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THE ECCLES-JORDAN TRIGGER CIRCUIT

WITH SPECIAL ATTENTION TO TRANSITION TIME

by

Rajeswari

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ABSTRACT

THE ECCLES-JORDAN TRIGGER CIRCUIT

WITH SPECIAL ATTENTION TO TRANSITION TIME

by Rajeswari

A detailed analytical study, confirmed by experiments, is made of the time of transition between states of the bistable Eccles-Jordan trigger circuit. The method of approach is different from that of previous workers in that (a) it makes use of a new equivalent circuit deduced from the differential equations describing the circuit, and (b) it is easily related graphically to the d-c voltage-current characteristics of the circuit.

Assuming straight-line tube characteristics, the differential equation with respect to time t satisfied by the voltage e between the plates of the two tubes is deduced to be of the form $a_1(d^2e/dt^2) + a_2(de/dt) + a_3e + a_4 = 0$, where a_1 is the product of a resistance and the square of a capacitance, a_2 is a capacitance, a_3 is a conductance, and a_4 is a current. The numerical values and in some cases the signs of a_1, a_2, a_3, and a_4 are different according to whether (1) both the tubes are carrying current, (2) only one tube is carrying current and grid current is not flowing, or (3) only one tube is carrying current and grid current is flowing.

It is found that, for typical circuit parameters, the effect of a_1 is negligible. Hence, as a first approximation, the equivalent circuit of only a resistance $1/a_3$ and a capacitance a_2 in parallel is used, and a study is made of the triggering action of the circuit when a rectangular voltage pulse is applied between the two plates in series with a finite resistance. The transition from one stable state to the other is divided into three intervals. The resistance $1/a_3$ has positive values during the

first and third intervals, while in the second interval, during which positive feedback is in control, it has a negative value. Thus, in the study of the transition, three differential equations of the first order, of the form $a_2(de/dt) + a_3e = a_4$, occur, where a_3 has positive values in the first and third intervals and a negative value in the middle interval, while a_2 is positive in all three intervals. The solution of these linear differential equations is very simple, and a complete solution is obtained by matching boundary conditions at the extremes of each interval. This study leads to the determination of (1) the time of transition, (2) minimum magnitude and minimum duration of the pulse necessary for triggering, and (3) optimum magnitude and optimum duration of the pulse for least time of transition.

The method is extended to the case where a_1 is not negligible. It is also shown how the same method can be applied to study the triggering action of the circuit by (1) a rectangular current pulse between one grid and the cathode, or (2) a rectangular voltage pulse in series with a finite resistance between one grid and the cathode.

ACKNOWLEDGEMENTS

The author wishes to express her sincere appreciation for the time devoted by her doctoral committee to examining this dissertation. She is especially grateful for the valuable suggestions and pertinent criticisms given by Professor W. G. Dow and Assistant Professor J. F. Cline at various stages of the work.

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LIST OF SYMBOLS

Symbol	Meaning	Page
R_1, R_2 . . .	Load resistances in the two plate circuits	2
E_{bb_1}, E_{bb_2} .	Plate supply voltages for the two tubes	2
E_{cc}	Grid supply voltage	2
i_{b_1}, i_{b_2} . .	Plate currents of the two tubes	9, 59
i_{c_1}, i_{c_2} . .	Grid currents of the two tubes	9, 59
i_{R_1}, i_{R_2} . .	Currents through resistors R_1 and R_2	9, 59
r_B	General symbol to represent the reciprocal of the slope of the i-e characteristic	11, 37
e_t, e_s . . .	Triggering voltages	11,42,48
E_1	Magnitude of triggering voltage pulse between the two plates	11
E_2	Magnitude of triggering voltage pulse in series with R_1	37
t	Instantaneous time	11,24,37
t_1	Duration of triggering voltage pulse	11, 37
r	Finite positive resistance in series with triggering pulse	11,42,48
C	Lumped capacitance between the two plates	11,37,42,77
i_C	Current through capacitance C	11,37,42
r_1, r_2, r_3'' .	Reciprocals of the slopes of different portions of the i-e characteristic	16
i_2, i_3'' . . .	Intercepts of the different portions of the i-e characteristic on the i axis	16
E_{1min}	Minimum magnitude of voltage pulse between the two plates necessary for triggering	26, 27
t_{1min}	Minimum duration of voltage pulse between the two plates necessary for triggering	26, 27
E_{2min}	Minimum magnitude of voltage pulse in series with R_1 necessary for triggering	35

Symbol	Meaning	Page
t_{1opt}	Optimum duration of voltage pulse between the two plates for least time of transition	29,30,31
t_r	Time of transition	22,24,30,31
T	Time constant Cr	31,84
r_o	General symbol to represent the reciprocal of the slope of the i-e characteristic	42
$C'^2r' = a_1$.	Element of dimension (capacitance)^{2}x(resistance) . .	42,74
i'	Current through element C'^2r'	42
i_2	External current flowing between G_2 and K	49
e_{b_1}, e_{b_2} . .	Instantaneous total plate voltages of the two tubes.	59
e_{c_1}, e_{c_2} . .	Instantaneous total grid voltages of the two tubes .	59
g_m	$\partial i_b/\partial e_e$ (e_b = constant) for $e_c < 0$	58
μ	$-\partial e_b/\partial e_c$ (i_b = constant) for $e_c < 0$	5
g_m''	$\partial i_b/\partial e_c$ (e_b = constant) for $e_c > 0$ and $e_b > e_c$ . .	58
μ''	$-\partial e_b/\partial e_c$ (i_b = constant) for $e_c > 0$ and $e_b > e_c$ . .	58
g_m'	$\partial i_b/\partial e_c$ (e_b = constant) for $e_c > 0$ and $e_b < e_c$ . .	61
μ'	$-\partial e_b/\partial e_c$ (i_b = constant) for $e_c > 0$ and $e_b < e_c$ . .	61
g_n''	$\partial i_c/\partial e_b$ (e_c = constant) for $e_c > 0$ and $e_b > e_c$ . .	58
μ_g''	$-\partial e_c/\partial e_b$ (i_c = constant) for $e_c > 0$ and $e_b > e_c$.	58
g_n'	$\partial i_c/\partial e_b$ (e_c = constant) for $e_c > 0$ and $e_b < e_c$ . .	61
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$C_{g_1p_1}$, $C_{g_2p_2}$ ($C_{g_1p_1} = C_{g_2p_2} = C_{gp}$)	Grid-plate capacitances of the two tubes	71
$C_{k_1g_1}$, $C_{k_2g_2}$ ($C_{k_1g_1} = C_{k_2g_2} = C_{kg}$)	Cathode-grid capacitances of the two tubes	71
$C_{k_1p_1}$, $C_{k_2p_2}$ ($C_{k_1p_1} = C_{k_2p_2} = C_{kp}$)	Cathode-plate capacitances of the two tubes	71

Symbol	Meaning	Page
$i_{g_1p_1}$, $i_{g_2p_2}$, $i_{k_1g_1}$, $i_{k_2g_2}$, $i_{k_1p_1}$, $i_{k_2p_2}$	. . Currents through the capacitances $C_{g_1p_1}$, $C_{g_2p_2}$, $C_{k_1g_1}$, $C_{k_2g_2}$, $C_{k_1p_1}$, and $C_{k_2p_2}$ respectively . .	71

CHAPTER 1

INTRODUCTION

A trigger circuit is an arrangement of circuit components which has two states of equilibrium for the same fixed supply voltages and circuit parameters. These two states of equilibrium are characterized by two sets of circuit currents. By small changes in parameters or by small changes in impressed controlling voltages, the circuit can be made to trigger from one state of equilibrium to the other. The change from one state to the other normally takes place very rapidly.

Trigger circuits have been used widely and form the basis of multivibrators, flip-flop circuits, counting circuits, etc. Three kinds of behavior of their two states of equilibrium are possible, (a) both may be quasi-stable as in multivibrators, (b) one may be stable and the other quasi-stable as in one-shot multivibrators, or (c) both may be stable as in flip-flop circuits, including the original Eccles-Jordan circuit[1] which is still widely used.

Figure 1.1 represents a simple form of the Eccles-Jordan circuit. The triggering action of this circuit depends on the fact that if the plate current of one tube is large, that in the other must be small, and vice versa. Suppose for an instant that there is an equilibrium condition in which both the tubes are carrying large plate currents. A slight increase of current in tube (2) will increase the voltage drop across R_2, and thus increase the negative grid voltage of tube (1), which in its turn decreases the plate current of tube (1). This decreases the voltage drop across R_1, which decreases the negative grid voltage of tube (2), and thus

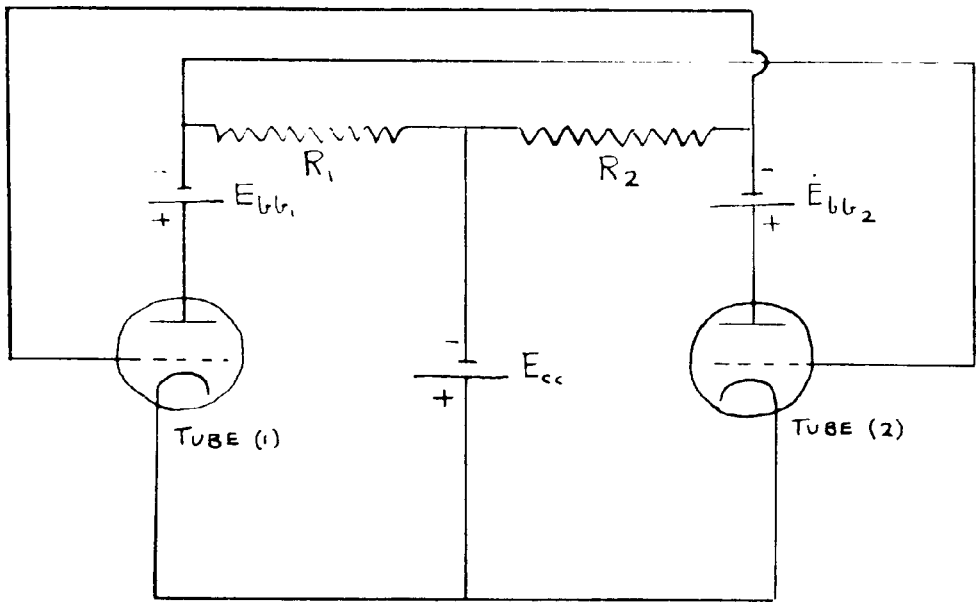


FIGURE 1.1

SIMPLE FORM OF ECCLES-JORDAN

TRIGGER CIRCUIT

increases the plate current of tube (2). Hence the action is cumulative, and it continues until tube (2) carries a large plate current and tube (1) carries a small plate current. The circuit is usually triggered from one state to the other by changing one or more voltages by the application of a pulse or other external influence.

Much work[2-11] has been done on the design problems in the application of trigger circuits, especially in connection with pulse generators, switching circuits, etc. Considerable work[12-18,29,30] has also been done toward determining the period of oscillation of these circuits and the wave shapes. However, there seems to have been a lack of attention paid to the switching time interval, i.e. the time of transition from one state to the other. It has usually been said that the change of state takes place instantaneously once the circuit reaches the critical point where positive feedback takes control. But certainly some time is taken in charging the interelectrode and stray capacitances which limit the speed of transition. Due to a need for an increase in the speed of operation of digital electronic computers, a better understanding of the time of transition seems to be important.

The time of transition can be divided into three intervals:

(1) the time from the instant the pulse is impressed to the instant positive feedback takes control;

(2) the interval during which positive feedback is in control, or the regenerative interval; and

(3) the time from the instant positive feedback loses control to the instant the circuit settles down to its new equilibrium state.

The first and third intervals determine the maximum repetition rate for applied pulses, while the second interval determines the maximum switching speed.

1.1 Statement of the Problem

In this dissertation, a study is made of the whole transition using a method that is somewhat different from that of previous workers. Assuming straight-line tube characteristics, it is shown that the whole circuit can be replaced by an equivalent circuit consisting of a parallel combination of a resistance, a capacitance, and a third element whose dimension is the product of a resistance and the square of a capacitance. This third element has the property that the current flowing through it is equal to the product of its (resistance)x(capacitance)^2 and the second derivative with respect to time of the voltage across it. The numerical values and signs of these three elements are different for the three intervals of the transition. Since the effect of the third element is negligible for typical circuit parameters, it is neglected as a first approximation. A study has been made of the triggering action of the simplified circuit, consisting of only a resistance and a capacitance in parallel, when a voltage pulse is applied to the circuit. This study leads to the determination of (1) the time of transition, (2) minimum magnitude and minimum duration of the pulse necessary for triggering, and (3) optimum magnitude and optimum duration of the pulse for least time of transition. The method is extended to the case when the third element is not negligible.

It is believed that the study gives a qualitative insight into the triggering action of the circuit and the transition between states, and is capable of yielding useful quantitative results if used judiciously.

1.2 Historical Background

The bistable vacuum tube circuit known as the Eccles-Jordan trigger circuit was first described by Eccles and Jordan[1] in 1919. Several

workers[2-11] have designed various modifications of this circuit to suit different purposes, while Reich[25] and others[4,9,10,26,27,28] have devoted considerable portions of their books to the description of trigger circuits and their applications.

A large portion[12-18,21,30] of the previous work done on trigger circuits and their modifications has been directed toward the determination of the wave shapes and the period of oscillation. Van der Pol[12,13,15] and others[17,29,30] have used the classical method of solving nonlinear differential equations involved in the circuit. In the work of Kiebert and Inglis[16] and the "Principles of Radar"[17] by M.I.T. Radar School, on the other hand, calculation of the waveforms and the period of oscillation is simplified by representing vacuum tubes as resistors in series with switches.

Some previous efforts have been made to study the time of transition. Sayre[19] has made an analysis of the regenerative interval of the transition between states of a conventional multivibrator (with capacitive coupling between the plate of one tube and the grid of the other and vice versa), taking into account two lumped capacitances C_1' and C_2' between the cathode and the grids of the two tubes, and assuming the mutual characteristics of the tubes as straight lines. His calculations lead to expressions for the electrode voltages as power series of time t. His analysis provides a figure of merit $g_m^2/C_1' C_2'$ in determining the switching speeds.

Williams, Aldrich, and Woodford[20] also have analyzed the regenerative interval for a conventional multivibrator, once more assuming the mutual characteristics of the tubes as straight lines. Their method consists in expressing mathematically the network equations and then solving differential equations for the grid voltages of the two tubes as functions of time.

They draw several conclusions, for instance that "the shape of the voltage in the transition interval should be independent of the triggering voltage". They also conclude, "There seems to be very little correlation between transition time and the time constants of the load resistances (in parallel with dynamic plate resistance) in combination with shunt capacity, although this has been suggested as a major factor by Kiebert and Inglis[16] and others." Like Sayre, they also give g_m/C as a figure of merit for the switching speed.

An analysis of the regenerative interval of the Eccles-Jordan circuit in which triggering is accomplished by introducing a voltage pulse in the lead to one grid, has been made by Tillman[21]. His method is similar to that of Williams, Aldrich, and Woodford, and consists in writing out the nodal equations of the circuit and getting a differential equation for the change in voltage with respect to time of either one anode or the other. This equation has been solved by considering the mutual characteristics of the tube as (1) linear and (2) parabolic. By studying what happens when the triggering voltage is removed after a short time, he shows that there is a minimum duration of the pulse for transition to take place, but there seems to be no mention of the fact that there is also a minimum magnitude of the pulse necessary for transition. Contrary to Williams et al., he states that the time constants of the load resistances in combination with shunt capacity figure prominently in the expressions relating the electrode potentials with time.

Ahmed[22,23] has made an analysis of the regenerative interval of the multivibrator as well as of the Eccles-Jordan circuit on similar lines, except that he has solved his differential equations by using Laplace transforms. He also gives g_m/C as a figure of merit for the switching speed, and

shows that there is a minimum duration of the pulse necessary for triggering.

Both Tillman and Williams et al. have suggested a rather tedious step-by-step analysis to give more accurate results.

All the authors mentioned above have only stated that for the study of the other two intervals of the transition time during which there is no positive feedback, some other more simple direct methods may be devised.

CHAPTER 2

BASIC PRINCIPLE OF TRIGGERING ACTION WHEN A RECTANGULAR VOLTAGE PULSE IS INTRODUCED BETWEEN THE PLATES OF THE TWO TUBES

2.1 Current-Voltage Characteristic Between the Two Plates, and Stable and Unstable Equilibrium Values of Voltage and Current

A simple form of the circuit is shown in Figure 2.1. The action of the circuit can be understood in an elementary way by studying its d-c current-voltage characteristic, which gives the relationship between the external current i that flows as the result of applying a direct voltage e between the two plates (X and Y). Such a characteristic is shown in Figure 2.2(a). The existence of this characteristic has been mentioned by Reich[24,25]. An analytical derivation of this characteristic, assuming straight-line tube characteristics (see Figure A.2(b)) is given in Appendix A, and is found to check reasonably well with the curve obtained experimentally. Figure 2.2(b) shows the analytically derived characteristic curve It is a simple matter to derive this characteristic point by point by using the actual tube characteristics.

Now consider the whole circuit as a resistance r_B, where r_B is given by the reciprocal of the slope of the current-voltage characteristic as given by Figure 2.2(b) and consequently has different values in the different regions of the characteristic. As a first approximation, all interelectrode and stray capacitances can be combined into a capacitance C in parallel with r_B as shown in Figure 2 3 (this is justified in Appendix B; Chapter 4 gives a study of the triggering action without making this approximation). It is shown below that when a voltage source e_t is connected from X to Y in series with a resistance r, there are three possible

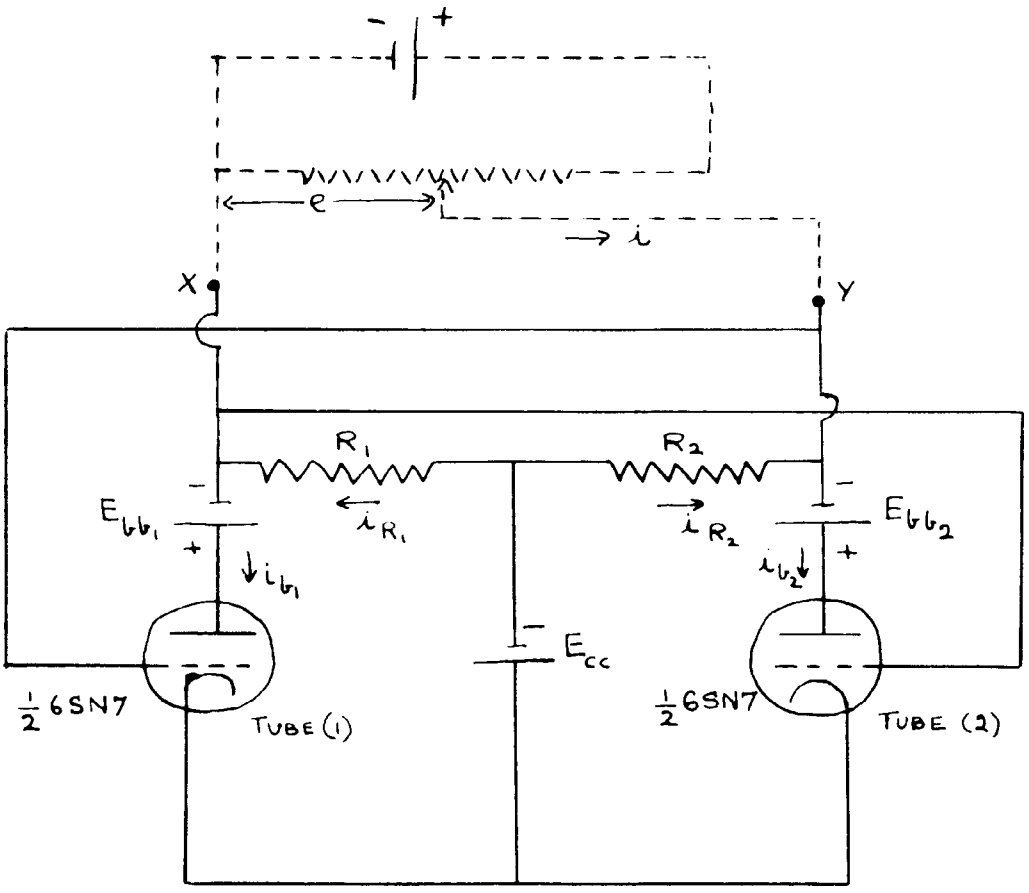
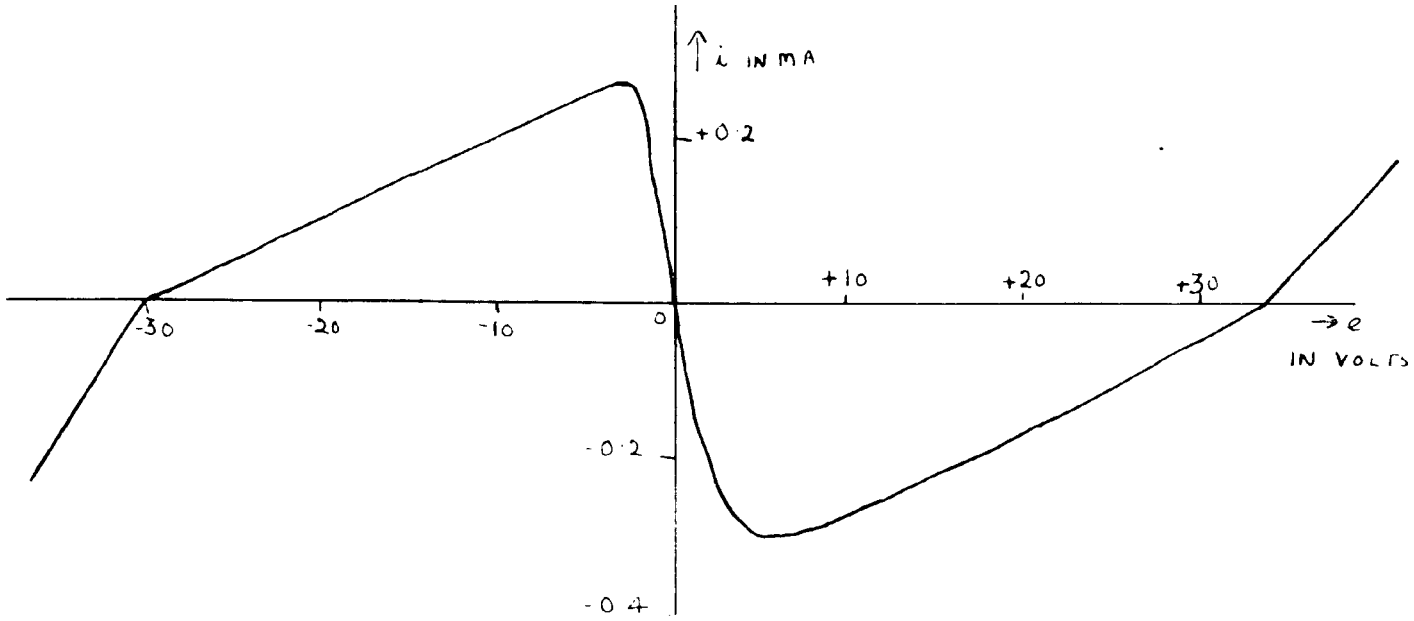
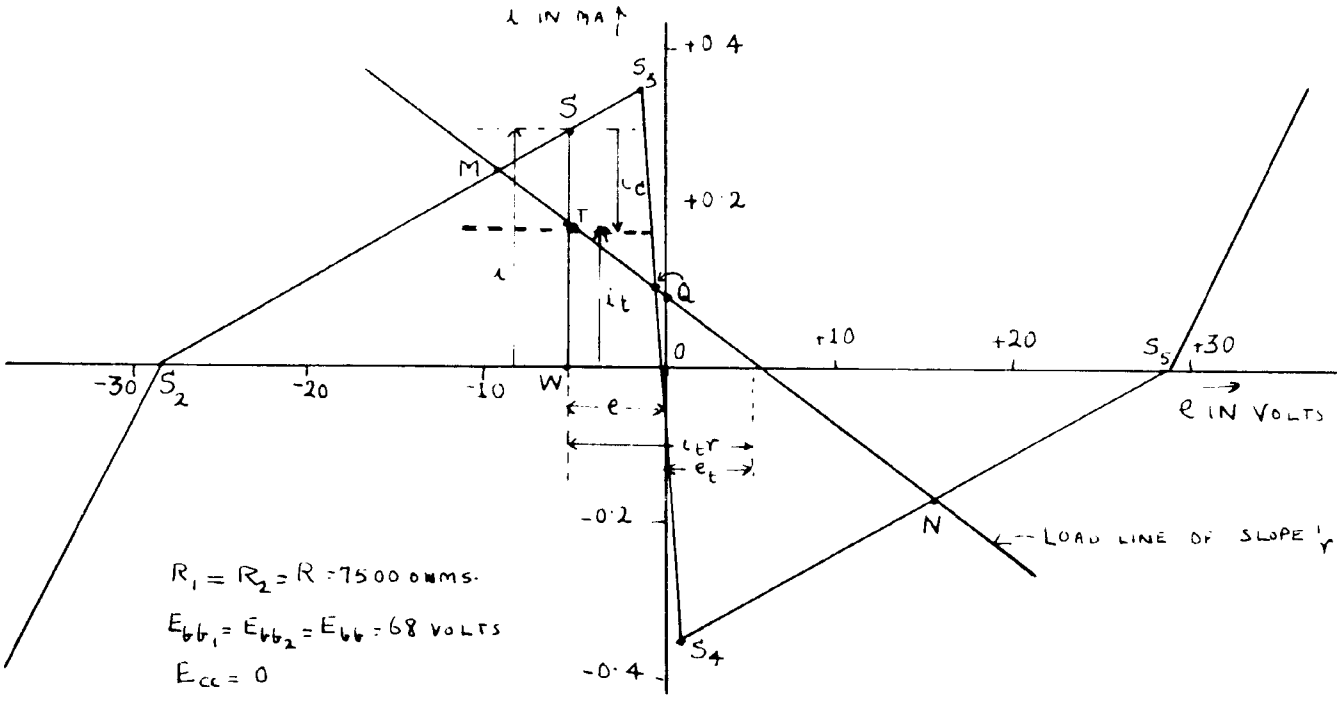


FIGURE 2.1

ECCLES–JORDAN TRIGGER CIRCUIT WITH A DIRECT VOLTAGE e BEING APPLIED BETWEEN THE TWO PLATES



(a)



(b)

FIGURE 2·2

VOLTAGE-CURRENT CHARACTERISTIC BETWEEN THE TWO PLATES

(a) Experimental (b) Analytically derived, assuming straight line tube characteristics. S_2S_3, S_3S_4, S_4S_5 represent the regions (2_2), (1), and (2_1) respectively.

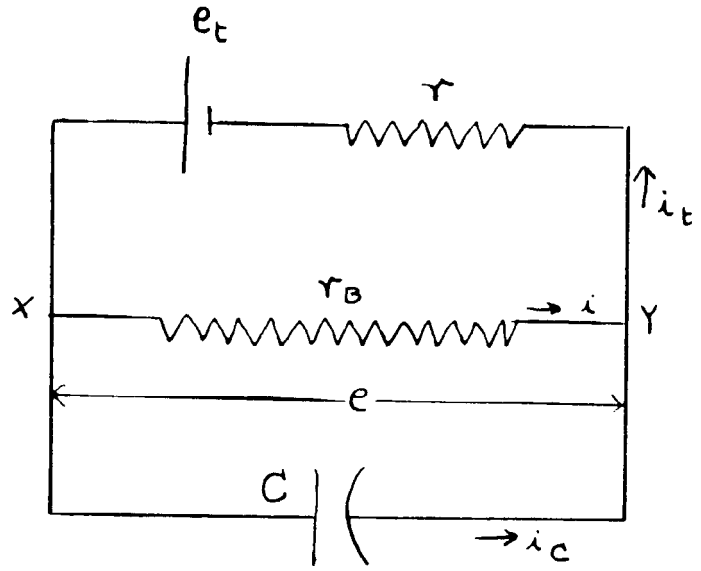


FIGURE 2.3

APPROXIMATE EQUIVALENT CIRCUIT BETWEEN THE TWO PLATES, CONNECTED TO A VOLTAGE SOURCE e_t IN SERIES WITH A RESISTANCE r.

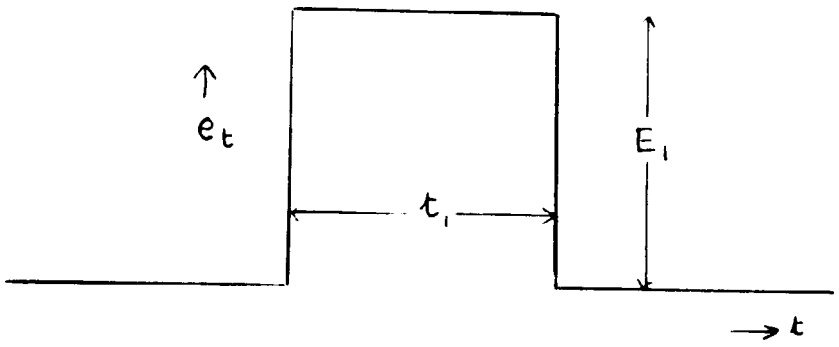


FIGURE 2.4

TRIGGERING PULSE BETWEEN THE TWO PLATES

equilibrium pairs of values of voltage and current. if r is greater than the reciprocal of the slope of the negative resistance part of the i-e characteristic. Of these three pairs, two are stable and one unstable.

In Figure 2.3,

$$e_t = i_t r + e \qquad (2.1a)$$

$$= i_t r + i r_B \qquad (2.1b)$$

$$i_t = i + i_C \qquad (2\ 2a)$$

$$= i + C \frac{de}{dt}. \qquad (2.2b)$$

Equations (2.1a) and (2.1b) describe the load line MQN of slope 1/r. This load line intersects the voltage-current characteristic at three points. M. Q, and N. M corresponds to tube (2) carrying current, N corresponds to tube (1) carrying current, and Q corresponds to both the tubes carrying current. The basic principles introduced in this thesis are illustrated by the following proof of the fact that M and N represent states of stable equilibrium and that Q represents a state of unstable equilibrium.

Suppose that there is some disturbance such that the condition of the circuit is represented by point S on the Figure 2.2(b) characteristic curve. Then, in this figure.

$$\overrightarrow{WS} = i \qquad (2\ 3)$$

$$\overrightarrow{WT} = i_t \ . \qquad (2.4)$$

From equation (2.2b),

$$C \frac{de}{dt} = i_t - i = \overrightarrow{WT} - \overrightarrow{WS} = \overrightarrow{ST} \ . \qquad (2.5)$$

$\overrightarrow{ST}$ is numerically negative Therefore de/dt is negative, and so e decreases until the condition of the circuit is represented by point M. There is no chance at all of e becoming e_Q. Similarly, if S lies between

Q and N, then e eventually becomes e_N and not e_Q. This shows that M and N are points of stable equilibrium. while Q is a point of unstable equilibrium.

Applying the method of Barkhausen[31] and others[32,33], this can also be proved analytically as follows:

On eliminating i_t and i_C between the two equations (2 1b) and (2.2b), the following differential equation for i can be obtained:

$$C\, r_B \frac{di}{dt} = -\left(1 + \frac{r_B}{r}\right) i + \frac{e_t}{R} \quad . \tag{2 6}$$

Let there be some disturbance such that i becomes $\bar{i} + \eta$, where $\bar{i}$ is the equilibrium value. Then,

$$C\, r_B \frac{d}{dt}(\bar{i} + \eta) = -\left(1 + \frac{r_B}{r}\right)(\bar{i} + \eta) + \frac{e_t}{r} \quad . \tag{2.7}$$

Because $\bar{i}$ is an equilibrium value,

$$-\bar{i}\left(1 + \frac{r_B}{r}\right) + \frac{e_t}{r} = 0 \tag{2.8}$$

and

$$\frac{d\bar{i}}{dt} = 0 \quad . \tag{2.9}$$

Therefore,

$$C\, r_B \frac{d\eta}{dt} = -\left(1 + \frac{r_B}{r}\right)\eta \quad . \tag{2.10}$$

A solution of the above differential equation for η is

$$\eta = K \epsilon^{-(1/C)(1/r_B + 1/r)t} \quad . \tag{2.11}$$

The circuit is stable if η decreases with increasing t, and unstable otherwise. η increases with t if $(1/C\ (1/r_B + 1/r)$ is negative. Therefore,

(a) If r_B is negative and r_B is greater than r, the circuit is stable.

(b) If r_B is negative and r_B is less than r, the circuit is unstable.

(c) If r_B is positive, the circuit is stable whether r_B is greater or less than r.

Thus it is seen that M and N are points of stable equilibrium and Q is unstable.

2.2 Description of the Triggering Action of the Circuit by a Rectangular Pulse

Let the voltage source e_t (Figure 2.3) give a rectangular pulse of magnitude E_1 and duration t_1 as shown in Figure 2.4. The triggering action of this pulse will now be described.

Before t = 0 when there is no pulse, $e_t = 0$, and the load line passes through the origin O as shown in Figure 2.5 (here Q coincides with O). For instance, let tube (2) be carrying current. Then M will represent the equilibrium values of current i and voltage e. If the circuit is to trigger, a voltage pulse is to be introduced so that N will become the new equilibrium point, and tube (1) will be carrying current. Therefore E_1 has to be positive in polarity and t_1 has to be greater than a certain value, as will be shown below.

Now e_t changes in zero time from zero to E_1. As a result, the load line moves from its zero position MN to its new position M'N' as shown in Figure 2.5.

Various possible sequences can occur:

(a) If E_1 and t_1 are large enough so that M'N' is to the right of point S_3 of the characteristic curve, there will be only one pair of equilibrium values of current and voltage given by point N'. Since the voltage across capacitance C in Figure 2.3 cannot change instantly, e cannot change instantly.

Therefore M moves up to point M' in zero time. Let t_1 be very large. C begins to discharge, and e moves on the characteristic curve until it finally comes very close to the equilibrium value N'. Now the circuit has triggered, and tube (1) is carrying current. The time it takes to get close to N depends on C, r, and the characteristic curve (this is analytically determined in the next section, 2.3).

(b) Let t_1 be just large enough so that, at time t_1, e is greater than $e_Q = 0$. Since at time t_1, e_t becomes zero, the load line comes back to its original position MN. As e is now greater than zero, it lies between e_Q and e_N, and the circuit will eventually come to the equilibrium point N. Therefore the circuit still triggers.

(c) Let t_1 be so small that at time t_1, e is still less than e_Q. Then when the load line comes back to its original position MN at time t_1, the circuit starts back toward its original equilibrium position M. Thus triggering does not take place.

(d) Let E_1 be so small that the load line M'N' is to the left of S_3. Then, however large t_1 is, e is always less than e_{S_3} and so lies between e_Q and e_M. Therefore e will eventually come back to e_M, and triggering does not take place.

(e) If E_1 is just large enough so that the line M'N' passes through S_3, as shown in the next section 2.3, it will take an infinite time for e to reach e_{S_3}. When at time t_1 the load line comes back to MN, e will still be less than e_{S_3}. Thus eventually the circuit will come back to M, and triggering does not take place.

(f) If E_1 is large enough so that M'N' is to the right of S_3, but t_1 is just large enough so that at time t_1 voltage e has reached e_Q, it will be shown in the next section that it takes an infinite time for e to leave

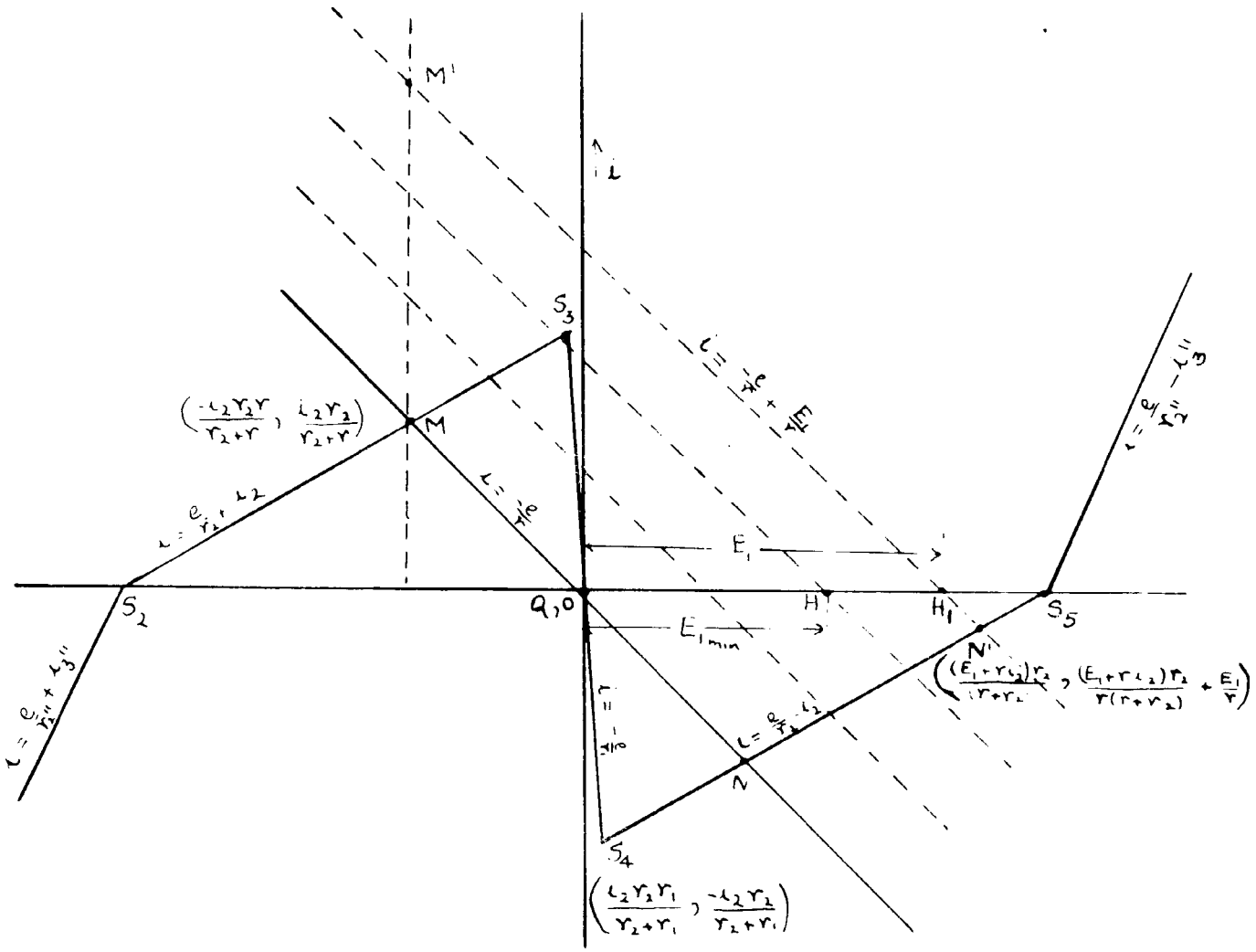


FIGURE 2.5

VOLTAGE-CURRENT CHARACTERISTIC BETWEEN THE TWO PLATES, WITH EQUATIONS WRITTEN FOR THE DIFFERENT STRAIGHT-LINE COMPONENTS.

r_1, r_2, r_3'', r, i_2, i_3'' are all positive numbers

$e_Q = 0$. This is an interesting case theoretically. However, there are always voltages due to noise in the circuit, and these noise voltages will cause e to start toward e_M or e_N, depending on the direction in which they act. So here triggering may or may not take place.

Hence, for triggering to take place, E_1 has to be greater than $\overrightarrow{OH}$ and t_1 has to be greater than the time it takes e to reach $e_Q = 0$.

The rest of this chapter will be devoted to an analytical treatment of the situation, including a determination of the time of transition, minimum magnitude and minimum duration of the pulse necessary for triggering, and optimum magnitude and duration for least time of transition.

2.3 Analytical Treatment of the Triggering Action

Figure 2.5 shows the characteristic curve as derived in Appendix A, with equations written for the straight-line portions in the different regions. Since the circuit is assumed to be symmetrical, the curve is antisymmetrical about the axes of coordinates. The point Q of intersection of the zero position MN of the load line with the negative resistance part S_3S_4 of the characteristic curve coincides with O, the origin of coordinates.

The rectangular voltage pulse shown in Figure 2.4 is applied in series with resistance $r > r_1$ (r_1 is positive), to the parallel combination of r_B and C (see Figure 2.3).

$$e_t = 0 \qquad \text{for} \qquad t \leq 0 \tag{2.12}$$

$$= E_1 \qquad \text{for} \qquad t = 0 \text{ to } t = t_1 \tag{2.13}$$

$$= 0 \qquad \text{for} \qquad t \geq t_1 \ . \tag{2.14}$$

To study the triggering action of this pulse, only regions (2_2), (1), and (2_1) need be considered.

Equations (2 la) and (2.2b) are:

and
$$e_t = i_t r + e \qquad (2\ la)$$

$$i_t = i + C\frac{de}{dt} \ . \qquad (2.2b)$$

By eliminating i_t between the above two equations,

$$C\frac{de}{dt} = \frac{e_t - e}{r} - i \ . \qquad (2.15)$$

In region (2_2), $i = e/r_2 + i_2$. so that

$$C\frac{de}{dt} = \frac{e_t - e}{r} - \left(\frac{e}{r_2} + i_2\right)$$

$$= -e\left(\frac{1}{r} + \frac{1}{r_2}\right) + \frac{e_t}{r} - i_2 \ . \qquad (2.16)$$

In region (1), $i = -e/r_1$. so that

$$C\frac{de}{dt} = \frac{e_t}{r} - \frac{e}{r} + \left(\frac{e}{r_1}\right) = -e\left(\frac{1}{r} - \frac{1}{r_1}\right) + \frac{e_t}{r} \ . \qquad (2.17)$$

In region (2_1), $i = e/r_2 - i_2$, so that

$$C\frac{de}{dt} = \frac{e_t}{r} - \frac{e}{r} - \left(\frac{e}{r_2} - i_2\right)$$

$$= -e\left(\frac{1}{r} + \frac{1}{r_2}\right) + \frac{e_t}{r} + i_2 \ . \qquad (2.1\square)$$

<u>From $t = 0$ to $t = t_1$</u>. $e_t = E_1$, and so equations (2.16), (2 17), and (2.1.) become

$$C\frac{de}{dt} + e\left(\frac{1}{r} + \frac{1}{r_2}\right) = \frac{E_1}{r} - i_2 \qquad (2.19)$$

$$C\frac{de}{dt} + e\left(\frac{1}{r} - \frac{1}{r_1}\right) = \frac{E_1}{r} \qquad (2.20)$$

$$C\frac{de}{dt} + e\left(\frac{1}{r} + \frac{1}{r_2}\right) = \frac{E_1}{r} + i_2 \ . \qquad (2.21)$$

<u>For $t > t_1$</u>. they become

$$C\frac{de}{dt} + e\left(\frac{1}{r} + \frac{1}{r_2}\right) = -i_2 \qquad (2.22)$$

$$C\frac{de}{dt} + e\left(\frac{1}{r} - \frac{1}{r_1}\right) = 0 \qquad (2.23)$$

$$C\frac{de}{dt} + e\left(\frac{1}{r} + \frac{1}{r_2}\right) = i_2 \ . \qquad (2.24)$$

Solutions of the above six differential equations (2.19), (2.20), (2.21), (2.22), (2.23), and (2.24) are respectively:

For $0 < t < t_1$:

$$e = K_{22}' \epsilon^{-(1/C)(1/r + 1/r_2)t} + \frac{(E_1 - ri_2)r_2}{r + r_2} \qquad (2.25)$$

$$e = K_1' \epsilon^{-(1/C)(1/r - 1/r_1)t} + \frac{E_1 r_1}{r_1 - r} \qquad (2.26)$$

$[(1/r - 1/r_1)$ is negative, since $r > r_1]$

$$e = K_{21}' \epsilon^{-(1/C)(1/r + 1/r_2)t} + \frac{(E_1 + ri_2)r_2}{r + r_2} \qquad (2.27)$$

For $t > t_1$:

$$e = K_{22}'' \epsilon^{-(1/C)(1/r + 1/r_2)t} - \frac{i_2 r_2 r}{r + r_2} \qquad (2.28)$$

$$e = K_1'' \epsilon^{-(1/C)(1/r - 1/r_1)t} \qquad (2.29)$$

$$e = K_{21}'' \epsilon^{-(1/C)(1/r + 1/r_2)t} + \frac{i_2 r_2 r}{r + r_2} \ , \qquad (2.30)$$

where ϵ = the natural logarithmic base. The constants K_{22}', K_1', K_{21}', K_{22}'', K_1'' and K_{21}'' of integration can be determined by the boundary conditions.

Let each of the six possible sequences mentioned in the last section now be considered.

(a) Let $E_1 > i_2 r_2 (r - r_1)/(r_2 + r_1)$, so that the load line for E_1 is to the right of S_3, and let t_1 be so large that at $t = t_1$, $e \simeq e_N$. (see Figure 2.6a). When $t = 0$,

$$e = e_M = \frac{-i_2 r_2 r}{r_2 + r} \quad . \tag{2.31}$$

Therefore in equation (2.25),

$$e_M = \frac{-i_2 r_2 r}{r_2 + r} = K_{22}' + \frac{(E_1 - r i_2) r_2}{r_2 + r} \quad . \tag{2.32}$$

Thus

$$K_{22}' = \frac{-i_2 r_2 r}{r_2 + r} - \frac{(E_1 - r i_2) r_2}{r_2 + r} = \frac{-E_1 r_2}{r_2 + r} \quad . \tag{2.33}$$

Therefore equation (2.25) can be written

$$e = \frac{-E_1 r_2}{r_2 + r} \epsilon^{-(1/C)(1/r + 1/r_2)t} + \frac{(E_1 - r i_2) r_2}{r_2 + r} \quad . \tag{2.34}$$

To find time $t = t_{S3}$ when $e = e_{S3}$, substitute $e = e_{S3} = -i_2 r_2 r/(r_2 + r_1)$ in equation (2.25).

$$\frac{-i_2 r_2 r_1}{r_2 + r_1} = \frac{-E_1 r_2}{r_2 + r} \epsilon^{-(1/C)(1/r + 1/r_2)t_{S3}} + \frac{(E_1 - r i_2) r_2}{r + r_2} \quad . \tag{2.35}$$

Therefore

$$t_{S3} = \frac{C r r_2}{r_2 + r} \log_\epsilon \left\{ \frac{(r_2 + r_1) E_1}{(r_2 + r_1) E_1 - i_2 (2 r_1 r + r_2 r + r_2 r_1)} \right\} \quad . \tag{2.36}$$

At $t = t_{S3}$, equation (2.26) replaces equation (2.25). By substituting $e = e_{S3}$ at $t = t_{S3}$ in equation (2.26), K_1' can be evaluated.;

$$K_1' = \left\{ \frac{-i_2 r_2 r_1 (r_1 - r) - E_1 r_1 (r_2 + r_1)}{(r_2 + r_1)(r_1 - r)} \right\} \left\{ \frac{(r_2 + r_1) E_1 - i_2 (2 r_1 r + r_2 r + r_1 r_2)}{(r_2 + r_1) E_1} \right\}^{-\frac{(r_1 - r) r_2}{(r_2 + r) r_1}} \tag{2.37}$$

Equation (2.26) applies until $e = e_{S4} = i_2 r_2 r_1/(r_2 + r_1)$. Let $t = t_{S4}$ when when $e = e_{S4}$. Time t_{S4} is given by

$$t_{S4} = \frac{r r_1 C}{r - r_1} \log_\epsilon \left\{ \left[\frac{i_2 r_2 r_1}{r_2 + r_1} - \frac{E_1 r_1}{r_1 - r} \right] \frac{1}{K_1'} \right\} \quad . \tag{2.38}$$

At $t = t_{S4}$, equation (2.27) replaces equation (2.26). Therefore by putting $e = e_{S4} = i_2r_2r_1/(r_2+r_1)$ when $t = t_{S4}$, and by substituting for t_{S4} from equation (2.38),

$$K_{21}' = \left\{\frac{i_2r_2r_1(r+r_2)-(E_1+ri_2)(r_2+r_1)r_2}{(r_2+r_1)(r+r_2)}\right\}\left\{\frac{i_2r_2r_1(r_1-r)+E_1r_1(r_2+r_1)}{K_1'(r_2+r_1)(r_1-r)}\right\}^{\alpha} \quad (2.39)$$

where

$$\alpha = \frac{-(r+r_2)Crr_1}{Crr_2(r_1-r)} = \frac{-(r+r_2)r_1}{(r_1-r)r_2} \quad . \quad (2.40)$$

Equation (2.26) then becomes

$$e = \left\{\frac{i_2r_2r_1(r+r_2)-(E_1+ri_2)(r_2+r_1)r_2}{(r_2+r_1)(r+r_2)}\right\}\left\{\frac{i_2r_2r_1(r_1-r)-E_1r_1(r_2+r_1)}{K_1'(r_2+r_1)(r_1-r)}\right\}^{\alpha} \epsilon^{-(r+r_2)t/Crr_2} + \frac{(E_1+ri_2)r_2}{r+r_2} \quad . \quad (2.41)$$

As $t \rightarrow \infty$, $e \rightarrow (E_1+ri_2)r_2/(r+r_2)$ $(= e_N')$. (2.42)

If t_1 is long enough, it can be assumed that $e = e_N'$ when $t = t_1$.

At $t = t_1$, $e_t = 0$, and so for $t > t_1$, equation (2.30) applies:

$$e = K_{21}'' \epsilon^{-(r+r_2)t/Crr_2} + \frac{i_2rr_2}{r+r_2} \quad . \quad (2.30)$$

K_{21}'' can be evaluated by putting $e = e_N'$ when $t = t_1$ in equation (2.30).

As $t \rightarrow \infty$, $e \rightarrow i_2rr_2/(r+r_2)$ $(= e_N)$. (2.43)

If it is important for e to come close to e_N as soon as possible, the case just discussed, where t_1 is longer than necessary to insure triggering, is not particularly useful.

Fig. 2.6(a) shows both e and e_t as functions of t for this case, and gives a clear picture of the triggering action.

(b) Let $E_1 > i_2r_2(r-r_1)/(r_2+r_1)$, and t_1 be just large enough to make e greater than e_Q $(= 0)$ at time t_1 (see Fig. 2.6b). When $t = t_1$, let $e =$

e_1, where $e_1 > e_Q$ and so is positive. Then in equation (2.26),

$$e_1 = K_1' \epsilon^{-(r_1-r)t_1/Crr_1} + \frac{E_1 r_1}{r_1-r} \quad . \qquad (2.44)$$

For $t > t_1$, equation (2.29) applies. By putting $e = e_1$ when $t = t_1$ in equation (2.2?), K_1'' can be evaluated.

$$e_1 = K_1'' \epsilon^{-(r_1-r)t_1/Crr_1} \quad . \qquad (2.45)$$

Therefore,

$$K_1'' = e_1 \epsilon^{+(r_1-r)t_1/Crr_1} > 0 \qquad (\text{since } e_1 > 0) \quad . \quad (2.46)$$

Thus,

$$e = e_1 \epsilon^{+(r_1-r)t_1/Crr_1} \epsilon^{-(r_1-r)t/Crr_1} \quad , \qquad (2.47)$$

e_1 is positive and with $-(r_1-r)$ positive, e increases and becomes more and more positive. Therefore e goes to e_{S_4} and cannot go back to e_{S_3}. Let $e = e_{S_4} = i_2 r_2 r_1/(r_2+r_1)$ when $t = t_{S_4}$. By substituting these values of e and t in equation (2.29) and solving for t_{S_4},

$$t_{S_4} = t_1 + \frac{Crr_1}{r-r_1} \log_\epsilon \left\{ \frac{i_2 r_2 r_1}{e_1(r_2+r_1)} \right\} \quad . \qquad (2.48)$$

For $t > t_{S_4}$, equation (2.30) applies; i.e.,

$$e = K_{21}'' \epsilon^{-(r+r_2)t/Crr_2} + \frac{i_2 r_2 r}{r_2 + r} \quad . \qquad (2.30)$$

By putting $e = e_{S_4} = i_2 r_2 r_1/(r_2+r_1)$ when $t = t_{S_4}$, K_{21}'' can be evaluated.

$$K_{21}'' = \frac{i_2 r_2^2 (r_1-r)}{(r_2+r_1)(r_1+r)} \left\{ \frac{i_2 r_2 r_1}{e_1(r_2+r_1)} \right\}^{-r_1(r+r_2)/(r_1-r)r_2} \epsilon^{(r+r_2)/Crr_2} \quad . \qquad (2.49)$$

For all practical purposes, the time of transition t_r can be defined as the time from the instant the pulse is impressed to the instant when e has reached 0.98 e_N or 1.02 e_N, as the case may be.

In this case, if t_2 is the time from the instant t_1 to the time e has reached 0.98 e_N,

$$t_2 = (t_{S_4} - t_1) + (t_{0.98e_N} - t_{S_4}) \qquad (2.50a)$$

$$= \frac{Crr_1}{r-r_1} \log_\epsilon\left[\frac{e_{S_4}}{e_1}\right] + \frac{Crr_2}{r+r_2} \log_\epsilon\left[\frac{e_{S_4} - \frac{(E_1+ri_2)r_2}{r+r_2}}{0.98\ e_N - \frac{(E_1+ri_2)r_2}{r+r_2}}\right] . \qquad (2.50b)$$

[from equations (2.29) and (2.30)]. Therefore, the time of transition t_r = $t_1 + t_2$ =

$$t_1 + \frac{Crr_1}{r-r_1} \log_\epsilon\left[\frac{e_{S_4}}{e_1}\right] + \frac{Crr_2}{r+r_2} \log_\epsilon\left[\frac{e_{S_4} - \frac{(E_1+ri_2)r_2}{r+r_2}}{0.98\ e_N - \frac{(E_1+ri_2)r_2}{r+r_2}}\right] . \qquad (2.51)$$

Since e_1 depends on E_1 and t_1. the total time of transition depends on E_1 and t_1 as well as on the circuit parameters.

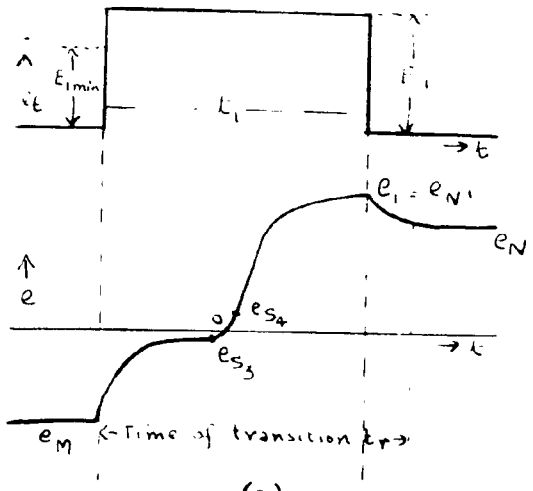
(c) Let t_1 be so short that at time t_1 the quantity e is still less than e_Q and hence e is still negative (see Fig. 2.6c). Put $e_{t_1} = -e_2$, where e_2 is positive. Then in equation (2.29), which applies for $t \quad t_1$,

$$K_1'' = -e_2\ \epsilon^{(r_1-r)t_1/Crr_1} . \qquad (2.52)$$

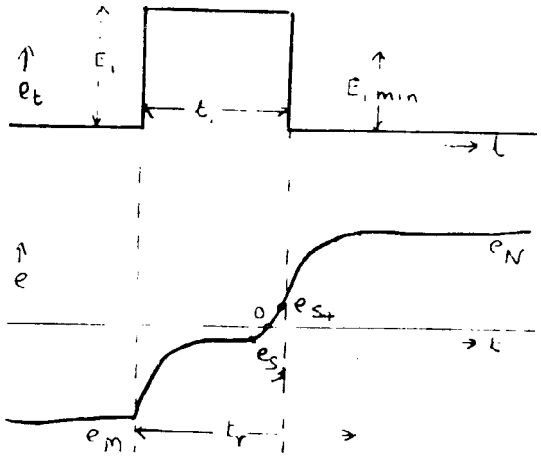
Therefore (2.29) becomes

$$e = -e_2\ \epsilon^{(r_1-r)t_1/Crr_1}\ \epsilon^{-(r_1-r)t/Crr_1} . \qquad (2.53)$$

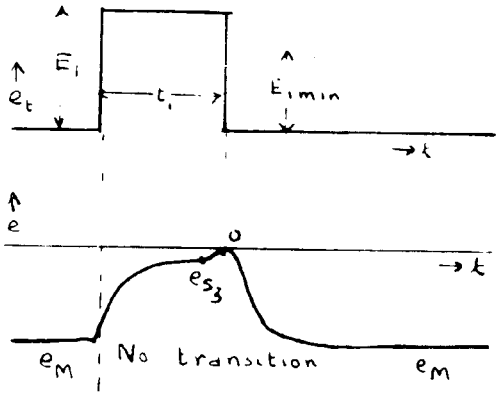
$-e_2\ \epsilon^{(r_1-r)t_1/Crr_1} < 0$ and $-(r_1-r) > 0$. Therefore e decreases and becomes more and more negative with increasing t. and e will eventually reach e_{S_3} and not e_{S_4}. For $e < e_{S_3}$, equation (2.28) applies, and e will come back to e_M finally. It is evident that no triggering action takes place.



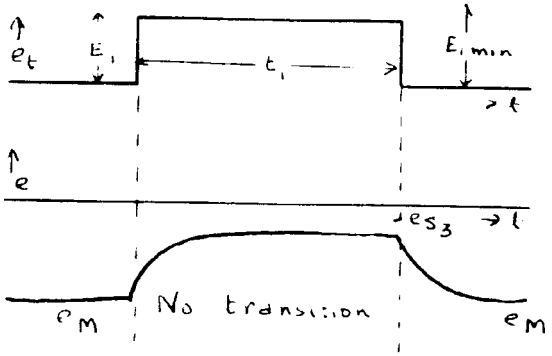
(a)



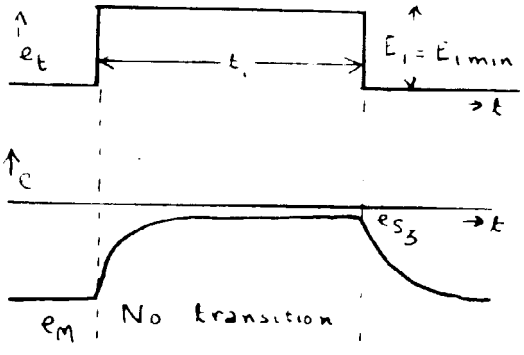
(b)



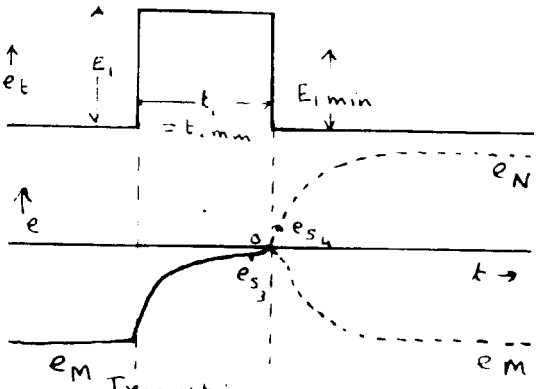
(c)



(d)



(e)



(f)

FIGURE 2.6

CURVES OF e_t AND e VERSUS t, SHOWING THE DIFFERENT POSSIBLE SEQUENCES THAT CAN OCCUR WHEN THE CIRCUIT IS TRIGGERED BY A RECTANGULAR PULSE BETWEEN THE TWO PLATES

(d) E_1 is so small that the load line M'N' is to the left of S_3, or $E_1 < i_2r_2(r-r_1)/(r_2+r_1)$ (see Fig. 2.6d). Equation (2.34) is

$$e = \frac{-E_1r_2}{r_2+r} \epsilon^{-(1/C)t(r_2+r)/r_2r} + \frac{(E_1-ri_2)r_2}{r+r_2} . \qquad (2.34)$$

Final value of $e = (E_1-ri_2)r_2/(r+r_2)$ and consequently

$$e < \left\{\frac{i_2r_2(r-r_1)}{r_2+r_1} - i_2r\right\} \frac{r_2}{r+r_2} . \qquad (2.54)$$

$$\left\{\frac{i_2r_2(r-r_1)}{r_2+r_1} - i_2r\right\} \frac{r_2}{r+r_2} = \frac{-i_2r_2r_1}{r_2+r_1} \qquad (2.55a)$$

$$= e_{S_3} . \qquad (2.55b)$$

Therefore the final value of $e < e_{S_3}$. As a result, e will never reach e_{S_3}, however large t_1 may be. Consequently when $e_t = 0$ for $t > t_1$, e starts toward e_M, and no triggering action takes place.

(e) E_1 is just large enough so that the load line M'N' passes through point S_3, or $E_1 = i_2r_2(r-r_1)/(r_2+r_1)$ (see Fig. 2.6e). Using the same arguments as in (d), it can be seen that e gets closer and closer to e_{S_3} but will never reach it. Therefore, however large t_1 may be, triggering cannot take place.

(f) If E_1 is large enough so that M'N' is to the right of S_3, but t_1 is just large enough so that at time t_1 the quantity e has reached $e_Q = 0$ (see Fig. 2.6f), or

$$E_1 > \frac{i_2r_2(r-r_1)}{r_2+r_1} , \qquad \text{and} \qquad e_{t1} = e_Q = 0 .$$

In equation (2.29), which holds good for $t > t_1$, by putting $e = 0$ when $t = t_1$,

$$0 = e_{t_1} = K_1'' \epsilon^{-(r_1-r)t/rr_1C} ; \qquad (2.57)$$

therefore

$$K_1'' = 0 \quad . \tag{2.58}$$

Thus equation (2.29) reduces to e = 0, which means that e stays at e_Q forever.

However, there are always voltages due to noise in the circuit. If these voltages make $e > e_Q$, that is, $e > 0$, triggering will occur and e will eventually go to e_N. On the other hand, if these noise voltages make $e < e_Q$, there is no triggering and e will eventually come back to e_M. Hence triggering may or may not take place, and so this condition is unreliable for practical purposes.

2.4 Minimum Magnitude and Minimum Duration of Pulse Necessary for Triggering

It is now evident that for triggering to take place the magnitude E_1 and the duration t_1 of the pulse should be as follows:

$$E_1 > \frac{i_2 r_2 (r - r_1)}{r_2 + r_1} \tag{2.59}$$

and

$$t_1 > t_Q \quad , \tag{2.60}$$

where

$$t_Q = \frac{Crr_1}{r - r_1} \log_\epsilon \left[\frac{E_1 r_1}{r - r_1} \left\{ \frac{(r_2 + r_1)(r_1 - r)}{-i_2 r_2 r_1 (r_1 - r) - E_1 r_1 (r_1 + r_2)} \right\} \cdot \left\{ \frac{(r_2 + r_1)E_1 - i_2(2r_1 r + r_2 r + r_2 r_1)}{(r_2 + r_1)E_1} \right\}^{(r_1 - r)r/(r_2 + r_1)r_1} \right] \tag{2.61}$$

2.5 Time of Transition Between States. Optimum Magnitude and Duration of Pulse for Least Time of Transition

It has already been shown that there is a minimum value of E_1, and that for a given value of $E_1 > E_{1min}$, there is a minimum value of $t_1 = t_{1min}$ which is necessary for triggering to take place. When these conditions are satisfied, the time of transition for different pairs of values of E_1 and t_1 will now be compared to obtain the optimum pair of values of E_1 and t_1 to give the least time of transition.

(a) Let e_1, which is the value of e at time t_1, lie between e_Q (= 0) and e_{S_4} (see Fig. 2.7a). According to the definition of the time of transition t_r in section 2.3,

$$t_r = (t_{S_3} - t_M) + (t_1 - t_{S_3}) + (t_{S_4} - t_1) + (t_{0.98e_N} - t_{S_4}) \quad . \quad (2.62)$$

From equation (2.25),

$$t_{S_3} - t_M = \frac{Crr_2}{r+r_2} \log_\epsilon \left[\frac{e_M - \frac{(E_1 - ri_2)r_2}{r+r_2}}{e_{S_3} - \frac{(E_1 - ri_2)r_2}{r+r_2}} \right] . \quad (2.63)$$

$(E_1 - ri_2)r_2/(r+r_2) > 0$, and so the larger E_1 is, the smaller $(t_{S_3} - t_M)$ is. From equation (2.26),

$$t_1 - t_{S_3} = \frac{Crr_1}{r-r_1} \log_\epsilon \left[\frac{e_1 + \frac{E_1 r}{r-r_1}}{e_{S_3} + \frac{E_1 r}{r-r_1}} \right] . \quad (2.64)$$

The smaller $E_1 r/(r-r_1)$ is, the smaller is $(t_1 - t_{S_3})$. Therefore, to make $(t_1 - t_{S_3})$ small, E_1 has to be made as small as possible. From equation (2.29),

$$t_{S_4} - t_1 = \frac{Crr_1}{r-r_1} \log_\epsilon \left[\frac{e_{S_4}}{e_1} \right] \quad (2.65)$$

and from equation (2.30),

$$t_{0.98e_N} - t_{S_4} = \frac{Crr_2}{r+r_2} \log_\epsilon \left[\frac{e_{S_4} - \frac{ri_2r_2}{r+r_2}}{0.98e_N - \frac{ri_2r_2}{r+r_2}} \right] \quad (2.66)$$

Therefore the time of transition is

$$\frac{Crr_2}{r+r_2} \log_\epsilon \left[\frac{e_M - \frac{(E_1-ri_2)r_2}{r+r_2}}{e_{S_3} - \frac{(E_1-ri_2)r_2}{r+r_2}} \right] + \frac{Crr_1}{r-r_1} \log_\epsilon \left[\frac{e_1 + \frac{E_1 r}{r-r_1}}{e_{S_3} + \frac{E_1 r}{r-r_1}} \right]$$

$$+ \frac{Crr_1}{r-r_1} \log_\epsilon \left[\frac{e_{S_4}}{e_1} \right] + \frac{Crr_2}{r+r_2} \log_\epsilon \left[\frac{e_{S_4} - \frac{ri_2r_2}{r+r_2}}{0.98e_N - \frac{ri_2r_2}{r+r_2}} \right] \quad (2.67)$$

For the particular circuit considered, $(e_1-e_{S_3})$ and $(e_{S_4}-e_1)$ are both very much less than $(e_{S_3}-e_M)$ and $(e_N-e_{S_4})$ (see Fig. 2.2). Therefore $(t_1-t_{S_3})$ and $(t_{S_4}-t_1)$ are both very much less than $(t_{S_3}-t_M)$ and $(t_N-t_{S_4})$. So, to make the time of transition small, it is necessary to make $(t_{S_3}-t_M)$ and $(t_N-t_{S_4})$ small. $(t_{S_3}-t_M)$ can be made small by making E_1 large. How $(t_N-t_{S_4})$ can be made sufficiently small will be considered in the next case (b).

(b) Let t_1 be so large that $e_1 = e_{t_1} = e_N$ (see Fig. 2.7b). Then the time of transition is exactly equal to t_1, because e will stay at e_N for $t > t_1$, since N is a stable equilibrium point. Thus the time of transition is less than the time of transition when $e_1 < e_{S_4}$.

(c) If t_1 is made too large so that $e_1 > e_N$ (see Fig. 2.7c), then according to the definition stated previously, the time of transition is the time it takes for e to come back to 1.02 e_N. Hence, in this case, the time of transition is again greater than t_1, and hence greater than that for the case where e_1 is exactly equal to e_N.

This discussion shows that the optimum value for t_1 is such that $e_1 = e_N$. As for E_1, the larger E_1 is, the smaller the time of transition, and it is difficult to optimize E_1. But not much will be gained by making E_1 too large, because the decrease in value of $(t_N - t_{S_4})$ and $(t_{S_3} - t_M)$ becomes less and less as E_1 increases. Therefore E_1 can be chosen to satisfy the particular purpose for which the circuit is used.

Figure 2.7 shows e and e_t as functions of t for these different cases, and it can be seen which case gives the least time of transition.

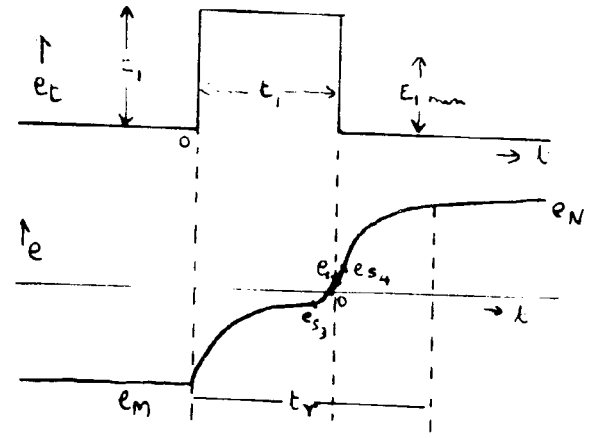
Figure 2.8 shows calculated curves of (a) t_{1min} and t_{1opt} versus E_1, (b) time of transition t_r versus E_1 for fixed t_1, and (c) t_r versus t_1 for fixed E_1.

For certain particular circuits, experimental evidence indicates that $(e_{S_4} - e_1)$ and $(e_1 - e_{S_3})$ are only slightly smaller than $(e_N - e_{S_4})$ and $(e_{S_3} - e_M)$, and may even be greater than these quantities. In that event, $(t_{S_4} - t_1)$ and $(t_1 - t_{S_3})$ are comparable in size to, or may even be greater than $(t_N - t_{S_4})$ and $(t_{S_3} - t_M)$. But it has been shown that to make $(t_N - t_{S_4})$ and $(t_1 - t_{S_3})$ small E_1 has to be made large, and to make $(t_{S_4} - t_1)$ and $(t_1 - t_{S_3})$ as small as possible E_1 has to be made small. Consequently, it is not possible to make both these pairs of times small simultaneously. Therefore, depending on the particular circuit, the value of E_1 must be chosen to make the sum of the four time intervals as small as possible.

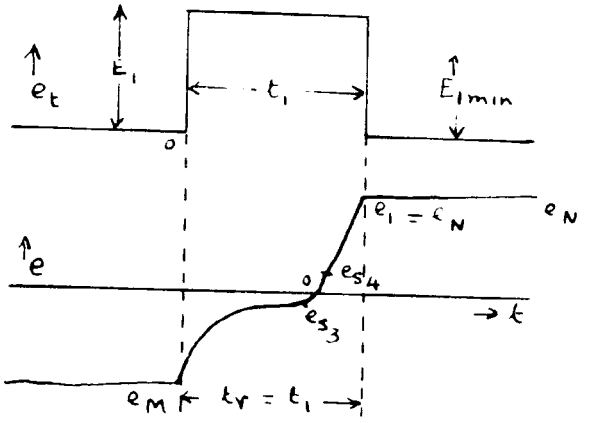
It is convenient to combine $(t_1 - t_{S_3})$ and $(t_{S_4} - t_1)$ so that the total time of transition consists of three main intervals:

(1) $(t_{S_3} - t_M)$ = the time taken to reach the point where positive feedback takes control.

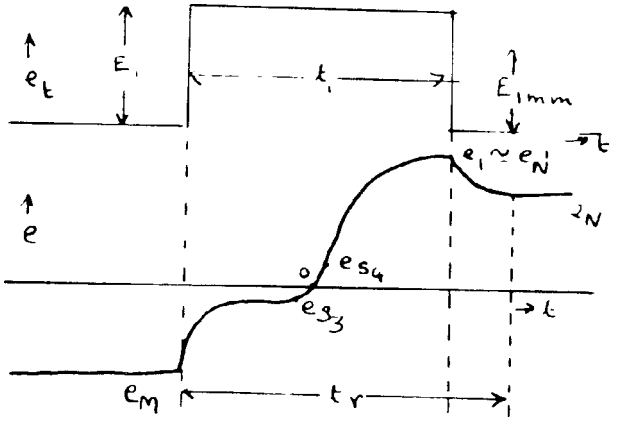
(2) $(t_{S_4} - t_{S_3})$ = the time during which positive feedback is in action, and consequently the most important factor in the



(a)
t_1 too short



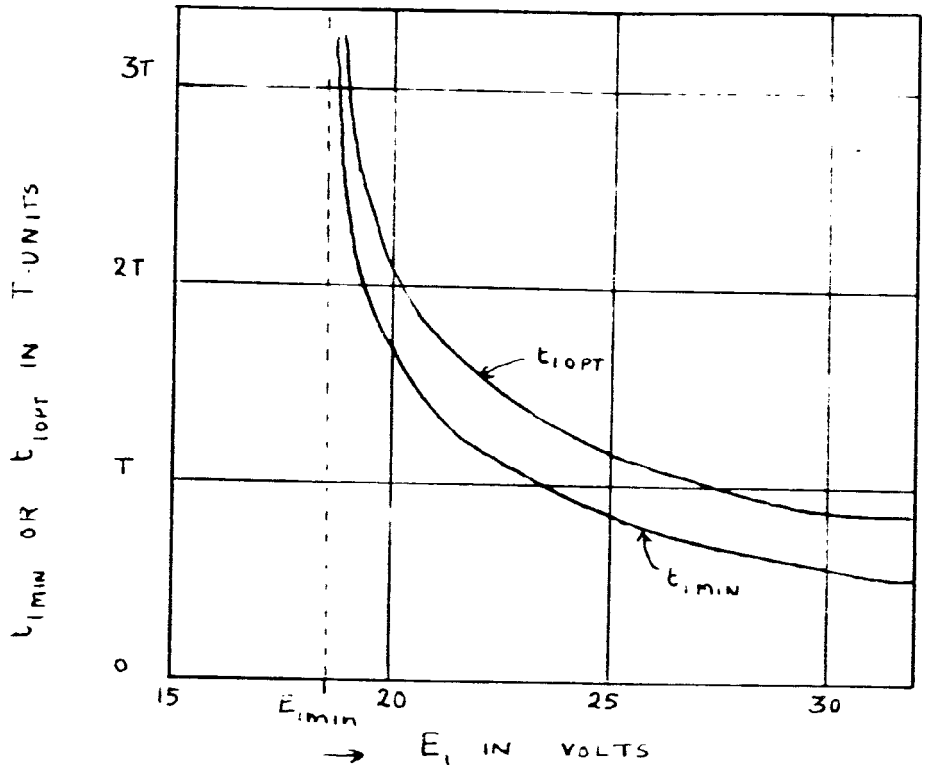
(b)
Optimum t_1



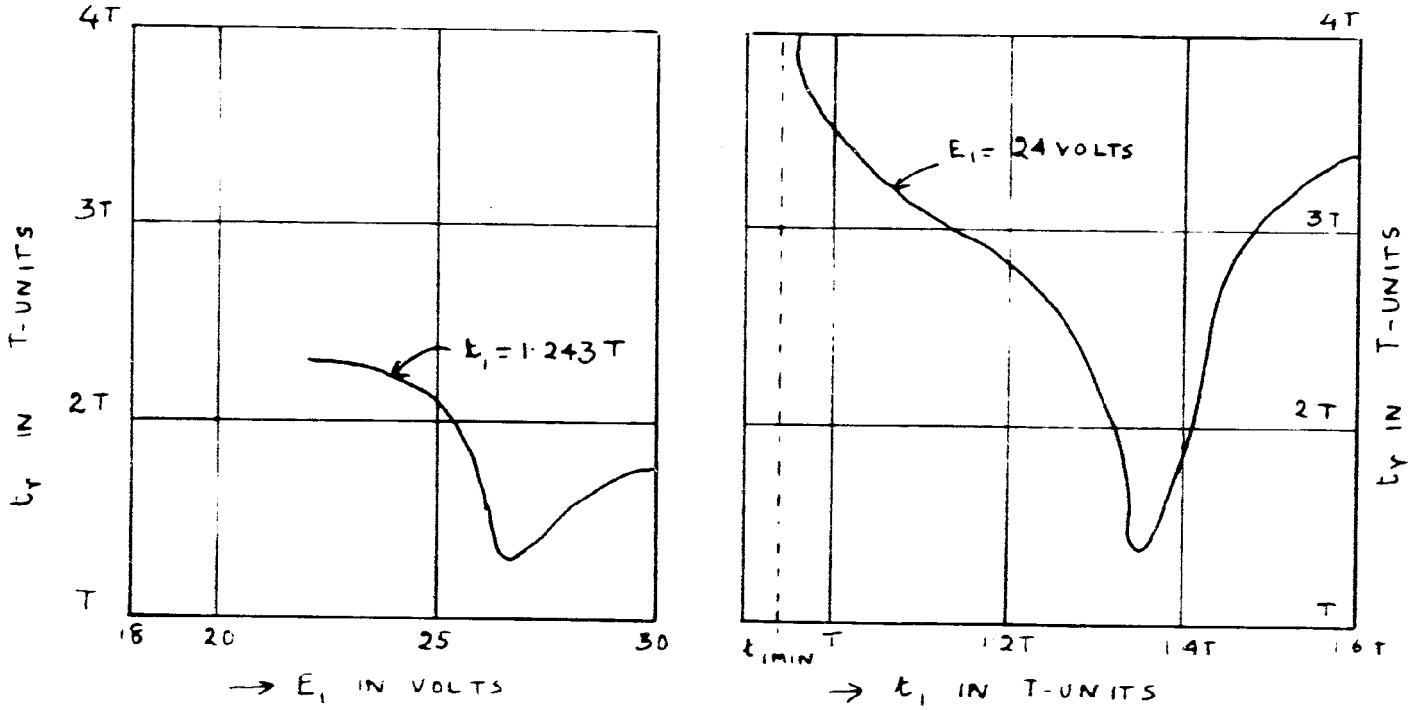
(c)
t_1 too long

FIGURE 2.7

CURVES OF e_t AND e VERSUS t TO ILLUSTRATE OPTIMUM DURATION OF PULSE



(a)



(b) (c)

FIGURE 2.8

CURVES OF (a) t_{1min} AND t_{1OPT} VERSUS E_1. (b) t_r VERSUS E_1 FOR FIXED t_1 (c) t_r VERSUS t_1 FOR FIXED E_1. $[T = Cr]$

switching action. This is the regenerative interval.

(3) $(t_N - t_{S_4})$ = the time taken to settle down to the new stable equilibrium state after positive feedback has lost control.

$(t_{S_3} - t_M)$ and $(t_N - t_{S_4})$ are the times which determine the maximum repetition rate of applied pulses that can be used. $(t_{S_4} - t_{S_3})$ is the actual switching time. The preceding discussion therefore shows that the repetition rate of applied pulses and the switching speed cannot be maximized simultaneously.

If the value of $r_1 = -2\,R/(\mu + g_m R - \mu g_m R)$ as derived in Appendix A is substituted in the expression for the regenerative interval $(t_{S_4} - t_{S_3})$, it can be seen that this time is approximately inversely proportional to g_m/C, and the values of the load resistances R_1 and R_2 ($R_1 = R_2 = R$) have very little effect on it. This is in agreement with the conclusions drawn by Sayre[19] and other previous workers[20,22]. However, this study does not agree with the conclusion of Williams, et al.[20] that "the shape of the voltage in the transition interval should be independent of the triggering voltage."

CHAPTER 3

THE TRIGGERING ACTION FOR A RECTANGULAR CURRENT PULSE INTRODUCED BETWEEN THE GRID AND CATHODE OF ONE TUBE

Chapter 2 deals with the triggering action when a rectangular voltage pulse in series with a large resistance is introduced between the two plates. This scheme is studied in detail because it provides the simplest approach for initial study. However, this is not the usual method used in practice to trigger the circuit. More usually the circuit is triggered by applying a pulse of the proper polarity between the cathode and grid (or plate) of one tube. (In the particular circuit considered, there is no difference between these two ways of triggering, since the grid of one tube is connected to the plate of the other.) In this chapter the triggering action is studied when a rectangular current pulse (a rectangular pulse from a source of theoretically infinite internal impedance) is introduced between the grid and cathode of one tube. Appendix C shows that such a rectangular current pulse introduced between G_2 and K is equivalent to a rectangular voltage pulse (a rectangular pulse from a source of zero internal impedance) introduced in series with the resistance R_1 (see Fig. C.1). By using this equivalence, the study of the triggering action when a rectangular voltage pulse is introduced in series with R_1 is now undertaken.

3.1 Description of the Triggering Action of the Circuit by a Rectangular Voltage Pulse in Series with R1

The circuit is shown in Fig. 3.1, where the instantaneous value of the voltage in series with R_1 is denoted as e_s.

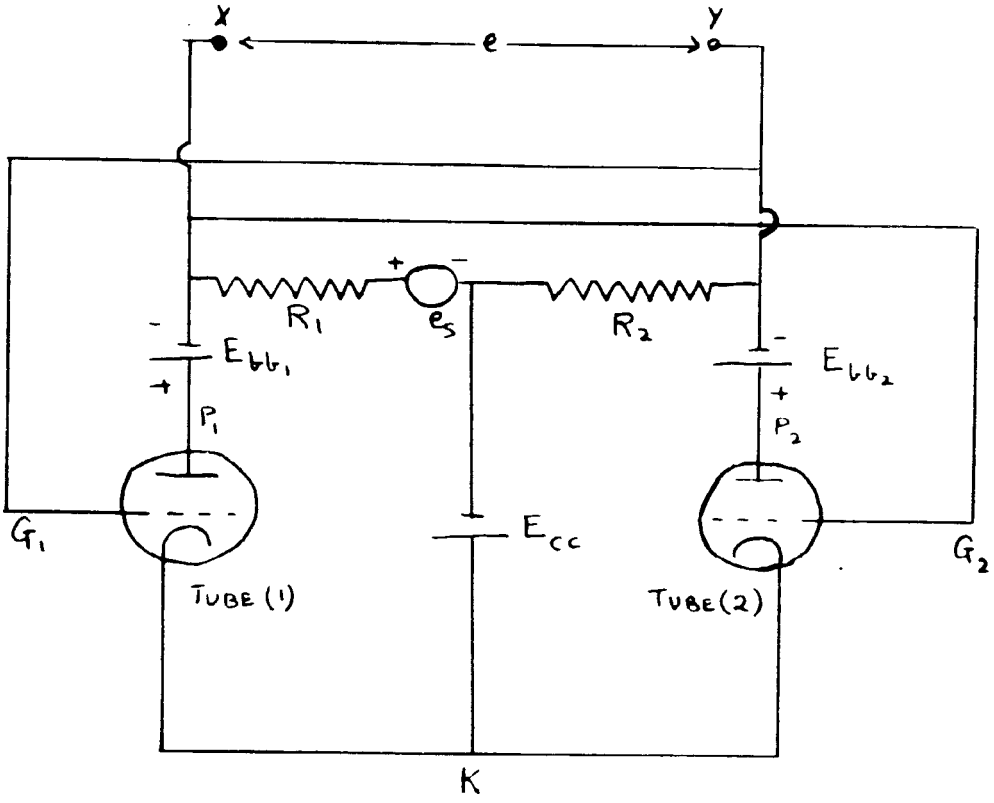


FIGURE 3.1

ECCLES-JORDAN CIRCUIT WITH VOLTAGE PULSE INTRODUCED IN SERIES WITH R_1

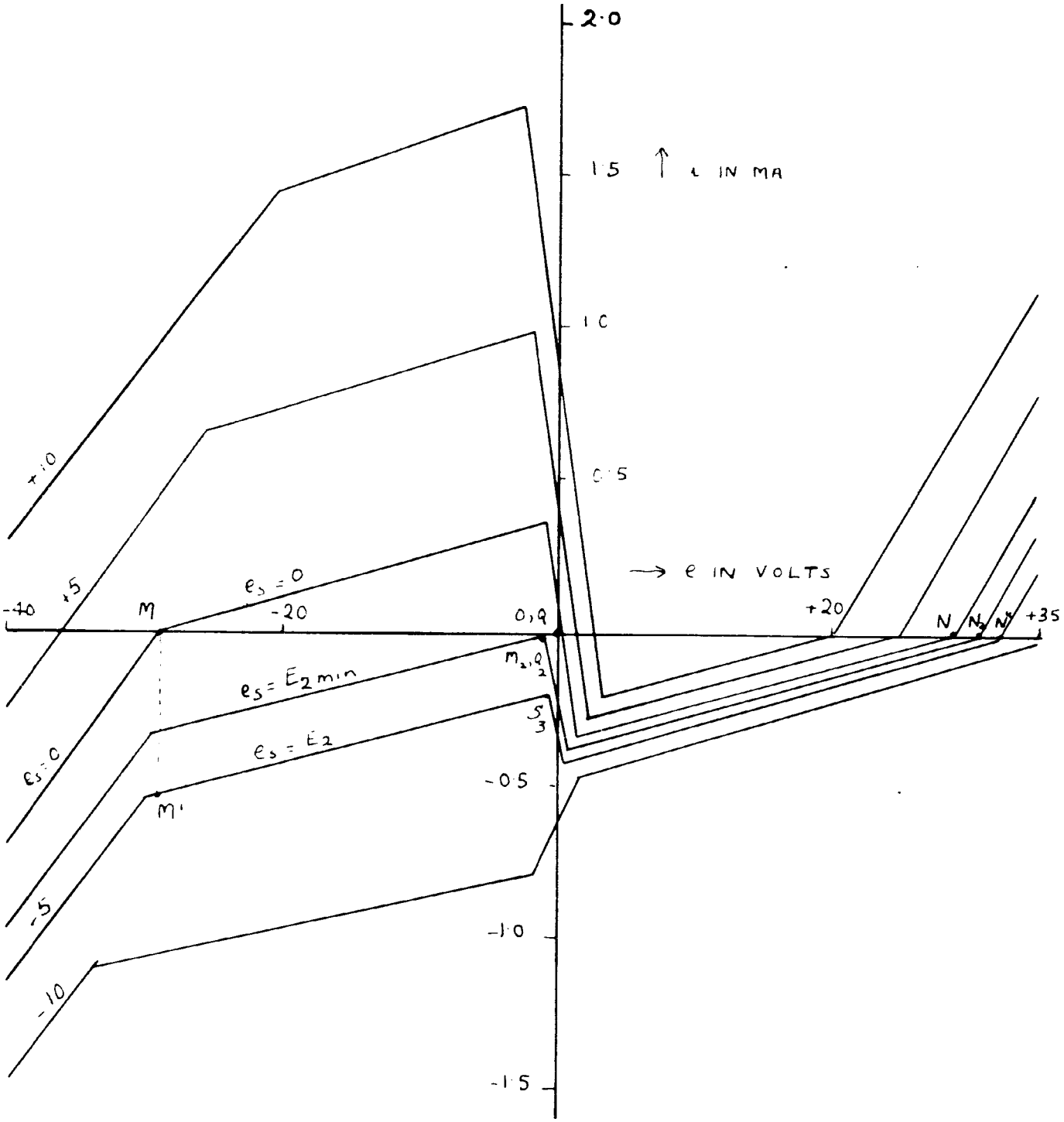


FIGURE 3·2

VOLTAGE-CURRENT CHARACTERISTICS BETWEEN THE TWO PLATES FOR DIFFERENT VALUES OF e_s, TO ILLUSTRATE THE TRIGGERING ACTION BY A PULSE IN SERIES WITH R_1

Figure 3.2 shows the family of i-e characteristic curves with e_s as a parameter The external resistance between the two plates (X and Y) is infinite, so that the load line is the e axis. Therefore, for any particular value of e_s. the two points of stable equilibrium of the circuit are given by the intersections of the positive resistance sections of the characteristic curve for that particular value of e_s with the e axis. To illustrate. when $e_s = 0$, M and N are the points of stable equilibrium and Q (or O' is the point of unstable equilibrium.

Let $e_s = 0$ initially and let M represent the initial stable state. If the circuit is to be triggered, e_s must be changed in such a way that the circuit changes over to the state of stable equilibrium represented by the point N. To achieve this, let e_s consist of a rectangular voltage pulse of magnitude E_2 and duration t_1 as shown in Fig. 3.3, applied in series with R_1. It is easy to see that this pulse has to be negative in order that the triggering can take place. By using the approximate equivalent circuit shown in Fig. 3.4, which is derived in Appendix B, the effect of the pulse on the circuit can be studied. The equivalent capacitance C in Fig. 3.4 is shown in Appendix B to be $2C_{gp} + (C_{kp}+C_{kg})/2$, and the value of the equivalent resistance r_B corresponds to the slope of the characteristic curve.

At t = 0 when the pulse is impressed, e_s changes suddenly from zero to E_2. As a result, the characteristic curve jumps from the $e_s = 0$ characteristic to the $e_s = E_2$ characteristic (see Fig. 3.2). Since the voltage e across capacitance C cannot change instantaneously, the current i jumps from its zero value at M to its value at M' on the $e_s = E_2$ characteristic curve.

Several possible sequences can occur as described below:

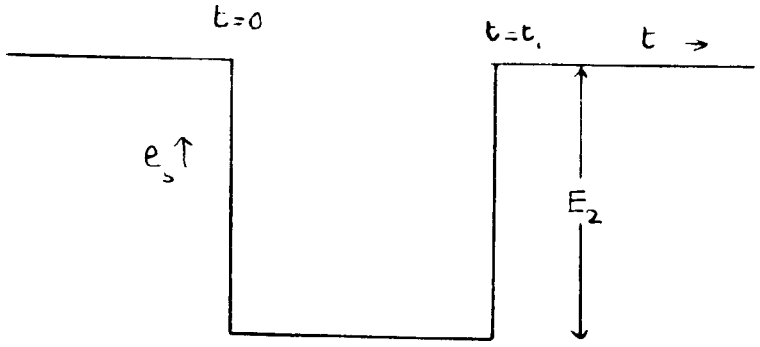


FIGURE 3.3

TRIGGERING PULSE IN SERIES WITH R_1

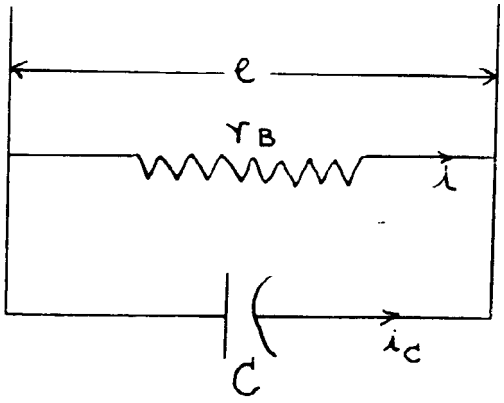


FIGURE 3.4

APPROXIMATE EQUIVALENT CIRCUIT BETWEEN THE TWO PLATES

(a) Let $|E_2|$ be large enough so that the E_2 characteristic curve intersects the e axis at only one point N", and let t_1 be large enough so that the transition to N" is completed before the pulse has ended. Under these circumstances the circuit triggers.

(b) Let $|E_2|$ be large enough so that the E_2 characteristic curve intersects the e axis at only one point N" and let t_1 be just large enough that at time t_1, e is greater than e_Q. Then, when e_s becomes zero again at time $t = t_1$, the characteristic curve comes back to its original $e_s = 0$ position. Since e cannot change at once because of the condenser, e is now greater than e_Q on this $e_s = 0$ characteristic. By arguments similar to those used in Chapter 2, e finally comes to e_N and cannot go back to e_M. Therefore the circuit still triggers.

(c) Let $|E_2|$ be large enough so that the E_2 characteristic curve intersects the e axis at only one point N", and let t_1 be small so that at time t_1 voltage e is still less than e_Q. When e_s becomes zero at $t = t_1$, e starts toward e_M, and triggering does not take place.

(d) Let t_1 be large, but let $|E_2|$ be small enough so that the $e_s = E_2$ characteristic curve intersects the e axis at three points M_1, Q_1, and N_1. Then, however large t_1 is, e eventually approaches e_{M_1} and not e_{N_1}. For $t > t_1$ when the characteristic curve comes back to the $e_s = 0$ position, e approaches e_M. The circuit therefore does not trigger.

(e) If $|E_2|$ is just large enough to permit the E_2 characteristic curve to touch the e axis at point M_2 (M_2 and Q_2 coincide in this case), it takes an infinite time for e to get to e_{M_2}. Finally, then, when e_s becomes zero at $t = t_1$, e will be still less than e_{Q_2} and so e returns to e_M. The result is no triggering.

(f) If $|E_2|$ is large enough so that the E_2 characteristic curve intersects the e axis at only one point N", but if t_1 is just large enough so that $e = e_Q$ at $t = t_1$, then theoretically e will stay at e_Q indefinitely. However, because of random noise voltages, e may become slightly greater than or slightly less than e_Q, and triggering may or may not result.

The above brief description shows that the method of treatment of the triggering action for the case where the pulse is introduced between grid and cathode is very similar in principle to the method described in Chapter 2. In Chapter 2 the introduction of the pulse changed the load line and the characteristic remained fixed; while in this case, the introduction of the pulse changes the characteristic curve and the load line remains fixed.

By examining Fig. 3.2. it can be seen that the minimum magnitude of the pulse is E_{2min} such that the E_{2min} characteristic just touches the e axis at point M_2 (or Q_2), and the minimum duration t_{1min} of the pulse is such that $e = e_Q$ at $t = t_{1min}$.

3.2 Analytical Treatment of the Triggering Action

The same analytical treatment that was used in Chapter 2 is applied here in the present case. The following differential equations must be solved in the different regions of the characteristic curve. They are identical with the differential equations (2.19), (2.20). (2.21), ... of Chapter 2 except for the constant terms:

In region (1) where both tubes carry current,

$$\frac{e(\mu + g_m R - g_m \mu R)}{2\mu R} + C\frac{de}{dt} + \frac{e_s}{2R} = 0 \quad . \qquad (3.1)$$

In region (2_1), where only tube (1) carries current and there is no grid current,

$$e\,\frac{\mu + g_m R}{R(g_m R + g_m R + 2\mu)} + C\,\frac{de}{dt} + \frac{-E_{bb} g_m R + e_s}{R(g_m R + g_m R + 2)} = 0 \quad . \qquad (3.2)$$

In region $(3_1'')$, where only tube (1) carries current and there is grid current,

$$e\left[\frac{\dfrac{g_m'' g_n''}{\mu'' \mu_g''} - g_m'' g_n'' + \dfrac{1}{R}\left(\dfrac{g_m''}{\mu''} + \dfrac{g_n''}{\mu_g''} + \dfrac{1}{R}\right)}{\dfrac{g_m''}{\mu''} + g_m'' + g_n'' + \dfrac{g_n''}{\mu_g''} + \dfrac{2}{R}}\right] + C\,\frac{de}{dt}$$

$$+\left[\frac{E_{bo}\left\{g_m'' g_n'' - \dfrac{g_m'' g_n''}{\mu'' \mu_g''} + \dfrac{1}{R}\left(g_n'' - \dfrac{g_m''}{\mu''}\right)\right\} + \dfrac{e_s}{R}\left(g_n'' + \dfrac{g_n''}{\mu_g''} + \dfrac{1}{R}\right)}{\dfrac{g_m''}{\mu''} + g_m'' + g_n'' + \dfrac{g_n''}{\mu_g''} + \dfrac{2}{R}}\right] = 0 \quad , \qquad (3.3)$$

where

$$C = 2\,C_{gp} + \frac{C_{kp} + C_{kg}}{2}$$

and

$$e_s = E_2 \quad \text{from } t = 0 \text{ to } t = t_1$$
$$= 0 \quad \text{for } t > t_1 \quad .$$

The solution of these differential equations with their boundary conditions can be carried out in the manner used in Chapter 2, and the results determine (1) the minimum magnitude and minimum duration of the pulse necessary for triggering, (2) the time of transition, and (3) the optimum magnitude and duration of the pulse which produces the shortest time of transition. The same arguments used to determine the optimum magnitude and duration of the pulse may also be applied in this case.

CHAPTER 4

A GENERALIZED STUDY OF TRIGGERING ACTION

In Chapters 2 and 3, a parallel combination of resistance and capacitance was used to represent the trigger circuit viewed between the two plate terminals. The value of the equivalent resistance was given by the reciprocal of the slope of the d-c characteristic curve, and the value of the capacitance was $2C_{gp} + (C_{kp} + C_{kg})/2$. This approximation can be used only for a symmetrical circuit and for certain typical circuit parameters. For a more general kind of circuit, which may be not symmetrical, the equivalent circuit has to be a parallel combination of a resistance, a capacitance, and a third element whose dimension is the product of a resistance and the square of a capacitance, as shown in Appendix B. It is also shown in Appendix B that the current through this third element is equal to the product of its (resistance)x(capacitance)^2 and the voltage across it. Not only does the resistance have different values in the different regions of the characteristic curve, but the capacitance and the third element also have different values in the different regions. The triggering action will now be studied, using this more general form of equivalent circuit.

4.1 Study of the Triggering Action When a Rectangular Voltage Pulse is Introduced Between the Two Plates, Using the More General Form of Equivalent Circuit

Figure 4.1 shows the d-c i-e characteristic between the two plates for the case of an asymmetrical circuit. Figure 4.2 shows the equivalent circuit consisting of the parallel combination of a resistance (= reciprocal of the slope of the characteristic curve), a capacitance (which is different

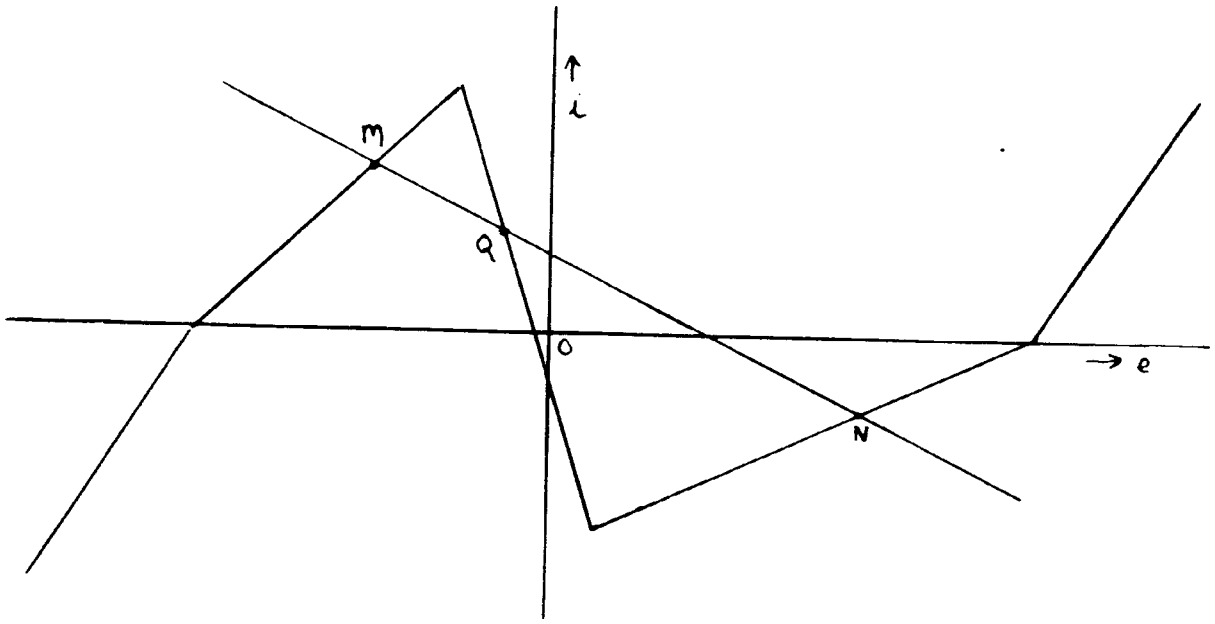


FIGURE 4.1

D-C VOLTAGE-CURRENT CHARACTERISTIC BETWEEN THE TWO PLATES FOR AN ASYMMETRICAL CIRCUIT

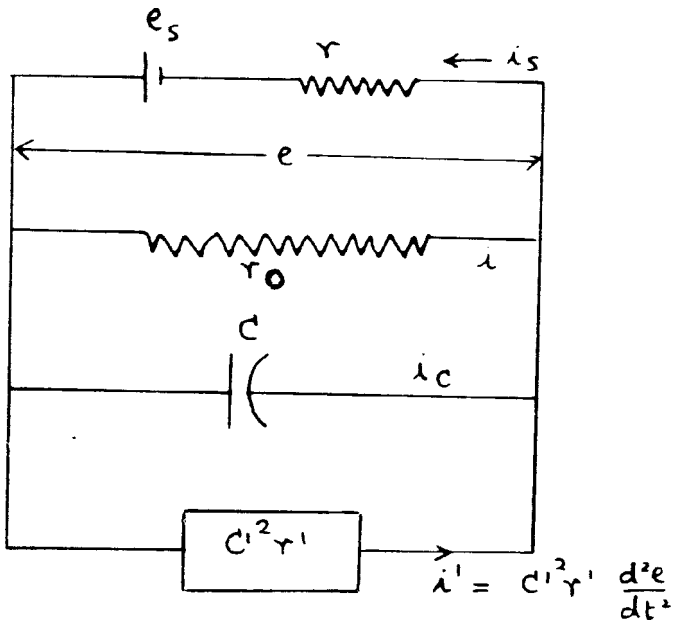


FIGURE 4.2

GENERAL FORM OF EQUIVALENT CIRCUIT, CONNECTED TO A VOLTAGE SOURCE e_s IN SERIES WITH A RESISTANCE r

for the different parts of the characteristic), and an element a_1 (= C'^2r', which is also different for the different parts of the characteristic), all connected to an external voltage source e_s in series with an external resistance r. The load line MQN for resistance r is drawn in Fig. 4.1. The points M, Q, and N represent the three states of equilibrium. It is first of all necessary to decide which of the three points M, Q, and N give a state of stable equilibrium and which unstable.

Using the results of Appendix B, it is evident that the differential equation satisfied by e is

$$C'^2r' \frac{d^2e}{dt^2} + C \frac{de}{dt} + \frac{e}{r_o} = \frac{e_s - e}{r} . \qquad (4.1a)$$

This may be rewritten as

$$C'^2r' \frac{d^2e}{dt^2} + C \frac{de}{dt} + e\left(\frac{1}{r_o} + \frac{1}{r}\right) = \frac{e_s}{r} . \qquad (4.1b)$$

This evidently is of the form

$$a_1 \frac{d^2e}{dt^2} + a_2 \frac{de}{dt} + a_3 e = a_4 , \qquad (4.2$$

where $a_1 = C'^2r'$

$a_2 = C$

$a_3 = 1/r_o + 1/r$

$a_4 = e_s/r$.

Let there be some disturbance such that e becomes $\bar{e} + \eta$. where $\bar{e}$ is an equilibrium value. Then

$$a_1 \frac{d^2}{dt^2}(\bar{e} + \eta) + a_2 \frac{d}{dt}(\bar{e} + \eta) + a_3 (\bar{e} + \eta) = a_4 . \qquad (4.3)$$

But since $\bar{e}$ is an equilibrium value,

$$a_3\bar{e} = a_4 \tag{4.4}$$

and

$$\frac{d^2\bar{e}}{dt^2} = \frac{d\bar{e}}{dt} = 0 \ . \tag{4.5}$$

Therefore

$$a_1 \frac{d^2\eta}{dt^2} + a_2 \frac{d\eta}{dt} + a_3 = 0 \ . \tag{4.6}$$

A solution of the above differential equation is

$$\eta = K_1 \epsilon^{m_1 t} + K_2 \epsilon^{m_2 t} \ , \tag{4.7}$$

where m_1 and m_2 are the roots of the algebraic equation

$$a_1 m^2 + a_2 m + a_3 = 0 \ . \tag{4.8}$$

Therefore

$$m_1 = \frac{-a_2 + \left|\sqrt{a_2^2 - 4a_1a_3}\right|}{2a_1} \tag{4.9}$$

$$m_2 = \frac{-a_2 - \left|\sqrt{a_2^2 - 4a_1a_3}\right|}{2a_1} \ . \tag{4.10}$$

The circuit is stable if η decreases with increasing t, and unstable otherwise.

In the positive resistance parts of the characteristic, a_1, a_2, and a_3 are all positive, and $(a_2^2 - 4a_1a_3)$ is positive for typical circuit parameters, and therefore $\left|\sqrt{a_2^2 - 4a_1a_3}\right| < a_2$. Hence m_1 is negative and m_2 is negative. Therefore $\eta = (K_1\epsilon^{m_1 t} + K_2\epsilon^{m_2 t})$ decreases with increasing t, and the circuit is stable.

In the negative resistance part of the characteristic, a_1 and a_2 are still positive, but a_3 is negative if $r > |r_o|$ and positive if $r < |r_o|$.

Therefore if $r < |r_o|$, the circuit is stable. If $r > |r_o|$, then $(a_2^2 - 4a_1a_3) > a_2^2$, so that $|\sqrt{(a_2^2 - 4a_1a_3)}| > a_2$. Therefore m_1 is positive and m_2 is negative, and consequently $\eta = K_1\epsilon^{m_1 t} + K_2\epsilon^{m_2 t}$ increases with increasing t for large values of t. Therefore the circuit is unstable if $r > |r_o|$.

The above analysis shows again that M and N, which are the points of intersection of the load line with the positive resistance parts of the characteristic, are stable whatever the value of r may be, and Q, which is the point of intersection of the negative resistance part of the characteristic, is stable if the absolute value of the negative resistance is greater than r, and unstable if it is less than r. When r is less than the absolute value of the negative resistance, the case is of no interest as far as the triggering action is concerned. Therefore in the study of the triggering action, r is taken to be greater than the reciprocal of the slope of the negative resistance part of the characteristic, and Q then is a point of unstable equilibrium.

Since M and N represent stable states of equilibrium and Q represents an unstable state of equilibrium the triggering action can be studied when the voltage source e_s gives a rectangular pulse of magnitude and duration t_1, just as in Chapter 2. Instead of solving first-order differential equations as in Chapter 2, here second-order differential equations have to be solved.

The differential equations (2.19), (2.20), (2.21), (2.22), (2.23), and (2.24) have to be replaced by equations of the form

$$a_1 \frac{d^2e}{dt^2} + a_2 \frac{de}{dt} + a_3 e = a_4 , \qquad (4.11)$$

where for the positive resistance parts of the characteristic a_1, a_2, and

a_3 are all positive. and for the negative resistance part of the characteristic a_1 and a_2 are still positive and a_3 is negative. Solution of the above differential e uation is of the form:

For the positive resistance part of the characteristic.

$$e = K_1 \epsilon^{-t/t_1} + K_2 \epsilon^{-t/t_2} . \qquad (4.12)$$

For the negative resistance part of the characteristic,

$$e = K_3 \epsilon^{-t/t_3} + K_4 \epsilon^{t/t_4} . \qquad (4.13)$$

t_1, t_2. t_3, and t_4 are all positive time constants, and K_1, K_2, K_3, and K_4 are constants of integration.

K_1, K_2. K_3. and K_4 can be determined by the following boundary conditions (see Appendix B):

(1) e has to be continuous at a boundary, and

(2) de/dt has to be continuous at a boundary.

By solving these differential equations, the time of transition, the minimum magnitude and minimum duration of pulse necessary for triggering and the optimum magnitude and duration to produce shortest time of transition can be determined in the same way as that used in Chapter 2.

It is interesting to note at this point that the concept of the characteristic curve and the load line can be used in this case in the same way as in the case where the equivalent circuit consists of only a resistance and a capacitance in parallel. By using this approach to the problem. it can be seen that the concepts of minimum magnitude and minimum duration of pulse necessary for triggering and of optimum magnitude and duration of pulse for shortest time of transition are not any different from the concepts derived in Chapter 2.

4.2 Study of the Triggering Action When a Rectangular Current Pulse is Introduced Between the Grid and Cathode of One Tube, Using the More General Form of Equivalent Circuit

From the preceding discussion, it is evident that the triggering action in this arrangement can be studied in the same way as in Chapter 3, the only difference being that here second-order differential equations occur instead of first-order differential equations.

4.3 Study of the Triggering Action When a Rectangular Voltage Pulse in Series with a Finite Resistance is Introduced Between the Grid and Cathode of One Tube

The circuit is shown in Fig. 4.3, where e_t denotes the instantaneous value of the voltage in series with a resistance r connected between G_2 and K.

When both the tubes are carrying current, equations (B.13) and (B.14) of Appendix B are:

$$C_{gp}\frac{d}{dt}(e) + \frac{e_s + E_{bb} - e_{b_1}}{R} + C_{kp}\frac{d}{dt}(-e_{b_1}) + C_{kg}\frac{d}{dt}(-e_{b_1}) - C_{gp}\frac{d}{dt}(-e) - g_m\left\{e - E_{bb} + e_{b_1}\left(1+\frac{1}{\mu}\right)\right\} = 0 \quad (4.14)$$

$$C_{gp}\frac{d}{dt}(-e) + \frac{E_{bb} - e - e_{b_1}}{R} + C_{kp}\frac{d}{dt}(-e-e_{b_1}) + C_{kg}\frac{d}{dt}(-e-e_{b_1}) - C_{gp}\frac{d}{dt}(e) - g_m\left\{-E_{bb} + \frac{e}{\mu} + e_{b_1}\left(1+\frac{1}{\mu}\right)\right\} = 0 \quad (4.15)$$

For $e_s = 0$, substituting $e_{b_1} = e_{c_2} + E_{bb}$ in equations (4.14) and (4.15), the sum and difference of equations of (4.14) and (4.15) are respectively:

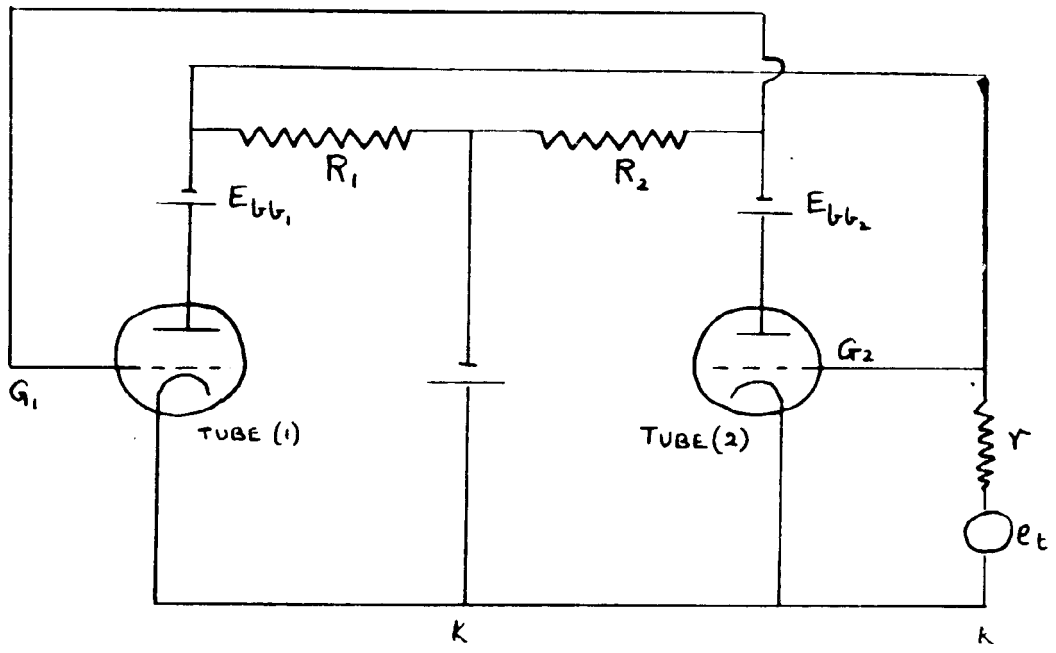
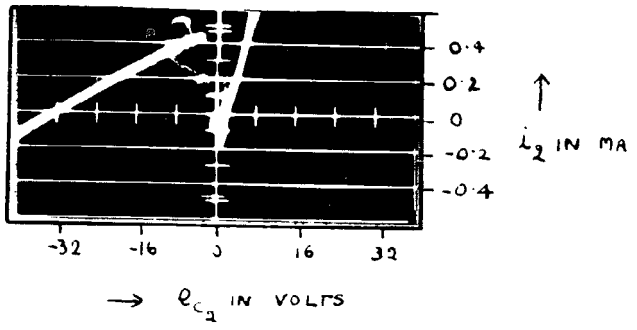


FIGURE 4.3

ECCLES-JORDAN CIRCUIT BEING TRIGGERED BY A VOLTAGE PULSE e_t IN SERIES WITH A FINITE RESISTANCE Y CONNECTED BETWEEN G_2 AND K

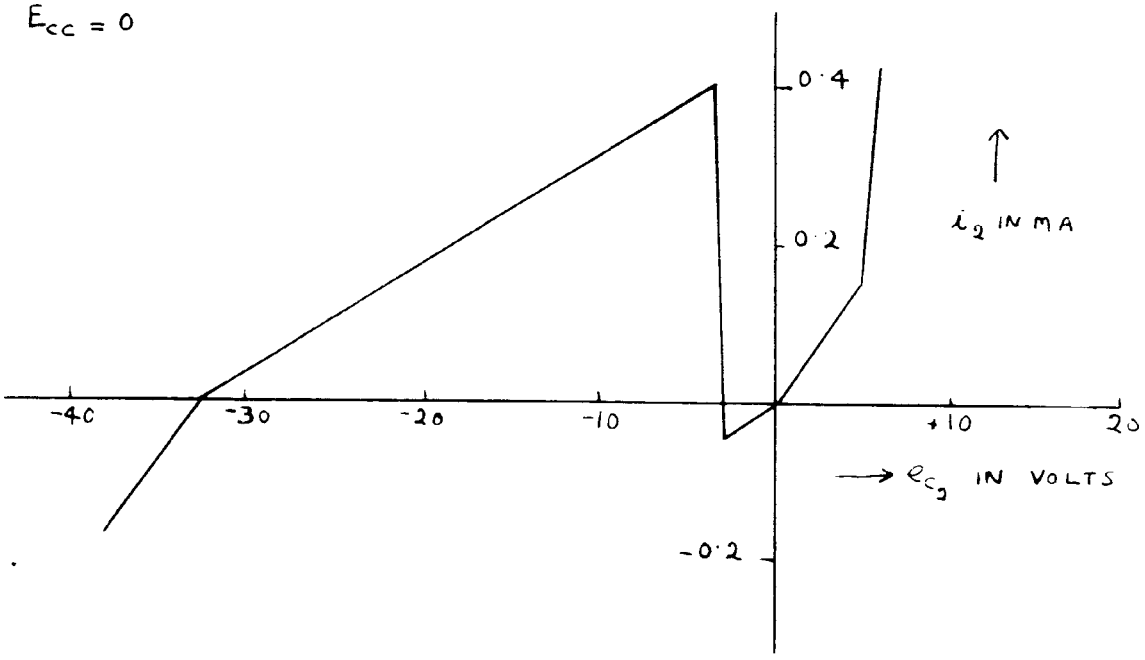


(a)

$R_1 = R_2 = R = 7500$ OHMS

$E_{bb_1} = E_{bb_2} = E_{bb} = 68$ VOLTS

$E_{cc} = 0$



(b)

FIGURE 4.4

VOLTAGE-CURRENT CHARACTERISTIC BETWEEN G_2 AND K

(a) Experimental (b) Analytically derived

$$e\left\{\frac{1}{2R} + \frac{\mu+1}{2\mu}\, g_m\right\} + \frac{C_{kg}+C_{kp}}{2}\frac{de}{dt} + e_{c_2}\left\{\frac{1}{R} + g_m\left(1+\frac{1}{\mu}\right)\right\}$$

$$+ (C_{kp}+C_{kg})\frac{de_{c_2}}{dt} + E_{bb}\frac{g_m}{\mu} = 0 \qquad (4.16)$$

and

$$e\left\{\frac{\mu + g_m R - \mu g_m R}{2\mu R}\right\} + \left\{2\,C_{gp} + \frac{C_{kp}+C_{kg}}{2}\right\}\frac{de}{dt} = 0\ . \qquad (4.17)$$

On eliminating e between equations (4.16) and (4.17),

$$\left\{\frac{\left(2C_{gp} + \frac{C_{kp}+C_{kg}}{2}\right)(C_{kp}+C_{kg})R\mu}{\mu + g_m R + \mu g_m R}\right\}\frac{d^2 e_{c_2}}{dt^2} + \left[\left\{\frac{(\mu + g_m R - \mu g_m R)(C_{kp}+C_{kg})}{2\mu R}\right.\right.$$

$$\left.\left. + (\mu + g_m R + \mu g_m R)\left(2C_{gp} + \frac{C_{kp}+C_{kg}}{2}\right)\right\}\frac{R\mu}{\mu + g_m R + \mu g_m R}\right]\frac{de_{c_2}}{dt}$$

$$+ \frac{\mu + g_m R - \mu g_m R}{2\mu R}\, e_{c_2} + \left[E_{bb}\frac{g_m}{2\mu}\left(\frac{\mu + g_m R - \mu g_m R}{\mu + g_m R + \mu g_m R}\right)\right] = 0\ , \qquad (4.18)$$

which is of the form

$$a_1\frac{d^2 e_{c_2}}{dt^2} + a_2\frac{de_{c_2}}{dt} + a_3\, e_{c_2} + a_4 = 0\ , \qquad (4.19)$$

where a_1 and a_2 are positive and a_3 is negative. This shows that when both the tubes are carrying current, the circuit between G_2 and K can be replaced by an equivalent circuit consisting of a parallel combination of a negative resistance, a capacitance, and an element of dimension (resistance)x(capacitance)^2.

In a similar manner it can be shown that, when only one tube is carrying current the equivalent circuit between G_2 and K is a parallel combination of a positive resistance, a capacitance, and an element having

the dimension (resistance)x(capacitance)^2. The values of these elements vary according to whether grid current is flowing or not, etc. Therefore a d-c characteristic curve exists between points G_2 and K which is of similar shape (but not symmetrical) to the one that exists between the two plates (see Fig. 4.4). The equivalent circuit that exists between G_2 and K for different sections of this characteristic curve consists of the same three elements (but of different numerical value) as exists between the two plates.

Therefore, when a rectangular voltage pulse in series with a finite positive resistance is introduced between G_2 and K, the triggering action of the circuit can be studied in exactly the same way as is done when such a pulse is introduced between the two plates, the only difference being the numerical values that occur in the differential equations and their boundary conditions. The value of the finite positive resistance in series with the pulse has to be greater than the reciprocal of the slope of the negative resistance part of the characteristic, in order that triggering can take place at all. All the results derived for the triggering action by a pulse between the two plates can be derived for this arrangement as well.

CHAPTER 5

EXPERIMENTAL VERIFICATION

To verify some of the preceding results experimentally, the circuit shown in Fig. 5.1 is set up, using the two halves of a 6SN7 tube, and with $R_1 = R_2 = R = 7500$ ohms, $E_{bb_1} = E_{bb_2} = 68$ volts, and $r = 15{,}000$ ohms.

The voltage source e_t produces alternating positive and negative rectangular pulses, which are obtained by mixing the outputs of two cathode-coupled one-shot multivibrators. The combined output is passed through a cathode follower circuit to make the internal impedance of the voltage source low.

Figure 5.2 shows oscillograms of e_t and e taken on a Tektronix 512 oscilloscope, whose vertical amplifier has a bandwidth of d-c to 2 mc and a rise time of 0.2 microsecond. Figures 5.2(a), (b), (c), and (d) correspond to Figs. 2.6(a), (b), (c), and (d). To show the details of the transition, large capacitances have been added to the experimental circuit to slow up the transition. Figure 5.3 shows the computed curves of e_t and e versus time t for the same circuit conditions as those for Fig. 2.6(a).

Figure 5.4 shows similar oscillograms of e_s and e, when e_s is introduced in series with R_1 (see Fig. 3.1). Figures 5.4(a), (b), and (d) correspond to the cases (a), (b), and (d) described in Chapter 3, while in Fig. 5.4(c) the triggering by the positive pulse corresponds to case (a) and that by the negative pulse corresponds to case (b).

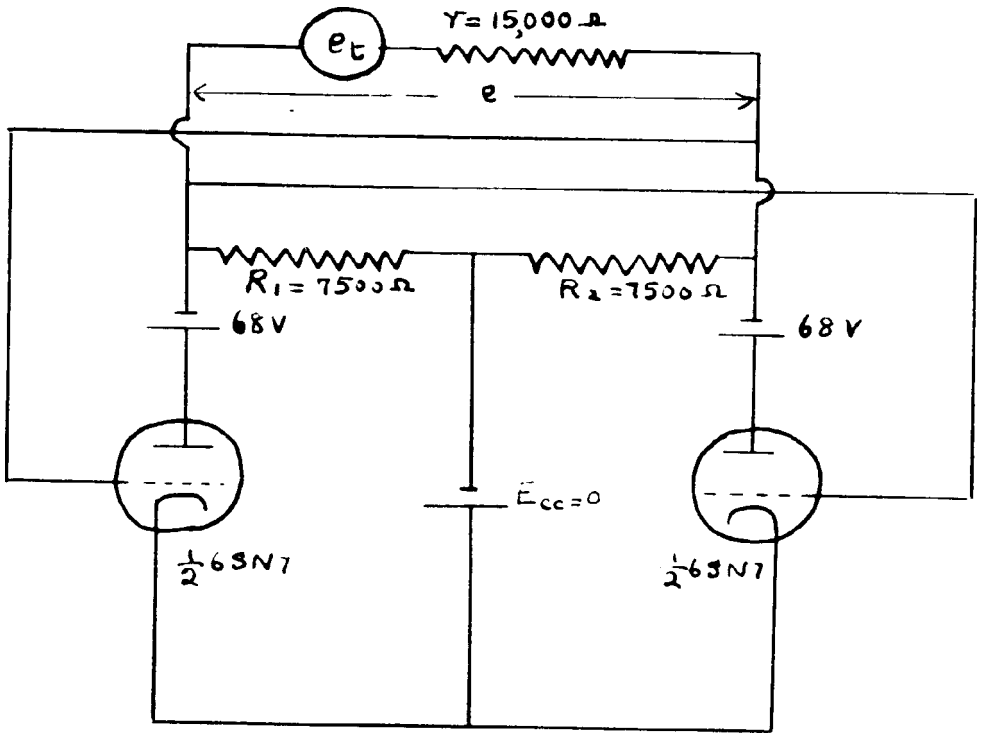
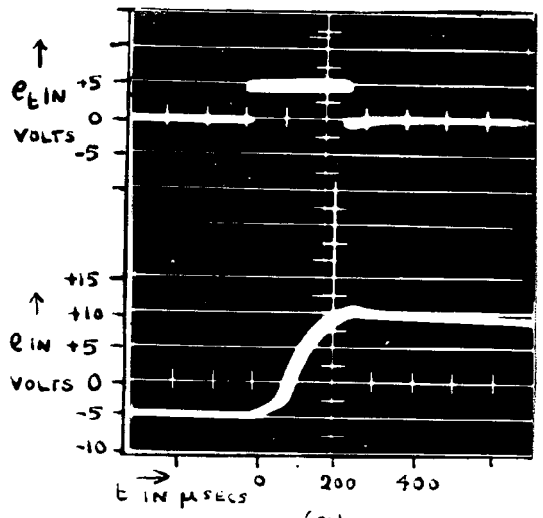


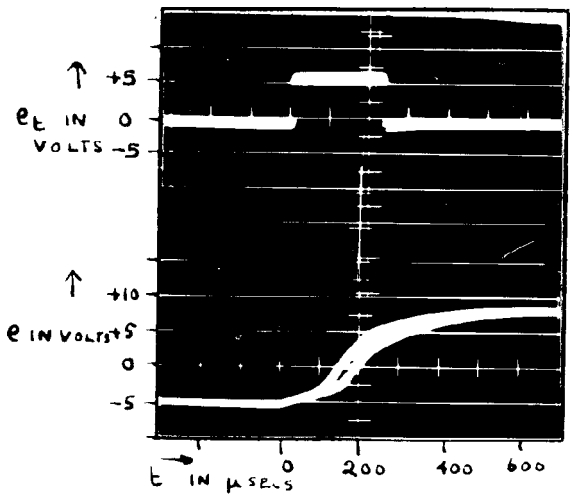
FIGURE 5.1

EXPERIMENTAL CIRCUIT



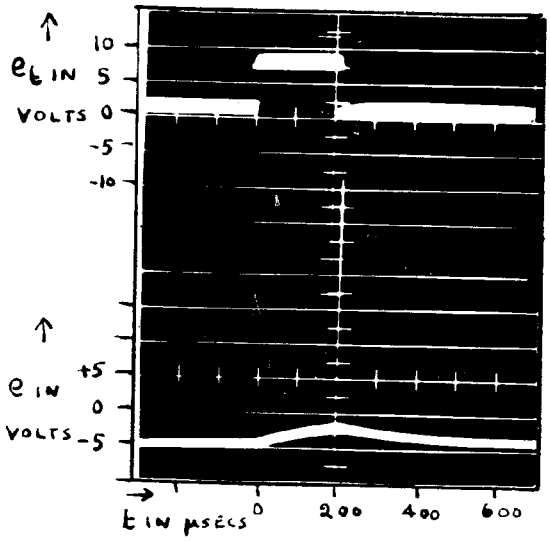
(a)

C = 2700 μμF



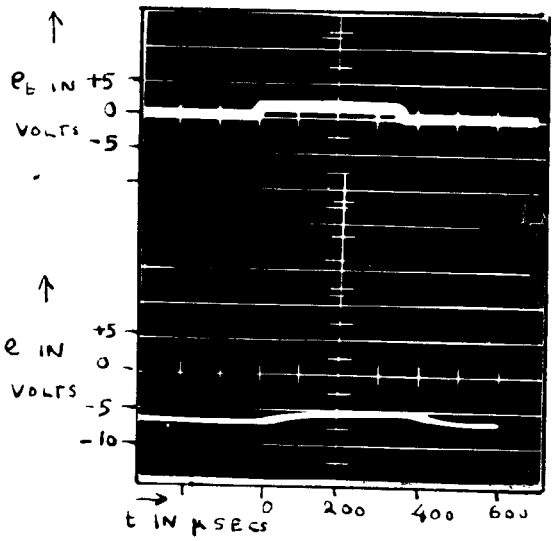
(b)

C = 6000 μμF



(c)

C = 8000 μμF



(d)

C = 2700 μμF

FIGURE 5-2

OSCILLOGRAMS OF e_t AND e, WHEN e_t IS INTRODUCED IN SERIES WITH r = 15,000 OHMS BETWEEN THE TWO PLATES

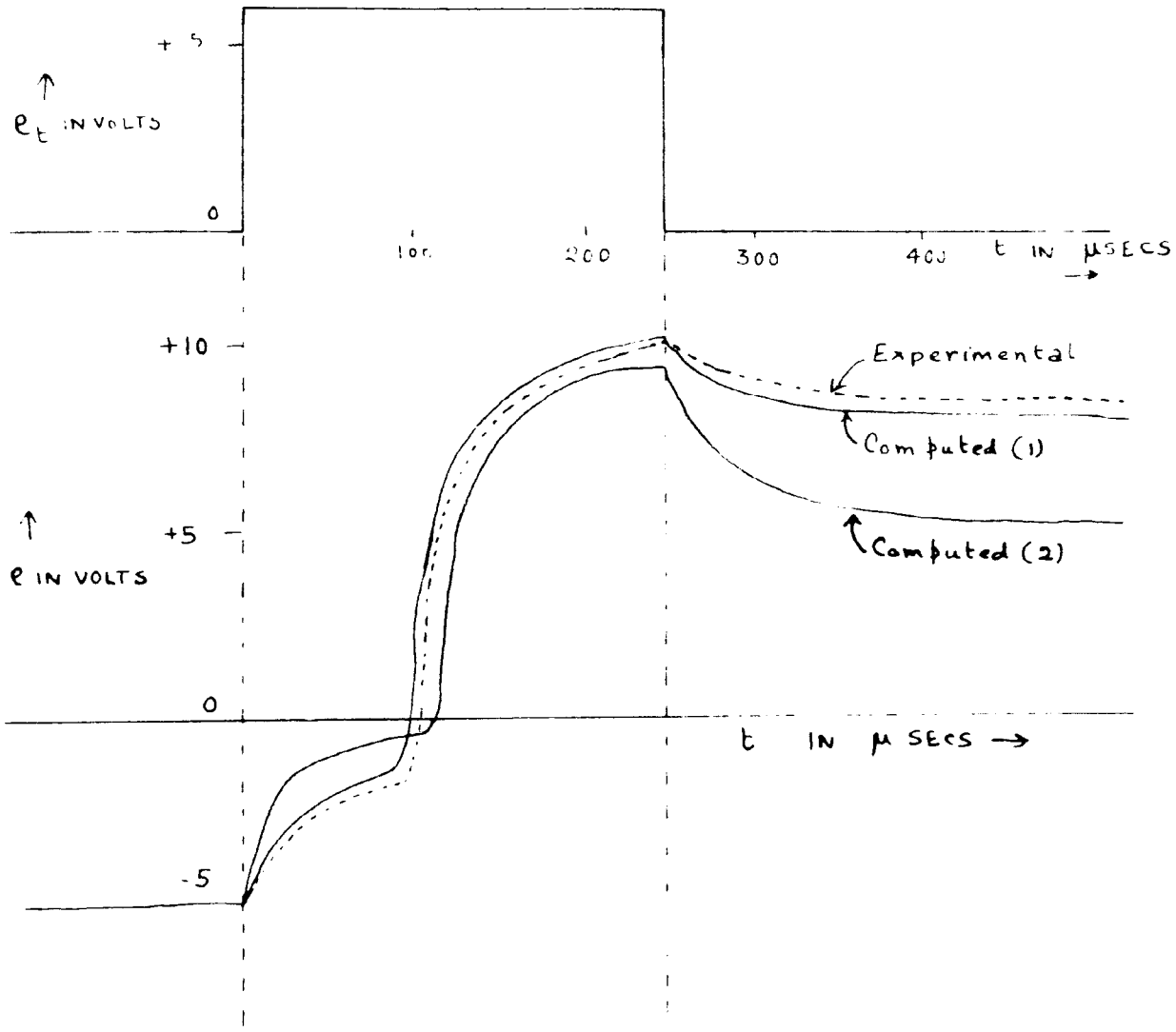
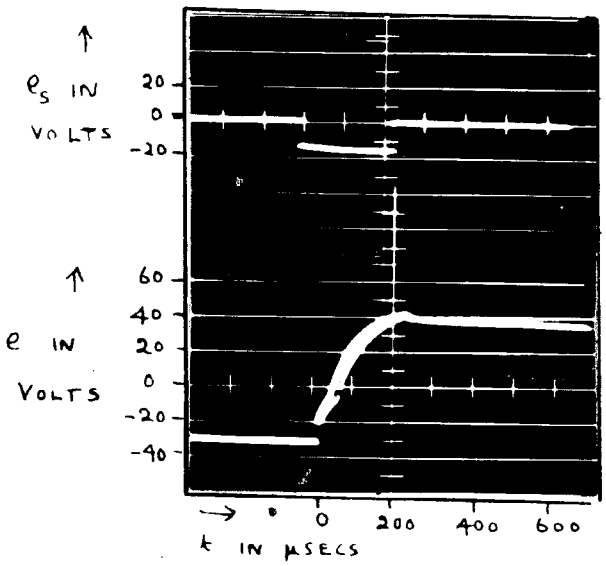


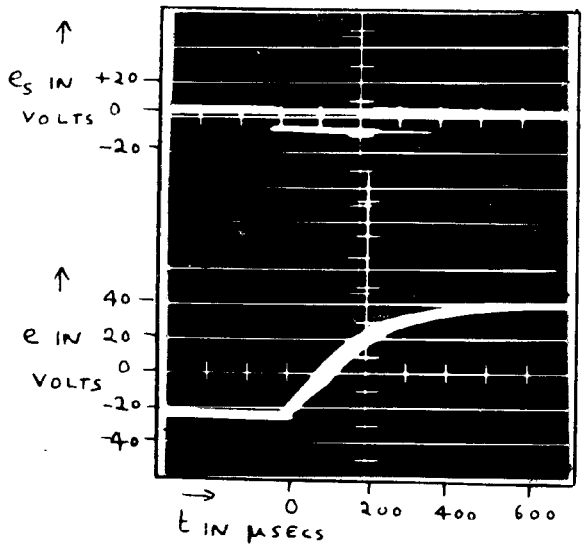
FIGURE 5·3

COMPUTED AND EXPERIMENTAL CURVES OF e VERSUS t, FOR GIVEN e_t AND THE FIG 5·2(a) CONDITIONS.

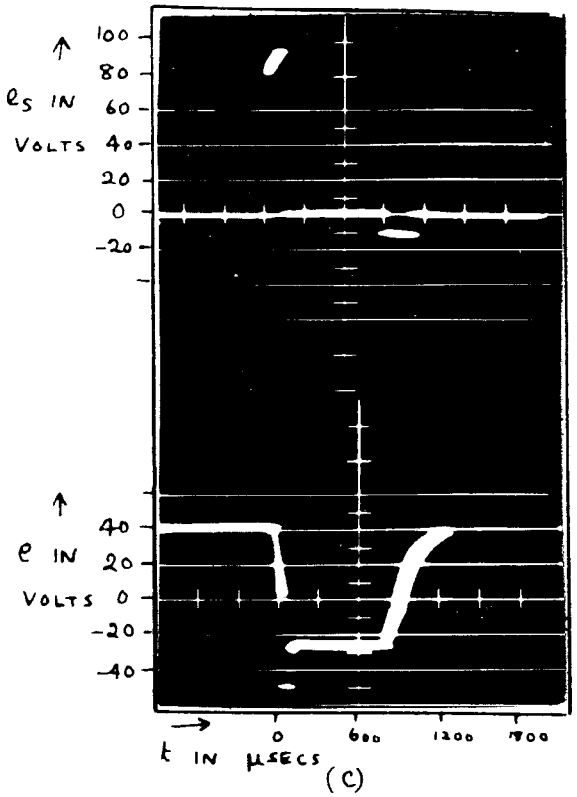
(1) Computed curve using actual i-e characteristic of Fig 2·2(a). (2) Computed curve using the idealized i-e characteristic of Fig 2·2(b). $C = 2700\ \mu\mu F$, $R = 7500\ \Omega$, $E_{bb} = 68$ volts.



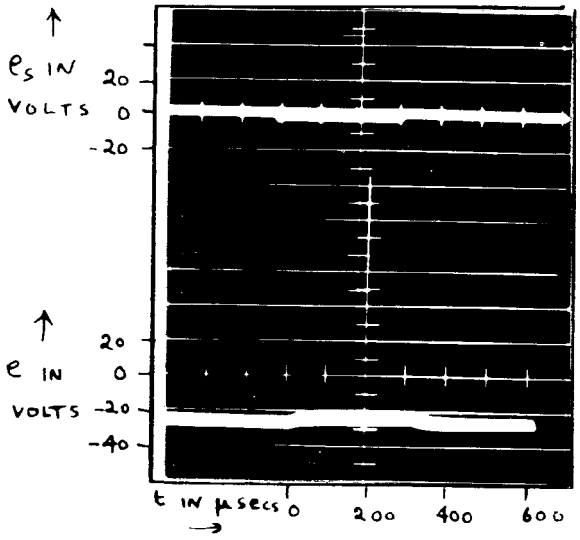
(a)
C = 1000μμF



(b)
C = 2700μμF



(c)
C = 2700μμF



(d)
C = 2700μμF

FIGURE 5.4

OSCILLOGRAMS OF e AND e_s, WHEN e_s IS INTRODUCED IN SERIES WITH R_1

CONCLUSIONS

This thesis has given consideration only to a simple form of the Eccles-Jordan trigger circuit. A method of approach different from that of previous workers is adopted in an attempt to facilitate the understanding of the triggering action of the circuit. The replacement of the whole circuit by an equivalent parallel combination of circuit elements not only simplifies the qualitative study of the triggering action but also helps to obtain some useful quantitative results.

Heavy demand for circuits with two stable states has resulted in several modifications of the Eccles-Jordan circuit. For instance, in a flip-flop circuit which is in common use, the grid of one tube is coupled to the plate of the other by a resistance in parallel with a small capacitance, and a single d-c voltage supply for the two tubes is used, the correct operating grid voltages being maintained by means of grid resistors. Many different methods of applying the triggering pulse are also in practice. Introduction of alternating positive and negative pulses between one grid and the cathode, and application of successive pulses of the same polarity simultaneously to the two grids, are only two of the several ways of triggering the circuit.

The method developed in this thesis for treating the problem is believed to be applicable with slight modifications to the study of variations of the bistable Eccles-Jordan circuit and its derivatives such as monostable and astable multivibrators.

APPENDIX A

ANALYTICAL DERIVATION OF THE D-C CURRENT-VOLTAGE CHARACTERISTICS BETWEEN THE PLATES OF THE TWO TUBES

Let a direct voltage e be applied between the terminals X and Y of the circuit shown in Fig. A.1. Let another constant direct voltage e_s be introduced in series with R_1 in the grid circuit of tube (2). Appendix C shows that a voltage pulse introduced in series with R_1 is equivalent to a current pulse introduced across the grid G_2 and the cathode. With e_s remaining fixed let e be varied and let i be the external current flowing between the terminals X and Y. The resulting i-e characteristic curve gives the d-c current-voltage characteristic between the plates of the two tubes. These characteristics are calculated for different values of e_s in the following manner.

A.1 Assumptions

(1) Identical tubes and symmetrical circuit; i.e.,

$$R_1 = R_2 = R, \quad E_{bb_1} = E_{bb_2} = E_{bb}, \quad E_{cc} = 0 .$$

(2) Straight-line tube characteristics (see Fig. A.2).

For the region of no grid current flow,

$$i_b = g_m \left(e_c + \frac{e_b}{\mu}\right) \qquad \text{for} \quad \left(e_c + \frac{e_b}{\mu}\right) > 0 \qquad \text{(A.1a)}$$

$$= 0 \qquad \text{for} \quad \left(e_c + \frac{e_b}{\mu}\right) < 0 . \qquad \text{(A.1b)}$$

For the region of grid current flow,

$$\left.\begin{aligned} i_b &= g_m'' \left(e_c + \frac{e_b}{\mu''}\right) \\ i_c &= g_n'' \left(e_b + \frac{e_c}{\mu_g''}\right) \end{aligned}\right\} \qquad \text{for} \quad e_b > e_c . \qquad \begin{matrix} \text{(A.2)} \\ \text{(A.3)} \end{matrix}$$

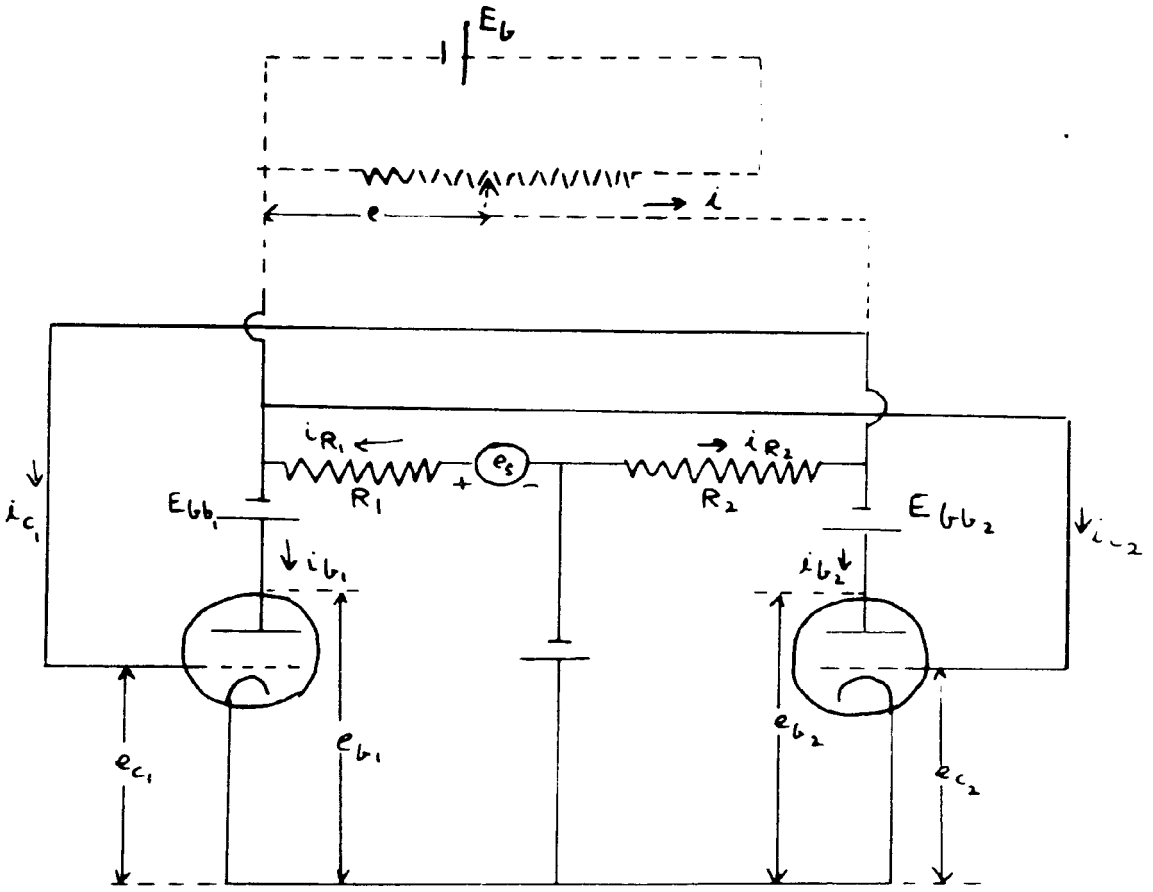
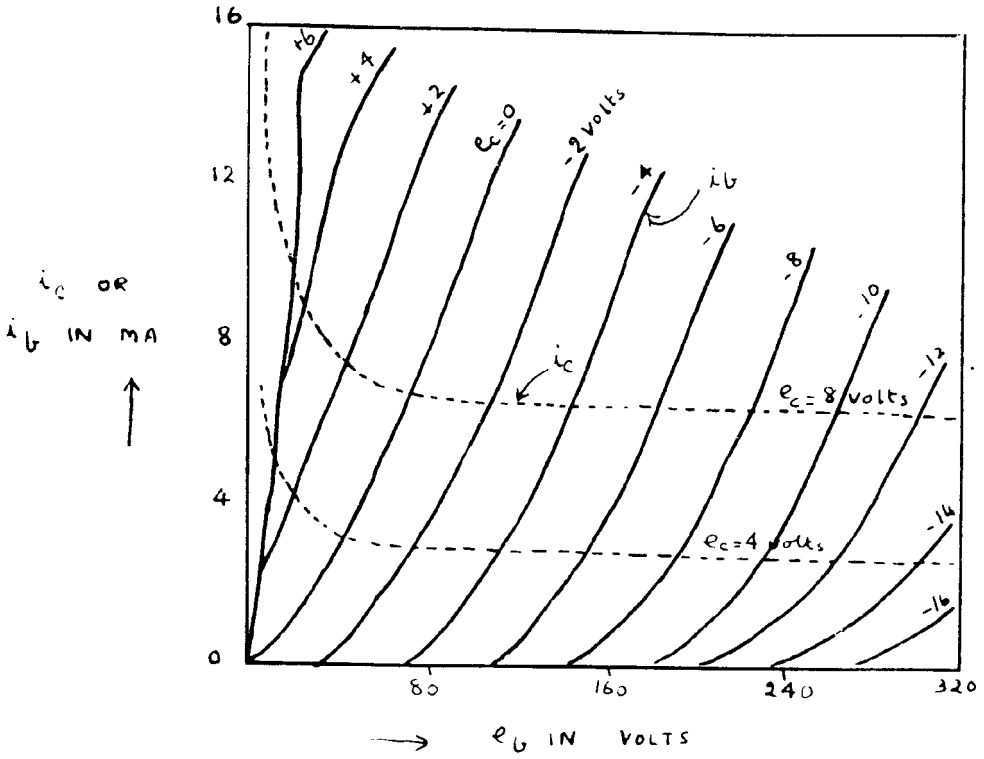


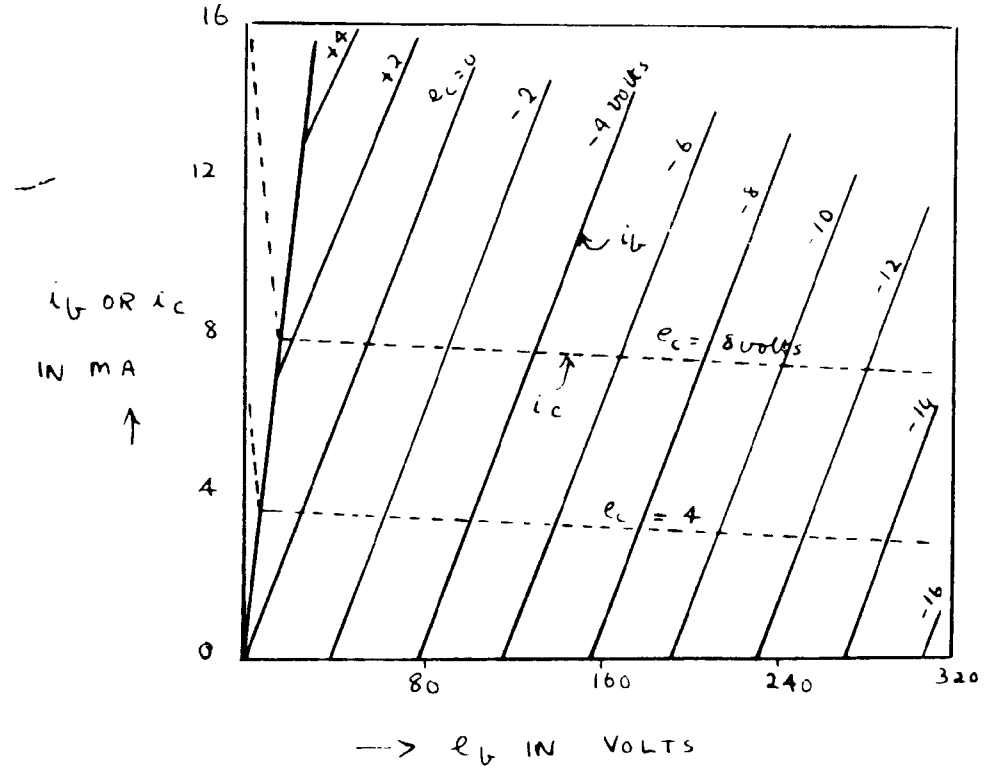
FIGURE A.1

SYMMETRICAL ECCLES-JORDAN CIRCUIT WHOSE i-e CHARACTERISTICS ARE DERIVED

$E_{bb_1} = E_{bb_2} = E_{bb}$, $R_1 = R_2 = R$, $E_{cc} = 0$



(a)



(b)

FIGURE A.2

TUBE CHARACTERISTICS. (a) Actual (b) Assumed ideal straight lines.

$$\left.\begin{aligned} i_b &= g_m' \left(\frac{e_b}{\mu'}\right) \\ i_c &= g_n' \left(e_b + \frac{e_c}{\mu_g'}\right) \end{aligned}\right\} \quad \text{for } e_b < e_c \ , \qquad (A.4)\ (A.5)$$

where

$$g_n = \frac{\partial i_c}{\partial e_b} \quad (e_c = \text{constant}) \qquad (A.6)$$

$$\mu_g = -\frac{\partial e_c}{\partial e_b} \quad (i_c = \text{constant}) \qquad (A.7)$$

$$g_m = \frac{\partial i_b}{\partial e_c} \quad (e_b = \text{constant}) \qquad (A.8)$$

$$\mu = -\frac{\partial e_b}{\partial e_c} \quad (i_b = \text{constant}) \qquad (A.9)$$

e_{b_1} = plate voltage of tube (1), i_{b_1} = plate current of tube (1)

e_{b_2} = plate voltage of tube (2), i_{b_2} = plate current of tube (2)

e_{c_1} = grid voltage of tube (1), i_{c_1} = grid current of tube (1)

e_{c_2} = grid voltage of tube (2), i_{c_2} = grid current of tube (2)

From Fig. A.1,

$$e_{b_2} - e_{b_1} = e_{c_1} - e_{c_2} = e \qquad (A.10)$$

$$i_{b_2} - i_{R_2} = i_{R_1} - i_{b_1} = i \qquad (A.11)$$

$$e_{c_2} = -E_{bb} + e_{b_1} \qquad (A.12)$$

$$e_{c_1} = e - E_{bb} + e_{b_1} \qquad (A.13)$$

$$e_{b_2} = e + e_{b_1} \qquad (A.14)$$

$$e_{R_1} = e_s + E_{bb} - e_{b_1} \qquad (A.15)$$

$$e_{R_2} = E_{bb} - e - e_{b_1} \qquad (A.16)$$

A.2 For Both Tubes Carrying Current

In this region, grid currents may have to be included for some particular circuit parameters and for some particular values of e_s and E_{bb}.

But for the particular circuit studied here, these values have been chosen so that the grid currents do not flow in this region.

$$i_{b_2} = g_m \left(e_{c_2} + \frac{e_{b_2}}{\mu}\right) = g_m \left\{-E_{bb} + \frac{e}{\mu} + e_{b_1}(1 + \frac{1}{\mu})\right\} \qquad (A.17)$$

$$i_{R_2} = \frac{e_{R_2}}{R} = \frac{E_{bb} - e - e_{R_1}}{R} \qquad (A.18)$$

$$i_{b_1} = g_m \left(e_{c_1} + \frac{e_{b_1}}{\mu}\right) = g_m \left\{e - E_{bb} + e_{b_1}(1 + \frac{1}{\mu})\right\} \qquad (A.19)$$

$$i_{R_1} = \frac{e_{R_1}}{\mu} = \frac{e_s + E_{bb} - e_{b_1}}{R} \quad . \qquad (A.20)$$

From Fig. A.1,

$$i_{b_2} - i_{R_2} = i_{R_1} - i_{b_1} = i \qquad (A.21a)$$

or

$$g_m \left\{-E_{bb} + \frac{e}{\mu} + e_{b_1}(1 + \frac{1}{\mu})\right\} - \frac{E_{bb} - e - e_{b_1}}{R}$$

$$= \frac{e_s + E_{bb} - e_{b_1}}{R} - g_m \left\{e - E_{bb} + e_{b_1}(1 + \frac{1}{\mu})\right\} \quad . (A.21b)$$

Solving for e_{b_1} in the above equation yields

$$e_{b_1} = \frac{\mu\{e_s + 2\,E_{bb}(g_m R + 1)\}}{2\{\mu + g_m R + \mu g_m R\}} - \frac{e}{2} \quad . \qquad (A.22)$$

Substituting this value of e_{b_1} in the expressions for i_{b_2} and i_{R_2} gives

$$i_{b_2} = g_m \left[-E_{bb} + \frac{e}{2\mu}(2 - \mu - 1) + \frac{(\mu+1)\{e_s + 2E_{bb}(g_m R+1)\}}{2\{\mu + g_m R + \mu g_m R\}}\right] \qquad (A.23)$$

$$i_{R_2} = \frac{E_{bb}}{R} - \frac{e}{2R} - \frac{\mu\{e_s + 2E_{bb}(g_m R + 1)\}}{2R\{\mu + g_m R + \mu g_m R\}} \quad . \qquad (A.24)$$

Therefore

$$i = i_{b_2} - i_{R_2} = \frac{e}{2\mu R}(\mu + g_m R - \mu g_m R) + \frac{e_s}{2R} \quad . \qquad (A.25)$$

Thus the i-e characteristic in region (1), where both the tubes carry current, is a straight line having a slope $di/de = (\mu + g_m R - \mu g_m R)/2\mu R$ which is negative if $\mu g_m R < \mu + g_m R$, and intersecting the e axis at

$$e = \frac{-e_s \mu}{\mu + g_m R - \mu g_m R} \quad .$$

(see Figs. 2.2 and A.3). The range of e in this region is

$$\frac{-\{2E_{bb} + (\mu+1)e_s\}\mu}{(\mu + g_m R + \mu g_m R)(\mu - 1)} < e < \frac{+\{2E_{bb} + (\mu+1)e_s\}\mu}{(\mu + g_m R + \mu g_m R)(\mu - 1)} \quad . \qquad (A.26)$$

A.3 For Only One Tube Carrying Current and No Grid Current

(a) Let tube (2) be carrying current and $i_{b_1} = 0$. Then

$$i = i_{b_2} - i_{R_2} = i_{R_1} \qquad (A.27a)$$

or

$$g_m\left\{-E_{bb} + \frac{e}{\mu} + e_{b_1}\left(1 + \frac{1}{\mu}\right)\right\} - \frac{E_{bb} - e - e_{b_1}}{R} = \frac{e_s + E_{bb} - e_{b_1}}{R} \quad . \qquad (A.27b)$$

By solving for e_{b_1} from the above equation, and by substituting in the expression for i,

$$i = \frac{e(\mu + g_m R) + E_{bb}(g_m R) + e_s(g_m R + \mu g_m R + \mu)}{R(g_m R + \mu g_m R + 2\mu)} \quad . \qquad (A.28)$$

Therefore the i-e characteristic in region (2_2 (see Figs. 2.2 and A.3), where only tube (2) is carrying current, is a straight line of positive slope

$$\frac{di}{de} = \frac{g_m R + \mu}{R(g_m R + \mu g_m R + 2\mu)}$$

and intersecting the e axis at

$$e = \frac{-\{E_{bb}(g_m R) + e_s(g_m R + \mu g_m R + \mu)\}}{\mu + g_m R} .$$

The range of e in this region is

$$\frac{-(E_{bb} g_m R - e_s \mu)}{\mu + g_m R} < e < \frac{-\{2E_{bb} + (\mu+1)e_s\}\mu}{(\mu + g_m R + \mu g_m R)(\mu - 1)} . \qquad (A.29)$$

(b) Let tube (1) be carrying current, and $i_{b_2} = 0$. Then

$$i = i_{R_1} - i_{b_1} = -i_{R_2} \qquad (A.30a)$$

or

$$\frac{e_s + E_{bb} - e_{b_1}}{R} - g_m \left\{ e - E_{bb} + e_{b_1}\left(1 + \frac{1}{\mu}\right)\right\} = -\frac{E_{bb} - e - e_{b_1}}{R} . \qquad (A.30b)$$

By solving for e_{b_1} and substituting in the expression for i,

$$i = \frac{e(\mu + g_m R) + E_{bb}(-g_m R) + \mu e_s}{R(g_m R + \mu g_m R + 2\mu)} . \qquad (A.31)$$

Therefore the i-e characteristic in the region (2_1) (see Figs. 2.2 and A.3), where tube (1) alone is carrying current, is a straight line of positive slope

$$\frac{di}{de} = \frac{\mu + g_m R}{R(g_m R + \mu g_m R + 2\mu)}$$

and which intersects the e axis at

$$e = \frac{E_{bb}(g_m R) - \mu e_s}{\mu + g_m R} .$$

The range of e in this region is

$$\frac{\{2E_{bb} + (\mu+1)e_s\}\mu}{(\mu + g_m R + \mu g_m R)(\mu - 1)} < e < \frac{E_{bb} g_m R - \mu e_s}{\mu + g_m R} . \qquad (A.32)$$

A.4 For Only One Tube Carrying Current and Grid Current Flowing

(a) Let i_{b_2} and i_{c_2} be flowing, and $i_{b_1} = 0$. Consider region (2_2), where i_{b_2} is flowing and $i_{b_1} = 0$ but there is no grid current. In this region,

$$e_{c_2} = e_{b_1} - E_{bb} \tag{A.33a}$$

$$= \frac{e_s\mu - e(\mu + g_m R) - E_{bb} g_m R}{g_m R + \mu g_m R + 2\mu} \quad . \tag{A.33b}$$

The grid current in tube (2) flows for $e_{c_2} > 0$, or for

$$e < \frac{e_s\mu - E_{bb} g_m R}{\mu + g_m R} \quad .$$

The derivation of the i-e characteristic when grid current i_{c_2} is flowing in tube (2) follows. Here $i_{b_1} = 0$ and $i_{c_1} = 0$. Therefore

$$i = i_{b_2} - i_{R_2} = i_{R_1} - i_{c_2} \quad . \tag{A.34}$$

In this region, as e becomes more and more negative, e_{c_2} becomes more and more positive, and e_{b_2} becomes more and more negative. Therefore, in regions where i_{c_2} is flowing, the situation can be represented approximately by the following equations:

For $e_{b_2} > e_{c_2}$ in region $(3_2'')$ (see Fig. A.3),

$$i_{b_2} = g_m'' \left(e_{c_2} + \frac{e_{b_2}}{\mu''} \right) \tag{A.35}$$

$$i_{c_2} = g_n'' \left(e_{b_2} + \frac{e_{c_2}}{\mu_g''} \right) \quad . \tag{A.36}$$

For $e_{b_2} < e_{c_2}$ in region $(3_2')$ (see Fig. A.3),

$$i_{b_2} = g_m' \left(\frac{e_{b_2}}{\mu} \right) \tag{A.37}$$

$$i_{c_2} = g_n' \left(e_{b_2} + \frac{e_{c_2}}{\mu_g'} \right) \tag{A.38}$$

In region $(3_2'')$, where $e_{b_2} > e_{c_2}$,

$$i = i_{b_2} - i_{R_2} = i_{R_k} - i_{c_2} \tag{A.39a}$$

or

$$\left[g_m'' \left\{ \frac{e + e_{b_1}}{\mu''} + (e_{b_1} - E_{bb}) \right\} \right] - \frac{E_{bb} - e - e_{b_1}}{R}$$

$$= \frac{e_s + E_{bb} - e_{b_1}}{R} - g_n'' \left\{ (e + e_{b_1}) + \frac{e_{b_1} - E_{bb}}{\mu_g''} \right\} . \tag{A.39b}$$

Solving for e_{b_1} from the above equation and substituting in the expression for i gives

$$i = \frac{1}{\frac{g_m''}{\mu''} + g_m'' + g_n'' + \frac{g_n''}{\mu_g''} + \frac{2}{R}} \left\{ e \left[\frac{g_m''}{\mu''} \frac{g_n''}{\mu_g''} - g_m'' g_n'' + \frac{1}{R} \left(\frac{g_n''}{\mu_g''} + \frac{g_m''}{\mu''} + \frac{1}{R} \right) \right] \right.$$

$$\left. + E_{bb} \left[\frac{g_m''}{\mu''} \frac{g_n''}{\mu_g''} - g_m'' g_n'' + \frac{1}{R} \left(\frac{g_m''}{\mu''} - g_n'' \right) \right] + \frac{e_s}{R} \left(\frac{g_m''}{\mu''} + g_m'' + \frac{1}{R} \right) \right\} , \tag{A.40}$$

which gives the straight-line portion of the i-e characteristic in region $(3_2'')$ (see Fig. A.3). The range of e in this region is

$$-E_{bb} < e < \frac{e_s \mu - E_{bb} g_m R}{g_m R + 1} . \tag{A.41}$$

Similarly in region $(3_2')$, where $e_{b_2} < e_{c_2}$ and consequently $e < -E_{bb}$, the i-e characteristic consists of still another straight line whose equation is

$$i = \frac{1}{\frac{g'_m}{\mu'} + g'_n + \frac{g'_n}{\mu'_g} + \frac{2}{R}} \left[e \left\{ \frac{g'_m}{\mu'} \frac{g'_n}{\mu'_g} + \frac{1}{R} \left(\frac{g'_m}{\mu'} + \frac{g'_n}{\mu'_g} + \frac{1}{R} \right) \right\} \right.$$

$$\left. + E_{bb} \left\{ \frac{g'_m}{\mu'} \frac{g'_n}{\mu'_g} + \frac{1}{R} \left(\frac{g'_m}{\mu'} - g'_n \right) \right\} + \frac{e_s}{R} \left(\frac{g'_m}{\mu'} + \frac{1}{R} \right) \right] . \qquad (A.42)$$

(b) Let currents i_{b_1} and i_{c_1} be flowing, with $i_{b_2} = i_{c_2} = 0$. Then

$$i = i_{R_1} - i_{b_1} = i_{c_1} - i_{R_2} . \qquad (A.43)$$

In this region, after i_{c_1} starts flowing, e becomes more and more positive, and consequently e_{b_1} becomes more and more negative and e_{c_1} becomes more and more positive. Therefore in regions where i_{c_1} is flowing, the situation can be represented approximately by the following equations:
For $e_{b_1} > e_{c_1}$ in region $(3''_1)$,

$$i_{c_1} = g''_n e_{b_1} + \frac{g''_n}{\mu''_g} e_{c_1} \qquad (A.44)$$

$$i_{b_1} = \frac{g''_m}{\mu''} e_{b_1} + g''_m e_{c_1} . \qquad (A.45)$$

For $e_{b_1} < e_{c_1}$ in region $(3'_1)$,

$$i_{b_1} = \frac{g'_m}{\mu'} e_{b_1} \qquad (A.46)$$

$$i_{c_1} = g'_n e_{b_1} + \frac{g'_n}{\mu'_g} e_{c_1} . \qquad (A.47)$$

Proceeding in a manner similar to the one used for regions $(3''_2)$ and $(3'_2)$, the straight-line portions of the characteristic for regions $(3''_1)$ and $(3'_1)$ can be represented by the following two equations:

$$i = \frac{1}{\frac{g''_m}{\mu''} + g''_m + g''_n + \frac{g''_n}{\mu''_g} + \frac{2}{R}} \left[e \left\{ \frac{g''_m}{\mu''} \frac{g''_n}{\mu''_g} - g''_m g''_n + \frac{1}{R}\left(\frac{g''_m}{\mu''} + \frac{g''_n}{\mu''_g} + \frac{1}{R} \right) \right\} \right.$$

$$\left. + E_{bb} \left\{ g''_m g''_n - \frac{g''_m}{\mu''} \frac{g''_n}{\mu''_g} + \frac{1}{R}\left(g''_n - \frac{g''_m}{\mu''} \right) \right\} + \frac{e_s}{R}\left(g''_n + \frac{g''_n}{\mu''_g} + \frac{1}{R} \right) \right] \qquad (A.48)$$

and

$$i = \frac{1}{\frac{g'_m}{\mu'} + g'_n + \frac{g'_n}{\mu'_g} + \frac{2}{R}} \left[e \left\{ \frac{g'_m}{\mu'} \frac{g'_n}{\mu'_g} + \frac{1}{R}\left(\frac{g'_m}{\mu'} + \frac{g'_n}{\mu'_g} + \frac{1}{R} \right) \right\} \right.$$

$$\left. + E_{bb} \left\{ \frac{1}{R}\left(g'_n - \frac{g'_m}{\mu'} \right) - \frac{g'_m}{\mu'} \frac{g'_n}{\mu'_g} \right\} + \frac{e_s}{R}\left(g'_n + \frac{g'_n}{\mu'_g} + \frac{1}{R} \right) \right] . \qquad (A.49)$$

These i-e characteristics with parameter e_s have been plotted in Fig. A.3, Fig. 3.2 of Chapter 3, and Fig. 2.2 of Chapter 2 (for $e_s = 0$) for some typical circuit parameters.

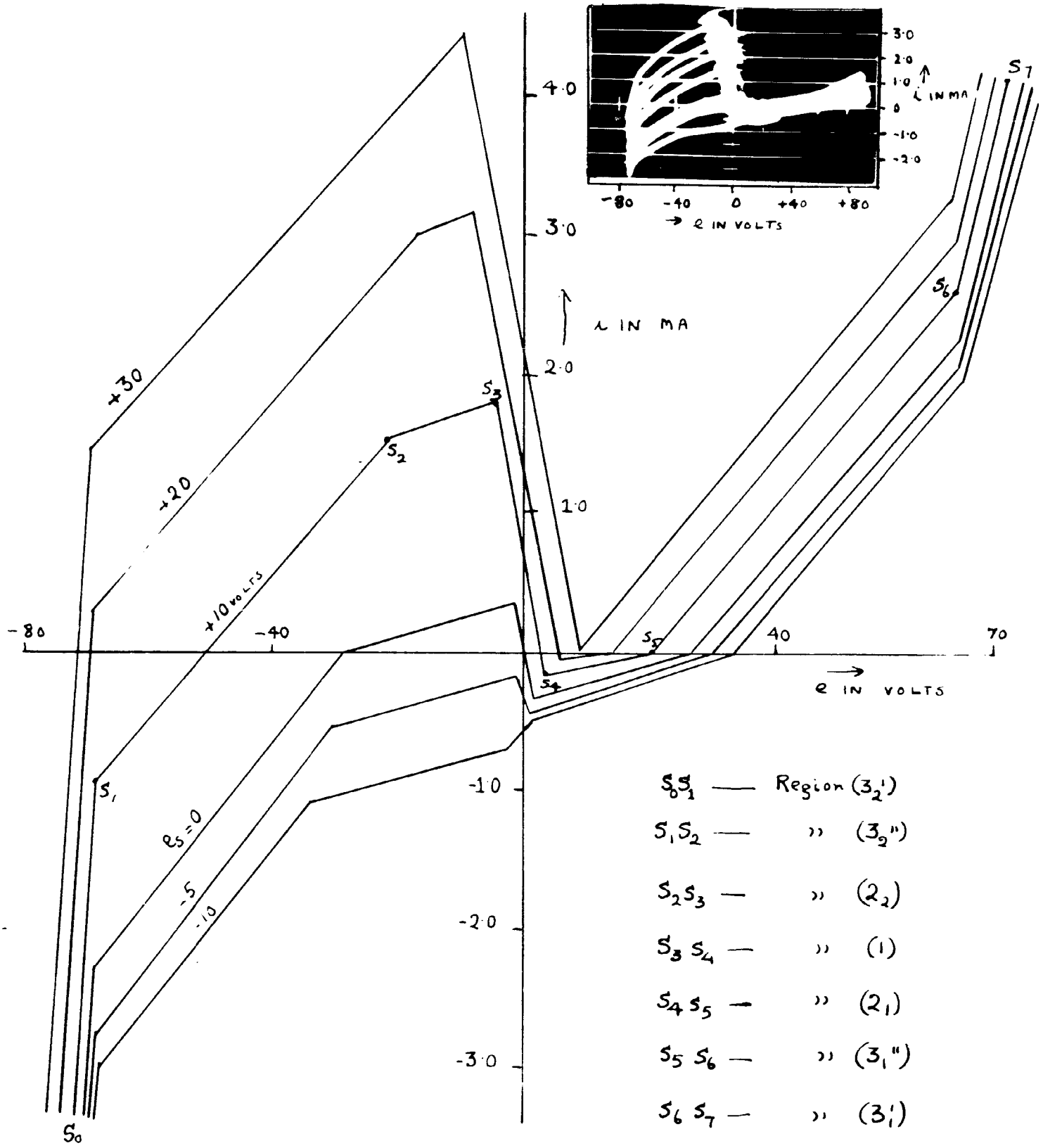


FIGURE A.3

ANALYTICALLY DERIVED VOLTAGE-CURRENT CHARACTERISTICS BETWEEN THE TWO PLATES, WITH e_s AS PARAMETER

The oscillogram shows the i-e characteristics for e_s = - 10, - 4, 0, + 6, + 10, + 15, + 23 volts.

APPENDIX B

PROOF OF EQUIVALENT CIRCUIT

In deriving the d-c current-voltage characteristics in Appendix A, the capacitances in the circuit were not considered. Fig. B.1 shows the same circuit with the six capacitances $C_{g_1p_1}$, $C_{g_1k_1}$, $C_{k_1p_1}$, $C_{g_2p_2}$, $C_{g_2k_2}$, and $C_{k_2p_2}$ connected between the several electrodes. The stray capacitances can be included in these six capacitances.

B.1 For Both Tubes Carrying Current

In this region, grid currents may have to be included for some particular circuit parameters and some particular values of e_s and E_{bb}. But for the particular circuit considered here, these values have been so chosen that grid currents do not flow in this region.

The nodal equations for the circuit at points X and Y are respectively,

$$i_{g_1p_1} + i_{R_1} + i_{k_1p_1} + i_{k_2g_2} - i_{g_2p_2} - i_{b_1} = 0 \qquad (B.1)$$

and

$$i_{g_2p_2} + i_{R_2} + i_{k_2p_2} + i_{k_1g_1} - i_{g_1p_1} - i_{b_2} = 0 \ . \qquad (B.2)$$

$$i_{b_1} = g_m\left(e_{c_1} + \frac{e_{b_1}}{\mu}\right) = g_m\left\{e - E_{bb} + e_{b_1}\left(1+\frac{1}{\mu}\right)\right\} \qquad (B.3)$$

$$i_{b_2} = g_m\left(e_{c_2} + \frac{e_{b_2}}{\mu}\right) = g_m\left\{- E_{bb} + \frac{e}{\mu} + e_{b_1}\left(1+\frac{1}{\mu}\right)\right\} \qquad (B.4)$$

$$i_{R_1} = \frac{e_s + E_{bb} - e_{b_1}}{R} \qquad (B.5)$$

$$i_{R_2} = \frac{E_{bb} - e - e_{b_1}}{R} \qquad (B.6)$$

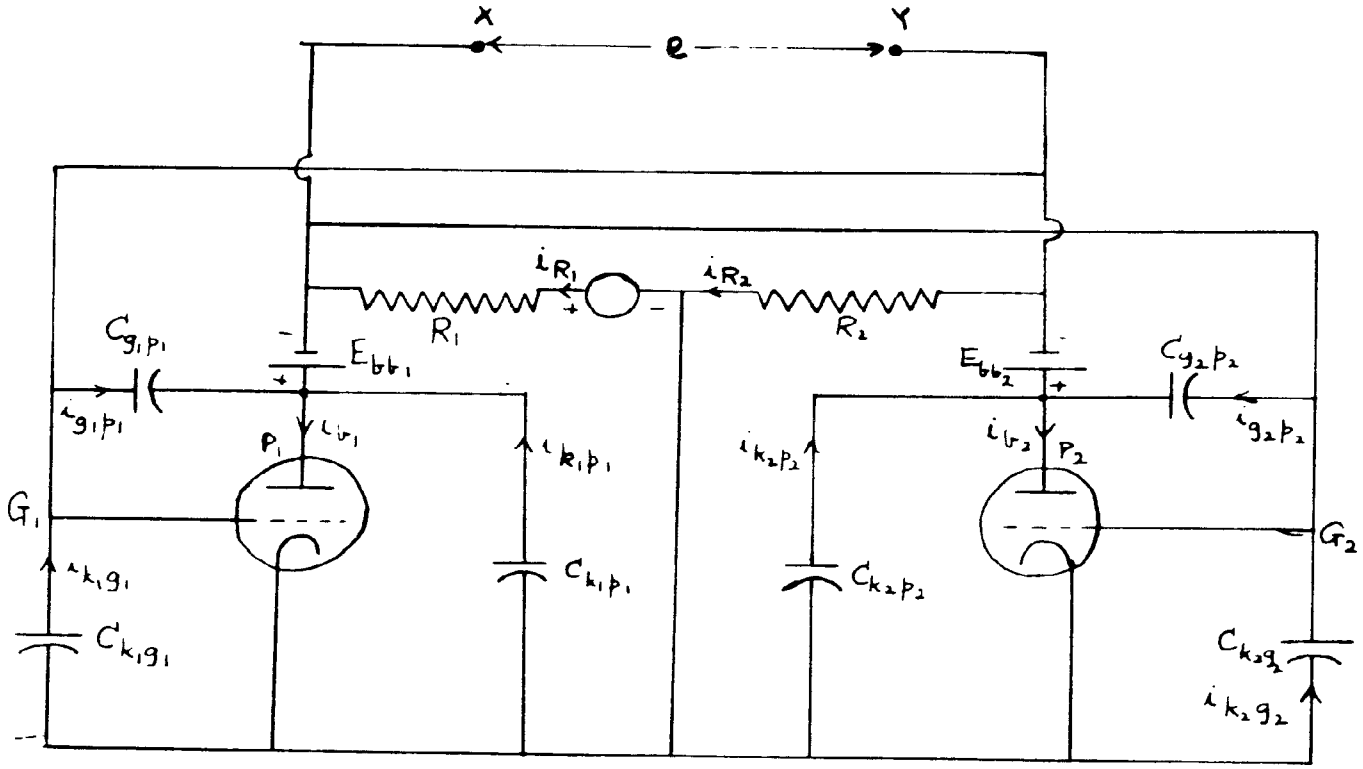


FIGURE B 1

ECCLES-JORDAN CIRCUIT, SHOWING THE INTERELECTRODE CAPACITANCES

For symmetry, $R_1 = R_2 = R$; $E_{bb_1} = E_{bb_2} = E_{bb}$; $C_{g_1p_1} = C_{g_2p_2} = C_{gp}$;

$C_{k_1p_1} = C_{k_2p_2} = C_{kp}$; $C_{k_1g_1}$ $C_{k_2g_2} = C_{kg}$.

$$i_{k_1p_1} = C_{k_1p_1} \frac{d}{dt}(-e_{b_1}) = C_{kp} \frac{d}{dt}(-e_{b_1}) \quad (B.7)$$

$$i_{k_1g_1} = C_{k_1g_1} \frac{d}{dt}(-e_{c_1}) = C_{kg} \frac{d}{dt}(-e - e_{b_1}) \quad (B.8)$$

$$i_{g_1p_1} = C_{g_1p_1} \frac{d}{dt}(e_{c_1} - e_{b_1}) = C_{gp} \frac{d}{dt}(e) \quad (B.9)$$

$$i_{k_2p_2} = C_{k_2p_2} \frac{d}{dt}(-e_{b_2}) = C_{kp} \frac{d}{dt}(-e - e_{b_1}) \quad (B.10)$$

$$i_{k_2g_2} = C_{k_2g_2} \frac{d}{dt}(-e_{c_2}) = C_{kg} \frac{d}{dt}(-e_{b_1}) \quad (B.11)$$

$$i_{g_2p_2} = C_{g_2p_2} \frac{d}{dt}(e_{c_2} - e_{b_2}) = C_{gp} \frac{d}{dt}(-e) \quad (B.12)$$

By substituting the above values of different currents in equations (B.1) and (B.2),

$$C_{gp} \frac{d}{dt}(e) + \frac{e_s + E_{bb} - e_{b_1}}{R} + C_{kp} \frac{d}{dt}(-e_{b_1}) + C_{kg} \frac{d}{dt}(-e_{b_1})$$

$$- C_{gp} \frac{d}{dt}(-e) - g_m \left\{ e - E_{bb} + e_{b_1}\left(1 + \frac{1}{\mu}\right) \right\} = 0 \quad (B.13)$$

and

$$C_{gp} \frac{d}{dt}(-e) + \frac{E_{bb} - e - e_{b_1}}{R} + C_{kp} \frac{d}{dt}(-e - e_{b_1}) + C_{kg} \frac{d}{dt}(-e - e_{b_1})$$

$$- C_{gp} \frac{d}{dt}(e) - g_m \left\{ -E_{bb} + \frac{e}{\mu} + e_{b_1}\left(1 + \frac{1}{\mu}\right) \right\} = 0 . \quad (B.14)$$

By subtracting (B.13) from (B.14) and dividing by 2,

$$\frac{e(\mu + g_m R - \mu g_m R)}{2\mu R} + \left[2C_{gp} + \frac{C_{kp} + C_{kg}}{2} \right] \frac{d}{dt}(e) + \frac{e_s}{2R} = 0 . \quad (B.15)$$

Equation (B.15) states that, looking at the circuit across X and Y, the whole circuit can be replaced by an equivalent circuit composed of a resistance of value $2\mu R/(\mu + g_m R - \mu g_m R)$ in parallel with a capacitance

$2C_{gp} + (C_{kp} + C_{kg})/2$. If the capacitance term is neglected, then

$$\frac{e(\mu + g_m R - \mu g_m R)}{2R} + \frac{e_s}{2R} = 0 , \qquad (B.16)$$

which is the same as equation (A.25) of Appendix A (with i = 0).

Therefore, when both the tubes are carrying current, looking across the two plates the circuit can be replaced by a parallel combination of a resistance whose value is the reciprocal of the slope of the d-c characteristic and a capacitance $C = 2C_{gp} + (C_{kp} + C_{kg})/2$.

B.2 For Only One Tube Carrying Current, and No Grid Current

Let $i_{b_2} = 0$. Then equation (B.15) becomes

$$\left[2C_{gp} + \frac{C_{kp} + C_{kg}}{2}\right]\frac{d}{dt}(e) + \frac{1}{2}(g_m - \frac{1}{R})e + \frac{g_m}{2}(1 + \frac{1}{\mu})e_{b_1} - \frac{1}{2}(g_m E_{bb} + \frac{e_s}{R}) = 0 \quad (B.17)$$

Adding equations (B.13) and (B.14) and dividing by 2 yields

$$\frac{e}{2}(g_m + \frac{1}{R}) + \left(\frac{C_{kg} + C_{kp}}{2}\right)\frac{d}{dt}(e) + \frac{e_{b_1}}{2}\left\{(1 + \frac{1}{\mu})g_m + \frac{2}{R}\right\}$$
$$+ (C_{kg} + C_{kp})\frac{d}{dt}(e_{b_1}) - \frac{E_{bb}}{2}(g_m + \frac{2}{R}) - \frac{e_s}{2R} = 0 . \qquad (B.18)$$

Eliminating e_{b_1} between equations (B.17) and (B.18) gives

$$\left[\left\{2C_{gp} + \left(\frac{C_{kp} + C_{kg}}{2}\right)\right\}\left(\frac{C_{kp} + C_{kg}}{2}\right)\left\{\frac{4\mu R}{g_m R + \mu g_m R + 2\mu}\right\}\right]\frac{d^2e}{dt^2}$$

$$+ \left[\left\{2C_{gp} + \left(\frac{C_{kp} + C_{kg}}{2}\right)\right\} + \left(\frac{C_{kp} + C_{kg}}{2}\right)\left(\frac{2\mu + g_m R - \mu g_m R}{2\mu + g_m R + \mu g_m R}\right)\right]\frac{de}{dt}$$

$$+ \left[\frac{g_m R + \mu}{R(g_m R + \mu g_m R + 2\mu)}\right]e + \left[\frac{-E_{bb} g_m R + \mu e_s}{R(g_m R + \mu g_m R + 2\mu)}\right] = 0 , \qquad (B.19)$$

which is of the form

$$a_1 \frac{d^2e}{dt^2} + a_2 \frac{de}{dt} + a_3 e = a_4 \quad . \qquad \text{(B 20)}$$

If the capacitances are neglected, equation (B 19) reduces to equation (A.31) of Appendix A (with i = 0)

The solution of equation (B.20) is of the form

$$e = K_1 \epsilon^{+m_1 t} + K_2 \epsilon^{+m_2 t} + \frac{a_4}{a_3} \quad , \qquad \text{(B.21)}$$

where K_1 and K_2 are constants of integration which can be determined by the boundary conditions. Also

$$m_1 = \frac{-a_2 - \left|\sqrt{a_2^2 - 4a_1a_3}\right|}{2a_1} \qquad \text{(B.22)}$$

$$m_2 = \frac{-a_2 + \left|\sqrt{a_2^2 - 4a_1a_3}\right|}{2a_1} \qquad \text{(B.23)}$$

a_1 = (a capacitance)^2 x (a resistance)

a_2 = a capacitance

a_3 = a conductance

a_4 = a current.

Therefore equation (B.20) states that, when only one tube is carrying current and there is no grid current, the circuit can be replaced across the two plates by a parallel combination of a resistance of value $1/a_3$, a capacitance of value a_2, and a third element a_1.

Since the voltage across the capacitance cannot change instantly, e cannot change instantly, which requires e to be continuous at a boundary. $a_1(d^2e/dt^2)$ is a current. Therefore de/dt cannot change instantly, because if it did d^2e/dt^2 would become infinite, and there would be infinite current in the element a_1, which is not possible. Therefore de/dt has to be

continuous at a boundary.

Hence the two boundary conditions for the differential equation (B.20) are:

(1) e has to be continuous at a boundary, and

(2) de/dt has to be continuous at a boundary.

If equation (B.20) is solved, using these boundary conditions and substituting numerical values, it can be seen that $|m_1| >> |m_2|$ and $|K_1| << |K_2|$ (see Appendix D). Hence approximately the solution of equation (B.20) is

$$e = K_2 \epsilon^{m_2 t} + \frac{a_4}{a_3} , \qquad (B.24)$$

where

$$m_2 \cong - \frac{a_3}{a_2} .$$

Therefore equation (B.20) is approximately

$$a_2 \frac{de}{dt} + a_3 e = a_4 . \qquad (B.25)$$

Thus even in this region, where only one tube is carrying current and there is no grid current, the circuit can be approximately replaced by a parallel combination of a resistance of value $(\mu + g_m R)/R(g_m R + \mu g_m R + 2\mu)$, which is the reciprocal of the slope of the d-c characteristic curve in this region and a capacitance of approximate value equal to $2C_{gp} + (C_{kp} + C_{kg})/2$.

B.3 For Only One Tube Carrying Current and Grid Current Flowing

Let $i_{b_2} = 0$ and i_{c_1} be flowing. Then equations (B.1) and (B.2) become

$$i_{g_1 p_1} + i_{R_1} + i_{k_1 p_1} + i_{k_2 g_2} - i_{g_2 p_2} - i_{b_1} = 0 \qquad (B.26)$$

$$i_{g_2 p_2} + i_{R_2} + i_{k_2 p_2} + i_{k_1 g_1} - i_{g_1 p_1} - i_{c_1} = 0 . \qquad (B.27)$$

Proceeding in the same manner as when there is no grid current, the differential equations satisfied by e in regions $(3_1')$ and $(3_1'')$ (see Fig. A.3) are:

$$\frac{1}{\frac{g_m''}{\mu''} + g_m'' + g_n'' + \frac{g_n''}{\mu_g''} + \frac{2}{R}} \left[\left\{ 4\left(\frac{C_{kp} + C_{kg}}{2}\right)\left(2C_{gp} + \frac{C_{kp} + C_{kg}}{2}\right)\right\} \frac{d^2e}{dt^2} \right.$$

$$+ \left\{ \left(\frac{C_{kg}+C_{kp}}{2}\right)\left(\frac{g_m''}{\mu''} - g_m'' - g_n'' + \frac{g_n''}{\mu_g''} + \frac{2}{R}\right) + \left(2C_{gp} + \frac{C_{kp}+C_{kg}}{2}\right)\left(\frac{g_m''}{\mu''} + g_m'' + g_n'' + \frac{g_n''}{\mu_g''} + \frac{2}{R}\right)\right\} \frac{de}{dt}$$

$$+ \left\{ \frac{g_m''}{\mu''}\frac{g_n''}{\mu_g''} - g_m''\, g_n'' + \frac{1}{R}\left(\frac{g_m''}{\mu''} + \frac{g_n''}{\mu_g''} + \frac{1}{R}\right)\right\} e$$

$$\left. + \left\{ E_{bb}\left(g_m'' g_n'' - \frac{g_m''}{\mu''}\frac{g_n''}{\mu_g''} + \frac{1}{R}\left[g_n'' - \frac{g_m''}{\mu''}\right]\right) + \frac{e_s}{R}\left(g_n'' + \frac{1}{R} + \frac{g_n''}{\mu_g''}\right)\right\}\right] = 0 \quad . \qquad (B.28)$$

and

$$\frac{1}{\frac{g_m'}{\mu'} + g_n' + \frac{g_n'}{\mu_g'} + \frac{2}{R}} \left[\left\{ 4\left(2C_{gp} + \frac{C_{kp} + C_{kg}}{2}\right)\left(\frac{C_{kp} + C_{kg}}{2}\right)\right\} \frac{d^2e}{dt^2} \right.$$

$$+ \left(2C_{gp} + \frac{C_{kp}+C_{kg}}{2}\right)\left(\frac{g_m'}{\mu'} + g_n' + \frac{g_n'}{\mu_g'} + \frac{2}{R}\right) + \left(\frac{C_{kp}+C_{kg}}{2}\right)\left(\frac{g_m'}{\mu'} + \frac{g_n'}{\mu_g'} - g_n' + \frac{2}{R}\right)\Bigg\} \frac{de}{dt}$$

$$+ \left\{ \frac{g_m'}{\mu'}\frac{g_n'}{\mu_g'} + \frac{1}{R}\left(\frac{g_m'}{\mu'} + \frac{g_n'}{\mu_g'} + \frac{1}{R}\right)\right\} e$$

$$\left. + \left\{ E_{bb}\left(\frac{1}{R}\left[g_n' - \frac{g_m'}{\mu'}\right] - \frac{g_m'}{\mu'}\frac{g_n'}{\mu_g'}\right) + \frac{e_s}{R}\left(g_n' + \frac{g_n'}{\mu_g'} + \frac{1}{R}\right)\right\}\right] = 0 \quad . \qquad (B.29)$$

If the capacitances are neglected in equations (B.28) and (B.29), they reduce to equations (A.48) and (A.49) of Appendix A (with i = 0). By

substituting numerical values in these two equations, and using similar arguments to those used for equation (B.20), it can be shown that, in the grid current region also, the circuit can approximately be replaced between the two plates by a resistance whose value is given by the reciprocal of the slope of the characteristic in parallel with a capacitance of approximate value equal to $2C_{gp} + (C_{kp} + C_{kg})/2$.

Thus approximately the whole circuit between the two plates can be replaced in all regions of the characteristic curve by an equivalent circuit consisting of a parallel combination of a resistance given by the reciprocal of the slope of the characteristic curve and a capacitance of approximate value equal to $2C_{gp} + (C_{kp} + C_{kg})/2$.

It can be shown that this approximation can be used only for a symmetrical circuit and for certain typical circuit parameters. But for a more general type of circuit, the term $a_1(d^2e/dt^2)$ cannot be neglected, and a_2 is quite different for the different regions of the characteristic curve. Chapter 4 describes a method of studying the triggering action without making use of this approximation.

APPENDIX C

EQUIVALENCE OF A RECTANGULAR CURRENT PULSE BETWEEN GRID AND CATHODE TO A RECTANGULAR VOLTAGE PULSE IN SERIES WITH GRID

Figures C.1(a) and (b) show respectively a constant current i_s introduced between G_2 and K, and a constant voltage e_s introduced in series with R_1. The relationship which should hold between i_s and e_s so that both arrangements give the same e_{c_2} and i_{c_2}, and consequently the same effect on the whole circuit, will now be derived.

In Fig. C.1(a),

$$R_1 \, i_{R_1} = e_{c_2} \tag{C.1}$$

$$C_{k_2 g_2} \frac{de_{c_2}}{dt} = i_{c_2} \, . \tag{C.2}$$

Therefore

$$i_{c_2} = i_s + i_{R_1} + i_{k_2 g_2} \tag{C.3a}$$

$$= i_s + \frac{e_{c_2}}{R_1} + C_{k_2 g_2} \frac{de_{c_2}}{dt} \, . \tag{C.3b}$$

In Fig. C.1(b),

$$R_1 \, i_{R_1}' + e_s = e_{c_2}' \tag{C.4}$$

$$C_{k_2 g_2} \frac{de_{c_2}}{dt} = i_{k_2 g_2} \tag{C.5}$$

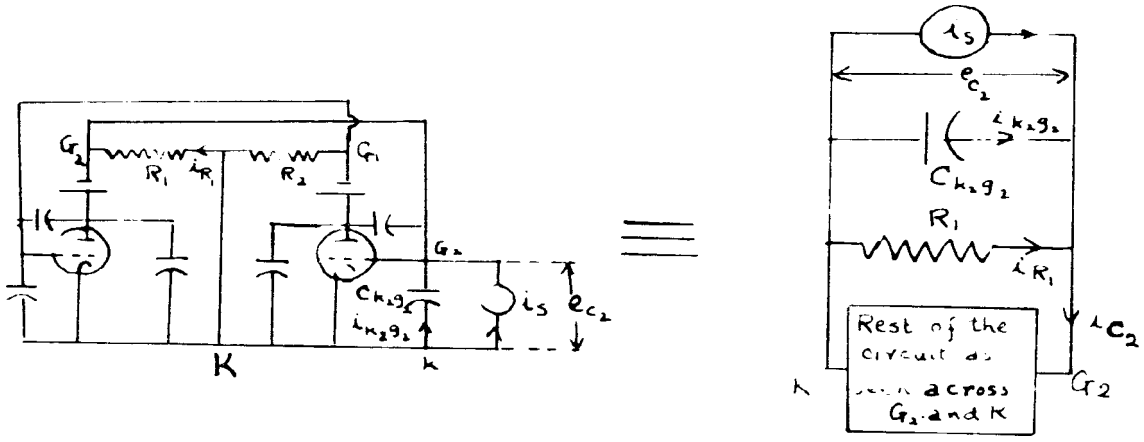
$$i_{c_2} = i_{R_1}' + i_{k_2 g_2}' \tag{C.6}$$

$$i_{R_1}' = \frac{e_{c_2} - e_s}{R_1} \, . \tag{C.7}$$

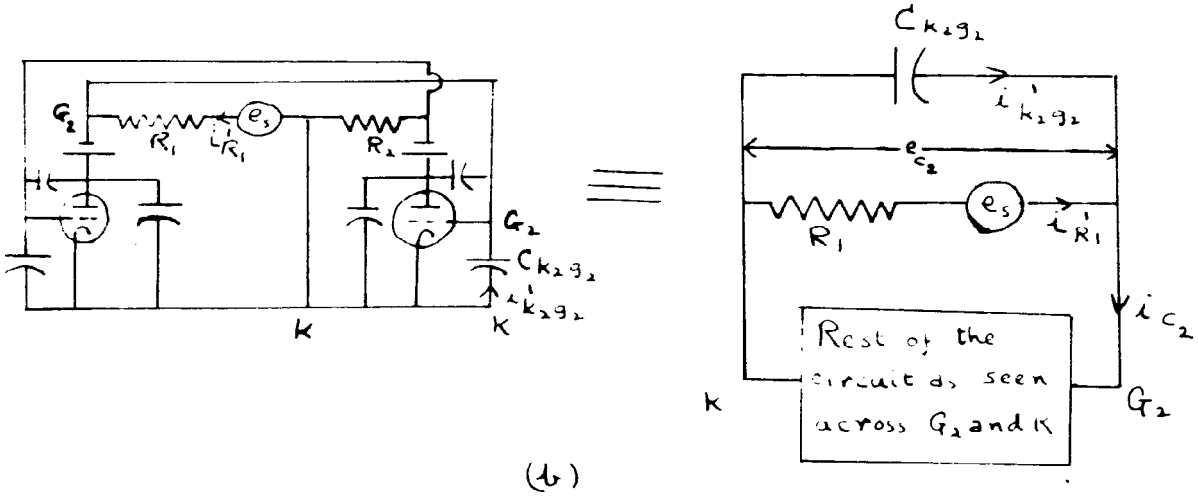
Therefore

$$i_{c_2} = \frac{e_{c_2} - e_s}{R_1} + C_{k_2 g_2} \frac{de_{c_2}}{dt} \tag{C.8a}$$

$$= - \frac{e_s}{R_1} + \frac{e_{c_2}}{R_1} + C_{k_2 g_2} \frac{de_{c_2}}{dt} \, . \tag{C.8b}$$



(a)



(b)

FIGURE C.1

ECCLES-JORDAN CIRCUIT WITH (a) CURRENT PULSE BETWEEN G_2 AND K (b) VOLTAGE PULSE IN SERIES WITH R_1

By comparing equations (C.3b) and (C.8b),

$$i_s = -\frac{e_s}{R_1} \quad \text{or} \quad e_s = -i_s R_1 \ . \tag{C.9}$$

Then

$$i_{k_2 g_2} = C_{k_2 g_2} \frac{de_{c_2}}{dt} = i_{k_2 g_2}' \ . \tag{C.10}$$

Hence if a constant voltage e_s is introduced in series with R_1, and if $e_s = -i_s R_1$, then the values of e_{c_2} and i_{c_2} obtained are the same as the values of e_{c_2} and i_{c_2} which result if a constant current i_s is introduced between G_2 and K. Since the rest of the circuit between G_2 and K is the same in both the arrangements, it is evident that the effect of the two arrangements on the rest of the circuit between points G_2 and K is the same. Therefore the introduction of a rectangular current pulse of magnitude i_s and duration t_1 between G_2 and K is equivalent to the introduction of a voltage pulse of magnitude $e_s = -i_s R_1$ and duration t_1 in series with R_1.

APPENDIX D

TYPICAL NUMERICAL CALCULATIONS

D.1 Circuit Data

In the circuit investigated, shown in Fig. 2.1, the following circuit parameters have been used:

(1) Tubes (1) and (2) are the two identical halves of a 6SN7 tube.

(2) $R_1 = R_2 = 7500$ ohms.

(3) $E_{bb_1} = E_{bb_2} = 68$ volts.

(4) $E_{cc} = 0$.

In assuming identical straight-line tube characteristics (see Appendix A) for the two tubes, the following average values have been used for the tube parameters:

$\mu = 20$, $g_m = 2000$ μmhos;

$\mu' = 0$, $g_m' = 0$, $\frac{g_m'}{\mu'} = 1000$ μmhos, $\mu_g' = -3.2$, $g_n' = -800$ μmhos;

$\mu'' = 35$, $g_m'' = 3500$ μmhos, $\mu_g'' = -.01$, $g_n'' = -10$ μmhos;

$C_{gp} = 4$ $\mu\mu$F, $C_{kg} = 3$ $\mu\mu$F, $C_{kp} = 1.2$ $\mu\mu$F.

D.2 Calculation of the d-c i-e Characteristics

Using the results obtained in Appendix A, the slope di/de and the range of e in the different regions for $e_s = 0$ have been calculated as shown in Table D.1 at the top of the following page.

The i-e characteristics for different values of e_s have been calculated similarly and plotted in Figs. A.3, 3.2, and 2.2.

TABLE D.1

Computation of d-c i-e Characteristic Between the Two Plates for $e_s = 0$

Region	Reciprocal of the slope in ohms	Range in volts
(1)	- 1,131	$-0.45 < e < +0.45$
(2_1)	75,000	$+0.45 < e < +29.0$
(2_2)	75,000	$-29.0 < e < -0.45$
$(3_1'')$	16,666	$+29.0 < e < +68.0$
$(3_2'')$	16,666	$-68.0 < e < -29.0$
$(3_1')$	1,000	$+68.0 < e$
$(3_2')$	1,000	$e < -68.0$

D.3 Justification for Assuming Equivalent Circuit of Only a Resistance and a Capacitance in Parallel

In equation (B.19) of Appendix B, let, for instance, $e_s = +50$ volts.

$$a_1 = \left(2C_{gp} + \frac{C_{kp} + C_{kg}}{2}\right)\left(\frac{C_{kp} + C_{kg}}{2}\right)\left\{\frac{4R}{g_m R(1+\mu) + 2\mu}\right\}$$

$$= 3.38 \times 10^{4} \text{ ohms(farads)}^2 .$$

$$a_2 = \left[\left(2C_{gp} + \frac{C_{kp} + C_{kg}}{2}\right) + \frac{C_{kp} + C_{kg}}{2}\left(\frac{2\mu + g_m R - \mu g_m R}{2\mu + g_m R + \mu g_m R}\right)\right]$$

$$= 9 \times 10^{-12} \text{ farads} \cong \left(2C_{gp} + \frac{C_{kp} + C_{kg}}{2}\right).$$

$$a_3 = \frac{g_m R + \mu}{R(g_m R + \mu g_m R + 2\mu)} = 0.0000133 \text{ mhos} .$$

$$a_4 = \frac{E_{bb} g_m R - \mu e_s}{R(g_m R + \mu g_m R + 2\mu)} = 8 \times 10^{-6} \text{ amperes} .$$

Therefore equation (B.19) is

$$3.38 \times 10^{4} \frac{d^2e}{dt^2} + 9 \times 10^{-12} \frac{de}{dt} + 0.0000133\, e + 8 \times 10^{-6} = 0$$

and its solution is

$$e = K_1 \epsilon^{m_1 t} + K_2 \epsilon^{m_2 t} + \frac{a_4}{a_3} ,$$

where

$$m_1 = \frac{-a_2 - \left|\sqrt{a_2^2 - 4a_1a_3}\right|}{2a_1} = -2.68 \times 10^{-4}$$

$$m_2 = \frac{-a_2 + \left|\sqrt{a_2^2 - 4a_1a_3}\right|}{2a_1} = -1.5 \times 10^{-6} .$$

It is evident that $|m_1| >> |m_2|$.

$$e = K_1 \epsilon^{-2.68 \times 10^{-4} t} + K_2 \epsilon^{-1.5 \times 10^{-6} t} + 0.6 .$$

Using the initial conditions that when $t = 0$, $e = +29$ volts and $de/dt = 0$, K_1 and K_2 are found to be 0.001 and 28.4 respectively. It is obvious that $|K_1|$ is very much less than $|K_2|$. Hence

$$e \cong K_2 \epsilon^{-1.5 \times 10^{-6} t} + 0\,6 .$$

Therefore

$$e \cong K_2 \epsilon^{-ta_3/a_2} + \frac{a_4}{a_3} ,$$

which shows that the circuit can be approximately replaced by an equivalent circuit consisting of a resistance $1/a_3$ in parallel with a capacitance $a_2 = 2C_{gp} + (C_{kp} + C_{kg})/2$.

D.4 Computation of (1) Minimum Magnitude E_{1min} and Minimum Duration t_{1min} of Pulse Necessary for Triggering, (2) Time of Transition, and (3) Optimum Duration t_{1opt} of Pulse for Shortest Time of Transition

Let r = 50,000 ohms. From the calculated characteristic curves it can be seen that r_1 = 1131 ohms, r_2 = 75,000 ohms, and $i_2 = 3.87 \times 10^{-4}$ amperes.

$$e_M = \frac{-i_2 r_2 r}{r_2+r} = -11.61 \text{ volts}, \qquad e_N = +11.61 \text{ volts}.$$

$$e_{S_3} = \frac{-i_2 r_2 r_1}{r_2+r_1} = -0.45 \text{ volts}, \qquad e_{S_4} = +0.45 \text{ volts}.$$

$$(1)\quad E_{1min} = \frac{i_2 r_2 (r-r_1)}{r_2+r_1} = 18.63 \text{ volts}.$$

For E_1 = 22 volts,

$$t_{1min} = t_Q = t_0 = (t_{S_3} - t_M) + (t_0 - t_{S_3})$$

$$= \frac{Crr_2}{r+r_2} \log_\epsilon \left[\frac{e_M - \frac{(E_1 - ri_2)r_2}{r+r_2}}{e_{S_3} - \frac{(E_1 - ri_2)r_2}{r+r_2}} \right] + \frac{Crr_1}{r-r_1} \log_\epsilon \left[\frac{0 + \frac{E_1 r_1}{r-r_1}}{e_{S_3} + \frac{E_1 r_1}{r-r_1}} \right]$$

$$= 1.16\, T \ ,$$

where $T = C r$.

(2) For E_1 = 22 volts and $t_1 = 1.3428\, T$,

$$t_r = t_1 + (t_{0.98e_N} - t_1) \quad \text{(using the definition of } t_r \text{ given in Chapter 2)}$$

$$e_1 = K_{21}' \epsilon^{-(r+r_2)t_1/Crr_2} + \frac{(E_1 + ri_2)r_2}{r+r_2}$$

$$= 10.63 \text{ volts}.$$

Therefore

$$t_r = t_1 + (t_{0.98e_N} - t_1)$$

$$= t_1 + \frac{Crr_2}{r+r_2} \log_\epsilon \left[\frac{e_1 - \frac{ri_2r_2}{r+r_2}}{0.98e_N - \frac{ri_2r_2}{r+r_2}} \right] = 2.0948 \text{ T} .$$

(3) For E_1 = 22 volts,

$$t_{1opt} = t_{S_4} + (t_N - t_{S_4})$$

$$= t_{1min} + \frac{Crr_1}{r-r_1} \log_\epsilon \left[\frac{e_{S4} + \frac{E_1r_1}{r-r_1}}{0 + \frac{E_1r_1}{r-r_1}} \right] + \frac{Crr_2}{r+r_2} \log_\epsilon \left[\frac{e_{S4} - \frac{(E_1+ri_2)r_2}{r+r_2}}{e_N - \frac{(E_1+ri_2)r_2}{r+r_2}} \right]$$

$$= 1.16 \text{ T} + 0.0128 \text{ T} + 1.1728 \text{ T}$$

$$= 1.5408 \text{ T}$$

Similarly t_{1min}, t_{1opt}, and t_r are calculated for different values of E_1 and t_1, and curves of (a) t_{1min} and t_{1opt} versus E_1, (b) t_r versus E_1 for fixed t_1, and (c) t_r versus t_1 for fixed E_1 have been plotted in Fig. 2.8 of Chapter 2.

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