Homework 4 – Assigned: February 4, 2026, Due: February 11, 2026

Homework 4 includes all questions for problems 13 and 14.

Constructing Decision Trees – Value of [Perfect, Sampled] Information

13. An inventor has developed an electronic instrument to monitor the impurities in the metal produced by a smelting process. A company that markets this type of equipment is considering the purchase of the patent rights to this instrument for $40,000. The company feels that there is a chance of 1/5 that the device will become a successful seller. It is estimated that, if the device is successful, it will produce net revenues of $150,000 each year for the next 5 years. If the product is not a success, no revenues will be received. The company's interest rate is 15%.

(a) Draw a decision tree describing the decision alternatives and determine the best decision policy.
(b) If there is a market research group that can provide perfect information about the success of this product, what would be the most the company should be willing to pay for their service?
(c) Suppose the market research group can make a market survey that with probability 0.7 will give a favorable result if the device will be a success, and with probability 0.9 will give an unfavorable result if the device will not sell. How much would this survey be worth to the company?

14. An electronics firm is trying to decide whether or not to manufacture a new communications device. The decision to produce the device means an investment of $5 million. The demand for such a device is not known.
If demand is *high*, the company expects a return of $2.0 million each year for 5 years.
If the demand is *moderate*, the return will be $1.6 million each year for 4 years.
If the demand is *light* the return will be $0.8 million each year for 4 years.
It is estimated that the probability of a *light* demand is 0.1, the probability of *moderate* demand is 0.4 and the probability of a *high* demand is 0.5. Interest is 10%.

(a) On the basis of expected present-worth, should the company make the investment?
(b) Someone has proposed that a survey be taken to establish the actual demand to be experienced. What is the maximum possible value of such a survey?
(c) A survey is available which has the characteristics shown in the table. What is the most that the company should pay for this survey? If the survey costs $75,000, should the company pay to make this survey? What is the expected profit if the survey says unfavorable and the device is produced? Should the company produce the device if the survey says unfavorable?

		Survey results say *favorable* or *unfavorable* with the following probabilities:	
		Favorable	**Unfavorable**
If demand is:	**High**	0.9	0.1
	Moderate	0.5	0.5
	Low	0.0	1.0

Rock and Soil Testing – Bayes Theorem

2.13. A large dam is being planned, and the engineer is interested in the source of fine aggregate for the concrete. A likely source near the site is rather difficult to survey accurately. From surface indications and from a single test pit, the engineer believes that the magnitude of the source has the possible descriptions:

"S_1: 50% of adequate"; "S_2: Adequate"; or "S_3: 150% of possible demand".

Based on these observations, the engineer assigns the following probabilities for these states.

State	Probability
S_1: 50% of adequate	0.5
S_2: Adequate	0.4
S_3: 150% of demand	0.1
	1.0

Before ordering a second test pit, the engineer decides that the various likelihoods of the sample's possible indications (Z_1, Z_2, Z_3) from the test pit depend upon the (unknown) true state as follows:

$P[Z_i|S_j]$ = P [Sample Indication | State]

	State		
Sample indication Z_i	**S_1: 50% of adequate**	**S_2: Adequate**	**S_3: 150% of demand**
Z_1: favoring 50% of adequate	0.50	0.40	0.20
Z_2: favoring adequate	0.40	0.50	0.40
Z_3: favoring 150% of demand	0.10	0.10	0.40
	1.00	1.00	1.00

(a) Find the probabilities $P[Z_1]$, $P[Z_2]$, $P[Z_3]$ of observing the sample indications Z_1, Z_2, Z_3.
(b) Suppose that the second test pit is then dug, and it indicated Z_2 (favoring adequate). Compute the posterior probabilities of state given that the second test pit indicated Z_2.
(c) Suppose that a third test pit was then dug, and also indicated Z_2 (favoring adequate). Compute the posterior probabilities of state given that the second test pit indicated Z_2 and the third test pit indicated Z_2.
(d) Compare prior and posterior state probabilities.

2.22. A preliminary investigation of a site leads an engineer to state that the relative weights are 2 to 5 to 3 (respectively) that the unconfined compressive strength of the soil below is S_1: 800, S_2: 1000; or S_3: 1200 psf (the only three values considered possible, for simplicity). "Undisturbed" samples of the soil will be obtained by boring and tested to gain further information. Owing to the difficulties in obtaining such samples and owing to testing inaccuracies, the following frequencies of indicated strengths are considered applicable for each specimen:

$P[Z_i|S_j]$ = P [Indicated Strength | State]

	State (true strength)		
Indicated Strength, Z_i	**S_1: 800 psf**	**S_2: 1000 psf**	**S_3: 1200 psf**
Z_1: 800 psf	0.8	0.4	0.2
Z_2: 1000 psf	0.2	0.5	0.3
Z_3: 1200 psf	0.0	0.1	0.5
	1.0	1.0	1.0

Suppose that the engineer calls for a sampling plan of two independent specimens.

(a) Find the conditional probabilities of each of the possible outcomes of this sample of size two, given that the true strength is S_3: 1200 psf.
(b) Suppose that the results of the sampling were "one specimen indicating Z_2: 1000 psf" and "one specimen indicating Z_1: 800 psf". Find the engineer's posterior probabilities of the strength.
(c) Suppose that after these two specimens, the engineer continued sampling and found an uninterrupted sequence of specimens indicating Z_3: 1200 psf. After how many specimens could he stop:
    (i) Confident that the strength was not actually S_1: 800?
    (ii) At least "90% confident" that the true strength was S_3: 1200?