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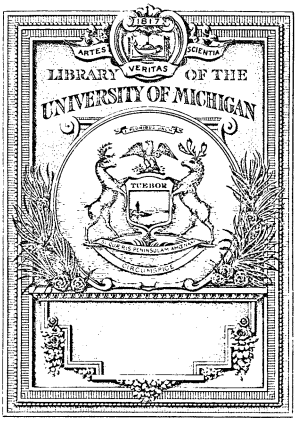
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ON THE

COEFFICIENT OF EXPANSION OF QUARTZ

BY

HARRISON McALLISTER RANDALL

THESIS

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ON THE COEFFICIENT OF EXPANSION OF QUARTZ.

BY HARRISON MCALLISTER RANDALL.

THE important part played by quartz in many scientific investigations has made a determination of its physical constants a matter of considerable importance. Among the determinations of the coefficient of thermal expansion, those by Fizeau[1] in 1864, Benoit[2] in 1888, Reimerdes[3] in 1896 and Scheel[4] in 1902 may be cited as the most important. The optical method devised and used by Fizeau has formed the basis of the methods employed in all the subsequent investigations. The range of temperature employed by Fizeau extended from 2° to 60°, that of Benoit from 6° to 80°, that of Reimerdes from 5° to 220°, and that of Scheel from 16° to 100° C. In addition to these investigations, there have been made rough determinations, intended simply to show the characteristics of the expansion of quartz by Le Chatelier[5] 1889, using a photographic method, and by Sahmen and Tammann[6] in 1903 by means of a self-registering dilatograph. Le Chatelier's observations extended from 15° to 1000°, and those of Sahmen and Tammann to 700°.

In the present investigation the method of Fizeau has been employed for a range of temperature of approximately 500° C. The series of determinations first to be described was made in 1901, and, for reasons to be stated subsequently, it seemed desirable to repeat the work with such changes in the apparatus as would insure the elimination of certain errors possible with the first arrangement. Opportunity to do this was not found till the present year, when a redetermination under quite a different set of conditions was made.

[1] Ann. de Chem. et de Phys., 2, 1864, p. 143; 8, 1866, p. 335; Comp. Rend., 58, p. 923; 62, p. 1133.

[2] Trav. et Mém. du Bureau Int. Poids et Mes, I., 1881, VI., 1888.

[3] Inaug. Diss., Jena, 1896.

[4] Ann. der. Physik, 9, 1902, pp. 837–853.

[5] Comp. Rend., 108, p. 1046.

[6] Ann. der Physik, 10, 1903, p. 879–889.

METHOD AND APPARATUS.

The optical method for measuring small expansions by the displacement of interference fringes, used in this investigation, is a modification of the method used by Fizeau, and later employed by Benoit. The apparatus (Fig. 1) for the measurement of the expansion of quartz is that first suggested by Pulfrich,[1] and used by Rei-

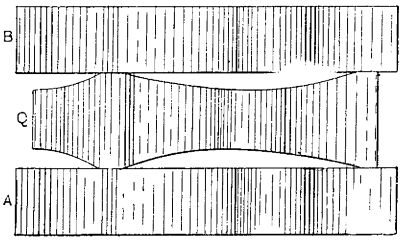

Fig. 1.

merdes in his work on the coefficient of expansion of quartz. *A*, the bed-plate, is a quartz plate 38 mm. in diameter and 7 mm. thick. Its upper surface, cut perpendicular to the principal axis of the quartz, is plane polished. The lower surface, cut parallel to the upper, is left unpolished. *Q* is a quartz ring approximately 1 cm. long, 34 mm. in external diameter and 25 mm. in internal diameter. It has been made with its axis accurately parallel to the principal axis of the quartz. Each of its ends has been ground away so as to leave three equidistant feet as shown in the figure. This ring is placed upon plate *A* and upon the ring plate *B* is placed. Plate *B* is of the same material and size as plate *A* and its surfaces, plane polished, and perpendicular to the principal axis of the quartz are inclined at an angle of 20′. There is a fourth quartz plate, *C*, about 2 mm. thick, and with a diameter of 2.2 cm. cut with its upper surface perpendicular to the principal axis of the quartz. This surface is plane polished while the under surface, unpolished, has been ground away so as to form three feet equidistant from the original surface. Plate *C* is placed inside the quartz ring upon plate *A*. One of the feet of plate *C* was ground away until the upper surface of the plate and the lower surface of *B* were properly inclined to

[1] Zeitschr. für Krystall., 31, p. 372, 1899.

give interference fringes of the desired width. These fringes were found to be perfectly straight and equidistant, showing that the reflecting surfaces were truly plane. The quartz ring Q and the small quartz plate C were furnished by Carl Zeiss, Jena, and the two larger plates A and B, by Breashear. For convenience of observation, the quartz plate B bears at the center of its lower surface a small circle which is sharply focussed by the telescope and furnishes a fixed point with reference to the fringe system. Each dark band of the fringe system represents a definite thickness of the air layer between the two reflecting surfaces. The thickness of this air layer under any dark band differs from its thickness under an adjacent dark band by $\frac{1}{2}\lambda$, where λ is the wave-length of the light used. When the system has been heated, the two reflecting surfaces are separated owing to the expansion of Q and each band, representing as it does a definite thickness of the air layer, has to move toward the thin edge of the air layer. The number of such bands passing the small circle on the cover plate represents the change in thickness of the air layer directly under the circle and thus measures the expansion of the quartz ring Q. The total expansion of the quartz for any temperature interval $(t_2 - t_1)$ may be represented by $f \times \lambda/2$ where f is the number of dark bands passing the small reference circle of the cover plate, and λ the wave-length of the light used. The coefficient of expansion, α, is therefore given by the equation

$$\alpha = \frac{f \cdot \frac{\lambda}{2}}{L(t_2 - t_1)}$$

where L is the effective length of the quartz, that is, the length of the ring minus the length of the thin plate inside the ring.

Apparatus.

The apparatus was set up in one of the basement rooms of the Physical Laboratory, a brick pier, surmounted by a sandstone slab, served as a support for part of the apparatus, while the remainder was supported directly by the asphalt floor. Fig. 2 shows a cross-section of the apparatus arranged for work. D represents an Abbé-Fizeau interferometer, P, a porcelain tube with a brass chamber

attached to its lower end in which the expansion apparatus shown in Fig. 1, is placed; O, an oven heated electrically and U, an adjustable support for the oven. The illuminating apparatus is not shown in the figure.

THE INTERFEROMETER.

The first Abbé dilatometer was made in 1884 and is described by Weidmann.[1] The instrument used in these measurements was made by Carl Zeiss, Jena, and has the many improvements introduced by Pulfrich,[2] and owing to its wide range of applications it may be more justly termed an interferometer. It consists of an autocollimating telescope mounted upon a strong pedestal. Light from a point source is focussed upon a small totally-reflecting prism, p, (Fig. 2), which is set into the telescope tube through a small opening, and occupies practically half the field. There are adjustments by which this prism may be turned about a vertical axis or about a horizontal axis perpendicular to the length of the telescope. From this prism as a source, the light is reflected to the lens L_1, which is mounted in a tube sliding within the telescope tube, and is so adjusted that the prism p is at its principal focus. The light emerging from L_1 is then a parallel beam reflected downward by the totally reflecting prism T. Prism T is mounted in a tube sliding upon the outside of the telescope tube and can consequently be turned to reflect the light in any direction perpendicular to the axis of the telescope. It can also be pushed in or out in a line with the telescope tube through a considerable range.

The light, after traversing the tube P, falls perpendicularly upon the interference apparatus and interference fringes are produced. If the reflecting surfaces of the interference apparatus were situated in the focal plane of the lens L_1, the rays proceeding from the interference fringes, considered as objects, would, after passing through the lens L_1, be parallel. Passing the free half of the telecope tube at the prism p, they would be brought to a focus at N in the focal plane of the lens L_2. The size of the heating apparatus makes it impossible to have the reflecting surfaces of the quartz apparatus in the focal plane of the lens L_1. They have to be at a

[1] Wied. Ann., 38, p. 453, 1889.

[2] Zeitschrift für Instrumentenkunde, 1893, pp. 365, 401, 435; 1898, p. 261.

little greater distance. The lens L_2 has sufficient adjustment, however, to bring the rays to a focus at N where is placed an adjustable horizontal slit. The image formed at N is viewed through the lens L_3. If the fringes do not appear vertical, they may be made so by rotating the Dove's prism E, using the spindles provided for that purpose. When light of two or more wave-lengths is used, the

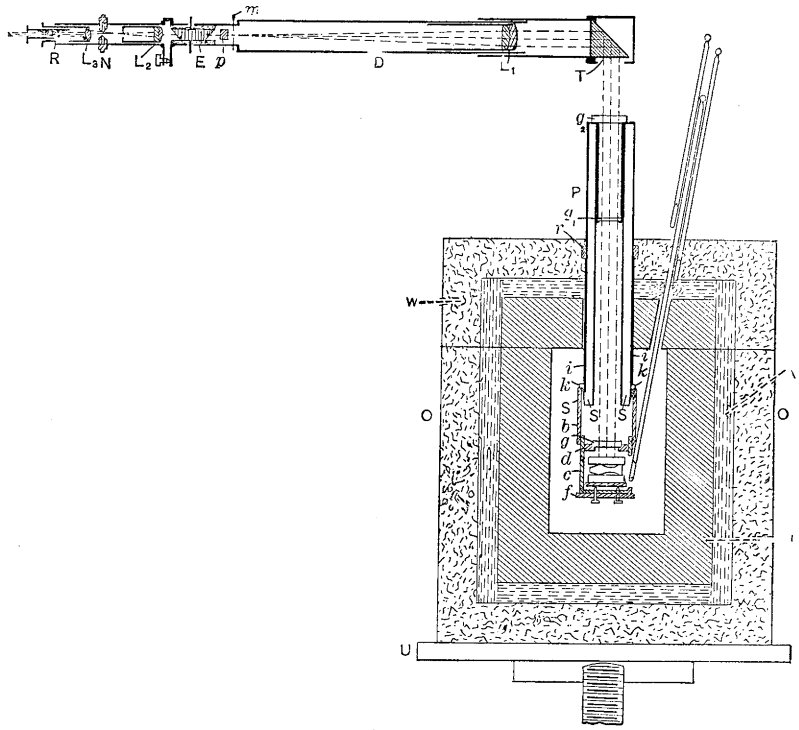

Fig. 2.

fringes due to each wave-length can be observed by inserting in the eye-piece the direct-vision spectroscope R so that the spectral colors lie one above another. On closing the adjustable slit, N, the spectral images of this slit now appear as horizontal bands whose width depends on the width of the slit, and are crossed by vertical interference bands. An iris diaphragm, m, placed close by the prism p on the side towards the lens L_1 is operated by a spindle projecting through a slit in the tube.

Immediately behind the slit N are situated two vertical cross-

wires, the distance between them being a little less than the apparent diameter of the small circle in the center of the quartz cover plate. The observing telescope is so mounted on the collimating tube as to have two independent angular movements, one vertical, the other horizontal. By the vertical movement it may be so tilted as to receive light from that part of the cover plate *B*, immediately about the reference circle. In this case the image of the reference circle appears in each of the colored bands. With the horizontal movement the cross-wires can be made to move apparently across the field parallel to themselves. The amount of this latter movement is measured by means of a micrometer screw, the head of which is divided into 100 equal divisions.

THE ILLUMINATING APPARATUS.

Light from an *H* form Geissler tube is focussed by a lens upon the prism *p*. The tube and the lens, so mounted as to admit of easy adjustment, are carried by an arm extending out at right angles to the collimating tube. The Geissler tube contains a drop of mercury and hydrogen gas under a pressure of a few millimeters. A slight warming of the mercury globule causes the formation of a sufficient amount of mercury vapor to produce brilliant mercury lines.

THE SUPPORTING TUBE P (FIG. 2).

The tube which supports the quartz apparatus in the oven is of very hard white porcelain. It is 27 cm. long, has an internal diameter of 2.7 cm. and its walls are 9 mm. thick. Tube *P* is supported in a vertical position directly beneath the totally reflecting prism *T* of the interferometer. A thick brass ring is closely fitted around the top and into this ring and at right angles to the tube is screwed an iron rod which in turn is clamped to the stone pier. This method of support proved entirely satisfactory, being sufficiently rigid and yet permitting easy change of position of the tube. At the lower end of the tube *P* is a shoulder, *s*, which serves to support the brass chamber in which the quartz apparatus is placed.

A brass ring is fitted around the tube *P* directly above the shoulder *s*, its thickness permitting it to extend slightly beyond the shoulder. The brass tube *b* is fastened by screws to this ring.

Tube *b* is 6 cm. in diameter and 5.5 cm. long. Into it is screwed a second tube *c*, 5.4 cm. long and 5.4 cm. in diameter. A thick brass ring *d* of internal diameter 2.5 cm., is fitted closely into the top of tube *c*. A shoulder is turned in the upper surface of ring *d*, around the inside of the proper diameter to hold tightly a circular glass plate, *g*. A bottom, *f*, is screwed into the tube *c*, through which project three equidistant leveling screws. A thin brass plate rests on the top of these leveling screws. To keep the brass chamber rigidly firm during all the temperature changes the following device was adopted. At a distance of 9 cm. above the shoulder *s* a brass ring *r* is fitted around the tube *P*. This ring is held fast by screws extending slightly into holes drilled in tube *P*. Around the top of the brass ring which rests on the shoulder *s* are placed several rings of asbestos. A thin walled tube *i* is cut just the right length to press firmly on the ring *k*, when put in place between the rings *r* and *k*. This pressure, which holds the supported chamber in position, is maintained upon increase of temperature by the expansion of the tube *i*. The asbestos rings, by their slight yielding, prevent rupture of the porcelain tube. Between the brass disks *d* and *f*, the tube *c* is half cut away so as to form a large window for the introduction of the quartz apparatus. In addition to the glass plate *g*, two other plates g_1 and g_2 are inserted, as shown in the figure, for the exclusion of cold air from above and the prevention of convection currents from below.

The Oven.

The central portion of the electrical heating apparatus consists of a cylindrical cup of cast iron, *I*, 23.8 cm. high, 22 cm. in external diameter, 5 cm. thick on sides and bottom, and weighing, with its two semicircular iron cover plates, about 140 pounds. Upon a sheet of asbestos paper placed around the cylinder is wound, non-inductively, No. 28 iron wire, the turns being about 3 mm. apart. A second sheet of asbestos is wound around the cylinder on top of the first layer of wire, and upon this is wound a second layer of wire of the same number of turns as the first, and in the same way a third layer is added above the second. Asbestos, *A*, is then wound around the cylinder to a thickness of two centimeters. On

the elevating stand, *U*, is placed a layer of asbestos 34 cm. in diameter and 1.5 cm. thick; upon this are placed three equidistant porcelain insulators about 4 cm. high, which carry a second asbestos layer 3 cm. thick and 22 cm. in diameter, upon which is set the iron cylinder. The space about the insulators is loosely packed with mineral wool, *W*. A sheet-iron cylinder 34 cm. in diameter, and equal in height to the combined height of the insulators and cast-iron cylinder, is next placed symmetrically around the iron cylinder, and the 4 cm. air space between is loosely packed with the mineral wool.

The sheet-iron cylinder is protected on the outside by a few layers of asbestos. The semicircular cover-plates of the iron cylinder are 5 cm. thick, pierced at the center to admit the porcelain tube freely, and are planed smooth on their under sides, as is also the top of the cylinder, to ensure perfect contact. One of the two plates has a radial slot for the passage of the thermometers. The sheet-iron cylinder is surmounted by a divided cap similar in shape to the semicircular cover-plates, and provided with asbestos and mineral-wool insulation corresponding to that in the other parts of the apparatus.

The conditions of the experiment require that the temperature of the quartz apparatus be maintained constant for long intervals of time. The oven seems well adapted to this end, since the large mass of metal employed undoubtedly renders the distribution of heat more uniform and tends to minimize sudden slight changes of temperature due to small variations in the heating current. A thermometer placed in a hole drilled in one of the cover-plates showed that while the temperature of the cover lagged behind the temperature of the air in the interior of the cylinder during the heating, it very quickly assumed practically the same temperature as soon as the latter was maintained constant. The excellent thermal connection between the covers and the cylinder is probably of value in compensating for loss of heat through the porcelain tube, since the covers encircle the tube closely at its middle point, and this part is quite likely to be the point of highest temperature in the tube.

The excellence of the heat insulation is shown by the fact that the outside of the sheet-iron cylinder was not uncomfortably warm for the hand, when the temperature inside was 500° C. An asbestos

screen, mounted half way between the top of the oven and the telescope of the interferometer, afforded ample protection against disturbing effects due to heating the interferometer.

The current used for heating was the 220-volt direct current furnished by the University lighting and power plant. For low temperatures and small currents a milliammeter was in circuit, to measure and aid in regulating the current with the adjustable resistances. For higher temperatures an ammeter reading directly to tenths was used.

The support which carries the oven has a vertical adjustment with a screw movement sufficient to lift the oven from a position several centimeters below the interference chamber, to one which brings the interference chamber to the center of the iron cylinder. The support is rigid enough to prevent all lateral movement and carries the binding posts to which the ends of the heating coils are attached.

THERMOMETERS.

Four thermometers were used for the complete range of temperature. From room temperature to 200° C. two thermometers made by Haak, Jena, were used. They read directly to $\frac{1}{5}$° and were calibrated at the Reichsanstalt in 1898. The scales of these thermometers extend from — 20° C. to 105° C., and from 90° C. to 210° C., respectively. A third thermometer, having a scale from 0° C. to 360° C., graduated in degrees, was used for the range between 200° C. and 350° C. This thermometer was calibrated at the Reichsanstalt in 1899. The fourth thermometer, graduated in degrees, is provided with a short scale at 0° C., and, after an enlargement of the bore, extends from 195° C. to 550° C. This thermometer was calibrated at the Reichsanstalt in 1897. During the progress of the investigation, the fixed points of these thermometers were determined a number of times and the necessary corrections applied.

THE SPHEROMETER.

The spherometer used to measure the lengths of the quartz ring and the small quartz plate, was made by the Geneva Society and reads directly to 0.001 mm.

METHOD OF SETTING UP APPARATUS.

The Geissler tube is adjusted in its holder until a bright image about 2 mm. in diameter is formed by the focussing lens upon the face of the prism p. The adjustment is properly made when the lens L_1, viewed through the prism T turned sidewise, is seen to be uniformly illuminated. Finally, after the apparatus is all set up, quite often it is possible to improve the appearance of the fringes by a slight change in the position of the tube, using the slow motion attachment provided for this purpose. The porcelain tube P is clamped in a vertical position directly below the prism T, and the bed-plate A placed in position on the thin brass plate supported by the leveling screws. To cause the beam of light incident upon A to retrace its path after reflection, the various adjustments possible are tried in turn. These adjustments consist in leveling the plate A by means of its supporting screws, the sliding of the interferometer as a whole on the pier, the tilting of the instrument by means of its leveling screws, and the combined turning and sliding in or out of the tube containing the totally reflecting prism T. When properly adjusted the circular field will be uniformly illuminated.

The quartz pieces having been carefully cleaned, the small quartz-plate is placed in the center of the bed-plate and the quartz ring symmetrically around it. The quartz cover-plate, with the reference circle down, is placed on top of the ring. While the quartz pieces are being thus arranged, care is taken to note their relative positions with respect to one another, and to see on which side the thin edge of the air layer lies. When viewed through the instrument, interference fringes are usually visible; if not, a slight adjustment of the interferometer will invariably produce them. When the adjustments are made and the fringes appear erect and at their brightest, readings are taken upon two adjacent dark bands to determine the width of the fringes. To avoid all possibility of particles of dust or lint being between any of the surfaces supposed to be in contact, the quartz apparatus is repeatedly taken down, the parts cleaned and remounted in the same relative positions, the width of the bands being measured each time. When several successive settings give the same band width this source of error is supposed to have been avoided.

It is then necessary to get the reference circle in the center of the field, so that, should it become necessary to erect the fringes, this circle will rotate about its own center. This is most readily done by sliding the interferometer slightly about on the pier, though sometimes it becomes necessary to move the quartz apparatus in the brass chamber.

The direct-vision spectroscope is then placed in the observing telescope and adjusted till the vertical cross-wires are most distinct. The slit is narrowed till the spectral colors, when separated, appear as horizontal bands of about the width of the reference circle and crossed by equidistant dark lines. If the reference circle does not lie within these bands, it can be brought there by the vertical adjustment of the observing telescope.

To avoid any accidental fall of the quartz apparatus through the window into the iron cylinder, there is fastened over the window a piece of wire gauze in which slits 1 cm. wide and equal in length to the height of the window are cut. There is left between these slits just enough of the gauze to hold the piece together, so that this protection in no way interferes with the free circulation of the air between the brass chamber and the cylinder.

The thermometer is next adjusted with its bulb on a level with the quartz apparatus and opposite an opening in the gauze. At first the thermometer hung vertically. In this position the bulb was about 2.5 cm. from the quartz expansion apparatus. Later the thermometer was tilted so that its bulb entered the edge of the window through one of the slits in the gauze. In this position the bulb was within 1.5 cm. of the quartz. No change in the results was apparent from the change in position of the bulb of the thermometer. The thermometer is supported by suitable clamps fastened to the stone slab of the pier.

The heating apparatus is raised until the chamber containing the quartz is symmetrically situated within it. The covers previously described are placed in position and the length of the thermometer stem in the insulation noted. A second thermometer is mounted next the first with its bulb half through the insulation to give the average temperature of the mercury thread in the insulation. A glass tube, closed at its upper end, about 3 cm. in diameter, to

shut off air currents, is put about the thermometers, each of which is provided with a small thermometer to get the average temperature of the mercury columns above the insulation. The mounting of the apparatus is then complete. Generally it can be accomplished without any change in the position of the fringes and reference circle with respect to the vertical cross-wires, thus showing that the quartz apparatus has not been disturbed.

The apparatus is then left to stand over night and acquire a stationary temperature. The next morning at intervals of a quarter of an hour, the temperature of the quartz apparatus is determined and micrometer readings on the position of some one of the dark bands are taken. If for one hour these do not change materially, a set of micrometer readings is taken on the fringe system in the manner shortly to be described. The thermometers are read as is also the barometer. A current of about 7 amperes is then sent through the heating coils, an ammeter and adjustable resistance being in the circuit. With this current the temperature of the oven increases 20° C. in about half an hour. The current is then turned off till the thermometer shows that the temperature inside has become stationary, then a current which will maintain this temperature is found by trial. This current is kept constant for three or more hours. During the last two hours the fringes generally appear quite stationary and micrometer readings are taken every quarter of an hour on some dark band. When the position of the band has not changed for an hour the condition of equilibrium is considered to have been attained and a second set of readings similar to the set taken at the lower temperature is made. Heat is again applied to the oven and the process repeated.

METHOD OF DETERMINING THE NUMBER OF DISPLACED FRINGES.

First Method — by Direct Observation. — The displacement of the fringes means the number of dark bands and fractions of a band passing the reference circle on the under side of the cover-plate during any temperature interval. Upon heating the apparatus the expansion is generally so regular that by illuminating the field about every half minute the passage of the dark bands through the center of the reference circle can be followed by the eye and counted. If

the time of the passage of each dark band through the reference circle is recorded, there is little danger of mistake. In fact, having determined the period of the passage from the crossing of the first few bands, it is necessary only to look at the end of each period, so regular are the time intervals. Beside the number of whole bands counted during any temperature interval, $t_2 - t_1$, there are generally fractions of a band to be added or subtracted for each end of the interval, as the bands do not ordinarily come to rest in the center of the reference circle. These fractions are determined by the aid of the micrometer attachment. While the bands are stationary at the lower temperature, t_1, micrometer readings are taken on the positions of the five bands nearest the reference circle. These readings are denoted in the table as l_1, l_2, l_3, l_4, l_5 respectively, l_1 being the reading of the band located farthest to the left of the circle (Fig. 3). A reading is also taken upon the center of the reference circle. In the table this reading is denoted by l_0. The average of the readings on the five bands gives a very accurate reading for the position of the central band of the system, the one nearest the center of the reference circle. The difference between this average reading for the central band, and the reading on the circle, gives, in scale divisions, the distance the band is from the center of the reference circle. This distance OC divided by the width of a band gives the fraction by which the dark line is distant from the center of the reference circle. The width of a band is given by the expression $b = \frac{1}{6}[(l_5 + l_4) - (l_1 + l_2)]$. This fraction may have either a positive or negative sign and never exceeds $\frac{1}{2}$ in magnitude. If care is taken to mount the quartz apparatus so that, with increasing temperature, the fringes move in the same direction as the vertical cross-wires move when the micrometer is turned to the larger numbers, then the above method for determining the magnitude of the fraction will give also its proper sign. If the convention be adopted that the initial conditions are those at the

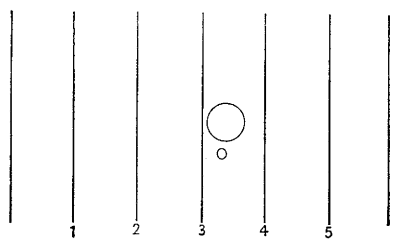

Fig. 3.

lower temperature, t_1, and the final conditions are those at the higher temperature, t_2, then the method works equally well for observations taken with rising or falling temperatures. Similarly when the second temperature limit, t_2, has been attained, and the fringes become stationary, the resulting fraction of a band with its proper sign is determined. The total displacement is given by the expression $f = M + \delta_{t_2} - \delta_{t_1}$ in which M represents the number of whole bands observed and δ_{t_2} and δ_{t_1} the fractions of a band at the temperatures t_2 and t_1 respectively.

Second Method — by Computations. — It is possible, as has been shown by Abbé,[1] to compute the number of displaced bands corresponding to any temperature interval, providing a second set of readings similar to those described in the first method is taken on fringes of another wave-length. As the green light of mercury vapor, $\lambda = 546\ \mu\mu$, is generally the most satisfactory to work with, it is always one of the two used. With quartz, when the air film is relatively thick, the only other color giving satisfactory fringes is the yellow due to mercury vapor, $\lambda_1 = 578.8\ \mu\mu$ and $\lambda_2 = 576.8\ \mu\mu$. This in reality gives two fringe systems which so completely overlap as to appear almost as a single system. With varying thickness of air film these two fringe systems exhibit the periodic variation in visibility first observed by Fizeau in the case of the sodium lines. At intervals of about 300 fringes, more accurately for each change in thickness of air film of 0.07924 mm., the combined fringe system exhibits maximum distinctness. In each of these regions of maximum distinctness there are about 100 bands of mean wave-length $577.8\ \mu\mu$ suitable for measurement. In the interval between these positions it becomes necessary to use the fringe system of one wave-length separately. The one corresponding to λ_2 has been used, and enough of this system appears on the edge toward the green to permit of accurate readings, although it is somewhat more difficult. Accordingly in the work with quartz three wave-lengths were used. That of the green is represented by λ_0, that of the average yellow, and the shorter yellow by λ as the case may be.

The change in the thickness of the air layer for a change in temperature, $t_2 - t_1$, may be represented by

[1] Wied. Ann., 38, 1889, p. 473.

$$[M_0 + \delta_{0t_2} - \delta_{0t_1}] \frac{\lambda_0}{2},$$

where the letters have the same significance as in the first method, the subscript o indicating that the green was used. Similarly

$$[M + \delta_{t_2} - \delta_{t_1}] \frac{\lambda}{2}$$

is an expression for the same change in thickness of the air film, when one of the yellow spectral colors is used. Placing these expressions equal,

$$[M_0 + \delta_{0t_2} - \delta_{0t_1}] \frac{\lambda_0}{2} = [M + \delta_{t_2} - \delta_{t_1}] \frac{\lambda}{2},$$

there is obtained

$$M_0 \frac{\lambda_0}{\lambda} = M + \delta_{t_2} - \delta_{t_1} - (\delta_{0t_2} - \delta_{0t_1}) \frac{\lambda_0}{\lambda}$$

or

$$M_0 \mu = M + \delta_{t_2} - \delta_{t_1} - (\delta_{0t_2} - \delta_{0t_1}) \mu \text{ where } \mu = \frac{\lambda_0}{\lambda}.$$

$$M_0 \mu = M + \rho \text{ where } \rho = \delta_{t_2} - \delta_{t_1} - (\delta_{0t_2} - \delta_{0t_1}) \mu.$$

That is to say, the number of whole green bands which have passed when reduced to an equivalent number of another color, will equal a number of bands plus a fraction of a band. A knowledge of the ratio between the wave-lengths of the colors used, λ_0/λ, enables one to compute the number of bands and the fraction of a band of the second color which corresponds to every whole number of green bands. In the following table, the columns headed M_0 contain the whole numbers of green bands. The maximum displacement of the green fringes for any temperature interval of 20° C. is not more than thirteen bands. The columns headed M and ρ contain respectively the corresponding whole number of yellow bands and fraction of a yellow band. It will be noticed that the fractions in the ρ columns do not repeat their values until eighteen green bands have been used. So, up to this point, each fraction is characteristic of a definite number of green bands. These fractions, of course, are all positive.

In the expression $M_0\mu = M + \rho$, ρ is the fraction of a band of the

TABLE I.

Change in Thickness of Air Film in mm.	Green.	Yellow (Average) $\mu = 0.945$.		Yellow (Upper) $\mu = 0.9466$.		Change in Thickness of Air Film in mm.	Green.	Yellow (Average) $\mu = 0.945$.		Yellow (Upper) $\mu = 0.9466$.	
	M_0	M	ρ	M	ρ		M_0	M	ρ	M	ρ
0	0	0	0	0	0	0.0055	20	18	.90	18	.93
	1	0	.94	0	.94		21	19	.84	19	.88
	2	1	.89	1	.89		22	20	.79	20	.83
	3	2	.83	2	.84		23	21	.73	21	.77
	4	3	.78	3	.78		24	22	.68	22	.72
0.0014	5	4	.72	4	.73	0.0068	25	23	.62	23	.67
	6	5	.67	5	.68		26	24	.57	24	.61
	7	6	.61	6	.63		27	25	.51	25	.56
	8	7	.56	7	.57		28	26	.46	26	.50
	9	8	.50	8	.52		29	27	.40	27	.45
0.0027	10	9	.45	9	.47	0.0082	30	28	.35	28	.40
	11	10	.39	10	.41		31	29	.29	29	.34
	12	11	.34	11	.36		32	30	.24	30	.29
	13	12	.28	12	.31		33	31	.18	31	.24
	14	13	.23	13	.25		34	32	.13	32	.18
0.0040	15	14	.17	14	.20	0.0096	35	33	.07	33	.13
	16	15	.12	15	.15		36	34	.02	34	.08
	17	16	.06	16	.09		37	34	.96	35	.02
	18	17	.01	17	.04		38	35	.91	35	.97
	19	17	.95	17	.98		39	36	.85	36	.92

second color which has to be added to a whole number of bands of that color, M, in order to be equivalent to a whole number of green bands, M_0. ρ, therefore, must always have some one of the values found in columns ρ, as they are the only fractions which can be added to a whole number of yellow bands so as to give an equivalent number of green bands. ρ computed from $\rho = \delta_{t_2} - \delta_{t_1} - (\delta_{0\,t_2} - \delta_{0_{t_1}})\mu$ will be found, therefore, in columns ρ of the table, and the whole number of bands of the two colors used, M_0 and M, are opposite the fraction. The two numbers M_0 and M, being known, the total expansion in green half-wave-lengths is $M_0 + \delta_{0_{t_2}} - \delta_{0_{t_1}}$. The same expansion in yellow half-wave-lengths is $M + \delta_{t_2} - \delta_{t_1}$. This latter value is reduced to the equivalent expression in green half-wave-lengths, and the average of the two expansions is considered the true expansion.

Occasionally the sign of ρ computed from the formula

$$\rho = \delta_{t_2} - \delta \ - (\delta_{0t_2} - \delta_{0\,t_1})\,\mu$$

is negative and as no negative fractions are found in the table, an equivalent positive fraction has to be found. If ρ is a negative quantity, $M_0\mu = M - \rho$. This may be written $M_0\mu = (M - 1) + (1 - \rho)$. The fraction $(1 - \rho)$ is now positive and may be found in the table. The number found opposite it in the column containing the whole number of yellow bands equals $M - 1$. M therefore, which is the quantity desired, equals this number plus 1.

CORRECTIONS FOR CHANGE IN REFRACTIVE INDEX OF AIR FILM.

As the index of refraction of the air layer between the two reflecting surfaces changes with a change of temperature, and also with an alteration of the atmospheric pressure, a suitable correction must be made upon the observed number of fringes displaced. With a rise in the temperature, the air becomes less dense, the wave-lengths of light used become correspondingly longer, and therefore fewer in number for a definite distance between the reflecting surfaces. The observed number of fringes is therefore too small, and this correction is always positive. Pulfrich[1] has shown that this correction may be computed by the formula

$$K_1 = d(t_2 - t_1)\frac{b_1}{760} \cdot \frac{1}{1 + \alpha t_1} \cdot \frac{1}{1 + \alpha t_2}\left[2\frac{(N - 1)\alpha}{\lambda}\right],$$

in which K_1 represents the correction, d the thickness of the air layer, b_1 the barometric pressure observed at the lower temperature, t_1, α the coefficient of expansion of air, and N its index of refraction under standard conditions.

The second correction, due to a change in atmospheric pressure, may evidently be positive or negative. If the pressure increases during a temperature interval, the corresponding increase in density of the air layer will result in shorter wave-lengths and more of them. Accordingly in this case the correction is negative, while a decrease in barometric pressure will give a positive correction. This is shown by Pulfrich to be

$$K_2 = d(b_2 - b_1)\frac{1}{1 + \alpha t_2}\left[2\frac{N - 1}{\lambda} \cdot \frac{1}{760}\right].$$

[1] Zeitschr. für Instr., 1893, p. 455.

The entire correction may therefore be represented by

$$K = K_1 + K_2 = d(t_2 - t_1)\frac{b_1}{760}\cdot\frac{1}{1+\alpha t_1}\cdot\frac{1}{1+\alpha t_2}\left[2\frac{(N-1)\alpha}{\lambda}\right]$$
$$+ d(b_2 - b_1)\cdot\frac{1}{1+\alpha t_2}\left[2\frac{N-1}{\lambda}\cdot\frac{1}{760}\right].$$

These corrections amount to a considerable fraction of a wave-length when the thickness of the air layer is large.

THE DEGREE OF ACCURACY ATTAINABLE.

The formula expressing the value of the coefficient of expansion

$$\alpha = \frac{f_0\cdot\frac{\lambda_0}{2}}{L(t_2 - t_1)}$$

shows that the value of α depends upon four measurable quantities, f_0, the number of displaced fringes, L, the length of the object whose expansion is to be measured, and thetwo temperatures, t_2 and t_1.

In the expression

$$f_0 = M_0 + \delta_{0t_2} - \delta_{0t_1},$$

the δ's can be determined with an average error of $\pm$ 0.015 and f therefore with a mean error of $\pm$ 0.02. The error in the value of α resulting from this error is $\pm$ 3.5×10^{-8}. The effect of the errors of observation in the measurement of L and the t's upon α will vary with the magnitude of f_0 if the temperature intervals $t_2 - t_1$, be assumed a constant, as 20° C. At room temperatures f_0 is approximately 4.3 and at 500° C., 13.3. The error in the measurement of L is $\pm$ 0.001 mm. and the corresponding error in α at room temperature is $.1 \times 10^{-8}$ and at 500° C. is $.3 \times 10^{-8}$. At the lower temperatures where the thermometers graduated in fifths are used, the difference in the temperatures, $t_2 - t_1$, can be determined with an error of $\pm$ 0°.05 resulting in an error in α of 1.9×10^{-8}. When the thermometers graduated in degrees are used, the interval, $t_2 - t_1$, can be determined with a mean error of $\pm$ 0.2°. This results in an error in α of 23×10^{-8} at the upper temperature limit of 500° C.

With the platinum resistance thermometer, to be described later, the temperature intervals can be determined throughout the entire temperature range of 500° C., with a mean error of ± 0.05°. This gives an error in α at 500° C. of $6. \times 10^{-8}$. The total effect of the errors of observation upon the result is therefore $E = \sqrt{E_f^2 + E_L^2 + E_t^2}$. At room temperature this amounts to 3.9×10^{-8}. At 500° C. with the mercury thermometers the total error is 23.2×10^{-8}, while with the platinum resistance thermometer this is reduced to 6.9×10^{-8}.

Example of a Complete Observation and Computation.

Length of quartz ring, 9.8426 mm.
Length of quartz cylinder, 2.0900 mm.
Effective length of quartz expanding, 7.7526 mm.
Thickness of air layer, 7.7526 mm.

$t_1 = 489.5°$ C. $\qquad b_1 = 740.0$ mm.

	Green.			Yellow (Upper).	
l_1	74	76	l_1	19	20
l_2	172	171	l_2	119	118
l_3	264	261	l_0	183	185
l_0	284	284	l_3	218	216
l_4	351	352	l_4	309	308
l_5	442	440	l_5	406	404
l_c	260.6	260.0	l_c	214.2	213.2
OC	−23.4	−24.0	OC	+31.2	+28.2
b	91.1	90.8	b	96.1	95.7
δ	−.256	−.264	δ	+.325	+.295
Ave. δ	−.260			+.310	

$t_2 = 509.6°$ $\qquad b_2 = 741.0$ mm.

	Green.			Yellow (Upper).	
l_1	90	94	l_1	69	67
l_2	181	185	l_2	167	167
l_0	275	277	l_3	260	258
l_3	279	280	l_0	275	276
l_4	363	366	l_4	354	356
l_5	447	450	l_5	449	450
l_c	272.0	275.0	l_c	259.8	259.6
OC	−3.0	−2.0	OC	−15.2	−16.4
b	89.8	89.5	b	94.5	95.3
δ	−.033	−.022	δ	−.161	−.172
Ave. δ	−.027		Ave. δ	−.166	

$$\delta_{0t_2} = -.027 \quad \delta_{0t_1} = -.260 \quad \delta_{t_2} = -.166 \quad \delta_{t_1} = +.310$$

$$t_2 - t_1 = 20.1° \quad \frac{t_2 + t_1}{2} = 499.5° \quad b_2 - b_1 = +1.0 \text{ mm.}$$

$$\rho = (\delta_{t_2} - \delta_{t_1}) - (\delta_{0t_2} - \delta_{0t_1})\mu = -.696$$

$$\rho + 1 = +.304 \text{ (Table .31)} \quad M_0 = 13 \quad M - 1 = 12 \quad M = 13$$

$$f_0 = M_0 + \delta_{0t_2} - \delta_{0t_1} = 13.233 \quad f = M + \delta_{t_2} - \delta_{t_1} = 12.524$$

$$\text{Ave. } f_0 = \left\{ \begin{array}{r} \frac{12.524}{.9466} = 13.231 \\ 13.233 \end{array} \right\} \cdot 13.232.$$

Correction to f_0 due to change in index of refraction of air.

$K_1 = +.075 \quad K_2 = +.004 \quad K = +.079 \quad f_0' = f_0 + K = 13.311.$

$$\alpha = \frac{f_0' \cdot \frac{\lambda_0}{2}}{L(t_2 - t_1)} = 2332 \times 10^{-8} \text{ at } 499.5°.$$

RESULTS OF OBSERVATIONS.

The results obtained at this time are given in Table II. The fourth column contains the value of α derived from the observations multiplied by 10^8. These results are represented in the plot, Fig. 5, by the circles.

SECOND PART.

As was mentioned in the introduction, certain considerations seemed to render desirable a repetition of the work just described. First no data was obtained in the temperature interval 350° to 420° C. as 420° were the lowest reading on the fourth thermometer appearing above the insulation of the oven. Second, as shown earlier, the error in measuring the temperature interval may be large in comparison with the error in the other measurements. This error is inherent in the use of the mercury thermometer, and combines the error arising from the constantly changing zero point, with those due to scale reading, and to uncertainties of stem and calibration corrections. Third, there may be an actual difference in temperature between the quartz apparatus and the thermometer

TABLE II.

t_1	t_2	$\frac{t_1+t_2}{2}$	$\alpha \times 10^8$
152.9	168.2	160.5	959.4
178.4	196.4	187.4	1,008
178.4	213.9	196.1	1,024
196.4	213.9	205.1	1,044
223.1	257.7	240.4	1,092
249.5	272.9	261.2	1,138
257.8	282.8	270.2	1,151
272.9	290.9	281.9	1,182
282.8	302.1	292.4	1,218
285.7	306.4	296.0	1,227
285.7	313.8	299.7	1,247
290.9	313.9	302.4	1,247
302.1	319.8	310.9	1,281
306.4	327.5	316.9	1,289
306.3	327.6	316.9	1,294
319.8	336.6	328.2	1,339
329.8	347.0	338.4	1,363
330.9	345.3	338.1	1,368
327.5	345.3	336.4	1,355
331.2	356.2	343.7	1,381
343.2	356.2	349.7	1,400
409.3	431.3	420.3	1,643
416.4	431.3	423.9	1,689
416.4	438.5	427.5	1,690
423.5	446.3	434.9	1,722
423.5	450.0	436.7	1,746
438.5	460.9	449.7	1,805
449.1	471.2	460.1	1,878
450.9	467.2	459.1	1,885
449.1	482.2	465.7	1,901
460.9	482.2	471.6	1,934
464.9	489.4	477.2	1,932
477.1	499.2	488.2	2,168
489.5	499.2	494.3	2,186
489.4	509.6	499.5	2,331
489.5	509.6	499.6	2,332
499.2	509.6	504.4	2,469

bulb as the former is within a brass chamber at the end of a tube extending out of the oven and the latter outside the chamber at a distance of 1.5 to 2.5 cm.

CHANGES IN APPARATUS.

In the redetermination a platinum resistance thermometer was substituted for the mercury thermometers and an entirely different method adopted for supporting the quartz apparatus within the oven.

As the laboratory possessed a large Post Office box made by Nalder Bros. which was known to be in excellent adjustment, the platinum thermometer constructed was one adapted to that method of measuring resistance. The terminals of the fine platinum wire, wound on the usual rectangular mica frame, were gold soldered to heavy double platinum leads. These leads, insulated in the stem of the thermometer by mica disks, were soldered externally to heavy copper leads terminating in mercury cups, so arranged that either pair of leads separately, or the leads together with the fine coil, could be put in the arm of the box for measurement by connecting the proper cups by means of heavy copper links. This thermometer was calibrated as recommended by Callendar and Griffiths,[1] and was capable, together with its accessory electrical apparatus, of measuring a temperature variation of 0.01°. In a determination, the resistance of the thermometer leads was first measured, then the resistance of the entire thermometer, followed by the interferometer readings. The resistance of the entire thermometer was again determined, and then the resistance of the leads. The temperatures thus obtained before and after the interferometer readings, which consumed about fifteen minutes, seldom varied by more than 0.02° or 0.03° C.

The arrangement for mounting the quartz apparatus is shown in Fig. 4. It consists of a cylindrical brass chamber, firmly supported by two solid porcelain rods 50 cm. long and 8 mm. in diameter, clamped at their upper ends to an iron bar, this bar being supported by clamps and rods to the pier. The wall of this chamber is formed by a solid brass cylinder, 6 cm. long, 7.7 cm. in diameter and 3 mm. thick, which fits tightly into a groove turned into the brass plate serving as a bottom. The cover, by means of a similar groove fits tightly on the cylinder. This cover plate is furnished with three circular apertures, the central one being fitted tightly with a thick

[1] Phil. Trans. Roy. Soc., 1891, A, p. 141.

glass disk, serves as a window for admitting the beam of light, and the two on opposite sides of this are for the introduction of the thermometers.

To mount the quartz within the chamber, the thermometers are first run through the holes in the cover, and then the brass cylinder and cover are lifted to a sufficient height and held by any convenient support. The quartz is then mounted on the brass disk supported by the leveling screws through the base. Two thin brass rods (not shown in figure) are screwed firmly to the bottom of the chamber near the edge. These rods are bent at right angles and so adjusted that the thermometers fastened to them just escape contact with the quartz. When the wall and top of this chamber are lowered into place, the quartz and the thermometers are completely surrounded by the brass chamber with the single exception of the thick glass window at the top. This chamber is in turn completely surrounded by the air of the oven. Thus the chimney-like effect due to the porcelain tube of the earlier arrangement is avoided, as is also the questionable arrangement of the large window with the thermometer bulb outside. As it is possible to maintain the temperature of the air of the oven surrounding the brass chamber constant, to within a tenth of a degree, for intervals of half an hour to an hour, it would seem that any possible difference in temperature between the quartz and the thermometers due to a different rate of following temperature changes, would be extremely small. To introduce the beam of light into the oven, a tube, provided with three glass plates to prevent air currents, is mounted in the insulation directly above the center of the brass chamber.

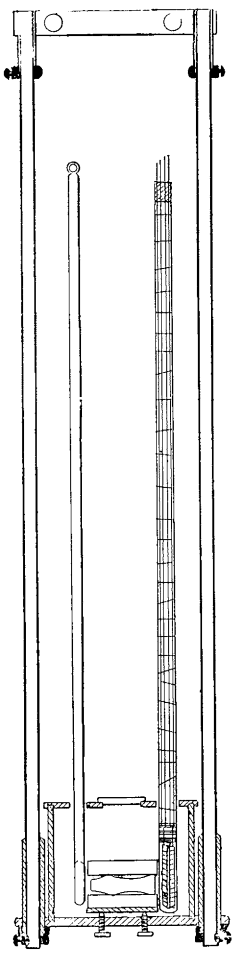

Fig. 4.

As an additional protection for the interferometer against heating effects from the oven, two asbestos shields were mounted between the instrument and the oven. A thermometer, in contact with the interferometer tube directly over the oven, was read with each set of observations. The results show that the temperature of the interferometer varied from a minimum of 1.2° C. to a maximum of 2.0° C., above that of the room while the room temperature varied during the entire course of the investigation 2.7° C. It may be concluded therefore, that the heat from the oven does not produce measurable disturbing effects.

RESULTS OF LATER OBSERVATIONS.

Two series of measurements were made with the apparatus just described. One of them comprised a few determinations between room temperature and 100° C., in which both platinum resis-

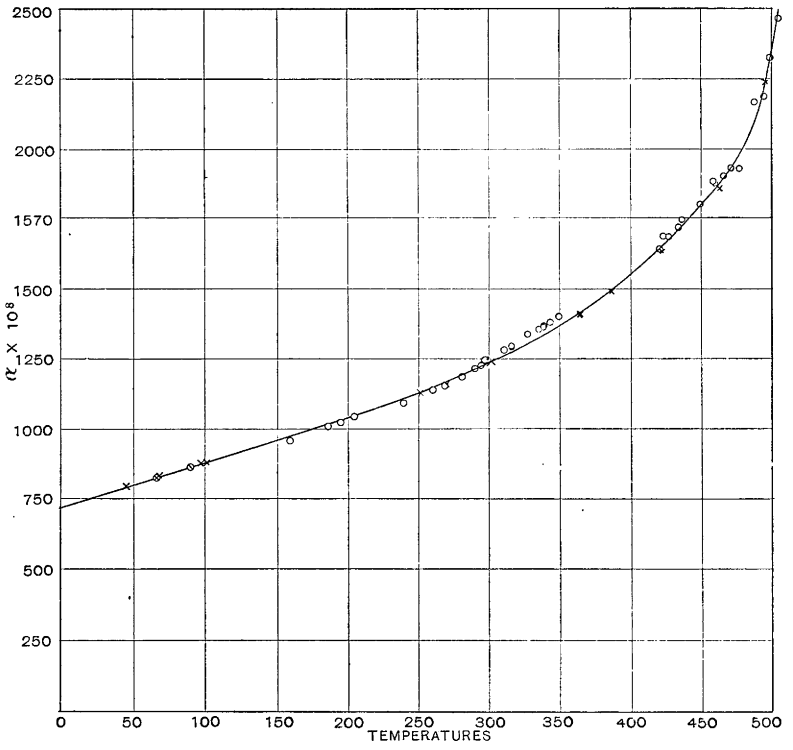

Fig. 5.

tance thermometer and standard mercury thermometer were employed. The results obtained are contained in Table III. and are shown in the plot by the combined cross and circle. The two thermometers agreed to the limits within which the mercury thermometer could be read. A more extended set of results was obtained extending to 500° C., in which the resistance thermometer only was used. The results are contained in Table IV., and are indicated in the plot by the crosses. The results obtained in 1901, as shown by the circles, are in substantial agreement with the recent redetermination, the region between 300° C. and 350° C. showing the only marked difference. Here the results obtained by the mercury thermometer are perceptibly higher. As the thermometer used in this region had unfortunately been broken, no opportunity was offered for discovering the cause of this difference.

TABLE III.

t_1	t_2	$\frac{t_1+t_2}{2}$	$\alpha \times 10^8$	$\alpha' \times 10^8$	$(\alpha - \alpha') \times 10^8$
56.54	78.56	67.55	825.8	826.4	+0.6
78.56	103.73	91.15	864.2	864.6	+0.4

TABLE IV.

36.24	51.98	44.11	790.0	788.5	− 1.5
51.98	85.35	68.66	830.6	828.3	− 2.3
21.44	173.23	97.83	875.6	875.4	− .2
85.35	114.81	100.08	877.8	880.3	+ 2.5
241.78	260.62	251.20	1129.5	1127.8	− 1.7
260.62	279.88	270.25	1156	1161	+ 5.0
279.88	328.62	304.25	1244	1233	−11.0
328.62	401.20	364.91	1408	1409	+ 1.0
328.62	399.43	364.02	1406	1407	+ 1.0
328.62	442.50	385.56	1490	1486	− 4.0
399.43	442.50	420.97	1639	1639	0.0
446.67	480.01	463.34	1862	1865	+ 3.0
487.39	504.58	495.98	2241		

Up to 250° C. the results may be represented by the equation $\alpha_t = (717.0 + 1.620t) \times 10^{-8}$. Between 250° and 470° C. the expression $\alpha_t = [1125.0 + 1.65(t - 250) + 0.00566(t - 250)^2 + 0.0000134(t - 250)^3] \times 10^{-8}$ represents the results quite closely.

The sharp upward turn of the curve in the neighborhood of 500° C. renders it impracticable to represent the results further by an equation. The values of α_t derived by means of these equations are contained in the column headed $\alpha' \times 10^8$ in the tables III. and IV. The last column of these tables contains the differences between α and α' in units of the eighth decimal place, and indicates how closely the equations may be expected to represent the results determined by direct measurement.

COMPARISON OF RESULTS WITH THOSE OF OTHER OBSERVERS.

The following table contains the results obtained in the investigations mentioned in the introduction.

$$\text{Fizeau } \alpha_t = 10^{-8}(710 + 1.770t).$$
$$\text{Benoit } \alpha_t = 10^{-8}(716.1 + 1.602t).$$
$$\text{Reimerdes } \alpha_t = 10^{-8}(692.5 + 1.689t).$$
$$\text{Scheel } \alpha_t = 10^{-8}(714.4 + 1.630t).$$

while from my observations for the interval 0° to 250° we have $\alpha_t = 10^{-8}(717.0 + 1.620t)$.

A comparison of these expressions shows that the results obtained in this investigation agree very closely with those of Benoit and Scheel. If the expansion for 1 meter be computed for intervals of temperature of 50° and 100°, the following values are obtained.

Benoit	(0°–50°)	378.1 μ	(0°–100°)	796.2 μ
Scheel		377.6		795.9
Randall		378.7		798.0

The differences in each case lie well within the limits of the experimental error.

As far as is known no accurate determinations of the coefficient of expansion of quartz beyond 220° have previously been made. Mallard and Le Chatelier[1] have made a rough determination of the expansion of quartz cut parallel to the principal axis up to 1,000° C. By means of two cameras properly mounted they were able to photograph the ends of a quartz rod 100 mm. long, both before and after it had been heated through a definite range of temperature. As the plates were ruled in $\frac{1}{10}$ mm. divisions, the expansions could

[1] Comp. Rendus, No. 108, p. 1046.

be read off directly from the plates, hundredths of a mm. being estimated with some degree of accuracy. Their results are given in the following table:

Temperature.	First Determination.	Second Determination.
15°	0	0
270°	0.20 mm.	0
480°	0.53 "	0.55
570°	0.93 "	0.93
660°	0.95 "	0.99
750°	0.95 "	0.95
910°	0.90 "	0.87
990°	0.90 "	0.86
1060°	0.90 "	0.89

The numbers in columns 2 and 3 are the expansions expressed in mm. when the change of temperature is from 15° C. to the temperature opposite a given expansion. The table shows that a great change in the rate of expansion occurs between the temperatures 480° C. and 570° C. Le Chatelier calls attention to this in the following statement: "These measurements on the expansion of quartz seem to indicate a very rapid transformation in the properties of quartz without proving it in an indisputable manner. We may also represent the results of the experiment by assuming between 400° and 600° C., a very rapid but continuous increase in the expansion which leads to a curve presenting a point of inflection, the tangent at which will be sensibly vertical, a very improbable hypothesis, it is true. It would be interesting to verify this point of sudden transformation by the study of phenomena subject to more accurate measurement than that of expansion by heat. At the same temperature, 570° C., the circular polarization, the double refraction, the electric conductivity, the specific heat of quartz should experience equally rapid changes."

This agrees well with the rather sudden rise in value of the coefficient after 480° C., as shown by the upward turn of the plotted curve, and if the expansions of a rod of quartz 100 mm. long be computed for the temperature ranges 15° — 270° and 15° — 480° by means of the two equations resulting from this investigation, they are found to be .24 mm. and .55 mm. respectively, agreeing well with those obtained by Le Chatelier. Sahmen and Tammann,[1]

[1] Ann. der Physik und Chemie, 10, 1903, p. 879.

working with two samples of quartz, find expansions for these intervals of approximately .22 mm. and .61 mm. and .26 mm. and .89 mm. respectively, results differing from each other by more than observational error and both considerably larger at the higher temperature than the results of Le Chatelier, or those of the author.

To Professor Reed, under whose immediate direction this investigation was conducted, the writer wishes to express his grateful thanks for the interest shown and the many valuable suggestions offered during the progress of the work.

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