Homework 1 – Assigned: January 14, 2026, Due: January 21, 2026

Events and Probability

1. Among the digits {1,2,3,4,5} first one is chosen and then a second selection is made among the remaining four digits.
   (a) Show the complete sample space S of all twenty possible simple events (i,j), S={(i,j), i = 1st digit, j = 2nd digit}.

   Assume that all twenty possible simple events (i,j) have the same probability. Find the probability that an odd digit will be selected:
   (b) The first time
   (c) The second time
   (d) Both times.
2. Two dice are thrown, one red and one blue.
   (a) Show the complete sample space S of all 36 possible simple events {(r,b)}

   Let A be the event that the sum of the faces is odd, B the event of at least one ace (i.e., the number "one").
   (b) Show the simple events (r,b) that belong to the compound events $A \cap B$, $A \cup B$, $A \cap B'$.
   (c) Find the probabilities of the compound events $A \cap B$, $A \cup B$, $A \cap B'$.
3. A coin is tossed until for the first time the same result appears twice in succession.
   We are interested in the number of required tosses, *N*.
   The PMF of *N* is $P[N = n] = (1/2)^{n-1}$, where n=2,3,…
   (a) Find $P[N \leq 5]$
   (b) Write $P[N = even]$ as an expanded sum of probabilities.
   (c) Write $P[N = odd]$ as an expanded sum of probabilities.
   (d) Compare your answers to (b) and (c).
   (e) Based on (d) find $P[n = even]$ and $P[n = odd]$.
4. You require highly trained welders on a project. If you carefully pick trainees from available welders, you estimate the probability *P*[welder qualifies] = 0.5.
   (a) If you trained 6 welders, what is the probability of qualifying exactly 2 of them?
   (b) If you trained 6 welders, what is the probability of qualifying at least 2 of them?
   (c) How many welders must you train to have a 90% probability of having at least one qualified welder?
   (d) How many would you need to train to be 70% sure of qualifying at least 2 of them?
   (e) If you trained that many, what is the probability that at least one would qualify?

   Note: 1. $P[A] = 1 - P[A']$
   2. A trial-and-error solution may sometimes be appropriate.

5. You need to estimate the daily production of a shovel from a gravel pit. You have dug a test pit but are still unsure of the strength of cementation between the stones. You estimate the following probabilities for the occurrence of cementation levels c_j

$$P[c_1] = 0.2,\ P[c_2] = 0.4,\ P[c_3] = 0.3,\ P[c_4] = 0.1.$$

For each possible level of cementation c_j, you estimate the daily production x_i to follow the probabilities below.

$P[x_i \mid c_j]$	Cementation level, c_j			
Output, x_i (cu yd/day)	1	2	3	4
$x_1 = 800$	0.1	0.2	0.3	0.4
$x_2 = 900$	0.2	0.3	0.3	0.3
$x_3 = 1000$	0.4	0.3	0.3	0.2
$x_4 = 1100$	0.3	0.2	0.1	0.1

Describe the events in terms of x_i and c_j and calculate the probabilities for the following events:

(a) Given that cementation level is c_3, production is 900 cu. yd. or greater.
(b) Cementation is c_2 and production is 1000 cu. yd.
(c) Cementation is (c_1 or c_3) and production is 900 cu. yd.?
(d) Cementation is (c_1 or c_2) and production is *at least* 1000 cu. yd.?
(e) Production is 1000 cu. yd.