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DIRECTIONAL DYNAMICS OF THE
TRACTOR-SEMITRAILER VEHICLE

by

Philip Martin Leucht

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ABSTRACT

DIRECTIONAL DYNAMICS OF THE TRACTOR-SEMITRAILER VEHICLE

by

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This paper describes an analytical study of the directional dynamics of a tractor-semitrailer vehicle. A hybrid computer simulation was developed to describe the longitudinal, lateral, and yawing motions of a three-axle tractor-semitrailer on a flat road surface. Idealized steering and braking inputs were applied to the simulation to determine the influence of (1) design parameters, (2) operating conditions, and (3) the environment on the directional behavior of the vehicle under study.

Results obtained in the computer study indicated that the probability of developing an undesirable response in a combined steering and braking maneuver was increased by increasing the forward velocity, the load on the semitrailer, and the slipperiness of the road surface. The variation in loading condition was identified as a major cause of undesirable directional response with a fixed brake-torque distribution. Brake-system design characteristics used to

improve the system behavior were the brake-torque distribution, load-sensitive brake-torque control, and brake application times. Longitudinal and lateral brake-torque variations from a baseline distribution had the overall effect of decreasing the combined steering and braking performance of the vehicle. Increasing the wheelbase of the tractor and the amount of friction at the fifth-wheel were most effective in reducing the tendency of the vehicle to jackknife in a combined steering and braking maneuver.

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CHAPTER 1

IDENTIFICATION OF THE PROBLEM

1.1 Introduction

One of the vitally important aspects of our society is the transport of people and goods from place to place on our roads and highways. The importance of safety to the operation of one particular type of vehicle, the tractor-semitrailer combination, is vital not only with respect to the driver and his costly vehicle, but also with respect to other vehicles, property, and people near it. Thus, the functional performance of a tractor-semitrailer vehicle as it is affected by design characteristics, environment, loading condition, and steering and braking inputs is a valid area of research in an engineering sense as well as in a highway safety sense.

1.2 History

A brief historical discussion of research on the dynamic behavior of tractor-trailer (and automobile-trailer) vehicles is essential in developing a basis for this work. Documented research over a period of more than thirty years shows the effects of applying mathematical techniques and the computer to increasingly sophisticated and more complete linear and nonlinear models. These previous works have

increased the understanding of the system dynamics involved and have led to the conception of this investigation.

Huber and Dietz [1]¹ realized the significance of lateral oscillations of trailers to traffic safety. Using model and full scale road tests, they indicated various ways to decrease the lateral motions of full trailers. The effects favorable to suppressing trailer oscillations were a longer wheelbase, a laterally rigid tow point, damping the drawbar about the kingpin, and using only the trailer brakes to stop the vehicle.

Ziegler investigated the stability of tractor-trailer vehicles with a constant forward velocity on a flat surface in Reference [2] and with braking on a longitudinally sloped surface in Reference [3]. The lateral forces developed by the tire on the road were assumed to be static friction forces and the application of the Hurwitz criterion for stability indicated that self-excited oscillations were developed beyond a critical forward velocity.

Williams [4] considered an articulated vehicle having a constant forward speed on a flat road surface and tire forces which were a function of the lateral displacement of

¹Numbers in brackets designate References in the List of References.

the tire relative to the rim. Factors which increased stability were a smaller radius of gyration of the trailer, a longer distance between the kingpin and the trailer axle, and a split live axle with a slipping clutch.

Slibar and Paslay [5] studied the lateral behavior of a semitrailer having a constant forward velocity. The kingpin was assumed to have a small periodic lateral velocity and displacement, and the frequency of the lateral oscillation was proportional to the forward velocity. The results of this analysis showed that the damping of the lateral oscillations was inversely proportional to the forward velocity and that self-excited motion never occurred. In Reference [6] the motion of a tractor was treated when a sinusoidal force and moment excitation acted at the fifth-wheel. The heading angle and the steering angle required to keep the velocity of the center of mass constant, and without a lateral component, were determined.

Clark and Segel [7] analyzed a two-axle tractor and a single-axle trailer with three steering configurations (front wheel, rear wheel, and articulated tractor). Four degrees of freedom (forward, lateral, and yaw motions of the tractor and relative yaw motion of the trailer) were used to describe the motions of the vehicle in a flat, steady-state turn. For a restricted range of lateral

acceleration, calculations indicated that: (1) with a fixed steer angle, the thrust needed to maintain a given velocity increases as the fourth power of velocity; (2) steering geometry has no influence on the speed-thrust relationship; and (3) for a constant radius turn, the understeer¹ vehicle requires less incremental thrust versus velocity than the oversteer² vehicle. In addition, with increasing velocity the center of rotation moves forward and out for an understeer vehicle and forward and in for an oversteer vehicle.

Jindra [8] studied the handling of a tractor-semitrailer with a constant forward velocity in a flat turn by means of a three-degree-of-freedom model of the vehicle (lateral and yaw motions of the tractor and relative yaw motion of the trailer). For the oversteer-vehicle configuration chosen, numerical calculations showed that the four roots of the characteristic equation were real at low speeds and that two roots combined into a complex pair at higher speeds. In addition, the vehicle exhibited less oversteer with increasing velocity if the center of mass of the tractor or the fifth-wheel is closer to the front axle of the tractor, or if the load on the vehicle is reduced.

¹A vehicle is understeer if the change in steer angle per unit change in steady-state lateral acceleration is greater than the ratio of the wheelbase and the square of the forward velocity.

²A vehicle is oversteer if the change in steer angle per unit change in steady-state lateral acceleration is less than the ratio of the wheelbase and the square of the forward velocity.

Jindra [9] considered the handling of a tractor and full trailer with a constant forward velocity in a steady flat turn. A fourth degree of freedom was necessary to describe the front axle and drawbar unit (dolly) of the trailer in addition to the three for the rest of the trailer and the tractor. The results of the sample calculations for an oversteer baseline vehicle indicated that the characteristic equation had six negative real roots at low speeds and two negative real roots plus two conjugate complex pairs at higher speeds. Damping of the lateral oscillations of the trailer, which decreases with increasing forward velocity, was increased with a trailer which has a longer tow pin to front axle distance, a longer wheelbase, a smaller yaw inertia, or a smaller load.

In Reference [10] Jindra reported on an analog computer study of the tractor, semitrailer, and full-trailer combination traveling at constant velocity on a flat road. Five degrees of freedom were used to describe the vehicle as a linear dynamic system. The computer study demonstrated that a decrease in yaw damping and an increase in oscillatory response occurred with increasing forward velocity. Increasing the wheelbase of the full trailer or the distance between the kingpin and the axle of the semitrailer were strong stabilizing factors. The most unfavorable loading condition consisted of a tractor towing an empty semitrailer and a loaded full trailer.

Ellis [11, 12] investigated the dynamics of a tractor-semitrailer vehicle in a flat turn while braking. Four degrees of freedom were used to describe the forward, lateral, and yaw motions of the tractor and the relative yaw motion of the trailer. The analysis included the nonlinear effects of braking and slip angle on the magnitude of the lateral tire forces. Digital computer solutions indicated a violent directional instability¹ when the rear tractor wheels were locked, a less violent directional instability² when the trailer wheels were locked, and stability but loss of steering control when the front tractor wheels were locked.

In Reference [13] Ellis reduced the equations of motion given in Reference [11] to a linear system by assuming that the forward velocity was constant, the lateral tire forces were linear functions of their slip angles, the articulation angle was small, and all products of variables were negligible. Numerical solutions of the characteristic equation showed that a forward fifth-wheel position was more stable and a slightly damped oscillatory motion developed if the distance

¹This violent instability, often termed jackknifing, is characterized by a divergent tractor yawing velocity and sideslip angle.

²The moderate form of instability, called trailer swing, is characterized by a negative trailer angle in a left turn indicating that the trailer may extend out of its traffic lane.

between the center of mass and the axle of the trailer was reduced.

Hales [14] investigated the lateral stability and control of a simplified articulated vehicle subjected to braking and steering disturbances. Four degrees of freedom were used to describe the forward, lateral, and yaw motions of the tractor and the articulation angle between the tractor and the semitrailer. A linear system of equations was obtained by eliminating all terms involving the product of two or more variables and by assuming that the lateral tire forces were linear functions of their slip angles. Examination of the characteristic equation indicated that static instability at all forward velocities was prevented by having the cornering coefficient¹ of the rear tires on the tractor greater than that of the front tires and the ratio of braking force on the front axle to the braking force on the rear axle greater than the ratio of corresponding static axle loads. An analog computer simulation of the equations of motion demonstrated that the yaw velocity response of the trailer to a step input of steer angle became more

¹The cornering coefficient of a tire is the ratio of the cornering stiffness to the vertical load. The cornering stiffness is the negative of the derivative of the lateral force with respect to the slip angle at a slip angle of zero degrees.

oscillatory if the cornering stiffness of the tires on the trailer was reduced or if the forward velocity was increased. Furthermore, if the braking forces on the wheels of the trailer were increased enough a stable configuration became divergently unstable.

In Reference [15] Hales reported on additional analytical work using the linear model employed in Reference [14]. The effect of locked wheels on the stability of tractor-semitrailer vehicles was studied by reducing the cornering stiffness of each locked wheel to zero. Numerical solutions of the characteristic equation for an otherwise stable vehicle showed that locked wheels produced instability in all cases except when the front wheels of the tractor were locked. The most serious deterioration in stability occurred when the rear wheels of the tractor were locked. Various combinations of wheels were locked while an experimental vehicle was placed in a slight turn on a smooth wet surface. These tests verified that instability exists when the rear wheels on the tractor, the trailer wheels, and both the rear wheels on the tractor and the trailer wheels lock. Instability did not occur when the front wheels on the tractor were locked even if all other wheels on the vehicle were locked.

Extensive field tests examining the lateral behavior of articulated vehicles during braking are discussed in Reference [16]. These tests verified theoretical predictions of a stable response with the front wheels of the tractor locked, a violent instability with the rear wheels of the tractor locked, and unstable trailer swing with the trailer wheels locked. In addition, stability was achieved during braking when wheel locking did not occur, when the front wheels on the tractor locked first, and when load proportioning valves altered the brake-force distribution according to the loading condition. Further studies employing the same linear vehicle model used in Reference [14] showed that either friction or a torsional spring at the fifth-wheel coupling had a stabilizing effect, but that these torques reduced the response of the vehicle to steering inputs. The existence of viscous damping at the fifth-wheel reduced the rate of divergence in an instability but tended to make the response of the vehicle to steering oscillatory when the system was otherwise stable.

Hales [17] summarized the results of theoretical, numerical, and experimental work on the lateral behavior of an articulated vehicle with and without braking. Two types of instability were identified for a vehicle without braking:

divergent instability occurred at a certain forward velocity when the cornering coefficient of the rear tires on the tractor was less than that of the front tires, and oscillatory instability occurred if the lateral stiffness of the tires on the trailer was reduced sufficiently. Vehicle response studies showed that locking the rear wheels of the tractor produced jackknifing, locking the trailer wheels produced trailer swing, and locking the front wheels on the tractor produced a stable vehicle which couldn't be steered. Methods considered to eliminate instability during braking were (1) a braking system which proportions the braking effort on each axle according to the axle load, (2) a braking system which prevents wheels from locking, and (3) the use of friction at the fifth-wheel.

Huizenga [18] analyzed a tractor-semitrailer combination as a linear dynamic system with three degrees of freedom. For the particular vehicle configuration chosen, numerical calculation of the roots of the characteristic equation indicated that the velocity to initiate instability was increased by a smaller trailer moment of inertia, a smaller trailer mass, a longer kingpin-to-trailer-axle distance, a shorter distance between the kingpin and the center of mass of the trailer, and increasing the cornering stiffness of the trailer tires. Experiments (with an automobile-trailer

combination) which varied the moment of inertia of the trailer provided a means of comparing the theoretical and experimental values of forward velocity at which instability was found to occur.

Schmid [19] evaluated the directional stability of a tractor with one or two semitrailers. The velocity, side-slip angle, and yawing motion of each unit; the constraint equations at each frictionless coupling point; and the effect of tractive force on the lateral tire forces were considered in the formulation of the linearized equations of motion. An examination of the characteristic equation showed that having the cornering coefficient of the rear tires on the tractor greater than that of the front tires was a stabilizing effect. The behavior of the articulated vehicle during braking was evaluated in terms of the forces and torques on the vehicle. Stability was increased by utilizing load dependent braking, having a larger trailer moment of inertia, preventing wheel locking, and by having a fifth-wheel coupling where the pin is in front of the load point.

In Reference [20] Bundorf analyzed the directional control behavior of an automobile-trailer combination operating at a constant forward velocity with steering and crosswind inputs. A parameter study of simplified vehicle systems with only the yaw motion of the trailer, or with

vehicle sideslip and trailer yaw motions, indicated that a high trailer mass and inertia, low trailer tire cornering stiffness, and high vehicle speed contributed to oscillatory instability. A forward hitch point location, low hitch load, and low trailer mass resulted in a minimum change in steady-state lateral acceleration gain from that of the automobile without the trailer. Analog computer solutions, with a three-degree-of-freedom model representing the yaw and sideslip motions of the automobile and rotation of the trailer about the hitch point, showed that a rearward trailer center of gravity location led to oscillatory instability, a forward hitch point decreased vehicle response time, and a towing angle constraint spring improved path response. A crosswind input, which turned the automobile without a trailer in the same direction as the wind, turned the automobile with a trailer the opposite way, into the wind.

Bundorf and Schulte [21] studied the directional dynamics of the automobile-trailer combination using pole-zero plots of the yawing velocity transfer function. The pole-zero plots with the vehicle model and parameter variations of Reference [20] were used to reach the same conclusions obtained in Reference [20] by comparing transient responses. This technique was suggested as a means of determining

design parameters which would improve the response of the automobile-trailer system.

Reference [22] examined the stability of automobile-trailer vehicles both experimentally and analytically. Road tests indicated substantial qualitative agreement between computed and measured values for the damping coefficient of periodic trailer oscillations. Factors which led to increased stability were a heavy car with a large yaw inertia and laterally stiff tires, and a light trailer with a long hitch to axle distance and laterally stiff tires.

Dugoff and Ehrlich [23] investigated the influence of motion-dependent aerodynamic forces and moments on the stability of a tractor-semitrailer with a constant forward velocity. It was found that the aerodynamic forces and moments influenced oscillatory and non-oscillatory stability and had a stabilizing or destabilizing effect depending on the operating characteristics of the vehicle.

Mikulcik [24] developed a digital computer simulation of the three-axle tractor-semitrailer combination having pitch, roll, and vertical motions in addition to the lateral, longitudinal, and yaw motions. In the computer study a ramp input of brake force was applied to an axle during a steady turn. Braking the front tractor wheels led to an

uncontrollable maneuver without path curvature, braking the rear tractor wheels led to jackknifing, and braking the rear tractor wheels led to trailer swing. Viscous, Coulomb, and velocity-squared damping were observed to slow up jackknifing but not prevent it when braking was applied to the rear tractor wheels.

Leucht [25] simulated the directional dynamics of a tractor-semitrailer vehicle during braking with a nonlinear four-degree-of-freedom model (longitudinal, lateral, and yawing motions of the tractor and the rotation of the trailer relative to the tractor). The variation in loading condition was identified as a major cause of undesirable directional response under normal commercial operation with a fixed brake-torque distribution. The study showed that the brake-torque distribution, load-sensitive brake-torque control, and brake application times are design characteristics which can be used to improve the overall safe behavior of the real vehicle system.

The cited references have studied: (1) the stability of the vehicle system as characterized by the roots of the characteristic equation, (2) the effect of various parameter changes on the response of a linear vehicle model to steering inputs, or (3) the response of a nonlinear vehicle

model to braking in a turn with the wheels on different axles locked. Linear systems of equations were sufficient in the study of vehicle response to steering inputs which result in moderate lateral accelerations. Nonlinear systems were required in the study of vehicle response to combined steering and braking inputs. The overall result of research on the directional dynamics of the tractor-trailer vehicle has been to determine the effect of basic vehicle characteristics and braking forces which cause the locking of the wheels on specific axles.

1.3 Objective of the Research Program

The objective of this research program was to investigate the directional dynamics of the tractor-semitrailer vehicle as it is affected by design parameters, environmental conditions, and operating characteristics. To accomplish this task a mathematical model describing the longitudinal, lateral, and yawing motions of the vehicle was developed and simulated on a hybrid computer. Features of the model included: (1) the absence of small motion restrictions, (2) the consideration of the non-stationary vertical load on each tire in determining the longitudinal and lateral tire forces, and (3) the use of tire characteristics which simulated the forces produced for both rolling and locked wheels. The computer study (many solutions of the equations

of motion for various parameter changes and steering and braking inputs) was structured around a recognition of the following facts:

1. The basic characteristics of the vehicle are determined by size and weight regulations (issued by the various states) as well as economic and other practical design considerations.

2. Vehicle usage results in uniform and non-uniform load variations from completely full to completely empty.

3. The tractor-semitrailer vehicle must interact with other cars and trucks which have superior handling and braking abilities without causing undesirable or dangerous situations.

The discussion and graphic presentation of the results of the computer study is intended to provide a demonstration of the cause and effect of certain known and unknown phenomena and point out areas where design improvements could improve the overall behavior of the real vehicle system.

CHAPTER 2

FORMULATION

2.1 Vehicle Model and Simplifying Assumptions

The vehicle system examined in this investigation is diagramed in Figure 2.1.¹ It consists of two rigid bodies which translate parallel to a smooth, level surface and rotate about an axis perpendicular to this surface. The fifth-wheel coupling, which allows relative rotation of the two bodies, transmits lateral and longitudinal forces as well as a moment due to friction. Aerodynamic forces and moments; the rotational inertia of the wheels; and the rolling, pitching, and vertical motions of the sprung masses are neglected.

The external forces acting on the vehicle model are the normal and frictional forces applied at the tire-road interface. The three moments which are created by moving the force system from its actual point of application to the center of tire contact with the road are assumed to have a negligible effect on the lateral and longitudinal motions of the vehicle and are disregarded. The total frictional shear force in the plane of the road can be resolved into two

¹Symbols used in Chapter 2 are defined in Table 2-1.

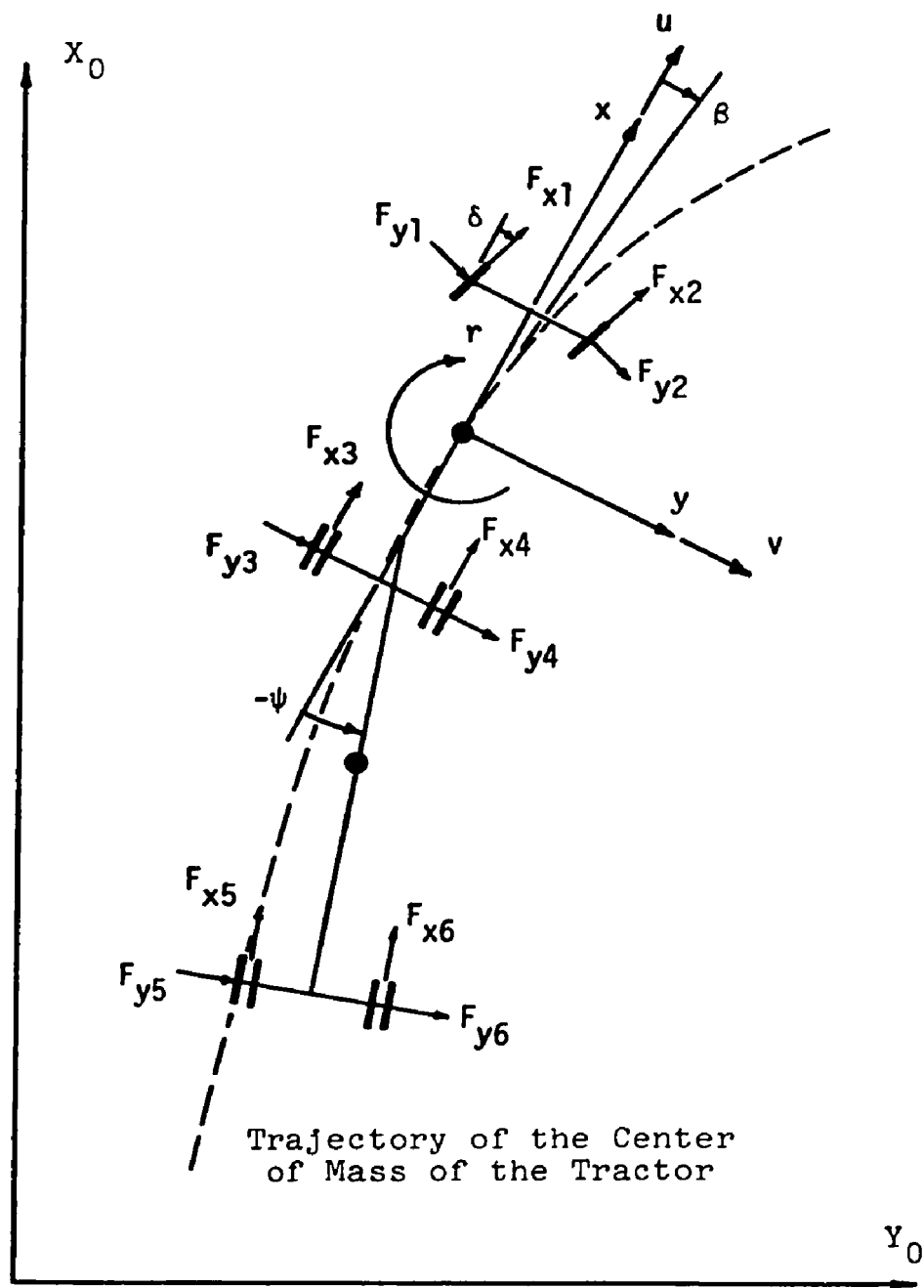


Fig. 2.1 - Tractor-semitrailer vehicle model

TABLE 2-1

SYMBOLS

W_1	= weight of the tractor
W_2	= weight of the semitrailer
m_1	= mass of the tractor
m_2	= mass of the semitrailer
I_1	= moment of inertia of the tractor
I_2	= moment of inertia of the semitrailer
a	= distance between the front axle and the center of mass of the tractor
b	= distance between the rear axle and the center of mass of the tractor
h	= distance between the fifth-wheel and the center of mass of the tractor
c	= distance between the fifth-wheel and the center of mass of the semitrailer
d	= distance between the axle and the center of mass of the semitrailer
z_0	= height of the fifth-wheel above the ground
z_1	= height of the center of mass of the tractor above the ground
z_2	= height of the center of mass of the semitrailer and the load above the ground
x, y, z	= axes of a rectangular coordinate system
u, v, w	= velocity components in the x, y , and z directions
p, q, r	= angular velocity components about the x, y , and z axes

TABLE 2-1 (Continued)

X_O, Y_O	= position coordinates of the center of mass of the tractor
F_x, F_y, F_z	= forces in the x, y, and z directions
M_x, M_y, M_z	= moments about the x, y, and z axes
γ_1	= portion of total lateral load transfer on the front axle of the tractor
γ_2	= portion of total lateral load transfer on the rear axle of the tractor
μ	= maximum tire-road interface coefficient of friction
a_x	= longitudinal acceleration of the center of mass of the tractor
a_y	= lateral acceleration of the center of mass of the tractor
ψ	= trailer angle (rotation of the trailer with respect to the tractor)
R	= equivalent radius of the fifth-wheel
μ_5	= fifth-wheel friction coefficient
X	= longitudinal force at the fifth-wheel
Y	= lateral force at the fifth-wheel
δZ	= longitudinal load transfer onto the fifth-wheel
T	= friction torque at the fifth-wheel
η, ξ	= polar coordinates
ϵ	= pressure on the fifth-wheel
ϕ	= magnitude of fifth-wheel friction torque

TABLE 2-1 (Continued)

δ	= front wheel steer angle
g	= 32.2 ft/sec ²
p_s	= brake system air pressure
E	= braking efficiency
F_{xi}	= longitudinal shear force on the tire or dual tires ^a
F_{yi}	= lateral shear force on the tire or dual tires
F_{zi}	= vertical load on the tire or dual tires
$\bar{F}_{zi}$	= static load on the tire or dual tires
$\bar{F}'_{zi}$	= static load on a tire or half the static load on dual tires
δF_{zi}	= longitudinal load transfer onto the tire
ΔF_{zi}	= lateral load transfer onto the tire
α_i	= slip angle of the tire
$\bar{\alpha}_i$	= normalized slip angle of the tire
v_i, σ_i	= experimental tire coefficients
C_i	= cornering stiffness of the tire
$f(\alpha_i, \bar{F}'_{zi})$	= normalized lateral force characteristic
L_i	= logic variable for wheel i
t_j	= half the lateral distance between the centers of tire contact on axle j ^b
TD_j	= application time of the brakes on axle j after the initial steering input
BT_j	= brake torque on axle j divided by total brake torque

TABLE 2-1 (Continued)

λ_k	= inertia force at the center of mass of the semitrailer in the k direction ^c
k_j	= ratio of the actual static axle load to the fully loaded static axle load
p_{ai}	= air pressure at the brake actuator
u_i	= longitudinal velocity of wheel i
v_i	= lateral velocity of wheel i
E_{ref}	= reference voltage of the computer
q_n	= a variable
p_n	= a coefficient
E_{q_n}	= amplifier voltage representing q_n
β	= sideslip angle of the center of mass of the tractor
ρ	= radius of curvature of the center of mass of the tractor
θ	= heading angle of the tractor
t	= time
Δt	= sampled time interval

^aSubscripts 1 through 4 correspond to the left-front, right-front, left-rear, and right-rear wheels of the tractor. Subscripts 5 and 6 correspond to the left and right trailer wheels.

^bSubscripts 1 through 3 correspond to the front axle of the tractor, the rear axle of the tractor, and the trailer axle.

^cSubscripts 1 and 2 correspond to the longitudinal and lateral axes of the semitrailer. Subscripts 3 and 4 correspond to the longitudinal and lateral axes of the tractor.

components as shown in Figure 2.2: a lateral force perpendicular to the plane of the wheel and a longitudinal force in the direction of wheel heading. In addition, dual tires are assumed to produce shear forces which are twice the forces generated by a single tire operating at one half the vertical load on the dual tires. A more complete discussion of the relations between the tire operating conditions and the resulting frictional forces is left to a separate section.

In addition to supporting the weight of the vehicle, the vertical force on the tire has a large influence on the magnitude of lateral and longitudinal shear forces that can be developed. The vertical load on each tire is determined by the static load plus any instantaneous change resulting from maneuvers of a vehicle with a center of mass above the ground plane. In this study, these load transfers were not determined as a function of the displacement and velocity of the suspension components, but rather as a quasi-static function of the acceleration of the center of gravity of each mass element. In this manner, it was possible to simulate the major contribution of pitch and roll to the directional response of the vehicle without having to treat these motions as degrees of freedom.

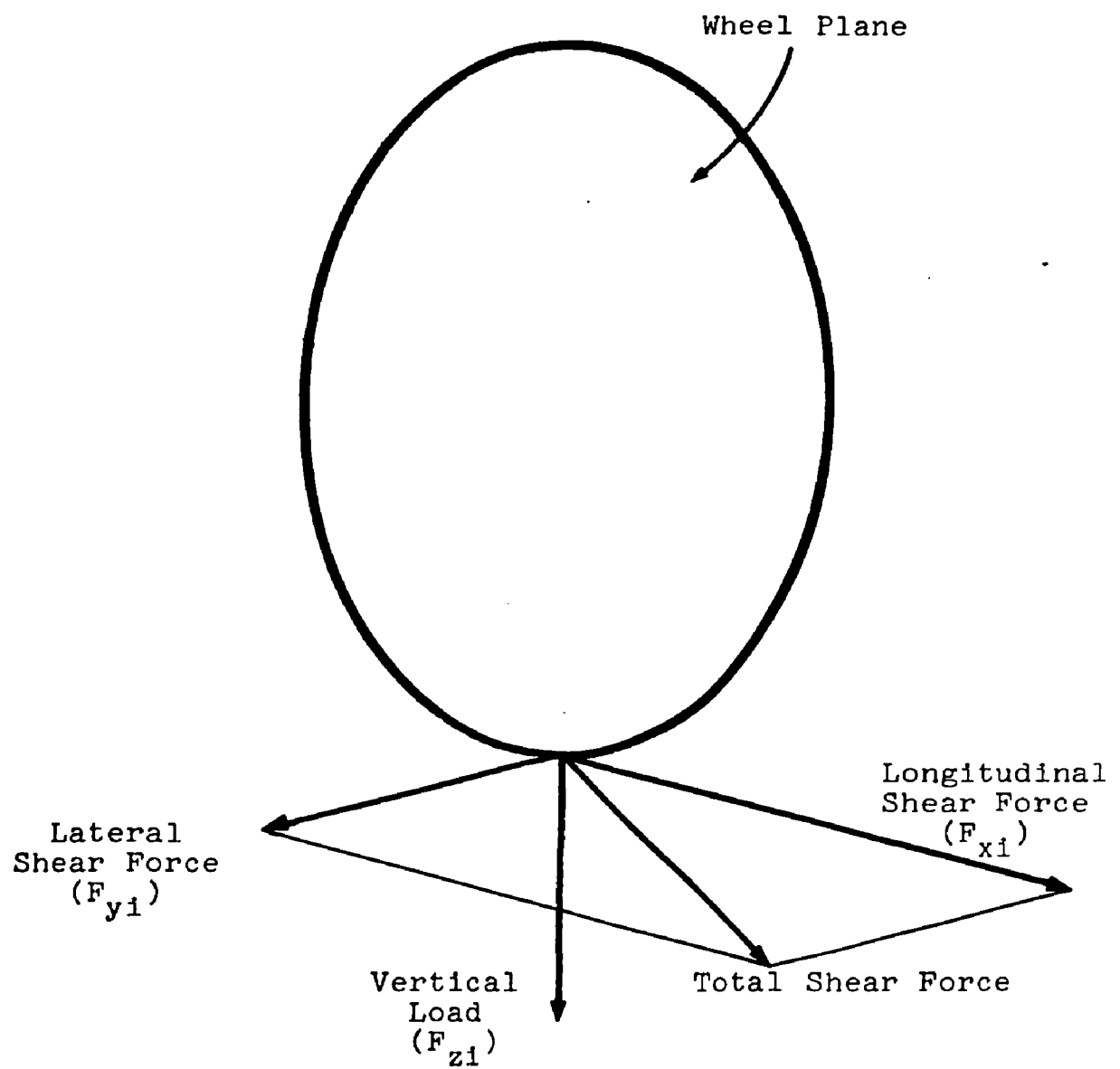


Fig. 2.2 - Tire forces

2.2 Equations of Motion

The equations of motion for the tractor-semitrailer vehicle are derived in the following manner: (1) consider each rigid body separately, (2) define a coordinate system fixed to each body with the origin at the center of mass, (3) identify the pertinent variables needed to describe the motion of each body, and (4) apply these conditions to equations describing the general motion of a rigid body which is translating and rotating in space.

The xyz coordinate system is fixed to the tractor with its origin at the center of mass, the x-axis in the forward direction, the y-axis out the right side parallel to the ground, and the z-axis into the ground. The velocity of the center of mass of the tractor has components u , v , and w in the x , y , and z directions and the angular velocity of the tractor has components p , q , and r about the x , y , and z axes. The $x'y'z'$ coordinate system is fixed to the trailer with its origin at the center of mass, the x' -axis in the forward direction, the y' -axis out the right side parallel to the ground plane, and the z' -axis into the ground. The velocity of the center of mass of the semitrailer has components u' , v' , and w' in the x' , y' , and z' directions and the angular velocity of the semitrailer has components

p' , q' , and r' about the x' , y' , and z' axes. Since both masses are assumed to translate parallel to the ground plane and rotate about an axis perpendicular to this flat surface, the velocities perpendicular to the ground (w and w'), the angular velocities about the longitudinal and lateral axes (p , q , p' , and q'), and the derivatives with respect to time of these six variables are zero.

The six equations which describe the general motion of a rigid body translating and rotating relative to an inertial reference are derived as Equations (A-18) through (A-20) and (A-28) through (A-30) in Appendix A. These equations, repeated for convenience as (2-1) through (2-6), apply to a rigid body in which an orthogonal xyz coordinate system is fixed with the origin located at the center of mass.

$$m(\dot{u} + wq - vr) = F_x \quad (2-1)$$

$$m(\dot{v} + ur - wp) = F_y \quad (2-2)$$

$$m(\dot{w} + vp - uq) = F_z \quad (2-3)$$

$$\begin{aligned} I_{xx}\dot{p} + (rp - \dot{q})I_{xy} - (\dot{r} + pq)I_{zx} + (r^2 - q^2)I_{yz} \\ + qr(I_{zz} - I_{yy}) = M_x \end{aligned} \quad (2-4)$$

$$\begin{aligned} I_{yy}\dot{q} + (pq - \dot{r})I_{yz} - (\dot{p} + qr)I_{xy} + (p^2 - r^2)I_{zx} \\ + rq(I_{xx} - I_{zz}) = M_y \end{aligned} \quad (2-5)$$

$$\begin{aligned} I_{zz}\dot{r} + (qr - \dot{p})I_{zx} - (\dot{q} + rp)I_{yz} + (q^2 - p^2)I_{xy} \\ + pq(I_{yy} - I_{xx}) = M_z \end{aligned} \quad (2-6)$$

The quantities F_i and M_i represent the sum of the external forces in the i -direction and the moments about the i -axis. The dot over a variable indicates the derivative with respect to time, m is the mass, I_{ii} is the mass moment of inertia about the i -axis, and I_{ij} is the mass product of inertia about the i - and j -axes.

On assuming the motion to be planar, the six equations reduce to three equations whose integration will yield the longitudinal, lateral, and yawing motions of a rigid body traversing a smooth half space. Three equations are required to describe the motion of the tractor and three for the trailer. Thus,

$$m_1(\dot{u} - vr) = F_x \quad (2-7)$$

$$m_1(\dot{v} + ur) = F_y \quad (2-8)$$

$$I_1 \dot{r} = M_z \quad (2-9)$$

$$m_2(\dot{u}' - v'r') = F_{x'} \quad (2-10)$$

$$m_2(\dot{v}' + u'r') = F_{y'} \quad (2-11)$$

$$I_2 \dot{r}' = M_{z'} \quad (2-12)$$

On expanding the force and moment summations in terms of the external forces existing at the tire-road interface (see Figure 2.1) and at the fifth-wheel, one obtains:

$$m_1(\dot{u} - vr) = (F_{x1} + F_{x2}) \cos \delta + (F_{x3} + F_{x4}) - (F_{y1} + F_{y2}) \sin \delta + X \quad (2-13)$$

$$m_1(\dot{v} + ur) = (F_{x1} + F_{x2}) \sin \delta + (F_{y1} + F_{y2}) \cos \delta + (F_{y3} + F_{y4}) + Y \quad (2-14)$$

$$I_1 \dot{r} = a(F_{x1} + F_{x2}) \sin \delta + t_1(F_{x1} - F_{x2}) \cos \delta + t_2(F_{x3} - F_{x4}) + a(F_{y1} + F_{y2}) \cos \delta - hY - t_1(F_{y1} - F_{y2}) \sin \delta - b(F_{y3} + F_{y4}) + T \quad (2-15)$$

$$m_2(\dot{u}' - v'r') = (F_{x5} + F_{x6}) - (X \cos \psi + Y \sin \psi) \quad (2-16)$$

$$m_2(\dot{v}' + u'r') = (F_{y5} + F_{y6}) + (X \sin \psi - Y \cos \psi) \quad (2-17)$$

$$I_2 \dot{r}' = t_3(F_{x5} - F_{x6}) - d(F_{y5} + F_{y6}) - T + c(X \sin \psi - Y \cos \psi) \quad (2-18)$$

where X and Y are the forces in the x and y directions at the fifth-wheel and T is the torque due to friction at the fifth-wheel.

The constraint equations relating the primed variables to the unprimed variables at the fifth-wheel are

$$r' = r + \dot{\psi} \quad (2-19)$$

$$u' = u \cos \psi + (v - hr) \sin \psi \quad (2-20)$$

$$v' + cr' = -u \sin \psi + (v - hr) \cos \psi \quad (2-21)$$

where h and c are the distances from the fifth-wheel to the center of mass of the tractor and the semitrailer.

The primed variables are eliminated from Equations (2-16) through (2-18) by first developing Equations (2-22) through (2-24) from the constraint equations and their derivatives.

$$\dot{r}' = \dot{r} + \ddot{\psi} \quad (2-22)$$

$$\begin{aligned} \dot{u}' - v'r' &= (\dot{u} - vr + hr^2)\cos \psi + (\dot{v} + ur - hr\dot{r})\sin \psi \\ &\quad + c(\dot{r} + \ddot{\psi})^2 \end{aligned} \quad (2-23)$$

$$\begin{aligned} \dot{v}' + u'r' &= -(\dot{u} - vr + hr^2)\sin \psi + (\dot{v} + ur - hr\dot{r})\cos \psi \\ &\quad - c(\dot{r} + \ddot{\psi}) \end{aligned} \quad (2-24)$$

On substituting the above relationships into Equations (2-16) through (2-18), three equations of motion are obtained for the semitrailer in terms of the unprimed variables (u , v , r , and ψ):

$$\begin{aligned} m_2(\dot{u} - vr + hr^2)\cos \psi + m_2(\dot{v} + ur - hr\dot{r})\sin \psi + cm_2(\dot{r} + \ddot{\psi})^2 \\ = (F_{x5} + F_{x6}) - (X \cos \psi + Y \sin \psi) \end{aligned} \quad (2-25)$$

$$\begin{aligned} -m_2(\dot{u} - vr + hr^2)\sin \psi + m_2(\dot{v} + ur - hr\dot{r})\cos \psi - cm_2(\dot{r} + \ddot{\psi}) \\ = (F_{y5} + F_{y6}) + (X \sin \psi - Y \cos \psi) \end{aligned} \quad (2-26)$$

$$I_2(\dot{r} + \ddot{\psi}) = t_3(F_{x5} - F_{x6}) - d(F_{y5} + F_{y6}) - T + c(X \sin \psi - Y \cos \psi) \quad (2-27)$$

Before eliminating the forces X and Y from the equations of motion, it is convenient to define the inertial forces λ_1 and λ_2 acting at the center of mass of the semitrailer in the x' and y' directions.

$$\lambda_1 = -m_2(\dot{u} - vr + hr^2)\cos \psi - m_2(\dot{v} + ur - hr\dot{r})\sin \psi - cm_2(r + \dot{\psi})^2 \quad (2-28)$$

$$\lambda_2 = m_2(\dot{u} - vr + hr^2)\sin \psi - m_2(\dot{v} + ur - hr\dot{r})\cos \psi + cm_2(\dot{r} + \ddot{\psi}) \quad (2-29)$$

It is also useful to write these inertial forces at the center of mass of the semitrailer as λ_3 acting in the x direction and λ_4 acting in the y direction.

$$\lambda_3 = -m_2(\dot{u} - vr + hr^2) - cm_2(r + \dot{\psi})^2 \cos \psi - cm_2(\dot{r} + \ddot{\psi}) \sin \psi \quad (2-30)$$

$$\lambda_4 = -m_2(\dot{v} + ur - hr\dot{r}) - cm_2(r + \dot{\psi})^2 \sin \psi + cm_2(\dot{r} + \ddot{\psi}) \cos \psi \quad (2-31)$$

Equations (2-25) and (2-26) are then rewritten as

$$X \cos \psi + Y \sin \psi = \lambda_1 + (F_{x5} + F_{x6}) \quad (2-32)$$

$$-(X \sin \psi - Y \cos \psi) = \lambda_2 + (F_{y5} + F_{y6}) \quad (2-33)$$

Solving Equations (2-32) and (2-33) for X and Y gives

$$X = \lambda_3 + (F_{x5} + F_{x6}) \cos \psi - (F_{y5} + F_{y6}) \sin \psi \quad (2-34)$$

$$Y = \lambda_4 + (F_{x5} + F_{x6}) \sin \psi + (F_{y5} + F_{y6}) \cos \psi \quad (2-35)$$

Equations (2-34) and (2-35) are used to eliminate the forces X and Y and, since the steer angle is always very small, $\sin \delta$ and $\cos \delta$ are approximated by δ and 1.0, respectively. Equations (2-13) through (2-15) and (2-27) are then rewritten as Equations (2-36) through (2-39). These equations describe the longitudinal, lateral, and yawing motions of the assumed tractor-trailer constrained to traverse a flat surface.

$$\begin{aligned} m_1(\dot{u} - vr) - \lambda_3 &= (F_{x1} + F_{x2}) + (F_{x3} + F_{x4}) \\ &+ (F_{x5} + F_{x6}) \cos \psi - (F_{y1} + F_{y2})\delta \\ &- (F_{y5} + F_{y6}) \sin \psi \end{aligned} \quad (2-36)$$

$$\begin{aligned} m_1(\dot{v} + ur) - \lambda_4 &= (F_{x1} + F_{x2})\delta + (F_{x5} + F_{x6}) \sin \psi \\ &+ (F_{y1} + F_{y2}) + (F_{y3} + F_{y4}) + (F_{y5} + F_{y6}) \cos \psi \end{aligned} \quad (2-37)$$

$$\begin{aligned}
I_1 \ddot{r} + h\lambda_4 = & a(F_{x1} + F_{x2})\delta + t_1(F_{x1} - F_{x2}) \\
& + t_2(F_{x3} - F_{x4}) - h(F_{x5} + F_{x6}) \sin \psi + a(F_{y1} + F_{y2}) \\
& - t_1(F_{y1} - F_{y2})\delta - b(F_{y3} + F_{y4}) - h(F_{y5} + F_{y6}) \cos \psi \\
& + T
\end{aligned} \tag{2-38}$$

$$\begin{aligned}
I_2(\ddot{r} + \ddot{\psi}) + c\lambda_2 = & t_3(F_{x5} - F_{x6}) - (c + d)(F_{y5} + F_{y6}) \\
& - T
\end{aligned} \tag{2-39}$$

2.3 Vertical Load on Each Tire

The instantaneous vertical load on each tire, F_{zi} , has a substantial influence on the magnitude of the longitudinal and lateral shear forces which can be developed. Therefore, it is desirable that the vertical load be known for each tire as a function of time. Such a determination must necessarily be approximate since the model does not allow for the pitch, roll, and vertical motions which deform the suspension components and alter the vertical tire loads. In this study, the vertical load on each tire or pair of dual tires is determined by the static load plus any instantaneous change as a result of longitudinal and lateral load transfer.¹

¹Longitudinal load transfer is the change in vertical load on an axle due to the longitudinal acceleration of the vehicle. Lateral load transfer is the change in vertical load between the right and left wheels on any axle due to the lateral acceleration of the vehicle.

The static loads on each tire or pair of dual tires are:

$$\bar{F}_{z1} = \frac{bW_1}{2(a+b)} + \frac{d(b-h)W_2}{2(a+b)(c+d)} \quad (2-40)$$

$$\bar{F}_{z2} = \frac{bW_1}{2(a+b)} + \frac{d(b-h)W_2}{2(a+b)(c+d)} \quad (2-41)$$

$$\bar{F}_{z3} = \frac{aW_1}{2(a+b)} + \frac{d(a+h)W_2}{2(a+b)(c+d)} \quad (2-42)$$

$$\bar{F}_{z4} = \frac{aW_1}{2(a+b)} + \frac{d(a+h)W_2}{2(a+b)(c+d)} \quad (2-43)$$

$$\bar{F}_{z5} = \frac{cW_2}{2(c+d)} \quad (2-44)$$

$$\bar{F}_{z6} = \frac{cW_2}{2(c+d)} \quad (2-45)$$

In calculating the longitudinal and lateral load transfer onto the semitrailer wheels, it is assumed that the fifth-wheel does not transmit roll or pitching moments between the tractor and the trailer. In other words, only a frictional yaw moment and three forces (longitudinal, lateral, and vertical) exist at the fifth-wheel. Using the free-body diagram of Figure 2.3 and taking moments about longitudinal axes in the ground plane through the center of the right and left trailer wheels, the lateral load transfer onto the left and right trailer wheels is found to be

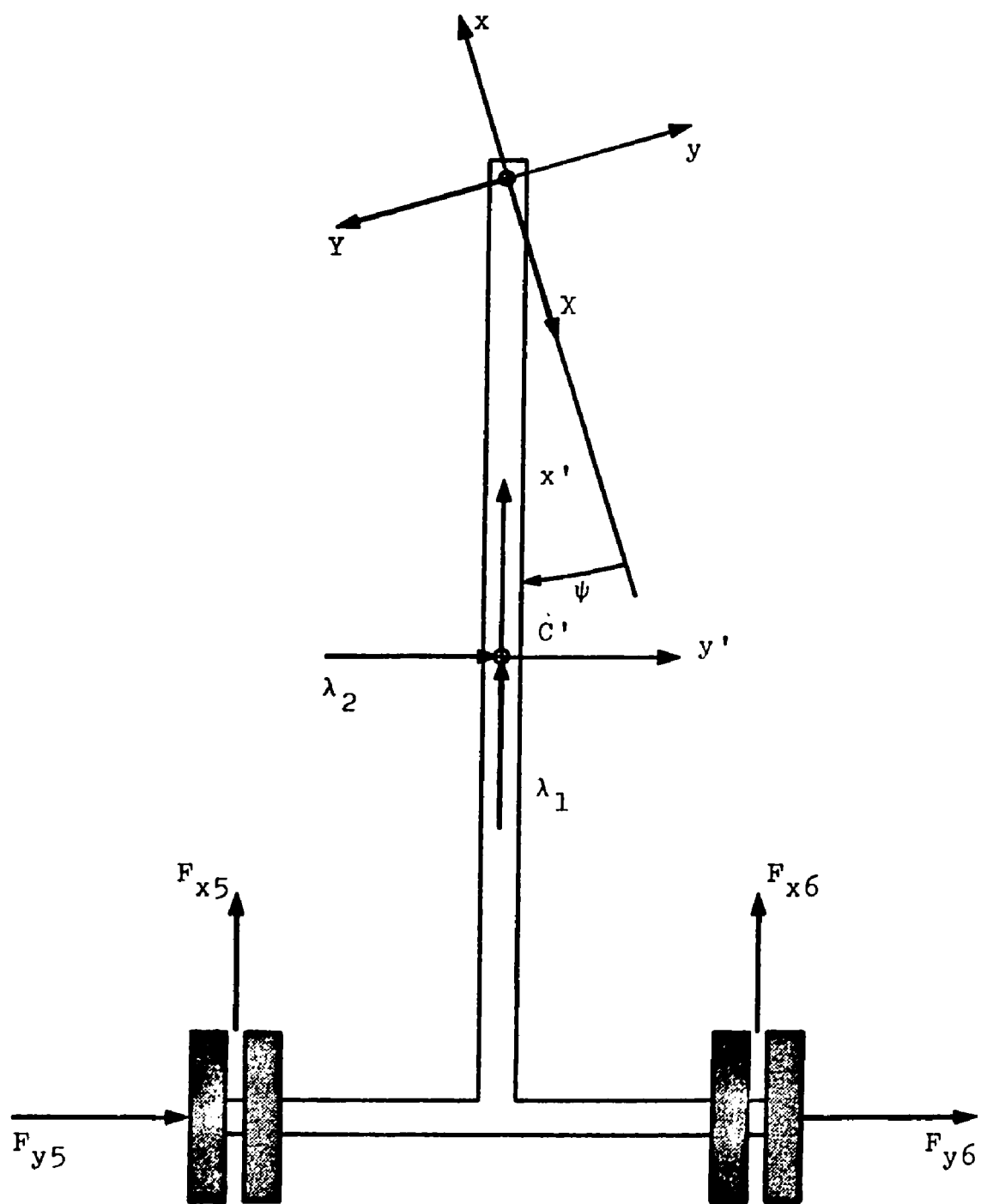


Fig. 2.3 - Free-body diagram of the semitrailer

$$\Delta F_{z5} = - \frac{z_2}{2t_3} \lambda_2 - \frac{z_0}{2t_3} (X \sin \psi - Y \cos \psi) \quad (2-46)$$

and

$$\Delta F_{z6} = \frac{z_2}{2t_3} \lambda_2 + \frac{z_0}{2t_3} (X \sin \psi - Y \cos \psi) \quad (2-47)$$

where z_0 and z_2 are the heights of the fifth-wheel and the center of mass of the semitrailer above the ground. Using Equation (2-33) to eliminate X and Y gives

$$\Delta F_{z5} = \frac{(z_2 - z_0)}{2t_3} \lambda_2 + \frac{z_0}{2t_3} (F_{y5} + F_{y6}) \quad (2-48)$$

and

$$\Delta F_{z6} = \frac{(z_2 - z_0)}{2t_3} \lambda_2 - \frac{z_0}{2t_3} (F_{y5} + F_{y6}) \quad (2-49)$$

On taking moments about lateral axes in the ground plane located under the fifth-wheel and under the trailer axle, the longitudinal load transfer is found to be

$$\delta F_{z5} = \delta F_{z6} = - \frac{z_2}{2(c + d)} \lambda_1 + \frac{z_0}{2(c + d)} (X \cos \psi + Y \sin \psi) \quad (2-50)$$

and

$$\delta Z = \frac{z_2}{(c + d)} \lambda_1 - \frac{z_0}{(c + d)} (X \cos \psi + Y \sin \psi) \quad (2-51)$$

Using Equation (2-32) to eliminate X and Y gives

$$\delta F_{z5} = \delta F_{z6} = - \frac{(z_2 - z_0)}{2(c + d)} \lambda_1 + \frac{z_0}{2(c + d)} (F_{x5} + F_{x6}) \quad (2-52)$$

and

$$\delta Z = \frac{(z_2 - z_0)}{(c + d)} \lambda_1 - \frac{z_0}{(c + d)} (F_{x5} + F_{x6}) \quad (2-53)$$

The moment on the tractor due to inertial forces about any longitudinal axis in the ground plane is expressed by

$$M_x = z_0 Y - z_1 m_1 (\dot{v} + ur) \quad (2-54)$$

where z_1 is the height of the center of mass of the tractor above the ground. This moment is distributed to the front and rear wheels of the tractor according to the roll rate distribution of the tractor.¹ The roll rate distribution is the division total roll rate between the front and rear suspensions and is expressed by γ_1 and γ_2 , respectively. Using the free-body diagram of Figure 2.4 and Equation (2-54), the lateral load transfer onto each of the tractor wheels is determined by the following equations:

¹The roll rate is the change in the restoring couple exerted by the suspension at either axle on the sprung mass of the tractor per unit change in roll angle of the sprung mass about a longitudinal axis. The total roll rate of the tractor is the sum of the separate roll rates of all suspensions.

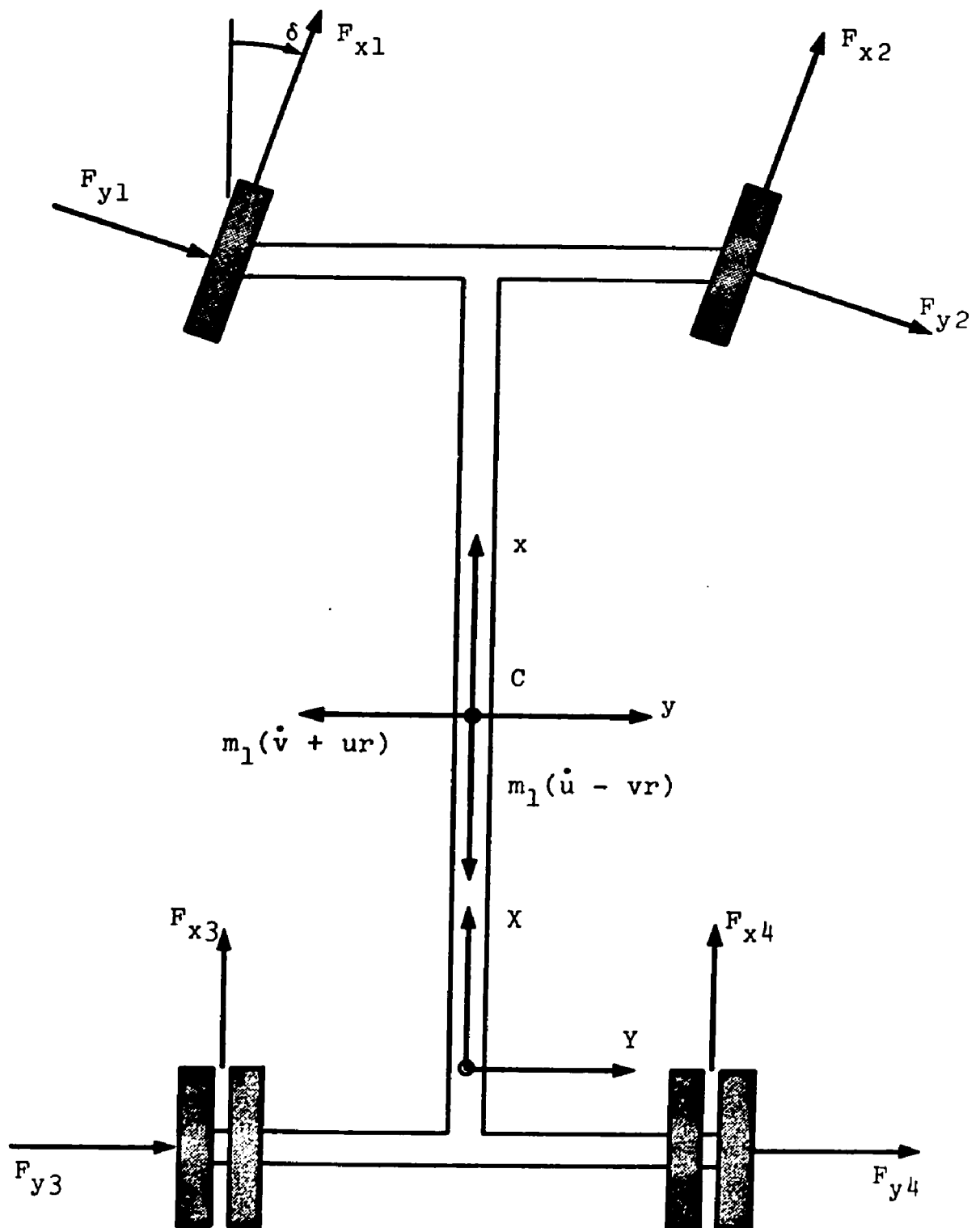


Fig. 2.4 - Free-body diagram of the tractor

$$\Delta F_{z1} = - \frac{\gamma_1 z_0}{2t_1} Y + \frac{\gamma_1 z_1 m_1}{2t_1} (\dot{v} + ur) \quad (2-55)$$

$$\Delta F_{z2} = \frac{\gamma_1 z_0}{2t_1} Y - \frac{\gamma_1 z_1 m_1}{2t_1} (\dot{v} + ur) \quad (2-56)$$

$$\Delta F_{z3} = - \frac{\gamma_2 z_0}{2t_2} Y + \frac{\gamma_2 z_1 m_1}{2t_2} (\dot{v} + ur) \quad (2-57)$$

$$\Delta F_{z4} = \frac{\gamma_2 z_0}{2t_2} Y - \frac{\gamma_2 z_1 m_1}{2t_2} (\dot{v} + ur) \quad (2-58)$$

where Y is defined by Equation (2-35). By taking moments about lateral axes located in the ground plane under the rear and front tractor axles, the longitudinal load transfers are found to be

$$\delta F_{z1} = \delta F_{z2} = \frac{(a + h)}{2(a + b)} \delta Z + \frac{z_0}{2(a + b)} X - \frac{z_1 w_1}{2(a + b)} (\dot{u} - vr) \quad (2-59)$$

$$\delta F_{z3} = \delta F_{z4} = \frac{(b - h)}{2(a + b)} \delta Z - \frac{z_0}{2(a + b)} X + \frac{z_1 w_1}{2(a + b)} (\dot{u} - vr) \quad (2-60)$$

where X and δZ are defined by Equations (2-34) and (2-51), respectively.

The instantaneous vertical load on any tire or pair of dual tires can be expressed as

$$F_{zi} = \bar{F}_{zi} + \delta F_{zi} + \Delta F_{zi} \quad (2-61)$$

where the subscript i refers to the particular wheel. The vertical loads, calculated in this manner, are approximate in a transient but are exact during steady acceleration. In order to simplify the analysis, these approximate loads are used in determining the longitudinal and lateral shear forces that are developed by the tires.

2.4 Tire Slip Angles

The slip angle of a tire is the angle, shown in Figure 2.5, between the direction of wheel travel and the direction of wheel heading. It is the primary variable in the generation of a lateral force on the tire and it is defined as

$$\tan (\alpha_1 + \delta_1) = \frac{v_1}{u_1} \quad (2-62)$$

where α_1 = slip angle of the tire
 δ_1 = steer angle of the wheel
 u_1 = longitudinal velocity of the wheel
 v_1 = lateral velocity of the wheel

The velocity of each wheel is given by the vector sum of the velocity of the center of mass and the vector product of the angular velocity and the position vector of the wheel relative to the center of mass.¹ Using Figure 2.1 and the

¹The longitudinal component of the position vector is the distance between the axle and the center of mass. The lateral component of the position vector is equal to one half of the lateral distance between the centers of tire contact on the axle.

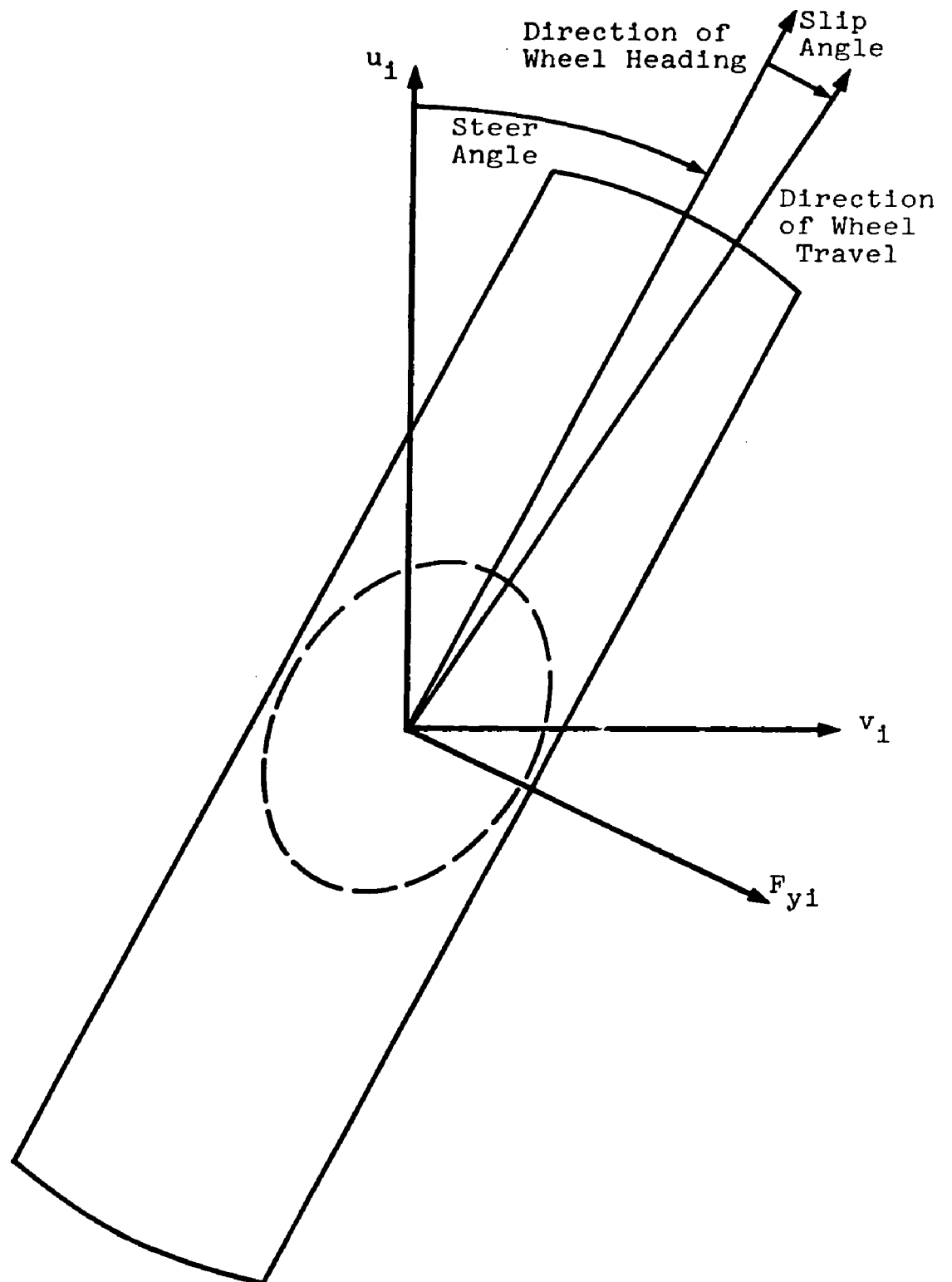


Fig. 2.5 - Slip angle of a tire

longitudinal and lateral distances between the wheel and the center of mass (defined in Table 2-1), the slip angles for each tire or dual tires are

$$\tan (\alpha_1 + \delta) = \frac{v + ar}{u + t_1 r} \quad (2-63)$$

$$\tan (\alpha_2 + \delta) = \frac{v + ar}{u - t_1 r} \quad (2-64)$$

$$\tan \alpha_3 = \frac{v - br}{u + t_2 r} \quad (2-65)$$

$$\tan \alpha_4 = \frac{v - br}{u - t_2 r} \quad (2-66)$$

$$\tan \alpha_5 = \frac{v' - dr'}{u' + t_3 r'} \quad (2-67)$$

$$\tan \alpha_6 = \frac{v' - dr'}{u' - t_3 r'} \quad (2-68)$$

If (1) the tangent of the angle is replaced by the angle itself (a 4 percent error at 20°), (2) the product of the wheel track and yaw rate divided by two is negligible in comparison to the longitudinal velocity, and (3) the primed variables are eliminated with Equations (2-19) through (2-21), the following expressions for slip angle are obtained:

$$\alpha_1 = \alpha_2 = \frac{v + ar}{u} - \delta \quad (2-69)$$

$$\alpha_3 = \alpha_4 = \frac{v - br}{u} \quad (2-70)$$

$$\alpha_5 = \alpha_6 = \frac{(v - hr)\cos \psi - (c + d)(r + \dot{\psi}) - u \sin \psi}{u \cos \psi + (v - hr)\sin \psi} \quad (2-71)$$

2.5 Tire Mechanics Considerations

The frictional shear force in the plane of the road is a function of many factors including the instantaneous vertical load, the frictional characteristics of the tire-road interface, the contamination of the road surface, the elastic deformation of the tire, and the relative motion between the tread and the road. In Figure 2.2 the total friction force in the plane of the road is shown resolved into two components: F_{x1} , due to wheel torque and F_{y1} , due to lateral deformation.

The lateral force characteristic of a free-rolling tire (the applied wheel torque is zero) is primarily a nonlinear function of slip angle and vertical load. Figure 2.6 shows an example of this characteristic for a 10.00-22 truck tire. A simple analytical function, similar to that of Hoffman [26], has been used to fit the experimental data points (shown by the symbol $\times$) and to plot lines of constant slip angle and vertical load. The assumed functional relationship is

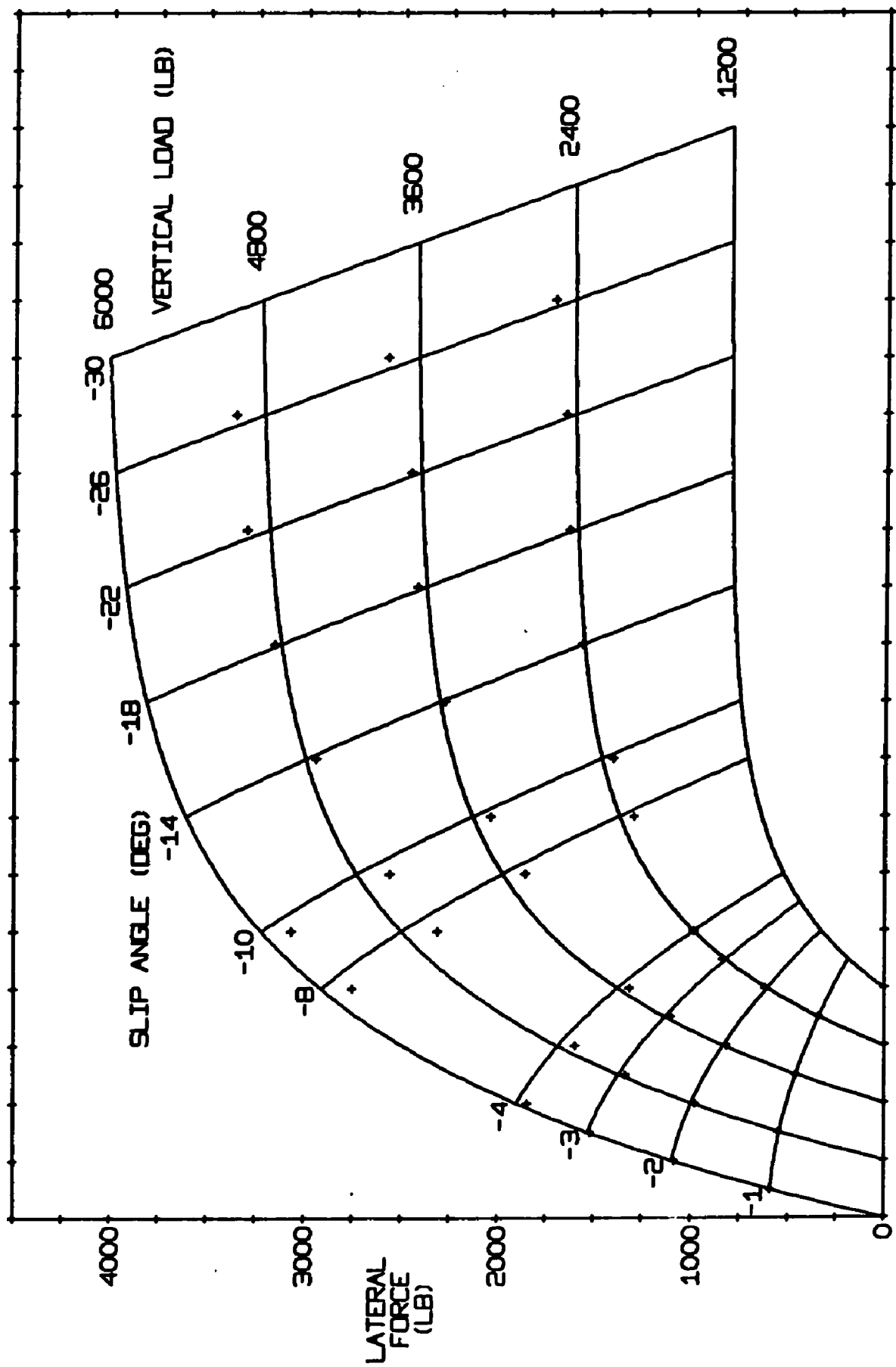


FIG. 2-6 - LATERAL FORCE VERSUS SLIP ANGLE AND VERTICAL LOAD
FOR A 10-00-22 TRUCK TIRE

$$F_{y1} = -\mu F_{z1} [1.0 - \exp(-|\bar{\alpha}_1|)] [\bar{\alpha}_1/|\bar{\alpha}_1|] \quad (2-72)$$

$$\bar{\alpha}_1 = \frac{C_1 \alpha_1}{\mu F'_{z1}} \quad (2-73)$$

$$\frac{C_1}{F'_{z1}} = v_1 + \sigma_1 F'_{z1} \quad (2-74)$$

where μ is the maximum coefficient of friction between the tire and the road, C_1 is the cornering stiffness of the tire, and α_1 is the slip angle. The constants v_1 and σ_1 were determined by a least squares fit of the data points at a slip angle of -1.0 degree. The maximum coefficient of friction is assumed to be that value which minimizes the total squared error between the calculated curve and all the remaining points. This procedure generalizes the limited information given by 32 data points (i.e., at four vertical loads and various slip angles) so that the lateral force is defined at any slip angle and any load. In addition, the functional relationship can be readily altered (by means of the quantities v_1 , σ_1 , and μ) to account for changes in lateral stiffness and the tire-road friction level. Figure 2.7 shows lateral force versus slip angle calculated with Equations (2-72) through (2-74) for a vertical load of 3000 pounds and friction coefficients of 0.3, 0.5, and 0.7.

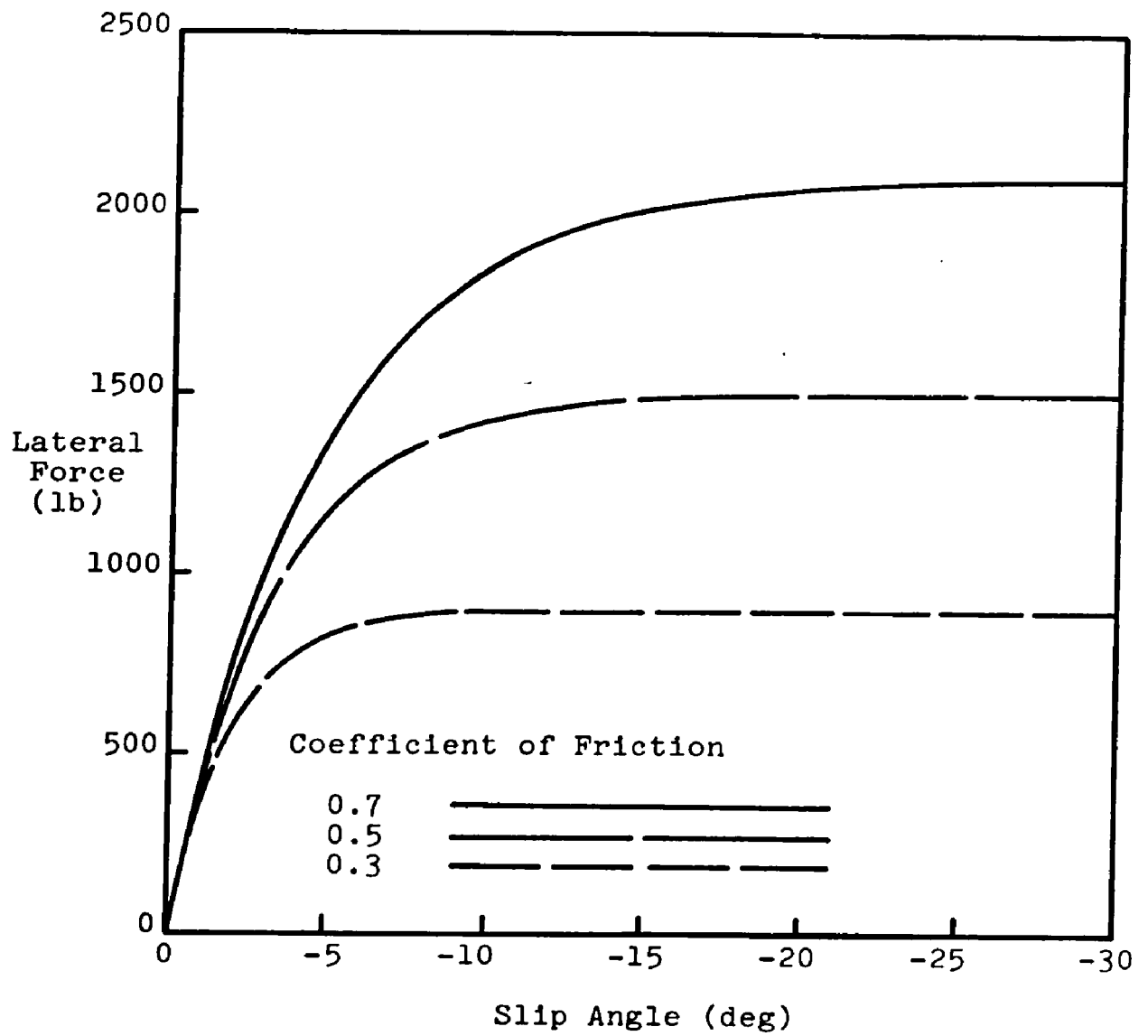


Fig. 2.7 - Lateral force versus slip angle for various friction coefficients

By means of the lateral force characteristic defined by Equations (2-72) through (2-74), a normalized lateral force characteristic has been defined (somewhat differently than Nordeen [27]) as the ratio of the lateral force produced by a tire operating at the static load divided by the product of the coefficient of friction and the static load. The normalized lateral force characteristic, which is a function of slip angle and the static load on the tire, is given as

$$f(\alpha_1, \bar{F}'_{z1}) = \frac{F_{y1}}{\mu \bar{F}'_{z1}} = - [1.0 - \exp(-|\bar{\alpha}_1|)] [\bar{\alpha}_1 / |\bar{\alpha}_1|] \quad (2-75)$$

where $\bar{F}'_{z1}$ is the static load on the tire. A reasonable approximation of lateral force on any tire or pair of dual tires as a function of slip angle and vertical load is formed by the following product

$$F_{y1} = \mu F_{z1} f(\alpha_1, \bar{F}'_{z1}) \quad (2-76)$$

where the instantaneous vertical load, F_{z1} , is given by Equation (2-61).

In this study, brake forces of varying levels with selected distributions of torque and time delays are applied in a step manner to each axle. In order to simplify the complex mechanics of the braked wheel, the wheel is assumed to roll as long as the applied brake torque is equivalent to

a brake force which is less than or equal to the product of the coefficient of friction, the instantaneous vertical load, and the cosine of the slip angle. This condition (that the braked wheel is rolling) is expressed as

$$- F_{x1}^* \leq \mu F_{z1} \cos \alpha_1 \quad (2-77)$$

where F_{x1}^* is the desired brake force on the wheel. The longitudinal and lateral forces acting on the wheel are

$$F_{x1} = F_{x1}^* \quad (2-78)$$

$$F_{y1} = \mu F_{z1} f(\alpha_1, \bar{F}_{z1}') \quad (2-79)$$

Whenever the desired brake forces exceeds the maximum frictional capability of the tire-road interface Equation (2-77) is not satisfied and the wheel is assumed to have locked. A sliding condition is then assumed to exist yielding the following relationships for F_{x1} and F_{y1} :

$$F_{x1} = - \mu F_{z1} \cos \alpha_1 \quad (2-80)$$

$$F_{y1} = - \mu F_{z1} \sin \alpha_1 \quad (2-81)$$

Equations (2-78) and (2-79) neglect interactive effects and approximate the combined braking and lateral forces of a

rolling wheel. In other words, the lateral force characteristic is assumed to be independent of the tractive force developed and equal to the free-rolling characteristic as long as the desired brake force is less than the maximum capability of the tire-road interface. Equations (2-80) and (2-81) represent the locked wheel condition where the total frictional shear force on the tire is opposite to the direction of motion and is equal to the product of the maximum coefficient of friction and the instantaneous vertical load.

Available data show that this tire model is substantially valid for operation at low slip angles with moderate braking forces. The model should be reasonably accurate when the wheel is locked and the tire is sliding. This treatment of the tire mechanics is most inadequate, however, for sustained operation at large slip angles and peak braking force. In this investigation, which does not simulate a control system to maintain the brake force near its maximum value,¹ the wheel is assumed to have locked if the applied

¹The typical longitudinal shear force characteristic shows that a maximum force does not occur with a locked wheel but occurs when the wheel is turning about 10 to 20 percent slower than a free-rolling wheel. A control system, which modulates the applied brake pressure, is required to prevent the wheel from locking and produce a maximum longitudinal force.

input does not satisfy Equation (2-77) and the amount of time which could be spent with a large slip angle and peak braking force is negligible. The approximate tire-mechanics model is based on the passenger car tire data from References [28-31] and the limited truck-tire data in Reference [32] for the combined generation of longitudinal and lateral forces. At a slip angle of 4 degrees the truck-tire data indicates virtually no decrease in lateral force for braking forces which are less than one half the vertical load on the tire.

2.6 Brake-Torque Distribution

The brake-torque generated at each wheel of a tractor-semitrailer vehicle is a function of the air pressure supplied to each wheel and the brake torque produced by a given line pressure. The spatial distribution of brake torque is a design parameter of the vehicle and is dependent upon the physical characteristics of the brake system components. Ideally, the brake torque should be distributed to the wheels in the same manner as the vertical load. In other words, each wheel should develop the same ratio of brake force to wheel load. Since the vertical load on each wheel varies as a function of the acceleration of the vehicle, a fixed brake-torque distribution cannot produce the same

ratio of brake force to wheel at more than one deceleration with a given loading condition. Therefore, the selection of a brake-torque distribution is a compromise. The rationale behind the brake-torque distributions used in the parameter study is discussed in the following paragraphs.

Almond [33] advocates static brake balance for the full vehicle under all loading conditions. In other words, the brake-torque distribution is equivalent to the static weight distribution of the full vehicle. The braking performance of the baseline vehicle with this fixed brake-torque distribution can be evaluated with a braking efficiency diagram as in Reference [34]. Braking efficiency, E , is defined as

$$E = \frac{a_x/g}{\mu} \quad (2-82)$$

where a_x is the maximum deceleration achieved in a straight stop without the lock-up of any wheels and g is 32.2 ft/sec^2 . Braking efficiency, plotted versus the coefficient of friction, is a measure of how well the frictional capability of the tire-road interface is being utilized prior to wheel lock-up. Thus, Figure 2.8 shows that the baseline vehicle with brake torque distributed at each axle in proportion to the static load on the axle has an efficiency in excess of 0.78, 0.71, and 0.48 for the full, half-full, and empty

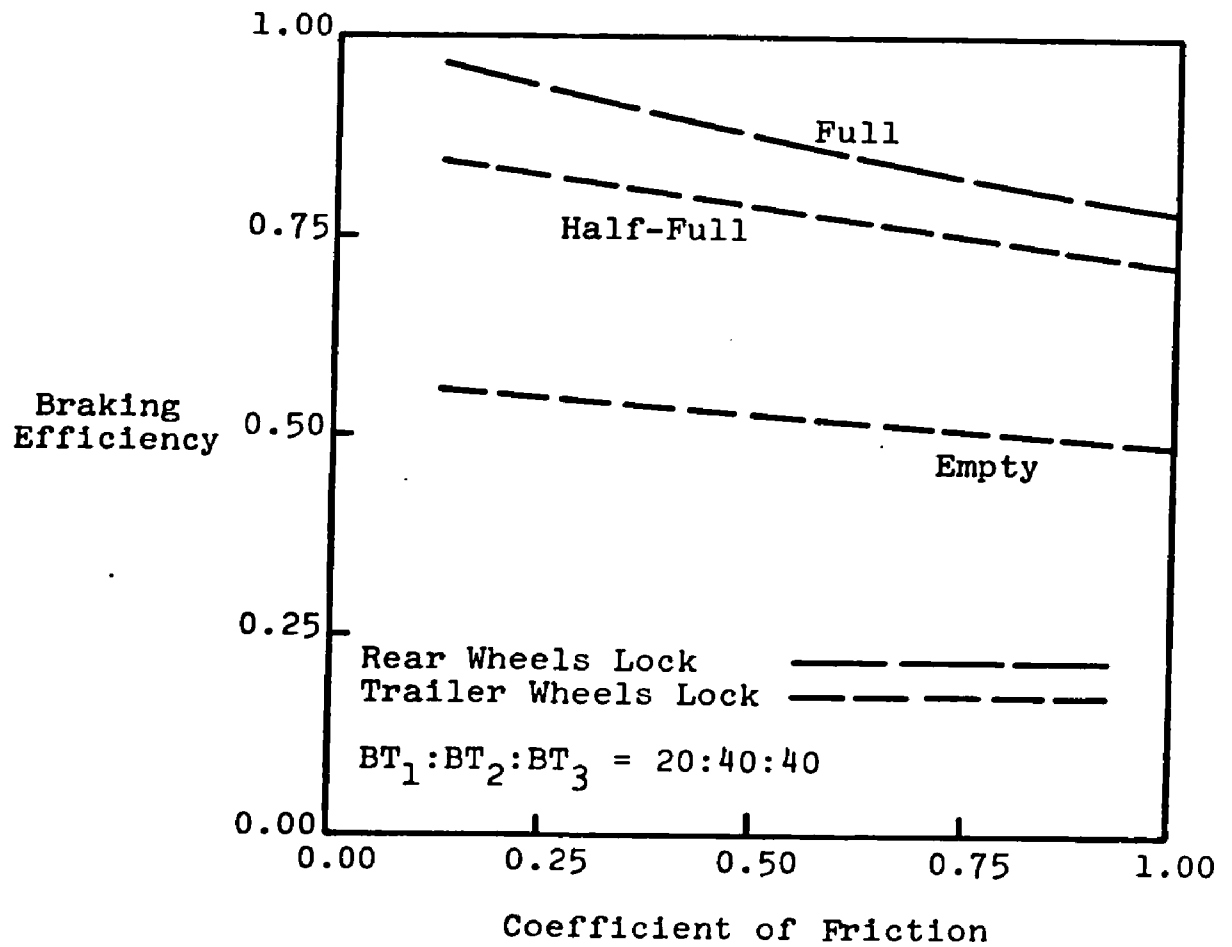


Fig. 2.8 - Braking efficiency of baseline vehicle with static brake-torque distribution

loading conditions. The rear wheels on the tractor lock first on any surface for the full vehicle and the trailer wheels lock first on any surface for the half-full and empty loading conditions.

The baseline brake-torque distribution used in this investigation is equivalent to the dynamic weight distribution of the fully loaded vehicle at a deceleration of 0.3g. This distribution, after Morse [35], is a compromise which attempts to account for the dynamic load transfer without braking the front wheels excessively on low friction surfaces. Figure 2.9 shows that the baseline vehicle with the baseline brake-torque distribution has an efficiency in excess of 0.81, 0.77, and 0.53 for the full, half-full, and empty loading conditions. The full vehicle has an efficiency of 1.0 (all wheels lock simultaneously) on a surface with a coefficient of friction of 0.3. The front wheels of the tractor lock first on a surface with a lower coefficient of friction and the rear wheels of the tractor lock first on a surface with a higher coefficient of friction. In contrast, the trailer wheels on the half-full and empty vehicles lock first regardless of the frictional capability of the tire-road interface.

The overall braking efficiency of a partially loaded or empty vehicle can be improved by load-sensitive brake-torque control. With this type of control the air pressure to the actuator, p_{a1} , is proportioned according to the actual static axle load. Thus,

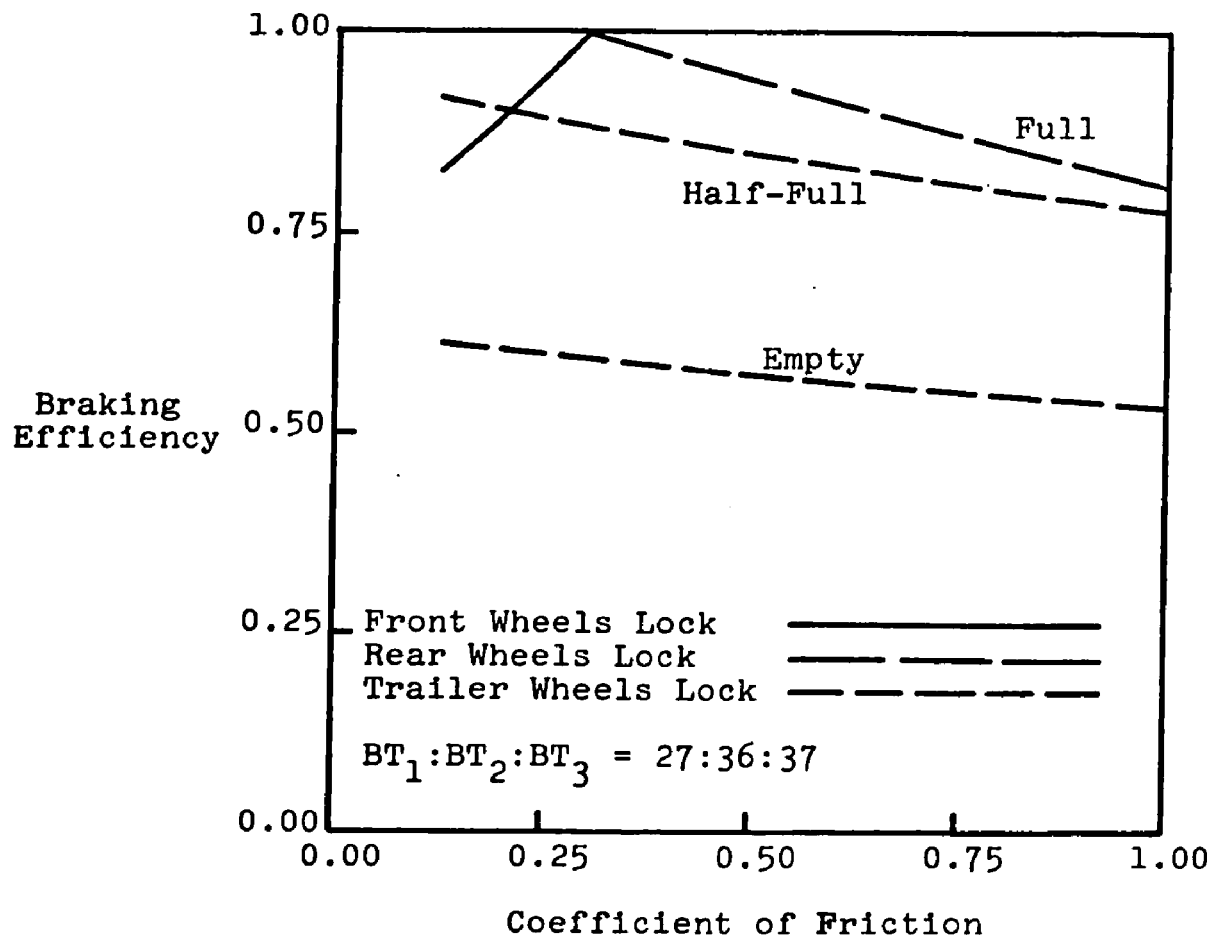


Fig. 2.9 - Braking efficiency of baseline vehicle with baseline brake-torque distribution

$$p_{ai} = k_j p_s \quad (2-83)$$

where p_s is the system air pressure and k_j is the ratio of the actual static axle load to the fully loaded static axle load. Figure 2.10 shows that the braking efficiency is greater than 0.80, 0.76, and 0.69 for the full, half-full, and empty baseline vehicle with load-sensitive brake-torque control. In addition, the partially loaded and empty vehicles behave more like the fully loaded vehicle in terms of which wheels lock first on a given surface.

2.7 Fifth-Wheel Friction Torque

In calculating the torque due to friction at the fifth-wheel, the fifth-wheel is assumed to be equivalent to a circular plate of radius R with a constant pressure distribution equal to the static load divided by the area of the plate. With reference to the geometry defined in Figure 2.11, the friction torque is

$$T = \frac{\dot{\psi}}{|\dot{\psi}|} \int_0^R \int_0^{2\pi} \mu_5 \epsilon \eta^2 d\xi d\eta \quad (2-84)$$

where μ_5 is coefficient of friction at the fifth-wheel and ϵ is the constant normal pressure distribution as given by

$$\epsilon = \frac{dW_2}{(c + d)\pi R^2} \quad (2-85)$$

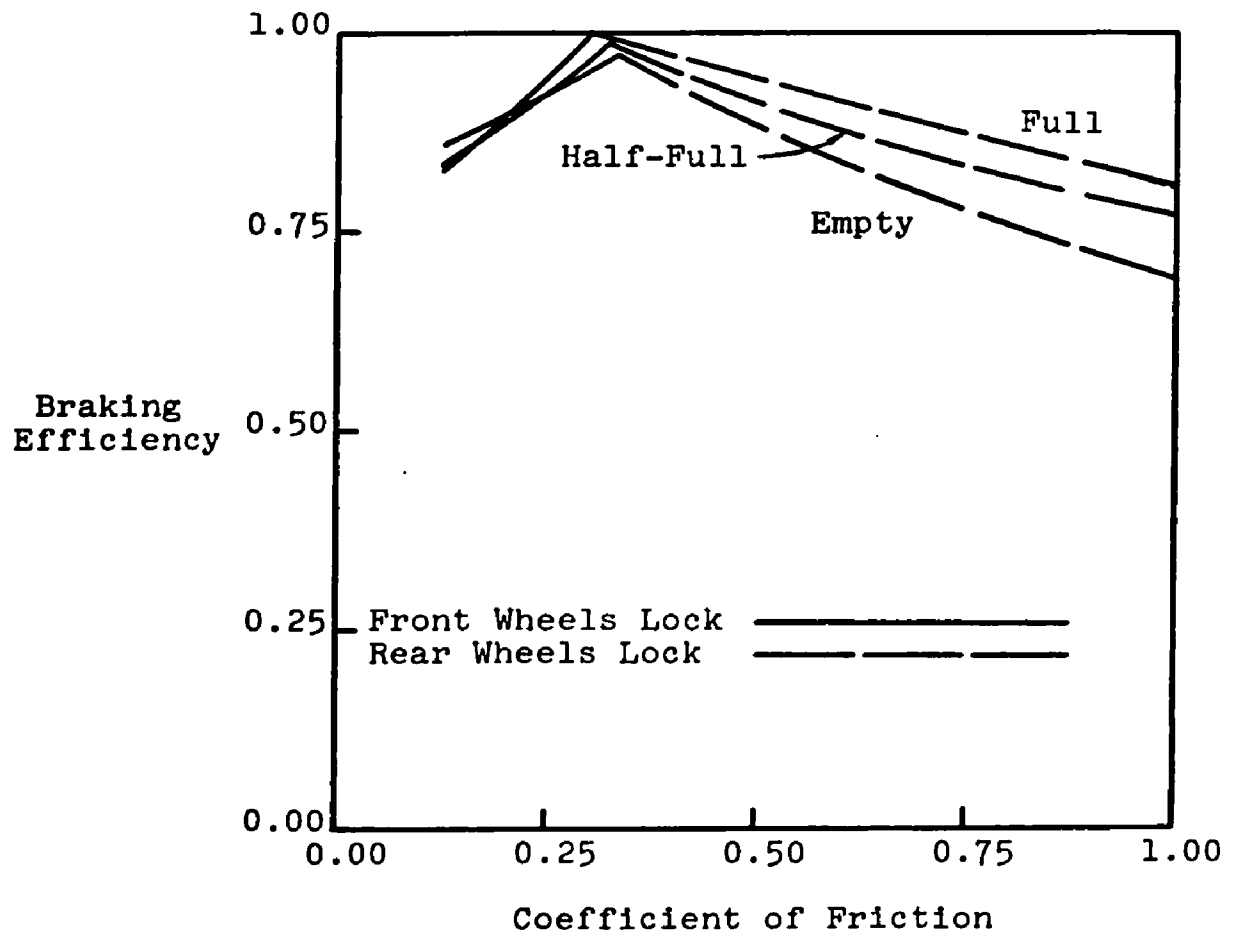


Fig. 2.10 - Braking efficiency of baseline vehicle with load-sensitive brake-torque control

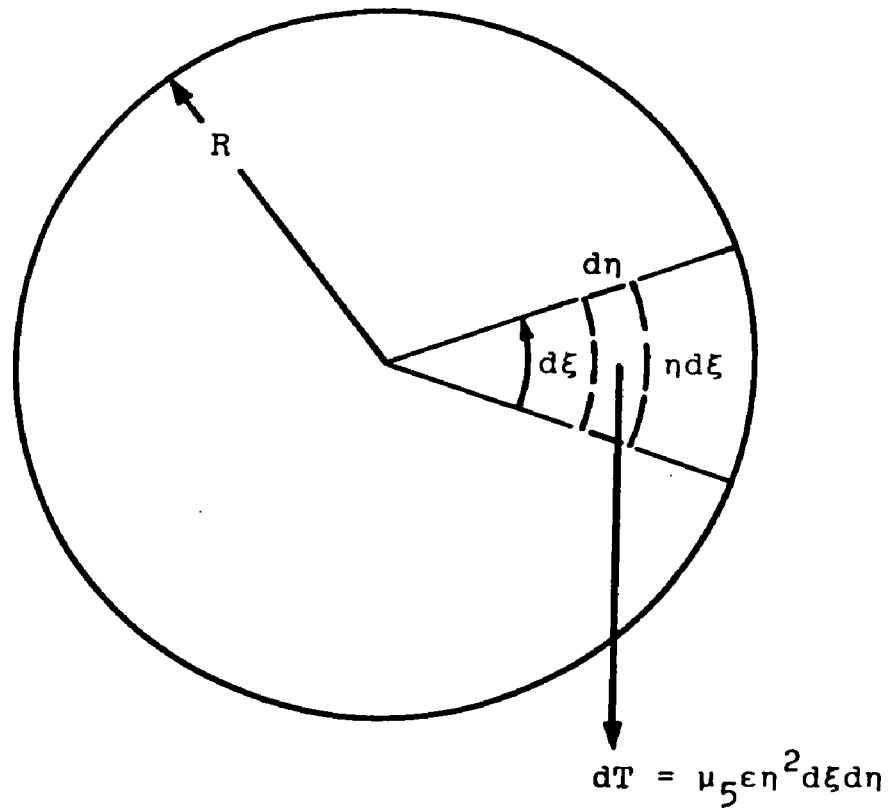


Fig. 2.11 - Assumed fifth-wheel geometry

Integrating Equation (2-84) with respect to ξ and η gives

$$T = \phi \frac{\dot{\psi}}{|\dot{\psi}|} \quad (2-86)$$

where the magnitude of the fifth-wheel friction torque is given by

$$\phi = \frac{2\mu_5 dW_2 R}{3(c + d)} \quad (2-87)$$

The baseline value for μ_5 is 0.05 which represents the coefficient of friction between greased steel plates.

CHAPTER 3

SOLUTION

3.1 General Description

A hybrid computer system, diagramed in Figure 3.1, was used to solve the equations of motion formulated in Chapter 2. The computer system consisted of a large general purpose analog computer, a small general purpose digital computer, and a data and control linkage between the two computers. An extensive operating system was developed to mechanize and analyze the results of the simulation of the tractor-semitrailer vehicle. The functional configuration of the operating system is shown in Figure 3.2 and a description of all digital programs and subroutines is given in Appendix F.

The plotting program (PML30 in Figure 3.2) is used to recreate selected solutions which have been stored on magnetic tape. The solutions are selected with the aid of a tabulated listing of vehicle response, as well as the output that is recorded on a strip chart recorder, as a means of demonstrating the effect of various input and parameter changes. The digital plotter produces plots in which time histories (namely, longitudinal acceleration, lateral acceleration, and sideslip angle of the center of mass of the tractor, yawing velocity of the tractor; and the

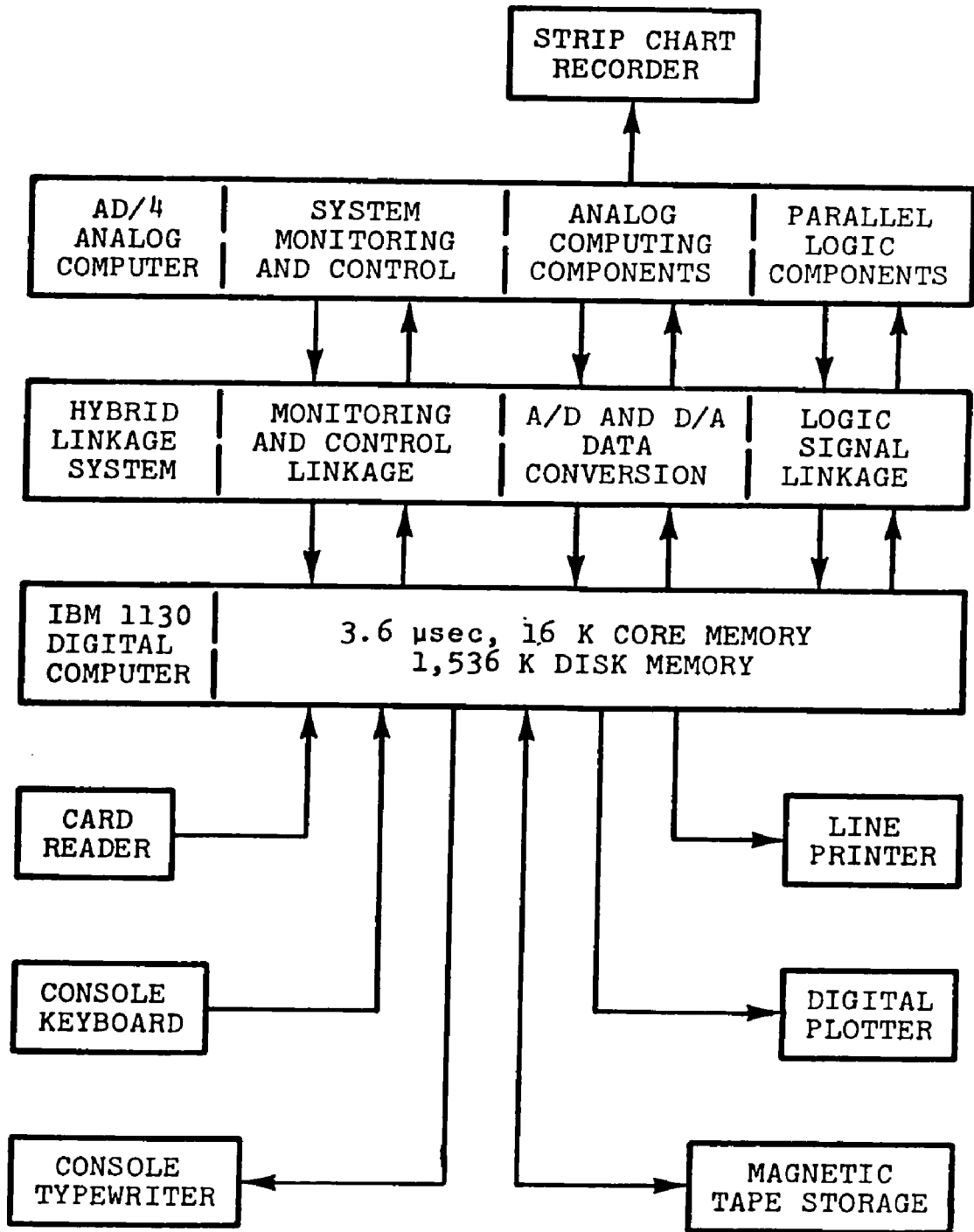


Fig. 3.1 - Hybrid computer system

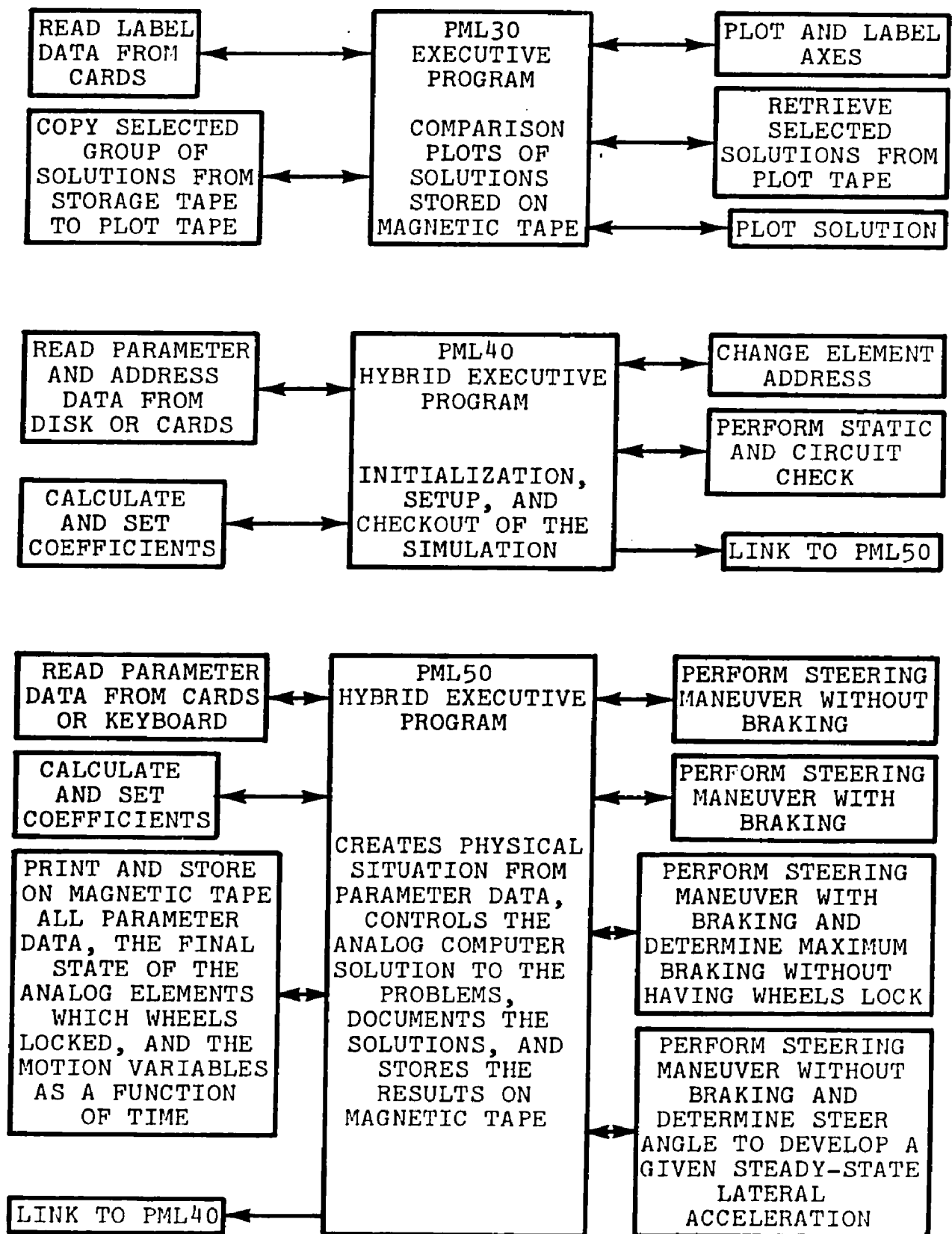


Fig. 3.2 - Digital computer programs

trailer angle) can be compared for two or three separate solutions. Examples of these solution comparisons are given in Chapter 4.

The setup program (PML40 in Figure 3.2) is used to initialize, setup, and checkout the analog simulation of the tractor-semitrailer vehicle. It calculates all coefficients from parameter data, sets the coefficient devices, calculates static check and circuit check voltages, and performs the static and circuit checks. Static check voltages are determined by solving the equations of motion using an arbitrary set of initial conditions and control inputs. A complete listing of the actual voltage and the static check voltage for each amplifier is automatically provided. If all of the corresponding voltages agree, then the computer circuit actually simulates the mathematical model. If they do not agree, the listing of the actual amplifier output voltages and the circuit check voltages, which were calculated from the actual voltages of the input elements, identifies the malfunctioning components. These checks can be repeated until all hardware problems are corrected.

The computation program (PML50 in Figure 3.2) is used to create, solve, document, and store the results of a physical problem. Parameters which define the physical problem or situation and the vehicle characteristics are changed by reading punched computer cards. The new

parameter values are then used to calculate the coefficients and reset the coefficient devices of the analog computer. The analog computer solves the set of nonlinear differential equations describing the motions of the specified vehicle as it responds to the steering and braking inputs applied by the digital computer. Variables describing the motion of the vehicle are sampled, entered into core memory, and used by the digital computer to calculate the radius of curvature, the sideslip angle, and the position of the center of mass of the tractor as a function of time. The time histories of the sampled response variables and the calculated response variables, as well as the current value of all parameters, the final value of all amplifiers, and the location of wheels which locked during the maneuver are printed out in tabular form and stored on magnetic tape.

3.2 Analog Computer Aspects

Since standard simulation techniques have been used to produce the computer model of the tractor-semitrailer vehicle, only certain elements of the total simulation will be discussed in detail. The normalized variables (ratio of the variable to the maximum value of the variable) used in the simulation are defined in Appendix B. The coefficient-device equations, which determine the settings of the

coefficient devices of the analog computer, are defined in terms of the vehicle parameters in Appendix C. The computer circuit equations, given in Appendix D, are the equations being solved by each amplifier of the analog computer. The complete analog computer circuit diagram is given in Appendix E. The symbols used in the analog computer diagrams are those advocated in Reference [36] with a few exceptions which are defined in Figure E.1.

The computer simulation of the tire mechanics formulated in Chapter 2 is a vital part of the total vehicle simulation. The analog computer circuit which calculates the forces on one wheel, the left front wheel of the tractor, is used as an example.

On normalizing Equation (2-69), the slip angle of the left front wheel of the tractor is given as

$$\frac{\alpha_l}{\alpha_m} = \frac{\left(\frac{v_m}{u_m \alpha_m}\right) \left(\frac{v}{v_m}\right) + \left(\frac{ar_m}{u_m \alpha_m}\right) \left(\frac{r}{r_m}\right)}{\left(\frac{u}{u_m}\right)} - \frac{\delta}{\alpha_m} \quad (3-1)$$

where the subscript m is used to indicate the maximum value of the variable.¹ The analog computer circuit which performs the calculation is shown in Figure 3.3.

¹Symbols used in Chapter 3 are defined in Table 2-1.

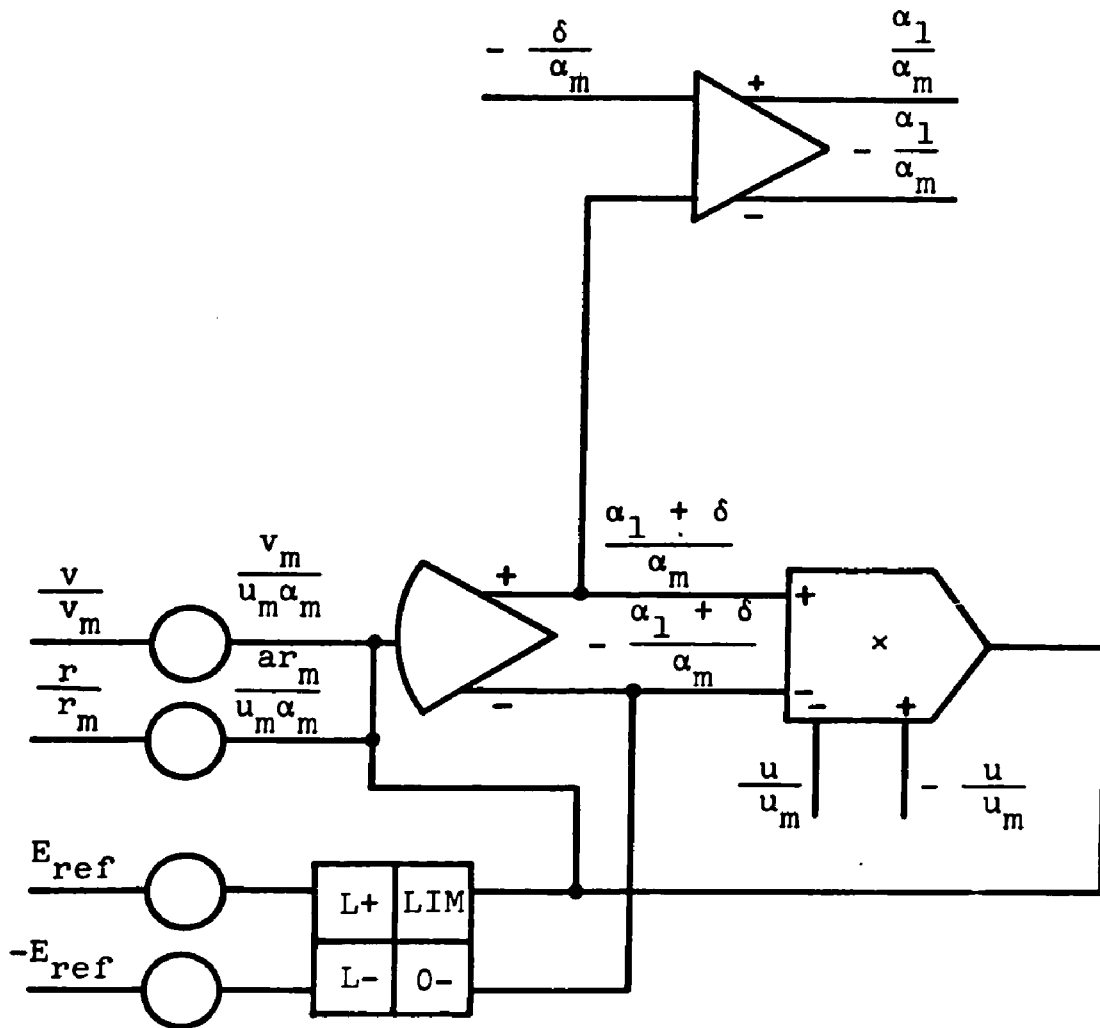


Fig. 3.3 - Slip angle of the left front wheel

The lateral force characteristics for a rolling and a locked wheel as a function of slip angle and static load are determined by the circuit of Figure 3.4. The equations mechanized are

$$\left(\frac{\bar{\alpha}_1}{\bar{\alpha}_m}\right) = \frac{(\nu_1 + \sigma_1 \bar{F}_{z1}) \alpha_m}{\mu \bar{\alpha}_m} \left(\frac{\alpha_1}{\alpha_m}\right) \quad (3-2)$$

and

$$f(\bar{\alpha}_1, \bar{F}_{z1}) = - [1.0 - \exp(-|\bar{\alpha}_1|)] \frac{\bar{\alpha}_1}{|\alpha_1|} \quad (3-3)$$

The graphic symbols labeled S, C, and K are used to indicate the functions sine, cosine, and $[1.0 - \exp(-|q|)](q/|q|)$.

The longitudinal and lateral shear forces on the left front wheel of the tractor are determined by the circuit diagramed in Figure 3.5. The braking force divided by two times the static load on the tire is set into the initial register of a digital to analog converter (DAC) by the digital computer. The digital computer updates the coefficient into the final register at the appropriate time during the solution of the problem. In other words, the braking force is zero until the specified brake application time when it assumes the desired value. The desired value is limited to 1.01 times the longitudinal friction limit of the tire-road interface. The longitudinal friction limit is the product

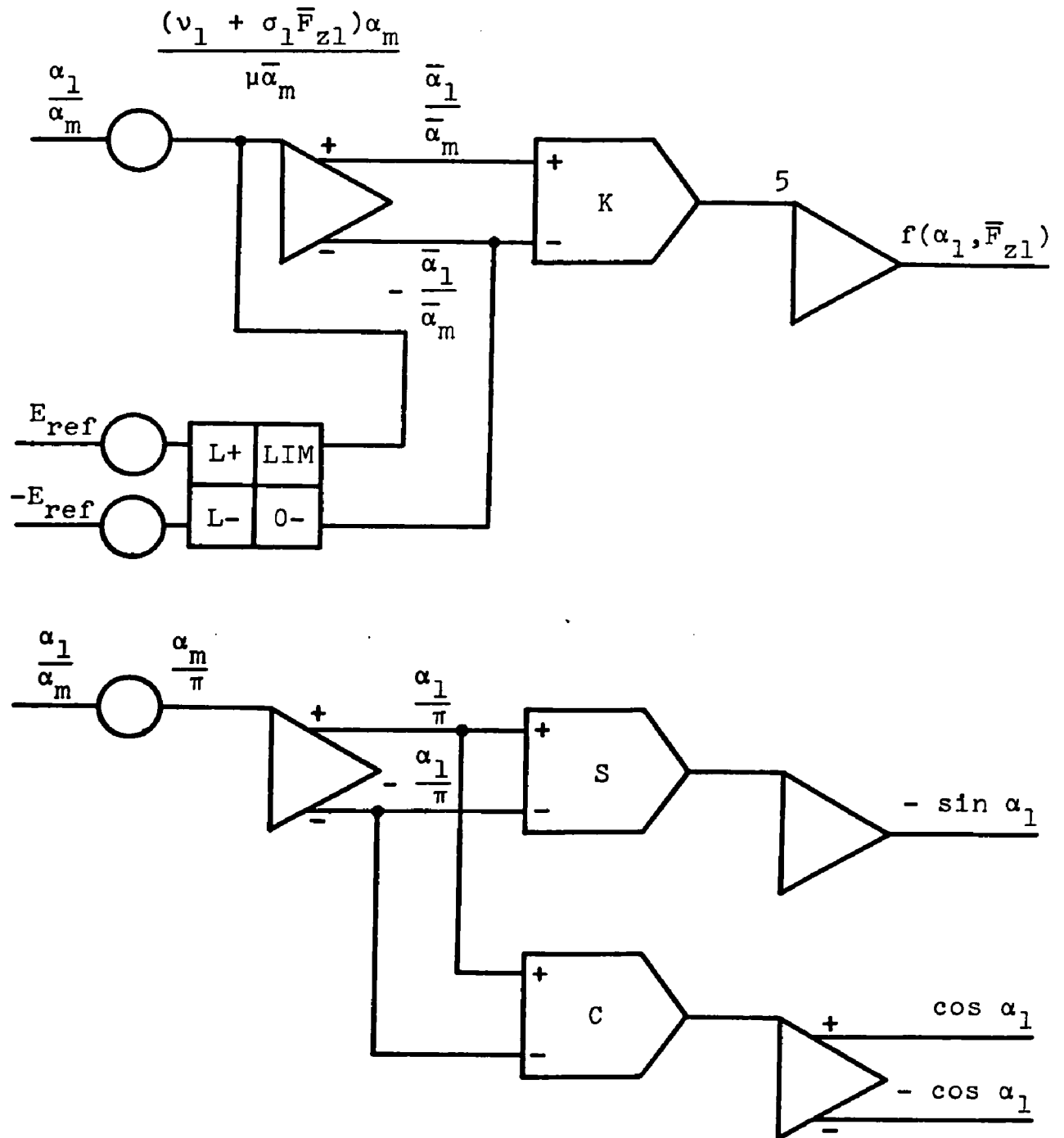


Fig. 3.4 - Rolling and locked wheel lateral force characteristic of the left front wheel

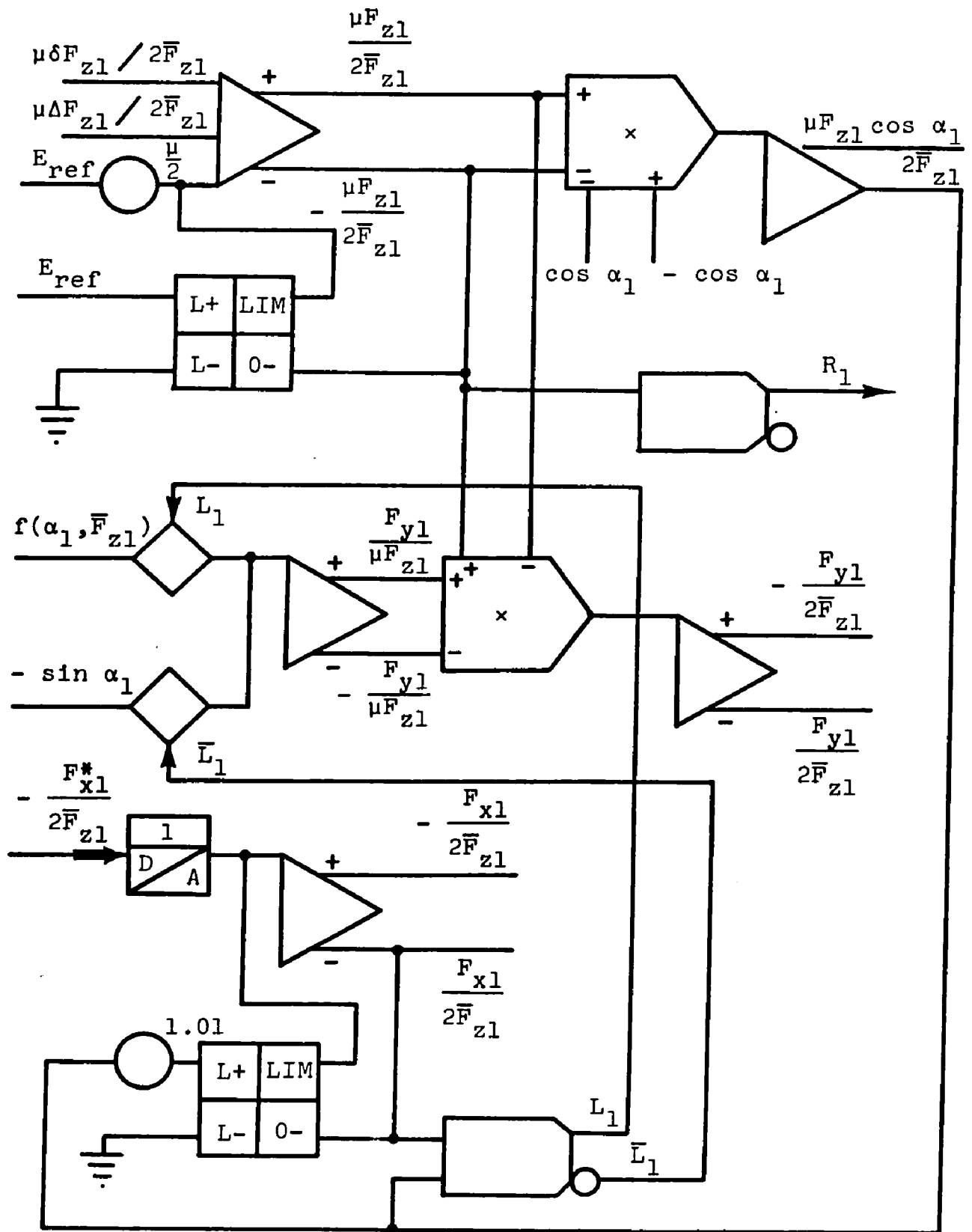


Fig. 3.5 - Tire forces on the left front wheel

of the coefficient of friction, the ratio of the instantaneous vertical load to twice the static load, and the cosine of the slip angle. The coefficient 1.01 is necessary for the operation of a comparator which subtracts the braking force from the longitudinal friction limit. The comparator logic output, L_1 , and its complement, $\bar{L}_1$, control the switches to the lateral force characteristic. This characteristic changes from the rolling state, Equation (3-4), to the locked wheel state, Equation (3-5), whenever the desired braking force exceeds the longitudinal friction limit.

$$\frac{F_{y1}}{\mu F_{z1}} = f(\alpha_1, \bar{F}_{z1}), \quad L_1 = 1 \quad (3-4)$$

$$\frac{F_{y1}}{\mu F_{z1}} = -\sin \alpha_1, \quad L_1 = 0 \quad (3-5)$$

Normalizing Equation (2-61) gives the instantaneous vertical load on the left front wheel of the tractor as

$$\frac{\mu F_{z1}}{2\bar{F}_{z1}} = \frac{\mu}{2} + \frac{\mu \delta F_{z1}}{2\bar{F}_{z1}} + \frac{\mu \Delta F_{z1}}{2\bar{F}_{z1}} \quad (3-6)$$

Finally, the lateral force on the left front wheel of the tractor is obtained as indicated below

$$\frac{F_{y1}}{2\bar{F}_{z1}} = \left(\frac{\mu F_{z1}}{2\bar{F}_{z1}} \right) \cdot \left(\frac{F_{y1}}{\mu F_{z1}} \right) \quad (3-7)$$

The friction torque, T , at the fifth-wheel coupling is simulated by means of the analog computer circuit shown in Figure 3.6. When the absolute value of the relative velocity, $\dot{\psi}$, between the tractor and the trailer is greater than or equal to 1.0 percent of the maximum yawing velocity, the simulated friction torque is expressed as

$$T = \phi \cdot (\dot{\psi}/|\dot{\psi}|) \quad (3-8)$$

where ϕ is the magnitude of the fifth-wheel friction torque defined by Equation (2-87) and $\dot{\psi}$ is the derivative of the trailer angle. When the absolute value of the relative velocity between the tractor and the trailer is less than 1.0 percent of the maximum yawing velocity, the simulated friction torque is expressed as

$$T = \phi \cdot (100 \dot{\psi}/r_m) \quad (3-9)$$

where r_m is the maximum yawing velocity. As Equation (3-9) indicates, when the relative velocity between the tractor and the trailer is zero, the friction torque, T , is also zero.

The continuous solution of six nonlinear algebraic equations is required as a result of neglecting the rolling,

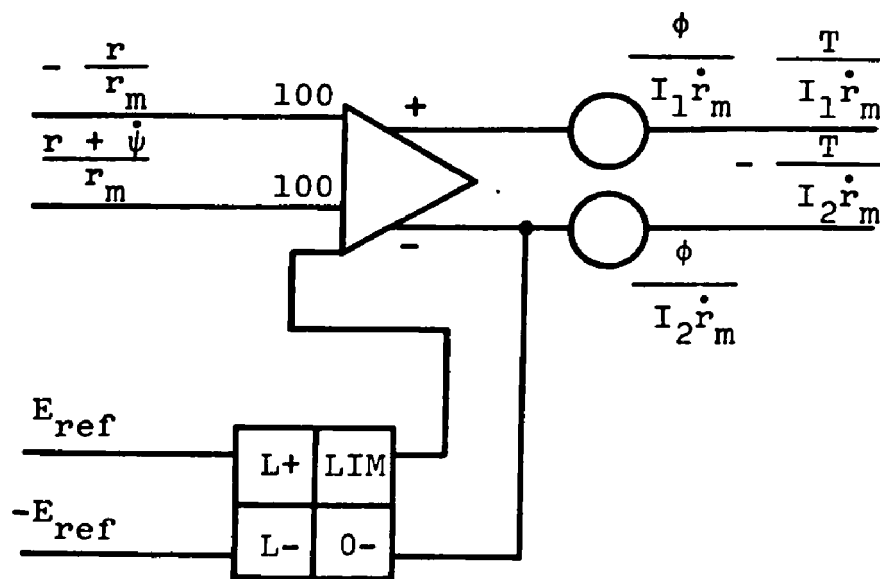


Fig. 3.6 - Fifth-wheel friction

pitching, and vertical motions of the sprung mass in formulating the problem. An approximate calculation of the longitudinal, lateral, and yaw accelerations of the center of mass of the tractor; the yaw acceleration of the semi-trailer; and the vertical load on the left and right wheels of the trailer was used to stabilize the solution of these algebraic equations on the analog computer. To illustrate the technique, consider Equation (3-10) where q_m is a function of the variables q_n .

$$q_m = \sum_{n=1}^j p_n q_n, \quad n \neq m \quad (3-10)$$

Instead of solving Equation (3-10) for q_m , Equation (3-11) is solved for q_m .

$$\frac{1}{1000} \frac{dq_m}{dt} + q_m = \sum_{n=1}^j p_n q_n, \quad n \neq m \quad (3-11)$$

The variable q_m is within 1.0 percent of its correct value in less than 5 ms for a step change in the right side of Equation (3-11). The computer circuit used to apply this technique is given in Figure 3.7.

3.3 Digital Computer Aspects

In addition to the control, setup, checkout, documentation, storage, and plotting functions, the digital computer performs other vital tasks. These tasks include (1) sampling

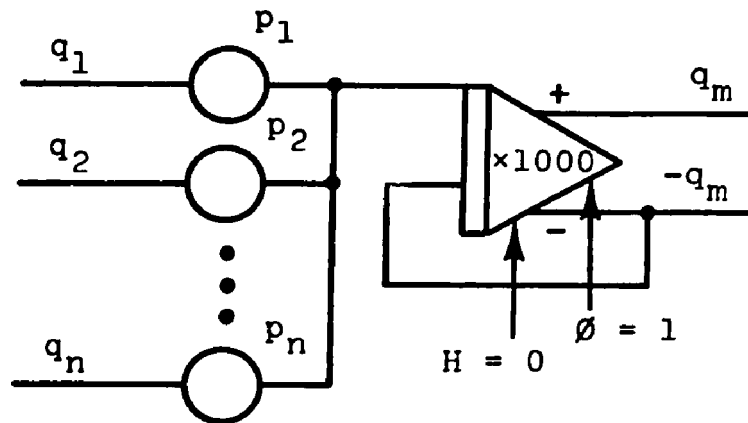


Fig. 3.7 - Approximate solution of algebraic equations

response variables, (2) providing the transport delays in the braking inputs, (3) calculating the sideslip angle and radius of curvature of the center of mass of the tractor, and (4) numerically integrating the position of the center of mass of the tractor.

The execution of the digital program must be related to the analog solution of the problem in order to accomplish transport delays and periodic sampling of analog variables. The digital computer continuously tests a hybrid sense line which is set by the digital logic of the analog computer 400 times during the 8 second solution time. When the line is on, the digital computer turns it off; samples, converts, and transmits the 9 analog signals listed in Table 3-1; and stores them in the solution array. The digital computer also updates the DAC coefficients which represent the braking forces on the vehicle at the appropriate times during the filling of the solution array. Updating the DAC coefficients at the time specified in the parameter data simulates the individual brake force application on each axle.

TABLE 3-1

ANALOG VARIABLES SAMPLED BY THE DIGITAL COMPUTER

Longitudinal velocity of the center of mass of the tractor	u
Lateral velocity of the center of mass of the tractor	v
Yawing velocity of the tractor	r
Yawing velocity of the trailer	$r + \dot{\psi}$
Heading angle of the tractor	θ
Trailer angle	ψ
Time	t
Longitudinal acceleration of the center of mass of the tractor	$\dot{u} - vr$
Lateral acceleration of the center of mass of the tractor	$\dot{v} + ur$

The sideslip angle, β , is defined as the angle between the x-axis of the tractor and the velocity vector of the center of mass of the tractor (see Figure 2.1). The digital computer calculates the sideslip angle as a function of time with Equation (3-12) using the values of u and v which are stored in the solution array.

$$\beta = \tan^{-1} \left(\frac{v}{u} \right) \quad (3-12)$$

The instantaneous velocity of the center of mass of the tractor in the X_0 and Y_0 directions (see Figure 2.1) is given by

$$\frac{dX_0}{dt} = u \cos \theta - v \sin \theta \quad (3-13)$$

$$\frac{dY_0}{dt} = u \sin \theta + v \cos \theta \quad (3-14)$$

The digital computer, using the values of u , v , θ , and t which are stored in the solution array, determines the position of the center of mass of the tractor as a function of time with Equations (3-15) and (3-16).

$$X_0(t) = X_0(t - \Delta t) + \frac{1}{2} \left(\frac{dX_0(t)}{dt} + \frac{dX_0(t - \Delta t)}{dt} \right) \Delta t \quad (3-15)$$

$$Y_0(t) = Y_0(t - \Delta t) + \frac{1}{2} \left(\frac{dY_0(t)}{dt} + \frac{dY_0(t - \Delta t)}{dt} \right) \Delta t \quad (3-16)$$

The sample time interval, Δt , is the time between successive values of the solution time, t .

The radius of curvature of the center of mass of the tractor is defined as the distance between the center of mass of the tractor and the point about which the center

of mass is instantaneously rotating. The radius of curvature, from Reference [37], is defined as

$$\rho = \frac{[1 + (\frac{dY_0}{dX_0})^2]^{3/2}}{\frac{d^2Y_0}{dX_0^2}} \quad (3-17)$$

where X_0 and Y_0 are the position coordinates of the center of mass of the tractor. The equations for X_0 and Y_0 are parametric equations in terms of t and can be expressed as

$$X_0 = X_0(t) \quad (3-18)$$

$$Y_0 = Y_0(t) \quad (3-19)$$

If (1) $Y_0(X_0)$ is a differentiable function of X_0 , (2) $X_0(t)$ is a differentiable function of t , and (3) the derivative of X_0 with respect to t is not zero, then the first and second derivatives of Y_0 with respect to X_0 are expressed as

$$\frac{dY_0}{dX_0} = Y'_0 = \frac{dY_0/dt}{dX_0/dt} \quad (3-20)$$

$$\frac{d^2Y_0}{dX_0^2} = \frac{dY'_0/dt}{dX_0/dt} \quad (3-21)$$

Carrying out the operations indicated by Equations (3-20) and (3-21) with the velocities of the center of mass as expressed by Equations (3-13) and (3-14) and substituting the results into Equation (3-17), one obtains:

$$\rho = \frac{(u^2 + v^2)^{3/2}}{u(\dot{v} + ur) - v(\dot{u} - vr)} \quad (3-22)$$

The digital computer uses Equation (3-22) and the values of u , v , $\dot{v} + ur$, and $\dot{u} - vr$ which are stored in the solution array to calculate the radius of curvature as a function of time.

CHAPTER 4

RESULTS

4.1 Vehicle Response to Steering

Step inputs of steer angle are applied to the tractor-semitrailer vehicle in order to determine its transient response to steering while traveling at a constant velocity u_0 .¹ The parameters that define the physical characteristics of the baseline vehicle are given in Table 4-2. Table 4-3 lists the parameters which change for different loading conditions.

Figure 4.1 shows the transient response of the full, half-full, and empty vehicles making a left hand turn at 50 MPH. The steer angle required to give each loading condition the same steady-state lateral acceleration (0.1 g) is applied as a step input. The variables plotted are, from top to bottom, longitudinal acceleration, lateral acceleration, sideslip angle, yawing velocity, and the trailer angle. The longitudinal acceleration of the center of mass of the tractor remains at zero during this maneuver at a constant 50 MPH. The lateral acceleration of the center of mass of the tractor has an initial step due to the step input of steer angle and a slow

¹Symbols used in Chapter 4 are defined in Table 4-1.

TABLE 4-1
SYMBOLS USED IN CHAPTER 4

W_1	=	weight of tractor
W_2	=	weight of semitrailer
I_1	=	moment of inertia of tractor
I_2	=	moment of inertia of semitrailer
a	=	distance between the front axle and the center of mass of the tractor
b	=	distance between the rear axle and the center of mass of the tractor
h	=	distance between the fifth-wheel and the center of mass of the tractor
c	=	distance between the fifth-wheel and the center of mass of the semitrailer
d	=	distance between the axle and the center of mass of the semitrailer
z_0	=	height of fifth-wheel above the ground
z_1	=	height of the center of mass of the tractor above the ground
z_2	=	height of the center of mass of the semitrailer above the ground
γ_1	=	portion of the total lateral load transfer on the front axle of the tractor
γ_2	=	portion of the total lateral load transfer on the rear axle of the tractor
μ	=	maximum tire-road interface coefficient of friction
μ_5	=	fifth-wheel friction coefficient

TABLE 4-1 (Continued)

R	=	equivalent radius of fifth-wheel
t_j	=	half the lateral distance between centers of tire contact on axle j^a
TD_j	=	application time of the brakes on axle j after the initial steering input
BT_j	=	brake torque on axle j divided by the total brake torque
a_x	=	longitudinal acceleration of the center of mass of the tractor
a_y	=	lateral acceleration of the center of mass of the tractor
a_t	=	normalized total acceleration
g	=	32.2 ft/sec^2
u_0	=	initial forward velocity
β_0	=	zero speed sideslip angle of the center of mass of the tractor
δ	=	front wheel steer angle
ϕ	=	magnitude of the fifth-wheel friction

^aSubscripts 1 through 3 correspond to the front axle of the tractor, the rear axle of the tractor, and the semitrailer axle.

TABLE 4-2
BASELINE VEHICLE PARAMETERS

W_1	(lb)	12,000
W_2	(lb)	33,000
I_1	(ft-lb-sec ²)	12,000
I_2	(ft-lb-sec ²)	74,000
a	(ft)	3.75
b	(ft)	6.25
h	(ft)	5.25
c	(ft)	12.00
d	(ft)	10.00
z_0	(ft)	4.00
z_1	(ft)	3.00
z_2	(ft)	5.16
γ_1		0.40
γ_2		0.60
μ		0.75
μ_5		0.05
R	(ft)	1.58
t_1	(ft)	3.30
t_2	(ft)	3.00
t_3	(ft)	3.00

TABLE 4-3
LOADING CONDITION PARAMETERS

LOADING CONDITION	W_2 (lb)	I_2 (ft-lb-sec ²)	c (ft)	d (ft)	z_2 (ft)
Empty	7,500	18,000	13.20	8.80	4.10
1/4 Full	13,875	32,000	12.50	9.50	4.29
1/2 Full	20,250	46,000	12.22	9.78	4.57
3/4 Full	26,625	60,000	12.10	9.90	4.86
Full	33,000	74,000	12.00	10.00	5.16
1/2 Full (Forward)	20,250	38,000	7.66	14.34	4.97
1/2 Full (Rearward)	20,250	30,000	15.64	6.36	4.97

LOADING CONDITION

STEP STEER INPUT

FULL

-0.35 DEGREES

HALF-FULL

-0.44 DEGREES

EMPTY

-0.50 DEGREES

COEFFICIENT OF FRICTION = 0.75

FORWARD VELOCITY = 50 MPH

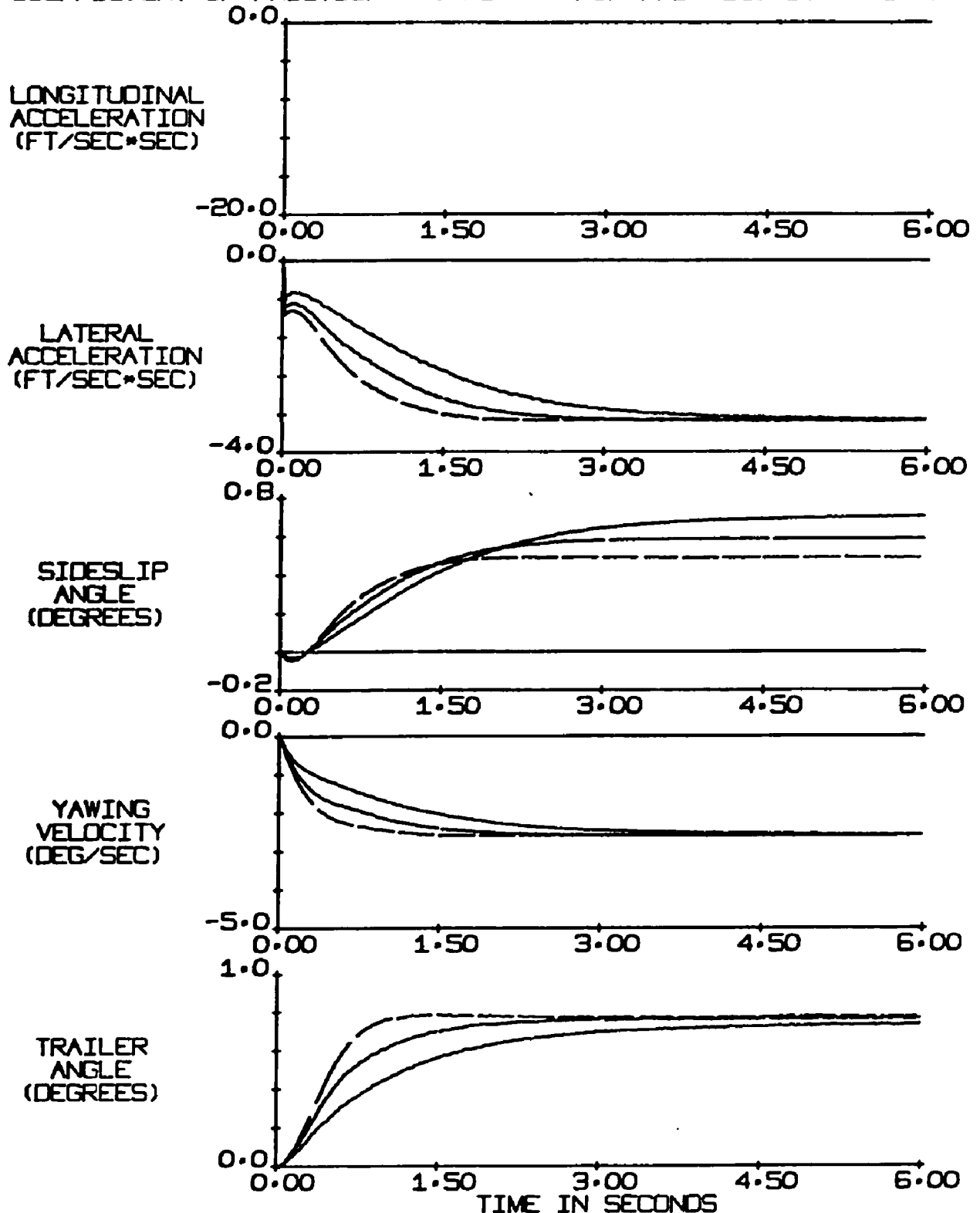


FIG. 4.1 - EFFECT OF LOADING CONDITION ON TRANSIENT RESPONSE

rise to its steady-state value. The sideslip angle of the center of mass of the tractor has a positive steady value after developing a small negative angle during the initial part of the transient response. The negative angle indicates that the rear wheels of the tractor are tracking inside the front wheels whereas the positive sideslip angle indicates that the rear wheels are tracking outside the front wheels of the tractor. The yawing velocity of the tractor develops a constant negative value and the trailer angle gradually acquires a small positive value which is characteristic of a tractor-semitrailer vehicle in a moderate left-hand turn.

The transient response of the vehicle can be characterized by (1) the yaw velocity response time, (2) the lateral acceleration response time, and (3) the sideslip angle response time. Response time is defined herein as the time required for the variable to reach 90 percent of its steady-state value. The lateral acceleration gain describes the steady-state behavior of the vehicle and is defined as the ratio of the steady-state lateral acceleration and the steer angle input. The effect of loading condition on the value of these four quantities is given in Table 4-4. Note that the response times and the lateral acceleration gain increase as the load in the semitrailer increases or if the load is moved forward.

TABLE 4-4

EFFECT OF LOADING CONDITION ON VEHICLE RESPONSE

LOADING CONDITION	YAW (sec)	RESPONSE TIMES ^a		LAT. ACCEL. GAIN (ft/sec ² /deg)
		LAT. ACCEL. (sec)	SIDESLIP (sec)	
Empty	0.72	1.16	1.35	6.57
1/4 Full	1.00	1.40	1.63	7.03
1/2 Full	1.30	1.70	1.98	7.45
3/4 Full	1.63	2.06	2.24	8.38
Full	2.24	2.60	2.88	9.58
1/2 Full (Forward)	2.19	2.49	2.82	8.75
1/2 Full (Rearward)	1.04	1.41	1.66	7.06

^aThe time required to reach 90 percent of the steady-state value in a 0.1 g turn at 50 MPH.

Table 4-5 shows the effect of parameter variations on the behavior of the full vehicle. Moving the fifth-wheel forward and increasing the wheelbase of the tractor decreases the response times and the lateral acceleration gain. Increasing the distance between the fifth-wheel and the trailer axle increases the response times but does not change the lateral acceleration gain. Increasing the fifth-wheel friction decreases lateral acceleration gain and substantially increases the response times.

4.2 Vehicle Response to Braking in a Turn

In Figure 4.2, the response of a fully loaded vehicle (with a brake-torque distribution of 20:40:40), with the brakes applied to produce a deceleration of 0.2 and 0.4 g, is contrasted to that of the unbraked vehicle. The application times of the front, rear, and trailer brakes (TD_1 , TD_2 , and TD_3) are 2.25, 2.30, and 2.35 seconds, respectively. Braking the vehicle increases the vertical load on the front tires due to longitudinal load transfer. Since the steer angle is fixed, the increased vertical load increases the lateral steering forces generated by the front tires and results in increased lateral acceleration, sideslip angle, and yawing velocity of the center of mass of the tractor. The lateral acceleration, sideslip angle, and yawing velocity of the tractor eventually reach a maximum and then decrease as the

TABLE 4-5

EFFECT OF PARAMETER VARIATIONS ON THE FULL VEHICLE

PARAMETER VARIATION	RESPONSE TIMES ^a			LAT. ACCEL. GAIN (ft/sec ² /deg)
	YAW (sec)	LAT. ACCEL. (sec)	SIDESLIP (sec)	
Baseline	2.24	2.60	2.88	9.58
a+b= 11 ft	1.83	2.24	2.54	8.64
a+b= 12 ft	1.64	2.10	2.35	7.98
c+d= 23 ft	2.29	2.69	2.96	9.58
c+d= 24 ft	2.34	2.76	3.04	9.58
ϕ =2375 ft-lb	4.10	4.34	4.81	9.17
ϕ =3958 ft-lb	5.19	5.47	5.79	8.56
h= 4.75 ft	1.59	2.06	2.28	8.20
h= 5.00 ft	1.86	2.28	2.52	8.86

^aThe time required to reach 90 percent of the steady-state value in a 0.1 g turn at 50 MPH.

VEHICLE DECELERATION

0.0 G

0.2 G

0.4 G

TD₁:TD₂:TD₃ = 2.25:2.30:2.35

FULLY LOADED VEHICLE

INITIAL VELOCITY = 50 MPH

STEER INPUT = -0.35 DEG

COEF. OF FRICTION = 0.75

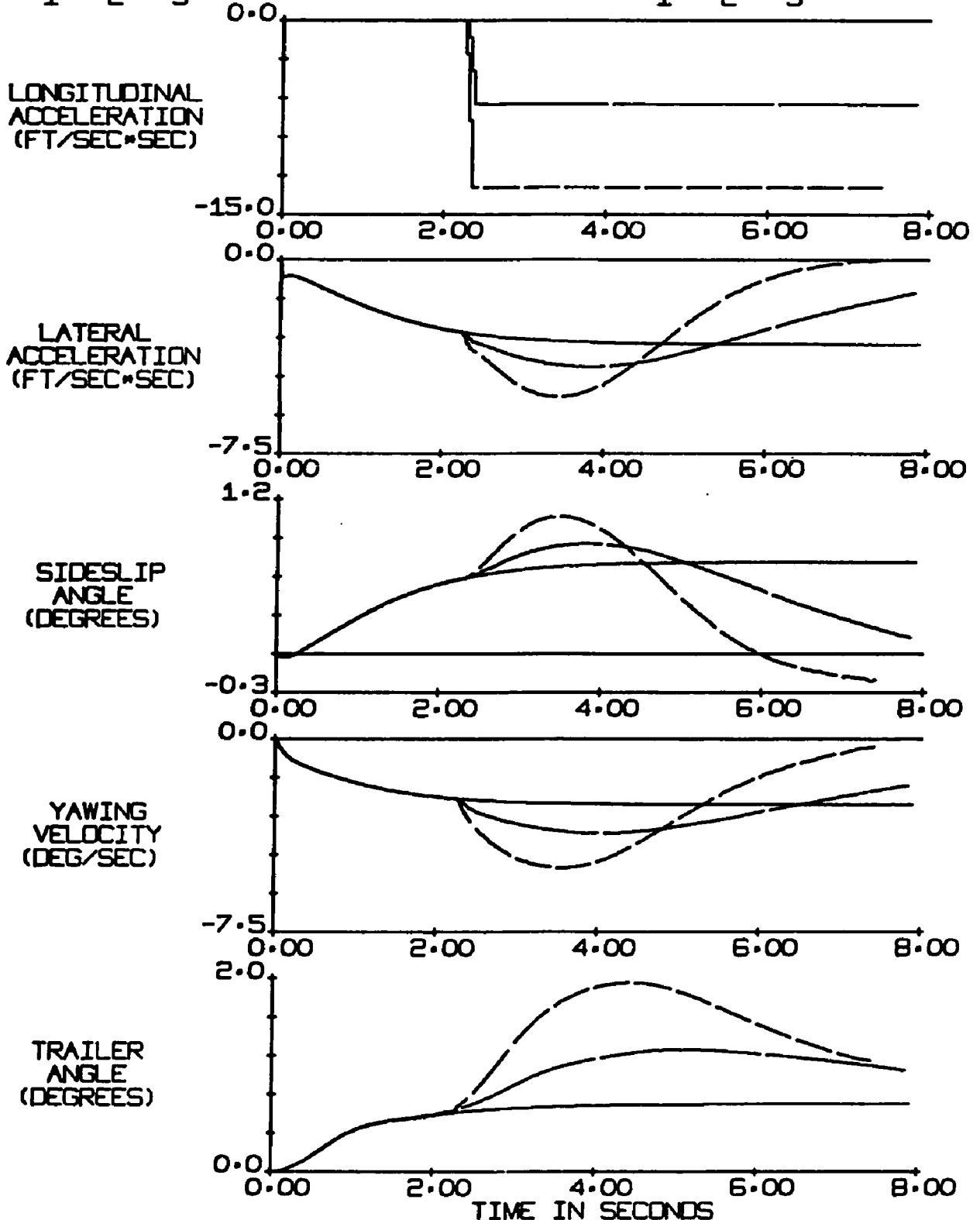
BT₁:BT₂:BT₃ = 20:40:40

FIG. 4.2 - EFFECT OF DECELERATION ON TRANSIENT RESPONSE

forward velocity of the tractor decreases. The lateral acceleration and yawing velocity of the tractor become zero when the vehicle comes to a stop and the sideslip angle goes to the zero speed Ackerman value, namely, $\beta_0 = [b/(a+b)]\delta$. Additional variables from the same solutions are plotted in Figure 4.3 versus the longitudinal displacement of the center of mass of the tractor.¹ As more severe braking is applied to increase the longitudinal deceleration, the turning of the vehicle is accentuated as is shown by the heading angle, lateral displacement, and path curvature of the tractor mass center.

4.3 Effect of Loading Condition on Vehicle Response

The maximum lateral acceleration developed at the tractor mass center can be used as a measure of the influence of braking on directional response. The effect of deceleration on the maximum lateral acceleration of the tractor for the full, half-full, and empty loading conditions is demonstrated in Figure 4.4. Initially, the vehicles have a forward velocity of 50 MPH and a steady-state lateral acceleration of 0.1 g on a flat surface with a friction coefficient of 0.75. The brakes are applied on the front,

¹The longitudinal displacement of the center of mass of the tractor is the distance traveled in the direction of initial heading.

VEHICLE DECELERATION

0.0 G

0.2 G

0.4 G

TD₁:TD₂:TD₃ = 2.25:2.30:2.35

FULLY LOADED VEHICLE

INITIAL VELOCITY = 50 MPH

STEER INPUT = -0.35 DEG

COEF. OF FRICTION = 0.75

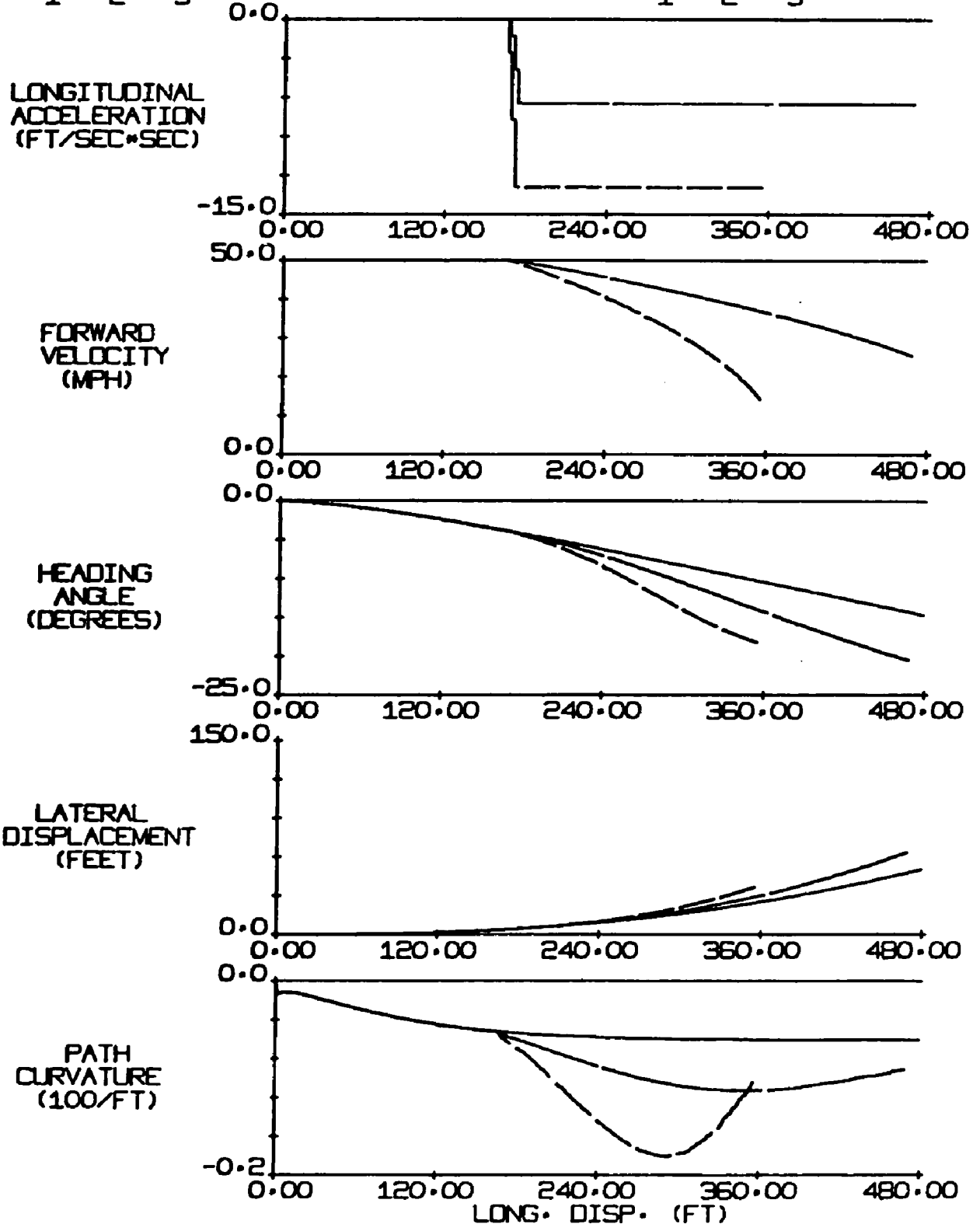
BT₁:BT₂:BT₃ = 20:40:40

FIG. 4.3 - EFFECT OF DECELERATION ON TRANSIENT RESPONSE

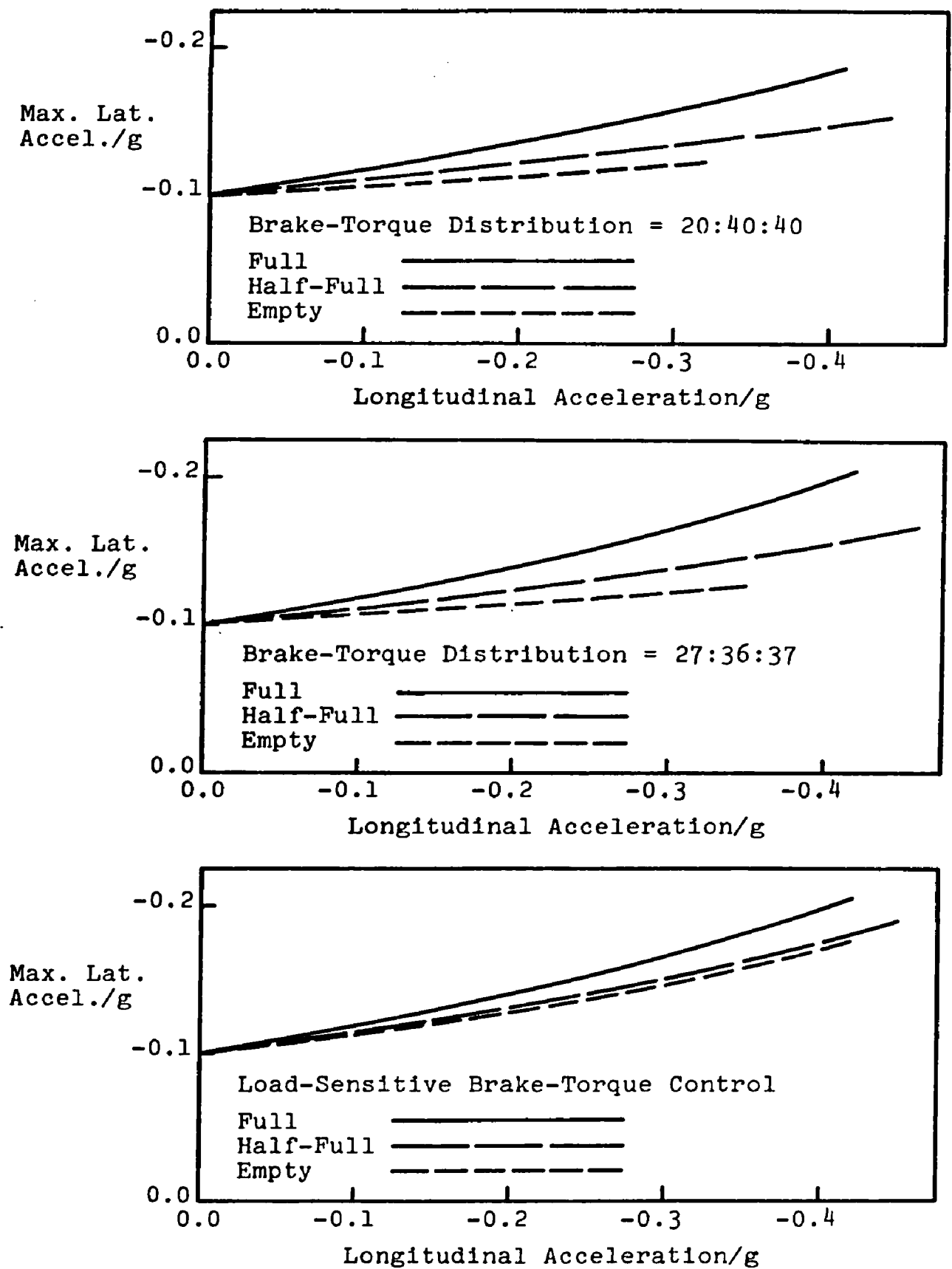


Fig. 4.4 - Effect of loading condition on the tractor's maximum lateral acceleration

rear, and trailer axles at 0.32, 0.36, and 0.40 seconds, respectively. The maximum lateral acceleration developed during the maneuver is plotted versus longitudinal acceleration for different brake-torque distributions.¹ In the upper part, the brakes are applied with a fixed brake-torque distribution of 20:40:40 which is the same as the static weight distribution of the full vehicle. In the middle section, brake torque is distributed as 27:36:37 which is the same as the dynamic weight distribution of the full vehicle at a deceleration of 0.3 g. In the lower section of Figure 4.4, the brake-torque distribution is altered from 27:36:37 on the full vehicle according to the ratio of the actual static axle load to the fully loaded static axle load. In all cases, the more lightly loaded vehicle exhibits a less severe response (lower maximum lateral acceleration) for the same rate of deceleration. This behavior is the result of three effects: (1) as the load is decreased the height of the center of mass of the trailer decreases which diminishes the longitudinal load transfer onto the tractor, (2) as the load is decreased the lateral acceleration gain is reduced, and (3) the trailer brakes on the vehicles

¹The maximum lateral acceleration developed is plotted for longitudinal accelerations up to the maximum that can be achieved without any wheel locking.

described by the upper and middle sections of Figure 4.4 do more than their share of the braking since the brake-torque distribution is not changed as the trailer load is reduced.

4.4 Effect of the Brake-Torque Distribution on Vehicle Response

The effect of brake-torque distribution on the lateral response of a vehicle to braking inputs can be evaluated with (1) the maximum longitudinal acceleration developed without any wheel locking, a_{xm} ; (2) the maximum lateral acceleration associated with this deceleration, a_{ym} ; and (3) the normalized total acceleration, a_{tm} , which is defined as

$$a_{tm} = [(a_{xm}/g)^2 + (a_{ym}/g)^2]^{1/2}/\mu \quad (4-1)$$

Table 4-6 tabulates the values of a_{xm} , a_{ym} , and a_{tm} developed when braking was initiated in a steady, 0.1 g turn with a coefficient of friction of 0.75. In using these values to evaluate the effect of brake-torque distribution it is assumed that (1) larger values of a_{xm} , a_{ym} , and a_{tm} are more desirable; (2) a driver could always develop a larger deceleration and a smaller lateral acceleration by decreasing the applied steer angle; and (3) the normalized total acceleration quantifies vehicle usage of the available

TABLE 4-6
EFFECT OF BRAKE-TORQUE DISTRIBUTION
ON LATERAL RESPONSE

LOADING CONDITION	BRAKE-TORQUE DISTRIBUTION	a_{xm}/g	a_{ym}/g	a_{tm}
Full	20:40:40	-0.41	-0.186	0.601
1/2 Full	20:40:40	-0.44	-0.154	0.622
Empty	20:40:40	-0.32	-0.121	0.457
Full	27:36:37	-0.42	-0.206	0.625
1/2 Full	27:36:37	-0.46	-0.167	0.653
Empty	27:36:37	-0.35	-0.126	0.496
Full	27:36:37	-0.42	-0.206	0.625
1/2 Full	35:34:31	-0.45	-0.189	0.652
Empty	50:30:20	-0.42	-0.176	0.608

friction. Thus, referring to Table 4-6, it is clear that the brake-torque distribution of 27:36:37 is an improvement over 20:40:40 for all loading conditions. In addition, the introduction of load-sensitive brake-torque control (brake-torque distribution altered according to the static loading condition of the vehicle) insures that all loading conditions exhibit a similar lateral acceleration response, deceleration without any wheel locking, and utilization of the available friction. In other words, the brake-torque distribution and load-sensitive brake-torque control are design characteristics which can be used to improve the overall behavior of the vehicle under various loading conditions.

The lateral response¹ of the full, half-full, and empty vehicles to an attempted deceleration of 0.5 g is given in Figures 4.5 and 4.6 for brake-torque distributions of 20:40:40 and 27:36:37, respectively. Initially, the vehicles are in a turn at 50 MPH on a surface with a coefficient of friction of 0.75 and the steady-state lateral acceleration is 0.1 g. The brakes are applied at 0.32, 0.36, and 0.40 seconds on the front, rear, and trailer axles.

¹The trailer angle and the lateral acceleration, sideslip angle, and yawing velocity of the tractor mass center are used to evaluate lateral response.

LOADING CONDITION	LOCKED WHEELS	STEER INPUT
FULL	1,3,4,5	-0.35 DEG
HALF-FULL	3,4,5	-0.44 DEG
EMPTY	3,5,6	-0.50 DEG

$TD_1:TD_2:TD_3 = 0.32:0.36:0.40$
 $BT_1:BT_2:BT_3 = 20:40:40$

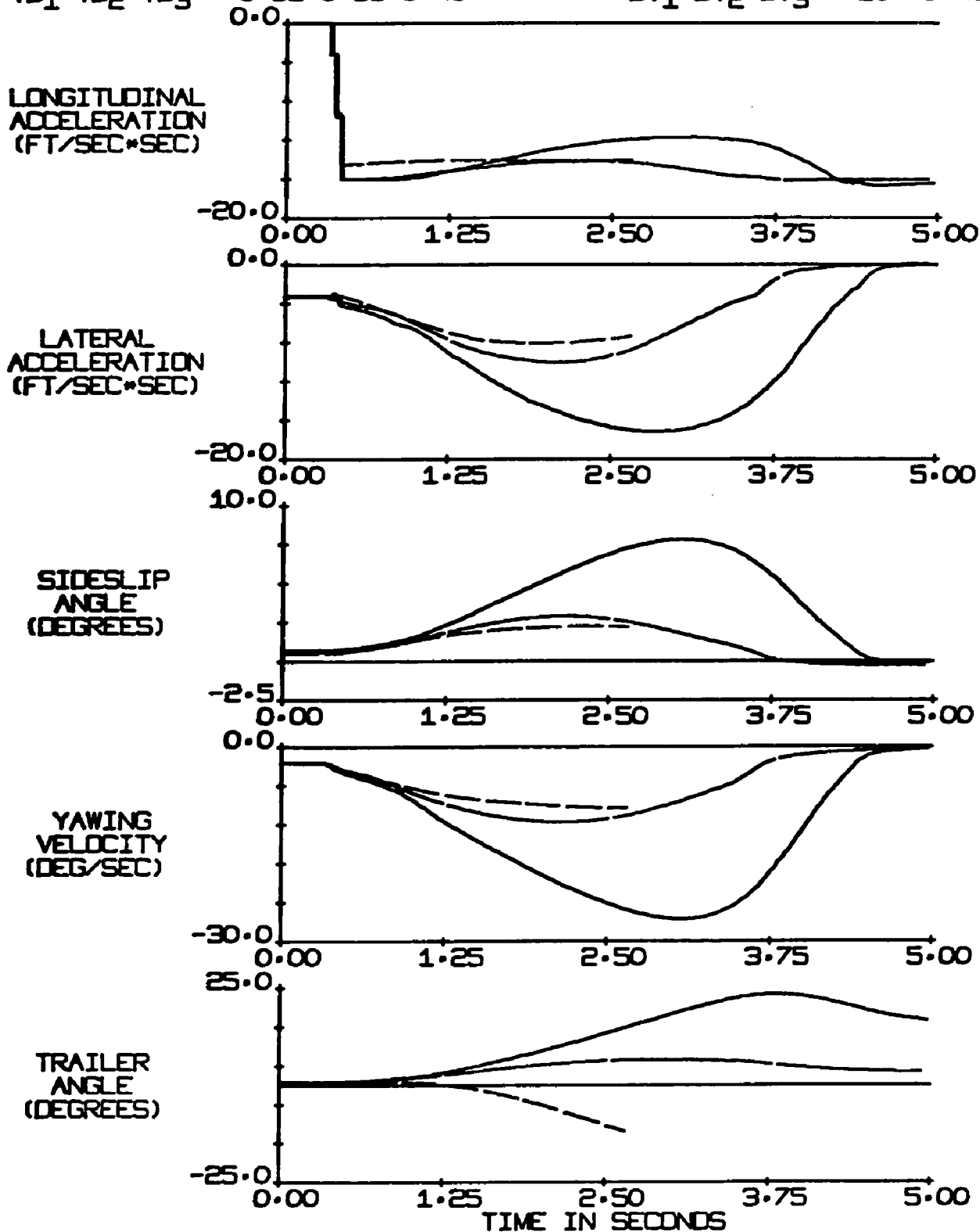


FIG. 4.5 - TRANSIENT RESPONSE TO BRAKING IN A TURN

LOADING CONDITION

FULL
HALF-FULL
EMPTY

LOCKED WHEELS

1,3,4,5
3,4,5
3,5,6

STEER INPUT

-0.35 DEG
-0.44 DEG
-0.50 DEG

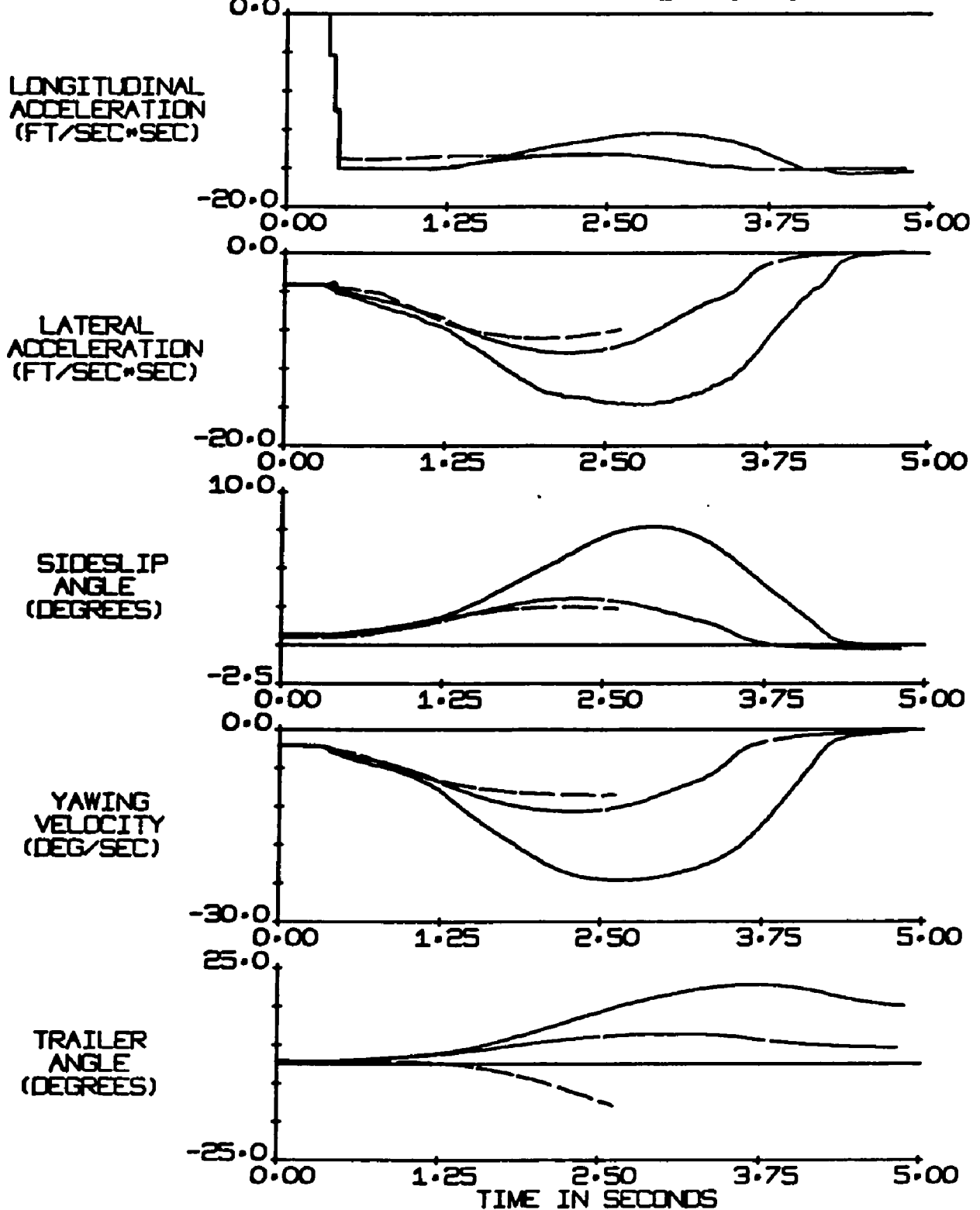
 $TD_1:TD_2:TD_3 = 0.32:0.36:0.40$
 $BT_1:BT_2:BT_3 = 27:36:37$


FIG. 4.6 - TRANSIENT RESPONSE TO BRAKING IN A TURN WITH THE BASELINE BRAKE-TORQUE DISTRIBUTION

The full vehicle in Figure 4.5, with wheels locked for more than 3.8 seconds, develops a more severe lateral response than the full vehicle in Figure 4.6, with wheels locked for less than 3.3 seconds.¹ This behavior, in contrast to that predicted by Figure 4.4,² is evidence that small changes in the brake-torque distribution, which alter the initiation and duration times of locked wheels, can adversely effect the performance of the total system. The half-full vehicles have a less severe lateral response than the full vehicles even though wheels are locked during the response for more than 3.6 seconds in Figure 4.5 and 3.0 seconds in Figure 4.6. The empty vehicles in Figures 4.5 and 4.6 both exhibit trailer swing due to the locking of the left and right trailer wheels.

In Figure 4.7, load-sensitive brake-torque control alters the 27:36:37 distribution such that all loading conditions have the same basic behavior in terms of lateral

¹The lateral response of the full vehicles in Figures 4.5 and 4.6 is termed a severe lateral response rather than jackknifing which is reserved for characterizing a divergent tractor yawing velocity and sideslip angle as shown in Figure 4.15.

²In Figure 4.4., the vehicle develops a larger maximum lateral acceleration with a brake-torque distribution of 27:36:37 than with 20:40:40.

LOADING CONDITION

LOCKED WHEELS

STEER INPUT

FULL

1,3,4,5

-0.35 DEG

HALF-FULL

1,3,4,5

-0.44 DEG

EMPTY

3,4,5

-0.50 DEG

 $TD_1:TD_2:TD_3 = 0.32:0.36:0.40$

COEFFICIENT OF FRICTION = 0.75

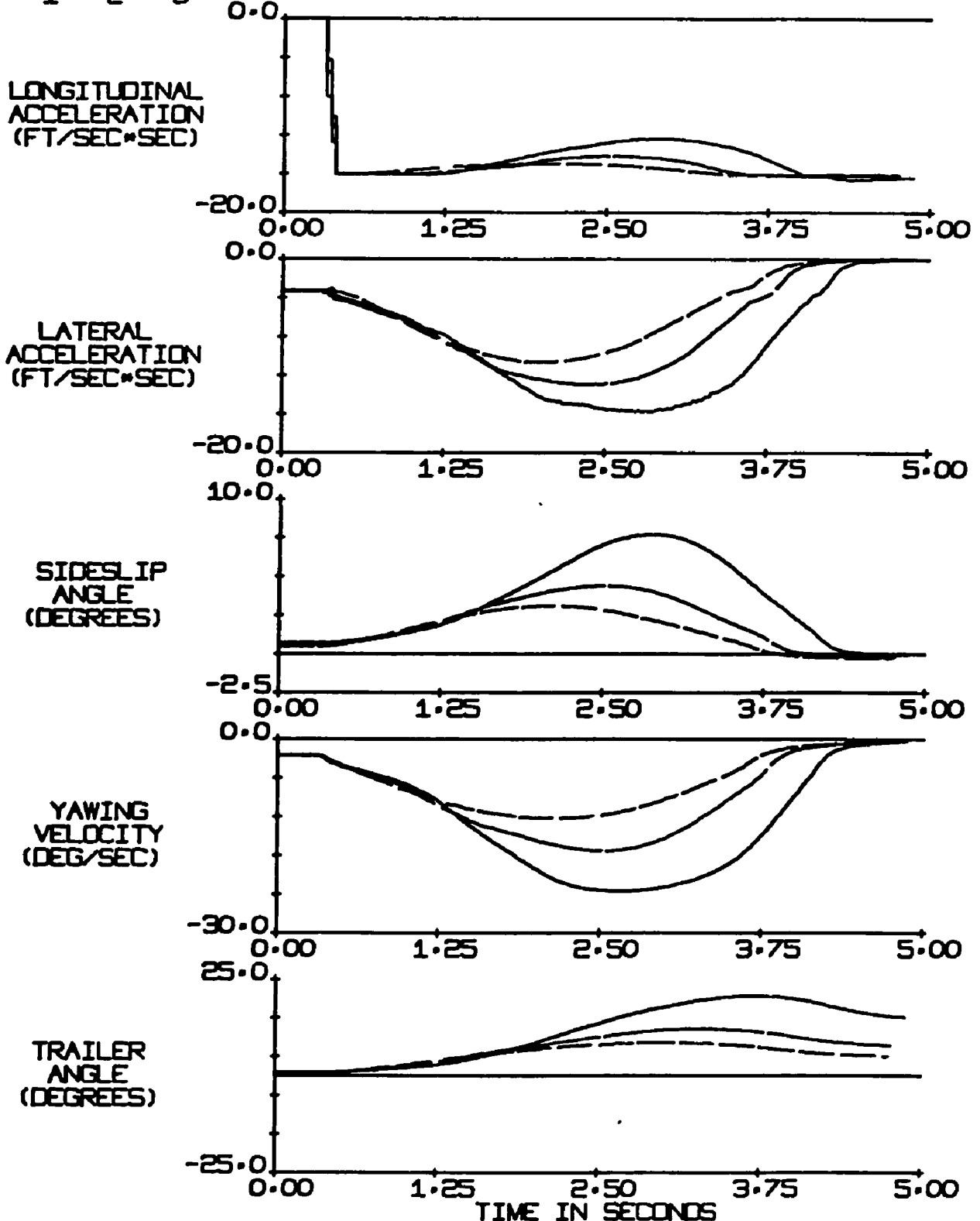


FIG. 4.7 - TRANSIENT RESPONSE TO BRAKING IN A TURN WITH LOAD-SENSITIVE BRAKE-TORQUE CONTROL

response. In addition, the empty vehicle with load-sensitive brake-torque control does not develop trailer swing as it does without it in Figures 4.5 and 4.6.

The attempted deceleration of 0.5 g was intentionally chosen to produce locked wheels and the severe lateral maneuvers which are developed. Furthermore, it is assumed that a severe lateral response (but not a divergent response), such as those generated by the full vehicles in Figures 4.5 and 4.6, could be made less severe by the driver reducing the applied steer angle. In other words, the driver could maintain the rate of deceleration with a less severe lateral response by reducing the steer angle, whereas, he must reduce the rate of deceleration to prevent trailer swing.

4.5 Effect of the Initial Lateral Acceleration on Vehicle Response

The ability of the tractor-semitrailer vehicle to utilize the available friction when subjected to braking at various initial lateral accelerations is evaluated with Figure 4.8. The brakes were applied at 0.32, 0.36, and 0.40 seconds on the front, rear, and trailer axles and the coefficient of friction was assumed to be 0.75. The maximum normalized total acceleration without any wheel locking is plotted versus the initial steady-state lateral acceleration for three loading conditions and different brake-torque

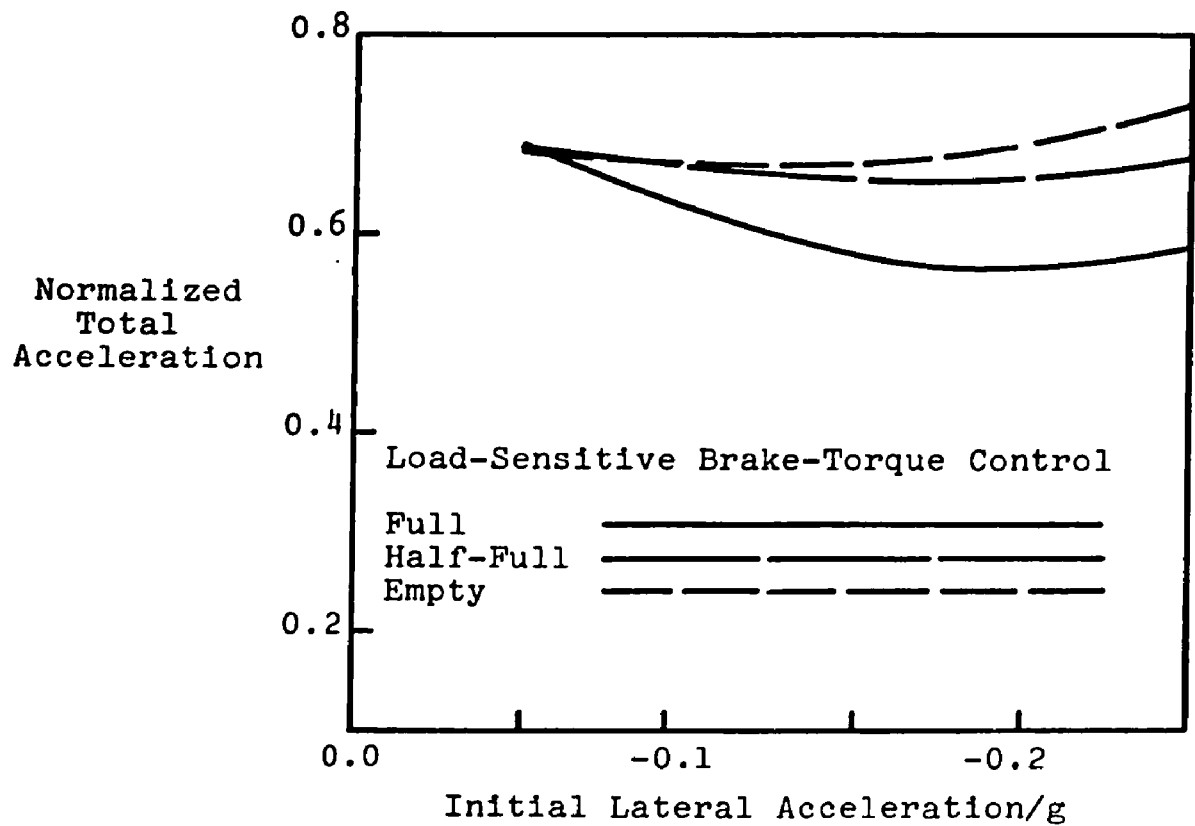
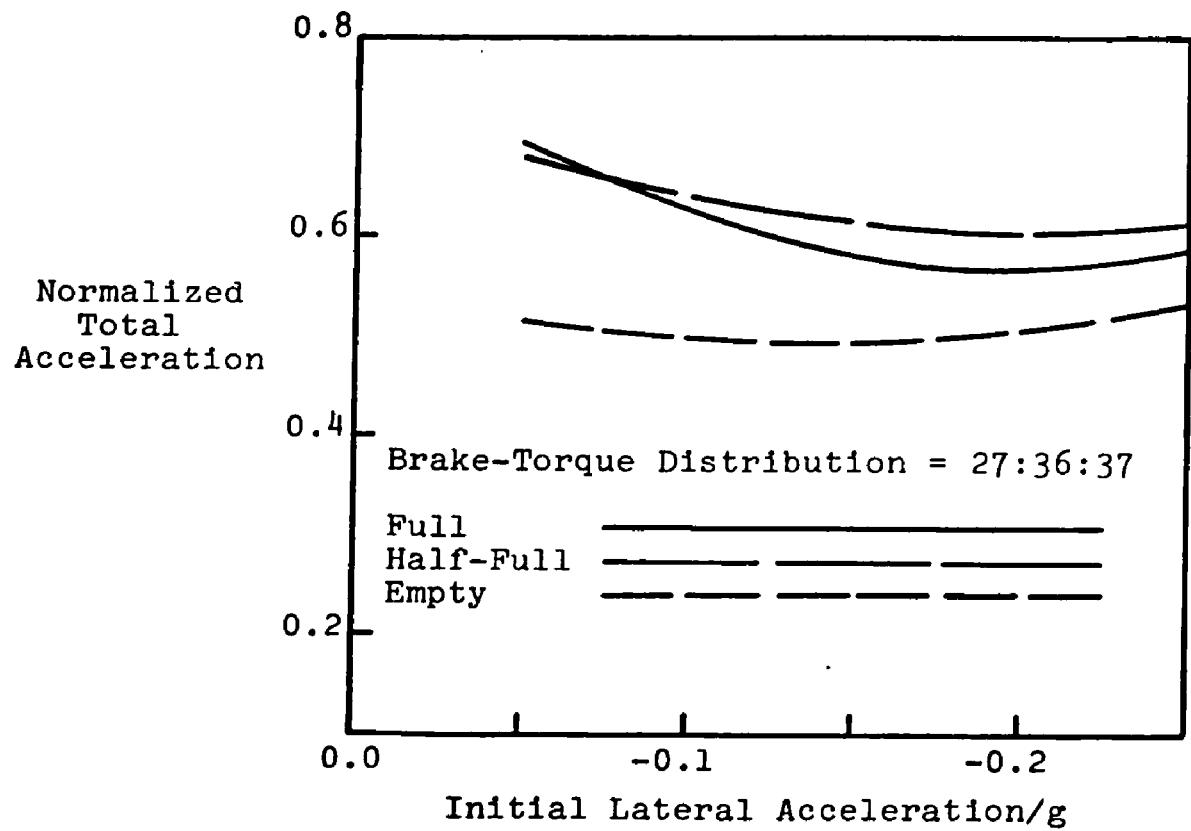


Fig. 4.8 - Effect of initial lateral acceleration on the normalized total acceleration

distributions. The figure indicates that sensitivity to the initial lateral acceleration when the brakes are applied is increased by increasing the load on the trailer. In addition, by using load-sensitive brake-torque control, the partially loaded vehicle behaves as well as or better than the full vehicle for any initial lateral acceleration.

4.6 Effect of Lateral Brake-Torque Variations on Vehicle Response

The effect of systematic variations in brake torque balance between the right and left sides of the vehicle on the overall performance of the vehicle has been studied. The fully loaded vehicle is, initially, in a steady turn at 50 MPH with a lateral acceleration of 0.1 g. The brakes are applied at 0.32, 0.36, and 0.40 seconds on the front, rear, and trailer axles and the assumed coefficient of friction is 0.75. The maximum longitudinal acceleration developed without any wheel locking, the maximum lateral acceleration associated with this deceleration, and the normalized total acceleration were determined for the baseline brake-torque distribution of 27:36:37. These accelerations (indicated by the symbols a_{xm}^* , a_{ym}^* , and a_{tm}^*) are used to normalize the values of a_{xm} , a_{ym} , and a_{tm} that are obtained for an assumed imbalance in the lateral distribution of brake torque. The brake-torque distributions are offset

from the baseline distribution by lowering and then raising the brake torque on the left side of each axle by 10 and 20 percent and adding the difference to the right side. The results are presented in Figure 4.9 where the bars represent the ratios (a_{xm}/a_{xm}^* , a_{ym}/a_{ym}^* , and a_{tm}/a_{tm}^*) of the maximum accelerations.

As a result of this portion of the investigation, it appears that the ability of the vehicle to stop in a combined steering and braking maneuver is decreased by lateral imbalance in brake torque. Reduced performance is indicated by the value of 0.984 for the average of the twelve normalized total acceleration ratios. Computing the three average normalized total acceleration ratios of the four variations on each axle indicates that the vehicle is most sensitive to lateral imbalance in brake torque on the rear axle of the tractor. Computing the two average normalized total acceleration ratios of the six variations at 10 and 20 percent from the baseline distribution indicates that the overall performance decreases as the amount of lateral imbalance increases.

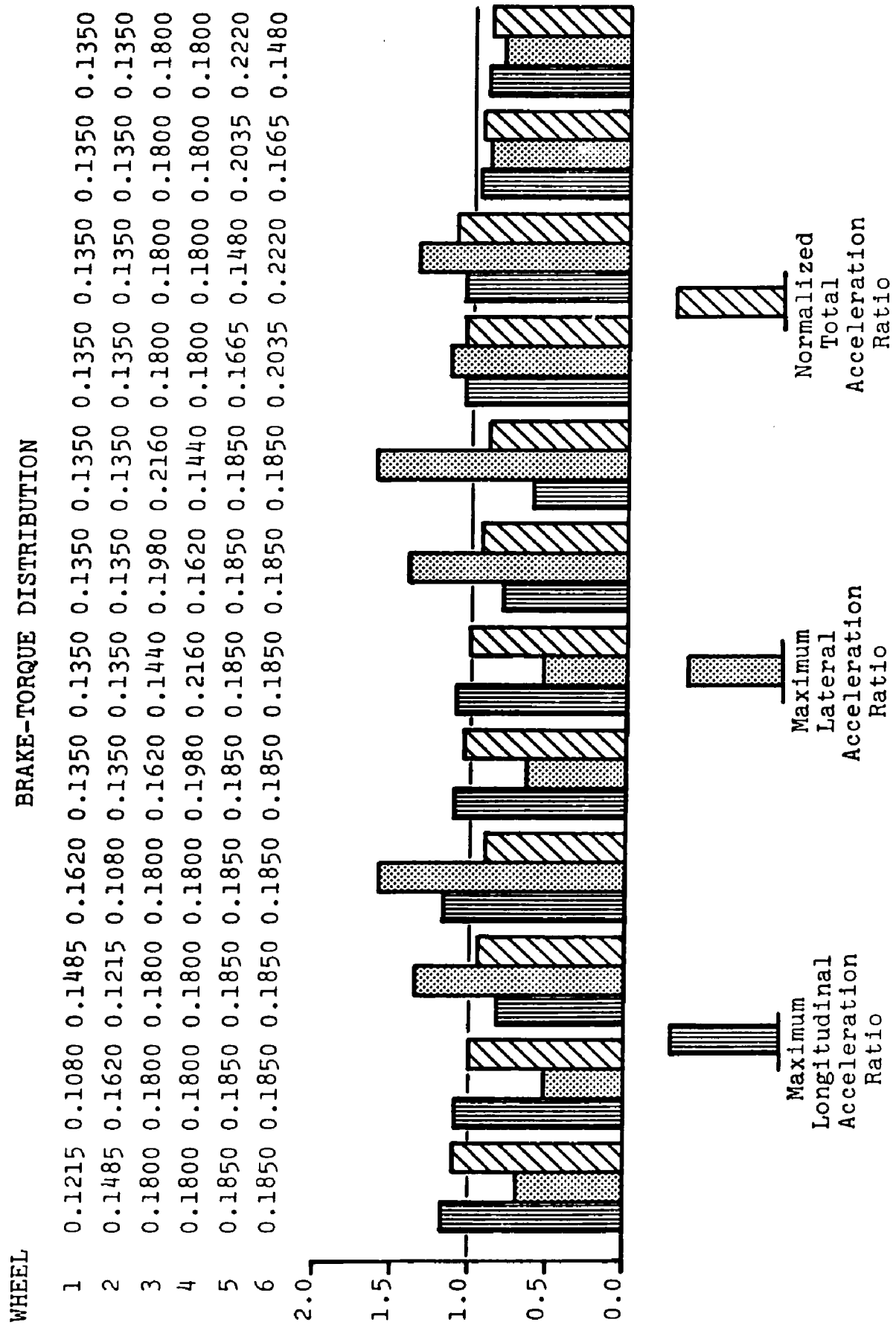


Fig. 4.9 - Effect of lateral brake-torque variations on acceleration ratios

4.7 Variations in the Longitudinal Distribution of Brake Torque

The results of systematic variations in the longitudinal distribution of brake torque from the baseline distribution of 27:36:37 are summarized in Figure 4.10. Using the same initial conditions as were assumed in Section 4.6, the brake torque on each axle is reduced and then raised by 10 and 20 percent. The average of all twelve normalized total acceleration ratios is 0.971 indicating that longitudinal variations will, in general, decrease the performance of the vehicle when combined steering and braking is required. Only one distribution (28.58 percent on the front, 38.12 percent on the rear, and 33.30 percent on the trailer axle) indicates an improved performance over the baseline distribution by having all three acceleration ratios (a_{xm}/a_{xm}^* , a_{ym}/a_{ym}^* , and a_{tm}/a_{tm}^*) greater than 1.0. Figure 4.11 compares the lateral responses produced by this distribution with two other distributions on a fully loaded vehicle in an attempted 0.5 g stop. The lateral response of the vehicle with the baseline distribution of 27:36:37, which has wheels locked for less than 3.3 seconds, is less severe than with a distribution of 28.58:38.12:33.30, which has wheels locked for 3.75 seconds, or with a distribution of 20:40:40, which has wheels locked for more than 3.8 seconds.

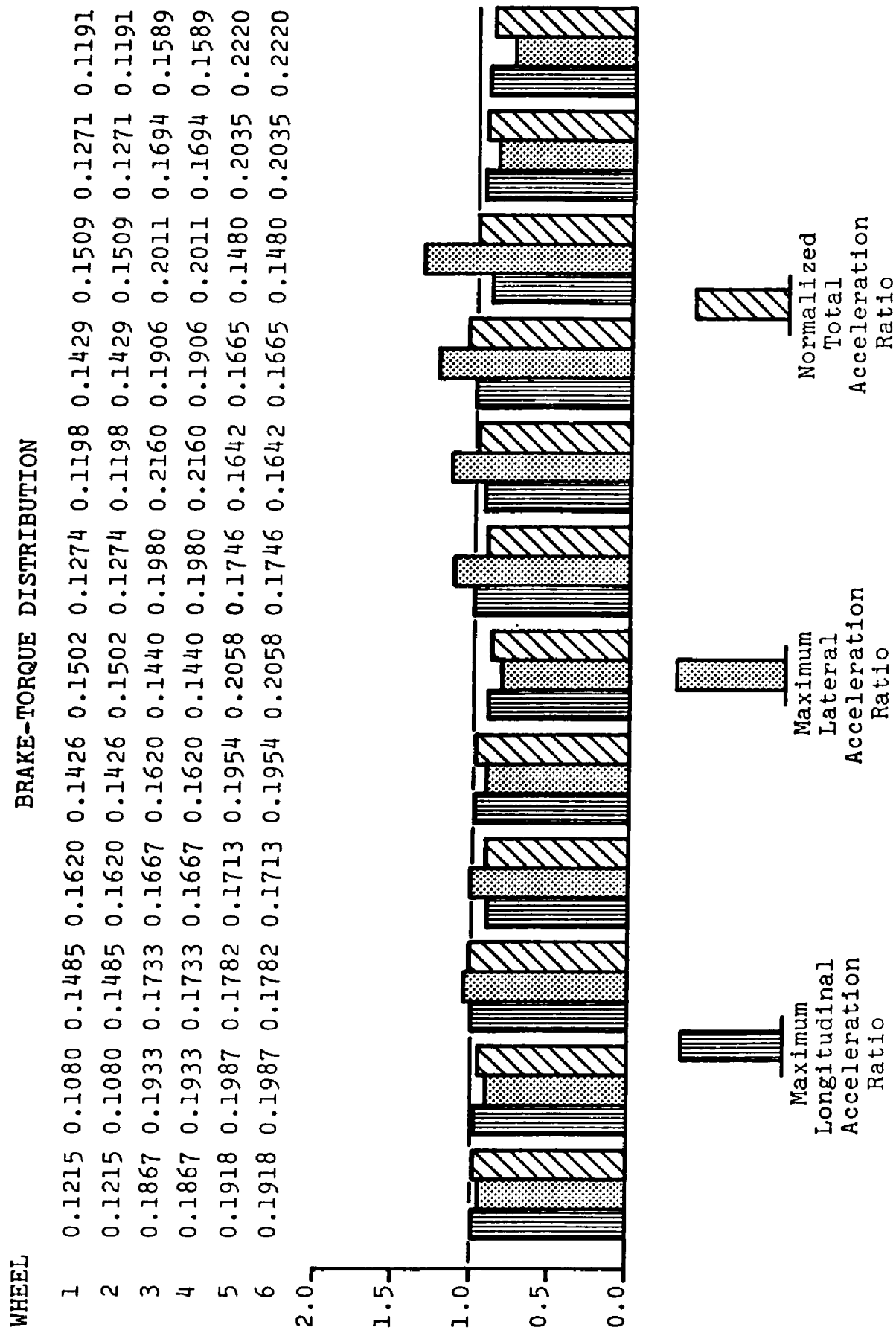


Fig. 4.10 - Effect of longitudinal brake-torque variations on acceleration ratios

BRAKE-TORQUE DISTRIBUTION	LOCKED WHEELS	STEER INPUT
20/40/40	1,3,4,5	-0.35 DEG
27/36/37	1,3,4,5	-0.35 DEG
29/38/33	1,3,4,5	-0.35 DEG

$TD_1:TD_2:TD_3 = 0.32:0.36:0.40$
 COEFFICIENT OF FRICTION = 0.75

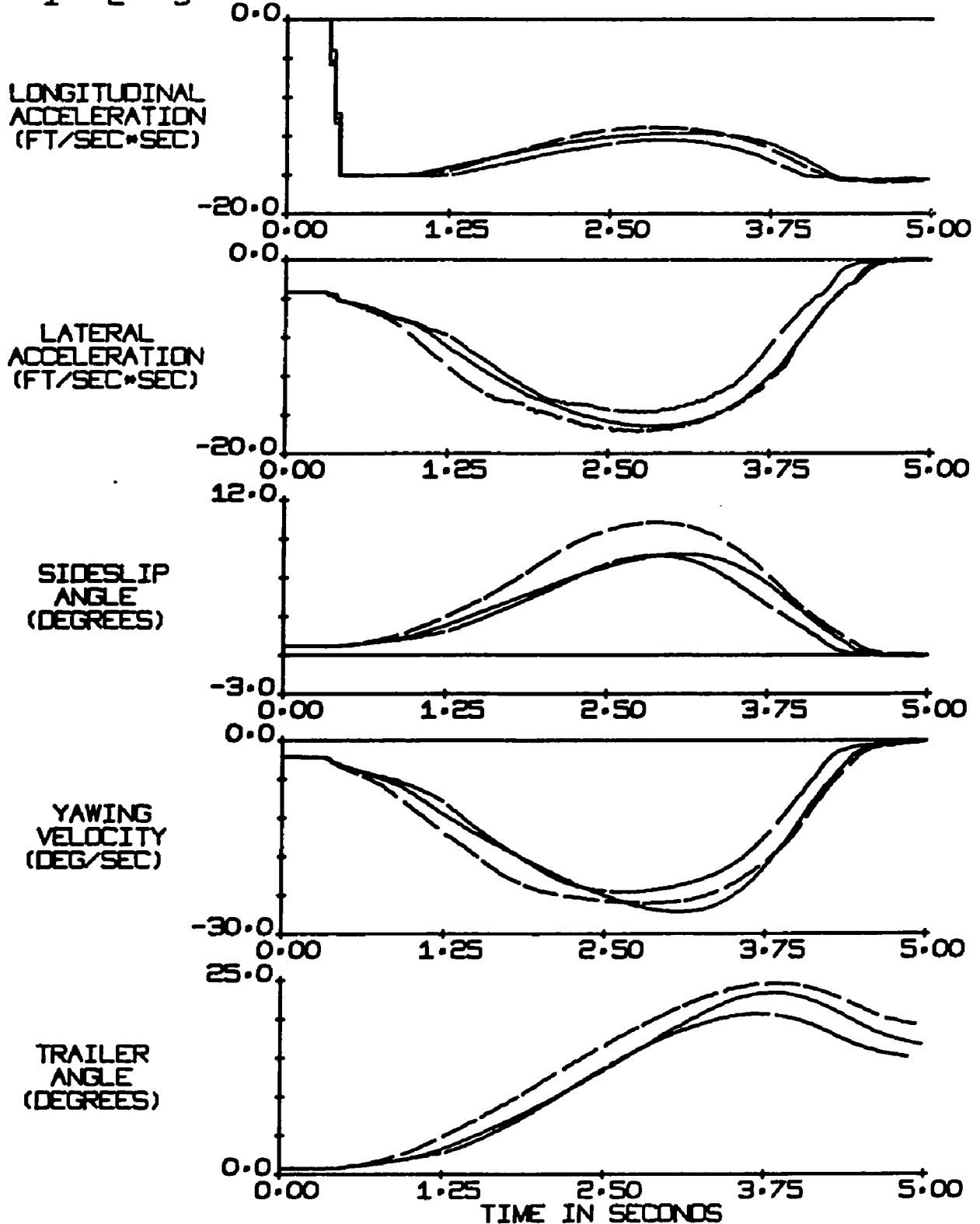


FIG. 4.11 - TRANSIENT RESPONSE OF THE FULL VEHICLE TO BRAKING IN A TURN

4.8 Effect of Load Distribution on Vehicle Response

The effect of load distribution on the response of the half-full vehicle to an attempted deceleration of 0.5 g (on a surface with a coefficient of friction of 0.75) is shown in Figure 4.12. The baseline brake-torque distribution is applied at 0.32, 0.36, and 0.40 seconds while the vehicle is in a steady 0.1 g lateral acceleration turn at 50 MPH. In comparison to the uniform distribution, the severity of the lateral response is reduced by having forward and rearward load distributions but the forward distribution does show trailer swing caused by the locking of the trailer wheels. Using load-sensitive brake-torque control increases the lateral response of the tractor for three load distributions shown in Figure 4.13 but it does prevent trailer swing as shown in Figure 4.12.¹ Figure 4.13 also shows that the lateral response increases in severity in the same order as lateral acceleration gain (see Table 4-4) when the brake torque is distributed in the same ratio as the static load.

¹In Section 4.4 it was assumed that the driver could always reduce lateral response by decreasing the applied steer angle.

LOAD DISTRIBUTION

LOCKED WHEELS

STEER INPUT

UNIFORM

FORWARD

REARWARD

3,4,5

5,6

3,4

-0.44 DEG

-0.39 DEG

-0.47 DEG

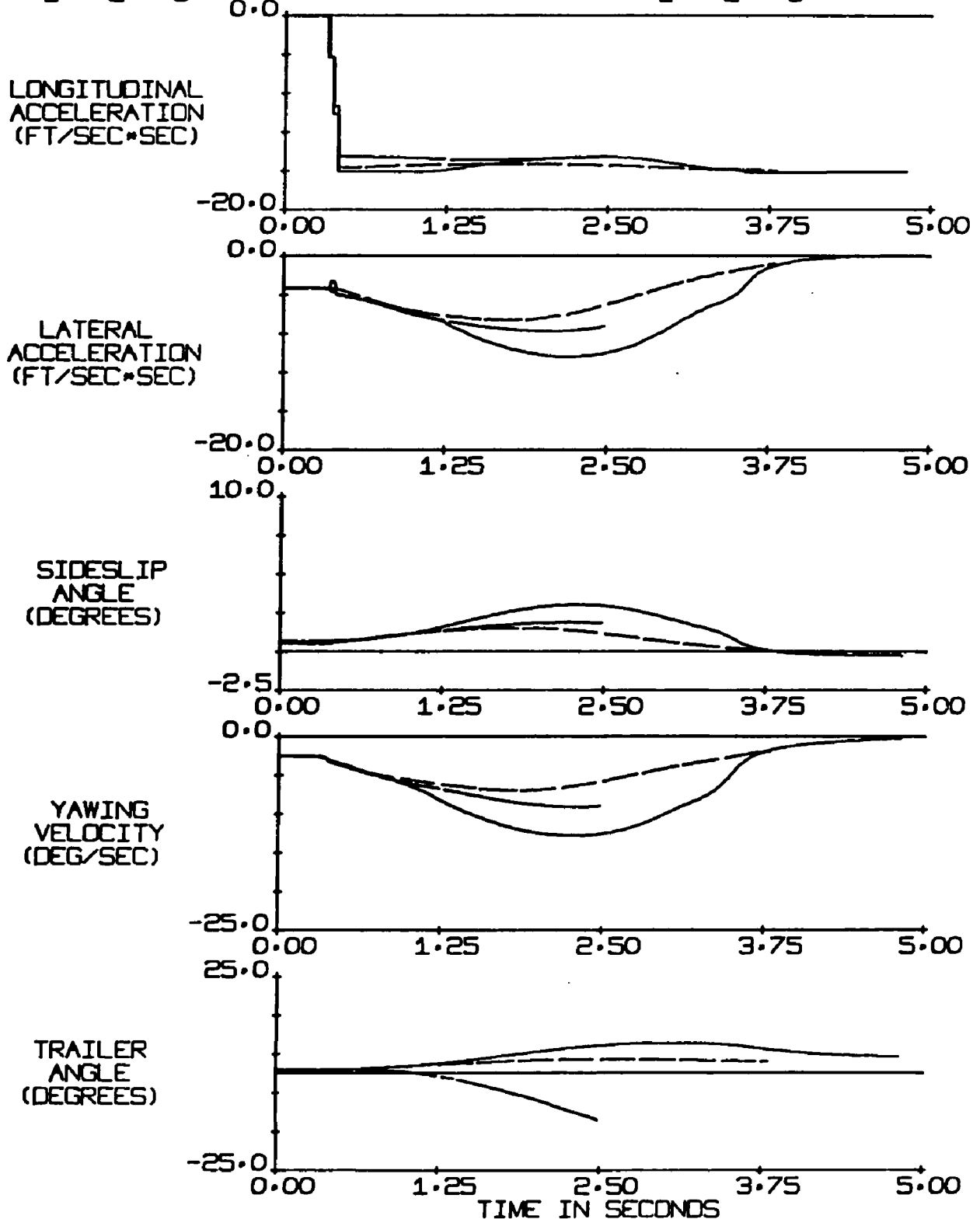
TD₁:TD₂:TD₃ = 0.32:0.36:0.40BT₁:BT₂:BT₃ = 27:36:37

FIG. 4.12 - EFFECT OF LOAD DISTRIBUTION ON TRANSIENT RESPONSE TO BRAKING IN A TURN WITH A HALF-FULL VEHICLE

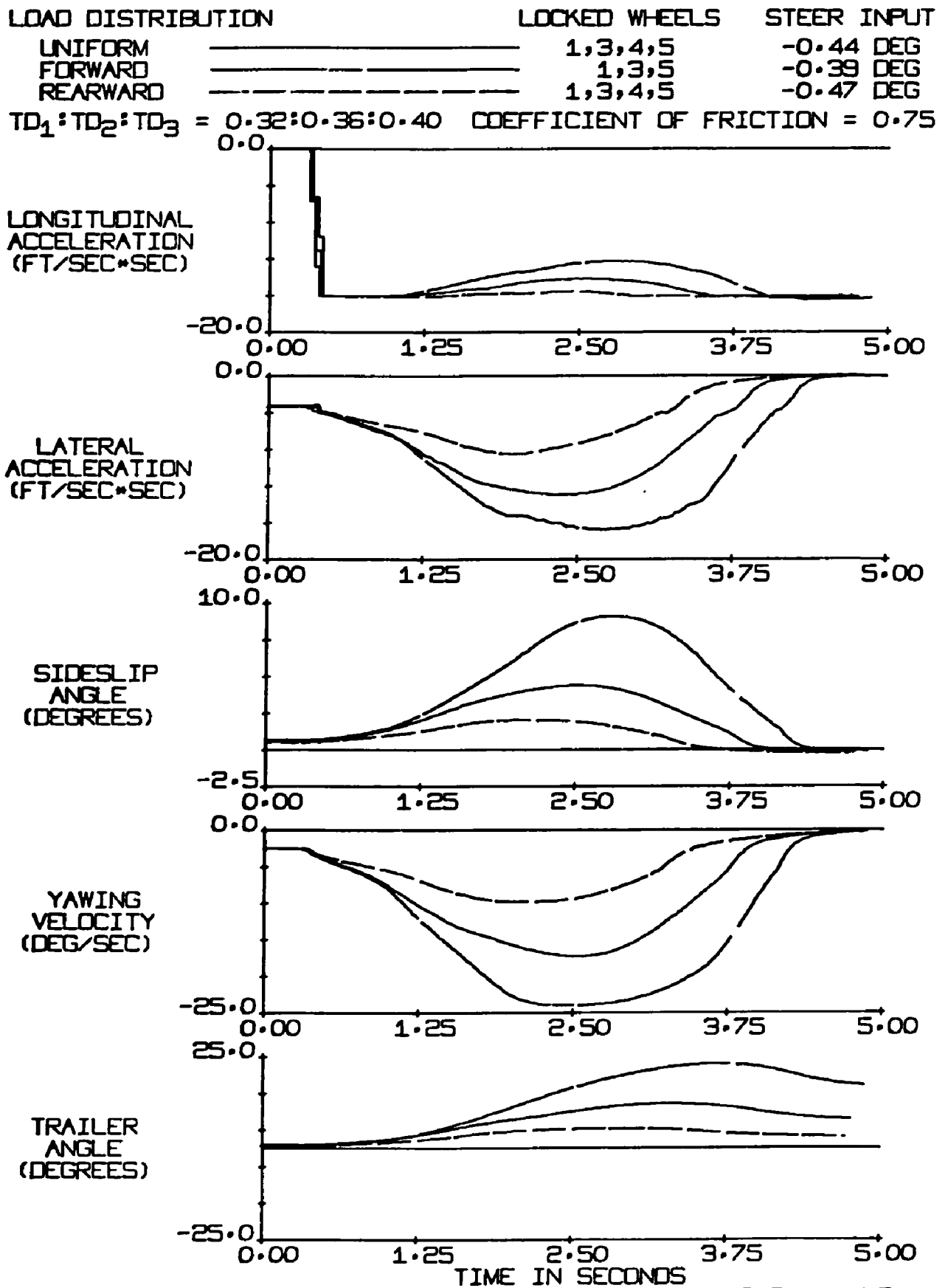


FIG. 4.13 - EFFECT OF LOAD DISTRIBUTION ON BRAKING A HALF-FULL VEHICLE WITH LOAD-SENSITIVE BRAKE-TORQUE CONTROL

4.9 Brake-Torque Application Time

Brake-torque application times have an important effect on the lateral response of the tractor-semitrailer vehicle. The maximum lateral acceleration of a fully loaded vehicle with (1) an initial velocity of 50 MPH on a surface with a coefficient of friction of 0.75, (2) an initial steady-state lateral acceleration of 0.15 g, and (3) a brake-torque distribution of 20:40:40 is plotted in Figure 4.14 as a function of the order of brake application and brake timing. Three brake application sequences, with 0.1 and 0.2 second time delays between axles, are considered: (1) the front brakes of the tractor are applied first, then the rear brakes, and finally the trailer brakes; (2) all brakes are applied at the same time; and (3) the trailer brakes are applied first, then the front brakes of the tractor, and finally the rear brakes of the tractor. Applying the trailer brakes first generates an initial longitudinal retarding force on the tractor which decreases the load on the front tires and the steering forces that they produce. Thus, it appears preferable that the trailer brakes be applied first in comparison with all brakes applied at once or in a front to rear sequence. In addition, an increase in the time delay interval increases the degree to which application of the trailer brakes first or last has an influence on the lateral acceleration of the tractor.

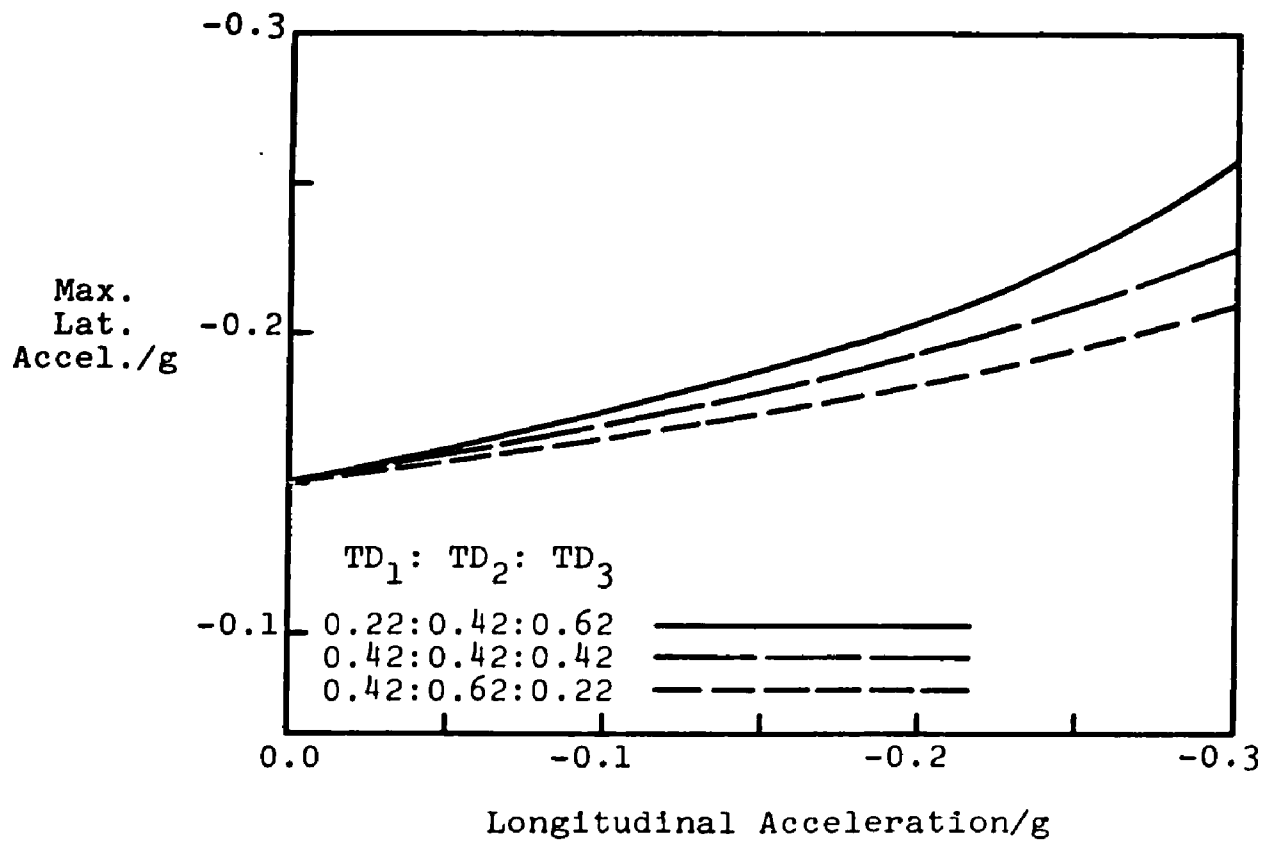
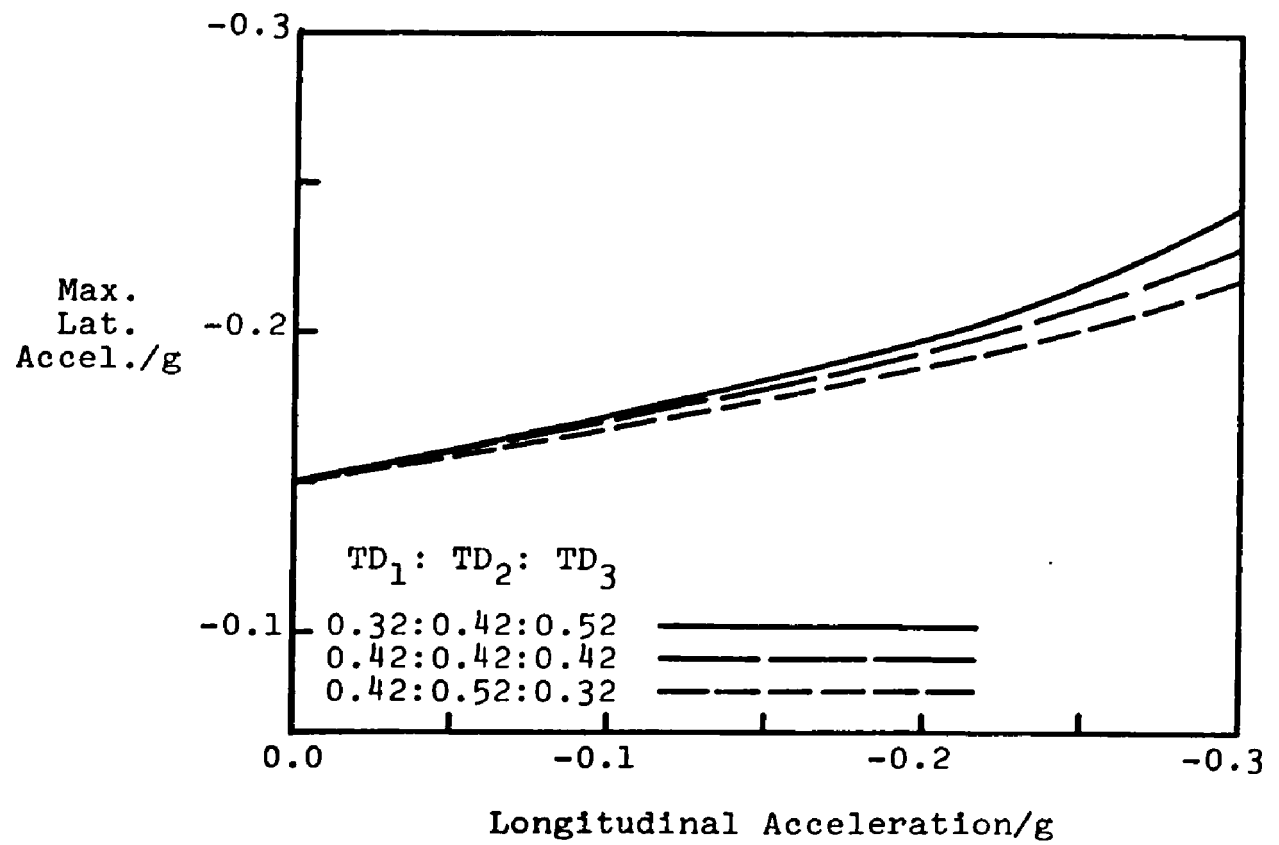


Fig. 4.14 - Effect of brake application time on the tractor's maximum lateral acceleration

4.10 Forward Velocity

Table 4-7 summarizes the effect of forward velocity on the transient response of the full vehicle. Step steer inputs which develop a steady-state lateral acceleration of 0.1 g are applied to the vehicle. Increasing the forward velocity increases the yawing velocity, lateral acceleration, and sideslip angle response times and the lateral acceleration gain. The increase in lateral acceleration gain with velocity indicates that the lateral response of the vehicle will increase substantially with velocity when braked from an initial steady 0.1 g turn. This situation is verified by Figure 4.15 for the fully loaded vehicle braked at a nominal 0.5 g deceleration with a brake-torque distribution of 20:40:40. In a stop from 40 MPH a very mild lateral response is developed whereas a very severe response is produced at 50 MPH and a divergent response (often termed jackknifing) at 60 MPH.

TABLE 4-7

EFFECT OF FORWARD VELOCITY ON RESPONSE TO STEERING

FORWARD VELOCITY (MPH)	YAW (Sec)	LAT. ACCEL. (Sec)	SIDESLIP (Sec)	LAT. ACCEL. GAIN (ft/sec ² /deg)
40	1.93	2.20	2.84	6.01
50	2.24	2.60	2.88	9.58
60	2.32	2.93	3.09	13.90

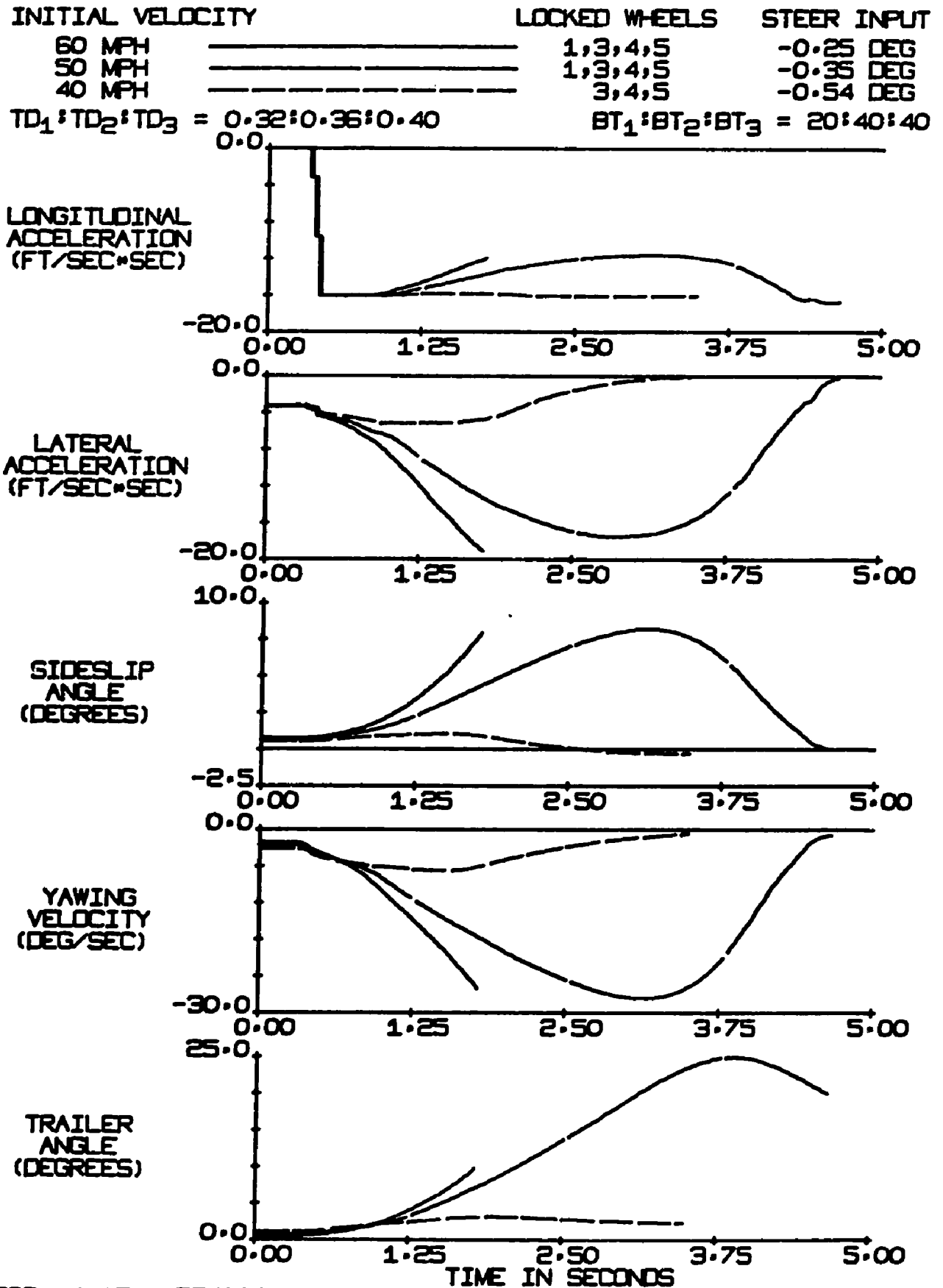


FIG. 4.15 - EFFECT OF INITIAL VELOCITY ON TRANSIENT RESPONSE TO BRAKING IN A TURN

4.11 Tractor Wheelbase

Since increasing the wheelbase of the tractor decreases the lateral acceleration gain (from Table 4-5), its lateral response to braking in a 0.1 g turn should also decrease. Figure 4.16 shows that the lateral response of the full vehicle is substantially reduced as the wheelbase is increased from 10 to 11 and 12 feet. This reduction in lateral response indicates that a longer wheelbase is desirable even though steer angles and steering effort would be increased in order to perform the same maneuver. Figure 4.16 also suggests that lateral acceleration gain, determined from a steady-state constant velocity analysis, can be used in predicting the effect of parameter changes on lateral response during braking.

4.12 Fifth-Wheel Friction

An increase in friction at the fifth-wheel also decreases the lateral acceleration gain of the fully loaded vehicle as shown in Table 4-5. Figure 4.17 shows that the fully loaded vehicle exhibits a more moderate response to braking as the fifth-wheel friction is increased.¹ While friction at the fifth-wheel does increase the steer angles and steering

¹The baseline friction value for the fifth-wheel is 0.05 which represents the coefficient of friction between greased plates.

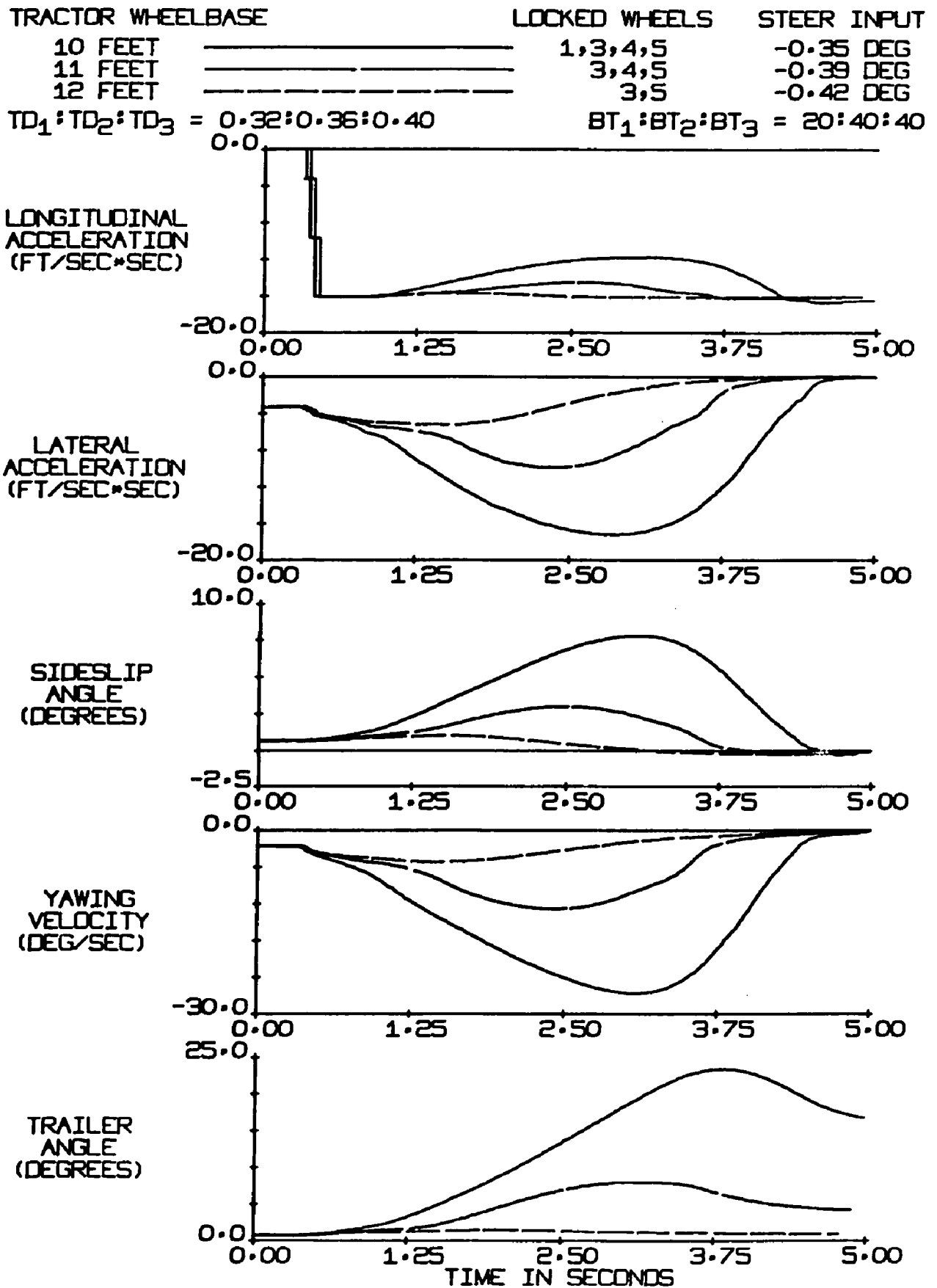


FIG. 4.16 - EFFECT OF TRACTOR WHEELBASE ON TRANSIENT RESPONSE TO BRAKING IN A TURN

FIFTH-WHEEL FRICTION

791.7 FT-LB

1583.3 FT-LB

2375.0 FT-LB

TD₁:TD₂:TD₃ = 0.32:0.36:0.40

LOCKED WHEELS

1,3,4,5

3,4,5

3,4,5

STEER INPUT

-0.35 DEG

-0.36 DEG

-0.37 DEG

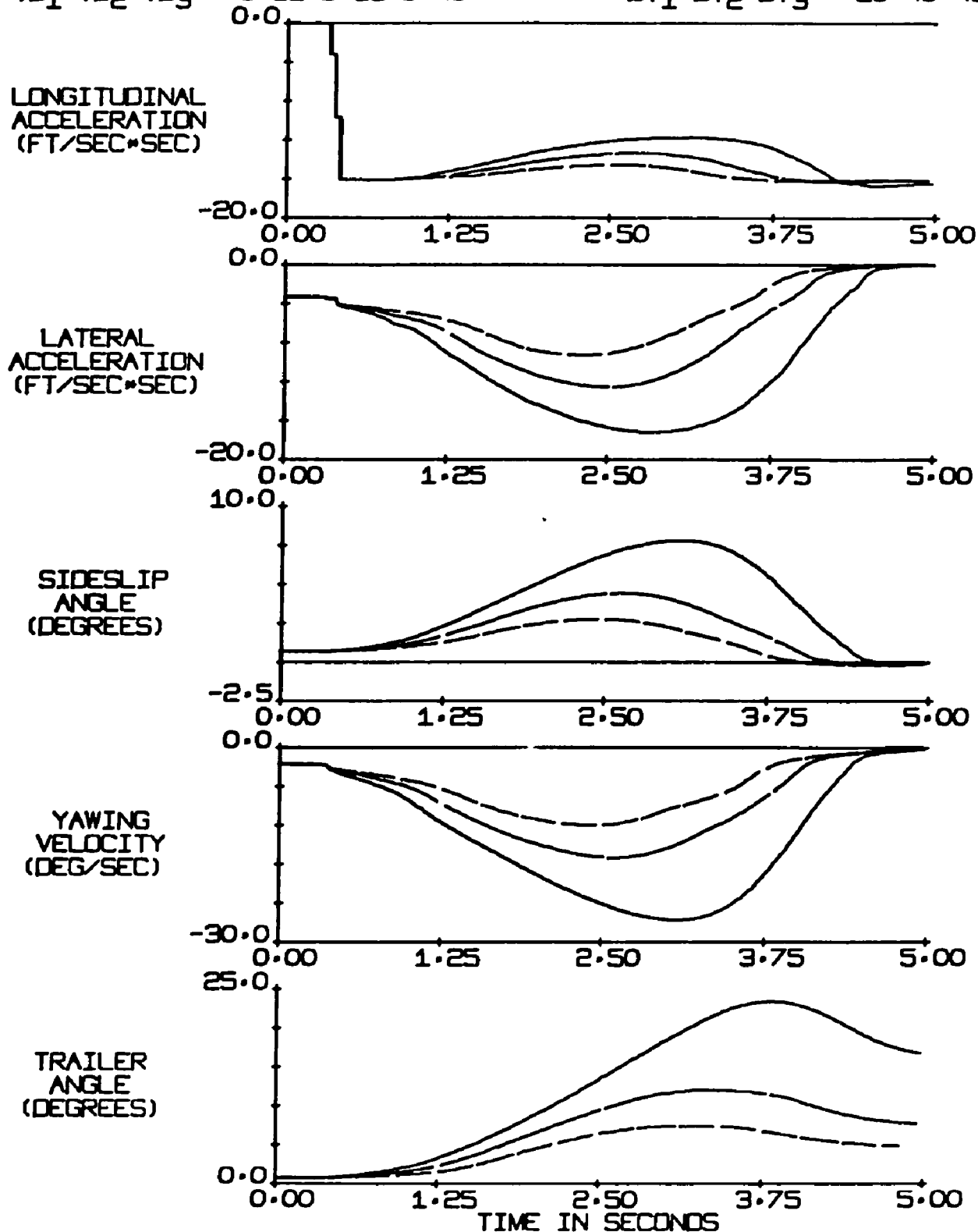
BT₁:BT₂:BT₃ = 20:40:40

FIG. 4.17 - EFFECT OF FIFTH-WHEEL FRICTION ON TRANSIENT RESPONSE TO BRAKING IN A TURN

effort required and cannot prevent instability in all situations, moderate amounts of friction do have a beneficial effect on the lateral response of the vehicle during braking.

4.13 Tire-Road Friction

The maximum coefficient of friction of the tire-road interface is used to introduce the condition of the road into the simulation. The magnitude of this coefficient has a substantial effect on the longitudinal and lateral tire forces that can be developed and the resulting steering and braking performance of the vehicle. In evaluating the effect of tire-road friction, it is assumed that vehicles are driven at the same speeds on the same roads regardless of the friction available. Thus, an initial velocity of 50 MPH in a 0.1 g turn is used in comparing the effect of three different friction coefficients (0.25, 0.50, and 0.75). Figure 4.18 compares the lateral response of the fully loaded vehicle (with a brake-torque distribution of 20:40:40) braked to achieve a deceleration, in g units, equal to 60 percent of the roadway friction coefficient. In this manner, the attempted deceleration is normalized with respect to the available friction. Figure 4.18 shows that essentially the same type of behavior is produced on all three surfaces with the exception that the speed with which the response develops

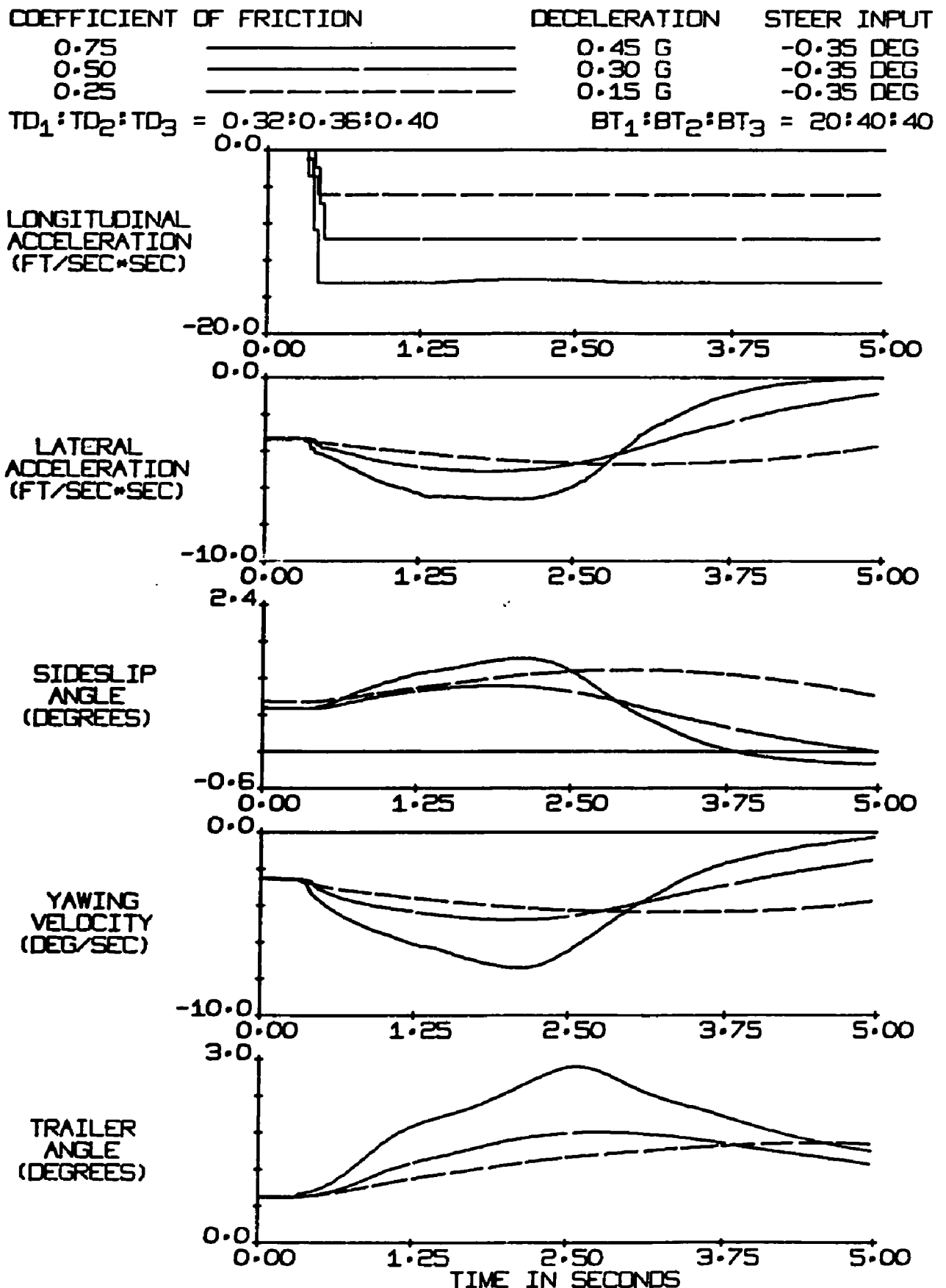


FIG. 4.18 - EVALUATION OF TIRE-ROAD FRICTION WITH THE FULL VEHICLE BRAKED AT 0.6 G TIMES THE COEFFICIENT OF FRICTION

is dependent on the level of longitudinal deceleration and the resulting rate of decrease in velocity. A more revealing evaluation of the effect of friction is made if the initial and the maximum lateral acceleration of the tractor mass center is divided by μ and g as in Table 4-8. The normalized initial lateral acceleration indicates that the severity of the turn increases as the friction decreases. In braking the vehicle, the normalized maximum lateral acceleration also increases as friction decreases, but not as sharply as the normalized initial lateral acceleration.

TABLE 4-8

EFFECT OF TIRE-ROAD FRICTION ON LATERAL RESPONSE

TIRE-ROAD FRICTION COEFFICIENT (μ)	ATTEMPTED NORMALIZED LONGITUDINAL ACCELERATION ($a_x/\mu g$)	NORMALIZED INITIAL LATERAL ACCELERATION ($a_{y0}/\mu g$)	NORMALIZED MAXIMUM LATERAL ACCELERATION ($a_{ym}/\mu g$)
0.75	-0.6	-0.133	-0.274
0.50	-0.6	-0.200	-0.318
0.25	-0.6	-0.400	-0.588

The response of the empty vehicle, with the same initial conditions and brake-torque distribution, when braked to achieve a normalized longitudinal acceleration of -0.6 is shown in Figure 4.19. Trailer swing is produced on all three surfaces, but, as Table 4-9 indicates, it is more serious when it occurs on a surface with a lower coefficient of friction.

COEFFICIENT OF FRICTION	LOCKED WHEELS	STEER INPUT
0.75	3,5,6	-0.50 DEG
0.50	3,5,6	-0.50 DEG
0.25	3,5,6	-0.50 DEG

$TD_1:TD_2:TD_3 = 0.32:0.36:0.40$
 $BT_1:BT_2:BT_3 = 20:40:40$

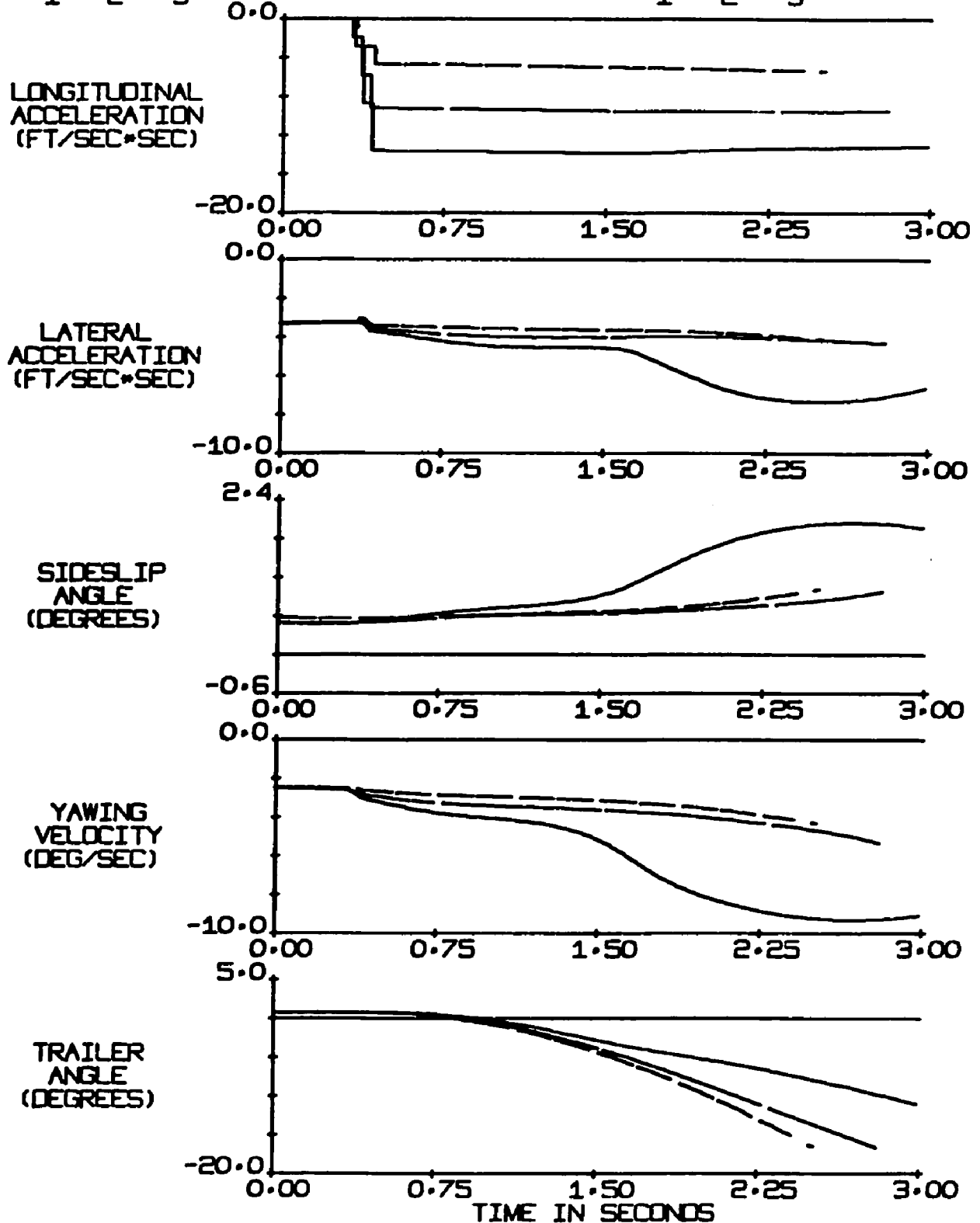


FIG. 4.19 - EVALUATION OF TIRE-ROAD FRICTION WITH THE EMPTY VEHICLE BRAKED AT 0.6 G TIMES THE COEFFICIENT OF FRICTION

TABLE 4-9

EFFECT OF TIRE-ROAD FRICTION ON TRAILER SWING

TIRE-ROAD FRICTION COEFFICIENT (μ)	INITIAL FORWARD VELOCITY (u_o)	FORWARD VELOCITY WHEN THE TRAILER ANGLE IS -10.0 DEGREES (u)
0.75	50 MPH	26.8 MPH
0.50	50 MPH	38.4 MPH
0.25	50 MPH	44.4 MPH

CHAPTER 5

CONCLUSION

5.1 Summary

A hybrid computer simulation has been used to study the directional dynamics of a tractor-semitrailer vehicle. A mathematical model was developed to describe the longitudinal, lateral and yawing motions of a three-axle tractor-semitrailer on a flat road surface. Idealized steering and braking inputs were applied to the simulation to determine the influence of (1) design parameters, (2) operating conditions, and (3) the environment on the directional behavior of the vehicle under study. Results obtained in the computer study were discussed and presented graphically in order to evaluate the relative importance of each effect.

The response to step steering was used to evaluate the influence of design parameters on transient response and steady turning behavior as measured by response times and lateral acceleration gain, respectively. Decreasing the forward velocity, moving the fifth-wheel forward, increasing the wheelbase of the tractor, decreasing the load on the semitrailer, and moving the load to the rear of the trailer decreased the lateral acceleration gain and the response times. Increasing the friction at the fifth-wheel decreased the lateral acceleration gain and increased the response time of the vehicle significantly.

The effects of brake-system parameter changes on directional response were evaluated by braking the vehicle from a steady turn with a fixed steer angle. Increased braking led to a sharper turn and an increased lateral response as measured by the yawing velocity, lateral acceleration, and sideslip angle of the tractor. The variation in loading condition was identified as a major cause of undesirable directional response with a fixed brake-torque distribution. Brake-system design characteristics used to improve the system behavior were the brake-torque distribution, load-sensitive brake-torque control, and brake application times. Longitudinal and lateral brake-torque variations from a baseline distribution had the overall effect of decreasing the performance of the vehicle in a combined steering and braking maneuver without any wheels locking.

The effect of the forward velocity and friction at the tire-road interface on combined steering and braking was also studied by braking the vehicle in a steady-state turn. An increase in the forward velocity of the tractor increases the gain of the system resulting in a more severe lateral response being exhibited at higher velocities. The vehicle developed (1) a slower lateral response to braking, (2) more severe stable responses, and (3) more serious unstable responses on a surface with a lower coefficient of friction.

The ability of changes in basic design parameters to mitigate against producing an undesirable lateral response while braking in a steady turn was also examined. Increasing the wheelbase of the tractor and the amount of friction at the fifth-wheel were most effective in reducing the tendency for jackknifing in a combined steering and braking maneuver.

5.2 Suggestions for Future Research

There are many additional vehicle characteristics, service factors, and specified control inputs which could be investigated with only minor modifications to the present model. Some of the areas which should be explored are:

- (1) Use the three-axle model to approximate three other vehicle configurations, namely:
a tandem axle tractor with a single axle trailer, a two-axle tractor with a tandem axle trailer, and a tandem axle tractor with a tandem axle trailer.
- (2) Study the effect of other safety related service factors such as imbalance in the lateral distribution of load, loss of lateral force on a specific wheel due to catastrophic tire failure, and prescribed brake system failures.

- (3) Make additional safety related fixed control studies with prescribed braking and steering inputs such as braking during a lane change.
- (4) Introduce a simple "driver" model which would attempt to maintain a constant path curvature of the tractor mass center during braking. Even a simple representation of the driver would be an important addition to the fixed control studies which have been performed.

A critical examination of the simulation indicates that certain parts could be improved and logical extensions which would be desirable. These improvements and extensions would be made possible through the use of judicious engineering simulation techniques, additional simplifications where appropriate, and/or a larger computer system. Three aspects which should be considered in developing an improved model are:

- (1) It would be desirable to improve the present quasi-static determination of the vertical load and the longitudinal and lateral shear forces which are subsequently developed on each tire by including pitch and roll as degrees of freedom.

- (2) It would be desirable to include the rotational degree of freedom for each wheel so that the effects of adaptive braking (control systems which prevent wheels from locking) on directional response could be studied.
- (3) Additional experimental truck-tire data should be obtained so that an improved tire model could be developed and used to determine longitudinal and lateral forces as a function of the vertical load, longitudinal and lateral slip, and the friction available at the tire-road interface.

Finally, actual vehicle tests should be performed in order to verify the analytical studies which have been performed.

5.3 Recommendations

Based on the results of this study, the following recommendations appear warranted for improving the overall safe behavior of real tractor-semitrailer vehicles.

- (1) Ideally, brake torque should be distributed in the same proportion as the dynamic weight distribution of the vehicle. If a single fixed brake-torque distribution is

used, it should be related to the dynamic weight distribution of the fully loaded vehicle at some moderate deceleration. If load-sensitive brake-torque control is used, it should be related to the dynamic weight distribution of each loading condition at some moderate deceleration.

- (2) It is desirable to have the brakes on the trailer axle applied first so that the force of the trailer on the tractor tends to extend the vehicle rather than tending to increase the trailer angle.
- (3) A longer tractor wheelbase is desirable in order to decrease the lateral acceleration gain, reduce longitudinal load transfer onto the front axle, and decrease the response time of the vehicle to steering inputs.
- (4) Moderate amounts of fifth-wheel friction are desirable in order to suppress large articulations at the fifth-wheel during braking.
- (5) The vehicle industry should standardize the design of brake-torque distributions so that desirable distributions are obtained when various tractors and trailers are mixed in service.

This analysis of the tractor-semitrailer vehicle was designed to produce findings which would lead to "safer" vehicles on our roads and highways. Simulation provides a means of testing design concepts in a safe, inexpensive, and time-saving manner without depending on the costly trial-and-error methods of extensive physical testing.

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APPENDIX A

RIGID BODY EQUATIONS OF MOTION

The most general displacement of a rigid body is equivalent to a translation followed by a rotation about a base point [38]. The base point is any point in the body. The translation involved will be altered by a change of base point but the rotation is independent of the choice of this point.

Consider a rigid body with a fixed point Q ; let the body rotate through an infinitesimal angle about an axis through Q .¹ This infinitesimal rotation is represented by the infinitesimal vector $\delta\vec{a}$ which has a magnitude equal to the angle and its sense is such that the rotation is positive with respect to it. In the rotation $\delta\vec{a}$ a particle P not on the axis of rotation moves perpendicular to the plane containing the vectors $\delta\vec{a}$ and $\vec{R}$ (the position vector of particle P relative to Q) and in the same direction as the vector product $\delta\vec{a} \times \vec{R}$. The magnitude of the displacement is $|\delta\vec{a}| |\vec{R}| \sin \theta$ in Figure A.1. Since the vector product is equivalent to the magnitude and direction of the displacement, the displacement is given as

$$\delta\vec{R} = \delta\vec{a} \times \vec{R} \quad (A-1)$$

¹Symbols used in Appendix A are defined in Table A-1.

TABLE A-1
SYMBOLS USED IN APPENDIX A

$\delta \vec{a}$	=	infinitesimal rotation vector
$\vec{S}, \vec{R}, \vec{R}_C$	=	position vectors
θ	=	an angle
$\delta \vec{s}$	=	infinitesimal translation vector
$\vec{v}$	=	velocity vector
$\vec{\omega}$	=	angular velocity vector
t	=	time
x, y, z	=	axes of a rectangular coordinate system fixed to a rigid body
x_0, y_0, z_0	=	axes of an inertial coordinate system
$\vec{i}, \vec{j}, \vec{k}$	=	unit vectors associated with the xyz coordinate system
$\vec{A}$	=	a vector
$\vec{F}$	=	sum of the external forces on B
$\vec{M}_O$	=	sum of the moments on B about point O
u, v, w	=	components of the velocity vector in the xyz directions
p, q, r	=	components of the angular velocity vector about the xyz axes
m	=	mass of body B
I_{11}	=	mass moment of inertia about the 1-axis
I_{1j}	=	mass product of inertia about the 1- and j- axis

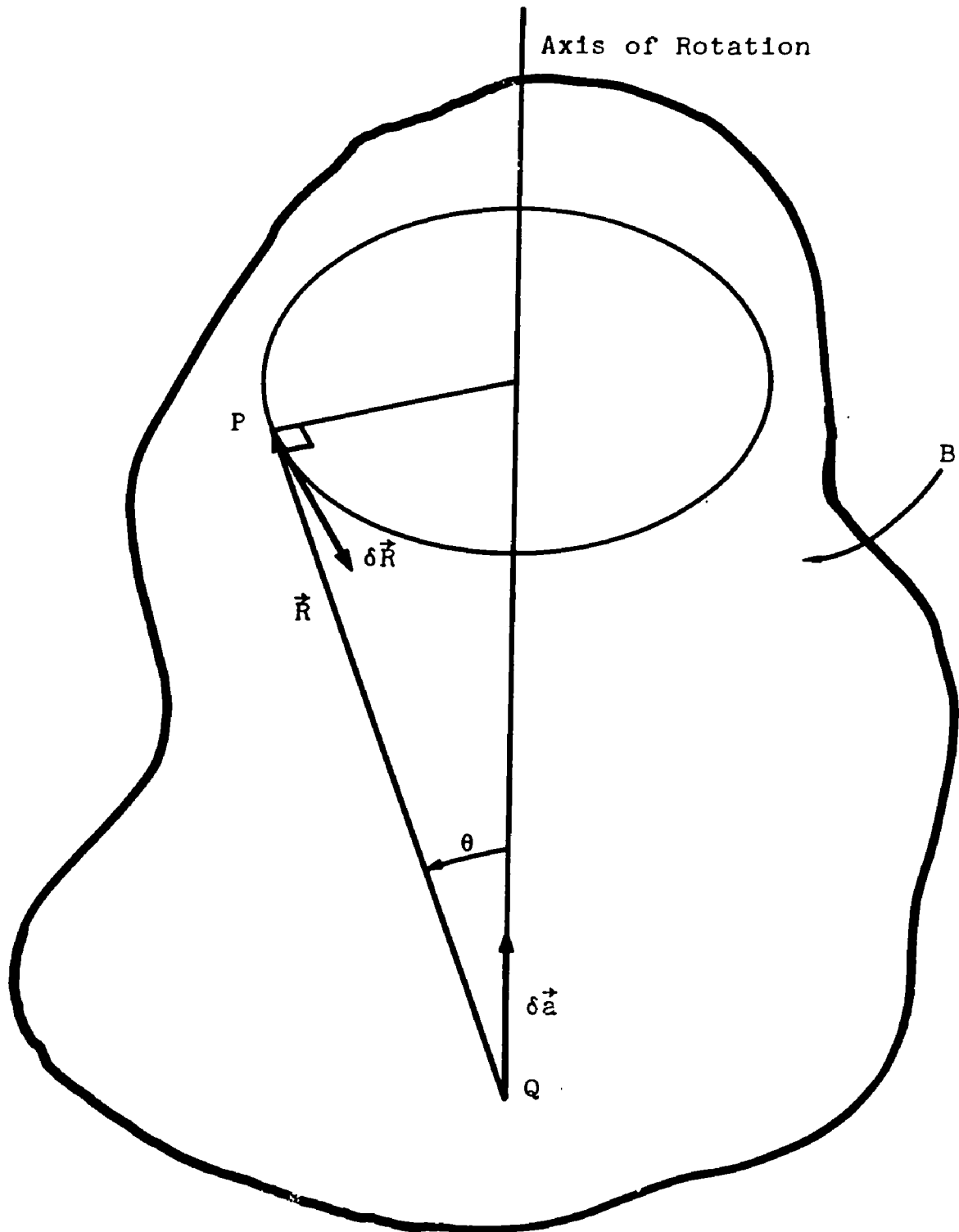


Fig. A.1 - Rotation of rigid body B

Thus, the vector displacement of any particle P arising from a general infinitesimal displacement of the rigid body is given by

$$\delta \vec{p} = \delta \vec{s} + \delta \vec{a} \times \vec{R} \quad (\text{A-2})$$

where $\delta \vec{p}$ is the general infinitesimal displacement of the particle, $\delta \vec{s}$ is the infinitesimal translation of the base point, $\delta \vec{a}$ is the infinitesimal rotation, and $\vec{R}$ is the position vector of the particle relative the the base point.

Next, select a particle Q as a base point in a rigid body moving in a general manner and denote its velocity as $\vec{V}_Q$. In an infinitesimal time dt , the displacement of any particle P of the body is

$$d\vec{s}_P = \vec{V}_Q dt + d\vec{a} \times \vec{R} \quad (\text{A-3})$$

where $\vec{R}$ is the position vector of P relative to Q . Hence, the velocity of particle P is given as

$$\vec{V}_P = \vec{V}_Q + \vec{\omega} \times \vec{R} \quad (\text{A-4})$$

where $\vec{\omega} = d\vec{a}/dt$. This velocity consists of two parts: the velocity $\vec{V}_Q$ of the base point, and the velocity of P relative to Q , viz., $\vec{\omega} \times \vec{R}$. If the base point Q is changed, the translation $\vec{V}_Q dt$, is changed, but the rotation $d\vec{a}$ remains the same. It follows that the angular velocity vector $\vec{\omega}$ relates to the whole body and is a free vector since it does not depend on the

base point.

Let an orthogonal coordinate system xyz be fixed to a rigid body B which is translating and rotating with an angular velocity $\vec{\omega}$, relative to an inertial reference frame $x_0y_0z_0$. The unit vectors $\vec{i}$, $\vec{j}$, and $\vec{k}$ are associated with the x , y , and z axes, respectively. Then in Figure A.2, any vector $\vec{A}$ can be expressed as

$$\vec{A} = A_x \vec{i} + A_y \vec{j} + A_z \vec{k} \quad (\text{A-5})$$

In differentiating $\vec{A}$ with respect to time, one must differentiate the vectors $\vec{i}$, $\vec{j}$, and $\vec{k}$ as well as A_x , A_y and A_z since their direction changes even though their magnitude does not change. Thus,

$$\begin{aligned} \frac{d\vec{A}}{dt} = & \frac{dA_x}{dt} \vec{i} + \frac{dA_y}{dt} \vec{j} + \frac{dA_z}{dt} \vec{k} \\ & + A_x \frac{d\vec{i}}{dt} + A_y \frac{d\vec{j}}{dt} + A_z \frac{d\vec{k}}{dt} \end{aligned} \quad (\text{A-6})$$

Since $\vec{i}$, $\vec{j}$, and $\vec{k}$ are vectors fixed in a rigid body which rotates with angular velocity $\vec{\omega}$, then these points can be considered position vectors of particles C , D , and E of this body relative to a base point Q . The derivatives of the position vectors are the velocities of C , D , and E relative to Q . Applying Equation (A-4) gives the time derivative of each of these fixed position vectors as

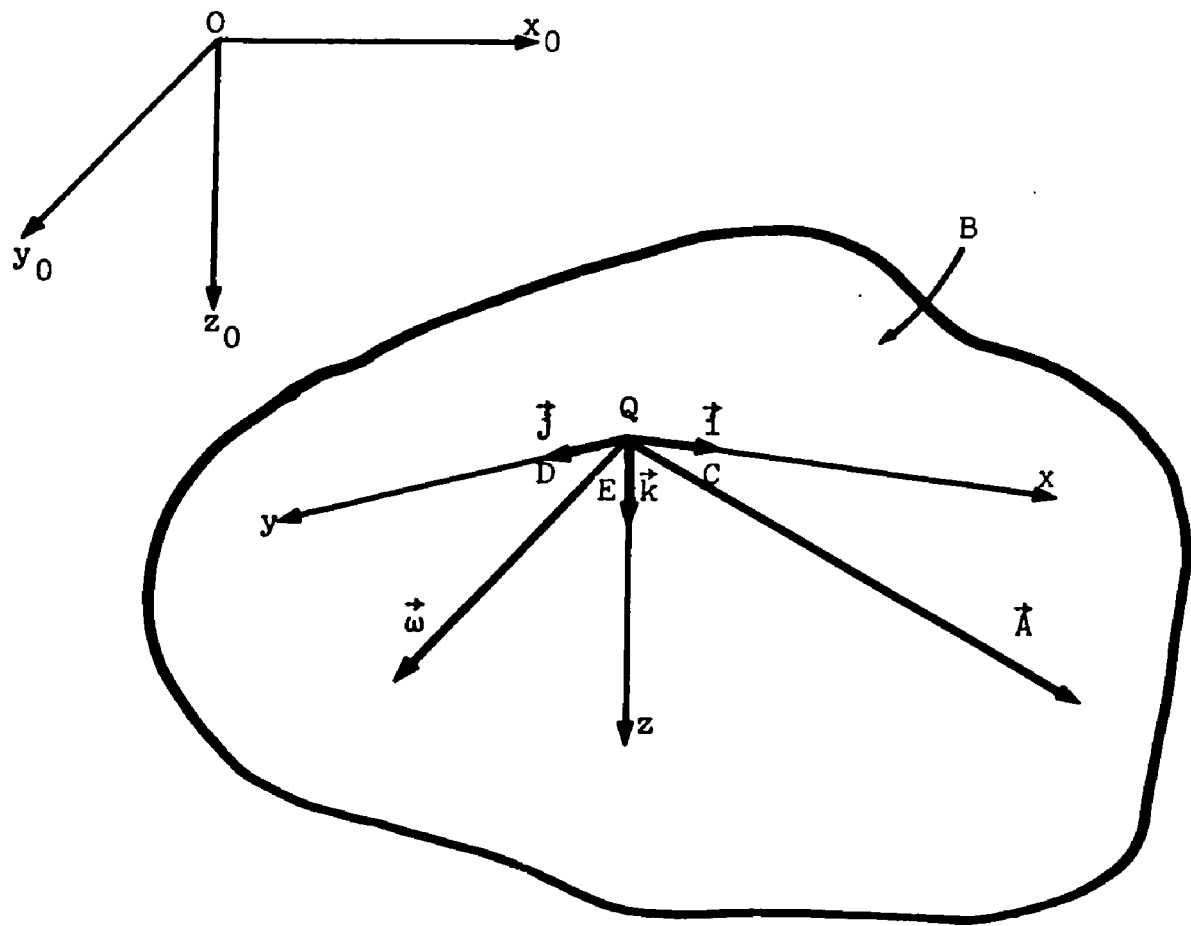


Fig. A.2 - Rigid body B translating and rotating relative to an inertial reference frame

$$\frac{d\vec{i}}{dt} = \vec{\omega} \times \vec{i}, \quad \frac{d\vec{j}}{dt} = \vec{\omega} \times \vec{j}, \quad \text{and} \quad \frac{d\vec{k}}{dt} = \vec{\omega} \times \vec{k} \quad (\text{A-7})$$

Combining Equations (A-6) and (A-7) gives the time derivative of a vector $\vec{A}$ defined in terms of a moving axis system as

$$\frac{d\vec{A}}{dt} = \frac{dA_x}{dt} \vec{i} + \frac{dA_y}{dt} \vec{j} + \frac{dA_z}{dt} \vec{k} + \vec{\omega} \times \vec{A} \quad (\text{A-8})$$

An arbitrary rigid body B, shown in Figure A.3, is translating and rotating in space. An orthogonal xyz coordinate system is fixed to B with the origin, C, located at the center of mass. The equations of motion for B are developed from Newton's Second Law expressed as

$$\vec{F} = \frac{d}{dt} \int_B \frac{d\vec{S}}{dt} dm \quad (\text{A-9})$$

$$\vec{M}_O = \frac{d}{dt} \int_B \vec{S} \times \frac{d\vec{S}}{dt} dm \quad (\text{A-10})$$

where $\vec{F}$ and $\vec{M}_O$ are the sum of the external forces and moments on body B and $\vec{S}$ is a vector from an inertial point O, the origin of the $x_0y_0z_0$ axes, to an elemental mass dm . Equation (A-9) equates the time rate of change of linear momentum of body B to the external forces in three orthogonal directions. Equation (A-10) equates the time rate of change of angular momentum to the external moments on the body about the same orthogonal directions.

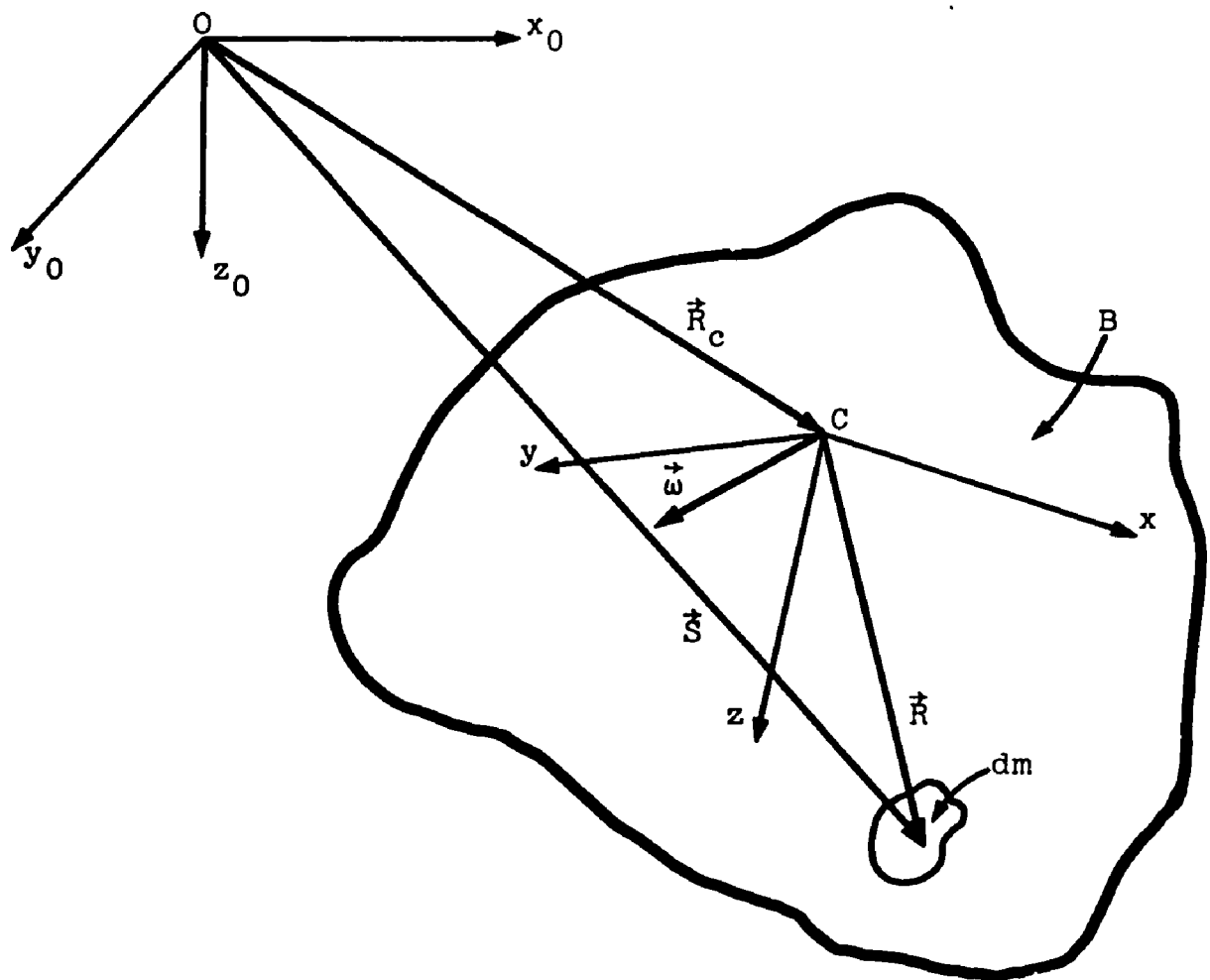


Fig. A.3 - Rigid body B

The vector $\vec{S}$ is defined in Equation (A-11) as the sum of the position vectors of the center of mass and of the elemental mass relative to the center of mass.

$$\vec{S} = \vec{R}_C + \vec{R} \quad (\text{A-11})$$

Differentiating $\vec{S}$ as in Equation (A-8) gives

$$\frac{d\vec{S}}{dt} = \vec{v}_C + \frac{dR_x}{dt} \vec{i} + \frac{dR_y}{dt} \vec{j} + \frac{dR_z}{dt} \vec{k} + \vec{\omega} \times \vec{R} \quad (\text{A-12})$$

where $\vec{v}_C$ is the derivative with respect to time of the position vector of the center of mass and $\vec{\omega}$ is the angular velocity vector of body B. Combining Equations (A-9) and (A-12) in a more convenient form

$$\begin{aligned} \vec{F} = \frac{d}{dt} \left(\vec{v}_C \int_B dm + \int_B \left(\frac{dR_x}{dt} \vec{i} + \frac{dR_y}{dt} \vec{j} \right. \right. \\ \left. \left. + \frac{dR_z}{dt} \vec{k} \right) dm + \vec{\omega} \times \int_B \vec{R} dm \right) \end{aligned} \quad (\text{A-13})$$

where the vectors $\vec{v}_C$ and $\vec{\omega}$ have been brought out of the integrand since they have no functional dependence on the distribution of the mass. The first integral in Equation (A-13) is the mass of the body and the third integral vanishes since the mass moment about the center of mass is zero. The second integral is also zero since the body is rigid and the components of the position vector to any elemental mass do not change with time. This reduces

Equation (A-13) to the following

$$\vec{F} = \frac{d}{dt} (m\vec{V}_C) \quad (\text{A-14})$$

The velocity of the center of mass and the angular velocity of the body are defined in terms of a moving axis system xyz as

$$\vec{V}_C = u\vec{i} + v\vec{j} + w\vec{k} \quad (\text{A-15})$$

$$\vec{\omega} = p\vec{i} + q\vec{j} + r\vec{k} \quad (\text{A-16})$$

Differentiating the velocity vector of the center of mass as in Equation (A-8) and substituting the results into (A-14) gives

$$\vec{F} = m(\dot{u}\vec{i} + \dot{v}\vec{j} + \dot{w}\vec{k} + \vec{\omega} \times \vec{V}_C) \quad (\text{A-17})$$

where the dot over a variable indicates the derivative with respect to time. The scalar components of Equation (A-17) are

$$F_x = m(\dot{u} + wq - vr) \quad (\text{A-18})$$

$$F_y = m(\dot{v} + ur - wp) \quad (\text{A-19})$$

$$F_z = m(\dot{w} + vp - uq) \quad (\text{A-20})$$

where F_x , F_y , and F_z are the sum of the external forces on B in the x , y , and z directions, respectively.

Combining Equations (A-10) and (A-12) in a more useful form gives

$$\begin{aligned}
 \vec{M}_O = \frac{d}{dt} \left(\vec{R}_C \times \vec{V}_C \int_B dm + \vec{R}_C \times \int_B \left(\frac{dR_x}{dt} \vec{i} \right. \right. \\
 + \frac{dR_y}{dt} \vec{j} + \frac{dR_z}{dt} \vec{k} \Big) dm + \vec{R}_C \times (\vec{\omega} \times \int_B \vec{R} dm) \\
 - \vec{V}_C \times \int_B \vec{R} dm + \int_B \vec{R} \times \left(\frac{dR_x}{dt} \vec{i} + \frac{dR_y}{dt} \vec{j} \right. \\
 \left. \left. + \frac{dR_z}{dt} \vec{k} \right) dm + \int_B \vec{R} \times (\vec{\omega} \times \vec{R}) dm \right) \quad (A-21)
 \end{aligned}$$

The vectors $\vec{V}_C$ and $\vec{\omega}$ have been brought out of the integrand as they have no functional dependence on the mass distribution. The first integral in Equation (A-21) is the mass of the body and the third and fourth integrals, being the mass moment about the center of mass, are zero. The second and fifth integrals are also zero since the components of a vector from the center of mass to any particle in a rigid body do not vary with time. Equation (A-21) reduces to

$$\vec{M}_O = \frac{d}{dt} \left(m \vec{R}_C \times \vec{V}_C + \int_B \vec{R} \times (\vec{\omega} \times \vec{R}) dm \right) \quad (A-22)$$

Equation (A-23) is formed by integrating the moment of each infinitesimal force $d\vec{F}$ on the body about point O and using the definition for $\vec{S}$ given in Equation (A-11).

$$\int_B \vec{S} \times d\vec{F} = \vec{R}_C \times \vec{F} + \int_B \vec{R} \times d\vec{F} \quad (\text{A-23})$$

The left side of Equation (A-23) is $\vec{M}_O$ and the first term on the right side of Equations (A-22) and (A-23) are equivalent by Equation (A-14). Equation (A-22) is rewritten as Equation (A-24) in terms of the center of mass rather than an inertial point by defining the second term on the right side of Equation (A-23) as $\vec{M}_C$ (the moment of all infinitesimal forces about the center of mass).

$$\vec{M}_C = \frac{d}{dt} \left(\int_B \vec{R} \times (\vec{\omega} \times \vec{R}) dm \right) \quad (\text{A-24})$$

The position vector $\vec{R}$ to an infinitesimal mass relative to the center of mass is defined as

$$\vec{R} = x\vec{i} + y\vec{j} + z\vec{k} \quad (\text{A-25})$$

The vector triple product in Equation (A-24) is expanded as follows:

$$\begin{aligned} \int_B \vec{R} \times (\vec{\omega} \times \vec{R}) dm &= \vec{i} \left(p \int_B (y^2 + z^2) dm - q \int_B xy dm - r \int_B zx dm \right) \\ &\quad + \vec{j} \left(q \int_B (z^2 + x^2) dm - r \int_B yz dm - p \int_B xy dm \right) \\ &\quad + \vec{k} \left(r \int_B (x^2 + y^2) dm - p \int_B zx dm - q \int_B yz dm \right) \end{aligned} \quad (\text{A-26})$$

The integrals on the right side of Equation (A-26) are the mass moments and products of inertia and each is replaced with the symbol I_{ii} (representing the mass moment of inertia about the i -axis) or I_{ij} (representing the mass product of inertia about the i - and j -axes). Equation (A-24) is then rewritten as

$$\begin{aligned} \vec{M}_C = \frac{d}{dt} \left((pI_{xx} - qI_{xy} - rI_{xz})\vec{i} + (pI_{yx} + qI_{yy} \right. \\ \left. - rI_{yz})\vec{j} + (-pI_{zx} - qI_{zy} + rI_{zz})\vec{k} \right) \end{aligned} \quad (A-27)$$

Since the mass moments and products of inertia are defined with respect to an xyz axis system fixed to the body, they are invariant with time and the vector differentiation can be performed as in Equation (A-8) to give

$$\begin{aligned} M_x = \dot{p}I_{xx} + (rp - \dot{q})I_{xy} - (\dot{r} + pq)I_{zx} + (r^2 - q^2)I_{yz} \\ + qr(I_{zz} - I_{yy}) \end{aligned} \quad (A-28)$$

$$\begin{aligned} M_y = \dot{q}I_{yy} + (pq - \dot{r})I_{yz} - (\dot{p} + qr)I_{xy} + (p^2 - r^2)I_{zx} \\ + rp(I_{xx} - I_{zz}) \end{aligned} \quad (A-29)$$

$$\begin{aligned} M_z = \dot{r}I_{zz} + (qr - \dot{p})I_{zx} - (\dot{q} + rp)I_{yz} + (q^2 - p^2)I_{xy} \\ + pq(I_{yy} - I_{xx}) \end{aligned} \quad (A-30)$$

where M_x , M_y , and M_z are the sum of the moments of the external forces on B about the x, y, and z axes, respectively.

APPENDIX B

DEFINITION OF NORMALIZED VARIABLES

The variables which define the problem formulated in Chapter 2 are normalized as they are used in the computer circuit diagrams of Appendix E. The normalized variable is the ratio of the variable to its maximum value. It is equivalent to the ratio of the computer voltage which represents it and the reference voltage. For example, the normalized longitudinal velocity of the center of mass of the tractor is $(u/u_m) \equiv (E_u/E_{ref})$ where E_u is the output voltage of the amplifier representing the longitudinal velocity, E_{ref} is the reference voltage of the computer, and the subscript m is used to indicate the maximum value of the variable.¹ The normalized variables are defined as $AV(I)$ where I refers to the equation number below and to the identification number of the amplifier which represents the variable. The value of L_1 is one (1) if wheel 1 is rolling and zero (0) if it is locked.

¹Symbols used in Appendix B are defined in Table 2-1.

$$AV(1) = \frac{\dot{u} - vr}{\dot{u}_m} \quad (B-1)$$

$$AV(2) = \frac{u}{u_m} \quad (B-2)$$

$$AV(3) = \frac{\dot{v} + ur}{\dot{u}_m} \quad (B-3)$$

$$AV(4) = \frac{v}{v_m} \quad (B-4)$$

$$AV(5) = \frac{\dot{r}}{\dot{r}_m} \quad (B-5)$$

$$AV(6) = \frac{r}{r_m} \quad (B-6)$$

$$AV(7) = \frac{\dot{r} + \ddot{\psi}}{\dot{r}_m} \quad (B-7)$$

$$AV(8) = \frac{r + \dot{\psi}}{r_m} \quad (B-8)$$

$$AV(9) = \frac{\psi}{\pi} \quad (B-9)$$

$$AV(10) = \frac{t - t_m/2}{t_m/2} \quad (B-10)$$

$$AV(11) = - \frac{F_{x1} + F_{x2}}{2\bar{F}_{z1}} \quad (B-11)$$

$$AV(12) = \left(\frac{F_{x1} + F_{x2}}{2\bar{F}_{z1}} \right) \delta \quad (B-12)$$

$$AV(13) = - \frac{F_{x5} + F_{x6}}{2\bar{F}_{z5}} \quad (B-13)$$

$$AV(14) = - \frac{F_{x5} + F_{x6}}{2\bar{F}_{z5}} \sin \psi \quad (B-14)$$

$$AV(15) = - \frac{F_{x5} + F_{x6}}{2\bar{F}_{z5}} \cos \psi \quad (B-15)$$

$$AV(16) = \frac{\delta}{\alpha_m} \quad (B-16)$$

$$AV(17) = \delta \quad (B-17)$$

$$AV(18) = - \frac{F_{y1} - F_{y2}}{2\bar{F}_{z1}} \quad (B-18)$$

$$AV(19) = \sin \psi \quad (B-19)$$

$$AV(20) = \cos \psi \quad (B-20)$$

$$AV(21) = \frac{\mu F_{z1}}{2\bar{F}_{z1}} \cos \alpha_1 \quad (B-21)$$

$$AV(22) = \frac{\alpha_1 + \delta}{\alpha_m} = \frac{\alpha_2 + \delta}{\alpha_m} \quad (B-22)$$

$$AV(23) = \frac{\bar{\alpha}_1}{\bar{\alpha}_m} = \frac{\bar{\alpha}_2}{\bar{\alpha}_m} \quad (B-23)$$

$$AV(24) = - \sin \alpha_1 = - \sin \alpha_2 \quad (B-24)$$

$$AV(25) = \cos \alpha_1 = \cos \alpha_2 \quad (B-25)$$

$$AV(26) = \frac{\alpha_3}{\alpha_m} = \frac{\alpha_4}{\alpha_m} \quad (B-26)$$

$$AV(27) = \frac{\alpha_3}{\pi} = \frac{\alpha_4}{\pi} \quad (B-27)$$

$$AV(28) = -\sin \alpha_3 = -\sin \alpha_4 \quad (B-28)$$

$$AV(29) = \cos \alpha_3 = \cos \alpha_4 \quad (B-29)$$

$$AV(30) = \frac{v - hr}{u_m} \quad (B-30)$$

$$AV(31) = \frac{\alpha_5}{\alpha_m} = \frac{\alpha_6}{\alpha_m} \quad (B-31)$$

$$AV(32) = \left(\frac{u}{u_m} \cos \psi + \frac{v - hr}{u_m} \sin \psi \right) \alpha_m \quad (B-32)$$

$$AV(33) = \frac{\alpha_5}{\pi} = \frac{\alpha_6}{\pi} \quad (B-33)$$

$$AV(34) = -\sin \alpha_5 = -\sin \alpha_6 \quad (B-34)$$

$$AV(35) = \cos \alpha_5 = \cos \alpha_6 \quad (B-35)$$

$$AV(36) = \frac{\alpha_1}{\pi} = \frac{\alpha_2}{\pi} \quad (B-36)$$

$$AV(37) = \frac{F_{y1}}{\mu F_{z1}} \quad (B-37)$$

$$AV(38) = \frac{F_{y3}}{\mu F_{z3}} \quad (B-38)$$

$$AV(39) = \frac{F_{y5}}{\mu F_{z5}} \quad (B-39)$$

$$AV(40) = - \frac{F_{x1}}{2\bar{F}_{z1}} \quad (B-40)$$

$$AV(41) = - \frac{F_{x2}}{2\bar{F}_{z2}} \quad (B-41)$$

$$AV(42) = - \frac{F_{x3}}{2\bar{F}_{z3}} \quad (B-42)$$

$$AV(43) = - \frac{ur}{u_m r_m} \quad (B-43)$$

$$AV(44) = \frac{vr}{v_m r_m} \quad (B-44)$$

$$AV(45) = \left(\frac{r + \dot{\psi}}{r_m} \right)^2 \quad (B-45)$$

$$AV(46) = \left(\frac{r}{r_m} \right)^2 \quad (B-46)$$

$$AV(47) = \left(\frac{r + \dot{\psi}}{r_m} \right)^2 \sin \psi \quad (B-47)$$

$$AV(48) = - \frac{2(\dot{r} + \ddot{\psi})}{\dot{r}_m} \cos \psi \quad (B-48)$$

$$AV(49) = - \frac{\lambda_4}{m_2 \dot{u}_m} \quad (B-49)$$

$$AV(50) = \left(\frac{r + \dot{\psi}}{r_m} \right)^2 \cos \psi \quad (B-50)$$

$$AV(51) = \frac{2(\dot{r} + \ddot{\psi})}{\dot{r}_m} \sin \psi \quad (B-51)$$

$$AV(52) = -\frac{\lambda_3}{m_2 \dot{u}_m} \quad (B-52)$$

$$AV(53) = \frac{\mu F_{z6}}{2\bar{F}_{z6}} \cos \alpha_6 \quad (B-53)$$

$$AV(54) = -\frac{\lambda_2}{m_2 \dot{u}_m} \quad (B-54)$$

$$AV(55) = \frac{\mu F_{z5}}{2\bar{F}_{z5}} \quad (B-55)$$

$$AV(56) = -\frac{\lambda_1}{m_2 \dot{u}_m} \quad (B-56)$$

$$AV(57) = \frac{\mu F_{z2}}{2\bar{F}_{z2}} \quad (B-57)$$

$$AV(58) = \frac{\mu \Delta F_{z3}}{2\bar{F}_{z3}} = -\frac{\mu \Delta F_{z4}}{2\bar{F}_{z4}} \quad (B-58)$$

$$AV(59) = \frac{\mu \Delta F_{z5}}{2\bar{F}_{z5}} = -\frac{\mu \Delta F_{z6}}{2\bar{F}_{z6}} \quad (B-59)$$

$$AV(60) = \frac{\mu \delta F_{z1}}{2\bar{F}_{z1}} = \frac{\mu \delta F_{z2}}{2\bar{F}_{z2}} \quad (B-60)$$

$$AV(61) = \frac{\mu \delta F_{z3}}{2\bar{F}_{z3}} = \frac{\mu \delta F_{z4}}{2\bar{F}_{z4}} \quad (B-61)$$

$$AV(62) = \frac{\mu \delta F_{z5}}{2\bar{F}_{z5}} = \frac{\mu \delta F_{z6}}{2\bar{F}_{z6}} \quad (B-62)$$

$$AV(63) = - \frac{F_{y1} + F_{y2}}{2\bar{F}_{z1}} \quad (B-63)$$

$$AV(64) = - \frac{F_{y3} + F_{y4}}{2\bar{F}_{z3}} \quad (B-64)$$

$$AV(65) = - \frac{F_{y5} + F_{y6}}{2\bar{F}_{z5}} \quad (B-65)$$

$$AV(66) = - \left(\frac{F_{y1} + F_{y2}}{2\bar{F}_{z1}} \right) \delta \quad (B-66)$$

$$AV(67) = \frac{\mu F_{z1}}{2\bar{F}_{z1}} \quad (B-67)$$

$$AV(68) = - \left(\frac{F_{y5} + F_{y6}}{2\bar{F}_{z5}} \right) \sin \psi \quad (B-68)$$

$$AV(69) = - \left(\frac{F_{y5} + F_{y6}}{2\bar{F}_{z5}} \right) \cos \psi \quad (B-69)$$

$$AV(70) = \frac{u}{u_m} \cos \psi + \frac{v - hr}{u_m} \sin \psi \quad (B-70)$$

$$AV(71) = \frac{\mu F_{z5}}{2\bar{F}_{z5}} \cos \alpha_5 \quad (B-71)$$

$$AV(72) = \frac{F_{y6}}{\mu F_{z6}} \quad (B-72)$$

$$AV(73) = \frac{\theta}{\pi} \quad (B-73)$$

$$AV(74) = - \frac{F_{x4}}{2\bar{F}_{z4}} \quad (B-74)$$

$$AV(75) = - \frac{F_{x5}}{2\bar{F}_{z5}} \quad (B-75)$$

$$AV(76) = - \frac{F_{x6}}{2\bar{F}_{z6}} \quad (B-76)$$

$$AV(77) = \frac{\mu F_{z6}}{2\bar{F}_{z6}} \quad (B-77)$$

$$AV(78) = \frac{\mu \Delta F_{z1}}{2\bar{F}_{z1}} = - \frac{\mu \Delta F_{z2}}{2\bar{F}_{z2}} \quad (B-78)$$

$$AV(79) = f(\alpha_1, \bar{F}'_{z1}) = f(\alpha_2, \bar{F}'_{z2}) \quad (B-79)$$

$$AV(80) = \frac{F_{y2}}{\mu F_{z2}} \quad (B-80)$$

$$AV(81) = f(\alpha_3, \bar{F}'_{z3}) = f(\alpha_4, \bar{F}'_{z4}) \quad (B-81)$$

$$AV(82) = \frac{\mu F_{z3}}{2\bar{F}_{z3}} \cos \alpha_3 \quad (B-82)$$

$$AV(83) = f(\alpha_5, \bar{F}'_{z5}) = f(\alpha_6, \bar{F}'_{z6}) \quad (B-83)$$

$$AV(84) = \frac{\mu F_{z2}}{2\bar{F}_{z2}} \quad (B-84)$$

$$AV(85) = \frac{F_{y4}}{\mu F_{z4}} \quad (B-85)$$

$$AV(86) = \frac{\mu F_{z4}}{2\bar{F}_{z4}} \cos \alpha_4 \quad (B-86)$$

$$AV(87) = \frac{\mu F_{z3}}{2\bar{F}_{z3}} \quad (B-87)$$

$$AV(88) = \frac{\mu F_{z4}}{2\bar{F}_{z4}} \quad (B-88)$$

$$AV(89) = \begin{cases} 1 & \text{if } \frac{\psi}{r_m} \geq 0.01 \\ 100 \frac{\dot{\psi}}{r_m} & \text{if } -0.01 < \frac{\dot{\psi}}{r_m} < 0.01 \\ -1 & \text{if } \frac{\psi}{r_m} \leq -0.01 \end{cases} \quad (B-89)$$

$$AV(90) = \cos \psi \quad (B-90)$$

$$AV(91) = \frac{\alpha_1}{\alpha_m} = \frac{\alpha_2}{\alpha_m} \quad (B-91)$$

$$AV(92) = \frac{|\alpha_3|}{\alpha_m} = \frac{|\alpha_4|}{\alpha_m} \quad (B-92)$$

$$AV(93) = \frac{|\alpha_5|}{\alpha_m} = \frac{|\alpha_6|}{\alpha_m} \quad (B-93)$$

$$AV(94) = \frac{\delta}{\alpha_m} \quad (B-94)$$

$$AV(95) = -0.05 \cdot L_1 - 0.10 \cdot L_2 - 0.20 \cdot L_3 - 0.40 \cdot L_4 - 0.80 \cdot L_5 + 1.60 \cdot L_6 \quad (B-95)$$

$$AV(96) = - \left(\frac{F_{y1} - F_{y2}}{2\bar{F}_{z1}} \right) \delta \quad (B-96)$$

$$AV(97) = \frac{\psi}{\psi_m} \quad (B-97)$$

APPENDIX C
COEFFICIENT-DEVICE EQUATIONS

The physical characteristics of the analog computer model are altered by means of the potentiometers. The physical parameters of the vehicle are used to calculate the coefficients for these devices. The coefficients are defined as PC(I) where I refers to the equation number below and to the identification number of the corresponding potentiometer in the computer circuit of Appendix E.¹

$$PC(1) = \frac{m_2}{2m_1} \quad (C-1)$$

$$PC(2) = \frac{2\bar{F}_{z1}}{m_1 \dot{u}_m} \quad (C-2)$$

$$PC(3) = \frac{2\bar{F}_{z1}}{m_1 \dot{u}_m} \quad (C-3)$$

$$PC(4) = \frac{2\bar{F}_{z5}}{m_1 \dot{u}_m} \quad (C-4)$$

$$PC(5) = \frac{2\bar{F}_{z5}}{m_1 \dot{u}_m} \quad (C-5)$$

¹Symbols used in Appendix C are defined in Table 2-1.

$$PC(6) = \frac{\dot{u}_m}{10\tau\dot{u}_m} \quad (C-6)$$

$$PC(7) = \frac{v_m r_m}{10\tau u_m} \quad (C-7)$$

$$PC(8) = 0.05 \quad (C-8)$$

$$PC(9) = \frac{m_2}{2m_1} \quad (C-9)$$

$$PC(10) = \frac{2\bar{F}_{z1}}{m_1\dot{u}_m} \quad (C-10)$$

$$PC(11) = \frac{2\bar{F}_{z1}}{m_1\dot{u}_m} \quad (C-11)$$

$$PC(12) = \frac{2\bar{F}_{z3}}{m_1\dot{u}_m} \quad (C-12)$$

$$PC(13) = \frac{2\bar{F}_{z5}}{m_1\dot{u}_m} \quad (C-13)$$

$$PC(14) = \frac{2\bar{F}_{z5}}{m_1\dot{u}_m} \quad (C-14)$$

$$PC(15) = \frac{\dot{u}_m}{10\tau v_m} \quad (C-15)$$

$$PC(16) = \frac{u_m r_m}{10\tau v_m} \quad (C-16)$$

$$PC(17) = 0.5 \quad (C-17)$$

$$PC(18) = \frac{hm_2\dot{u}_m}{2I_1\dot{r}_m} \quad (C-18)$$

$$PC(19) = \frac{2h\bar{F}_{z1}}{I_1 \dot{r}_m} \quad (C-19)$$

$$PC(20) = \frac{b\bar{F}_{z3}}{I_1 \dot{r}_m} \quad (C-20)$$

$$PC(21) = \frac{2h\bar{F}_{z5}}{I_1 \dot{r}_m} \quad (C-21)$$

$$PC(22) = \frac{2h\bar{F}_{z5}}{I_1 \dot{r}_m} \quad (C-22)$$

$$PC(23) = \frac{2a\bar{F}_{z1}}{I_1 \dot{r}_m} \quad (C-23)$$

$$PC(24) = \frac{(c + d)\bar{F}_{z5}}{I_2 \dot{r}_m} \quad (C-24)$$

$$PC(25) = 0.025 \quad (C-25)$$

$$PC(26) = 0.02 \quad (C-26)$$

$$PC(27) = \frac{\dot{r}_m}{20\tau r_m} \quad (C-27)$$

$$PC(28) = 0.10 \quad (C-28)$$

$$PC(29) = \frac{r_m}{10\tau\psi_m} \quad (C-29)$$

$$PC(30) = \frac{cm_2 \dot{u}_m}{2I_2 \dot{r}_m} \quad (C-30)$$

$$PC(31) = \frac{(c + d)\bar{F}_{z5}}{I_2 \dot{r}_m} \quad (C-31)$$

$$PC(32) = 1.5 \quad (C-32)$$

$$PC(33) = 1.5 \quad (C-33)$$

$$PC(34) = \frac{\dot{r}_m}{20\tau r_m} \quad (C-34)$$

$$PC(35) = 0.15 \quad (C-35)$$

$$PC(36) = \frac{r_m}{10\tau\psi_m} \quad (C-36)$$

$$PC(37) = 0.30 \quad (C-37)$$

$$PC(38) = \frac{v_m}{u_m \alpha_m} \quad (C-38)$$

$$PC(39) = \frac{ar_m}{u_m \alpha_m} \quad (C-39)$$

$$PC(40) = 1.5 \quad (C-40)$$

$$PC(41) = 1.5 \quad (C-41)$$

$$PC(42) = \frac{\alpha_m}{\pi} \quad (C-42)$$

$$PC(43) = \frac{v_m}{u_m \alpha_m} \quad (C-43)$$

$$FC(44) = \frac{br_m}{u_m \alpha_m} \quad (C-44)$$

$$PC(45) = 1.5 \quad (C-45)$$

$$PC(46) = 1.5 \quad (C-46)$$

$$PC(47) = \frac{(c + d)r_m}{u_m} \quad (C-47)$$

$$PC(48) = \frac{v_m}{u_m} \quad (C-48)$$

$$PC(49) = \frac{hr_m}{u_m} \quad (C-49)$$

$$PC(50) = \frac{\alpha_m}{\pi} \quad (C-50)$$

$$PC(51) = \frac{cr_m^2}{\dot{u}_m} \quad (C-51)$$

$$PC(52) = \frac{c\dot{r}_m}{2\dot{u}_m} \quad (C-52)$$

$$PC(53) = \frac{cr_m^2}{\dot{u}_m} \quad (C-53)$$

$$PC(54) = \frac{c\dot{r}_m}{2\dot{u}_m} \quad (C-54)$$

$$PC(55) = \frac{hr_m^2}{\dot{u}_m} \quad (C-55)$$

$$PC(56) = 1.01 \quad (C-56)$$

$$PC(57) = \frac{h\dot{r}_m}{\dot{u}_m} \quad (C-57)$$

$$PC(58) = \frac{m_2}{2m_1} \quad (C-58)$$

$$PC(59) = \frac{hm_2\dot{u}_m}{2I_1\dot{r}_m} \quad (C-59)$$

$$PC(60) = \frac{b\bar{F}_{z3}}{I_1\dot{r}_m} \quad (C-60)$$

$$PC(61) = \alpha_m \quad (C-61)$$

$$PC(62) = \frac{\mu}{2} \quad (C-62)$$

$$PC(63) = \frac{\mu\gamma_2 z_1 m_1 \dot{u}_m}{4t_2 \bar{F}_{z3}} \quad (C-63)$$

$$PC(64) = \frac{\mu\gamma_2 z_0 \bar{F}_{z5}}{2t_2 \bar{F}_{z3}} \quad (C-64)$$

$$PC(65) = \frac{\mu\gamma_2 z_0 \bar{F}_{z5}}{2t_2 \bar{F}_{z3}} \quad (C-65)$$

$$PC(66) = \frac{\mu\gamma_2 z_0 m_2 \dot{u}_m}{4t_2 \bar{F}_{z3}} \quad (C-66)$$

$$PC(67) = \frac{\mu(z_2 - z_0)m_2 \dot{u}_m}{4t_3 \bar{F}_{z5}} \quad (C-67)$$

$$PC(68) = \frac{\mu z_0}{2t_3} \quad (C-68)$$

$$PC(69) = \frac{\mu z_1 m_1 \dot{u}_m}{4(a + b)\bar{F}_{z1}} \quad (C-69)$$

$$PC(70) = \frac{\mu z_0 m_2 \dot{u}_m}{4(a + b)\bar{F}_{z1}} \quad (C-70)$$

$$PC(71) = \frac{\mu z_0 \bar{F}_{z5}}{2(a+b)\bar{F}_{z1}} \quad (C-71)$$

$$PC(72) = \frac{\mu z_0 \bar{F}_{z5}}{2(a+b)\bar{F}_{z1}} \quad (C-72)$$

$$PC(73) = \frac{\mu z_0 (b-h)\bar{F}_{z5}}{2(a+b)(c+d)\bar{F}_{z1}} \quad (C-73)$$

$$PC(74) = \frac{\mu(z_2 - z_0)(b-h)m_2 \dot{u}_m}{4(a+b)(c+d)\bar{F}_{z1}} \quad (C-74)$$

$$PC(75) = \frac{\mu z_1 m_1 \dot{u}_m}{4(a+b)\bar{F}_{z3}} \quad (C-75)$$

$$PC(76) = \frac{\mu z_0 m_2 \dot{u}_m}{4(a+b)\bar{F}_{z3}} \quad (C-76)$$

$$PC(77) = \frac{\mu z_0 \bar{F}_{z5}}{2(a+b)\bar{F}_{z3}} \quad (C-77)$$

$$PC(78) = \frac{\mu z_0 \bar{F}_{z5}}{2(a+b)\bar{F}_{z3}} \quad (C-78)$$

$$PC(79) = \frac{\mu z_0 (a+h)\bar{F}_{z5}}{2(a+b)(c+d)\bar{F}_{z3}} \quad (C-79)$$

$$PC(80) = \frac{\mu(z_2 - z_0)(a+h)m_2 \dot{u}_m}{4(a+b)(c+d)\bar{F}_{z3}} \quad (C-80)$$

$$PC(81) = \frac{\mu(z_2 - z_0)m_2 \dot{u}_m}{4(c+d)\bar{F}_{z5}} \quad (C-81)$$

$$PC(82) = \frac{\mu z_0}{2(c+d)} \quad (C-82)$$

$$PC(83) = \frac{\mu}{2} \quad (C-83)$$

$$PC(84) = \frac{2\bar{F}_{z3}}{m_1 \dot{u}_m} \quad (C-84)$$

$$PC(85) = \frac{2\bar{F}_{z3}}{m_1 \dot{u}_m} \quad (C-85)$$

$$PC(86) = 1.5 \quad (C-86)$$

$$PC(87) = 1.5 \quad (C-87)$$

$$PC(88) = \frac{\alpha_m}{\pi} \quad (C-88)$$

$$PC(89) = \frac{2t_3 \bar{F}_{z5}}{I_2 \dot{r}_m} \quad (C-89)$$

$$PC(90) = \frac{2t_3 \bar{F}_{z5}}{I_2 \dot{r}_m} \quad (C-90)$$

$$PC(91) = \frac{m_2}{2m_1} \quad (C-91)$$

$$PC(92) = \frac{2t_1 \bar{F}_{z1}}{I_1 \dot{r}_m} \quad (C-92)$$

$$PC(93) = \frac{2t_1 \bar{F}_{z1}}{I_1 \dot{r}_m} \quad (C-93)$$

$$PC(94) = \frac{\mu \gamma_1 z_1 m_1 \dot{u}_m}{2t_1 \bar{F}_{z1}} \quad (C-94)$$

$$PC(95) = \frac{2t_2 \bar{F}_{z3}}{I_1 \dot{r}_m} \quad (C-95)$$

$$PC(96) = \alpha_m \quad (C-96)$$

$$PC(97) = \frac{2t_2 \bar{F}_{z3}}{I_1 \dot{r}_m} \quad (C-97)$$

$$PC(98) = \frac{r_m}{10\tau\pi} \quad (C-98)$$

$$PC(99) = 1.01 \quad (C-99)$$

$$PC(100) = \frac{\mu}{2} \quad (C-100)$$

$$PC(101) = 1.5 \quad (C-101)$$

$$PC(102) = 1.5 \quad (C-102)$$

$$PC(103) = 1.01 \quad (C-103)$$

$$PC(104) = 1.01 \quad (C-104)$$

$$PC(105) = \frac{\mu\gamma_1 z_0 \bar{F}_{z5}}{2t_1 \bar{F}_{z1}} \quad (C-105)$$

$$PC(106) = \frac{\mu}{2} \quad (C-106)$$

$$PC(107) = \frac{\mu\gamma_1 z_0 \bar{F}_{z5}}{2t_1 \bar{F}_{z1}} \quad (C-107)$$

$$PC(108) = \frac{\mu}{2} \quad (C-108)$$

$$PC(109) = \frac{\mu\gamma_1 z_0^m \dot{u}_m}{4\bar{F}_{z1}} \quad (C-109)$$

$$PC(110) = \frac{\mu}{2} \quad (C-110)$$

$$PC(111) = 1.01 \quad (C-111)$$

$$PC(112) = 1.01 \quad (C-112)$$

$$PC(113) = \frac{cm_2 \dot{u}_m}{2I_2 \dot{r}_m} \quad (C-113)$$

$$PC(114) = \frac{\phi}{I_1 \dot{r}_m} \quad (C-114)$$

$$PC(115) = \frac{\phi}{I_2 \dot{r}_m} \quad (C-115)$$

$$PC(116) = \frac{\dot{r}_m}{20\tau r_m} \quad (C-116)$$

$$PC(117) = \frac{\dot{r}_m}{20\tau r_m} \quad (C-117)$$

$$PC(118) = \frac{(\nu_1 + \sigma_1 \bar{F}'_{z1})\alpha_m}{\mu \bar{\alpha}_m} \quad (C-118)$$

$$PC(119) = \frac{(\nu_3 + \sigma_3 \bar{F}'_{z3})\alpha_m}{\mu \bar{\alpha}_m} \quad (C-119)$$

$$PC(120) = \frac{(\nu_5 + \sigma_5 \bar{F}'_{z5})\alpha_m}{\mu \bar{\alpha}_m} \quad (C-120)$$

$$PC(121) = 1.5 \quad (C-121)$$

$$PC(122) = 1.5 \quad (C-122)$$

$$PC(123) = 0.1 \quad (C-123)$$

$$PC(124) = 0.2 \quad (C-124)$$

$$PC(125) = 0.4 \quad (C-125)$$

$$PC(126) = 0.8 \quad (C-126)$$

$$PC(127) = \frac{2t_1 \bar{F}_{z1}}{I_1 \dot{r}_m} \quad (C-127)$$

$$PC(128) = \frac{\psi_m}{\pi} \quad (C-128)$$

$$PC(129) = 1.6 \quad (C-129)$$

APPENDIX D

ANALOG COMPUTER CIRCUIT EQUATIONS

Using the normalized variables defined in Appendix B and the coefficients defined in Appendix C, the problem equations formulated in Chapter 2 are rewritten as the analog computer circuit equations. Symbols used in Appendix D are defined in Table 2-1.

$$\begin{aligned} AV(1) = & - (PC(1) + PC(58)) \cdot AV(52) + PC(2) \cdot AV(66) \\ & - PC(3) \cdot AV(11) - PC(4) \cdot AV(15) + PC(5) \cdot AV(68) \\ & - PC(84) \cdot AV(42) - PC(85) \cdot AV(74) \end{aligned} \quad (D-1)$$

$$AV(2) = \frac{u_0}{u_m} + 10 \int_0^t (PC(6) \cdot AV(1) + PC(7) \cdot AV(44)) dt \quad (D-2)$$

$$\begin{aligned} AV(3) = & - (PC(9) + PC(91)) \cdot AV(49) - PC(10) \cdot AV(63) \\ & + PC(11) \cdot AV(12) - PC(12) \cdot AV(64) \\ & - PC(13) \cdot AV(69) - PC(14) \cdot AV(14) \end{aligned} \quad (D-3)$$

$$\begin{aligned} AV(4) = & \frac{v_0}{v_m} + 10 \int_0^t (PC(10) \cdot AV(3) \\ & + PC(16) \cdot AV(43)) dt \end{aligned} \quad (D-4)$$

$$\begin{aligned}
AV(5) = & (PC(18) + PC(59)) \cdot AV(49) - PC(19) \cdot AV(63) \\
& + (PC(20) + PC(60)) \cdot AV(64) + PC(21) \cdot AV(14) \\
& + PC(22) \cdot AV(69) + PC(63) \cdot AV(12) \\
& - PC(92) \cdot AV(40) + PC(93) \cdot AV(41) - PC(95) \cdot AV(42) \\
& + PC(97) \cdot AV(74) + PC(127) \cdot AV(96) \\
& + PC(114) \cdot AV(89)
\end{aligned} \tag{D-5}$$

$$AV(6) = \frac{r_0}{r_m} + 10 \int_0^t (PC(27) + PC(116)) \cdot AV(5) dt \tag{D-6}$$

$$\begin{aligned}
AV(7) = & (PC(30) + PC(113)) \cdot AV(54) + (PC(31) \\
& + PC(24)) \cdot AV(65) - PC(89) \cdot AV(75) \\
& + PC(90) \cdot AV(76) - PC(115) \cdot AV(89)
\end{aligned} \tag{D-7}$$

$$AV(8) = \frac{r_0 + \dot{\psi}_0}{r_m} + 10 \int_0^t (PC(34) + PC(117)) \cdot AV(7) dt \tag{D-8}$$

$$AV(9) = PC(128) \cdot AV(97) \tag{D-9}$$

$$AV(10) = -1 + 10 \int_0^t PC(25) dt \tag{D-10}$$

$$AV(11) = AV(40) + AV(41) \tag{D-11}$$

$$AV(12) = -AV(11) \cdot AV(17) \tag{D-12}$$

$$AV(13) = AV(75) + AV(76) \quad (D-13)$$

$$AV(14) = AV(13) \cdot AV(19) \quad (D-14)$$

$$AV(15) = AV(13) \cdot AV(20) \quad (D-15)$$

$$AV(16) = AV(94) \quad (D-16)$$

$$AV(17) = PC(61) \cdot AV(16) \quad (D-17)$$

$$AV(18) = - AV(37) \cdot AV(67) + AV(80) + AV(84) \quad (D-18)$$

$$AV(19) = S[AV(9)] \quad (D-19)$$

$$AV(20) = C[AV(9)] \quad (D-20)$$

$$AV(21) = AV(25) \cdot AV(67) \quad (D-21)$$

$$AV(22) = \frac{PC(38) \cdot AV(4) + PC(39) \cdot AV(6)}{AV(2)} \quad (D-22)$$

$$AV(23) = 10 \cdot PC(118) \cdot AV(91) \quad (D-23)$$

$$AV(24) = - S[AV(36)] \quad (D-24)$$

$$AV(25) = C[AV(36)] \quad (D-25)$$

$$AV(26) = \frac{PC(43) \cdot AV(4) - PC(44) \cdot AV(6)}{AV(2)} \quad (D-26)$$

$$AV(27) = PC(88) \cdot AV(26) \quad (D-27)$$

$$AV(28) = -S[AV(27)] \quad (D-28)$$

$$AV(29) = C[AV(27)] \quad (D-29)$$

$$AV(30) = PC(48) \cdot AV(4) - PC(49) \cdot AV(6) \quad (D-30)$$

$$AV(31) = \frac{-AV(2) \cdot AV(19) - PC(47) \cdot AV(8) + AV(20) \cdot AV(30)}{AV(32)} \quad (D-31)$$

$$AV(32) = PC(96) \cdot AV(70) \quad (D-32)$$

$$AV(33) = PC(50) \cdot AV(31) \quad (D-33)$$

$$AV(34) = -S[AV(33)] \quad (D-34)$$

$$AV(35) = C[AV(33)] \quad (D-35)$$

$$AV(36) = PC(42) \cdot AV(91) \quad (D-36)$$

$$AV(37) = \begin{cases} AV(79) & \text{if } L_1 = 1 \\ AV(24) & \text{if } L_1 = 0 \end{cases} \quad (D-37)$$

$$AV(38) = \begin{cases} AV(81) & \text{if } L_3 = 1 \\ AV(28) & \text{if } L_3 = 0 \end{cases} \quad (D-38)$$

$$AV(39) = \begin{cases} AV(83) & \text{if } L_5 = 1 \\ AV(34) & \text{if } L_5 = 0 \end{cases} \quad (D-39)$$

$$AV(40) = - \frac{F^*_{x1}}{2\bar{F}_{z1}} \quad (D-40)$$

$$AV(41) = - \frac{F^*_{x2}}{2\bar{F}_{z2}} \quad (D-41)$$

$$AV(42) = - \frac{F^*_{x3}}{2\bar{F}_{z3}} \quad (D-42)$$

$$AV(43) = - AV(2) \cdot AV(6) \quad (D-43)$$

$$AV(44) = AV(4) \cdot AV(6) \quad (D-44)$$

$$AV(45) = AV(8) \cdot AV(8) \quad (D-45)$$

$$AV(46) = AV(6) \cdot AV(6) \quad (D-46)$$

$$AV(47) = AV(19) \cdot AV(45) \quad (D-47)$$

$$AV(48) = 2 \cdot AV(7) \cdot AV(20) \quad (D-48)$$

$$AV(49) = AV(3) - PC(57) \cdot AV(5) + PC(51) \cdot AV(47) \quad (D-49)$$

$$AV(50) = AV(20) \cdot AV(45) \quad (D-50)$$

$$AV(51) = 2 \cdot AV(7) \cdot AV(19) \quad (D-51)$$

$$AV(52) = AV(1) + PC(53) \cdot AV(50) + PC(54) \cdot AV(51) \\ + PC(55) \cdot AV(46) \quad (D-52)$$

$$AV(53) = AV(35) \cdot AV(77) \quad (D-53)$$

$$AV(54) = AV(20) \cdot AV(49) - AV(19) \cdot AV(52) \quad (D-54)$$

$$AV(55) = PC(110) + AV(59) + AV(62) \quad (D-55)$$

$$AV(56) = AV(20) \cdot AV(52) + AV(19) \cdot AV(49) \quad (D-56)$$

$$AV(57) = AV(25) \cdot AV(84) \quad (D-57)$$

$$AV(58) = PC(63) \cdot AV(3) + PC(64) \cdot AV(14) \\ + PC(65) \cdot AV(69) + PC(66) \cdot AV(49) \quad (D-58)$$

$$AV(59) = PC(67) \cdot AV(54) - PC(68) \cdot AV(65) \quad (D-59)$$

$$AV(60) = - PC(69) \cdot AV(1) - PC(70) \cdot AV(52) \\ + PC(71) \cdot AV(68) - PC(72) \cdot AV(15) \\ + PC(73) \cdot AV(13) - PC(74) \cdot AV(56) \quad (D-60)$$

$$\begin{aligned} AV(61) = & PC(75) \cdot AV(1) + PC(76) \cdot AV(52) - PC(77) \cdot AV(68) \\ & + PC(78) \cdot AV(15) + PC(79) \cdot AV(13) \\ & - PC(80) \cdot AV(56) \end{aligned} \quad (D-61)$$

$$AV(62) = PC(81) \cdot AV(56) - PC(82) \cdot AV(13) \quad (D-62)$$

$$AV(63) = - AV(37) \cdot AV(67) - AV(80) \cdot AV(84) \quad (D-63)$$

$$AV(64) = - AV(38) \cdot AV(87) - AV(85) \cdot AV(88) \quad (D-64)$$

$$AV(65) = - AV(39) \cdot AV(55) - AV(72) \cdot AV(77) \quad (D-65)$$

$$AV(66) = AV(17) \cdot AV(63) \quad (D-66)$$

$$AV(67) = PC(62) + AV(60) + AV(78) \quad (D-67)$$

$$AV(68) = AV(19) \cdot AV(65) \quad (D-68)$$

$$AV(69) = AV(20) \cdot AV(65) \quad (D-69)$$

$$AV(70) = AV(2) \cdot AV(20) + AV(19) \cdot AV(30) \quad (D-70)$$

$$AV(71) = AV(35) \cdot AV(55) \quad (D-71)$$

$$AV(72) = \begin{cases} AV(83) & \text{if } L_6 = 1 \\ AV(34) & \text{if } L_6 = 0 \end{cases} \quad (D-72)$$

$$AV(73) = \frac{\theta_0}{\pi} + 10 \int_0^t PC(98) \cdot AV(6) dt \quad (D-73)$$

$$AV(74) = - \frac{F_{x4}^*}{2\bar{F}_{z4}} \quad (D-74)$$

$$AV(75) = - \frac{F_{x5}^*}{2\bar{F}_{z5}} \quad (D-75)$$

$$AV(76) = - \frac{F_{x6}^*}{2\bar{F}_{z6}} \quad (D-76)$$

$$AV(77) = PC(83) - AV(59) + AV(62) \quad (D-77)$$

$$AV(78) = PC(94) \cdot AV(3) + PC(105) \cdot AV(14) \\ + PC(107) \cdot AV(69) + PC(109) \cdot AV(49) \quad (D-78)$$

$$AV(79) = K[AV(23)] \quad (D-79)$$

$$AV(80) = \begin{cases} AV(79) & \text{if } L_2 = 1 \\ AV(24) & \text{if } L_2 = 0 \end{cases} \quad (D-80)$$

$$AV(81) = K[AV(92)] \quad (D-81)$$

$$AV(82) = AV(29) \cdot AV(87) \quad (D-82)$$

$$AV(83) = K[AV(93)] \quad (D-83)$$

$$AV(84) = PC(100) + AV(60) - AV(78) \quad (D-84)$$

$$AV(85) = \begin{cases} AV(81) & \text{if } L_4 = 1 \\ AV(28) & \text{if } L_4 = 0 \end{cases} \quad (D-85)$$

$$AV(86) = AV(29) \cdot AV(88) \quad (D-86)$$

$$AV(87) = PC(106) + AV(58) + AV(61) \quad (D-87)$$

$$AV(88) = PC(108) - AV(58) + AV(61) \quad (D-88)$$

$$AV(89) = 100(AV(8) - AV(6)) \quad (D-89)$$

$$AV(90) = AV(20) \quad (D-90)$$

$$AV(91) = AV(22) - AV(16) \quad (D-91)$$

$$AV(92) = 10 \cdot PC(119) \cdot AV(26) \quad (D-92)$$

$$AV(93) = 10 \cdot PC(120) \cdot AV(31) \quad (D-93)$$

$$AV(94) = \frac{\delta}{\alpha_m} \quad (D-94)$$

$$AV(95) = -PC(8) \cdot L_1 - PC(123) \cdot L_2 - PC(124) \cdot L_3 \\ - PC(125) \cdot L_4 - PC(126) \cdot L_5 + 1.6 \cdot L_6 \quad (D-95)$$

$$AV(96) = AV(17) \cdot AV(18) \quad (D-96)$$

$$AV(97) = \frac{\psi_0}{\psi_m} + 10 \int_0^t (PC(36) \cdot AV(8) - PC(29) \cdot AV(6)) dt \quad (D-97)$$

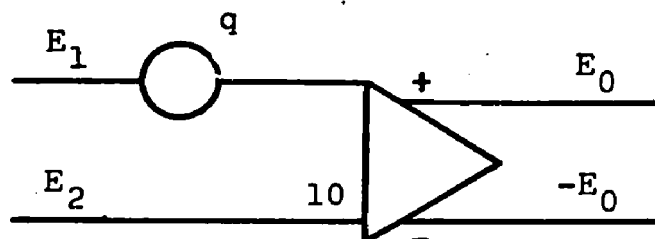
APPENDIX E

ANALOG COMPUTER CIRCUIT

The circuits which interconnect the operational devices of the analog computer are given in Figures E.2 through E.7. The graphic symbols used are those advocated by Reference [36] with a few exceptions which are defined in Figure E.1. Other symbols used are defined in Tables E-1 and E-2. The truth tables for analog mode control and individual integrator mode control are given in Tables E-3 and E-4, respectively.

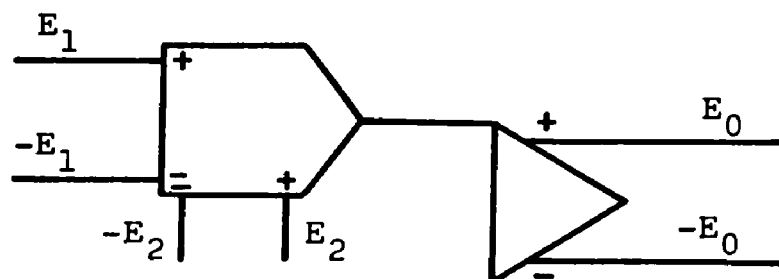
The graphic symbols and operating characteristics of the bipolar amplifier, multiplier, hard limiter, reference voltage source, and flip-flop are given in Figure E.1. The bipolar amplifier, used in (a) as a summer, has an inverting amplifier connected to its output so that both polarities of the problem variable are available. Inputs which enter at the top or bottom of the amplifier symbol, such as the one from the current potentiometer,¹ are current or summing junction inputs. All amplifier gains other than unity are shown at the point where the input enters. The multiplier, shown in (b), requires both polarities of the input

¹The current potentiometers have an input resistor in series with the arm.



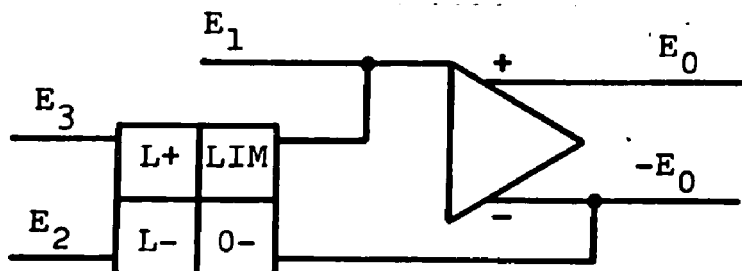
$$E_0 = q \cdot E_1 + 10 \cdot E_2$$

(a) - Bipolar Amplifier



$$E_0 = E_1 \cdot E_2 / E_{ref}$$

(b) - Multiplier

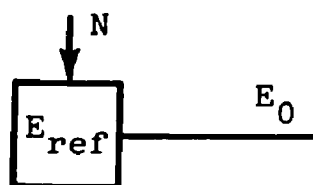


$$E_0 = E_3 \quad (E_1 > E_3)$$

$$E_0 = E_1 \quad (E_2 < E_1 < E_3)$$

$$E_0 = E_2 \quad (E_1 < E_2)$$

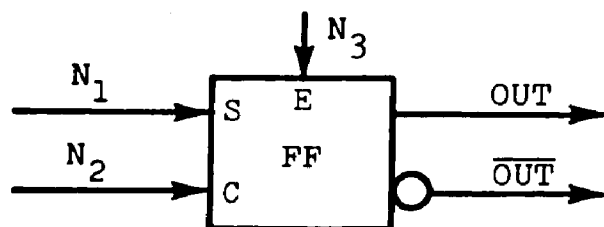
(c) - Hard Limiter



$$E_0 = E_{ref} \quad (N = 1)$$

$$E_0 = 0 \quad (N = 0)$$

(d) - Reference Voltage



(e) - Flip-Flop

Fig. E.1 - Graphic symbols used in computer circuits

TABLE E-1

SYMBOLS USED IN APPENDIX E

$AV(I)$	=	normalized problem variable for amplifier I
I	=	identification number of the amplifier
E_j	=	analog computer voltage
E_{ref}	=	reference voltage of the computer
u	=	forward velocity of the tractor
v	=	lateral velocity of the center of mass of the tractor
r	=	yaw velocity of the tractor
ψ	=	trailer angle
t	=	time
θ	=	heading angle of the tractor
δ	=	front wheel steer angle
q	=	potentiometer coefficient
S	=	indicates sine function
C	=	indicates cosine function
K	=	indicates normalized lateral force function
$\times$	=	multiplier symbol
$\times^2$	=	dual $\times^2$ diode function generator symbol

TABLE E-2
LOGIC SYMBOLS

W	=	logic one for static check or for a steady-state turn
L ₁	=	logic one if wheel 1 is rolling and logic zero if it is locked ^a
P	=	a logic one pulse every 10 ms
SYS OP	=	logic one when the analog mode "operate" is selected by the control system
SYS HLD	=	logic one when the analog mode "hold" is selected by the control system
DGC	=	logic one when digital control line is on
DGS	=	logic one sets digital sense line
C OP	=	analog mode control of the computer
C HLD	=	analog mode control of the computer
Ø	=	integrator mode control
H	=	integrator mode control
R _j	=	logic one to put analog mode in hold
OP	=	logic one if the analog mode is operate
HLD	=	logic one if the analog mode is hold
S	=	set input of flip-flop
C	=	clear input of flip-flop
E	=	enable input of flip-flop

^aSubscripts 1 through 4 correspond to the left-front, right-front, left-rear and right-rear wheels of the tractor. Subscripts 5 and 6 correspond to the left and right trailer wheels.

TABLE E-3
ANALOG MODE CONTROL

MODE	C OP	C HLD
IC	0 ^a	0
HOLD	0	1 ^b
OPERATE	1	0
HOLD	1	1

TABLE E-4
INDIVIDUAL INTEGRATOR MODE CONTROL

MODE	Ø	H
IC	0	0
HOLD	0	1
OPERATE	1	0
HOLD	1	1

^aIndicates logic zero.

^bIndicates logic one.

variables. When the multiplier is patched to the summing junction of a bipolar amplifier the product of the positive (+) multiplier inputs is available at the positive (+) output of the amplifier. The hard limiter, used in (c) with a summing amplifier, unloads the feedback path so that it appears to have zero impedance beyond the limiting value. A reference voltage source controlled by a logic signal, N, is shown in (d). The operation of the flip-flop, (e) in Figure E.1, is described by Table E-5. The flip-flop changes state according to the inputs at one μ sec intervals when the enable signal, E, is logic one.

TABLE E-5
FLIP-FLOP TRUTH TABLE

S	C	Before Enabled Clock Pulse		After Enabled Clock Pulse	
		OUT	$\overline{\text{OUT}}$	OUT	$\overline{\text{OUT}}$
0	0	1	0	1	0
0	0	0	1	0	1
1	0	* ^a	*	1	0
0	1	*	*	0	1
1	1	0	1	1	0
1	1	1	0	0	1

^aState of flip-flop before enabled clock pulse does not effect state after enabled clock pulse.

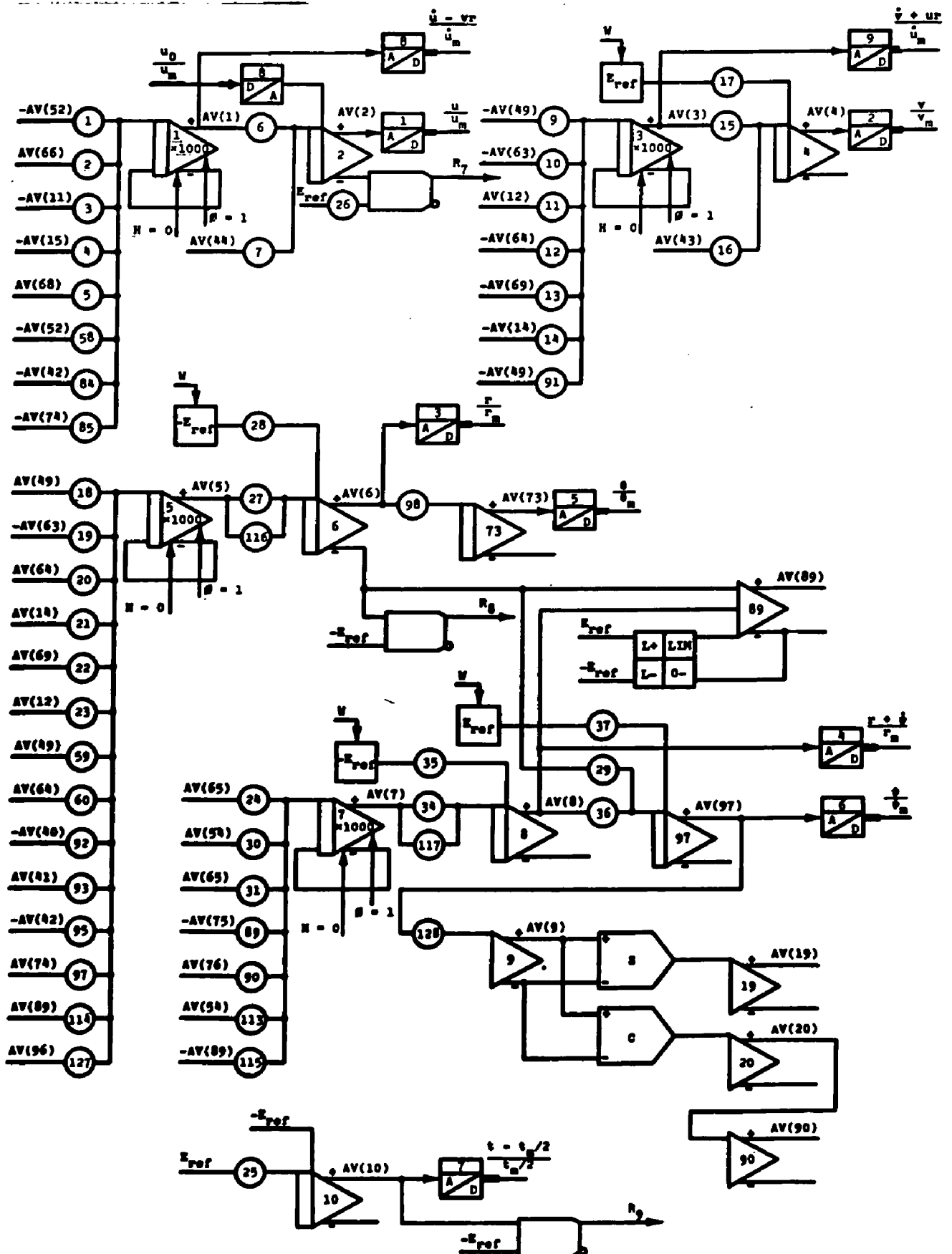


Fig. E.3 - Response variables

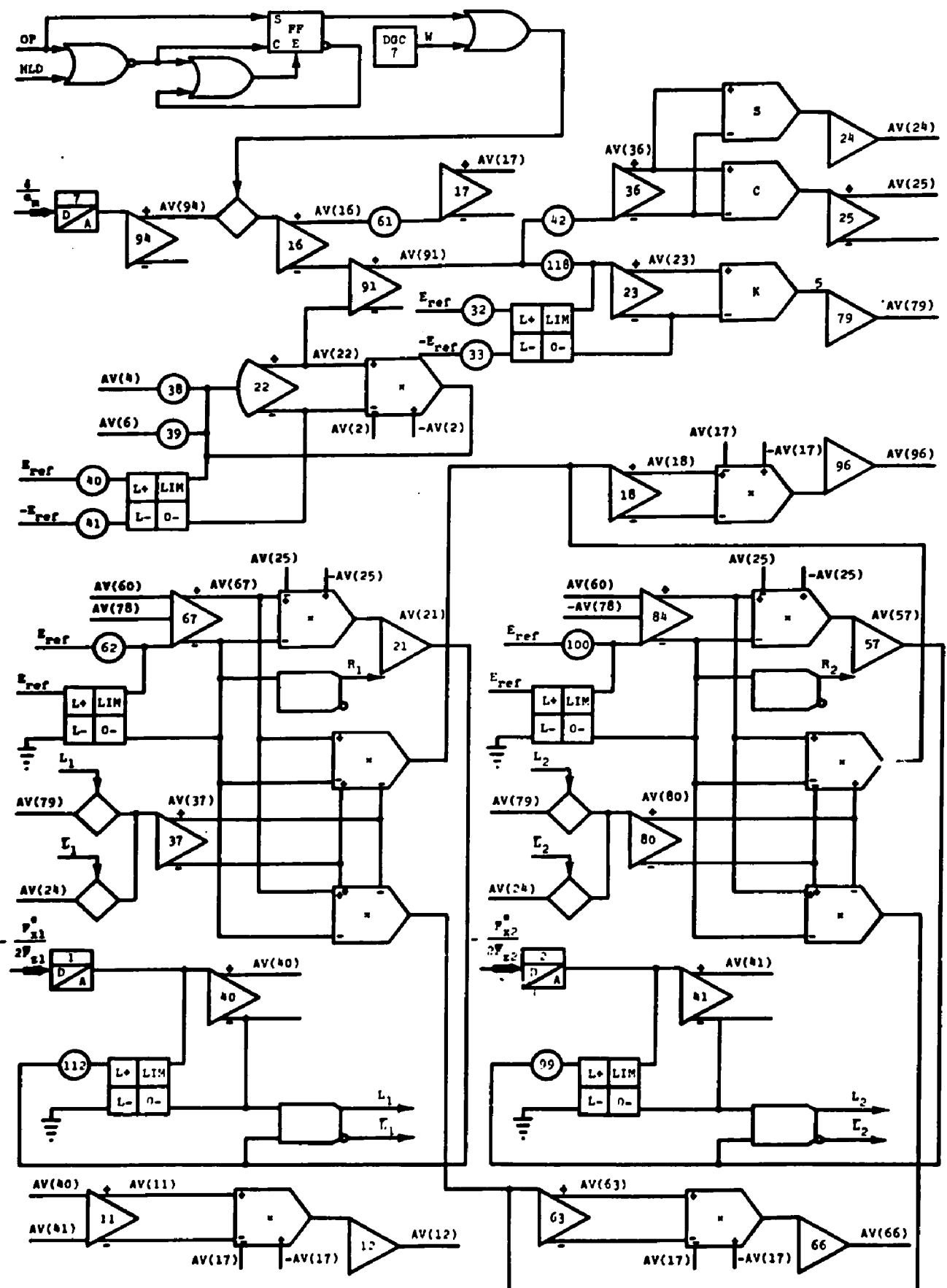


Fig. E.4 - Tire forces on the front wheels of the tractor

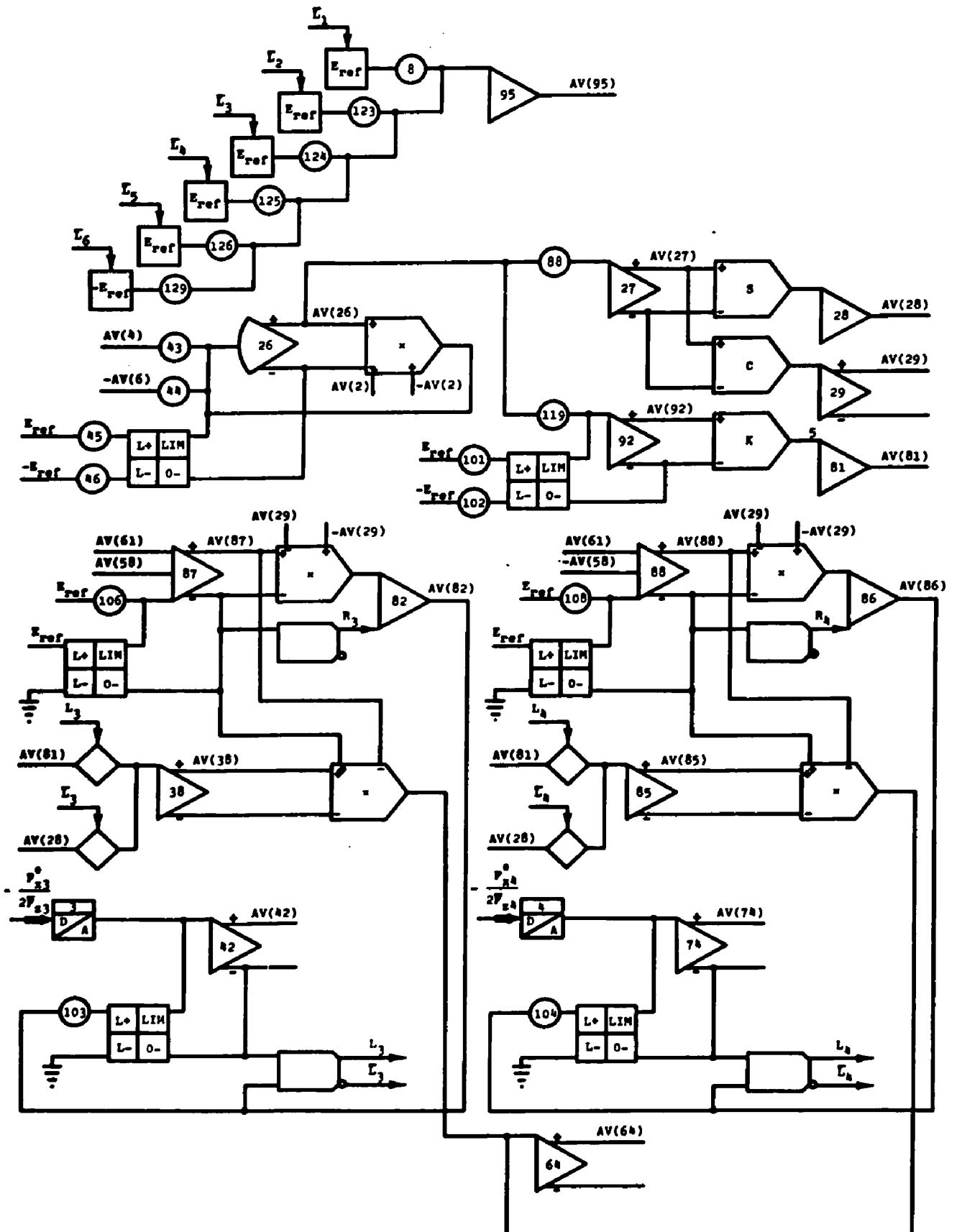


Fig. E.5 - Tire forces on the rear wheels of the tractor

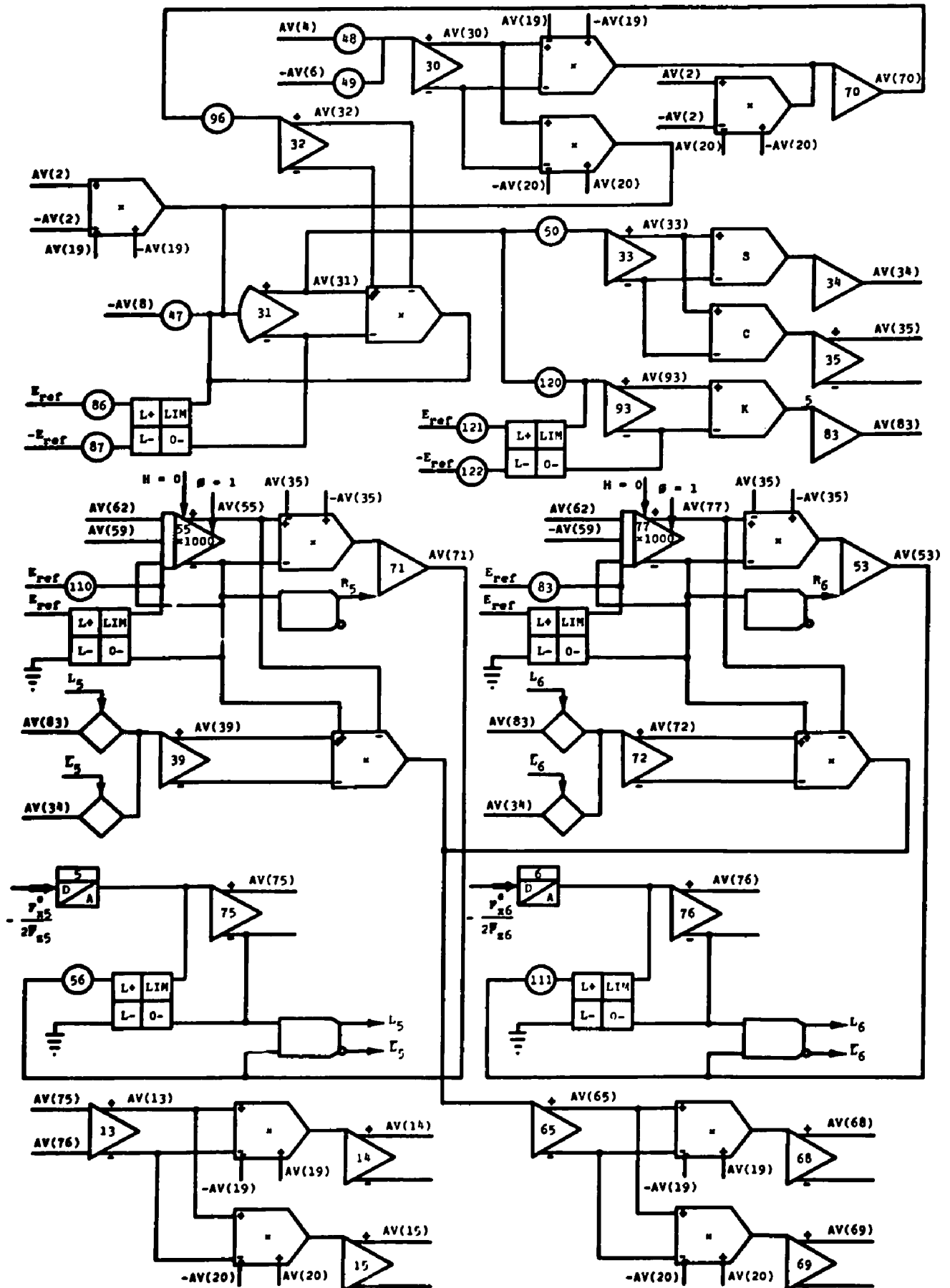


Fig. E.6 - Tire forces on the semitrailer wheels

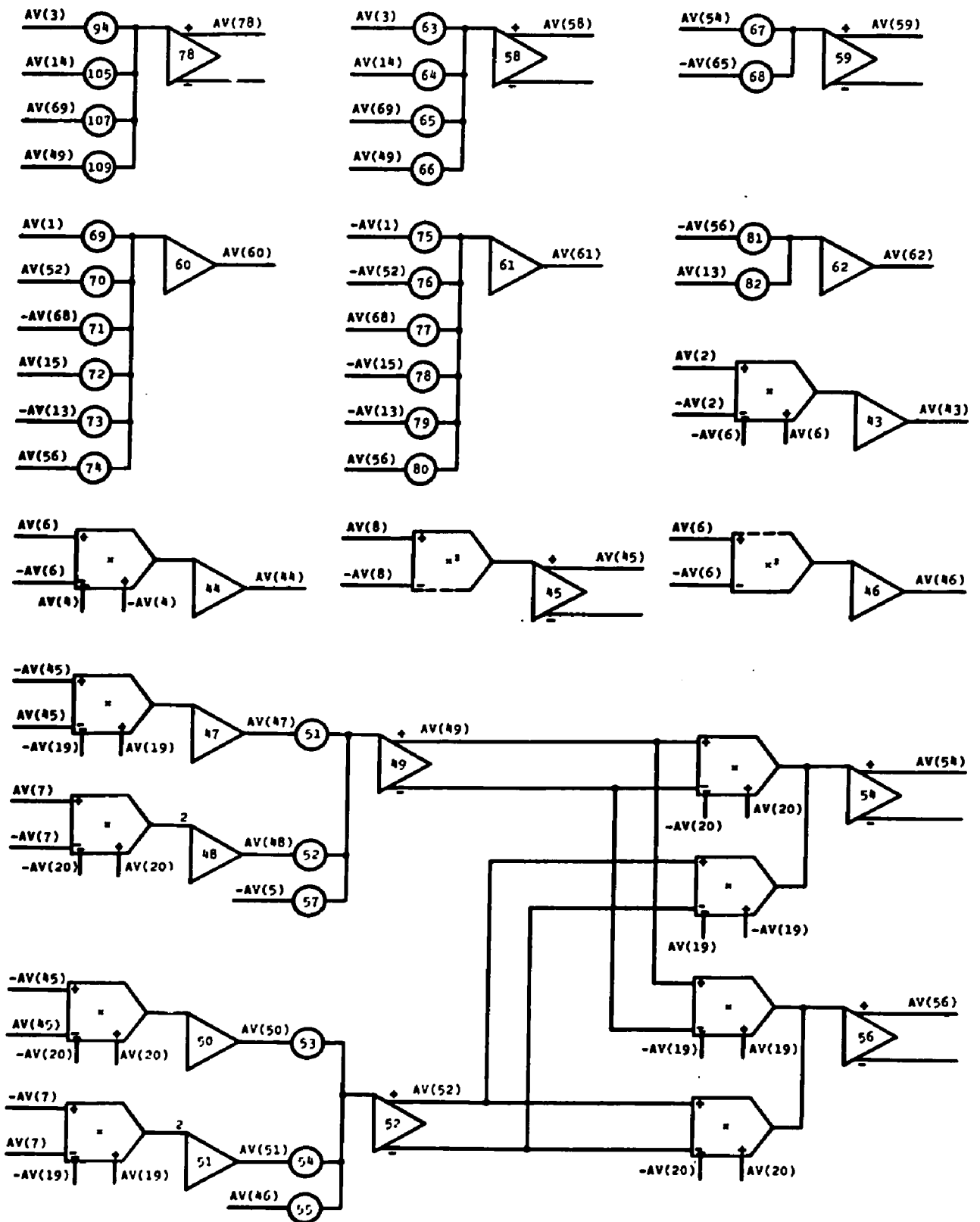


Fig. E.7 - Load transfer and semitrailer inertia forces

APPENDIX F

TRACTOR-TRAILER SIMULATION SYSTEM

This Appendix describes the three executive programs and nineteen subroutines which constitute the Tractor-Trailer Simulation System. The three executive programs are used to setup and checkout the simulation, operate the simulation to obtain solutions, and plot the results of selected solutions. A common area is defined by the programs so that all address information, parameter data, internal program switches, and solution results are accessible by each program and subroutine. The programs are written in Fortran and they use the standard subprograms available with the IBM 1130 Disk Monitor Operating System and the Hybrid Communication Routines which interface the programs with the hybrid hardware.

The executive program PML30 plots the results of one, two, or three solutions in a single figure. The console keyboard is used to set the control variable which selects the function that is performed. The input data required for each figure includes the identification numbers of the solutions to be plotted and the amplitude scaling information for each axis. Table F-1 lists the functions performed and the subroutines called by each value of the control variable I.

TABLE F-1

PML30 EXECUTIVE PLOTTING PROGRAM

I	Function Performed	Subroutines Called
0	Initialization	PML31
1	Plot one solution	PML32, PML33, PML34
2	Plot two solutions	PML32, PML33, PML34
3	Plot three solutions	PML32, PML33, PML34
4	Not used	-
5	Copy to scratch tape	PML35
6	Not used	-
7	Not used	-
8	Not used	-
9	Exit to monitor system	EXIT

The hybrid executive program PML40 is used to initialize, setup, and checkout the simulation. The program produces a listing of the static check, circuit check, and coefficient device settings which is used in correcting analog hardware problems. The control variable which selects the function to be executed is entered from the console keyboard. Table F-2 lists the functions performed and the subroutines called by each value of the control variable I.

TABLE F-2

PML40 HYBRID SETUP PROGRAM

I	Function Performed	Subroutines Called
0	Initialization	PML59
1	Calculate and set coefficients	PML53, PML54
2	Static and circuit check	PML58, PML65, PML66
3	Not used	-
4	Calculate and reset coefficients	PML53, PML54
5	Link to PML50	LINK
6	Write parameters in disk file	-
7	Change address and reset coefficients	PML59, PML53, PML54
8	Change parameters	PML52
9	Exit to monitor system	EXIT

The hybrid executive program PML50 creates physical problems, executes the solutions, documents them, and stores the results on magnetic tape. The program is called by PML40 when the static and circuit checks indicate the simulation is operational. The functions performed are executed by entering a control variable I through the console keyboard. The constant velocity, braking, or maximum deceleration maneuvers may be repeated J times with different parameters by setting an appropriate console sense switch and entering the value of the control variable J through the console keyboard. Table F-3 lists the functions performed and the sub-

routines called by each value of the control variable I.

TABLE F-3

PML50 HYBRID COMPUTATION PROGRAM

I	Function Performed	Subroutines Called
0	Enter continuous solution	-
1	Change parameters and reset	PML52, PML53, PML54
2	Constant velocity maneuver	PML55, PML56, PML68
3	Braking maneuver	PML55, PML56, PML70
4	Link to PML40	LINK
5	Set control variable J	-
6	Set solution number	-
7	Set lateral acceleration	PML72
8	Determine maximum deceleration	PML73, PML74
9	Exit to monitor system	EXIT

Subroutine PML31 sets the logic unit numbers for the input and output devices and reads the labels for the figure from punched cards.

Subroutine PML32 searches the magnetic tape for the file of the solution to be plotted. The complete file, containing the results stored by subroutine PML56, is read into core memory.

Subroutine PML33 draws the axes and labels the figure with a summary of the solutions to be plotted.

Subroutine PML34 plots the results of a given solution in the figure drawn by subroutine PML33.

Subroutine PML35 copies a series of selected solutions from the main storage tape to a scratch tape.

Subroutine PML52 changes the physical parameters used in the simulation. The digital computer changes the parameter values stored in the data array by reading punched data cards until a blank card terminates the data entry phase. If only blank cards are read the computer uses the console keyboard for data entry. The identification number of the next problem solution, the values of the parameters which have just been altered, and the value of all the parameters are listed on the line printer as part of the documentation of the next solution.

Subroutine PML53 calculates the value of each coefficient in the coefficient array from the current parameter data. The coefficient array is used to set the coefficient devices of the analog computer.

Subroutine PML54 sets all coefficient devices whose current value does not agree with the calculated value. If any coefficient value is changed, the line printer lists the calculated value, the actual value, and the absolute value of the error for every coefficient device as part of the documentation. If any coefficient device does not set within a specified tolerance which is stored in the parameter data, the digital computer pauses until the user can correct the situation.

Subroutine PML55 lists the solution identification number, the final value of each amplifier, and which wheels locked during the solution on the line printer. It also provides a tabulation of the trailer angle and the forward velocity, sideslip angle, yawing velocity, longitudinal acceleration, and lateral acceleration of the tractor mass center as a function of time.

Subroutine PML56 tabulates the yawing velocity of the trailer and the radius of curvature, lateral velocity, heading angle, and the position of the tractor mass center as a function of time on the line printer. This subroutine also stores the solution identification number, the parameter data, the final value of each amplifier, which wheels locked, the solution array, and the position-curvature array on magnetic tape.

Subroutine PML58 calculates and performs the circuit check of the analog simulation. The voltage expected on each amplifier is calculated with the circuit equations from Appendix D and the current value of each input element. In order to identify the components which are not working, the line printer lists the circuit check voltage, the actual voltage, and the absolute value of the error for each amplifier.

Subroutine PML59 reads the last solution identification number, the analog element addresses, and the parameter data from a disk file or from punched cards if changes are necessary. It also initializes the common

area, lists the parameter data, and selects the tape unit for storage of the problem solutions.

Subroutine PML65 initializes all interface functions to the zero state and sets the DAC's (digital to analog converters) to their static check values.

Subroutine PML66 calculates and performs the static check of the analog simulation. The static check voltages are calculated by solving the equations of motion with the current initial conditions and control inputs. The line printer lists the static check voltage, the actual voltage, and the absolute value of the error for each amplifier.

Subroutine PML68 applies a step input of steer angle at the beginning of a constant velocity solution. Nine variables (time; trailer angle; yawing velocity of the trailer; and the forward velocity, lateral velocity, yawing velocity, heading angle, longitudinal acceleration, and lateral acceleration of the tractor mass center) are sampled 400 times during the solution and stored in the solution array. The subroutine also reads and stores the final value of each amplifier.

Subroutine PML70 applies the brakes on each axle as a step input at specified times during the solution. The solution may be initiated with a step input of steer angle or with the vehicle in a constant velocity, steady-state turn. The subroutine samples and stores the response variables in the solution array, reads

and stores the final value of each amplifier, and determines which wheels have locked during the solution.

Subroutine PML72 calculates the steer angle required to develop a given steady-state lateral acceleration in a constant velocity solution. It also determines the initial conditions necessary to start a braking maneuver in a specified steady-state turn.

Subroutine PML73 calculates the brake-torque distribution and the brake application times from the parameter data. It also initializes the hardware for the braking maneuvers of PML74.

Subroutine PML74 repeats braking maneuvers at increasing levels of deceleration in order to determine the maximum deceleration that can be achieved without locking any wheel. It summarizes the results of each solution by tabulating the maximum sideslip angle, yawing velocity, heading angle, longitudinal acceleration, and lateral acceleration of the center of mass of the tractor. It also lists which wheels locked, the maximum and minimum trailer angles, the vehicle configuration, the brake-torque distribution, the steer angle, and the initial lateral acceleration of the center of mass of the tractor.

APPENDIX G

STEADY TURNING OF A TRUCK-TRACTOR

A simplified yaw plane model without a roll degree of freedom was used to analyze the directional response of a three-axle truck-tractor. The equations of motion are formulated and used to predict steady-state turning rates at various forward velocities. Plots of yawing velocity as a function of average front wheel steer angle and forward velocity are compared with the measured values of yawing velocity from actual vehicle tests.

The vehicle system used to represent the truck-tractor is diagrammed in Figure G.1. It consists of a rigid body which translates parallel to a smooth, level surface and rotates about an axis perpendicular to this surface. Aerodynamic forces and moments; the rotational inertia of the wheels; and the rolling, pitching, and vertical motions of the sprung mass are neglected. Additional assumptions concerning the external tire forces on the vehicle and other simplifications are discussed in Chapter 2. Equations (2-7) through (2-9) describe the longitudinal, lateral, and yawing motions of the assumed truck-tractor constrained to traverse a flat surface. The three equations are written in terms of the external forces which exist at the tire-road inter-

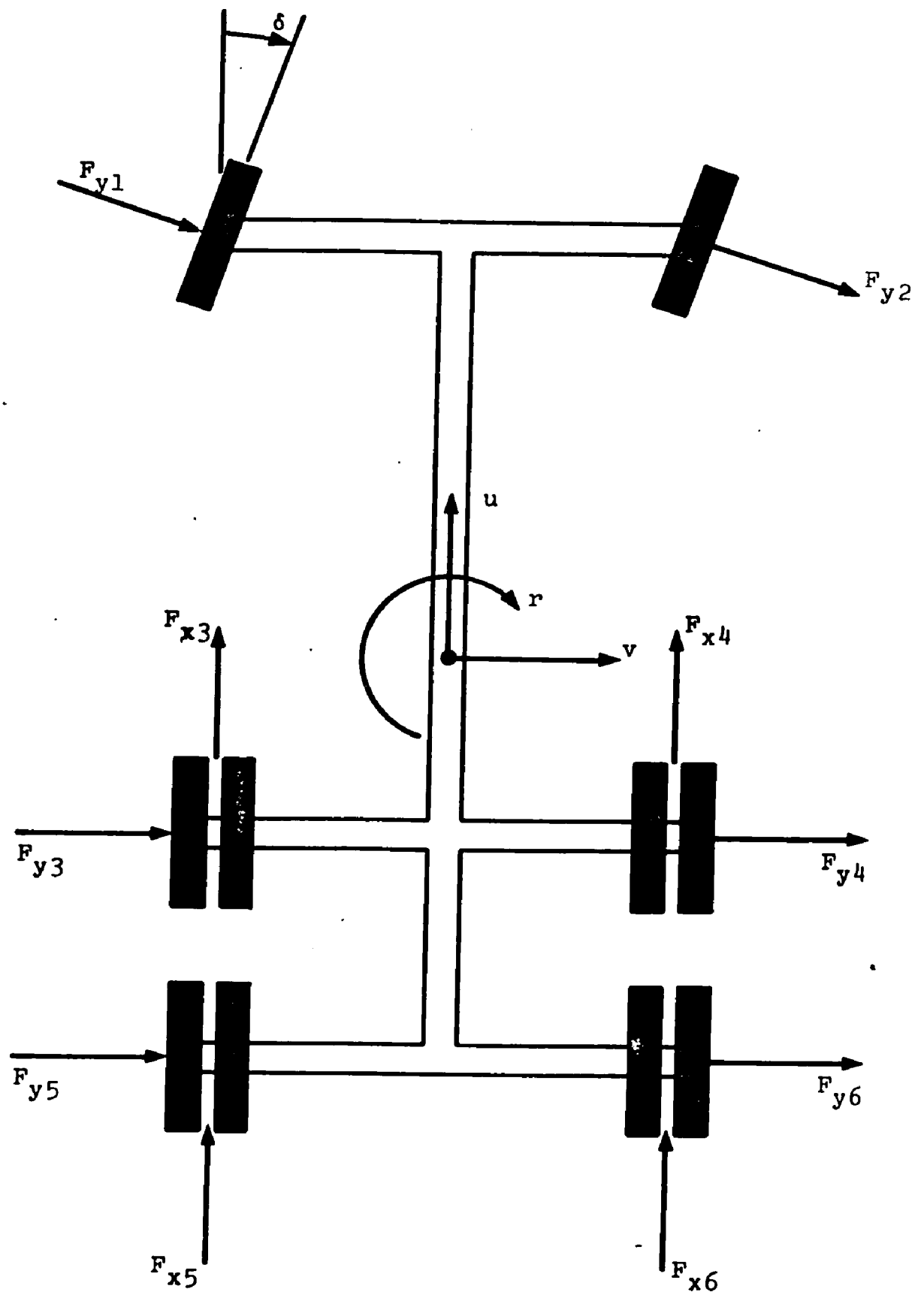


Fig. G.1 - Truck-tractor vehicle model

face as shown in Figure G.1.¹

$$m_1(\dot{u} - vr) = -(F_{y1} + F_{y2})\sin \delta + F_{x3} + F_{x4} + F_{x5} + F_{x6} \quad (G-1)$$

$$m_1(\dot{v} + ur) = (F_{y1} + F_{y2})\cos \delta + F_{y3} + F_{y4} + F_{y5} + F_{y6} \quad (G-2)$$

$$I_1\dot{r} = a(F_{y1} + F_{y2})\cos \delta - b(F_{y3} + F_{y4}) - e(F_{y5} + F_{y6}) \quad (G-3)$$

The steady-state response is determined by setting the derivatives equal to zero and rewriting Equations (G-1) through (G-3) as

$$(F_{y1} + F_{y2})\sin \delta - m_1vr = F_{x3} + F_{x4} + F_{x5} + F_{x6} \quad (G-4)$$

$$m_1ur = (F_{y1} + F_{y2})\cos \delta + F_{y3} + F_{y4} + F_{y5} + F_{y6} \quad (G-5)$$

$$0 = a(F_{y1} + F_{y2})\cos \delta - b(F_{y3} + F_{y4}) - e(F_{y5} + F_{y6}) \quad (G-6)$$

The tire slip angles, vertical loads, and lateral forces are defined as in Chapter 2 by the following equations.

¹Symbols used are defined in Table G-1.

TABLE G-1

SYMBOLS USED IN APPENDIX G

W_1	= weight of truck-tractor
m_1	= mass of truck-tractor
g	= 32.1725 ft/sec ²
u, v	= longitudinal and lateral velocity of the center of mass
r	= yawing velocity of the tractor
δ	= average front wheel steer angle
a	= distance between the center of mass and front axle
b	= distance between the center of mass and middle axle
e	= distance between the center of mass and rear axle
z_1	= height of the center of mass above the ground
γ_1	= portion of the total lateral load transfer on the front axle
γ_2	= portion of the total lateral load transfer on the middle axle
γ_3	= portion of the total lateral load transfer on the rear axle
μ	= maximum tire-road interface coefficient of friction

TABLE G-1 (continued)

F_{x1}	= longitudinal shear force on the tire or dual tires ^a
F_{y1}	= lateral shear force on the tire or dual tires
F_{z1}	= vertical load on the tire or dual tires
$\bar{F}_{z1}$	= static load on the tire or dual tires
ΔF_{z1}	= lateral load transfer onto the tire
α_1	= slip angle of the tire
v_1, σ_1	= experimental tire coefficients
C_1	= cornering stiffness of the tire
$f(\alpha_1, \bar{F}_{z1})$	= normalized lateral force characteristic
t_j	= half the lateral distance between the centers of tire contact on axle j ^b
F'_{z1}	= vertical load on the tire or half the load on dual tires

^aSubscript 1 through 6 correspond to the left-front, right-front, left-middle, right-middle, left-rear, and right-rear wheels of the tractor.

^bSubscripts 1 through 3 correspond to the front, middle, and rear axle of the tractor.

$$\alpha_1 = \alpha_2 = \frac{v + ar}{u} - \delta \quad (G-7)$$

$$\alpha_3 = \alpha_4 = \frac{v - br}{u} \quad (G-8)$$

$$\alpha_5 = \alpha_6 = \frac{v - er}{u} \quad (G-9)$$

$$F_{z1} = \bar{F}_{z1} + \Delta F_{z1} \quad (G-10)$$

$$\Delta F_{z1} = - \Delta F_{z2} = \frac{\gamma_1 z_1 m_1}{2t_1} \text{ ur} \quad (G-11)$$

$$\Delta F_{z3} = - \Delta F_{z4} = \frac{\gamma_2 z_1 m_1}{2t_2} \text{ ur} \quad (G-12)$$

$$\Delta F_{z5} = - \Delta F_{z6} = \frac{\gamma_3 z_1 m_1}{2t_3} \text{ ur} \quad (G-13)$$

$$F_{y1} = - \mu F_{z1} [1.0 - \exp(-|\bar{\alpha}_1|)] \frac{\bar{\alpha}_1}{|\bar{\alpha}_1|} \quad (G-14)$$

$$\bar{\alpha}_1 = \frac{C_1 \alpha_1}{\mu F'_{z1}} \quad (G-15)$$

$$\frac{C_1}{F'_{z1}} = v_1 + \sigma_1 F'_{z1} \quad (G-16)$$

The maximum lateral force that can be developed by a free-rolling tire (the applied wheel torque is zero) is defined by Equation (G-17) as the product of the coefficient of friction and the instantaneous vertical load.

$$(F_{y1})_{\max} = \mu F_{z1} \quad (G-17)$$

The maximum lateral force is assumed to be reduced by the application of a tractive force, F_{x1} , according to Equation (G-18) from Reference [39].

$$(F_{y1})_{\max} = \mu F_{z1} \left[1.0 - \left(\frac{F_{x1}}{\mu F_{z1}} \right)^2 \right]^{1/2} \quad (G-18)$$

This equation implies that the resultant force determined by the longitudinal and lateral shear force components can never exceed the product of the coefficient of friction and the instantaneous vertical load. After replacing the maximum lateral force of Equation (G-17) with that of Equation (G-18), Equations (G-14) and (G-15) are rewritten as

$$F_{y1} = - \mu F_{z1} \left[1.0 - \left(\frac{F_{x1}}{\mu F_{z1}} \right)^2 \right]^{1/2} \times [1.0 - \exp(-|\bar{\alpha}_1|)] \frac{\bar{\alpha}_1}{|\alpha_1|} \quad (G-19)$$

$$\bar{\alpha}_1 = \frac{C_1 \alpha_1}{\mu F'_{z1} \left[1.0 - \left(\frac{F_{x1}}{\mu F_{z1}} \right)^2 \right]^{1/2}} \quad (G-20)$$

Finally, each pair of dual wheels is assumed to develop the same tractive force such that Equation (G-4) is satisfied.

Measurements of the steady-state yawing velocity produced by various steer angles were made at nominal

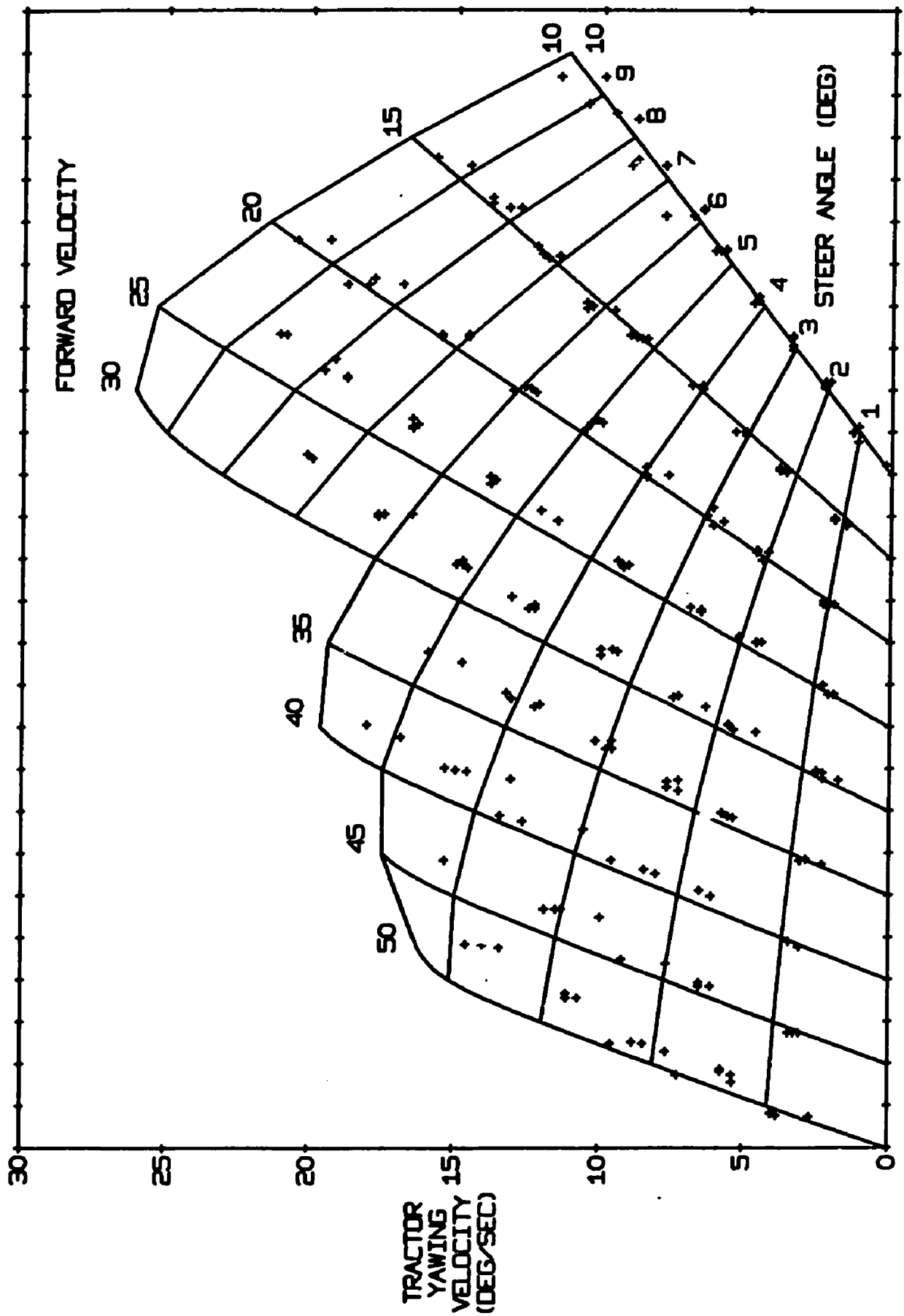


FIG. G-2 - TRACTOR YAWING VELOCITY VERSUS STEER ANGLE AND FORWARD VELOCITY

TABLE G-2

MEASURED PARAMETER VALUES FOR AN
INTERNATIONAL HARVESTER TRUCK-TRACTOR

W_1	=	20,777.5 lb
a	=	7.425 ft
b	=	2.358 ft
e	=	6.617 ft
$\bar{F}_{z1}, \bar{F}_{z2}$	=	3,918.75 lb
$\bar{F}_{z3}, \bar{F}_{z4}$	=	3,227.5 lb
$\bar{F}_{z5}, \bar{F}_{z6}$	=	3,242.5 lb
γ_1	=	0.101
γ_2	=	0.461
γ_3	=	0.438
z_1	=	2.984 ft
t_1	=	3.308
t_2	=	3.000 ft
t_3	=	3.000 ft

forward velocities from 5 to 50 MPH. The 238 experimental data points from these tests are plotted as a function of two variables in each of Figures G.2 through G.4. The abscissa acts as a dummy axis where the distance between tick marks represents one degree of steer angle. The origin for each set of experimental data points is displaced two tick marks to the right for each 5 MPH decrease in forward velocity from 50 MPH. A digital computer was used to solve Equations (G-5) and (G-6) for the steady-state yawing velocity as a function of the steer angle at nine forward velocities. The results of these calculations are also plotted in Figures G.2 through G.4 in the same manner as the experimental data points. The distance parallel to the ordinate between each experimental point and the calculated value at the same velocity is a measure of the difference in yawing velocity between the predicted value and the experimental value at a given steer angle. The tractive forces necessary to sustain a given forward velocity and their effect on the lateral tire forces have been neglected in the analytical results shown in Figures G.2 and G.3.

Figure G.2 compares the experimental data points with the yawing velocity predicted by the solution of Equations (G-5) and (G-6) using the measured parameter values given in Table G-2. Substantial agreement exists between the analytical calculations and the experimental points for low forward velocities at any steer angle

and for any forward velocity with small steer angles. Significant deviations do occur at higher velocities and large steer angles.

In Figure G.3 the cornering stiffness of the front tires is reduced by ten percent and the stiffness of the tires on the middle and rear axles is increased slightly such that the total lateral stiffness of all the tires has remained the same. This same technique is used by Bundorf [20] in order to compensate for the roll understeer deleted by the approximate treatment of the vehicle without a roll degree of freedom. Figure G.3 shows substantial agreement between the predicted yawing velocity and the experimental yawing velocity measurements at all velocities and steer angles considered.

The final comparison includes the effect of tractive force on the yawing velocity response. Figure G.4 shows that the net difference between calculated and measured values is decreased very slightly by the introduction of tractive effects to the vehicle model.

These comparisons of a simplified yaw plane model with experimental test data have demonstrated substantial qualitative agreement. Furthermore, the ability of the analytical model to exhibit real vehicle behavior can be increased by including tractive effects and by altering the cornering stiffness of the tires without adding additional degrees of freedom.

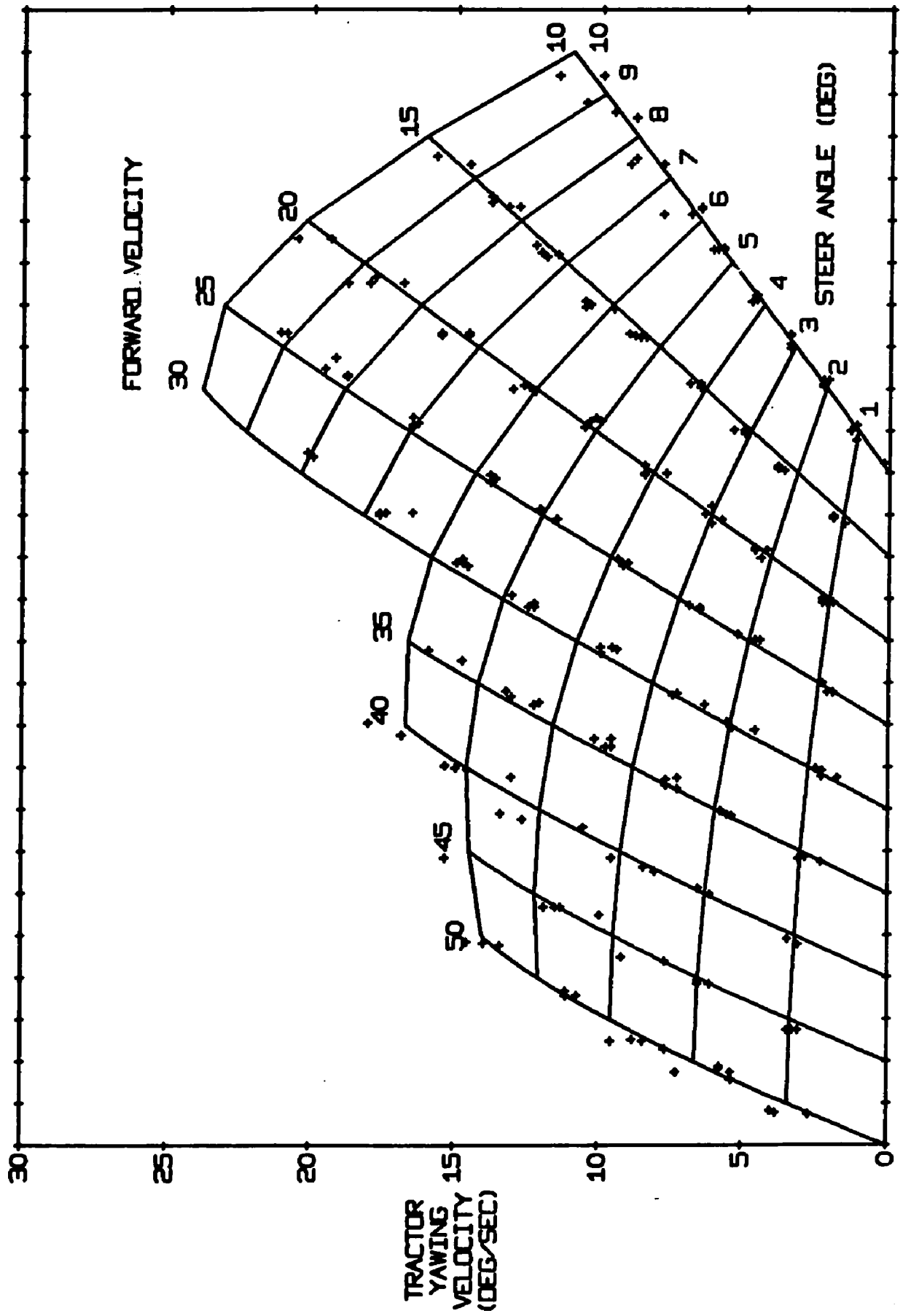


FIG. G-3 - TRACTOR YAWING VELOCITY VERSUS STEER ANGLE AND FORWARD VELOCITY

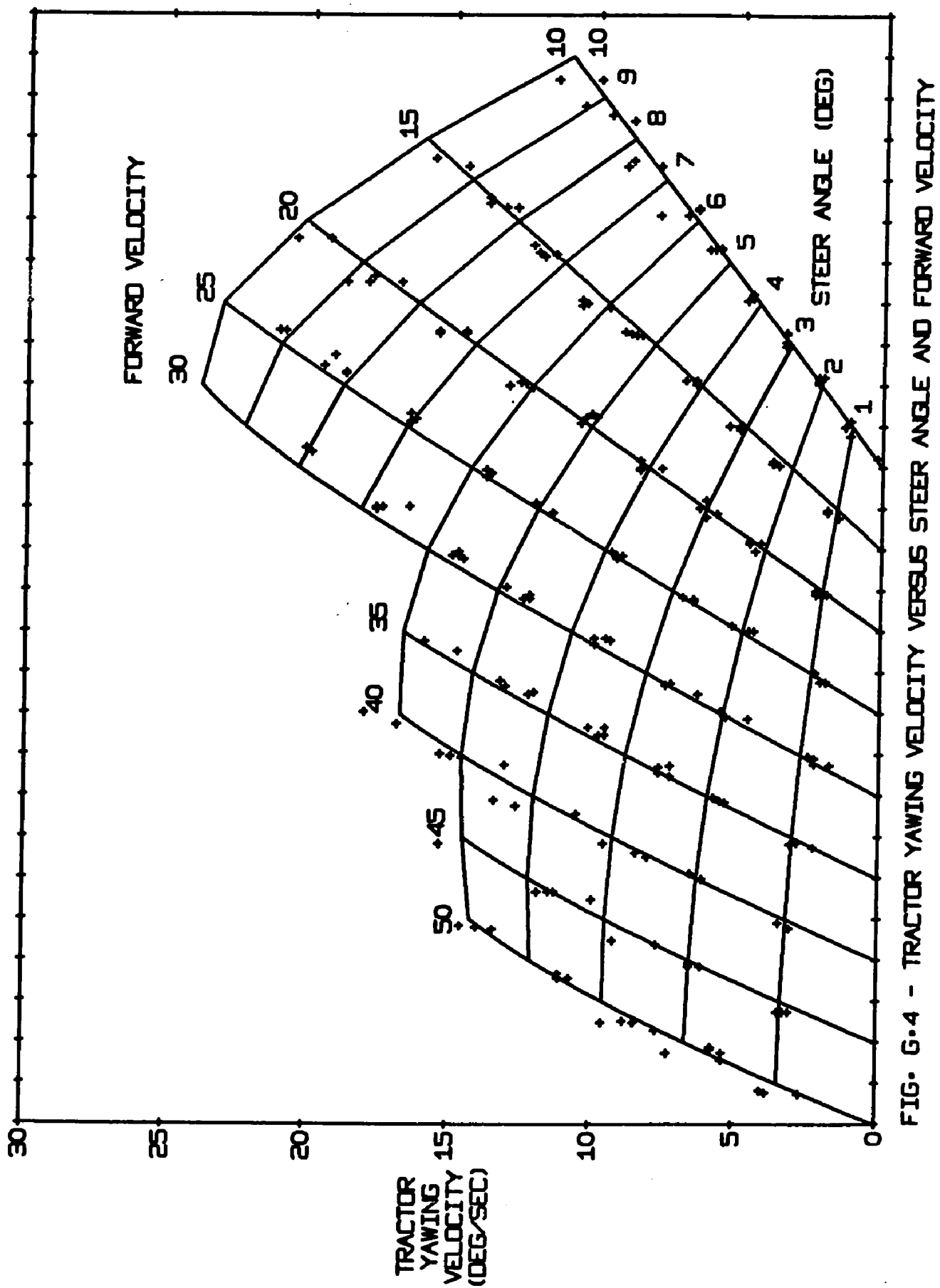


FIG. G.4 - TRACTOR YAWING VELOCITY VERSUS STEER ANGLE AND FORWARD VELOCITY