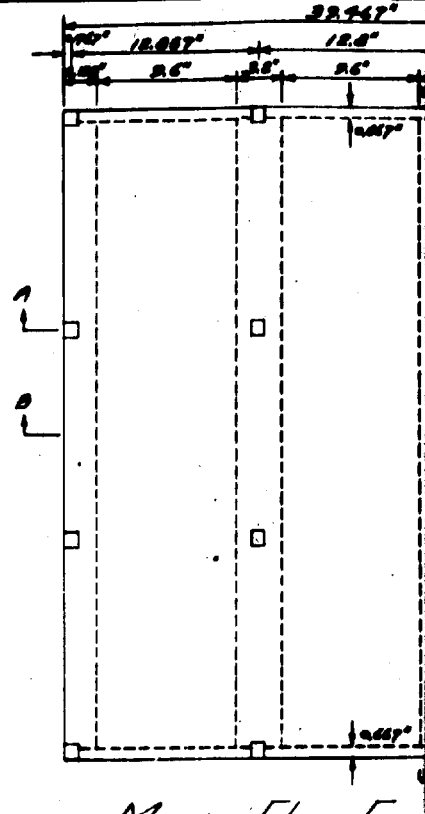
**Elevation and Perspective of the Model**

**BUILDING MODEL**  
**SLAB-BAND CONSTRUCTION**  
 DEPT. OF CIVIL ENG.  
 UNIV. OF MICH.  
 ANN ARBOR, MICH.

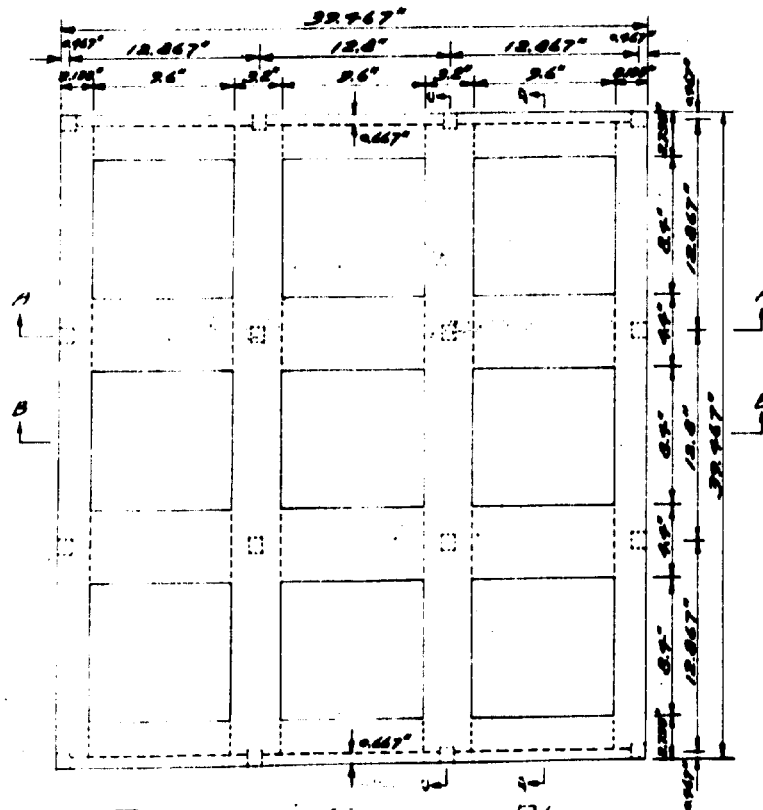
*SHEET No: 1*  
*DETAIL: ELEVATION & PERSPECTIVE*  
*DRAWN BY:*  
*APPROVED BY:*  
*DATE: July 1950*



*BOTTOM FLOOR FRAMING PLAN*



*MIDDLE FLOOR FRAMING PLAN*



*TOP FLOOR FRAMING PLAN*

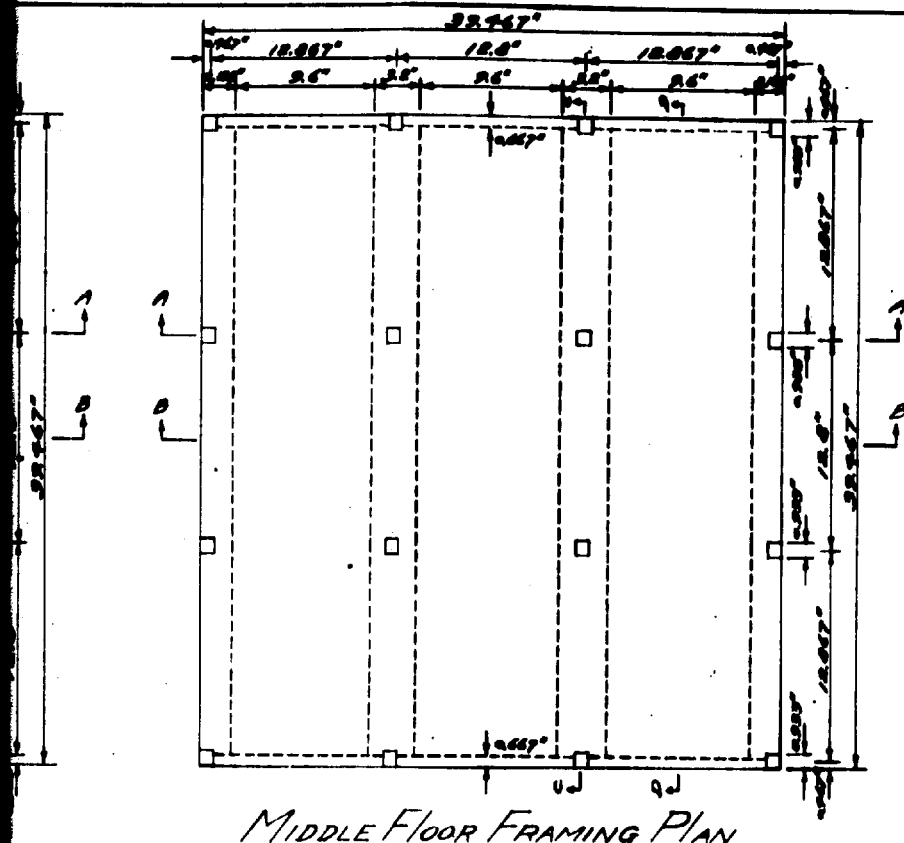
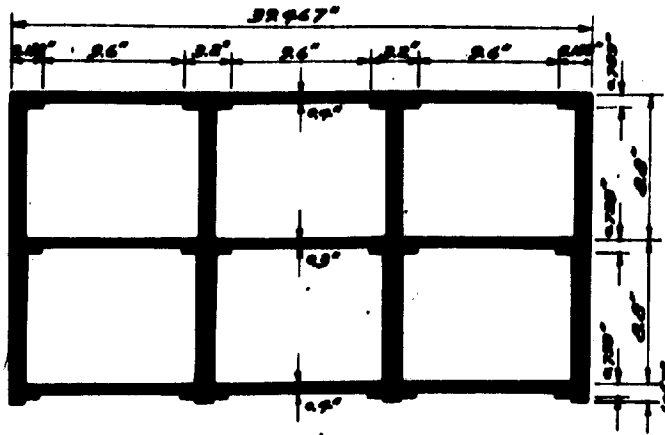


Figure 3.03.

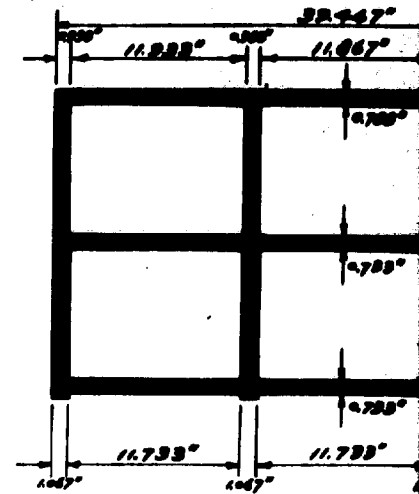
Floor Framing Plans of the Model

BUILDING MODEL  
SLAB-BAND CONSTRUCTION  
DEPT. OF CIVIL ENG.  
UNIV. OF MICH.  
ANN ARBOR, MICH.

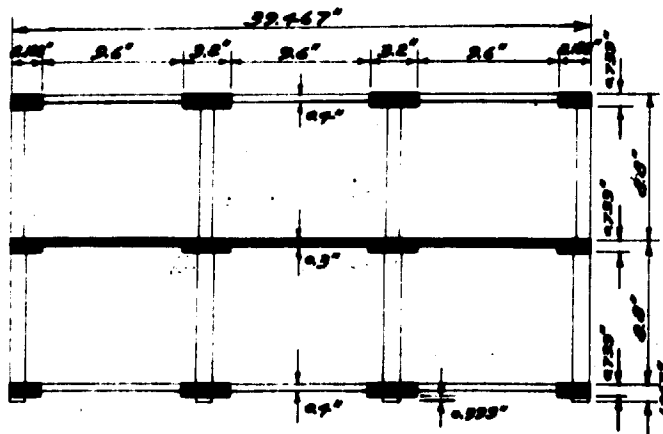
SHEET No: 2  
DETAIL: FLOOR FRAMING PLANS  
DRAWN BY:  
APPROVED BY:  
DATE: July 1950



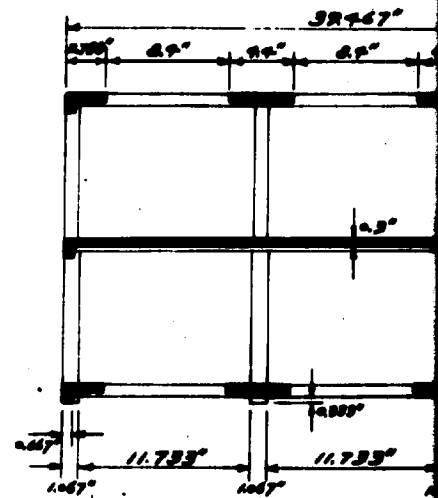
SECTION A-A



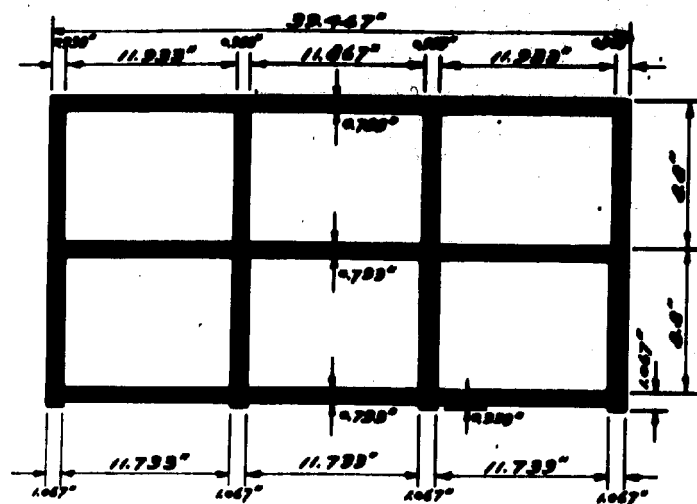
SECTION C



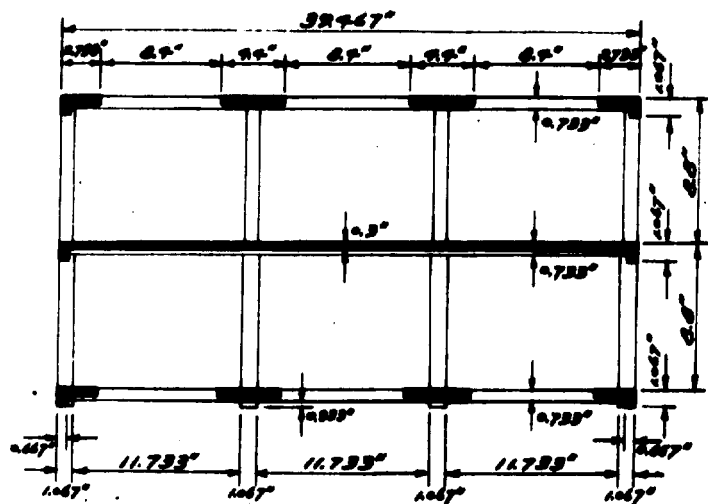
SECTION B-B



SECTION D



SECTION C-C



SECTION D-D

Figure 3.04.

Sections of the Model

BUILDING MODEL  
SLAB-BAND CONSTRUCTION  
DEPT. OF CIVIL ENG.  
UNIV. OF MICH.  
ANN ARBOR, MICH.

SHEET No. 3  
DETAIL SECTIONS  
DRAWN BY:  
APPROVED BY:  
DATE: July 1950

### 3.3.1. Size

For best results the model should be as large as possible.

The size is limited by

- A. maximum available size of Plexiglas sheets;
- B. maximum size of sheet that can be milled in our machine shop; and
- C. the difficulty of installing strain gages if too small a model is used.

It was finally decided that a  $\frac{1}{15}$  scale model satisfied all controlling conditions. The dimensions of the model are shown in figures 3.02, 3.03, and 3.04.

### 3.3.2. Number of Stories

Although a four story building frame was used in the analysis, a model of that height would be not only too expensive but also difficult for installing strain gages, fixing wires, applying load and so forth. In a two-story model the middle floor corresponds closely to any interior floor. Therefore a two-story model was used in which the middle floor was the loaded floor.

### 3.3.3. Floors

Only the middle floor of the model was to be tested and therefore it was made most carefully. This floor was made from a whole sheet of Plexiglas  $48" \times 60" \times \frac{3}{4}"$  except at the spandrel beams where two additional pieces of Plexiglas of quarter inch thickness were cemented to obtain the required depth of the spandrel beams. The sheet of Plexiglas was machined and polished to the required dimensions of the middle floor as described in section 3.4.

The upper and the lower floors of this model should be made the same as the middle floor. This was not done for two reasons. First, the total cost of material and milling for one floor was more than \$500. Second, having a solid floor above the loaded floor would make loading very difficult because of restricted space. Therefore, the top and bottom floors were made as a lattice of wide shallow beams of the same stiffness as the slab-band construction. This gives easy access to the middle floor for placing strain gages, arranging lead wires, and applying load. Figures 3.03 and 3.04 show the structural arrangement of the elements of these floors.

#### 3.3.4. Columns

The sixteen columns used between middle and lower floors were machined and polished to the required dimension and then cemented to the middle floor one by one. The lower floor was then cemented to the sixteen columns in one operation. The same procedure was used in assembling the upper half of the model.

#### 3.3.5. Tolerance

All the separate members of the model are accurate in size to five thousandths of an inch.

### 3.4. Making the Model

The model was made according to the following procedure.

#### 3.4.1. Storing the Plexiglas Sheet

It is best to store the Plexiglas sheets vertically on edge

with masking paper in place. Sagging or bowing of the sheets will occur when they are placed at large angle with vertical for a long period of time. In case sagging or bowing does occur to the sheets it can be remedied by laying the sheets flat for a few days. Plexiglas for this model was stored vertically until used.

#### 3.4.2. Layout of Elements

The middle or loaded floor is the most important part of the model. As previously mentioned, it was milled from a single sheet of Plexiglas except for narrow strips on the webs of the spandrel beams. The outlines of slab-bands and spandrel beams were laid out on the masking paper as guide lines for the milling operation.

The columns were not laid out to size before milling. The machinist cut them to length and milled them to the required size without the necessity of preliminary layout.

The grid of slab-bands and slabs for the top and bottom floors were laid out for cutting as separate members. They were later assembled into a grid as shown in figure 3.03.

#### 3.4.3. Cutting and Machining

That portion of Plexiglas which was used for the middle floor of the model was cut by hand saw from the large sheet. Because the largest milling machine in the machine shop of the University was not wide enough to hold that piece of Plexiglas firmly a frame made of steel bars was built on the table of that milling machine and was securely fastened to the table. The Plexiglas sheet was then laid on the frame and was clamped to the latter with C-clamps. There was no relative motion between the table of the machine and the Plexiglas sheet and the

sheet was horizontal. Cutters of suitable sizes were used to mill the sheet to the required dimensions of the middle floor. As Plexiglas is a poor conductor of heat and tends to soften if frictional heat is generated during machining operations proper speed and feed were used to avoid this. Compressed air was used not only to blow away chips produced by cutters during operations but also to cool the cutters. Pieces of Plexiglas of  $\frac{1}{4}$ " thickness were cemented to the sheet to form the webs of the spandrel beams. The spandrels were then milled to the required size. Slots were also made in the spandrel beams for the insertion of exterior and corner columns under the slab-bands. See Figure 3.05. The machining of the middle floor was the most difficult part of the model construction.

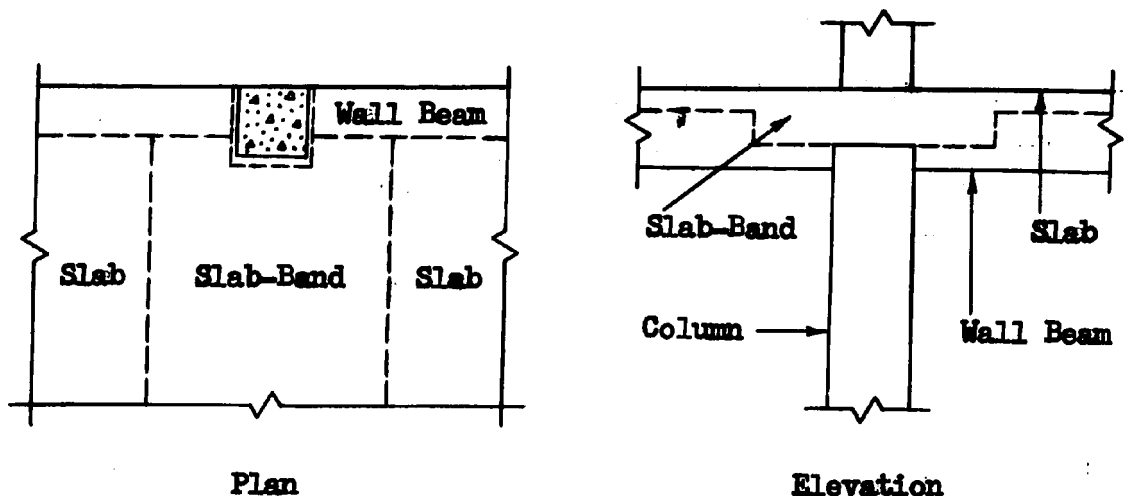


Figure 3.05.

#### Slot in Spandrel Beam for the Insertion of Exterior Column

The members of the upper and lower floors and columns were machined to size without difficulty.

The tolerance for the machine work was set at  $\pm 0.005$ . In a 4-foot wide slab-band for the prototype this is equivalent to a variation in width of 0.075". This is much closer than in actual construction,

### 3.4.4. Finishing

Machined or scratched surfaces and sawed edges of Plexiglas were finished to a high polish by sanding or polishing. The following is the process to accomplish this.

A. The Plexiglas pieces were cleaned first.

B. Sanding was used only where surface imperfections were too deep to be removed by light buffing. The finest sandpaper was used and it was soaked in water or oil a few minutes before using. It was necessary to use gentle pressure and plenty of water or oil while sanding in order to reduce the heat caused by friction. It was also important to sand with rotating motion although scratches or humps on the Plexiglas surfaces were straight.

C. Machine buffing with light pressure can remove most of surface imperfections caused by machining or scratching. The buffing wheel should be free of dirt and grit to avoid seriously scratching the surface during the polishing operations. In order to prevent excessive heat generated by buffing it was necessary to keep the buffing wheel in motion relative to the Plexiglas piece, to change the direction of buffing constantly and to keep pressure against the wheel to a minimum. The buffs used in this job were made of stitched cotton and had diameters of 6" and 8" respectively. A kind of abrasive compound was applied to the buffs before polishing.

### 3.4.5. Assembly of Model

#### A. Cement

Plexiglas members can be united by either special cement or heat welding. Since the welding method requires special set-up to perform the operation the cementing method was used for this model. Cement I-A was used. It is the preferred cement for Plexiglas I-A where strength is

a factor. This cement is a mixture of equal parts by volume of monomeric methyl methacrylate (inhibited with 0.006% hydroquinone by weight) and methylene dichloride. The container of the cement was not opened until the cement was needed and was stored in refrigerator when it was not used. Three capsules of catalyst per pound of cement was added to the cement to counteract the inhibitor. As the cement is a low boiling solvent the container was tightly closed and the exposed soak tank was covered when not in use for the sake of economy and safety.

#### B. Preparation of the Joint

The surfaces to be cemented were left polished and fit accurately without forcing. Care was taken that the edges were not rounded during polishing by buffing.

#### C. Cementing the Joints

When an individual column was cemented to the middle floor that end of the column to be cemented was masked about  $\frac{1}{16}$ " from the end surface by scotch tape in order to confine the softening action of the cement to the area of the joint. The masked end was then immersed in cement in the soak tank. The tape was removed as soon as the column was removed from the soak container after ten minutes of soaking.

When the slab-bands and slabs of the upper and lower floors had been assembled and when either of these floors was cemented to sixteen columns at the same time the soak method of cementing could not be used on account of the amount of cement and size of container needed but the following modified method was used. The ends of several pieces to be cemented were masked in such a way that half of the width of the scotch tape was beyond the surfaces to be cemented to form a dam. These ends of the pieces were held up and cement was poured on. After ten minutes elapsed part of excess cement left at these ends was removed

and so was the scotch tape.

The pieces were assembled right after the application of cement. The pieces were held together gently for a half minute before applying pressure in order to allow a cushion to be formed on the dry piece. The pressure needed to hold the pieces together without squeezing out the cushion was about ten pounds per sq. in. and this intensity of pressure was used in this assemblage by means of weight. An electric bulb was used for heat treatment at the places of assembly in order to speed the curing of the cement.

## CHAPTER 4

### TESTS OF SPECIMENS OF PLEXIGLAS

#### 4.1. General

The tests of specimens of the very same material as the model were equally as important as the tests of the model itself because certain constants such as modulus of elasticity and Poisson's ratio determined by testing specimens are required for computing stresses in the model. It is therefore obvious that the accuracy of the whole results of the model tests will depend partly upon the accuracy of these constants. Furthermore the behavior of the strain gages on Plexiglas should be ascertained before model tests in order to develop a testing procedure that will yield the greatest accuracy in the results.

All the specimens were cut from the sheet of Plexiglas from which the middle floor of the model was made. Specimens No. 1 and No. 2 were used for determining the modulus of elasticity for axial tension and Poisson's ratio. Their dimensions are as shown in figure 4.01. Specimens No. A and No. B were used for determining the modulus of elasticity in bending. Their dimensions are as shown in figure 4.02.

There are three constants needed in converting strains from model tests into stresses as shown in section 7.1. They are the modulus of elasticity determined from tensile flexural strains, the modulus of

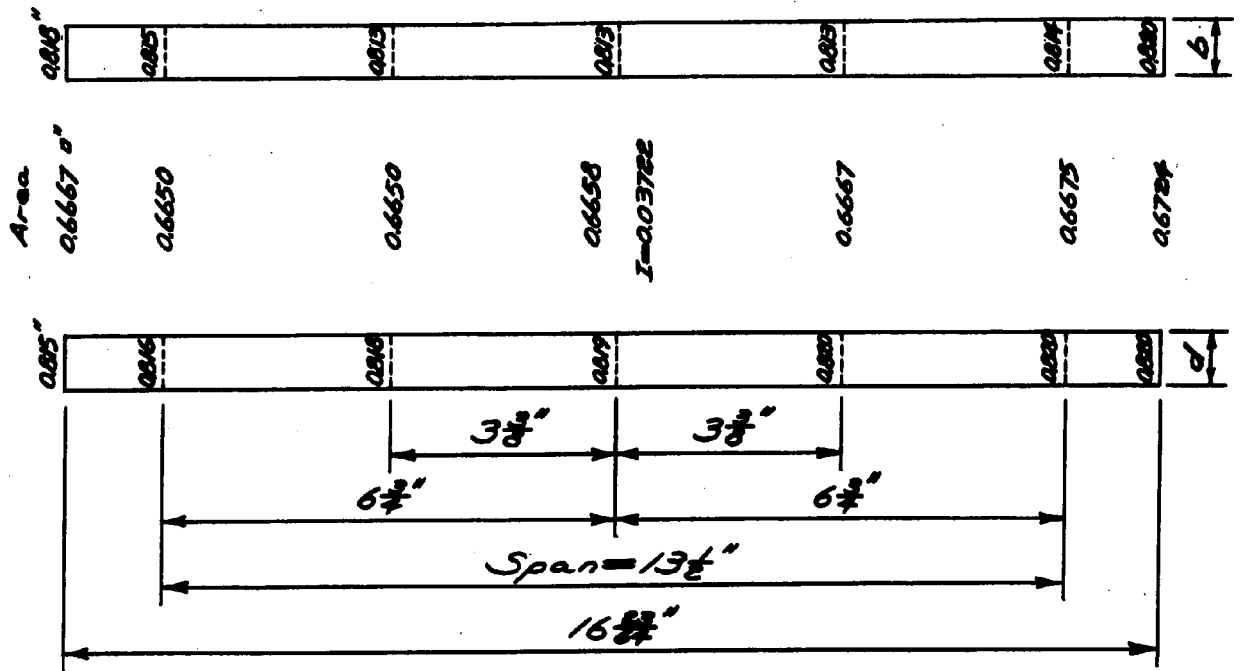
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[illegible]

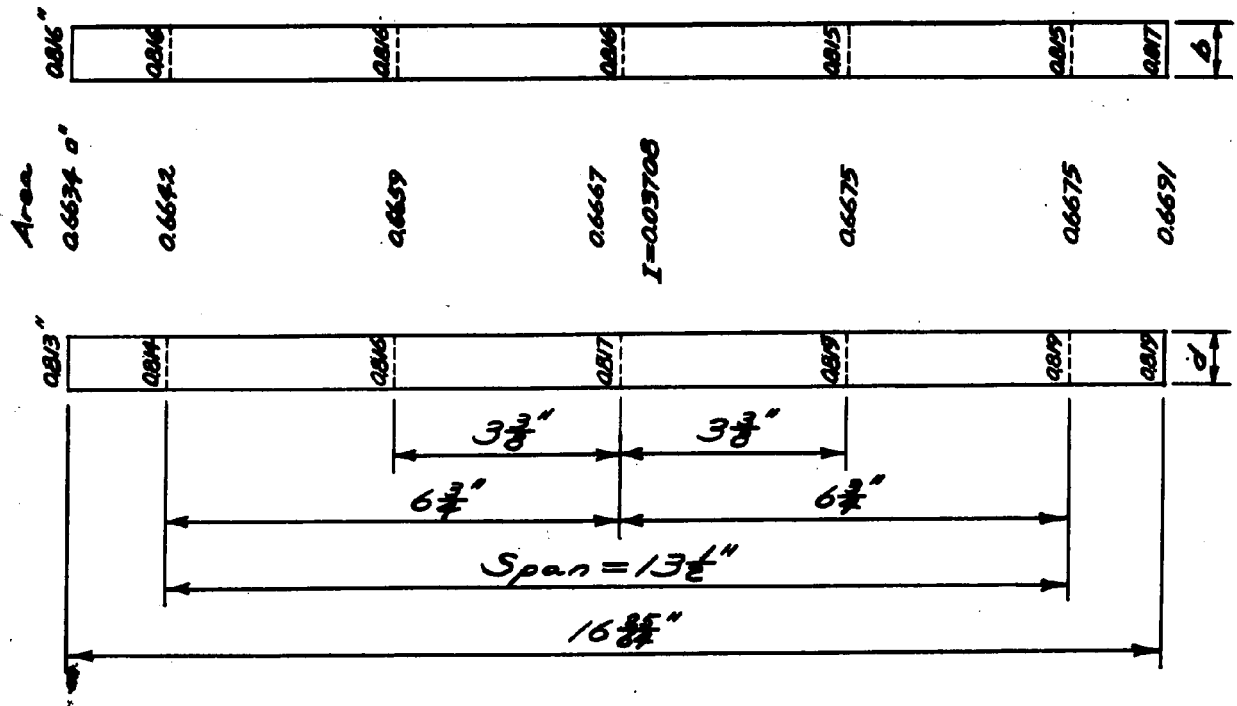
$$\text{Area}_3 = 0.8278$$

**Figure 4.01.**

### Dimensions of Specimens No. 1 and No. 2



Specimen No. A



Specimen No. B

Figure 4.02.

Dimensions of Specimens No. A and No. B

elasticity determined from compressive flexural strains, and Poisson's ratio. These constants were determined as described in sections 4.2 through 4.4. In addition to these, other tests were made in order to show the relationship between measured strain and time elapsed after application of load. Reference is made to section 4.5 for the full description of this. Section 4.6 is devoted to the tests which were designed to show the change in a strain reading on a Plexiglas specimen as the bridge circuit was energized for varying time.

Because the specimens were not large and the load needed in testing was 200 lbs. at most, lead shots of  $\frac{1}{16}$ " average diameter or steel plates of about 2.30 lbs. each were used for loading instead of a standard testing machine.

#### 4.2. Modulus of Elasticity Determined from Tensile Flexural Strains

The modulus of elasticity for using with measured tensile strains was determined in the following way. Specimen B was used.

##### 4.2.1. Arrangement for The Testing

Figure 4.03 shows the arrangement of the specimen, its supporting system and the loading system for this series of tests.

##### 4.2.2. Testing Procedure

A. The tests Nos. 1, 2, and 3 were carried out as follows.

- a. Wires from active gage at bottom of specimen and from compensating gage were connected to the bridge.
- b. The zero gage reading was taken after the bridge switch had been on for 10 minutes.

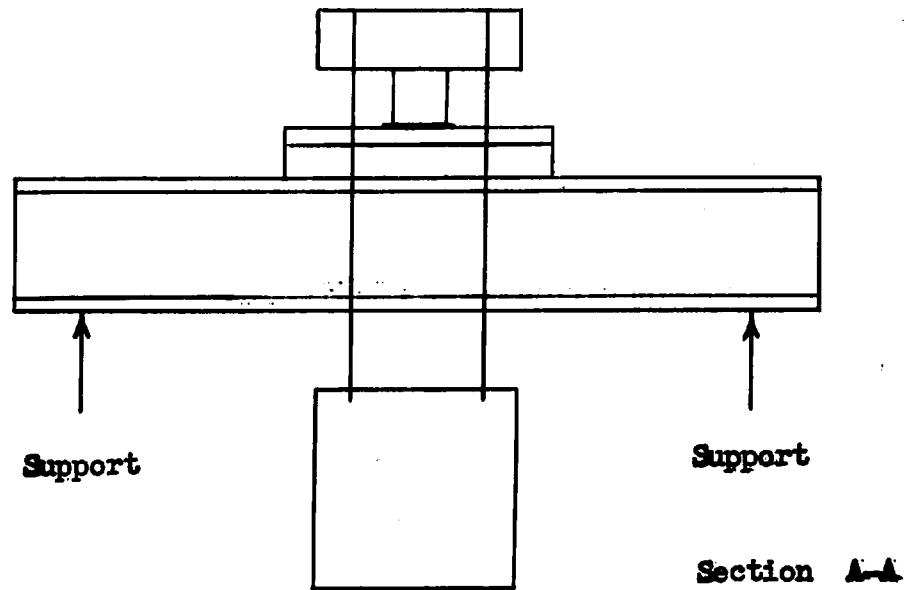
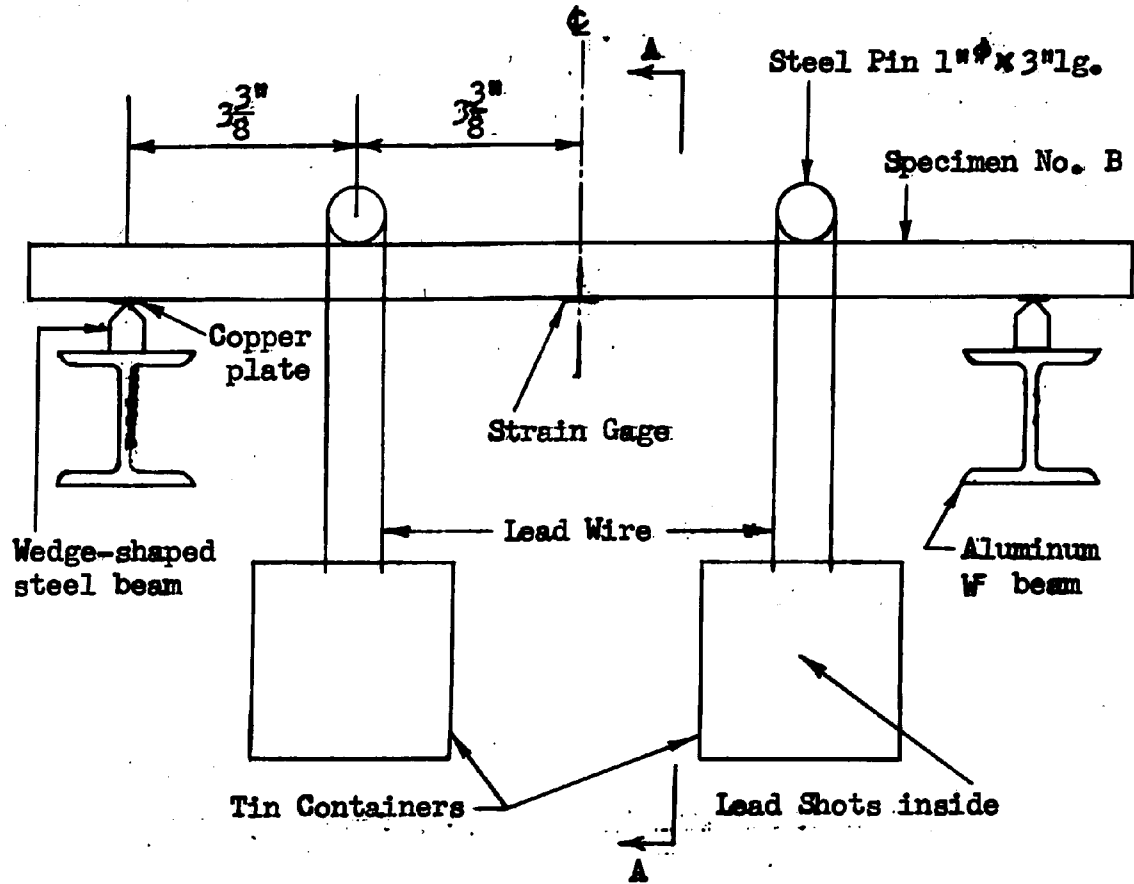


Figure 4.03.

Arrangement for Testing of Bending

c. The loading system including steel pins and tin containers were put at the quarter points of the specimen. The loading system weighed two pounds.

d. The strain reading was taken after the load system was in place for 15 minutes.

d. Lead shot was added to the pans in increments of  $\frac{1}{2}$  pound per container and strain was recorded for each increment of five minutes after loading. The load was increased until there was 12 pounds at each quarter point.

B. Test No. 4 was different from the first three tests in the following points.

a. The loading system was 1.04 lbs. at center of the specimen and the second load was 0.46 lbs.

b. The second gage reading was taken 14 minutes after the first one and the subsequent time interval used for applying load and taking reading was four minutes.

c. The total load applied at center of the specimen was 23 lbs. and the test lasted 3 hours 20 minutes.

C. Test No. 5 differed from the first three tests in the following points.

a. The loading system was 1.023 lbs. at center of the specimen and the subsequent increment of load was quarter pound.

b. The time interval between two successive application of loads was three minutes.

c. The total load applied at center of the specimen was 14.773 lbs. and the test lasted 3 hours 10 minutes.

### 4.2.3. Test Results

The test results are shown in tables 4.01 to 4.05.

### 4.2.4. Calculation of Modulus of Elasticity

#### A. Unit Stress

Bending moment at center of specimen due to half pound load at each quarter point  $= \frac{1}{2} \times \frac{L}{4} = \frac{13.5}{8} = 1.6875''\#$

Bending moment at center of specimen due to half pound load at center of span  $= \frac{1}{4} \times \frac{L}{2} = \frac{13.5}{8} = 1.6875''\#$

Moment of inertia at center of the specimen  $= 0.03708 \text{ in.}^4$

Centroid distance  $= \frac{d}{2} = \frac{0.817}{2} = 0.4085''$

Unit stress  $S = \frac{Mc}{I} = \frac{1.6875 \times 0.4085}{0.03708} = 18.5907 \text{ #/in.}^2$

#### B. Average Tensile Strain $\bar{\delta}$ and Standard Deviation $\sigma$ .

Table 4.06 lists data summarized from tables 4.01 to 4.05 and needed for computing  $\bar{\delta}$  and  $\sigma$ . The first column n gives the number of strains measured in tests 1 to 5. The second column  $\delta$  shows the tensile strains measured at the center of the specimen due to half pound load at each quarter point of the span for tests 1 to 3 and due to a half pound load at center of the span for tests 4 and 5. The strains from all five tests are listed together because they should be the same since the bending moment at the center line due to one load increment is the same for both the quarter point and center line loading.

$$\bar{\delta} = \frac{\sum n\delta}{\sum n} = \frac{6174}{141} = 43.7872 \text{ microinches.}$$

$$\sigma = \sqrt{\frac{\sum \delta^2}{n} - \bar{\delta}^2} = \sqrt{1922.81 - 1917.32} = \sqrt{5.49} = 2.3431 \text{ microinches.}$$

#### C. Modulus of Elasticity

$$E_r = \frac{S}{\bar{\delta}} = \frac{18.5907}{\frac{43.7872}{1,000,000}} = 424,570 \text{ #/in.}^2$$

Table 4.01.

Bending Test ——— Test No. 1

| Increment<br>of<br>Load<br>lbs. | Total<br>Load<br>lbs. | Increment<br>of<br>Time<br>min. | Total<br>Time<br>min. | Tension<br>Strain<br>Gage<br>Reading<br>microinch | Tension<br>Strain<br>microinch | Total<br>Tension<br>Strain<br>microinch |
|---------------------------------|-----------------------|---------------------------------|-----------------------|---------------------------------------------------|--------------------------------|-----------------------------------------|
| 0                               | 0                     | 0                               | 0                     | —                                                 | —                              | —                                       |
| 0                               | 0                     | 1                               | 1                     | 375                                               | —                              | —                                       |
| 0                               | 0                     | 9                               | 10                    | 408                                               | —                              | —                                       |
| 1.0                             | 1.0                   | 15                              | 25                    | 518                                               | +110                           | + 110                                   |
| 0.5                             | 1.5                   | 5                               | 30                    | 563                                               | + 35                           | + 155                                   |
| 0.5                             | 2.0                   | 5                               | 35                    | 607                                               | + 44                           | + 199                                   |
| 0.5                             | 2.5                   | 5                               | 40                    | 650                                               | + 43                           | + 242                                   |
| 0.5                             | 3.0                   | 5                               | 45                    | 694                                               | + 44                           | + 286                                   |
| 0.5                             | 3.5                   | 5                               | 50                    | 738                                               | + 44                           | + 330                                   |
| 0.5                             | 4.0                   | 5                               | 55                    | 785                                               | + 47                           | + 377                                   |
| 0.5                             | 4.5                   | 5                               | 60                    | 827                                               | + 42                           | + 419                                   |
| 0.5                             | 5.0                   | 5                               | 65                    | 870                                               | + 43                           | + 462                                   |
| 0.5                             | 5.5                   | 5                               | 70                    | 913                                               | + 43                           | + 505                                   |
| 0.5                             | 6.0                   | 5                               | 75                    | 955                                               | + 42                           | + 547                                   |
| 0.5                             | 6.5                   | 5                               | 80                    | 1000                                              | + 45                           | + 592                                   |
| 0.5                             | 7.0                   | 5                               | 85                    | 1045                                              | + 45                           | + 637                                   |
| 0.5                             | 7.5                   | 5                               | 90                    | 1088                                              | + 43                           | + 680                                   |
| 0.5                             | 8.0                   | 5                               | 95                    | 1135                                              | + 47                           | + 727                                   |
| 0.5                             | 8.5                   | 5                               | 100                   | 1180                                              | + 45                           | + 772                                   |
| 0.5                             | 9.0                   | 5                               | 105                   | 1223                                              | + 43                           | + 815                                   |
| 0.5                             | 9.5                   | 5                               | 110                   | 1269                                              | + 46                           | + 861                                   |
| 0.5                             | 10.0                  | 5                               | 115                   | 1312                                              | + 43                           | + 904                                   |
| 0.5                             | 10.5                  | 5                               | 120                   | 1358                                              | + 46                           | + 950                                   |
| 0.5                             | 11.0                  | 5                               | 125                   | 1401                                              | + 43                           | + 993                                   |
| 0.5                             | 11.5                  | 5                               | 130                   | 1446                                              | + 45                           | +1038                                   |
| 0.5                             | 12.0                  | 5                               | 135                   | 1493                                              | + 47                           | +1085                                   |
| -12.0                           | 0                     | 5                               | 140                   | 485                                               | —                              | —                                       |
| 0                               | 0                     | 5                               | 145                   | 472                                               | —                              | —                                       |
| 0                               | 0                     | 5                               | 150                   | 467                                               | —                              | —                                       |
| 0                               | 0                     | 5                               | 155                   | 464                                               | —                              | —                                       |
| 0                               | 0                     | 5                               | 160                   | 459                                               | —                              | —                                       |

Table 4.02.

Bending Test ——— Test No. 2

| Increment<br>of<br>Load<br>lbs. | Total<br>Load<br>lbs. | Increment<br>of<br>Time<br>min. | Total<br>Time<br>min. | Tension<br>Strain<br>Gage<br>Reading<br>microinch. | Tension<br>Strain<br>microinch | Total<br>Tension<br>Strain<br>microinch |
|---------------------------------|-----------------------|---------------------------------|-----------------------|----------------------------------------------------|--------------------------------|-----------------------------------------|
| 0                               | 0                     | 0                               | 0                     | —                                                  | —                              | —                                       |
| 0                               | 0                     | 1                               | 1                     | 385                                                | —                              | —                                       |
| 0                               | 0                     | 9                               | 10                    | 407                                                | —                              | —                                       |
| 1.0                             | 1.0                   | 15                              | 25                    | 511                                                | +104                           | + 104                                   |
| 0.5                             | 1.5                   | 5                               | 30                    | 555                                                | + 44                           | + 148                                   |
| 0.5                             | 2.0                   | 5                               | 35                    | 596                                                | + 41                           | + 189                                   |
| 0.5                             | 2.5                   | 5                               | 40                    | 637                                                | + 41                           | + 230                                   |
| 0.5                             | 3.0                   | 5                               | 45                    | 680                                                | + 43                           | + 273                                   |
| 0.5                             | 3.5                   | 5                               | 50                    | 723                                                | + 43                           | + 316                                   |
| 0.5                             | 4.0                   | 5                               | 55                    | 759                                                | + 36                           | + 352                                   |
| 0.5                             | 4.5                   | 5                               | 60                    | 807                                                | + 48                           | + 400                                   |
| 0.5                             | 5.0                   | 5                               | 65                    | 852                                                | + 45                           | + 445                                   |
| 0.5                             | 5.5                   | 5                               | 70                    | 896                                                | + 44                           | + 489                                   |
| 0.5                             | 6.0                   | 5                               | 75                    | 940                                                | + 44                           | + 533                                   |
| 0.5                             | 6.5                   | 5                               | 80                    | 985                                                | + 45                           | + 578                                   |
| 0.5                             | 7.0                   | 5                               | 85                    | 1027                                               | + 42                           | + 620                                   |
| 0.5                             | 7.5                   | 5                               | 90                    | 1071                                               | + 44                           | + 664                                   |
| 0.5                             | 8.0                   | 5                               | 95                    | 1113                                               | + 42                           | + 706                                   |
| 0.5                             | 8.5                   | 5                               | 100                   | 1155                                               | + 42                           | + 748                                   |
| 0.5                             | 9.0                   | 5                               | 105                   | 1197                                               | + 42                           | + 790                                   |
| 0.5                             | 9.5                   | 5                               | 110                   | 1240                                               | + 43                           | + 833                                   |
| 0.5                             | 10.0                  | 5                               | 115                   | 1283                                               | + 43                           | + 876                                   |
| 0.5                             | 10.5                  | 5                               | 120                   | 1327                                               | + 44                           | + 920                                   |
| 0.5                             | 11.0                  | 5                               | 125                   | 1370                                               | + 43                           | + 963                                   |
| 0.5                             | 11.5                  | 5                               | 130                   | 1413                                               | + 43                           | +1006                                   |
| 0.5                             | 12.0                  | 5                               | 135                   | 1460                                               | + 47                           | +1053                                   |
| -12.0                           | 0                     | 5                               | 140                   | 467                                                | —                              | —                                       |
| 0                               | 0                     | 5                               | 145                   | 453                                                | —                              | —                                       |
| 0                               | 0                     | 5                               | 150                   | 449                                                | —                              | —                                       |
| 0                               | 0                     | 5                               | 155                   | 447                                                | —                              | —                                       |
| 0                               | 0                     | 5                               | 160                   | 445                                                | —                              | —                                       |

Table 4.03.

Bending Test ——— Test No. 3

| Increment<br>of<br>Load<br>lbs. | Total<br>Load<br>lbs. | Increment<br>of<br>Time<br>min. | Total<br>Time<br>min. | Tension<br>Strain<br>Gage<br>Reading<br>microinch | Tension<br>Strain<br>microinch | Total<br>Tension<br>Strain<br>microinch |
|---------------------------------|-----------------------|---------------------------------|-----------------------|---------------------------------------------------|--------------------------------|-----------------------------------------|
| 0                               | 0                     | 0                               | 0                     | —                                                 | —                              | —                                       |
| 0                               | 0                     | 1                               | 1                     | 388                                               | —                              | —                                       |
| 0                               | 0                     | 9                               | 10                    | 407                                               | —                              | —                                       |
| 1.0                             | 1.0                   | 15                              | 25                    | 520                                               | +113                           | + 113                                   |
| 0.5                             | 1.5                   | 5                               | 30                    | 563                                               | + 43                           | + 156                                   |
| 0.5                             | 2.0                   | 5                               | 35                    | 605                                               | + 42                           | + 198                                   |
| 0.5                             | 2.5                   | 5                               | 40                    | 645                                               | + 40                           | + 238                                   |
| 0.5                             | 3.0                   | 5                               | 45                    | 686                                               | + 41                           | + 279                                   |
| 0.5                             | 3.5                   | 5                               | 50                    | 727                                               | + 41                           | + 320                                   |
| 0.5                             | 4.0                   | 5                               | 55                    | 767                                               | + 40                           | + 360                                   |
| 0.5                             | 4.5                   | 5                               | 60                    | 810                                               | + 43                           | + 403                                   |
| 0.5                             | 5.0                   | 5                               | 65                    | 850                                               | + 40                           | + 443                                   |
| 0.5                             | 5.5                   | 5                               | 70                    | 892                                               | + 42                           | + 485                                   |
| 0.5                             | 6.0                   | 5                               | 75                    | 932                                               | + 40                           | + 525                                   |
| 0.5                             | 6.5                   | 5                               | 80                    | 975                                               | + 43                           | + 568                                   |
| 0.5                             | 7.0                   | 5                               | 85                    | 1015                                              | + 40                           | + 608                                   |
| 0.5                             | 7.5                   | 5                               | 90                    | 1059                                              | + 44                           | + 652                                   |
| 0.5                             | 8.0                   | 5                               | 95                    | 1102                                              | + 43                           | + 695                                   |
| 0.5                             | 8.5                   | 5                               | 100                   | 1143                                              | + 41                           | + 736                                   |
| 0.5                             | 9.0                   | 5                               | 105                   | 1185                                              | + 42                           | + 778                                   |
| 0.5                             | 9.5                   | 5                               | 110                   | 1227                                              | + 42                           | + 820                                   |
| 0.5                             | 10.0                  | 5                               | 115                   | 1269                                              | + 42                           | + 862                                   |
| 0.5                             | 10.5                  | 5                               | 120                   | 1313                                              | + 44                           | + 906                                   |
| 0.5                             | 11.0                  | 5                               | 125                   | 1355                                              | + 42                           | + 948                                   |
| 0.5                             | 11.5                  | 5                               | 130                   | 1396                                              | + 41                           | + 989                                   |
| 0.5                             | 12.0                  | 5                               | 135                   | 1442                                              | + 46                           | +1035                                   |
| -12.0                           | 0                     | 5                               | 140                   | 459                                               | —                              | —                                       |
| 0                               | 0                     | 5                               | 145                   | 450                                               | —                              | —                                       |
| 0                               | 0                     | 5                               | 150                   | 441                                               | —                              | —                                       |
| 0                               | 0                     | 5                               | 155                   | 437                                               | —                              | —                                       |
| 0                               | 0                     | 5                               | 160                   | 435                                               | —                              | —                                       |

Table 4.04.

Bending Test ——— Test No. 4

| Increment<br>of<br>Load<br>lbs. | Total<br>Load<br>lbs. | Increment<br>of<br>Time<br>min. | Total<br>Time<br>min. | Tension<br>Strain<br>Gage<br>Reading<br>microinch | Tension<br>Strain<br>microinch | Total<br>Tension<br>Strain<br>microinch |
|---------------------------------|-----------------------|---------------------------------|-----------------------|---------------------------------------------------|--------------------------------|-----------------------------------------|
| 0                               | 0                     | 0                               | 0                     | —                                                 | —                              | —                                       |
| 0                               | 0                     | 10                              | 10                    | 113                                               | —                              | —                                       |
| 1.04                            | 1.04                  | 14                              | 24                    | 201                                               | -88                            | + 88                                    |
| 0.46                            | 1.50                  | 4                               | 28                    | 244                                               | +43                            | + 131                                   |
| 0.50                            | 2.00                  | 4                               | 32                    | 288                                               | +44                            | + 175                                   |
| 0.50                            | 2.50                  | 4                               | 36                    | 325                                               | +37                            | + 212                                   |
| 0.50                            | 3.00                  | 4                               | 40                    | 369                                               | +44                            | + 256                                   |
| 0.50                            | 3.50                  | 4                               | 44                    | 412                                               | +43                            | + 299                                   |
| 0.50                            | 4.00                  | 4                               | 48                    | 455                                               | +43                            | + 342                                   |
| 0.50                            | 4.50                  | 4                               | 52                    | 499                                               | +44                            | + 386                                   |
| 0.50                            | 5.00                  | 4                               | 56                    | 539                                               | +40                            | + 426                                   |
| 0.50                            | 5.50                  | 4                               | 60                    | 583                                               | +44                            | + 470                                   |
| 0.50                            | 6.00                  | 4                               | 64                    | 627                                               | +44                            | + 514                                   |
| 0.50                            | 6.50                  | 4                               | 68                    | 674                                               | +47                            | + 561                                   |
| 0.50                            | 7.00                  | 4                               | 72                    | 715                                               | +41                            | + 602                                   |
| 0.50                            | 7.50                  | 4                               | 76                    | 763                                               | +48                            | + 650                                   |
| 0.50                            | 8.00                  | 4                               | 80                    | 804                                               | +41                            | + 691                                   |
| 0.50                            | 8.50                  | 4                               | 84                    | 849                                               | +45                            | + 736                                   |
| 0.50                            | 9.00                  | 4                               | 88                    | 892                                               | +43                            | + 779                                   |
| 0.50                            | 9.50                  | 4                               | 92                    | 939                                               | +47                            | + 826                                   |
| 0.50                            | 10.00                 | 4                               | 96                    | 979                                               | +40                            | + 866                                   |
| 0.50                            | 10.50                 | 4                               | 100                   | 1023                                              | +44                            | + 910                                   |
| 0.50                            | 11.00                 | 4                               | 104                   | 1070                                              | +47                            | + 957                                   |
| 0.50                            | 11.50                 | 4                               | 108                   | 1115                                              | +45                            | +1002                                   |
| 0.50                            | 12.00                 | 4                               | 112                   | 1159                                              | +44                            | +1046                                   |
| 0.50                            | 12.50                 | 4                               | 116                   | 1202                                              | +43                            | +1089                                   |
| 0.50                            | 13.00                 | 4                               | 120                   | 1248                                              | +46                            | +1135                                   |
| 0.50                            | 13.50                 | 4                               | 124                   | 1291                                              | +43                            | +1178                                   |
| 0.50                            | 14.00                 | 4                               | 128                   | 1336                                              | +45                            | +1223                                   |
| 0.50                            | 14.50                 | 4                               | 132                   | 1380                                              | +44                            | +1267                                   |
| 0.50                            | 15.00                 | 4                               | 136                   | 1427                                              | +47                            | +1314                                   |
| 0.50                            | 15.50                 | 4                               | 140                   | 1471                                              | +44                            | +1358                                   |
| 0.50                            | 16.00                 | 4                               | 144                   | 1517                                              | +46                            | +1404                                   |
| 0.50                            | 16.50                 | 4                               | 148                   | 1559                                              | +42                            | +1446                                   |
| 0.50                            | 17.00                 | 4                               | 152                   | 1605                                              | +46                            | +1492                                   |
| 0.50                            | 17.50                 | 4                               | 156                   | 1653                                              | +48                            | +1540                                   |
| 0.50                            | 18.00                 | 4                               | 160                   | 1699                                              | +46                            | +1586                                   |
| 0.50                            | 18.50                 | 4                               | 164                   | 1745                                              | +46                            | +1632                                   |
| 0.50                            | 19.00                 | 4                               | 168                   | 1789                                              | +44                            | +1676                                   |
| 0.50                            | 19.50                 | 4                               | 172                   | 1837                                              | +48                            | +1724                                   |
| 0.50                            | 20.00                 | 4                               | 176                   | 1883                                              | +46                            | +1770                                   |
| 0.50                            | 20.50                 | 4                               | 180                   | 1929                                              | +46                            | +1816                                   |

Table 4.04. (continued)

Bending Test ——— Test No. 4

| Increment of Load lbs. | Total Load lbs. | Increment of Time min. | Total Time min. | Tension Strain Gage Reading microinch | Tension Strain microinch | Total Tension Strain microinch |
|------------------------|-----------------|------------------------|-----------------|---------------------------------------|--------------------------|--------------------------------|
| 0.50                   | 21.00           | 4                      | 184             | 1977                                  | +48                      | +1864                          |
| 0.50                   | 21.50           | 4                      | 188             | 21                                    | +44                      | +1908                          |
| 0.50                   | 22.00           | 4                      | 192             | 65                                    | +44                      | +1952                          |
| 0.50                   | 22.50           | 4                      | 196             | 110                                   | +45                      | +1997                          |
| 0.50                   | 23.00           | 4                      | 200             | 149                                   | +39                      | +2036                          |

Table 4.05.

Bending Test ——— Test No. 5

| Increment of Load lbs. | Total Load lbs. | Increment of Time min. | Total Time min. | Tension Strain Gage Reading microinch | Tension Strain microinch | Total Tension Strain microinch |
|------------------------|-----------------|------------------------|-----------------|---------------------------------------|--------------------------|--------------------------------|
| 0                      | 0               | 0                      | 0               | —                                     | —                        | —                              |
| 0                      | 0               | 10                     | 10              | 1087                                  | —                        | —                              |
| 1.023                  | 1.023           | 15                     | 25              | 1170                                  | +83                      | + 83                           |
| 0.25                   | 1.273           | 3                      | 28              | 1197                                  | +27                      | + 110                          |
| 0.25                   | 1.523           | 3                      | 31              | 1221                                  | +24                      | + 134                          |
| 0.25                   | 1.773           | 3                      | 34              | 1243                                  | +22                      | + 156                          |
| 0.25                   | 2.023           | 3                      | 37              | 1267                                  | +24                      | + 180                          |
| 0.25                   | 2.273           | 3                      | 40              | 1289                                  | +22                      | + 202                          |
| 0.25                   | 2.523           | 3                      | 43              | 1310                                  | +21                      | + 223                          |
| 0.25                   | 2.773           | 3                      | 46              | 1330                                  | +20                      | + 243                          |
| 0.25                   | 3.023           | 3                      | 49              | 1351                                  | +21                      | + 264                          |
| 0.25                   | 3.273           | 3                      | 52              | 1372                                  | +21                      | + 285                          |
| 0.25                   | 3.523           | 3                      | 55              | 1393                                  | +21                      | + 306                          |
| 0.25                   | 3.773           | 3                      | 58              | 1416                                  | +23                      | + 329                          |
| 0.25                   | 4.023           | 3                      | 61              | 1438                                  | +22                      | + 351                          |
| 0.25                   | 4.273           | 3                      | 64              | 1458                                  | +20                      | + 371                          |

Table 4.05. (continued)

Bending Test ——— Test No. 5

| Increment<br>of<br>Load<br>lbs. | Total<br>Load<br>lbs. | Increment<br>of<br>Time<br>min. | Total<br>Time<br>min. | Tension<br>Strain<br>Gage<br>Reading<br>microinch | Tension<br>Strain<br>microinch | Total<br>Tension<br>Strain<br>microinch |
|---------------------------------|-----------------------|---------------------------------|-----------------------|---------------------------------------------------|--------------------------------|-----------------------------------------|
| 0.25                            | 4.523                 | 3                               | 67                    | 1479                                              | +21                            | + 392                                   |
| 0.25                            | 4.773                 | 3                               | 70                    | 1502                                              | +23                            | + 415                                   |
| 0.25                            | 5.023                 | 3                               | 73                    | 1523                                              | +21                            | + 436                                   |
| 0.25                            | 5.273                 | 3                               | 76                    | 1545                                              | +22                            | + 458                                   |
| 0.25                            | 5.523                 | 3                               | 79                    | 1567                                              | +22                            | + 480                                   |
| 0.25                            | 5.773                 | 3                               | 82                    | 1589                                              | +22                            | + 502                                   |
| 0.25                            | 6.023                 | 3                               | 85                    | 1612                                              | +23                            | + 525                                   |
| 0.25                            | 6.273                 | 3                               | 88                    | 1633                                              | +21                            | + 546                                   |
| 0.25                            | 6.523                 | 3                               | 91                    | 1658                                              | +25                            | + 571                                   |
| 0.25                            | 6.773                 | 3                               | 94                    | 1679                                              | +21                            | + 592                                   |
| 0.25                            | 7.023                 | 3                               | 97                    | 1702                                              | +23                            | + 615                                   |
| 0.25                            | 7.273                 | 3                               | 100                   | 1723                                              | +21                            | + 636                                   |
| 0.25                            | 7.523                 | 3                               | 103                   | 1744                                              | +21                            | + 657                                   |
| 0.25                            | 7.773                 | 3                               | 106                   | 1768                                              | +24                            | + 681                                   |
| 0.25                            | 8.023                 | 3                               | 109                   | 1791                                              | +23                            | + 704                                   |
| 0.25                            | 8.273                 | 3                               | 112                   | 1814                                              | +23                            | + 727                                   |
| 0.25                            | 8.523                 | 3                               | 115                   | 1836                                              | +22                            | + 749                                   |
| 0.25                            | 8.773                 | 3                               | 118                   | 1857                                              | +21                            | + 770                                   |
| 0.25                            | 9.023                 | 3                               | 121                   | 1880                                              | +23                            | + 793                                   |
| 0.25                            | 9.273                 | 3                               | 124                   | 1904                                              | +24                            | + 817                                   |
| 0.25                            | 9.523                 | 3                               | 127                   | 1927                                              | +23                            | + 840                                   |
| 0.25                            | 9.773                 | 3                               | 130                   | 1949                                              | +22                            | + 862                                   |
| 0.25                            | 10.023                | 3                               | 133                   | 1971                                              | +22                            | + 884                                   |
| 0.25                            | 10.273                | 3                               | 136                   | 1995                                              | +24                            | + 908                                   |
| 0.25                            | 10.523                | 3                               | 139                   | 1019                                              | +24                            | + 932                                   |
| 0.25                            | 10.773                | 3                               | 142                   | 1041                                              | +22                            | + 954                                   |
| 0.25                            | 11.023                | 3                               | 145                   | 1062                                              | +21                            | + 975                                   |
| 0.25                            | 11.273                | 3                               | 148                   | 1082                                              | +20                            | + 995                                   |
| 0.25                            | 11.523                | 3                               | 151                   | 1107                                              | +25                            | +1020                                   |
| 0.25                            | 11.773                | 3                               | 154                   | 1131                                              | +24                            | +1044                                   |
| 0.25                            | 12.023                | 3                               | 157                   | 1152                                              | +21                            | +1065                                   |
| 0.25                            | 12.273                | 3                               | 160                   | 1177                                              | +25                            | +1090                                   |
| 0.25                            | 12.523                | 3                               | 163                   | 1199                                              | +22                            | +1112                                   |
| 0.25                            | 12.773                | 3                               | 166                   | 1221                                              | +22                            | +1134                                   |
| 0.25                            | 13.023                | 3                               | 169                   | 1243                                              | +22                            | +1156                                   |
| 0.25                            | 13.273                | 3                               | 172                   | 1267                                              | +24                            | +1180                                   |
| 0.25                            | 13.523                | 3                               | 175                   | 1290                                              | +23                            | +1203                                   |
| 0.25                            | 13.773                | 3                               | 178                   | 1313                                              | +23                            | +1226                                   |
| 0.25                            | 14.023                | 3                               | 181                   | 1336                                              | +23                            | +1249                                   |
| 0.25                            | 14.273                | 3                               | 184                   | 1359                                              | +23                            | +1272                                   |
| 0.25                            | 14.523                | 3                               | 187                   | 1380                                              | +21                            | +1293                                   |
| 0.25                            | 14.773                | 3                               | 190                   | 1403                                              | +23                            | +1316                                   |

Table 4.06.

Computation of Average Tensile Strain and Standard Deviation

| n   | $\delta$ | $n\delta$ | $\delta^2$ | $n\delta^2$ |
|-----|----------|-----------|------------|-------------|
| 1   | 36       | 36        | 1296       | 1296        |
| 1   | 37       | 37        | 1369       | 1369        |
| 1   | 39       | 39        | 1521       | 1521        |
| 7   | 40       | 280       | 1600       | 11200       |
| 12  | 41       | 492       | 1681       | 20172       |
| 17  | 42       | 714       | 1764       | 29988       |
| 24  | 43       | 1032      | 1849       | 44376       |
| 29  | 44       | 1276      | 1936       | 56144       |
| 16  | 45       | 720       | 2025       | 32400       |
| 13  | 46       | 598       | 2116       | 27508       |
| 13  | 47       | 611       | 2209       | 28717       |
| 6   | 48       | 288       | 2304       | 13824       |
| 1   | 51       | 51        | 2601       | 2601        |
| 141 |          | 6174      |            | 271116      |

## 4.3. Modulus of Elasticity Determined from Compressive Flexural Strains

Three tests, Nos. 6, 7, and 8, were made for the determination of modulus of elasticity for use with measured compressive strains. The arrangement and procedure of the testing were exactly as those for the tests Nos. 1 to 3 as described in sections 4.2.1 and 4.2.2. The results of tests Nos. 6, 7, and 8 are given in tables 4.07 to 4.09.

The unit stress in the top fiber at the center of the span due to a half pound load applied at each quarter point of the span of the specimen was 18.5907 #/in.<sup>2</sup> as determined in section 4.2.4.A. The average compressive strain  $\delta$  due to one increment of load can be calculated from the data in table 4.10 which is compiled from tables 4.07 to 4.09. In table 4.10 the first column n gives the number of strains measured and

Table 4.07

Bending Test ——— Test No. 6

| Increment<br>of<br>Load<br>lbs. | Total<br>Load<br>lbs. | Increment<br>of<br>Time<br>min. | Total<br>Time<br>min. | Compression<br>Strain<br>Gage<br>Reading<br>microinch | Compression<br>Strain<br>microinch | Total<br>Compression<br>Strain<br>microinch |
|---------------------------------|-----------------------|---------------------------------|-----------------------|-------------------------------------------------------|------------------------------------|---------------------------------------------|
| 0                               | 0                     | 0                               | 0                     | —                                                     | —                                  | —                                           |
| 0                               | 0                     | 1                               | 1                     | 1642                                                  | —                                  | —                                           |
| 0                               | 0                     | 9                               | 10                    | 1670                                                  | —                                  | —                                           |
| 1.0                             | 1.0                   | 15                              | 25                    | 1625                                                  | -45                                | - 45                                        |
| 0.5                             | 1.5                   | 5                               | 30                    | 1588                                                  | -37                                | - 82                                        |
| 0.5                             | 2.0                   | 5                               | 35                    | 1551                                                  | -37                                | -119                                        |
| 0.5                             | 2.5                   | 5                               | 40                    | 1512                                                  | -39                                | -158                                        |
| 0.5                             | 3.0                   | 5                               | 45                    | 1470                                                  | -42                                | -200                                        |
| 0.5                             | 3.5                   | 5                               | 50                    | 1430                                                  | -40                                | -240                                        |
| 0.5                             | 4.0                   | 5                               | 55                    | 1390                                                  | -40                                | -280                                        |
| 0.5                             | 4.5                   | 5                               | 60                    | 1350                                                  | -40                                | -320                                        |
| 0.5                             | 5.0                   | 5                               | 65                    | 1311                                                  | -39                                | -359                                        |
| 0.5                             | 5.5                   | 5                               | 70                    | 1267                                                  | -44                                | -403                                        |
| 0.5                             | 6.0                   | 5                               | 75                    | 1226                                                  | -41                                | -444                                        |
| 0.5                             | 6.5                   | 5                               | 80                    | 1186                                                  | -40                                | -484                                        |
| 0.5                             | 7.0                   | 5                               | 85                    | 1146                                                  | -40                                | -524                                        |
| 0.5                             | 7.5                   | 5                               | 90                    | 1104                                                  | -42                                | -566                                        |
| 0.5                             | 8.0                   | 5                               | 95                    | 1063                                                  | -41                                | -607                                        |
| 0.5                             | 8.5                   | 5                               | 100                   | 1024                                                  | -39                                | -646                                        |
| 0.5                             | 9.0                   | 5                               | 105                   | 981                                                   | -43                                | -689                                        |
| 0.5                             | 9.5                   | 5                               | 110                   | 940                                                   | -41                                | -730                                        |
| 0.5                             | 10.0                  | 5                               | 115                   | 898                                                   | -42                                | -772                                        |
| 0.5                             | 10.5                  | 5                               | 120                   | 856                                                   | -42                                | -814                                        |
| 0.5                             | 11.0                  | 5                               | 125                   | 813                                                   | -43                                | -857                                        |
| 0.5                             | 11.5                  | 5                               | 130                   | 771                                                   | -42                                | -899                                        |
| 0.5                             | 12.0                  | 5                               | 135                   | 729                                                   | -42                                | -941                                        |
| -12.0                           | 0                     | 5                               | 140                   | 1681                                                  | —                                  | —                                           |
| 0                               | 0                     | 5                               | 145                   | 1692                                                  | —                                  | —                                           |
| 0                               | 0                     | 5                               | 150                   | 1699                                                  | —                                  | —                                           |
| 0                               | 0                     | 5                               | 155                   | 1705                                                  | —                                  | —                                           |
| 0                               | 0                     | 5                               | 160                   | 1707                                                  | —                                  | —                                           |

Table 4.08.

Bending Test ——— Test No. 7

| Increment<br>of<br>Load<br>lbs. | Total<br>Load<br>lbs. | Increment<br>of<br>Time<br>min. | Total<br>Time<br>min. | Compression<br>Strain<br>Gage<br>Reading<br>microinch | Compression<br>Strain<br>microinch | Total<br>Compression<br>Strain<br>microinch |
|---------------------------------|-----------------------|---------------------------------|-----------------------|-------------------------------------------------------|------------------------------------|---------------------------------------------|
| 0                               | 0                     | 0                               | 0                     | —                                                     | —                                  | —                                           |
| 0                               | 0                     | 1                               | 1                     | 1661                                                  | —                                  | —                                           |
| 0                               | 0                     | 9                               | 10                    | 1684                                                  | —                                  | —                                           |
| 1.0                             | 1.0                   | 15                              | 25                    | 1635                                                  | -49                                | - 49                                        |
| 0.5                             | 1.5                   | 5                               | 30                    | 1595                                                  | -40                                | - 89                                        |
| 0.5                             | 2.0                   | 5                               | 35                    | 1553                                                  | -42                                | -131                                        |
| 0.5                             | 2.5                   | 5                               | 40                    | 1513                                                  | -40                                | -171                                        |
| 0.5                             | 3.0                   | 5                               | 45                    | 1471                                                  | -42                                | -213                                        |
| 0.5                             | 3.5                   | 5                               | 50                    | 1432                                                  | -39                                | -252                                        |
| 0.5                             | 4.0                   | 5                               | 55                    | 1387                                                  | -45                                | -297                                        |
| 0.5                             | 4.5                   | 5                               | 60                    | 1348                                                  | -39                                | -336                                        |
| 0.5                             | 5.0                   | 5                               | 65                    | 1308                                                  | -40                                | -376                                        |
| 0.5                             | 5.5                   | 5                               | 70                    | 1268                                                  | -40                                | -416                                        |
| 0.5                             | 6.0                   | 5                               | 75                    | 1226                                                  | -42                                | -458                                        |
| 0.5                             | 6.5                   | 5                               | 80                    | 1186                                                  | -40                                | -498                                        |
| 0.5                             | 7.0                   | 5                               | 85                    | 1147                                                  | -39                                | -537                                        |
| 0.5                             | 7.5                   | 5                               | 90                    | 1107                                                  | -40                                | -577                                        |
| 0.5                             | 8.0                   | 5                               | 95                    | 1067                                                  | -40                                | -617                                        |
| 0.5                             | 8.5                   | 5                               | 100                   | 1025                                                  | -42                                | -659                                        |
| 0.5                             | 9.0                   | 5                               | 105                   | 986                                                   | -39                                | -698                                        |
| 0.5                             | 9.5                   | 5                               | 110                   | 945                                                   | -41                                | -739                                        |
| 0.5                             | 10.0                  | 5                               | 115                   | 903                                                   | -42                                | -781                                        |
| 0.5                             | 10.5                  | 5                               | 120                   | 861                                                   | -42                                | -823                                        |
| 0.5                             | 11.0                  | 5                               | 125                   | 820                                                   | -41                                | -864                                        |
| 0.5                             | 11.5                  | 5                               | 130                   | 780                                                   | -40                                | -904                                        |
| 0.5                             | 12.0                  | 5                               | 135                   | 740                                                   | -40                                | -944                                        |
| -12.0                           | 0                     | 5                               | 140                   | 1686                                                  | —                                  | —                                           |
| 0                               | 0                     | 5                               | 145                   | 1695                                                  | —                                  | —                                           |
| 0                               | 0                     | 5                               | 150                   | 1700                                                  | —                                  | —                                           |
| 0                               | 0                     | 5                               | 155                   | 1706                                                  | —                                  | —                                           |
| 0                               | 0                     | 5                               | 160                   | 1711                                                  | —                                  | —                                           |

Table 4.09.

Bending Test ——— Test No. 8

| Increment<br>of<br>Load<br>lbs. | Total<br>Load<br>lbs. | Increment<br>of<br>Time<br>min. | Total<br>Time<br>min. | Compression<br>Strain<br>Gage<br>Reading<br>microinch | Compression<br>Strain<br>microinch | Total<br>Compression<br>Strain<br>microinch |
|---------------------------------|-----------------------|---------------------------------|-----------------------|-------------------------------------------------------|------------------------------------|---------------------------------------------|
| 0                               | 0                     | 0                               | 0                     | —                                                     | —                                  | —                                           |
| 0                               | 0                     | 1                               | 1                     | 1690                                                  | —                                  | —                                           |
| 0                               | 0                     | 9                               | 10                    | 1706                                                  | —                                  | —                                           |
| 1.0                             | 1.0                   | 15                              | 25                    | 1661                                                  | -45                                | - 45                                        |
| 0.5                             | 1.5                   | 5                               | 30                    | 1622                                                  | -39                                | - 84                                        |
| 0.5                             | 2.0                   | 5                               | 35                    | 1580                                                  | -42                                | -126                                        |
| 0.5                             | 2.5                   | 5                               | 40                    | 1539                                                  | -41                                | -167                                        |
| 0.5                             | 3.0                   | 5                               | 45                    | 1496                                                  | -43                                | -210                                        |
| 0.5                             | 3.5                   | 5                               | 50                    | 1455                                                  | -41                                | -251                                        |
| 0.5                             | 4.0                   | 5                               | 55                    | 1412                                                  | -43                                | -294                                        |
| 0.5                             | 4.5                   | 5                               | 60                    | 1372                                                  | -40                                | -334                                        |
| 0.5                             | 5.0                   | 5                               | 65                    | 1330                                                  | -42                                | -376                                        |
| 0.5                             | 5.5                   | 5                               | 70                    | 1289                                                  | -41                                | -417                                        |
| 0.5                             | 6.0                   | 5                               | 75                    | 1248                                                  | -41                                | -458                                        |
| 0.5                             | 6.5                   | 5                               | 80                    | 1206                                                  | -42                                | -500                                        |
| 0.5                             | 7.0                   | 5                               | 85                    | 1166                                                  | -40                                | -540                                        |
| 0.5                             | 7.5                   | 5                               | 90                    | 1124                                                  | -42                                | -582                                        |
| 0.5                             | 8.0                   | 5                               | 95                    | 1085                                                  | -39                                | -621                                        |
| 0.5                             | 8.5                   | 5                               | 100                   | 1043                                                  | -42                                | -663                                        |
| 0.5                             | 9.0                   | 5                               | 105                   | 1003                                                  | -40                                | -703                                        |
| 0.5                             | 9.5                   | 5                               | 110                   | 960                                                   | -43                                | -746                                        |
| 0.5                             | 10.0                  | 5                               | 115                   | 920                                                   | -40                                | -786                                        |
| 0.5                             | 10.5                  | 5                               | 120                   | 879                                                   | -41                                | -827                                        |
| 0.5                             | 11.0                  | 5                               | 125                   | 837                                                   | -42                                | -869                                        |
| 0.5                             | 11.5                  | 5                               | 130                   | 797                                                   | -40                                | -909                                        |
| 0.5                             | 12.0                  | 5                               | 135                   | 755                                                   | -42                                | -951                                        |
| -12.0                           | 0                     | 5                               | 140                   | 1692                                                  | —                                  | —                                           |
| 0                               | 0                     | 5                               | 145                   | 1703                                                  | —                                  | —                                           |
| 0                               | 0                     | 5                               | 150                   | 1708                                                  | —                                  | —                                           |
| 0                               | 0                     | 5                               | 155                   | 1711                                                  | —                                  | —                                           |
| 0                               | 0                     | 5                               | 160                   | 1713                                                  | —                                  | —                                           |

Table 4.10.

Computation of Average Compressive Strain and Standard Deviation

| n  | $\delta$ | $n\delta$ | $\delta^2$ | $n\delta^2$ |
|----|----------|-----------|------------|-------------|
| 2  | 37       | 74        | 1369       | 2738        |
| 9  | 39       | 351       | 1521       | 13689       |
| 19 | 40       | 760       | 1600       | 30400       |
| 10 | 41       | 410       | 1681       | 16810       |
| 19 | 42       | 798       | 1764       | 33516       |
| 5  | 43       | 215       | 1849       | 9245        |
| 1  | 44       | 44        | 1936       | 1936        |
| 1  | 45       | 45        | 2025       | 2025        |
| 66 |          | 2697      |            | 110359      |

the second column shows the compressive strains measured at the center of the specimen for one load increment.

$$\bar{\delta} = \frac{\sum n\delta}{\sum n} = \frac{2697}{66} = 40.8636 \text{ microinches}$$

$$\sigma = \sqrt{\frac{\sum \delta^2}{n} - \bar{\delta}^2} = \sqrt{1672.11 - 1669.84} = \sqrt{2.27}$$

$$= 1.5067 \text{ microinches}$$

$$E_c = \frac{S}{\bar{\delta}} = \frac{18,5907}{\frac{40.8636}{1,000,000}} = 454,950 \text{ \#/in}^2$$

#### 4.4. Poisson's Ratio

The Poisson's ratio was determined in the following way.

##### 4.4.1. Specimen and Strain Gages

The specimen No. 2 as shown in figure 4.01. was used in these tests. The specimen had nearly constant width, depth and cross-sectional area in a length of two inches at its middle. The specimen had a hole

near one of its ends which was used to support the loading system.

There were four strain gages installed on the specimen. Two gages were on each face of the specimen. They were located as shown in figure 4.04.

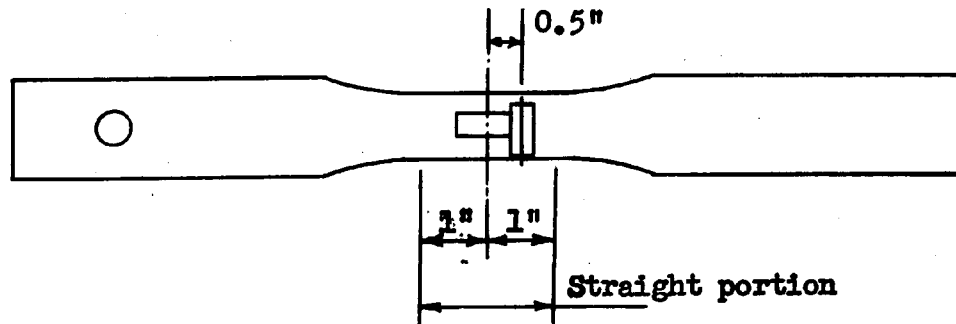


Figure 4.04.

#### Locations of Strain Gages on Specimen No. 2

Those two gages, one at each face of the specimen, which were in the direction of the axis of the specimen were connected in series to the strain indicator and so were the other two gages in the direction perpendicular to that of the axis of the specimen. The strains measured according to this wiring were free from any error due to the possibility that the specimen might be eccentrically supported or loaded.

The longitudinal and the transverse strains measured were not at the same point because the centroids of the grids of the gages were half inch apart. The error from this source is negligible.

#### 4.4.2. Arrangement for The Testing

Figure 4.05 shows the arrangement of the specimen, its supporting system and the loading system for this series of tests.

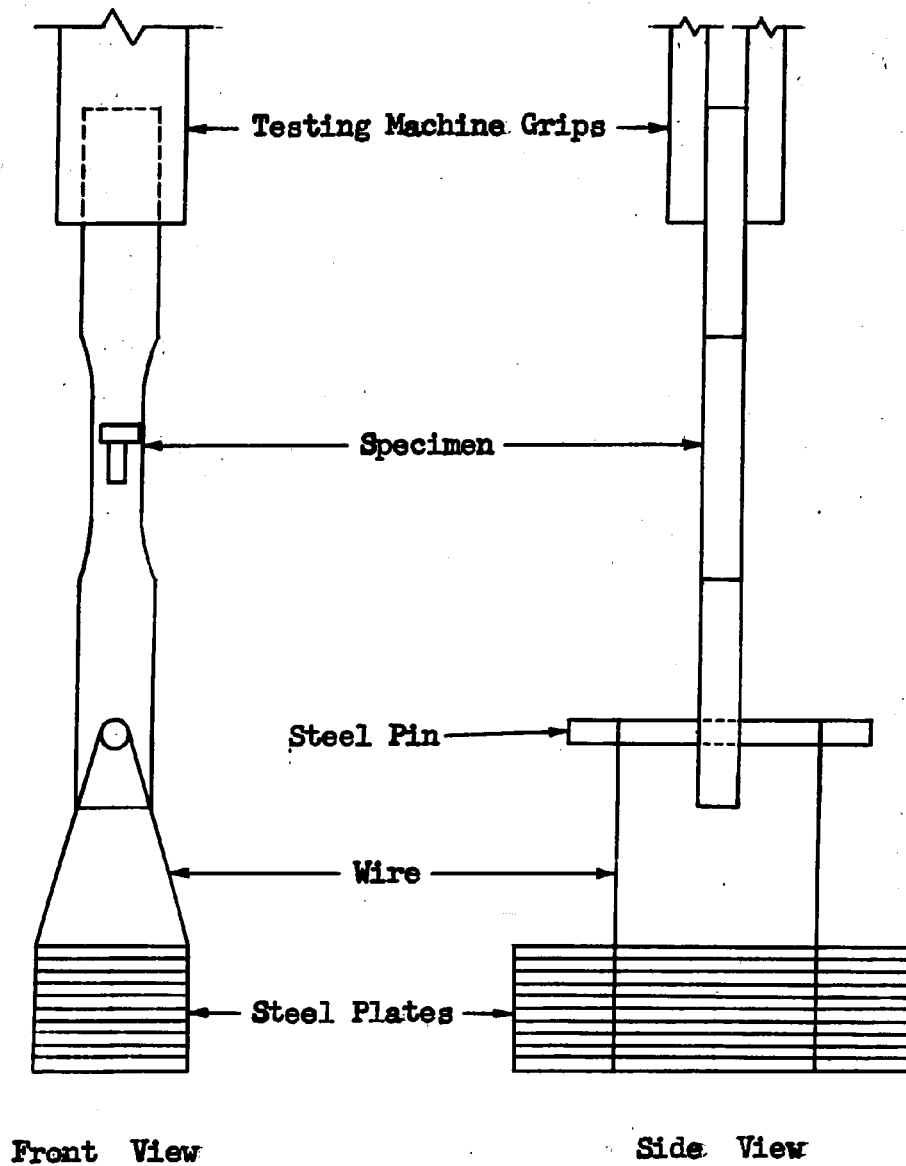


Figure 4.05.

#### Testing Arrangement for Determining Poisson's Ratio

##### 4.4.3. Testing Procedure

Three tests, Nos. 9, 10 and 11, were made. Each test was carried out as follows.

A. The top of the specimen was clamped to the Olsen and Titus Testing Machine and the loading frame was attached to its bottom one hour before the test.

B. Two gages used to measure longitudinal strains were connected in series. The other two gages used to measure transverse strains and two compensating gages were also connected in series respectively.

C. The power switch of the strain indicator was turned on.

D. The initial gage readings were read at the end of two minutes.

E. Four steel plates of about 9.30 lbs. were added to the loading frame.

F. The second readings were taken at two minutes after the first one.

G. The process of adding loads and taking readings was repeated for twenty load increments.

H. All the steel plates were removed immediately after the end of the test.

#### 4.4.4. Test Results

The test results are shown in tables 4.11 to 4.13.

#### 4.4.5. Calculation of Poisson's Ratio

Table 4.14 lists data compiled from tables 4.11 to 4.13 and needed for computing average Poisson's ratio  $\bar{\nu}$  and standard deviation  $\sigma$ . The first column  $n$  and the second column  $\nu$  in table 4.14 are the number and magnitude of Poisson's ratio respectively as listed in the last column of tables 4.11 to 4.13 for one load increment.

$$\bar{\nu} = \frac{\sum n\nu}{\sum n} = \frac{21.637539}{57} = 0.3796$$

$$\sigma = \sqrt{\frac{\sum \nu^2}{n} - \bar{\nu}^2} = \sqrt{0.144571 - 0.144101} = \sqrt{0.000470} = 0.0217$$

Table 4.11

Test of Poisson's Ratio ——— Test No. 9

| A       | B      | C | D  | E    | F   | G    | H   | I   | J    | K      |
|---------|--------|---|----|------|-----|------|-----|-----|------|--------|
| 0       | 0      | 0 | 0  | 632  | —   | —    | 909 | —   | —    | —      |
| 9.34    | 9.34   | 2 | 2  | 657  | +25 | + 25 | 896 | -13 | - 13 | 0.5200 |
| 9.32    | 18.66  | 2 | 4  | 682  | +25 | + 50 | 883 | -13 | - 26 | 0.5200 |
| 9.29    | 27.95  | 2 | 6  | 707  | +25 | + 75 | 871 | -12 | - 38 | 0.4800 |
| 9.34    | 37.29  | 2 | 8  | 733  | +26 | +101 | 861 | -10 | - 48 | 0.3846 |
| 9.33    | 46.62  | 2 | 10 | 759  | +26 | +127 | 851 | -10 | - 58 | 0.3846 |
| 9.36    | 55.98  | 2 | 12 | 787  | +28 | +155 | 840 | -11 | - 69 | 0.3929 |
| 9.29    | 65.27  | 2 | 14 | 811  | +24 | +179 | 830 | -10 | - 79 | 0.4167 |
| 9.32    | 74.59  | 2 | 16 | 837  | +26 | +205 | 820 | -10 | - 89 | 0.3846 |
| 9.33    | 83.92  | 2 | 18 | 862  | +25 | +230 | 809 | -11 | -100 | 0.4400 |
| 9.38    | 93.30  | 2 | 20 | 889  | +27 | +257 | 799 | -10 | -110 | 0.3704 |
| 9.36    | 102.66 | 2 | 22 | 915  | +26 | +283 | 788 | -11 | -121 | 0.4231 |
| 9.37    | 112.03 | 2 | 24 | 941  | +26 | +309 | 778 | -10 | -131 | 0.3846 |
| 9.35    | 121.38 | 2 | 26 | 968  | +27 | +336 | 768 | -10 | -141 | 0.3704 |
| 9.33    | 130.71 | 2 | 28 | 995  | +27 | +363 | 758 | -10 | -151 | 0.3704 |
| 9.32    | 140.03 | 2 | 30 | 1020 | +25 | +388 | 748 | -10 | -161 | 0.4000 |
| 9.35    | 149.38 | 2 | 32 | 1046 | +26 | +414 | 737 | -11 | -172 | 0.4231 |
| 9.34    | 158.72 | 2 | 34 | 1072 | +26 | +440 | 727 | -10 | -182 | 0.3846 |
| 9.37    | 168.09 | 2 | 36 | 1098 | +26 | +466 | 717 | -10 | -192 | 0.3846 |
| 9.34    | 177.43 | 2 | 38 | 1124 | +26 | +492 | 707 | -10 | -202 | 0.3846 |
| 9.27    | 186.70 | 2 | 40 | 1150 | +26 | +518 | 697 | -10 | -212 | 0.3846 |
| -186.70 | 0      | 4 | 44 | 654  | —   | —    | 889 | —   | —    | —      |
| 0       | 0      | 4 | 48 | 645  | —   | —    | 890 | —   | —    | —      |
| 0       | 0      | 4 | 52 | 640  | —   | —    | 890 | —   | —    | —      |
| 0       | 0      | 4 | 56 | 637  | —   | —    | 890 | —   | —    | —      |
| 0       | 0      | 4 | 60 | 635  | —   | —    | 889 | —   | —    | —      |
| 0       | 0      | 4 | 64 | 634  | —   | —    | 889 | —   | —    | —      |
| 0       | 0      | 4 | 68 | 633  | —   | —    | 889 | —   | —    | —      |

- A: Increment of load in lbs.  
 B: Total load in lbs.  
 C: Increment of time in min.  
 D: Total time in min.  
 E: Longitudinal strain gage reading in microinch.  
 F: Longitudinal strain in microinch.  
 G: Total longitudinal strain in microinch.  
 H: Transverse strain gage reading in microinch.  
 I: Transverse strain in microinch.  
 J: Total transverse strain in microinch.  
 K: Poisson's ratio  $\mu$ .

Table 4.12.

Test of Poisson's Ratio ——— Test No. 10

| A      | B      | C | D  | E    | F   | G    | H   | I   | J    | K      |
|--------|--------|---|----|------|-----|------|-----|-----|------|--------|
| 0      | 0      | 0 | 0  | 632  | —   | —    | 886 | —   | —    | —      |
| 9.34   | 9.34   | 2 | 2  | 658  | +26 | + 26 | 885 | -11 | - 11 | 0.4231 |
| 9.32   | 18.66  | 2 | 4  | 683  | +25 | + 51 | 875 | -10 | - 21 | 0.4000 |
| 9.29   | 27.95  | 2 | 6  | 709  | +26 | + 77 | 865 | -10 | - 31 | 0.3846 |
| 9.34   | 37.29  | 2 | 8  | 734  | +25 | +102 | 856 | - 9 | - 40 | 0.3600 |
| 9.33   | 46.62  | 2 | 10 | 760  | +26 | +128 | 846 | -10 | - 50 | 0.3846 |
| 9.36   | 55.98  | 2 | 12 | 787  | +27 | +155 | 837 | - 9 | - 59 | 0.3333 |
| 9.29   | 65.27  | 2 | 14 | 812  | +25 | +180 | 828 | - 9 | - 68 | 0.3600 |
| 9.32   | 74.59  | 2 | 16 | 838  | +26 | +206 | 818 | -10 | - 78 | 0.3846 |
| 9.33   | 83.92  | 2 | 18 | 863  | +25 | +231 | 809 | - 9 | - 87 | 0.3600 |
| 9.38   | 93.30  | 2 | 20 | 889  | +26 | +257 | 799 | -10 | - 97 | 0.3846 |
| 9.36   | 102.66 | 2 | 22 | 916  | +27 | +284 | 789 | -10 | -107 | 0.3704 |
| 9.37   | 112.03 | 2 | 24 | 943  | +27 | +311 | 779 | -10 | -117 | 0.3704 |
| 9.35   | 121.38 | 2 | 26 | 968  | +25 | +336 | 769 | -10 | -127 | 0.4000 |
| 9.33   | 130.71 | 2 | 28 | 995  | +27 | +363 | 759 | -10 | -137 | 0.3704 |
| 9.32   | 140.03 | 2 | 30 | 1020 | +25 | +388 | 749 | -10 | -147 | 0.4000 |
| 9.35   | 149.38 | 2 | 32 | 1046 | +26 | +414 | 739 | -10 | -157 | 0.3846 |
| 9.33   | 158.71 | 2 | 34 | 1072 | +26 | +440 | 729 | -10 | -167 | 0.3846 |
| 9.37   | 168.08 | 2 | 36 | 1099 | +27 | +467 | 719 | -10 | -177 | 0.3704 |
| 9.34   | 177.42 | 2 | 38 | 1126 | +27 | +494 | 709 | -10 | -187 | 0.3704 |
| 9.27   | 186.69 | 2 | 40 | 1151 | +25 | +519 | 699 | -10 | -197 | 0.4000 |
| 186.69 | 0      | 4 | 44 | 653  | —   | —    | 889 | —   | —    | —      |
| 0      | 0      | 4 | 48 | 646  | —   | —    | 891 | —   | —    | —      |
| 0      | 0      | 4 | 52 | 641  | —   | —    | 891 | —   | —    | —      |
| 0      | 0      | 4 | 56 | 638  | —   | —    | 891 | —   | —    | —      |
| 0      | 0      | 4 | 60 | 636  | —   | —    | 890 | —   | —    | —      |
| 0      | 0      | 4 | 64 | 635  | —   | —    | 890 | —   | —    | —      |
| 0      | 0      | 4 | 68 | 633  | —   | —    | 890 | —   | —    | —      |
| 0      | 0      | 4 | 72 | 632  | —   | —    | 890 | —   | —    | —      |

A: Increment of load in lbs.  
 B: Total load in lbs.  
 C: Increment of time in min.  
 D: Total time in min.  
 E: Longitudinal strain gage reading in microinch.  
 F: Longitudinal strain in microinch.  
 G: Total longitudinal strain in microinch.  
 H: Transverse strain gage reading in microinch.  
 I: Transverse strain in microinch.  
 J: Total transverse strain in microinch.  
 K: Poisson's ratio  $\mu$ .

Table 4.13.

Test of Poisson's Ratio ——— Test No. 11

| A       | B      | C | D  | E    | F   | G    | H   | I   | J    | K      |
|---------|--------|---|----|------|-----|------|-----|-----|------|--------|
| 0       | 0      | 0 | 0  | 619  | —   | —    | 896 | —   | —    | —      |
| 9.33    | 9.33   | 2 | 2  | 644  | +25 | + 25 | 886 | -10 | - 10 | 0.4000 |
| 9.32    | 18.65  | 2 | 4  | 670  | +26 | + 51 | 877 | - 9 | - 19 | 0.3462 |
| 9.29    | 27.94  | 2 | 6  | 695  | +25 | + 76 | 868 | - 9 | - 28 | 0.3600 |
| 9.34    | 37.28  | 2 | 8  | 721  | +26 | +102 | 859 | - 9 | - 37 | 0.3462 |
| 9.32    | 46.60  | 2 | 10 | 747  | +26 | +128 | 849 | -10 | - 47 | 0.3846 |
| 9.36    | 55.96  | 2 | 12 | 773  | +26 | +154 | 839 | -10 | - 57 | 0.3846 |
| 9.28    | 65.24  | 2 | 14 | 799  | +26 | +180 | 830 | - 9 | - 66 | 0.3462 |
| 9.31    | 74.55  | 2 | 16 | 825  | +26 | +206 | 821 | - 9 | - 75 | 0.3462 |
| 9.32    | 83.87  | 2 | 18 | 851  | +26 | +232 | 811 | -10 | - 85 | 0.3846 |
| 9.37    | 93.24  | 2 | 20 | 877  | +26 | +258 | 802 | - 9 | - 94 | 0.3462 |
| 9.36    | 102.60 | 2 | 22 | 903  | +26 | +284 | 792 | -10 | -104 | 0.3846 |
| 9.36    | 111.96 | 2 | 24 | 929  | +26 | +310 | 782 | -10 | -114 | 0.3846 |
| 9.34    | 121.30 | 2 | 26 | 956  | +27 | +337 | 772 | -10 | -124 | 0.3704 |
| 9.33    | 130.63 | 2 | 28 | 982  | +26 | +363 | 762 | -10 | -134 | 0.3846 |
| 9.31    | 139.94 | 2 | 30 | 1009 | +27 | +390 | 752 | -10 | -144 | 0.3704 |
| 9.35    | 149.29 | 2 | 32 | 1035 | +26 | +416 | 743 | - 9 | -153 | 0.3462 |
| 9.33    | 158.62 | 2 | 34 | 1061 | +26 | +442 | 733 | -10 | -163 | 0.3846 |
| 9.36    | 167.98 | 2 | 36 | 1089 | +28 | +470 | 723 | -10 | -173 | 0.3571 |
| 9.33    | 177.31 | 2 | 38 | 1115 | +26 | +496 | 714 | - 9 | -182 | 0.3462 |
| 9.26    | 186.57 | 2 | 40 | 1141 | +26 | +522 | 704 | -10 | -192 | 0.3846 |
| -186.57 | 0      | 4 | 44 | 642  | —   | —    | 896 | —   | —    | —      |
| 0       | 0      | 4 | 48 | 633  | —   | —    | 899 | —   | —    | —      |
| 0       | 0      | 4 | 52 | 630  | —   | —    | 899 | —   | —    | —      |
| 0       | 0      | 4 | 56 | 628  | —   | —    | 899 | —   | —    | —      |
| 0       | 0      | 4 | 60 | 626  | —   | —    | 897 | —   | —    | —      |
| 0       | 0      | 4 | 64 | 625  | —   | —    | 898 | —   | —    | —      |
| 0       | 0      | 4 | 68 | 623  | —   | —    | 897 | —   | —    | —      |
| 0       | 0      | 4 | 72 | 623  | —   | —    | 896 | —   | —    | —      |

A: Increment of load in lbs.  
 B: Total load in lbs.  
 C: Increment of time in min.  
 D: Total time in min.  
 E: Longitudinal strain gage reading in microinch.  
 F: Longitudinal strain in microinch.  
 G: Total longitudinal strain in microinch.  
 H: Transverse strain gage reading in microinch.  
 I: Transverse strain in microinch.  
 J: Total transverse strain in microinch.  
 K: Poisson's ratio  $\mu$ .

Table 4.14.

Computation of Average Poisson's Ratio and Standard Deviation

| n  | $\mu$    | $n\mu$    | $\mu^2$  | $n\mu^2$ |
|----|----------|-----------|----------|----------|
| 1  | 0.440000 | 0.440000  | 0.193600 | 0.193600 |
| 3  | 0.423077 | 1.269231  | 0.178994 | 0.536982 |
| 1  | 0.416667 | 0.416667  | 0.173611 | 0.173611 |
| 6  | 0.400000 | 2.400000  | 0.160000 | 0.960000 |
| 1  | 0.392857 | 0.392857  | 0.154337 | 0.154337 |
| 22 | 0.384615 | 8.461530  | 0.147929 | 3.254438 |
| 10 | 0.370370 | 3.703700  | 0.137174 | 1.371740 |
| 4  | 0.360000 | 1.440000  | 0.129600 | 0.518400 |
| 1  | 0.357143 | 0.357143  | 0.127551 | 0.127551 |
| 7  | 0.346154 | 2.423078  | 0.119823 | 0.838761 |
| 1  | 0.333333 | 0.333333  | 0.111111 | 0.111111 |
| 57 |          | 21.637539 |          | 8.240531 |

## 4.5. Relation between Strain and Time

Tests were made on a beam specimen to determine the effect, on the measured strain, of the time interval between switching power to the bridge circuit and taking the strain reading. These tests indicated that there was a slow progressive change in reading, especially after the specimen was under load. This change is apparently due to creep in the plastic, probably combined with local temperature effects at the gage.

It was decided that, except for the effect of creep, the drift of the gage reading could be compensated for by always reading a particular gage at the same time after applying load and also at the same time interval after switching power to that gage.

#### 4.6. Determination of Procedure of Using Compensating Gages in Model Tests

Three series of tests were made in order to determine the effect of time on the strain gage readings when a compensating gage was used with several active gages in succession with no load involved.

##### 4.6.1. First Series of Tests

Each one of the first series of tests was done in such a way that a compensating gage was used with seven active gages in succession. The power switch of the strain indicator was on during the entire test. Nine readings were taken for every active gage at one minute intervals except the first one which was forty-five seconds. Table 4.15 shows the test results.

It is interesting to note that the strains measured for the first active gage had an entirely different pattern than that for the rest of the active gages. The strains measured for all active gages after the first one had the same pattern, with large changes in strain during the first three minutes.

##### 4.6.2. Second Series of Tests

Seven tests were made in the second series. Every test was done in such a way that a compensating gage and an active gage were connected to the strain indicator and readings were taken every 15 seconds until three minutes elapsed. About a half hour later the test was repeated with the same compensating gage but a different active gage. Tables 4.16 and 4.17 show two of this series of tests.

One important fact revealed in these tests was that the difference between the strain from the first active gage and that

Table 4.15.

## Test for Using Compensating Gages

## First Series of Tests

(Data of one of four tests)

| Compensating<br>Gage | Active<br>Gage | Time<br>: " | Strain<br>Gage<br>Reading<br>microinch | Strain<br>microinch | Total<br>Strain<br>microinch |
|----------------------|----------------|-------------|----------------------------------------|---------------------|------------------------------|
| #2                   | #323           | 0 0         | —                                      | —                   | —                            |
|                      |                | 0 15        | 1830                                   | —                   | —                            |
|                      |                | 1 0         | 1822                                   | - 8                 | - 8                          |
|                      |                | 2 0         | 1816                                   | - 6                 | - 14                         |
|                      |                | 3 0         | 1811                                   | - 5                 | - 19                         |
|                      |                | 4 0         | 1806                                   | - 5                 | - 24                         |
|                      |                | 5 0         | 1801                                   | - 5                 | - 29                         |
|                      |                | 6 0         | 1799                                   | - 2                 | - 31                         |
|                      |                | 7 0         | 1797                                   | - 2                 | - 33                         |
|                      |                | 8 0         | 1794                                   | - 3                 | - 36                         |
|                      | #343           | 9 0         | —                                      | —                   | —                            |
|                      |                | 9 15        | 1718                                   | —                   | —                            |
|                      |                | 10 0        | 1770                                   | +52                 | + 52                         |
|                      |                | 11 0        | 1791                                   | +21                 | + 73                         |
|                      |                | 12 0        | 1800                                   | + 9                 | + 82                         |
|                      |                | 13 0        | 1805                                   | + 5                 | + 87                         |
|                      |                | 14 0        | 1808                                   | + 3                 | + 90                         |
|                      |                | 15 0        | 1809                                   | + 1                 | + 91                         |
|                      |                | 16 0        | 1809                                   | 0                   | + 91                         |
|                      |                | 17 0        | 1809                                   | 0                   | + 91                         |
|                      | #363           | 18 0        | —                                      | —                   | —                            |
|                      |                | 18 15       | 1640                                   | —                   | —                            |
|                      |                | 19 0        | 1696                                   | +56                 | + 56                         |
|                      |                | 20 0        | 1721                                   | +25                 | + 81                         |
|                      |                | 21 0        | 1733                                   | +12                 | + 93                         |
|                      |                | 22 0        | 1741                                   | + 8                 | +101                         |
|                      |                | 23 0        | 1747                                   | + 6                 | +107                         |
|                      |                | 24 0        | 1749                                   | + 2                 | +109                         |
|                      |                | 25 0        | 1751                                   | + 2                 | +111                         |
|                      |                | 26 0        | 1752                                   | + 1                 | +112                         |

Table 4.15. (continued)

| Compensating<br>Gage | Active<br>Gage | Time<br>: " |    | Strain<br>Gage<br>Reading<br>microinch | Strain<br>microinch | Total<br>Strain<br>microinch |
|----------------------|----------------|-------------|----|----------------------------------------|---------------------|------------------------------|
| #2                   | #314           | 27          | 0  | —                                      | —                   | —                            |
|                      |                | 27          | 15 | 1190                                   | —                   | —                            |
|                      |                | 28          | 0  | 1237                                   | +47                 | + 47                         |
|                      |                | 29          | 0  | 1255                                   | +18                 | + 65                         |
|                      |                | 30          | 0  | 1269                                   | +14                 | + 79                         |
|                      |                | 31          | 0  | 1273                                   | + 4                 | + 83                         |
|                      |                | 32          | 0  | 1277                                   | + 4                 | + 87                         |
|                      |                | 33          | 0  | 1280                                   | + 3                 | + 90                         |
|                      |                | 34          | 0  | 1281                                   | + 1                 | + 91                         |
|                      |                | 35          | 0  | 1282                                   | + 1                 | + 92                         |
|                      | #334A          | 36          | 0  | —                                      | —                   | —                            |
|                      |                | 36          | 15 | 1249                                   | —                   | —                            |
|                      |                | 37          | 0  | 1296                                   | +47                 | + 47                         |
|                      |                | 38          | 0  | 1316                                   | +20                 | + 67                         |
|                      |                | 39          | 0  | 1326                                   | +10                 | + 77                         |
|                      |                | 40          | 0  | 1331                                   | + 5                 | + 82                         |
|                      |                | 41          | 0  | 1334                                   | + 3                 | + 85                         |
|                      |                | 42          | 0  | 1337                                   | + 3                 | + 88                         |
|                      |                | 43          | 0  | 1339                                   | + 2                 | + 90                         |
|                      |                | 44          | 0  | 1340                                   | + 1                 | + 91                         |
|                      | #344           | 45          | 0  | —                                      | —                   | —                            |
|                      |                | 45          | 15 | 1437                                   | —                   | —                            |
|                      |                | 46          | 0  | 1489                                   | +52                 | + 52                         |
|                      |                | 47          | 0  | 1516                                   | +27                 | + 79                         |
|                      |                | 48          | 0  | 1528                                   | +12                 | + 91                         |
|                      |                | 49          | 0  | 1535                                   | + 7                 | + 98                         |
|                      |                | 50          | 0  | 1540                                   | + 5                 | +103                         |
|                      |                | 51          | 0  | 1543                                   | + 3                 | +106                         |
|                      |                | 52          | 0  | 1546                                   | + 3                 | +109                         |
|                      |                | 53          | 0  | 1547                                   | + 1                 | +110                         |
|                      | #374           | 54          | 0  | —                                      | —                   | —                            |
|                      |                | 54          | 15 | 1082                                   | —                   | —                            |
|                      |                | 55          | 0  | 1131                                   | +49                 | + 49                         |
|                      |                | 56          | 0  | 1157                                   | +26                 | + 75                         |
|                      |                | 57          | 0  | 1169                                   | +12                 | + 87                         |
|                      |                | 58          | 0  | 1176                                   | + 7                 | + 94                         |
|                      |                | 59          | 0  | 1181                                   | + 5                 | + 99                         |
|                      |                | 60          | 0  | 1183                                   | + 2                 | +101                         |
|                      |                | 61          | 0  | 1187                                   | + 4                 | +105                         |
|                      |                | 62          | 0  | 1189                                   | + 2                 | +107                         |

Table 4.16.  
 Test for Using Compensating Gages  
 Second Series of Tests  
 (Data of one of seven tests)

| Compen-<br>sating<br>Gage | Active<br>Gage | Time<br>: " |    | Strain<br>Gage<br>Reading<br>microinch | Strain<br>microinch | Total<br>Strain<br>microinch |
|---------------------------|----------------|-------------|----|----------------------------------------|---------------------|------------------------------|
| #3                        | #131           | 0           | 0  | —                                      | —                   | —                            |
|                           |                | 0           | 5  | 1115                                   | —                   | —                            |
|                           |                | 0           | 15 | 1112                                   | -3                  | -3                           |
|                           |                | 0           | 30 | 1111                                   | -4                  | -4                           |
|                           |                | 0           | 45 | 1108                                   | -3                  | -7                           |
|                           |                | 1           | 0  | 1104                                   | -4                  | -11                          |
|                           |                | 1           | 15 | 1101                                   | -3                  | -14                          |
|                           |                | 1           | 30 | 1097                                   | -4                  | -18                          |
|                           |                | 1           | 45 | 1095                                   | -2                  | -20                          |
|                           |                | 2           | 0  | 1091                                   | -4                  | -24                          |
|                           |                | 2           | 15 | 1090                                   | -1                  | -25                          |
|                           |                | 2           | 30 | 1088                                   | -2                  | -27                          |
|                           |                | 2           | 45 | 1087                                   | -1                  | -28                          |
|                           |                | 3           | 0  | 1083                                   | -4                  | -32                          |
| #3                        | #132           | 0           | 0  | —                                      | —                   | —                            |
|                           |                | 0           | 5  | 399                                    | —                   | —                            |
|                           |                | 0           | 15 | 397                                    | -2                  | -3                           |
|                           |                | 0           | 30 | 393                                    | -4                  | -6                           |
|                           |                | 0           | 45 | 391                                    | -2                  | -8                           |
|                           |                | 1           | 0  | 389                                    | -2                  | -10                          |
|                           |                | 1           | 15 | 386                                    | -3                  | -13                          |
|                           |                | 1           | 30 | 382                                    | -4                  | -17                          |
|                           |                | 1           | 45 | 380                                    | -2                  | -19                          |
|                           |                | 2           | 0  | 378                                    | -2                  | -21                          |
|                           |                | 2           | 15 | 375                                    | -3                  | -24                          |
|                           |                | 2           | 30 | 373                                    | -2                  | -26                          |
|                           |                | 2           | 45 | 371                                    | -2                  | -28                          |
|                           |                | 3           | 0  | 370                                    | -1                  | -29                          |

Table 4.17.  
 Test for Using Compensating Gages  
 Second Series of Tests  
 (Data of one of seven tests)

| Compen-<br>sating<br>Gage | Active<br>Gage | Time<br>: " |    | Strain<br>Gage<br>Reading<br>microinch | Strain<br>microinch | Total<br>Strain<br>microinch |
|---------------------------|----------------|-------------|----|----------------------------------------|---------------------|------------------------------|
| #7                        | #161           | 0           | 0  | ---                                    | ---                 | ---                          |
|                           |                | 0           | 5  | 872                                    | ---                 | ---                          |
|                           |                | 0           | 15 | 872                                    | 0                   | 0                            |
|                           |                | 0           | 30 | 872                                    | 0                   | 0                            |
|                           |                | 0           | 45 | 871                                    | -1                  | -1                           |
|                           |                | 1           | 0  | 870                                    | -1                  | -2                           |
|                           |                | 1           | 15 | 869                                    | -1                  | -3                           |
|                           |                | 1           | 30 | 867                                    | -2                  | -5                           |
|                           |                | 1           | 45 | 864                                    | -3                  | -8                           |
|                           |                | 2           | 0  | 861                                    | -3                  | -11                          |
|                           |                | 2           | 15 | 860                                    | -1                  | -12                          |
|                           |                | 2           | 30 | 858                                    | -2                  | -14                          |
|                           |                | 2           | 45 | 857                                    | -1                  | -15                          |
|                           |                | 3           | 0  | 855                                    | -2                  | -17                          |
| #7                        | #162           | 0           | 0  | ---                                    | ---                 | ---                          |
|                           |                | 0           | 5  | 680                                    | ---                 | ---                          |
|                           |                | 0           | 15 | 679                                    | -1                  | -1                           |
|                           |                | 0           | 30 | 680                                    | +1                  | 0                            |
|                           |                | 0           | 45 | 680                                    | 0                   | 0                            |
|                           |                | 1           | 0  | 679                                    | -1                  | -1                           |
|                           |                | 1           | 15 | 678                                    | -1                  | -2                           |
|                           |                | 1           | 30 | 675                                    | -3                  | -5                           |
|                           |                | 1           | 45 | 673                                    | -2                  | -7                           |
|                           |                | 2           | 0  | 671                                    | -2                  | -9                           |
|                           |                | 2           | 15 | 670                                    | -1                  | -10                          |
|                           |                | 2           | 30 | 670                                    | 0                   | -10                          |
|                           |                | 2           | 45 | 669                                    | -1                  | -11                          |
|                           |                | 3           | 0  | 667                                    | -2                  | -13                          |

Table 4.18.  
 Test for Using Compensating Gages  
 Third Series of Tests  
 (Data of one of seven tests)

| Compensating<br>Gage | Active<br>Gage | Time<br>:    " |    | Strain<br>Gage<br>Reading<br>microinch | Strain<br>microinch | Total<br>Strain<br>microinch |
|----------------------|----------------|----------------|----|----------------------------------------|---------------------|------------------------------|
| #7                   | #161           | 0              | 0  | —                                      | —                   | —                            |
|                      |                | 0              | 5  | 852                                    | —                   | —                            |
|                      |                | 0              | 15 | 853                                    | +1                  | +1                           |
|                      |                | 0              | 30 | 853                                    | 0                   | +1                           |
|                      |                | 0              | 45 | 852                                    | -1                  | 0                            |
|                      |                | 1              | 0  | 851                                    | -1                  | -1                           |
|                      | #162           | 11             | 0  | —                                      | —                   | —                            |
|                      |                | 11             | 5  | 653                                    | —                   | —                            |
|                      |                | 11             | 15 | 657                                    | +4                  | +4                           |
|                      |                | 11             | 30 | 658                                    | +1                  | +5                           |
|                      |                | 11             | 45 | 658                                    | 0                   | +5                           |
|                      |                | 12             | 0  | 657                                    | -1                  | +4                           |
|                      | #163           | 22             | 0  | —                                      | —                   | —                            |
|                      |                | 22             | 5  | 1333                                   | —                   | —                            |
|                      |                | 22             | 15 | 1332                                   | -1                  | -1                           |
|                      |                | 22             | 30 | 1331                                   | -1                  | -2                           |
|                      |                | 22             | 45 | 1330                                   | -1                  | -3                           |
|                      |                | 23             | 0  | 1329                                   | -1                  | -4                           |
|                      | #164           | 33             | 0  | —                                      | —                   | —                            |
|                      |                | 33             | 5  | 1105                                   | —                   | —                            |
|                      |                | 33             | 15 | 1103                                   | -2                  | -2                           |
|                      |                | 33             | 30 | 1101                                   | -2                  | -4                           |
|                      |                | 33             | 45 | 1100                                   | -1                  | -5                           |
|                      |                | 34             | 0  | 1100                                   | 0                   | -5                           |

from the second active gage after the same time interval was only a few microinches.

#### 4.6.3. Third Series of Tests

The third series comprised seven tests. One compensating gage was used with four active gages in every test. One minute was allotted to one active gage and the time interval between two active gages was ten minutes. The power switch of the strain indicator was turned off during these ten minutes. The results of one of the tests are given in table 4.18.

It is clearly shown in the table that there was no big change of strains for this method of using compensating and active gages.

#### 4.6.4. Conclusion

From the results of these series of tests the following procedure for using compensating gages in model tests was devised.

A. The power switch of the strain indicator was turned on and the first compensating gage and the first active gage were switched in. The reading was taken after one minute.

B. After the reading the switches were turned to the second compensating gage and the second active gage. The reading was again made after one minute.

C. This process was repeated until the eighth, the last, compensating gage was used. Then the first compensating gage was used again.

D. All the compensating gages were repeatedly used in this way until the last active gage was measured.

## CHAPTER 5

### WORK PREPARATORY TO TESTING

#### 5.1. Marking Locations of SR-4 Strain Gages on the middle floor of the model

It was not very easy to mark the exact positions of the strain gages on the top of the middle floor of the model because the surface of the Plexiglas is smooth. However, the method as described below was quite successful.

A. A china marking pencil of red color was first used to draw lines of about one-quarter inch wide in the directions and at the places where the strain gages would be installed.

B. A 2H-pencil with neither too sharp nor dull point was used next to draw lines through red marks for the exact directions and locations of the strain gages. There were neither black pencil marks nor scratches caused by pencil on the top of the floor and actually the position of any strain gage was marked by two lines perpendicular to each other within the red mark.

C. The red mark was then erased from an area large enough to accomodate the gage. The gage was installed in the cleared area using the marks outside the cleared area as guides.

D. The scale of the model was not large and therefore the

positions of the strain gages were carefully and accurately located.

## 5.2. Selecting Strain Gages

The primary concern in selecting type of SR-4 strain gages used in this project was the gage length because of the limited space of the testing floor in the model. The SR-4 strain gages of the type A-7 with a gage length of  $\frac{1}{4}$  in. was finally selected. The figure 5.01 shows the dimensions of this type of gage.

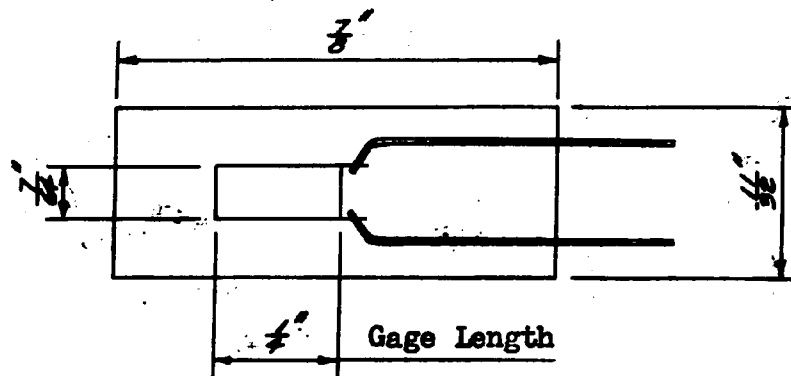


Figure 5.01.

Dimensions of Type A-7 Strain Gage

## 5.3. Installation of Strain Gages

The strain gages are very sensitive and accurate devices to measure stresses in the structures but the results obtained by using them are accurate only if every strain gage is so bonded to the test structure that the sensitive element of the gage is strained exactly as the test structure is strained at the point where the gage is attached. As a result the method of installation of strain gages to the test floor of the model is very important and the procedure for installing gages as outlined below was followed strictly.

### 5.3.1. Preparing The Surface

The top surface of the test floor of the model was very smooth. Because the very smooth surface provided a bad base for bonding of strain gages the places on the test floor where the gages would be installed were therefore roughened by very fine sand paper. As a guide to the spot where it should be roughened a piece of galvanized steel plate with a rectangular hole of the size a little bit larger than that of the A-7 gage was used. It is also important to notice that the sanding was done lightly so that its effect upon the thickness of the floor of the model was very small.

### 5.3.2. Cleaning

The spots where the gages were to be installed were cleaned carefully. The powder which was left after sanding was removed with fine and soft brush and those spots were kept untouched until the next step which followed in few minutes.

### 5.3.3. Applying SR-4 Pre-coat

The SR-4 pre-coat was used for providing an adhesive surface for the gage. A soft brush was used to paint that spot with a single thin coat of pre-coat. At least fifteen minutes elapsed before the next step, to allow the pre-coat to dry.

### 5.3.4. Marking Center Lines on the Gage

The center lines of the gage wire of a gage, one of which is parallel to the direction of the gage wire and the other perpendicular to it, were marked with pencil in order that the gage could be oriented at the exact spot on the test floor as described in section 5.1.

### 5.3.5. Cementing

A liberal coat of SR-4 cement was applied to the back of the gage with a soft brush. The gage was then applied at the required spot on the test floor at once. One of the advantages of using Plexiglas as material for this kind of model is its transparency. When an electric bulb is put under the test floor it could be detected readily whether there was any air pocket between the gage and the test floor. In order to squeeze out the excess cement the thumb was used to press the gage to the surface gently. It was not advisable to squeeze all of the excess cement out.

### 5.3.6. Applying Weight

Because there was no felt attached to the SR-4 A-7 gage a loose piece of felt of the size a little bit smaller than that of the gage was used to cover the gage temporarily for protecting it from the weight used in this step. Right after applying the felt a weight of about one pound was applied to the gage. The weight was removed after one hour.

### 5.3.7. Covering with Felt

As mentioned above there is no felt attached to the SR-4 A-7 gages and it was desirable to cover the gage with felt after it was in its designed position in order to protect it. The size of the felt was the same as that of the gage and it was glued to the gage with SR-4 cement.

### 5.3.8. Air-drying

It was impracticable to use heat drying for the gages for this

project and therefore air-drying was used. The laboratory was heated at the temperature of about 70°F. at the time of installation of the gages. The gages were cured at laboratory temperature.

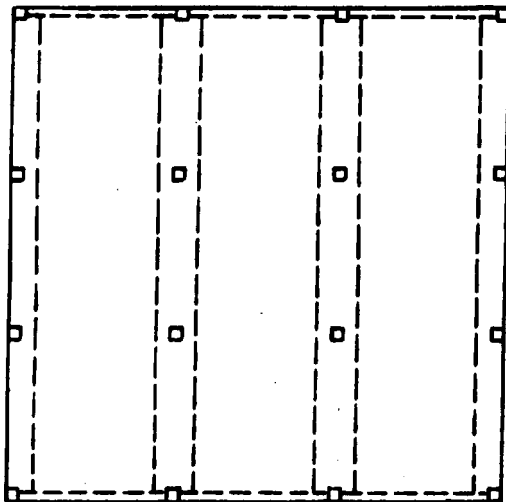
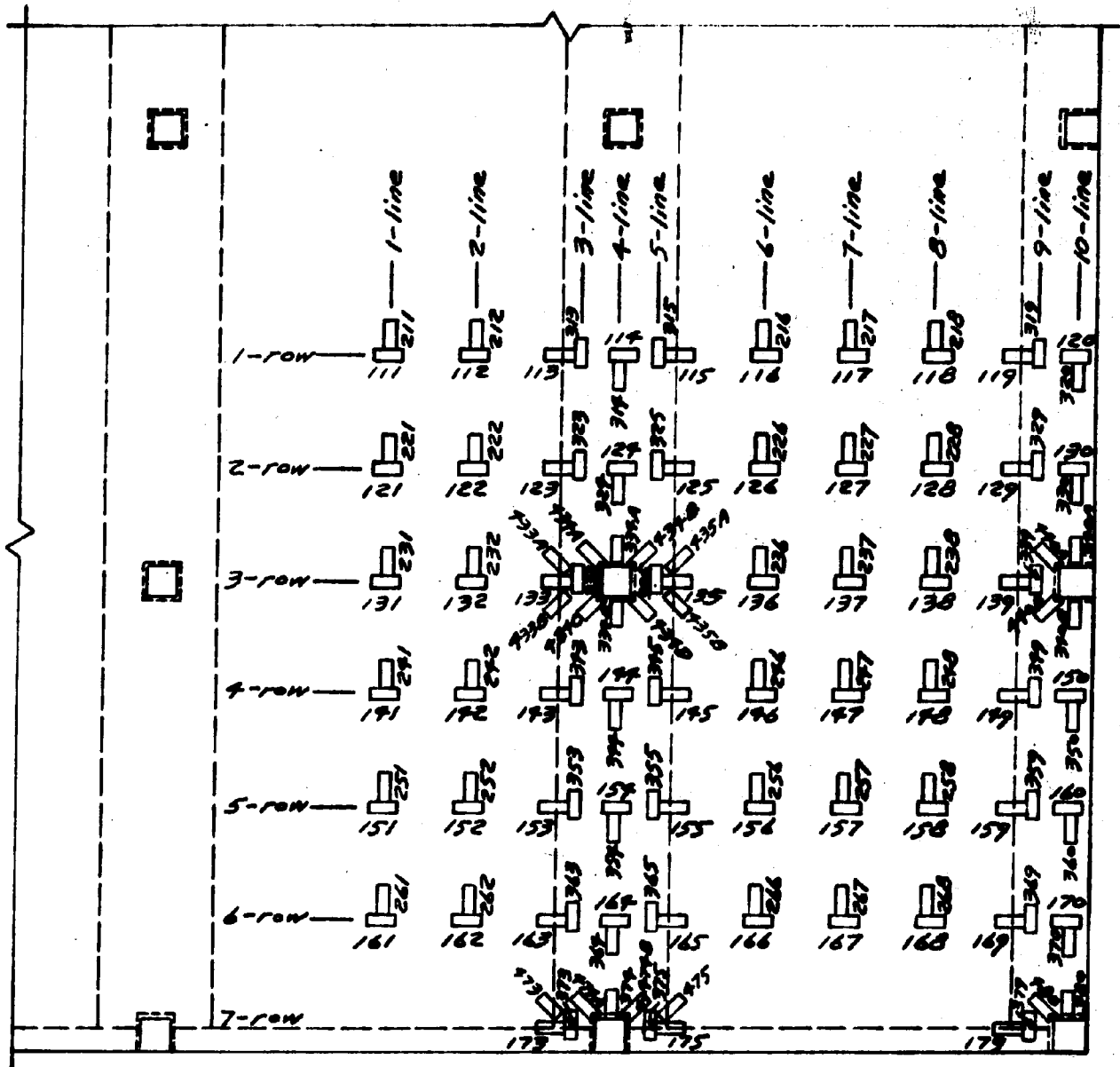
#### 5.4. Numbering the Strain Gages

There were one hundred fifty-one strain gages installed on the middle floor of the model. It was of great importance to have a systematic way to name all the gages in order to avoid confusion. The following was the system to be used.

- A. Every gage was given a number of three figures.
- B. The first figure of each gage number was either 1, 2, 3 or 4. These had the following meanings. The first figure 1 indicated a gage oriented in the direction of the slab span. It may be located on either slab or slab-band. The first figure 2 indicated a gage on the slab and in the direction normal to the slab span. The first figure 3 indicated a gage on the slab-band in the direction of its span. The first figure 4 indicated a gage located diagonally near a column.
- C. The second figure of each gage number (from 1 to 7) indicated the row number. The rows began at the middle of the building length and were in the direction of the slab span.
- D. The third number (from 1 to 9, and 10) indicated the line in which the gage appeared. The lines were in the direction of the slab-bands.

The diagram 5.01 shows the gages numbers according to this numbering system.

As an example, consider gage No. 138. It is located in the direction of the slab span at the intersection of row 3 and line 8.



KEY

Diagram 5.01.  
Numbering the Strain Gages

### 5.5. Strain Gage Wiring

Number 20 single conductor wire was used for lead wires. In order to avoid confusion the lead wires were color coded. The lead wires from the gages were led through a wood clamp located at the edge of the model to prevent any pull on the wires from damaging a gage. There were two such clamps on adjacent sides of the model.

There were four large switch boxes used for connecting 151 active strain gages and a small one for connecting 8 compensating gages. The wires from the strain gages were connected to the boxes according to the ordinal number order of the gages in order to avoid mistakes and confusion.

### 5.6. Arrangement of Wiring on the Model Floor

The lead wires should be held on the top of the middle floor of the model by some adhesive material. Scotch tape, adhesive tape, masking tape and rubber cement were tried. The masking tape applied along the whole length of the wire within the model proved to be the best.

### 5.7. Filling the Spaces between Wires and Strain Gages

The spaces between wires and between wires and strain gages and the remaining area of the floor surface without gages were filled or covered with hard rubber in order to provide an intermediate layer of uniform thickness between the floor surface and the gum rubber applied above this layer. This layer of uniform thickness protected the gages from the applied load and also transmitted the load to the



Figure 5.02.

Work Preparatory to Testing



Figure 5.03.

Work Preparatory to Testing

floor as uniformly as possible. Both figures 5.02 and 5.03 show the preparatory work at this stage.

#### 5.8. Covering of Middle Floor

It was necessary to cover the entire floor above the intermediate layer with a layer of gum rubber of  $\frac{1}{8}$  in. thickness in order to protect the strain gages, wires and filling rubbers from being damaged by the process of applying and removing loads. This top layer of gum rubber was sealed at junctions and at edges and it provided a neat and flat surface.

#### 5.9. Wooden Forms to Confine Uniform Live Load in A Certain Position

A wooden frame was used to confine uniform live load in a certain part of the middle floor of the model. It consisted of vertical plates and horizontal bars.

The vertical plates were 4 in. high and  $\frac{1}{4}$  in. shorter than the clear spacing of columns. There was one soft rubber strip glued to each vertical edge of every plate and when the vertical plate was placed between two columns the rubber was very near to the columns but did not touch them. There were horizontal lines spaced at  $\frac{1}{16}$  in. apart marked on the inner vertical face of every plate in order to provide a guide for leveling steel shot used as live load.

The horizontal bars with slots at their bottoms were used to hold vertical plates in position. There were notches in these bars at the places of columns so that the bars did not touch the columns either.

There were tie wires attached to the lower parts of the

vertical plates on opposite sides of the frame to prevent the plates from spreading at their bottoms when load was applied.

#### 5.10. Compensating Gages

Eight gages mounted on pieces of Plexiglas were used as compensating gages for the model tests. These pieces were put on the bottom floor of the model directly under that portion of the middle floor where 151 active gages were installed. It was also important to cover these compensating gages and the surrounding area with felt, hard rubber and gum rubber in the same way that the middle floor of the model was covered in order to insure that the active and compensating gages were subject to the same temperature conditions.

## CHAPTER 6

### TESTS OF THE MODEL

#### 6.1. Procedure of Testing

The procedure of testing described herein was followed in order to get accurate results and to avoid mistakes. It was based partly on the tests of specimens and partly on the preliminary tests of the model. This procedure was followed for each test.

##### 6.1.1. Fixing the Wooden Forms

Wooden forms were placed around the panel or panels in which the loads were applied later. This operation was done one day before the test because it was very likely that the installation of the wooden forms on the model would affect the strain gage readings if it was done shortly before test. The following few things should be noticed.

A. The inside surfaces of the wooden forms should be placed accurately along the surfaces passing through the center lines of columns in row.

B. The lower part of the wooden forms should be held in position against spreading by lead wires when and after the steel balls are applied.

C. The wooden forms should not be in contact with the columns

so that they do not offer restraint to the columns.

D. The gaps between the wooden forms and the columns should not be large so that steel balls will not leak through after they are put in the forms.

#### 6.1.2. Location of Compensating Gages

Five pieces of Plexiglas on which eight compensating gages were installed were put directly under that quarter of the middle floor of the model on which 151 strain gages were situated. It was safe to assume that the temperature was uniform in this small space. Even if the temperature changed in this space during testing the active and the compensating gages were equally affected and consequently the strain gage readings were not influenced because the compensating gages automatically compensated the effect due to change of temperature. However, care was still taken that all doors and windows of the room in which the model was situated were closed from one day before test to the end of the test. In this way the room temperature was nearly constant during test and the change of temperature was thus kept to a minimum.

#### 6.1.3. Order of Using Compensating Strain Gages

A. Every active strain gage was used with a certain compensating strain gage.

B. Any one compensating strain gage was not used continuously. The reason for this was clearly explained in the tests described in section 4.6.

C. All the compensating strain gages were so arranged that the time interval between two consecutive applications of the same

compensating strain gage was at least seven minutes or better still more than ten minutes. The test results shown in section 4.6 explained this.

D. In order to avoid confusion and mistakes the compensating strain gages were used from No. 1 to No. 8 consecutively. The minimum time interval allotted to one gage reading was one minute.

#### 6.1.4. Order of Taking Strain Gage Readings

A. After the purpose of a certain test was decided those active strain gages which were useful for that test, and consequently whose strains would be measured, were selected.

B. These active strain gages were recorded in the data book in the order according to their respective numbers given in section 5.4. This not only simplified the operation of switching but also helped to avoid mistakes.

#### 6.1.5. Details of Instrument Arrangement and Wiring

A. The SR-4 portable strain indicator was put on a table next to the model.

B. The batteries were checked.

C. The gage factor was set at 1.96.

D. The range extender was set at 0.

E. The active gages were connected as follows:

a. One wire, which picked up one lead from each active gage, was taken directly to the instrument (bridge).

b. The other lead from each gage was connected to its proper terminal on a switch box. The switch box was then connected to the instrument (bridge).

F. The Compensating Gages were connected as follows:

- a. One wire, picking up one lead from each compensating gage, was connected to the instrument.
- b. The other lead from each gage was connected to its terminal on a special switch box used for compensating gages. This switch box was in turn connected to the instrument.

6.1.6. Procedure for Taking Strain Gage Readings.

The following procedure was used for making each measurement of strain.

A. Select the desired active gage and the proper compensating gage. Switch them into the bridge circuit.

B. Turn on the bridge power switch and at the same time start a stop watch. Balance the bridge after the time interval chosen for the particular test. This interval was a minimum of 30 seconds. The time interval for the various tests are discussed in sections 4.5 and 4.6.

C. Switch to the next active and compensating gage, turn on bridge power and start the stop watch.

D. Record the bridge reading for previous gage while waiting for time to balance the bridge.

E. Repeat for succeeding gages.

6.1.7. Loading Conditions

Both slab and slab-bands in the model are continuous for three spans. In order to obtain maximum stresses at supports and at mid-span it is necessary to apply live load on alternate spans and on adjacent spans. The following loading schedule was used to obtain maximum

stresses in slab and slab-bands.

- A. Test No. 1 — Slab: Two exterior spans loaded.
- B. Test No. 2 — Slab: Two adjacent spans loaded.
- C. Test No. 3 — Slab: Interior span loaded.
- D. Test No. 4 — Slab-bands: Two exterior spans loaded.
- E. Test No. 5 — Slab-bands: Two adjacent spans loaded.
- F. Test No. 6 — Slab-bands: Interior span loaded.

Extent of loading is shown on small sketches in diagrams 6.01 to 6.06.

#### 6.1.8. Application of Load

A. Steel shot used as load was weighed in a balance which was graduated to one-fiftieth of a pound. The accuracy of this balance was enough for this purpose.

B. Weighed twenty pounds of steel shot and poured them gently in one of the panels to be loaded. Did the same thing for every one of these panels.

C. Added another twenty pounds of steel shot in every one of these panels. Therefore the total load in every one of these loaded panels was forty pounds. The reason that the load was applied in this manner was to avoid sudden application of heavy load in one panel before others were loaded.

D. Levelled the steel shot in order to insure that the load was uniformly distributed on the floor. The horizontal lines at the inner surfaces of the wooden forms were used as guide for leveling the steel shot.

E. Waited one to two hours before taking final strain gage reading. This interval of time was used to allow the model to set

under the load.

#### 6.1.9. Removing Load

As soon as the gage readings were completed steel shot was removed. It was intended that the possible permanent set in the material of the model due to long period of loading was to be avoided.

#### 6.2. Test Results

In order to present the test results clearly diagrams are used instead of tables. Refer to diagrams 6.01 to 6.06. In the diagrams the structural members on the tested floor are shown, together with columns above and below it and the positions of all strain gages.

Those strain gages which were used in a certain test are shaded in the diagrams. The strain measured for a certain gage is recorded next to the indicated position of that gage. A plus sign before the value of the strain denotes tension and a minus sign denotes compression.

##### 6.2.1. Test No. 1

This test was intended primarily to ascertain the maximum stresses in the slab around the middle of its exterior spans. It was also intended that this test would show the manner of stress distribution in the slab as well as in the slab-bands for this loading condition.

The loaded area is indicated in diagram 6.01. Applied load was 40 lbs. per panel.

The length of time allotted for one strain gage reading was

1 minute and the interval of time between loading steel shot and taking final strain gage readings was 50 minutes.

The results of this test are shown on diagram 6.01.

#### 6.2.2. Test No. 2

This test was intended primarily to ascertain the maximum stresses in the slab around and at the interior support. It was also intended that this test would show the manner of stress distribution in the slab as well as in the slab-bands for this loading condition.

The loaded area is indicated in diagram 6.02. Applied load was 40 lbs. per panel.

The length of time allotted for one strain gage reading was 1 minute 30 seconds and the interval of time between loading steel shot and taking final strain gage readings was 1 hour 40 minutes.

The results of this test are shown on diagram 6.02.

#### 6.2.3. Test No. 3

This test was intended primarily to ascertain the maximum stresses in the slab around the middle of its interior span. It was also intended that this test would show the manner of stress distribution in the slab in its interior span as well as in part of the interior slab-band for this loading condition.

The loaded area is indicated in diagram 6.03. Applied load was 40 lbs. per panel.

The length of time allotted for one strain gage reading was 2 minutes and the interval of time between loading steel shot and taking final strain gage readings was 1 hour 10 minutes.

The results of this test are shown on diagram 6.03.

#### 6.2.4. Test No. 4

This test was intended primarily to ascertain the maximum stresses in both the interior and the exterior slab-bands around the middle of their exterior spans. It was also intended that this test would show the manner of stress distribution in both slab-bands in their exterior spans for this loading condition.

The loaded area is indicated in diagram 6.04. Applied load was 40 lbs. per panel.

The length of time allotted for one strain gage reading was 1 minute.

The results of this test are shown on diagram 6.04.

#### 6.2.5. Test No. 5

This test was intended primarily to ascertain the maximum stresses in both the interior and the exterior slab-bands around their interior supports. It was also intended that this test would show the manner of stress distribution in both slab-bands for this loading condition.

The loaded area is indicated in diagram 6.05. Applied load was 40 lbs. per panel.

The length of time allotted for one strain gage reading was 2 minutes and the interval of time between loading steel shot and taking final strain gage readings was 1 hour 50 minutes.

The results of this test are shown on diagram 6.05.

#### 6.2.6. Test No. 6

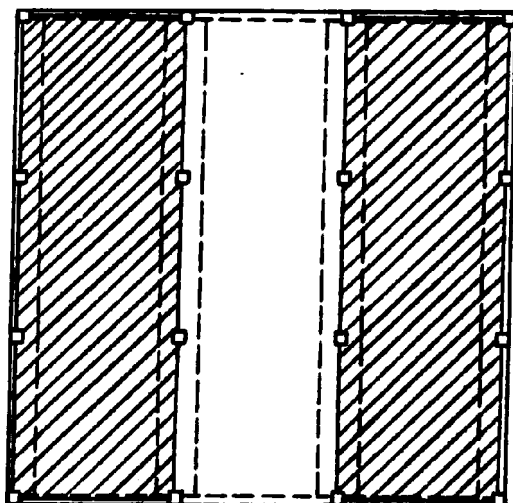
This test was intended primarily to ascertain the maximum stresses in both the interior and the exterior slab-bands around the

middle of their interior spans. It was also intended that this test would show the manner of stress distribution in both slab-bands in their interior spans for this loading condition.

The loaded area is indicated in diagram 6.06. Applied load was 40 lbs. per panel.

The length of time allotted for one strain gage reading was 2 minutes and the interval of time between loading steel shot and taking final strain gage readings was 1 hour 30 minutes.

The results of this test are shown on diagram 6.06.



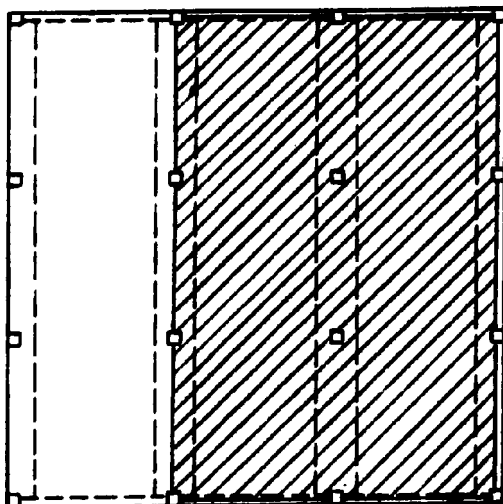
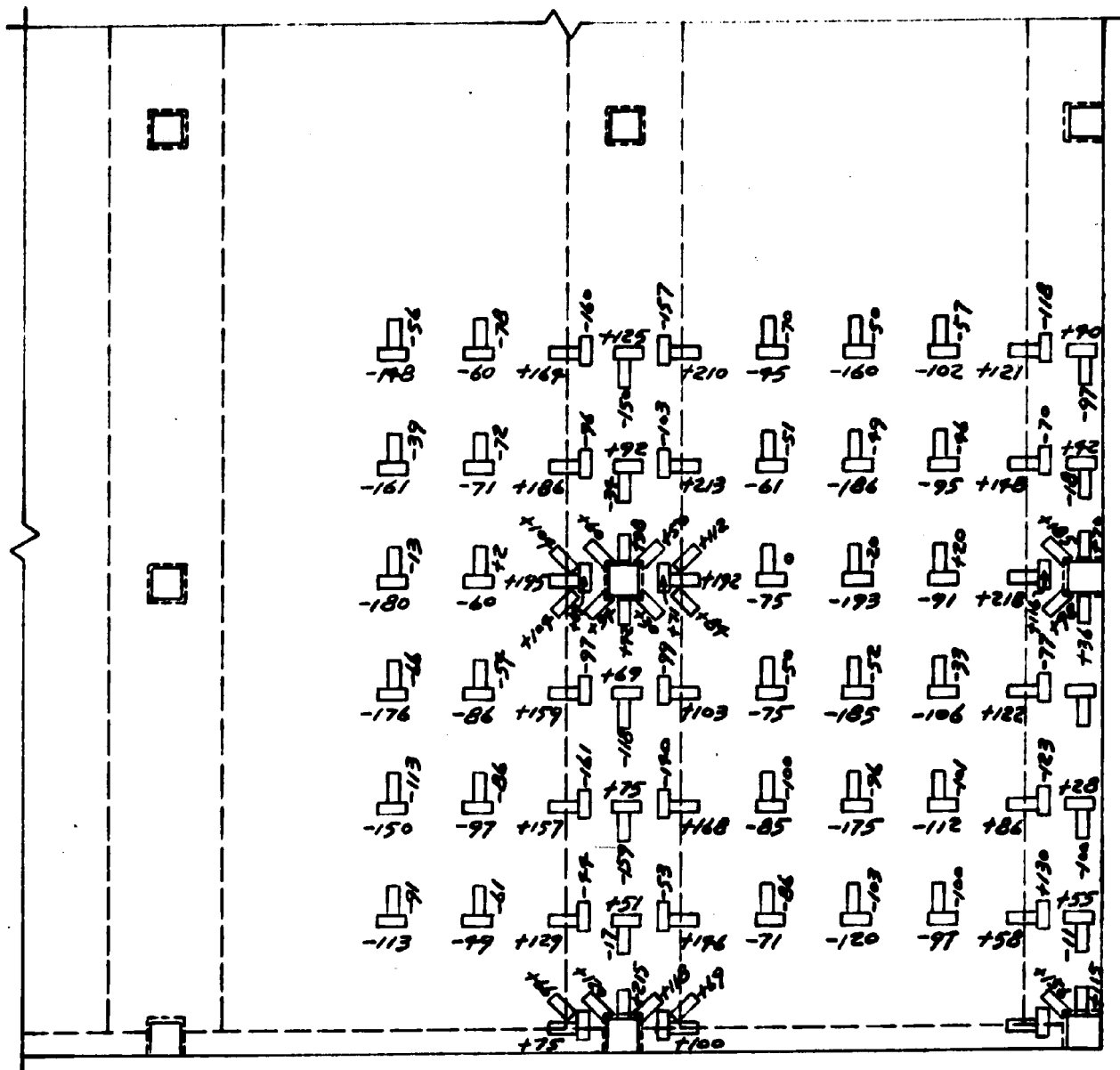
**Loaded Areas Are Shaded**

Diagram 6.01.

Strains in the Top Fibers

(Strains are in micro-inches)

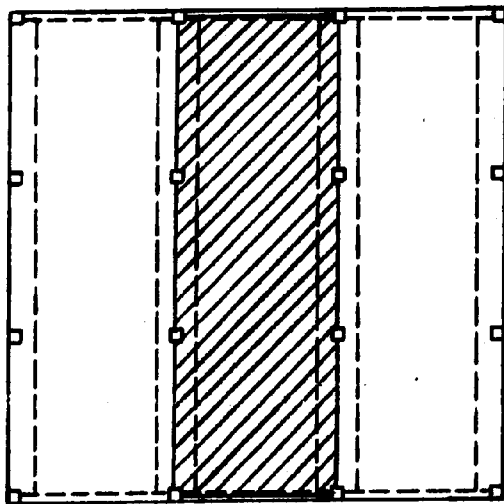
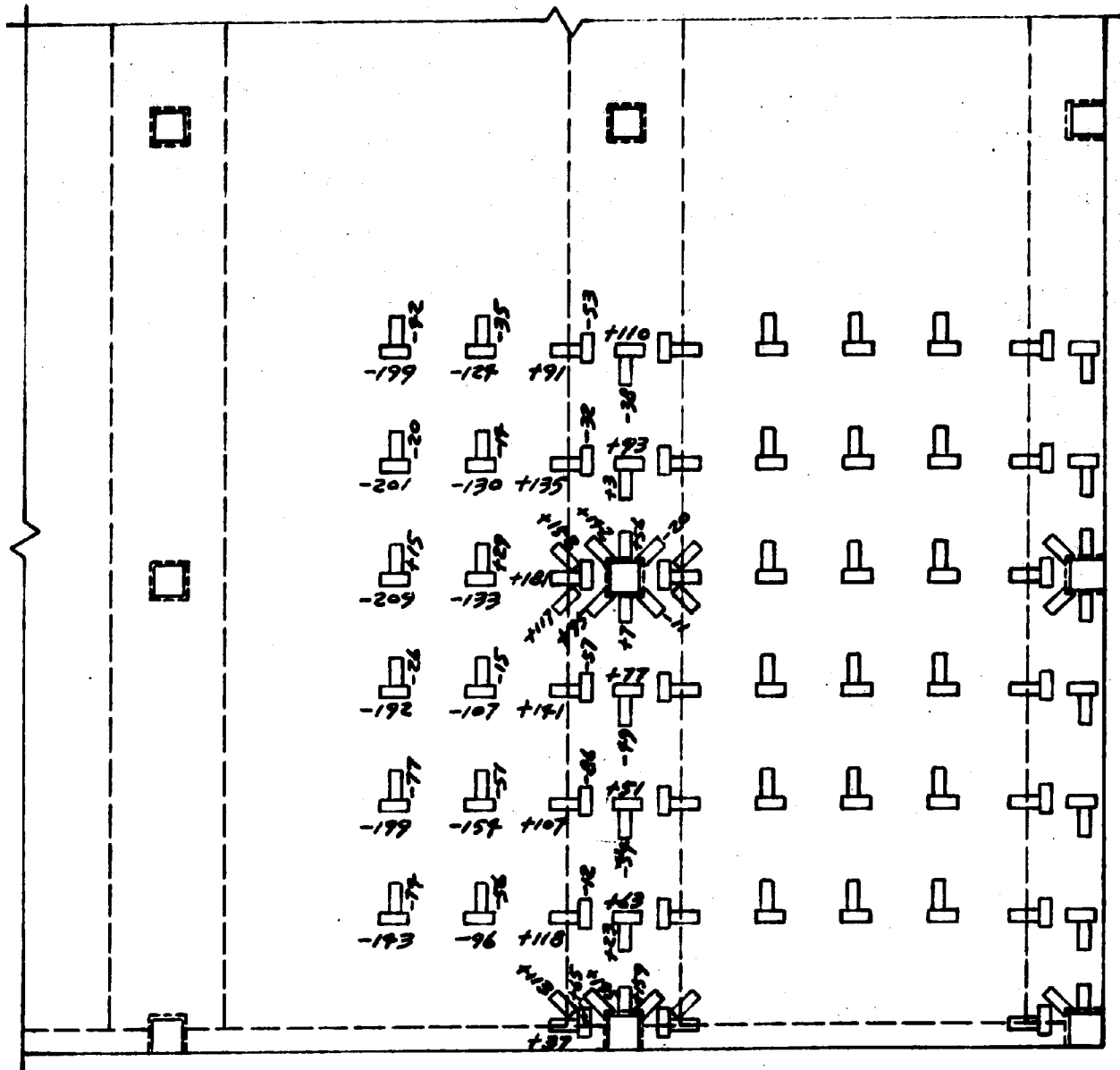
Loading Condition No. 1



KEY

Loaded Areas Are Shaded

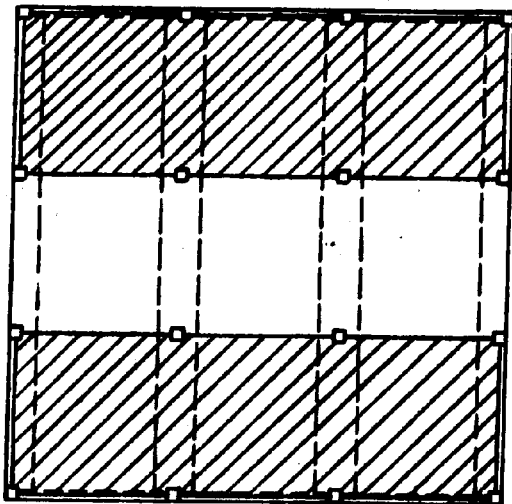
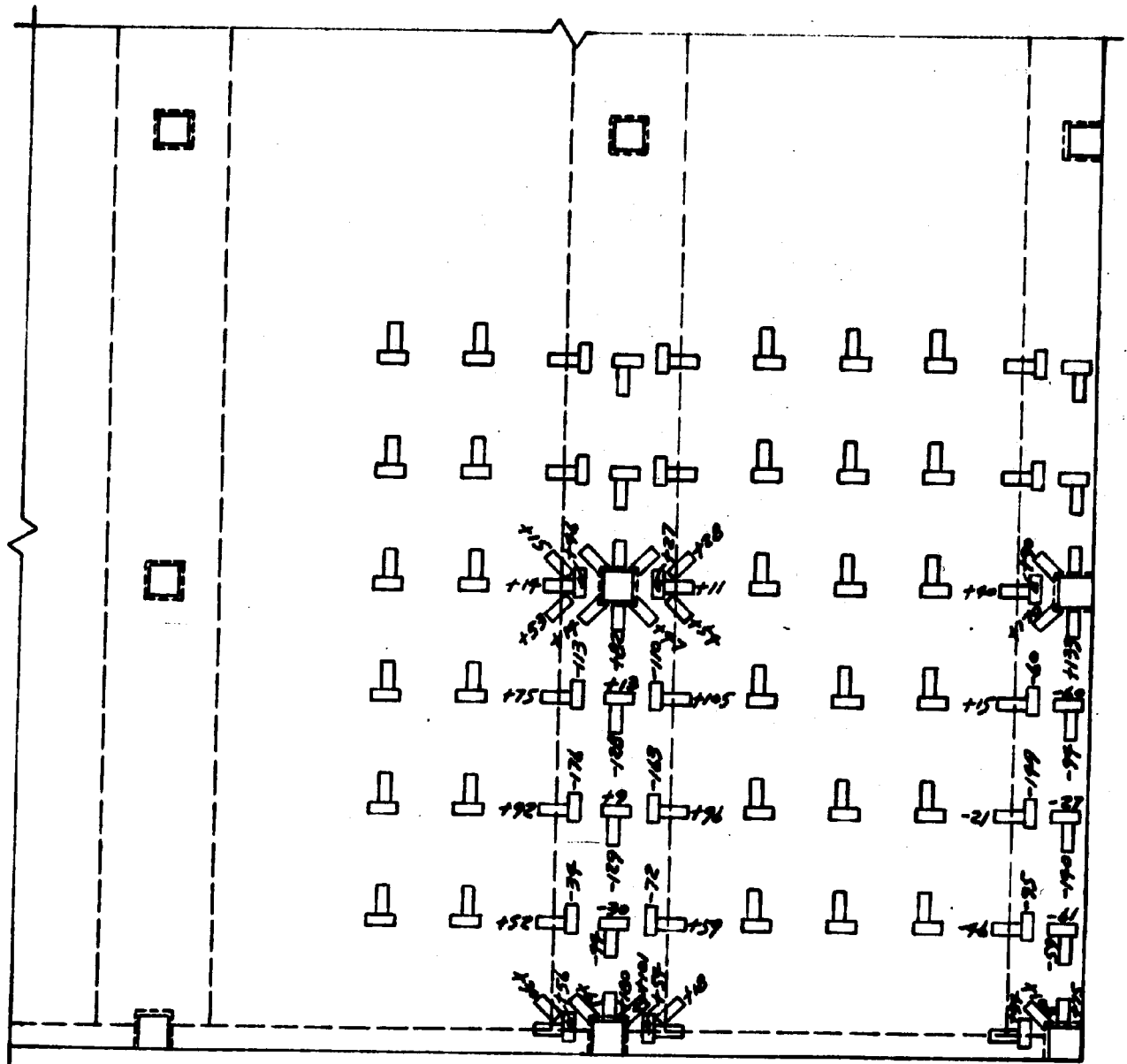
Diagram 6.02.  
 Strains in the Top Fibers  
 (strains are in micro-inches)  
 Loading Condition No. 2



KEY

Loaded Areas Are Shaded

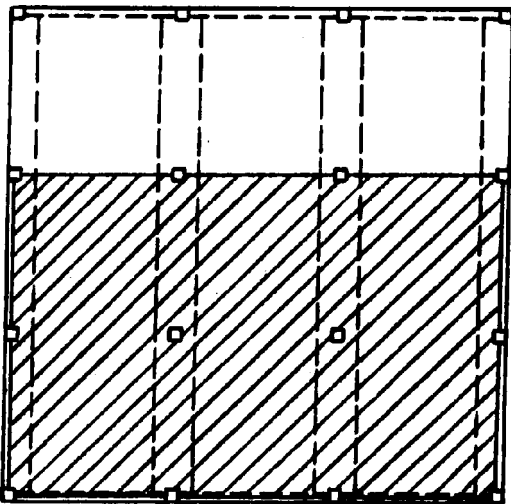
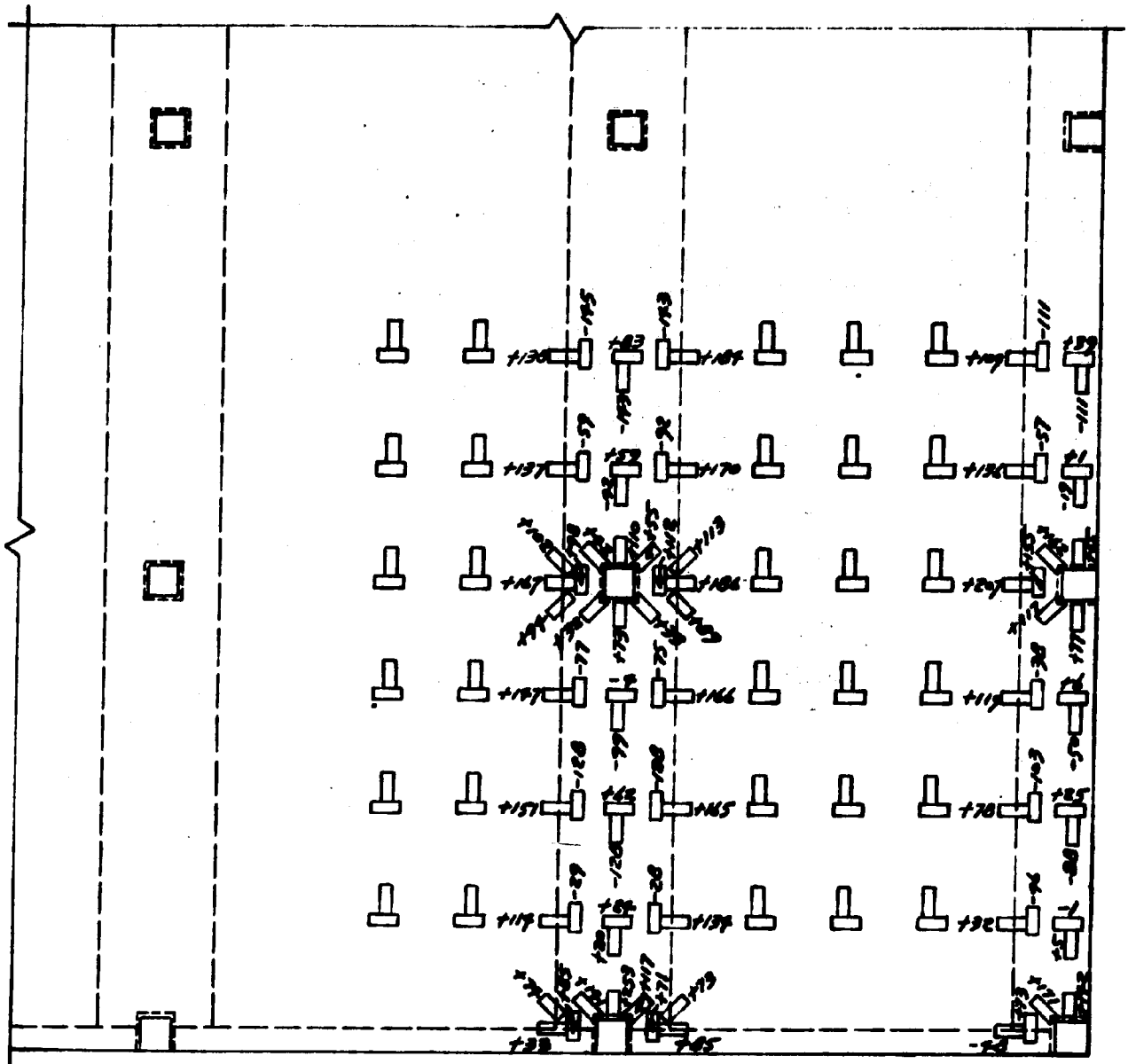
Diagram 6.03.  
 Strains in the Top Fibers  
 (Strains are in micro-inches)  
 Loading Condition No. 3



KEY

Loaded Areas Are Shaded

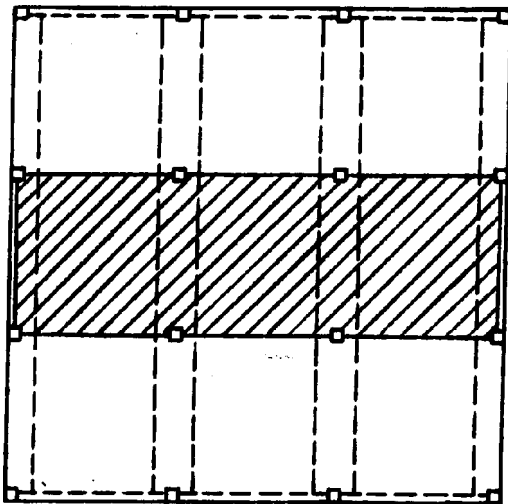
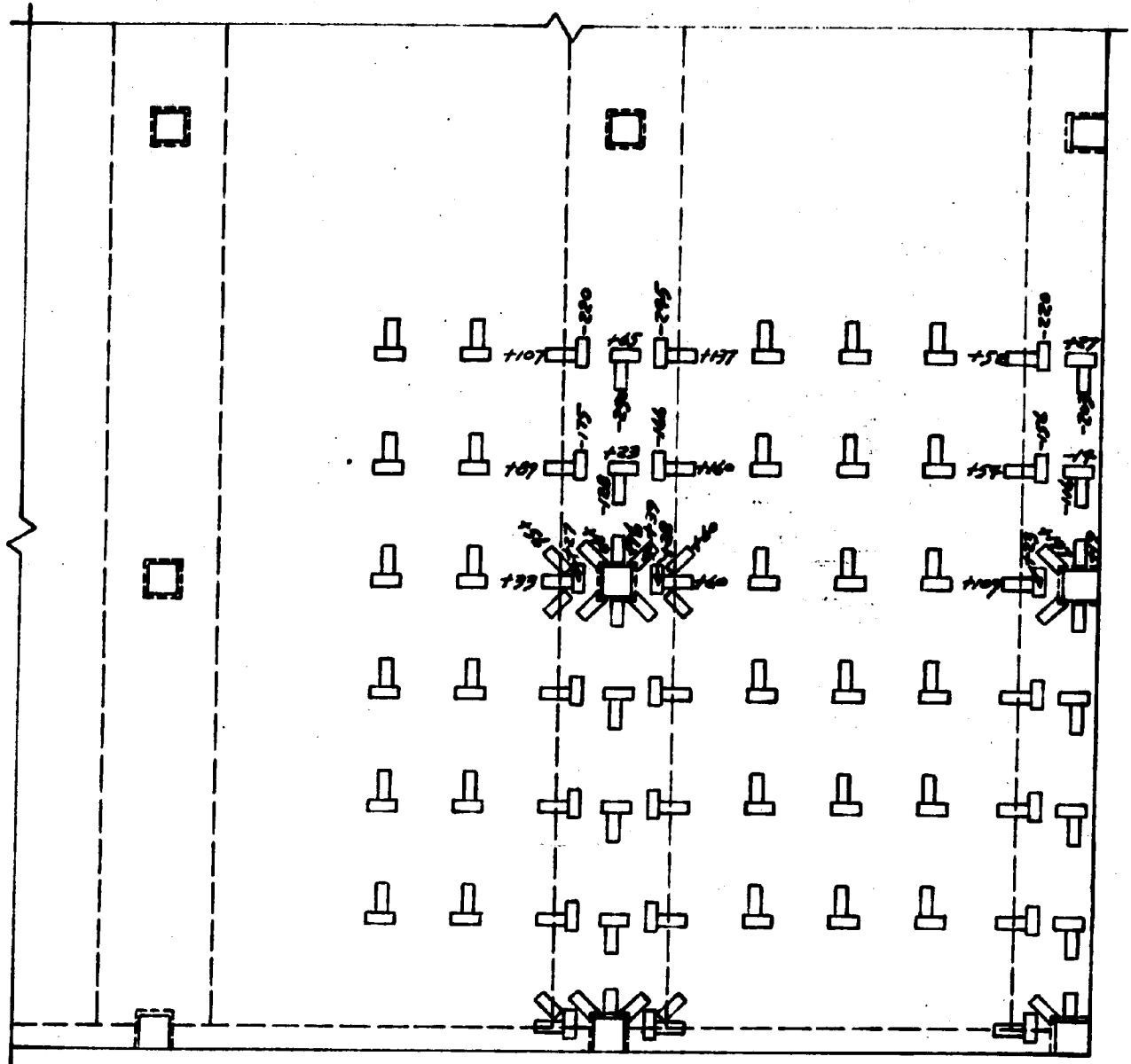
Diagram 6.04.  
 Strains in the Top Fibers  
 (Strains are in micro-inches)  
 Loading Condition No. 4



KEY

Loaded Areas Are Shaded

Diagram 6.05.  
 Strains in the Top Fibers  
 (Strains are in micro-inches)  
 Loading Condition No. 5



KEY

Loaded Areas Are Shaded

Diagram 6.06  
 Strains in the Top Fibers  
 (Strains are in micro-inches)  
 Loading Condition No. 6

## CHAPTER 7

### STRESSES IN THE MODEL ACCORDING TO TESTS

#### 7.1. The Relationship between Stresses and Strains

It is the intention, in this section, to ascertain the stresses along two perpendicular axes at a point in an elastic material based on strains measured along the same axes. The following notations will be needed in the derivation of formulas.

$x$  and  $y$ : Two perpendicular axes through the point

$S$ : Unit stress

$\epsilon$ : Unit strain

$E_t$ : Modulus of elasticity in tension

$E_c$ : Modulus of elasticity in compression

$\nu$ : Poisson's ratio

The numerical values of the last three terms were determined in Chapter 4 for the Plexiglas which was used as the material of the model tested.

$$E_t = 424,600 \text{ \#/in.}^2$$

$$E_c = 454,900 \text{ \#/in.}^2$$

$$\nu = 0.3796$$

## 7.1.1. Tension along Both Axes

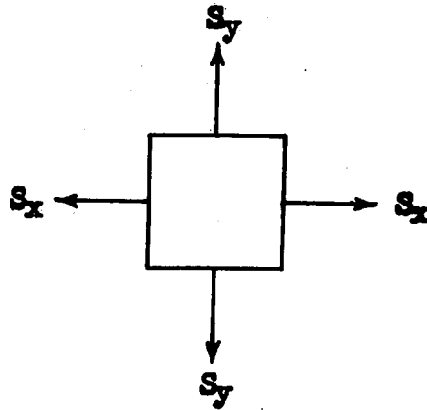


Figure 7.01.

## A Unit Block under Tension

In Figure 7.01 a unit block is shown under tension along two perpendicular axes  $x$  and  $y$ . The unit strain along the  $x$ -axis caused by unit stress  $S_x$  is  $\frac{S_x}{E_t}$  and that caused by unit stress  $S_y$  is  $\nu \frac{S_y}{E_t}$ . The former is elongation while the latter is shortening. The sum,  $\epsilon_x$ , of these two components is

$$\epsilon_x = \frac{S_x}{E_t} - \nu \frac{S_y}{E_t} \quad (7.01)$$

The total strain along the  $y$ -axis caused by  $S_x$  and  $S_y$  is obtained by the same reasoning as above.

$$\epsilon_y = \frac{S_y}{E_t} - \nu \frac{S_x}{E_t} \quad (7.02)$$

Solve (7.01) and (7.02) for  $S_x$  and  $S_y$ :

$$\nu \times (7.02) \quad \nu \epsilon_y = \nu \frac{S_y}{E_t} - \nu^2 \frac{S_x}{E_t} \quad (7.03)$$

$$\begin{aligned} (7.01) + (7.03) \quad \epsilon_x + \nu \epsilon_y &= \frac{S_x}{E_t} - \nu^2 \frac{S_x}{E_t} \\ &= \frac{S_x}{E_t} (1 - \nu^2) \end{aligned}$$

$$S_x = \frac{E_t}{1 - \nu^2} (\epsilon_x + \nu \epsilon_y) \quad (7.04)$$

$$\mu \times (7.01) \quad \mu \epsilon_x = \mu \frac{S_x}{E_t} - \mu^2 \frac{S_y}{E_t} \quad (7.05)$$

$$(7.02) + (7.05) \quad \epsilon_y + \mu \epsilon_x = \frac{S_y}{E_t} - \mu^2 \frac{S_y}{E_t} \\ = \frac{S_y}{E_t} (1 - \mu^2)$$

$$S_y = \frac{E_t}{1 - \mu^2} (\epsilon_y + \mu \epsilon_x) \quad (7.06)$$

When the values of  $E_t$  and  $\mu$  for the Plexiglas are used equation (7.04) becomes

$$S_x = \frac{424600}{0.8559} (\epsilon_x + 0.3796 \epsilon_y) \\ = 496,100 \times (\epsilon_x + 0.3796 \epsilon_y) \quad (7.07)$$

and equation (7.06) becomes

$$S_y = \frac{424600}{0.8559} (\epsilon_y + 0.3796 \epsilon_x) \\ = 496,100 \times (\epsilon_y + 0.3796 \epsilon_x) \quad (7.08)$$

#### 7.1.2. Compression along Both Axes

The following equations are derived by the same reasoning as above.

$$S_x = -531,500 \times (\epsilon_x + 0.3796 \epsilon_y) \quad (7.09)$$

$$S_y = -531,500 \times (\epsilon_y + 0.3796 \epsilon_x) \quad (7.10)$$

#### 7.1.3. Tension along x-axis and Compression along y-axis

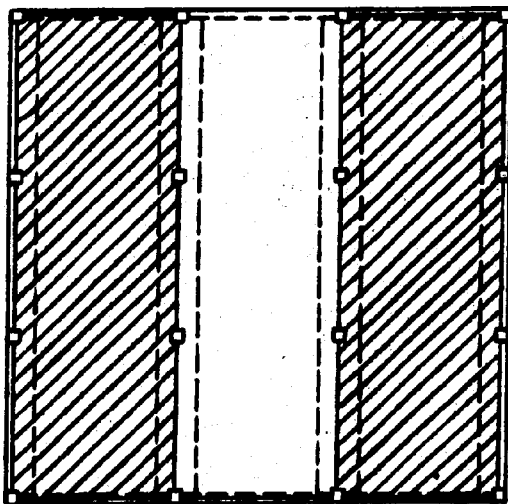
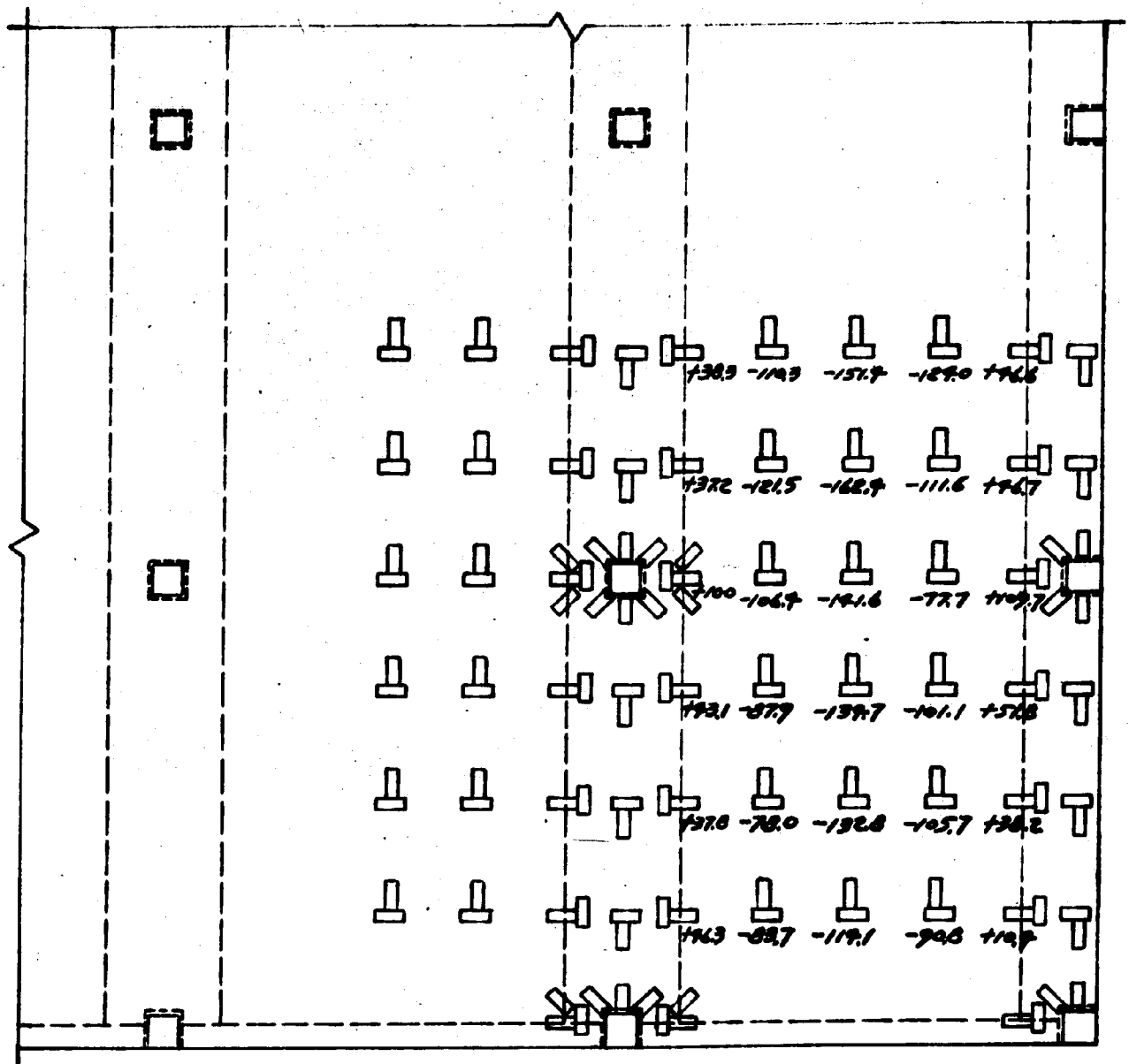
$$S_x = 496,100 \times (\epsilon_x + 0.3796 \epsilon_y) \quad (7.11)$$

$$S_y = -531,500 \times (\epsilon_y + 0.3796 \epsilon_x) \quad (7.12)$$

#### 7.1.4. Compression along x-axis and Tension along y-axis

$$S_x = -531,500 \times (\epsilon_x + 0.3796 \epsilon_y) \quad (7.13)$$

$$S_y = 496,100 \times (\epsilon_y + 0.3796 \epsilon_x) \quad (7.14)$$



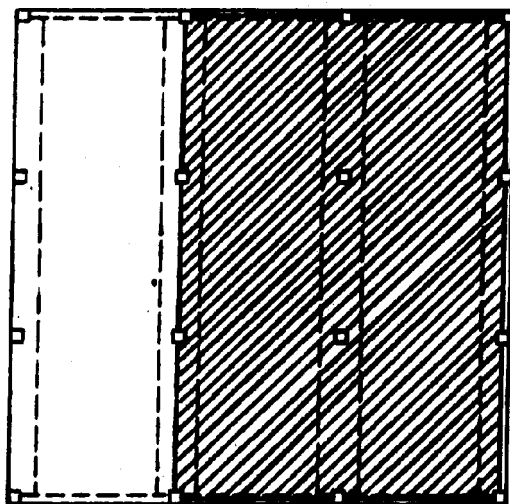
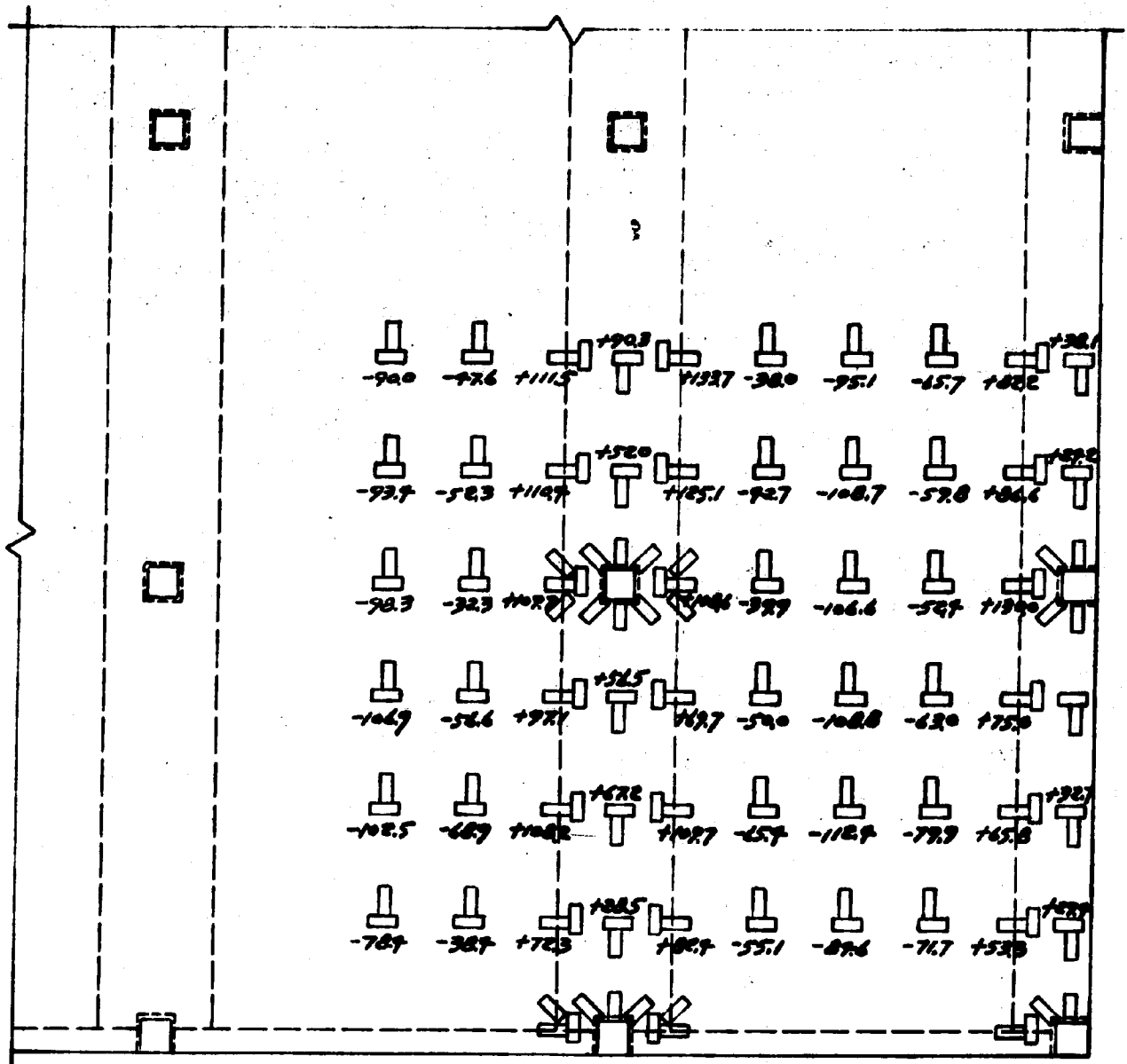
KEY

Loaded Areas Are Shaded

Diagram 7.01.

Stresses in the Top Fibers of the Slab  
in the Direction of the Slab Span  
(Stresses are in lbs. per sq. in.)

Loading Condition No. 1



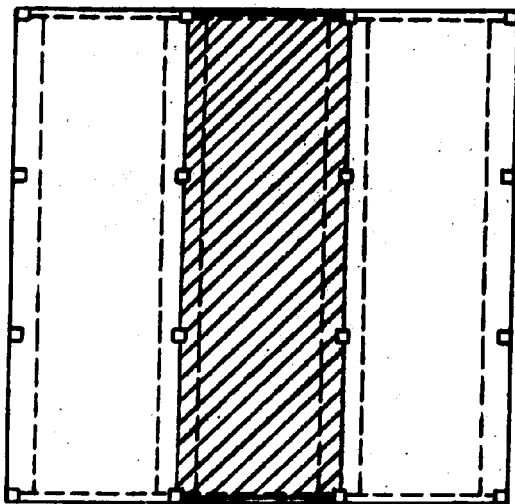
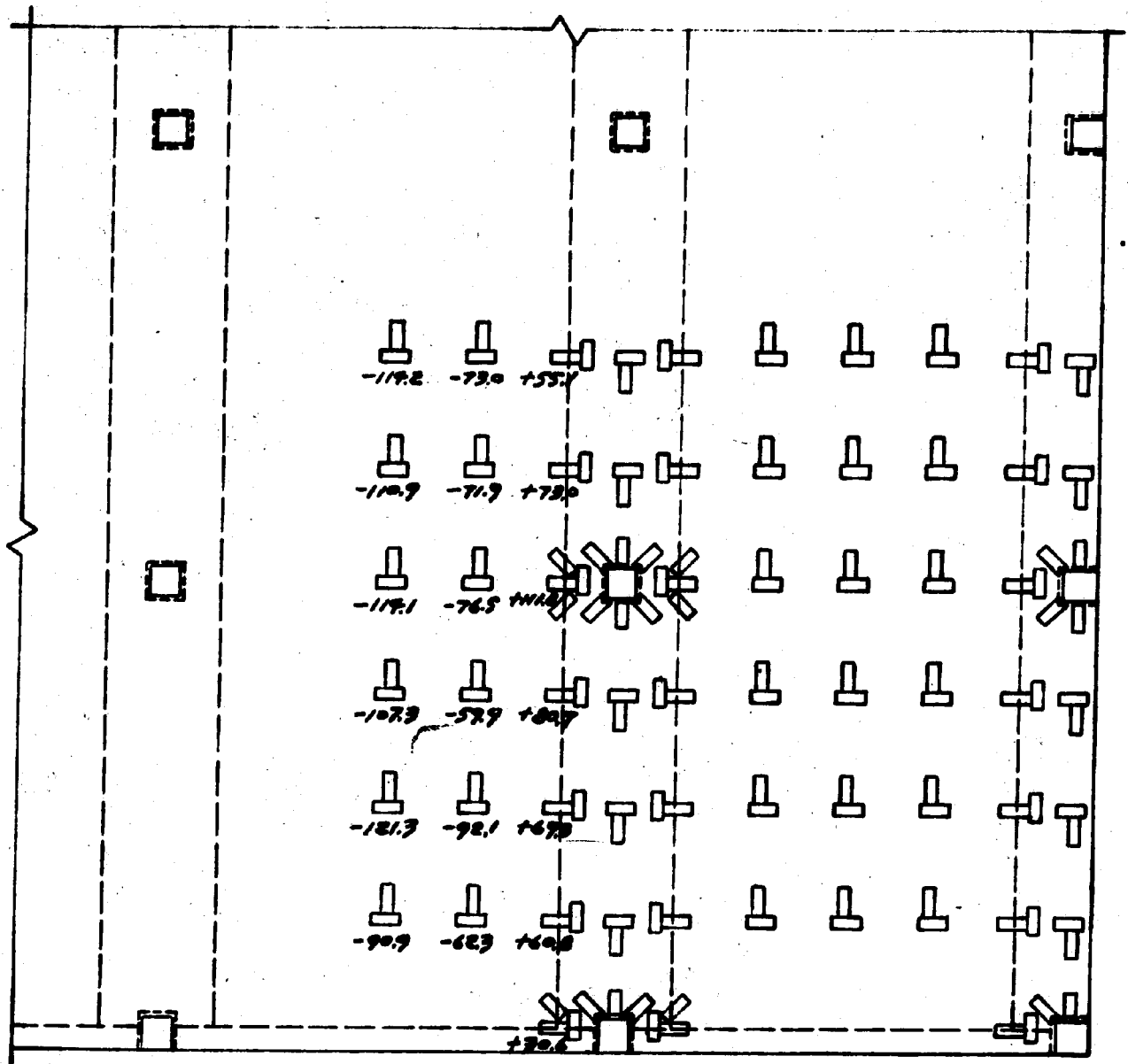
KEY

Loaded Areas Are Shaded

Diagram 7.02.

Stresses in the Top Fibers of the Slab  
in the Direction of the Slab Span  
(Stresses are in lbs. per sq. in.)

Loading Condition No. 2



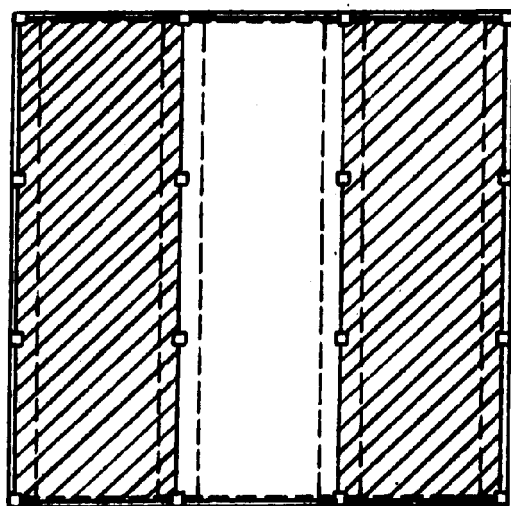
KEY

Loaded Areas Are Shaded

Diagram 7.03.

Stresses in the Top Fibers of the Slab  
in the Direction of the Slab Span  
(Stresses are in lbs. per sq. in.)

Loading Condition No. 3

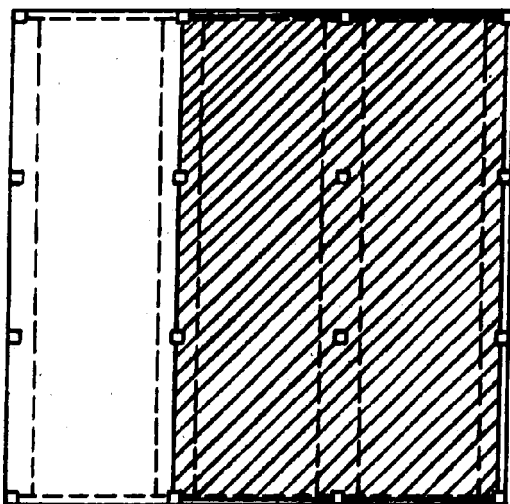
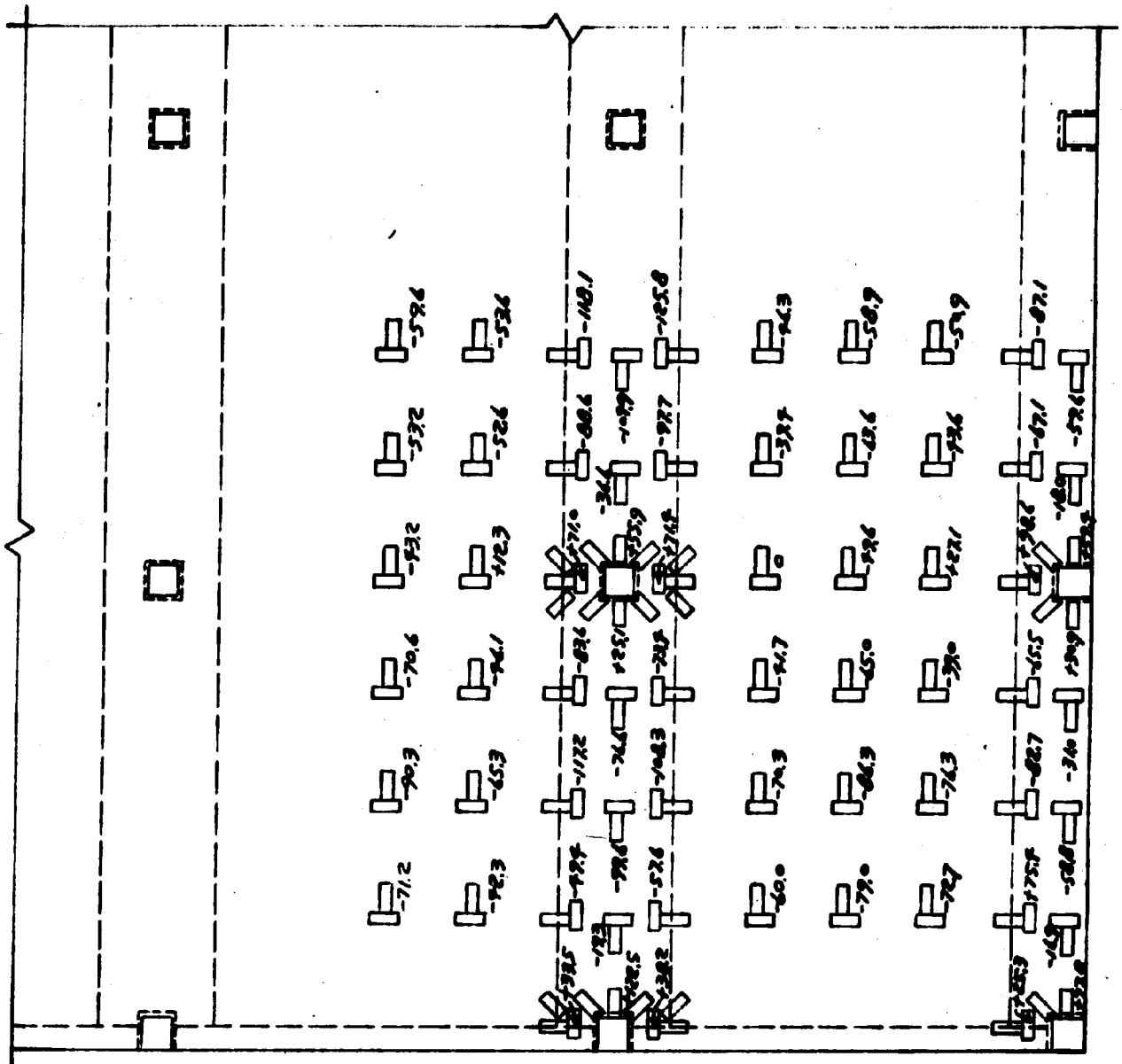


**Loaded Areas Are Shaded**

**Stresses in the Top Fibers of the Slab  
in the Direction Perpendicular to That  
of the Slab Span**

(Stresses are in lbs. per sq. in.)

### Loading Condition No. 1



KEY

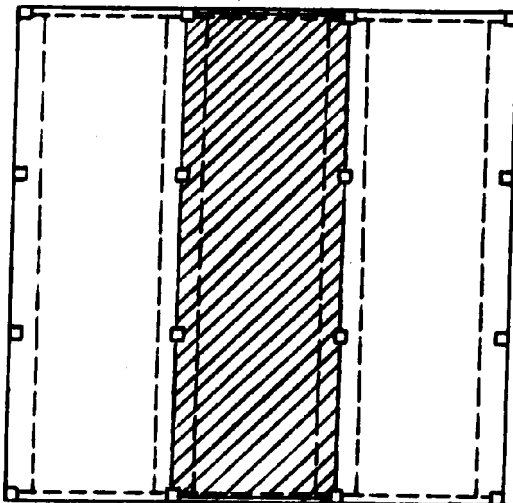
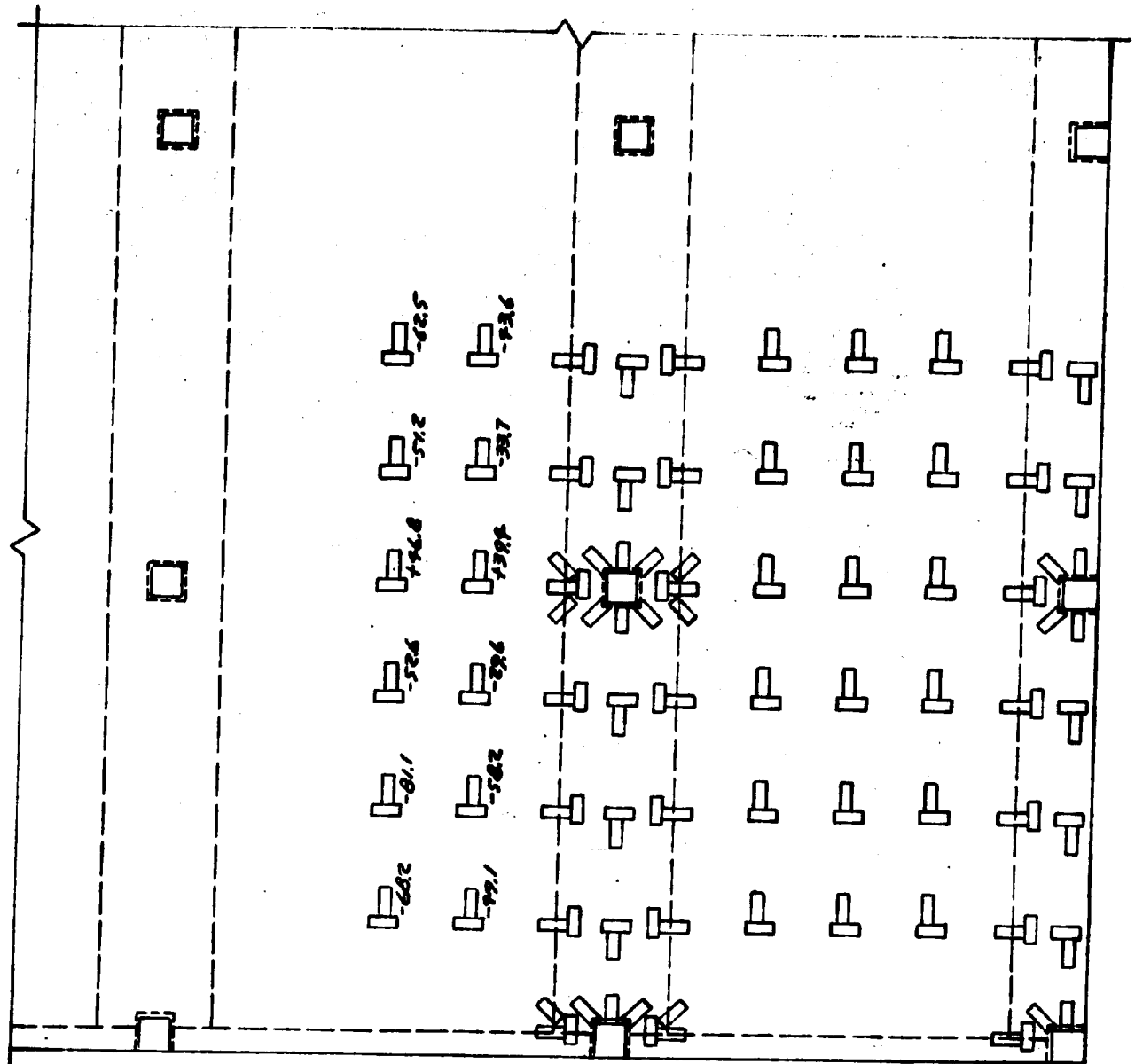
Loaded Areas Are Shaded

Diagram 7.05.

Stresses in the Top Fibers of the Slab  
in the Direction Perpendicular to That  
of the Slab Span

(Stresses are in lbs. per sq. in.)

Loading Condition No. 2



KEY

Loaded Areas Are Shaded

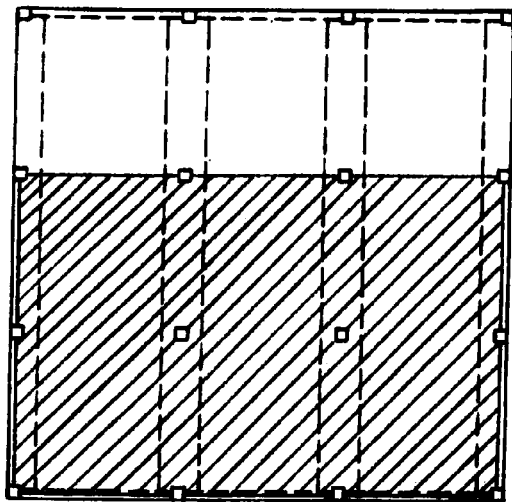
Diagram 7.06.

Stresses in the Top Fibers of the Slab  
in the Direction Perpendicular to That  
of the Slab Span

(Stresses are in lbs. per sq. in.)

Loading Condition No. 3





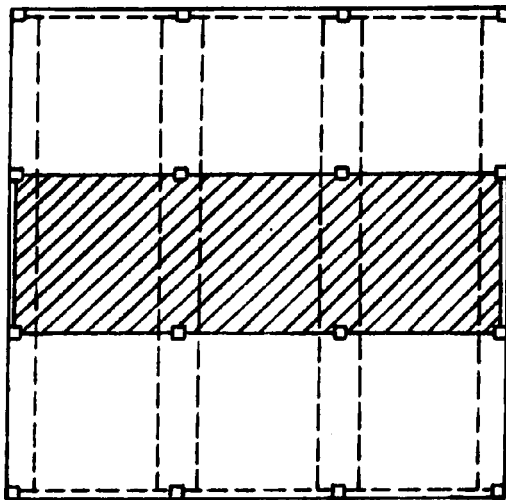
**Loaded Areas Are Shaded**

Diagram 7.08.

Stresses in the Top Fibers  
of the Slab-Bands

(Stresses are in lbs. per sq. in.)

Loading Condition No. 5



**Loaded Areas Are Shaded**

Diagram 7.09.

Stresses in the Top Fibers  
of the Slab-Bands

(Stresses are in lbs. per sq. in.)

Loading Condition No. 6

## 7.2. The Stresses in the Slabs and the Slab-Bands of the Model

The strains at various points on the top of the middle floor of the model due to different loading conditions were determined by tests and the results are shown on diagrams 6.01 through 6.06. Two strains were measured at every spot, one of which was parallel to the span of the slab or the slab-band and the other perpendicular to it. The stresses in the direction of the strains  $\epsilon_x$  and  $\epsilon_y$  at any point can be determined by equations (7.07) through (7.14). The equations to be used for stresses at any point depend upon the combination of strains occurring at that point.

The results of the computation of stresses are shown on diagrams 7.01 through 7.09. The first group, which consists of the first three diagrams, is for the stresses in the slab in the direction of the slab span while the second group consisting of diagrams 7.04, 7.05 and 7.06 is for the stresses in the slab in the direction perpendicular to that of the slab span. The third group, Diagrams 7.07, 7.08 and 7.09, is for the longitudinal stresses in the slab-bands.

## 7.3. Discussion of the Stresses in the Slabs and the Slab-Bands

In this section the possible sources of errors in the test results will be discussed, followed by detailed discussions of the stresses in the slabs and the slab-bands as obtained in section 7.2.

### 7.3.1. Sources of Errors

Although great care was taken in making the model, in fixing strain gages and other accessories and in testing, errors due to unforeseen and unavoidable causes may still be present in the results.

The following is a discussion, item by item, of various possible sources of errors.

#### A. Properties of the Material of the Model

The Plexiglas is assumed to be a homogeneous, isotropic and elastic material and to follow Hooke's law. Actually, its properties do not conform exactly to those assumptions. This was shown in Chapter 4 where the values of the modulus of elasticity  $E_t$  and  $E_c$  and Poisson's ratio  $\nu$  were reported.

#### B. Errors in Construction of Model

In making the model a dimensional tolerance of  $\pm 0.005$  inches was used for all elements. As was previously noted, this is proportionately a much greater accuracy than could be obtained in the construction of the prototype, therefore errors from this source are negligible.

The columns at the ends of the building model occur at the intersections of slab-bands and spandrel beam. Since the spandrel beam is deeper than the slab-band and narrower than the column it was necessary to cut a notch in the outer edge of the spandrel at each column to allow the column to come up under the slab-band. This detail is shown in Figure 3.05. In four instances this notch was cut too wide and consequently, within the notch, one face of the column is not in contact with the spandrel beam. Although this mistake introduces errors it is not important in this study since the spandrel beams are not being investigated. It is believed that the effect of this mistake on the action of slabs and slab-bands must be minor, even in the exterior panels.

The parts of the model were connected by means of Cement I-A as described in Chapter 3. The connections of the columns to the middle floor were made one at a time and were very carefully controlled. The parts of the top and bottom floors were then assembled. Finally the

entire top floor was cemented to the 16 columns in one operation. The bottom floor was connected also in one operation. The operation of cementing 16 columns at one time cannot be as well controlled as single connections. Therefore the joints at the middle floor are probably stronger than the joints at top and bottom floors. Any error arising from this condition is believed to be negligible because the level of loading used in the tests was not above the working load.

#### C. Distance between Centers of Two Adjoining Strain Gages

Type A-7 strain gages, gage length  $\frac{1}{4}$  inch, were used for measuring strains on the model. For strains along two axes the gages were placed as close together as possible ( $\frac{1}{8}$  inch center to center) and  $90^\circ$  to each other. On the prototype this is equivalent to locating the gages 7.5 inches apart. The resulting error is not serious because the difference in location of gages is in a direction normal to the slab span. However, on the slab-bands the error is in the direction of the span. But since the beam span is greater than the slab span it is thought that this error is not significant.

#### D. Seal between Columns and Bulkheads for Loading

The fine steel shot used for loading was retained in the desired areas by wood bulkheads between the columns. The seal between bulkheads and columns was of  $1/8$ -thick rubber in order to minimize any pressure between bulkhead and column. It was noticed that some of the smaller pieces of shot worked their way between the column and the rubber bulkhead seal. It is therefore possible that the columns were to some extent restrained.

#### E. Load Distribution

There were several mediums between the applied load (steel shot) and the top surface of the tested floor of the model. They

consisted of strain gages, felt, electric wires, masking tape, hard rubber and gum rubber. The strain gages were installed first and were covered with felt to protect them. The electric wires were connected to the strain gages at one of their ends and to the switch boxes outside the model at the other end and they were held on the top of the floor by means of masking tape. The spaces between wires and between wires and strain gages were filled with hard rubber. At the top of all these medium a sheet of gum rubber was used to cover the whole area of the floor. Although there were different mediums between the tested floor and the gum rubber, the load that was uniformly distributed on the gum rubber can be considered as uniformly distributed on the floor tested without any appreciable error.

#### F. Change of Temperature

When the surrounding conditions are identical for the active and the compensating strain gages the change of temperature during test will not affect the changes of strains measured. Moreover the change of temperature during one whole test was only few degrees Fahrenheit and therefore the chance was very small that the measured changes of strains contained any error due to change of temperature.

#### G. Time Factor in Taking Strain Gage Readings

The influence of the time factor on strain gage readings was discussed in Chapter 4 along with the results of tests illustrating the time effect. In that chapter, also, a time schedule was worked out for the use of compensating gages and for the time interval between consecutive strain readings for the purpose of minimizing the effect of time on the readings. Since that time schedule was rigidly followed during the model tests it is believed that any error from this source is small enough to be neglected.

## H. Settlement of Supports

The model was laid on a flat piece of plywood which was supported by a level table made of heavy timber. Aluminum shims were used under the columns to equalize the bearing of the columns on the support. Thus there was very little possibility that the supports of the model settled under the load applied to the model.

### 7.3.2. Stresses in the Slab in the Direction of the Slab Span

This is the discussion of the stresses in the slab in the direction of the slab span. These stresses were calculated in the last section and they were shown in the diagrams 7.01, 7.02 and 7.03.

Attention is called to the fact that all the stresses to be discussed in this section are on the top of the slab in the direction of the slab span except as noted. See diagram 5.01 for designation of gages.

#### A. Test No. 1

Reference is made to diagram 7.01 for this discussion.

a. Both of the two exterior panels of the slab were loaded. This loading was used for producing absolute maximum positive bending moment at any point of the slab in the exterior span due to specified uniform load and the compressive stresses shown on diagram 7.01 are absolute maximum stresses for those respective points. The terms "Absolute maximum moment" and "Absolute maximum stress" at one point are defined as those which are the maximum of all the moments and all the stresses respectively produced at one point due to different possible loading conditions.

b. For a strip of slab in the exterior span the maximum positive bending moment does not occur at the middle of the slab span and the stress measured by means of strain gage which is at the middle of the slab span was not the maximum stress along that strip.

However, it is impossible to locate the experimental spot of maximum positive bending moment before test. Moreover, the location of the maximum positive bending moment is usually not very far away from the center of the span and it is also true that the difference between stresses at center of span and at place of maximum positive bending moment is not large and therefore it is considered that the compressive stress measured at the center of the span is the maximum compression along that strip.

c. Although these absolute maximum stresses at the quarter points of the exterior slab, i. e., along the 6- and 8-lines, are not used in the specification of design they are nevertheless useful in determining the zero-stress contour lines.

d. The tensile stresses measured at the edges of the exterior and interior slab-bands, i. e., along the 5- and 9-lines are not absolute maximum values. They are not useful in design except where they are used to locate the zero-stress contour lines.

e. The zero-stress contour lines in the slab as shown in diagram 7.14 are needed to determine the lengths of the positive reinforcing steel in reinforced concrete floor construction.

f. Along the loaded edges of the interior and the exterior slab-bands, i. e., along the 5- and the 9-lines the stresses near the columns at the locations 135 and 139 are larger than those at other locations along the same edges of the respective slab-bands. This can be explained as follows. The slab is supported and restrained by the slab-band but the slab-band itself tilts in a certain degree toward the loaded side when one side of its center line is only loaded. The tilt of the slab-band causes the release of negative bending moment in certain degree in the slab at the edge of the slab-band and consequently the stresses at those

places are decreased. On the other hand for that part of the slab-band which is around its supporting columns the columns tend to prevent the tilt of the slab-band and therefore the negative moment in the slab at the edge of the slab-band near the supporting columns only decreases a small amount. Thus the stresses in the slab at the edge of the slab-band are larger near the supports of the slab-band than those near its center.

#### B. Test No. 2

Reference is made to diagram 7.02 for this discussion.

- a. Two adjacent panels of the slab were loaded. This loading was used for producing absolute maximum negative bending moments in the slab along the edges of the supporting slab-bands due to specified uniform load and the tensile stresses shown on diagram 7.02 are absolute maximum stresses along those edges.
- b. The tensile stresses at top of the slab along the edge of the interior slab-band in the exterior span, i. e., along the 5-line, were larger than those at the corresponding points along the other edge of the interior slab-band in the interior span, i. e., along the 3-line.
- c. The slab stresses on the center line of the interior slab-band, i. e., along the 4-line, and those along the 10 -line of the exterior slab-band were absolute maximum stresses at their respective points but they were smaller than those at the corresponding points along the edges of their respective slab-band. The moments along the center line, i. e., along the 4-line, of the interior slab-band in the direction of the slab span are larger than those at the corresponding points along the edges of the same slab-band but the stresses along the 4-line are smaller than those along the edge lines. The additional thickness in the slab-band accounts for this reduction in stress.
- d. The compressive stresses obtained in this test were not absolute.

maximum stresses but those along the 2-, 6- and 8-lines are useful in determining the zero-stress contour lines.

e. The zero-stress contour lines in the slab as shown on diagram 7.15 are useful in determining the lengths of the negative reinforcing steel extending into the slab from the slab-bands in reinforced concrete floor construction.

f. The tensile stress in the direction of the slab span at the center of the exterior span of the interior slab-band, i. e., at location 154, is larger than those in the same direction at quarter points of the same slab-band span, i. e., at locations 144 and 164. The reason for this is as follows. The compressive stress in the direction of the slab-band span at location 154 is larger than those at locations 144 and 164, and therefore this larger compressive stress at location 154 induces more tensile stress at the same location in the direction of slab span than those at locations 144 and 164. This same reason is applied to the tensile stresses at locations 114, 120 and 160 as compared to those at locations 124, 130 and 170 respectively.

g. Along the edge of the exterior slab-band the stress at location 139 near the supporting column was larger than those at the quarter points, i. e., at locations 129 and 149, and the latter were in turn larger than those at the centers of the slab-band spans, i. e., at locations 119 and 159. The reason for this was explained in section 7.3.2.A.f.

h. When both sides of the interior slab-band are loaded there is not much difference between stresses in the slab along the same edge of the interior slab-band except the one near the wall beam and another one at location 145 which is apparently in error. This is quite different from the case described in section 7.3.2.A.f. For example the stress at location 133 near the supporting column was practically the same as the

stresses at other locations along the same edge of the interior slab-band except the one at location 163 near the wall beam. This is due to the fact that when panels on both sides of the interior slab-band are loaded the whole width of this slab-band can not tilt to one side. In other words, one half width of the slab-band prevents the other half from tilting. From the test data it is clear that the presence of the column at location 134 did not affect the stresses along the edges of the interior slab-band for loads on both of its sides.

i. The stresses along the 6-row are smaller than those along rows further from the wall beam. The reason for this phenomenon is that a great deal of load on the slab along the wall beam flows to the wall beam instead of going to the slab-bands and consequently the stresses in the slab in the direction of its span near the wall beam are smaller than those at the corresponding points further from the wall beam.

### C. Test No. 3

Reference is made to diagram 7.03 for this discussion.

- a. The center panel of the slab was loaded. This loading was used for producing absolute maximum positive moment at points in the interior slab span due to specified uniform load. The compressive stresses recorded on diagram 7.03 are absolute maximum stresses at those points.
- b. Because of the symmetrical frame and the symmetrical loading the stresses measured at the center of the interior slab must be the maximum stresses at those points.
- c. Although these absolute maximum stresses at the quarter points of the interior slab, i. e., along the 2-line, are not used in the specification of design, they are useful in determining the zero-stress contour line.
- d. The tensile stresses along the interior edge of the interior slab-

band, i. e., along the 3-line, are not absolute maximum stresses but they are useful in determining the zero-stress contour line.

e. The zero-stress contour lines in the slab as shown in diagram 7.16 are useful in determining the points of bending of positive reinforcing steel.

f. Along the interior slab-band, i. e., along the 3-line, the stress at the location 133 near the supporting column was larger than those at the quarter points, locations 123 and 143, and the latter were in turn larger than those at the centers of the slab-band spans, locations 113 and 153. The reason for this was explained in section 7.3.2.A.f.

g. When two strips of slab having different end restraint are loaded with a load of the same intensity the strip having more restraint at the ends gives less compression at its top at mid-span and more tension at its top near the ends. For this reason one might think the stress at the location 131 should be much smaller than those at locations 111, 121, 141 and 151 because the restraint at the ends of the strip along the 3-row is much larger than those at the corresponding points along the other rows. The fact is that although this slab was uniformly loaded the strip along the columns, the 3-row, has more rigidity than other strips and therefore the former takes more load than the others. Consequently the stress at the location 131 becomes larger than at the corresponding points of other strips.

h. The stresses along the 6-row were smaller than those at the corresponding points along other rows which are parallel to the 6-row. The reason for this has been explained in section 7.3.2.B.i.

### 7.3.3. Stresses in the Slab in the Direction Perpendicular to That of the Slab Span

This is the discussion of the stresses on top of the slab in the direction perpendicular to that of the slab span. These stresses were calculated in section 7.2. and were shown in diagrams 7.04, 7.05 and 7.06.

Attention is called to the fact that the stresses considered in this section were not absolute maximum stresses at their respective points. The loading conditions used were not selected for the purpose of studying maximum stress normal to the slab span. This discussion of such stresses is included to illustrate the general action. The interrelation of these stresses also demonstrates the uniformity of action of the model.

Reference is made to the test results shown in diagram 7.05 for test No. 2.

a. From a. to d. the discussion is limited to the stresses along the 1-line except as noted.

The stress at location 211 was the largest compression in the interior span along the 1-line because it was located at the center of the span.

b. The stress at location 231 was the smallest compression along the 1-line and that at location 232 was the only tension along the 2-line.

This demonstrates clearly that the slab along the 3-row acted as an intermediate to transmit load from adjacent slab to the slab-band and column although the slab along the 3-row had the same thickness as those along other rows.

c. The stress at location 251 was the largest compression along the 1-line because it was located at the center of the exterior span.

d. The stress at location 261 was larger than that at location 241 because the former was located at the exterior quarter point in the exterior bay.

- e. The discussion of stresses along 2-, 6-, 7- and 8-lines, respectively, is the same as for those in a. to d., inclusive.
- f. The stresses along the 2-line were smaller than those along the 1-line at corresponding points. The stresses along the 6- and 8-lines were similarly related to the stresses along the 7-line. The reason for this is that the 2-, 6- and 8-lines were nearer to the slab-bands than the 1- and 7-lines.
- g. The stresses along the 8-line were larger than the corresponding stresses along the 6-line because the 8-line was near the exterior slab-band and the 6-line was near the interior slab-band.
- h. Reference is made to a. to d. of this section and section 7.3.4. and 7.3.5. for the discussion of stresses in the slab-bands.

The test results shown in diagram 7.06 for Test No. 3 indicate action similar to that in diagram 7.05.

#### 7.3.4. Stresses in the Interior Slab-Band

This is the discussion of the stresses on top of the interior slab-band in the direction of the slab-band span. These stresses were calculated in section 7.2. and are shown in the diagrams 7.07, 7.08 and 7.09. See diagram 5.01 for designation of gages.

##### A. Test No. 4

Reference is made to diagram 7.07.

- a. The two exterior spans were loaded. This loading produced maximum positive bending moments in the exterior spans of the slab-band. The compressive stresses shown on diagram 7.07 are maximum stresses at their respective points.
- b. The maximum positive bending moment in the exterior span of the slab-band did not occur at the middle of the slab-band span but the compressive

stresses measured at gages 353 and 355 may be considered as the maximum compressive stresses in the exterior span for the reason explained in section 7.3.2.A.b.

c. Although these absolute maximum stresses at the quarter points in the exterior span of the slab-band, i. e., along the 4- and 6-rows, are not used in the specification for design, they are nevertheless useful in determining the zero-stress contour lines.

d. The tensile stresses measured near the columns, i. e., along the 3- and 7-rows, are not absolute maximum values. They are not useful except to locate the zero-stress contour line.

e. The zero-stress contour lines in the slab-band, as shown in diagram 7.17, are useful in determining the points of bending of positive reinforcing steel.

f. The interior slab-band can be divided into two parts according to its structural behavior. The middle strip was along the columns supporting this slab-band and was directly restrained by columns. The two side strips were located beyond the columns and were restrained by wall beams at their exterior ends and overhung the columns at their interior supports. The model tests showed that the measured tensions at the supports in the middle strip were greater than those in the side strip. At midspan the measured compression was smaller in the middle strip than those in the side strip.

g. The stresses along the 7-row were larger than those along the 3-row at the corresponding points. The reason for this was that the exterior column and wall beams offered more restraint to the slab-band than the interior column.

#### B. Test No. 5

Reference is made to diagram 7.08.

- a. Two adjacent spans were loaded. This loading was used for producing absolute maximum negative bending moments in the slab-band at the supports. The tensile stresses shown on diagram 7.08 are absolute maximum stresses at those places.
- b. Although the stresses at the quarter points in the exterior and interior spans of the slab-band, i. e., along the 2-, 4- and 6-rows, are not used in the specification of design, they are nevertheless useful in determining the zero-stress contour lines.
- c. The zero-stress contour lines in the slab-band, as shown in diagram 7.18, are useful in determining length of negative reinforcement over supports.
- d. Same as 7.3.4.A.f.
- e. The stresses in the side strip along the 5-line were larger than those at the corresponding points in the other side strip along the 3-line. The reason is that the side strip along the 5-line was in the exterior panel of the slab while the other side strip was in the interior panel of the slab.
- f. The stresses at gages 333 and 335 were larger than those at gages 373 and 375 because both the exterior and the interior spans were loaded.
- g. The stress at gage 374 was larger than that at gage 334B because the exterior column assisted by wall beams offered more restraint to that end of the middle strip than the interior column did to the other end of the same strip.

#### C. Test No. 6

Reference is made to diagram 7.09.

- a. The center span was loaded. This loading was used for producing absolute maximum positive moment at points in the middle span of the interior slab-band. The compressive stresses recorded on diagram 7.09

are absolute maximum stresses at those points.

b. Because of the symmetrical frame and the symmetrical loading the stresses measured in the middle span of the interior slab-band must be the maximum stresses at those points.

c. Although these absolute maximum stresses at the quarter points of the interior slab-band, i. e., along the 2-row, are not used in the specification of design, they are useful in determining the zero-stress contour line.

d. The tensile stresses near the column, i. e., along the 3-row, are not absolute maximum stresses but they are useful in determining the zero-stress contour line.

e. The zero-stress contour lines in the slab-band, as shown in diagram 7.19, are useful in determining the points of bending of positive reinforcing steel.

f. Because of difference in degree of restraint the stress at gage 334A was larger than those at gages 333 and 335.

g. Same as 7.3.4.B.e.

#### 7.3.5. Stresses in the Exterior Slab-Band

The is the discussion of the stresses on top of the exterior slab-band in the direction of the slab-band span. These stresses are calculated in section 7.2. and are shown in the diagrams 7.07, 7.08 and 7.09. See diagram 5.01 for designation of gages.

##### A. Test No. 4

Reference is made to diagram 7.07.

a. Same as section 7.3.4.A.a.

b. The compressive stress measured at gage 159 may be considered as the maximum compressive stress in the exterior span for the reason explained

in section 7.3.2.A.b.

c. Same as section 7.3.4.A.c.

d. Same as section 7.3.4.A.d.

e. Same as section 7.3.4.A.e.

f. The exterior slab-band can be divided into two parts according to its structural behavior. The column strip is that part of the exterior slab-band width which frames directly into the columns, i. e., does not overhang the columns. The side strip was located beyond the columns and was restrained by the wall beam at its exterior end and overhung the columns at its interior supports. The model tests showed that the measured tensions at the supports in the column strip were greater than those in the side strip. At midspan the measured compression was smaller in the column strip than that in the side strip.

#### B. Test No. 5

Reference is made to diagram 7.08.

a. Same as section 7.3.4.B.a.

b. Same as section 7.3.4.B.b.

c. Same as section 7.3.4.B.c.

d. Same as section 7.3.5.A.f.

e. The stress at gage 339 was larger than that at gage 379 because both the exterior and the interior spans were loaded.

f. The stress at gage 380 was larger than that at gage 340B because the corner column assisted by wall beam offered more restraint to that end of the column strip than the interior column did to the other end of the same strip.

#### C. Test No. 6

Reference is made to diagram 7.09.

a. Same as section 7.3.4.C.a.

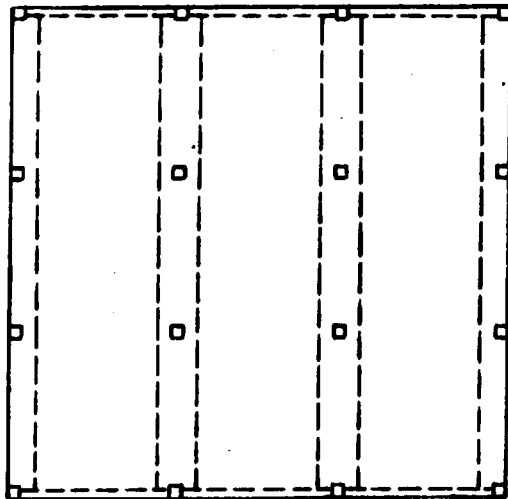
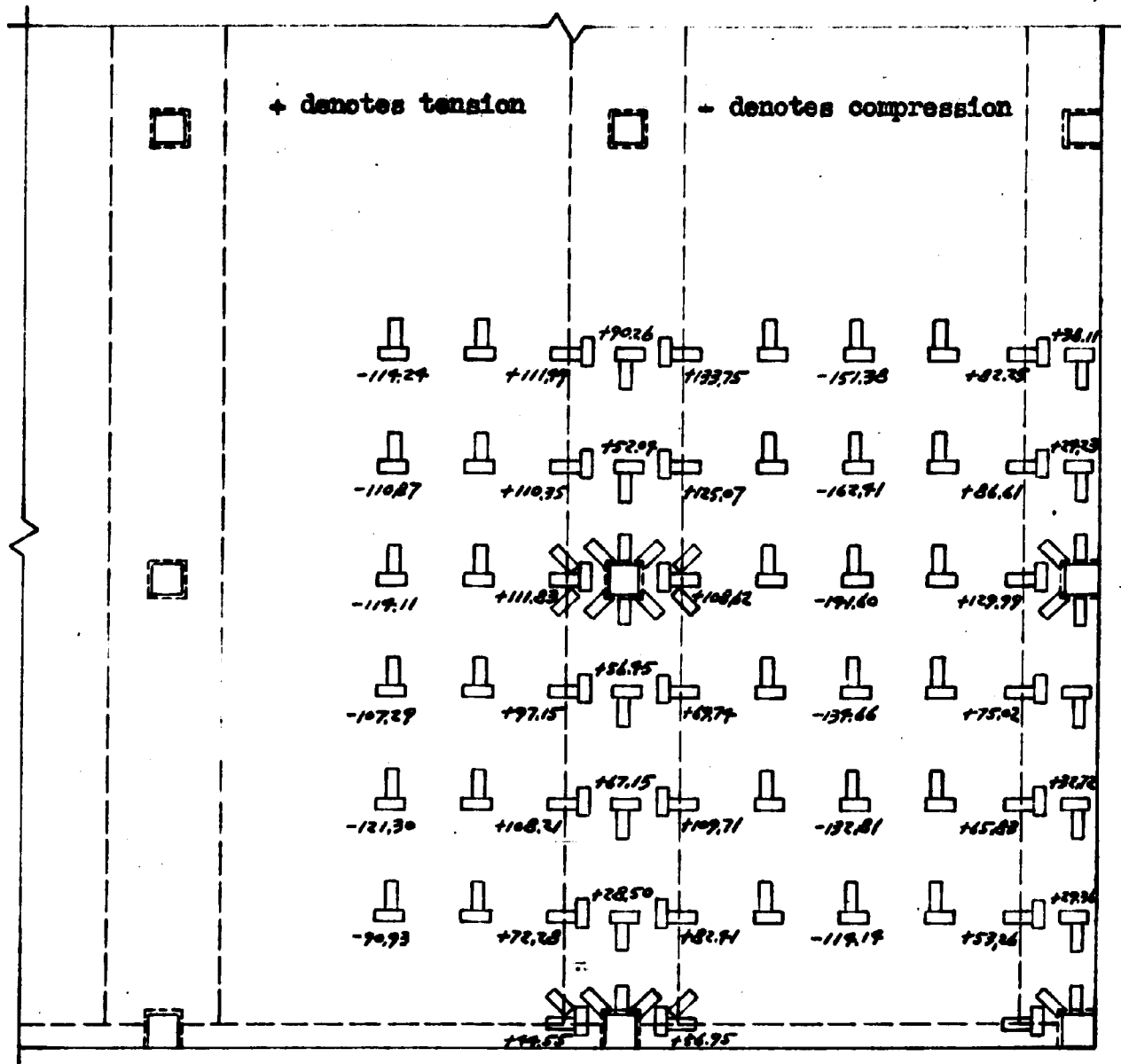
- b. Same as section 7.3.4.C.b.
- c. Same as section 7.3.4.C.c.
- d. Same as section 7.3.4.C.d.
- e. Same as section 7.3.4.C.e.
- f. Same as section 7.3.5.A.f.

#### 7.4. Maximum and Minimum Stresses

The maximum stress at any point is here defined as the largest extreme fiber stress at that point for all load conditions. The minimum stress at the point is either the smallest extreme fiber stress of same sign or the largest extreme fiber stress of opposite sign. The minimum stress is considered only when its sign is opposite to that of the maximum stress.

In the design of slabs and slab-bands it is necessary to determine the maximum bending moments for all critical sections. In order to determine these maximum moments from model tests it is necessary to first determine maximum stresses. In section 7.5. formulas will be derived for the relationship between bending moment coefficients and measured stresses for the cases under consideration in this thesis.

In the diagram 7.10 the maximum stresses from diagrams 7.01, 7.02 and 7.03 at the critical sections in the slab are listed. Similarly, maximum stresses from diagrams 7.07, 7.08 and 7.09 at the critical sections in the slab-bands are listed in diagram 7.11. Those values will be used in computing moment coefficients in section 7.6.



KEY

Loaded Areas Are Shaded

Diagram 7.10.

Maximum Stresses in the Top Fibers of the  
Slab in the Direction of the Slab Span  
(Stresses are in lbs. per sq. in.)  
Loading Conditions Nos. 1, 2 & 3

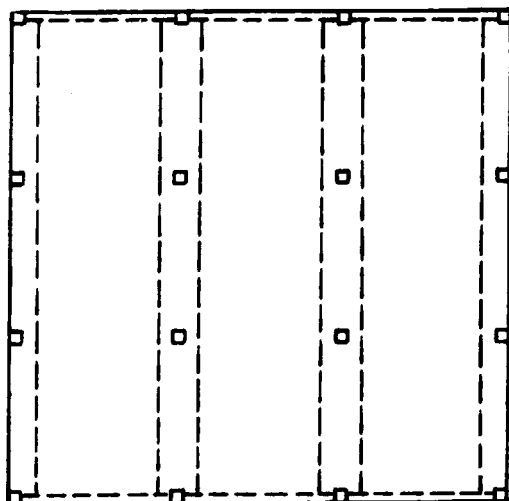


Diagram 7.11.

**Maximum Stresses in the Top Fibers of  
the Slab-Bands in the Direction  
of the Slab-Band Span**

(Stresses are in lbs. per sq. in.)

**Loading Conditions Nos. 4, 5 & 6**

**KEY**

## 7.5. The Relationship between Bending Moment Coefficient and Fiber Stress

It is the intention of this section to find the bending moment coefficient  $C$  in terms of the fiber stress  $f$  calculated from testing results.

### 7.5.1. Notations

|              |                                                                                                             |
|--------------|-------------------------------------------------------------------------------------------------------------|
| subscript m: | Anything pertaining to the model                                                                            |
| M:           | Bending moment at a certain section of a member under consideration                                         |
| C:           | Bending moment coefficient                                                                                  |
| w:           | Loads applied during tests in lbs. per sq. in. for slab<br>or in lbs. per in. of span length for slab-bands |
| L:           | Span length of the member considered                                                                        |
| f:           | Extreme fiber stress at the same section as M                                                               |
| I:           | Moment of inertia at the section considered                                                                 |
| c:           | Distance from the neutral axis to the extreme fiber stress at the same section as M                         |

### 7.5.2. General Formula for C

The equation for bending moment can be expressed in terms of  $wL^2$  as

$$B. M. = CwL^2 \quad (7.15)$$

and the equation for resisting moment can be expressed in terms of  $f$ ,  $I$  and  $c$  as thus:

$$R. M. = f \frac{I}{c} \quad (7.16)$$

These two equations must be equal at the same section of the member under consideration

$$CwL^2 = f \frac{I}{c}$$

or  $C = \frac{1}{wL^2} \times \frac{I}{c} \times f$  (7.17)

## 7.5.3. C for Slab

$$w = \frac{40}{12.8^2} \text{ \#/in.}^2$$

$L$  = Clear span of slab, i. e., the distance between edges of two adjacent slab-bands

$$= 9.6''$$

$$\frac{I}{c} = 0.015$$

Substituting these values in equation (7.17), we get

$$C = \frac{12.8^2}{40 \times 9.6^2} \times 0.015 \times f_m$$

$$= 0.000667 f_m \quad (7.18)$$

## 7.5.4. C for the Interior Slab-Band

$$w = \frac{40}{12.8} \text{ \#/in.}$$

$$L = 12.8''$$

$$\frac{I}{c} = \frac{bd^2}{6} = \frac{3.2 \times 0.733^2}{6}$$

Substituting these values in (7.17), we get

$$C = \frac{12.8}{40 \times 12.8^2} \times \frac{3.2 \times 0.733^2}{6} \times f_m$$

$$= 0.00056 f_m \quad (7.19)$$

## 7.5.5. C for the Exterior Slab-Band

$$w = \frac{40}{2 \times 12.8} \text{ \#/in.}$$

$$L = 12.8''$$

$$\frac{I}{c} = \frac{bd^2}{6} = \frac{1}{6} \times \frac{32}{15} \times 0.733^2$$

Substituting these values in (7.17), we get

$$\begin{aligned} C &= \frac{12.8}{20 \times 12.8^2} \times \frac{32}{90} \times 0.733^2 \times f_m \\ &= 0.000746 f_m \end{aligned} \quad (7.20)$$

## 7.6. The Bending Moment Coefficients C for Slab and Slab-Bands

Formulas have been derived in section 7.5. for the bending moment coefficients C in terms of extreme fiber stress f determined by model tests. C-values are calculated according to maximum values of f which can be found on diagrams 7.10 and 7.11. Diagrams 7.12 and 7.13 show C-values for slab and slab-bands, respectively, and these C-values can also be found in tabular form in tables 10.01 and 10.03. Reference is made to sections 10.4.1. and 10.4.2. for definitions of typical panels and panel strips.

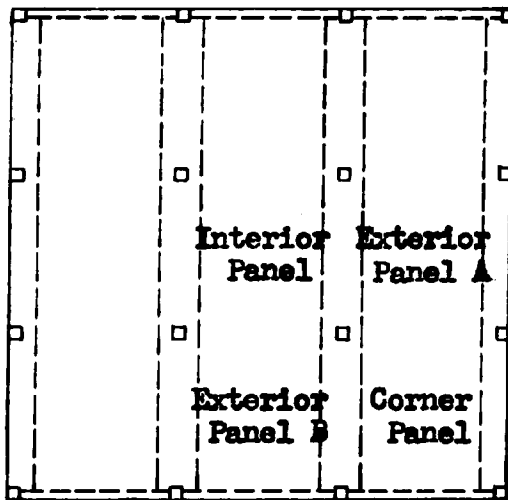
## 7.7. Stress Contours

The stress contours are useful in visualizing the stress distribution in a floor system and in determining the points of bend and the ends of reinforcing steel when the floor system is made of reinforced concrete.

### 7.7.1. General

It was impracticable to install strain gages all over the tested floor of the model not only because of the cost of the gages but also because of the limited space between gages for the lead wires. Sixty-

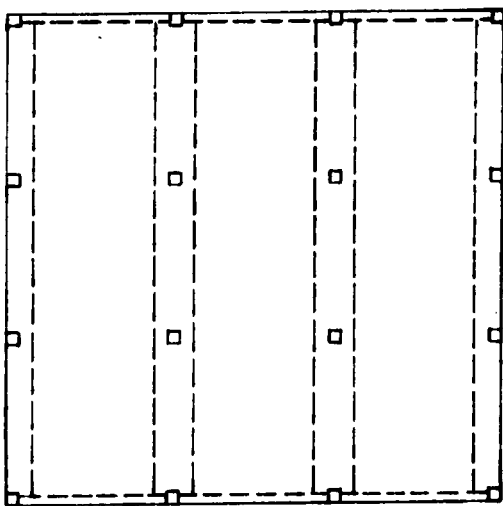
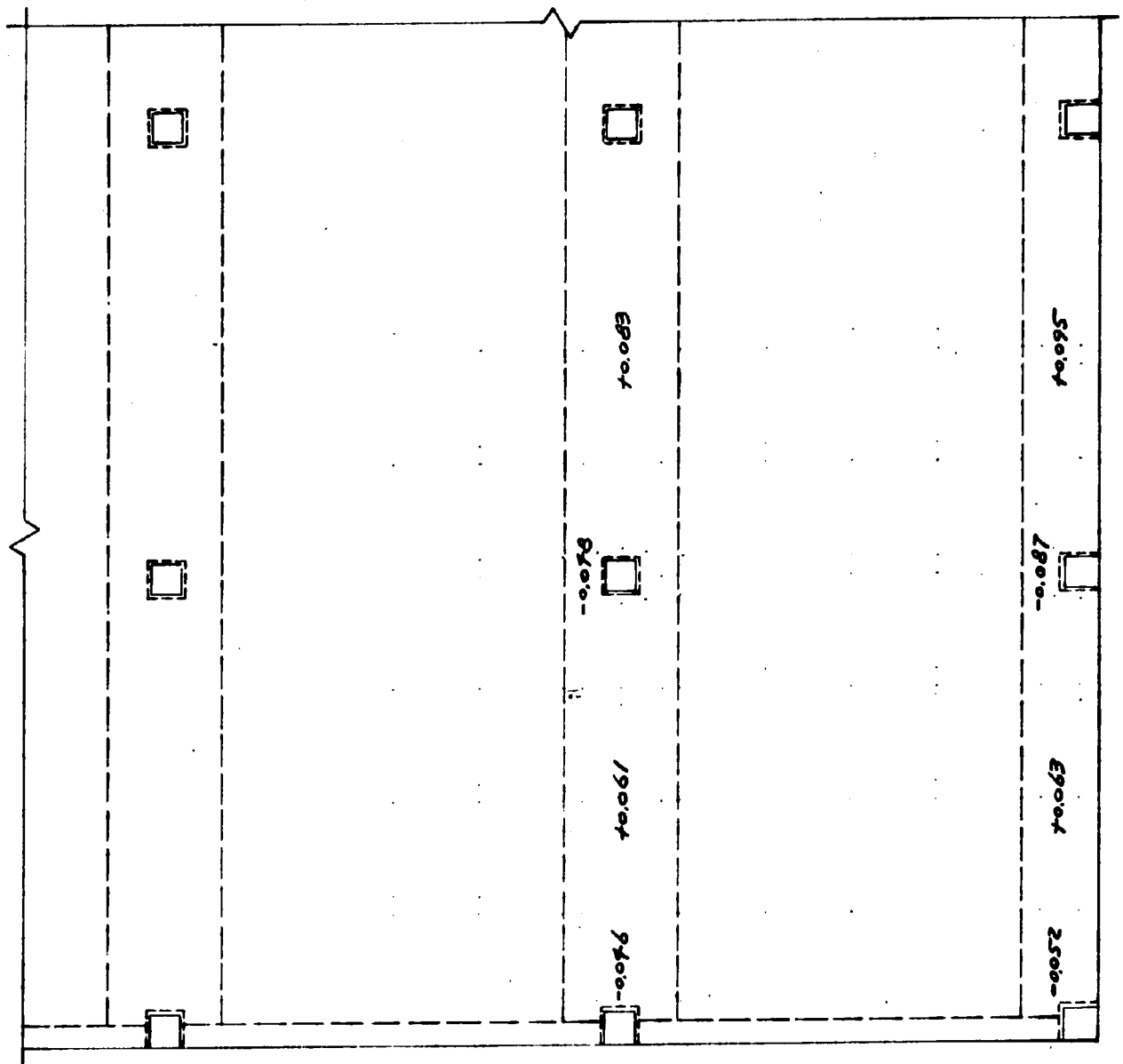
|                            |        |        |        |        |        |
|----------------------------|--------|--------|--------|--------|--------|
| $\frac{1}{2}$ Column Strip | +0.075 | -0.079 | -0.078 | +0.102 | -0.073 |
| Middle Strip               | +0.075 | -0.079 | -0.087 | +0.105 | -0.057 |
| $\frac{1}{2}$ Column Strip | +0.075 | -0.079 | -0.078 | +0.102 | -0.073 |
| $\frac{1}{2}$ Column Strip | +0.077 | -0.073 | -0.073 | +0.092 | -0.069 |
| Middle Strip               | +0.077 | -0.067 | -0.069 | +0.086 | -0.074 |
| $\frac{1}{2}$ Column Strip | +0.061 | -0.079 | -0.053 | +0.076 | -0.036 |



KEY

Diagram 7.12.  
Moment Coefficients for Slab

$$M = C_m W L^2$$



KEY

Diagram 7.13.

Moment Coefficients for Slab-Bands

$$M = C_m w L^2$$

seven points on one quarter of the floor area were selected as the control points for measuring strains. The stresses found were recorded on diagrams such as on diagrams 7.01 through 7.09. From these diagrams one can visualize the general picture of the stress distribution on a certain part of the tested floor due to a certain load. The best way to show the stress distribution clearly is to draw the stress contours which are defined as lines of equal stresses, i. e., all the points along a stress contour have stresses of same magnitude. Such contours are shown on diagrams 7.14 to 7.19.

#### 7.7.2. Characteristics of Stress Contours

The stress contours have the following characteristics.

- A. All points along a stress contour have stresses of the same magnitude.
- B. Stress contours never cross each other.
- C. Every stress contour forms a closed line. If it does not close at the horizontal surface of the slab it continues and closes at the vertical surfaces at the edges of the floor considered.
- D. The closer the stress contours are spaced the more rapid the stresses change and vice versa.
- E. The area bounded by the innermost contour is the place of either the highest tension or the highest compression.

#### 7.7.3. To Locate Contour Lines

It is not correct or accurate to use straight-line interpolation to find the location of a certain stress between two points of known stresses because the moment diagram is a parabolic curve and therefore parabolic variation will be used to find the point required.

**Example:**

Refer to diagram 5.01 for the locations of points. For the stresses produced by the third loading condition as recorded on diagram 7.03,  $f_{111} = -114.2$ ,  $f_{112} = -73.0$  and  $f_{113} = +55.1$ . Stresses are in lbs. per sq. in. It is required to find the location of the 100-contour line. The best way to locate this contour line is by the following graphical method.

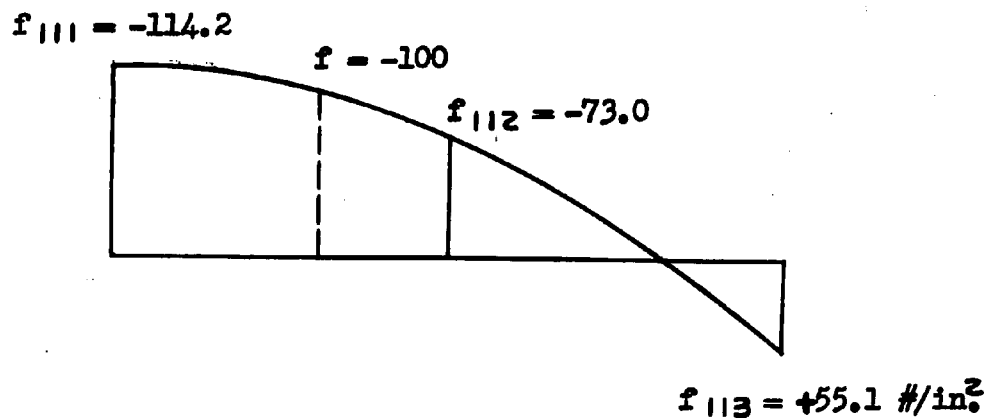


Figure 7.02.

To Locate Contour Line

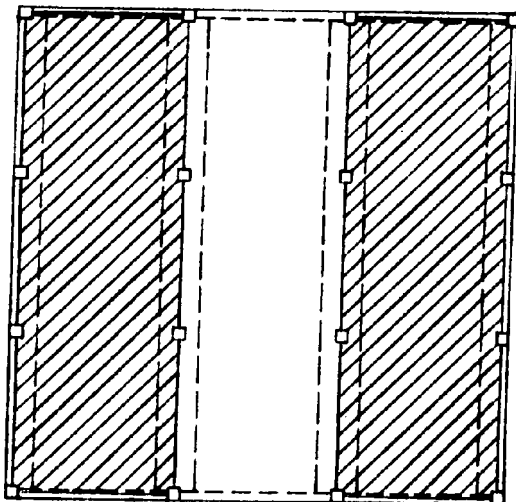
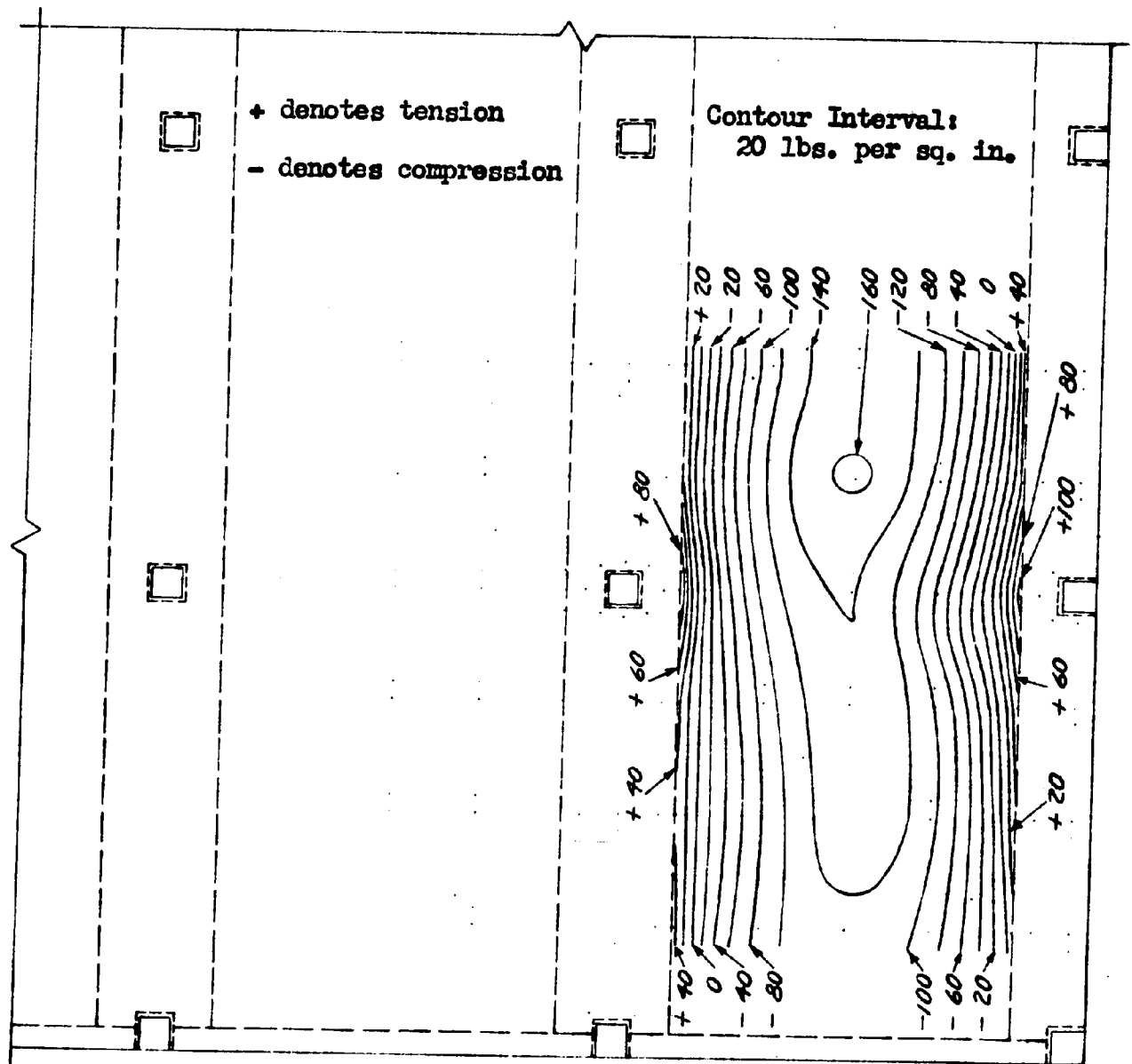
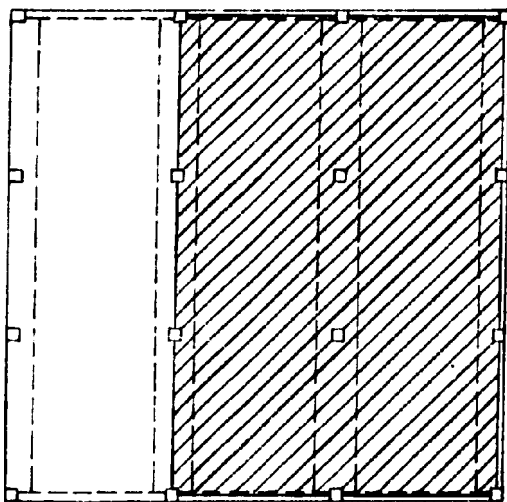
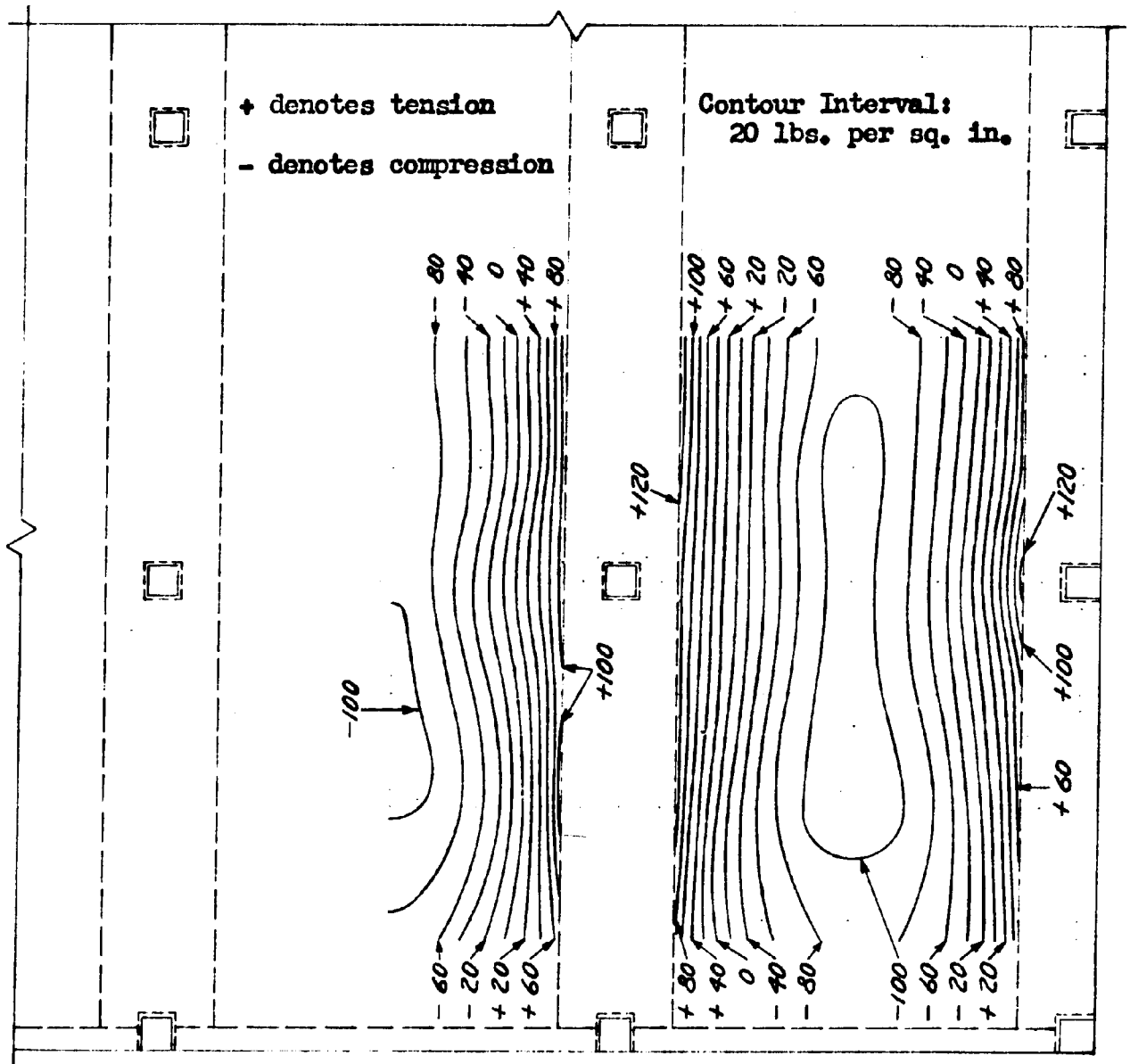


Diagram 7.14.

Stress Contours for the Slab

(Stresses are in lbs. per sq. in.)

Loading Condition No. 1



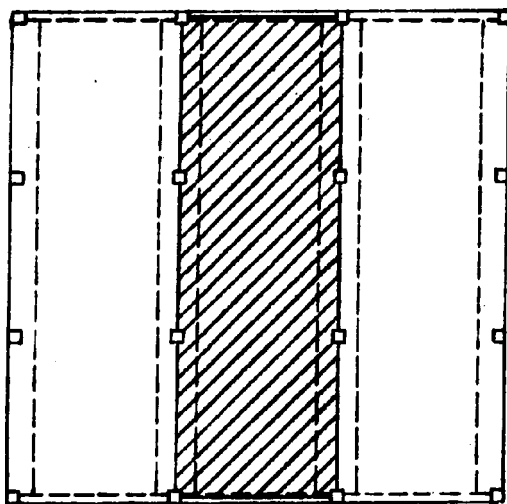
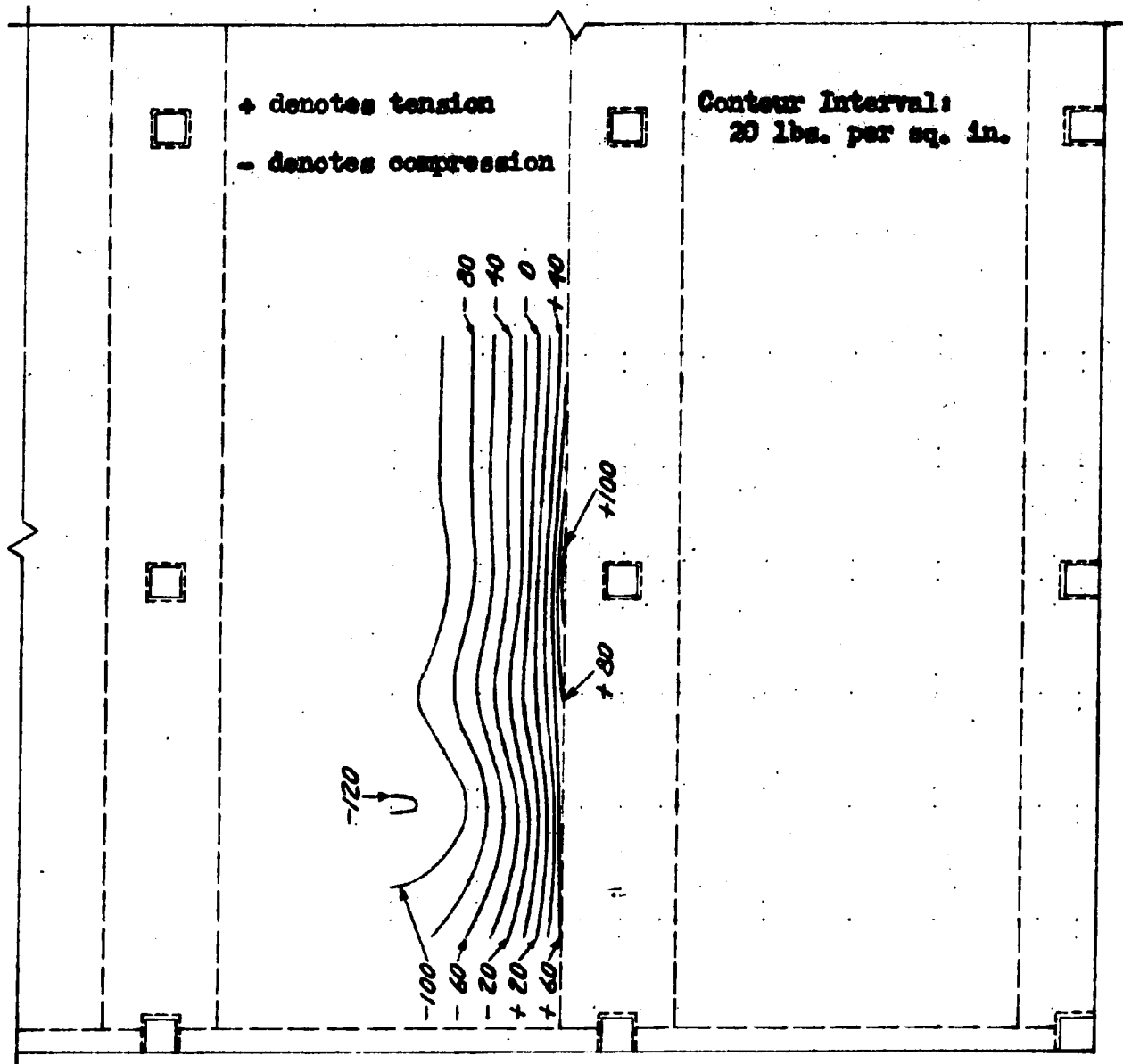
KEY

Loaded Areas Are Shaded

Diagram 7.15.

Stress Contours for the Slab  
(Stresses are in lbs. per sq. in.)

Loading Condition No. 2



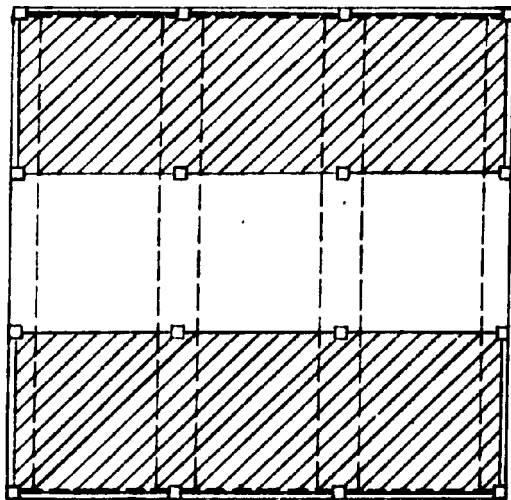
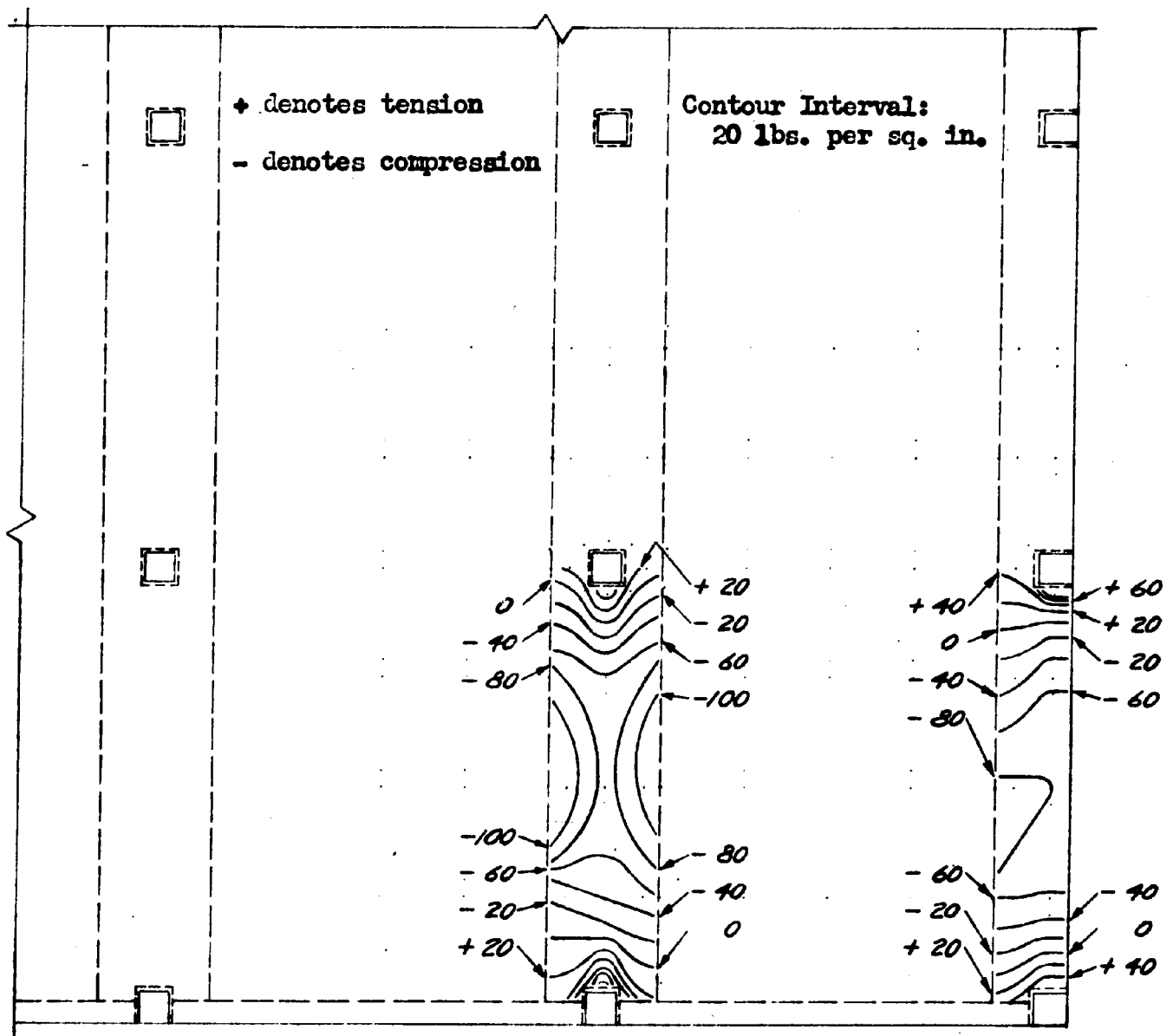
KEY

Loaded Areas Are Shaded

Diagram 7.16.

Stress Contours for the Slab  
(Stresses are in lbs. per sq. in.)

Loading Condition No. 3



KEY

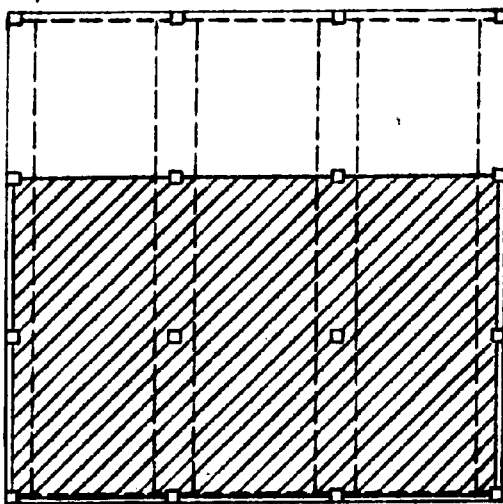
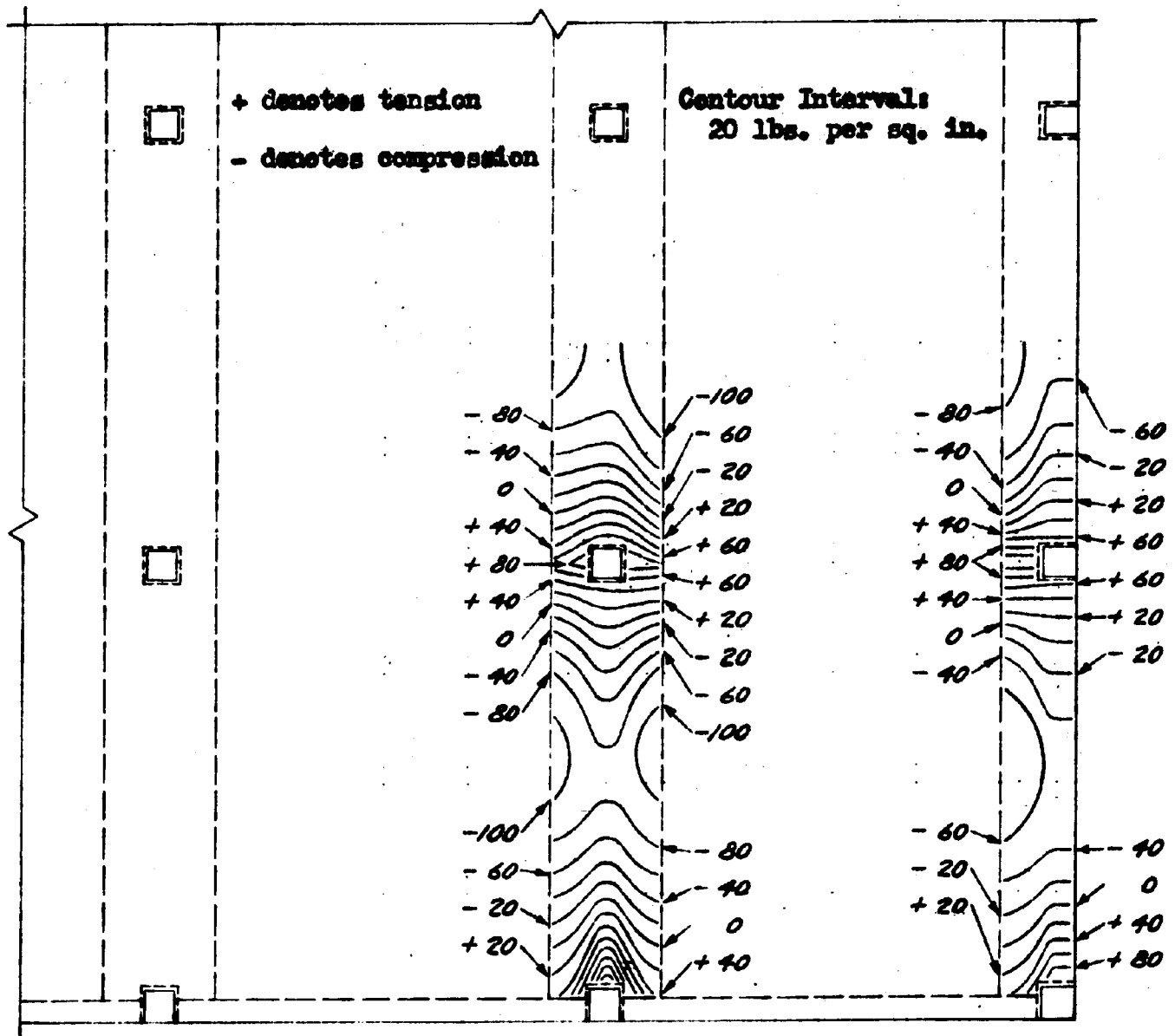
Loaded Areas Are Shaded

Diagram 7.17.

Stress Contours for the Slab-Bands

(Stresses are in lbs. per sq. in.)

Loading Condition No. 4



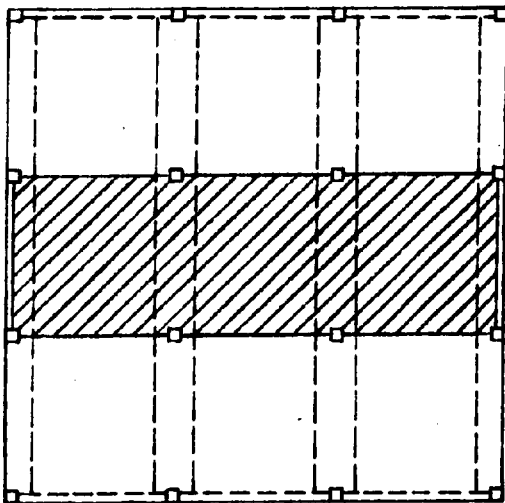
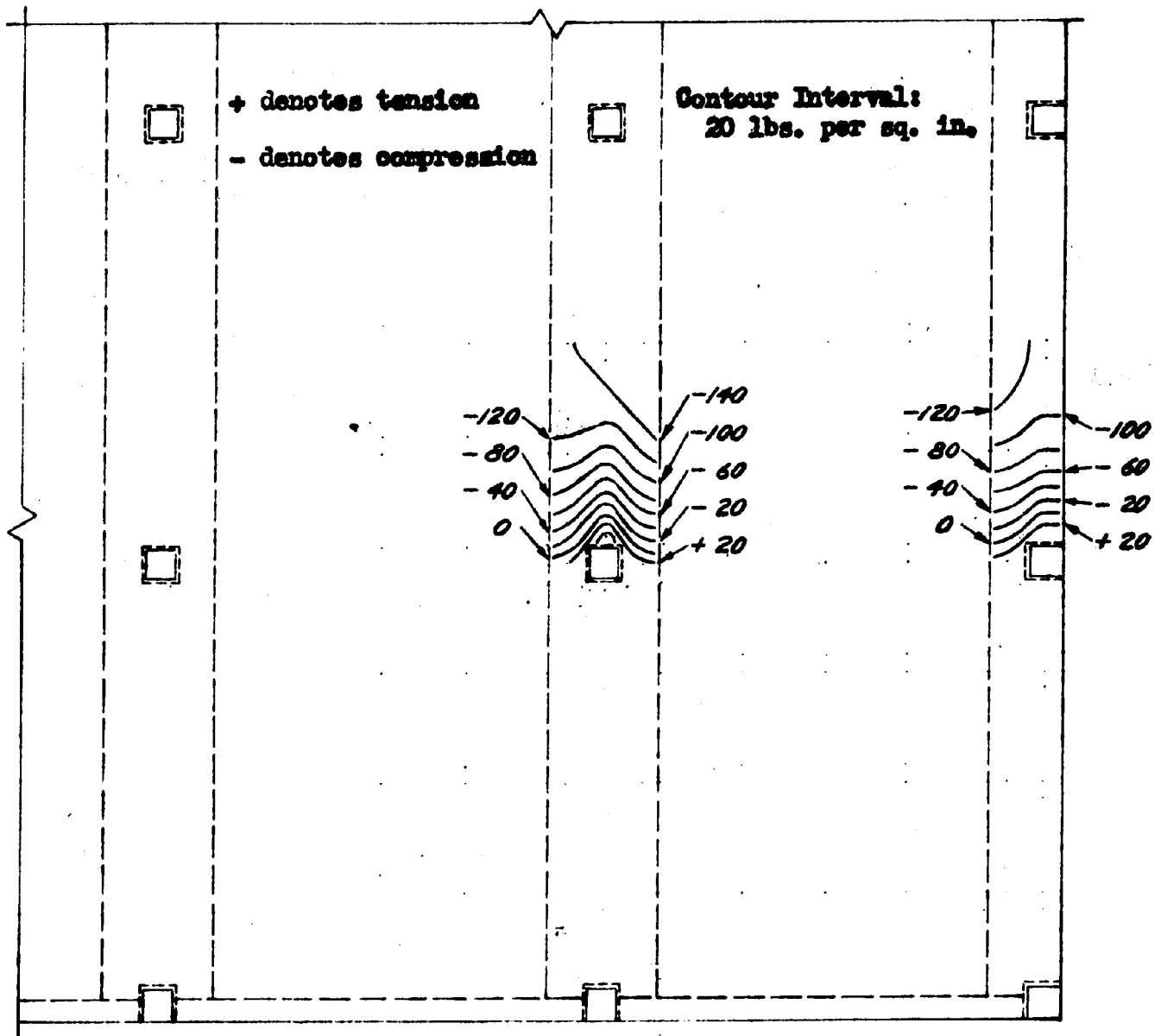
KEY

Loaded Areas Are Shaded

Diagram 7.18.

Stress Contours for the Slab-Bands  
(Stresses are in lbs. per sq. in.)

Loading Condition No. 5



KEY

Loaded Areas Are Shaded

Diagram 7.19.

Stress Contours for the Slab-Bands  
(Stresses are in lbs. per sq. in.)

Loading Condition No. 6

## CHAPTER 8

### THEORETICAL ANALYSIS OF THE BUILDING FRAME

#### 8.1. Method of Analysis

The prototype of the slab and slab-band floor system used in the model studies will be analyzed in this chapter. The analysis is made according to the ACI Code 1951 using the simplifying assumptions allowed by the code.

##### 8.1.1. Bents

In analyzing this building frame four bents are used in order to get the maximum stresses in the slab and the interior and the exterior slab-bands.

The first bent is in the direction of the slab span, and consists of a strip of slab and slab-bands of a full panel width normal to the slab span, and the columns one storey above and below the center line of this strip. The analysis of this bent gives stresses in the slab and slab-band in the direction of the slab span.

The second bent is in the direction of the slab-bands. It consists of a full interior slab-band, plus one-half the clear slab span to the adjacent slab-band on each side and the columns one

storey above and below the center line of this strip. The analysis of this bent gives the maximum stresses in the interior slab-band.

The third bent is in the direction of the slab-band at the exterior row of columns. It consists of a full exterior slab-band, half of the slab to the adjacent interior slab-band and a row of exterior columns extending one storey above and below. The purpose of analyzing this bent is for the stresses in the exterior slab-band.

The fourth bent consists of a series of spandrel beams with columns directly above and below them. The purpose of analyzing this bent is for the stresses in the spandrel beams.

#### 8.1.2. Assumptions

The following assumptions will be used in the analysis.

A. The remote ends of the columns directly above or below the floor under consideration are assumed to be fixed. This is in accordance with the ACI Code.

B. The approximate values of moments of inertia of all the structural members involved in the analysis of the frame should be known at the start of this analysis. In order to do this the approximate dimensions of the concrete cross-sections of the various members are estimated as accurately as possible and then the moments of inertia are calculated as follows:

a. Moments of inertia for slabs, slab-bands and spandrel beams are equal to  $\frac{bd^3}{12}$  where b is the width and d is the effective depth of any such member considered.

b. Moments of inertia for tied columns are equal to  $\frac{bd^3}{12}$  where b is one dimension of the column which is perpendicular to the direction

of the span considered and  $d$  is the other dimension of the column. It equals  $\frac{b^4}{12}$  when the column is square. Although the reinforcing steel is not included in the computation of moment of inertia in this way, its effect has been partly taken care of by these formulas. The flanges of the slab-bands are neglected in computing the moments of inertia.

C. The joints between slab-bands and columns are assumed to be rigid.

D. The change in length of slabs, slab-bands, spandrel beams and columns due to direct stress and deflections due to shear are neglected.

E. It is assumed that there is no settlement of the supporting columns.

### 8.1.3. Loadings

The vertical loads acting on the structure consist of dead and live loads. The dead loads are the weights of the structural members, the floor covering material and the ceiling and the equivalent uniform load due to partition walls. The average value of the live load for structures such as office, school, apartment and hospital buildings is taken as 80 lbs. per sq. ft. of floor surface. The maximum moments and shears will occur at certain sections of the building frame under investigation according to the loading conditions as described below.

A. The maximum positive moment occurs near the center of a span when this span and the alternate spans are loaded.

B. The maximum negative moment occurs at the face of any support when the two adjacent spans are loaded in the case of the

section being at interior support, and when the adjacent and the alternate spans are loaded in the case of the section being at exterior support.

C. The maximum shear occurs at the face of the support when the same spans are loaded as in the case described in the last paragraph for the maximum negative moment at the same place.

## 8.2. Slope-Deflection Equations for The Slab And The Slab-Bands

The slab is supported by slab-bands which are wide and shallow so that when the slab is analyzed, the slab-bands may be considered as part of the slab. In this way the slab is thickened at its ends in each span and the slab is assumed to be supported along the center lines of the interior slab-bands and along the line joining the centers of the exterior columns along the exterior slab-bands. In the following sections typical slope-deflection equations, in which several constants will be needed in the moment-distribution method for the slab analysis, will be derived for one slab span.

### 8.2.1. The General Slope-Deflection Equations

If a and b are two ends of this slab then the end moments caused by certain loads on this slab can be expressed in the following general equations.

$$M_{ab} = \frac{EI_0}{L} \left[ C_1 \theta_a + C_2 \theta_b \pm (C_1 + C_2) \frac{\Delta}{L} \right] \pm M_{Fab} \quad (8.01)$$

$$M_{ba} = \frac{EI_0}{L} \left[ C_2 \theta_a + C_3 \theta_b \pm (C_2 + C_3) \frac{\Delta}{L} \right] \pm M_{Fba} \quad (8.02)$$

If the end a of the slab is hinged then the moment at the end b is expressed in the following form.

$$M_{ba} = \frac{EI_0}{L} \left[ \left( C_3 - \frac{C_2^2}{C_1} \right) \left( \theta_b \pm \frac{\Delta}{L} \right) \right] + M_{Fba} - \frac{C_2}{C_1} M_{Fab} \quad (8.03)$$

in which  $E$  = Modulus of elasticity of the material of the member considered.

$I_0$  = Moment of inertia of the slab section.

$I$  of the slab-band portion can be expressed in terms of  $I_0$ .

$L$  = Span of the slab. It is the distance between center lines of the supporting columns along the slab-bands.

$$\frac{EI_0}{L} = K_0$$

$C_1$ ,  $C_2$  and  $C_3$  = Constants for the member considered.

$\theta_a$  = Angle of rotation of the tangent at end a.

$\theta_b$  = Angle of rotation of the tangent at end b.

$\Delta$  = Relative displacement of the ends a and b.

$M_{Fab}$  = Fixed-end moment at end a.

$M_{Fba}$  = Fixed-end moment at end b.

### 8.2.2. The Values of $C_1$ , $C_2$ and $C_3$

When  $\beta$ 's have the following meanings

$\beta_{a1}$  and  $\beta_{b1}$  = Angles of rotation at ends a and b respectively due to unit moment at end a, with end b pinned

$\beta_{a2}$  and  $\beta_{b2}$  = Angles of rotation at ends a and b respectively due to unit moment at end b, with end a pinned

then the end moments can be expressed as follows:

$$M_{ab} = \frac{\beta_{b2}}{\Delta} \theta_a + \frac{\beta_{b1}}{\Delta} \theta_b \quad (8.04)*$$

$$M_{ba} = \frac{\beta_{b1}}{\Delta} \theta_a + \frac{\beta_{a1}}{\Delta} \theta_b \quad (8.05)*$$

in which  $\Delta = \beta_{a1}\beta_{b2} - \beta_{b1}^2$

Using  $K_0 = \frac{EI_0}{L}$  and comparing equations (8.01) with (8.04) and (8.02) with (8.05), we get

$$C_1 K_0 = \frac{\beta_{b2}}{\Delta} \quad C_2 K_0 = \frac{\beta_{b1}}{\Delta} \quad C_3 K_0 = \frac{\beta_{a1}}{\Delta}$$

In Figure 8.01 the dimensions of the slab are shown. The end rotations  $\beta_{a1}$  and  $\beta_{b1}$  due to a unit moment at end a are found by the conjugate beam method as follows. Assume a conjugate beam of the same span, simply supported and loaded with the  $\frac{M}{EI}$  diagram caused by a moment of 16 at end a. The reactions of the conjugate beam divided by 16 are then equal to the rotations  $\beta_{a1}$  and  $\beta_{b1}$  respectively for the real beam.

A. For Unit Moment at Left End

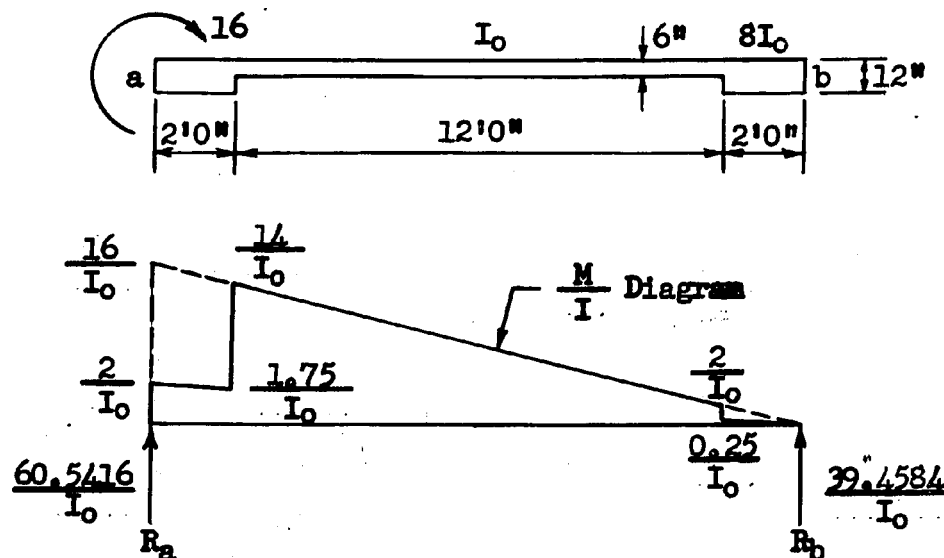


Figure 8.01.

Rotation of End Tangents due to  $M = 16$  at Left End

$$\begin{aligned}
 16R_a &= \left[ \frac{1.75}{I_o} \times 2 \times 15 \right] + \left[ \frac{0.25}{I_o} \times 2 \times \frac{1}{2} \times (14 + \frac{2}{3} \times 2) \right] + \left[ \frac{2}{I_o} \times 12 \times 8 \right] \\
 &\quad + \left[ \frac{1}{2} \times \frac{12}{I_o} \times 12 \times (2 + \frac{2}{3} \times 12) \right] + \left[ \frac{1}{2} \times \frac{0.25}{I_o} \times 2 \times (\frac{2}{3} \times 2) \right] \\
 &= \frac{52.5}{I_o} + \frac{3.833}{I_o} + \frac{192}{I_o} + \frac{720}{I_o} + \frac{0.333}{I_o} \\
 &= \frac{968.666}{I_o}
 \end{aligned}$$

$$R_a = \frac{60.5416}{I_o}$$

$$\begin{aligned}
 16R_b &= \left[ \frac{1.75}{I_o} \times 2 \times 1 \right] + \left[ \frac{1}{2} \times \frac{0.25}{I_o} \times 2 \times (\frac{1}{3} \times 2) \right] + \left[ \frac{2}{I_o} \times 12 \times 8 \right] \\
 &\quad + \left[ \frac{1}{2} \times \frac{12}{I_o} \times 12 \times (2 + 12 \times \frac{1}{3}) \right] + \left[ \frac{1}{2} \times \frac{0.25}{I_o} \times 2 \times (14 + \frac{1}{3} \times 2) \right] \\
 &= \frac{3.5}{I_o} + \frac{0.167}{I_o} + \frac{192}{I_o} + \frac{432}{I_o} + \frac{3.667}{I_o} \\
 &= \frac{631.334}{I_o}
 \end{aligned}$$

$$R_b = \frac{39.4584}{I_o}$$

$$\text{Check: } \frac{60.5416}{I_o} + \frac{39.4584}{I_o} = \frac{100}{I_o}$$

$$\begin{aligned}
 &(\frac{1.75}{I_o} \times 2) + (\frac{1}{2} \times \frac{0.25}{I_o} \times 2) + (\frac{2}{I_o} \times 12) + (\frac{1}{2} \times \frac{12}{I_o} \times 12) + (\frac{1}{2} \times \frac{0.25}{I_o} \times 2) \\
 &= \frac{3.5}{I_o} + \frac{0.25}{I_o} + \frac{24}{I_o} + \frac{72}{I_o} + \frac{0.25}{I_o} \\
 &= \frac{100}{I_o}
 \end{aligned}$$

For a unit moment at a and for any span L

$$\theta_{a1} = \frac{60.5416}{I_o} \times \frac{1}{16} \times \frac{L}{16} = \frac{0.236491L}{I_o}$$

$$\theta_{b1} = \frac{39.4584}{I_o} \times \frac{1}{16} \times \frac{L}{16} = \frac{0.154134L}{I_o}$$

## B. For Unit Moment at Right End

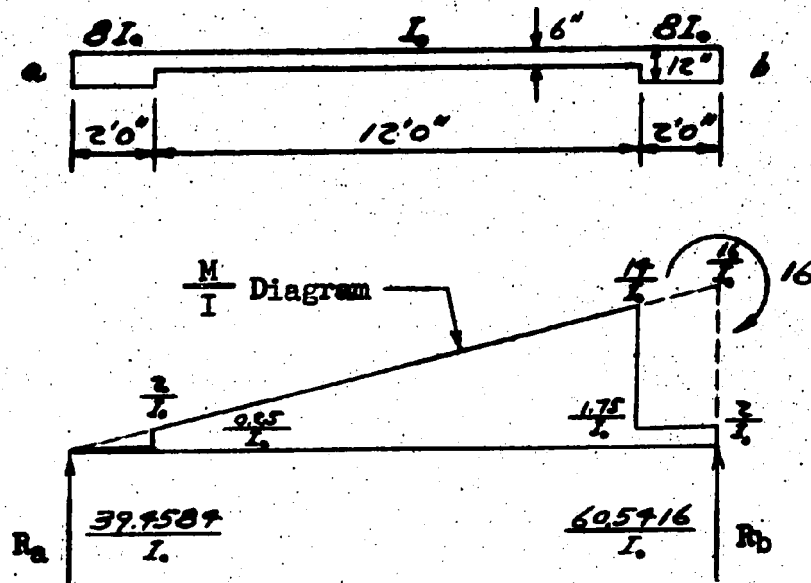


Figure 8.02.

Rotation of End Tangents due to  $M = 16$  at Right End

Using the same method as above, we get

$$\beta_{a2} = \beta_{b1} = \frac{0.154134L}{I_0}$$

$$\beta_{b2} = \beta_{a1} = \frac{0.236491L}{I_0}$$

C. C - Values

$$\begin{aligned} A &= \beta_{a1}\beta_{b2} - \beta_{b1}^2 = \left(\frac{0.236491L}{I_0}\right)^2 - \left(\frac{0.154134L}{I_0}\right)^2 \\ &= \frac{0.0559268L^2}{I_0^2} - \frac{0.0237574L^2}{I_0^2} \\ &= \frac{0.0321694L^2}{I_0^2} \end{aligned}$$

Substituting the values calculated above into equations (8.04) and (8.05), we get

$$M_{ab} = \frac{\frac{0.236491L}{I_o}}{\frac{0.0321694L^2}{I_o^2}} \theta_a + \frac{\frac{0.154134L}{I_o}}{\frac{0.0321694L^2}{I_o^2}} \theta_b$$

$$= \frac{EI_o}{L} (7.35 \theta_a + 4.79 \theta_b)$$

$$M_{ba} = \frac{\frac{0.154134L}{I_o}}{\frac{0.0321694L^2}{I_o^2}} \theta_a + \frac{\frac{0.236491L}{I_o}}{\frac{0.0321694L^2}{I_o^2}} \theta_b$$

$$= \frac{EI_o}{L} (4.79 \theta_a + 7.35 \theta_b)$$

Therefore the values of C's are

$$C_1 = 7.35$$

$$C_2 = 4.79$$

$$C_3 = 7.35$$

### 8.2.3. The Fixed-End Moments

When the  $\alpha$ 's are defined as the angles of rotation at ends a and b respectively due to actual loads on the slab then the fixed-end moments can be expressed as follows.

$$M_{Fab} = C_1 K_o (-\alpha_a) + C_2 K_o (-\alpha_b) \quad (806)^*$$

$$M_{Fba} = C_2 K_o (-\alpha_a) + C_3 K_o (-\alpha_b) \quad (807)^*$$

The method of elastic weights will be again used in order to find the values of  $\alpha$ 's. In Figure 8.03 both the M-diagram and the  $\frac{M}{I}$  diagram are shown.

\* L. C. Maugh, "Statically Indeterminate Structures," page 135.

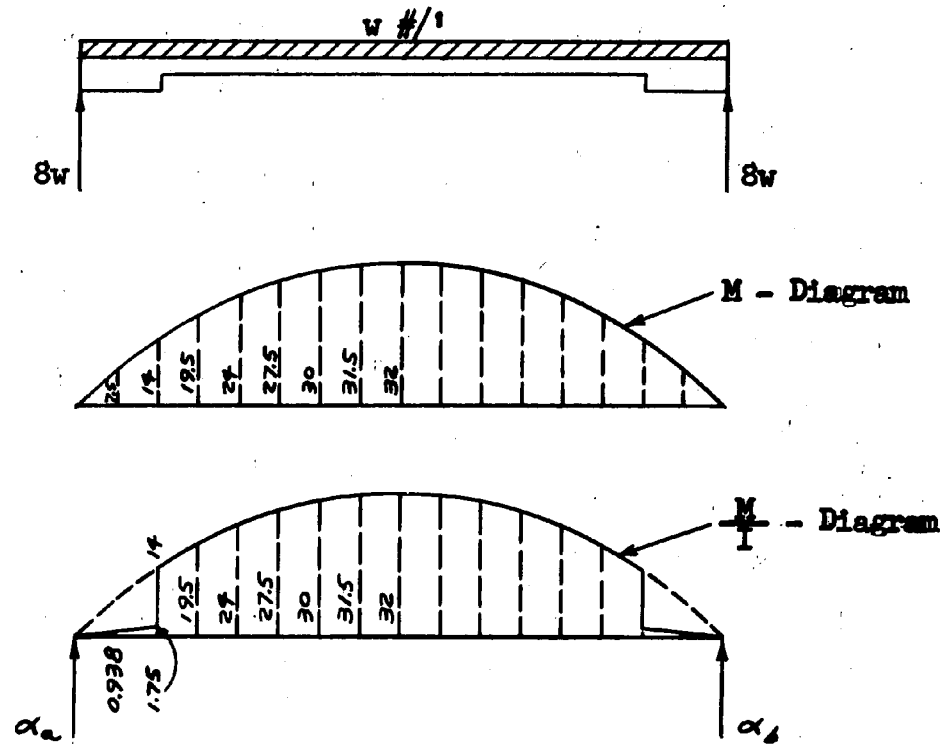


Figure 8.03.

Rotation of End Tangents due to Actual Load on the Slab

$$\begin{aligned}
 \alpha_a &= \left\{ \frac{2}{3} \times 32 \times 16 - 2 \left[ \left( \frac{1}{2} \times 6.562 \times 1 \right) + \left( \frac{6.562 + 12.25}{2} \times 1 \right) \right] \right\} \times \frac{8}{16} \times \frac{W}{EI_0} \\
 &= \{ 341.333 - 25.374 \} \times \frac{1}{2} \times \frac{W}{EI_0} \\
 &= 157.98 \frac{W}{EI_0}
 \end{aligned}$$

$$\alpha_b = -157.98 \frac{W}{EI_0}$$

Substituting the values of  $C_1$ ,  $C_2$ ,  $C_3$ ,  $K_0$ ,  $\alpha_a$ , and  $\alpha_b$  in equations 8.06 and 8.07 the fixed end moments are:

$$\begin{aligned}
 M_{Fab} &= C_1 K_0 (-\alpha_a) + C_2 K_0 (-\alpha_b) \\
 &= \frac{EI_0}{L} \left[ 7.35 \times \left( -157.98 \frac{W}{EI_0} \right) + 4.79 \times 157.98 \frac{W}{EI_0} \right] \\
 &= \left[ -1161 + 757 \right] \frac{W}{L}
 \end{aligned}$$

$$= - 404 \frac{W}{16}$$

$$= - 25.25 W$$

$$M_{Fba} = C_2 K_o (-\alpha_a) + C_3 K_o (-\alpha_b)$$

$$= \frac{EI_o}{L} \left[ 4.79 \times (-157.98 \frac{W}{EI_o}) + 7.35 \times 157.98 \frac{W}{EI_o} \right]$$

$$= [-757 + 1161] \times \frac{W}{16}$$

$$= + 25.25 W$$

#### 8.2.4. The Complete Slope-Deflection Equations

With the values of C's and M's known the equations (8.01), (8.02) and (8.03) can be rewritten as follows:

A. For the member with both ends restrained

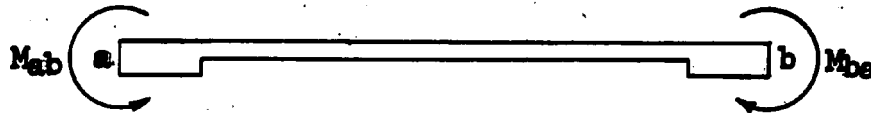


Figure 8.04.

Member with Both Ends Restrained

$$M_{ab} = \frac{EI_o}{L} \left[ 7.35 \theta_a + 4.79 \theta_b + 12.14 \frac{\Delta}{L} \right] - 25.25 W \quad (8.08)$$

$$M_{ba} = \frac{EI_o}{L} \left[ 4.79 \theta_a + 7.35 \theta_b + 12.14 \frac{\Delta}{L} \right] + 25.25 W \quad (8.09)$$

$$\begin{aligned} \text{Carry-over factor} &= \frac{C_2}{C_1} = \frac{C_2}{C_3} = \frac{4.79}{7.35} \\ &= 0.652 \end{aligned}$$

B. For the member with one end hinged

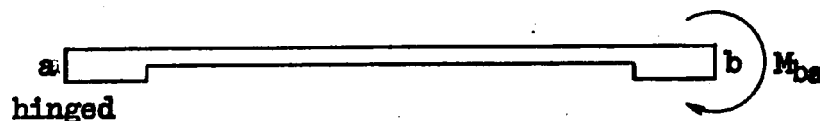


Figure 8.05.

Member with One End Restrained



### 8.3.1. Compression Combined with Bending Moment

In Figure 8.06 a reinforced concrete member under the influence of the external axial load and the bending moment and with the stresses developed in concrete and steel because of the loads is diagrammatically shown. All the symbols on the diagram except  $f_t$  are conventional and they do not require definition. The symbol  $f_t$  designates the extreme fiber stress at the tension side of the concrete member for the uncracked section.

The total compressive stress  $C_c$  developed in the concrete and the total tensile stress  $T_s$  developed in the steel due to external force and bending moment can be expressed in the following formulas.

$$C_c = \frac{1}{2} f_c b k d \quad (8.11)$$

$$T_s = A_s f_s \quad (8.12)$$

because  $A_s = p b d$

$$\text{and } \frac{f_s}{n f_c} = \frac{d - k d}{k d} \quad \text{or } f_s = n f_c \frac{1 - k}{k} \quad (8.13)$$

$$\text{therefore } T_s = p b d n f_c \frac{1 - k}{k} \quad (8.14)$$

#### A. Fiber Stresses

Referring to Figure 8.06 this member is balanced by three forces,  $P$ ,  $T_s$  and  $C_c$ . The weight of the member is included in the term  $P$  when the member is vertical and it need not be considered when the member is horizontal. The equilibrium condition  $\sum V = 0$  gives:

$$P + T_s = C_c$$

$$\text{or } P = C_c - T_s \quad (8.15)$$

Substituting equations (8.11) and (8.14) in equation (8.15), we get

$$P = \frac{1}{2}f_c b k d - p b d n f_c \frac{1 - k}{k} \quad (8.16)$$

$$P k = f_c b d \left[ \frac{1}{2}k^2 - p n (1 - k) \right]$$

$$f_c = \frac{P k}{b d \left[ \frac{1}{2}k^2 - p n (1 - k) \right]} \quad (8.17)$$

Substituting equation (8.17) in equation (8.13), we get

$$f_s = \frac{n P (1 - k)}{b d \left[ \frac{1}{2}k^2 - p n (1 - k) \right]} \quad (8.18)$$

Furthermore  $\frac{f_t}{f_c} = \frac{D - k d}{k d}$

$$f_t = f_c \frac{D - k d}{k d} \quad (8.19)$$

Substituting equation (8.17) in equation (8.19), we get

$$f_t = \frac{P (D - k d)}{b d^2 \left[ \frac{1}{2}k^2 - p n (1 - k) \right]} \quad (8.20)$$

#### B. Neutral Axis

If the axial load  $P$  is moved away from the center of the member to a new location with an eccentricity of  $e = \frac{M}{P}$  then this eccentrically applied load  $P$  will produce the same effect on the member as the original combined axial load and bending moment do. In this way  $M$  equals  $Pe$ .

Because the force system acting on this member is in equilibrium, the summation of moments about the center of the member must be equal zero. Referring to Figure 8.06 the following equation is established.

$$M - C_c \times \text{Moment arm} - T_s \times \text{Moment arm} = 0$$

$$\text{or} \quad M = C_c \times \text{moment arm} + T_s \times \text{moment arm} \quad (8.21)$$

in which  $M = Pe = \text{External bending moment applied to the member}$

$$\begin{aligned} C_c \times \text{moment arm} &= \text{Moment of total compressive force about center} \\ &= \frac{1}{2}f_c b k d \times \left( \frac{D}{2} - \frac{k d}{3} \right) \end{aligned}$$

$T_s \times \text{moment arm} = \text{Moment of total tensile force in steel}$   
 about **center of the member**

$$= pbdnf_c \frac{1-k}{k} \times (d - \frac{D}{2})$$

Equation (8.21) can be rewritten as follows:

$$Pe = \frac{1}{2}f_c bkd \times (\frac{D}{2} - \frac{kd}{3}) + pbdnf_c \frac{1-k}{k} \times (d - \frac{D}{2}) \quad (8.22)$$

Substituting equation (8.16) in equation (8.22), we get

$$\begin{aligned} & \left[ \frac{1}{2}f_c bkd - pbdnf_c \frac{1-k}{k} \right] e \\ &= \frac{1}{2}f_c bkd \times (\frac{D}{2} - \frac{kd}{3}) + pbdnf_c \frac{1-k}{k} \times (d - \frac{D}{2}) \end{aligned}$$

The final simplified form of this equation is

$$\begin{aligned} dk^3 + (3e - 1.5D) k^2 + 6pn (e + d - \frac{D}{2})k \\ - 6pn (e + d - \frac{D}{2}) = 0 \end{aligned} \quad (8.23)$$

$$\begin{aligned} \text{or} \quad k^3 + \frac{3e - 1.5D}{d} k^2 + \frac{6pn}{d} (e + d - \frac{D}{2})k \\ - \frac{6pn}{d} (e + d - \frac{D}{2}) = 0 \end{aligned} \quad (8.24)$$

### 8.3.2. Tension Combined with Bending Moment

In deriving the equations for the fiber stresses and  $k$ -value for a reinforced concrete member due to axial tension combined with bending moment the same line of reasoning will be used here as that in the last section. The summation of all the vertical forces must be equal to zero because of the equilibrium of the force system.

$$P + C_c = T_s$$

$$\text{or} \quad P = T_s - C_c \quad (8.25)$$

Other fundamental equations are the same as those in the last section.

The process of derivation of equations are omitted and only the results are given below.

$$f_c = \frac{Pk}{bd \left[ pn (1 - k) - \frac{1}{2}k^2 \right]} \quad (8.26)$$

$$f_s = nf_c \frac{1 - k}{k} \quad (8.27)$$

$$f_t = \frac{P (D - kd)}{bd^2 \left[ pn (1 - k) - \frac{1}{2}k^2 \right]} \quad (8.28)$$

$$\begin{aligned} dk^3 - (3e + 1.5D) k^2 + 6pn \left( d - \frac{D}{2} - e \right) k \\ - 6pn \left( d - \frac{D}{2} - e \right) k = 0 \end{aligned} \quad (8.29)$$

$$\begin{aligned} k^3 - \frac{3e + 1.5D}{d} k^2 + \frac{6pn}{d} \left( d - \frac{D}{2} - e \right) k \\ - \frac{6pn}{d} \left( d - \frac{D}{2} - e \right) = 0 \end{aligned} \quad (8.30)$$

#### 8.4. Example of Analysis

In this section a complete example of the theoretical analysis of a bent is presented. The bent used is the first one as described in section 8.1.1.

##### 8.4.1. General Information

The dimensions of various members are assumed first and then the moments of inertia of these members are calculated according to the formulas shown in section 8.1.2.B.

The stiffness factor  $k$  of a member is its moment of inertia divided by its length or height which is the distance from center to center of joints.

The value of  $C$  is 4 for the columns and 7.35 for the slab and the slab-bands. The carry-over factor, 0.652, is obtained by dividing  $C_2$  by  $C_1$  or by  $C_3$  as shown in section 8.2.4.

The distribution and carry-over factors for the method of moment distribution are calculated in the usual way except that the  $C$ -value for the slab is modified for the case of symmetrical loading ( $\theta_a = -\theta_b$ ) and also for the anti-symmetrical condition ( $\theta_a = \theta_b$ ). This simplifies the moment distribution method.

The dead load includes the weights of slab, equivalent slab-band, floor covering, partition wall and ceiling and the live load is 80 lbs. per sq. ft. floor surface for the building proposed in chapter 2.

The fixed-end moments are expressed in terms of  $w$  which is the load per ft. of span and the end moments of members after the moment distribution will be expressed in terms of  $w$  too. In this way the end moments can be calculated for any load  $w$  instead of being confined to one definite load.

#### 8.4.2. Moment Distribution

The end moments of members are obtained by the method of moment distribution with load in one panel at a time.

One of the exterior panels is loaded first. Although the frame is symmetrical, the loading is not and therefore a correction for side sway due to unsymmetrical loading should be made.

Because of symmetry of the structure it is necessary to load only one exterior panel to obtain end moments for load in either exterior panel.

The moment distribution for the load in the middle panel is simple because both the frame and the loading are symmetrical.

#### 8.4.3. End Moments, Direct Stresses and Shears

The end moments for live loads in any two panels can be obtained by combining the appropriate results from the moment distribution in last section. The numerical values of live load end moments are obtained by substituting the value of  $w$  into the expressions for end moments obtained by moment distribution. In making comparisons with results of the model tests only the live load moments should be used.

The reactions, shears and direct stresses are calculated by statics using the calculated end moments and the applied loads.

The bending moment at any point in the slab is then calculated from the known shear, end moment and load.

#### 8.4.4. Neutral Axes and Stresses

The neutral axis for combined bending and direct stress as well as the stresses in concrete and steel at a number of sections were calculated. These calculations were made for the sections indicated in Figure 8.24.

The constants needed in locating the neutral axis for combined stress are given in table 8.01 and the calculations of the  $k$ -values is shown in table 8.02. Table 8.03 shows the calculation of stresses,  $f_c$  and  $f_s$ , according to equations (8.17) and (8.18). Moments of inertia at different sections along the slab are listed in table 8.04. They will be used in the formulas for transforming the stresses in the model into that in the prototype.

## 8.4.1.1 General Information

## A. Moments of Inertia

Upper Columns

$$I_{uc} = \frac{14 \times 14^3}{12} = 3,201$$

Lower Columns

$$I_{lc} = \frac{16 \times 16^3}{12} = 5,463$$

Slab

$$d = 4.5 - \left(\frac{1}{4} + \frac{3}{4}\right) = 3.5"$$

$$I_s = \frac{16 \times 12 \times 3.5^3}{12} = 686$$

## B. Stiffness Factors

Upper Columns

$$K_{uc} = \frac{3201}{10 \times 12} = 26.68$$

Lower Columns

$$K_{lc} = \frac{5463}{10 \times 12} = 45.5$$

Slab

$$K_s = \frac{686}{16 \times 12} = 3.58$$

## C. C - Values

Upper Columns

$$C_{uc} = 4$$

Lower Columns

$$C_{lc} = 4$$

Slab

$$C_1 = 7.35, C_2 = 4.79$$

## D. Distribution and Carry-over Factors

## a. Loading Unsymmetrical

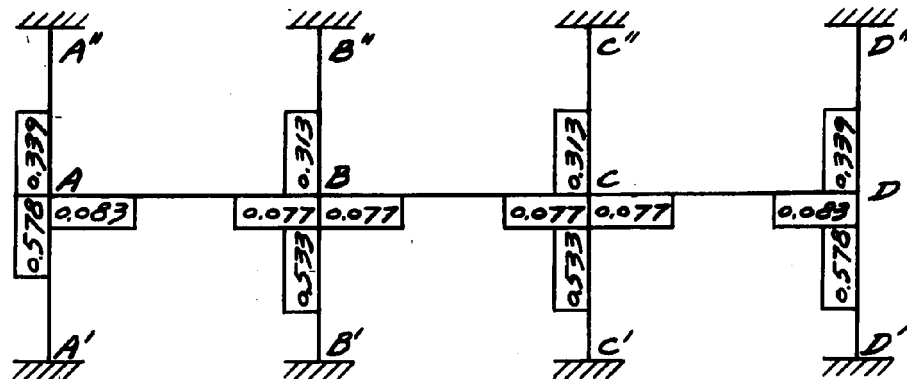


Figure 8.07

Distribution Factors for Unsymmetrical Loading

## At Joint A

| Member | C    | K     | CK          | r            | carry-over |
|--------|------|-------|-------------|--------------|------------|
| AA''   | 4    | 26.68 | 106.7       | 0.339        | 0.5        |
| AA'    | 4    | 45.5  | 182         | 0.578        | 0.5        |
| AB     | 7.35 | 3.58  | <u>26.3</u> | <u>0.083</u> | 0.652      |
|        |      |       | 315.0       | 1.000        |            |

## At Joint B

| Member | C    | K     | CK          | r            | carry-over |
|--------|------|-------|-------------|--------------|------------|
| BB''   | 4    | 26.68 | 106.7       | 0.313        | 0.5        |
| BB'    | 4    | 45.5  | 182         | 0.533        | 0.5        |
| BA     | 7.35 | 3.58  | 26.3        | 0.077        | 0.652      |
| BC     | 7.35 | 3.58  | <u>26.3</u> | <u>0.077</u> | 0.652      |
|        |      |       | 341.3       | 1.000        |            |

b. r - Values used in correcting for sidesway

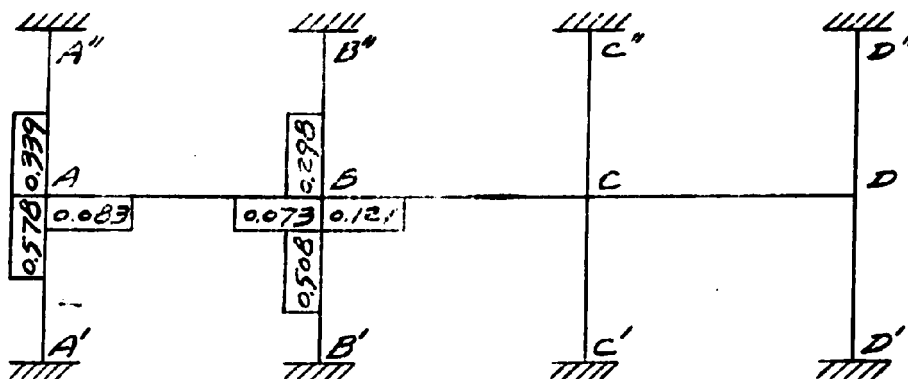


Figure 8.08

Distribution Factors for Correcting for Sidesway

At Joint B

| Member           | C             | K     | CK          | r            | carry-over |
|------------------|---------------|-------|-------------|--------------|------------|
| BB <sup>''</sup> | 4             | 26.68 | 106.7       | 0.298        | 0.5        |
| BB <sup>'</sup>  | 4             | 45.5  | 182         | 0.508        | 0.5        |
| BA               | 7.35          | 3.58  | 26.3        | 0.073        | 0.652      |
| BC               | 7.35          | 3.58  | <u>43.5</u> | <u>0.121</u> | 0          |
|                  | <u>+ 4.79</u> |       |             |              |            |
|                  | = 12.14       |       | 358.5       | 1.000        |            |

c. r - Values for symmetrical vertical load

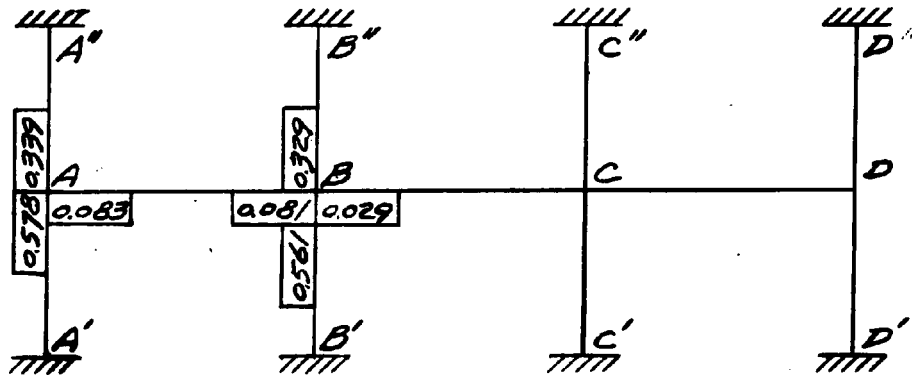


Figure 8.09

Distribution Factors for Symmetrical Vertical Loading

At Joint B

| Member           | C             | K     | CK         | r            | carry-over |
|------------------|---------------|-------|------------|--------------|------------|
| BB <sup>''</sup> | 4             | 26.68 | 106.7      | 0.329        | 0.5        |
| BB <sup>'</sup>  | 4             | 45.5  | 182        | 0.561        | 0.5        |
| BA               | 7.35          | 3.58  | 26.3       | 0.081        | 0.652      |
| BC               | 7.35          | 3.58  | <u>9.2</u> | <u>0.029</u> | 0          |
|                  | <u>- 4.79</u> |       |            |              |            |
|                  | = 2.56        |       | 324.2      | 1.000        |            |

## E. Loads

## a. Dead load

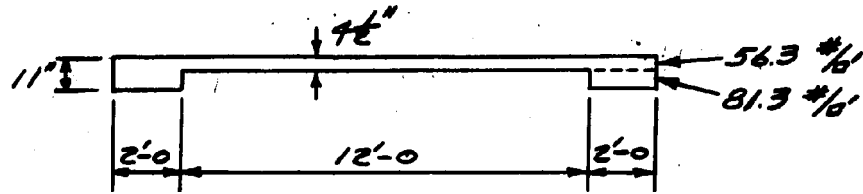


Figure 8.10

## Section of Slab in One Panel

F.E.M. due to weight of haunch

$$= - \frac{81.3 \times 2^2}{12 \times 16^2} \times [3 \times 2^2 - 8 \times 2 \times 16 + 6 \times 16^2 + 4 \times 16 - 3 \times 2]$$

$$= - 144.7 \text{ #}$$

F.E.M. due to a uniform load  $w$ .

$$= - \frac{w l^2}{12} = - \frac{w \times 16^2}{12}$$

$$= - 21.33 w \text{ #}$$

Equivalent unit uniform load due to wt. of haunches

$$- 21.33 w = - 144.7$$

$$w = 6.775 \text{ #/ft}$$

Unit dead load:

|                      |           |
|----------------------|-----------|
| Partition Wall       | 20 #/ft   |
| Wood Floor Surface   | 3         |
| Concrete Slab        | 56.3      |
| Equivalent Slab-band | 6.8       |
| Ceiling              | 10        |
|                      | <hr/>     |
|                      | 96.1 #/ft |
| say                  | 97 #/ft   |

$$\begin{aligned}
 W_D &= \text{Uniform dead load in lbs. per ft. of span} \\
 &= 97 \times 16 \\
 &= 1,552 \text{ \#/ft.}
 \end{aligned}$$

b. Live load

$$\begin{aligned}
 W_L &= \text{Uniform live load in lbs. per ft. of span} \\
 &= 80 \times 16 \\
 &= 1,280 \text{ \#/ft.}
 \end{aligned}$$

F. Fixed End Moment

$$\begin{aligned}
 \frac{1}{12} WL^2 &= \frac{1}{12} \times W \times 16^2 \\
 &= 21.33 W' \text{ \#}
 \end{aligned}$$

8.4.2'. Moment Distribution

A. Load w on the Left Exterior Span.

When the building frame is unsymmetrically loaded, its joints undergo both rotation and translation. The end moments on the members of the frame are found in the following way.

a. The load is applied on the left exterior span and the end moments are calculated by the method of moment distribution assuming that there are two auxiliary forces  $F_1$  and  $F_2$  at joints  $A''$  and  $A$  respectively to prevent the sidesway of the frame (See Figure 8.12).

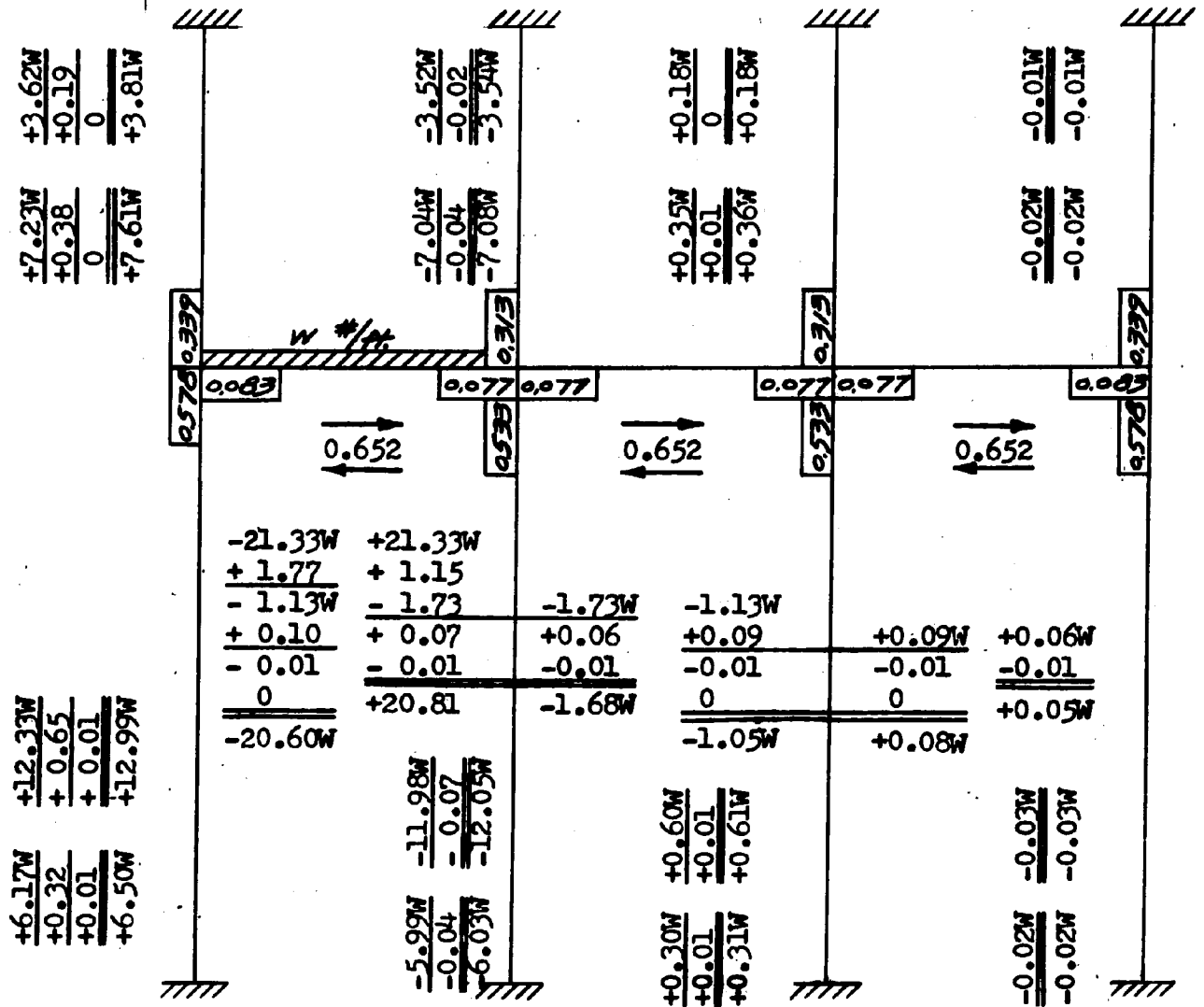


Figure 8.11

End Moments

Calculate  $F_1$  and  $F_2$ :

$$\frac{7.61w + 3.81w}{10} = 1.142w = V_{AA}''$$

$$\frac{7.08w + 3.54w}{10} = 1.062w = V_{BB}''$$

$$\frac{0.36w + 0.18w}{10} = 0.054w = V_{CC}''$$

$$\frac{0.02w + 0.01w}{10} = 0.003w = V_{DD}''$$

$$\frac{12.99w + 6.50w}{10} = 1.949w = V_{AA}'$$

$$\frac{12.05w + 6.03w}{10} = 1.808w = V_{BB}'$$

$$\frac{0.61w + 0.31w}{10} = 0.092w = V_{CC}'$$

$$\frac{0.03w + 0.02w}{10} = 0.005w = V_{DD}'$$

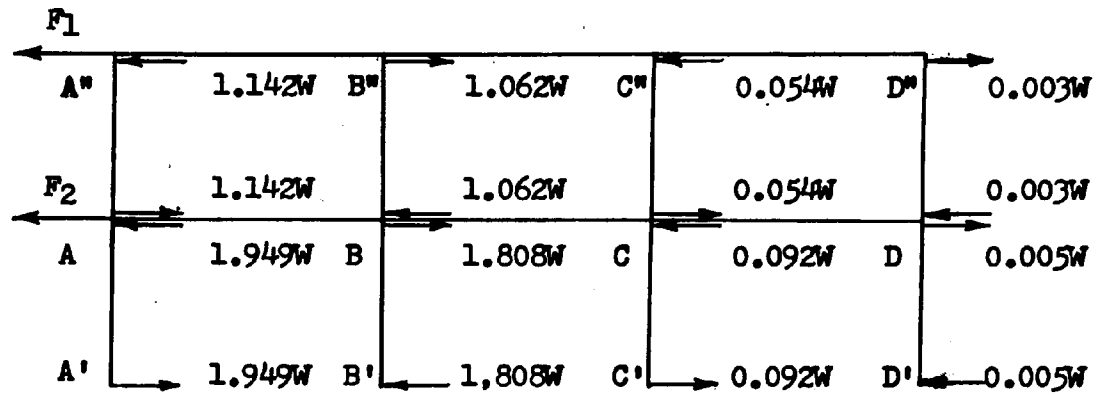


Figure 8.12

Auxiliary Forces  $F_1$  and  $F_2$ 

$$F_1 = 1.142 w - 1.062 w + 0.054 w - 0.003 w$$

$$= 0.131 w$$

$$F_2 = 1.949 w - 1.808 w + 0.092 w - 0.005 w$$

$$- 1.142 w + 1.062 w - 0.054 w + 0.003 w$$

$$= 0.097 w$$

b. The end moments obtained in Figure 8.11 shall be corrected because of sidesway. Figure 8.13 shows diagrammatically the displacements of the joints with the amounts exaggerated. It is assumed first that the top two floors are translated a distance  $\Delta_1$  to the right with respect to the bottom floor. The fixed-end moments in the members  $AA'$ ,  $BB'$ ,  $CC'$ , and  $DD'$  are calculated and the moments in the members are obtained by the method of moment distribution as shown in Figure 8.14. The forces  $F_1'$  and  $F_2'$  at joints  $A''$  and  $A$  respectively due to the displacement  $\Delta_1$  are shown in Figure 8.15.

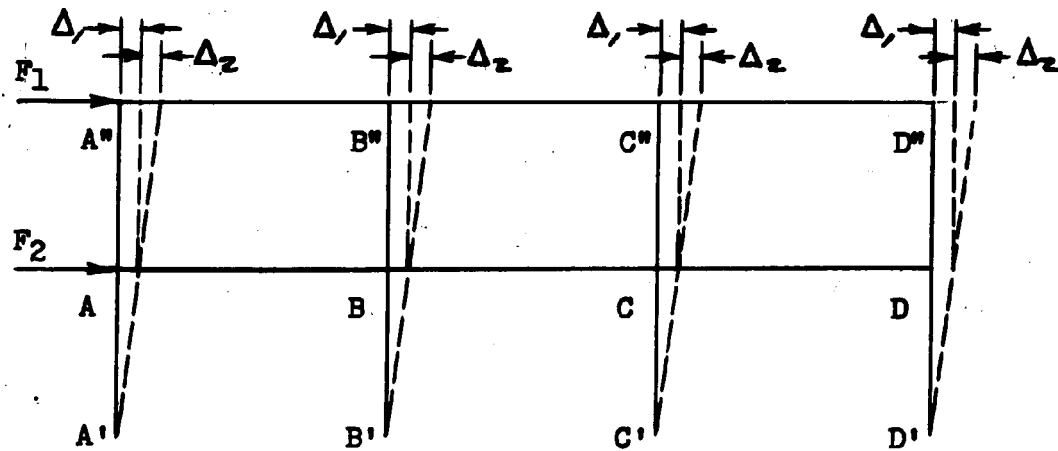


Figure 8.13

Sidesway  $\Delta_1$  and  $\Delta_2$ 

$$M_{FAA'} = M_{FA'A} = -6 \frac{EI}{L} \times \frac{\Delta_1}{L} = -6 \times 1 \times 45.5 \times \frac{\Delta_1}{10}$$

$$= -27.30 \Delta_1$$

$$M_{FBB'} = M_{FB'B} = -27.30 \Delta_1$$

$$M_{FCC'} = M_{FC'C} = -27.30 \Delta_1$$

$$M_{FDD'} = M_{FD'D} = -27.30 \Delta_1$$

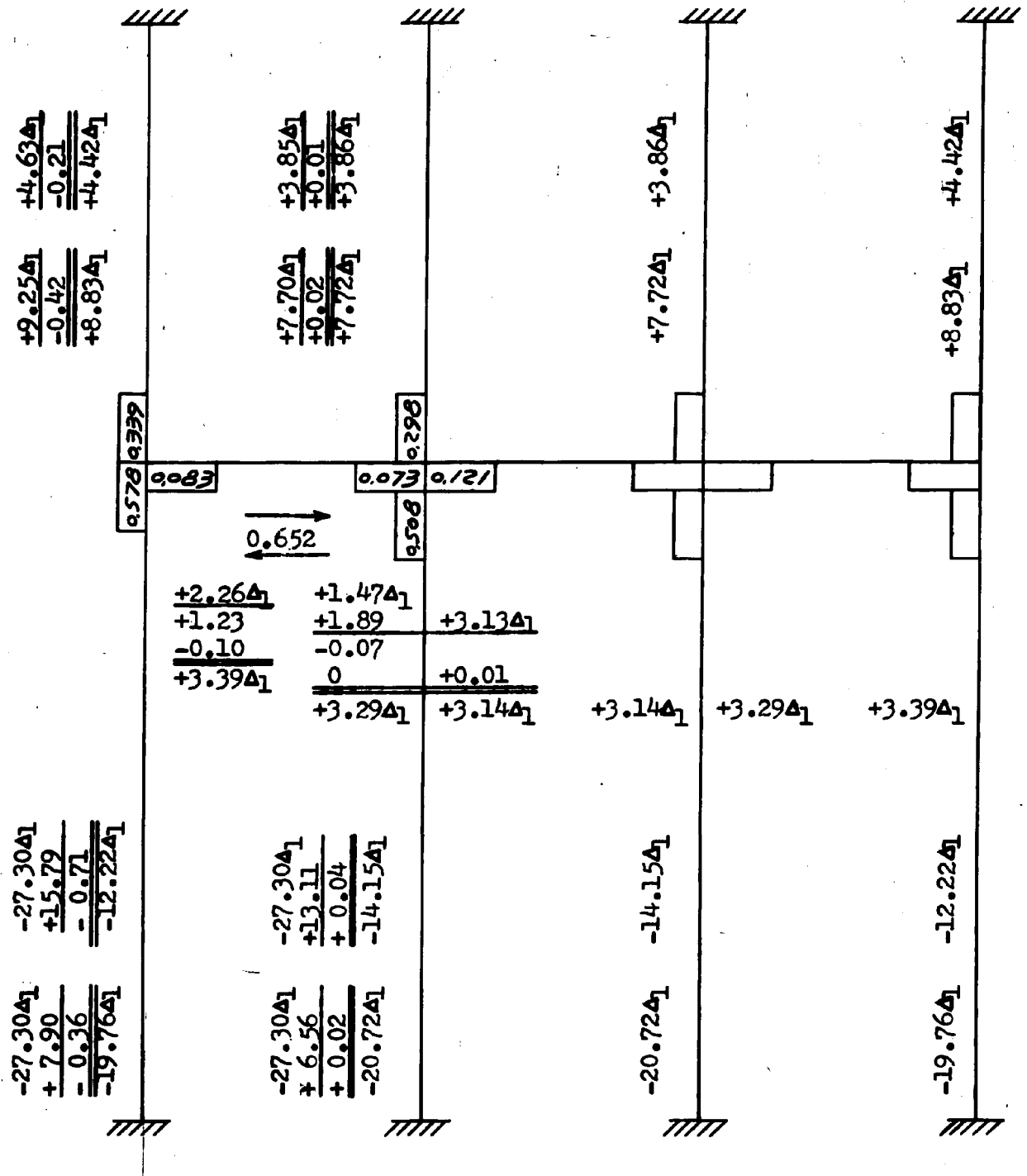


Figure 8.14

Moment Distribution for Displacement  $\Delta_1$ 

$$\frac{8.83\Delta_1 + 4.42\Delta_1}{10} = 1.325\Delta_1$$

$$\frac{7.72\Delta_1 + 3.86\Delta_1}{10} = 1.158\Delta_1$$

$$\frac{12.22\Delta_1 + 19.76\Delta_1}{10} = 3.198\Delta_1$$

$$\frac{14.15\Delta_1 + 20.72\Delta_1}{10} = 3.487\Delta_1$$

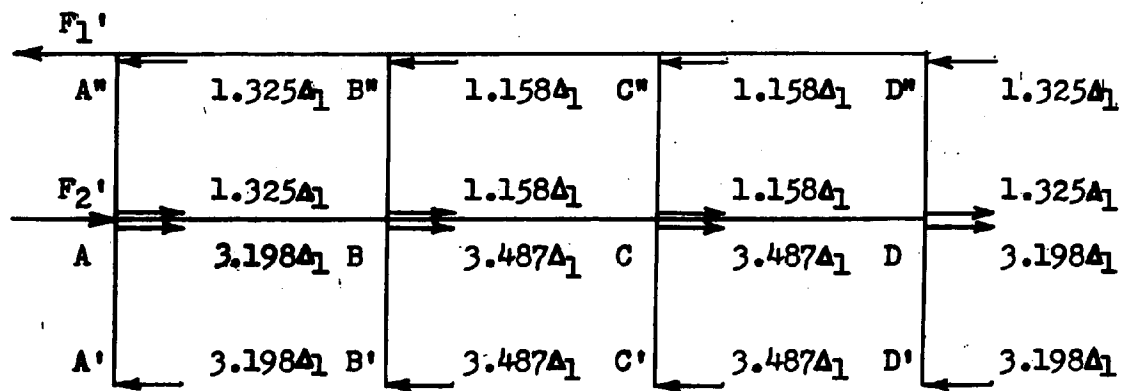


Figure 8.15

Auxiliary Forces  $F_1'$  and  $F_2'$

$$\begin{aligned} F_1' &= 1.325 \Delta_1 + 1.158 \Delta_1 + 1.158 \Delta_1 + 1.325 \Delta_1 \\ &= 4.966 \Delta_1 \end{aligned}$$

$$\begin{aligned} F_2' &= 1.325 \Delta_1 + 1.158 \Delta_1 + 1.158 \Delta_1 + 1.325 \Delta_1 \\ &\quad + 3.198 \Delta_1 + 3.487 \Delta_1 + 3.487 \Delta_1 + 3.198 \Delta_1 \\ &= 18.336 \Delta_1 \end{aligned}$$

It is assumed next that the top floor is displaced a distance  $\Delta_2$  to the right with the other floors fixed. The end moments of the members and the forces  $F_1''$  and  $F_2''$  due to this displacement of joints are shown in Figures 8.16 and 8.17 respectively.

$$\begin{aligned} M_{FAA''} &= M_{FA''A} = -6 \frac{EI}{L} \times \frac{\Delta_2}{L} = -6 \times 1 \times 26.68 \times \frac{\Delta_2}{10} \\ &= -16.01 \Delta_2 \end{aligned}$$

$$M_{FBB''} = M_{FB''B} = -16.01 \Delta_2$$

$$M_{FCC''} = M_{FC''C} = -16.01 \Delta_2$$

$$M_{FDD''} = M_{FD''D} = -16.01 \Delta_2$$

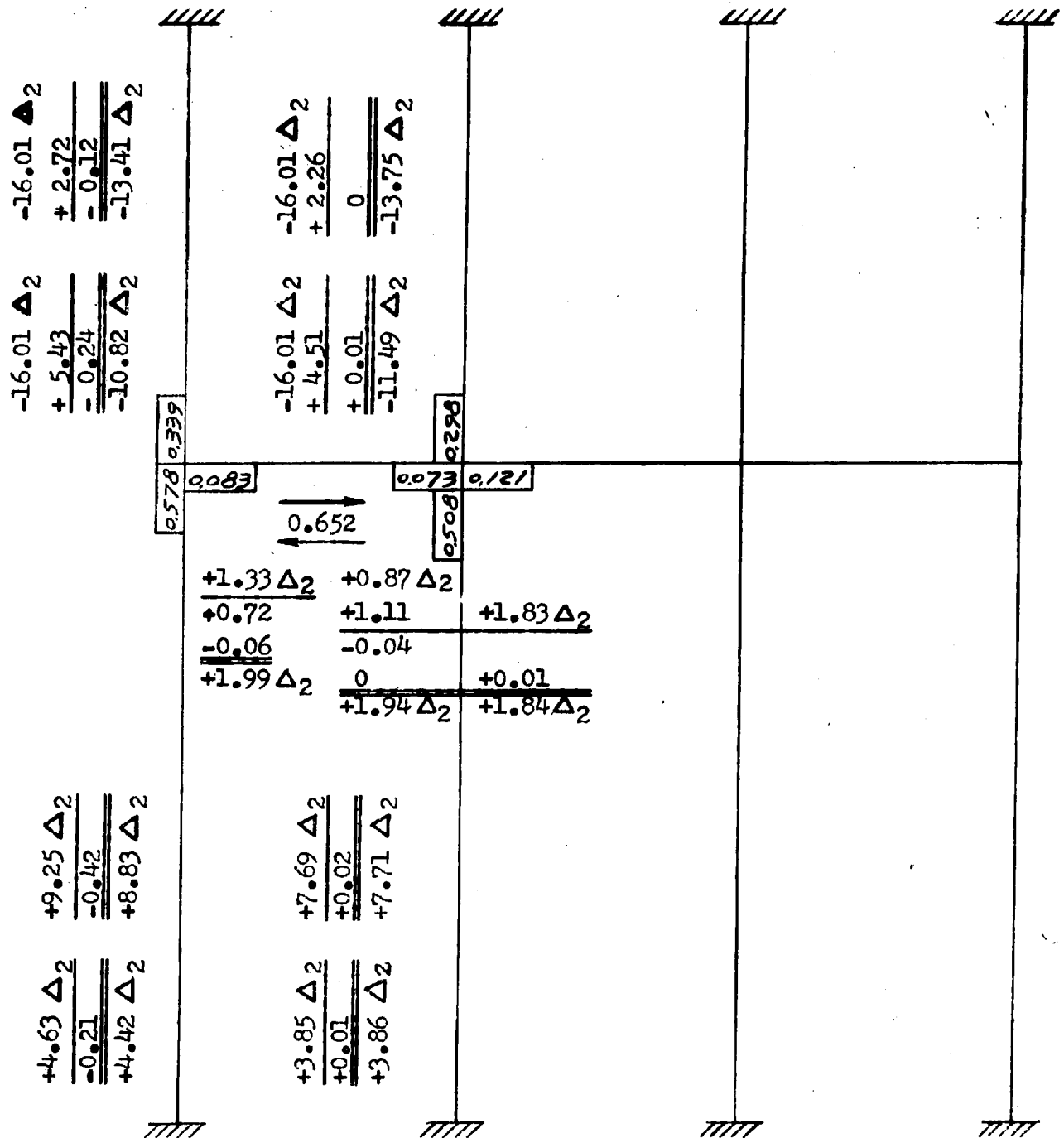


Figure 8.16

Moment Distribution for Displacement  $\Delta_2$ 

$$\frac{10.82 \Delta_2 + 13.41 \Delta_2}{10} = 2.423 \Delta_2$$

$$\frac{11.49 \Delta_2 + 13.75 \Delta_2}{10} = 2.524 \Delta_2$$

$$\frac{8.83 \Delta_2 + 4.42 \Delta_2}{10} = 1.325 \Delta_2$$

$$\frac{7.71 \Delta_2 + 3.86 \Delta_2}{10} = 1.157 \Delta_2$$

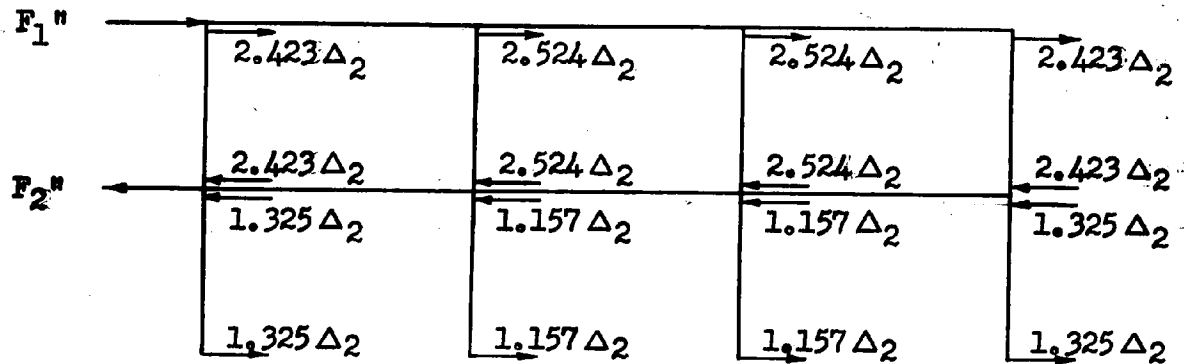


Figure 8.17.

Auxiliary Forces  $F_1''$  and  $F_2''$

$$\begin{aligned} F_1'' &= 2.423\Delta_2 + 2.524\Delta_2 + 2.524\Delta_2 + 2.423\Delta_2 \\ &= 9.894\Delta_2 \end{aligned}$$

$$\begin{aligned} F_2'' &= (2.423\Delta_2 + 2.524\Delta_2 + 1.325\Delta_2 + 1.157\Delta_2) \times 2 \\ &= 14.858\Delta_2 \end{aligned}$$

The actual values of  $\Delta_1$  and  $\Delta_2$  due to the load on the left exterior span can be obtained by considering the fact that there are no external horizontal loads at the joints  $A''$  and  $A$  and, therefore, the following formulas can be established.

The actual end moments on the members due to the load on the left exterior span are obtained by adding algebraically the corresponding terms in figures 8.11., 8.14. and 8.16. after the values of  $\Delta_1$  and  $\Delta_2$  are inserted in figures 8.14. and 8.16. Figure 8.18. gives the final end moments for loading at left span.

$$F_1' + F_1'' + F_1 = 0.$$

$$-4.966\Delta_1 + 9.894\Delta_2 - 0.131w = 0$$

$$-\Delta_1 + 1.99235\Delta_2 = 0.0263794w \quad (1)$$

$$F_2' + F_2'' + F_2 = 0$$

$$18.336\Delta_1 - 14.858\Delta_2 - 0.097w = 0$$

$$\Delta_1 - 0.81032\Delta_2 = 0.0052901w \quad (2)$$

Solving equations (1) and (2) we have

$$\Delta_1 = 0.027 w$$

$$\Delta_2 = 0.02679 w$$

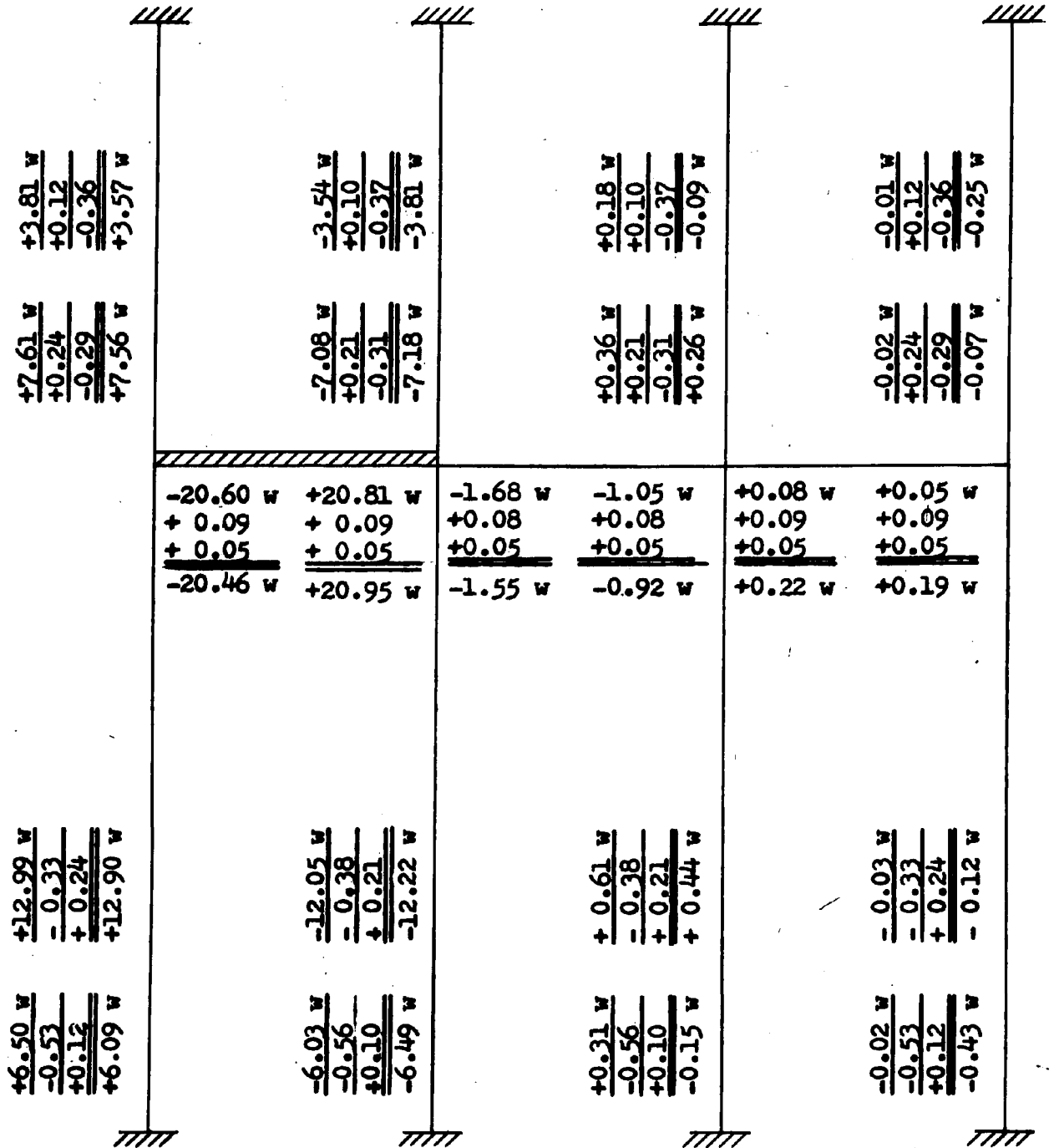


Figure 8.18

End Moments for Loading at Left Span

Finally the following check is made to show that there is no external horizontal force at joints A" and A.

$$\frac{7.56w + 3.57w}{10} = 1.113w = V_{AA''}$$

$$\frac{7.18w + 3.81w}{10} = 1.099w = V_{BB''}$$

$$\frac{0.26w - 0.09w}{10} = 0.017w = V_{CC''}$$

$$\frac{0.07w + 0.25w}{10} = 0.032w = V_{DD''}$$

$$\frac{12.90w + 6.09w}{10} = 1.899w = V_{AA'}$$

$$\frac{12.22w + 6.49w}{10} = 1.871w = V_{BB'}$$

$$\frac{0.44w - 0.15w}{10} = 0.029w = V_{CC'}$$

$$\frac{0.12w + 0.43w}{10} = 0.055w = V_{DD'}$$

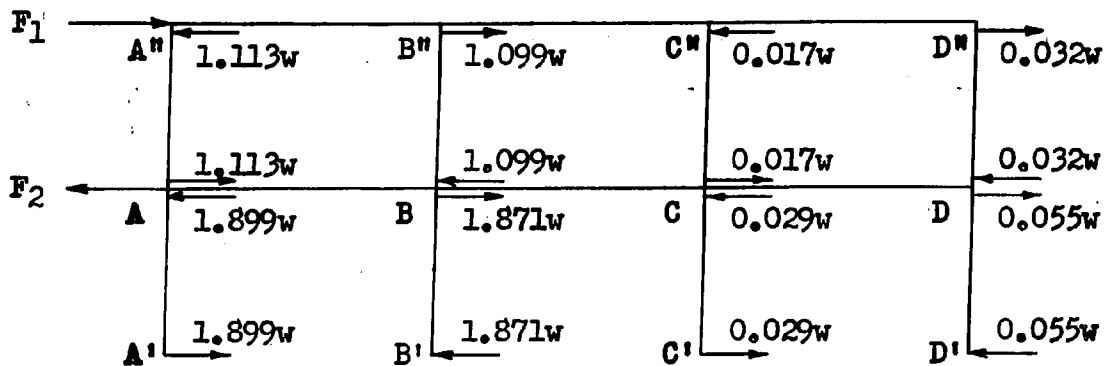


Figure 8.19.

#### Equilibrium of Horizontal Forces

$$F_1 = 1.113w - 1.099w + 0.017w - 0.032w$$

$$= -0.001w$$

$$\approx 0$$

$$F_2 = 1.899w - 1.871w + 0.029w - 0.055w$$

$$- 1.113w + 1.099w - 0.017w + 0.032w$$

$$= 0.003w$$

$$\approx 0$$

## B. Load on the Center Span

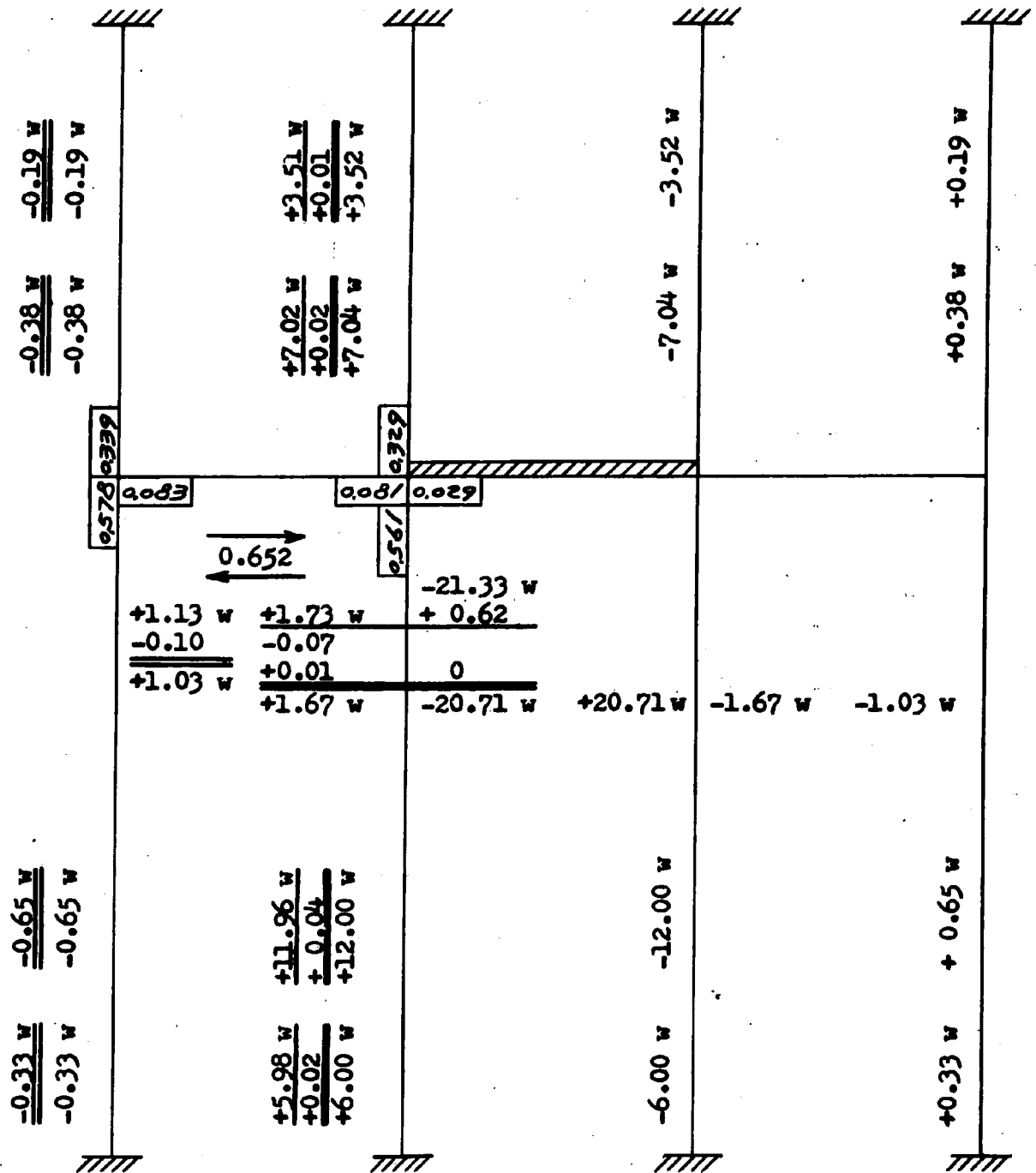


Figure 8.20.

End Moments for Loading at Middle Span

### C. Load on the right exterior span

Because of symmetry of the building frame this load condition and resulting moments are the exact reverse of case of load on the left span.

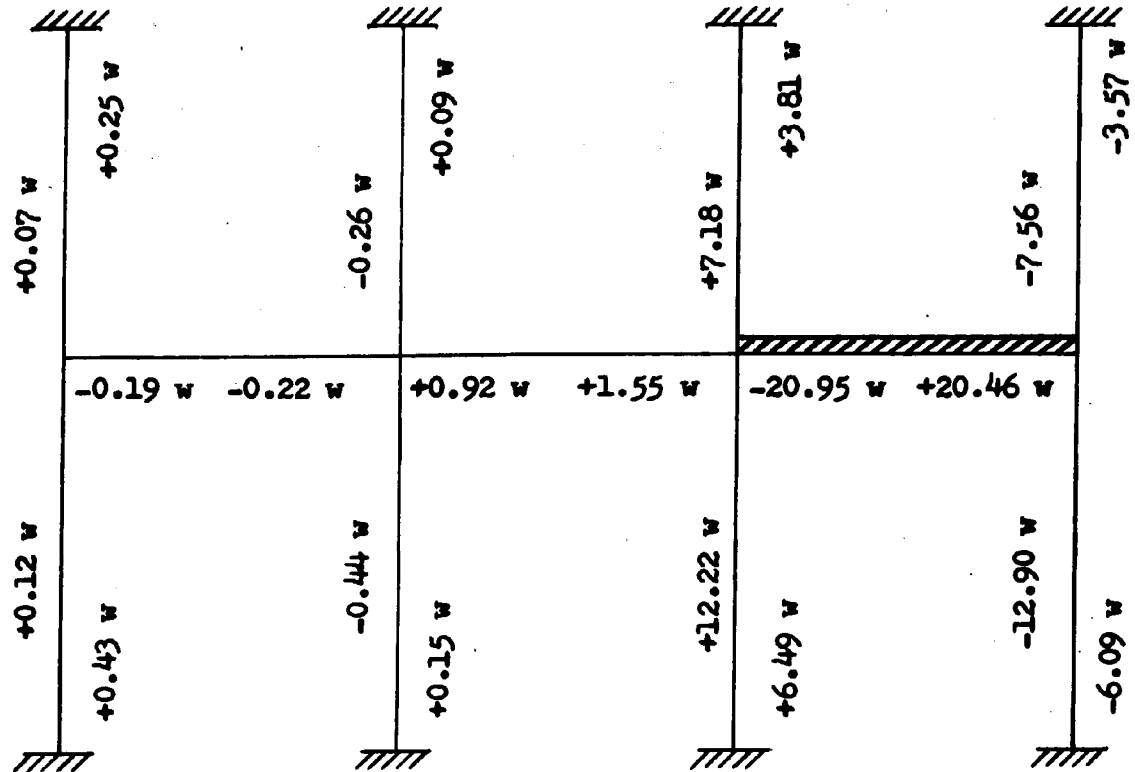


Figure 8.21.

End Moments for Loading at Right Span

### 8.4.3'. End Moments and Shears Due to Live Load

Live loads on both exterior spans will be illustrated.

Figure 8.22 shows the end moments of the members due to live loads on both exterior spans. Reference is made to Figures 8.18 and 8.21 for the end moments due to load on one exterior span only.

Figure 8.23 shows the force diagram for this frame due to live load of 1,280 lbs. per ft. of span and Figure 8.24 shows the bending moments at specified points.

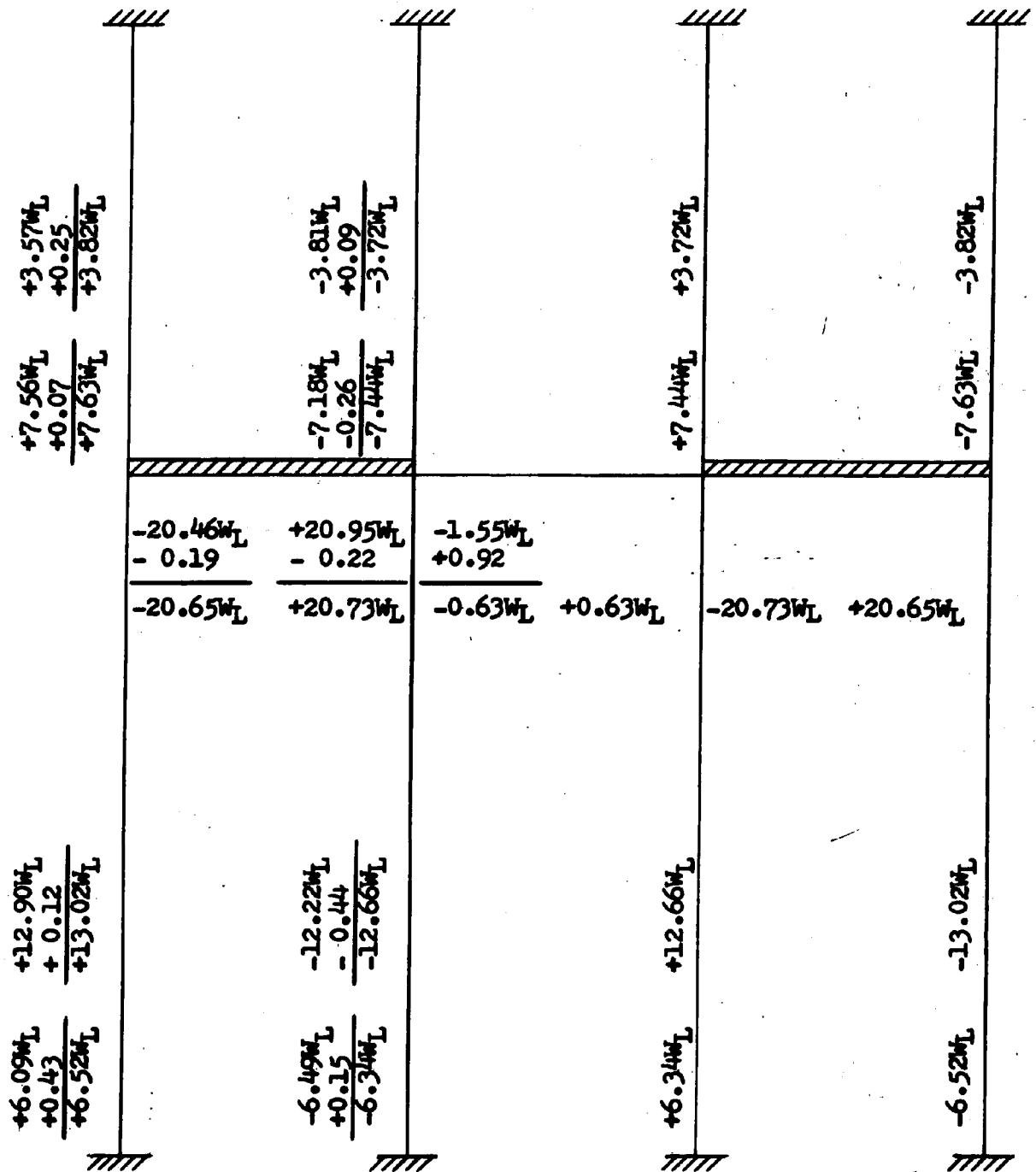


Figure 8.22.

End Moments for L. L. on Exterior Spans

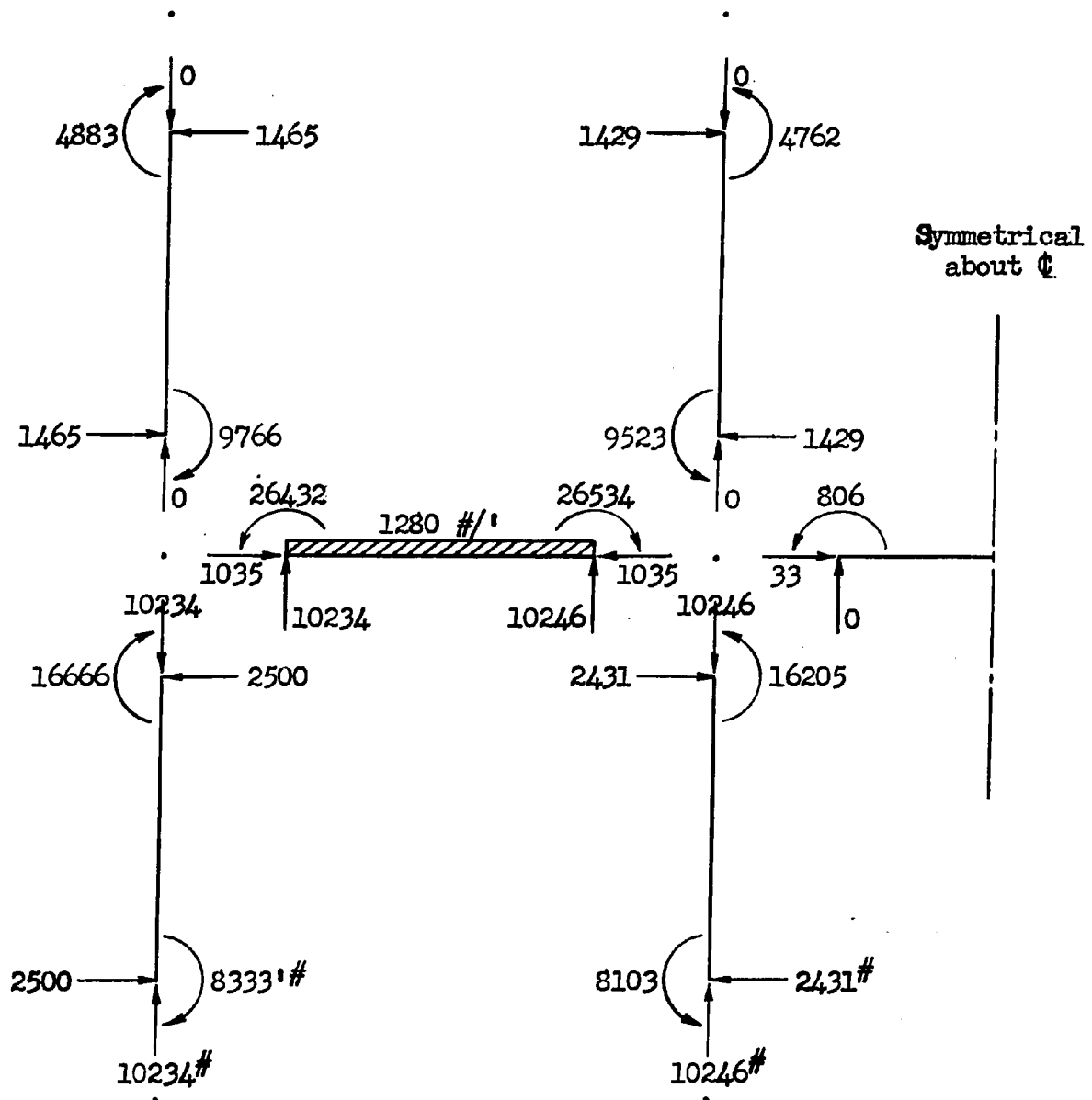


Figure 8.23.

Force Diagram for L. L. on Exterior Spans

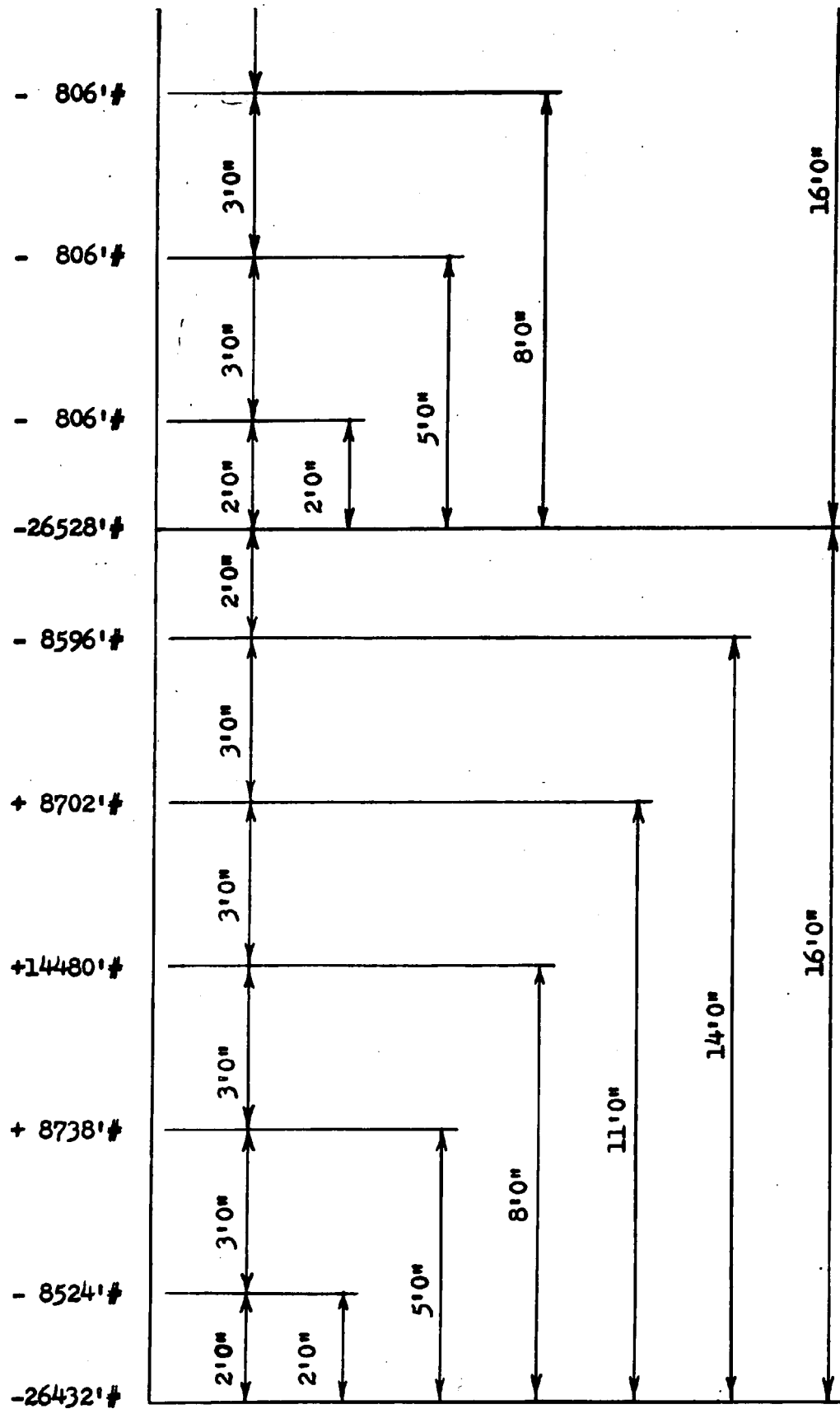


Figure 8.24.  
Bending Moments in the Slab Due to Live Load  
on Both Exterior Spans

## 8.4.4 Neutral Axis and Unit Stresses

Table 8.01 shows necessary data used to calculate neutral axis which is given in Table 8.02 and unit stresses which are given in Table 8.03. Table 8.04 gives moments of inertia which will be used in formulas in section 9.1.

Table 8.01.

Data Used in Calculating  $k$ ,  $f_c$  and  $f_s$ 

| Location<br>of<br>Section<br>i | M<br># | M<br>"  | P<br># | e<br>" | D<br>" | d<br>" | p        | n  |
|--------------------------------|--------|---------|--------|--------|--------|--------|----------|----|
| Exterior Panel                 |        |         |        |        |        |        |          |    |
| 0                              | -26432 | -317184 | -1035  | 306.46 | 11.0   | 10.0   | 0.002500 | 10 |
| 2.0                            | - 8524 | -102288 | -1035  | 98.83  | 4.5    | 3.5    | 0.007143 | 10 |
| 5.0                            | - 8738 | -104856 | -1035  | 101.31 | 4.5    | 3.5    | 0.009524 | 10 |
| 8.0                            | -14480 | -173760 | -1035  | 167.88 | 4.5    | 3.5    | 0.009524 | 10 |
| 11.0                           | - 8702 | -104424 | -1035  | 100.89 | 4.5    | 3.5    | 0.009524 | 10 |
| 14.0                           | - 8596 | -103152 | -1035  | 99.66  | 4.5    | 3.5    | 0.007143 | 10 |
| 16.0                           | -26528 | -318336 | -1035  | 307.57 | 11.0   | 10.0   | 0.002500 | 10 |
| Interior Panel                 |        |         |        |        |        |        |          |    |
| 0                              | -806   | -9672   | -33    | 293.09 | 11.0   | 10.0   | 0.002500 | 10 |
| 2.0                            | -806   | -9672   | -33    | 293.09 | 4.5    | 3.5    | 0.007143 | 10 |
| 5.0                            | -806   | -9672   | -33    | 293.09 | 4.5    | 3.5    | 0.009524 | 10 |
| 8.0                            | -806   | -9672   | -33    | 293.09 | 4.5    | 3.5    | 0.009524 | 10 |

Table 8.02.  
Calculation of Neutral Axis

| Location<br>of<br>Section | $dk^3 + (3e - 1.5D)k^2 + 6pn(e + d - \frac{D}{2})k - 6pn(e + d - \frac{D}{2}) = 0$ $Ak^3 + Bk^2 + Ck - D = 0$ |         |          |          |       |
|---------------------------|---------------------------------------------------------------------------------------------------------------|---------|----------|----------|-------|
|                           | A                                                                                                             | B       | C        | D        | k     |
| Exterior Panel            |                                                                                                               |         |          |          |       |
| 0                         | 10.0                                                                                                          | 902.874 | 46.6437  | 46.6437  | 0.203 |
| 2.0                       | 3.5                                                                                                           | 289.737 | 42.8919  | 42.8919  | 0.317 |
| 5.0                       | 3.5                                                                                                           | 297.180 | 58.6069  | 58.6069  | 0.356 |
| 8.0                       | 3.5                                                                                                           | 496.902 | 96.6499  | 96.6499  | 0.354 |
| 11.0                      | 3.5                                                                                                           | 295.929 | 58.3686  | 58.3686  | 0.356 |
| 14.0                      | 3.5                                                                                                           | 292.242 | 43.2497  | 43.2497  | 0.317 |
| 16.0                      | 10.0                                                                                                          | 906.213 | 46.8107  | 46.8107  | 0.203 |
| Interior Panel            |                                                                                                               |         |          |          |       |
| 0                         | 10.0                                                                                                          | 862.773 | 44.6387  | 44.6387  | 0.203 |
| 2.0                       | 3.5                                                                                                           | 872.523 | 126.1487 | 126.1487 | 0.315 |
| 5.0                       |                                                                                                               |         |          |          |       |
| 8.0                       |                                                                                                               |         |          |          |       |

Table 8.03.

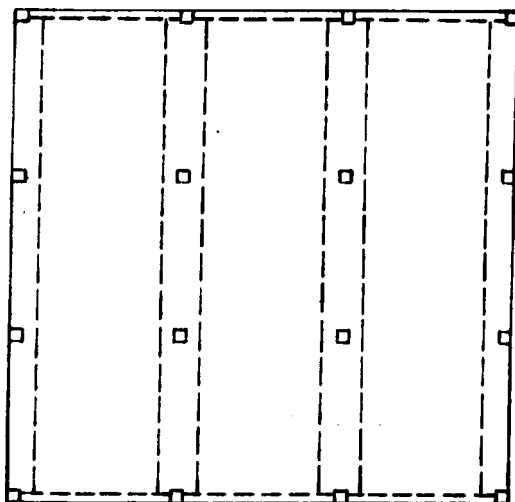
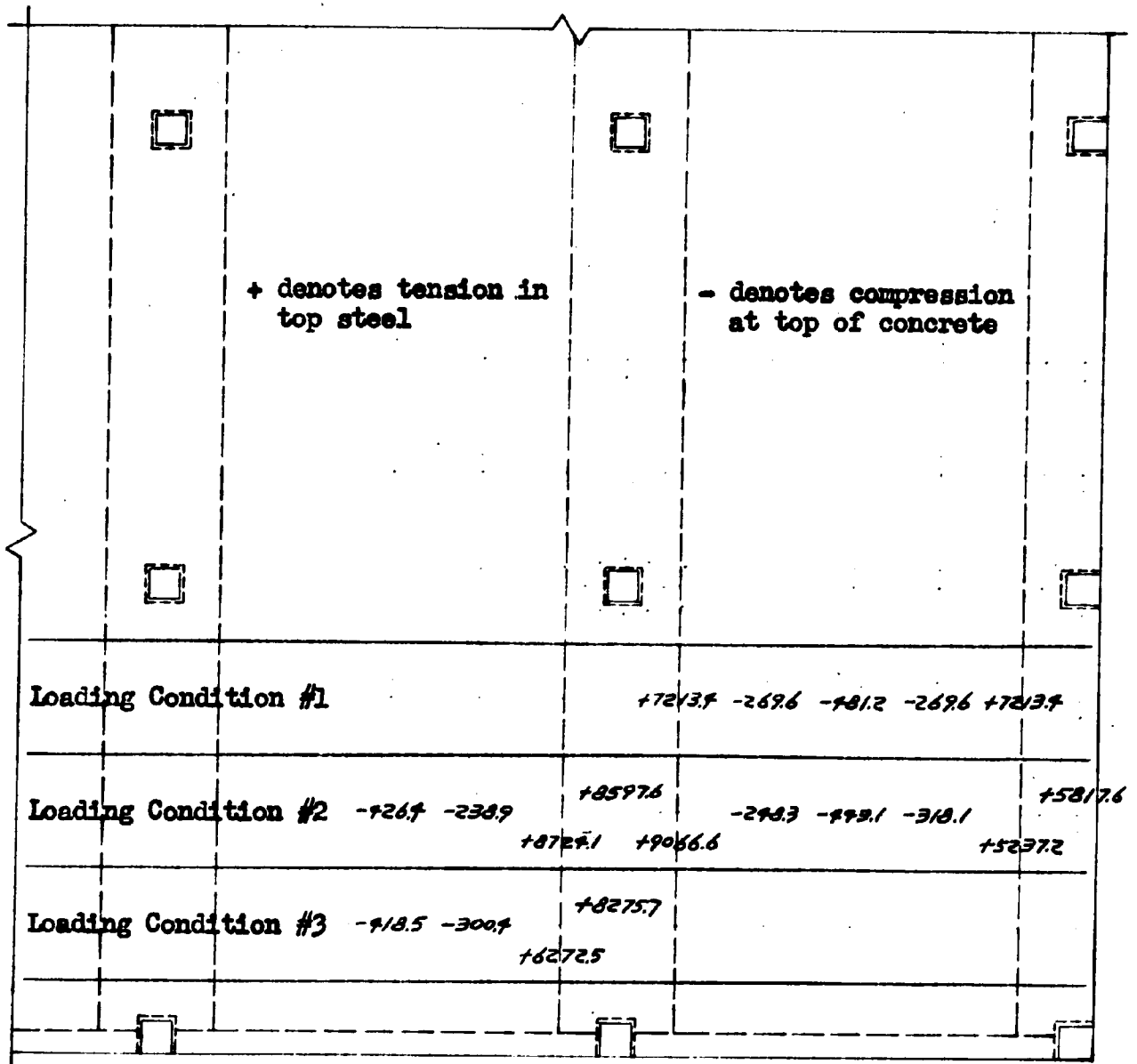
Calculation of Unit Stresses  $f_c$  and  $f_s$ 

| Location of Section | $f_c = \frac{Pk}{bd[\frac{1}{2}k^2 - pn(1-k)]} = \frac{E}{F}$ |        |        | $f_s = \frac{nP(1-k)}{bd[\frac{1}{2}k^2 - pn(1-k)]} = \frac{G}{H}$ |         |         |
|---------------------|---------------------------------------------------------------|--------|--------|--------------------------------------------------------------------|---------|---------|
|                     | E                                                             | F      | $f_c$  | G                                                                  | H       | $f_s$   |
| Exterior Panel      |                                                               |        |        |                                                                    |         |         |
| 0                   |                                                               |        |        | 8248.95                                                            | 0.45662 | 18065.1 |
| 2.0                 |                                                               |        |        | 7069.05                                                            | 0.97965 | 7215.9  |
| 5.0                 | 368.46                                                        | 1.3664 | 269.65 |                                                                    |         |         |
| 8.0                 | 366.39                                                        | 0.7614 | 481.22 |                                                                    |         |         |
| 11.0                | 368.46                                                        | 1.3664 | 269.65 |                                                                    |         |         |
| 14.0                |                                                               |        |        | 7069.05                                                            | 0.97965 | 7215.9  |
| 16.0                |                                                               |        |        | 8248.95                                                            | 0.45662 | 18065.1 |
| Interior Panel      |                                                               |        |        |                                                                    |         |         |
| 0                   |                                                               |        |        | 263.01                                                             | 0.45662 | 576.0   |
| 2.0                 |                                                               |        |        | 226.05                                                             | 0.45894 | 638.2   |
| 5.0                 |                                                               |        |        |                                                                    |         |         |
| 8.0                 |                                                               |        |        |                                                                    |         |         |

Table 8.04.

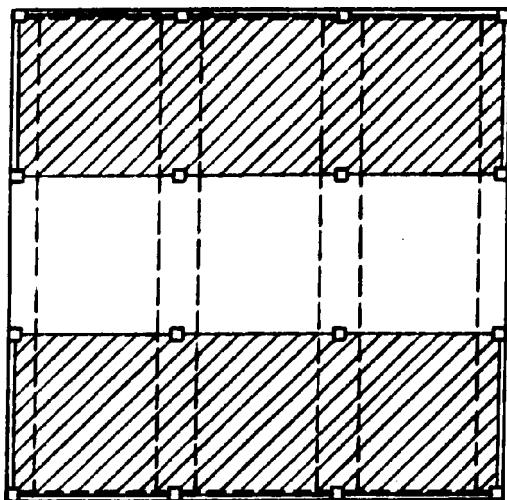
Moments of Inertia.

[illegible]



KEY

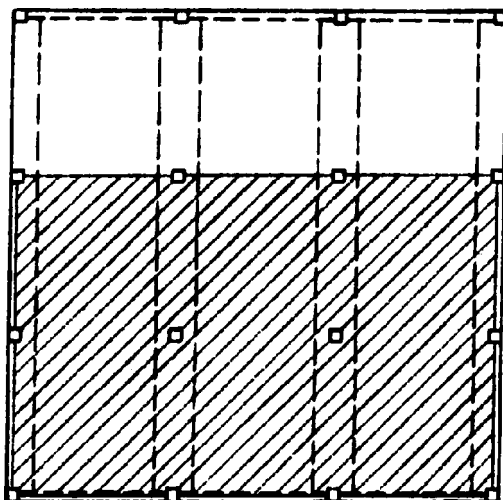
Diagram 8.01.  
Stresses in the Slab in the Direction  
of the Slab Span Obtained from  
Theoretical Analysis  
(Stresses are in lbs. per sq. in.)  
Loading Conditions Nos. 1, 2 & 3



**Loaded Areas Are Shaded**

**Stresses in the Slab-Bands in the  
Direction of the Slab-Band Span  
Obtained from Theoretical Analysis  
(Stresses are in lbs. per sq. in.)**

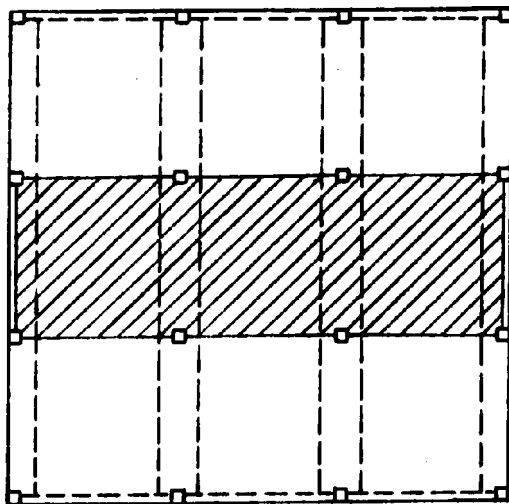
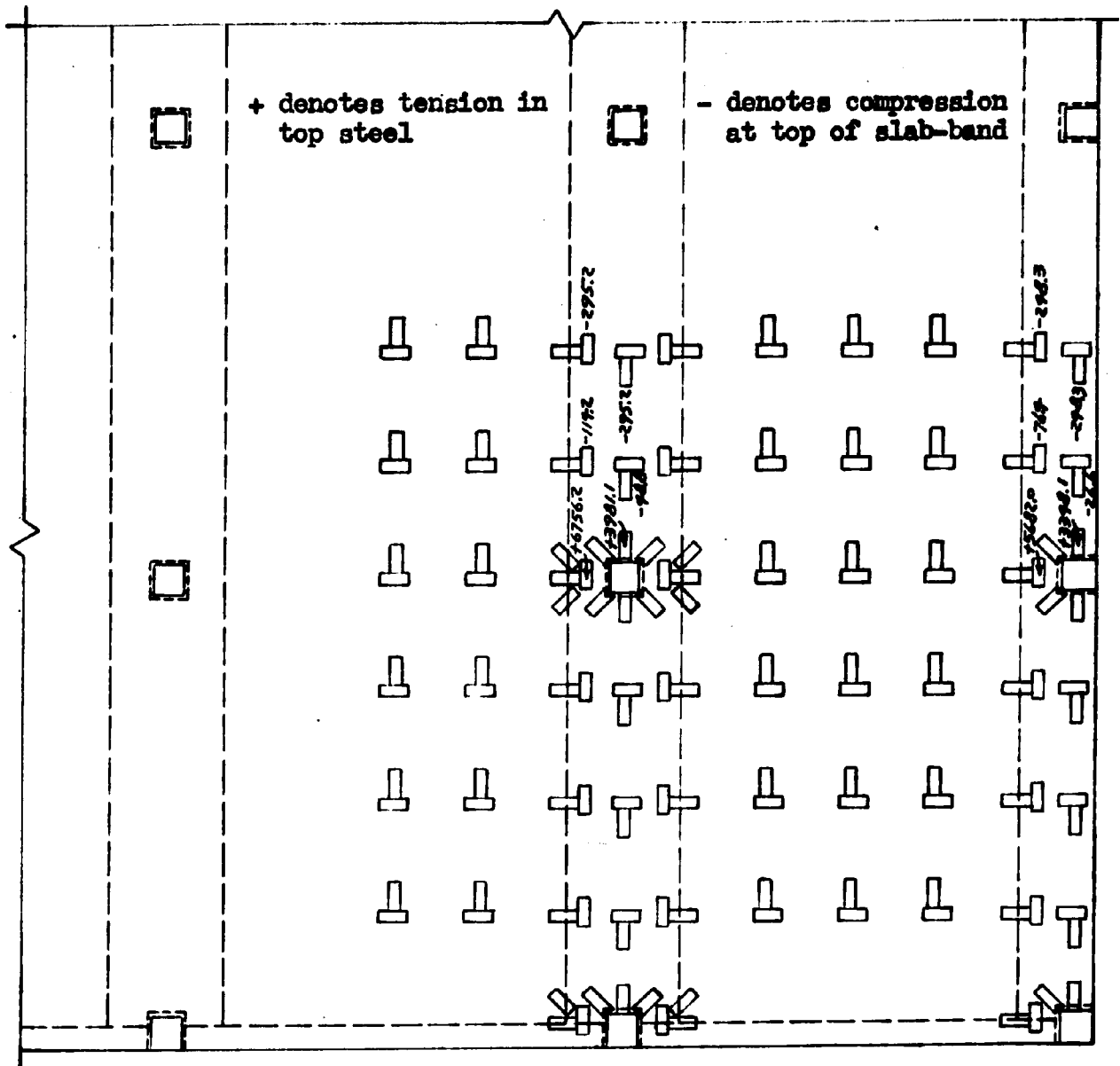
### Loading Condition No. 4



**Loaded Areas Are Shaded**

Stresses in the Slab-Bands in the  
Direction of the Slab-Band Span  
Obtained from Theoretical Analysis  
(Stresses are in lbs. per sq. in.)

### Loading Condition No. 5



KEY

Loaded Areas Are Shaded

Diagram 8.04.

Stresses in the Slab-Bands in the  
Direction of the Slab-Band Span  
Obtained from Theoretical Analysis  
(Stresses are in lbs. per sq. in.)

Loading Condition No. 6

### 8.5. Results of Analysis

For simplicity of presentation and clearness of results only the compressive stresses at the top of the concrete and the tensile stresses in the top steel are given in diagrams 8.01 through 8.04. The method and process of analysis of all the three bents are more or less as the one given in last section.

In diagram 8.01 the stresses shown are for the slab under three different loading conditions Nos. 1, 2, and 3. The stresses in the interior and exterior slab-bands are given in diagrams 8.02, 8.03 and 8.04 with the different loading conditions indicated in the respective diagrams.

### 8.6. Discussion of the Theoretical Results

The results obtained from the frame analysis described in section 8.1 and carried out in section 8.5 contain some possible errors of small magnitude. The following is a discussion of sources of those errors.

A. The remote ends of the columns of the frame were assumed as fixed. Actually those ends rotated in a small degree at the time that loads were applied at the middle floor because both the adjacent columns directly above or below those under consideration and the slab-bands at the upper and lower floors were not large enough to prevent the remote ends of the columns from rotation. However, this assumption caused but little effect on the bending moments in the members of the middle floor.

B. For the theoretical analysis as well as for practical designs the moments of inertia for members which had not yet been designed were needed and they were calculated according to the assumed concrete cross-sections with steel neglected. The difference between this section and the one which should be considered is an error which is inherent in all reinforced concrete design. It is of unknown magnitude.

C. The change in length of different members due to direct stresses and the deflection due to shear affected the stress conditions in the members but they were too small to be considered.

D. For the theoretical analysis of the slab a bent in the direction of the slab span was taken. There was a row of interior and exterior columns along the center line of the bent and the width of the bent equaled the panel length in the direction perpendicular to that of the bent. In the analysis it was assumed that the columns restrained the whole width of the bent. Actually it was not quite so because the dimension of the columns was only one-twelfth of the width of the bent. The columns offered the most restraint to the slab and the slab-bands at the center line of the bent and the least restraint to them at the two edges of the bent.

E. The supporting columns of interior slab-band are located at the middle of the slab-band width. The two side strips of the interior slab-band at the interior columns overhang the columns while the entire ends of the slab-band are restrained at the exterior support by both columns and spandrel beams. It is therefore evident that the middle strip of the interior slab-band is different from the side strips in the structural behavior. The exterior slab-band is the same as the interior one except the former only has one over-hanging

side strip at the columns. In the theoretical analysis the columns were considered to restrain the whole width of the slab-band and therefore the entire width of the slab-band was considered as uniform in structural action.

F. At the points of positive bending moment along the slab-bands the rectangular slab-band section was considered in calculating stresses. Actually the section is a tee beam or a wider rectangular beam since part of the slab acts as flange.

## CHAPTER 9

### COMPARISON OF EXPERIMENTAL AND THEORETICAL RESULTS

The following terms used in this chapter are defined as:

**Model Stresses:** Stresses in the model obtained by tests

**Prototype Stresses:** Stresses in the prototype obtained from results of model tests by use of conversion formulas

**Calculated Stresses:** Stresses in the prototype calculated by approximate methods of frame analysis

#### 9.1. The Relation between Stresses in the Model and in the Prototype

The stresses at certain locations in the model due to different loading conditions have been determined by tests and formulas as described in Chapter 7. In this section formulas will be developed in order to convert these measured stresses in the model into the equivalent stresses in the prototype at the corresponding points. In section 9.4. these equivalent stresses will then be compared with those stresses at corresponding points determined by theoretical analysis.

The notations used in this section have the following meanings:

**subscript A:** Anything pertaining to the left end of the member considered

subscript B: Anything pertaining to the right end of the same member

subscript p: Anything pertaining to the prototype

subscript m: Anything pertaining to the model

f: Fiber stress  $f_p$ : Fiber stress in the prototype

$f_m$ : Fiber stress in the model

M: Bending moment

c: Distance from the neutral axis of the section to the fiber considered

I: Moment of inertia

L: Span length between centers of supports

w: Load per unit length of span

R: Reaction

x: Distance between left support and the section considered

a:  $x/L$

$\alpha$  and  $\beta$ : Moment coefficients

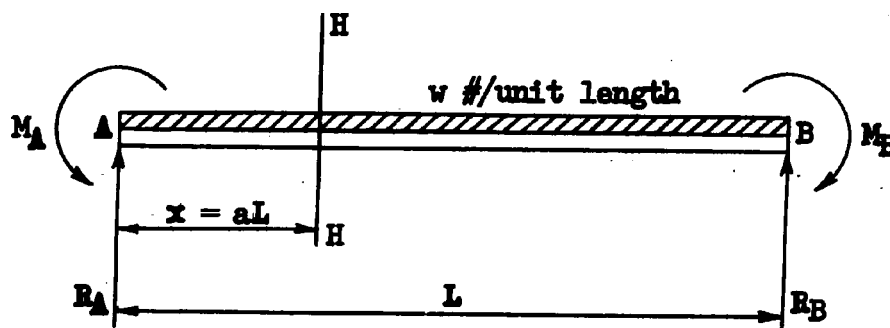


Figure 9.01.

#### A. Uniformly Loaded Beam with Restrained Ends

In Figure 9.01 is shown a member AB loaded with uniform load and continuous or restrained at both ends. The bending moment at section H-H is

$$M = R_A x - M_A - \frac{1}{2} w x^2$$

and the left reaction is

$$R_A = (M_A + \frac{1}{2}wL^2 - M_B) \times \frac{1}{L}$$

The end moments can be expressed in terms of  $wL^2$  and therefore

$$M_A = \alpha wL^2$$

$$M_B = \beta wL^2$$

The term  $\alpha$  or  $\beta$  at one end of a member in the model is the same as at that end of the corresponding member in the prototype, because both the model and the prototype have the same construction except the latter is many times larger than the former.

The stress at any spot of the slab or the slab-band in the prototype is expressed as follows:

$$\begin{aligned} f_p &= \frac{M_p c_p}{I_p} = \frac{M_p}{I_p / c_p} = \frac{R_{Ap} x_p - M_{Ap} - \frac{1}{2} w_p x_p^2}{I_p / c_p} \\ &= \frac{[(M_{Ap} + \frac{1}{2} w_p L_p^2 - M_{Bp}) \times \frac{1}{L_p} \times x_p] - M_{Ap} - \frac{1}{2} w_p x_p^2}{I_p / c_p} \\ &= \frac{[(\alpha w_p L_p^2 + \frac{1}{2} w_p L_p^2 - \beta w_p L_p^2) \times \frac{1}{L_p} \times a L_p] - \alpha w_p L_p^2 - \frac{1}{2} w_p (a L_p)^2}{I_p / c_p} \\ &= \frac{w_p L_p^2 \times [(\alpha a + \frac{1}{2} a - \beta a) - \alpha - \frac{1}{2} a^2]}{I_p / c_p} \end{aligned} \quad (9.01)$$

The stress at the corresponding spot in the model is expressed as follows:

$$\begin{aligned} f_m &= \frac{M_m c_m}{I_m} = \frac{M_m}{I_m / c_m} = \frac{R_{Am} x_m - M_{Am} - \frac{1}{2} w_m x_m^2}{I_m / c_m} \\ &= \frac{[(M_{Am} + \frac{1}{2} w_m L_m^2 - M_{Bm}) \times \frac{1}{L_m} \times x_m] - M_{Am} - \frac{1}{2} w_m x_m^2}{I_m / c_m} \end{aligned}$$

$$\begin{aligned}
&= \frac{[(\alpha w_m I_m^2 + \frac{1}{2} w_m I_m^2 - \beta w_m I_m^2) \times \frac{1}{I_m} \times a I_m] - \alpha w_m I_m^2 - \frac{1}{2} w_m (a I_m)^2}{I_m/c_m} \\
&= \frac{w_m I_m^2 \times [(\alpha a + \frac{1}{2} a - \beta a) - \alpha - \frac{1}{2} a^2]}{I_m/c_m} \quad (9.02)
\end{aligned}$$

The relation between  $f_p$  and  $f_m$  can be obtained by dividing (9.01) by (9.02)

$$\begin{aligned}
\frac{f_p}{f_m} &= \frac{w_p I_p^2 \times [(\alpha a + \frac{1}{2} a - \beta a) - \alpha - \frac{1}{2} a^2]}{I_p/c_p} \times \frac{I_m/c_m}{w_m I_m^2 \times [(\alpha a + \frac{1}{2} a - \beta a) - \alpha - \frac{1}{2} a^2]} \\
&= \frac{w_p}{w_m} \times \frac{I_p^2}{I_m^2} \times \frac{I_m/c_m}{I_p/c_p} \quad (9.03)
\end{aligned}$$

In which  $w_p$  = Load in lbs. per ft. of span for the prototype

$$= 80 \text{ lbs. per sq. ft.} \times 16 \text{ ft.}$$

$w_m$  = Load in lbs. per in. of span for the model

$$= \frac{40}{12.8^2 - 0.933^2} \times 12.8$$

$I_p$  = Span of the member in ft. for the prototype

$$= \frac{15}{12} I_m$$

$I_m$  = Span of the member in in. for the model

Substitute these values in (9.03)

$$\begin{aligned}
\frac{f_p}{f_m} &= \frac{80 \times 16}{\frac{40}{12.8^2 - 0.933^2} \times 12.8} \times \frac{(\frac{15 I_m}{12})^2 \times 12}{I_m^2} \times \frac{I_m/c_m}{I_p/c_p} \\
&= \frac{80 \times 16 \times 15^2 \times 12 \times (12.8^2 - 0.933^2)}{40 \times 12.8 \times 12^2} \times \frac{I_m/c_m}{I_p/c_p} \\
&= 7639.22 \times \frac{I_m/c_m}{I_p/c_p} \quad (9.04)
\end{aligned}$$

$$\text{or} \quad f_p = 7639.22 \times \frac{I_m/c_m}{I_p/c_p} \times f_m \quad (9.05)$$

The formula 9.03 can be used for a model of any scale, assuming that both model and prototype are made of homogeneous, elastic materials. Formula 9.05 is for the scale relation used in these tests and for homogeneous elastic materials. For the case under consideration the prototype is built of reinforced concrete and therefore these formulas must be rewritten using the properties of the transformed section for the prototype:

$$\frac{f_{pc}}{f_m} = \frac{w_p}{w_m} \times \frac{L_p^2}{L_m^2} \times \frac{\frac{I_m}{C_m}}{\frac{I_{pc}}{C_{pc}}} \quad (9.06)$$

$$f_{pc} = 7639.22 \times \frac{\frac{I_m}{C_m}}{\frac{I_{pc}}{C_{pc}}} \times f_m \quad (9.07)$$

In which  $f_{pc}$  = The unit stress of the extreme concrete fiber at the section of the prototype considered

$I_{pc}$  = The moment of inertia of the transformed section

$C_{pc}$  = The distance from the neutral axis to the extreme concrete fiber

As for the stress  $f_{ps}$  in the steel of the prototype it can be derived from the above formula (9.07). The relationship between  $f_{ps}$  and  $f_{pc}$  can be expressed as follows:

$$\frac{\frac{f_{ps}}{n}}{f_{pc}} = \frac{C'_{pc}}{C_{pc}}$$

$$\text{or} \quad f_{ps} = n f_{pc} \frac{C'_{pc}}{C_{pc}} \quad (9.08)$$

In which  $C'_{pc}$  is the distance from the neutral axis to the centroid of the steel in the reinforced concrete section considered. Substitute the value of  $f_{pc}$  from formula (9.07) into formula (9.08), using  $n = 10$ .

$$\begin{aligned}
 f_{ps} &= 10 \times 7639.22 \times \frac{\frac{I_m}{C_m}}{\frac{I_{pc}}{C_{pc}}} \times f_m \times \frac{1}{\frac{C_{pc}}{C'_{pc}}} \\
 &= 76392.2 \times \frac{\frac{I_m}{C_m}}{\frac{I_{pc}}{C'_{pc}}} \times f_m \quad (9.09)
 \end{aligned}$$

The stresses in the prototype can be calculated by means of formulas (9.03) and (9.06) for the general case or by use of formulas (9.05), (9.07) and (9.09) for the special case under consideration in this thesis. In applying the formulas (9.05), (9.07) and (9.09) the factors  $I$  and  $c$  are known quantities and  $f_m$  is obtained through tests and formulas as shown in section 7.1.

## 9.2. The Relation Between Prototype Stress and Calculated Stress for Homogeneous Materials

In this section relations are developed between the prototype stresses  $f_p$  and the calculated stresses  $F_p$  for homogeneous material. As previously defined, the term "calculated stress" means the stress in the phototype calculated using approximate methods of frame analysis.

The formula (9.05) is

$$f_p = 7639.22 \times \frac{\frac{I_m}{C_m}}{\frac{I_p}{C_p}} \times f_m \quad (9.05)$$

and 
$$F_p = \frac{M}{\frac{I_p}{C_p}} \quad (9.10)$$

The ratio of  $f_p$  to  $F_p$  is obtained by dividing formula (9.05) by formula (9.10).

$$\begin{aligned}
 \frac{f_p}{F_p} &= \frac{7639.22 \times \frac{I_m}{C_m} \times f_m}{\frac{M}{\frac{I_p}{C_p}}} \\
 &= \frac{7639.22 \times \frac{I_m}{C_m} \times f_m}{M} \quad (9.11)
 \end{aligned}$$

The terms  $\frac{I_p}{C_p}$  in both formulas (9.05) and (9.10) are the same because the same section of the same member in the prototype is considered. The terms in the formula (9.11) have the meanings as defined in section 9.1.

For the case of perfect test results and perfect methods of analysis the ratio  $\frac{f_p}{F_p}$  would be unity. A value of unity could also mean that both the model tests and the calculations involved the same percentage of net error. It should be expected that the ratio  $\frac{f_p}{F_p}$  will either be larger or smaller than unity as a net result of all errors in testing and in the frame analysis.

#### Example 9.01

For the purpose of illustrating the application of formula (9.11) let the ratio  $\frac{f_p}{F_p}$  at point 111 due to the third loading condition be required. The location of this point is at the center of the interior panel of the slab.

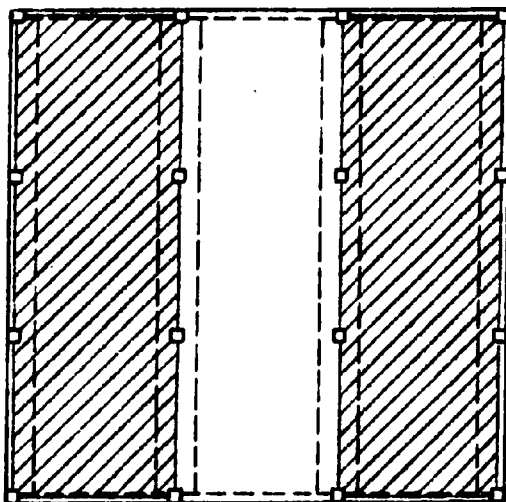
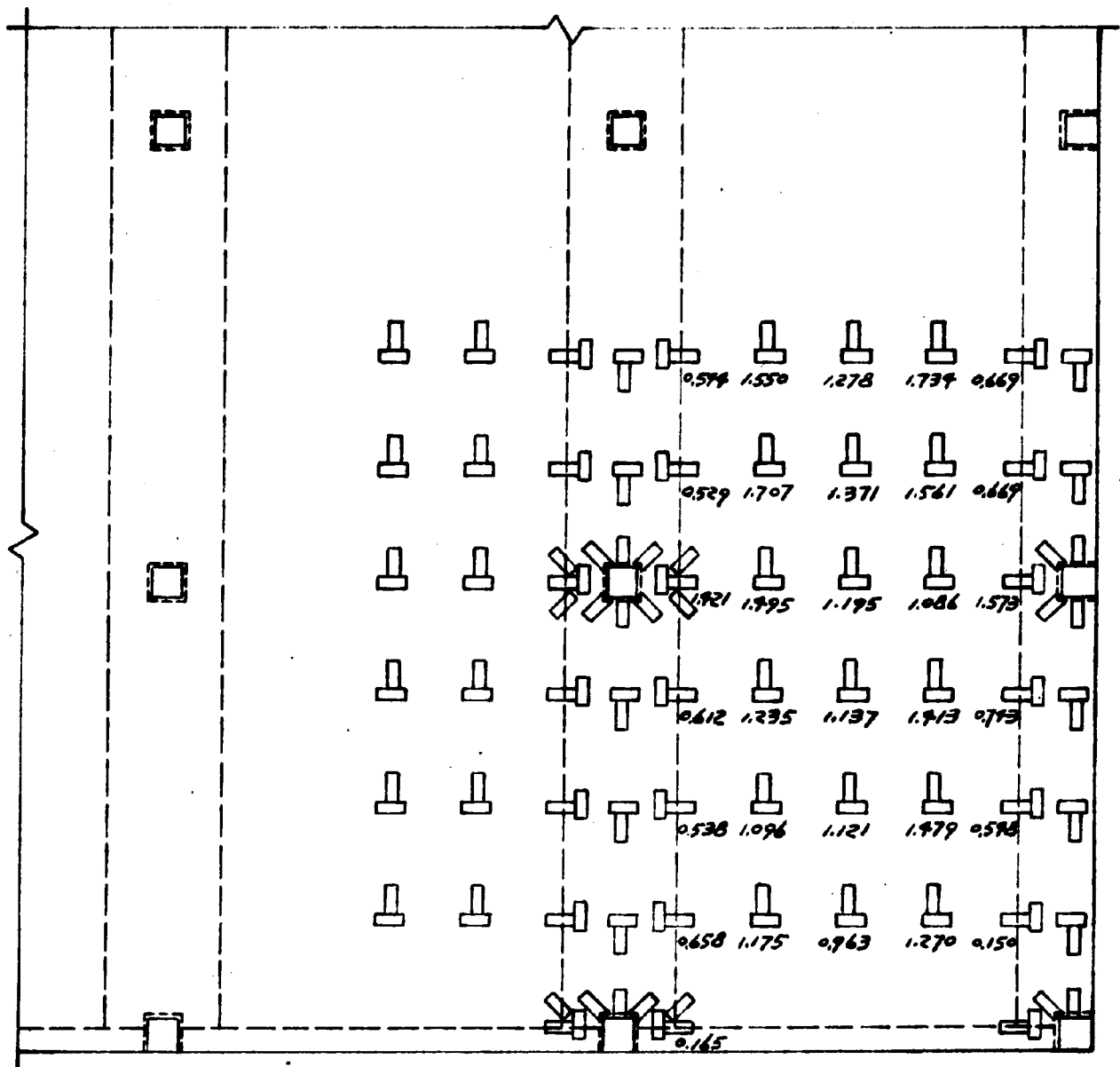
$$I_m = \frac{12.8 \times 0.3^3}{12} = 0.0288 \text{ in.}^4$$

$$c_m = \frac{1}{2} \times 0.3 = 0.15 \text{ in.}$$

$$\frac{I_m}{c_m} = \frac{0.0288}{0.15} = 0.192 \text{ in.}^3$$

$$f_m = 114.24 \text{ \#/in.}^2$$

$$M = 173,412 \text{ \#}$$



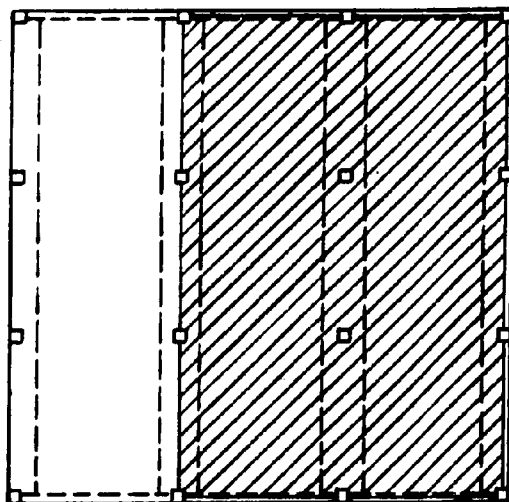
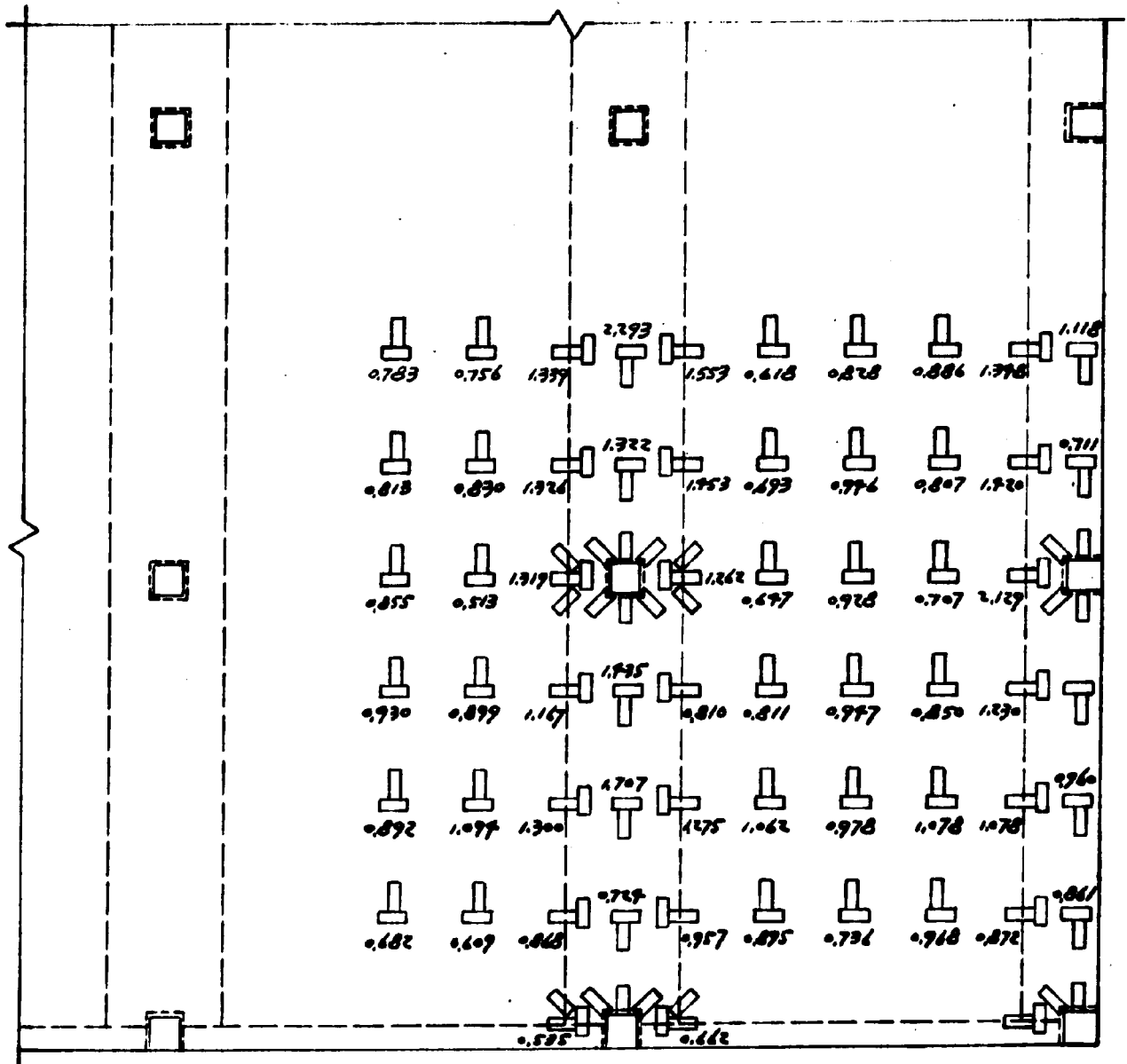
KEY

Loaded Areas Are Shaded

Diagram 9.01.

Ratio  $\frac{f_p}{F_p}$  for the Slab $f_p$  = Prototype Stress $F_p$  = Calculated Stress

Loading Condition No. 1



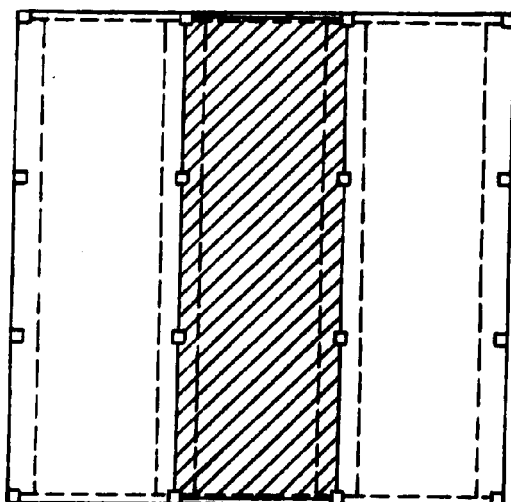
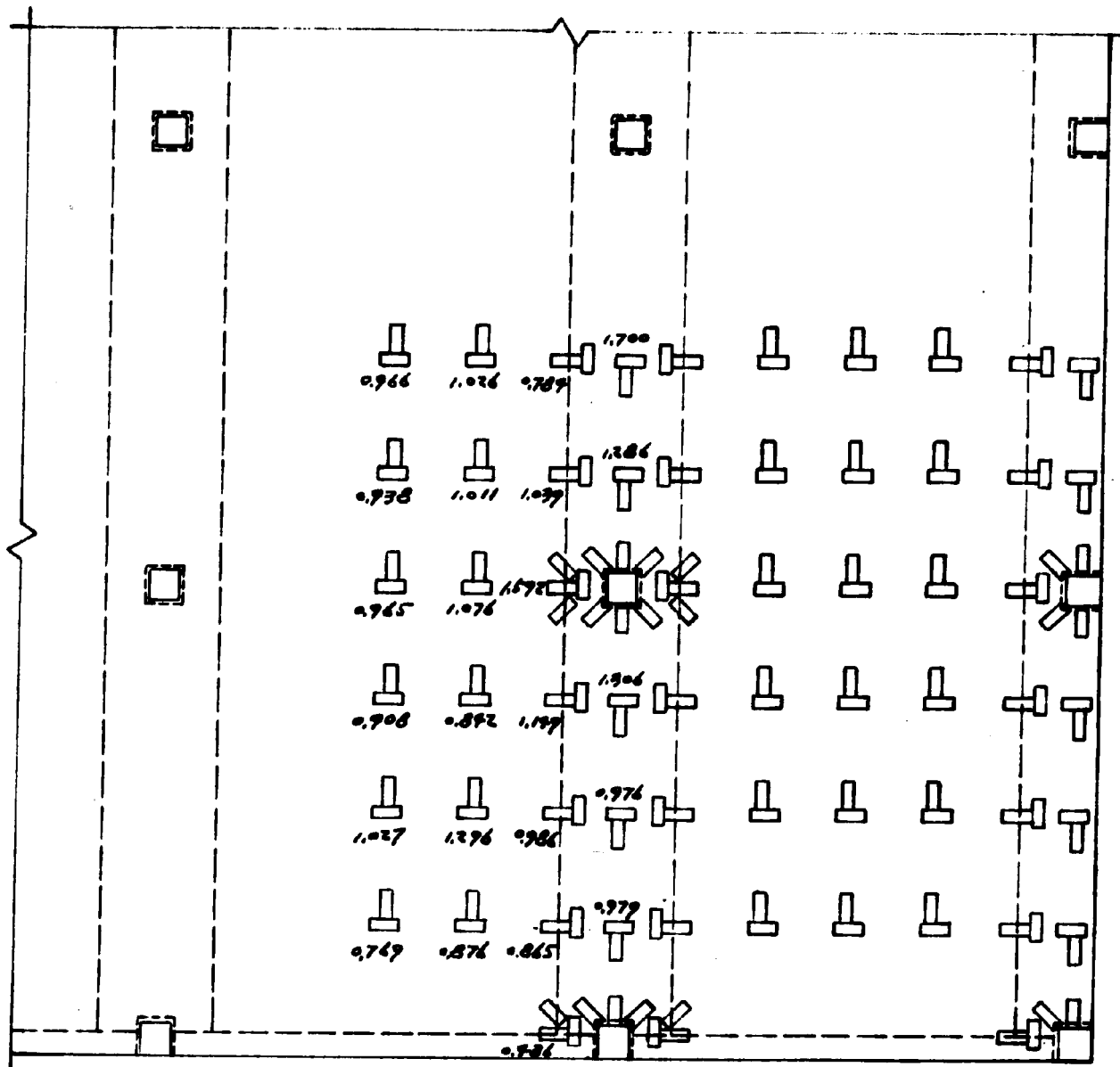
KEY

Loaded Areas Are Shaded

Diagram 9.02.

Ratio  $\frac{f_p}{F_p}$  for the Slab $f_p$  = Prototype Stress $F_p$  = Calculated Stress

Loading Condition No. 2



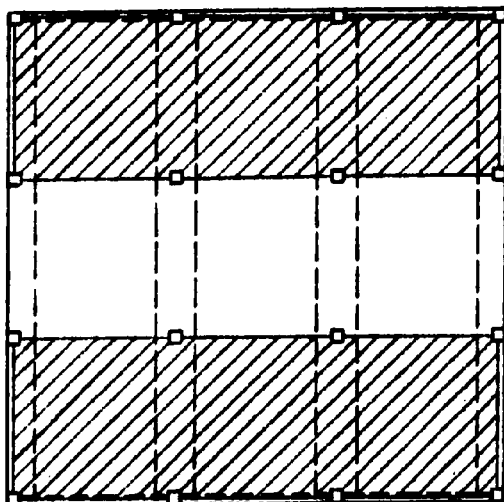
KEY

Loaded Areas Are Shaded

Diagram 9.03.

Ratio  $\frac{f_p}{F_p}$  for the Slab $f_p$  = Prototype Stress $F_p$  = Calculated Stress

Loading Condition No. 3

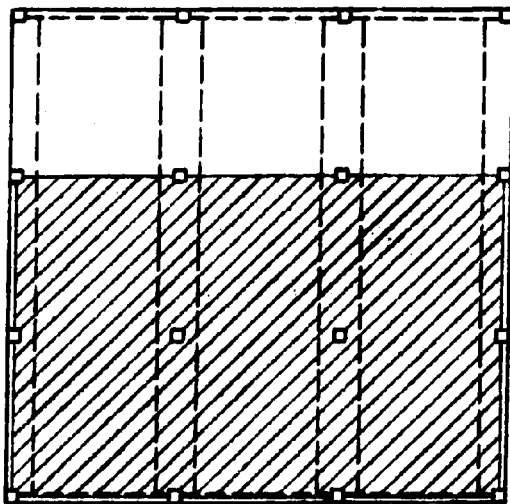


**Loaded Areas Are Shaded**

Ratio  $\frac{F_D}{F_P}$  for the Slab-Bands

$F_p$  = Calculated Stress

### Loading Condition No. 4

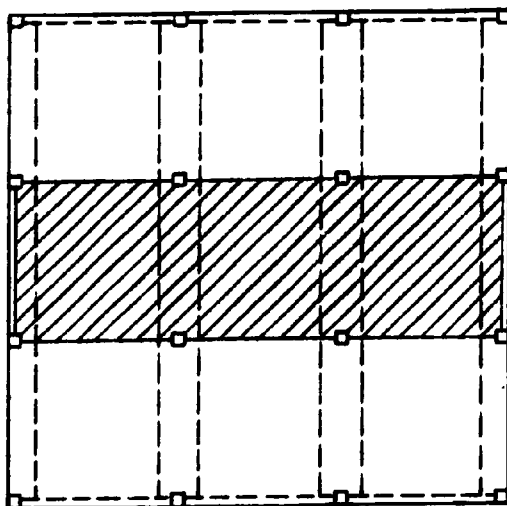
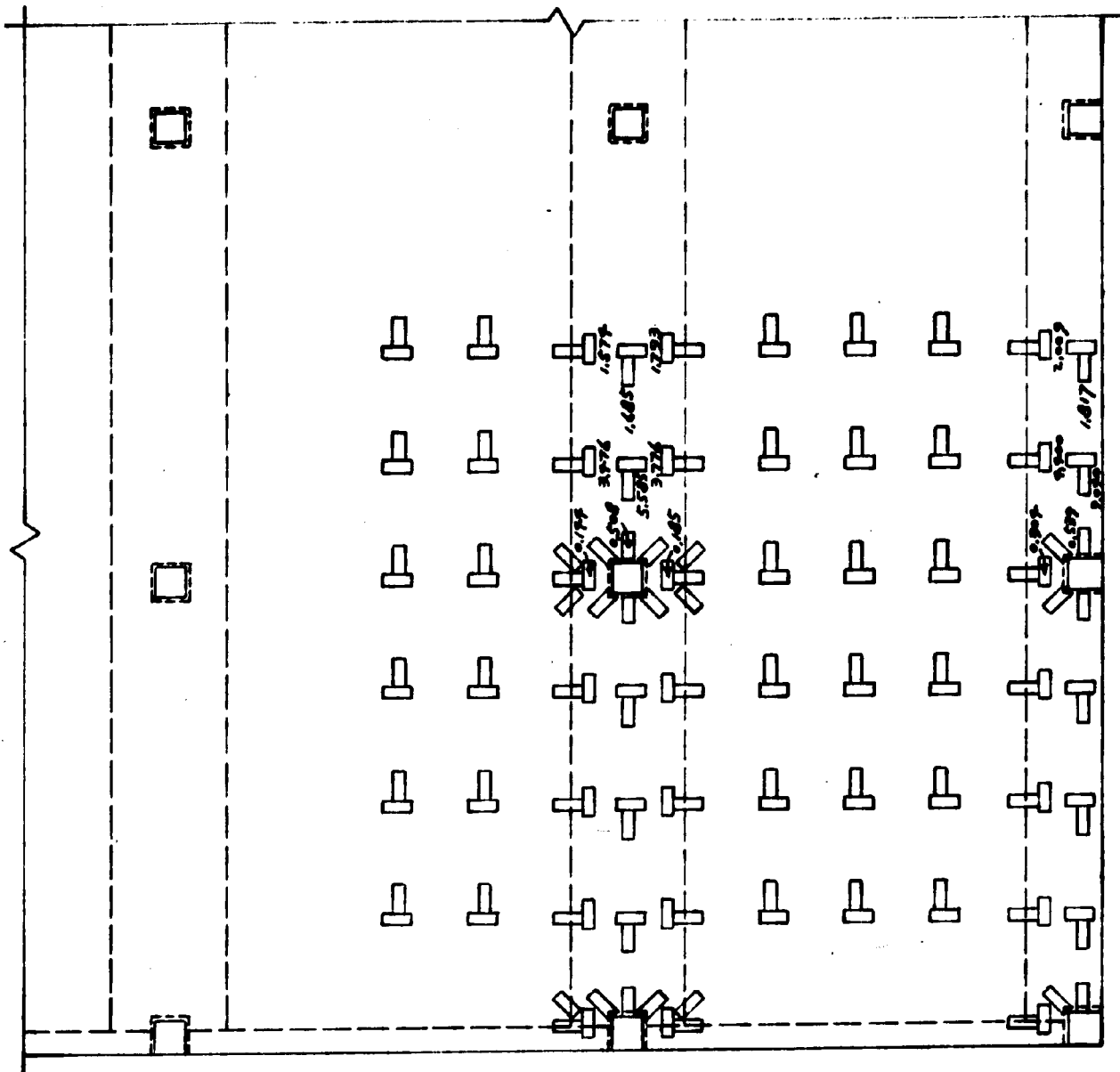


**Loaded Areas Are Shaded**

Ratio  $\frac{f_p}{F_p}$  for the Slab-Bands

**$F_p$  = Calculated Stress**

### Loading Condition No. 5



KEY

Loaded Areas Are Shaded

Diagram 9.06.

Ratio  $\frac{f_p}{F_p}$  for the Slab-Bands $f_p$  — Prototype Stress $F_p$  — Calculated Stress

Loading Condition No. 6

Substitute these values into formula (9.11), we get

$$\frac{f_p}{F_p} = \frac{7639.22 \times 0.192 \times 114.24}{173,412}$$

$$= 0.966$$

The diagrams 9.01, 9.02, and 9.03 show the  $\frac{f_p}{F_p}$  values at different points due to different loadings in the slab while the diagrams 9.04, 9.05 and 9.06 show those in the slab-bands.

### 9.3. The Stresses in Slab and Slab-bands of the Reinforced Concrete Building as Calculated from the Results of Model Tests

The compressive stresses in the concrete and the tensile stresses in the steel in the slab and the slab-bands of the prototype reinforced concrete building according to the results of tests of model can be calculated by means of formulas (9.07) and (9.09). For the purpose of illustrating the application of these formulas the following two examples are given.

#### Example 9.02

The point lll at the center of the interior panel of the slab is considered and the maximum concrete fiber stress which was produced by loading condition No. 3 is required.

$$I_m = \frac{12.8 \times 0.3^3}{12} = 0.0288 \text{ in.}^4$$

$$c_m = \frac{1}{2} \times 0.3 = 0.15 \text{ in.}$$

$$\frac{I_m}{c_m} = \frac{0.0288}{0.15} = 0.192$$

$$f_m = 114.24 \text{ \#/in.}^2$$

$$I_{pc} = 448.905$$

$$C_{pc} = 1.239$$

$$\frac{I_{pc}}{C_{pc}} = 362.31$$

Substituting these values into formula (9.07), we get

$$\begin{aligned} f_{pc} &= 7639.22 \times \frac{0.192}{362.31} \times 114.24 \\ &= 462.5 \text{ \#/in.}^2 \end{aligned}$$

#### Example 9.03

The point 113 at the edge of the interior slab-band in the interior panel of the slab is considered and the maximum steel stress which was produced by loading condition No. 2 is required.

$$I_m = 0.0288 \text{ in.}^4$$

$$c_m = 0.15 \text{ in.}$$

$$\frac{I_m}{c_m} = 0.192 \text{ in.}^3$$

$$f_m = 111.49 \text{ \#/in.}^2$$

$$I_{pc} = 361.685$$

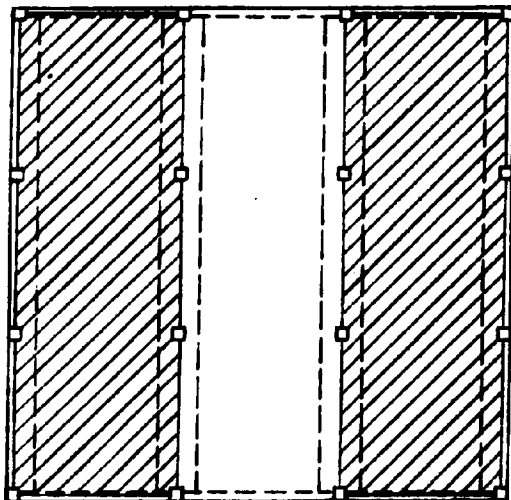
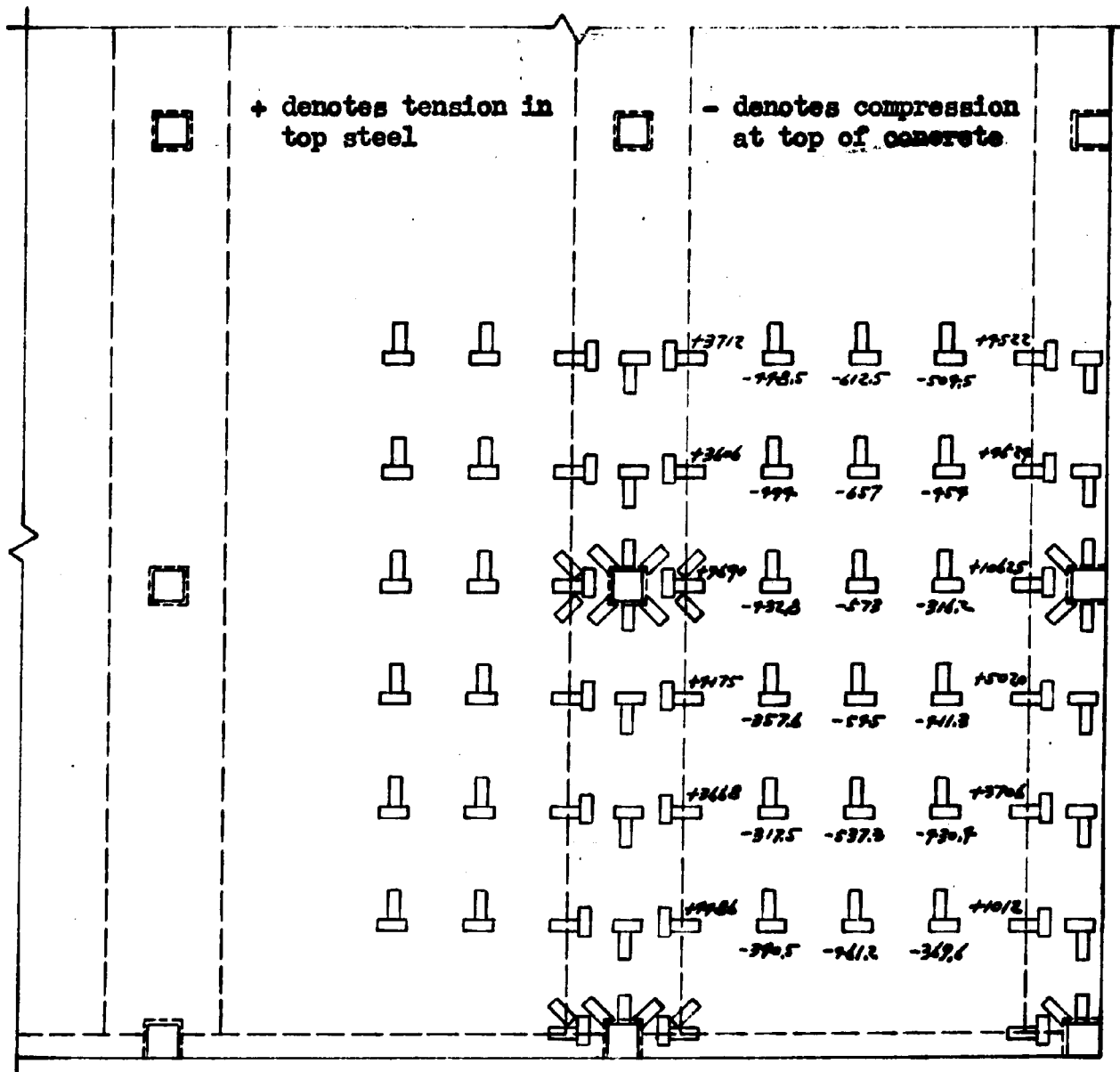
$$C'_{pc} = 2.394$$

$$\frac{I_{pc}}{C'_{pc}} = \frac{361.685}{2.394} = 150.9$$

Substituting these values into formula (9.09), we get

$$\begin{aligned} f_{ps} &= 76392.2 \times \frac{0.192}{150.9} \times 111.49 \\ &= 10,820 \text{ \#/in.}^2 \end{aligned}$$

The compression in the concrete and the tension in the reinforcing steel of the slab and the slab-bands of the prototype due to six different loading conditions were calculated by formulas (9.07) and (9.09) as described above and are recorded on the diagrams 9.07 through 9.12.



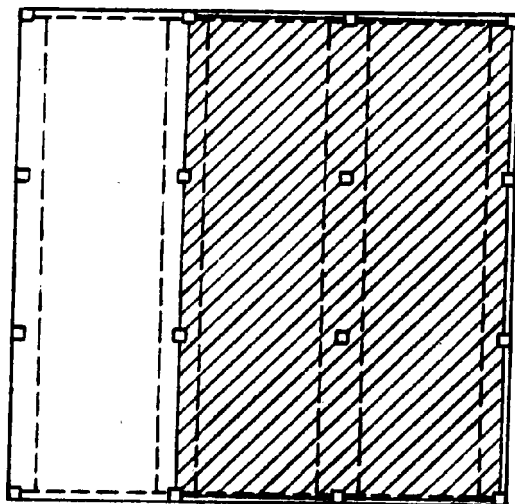
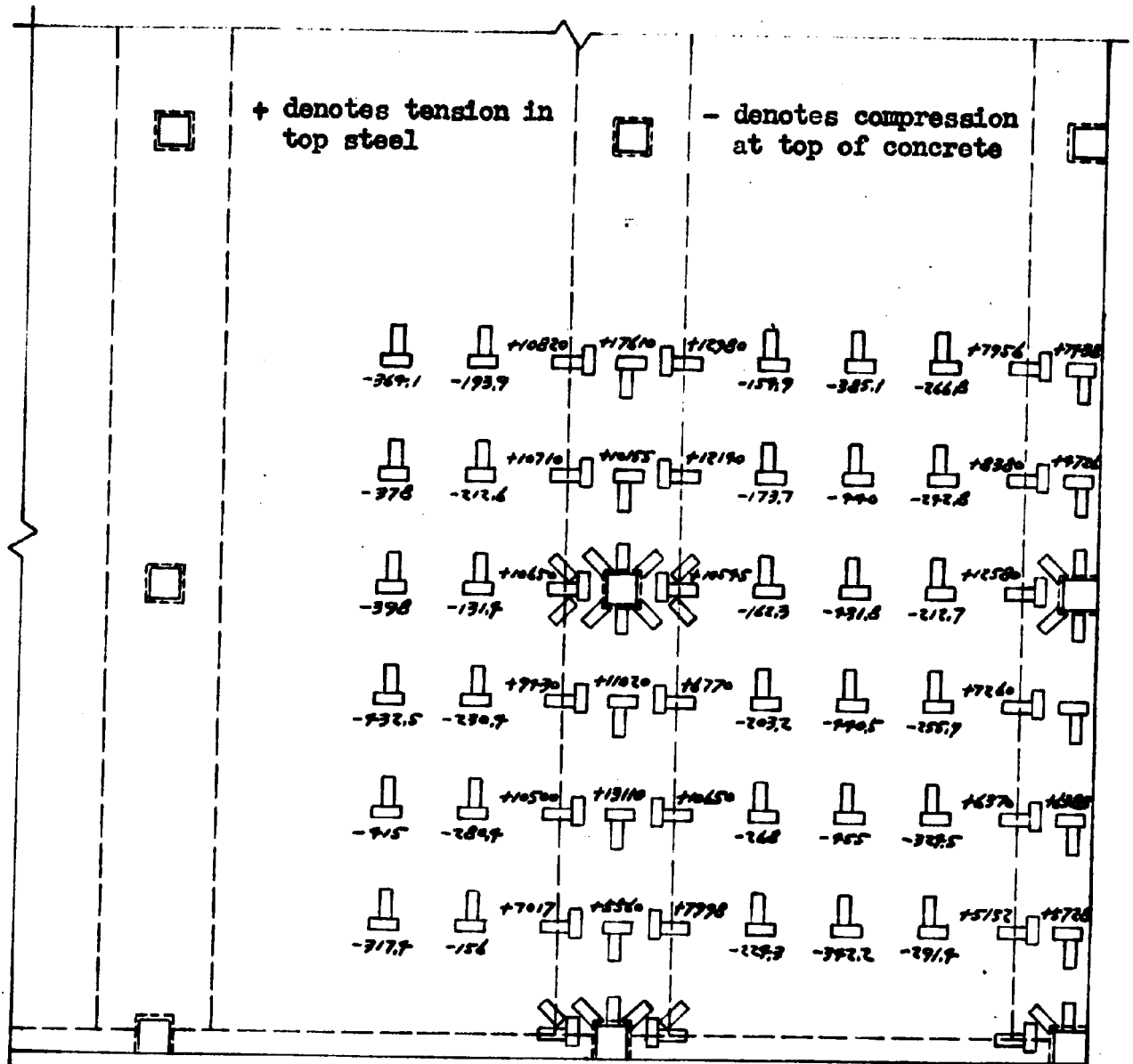
KEY

Loaded Areas Are Shaded

Diagram 9.07.

Prototype Stresses in the Slab  
(Stresses are in lbs. per sq. in.)

Loading Condition No. 1



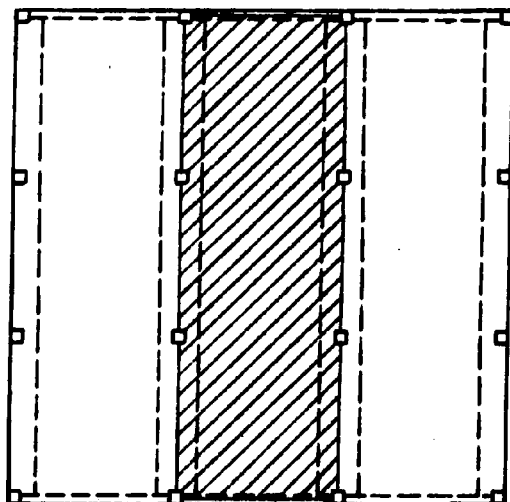
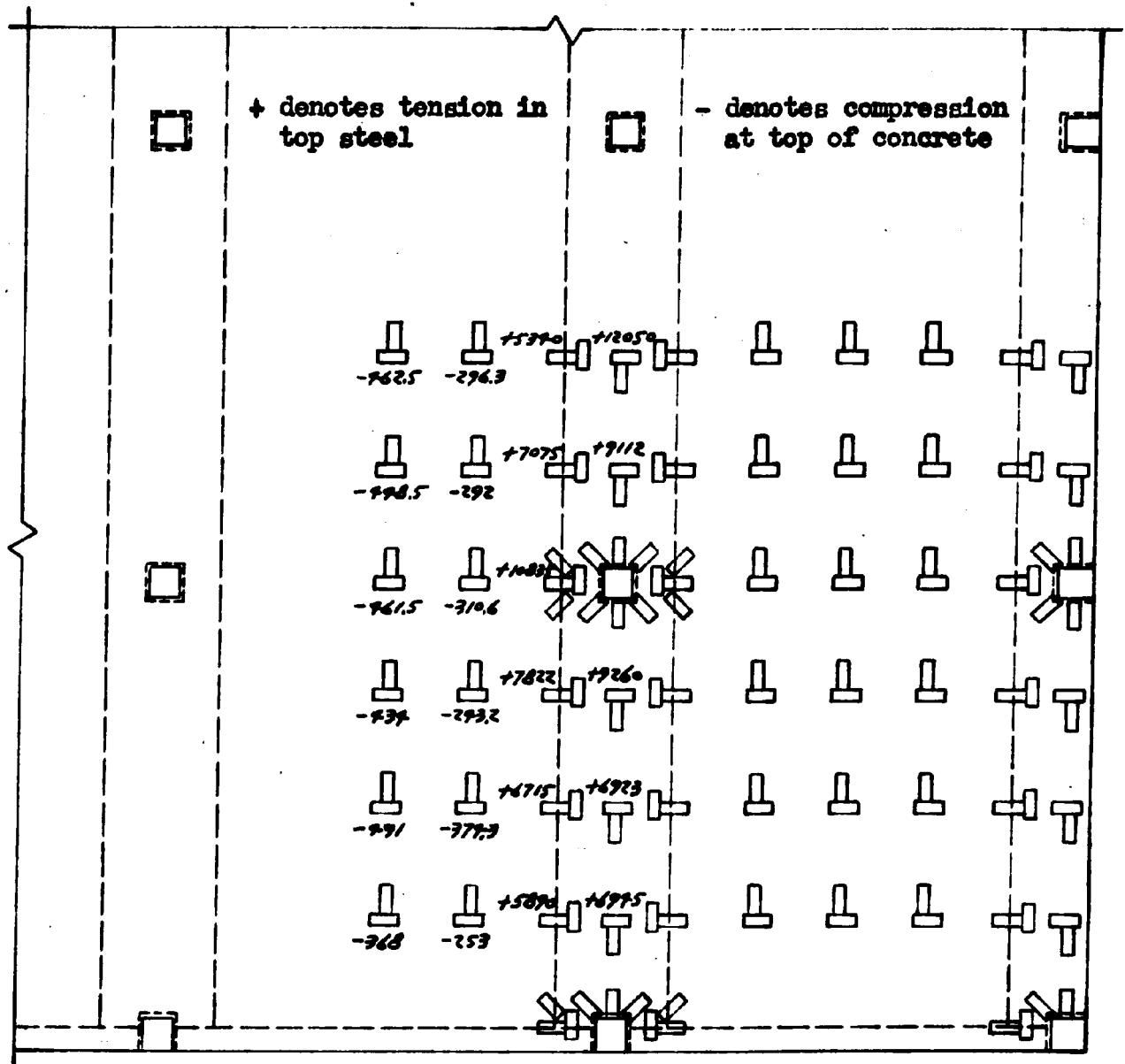
KEY

Loaded Areas Are Shaded

Diagram 9.08.

Prototype Stresses in the Slab  
(Stresses are in lbs. per sq. in.)

Loading Condition No. 2



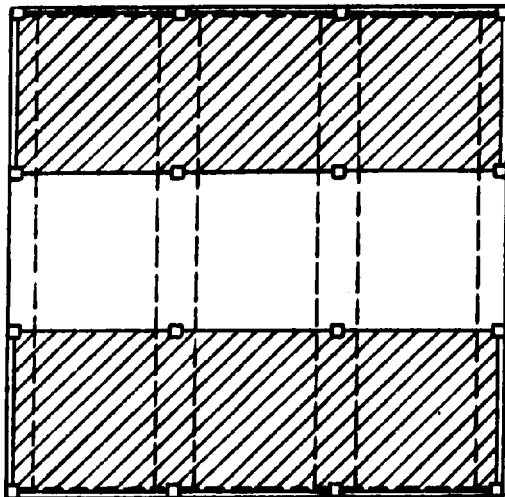
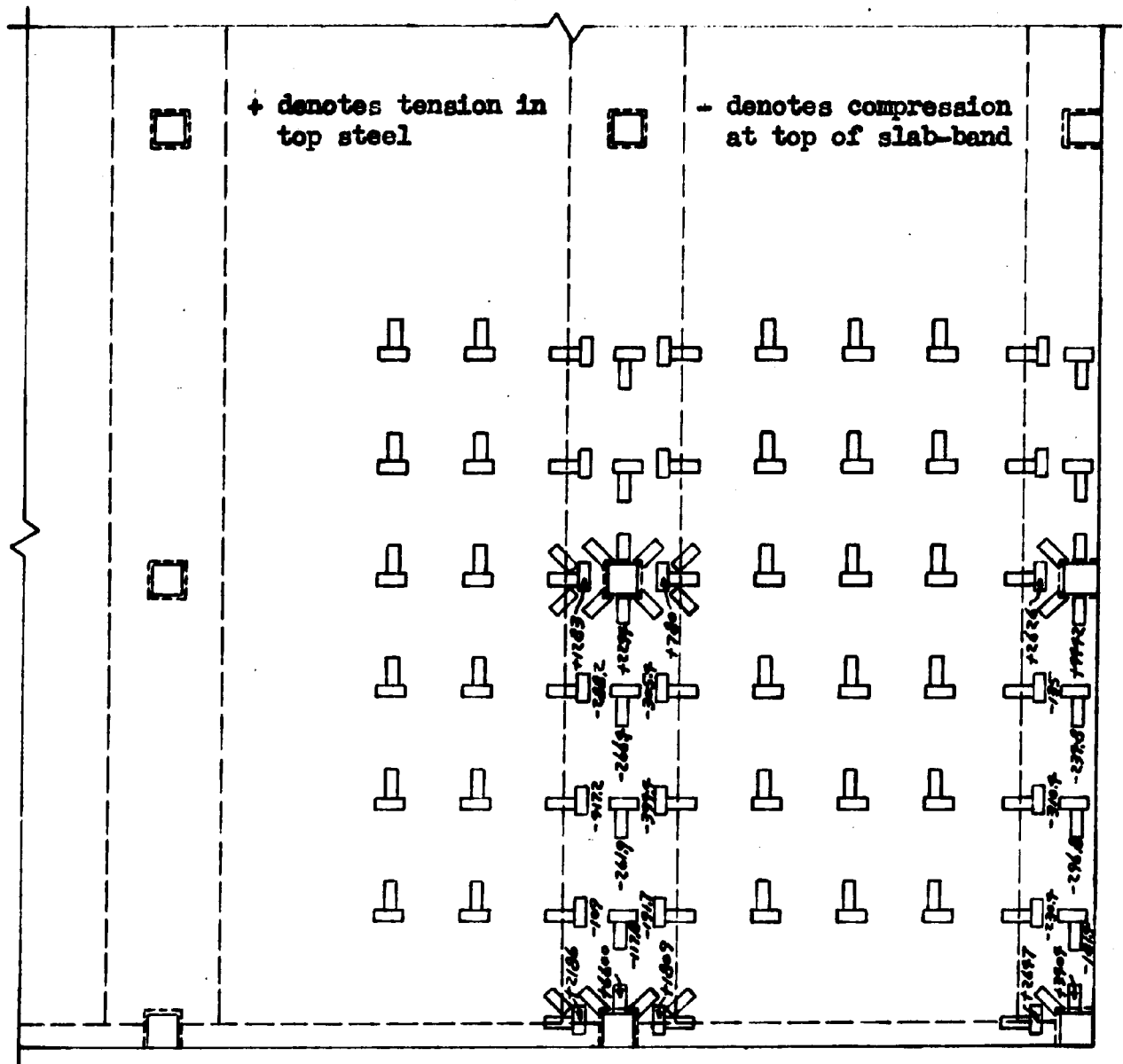
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Loaded Areas Are Shaded

Diagram 9.09.

Prototype Stresses in the Slab  
(Stresses are in lbs. per sq. in.)

Loading Condition No. 3



KEY

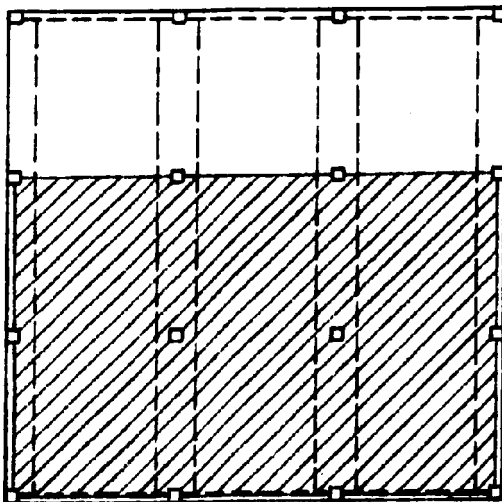
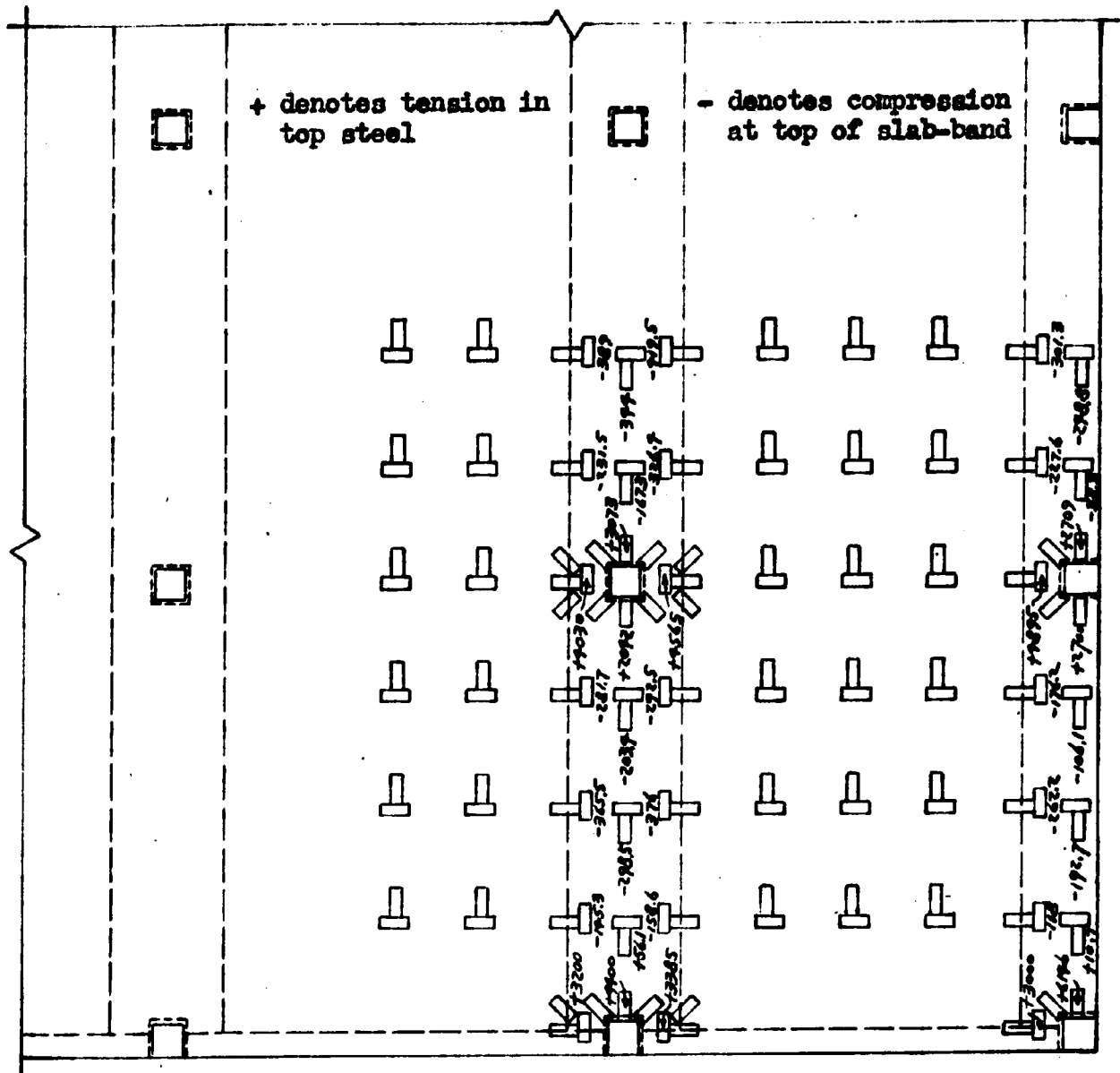
Loaded Areas Are Shaded

Diagram 9.10.

Prototype Stresses in the Slab-Bands

(Stresses are in lbs. per sq. in.)

Loading Condition No. 4



KEY

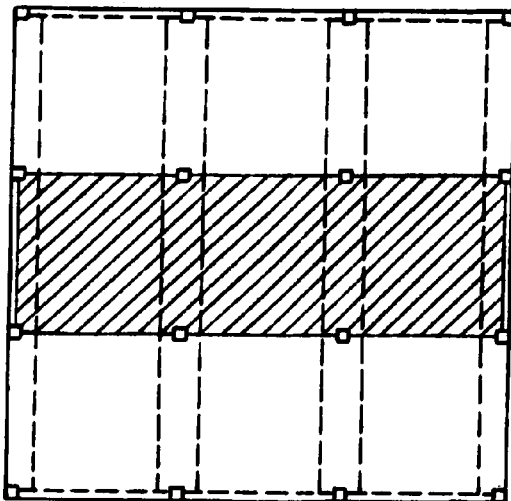
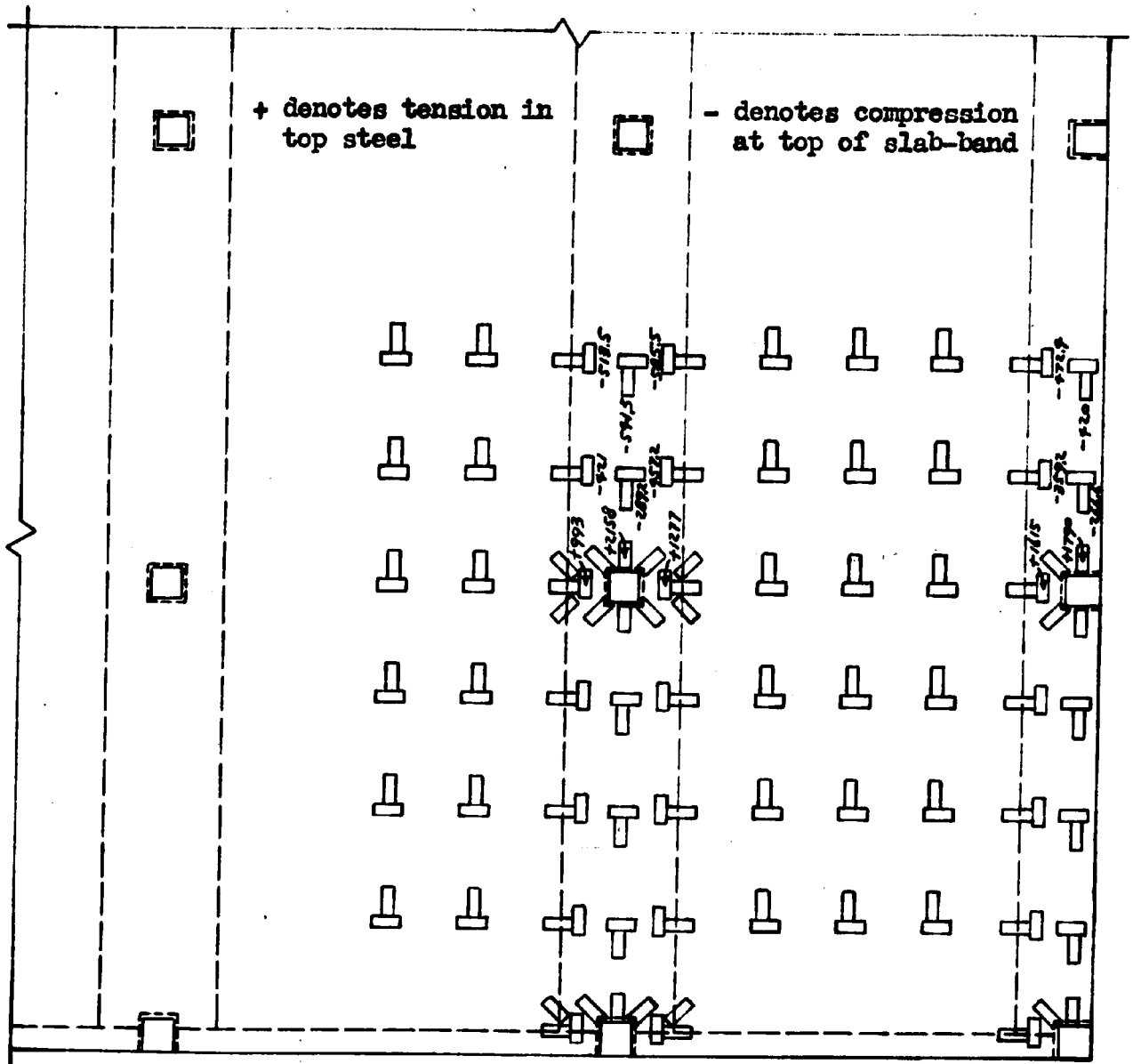
Loaded Areas Are Shaded

Diagram 9.11.

Prototype Stresses in the Slab-Bands

(Stresses are in lbs. per sq. in.)

Loading Condition No. 5



KEY

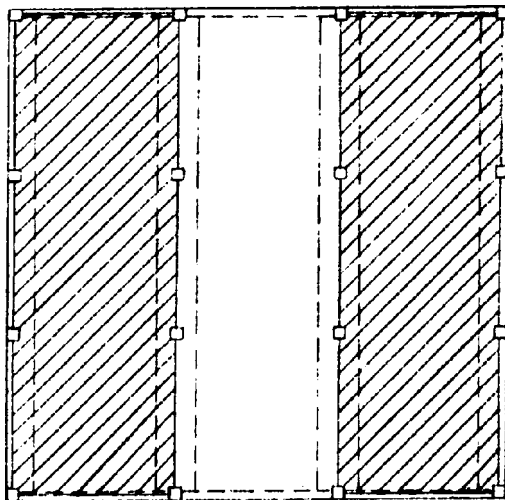
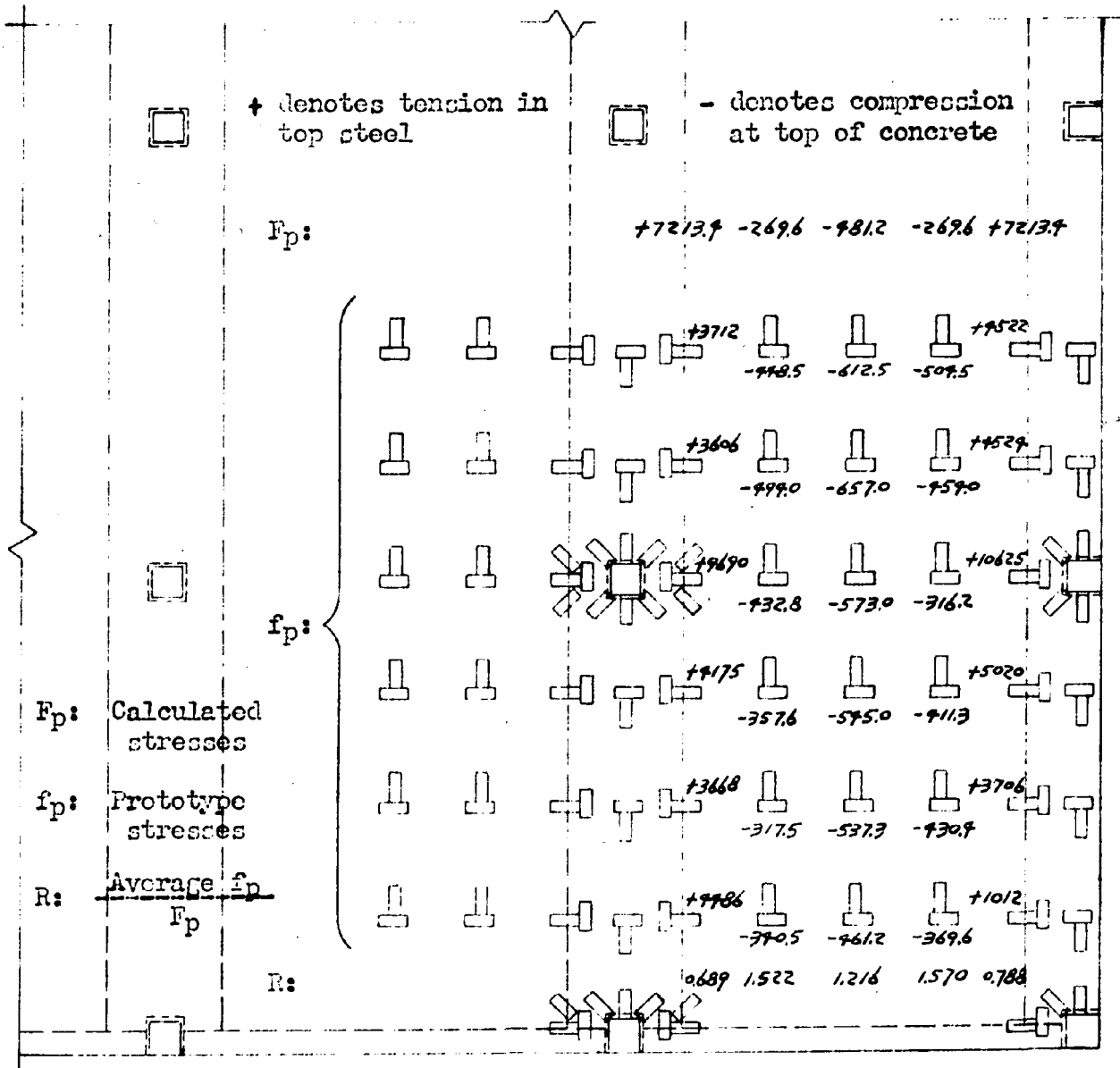
Loaded Areas Are Shaded

Diagram 9.12.

Prototype Stresses in the Slab-Bands

(Stresses are in lbs. per sq. in.)

Loading Condition No. 6



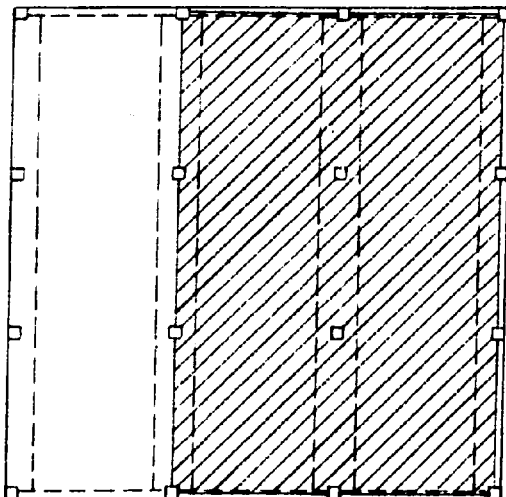
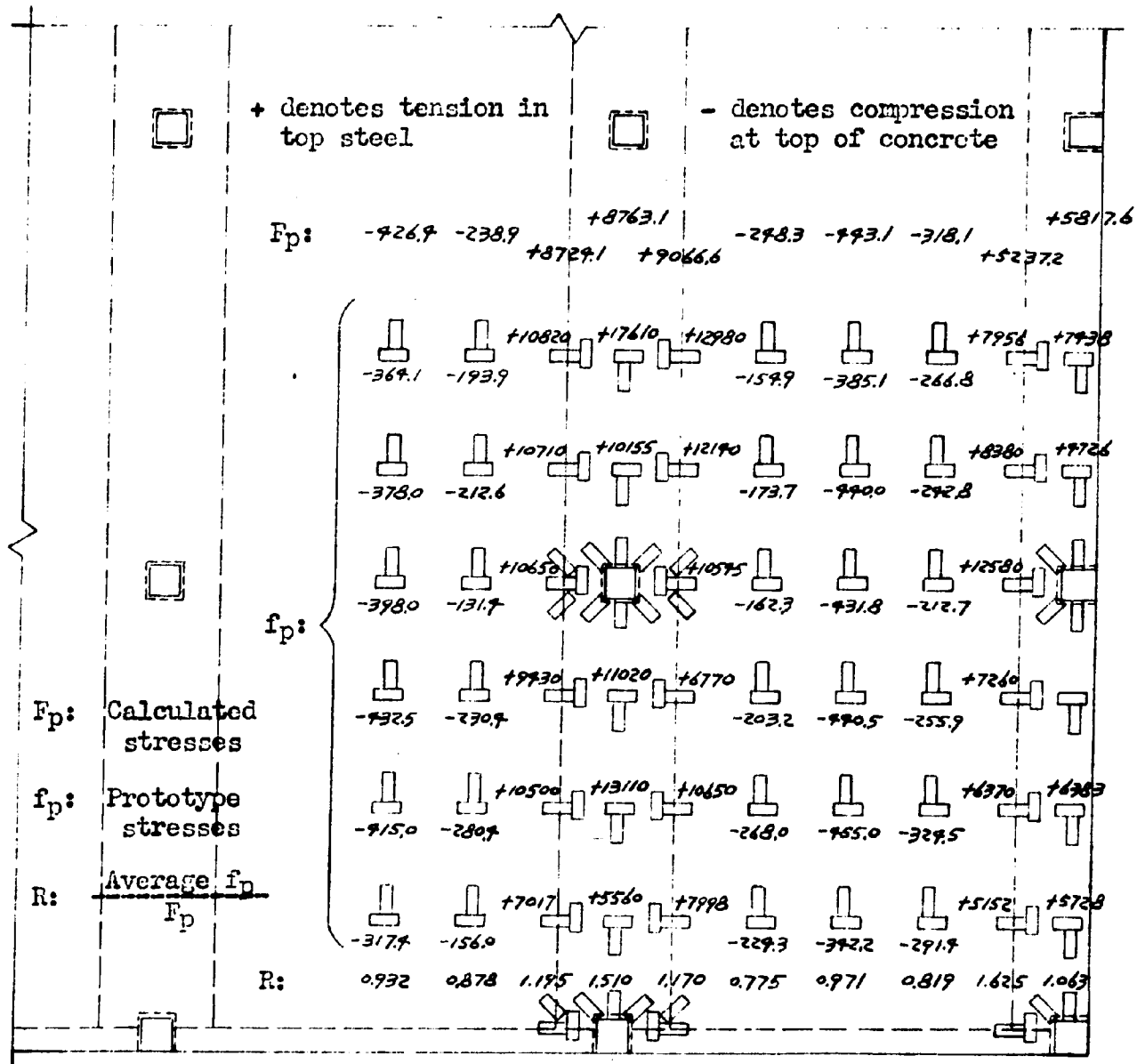
KEY

Loaded Areas Are Shaded

Diagram 9.13.

Comparison of Prototype Stress and  
Calculated Stress in the Slab  
(Stresses are in lbs. per sq. in.)

Loading Condition No. 1



KEY

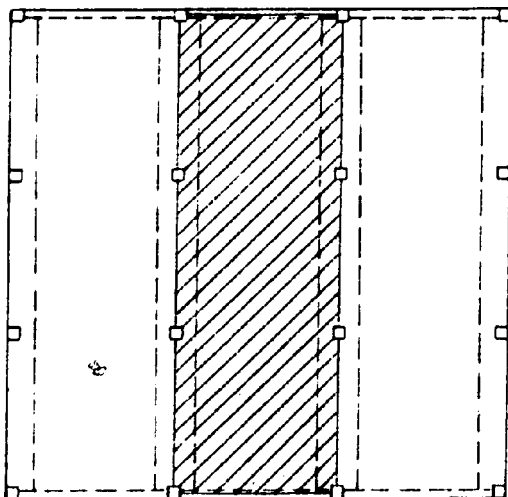
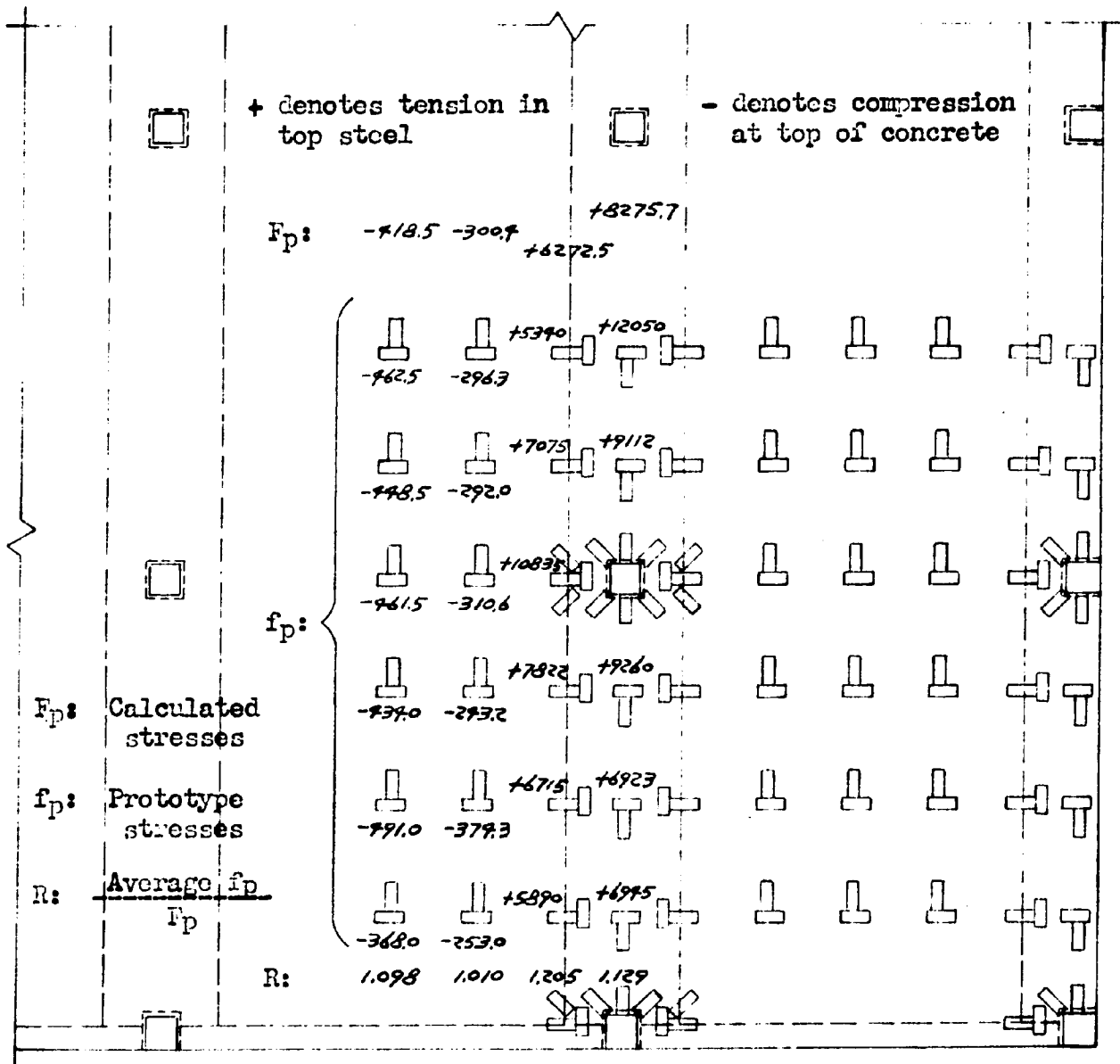
Loaded Areas Are Shaded

Diagram 9.14.

Comparison of Prototype Stress and  
 Calculated Stress in the Slab

(Stresses are in lbs. per sq. in.)

Loading Condition No. 2



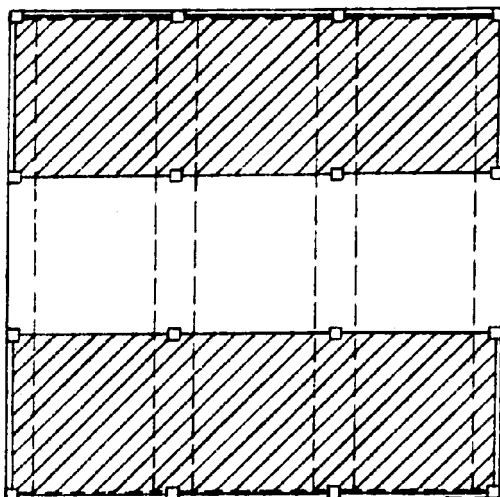
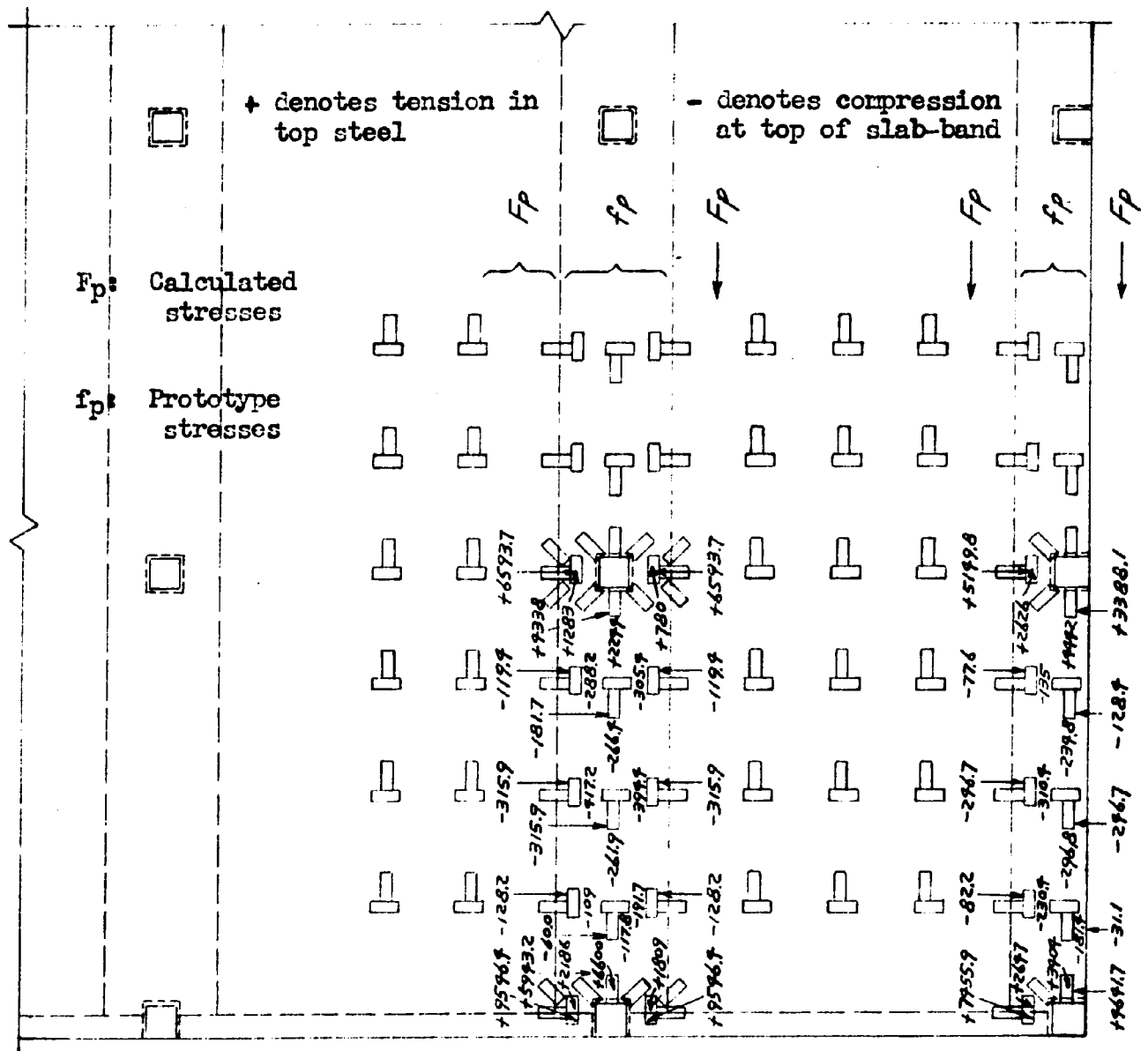
KEY

Loaded Areas Are Shaded

Diagram 9.15.

Comparison of Prototype Stress and  
 Calculated Stress in the Slab  
 (Stresses are in lbs. per sq. in.)

Loading Condition No. 3



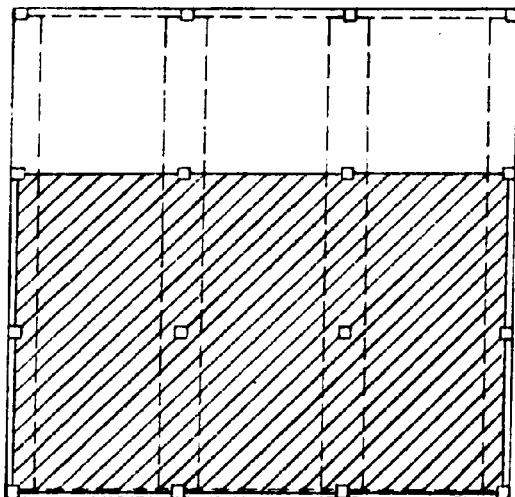
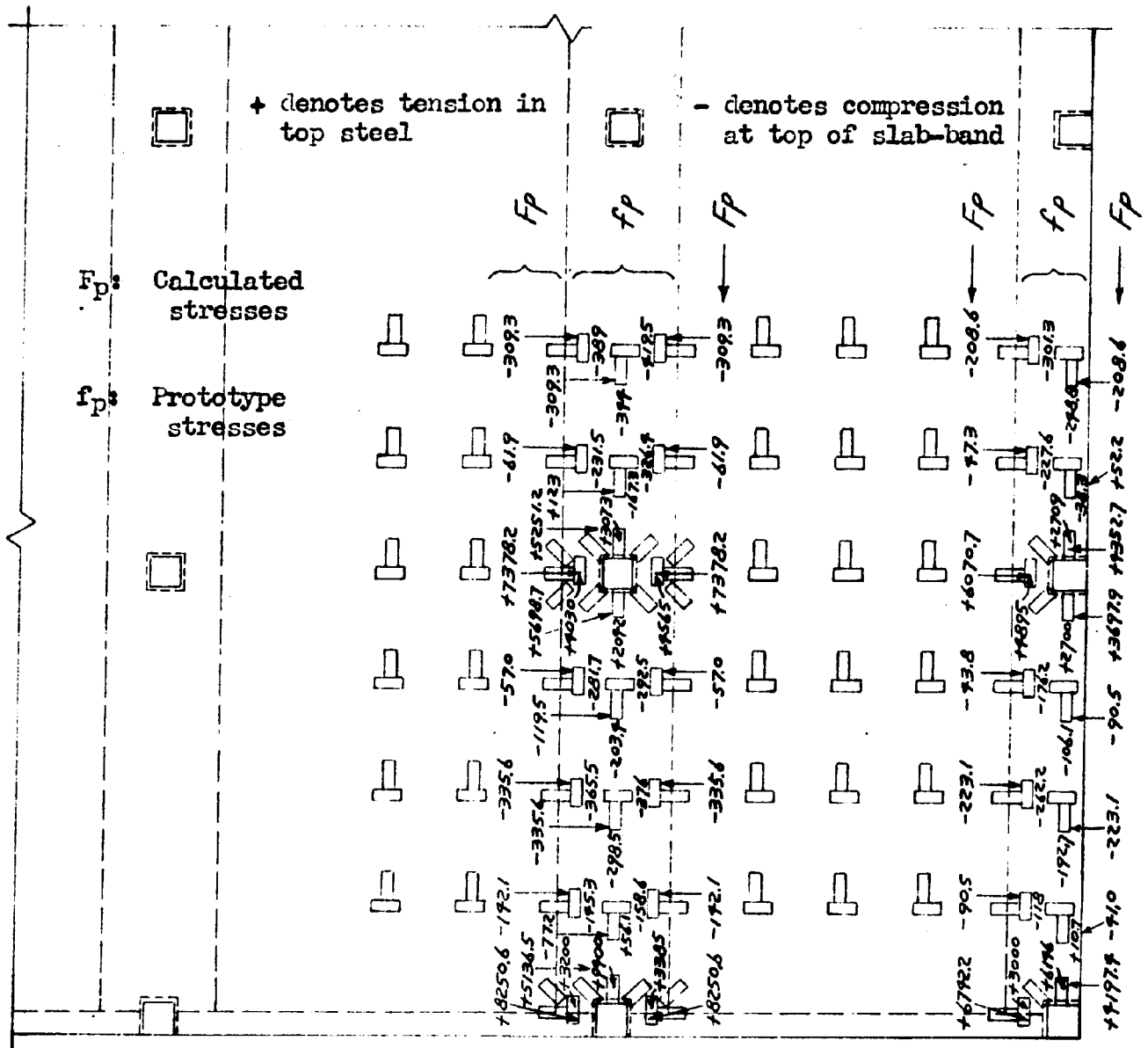
KEY

Loaded Areas Are Shaded

Diagram 9.16.

Comparison of Prototype Stress and  
Calculated Stress in the Slab-Bands  
(Stresses are in lbs. per sq. in.)

Loading Condition No. 4



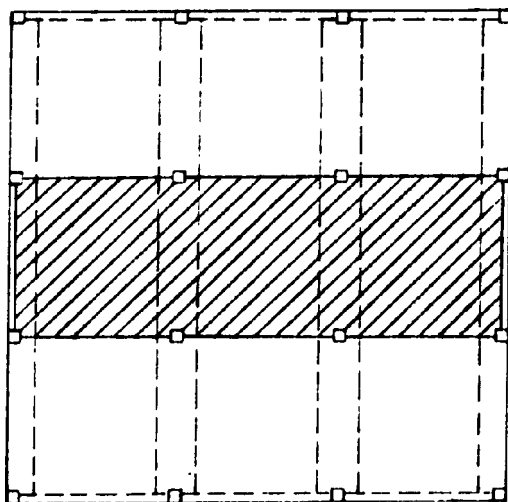
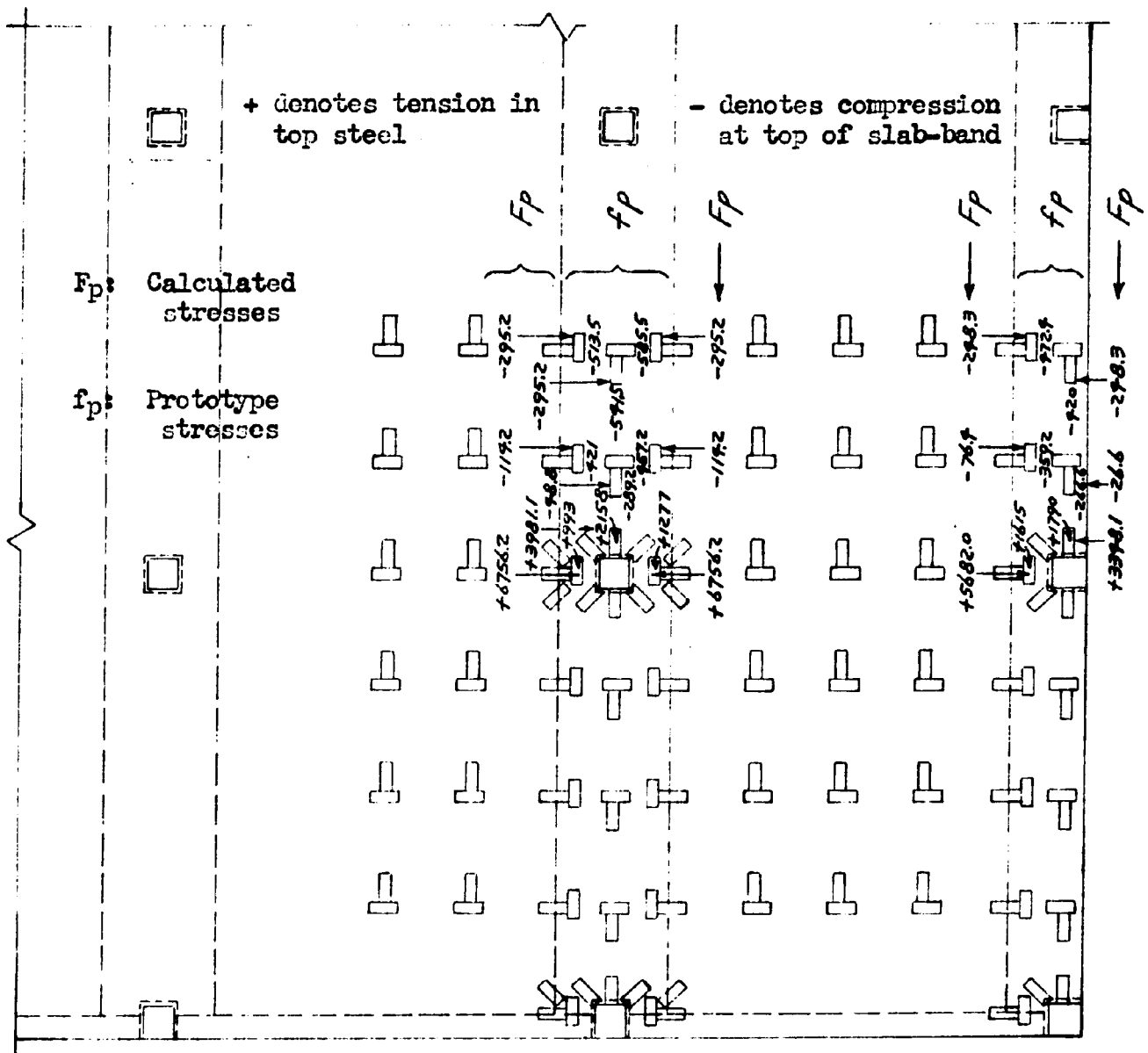
KEY

Loaded Areas Are Shaded

Diagram 9.17.

Comparison of Prototype Stress and  
 Calculated Stress in the Slab-Bands  
 (Stresses are in lbs. per sq. in.)

Loading Condition No. 5



KEY

Loaded Areas Are Shaded

Diagram 9.18.

Comparison of Prototype Stress and  
Calculated Stress in the Slab-Bands  
(Stresses are in lbs. per sq. in.)

Loading Condition No. 6

#### 9.4. Comparison of Experimental and Theoretical Results

The stresses in the slab and the slab-bands of the prototype calculated from the results of model tests are recorded on diagrams 9.07 through 9.12 while the stresses calculated by frame analysis at the corresponding points of the same building are given on diagrams 8.01 through 8.04. For ease of comparison both the prototype and calculated stresses are recorded together on diagrams 9.13 through 9.18.

The following is the comparison:

##### 9.4.1. Stresses in the Slab in the Direction of the Slab Span

Refer to section 7.3.2. before reading the discussion below.

###### A. Test No. 1.

Reference is made to diagram 9.13 for this discussion.

- a. Along 5-line the calculated stress is smaller than the prototype stress at location 135 and is larger than the prototype stresses at other locations. The same comparison can be applied to the stresses along the 9-line. The reason for this was fully explained in sections 7.3.2.A. and 8.6.D.
- b. Along the 1-, 2-, 4-, 5-, and 6-rows the slab at the edges of the slab-bands rotated toward the center of the exterior span without much restraint offered by supporting columns when only the exterior slab span was loaded. The slab stresses along the edges of the slab-bands were reduced and consequently the stresses at the center of the slab were increased. As a result of this fact, the prototype stress along the 7-line is larger than the calculated stress along the same line with one exception at location 167 which was near the wall beam.

c. For the same reason as described in b the calculated stress is smaller than the prototype stresses along the 6-line as well as along the 8-line.

d. For clear comparison the ratio of the average prototype stress to the calculated stress along any one of the 5-, 6-, 7-, 8-, and 9-lines was calculated and recorded at the bottom of the diagram. The prototype stress nearest to the wall beam was not included in the calculation of the average prototype stress.

#### B. Test No. 2

Reference is made to diagram 9.14 for this discussion

a. With a few exceptions, the prototype stresses on the top of the slab were smaller than the corresponding calculated stresses. The prototype stresses on the top of the slab-bands (in the direction of the slab-span) were larger than the corresponding calculated stresses. This means that the slab-band furnishes more restraint to the slab than is indicated by the frame analysis.

b. Along each line the ratio of average  $f_p$  to  $F_p$  was calculated and recorded on the diagram 9.14. The average  $f_p$  value was obtained by considering all the prototype stresses along the line considered except the one which was nearest to the wall beam. These ratios  $\frac{f_p}{F_p}$  showed the differences between prototype stresses and calculated stresses. The difference is small if the ratio is near one and it is large if the ratio is quite different from one.

c. Along 1-line all the prototype stresses were close to the calculated stress except that one at location 161 which was near the wall beam.

The ratio  $\frac{f_p}{F_p}$  at location 141 was 1.014 which indicates a very good agreement between model studies and calculated values.

- d. Along 2-line the ratio  $\frac{f_p}{F_p}$  at location 142 was 1.036 while that at location 132 was 0.55. As a whole the ratio of average  $f_p$  to  $F_p$  was 0.878.
- e. Along 3-line the prototype stresses were nearly uniform and were larger than the calculated stress except that one at location 163 which was near the wall beam. The average prototype stress was nearly 20 per cent larger than the calculated stress.
- f. Along 4-line the prototype stress at location 114 was larger than others and it doubled the calculated stress. The reason for this has been described in section 7.3.2.B.f.
- g. Along 5-line the prototype stresses were larger than calculated stress except those at locations 145 and 165. The prototype stress at location 145 was unreasonably low as compared to others and that at location 165 was near the wall beam.
- h. Along 6-line any of the prototype stress in the exterior panel was about two-thirds of the calculated stress. This was due to the fact that the prototype stresses along the 5-line were larger than the calculated stress. The prototype stress at location 156 was the only one which was larger than the calculated stress along the 6-line.
- i. Along the 7-line the prototype stresses at locations 127, 137, 147, and 157 were very close to the calculated stress. The prototype stress at location 117 was 13.1 per cent smaller than the calculated stress and that at location 167 was small too because of its location near the wall beam.
- j. Along the 8-line the prototype stresses at locations 118, 128, 138, and 148 were about 75 per cent of the calculated stress. This may be explained by noting that the negative bending moments along the

9-line (indicated by measured strains) were larger than the calculated negative bending moment along the same line. The only prototype stress which was larger than the calculated stress along the 8-line was at location 158 and it was 2 per cent larger than the calculated stress.

k. Along 9-line all the prototype stresses were larger than the calculated stress except that one which was nearest to the wall beam. Special attention should be given to the location 139 where the prototype stress was the largest of all. This was due to the restraint of the slab by columns and exterior slab-band.

l. Along 10-line the calculated stress was smaller than the prototype stresses at locations 120 and 160 but larger than those at locations 130 and 170. This was reasonable because the locations 120 and 160 are at the middle of slab-band spans while locations 130 and 170 are at quarter points of the slab-band spans.

### Test No. 3

Reference is made to diagram 9.15 for this discussion

a. Along 1-line the prototype stress is larger than the calculated stress except that at location 161. Because only the center span was loaded these two interior slab-bands twisted toward the center span of the slab and therefore there was more positive bending moment along the 1-line. Excluding the location 161 the minimum  $\frac{f_p}{F_p}$  along the 1-line was 1.037 at location 141 and the maximum  $\frac{f_p}{F_p}$  was 1.172 at location 151. The ratio of average  $f_p$  of values at locations 111 to 151 inclusive to  $F_p$  was 1.098.

b. Along 2-line the calculated stress was nearly equal to the prototype stresses at location 112, 122, and 132. The  $\frac{f_p}{F_p}$  value which was nearest to unity was 0.986 at location 112. The ratio  $\frac{f_p}{F_p}$  at

location 162 was 0.842 because this location is near to the wall beam. The ratio of  $\frac{\text{average fp}}{F_p}$  for points along this line except location 162 was 1.01.

c. Along 3-line the ratio of  $\frac{\text{average fp}}{F_p}$  for points from 113 to 153 inclusive was 1.205. This large ratio was principally due to the large fp value at location 133 which is near the column.

d. Along 4-line the ratio of average  $\frac{fp}{F_p}$  for points from 114, 124, 144, and 154 was 1.129. The reason for the large stress at location 141 was explained in section 7.3.2.B.f.

e. The average of  $\frac{fp}{F_p}$  values along 6-row was 0.875 which was very reasonable.

#### 9.4.2. Stresses in the Interior Slab-Band

Refer to section 7.3.4. before reading the discussion below.

##### A. Test No. 4

Reference is made to diagram 9.16 for this discussion

a. The calculated stress along 7-row was much larger than the prototype stresses at locations 373 and 375. The reason for this is that the columns were assumed to restrain the whole width of the slab-band in the theoretical frame analysis but actually only wall beams restrained two side strips of the slab-band. The wall beams, of course, were not as stiff as columns.

b. The prototype stress at location 374 was larger than the calculated stress because that part of the slab-band was restrained by two columns. In the frame analysis these two columns were assumed to restrain the whole width of the slab-band.

c. At location 354 the prototype stress was smaller than the calculated stress because along the middle strip of the slab-band there was more

end restraint than was estimated in the analysis.

d. Along the side strips of the slab-band there was less actual restraint at both supporting ends than the restraint assumed in the frame analysis and therefore there was more actual positive bending moments at the centers of two side strips than calculated. This partly explains the fact that the prototype stresses at locations 353 and 355 were larger than the calculated stress and that the prototype stresses at locations 343, 345, and 365 were larger than the calculated stresses.

e. At the interior support of the slab-band it was assumed in the frame analysis that the interior columns restrained the whole width of the slab-band. Thus the calculated steel stress at the support was 6594 #/sq in the exterior span and 716 #/sq in the middle span. Actually two side strips of the interior slab-band overhang from the interior columns and the prototype stresses at locations 333 and 335 were between these values since the gauge was on the center line between columns.

f. The prototype stresses at locations 334A and 334B were smaller than the calculated stresses at the corresponding points. This indicated that there was not as much end restraint of the slab-band near the column as was assumed in the frame analysis.

g. Because there was less actual negative bending moment at location 334B than calculated at the same spot, the actual positive bending moment at location 344 was therefore larger than the calculated one at the same spot.

#### B. Test No. 5

Reference is made to diagram 9.17 for this discussion

a. - d. Same as a. - d. in A for test No. 4.

- e. The prototype stress at location 364 was tension while the calculated stress was compression. This was due to the large actual negative bending moment at the end of the middle strip.
- f. The calculated stress along 6-row was smaller than the prototype stresses at location 363 and 365 and that along 4-row was also smaller than the prototype stresses at location 343 and 345. The reason for this was explained in section 9.4.A.d.
- g. The prototype stresses at locations 333 and 335 were smaller than the calculated stresses along 3-row. The reason for this was that the side strips of the slab-band overhang from the column but in the frame analysis the whole width of the slab-band was assumed to be restrained by columns.
- h. Same as f. in section 9.4.2.A.
- i. Because of smaller actual negative bending moments at ends of middle strip in the center span of the slab-band than the calculated negative bending moments at the same places, the actual positive bending moments in the middle strip were consequently larger than the calculated positive bending moments at the corresponding locations.
- j. Same as g. in section 9.4.2.A.
- k. The prototype stresses at locations 313 and 315 were larger than the calculated stresses. It was also true that the prototype stresses at locations 323 and 325 were larger than the calculated ones. The reason is the same as that explained in section 9.4.2.A.d.

#### C. Test No. 6

Reference is made to diagram 9.18 for this discussion

- a. The prototype stresses at locations 333 and 335 were smaller than the calculated stress for the reason that was explained in section 9.4.2.B.g.

- b. The explanation in section 9.4.2.A.f. is also applied here for the fact that the prototype stress at location 334A was smaller than the calculated stress.
- c. The prototype stresses at locations 324 and 314 were larger than the calculated ones at the corresponding locations because of the reason explained in section 3.4.2.B.i..
- d. Same as section 9.4.2.B.k.

#### 9.4.3. Stresses in the Exterior Slab-Band

Refer to section 7.3.5. before the discussion below

##### A. Test No. 4

Reference is made to diagram 9.16 for this discussion

- a. The prototype stress at location 379 was much smaller than the calculated stress. The reason for this was explained in section 9.4.2.A.a.
- b. The prototype stresses at locations 349, 359, and 369 were larger than the corresponding calculated stresses because of the reason explained in section 9.4.2.A.d.
- c. The calculated stresses at location 339 were 5150 #/in<sup>2</sup> in the exterior span and 396 #/in<sup>2</sup> in the middle span while the prototype stress at the same location was 2626 #/in<sup>2</sup> which was a value between these two calculated stresses.
- d. Along the column strip of the exterior slab-band all the prototype stresses were larger than the corresponding calculated stresses except that one at location 380.

##### B. Test No. 5

Reference is made to diagram 9.17 for this discussion

- a. Because of the reason explained in section 9.4.2.A.a. the prototype stress at location 379 was much smaller than the calculated stress.
- b. Because of the reason explained in section 9.4.2.A.b. the prototype stress at location 380 was larger than the calculated stress.
- c. Because of the reason explained in section 9.4.2.A.d. the prototype stresses at locations 349, 359, and 369 were larger than the corresponding calculated stresses.
- d. Because of the large actual negative bending moment at location 380, the prototype stress at location 370 was tension while the calculated stress was compression.
- e. Because of the same reason as in d. the prototype stress at location 360 was smaller than the calculated stress.
- f. The prototype stresses at locations 340A and 340B were smaller than the corresponding calculated stresses. This indicated that there was not as much restraint in the column strip of the exterior slab-band near the intermediate support as it was assumed in the frame analysis.
- g. For the reason given in f. the prototype stresses at locations 320 and 350 were larger than the corresponding calculated stresses and the prototype stress at location 330 was compression instead of tension which was the nature of the calculated stress at that spot.
- h. Because of the reason explained in section 9.4.2.B.g. the prototype stress at location 339 was smaller than the calculated stresses, one of which was at the exterior span and the other was in the middle span.
- i. Because of the reason given in section 9.4.2.A.d. the prototype stresses at locations 319 and 329 were larger than the corresponding calculated stresses.

### C. Test No. 6

Reference is made to diagram 9.18 for this discussion

- a. For the reason explained in section 9.4.2.B.g. the prototype stress at location 339 was smaller than the calculated stress.
- b. Same as section 9.4.3.B.f.
- c. For the reason given in section 9.4.3.B.f. the prototype stresses at location 320 and 330 were larger than the corresponding calculated stresses.
- d. Same as section 9.4.3.B.1.

### 9.5. Fundamental Reasons for the Discrepancies Between Prototype Stresses and Calculated Stresses

The comparison has been made in the last section of the results by model test and those by frame analysis. There were some discrepancies between the results of these two methods and it is the intention of this section to show the fundamental reasons for these discrepancies. Refer first to the section 7.3.1. for the general discussion of the testing results and to the section 8.6 for the discussion of theoretical results.

A. The results obtained from the model tests were already affected by change of lengths of the slab, the slab-bands, the wall beams and the columns and deflection due to shear, while in the theoretical analysis these factors were neglected.

B. Although there are holes in the slabs of the upper and lower floor systems of the model, the thickness of the remaining slabs were increased in order to provide the same moment of inertia as the

solid slab. Incidentally, the reasons for providing holes on these floors were explained in Chapter 3. When the middle floor was loaded during tests the remote ends of the columns were not fixed as assumed in the theoretical analysis and rotated in the way that they would rotate in an actual reinforced concrete building under the equivalent loading conditions.

C. It is difficult to define exactly the action of the slab and slab-bands. This case is further complicated by the presence of columns used to support the slab-bands. Any strip in the direction of the slab span consisting slab and slab-bands differs in structural action from neighboring strips. For example the strip along 1-row consists only of slab and slab-bands while the strip along 3-row consists of columns in addition to slab and slab-bands. Another example is that the strip along the 6-row is quite different from the others because the former is near the edge of the building and the spandrel beams. For a complicated case like this the only way to get a reasonably accurate picture of the magnitude and manner of distribution of stresses is by means of model tests such as those discussed in this paper. Furthermore, it is evident that the assumption in the theoretical analysis that any column in the bent restrains the whole width of the bent is obviously far from the truth.

D. The support conditions of the slab-bands are rather complicated. For the interior slab-bands at the interior columns the middle part of the slab-band is directly supported by the columns while its edges overhang from the columns and at the places of exterior columns the middle part of the slab-band is supported by the columns while its edges are restrained or supported by deep spandrel beams.

For the exterior slab-band the case is nearly the same. Because the model was made to represent exactly these end conditions the results obtained from the testing of the model should give an accurate picture of magnitude and manner of distribution of stresses in the slab-band.

As for the theoretical analysis the whole width of the slab-band was assumed to be supported by the columns and the restraint at the ends of the slab-band by the spandrel beams was neglected. In a case like this it is usually difficult to get the right numerical factors to use in the theoretical analysis. The manner in which the slab-bands are supported by columns and the presence of the spandrel beams at the ends of the slab-band as discussed above are the main factors for the difference between stresses obtained by tests and those obtained by theoretical analysis.

E. The slab and the slab-bands are an entity. Although the slab-bands are used to support the slab, the slab also helps the slab-bands to resist the bending moments in the direction of the slab-bands. In the model tests this combined action of the slab-bands and the slab was revealed by the results obtained. In the frame analysis the help offered by the slab in the analysis of the slab-bands was neglected.

## 9.6. Summary

According to the discussions in sections 7.3.1. and 8.6. and the comparisons in section 9.5. the conclusion has been reached that the results obtained by testing of model as given in Chapter 7 are a better basis for the design of slab-band floor construction than

those obtained by frame analysis as given in Chapter 8. In the next chapter a complete specification will be given for designing this kind of floor system and in Chapter 11 a typical example of designing this floor system will be given.

## CHAPTER 10

### SPECIFICATION SUGGESTED FOR THE DESIGN OF SLAB-BAND FLOOR CONSTRUCTION

#### 10.1. General

"Slab-Band Floor Construction" is that kind of floor system in which the reinforced concrete slab is built monolithically with shallow and wide supporting beams (slab-bands) which are placed in one direction only. They are in turn supported by columns monolithically built with the beams. Conventional spandrel beams are used at the ends of the building normal to the slab-bands. Slab-bands could be used in place of conventional spandrel beams but the latter were used in the model, the tests of which form the basis for this specification. The function of the spandrel beams is not only to support the curtain walls at the ends of the building but also for bracing the whole structure in that direction.

The method of design of this slab-band floor construction presented in this chapter is based mainly on experimental results obtained from model tests. Coefficients are introduced for calculating design bending moments and shears. As the model tested is limited in some respects, certain limitations as described in the following section should be observed in order to apply these coefficients to design.

## 10.2. Limitations

Since the design requirements in this chapter are based on tests of a single model they should be applied to structures having approximately the same proportions as the model. They should not be applied to floors of far different proportions without additional information.

The general proportions of the model were as follows:

- A. Ratio of depth of slab to depth of slab-band = 0.41
- B. Ratio of depth of slab-band to width of slab-band = 0.23
- C. Ratio of clear span of slab to span center to center of slab-bands = 0.75
- D. Ratio of width of slab-band to distance center to center of slab-bands = 0.25
- E. Column spacing the same in both directions
- F. The model had three bays in each direction
- G. The exterior slab-bands were half the width of the interior slab-bands plus half the column width
- H. The exterior columns were in line with the outside of the exterior slab-band. That slab-band was therefore supported eccentrically.

## 10.3. Notation

The following notation is used in this specification for slab-band floor construction:

M = The positive or negative bending moment at the principal design sections in either the middle or the column strip or in the

slab-band. For slabs,  $M$  is moment in ft. lbs. per foot width of slab. For slab-bands,  $M$  is moment in the slab-band in ft. lbs.

$V$  = The external shear

$C_m$  = The coefficient for bending moment.  $M = C_m w L^2$

$C_v$  = The coefficient for shear.  $V = C_v w L$

$w$  = Uniformly distributed load (D.L. + L.L.) in lb. per sq. ft. for slab or in lbs. per ft. for slab-bands.

$L$  = The clear span for slabs (between edges of slab-bands) or the span center to center of columns for slab-bands.

#### 10.4. Design of Slab

The following is for the design of slab.

##### 10.4.1. Typical Panels

The floor system of the slab-band construction for a building with three or more panels in each direction can be classified into four typical panels according to their location.

##### A. Interior Panel

The panel is continuous at all four edges.

##### B. Exterior Panel A

One edge of the panel is discontinuous and this discontinuous edge is perpendicular to the direction of the span of the slab.

##### C. Exterior Panel B

One edge of the panel is discontinuous and this discontinuous edge is parallel to the direction of the span of the slab.

#### D. Corner Panel

The panel is located at the corner of the building and has two discontinuous edges.

#### 10.4.2. Panel Strips

For any one of the four typical panels it is considered that the panel consists of two strips, the middle and the column strips, which are in the direction of the slab span.

The middle strip having a width to be equal to half of the panel width occupies the center part of the width of the panel.

The column strip is made up of two strips, each one quarter of a panel in width and located on either side of the middle strip.

#### 10.4.3. Principal Design Sections

The principal design sections are the critical sections for moment calculation and are located as follows:

The principal design section for positive bending moment is the section at the center line of the slab span for both the middle strip and the column strip.

The principal design sections for negative bending moment in the slab are the sections at the edges of the slab-bands.

#### 10.4.4. Bending Moment

The positive and the negative bending moments for both the middle and the column strips in any one of the four typical panels shall be calculated from the following formula:

$$M = C_m W L^2$$

Table 10.01.  
Moment Coefficients for Slab

$$M = C_m W L^2$$

| Panels           | Strips                                           | +M at Center of Slab | -M at Continuous Edge | -M at Discontinuous Edge |
|------------------|--------------------------------------------------|----------------------|-----------------------|--------------------------|
| Interior Panel   | Middle Strip                                     | 0.075                | 0.074                 | —                        |
|                  | Column Strip                                     | 0.075                | 0.074                 | —                        |
| Exterior Panel A | Middle Strip                                     | 0.105                | 0.087                 | 0.057                    |
|                  | Column Strip                                     | 0.102                | 0.078                 | 0.073                    |
| Exterior Panel B | Half Column Strip Connected with Interior Panel  | 0.074                | 0.073                 | —                        |
|                  | Middle Strip                                     | 0.074                | 0.067                 | —                        |
|                  | Half Column Strip Adjacent to Discontinuous Edge | 0.061                | 0.049                 | —                        |
| Corner Panel     | Half Column Strip Connected with Adjacent Panel  | 0.092                | 0.073                 | 0.069                    |
|                  | Middle Strip                                     | 0.086                | 0.069                 | 0.044                    |
|                  | Half Column Strip Adjacent to Discontinuous Edge | 0.076                | 0.055                 | 0.036                    |

in which  $C_m$  is the moment coefficient which can be found in Table 10.01 for the bending moment at the section considered.

#### 10.4.5. External Shear

The shear at the ends of the slab for both the middle and the column strips in any one of the four typical panels shall be calculated from the following formula:

$$V = C_v wL$$

in which  $C_v$  is the shear coefficient which can be found in Table 10.02 for the external shear at the section considered.

Table 10.02  
Shear Coefficients for Slab

$$V = C_v wL$$

|                                    | Along Interior<br>Slab-Band | Along Exterior<br>Slab-Band |
|------------------------------------|-----------------------------|-----------------------------|
| Interior Panel<br>Exterior Panel B | 0.50                        | ---                         |
| Exterior Panel A<br>Corner Panel   | 0.53                        | 0.48                        |

#### 10.4.6. Slab Thickness

The slab thickness shall be based on the maximum bending moment or the maximum shear, whichever controls.

The required slab thickness based on moment is

$$d_m = \sqrt{\frac{M}{Kb}}$$

in which  $d$  = Effective depth

$M$  = Maximum bending moment

$K = \frac{1}{2}f_c k j$  for a balanced rectangular section

$b$  = Width of strip = 12"

The slab thickness based on shear is

$$d_v = \frac{V}{vb_j}$$

in which  $V$  = Maximum external shear

$v$  = Allowable shearing unit stress

$$= 0.03 f_c'$$

$j$  = Ratio of resisting moment arm to effective depth

The concrete protection for the steel reinforcement in the slab shall not be less than three-quarters inch.

#### 10.4.7. Main Reinforcement

The amount of steel required at any section considered is determined by the following formula

$$A_s = \frac{M}{f_s j d}$$

in which  $M$  is expressed in inch-pounds per foot width and other terms have the conventional meanings.  $A_s$  may be determined by the following simpler formula

$$A_s = \frac{M}{a d}$$

in which  $M$  is expressed in ft.-kips,  $a = \frac{fsj}{12000}$  and other terms have the conventional meanings.

The minimum amount of steel reinforcement in the slab shall be  $0.0025bd$ .

#### 10.4.8. Shrinkage and Temperature Reinforcement

Shrinkage and temperature reinforcement shall be provided in the slab in the direction perpendicular to that of the main reinforcement. The area of such reinforcement shall be not less than  $0.002bd$ .

#### 10.4.9. Arrangement of Slab Steel

##### A. Positive Steel

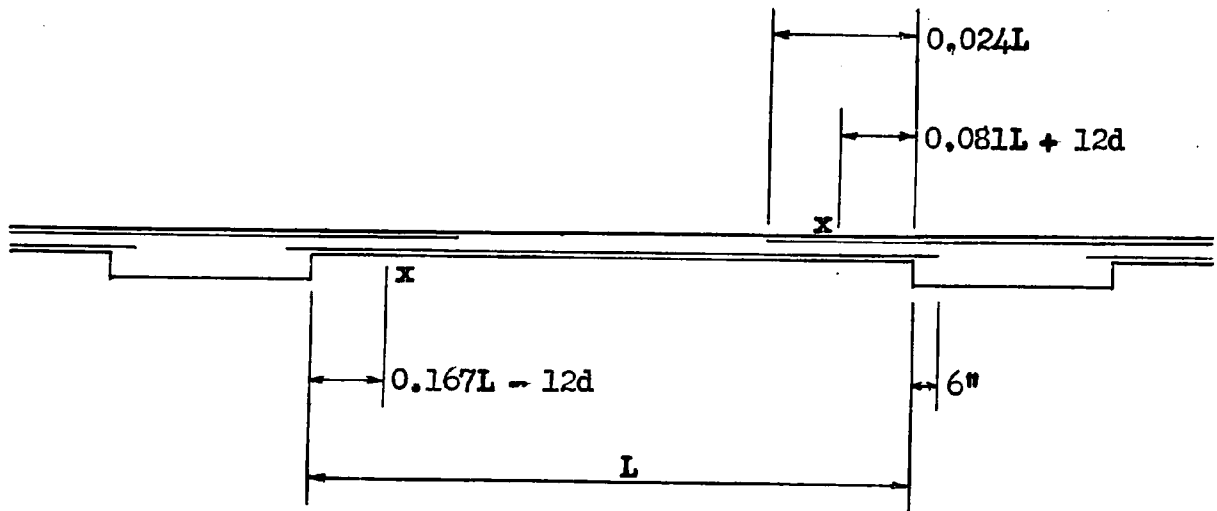
One half of the positive reinforcement may be stopped at the place which is 12 bar diameters beyond the point at which it is no longer needed to resist stress. The remaining one half of the positive reinforcement shall be extended along the same face of the slab into the slab-band a distance of 6 inches.

##### B. Negative Steel

One half of the negative reinforcement may be stopped at the place which is 12 bar diameters beyond the point at which it is no longer needed to resist stress. The remaining one half of the negative reinforcement shall be extended beyond the extreme position of the point of inflection a distance of one-sixteenth of the clear span length.

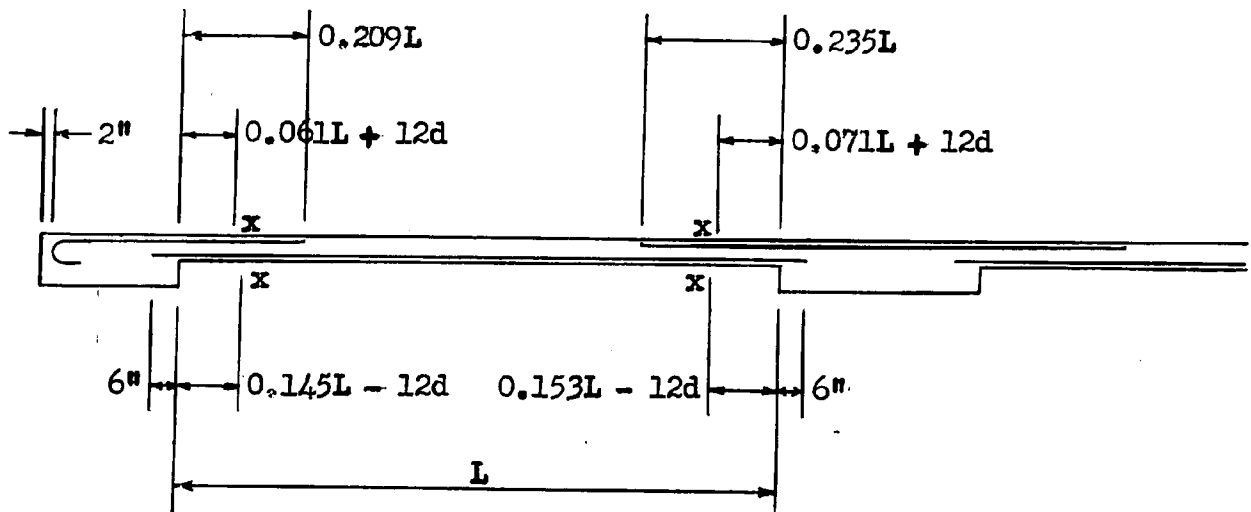
Figure 10.01 shows the arrangement of slab steel in interior, exterior and corner panels.

For Interior Panel and Exterior Panel B



$x$ : The point where half of the reinforcement may be stopped.

For Exterior Panel A and Corner Panel



$d$ : Bar diameter

Figure 10.01.

Minimum Length of Slab Reinforcement

#### 10.4.10. Holes in Slab

Holes of any size may be cut through a slab if provision is made to take care of the positive and negative bending moments in a panel as required in section 10.4.4. without exceeding the allowable unit stresses as given in ACI Building Code sections 305 and 306.

#### 10.5. Design of Slab-Bands

The following is for the design of slab-bands.

##### 10.5.1. Classification of Slab-Bands

Slab-bands are classified into two categories, namely interior and exterior slab-bands. The interior slab-band supports loads from slabs on both sides while an exterior slab-band receives the load from slab on one side only.

##### 10.5.2. Principal Design Sections

The principal design sections are the critical sections for moment calculation and are located as follows:

The principal design section for positive bending moment is at the mid-span of the slab-bands.

The principal design sections for negative bending moment are the sections at the edges of the supporting columns across the whole slab-bands.

##### 10.5.3. Bending Moment

The positive and the negative bending moments for both the interior and the exterior slab-bands shall be calculated from the

following formula:

$$M = C_m w L^2$$

in which  $C_m$  is the moment coefficient which can be found in Table 10.03 for the section considered.

Table 10.03.

Moment Coefficients for Slab-Bands

$$M = C_m w L^2$$

| Location              |               | +M at Center<br>of Span | -M at conti-<br>nuous End | -M at discon-<br>tinuous End |
|-----------------------|---------------|-------------------------|---------------------------|------------------------------|
| Interior<br>Slab-Band | Interior Span | 0.083                   | 0.048                     | -----                        |
|                       | Exterior Span | 0.061                   | 0.048                     | 0.046                        |
| Exterior<br>Slab-Band | Interior Span | 0.095                   | 0.087                     | -----                        |
|                       | Exterior Span | 0.063                   | 0.087                     | 0.052                        |

#### 10.5.4. Thickness of Slab-Band

The thickness of slab-band shall be based on the maximum bending moment and is determined by the following formula:

$$d = \sqrt{\frac{M}{Kb}}$$

in which  $d$  = Effective depth

$M$  = Maximum bending moment

$K = \frac{1}{2}f_{ck}$  for a balanced rectangular section

$b$  = Width of the slab-band

The concrete protection for the steel reinforcement in the slab-band shall not be less than three-fourths inch and two outmost bars, one on each side, of the slab-band shall be at least one and one-half inches from the vertical faces of the slab-band.

#### 10.5.5. Steel Reinforcement

Same as section 10.4.7.

#### 10.5.6. Arrangement of Steel

The positive steel reinforcement is uniformly arranged at the bottom of the slab-band and the negative steel reinforcement is also uniformly distributed over interior supports. Sixty per cent of the negative reinforcement over the exterior end of the interior slab-band shall be placed at the middle third of the width of the slab-band and the other forty per cent is uniformly distributed at two outer thirds.

Two-thirds of the negative reinforcement over the exterior end of the exterior slab-band shall be placed in that half of the slab-band width which is directly supported by corner column.

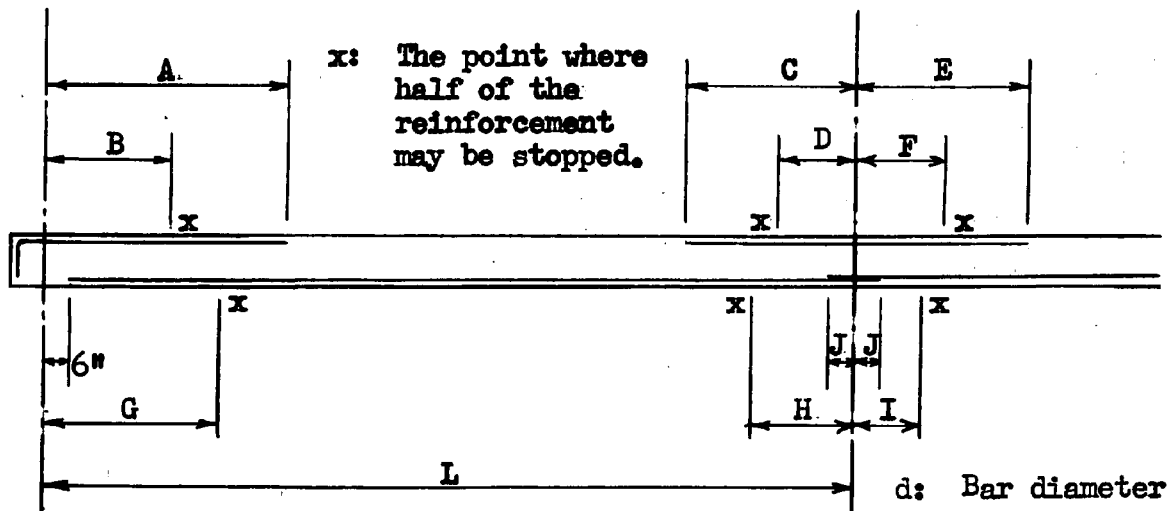


Figure 10.02.

## Minimum Length of Slab-Band Reinforcement

|          | Interior Slab-Band | Exterior Slab-Band |
|----------|--------------------|--------------------|
| <b>A</b> | $0.299L$           | $0.281L$           |
| <b>B</b> | $0.133L + 12d$     | $0.126L + 12d$     |
| <b>C</b> | $0.207L$           | $0.246L$           |
| <b>D</b> | $0.055L + 12d$     | $0.072L + 12d$     |
| <b>E</b> | $0.211L$           | $0.247L$           |
| <b>F</b> | $0.083L + 12d$     | $0.055L + 12d$     |
| <b>G</b> | $0.264L - 12d$     | $0.191L - 12d$     |
| <b>H</b> | $0.173L - 12d$     | $0.209L - 12d$     |
| <b>I</b> | $0.154L - 12d$     | $0.169L - 12d$     |
| <b>J</b> | $6"$               | $0$                |

Figure 10.02 shows the arrangement of steel reinforcement in both interior and exterior slab-bands.

#### 10.5.7. Shearing Stress

The shearing unit stress  $v$ , as a measure of diagonal tension, in the slab-bands shall be computed by the following formula:

$$v = \frac{V}{bjd}$$

in which  $V$  = External shear in pounds which is the total load that the section considered shall resist.

$b$  = Width of the section considered, in inches.

$j$  = Ratio of resisting moment arm to effective depth.

$d$  = Effective depth, in inches.

The vertical section considered critical for diagonal tension in slab-band lies at a distance  $t - 1\frac{1}{2}"$  beyond the edge of the supporting column and parallel to and concentric with the column faces. Here  $t$  is the total depth of the slab-band. When the width of the slab-band is equal to or less than the column width plus  $2t$ , only the width of the slab-band is used for  $b$  in the above formula.

## CHAPTER 11

### COMPARATIVE DESIGN OF FOUR KINDS OF FLOOR CONSTRUCTION

#### 11.1. General

In order to compare the relative economy of slab-band floor construction with three of the most common floor systems, one-way slab, two-way slab and flat slab, designs are made to show the amounts of concrete and steel needed for each one of the four floor constructions.

Only one typical interior panel on a typical floor of a building of light-load type will be designed. The architectural layout of the panel is the same for all four floor systems and this panel is a square one with a panel length of 16'0" center to center of supporting columns.

The live load used is 60 lbs. per sq. ft. which is an average intensity for buildings of this type. The equivalent partition wall load is 20 lbs. per sq. ft. of floor surface. Loads due to ceiling and flooring are chosen at 5 lbs. per sq. ft. each. Concrete is 150 lbs. per cu. ft.

#### 11.2. Materials Used

Quality of concrete and reinforcing steel used in the design examples as well as allowable unit stresses and design coefficients are

given below.

### 11.2.1. Concrete

The concrete assumed in these designs has a 28-day compressive strength of  $f_c^1 = 3,000$  p.s.i.

### 11.2.2. Reinforcing Steel

Rail-steel, billet-steel or axle-steel concrete reinforcement bars of intermediate grade is assumed. The bars are A-305 type deformed bars.

### 11.2.3. Allowable Unit Stresses

#### A. Concrete

#### a. Flexure: $f_c$

$$\begin{aligned} f_c &= \text{Allowable unit flexural stress of concrete in compression} \\ &= 0.45 f_c^1 \\ &= 1350 \text{ \#/in.}^2 \end{aligned}$$

#### b. Shear: $v$ (as a measure of diagonal tension)

$$\begin{aligned} v_c &= \text{Allowable unit shearing stress of concrete in slab-band,} \\ &\quad \text{one-way slab, two-way slab and beams} \\ &= \text{Allowable unit shearing stress of concrete in flat slabs} \\ &\quad \text{at a distance } d \text{ from edge of column capital, drop panel} \\ &\quad \text{or face of column} \\ &= 0.03 f_c^1 \\ &= 90 \text{ \#/in.}^2 \\ v &= \text{Allowable unit shearing stress in beams with stirrups} \\ &\quad \text{placed perpendicular to the longitudinal reinforcement} \\ &= 0.08 f_c^1 \end{aligned}$$

$$= 240 \text{ \#/in.}^2$$

c. Bond: u

u = Allowable unit bond stress for top horizontal deformed bars  
which are so placed that more than 12 in. of concrete is  
cast in the member below the bar

$$= 0.07 f_c'$$

$$= 210 \text{ \#/in.}^2$$

u = Allowable unit bond stress for all other deformed bars

$$= 0.10 f_c'$$

$$= 300 \text{ \#/in.}^2$$

B. Reinforcing Steel

$f_s$  = Allowable tensile unit stress in longitudinal reinforcement

$$= 20,000 \text{ \#/in.}^2$$

$f_s$  = Allowable compressive unit stress in vertical column  
reinforcement

$$= 16,000 \text{ \#/in.}^2$$

$f_v$  = Allowable tensile unit stress in web reinforcement

$$= 20,000 \text{ \#/in.}^2$$

#### 11.2.4. Constants for Balanced Design

The constants which have conventional meanings in reinforced concrete for balanced design of rectangular sections are listed below for the materials used.

$$K = 236$$

$$k = 0.403$$

$$j = 0.866$$

$$p = 0.0136$$

$$a = 1.44$$

## 11.3. Design of Slab-Band Floor

## 11.3.1. Slab

|           |                |                        |
|-----------|----------------|------------------------|
| A. Loads: | D.L. Ceiling   | 5                      |
|           | Partition Wall | 20                     |
|           | Flooring       | 5                      |
|           | Wt. of Slab    | 57                     |
|           | Total D.L.     | 87                     |
|           | L.L.           | 60                     |
|           | Total Load     | 147 #/ft. <sup>2</sup> |

B. Span:  $L = 16'0''$  c. to c. of supports  
 $L' = \text{Clear Span} = 16'0'' - 4'0''$   
 $= 12'0''$

C. Shear:  $\frac{1}{2}wL' = \frac{1}{2} \times 147 \times 12$   
 $= 882 \text{ \#/ft. width of slab}$

D. B.M.: In both middle and column strips  
 $+M$  at center of interior slab  $= 0.075 \times 147 \times 12^2$   
 $= 1588 \text{ \#/ft. of slab}$   
 $-M$  at face of interior slab-band  
 $= 0.074 \times 147 \times 12^2$   
 $= 1567 \text{ \#/ft. width of slab}$   
 $+M$  at center of middle strip in exterior slab A  
 $= 0.105 \times 147 \times 12^2$   
 $= 2223 \text{ \#/ft. width of slab}$

## E. Thickness of Slab:

Based on B.M.

$$d^2 = \frac{M}{K_b} = \frac{2223 \times 12}{236 \times 12} = 9.42$$

$$d = 3.07''$$

$$\text{Total depth} = 3.07 + 0.25 + 0.75 = 4.07''$$

Based on Deflection (usual practice)

$$\frac{L^4}{t} \text{ max.} = 32$$

$$\text{Min. total depth} = \frac{L^4}{32} = \frac{12 \times 12}{32} = 4.5''$$

Use  $4\frac{1}{2}''$  as total depth of the slab

$$d = 4.5 - 0.75 - 0.25 = 3.5''$$

#### F. Main Reinforcement

$$A_s \text{ min.} = 0.0025bd = 0.0025 \times 12 \times 3.5$$

$$= 0.105 \text{ in}^2/\text{ft. width of slab}$$

$$+A_s = \frac{M}{ad} = \frac{1.588}{1.44 \times 3.5}$$

$$= 0.315 \text{ in}^2/\text{ft. width of slab}$$

Use 25 - #4 Bottom straight per panel width

$$-A_s = \frac{1.567}{1.44 \times 3.5}$$

$$= 0.311 \text{ in}^2/\text{ft. width of slab}$$

Use 25 - #4 Top straight total over one support per panel width

$$+A_s \text{ or } -A_s = 0.3125 \text{ in}^2/\text{ft. width of slab}$$

$$\Sigma o = 2.454 \text{ "/ft. width of slab}$$

$$p = 0.00744$$

$$j = 0.894$$

#### G. Temperature Reinforcement

$$A_s = 0.002bd = 0.002 \times 12 \times 3.5$$

$$= 0.084 \text{ in}^2/\text{ft. width of slab}$$

Use 9 - #3 Straight

#### H. Diagonal Tension

$$v = \frac{V}{bjd} = \frac{882}{12 \times 0.894 \times 3.5} = 23.5 \text{ \#/in}^2$$

#### I. Bond

$$u = \frac{V}{\Sigma ojd} = \frac{882}{2.454 \times 0.894 \times 3.5} = 114.9 \text{ \#/in}^2$$

## 11.3.2. Slab-Band

A. Loads: From slab: D.L.:  $87 \times 16 = 1392$

L.L.:  $60 \times 16 = 960$

Wt. of slab-band web:  $\frac{48 \times (9 - 1.5)}{144} \times 150 = 225$

Total Load in #/ft.  $2577$

$\frac{L.L.}{D.L.} = 0.594$

B. Span:  $L = 16'0''$  c. to c. of supports

C. Shear: Total load in one panel  $= 2577 \times 16 = 41232\#$

Unit dead and live load on slab-band  $= 87 + 57 + 60$   
 $= 204 \#/\text{ft}^2$

Shear section around column  $= b \times b$

$= [14 + 2 \times (9 - 1.5)] \times [14 + 2 \times (9 - 1.5)]$

$= 29'' \times 29''$

Load within the shear section  $= 204 \times \left(\frac{29}{12}\right)^2$   
 $= 1033\#$

Total shear at shear section around the column

$= 41232 - 1192$

$= 40040\#$

D. B.M.  $+M$  at center of interior span  $= 0.083wL^2$

$= 0.083 \times 2577 \times 16^2$

$= 54750'\#$

$-M$  at faces of interior supports  $= 0.048wL^2$

$= 0.048 \times 2577 \times 16^2$

$= 31660'\#$

## E. Size of Slab-Band

$bd^2 = \frac{M}{K} = \frac{54750 \times 12}{236} = 2785$

If  $b = 48''$  then  $d = 7.62''$

Total depth required  $= 7.62 + 0.75 + 0.375 = 8.75''$

Use 48" x 9"

$$d = 9 - 0.75 - 0.375 = 7.87" \text{ for bottom steel}$$

$$d = 9 - 0.75 - 0.5 - 0.375 = 7.37" \text{ for top steel}$$

#### F. Steel Reinforcement

$$A_s \text{ min.} = 0.0025bd = 0.0025 \times 48 \times 7.87 \\ = 0.95 \text{ in}^2$$

$$+A_s = \frac{M}{ad} = \frac{54.75}{1.44 \times 7.87} = 4.83 \text{ in}^2$$

Use 11 - #6 Bottom straight

$$-A_s = \frac{31.66}{1.44 \times 7.37} = 2.98 \text{ in}^2$$

Use 7 - #6 Top straight total over one support

$$-A_s \text{ provided} = 3.08 \text{ in}^2$$

$$p = 0.008705$$

$$j = 0.858$$

#### G. Diagonal Tension

$$v = \frac{V}{bjd} = \frac{40040}{4 \times 29 \times 0.858 \times 7.37} \\ = 54.6 \text{ \#/in}^2$$

### 11.3.3. Column

#### A. Loads:

|            |                    |                                                        |   |       |        |
|------------|--------------------|--------------------------------------------------------|---|-------|--------|
| Roof:      | From roof slab:    | $99 \times 16^2$                                       | = | 25344 |        |
|            | From slab-band web | $\frac{48 \times (9 - 4.5)}{144} \times 150 \times 16$ | = | 3600  |        |
|            | Wt. of column      | $\frac{12 \times 12}{144} \times 150 \times 9$         | = | 1350  |        |
|            |                    |                                                        |   | 30294 | 30294# |
| 6th Floor: | From floor slab    | $147 \times 16^2$                                      | = | 37632 |        |
|            | From slab-band web | $\frac{48 \times (9 - 4.5)}{144} \times 150 \times 16$ | = | 3600  |        |
|            | Wt. of column      | $\frac{12 \times 12}{144} \times 150 \times 9$         | = | 1350  |        |
|            |                    |                                                        |   | 42582 | 42582  |

|                            |                                                        |   |       |         |
|----------------------------|--------------------------------------------------------|---|-------|---------|
| 5th Floor: From floor slab | $147 \times 16^2$                                      | = | 37632 |         |
| From slab-band web         | $\frac{48 \times (9 - 4.5)}{144} \times 150 \times 16$ | = | 3600  |         |
| Wt. of column              | $\frac{14 \times 14}{144} \times 150 \times 9$         | = | 1838  |         |
|                            |                                                        |   | 43070 | 43070   |
| Total load                 |                                                        |   |       | 115946# |

## B. Size of Column

14" x 14" tied column

## C. Steel Reinforcement

4 - #7 vertical bars

10 - #3 ties @ 12"

## 11.3.4. Amount of Concrete

|            |                                             |   |                        |
|------------|---------------------------------------------|---|------------------------|
| Slab:      | $\frac{4.5}{12} \times 16^2$                | = | 96.00                  |
| Slab-Band: | $\frac{48 \times (9 - 4.5)}{144} \times 16$ | = | 24.00                  |
| Column:    | $\frac{14 \times 14}{144} \times 9$         | = | 12.25                  |
|            |                                             |   | 132.25 ft <sup>3</sup> |

## 11.3.5. Amount of Steel

## A. In Slab

|                    |                   |             |       |
|--------------------|-------------------|-------------|-------|
| Top Steel:         | 13 - #4 x 9'10"   | } Alternate |       |
|                    | 12 - #4 x 7'0"    |             |       |
|                    | 13 x 9.83 x 0.668 | =           | 85.4  |
|                    | 12 x 7.00 x 0.668 | =           | 56.2  |
| Bottom Steel:      | 13 - #4 x 13'0"   | } Alternate |       |
|                    | 12 - #4 x 9'0"    |             |       |
|                    | 13 x 13.0 x 0.668 | =           | 112.9 |
|                    | 12 x 9.0 x 0.668  | =           | 72.2  |
| Temperature Steel: | 9 - #3 x 16'0"    |             |       |

$$9 \times 16.0 \times 0.376 = 54.2$$

|                      |       |        |
|----------------------|-------|--------|
| Total Steel in Slab: | 380.9 | 380.9# |
|----------------------|-------|--------|

#### B. In Slab-Band

Top Steel:            4 - #6 x 6'9" Str.

                         3 - #6 x 4'3" Str.

$$4 \times 6.75 \times 1.502 = 40.6$$

$$3 \times 4.25 \times 1.502 = 19.2$$

Bottom Steel:        6 - #6 x 17'0" Str.

                         5 - #6 x 12'8" Str.

$$6 \times 17.0 \times 1.502 = 153.2$$

$$5 \times 12.67 \times 1.502 = 95.1$$

|                           |       |       |
|---------------------------|-------|-------|
| Total Steel in Slab-Band: | 308.1 | 308.1 |
|---------------------------|-------|-------|

#### C. In Column

Vertical Steel:      4 - #7 x 11'3"

$$4 \times 11.25 \times 2.044 = 92.0$$

Ties:                    10 - #3 x 4'4"

$$10 \times 4.33 \times 0.376 = 16.3$$

|                       |       |       |
|-----------------------|-------|-------|
| Total Steel in Column | 108.3 | 108.3 |
|-----------------------|-------|-------|

|                           |  |        |
|---------------------------|--|--------|
| D. Total Amount of Steel: |  | 797.3# |
|---------------------------|--|--------|

In the following three sections, sections 11.4 to 11.6 inclusive, three different kinds of floor slabs are designed according to "The ACI Building Code" (ACI 318 - 56). For simplicity in presenting the design computation procedure is omitted and only results are given.

## 11.4. Design of One-Way Slab Floor

## 11.4.1. Sizes

Slab thickness: 6"  
 Size of beams: 12" x 19"  
 Size of columns: 14" x 14"

## 11.4.2. Steel Reinforcement

## A. In Slab

|                    |                      |                                        |
|--------------------|----------------------|----------------------------------------|
| Top Steel:         | 19 - #4 x 10'3" Str. | } Alternate, total<br>over one support |
|                    | 19 - #4 x 5'3" Str.  |                                        |
| Bottom Steel:      | 13 - #4 x 16'0" Str. | } Alternate                            |
|                    | 13 - #4 x 8'6" Str.  |                                        |
| Temperature Steel: | 11 - #4 x 16'0" Str. |                                        |

## B. In Beam

Top Steel: 4 - #7 x 8'7" Str., total over one end  
 Bottom Steel: 4 - #6 x 15'10" Str.  
 Stirrups: 3 - #3 x 4'1" U-shaped, each end

## C. In Column

Vertical Steel: 4 - #7 x 12'1"  
 Ties: 11 - #3 x 4'4"

## 11.4.3. Amount of Materials for One Panel

A. Concrete: 157.8 ft<sup>3</sup>  
 B. Steel: 818.4 lbs.

## 11.5. Design of Two-Way Slab Floor

### 11.5.1. Sizes:

Slab thickness:  $4\frac{1}{2}"$   
 Size of beams:  $10" \times 17"$   
 Size of columns:  $14" \times 14"$

### 11.5.2. Steel Reinforcement

#### A. In Slab

|               |                     |                       |
|---------------|---------------------|-----------------------|
| Top Steel:    | 11 - #4 x 8'3" Str. | } Alternate, each way |
|               | 10 - #4 x 4'6" Str. |                       |
| Bottom Steel: | 8 - #4 x 16'2" Str. | } Alternate, each way |
|               | 8 - #4 x 8'6" Str.  |                       |

#### B. In Beams

Top Steel: 3 - #7 x 8'7" Str., total over one end  
 Bottom Steel: 2 - #7 x 15'10" Str.  
 Stirrups: 2 - #3 x 3'7" U-shaped, each end

#### C. In Column

Vertical Steel: 4 - #7 x 11'11"  
 Ties: 11 - #3 x 4'4"

### 11.5.3. Amount of Materials for One Panel

A. Concrete:  $135.43 \text{ ft.}^3$   
 B. Steel: 805.7 lbs.

## 11.6. Design of Flat Slab (without drop panel and without capital)

### 11.6.1. Sizes

Slab thickness:  $7\frac{1}{4}"$

Size of columns:  $14" \times 14"$

### 11.6.2. Steel Reinforcement

#### A. In Slab

|                  |                                  |   |                     |
|------------------|----------------------------------|---|---------------------|
| In Column Strip: | 9 - #4 x 9'8" Top Str.           | } | Alternate, each way |
|                  | 9 - #4 x 8'0" Top Str.           |   |                     |
|                  | 5 - #4 x 15'6" Bot. Str.         | } | Alternate, each way |
|                  | 4 - #4 x 12'0" Bot. Str.         |   |                     |
| In Middle Strip: | 7 - #4 x 8'0" Top Str., each way |   |                     |
|                  | 4 - #4 x 15'6" Bot. Str.         | } | Alternate, each way |
|                  | 3 - #4 x 11'3" Bot. Str.         |   |                     |

#### B. In Column

Vertical Steel: 4 - #7 x 11'2"

Ties: 10 - #3 x 4'4"

### 11.6.3. Amount of Materials for One Panel

A. Concrete: 166.95 ft.<sup>3</sup>

B. Steel: 689.5 lbs.

11.7. Comparison of Amounts of Concrete and Steel Used in Four Types  
of Floor Constructions

Table 11.01. shows a summary of amounts of concrete and steel needed in one typical interior panel, floor plus column, for the four Types of floor constructions as designed in sections 11.3. to 11.6. inclusive.

Table 11.01.  
Comparison of Amounts of Concrete and  
Steel Used in Four Types of Floor Constructions

| Type of Floor System                                   | Total Material for One Typical Interior Panel |               |
|--------------------------------------------------------|-----------------------------------------------|---------------|
|                                                        | Concrete<br>ft. <sup>3</sup>                  | Steel<br>lbs. |
| Slab-Band                                              | 132.25                                        | 791.3         |
| One-Way Slab                                           | 157.80                                        | 818.4         |
| Two-Way Slab                                           | 135.43                                        | 805.7         |
| Flat Slab<br>(without capital)<br>(without drop panel) | 166.95                                        | 689.5         |

## CHAPTER 12

### CONCLUSION

The use of Plexiglas as the material of the model and the use of strain gages on Plexiglas proved to be successful. This is truly in general for building models especially if temperature effects are minimized.

As the adoption of slab-band floor constructions in buildings has increased constantly it is of utmost importance to have a standard specification for designing this kind of floor system. The specification given in chapter 10 is intended for this purpose. It is based upon tests of a model which has definite proportions of the structural members. More models with different proportions should be tested in order to make less limitations and to broaden the sphere of application of the suggested specification.

The specification given in chapter 10 is simple to apply to the design of slab-band floor system. In some instances when the proportions of the building frame fall beyond the limitations of the specification it is better to use frame analysis for design with modifications based on the comparisons of stresses obtained by test and those obtained by frame analysis. These comparisons are made in section 9.4.

For the same architectural layout, limited cost comparisons

indicate that slab-band floor construction is more economical in total cost of concrete and steel than conventional floor systems under the following conditions.

1. Slab-band is more economical than one-way slab floor construction for all loads and spans.

2. Slab-band is more economical than two-way slab floor construction for all loads and spans.

3. Slab-band is more economical than flat slab floor construction without capitals and without drop panels for all loads and spans if the shearing unit stress in the slab around column is within the value specified by A.C.I. Building Code.

In addition, the slab-band construction saves wall materials, Length of pipes, conduits etc. because it results in a shallower floor system than with slab and beam construction.

It is not the intention of the author to say that the slab-band floor is superior to all others but the author does feel that this floor construction is superior to others in some respects and it can be used to advantage in a wide range of structures.

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