Homework 10 – Assigned: March 25, 2026, Due: April 1, 2026

Construction Risk Analysis Using PERT Formulas

Contractor I expects a profit of 10% over costs on work with no risk. He wishes to bid a section of highway containing a single bridge over a stream. He estimates the following costs:

i	Cost Item C_i	Estimated Values for Cost Item C_i (\$)		
		Most Optimistic	Most Likely	Most Pessimistic
1	Roadway	1,800,000	2,000,000	2,200,000
2	Bridge foundation	1,000,000	1,200,000	2,000,000
3	Bridge superstructure	450,000	500,000	600;000

For each cost item, use the PERT formulas to find m_i and σ_i and then calculate σ_i^2.

(a) What is the range estimate $m \pm \sigma$ for the total cost of the entire project? (*)

(b) What would the contractor ask as a lump sum price in order to receive an expected profit equal to 10% of expected cost? (This question has nothing to do with utilities.)

(c) What is the contractor's utility of receiving with certainty a profit equal to 10% of the expected cost?

(d) What lump sum would the contractor bid to receive an *expected utility* equal to the answer to (c)? Compare this to the answer of (b).

(e) Suppose that Contractor I has already signed a contract for the price in (d). Up to how much would he be willing to pay a subcontractor to construct the bridge (both the foundation and superstructure) while Contractor I constructs only the roadway?

(f) Calculate (a) if the correlation coefficients are estimated to be: (*)

C_i, C_j	Cost Items	Correlation Coefficient: $\rho_{i,j}$
C_1, C_2	Roadway, Foundation	$\rho_{1,2} = 0.5$
C_1, C_3	Roadway, Superstructure	$\rho_{1,3} = 0.5$
C_2, C_3	Foundation, Superstructure	$\rho_{2,3} = 0.85$

(g) Calculate (d) given (f).

(h) Calculate (e) given (f).

(*) The *variance* of a sum of cost items is the sum of *variances* plus (any) *covariance* terms. To discretize a symmetrical distribution (e.g., for the total project cost) use 5 branches representing outcomes that are (-2, -1, 0, 1, 2) standard deviations away from the mean.

If you wish, you may solve this problem in Excel and use the add-in function YValue (available on the CEE631 website on Canvas) to automate finding the utility of money from a table.

Construction Risk Analysis Using Perry and Greig Formulas

Rework *all* questions for the previous problem for the construction of a roadway and bridge. This time use the Perry and Greig formulas instead of the PERT formulas. The necessary percentiles and modes for estimating the distributions of the project's cost items are shown below.

		Estimated Values for Cost Item C_i (\$)		
i	Cost Item C_i	p_5	M	p_{95}
1	Roadway	1,890,000	2,000,000	2,110,000
2	Bridge foundation	1,077,000	1,200,000	1,620,000
3	Bridge superstructure	470,000	500,000	550,000

Use the Perry and Greig formulas:

(a) To find m_i and σ_i for each cost item.

(a) To find the *expected utilities* for profit lotteries (see hint below).

Compare your answers and ease of calculations to those using PERT formulas.

Hint: In this problem DO NOT discretize the distribution for the total project cost using 5 branches representing outcomes that are (-2σ, -1σ, 0σ, 1σ, 2σ) away from the mean m.

Instead, find and use the percentiles for total cost, for profit, and for utility as described below and in the class notes.

For a *symmetrical* distribution (e.g., for the total project cost):

$$m = \text{mean} = \text{mode} = \text{median} = p_{50}$$

Use the Perry and Greig formulas to back-calculate the percentiles p_5 and p_{95} for the total cost (using the expected value m and the standard deviation σ of the total cost):

$$p_5 = m - \frac{3.25s}{2} \text{ and } p_{95} = m + \frac{3.25s}{2}$$

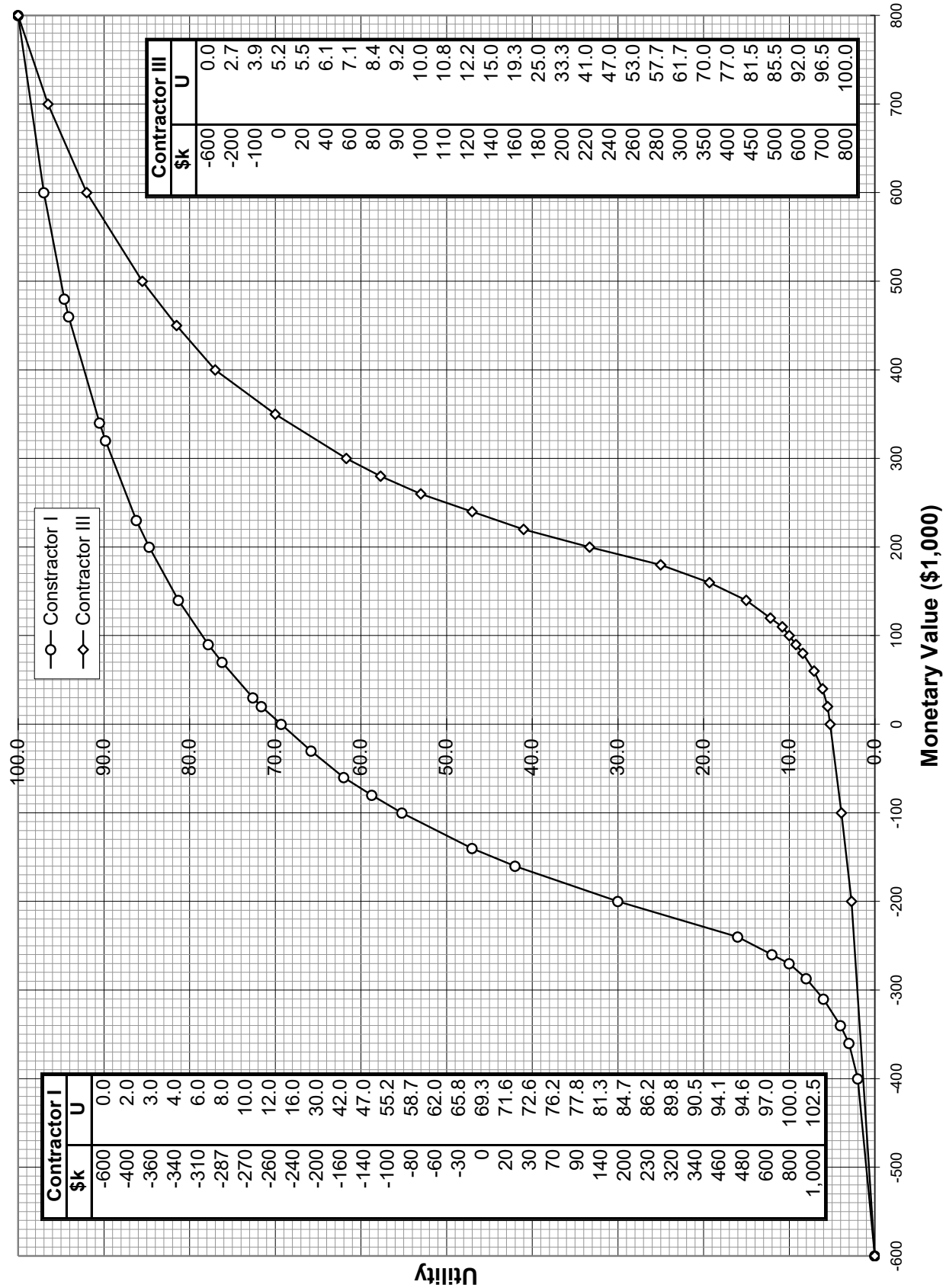
From the percentiles of cost, find the percentiles of profit, and the percentiles of the utility of profit, and then calculate the *expected utility of profit* using the formula:

$$E[u] = 0.63u(p_{50}) + 0.185[u(p_5) + u(p_{95})]$$

(The results will be quite accurate.)

Consult your notes and use the correct formula for computing *the variance of a sum of random variables*. Remember, that the *variance* of a sum of cost items is the sum of *variances* plus (any) *covariance* terms. (I.e., adding the standard deviations of cost items *does not* give the standard deviation of the total cost.)

If you wish, you may solve this problem in Excel and use the add-in function YValue (available on the CEE631 website on Canvas) to automate finding the utility of money from a table.



Contractor I	
$k	U
-600	0.0
-400	2.0
-360	3.0
-340	4.0
-310	6.0
-287	8.0
-270	10.0
-260	12.0
-240	16.0
-200	30.0
-160	42.0
-140	47.0
-100	55.2
-80	58.7
-60	62.0
-30	65.8
0	69.3
20	71.6
30	72.6
70	76.2
90	77.8
140	81.3
200	84.7
230	86.2
320	89.8
340	90.5
460	94.1
480	94.6
600	97.0
800	100.0
1,000	102.5

Contractor III	
$k	U
-600	0.0
-200	2.7
-100	3.9
0	5.2
20	5.5
40	6.1
60	7.1
80	8.4
90	9.2
100	10.0
110	10.8
120	12.2
140	15.0
160	19.3
180	25.0
200	33.3
220	41.0
240	47.0
260	53.0
280	57.7
300	61.7
350	70.0
400	77.0
450	81.5
500	85.5
600	92.0
700	96.5
800	100.0