Homework 12– Assigned: April 8, 2026, Due: April 15, 2026

1. Competitive Bidding Analysis –Number of Opponents is Uncertain

The table below shows the costs estimated by a contractor for the past 30 projects and the bid prices submitted by his opponents. (The table does not show the contractor's own bid price. Also, each of the columns B_i does not show the bid prices of any particular opponent.)

The contractor is about to bid a new job but does not know how many opponents K will also bid on this job. His only information is that given in the table below. Use the relative frequencies for $K=k$ from past projects in the table below to estimate the probabilities $P[K=k]$ for k = 1, 2, …, 7 opponents and use them to find the optimum bid-to-cost ratio for the new job he is about to bid based on:

a. Friedman's formula
b. Gates' formula

Prepare one graph that shows $P[Win|FBC]$ vs %Markup (= x_0-1 = FBC-1) for both methods (Friedman and Gates) and one graph that shows $E[V|FBC]/E[C]$ vs %Markup (= x_0-1 = FBC-1) for both methods. Compare your graphs and answers to those from the previous homework and explain the differences.

Project No.	Est. Cost ($1,000)	Opponent's Bids ($1,000)						
		B1	B2	B3	B4	B5	B6	B7
1	96.90	103.10	102.47					
2	555.06	584.76	583.00	575.08	577.94	586.96		
3	244.36	264.09	256.49					
4	988.52	1,043.98	1,034.26	1,041.55	1,053.69	1,089.32	1,088.51	
5	718.80	760.55	755.44	796.62				
6	308.28	348.86	327.42					
7	201.25	203.19	208.69	205.20				
8	481.50	484.05	488.08	505.08	491.55	488.09		
9	413.11	447.48	438.29	410.10				
10	293.83	318.42	313.43	304.15	304.38	315.17	311.00	
11	1,065.74	1,233.36	1,095.98	1,125.79	1,106.78	1,145.66	1,158.19	1,098.14
12	932.40	976.85	922.25					
13	466.62	488.52	490.36					
14	1,219.19	1,247.89	1,209.79	1,322.60	1,209.29			
15	254.43	274.48	266.46	261.32	256.49	264.81	258.64	
16	193.17	207.90	209.48	200.14	200.22	207.27		
17	767.21	822.12	856.44	841.46				
18	107.88	120.33						
19	499.80	517.44	528.02	545.66	522.93			
20	645.42	743.37	748.02	722.85	746.11	757.05	684.55	673.60
21	887.86	911.60	872.73	948.75				
22	615.49	680.39						
23	520.13	605.88	538.92	558.58	600.05	600.11	559.87	
24	200.78	211.21	207.76	209.96	215.91	205.56	199.29	
25	854.25	894.20	888.02	974.47	930.52	948.32	985.72	877.85
26	444.75	469.06	476.93	497.40	496.12	463.76	465.41	456.09
27	650.28	676.75	657.36	700.33	714.47	719.19	685.65	
28	954.91	965.10	1,004.30					
29	900.62	895.40	881.48	971.27	912.46	928.46	958.39	940.30
30	834.52	920.71	919.70	897.39	900.09	890.63	890.74	

Multiattribute Utility

20.2. *General II*

Given $$U_x(X) = \frac{X(X+6)}{160} \qquad 0 \le X \le 10$$

$$U_y(Y) = \frac{(Y^{1/2} - 5)}{5} \qquad 25 \le Y \le 100$$

and the following indifference statements:

$$(10, 25) \sim [(10, 100),\ 0.6;\ (0, 25)]$$

$$(0, 100) \sim [(10, 100),\ 0.6;\ (0, 25)]$$

(*a*) Give the value of $U(X,Y)$ for all four corner points for which the multiattribute utility is defined.
(*b*) Find by interpolation $U(4, 25)$; $U(4, 100)$.
(*c*) Write the general formula for $U(X,Y)$.
(*d*) Solve for the value of K in the formula.
(*e*) Calculate $U(4,100)$ by using the formula.

20.7. *Developer*

A developer reviewing the design of a \$20M building faces a tradeoff between construction costs and energy consumption. The current design requires 10 million btu/yr of heating and cooling power. Many modifications such as the addition of awnings, improved insulation, or solar panels could save up to 7 billion btu/yr, at a cost of \$4M. Since the range of possibilities and the uncertainties are rather confusing, she wants you to help sort out her preferences.

From interviews you determine that, for building costs alone, she is indifferent between

\$22.5M and (\$24M, 0.5; \$20M)
\$23.4M and (\$24M, 0.5; \$22.5M)
\$21.5M and (\$22.5M, 0.5; \$20M)

regardless of the level of energy used. Similarly, for energy measured in millions of btu/yr, she is indifferent between

5 and (10, 0.5; 3)
3.75 and (5, 0.5; 3)
7 and (10, 0.5; 5)

regardless of the level of costs. For combinations of costs and energy, she is indifferent between

(\$20M, 10) = the current plan
and [(\$24M, 10), 0.2; (\$20M, 3)]

and also

(\$24M, 3) = maximum energy conservation
and [(\$24M, 10), 0.3; (\$20M, 3)]

(*a*) Solve the indifference statements for their implied utility.
(*b*) Determine which plan the developer is apt to prefer:

Plan	Cost (\$M)	Energy use
A. Current design	20	10
B. High insulation	21	7
C. Solar heating	22.5	5
D. Sheltered windows	23	4
E. Maximum conservation	24	3

(*c*) Would the developer prefer the current design or the alternative [(22.5, 7), 0.6; (22.4, 3.75)]?
(*d*) Find K by applying the general formula and check the results from part (*b*).