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TARGETS.**

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MULTIPARTICLE PRODUCTION IN THE INTERACTIONS OF CHARGED
COSMIC RAY HADRONS WITH NUCLEAR TARGETS

by
Palahalli Ramaiya Viswanath

A dissertation submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
(Physics)
in The University of Michigan
1974

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I started studying physics at the University of Mysore in Bangalore and my initiation into research began at the Tata Institute of Fundamental Research in Bombay. I remember my old Indian colleagues in the Extensive Air Shower group with gratitude and affection. I would also like to thank the teachers and students at Bangalore, Bombay, Ann Arbor and Madison, from whom I learnt a substantial amount of physics. These places also provided me with friends in other walks of life, who gave me various insights into life and into myself; life, indeed, would have been difficult without these friends.

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CHAPTER I

INTRODUCTION

A. Motivation

One of the most important problems in physics during the last few decades has been the search for a description of elementary particles and their interactions. While a better understanding has been achieved of processes involving photons and electrons, the nature of the strong interaction processes involving hadrons has been very elusive. Hadron-hadron interactions, and in particular proton-proton interactions, have been the subject of extensive study during the last two decades. The multiple production of particles is the main process in high energy proton collisions since the cross-section for multiple production accounts for about 75% of the total cross-sections. The earliest information about multiparticle production in p-p collisions came from cosmic ray experiments which suffered from an incomplete knowledge of the important parameters in the collision process. But the results from these cosmic ray experiments served as guides or hints for subsequent experiments at accelerators where the parameters were controllable and the precision was of much higher order.

The general philosophy of the earlier cosmic ray experiments was to understand p-p interactions by a study of hadron-nucleus interactions (1 - 7). Lower energy experiments ($E < 10^{14}$ ev) were done with cloud chamber and emulsion techniques. While the events in emulsion experiments were classified as p-p or p-A (proton-nucleus) interactions by means of several stipulations, the experiments with the cloud chambers used targets of light nuclei. The higher energy experiments dealt with the phenomena of Extensive Air Showers where the target was the air nucleus.

The necessity of doing high energy physics with cosmic ray beams and pure proton targets was stressed at the Case Institute Conference (8) in 1964. The first experiment with cosmic ray protons and a proton target was conducted in 1968 at Echo Lake, Colorado, and the results have been well documented (9-15). Apart from providing the first systematic data on p-p interactions, this experiment was also of considerable influence in the recent growth of interest in multiparticle production processes (16-22).

In spite of the nuclear targets in the earlier cosmic ray experiments, they had been treated and analyzed as though the targets were protons and thus there was no systematic data on proton-nucleus interactions. It has been realized in the past few years, that a detailed study of proton-nucleus interactions is relevant to both high energy physics and cosmic ray

physics. Several authors have stressed that the more fundamental p-p interaction could be understood better by using the nucleus as a microscopic laboratory (23-30). Extensive Air Shower and other cosmic ray phenomena could be more clearly interpreted with the knowledge gained from a study of proton-nucleus interactions (31-40).

After the conclusion of the p-p experiment, a new series of experiments was started by the Echo Lake group in 1969. The liquid hydrogen target was replaced by targets of Al, Fe, Sn and Pb. The motivation was to study the differences in interaction characteristics with the variation of the nuclear size of the target. Thus, the aim differed very markedly from those of the earlier cosmic ray experiments. The thesis to be presented here deals with these Echo Lake hadron-nucleus experiments. In the remainder of this chapter, a brief review of proton-proton collisions and proton-nucleus collisions and their relevance to the study of high energy physics and cosmic ray physics will be presented. The second chapter deals with the experimental apparatus and the data reduction. The various experimental biases and Monte Carlo programs developed for the correction of these systematic biases are presented in the third chapter. The corrected results on the average charged multiplicities and the angular distributions are presented in the final chapter. Comparisons with other data and theoretical predictions are also given in the final chapter.

B. Previous Data on Hadron-Nucleus Interactions

Hadron-nucleus interactions can be represented by

$$h + A \rightarrow x + y + z + \dots + A' \quad (\text{I.1})$$

where the beam particle is represented by h , the initial and the final states of the target nucleus by A and A' and the final state particles by x , y and z . In coherent production reactions, the initial and the final state of the target nucleus are identical. For our purposes, we will deal only with reactions where there is no knowledge of the final state of the nucleus. The discussion will be further limited to the multiplicity distributions and the angular distributions of the fairly relativistic particles in the final state. Most of the cosmic ray experiments and some recent exposures of emulsions at Serpukhov and FNAL belong in this particular category.

1) Techniques: Most of the data are from emulsions exposed to either the cosmic ray beam or the beam at accelerators. The targets in emulsions are composed of three groups of elements: a) hydrogen (the mass number $A = 1$), b) carbon, nitrogen and oxygen (mean atomic weight of 14) and c) silver and bromine (mean atomic weight of 94). The observed tracks are labelled according to their ionizations: a) black tracks with $\beta (= v/c) < 0.3$, b) grey tracks with $.3 \leq \beta < .7$ and c) shower tracks with $\beta > 0.7$. The black and grey tracks are lumped together and the multiplicity of these so-called heavy

prongs (N_h) is observed to be independent of the incident energy. Nuclear fragmentation results in the appearance of these heavy tracks and a low or zero value of N_h was taken as an indication of 'hydrogen-like interactions'. This assumption was refined later to include low value for the multiplicity of shower tracks (N_s) as a criterion for p-p interactions. Interactions are also classified as belonging to the light nuclei or the heavy nuclei depending on their N_h values. Another way to treat the interactions in the emulsions is to weigh the probabilities (here the cross-sections are assumed to be proportional to $A^{2/3}$) for collisions with nuclei of different atomic weight and treat the target as a composite nucleus of average atomic weight 69. The latter approach is more relevant in the light of our interest in hadron-nucleus interactions.

Apart from the uncertainty about the nature of the target, emulsion experiments suffer from other incomplete information when they are done with cosmic ray beams. The energies are determined with the aid of the observed angular distributions and with certain theoretical conjectures regarding the mechanism of the collision process. The nature of the projectile is also uncertain since cosmic ray beams are basically admixtures of protons and pions and identification of these particles is not possible in emulsion experiments.

Data on these interactions are also available from experiments which employed scintillation counters and either

cloud chambers or spark chambers. As opposed to the emulsion experiments, this class of experiments has almost always used pure targets. Targets of low atomic weight have been characteristic of these experiments and the analysis procedures were heavily influenced by the methods used in the emulsion experiments. The comments made earlier about the uncertainties in using cosmic ray beams also applies to this class of experiments.

2) Average charged multiplicities: The most reliable emulsion data are from emulsions exposed to proton beams at accelerators. The average charged multiplicities as measured by Hebert et al.(41), Babecki et al.(42), and Florian et al.(43) and Gurtu et al.(56) are shown in Table I. Some of the earlier measurements done by Lord et al.(44), Meyer et al.(45), and Gierula and Wolter (46) are also shown in the table. This table also shows R, the ratio of the average charged multiplicity in proton-nucleus interactions to the average charged multiplicity in p-p interactions.

$$R = \frac{(\bar{n}_c)_{p-A}}{(\bar{n}_c)_{p-p}} \quad (I.2)$$

A cloud chamber was used to study cosmic ray interactions in Carbon and Brass by Lal et al (47). A carbon target was also used by the Echo Lake group before the experiment with the liquid hydrogen target and after the experiments with the metallic targets. These experiments, as reported in the doctoral theses

of K. N. Erickson (12) and R. J. Wilkes (15), used a spark chamber for the study of the interactions. Results of Lal, et al, Erickson and Wilkes are shown in Table I along with the other emulsion results.

Wilkes found a weak dependence of the average charged multiplicity on the atomic weight of the target. His results on the carbon target, along with the emulsion results, suggest a value of 0.13 for X where X is defined by

$$R \propto A^X \quad (I.3)$$

The ratio R is also remarkably independent of energy at energies greater than 100 GeV.

3) Angular distributions: Angular distributions have also been studied by many of the authors referred to in Table I. Systematic studies of angular distribution characteristics in hadron-emulsion nucleus collisions were also done by Rybicki and Wolter (48). Angular distributions are usually presented as $\log \tan \theta$ plots, where θ is the angle between the direction of the incident proton and the shower particle. This variable is proportional to the rapidity when the secondary is a relativistic particle. Hebert et al. (41), by comparing the distributions in emulsions and hydrogen, observed that the angular distribution has a strong anisotropy in the center of mass system. The whole excess in the multiplicity in relation to hydrogen is in the target hemisphere. Lal et al., also found

TABLE I
AVERAGE CHARGED MULTIPLICITIES IN
HADRON-NUCLEUS COLLISIONS

Energy in GeV	Target (Mass Number)	Average Charged Multiplicity	R ^a	Reference
50	C (12)	5.60 ± 0.30	1.02 ± 0.06	47
50	Cu (64)	7.80 ± 0.50	1.43 ± 0.14	47
200	C (12)	6.45 ± 0.42	1.10 ± 0.08	12 ^b
290	C (12)	7.17 ± 0.58	1.15 ± 0.11	12
400	C (12)	7.57 ± 1.26	1.16 ± 0.21	12
120	C (12)	10.13 ± 0.42	1.48 ± 0.09	15
173	C (12)	11.44 ± 0.64	1.55 ± 0.10	15
379	C (12)	12.28 ± 0.83	1.40 ± 0.11	15
200	W (184)	16.80 ± 3.80	2.19 ± 0.56	43
27	Em (69)	6.20 ± 0.20	1.49 ± 0.20	45
30	Em (69)	6.96 ± 0.07	1.50 ± 0.20	44
67	Em (69)	9.73 ± 0.23	1.65 ± 0.04	42
200	Em (69)	13.31 ± 0.28	1.73 ± 0.04	42
200	Em (69)	12.90 ± 0.40	1.68 ± 0.06	41
200	Em (69)	13.00 ± 0.40	1.71 ± 0.06	56
200	Em (69)	13.90 ± 0.60	1.82 ± 0.12	43
1000	Em (69)	19.16 ± 1.85	1.71 ± 0.31	46

^a $R = \frac{(\bar{n}_c)_{p-A}}{(\bar{n}_c)_{p-p}}$; a fit to the latest ISR data has been used

to compute the average charged multiplicity in p-p collisions. The errors on R are statistical errors.

^b Since the spark chamber inefficiencies were not considered in this experiment, the value of R is possibly lower than the true R.

that the average number of secondaries in the forward direction was the same with their two targets, carbon and brass. Eighty percent of the interactions observed by Rybicki and Wolter (48) in emulsion targets displayed the same angular distribution characteristics as was expected from a hydrogen target. From this observation, the relatively negligible role of secondary interactions in the nucleus could be inferred since the probability of interaction with the hydrogen nucleus in the emulsion was only 5%. The leading particle effect, which is characteristic of p-p collisions, was also seen with nuclear targets (5). This would lead to the conclusion that the inelasticity, i.e., the fraction of the energy given to the secondaries, is independent of nuclear size of the target.

The main inference from these results is that the average charged multiplicity and the angular distribution characteristics change very slowly with the nuclear size of the target.

C. Nuclear Targets and High Energy Physics

1) Motivation for the study of hadron-nucleus interactions: When a complex nucleus is used as a target, it might appear as though the complexity is added without providing any additional insight into the nature of strong interactions. In the past few years, many authors (23-30) have stressed the fact that use of nuclear targets can contribute to a better understanding of the processes in high energy physics. There are

two main reasons for the pursual of hadron-nucleus studies:

a) When a hadron collides with a nucleon in the nucleus, the projectile and/or the target is transmuted to an intermediate state. These intermediate states cannot be studied directly since they are highly unstable. Classic examples of these states are vector mesons and some properties of these mesons have been known mainly through studies of coherent reaction in nuclei (23).

b) The second reason has its genesis in the recent phenomenal growth of interest in multiparticle production in p-p interactions. This impetus comes from the fact that it is difficult to study many particle correlations in p-p interactions. Many models have been put forth to explain multiparticle phenomena and with the knowledge of correlations one could possibly choose from these different models. A supplementary tool would be the study of proton-nucleus interactions since reactions in nuclei offer unique possibilities of studying the space-time development of the initial production process. Gottfried's (23) analogy that the nucleus is really a very dense bubble chamber is very revealing in this context. Thus, it has been suggested that a study of the variation of the interaction characteristics with the mass number of the target nucleus will permit us a glimpse of the underlying mechanism that is not obvious in the fundamental p-p interactions.

2) Experimental data on p-p interactions: An understanding of the experimental results on p-p collisions is necessary for a comparison of the results on p-nucleus collisions. Most of the information on p-p interactions has come from the experiments done with the proton beams available from the new generation of accelerators at Serpukhov, National Accelerator Laboratory (NAL) and the CERN Intersecting Storage Rings (ISR). The energies covered by these facilities range from 50 GeV to 1500 GeV. The average properties of these interactions come from 'inclusive' reactions, where 'inclusive', as opposed to exclusive, is a reaction where not all the reaction products are observed. References 16-22 are recent articles on the properties of proton-proton interaction.

a) Total cross-sections: Recent experiments have shown that σ_{tot} increases by about 10% over the ISR energy range. σ_{tot} and σ_{el} vary appreciably with energy, falling from 2 to 20 GeV and then rising again above 100 GeV, whereas σ_{inel} rises monotonically from threshold.

b) Average charged multiplicity ($\bar{n}_c$): The most recent fit to the variation of n_c with energy is given by

$$\bar{n}_c = -3.8 + 1.88 \log s + 6.4/s^{0.5} \quad (I.4)$$

where s , the square of centre of mass energy, is in GeV. At higher energies, Extensive Air Shower experiments suggest a power law with an exponent greater than 0.25 and lesser than 0.50.

- c) Topological cross-sections ($\sigma(n)$): For each value of n , $\sigma(n)$ rises from threshold and reaches a maximum and then seems to drop off with increasing energy. The energy of the maximum increases with increasing n . In the energy range covered so far, a drop off can be noticed only for $n=2$ and $n=4$.
- d) Multiplicity distributions: Experimental distributions are not explained by any one simple Poisson distribution. Different types of distributions are seen at different energies. The distribution is narrower than Poisson below 70 GeV, reaches a Poisson distribution at about 70 GeV and becomes broader at NAL energies. Several empirical formulae have been proposed to explain these distributions (22).
- e) Transverse momentum distributions: It had been known for a long time that this distribution falls off rather steeply leading to small value of average transverse momentum ($\bar{p}_t \approx 0.35$ GeV/c). But according to recent experiments at ISR, pions with large p_t occur by several magnitudes more frequently than predicted by an e^{-6p_t} distribution. Also at high p_t , pions lose the dominant role they have at low values of p_t and heavier particles become more important.
- f) Scaling and rapidity distributions: The concept of scaling, which has attained great importance in the recent years, refers to the asymptotic energy independence of inclusive reactions. A typical single particle inclusive reaction is symbolized by

$$p + p \rightarrow \pi + \text{anything}$$

with momenta

$$p_a + p_b \rightarrow q + \dots\dots\dots$$

Two popular variables are

$$\text{Feynman's } x = 2 q_{||}/s^{\frac{1}{2}}$$

$$\text{and rapidity } y = \frac{1}{2} \log ((E_q + q_{||})/(E_q - q)).$$

For particles where $(q_{||} \gg q_{\perp} \gg m_q)$, $y = \log \tan \theta/2$ and under a Lorentz transformation along the direction of the incident particle, all rapidities are shifted by a constant amount. The rapidity distributions can be roughly subdivided into three regions: target fragmentation region, beam fragmentation region and central region. The central region, which is the intermediate region, is also called the pionization region. In the x variable, scaling predicts

$$E_q \, d\sigma/dq^3 \propto f(q_{\perp}, x) \text{ independent of } s.$$

In particular, Feynman (the originator of the scaling concept in these applications) suggested that the rapidity distributions should have a central plateau. Scaling is now well established, to an accuracy of about 10%, in most of the interactions over the ISR energy range.

g) Correlations: Single particle inclusive reactions reveal only a limited part of the underlying dynamics. A step further in learning more about the dynamics is the study of correlations which might exist among the secondaries. The simplest case, next to the single particle distribution, is the study of two

particle distributions and correlations. In the case of rapidity correlations, two types of correlations are important: short range correlations (SRC) and long range correlations. If particles have their rapidities very close ($\Delta y < 2$), the correlation is termed a short range correlation. The rapidity space is evenly populated in case of a short range correlation. The observation of a particle with a large value of x (say, $x \gg 0.9$) and a cluster of particles at small x ($x \sim 0$) would be the observation of a long range effect. In general, events with SRC will have large multiplicities and low multiplicities are characteristic of long range effects. Recent experiments at ISR have demonstrated the dominance of SRC among high multiplicity events.

3) Theoretical models for p-p interactions: A variety of models have been put forth for the explanation of p-p interactions, but three classes of models are more important.

a) Older models: These models, which were put forth two decades ago, had been quite successful in the interpretation of the early cosmic ray data. Fermi's Statistical Model and Landau's Hydrodynamical Model are the most important older models. Fermi's model, which was later refined by Hagedorn and others, uses the statistical approach since the number of particles produced is quite large. All conclusions are drawn from the thermodynamics of a volume V in which the equilibrium is established. The variation of multiplicity with energy is given

by

$$n \propto E^{\frac{1}{4}} \quad (I.6)$$

where E is the laboratory energy of the proton. While this variation had been noted in the earlier experiments, the wrong particle composition was the weak point of the theory. An additional defect was the prediction of angular isotropy in the centre of mass system. Pomeranchuk pointed out that the volume V , which was kept constant in Fermi's theory, was in fact dependent on the multiplicity. In 1955, Landau noticed the inadequacy of the statistical approach and interpreted the collision from a hydrodynamical approach. In his model, two hadronic discs meet and coalesce into a single body which expands until an optimum volume is reached in which there is no strong interaction between the particles. After this stage is reached, the particles in the system fly outward. This theory also predicts a variation of multiplicity with energy which is similar to the variation in Fermi's model. The progress made by Landau's model was only in predicting the correct particle composition, since it also did not predict correct angular distributions. Some of the later versions of these models did predict correct angular distributions.

b) Multiperipheral models (MPM): These models have their origin in the relatively successful one particle exchange model (OPE), which had been put forth for the interpretation of quasi two body reactions. Figure 1 shows a schematic representation

TABLE II

PREDICTIONS OF DIFFERENT MODELS FOR PROTON-PROTON INTERACTIONS

Property	Multiperipheral Models	Diffraction Models	Experimental Data
Scaling	Valid	Valid	Valid up to ISR Energies
Average Multiplicity	$A + B \log s$	No Prediction	$A + B \log S + C/s^2$
Topological $\sigma^-(n)$ Cross-sections as $s \rightarrow \infty$	Goes to Zero	Attains a Constant Value	At NAL Energies Still Falling
Multiplicity Distributions	Poissonian (only a rough approximation)	$1/n^2$ if $\bar{n} \propto \log s$ is assumed	Varies with Energy (Broader than Poisson at NAL Energies)
Rapidity distributions	Plateau at Central Region	Valley at Centre; Plateau can be Accomodated	Plateau at Central Region
Correlations	Short Range Correlations Between Particles Close to Each Other in the Multiperipheral Chain	Short Range Correlation for Particles in Clusters	Dominant Short Range Component

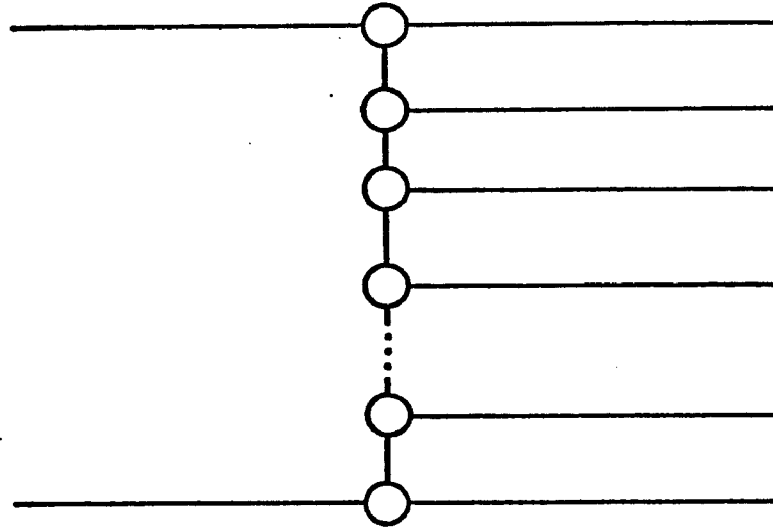


Fig. 1. Particle Production in P-P Collisions According to Multiperipheral Models

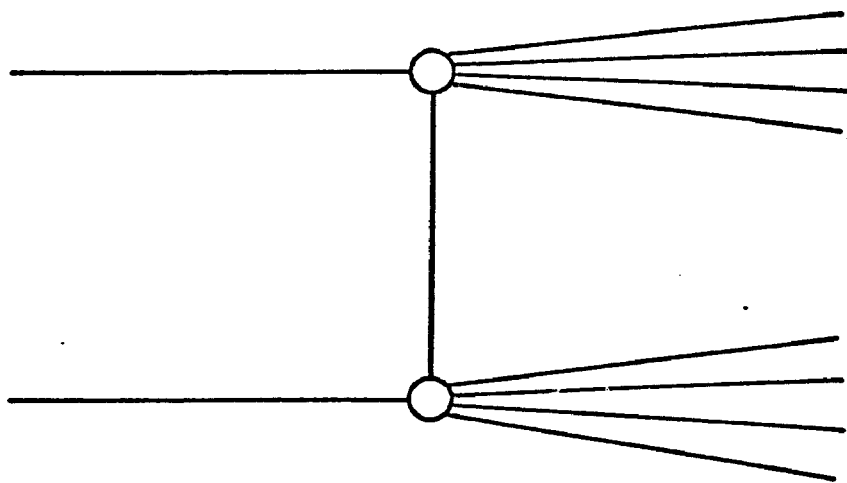


Fig. 2. Particle Production in P-P Collisions According to Diffraction Dissociation Models

of this model. The secondary particles are produced instantaneously and come off a multiperipheral chain. The center of mass longitudinal momenta of the secondary particles tend to be continuously distributed. The particles at the end of the chain have the tendency to keep the direction and also carry a large fraction of the energy. Table II shows the predictions of these models for p-p interactions.

c) Diffractive Fragmentation Models (DFM): In these models, the secondary particles are considered as fragments of two incident particles. These models evolved from the explanation of diffraction scattering phenomena. Figure 2 shows schematic representations of different classes of diffractive models. The particles are subdivided into particles which go forward in the centre of mass system and particles which go backward. The backward and forward tendencies become stronger with increase of energy. Table II shows the predictions of these models. From Table II, which also shows the corresponding experimental data, it is seen that neither the multiperipheral models nor the diffractive models by themselves can explain all the experimental data. To correct for this situation, models have been proposed in which both the mechanisms have been invoked. In a typical two component model, the contribution of the diffractive component is about 20% and is independent of the energy.

4. Theoretical models for proton-nucleus interactions:

As opposed to the great variety of theoretical models for

proton-proton collisions, there have been very few models for proton-nucleus interactions.

a) Earlier models: The motivation for these models came from a necessity to relate the energy determination from the angular distribution in a p-nucleus collision to the true energy in a p-p collision. One of the most important concepts in all these models is the idea of Intra Nuclear Cascade (INC). In an INC, pions and nucleons from the first collision in the nucleus undergo subsequent collisions with target nucleons which results in a cascade. An auxiliary concept in these models is the average number of collisions that is possible for a beam particle inside the nucleus. A formula relating the average number of collisions and the nuclear size will be given at a later stage. Messel (2) had already shown in 1952 that INC mechanism could explain the results on p-nucleus interactions at low energies. With the increase in multiplicity with energy, the INC mechanism gave too many final state particles at higher energies. The variation of the simple INC model was the tunnel model, where the cascade was limited to a central cylindrical region inside the nucleus, because of the smaller emission angles of the secondaries. A further variation of the tunnel model was the funnel model, where the cross-sectional area of the tunnel increases and assumes a funnel shape. As an extension of Fermi's statistical model, Cocconi considered the collision of the incoming nucleon with

a lump of n nucleons, where n is the average number of nucleons involved in the collision. All these models can be seen only in a historical vein, since these models did not have any detailed predictions. A good review of all these earlier models has been given by Hayakawa (2). The main inference from these models for the present discussion is the failure of the INC mechanism in explaining the data at higher energies.

b) Multiperipheral models: Since particles are formed instantaneously in MPM, these models lead very naturally to cascade phenomena when extended to p-nucleus interactions. Fisband and Trefil (26), Dar and Vary (27), Subramanian (25) and Gottfried (23) have all shown that this class of models does not explain the observed data. Gottfried has further shown that even a restricted version of the cascade involving only the leading particle also results in too many final state particles. A schematic picture of particle production in MPM is shown in Figure 3. Dar and Vary, with their version of MPM, predict a strong dependence of the charged multiplicity on both the mass number and the energy of the beam particle. A linear dependence of the multiplicity on energy is predicted by them for heavy nuclei. In the incoherent production model (IPM), which is the MPM version of Fishbane and Trefil, average charged multiplicity is given by

$$\bar{n}_c = A + B \ln s \quad (I.7)$$

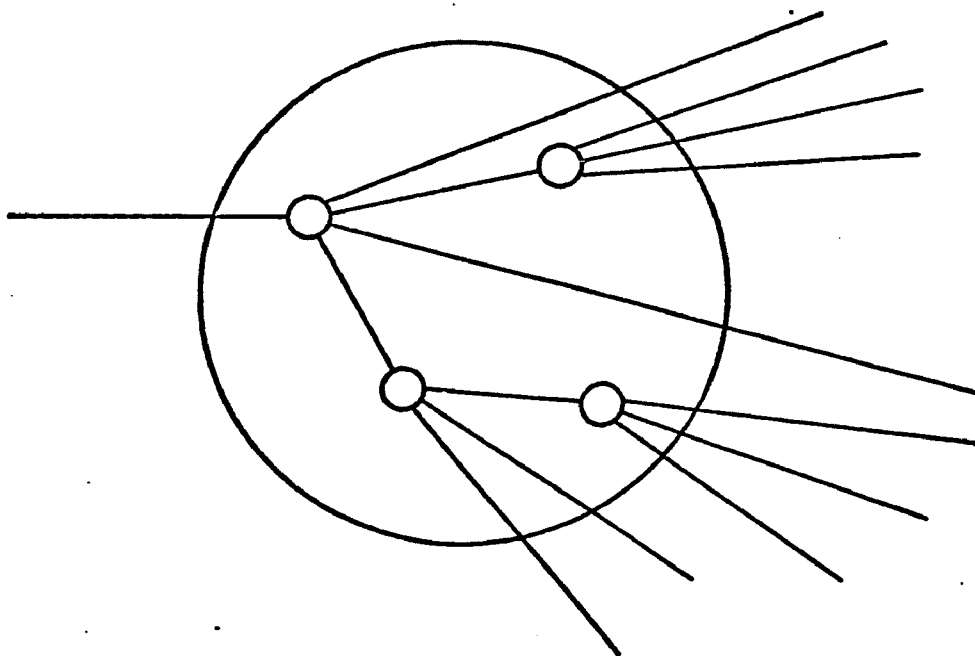


Fig. 3. Particle Production in P-Nucleus Collisions
According to Multiperipheral Models

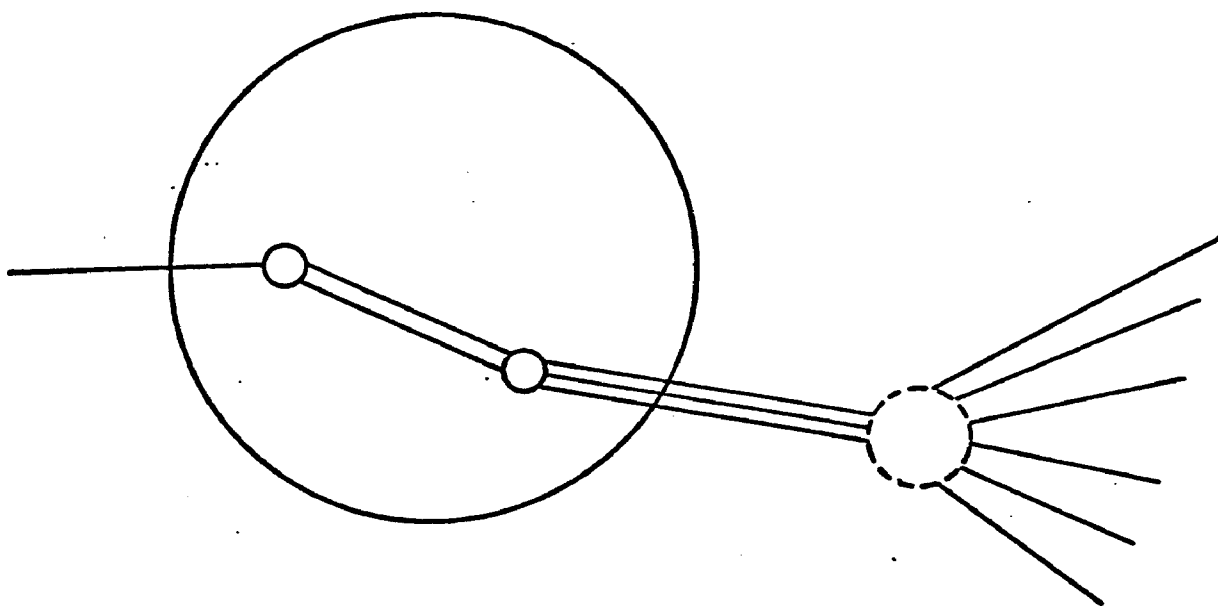


Fig. 4. Particle Production in P-Nucleus Collisions
According to Diffraction Dissociation Models

where s is the square of the centre of mass energy and A and B are larger than the fits for p - p collisions. Further, their charged multiplicities are independent of the mass number for mass number greater than 10. Table III shows some of the predictions of these models.

c) Diffractional Fragmentation Models: In these models the excited state of the projectile proceeds through the nucleus exciting other target nucleons in its path. The target excited states are rather stationary and decay inside the nucleus. Because of time dilation, the projectile excited state escapes from the nucleus and decays outside the nucleus. The essential characteristics of these models are low multiplicities and the logarithmic variation of the multiplicity with energy. The observed angular distribution is also qualitatively explained by these models. The predictions of these models are shown in Table III. The ratio R is independent of energy in these models, but Gottfried has pointed out that this is superficial. With increasing energy, the target excited states decay into higher energy secondaries which might lead to intra-nuclear cascades. A schematic picture of the particle production in these models is given in Figure 4.

d) Other models: Feinberg (3) has pointed out that proton-nucleus collisions are more suitable than p - p collisions for a hydrodynamical approach. According to him, the formation of an overall statistical system, which is the basic condition for

TABLE III

PREDICTIONS OF DIFFERENT MODELS FOR PROTON-NUCLEUS INTERACTIONS*

Property	Multiperipheral Models	Diffraction Models	Energy Flux Cascade Model	Experimental Data
Variation of R with Atomic Wt.	Strongly Dependent (others) Independent of A for $A > 10$ (Fishbane and Trefil)	$n_0 = \frac{1}{2}(1 + \frac{1}{3}A^{1/3})$ (Fishbane and Trefil) $= \frac{1}{2}(1 + \sqrt[3]{A})$ (Gottfried)	$R = 1 + 1/3(\sqrt[3]{A} - 1)$	$\propto A^{0.13}$
Variation of R with Energy	Strong Variation	No Variation	No Variation	No Variation
Leading Particle Effect	No	Yes	No Prediction	Yes
Rapidity Distribution in Forward Direction	Not Similar to p-p Case (others) Similar to p-p case (Fishbane and Trefil)	Similar to p-p Interactions	Similar to p-p Interactions	Similar to p-p Interactions
Rapidity Distribution in the Backward Direction	Large Increase Compared to p-p Interactions	Increase But Smaller than in Case of Multiperipheral Models	Increase Compared to p-p	Increase Compared to p-p

* Many authors have examined both Multiperipheral models and Diffractive Fragmentation and when the predictions are different, the individual predictions have been given.

the applicability of hydrodynamical ideas, is well satisfied in proton-nucleus collisions. In fact, Landau's model has predictions for multiplicities on nuclear targets and the variation of R with the mass number is given by

$$R \propto A^{0.19}. \quad (1.8)$$

Landau's model also predicts an average shift in rapidity towards target fragmentation regions. Gottfried (23) has recently proposed a model which is quite close in spirit to Landau's model. The energy-momentum flux of the hadronic matter is the essential variable that determines the early evolution in the model. The energy momentum flux proceeds through the nucleus and then decays outside the nucleus. For each extra collision in the nucleus, the multiplicity increases by $1/3 \bar{n}_{c(p-p)}$. Thus, the prediction for R is

$$R = 1 + 1/3 (\bar{\nu}_A - 1) \quad (I.9)$$

where $\bar{\nu}_A$ is the average number of collisions inside the nucleus. For an uniform density distribution for the nucleus (23),

$$\bar{\nu}_A = 4/3 r/\lambda (1 + \lambda^2/2r^2) \quad (I.10)$$

where r is the radius of the nucleus and λ is the inelastic mean free path. This formula holds good only when $A^{2/3} \gg 1$ (for example, $\bar{\nu}_A = 3.2$ for $A = 69$, when λ corresponds to a σ_{inel} of 32 mbarns). The observed angular distribution is

also explained by this Energy Flux Cascade model (EFC) but the absence of the leading particle may be the weak point of the model.

Some amount of caution is necessary when the experimental results are compared with the predictions of different models. While all the models use the concept of the average number of collisions ($\bar{\nu}_A$), they differ in the formulation for $\bar{\nu}_A$. Equation (I.10) is the only explicit formula for in the recent literature. For the same $\bar{\nu}_A$, the ratio R in Gottfried's version of the Diffractive Fragmentation model is higher than R in the EFC model. For example, with a $\bar{\nu}_A$ of 3, EFC model has 1.67 for R ; the DF model gives an R of 2.0 with the same $\bar{\nu}_A$. An earlier version of the Diffractive Fragmentation model by Fishbane and Trefil predicted a value of 1.4 for $\bar{\nu}_A$ of 3. From Table III, where the predictions of some of these models are compared with the available data, it is apparent that the Diffractive Fragmentation models and the EFC model explain most of the data.

D. Importance of Nuclear Targets in Cosmic Ray Phenomena

Most cosmic ray phenomena involve particles which are the reaction products of several collisions of cosmic ray primaries and secondaries with air nuclei in the atmosphere. A significant percent of the primaries are not protons and majority of the secondaries are charged pions and the decay products of the neutral pions. Therefore, the types of interactions that

are present in the cosmic ray phenomena are hadron-nucleus interactions and nucleus-nucleus interactions. The conclusions drawn from cosmic rays for p-p interactions are always affected by the fact that the target, and sometimes even the projectile, is not a proton. We will discuss two phenomena in which the effect of nuclear targets is quite considerable.

1) Extensive Air Showers: The particles observed in Air Showers (EAS) are the cumulative products of many hadron-nucleus collisions in the atmosphere. By observation of different properties of these showers, conclusions were drawn about the primary interaction in the upper layers of the atmosphere. By observation of different properties of these showers, conclusions were drawn about the primary interaction in the upper layers of the atmosphere. Recent studies of several parameters have indicated that the showers are developing faster at higher energies. A logarithmic variation of the multiplicity with energy is too slow to account for the observed shower development (38). The type of variation that has been proposed is a power law with an exponent greater than 0.25 and lesser than 0.50. Several authors (31, 32, 33) have computed the multiplicities recognizing the importance of nuclear targets and the presence of heavy primaries and possible changes in the interaction characteristics at high energies (like rising cross-sections and nucleon-antinucleon production). Gaisser et al. (31), invoke all these effects but do not get an agreement with the data. Greider (32) gets a value of 0.88 for X

using a naive intranuclear cascade model and a value of 0.27 by letting only the leading particle cascade in the nucleus. The lateral distribution of shower particles, which is an important parameter in EAS, is a reflection of the angular distribution and is also possibly affected by the presence of nuclear targets.

2) Charge ratio of muons (μ^+/μ^-): The charged ratio of muons is an important factor in distinguishing between different models of high energy interactions. Experimentally, this ratio is known to be between 1.25 and 1.30 and is almost constant from 10 GeV to 4000 GeV. The conventional way to deduce this ratio is to assume a charge ratio for pions in the primary interaction and from ISR experiments, this charge ratio is known to be between 1.15 and 1.17. By taking into account the different types of primaries in the atmosphere, the muon charge ratio has been computed by Frazer et al.(34), Morrison and Elbert (35), Garaffo et al.(36), and Adair (37). Without considering the effect of nuclear targets, Frazer et al, get a value of 1.56 and Garaffo et al, get a value of 1.58. With nuclear effects in their computations, Adair gets 1.53, Garaffo et al, get 1.36 and Morrison and Elbert get 1.26.

While we have discussed only two aspects of cosmic ray phenomena, the importance of nuclear targets for many cosmic ray phenomena has been stressed by the various rapporteurs in the most recent International Conference on Cosmic Rays held at Denver (38, 39, 40).

CHAPTER II

THE EXPERIMENT AND THE DATA REDUCTION

A. Beam

The experiment was conducted at Echo Lake, Colorado, at an altitude of 3230 meters elevation which corresponded to a depth of 715 g/cm² in the atmosphere. The beam consisted of unaccompanied charged hadrons of cosmic rays and from earlier studies the flux of these hadrons was known to be

$$N(>E) = 6 \times 10^{-3} E^{-2 \pm .2} \text{ cm}^2 \text{ sr sec}^{-1} \quad (\text{II.1})$$

where $N(>E)$ is the flux of hadrons of energy greater than E . This flux consisted mainly of protons and pions and an indirect estimate of the flux of the pions was obtained by a study of the charge to neutral ratio of hadron triggers. The pion flux was determined to be 30% of the proton flux and was a function of this particular experimental setup.

B. The Apparatus

Figure 5 gives an overall view of the experimental arrangement.

1) The top counter: This was the scintillation counter placed above all the other equipment in the experiment.

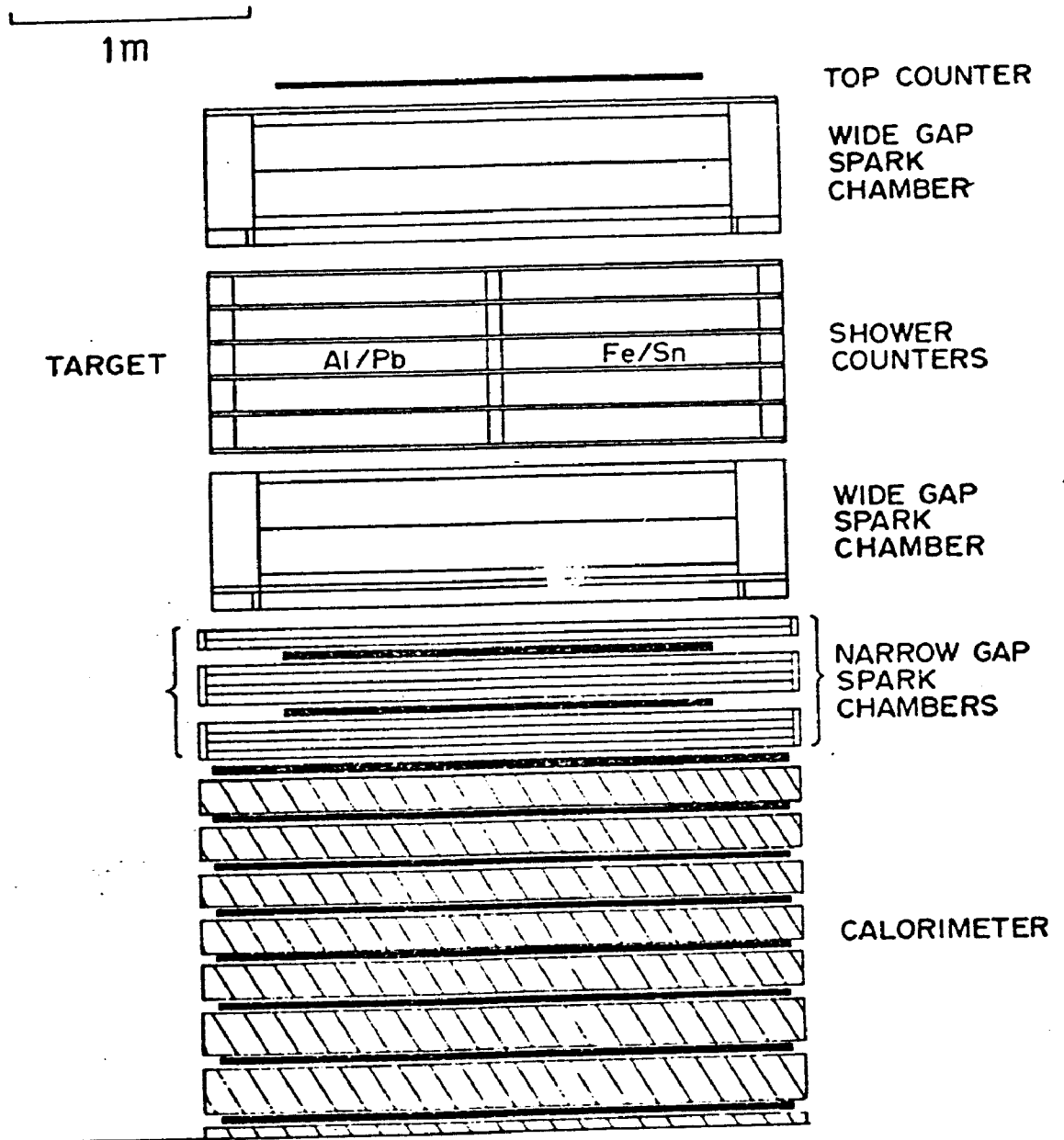


Fig. 5. The Experimental Apparatus

The area of this scintillation counter was $1.83 \times 1.83 \text{ m}^2$ and employed eight photomultipliers. This counter served as the sampler of the beam particles and was useful in the elimination of neutral particles and particles entering from outside the fiducial geometry.

2) The wide gap spark chambers: The wide gap spark chambers have become useful tools in high energy research and detailed descriptions and discussions of the chambers used in this experiment are given in the doctoral thesis of B.W. Loo(14). Two wide gap spark chambers were used, one above the target to record the incident particle and one below to record the interactions. Each chamber had two 20.3 cm gaps separated by a central foil of .0051 cm Aluminium. The area of the chambers was $2 \times 2 \text{ m}^2$ and the sides of the chambers were made of 1.22 cm thick glass. Standard spark chamber neon ($\sim 85\%$ neon and $\sim 15\%$ helium) was used in these chambers and this gas was continuously recirculated through a calcium oven at 300°C and a molecular sieve at room temperature. This assured gas purity at all stages of the experiment. The spark chambers were pulsed by an eight stage Marx generator with a delay of about 150 nsec between the pulse from the electronics logic and the high voltage pulse on the chambers. An evaluation of the performance of these chambers will be made in a later section.

3) The shower counters: To eliminate the hadrons accompanied by air showers, an array of shower counters was

arranged around the target in its mid plane. This array, which was rectangular, had an area of 7.68 m^2 and 24 photomultipliers and scintillators were used in these shower counters. Very small showers or events with one or more charged particle in geometry but none in the shower counters were rejected in the process of scanning the spark chamber film. A veto from these counters for the final trigger was considered as an indication of an unaccompanied hadron.

4) Targets: The main difference between this experiment and the hydrogen target experiment was in the nature of the targets. The liquid hydrogen target was removed in August, 1969, and was replaced initially by targets of Aluminium and Iron. In March, 1970, the Aluminium and Iron targets were replaced by targets of Lead and Tin. All these targets were in the form of six thin plates, each about 3% interaction mean freepath thick. The spacing between the plates was about 15 cms and the area of the plates was $105 \times 210 \text{ cm}^2$. The spacing was provided by $160 \times 160 \text{ cm}^2$ frames of Aluminium channel and beams which were bolted to the plates to form a rigid assembly. Table IV gives the details of the target geometries and properties. Two runs were taken, one with targets of Al and Fe plates side by side and the other with targets of Sn and Pb plates side by side.

TABLE IV
TARGET PROPERTIES

Target Element	Thickness of One Plate GM/CM ²	Thickness of the Total Target GM/CM ²	Rad Lengths
Al	3.429	20.674	0.857
Fe	4.997	29.984	2.157
Sn	4.642	27.851	3.133
Pb	5.405	32.430	4.974

5) The ionization calorimeter: As an alternative to the ambiguous methods of energy determination in emulsion experiments, the counter and cloudchamber physicists pioneered in the measurement of hadronic cascades in heavy materials which resulted in the so-called ionization calorimeter or total absorption spectrometer. Here the energy of the incident particle is largely converted into ionization which is sampled by several scintillation counters placed at convenient locations inside the material (50). This ionization was then compared to the ionization produced by a minimum ionizing particle such as a muon and a measure of the incident energy was obtained. The upper 200 gms of this calorimeter included ten narrow gap spark chambers in which the initial development of cascades could be observed. Each of these chambers was constructed from two 203 x 254 cm² copperclad iron plates of 1.37 cms thickness, one of which was on either side of a 1.91 cm gap. These spark

chambers were pulsed in parallel with the wide gap spark chambers and were also photographed. The total amount of iron in the calorimeter was 1170 g/cm^2 . In total, ten layers of plastic scintillators were used in the calorimeter stack and the separation between the layers in the lower part of the calorimeter was one proton interaction mean freepath. Eight photomultipliers were used for each layer and the output voltages from all these tubes were summed, which was then an indication of the total amount of light observed in the layer. A measure of the visible energy was obtained by summing the signals from each layer.

6) Trigger and electronics: A discussion of the electronics is facilitated by classifying the electronic circuits into two parts: triggering electronics and recording electronics.

A normal charged hadron trigger required a charged particle in the top counter unaccompanied by any air showers and which deposited an energy greater than the nominal threshold energy. A muon trigger which was used for calibration purposes (to be discussed in a later section) required a charged particle in the topmost layer, the first layer and the tenth layer of the calorimeter and no accompanying air showers. The triggering electronics was used to comply with these requirements and is shown in the Figure 6. The summed outputs from the top counter and the summed outputs of the shower counters were

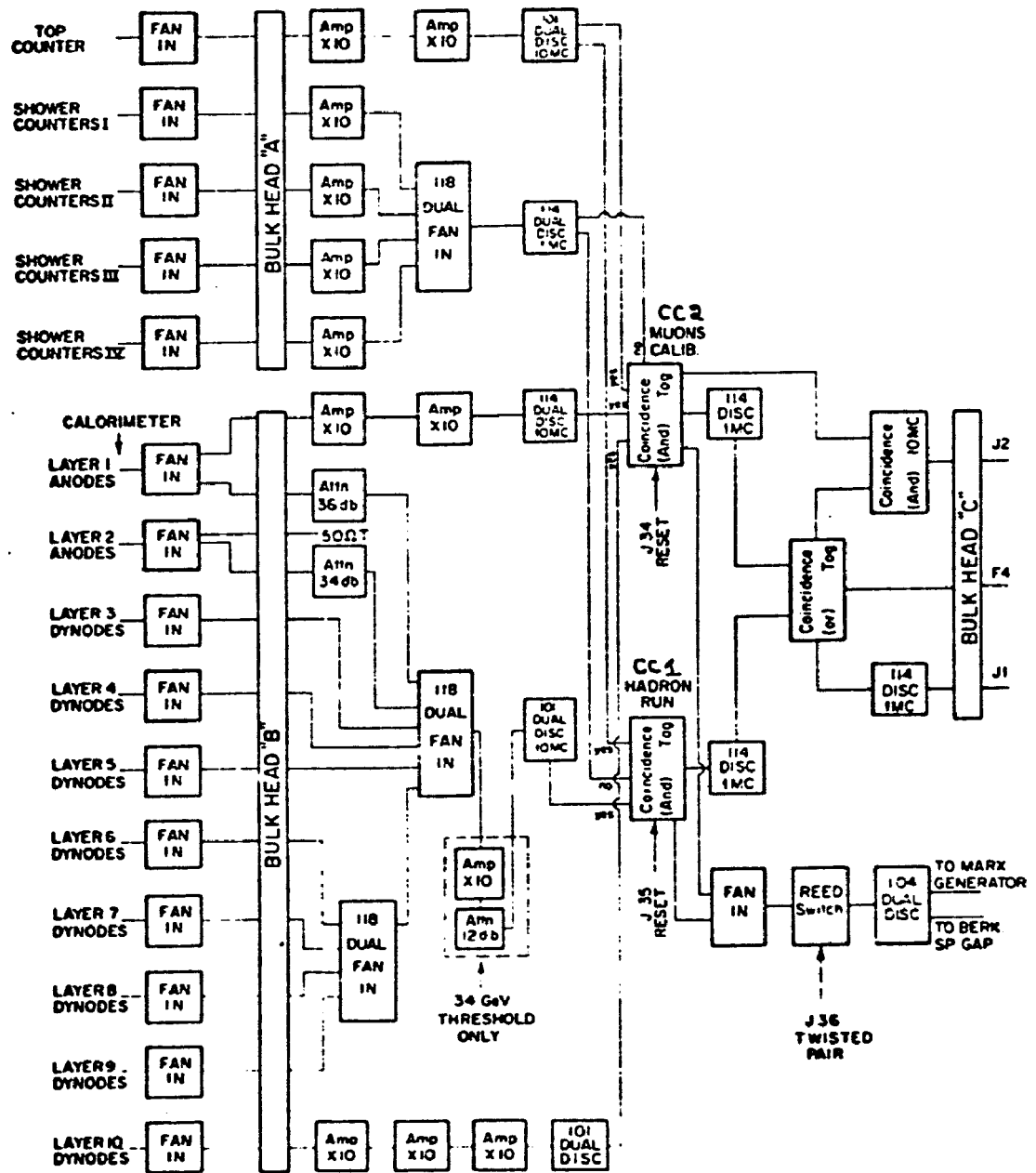


Fig. 6. Block Diagram of the Triggering Electronics

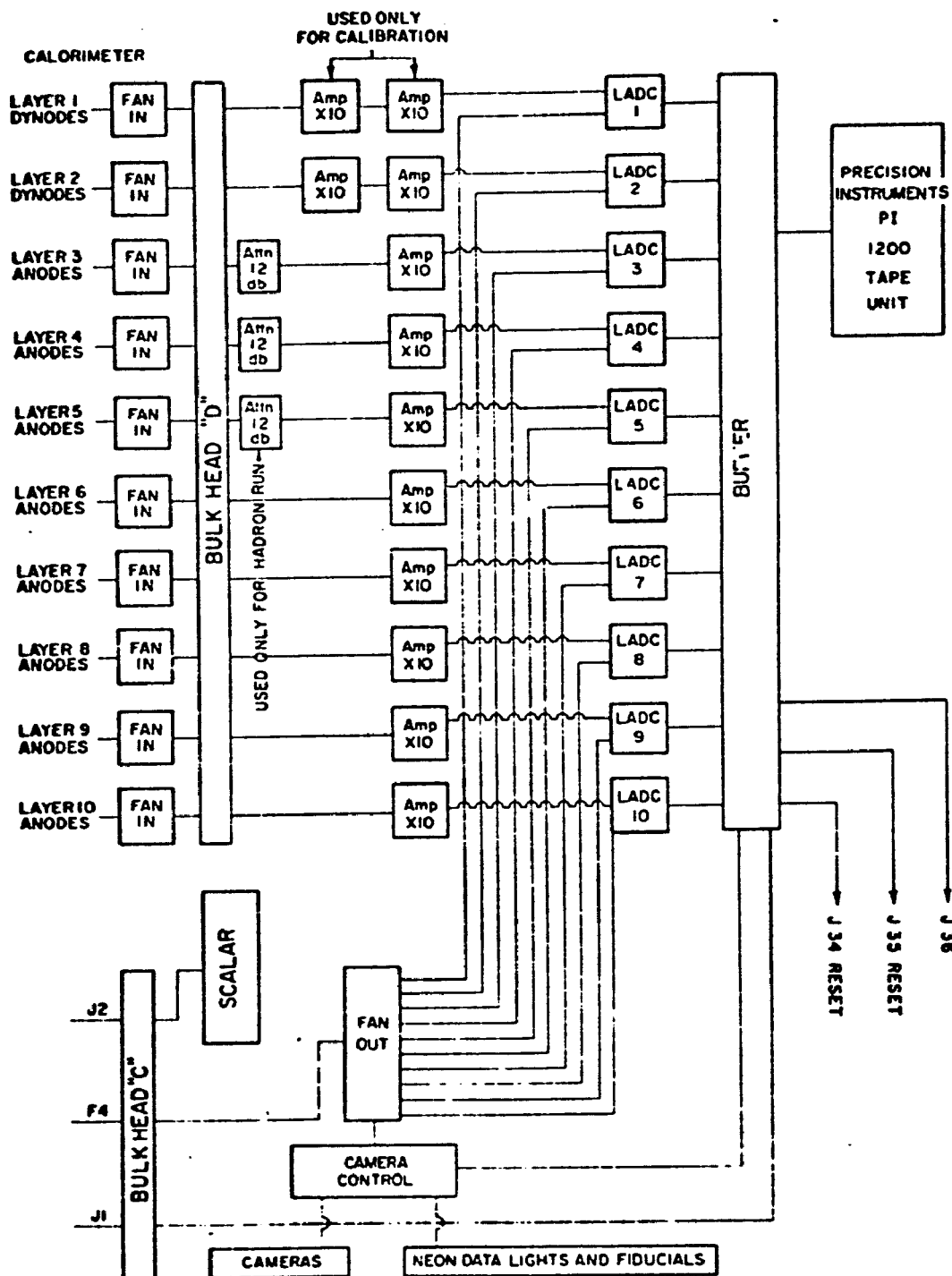


Fig. 7. Block Diagram of the Recording Electronics

amplified separately and sent into two discriminator units and the discriminated outputs were fed into the 'yes' and 'no' inputs respectively of a coincidence unit CC1. The summed outputs from the photomultiplier dynodes of layers 3-10 and the attenuated summed outputs of layers 1 and 2 (RCA 6810 A photomultipliers which were used in layers 1 and 2 had more gain than EMI 4518 photomultipliers used in the other layers) were fed into a 'fan in', the output of which was fed into a discriminator. The output of the 'fan in' was a measure of the energy recorded by the calorimeter and the discriminator allowed the variation of the energy threshold. This discriminated pulse was fed into another 'yes' input of the coincident circuit CC1 and the output of CC1 was the so-called hadron trigger. For the muon trigger, the pulses from the top counter, the first layer in the calorimeter and the tenth layer in the calorimeter were fed into separate 'yes' inputs of a coincidence circuit CC2 which also had a 'no' input to which the shower counter pulses were fed. The output of CC1 (CC2 for the muon trigger) was used for pulsing the Marx generators and gating the circuits in recording the data.

The recording electronics is shown in Figure 7. The recording of the data involved the recording of the pulses from the ten layers of the calorimeter, the recording of the event, time and date, and photographing the events in the spark chambers. The dynode signal from the first two layers

and the anode signals from the other layers were fed separately into ten logarithmic analogue digital converters (LADCs). The main function of these LADCs was to provide an output channel number which was proportional to the logarithm of the input pulse area. The trigger mentioned in the previous paragraph furnished a gate pulse to these LADCs. The LADC outputs and other information (event number, etc.) were recorded on a magnetic tape for Al/Fe runs and on the film for Sn/Pb runs. The trigger pulse also controlled the camera units and the fiducial lights. Each event in the spark chambers, along with the other informations, was photographed in 90° stereo using two 35mm Flight Research cameras. A system of front faced mirrors was used to arrange the information on the film with a demagnification of 65:1.

The experimental apparatus had a useful admittance of 0.94 m^2 steradian and the rate of hadron triggers of energy greater than 84 Gev was 20 per hour. In contrast, the rate of muons was 4000 per minute.

C. Scanning, Measuring and Reconstruction of Events

1) Nature of events: The hadronic processes of interest in this experiment fell into two categories: non-interacting events and interacting events. In a non-interacting event, a charged hadron traversed the whole length of the target without any interaction. These events were also called

'straight through events'. An event was so classified when there was only one track in both the wide gap spark chambers and when the track in the lower chamber could be related to the track in the upper chamber in both the views. Interacting events were events where there was an inelastic interaction of the beam particle in the target resulting in the production of particles. An event was labelled as an interacting event when there was only one track in the upper chamber and more than one track in the lower chamber and when the lower chamber tracks could be related to the single track in the upper chamber in both the views.

2) Scanning of events: The purpose of scanning was to select these interesting events from all the film frames collected during the runs and both the stereo views were consulted for this purpose. When charged particles pass through matter, non-hadronic processes are possible resulting in the simulation of interacting events. The main contamination came from a charged particle knocking off an electron giving rise to pseudo-two prong events. Pseudo-two prong events with a low energy electron could be recognized easily on the scanning table since the electron suffered considerable amount of multiple scattering in the thin electrode of the lower wide gap spark chamber. When the track did not show any appreciable scattering, the penetration of the track in the first four gaps of the iron plate spark chamber was studied. When the track did not penetrate deeper than three gaps, it was considered to be due to

an electron. Since the performance of the iron plate spark chamber was erratic and did deteriorate with time, one had to resort to other methods after the reconstruction procedure to reject these non-hadronic events. These methods, to be discussed in Appendix B, were also used to reject non-hadronic events with more than two prongs. The kinky tracks in interacting events were identified as due to the production of electrons by secondaries and were rejected. Each interacting event was assigned a 'scan multiplicity' which was the number of secondaries in the interaction. If one view showed more tracks than the other view, the 'scan multiplicity' was given by the number of tracks in the view which showed more tracks. Table V gives the number of total frames and the number of interacting and non-interacting events selected for measuring. The multiplicity distribution of events selected for measuring is shown in the Table VI.

3) Measurement: The non-interacting events were measured on a flying spot digitizer connected online to a DDP 124 computer. This system, called MASS (Michigan Automatic Scanning System), had a resolution of $10\ \mu$ on film and measured eight points in each gap of the two wide gap spark chambers. Measurements along the tracks, the event number and the fiducials were recorded separately for each view on a magnetic tape. Later both views were merged, considering events only when they had been measured properly in both the views. The interacting

TABLE V
RUN SUMMARY

	Al/Fe Run	Sn/Pb Run
Total number of frames	77156	86300
Number of frames selected for scanning	61347	67880
Number of events selected for measuring		
a) Non-interacting events	32440	35344
b) Interacting events	2403	2072
Number of events after reconstruction		
a) Non-interacting events	16400	14933
b) Interacting events	2074	1809

TABLE VI
MULTIPLICITY DISTRIBUTION OF EVENTS
SELECTED FOR MEASURING

Multiplicity	Al/Fe Run	Sn/Pb Run
$2 \leq n \leq 5$	1235	1299
$6 \leq n \leq 10$	702	349
$11 \leq n \leq 15$	346	219
$16 \leq n \leq 20$	106	117
$21 \leq n \leq 25$	13	64
$n > 25$	1	24

events were measured at the University of Wisconsin on a microscope which had a least count of $1\ \mu$. Eight points were measured along each track and in each gap in both the views. The measurement information was recorded on a magnetic tape via a CDC 924 computer.

4) Optics program: Prior to the actual reconstruction of events, corrections for optical distortions had to be determined for each measured point. A fiducial plate containing 32 equally spaced fiducial marks and 8 fiducials on the glass of the spark chambers were measured with a $1\ \mu$ resolution microscope. A χ^2 fit was performed with these measurements to determine the coefficients of a second order polynomial correction function. The minimization of this χ^2 was done by using GAUSHAUS, a subroutine which provided a least squares estimate of the parameters using an iterative technique. Later the measured points were rotated, translated and applied a differential magnification for purposes of proper orientation with respect to the optical axis. The real coordinate system used in the experiment had its origin midway between the centres of two wide gap spark chambers. The diagram of the coordinate system used to describe the experiment is shown in Figure 8.

5) Cosmic program: This program, which controlled the actual reconstruction of events, used several subroutines. INTAPE was a subroutine which provided the complete measurement records after calling several other subroutines which picked up different bits of information from the measuring tape. The

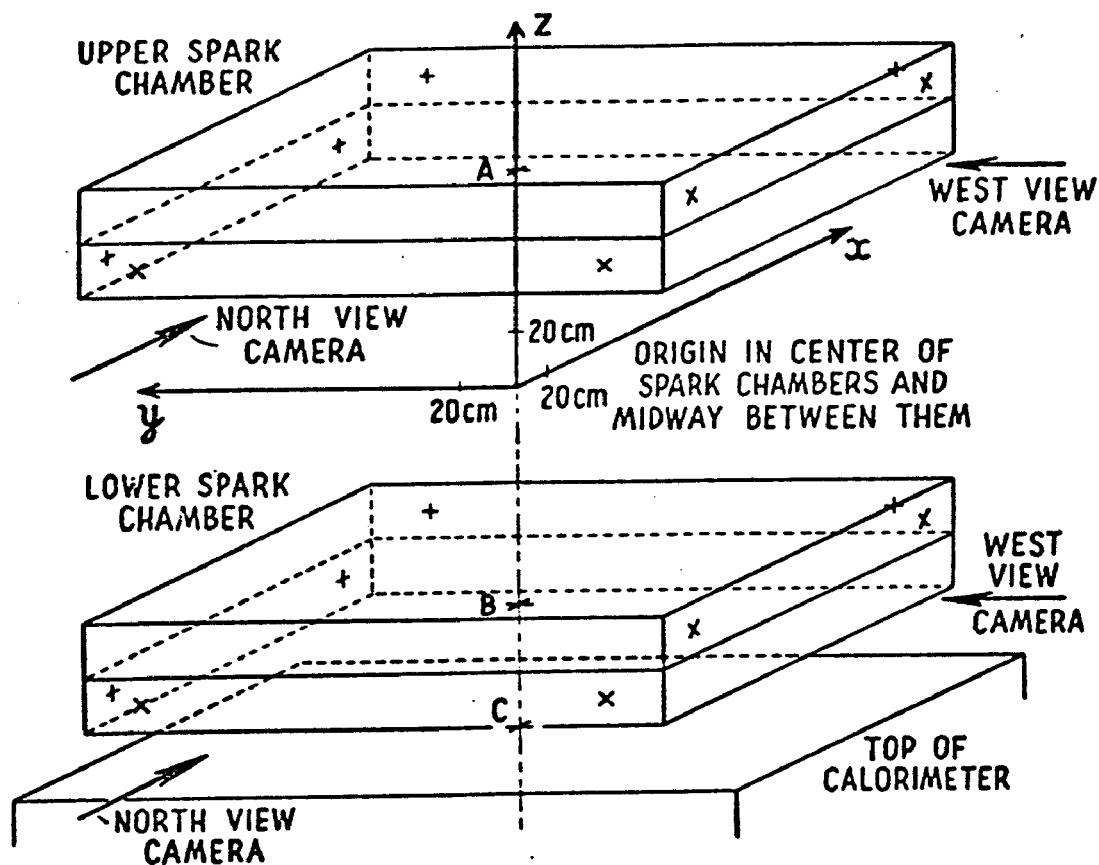


Fig. 8. Diagram of the Co-ordinate System Used to Describe the Experiment

program READ after having called INTAPE to provide a record, separated and stored the track and fiducial measurements. As the first step in reconstruction, READ called a) FITLIN which performed preliminary straight line fits after eliminating undesirable points and b) ROT which transformed measurements from one frame to another by least square fitting fiducial measurements to assigned positions and which called CORREK to compute constant correction terms for each measurement point by interpolation from standard fiducial positions. With this information, READ rejected the data if there were excessive fiducial errors or too many bad points on a track. For good data, READ projected the measured points as rays into the chambers. At this stage, the main program called FITONE which used non-linear least squares algorithm to find parameters for the line which is the best fit to the rays set by READ and made extensive use of SETA which set up the relevant matrices. FITONE was called again for the lower chamber if the event was a non-interacting event. For interacting events, INTER was called which performed vertex fits with all the information given by the preceding programs. Finally, the program OUTPUT was called to set up the output records and to print the necessary output information. The details of the function used for minimization, etc., are given in reference 13. The events that could not be reconstructed the first time were remeasured and reconstructed again. Some of the events could not be reconstructed even after three or four remeasures. The factors that were responsible

were very faint tracks, bad fiducials and overall poor quality of those particular film frames. A discussion of the reconstruction efficiency is given in the Appendix A.

D. Energy Determination

A brief description of the calorimeter and the recording of the layer outputs was given in an earlier section. A description of the energy determination from the layer outputs is given below.

1) Conversion of the observed ionization to the number of minimum ionizing particles: This conversion was done through two calibration procedures. In the LADC calibration, which was conducted with a pulser and a precision attenuator, a standard pulse was attenuated in increments of 5 db over a range of 0 to 80 db and LADC outputs at each setting were recorded. In the MU calibration, the system was triggered by about 1000 muons and the summed voltages from each layer were amplified and fed into LADCs whose binary output was recorded. The LADC calibration provided the attenuation corresponding to a standard pulse and the MU calibration gave the voltage of a muon pulse after amplification. From these two calibrations, which were conducted periodically, the attenuation corresponding to an unamplified muon pulse was determined. Similarly the attenuation corresponding to a hadron pulse was also determined. The number of equivalent muons, N , was given by

$$N = 2^T \quad (\text{II.2})$$

where

$$T = (db_{\mu} - db_h)/6.02.$$

The attenuation corresponding to a single unamplified muon pulse was given by db_{μ} and db_h was the similar quantity for a single hadron. Table VII gives the number of equivalent muons for some values of LADC channel numbers for layer 3.

TABLE VII
NUMBER OF EQUIVALENT MUONS
vs
LADC CHANNEL NUMBER

LADC Channel Number	Number of Equivalent Muons
16	0.4
32	19.0
48	65.7
64	172.6
80	405.4
96	969.5
112	2381.3
128	6182.5

2) Corrections for the observed number of equivalent muons: The observed number of equivalent muons had to be corrected for two effects connected with the thickness of the scintillator. Since the total amount of light from a layer depended on the thickness of the scintillator and the location

of the particle traversal, the observed light was corrected for the relative brightness of the observed light from various parts of the layer. Since layers 1, 2 and 10 used better scintillators (thickness 1.91 cm), the observed number was not corrected for the outputs from these layers. For the other layers (thickness 3.81 cm), following correction was applied.

$$N_c = N_0 (1 - 3.70 \times 10^5 \times X^2) \quad (\text{II.3})$$

where N_c was the corrected number of equivalent muons and N_0 was the observed number when the extrapolated trajectory of the particle was at a distance X from the centre. The second correction arises because the ionization was sampled by the scintillators and not by the material of the calorimeter. This transition effect (51), which is smaller for thinner scintillators, results in the underestimation of the energy. The observed number of equivalent muons was multiplied by a factor f which had a value between 1.05 and 1.10.

3) Determination of the visible energy: E_v , the visible energy was given by

$$E_v = \int \beta N(\beta, t) d\beta dt \quad (\text{II.4})$$

where $N(\beta, t)$ was the corrected number of equivalent muons with specific ionization β at a depth t in Iron. For a finite number of probes, this integral was approximated by a sum over all the layers and was given by

$$E_v = \beta \left\{ \frac{N_B w_B}{2} + \sum_{i=8+1}^{10} \frac{(N_{i-1} + N_i) w_i}{2} \right\} \quad (\text{II.5})$$

where β was the average energy loss of a 2 GeV muon in Iron and was equal to $1.72 \text{ MeV/gm cm}^{-2}$, N_i was the corrected number of equivalent muons in the i^{th} layer and W_i was the thickness of Iron between the i^{th} scintillator and $(i-1)$ scintillator. The first layer included in the sum $i = b$ was the first layer from the top where the number of equivalent muons was greater than 5. W_b was the depth of this first layer from the top of the calorimeter. In case of the interacting events, W_b included the thickness of the target. An equivalent muon tape was generated for all the events and the final data tape was obtained by merging the reconstruction tape and the muon tape. (Due to parity errors on the tape used with the apparatus for the recording of the observed ionization, about 30% of the events in the Al-Fe run did not have useful recorded energies.) This energy was only the visible energy and the corrections for systematic underestimation of the energy will be discussed in the next chapter.

CHAPTER III

EXPERIMENTAL BIASSES AND MONTE CARLO CORRECTIONS

Because of the limitations of the three main components of the apparatus--the target, the wide gap spark chambers and the calorimeter--there were several experimental biases. Some of these biases could be singled out at different stages of the analysis and the events were either rejected or corrected for these biases and a final sample of good events was obtained. The other biases were difficult to identify in each event and average corrections were obtained with a Monte Carlo model for the primary p-nucleus interaction and the experimental response. A discussion of these biases will precede the discussion of the Monte Carlo program.

A. Overestimation of the True Multiplicity

1) The observed vertex distribution of the events:

The overestimation of the multiplicity depends upon the amount of the matter that particles traverse after leaving the vertex of the interaction. The digitization of the events provided each event with a vertex (x , y , and z) and z distribution of the vertices for all the digitized events is given in the Table VIII. The observed distribution was very different from

the ideal distribution which would have had almost the same number of events from each plate. More than half of the events were from the bottom plates and the number of events from the bottom plates increased with the atomic weight of the target. This observed distribution was a result of the energy underestimation and will be considered in a later section. Because of this observed distribution, the overestimation turned out to be considerably smaller than in the case of the ideal distribution.

2) π^0 decay and conversion of photons: According to the conservation of isospin, $1/3$ of the produced particles are π^0 's and these π^0 's decay instantaneously into two photons. The conversion probability for a photon in a single plate, the total conversion probability for an ideal vertex distribution and for the observed distribution are shown in Table IX.

TABLE VIII
OBSERVED VERTEX DISTRIBUTION FOR ALL
DIGITIZED INTERACTING EVENTS

Target	Plate Location of the Vertex						Total Number of Inter- Acting Events
	1	2	3	4	5	6	
Al	34.0 ^a	19.5	16.7	14.9	9.6	5.3	1100
Fe	40.9	22.7	15.1	9.4	7.9	4.0	1024
Sn	56.2	21.8	7.4	6.8	5.6	2.2	933
Pb	63.9	18.0	7.9	5.5	2.9	2.3	866

^a The total number of interacting events was normalized to 100 to get the distribution.

TABLE IX
PAIR PRODUCTION PROBABILITIES

Target	For a Single Plate	For the Ideal Vertex Distribution	For the Observed Vertex Distribution
Al	0.1051	0.3098	0.2419
Fe	0.2450	0.5816	0.3939
Sn	0.3371	0.7001	0.5067
Pb	0.4759	0.8210	0.5837

TABLE X
SECONDARY INTERACTION PROBABILITIES

Target	For a Single Plate	For the Ideal Vertex Distribution	For the Observed Vertex Distribution
Al	0.0360	0.1170	0.0889
Fe	0.0396	0.1278	0.0729
Sn	0.0298	0.0978	0.0576
Pb	0.0293	0.0960	0.0406

TABLE XI
PROBABILITIES OF DELTA RAY PRODUCTION BY SECONDARIES

Target	For the Ideal Vertex ^a Distribution	For the Observed Vertex Distribution
Al	0.0031	0.0035
Fe	0.0024	0.0034
Sn	0.0014	0.0025
Pb	0.0007	0.0019

^a The minimum energies for electrons to simulate straight tracks are discussed in Appendix B.

3) Secondary interactions: The probability for secondary interaction for a charged hadron is about 3% in each plate. The total probability for secondary interaction increases with the charged multiplicity of the created particles and the height of the primary vertex. Table X shows the total secondary interaction probability for a single charged hadron in the case of an ideal distribution and the observed vertex distribution.

4) Delta ray production by secondaries: The production of delta ray electrons by primaries has been discussed in the section on scanning and in the appendix B. The minimum energies needed for the electron to appear as a straight track was given in Table B1. The total probability of a charged secondary producing delta rays of energies greater than the minimum energy is shown in the Table XI for the ideal and the observed vertex distributions. Unlike the other two effects, this process is more pronounced for the observed vertex distribution than the ideal distribution because electrons travel lesser amount of matter in the observed case and the minimum energies are lower than in the ideal case.

B. Underestimation of the Multiplicity

There were three means by which tracks missed observation.

1) Loss of low energy particles: The low energy particles with this apparatus were nucleons which were the evaporation products of a nucleus in a nuclear interaction and low

energy pions. The approximate energies for a particle to penetrate the target plates is given in the Table XII. The number of evaporation products--the so-called black tracks with an average energy of 25 MeV and grey tracks with an average energy of 160 MeV--is independent of the incident energy. From the Table XII, it is evident that these particles will be stopped by the target plates. The particles which are not stopped completely by the target would lose considerable energy by ionization and would suffer large multiple scattering in the target plates. The amount of multiple scattering for the black and grey tracks is shown in the Table XII. Thus, most of the low energy particles, whether they be nucleons or pions, will be at large angles and will miss the effective area of the spark chambers altogether.

TABLE XII
MINIMUM ENERGIES FOR TARGET PENETRATION AND
MULTIPLE SCATTERING ANGLES FOR LOW ENERGY NUCLEONS

	Thickness	Al	Fe	Sn	Pb
Minimum kinetic energy needed for complete penetration for					
a) proton ^a	1 plate	55.00	62.00	--	50.00
	all plates	151.00	175.00	--	145.00
b) pion	1 plate	24.00	28.00	--	23.00
	all plates	71.00	85.00	--	69.00
Average multiple scattering angle for					
a) black track ^b	1 plate	99.18	14.55	17.53	22.10
	all plates	22.50	35.65	42.95	54.14
b) grey track	1 plate	1.39	2.21	2.66	3.35
	all plates	3.41	5.40	6.51	8.20

^a The energies are in MeV and the minimum energies were not available for the Sn target.

^b These are projected angles and are given in degrees.

2) Limited resolution: When particles were created very close to each other, only one track was observed (because of limited resolution). In this experiment it was found that sparks less than 2 mm apart in the lower chamber were not resolved. This separation corresponded to an angular divergence of 2 mr projected angle and the probability of missing a track in both views was determined to be about 0.3%.

3) Angular and multiparticle inefficiencies of the wide gap spark chambers: The average charged multiplicities in the Echo Lake p-p experiment (9), which were lower than the values found in the accelerator experiments were corrected later for the multiparticle inefficiencies of the wide gap spark chambers (52) and the agreement between the corrected multiplicities and the accelerator multiplicities was much closer than it was before. This analysis had been conducted by studying the inefficiencies of the upper and the lower gaps of the lower spark chamber. A complimentary, and possibly complete, approach would be to consider the overall efficiency as a superposition of the multiparticle inefficiency on the angular inefficiency. The angular inefficiency would be the only source of inefficiency when there was only one particle present in the chamber, the evidence and the other details given in the Appendix C, indicated that the available energy was used up mostly by the tracks that were parallel to the field direction at the expense of tracks which were at large angles

to the field direction. The efficiency function given in the appendix was applied to all the tracks generated by the Monte Carlo models which will be discussed in a later section.

C. Energy Underestimation

The visible energy given by the equation (II.5) differed from the true energy because of unsampled energy losses.

1) Nuclear disintegration energy: When a charged hadron interacts with a nucleus, the struck nucleus disintegrates into nuclear fragments and this disintegration energy has an average value of 800 MeV (53). G. Demeester (13) has estimated that half of this energy is measured by the scintillators. In a hadronic cascade, the total energy going into nuclear disintegrations is a function of the number of nuclear interactions which in turn is a function of both the incident energy and the amount of matter available for the development of the cascade. Monte Carlo calculations on the unsampled energy losses were conducted by W. V. Jones (54) for a shallow spectrometer and for the calorimeter in this experiment. The results of the Monte Carlo were parametrized by R. J. Wilkes (15) and the results of the parametrizations are shown in the Table XIII for some values of the incident energy and t , the amount of matter traversed by the particle after its first interaction. At constant t , F_{dis} , the fraction going into nuclear disintegrations, decreases with increasing incident energy; at constant incident energy, F_{dis} increases with increasing t .

2) Energy leaking out of bottom of the calorimeter:

For any finite calorimeter, there is always the possibility that the cascade is not completely contained by the amount of matter in the calorimeter and some energy leaks out of the bottom. For average shower development at large enough depths into the calorimeter, exponential absorption prevails and the energy leaking out of the bottom is given by

$$C(L) = N_L \Lambda \quad (\text{III.1})$$

where L is the last layer traversed by the extrapolated incident track, N_L is the corrected number of equivalent muons (section) in the last layer and Λ is the attenuation length and is equal to 240 gms/cm^2 . While this factor could be added to the visible energy for hadrons which did not interact in the target, the interacting events posed a problem. In the latter case, there were many particles impinging on the calorimeter at different angles and their cascades would exit from the calorimeter at different levels resulting in many 'last' layers. The results of the parametrization on F_{out} , the fraction of the energy leaking out of the bottom, are shown in the Table XIII for some values of the incident energy and t . At constant t , F_{out} increases with increasing incident energy; at constant incident energy, F_{out} decreases with increasing t .

The total unmeasured energy in a non-interacting event is given by

$$E_{um} = E_o (F_{out} + F_{dis}/2) \quad (III.2)$$

where E_o is the incident energy. The total fraction which is unsampled is given in Table XIII for some values of incident energy and t and plotted in Figure 9. L. W. Jones, et al (55) have determined that $28 \pm 5\%$ of the energy is not sampled at an incident energy of 300 GeV. Their calorimeter was 64 r.l. thick which would correspond to a t of ~ 53 r.l.

3) Additional energy losses for an interacting event:

In an interacting event, some of the secondaries do not enter the calorimeter at all and their contribution to the observed energy is obviously zero. Some of the particles exit the calorimeter before they interact and some decay before they can interact. The photons from decay of created π^0 's can lose energy in the target material before entering the calorimeter. The property of the interaction which is responsible for some of these losses is the angular distribution of the event. Table XIX shows the effect of angular distribution on the energy recorded in the calorimeter. The angular distribution characteristics are displayed by the variable η_p ($\eta_p = \log_{10} \tan \theta_p$) where θ_p is the projected angle of a secondary with respect to the primary beam particle direction. Events were classified as 'below threshold energy events' and 'above threshold energy event'. The average angle of a track in a 'below threshold energy event' was greater than in an 'above threshold energy event'. Also there were more tracks at large

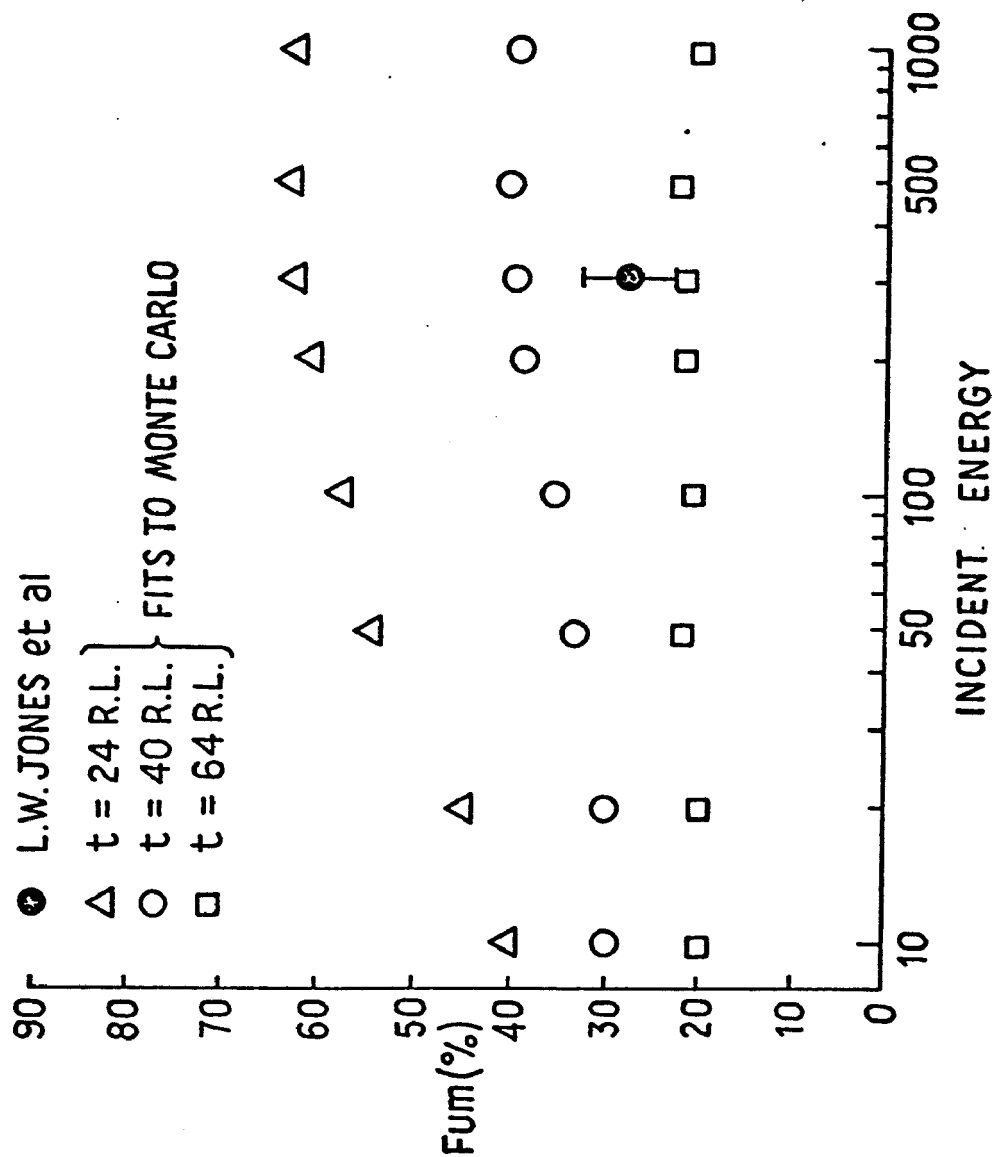


Fig. 9. Unsamped Energy for a Non-Interacting Event in the Calorimeter

TABLE XIII
UNMEASURED ENERGY IN A CALORIMETER
FOR A NON-INTERACTING EVENT

Incident Energy in GeV	t^a in rad len	F_{dis}^b	F_{out}	F_{um}
50	24	17.0	46.0	54.5
	40	25.0	21.0	33.5
	64	34.0	5.0	22.0
300	24	10.0	58.0	63.0
	40	16.0	32.0	40.0
	64	24.0	10.0	22.0
500	24	8.0	59.0	63.0
	40	16.0	33.0	40.5
	64	23.0	11.0	22.5

^a t is the amount of matter traversed by the particle after its first interaction in the calorimeter.

^b F_{dis} , F_{out} and F_{um} have been given as the fractions of the incident energy. F_{dis} is the fraction going into nuclear disintegrations, F_{out} is the fraction leaking out of the bottom and F_{um} is the total unmeasured fraction.

TABLE XIV
ANGULAR DISTRIBUTION EFFECTS IN THE
ENERGY UNDERESTIMATION

	Below Threshold Events	Above Threshold Events
Fraction of Tracks With $\eta_p > -1.0$ in		
a) Al target	0.36	0.25
b) Fe target	0.41	0.26
Average η_p in		
a) Al target	-1.37	-1.54
b) Fe target	-1.35	-1.50

angles for 'below threshold energy events'. Thus energy underestimation was larger for events with more large angle particles. This was one of the major factors responsible for the observed vertex distribution discussed earlier.

D. The Final Data Sample and The Observed Charged Multiplicities

The total number of digitized events was given in the Table VIII for all the targets and cuts had to be made on this raw sample to get the final sample of good events. While the biases discussed in the previous sections could affect any event, the distortion of the data due to these biases was more serious in certain types of events. Thus, whenever the possibility of distortion of the final results was very high, the events were completely rejected.

1) Out of geometry events: When the extrapolation of the primary did not extend to the eighth layer in the calorimeter, the event was rejected as an out of geometry event. Due to spark chamber inefficiencies, the observed multiplicity was smaller for a primary with a large zenith angle than for one with smaller zenith angles (Appendix C). Also an out of geometry particle deposited less energy than an in-geometry particle. When the extrapolation of the primary track crossed from one target to another, the event was rejected because of confusion resulting from the possibility of secondary vertices in another target material.

2) Events due to non-hadronic processes: As discussed in Appendix B, these processes distort the true distributions by contributing mostly low multiplicity events. These events were also rejected by the methods discussed in Appendix B.

3) Events with incorrect energies: Because of the problems with electronics, some events were recorded with incorrect energies. A significant fraction ($\sim 30\%$) of the events in Al-Fe run had to be rejected because of the problems with the magnetic tape used to record the energy. The nominal threshold energy demanded in the trigger was different from the true energy because each layer output was not used explicitly in the trigger. This bias setting was expected to affect events with recorded energy less than 1.5 times the trigger threshold energy. Therefore, events with energy less than 1.5 times the threshold energy were rejected. Events in which there were signs of obvious misbehaviour of recording electronics, such as unrealistic equivalent number of muons in some layers, were also rejected.

4) The observed charged multiplicities: For the final sample, which was obtained after rejection of the events, the energy of each event was increased by an amount which corresponded to 44% unsampled energy losses (Appendix E). The energy bins used for the data were 50% wide and contiguous. Specifically, there were three energy bins: 100 GeV to 200 GeV, 200 GeV to 400 GeV and 400 to 800 GeV. The observed final charged multiplicity distributions for all the targets and the two energy bins is given in Table XV. The average charged

TABLE XV

OBSERVED MULTIPLICITY DISTRIBUTIONS

Charged Multi- plicity	Al		Fe		Sn		Pb	
	160 ^a GeV	260 GeV	160 GeV	260 GeV	160 GeV	260 GeV	160 GeV	260 GeV
2	4	2	3	1	3	2	4	2
3	4	3	1	2	4	3	2	2
4	6	4	3	5	1	3	3	0
5	8	9	8	5	4	2	2	2
6	5	5	9	5	7	3	2	1
7	8	7	5	3	1	3	0	1
8	8	10	4	4	3	5	3	5
9	9	2	3	9	2	1	0	2
10	3	5	4	7	2	3	2	5
11	6	7	6	4	2	3	2	5
12	0	3	2	3	3	6	1	3
13	4	3	4	1	4	2	6	1
14	4	1	2	1	3	1	3	5
15	0	3	3	5	0	3	0	0
16	0	0	1	6	4	0	2	1
17	0	1	0	4	2	4	0	2
18	1	0	0	2	2	3	4	0
19	0	1	0	0	1	0	1	0
20	0	1	1	0	0	0	0	0
20	0	1	1	1	1	2	2	2

^a The energies are average energies.

multiplicities were computed using

$$\bar{n}_{cj} = \frac{\sum_{i=2} i N_{ij}}{\sum_{i=2} N_{ij}} \quad (\text{III.3})$$

where N_{ij} was the number of events with i prongs in the j^{th} energy bin. The values of these raw observed average charged multiplicities are shown in Table XVI. The errors shown represent the statistical error given by

$$\sigma_{\bar{n}} = \left[\frac{\bar{n}_{cj}^2 - \bar{n}_{cj}^2}{N_{cj}} \right]^{1/2} \quad (\text{III.4})$$

TABLE XVI

OBSERVED AVERAGE CHARGED MULTIPLICITIES

Energy in GeV	Al	Fe	Sn	Pb
160	7.67 $\pm$ 0.42	8.66 $\pm$ 0.53	9.77 $\pm$ 0.73	10.42 $\pm$ 0.75
260	8.55 $\pm$ 0.51	10.28 $\pm$ 0.61	10.72 $\pm$ 0.75	10.54 $\pm$ 0.77
520	8.08 $\pm$ 1.05	9.96 $\pm$ 1.05	9.09 $\pm$ 1.56	---

The raw data showed an increase in average charged multiplicity with atomic weight for the first two energy bins. At higher energies, and hence higher multiplicities, spark chamber inefficiencies played a very significant role in distorting the observed results.

E. Monte Carlo Models for Proton-Nucleus Interactions

The most suitable approach for correction of the systematic biases in the observed distributions was through Monte Carlo models. Generation of the primary proton-nucleus interactions and simulation of the instrumental response formed the two major constituents of the correction program.

A model for p-target nucleon collision is a necessary input in the consideration of p-nucleus collisions and a Monte Carlo model which incorporated the general features of the multiperipheral models was used for each p-nucleon collision. This model for p-p collisions has been described in Appendix D. Any model for p-nucleus collision has to predict, at least qualitatively, the inferences reached in earlier experiments. These inferences, which were described in detail in the first chapter, were the slow increase of the multiplicity with atomic weight, with the major part of the increase coming in the backward directions. All the models to be discussed here considered the nucleus as composed of free nucleons and nuclear effects like multiple scattering, etc., were completely ignored. The number of collisions possible for a beam proton in the nucleus was limited arbitrarily to a small number. This constraint was needed because a full fledged intranuclear cascade would result in too many final state particles. In all these models, C_{eff} which was a measure of the number of collisions was limited to be between one and six.

The incident particle was always chosen to be a cosmic ray proton with its differential energy spectrum given by an $E^{-2.8}$ (II.1) distribution and its zenith angle chosen from a $\cos^{7.6} \theta$ distribution. The azimuth and the x, y co-ordinates of the vertex were chosen from a random distribution. The z co-ordinate of the vertex was chosen to be compatible with the interaction mean free path in the target.

1) Model with $C_{eff} = 1$: The results of many of the earlier experiments had indicated that the majority of the collisions were with a single nucleon inside the nucleus. In this model, p-nucleus collisions were treated as p-p collisions but with an increase in multiplicity given by

$$n_c(p-A) = n_c(p-p) \times A^{0.15} . \quad (III.5)$$

While this model gave reasonable average multiplicities, the increase of multiplicity was mainly in the medium angle regions as opposed to the experimental observations. Figure 10 shows η_p plots for p-p collisions and for p-Pb collisions with this model. While 48.33% of the particles were in the backward hemisphere in p-p collisions, only 44.98% were backward for p-Pb collisions. Here 'backward' and 'forward' refer to the hemispheres in p-p center of mass system. Since this model did not duplicate the observed angular distributions, this model was not considered for further analysis.

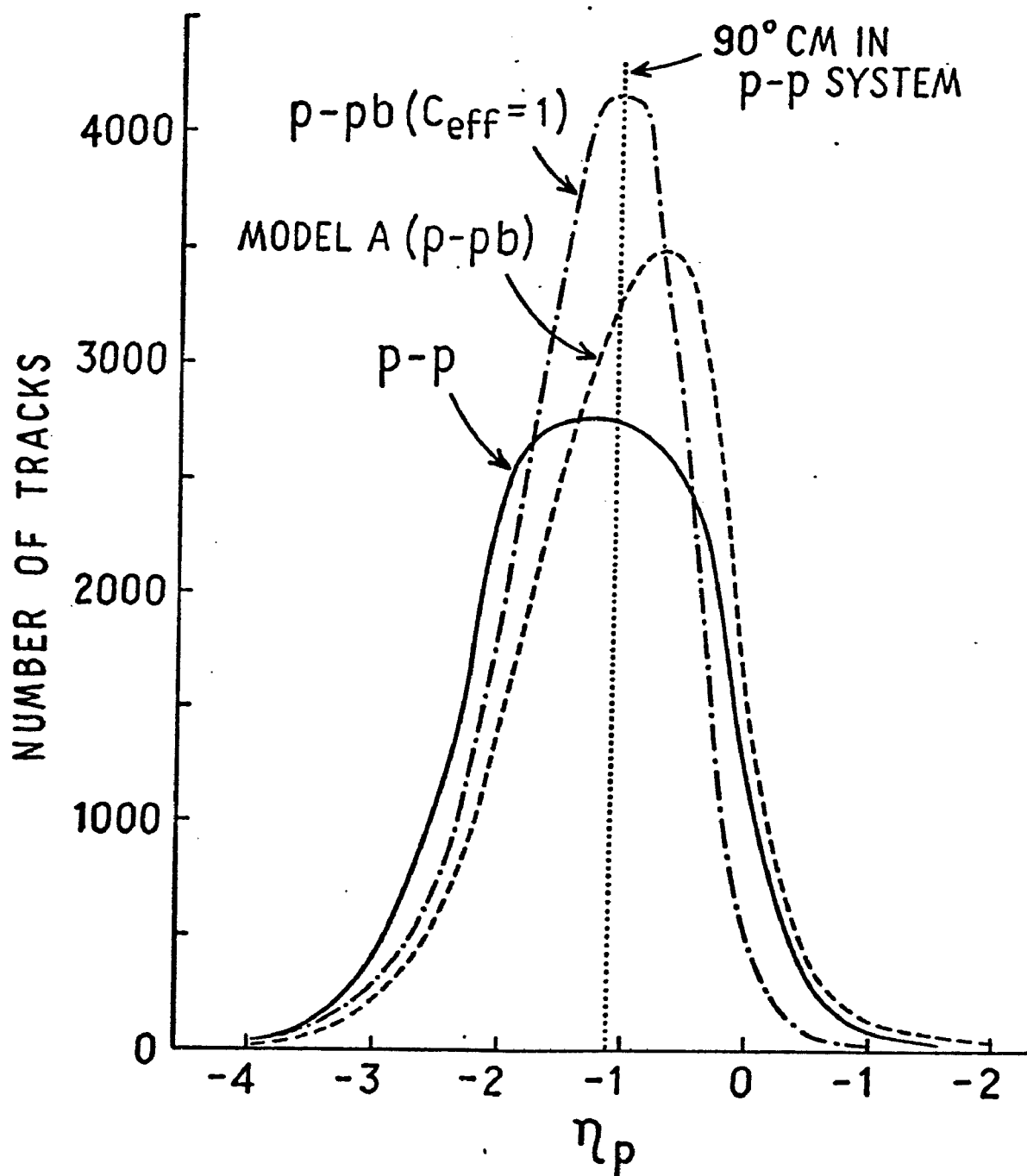


Fig. 10. η_p Plots for the Monte Carlo Models

TABLE XVII
COLLISION PROBABILITIES IN THE MONTE CARLO MODELS

Number of Collisions	Probabilities			
	Al	Fe	Sn	Pb
1	1.0000	1.0000	1.0000	1.0000
2	0.8100	0.9100	0.9550	0.9750
3	0.5000	0.6650	0.7850	0.9000
4	0.2450	0.3750	0.5450	0.7250
5	0.0350	0.1300	0.2700	0.5000
6	0.0025	0.0300	0.0800	0.2750

2) Models with $C_{eff} \geq 1$ (Models A and B): These models used a modified and mainly utilitarian version of the Intra Nuclear Cascade mechanism. Only the primary particle was allowed to have collisions inside the nucleus and half of these collisions were with a target neutron. The minimum energy for the beam proton to interact in the nucleus was taken to be 5 GeV. The probability for each collision was a function of the nuclear size of the target and was taken from an interpolation to the calculations of Fishbane and Trefil (26). These probabilities are displayed in Table XVII for all the targets. The momentum conservation for each collision inside the nucleus and the overall momentum conservation for the beam particle in the nucleus was maintained. The angular distributions of all the subsequent collisions were obtained with respect to the beam particle and also with respect to the

experimental co-ordinate system. In Model A, the maximum number of collisions was limited to six. In Model B, the maximum number was limited to five and the results were obtained by taking in each case one less interaction than in Model A.

The average number of collisions resulting from these assumptions and the dependence of the multiplicity on atomic weight are shown in Table XVIII for both Models A and B. The dependences, though different from each other, were still compatible with the limited data reported in the first chapter. An η p plot also shows the experimentally observed enhancements at large angles. A quantitative description of the results on angular distributions is given in Table XIX. The fraction of backward (i.e., in p-p centre of mass system) tracks increased with atomic weight in both Model A and Model B.

While the agreement with the data generated confidence in these Monte Carlo models, one should note that this agreement was rather fortuitous. Since the excited states of the nucleus were not considered, the emulsion results on heavy tracks could not be reproduced. The leading particle effect could not be duplicated for many events. The use of Poisson statistics for the multiplicity distributions in p-p collisions was in disagreement with the NAL-ISR data. But, these defects of the models were not expected to affect the corrections in any significant way. Since heavy tracks were not recorded by the experimental set up, the absence of nuclear evaporation

TABLE XVIII
AVERAGE NUMBER OF COLLISIONS AND A DEPENDENCE
IN THE MONTE CARLO MODELS

Model	Energy	Al	Fe	Sn	Pb	X^a
A	160 GeV	2.27	2.62	3.00	3.58	0.158 ± 0.005
	260 GeV	2.35	2.72	3.21	3.67	0.162 ± 0.007
B	160 GeV	1.50	1.70	2.14	2.54	0.100 ± 0.009
	260 GeV	1.49	1.74	2.37	2.60	0.120 ± 0.010

^a X is defined by $R \propto A^X$.

TABLE XIX
MONTE CARLO RESULTS ON ANGULAR
DISTRIBUTIONS

Property	Model	H	Al	Fe	Sn	Pb
Fraction of Backward Tracks ^a	B	48.33	53.78	55.55	57.27	60.47
	A	48.33	59.03	60.77	61.99	65.13
Average Log $\tan \theta_p (\eta_p)$	B	-1.12	-1.02	-0.99	-0.96	-0.91
	A	-1.12	-0.93	-0.91	-0.89	-0.84

^a At 160 GeV, the tracks with $\eta_p > -1.12$ were considered to be backward in p-p centre of mass. The total number of tracks was normalized to 100 in each case.

products in the results will not affect the corrections. The leading particle effect and the multiple scattering effects would act to increase the fraction of particles at large angles. Since the spark chambers have been shown to be insensitive to particles at large angles (Appendix C) these effects should not affect the corrections. The only conceivable effect would be on the simulation of the energy response by the calorimeter, since the energy deposited was strongly dependent on the angular distributions (Appendix E).

F. Corrections for Instrumental Response

The aim of the simulation program was to correct for systematic biases in the final sample of good events. Since the final sample excluded out of geometry events and events due to non-hadronic processes and had their energies corrected for unsampled energy losses, these biases were not considered in the simulation program. Though an attempt was made to simulate the energy underestimation (Appendix E), the results were not used in the main simulation program. The simulation program used the events generated by the p-nucleus Monte Carlo models which had been stored on a magnetic tape.

Specifically, the program considered two major biases-- the overestimation and the underestimation of the multiplicity-- for in-geometry events with energy greater than the threshold energy. The secondary prongs from the primary event were followed down from the vertex of the interaction through the

remaining target plates. The charged particles were allowed secondary interactions if they had energies greater than 5 GeV. When the particle interacted, the tertiary particles were assigned momenta and energies just like in the primary Monte Carlo models. The photons from the decay of π^0 's were allowed to produce electron pairs if the angle of the neutral pion was less than the cut-off angle mentioned in Appendix C. Since the photon energies were generally high, it was assumed that only one track would be observed because of limited resolution. Delta ray production by secondaries was not considered explicitly since the probability of production of high energy electrons which would simulate straight tracks was quite small (Table XI).

Once all the creation processes were finished, the biases which underestimated the multiplicity were considered. All particles which were at angles greater than the cut-off angle to the field direction were not counted. Particles which were closer than 3 mrad to each other were considered as contributing only one observable prong. The energies of the electrons from pair production were checked against the minimum electron energies in Table B1. If the energies were lower than the minimum energies, the electrons were not counted.

For each event, the multiplicity of the primary interaction, the final observed multiplicity and the energy were recorded. The energy bins used in the data analysis were also used for these Monte Carlo events.

TABLE XX

MONTE CARLO CORRECTION PROCEDURES

The Vertex Plate, k	C_{jk}^a Model A	C_{jk} Model B	n_{cjk} Data (obsvd)	n_{cjk} Corr. A	n_{cjk} Corr. B
1	1.16	1.21	---	---	---
2	1.28	1.24	9.00	11.52	11.14
3	1.27	1.42	7.92	12.52	11.25
4	1.43	1.40	7.75	11.05	10.83
5	1.40	1.39	8.92	12.52	12.40
6	1.74	1.63	6.62	11.52	10.80
all	1.47	1.46	7.67	11.31	11.20

^a C_{jk} was the correction factor for the k^{th} plate in the j^{th} energy bin. Here j corresponds to 1, the lowest energy bin.

TABLE XXI

MONTE CARLO CORRECTION FACTORS

Target	Energy in GeV	Correction Factors	
		Model A	Model B
Al	160	1.47	1.46
	260	1.48	1.53
	520	1.67	1.80
Fe	160	1.41	1.42
	260	1.29	1.28
	520	1.59	1.64
Sn	160	1.39	1.33
	260	1.41	1.41
	520	1.96	1.79
Pb	160	1.42	1.41
	260	1.51	1.45

TABLE XXII

COMPOSITION OF FINAL STATE PARTICLES

Fraction of Particles From:	Al	Fe	Sn	Pb
a) Primary Interaction	0.8180	0.7900	0.7180	0.7330
b) Secondary Interactions	0.0590	0.0400	0.0460	0.0360
c) Conversion of Photons	0.1230	0.1900	0.2360	0.2310

The correction procedures will be illustrated by considering the raw data in the 100 GeV to 200 GeV energy bin for the Aluminium target. C_{jk} was the correction factor given by the simulation programs for the k^{th} plate in the j^{th} energy bin. Specifically,

$$C_{jk} = \frac{(\bar{n}_{c_{jk}}) \text{ for the primary interaction}}{(\bar{n}_{c_{jk}}) \text{ after the biases}} \quad (\text{III.6})$$

where $\bar{n}_{c_{jk}}$ was the average charged multiplicity from the k^{th} plate in the j^{th} energy bin. The raw average charged multiplicity from the k^{th} plate in the j^{th} energy bin was multiplied by C_{jk} to get the corrected charged multiplicity. These values are displayed in Table XX for both Models A and B. The final corrected charged multiplicity was obtained by considering the individual multiplicities from all the plates. The values of C_j , the correction factor thus derived for the whole energy bin, are given in the Table XXI for all the targets. C_{jk} decreased with the height of the vertex since the loss of particles was compensated by the creation of new particles.

While the average multiplicities in the data showed an increase with the height of the vertex, the corrected average multiplicities were not functions of the vertex plate. While the corrections were quite model independent, they were dependent on the energy and the observed vertex distribution. For the lowest energy bin, the corrections for all the targets were almost

equal; whereas for an ideal vertex distribution, the corrections for Al and Pb would have been very different. The observed particles in the final state included particles from primary interactions, from secondary interactions and electrons from γ conversions. Table XXII shows the composition of the final state particles as given by Model A. In all targets, at least 70% of the observed prongs were from the primary interactions.

CHAPTER IV

RESULTS AND CONCLUSIONS

A. Charged Multiplicities

1) The corrected average charged multiplicities:

The observed average charged multiplicities and the Monte Carlo correction factors were given in Tables XV and XXI respectively. Since the correction factors were quite independent of the models, the observed average charged multiplicities were multiplied by the mean of the correction factors given by Models A and B to get the final average charged multiplicities. The corrected average charged multiplicities are given in Table XXIII. The errors are statistical and reflect uncertainties in both the data and the Monte Carlo models.

2) The variation of R with the mass number: The values of the ratio R for all the targets at three values of energy are given in Table XXIV. The best fits to the variation of R with A, the mass number, are also given in that table. The variation of R with A at 160 GeV is shown in Figure 11 along with the carbon data of Wilkes (15). While the errors in the highest energy bin are rather large, the

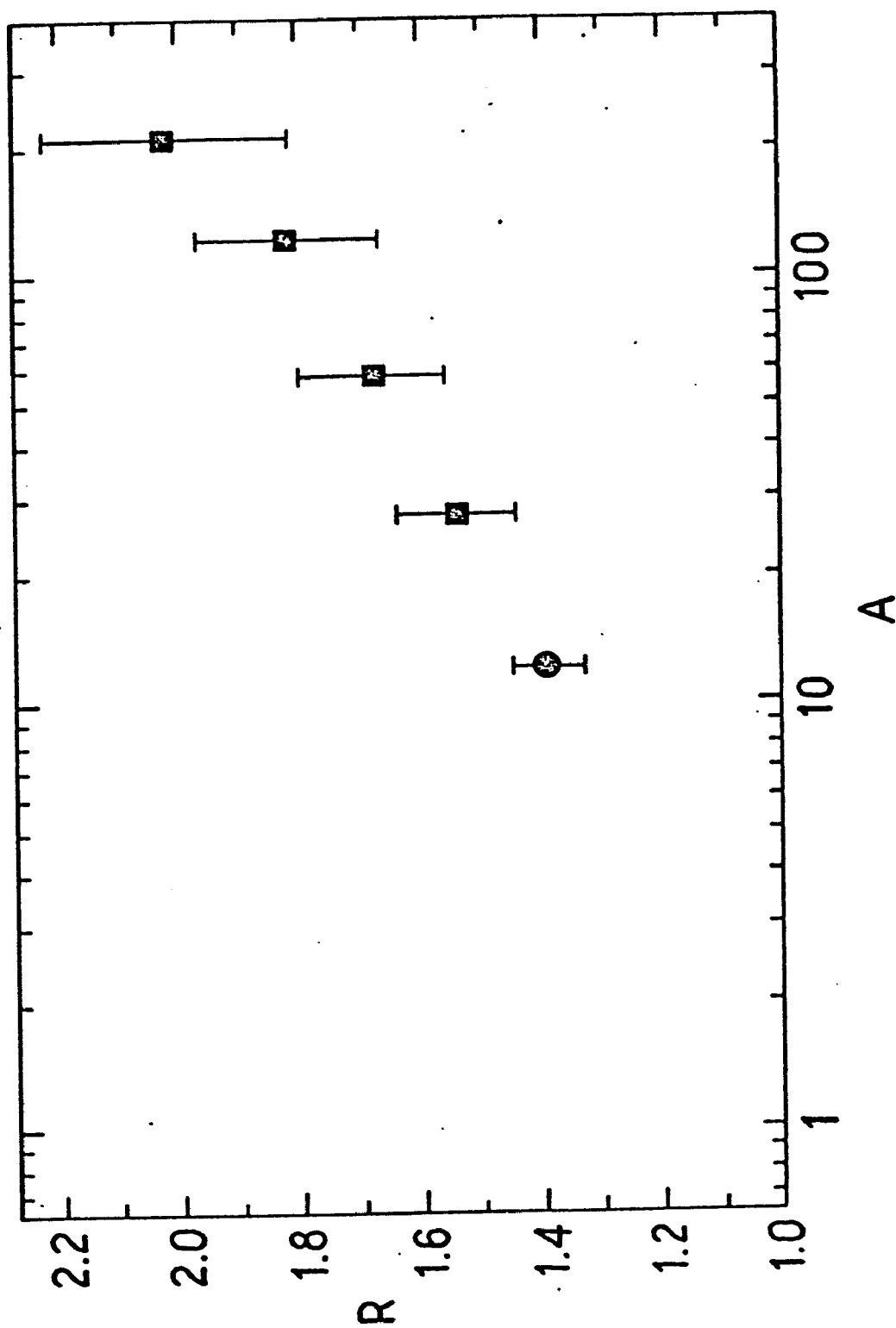


Fig. 11. Variation of R with the Mass Number

equation

$$R \propto A^{0.129 + 0.004} \quad (\text{IV.1})$$

is in agreement with the whole data. It should be stressed that the raw charged multiplicities also increased with the mass number for the targets and for the energy bins where the biases were minimal. This value of the exponent is in good agreement with the value of 0.13 given by Wilkes (15) for the carbon target and by Gurtu et al. (56) for a composite emulsion target.

3) The variation of R with energy: Though the value of R does not show any significant increase with energy, a very slight increase with energy cannot be ruled out. A constant R is in agreement with the results on carbon and emulsion targets. Table XXV shows the dependence of the average charged multiplicity on the energy of the beam particle. This dependence on energy is in agreement, within errors, with an $E^{\frac{1}{4}}$ power law. The large errors in the highest energy bin do not warrant either a sophisticated parametrization or a detailed comparison with the logarithmic power law which is the best fit to the p-p data. Further, the concept of centre of mass energy, which is used in the fits to the p-p data, is ambiguous in case of nuclear targets.

4) Multiplicity distributions: Since the observed multiplicity distributions were not corrected independent of

TABLE XXIII
CORRECTED AVERAGE CHARGED MULTIPLICITIES

Energy in GeV	Al	Fe	Sn	Pb
160	11.25 ± 0.76	12.25 ± 0.89	13.31 ± 1.13	14.75 ± 1.41
260	12.87 ± 1.18	13.21 ± 1.16	15.14 ± 1.40	15.61 ± 1.51
520	14.02 ± 2.49	16.05 ± 2.90	17.05 ± 3.60	---

TABLE XXIV
R AND THE A DEPENDENCE OF R

Energy in GeV	Al	Fe	Sn	Pb	X^a
160	1.54 ± 0.10	1.68 ± 0.12	1.82 ± 0.15	2.02 ± 0.20	0.129 ± 0.004
260	1.58 ± 0.10	1.62 ± 0.12	1.86 ± 0.15	1.92 ± 0.18	0.129 ± 0.004
520	1.50 ± 0.26	1.72 ± 0.31	1.82 ± 0.35	---	0.128 ± 0.011

^a X is defined by $R \propto A^X$.

TABLE XXV
THE DEPENDENCE OF CHARGED MULTIPLICITY ON ENERGY

Target	C^a	X^a
Al	3.67	0.22 ± 0.06
Fe	4.22	0.21 ± 0.06
Sn	4.28	0.22 ± 0.07

^a C and X are defined by the relation $\bar{n}_c = C E^X$ where $\bar{n}_c$ is the average charged multiplicity and E is the energy in GeV.

the average charged multiplicities, only an indirect estimate of the moments of the multiplicity distributions was possible. The values of D and f_2 , where D is defined by

$$D = (\bar{n}_c^2 - \bar{n}_c^2)^{\frac{1}{2}} \quad (\text{IV.2})$$

and f_2 is defined by

$$f_2 = D^2 - \bar{n}_c \quad (\text{IV.3})$$

are given in Table XXVI for the lowest energy bin, where the average energy was 160 GeV. The results on targets of proton, carbon and emulsion are also shown in the table. Both D and f_2 increased with the mass number of the target. The variation of D with the multiplicity is found to be faster for our data than for p-p collisions. Though no quantitative estimates could be made because of the large errors in the highest energy bin, the increase in D with n for the Aluminium target was higher than the increase for the Tin target, both being higher than the increase for a pure proton target. This trend was also noticed by Babecki et al., when the interactions in emulsion were classified according to their N_h values. While the ratio of diffractive component to the multiperipheral component could be predicted for p-p collisions using the moments of the multiplicity distributions, it is difficult to understand the results presented here or the results on the emulsion targets with typical two component models.

TABLE XXVI
MOMENTS OF THE MULTIPLICITY DISTRIBUTIONS

Target	D	f_2	$D/\bar{n}_c$
H ^a	3.68	6.24	0.50
C ^b	5.28 ± 0.76	16.47 ± 5.52	0.46
Al	6.40 ± 0.84	29.71 ± 11.40	0.56
Fe	6.89 ± 0.93	35.22 ± 13.78	0.56
Em ^c	8.34 ± 0.26	56.19 ± 4.09	0.63
Em	8.50 ± 0.50	---	0.65
Sn	7.91 ± 1.05	49.26 ± 17.70	0.59
Pb	8.91 ± 1.20	64.63 ± 23.23	0.60

^a For p-p collisions, D was calculated from $D = 0.585 (n-1)(22)$.

^b The carbon results are from Wilkes (15) and are at 170 GeV.

^c The emulsion results are from References 42 and 56 and are at 200 GeV.

5) Comparison with theoretical predictions: The values of R at 160 GeV are compared with the theoretical predictions in Figure 12. The emulsion results are at 200 GeV and are from Hebert et al (41), Babecki et al.(42), and Florian et al.(43), and the carbon results are from Wilkes (15). (The more recent emulsion results of Gurtu et al.(56), at 200 GeV are not shown in the figure since it basically duplicates the other results.) The predictions of the Energy Flux Cascade Model (23) and the Coherent Production Model (26) are shown in the figure. The results of this experiment and the experiments on carbon and emulsion agree well with the

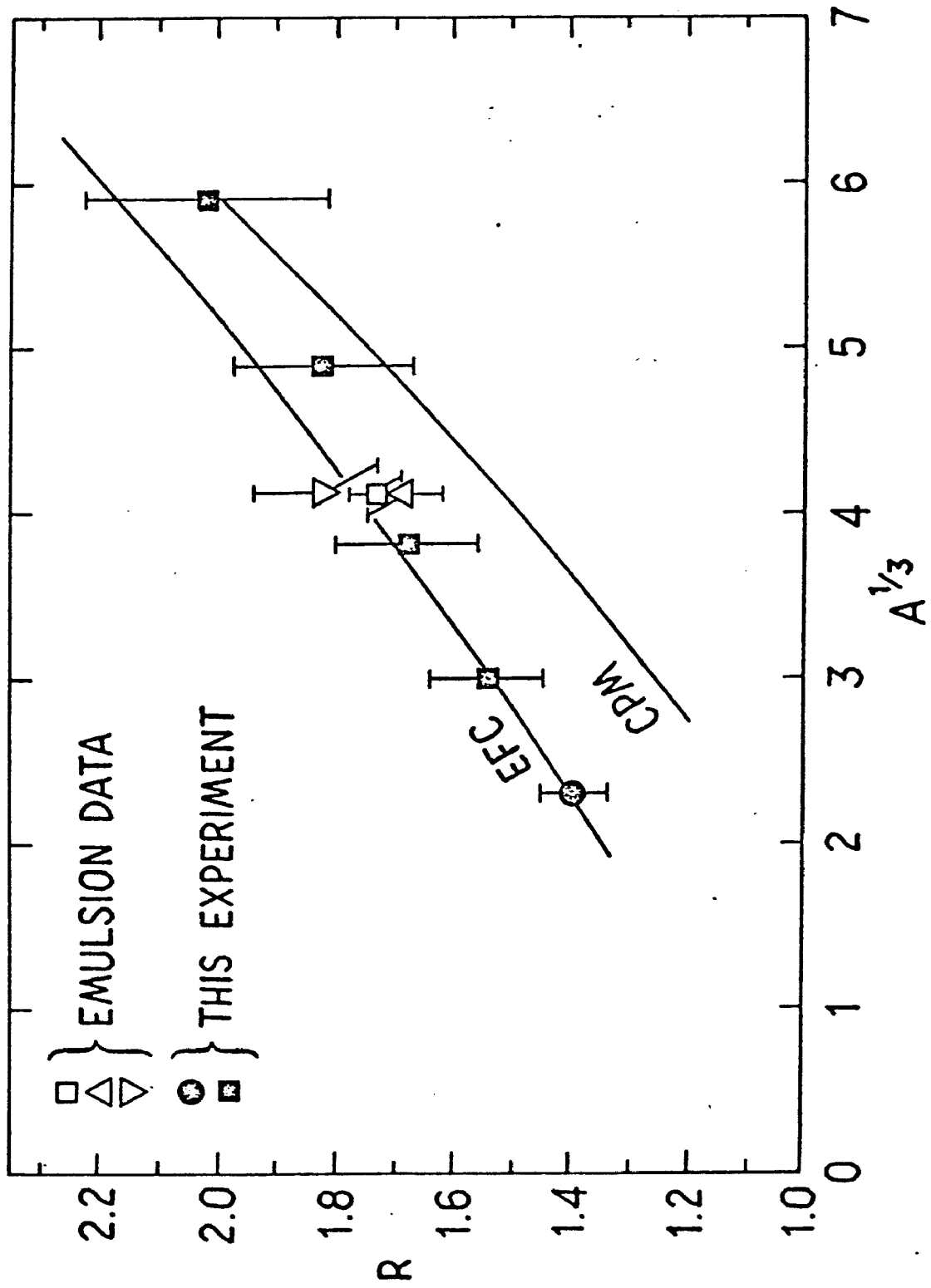


Fig. 12. Comparison of R with Theoretical Predictions

predictions of the EFC model. The predictions of the Coherent Production Model do not agree with the results on targets of lower mass number. The predictions of the diffractive fragmentation model as proposed by Gottfried (23) and of the recent two phase model of Fishbane and Trefil (26) predict higher values of R than in the EFC model. (The importance of using same nuclear inputs in the calculation of $\bar{\nu}_A$ cannot be overstressed and in the present comparisons, $\bar{\nu}_A$ was calculated using equation (I.10) which was proposed by Gottfried.) Landau's hydrodynamical model (29) predicts a value of 0.19 for the exponent and is much higher than the exponent observed in this experiment. Our results are also in disagreement with the predictions of the incoherent production model of Fishbane and Trefil (26), which predicts a flat dependence of multiplicity on mass number and with predictions of naive versions of multiperipheral models.

B. Angular Distributions

1) Normalized η_p plots: Because the tracks in the two orthogonal views could not be correlated, the angular distributions are presented in terms of the variable θ_p , the laboratory angle of emission of a secondary relative to the primary direction, projected onto a plane containing the particle trajectory. Data from the two orthogonal views were considered as independent. Specifically, the variable used

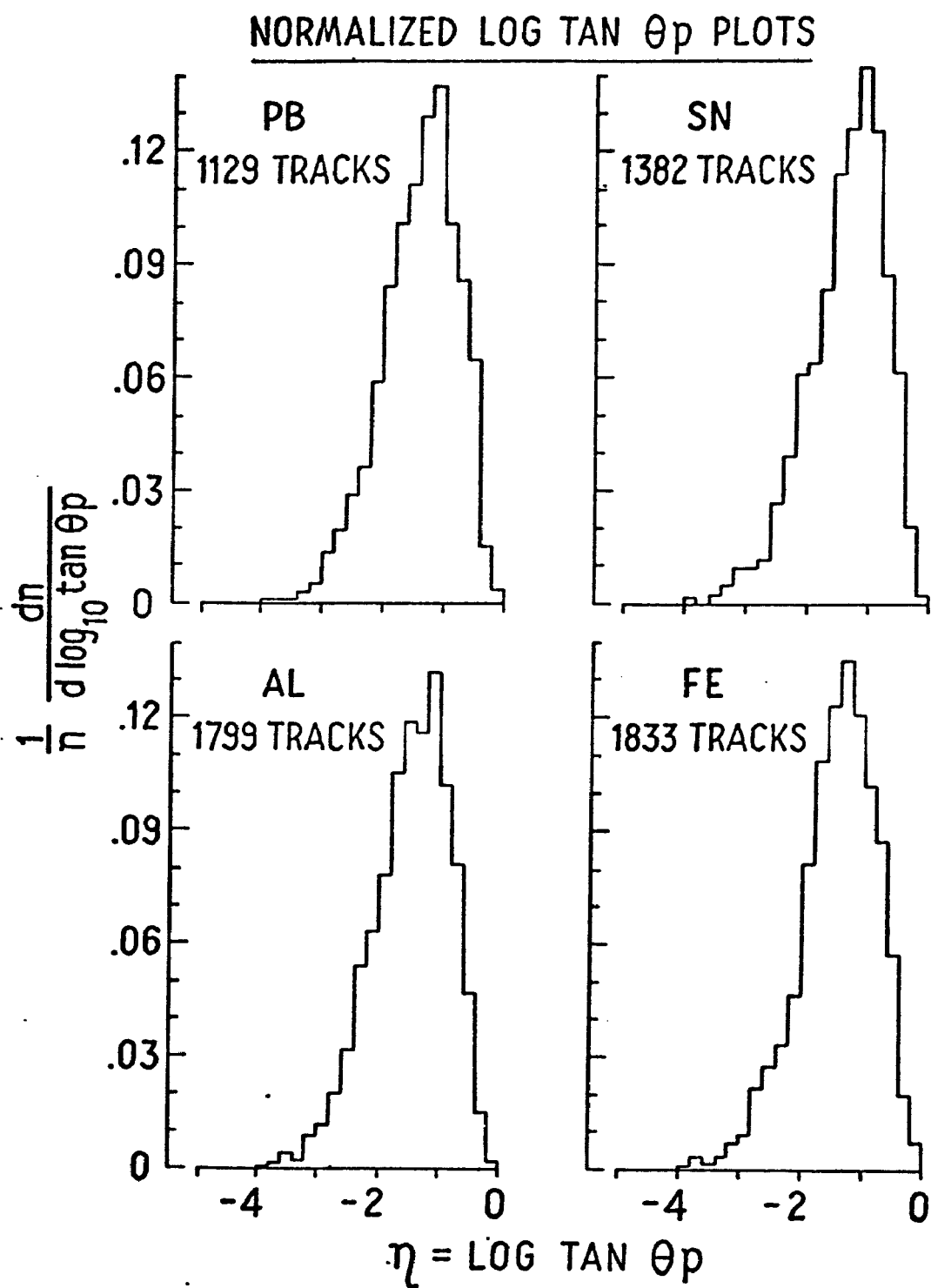


Fig. 13. Normalized η_p Plots for all Targets

was η_p where ($\eta_p = \log_{10} \tan \theta_p$) and which is proportional to the rapidity when the particles are relativistic. Figure 13 shows the normalized η_p plots for all the targets at an average energy of 160 GeV. The final sample of good events was used to get the distributions and normalization was necessary for proper comparison of the data. There are several effects that bias these distributions:

- a) Experimental angular resolution of 3 mrad
- b) Inability to resolve close tracks
- c) Contamination from prongs due to the processes which were responsible for the overestimation of the multiplicity, and
- d) the limited angular efficiency of the spark chamber.

The Monte Carlo predictions on the composition of the final state particles were given in Table XXII. The electron contamination was more serious for heavier targets; the contamination from the secondary tracks appeared mostly at large angles. The spark chamber inefficiencies also became more pronounced at large angles. In spite of these biases, 70 - 80% of the observed tracks were due to the primary interactions.

2) The variation of the observed normalized charged multiplicities in different angular regions: Because of the experimental biases, the normalized η_p plots display a certain superficial similarity. The differences become more obvious when certain sections of these plots are compared among

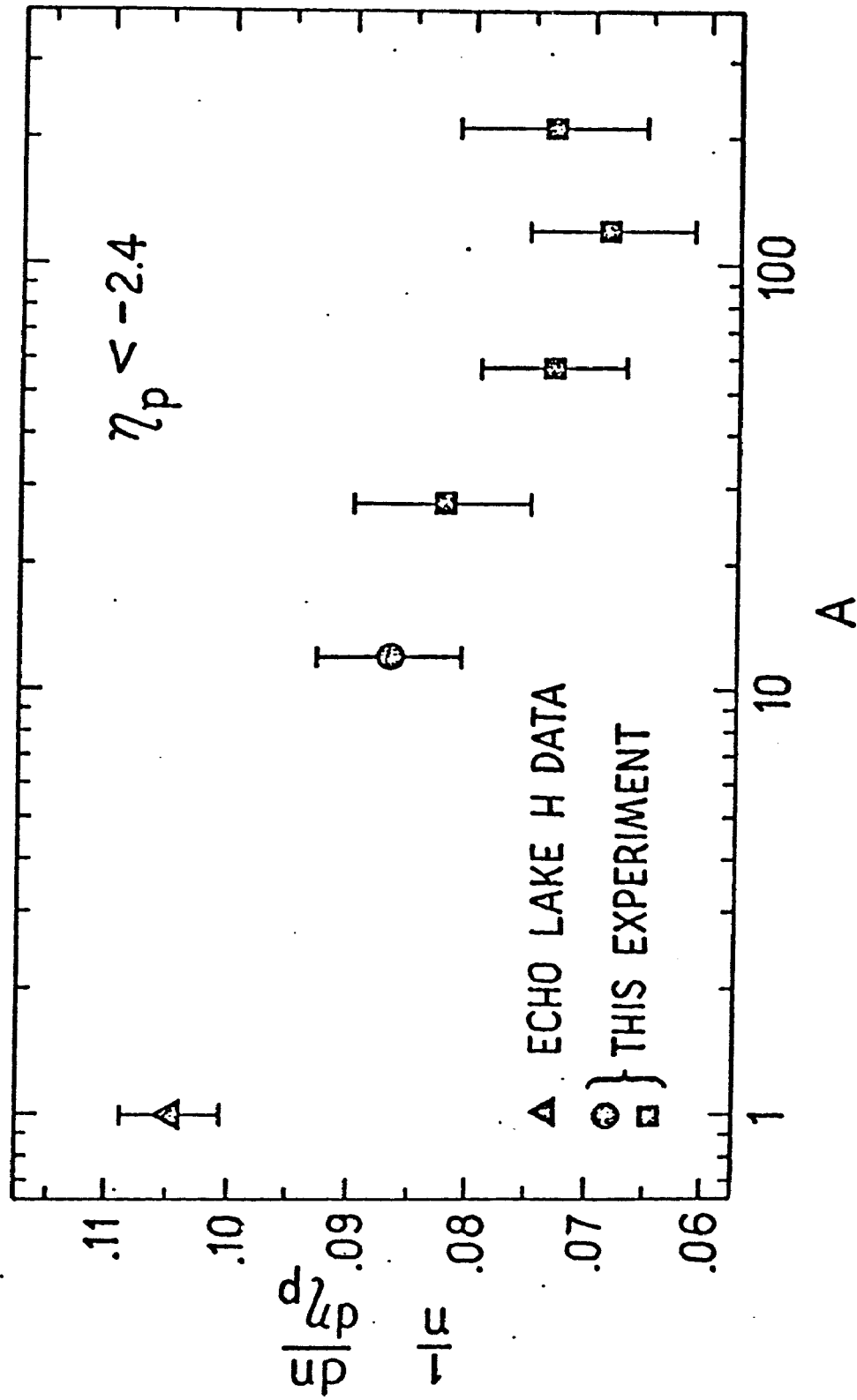


Fig. 14. Variation of Normalized Multiplicity with the Mass Number in the Extreme Forward Direction

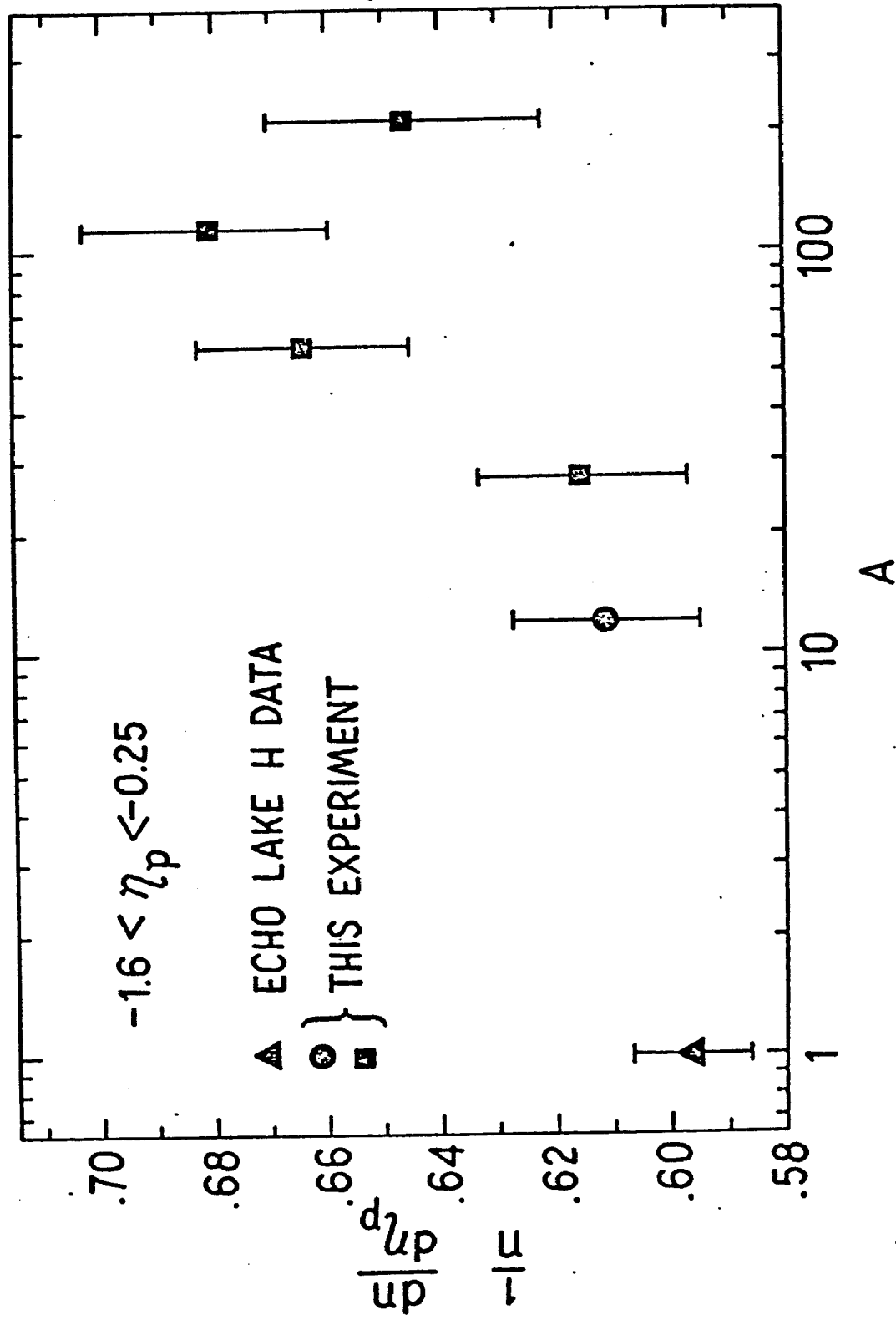


Fig. 15. Variation of Normalized Multiplicity with the Mass Number in the Large Angle Region

TABLE XXVII
OBSERVED NORMALIZED MULTIPLICITIES IN TWO ANGULAR REGIONS

Target	$\eta_P \leq -2.4$	$-1.6 < \eta_P < -0.25$
H	10.47 ± 0.41^a	59.70 ± 1.10
C	8.72 ± 0.60	61.10 ± 1.60
Al	8.23 ± 0.67	61.50 ± 1.80
Fe	7.31 ± 0.63	66.40 ± 1.90
Sn	6.87 ± 0.70	68.10 ± 2.20
Pb	7.35 ± 0.80	64.60 ± 2.40

^a The total multiplicity has been normalized to 100 particles.

TABLE XXVIII
CHARGED MULTIPLICITIES IN DIFFERENT ANGULAR REGIONS

Target	$\eta_P \leq -2.2$	$\eta_P \leq -1.12$	$\eta_P > -1.12$
H	0.95 ± 0.06	3.74 ± 0.17	3.59 ± 0.20
C	1.04 ± 0.10	4.82 ± 0.32	5.58 ± 0.50
Al	1.06 ± 0.11	4.75 ± 0.31	6.49 ± 0.61
Fe	0.93 ± 0.10	5.14 ± 0.38	7.11 ± 0.74
Sn	1.08 ± 0.14	5.45 ± 0.49	7.86 ± 0.96
Pb	1.14 ± 0.16	6.19 ± 0.55	8.56 ± 1.11

TABLE XXIX
R FOR DIFFERENT ANGULAR REGIONS AT 160 GeV

Target	Forward Direction	Backward Direction	Over All Angles
Al	1.27 ± 0.15	1.81 ± 0.20	1.54 ± 0.10
Fe	1.37 ± 0.18	1.98 ± 0.33	1.68 ± 0.12
Sn	1.46 ± 0.20	2.19 ± 0.41	1.82 ± 0.15
Pb	1.65 ± 0.24	2.38 ± 0.47	2.02 ± 0.20

each other and with the data on the angular distributions in other Echo Lake experiments, i.e., the p-p experiment (9) and the p-C experiment (12). These experiments used the same apparatus and were prone to similar biases. The tracks from all these η_p plots were binned into several η_p bins according to their η_p values. Specifically, two bins were considered; one with $\eta_p \leq -2.4$ which corresponded to the extreme forward direction, another where η_p was between -1.6 and 0.25 and which corresponded to the tracks at large angles. The fraction of the particles in these angular regions, which corresponded to the observed normalized charged multiplicities, was plotted vs. the mass number in Figures 14 and 15 and are also shown in Table XVII. The normalized charged multiplicity in the forward direction showed a monotonic decrease with the mass number; while the increase in the normalized multiplicity at large angles is evident from Figure 15, the experimental biases mask and distort the true increases.

3) The variation of the charged multiplicity in different angular regions: Since the loss of particles occurred only at large angles, the observed multiplicities at small angles are not expected to be different from the true values. Thus, the observed multiplicity at small angles, which was the product of the total observed multiplicity and the observed normalized multiplicity, was assumed to be equal to the true multiplicity. (at 160 GeV, 90° in p-p centre of mass system would correspond to η of -0.97 and η_p of -1.12. A η_p of

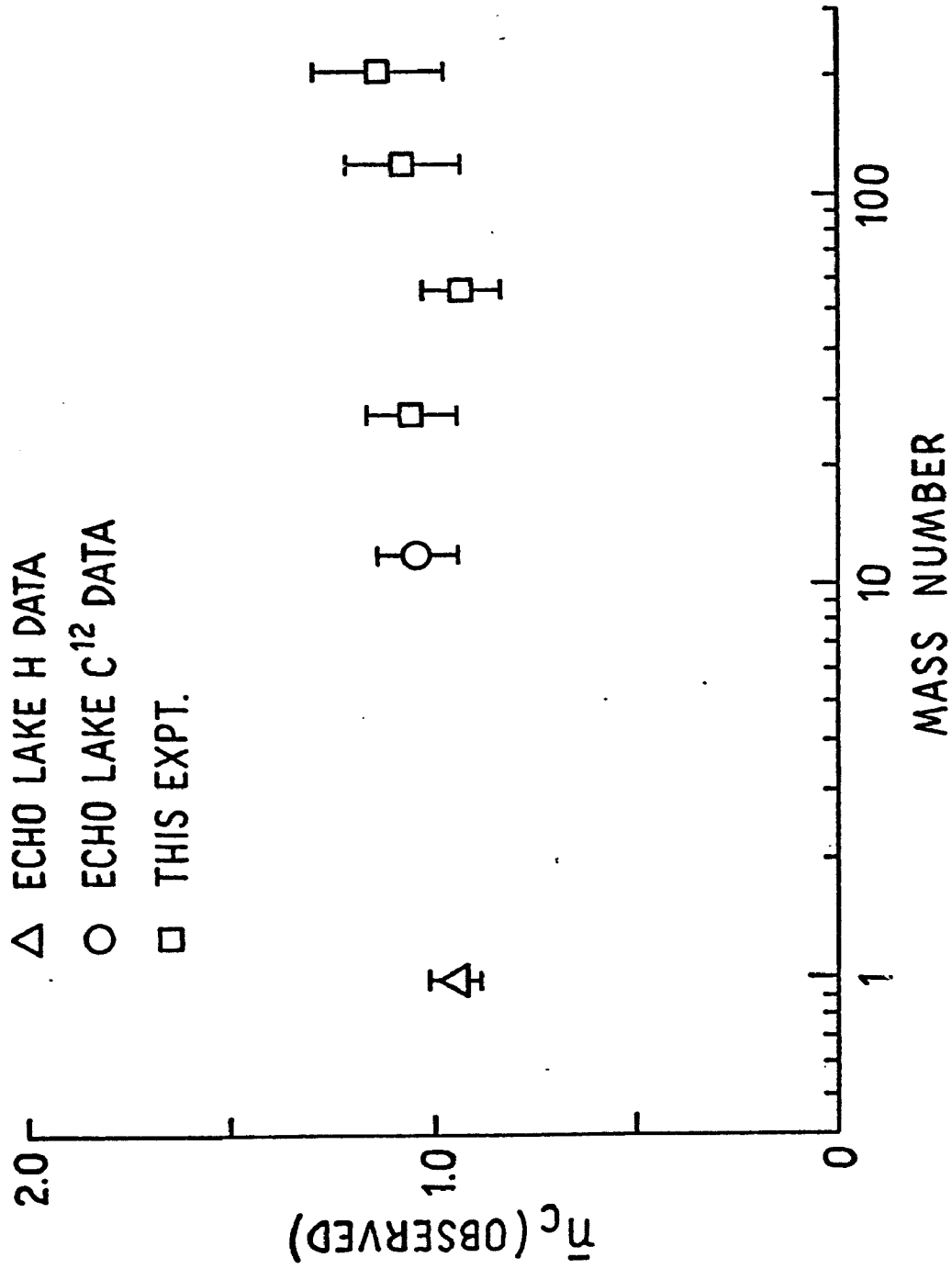


Fig. 16. Variation of the Charged Multiplicity with the Mass Number in the Extreme Forward Direction

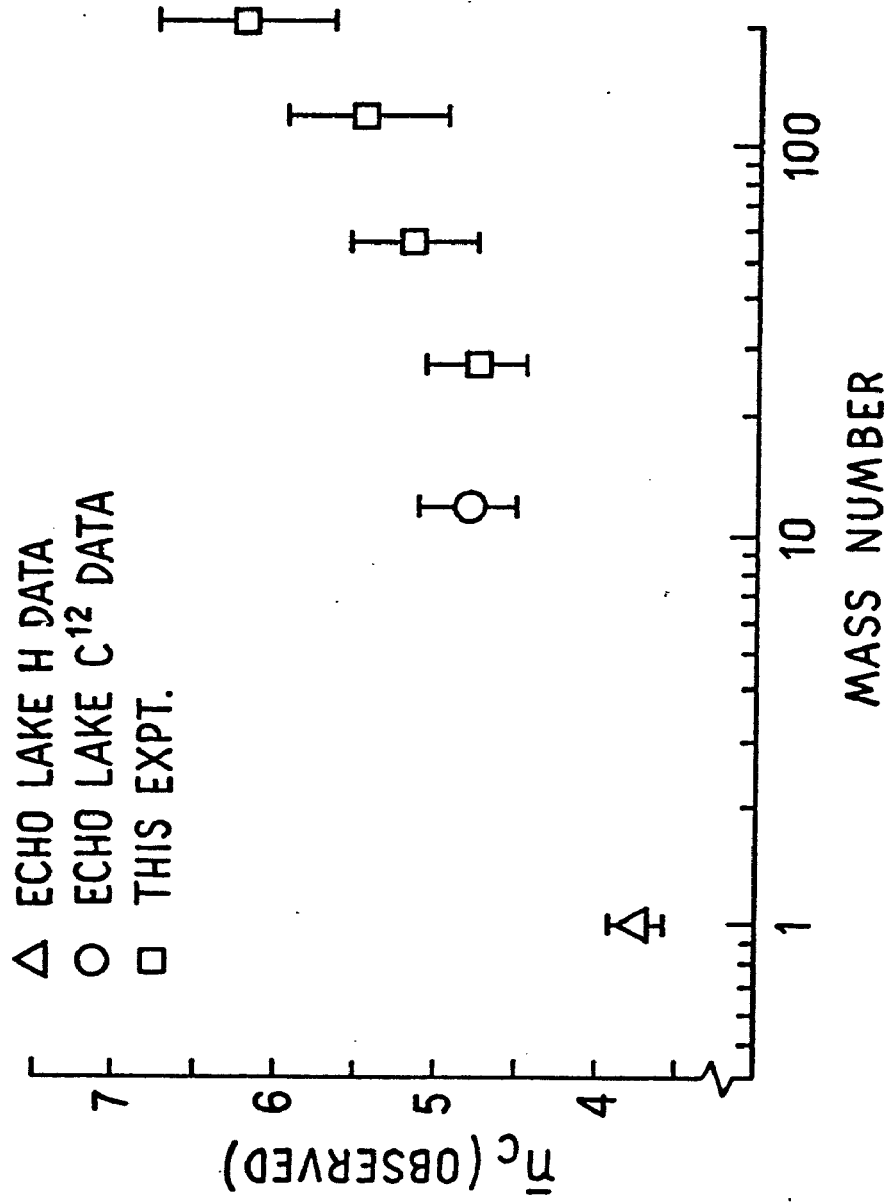


Fig. 17. Variation of the Charged Multiplicity with the Mass Number in the Forward Direction

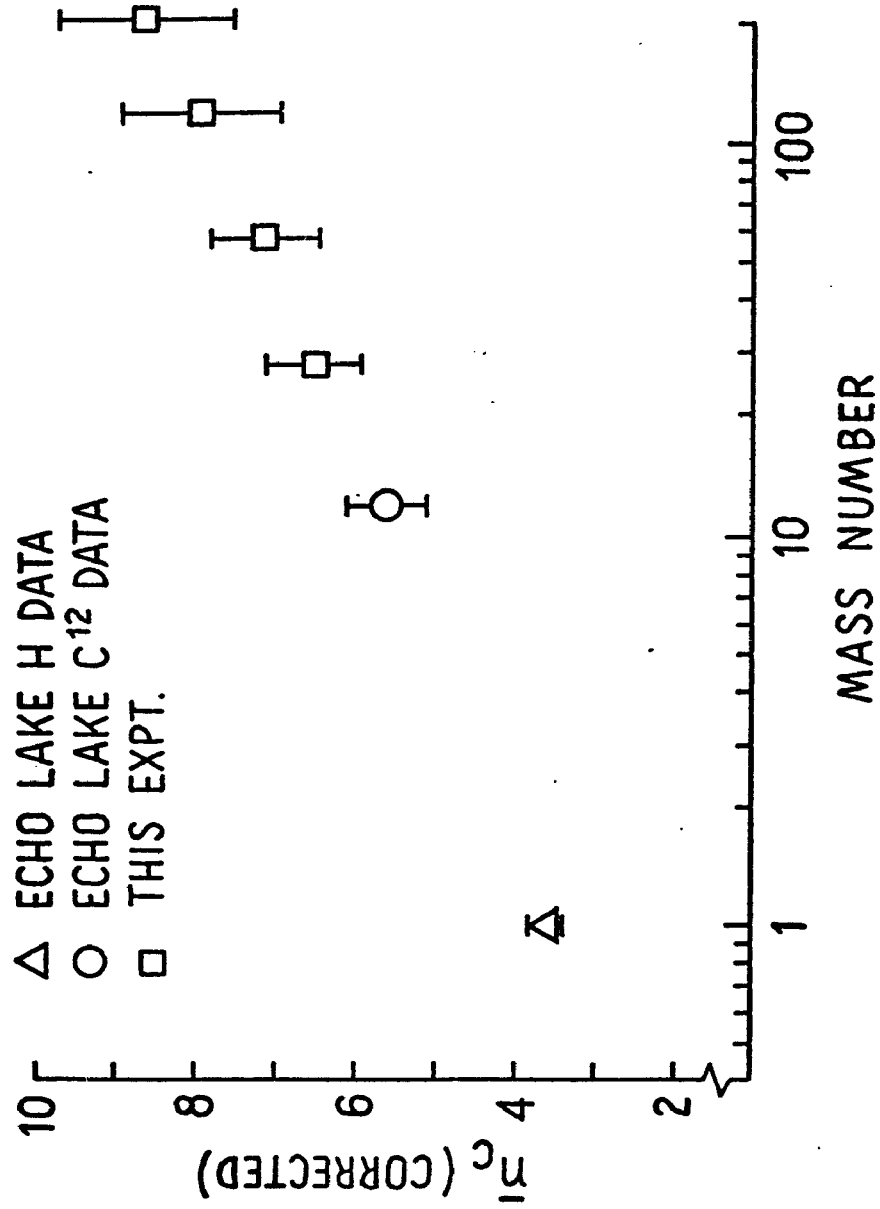


Fig. 18. Variation of the Charged Multiplicity with the Mass Number in the Backward Direction

-1.12 would correspond to about 4.5° with respect to the beam direction and the efficiency at $\sim 4.5^\circ$ is almost 100% even when the beam is not vertical.) Table XXVIII shows the observed multiplicities for the extreme forward direction and for the whole forward direction. The true multiplicity in the backward direction was obtained by subtracting the observed multiplicity in the forward direction from the total corrected multiplicity. These corrected multiplicities are also given in the Table XXVIII. (The corrected multiplicity for a proton target was taken from the fit to the accelerator p-p data and for the carbon target from the data of Wilkes.) Figures 16, 17 and 18 show the variation of the multiplicity with the mass number for all three regions. From these figures, it is apparent that the variation is faster in the backward direction. The slope of the variation in the backward direction is given by 0.167 ± 0.008 , whereas it is 0.084 ± 0.006 in the forward direction. The average multiplicity in the very forward direction is constant indicating the presence of the leading particles in all the targets. The values of R in both the forward direction and backward direction are given in Table XXIX for the metallic targets; for comparison, the values of R over all the angular regions at 160 GeV are also shown in Table XXIX.

4) Profile of the momentum distributions: At a fixed charged multiplicity, the observed secondary prongs were ordered by their individual η_p values and plotted separately.

Average Interval is the Difference Between the Average Values of the Ordered $\log_{10} \tan \theta_P$ Plots.

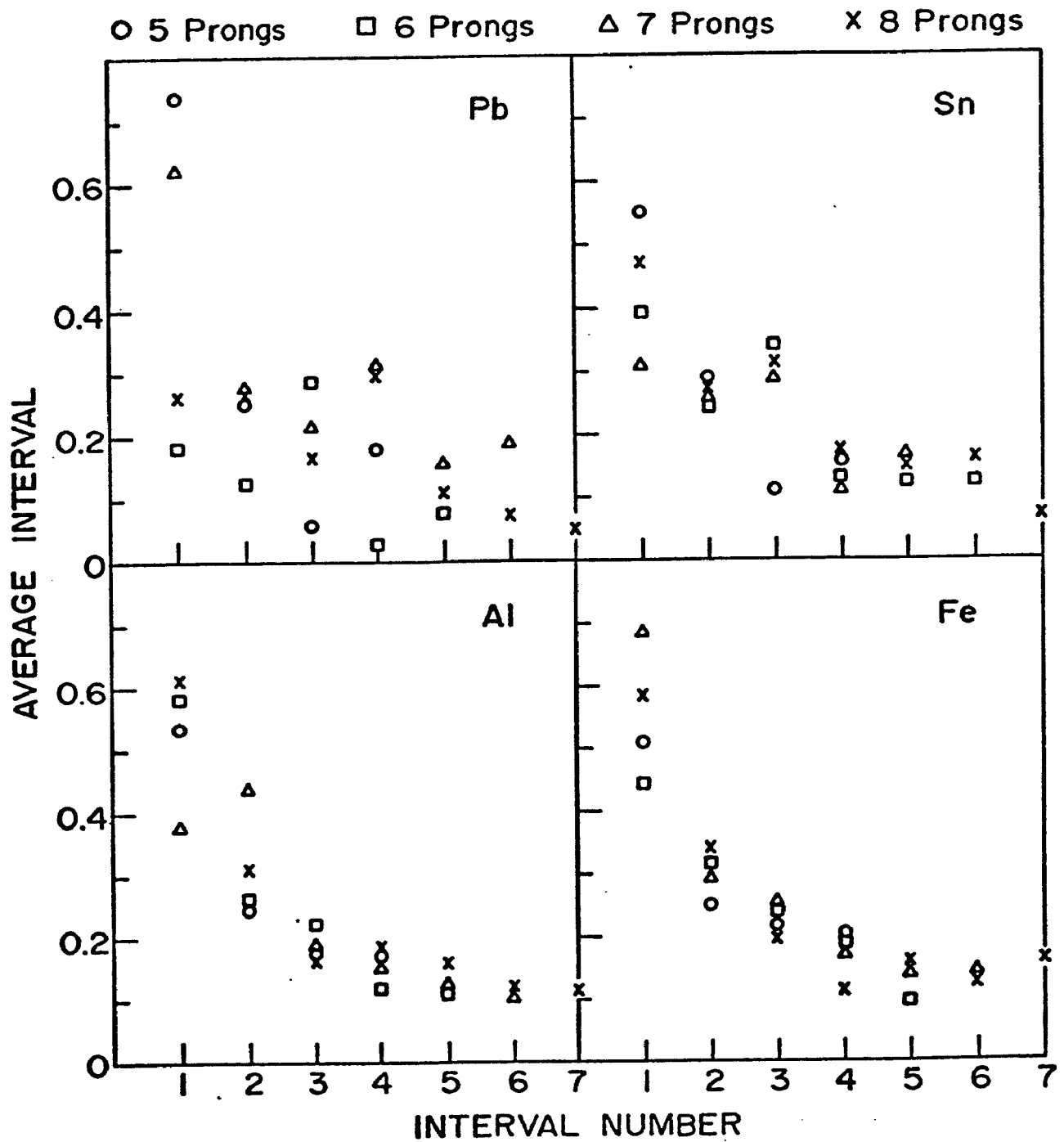


Fig. 19. Average Interval of Ordered $\log \tan \theta$ Plots vs. the Interval Number

The first plot contained the track from each event with the smallest η_p , the next plot contained the track from each event with the second smallest η_p and so on. The differences between the average values of these distributions, the so-called average interval, was plotted against the interval number. These plots are shown in Figure 19 for five, six, seven and eight prong events. The value of the average interval for the first interval number is significantly larger than the values for other interval numbers; the existence of a leading particle can be ascertained from this observation. For higher intervals, the distribution is more uniform indicating the presence of secondaries with similar momentum.

5) Comparison with theoretical predictions: All models predict an increase of multiplicity in the backward direction, which is also seen in this experiment. In general, the multiperipheral models predict larger increases in the backward direction compared to the other models. From Table XXIX, relative to hydrogen the increase in charged multiplicity is about 2.2 for the Tin target in the backward direction. While one also observes an increase in the forward direction multiplicity, this increase is very small. This observation is also in agreement with the predictions of most of the models. The leading particle effect seen in this experiment is predicted only by diffractive models. Thus, these results on angular distributions are consistent with the qualitative predictions of most of the models.

C. Conclusions

While the presence of pure targets in this experiment had a decided advantage over the ambiguous, but ingenious, use of the emulsion targets, this experiment was also prone to several experimental biases. The single major bias was due to the angular and multiparticle inefficiencies of the spark chamber, which resulted in an underestimation of the multiplicity. The spark chamber inefficiencies, along with biases due to extraneous processes, prevented an unambiguous quantification of the variation of angular distribution characteristics with the mass number of the target nucleus. The data in the higher energy bins were affected by underestimation of the energy by the calorimeter, which often resulted in the loss of those events. While the Monte Carlo approach was the only reasonable option for correction of the data, the pitfalls in this approach are obvious. Further, the analysis was conducted by ignoring the fraction of the pions in the beam.

In spite of these shortcomings, several significant conclusions were reached in this experiment:

1) R , the ratio of the charged multiplicity in proton-nucleus collisions to the charged multiplicity in proton-proton collision, varied only slowly with the mass number of the target nucleus. The variation was given by

$$R \propto A^{0.129 \pm 0.004}.$$

The value of this slope was in agreement with all the reliable data at present.

2) There was no significant variation of R with energy of the incident beam particle. Thus, the variation of charged multiplicity with energy was similar to that of p - p collisions.

3) Traditional models, like the extensions of the multiperipheral models and the diffraction dissociation models, did not explain our results. The EFC model of K. Gottfried gave the best fit to our data.

4) The fraction of particles at large angles increased with the mass number of the target nucleus; R for large angles was much higher than R over all angles. Thus, the major increase in multiplicity was due to particles at large angles.

5) The presence of a leading particle in all these targets was an indication of a very slow variation of inelasticity with the mass number.

Thus, the most important conclusion of the experiment was the very slow variation of the interaction characteristics with the mass number of the target nucleus. While this conclusion had been reached in the earlier experiments, this experiment was the first systematic effort to effectively quantify the earlier conclusions. We hope that more detailed experiments at accelerators will exploit the nucleus as a microscopic laboratory and shed more light on the space-time development of the final state of the proton-proton interaction.

APPENDIX A

THE RECONSTRUCTION EFFICIENCY

The overall poor quality of some of the picture frames prevented the reconstruction of these events, even after four remeasures. Table A1 gives the reconstruction efficiencies for the interacting events. These efficiencies were poorer for high multiplicity events and even a partial reconstruction was not possible. It would be difficult to make a quantitative estimate of this inefficiency on the final results except to remark that it would affect mostly the higher energy bins and the data from the lead target. As shown in the Table XIV, the probability of these events with any tracks at large angles satisfying the energy requirements was rather low. Thus, the energy underestimation was a far more serious bias against high multiplicity events than these moderate reconstruction inefficiencies.

TABLE A1
RECONSTRUCTION EFFICIENCIES

	Al-Fe Run	Sn-Pb Run
Reconstruction Efficiencies for Events With		
a) Low Multiplicity ($\bar{n}_t < 10$)	0.92	0.93
b) High Multiplicity ($\bar{n}_t > 10$)	0.75	0.71
Ratio of High Multiplicity to Low Multiplicity Events		
a) Before Reconstruction	0.24	0.24
b) After Reconstruction	0.19	0.18
Ratio of Average Multiplicity of the Events Before Recon- struction to After Recon- struction		
	0.95	0.81

APPENDIX B

EVENTS DUE TO NON-HADRONIC PROCESSES

Charged hadrons while traversing the target can knock off electrons (delta rays) from the target material. The presence of this electron along with the hadron which does not interact in the target results in the simulation of a two prong interacting event. When the electron has higher energies it can start a small cascade in the target which results in the simulation of an interacting event with more than two prongs. As noted in the section on scanning, tracks due to low energy (≤ 50 MeV) electrons could be identified on the scanning table. The thickness of the target also contributed to the absorption and scattering of electrons if they originated in the top plates. The minimum energy for an electron to appear as a straight track after traversing the matter in the target was a function of the plate number and the target properties and is given in the Table B1. These minimum energies were computed assuming only radiation losses which could be justified since the critical energies for all these elements are lower than 50 MeV. The probability for an electron created in the top plate to appear as a straight track was quite small.

TABLE B1
MINIMUM ELECTRON ENERGIES FOR SIMULATION
OF A STRAIGHT TRACK

Target	Origin of the Electron					
	1	2	3	4	5	6
Al	57	66	76	88	102	117
Fe	72	102	146	210	300	430
Sn	84	141	238	401	676	1139
Pb	114	262	599	1372	3140	6195

^a Bottommost plate is assigned a plate number 1 and the topmost plate is assigned a number 6. All the energies are in MeV.

A Monte Carlo program was undertaken to assess the contribution of the events with electrons of energy greater than 50 MeV to the observed distribution. Some of the important features of this program are presented below.

15000 charged primaries were followed through the six plates of all the targets. The probability of generating a delta ray (58) was given by

$$P = .15 (Z/A) (X/E) \quad (B.1)$$

where Z and A were the atomic number and the mass number respectively of the target, X was the amount of matter in each plate and E was the minimum energy to penetrate one single plate. The energy spectrum of the generated electrons had a

slope of -2. The maximum number of electrons that a hadron could generate was limited to six, i.e., one in each plate. Each electron was allowed to undergo bremsstrahlung and each created photon was allowed pair production. The electron pair thus produced was also allowed to undergo bremsstrahlung. The particle creation processes stopped with these second generation photons. The energy loss in each process was calculated and particles with energy less than 50 MeV were not counted. This program did not assign any angles to these particles. The number of electrons from each plate was added to give the number of electrons in an event.

The results of this Monte Carlo program are shown in Table B2. The probabilities for simulation of a two prong event are smaller for the heavier targets because of high energies needed for electrons to appear as straight tracks. For detection of these events with higher energy electrons, two separate, but sometimes supplementary, methods were employed after the digitization of the events.

1) The χ^2 given by the reconstruction program was a measure of the best fit to the tracks. A non-hadronic event had a high χ^2 since straggling effects were more important for electrons. All events with low prong numbers and bad fits were rejected with the assumption that these tracks were due to electrons.

2) The calorimeter responses were different for non-interacting hadrons and hadrons which interacted in the target.

TABLE B2
 PROBABILITY OF SIMULATION OF AN INTERACTING
 EVENT BY NON-HADRONIC PROCESSES

Target	Multiplicity of the Event		
	$n = 2$	$2 < n \leq 4$	$n > 4$
Al	2.78×10^{-2}	5.33×10^{-4}	---
Fe	2.87×10^{-2}	2.86×10^{-3}	2.00×10^{-4}
Sn	1.94×10^{-2}	2.40×10^{-3}	6.67×10^{-4}
Pb	1.65×10^{-2}	2.47×10^{-3}	6.67×10^{-4}

TABLE B3
 CALORIMETER RESPONSES TO A TRUE INTERACTING EVENT
 AND AN EVENT DUE TO NON-HADRONIC PROCESSES

Calorimeter Layer	Number of Equivalent Muons	
	in a True Interacting Event	in an Event Due to Non-hadronic Processes
1	99	14
2	155	16
3	127	12
4	135	1
5	31	57
6	19	137
7	2	183
8	2	167
9	9	19
10	0	2

TABLE B4
PERCENTAGE OF EVENTS REJECTED DUE TO
NON-HADRONIC PROCESSES

Target	Due to Reconstruction Criterion	Due to Calorimeter Response
Al	2.8	4.5
Fe	2.3	4.9
Sn	9.0	11.1
Pb	11.5	13.5

These differences could not be explained by assuming fluctuation in the shower development or in the behaviour of the LADCs employed in the calorimeter layers. The total probability for any interaction before the third layer of the calorimeter was about 60% for one hadron whereas it was about 94% for three charged hadrons. Interacting events in which the calorimeter response was similar to that of a straight-through event were rejected as being due to non-hadronic processes. The assumption was that the electrons from non-hadronic events could not start a measurable cascade in the calorimeter and was similar to the criterion of penetration in the iron spark chambers mentioned in the section on scanning. Table B3 shows the responses of all the ten layers to an interacting event and an event rejected as due to non-hadronic processes. Both events had three straight tracks, had similar energies and had their vertex in the iron target. To reduce the probability of any bias stemming from this criterion, this criterion was applied to all interacting events. Table B4 shows the percent of events rejected due to non-hadronic processes. It has to be noted that the narrow gap spark chambers were more efficient in the Al/Fe run than in the Sn/Pb run.

APPENDIX C

THE PERFORMANCE OF THE WIDE GAP SPARK CHAMBERS

A major effort was undertaken to understand the performance of the wide gap spark chambers since it was the most important experimental bias in the apparatus. The reaction products were observed as sparks in these chambers which had two gaps each of 20 cm spacing and $2 \times 2 \text{ m}^2$ area. Two views, corresponding to 90° stereo, were recorded for each event. The charged multiplicity for each event was determined to be the maximum number of prongs in either stereo view. It was enough for a track to be present in either gap to contribute to the recorded multiplicity.

There were several factors which indicated that the efficiency of the chamber decreased with an increase in the angle a track made with the field direction and with an increase in the number of tracks in the chamber.

1) The visible intensity of a track was maximum when it was parallel to the field direction and large angle tracks were frequently very faint.

2) The observed zenith angle distribution of the primaries was different from the expected distribution at the Echo Lake altitude. According to Hayakawa (2), the zenith angle distribution at 710 gms/cm^2 is given by a $\cos^{7.63} \theta$

TABLE C1
VARIATION OF OBSERVED CHARACTERISTICS
OF INTERACTIONS WITH ZENITH ANGLE

Average Zenith Angle of the Primary in Degrees	Average Charged Multiplicity	Maximum Projected Angle of a Secondary WRT the Primary
3.0	11.00	10° 20'
9.0	8.86	12° 50'
15.0	7.76	13° 30'
21.0	7.21	16° 50'
27.0	6.83	18° 20'

TABLE C2
OBSERVED INEFFICIENCIES FOR UPPER AND LOWER GAPS

Multiplicity	U p p e r		L o w e r	
	Al-Fe	Sn-Pb	Al-Fe	Sn-Pb
2	.0760	.0283	.1616	.2448
3	.1111	.0731	.2664	.2485
4	.1195	.1245	.3135	.3245
5	.1676	.0981	.3081	.3236
6	.1448	.1598	.3541	.3584
7	.1525	.1219	.3606	.4286
8	.1647	.1143	.3786	.4574
9	.2081	.1567	.3516	.4422
10	.1713	.1473	.4169	.4527
11	.2035	.1223	.3570	.5087
12	.1905	.1345	.3259	.4670
13	.1589	.1539	.3357	.4193
14	.1896	.1013	.3859	.5810
15	.2798	.1411	.3978	.4778

distribution. Figure 20 shows the observed zenith angle distribution for non-interacting events and Monte Carlo events generated with the expected distribution. The observed events were in geometry and had energies greater than 100 GeV and the Monte Carlo events were also required to satisfy these specifications. The observed distribution was a superposition of the experimental biases on the incoming distribution. Primaries with large zenith angles often would not trigger the apparatus because of low energy deposition in the calorimeter but this effect was negligible for in geometry events. Charged pions could trigger the apparatus but their fraction was quite small. So, the main experimental bias which affected the zenith angle distribution was the inefficiency of the chambers for large angle tracks.

3) Table C1 shows the variation of the observed characteristics of interacting events with the zenith angle of the primary in case of the Pb target. The average charged multiplicity decreased with increasing zenith angle because the number of tracks at large angles to the field direction increased with increase in zenith angle. The maximum projected angle of a secondary with respect to the beam direction increased with the zenith angle of the primary. The largest angle track was at large angle to the field direction in case of a vertical beam particle and the probability of observation of this track was quite small. When the beam was not

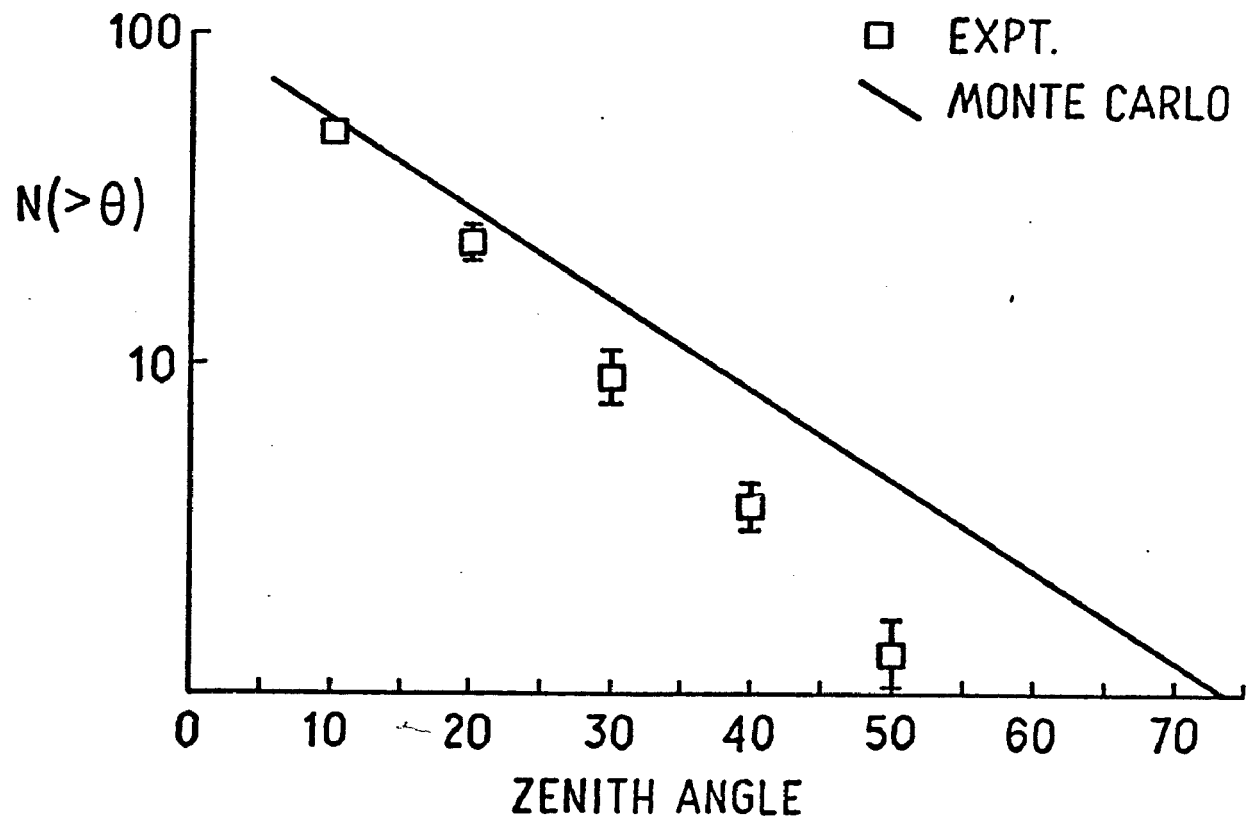


Fig. 20. Expected and Observed Zenith Angle Distribution

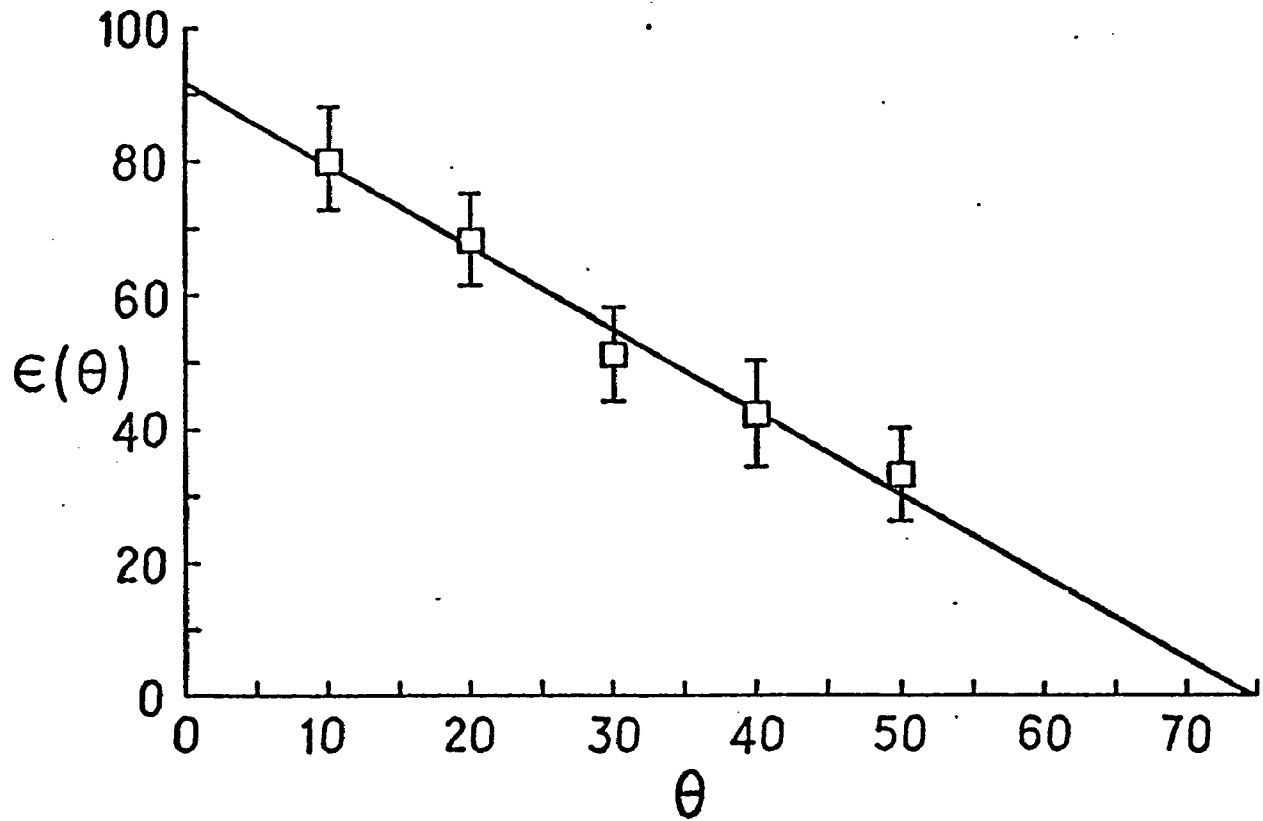


Fig. 21. Angular Efficiency of a Single Gap in the Spark Chamber

vertical, the probability of observing the same track was higher because it would be at small angle to the field direction. The maximum projected angle in the near vertical bin (small zenith angles) also gave a measure of the largest angle recorded with respect to the field direction. In this experiment, the maximum angle was 22.5° .

4) A study of the gap efficiency as a function of the prong number in each gap showed that the efficiency decreased with increasing multiplicity. The observed inefficiency of a gap was defined as the number of tracks visible in the other gap and not seen to extrapolate through that gap divided by the total recorded multiplicity. These observed inefficiencies, which were lower limits to the true inefficiencies, are shown in Table C2. An earlier analysis used these gap inefficiencies to correct the Echo Lake p-p data (52).

All these observations were used in the deduction of the spark chamber inefficiencies. A measure of the angular inefficiency for a single particle was obtained by comparing the observed and expected zenith angle distributions. The observed distribution had been obtained by demanding at least one track in both upper and lower spark chambers. Thus, the observed efficiency was the result of asking for tracks in at least two gaps when the total number of gaps was four. The single gap efficiency, which was deduced from these observed efficiencies, is shown in Figure 21 as a function of θ , the

angle the track made with the field direction. From the best fit to these points, it can be seen that the efficiency becomes zero at about 75° and is about 91% at 0° . (Earlier analysis for the p-p data had assumed 92% efficiency for the upper gap and 88% for the lower gap irrespective of the angle (52) .)

When there were more particles in the chamber, the available energy had to be shared by all the particles. An attempt was made to understand the sharing of the energy by using some of the observations which suggested that the energy was used up mostly by the tracks which were parallel to the field direction at the expense of the tracks at large angles. A model was adopted in which particles shared the energy equally as long as they were within a certain cut-off angle and no energy was available to tracks at angles beyond the cut-off angle. 22.5° was taken to be the cut-off angle beyond which tracks were not visible. The single particle efficiency function was used for tracks within the cut-off angle. When this model was used with the Monte Carlo model for p-p collisions (Appendix D), it reproduced the observed multiplicities in the Echo Lake p-p experiment. The model also reproduced the variation in the interaction characteristics with the zenith angle of the primary.

In spite of these inefficiencies, it should be noted that of the initial sample of 4000 interactions, over 100 had observed multiplicities of greater than 20 (Table VI). The

maximum number of the observed prongs was 30 in Pb and 40 in Fe. According to Allkoffer (59), a track at 5° to the field direction was six times more luminous than a track at 30° ; the multitrack efficiency in that 10 cm gap spark chamber was zero at angles greater than 50° . Though our efficiency functions were in qualitative agreement with the data about spark chamber performances, one should note that these were deduced from many observations, which by themselves might have been biased. For example, the single particle angular efficiency was derived by assuming that the fraction of the pions in the beam was negligible. The assumption about a cut-off angle for multiparticle events is also not realistic. Despite these biases, this procedure was the only reasonable option in the present analysis.

APPENDIX D

A MONTE CARLO MODEL FOR P-P COLLISIONS

A Monte Carlo model had been successfully used in the correction of the data taken in the Echo Lake p-p experiment and was used in the present data analysis for the generation of each p-target nucleon interaction. This model has been described in detail elsewhere (9) and only some of the important features will be mentioned here.

1) The average charged multiplicity was allowed to vary as

$$n_c = A + B \ln s + C/s^2 \quad (D.1)$$

where s is the square of the center of mass energy. This formula was taken from Reference 49 and is in agreement with the latest NAL and ISR results.

2) The number of created particles was described by a Poissonian distribution whose average multiplicity was taken to be

$$n = (n_c - 1.4)1.5. \quad (D.2)$$

All the created particles were assumed to be pions and a factor of 1.5 was necessary to include the neutral pions. The average number of protons in the final state was taken to be 1.4.

3) The centre of mass distribution of the momenta of the pions was given by

$$d^3N d f(x, s) e^{-b p_t^2} d^3p / E \quad (D.3)$$

where b was chosen to correspond to an average p_t of 0.35 GeV/c. The transverse momenta for all the particles was chosen from an exponential distribution and the longitudinal momenta for the pions from a $1/E$ distribution. After having assigned momenta and energy to the created particles, the equations of conservation of energy and momenta were solved to get the longitudinal momenta for the nucleons. The momenta and the energies were then transformed into the laboratory system, where the angles of the secondaries were with respect to the direction of the beam particle. A rotation was also done from the beam co-ordinates to the experimental co-ordinates.

4) The charges of the final state resulted as if the particles were produced multiperipherally via an alternating isospin exchange ($I = 0, 1$). The Monte Carlo model for the p-p experiment had used a pure ($I = 1$) exchange model but the alternating isospin exchange model was preferred since it gave the correct number of final state nucleons.

This model produced results which were compatible with the available data on p-p collisions. Wherever a comparison of the models for the p-nucleus collisions and the model for p-p collision was needed, the results of this Monte Carlo model have been given.

APPENDIX E
SIMULATION OF THE ENERGY UNDERESTIMATION
BY THE CALORIMETER

The aim of the simulation was to understand the energy bias in the experiment and hence, the observed distribution of the event vertices. For the primary interaction, the simulation program used the events generated by the Monte Carlo model A. The secondary prongs were followed downward from the vertex of the interaction through the remaining target material and the calorimeter. In the case of a secondary interaction, the tertiary prongs were traced through the experimental apparatus. When the extrapolation of the particle trajectory did not enter the calorimeter, the energy deposited was considered to be zero. No attempt was made to simulate either the hadronic cascade or the electromagnetic cascade. For photons, the energy lost in the target and the energy not deposited in the calorimeter was taken from a parametrization by R. J. Wilkes (15) to the electromagnetic cascade theory of B. Rossi (60). The charged hadrons which entered the calorimeter were assigned interaction sites based on the interaction mean free path of the proton. When the hadrons interacted outside the calorimeter or when they decayed (only for pions) inside the calorimeter or when the

amount of matter available for the cascade was less than one radiation length, the energy deposited was considered to be zero. The total amount of matter traversed by the particle in the iron was calculated after subtracting the amount traversed in the scintillator material. The fraction of energy going into nuclear disintegrations and the energy leaking out of the bottom were calculated by the parametrizations mentioned in the section on energy underestimation. Thus the amount of energy which was deposited by each particle was calculated and added up to get the final energy of the event.

Table E1 shows the results of this simulation program. An interacting event in the Pb target deposited lesser energy than an event in the Al target. Also, the energy loss was more when the vertex was in the top plates. The fraction of particles not contributing any energy increased with the atomic weight of the target. The energy deposited as a function of charged multiplicity is shown in Table E2 for Al and Pb targets. The energy deposited showed a decrease with the multiplicity of the event. The distribution of the fraction of the energy deposited is shown in Table E3. The peak of the distribution was between 60% and 80% for both the targets.

While all the qualitative features mentioned in the section on energy underestimation could be reproduced, the program failed to simulate the actual observed vertex distributions. While the simulation program showed more events from

TABLE E1
ENERGY UNDERESTIMATION FOR INTERACTING EVENTS

	Al	Fe	Sn	Pb
Fraction of Events With Energy Greater Than Threshold Energy	0.5045	0.4730	0.4680	0.4451
Unsampled Energy as a Fraction of the Incident Energy	0.3684	0.4028	0.4069	0.4391
Number of Particles Not Contributing Any Energy as a Fraction of the Total Number of Particles	0.4737	0.4922	0.4976	0.5232
Fractional Energy Loss with the Vertex in				
a) Topmost Plate	0.3871	0.4702	0.4574	0.5729
b) Bottommost Plate	0.2657	0.2600	0.2903	0.3273

TABLE E2
FRACTIONAL ENERGY DEPOSITED vs CHARGE MULTIPLICITY

Charged Multiplicity	Al	Pb
$2 \leq n \leq 5$	0.68	0.66
$6 \leq n \leq 10$	0.68	0.63
$11 \leq n \leq 15$	0.63	0.56
$16 \leq n \leq 20$	0.60	0.56
$21 \leq n \leq 25$	0.64	0.54
$26 \leq n \leq 30$	0.55	0.53

TABLE E3
DISTRIBUTION OF FRACTIONAL ENERGY DEPOSITED
IN THE CALORIMETER

f, The Fractional Energy* Deposited in the Calorimeter	Al	Pb
$0 \leq f \leq 20$	17.7	19.4
$20 < f \leq 40$	3.2	4.0
$40 < f \leq 60$	5.9	10.3
$60 < f \leq 80$	41.9	47.7
$80 < f \leq 100$	31.3	18.1

* Both the total energy and the total number of events have been normalized to 100.

the bottom plates, the number of events from the top plates was still considerable and much higher than the observed values (Table VI). This failure to reproduce the observed vertex distributions was a reflection on the simulation program which was a superposition of many Monte Carlo models. While it has been shown that the biases in the target and the spark chamber were relatively insensitive to the nature of the Monte Carlo models for the primary interaction (Table XXI), a correct angular distribution becomes more important in the simulation of the calorimeter response. The charge to neutral ratio of the secondaries is also more important because a photon deposits almost all of its energy compared to a charged hadron. Apart from the parameters of the interactions, a calibration of the calorimeter device for interacting events has not yet been done with an accelerator beam and the correctness of the ideas on the unsampled energy losses have still to be tested. Thus, the observed vertex distributions were taken for granted and the energy of each event was multiplied by a factor which corresponded to an energy underestimation of 44%.

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