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NONLINEAR INTERACTION THEORY FOR CROSSED-FIELD
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by
Thomas Andrew DeMassa

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Doctoral Committee:

Professor Joseph E. Rowe, Chairman
Associate Professor George I. Haddad
Assistant Professor Noel J. Hicks
Professor Gunnar Hok
Associate Professor Ronald J. Lomax
Doctor Nino A. Masnari

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A generalized nonlinear electron-wave interaction theory is developed for the crossed-field emitting-sole amplifier in order to explain the physical phenomena involved in the interaction and ultimately to provide a basis for design. The system equations are nonlinear integro-differential equations developed from the Lorentz force equation, the continuity equation, Poisson's equation and the one-dimensional equivalent transmission-line equation.

The effects of d-c, r-f and space-charge forces are taken into account and secondary emission from the sole electrode is included in the theory. Space-charge forces are evaluated using the Green's function for space-charge flows in a rectangular geometry. Solution of the system equations on a digital computer involves following a set of N charge groups through the interaction region using electron phase position and interaction length as the independent variables. The number of system equations then becomes $4N + 2$.

The large-signal charge group and r-f wave variables as calculated using the $4N + 2$ equations are illustrated in graphical form for various d-c and r-f conditions. These graphs display the positions and velocities of the electrons in the interaction region and the amplitude and phase of the r-f wave as functions of longitudinal distance. The d-c parameters are chosen in most cases to correspond to synchronous operation with the average electron drift velocity equal to the phase velocity of the unperturbed r-f wave. At the same time, however, these parameters are chosen for d-c operation above, below and at the magnetron cutoff condition (the d-c crossed-field condition at which electrons emitted with zero initial velocities are just able to graze the anode). Solutions for operation above, below, and at cutoff are obtained and it is observed that below cutoff operation results in higher amplifier gain. An enhancement in amplifier gain is also observed for larger initial amplitude r-f signals.

An approximate set of equations is derived by neglecting space-charge forces and assuming the gain parameter D to be small. This set of equations is then made amenable to solution by further assuming that the electrons coalesce into charge groups called "hard-kernel bunches". This assumption implies that the bunching process has proceeded to the ultimate state after which maximum energy transfer takes place as the ideal bunches approach and withdraw from the circuit. Solutions for various sets of d-c parameters, using this "hard-kernel-bunch" approximation, are presented. It is found that the gain of the amplifier is increased when more charge is injected into the interaction region. In addition, the solutions are found to be in good agreement with those obtained from the $4N + 2$ system equations.

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CHAPTER I. INTRODUCTION

1.1 Crossed-Field Microwave Devices

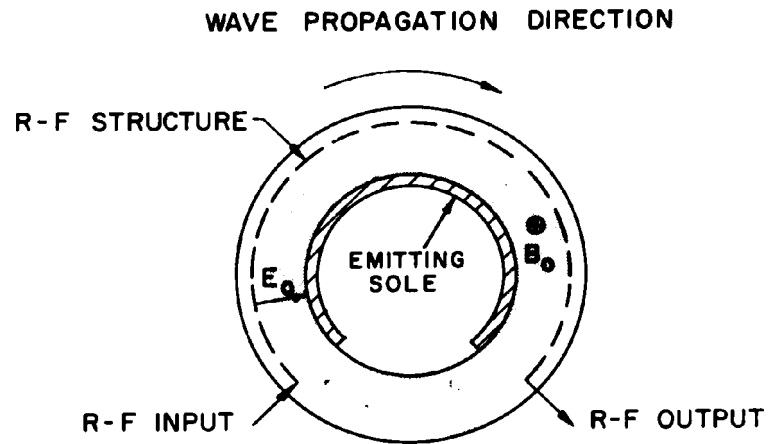
Crossed-field devices (also called magnetron-type or simply M-type devices) are those in which the electric field and the d-c magnetic field are mutually perpendicular. The first use for this type of device was as a generator of microwave power, and the first such device was called a magnetron oscillator. The interaction process that takes place in a magnetron is the same as that which takes place in all crossed-field devices. Namely, electron potential energy is converted into r-f energy. However, there are significant differences between magnetron oscillators and crossed-field amplifiers. The major difference is that the oscillator generates microwave energy, whereas the amplifier enhances this energy by amplifying an input r-f signal. Another difference is that the r-f circuit of the magnetron is reentrant while that for the amplifier is nonreentrant; thus the magnetron is permitted to support a standing r-f wave, whereas the amplifier supports a traveling r-f wave. Nonetheless, since the energy conversion process for both the oscillator and the amplifier is internally the same, a general description of crossed-field electron-wave interaction is appropriate.

Before discussing the interaction process, it is convenient at this point to describe the various types of crossed-field amplifiers or CFA's (the abbreviation CFA will be used interchangeably with crossed-field amplifier throughout this dissertation). There are two basically different types of CFA's; the injected-beam amplifier and

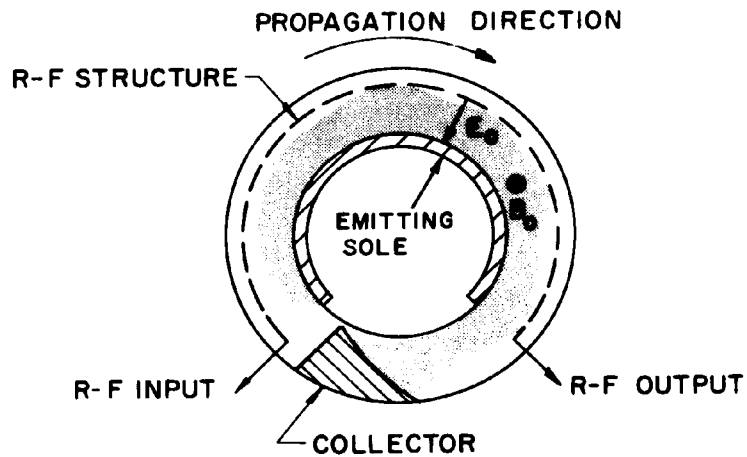
the emitting-sole or distributed-emission amplifier. The injected-beam amplifier differs from the emitting-sole amplifier in that the electron-gun region is separated from the interaction region. Figures 1.1 and 1.2 display these two types of amplifiers in both the linear and cylindrical forms in which they are constructed. A special feature which the cylindrically built tube possesses is the possibility of beam reentrance (the electrons are able to traverse the interaction region more than once) as shown in Fig. 1.1a. This is advantageous in the emitting-sole tube because with beam reentrance the electrons emitted near the r-f output end of the sole are able to interact more completely with the slow wave until collection on the r-f circuit occurs. This greater interaction necessarily results in higher electronic efficiency. However, a reentrant beam also produces feedback and hence undesired oscillations.

Injected-beam amplifiers, illustrated in Fig 1.2, do not use a reentrant beam for the simple reason that the interaction length between the r-f input and output is already long enough to provide almost complete electron-wave interaction. Despite this, injected-beam amplifiers are frequently built in cylindrical form in order to achieve a compact magnet system. A family tree of these "M" (magnetron)-type traveling-wave amplifiers is shown in Fig. 1.3, illustrating the various kinds of crossed-field amplifiers.

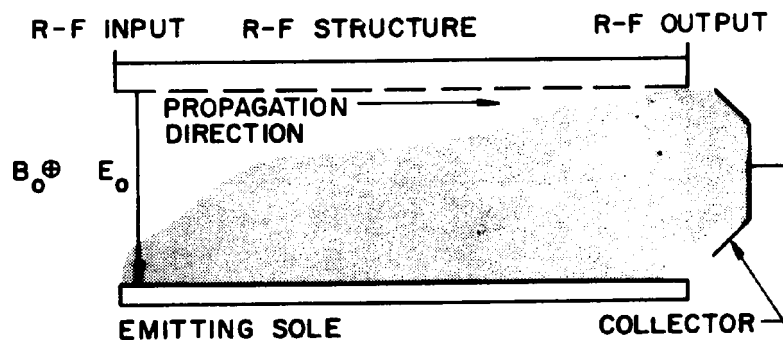
While discussing reentrant beams, it is important to also consider the reentrant circuit. This type of circuit is one which closes back upon itself, and is used in magnetron oscillators, but not in CFA's. This is because a resonant transmission circuit results and only a wave with frequency inversely proportional to an



a) CYLINDRICAL REENTRANT EMITTING-SOLE AMPLIFIER

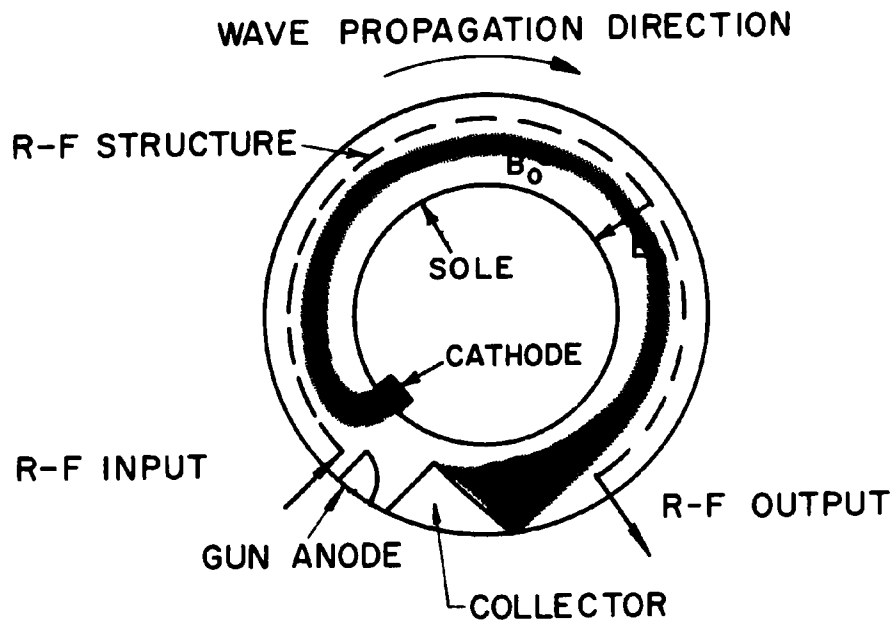


b) CYLINDRICAL NONREENTRANT EMITTING-SOLE AMPLIFIER

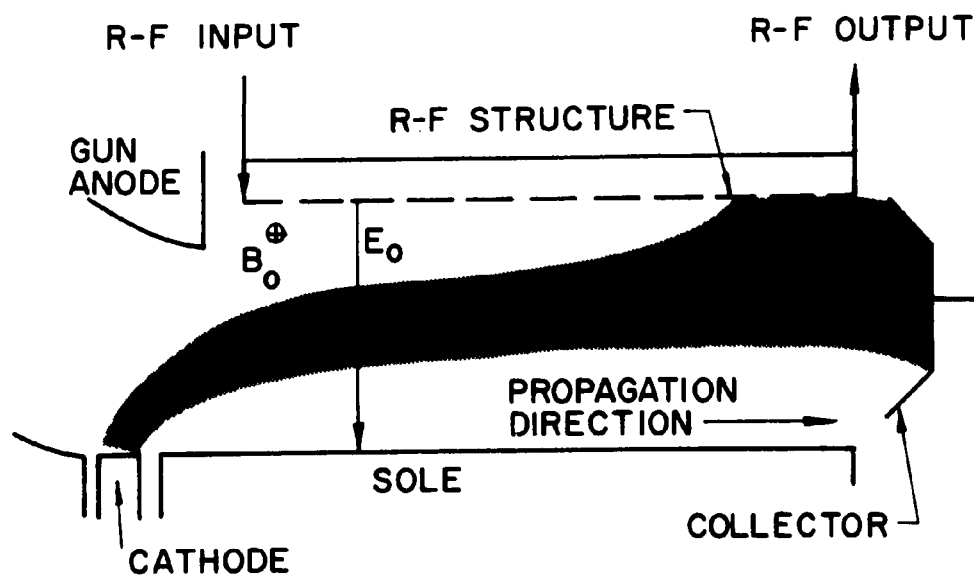


c) LINEAR EMITTING-SOLE AMPLIFIER

FIG. 1.1 CROSSED-FIELD, EMITTING-SOLE, TRAVELING-WAVE AMPLIFIERS.



a) CYLINDRICAL INJECTED-BEAM AMPLIFIER



b) LINEAR INJECTED-BEAM AMPLIFIER

FIG. 1.2 CROSSED-FIELD, INJECTED-BEAM, TRAVELING-WAVE AMPLIFIERS.

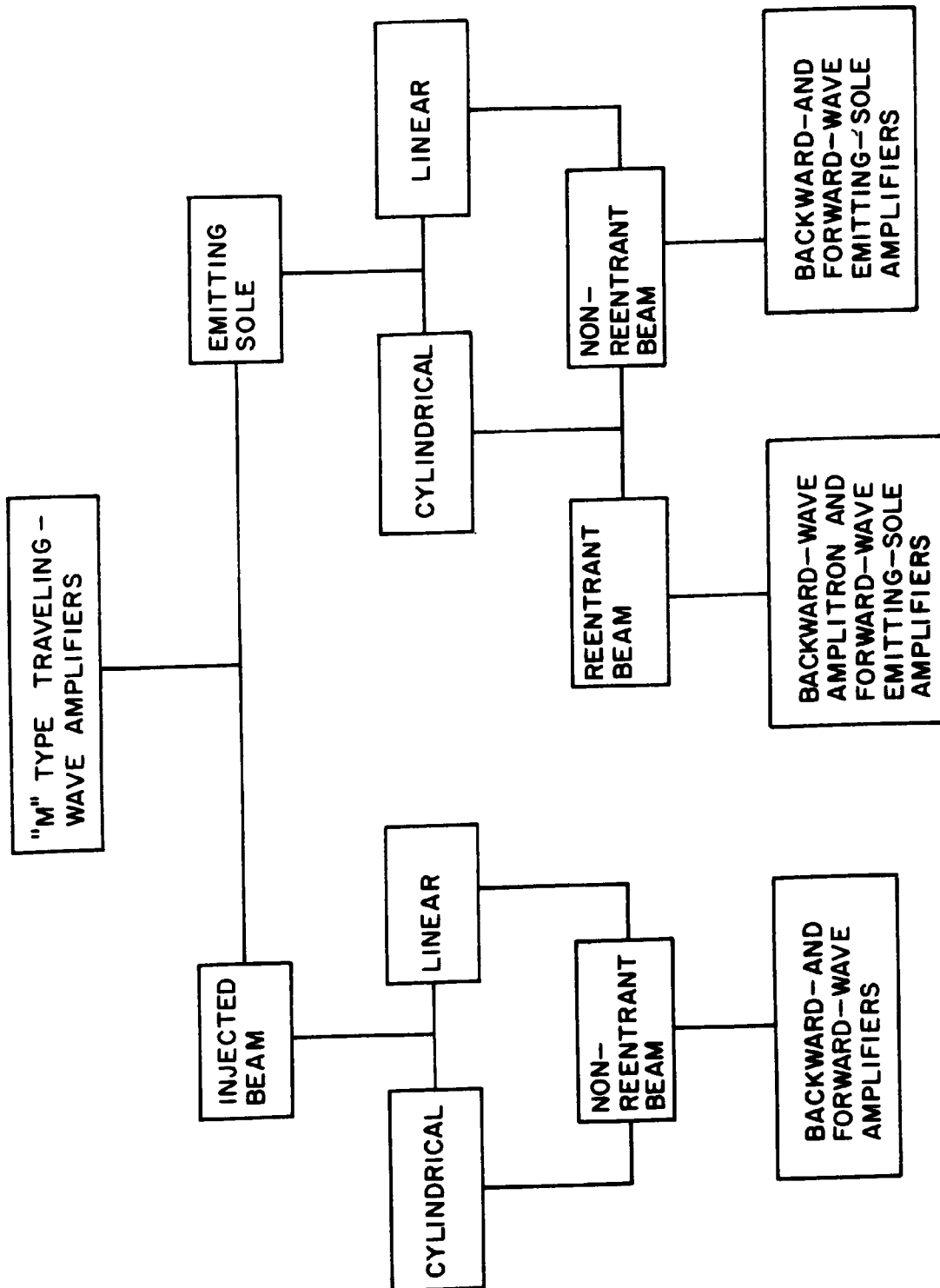


FIG. 1.3 FAMILY TREE OF "M" -TYPE TRAVELING-WAVE AMPLIFIERS.

integral number of wavelengths around the closed path can exist in a steady state. This is acceptable in the magnetron because it is essentially a single frequency device, whereas in an amplifier, wideband frequency operation is desirable.

1.2 Crossed-Field Interaction Mechanism

The mechanism which produces r-f amplification in a crossed-field microwave device is most easily described by considering the linear form of the tube. Linear crossed-field geometries are shown in Figs. 1.1c and 1.2b. The cylindrical form of a crossed-field amplifier can also be analyzed using this linear geometry when the sole-to-circuit spacing is much less than the sole radius. As can be seen in the figures the d-c magnetic field B_0 , the d-c electric field E_0 , and the direction of wave propagation (z-direction) are all mutually orthogonal.

The cathode or sole electrode is a source of electrons and when no r-f wave is present these electrons follow cycloidal trajectories which are calculable when the d-c field and the initial coordinates of the electrons are known.

R-f wave propagation is supported by the r-f structure (or anode in the diode geometry of Fig. 1.2b), which slows the wave to a speed comparable to that of the electrons. This structure must also be capable of dissipating d-c power since electrons are collected on it. In the absence of electrons, the output energy of the r-f wave will be less than the input energy because of resistive heating of the circuit by the r-f currents.

A simplified description of the electron-wave interaction process will now be given neglecting space-charge fields. The average net longitudinal drift velocity of all of the electrons under these conditions

is the same and is equal to the ratio of the d-c electric field to the d-c magnetic field ($v_d = E_0/B_0$). In order for efficient interaction to take place, the phase velocity of the r-f wave must be nearly synchronous with this drift velocity, i.e., the two velocities must be approximately equal. Under synchronous conditions and in a coordinate system moving at the synchronous velocity, the electrons begin to move in a direction perpendicular to the r-f electric field ($\vec{E}_{rf} \times \vec{B}_0$). This motion results in the formation of electron bunches (one in each cycle of the r-f wave). The movement of these bunches is against a retarding r-f field and hence toward the r-f circuit into a region of lower potential energy. As the bunches move toward the circuit a stronger interaction occurs due to the larger r-f field closer to the circuit. The bunches remain in proper phase with the r-f field and thus give up potential energy to the r-f wave as they move toward the circuit when the average electron longitudinal velocity and the r-f wave velocity are synchronous. On the other hand, electrons in the accelerating r-f field region, instead of bunching, become rarefied. These "unfavorable" electrons gain potential energy from the r-f wave and approach the sole electrode. In so doing, these electrons move into a weaker r-f field region and interact less strongly with the r-f wave than the "favorable" bunched electrons. Hence, an exchange of energy takes place between the potential energy of the electrons and the r-f energy of the wave. This exchange yields a net gain in r-f energy, because the retarded electrons give up more potential energy to the r-f wave than the accelerated electrons extract.

Hence, crossed-field devices, unlike their traveling-wave amplifier counterparts, possess an inherent mechanism for maintaining bunches of electrons in favorable r-f phase positions. This is because

kinetic energy is not extracted from the electrons and hence the favorably phased electrons do not slip out of the retarding field region of the wave. The conversion of potential energy leads to theoretical electronic efficiencies of about 90 percent with practically realizable efficiencies ranging from 50 to 75 percent. This high conversion efficiency is one of the chief advantages to be gained by using crossed-field devices.

1.3 Previous Studies of Crossed-Field Microwave Devices

The early work done by crossed-field device researchers was concentrated on the magnetron. Probably the most significant work was contributed by Hull¹ in 1921 when he laid the foundation for the theory of operation of cylindrical magnetron oscillators, which included the very important derivation of the magnetron cutoff condition. Another significant advance was contributed by Hartree and Buneman² who developed an expression for the lowest voltage, for a given magnetic field value, at which the magnetron oscillations can start (the Hartree condition). Further understanding of the magnetron was also contributed by Tibbs, Wright and Hartree³ (1948) and later by workers at Raytheon⁴ (1959) through their study of electron trajectories under the influence of space-charge forces. However, using a method of successive approximations these researchers found that the electron trajectories often did not converge to self-consistent solutions. Contributions toward explaining magnetron behavior were also made by Welch⁵ (1953) and later by Hull⁶ (1958) who both used simplifying stream assumptions to obtain useful results.

Because of the success achieved with magnetron oscillators, researchers decided to attempt to use the crossed-field device as an

amplifier. Shortly after World War II, experiments^{7,8} on both the linear and nonreentrant cylindrical crossed-field distributed-emission tubes were performed, but very low gain and efficiency were obtained. The conversion from an oscillator to an amplifier was achieved by using a slow-wave propagating structure instead of resonant cavities.

Due to the lack of success in this early work, researchers were prompted to consider injected-beam amplifiers. The CSF group⁹ in France initiated injected-beam crossed-field amplifier research in 1952. Pierce¹⁰ also began work in this area and was the first to derive a small-signal theory, although his theory considered only thin beams. Later, MacFarlane and Hay¹¹ studied the thick beam case and observed the presence of growing waves in a slipping stream (the diocotron effect). Small-signal theories for the injected-beam device were also derived by Dombrowski¹² and Gould¹³ who considered the effects of space-charge forces. However, these small-signal theories are inadequate for explaining the characteristics of saturated amplifiers which inherently operate at large-signal levels.

The first quasi-large signal injected-beam analysis was that of Feinstein and Kino¹⁴ (this theory also applies to the emitting-sole amplifier since a beam configuration is not specified). By neglecting space-charge fields and assuming beam-wave synchronism along with adiabatic motion (neglect of acceleration terms in the force equations), they were able to obtain solutions for the electron trajectories in closed analytic form. Then by numerical integration over these electron trajectories a prediction of the r-f wave growth as a function of longitudinal distance was obtainable. However, because of the

adiabatic assumption, this theory represents a good approximation only for small interaction parameter D values and a large ratio of cyclotron to r-f frequency ω_c/ω .

Another large-signal analysis accounting for nonsynchronism between the beam and the wave was derived by Sedin¹⁵ although, again only results for small D were obtained.

In 1960, however, a very general large-signal injected-beam theory was developed by Gandhi and Rowe¹⁶. Through use of the Lagrangian method of analysis, six nonlinear integro-differential equations were obtained and were solved by numerical methods. This analysis considered space-charge forces, nonsynchronism, and non-adiabatic motion. As a result of this theory solutions for both large and small D values were obtained. Rowe and Gandhi found that space-charge forces enhanced the amplifier gain. Volkholz¹⁷ extended this theory to include circuit tapers in an attempt to optimize gain and efficiency. An improvement of up to 2 db was achieved with the tapered circuit. Volkholz also obtained closed-form solutions by assuming that all of the charge coalesced into hard-kernel bunches. This approach was seen to agree closely with that of Gandhi and Rowe¹⁶.

Recently, renewed effort has been directed toward achieving higher gain crossed-field emitting-sole amplifiers. This new interest was encouraged because of the fact that the emitting-sole amplifier is suitable for much higher power operation than the injected-beam amplifier. This is a result of the availability of many more electrons in the interaction region which results in greater current and hence greater power. Thus, if a means of improving the saturation gain of

this high-power device were achieved, a satisfactory large-signal high-power amplifier could be obtained.

In 1957, research at Raytheon evolved a device called the platinotron¹⁸, which is a cylindrical distributed-emission reentrant beam device that can be used either as an amplifier or oscillator. This device differs from the conventional crossed-field amplifier in that it operates with backward-wave rather than forward-wave interaction. Experimental gains of 6 to 10 db (at high power levels) have been achieved when the tube is used as an amplifier (it is then called the ampliton).

A theory of the ampliton has been developed by Dombrowski¹⁹. He obtains relations which form the counterpart of Pierce's traveling-wave tube circuit equation by using series expressions for the amplitude and phase of the forward and backward traveling waves. A Fourier expansion of the space-charge induced wave (the sum of the forward and backward traveling waves) and the input-signal wave yields an expression for the phase angle between these two waves by comparing the kth harmonics of each. From this relation the gain of the ampliton is obtained and the results range from 10 to 30 db for lossless circuits. However, this theory omits the detailed description of electron stream behavior and hence would be inadequate for a complete description of emitting-sole amplifiers.

Experimental research using a forward-wave reentrant beam emitting-sole amplifier has been carried out by the SFD Laboratories. Amplifiers have been built and operated in both the L and X frequency bands. The L-band research²⁰ has resulted in the construction of an amplifier with 15 db gain for input powers of 2 to 5 kw. However, the

X-band work²¹ has not been as successful, since the device has achieved only 10 db maximum gain.

Emitting-sole theories involving nonreentrant electron beams have also been developed. One of these is that of Chen²², who extended Buneman's small-signal or linear magnetron theory to the magnetron amplifier and calculated a gain figure of 1.4 db/cm. This gain was measured to be 1.1 db/cm under small-signal conditions. However, an extension of this theory to include large r-f signals would not be valid because in the large-signal region the amplifier does not behave linearly.

Hull²³ has also given renewed attention to the linear emitting-sole nonreentrant beam device. His latest design theory, which is heuristic, is based on a magnetron comparison technique, i.e., equivalent magnetron oscillators in which the fields are identical to those in the amplifier are assumed to represent the interaction region. An extension²⁴ of this work has included a magnetic field taper which would affect the electric field space-charge depression and keep the total electron drift velocity the same as that of the r-f wave. The name coined for this tapered B-field emitting-sole amplifier is the dematron, and experimental results have yielded gains of 6 to 15 db.

Another large-signal theory for the emitting-sole amplifier was derived by Monosov²⁵. In this analysis a single integro-differential equation is obtained from an energy balance equation relating the electron's kinetic and potential energy to the r-f energy and joule losses. This equation is solved by numerical methods and a distribution of the r-f potential amplitude as a function of longitudinal distance

showing saturation characteristics is presented. However, this theory neglects the change in the phase velocity of the r-f wave which occurs when an electron stream perturbs the wave. Also, the theory is only valid for $\omega_c/\omega \gg 1$ and zero space charge.

Finally, the first theoretical large-signal analysis of the nonreentrant beam emitting-sole amplifier considering space-charge forces and including an analysis of electron trajectories was recently done by Yu et al., at Litton³⁴. The results of this work are similar to those presented in this thesis. The analyses are different in that Yu et al. use time as the independent variable (time dependent analysis), whereas this analysis uses longitudinal distance. Yu also computes the space-charge forces differently. In each analysis useful results are obtained by assuming that a finite number of electrons are present. However, both theories are limited by computer capacity, which limits the number of electrons that can be tracked through the interaction region. When the time analysis is used, only about 400 primary electrons and 2500 secondary electrons can be followed, while in this presentation about 2700 primaries and the same number of secondaries are possible.

A complete summary of Yu's theory and a comparison with the theory presented in this dissertation are given in Appendix F.

1.4 Outline of the Present Study

This investigation considers a large-signal analysis of crossed-field emitting-sole amplifiers. The purpose of this work is mainly to develop a physical understanding of the interaction process which takes place. Since the nonlinear equations representing the crossed-field

electron-wave interaction are very complicated, computer methods are used to obtain the solutions on a case-by-case basis. By varying the d-c parameters, such as the electrostatic field, the magnetic field, and the sole-to-anode spacing, the gain of the amplifier is improved. This procedure can therefore be thought of as a computer design method whose results can be used to develop practical devices.

Chapter II contains the derivation of a set of large-signal equations with most of the detail relegated to the appendices. The Lagrangian approach is used in which each electron is followed from initial coordinates through the interaction region until collection on an electrode. Thus, the problem is set up as an initial-value problem. The continuous equations obtained are nonlinear integro-differential equations, and thus an exact analytic solution is impossible. These equations must either be solved approximately or by digital computer techniques.

The final form of the equations is presented in difference form in Chapter III. The numerical method used is described in symbolic form and again the details are relegated to the appendices. The special numerical integration procedure used for the space-charge and induced-current integrals is also described along with the initial conditions for the dependent variables.

Numerical solutions neglecting space-charge forces are presented and discussed in Chapter IV. These include solutions for the emitting-sole amplifier, both with and without secondary emission. Solutions for various initial r-f wave amplitudes are also discussed.

In Chapter V the computer solutions including space-charge fields are presented for the emitting-sole amplifier and also a

multiple-cathode amplifier. The d-c parameters chosen in most of the cases satisfy the synchronism condition, which stipulates that the unperturbed phase velocity of the r-f wave (v_o) is equal to the average longitudinal drift velocity of the electrons ($v_d = E_o/B_o$). The effects of nonsynchronous operation are also discussed. By changing ω_c/ω and quantities dependent upon this input variable, three sets of d-c parameters for operation above, at and below d-c magnetron cutoff are obtained. The solutions for these cases are presented and discussed. Also in Chapter V, the effects on the gain of the amplifier are investigated when a magnetic field taper (a linear decrease with longitudinal distance) is applied.

Chapter VI contains a simplification of the general large-signal equations using the hard-kernel-bunch assumption. A closed-form solution is obtained for the hard-kernel-bunch trajectories and the resulting r-f wave amplitude. Although this is an idealized assumption, it does lend insight to the actual phenomenon taking place in a crossed-field emitting-sole amplifier.

The final chapter summarizes the conclusions reached in the preceding chapters. Suggestions for further study are also given.

CHAPTER II. DERIVATION OF THE LARGE-SIGNAL EQUATIONS

2.1 Introduction

The analytical treatment carried out in this dissertation is aimed at presenting a satisfactory explanation of the interaction which takes place in the crossed-field emitting-sole amplifier. Figure 2.1 displays the assumed linear geometry for this device. The large-signal equations for this system are derived using a Lagrangian approach in which each electron is followed through the interaction region from initial coordinates until collection on an electrode. The solutions obtained from these equations will provide the necessary information needed for the physical interpretation of the phenomena taking place.

The fundamental equations which are to be used to describe the interaction process in crossed-field emitting-sole devices are:

1. The one-dimensional equivalent transmission-line equation,
2. The Lorentz force equation,
3. The continuity equation, and
4. Poisson's equation.

These same equations were used by Gandhi¹⁶ in his large-signal analysis of injected-beam crossed-field devices. The similarity is to be expected since the emitting-sole device simply has a modified source of electrons while the field configurations are the same in both devices. The above equations are therefore the starting point for a Lagrangian analysis of crossed-field devices. From this point, however, modifications are made in each theory to fit the particular device.

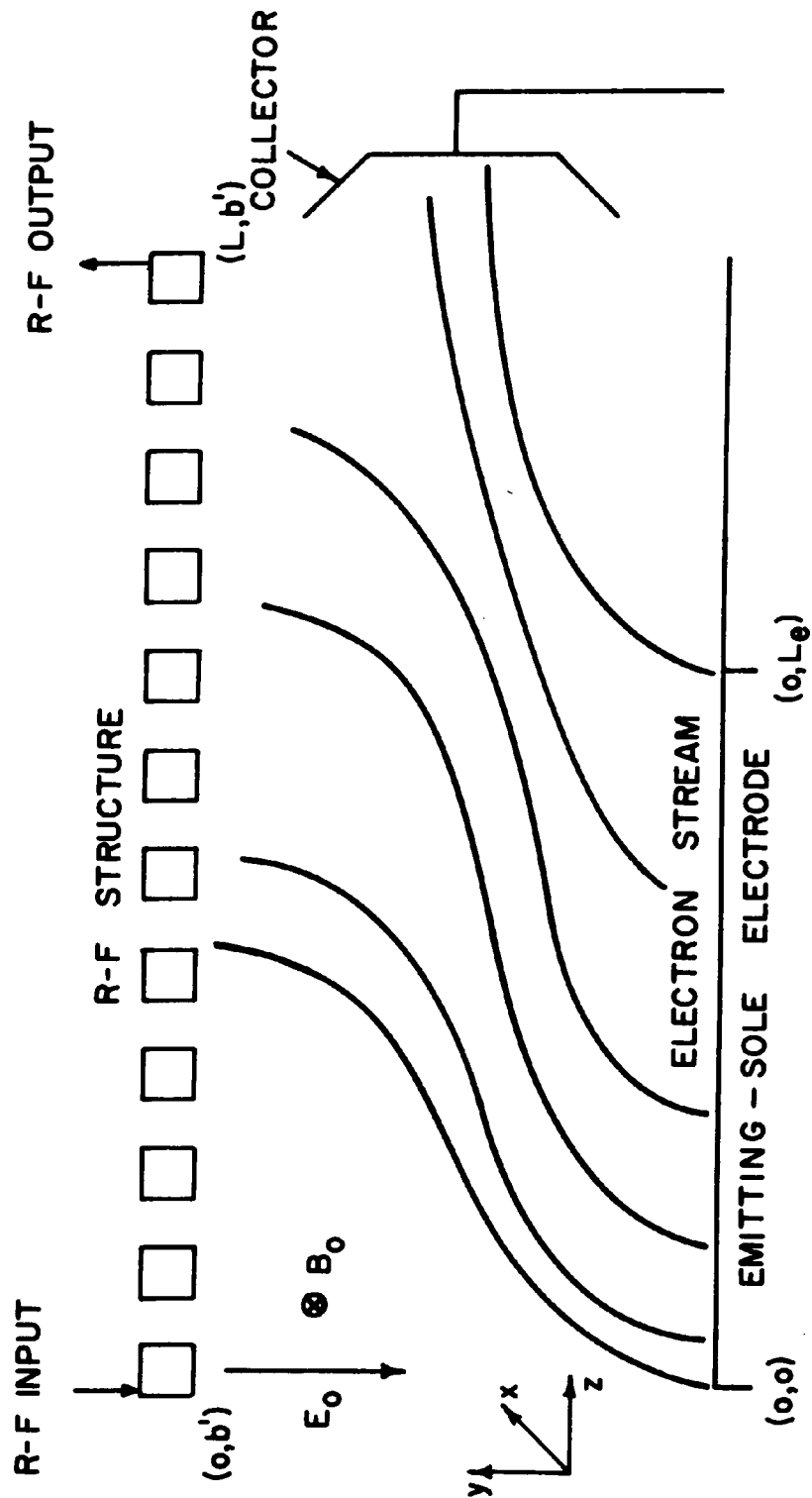


FIG. 2.1 CROSSED-FIELD EMITTING-SOLE DEVICE.

This theory differs from that of Gandhi in the following respects:

1. A new set of normalizing parameters is used, including different initial conditions for the electron coordinates.
2. A derived expression for the "input plane" charge density is used instead of the ratio of beam current density to beam velocity.
3. A different electron coordinate transformation is used.
4. An emission model must be considered in the emitting-sole amplifier analysis because the gun region and the interaction region are no longer separate.
5. Secondary electrons from the sole are taken into account.

A particle dynamic treatment called Lagrangian mechanics (to be discussed in greater detail later) is used in conjunction with the basic equations. The final equations obtained are nonlinear integro-differential equations which are not amenable to exact closed-form solution. The continuous system equations will be derived in this chapter.

2.2 Assumptions Used in the Analysis

The following assumptions are made in this analysis:

1. Nonrelativistic mechanics are utilized ($v^2/c^2 \ll 1$).
2. The r-f magnetic field force on an electron is neglected (the guide wavelength is much less than that of free space).
3. A linear (Fig. 2.1) two-dimensional model is used ($\partial/\partial x = 0$).
4. The magnetic field is confined to the x-direction and is uniform.
5. Electron-wave interaction is neglected with all except one space harmonic of the r-f wave.

6. In the charge density derivation a Maxwellian distribution in the x- and z-emission velocities and a half-Maxwellian distribution in the y-emission velocity are used. The final expression is the ratio of thermionic emission current density to average emission velocity.

7. Particle-particle collisions are neglected.

8. The r-f structure is terminated in its characteristic impedance so that there are no reflected waves.

2.3 Lagrange Variables

In an Eulerian approach for M- and O-type devices, the hydrodynamic equation of continuity is utilized. This theory is inadequate in the case where individual electrons overtake one another as is the case in an emitting-sole device. This type of interaction (electron crossings) has been discussed by Gandhi¹⁶ and Rowe^{26,27} and a further discussion is not necessary here, except to mention that when electron flight lines intersect, a particle-dynamic treatment must be used in place of the Eulerian approach in order to treat the multivalued (more than 2) functions which arise.

The reformulation of the problem, therefore, consists of eliminating this flight-line crossing difficulty. This can be done by introducing new independent variables which will be called Lagrange variables. The independent variables to be used are the initial position of an electron z_0 , initial phase of an electron relative to the r-f wave Φ_0 , and longitudinal position along the tube z . These quantities will be normalized with respect to the phase velocity (v_0) of the "cold" circuit wave, since a single average beam velocity cannot be defined in the case of a crossed-field emitting-sole device. The physical interpretation of this transformation of coordinates is a shift

from a stationary coordinate system to one moving with the undisturbed phase velocity v_0 . This causes the dependent variables to vary more slowly with both time and distance.

Normalized independent variables are defined as follows:

$$\phi_0 = [\phi]_{(z_0, t_0)} , \quad (2.1)$$

$$q_0 = [q]_{(z_0, t_0)} \quad (2.2)$$

and

$$q \triangleq \frac{D\omega z}{v_0} , \quad (2.3)$$

where z_0 is the longitudinal position at which an electron is emitted, t_0 is the time of emission, D is the gain parameter to be defined later, and ω is the radian frequency of the r-f wave. Physically, ϕ_0 is the entrance phase position of an electron relative to the r-f wave, q is the longitudinal distance proportional to the number of circuit wavelengths and q_0 is the initial longitudinal position of an electron.

Along with these independent variables, it is convenient to define the dependent velocity and phase variables in the following manner:

$$u_y \triangleq \frac{\omega_c}{\omega} \frac{1}{v_0} \frac{dy}{dt} , \quad (2.4)$$

$$\frac{dz}{dt} \triangleq v_0 (1 + 2Du_z) \quad (2.5)$$

and

$$\phi \triangleq \frac{\omega z}{v_0} - \omega t - \theta_c(z) , \quad (2.6)$$

where ω_c is the cyclotron frequency, u_y and u_z are the dimensionless y and z electron velocities, Φ is the phase position of an electron in the stream relative to the r-f wave, and $\theta_c(z)$ is a perturbing quantity representing a change in phase position due to interaction with the beam, i.e., with no beam present $\theta_c = 0$ and $\Phi = \omega z/v_0 - \omega t$. Figure 2.2 is a flight-line diagram or phase diagram indicating the way in which Eq. 2.6 was obtained. The curve marked $u(z)$ represents a flight line for a beam of particles moving with velocity $u(z)$, while the curves marked v_0 and $v(z)$ represent a constant and varying wave velocity respectively. It has been shown²⁷ that Φ has an even simpler physical meaning. It is the phase angle of the r-f traveling wave at any q-plane.

2.4 Circuit Equations

In this section two circuit equations will be derived from the one-dimensional transmission-line equation. This equation is a second-order partial-differential equation relating the r-f potential along the slow-wave circuit to the induced charge on this circuit due to the presence of the electron beam. It is used as a substitute for Maxwell's field equations in order to make use of the well-known transmission-line techniques to solve the r-f field problem. This substitution has been verified by Schelkunoff²⁸ for general cases consisting of TE and TM mode waveguide propagation, and is valid in this case when it is assumed that only one space harmonic of the total field is involved in the interaction.

The transmission-line equation represents the circuit interaction as an equivalent network consisting of distributed L, R and C elements. Both forward- and backward-wave circuits can be analyzed in this manner,

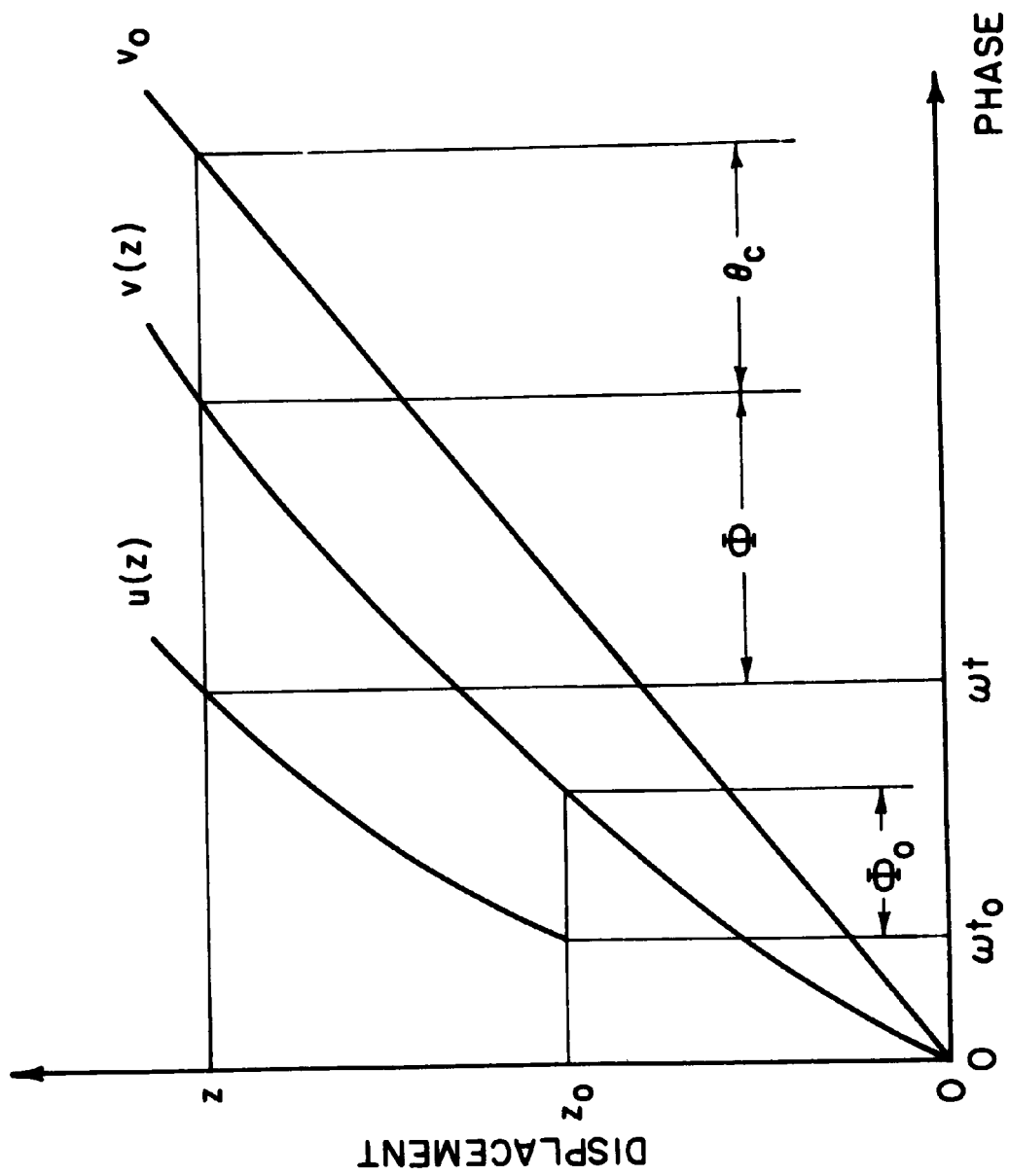


FIG. 2.2 FLIGHT-LINE DIAGRAM FOR ELECTRON AND WAVE VELOCITIES.

the phase velocity and impedance in the forward-wave case being given by

$$v_o = \frac{1}{\sqrt{L_f C_f}} , \quad (2.7)$$

$$Z_o = \sqrt{L_f / C_f} \quad (2.8)$$

and for the backward-wave circuit

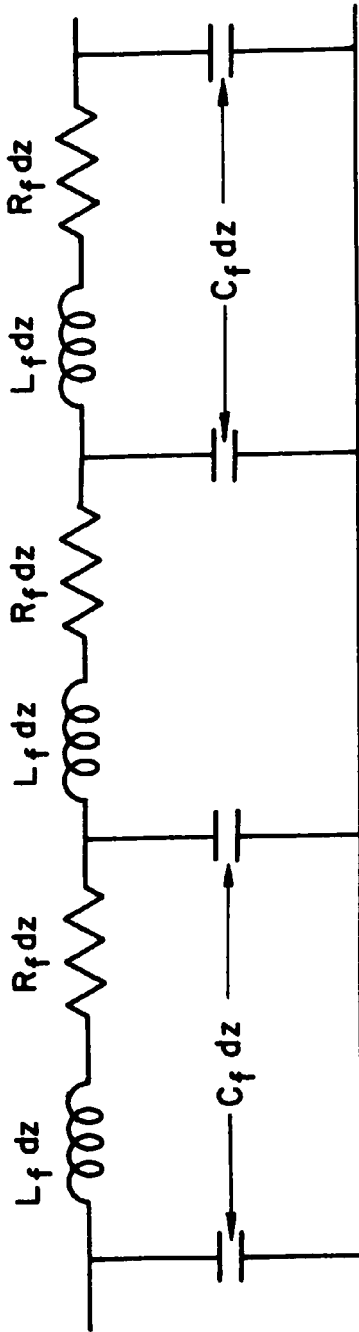
$$v_o = -\omega^2 \sqrt{L_b C_b} , \quad (2.9)$$

$$Z_o = \sqrt{L_b / C_b} . \quad (2.10)$$

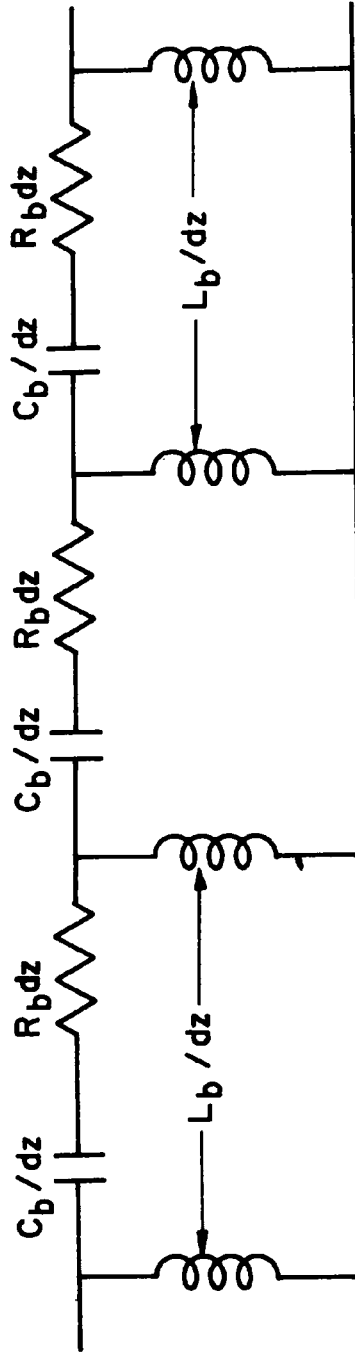
The equivalent networks are illustrated in Fig. 2.3 with resistance elements appearing in each case to account for loss in the circuit. The differential equations for the r-f potential in both the backward- and forward-wave cases can be written as²⁷

$$\begin{aligned} \frac{\partial^2 V_{rf}(z,t)}{\partial t^2} - v_o^2 \frac{\partial^2 V_{rf}(z,t)}{\partial z^2} + 2\omega D d \frac{\partial V_{rf}(z,t)}{\partial t} = \pm v_o Z_o \frac{\partial^2 \rho(z,t)}{\partial t^2} \\ \pm 2\omega D d v_o Z_o \frac{\partial \rho(z,t)}{\partial t} . \quad (2.11) \end{aligned}$$

In Eq. 2.11 the upper sign represents forward-wave interaction and the lower sign represents backward-wave interaction. The resistance was eliminated by defining the loss factor d as $R/L \triangleq 2\omega D d$. The inhomogeneous terms on the right side of Eq. 2.11 are the driving terms with $\rho(z,t)$ representing the induced charge on the circuit created by the electrons in the "stream".



a) FORWARD-WAVE DEVICES



b) BACKWARD-WAVE DEVICES

L_f , L_b = Forward- and Backward-Wave Inductance per Length and Times Length, Respectively.
 C_f , C_b = Forward- and Backward-Wave Capacitance per Length and Times Length, Respectively.
 R_f , R_b = Forward- and Backward-Wave Resistance per Length.

FIG. 2.3 EQUIVALENT CIRCUITS FOR FORWARD- AND BACKWARD-WAVE DEVICES.

To evaluate these driving terms the continuity equation is used, and must be viewed in a form consistent with Lagrangian mechanics. The form to be used arises directly from the conservation of charge which is written as follows:

$$\rho(y,z,t)|dydz| = \rho_o(y_o,z_o) \sum |dy_o dz_o| , \quad (2.12)$$

where the absolute value signs imply that ρ and ρ_o have the same signs, and where $\rho(y,z,t)|dydz|$ is the charge at some position (y,z) at a time $t > 0$ which emanated from that at the input plane (the emitting sole) given by $\rho_o(y_o,z_o) \sum |dy_o dz_o|$. The summation over initial elements of area $|dy_o dz_o|$ accounts for the fact that the space-charge density in several initial elements contribute to the charge density at $|dydz|$ and time t . Also, $\rho_o(y_o,z_o)$ is outside the summation sign because it is assumed to be constant along the entire sole. To obtain $\rho(y,z,t)$ in terms of $\rho_o(y_o,z_o)$ the Jacobian of the transformation from (y_o,z_o) to (y,z) is obtained. Equation 2.12 can be rewritten as

$$\rho(y,z,t) = \rho_o(y_o,z_o) \sum \left| J \left(\frac{y_o,z_o}{y,z} \right) \right| , \quad (2.13)$$

where the Jacobian for one element is given by

$$J = \left(\frac{\partial y_o}{\partial y} \frac{\partial z_o}{\partial z} - \frac{\partial y_o}{\partial z} \frac{\partial z_o}{\partial y} \right) . \quad (2.14)$$

The partial derivatives appearing in this equation will now be evaluated individually to obtain a final expression for J .

From the definitions of Φ and dz/dt the partial derivative $\partial z_o/\partial z$ can be rearranged as

$$\frac{\partial z_o}{\partial z} = \frac{\partial z_o/\partial t}{\partial z/\partial t} = \frac{-\omega/v_o (\partial z_o/\partial \Phi_o) (\partial \Phi_o/\partial \Phi)}{1 + 2Du_z} . \quad (2.15)$$

By definition of Φ , $\Phi_o \triangleq \omega z_o/v_o - \omega t_o - \theta_c(z_o)$, and hence by partial differentiation, Eq. 2.15 becomes

$$\frac{\partial z_o}{\partial z} = \frac{-\partial \Phi_o/\partial \Phi}{(1 + 2Du_z) \{1 - D[\partial \theta_c(q_o)/\partial q_o]\}} , \quad (2.16)$$

where $q_o/D\beta_o$ has been substituted for z_o .

Next, by the chain rule,

$$\frac{\partial y_o}{\partial y} = \frac{\partial y_o}{\partial z_o} \frac{\partial z_o}{\partial y} , \quad (2.17)$$

and by using the trajectory equation, given by

$$y = y_o + \int_0^t \frac{dy}{dt} dt = y_o + \int_0^z \frac{u_y dz}{\omega_c/\omega (1 + 2Du_z)} , \quad (2.18)$$

or in differential form

$$\frac{\partial y}{\partial z} = \frac{u_y}{\omega_c/\omega (1 + 2Du_z)} , \quad (2.19)$$

Eq. 2.17 can be rewritten as

$$\frac{\partial y_o}{\partial y} = \frac{u_{y_o}}{\omega_c/\omega (1 + 2Du_{z_o})} \frac{\partial z_o}{\partial y} . \quad (2.20)$$

Finally, the partial derivative $\partial y_o / \partial z$ is rearranged as

$$\frac{\partial y_o}{\partial z} = \frac{(\partial y_o / \partial \Phi_o)(\partial \Phi_o / \partial \Phi)(\partial \Phi / \partial t)}{\partial z / \partial t} ,$$

but Φ_o is not a function of y_o , and therefore

$$\frac{\partial y_o}{\partial z} \equiv 0 . \quad (2.21)$$

Substitution of Eqs. 2.16, 2.20 and 2.21 into Eq. 2.14 yields the following value for the Jacobian:

$$|J| = \frac{\omega}{\omega_c} v_r \left| \frac{1}{1 + 2Du_z} \right| \left| \frac{\partial z_o}{\partial y} \frac{\partial \Phi_o}{\partial \Phi} \right| , \quad (2.22)$$

where

$$v_r \triangleq \left| \frac{u_{y_o}}{1 + 2Du_{z_o}} \right| \left| \frac{1}{1 - D[\partial \theta_c(q_o) / \partial q_o]} \right| .$$

When Eq. 2.22 is combined with Eq. 2.13 the desired continuity of charge equation is obtained:

$$\rho(y, z, t) = \rho_o(y_o, z_o) \sum \frac{\omega}{\omega_c} v_r \left| \frac{1}{1 + 2Du_z} \right| \left| \frac{\partial z_o}{\partial y} \frac{\partial \Phi_o}{\partial \Phi} \right| . \quad (2.23)$$

To obtain $\rho(z, t)$, the induced charge on the circuit, Ramo's induced current theorem²⁹ is used. This can be formulated to yield the following expression for the charge induced on the r-f structure for a rectangular interaction configuration

$$\rho(z,t) = h \int_0^{b'} \rho(y,z,t) \psi(y) dy, \quad (2.24)$$

where $\psi(y) \triangleq \sinh \beta y / \sinh \beta b'$, b' is the sole-to-circuit spacing, h is the width of the emitting sole and β is the propagation constant of the r-f wave. The coupling function $\psi(y)$ indicates the coupling of charge in the stream to the r-f circuit and also measures the falloff of r-f potential away from the r-f circuit. By using Eq. 2.23, $\rho(z,t)$ may be transformed to a function of Φ and z and is given by

$$\rho(\Phi, z) = \frac{\omega}{\omega_c} h \int_0^{b'} \rho_o(y_o, z_o) \sum \left| \frac{v_r}{1 + 2Du_z(z_o, \Phi_o, z)} \right| \cdot \left| \frac{\partial z_o}{\partial y} \frac{\partial \Phi_o}{\partial \Phi} \right| \psi(y) dy. \quad (2.25)$$

The usual assumption of large interaction of the "stream" electrons with the fundamental component of the total field and negligible interaction with all other harmonics will now be employed. Thus, expanding the charge density into a Fourier series in the periodic variable Φ yields

$$\rho(\Phi, z) = \sum_{n=1}^{\infty} (a_n \sin n\Phi + b_n \cos n\Phi), \quad (2.26)$$

where the coefficients are given by

$$a_n = \frac{1}{\pi} \int_0^{2\pi} \rho(\Phi, z) \sin n\Phi d\Phi \quad (n = 1, 2, \dots) \quad (2.27)$$

and

$$b_n = \frac{1}{\pi} \int_0^{2\pi} \rho(\Phi, z) \cos n\Phi d\Phi \quad (n = 1, 2, \dots) \quad (2.28)$$

By neglecting all harmonics except the fundamental, Eq. 2.26 becomes

$$\rho(\Phi, z) = \rho_1(\Phi, z) = a_1 \sin \Phi + b_1 \cos \Phi \quad (2.29)$$

Substitution of Eq. 2.25 into Eqs. 2.27 and 2.28 (for $n = 1$) yields

$$a_1 = \frac{\omega}{\omega_c} \frac{h}{\pi} \int_0^{2\pi} \sin \Phi \int_0^{b'} \rho_o \sum \left| \frac{v_r}{1 + 2Du_z(y_o, \Phi_o, z)} \right| \left| \frac{\partial z_o}{\partial y} \frac{\partial \Phi_o}{\partial \Phi} \right| \cdot \psi(y) dy d\Phi \quad (2.30)$$

and

$$b_1 = \frac{\omega}{\omega_c} \frac{h}{\pi} \int_0^{2\pi} \cos \Phi \int_0^{b'} \rho_o \sum \left| \frac{v_r}{1 + 2Du_z(y_o, \Phi_o, z)} \right| \left| \frac{\partial z_o}{\partial y} \frac{\partial \Phi_o}{\partial \Phi} \right| \cdot \psi(y) dy d\Phi \quad (2.31)$$

When the indicated summations are carried out and the normalized distance variables $q_o = D(\omega/v_o)z_o$ and $p = y/b'$ are inserted, the fundamental coefficients a_1 and b_1 become

$$a_1 = \frac{\omega}{\omega_c} \frac{h}{\pi} \frac{1}{D\beta_o} \int_0^{2\pi} \sin \Phi \int_0^q \rho_o \left| \frac{v_r}{1 + 2Du_z(y_o, \Phi_o, z)} \right| \psi(p) dq_o d\Phi_o \quad (2.32)$$

and

$$b_1 = \frac{\omega}{\omega_c} \frac{h}{\pi} \frac{1}{D\beta_o} \int_0^{2\pi} \cos \Phi \int_0^q \rho_o \left| \frac{v_r}{1 + 2Du_z(y_o, \Phi_o, z)} \right| \psi(p) dq_o d\Phi_o \quad (2.33)$$

where $\beta_o \triangleq \omega/v_o$, and

$$\psi(p) \triangleq \frac{\sinh \beta b' p}{\sinh \beta b'} . \quad (2.34)$$

Substituting the fundamental component of induced space-charge density into the general transmission-line expression, Eq. 2.11, for propagation in the z-direction, and changing to the longitudinal normalized variable q yields

$$\frac{\partial^2 V_{rf}(q, \Phi)}{\partial t^2} - \omega^2 D^2 \frac{\partial^2 V_{rf}(q, \Phi)}{\partial q^2} + 2\omega D d \frac{\partial V_{rf}(q, \Phi)}{\partial t} = \pm v_o Z_o \left[\frac{\partial^2 \rho_1(q, \Phi)}{\partial t^2} + 2\omega D d \frac{\partial \rho_1(q, \Phi)}{\partial t} \right] . \quad (2.35)$$

This equation may be put into a more convenient form by defining the r-f potential in terms of a normalized amplitude $A_{rf}(q)$ and phase $\Phi(q, t)$ by the relation

$$V_{rf}(q, \Phi) \triangleq \text{Re} \left[\frac{Z_o I_o}{D} A_{rf}(q) \exp(-j\Phi) \right] , \quad (2.36)$$

where Z_o = characteristic impedance of the r-f circuit,

I_o = thermionic emission current at the sole,

$D \triangleq [(\omega_c/\omega)(I_o Z_o/2V_{oc})]^{1/2}$, the gain parameter of the tube, and

$V_{oc} = v_o^2/2\eta$, the d-c voltage equivalent of the unperturbed wave velocity (where η is the charge-to-mass ratio of an electron).

Substituting Eq. 2.36 along with the integral expression for ρ_1 into Eq. 2.35 and equating the coefficients of the orthogonal functions $\sin \Phi$ and $\cos \Phi$ to zero yields two integro-differential equations as follows:

$$\begin{aligned}
& \frac{1}{D} \frac{dA_{rf}}{dq} \left(1 - D \frac{d\theta_c}{dq} \right) + A_{rf} \left(\frac{d}{D} - \frac{1}{2} \frac{d^2\theta_c}{dq^2} \right) \\
& = \pm \frac{\omega/\omega_c (v_o/v_c)}{2\pi\beta_o L_e D^2} \left[\int_0^{2\pi} \sin \Phi \int_0^q \frac{v_r |\rho'_o|}{|1 + 2Du_z|} \psi dq_o d\Phi_o \right. \\
& \quad \left. - 2Dd \int_0^{2\pi} \cos \Phi \int_0^q \frac{v_r |\rho'_o|}{|1 + 2Du_z|} \psi dq_o d\Phi_o \right] , \quad (2.37)
\end{aligned}$$

$$\begin{aligned}
& \frac{d^2 A_{rf}}{dq^2} + \frac{A_{rf}}{D^2} \left[1 - \left(1 - D \frac{d\theta_c}{dq} \right)^2 \right] \\
& = \mp \frac{\omega/\omega_c (v_o/v_c)}{\pi\beta_o L_e D^2} \left[\int_0^{2\pi} \cos \Phi \int_0^q \frac{v_r |\rho'_o|}{|1 + 2Du_z|} \psi dq_o d\Phi_o \right. \\
& \quad \left. + 2Dd \int_0^{2\pi} \sin \Phi \int_0^q \frac{v_r |\rho'_o|}{|1 + 2Du_z|} \psi dq_o d\Phi_o \right] , \quad (2.38)
\end{aligned}$$

where $v_c \triangleq (2kT/m)^{1/2}$ and the negative sign has been extracted from ρ_o .

Also, the equation

$$\rho_o = - | (J_{th}/v_c) \rho'_o | , \quad (2.39)$$

where $J_{th} \triangleq I_o/hL_e$, derived in Appendix A, has been substituted. Equations 2.37 and 2.38 are the circuit equations in final form which must be coupled with four other relations between the dependent variables in order to solve for the six unknowns A_{rf} , θ_c , Φ , u_y , u_z and p . Two other equations will be derived from the Lorentz force equation, and the remaining two equations will be obtained from the defining relations of the dependent variables.

2.5 Force Equations

The motion of charged particles in electric and magnetic fields is described by the Lorentz force equation

$$\frac{d\vec{v}}{dt} = \eta(-\nabla V_T + \vec{v} \times \vec{B}) \quad , \quad (2.40)$$

where $\vec{v}$ is the velocity vector of a charged particle, t is time, ∇V_T is the gradient of the total potential and $\vec{B}$ is the magnetic field vector. Applying Eq. 2.40 to the specific case of electrons moving in a two-dimensional electromagnetic field including the space-charge field yields the following Eulerian equations of motion:

$$\frac{dv_z}{dt} = |\eta| \frac{\partial V}{\partial z} + \omega_c v_y - |\eta| E_{sc-z} \quad (2.41)$$

and

$$\frac{dv_y}{dt} = |\eta| \frac{\partial V}{\partial y} - \omega_c v_z - |\eta| E_{sc-y} \quad , \quad (2.42)$$

where E_{sc-y} and E_{sc-z} are the y- and z-components of the space-charge field, v_y and v_z are the y- and z-components of velocity, and V is the sum of d-c and r-f potentials.

The independent variables appearing in these equations are time and displacement which correspond to the Eulerian or small-signal variables. To transform these equations to the Lagrangian system, distance and time are eliminated in terms of normalized distances (p and q) and the initial phase (Φ_0). Also, the dependent phase variable Φ and normalized y- and z-velocities will be used.

To transform Eq. 2.41, note by the properties of differentiation that

$$\frac{dv_z}{dt} = \frac{\partial v_z}{\partial q} \frac{dq}{dt} = \frac{d^2 z}{dt^2} \quad (2.43)$$

and also from Eq. 2.5 that

$$\frac{dq}{dt} = \omega D [1 + 2Du_z(z_o, \Phi_o, z)] \quad (2.44)$$

Thus, from Eq. 2.5,

$$\frac{\partial v_z}{\partial q} = 2v_o D \frac{\partial u_z}{\partial q} \quad (2.45)$$

and therefore from Eqs. 2.43 through 2.45

$$\frac{dv_z}{dt} = 2v_o \omega D^2 \frac{\partial u_z}{\partial q} (1 + 2Du_z) \quad (2.46)$$

Substituting Eq. 2.46 into Eq. 2.41 and eliminating z in terms of q yields

$$\frac{\partial u_z}{\partial q} (1 + 2Du_z) = \frac{1}{2v_o \omega D^2} \left(\frac{D\omega|\eta|}{v_o} \frac{\partial V}{\partial q} + \omega_c v_y - |\eta| E_{sc-z} \right) \quad (2.47)$$

To transform Eq. 2.42, the normalized y -velocity (Eq. 2.4) is used. Making a change in the independent variable, differentiation of the y -velocity and substitution of the normalized form yields

$$\frac{dv_y}{dt} = \frac{\partial v_y}{\partial q} \frac{dq}{dt} = \frac{v_o \omega^2 D}{\omega_c} \frac{\partial u_y}{\partial q} (1 + 2Du_z) \quad (2.48)$$

Substitution of Eqs. 2.5 and 2.48 into Eq. 2.42 yields

$$(1 + 2Du_z) \left[\frac{\partial u_y}{\partial q} + \left(\frac{\omega_c}{\omega} \right)^2 \frac{1}{D} \right] = \frac{|\eta|}{b'} \left(\frac{\omega_c}{v_o \omega^2 D} \right) \frac{\partial V}{\partial q} - \frac{\omega_c |\eta|}{v_o \omega^2 D} E_{sc-y} , \quad (2.49)$$

where a transformation of independent variables ($p = y/b'$) has also been carried out.

The scalar potential V is the sum of both the d-c and r-f components and in general is given by

$$V(y, z, t) = V_{dc}(y, z) + V_{rf}(y, z, t) . \quad (2.50)$$

This will now be obtained in normalized form before substitution into Eqs. 2.47 and 2.49.

The r-f potential can be written as

$$V_{rf}(z, t) = V(z, t) \psi(y) , \quad (2.51)$$

where

$$\psi(y) = \frac{\sinh \beta y}{\sinh \beta b'} \quad (2.52)$$

as can be seen if the wave equation is solved assuming that a slow r-f wave is propagating in the z -direction along a rectangular structure with propagation constant β . In terms of the normalized distance variables p and q , and the phase Φ , the total r-f potential can be normalized as in Eq. 2.36; that is,

$$V_{rf}(p, q, \Phi) \triangleq \text{Re} \left[\frac{Z_o I_o}{D} A_{rf}(q) \psi(p) \exp(-j\Phi) \right] . \quad (2.53)$$

The d-c potential $V_{dc}(y, z)$ can be similarly written as a product of functions. The d-c potential is assumed to be a function of z , as well as y , to allow for a variation in the d-c potential with longitudinal distance. The z -variation of the d-c potential, which is considered to be a known quantity, will be denoted as $A_{dc}(z)$ and the normal y -variation by $\xi(y)$. For a linear untapered geometry A_{dc} is a constant dependent upon the anode potential and $\xi(y) = p$. Thus, the d-c potential can be defined in general as

$$V_{dc}(y, z) \triangleq \frac{Z_o I_o}{D} A_{dc}(z) \xi(y) , \quad (2.54)$$

or in terms of normalized distance variables,

$$V_{dc}(p, q) \triangleq \frac{Z_o I_o}{D} A_{dc}(q) \xi(p) . \quad (2.55)$$

Substituting Eqs. 2.55 and 2.53 into Eq. 2.50 yields

$$V(p, q, \Phi) = \frac{Z_o I_o}{D} \left[A_{dc}(q) \xi(p) + A_{rf}(q) \psi(p) \cos \Phi \right] . \quad (2.56)$$

Substituting this expression into Eqs. 2.47 and 2.49 and using the explicit expressions for the space-charge-field terms as derived in Appendix B yields

$$\begin{aligned} \frac{\partial u_z}{\partial q} (1 + 2Du_z) &= \frac{\omega}{2\omega_c} \left[\frac{dA_{dc}}{dq} \xi + \frac{dA_{rf}}{dq} \psi \cos \Phi - \frac{A_{rf}}{D} \left(1 - D \frac{d\theta_c}{dq} \right) \right. \\ &\quad \left. + \psi \sin \Phi \right] + \frac{u_y}{2D^2} + \frac{(\omega_{pc}/\omega)(\omega_{pc}/\omega_c) |\rho'_o| v_r}{\pi \beta_o b' D^3} I_z \end{aligned} \quad (2.57)$$

and

$$(1 + 2Du_z) \left[\frac{\partial u_y}{\partial q} + \left(\frac{\omega_c}{\omega} \right)^2 \frac{1}{D} \right] = \frac{1}{\beta_o b'} \left\{ A_{dc} \frac{d\xi}{dp} + \beta_o b' A_{rf} \cos \Phi \right. \\ \left. \cdot \left(1 - D \frac{d\theta_c}{dq} \right) \frac{\cosh \left[\beta_o b' \left(1 - D \frac{d\theta_c}{dq} \right) p \right]}{\sinh \left[\beta_o b' \left(1 - D \frac{d\theta_c}{dq} \right) \right]} \right\} - \frac{2 \left(\frac{\omega_{pc}}{\omega} \right)^2 |\rho_o'| v_r}{\pi \beta_o b' D^2} I_y . \quad (2.58)$$

Equations 2.57 and 2.58 are the final continuous form of the large-signal force equations.

2.6 Relations Between Dependent Variables

Thus far, four equations have been derived containing the unknowns A_{rf} , Φ , θ_c , u_z , u_y and p . Two more independent equations are needed and may be obtained from the definitions of these variables in terms of one another. One equation is obtained by differentiating the dependent variable Φ with respect to the time t ; that is, Eq. 2.6 is rearranged as

$$q = D(\Phi + \omega t + \theta_c) \quad (2.59)$$

and, by differentiating with respect to t , the result is

$$\frac{dq}{dt} = D \left(\frac{\partial \Phi}{\partial q} \frac{dq}{dt} + \omega + \frac{d\theta_c}{dq} \frac{dq}{dt} \right) . \quad (2.60)$$

However

$$\frac{dq}{dt} \triangleq \omega D(1 + 2Du_z) , \quad (2.61)$$

and therefore substitution of Eq. 2.61 into Eq. 2.60 gives

$$\frac{\partial \Phi}{\partial q} + \frac{d\theta_c}{dq} = \frac{2u_z}{1 + 2Du_z} . \quad (2.62)$$

To obtain the second equation relating the dependent variables, note that the vertical position can be written as follows:

$$y = y_o + \int_0^t \frac{dy}{dt} dt . \quad (2.63)$$

Substitution of the normalized variables p and u_y , and eliminating t in terms of q , gives

$$p = p_o + \frac{1}{\beta_c b'D} \int_0^q \frac{u_y dq}{1 + 2Du_z} , \quad (2.64)$$

where β_c is defined as ω_c/v_o . By combining Eqs. 2.62 and 2.64 with the two circuit equations and the two force equations, the two-dimensional emitting-sole problem can be solved.

2.7 Large-Signal Equations for the Emitting-Sole Device

The following six equations are continuous equations containing A_{rf} , Φ , θ_c , u_z , u_y and p as continuous dependent variables, and are the complete set of large-signal equations for the emitting-sole device.

Force Equations

$$\frac{\partial u_z}{\partial q} (1 + 2Du_z) = \frac{\omega}{2\omega_c} \left[\frac{dA_{dc}}{dq} \xi + \frac{dA_{rf}}{dq} \psi \cos \Phi - \frac{A_{rf}}{D} \left(1 - D \frac{d\theta_c}{dq} \right) \right. \\ \left. \cdot \psi \sin \Phi \right] + \frac{u_y}{2D^2} + \frac{(\omega_{pc}/\omega)(\omega_{pc}/\omega_c) |\rho'_o| v_r}{\pi \beta_o b' D^3} I_z, \quad (2.65)$$

$$(1 + 2Du_z) \left[\frac{\partial u_y}{\partial q} + \left(\frac{\omega_c}{\omega} \right)^2 \frac{1}{D} \right] = \frac{1}{\beta_o b'} \left\{ A_{dc} \frac{d\xi}{dp} + \beta_o b' A_{rf} \cos \Phi \right. \\ \left. \cdot \left(1 - D \frac{d\theta_c}{dq} \right) \frac{\cosh \left[\beta_o b' \left(1 - D \frac{d\theta_c}{dq} \right) p \right]}{\sinh \left[\beta_o b' \left(1 - D \frac{d\theta_c}{dq} \right) \right]} \right\} - \frac{2 \left(\frac{\omega_{pc}}{\omega} \right)^2 |\rho'_o| v_r}{\pi \beta_o b' D^2} I_y. \quad (2.66)$$

Circuit Equations

$$\frac{d^2 A_{rf}}{dq^2} + \frac{A_{rf}}{D^2} \left[1 - \left(1 - D \frac{d\theta_c}{dq} \right)^2 \right] = \mp \frac{\omega/\omega_c (v_o/v_c)}{\pi \beta_o L_e D^2} \\ \cdot \int_0^{2\pi} \int_0^q \frac{v_r |\rho'_o| \psi(p)}{|1 + 2Du_z|} (\cos \Phi + 2Dd \sin \Phi) dq_o d\Phi_o, \quad (2.67)$$

$$\frac{1}{D} \frac{dA_{rf}}{dq} \left(1 - D \frac{d\theta_c}{dq} \right) + A_{rf} \left(\frac{d}{D} - \frac{1}{2} \frac{d^2 \theta_c}{dq^2} \right) = \pm \frac{\omega/\omega_c (v_o/v_c)}{2\pi \beta_o L_e D^2} \\ \cdot \int_0^{2\pi} \int_0^q \frac{v_r |\rho'_o| \psi(p)}{|1 + 2Du_z|} (\sin \Phi - 2Dd \cos \Phi) dq_o d\Phi_o. \quad (2.68)$$

Relations Between Dependent Variables

$$\frac{\partial \Phi}{\partial q} + \frac{\partial \theta_c}{\partial q} = \frac{2u_z}{1 + 2Du_z}, \quad (2.69)$$

$$p = p_o + \frac{1}{\beta_c b' D} \int_0^q \frac{u_y dq}{1 + 2Du_z}. \quad (2.70)$$

The above equations are solved using a digital computer and the equations must therefore be transformed into difference form. This will be carried out in the next chapter. The next section discusses the initial conditions imposed on the dependent variables.

2.8 Initial Value Problem

The six nonlinear integro-differential equations are to be solved in conjunction with specified boundary conditions or initial conditions. Since the nonlinearities of the equations necessitate the use of a digital computer, and since a boundary-value problem would require a trial and error method of solution, an initial value problem will be specified and eventually solved.

The differential equations themselves dictate the number of initial conditions which must be specified; that is, if the first derivative of a dependent variable appears in any of the equations, only the initial value of the variable need be specified, while if the second derivative of a variable appears, both the variable and its first derivative must be known initially. Thus initial values must be given for A_{rf} and θ_c and their derivatives while only the initial values are needed for the variables u_y , u_z , p and Φ .

2.8.1 Initial R-f Conditions. The magnitude of the input r-f signal is given by

$$A_{rf} \Big|_{q=0} = A_{rf_0} . \quad (2.71)$$

Also, since interaction between the electron and the wave has not yet occurred, there is no perturbation of the traveling wave at the input by the beam and hence

$$\theta_c \Big|_{q=0} = 0 . \quad (2.72)$$

To determine the remaining initial r-f conditions on A'_{rf} and θ'_c , recall the definition of the r-f potential:

$$V_{rf}(y, z, t) \triangleq \text{Re} \left\{ \frac{I_o Z_o}{D} \frac{\sinh \beta y}{\sin \beta b} A_{rf}(z) \exp[-j\Phi(z, t)] \right\}$$

or at the input

$$V_{rf}(y, z, t) = \text{Re} \left\{ V_{in} \frac{\sinh \beta y}{\sinh \beta b} \exp[j(\omega t - \beta_o z)] \right\} , \quad (2.73)$$

where V_{in} is the peak r-f input voltage. Differentiating with respect to z , setting $z = 0$ and equating the above two expressions yields

$$-j\beta_o V_{in} = \frac{I_o Z_o}{D} \left(\frac{dA_{rf}}{dz} - jA_{rf} \frac{\partial \Phi}{\partial z} \right) \Big|_{z=0} . \quad (2.74)$$

Equating the real part of this equation to zero and substituting q for z gives

$$\frac{dA_{rf}}{dq} \Big|_{q=0} = 0 , \quad (2.75)$$

which is the second condition needed for A_{rf} . Similarly, equating imaginary parts in Eq. 2.74 yields

$$\frac{\partial \Phi}{\partial z} = \frac{\beta_o V_{in}}{I_o Z_o A_{rf_o} / D} = \beta_o . \quad (2.76)$$

Also from the definition of Φ

$$\frac{\partial \Phi}{\partial z} = \frac{\omega}{v_o} - \frac{d\theta_c}{dz} , \quad (2.77)$$

and by equating Eqs. 2.76 and 2.77, and substituting q for z ,

$$\left. \frac{d\theta_c}{dq} \right|_{q=0} = 0 , \quad (2.78)$$

which is the other initial condition required for θ_c .

2.8.2 Initial Electron Coordinates. Each electron in the interaction region between the emitting sole and the anode begins its flight at the sole electrode. The trajectory of this flight is characterized by position and velocity coordinates, and the initial values of these quantities must be specified in order to complete the statement of an initial value problem.

The initial position of an electron in the selected normalized coordinates is given by

$$(p, q, \Phi)_i = (0, q_o, \Phi_o) . \quad (2.79)$$

The quantities q_o and ϕ_o are labels for each electron, and represent the relative longitudinal entrance position and the relative time of entry, respectively, for each electron.

The magnitude of the average emission velocity of an electron is given by

$$|\vec{v}|_i = |\vec{J}_{th}/\rho_o| , \quad (2.80)$$

where $\vec{J}_{th}$ is the thermionic emission current and ρ_o is the charge density at the sole (as given in Appendix A). This velocity is small compared to the average d-c field electron velocity obtained in the interaction region.

In order to separate the total initial velocity into vertical and longitudinal components, electrons are assumed to be emitted in four directions, distributed symmetrically about the normal as shown in Fig. 2.4. The magnitude of the total initial velocity is assumed to be the same in each of these directions. However the y- and z-components vary with the direction of emission as follows:

$$(v_y, v_z)_i = (|\vec{v}|_i \cos \Omega, |\vec{v}|_i \sin \Omega) , \quad (2.81)$$

where the angle Ω takes on the values $-\beta\alpha$, $-\alpha$, α and $\beta\alpha$ to achieve symmetric emission. Thus the normalized initial electron velocities are given by

$$(u_y, u_z)_i = \left(\frac{\omega_c}{\omega} \frac{|\vec{v}|_i}{v_o} \cos \Omega, \frac{|\vec{v}|_i \sin \Omega - v_o}{2Dv_o} \right) . \quad (2.82)$$

A further discussion of the initial electron velocities is given in Appendix D.

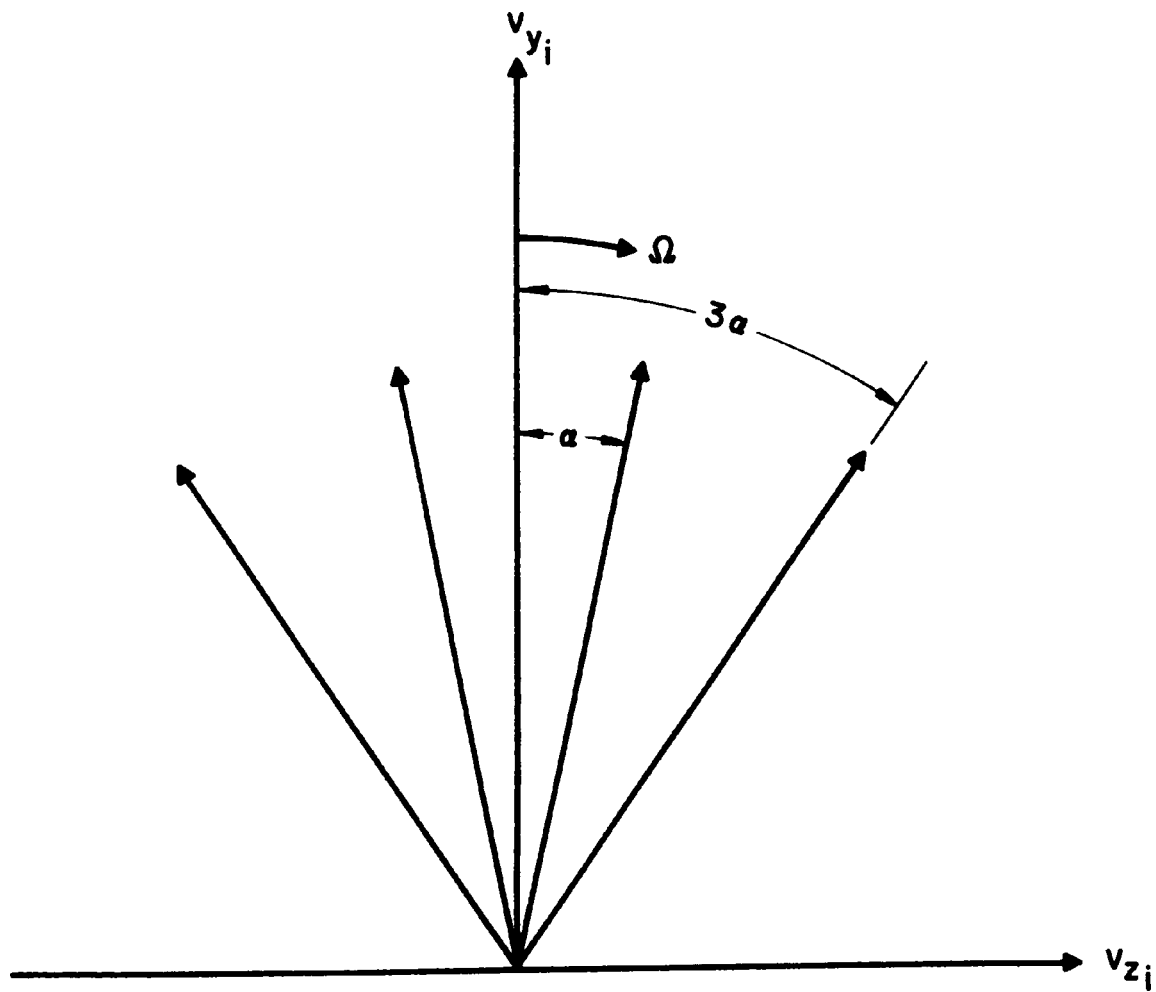


FIG. 2.4 ELECTRON EMISSION DIRECTIONS.

Finally, the initial value of the phase Φ of an electron relative to the r-f wave must be specified. This initial phase is labeled Φ_0 and in the continuous case is given by

$$\Phi_0 = \frac{\omega z_0}{v_0} - \omega t_0 - \theta_c(z_0) \quad , \quad (2.83)$$

where z_0 is the initial longitudinal coordinate of the electron and t_0 is its entry time. The digital computer model specifies this quantity more explicitly and this will be discussed in the next chapter.

The initial value problem is thus completely specified by the six integro-differential equations and the above initial conditions.

CHAPTER III. DESCRIPTION OF THE DIGITAL COMPUTER SOLUTION METHOD

3.1 Introduction

The six integro-differential equations for the emitting-sole device have been presented in the previous chapter. These equations are very nonlinear and thus an exact analytical solution cannot be carried out.

There are a number of numerical methods which can be used to solve a set of nonlinear equations. One of the most obvious is to replace the nonlinear portions of the problem by linear ones. Thus, nonlinear problems can be solved by breaking them up into "piecewise-linear" problems, a method which has been widely used in the analysis of vacuum tube circuits. Such a process is carried out by making power series expansions of the dependent variables around a starting point, and then obtaining the solution for that starting point while ignoring second- and higher-order terms. A new starting point is then obtained and the process can be carried on as before. This method of solution can be employed regardless of the degree of nonlinearity as long as the increments in the power series expansions are chosen to be sufficiently small. This is the method of solution which will be used to solve the emitting-sole equations. Since the degree of nonlinearity is quite large for this problem, very small increments must be used in the expansions. This means that many iterations are necessary and the use of a digital computer is warranted. The IBM-7090 data processing system was the machine available for this purpose.

The numerical calculations will be carried out by also assuming a finite number of charge groups or electrons to be present (the words "charge group" and "electron" will be used interchangeably throughout this dissertation to denote one of the assumed finite number of groups of charged particles). These electrons are actually rods of charge in the selected two-dimensional geometry.

3.2 Explanation of Dependent Variable Indices

Keeping track of the dependent variables for each electron (charge group) in the discrete problem requires an efficient method of symbolism. The method used is basically one of indexing. Each electron has four dependent variables (called per electron quantities) which are normalized y- and z-directed velocities, normalized height p and phase ϕ . In the exact analysis these functions are continuous, but due to the assumption of a finite number of electrons they now become discrete. The discrete variables will be given three subscripts and will appear as follows:

$$u_{y_{i,j,k}}^{}, \quad u_{z_{i,j,k}}^{}, \quad \phi_{i,j,k} \quad \text{and} \quad p_{i,j,k},$$

where the index i indicates evaluation of the function at the ith q-plane, the index j indicates that the function is to represent an "electron" emitted from the jth emission point, and k stands for the kth electron out of the total which is emitted from each emission point. The emission points are numbered in order from $j = 1$ to M as one progresses along the tube in the longitudinal direction, while the electrons from each emission point are numbered from $k = 1$ to m. Thus,

for example, for the thirteenth electron from the third emission point at a particular q-plane (the i th), the per electron quantities are given by

$$u_{y_{i,3,13}} , u_{z_{i,3,13}} , \phi_{i,3,13} \text{ and } p_{i,3,13} .$$

3.3 Large-Signal Equations in Difference Form

The method of solution for the emitting-sole device equations, that is, a piecewise-linear solution with very small increments for each linear segment, has been outlined above. To apply the method and obtain the equations in difference form, it is necessary to substitute the familiar Taylor series expansion of a function $F(x)$ about a point $x = x_0$ for each of the unknowns. The exact expansion is given by

$$F(x) = F(x_0) + F'(x_0)(x - x_0) + \frac{1}{2!} F''(x_0)(x - x_0)^2 + \dots \\ + \frac{1}{n!} F^n(x_0)(x - x_0)^n + \dots \quad (3.1)$$

Considering only the first two terms, the expansion becomes

$$F(x) \approx F(x_0) + F'(x_0)(x - x_0) , \quad (3.2)$$

or by transforming to discrete notation,

$$F_{l+1} = F_l + F'_l \Delta x . \quad (3.3)$$

Thus using the notation of the previous section, the dependent variables at the (i+1)th q-plane may be written in terms of the ith q-plane variables as follows:

$$A_{rf_{i+1}} = A_{rf_i} + A'_{rf_i} \Delta q , \quad (3.4)$$

$$\theta_{c_{i+1}} = \theta_{c_i} + \theta'_{c_i} \Delta q , \quad (3.5)$$

$$u_{y_{i+1,j,k}} = u_{y_{i,j,k}} + u'_{y_{i,j,k}} \Delta q , \quad (3.6)$$

$$u_{z_{i+1,j,k}} = u_{z_{i,j,k}} + u'_{z_{i,j,k}} \Delta q , \quad (3.7)$$

$$\phi_{i+1,j,k} = \phi_{i,j,k} + \phi'_{i,j,k} \Delta q \quad (3.8)$$

and

$$p_{i+1,j,k} = p_{i,j,k} + p'_{i,j,k} \Delta q , \quad (3.9)$$

where the primes refer to derivatives with respect to the independent variable q. Substitution of these expansions into the continuous emitting-sole device equations and solving for the (i+1)th variables yields the following discrete equations.

Circuit Equations

$$A_{rf_{i+1}} = \frac{A_{rf_i}}{D} \left[-2\Delta q (\theta_{c_{i+1}} - \theta_{c_i}) + D(\theta_{c_{i+1}} - \theta_{c_i})^2 \right] + \Delta q^2 \Lambda_i + 2A_{rf_i} - A_{rf_{i-1}}, \quad (3.10)$$

$$A_{rf_{i+1}} = A_{rf_i} + \frac{-A_{rf_i} \left[d - \frac{D}{2} \left(\frac{\theta_{c_{i+1}} - 2\theta_{c_i} + \theta_{c_{i-1}}}{(\Delta q)^2} \right) \right] + \lambda_i D}{1 - D \left(\frac{\theta_{c_{i+1}} - \theta_{c_i}}{\Delta q} \right)} \Delta q, \quad (3.11)$$

where Λ and λ represent the induced current terms defined by

$$\Lambda_i \triangleq \mp K_1 \left(\int_0^{2\pi} \int_0^{q_i} \cos \phi_{i,j,k} \frac{|\rho'_o|}{|1 + 2Du_{z_{i,j,k}}|} \psi_{i,j,k}^{dq_{o,j,k}} \phi_{o,j,k}^{d\phi_{o,j,k}} + 2Dd \int_0^{2\pi} \int_0^{q_i} \sin \phi_{i,j,k} \frac{|\rho'_o|}{|1 + 2Du_{z_{i,j,k}}|} \psi_{i,j,k}^{dq_{o,j,k}} \phi_{o,j,k}^{d\phi_{o,j,k}} \right) \quad (3.12)$$

and

$$\lambda_i \triangleq \pm \frac{K_1}{2} \left(\int_0^{2\pi} \int_0^{q_i} \sin \phi_{i,j,k} \frac{|\rho'_o|}{|1 + 2Du_{z_{i,j,k}}|} \psi_{i,j,k}^{dq_{o,j,k}} \phi_{o,j,k}^{d\phi_{o,j,k}} - 2Dd \int_0^{2\pi} \int_0^{q_i} \cos \phi_{i,j,k} \frac{|\rho'_o|}{|1 + 2Du_{z_{i,j,k}}|} \psi_{i,j,k}^{dq_{o,j,k}} \phi_{o,j,k}^{d\phi_{o,j,k}} \right), \quad (3.13)$$

where

$$K_1 \triangleq \frac{v_r (\omega/\omega_c) (v_o/v_c)}{\pi \beta_o L_e D^2} .$$

Force Equations

$$\begin{aligned} u_{z_{i+1},j,k} = & u_{z_{i,j,k}} + \frac{\Delta q}{1 + 2Du_{z_{i,j,k}}} \left\{ \frac{\omega}{2\omega_c} \left[\left(\frac{A_{dc_{i+1}} - A_{dc_i}}{\Delta q} \right) \xi_{i,j,k} \right. \right. \\ & + \left(\frac{A_{rf_{i+1}} - A_{rf_i}}{\Delta q} \right) \psi_{i,j,k} \cos \phi_{i,j,k} - \frac{A_{rf_i}}{D} \left(1 - D \left\{ \frac{\theta_{c_{i+1}} - \theta_{c_i}}{\Delta q} \right\} \right) \\ & \left. \left. \cdot \psi_{i,j,k} \sin \phi_{i,j,k} \right] + \frac{u_{y_{i,j,k}}}{2D^2} + \frac{(\omega_{pc}/\omega) (\omega_{pc}/\omega_c) |\rho_o'| v_r}{\pi \beta_o b' D^3} I_{z_i} \right\} \quad (3.14) \end{aligned}$$

and

$$\begin{aligned} u_{y_{i+1},j,k} = & u_{y_{i,j,k}} - \left(\frac{\omega_c}{\omega} \right)^2 \frac{\Delta q}{D} + \frac{\Delta q}{1 + 2Du_{z_{i,j,k}}} \left\{ \frac{1}{\beta_o b'} \left[A_{dc} \left(\frac{d\xi}{dp} \right)_{i,j,k} \right. \right. \\ & + \beta_o b' A_{rf_i} \cos \phi_{i,j,k} \left(1 - D \left\{ \frac{\theta_{c_{i+1}} - \theta_{c_i}}{\Delta q} \right\} \right) \phi_{i,j,k} \left. \right] \\ & \left. - \frac{2|\rho_o'| v_r (\omega_{pc}/\omega)^2}{\pi \beta_o b' D^2} I_{y_i} \right\} , \quad (3.15) \end{aligned}$$

where I_{y_i} and I_{z_i} are defined in Appendix B, and where

$$\psi_{i,j,k} \triangleq \frac{\sinh \left\{ \beta_o b' \left[1 - D \left(\frac{\theta_{c_{i+1}} - \theta_{c_i}}{\Delta q} \right) \right] p_{i,j,k} \right\}}{\sinh \left\{ \beta_o b' \left[1 - D \left(\frac{\theta_{c_{i+1}} - \theta_{c_i}}{\Delta q} \right) \right] \right\}} \quad (3.16)$$

and

$$\varphi_{i,j,k} \triangleq \frac{\cosh \left\{ \beta_o b' \left[1 - D \left(\frac{\theta_{c_{i+1}} - \theta_{c_i}}{\Delta q} \right) \right] p_{i,j,k} \right\}}{\sinh \left\{ \beta_o b' \left[1 - D \left(\frac{\theta_{c_{i+1}} - \theta_{c_i}}{\Delta q} \right) \right] \right\}} \quad (3.17)$$

Relations Between Dependent Variables

$$\phi_{i+1,j,k} = \phi_{i,j,k} + \left(\frac{2u_{z_{i,j,k}}}{1 + 2Du_{z_{i,j,k}}} \right) \Delta q - \theta_{c_{i+1}} + \theta_{c_i} \quad (3.18)$$

and

$$p_{i+1,j,k} = p_{i,j,k} + \frac{\Delta q}{\beta_c b' D} \left(\frac{u_{y_{i,j,k}}}{1 + 2Du_{z_{i,j,k}}} \right) \quad (3.19)$$

To obtain the general equations above, it was necessary to assume that the entering charge consists of a finite number of electrons given by N . This means that the six difference-form equations now represent $(4N+2)$ equations ($2N$ force equations, $2N$ relations between dependent electron variables and 2 circuit equations).

The $(4N+2)$ difference-form equations presented above are not quite in soluble form, since there is no explicit expression for the

dependent variable $\theta_{c_{i+1}}$. This is easily obtained by eliminating $A_{rf_{i+1}}$ from the two circuit equations. The result upon solving for $\theta_{c_{i+1}}$ is the following cubic equation:

$$\theta_{c_{i+1}}^3 + \epsilon_1 \theta_{c_{i+1}}^2 + \epsilon_2 \theta_{c_{i+1}} = \theta_{c_i}^3 + \epsilon_3 \theta_{c_i}^2 + \epsilon_4 \theta_{c_i} + \epsilon_5, \quad (3.20)$$

where $\epsilon_1 \triangleq - \left(\frac{3\Delta q}{D} + 3\theta_{c_i} \right),$

$$\epsilon_2 \triangleq \Delta q^2 \left(\frac{2}{D^2} + \frac{\Lambda_i}{A_{rf_i}} \right) + 6 \frac{\theta_{c_i}}{D} \Delta q + 3\theta_{c_i}^2 - \frac{A_{rf_{i-1}}}{A_{rf_i}} + \frac{3}{2},$$

$$\epsilon_3 \triangleq \frac{3\Delta q}{D},$$

$$\epsilon_4 \triangleq \Delta q^2 \left(\frac{2}{D^2} + \frac{\Lambda_i}{A_{rf_i}} \right) - \frac{A_{rf_{i-1}}}{A_{rf_i}} + 2$$

and

$$\epsilon_5 \triangleq \Lambda_i \frac{\Delta q^3}{A_{rf_i} D} + \Delta q^2 \left(\frac{d}{D} - \frac{\lambda_i}{A_{rf_i}} \right) + \frac{\Delta q}{D} \left(1 - \frac{A_{rf_{i-1}}}{A_{rf_i}} \right) - \frac{\theta_{c_{i-1}}}{2}.$$

This equation is now used in place of the second circuit equation and is solved by using the well-known Newton-Raphson process.

With initial values for the dependent variables given at the i th q -plane, the six difference equations, Eqs. 3.10, 3.14, 3.15, 3.18, 3.19 and 3.20, can be solved for the six dependent variables at the $(i+1)$ th q -plane. The procedure that the computer follows to obtain these solutions can be explained as follows. The two induced-current integrals are computed first, thus enabling solution of Eq. 3.20 for $\theta_{c_{i+1}}$ and hence $A_{rf_{i+1}}$ from Eq. 3.10. Next, the space-charge integrals

are calculated, thereby enabling $u_{z_{i+1,j,k}}$ and $u_{y_{i+1,j,k}}$ to be obtained from the force equations. Finally, the variables $\Phi_{i+1,j,k}$ and $P_{i+1,j,k}$ are calculated from Eqs. 3.18 and 3.19. The index i is then increased by one and the process is carried out again, continuing until a suitable number of q -planes have been passed. A simplified flow diagram for this process is shown in Fig. 3.1.

3.4 Computer Simulation of Electron Emission

3.4.1 Primary Electron Emission and Subsequent Calculations.

In the emitting-sole amplifier, an r-f wave is propagated along the slow-wave r-f structure and the fields of this wave interact with electrons emitted from the sole. The digital computer simulation of this amplifier is displayed in Fig. 3.2. The emitting sole is represented in the figure by a plane at $p = 0$ and the r-f structure is represented by a plane at $p = 1.0$ (recall that $p \triangleq y/b'$). The r-f signal is injected at $q = 0$ and extracted at $q = q_L$.

In order to simulate uniform emission of electrons, M groups of primary electrons (those which are not emitted due to backbombarding electrons) are emitted from discrete points which are uniformly spaced along the sole from $(q_0, p_0) = (0, 0)$ to $(q_0, p_0) = (q_e, 0)$. The triangles along the sole in Fig. 3.2 represent the M emission points. From each of these M emission points there are $m/4$ electrons injected in each of the four different directions of emission, but with the same total magnitude of velocity as discussed in the previous chapter.

The initial phase of the primary electrons from the first emission point is selected by distributing these electrons uniformly over one cycle of the r-f wave. Only this particular cycle of the wave

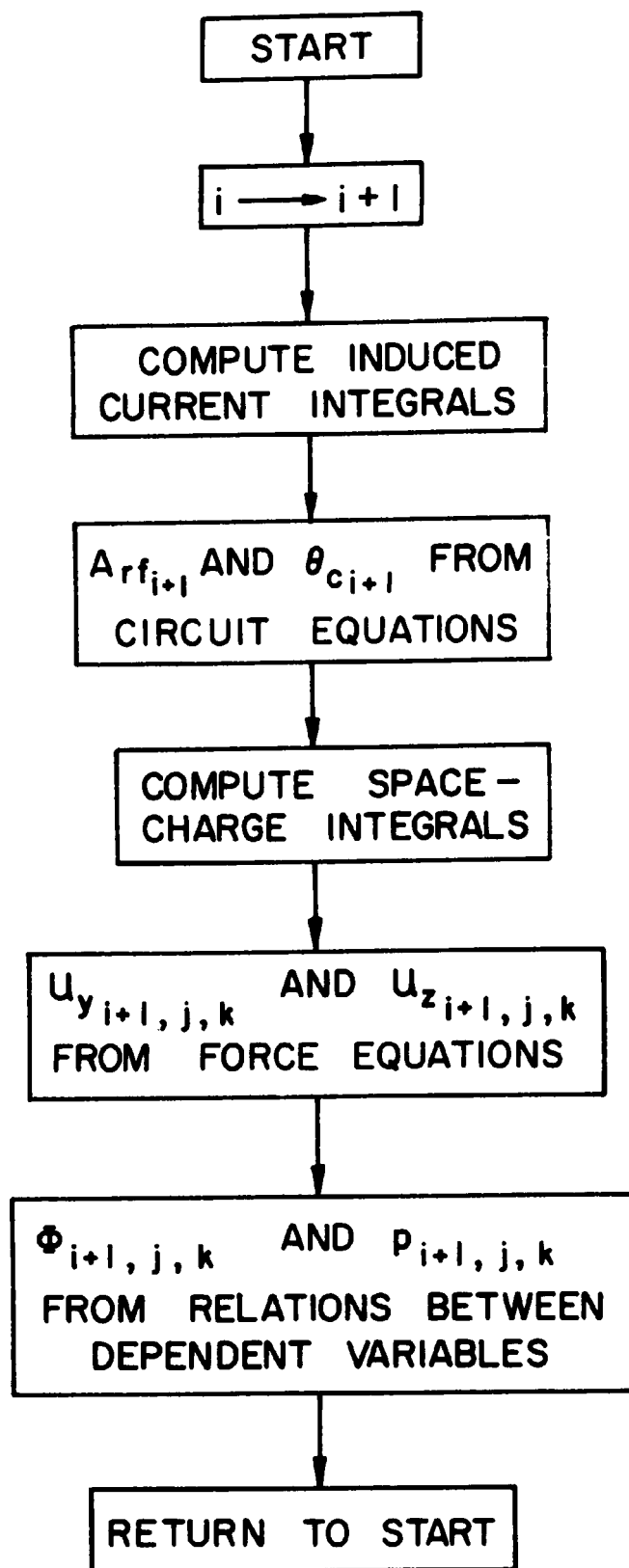


FIG. 3.1 FLOW DIAGRAM FOR SOLUTION OF THE EMITTING-SOLE EQUATIONS.

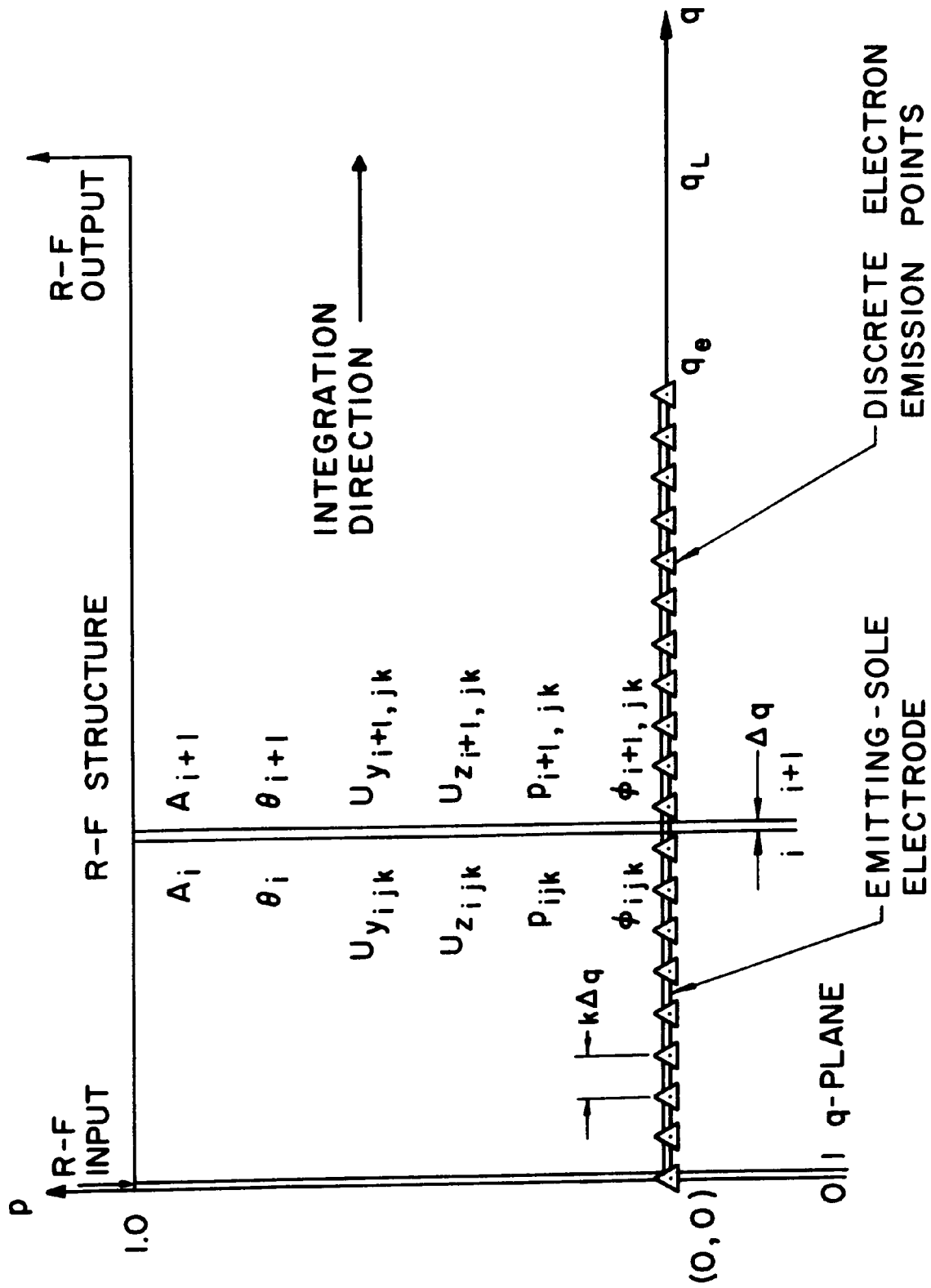


FIG. 3.2 DIGITAL COMPUTER EMITTING-SOLE AMPLIFIER MODEL.

is then followed in the calculations and all succeeding emission points also distribute m electrons uniformly over this cycle. For convenience, the wave cycle of 0 to 2π is chosen, and thus the initial value of Φ for each electron is given by

$$\Phi_{0,j,k} = \left(\frac{2\pi}{m} \right) j_k, \quad (3.21)$$

where

$$j_k = 0, 1, 2, \dots, m-1$$

respectively for

$$k = 0, 1, 2, \dots, m-1.$$

The succeeding values of phase as well as u_y , u_z and p for each electron are obtained from the difference-form equations as the calculations proceed in an iterative manner in the $+q$ -direction. Calculations are carried out at q -planes separated by a distance Δq and each q -plane calculation depends upon the variables at the previous q -plane.

When the calculated phase of an electron leaves the 0 to 2π range, this phase value is normalized by adding or subtracting 2π , (depending upon whether the phase value becomes less than zero or greater than 2π). Thus it is assumed that an identical interaction takes place in every cycle of the r - f wave, and that a mirror electron therefore replaces the one which leaves the selected cycle of the r - f wave.

It is also assumed that electrons are collected on the anode. That is, when the p value of an electron exceeds 1.0 it is eliminated from the computer problem by setting its velocity and position equal to

zero. On the other hand, when an electron returns to the cathode ($p = 0$), secondary electrons may be emitted, but this will be discussed in the next section.

Reiterating, one cycle of the r-f wave is followed in the longitudinal direction and as each emission point is passed, m electrons are injected with a uniform distribution in phase over this cycle of the r-f wave. Hence, the charge builds up in the longitudinal direction as the wave progresses from input to output, and the effects of this on all six dependent variables are computed by using the difference-form equations.

3.4.2 Secondary Emission Model. In general, secondary electrons are emitted from the cathode due to the bombardment of the cathode by other particles. In the specialized case of an emitting-sole amplifier, electrons originally emitted by thermal excitation are able to return to the cathode because of the bending effect of the d-c magnetic field. When they strike the sole electrode, secondary electrons may be emitted and these particles must also be included in the analysis of this device.

The secondary emission model to be used must precisely fix the following four quantities:

1. The energy distribution of secondary electrons,
2. The secondary emission ratio δ (the ratio of secondary to primary electrons),
3. The angle of emission of a secondary electron, and
4. The phase of the secondary with respect to the wave.

These quantities will now be discussed and specified individually.

The general shape of the energy distribution of secondary electrons is shown in Fig. 3.3^{30,31} and has been the subject of many investigations. The ordinate in the figure is a measure of the relative number of secondaries emitted and the abscissa is the energy of these secondaries. The curve shows that more than 90 percent of all secondaries have energy values of less than 20 electron-volts with a maximum near ten. The spike at the right of the curve indicates secondary electron energy equal to that of the primary electrons, and hence consists of scattered primary electrons. It is found experimentally that this spike contains less than three percent of the total secondary electrons and will be neglected in this model. Also, since most of the electrons are emitted with energies near 10 eV, it will be assumed that the energy distribution of secondary electrons is independent of the primary energy and that all secondaries are injected with an energy of 10 eV (this is about ten times greater than the energy of the thermally emitted electrons).

The secondary emission ratio δ has also been investigated quite extensively and has been found to vary with incident electron energy. The emission ratio is also a function of the material being bombarded. The emitting sole in this investigation is assumed to be an impregnated cathode consisting of porous tungsten impregnated with barium. The emission ratio curves for tungsten and barium are displayed in Fig. 3.4³². The composite curve will fall between these and will be assumed to be flat with $\delta = 1.0$ (see Fig. 3.4).

The emission angle for secondary electrons follows a cosine distribution curve³², but for simplicity a single emission direction will

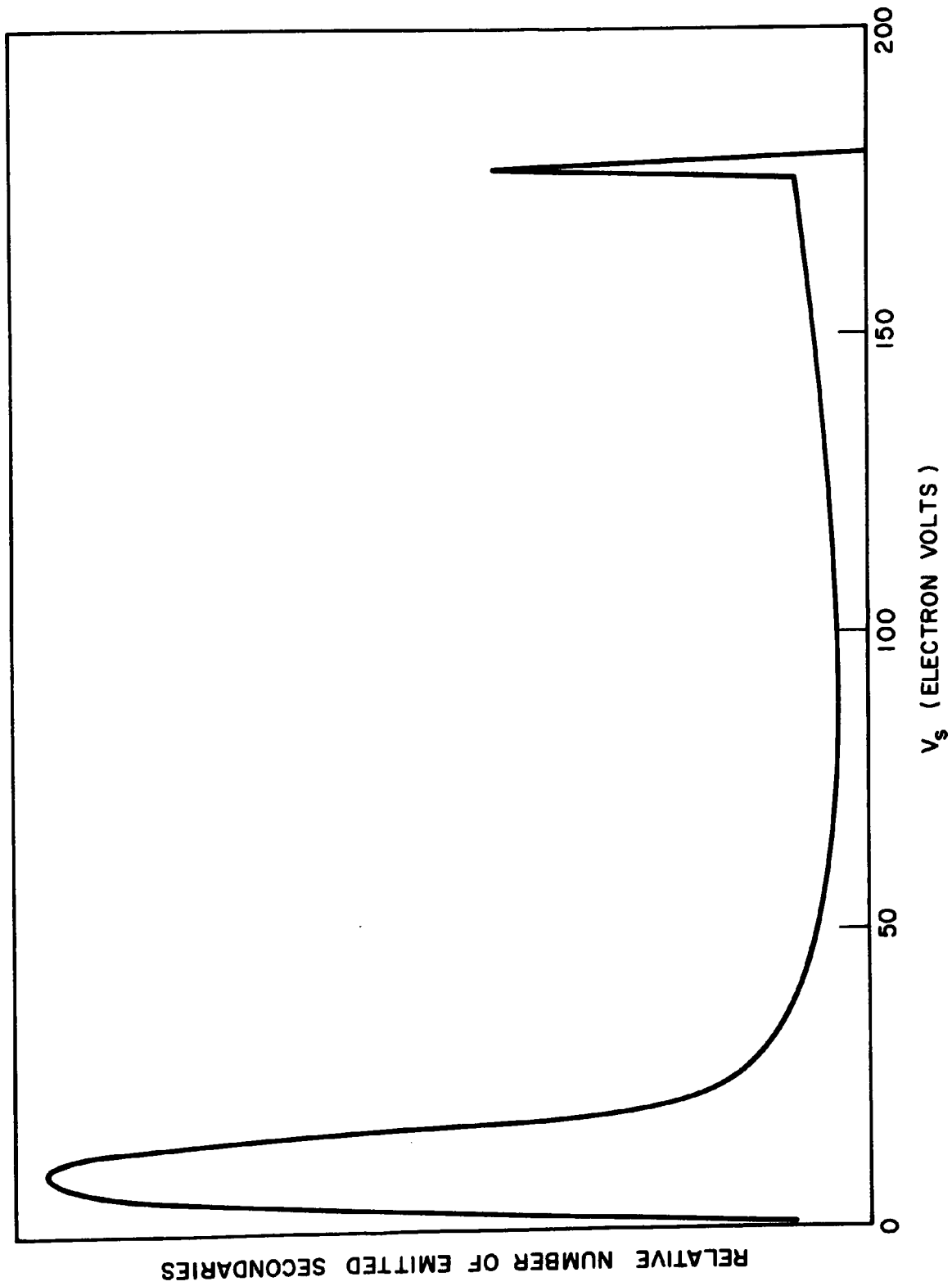


FIG. 3.3 ENERGY DISTRIBUTION OF SECONDARY ELECTRONS.

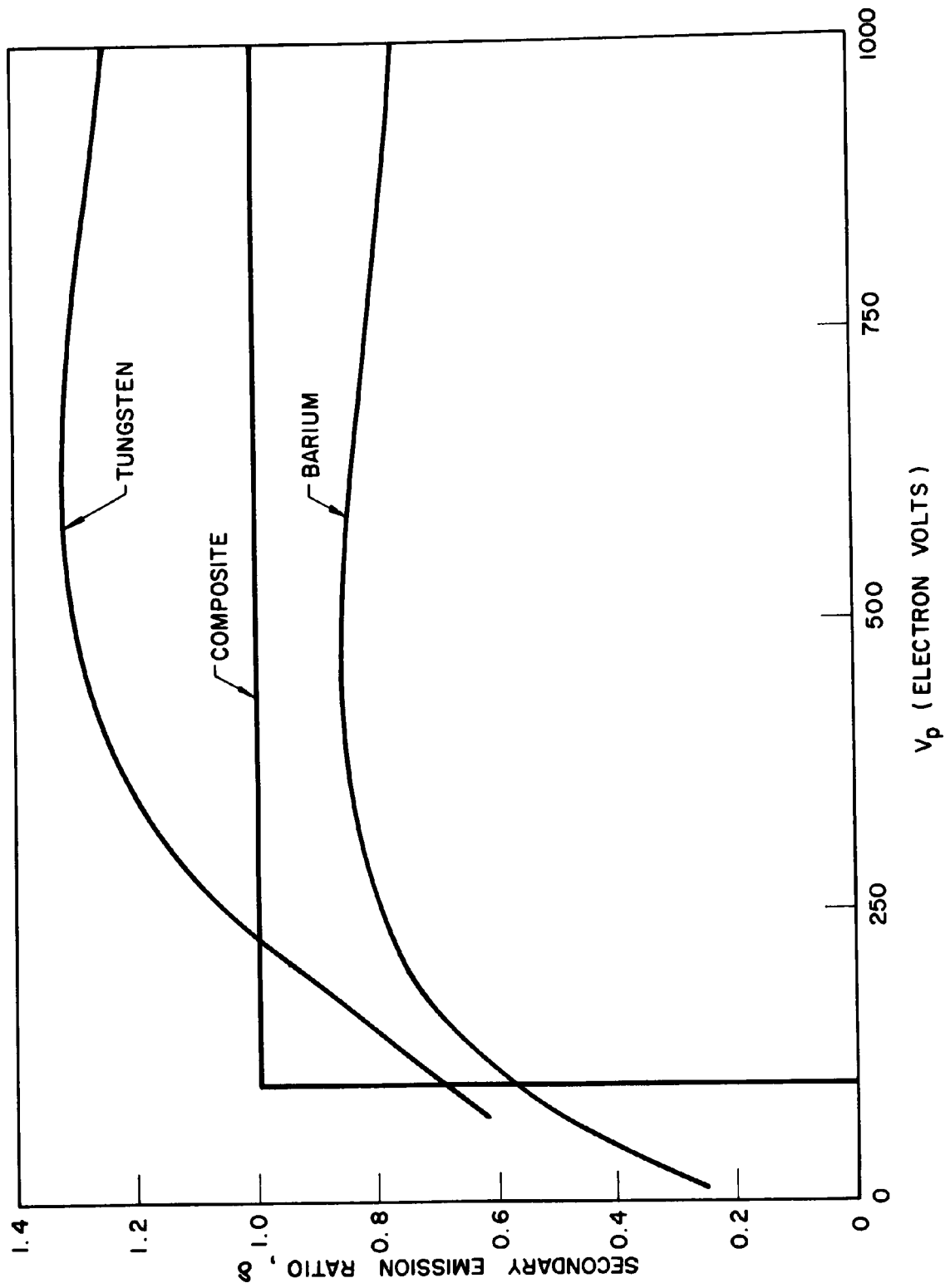


FIG. 3.4 THE VARIATION IN SECONDARY EMISSION RATIO WITH PRIMARY ELECTRON ENERGY.

be assumed. This is chosen to be 10 degrees, which makes the normal component of initial velocity greater than the tangential component.

Finally, the initial phase of a secondary electron will be assumed to be equal to that of the incident primary electron.

3.4.3 Computer Procedure to Account for Secondary Emission. In the computer program all electron trajectories are followed, and as previously mentioned the per electron quantities are calculated in a step-by-step piecewise-linear procedure. When an electron returns to the cathode, its incident energy is computed and it will be assumed that a secondary is produced if the primary energy is greater than 100 eV (one secondary for each primary because of the assumption $\delta = 1.0$). Regardless of whether a secondary is produced or not, the primary electron is removed from the problem. The emitted secondary electrons are followed through the device and are treated in the same way as primary electrons upon returning to the cathode. Thus, in summary, all secondaries are injected at an angle of 10 degrees, with an energy of 10 eV, and at a phase equal to that of the incident electron.

3.5 Integration of Induced-Current Integrals

The inhomogeneous terms appearing in the circuit equations are the induced-current terms which have been defined previously as Λ_1 and λ_1 . By inspecting Eqs. 3.12 and 3.13, it can be seen that only two induced-current integrals require evaluation in order to calculate Λ_1 and λ_1 . These integrals are given by

$$F_{s_1} = \int_0^{2\pi} \int_0^{q_1} \sin \phi_{1,j,k} \frac{|\rho'_0| \psi_{1,j,k}}{|1 + 2Du_{z1,j,k}|} dq_{o,j,k} d\phi_{o,j,k} \quad (3.22)$$

and

$$F_{c_1} = \int_0^{2\pi} \int_0^{q_1} \cos \phi_{1,j,k} \frac{|\rho'_0| \psi_{1,j,k}}{|1 + 2Du_{z_{1,j,k}}|} dq_{0,j,k} d\phi_{0,j,k} \quad (3.23)$$

In order to discuss the evaluation of these equations more efficiently, a single integral is defined to represent both as follows:

$$F_i = \int_0^{2\pi} \int_0^{q_1} f(\phi_{0,j,k}, q_{0,j,k}, q_1) dq_{0,j,k} d\phi_{0,j,k} \quad (3.24)$$

The evaluation of this double integral on the computer is accomplished using both Simpson's rule and the trapezoidal rule. Simpson's integration rule, which is more accurate than the trapezoidal rule but also more time consuming, is given by the following formula:

$$\begin{aligned} \int_{x_0}^{x_{2m}} g(x) dx &= \frac{h_1}{3} \left[g(x_0) + 4g(x_1) + 2g(x_2) + 4g(x_3) \right. \\ &\quad \left. + \dots + 4g(x_{2m-1}) + g(x_{2m}) \right] - \frac{mg^{(4)}(x)h_1^5}{90}, \quad (3.25) \end{aligned}$$

while the trapezoidal method represents the integral as

$$\begin{aligned} \int_{y_0}^{y_m} g(y) dy &= h_2 \left[\frac{1}{2} g(y_0) + g(y_1) + g(y_2) \right. \\ &\quad \left. + \dots + g(y_{m-1}) + \frac{1}{2} g(y_m) \right] - \frac{mg^{(2)}(y)h_2^3}{2}, \quad (3.26) \end{aligned}$$

where $x_0, x_1, \dots, x_{2m}$ and $y_0, y_1, \dots, y_m$ are equally spaced in the x- and y-directions respectively, and

$$\begin{aligned} h_1 &\triangleq \text{distance between equally spaced points in x,} \\ h_2 &\triangleq \text{distance between equally spaced points in y,} \\ g^{(2)}(y) &\triangleq \text{value of the second derivative with respect to y and} \\ g^{(4)}(x) &\triangleq \text{value of the fourth derivative with respect to x.} \end{aligned}$$

The last term in Eqs. 3.25 and 3.26 is the error term and is neglected in the actual calculations.

The integration over ϕ_0 is carried out first using Simpson's rule and the range 0 to 2π is subdivided into m parts. Expanding according to Eq. 3.25 yields

$$\begin{aligned} F_i = & \int_0^{q_i} \frac{2h_1}{3} \left[f(0, q_{0,j,0}, q_i) + 2f(h_1, q_{0,j,1}, q_i) + f(2h_1, q_{0,j,2}, q_i) \right. \\ & \left. + 2f(3h_1, q_{0,j,3}, q_i) + \dots + 2f([m-1]h_1, q_{0,j,m-1}, q_i) \right] dq_{0,j,k} \quad , \quad (3.27) \end{aligned}$$

where $h_1 = 2\pi/m$ and the following relationship has been used:

$$f(0, q_{0,j,0}, q_{i,j,k}) = f(2\pi, q_{0,j,m}, q_{i,j,k}) \quad . \quad (3.28)$$

The integration over q_0 is now carried out using the trapezoidal rule. Expanding Eq. 3.27 according to Eq. 3.26 yields

$$\begin{aligned}
 F_i = & 2 \frac{h_1 h_2}{3} \left\{ \frac{1}{2} [f(0, q_{o,1,o}, q_i) + 2f(h_1, q_{o,1,1}, q_i) + f(2h_1, q_{o,1,2}, q_i) + \right. \\
 & + \dots + 2f([m-1]h_1, q_{o,1,m}, q_i)] \\
 & + [f(0, q_{o,2,o}, q_i) + 2f(h_1, q_{o,2,1}, q_i) + f(2h_1, q_{o,2,2}, q_i) + \\
 & + \dots + 2f([m-1]h_1, q_{o,2,m}, q_i)] \\
 & + [f(0, q_{o,3,o}, q_i) + 2f(h_1, q_{o,3,1}, q_i) + f(2h_1, q_{o,3,2}, q_i) + \\
 & + \dots + 2f([m-1]h_1, q_{o,3,m}, q_i)] \\
 & + \dots \\
 & + \frac{1}{2} [f(0, q_{o,M,o}, q_i) + 2f(h_1, q_{o,M,1}, q_i) + f(2h_1, q_{o,M,1}, q_i) + \\
 & \left. + \dots + 2f([m-1]h_1, q_{o,M,m}, q_i)] \right\}, \quad (3.29)
 \end{aligned}$$

where h_2 = the distance between emission points.

The group of terms enclosed by the first pair of square brackets corresponds to the first emission point. The group enclosed by the second pair corresponds to the second emission point, and so on. Thus, if only one emission point has been passed, the integral is equal to the first term in the square brackets. If two emission points have been passed, F_i is the sum of the first and second square-bracketed terms.

Finally, when all M emission points have been passed, F_i has the total value indicated in Eq. 3.29.

The summations indicated in Eq. 3.29 are carried out at each q -plane and begin at the first q -plane by calculating the f functions (the integrands of Eqs. 3.22 and 3.23) for the m electrons which have been emitted from the first emission point and are present at this particular plane ($q = 0$). At $q = 0$ the initial values of phase and velocity are used to calculate these functions. At later q -planes, but when still only one emission point has been passed, the values of phase and velocity used in the calculations of the f functions are the values from the preceding q -plane.

As q increases and new emission points are passed, each of these is accounted for by adding m additional quantities to the summation (or one square-bracketed term). With the values of electron phase and velocity taken from the preceding q -plane, the f functions are calculated at this i th plane and again summed using Eq. 3.29, thus completing the integration.

3.6 Integration of Space-Charge Integrals

Poisson's equation is solved in Appendix B for the space-charge fields by using a Green's function technique²⁷. Appendix C extends this space-charge-field calculation by defining weighting functions, which are infinite summations arising in the integrands of the y - and z -space-charge forces. A thorough discussion of these weighting functions appears in Appendix C.

The weighting functions for various vertical positions and electron phase positions were calculated in a separate computer program

in order to save computer time. The results were then tabulated on IBM cards. Thus, in the main digital computer program the appropriate weighting function for each particular electron (determined by its vertical position and phase) is obtained by interpolation of these weighting function tables and a separate calculation for each electron is not necessary.

When the space-charge-weighting functions have been obtained the integrals are calculable. The space-charge integrals, defined by

$$I_{y_i} = \int_0^{2\pi} \int_0^{q_i} dq_{o,j,k} d\phi_{o,j,k} \frac{F_{y_{i,j,k}}}{|1 + 2Du_{z_{i,j,k}}|} \quad (3.30)$$

and

$$I_{z_i} = \int_0^{2\pi} \int_0^{q_i} dq_{o,j,k} d\phi_{o,j,k} \frac{F_{z_{i,j,k}}}{|1 + 2Du_{z_{i,j,k}}|} , \quad (3.31)$$

are evaluated by using essentially the same procedure as that used for the induced-current integral calculations. That is, the arguments of each integral are calculated for all electrons present at a particular q-plane and then Simpson's rule and the trapezoidal rule are used to evaluate the integral over $\phi_{o,j,k}$ and $q_{o,j,k}$, respectively. The only difference in the integration methods is that in the space-charge calculation the values of the space-charge-weighting functions $F_{y_{i,j,k}}$ and $F_{z_{i,j,k}}$ for each electron are interpolated from the weighting function tables by using the calculated per electron quantities,

while in the induced-current calculations the f functions are calculated without interpolation.

CHAPTER IV. AMPLIFIER SOLUTIONS NEGLECTING SPACE-CHARGE-FIELD EFFECTS

4.1 Introduction

The continuous equations describing large-signal electron-wave interaction in crossed-field emitting-sole amplifiers have been derived in Chapter II. The difference forms of these equations, suitable for solution on a large-scale digital computer, were developed in the last chapter. These equations together with appropriate initial conditions are now in a form amenable to solution. The first set of solutions to be presented are those for which space-charge forces are assumed to be negligible compared with r-f and d-c forces. Thus, in this chapter

$$I_{y_{i,j,k}} \equiv I_{z_{i,j,k}} \equiv 0 \quad . \quad (4.1)$$

In the first set of solutions presented, secondary emission will also be neglected. This effect will then be included and the differences will be discussed. Lossless circuits are also assumed.

4.2 Specification of Computer Quantities

The quantities to be discussed in this section are the number of electrons per emission point m , the number of emission points M , the integration increment Δq , and the distance between primary electron emission points. A general description of these quantities was given in Chapter III in Section 3.4.1. As stated in that chapter, the discrete electron model which best represents a continuously emitting cathode is a model with uniformly distributed emission points. Ideally

these points should be located as close to one another as possible and a large number of electrons should be used for greatest computational accuracy. However, the maximum number of usable electrons is limited by the computer storage capacity. The capacity of the IBM-7090 digital computer used in this investigation is approximately equal to 2700 primary electrons. Thus, there is an upper limit to the number of emission points as well as the number of electrons per emission point which may be used.

As previously established by Rowe²⁸ and later by Gandhi¹⁶, a distribution of 32 electrons over one cycle of the r-f wave yields sufficient accuracy in the calculations. Thus, in the emitting-sole solution, m is chosen to be 32, i.e., 32 electrons are injected per emission point. The number m and the computer capacity therefore determine the number of emission points. Consequently this is chosen to be $M = 81$.

The integration increment, Δq , must also be decided a priori. This must be a compromise between computer time consumption and accuracy. If too large an increment is selected the accuracy will be poor because of the fact that electrons are emitted almost vertically (small thermal initial velocities) and a large value of Δq will result in too high a trajectory, i.e., very rapid collection on the anode. On the other hand, too small an increment leads to excessive computer time and also introduces undesirable round-off errors. A good compromise was found to be $\Delta q = 0.005$ and this value has been used in all of the cases. A comparison with the ballistics of a d-c electron provides a check on the computer accuracy.

Finally, the emission point spacing must be specified. It was found that a good overlapping of trajectories (more than 10 emission points per cycloidal length) is obtained with a spacing of $10 \Delta q$ and this value is therefore used.

4.3 Normalized Computer Input Parameters

In addition to the specification of the initial values of the dependent variables (as discussed in Chapter II) and the above-mentioned computer quantities, the following dimensionless d-c parameters must also be specified:

- $A_{dc} \triangleq$ the normalized d-c voltage amplitude,
- $\beta_0 b' \triangleq$ the normalized sole-to-anode spacing,
- $\beta_0 L_e \triangleq$ the normalized emitting-sole length,
- $D \triangleq$ the gain parameter,
- $v_0/v_c \triangleq$ the ratio of constant wave velocity to characteristic thermionic velocity,
- $\omega_c/\omega \triangleq$ the ratio of cyclotron frequency to r-f frequency, and
- $\omega_{pc}/\omega \triangleq$ the ratio of electron plasma frequency at the cathode to r-f frequency.

These quantities are constants which determine the physical dimensions, and d-c fields and velocities which are appropriate for the emitting-sole amplifier.

4.4 Solutions Neglecting Secondary Emission

4.4.1 Large-Signal Electron Ballistics. A study of the electron ballistics for any electron-wave interaction process is a very important part of the theoretical analysis. To obtain a full understanding of such a process, it is necessary to have a knowledge of the effects of

the wave on the electrons as well as a knowledge of the effects of the electrons on the wave. In this section the electron ballistics in the emitting-sole amplifier will be discussed, neglecting space-charge and secondary emission effects in order to determine the effect of the wave on the electron motion. In the next section the action of the beam space charge in amplifying an r-f voltage on the structure will be discussed.

Equations 3.14 through 3.18 are the difference-form equations whose solutions yield the normalized position, p , the normalized velocities, u_y and u_z , and the phase position, Φ , of each electron. Results have been obtained for the following set of parameters: $A_{dc} = 20$, $\beta_0 b' = 2$, $\beta_0 L_e = 40$, $v_0/v_c = 57.5$, $\omega_c/\omega = 1$, $\delta = 0$, and $\omega_{pc}/\omega = 0$. These variables were chosen to satisfy two conditions; namely, the d-c magnetron cutoff condition ($\omega_c b' = \sqrt{2|\eta|V_a}$) and synchronous operation ($v_0 = v_d = E_0/B_0$). The second condition is used because as discussed in Chapter I the average velocity of the electrons must be approximately equal to that of the wave in order to obtain efficient electron-wave interaction. Solutions for various values of the initial r-f voltage amplitude are also obtained. Electron phase positions, trajectories and velocities for these cases are shown in Figs. 4.1 through 4.15. It should be noted that the electron quantities have been plotted for the first emission point only. This is because the r-f amplitude did not grow appreciably, and thus the trajectories from the various other emission points were the same except for small perturbations.

Figures 4.1, 4.2 and 4.3 show the variation of electron phase position as a function of longitudinal distance with Φ_0 as a parameter

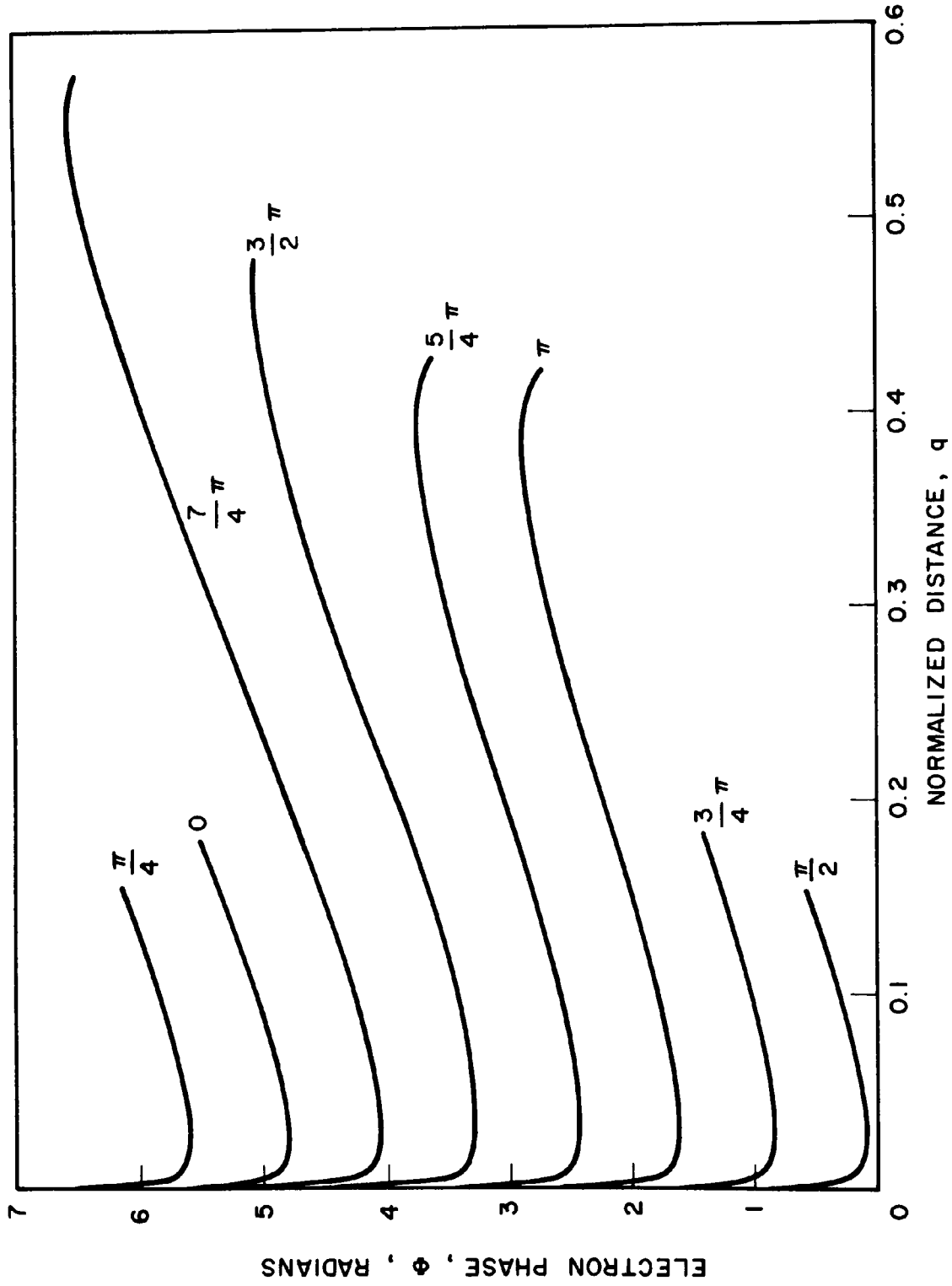


FIG. 4.1 PHASE POSITION VS. NORMALIZED DISTANCE WITH ϕ_0 AS A PARAMETER AND $A_{rf}/A_{dc} = 1/4$.
 ($A_{rf} = 5$, $D = 0.1$, $\omega_c/\omega = 1$, $A_{dc} = 20$, $\beta_{b'} = 2$, $\beta_{oe} = 40$)

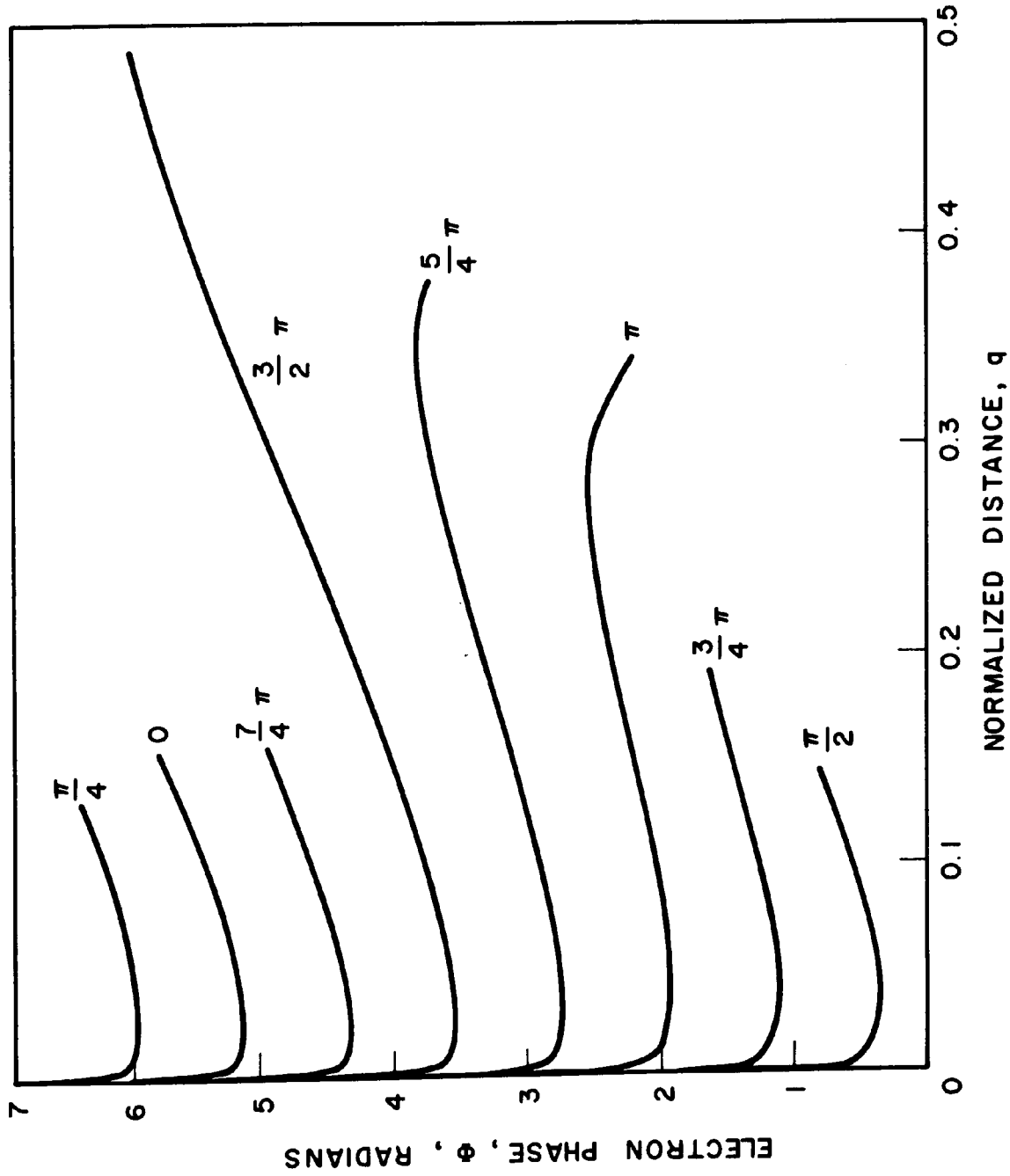


FIG. 4.2 PHASE POSITION VS. NORMALIZED DISTANCE WITH ϕ_0 AS A PARAMETER AND $A_{rf0}/A_{dc} = 1/2$.

($A_{rf0} = 10$, $D = 0.1$, $\omega_c/\omega = 1$, $A_{dc} = 20$, $\beta_0 b' = 2$, $\beta_0 L_e = 40$)

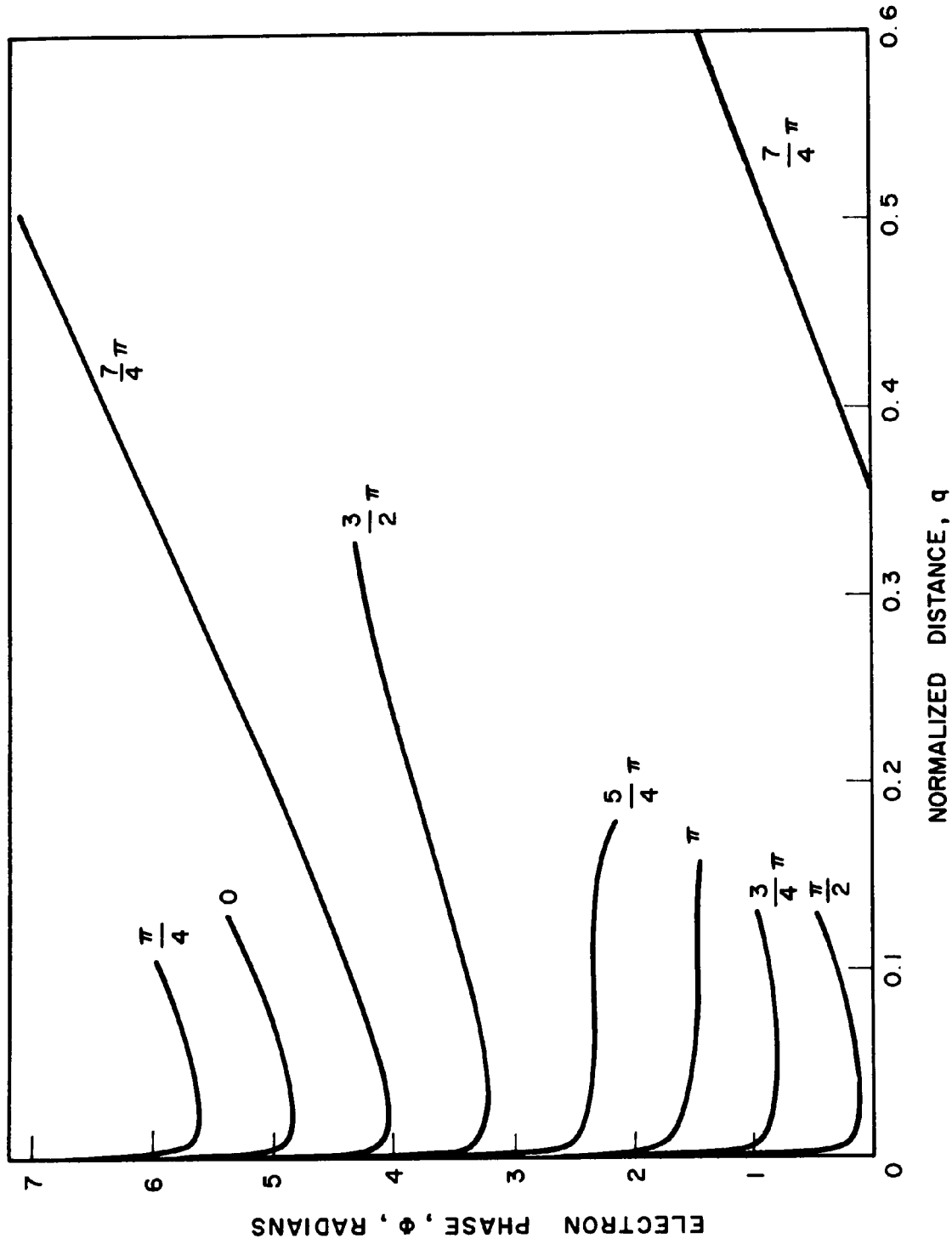


FIG. 4.3 PHASE POSITION VS. NORMALIZED DISTANCE WITH ϕ_0 AS A PARAMETER AND $A_{rf_0}/A_{dc} = 1$.
 $(A_{rf_0} = 20, D = 0.1, \omega_c/\omega = 1, A_{dc} = 20, \beta_0 b' = 2, \beta_0 L_0 = 40)$

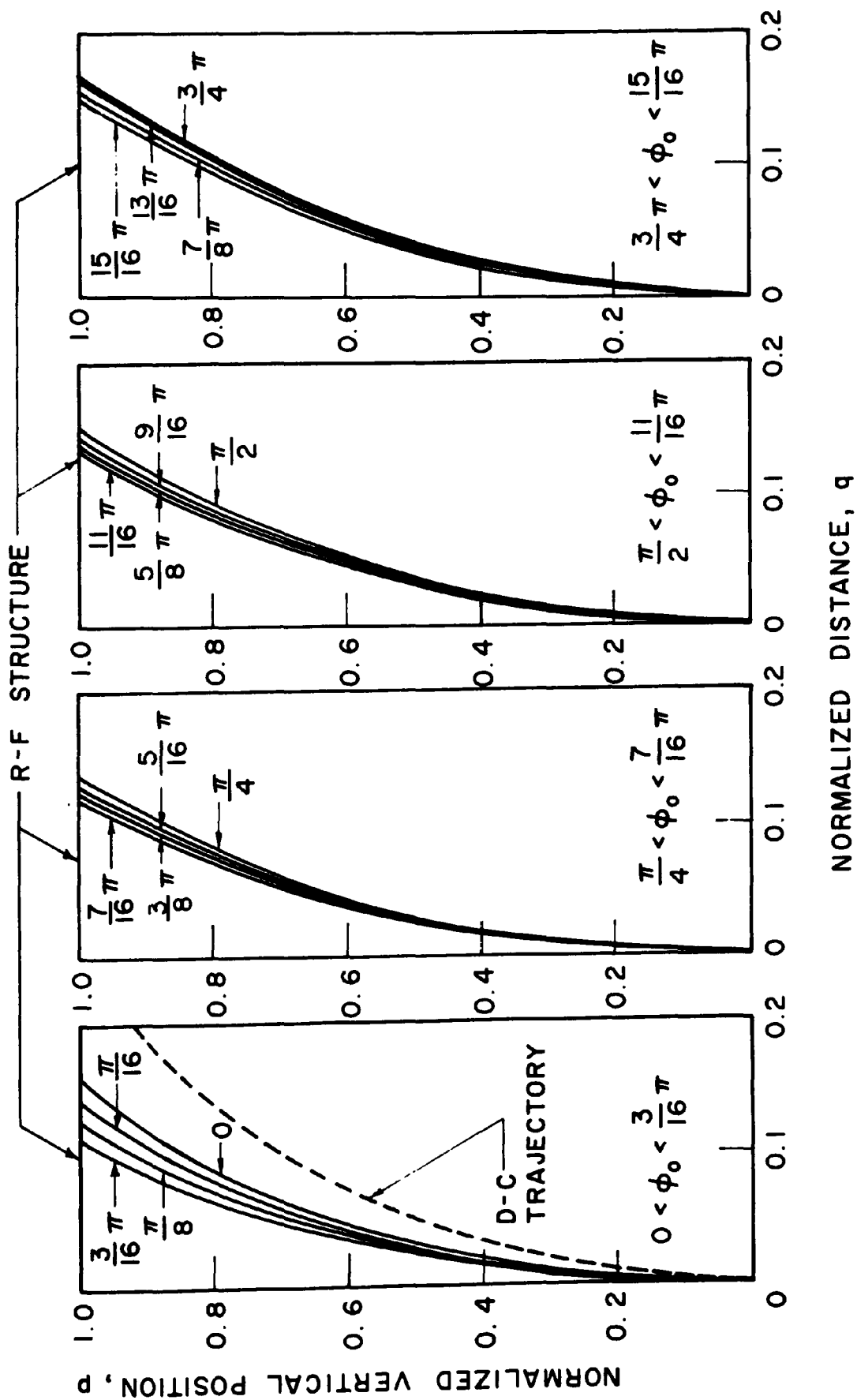


FIG. 4.4 ELECTRON TRAJECTORIES (0 to $15\pi/16$) FOR THE FIRST EMISSION POINT WITH ϕ_0 AS A PARAMETER.

$$(A_{rf_0} = 10, D = 0.1, \omega_c/\omega = 1, A_{dc} = 20, \beta_{0b'} = 2, \beta_{0L_e} = 40)$$

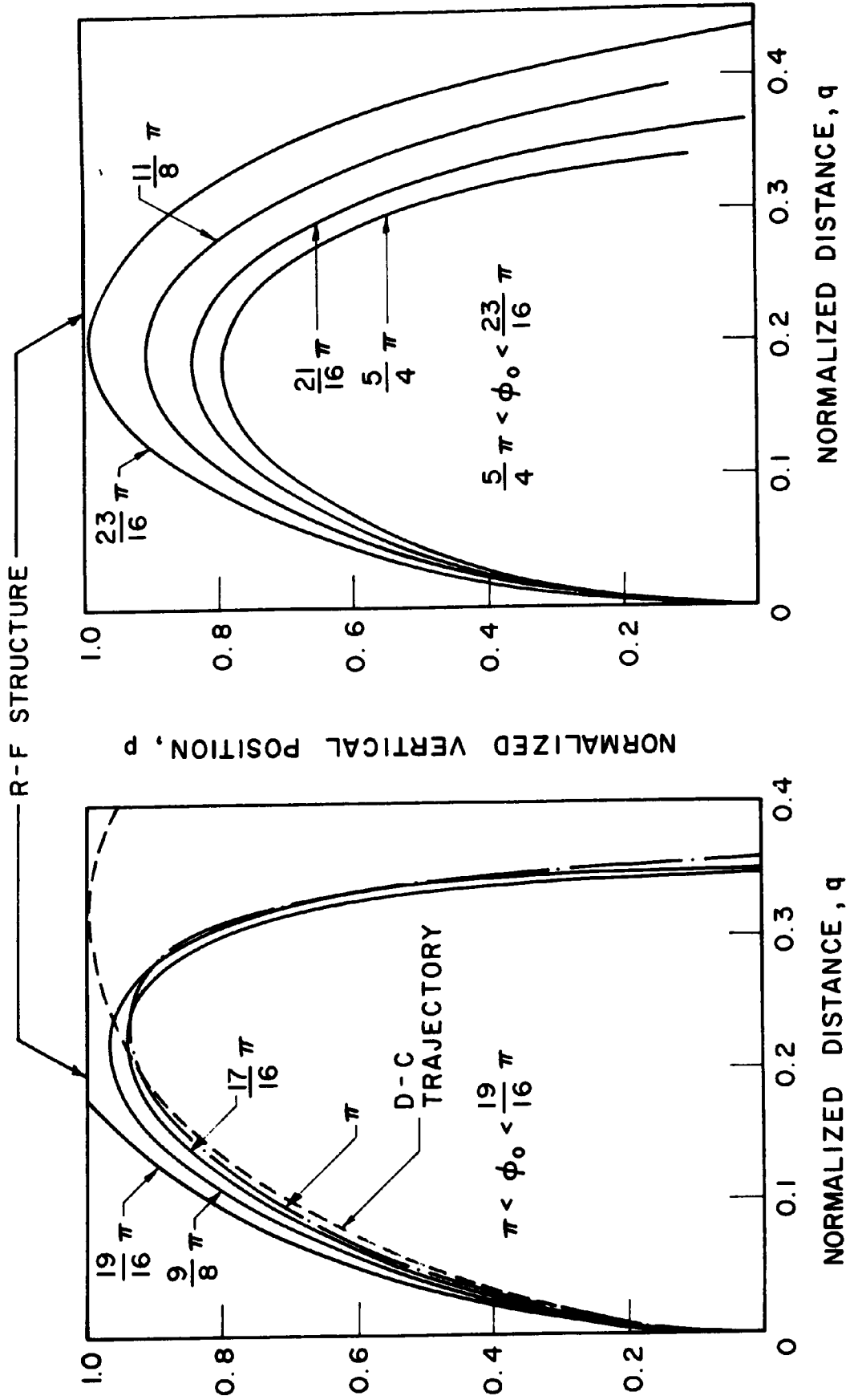


FIG. 4.5 ELECTRON TRAJECTORIES (π to $23\pi/16$) FOR THE FIRST EMISSION POINT WITH ϕ_0 AS A PARAMETER.

$$(A_{rf} = 10, D = 0.1, \omega_c/\omega = 1, A_{dc} = 20, \beta_0 b' = 2, \beta_0 L_e = 40)$$

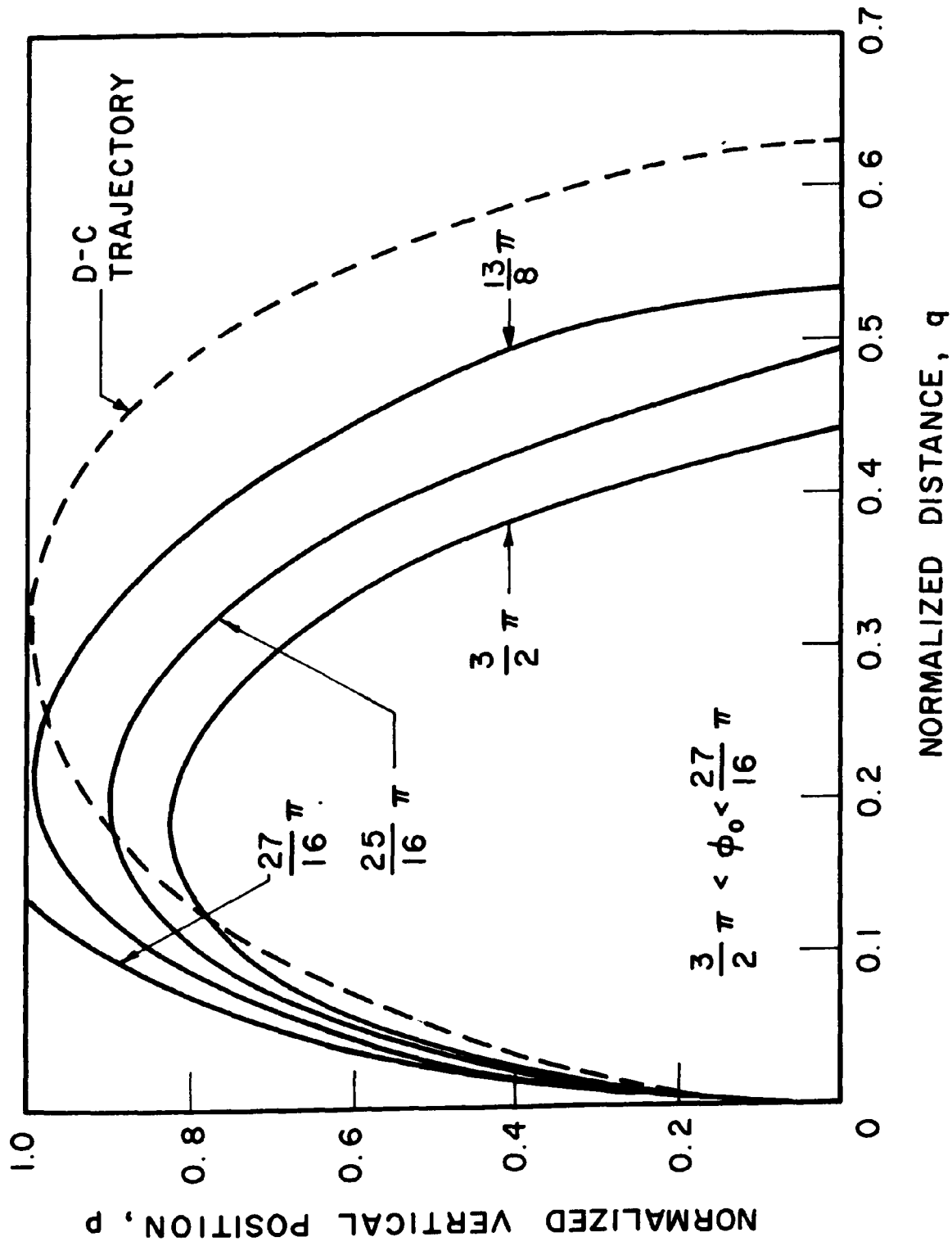


FIG. 4.6 ELECTRON TRAJECTORIES ($3\pi/2$ to $27\pi/16$) FOR THE FIRST EMISSION POINT WITH ϕ_0 AS A PARAMETER.

($A_{rf_0} = 10$, $D = 0.1$, $\omega_c/\omega = 1$, $A_{dc} = 20$, $\beta_0 b' = 2$, $\beta_0 I_e = 40$)

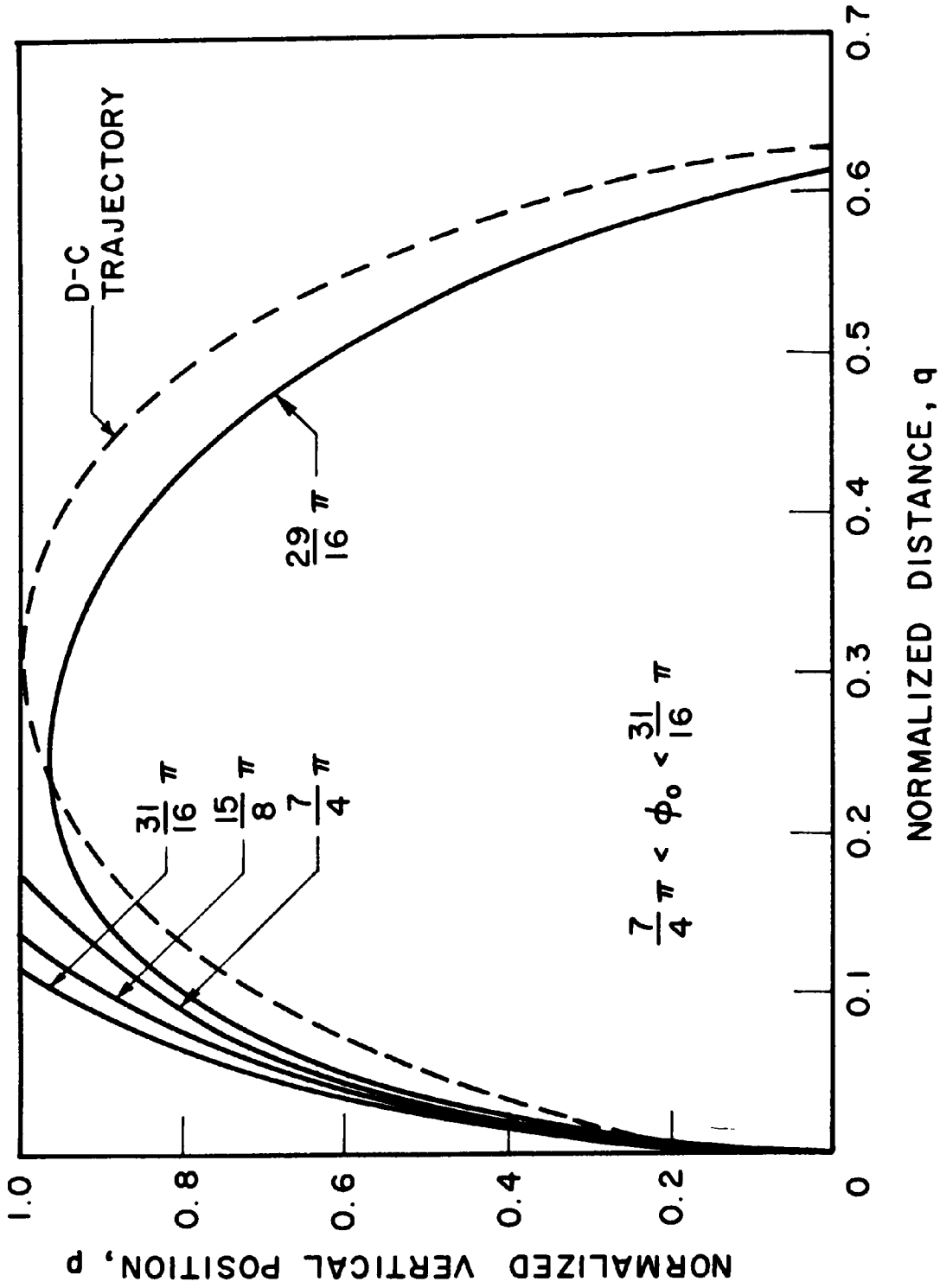


FIG. 4.7 ELECTRON TRAJECTORIES ($7\pi/4$ to $31\pi/16$) FOR THE FIRST EMISSION POINT WITH ϕ_0 AS A PARAMETER.

($A_{rf_0} = 10$, $D = 0.1$, $\omega_c/\omega = 1$, $A_{dc} = 20$, $\beta_{ob'} = 2$, $\beta_{oe} = 40$)

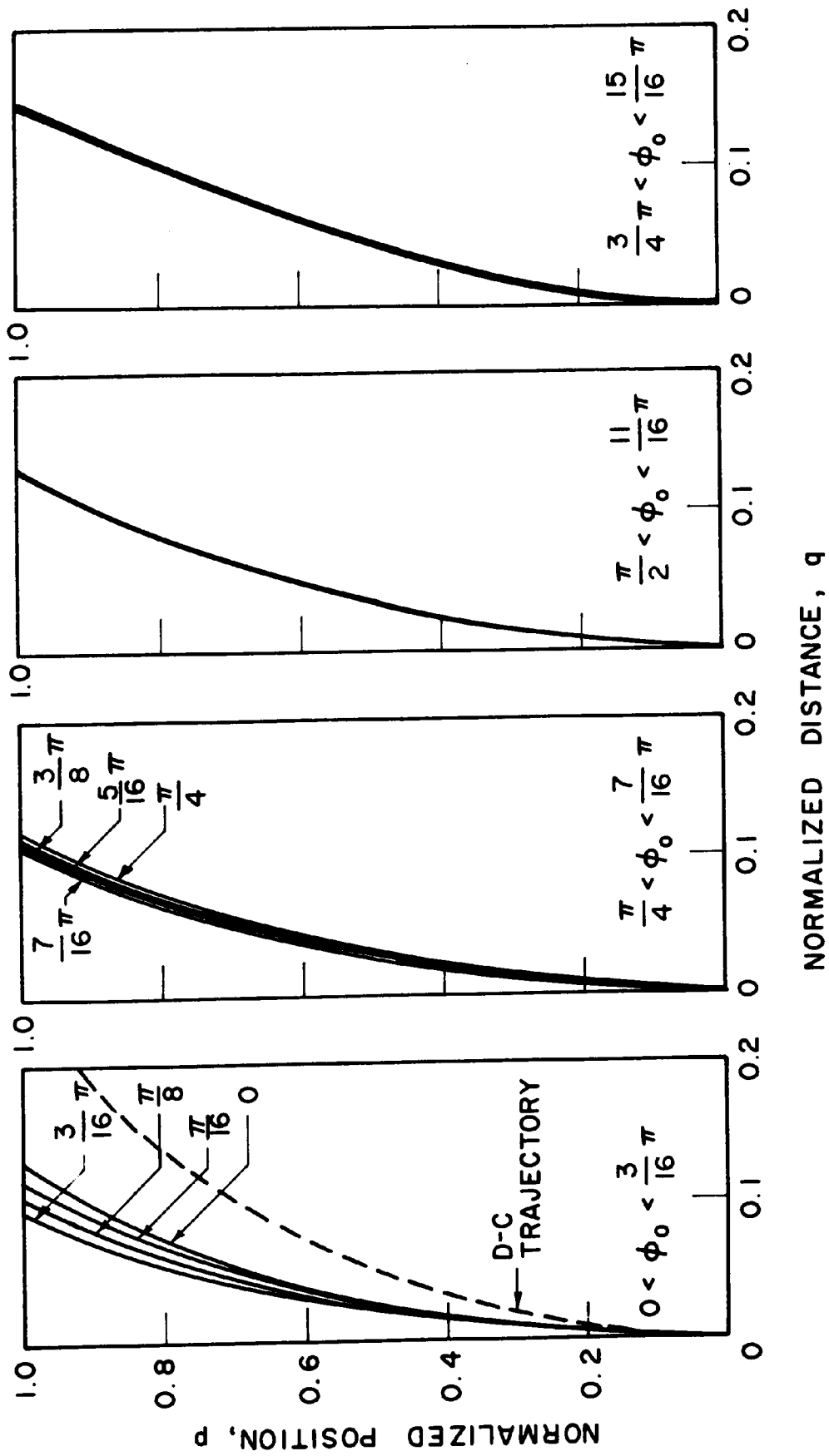


FIG. 4.8 ELECTRON TRAJECTORIES (0 to $15\pi/16$) FOR THE FIRST EMISSION POINT WITH ϕ_0 AS A PARAMETER.

($A_{rf_0} = 20$, $D = 0.1$, $\omega_c/\omega = 1$, $A_{dc} = 20$, $\beta_0 b' = 2$, $\beta_0 L_e = 40$)

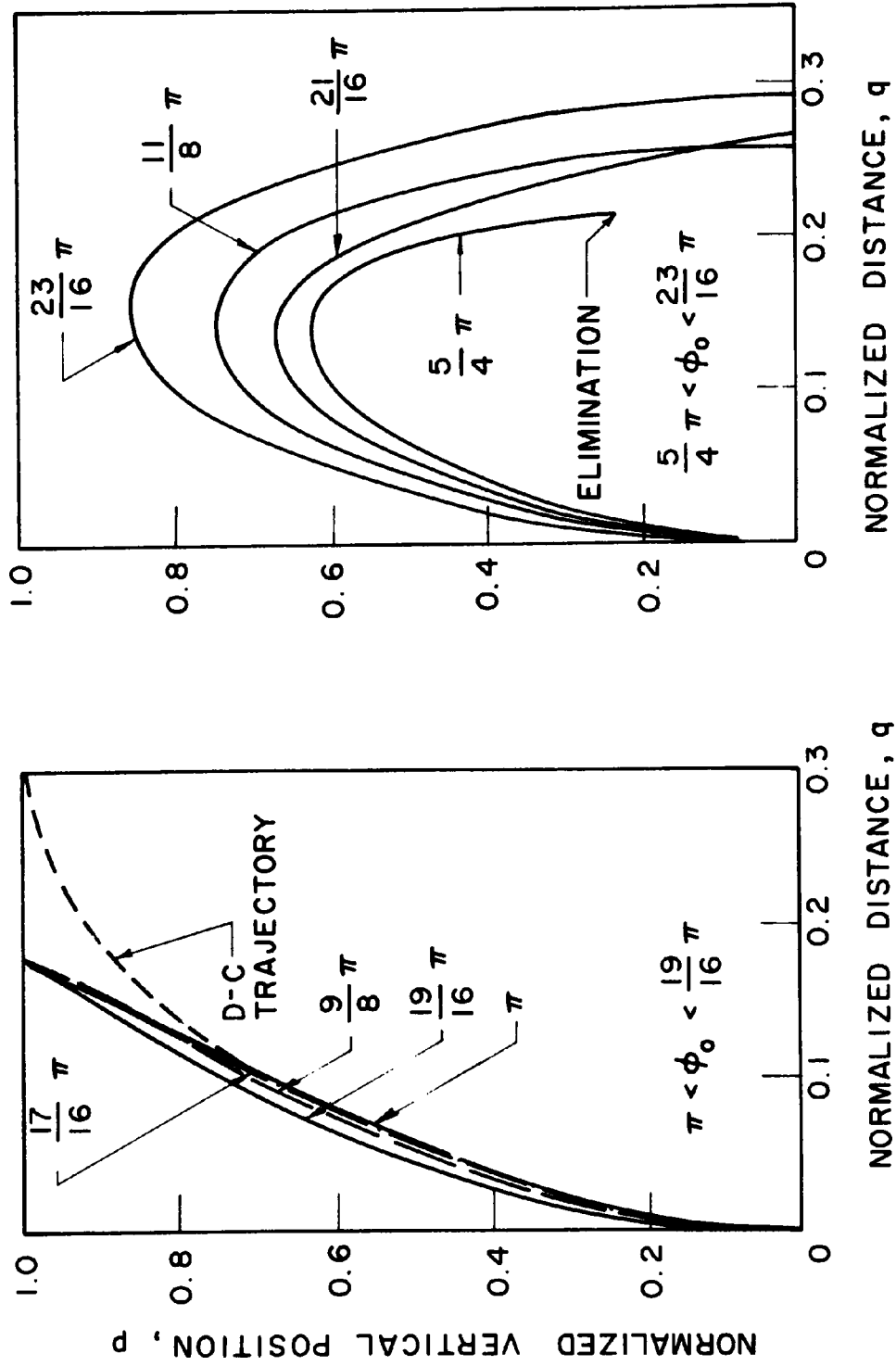


FIG. 4.9 ELECTRON TRAJECTORIES (π TO $23\pi/16$) FOR THE FIRST EMISSION POINT WITH ϕ_0 AS A PARAMETER.

($A_{rf_0} = 20$, $D = 0.1$, $\omega_c/\omega = 1$, $A_{dc} = 20$, $\beta_0 b' = 2$, $\beta_0 L_e = 40$)

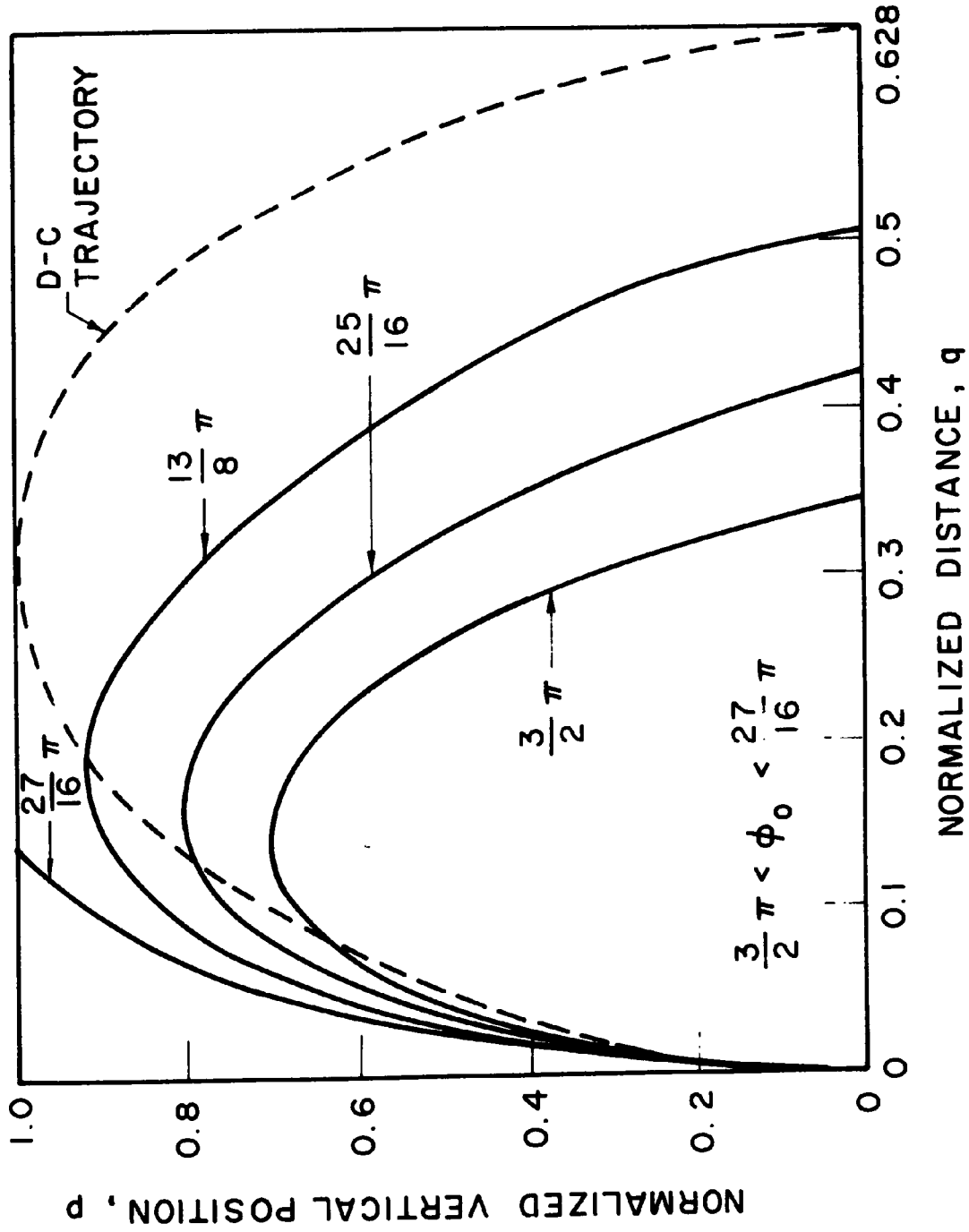


FIG. 4.10 ELECTRON TRAJECTORIES ($3\pi/2$ to $27\pi/16$) FOR THE FIRST EMISSION POINT WITH ϕ_0 AS A PARAMETER.

$$(A_{rf_0} = 20, D = 0.1, \omega/\omega_c = 1, A_{dc} = 20, \beta_{0b'} = 2, \beta_{0L_e} = 40)$$

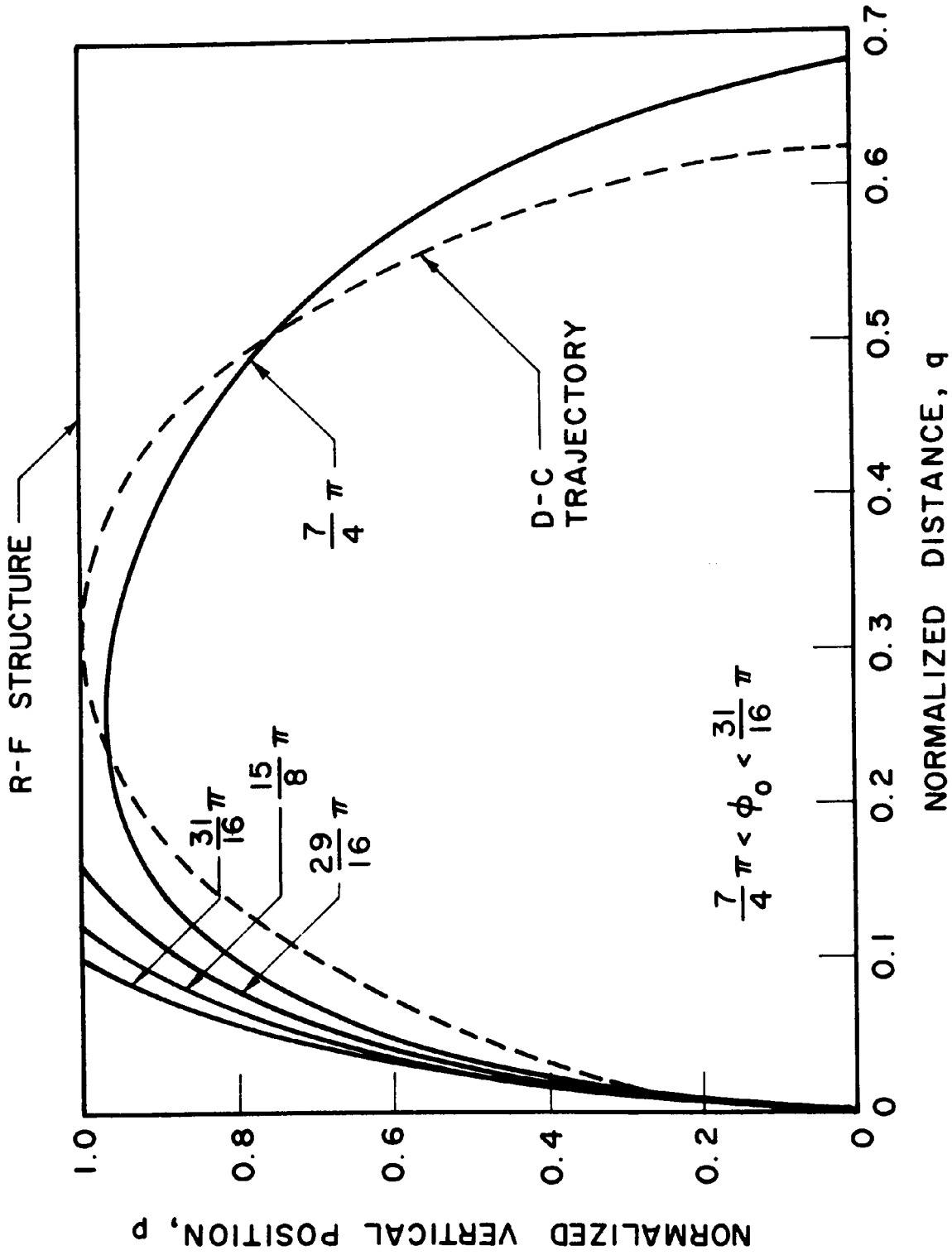


FIG. 4.11 ELECTRON TRAJECTORIES ($7\pi/4$ to $31\pi/16$) FOR THE FIRST EMISSION POINT WITH ϕ_0 AS A PARAMETER.

($A_{rf_0} = 20$, $D = 0.1$, $\omega_c/\omega = 1$, $A_{dc} = 20$, $\beta_{ob'} = 2$, $\beta_{oe} = 40$)

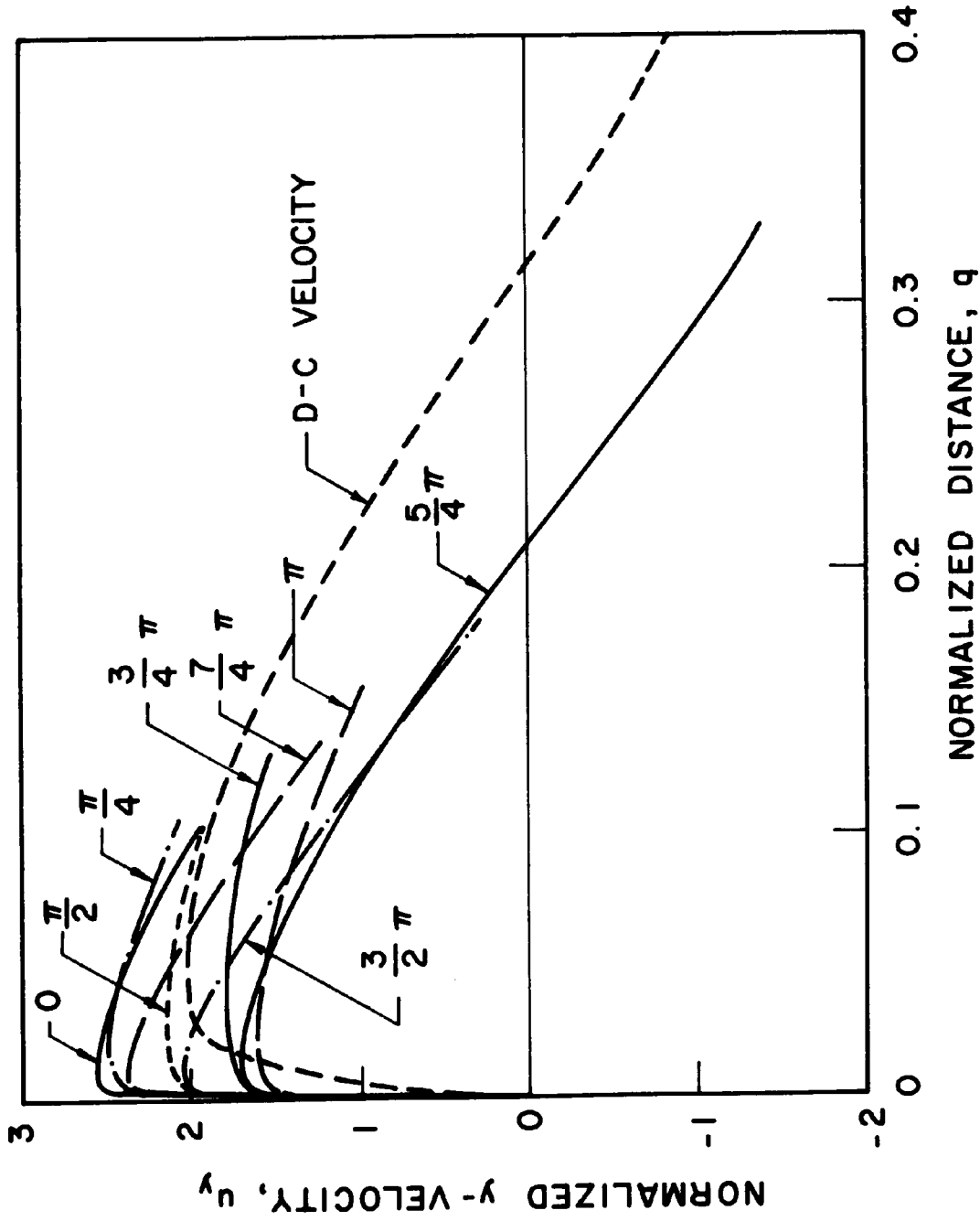


FIG. 4.12 NORMALIZED VERTICAL VELOCITY VS. DISTANCE WITH ϕ_0 AS A PARAMETER AND $A_{rf_0}/A_{dc} = 1/2$.

($A_{rf_0} = 10$, $D = 0.1$, $\omega_c/\omega = 1$, $A_{dc} = 20$, $\beta_0 b' = 2$, $\beta_0 L_e = 40$)

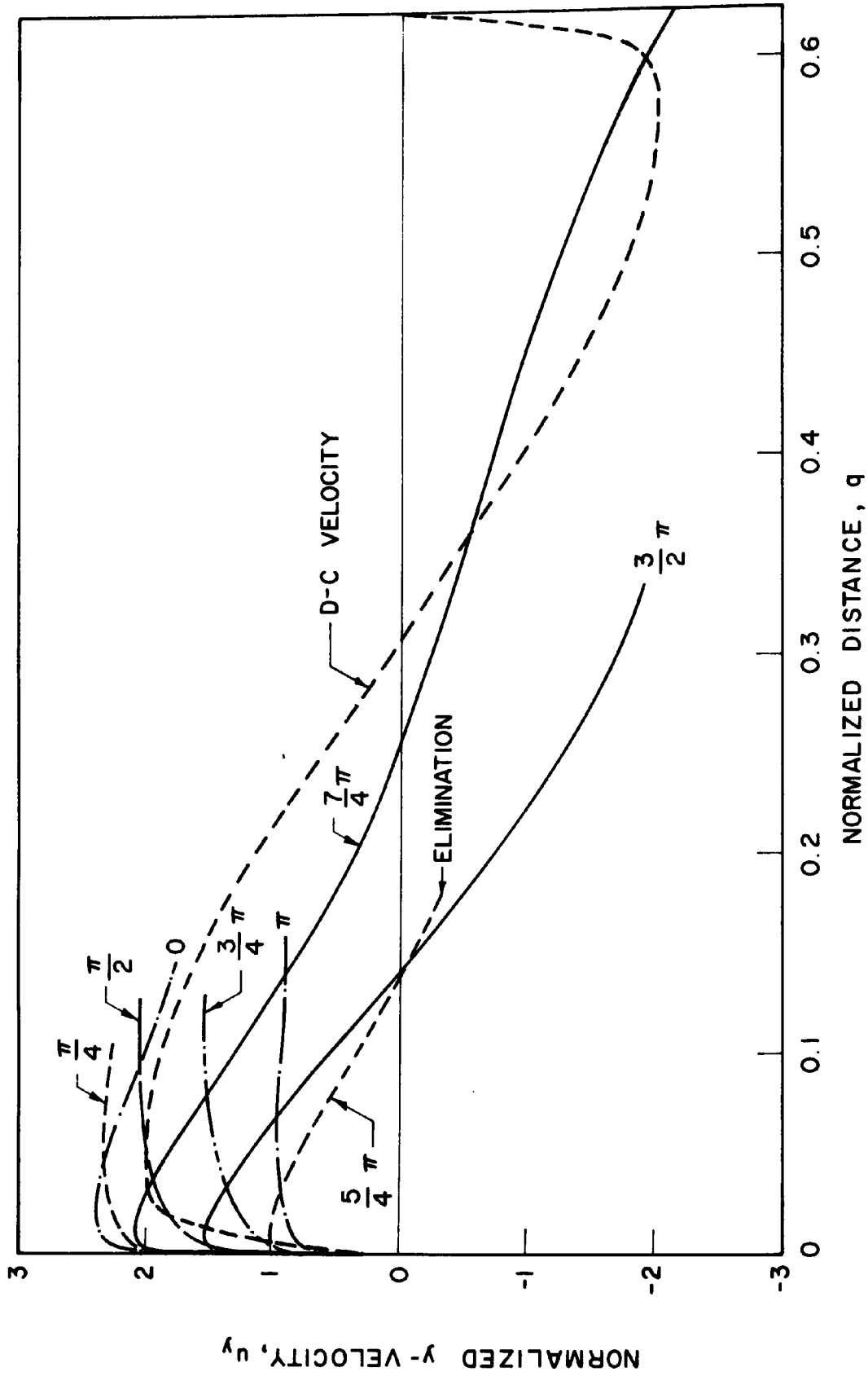


FIG. 4.13 NORMALIZED VERTICAL VELOCITY VS. DISTANCE WITH ϕ_0 AS A PARAMETER AND $A_{rf}/A_{dc} = 1$.

($A_{rf} = 20$, $D = 0.1$, $\omega_c/\omega = 1$, $A_{dc} = 20$, $\beta_0 b' = 2$, $\beta_0 L_e = 40$)

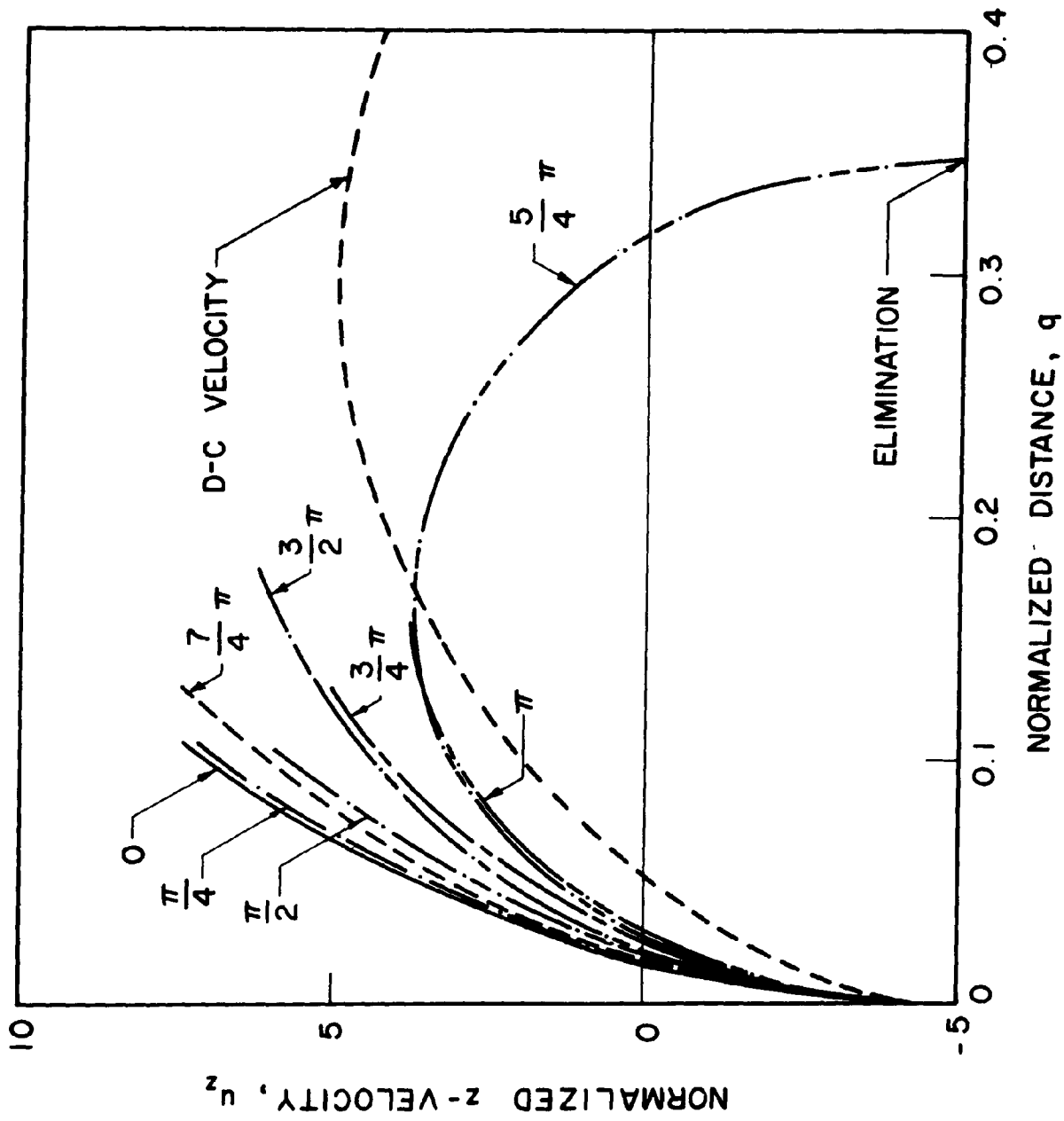


FIG. 4.14 NORMALIZED LONGITUDINAL VELOCITY VS. DISTANCE WITH ϕ_0 AS A PARAMETER AND $A_{rf_0}/A_{dc} = 1/2$.
 $(A_{rf_0} = 10, D = 0.1, \omega_c/\omega = 1, A_{dc} = 20, \beta_0 b' = 2, \beta_0 I_e = 40)$

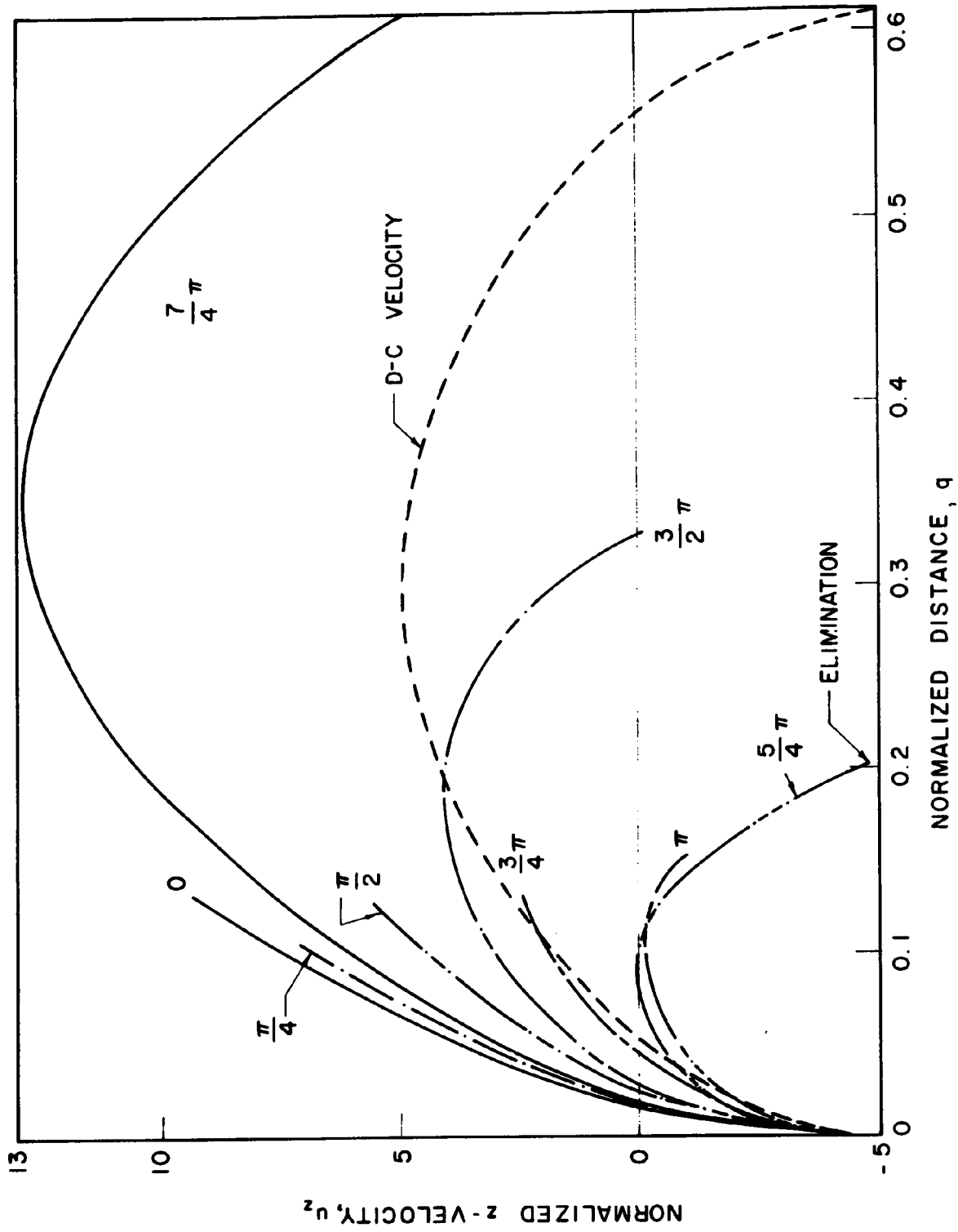


FIG. 4.15 NORMALIZED LONGITUDINAL VELOCITY VS. DISTANCE WITH ϕ_0 AS A PARAMETER AND $A_{rf_0}/A_{dc} = 1$.
 ($A_{rf_0} = 20$, $D = 0.1$, $\omega_c/\omega = 1$, $A_{dc} = 20$, $\beta_{ob} = 2$, $\beta_{oe} = 40$)

for $A_{rf_0} = 5, 10$ and 20 , respectively. These curves were calculated for one pair of electron velocity components, namely, those with an injection angle of -30 degrees with respect to a vertical line projected normal to the cathode (see Appendix D). Thus the curves represent one fourth of the electrons emitted from the first emission point, while the other three fourths of these electrons are omitted for reasons of clarity. As can be seen from the curves, some bunching about the phase position $\phi = \pi/2$ occurs for the case of $A_{rf_0} = 20$, a lesser amount for the case of $A_{rf_0} = 10$, and no bunching occurs for the case of $A_{rf_0} = 5$. The gain vs. distance curves for these three cases agree with these facts in that there is no gain for the case in which $A_{rf_0} = 5$, and only a slight amount in the other two cases. This will be discussed further in the next section.

The first emission point electron trajectories for the cases of $A_{rf_0} = 10$ and $A_{rf_0} = 20$ are shown in Figs. 4.4 through 4.11. Four electrons per graph have been used in order to distinguish the trajectories from one another. Each of these electrons on one graph also has a different initial velocity which corresponds to emission angles of $-30, -10, 10$ and 30 degrees with respect to a normal to the cathode (the exact values are given in Appendix D). Figures 4.4 to 4.11 also display a zero initial velocity d-c trajectory, i.e., a trajectory without r-f fields. It can be noted from these figures that the r-f field decelerates electrons with phase positions near $\pi/2$ causing collection of these electrons upon the anode in a shorter distance than the grazing distance of the d-c trajectory. Similarly, it is observed that the r-f field accelerates electrons with phase positions near $3\pi/2$ resulting in shorter backbombardment lengths for the r-f

trajectories as compared to the d-c trajectory. The slight discrepancy which is observed near the emission point is due to the small thermionic velocity which is assumed in the r-f solutions. Also depicted in the trajectory figures is the fact that some of the electrons give up energy to the r-f wave while others extract energy from it. This is observed by recalling from the discussion of Chapter I that electrons collected on the r-f structure give up energy to the r-f wave, while those returning to the cathode extract r-f energy. In Figs. 4.4 through 4.11 it can be seen that, in general, electrons with Φ_0 values between 0 and π approach the r-f circuit while those with Φ_0 between π and 2π recede from the circuit. This indicates the existence of both favorably and unfavorably phased electrons as in the injected-beam crossed-field amplifier. The actual energy conversion mechanism which is occurring can be explained as follows. Upon approaching the r-f circuit, electrons enter regions of higher electric potential and therefore lower potential energy and hence these electrons must suffer a decrease in potential energy. On the other hand, electrons which approach the sole must gain potential energy. However, the electrons which recede from the r-f circuit interact with weaker r-f fields which are present closer to the cathode. Thus, electrons which approach the circuit are able to interact more effectively with the r-f wave and a direct conversion of d-c potential energy to r-f energy is achieved since the kinetic energy of the electrons is not modified significantly.

Figure 4.8 shows that electrons with Φ_0 values near $\pi/2$ follow similar trajectory paths. This implies bunching in coordinate space since bunching in phase-coordinate space is occurring (see Fig. 4.3).

The trajectories for $A_{rf_0} = 10$ and Φ_0 values near $\pi/2$, seen in Fig. 4.4, show only a slight similarity and hence imply that for this case less coordinate-space bunching is occurring. This again indicates that a greater interaction is occurring in the cases with a larger A_{rf_0} . Finally the trajectory curves also show that not only do electron flight lines cross for electrons from different emission points, but they also cross for electrons emitted from the same emission points, which thus substantiates the necessity of using a Lagrangian analysis.

The normalized velocity variations with longitudinal distance are displayed in Figs. 4.12 through 4.15. The figures indicate the large initial acceleration that the electrons experience at emission (a correspondingly large acceleration is also observed for the d-c trajectory). Thus, each electron is accelerated from its initially small thermal velocity to its large d-c velocity almost immediately after emission. This effect is more apparent in the y-velocity curves than in the z-velocity curves. It is to be noted that the electrons attain a maximum y-velocity quite rapidly, and then this velocity decreases, indicating an apparent deceleration. However, such behavior does not necessarily imply deceleration of these electrons since the z-velocity may be increasing. Calculations of the change in total d-c energy (kinetic energy plus potential energy) of an electron in a distance Δq show a decrease for electrons with initial phases near $\pi/2$ and an increase for electrons with initial phases near $3\pi/2$ thus indicating an exchange of energy with the r-f wave.

It should be mentioned here that one further assumption was necessary in order to eliminate a computer problem which arose. When

the initial r-f voltage amplitude is large (of the same order of magnitude as the d-c voltage amplitude), one or two electrons from a single emission point (about 3 to 6 percent of the total charge) are influenced to such an extent that their trajectories actually form a loop, causing these electrons to begin traveling in the negative z-direction. This is recognized when u_z becomes less than $-1/2D$ and occurs in Figs. 4.14 and 4.15 for the electrons with $\Phi_0 = 5\pi/4$. However, the computer simulation of the emitting-sole problem dictates that the electrons must be moving in the positive z-direction since this is the direction of coordinate integration. It is therefore necessary to eliminate these electrons from the program. This can be justified because these electrons are few in number and close to the cathode where the r-f field is weak. Therefore their absence does not appreciably influence the r-f quantities.

Figures 4.14 and 4.15 also show that the maximum z-velocity occurs at the top of each trajectory as it should. This maximum varies with the initial phase of the electrons and is seen to range from approximately v_0 to $3v_0$. For d-c fields only, and zero emission velocities, the maximum z-velocity would have been $2v_0$ at the peak of the trajectory.

4.4.2 Results for the R-f Circuit Variables. Although the results thus far seem fairly favorable, with noticeable electron bunching in both coordinate and phase-coordinate spaces, the gain obtained in solving the equation for $A_{rf}(q)$ is rather disappointing. Figure 4.16 shows the variation in r-f voltage amplitude with distance for three values of the initial amplitude. The saturation gain for the two cases which display growth is only about 3 db. The curves

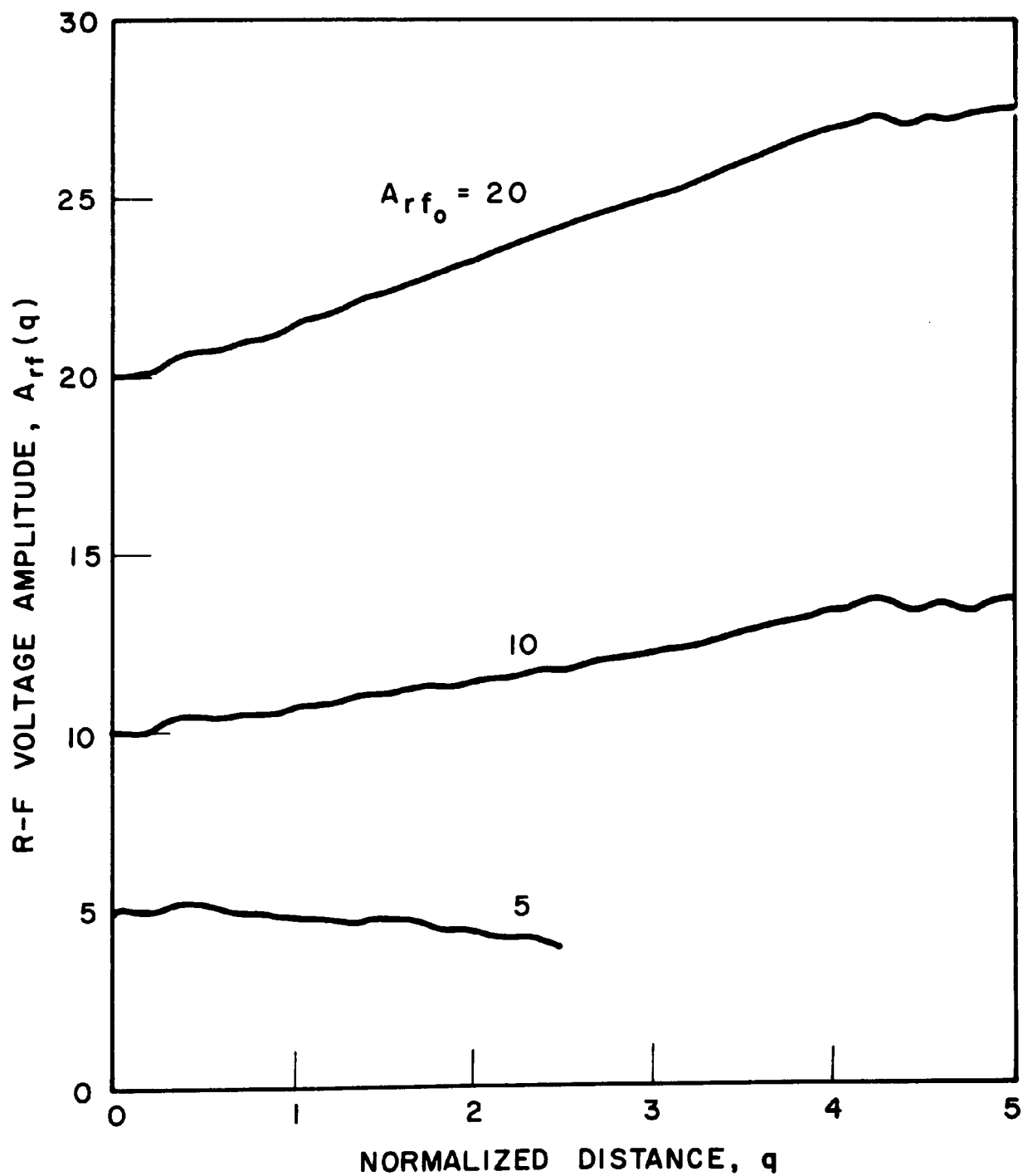


FIG. 4.16 VARIATION IN THE R-F VOLTAGE AMPLITUDE WITH LONGITUDINAL DISTANCE.

($D = 0.1$, $\omega_c/\omega = 1$, $A_{dc} = 20$, $\beta_o b' = 2$, $\beta_o L_e = 40$)

also show that the growth rate, dA_{rf}/dq , is larger for a larger initial r-f voltage amplitude. This indicates that a stronger electron-wave interaction occurs for larger initial amplitudes. Physically, this is quite logical because electron trajectories obviously can be modified more effectively by large r-f fields than by small fields. Thus electrons can be decelerated more easily by large fields, indicating that more potential energy is extractable under these conditions. It is also observed in Fig. 4.16 that a minimum initial r-f voltage amplitude is required if gain is to occur at all. This is in agreement with the results obtained by Yu et al.³⁴.

The phase angle $\theta_c(q)$ is defined as the phase angle between the actual wave on the r-f circuit and the unperturbed traveling wave which has phase velocity v_o . Figure 4.17 shows the variation of $\theta_c(q)$ as a function of longitudinal distance with A_{rf0} used as a parameter. The slope of these curves ($d\theta_c/dq \triangleq \theta'_c$) is an indication of the amount of stream-loading upon the r-f wave, in that the propagation constant of the wave is given by $\beta = \beta_o(1 - D\theta'_c)$. From this equation, the phase velocity of the r-f wave can be written as

$$v_{ph} = \frac{v_o}{1 - D\theta'_c} \quad (4.2)$$

If θ_c increases with q , θ'_c is positive and v_{ph} becomes greater than v_o . On the other hand, if θ_c decreases with q , θ'_c will be negative and v_{ph} will be less than v_o . Since v_o is chosen to be equal to the average drift velocity of the electrons ($v_o = E_o/B_o$, the synchronous condition) a nonzero θ'_c indicates that operation is further from synchronism. Thus, the observed variations in Fig. 4.17 are physically

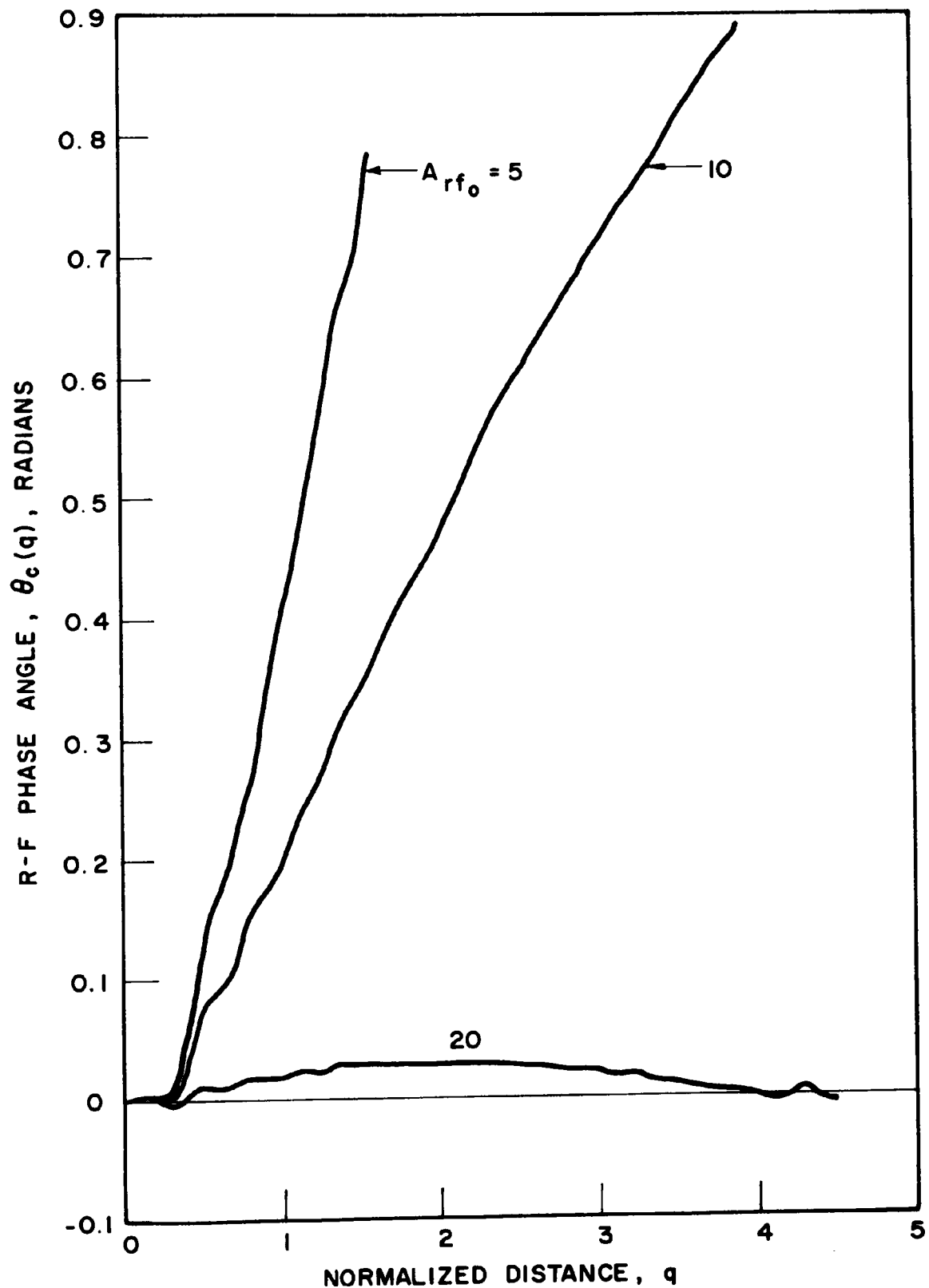


FIG. 4.17 VARIATION IN THE PHASE ANGLE OF THE R-F WAVE WITH LONGITUDINAL DISTANCE. ($D = 0.1$, $\omega_c/\omega = 1$, $A_{dc} = 20$, $\beta_o b' = 2$, $\beta_o L_e = 40$)

correct, showing θ'_c largest for $A_{rf_0} = 5$ (no gain) and smallest for $A_{rf_0} = 20$ (largest gain).

Note that both Figs. 4.16 and 4.17 display ripple variations. These variations are seen to be most prominent for small q and large q . They are therefore thought to be due to an insufficient number of electrons being used to calculate the induced current integrals, thus upsetting the averaging process. The reason for this insufficient number is that for small q not enough electron emission points have been passed, and for large q many electrons have been collected on the electrodes and eliminated from the problem.

4.5 Solution Considering Secondary Emission

In Section 4.4.1 the types of trajectories (including curves of Φ , p , u_y and u_z vs. q) obtained without secondary emission are described. In this section the more interesting cases including secondary emission will be presented. Also, a comparison of the r-f quantities for the cases both with and without secondary emission will be given.

4.5.1 Secondary Emission Trajectories. Figures 4.18 through 4.22 display the electron trajectories and velocity variations for electrons emitted from the first emission point, neglecting space charge but with secondary emission included. The d-c parameters chosen for this case are the same as those in the previous section, except that now $\delta = 1.0$. It should be noted that the trajectories for this case are almost identical to those for the zero secondary emission case except that here secondaries are emitted when primary electrons strike the sole electrode.

For clarity, the p-curves shown in Fig. 4.18 include only one fourth of the total number of electrons emitted from the first emission

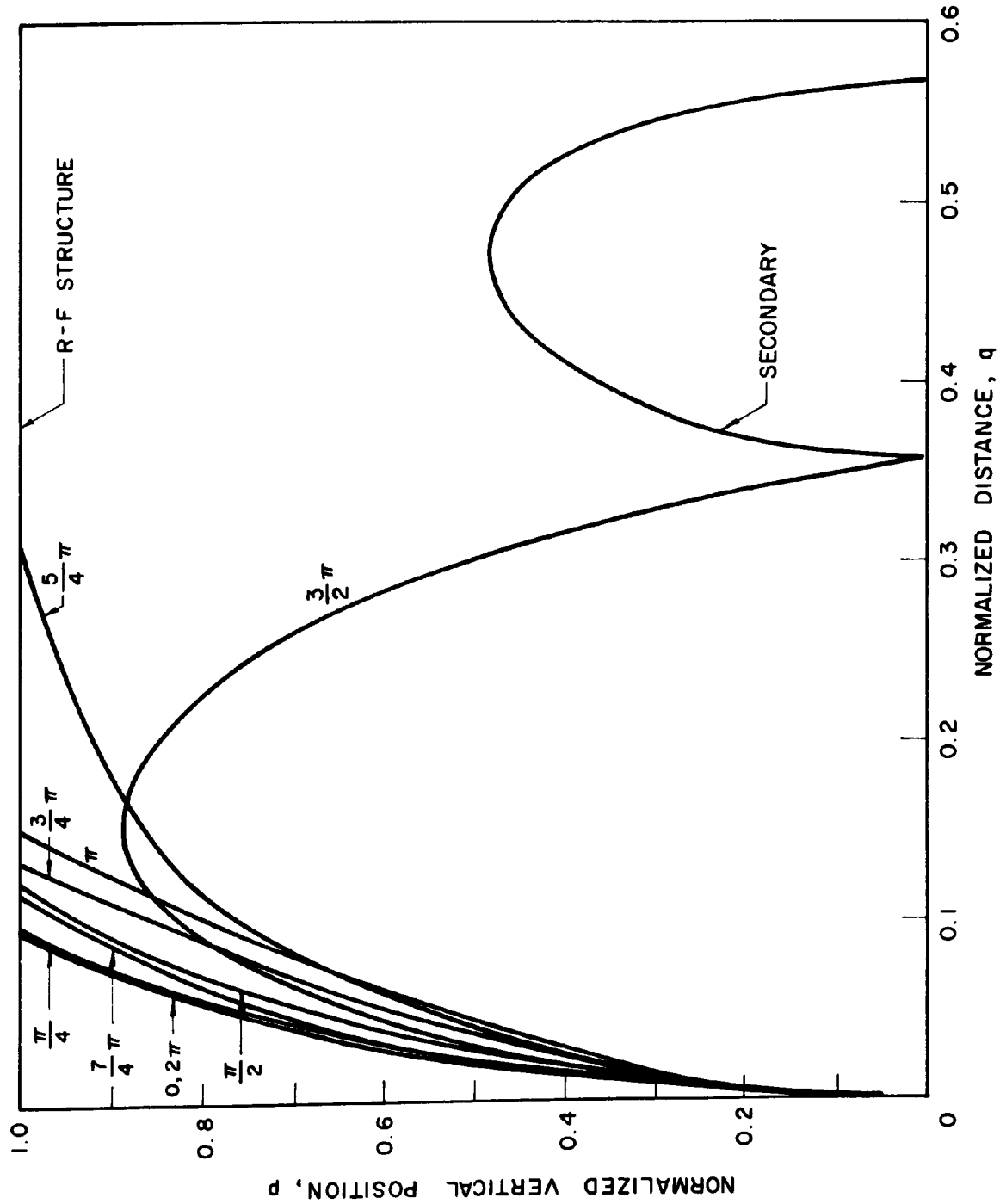


FIG. 4.18 ELECTRON TRAJECTORIES FOR THE FIRST EMISSION POINT WITH ϕ_0 AS A PARAMETER INCLUDING SECONDARY ELECTRONS. ($D = 0.1$, $\omega_c/\omega = 1$, $\delta = 1$, $A_{rf}/A_{dc} = 1$, $\beta_0 b' = 2$, $\beta_0 L_e = 40$)

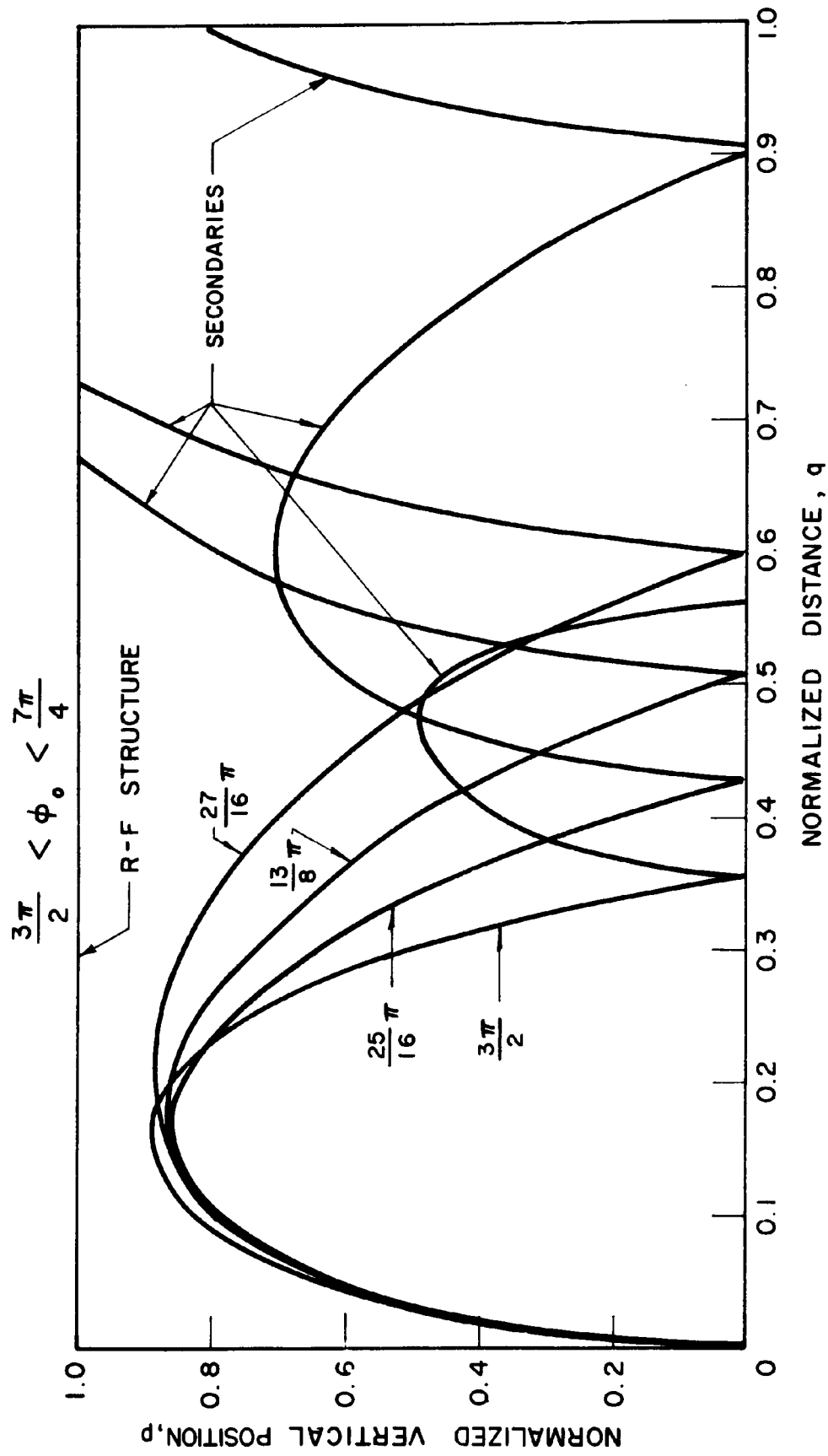


FIG. 4.19 ELECTRON TRAJECTORIES ($3\pi/2$ TO $7\pi/4$) IN THE ACCELERATION PHASE OF THE R-F WAVE INCLUDING

SECONDARY ELECTRONS. ($D = 0.1$, $\omega/\omega_c = 1$, $\delta = 1$, $A_{rf}/A_{dc} = 1$, $\beta_0 b' = 2$, $\beta_0 L_e = 40$)

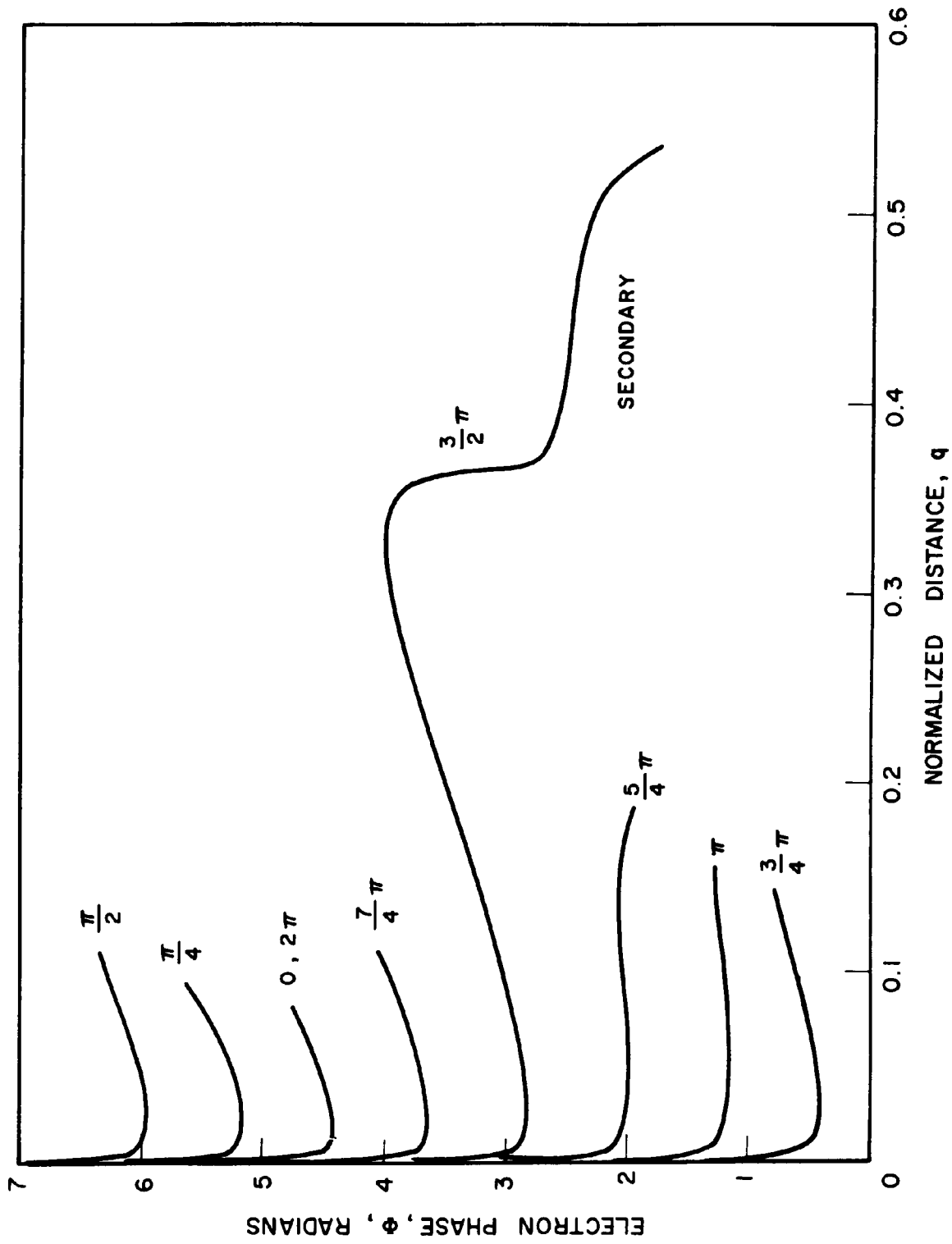


FIG. 4.20 PHASE POSITION VS. LONGITUDINAL DISTANCE WITH ϕ_0 AS A PARAMETER INCLUDING SECONDARY

ELECTRONS. ($D = 0.1$, $\omega_c/\omega = 1$, $\delta = 1$, $A_{rf}/A_{dc} = 1$, $\beta_{bo} = 1$, $\beta_{bo} L_e = 40$)

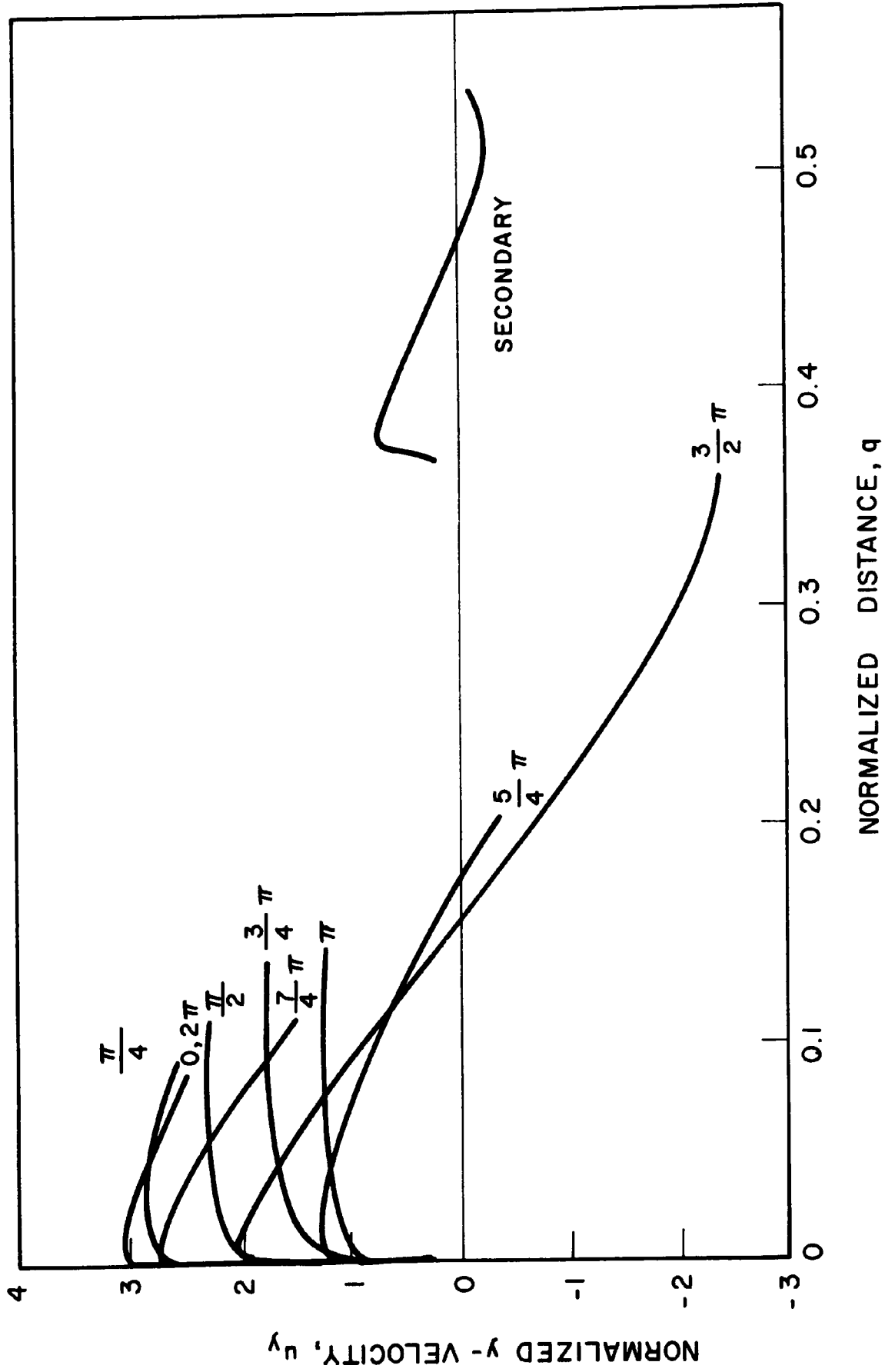


FIG. 4.21 NORMALIZED VERTICAL VELOCITY VS. LONGITUDINAL DISTANCE WITH ϕ_0 AS A PARAMETER INCLUDING SECONDARY ELECTRONS. ($D = 0.1$, $\omega_c/\omega = 1$, $\delta = 1$, $A_{rf}/A_{dc} = 1$, $\beta_{ob}' = 2$, $\beta_{oe} = 40$)

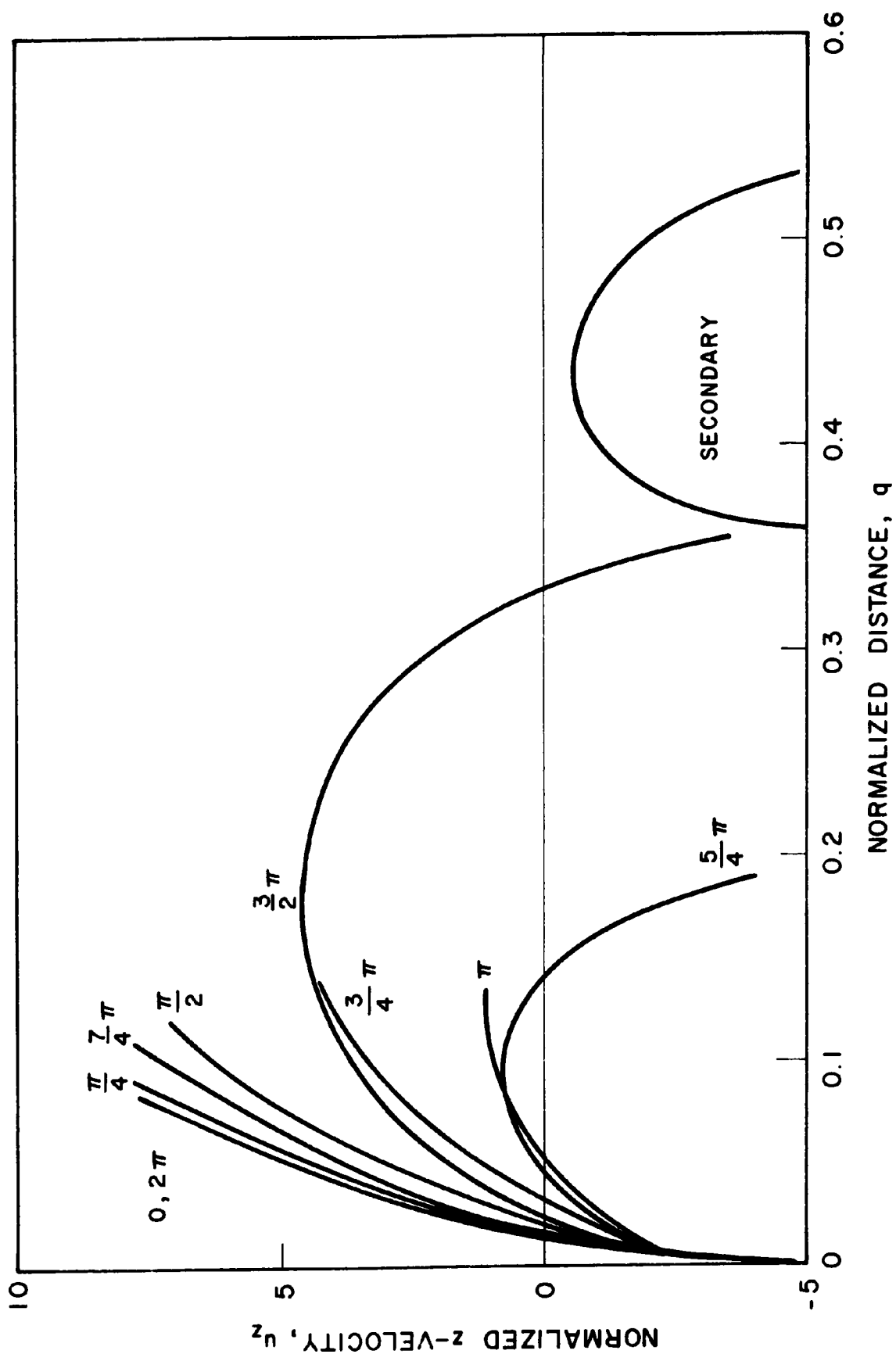


FIG. 4.22 NORMALIZED LONGITUDINAL VELOCITY VS. LONGITUDINAL DISTANCE WITH ϕ_0 AS A PARAMETER INCLUDING SECONDARY ELECTRONS. ($D = 0.1$, $\omega_c/\omega = 1$, $\delta = 1$, $A_{rf}/A_{dc} = 1$, $\beta_{ob} = 1$, $\beta_{oe} = 40$)

point. Also, as in Section 4.4.1, the initial electron velocity components used for the calculation of these curves are for an injection angle of $\Omega = -30$ degrees. The parameter used is Φ_0 and it ranges from 0 to 2π in steps of $\pi/4$. As previously observed in the zero secondary emission case, the trajectories indicate collection of electrons on the r-f circuit if the initial phase of an electron is in the range 0 to π , whereas the electrons with initial phase values between π and 2π are returned to the sole. Figure 4.19 shows this acceleration effect for electrons near the maximum accelerating r-f field ($\Phi_0 = 3\pi/2$), which causes them to return to the sole and emit secondaries. This curve also displays the large number of trajectory crossings which are occurring.

The variation of electron phase relative to the r-f wave as a function of longitudinal distance is shown in Fig. 4.20. Again, Φ_0 is the parameter and only one fourth of the electrons from the first emission point are plotted. It should be noted that bunching is occurring about the phase position $\Phi = \pi/2$. Since this is the maximum deceleration phase of the r-f wave, it would appear that energy is being extracted from these bunching electrons. This indeed is the case and will be discussed in the section on r-f voltage amplitude.

The curves for u_y and u_z for one fourth of the electrons emitted from the first emission point are shown in Figs. 4.21 and 4.22. The same variations as previously obtained for the zero secondary emission case are obtained here. However, one difference is evident and that is that a jump in velocity occurs when a secondary electron is emitted. The electron with $\Phi_0 = 3\pi/2$ is an example of this velocity discontinuity.

This, nonetheless, is entirely reasonable since the energy distribution of secondary electrons is independent of primary energy.

4.5.2 Results for the R-f Wave Variables. Since the optimum growth rate and therefore gain was achieved for the zero secondary emission case when A_{rf_0} was chosen to be equal to A_{dc} , only this particular case has been investigated here. Figure 4.23 shows the variation in r-f voltage amplitude with distance. This figure indicates that the deceleration and bunching of the electrons as described in the previous section has produced an increase in r-f energy, which is attributable to a decrease in electron potential energy due to the constant average kinetic energy of the electrons. Although growth of the r-f amplitude is achieved in the zero space-charge case, both with and without secondary emission, the saturation gain is still disappointing. The saturation gain for the secondary emission case is about 3.3 db and without these "extra" electrons it is only about 2.6 db. This increase can be explained by realizing that more electrons, namely secondaries, are able to give up their potential energy to the wave. The effect of a higher secondary emission ratio ($\delta > 1$) should also lead to greater saturation gain.

The curve of $\theta_c(q)$ vs. q for this secondary emission case is shown in Fig. 4.24. Again, as discussed in Section 4.4.2, the important quantity to note from this curve is the slope, $\theta'_c(q)$. As can be seen from Fig. 4.24, the slight variations in θ_c imply that operation is near synchronism and thus gain in the r-f voltage amplitude results.

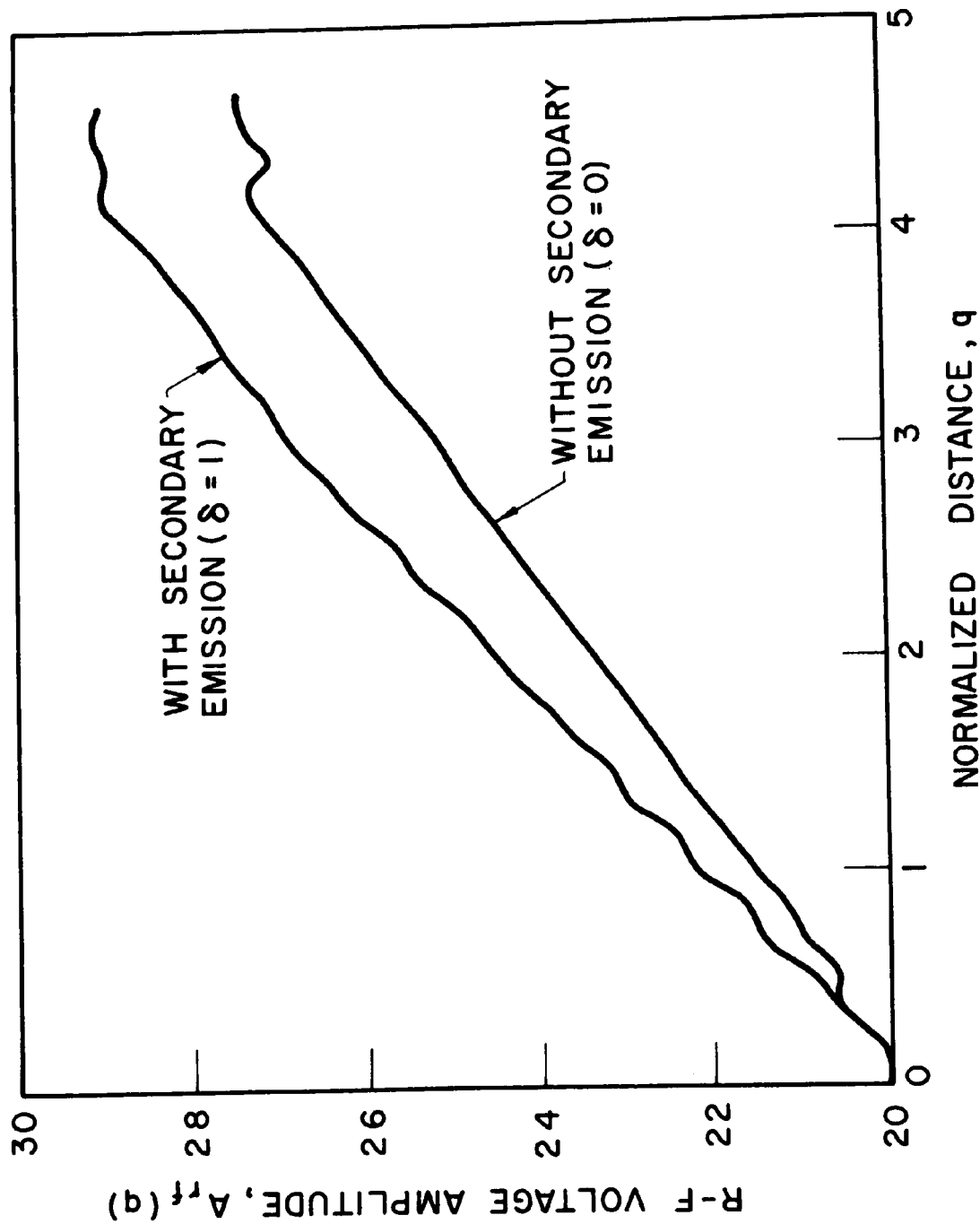


FIG. 4.23 EFFECT OF SECONDARY EMISSION UPON THE R-F VOLTAGE AMPLITUDE.

($D = 0.1$, $\omega_c/\omega \approx 1$, $A_{rf}/A_{dc} = 1$, $\beta_o b' = 2$, $\beta_o L_e = 40$)

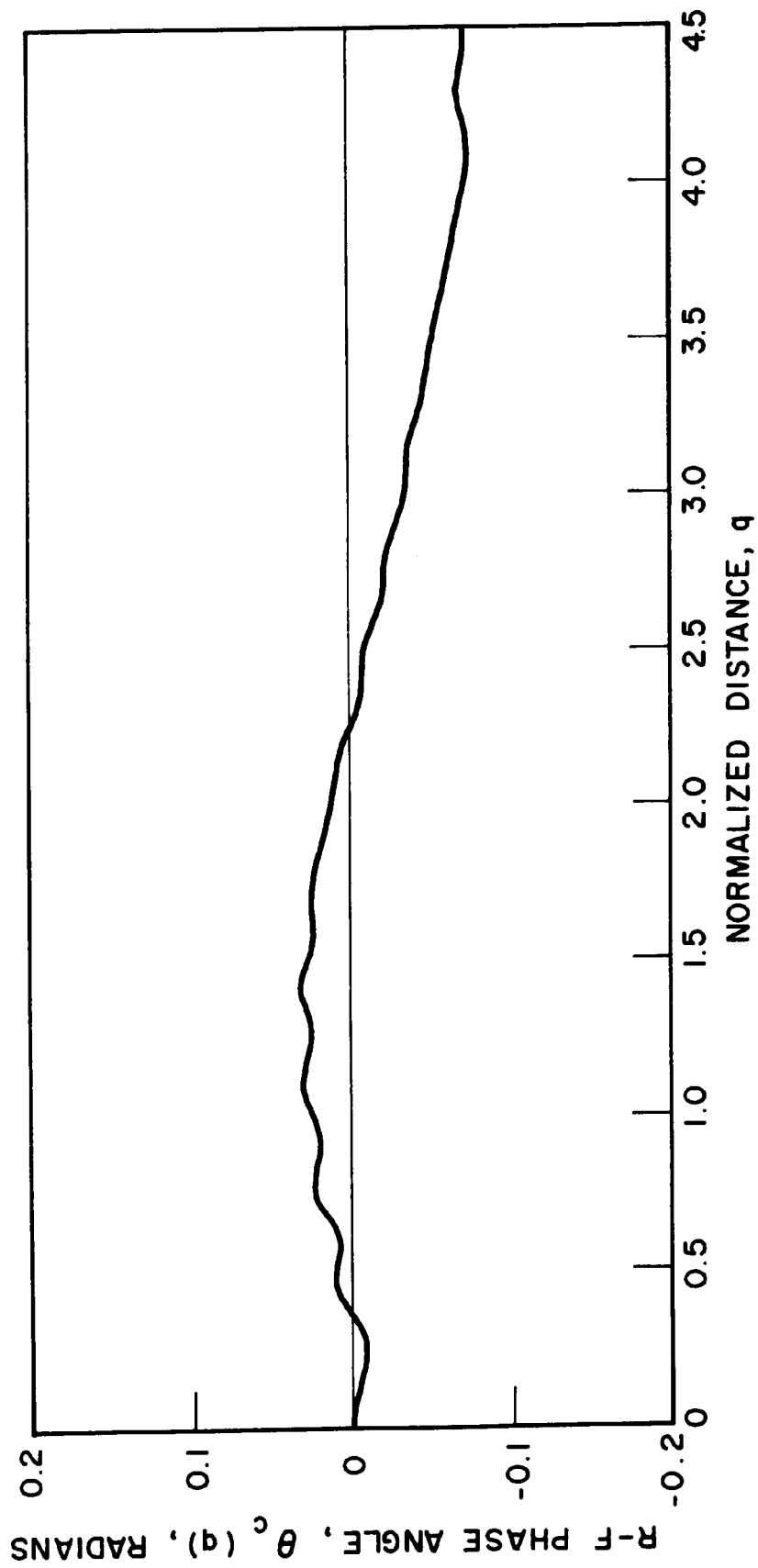


FIG. 4.24 VARIATION IN THE R-F PHASE ANGLE WITH LONGITUDINAL DISTANCE.

$$(D = 0.1, \omega_c/\omega = 1, A_{rf}/A_{dc} = 1, \beta_o b' = 2, \beta_o L_o = 40)$$

CHAPTER V. AMPLIFIER SOLUTIONS INCLUDING SPACE-CHARGE-FIELD EFFECTS

5.1 Introduction

The computer solutions to be presented in this chapter also were obtained from the difference-form equations of Chapter III. In these cases, however, the space-charge fields are no longer assumed to be negligible and thus the space-charge force terms are included in the dynamic equations of motion. These terms are included by giving a finite value to the parameter ω_{pc}/ω . The definition of ω_{pc} is

$$\omega_{pc} \triangleq 2 \left| \frac{\eta J_{th}}{\epsilon_o v_c} \right| , \quad (5.1)$$

where η is the charge-to-mass ratio of an electron, J_{th} is the thermionic emission current density, ϵ_o is the dielectric constant of free space and v_c is the characteristic thermionic velocity given by $(2kT/m)^{1/2}$. The values chosen for the ratio of cathode plasma frequency to r-f radian frequency, ω_{pc}/ω , correspond to thermionic current density values which can easily be obtained from an impregnated tungsten cathode at temperatures ranging from 1050°K to 1200°K and r-f signals in the S-band frequency range. Under these conditions the parameter ω_{pc}/ω may have values ranging from one to ten. For X-band frequency operation and the same cathode temperature this parameter would necessarily be reduced.

The same emission model as used in Chapter IV is used in this chapter to obtain the emitting-sole amplifier solutions, although a different model is used in the case of the multiple-cathode amplifier solutions (this will be discussed in Section 5.4.1). In all cases

32 electrons are emitted from each emission point and there are 81 emission points uniformly spaced a distance of $10 \Delta q$ apart from one another, where Δq is the integration increment ($\Delta q = 0.005$). Also, the normalized computer input parameters are defined in the same manner as those in Section 4.3.

5.2 Emitting-Sole Amplifier Solutions

5.2.1 Selection of the D-c Parameters. In specifying the d-c parameters, the following two conditions are to be satisfied:

$$v_o = V_a / (b'B) = v_d \quad (5.2)$$

and

$$\omega_c b' = K \sqrt{2|\eta|V_a} \quad (5.3)$$

Equation 5.2 is the synchronism condition which stipulates that the unperturbed phase velocity v_o of the fundamental r-f wave component is exactly equal to the average drift velocity v_d of the electrons. It has been found^{19,23,24} that when this relation is approximately satisfied an effective electron-wave interaction takes place. However, when space-charge forces are considered the situation becomes more complicated than the first d-c relationship indicates and a thorough discussion of this will be presented in this chapter. The second relation, Eq. 5.3, can be recognized as the magnetron cutoff condition for a parallel-plane geometry with crossed d-c fields when K is set equal to unity. For $K > 1$ the electrons emitted from the zero potential planar electrode describe a cycloid between this electrode and the higher potential electrode, whereas for $K < 1$ the electrons again exhibit cycloidal motion but in this case the trajectories are high enough to result in electron interception on the higher potential

electrode. This situation is altered when the higher d-c potential electrode is an r-f slow-wave structure and an r-f signal is applied. However, the constant K retains its significance in that its value gives an indication of the vertical position of the majority of electrons, i.e., for $K < 1$ the electrons will approach the r-f structure more closely than for $K > 1$. In the next section trajectory solutions are obtained for values of K greater than, less than and equal to unity. These values of K correspond respectively to d-c operation above, below and at the magnetron cutoff condition.

5.2.2 Large-Signal Electron Ballistics. The large-signal electron ballistics to be discussed in this chapter include the effects of the crossed d-c electric and magnetic fields, the r-f field and the space-charge field. The electron variables p , Φ , u_y and u_z in this case are similar to those obtained in Chapter IV in which the space-charge fields were neglected. However, variations are present as will be seen.

The first variable to be discussed is the phase parameter Φ , which indicates the phase position of an electron relative to the r-f wave. Figures 5.1 through 5.3 show the variation in Φ with longitudinal distance for the electrons emitted from the first emission point. The computer input parameters used for these cases include $D = 0.1$, $\delta = 1$, $\beta_o b' = 2$, $\beta_o L_e = 40$ and $A_{rf_o}/A_{dc} = 1$. Also, each figure exhibits the solutions for a different value of ω_c/ω . As in Chapter IV only one fourth of the electrons from this emission point are displayed for an injection angle $\Omega = -30$ degrees (see Appendix D).

Figure 5.1 shows the phase vs. distance variation for electrons with a value of magnetic field above the d-c cutoff value,

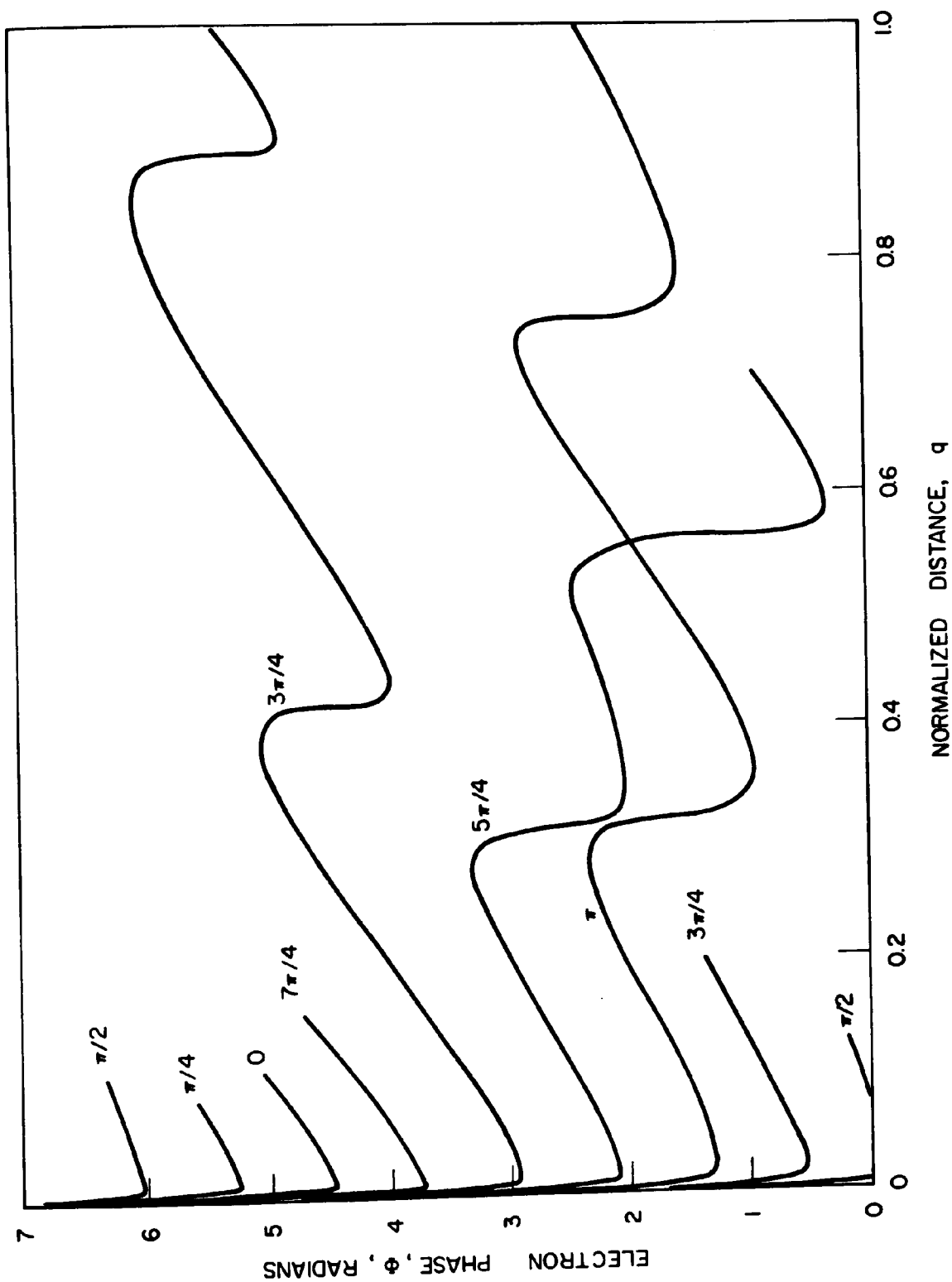


FIG. 5.1 PHASE POSITION VS. LONGITUDINAL DISTANCE WITH ϕ_0 AS A PARAMETER AND $\omega_c/\omega = 1.5$.

$$(\omega_{pc}/\omega = 5, \delta = 1, A_{rf}/A_{dc} = 1, D = 0.1, \beta_0 b' = 2, \beta_0 L_e = 40)$$

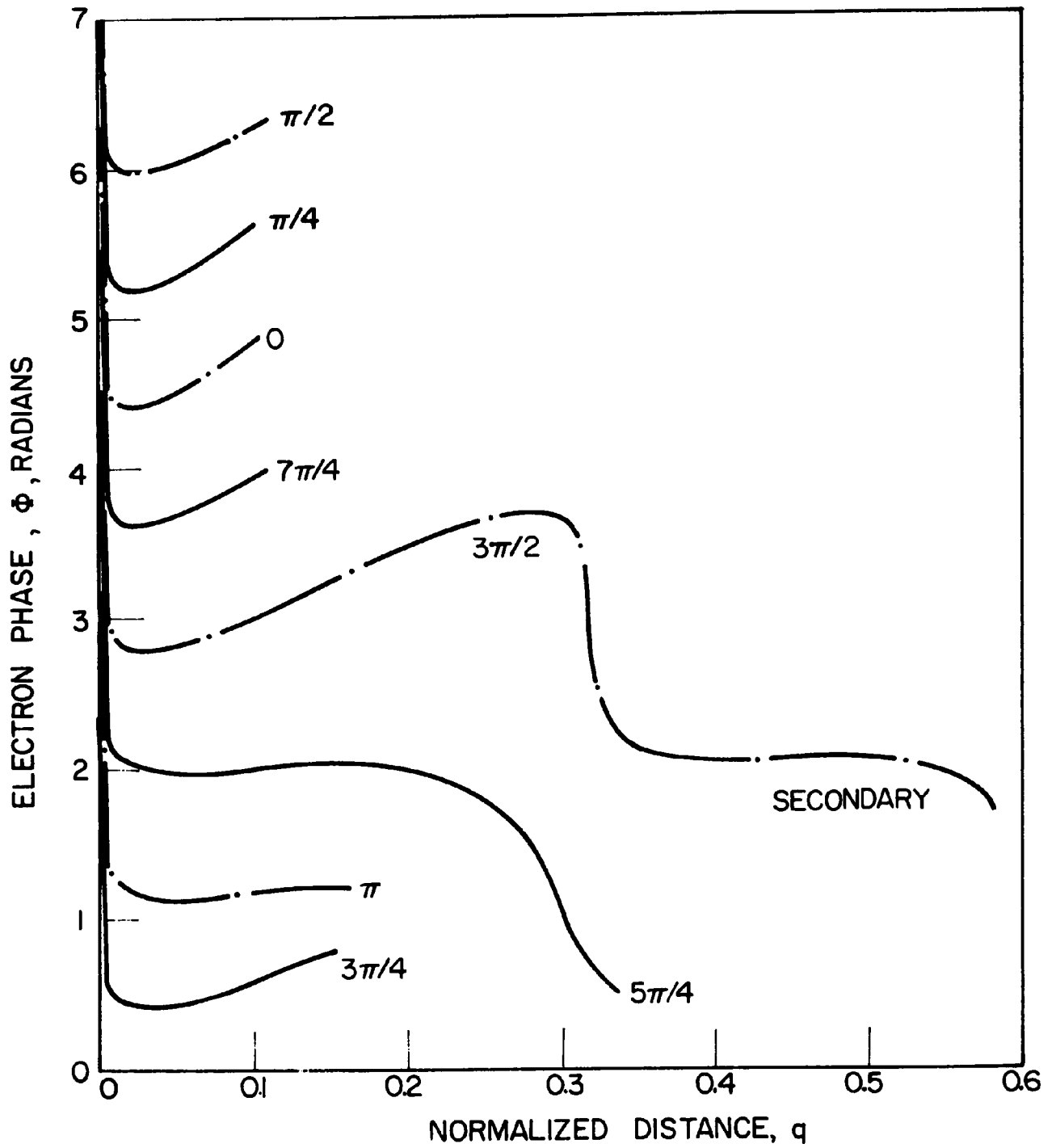


FIG. 5.2 PHASE POSITION VS. LONGITUDINAL DISTANCE WITH ϕ_0 AS A
 PARAMETER AND $\omega_c/\omega = 1$. ($\omega_{pc}/\omega = 5$, $\delta = 1$, $A_{rf0}/A_{dc} = 1$,
 $D = 0.1$, $\beta_0 b' = 2$, $\beta_0 L_e = 40$)

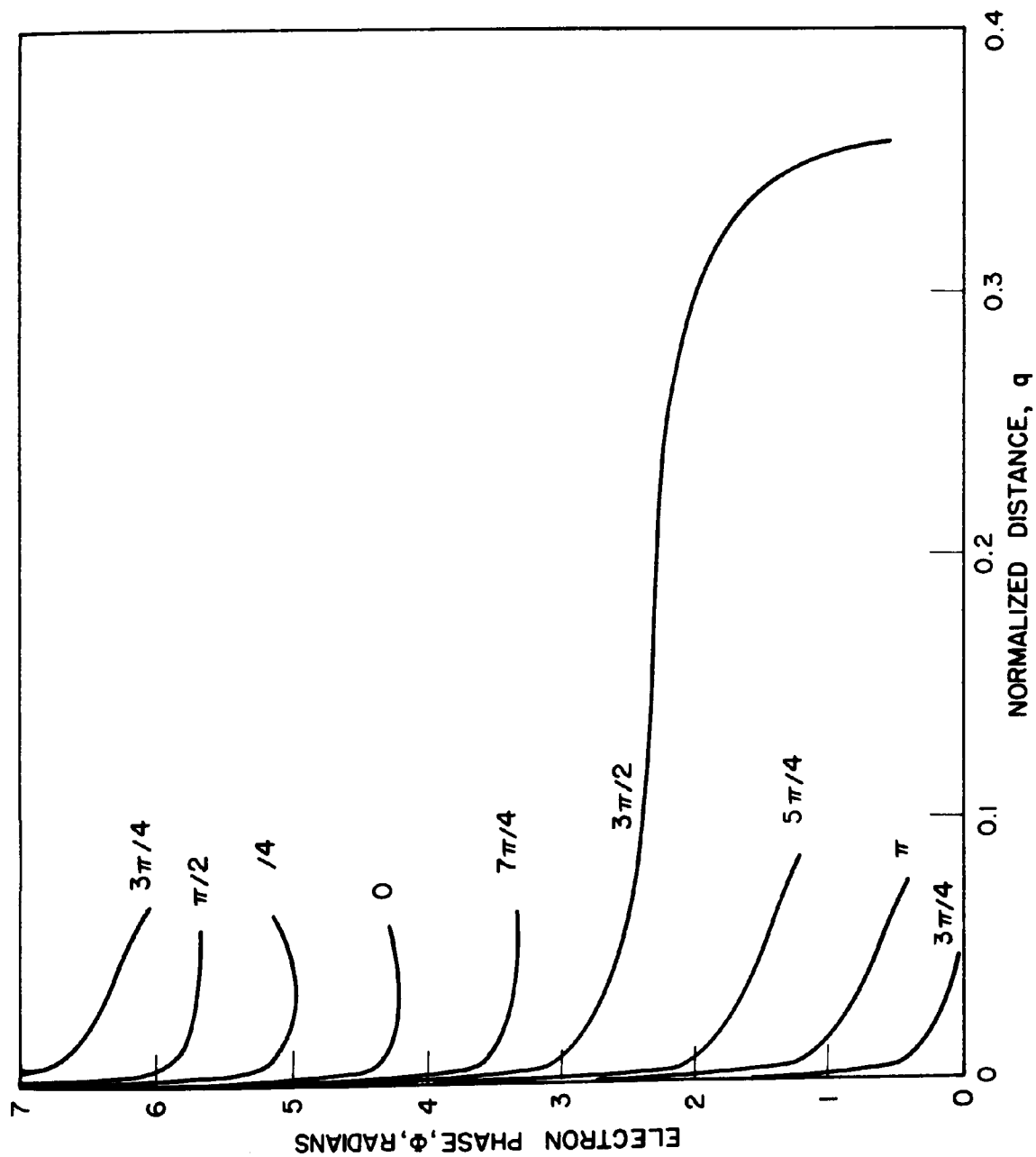


FIG. 5.3 PHASE POSITION VS. LONGITUDINAL DISTANCE WITH ϕ_0 AS A PARAMETER AND $\omega_c/\omega = 0.5$.

$$(\omega_{pc}/\omega = 5, \delta = 1, A_{rf}/A_{dc} = 1, D = 0.1, \beta_0 b' = 2, \beta_0 L_e = 40)$$

$\omega_c/\omega = 1.5$ ($K^2 = 1.5$). The figure indicates that there is only a slight amount of bunching occurring in the maximum deceleration phase of the r-f field, i.e., the curves are very nearly parallel to one another and are only very slightly converging toward the phase position $\Phi = \pi/2$ as a function of distance. The phase vs. distance curves for a reduced magnetic field equal to the cutoff value ($\omega_c/\omega = 1$ and $K = 1$) are displayed in Fig. 5.2. These paths show a slightly greater tendency to converge into the region of maximum decelerating field. Finally, Fig. 5.3 indicates the phase vs. distance variations for electrons subjected to a magnetic field value which is less than the cutoff value ($\omega_c/\omega = 0.5$ and $K^2 = 0.5$). This figure shows greater bunching tendencies than those observed in Figs. 5.1 and 5.2. It should also be noted that the phase values for each electron show divergent tendencies away from the phase region of maximum acceleration, i.e., the phase position $\Phi = 3\pi/2$. The results of convergence toward $\Phi = \pi/2$ and divergence away from $\Phi = 3\pi/2$ were also obtained in Chapter IV when space-charge forces were neglected.

The physical significance of these results is that, due to bunching in the decelerating field region as the electrons approach the slow-wave structure, there is a conversion of electron potential energy into r-f energy which results in a growth of the r-f wave amplitude. Under ideal conditions all of the electrons would form a bunch in the maximum decelerating field region and become rarified in the maximum accelerating field region. Furthermore, this bunch remains in synchronism ($v_d = v_o$) with the r-f wave and approaches the r-f structure, thus giving up its potential energy to the r-f wave. This would then lead

to a large amplification of the r-f signal. Such an ideal behavior, however, is never actually achieved, although even a slight amount of bunching as observed in Figs. 5.1 through 5.3 will be shown to be sufficient to produce r-f amplification. Also, as expected, it will be shown that larger r-f growth rates and greater amplification is obtained for the cases that display better bunching tendencies, i.e., when convergence about $\phi = \pi/2$ is achieved in shorter longitudinal distances.

The phase position variations with longitudinal distance displayed in Figs. 5.1 through 5.3 demonstrate a greater amount of bunching in shorter distances than was observed in Chapter IV for the zero space-charge-field cases. This implies a greater r-f amplitude growth, as will be discussed in the next section. Thus, the inclusion of space-charge forces tends to aid the bunching process. A comparison of the growth rates for these three cases will be given in the next section.

The first emission point electron trajectories for the same three cases of d-c magnetic field ($\omega_c/\omega = 1.5, 1$ and 0.5 , or $K^2 = 1.5, 1$ and 0.5) are shown in Figs. 5.4 through 5.6. It is seen that the trajectories for cases with space-charge fields included are similar to the zero space-charge trajectories of Chapter IV. However, variations are present as can be seen in Fig. 5.5 by noting the path of the electron with $\phi_0 = 5\pi/4$. This type of trajectory did not occur in the zero space-charge cases, and as observed in Fig. 5.2 the space-charge force on this electron has forced it into the deceleration phase of the r-f wave and hence to the r-f circuit.

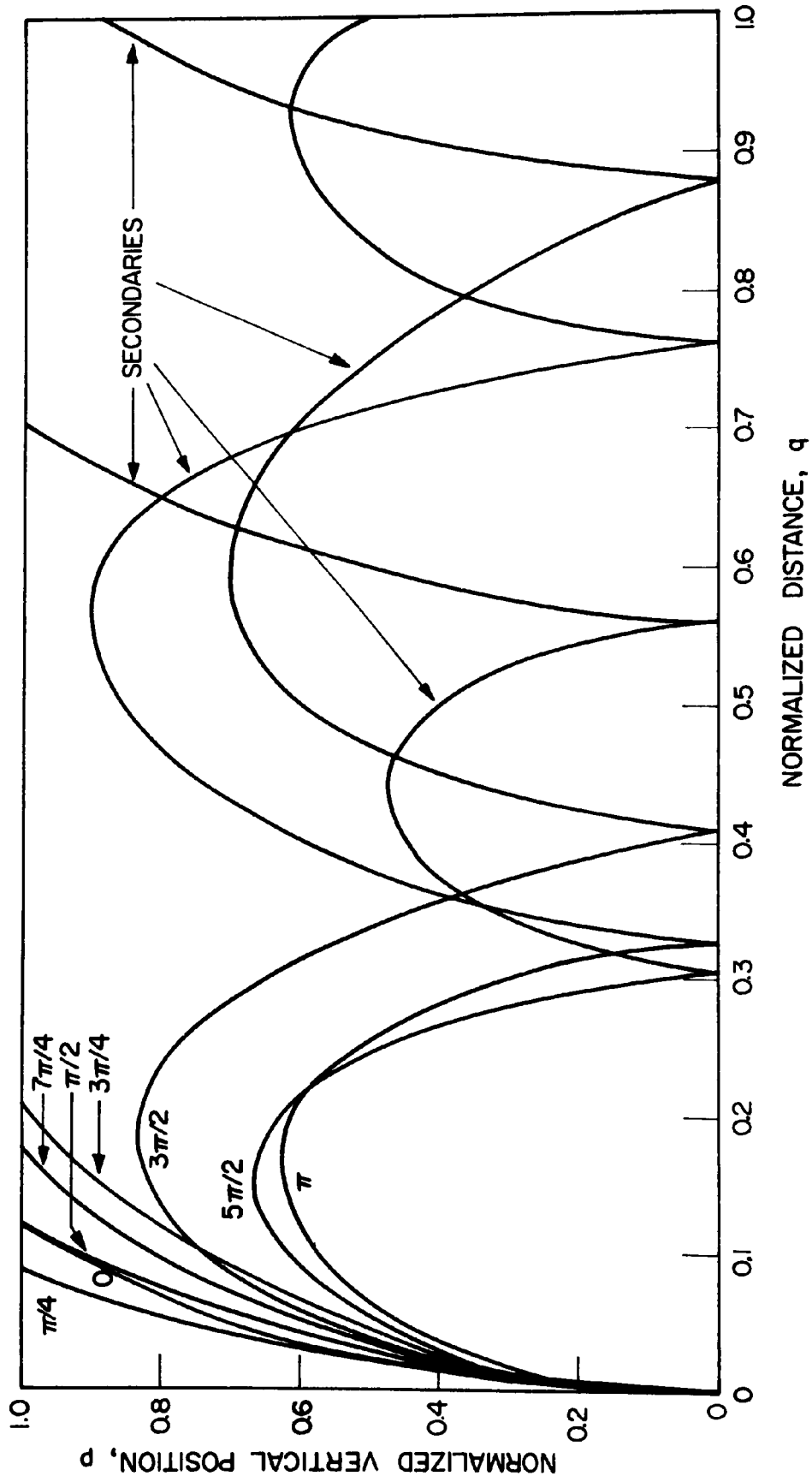


FIG. 5.4 ELECTRON TRAJECTORIES FOR ABOVE CUTOFF OPERATION WITH ϕ_0 AS A PARAMETER.

$$(\omega_c/\omega = 1.5, \omega_{pc}/\omega = 5, \delta = 1, A_{rf}/A_{dc} = 1, \beta_{ob} = 2, \beta_{oe} = 40)$$

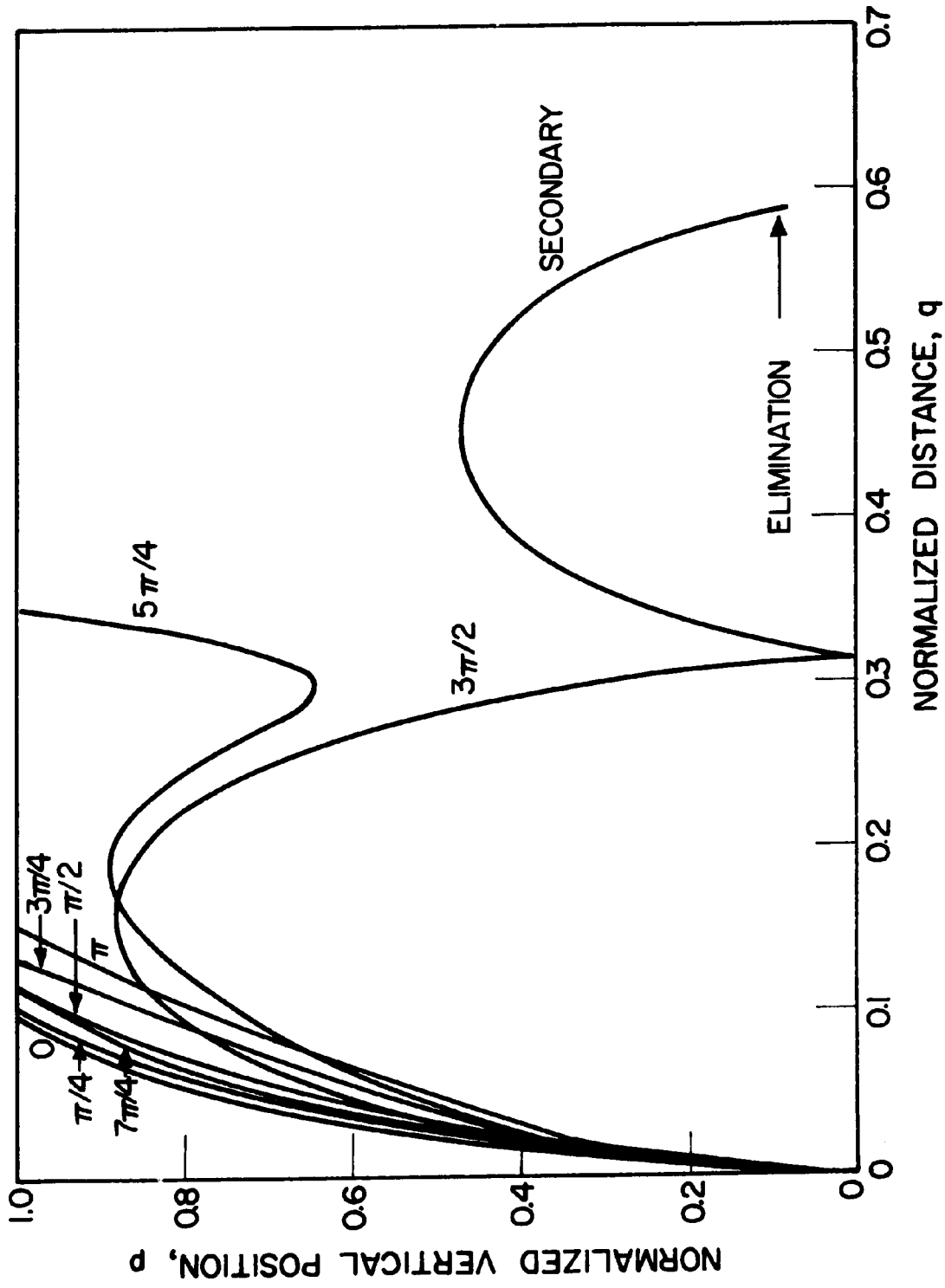


FIG. 5.5 ELECTRON TRAJECTORIES FOR OPERATION AT CUTOFF WITH ϕ_0 AS A PARAMETER.

$$(\omega_c/\omega = 1.0, \omega_{pc}/\omega = 5, \delta = 1, A_{rf}/A_{dc} = 1, \beta b' = 2, \beta I_0 e = 40)$$

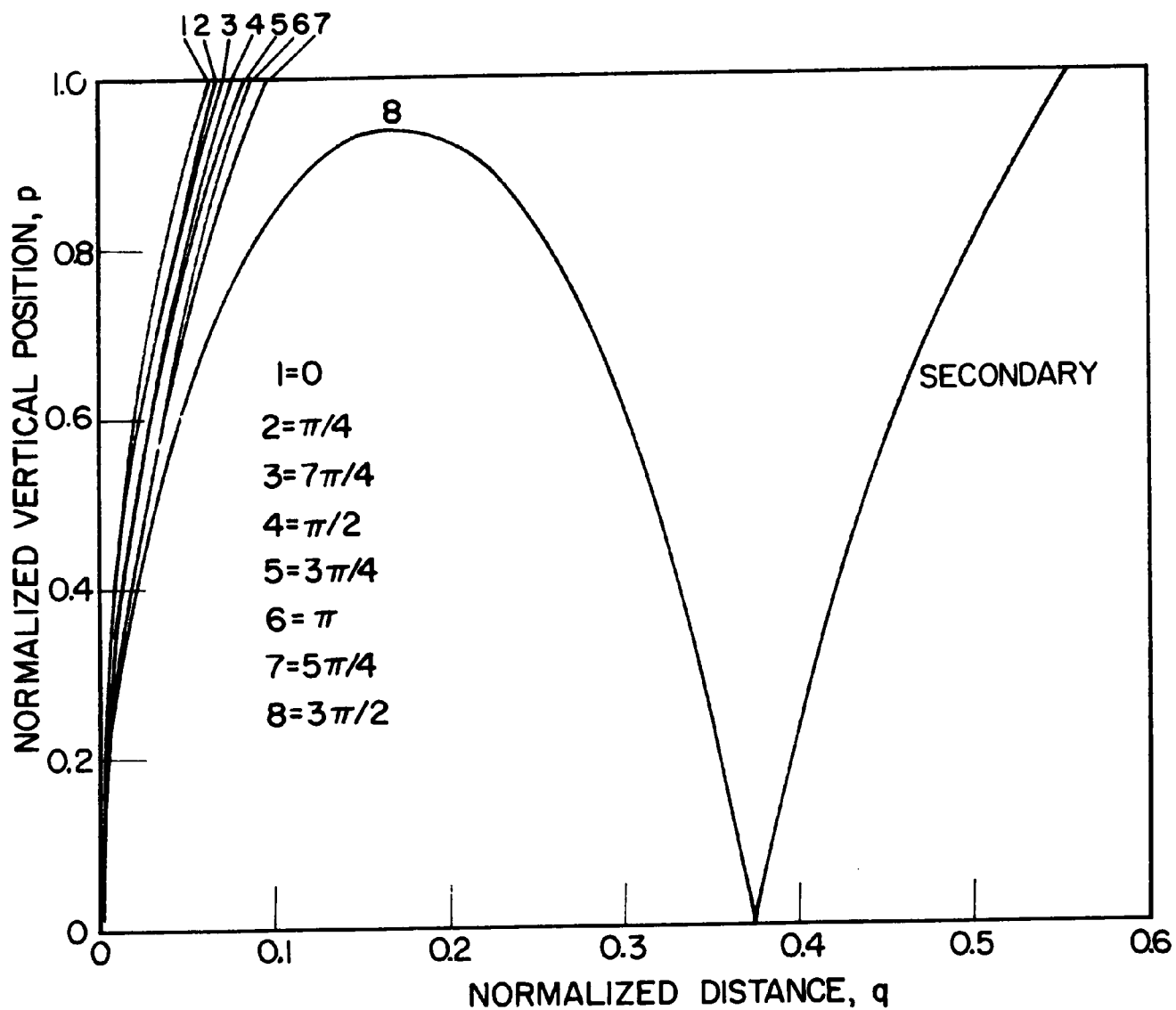


FIG. 5.6 ELECTRON TRAJECTORIES FOR BELOW CUTOFF OPERATION WITH ϕ_0 AS A PARAMETER. ($\omega_c/\omega = 0.5$, $\omega_{pc}/\omega = 5$, $\delta = 1$, $A_{rf0}/A_{dc} = 1$, $\beta_0 b' = 2$, $\beta_0 L_e = 40$)

Each of the curves illustrates the fact that electrons with initial phase values relative to the r-f wave of 0 to π are in general collected on the r-f circuit. On the other hand, electrons with phase values between π and 2π are in general forced to return to the sole electrode and produce secondary electrons.

It should be noted, however, that for magnetic field values below the cutoff value ($\omega_c/\omega = 0.5$ and $K^2 = 0.5$), the electrons experience weaker downward magnetic field forces and thus some of the electrons with initial phase values in the accelerating field region ($\Phi = \pi$ to 2π) are collected on the r-f structure and do not back-bombard the sole (see Fig. 5.4) as they would if the magnetic field were stronger. Also, for d-c magnetic fields above the cutoff value ($\omega_c/\omega = 1.5$ and $K^2 = 1.5$), it is observed that some of the electrons with phase values in the decelerating phase region of the r-f wave are subjected to larger downward magnetic field forces and are not collected on the r-f structure but instead return to the sole and produce secondary electrons. Thus as a result of a larger magnetic field it can be concluded that lower vertical positions of the trajectories are obtained on the average. This conclusion will be used to formulate an argument to partially explain the gain results in the next section.

The same problem as discussed in Chapter IV of electrons reversing their directions when they return to the sole also occurs here. It should be recalled that electron motion in the negative z-direction is inconsistent with the direction of integration used in the numerical solution method. Hence to eliminate this problem, backward-traveling electrons are removed in the computer analysis by

setting their coordinates and velocities equal to zero. An example of this is shown in Fig. 5.5. It is observed that the secondary electron produced by the primary electron with $\Phi_0 = 3\pi/2$ is eliminated from the computer analysis when this secondary electron reverses its longitudinal direction close to the sole. The justification for the elimination of this electron and others similar to it is again that this electron is quite far from the circuit, in an unfavorable phase position, and hence its absence does not appreciably influence the r-f quantities.

When the r-f amplitude grows with longitudinal distance due to a strong electron-wave interaction, the trajectories for electrons emitted near the r-f input end of the amplifier will be significantly different from those emitted near the r-f output end of the amplifier. This is because larger r-f fields are then acting upon the electrons. Since the gain achieved by this amplifier is not very large, a good comparison between different emission-point trajectory solutions can be made by comparing those from the first emission point with those from the last. Figures 5.7 and 5.8 display the phase of the electrons relative to the r-f wave and the vertical position of the electrons, respectively, for one fourth of the electrons emitted from the 81st emission point. The d-c parameters used in these figures are those for magnetron cutoff operation, i.e., $\omega_c/\omega = 1$ and $K^2 = 1$.

Again it is observed that these trajectory results are quite similar to those previously discussed. However, several differences between the trajectories for the first emission point and those for the 81st can be noted by careful inspection. First of all, the trajectories for the 81st emission point are shorter, i.e., collection

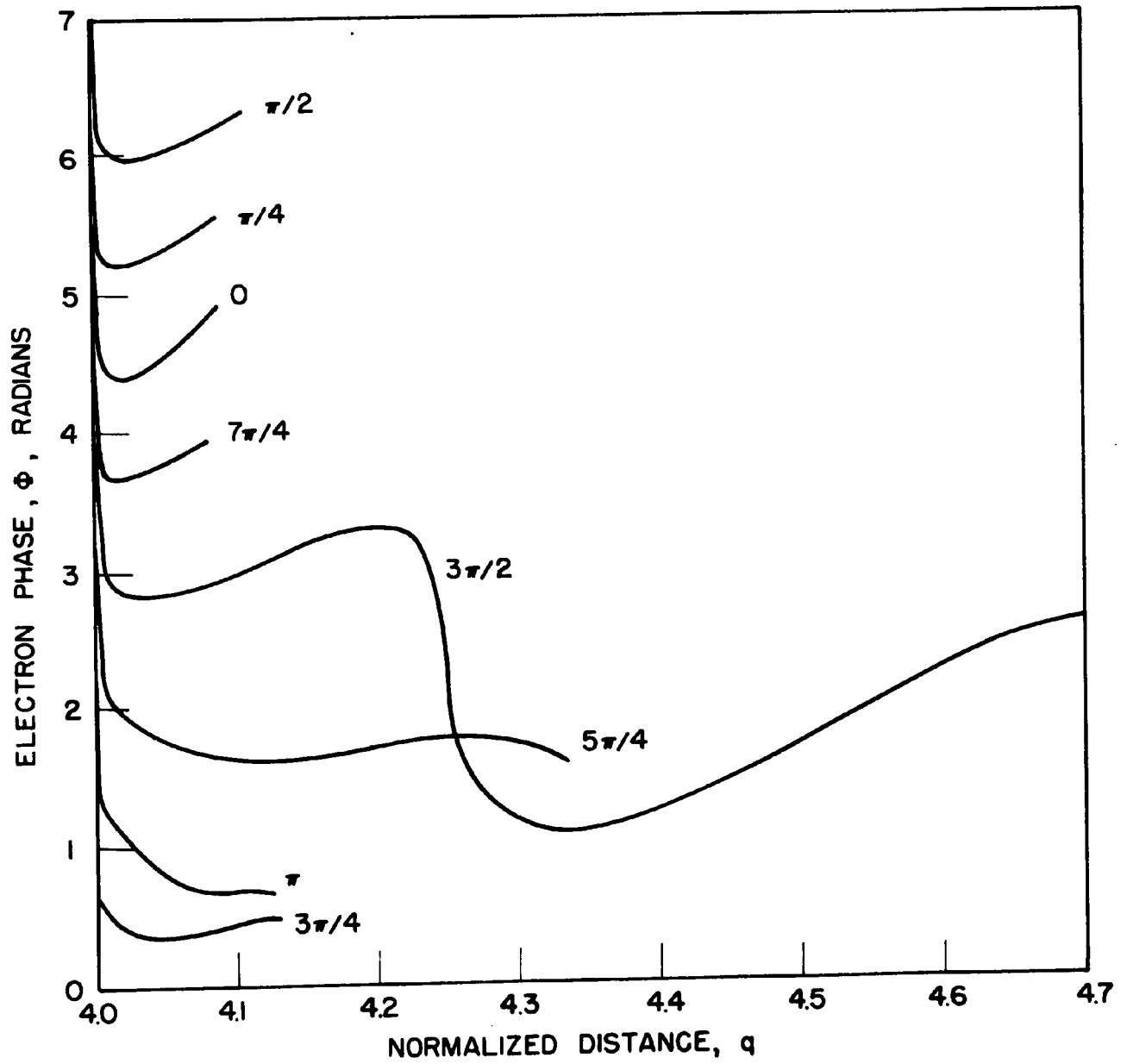


FIG. 5.7 ELECTRON PHASE VS. LONGITUDINAL DISTANCE FOR THE 81ST EMISSION POINT. ($\omega_c/\omega = 1$, $\omega_{pc}/\omega = 5$, $\delta = 1$, $A_{rf_0}/A_{dc} = 1$, $\beta_0 b' = 2$, $\beta_0 L_e = 40$)

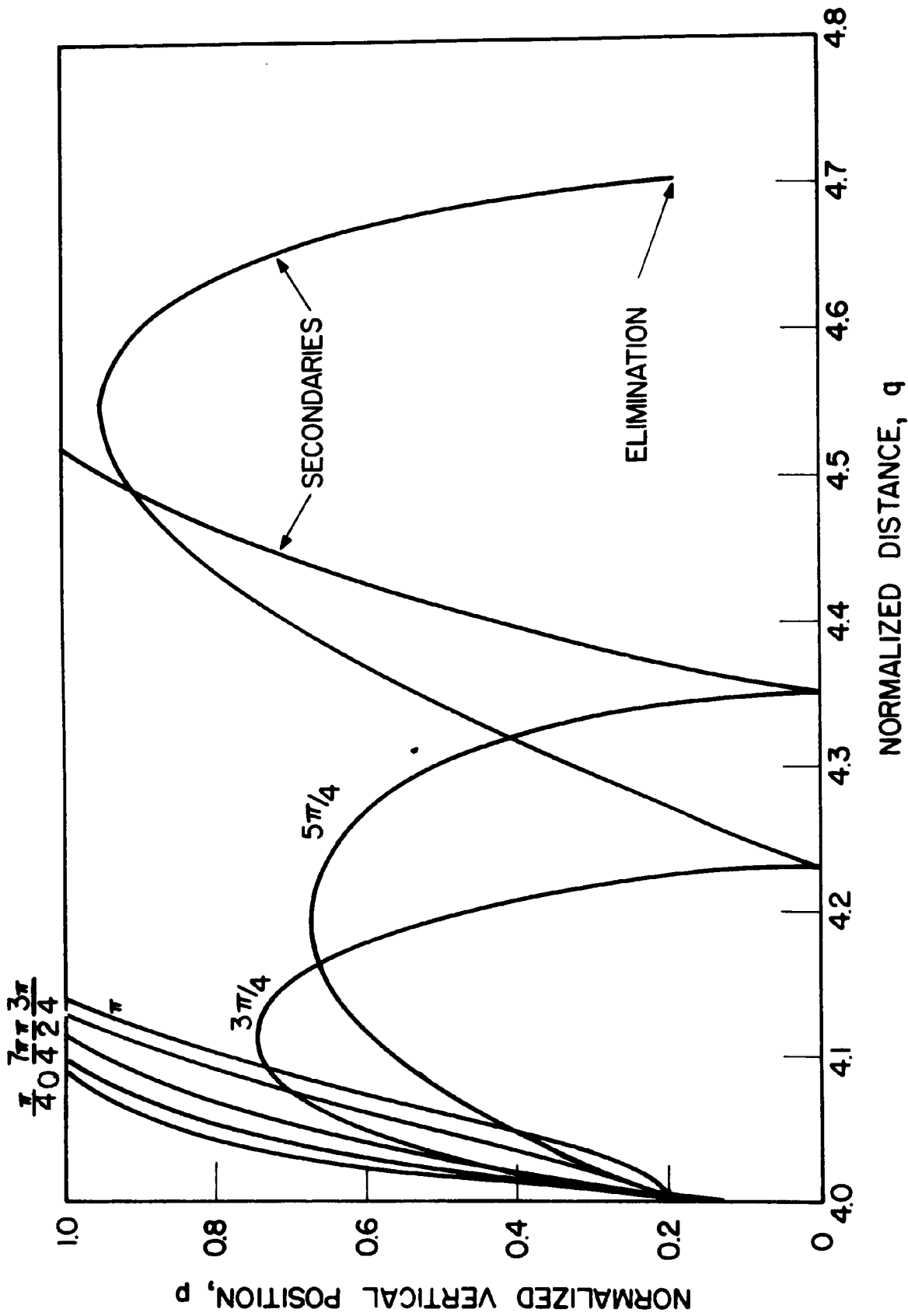


FIG. 5.8 NORMALIZED VERTICAL POSITION VS. LONGITUDINAL DISTANCE FOR THE 81ST EMISSION POINT.

$$(\omega_c/\omega = 1, \omega_{pc}/\omega = 5, \delta = 1, A_{rf}/A_{dc} = 1, \beta_{ob'} = 2, \beta_{oe} = 40)$$

occurs on the r-f slow-wave structure in shorter longitudinal distances (from $1/4$ to 1 circuit wavelength) and the backbombardment longitudinal lengths for electrons which return to the emitting sole are shorter (from $1/2$ to $3/2$ circuit wavelengths). Also, it can be observed that the phase positions of the electrons for the 81st emission point are slightly different from those of the first. The electrons with initial phase positions between 0 and π for the 81st emission point have phase trajectory solutions slightly greater than those for the first. Similarly the phase trajectory solutions for initial phase values between π and 2π are in general lower for the 81st emission point. These results are due directly to the larger r-f voltage amplitude at this position. The observed bunching, nonetheless, remains about the same.

The normalized velocity variations with longitudinal position for cutoff operation ($\omega_c/\omega = 1$ and $K = 1$) are displayed in Figs. 5.9 and 5.10. For brevity, only this particular case will be examined because the inclusion of the other two cases would not lead to any significantly different conclusions. The behavior of these dependent variables is also characteristic of the previous cases: a large initial acceleration just after emission and jump discontinuities in the velocities when secondary electrons are produced. The large acceleration values are due to the fact that the electrons are emitted with small thermionic velocities (about one tenth the value obtainable due to the d-c fields) and are accelerated over short distances by the d-c electric fields. The jump discontinuities in the velocities are obtained because the secondary electron energy was assumed independent of the primary energy.

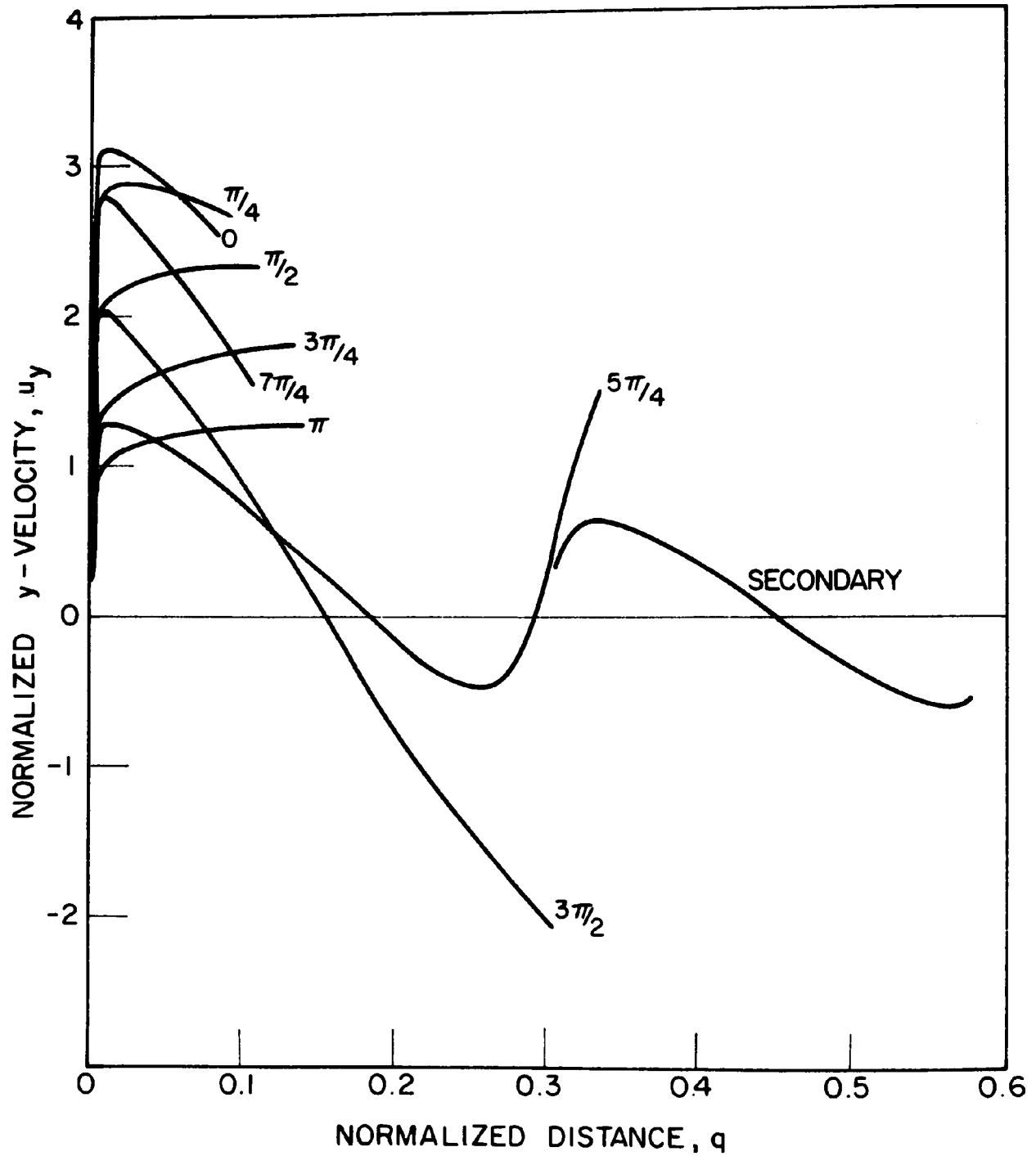


FIG. 5.9 NORMALIZED VERTICAL VELOCITY VS. LONGITUDINAL DISTANCE FOR THE 81ST EMISSION POINT. ($\omega_c/\omega = 1$, $\omega_{pc}/\omega = 5$, $\delta = 1$, $A_{rf_0}/A_{dc} = 1$, $\beta_0 b' = 2$, $\beta_0 L_e = 40$)

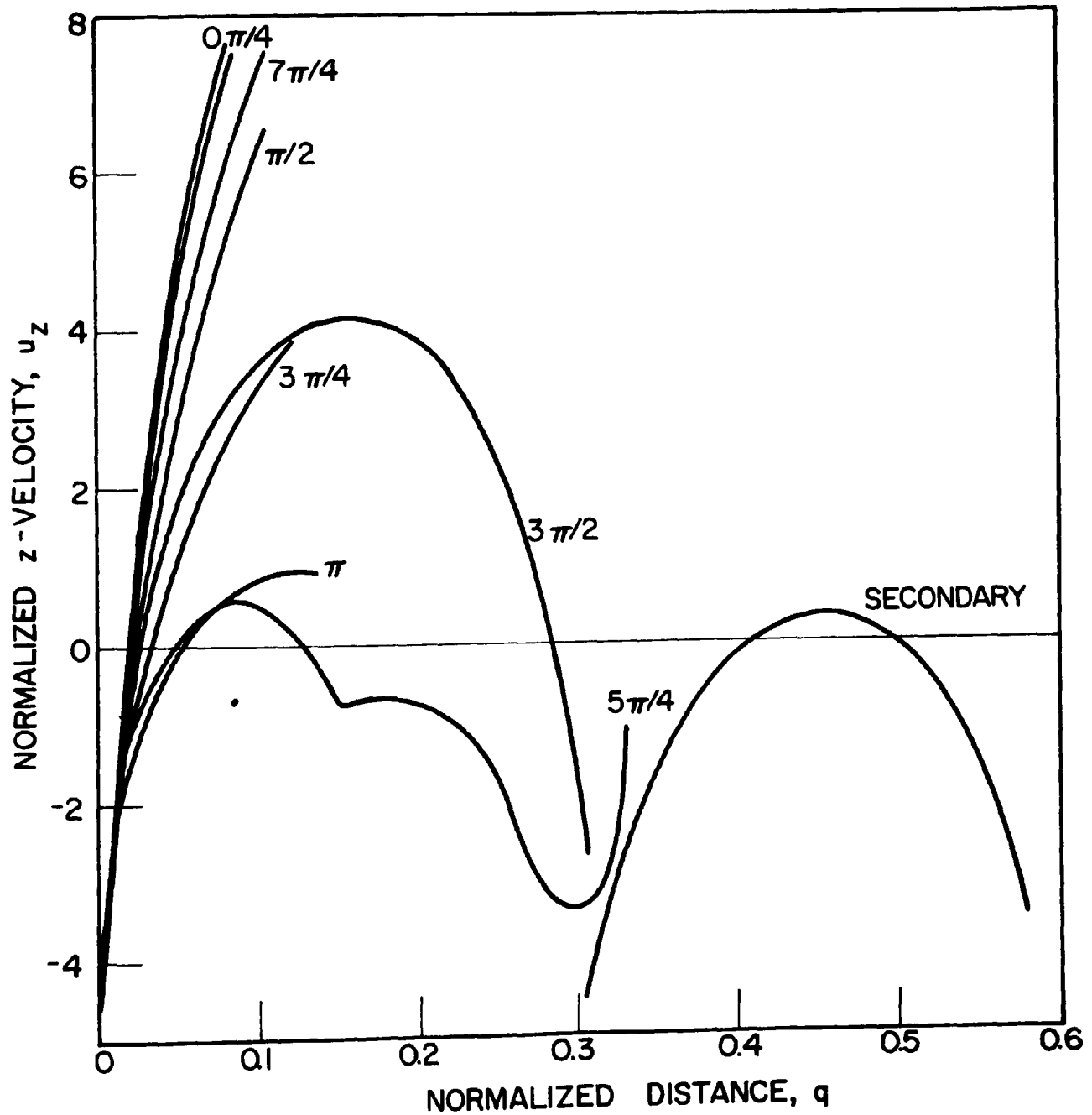


FIG. 5.10 NORMALIZED LONGITUDINAL VELOCITY VS. LONGITUDINAL DISTANCE FOR THE 81ST EMISSION POINT. ($\omega_c/\omega = 1$, $\omega_{pc}/\omega = 5$, $\delta = 1$, $A_{rf0}/A_{dc} = 1$, $\beta_0 b' = 2$, $\beta_0 L_e = 40$)

5.2.3 Results for the R-f Circuit Variables. The solutions obtained for the r-f gain of the emitting-sole amplifier when space-charge forces are included and when the computer input parameters are given by $\omega_{pc}/\omega = 5$, $D = 0.1$, $\delta = 1$, $A_{rf0}/A_{dc} = 1$, $\beta_0 b' = 2$ and $\beta_0 L_e = 40$ are shown in Fig. 5.11. These results are displayed using the d-c magnetic field as a parameter. It can be seen from this figure that larger growth rates and hence greater saturation gain occurs for smaller values of the d-c magnetic field and as the d-c magnetic field is increased, the growth rate and saturation gain decrease. This result is due to the fact that for larger magnetic field values the major beam position is closer to the emitting sole and hence interacts with weaker longitudinal r-f fields resulting in smaller r-f amplitude growth. On the other hand, for smaller magnetic fields, the majority of electrons have average vertical positions which are closer to the r-f circuit and hence are able to interact more strongly with the larger longitudinal r-f fields which are present there. Thus, larger r-f amplitude growth rates and saturation gain are achieved for a d-c magnetic field below the cutoff value than for one above this value.

The resulting solutions from the time analysis by Yu³⁴ show an equivalent conclusion. Yu deduces from his solutions that a larger saturation gain is achieved from larger anode potentials when the magnetic field and the wave velocity are held constant. This is exactly equivalent to the statement above and can be seen by recalling Eq. 5.3. From this equation it is observed that increasing V_a while leaving ω_c , b' and v_0 unchanged is identical to decreasing ω_c while leaving V_a , b' and v_0 unchanged. In other words, $K > 1$ implies

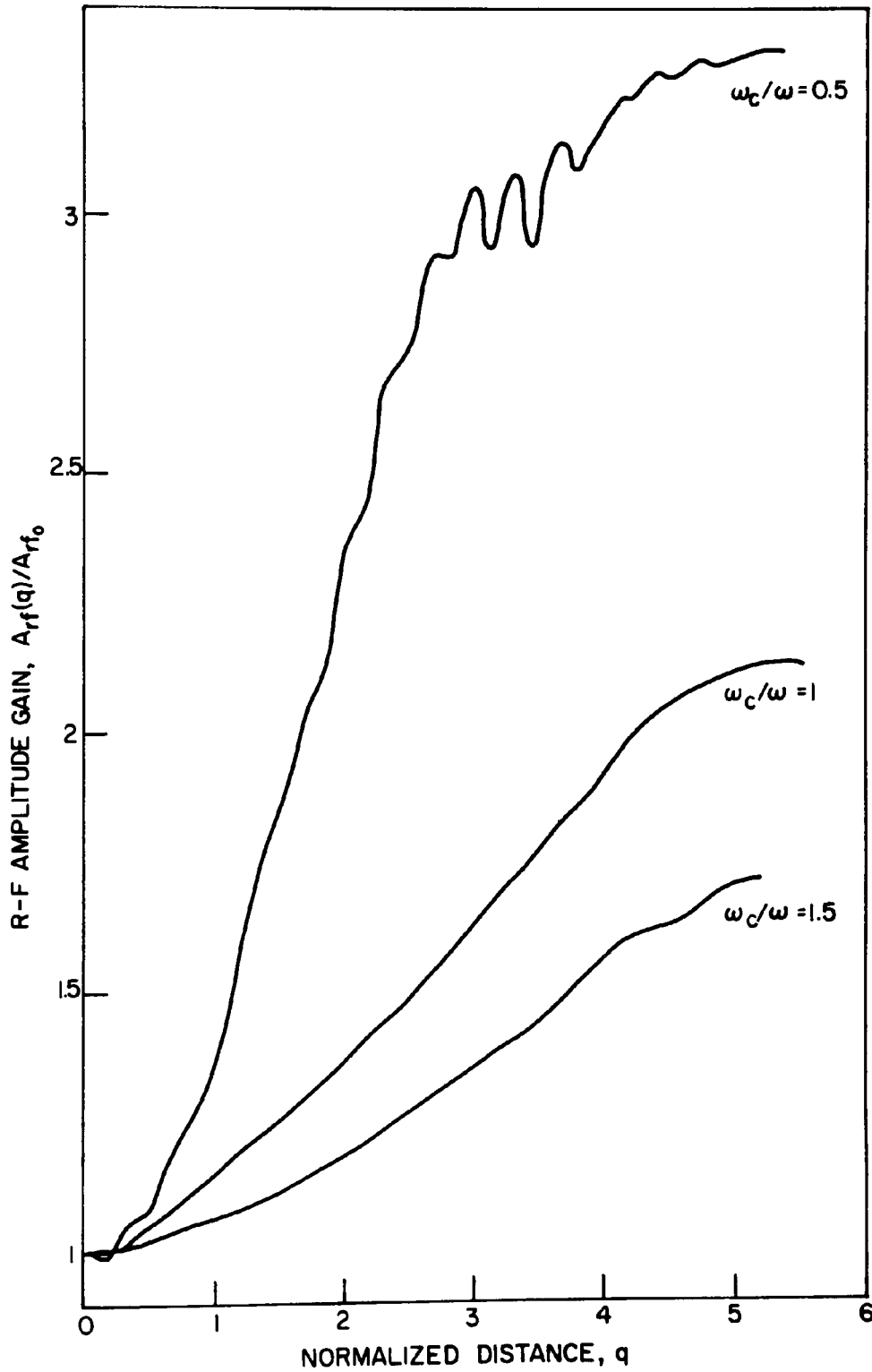


FIG. 5.11 R-F AMPLITUDE GAIN VS. LONGITUDINAL DISTANCE WITH ω_c/ω AS THE PARAMETER. ($\omega_{pc}/\omega = 5$, $D = 0.1$, $\delta = 1$, $A_{rf0}/A_{dc} = 1$, $\beta_0 b' = 2$, $\beta_0 L_e = 40$)

that either V_a is increased above the cutoff value of d-c potential or ω_c is decreased below the cutoff value of magnetic field. Excellent agreement between Yu's results (see Appendix F) and the results presented here are therefore observed.

Figure 5.12 gives a comparison of the solutions obtained with and without space-charge forces. The d-c parameters are chosen to satisfy the d-c magnetron cutoff condition ($\omega_c/\omega = 1$ and $K = 1$). The curves indicate that with space-charge forces included a larger growth rate and hence larger saturation gain is achieved than for the zero space-charge solution of Chapter IV. This result is due to the more efficient bunching process which is taking place when the space-charge fields are included. A similar increase in saturation gain was also obtained by Gandhi in his large-signal analysis of injected-beam devices, so these results are also in agreement with previous work. It appears as though the electrons are being modulated more effectively by the space-charge forces and therefore better beam focusing is occurring. Note that the saturation gain for the space-charge case is now about 7 db, whereas for zero space charge only 3 db was obtained.

The variations in the r-f phase angle θ_c representing a perturbation in the phase of an electron relative to the wave are displayed in Fig. 5.13 with the magnetic field as a parameter. It should be recalled that this dependent variable gives an indication of the amount of stream-loading upon the r-f wave and the phase velocity of the r-f wave when written in terms of this quantity is given by $v_{ph} = v_o/[1 - D\theta'_c(q)]$. To explain the variations observed in Fig. 5.13 an extension of the argument given in Chapters I and IV concerning synchronism will be presented.

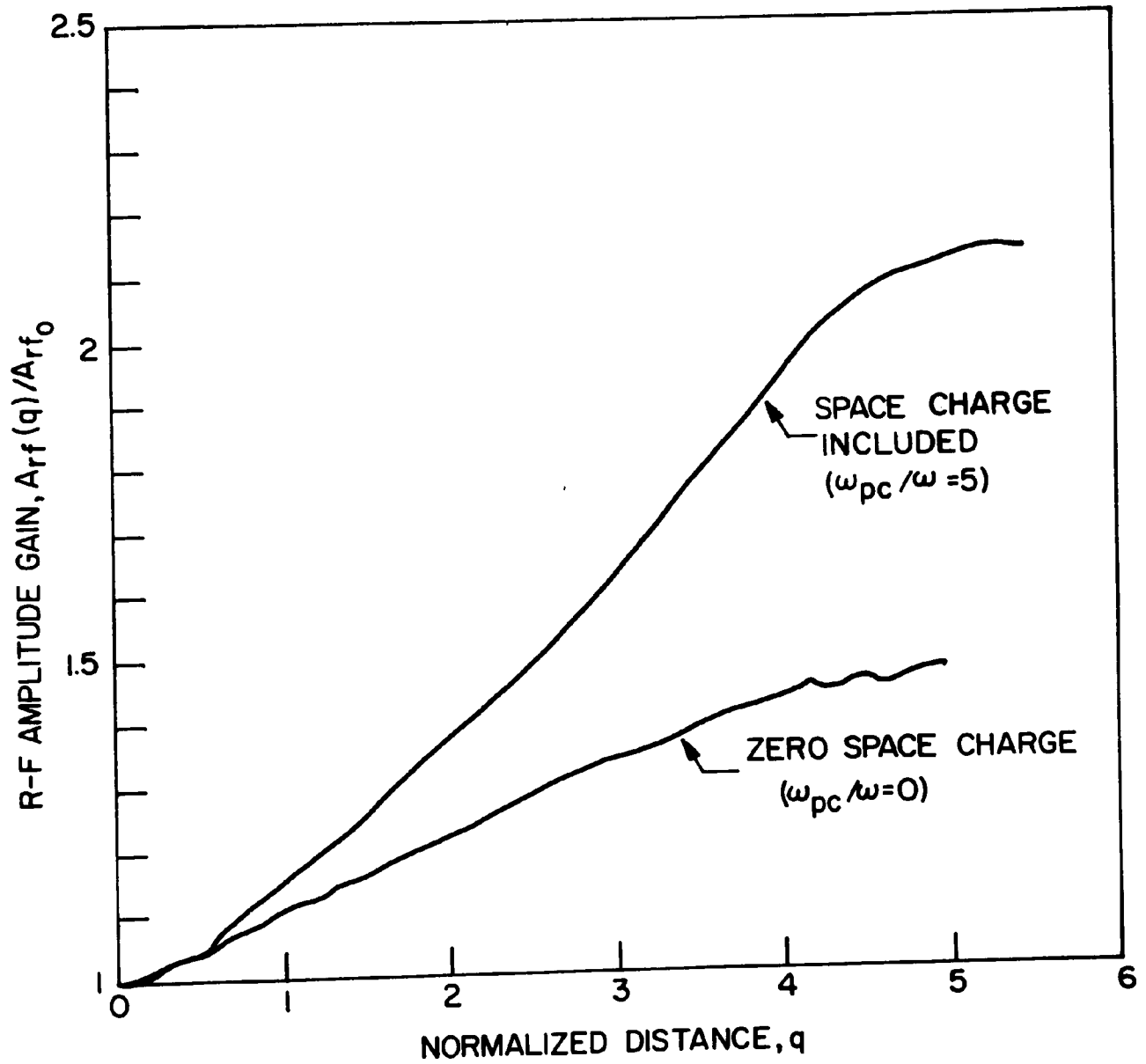


FIG. 5.12 VARIATION IN R-F AMPLITUDE GAIN WITH SPACE CHARGE AS THE
 PARAMETER. ($\omega_c/\omega = 1$, $D = 0.1$, $\delta = 1$, $A_{rf0}/A_{dc} = 1$, $\beta_o b' = 2$,
 $\beta_o L_e = 40$)

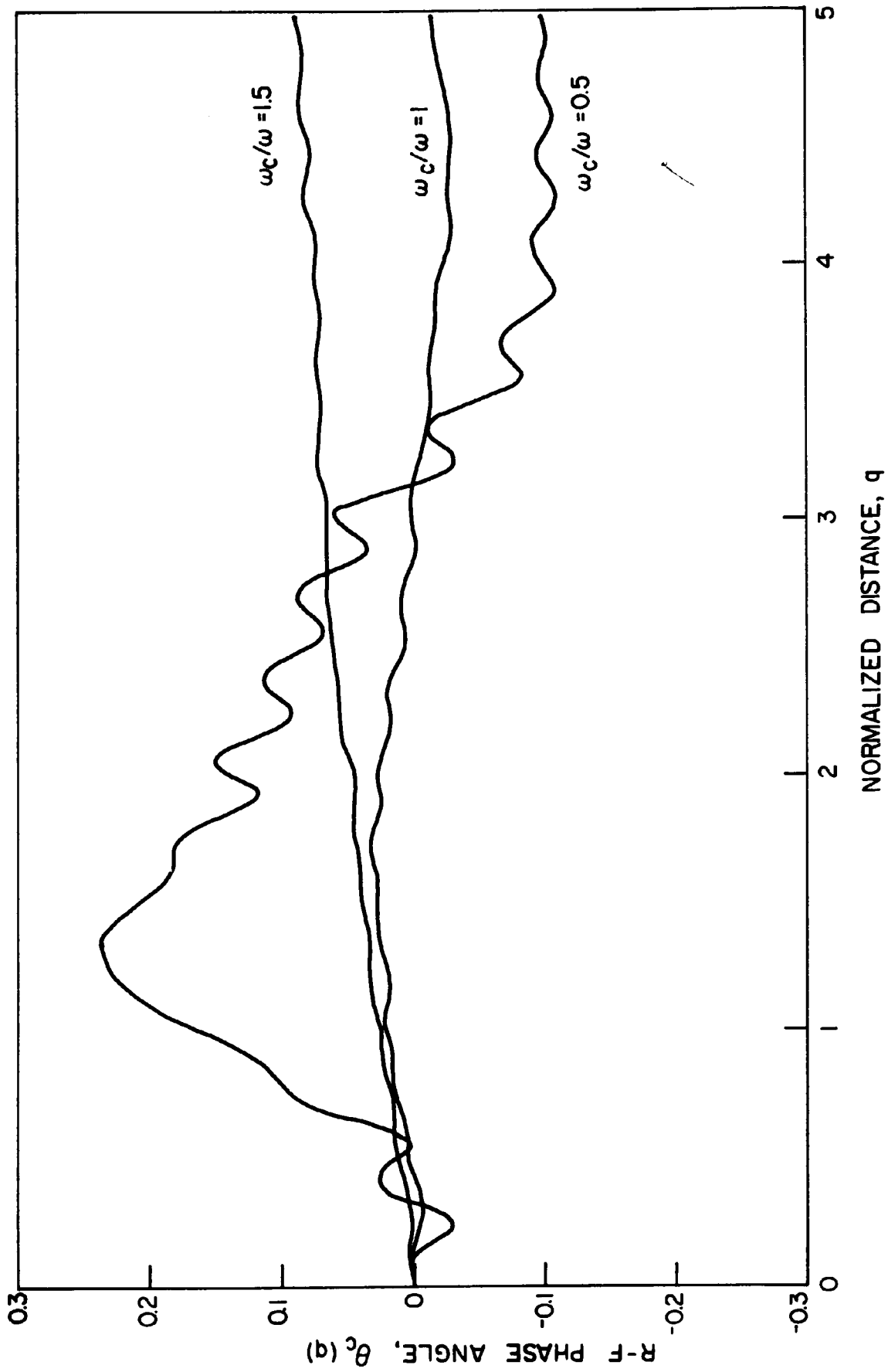


FIG. 5.13 VARIATION IN THE R-F PHASE ANGLE WITH LONGITUDINAL DISTANCE WITH ω_c/ω AS THE PARAMETER.

$$(\omega_{pc}/\omega = 5, D = 0.1, \delta = 1, A_{rf}/A_{dc} = 1, \beta_{ob'} = 2, \beta_{oe} = 40)$$

It has been mentioned previously that for the zero space-charge-force case the greatest electron-wave interaction occurs when nearly synchronous conditions exist between the electrons and the r-f wave, i.e., $v_{ph} \approx v_d$, where v_d is the average drift velocity of the electrons given by E_0/B_0 , and where E_0 and B_0 are the d-c crossed electric and magnetic fields, respectively. The synchronism condition is necessary because continued operation away from synchronism would force the electrons and the wave to remain out of phase with one another which would result in an attenuated rather than amplified r-f wave. Thus, when $D\theta'_c(q)$ is small compared to one, the zero space-charge synchronous condition becomes $v_0 = v_d$ which is one of the two stipulated conditions for the d-c parameters (see Eq. 5.2).

However, when space-charge fields are included in the analysis the above discussion of synchronism is over-simplified. The following description of the more complicated interaction is that of Hull et al.²⁴. Their calculations have shown that the electrons which bunch (those which give up potential energy to the r-f wave) in the retarding phase of the r-f field move in a rotary motion about the center of the bunch in addition to the longitudinal drift motion. Furthermore as bunching progresses, the main portion of the bunch tends to remain in synchronism with the r-f wave, while the portion near the sole electrode drops backward in phase due to the space-charge fields. The result is a decrease in the average electron drift velocity and hence the electrons gradually depart from synchronism with the r-f wave. In order to maintain synchronism either the phase velocity of the wave must be decreased or the average electron drift velocity must be increased. In Section 5.3, solutions with a linear

decrease in the magnetic field which produces an increase in v_d are presented and the results are seen to produce larger saturation gain. However, in Fig. 5.13 it can be seen that for smaller values of magnetic field an inherent mechanism causes v_{ph} to decrease, i.e., the curves indicate that the values of θ'_c are in general more negative for smaller magnetic field values. This is due to greater stream-loading upon the r-f wave. From the argument presented above, a greater saturation gain should and does occur for smaller magnetic field values as was displayed in Fig. 5.13.

Another reason for obtaining higher gain for smaller ω_c/ω can be seen by observing the gain parameter D. This parameter is a constant in each of the above solutions and was chosen equal to 0.1. The definition of D is

$$D^2 \triangleq \frac{\omega_c}{\omega} \frac{I_o Z_o}{2V_{oc}} \quad (5.4)$$

Now if D is to remain constant for a decrease in ω_c/ω , one of the other parameters must be changed. However, if the characteristic impedance of the r-f circuit, Z_o , and the voltage equivalent of the circuit phase velocity, V_{oc} , are kept constant, the thermionic emission current, I_o , must be increased when ω_c/ω is decreased in order to keep D the same. Hence under these conditions as ω_c/ω decreases and D is kept constant, the r-f saturation gain should increase due to a greater amount of emission current. With more current the electron-wave interaction should be stronger and therefore more gain should be possible.

It should be noticed that in Figs. 5.11 and 5.13 a periodic perturbation in the parameters $A_{rf}(q)$ and $\theta_c(q)$ is occurring. These

are presumed to be due to the collection of electrons on the anode, which results in a disturbance in the averaging process being carried out in the circuit integrals and the space-charge integrals used in the calculations. The results of Gandhi¹⁶ for the injected-beam crossed-field amplifier also indicated similar perturbations.

In an attempt to increase the saturation gain of the amplifier, the gain parameter D was increased to a value of 0.125. The computer solutions for $A_{rf}(q)$ and $\theta_c(q)$ for this case are displayed in Figs. 5.14 and 5.15. Figure 5.14 indicates that increasing D has little effect upon the saturation gain. This is apparently because D was only increased by a small amount. However for larger values of D the resulting interaction causes a significant number of electrons to reverse their longitudinal directions and travel in a direction opposite to that of the traveling wave. Since this direction, as mentioned in Chapter IV, is inconsistent with the computer simulation of the problem, incorrect solutions result and therefore solutions for larger values of D were not investigated.

Figures 5.14 and 5.15 show similar results to those obtained with $D = 0.1$ and the other parameters unchanged. Figure 5.14 shows that larger r-f growth rates and saturation gain are achieved for a smaller magnetic field value. Also, Fig. 5.15 again displays the result that the phase velocity of the traveling wave is more nearly equal to the average electron drift velocity (with space-charge fields) for a magnetic field value of $\omega_c/\omega = 0.5$. Thus for this case operation closer to synchronism is occurring which results in a greater conversion of d-c potential energy to r-f energy which leads to greater gain.

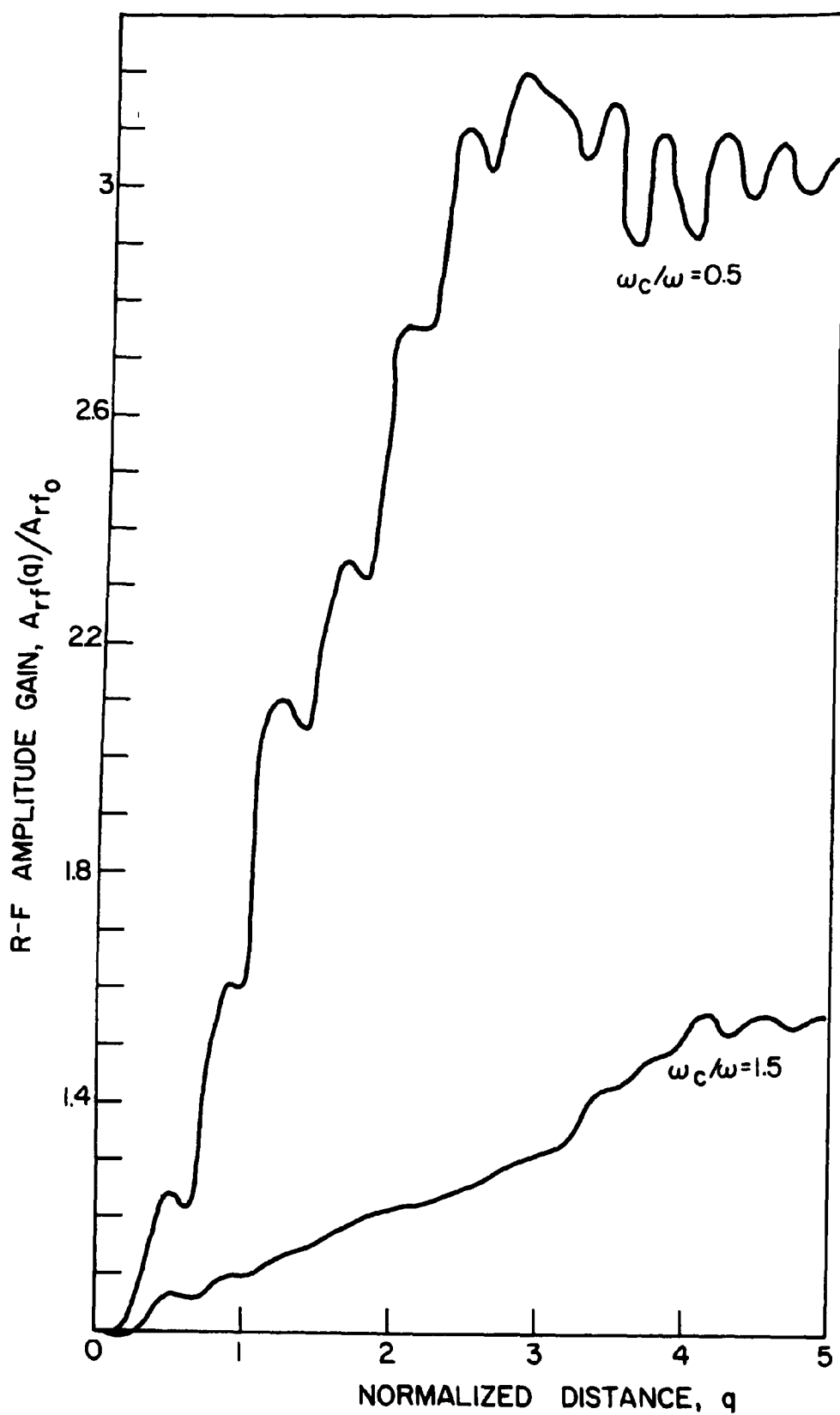


FIG. 5.14 R-F AMPLITUDE GAIN VS. LONGITUDINAL DISTANCE WITH $D = 0.125$.

$$(\omega_{pc}/\omega = 5, \delta = 1, A_{rf0}/A_{dc} = 1, \beta_o b' = 2, \beta_o L_e = 40)$$

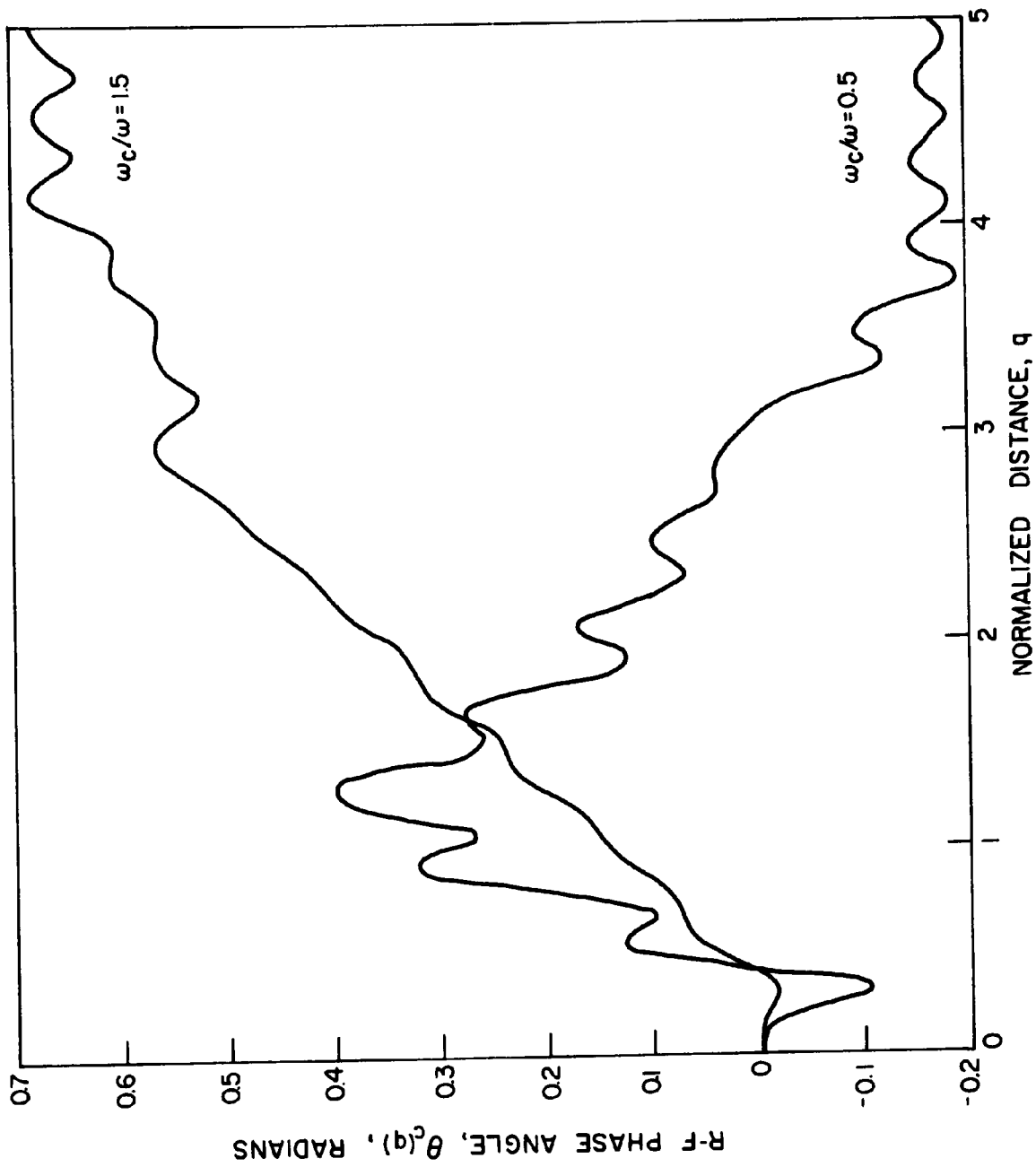


FIG. 5.15 VARIATION IN THE R-F PHASE ANGLE WITH LONGITUDINAL DISTANCE WITH $D = 0.125$. ($\omega_{pc}/\omega = 5$,

$$\delta = 1, A_{rf}/A_{dc} = 1, \beta_o b' = 2, \beta_o L_e = 40)$$

5.3 Emitting-Sole Amplifier Solutions with a Tapered Magnetic Field

5.3.1 Discussion of the Magnetic Field Taper. As discussed in the previous section, electron-wave synchronism is necessary in order to obtain appreciable interaction. However, when space-charge forces are considered, the synchronous condition is altered from that of the zero space-charge case and the electrons begin to slip out of phase with the r-f wave. Hence synchronizing the wave velocity v_o with E_o/B_o at the r-f input end of the amplifier is not sufficient to maintain approximate synchronism over the entire interaction length. The decrease in the electron drift velocity near the cathode can be countered by applying variable d-c fields in either of two different ways. One method would be to gradually decrease the sole-to-anode spacing with longitudinal distance and thus increase the d-c electric field. Another method would be to gradually decrease the d-c magnetic field. The result of either (or both) of these d-c field tapers applied in the correct manner would then adjust the average electron drift velocity such that it remains in synchronism with the r-f wave. Hull et al.²⁴ have found experimentally that a linear decrease in the d-c magnetic field with longitudinal distance does in fact produce larger r-f wave amplification and hence it was decided to investigate this result theoretically using the Lagrangian analysis.

In order to carry out this study a linear d-c magnetic field taper is used. Thus, the following equation replaces the previously constant parameter ω_c/ω :

$$\omega_c(q)/\omega = (\omega_c/\omega)_o [1 - \alpha(q-q_o)] ,$$

where $(\omega_c/\omega)_0$ is the ratio of cyclotron to r-f frequency at the r-f input end of the tube, α is the slope of the linear taper and q_α is the longitudinal position at which the d-c magnetic field begins to decrease. For $q < q_\alpha$, the d-c magnetic field has the same value as that existing at the r-f input of the amplifier. Due to this taper in the x-component of magnetic field, a z-component of magnetic field must also exist in order to satisfy Maxwell's equations. However, in practical devices the magnet system can be aligned in such a manner that this z-component is negligible and thus the forces due to this component are neglected.

5.3.2 Results for the R-f Circuit Variables. It will be noticed from the solutions for the r-f potential amplitude $A_{rf}(q)$ that the d-c magnetic field tapers did not enhance the gain in all of the cases investigated. Figure 5.16 is an example of this behavior with the initial ($q = 0$) d-c parameters satisfying the magnetron cutoff condition. This variable magnetic field solution is for the case of a linear taper applied over the entire length of the tube ($q_\alpha = 0$ and $\alpha = 0.125$). The figure indicates that the growth in r-f amplitude is more rapid for the linear taper case than for the case without this taper for values of q less than three. However, the amplifier with a tapered magnetic field eventually saturates at a lower value of the r-f amplitudes and therefore for a tube length of $q_L = 5$, less gain is obtained.

The following is an explanation of this result. From Fig. 5.16 it can be observed that the taper was aiding the amplitude growth from $q = 0$ out to approximately $q = 3$, after which growth continued to occur but at approximately the same rate as that for the untapered tube. From Fig. 5.17, it can be seen that just prior to the value

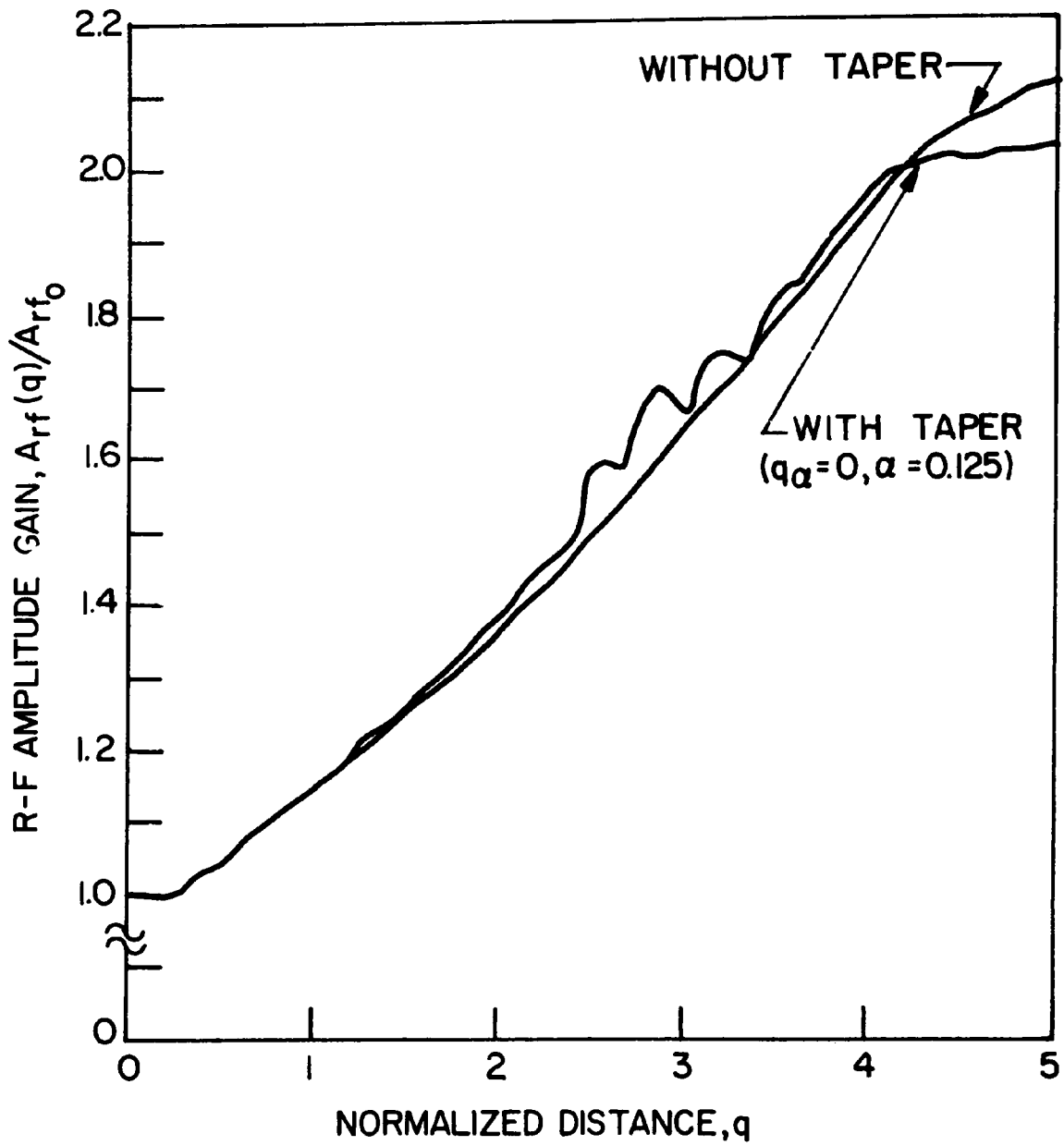


FIG. 5.16 R-F AMPLITUDE GAIN VS. LONGITUDINAL DISTANCE WITH AND WITHOUT A MAGNETIC FIELD TAPER. ($\omega_c/\omega = 1$, $\omega_{pc}/\omega = 5$, $D = 0.1$, $\delta = 1$, $A_{rf_0}/A_{dc} = 1$, $\beta_o b' = 2$, $\beta_o L_e = 40$)

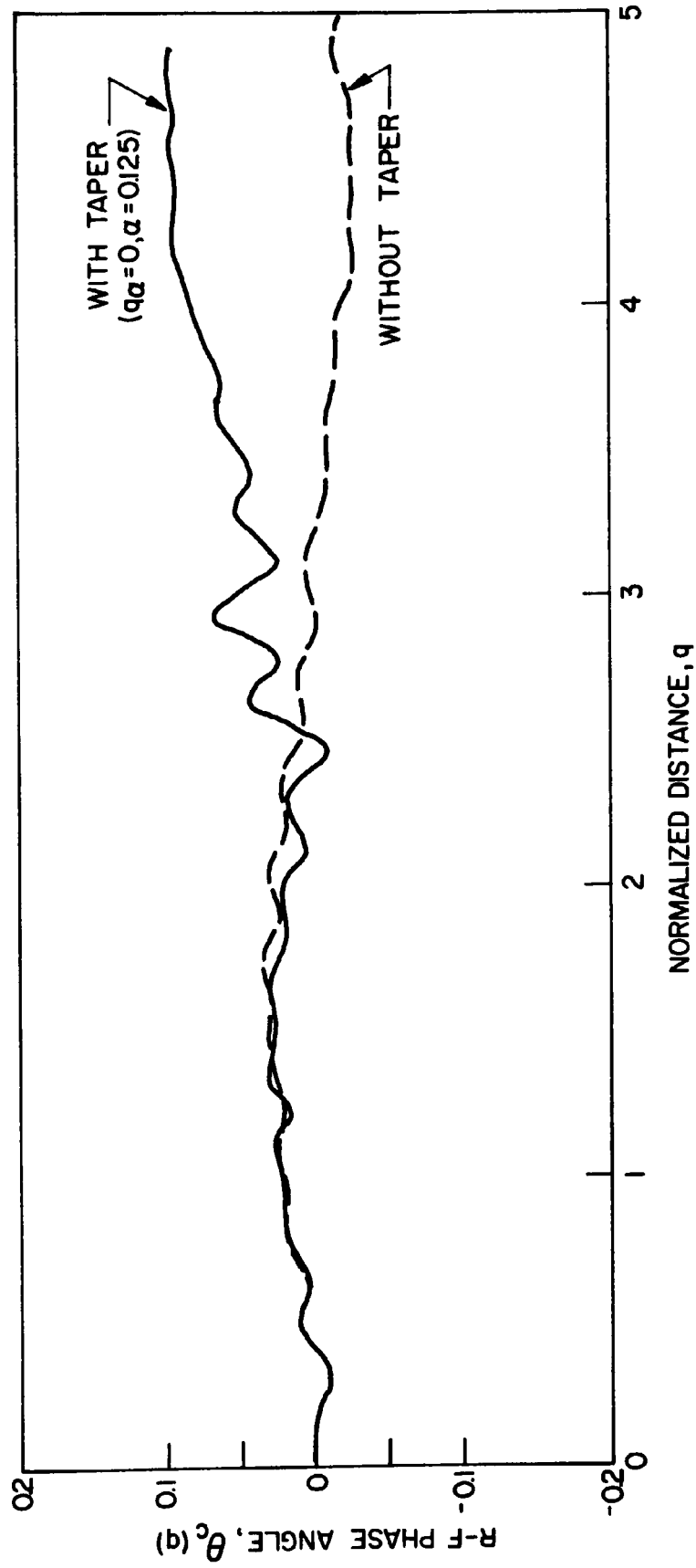


FIG. 5.17 VARIATION IN THE R-F PHASE ANGLE WITH LONGITUDINAL DISTANCE WITH AND WITHOUT A MAGNETIC FIELD

TAPER. $[(\omega_c/\omega)_0 = 1, \omega_{pc}/\omega = 5, D = 0.1, \delta = 1, A_{rf}/A_{dc} = 1, \beta_0 b' = 2, \beta_0 L_e = 40]$

$q = 3$, the r-f phase angle $\theta_c(q)$ is perturbed by the electron-wave interaction and begins to increase. This increase necessarily forces $\theta'_c(q)$ to also increase (positively) and hence the phase velocity of the wave increases. Therefore the effects of increasing the average drift velocity of the electrons by tapering the d-c magnetic field are being canceled by an increase in the phase velocity of the wave. Thus the taper is not beneficial in this case over the entire length since the interaction forces the phase velocity to increase and as a result the amplifier saturates at a smaller value of $A_{rf}(q)$.

In an effort to increase the r-f saturation amplitude at the output, the magnetic field taper was shifted to a value of $q_\alpha = 2$, leaving the slope of the taper unchanged. These results are shown in Figs. 5.18 and 5.19. It is observed that an increase in the r-f saturation amplitude has been achieved, but the amount of increase is still disappointing.

A comparison of the $\theta_c(q)$ solutions with and without this magnetic field taper is shown in Fig. 5.19. From this figure the reason for the increase in r-f saturation amplitude becomes evident. It is observed that with the taper $\theta_c(q)$ decreases slightly with increasing longitudinal distance and remains below the value without a taper. Thus the phase velocity of the r-f wave decreases slightly due to this perturbation caused by the interaction, while the average electron drift velocity is being increased and approaches that of the wave via the magnetic field taper. Thus, operation closer to synchronism is being achieved when this magnetic field taper is applied and a larger r-f saturation amplitude results.

A solution with a larger magnetic field taper ($\alpha = 0.25$) applied at $q_\alpha = 2$ was also obtained. However, this did not yield a further

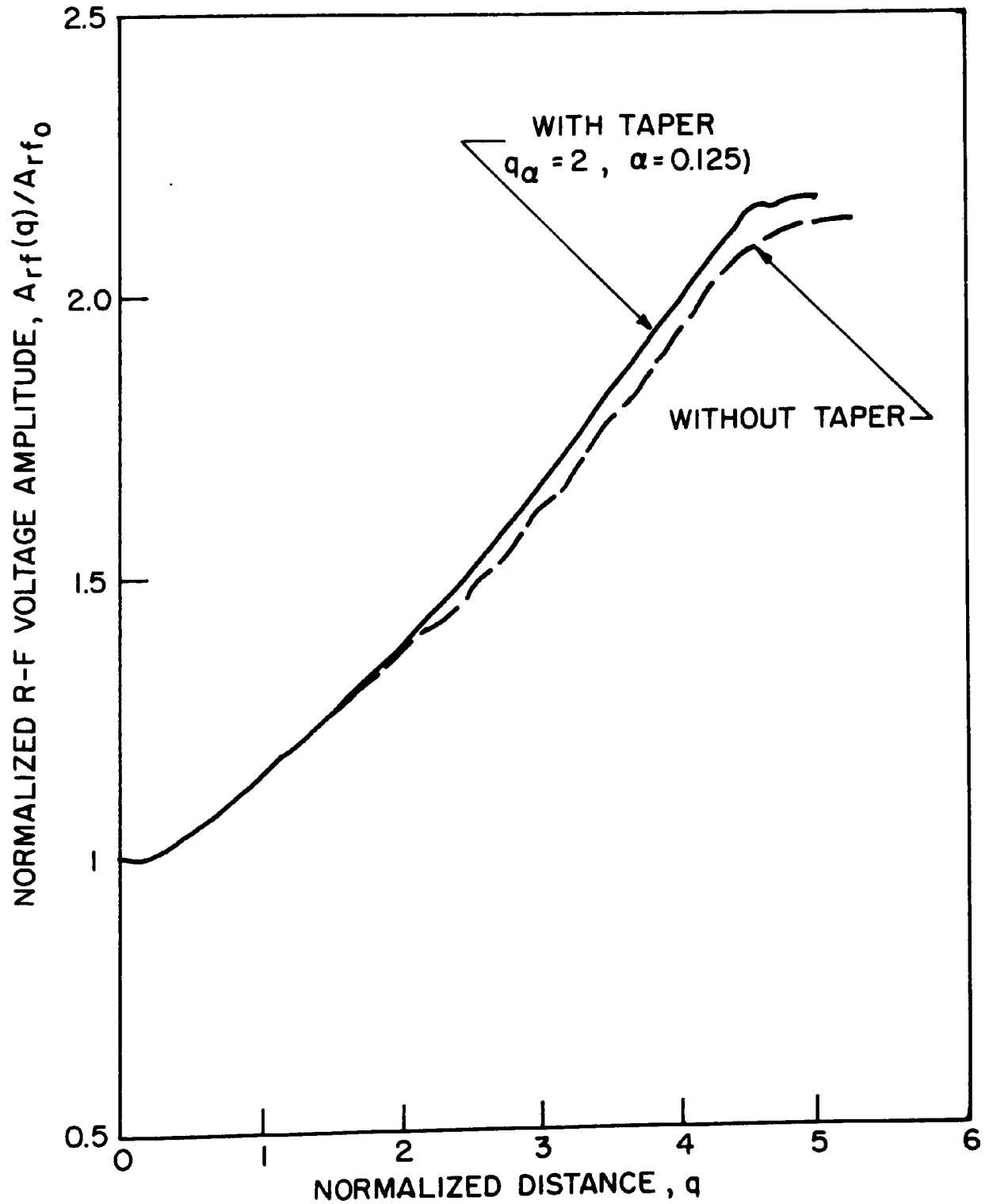


FIG. 5.18 R-F AMPLITUDE GAIN VS. LONGITUDINAL DISTANCE. ($\omega_c/\omega = 1$,
 $\omega_{pc}/\omega = 5$, $D = 0.1$, $\delta = 1$, $A_{rf0}/A_{dc} = 1$, $\beta_o b' = 2$, $\beta_o L_e = 40$)

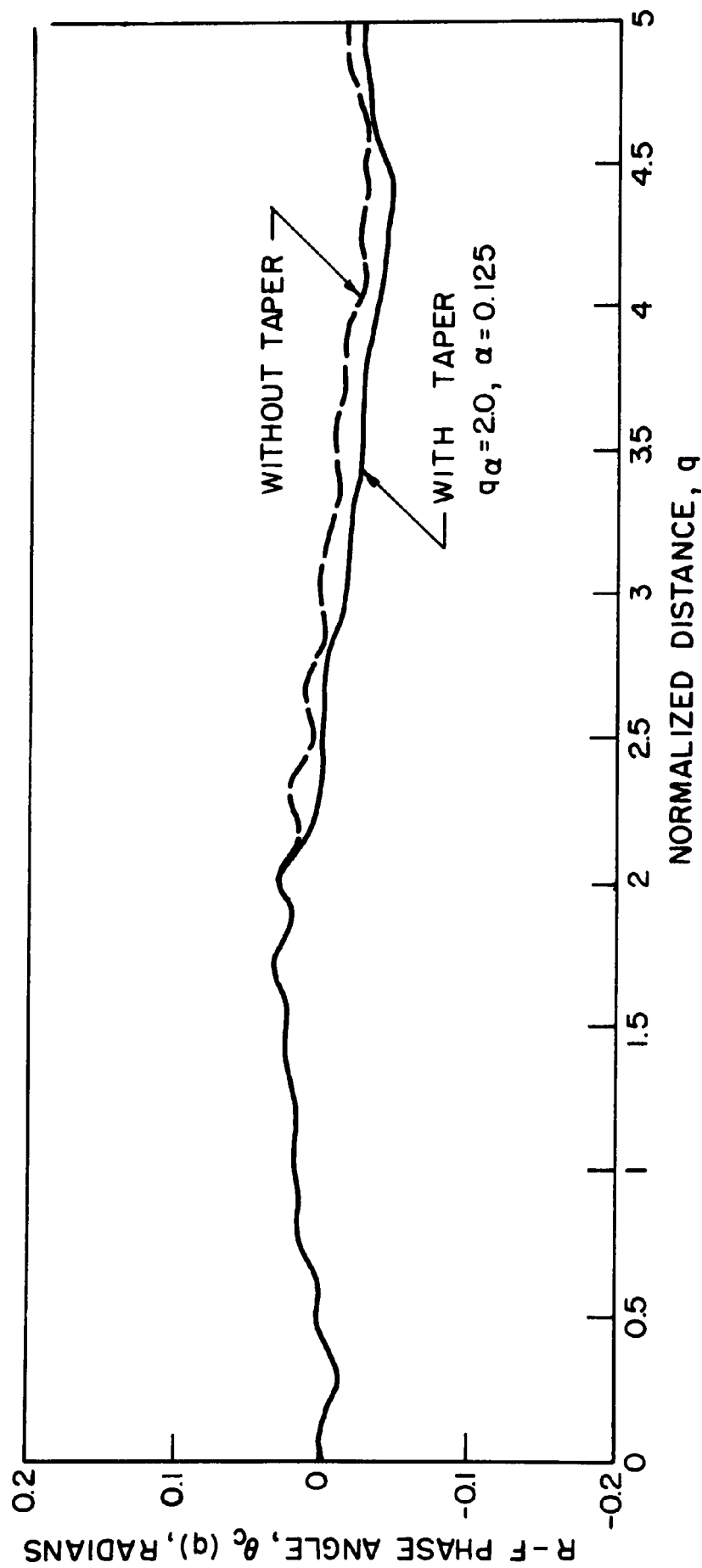


FIG. 5.19 VARIATION IN THE R-F PHASE ANGLE WITH LONGITUDINAL DISTANCE. ($\omega_c/\omega = 1$, $\omega_{pc}/\omega = 5$, $D = 0.1$, $\delta = 1$, $A_{rf0}/A_{dc} = 1$, $\beta_{ob'} = 2$, $\beta_{oe} = 40$)

increase in the r-f saturation amplitude and hence the results are not presented.

Further comparisons for other sets of d-c parameters are given in Figs. 5.20 through 5.23. These solutions were obtained for $D = 0.125$ and with initial ($q = 0$) d-c operation above and below the magnetron cutoff condition. The linear magnetic field taper is applied at $q_{\alpha} = 0$ and the slope of the taper used is $\alpha = 0.125$. Again the solutions without a magnetic field taper are included in the figures for comparison.

In Figs. 5.20 and 5.22 for d-c magnetic field values below and above the cutoff value, respectively, significant enhancement in the saturation r-f amplitude is observed. For below cutoff operation, the r-f saturation gain values are approximately 11 db with the taper and 9.5 db without the taper. A similar increase is shown in Fig 5.22 for above cutoff operation. The r-f saturation gain values are about 7 db with the taper and 4 db without the taper.

The enhancement in r-f output signal level in both cases is attributed in part to operation closer to synchronism between the electrons and the wave. Figures 5.21 and 5.23 show justification for this conclusion. Both of these figures indicate that the $\theta_c(q)$ solutions for the cases with a magnetic field taper are less than those without a taper. Again this implies that the interaction has perturbed the phase velocity of the r-f wave in such a manner that a decrease (however slight) has been brought about. At the same time, however, an increase in the average drift velocity of the electrons has been accomplished. The end result of these two alterations is operation

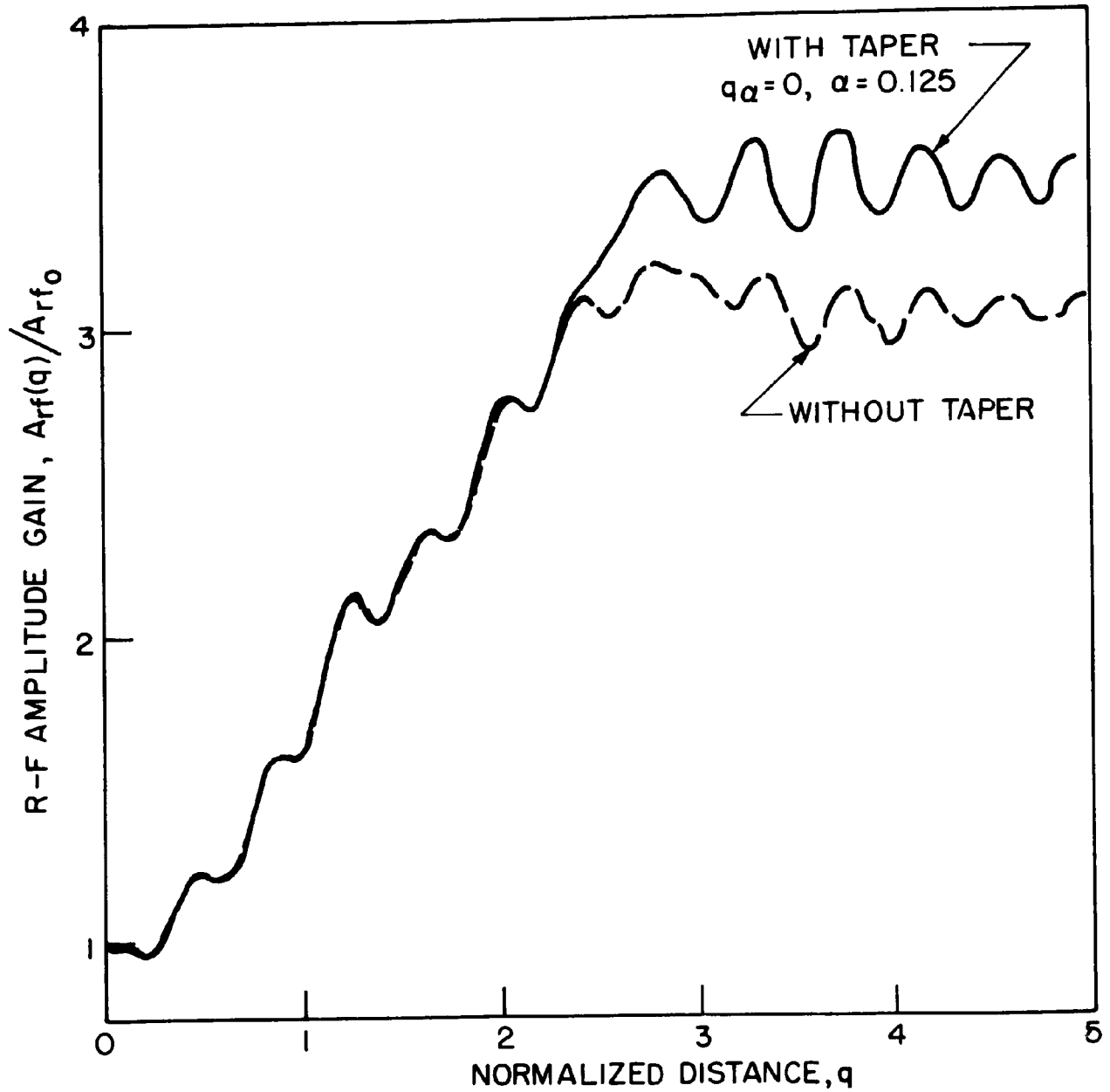


FIG. 5.20 R-F AMPLITUDE GAIN VS. LONGITUDINAL DISTANCE WITH AND WITHOUT A MAGNETIC FIELD TAPER AND $(\omega_c/\omega)_0 = 0.5$.

$$(D = 0.125, \delta = 1, A_{rf0}/A_{dc} = 1, \beta_0 b' = 2, \beta_0 L_e = 40)$$

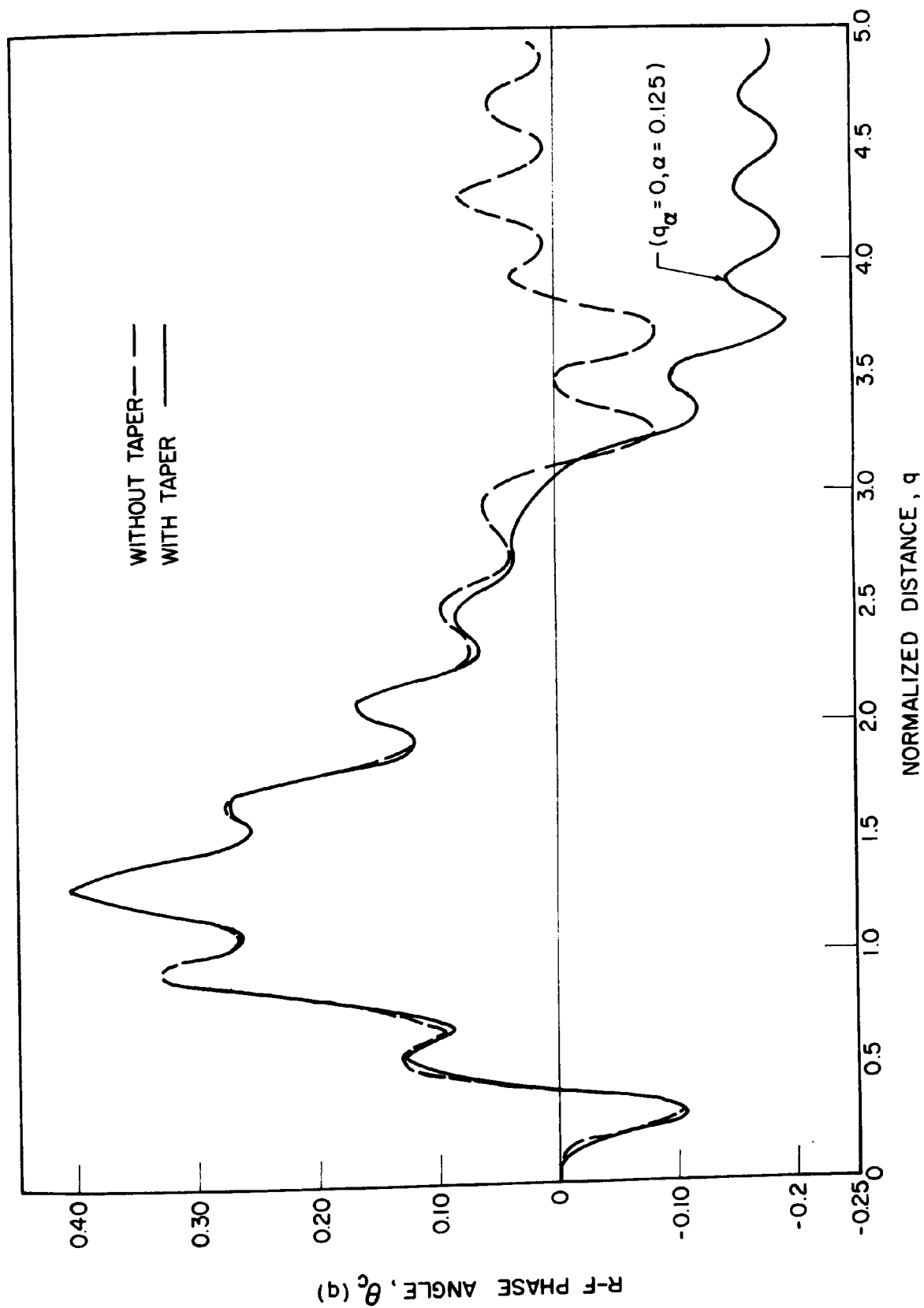


FIG. 5.21 R-F PHASE ANGLE VS. LONGITUDINAL DISTANCE WITH AND WITHOUT A MAGNETIC FIELD TAPER AND $(\omega_c/\omega)_0 =$

0.5. ($D = 0.125$, $\delta = 1$, $A_{rf}/A_{dc} = 1$, $\beta_0 b' = 2$, $\beta_0 L_e = 40$)

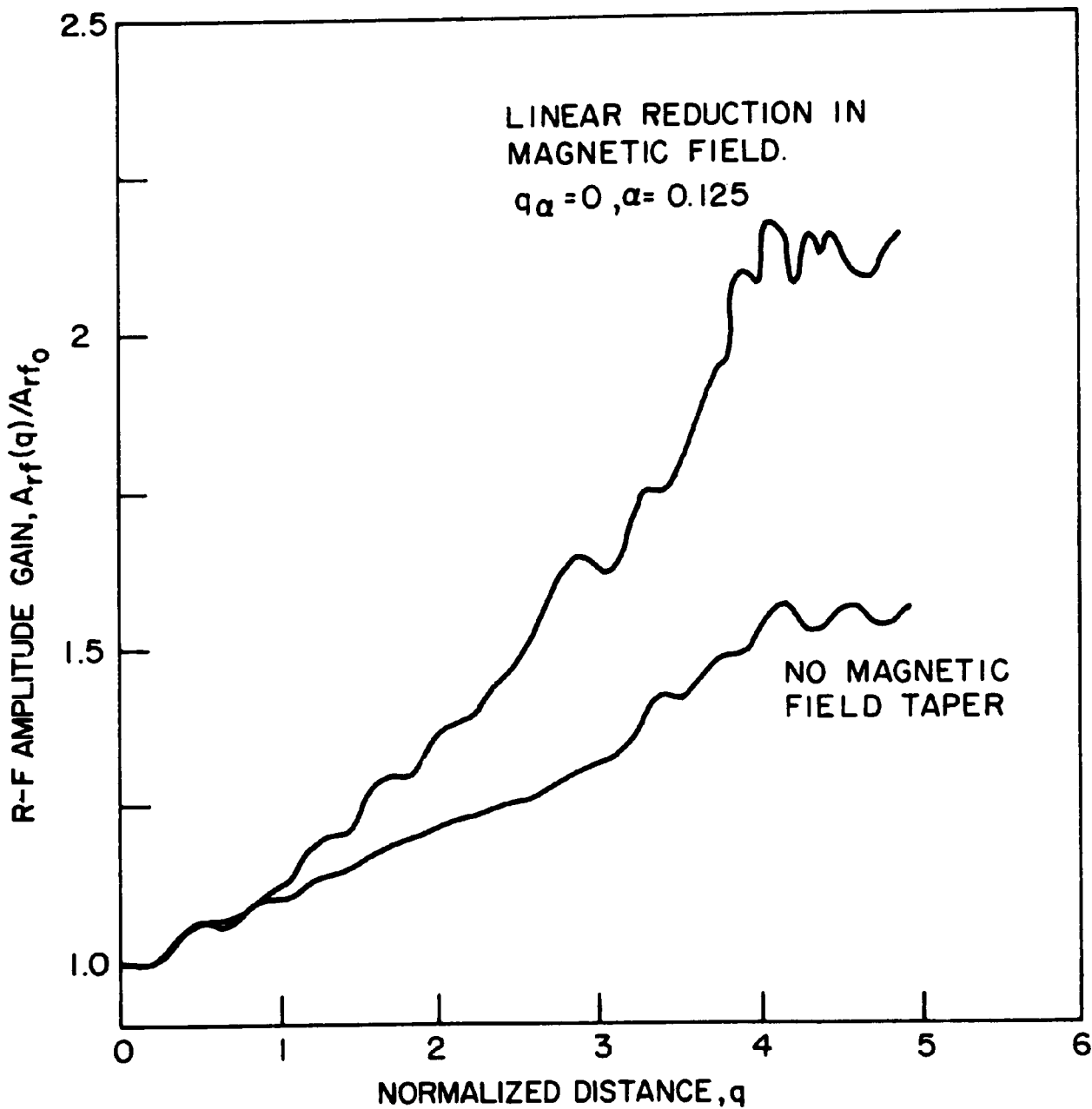


FIG. 5.22 R-F AMPLITUDE GAIN VS. LONGITUDINAL DISTANCE WITH AND WITHOUT A MAGNETIC FIELD TAPER AND $(\omega_c/\omega)_0 = 1.5$. ($\omega_{pe}/\omega = 5$, $D = 0.125$, $\delta = 1$, $A_{rf0}/A_{dc} = 1$, $\beta_0 b' = 2$, $\beta_0 L_e = 40$)

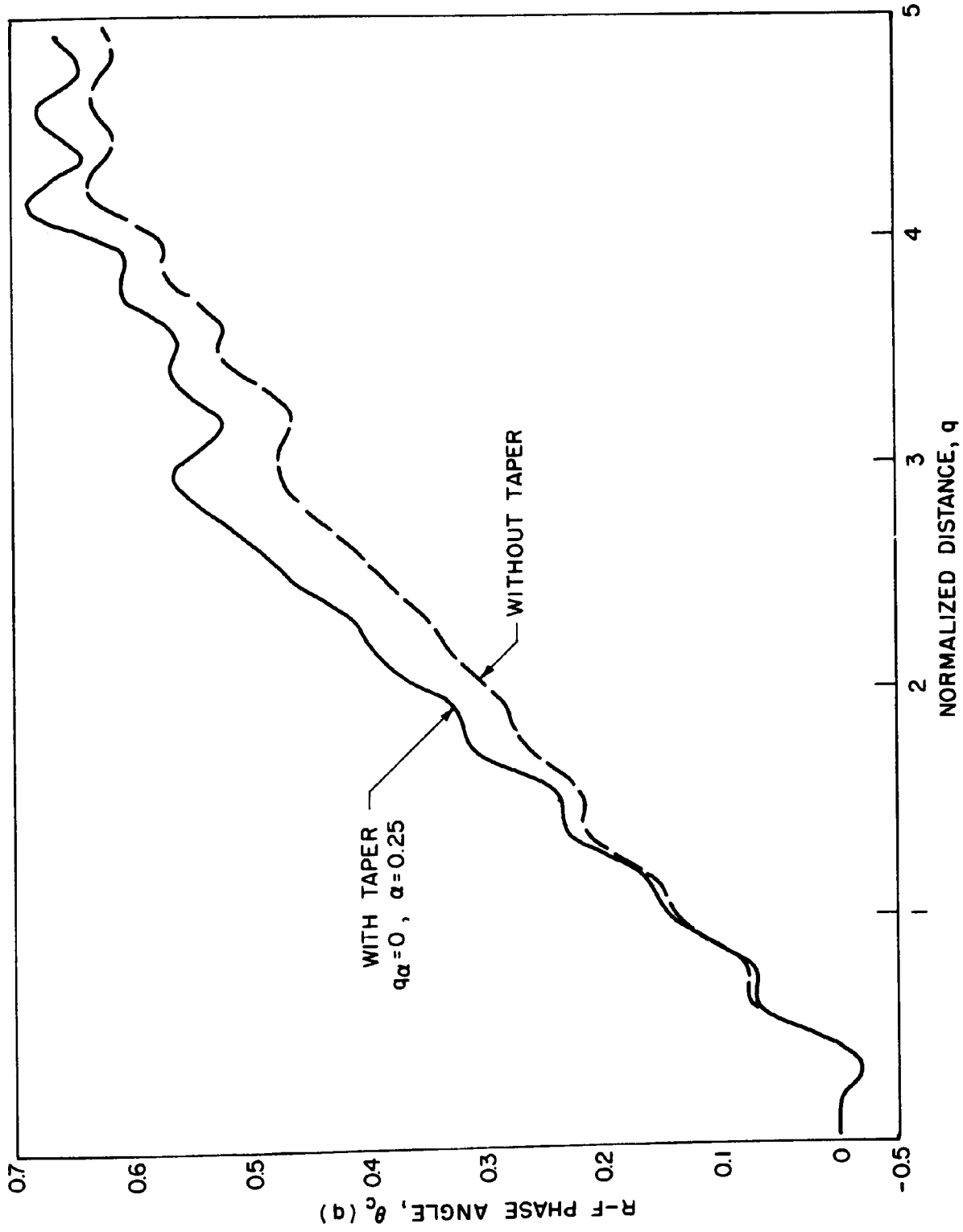


FIG. 5.23 R-F PHASE ANGLE WITH AND WITHOUT A MAGNETIC FIELD TAPER AND $(\omega_c/\omega)_0 = 1.5$.

$$(\omega_{pc}/\omega = 5, D = 0.125, \delta = 1, A_{rf}/A_{dc} = 1, \beta_o b' = 2, \beta_o L_e = 40)$$

closer to synchronism (the electrons and the wave are more nearly in phase with one another) and hence higher gain occurs.

In addition to the synchronism argument predicting higher gain presented above, it will be shown in the next section that a linear decrease in the d-c magnetic field forces the electron trajectories to be closer to the r-f structure. This results in a larger interaction and is another reason for the resulting larger gain when a magnetic field taper is applied.

5.3.3 Effects of a Variable Magnetic Field Upon the Trajectories.

The inclusion of a linear decrease in the d-c magnetic field with longitudinal distance forces the trajectory solutions to be different from those obtained with a constant d-c magnetic field. However, since the taper is applied gradually the trajectory solutions for the first few emission points remain essentially the same for both cases. Therefore the first emission point trajectories to be presented will represent both the tapered and untapered cases. The differences in the trajectories at the other end of the amplifier, where the application of a magnetic field taper has significantly changed the magnetic field value from the constant value, will be illustrated by comparing the solutions for the 81st emission point.

In this section the trajectory solutions for the cases with $D = 0.125$ and d-c operation above and below cutoff will be discussed and the results obtained with and without a magnetic field taper will be compared. The linear taper considered here is specified by $q_\alpha = 0$ and $\alpha = 0.125$.

Figures 5.24 and 5.25 show the electron phase and position solutions obtained for input ($q = 0$) d-c operation below the

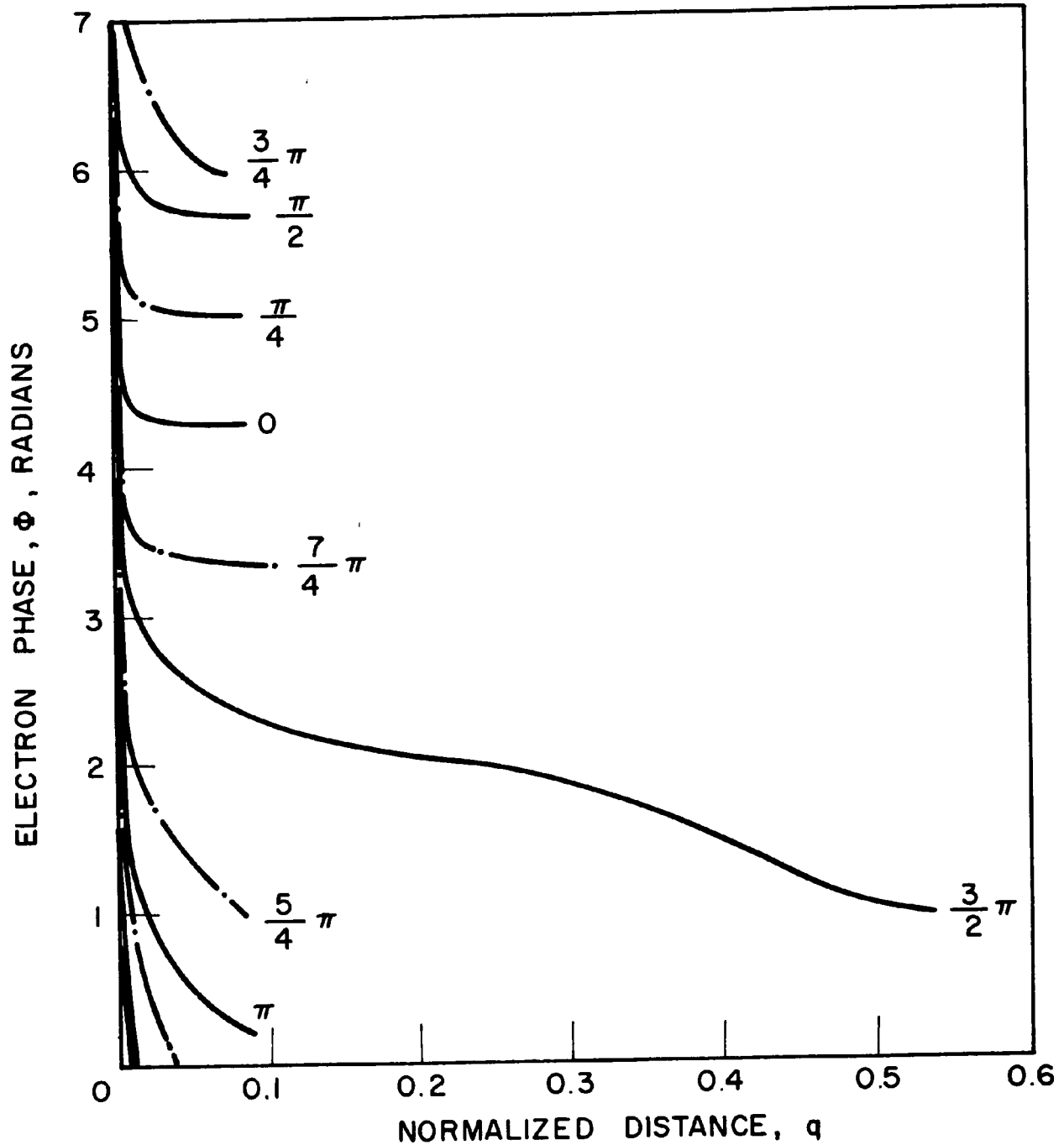


FIG. 5.24 ELECTRON PHASE VS. LONGITUDINAL DISTANCE FOR THE FIRST EMISSION POINT. ($\omega_c/\omega = 0.5$, $\omega_{pc}/\omega = 5$, $D = 0.125$, $\delta = 1$, $A_{rf0}/A_{dc} = 1$, $\beta_o b' = 2$, $\beta_o L_e = 40$)

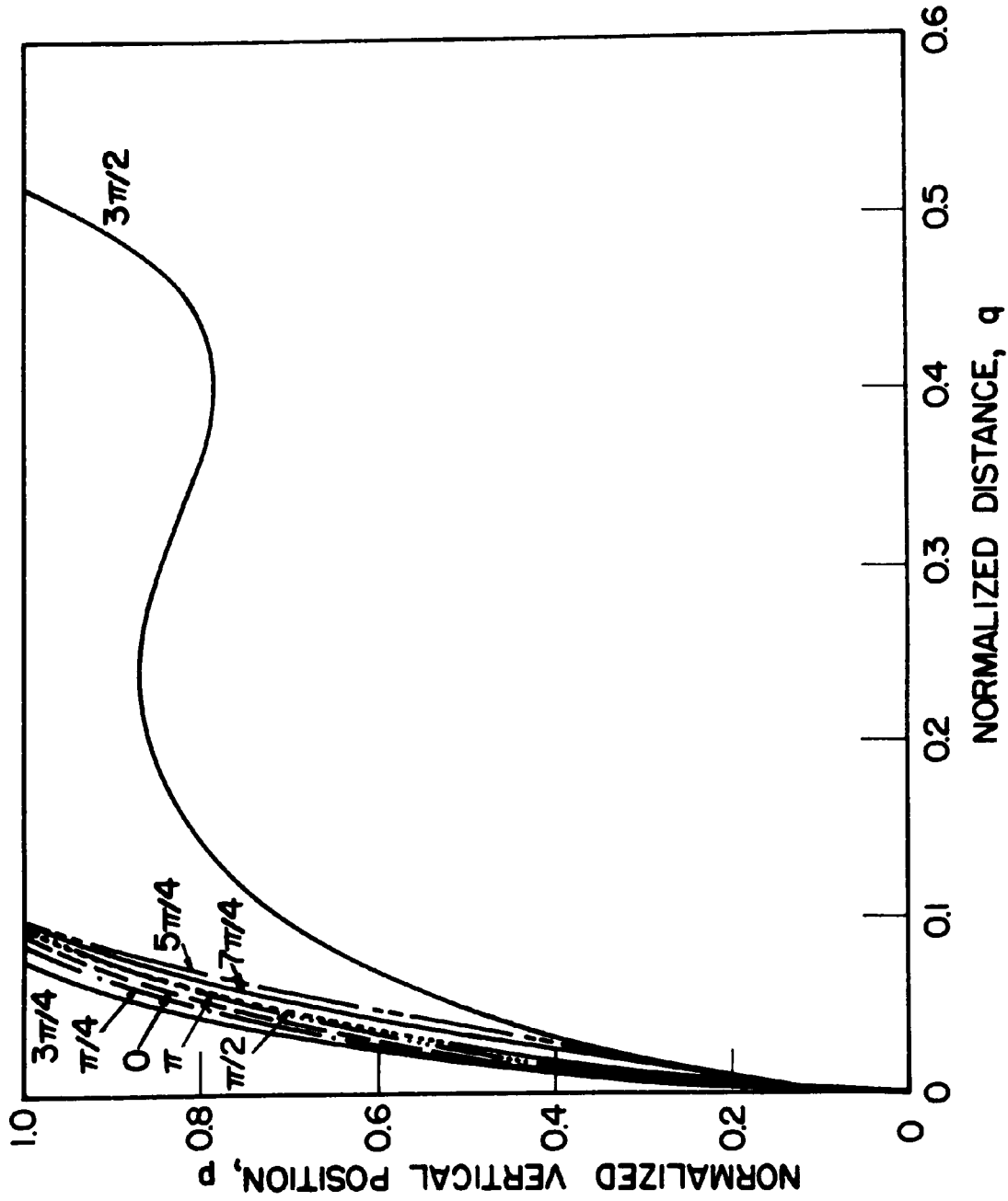


FIG. 5.25 NORMALIZED VERTICAL POSITION VS. LONGITUDINAL DISTANCE FOR THE FIRST EMISSION POINT. ($\omega_c/\omega = 0.5$, $\omega_{pc}/\omega = 5$, $D = 0.125$, $\delta = 1$, $A_{rf}/A_{dc} = 1$, $\beta_{ob'} = 2$, $\beta_{oe} = 40$)

magnetron cutoff value ($\omega_c/\omega = 0.5$ and $K^2 = 0.5$). These figures display the variations in phase and vertical position, respectively, for the electrons emitted from the first emission point. Higher gain for this case (below cutoff), as indicated in the previous section, is partially attributed to motion such as that displayed by the electron with $\Phi_0 = 3\pi/2$ (Fig. 5.25). This electron was initially in the phase position of maximum acceleration, but its trajectory indicates that deceleration has taken place, and hence a conversion of some of this electron's energy into r-f energy has occurred.

It should be observed that the interaction occurring here (bunching about $\Phi = \pi/2$ and divergence away from $\Phi = 3\pi/2$) is more striking than that obtained in Section 5.2.2 for the below cutoff case. Apparently the larger interaction parameter D is responsible for this.

Figure 5.26 shows the variations in phase position and vertical coordinate for the below cutoff case for the electrons from the 81st emission point with and without a magnetic field taper. It is observed that all of the trajectories are noticeably shorter for this emission point than those from the first. Because of this shortness, bunching about $\Phi = \pi/2$ is not easily detectable, but divergence away from $3\pi/2$ is still noticeable. By observing Fig. 5.26 carefully, it can be noticed that electrons with initial phase values between 0 and π for the taper case are collected on the r-f structure in shorter distances than those without a taper. This result is not only due to the weaker magnetic field, but also to the stronger r-f field at this longitudinal position. On the other hand, the most strongly accelerated electron ($\Phi_0 = 3\pi/2$) returns to the sole in a shorter distance for the tapered

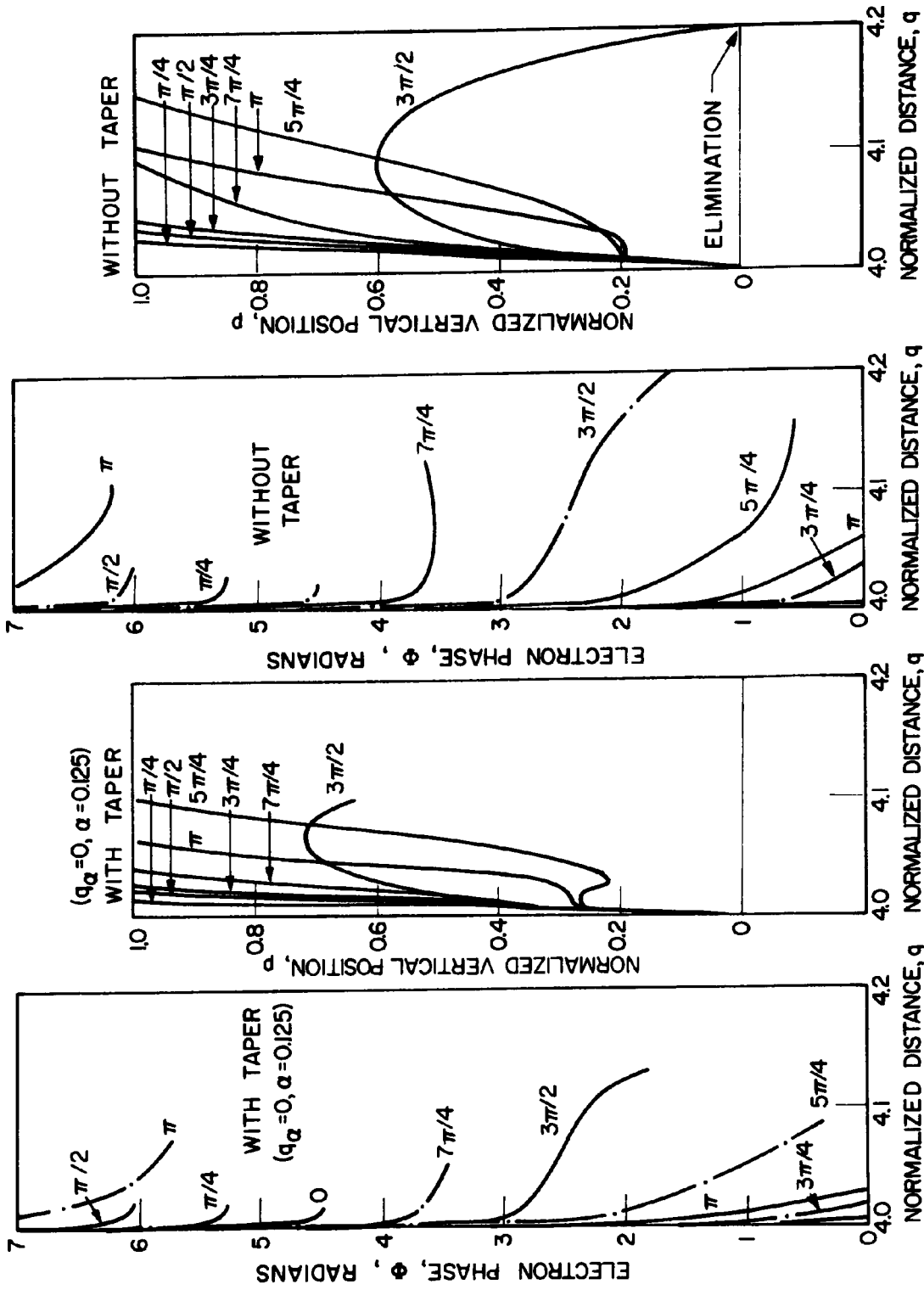


FIG. 5.26 ELECTRON PHASE AND VERTICAL POSITION VS. LONGITUDINAL DISTANCE WITH AND WITHOUT A MAGNETIC

FIELD TAPER FOR THE 81ST EMISSION POINT. [$(\omega_c/\omega)_0 = 0.5, \omega_{pc}/\omega = 5, D = 0.125, \delta = 1,$

$$A_{rf}/A_{dc} = 1, \beta_{ob}' = 2, \beta_{oe}^L = 40]$$

case. This is because the acceleration imparted to this electron by the r-f field overcomes the effect of the weaker magnetic field.

The phase and vertical position solutions for above cutoff operation ($\omega_c/\omega = 1.5$ and $K^2 = 1.5$) and $D = 0.125$ are shown in Figs. 5.27 through 5.29. Again the first and 81st emission point trajectories are presented for comparison. In Fig. 5.27 it is observed that the bunching process for the electrons from the first emission point is weaker than that obtained in the above cutoff case. This figure also shows that the longitudinal distances of electron collection on the r-f structure are longer for this case due to the stronger magnetic field. These results are in agreement with those obtained for $D = 0.10$.

Figures 5.28 and 5.29 indicate the solutions for above cutoff operation for the 81st emission point with and without a magnetic field taper, respectively. Again the electrons initially in the decelerating phase of the r-f wave reach the r-f structure in a shorter distance for the case with a variable magnetic field. Also, the backbombardment lengths for the accelerated electrons with a magnetic field taper applied are shorter. This result is again due to the larger r-f amplitude at this longitudinal position which overcomes the effects of the weaker magnetic field.

It can be concluded from the trajectory solutions presented above that a decrease in the magnetic field with longitudinal distance results in higher average electron vertical positions. Furthermore, since the r-f longitudinal field is stronger closer to the r-f structure, a stronger interaction with the electrons occurs, resulting in greater amplification. Thus, the application of a magnetic field

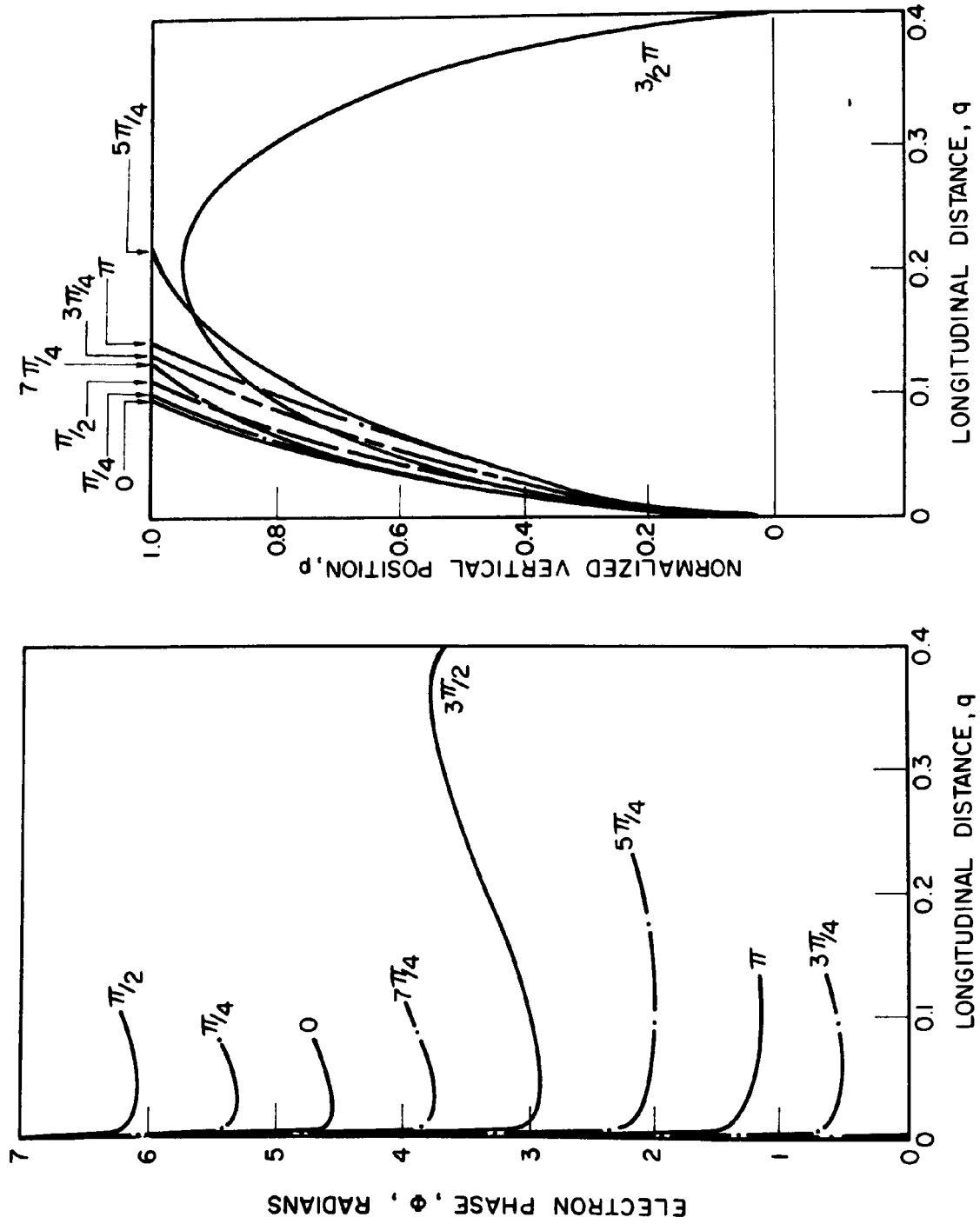


FIG. 5.27 ELECTRON PHASE AND VERTICAL POSITION VS. LONGITUDINAL DISTANCE FOR THE FIRST EMISSION POINT. ($\omega_c/\omega = 1.5$, $\omega_{pc}/\omega = 5$, $D = 0.125$, $\delta = 1$, $A_{rf}/A_{dc} = 1$, $\beta_{ob} = 2$, $\beta_{oe} = 40$)

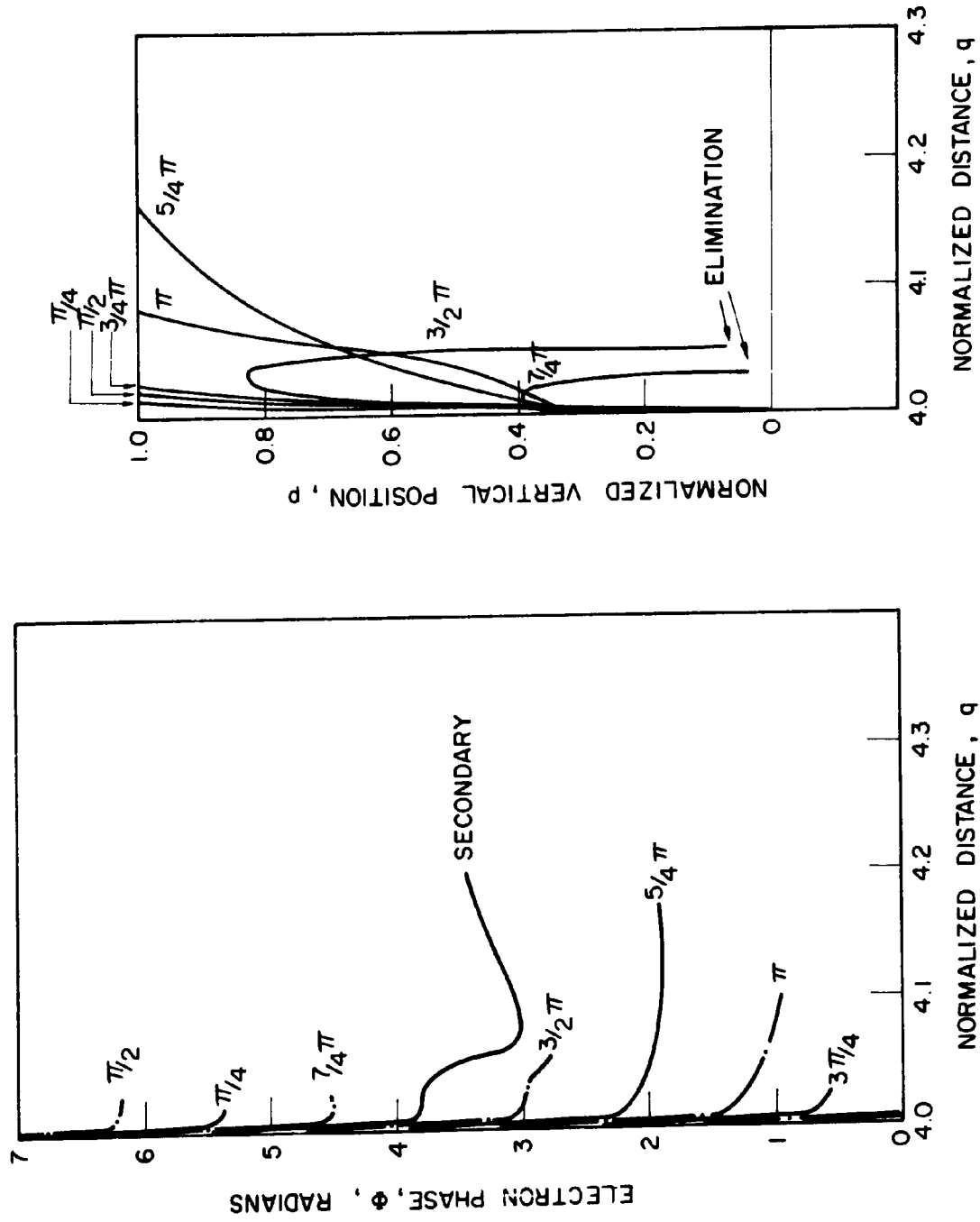


FIG. 5.28 ELECTRON PHASE AND VERTICAL POSITION VS. LONGITUDINAL DISTANCE WITH A MAGNETIC FIELD TAPER

FOR THE 81ST EMISSION POINT. $[(\omega_c/\omega)_0 = 1.5, \omega_{p0}/\omega = 5, D = 0.125, \delta = 1, A_{rf0}/A_{dc} = 1,$

$$\beta_{0e} = 2, \beta_{0e} L_e = 40]$$

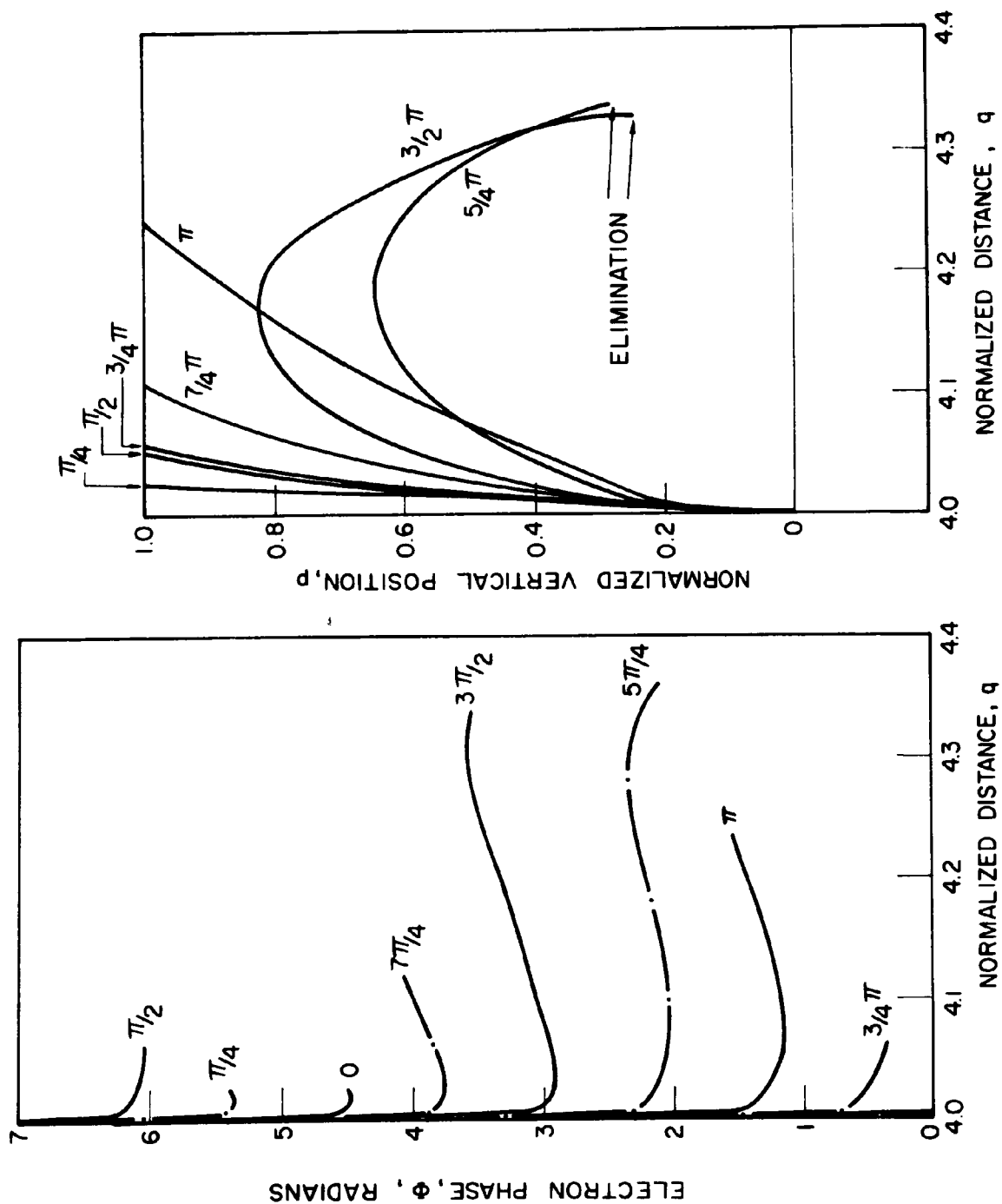


FIG. 5.29 ELECTRON PHASE AND VERTICAL POSITION VS. LONGITUDINAL DISTANCE WITH A CONSTANT D-C MAGNETIC

FIELD FOR THE 81ST EMISSION POINT. ($\omega_c/\omega = 1.5$, $\omega_{pc}/\omega = 5$, $D = 0.125$, $\delta = 1$, $A_{rf}/A_{dc} = 1$,

$\beta_{0e} = 2$, $\beta_{0e} = 40$)

taper not only leads to operation closer to synchronism but also to a greater electron-wave interaction.

5.4 The Multiple-Cathode Amplifier Solution

The multiple-cathode amplifier is a degenerate case of the emitting-sole amplifier in that the electron emission is staggered. In order to simulate this emission using the digital computer, electrons are emitted uniformly over a short longitudinal distance with no emission over the next short distance, and then again there is uniform emission and so on depending upon the number of cathodes that the amplifier is assumed to have. As in the emitting-sole solutions, there are 81 emission points assumed and 32 electrons are emitted from each point. In this case, however, the emission points are necessarily more closely spaced since the same number of electrons is used and the emission length has been shortened.

5.4.1 Computer Input Parameters and the Digital Computer Emission Model. The computer input parameters selected for this case are the same as those used previously in this chapter for the emitting-sole amplifier operating at cutoff, except that the emitting portion of the sole is now shortened ($\beta_o L_e = 8$ instead of 40). The secondary emission ratio has not been changed since portions of the sole other than the cathode also emit secondaries.

The emission model selected consists of four cathodes distributed uniformly along the sole. The first three cathodes have 20 separate emission points spaced equidistantly ($2\Delta q$, where $\Delta q = 0.005$). However, the fourth cathode has 21 emission points (distributed in the same manner) in order to keep the total number of electrons the same

as that used in the emitting-sole solutions. Again, 32 electrons are emitted from each point.

5.4.2 Solution with the Magnetic Field at the Cutoff Value.

For brevity, only the coordinate trajectories for the first and second cathode will be displayed and discussed, and the variations for the other electron variables will be omitted because no new information would be added by including them. Figure 5.30 shows the trajectories of some of the electrons emitted from the first, tenth, 20th and 21st emission points. For the first and 21st emission points, one fourth of the electrons are shown, while for the tenth and 20th emission points only the primary electrons which produce several secondaries are displayed for reasons of clarity. It can be observed that the trajectories in Fig. 5.30 are very similar to those obtained for the emitting-sole amplifier. Furthermore, acceleration of electrons with initial phase values between 0 and π (leading to collection on the r-f circuit) and deceleration of electrons with phase values between π and 2π (causing backbombardment of the sole electrode) is again observed. This implies the direct conversion of d-c potential energy to r-f energy since the electrons which approach the r-f circuit are able to give up more potential energy than can be gained by the electrons receding from the r-f circuit. This is because the r-f fields are strongest in regions closer to the r-f structure, which results in a stronger electron-wave interaction between the r-f wave and the electrons which approach the r-f circuit than that which occurs between the r-f wave and the electrons which recede from it.

It is further observed in Fig. 5.30 that the electron trajectories of the 21st emission point display greater deceleration and

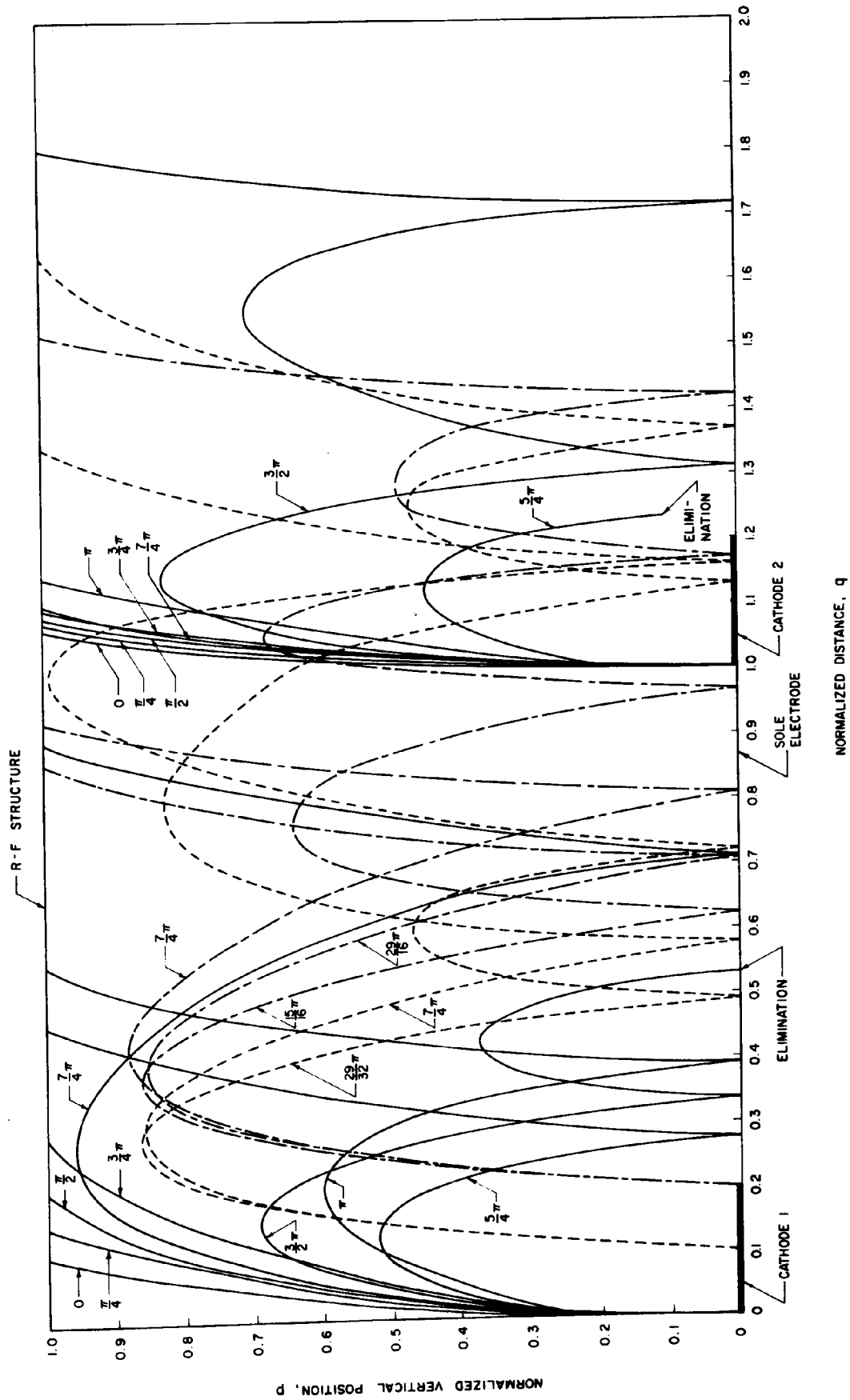


FIG. 5.30 ELECTRON TRAJECTORIES FOR ELECTRONS EMITTED FROM THE FIRST AND SECOND CATHODES WITH ϕ_0 AS A

PARAMETER. ($\omega_c/\omega = 1$, $\omega_{pc}/\omega = 5$, $\delta = 1$, $A_{rf}/A_{dc} = 1$, $D = 0.1$, $\beta_{b'} = 2$, $\beta_{0e} = 8$)

acceleration effects than those obtained for the first emission point, i.e., the decelerated electrons emitted from the 21st emission point have shorter longitudinal collection lengths than those obtained for the first emission point and the cathode backbombardment lengths of the accelerated electrons are also shorter. The reason for this is that the r-f amplitude is increasing with longitudinal distance and hence stronger r-f fields are accelerating and decelerating the electrons at larger longitudinal q-positions. This effect was also observed in the emitting-sole amplifier solutions. Secondary electron trajectories are also shown in Fig. 5.30 and it is seen that these are also very similar to the secondary trajectories obtained previously.

Figures 5.31 and 5.32 are the computer solutions for $A_{rf}(q)$ and $\theta_c(q)$ for the assumed four-cathode amplifier. In these figures the cathodes are represented by dark lines along the abscissa. The $A_{rf}(q)$ curve shows significant growth with a saturation gain value of 5.5 db. For the same set of input parameters (except $\beta_o L_e = 8$) and the same number of emitted electrons, the result for the emitting-sole amplifier showed a resulting saturation gain of 7 db. This difference can be attributed to a weaker interaction in the multiple-cathode amplifier due to a lower total current, since the emission current densities were chosen to be the same but the emission area for the multiple-cathode amplifier is smaller.

The variation in r-f phase angle, Fig. 5.32, implies that the perturbed circuit wave velocity is at times greater than v_d and at other times less than v_d . However, since $v_o = v_d$, this type of fluctuation corresponds to operation both above and below synchronism with an r-f wave velocity oscillating approximately sinusoidally about v_o .

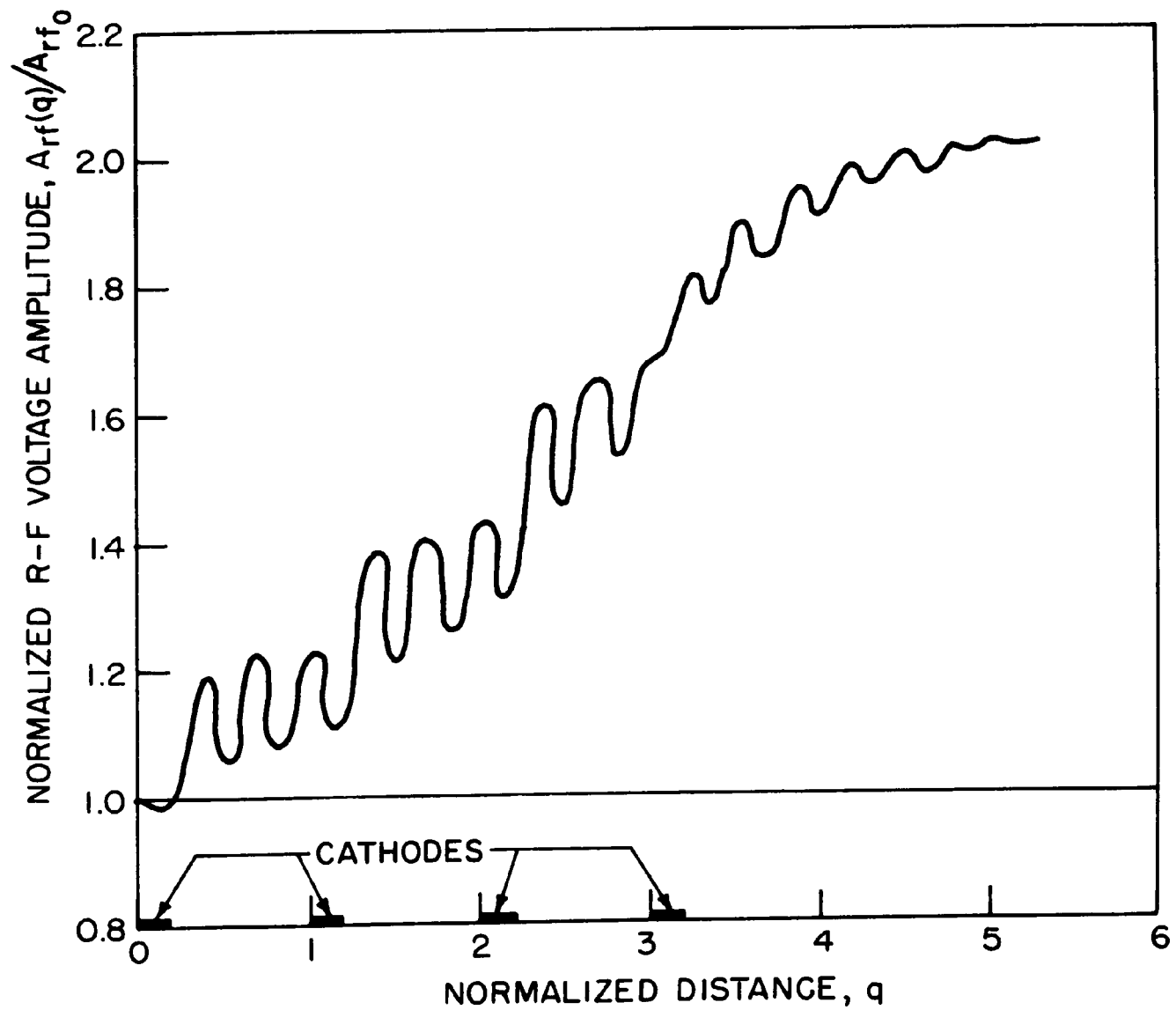


FIG. 5.31 NORMALIZED R-F VOLTAGE VARIATION VS. LONGITUDINAL DISTANCE FOR THE MULTIPLE-CATHODE AMPLIFIER. ($\omega_c/\omega = 1$, $\omega_{pc}/\omega = 5$, $D = 0.1$, $\delta = 1$, $A_{rf0}/A_{dc} = 1$, $\beta_o b' = 2$, $\beta_o L_e = 8$)

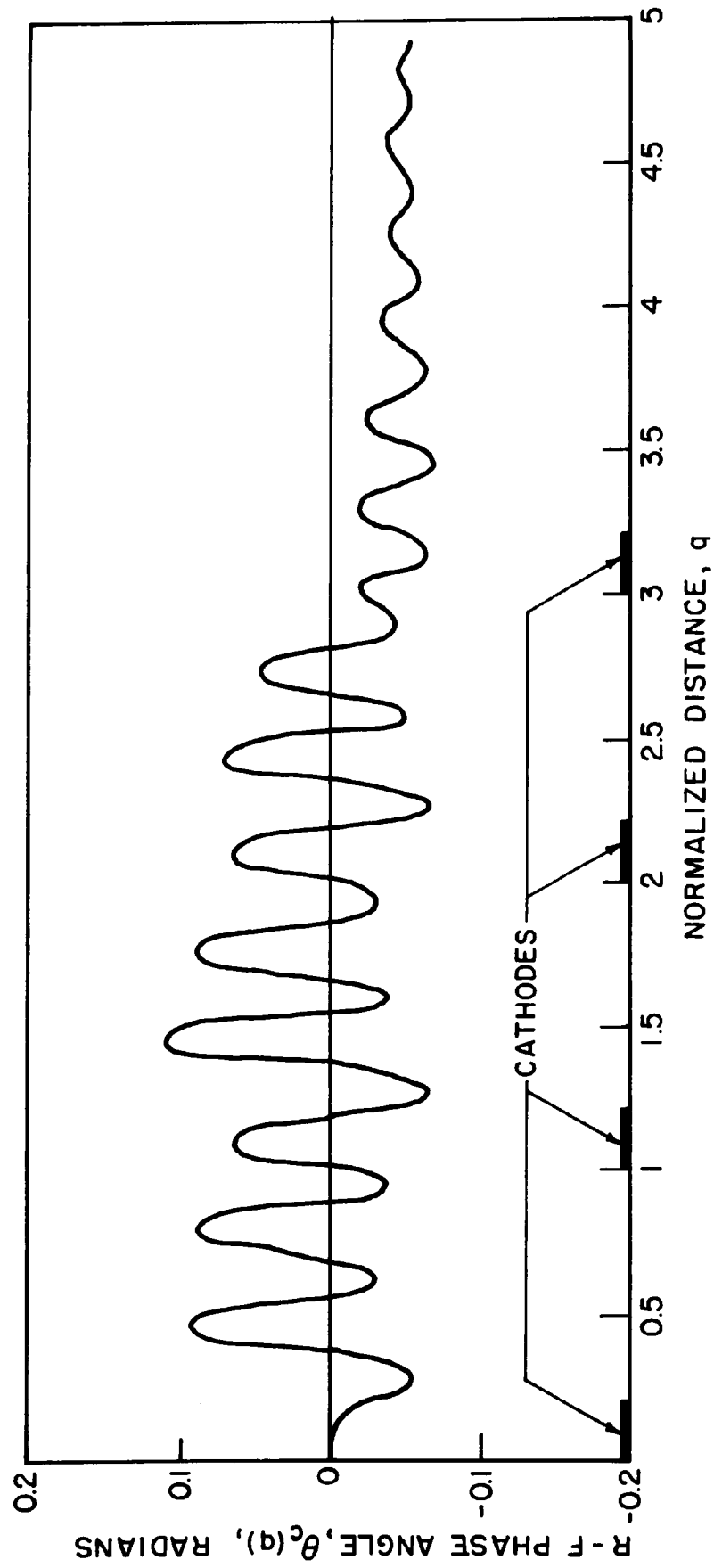


FIG. 5.32 VARIATION IN R-F PHASE ANGLE WITH LONGITUDINAL DISTANCE FOR THE MULTIPLE-CATHODE AMPLIFIER.

($\omega_c/\omega = 1$, $\omega_{pc}/\omega = 5$, $D = 0.1$, $\delta = 1$, $A_{rf}/A_{dc} = 1$, $\beta_o b' = 2$, $\beta_o L_e = 8$)

Both Fig. 5.31 and Fig. 5.32 exhibit periodic fluctuations. Since there are 16 minima in a normalized distance $q = 5$, the average normalized wavelength of the fluctuation is $\lambda_q = 5/16 = 0.3125$. This value is not constant, but decreases slightly for increasing q .

The above wavelength will now be shown to be approximately equal to half the d-c backbombardment length. Under d-c conditions only, i.e., when space-charge and r-f forces are omitted, and for zero initial electron velocities, the backbombardment length is given by $x_0 = \pi y_{\max}$, where $y_{\max}$ is the maximum vertical position of an electron. For d-c magnetron cutoff operation this becomes $x_0 = \pi b'$, where b' is the sole-to-anode spacing. In terms of the normalized coordinate q , this becomes

$$q_{x_0} = \beta_0 D x_0 = \pi \beta_0 b' D ,$$

or since $\beta_0 b' = 2$ and $D = 0.1$,

$$q_{x_0} = 0.628 .$$

Thus, the period of oscillation for the r-f variables corresponds to half the d-c backbombardment length. A physical argument consistent with this result can be formulated as follows.

As discussed previously, the electrons in a crossed-field device have cycloidal trajectories with an average drift velocity in the longitudinal direction given by $v_d = E_0/B_0$. Figure 5.33 shows qualitatively the common cycloidal trajectory for an electron emitted with zero initial velocity in the presence of d-c fields only. This trajectory can also be considered as the average path for electrons in the presence of an r-f wave, recalling that some electrons are accelerated while others are decelerated with the unaffected electrons (the d-c

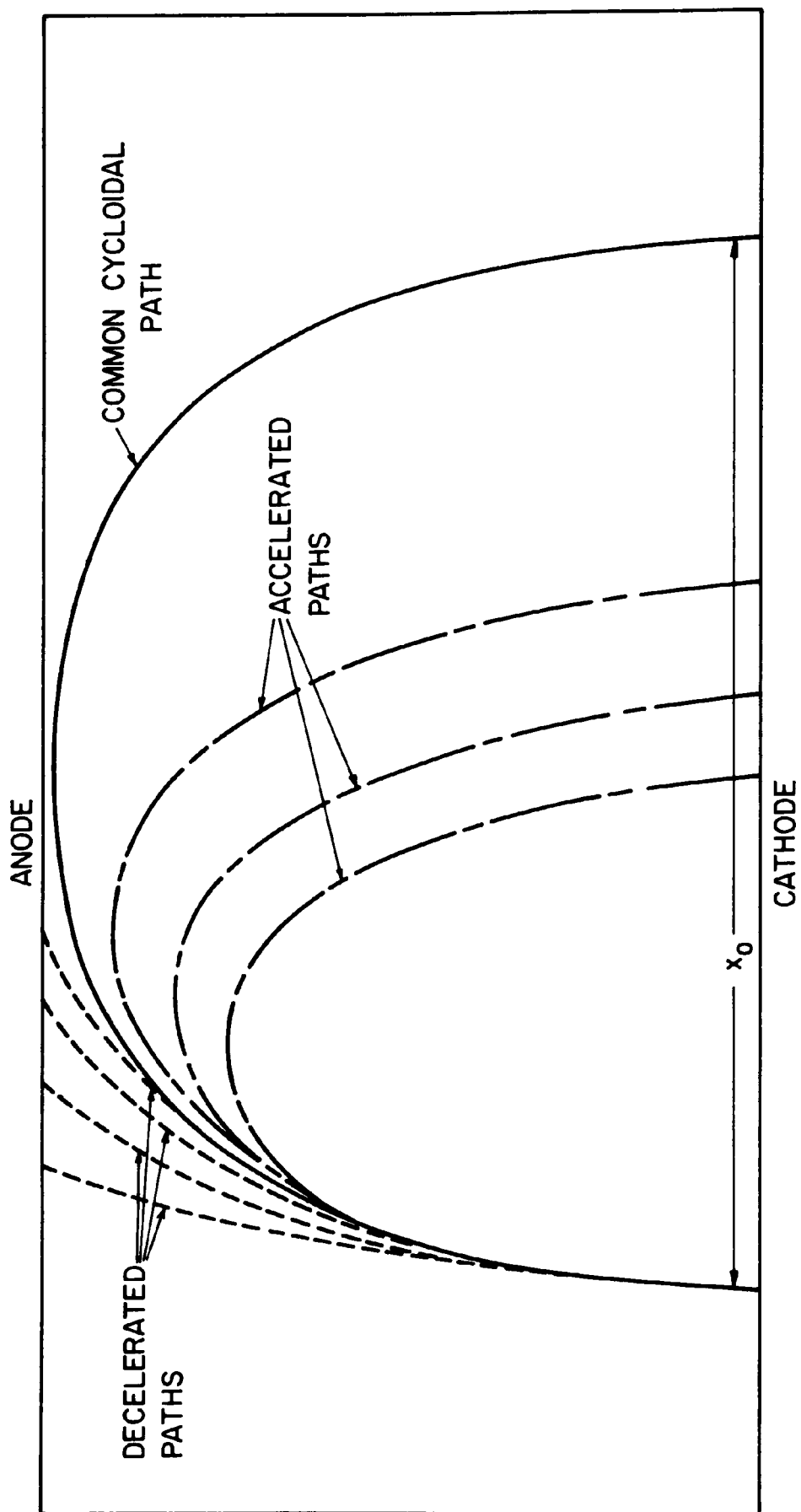


FIG. 5.33 ELECTRON TRAJECTORIES IN A CROSSED-FIELD DEVICE.

trajectories) being the dividing line between these groups. R-f acceleration and deceleration paths are also shown in Fig. 5.33. Now, the decelerated electrons are absorbed on the anode in a distance of less than $x_o/2$, while the accelerated electrons return to the cathode in a distance greater than $x_o/2$ but less than x_o . This collection of decelerated electrons disturbs the averaging process used to calculate $A_{rf}(q)$ and $\theta_c(q)$ and thus produces fluctuations. The first maximum in $A_{rf}(q)$ therefore is caused by the decelerated primary electrons converting electron potential energy into r-f energy. The first minimum, on the other hand, is brought about by accelerated primary electrons, and the extraction of energy from the r-f wave. After this initial energy conversion process, the fluctuations are carried on by secondary electrons. It should be observed that the fluctuations are largest immediately after each cathode and then gradually taper off. This is due to the presence of more primary electrons than secondaries, which results in a greater disturbance in the averaging process and hence greater fluctuations.

Further evidence for this argument is the fact that the period of fluctuation decreases with increasing q . This is caused by the growth in the r-f wave amplitude, causing greater acceleration and deceleration of the electrons in shorter distances.

5.5 Summary

In this chapter the digital computer solutions of the large-signal nonlinear equations of Chapter II including the effects of space-charge fields were presented. The following characteristics were observed:

1. The inclusion of space-charge fields enhances the bunching process and results in larger r-f voltage amplitude growth rates and saturation gain than were obtainable in the zero space-charge-field cases.

2. A constant d-c magnetic field value below the d-c magnetron cutoff value was seen to yield greater r-f amplification than was obtained with a value above cutoff. The reason for this result was attributed to a greater electron-wave interaction taking place for a weaker d-c magnetic field. When a weaker magnetic field is present, the electrons are able to approach the r-f structure more closely and interact more strongly with the larger r-f longitudinal field which is present closer to the structure.

3. It was found that an improvement in the gain of the amplifier is achieved when the d-c magnetic field is linearly reduced in the longitudinal direction. It was concluded that operation closer to synchronism and higher average vertical electron positions are responsible for this.

4. For d-c cutoff operation the multiple-cathode amplifier did not achieve as much gain as the emitting-sole amplifier for the same tube length and number of charge groups. This is a result of having less cathode area and hence less total current in the interaction region.

6.1 Approximate Equations from the Lagrangian Formulation

The large-signal equations for the crossed-field emitting-sole amplifier, as given in Chapter II, can be considerably simplified when space-charge forces are neglected and the interaction parameter is considered very small compared to one. Under the small-D assumption, it has been shown by Gandhi¹⁶ that the adiabatic equations of motion may be used (actually, Gandhi found that the adiabatic approximation is valid when $(\omega_c/\omega D) \gg 1$), i.e., the acceleration terms in the force equations may be neglected. The trajectories obtained by solving the adiabatic equations will then be average paths resulting in the elimination of cycloidal motion, which is of no importance in the present analysis.

Thus, for weak interaction ($D \ll 1$), zero space-charge forces, and also for lossless circuits ($d = 0$), the crossed-field emitting-sole equations become

$$1 + 2Du_z = [D(\omega/\omega_c)^2/(\beta_o b')] [A_{dc} + \beta_o b' A_{rf}(q) \phi(p) \cos \Phi] , \quad (6.1)$$

$$u_y = (\omega/\omega_c) D [A_{rf}(q) \psi(p) \sin \Phi - D \frac{dA_{rf}(q)}{dq} \psi(p) \cos \Phi] , \quad (6.2)$$

$$A_{rf}(q) \frac{d\theta_c(q)}{dq} = - C_1 \int_0^q \int_0^{2\pi} \frac{\psi(p) \cos \Phi}{|1 + 2Du_z|} dq_o d\Phi_o , \quad (6.3)$$

$$A'_{rf}(q) = C_1 \int_0^q \int_0^{2\pi} \frac{\psi(p) \sin \Phi}{|1 + 2Du_z|} dq_o d\Phi_o , \quad (6.4)$$

$$\frac{\partial \Phi}{\partial q} = \frac{2u_z}{1 + 2Du_z} \quad (6.5)$$

and

$$\frac{dp}{dq} = \frac{u_y}{(\omega_c/\omega) \beta_o b' D(1 + 2Du_z)} \quad (6.6)$$

where

$$c_1 \triangleq \frac{v_r(v_o/v_c) |\rho_o'|}{2\pi(\omega_c/\omega) \beta_o L_e D} \quad ,$$

and the other parameters have been defined in Chapter II.

6.2 The HKB Equations from the Lagrangian Formulation

Although the above equations do not seem to be soluble in their present form, an approximate solution may be obtained by assuming that the electrons in the interaction region of the emitting-sole amplifier coalesce into giant charge groups called hard-kernel bunches¹⁷ (HKB). This assumption dictates that the charge is concentrated at particular phase positions in each period of the r-f wave. These phase positions are naturally chosen to be $\Phi_1 = \pi/2$, the phase region of maximum decelerating electric field, and $\Phi_2 = -(\pi/2)$, the maximum accelerating electric field region. This favorable and unfavorable bunch-pair model corresponds to the actual bunching of the decelerated electrons about $\Phi_1 = \pi/2$ and it accounts for the accelerated electrons which return to the sole by assuming another bunch at $\Phi_2 = -(\pi/2)$. The HKB approximation therefore implies that the bunching process has proceeded to the ultimate state after which potential energy transfer takes place as the ideal bunches approach and recede from the circuit.

When the hard-kernel-bunch (HKB) assumption is used and synchronism is assumed between the r-f wave and the d-c fields ($v_o = E_o/B_o$), Eqs. 6.1 to 6.6 reduce to

$$u_{y_{1,2}} = \pm D(\omega/\omega_c) \psi(p_{1,2}) A_{rf}(q) , \quad (6.7)$$

$$A'_{rf}(q) = 2\pi C_1 q [\psi(p_1) - \psi(p_2)] , \quad (6.8)$$

$$\phi_{1,2} = \pm (\pi/2) \quad (6.9)$$

and

$$p'_{1,2} = \frac{u_{y_{1,2}}}{(\omega_c/\omega) \beta_o b' D} , \quad (6.10)$$

where the primed functions imply differentiation with respect to q . Also, the upper sign and the subscript 1 in Eqs. 6.7, 6.9 and 6.10 refer to the bunch at $\phi_1 = \pi/2$ and the lower sign along with the subscript 2 in each equation refers to the bunch at $\phi_2 = -(\pi/2)$. It should be noted from Eq. 6.1 and the synchronism assumption that $u_z = 0$, implying that the energy conversion process excludes kinetic energy transfer. It should also be noted from Eq. 6.3 that $\theta'_c = 0$ when the HKB approximation is applied. However this was implied by the assumption of weak interaction and therefore this result is not surprising. Eliminating u_y from the above set of equations yields the following system of equations:

$$A'_{rf}(q) = 2\pi C_1 q [\psi(P_1) - \psi(P_2)] \quad (6.11)$$

and

$$P'_{1,2} = \pm C_2 A_{rf}(q) \psi(P_{1,2}) , \quad (6.12)$$

where $P_{1,2} \triangleq \beta_o b' p_{1,2}$ and $C_2 \triangleq (\omega/\omega_c)^2$. An identical set of equations is derived in Appendix E using an energy analysis. The solution of these equations for $P_1(q)$, $P_2(q)$ and $A_{rf}(q)$ is presented in the next section.

6.3 The HKB Model and Solutions

This section contains several solutions of the HKB equations for various operating conditions. These conditions, as in Chapter V, are for synchronous operation and also for operation above, below and at the magnetron cutoff value of crossed d-c fields. The same parameter values are used in these calculations as were used in Chapter V for the cases with $D = 0.10$ and $\omega_c/\omega = 0.5, 1.0$ and 1.5 .

Solution of the HKB equations requires the stipulation of a bunch model. This model must simulate the bunching process as closely as possible. This cannot be accomplished by only one pair of hard-kernel bunches, since this pair would have to represent the entire set of electrons emitted from the entire length of the cathode. A more accurate model is obtained by assuming that several HKB pairs interact with the r-f wave. These pairs are assumed to be distributed uniformly in the longitudinal direction and their effects upon the r-f wave are then individually determined (for $q > q_o^k$) from the following equation:

$$A_{rf}'(q) = 2\pi C_1 \sum_{k=1}^N (q - q_o^k) [\psi(P_1^k) - \psi(P_2^k)] , \quad (6.13)$$

where N is the number of HKB pairs and q_o^k is the initial longitudinal position of the k th HKB pair. Equation 6.13 is the same as Eq. 6.11 except that a summation is used over all of the assumed HKB pairs. The trajectory equation for each bunch pair (assuming several bunch pairs) is then given by

$$\frac{dP_{1,2}^k}{dq} = \pm C_2 A_{rf}(q) \psi(P_{1,2}^k) , \quad (6.14)$$

where $k = 1, 2, 3, \dots, N$.

The solutions for the r-f amplitude for three different values of the d-c magnetic field and with the total number of assumed bunch pairs as a parameter are shown in Figs. 6.1 to 6.3. In each case the bunches were distributed uniformly over a length $q_e = 4$. These figures demonstrate that the inclusion of a greater number of bunch pairs enhances the gain of the amplifier. This result was not unexpected, since increasing the number of bunch pairs implies increasing the total charge in the interaction region and hence the emission current. As a result of a larger current a stronger electron-wave interaction is then possible which leads to greater gain. The reason for greater gain when the injection charge is increased is simply that more electron potential energy is made available for conversion. In a practical device this increase in current could be achieved by increasing the primary or the secondary electron emission from the cathode. Figures 6.1 to 6.3 also illustrate the result that for the same number of hard-kernel-bunch pairs, a d-c magnetic field value ($\omega_c/\omega = 0.5$) below the d-c magnetron cutoff value results in the largest r-f amplification. This is due to a stronger interaction which results under these d-c field conditions and will be discussed more completely in connection with the bunch-pair trajectories.

Trajectory solutions for the 20 and 40 bunch models are indicated in Figs. 6.4 and 6.5 for the separate cases of operation below ($\omega_c/\omega = 0.5$), above ($\omega_c/\omega = 1.5$) and at ($\omega_c/\omega = 1.0$) the magnetron cutoff value of the d-c magnetic field. Similar results were also obtained for the 60 and 80 bunch-pair models, but for brevity are not included here. In the 20 bunch model, the favorable and unfavorable kernels of charge are injected at longitudinal positions corresponding to $q = 0, 0.2$,

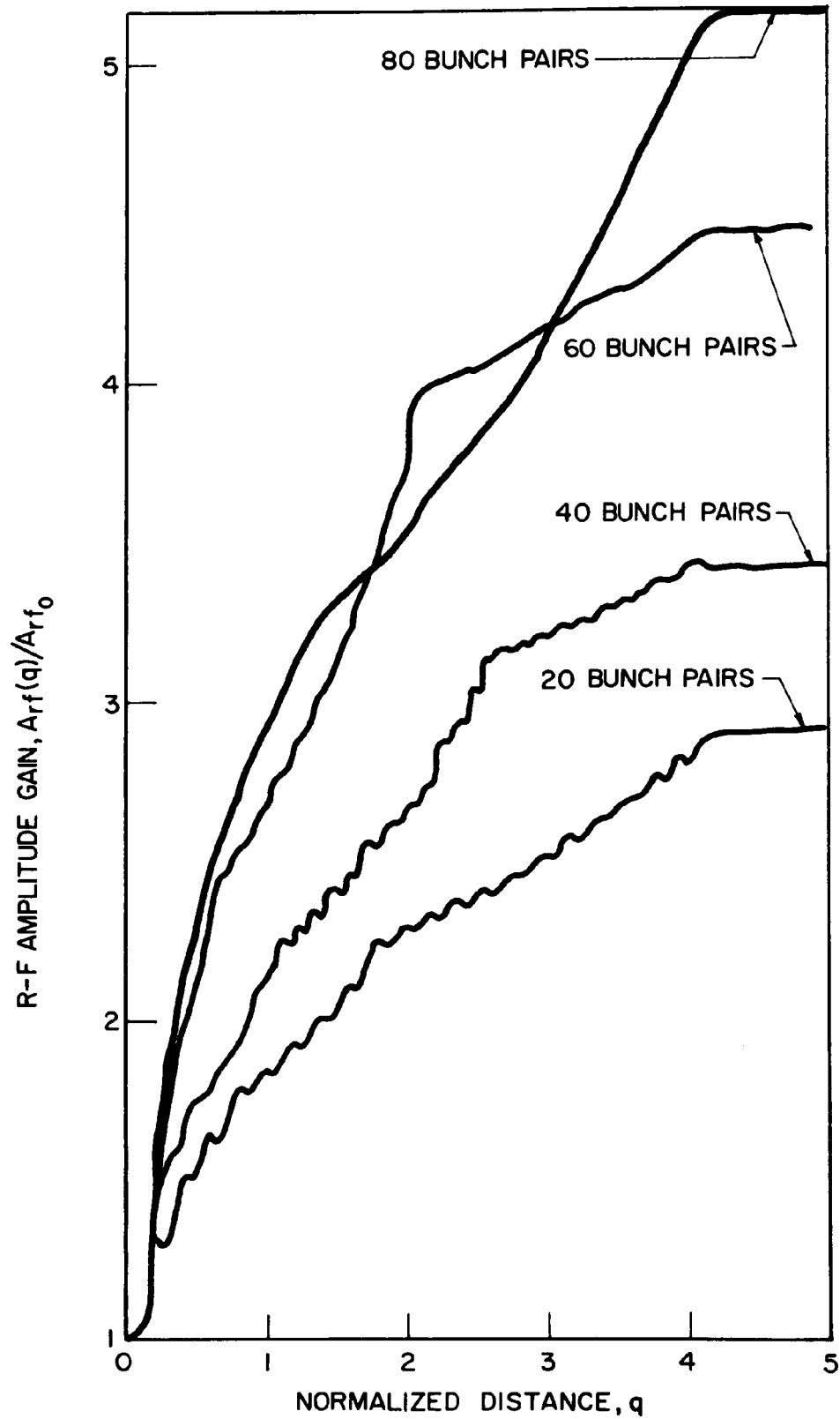


FIG. 6.1 AMPLITUDE GAIN OF THE R-F WAVE VS. LONGITUDINAL DISTANCE WITH ELECTRON BUNCH PAIRS AS THE PARAMETER AND $\omega_c/\omega = 0.5$. ($D = 0.1$, $A_{rf0}/A_{dc} = 1$, $\beta_0 b' = 2$, $\beta_0 L_e = 40$)

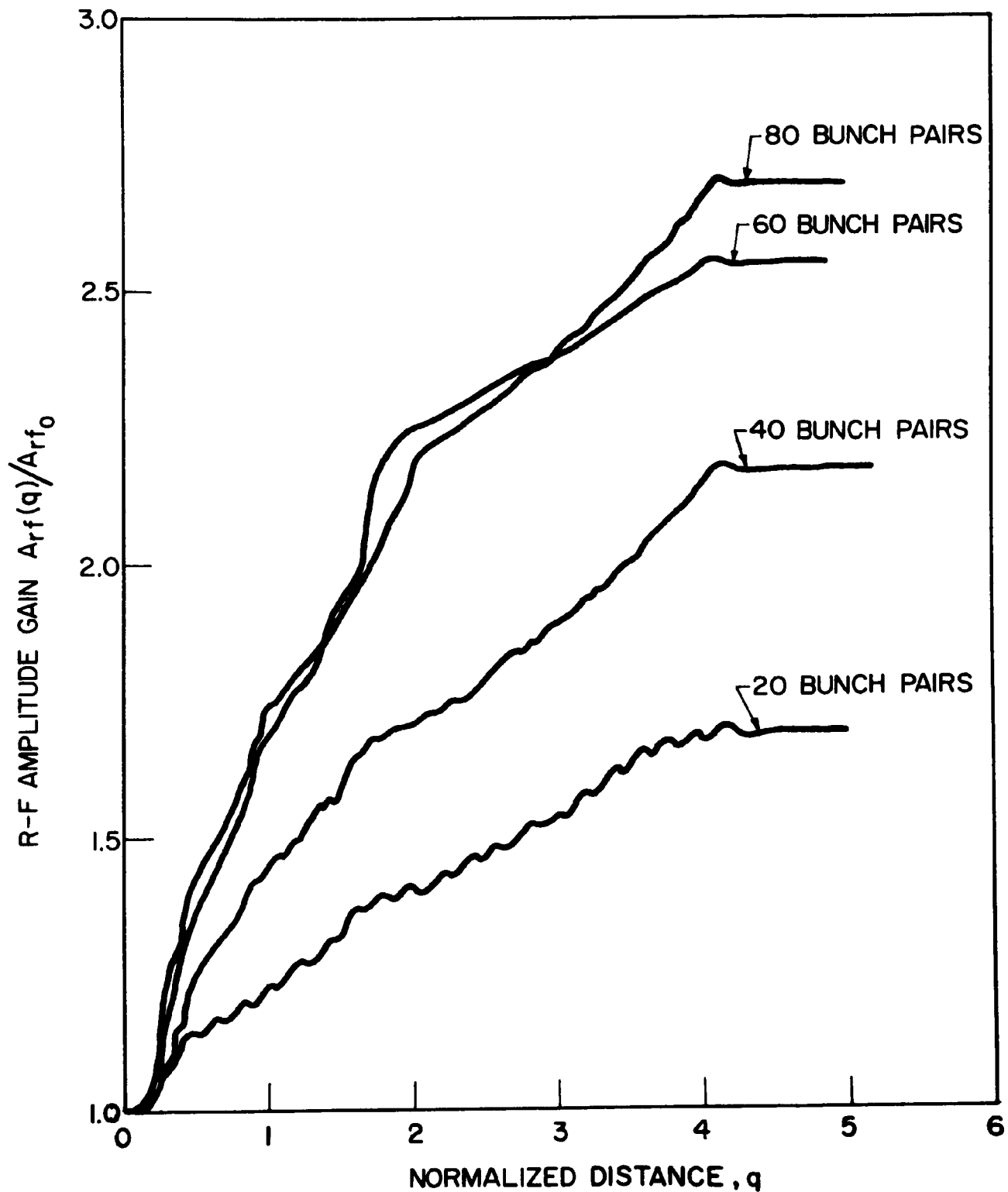


FIG. 6.2 AMPLITUDE GAIN OF THE R-F WAVE VS. LONGITUDINAL DISTANCE WITH ELECTRON BUNCH PAIRS AS THE PARAMETER AND $\omega_c/\omega = 1$. ($D = 0.1$, $A_{rf0}/A_{dc} = 1$, $\beta_o b' = 2$, $\beta_o L_e = 40$)

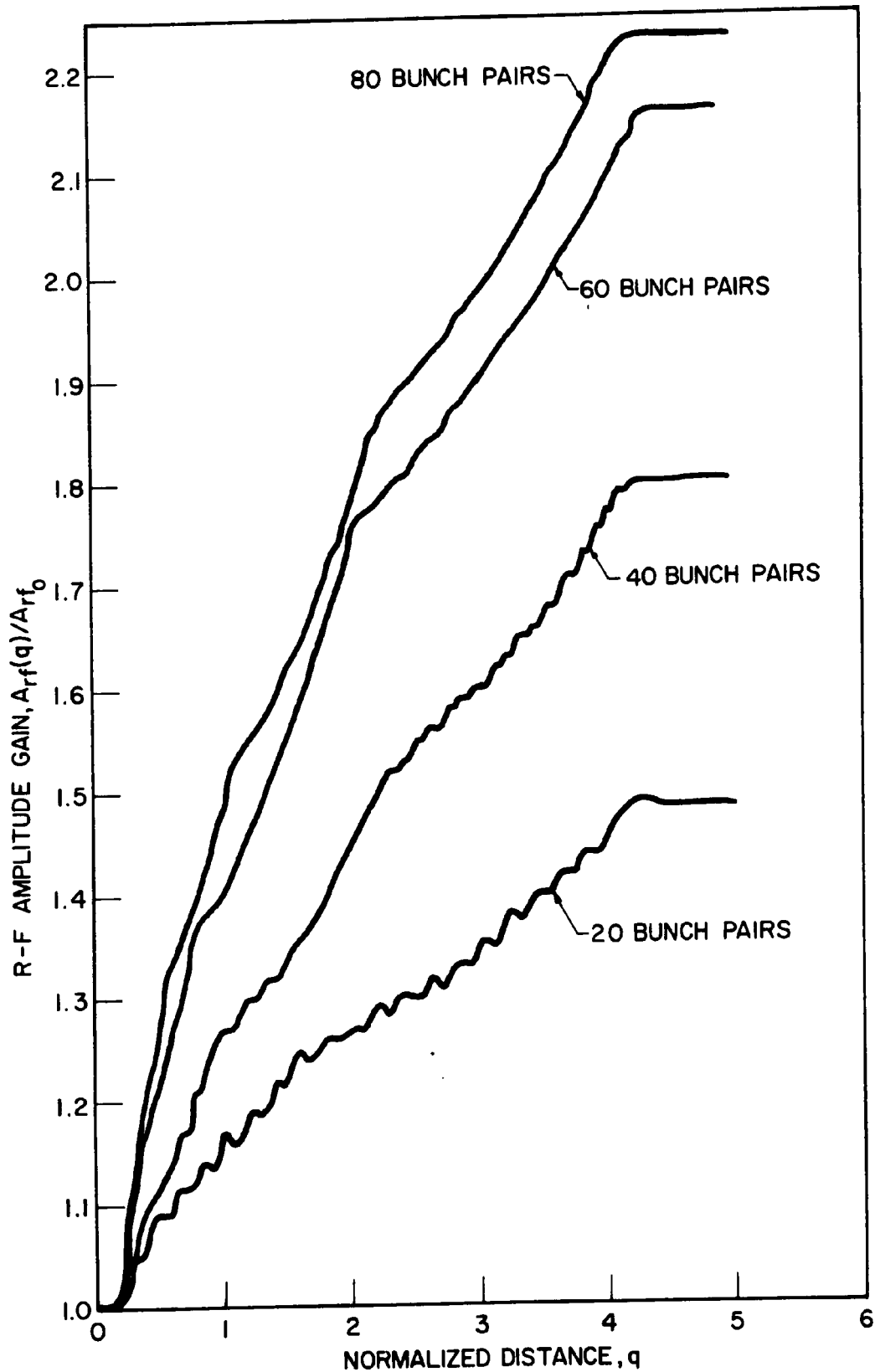


FIG. 6.3 AMPLITUDE GAIN OF THE R-F WAVE VS. LONGITUDINAL DISTANCE WITH ELECTRON BUNCH PAIRS AS THE PARAMETER AND $\omega_c/\omega = 1.5$. ($D = 0.1$, $A_{rf0}/A_{dc} = 1$, $\beta_o b' = 2$, $\beta_o L_e = 40$)

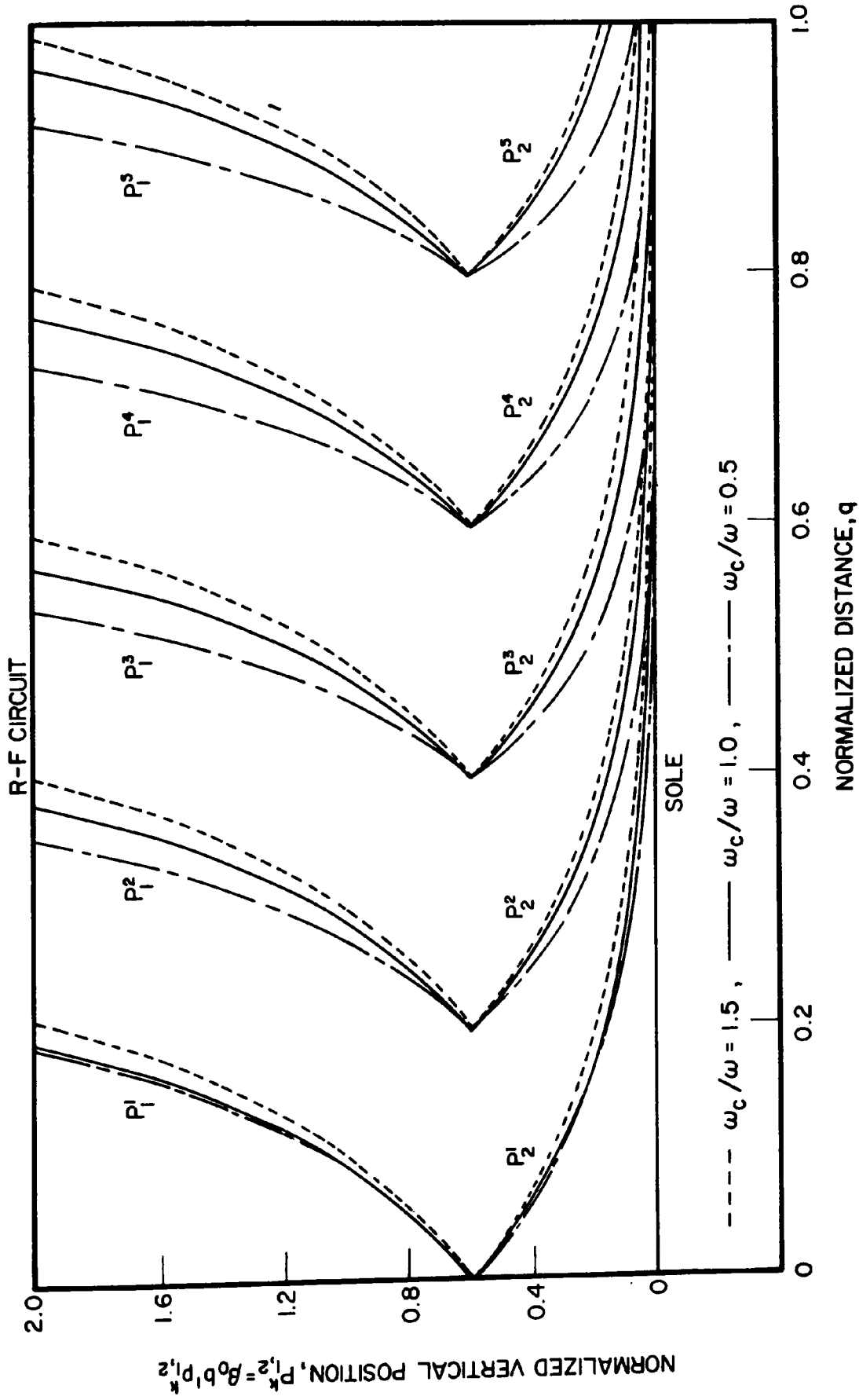


FIG. 6.4 HARD-KERNEL-BUNCH TRAJECTORIES FOR OPERATING CONDITIONS ABOVE, BELOW AND AT THE MAGNETRON CUT-OFF VALUE OF THE D-C MAGNETIC FIELD FOR 20 BUNCH PAIRS. ($D = 0.1$, $A_{rf_0} / A_{dc} = 1$, $\beta_0 b' = 2$, $\beta_0 L_e = 40$)

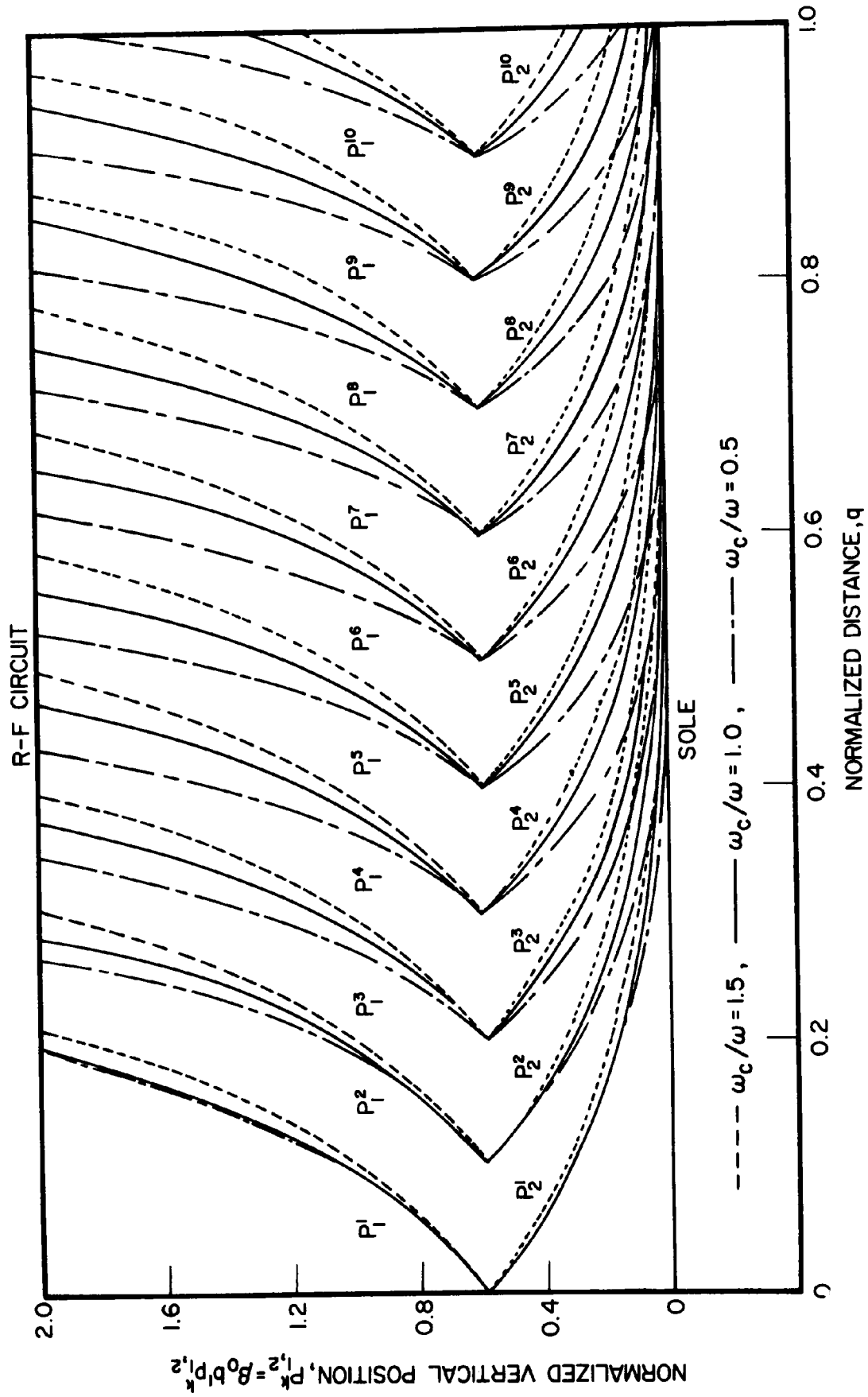


FIG. 6.5 HARD-KERNEL-BUNCH TRAJECTORIES FOR OPERATING CONDITIONS ABOVE, BELOW AND AT THE MAGNETRON CUT-OFF VALUE OF THE D-C MAGNETIC FIELD FOR 40 BUNCH PAIRS. ($D = 0.1$, $A_{rf}/A_{dc} = 1$, $\beta_0 b' = 2$, $\beta_0 L_e = 40$)

0.4, ..., 3.8, but only the first five hard-kernel-bunch pairs are shown in Fig. 6.4 in order to compare these trajectories with those of Chapters IV and V. Figure 6.5 shows the trajectory solutions for the 40 bunch-pair case in which the distance between initial bunch positions is now 0.1 instead of 0.2 as in the 20 bunch case and hence 10 bunch pairs are shown in the range of q values 0 to 1 (one unit in q corresponds to approximately two circuit wavelengths). For the 60 and 80 bunch-pair models this spacing was 0.07 and 0.05, respectively. It can be seen that the favorable bunches in each case are collected on the r-f circuit over very short distances. This corresponds to the rapid collection of the electrons with initial phases between 0 and π which was observed in Chapters IV and V. Also, the unfavorable bunches are seen to approach the sole as did the electrons in Chapters IV and V which had initial phase values between π and 2π .

This motion is due to the decelerating and accelerating r-f forces on the electrons with initial phase positions in the decelerating and accelerating phase of the r-f wave. Thus, since the magnetic field force is proportional to velocity, a favorable electron bunch, which is in the maximum decelerating phase of the r-f wave ($\phi = \pi/2$), experiences an increase in total force directed toward the r-f structure. This is due to a decrease in the vertical magnetic field force on this bunch. The favorable bunches are hence strongly decelerated by the r-f field and are therefore collected on the r-f structure. On the other hand, the unfavorable electron bunches have phase positions corresponding to the maximum accelerating phase of the r-f wave and thus approach the sole due to an increase in the magnetic field force toward this electrode.

It is also observed in comparing the trajectories of Fig. 6.4 with those of Fig. 6.5 that for the 40 bunch-pair case the favorable electron bunches approach the r-f structure in shorter longitudinal distances. This becomes more evident as q increases and can be observed by comparing trajectories which are injected at the same q location, such as the P_1^9 trajectories of Fig. 6.5 with the P_1^5 trajectories of Fig. 6.4. This result is due to the greater effect of the stronger r-f field which prevails for the cases with more electron bunches. This increase in r-f field for a greater number of bunch pairs was observed as an increase in r-f potential amplitude and was displayed in Figs. 6.1 to 6.3. Hence, because of a stronger r-f field, the favorable bunches are decelerated more effectively (in shorter longitudinal distances). In turn, the r-f wave receives more energy (for the same longitudinal distance) as these bunches approach the r-f structure via potential to r-f energy conversion. Thus, two mechanisms are causing greater amplification of the r-f wave when greater numbers of bunch pairs are assumed. First, more charge is being injected into the interaction region, thereby increasing the potential energy available for conversion. And second, larger r-f fields are produced as a result of this greater current which enables a more effective energy conversion process to take place.

It should be noted that the initial vertical position of the bunches in Figs. 6.4 and 6.5 is chosen to be the same for all bunch pairs and equal to 0.6. This value was selected because it yielded the most favorable results, i.e., when a larger initial vertical position was selected the unfavorable electron bunches had too great an effect upon the r-f amplitude which resulted in very little gain. On the

other hand, a smaller value of initial vertical position yielded unreasonably higher gain.

These results are explained as follows. In the derivation of the HKB equations it was implicitly assumed that each bunch contained the same amount of charge, i.e., each bunch was weighed equally in Eq. 6.13. However, in the actual device there is a transfer of electrons from the unfavorable (π to 2π) to the favorable (0 to π) portion of a cycle of the r-f wave. This implies that more charge should reside in the favorable bunches than in the unfavorable bunches. However, a mixing of electrons has not been accounted for in this HKB model and the effect of this upon the r-f wave amplitude is directly related to the selection of the initial vertical position of the bunches. Thus, by using an initial value of $P_{1,2}^k = 0.6$, the unfavorable bunches are, in effect, being assigned smaller charge values than the favorable bunches and their effects are hence being accounted for in a more correct manner.

The trajectories of Figs. 6.4 and 6.5 agree physically with the magnetron cutoff condition. It can be seen that the favorable HKB trajectories for operation below cutoff ($\omega_c/\omega = 0.5$) reach the r-f circuit in the shortest distances, while those for operation above cutoff ($\omega_c/\omega = 1.5$) reach the r-f circuit in the longest distance and the trajectories for operation at cutoff ($\omega_c/\omega = 1.0$) have collection distances between these values. This result is due to the weaker magnetic field for smaller values of ω_c/ω , which increases the total force on an electron directed toward the r-f structure.

Figure 6.6 indicates the variation in the gain of $A_{rf}(q)/A_{rf_0}$ of this amplifier with the d-c magnetic field as a parameter. In this figure the HKB approximate solutions for 40 bunch pairs are shown along

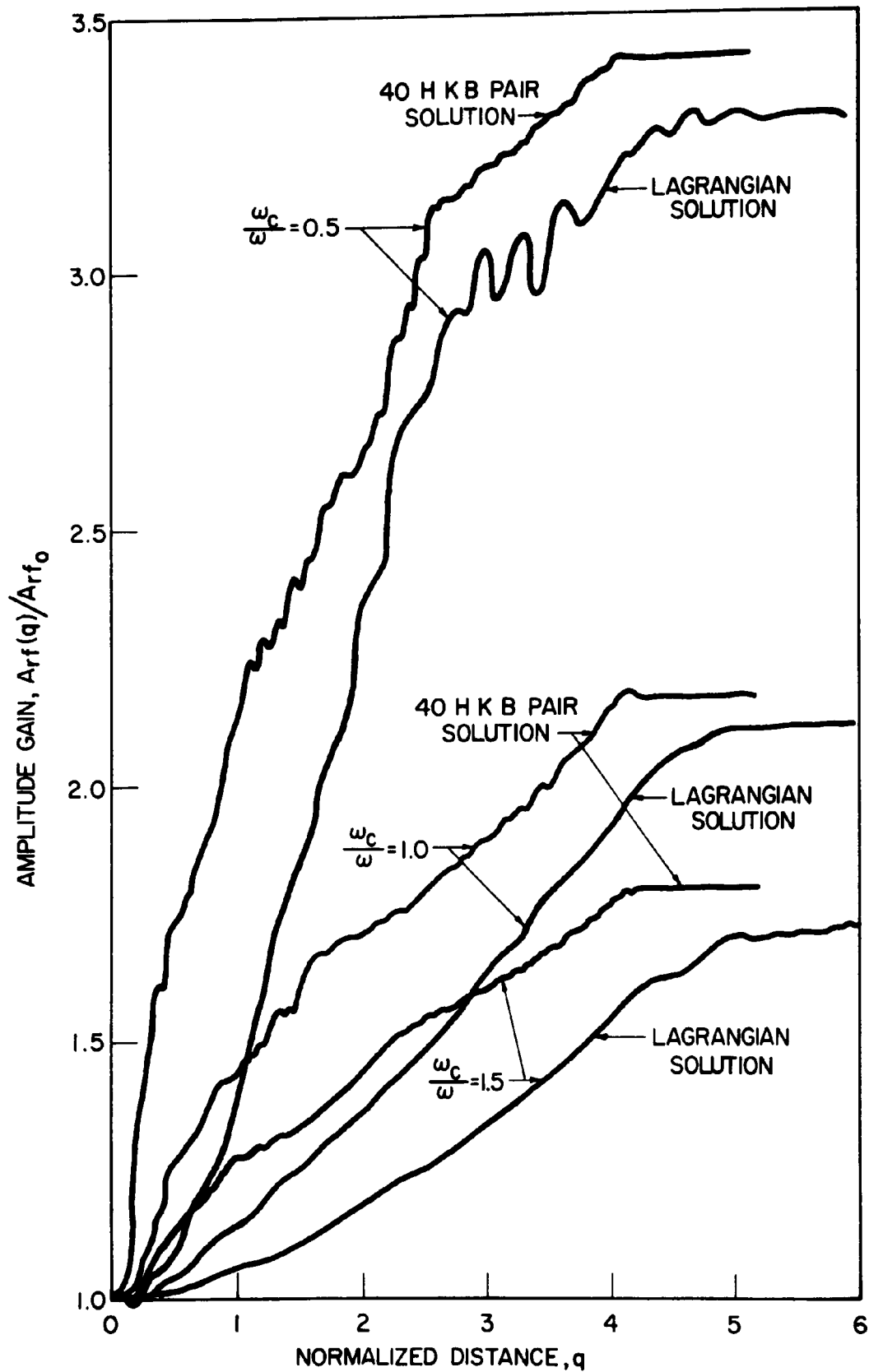


FIG. 6.6 AMPLITUDE GAIN OF THE R-F WAVE VS. LONGITUDINAL DISTANCE WITH THE D-C MAGNETIC FIELD AS A PARAMETER. ($D = 0.1$, $A_{rf0}/A_{dc} = 1$, $\beta_0 b' = 2$, $\beta_0 L_e = 40$)

with the Lagrangian solutions of Chapter V in order to compare the results. It is observed that this bunch-pair model is in very good agreement with the digital computer solutions of Chapter V. Again it is seen that the d-c field conditions for operation below cutoff ($\omega_c/\omega = 0.5$) yield the best gain and growth rates, while for larger values of ω_c/ω the gain and growth rates are reduced. The physical significance of this result, as stated more completely in Chapter V, is that for a smaller value of d-c magnetic field the electrons are able to interact more strongly with the r-f fields because they are closer, on the average, to the r-f structure where the r-f fields are stronger.

Figure 6.6 also shows that the initial amplitude growth rates obtained using the HKB approximation are larger than those obtained using the difference-form equations. This is because the first HKB pair is injected at the left-hand end of the emitting sole ($q = 0$) and in the actual device bunching does not occur until the wave has interacted with the electrons for a finite (nonzero) distance.

The rippling effect seen in the r-f amplitude figures is due to the abrupt collection of favorable bunches on the r-f circuit. A similar variation was also observed in Chapters IV and V and was also due to collection of the electrons.

Although this approximate analysis is very hypothetical, it lends physical understanding to the interaction process taking place. Furthermore, it generally agrees with the computer solutions obtained in the previous chapters.

CHAPTER VII. SUMMARY, CONCLUSIONS AND SUGGESTIONS FOR FURTHER RESEARCH

7.1 Summary and Conclusions

A generalized two-dimensional electron-wave interaction theory has been developed in the previous chapters for the crossed-field emitting-sole device. The basic equations used in the analysis were (1) the Lorentz equations of motion, (2) the one-dimensional equivalent transmission-line equation, (3) Poisson's equation, and (4) the continuity of charge equation. The effects of d-c, r-f and space-charge forces were taken into account and also secondary emission from the sole electrode was included in the theory.

The space-charge fields were calculated from the three-dimensional Green's function for a rectangular geometry. Simplified two-dimensional integrals were then obtained for the space-charge fields by defining weighting functions and averaging over the magnetic field direction. The final integral expressions were then obtained by using a transformation from final electron coordinates to initial electron coordinates.

In order to investigate the effect of the electrons upon the r-f wave, it was necessary to obtain an expression for the induced charge on the r-f slow-wave structure. From induced current considerations, this charge was obtained in integral form and the integrals were called the circuit integrals. These integrals are essentially averages over the total charge in the interaction region. The same transformation as that used in the space-charge integrals was also used to obtain the final induced-charge expressions. -

The nonlinear integro-differential equations were then derived from the basic equations using the integral expressions for the

space-charge fields and the induced circuit charge. These equations were solved by numerical methods assuming that N electrons were emitted into the interaction region. The problem then consisted of solving $4N+2$ equations. Numerical solutions were obtained for different values of the gain parameter D and the magnetic field parameter ω_c/ω for cases with and without secondary emission and also with and without space charge. The numerical solutions were presented in graphical form and the analysis of the results allowed various conclusions to be reached. These will now be reiterated.

First of all, it was noticed for the zero space-charge cases that the bunching process and the r-f amplitude gain were dependent upon the initial r-f voltage amplitude. For low input r-f signal levels, very little bunching occurred and no gain was obtained. In fact, for an initial r-f amplitude equal to one fourth the d-c amplitude, the r-f voltage amplitude was attenuated with distance and actually decreased from that at the input. It was therefore concluded that a minimum initial r-f voltage amplitude is necessary in order for gain to occur at all. This is because large r-f fields are needed to bunch the electrons and create an effective electron-wave interaction in the crossed-field emitting-sole device.

It was also demonstrated that when secondary electron emission (with a secondary emission ratio of one) is included, the gain of the amplifier is increased above that obtained without secondaries. This indicates that the unfavorably phased electrons, those which are accelerated by the r-f wave and collected on the sole, will eventually produce favorably phased secondary electrons which convert their potential energy to the r-f wave and therefore enhance the gain.

The effect of including the space-charge fields was a further increase in the gain of the amplifier. The electron trajectories with space-charge forces included also showed better bunching characteristics. Thus, it was concluded that the space-charge forces improve the phase focusing of the electrons which results in greater energy conversion to the r-f wave.

Upon comparing the solutions for the values of constant d-c magnetic field above, below and at the d-c magnetron cutoff value, it was found that below cutoff operation resulted in the largest r-f growth rates and saturation gain, and also the best bunching properties. Furthermore, using smaller values of constant d-c magnetic field was shown to be equivalent to using larger d-c anode potentials. Since the electrons under these conditions are able to approach the circuit more closely, it was concluded that the higher gain could be attributed to a greater coupling of electrons to the circuit.

In an endeavor to optimize the gain of the amplifier a linear decrease in the d-c magnetic field with longitudinal distance was introduced into the nonlinear equations. The purpose of this taper in magnetic field was to counteract the effects of space-charge potential depression and so keep the average electron drift velocity approximately equal to the unperturbed wave velocity. This is the synchronous condition under which greatest electron-wave interaction takes place. By using a linear magnetic field taper over either the partial or entire lengths of the tubes, an enhancement in the gain of the amplifier was achieved. A similar enhancement in the bunching properties was again obtained for the cases of higher gain. It was also noticed that in addition to allowing the operation to be more nearly synchronous, the

application of a magnetic field taper also forced the electron trajectories to approach the r-f structure more closely which resulted in a greater interaction.

Solutions for two values of the gain parameter D were obtained for d-c operation above and below the magnetron cutoff condition. However, this change had little effect on the gain and trajectory characteristics, and further increases in this parameter were prohibited by the numerical method of solution.

Theoretical solutions for a multiple-cathode amplifier were also obtained from the integro-differential equations by altering the electron emission model. This emission model consisted of uniform emission of electrons from parts of the cathode and no emission from other parts. Four emission portions were assumed to be equidistantly spaced along the sole electrode. The solutions indicated that r-f amplitude gain is achieved by this amplifier; however, the gain was less than that obtained for the emitting-sole amplifier. This result was due to a lower total current value than that obtainable in the emitting-sole amplifier, which leads to a weaker electron-wave interaction.

Finally, approximate solutions of the nonlinear integro-differential equations were presented. Under the assumptions of small interaction parameter D and negligible r-f space-charge forces, the equations were considerably simplified. Furthermore, by assuming that the electronic charge in the interaction region coalesces into hard-kernel bunches, closed-form solutions were obtained for the bunch trajectories and the amplifier gain. Inspection of the approximate trajectory solutions indicated that the favorably phased ($\Phi = \pi/2$)

electron bunches are decelerated and approach the r-f circuit, while the unfavorably phased ($\Phi = -\pi/2$) electron bunches are accelerated and approach the sole. At the same time an approximate solution for the r-f voltage amplitude showed a significant increase with longitudinal distance. These results imply the conversion of the potential energy of the bunches into the r-f energy of the wave.

The approximate solutions yielded very good agreement with the numerical solutions obtained with a digital computer, and verified the results which predicted larger amplification for d-c magnetic field values below the cutoff value. It was also found that an increase in the amount of injected charge per wavelength of the r-f wave increases the gain of the amplifier. This increase in charge could be obtained from a cathode with a higher secondary emission ratio or by simply increasing the temperature of the cathode.

In conclusion, the HKB analysis provided a quick method of solution and was found to be helpful in gaining better physical understanding of the interaction process.

7.2 Experimental Results

Experimental research on the crossed-field emitting-sole amplifier has recently been performed by Hess³⁹. Although this work is still incomplete the results thus far appear to be very promising. The following list is a summary of the results of this experimental work:

1. For a power input of 2.5 kw a net power gain of 18 db was achieved for an r-f frequency of 8.3 GHz, and more than 13 db net gain was realized over the frequency range of 8.3 to 8.8 GHz.

2. For a power input of 20 kw, a net power gain of 9 db was achieved for an r-f frequency of 8.3 GHz, and more than 7 db net gain was realized over the frequency range of 8.3 to 8.8 GHz.

3. For values of ω_c/ω between 1.25 and 1.6 the power gain was observed to increase slightly (approximately 2 db) with an increase in ω_c/ω which is in opposition to the predictions of this theory. The increased gain observed when ω_c/ω is increased beyond one may stem from a nonuniformity in the electron motion not considered in the present theory.

7.3 Suggestions for Further Research

Since the application of a linear decrease in the d-c magnetic field in the longitudinal direction was shown to improve the gain of the emitting-sole amplifier, it would be of considerable interest to obtain additional solutions for many different types of magnetic field tapers. Linear quadratic and higher order magnetic field variations can easily be incorporated into the computer analysis.

It would also be of interest to include magnetic field variations in the HKB analysis. However, an extension of the analysis presented herein would be necessary, since the phase position of the hard-kernel bunches would now vary with longitudinal distance. Thus, solutions could be obtained by using the expanded HKB method to find an optimum taper and then using the computer method to improve the accuracy of the solution with the inclusion of space-charge fields.

In a further attempt to enhance the gain, the application of a magnetic field variation in the vertical direction could also be investigated. This could consist of a stronger d-c magnetic field

closer to the emitting sole which would result in the removal of unfavorably phased electrons in shorter distances. Also, more secondary electrons could then be produced and the possibility of a greater energy transfer to the r-f wave could conceivably result.

The combination of injected-beam emission with emitting-sole emission in a single device could also be studied by using this analysis. This might also bear fruitful results in that the higher gain characteristic of the injected-beam amplifier and the greater availability of current from the emitting-sole amplifier could understandably be combined into one amplifier by using both types of emission. Along these same lines, the reentrant-beam tubes could also be analyzed (ever though this analysis has considered a linear geometry) by injecting electrons at the r-f input with velocities and coordinates identical to those for electrons at the r-f input. This type of analysis would be valid for $L \gg b'$. Furthermore, with the inclusion of a reentrant beam, a study of backward-wave devices, such as the amplitron and stabilotron, could also be carried out.

APPENDIX A. CHARGE DENSITY AT THE SOLE ELECTRODE

To obtain a closed-form expression for the charge density $\rho(y,z,t)$ in the interaction region, taking the d-c, r-f and space-charge fields into account, requires the solution of an extremely complex system of equations. A somewhat simplified approach to the problem starts from a statement of the conservation of charge for the interaction region. This is given symbolically by

$$\rho(y,z,t) |dydz| = \sum \rho_0(y_0, z_0, t_0) |dy_0 dz_0| , \quad (A.1)$$

where $\rho(y,z,t)$ is the charge density at position (y,z) and time t which emanated from the elements of area $|dy_0 dz_0|$ along the sole at a different time t_0 . Assuming that the charge density along the emitting sole is the same function of y_0 and z_0 and independent of time, Eq. A.1 may be rewritten as

$$\rho(y,z,t) |dydz| = \rho_0(y_0, z_0) \sum |dy_0 dz_0| . \quad (A.2)$$

A closed-form expression for $\rho(y,z,t)$ is now no longer necessary. Instead, only the d-c charge density $\rho_0(y_0, z_0)$ at the input plane (the emitting sole) need be obtained. This will now be carried out using a density function method.

The density of electrons in six-dimensional phase space (three-dimensional position and velocity spaces combined),

$$(\vec{x}, \vec{v}) = (x, y, z, v_x, v_y, v_z) , \quad (A.3)$$

is given by

$$F(\vec{x}, \vec{v}) = N(m/2kT)^{3/2} \exp(-\mathcal{E}/kT) , \quad (A.4)$$

$$\text{where } \mathcal{E} = (m/2) |\vec{v}|^2 - |e| \bar{V}(x,y,z), \quad (\text{A.5})$$

k = Boltzmann's constant,

N = total number of electrons per cubic meter at the cathode surface,

m = mass of an electron,

T = temperature of the cathode

and where $\bar{V}$ is the potential acting on an electron.

Equation A.4 is the well-known distribution function of Maxwell-Boltzmann statistics³³, where physically F represents the expected value of the number of electrons per (meter)³ and per (meter/sec)³.

Since the concentrations of particles N have not been tabulated as thoroughly as the emission current densities J_{th} have been for various cathode materials and temperatures, N will be eliminated in favor of J_{th} .

The electrons leaving the surface of an emitter (in the x-z plane) produce an element of current on this cathode of density

$$|dJ_{th}| = e v_y dN. \quad (\text{A.6})$$

By integration, the average current density of emission from the cathode in the y-direction is given by

$$\begin{aligned} J_{th} &= \int_{-\infty}^{\infty} dv_x \int_{-\infty}^{\infty} dv_z \int_0^{\infty} dv_y v_y eN \left(\frac{m}{2\pi kT} \right)^{3/2} \exp \frac{-m}{2kT} (v_x^2 + v_y^2 + v_z^2) \\ &= \frac{1}{2} eN \left(\frac{2kT}{\pi m} \right)^{1/2}. \end{aligned}$$

Solving for N yields

$$N = \frac{2J_{th}}{e} \left(\frac{\pi m}{2kT} \right)^{1/2}. \quad (\text{A.7})$$

The distribution function for electrons emitted from an infinite plane surface into a constant magnetic field in the x-direction and a general potential distribution (electric field in all directions) can therefore be written as

$$F = N' \exp \left[\frac{eV(x,y,z)}{kT} - \frac{m}{2kT} (v_x^2 + v_y^2 + v_z^2) \right] , \quad (A.8)$$

where

$$N' = 2 \left(\frac{m}{2kT} \right)^2 \frac{J_{th}}{\pi e} . \quad (A.9)$$

The function F , appearing in Eq. A.8, is the distribution function for an infinite plane emitting surface. The charge density (coulombs per cubic meter) for an emitting-sole device may now be calculated by integrating this function over the existing rectangular velocities. The problem that now arises however is that all velocities are not possible for the electrons in an emitting-sole device, i.e., the electrons will be obstructed by the cathode and anode, and in order to obtain the charge density the proper velocity limits must be used in the integration. The velocity limits are obtained by performing one integration of the equations of motion.

To obtain the limits of integration, it will be assumed that the space-charge fields are negligible. The velocity limits are then obtainable from the Lorentz equations by performing one integration. Lindsay³⁶ has carried out this analysis for the same parallel plane crossed-field geometry. His results yield the following equations for the boundary limits of the electron velocities when the potential is only a function of y :

$$v_y^2 = (\omega_c b' - \omega_c y)(\omega_c b' - \omega_c y + 2v_z) + 2|\eta|[V(y) - V(b')] \quad (A.10)$$

and

$$v_y^2 = \omega_c y (\omega_c y - 2v_z) + 2|\eta| [V(y) - V(0)] \quad . \quad (A.11)$$

The solutions of Eqs. A.10 and A.11 will be denoted as $v_I(y)$ and $v_{II}(y)$, respectively. These solutions are plotted in Fig. A.1 in the (v_y, v_z) plane for a particular value of y . This figure indicates the regions in which electrons may exist and those which are not allowed due to the obstructions caused by the electrodes. The intersections with the v_y - and v_z -axes are given by

$$a_I = \{ \omega_c^2 (b' - y)^2 + 2\eta [V(y) - V(b')] \}^{1/2} \quad ,$$

$$a_{II} = \{ (\omega_c y)^2 + 2\eta [V(y) - V(0)] \}^{1/2} \quad ,$$

$$b_I = \frac{-\eta [V(y) - V(b')]}{\omega_c (b' - y)} - \frac{\omega_c (b' - y)}{2}$$

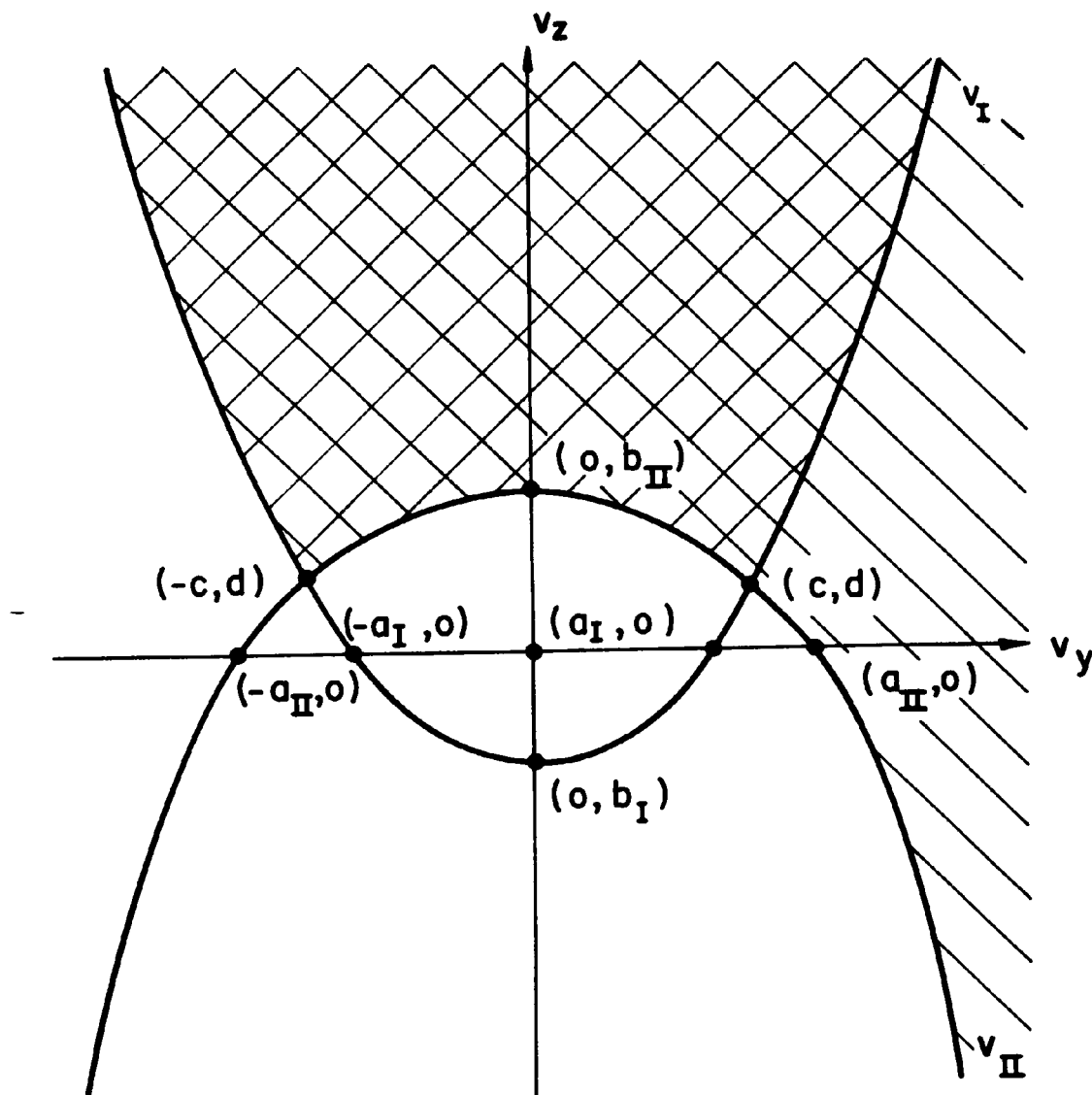
and

$$b_{II} = \frac{\eta [V(y) - V(0)]}{\omega_c y} + \frac{\omega_c y}{2} \quad .$$

The two-dimensional charge distribution between the two parallel electrodes will now be obtained by integrating the distribution function over the allowed electron velocities. To include the secondary emission electrons in this calculation, which Lindsay has omitted, the charge density is written as the sum of two integrals as follows:

$$\rho(y) = \rho_1(y) + \rho_2(y)$$

or



ALL REGIONS OF POSSIBLE ELECTRON ORBITS.



THE REGION IN WHICH ELECTRONS RETURN TO THE CATHODE.



THE REGION OF UNREALIZABLE ELECTRON ORBITS.

FIG. A.1 SOLUTIONS OF THE LIMITING TRAJECTORY EQUATIONS ILLUSTRATING THE REGIONS OF REALIZABLE ELECTRON VELOCITIES AT A PARTICULAR y -PLANE.

$$\rho(y) = -|e| \int_{\vec{v} \text{ allowable}} \int \int \int dv_x dv_y dv_z F(y, \vec{v}) - |e| \delta \int_{\vec{v} \text{ returning}} \int \int \int dv_x dv_y dv_z F(y, \vec{v}). \quad (\text{A.12})$$

The first integral is an integration of the density function over all of the allowable electron velocities between the two electrodes (represented by all of the shaded regions in Fig. A.1). This is the primary electron contribution to the total charge density. The second integral is an integration over only those electrons which return to the cathode (illustrated by the double cross-hatched region in Fig. A.1). This integral represents the secondary electron charge density when it is multiplied by the secondary emission ratio δ . Writing ρ_1 and ρ_2 separately with the correct velocity limits yields

$$\rho_1 = -|e| \int_{-\infty}^{\infty} dv_x \left(\int_{-\infty}^d dv_z \int_{v_{IIy}}^{\infty} dv_y F + \int_d^{\infty} dv_z \int_{-v_{Iy}}^{\infty} dv_y F - 2 \int_d^{b_{II}} dv_z \int_0^{v_{IIy}} dv_y F \right) \quad (\text{A.13})$$

and

$$\rho_2 = -|e| \delta \int_{-\infty}^{\infty} dv_x \left(\int_d^{\infty} dv_z \int_{-v_{Iy}}^{v_{Iy}} dv_y F - 2 \int_d^{b_{II}} dv_z \int_0^{v_{IIy}} dv_y F \right), \quad (\text{A.14})$$

where

$$d \triangleq (\eta/\omega_c b') [V(b') - V(0)] + \omega_c [y - (b'/2)] .$$

Substituting the distribution function F , integrating over v_x and v_y (using the transformation $\vec{u} = (m/2kT)^{1/2} \vec{v}$) and combining ρ_1 and ρ_2 yields

$$\begin{aligned} \rho(y) = & A(y) \left[\sqrt{\pi} - \int_{-\infty}^{d/v_c} du_z \exp\left(-u_z^2\right) \operatorname{erf}\left(v_{II_y}/v_c\right) + (1 + 2\delta) \right. \\ & \cdot \int_{d/v_c}^{\infty} du_z \exp\left(-u_z^2\right) \operatorname{erf}\left(v_{I_y}/v_c\right) - 2(1 + \delta) \int_{d/v_c}^{b_{II}/v_c} du_z \\ & \left. \cdot \exp\left(-u_z^2\right) \operatorname{erf}\left(v_{II_y}/v_c\right) \right] , \quad (\text{A.15}) \end{aligned}$$

where the following definitions have been made:

$$\begin{aligned} v_c & \triangleq (2kT/m)^{1/2}, \text{ the characteristic velocity,} \\ A(y) & \triangleq - (J_{th}/v_c) \exp[eV(y)/kT] \text{ and} \\ \operatorname{erf}(x) & \triangleq 2/\sqrt{\pi} \int_0^x \exp(-r^2) dr. \end{aligned}$$

To obtain the charge density at the input plane (the sole is the input plane), y is set equal to zero in Eq. A.15, yielding

$$\begin{aligned} \rho_o = & - \frac{J_{th}}{v_c} \left[\sqrt{\pi} + (1 + 2\delta) \int_{d(0)/v_c}^{\infty} du_z \exp\left(-u_z^2\right) \right. \\ & \left. \cdot \operatorname{erf}\left(\sqrt{\omega_c b' (\omega_c b' + 2v_c u_z)} - 2|\eta|V(b')/v_c\right) \right] , \quad (\text{A.16}) \end{aligned}$$

where $V(0)$ has been assumed to be zero. Equation A.16 can now be written in terms of two dimensionless charge densities as follows:

$$\rho_o = - \frac{J_{th}}{v_c} (\rho_o' + \rho_o'' + 2\delta\rho_o'') \quad . \quad (A.17)$$

The definition of ρ_o' and ρ_o'' can be seen by comparing Eq. A.16 with A.17. The first term in this expression represents the charge density due to thermionic emission, the second term is the additional charge density due to electrons which have been returned to the sole by the magnetic field, and the last term is the additional charge density produced by secondary electrons. Equation A.17 is the final form for the input plane charge density.

The above expression represents an average one-dimensional result and therefore does not account for end effects. However, in the digital computer program where it has been assumed that only a finite number of charge groups are present, end effects are definitely important. This can be seen by noting that the actual starting point of the computer program calculations is at one end of the sole electrode. Thus, the above value of ρ_o at this position will be too large, since there have not yet been any electrons returned to the sole and, consequently, there are also no secondaries. Hence, in the computer program, the input plane charge density will be represented by only the first term in the above expression, and secondary emission will be accounted for in a different manner. This is discussed in Chapter III.

It is interesting to observe the results of carrying out the indicated integration in Eq. A.16. Figure A.2 indicates the variation in ρ_o'' for variations in the d-c potential, the d-c magnetic field and the cathode temperature. The velocity parameters appearing in the figure have the following definitions:

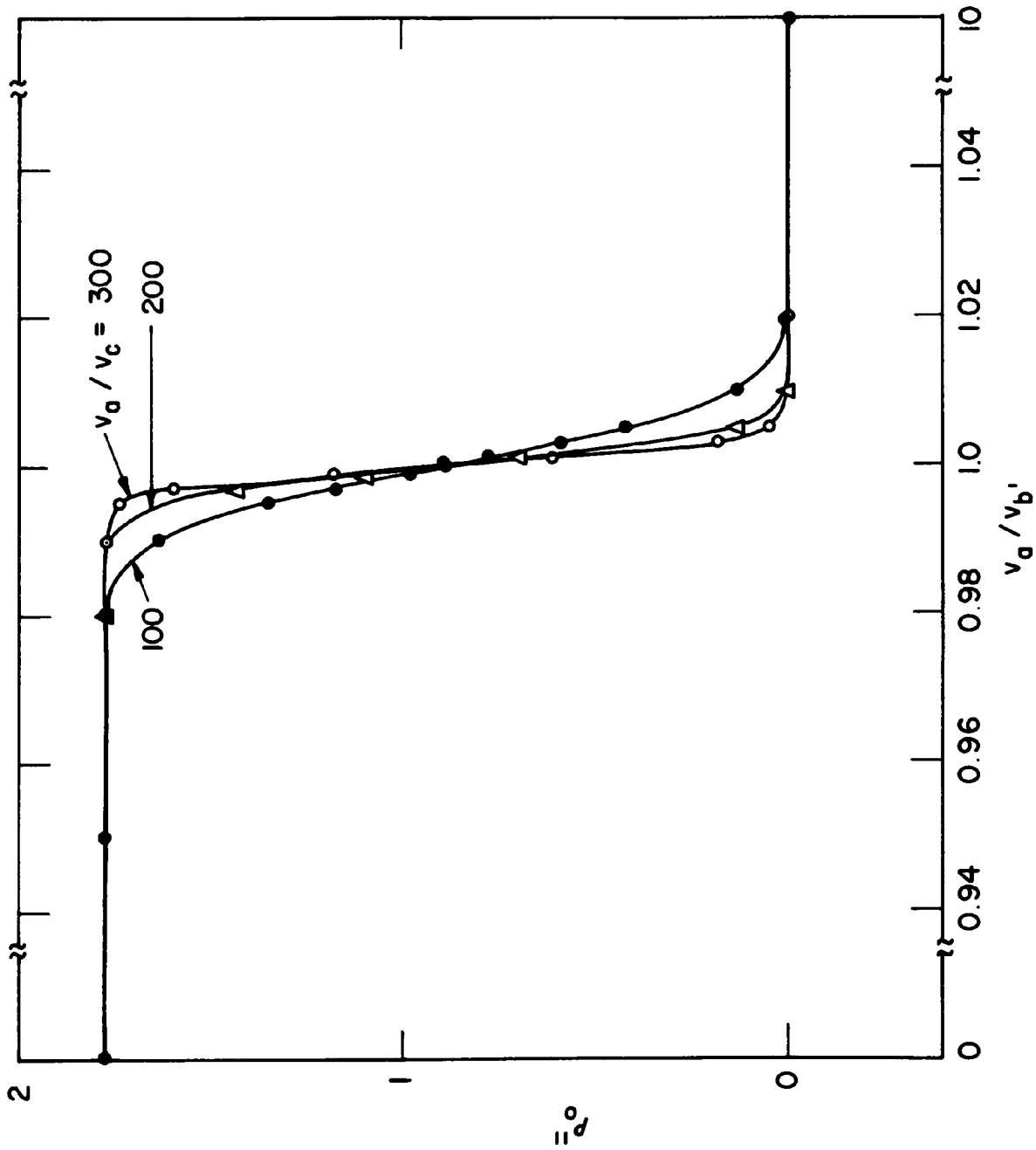


FIG. A.2 VARIATION OF NORMALIZED CHARGE DENSITY WITH D-C POTENTIAL,
MAGNETIC FIELD AND CATHODE TEMPERATURE.

$$v_a \triangleq [2|\eta|V(b')]^{1/2} ,$$

$$v_b' \triangleq \omega_c b' ,$$

and

$$v_c \triangleq (2kT/m)^{1/2} .$$

It is to be noted that all of the curves have an approximate step function variation; that is, ρ_o'' is approximately either $\sqrt{\pi}$ or zero. The significance of this variation may be seen from the cutoff condition for a linear magnetron.

The cutoff condition is derived from the Lorentz force equations for crossed d-c fields in a two-dimensional parallel plane geometry. The pertinent equations, given by

$$\frac{d^2y}{dt^2} = \frac{|\eta|V_a}{b'} - \omega_c \frac{dz}{dt} \quad (A.18)$$

and

$$\frac{d^2z}{dt^2} = \omega_c \frac{dy}{dt} , \quad (A.19)$$

are integrated once and used simultaneously to eliminate dz/dt , yielding

$$\left(\frac{dy}{dt}\right)^2 = \frac{2|\eta|V_a}{b'} y - \omega_c^2 y^2 , \quad (A.20)$$

where initial velocities at the cathode have been assumed to be zero.

The y-velocity is now set equal to zero at $y = b'$, yielding the cutoff relation

$$\omega_c b' = \sqrt{2|\eta|V_a} , \quad (A.21)$$

which is the condition for an electron to just graze the anode. For smaller values of ω_c , the magnetic field is said to be below the cutoff value.

Thus to explain the step function variation of ρ_0'' , it can be seen in Eq. A.17 that ρ_0'' is multiplied by the secondary emission ratio δ and when ρ_0'' is zero the contribution to the total charge density due to secondary emission must also be zero. From Fig. A.2 the condition for ρ_0'' to vanish is given by

$$v_a > v_b', \quad (\text{A.22})$$

or

$$\sqrt{2|\eta|V_a} > \omega_c b', \quad (\text{A.23})$$

which is the operating condition for a linear magnetron below cutoff. The fact that there is no contribution from secondary emission when this inequality holds is physically quite logical, because under these conditions of d-c potential and magnetic field, there are no electrons returning to the cathode; i.e., all electrons are collected on the anode. The charge density at the sole in this case consists of newly emitted electrons only.

When the inequality sign in Eq. A.23 is reversed, the inequality becomes

$$\sqrt{2|\eta|V_a} < \omega_c b', \quad (\text{A.24})$$

which represents operation above cutoff such that none of the electrons can reach the anode. In this case either the electrons return to the cathode or they drift in the longitudinal direction until they reach the collector at the end of the structure.

APPENDIX B. SPACE-CHARGE-FIELD EXPRESSIONS

The space-charge potential $V_{sc}(\vec{r})$ at any point $\vec{r} = (x, y, z)$ in a region of space must satisfy Poisson's equation

$$\nabla^2 V_{sc} = -\rho/\epsilon_0, \quad (B.1)$$

where ρ is the space-charge density, and ϵ_0 is the dielectric constant for free space. The solution of Eq. B.1 can be written in integral form by employing a Green's function as outlined by Rowe²⁷. The integral solution is given by

$$V_{sc}(\vec{r}) = \iiint \rho(\vec{r}') G(\vec{r}, \vec{r}') d\vec{r}', \quad (B.2)$$

where $V_{sc}(\vec{r})$ is the space-charge potential, and the integration extends over all regions containing space charge. The Green's function $G(\vec{r}, \vec{r}')$ or weighting function must satisfy

$$\nabla^2 G(\vec{r}, \vec{r}') = -\frac{\delta(\vec{r} - \vec{r}')}{\epsilon_0}, \quad (B.3)$$

along with appropriate boundary conditions. The general solution of Eq. B.3, which also satisfies the boundary conditions for the rectangular geometry ($V = 0$ at $x = 0, a$, and $V = 0$ at $y = 0, b'$) is given by

$$G(\vec{r}, \vec{r}') = \sum_{k=0}^{\infty} \sum_{m=0}^{\infty} A_{mk} \sin \frac{k\pi x}{a} \sin \frac{m\pi y}{b'} \cdot \exp \left[\frac{-\pi}{ab'} (m^2 a^2 + k^2 b'^2)^{1/2} |z - z'| \right] \quad (B.4)$$

The general A_{mk} constants are found by applying Gauss' theorem to a unit charge located at the point $\vec{r} = \vec{r}'$, yielding

$$G(\vec{r}, \vec{r}') = \frac{2}{\pi \epsilon_0} \sum_{k=0}^{\infty} \sum_{m=0}^{\infty} (m^2 a^2 + k^2 b'^2)^{-1/2} \sin \frac{k\pi x}{a} \sin \frac{k\pi x'}{a} \sin \frac{m\pi y'}{b'} \cdot \sin \frac{m\pi y}{b'} \exp \left[\frac{-\pi}{ab'} (m^2 a^2 + k^2 b'^2) |z - z'| \right] \quad (B.5)$$

By superposing these solutions, the solution of Poisson's equation for the general charge distribution $\rho(\vec{r})$ with rectangular geometry can be written as

$$V_{s-c}(\vec{r}) = \iiint \rho(\vec{r}') \left[\frac{2}{\pi \epsilon_0} \sum_{k=0}^{\infty} \sum_{m=0}^{\infty} (m^2 a^2 + k^2 b'^2)^{-1/2} \sin \frac{k\pi x}{a} \sin \frac{k\pi x'}{a} \cdot \sin \frac{m\pi y}{b'} \sin \frac{m\pi y'}{b'} \exp \left(\frac{-\pi}{ab'} (m^2 a^2 + k^2 b'^2)^{1/2} |z - z'| \right) \right] d\vec{r}' \quad (B.6)$$

To enforce the two-dimensional assumption made in the text, x is set equal to $a/2$ (the average value of x), and integrating over x' from $(a-h)/2$ to $(a+h)/2$, where h is the emitter width, yields

$$V_{sc}(y, z) = \frac{4a}{\pi^2 \epsilon_0} \iint dy' dz' \rho(y', z') \sum_{k=0}^{\infty} \sum_{m=0}^{\infty} (m^2 a^2 + k^2 b'^2)^{-1/2} \cdot \frac{\sin^2 \frac{k\pi}{2}}{k} \sin \frac{k\pi h}{2a} \sin \frac{m\pi y}{b'} \sin \frac{m\pi y'}{b'} \exp \left[-\frac{\pi}{ab'} (m^2 a^2 + k^2 b'^2)^{1/2} |z - z'| \right] . \quad (B.7)$$

The space-charge fields are now obtained by differentiating Eq. B.7 with respect to the observation coordinates y and z . Thus, by partial differentiation, the y - and z -components of the space-charge fields are

$$E_{sc-y} = - \frac{\partial V_{sc}}{\partial y} = \frac{4}{\pi \epsilon_0 \beta_o^2 b' \frac{\omega_c}{\omega} D} \int_0^{2\pi} d\phi'_0 \int_0^q dq'_0 \left| \frac{v_{r\rho_0}}{1 + 2Du_z} \right| \sum_{k=0}^{\infty} \sum_{m=0}^{\infty} \frac{m}{k} \cdot \left[m^2 + \left(\frac{kb'}{a} \right)^2 \right]^{-1/2} \sin^2 \frac{k\pi}{2} \sin \frac{k\pi h}{2a} \cos m\pi p \sin m\pi p' \cdot \exp \left\{ \frac{-\pi}{\beta_o b'} \left[m^2 + \left(\frac{kb'}{a} \right)^2 \right]^{1/2} |\phi - \phi'| \right\} \quad (B.8)$$

and

$$\begin{aligned}
 E_{sc-z} &= - \frac{\partial V_{sc}}{\partial z} \\
 &= \frac{-4}{\pi \epsilon_0 \beta_o^2 b' \frac{\omega_c}{\omega} D} \int_0^{2\pi} d\phi'_o \int_0^q dq'_o \left| \frac{v_{r_o}}{1 + 2Du_z} \right| \left| \text{sgn}(\Phi - \Phi') \sum_{k=0}^{\infty} \sum_{m=0}^{\infty} \frac{1}{k} \right. \\
 &\quad \cdot \sin^2 \frac{k\pi}{2} \sin \frac{k\pi h}{2a} \sin m\pi p \sin m\pi p' \exp \left\{ \frac{-\pi}{\beta_{ob'}} \left[m^2 + \left(\frac{kb'}{a} \right)^2 \right]^{1/2} |\Phi - \Phi'| \right\} , \\
 &\quad (B.9)
 \end{aligned}$$

where $\text{sgn}(\Phi - \Phi') = 1$ for $\Phi > \Phi'$ and -1 for $\Phi < \Phi'$.

A transformation to initial coordinates has also been carried out in Eqs. B.8 and B.9, and it has been assumed that $D\theta'_c \ll 1$ so that $\beta \cong \beta_o$.

It will be convenient to define space-charge-weighting functions

as

$$\begin{aligned}
 F_y(p, p', \Phi, \Phi') &\triangleq \sum_{m=0}^{\infty} \sum_{k=0}^{\infty} \frac{m}{k} \left[m^2 + k^2 \left(\frac{b'}{a} \right)^2 \right]^{-1/2} \sin^2 \frac{k\pi}{2} \sin \frac{k\pi h}{a} \\
 &\quad \cdot \cos m\pi p \sin m\pi p' \exp \left\{ \frac{-\pi}{a} \left[m^2 \left(\frac{a}{b'} \right)^2 + k^2 \right]^{1/2} \left| \frac{\Phi - \Phi'}{\beta_o} \right| \right\} \quad (B.10)
 \end{aligned}$$

and

$$\begin{aligned}
 F_z(p, p', \Phi, \Phi') &\triangleq \sum_{m=0}^{\infty} \sum_{k=0}^{\infty} \frac{1}{k} \sin^2 \frac{k\pi}{2} \sin \frac{k\pi h}{a} \sin m\pi p \sin m\pi p' \\
 &\quad \cdot \exp \left\{ \frac{-\pi}{a} \left[m^2 \left(\frac{a}{b'} \right)^2 + k^2 \right]^{1/2} \left| \frac{\Phi - \Phi'}{\beta_o} \right| \right\} . \quad (B.11)
 \end{aligned}$$

The calculation of these functions appears in Appendix C. Thus the space-charge fields may be written as

$$E_{sc-y} = \frac{4}{\pi \epsilon_0 \beta_0 b' \frac{\omega}{c}} \int_0^{2\pi} d\Phi'_0 \int_0^q dq'_0 \frac{|v_r \rho_0|}{|1 + 2Du_z|} F_y(p, p', \Phi, \Phi') \quad (B.12)$$

and

$$E_{sc-z} = \frac{-4}{\pi \epsilon_0 \beta_0 b' \frac{\omega}{c}} \int_0^{2\pi} d\Phi'_0 \int_0^q dq'_0 \frac{|v_r \rho_0|}{|1 + 2Du_z|} F_z(p, p', \Phi, \Phi') \quad (B.13)$$

Substituting for ρ_0 in terms of the dimensionless charge density defined in Appendix A

$$\rho_0 = - \left| \frac{J_{th}}{v_c} \rho'_0 \right|, \quad (B.14)$$

and defining the integrals,

$$I_y(p, \Phi) = \int_0^{2\pi} d\Phi'_0 \int_0^q dq'_0 \frac{1}{|1 + 2Du'_z|} F_y(p, p', \Phi, \Phi') \quad (B.15)$$

and

$$I_z(p, \Phi) = \int_0^{2\pi} d\Phi'_0 \int_0^q dq'_0 \frac{\text{sgn}(\Phi - \Phi')}{|1 + 2Du_z|} F_z(p, p', \Phi, \Phi') \quad (B.16)$$

yields the following expressions for the space-charge fields:

$$E_{sc-y} = K_2 I_y \quad (B.17)$$

and

$$E_{sc-z} = - K_2 I_z, \quad (B.18)$$

where

$$K_2 \triangleq \frac{2v_o \omega_{pc}^2 |\rho_o'| v_r}{\pi \beta_o b' |\eta| \omega_c D}$$

and the frequency ω_{pc} has been defined as

$$\omega_{pc}^2 \triangleq \frac{2|\eta| J_{th}}{\epsilon_o v_c} . \quad (B.19)$$

This will be called the cathode plasma frequency and Eqs. B.17 and B.18 are used in the derivation of the force equations.

APPENDIX C. SPACE-CHARGE-WEIGHTING FUNCTIONS

In the derivation of the two large-signal force equations in Chapter II, space-charge forces were accounted for by solving Poisson's equation using a Green's function. This Green's function solution is presented in Appendix B and resulted in the representation of the space-charge forces as definite integrals over doubly infinite summations. These infinite summations were called the space-charge-weighting functions and were defined as follows:

$$F_y(p, p', \Phi, \Phi') = \sum_{m=1}^{\infty} \sum_{k=1}^{\infty} \left(\frac{a}{b'} \right) \left(\frac{m}{k} \right) \left[\left(\frac{a}{b'} m \right)^2 + k^2 \right]^{-1/2} \sin^2 \frac{k\pi}{2} \sin \frac{k\pi h}{a} \\ \cdot \cos m\pi p \sin m\pi p' \exp \left\{ \frac{-\pi |\Phi - \Phi'|}{\beta_0 a} \left[\left(\frac{a}{b'} m \right)^2 + k^2 \right]^{1/2} \right\} \quad (C.1)$$

and

$$F_z(p, p', \Phi, \Phi') = \sum_{m=1}^{\infty} \sum_{k=1}^{\infty} \frac{1}{k} \sin^2 \frac{k\pi}{2} \sin \frac{k\pi h}{a} \sin m\pi p \sin m\pi p' \\ \cdot \exp \left\{ \frac{-\pi}{\beta_0 a} |\Phi - \Phi'| \left[\left(\frac{a}{b'} m \right)^2 + k^2 \right]^{1/2} \right\} , \quad (C.2)$$

where $a \triangleq$ width of the interaction region in the magnetic field direction,

$h \triangleq$ width of the emitting sole,

$p \triangleq y/b'$, normalized vertical coordinate of the observed electron,

$p' \triangleq y'/b'$, normalized vertical coordinate of a source electron,

$\Phi \triangleq$ r-f phase of the observed electron,

$\Phi' \triangleq$ r-f phase of a source electron and

$\beta_o \triangleq \omega_o/v_o$, phase constant of the cold circuit wave.

These weighting functions can be computed rapidly by utilizing a digital computer. The results for a typical set of normalized dimensions, given by $\beta_o a = 8.0$, $\beta_o h = 6.4$ and $\beta_o b' = 2$, are shown in Figs. C.2 through C.13. In order to carry out the calculations using the data above, it was necessary to divide the interaction region into sections (following a procedure suggested by Rowe²⁷). Since the functions F_y and F_z do not vary in the magnetic field direction (the integration over x' has been carried out and the average value of $x(x = a/2)$ was used to replace x), it was only necessary to subdivide the normalized vertical distance p . It was decided that eight subdivisions would yield sufficient accuracy and these divisions are shown and numbered in Fig. C.1. Each charge group can now be represented by a p (or p') depending upon which section it is in and these particular values are substituted into the space-charge-weighting functions, thus enabling their calculation. The weighting functions shown in Figs. C.2 through C.13 are labeled according to the numbered subdivisions in Fig. C.1 which they represent.

The results shown in the curves indicate a number of interesting characteristics. It is seen that the magnitudes of both F_y and F_z are greatest for two elements, represented by values of p and p' , which are closer together than for two which are farther apart. This is in agreement with the Coulomb force law since F_y and F_z are multiplicative factors of the y - and z -directed space-charge forces. It can also be seen that for an element represented by a given p , F_y has an opposite

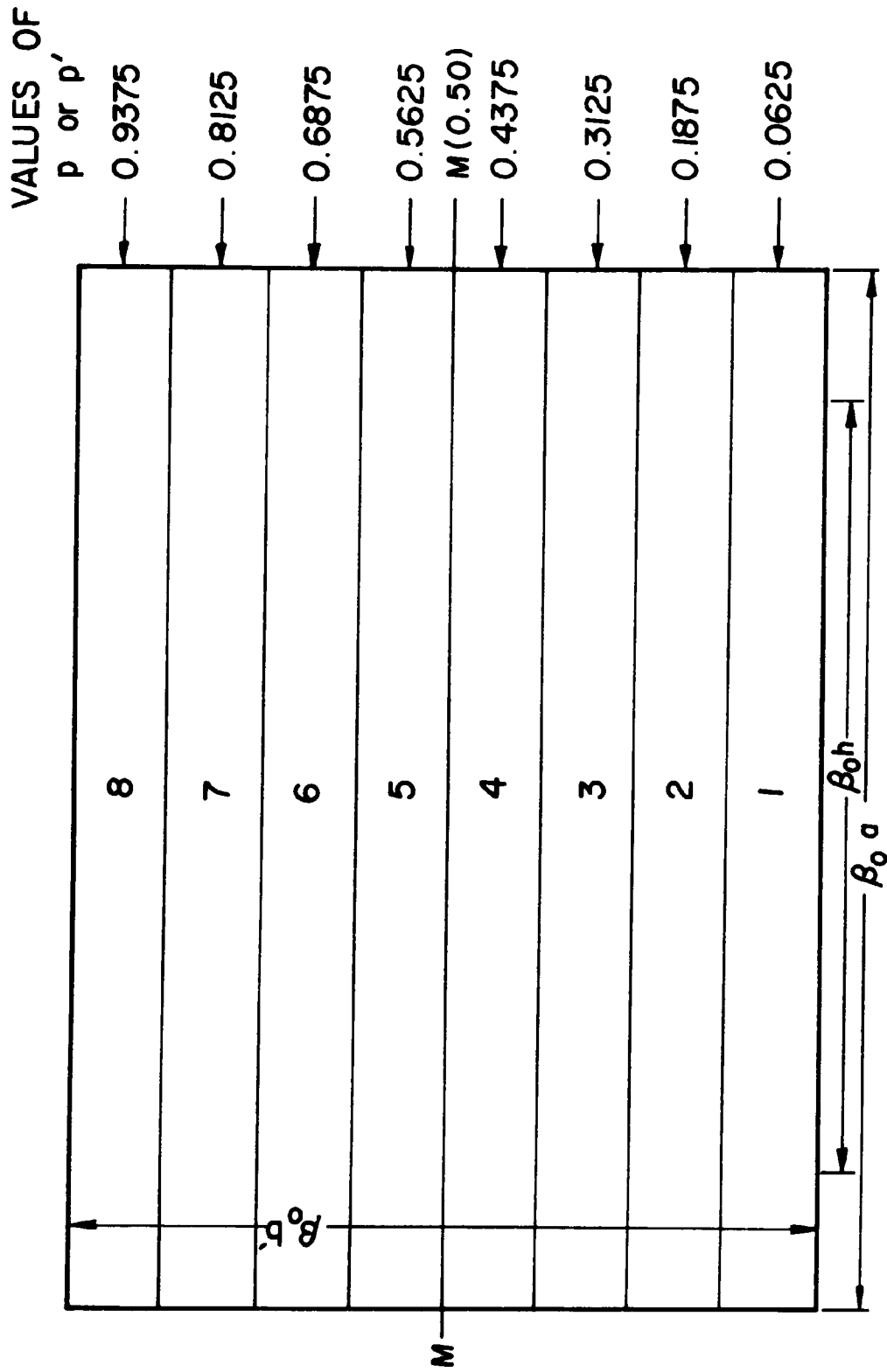


FIG. C.1 DIVISION OF INTERACTION SPACE FOR COMPUTATION OF THE SPACE-CHARGE-

WEIGHTING FUNCTIONS. ($\beta_0 b' = 2$, $\beta_0 a = 8$)

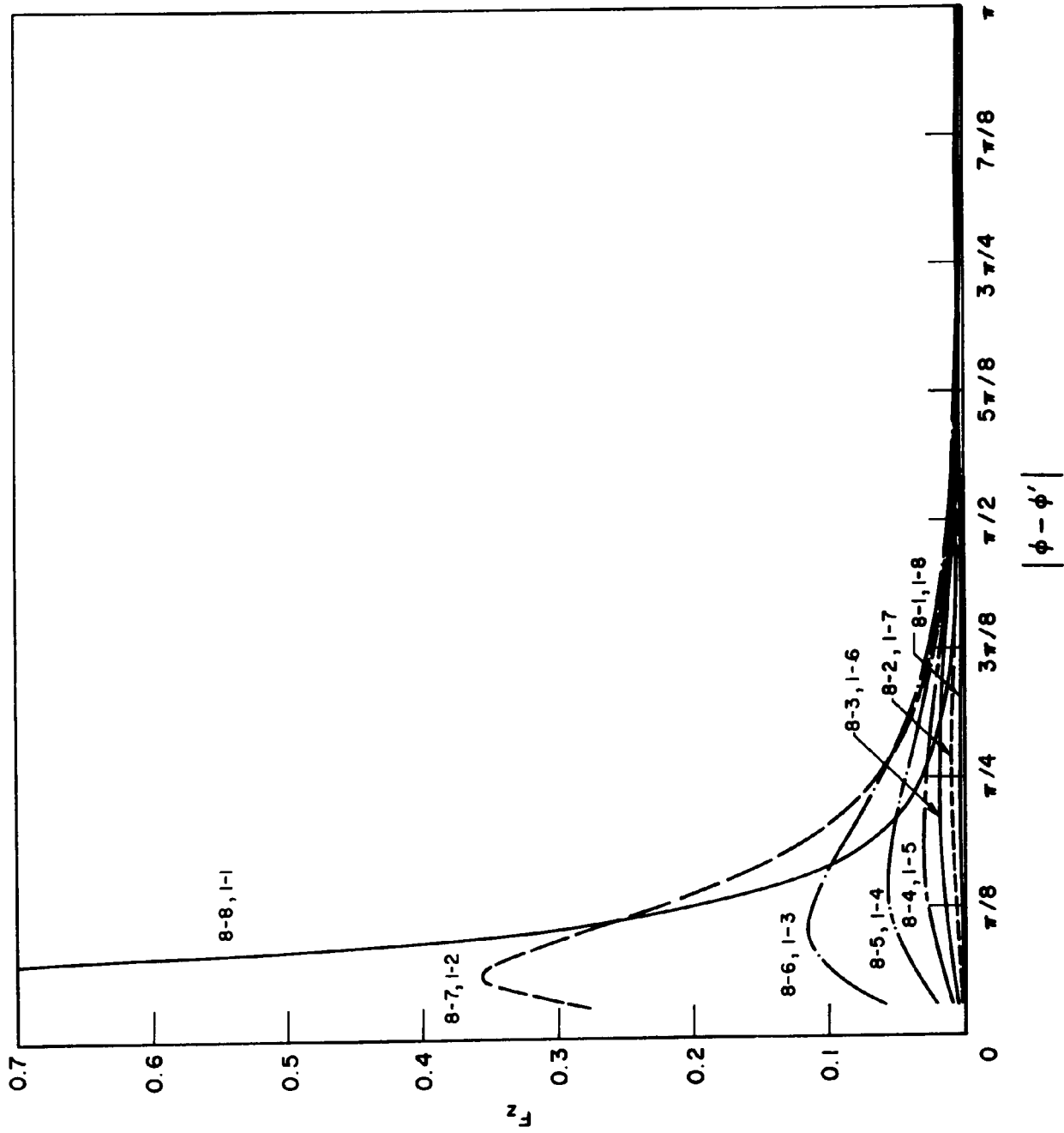


FIG. C.2 F_z SPACE-CHARGE-WEIGHTING FUNCTIONS. ($p = 0.0625, 0.9375$; p' VARIABLE)

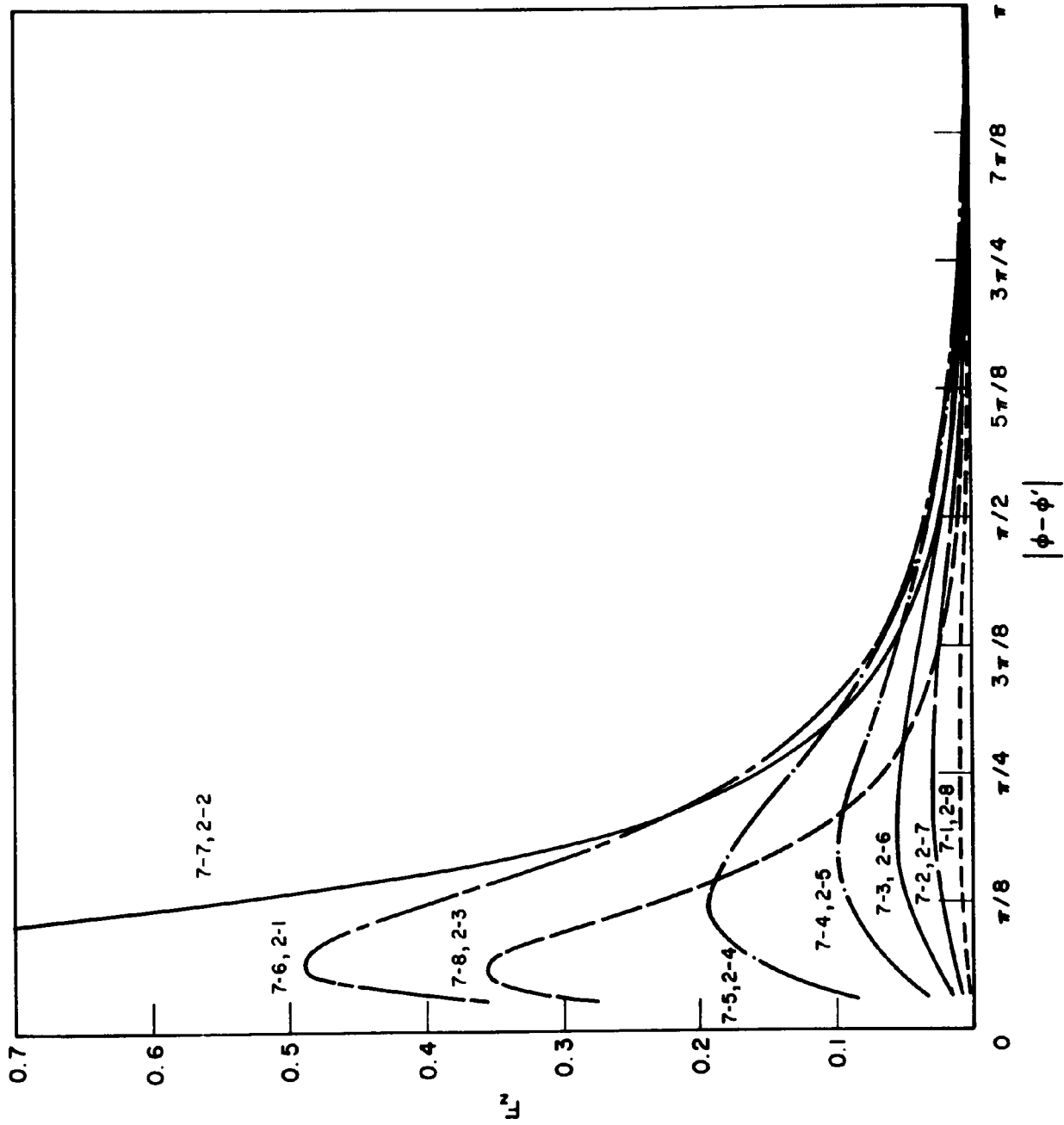


FIG. C.3 F_z SPACE-CHARGE-WEIGHTING FUNCTIONS. ($p = 0.1875, 0.8125$; p' VARIABLE)

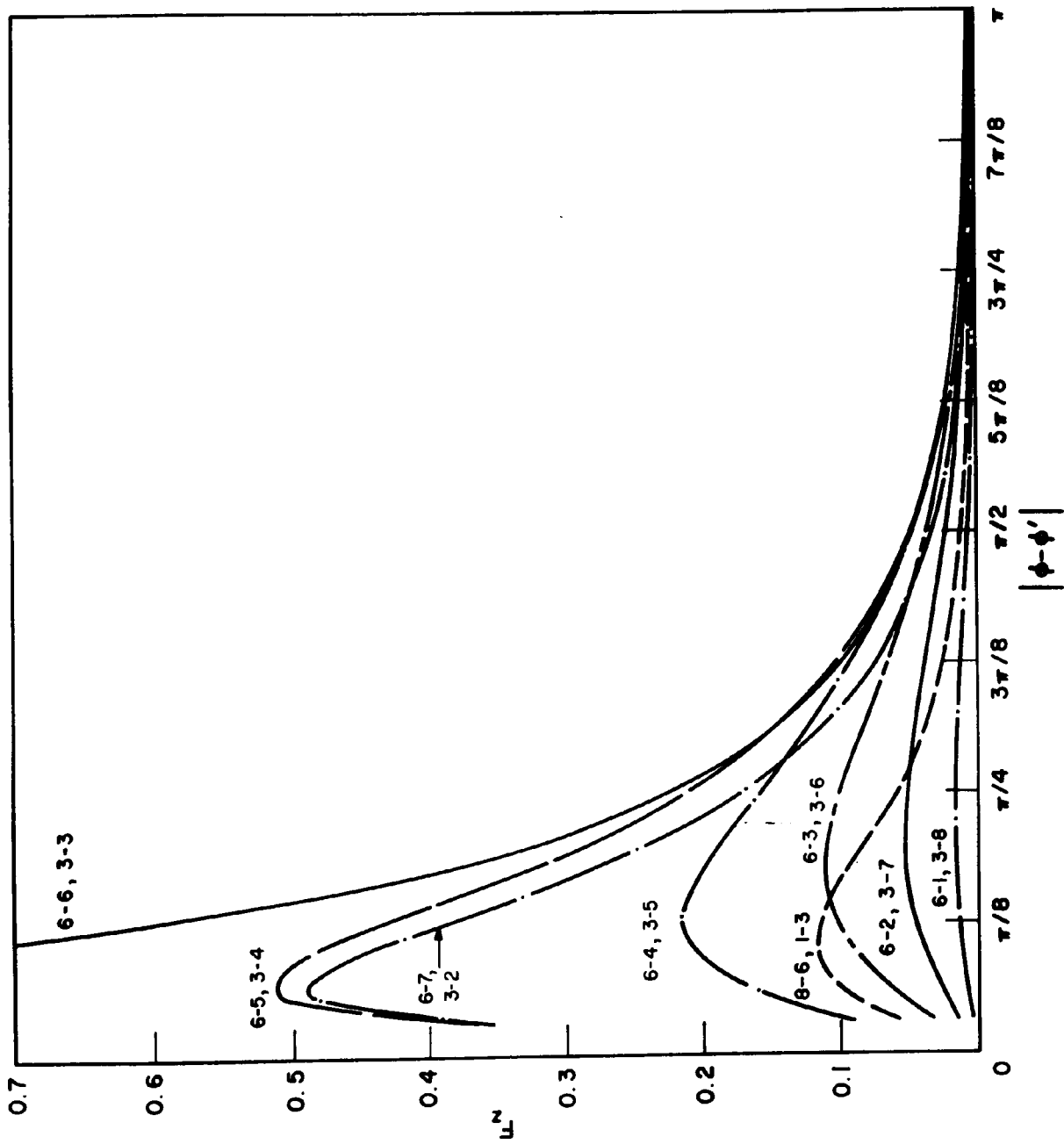


FIG. C.4 F_z SPACE-CHARGE-WEIGHTING FUNCTIONS. ($p = 0.3125$, $p' = 0.6875$; p' VARIABLE)

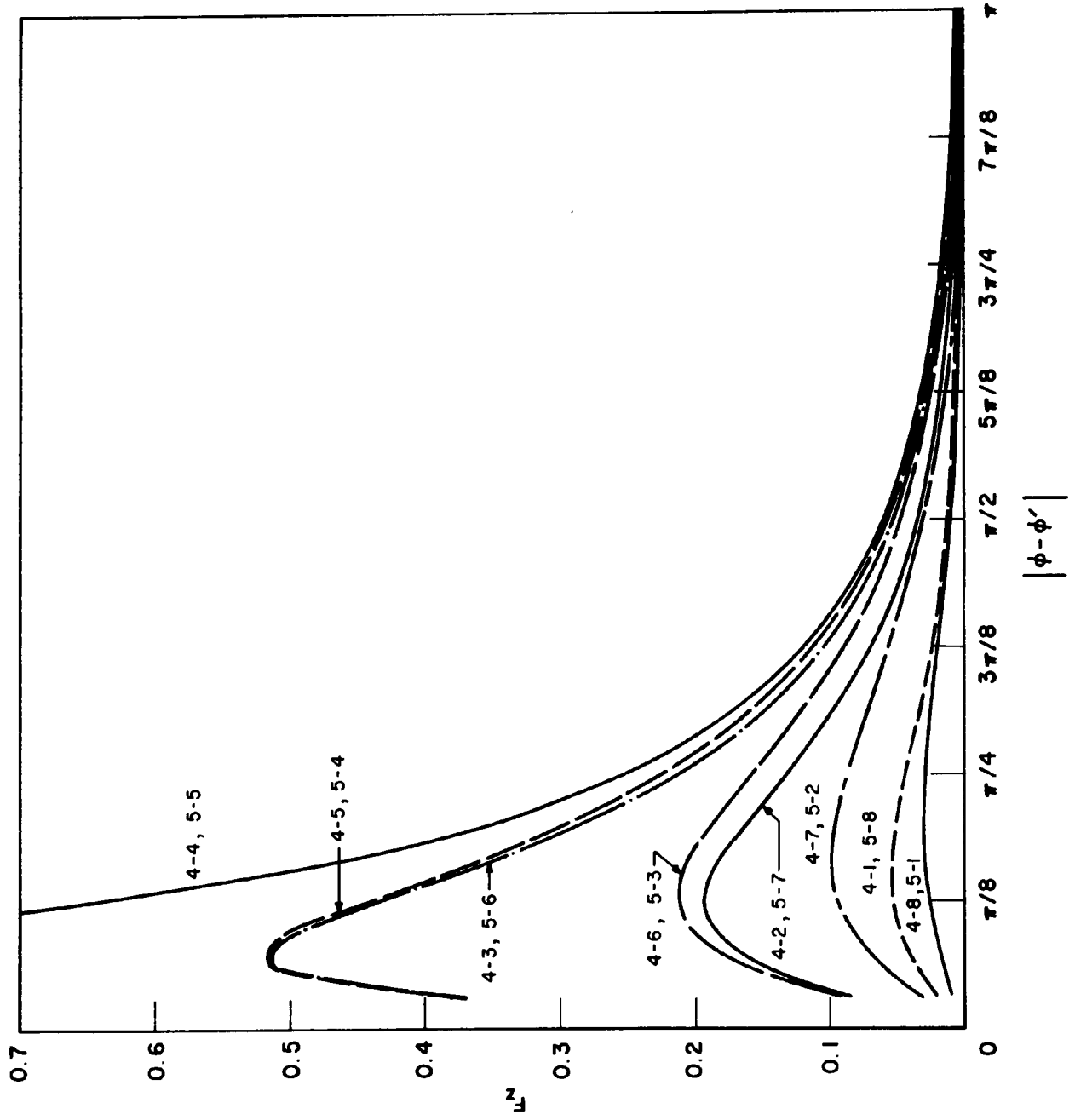


FIG. C.5 F_z SPACE-CHARGE-WEIGHTING FUNCTIONS. ($p = 0.4375$, $p' = 0.5625$; p' VARIABLE)

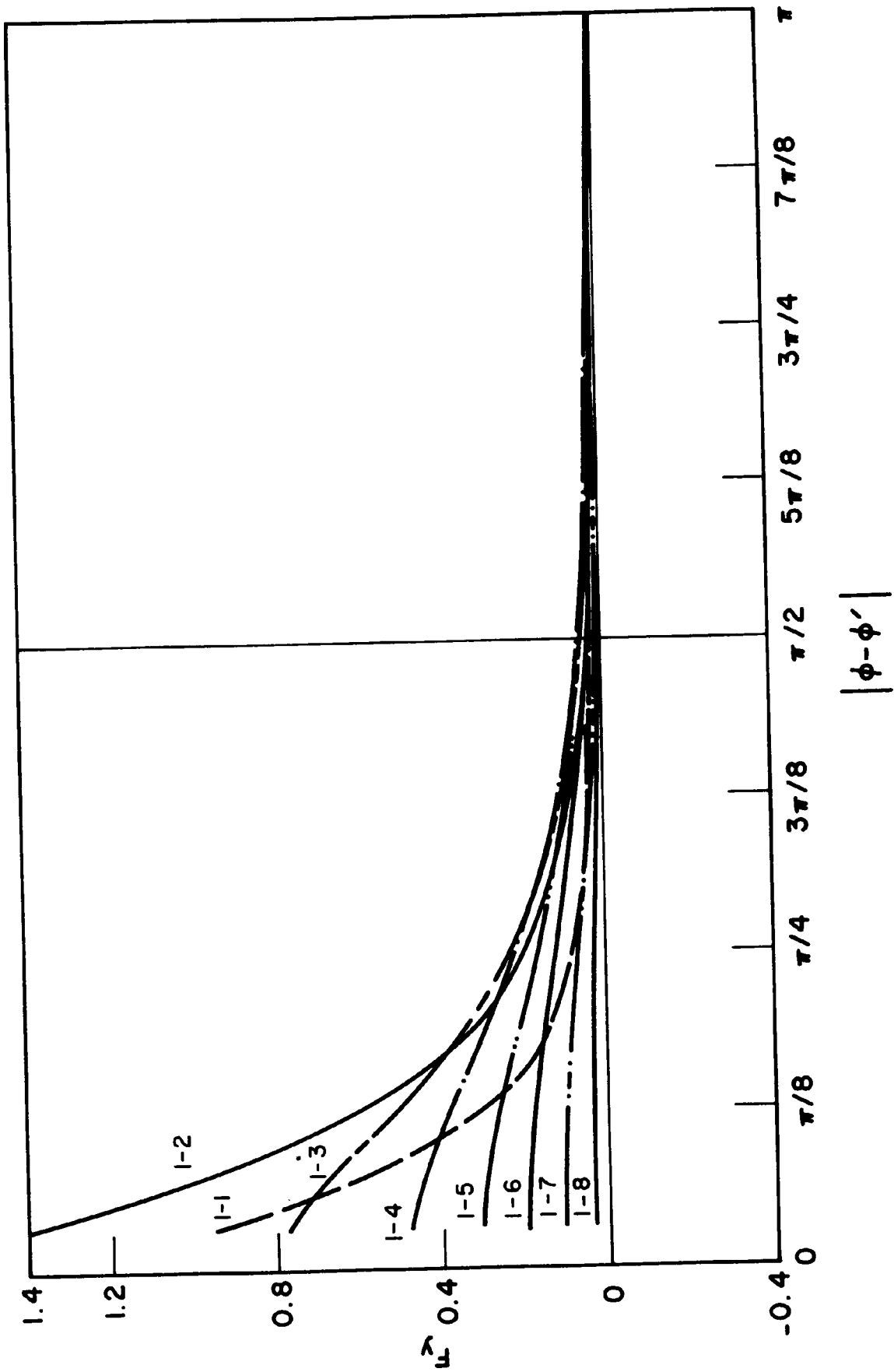


FIG. C.6 F_y SPACE-CHARGE-WEIGHTING FUNCTIONS. ($p = 0.0625$; p' VARIABLE)

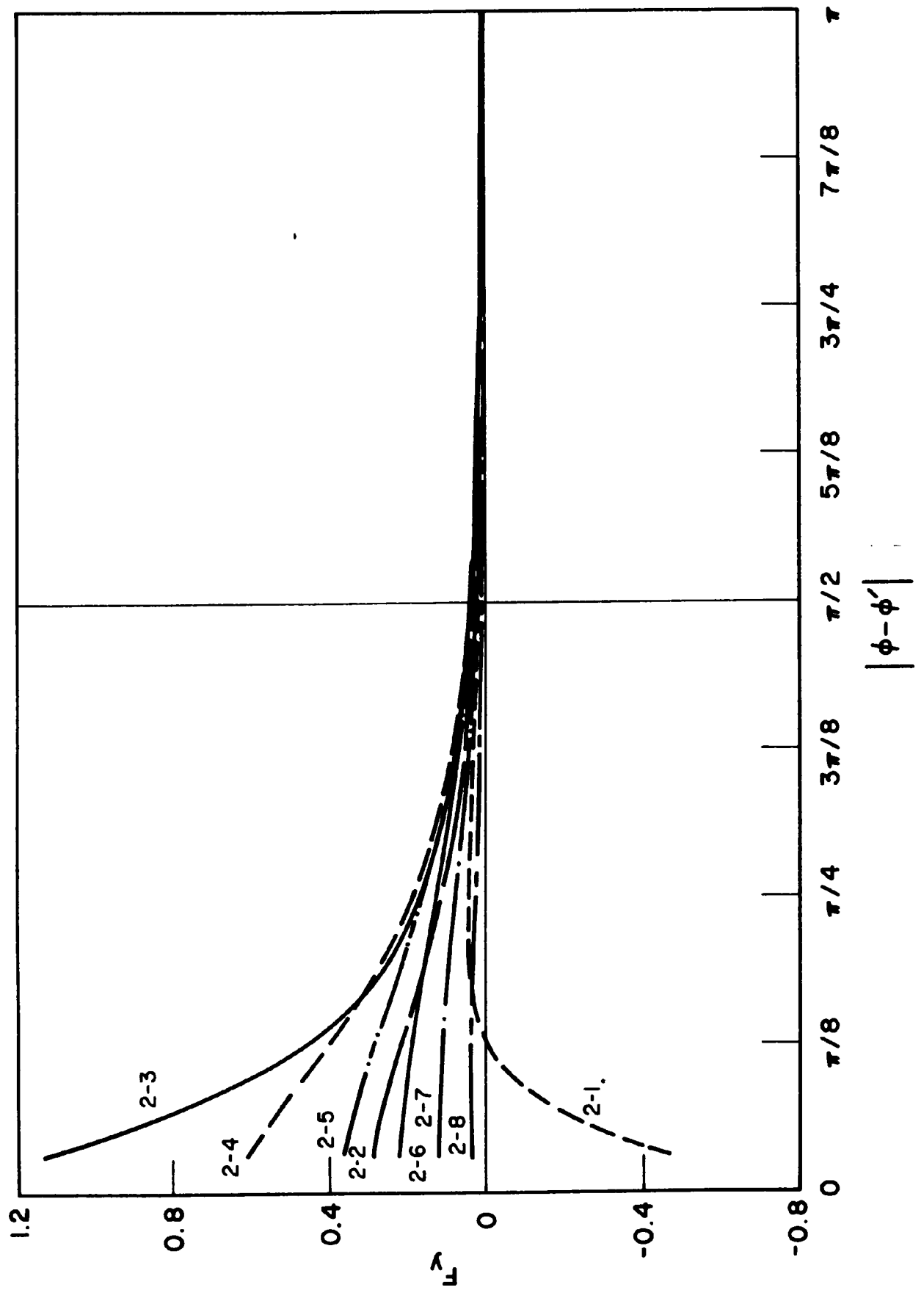


FIG. C.7 F_y SPACE-CHARGE-WEIGHTING FUNCTIONS. ($p = 0.1875$; p' VARIABLE)

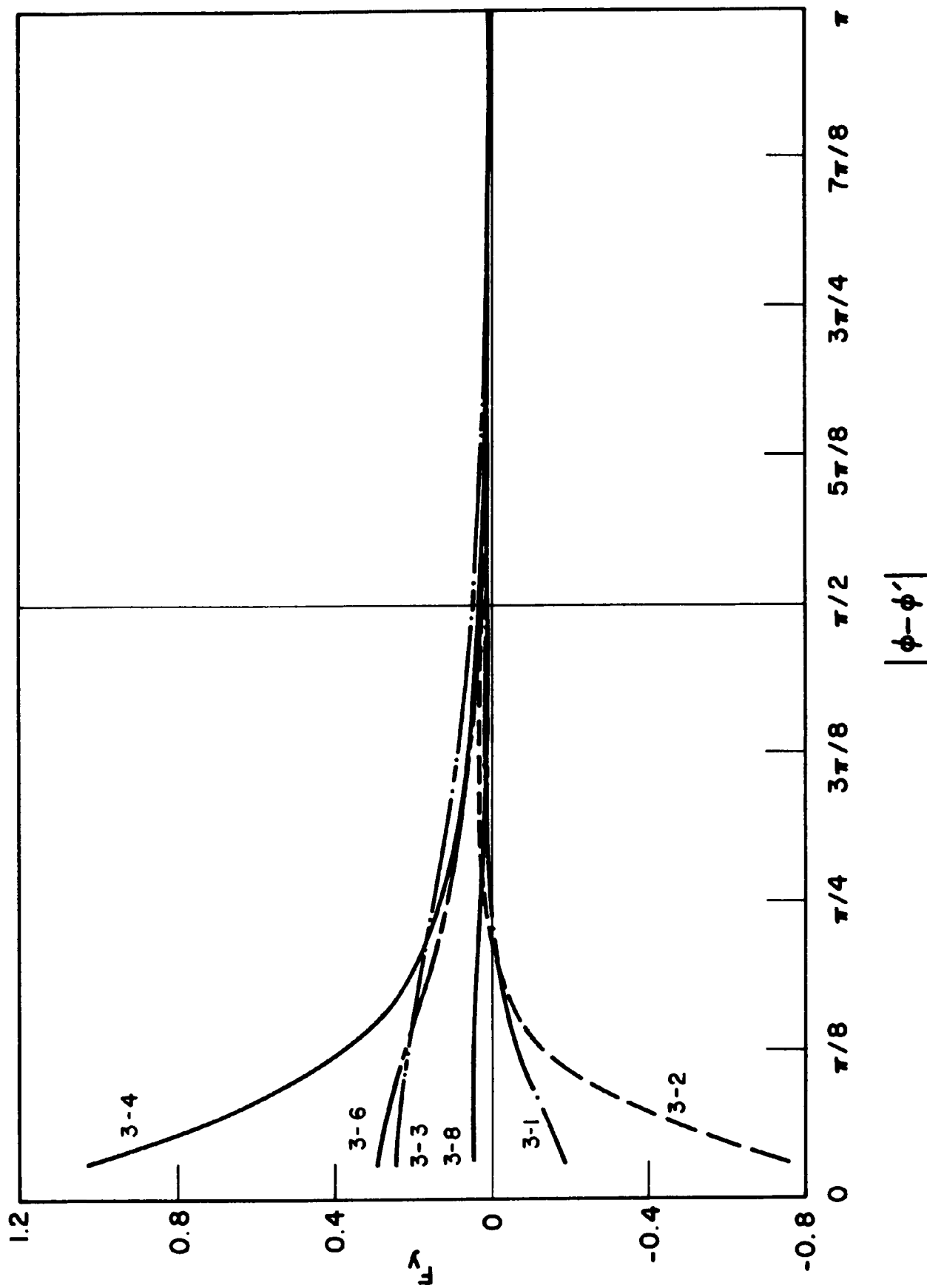


FIG. C.8 F_y SPACE-CHARGE-WEIGHTING FUNCTIONS. ($p = 0.3125$; p' VARIABLE)

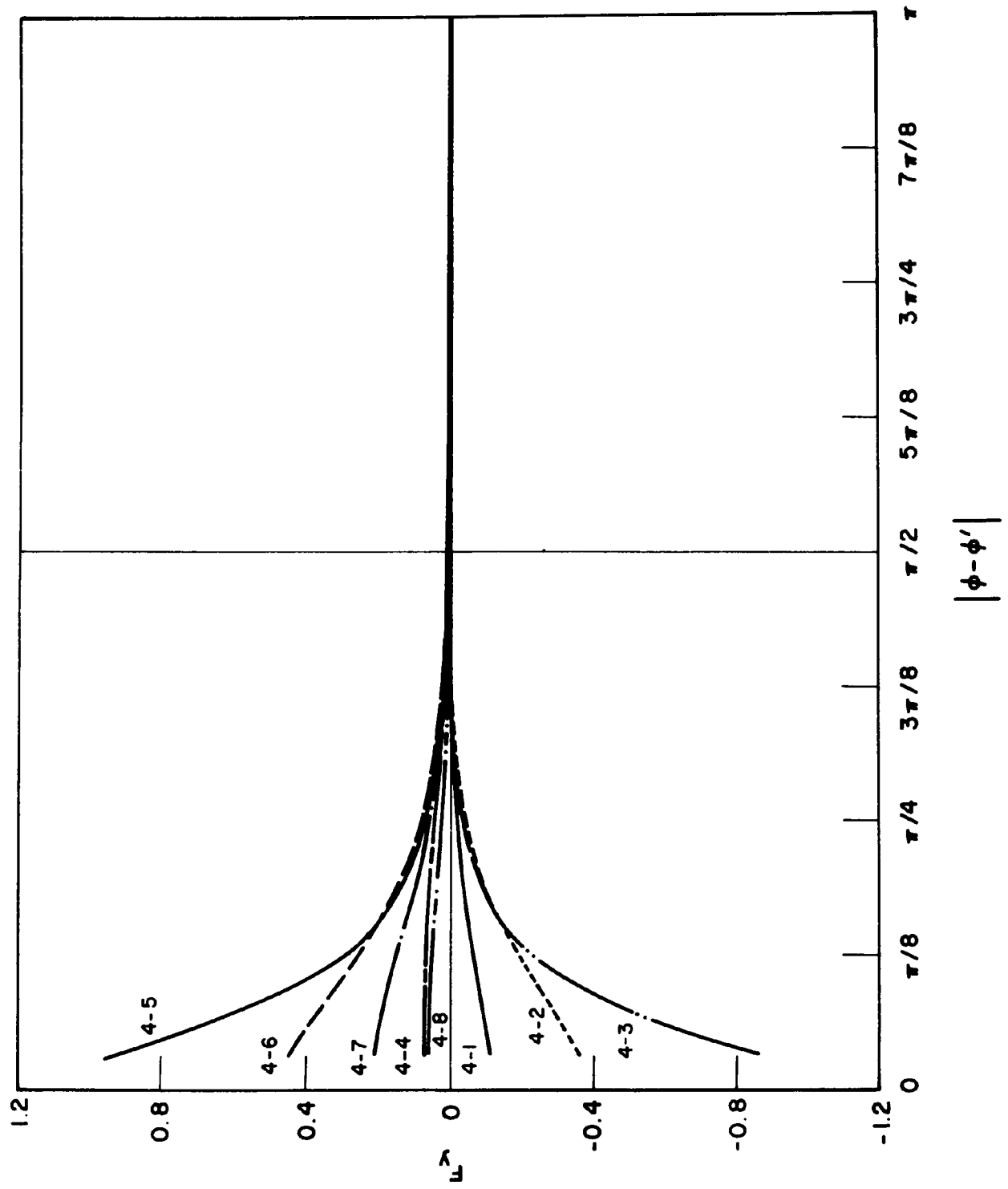


FIG. C.9 F_y SPACE-CHARGE-WEIGHTING FUNCTIONS. ($p = 0.4375$; p' VARIABLE)

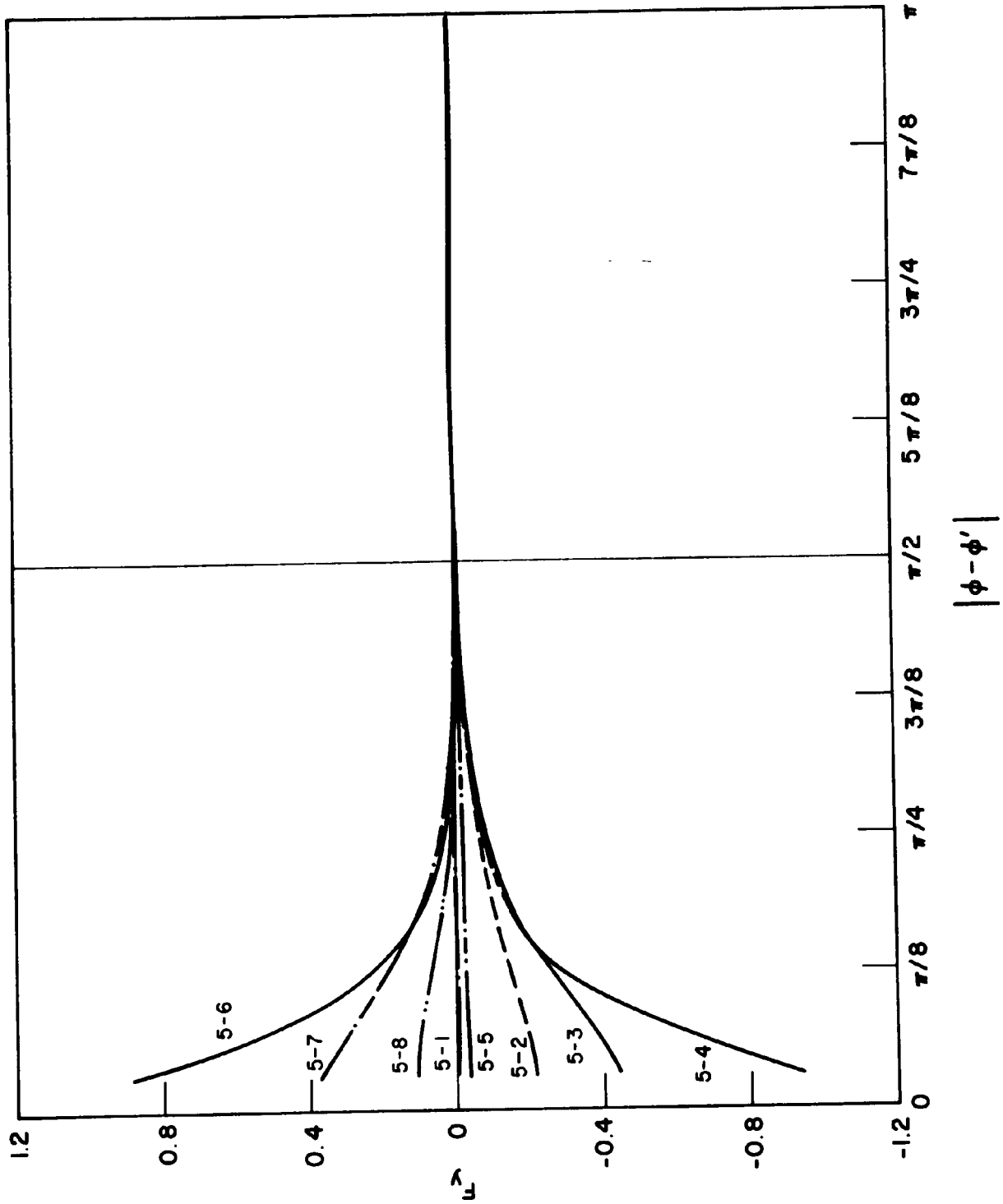


FIG. C.10 F_y SPACE-CHARGE-WEIGHTING FUNCTIONS. ($p = 0.5625$; p' VARIABLE)

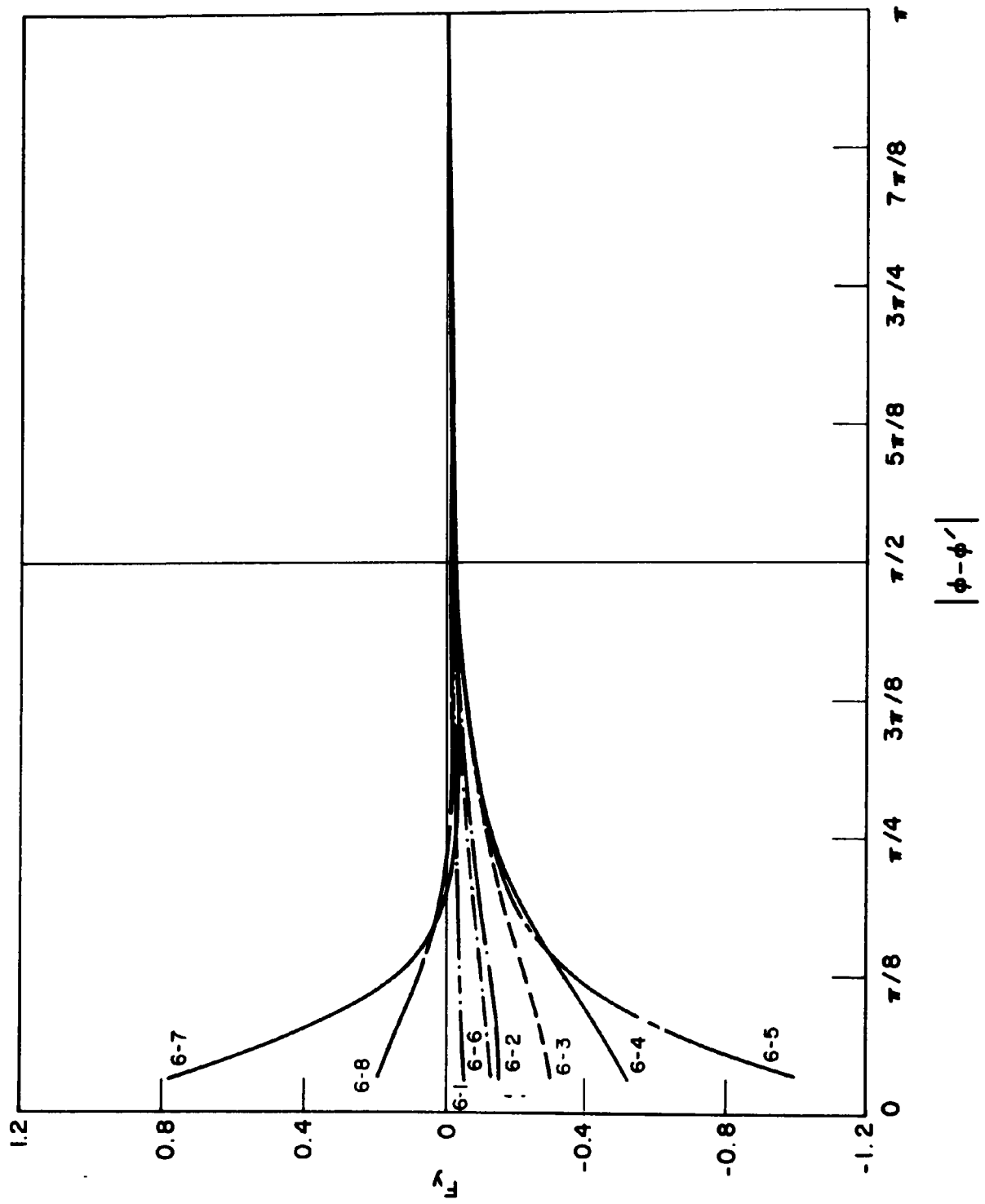


FIG. C.11 F_y SPACE-CHARGE-WEIGHTING FUNCTIONS. ($p = 0.6875$; p' VARIABLE)

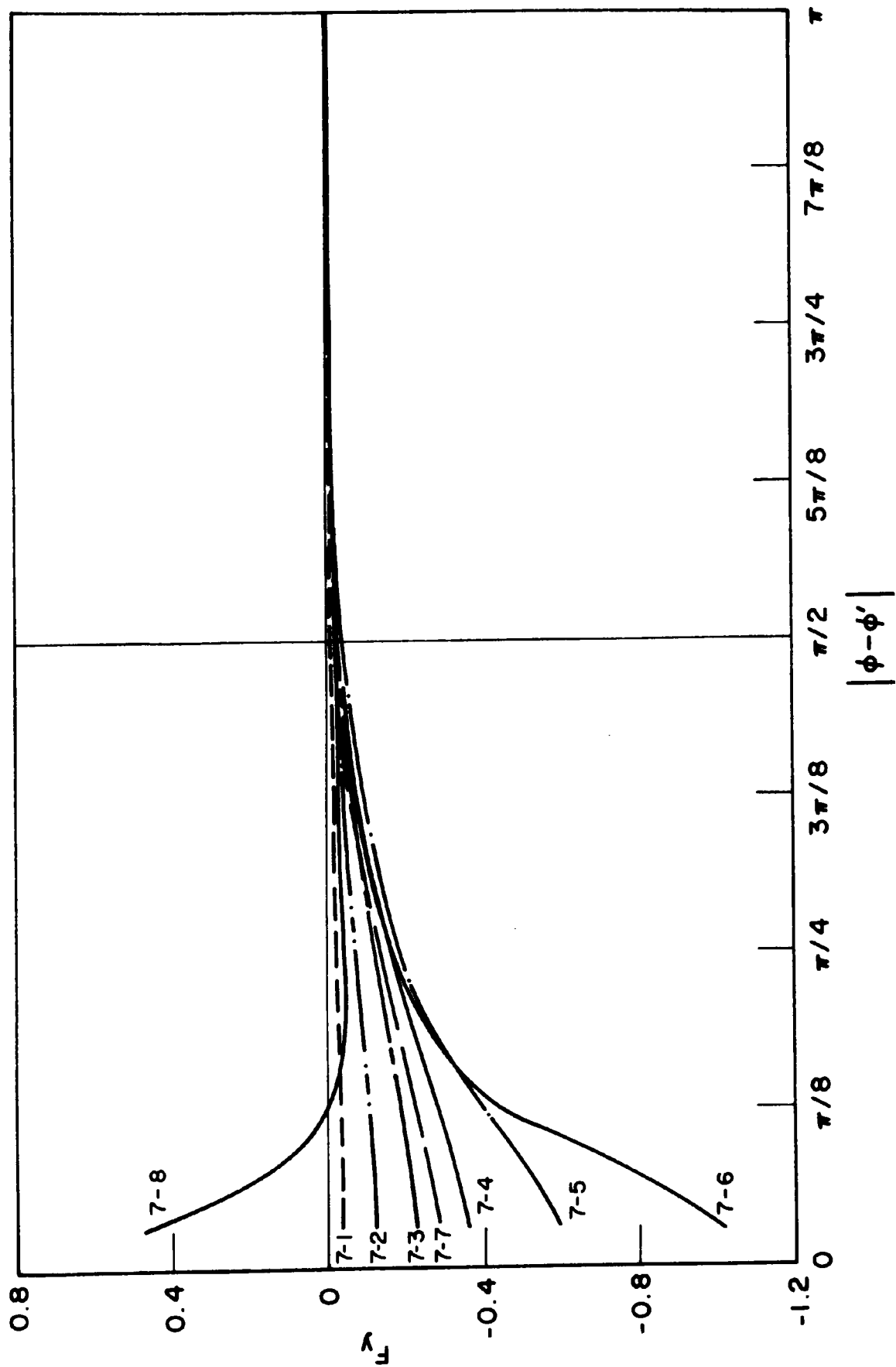


FIG. C.12 F_y SPACE-CHARGE WEIGHTING FUNCTIONS. ($p = 0.8125$; p' VARIABLE)

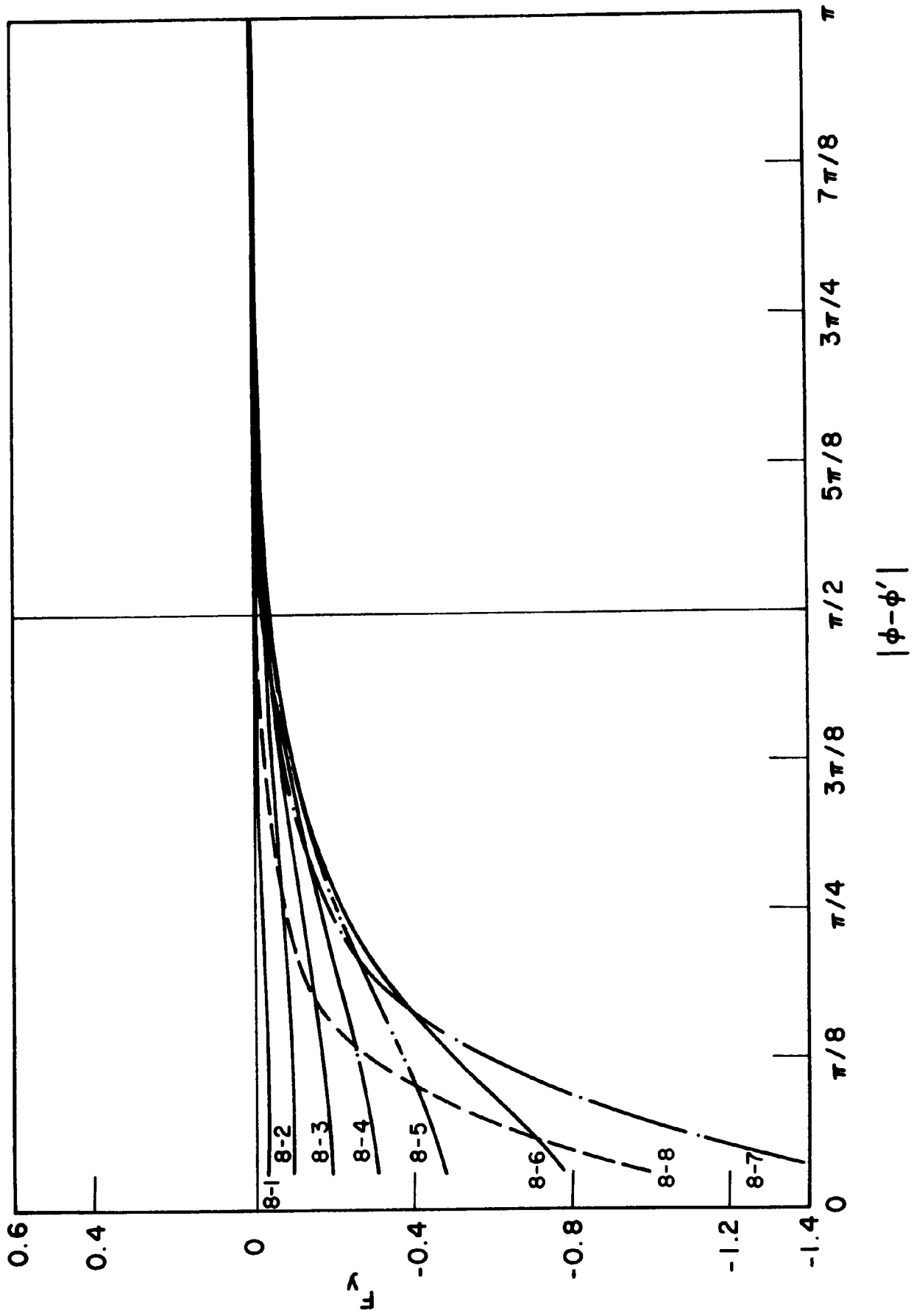


FIG. C.13 F_y SPACE-CHARGE-WEIGHTING FUNCTIONS. ($p = 0.9375$; p' VARIABLE)

sign for elements (represented by values of p') above and below this particular subdivision. Thus, the space-charge-weighting functions also give information regarding the direction of the space-charge forces.

The resulting values of F_y and F_z obtained from the computer showed a great deal of symmetry and antisymmetry about the midpoint M-M in Fig. C.1. The function F_z displayed complete symmetry about M-M, and in fact it was only necessary to compute F_z for half the values of p and p' . This symmetry property is apparent from Eq. C.2, since interchanging p and p' leaves F unchanged.

The function F_y showed approximate antisymmetry about M-M; that is, when two values of both p and p' equidistant above and below M-M are substituted into F_y , the magnitude of F_y remains approximately the same, while the sign is reversed. For example, F_y with $p = 0.8125$ and $p' = 0.0625$ (cell 7 on 1) is approximately equal in magnitude, but opposite in sign to F_y with $p = 0.1875$ and $p' = 0.9375$ (cell 2 on 8). The fact that the magnitudes are approximately equal can be seen by considering only the first few terms in Eqs. C.1 and C.2. From Eq. C.1, when only the first term is considered, the antisymmetry of F_y is seen immediately by observing the factor $\cos \pi p \sin \pi p'$. Since p and p' range from zero to one, the sine factor has symmetry about $p' = 0.50$ (M-M in Fig. C.1), while the cosine factor leads to antisymmetry about $p = 0.50$.

A small computer program, separate from the large-signal Lagrangian program, was used in calculating these weighting functions. The solutions were printed out on the usual tabulating paper to enable

plotting the curves in Figs. C.2 to C.13, and also on IBM cards to be used in the solution of the six large-signal emitting-sole equations. There were 128 cards punched, each of which contain 32 values of F_y or F_z corresponding to a single value for p and p' . Each card is called a table. In the main program at a particular q -plane, the computer is programmed to scan all of the tables when particular values of p , p' and $|\phi - \phi'|$ are given, and for these values obtain the appropriate weighting functions and thus the space-charge forces.

APPENDIX D. INITIAL VELOCITIES

The initial velocities of the electrons were discussed in Chapter II. It was assumed there that electrons are emitted in four different directions in the y-z plane and that these directions are distributed symmetrically about a normal to the cathode. It was further assumed that the magnitude of the total initial velocity is the same in each of these directions. However, if the normalized y- and z-directed velocities are computed from these assumed values and used in the computations, the resulting solutions are erroneous. This is because of the assumption that half of the electrons are injected with negative z-directed velocities. This assumption is inconsistent with the direction of integration used in the space-charge integrals as well as that used in the induced-current integration.

To eliminate this problem, initial z-velocities are first computed at a y-plane very close to the cathode, but far enough from it to make all initial z-velocities positive. The initial velocities at such a y-plane are derived using Poisson's equation and Lorentz's force equation along with the assumption that constant space-charge density exists near the cathode (this assumption is consistent with the derivation of the large-signal equations). For the assumed rectangular geometry, Poisson's equation is given by

$$\frac{\partial E_{sc-y}}{\partial y} = - \frac{|\rho_o|}{\epsilon_o} , \quad (D.1)$$

and the y- and z-component Lorentz force equations are given by

$$\frac{dv_y}{dt} = |\eta| (E_o - E_{sc-y}) - \omega_c v_z \quad (D.2)$$

and

$$\frac{dv_z}{dt} = \omega_c v_y \quad (D.3)$$

Integration of Eq. D.3 yields

$$v_z = v_z(0) + \omega_c y \quad (D.4)$$

Substituting this along with Eq. D.1 into Eq. D.2 and integrating yields

$$v_y^2 = v_y^2(0) + 2\{|\eta|E_o - \omega_c v_z(0)\}y - \left(\omega_c^2 - \frac{|\eta\rho_o|}{\epsilon_o}\right)y^2, \quad (D.5)$$

where it has been assumed that the emission material has infinite conductivity.

For a negative value of thermionic emission velocity in the z-direction, it can be seen from Eq. D.4 that v_z will be positive for values of y greater than $|v_z(0)/\omega_c|$. For a negative emission velocity $v_z(0)$, corresponding to a cathode temperature of 1200°K, and a magnetic field of 570 gauss, a y -value of 2×10^{-5} meters yields a positive v_z -velocity. This is less than one thousandth of the sole-to-anode spacing and is neglected in the computer calculations.

The initial electron velocity components used in the computer calculations are tabulated below for the given values of D and ω_c/ω .

Table D.1
Normalized Initial Velocities

Case	Ω	u_y	u_z
$D = 0.1, \omega_c/\omega = 0.5$	-30°	0.124	-4.865
	-10°	0.124	-4.8495
	10°	0.124	-4.8325
	30°	0.124	-4.8165
$D = 0.1, \omega_c/\omega = 1.0$	-30°	0.248	-4.865
	-10°	0.248	-4.8495
	10°	0.248	-4.8325
	30°	0.248	-4.8165
$D = 0.1, \omega_c/\omega = 1.5$	-30°	0.372	-4.865
	-10°	0.372	-4.8495
	10°	0.372	-4.8325
	30°	0.372	-4.8165
$D = 0.125, \omega_c/\omega = 0.5$	-30°	0.11	-3.905
	-10°	0.11	-3.894
	10°	0.11	-3.881
	30°	0.11	-3.870
$D = 0.125, \omega_c/\omega = 1.0$	-30°	0.22	-3.905
	-10°	0.22	-3.894
	10°	0.22	-3.881
	30°	0.22	-3.870
$D = 0.125, \omega_c/\omega = 1.5$	-30°	0.33	-3.905
	-10°	0.33	-3.894
	10°	0.33	-3.881
	30°	0.33	-3.870

These values were calculated by also assuming a cathode temperature of 1200°K, an r-f frequency corresponding to S-band, and emission angles Ω of -30° , -10° , 10° and 30° with respect to a line projected normal to the cathode (this corresponds to $\alpha = 10^\circ$ using the notation of Section 2.8.2).

APPENDIX E. DEVELOPMENT OF HKB EQUATIONS FROM AN ENERGY ANALYSIS

It will be shown in this section that exactly the same form of the HKB equations can be obtained from an energy analysis of the interaction. An analysis similar to that of Feinstein and Kino¹⁴, and Monosov²⁵ is used in conjunction with the HKB assumptions.

The following assumptions are made in this analysis:

1. Adiabatic equations of motion are assumed.
2. Space-charge forces are neglected.
3. Synchronism between the electrons and the wave are assumed.
4. The amplitude of the r-f wave is a slowly varying function of longitudinal distance.
5. Nonrelativistic mechanics are assumed.
6. Interaction occurs with only one space harmonic of the r-f wave.
7. There is negligible circuit loss.
8. Ideal hard-kernel bunches are assumed.
9. Kinetic energy transfer is neglected.

The general conservation of energy equation for a crossed-field continuously emitting-sole electron-wave interaction can be written as²⁵

$$dP_L(z)dz = - \int_0^z dz' \int_0^{2\pi} \frac{d\Phi}{2\pi} \frac{d}{dz} [W_V(z, z', \Phi_0) + W_T(z, z', \Phi_0)] - P_R(z) \quad , \quad (E.1)$$

where $P_L(z) \triangleq$ power flow in the direction of the load

$$[E^2(z)/(2\beta^2 K)],$$

$E(z) \triangleq$ the magnitude of the total r-f field,

$\beta \triangleq$ the propagation factor for the r-f wave,

- $K \triangleq$ the circuit impedance,
- $P_R(z) \triangleq$ joule power loss per unit length,
- $W_V(z, z', \Phi_0) d\Phi \triangleq$ potential energy of electrons in a length z emitted from a section of the cathode with length z' in the interval of phase Φ_0 to $\Phi_0 + d\Phi_0$ per unit length, and
- $W_T(z, z', \Phi_0) d\Phi \triangleq$ kinetic energy of electrons in a length z emitted from a section of the cathode with length z' in the interval of phase Φ_0 to $\Phi_0 + d\Phi_0$ per unit length.

The coordinate system selected for this analysis is shown in Fig. E.1. By neglecting the kinetic energy transfer between the electrons and the wave and by neglecting the joule power loss, Eq. E.1 becomes

$$\frac{E(z)}{\beta^2 K} \frac{dE}{dz} = - \frac{1}{2\pi} \int_0^z dz' \int_0^{2\pi} d\Phi \frac{d}{dz} [W_V(z, z', \Phi_0)] \quad (E.2)$$

An explicit function for $W_V(z, z', \Phi_0)$ will now be obtained.

The potential energy function $W_V(z, z', \Phi_0) d\Phi$ can be written as

$$W_V(z, z', \Phi_0) d\Phi = -qE_0 y d\Phi / \lambda \quad (E.3)$$

where λ is the wavelength of the r-f wave, or by inserting the conservation of charge ($q d\Phi = d\Phi_0 I_0 / v_0$),

$$W_V(z, z', \Phi_0) d\Phi = - \frac{I_0 E_0}{\lambda v_0} y d\Phi_0 \quad (E.4)$$

Differentiating with respect to z gives

$$\frac{dW_V}{dz} (z, z', \Phi_0) = - \frac{I_0 E_0}{\lambda v_0} \frac{dy}{dz} \quad (E.5)$$

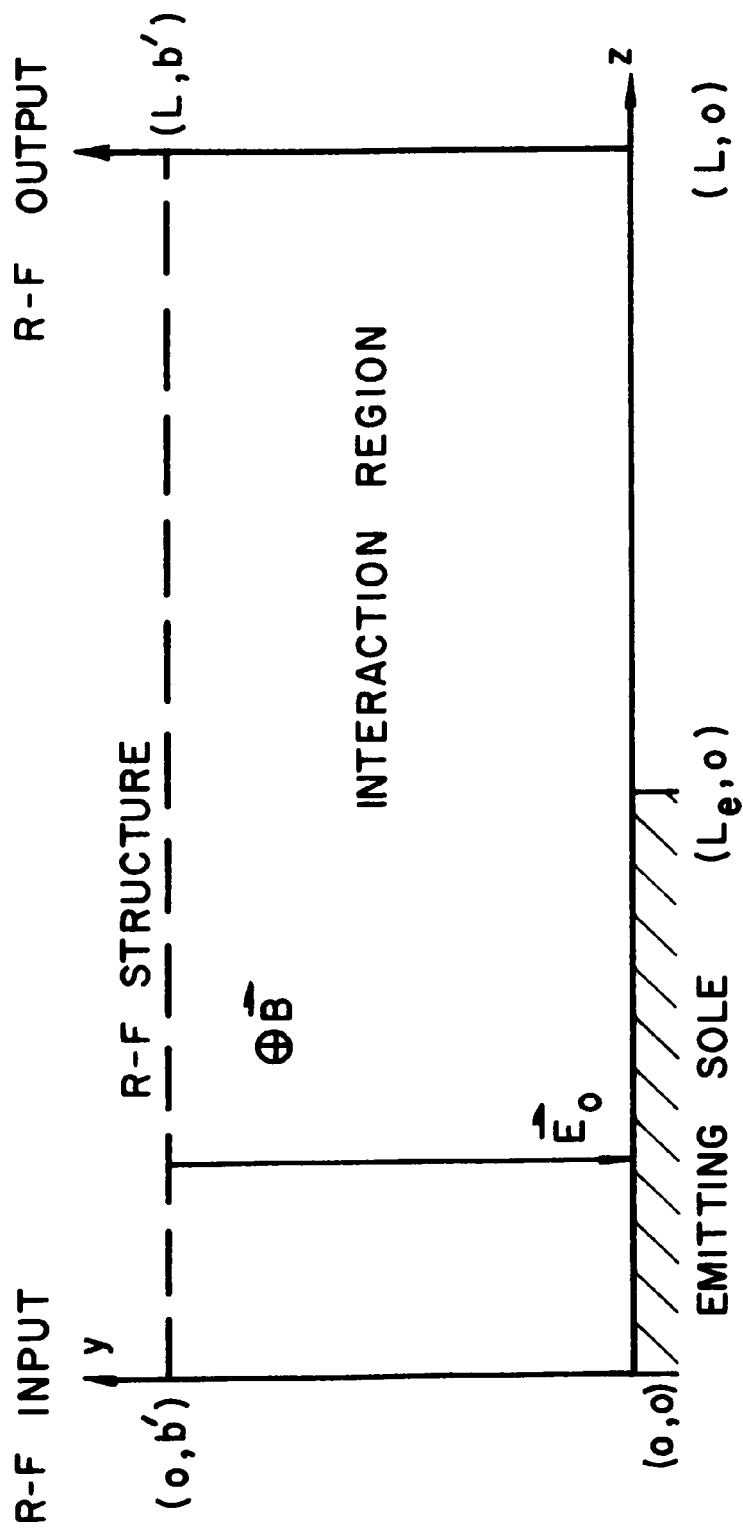


FIG. E.1 ENERGY ANALYSIS MODEL OF THE CROSSED-FIELD
EMITTING-SOLE AMPLIFIER.

Thus, Eq. E.2 becomes

$$\frac{E(z)}{\beta^2_K} \frac{dE(z)}{dz} = \frac{1}{2\pi} \int_0^z dz' \int_0^{2\pi} d\Phi \left(\frac{I_o E_o}{\lambda v_o} \frac{dy}{dz} \right) . \quad (E.6)$$

An expression for dy/dz will now be obtained from the Lorentz equations of motion and substituted into the above expression.

It should first be recalled that in the absence of an electron stream the r-f field in the interaction space of a crossed-field device is represented by an infinite summation of space harmonics. Each of these moves along the z-axis with different phase velocities v_n . For the assumed two-dimensional geometry of Fig. E.1, the components of the r-f field are given by

$$E_y(y, z, t) = \sum_{n=-\infty}^{\infty} A_n \frac{\cosh \beta_n y}{\sinh \beta_n b'} \cos(\omega t - \beta_n z) \quad (E.7)$$

and

$$E_z(y, z, t) = \sum_{n=-\infty}^{\infty} A_n \frac{\sinh \beta_n y}{\sinh \beta_n b'} \sin(\omega t - \beta_n z) . \quad (E.8)$$

When electrons are present in the interaction region, it is assumed that the r-f field has the same form except that the amplitudes A_n become slowly varying functions of the longitudinal coordinate z , i.e., the amplitude is given by

$$A_n = A_n(z) , \quad (E.9)$$

where

$$\frac{dA_n(z)}{dz} \ll \beta_n A_n(z) . \quad (E.10)$$

The equations of motion for electrons in the crossed-field interaction region of Fig. E.1, under the influence of an r-f field and neglecting the space-charge forces, can now be written as

$$\frac{d^2y}{dt^2} = |\eta|A(z) \frac{\cosh \beta y}{\sinh \beta b} \cos(\omega t - \beta z) - \omega_c \frac{dz}{dt} + |\eta|E_0 \quad (E.11)$$

and

$$\frac{d^2z}{dt^2} = -|\eta|A(z) \frac{\sinh \beta y}{\sinh \beta b} \sin(\omega t - \beta z) + \omega_c \frac{dy}{dt}, \quad (E.12)$$

where it has been assumed that only one space harmonic of the r-f wave is interacting with the electrons and the subscript n has therefore been omitted.

In order to simplify these equations the adiabatic assumption is introduced. Under this assumption the acceleration terms in the force equations (the left-hand sides of Eqs. E.11 and E.12) are neglected. This assumption results in an averaging over electron trajectories with an elimination of the cycloidal motion, and applying it to Eqs. E.11 and E.12 yields

$$\frac{dz}{dt} = v_d - \frac{|\eta|}{\omega_c} A(z) \frac{\cosh \beta y}{\sinh \beta b} \cos(\omega t - \beta z) \quad (E.13)$$

and

$$\frac{dy}{dt} = \frac{|\eta|}{\omega_c} A(z) \frac{\sinh \beta y}{\sinh \beta b} \sin(\omega t - \beta z), \quad (E.14)$$

where $v_d = E_0/B$.

Dividing Eq. E.13 into Eq. E.14 yields the following trajectory equation:

$$\frac{dy}{dz} = \frac{\frac{|\eta|}{\omega_c} A(z) \frac{\sinh \beta y}{\sinh \beta b'} \sin(\omega t - \beta z)}{v - \frac{|\eta|}{\omega_c} A(z) \frac{\cosh \beta y}{\sinh \beta b'} \cos(\omega t - \beta z)} . \quad (E.15)$$

The hard-kernel-bunch-pair model of Chapter VI is now applied. This assumption stipulates that the charge has become concentrated at two phase regions of one period of the r-f wave. These phase positions are chosen to be $\phi_1 = \pi/2$ and $\phi_2 = -(\pi/2)$, corresponding to the phase region of maximum decelerating r-f field and maximum accelerating r-f field, respectively. Two trajectory equations therefore result, one for the decelerated bunch of electrons and one for the accelerated bunch as follows:

$$\frac{dy_1}{dz} = \frac{A(z)}{E_0} \frac{\sinh \beta y}{\sinh \beta b'} \quad (E.16)$$

and

$$\frac{dy_2}{dz} = - \frac{A(z)}{E_0} \frac{\sinh \beta y}{\sinh \beta b'} . \quad (E.17)$$

Hence, carrying out the integration indicated in Eq. E.6 using the hard-kernel-bunch assumption yields

$$\frac{dA(z)}{dz} = \frac{\beta^2 K I_0}{\lambda v_0} \left(\frac{\sinh \beta y_1}{\sinh \beta b'} - \frac{\sinh \beta y_2}{\sinh \beta b'} \right) z , \quad (E.18)$$

where $A(z) = E(z)$ when only one space harmonic is present. By using the normalized variables of Chapter VI, Eqs. E.16 through E.18 can be written as

$$A'(q) = 2\pi C' q [\psi(P_1) - \psi(P_2)] \quad (E.19)$$

and

$$P'_{1,2} = \pm C'_2 A(q) \psi(P'_{1,2}) , \quad (E.20)$$

where $C'_1 \triangleq (\beta K I_0)/(\lambda v_0 D)$, $C'_2 = \beta b'/E_0$, and where the subscript 1 corresponds to the upper sign, while the subscript 2 corresponds to the lower sign. Although the constants appearing in this set of equations are not the same as those in Eqs. 6.11 and 6.12 due to the different defining symbols which were used, it can be seen that the form of these two sets of equations is identical.

APPENDIX F. COMPARISON WITH THE TIME-DEPENDENT ANALYSIS OF YU ET AL.

A large-signal crossed-field distributed-emission analysis has been carried out by Yu et al.³⁴ using time as an independent variable rather than longitudinal or interaction distance as used in this investigation. Essentially the same basic equations are solved in each analysis and thus similar results should be obtained. Yu's analysis is outlined and summarized in this appendix and comparisons with the present study are given.

The model used in the time-dependent analysis is shown in Fig. F.1. This theory considers the electron-wave interaction in one wavelength λ_r (as seen in Fig. F.1) of the r-f wave. As is observed, a linear two-dimensional interaction region in a moving y-z reference frame is used and it is assumed that all quantities are independent of x. The r-f slow-wave structure is located at $y = a$ and the emitting sole is at $y = 0$.

The expression for the location of the moving reference frame referred to a fixed coordinate system can also be used as a transformation equation from time to longitudinal distance. This integral expression is given by

$$l(t) = \int_0^t [\omega/\beta_r(t)] dt , \quad (F.1)$$

when only one space harmonic (the rth) component of the r-f wave is taken into consideration. The length $l(t)$ is shown in Fig. F.1 and the quantities ω and β_r denote the r-f radian frequency and phase constant of the rth harmonic component of the wave respectively. The integrand

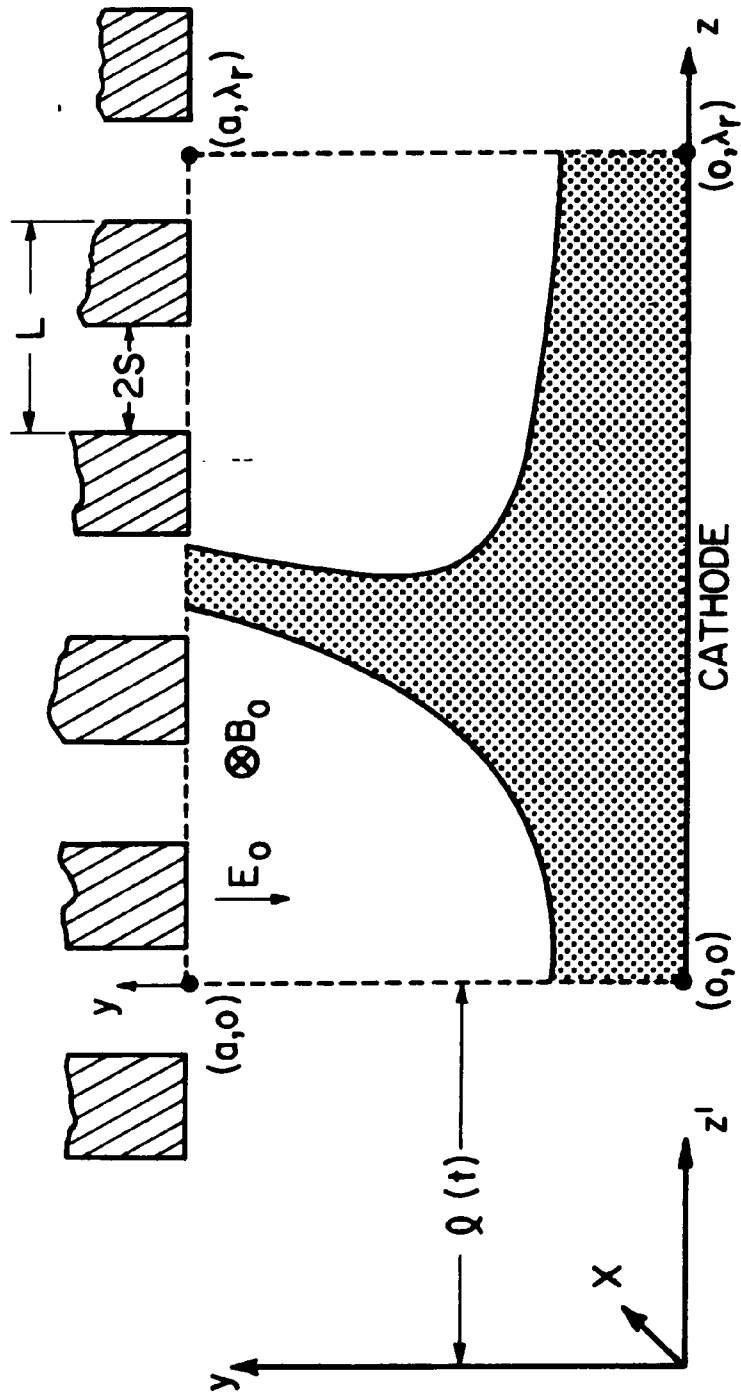


FIG. F.1 CROSSED-FIELD DISTRIBUTED-INTERACTION ANALYSIS MODEL³⁴.

in the above equation is the longitudinal phase velocity of this particular space harmonic and is in general a function of time. Equation F.1 will be used in comparing the r-f amplitude growth results which are obtained from the two analyses.

The motion of each electron, or rod of charge in the assumed linear two-dimensional model, is again determined by using the Lorentz equations of motion. By defining two complex quantities as

$$Z = y + jz \quad (F.2)$$

and

$$E = E_y + jE_z, \quad (F.3)$$

where y and z are the vertical and longitudinal coordinates of an electron, respectively, and E is the total electric field, the two component equations of motion are reduced to one equation given by

$$\frac{\partial^2 Z}{\partial t^2} - j\omega_c \frac{\partial Z}{\partial t} = - \left\{ \eta E + j \frac{\partial}{\partial t} [\omega/\beta_r(t)] \right\}. \quad (F.4)$$

The general solution of this equation is obtained by using the variation of parameters method, yielding

$$Z = Z_0 + (\dot{Z}_0/j\omega_c) \{ \exp [j\omega_c(t-t_0)] - 1 \}$$

$$- \left\{ \eta E + j \frac{d}{dt} [\omega/\beta_r(t)] \right\} \left\{ \frac{1 - \exp[j\omega_c(t-t_0)]}{\omega_c^2} - \frac{t-t_0}{j\omega_c} \right\}, \quad (F.5)$$

where E has been assumed to vary slowly with position and time.

Physically this is justified by assuming that only one harmonic of the wave interacts with the stream charge. This assumption is valid when the computer integration increment Δt is kept sufficiently small and

the coordinate mesh analysis is sufficiently fine. Equation F.5 is solved using a digital computer and yields the y- and z-coordinates as well as the y- and z-velocities for each electron. It is observed in the above equation that time is the independent variable. The use of Eqs. 2.65, 2.66, 2.69 and 2.70 in this dissertation, using longitudinal position as the independent variable, is equivalent to the usage of Eq. F.5 by Yu et al.

In order to determine the space-charge fields, Poisson's equation is solved for the postulated interaction region. However, a different method of analysis than the Green's function method presented in this report is utilized. The method used was first suggested by Hockney¹³ and consists of dividing the interaction region into rectangles and solving for the space-charge potential at the corners of each rectangle. This results in a system of equations for the potential functions. The method also uses a Fourier analysis to decouple these unknown potentials in the system of equations instead of inverting the system matrix. The specific mesh configuration used consists of 48 vertical divisions and 96 horizontal divisions and thus there are 48 x 96 potential function equations.

The effect of the electrons upon the wave is calculated in the time analysis by writing an expression for the work done by a small element of charge $\rho h dy dz$ due to an arbitrary displacement $\vec{\Delta s}$. This is given by

$$\Delta U = -\rho h dy dz \vec{E}_{rf} \cdot \vec{\Delta s} , \quad (F.6)$$

where h is the dimension of the interaction space in the direction of the d-c magnetic field and $\vec{E}_{rf}$ is the r-f electric field. This

expression is differentiated with respect to time and integrated over distance to yield the power generated per unit wavelength as follows:

$$P_g(t) = -(h/\lambda_r) \int_0^a dy \int_0^{\lambda_r} dz [\rho(E_y^{rf} \dot{y} + E_z^{rf} \dot{z})] , \quad (F.7)$$

where λ_r is the wavelength of the r th space harmonic of the wave and a is the sole-to-anode spacing. It should be noticed that this expression follows immediately from Poynting's power theorem³⁷ which is derived from Maxwell's equations. This is the specific term due to the motion of free charges. Furthermore, when it is assumed that only one space harmonic of the total r-f field is involved in the interaction it was explained previously (in Chapter II) that the equivalent transmission-line equation can be used as a substitute for Maxwell's equations. Hence, Eq. F.7 and the one-dimensional circuit equation (Eq. 2.11) are equivalent calculating procedures to determine the effects of the electrons upon the r-f wave.

The continuity of charge equation is also incorporated into the time analysis. This expression is given by

$$\rho|dydz| = \sum \rho_0|dy_0dz_0| \quad (F.8)$$

and relates the charge at position (y,z) and time t to the elements of charge emitted from the cathode at initial positions (y_0,z_0) and time t_0 . This relation is used to perform a transformation from final electron coordinates to initial coordinates as was also done in this text.

The above equations are solved numerically by Yu et al. utilizing a digital computer. The trajectory solutions obtained are "beam profiles" of the interaction at various times t , i.e., the trajectory positions of the electrons in the interaction region are read out of

the computer for several different values of t and each indistinguishable electron is represented as a point in the rectangular configuration of dimensions λ_r by a . Examples of the beam profiles obtained are shown in Fig. F.2 for various times T , where λ_c is defined as one cyclotron period. Notice that the electrons are bunching as time is increased. This is qualitatively equivalent to bunching with longitudinal distance as observed in this analysis. The equivalence is established by recalling Eq. F.1 and assuming that β_r is independent of time (this would be true for a weak interaction). Under this condition longitudinal position and time are linearly related and hence bunching in time is the same as bunching with distance except for a multiplicative constant. However, a qualitative comparison of the trajectories is not possible since individual electrons were not followed in the time-dependent analysis.

In order to compare the r-f gain results obtained in each analysis, the following summary of normalized potential parameters used in the time analysis are presented:

1. The normalized anode potential is defined as

$$\varphi_a \triangleq (2\eta V_a)/(a\omega_c)^2 .$$

2. The normalized synchronous potential, which is derived from the relation $E_0/B = \omega/\beta_r$, is given by

$$\varphi_s \triangleq (\omega/\omega_c)/(\pi a/\lambda_r) .$$

3. The normalized r-f potential is defined as

$$\varphi_{rf} = (2\eta V_{rf})/(a\omega_c)^2 .$$

$$T = 15.6\lambda_c$$

$$T = 31.3\lambda_c$$

$$T = 156.2\lambda_c$$

$$T = 313\lambda_c$$

$$T = 469\lambda_c$$

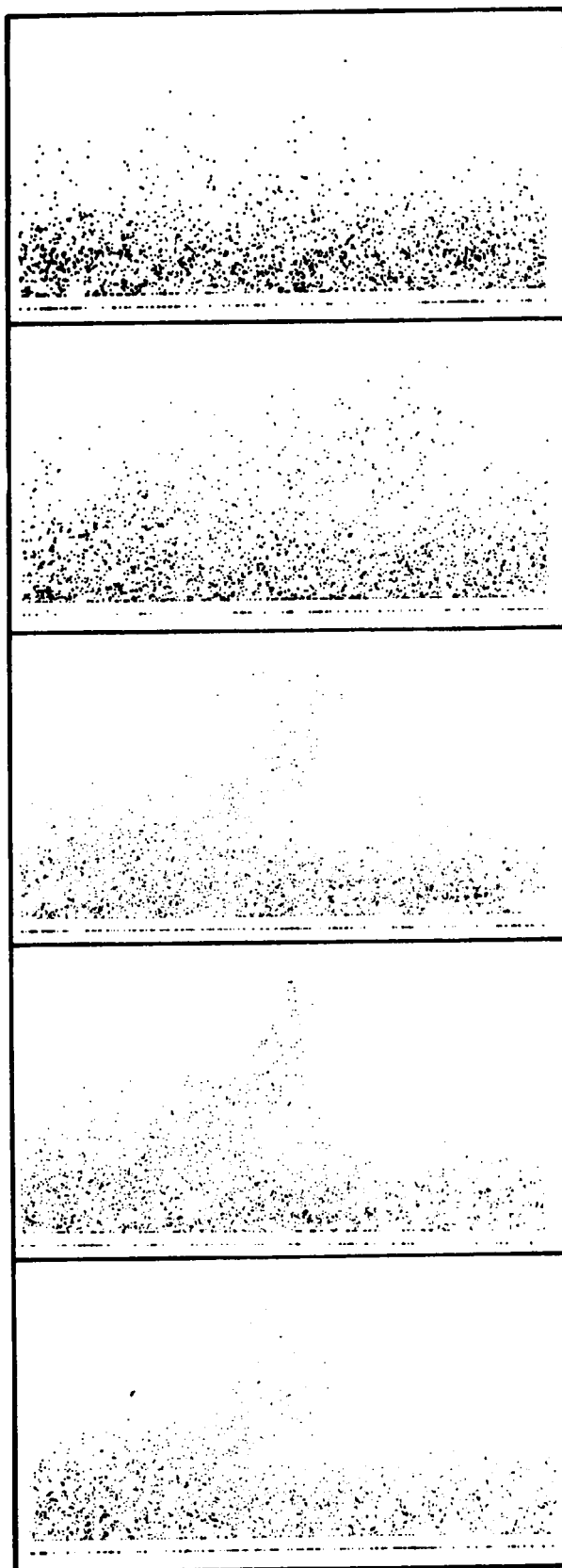


FIG. F.2 ELECTRON BEAM PROFILES. $(\omega/\omega_c = 0.706, 2a/\lambda_r = 0.879, \varphi_s = 0.512, \varphi_a = 0.480)^{34}$

In the above definitions V_{rf} is the sole-to-anode potential and the other parameters are as defined previously.

Figure F.3 indicates the normalized r-f power as a function of time with anode potential as a parameter as computed using the time-dependent analysis. This figure shows that increasing the d-c anode potential ϕ_a while keeping the d-c magnetic field ω_c/ω constant increases the gain of the amplifier as time increases. This variation is similar to keeping ϕ_a constant and decreasing ω_c/ω , since increasing the d-c electric field or decreasing the d-c magnetic field, while keeping the other parameters fixed, results in greater vertical forces on the electrons. Thus, under either of these conditions the electrons approach the r-f structure more closely where the r-f fields are stronger. Under the influence of stronger r-f fields the electrons are then decelerated and bunched more effectively and a greater energy conversion is achieved. In Chapter V (see Figs. 5.11 and 5.14), it was found that decreasing the d-c magnetic field did produce the anticipated larger r-f gain. However, these results were obtained as a function of longitudinal distance and an exact comparison with the results of the time-dependent analysis is therefore difficult. Nonetheless, longitudinal distance and time are related by Eq. F.1 which shows qualitatively that as time increases, longitudinal distance increases. Thus, the r-f gain solutions obtained in both of the analyses showing larger gains for similar d-c variations are seen to produce qualitatively equivalent results.

A further comparison of the r-f potential solutions in each theory can be made by observing Fig. F.4. This figure indicates, as in Fig. F.3, that increasing the d-c anode potential increases the

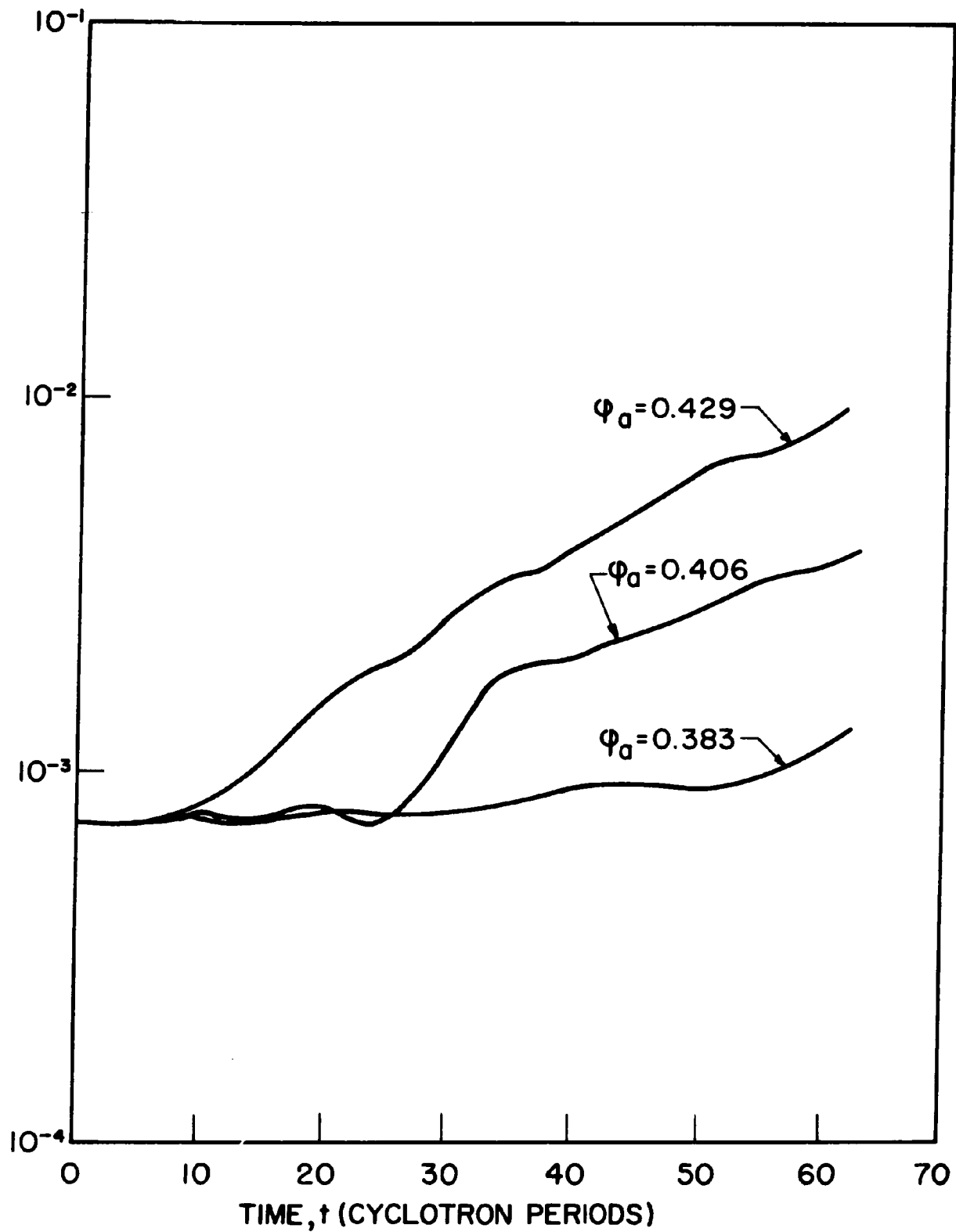


FIG. F.3 VARIATION IN R-F POWER VS. TIME WITH THE ANODE
POTENTIAL AS A PARAMETER. ($\omega/\omega_c = 0.319$, $2a/\lambda_r$
 $= 0.5$, $\phi_s = 0.406$)³⁴

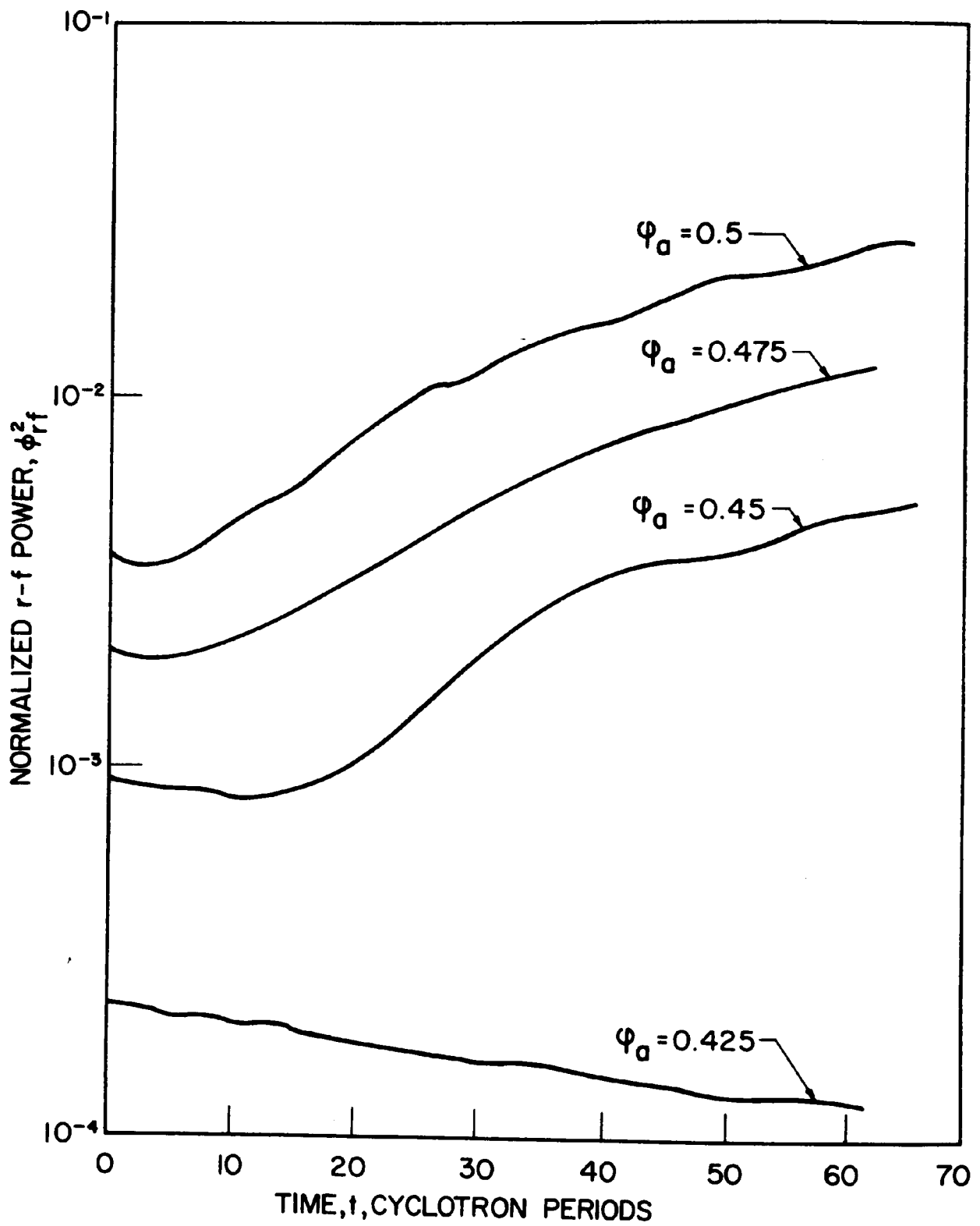


FIG. F.4 VARIATION OF R-F POWER VS. TIME WITH THE INPUT R-F AMPLITUDE AND THE ANODE POTENTIAL AS PARAMETERS.

$$(\omega/\omega_c = 0.795, 2a/\lambda_r = 1, \phi_s = 0.506)^{34}$$

amplifier gain. However, in addition to this, Fig. F.4 shows the same results as were deduced in Chapter IV (see Fig. 4.16), namely, that a minimum r-f input signal level is required if gain is to occur at all. This implies that in order to obtain a favorable electron-wave interaction, it is necessary to have large r-f potentials and hence fields. Under these conditions, as explained above, a stronger interaction is then possible.

Yu's solutions also include an investigation of nonsynchronous operation with the parameters adjusted such that $v_o \neq v_d$. However, the solutions indicate that near synchronous operation results in the greatest gain.

The theory and results of the time-dependent analysis have thus been outlined and compared with the present study. Electron bunching in the interaction region was evident in both analyses; however, a specific electron trajectory comparison was not possible because this information was not obtained in the time-dependent analysis. Comparisons of the solutions for the r-f amplitude for different d-c field conditions and various input r-f signal levels were also made and the results were observed to be equivalent.

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LIST OF SYMBOLS

$A_{dc}(q)$	Normalized d-c potential amplitude defined as a function of normalized distance q in Eq. 2.55.
$A_{dc}(z)$	Normalized d-c potential amplitude defined as a function of longitudinal distance z in Eq. 2.54.
$A_{rf}(q)$	Normalized amplitude of the r-f potential defined as a function of normalized longitudinal distance q by Eq. 2.36.
$A_{rf}(z)$	Normalized amplitude of the r-f potential defined as a function of longitudinal distance z by Eq. 2.73.
A_{rf_0}	Normalized amplitude of the r-f potential at the r-f input plane $q = 0$.
a	Width of the interaction region in the magnetic field direction, m.
B_0	D-c magnetic field intensity in the x-direction, webers/m ² .
b'	Sole-to-anode spacing, m.
C_b	Capacitance per section for a backward-wave structure, farads·m.
C_f	Capacitance per section for a forward-wave structure, farads/m.
D	Gain parameter of the tube defined by $D \triangleq [\omega_c / \omega(I_0 Z_0 / 2V_0)]^{1/2}.$
d	Loss parameter of the slow-wave circuit, $d = R/2\omega LD.$
E_0	D-c electric field between the sole and the anode, $E_0 = V_a/b'.$
E_y, E_z	y-directed and z-directed total electric field components.
$\vec{E}_{dc}$	D-c electric field, volts/m.
$\vec{E}_{sc}$	Space-charge electric field, volts/m.

E_{sc-y}, E_{sc-z}	y-directed and z-directed space-charge electric field components.
$\mathcal{E}$	The sum of the kinetic and potential energy of an electron defined by Eq. A.5.
F_y, F_z	y- and z-component space-charge weighting functions defined by Eqs. B.10 and B.11. — —
f	R-f frequency, cycles/sec.
h	Width of the emitting-sole electrode, m.
I_o	Thermionic emission current from the sole electrode given by $I_o = hL_e J_{th}$.
I_y, I_z	y- and z-component space-charge integrals defined by Eqs. B.15 and B.16.
J_{th}	Thermionic emission current density, amp/m ² .
k	Boltzmann's constant, joules/°K.
L	Length of the crossed-field tube from r-f input to r-f output, m.
L_e	Length of the emitting-sole electrode, m.
L_b	Self-inductance per section for a backward-wave structure, henries.m.
L_f	Self-inductance per section for a forward-wave structure, henries/m.
M	Number of electron emission points.
m	Number of electrons injected from each emission point.
p	Normalized vertical coordinate, $p = y/b'$.
p_o	Normalized vertical initial coordinate of an electron ($p_o = 0$).
$p_{1,2}$	Normalized vertical position of the favorable and unfavorable hard-kernel bunches given by $p_{1,2} = y_{1,2}/b'$.
$P_{1,2}$	Normalized vertical position of the favorable and unfavorable hard-kernel bunches given by $P_{1,2} = \beta_o b' p_{1,2}$.
q	Normalized longitudinal coordinate defined by $q = \frac{\Delta}{D} \beta_o z$.

q_0	Normalized initial longitudinal position of an electron.
q_L	Normalized longitudinal distance from the r-f input plane to the r-f output plane.
q_e	Normalized longitudinal length of the emitting-sole electrode.
q_α	Normalized longitudinal distance at which the d-c magnetic field begins to decrease linearly.
R_b	Resistance per section for a backward-wave structure, ohms/m.
R_f	Resistance per section for a forward-wave structure, ohms/m.
T	Temperature of the cathode, °K.
t	Time variable, sec.
t_0	Emission time of an electron, sec.
u_y	Normalized y-directed velocity parameter defined by $u_y \triangleq (\omega_c/\omega v_0)(dy/dt)$.
u_z	Normalized z-directed velocity parameter defined by $(dz/dt) \triangleq v_0(1+2Du_z)$.
u_{y0}	Initial y-directed electron velocity.
u_{z0}	Initial z-directed electron velocity.
$u_{y1,2}$	Normalized y-directed velocity parameters for the favorable and unfavorable hard-kernel bunches given in Eq. 6.7.
V_a	D-c sole-to-anode voltage, volts.
V_{in}	R-f voltage impressed upon the slow-wave circuit, volts.
V_{oc}	Voltage equivalent of the r-f wave phase velocity defined by $V_{oc} \triangleq v_0^2/2\eta$, volts.
$V(y,z,t)$	Scalar sum of the d-c and r-f potentials defined by Eq. 2.50, volts.

$V(p, q, \Phi)$	Scalar sum of the d-c and r-f potentials defined as a function of normalized coordinates by Eq. 2.56, volts.
$V_{SC}(y, z)$	Space-charge potential given by Eq. B.7, volts.
V_T	Total electric potential including the d-c, r-f and space-charge potentials, volts.
$V_{dc}(y, z)$	D-c potential defined by Eq. 2.54.
$V_{dc}(p, q)$	D-c potential in terms of normalized variables defined by Eq. 2.55.
$V_{rf}(z, t)$	R-f potential along the slow-wave structure, volts.
$V_{rf}(q, \Phi)$	R-f potential along the slow-wave structure defined as $V_{rf}(q, \Phi) = \text{Re}[(I_0 Z_0 / D) A_{rf}(q) \exp(-j\Phi)]$, volts.
$V_{rf}(p, q, \Phi)$	R-f potential in the interaction region defined as a function of normalized coordinates by Eq. 2.53, volts.
$\vec{v}$	Velocity vector of the electron, m/sec.
v_c	Characteristic velocity corresponding to the cathode temperature defined as $v_c \triangleq (2kT/m)^{1/2}$, m/sec.
v_d	Average electron drift velocity defined by $v_d = E_0 / B_0$, m/sec.
v_d^{SC}	Average electron drift velocity due to the d-c and space-charge fields defined as $v_d^{SC} = (\vec{E}_{dc} + \vec{E}_{sc}) / B_0$, m/sec.
v_o	Unperturbed circuit phase velocity of the r-f wave, m/sec.
v_{ph}	The phase velocity of the r-f wave defined by Eq. 4.2, m/sec.
v_r	Velocity coefficient defined by $v_r \triangleq u_{y_0} / (1 + 2Du_{z_0}) \cdot 1 / (1 - D(\partial\theta_c / \partial q)) $, m/sec.
v_x, v_y, v_z	Velocity components of the electron in three-dimensional velocity space, m/sec.
y	Vertical position of an electron, m.
y_o	Initial vertical position of an electron ($y_o = 0$).

Z_0	Characteristic impedance of the r-f structure, ohms.
z	Longitudinal coordinate, m.
z_0	Longitudinal initial position of an electron, m.
α	Emission angle as displayed in Fig. 2.4, radians.
α	Slope of the linear d-c magnetic field taper in the longitudinal direction.
β	Phase constant of the r-f wave given by $\beta = \partial\Phi/\partial z$, 1/m.
β_c	Cyclotron phase constant defined by $\beta_c \triangleq \omega_c/v_0$, 1/m.
β_0	Phase constant of the "cold" circuit wave defined by $\beta_0 \triangleq \omega/v_0$, 1/m.
δ	Secondary emission ratio.
ϵ_0	Dielectric constant of free space, farads/m.
η	Charge-to-mass ratio for the electron.
θ_c	Phase angle of the r-f potential relative to the electron.
λ, Λ	Induced current integrals defined by Eqs. 3.13 and 3.14, respectively.
$\xi(y)$	D-c potential parameter given by $\xi(y) = y/b'$.
$\xi(p)$	D-c potential parameter as a function of the — normalized vertical coordinate given by $\xi(p) = p$.
$\rho(y, z, t)$	Instantaneous charge density of the electron stream, coulombs/m ³ .
ρ_1	First forward space harmonic of the r-f induced charge, coulombs/m ³ .
$\rho(z, t)$	Induced charge on the r-f structure, coulombs/m.
ρ_0	Space-charge density at the entrance plane ($y = 0$) defined by $\rho_0 \triangleq - (J_{th}/v_c) \rho'_0$, coulombs/m ³ .

ρ'_0	Normalized charge density at the entrance plane ($y = 0$) given by $\rho'_0 = \sqrt{\pi}$.
$\Phi(z, t)$	Instantaneous phase of the fundamental r-f wave component relative to the stream defined by $\Phi \triangleq \omega z/v_0 - \omega t - \theta_c$.
Φ_0	Entrance phase of an electron.
$\psi(y)$	R-f coupling function defined by $\psi(y) \triangleq \sinh \beta y / \sinh \beta b'$.
$\psi(p)$	Normalized r-f coupling function defined by $\psi(p) \triangleq \sinh \beta_0 b' p / \sinh \beta_0 b'$.
ω	Angular frequency of the r-f wave, radians/sec.
ω_c	Cyclotron angular frequency defined by $\omega_c \triangleq \eta B_0$, radians/sec.
ω_{pc}	Cathode plasma frequency, defined by Eq. 5.1, radians/sec.
Ω	Emission angle of an electron with respect to the cathode normal.