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FROTH STRATIFICATION AND LIQUID MIXING
IN A BUBBLE TRAY COLUMN

by

Ronald D. Crozier

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Ronald D. Crozier

The hydraulic characteristics of both bubble-cap and sieve trays were studied in a 13- by 7.5-inch rectangular column. The bubble-cap plate had nine 1-1/2-inch bubble-caps on a 2-1/2-inch square pitch. Each cap had eighteen 1/8-inch by 3/4-inch slots, which extended to the tray floor. The sieve tray had 13 by 13 rows of 11/64-inch drilled holes on 7/16-inch square spacing. The air-water system was used for all runs.

The purpose of the investigation was to determine the effect of operating variables on the local density distribution in the froth and the degree of liquid mixing. The range of operating variables studied were liquid rates from 1 to 4.30 gal/min-inch of weir, gas superficial velocities from 0.6 to 4.5 ft/sec, and weir heights of 1, 2, 3, and 3-1/2 inches.

The fractional extinction of a light beam transmitted by the froth was found to be a function of local froth density. The local froth density on a tray was found to stratify into three layers. High speed photographs of the froth showed the three layers to be due to: (a) sheets of gas around caps, giving a local density near the pure liquid density; (b) break-up of the gas sheets into bubbles above the caps resulting in a variable density layer; and (c) coagulation of the bubbles formed into a constant density layer. The constant density layer was found to occur only if the clear liquid height in the active area of the tray was higher than the total cap height. The sieve tray always showed a constant froth layer. The specific interfacial area (ft^2/ft^3 of froth) available for mass transfer was found proportional to the froth layer density rather than the average bulk froth density, explaining discrepancies found in mass transfer rates determined by different investigators operating columns with clear liquid heights above and below cap height.

The relation between point (E_{OG}) and plate (E_{MV}) Murphree efficiencies, including the effect of mixing, was found to be:

$$E_{MV} = \left(\frac{1 + \gamma}{\lambda} \right) \left(\exp \frac{E_{OG} \lambda}{1 + \gamma} - 1 \right) \quad (23)$$

where $\lambda = mV/L$; m is the slope of equilibrium line; V is the gas rate; and L is the liquid rate.

The degree of liquid mixing was measured in the column using a dye tracer and colorimetric analysis. The relative mixing on the tray was found to increase slightly with increasing gas rate and decreasing liquid rate, but varied little from $\gamma = 1$ for the operating conditions studied, and γ , the mixing parameter, is defined in terms of unsteady state tracer decay as follows:

$$\mu(\theta) = C_o \exp \left(- \frac{\gamma L \theta}{A_c} \right)$$

where $\mu(\theta)$ = tracer concentration at time θ

C_o = constant

θ = elapsed time, secs.

A_c = clear liquid cross section, ft^2

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ABSTRACT

The hydraulic characteristics of both bubble-cap and sieve trays were studied in a 13- by 7.5-inch rectangular column. The bubble-cap plate had nine 1-1/2-inch bubble-caps on a 2-1/2-inch square pitch. Each cap had eighteen 1/8-inch by 3/4-inch slots, which extended to the tray floor. The sieve tray had 13 by 13 rows of 11/64-inch drilled holes on 7/16-inch square spacing.

The purpose of the investigation was to determine the effect of operating variables on the local density distribution in the froth and the degree of liquid mixing. The range of operating variables studied were liquid rates from 1 to 4.30 gal/min-inch of weir, gas superficial velocities from 0.6 to 4.5 ft/sec, and weir heights of 1, 2, 3, and 3-1/2 inches.

The fractional extinction of a light beam transmitted by the froth was found to be a function of local froth density. The local froth density on a tray was found to stratify into three layers. High speed photographs of the froth showed the three layers to be due to: (a) sheets of gas around caps, giving a local density near the pure liquid density; (b) break-up of the gas sheets into bubbles above the caps resulting in a variable density layer; and (c) coagulation of the bubbles formed into a constant density layer. The constant density layer was found to occur only if the clear liquid height in the active area of the tray was higher than the total cap height. The sieve tray always showed a constant froth layer. The specific interfacial area (ft²/ft³ of froth) available for mass transfer was found proportional to the froth layer density rather than the average bulk froth density, explaining discrepancies found in mass transfer rates determined by different investigators operating columns with clear liquid heights above and below cap height.

The froth heights and average bulk froth densities measured in this investigation, and data available in the literature, were found to correlate as follows, regardless of tray design:

$$\ln Z_f = 0.64 F + 0.25 L S^{-1/3} + 0.33 Z_w - 0.1 \quad (16)$$

and

$$-\ln \phi = 0.715 F + 0.45 \quad (17)$$

where Z_f is froth height, inches; Z_c is clear liquid height, inches; $\phi = Z_f/Z_c$, average bulk density; $F = V_s \sqrt{\rho G}$, ft/sec $\sqrt{\text{lb/cu.ft}}$; L is the liquid rate, gpm/inch; S is the active plate length, ft; and Z_w is weir height, inches.

A mixing parameter, γ , was derived from a modified Prandtl mixing length concept. The relation between point (EOG) and plate (E_{MV}) Murphree efficiencies, including the effect of mixing, was found to be:

$$E_{MV} = \left(\frac{1 + \gamma}{\lambda} \right) \left(\exp \frac{EOG \lambda}{1 + \gamma} - 1 \right) \quad (23)$$

where $\lambda = mV/L$; m is the slope of equilibrium line; V is the gas rate; and L is the liquid rate.

The degree of liquid mixing was measured in the column using a dye tracer and colorimetric analysis. The relative mixing on the tray was found to increase slightly with increasing gas rate and decreasing liquid rate, but vary little from $\gamma = 1$ for the operating conditions studied.

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INTRODUCTION

Gas-liquid contacting is the basic step in the recovery and purification of many industrially important substances. Although spray chambers, wetted wall towers and packed towers are used for a wide variety of small-scale processes, most large-scale purification of gases and liquids in the chemical and petroleum industries is done in tray-type contacting devices.

The hydraulic characteristics of bubble-cap and sieve trays have already been studied extensively with the object of predicting gas and liquid capacities of particular designs. The influence on mass transfer of the behavior of the froth mass on the tray is less completely understood.

The object of this investigation was to study, in a pilot scale tray, the properties of the gas-liquid froth that affect mass transfer, and the degree of liquid mixing on the tray. The basic equipment for this study consisted of a one-plate air-water simulator, equipped with a 13 x 7.5 inch rectangular tray containing nine 1-1/2 inch bubble caps laid out on 2-1/2 inch square pitch. A few runs were made with a sieve tray of a similar stability range to the bubble-cap tray.

The tray design used was chosen so that the results might be coordinated with mass transfer data from towers with identical tray layout used by the American Institute of Chemical Engineers Tray Efficiency Research Program at North Carolina State College and The University of Michigan.

Previous attempts at correlating mass transfer data as a function of hydraulic variables on a tray have indicated that the assumption of

constant froth density was inadequate to explain discrepancies found between the work of various investigators. To test the uniformity of the froth on a tray, a new technique, using light transmission, was developed to explore the local density in the tray froth. From point density data obtained with this method, the existence of three density layers on the tray under certain bubbling conditions was demonstrated.

A bubbling mechanism, based on photographic studies of the froth, is proposed, rationalizing the conditions which result in the existence or non-existence of froth density stratification.

The degree of liquid mixing on the test tray was studied using a dye tracer technique. A characterization factor for mixing was developed from a modified Prandtl-Taylor mixing length concept. Equations were derived using the proposed mixing factor to relate Murphree plate (E_{MV}) and point (E_{OG}) efficiencies. The proposed mixing factor and resultant equations have been tested using mass transfer data available in the literature.

General hydraulic data taken on the test tray included froth and clear liquid heights, but did not include hydraulic gradients along the tray, as the short test section was not appropriate to this type of measurement. Measured froth heights and bulk froth densities were empirically correlated with data available in the literature.

A detailed discussion of the above phenomena theories and data forms the major portion of this work.

REVIEW OF PREVIOUS WORK

Because of the complex and discontinuous nature of the mass transfer process on plate contactors, the design of this type of equipment usually involves the calculation of hypothetical equilibrium stages, which are then corrected for fractional attainment of equilibrium by means of a plate efficiency. The most useful efficiency term, from the designer's point of view, is the overall plate efficiency, which yields actual contacting stages as a simple multiple of ideal stages.

A comprehensive review of plate efficiency literature has been presented by Warzel (73). The following discussion emphasizes the hydraulic variables necessary in correlating plate efficiencies, and extends Warzel's survey on liquid mixing and general turbulence.

Efficiency Correlations

Overall Efficiencies

Empirical correlations of overall column efficiency have been proposed by Drickamer and Bradford (20), and O'Connell (50). Both these correlations relate the column efficiency to the feed viscosity. O'Connell's correlation also includes the relative volatility of the key component.

Point Efficiencies

As the overall column efficiency cannot be related to the actual rate process occurring, Murphree plate and point efficiencies have been proposed for a more fundamental correlation of mass transfer in tray columns. The point efficiency, E_{OG} , is defined as the fractional approach to equilibrium

attained by the gas and liquid at a point in a plate, assuming no vertical change in concentration of the liquid.

Recently, point efficiencies have been related to the rate of mass transfer, using both the Whitman (76) two-film theory and the Higbie (36) penetration theory as models. If it is assumed that the mechanism of contact occurring on a plate can be approximated by individual bubbles rising through the liquid, the two-film theory predicts the following relation:

$$E_{OG} = 1 - \exp \left(\frac{-PK_{Gat}}{G} \right) \quad (1)$$

Walter and Sherwood (72) empirically related the variables in equation (1) as follows:

$$\frac{PK_{Gat}}{G} = \frac{h}{(2.50 + 0.37/HP) \mu^{0.68} w_s^{0.33}} \quad (2)$$

where

H - Henry's law constant, lb moles/(cu. ft.)(atm)

and

w_s - slot width, inches

other variables defined in the nomenclature.

Essentially, this equation assumes that t/G is a constant function of the liquid submergence, h .

Geddes (28) later derived an efficiency relation directly from equation (1), using wetted wall data to evaluate K_G , and theoretical relations derived from a perfectly spherical bubble rising in still water to obtain "a" and "t". As Geddes pointed out, the approach represents an oversimplification, as the behavior of bubbles in an actual froth is very different from spherical bubbles in a still liquid.

Recently, Gerster et al. (1, 29, 30, 31, 32, 33, 34), have separated the liquid and gas phase resistances and proposed the following equation:

$$\frac{1}{-\ln(1 - E_{OG})} = \frac{G}{k_G a Z_V} + \frac{mG}{k_L a Z_V} = \frac{1}{N_G} + \frac{m G/L}{N_L} \quad (3)$$

where

Z_V - distance a gas bubble contacts the liquid,
or effective liquid depth.

The number of transfer units are calculated:

$$N_G = k_G a R T t_G = k'_G a t_G \quad (4a)$$

and

$$N_L = k'_L a t_L \quad (4b)$$

The gas contact time, t_G , is calculated from the relation:

$$t_G = (Z_f - Z_c)/V_s \quad (4c)$$

and the liquid contact time, t_L , from:

$$t_L = \epsilon/L = Z_c A_a/L \quad (4d)$$

Stone (62) has derived an efficiency correlation based on a similar mechanistic analysis (single bubbles rising through the liquid), but applying Higbie's(36) penetration theory rather than the two-film analogy. Stone derives the following equation:

$$E_{OG} = A B \quad (5a)$$

where

$$A = C_1 \frac{D_G Z}{V_s w_s^{2/3}} \quad (5b)$$

and

$$B = C_2 \frac{\sqrt{D_L} T \rho_L w_s^{1/3}}{D_G H} \quad (5c)$$

where the nomenclature corresponds to the one used in this work. The interfacial

area was assumed proportional to the slot width " w_s ". The variable, Z , apparently was intended to be froth height.*

Bakowski (8), based on high speed motion pictures of bubbling from a single cap, has proposed a different mechanism for the gas-liquid contacting on a tray. Noting that the bubbles formed at a submerged slot are sausage-shaped and inter-connected up to the surface of the liquid, Bakowski proposed that the gas contacts the liquid via a series of oblong channels rising from the individual slots. On this basis, the amount of mass transferred at a point in a tray can be calculated using mass transfer coefficients for wetted wall columns; the interfacial area is approximated from the height and perimeter of the channelette, and the contact time from the channelette height and gas velocity through the channelette. The final equation derived by Bakowski applies only to gas phase controlling systems.

As can be seen from the foregoing, both the two-film theory and the Danckwerts-Higbie penetration theory emphasize the importance in mass transfer of the gas-liquid contacting distance and the volumetric interfacial area, indicating the necessity of further study of these two variables.

Mixing on Trays

Effect of Mixing on Efficiencies

The point efficiencies obtained from the above correlations are not directly applicable to column design calculations. Overall column efficiency has been related to Murphree efficiencies by Lewis (44a) and Colburn (15)

* Stone does not define Z in his nomenclature table, but refers to it in the text as liquid height. From the following statement on page 118 of the thesis, it has been deduced that $Z = Z_f$: "These data [Klein (43)] reveal that for low weir heights the liquid height on the plate is roughly equal to twice that of the un-aerated downstream liquid height, which is calculated by adding the height of the weir crest to the weir height."

for reasonably idealized cases. The relation between point and Murphree efficiencies has been found to be a function of the degree of liquid mixing on the plate. The point and plate efficiencies are equal for perfect mixing (no concentration gradients). For no mixing or plug flow, Lewis (44b) has derived the following equation relating point and plate efficiency:

$$E_{MV} = \frac{1}{\lambda} [\exp (\lambda E_{OG}) - 1] \quad (6)$$

From this equation it can be seen that the greatest difference between E_{OG} and E_{MV} occurs for liquid phase controlling systems where λ is large. Only very recently have attempts been made to quantitatively relate liquid mixing in columns. Stone (62) has proposed an empirical correction to equation (6), where mixing is assumed inversely proportional to the liquid flow path.

The simplest model for characterizing mixing has been put forward by Oliver and Watson (51) and Warzel (73). These investigators proposed that mixing in a small column can be approximated by a hypothetical stream carrying liquid from the exit downcomer to the inlet weir. The degree of mixing is then indicated by the concentration jump at the inlet weir. They derive the following equation relating point and plate efficiencies:

$$E_{MV} = \frac{C}{\lambda} [\exp (\frac{\lambda E_{OG}}{C}) - 1] \quad (7)$$

where

$$C = \frac{x_i - x_o}{x_e - x_o}$$

and

$C = 1$ for no mixing and $C = \infty$ for perfect mixing.

Kirschbaum (41), Nord (49) and Gautreaux and O'Connell (27) proposed that the mixing of a long tray would have the same effect as the existence of a series of perfectly mixed pools along the length of the tray. Gautreaux

and O'Connell derive the following equation relating point and plate efficiency:

$$E_{MV} = \frac{1}{\lambda} \left[\left(1 + \frac{E_{OG} \lambda}{n} \right)^n - 1 \right] \quad (8)$$

where n - number of pools.

They postulated " n " to be a function of the liquid flow path length and the liquid and gas rates.

The most rigorous equation on mixing so far has been proposed by Anderson (4), who applied the eddy diffusivity concept to bubble trays and derived:

$$1 + \lambda E_{MV} = \frac{1 - A_1/2M}{(1 - A_1/A_2)\exp(1/2 A_1)} + \frac{1 - A_2/2M}{(1 - A_2/A_1)\exp(1/2 A_2)} \quad (9)$$

where

$$A_1 = M + \sqrt{M^2 + 4 MN}$$

$$A_2 = M - \sqrt{M^2 + 4 MN}$$

$$M = \frac{S L}{D_E Z_f d M_L \phi} = \frac{S L'}{D_E Z_c M_L}$$

$$N = \lambda E_{OG} \quad .$$

The nomenclature, otherwise, has been changed to conform with this work. The factor ϕ , froth specific gravity, has been added to the parameter M , to make M dimensionally consistent.

Measurement of Tray Mixing

Byfield (11), Stone (62), Brown (9) and Wharton (75) have measured eddy diffusivities in bubble trays. Over the narrow range of variables tested, they found eddy diffusivities of 70 to 150 ft²/hour.

A certain amount (1, 3, 73, 77) of concentration gradient data is available in the literature. In general, no attempt has been made in these studies to quantitatively relate the point efficiencies obtained to plate efficiencies also measured.

A considerable amount of published material is available on mixing in batch equipment. Most of the mixing parameters and correlations proposed are specific to propeller and jet mixing and are not applicable to bubble-cap columns. This is primarily a consequence of the more complex process occurring on a tray, where multiple gas jets are transferring energy to a moving liquid.

Of more interest to bubble tray mixing are the general theories of turbulence, such as the Prandtl-Taylor theory (63, 64, 65) and the Danckwerts (17) statistical description of mixing. These will be discussed in detail in the theoretical section on mixing.

Hydraulic Studies in Tray Columns

A number of studies have been made on the effects on tray capacity of hydraulic gradient, entrainment and flooding. The most extensive are (6, 7, 12, 14, 22, 38, 39, 52, 58, 59).

Very few published studies of hydraulic effects on mass transfer are available. The series by Gerster et al. (1, 29-34), and the theses of Ashby (5), Freshwater (24), Polich (53), Warzel (73) and Winslow (77) are the most extensive. Individual articles will be reviewed critically in the comparison of results from this investigation.

Bubbling from Submerged Orifices

A considerable amount of data on regular bubbling from simple capillary orifices have been published (10, 16, 19, 26, 35, 37, 40, 70). A good review of the equipment used and the data obtained is given by Hughes et al. (37). For small orifices and capillaries, Hughes et al. found a strong effect of air chamber volume on bubble volume and frequency, which probably explains discrepancies between different investigators. With the exception

of Johnson et al. (40), all these studies have been made with relatively small orifices discharging into a quiet liquid. Johnson et al. found no apparent difference between their data, taken with liquid cross-flow, and the data of other investigators, with the exception of differences explained by changes in the air chamber volume. Nearly all data published on bubble size and rise velocities have been taken with small single orifices located perpendicular to the direction of bubble rise. It is questionable whether this type of equipment approximates the conditions encountered in the bubbling from multiple vertical slots of bubble-cap trays.

Spells and Bakowski (60, 61) have published data on bubbling from single and multiple vertical slots. In their study of single slots they found two regimes of bubbling, which they designated as deep and shallow seal bubbling. In deep seal bubbling, the gas stream issuing from the slot broke up into individual bubbles with a definite frequency of formation; while in shallow seal bubbling, although chains of bubbles were visible, there was considerable leakage of gas directly from the slot to the liquid surface via the bubble chain. Their studies of multiple slot bubbling (a single complete bubble cap) indicated that over most commercial ranges of liquid seal and gas rate, the shallow mechanism occurred exclusively.

APPARATUS

The test equipment used in this study consisted of a one-plate simulator, identical in layout to the absorber used by Ashby (5) and Warzel (73), supplied with water from the mains and air from a blower. Both the air and water were not recirculated. Auxiliary measuring equipment will be described in detail.

Test Column

The test column consisted of a removable bubble-cap or sieve plate installed in an open top shell of identical inside dimensions to the absorber used by the A. I. Ch. E. Tray Efficiency Research Project.

The shell was constructed by the author and personnel of the Chemical and Metallurgical Engineering Department from 12 gage mild steel, and had a wind-box below the plate for the air supply and to house the outlet downcomer. Above the plate the shell extended 28 inches and was fitted with a Plexiglas front and back. A 1-1/2 inch galvanized coupling was welded into the inlet downcomer and an identical coupling was installed at the bottom of the outlet downcomer. The air supply entered through a 3 inch galvanized coupling welded into the middle of the back of the wind-box. Access to the wind-box was through a bolted cover over the front face. Both Plexiglas windows were bolted by means of a light frame directly to the shell. Pertinent dimensions and constructional details are given in Fig. 1.

The whole shell was supported at a convenient height by means of an angle-iron box frame rigidly bolted to the concrete floor. The angle-iron frame was made long enough to accomodate the instrument panel board.

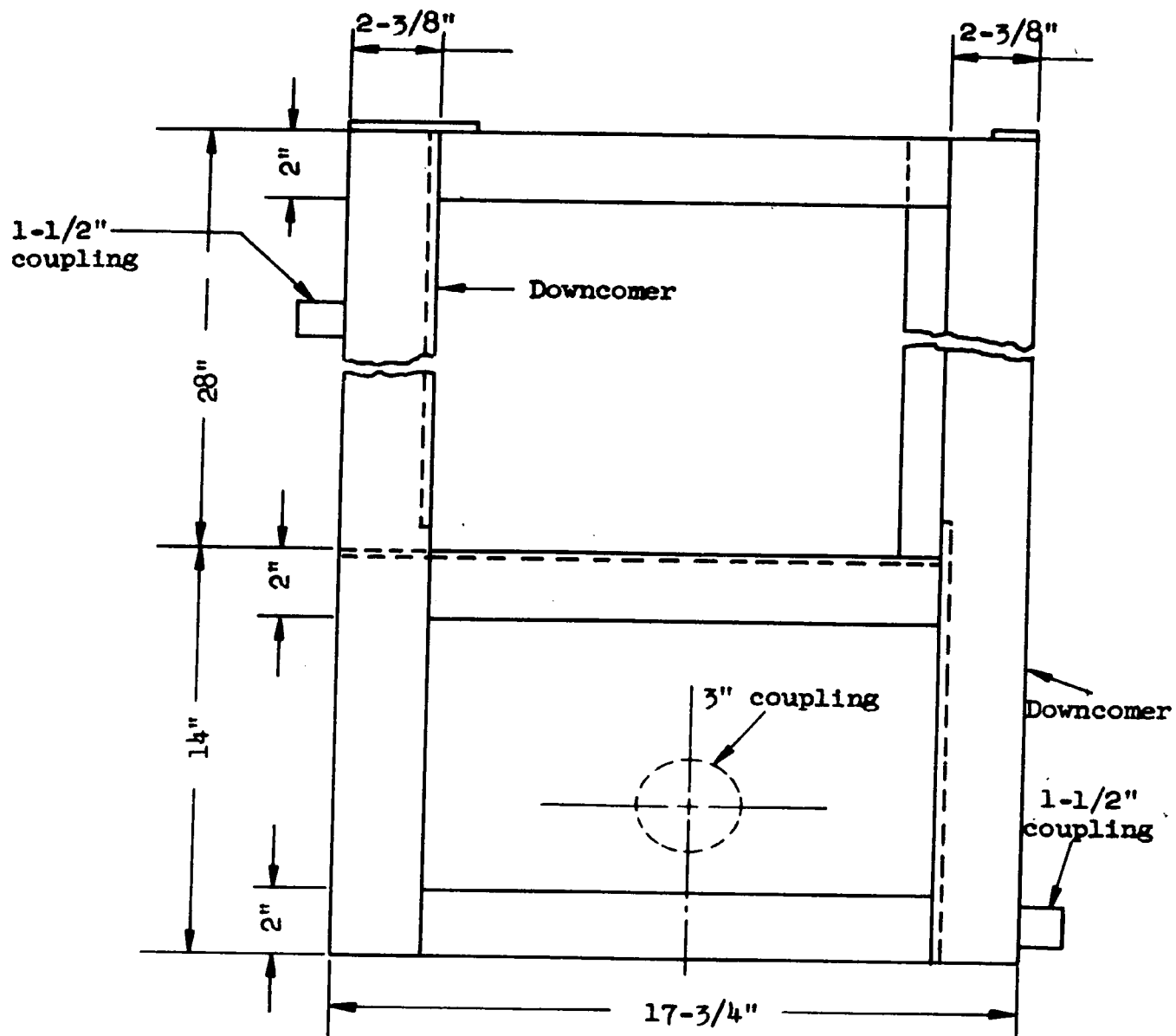


Fig. 1. Plan view of test column shell.

Bubble-Plate Design

Dimensions of the bubble-cap plate are shown in Fig. 2 and summarized in Table I. Figure 3 is a photograph of the actual plate, showing cap and riser details. Complete details of the construction of this plate are given by Warzel (73).

Sieve-Plate Design

A sieve tray was made of 16 gage brass sheet; the overall dimensions of the plate were identical to the bubble-plate, making the plates interchangeable.

The perforations consisted of 13 by 13 rows of drilled holes (with burrs ground off), $11/64$ inches in diameter, and laid out at $7/16$ inches on square spacing. The pertinent dimensions are given in Fig. 4. The total hole area was 0.027 sq. ft. The choice of tray was arbitrary, and was designed to give the same operating range as the bubble-cap tray. There was no weepage even with the combination of the highest liquid rate and smallest gas rate.

Weirs

A removable overflow weir was provided to permit adjustment of weir height from 1 to 3-1/2 inches. The removable weir was made of 16 gage brass and was channel-shaped to slip inside the downcomer. Liquid leakage around the weir was prevented by sealing all joints with electrical tape.

Splash Baffle

A splash baffle was installed 1 inch upstream of the overflow weir with the bottom of the baffle 1/2 inch above the top of the weir. A spare baffle from the A. I. Ch. E. absorber was used and consisted of a 16 gage Incoloy sheet 12-1/2 inches long by 7-3/8 inches wide. The baffle was

TABLE I

Characteristics of the Tray Layout

Cap	Diameter (O. D.)	1-1/2 inch
	Height	1-1/2 inch
	Metal Thickness	1/16 inch
Slot	Height	3/4 inch
	Width	1/8 inch
	Number, per cap	18
	Area, per cap	0.0117 sq. ft.
	Area, per plate	0.105 sq. ft.
	Area, fraction of bubbling area	0.171
Risers	Diameter (O. D.)	1 inch
	Diameter (I. D.)	7/8 inch
	Area, per cap	0.00417 sq. ft.
	Area, per plate	0.0375 sq. ft.
	Area, fraction of bubbling area	0.061
Weir (variable)	Length	7-3/8 inches
	Height	3-1/2, 3, 2 and 1 inches
Splash Baffle (variable)	Length	7-3/8 inches
	Clearance above tray floor	4, 3-1/2, 2-1/2, and 1-1/2 inches
Downcomer	Size	7-3/8 x 2-1/8 inches
	Area, cross sectional	0.137 sq. ft.
Bubbling Area		
(taken as space between downcomer and splash baffle)	Width	7-1/2 inches
	Length	11-13/16 inches
	Area	0.615 sq. ft.

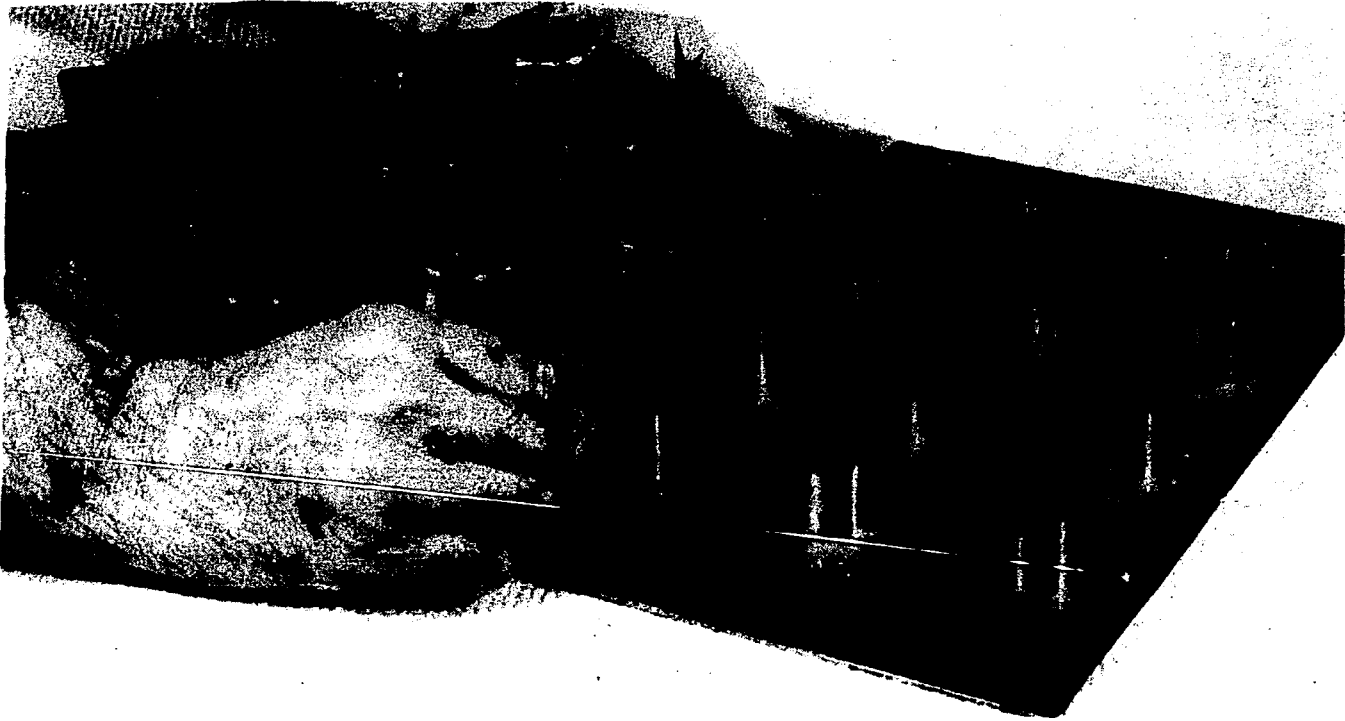


Fig. 3. Removable trays.

supported at the proper height by bolting to a support angle tack-welded to the back of the column. A foam rubber gasket between the front edge of the baffle and the window prevented leakage around the edge of the baffle.

Air Supply System

The air supply system included a separate blower, piping, a thermocouple, pressure tap and a rotameter. Both screw and flanged fittings were used with 3 inch galvanized pipe for the air supply.

Motor and Blower

Atmospheric air from the laboratory was drawn directly into the inlet of a two-lobe rotary blower, driven by a General Electric 7.5-horsepower, 230 volt, shunt-wound d-c motor, Model No. 5 B 294A14. Since the blower was obtained from Navy surplus and was apparently a special design, no exact

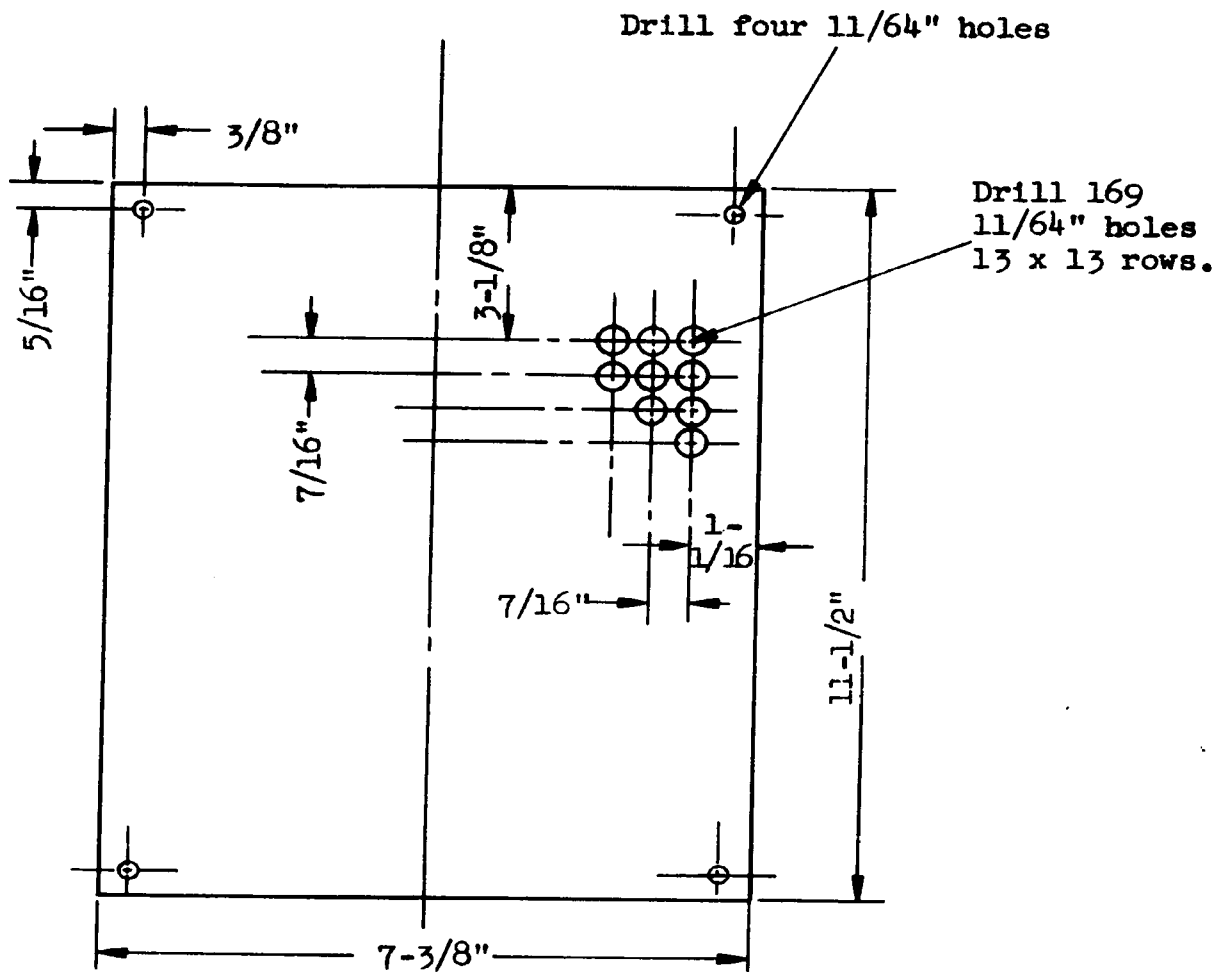


Fig. 4. Sieve tray layout.

specifications can be given, other than that it was built by the Sutorbilt Corp. and appeared to be similar to their blower No. 5-M, which has a displacement of 0.18 cubic feet of air per revolution, and a rated capacity of 115 cubic feet per minute at a discharge head of 4.9 psi.

The speed of the motor could be regulated by means of a rheostat in the field circuit, as well as by introducing resistance into the armature circuit. The resistor board for the latter circuit consisted of six fixed and two variable slide-wire resistors which could be wired in various parallel and series combinations to provide a wide range of resistance without overload.

Metering

Total gas flow to the column was metered by a size 12 Fischer and Porter Flowrator (Tube No. 12-LI25, Serial No. D8-1612), with a capacity of 0 - 200 standard cubic feet per minute of 0.877 specific gravity gas, at 14.7 psia and 60°F. The scale was graduated in increments of 2 scfm. The air supply system was used in a previous investigation by C. M. Thatcher (66). His calibration, which was made by material balances on oxygen-enriched air, was used in this study. Thatcher found the calibration method to be reproducible to ± 1.5 per cent.

Water Supply System

Ann Arbor city water was supplied to the test section through 1-1/2 inch galvanized piping on a once-through basis. The water discharged from the outlet downcomer into a 55 gallon drum which in turn discharged into a sewer. The drum compensated for the pressure drop of the line to the sewer and guarded against any unusual backing up of liquid in the downcomer.

Metering

Two identical size 8, series 700, Fischer and Porter Flowrators were used to meter the manually controlled water supply to the column. The first meter (Serial No. W 70-4024/2, tube No. B 9-27-10/70-G) was borrowed from Ashby's (5) equipment while awaiting delivery of the second meter, and was calibrated by Ashby by weighing the discharged water. This meter was used for all of the froth density runs. The mixing runs used a second (Serial No. 5601D1038B1) identical meter which was also calibrated by weighing the discharged water.

Auxiliary Equipment

Gas inlet, liquid inlet and liquid outlet temperatures were measured with iron-constantan thermocouples connected to a common cold junction by a 3 point Mallory selector switch. Millivolt readings were made with a Leeds and Northrup portable precision potentiometer (Model 8662, Serial No. 773919).

The pressure in the gas rotameter was measured with a 30 inch mercury-filled Meriam manometer. The gage pressure in the air-chamber was measured with a water-filled 30 inch Meriam manometer.

Clear liquid heights in the inlet and outlet calming section, and between the first and second row of caps on the tray, was measured with probes connected to a manometer board. The probes consisted of 4 mm glass tubing inserted into the tray through brass fittings screwed into the Plexiglas window. The tip of the probe was bent at a right angle and placed with the opening approximately 1/16 inch above the tray floor.

Average froth heights were measured visually by means of two scales taped to the Plexiglas window.

Light Transmission Measurements

The light transmitted by the froth for both the density and the mixing runs was measured with the same photocell and measuring circuit.

Photocell

An R. C. A. 931-A, 10-stage multiplier phototube, was the basic measuring device. The phototube was mounted upright in a 3 x 4 x 5 inch metal box. A schematic diagram of the circuit associated with the phototube is given in Fig. 5. The incident light beam to the photocell was collimated by two 1/8 inch pinholes located 12 inches apart and mounted in a 1/2 inch piece of black plastic pipe. All surfaces in the pipe and the phototube box were sprayed with a special light-absorbing matte black paint.

Power Supply and Measuring Circuit

A schematic wiring diagram is given in Fig. 6. The main power supply consisted of a number of 90 and 45 volt radio B batteries in series. The voltage output was remarkably constant over the 2 or 3 hours normally run at one time. The voltage output of the batteries was checked periodically with a portable potentiometer and minor adjustments made with the variable resistor in the circuit. No replacement of batteries was necessary over the 8 month period of the research.

The power supply was housed in a standard metal radio cabinet; auxiliary electrical controls for the lighting system and traversing motor were also included in the cabinet. A 6 volt a-c power system was included to energize pilot lights on all switches in the system.

Photocell Traversing System

The metal box housing the photocell was mounted on a power-driven vertical screw for automatic traversing. The traversing device consisted

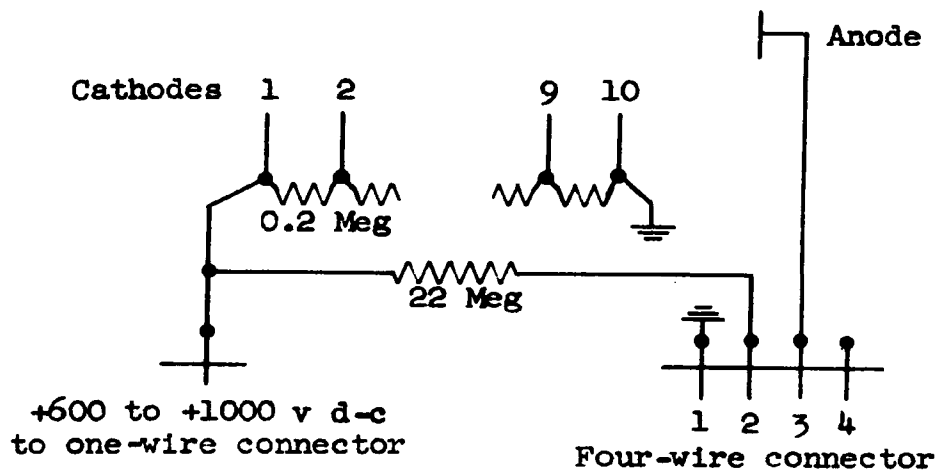


Fig. 5. Phototube circuit.

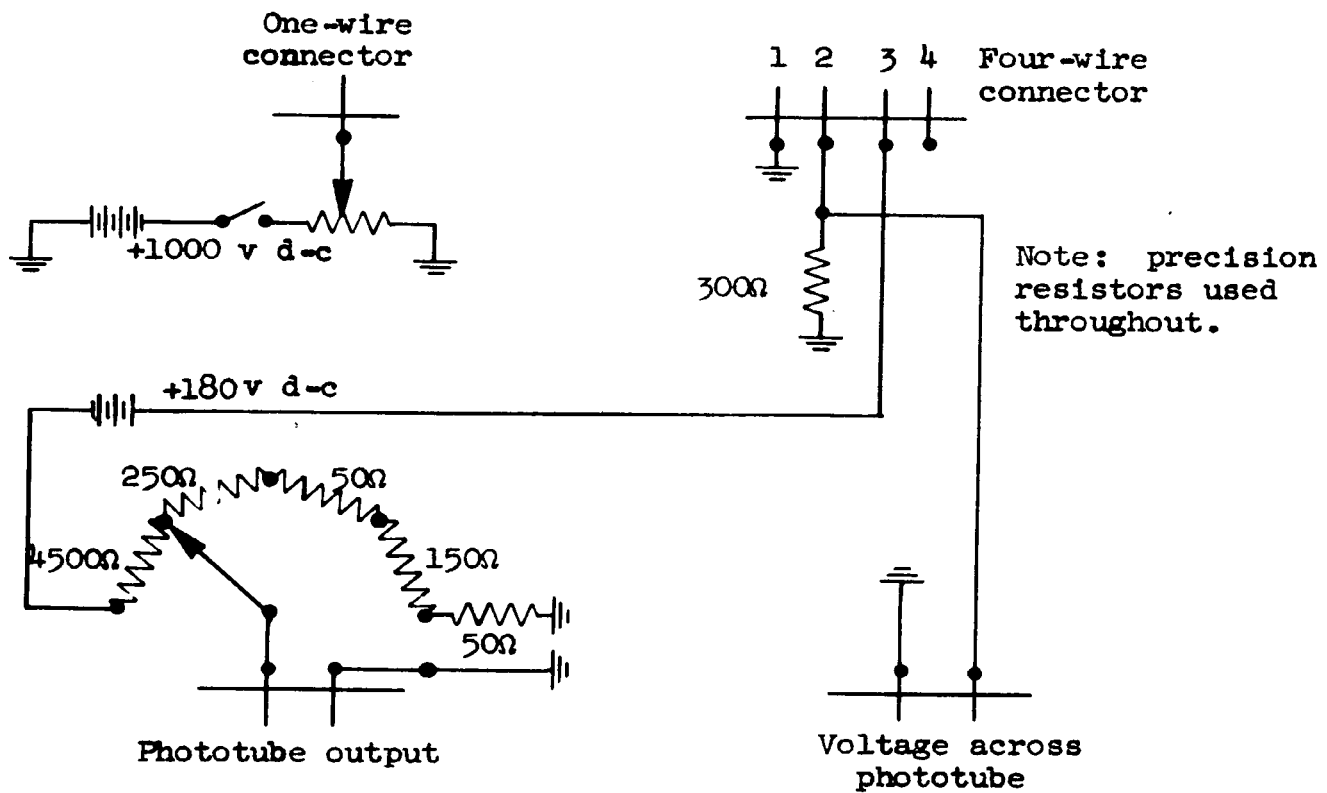


Fig. 6. Power supply and measuring circuit.

of a 36 inch screw obtained from a good quality "Sears" tripod screw bumper jack. The screw was mounted in a U-channel with ball bearings at the top and bottom, and the whole assembly was supported in the middle of a welded angle-iron structure; see Fig. 7. The whole structure was carefully leveled by means of four leveling screws in the base. Lateral movement of the photocell housing was eliminated by two arms with ball bearing wheels running in the two front vertical members of the angle-iron frame.

A small split-phase reversible a-c motor (salvaged from a motorized valve) raised and lowered the photocell at a steady rate of about 2 inches per minute. Two microswitches in the motor circuit stopped the photocell at the two ends of the traverse, allowing unattended operation during traversing.

The vertical position of both microswitches was adjustable and fixed the extent of the traverse. A two-pole double throw switch controlled the direction of rotation of the motor and actuated a pilot light which indicated when the motor circuit was energized, regardless of whether the microswitches were open or closed.

Light Sources

Two separate light sources were used for the froth density and mixing runs because of the different incident light intensities necessary for accurate measurements in the two different experiments.

Argon Tubes

Blue fluorescent tubes, specially made up by a local neon tube firm, were used as a light source in the froth density measurements. Two 38-inch long 8-mm glass tubes were filled with argon and a bit of mercury. The inside of the tubes was coated with a blue fluorescent powder to

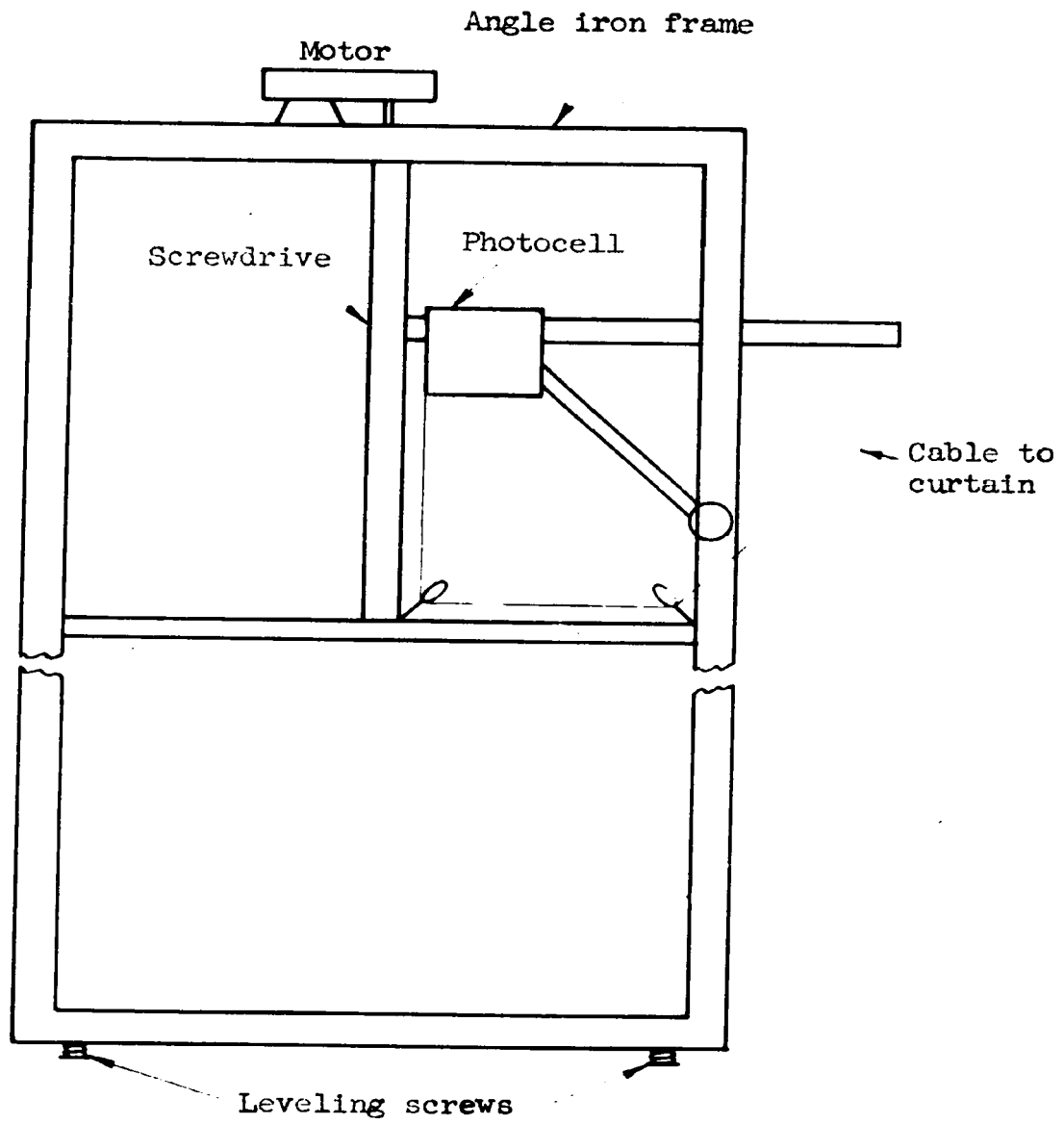


Fig. 7. Schematic diagram of traversing mechanism.

convert the mercury light to the blue range of the spectrum. With a voltage of approximately 10,000 volts a-c across the tubes, a blue light was given off with very little variation in intensity along the length of the tube. The light was not monochromatic, but quite sharply peaked in the blue region of the spectrum, which corresponded to maximum sensitivity of the phototube.

To maintain maximum uniformity of incident light in a vertical line along the back of the column, a sheet of opal glass was mounted over the rear Plexiglas window. With the exception of a 1/2-inch vertical stripe between the second and third row of caps, the opal glass was then painted black. Two argon tubes were mounted two inches away from, and on, either side of the clear stripe, with an aluminum reflector behind them. The whole lighting unit was then enclosed in a wooden box to reduce stray light around the equipment.

Because of multiple scattering in the froth, the above line source did not give completely reproducible results between the upward and downward traverses. To circumvent this, a traveling slit was devised. The slit consisted of a 2 x 1/2-inch hole cut in an ordinary cloth window blind. The roller from the blind (minus the ratchet stop) was installed in the bottom of the light box with guides holding the cloth against the column window. The blind was then raised and lowered by a system of wire cables and pulleys attached to the photocell box. The system worked very smoothly and was very easy to synchronize with the movement of the photocell.

Power Supply to Argon Lights

A constant voltage of approximately 10,000 a-c was supplied to the argon tubes by a standard 150 watt neon light high voltage transformer wired in series with a 250 watt Sola voltage stabilizing transformer. The feed voltage to the high voltage transformer was maintained at 115 volts ± 1.0 volts

for line voltages of 95 - 125 volts by the Sola transformer.

Tungsten Lights

Because of the strong absorption of light by the dye used in the mixing runs, more intense incident illumination in the green region of the spectrum was necessary. The light source used consisted of a standard pin-base 200-watt, 115-volt, projection lamp, mounted in the removable condenser system of an Argus C-300 slide projector. A 300-watt lamp could not be used because of excessive heating without a blower. A standard green filter (Klett Mfg. Co. Filter No. 54), transmitting wave lengths between 5200 and 5800 angstroms, was installed over the exit lens of the condenser system. To maintain constant light intensity throughout each run, the projector lamp was powered directly from the Sola constant voltage transformer.

The complete lighting structure was mounted, by means of laboratory clamps, on aluminum rods, allowing free movement of the source over the plane of the Plexiglas window.

Photocell Output Recording Devices

Two recorders were used during this investigation to measure the photocell output. For the froth density data a record of the transmitted light as a function of height above the tray was required, while the variation of the light intensity with time was of interest in the case of mixing.

X-Y Recorder

The variation of light intensity with height above the tray was recorded on a Leeds and Northrup (Speedomax, type G) x-y recording potentiometer. This type of recorder consists of two potentiometer circuits fed from d-c amplifiers. The dynamic response characteristic of both coordinates was about one second full scale, with a range of 0 to 50 millivolts in each case.

This type of recorder has a built-in filter system to cut out frequencies above 10 cycles/second, and thus maintain recorder stability. No trouble was caused by the 60 cycle flicker of the gas lamps.

In recording a run, the output from the appropriate sensitivity scale of the photocell was fed to the x-coordinate, and a voltage which varied with the position of the photocell relative to the bubble tray was fed to the y-coordinate.

The position-variable voltage was generated from a potentiometer circuit attached to the photocell carriage. This potentiometer consisted of a fine resistance wire, in a closed circuit with a dry cell, strung between the two platforms supporting the traversing mechanism. The voltage output of a brush contact attached to the photocell box was then proportional to the height of the photocell above the tray. Scale changes and standardization of this circuit could be made with a variable resistance installed in series with the resistance wire circuit.

High Speed Recorder

A Sanborn Model 60 recording galvanometer fitted with a d-c amplifier and pre-amplifier was used to record the photocell output as a function of time in the dye mixing runs. The Sanborn recorder is very versatile and has a number of sensitivity scales and chart speeds. Most of the runs were recorded at a chart speed of 2.5 mm per second and a sensitivity of 5 mv/cm. An auxiliary second timer was used continuously to double-check the chart velocity. The galvanometer will respond to frequencies as high as 80 cycles per second, and therefore caused trouble with 60 cycle pick-up. A standard filter circuit employing a choke and a condenser was installed in the photocell output line. The minimum value of condenser capacitance which would just eliminate the high frequency stray currents was found by trial and error and used for all the runs.

To check if there was any effect of electrical response on the system, a few dye decay runs were made with a standard Leeds and Northrup Speedomax recording potentiometer; no difference with the Sanborn was detected.

Dye Injection System

A solution of potassium permanganate was injected at a steady rate into a T connection between the water rotameter and the inlet downcomer. The injection point consisted of a 6 mm glass tube slightly drawn out at its end and inserted into the pipe via a brass tubing connection. A 5-gallon supply of permanganate solution was maintained on the floor above the test equipment set-up.

The solution was syphoned via rubber tubing to the injection point in the column; the syphoning height was approximately 20 feet. Quick cut-off of dye feed was made with a spring hose clamp. Variations in dye feed rate were made by constricting the diameter of the rubber tube with a screw hose clamp.

EXPERIMENTAL PROCEDURE

Start-Up Procedure

The start-up and operation of the column was identical for both the froth density and the mixing measurements. A complete set of runs at any weir height consisted of four gas rates with four liquid rates in each case. All runs were made after dark to avoid having to darken a section of the large laboratory.

For a typical evening's experiments, a gas rate and weir height were selected before starting the runs. The column front window was removed and the weir adjusted to the correct height. The rheostats in the field and armature of the blower motor were then adjusted to give the desired air rate, and the blower was started. The water was then turned on to a rate of about 20 gpm. While the major changes in the water and air temperature were occurring and the column coming to equilibrium, the amplifier in the recorder was warming up and adjustment of the auxiliary equipment was completed.

Column Operation

The warm-up period normally took about 20 minutes, at which time the liquid rate was adjusted to the lowest rate to be run in the series (8 gpm). The following measurements were then taken:

1. Gas and liquid rotameter readings
2. Froth height
3. Clear liquid heights
4. Pressure in gas rotameter and air chamber
5. Entering and exit liquid temperature
6. Gas entering temperature

The barometric pressure was recorded once or twice each evening. The froth heights and clear liquid heights were checked at the beginning and end of each run.

Rotameter Readings

Both the liquid and gas rotameter were read with the aid of a flashlight to the nearest half division. The liquid rate remained quite steady so that the rotameter could be accurately read to half a division (approximately 0.2 gpm). The air was supplied by a lobe blower which inherently pulsates, therefore the gas rotameter float never was completely steady. Because of the oscillation of air flow, the rotameter could not be read to closer than 2 cfm (approximately 2% accuracy).

Froth Height

The froth height was estimated visually with the aid of two tapes glued to the Plexiglas window on either side of the active bubbling area. The interface between the region where water was the continuous phase, and where gas was the continuous phase, was well defined for the higher gas and liquid rates (see Fig. 8). At the lower submergences and gas rates, a different bubbling regime occurs; in this case the bubbling action no longer appears frothy, but instead is similar to an inverted rainstorm.

The froth height was reproducible to between $1/4$ and $1/2$ inch in the froth regime, which corresponds to an accuracy of about 5%. In the drop regime the errors in measuring froth heights were probably as high as 25%.

Clear Liquid Heights

Manometer probes were located at three positions on the tray (in the inlet calming section, between the first and second row of caps, and in the outlet calming section) with probe openings parallel to the tray floor and

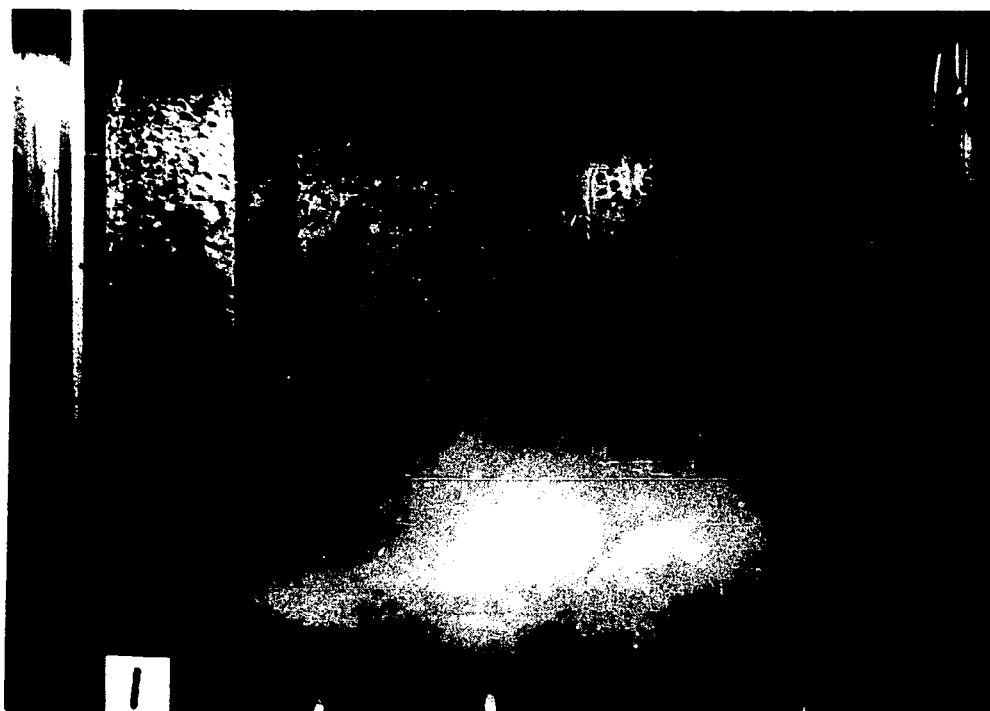
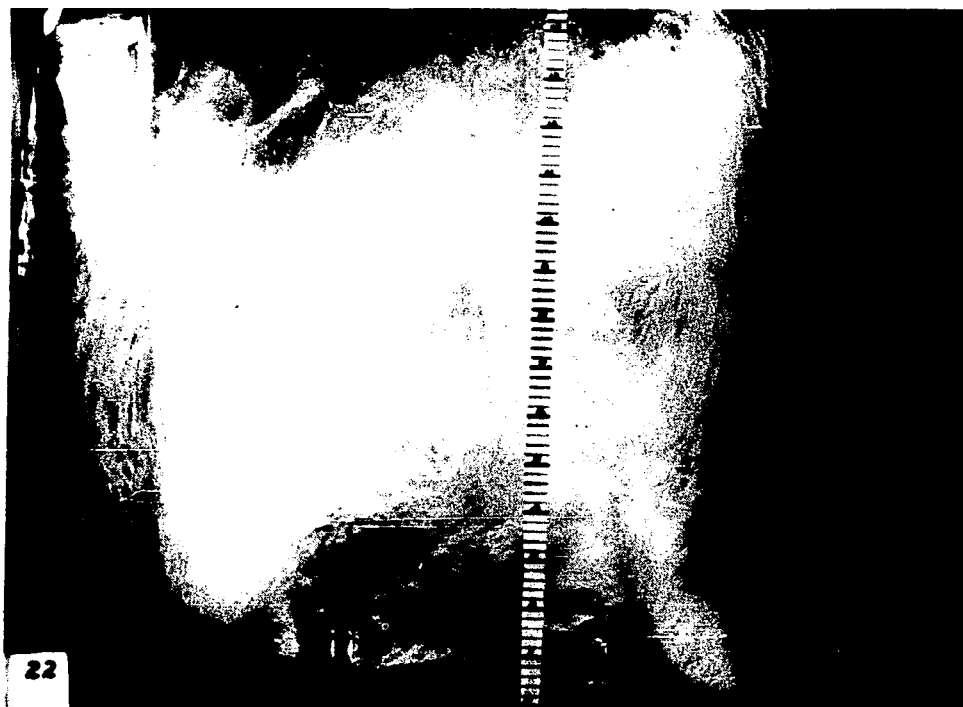


Fig. 8. General appearance of froth at high and low liquid rates. $F = 0.96$, $L = 1$ and $L = 5$ gpm/inch, 2-inch weir.

approximately 1/16 inch above it. The pressure indicated by the probe was measured on a special manometer board attached to the side of the window and reported as inches of equivalent clear liquid height. In general, the clear liquid heights were quite steady and could be read to 1/10 inch; occasionally, due to gas pulsation, cycling of the liquid holdup on the tray occurred, in which case an optically averaged clear liquid height was reported.

Pressure Measurements

The static pressure upstream from the gas rotameter was measured using a 30-inch mercury filled Meriam manometer; this pressure was used in calculating the gas mass flow rate. The static pressure in the air chamber was measured using an identical manometer filled with dyed water.

Temperature Measurements

Inlet and outlet liquid temperatures and the inlet gas temperature were measured using iron-constantan thermocouples and a common cold junction. The cold junction was melting ice in a wide-mouth Thermos jug. The inlet water thermocouple well was located just downstream of the liquid rotameter; the outlet liquid temperature was measured in the outlet calming section of the tray.

The inlet gas thermocouple was located downstream of the gas rotameter. Temperature measurements were made once each run.

Froth Density Traverses

Froth density traverses were initially made at two horizontal positions in the column, between the first and second, and the second and third, rows of caps respectively. No significant difference between the two positions was found, and as realigning the photocell each time was extremely tedious,

for most of the runs the traverse was made only between the second and third row of caps.

Start-up Procedure

As soon as the electrical measuring equipment had warmed up enough to stabilize, the gas and liquid flow to the tower was cut off and a blank traverse made with the photocell. During the blank traverse, the vertical alignment of the photocell traversing structure and was checked and adjusted if necessary. Adjustments consisted of leveling the structure by means of the leveling screws, and adjusting the length of the cable which moved the light slit in the back of the column to assure synchronization of the slit and the photocell collimating tube.

The photocell measurement of the incident light was also checked and the photocell output adjusted to a standard value by changing the high voltage rheostat. Day to day drift in the photocell output was thus compensated for. The standardized potential was measured with the portable potentiometer and maintained at this value by minor adjustment of the high voltage rheostat, throughout the evening's set of runs.

Photocell Traverse

After the standardization procedure, the column was started up and the microswitches controlling the extent of the traverse adjusted to the froth height under study. The position scale appropriate to the froth height was adjusted on the position-indicating potentiometer to give a simple relation between the chart paper and the distance traveled (usually 1 inch = 1 inch, or 1 inch = 2 inches of travel). The motor switch was then thrown to the "up" position.

While the photocell was traversing, other measurements such as gas and liquid temperatures were made. After completing the upward traverse,

the chart on the x-y recorder was moved down enough to cover a new traverse (the direction of motion of the chart reversed when the photocell reversed), and a downward traverse of the photocell recorded. For most runs two complete cycles of the photocell, resulting in four independent traverses, were made. The charts were then marked with the run number, incident light reading and distance scale.

Mixing Runs

The column start-up operating procedure was identical for the mixing runs as for the froth density runs. After turning on the Sanborn recorder and starting up the column, the dye supply on the floor above was checked. If necessary, a new supply was made up by dissolving 10 gms. of solid potassium permanganate in approximately 5 gallons of distilled water. The recorder was then balanced and adjusted to maximum sensitivity. Then the photocell and the light were located for the run, and the photocell output for pure froth (no dye) was adjusted to give nearly full scale deflection of the recorder.

A run was started by opening the dye-line valve and starting the recorder (at 2.5 mm/sec chart speed). The dye valve was closed when the recorder indicated the dye concentration in the froth had stopped increasing, and the recorder was left running till the light intensity again reached a maximum. Normally, duplicate measurements were made at four positions for each run. The positions tested were at bubble cap height (1-1/2 to 2 inches) between the first and second, and second and third, row of caps; and in the true froth at these same horizontal positions. The true froth reading was usually taken 2 to 4 inches below the total froth height to avoid the region affected by the irregular bounce of the froth interface.

The charts were then marked with the run number and the position at which the measurement was made. The code used to identify the positions were the letters "A" or "B", followed by a number. The letters B and A represent the horizontal position (between first and second, and second and third, row of caps respectively), and the number indicates the height above the tray in inches.

To check whether any peculiarity in response of the Sanborn recorder affected the results, a few runs were made using a standard Speedomax recorder with a chart speed of 3 inches/minute. No difference was detected.

HYDRAULIC CHARACTERISTICS OF TRAY COLUMNS

Variables Important in Mass Transfer

The most common method of evaluating the mass transfer process on a tray column is by means of the plate efficiency concept. The use of plate efficiencies stems from the standard practice of calculating columns using, as a first approximation, the assumption that the liquid and gas on a tray reach equilibrium. The fractional approach to the equilibrium actually attained is used to define a plate efficiency. Plate efficiencies are, therefore, an artificial concept and can only be approximately related to the actual mass transfer process occurring on a tray.

Many definitions of plate efficiencies have been proposed in the past. These have ranged from the completely thermodynamic, such as the free energy definition proposed by Miller (46); the easy efficiencies to measure, such as the temperature efficiency proposed by Nord (48); to the completely arbitrary definition of efficiency, such as the one proposed by Edminster (21) because of its ease in application to a particular graphical method of tower calculation.

Of the many efficiencies proposed, three are in common use: overall column efficiency, Murphree plate efficiency, and point or local efficiency. All three can be related to the mass transfer occurring on a tray. The number of assumptions necessary in relating overall column efficiency and Murphree plate efficiency to mass transfer makes the use of these two definitions impractical as fundamental correlating variables. The relation of point efficiencies to mass transfer requires only one major assumption — uniform liquid concentration vertically above each point on a tray — and is therefore the efficiency usually used in non-empirical correlations.

The most important correlations of plate efficiencies are mentioned in the section on literature review. From these correlations it was noted that the variables with the most direct effect on plate efficiency are the gas-liquid contact time and the interfacial area. The hydraulic gradient on a tray, and entrainment from a tray, have only secondary effects on plate efficiency. Hydraulic gradient, for example, has no effect per se on efficiency, but too great a gradient causes dumping of the upstream rows of caps, and the resultant liquid by-passing reduces overall tray efficiency.

The basic information necessary to calculate the amount of mass transferred at a point in a column, therefore, is: the mass transfer coefficient, the interfacial area, and the gas-liquid contacting time. See, for example, equations (4a) and (4b) derived by Gerster et al. (1).

Mass Transfer Coefficients

The prediction of mass transfer coefficients for surfaces having stable shapes is relatively straight forward. The coefficient has been found to be a function of the relative velocity of the two phases, a shape factor, and the physical properties of the system.

For a gas bubbling through a liquid, only the physical properties of the phases are reasonably well defined (as long as the interfacial temperature is known). Both the relative velocity and the interfacial shape can at best be described only approximately.

In analyzing mass transfer data, two mechanisms have been postulated: (a) individual bubbles rising through the liquid on the plate, and (b) gas flowing through channels connecting the bubble-cap slots to the liquid surface.

(a) Finite Bubbling.—Geddes (28), in the derivation of a plate efficiency correlation, took the bubbling mechanism literally. He calculated both the bubble size and its velocity from data on bubbling from single orifices into clear liquid. The resultant equations were used to calculate mass transfer coefficients from experimental data. The calculated mass transfer coefficients were then correlated empirically.

Walter and Sherwood (72) used the same analysis, but did not attempt to separate the mass transfer coefficient from the other variables, instead, they correlated the group $PK_{Ga}t/G$ empirically.

Gerster et al. (1, 29-34) re-wrote the mass transfer equations in terms of packed-column concepts of height and number of transfer units. By this method, number of transfer units could be calculated directly from efficiency data without any assumptions as to contact time, interfacial area, etc. The transfer units calculated in this way were then related to the mass transfer coefficient, based on concentration units, by the following equations:

$$N_G = k'_G a t_G \quad (4a)$$

and

$$N_L = k'_L a t_L \quad (4b)$$

(b) Bakowski Bubbling Mechanism.—Bakowski (8), based on high-speed movies of slot bubbling, has recently proposed that in a bubble-cap column the gas issuing from a slot does not form bubbles, but an interconnected, sausage-shaped, chain of bubbles, which allows the gas to travel directly from the slot to the liquid surface. Based on this analysis, Bakowski derived equations relating point efficiencies to mass transfer for gas phase controlling systems. The mass transfer coefficients were correlated using kinetic theory and wetted wall column data.

All methods of measuring mass transfer coefficients involve assumptions as to the contact mechanism, and results of different investigators will differ depending on the theoretical approach used.

Experimental plate efficiency measurements performed by the A. I. Ch. E. Tray Efficiency Research Project indicate that at F-factors ($F = V_s \sqrt{\rho_G}$)* greater than one, the mass transfer coefficient times specific interfacial area becomes constant. One can conclude, therefore, from equations (4a) and (4b), that for $F > 1$, the point efficiency is only a function of gas-liquid contact time.

Gas-Liquid Contact Time

Numerous methods have been proposed for calculating gas-liquid contact times; a few of these are compared, for a specific example of tray operation, in Table II.

Two general methods of calculating gas residence times in the froth are in use. The methods are:

- (a) Methods based on the average linear gas velocity; and
- (b) Methods based on rise velocities of single bubbles in a liquid.

(a) Methods Based on Linear Vapor Velocity.—If the gas held up in the froth is constant, a material balance on the gas shows that regardless of degree of mixing, the average residence time in the froth is the gas hold-up volume divided by the volumetric gas feed to the froth. This case is shown as (1a) in the accompanying table. If no mixing of the gas occurs in the froth, the height of the froth divided by the gas velocity in the froth is the contact time.

* F-factors, defined in terms of the superficial gas velocity based on the active bubbling area of the column, are normally used in describing tower gas rates. Their use stems from the observation that hydraulic data, such as flooding and froth heights, for different systems or for systems at different pressures, are correlated only if the gas density is taken into account.

TABLE II

Example of Gas-Liquid Contact Time

Run No. 24, bubble-cap tray

Liquid rate	L = 31.6 gpm
Linear gas velocity	$V_s = 2.66 \text{ ft/sec}$
Froth height	$Z_f = 1.37 \text{ ft}$
Clear liquid height	$Z_c = 0.54 \text{ ft}$
Weir height	$Z_w = 0.25 \text{ ft}$

Gas-Liquid Contact Times

(1) Based on linear gas velocity

(a) Stirred pot definition:

$$\begin{aligned} &(\text{gas hold-up in froth})/(\text{linear gas velocity}) = \\ &(Z_f - Z_c)/V_s = 0.313 \text{ secs} \end{aligned}$$

(b) Gas contacting froth at linear gas velocity based on cross-sectional area of tower unoccupied by liquid:

$$\begin{aligned} &(\text{froth height})/(\text{corrected linear gas velocity}) = \\ &Z_f(1 - \phi)/V_s = 0.313 \text{ secs} \end{aligned}$$

(c) Gas contacting froth at linear gas velocity:

$$\begin{aligned} &(\text{froth height})/(\text{linear gas velocity}) = \\ &Z_f/V_s = 0.518 \text{ secs} \end{aligned}$$

(2) Based on rise velocities of bubbles

(a) Velocity of rise of single bubbles through clear liquid:

$$v_s = 1 \text{ ft/sec from Van Krevelen (70)}$$

(i) Height of travel of bubble = froth height

$$Z_f/v_s = 1.38 \text{ secs}$$

(ii) Height of travel of bubble = clear liquid height

$$Z_c/v_s = 0.54 \text{ secs}$$

(b) Velocity of rise of bubbles based on chain bubbling:

$$v_s = 4.40 \text{ ft/sec from Van Krevelen (70)}$$

$$Z_f/v_s = 0.313 \text{ secs}$$

The average gas velocity in the froth is based on the tower cross-section unoccupied by liquid. This is Case (1b) in the table; as can be seen, it gives an identical value to Case (1a).

The degree of mixing of the gas affects only the age distribution of the bubbles in the froth, but not the average contact time. In the case of no mixing, every bubble formed remains in contact with the liquid the same amount of time. For perfect mixing, the age distribution is exponential; the lowest age is probably represented by gas that passes through the froth with its initial slot velocity, while the oldest bubble has infinite contact with the liquid as it recirculates and never escapes. Intermediate degrees of mixing will change the age distribution function, but not the average age.

A method that has been used to estimate the contact time is to assume the bubble to have the same velocity in the froth as in the empty column (Case 1c); this assumption yields higher contact times and does not have theoretical justification.

(b) Methods Based on Rise Velocities of Single Bubbles.--Geddes (28) assumed that the contact time could be estimated from the rate of rise of single bubbles through the height of the clear liquid on the tray. Case (2a) is an example of this method. The velocity of rise of single bubbles was estimated from a chart, summarizing the work of a number of investigators, reported by Van Krevelen and Hofstijzer (70).

Case (2b) is of interest, as it represents the same approach as the one above, except that the velocity of rise of a chain of bubbles is used instead of data for single bubble rise.

A correlation for the velocity of rise of bubbles in a chain was proposed by Van Krevelen et al. (70), based on their experimental data. The velocity of rise is reported to be equal to the volumetric flow rate divided by the gas hold-up per foot of column (liquid) height. For a unit area

of column, this can be expressed $v_s = V_s/(1 - \phi)$, and the contact time is then $t_G = Z_F(1 - \phi)/V_s$. This last expression is identical to Case (1b), and serves to confirm the use of the stirred pot analogy in calculating contact times.

Interfacial Area

Interfacial areas in liquid-liquid and gas-liquid batch mixing have been measured by Vermeulen et al. (71), and Trice et al. (69), using a light transmission technique; a number of investigators have studied interfacial areas in stable foams by the same technique. It has been shown (55) that light transmission measurements will yield data on interfacial area in unstable froths if the bubble or drop size is less than 1 mm in diameter; and in foams with any size bubbles, as long as the foams have a stability greater than a critical minimum.

No direct measurements of interfacial area in a tray column have been reported in the literature. Light transmission from a tray froth is not a measure of the interfacial area, as the bubble sizes are greater than 1 mm and the froth unstable.

Indirect methods of measuring interfacial area in froths do not give precise data due to the irregular shape of the bubbles.

Bubble volumes for bubbles formed at single orifices have been measured with precision by Van Krevelen et al. (70), Hughes et al. (37), and others (10, 16, 19, 26, 35, 40), and from these volumes interfacial areas were estimated by assuming spherical bubbles. High-speed motion pictures of bubbles rising in a liquid show continuous variation in shape (and interfacial area) of the bubbles as they rise. Therefore, areas estimated from spherical bubbles are at best a rough estimate.

Using the approximation of spherical bubbles, West et al. (74) attempted to measure the interfacial area in a sieve tray from the bubble volumes on formation. The volume at formation was estimated from stroboscopically measured bubble frequencies and known gas rates. West et al. point out that the bubbles, on leaving the surface, break up into random sizes, so that areas estimated from bubble formation sizes are very crude approximations to the actual interfacial area. Their data indicated a reduction in interfacial area with increasing gas rate.

Recently, Garner and Freshwater (25) have suggested that the residual pressure drop (defined as the difference between the dry pressure drop through the plate and the foam pressure at the middle of the open slot) is proportional to the interfacial area. Their measured values of reduced pressure in a 5 ft diameter tower increased with increasing gas rate, and varied randomly with liquid rate.

The Bubbling Process in Gas-Liquid Contacting

Single Orifice Bubbling

A number of papers in the literature report photographic studies of bubbles formed at single orifices and from bubble-caps. Pairing of bubbles leaving single orifices has been reported by Bryan and Garber (10). Van Krevelen et al. (70) found that at high gas rates the bubbles leaving an orifice formed a continuous chain, the velocity of rise of the chain being considerably higher than for single bubbles. Johnson et al. (40) found that in a single-orifice perforated tray with liquid cross-flow, the velocity of rise of single bubbles varied considerably for the same bubble over the path length followed.

Spells and Bakowski (60, 61) studied bubbling from single and multiple vertical slots. Two regimes of bubbling were observed photographically and designated shallow and deep seal bubbling. In shallow seal bubbling, the gas issuing from a slot essentially formed channels directly to the liquid surface, while deep seal bubbling consisted of actual bubble formation. All the runs they made with multiple slots (one complete bubble-cap) showed shallow seal bubbling.

The author has had occasion to see a high-speed movie of single-orifice bubbling taken at the Experimental Station of the E. I. du Pont de Nemours and Co. The orifice was $1/4$ inch in diameter and the apparatus was operated at gas velocities through the orifice as high as 50 ft/sec, and with a 2-inch liquid seal. At gas rates above a minimum, the bubbles were formed at irregular intervals, and in general left the orifice at a high enough velocity to penetrate the previous bubble which still was in the liquid. As many as four or five bubbles would form one large bubble in this manner. When the liquid seal was reduced to $3/4$ inch, the gas tended to jet directly from the orifice to the liquid surface even at gas velocities as low as 10 ft/sec; essentially, this would confirm the Spells and Bakowski shallow seal mechanism.

Multiple Orifice Bubbling

In attempting to extend experimental information on bubbling to actual tray column operation, two points should be kept in mind: studies on single orifices have been made at orifice velocities lower than normally used in columns; and the effect of bubble interaction between different orifices is difficult to evaluate.

West et al. (74) studied bubble frequencies and volumes in a sieve tray using a stroboscopic technique. They found that bubble diameters on formation were in general larger than the distance between perforations, indicating out

of phase bubbling between adjacent orifices, and that the bubbles tended to break up considerably as soon as they left the orifice. Spells and Bakowski's observation that bubbling from multiple slots tended to form channels to the liquid surface was tested in equipment where slots were adjacent but not facing each other. It appears that this channeling effect is suppressed when slots facing each other interact.

High speed movies at 800 frames per second were taken by Warzel (73) of the bubbling action in the 5-plate A. I. Ch. E. bubble-cap absorber at Michigan, and single exposures of approximately 2 micro-second duration were taken by the author in the one-plate simulator. In both cases linear gas velocities were around 2 ft/sec, and clear liquid heights were 2 inches.

Careful observation of the high-speed movies indicates that the bubbling mechanism in a multi-cap tray is intermediate between the Spells and Bakowski deep and shallow seal mechanism. The gas issuing from the slots forms a continuous sheet, completely surrounding each individual cap, and extending from the top of the slots to the top of the cap. As soon as the sheet leaves the protection of the cap wall, the turbulence in the tray liquid breaks it up into individual bubbles which then rise through the froth. The high-speed movie at 800 frames/second was not fast enough to stop the bubbling action so that no individual frames will give a print which will show the effect in a half-tone reproduction.

Figures 9 to 15 are reproductions of the 2 micro-second exposures. The flash, in these pictures, was placed at an angle to the Plexiglas face of the column to highlight the bubbles. Areas of the negatives gave very clear pictures of the bubble size in different regions of the froth. Among the caps, the pictures are not too good, as the caps were painted with light-absorbing black paint for the light transmission

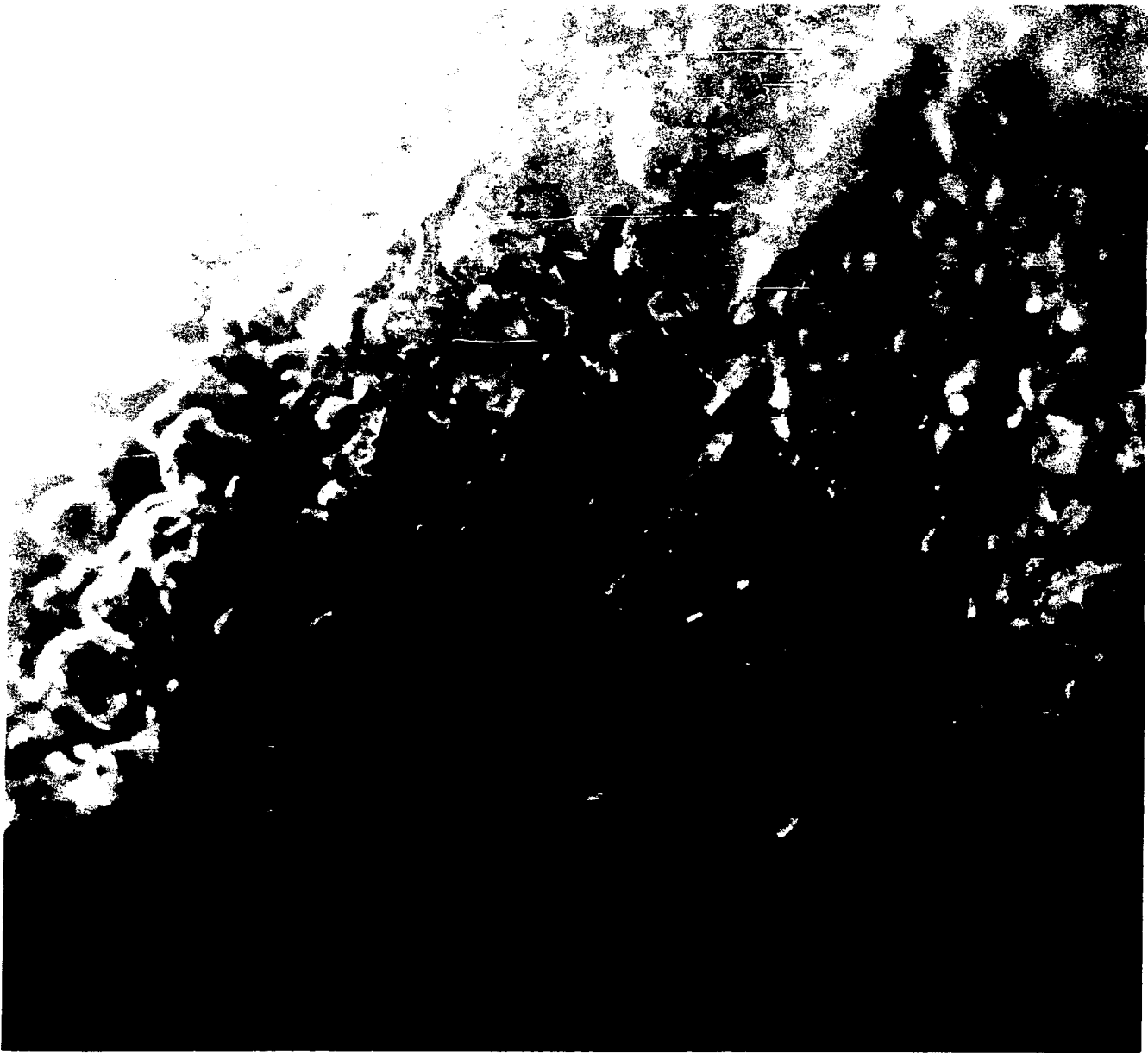


Fig. 9. Bubbling at bubble-cap height, 2 microsecond flash. Two gas columns and one sheet of gas are visible disintegrating at bubble-cap height. Area covered by photo approximately 3 x 3 inches. Two-inch weir. $F = 0.8$, $L = 2.16$ gpm/inch.



Fig. 10. Bubbling at bubble-cap height, 2 microsecond flash. Same gas rate as Fig. 9. Characteristic column of gas visible diagonally between caps. More uniform bubbles than Fig. 9. Area 3 x 3 inches, 2-inch weir. $F = 0.8$, $L = 21.6$ gpm/inch.

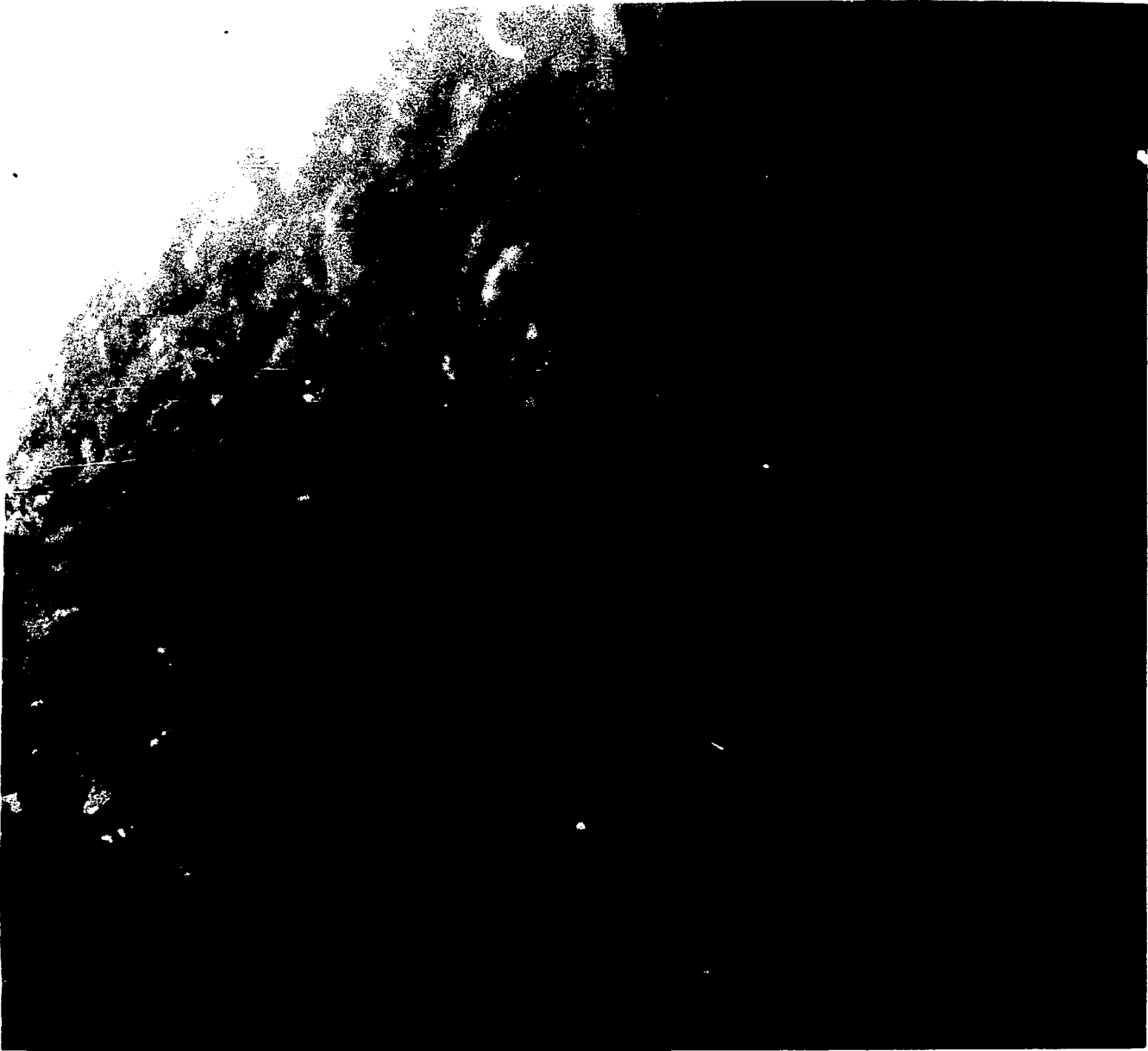


Fig. 11. Bubbling at bubble-cap height, higher gas rate than previous figures. Clearly marked column of gas between bubble-caps. Area 3 x 3 inches, 2 microsecond flash. $F = 1.0$, $L = 2.16$ gpm/inch, 2-inch weir.



Fig. 12. Froth area 5 inches from plate floor. Same negative as Fig. 11, same enlargement, 2 microsecond flash. Note uniformity in bubble size between Figs. 11 and 12. Area 3 x 3 inches. $F = 1.0$, $L = 2.16$ gpm/inch, 2-inch weir.

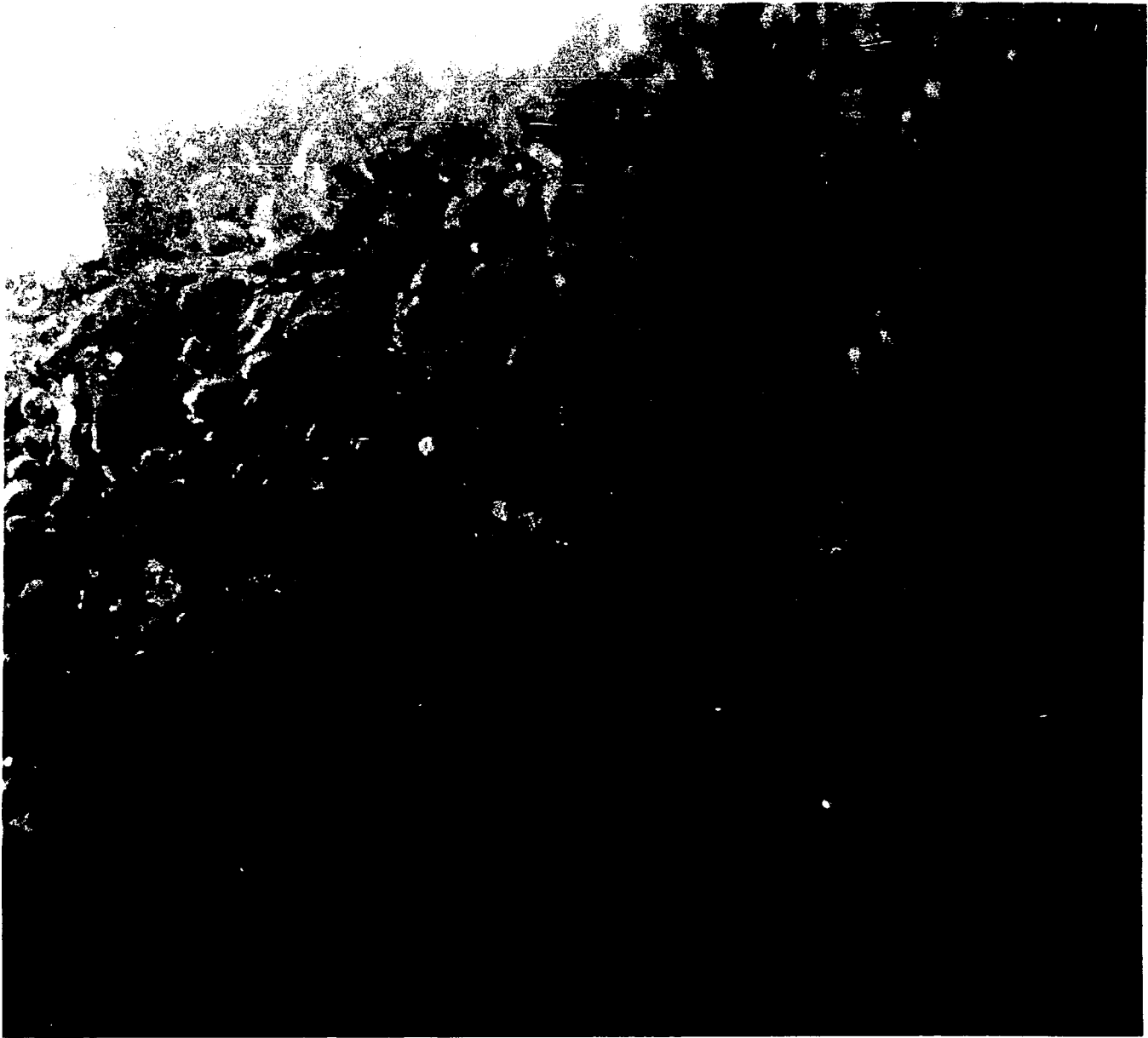


Fig. 13. Bubbling at bubble-cap height, 2 microsecond flash. Note extensive sheeting of gas breaking up at bubble-cap height. Area 4 x 4 inches. $F = 0.9$, $L = 1.08$ gpm/inch, 2-inch weir.



Fig. 14. Froth area 5 inches above plate; 2 microsecond flash. Same negative as Fig. 13, same enlargement. Area 4 x 4 inches. $F = 0.9$, $L = 1.08$ gpm/inch, 2-inch weir.

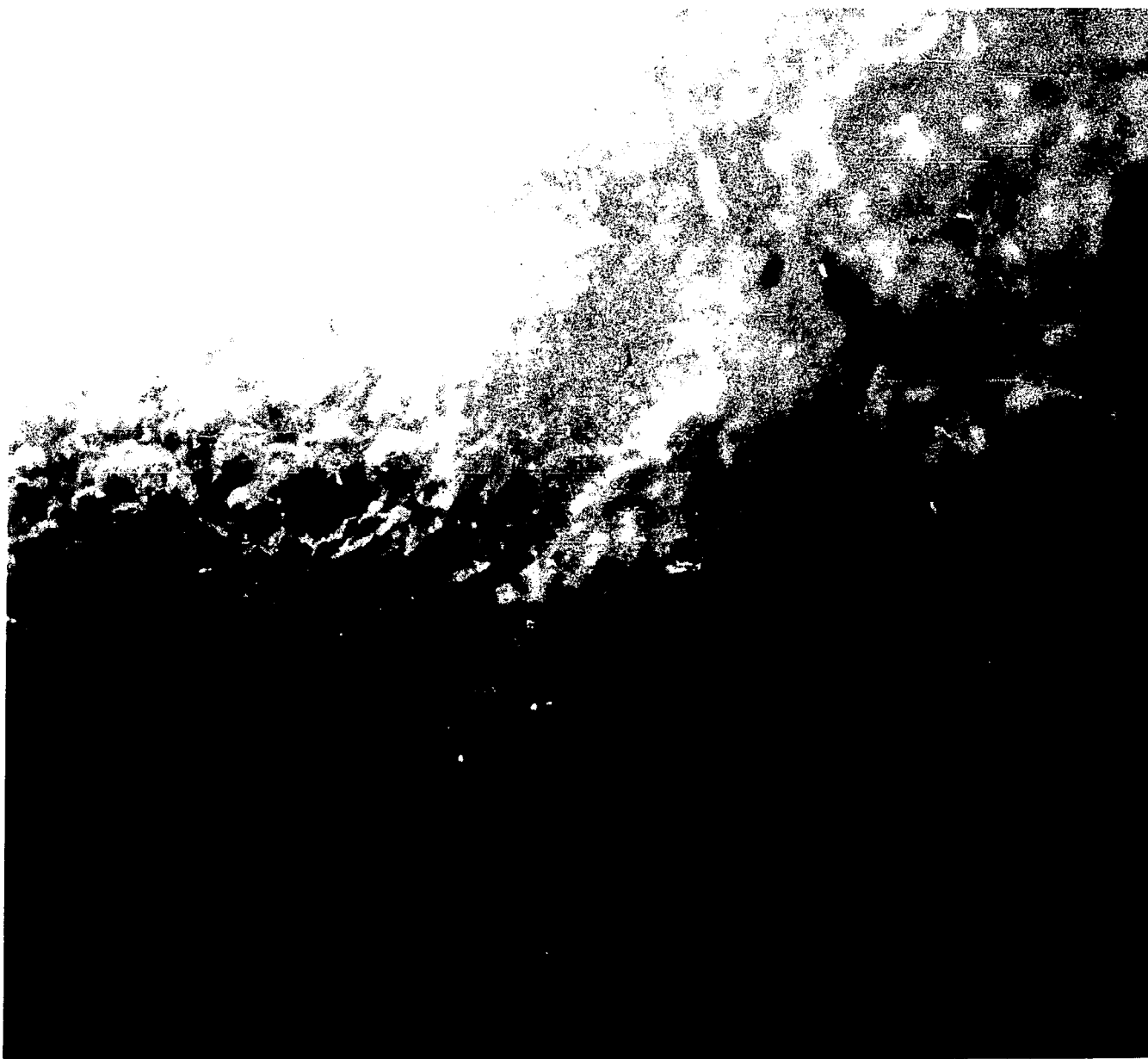


Fig. 15. Bubbling at bubble-cap height, 2 microsecond flash. Note uniformity of bubble size. Area 5 x 5 inches, $F = 1.1$, $L = 2.16$ gpm/inch, 2-inch weir.

measurements. The most striking conclusion brought out by the above figures is the extremely uniform size of the bubbles all the way through the froth. The figure captions draw attention to the details visible in each picture.

Froth Density Stratification

On the basis of the bubbling mechanism observed for multi-cap trays, the density of the gas liquid mixture would be expected to stratify into three regions, with the density probably decreasing with height.

The first region extends from the tray floor to cap height and consists of liquid separating the sheets of gas surrounding the caps; for convenience this region will be referred to as the columnette region. Above the cap height the gas sheets break up and the bubbles thus formed pick up a layer of liquid to carry up into the froth. This transition layer will be designated the froth formation layer. Above the froth formation layer, the bubbles of uniform size carry up an equal amount of liquid, with each bubble yielding a constant density layer. This layer will be designated the true froth layer, or simply the froth layer.

The existence of three layers will depend on the amount of liquid held up on the tray. For very low liquid hold ups on the tray (clear liquid height less than total cap height), only the bottom layer may exist, and the gas simply channels through the liquid.

To test the three layer hypothesis, measurement of the density distribution on the tray is necessary.

Measurement of Point Froth Densities

Miscellaneous Methods

The most direct method of measuring froth density is capture of a froth sample and measurement of the trapped liquid volume. The mechanical design

of a suitable trap presents considerable difficulty, and even if an effective trap were to be designed, the effect of a foreign body in the froth is difficult to evaluate. Any method employing a probe to measure the froth density is open to the above criticism of disturbing the froth.

Mitkevich (47) determined the density of soap foams by means of an empirically calibrated glass rod. The glass rod was placed upright in the foam, the time taken for the rod to fall being a measure of the foam density. This method essentially relates the viscosity of the foam to its density. Clark (13) measured the density of a soap foam by means of a conductivity probe. Apparently this method was successful because of the static nature of the foam; Warzel (73) attempted to measure the conductivity of the froth in a bubble-cap column by means of a pH probe, and could not get reproducible readings.

Measurement of Point Densities by Light Transmission

The decrease in the intensity of a light beam passing through a froth, in general, is assumed to be proportional to the bubble interfacial area present. This is true for stable foams, where the liquid film between bubbles is thin enough so that the bubbles are no longer round. In a normal froth, where drainage of liquid occurs on standing, Ross and Cutillas (55) have shown that the amount of light absorbed by a given thickness of froth is an inverse function of the volume of liquid held in the froth.

Ross (54) in 1943 showed from physical measurements in batch experiments that the drainage of a froth with time can be related by the equation:

$$\left(\frac{V_0}{V_t}\right)^2 = (1 + bt) \quad (10)$$

where

V_0 - volume of liquid in froth at time zero

V_t - volume of liquid in froth at time t

t - time elapsed

b - constant.

Later, Ross and Cutillas showed in batch experiments that the amount of light transmitted by a froth is a linear function with time, if the fraction transmitted squared is plotted versus time, i.e.,

$$t = b_1 + c_1(I/I_i)^2 \quad (11)$$

where b_1 and c_1 are constants.

Substituting in equation (10),

$$\left(\frac{V_0}{V_t}\right)^2 = 1 + b_2 + c_2(I/I_i)^2 \quad (12)$$

But for a given froth both the total volume occupied by the froth and the initial volume of liquid held by the froth is constant; therefore, the froth density can be defined:

$$\rho_f = \frac{\rho_L V_t}{V_F} \quad (13)$$

where V_F - froth volume.

Now, as ρ_L and V_F are constant for a given froth, we can substitute in equation (12):

$$\left(\frac{1}{\rho_f}\right)^2 = b + c(I/I_i)^2 \quad (14)$$

where b and c are constants dependent on the liquid and gas properties and the bubble size.

This relation breaks down when a single phase system is approached.

For the system air-water, $b = 1$, and equation (14) can be written:

$$\rho_f = \frac{1}{\sqrt{1 + c(I/I_1)^2}} \quad (15)$$

In their froth drainage experiments, Ross and Cutillas found the constants in the equation to be a function of bubble size; therefore, the same dependence on bubble size will be shown by "c."

The mechanism of light extinction in a froth is not due to particle scattering, but instead consists of reflection of the primary light beam from the bubble's inner surfaces. The angle made by these surfaces with the light beam is a function of the bubble size and the distance between adjacent bubbles. If the bubble size in a gas-liquid mixture is uniform, the density of the mixture will depend on the distance between the bubbles, and the light transmitted is also a function of the separation between bubbles.

If two froths transmit identical amounts of light, one cannot conclude that their density is equal, as the bubble size also controls the amount of light scattered from the primary beam (different bubble sizes change the value of "c"). The light transmission technique is capable, then, only of detecting changes in density of froths and not their absolute density. Experimentally it is, therefore, necessary to calibrate the probe in some independent way for every run.

The experimental technique used in studying froth stratification involved measuring the average froth density (by measuring froth height and liquid hold-up), at the same time as making a light traverse. Based on Figs. 9 to 15, the bubble size in the froth was assumed constant, and "c" evaluated by making the integrated point froth density equal to the measured average froth density.

Experimental Results

General Hydraulic Characteristics of the Test Column

The hydraulic studies in the one-plate simulator included tests of the nine-cap bubble tray (described in Table I) with and without a splash baffle, and the sieve tray (shown in Fig. 4) with a splash baffle. The liquid rates were varied from 8 to 32 gallons per minute, and the gas rates from 20 to 180 cubic feet per minute. All runs were made with an air-water system. Weir heights tested were 1, 2, 3 and 3-1/2 inches. The range of variables studied was designed to cover the range used in mass transfer studies by Ashby (5) and Warzel (73) in a 5-plate absorber with identical tray layout. The highest gas and liquid rates used were fixed, not because of column flooding, but because of the capacity of the auxiliary equipment.

Except at the highest rates, the test column behaved in a very similar manner to the 5-plate absorber. Due to the low liquid gradient across the tray, the bubble-cap tray was stable at all gas rates; the sieve tray tended to weep at the lowest gas rates. The column pulsated strongly at gas rates lower than $F = 0.3^*$; the pulsation was strong enough to oscillate the rotameter float over the full rotameter scale.

Froth Heights.—Froth heights were estimated visually with the aid of tapes glued to the front window of the column. The froth heights are reported in Figs. 16 to 18 as height from the tray floor to the interface where liquid becomes the discontinuous phase.

* Gas rates in general will be referred to in terms of F-factor, where $F = V_s \sqrt{\rho G}$ and V_s , the superficial gas velocity, is based on the area between the splash baffle and inlet downcomer, 0.615 sq. ft.

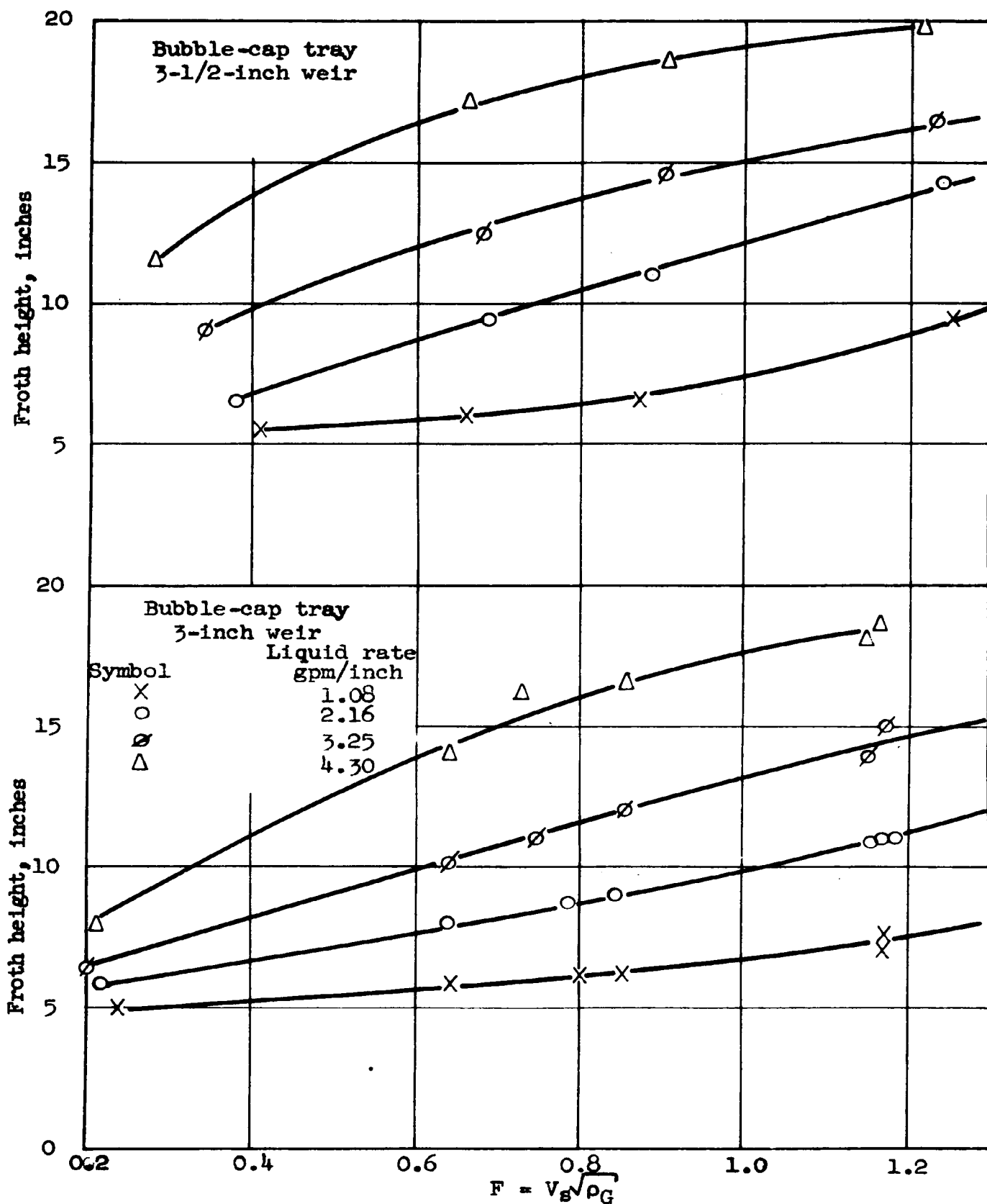


Fig. 16. Froth heights vs F-factor 3-inch and 3-1/2-inch weirs.

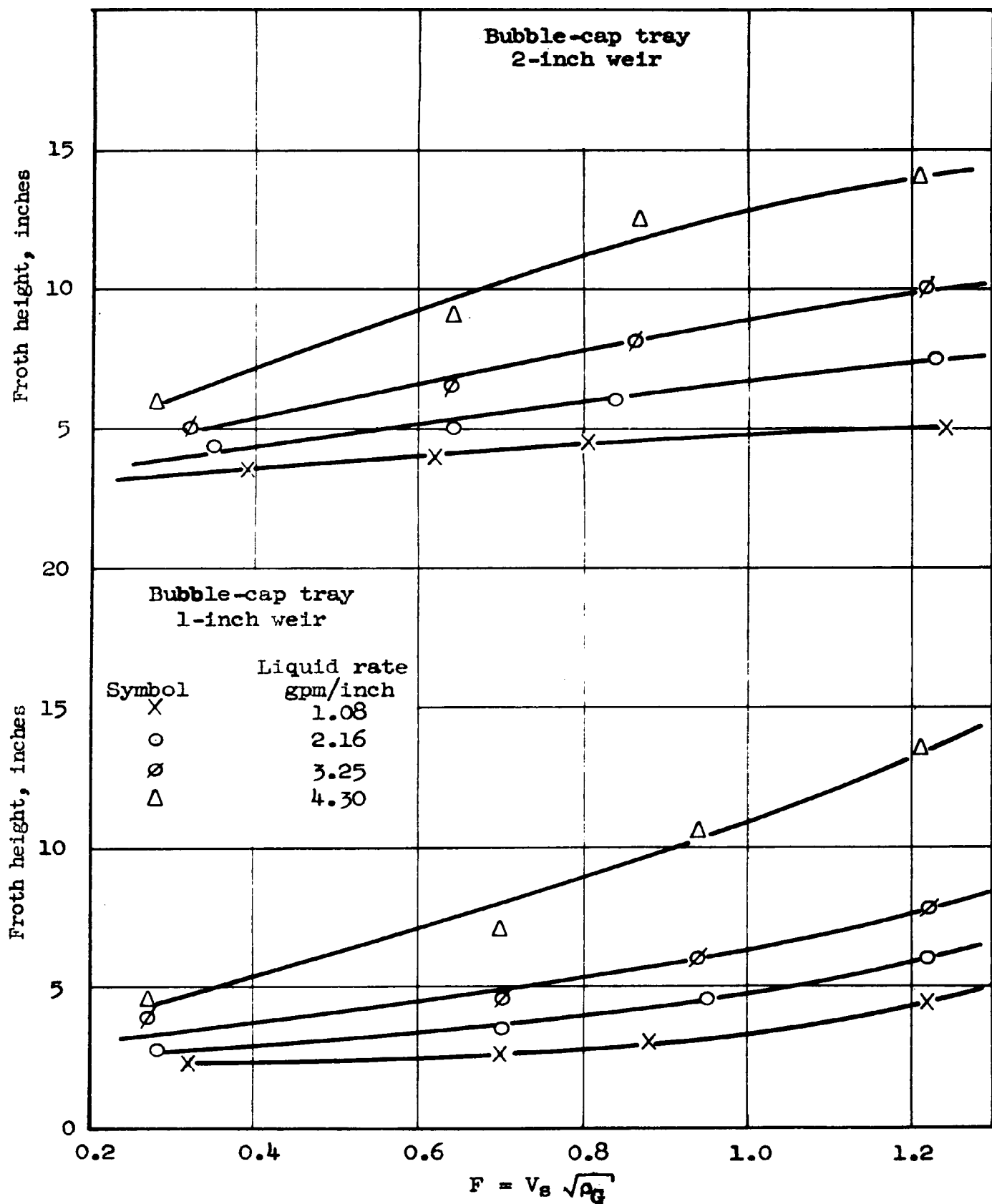


Fig. 17. Froth heights vs F-factor 2-inch and 1-inch weirs.

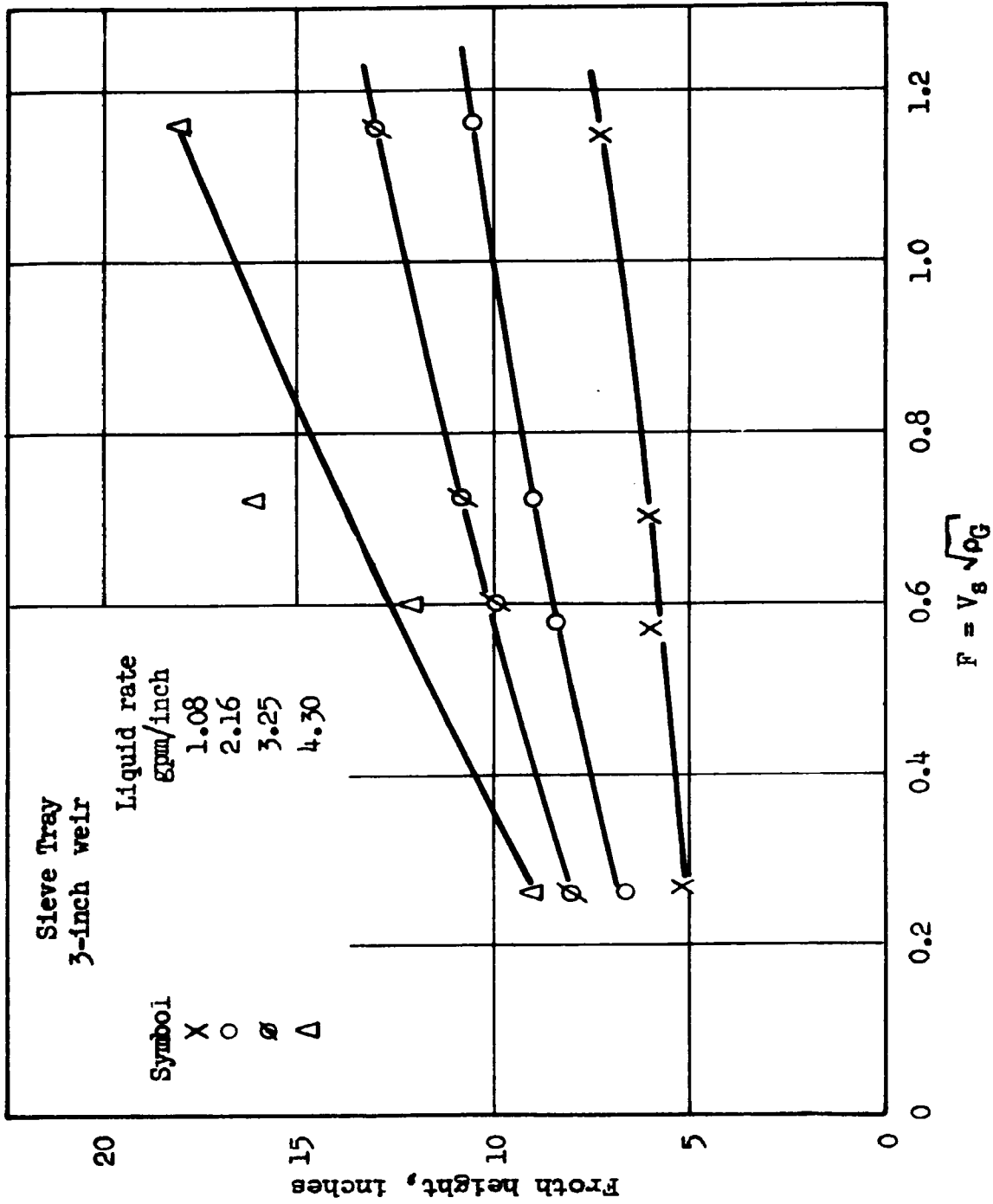


Fig. 18. Froth height vs F-factor sieve tray, 3-inch weir.

The interface between the froth and gas was quite distinct at the higher gas rates. At low submergences and gas rates a true froth no longer was present; instead, a considerable amount of coarse drops were blown out of the froth, making the interface indistinct. The gas and liquid rates at which drop formation occurred apparently corresponded to gas channeling through the liquid directly from the slot to the gas phase, and corresponded to a clear liquid height lower than the total height of the caps ($Z_c < 1.5$ inches).

The froth height data were correlated empirically by plotting Z_f versus each individual variable with all others held constant. An effect of plate length was detected from an examination of the data taken by Klein (43) on a 5-foot by 1-foot bubble-cap tray. Using Klein's data to determine the relative effect of plate length, the final correlation arrived at is as follows:

$$\ln Z_f = 0.64 F + 0.25 L S^{-1/3} + 0.33 Z_w - 0.1 \quad (16)$$

where the units are:

Z_f - froth height, inches

F - $V_s \sqrt{\rho_g}$, (ft/sec) $\sqrt{\text{lbs/cu.ft.}}$

L - liquid rate, gpm/inch of weir

S - plate length, taken as the distance between the outer edges of the end caps, ft.

Z_w - weir height, inches.

This correlation is shown graphically in Fig. 19, using data from the author's one-plate simulator.

The most important conclusion that can be drawn from the above correlation is the effect of plate length. This will be discussed more fully in the section, "Comparison with Other Investigators."

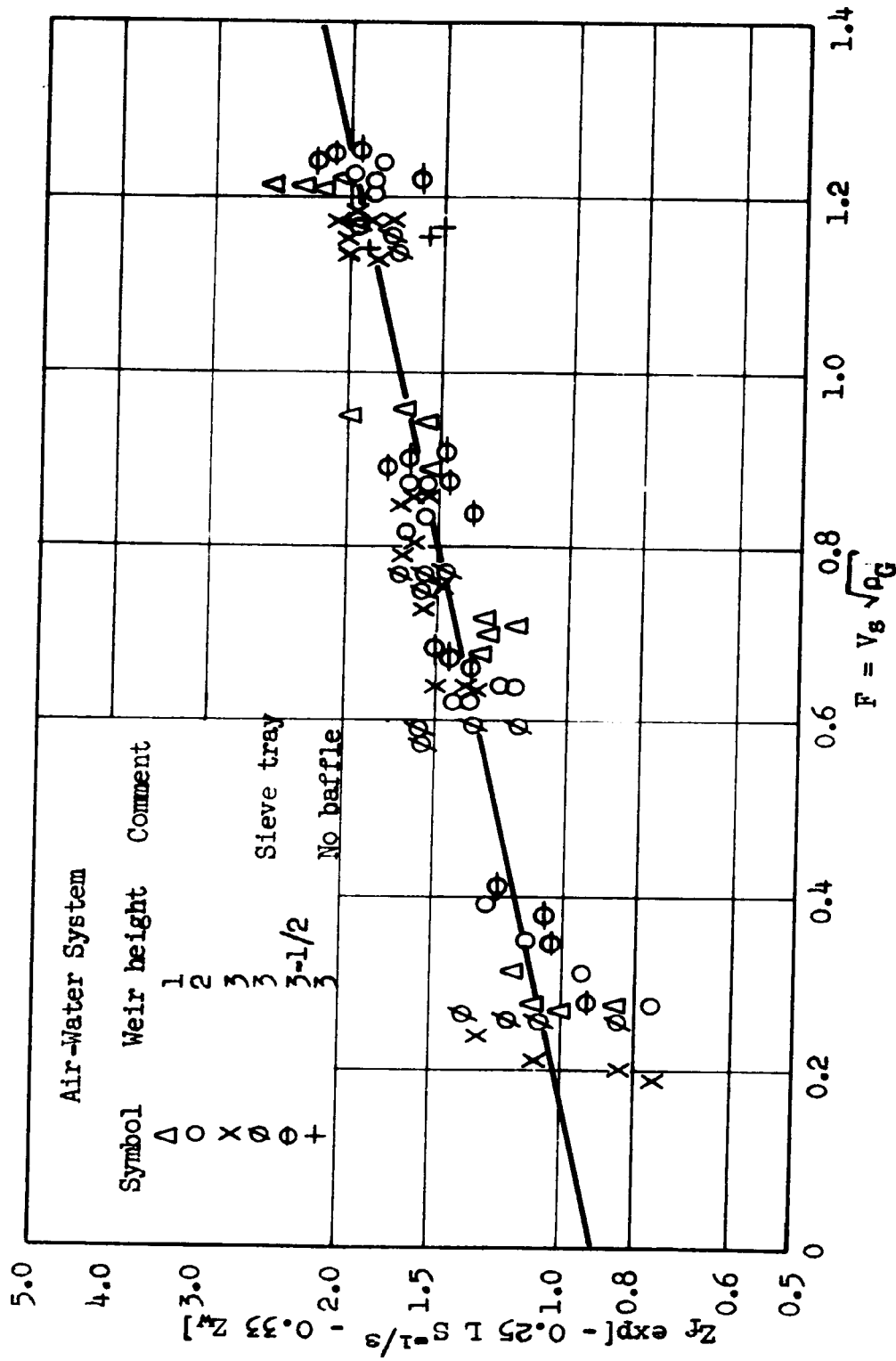


Fig. 19. Empirical froth height correlation.

Clear Liquid Heights.—Pressure tap measurements at three points on the tray were made for all froth density traverse runs; they were omitted in the mixing runs, as the probes interfered with the light transmission measurement.

The three probes were located as follows: in the inlet calming zone, between the first and second row of caps, and in the outlet calming section (positions C, B and O respectively). For the sieve tray measurements the probe design apparently was not adequate to measure the liquid height in the aerated portion of the tray as it probably picked up some gas and gave higher readings than the inlet and outlet calming sections.

Figures 20 to 24 show the data obtained in the outlet calming section and in the aerated region for the different weir heights studied. A general correlation of clear liquid heights was not attempted, as they seemed to be more sensitive to plate design than the froth heights. The aerated clear liquid height can be calculated from the correlated froth heights using an appropriate average froth density (ϕ).

In an operating column the aerated clear liquid height can be approximated from measured wet and dry pressure drop. The wet pressure drop is the sum of the dry pressure drop, the clear liquid height, and a residual pressure drop. The residual pressure drop varies from 0.1 to 0.4 inches, increasing with liquid and gas rate. Even without a correction for residual pressure drop, the clear liquid height can be obtained to approximately $\pm 10\%$ accuracy.

Average Froth Density.—These were calculated from froth height and clear liquid height data. Both heights were not corrected for slot opening, but instead measured directly from the tray floor. In a column with caps having no skirt clearance, the accuracy of the data does not justify

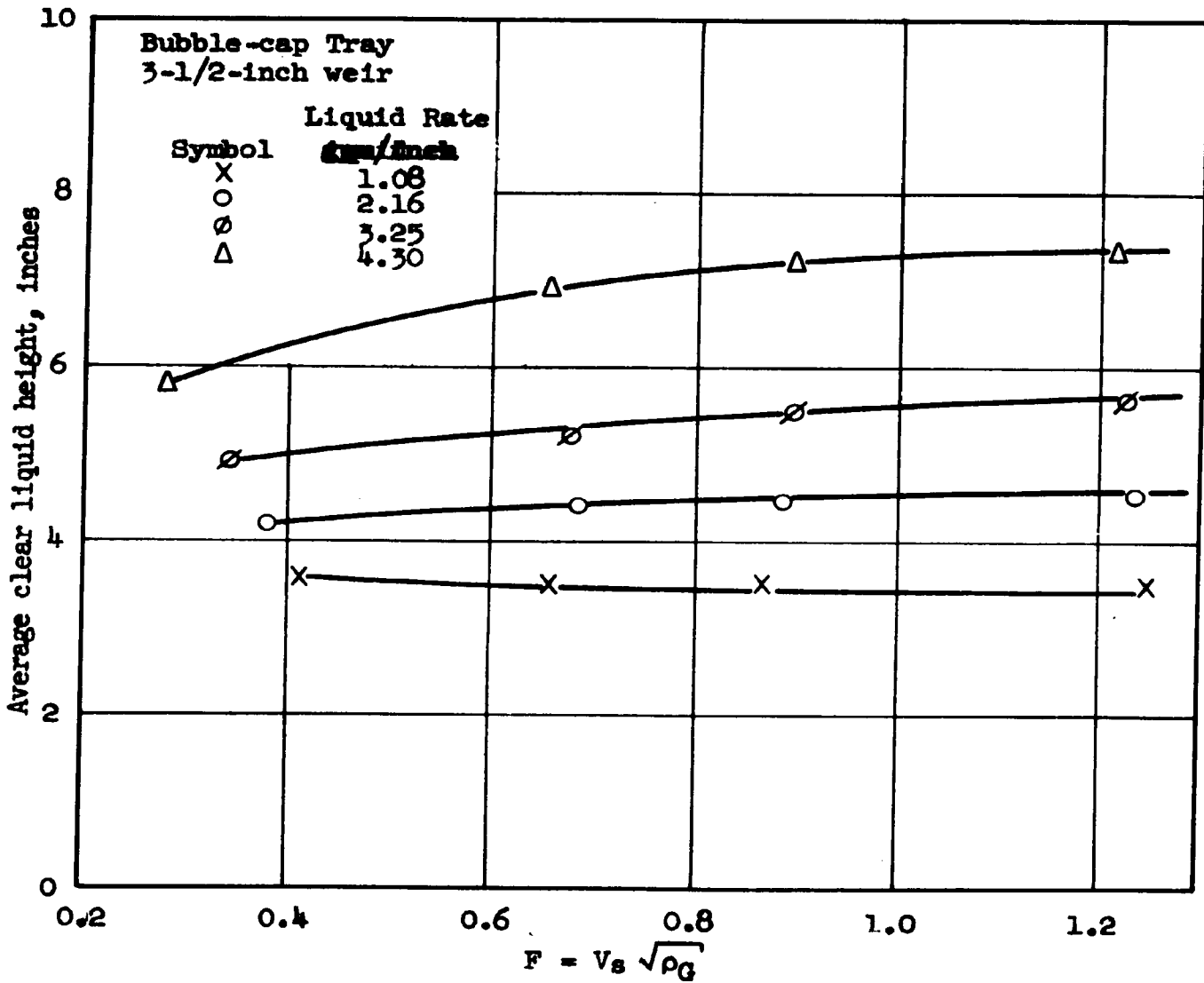


Fig. 20. Average clear liquid height vs F-factor, bubble-cap tray 3-1/2-inch weir.

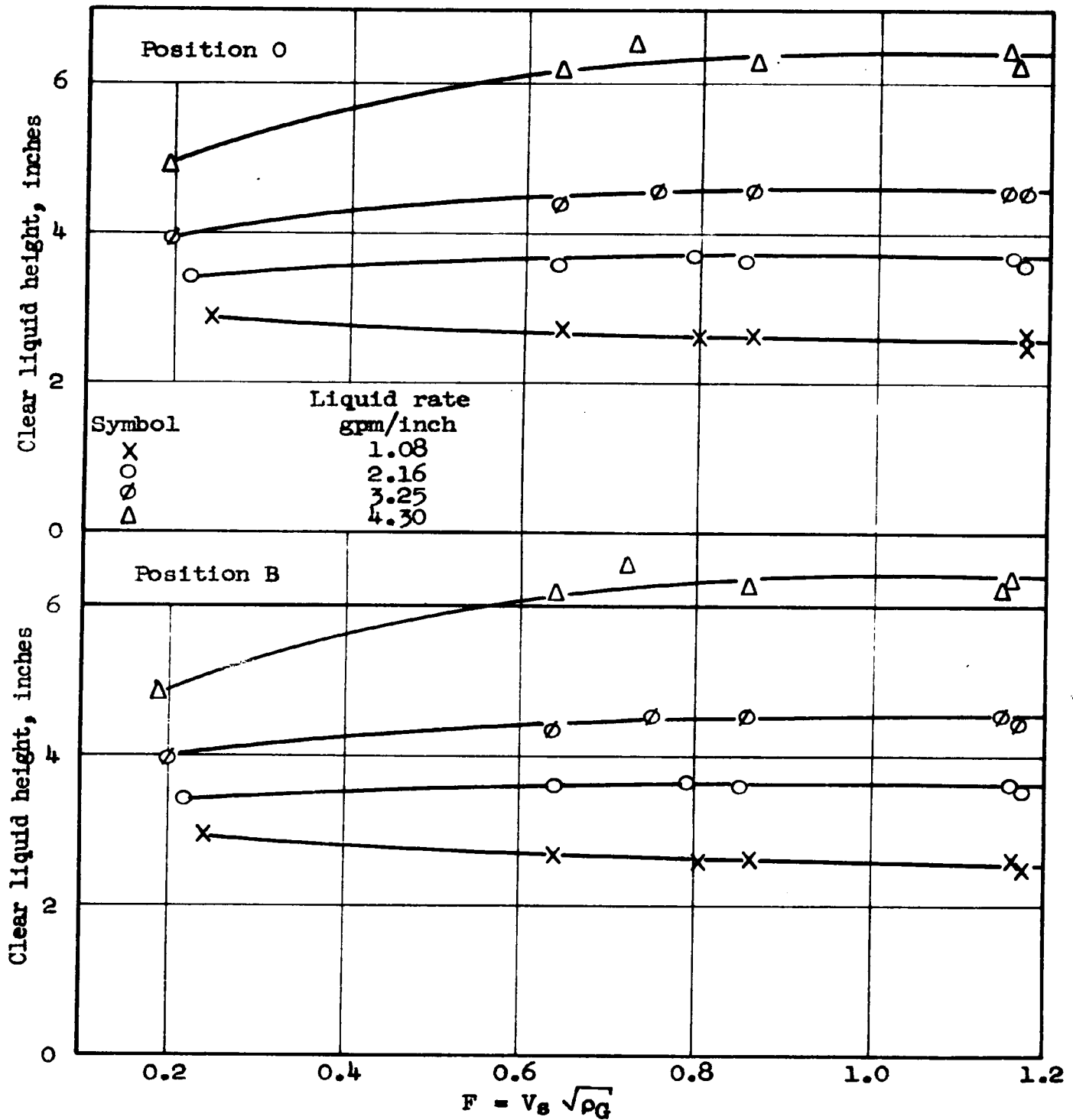


Fig. 21. Clear liquid height vs F-factor bubble-cap tray, 3-inch weir.

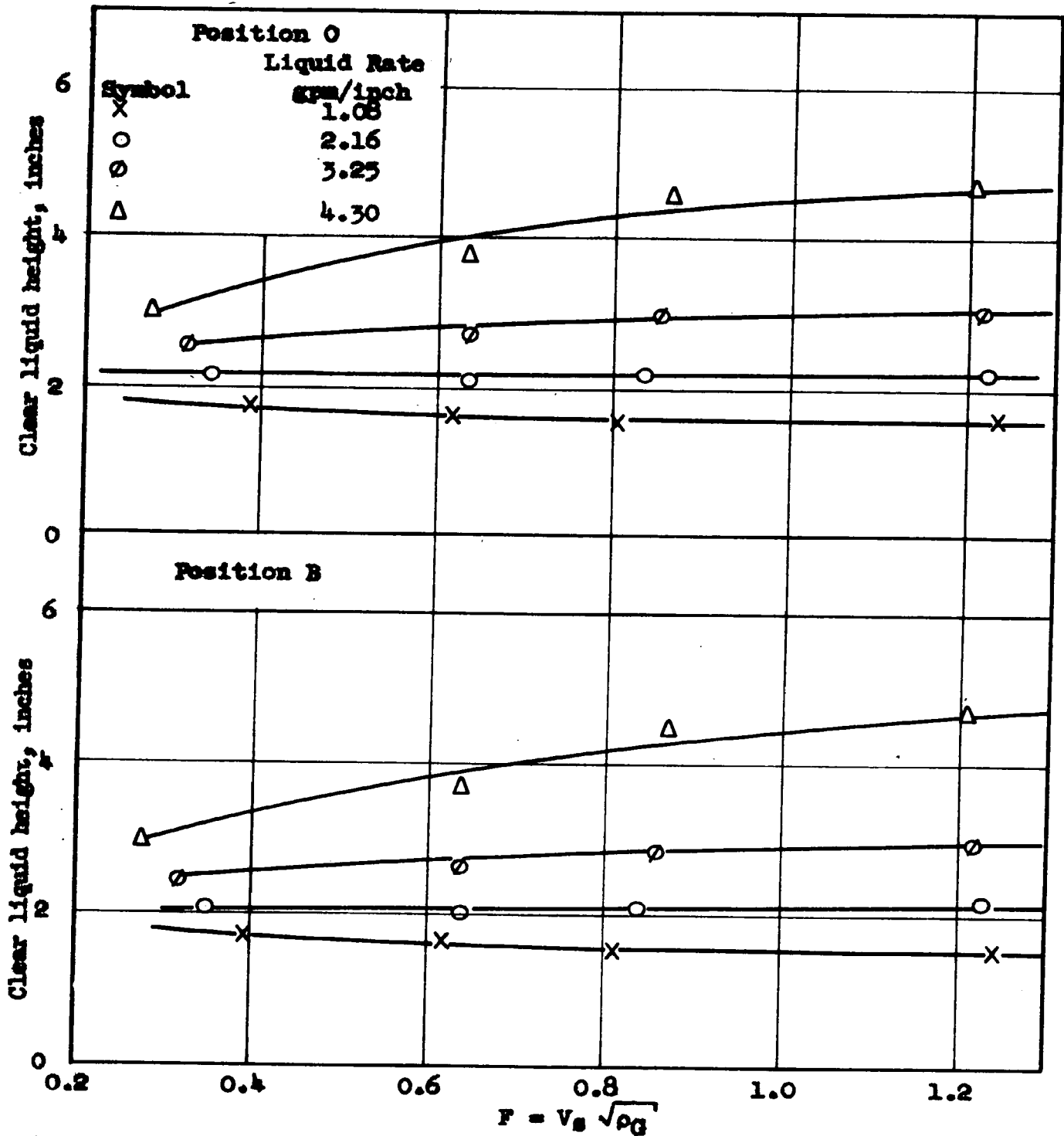


Fig. 22. Clear liquid height vs F-factor bubble-cap tray, 2-inch weir.

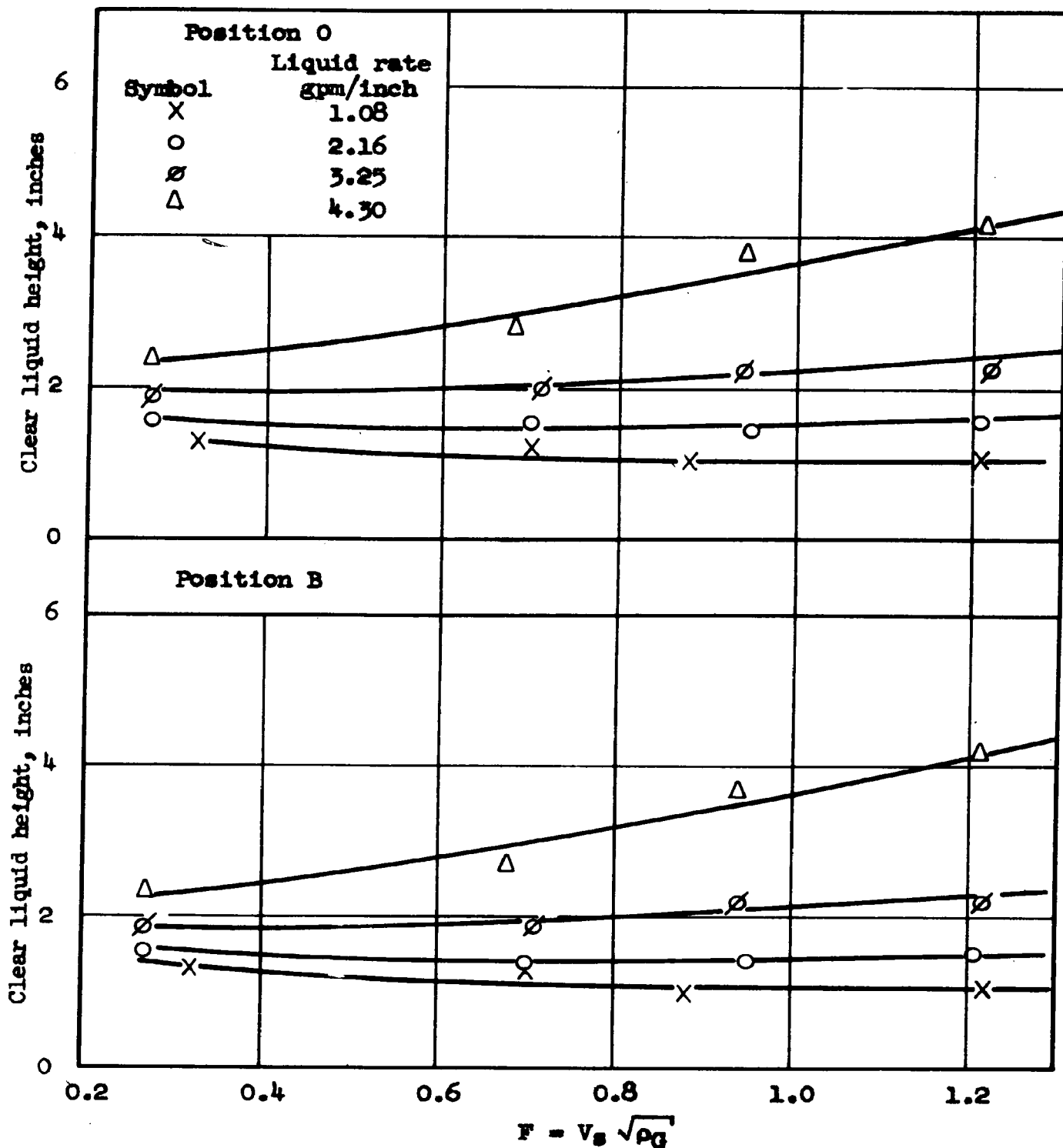


Fig. 23. Clear liquid height vs F-factor bubble-cap tray, 1-inch weir.

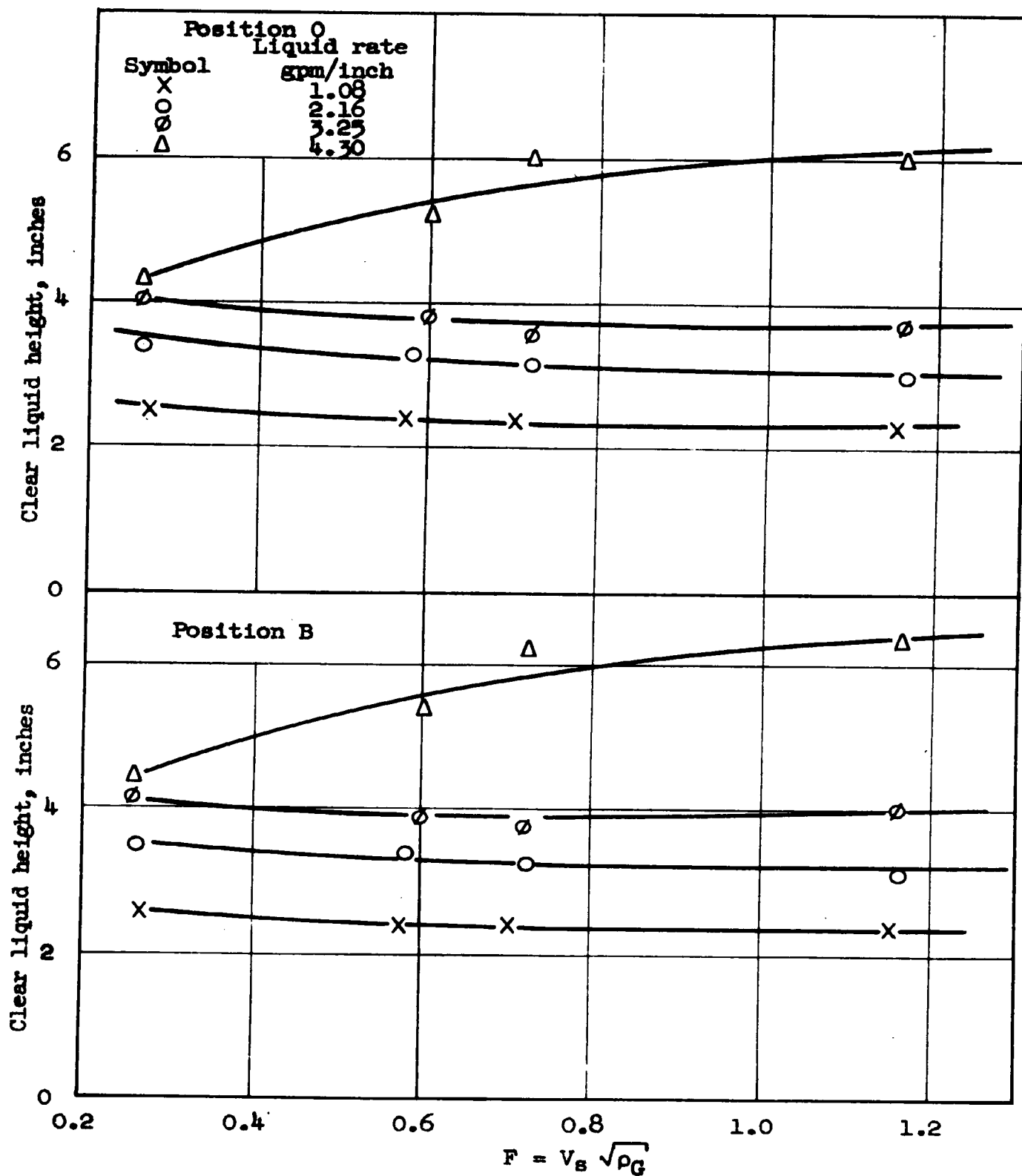


Fig. 24. Clear liquid height vs F-factor sieve tray, 3-inch weir.

correcting for slot opening in the few runs where the slots were not fully open. For caps with significant skirt clearance, it is recommended that both the froth height and clear liquid height be measured from the bottom of the slot.

The average froth density was found to be a function only of F-factor, with no detectable variation with liquid rate, weir height, or tray design.

The data are shown in Fig. 25, and are adequately represented by the equation:

$$- \ln \phi = 0.715 F + 0.45 \quad . \quad (17)$$

The scatter of the data is quite high ($\pm 30\%$), indicating that there is probably some effect of liquid rate, as the precision of the measurements of froth height and clear liquid height is about $\pm 20\%$ at worst. The difficulty in measuring froth and clear liquid heights with precision in this investigation made it impossible to determine the effect of liquid rate.

Froth Layer Stratification

To test the bubbling mechanism deduced from photographic studies, the density distribution in the froth was determined by means of light extinction measurements.

Nomenclature.—The following special nomenclature is used in discussing the density distribution in a froth:

Average froth density, ϕ - the density associated with the total froth volume and the total liquid hold-up on the tray, normally referred to as a specific gravity compared to pure water. Calculated by Z_f/Z_c .

Point froth density, ρ_f - the true specific gravity at a point in the froth; calculated from light transmission measurements.

Froth layer density, $\rho_{\overline{f}}$ - the true specific gravity of the constant density layer; calculated from light transmission measurements.

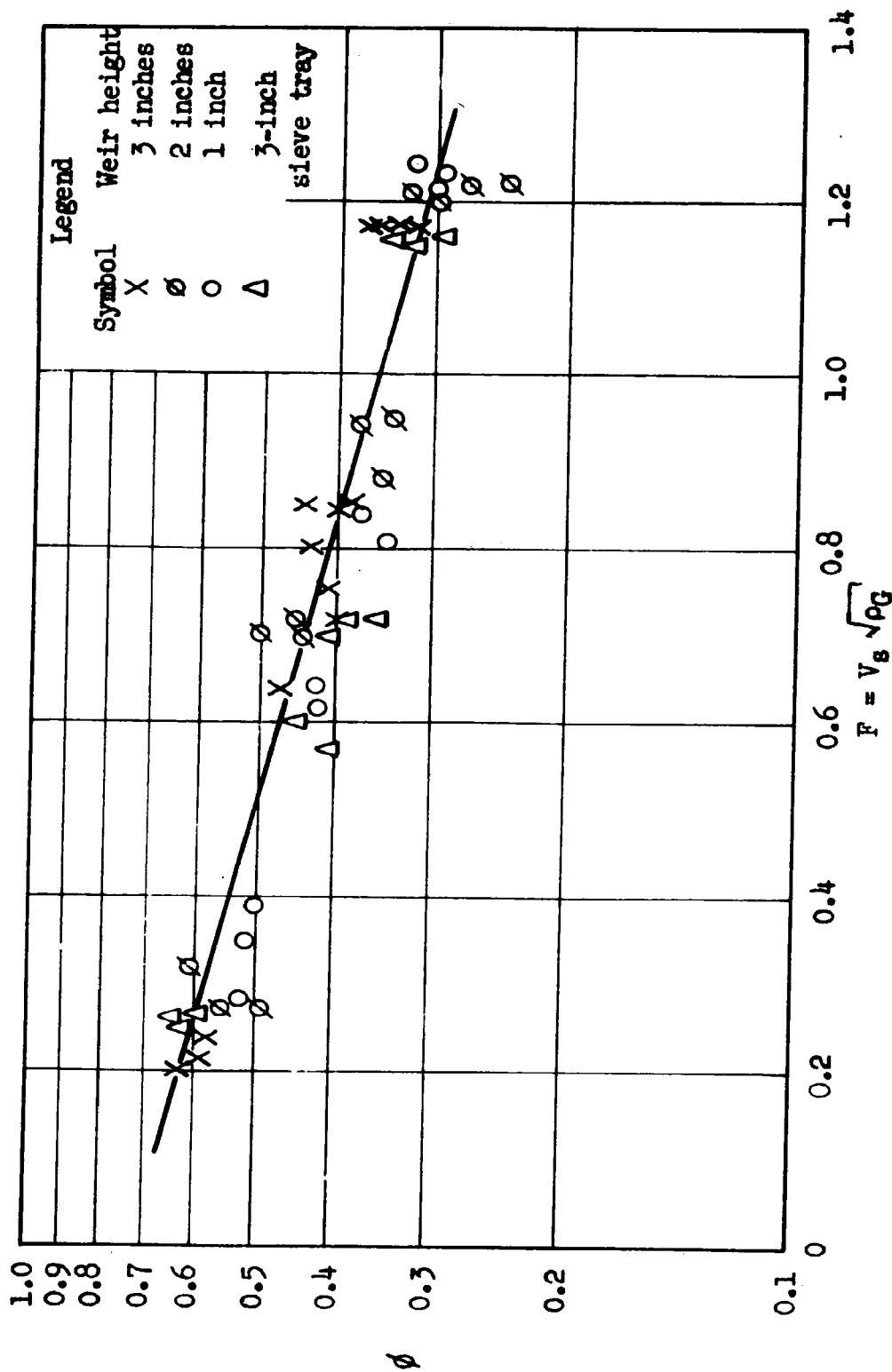


Fig. 25. Average froth density vs F-factor.

The term density is used throughout this work instead of specific gravity, as the system studied was air and water. The density of the air is small compared to water, and can be neglected.

Test of Measuring Technique.—Based on the froth drainage studies of Ross and Cutillas (55), the following relation between light transmission and point froth density for the air-water system has been derived:

$$\rho_f = [1 + c(I/I_1)^2]^{-1/2} \quad (15)$$

To test this relation, the point froth density of a number of froths with identical bubble size must be related to the light transmission of the froth. The greatest difficulty encountered in testing the above relation was the generation of a uniform froth where the point and average density were equal. A number of 4- x 4-inch sieve trays were tested without success. Even a tray with 1900, 1/64-inch holes generated a stratified froth. A uniform froth was finally successfully generated by means of a 1/2-inch thick porous brick diffuser.

The tests were conducted by blowing a measured amount of air through the test section, and measuring the light transmission, the liquid hold-up volume, and the froth volume. The uniformity of the froth was checked by means of a light traverse.

The data obtained, and a description of the test section, is given in Table IX in the Appendix. Figure 26 is a plot indicating the close agreement with equation (15). Of interest, in evaluating the performance of the tray column, is the dependence of the bubble size only on the gas rate, while the froth density varies with both gas rate and liquid hold-up.

Results of Photocell Traverses.—The density distribution in the froth was calculated by a trial and error graphical integration of the original light transmission traverses. A sample calculation is shown in detail in

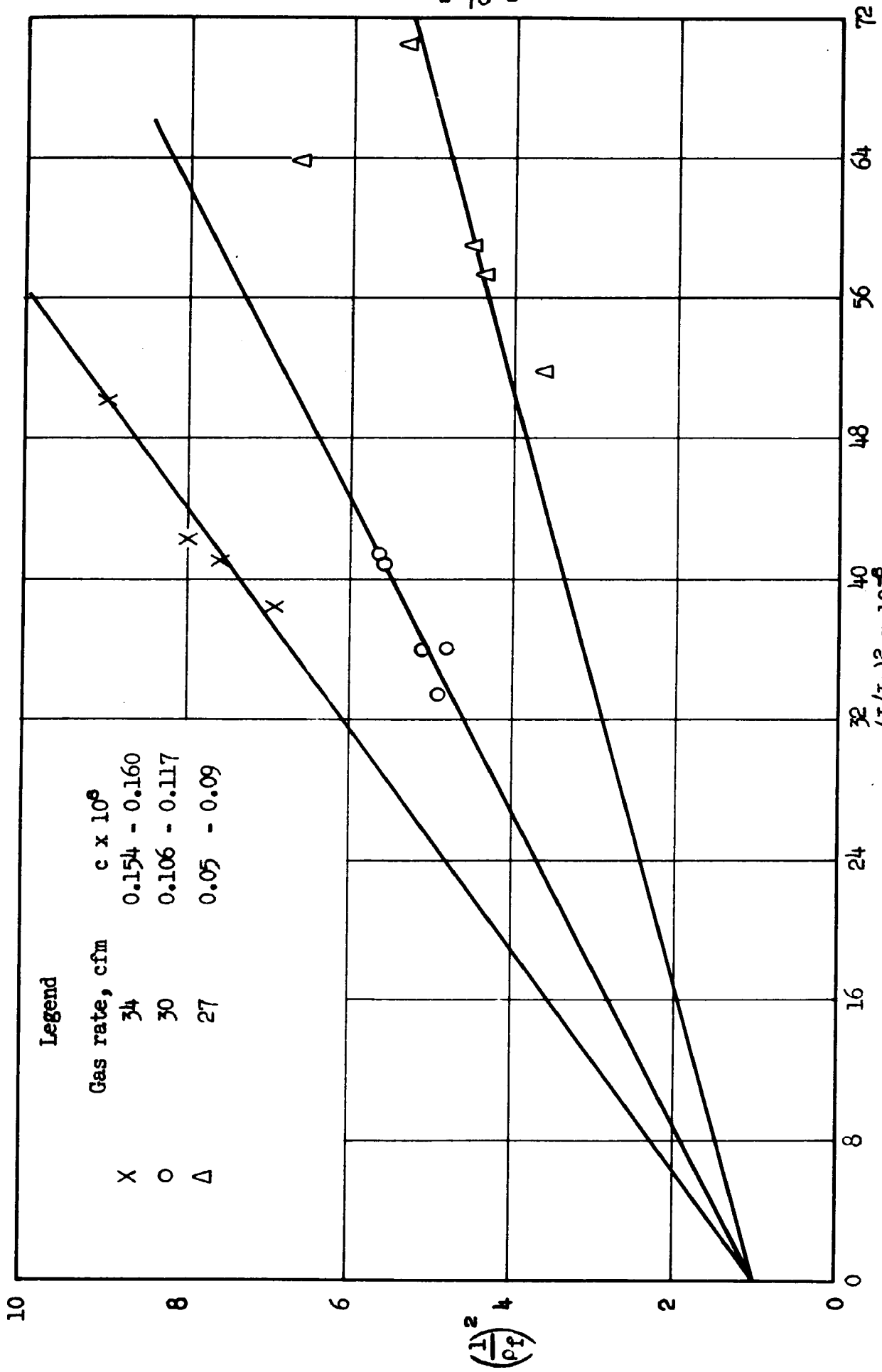


Fig. 26. Test of light transmission relation in special porous cell.

the Appendix. The trial and error calculation was rather time consuming, as the convergence was slow. The problem was, therefore, set up for calculation on a high-speed digital computer, and the University of Michigan computer (MIDAC) located at the Willow Run Research Center was used. The calculation took a total of two hours of computer time to complete. Spot checks of approximately 20% of the runs were made. The resultant density traverses and the tray operating conditions are given in Tables X, XI, XII and XIII of the Appendix. Figure 27 is a photographic copy of the photo-cell output chart for a typical experimental run.

The density traverse data clearly support the proposed three layer bubbling theory. A typical traverse for a high froth density run is shown in Fig. 28. At low gas and liquid rates the constant density region did not always appear. The existence of this region depended strongly on the submergence. Approximate operating conditions for the appearance of three layers is given in Fig. 29 for the bubble-cap trays tested. The sieve tray showed only two layers for all runs, the columnette and froth formation layers having merged into one. Every run showed a constant density layer.

The height at which the constant density layer started depended on the gas and liquid rates for the bubble-cap tray, and only on the liquid rate in the sieve tray. This height is plotted for the 2- and 3-inch weirs on the bubble-cap tray, and for the sieve tray, in Fig. 30. From a comparison of the total froth heights and the transition layer heights, it may be noted that the thickness of the constant density layer increases with gas and liquid rates. The change in thickness of the constant density froth layer with gas rate is shown in Figs. 31 and 32.

Correlation of Constant "c".—The constant "c" was evaluated for each run by trial and error, using equation (15) and the assumption that the

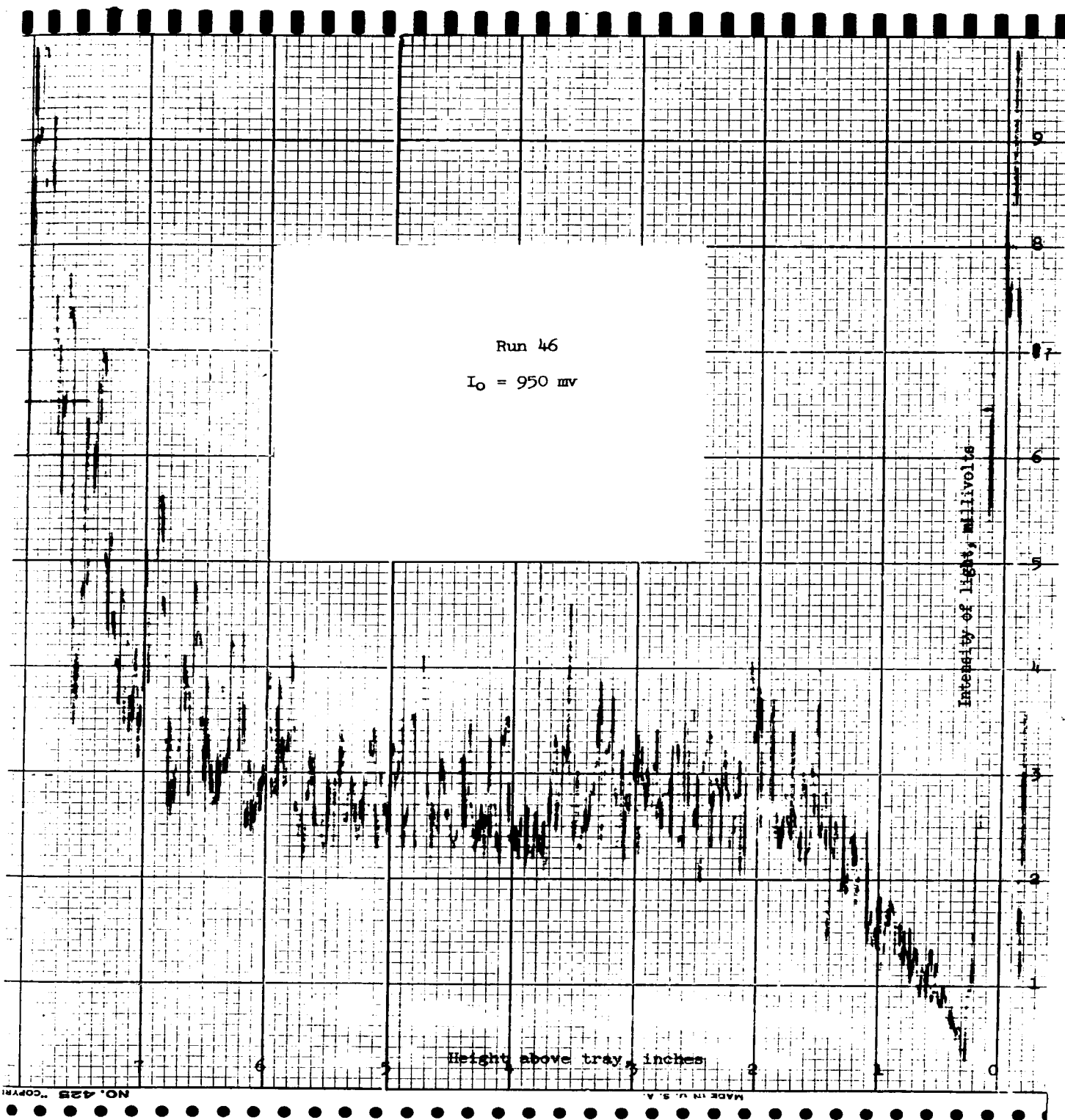


Fig. 27. Photograph of original photocell output chart.

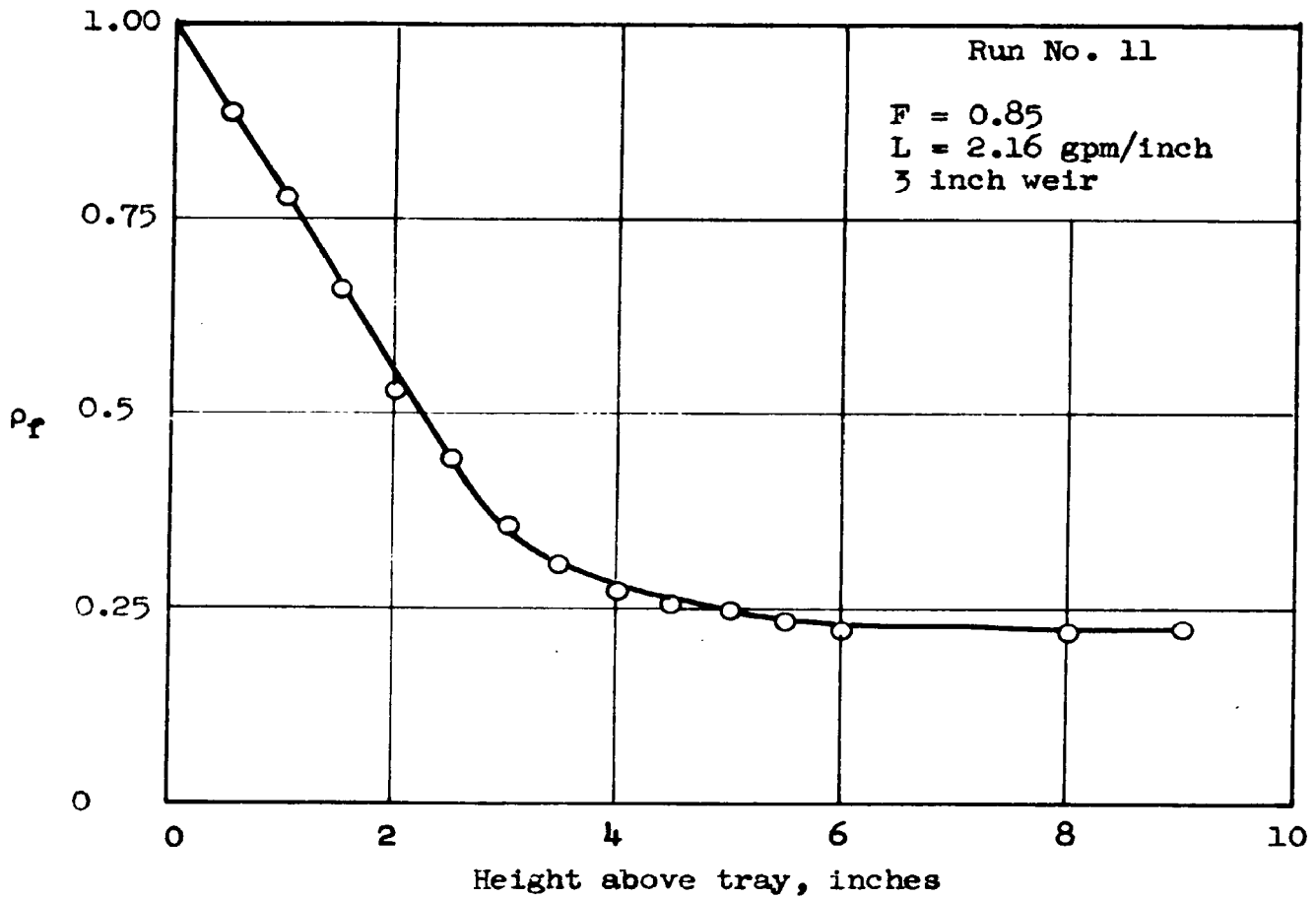


Fig. 28. Point froth density vs height above tray floor.

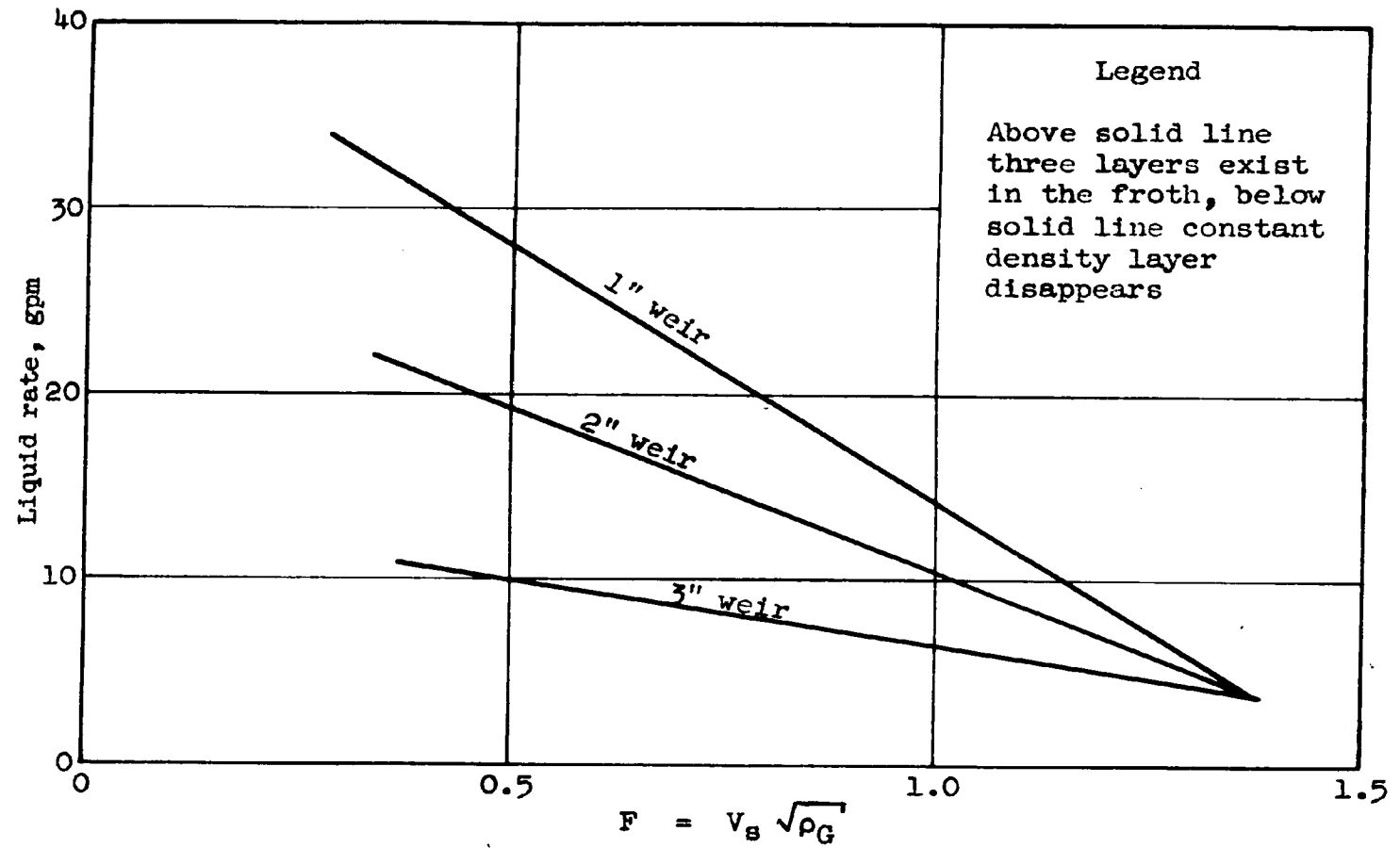


Fig. 29. Operating range for 1, 2 and 3 inch weirs where three density layers exist in the tray froth.

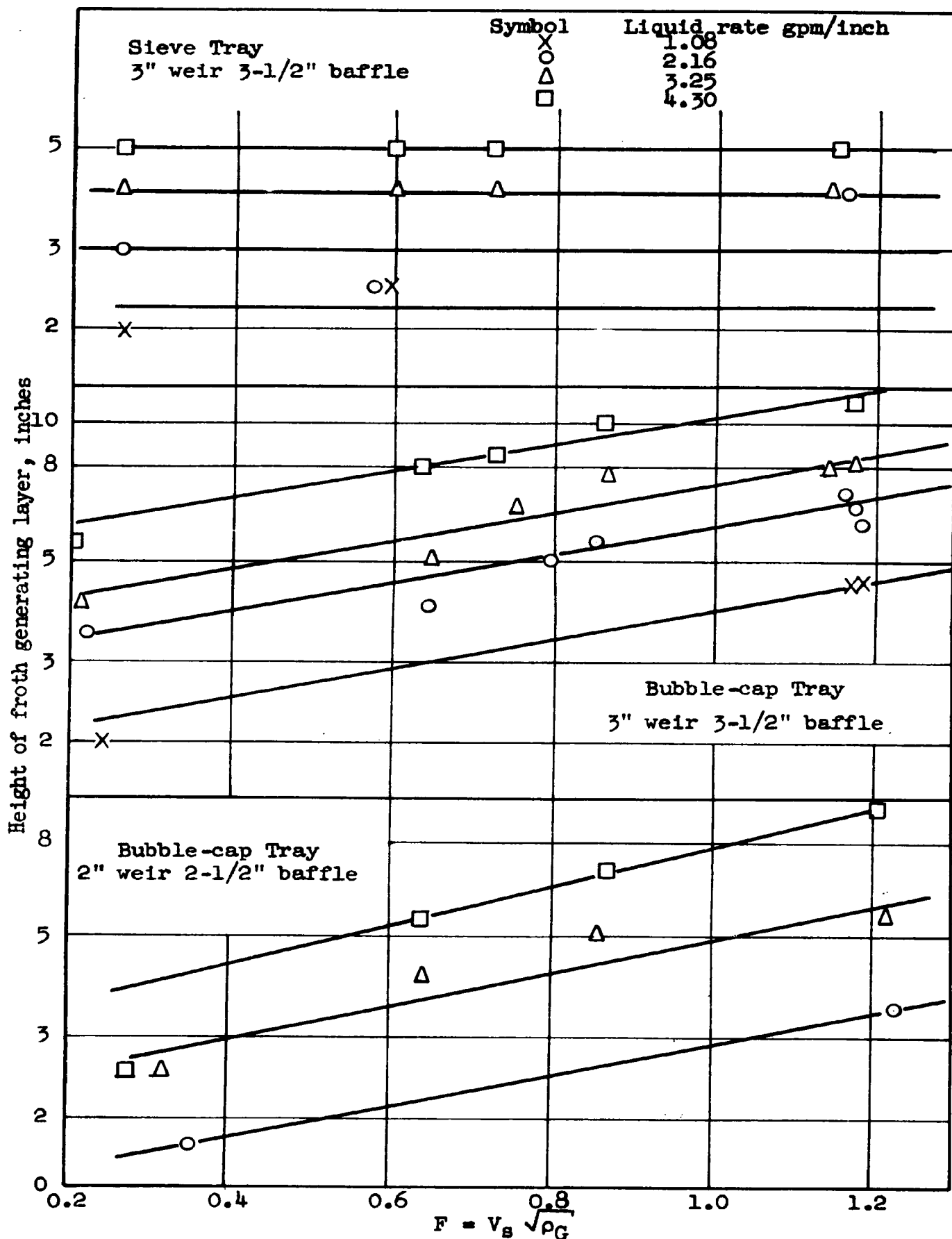


Fig. 30. Height of froth generating layer.

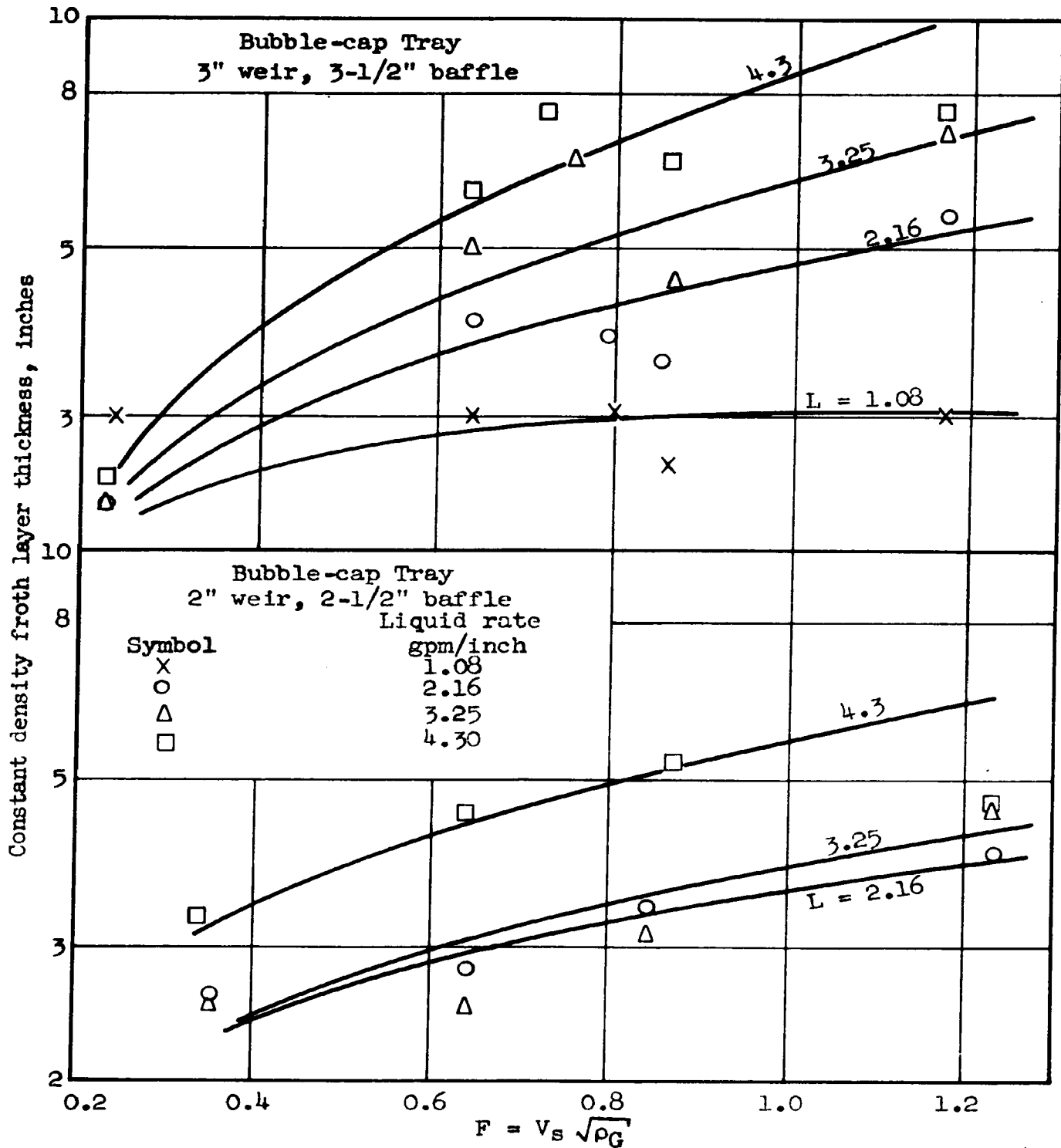


Fig. 31. Thickness of constant density froth layer, bubble-cap tray.

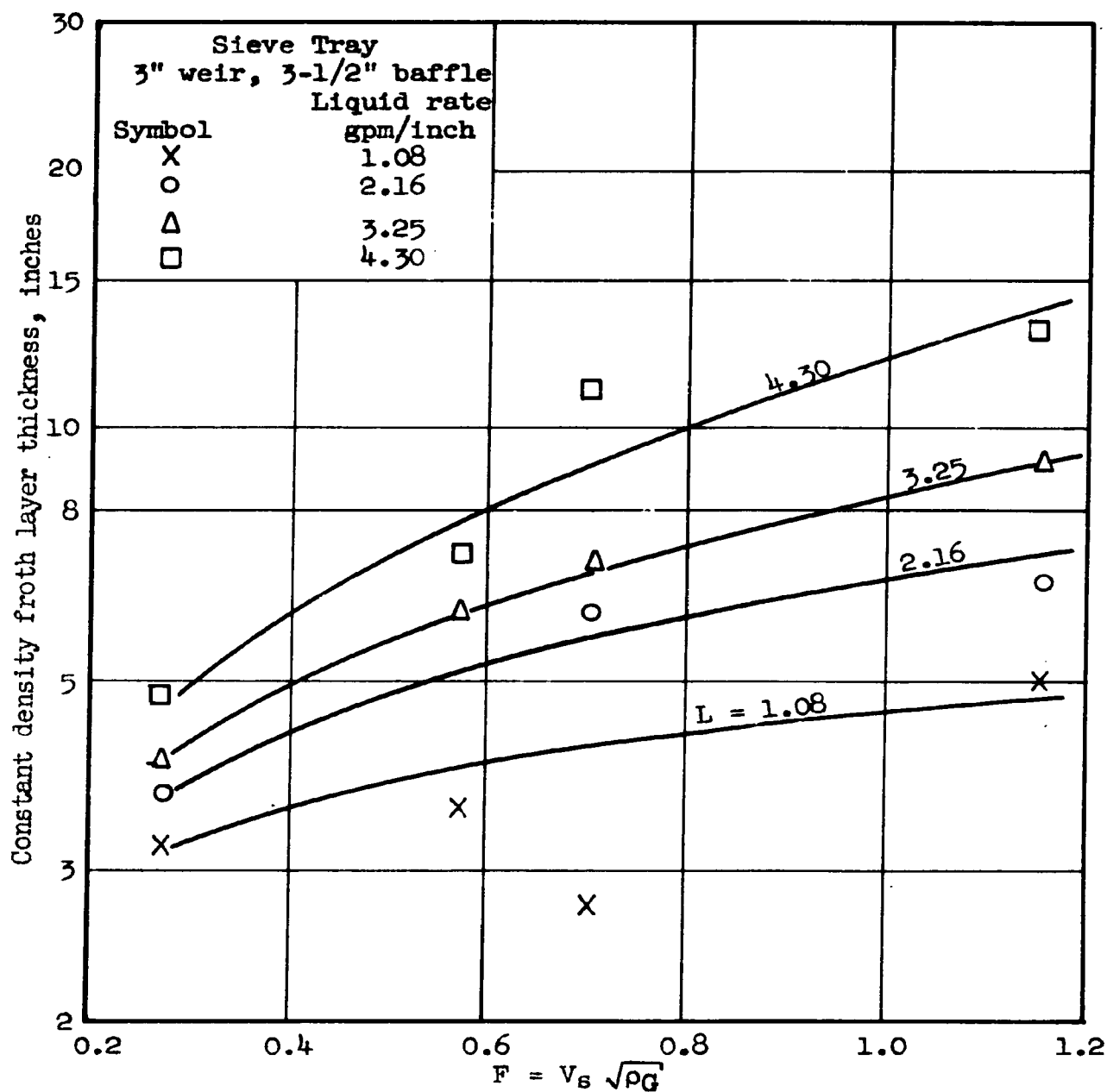


Fig. 32. Thickness of constant density froth layer, sieve tray.

bubble² size did not change in the different regions of the froth. This assumption was based primarily on photographic evidence. If the assumption is correct, the constant "c" should be a function only of the froth layer density, ρ_F . In Fig. 33, the constant "c" is plotted versus ρ_F for those runs which showed a constant froth layer. As can be seen, the correlation gives adequate support to the assumption of constant bubble size. The line drawn can be fitted by the equation:

$$c \times 10^{-6} = 0.1 (\rho_F)^{-2.17} \quad (18)$$

This relation is of interest as it indicates that the constant "c" has a simple proportionality with the interfacial area. This deduction is based on the previously discussed assumption that the froth layer density is a function only of the bubble size, and the fact that the interfacial area is a function of the square of the bubble size.

Correlation of Froth Layer Density.—The froth layer density, calculated from the light transmission traverses, was found to vary with both the gas rate and the liquid rate, unlike the average froth density which is a function of gas rate only.

The accuracy of measurement of the point froth densities is unfortunately low, as errors in measurement of the froth height, clear liquid height and the light transmission accumulate and affect the value of ρ_F strongly.

The trends in ρ_F with liquid and gas rates are shown in Fig. 34. The most accurate data are for the sieve tray, which has a simpler bubbling mechanism.

Interfacial Area.—The interfacial area of a gas-liquid mixture is a function of the bubble size and the number of bubbles per unit volume. West et al. (74) point out that the proportionality between interfacial area and bubble size is $[a \propto (3/r)(1 - \rho_F)]$, where "r" is the mean bubble radius in the

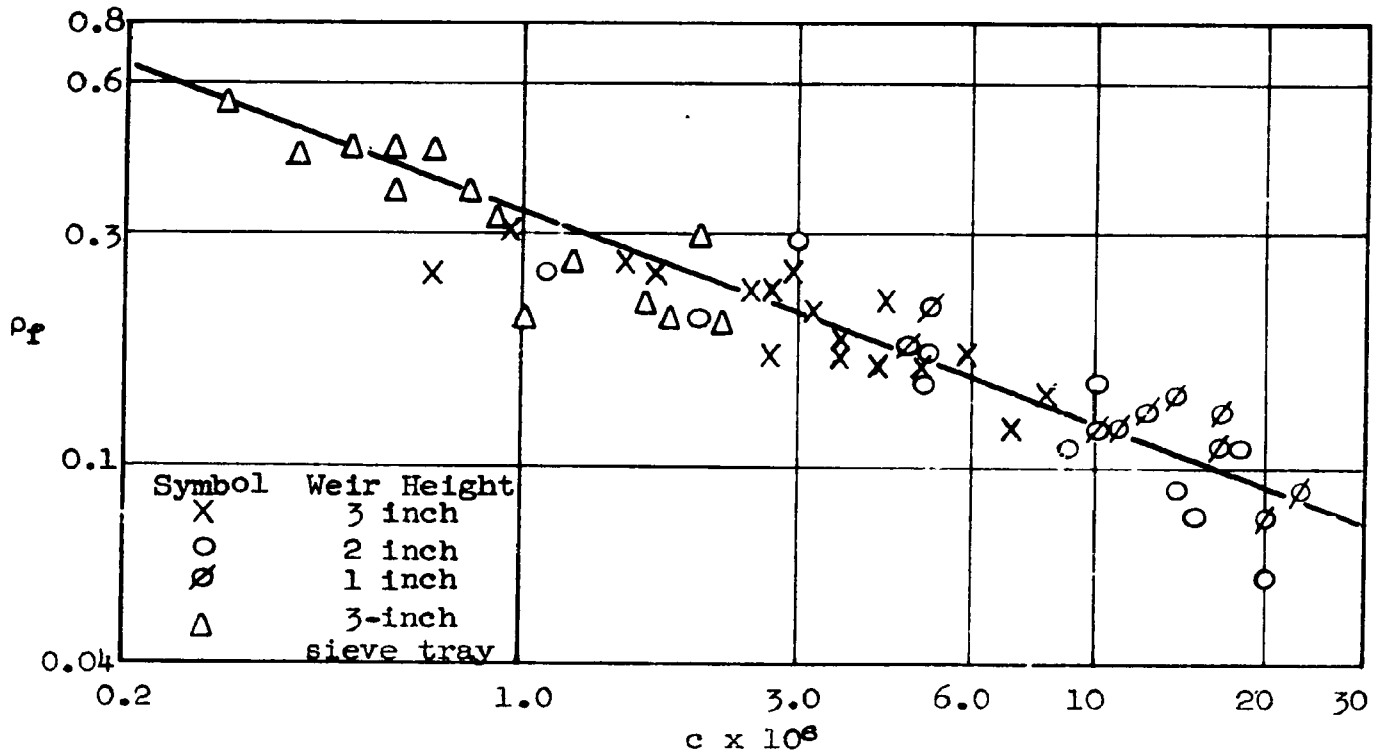


Fig. 33. Light transmission constant "c" vs froth layer density.

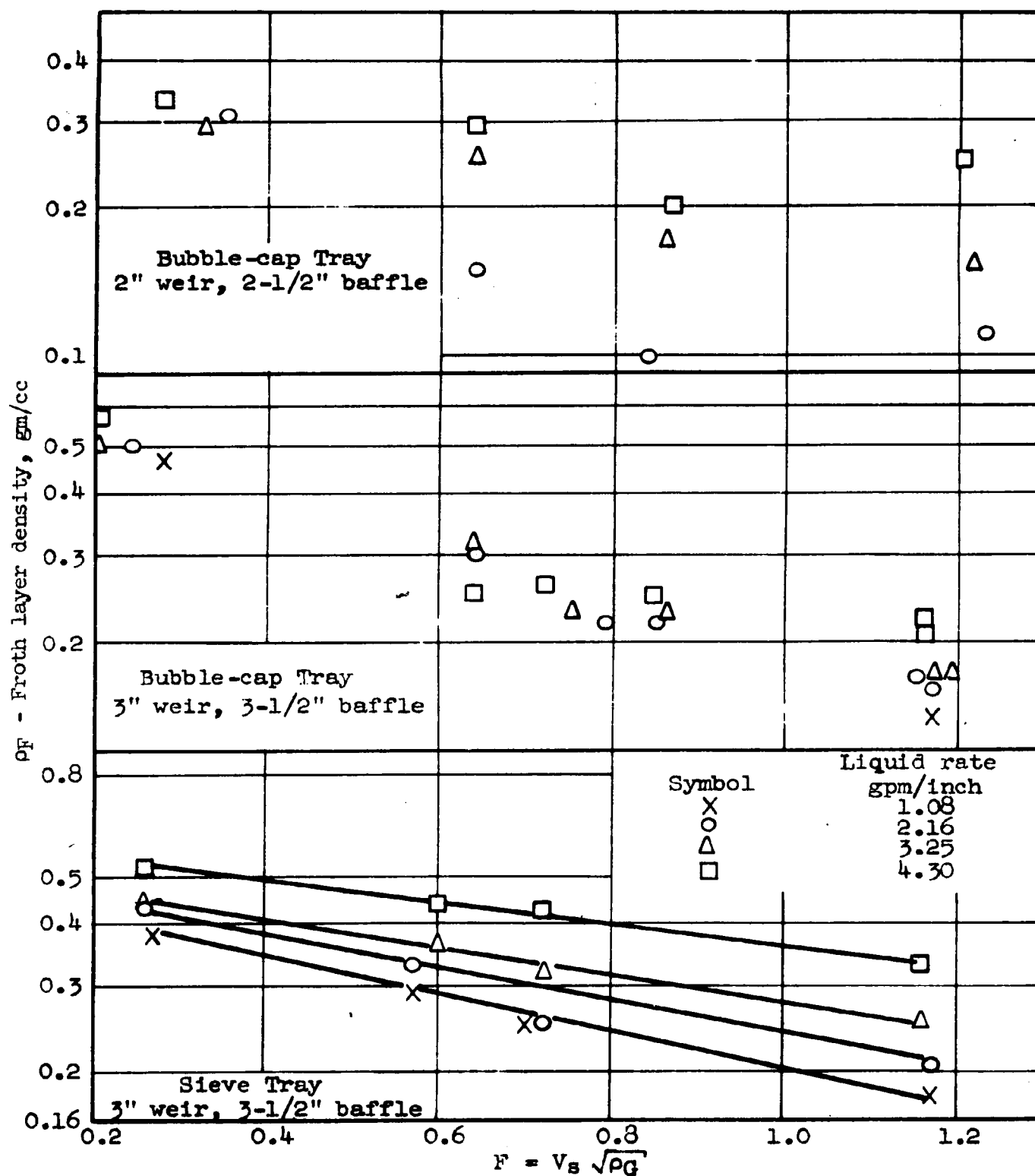


Fig. 34. Froth layer density vs gas and liquid rate.

froth. Now $\rho_F \propto 1/r$, and therefore

$$a \propto \rho_F(1 - \rho_F) = b \rho_F(1 - \rho_F) \quad (19)$$

Thus, the general trends in the magnitude of the interfacial area in a froth can be followed by comparison with the variation in froth layer density. One can therefore conclude that the interfacial area decreases with increasing gas rate and decreasing liquid rate.

Noting that the interfacial area is greater the smaller the bubble size, the variation of interfacial area with gas and liquid rates can be explained as follows: photographic evidence shows that the bubbles in a froth are generated by the breaking up of a sheet of gas rising from the bubble caps; due to surface tension forces, a greater input of energy per unit volume of gas is necessary to create smaller bubbles. In a bubble-cap tray the greatest energy input per unit volume of gas will be at low gas rates and high liquid rates, indicating that the bubble size should decrease with decreasing gas rate and increasing liquid rate.

Comparison of Results with Other Investigators

Froth Height Correlation

Froth height data is difficult to measure with accuracy and is not extensively available in the literature.

The plate length correction was determined by comparison of the author's data with the data of Klein (43). The length of liquid flow path was taken as the distance between the outer edges of the caps in the terminal and initial cap rows. Figures 35 and 36 show a comparison of equation (16) with data for columns with liquid paths ranging from 0.5 to 5.0 feet and operating at gas rates more than twice those used by the author. The agreement with equation (16) is shown to be good. Table III summarizes the main

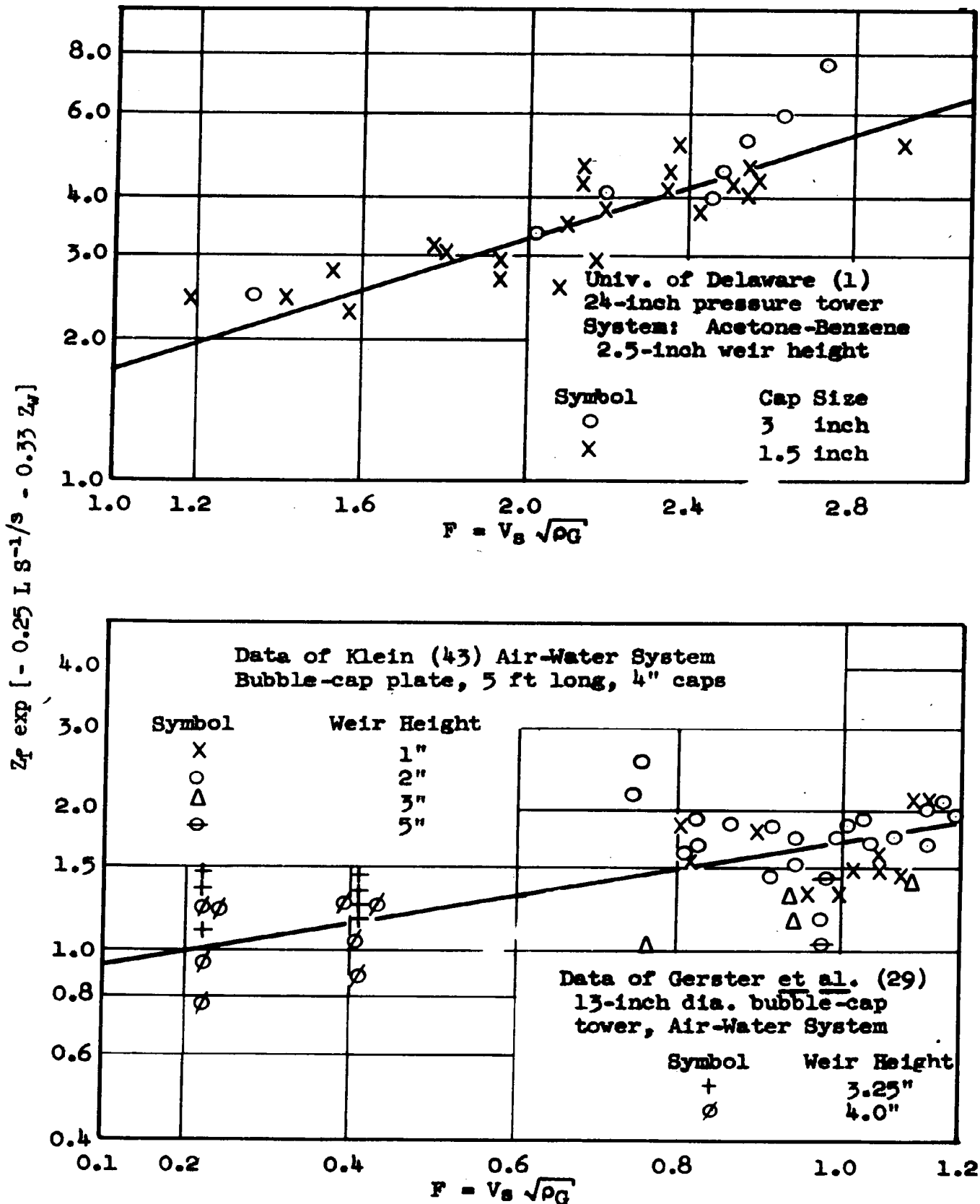


Fig. 35. Test of equation (16) with data of other investigators.

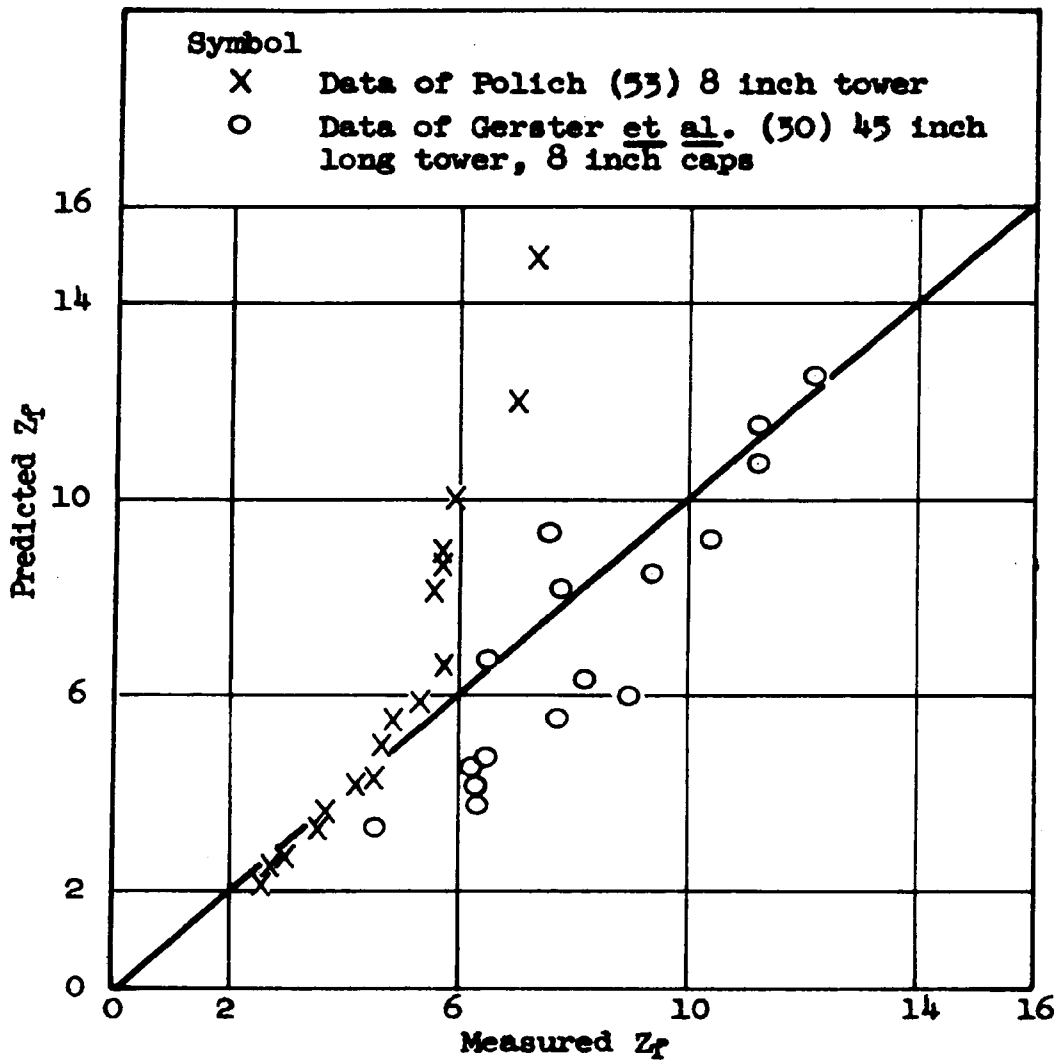


Fig. 36. Comparison of Z_f predicted by equation (16) and froth heights measured in an 8 inch round tower and a 45 inch rectangular tower.

dimensions of the columns used to obtain the data plotted in Figs. 35, 36 and 37.

TABLE III

Dimensions of Test Plates Used by Authors
in Figures 35 to 39

Reference	Column Diameter or Plate Over- all Length, ft.	Plate Length, "S" ft.	Cap Size, inches
Univ. of Delaware (1) pressure column	2 diameter 2 diameter	0.835 1.44	1-1/2 3
Gerster <u>et al.</u> (29)	1.08 diameter	0.5	1-5/8
Klein (43)	10 x 1.18	5	4
Ashby (5) and Warzel (73)	identical tray to that used by author		
Polich (53)	0.835 diameter	0.6	2.4
Gerster <u>et al.</u> (30)	13 diameter	3.75	8
Fractionation Research, Inc. (3)	4 diameter	2.4	4

Figure 37 compares equation (16) with data from the A. I. Ch. E. absorber at Michigan which has an identical tray layout to that used by the author. The data of Warzel (73) were taken with a system consisting mainly of air and water and with operating variables covering the same range as this investigation. The agreement is good.

The Ashby (5) data, also shown in Fig. 37, cover a number of systems. The data include the air-water system and are consistent with each other. They do not agree with data taken by Warzel (73) or that of the present investigation. It should be noted that Ashby operated the column at one weir height (1-1/2 inches) and one liquid rate (1.08 gpm/inch), and varied only

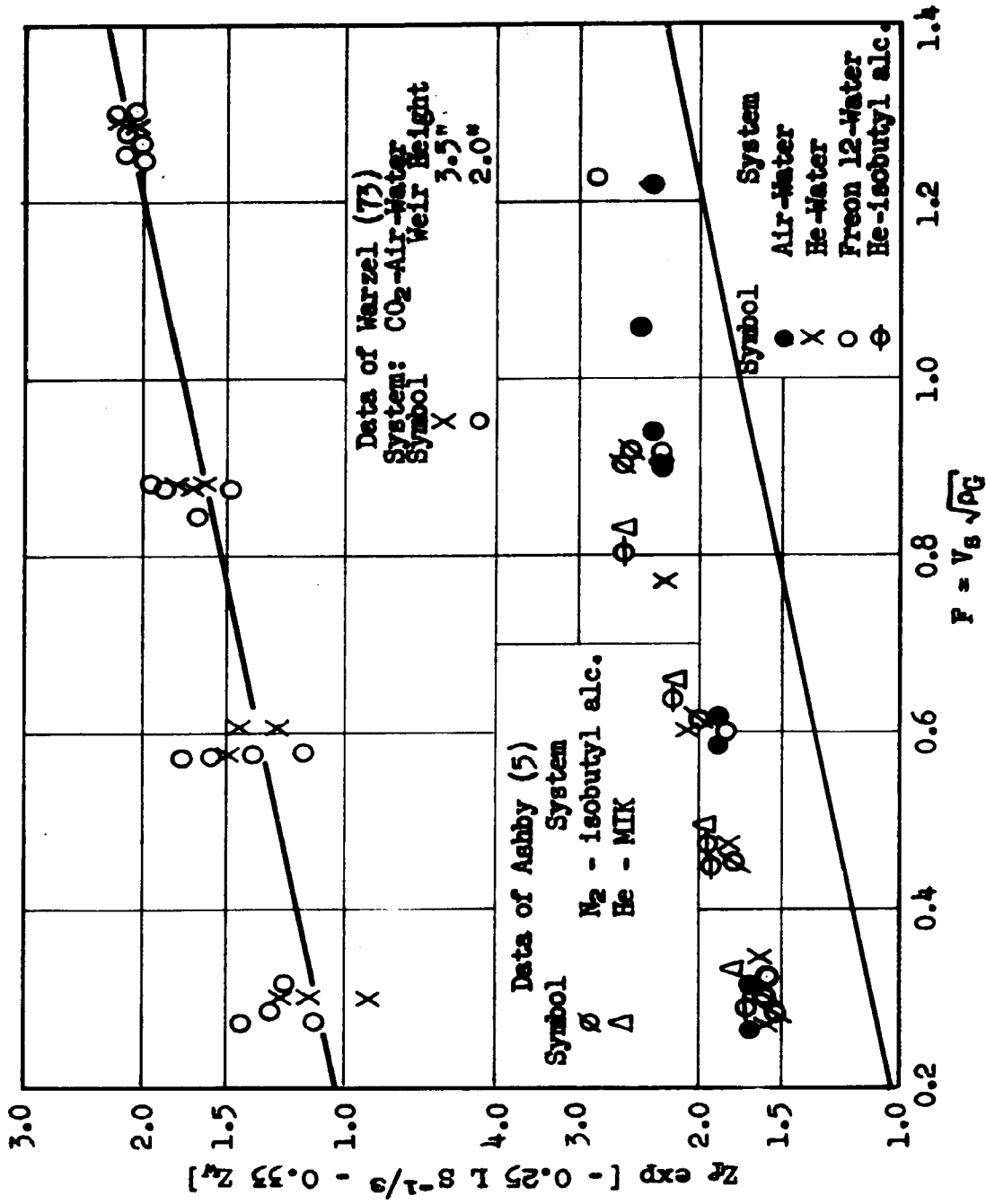


Fig. 37. Comparison of equation (16) with data with identical tray layout to the author's simulator.

the gas rate. The corresponding rates for the one-plate simulator were found (by light traverses) to result in the spray type froth without a constant density layer. The measurement of the froth height for this regime is open to considerable personal interpretation because of the lack of a clear froth-gas interface. As the air-water data is consistent with the data for the other systems, it may be concluded that there is no effect on froth height due to reasonable changes in system properties; this is, of course not true for changes in system properties caused by foam-producing impurities.

The wide variation in cap design used in obtaining the data shown in Figs. 35, 36 and 37 suggests that the gas slot velocity has no effect on the froth height. Data to determine the effect of slot velocity was taken in the A. I. Ch. E. absorber at Michigan (2) and is shown in Fig. 38. The system used was air and water with identical operating conditions to those used by Ashby (5) except that the cap slots were modified. The modification consisted of the following: (a) sealing the bottom half of all slots; (b) sealing alternate slots; and (c) cutting off the slot teeth completely. No effect of slot velocity on froth height or the mass transfer coefficients was found. Of considerable interest from the point of view of the bubbling mechanism is the operation of the caps without slots. The fact that no difference in the operation was detected for the caps without slots strongly supports the deduction that an annular sheet of gas forms around the caps and is only broken up into bubbles after leaving the protection of the cap wall.

Further study of the effect of the outlet splash baffle on froth height is necessary, as Hutchinson et al. (39) have pointed out that a splash baffle placed below the weir height will considerably increase the clear liquid height in the aerated region. The few runs made without a splash baffle showed

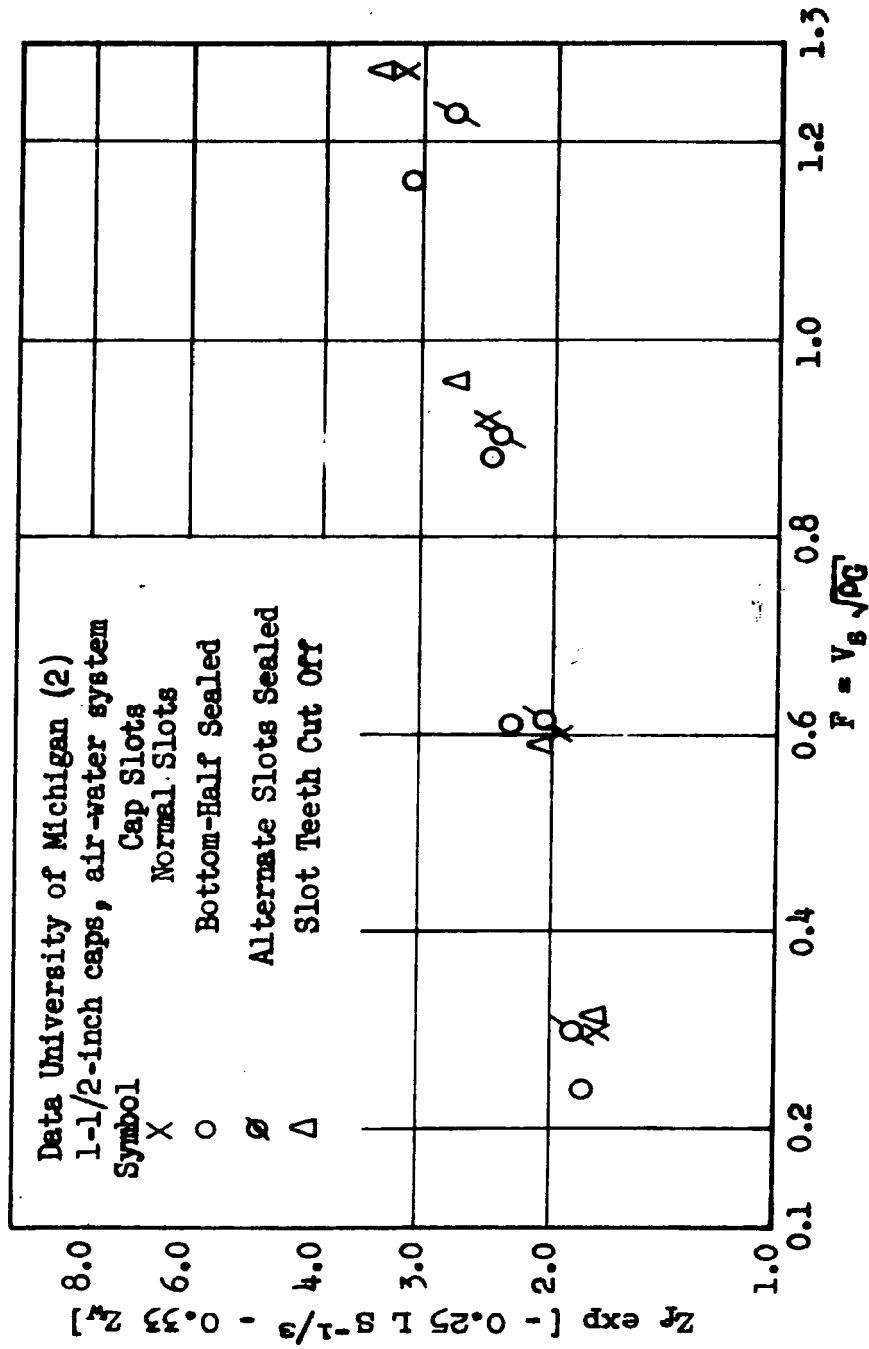


Fig. 38. Effect of slot velocity on froth height.

no significant effect on froth height of the splash baffle, even though at the high liquid rates the lower edge was below the clear liquid surface.

A general correlation of froth heights is of importance in interpreting mass transfer results and in predicting the hydraulic characteristics of a column. For example, it has been observed by the author and others (3) that columns with adequate liquid capacity in the downcomer flood when the froth height reaches the tray above.

Average Froth Density

The average froth density data were correlated by equation (17). This equation is compared to data available in the literature in Fig. 39. The details of the tray designs used are given in Table III. The data of Zenz (78) was taken on a number of sieve trays. Some of the scatter of Zenz's data is due to the fact it was read off from small-scale figures given in the original article. The scatter of all the data is considerable, but appears to be random above and below the curve represented by equation (17). The increased scatter at high F-factors is understandable, considering the difficulty in measuring froth heights and clear liquid heights in high pressure equipment.

A wide range in bubble-cap and sieve tray design, and a considerable number of gas-liquid systems are represented in Fig. 39; one can, therefore, conclude that the average specific gravity of the froth in any type of column is a function only of the F-factor. It has been found that the linear gas velocity will not correlate data for different systems at different pressures, and there is little doubt that the F-factor does do this adequately.

The above-mentioned correlation of average specific gravity can be used to calculate the froth height in a column if the clear liquid height in the aerated region of the tray is known or if the wet pressure drop can be

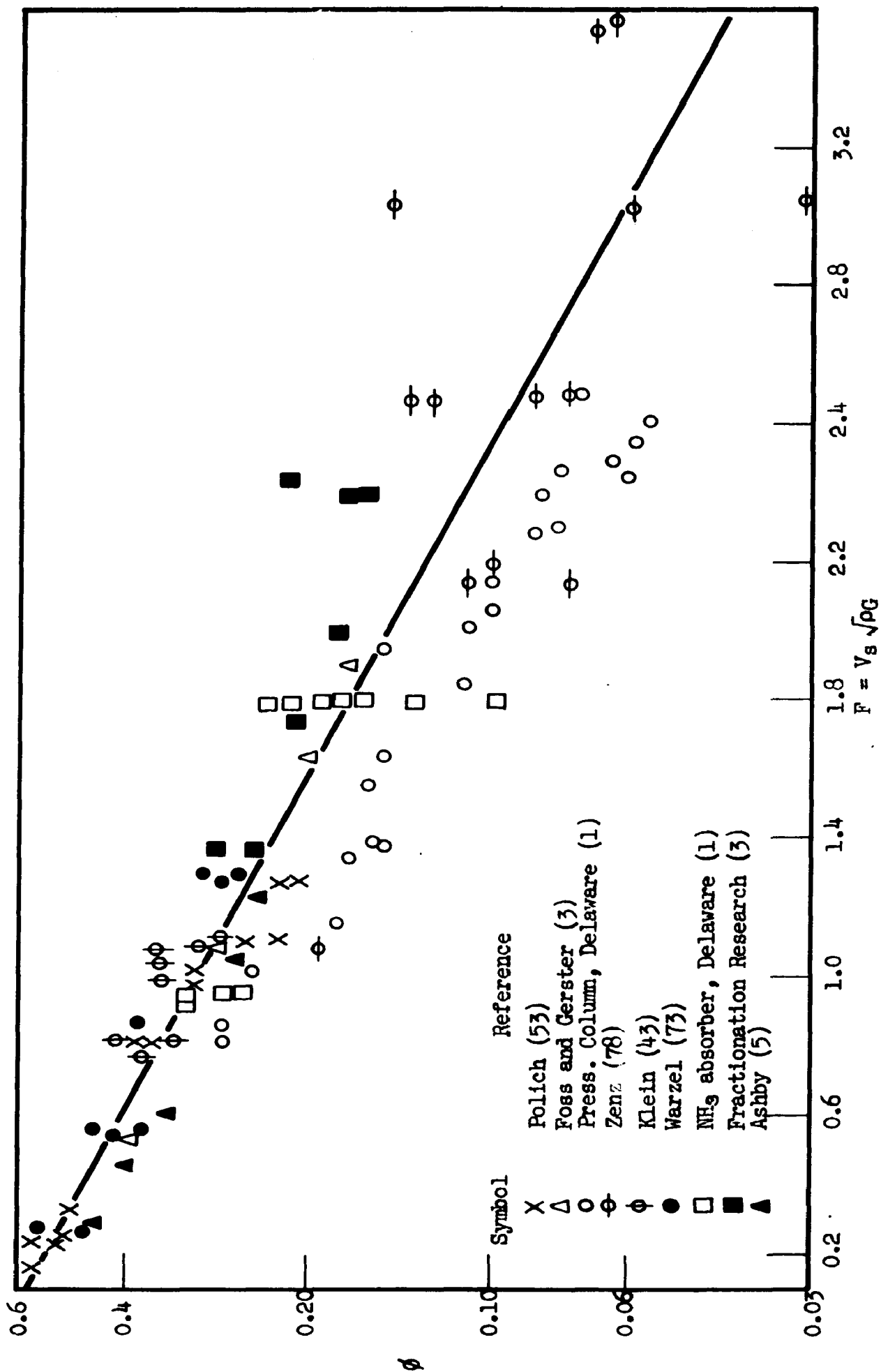


Fig. 39. Comparison of bulk froth density correlation, equation (17), with data of other investigators.

measured. This is a useful check of the froth height correlation, if mass transfer data from a column without visual access to the plate are being interpreted.

Froth Density Stratification

No measurements of local froth density have been found in the literature. Klein (43) and Freshwater (24) have made vertical pressure tap traverses in large trays. Freshwater's traverses unfortunately extend only a short distance into the froth and therefore do not give any information about the constant density region. Klein reported the pressure traverses for only one run in his thesis, the rest of his pressure traverse data apparently is no longer available.

The derivative of the point pressure with respect to height is the local froth density. The local pressures reported by Klein at four points along his tray are shown in Fig. 40. An equal area smoothed curve through the differenced data is shown in Fig. 41. As can be seen, the phenomena of froth stratification into three layers also occurs in a tray five feet long. The froth layer density calculated from this data agrees reasonably well with the data obtained at the same liquid and gas rates in the present study.

Application to Mass Transfer.—Ashby (5) and Warzel (73) measured mass transfer for gas phase controlling systems in an identical tray to that used in this investigation. Ashby's data were all taken on humidification systems with a 1-1/2-inch weir, and an 8 gpm liquid rate. Warzel's gas-film controlling data were taken on ammonia-air-water at weir heights of 2 and 3-1/2 inches.

From the results of the present study, it can be predicted that most of Ashby's data corresponded to clear liquid heights low enough to suppress the constant density layer, while Warzel's data were taken at rates giving

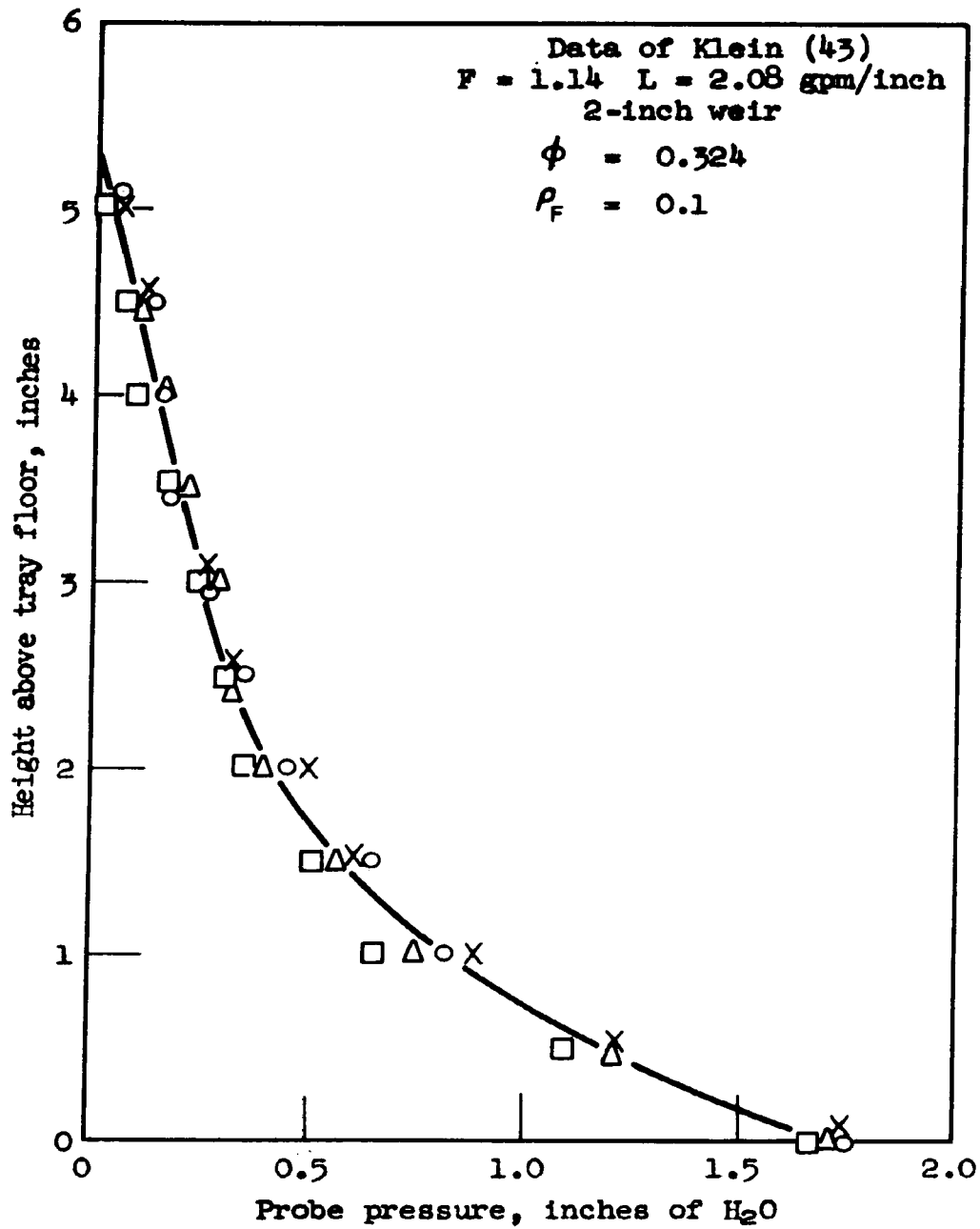


Fig. 40. Froth pressure traverse data of Klein (43), Run B-8.

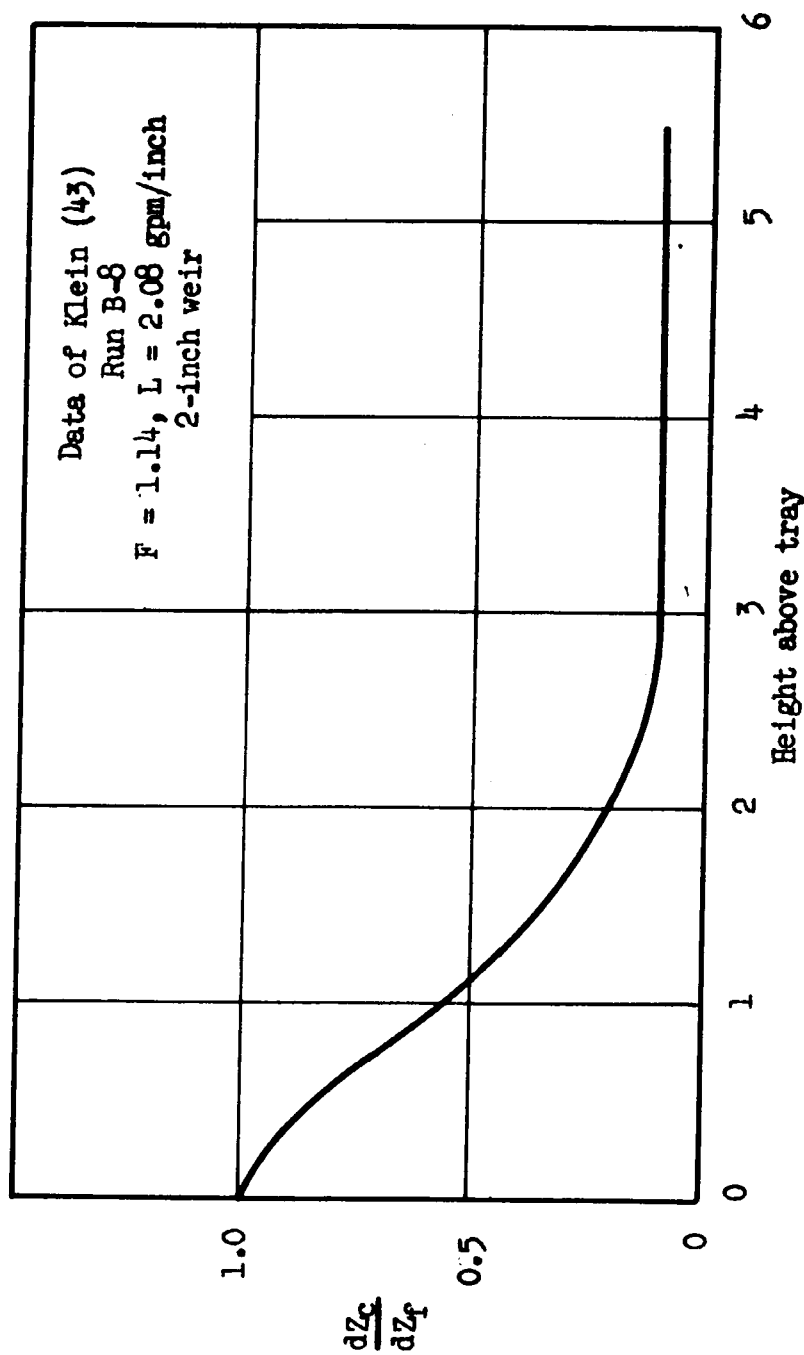


Fig. 41. Point density traverse calculated from local froth pressure.

a fully developed constant density layer. The interfacial area per unit volume of froth is greater, the thicker the constant density layer becomes, and therefore it would be expected that Warzel's efficiencies would be higher than Ashby's.

Ashby compared the number of transfer units, N_G , predicted by his correlation, with Warzel's data, and found Warzel's measured values 40% higher than predicted. This difference is too large to be attributed to experimental error, and is probably due to a considerable difference in interfacial area, which is quite conceivable if the constant density layer, normally contributing a sizeable percentage of the interfacial area, is missing in Ashby's experiments.

LIQUID MIXING

Previous Mixing Theories

The degree of liquid mixing in a bubble-cap tower is of great importance in the theoretical correlation and prediction of tower performance. With few exceptions, the efficiency of operation of plate contactors has been determined by measurement of inlet and outlet stream concentration. The resultant Murphree overall plate efficiency (E_{MV}) is specific to the tower tested. For large plates, efficiencies of over 100% are common. A more fundamental measure of performance is the degree of approach to equilibrium at a point, or the point efficiency, E_{OG} .

Effect of Mixing on Efficiencies

The relation between point and plate efficiencies was first studied by Lewis (44a), who found that the relation depends on the degree of liquid mixing on the plate. For perfect mixing, i.e., no concentration gradients, point and plate efficiency are identical. For no mixing, i.e., plug flow, the operating variables and the vapor-liquid equilibrium constant for the system relate the efficiencies as follows:

$$E_{MV} = \frac{L}{mV} \left[\exp\left(\frac{mV}{L}\right) E_{OG} - 1 \right] \quad (6)$$

The greatest difference between plate and point efficiencies occurs, therefore, with liquid phase controlling systems (m large) at low pressure. To avoid the difficulty of trying to evaluate point concentrations or the degree of mixing on a plate, most investigators have used small towers and systems where $\lambda = mV/L$ was small, and then assumed good mixing in the tower, or $E_{MV} = E_{OG}$. In general, towers are considered small if they are one foot or less in diameter. Two major drawbacks in assuming

$E_{MV} = E_{OG}$ are possible wall effects and incomplete mixing. Towers as small as 50 mm have been shown by Kirschbaum (42) to have concentration gradients.

An empirical correction for mixing in plate columns has been proposed by Stone (62). Based on one oxygen desorption run from a 5-foot long bubble-cap tray, and an estimate of the effect of end mixing in a sieve tray, Stone proposed the following modification of equation (6) to calculate Murphree plate efficiencies from point efficiencies:

$$E_{MV} = \frac{L}{mV} \left[\exp\left(\frac{E_{OGmV}}{L}\right) \left(\frac{S - s_1}{S}\right) - 1 \right] \quad (20)$$

where S is the total plate length, ft.

and s_1 is an end mixing correction

($s_1 = 1$ for bubble-cap trays and

$s_1 = 0.5$ for sieve trays).

Essentially, this equation proposes that mixing in a column is only a function of length of liquid travel and the type of tray involved. From the correction $s_1 = 1$ for bubble-cap trays, the equation predicts that towers less than one foot in diameter are perfectly mixed. Data obtained by the author and by Warzel (73) show that this is incorrect for a column with only three rows of caps on 2-1/2-inch square pitch (i.e., only 0.5 ft. of active liquid travel length).

Stone's empirical correlation could probably be made to fit actual mixing data if the correction factor s_1 were related to liquid and gas rates, weir height and plate length. Equation (20) in its present form has not been tested with experimental data.

The simplest non-empirical approach to characterize mixing in columns is the pool theory proposed by Kirschbaum (41) and Gautreaux and O'Connell (27). It assumes that the liquid concentration on the plate changes by

equal-length finite steps down the tray, each step or pool being perfectly mixed. Gautreaux and O'Connell derived the following equation relating point and plate efficiencies:

$$E_{MV} = \frac{L}{mV} \left[\left(1 + \frac{mV}{L} \frac{EOG}{n} \right)^n - 1 \right] \quad (8)$$

where n is the number of stages or pools.

As $n \rightarrow \infty$, equation (8) approaches the Lewis equation and for $n = 1$, point and plate efficiency are equal. This equation probably is adequate for treatment of commercial size equipment, though, as yet, data are not available for accurately predicting the number of pools. For experimental towers under two feet in diameter, this approach is probably an oversimplification. One of the main objections to the pool theory is the extensive concentration gradient data necessary for determining point efficiencies used in calculating the function n .

To circumvent the necessity of measuring concentration gradients in small columns, Warzel (73) and Oliver and Watson (51) have recently defined similar mixing parameters based on the concentration jump at the inlet weir. Essentially, this implies the assumption of a hypothetical stream carrying liquid from the exit downcomer to the inlet weir. They derive identical equations relating point and plate efficiencies; these equations are unfortunately in error due to an oversimplified boundary condition.* Attempts to correctly rederive the equations have resulted in unwieldy relations which include gas and liquid concentrations, and are therefore not of general applicability.

Turbulence Characteristics

The methods of characterizing mixing reviewed so far, essentially are discontinuous approximations of a continuous phenomenon.

* See Appendix A for details of their derivation.

A continuous description of mixing can be based on current eddy diffusivity theory. Eddy diffusivity is defined as the ratio of the net rate of mass transfer over the concentration gradient. Prandtl's simple theory of isotropic turbulence relates the eddy diffusivity to the deviating velocity (v') and to the eddy length (L') as follows:

$$D_E = \frac{1}{2} v' L' \quad (21)$$

Taylor's (63, 64, 65) statistical theory of isotropic turbulence results in:

$$D_E = v' L'_0 \quad (21a)$$

where L'_0 is a characteristic length. Therefore, both theories predict that the eddy diffusivity should be proportional to the deviating velocity and the eddy length.

Towle and Sherwood (67) and Towle and Seder (68) measured the turbulent diffusion of CO_2 and H_2 in ducts; eddy diffusivities varying between 80 - 140 $ft^2/hour$ were found for Reynold numbers ranging from 80,000 to 175,000. On the basis of the concentration traverses, Towle et al. concluded that the transfer of matter was proportional to the concentration gradient. In contrast, Schubauer (56), with measurements based on turbulent heat diffusion in a wind tunnel, concluded that the transfer was not proportional to the gradient.

The basic difference between these experiments was the distance over which the gradients were measured. Towle et al. covered the cross-section of the duct, while Schubauer made traverses in the wake of the heat source, two inches downstream.

Sherwood and Woertz (57) studied the turbulent diffusion of water vapor in various gases. They found that the data on eddy diffusion for the different

gases could be correlated if $D_E \rho_G$ was plotted versus Reynolds number. This amounts to finding that the eddy momentum per unit area is a function of Reynolds number.

Stone (62) and Byfield (11) report an eddy diffusivity of approximately 70 ft²/hour for a bubble-cap tower. Stone does not report either the liquid or gas rate or the weir height at which his measurement was made. He reports the eddy diffusivity to have been a function of the plate length near the inlet liquid side of the column.

Brown (9) measured eddy diffusivities in a bubble plate and Wharton (75) measured them in a sieve tray, the column in both cases being 5 feet long by 1 foot wide. Their data is shown in Fig. 42. The eddy diffusivities measured in both trays varied from 80 to 150 ft²/hour over the liquid and gas rate used.

Application of Turbulence to Efficiencies

Based on the concept of eddy diffusivities, Anderson (4) has derived the following equation relating point and plate efficiencies:

$$1 + E_{MV} \left(\frac{mV}{L} \right) = \frac{1 - A_1/2M}{(1 - A_1/A_2) \exp(1/2 A_1)} + \frac{1 - A_2/2M}{(1 - A_2/A_1) \exp(1/2 A_2)} \quad (9)$$

where $A_1 = M + \sqrt{M^2 + 4 MN}$

$A_2 = M - \sqrt{M^2 + 4 MN}$

$M = \frac{SL}{D_E Z_f d M_L \phi} = \frac{SL'}{D_E Z_c M_L}$

$N = E_{OG} \left(\frac{mV}{L} \right)$

and S - plate length

L' - liquid rate per foot column width, lb moles/hour ft

D_E - eddy diffusivity, ft²/hour

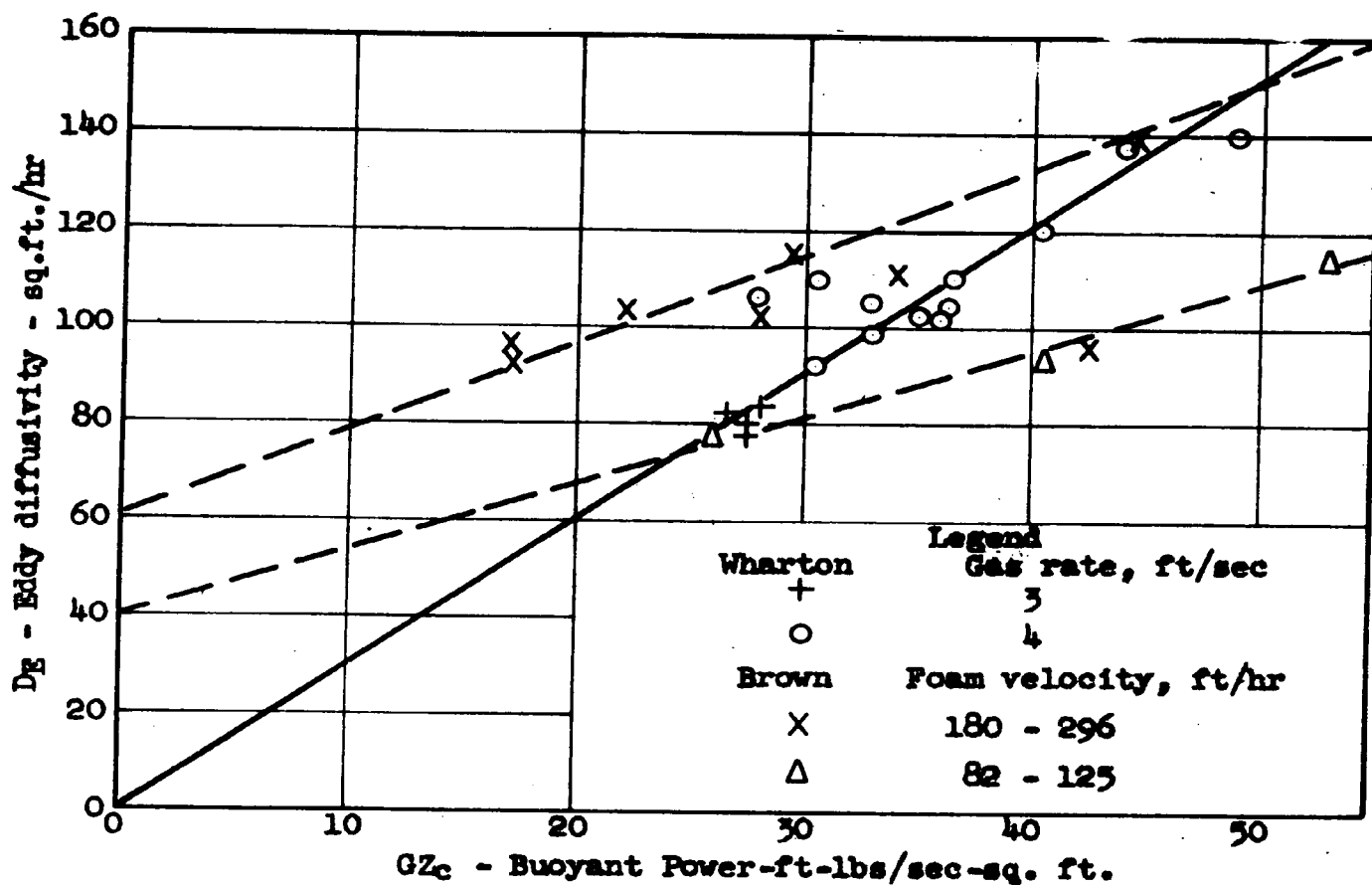


Fig. 42. Eddy diffusivity data of Wharton (75) for sieve trays and Brown (9) for bubble-cap trays.

Z_c - clear liquid height of plate

V - molar gas rate, lb moles/hour

m - slope of equilibrium curve

M_L - liquid molar density, lb moles/cu.ft.

The factor ϕ , froth specific gravity, has been added to the parameter M , as in the original derivation M was dimensionally incorrect.

Summarizing, methods proposed so far for characterizing and correcting for partial mixing in a plate column, with the exception of the Anderson (4) eddy diffusivity equation, are either empirical or discontinuous. An ideal mixing parameter should both result from a continuous model and require a minimum number of concentration measurements for its evaluation.

Derivation of a Mixing Model

To arrive at a suitable mixing model a careful study of the nature of turbulence is necessary. The characterization of turbulence depends considerably on the scale used as a reference. In atmospheric turbulence, for example, if the rate of travel of a high or low pressure area is taken as a mean velocity for the system, normal winds constitute the turbulence, the eddy length being a number of miles. If the observed wind velocity is taken as the system mean velocity, gusts with eddy lengths of a mile or so constitute the turbulent effect; and if these gusts are taken as the system mean velocity, the turbulent eddy length is measured in feet; and so on. In turbulent flow through pipes and ducts, the eddy lengths normally are fractions of an inch. These different scales of turbulence can be classified as macroscopic and microscopic turbulence. Isotropic turbulence implies statistically uniform deviating velocities in all directions.

The turbulence causing mixing in a plate column is probably both macroscopic and microscopic, and, because of the vapor cross-flow, probably non-isotropic. Visual observation of the small one-plate simulator used in this study has revealed two definite scales of turbulence. At moderate and high froth heights, the froth on the plate circulates in a doughnut-shaped eddy parallel to the plate extending over the whole column; each whorl in the doughnut has a diameter of approximately 6 inches. In a larger column this doughnut eddy probably constitutes the wall and end effects, and is a larger diameter doughnut with whorls of the same magnitude as in the small column. This type of turbulence probably explains Stone's (62) observation that a large plate acts as if approximately 0.5 ft 'at each end is perfectly mixed.

In the simulator, this doughnut eddy is apparent during normal operation. If dye is injected into the froth, smaller scale turbulence with eddies 1 to 2 inches in diameter is visible within the whorls of the large scale turbulence. Within the bubbles, and in the clear liquid between the bubbles, microscopic eddies probably exist.

The energy necessary for turbulent mixing in a column is probably derived mainly from the drag of the bubbles being pushed through the froth, only a minor part of the energy coming from conversion of the kinetic energy of the vapor into turbulence. This would indicate that eddies effective in mixing should have lengths of various bubble diameters (i.e., one to two inches).

Eddy diffusivities, measured by Brown (9) and Wharton (75) for a 5-foot long tower, were in the order of $100 \text{ ft}^2/\text{hour}$; if the deviating velocity is approximately the superficial vapor velocity in the empty tower (the bubble rise velocity should be higher than the linear vapor velocity),

the calculated eddy lengths measured were about 1/10 inch. Even if the deviating velocity be taken as low as the buoyant rise velocity of bubbles in clear liquid, the resulting calculated eddy lengths would be considerably smaller than the visually observed eddies.

A convenient mixing model should be a continuous representation of the continuous mixing process, should take into account the non-isotropic nature of the system, and ideally be easier to measure in an operating column than eddy diffusivities, that is, not require the determination of point efficiencies.

Proposed Mixing Model

Noting that the probable mechanism of mixing in a bubble-cap column is bodily lifting of fluid from the plate by the gas issuing from the slots or perforations, and not just from the turbulence generated, a modification of the Prandtl mixing length concept can be postulated.

Assuming that the net effect of the liquid spray is to carry material from position $(w + \Delta w)$ to position 'w' upstream to every point in the column, a material balance for element 'dw' in Fig. 45 is:

$$(y_n - y_{n-1})dV = d [Lx_n(w) + Kx_n(w + \Delta w)] \quad (22)$$

where K is the mixing stream flow rate.

To solve equation (22), a knowledge of the concentration at position $(w + \Delta w)$ relative to (w) is necessary.

Assuming:

- (a) $x_n(w + \Delta w) = (1 - k) x_n(w)$
- (b) the point efficiency (E_{OG}), the liquid rate (L), and the vapor rate (V), are all constant at all points on the tray;
- (c) the equilibrium relation can be represented by $y_n^* = m x_n + b$; and
- (d) the inlet gas is perfectly mixed;

then

$$(y_n - y_{n-1}) = \left(\frac{1}{\lambda} + \frac{\beta}{mV} \right) \frac{dy_n^*}{dw} \quad (22a)$$

where $\lambda = \frac{mV}{L}$ and $\beta = K(1 - k)$

The solution to this equation, with the boundary condition $y_n = (y_n)_0$ at $w = 0$ is:*

$$E_{MV} = \frac{1}{\lambda} (1 + \gamma) \left(\exp \frac{E_{OG} \lambda}{1 + \gamma} - 1 \right) \quad (23)$$

where $\gamma = \beta/L =$ mixing parameter.

This equation is plotted in Fig. 43.

A comparison of the equations resulting from the pool theory, the eddy diffusivity theory and the modified mixing length theory is given in Fig. 44. As can be seen from the plot, the mixing parameters n , M and γ are not simply related, but are secondary function of λ . Approximately, one can conclude that γ is some function of $1/M$ or $1/n$, i.e.:

$$\gamma \propto \frac{D_E Z_c d M_L}{S L} \propto \frac{1}{n} \quad (24)$$

For the system air-water in a sieve tray over a limited range of gas rates and liquid hold ups, Wharton (75) found the eddy diffusivities varying directly with the buoyant power, i.e., $D_E \propto Z_c G$.

It is probable that the liquid hold up and flow rate across the plate are important variables affecting the eddy diffusivity. Wharton varied Z_c by only 50%, and G by 25%; it would therefore not be contradictory to assume $D_E \propto Z_c G / Z_c^a d^b L^c$. Substituting this relation in equation (24):

$$\gamma \propto \frac{Z_c^2 G' d M_L}{(Z_c^a d^b L^c) S L} \quad (25)$$

* See Appendix A for details of solution.

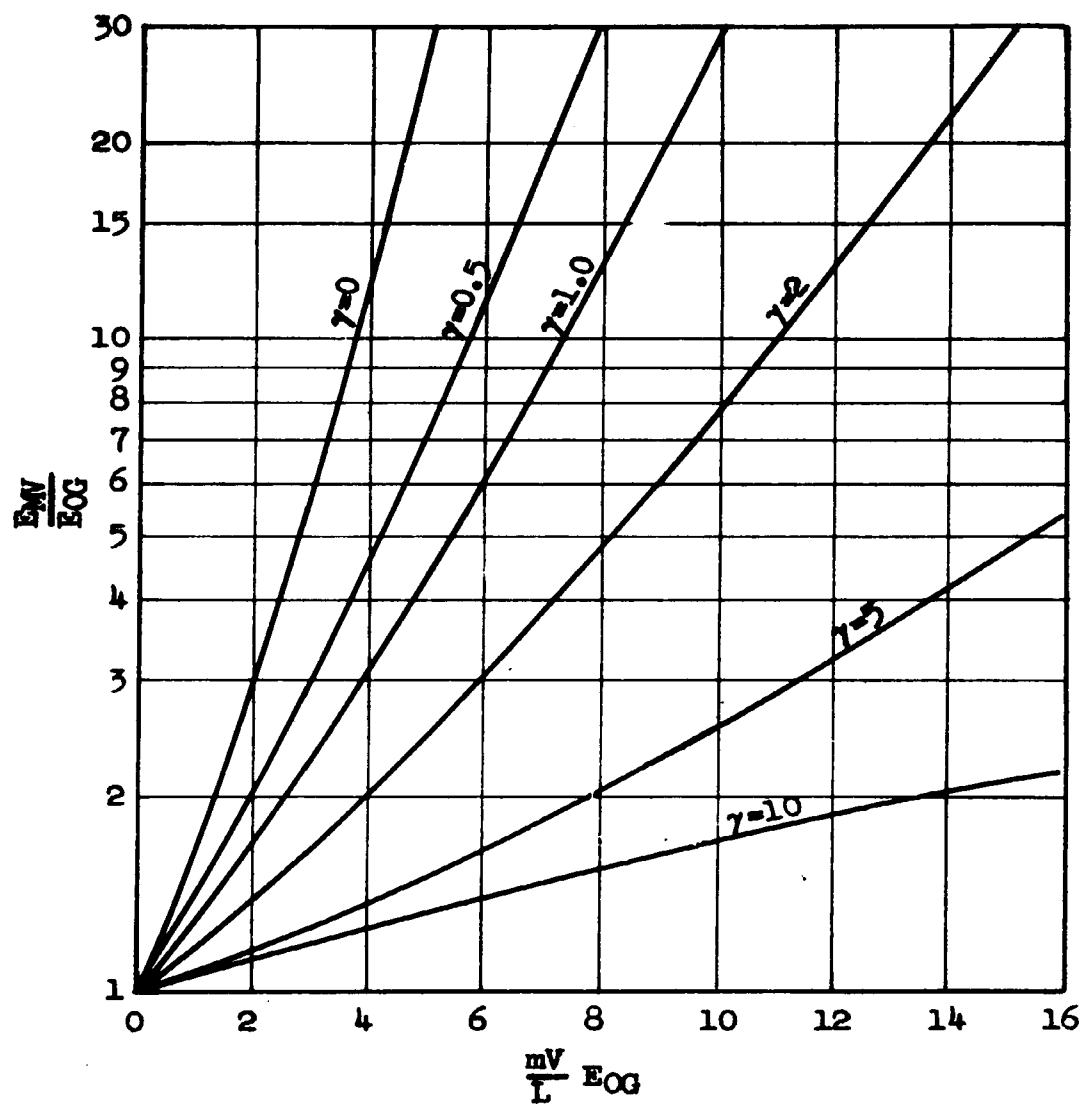


Fig. 43. Plot of proposed relation between point and plate efficiencies with γ mixing factor parameters.

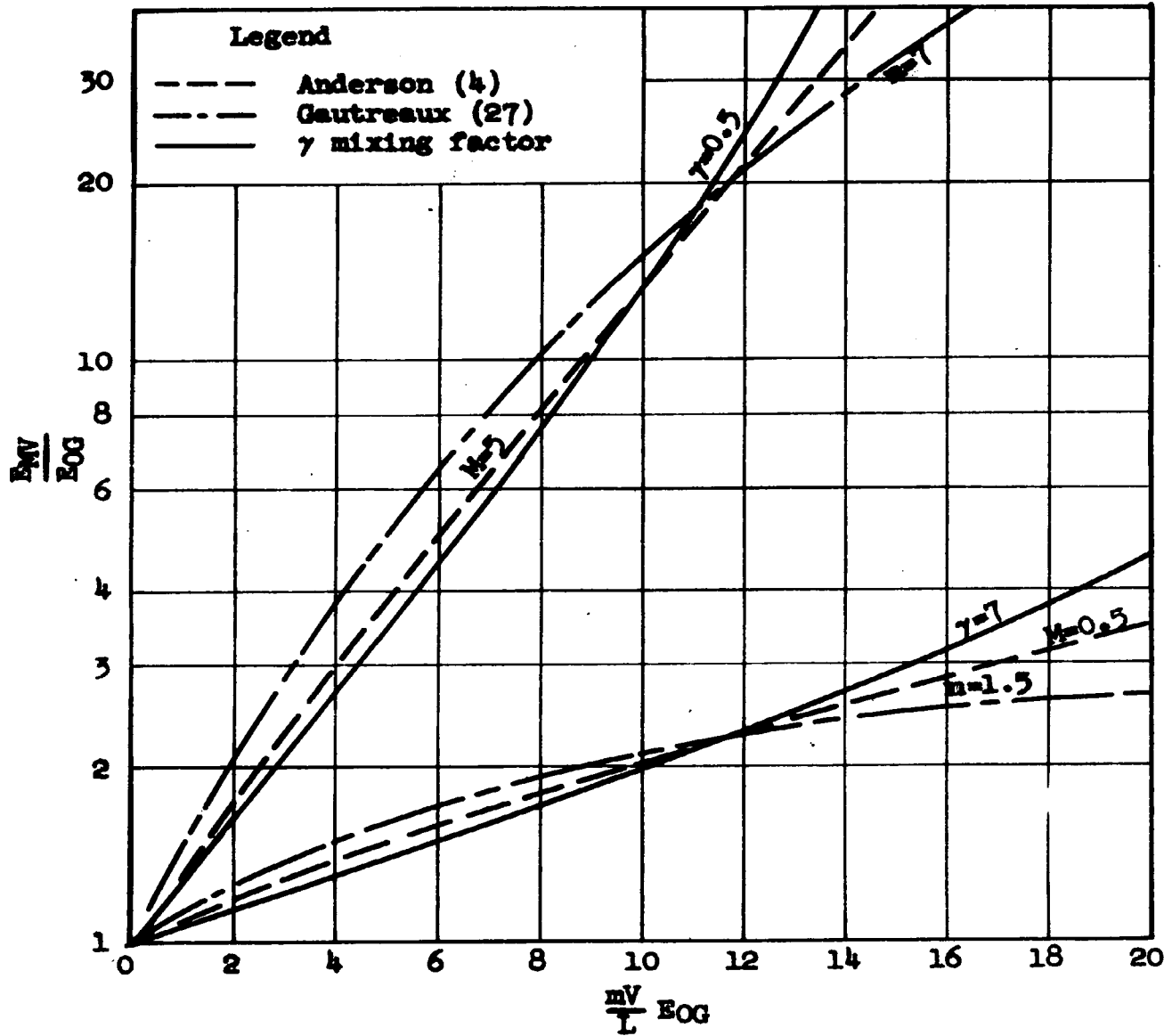


Fig. 44. Comparison of the effect of the Anderson (4), Gautreaux and O'Connell (27), and the proposed mixing parameter on the relation between point and plate efficiencies.

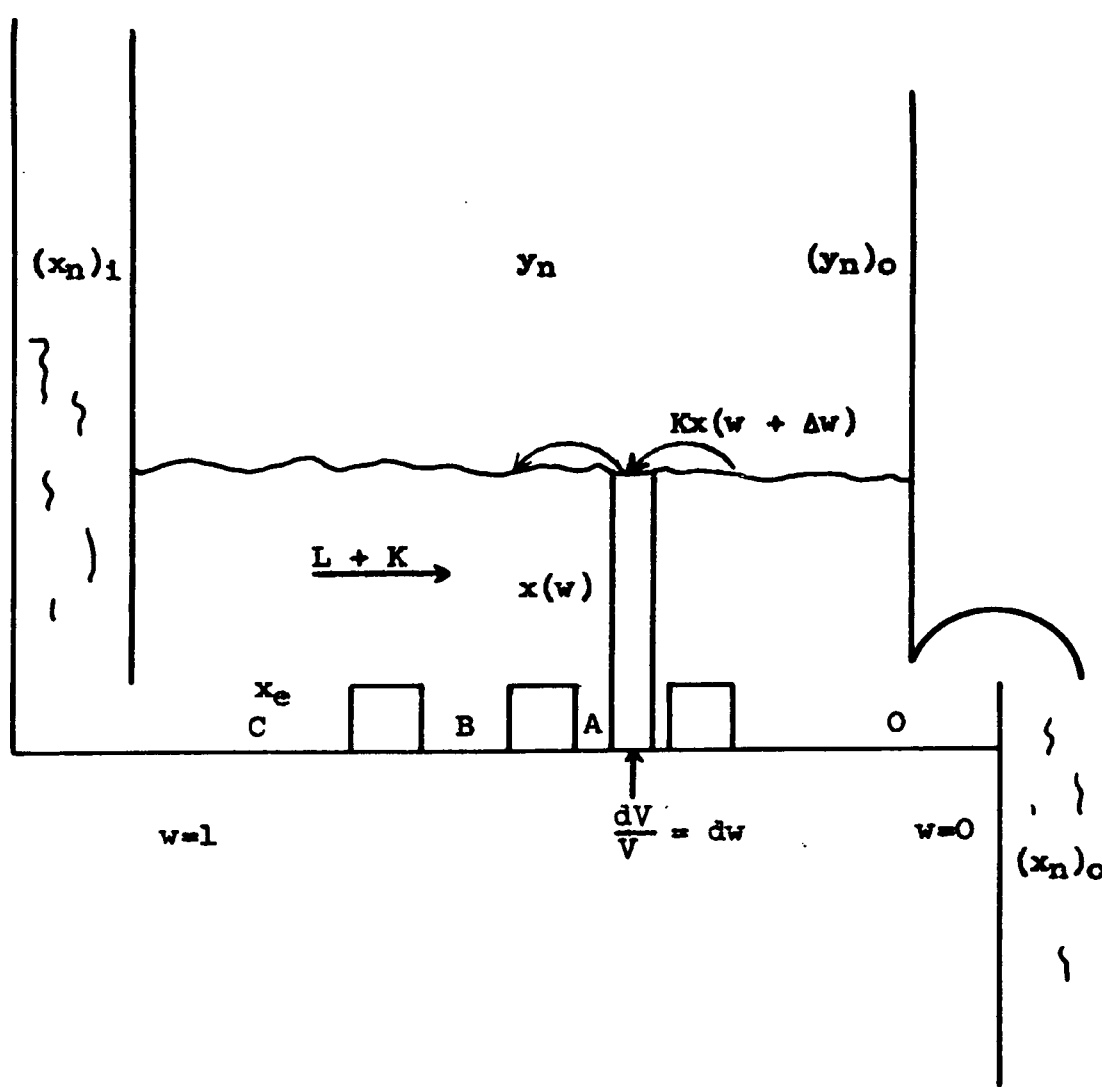


Fig. 45. Schematic diagram of column in operation.

For the system air-water in a given tower, M_L , d and S are constants and

$$\gamma \propto \frac{G}{Z_c^{a-2} L^{c+1}} \quad (25a)$$

For systems other than air and water, the major variables affecting the degree of mixing are probably the liquid viscosity, the difference in density of the vapor and of the liquid, and possibly the surface tension.

Measurement of the Mixing Parameter γ

The mixing parameter " γ " can be evaluated in a number of ways: (a) terminal liquid concentrations, (b) concentration gradients, and (c) unsteady state dye concentration measurements.

(a) The mixing parameter γ can be defined in terms of terminal liquid concentrations as follows:

From Fig. 45, an overall material balance for the plate is

$$[(y_n)_{ave} - y_{n-1}] V = -L(x_i - x_o) = -(L + K)(x_e - x_o)$$

or

$$L + K = \frac{L(x_i - x_o)}{x_e - x_o} \quad (26)$$

A material balance for the first element with concentration x_e is:

$$x_e = \frac{Lx_i + \beta x_e}{L + K} \quad (27)$$

Eliminating $(L + K)$ from equations (26) and (27):

$$\gamma = \beta/L = \left(\frac{x_i - x_o}{x_e - x_o} - \frac{x_i}{x_e} \right) \quad (28)$$

where x_i and x_o are the inlet and outlet downcomer liquid concentrations respectively, and x_e is the concentration on the downstream side of the inlet weir.

Unfortunately it is difficult to pick the position at which x_e should be evaluated if there is an appreciable length of calming section after the inlet weir; this will be dealt with more fully in the discussion on results.

(b) Stone (62) derives the following equation to relate point efficiencies and concentration gradients:

$$E_{OG} = \frac{L P S}{H V (S_2 - S_1)} \ln \frac{p_1 - H x_1}{p_1 - H x_2} \quad (29)$$

where

S - distance down the plate

and

p_1 - partial pressure of solute in entering gas.

Using a similar derivation, the equation including mixing is:

$$E_{OG} = \frac{L(1 + \gamma)PS}{H V (S_2 - S_1)} \ln \frac{p_1 - H x_1}{p_1 - H x_2} \quad (29a)$$

Therefore, from concentration gradient data and the plate efficiency E_{MV} , the mixing parameter γ can be calculated by trial and error using equations (23) and (29a).

(c) One possible method for determining " γ " by unsteady state tracer techniques is to inject a non-volatile dye into the inlet of the column at a steady rate and measure the rate of change of dye concentration at a point until the steady state dye concentration has been reached. Alternatively, after dye injection has been stopped, the rate of decay of the dye concentration can be measured.

A material balance for a differential element ' dw ' for the column shown schematically in Fig. 45 is:

Input to element	$LU(\theta, w) + KU(\theta, w + \Delta w)$
Out of element	$LU(\theta, w) + KU(\theta, w + dw) + \frac{\partial}{\partial w} [LU(\theta, w) + KU(\theta, w + \Delta w)]$
Accumulation	$A_c \frac{\partial U}{\partial \theta}$

where U - dimensionless concentration

and $A_c = Z_{cd}$ - clear liquid height above tray x width of tray.

The above material balance results in a partial differential-difference equation which is difficult to solve. The main difficulty in linearizing this equation is in choice of the boundary conditions.

Assuming:

$$U(\theta, w + \Delta w) = \begin{cases} 0 \text{ for dye injection} \\ 1 \text{ for dye decay} \end{cases} \text{ for } 0 < \theta < \frac{A_c(w + \Delta w)}{L + K}$$

$$(1 - k)U(\theta, w) \text{ for } \theta > \frac{A_c(w + \Delta w)}{L + K}$$

and

$$U(\theta, w + dw) = \begin{cases} U(\theta, w) \text{ for } 0 < \theta < \frac{A_c(w + \Delta w)}{L + K} \\ (1 - k)U(\theta, w) \text{ for } \theta > \frac{A_c(w + \Delta w)}{L + K} \end{cases}$$

and the boundary conditions for dye injection

$$\begin{cases} U(0, w) = 0 \\ U(\theta, 0) = 1 \end{cases} \text{ for } 0 < \theta < \frac{A_c(w + \Delta w)}{L + K}$$

and

$$\begin{cases} U(0, w) = \exp\left(-\frac{Kw}{L + K}\right) \\ U(\theta, 0) = 1 \end{cases} \text{ for } \theta > \frac{A_c(w + \Delta w)}{L + K}$$

the solution, using Laplace transforms, for dye injection is:*

$$U(\theta, w) = \left[\exp\left(-\frac{Kw}{L + K}\right) \right] \left[\exp\left(\frac{K(L + \beta)\theta}{A_c(L + K)}\right) \right] \quad (30a)$$

for $\frac{A_c w}{L + \beta} < \theta < 2 \frac{A_c w}{L + \beta}$.

The solution for dye decay with the boundary conditions

$$U(0, 0) = 1 \text{ and } \frac{\partial U(\theta, 0)}{\partial w} = \frac{K}{L + K}$$

is:

$$U(\theta, w) = \left[\exp\left(\frac{Kw}{L + K}\right) \right] \left[\exp\left(-\frac{K(L + \beta)\theta}{A_c(L + K)}\right) \right] \quad (31a)$$

for $\theta > \frac{A_c w}{L + \beta}$.

* See Appendix A for details of solution.

Now, if $\frac{\beta}{L + \beta} \approx \frac{K}{L + K}$, i.e., 'k' small, then:

$$U(\theta, w) = \left[\exp \left(- \frac{Kw}{L + K} \right) \right] \left[\exp \left(\frac{\beta}{A_c} \theta \right) \right] \quad (30)$$

for dye injection, and:

$$U(\theta, w) = \left[\exp \left(\frac{Kw}{L + K} \right) \right] \left[\exp \left(- \frac{\beta}{A_c} \theta \right) \right] \quad (31)$$

for dye decay.

Equations (30) and (31) show that if the dye concentration is plotted on semi-logarithmic paper versus time, θ , a straight line should result with slope $\frac{A_c}{\beta} = \frac{\Delta \theta}{\Delta \ln U}$.

Experimental Results

The unsteady state dye concentration in the previously described one-plate simulator was measured by colorimetric methods.

Validity of Beer's Law

To check the validity of Beer's law for potassium permanganate in a two-phase froth, the amount of green light transmitted by a series of measured steady state dye concentrations was determined. Dye was injected into the inlet of the simulator, operating at a liquid rate of 15.7 gallons per minute of water and 159 cfm of air. The dye concentration in the froth was allowed to reach steady state, at which time the amount of light transmitted was measured. The steady state rate of dye feed to the column was measured by means of a small calibrated rotameter.

Beer's law relates solute concentration exponentially to light transmitted, as follows:

$$I/I_0 = \exp [- K_1 C'] \quad (32)$$

where

I - light transmitted by solution

I₀ - light transmitted by pure solvent

C' - solute concentration

K₁ - a constant dependent on the system.

With concentration explicit, equation (32) becomes:

$$C' = \frac{1}{K_1} \ln \frac{I_0}{I} \quad (32a)$$

As can be seen from Fig. 46, a plot of $\ln I_0/I$ versus rotameter reading is a straight line, confirming that the permanganate solution obeys Beer's law even in a froth.

Mixing Measurements

The light transmitted by the clear froth and by the dyed froth fluctuated quite rapidly and widely, but the mean values obtained were reproducible. The background noise in the measuring circuit was great enough to mask the instantaneous variation of the dye concentration, making it impossible to apply the Danckwerts (17) statistical mixing theory.

Using the photocell and measuring circuit described in the section on apparatus, dye injection and decay concentrations were measured as a function of time. To check the uniformity of mixing in different parts of the plate, measurements were made between the first and second rows of caps (position B) and between the second and third row (position A), each at two vertical positions. The vertical heights above the plate were chosen, dependent on the uniformity of the froth, to minimize background noise due to discontinuous bubbling. Normally, the lower reading was made at approximately 2 inches above the plate, and the upper reading within 3 or 4 inches from the top of the froth.*

* In the data tables, the position of the dye decay runs has been coded; letters A and B denote the horizontal position, and the number following the code letter denotes the distance above the tray floor in inches.

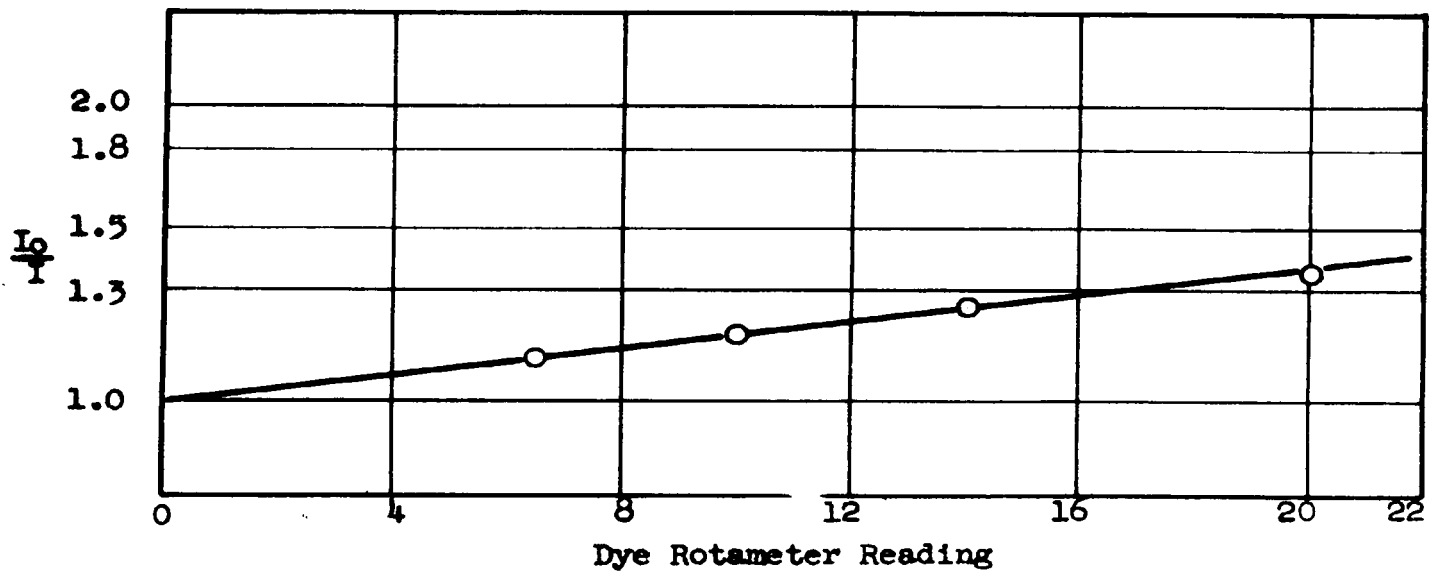


Fig. 46. Test of Beer's Law in a dynamic froth, Run R-17.

At any point in the column, for a given operating condition, equation (31) indicates that a plot of $(\ln U)$ versus elapsed time should be a straight line of slope (A_c/β) . In terms of light extinction, equation (31) can be written:

$$\ln\left(\frac{\ln I_0/I_f}{\ln I_0/I}\right) = \frac{\beta}{A_c} \theta + K_1 \quad (31b)$$

where

I_f - light transmitted at initial dye concentration

I_0 - light transmitted by clear froth

K_1 - a constant.

A representative plot of the variables in equation (31b) are shown in Fig. 47. The points plotted for the four positions are averages of two independent measurements in each case. The data fall closely on a straight line, indicating the validity of the mixing mechanism proposed.

Dye decay data were found to be much more reproducible than dye injection data, and were used exclusively in the proposed correlation. Dye injection data were probably less reproducible due to the difficulty in maintaining constant dye flow rate into the inlet stream, and good mixing in this stream, while dye decay was not dependent on the condition of the inlet stream.

For all the conditions tested in the column, negligible variation in mixing was found at the different positions.

The estimated accuracy of the dye decay data is $\pm 20\%$. The major source of error is in the measurement of the clear liquid height, which fluctuates due to the inherent instability of the bubbling action. The fluctuation in light intensity due to the non-homogeneity of the froth as a function of time contributes as much as 10% to the error in measuring the concentration.

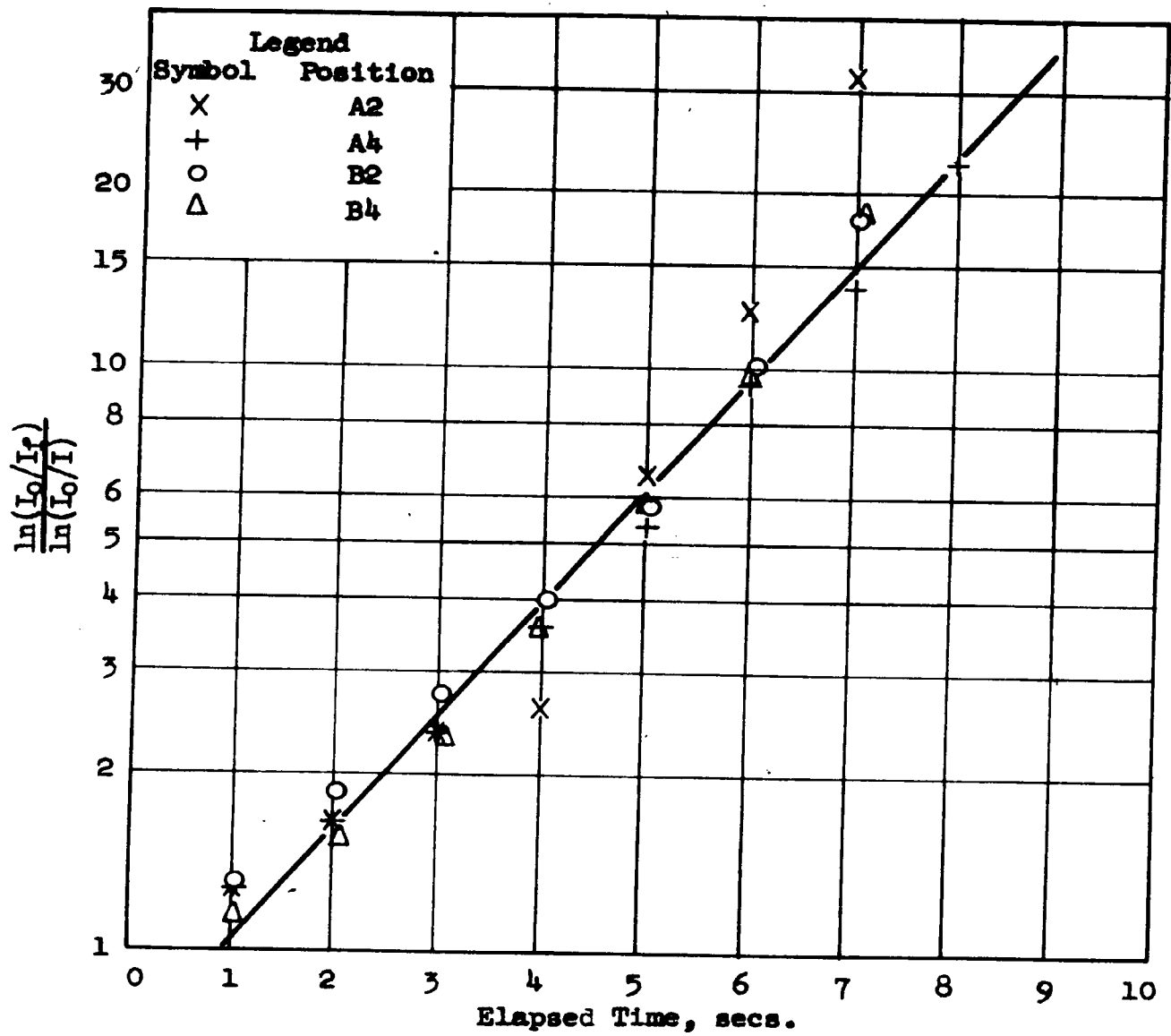


Fig. 47. Sample plot of dye decay data. Run R-55.

Only the air-water system was studied in these experiments; therefore, no conclusions can be drawn as to the effect of physical properties on mixing.

Correlation of Mixing Data

As suggested by an analysis of the Anderson (4) mixing parameter, and the type of turbulence in a tray column, the effect of operating variables on mixing is given by equation (25a):

$$\gamma \propto \frac{G}{Z_c^{a-2} L^{c+1}} \quad (25a)$$

This type of a correlation can also be justified from data available on power mixing of liquids.

Mixing studies (18, 23) in batch equipment have shown that the degree of mixing is a function of the power input to the system and to the geometry of the apparatus. In a tray tower the power input per unit volume is directly proportional to the gas velocity and volume, and inversely proportional to the liquid rate and holdup on the tray. The liquid holdup is a function of weir height and of liquid rate.

From a study of the liquid mixing data obtained, it was found that the most important variables affecting mixing are the holdup and liquid rate, with gas velocity as a minor variable.

As shown in Fig. 48, the mixing parameter " γ " has been empirically correlated by the equation

$$\gamma = 0.64 \left(\frac{\sqrt{V_s}}{Z_c^2 L} \right)^{0.074} \quad (33)$$

for operation including the splash baffle. Without splash baffle the correlation is

$$\gamma = 0.4 \left(\frac{\sqrt{V_s}}{Z_c^2 L} \right)^{0.074} \quad (34)$$

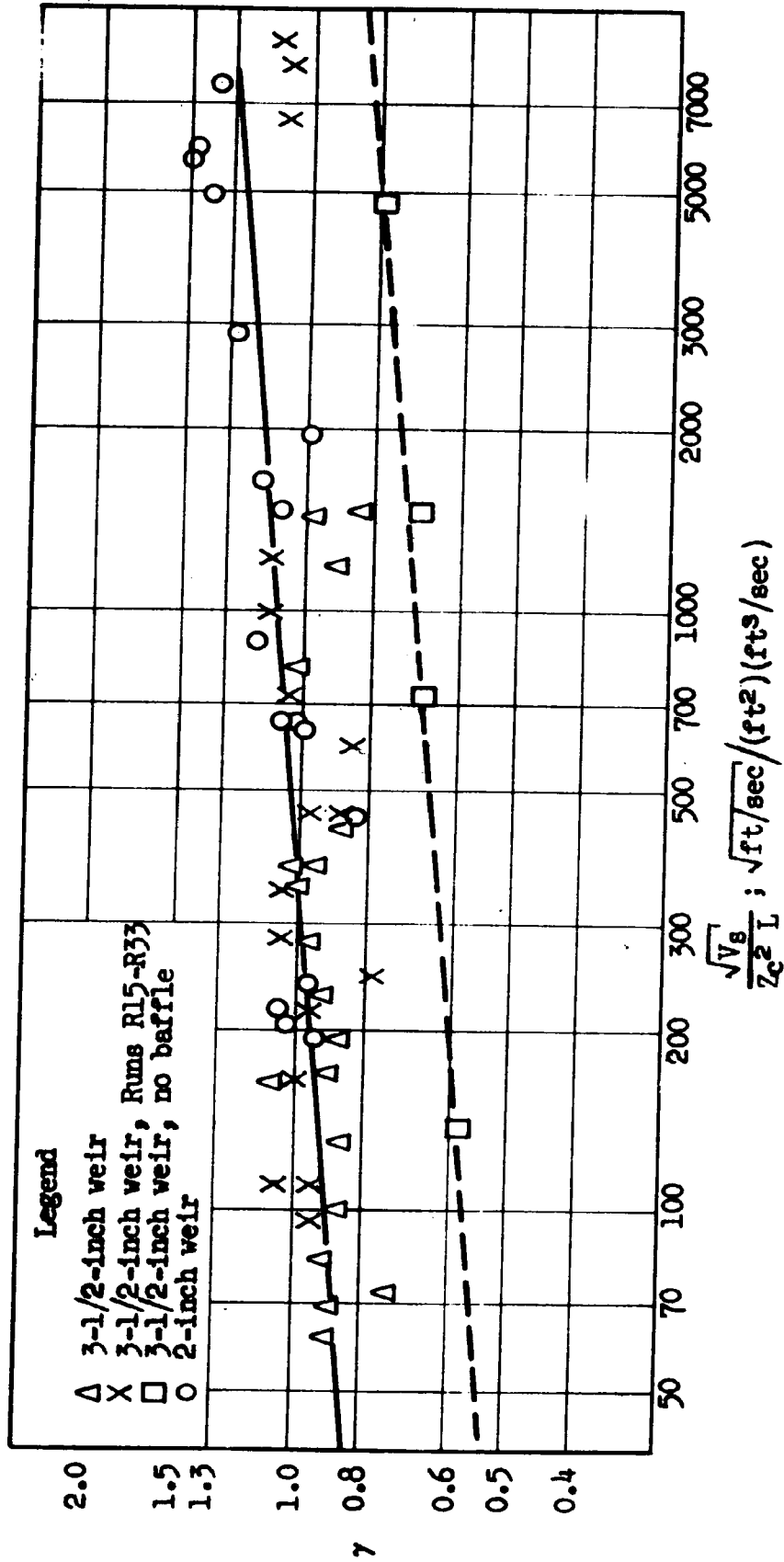


Fig. 48. Mixing correlation.

where

L - liquid rate per inch of weir, ft³/sec-inch

V_s - superficial velocity based on area between downcomer and splash baffle, ft/sec.

The range of operating variables studied were as follows:

Superficial velocity	0.96 to 4.8 ft/sec
Liquid rate	7.9 to 32 gpm
Clear liquid height	1.2 to 7.3 inches

Independent variation of clear liquid height was obtained by running at two weir heights, 2 and 3-1/2 inches.

Effect on mixing of tray layout has not been studied, except for four runs at the 3-1/2-inch weir where the outlet splash baffle was removed. The splash baffle was found to definitely promote mixing on the tray.

The most surprising result apparent from Fig. 48 is the very small variation in degree of mixing shown by the test column over a wide change in operating variables. It is felt that this is due to the strong influence of the end effects on the mixing of the liquid on the tray.

A major justification for studying this particular tray layout is the considerable amount of mass transfer data taken in identical columns by the A. I. Ch. E. tray efficiency research program at North Carolina State College and the University of Michigan. Interpretation of this data requires as complete a knowledge of the hydraulic characteristics of the tray as possible.

To apply the γ mixing factor to a column with a different liquid flow length from the one studied in this investigation, it is suggested that the following relation be used:

$$\gamma_2 = \gamma_1 \left(\frac{d_2}{S_2} \right) \quad (35)$$

where

d - tray width

S - active bubbling length.

Comparison of Results with Data of Other Investigators

As has been shown previously [equation (28)], the mixing parameter " γ " can be determined from terminal liquid concentrations in a column. The accuracy and generality of a mixing parameter calculated in this way depends to some extent on the tray layout. To apply equation (28), the tray must have an inlet weir and a short calming section upstream of the first row of caps. The jump concentration (x_e) must be measured on the downstream side of the weir near the plate floor. A number of samples have to be taken to obtain a valid average, as the concentration will fluctuate quite widely at this point.

An alternate check is possible from a comparison of measured Murphree plate efficiencies with point efficiencies calculated using equations (23) and (29a) from concentration profiles down the plate.

Stone (62) measured concentration gradients in 5-foot-long bubble-cap and sieve trays and calculated point efficiencies assuming plug flow. Unfortunately he did not take any data on overall efficiency and therefore his concentration gradient data cannot be used to evaluate the degree of mixing. A limited number of runs on oxygen desorption in a bubble-cap tray are sufficiently described by Gerster et al. (1) to calculate both point and plate efficiencies.

Three of Gerster's runs are shown in Table IV. As can be seen, the agreement between predicted and measured Murphree liquid efficiency is reasonable considering the difficulties in measuring point concentrations. The bubble tray used in Gerster's investigation had identical caps and layout to that used in this study, except that the column, instead of being a rectangular tower with a 6.5-inch active liquid flow path, was a 24-inch

diameter round tower with 5 rows of caps on 2-1/2-inch square spacing. The inlet weir to outlet weir distance was 16 inches.

TABLE IV
Oxygen Desorption Data of Gerster et al. (1)

Run No.	21	23	14
Liquid rate, lb moles/min.	30.5	21.1	21.1
Gas rate (estimated), lb moles/min.	0.197	1.38	1.38
Calculated mixing factor " γ "	0.8	0.88	0.88
Calculated E_{OG}	0.0126	0.00224	0.00332
Predicted E_{ML} (Plug flow)	0.505	0.650	0.733
Predicted E_{ML} (mixing)	<u>0.650</u>	<u>0.780</u>	<u>0.840</u>
Predicted E_{ML} (mixing and flow path correction)	<u>0.587</u>	<u>0.735</u>	<u>0.807</u>
Measured E_{ML}	<u>0.585</u>	<u>0.924</u>	<u>0.893</u>

Although the agreement is reasonably good, the data is not precise enough to confirm the applicability of the plate length correction to the mixing factor.

For this same tower Gerster et al. calculated " n " (the number of Gautreaux and O'Connell pools) for the system methyl-isobutyl-ketone-air-water, using point efficiencies calculated by their method of correlation, and measured plate efficiencies. The calculated " n " varied from about 3 to 9 for the range of operating variables covered; this range corresponds to γ mixing factors of 0.5 to 1.5, which is order of magnitude agreement with γ measured by the author.

Warzel (73), using an identical column to that in this investigation, measured concentration gradients for ammonia absorption in water from air. His data is reproduced in Table V. The last three entries compare measured Murphree plate efficiencies with point efficiencies predicted from concentration gradients. Here E_{OG} is the point efficiency assuming plug

TABLE V

Point Efficiencies Calculated from Concentration Profiles, Using Plug Flow [Equation (29)] and Flow Corrected for Mixing [Equation (29a)]
Air - NH₃ - Water Data of L. A. Warzel (73)

Run No.	A 19	A 31	A 18	A 26	A 27	A 17	A 16	A 15
G moles min	0.2934	0.0985	0.296	0.1019	0.298	0.3002	0.301	0.2037
L moles min	2.12	4.24	4.24	14.83	14.83	2.12	4.24	8.47
H in Hg mol frac	39.14	33.01	36.17	31.41	31.41	40.86	36.80	32.37
P in Hg	29.50	29.48	30.32	29.78	30.78	30.2	30.2	30.4
*Y ₀ NH ₃	88.81	159.6	111.3	245.8	107.4	88.4	109.4	164.0
"Hg								
P ₁ NH ₃	2.6	4.7	3.36	7.3	3.3	2.67	3.3	5.0
**x ₂ mol frac	4.63	0.65	2.04	0.12	0.19	4.59	0.26	0.36
P ₁ - Hx ₂	2.42	4.68	3.29	7.298	3.29	2.65	3.29	4.99
x _A	6.51	0.86	2.17	0.134	0.22	9.40	4.12	0.66
P ₁ - Hx _A	2.35	4.67	3.26	7.29	3.29	2.63	3.15	4.98
x _D	11.26	2.54	6.43	0.53	1.25	11.04	6.09	2.79
P ₁ - Hx _D	2.16	4.62	3.12	7.28	3.26	2.22	3.076	4.91
x _E	12.49	3.54	7.82	1.26	2.08	12.50	7.46	3.87
P ₁ - Hx _E	2.11	4.58	3.08	7.26	3.245	2.16	3.035	4.88
x ₁	12.85	3.95	8.29	1.69	2.20	12.67	7.83	3.91
P ₁ - Hx ₁	2.09	4.57	3.06	7.25	3.23	2.15	3.012	4.84
measured E _W x 10 ²	75.9	82.2	79.0	90.5	90.9	86.7	90.7	94.3
***E _{OG}	32.2	45.7	37.6	53.8	53.4	38.9	75.5	56.4
E _{OG}	66.0	82.8	74.0	91.5	90.7	77.0	142.5	98.5
γ	1.05	0.81	0.97	0.70	0.70	0.98	0.89	0.75
****C	5.3	2.34	3.36	1.35	2.12	4.95	4.24	3.17

* All NH₃ mole fractions multiplied by 10³.

** positions 2 - outlet of plate above; A - under downcomer; D - between row 1 and 2 of caps;

E - between row 2 and 3 of caps; 1 - between baffle and outlet downcomer.

*** E_{OG} calculated for plug flow; E_{OG} includes mixing factor γ.

**** C value calculated by Warzel.

flow, and E_{OG} is the point efficiency corrected by using mixing factors " γ " from dye decay measurements. The agreement between measured plate efficiencies and corrected point efficiencies is remarkable, especially as the concentration gradient used to calculate E_{OG} was measured over a distance of only 2.5 inches. Plots of mass transfer driving force versus distance on the bubble tray for five of Warzel's runs are shown in Fig. 49.

Warzel's concentration gradient data are of special interest as he used them in an attempt to characterize liquid mixing in his column. As pointed out earlier, Warzel's mixing parameter was defined in terms of the concentration jump at the inlet weir:

$$C = \frac{x_o - x_i}{x_o - x_e} .$$

As his column did not have an inlet weir, (x_e) was measured between the first and second row of caps. This position corresponds to the liquid contacting about 30% of the gas passing through the tray. Fig. 50 shows plots of concentration gradients for four of his runs on ammonia absorption. The square points represent values of (x_e) calculated using γ mixing factors predicted for the liquid and gas rates used for each run.

As can be seen, if the concentration gradient is assumed a straight line between the inlet and the first row of caps (not necessarily an accurate assumption), the correct position for measuring (x_e) is somewhere between the inner wall of the downcomer and the beginning of the bubbling area, about half way on the average. A definite conclusion that can be reached is that using the position between the first and second row of caps to evaluate (x_e) will give mixing factors which are 100% or more too high. As these high values of "C" were used to calculate the number of liquid transfer units, N_{OL} , in Warzel's investigation, his correlation of N_{OL} , in general, will be somewhat on the high side.

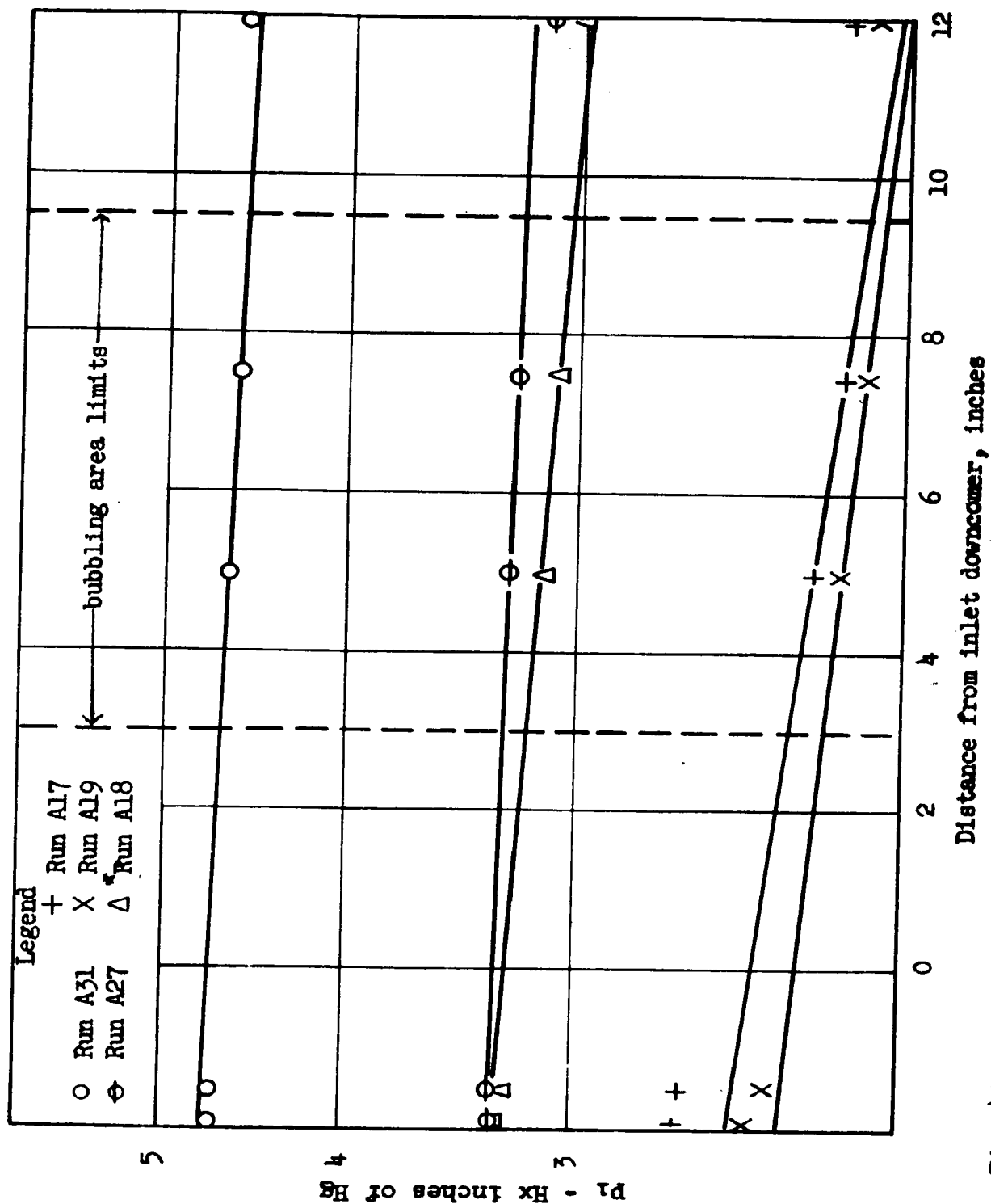


Fig. 49. Mass transfer driving force vs position on tray. Data of L. A. Warzel (73).

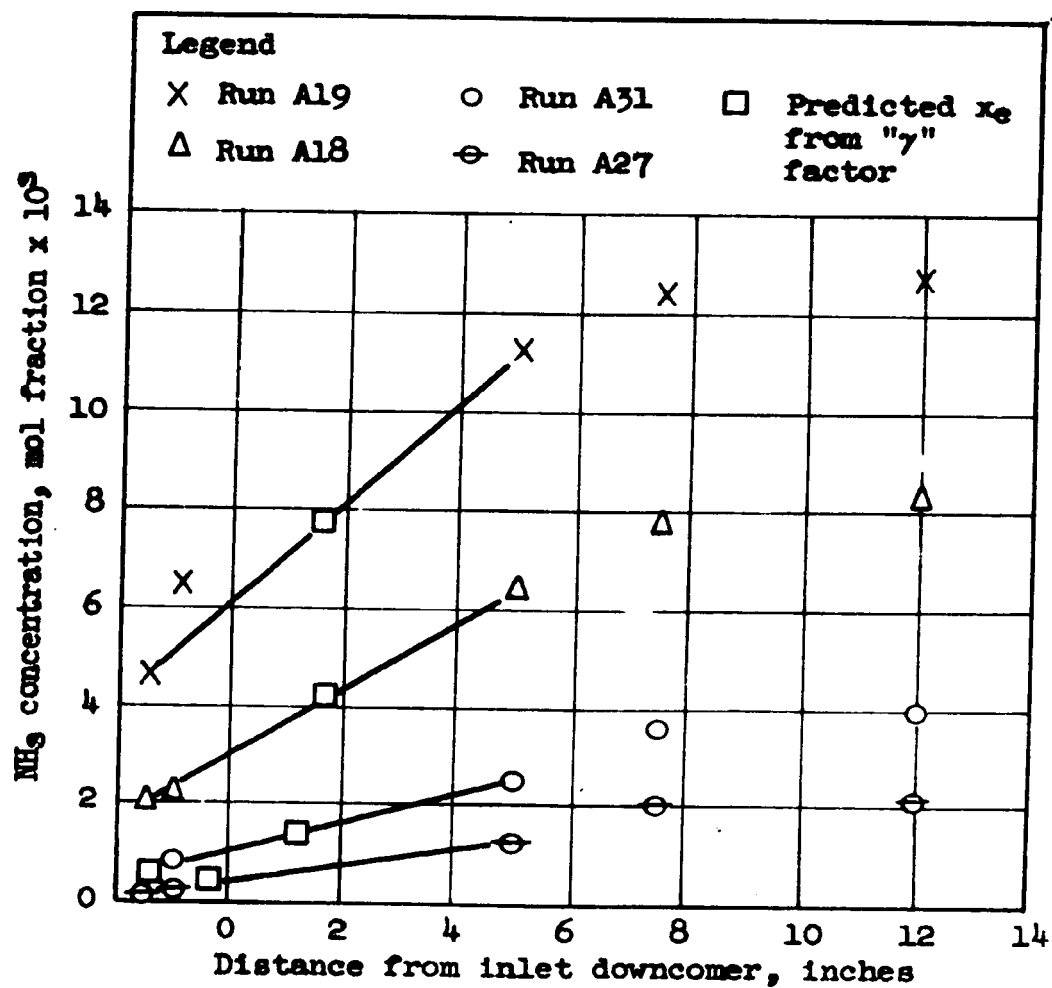


Fig. 50. Concentration gradient data L. A. Warzel (73).

Winslow (77) reports a considerable amount of concentration gradient data for a column with identical tray layout to the simulator used in this study. The column used by Winslow was operated without an exit splash baffle, and at liquid rates lower than the lowest used in this study. The tray design included an inlet weir. Table VI lists point and plate efficiencies measured by Winslow for a number of distillation systems. The point efficiencies were calculated by measuring point liquid concentrations and the corresponding point gas concentration. Complete concentration gradient data including the concentration at the inlet weir are reported. Warzel mixing factors, C , and γ mixing factors calculated from the inlet weir concentration, are considerably higher than the mixing factors that predict [using equation (23)] the measured E_{MV} from the measured point efficiencies.

Figure 51 is a plot of the predicted plate efficiencies using assumed γ mixing factors versus measured plate efficiencies; although there is considerable scatter, the accuracy is as good as would be expected of point concentration measurements. A comparison of the γ factors obtained from Winslow's data and Warzel's mixing factors, C , listed in Table V, indicate that Warzel's values are high by a factor of 2 or 3.

From an examination of the concentration data necessary to evaluate the mixing factor " γ ", it appears that using a non-volatile dye is a simpler and more reliable technique. The rate of dye decay need not be measured in the bubbling area proper, but could be measured by means of a small compact probe in the downcomer leaving the tray. Suitable probes can be designed specially for each particular application, or a probe such as proposed by Vermeulen et al. (71) can be applied.

TABLE VI
Evaluation of γ Mixing Factors from Measured Point and Plate Efficiencies

Data of Winslow (77)

Run No.	x_1	T_1 °C	Z_r "	Z_c "	Meas. EOG	Meas. EMV	Calc. EMV	L gpm
49	3.5	95.0	3.75	-	41.4	55.8	81.0	
51	7.4	94.0	3.75	-	48.1	69.2	68.5	
52	8.0	91.0	3.50	-	42.5	69.3	56.0	
53	6.7	94.0	3.50	-	44.3	53.5	62.5	
54	9.0	-	-	-	39.8	69.0	51.5	
55	4.1	88.0	3.50	-	47.3	67.3	85.0	
56	4.8	95.0	3.50	-	52.6	67.2	93.5	
57	6.6	93.0	3.50	-	47.8	60.4	71.0	
58	13.0	92.5	3.75	-	58.8	77.7	73.5	
59	38.0	86.0	3.25	-	67.5	79.7	73.5	
60	53.0	83.5	4.00	-	67.7	70.8	73.5	
61	58.0	80.0	4.25	-	80.6	86.8	89.3	
62	46.7	80.0	4.25	-	79.7	85.4	87.6	
64	58.0	80.0	-	-	71.6	79.3	76.5	
65	21.3	89.0	3.50	-	61.6	70.8	67.5	
66	25.4	89.0	3.50	-	65.8	78.0	71.5	
67	26.5	88.0	3.50	-	71.6	83.6	78.5	
68	28.0	89.0	3.25	-	71.5	85.9	78.0	
69	7.7	97.0	3.75	-	56.3	58.3	84.0	
15	78.5	58.0	5.00	1.75	72.7	89.7	86.5	
19	88.6	57.0	5.00	2.25	59.1	87.5	68.0	
22	89.0	57.0	5.00	2.25	66.0	64.4	77.0	
25	89.0	57.0	4.75	2.13	66.7	72.9	78.0	
27	90.1	57.5	4.75	2.25	61.4	80.0	71.2	
39	79.4	59.0	5.00	1.50	69.7	75.2	82.0	
5	90.8	80.0	4.75	2.75	70.2	83.9	84.0	
7	89.2	80.0	4.75	1.75	70.7	76.2	83.2	
9	95.4	79.0	4.75	1.63	10.0	38.5	11.0	
15	89.2	80.0	4.75	1.87	73.6	81.6	87.5	
17	81.8	82.5	4.75	2.13	79.6	88.7	89.0	
24	65.3	89.5	4.75	2.00	81.1	91.8	91.3	
36	18.2	104.0	4.25	2.00	91.3	126.0	168.0	
11	88.7	106.0	3.50	1.25	46.8	72.1	56.6	0.62
12	88.1	-	3.50	1.25	57.8	93.2	72.9	0.63
13	86.6	105.0	3.50	1.00	60.7	91.8	77.5	0.65
17	75.9	106.5	4.00	1.25	65.3	93.6	83.5	0.72
18	75.8	106.0	3.50	1.00	78.7	92.1	105.0	0.72
4	93.5	58.5	4.50	1.50	65.2	76.2	88.4	0.94
6	90.5	58.5	4.00	1.50	76.6	81.8	89.0	0.93
10	90.6	58.5	4.00	1.50	73.4	96.3	85.0	0.93
15	84.6	58.5	4.00	1.25	71.2	92.7	80.0	0.86
16	76.1	61.0	3.50	1.25	67.3	88.3	73.0	0.73

System: Ethanol-Water
Total Reflux, F = 0.57
L = 1.0 gpm, 3 inch weir
 x_1 = inlet mol % Ethanol
 T_1 = inlet temp °C
 γ = 1.0 assumed
Approx. μ = 0.4 centipoise

System: Acetone-Benzene
Total Reflux, F = 0.49
L = 1.0 gpm, 3 inch weir
 x_1 = mol % Acetone
 γ = 0.4 assumed
Approx. μ = 0.25 centipoise

System: Benzene-Acetic Acid
Total Reflux, F = 0.50
L = 0.89 gpm, 3 inch weir
 x_1 = mol % Benzene
 γ = 0.3 assumed
Approx. μ = 0.3 centipoise

System: Water-Acetic Acid
Total Reflux, F = 0.57
3 inch weir
 x_1 = mol % water
 γ = 0 no mixing

System: Acetone-Water
Total Reflux, F = 0.57
3 inch weir
 x_1 = mol % Acetone
 γ = 0.3
Approx. μ = 0.25 centipoise

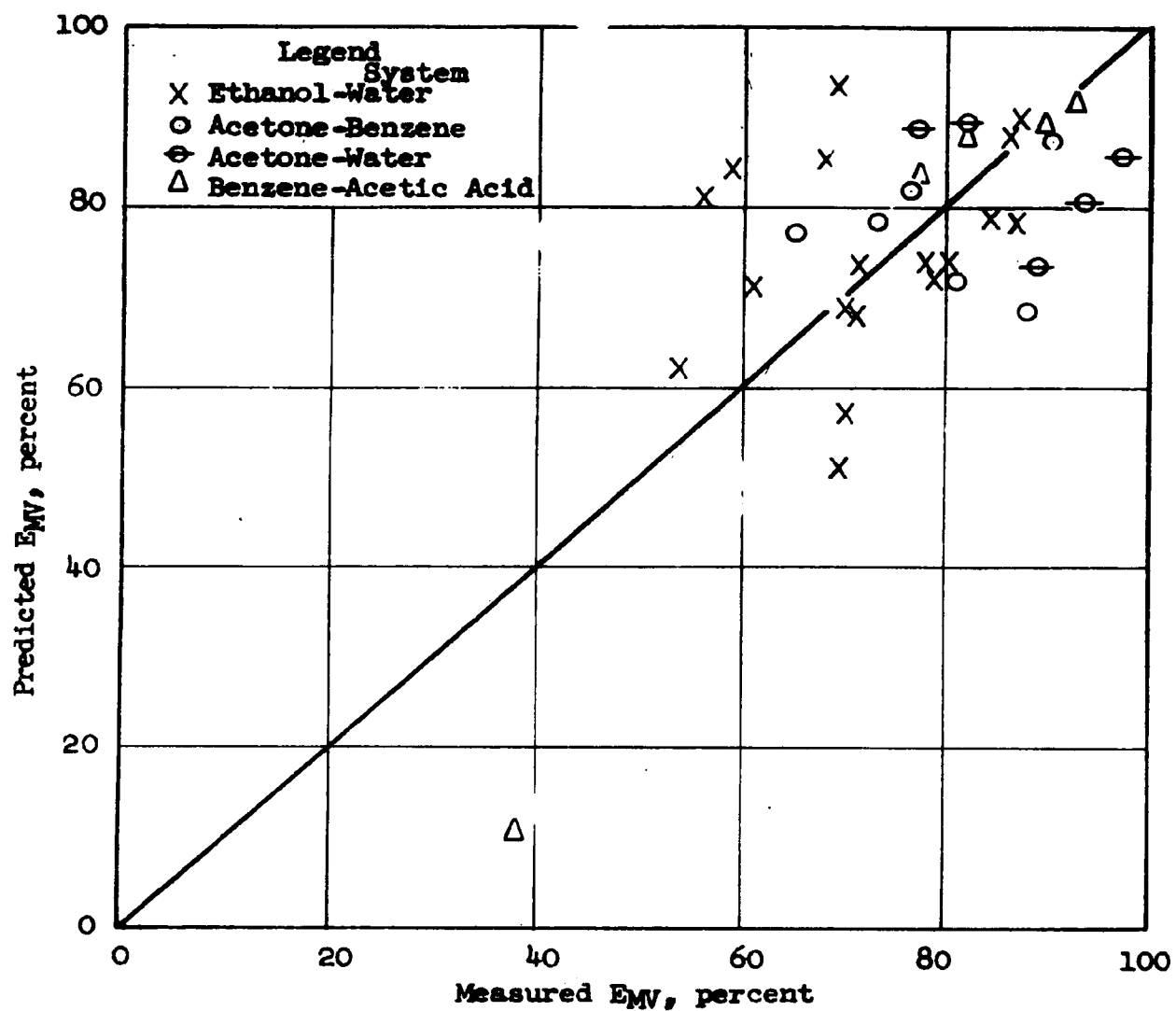


Fig. 51. Comparison of measured E_{MV} with E_{MV} predicted from measured E_{OG} and γ . Data of Winslow (77).

SUMMARY AND CONCLUSIONS

The hydraulic characteristics of both bubble-cap and sieve trays were studied in a rectangular column. The bubble-cap plate had nine 1-1/2-inch bubble caps on a 2-1/2-inch square pitch. Each cap had eighteen 1/8-inch by 3/4-inch slots, which extended to the tray floor. The plate was 7-1/2 inches wide and 13 inches long from the inlet downcomer to the outlet weir. A splash baffle was provided in front of the outlet weir. Both the front and back of the column was covered with Plexiglas to allow an unrestricted view of the bubbling action. The sieve tray had 13 by 13 rows of 11/64-inch drilled holes on 7/16-inch square spacing, resulting in a total hole area of 0.027 sq. ft.

The range of operating variables studied was as follows: weir heights from 1 to 3-1/2 inches, liquid rates from 8 to 32 gallons per minute, and gas rates from 30 to 180 cubic feet per minute. Data were obtained on the variation of local froth density, froth heights, clear liquid heights and relative mixing as a function of operating conditions.

The degree of extinction of light transmitted through a froth was demonstrated to be proportional to the local froth density. Three distinct density layers were detected from the local density measurements: a constant-density layer at bubble-cap height, a variable-density froth-generating layer above the caps, and a constant-density froth layer following the froth-generating layer. Sieve trays were found to generate only two froth layers: a variable-density froth-generating layer and a constant-density froth layer. The existence of the constant-density froth layer was found to be a function of the operating conditions. In general, this layer was suppressed when the clear liquid height was lower than the top of

the cap. The sieve tray showed a constant-density froth layer for all the operating conditions tested.

The three layer froth stratification was explained in terms of a bubbling mechanism deduced from high-speed photographs of the bubbling process. The gas leaving the slots in a bubble-cap (at normal operating slot velocities) does not form bubbles, instead it feeds directly into a more or less permanent annular gas envelope surrounding the top of the cap; directly above the cap, where the cap walls no longer protect the gas envelope, the cross-velocity and turbulence of the liquid break up the sheets of gas into individual bubbles of reasonably uniform size, the size depending on the gas and liquid rates; the bubbles then rise and produce a constant-density froth layer.

The average froth density was found to vary in a different manner than the froth layer density. It is proposed that the interfacial area available for mass transfer in the froth is more closely related to the froth layer density rather than the average froth density as has been assumed up to the present.

The suppression of the constant-density layer, resulting in less interfacial area, for clear liquid heights below cap height, explains the discrepancies found in mass transfer rates between different investigators operating columns with clear liquid heights above and below cap height.

To aid in the evaluation of mass transfer data from bubble-tray columns, generalized empirical correlations of froth height and average froth specific gravity have been developed. The correlation relating froth height to tray variables is:

$$\ln Z_f = 0.64 F + 0.25 L S^{-1/3} + 0.33 Z_w - 0.1 \quad (16)$$

where

Z_f - froth height, inches

F - $V_s \sqrt{\rho_G}$, ft/sec $\sqrt{\text{lb/cu.ft.}}$

L - liquid rate, gpm/inch

S - active plate length, ft.

Z_w - weir height, inches.

The average froth density, ϕ , was found to be a function only of the F-factor, as follows:

$$-\ln \phi = 0.715 F + 0.45 \quad (17)$$

The accuracy of the above two correlations is not high, as the precision of measurement of both the froth height and the derived quantity froth density is not good. The accuracy of both these relations is worst if the froth under study does not have a constant-density layer stabilizing it.

The degree of mixing in the experimental column was studied by means of unsteady state dye decay measurements. The dye concentration was related to elapsed times by colorimetric methods.

Based on a modified Prandtl mixing length mechanism, a relation between point and plate Murphree efficiencies is proposed, as follows:

$$E_{MV} = \left(\frac{1 + \gamma}{\lambda} \right) \left(\exp \frac{E_{OG} \lambda}{1 + \gamma} - 1 \right) \quad (23)$$

where

E_{MV} - Murphree plate efficiency

E_{OG} - point efficiency

λ - mV/L

m - slope of equilibrium line

V - gas rate

L - liquid rate

γ - proposed mixing factor.

The mixing factor, γ , can be determined from equation (23) or from tracer data.

Mixing factors, γ , were measured by means of dye tracer studies over the complete operating range of the column. The degree of mixing in the test column was found to vary only slightly with gas and liquid rate. The data could adequately be represented by:

$$\gamma = 0.64 \left(\frac{\sqrt{V_s}}{Z_c^2 L} \right)^{0.074} \quad (33)$$

for operation with a splash baffle, and by:

$$\gamma = 0.40 \left(\frac{\sqrt{V_s}}{Z_c^2 L} \right)^{0.074} \quad (34)$$

for operation without a splash baffle,

where:

L - liquid rate per inch of weir, ft³/sec-inch

V_s - superficial velocity of gas, ft/sec

Z_c - clear liquid height, inches.

The assumption of perfect mixing in a column with 0.5 ft of active plate length was found to be seriously in error.

It is suggested that the mixing parameter calculated from equations (33) and (34) be corrected for tray dimensions by the following relation:

$$\gamma_2 = \gamma_1 \left(\frac{d_2}{S_2} \right) \quad (35)$$

where

d - plate width

S - active tray length.

APPENDICES

APPENDIX A

MATHEMATICS OF MIXING IN A BUBBLE-CAP COLUMN

A number of attempts have been made to mathematically describe partial mixing in a bubble-cap column. The relation between Murphree point and plate efficiencies for the cases of perfect mixing and plug flow have been worked out by Lewis (44a), but no satisfactory treatment has been made for intermediate degrees of mixing. The accurate characterization of mixing and the interrelation of point and plate efficiency for a given degree of mixing is extremely important in distillation research, because even the small laboratory test columns normally used do not show perfect mixing.

Ideally, for small columns, any proposed mixing parameter should be easily characterized in terms of entrant and terminal liquid concentrations, and relate point and plate efficiencies by some relatively simple equation.

Recently, L. A. Warzel (73) and Oliver and Watson (51) have proposed similar mixing parameters defined in terms of the liquid concentration in the inlet and outlet downcomers— $(x_n)_i$ and $(x_n)_o$ respectively—and the concentration in the downstream side of the inlet weir (x_e). The definitions are as follows:

$$\text{Oliver and Watson } F = \frac{x_i - x_e}{x_i - x_o}$$

$$\text{L. A. Warzel } C = \frac{x_o - x_i}{x_o - x_e} .$$

The relation between the two mixing parameters is:

$$1 - F = \frac{1}{C} .$$

Both investigators derive the following relation between point (E_{OG}) and plate (E_{MV}) Murphree efficiencies for gas desorption:

$$E_{MV} = \frac{[\exp(E_{OG} \lambda/C) - 1]}{\lambda/C} \quad (7)$$

$$y_n = y_{n-1} + A \exp\left(\frac{EOG \lambda w}{C}\right) . \quad (38)$$

The constant "A" is evaluated from the boundary condition $y_n = (y_n)_0$ at $w = 0$. By replacing $(y_n)_{ave} = \int_0^1 y_n dw$ in the definition of plate efficiency $E_{MV} = [(y_n)_{ave} - y_{n-1}] / [(y_n^*)_0 - y_{n-1}]$, equation (7) is obtained. The error in this derivation is in the use of the boundary condition, as this boundary condition does not include the relation between the inlet and outlet liquid concentration.

A physical interpretation of equation (7) can be obtained by comparison with the plug flow case. Marshall and Pigford (45)(Page 10) derive the following differential equation for the plug flow case:

$$\frac{dy_n}{dw} - EOG \lambda y_n = - EOG \lambda y_{n-1} \quad (39)$$

whose solution is

$$E_{MV} = \frac{\exp(EOG \lambda) - 1}{\lambda} . \quad (6)$$

The only difference between the differential equations (39) and (37), and the integral equations (6) and (7) is the factor $1/C$ on λ ; i.e., equation (7) could equally well be derived from the plug flow case by redefining the liquid rate as CL instead of L . Therefore, equation (7), instead of representing the relation between point and plate efficiency for back mixing, represents a relation for plug flow when the liquid flow across the plate has been increased by the factor C . From a material balance the definition of C in terms of liquid concentration is $C = \frac{x_i}{x_e}$ and not $C = \frac{x_o - x_i}{x_o - x_e}$.

The definition of a mixing parameter in terms of recycle of the exit liquid is too discontinuous to give a simple relation between the point and plate efficiency in a bubble-cap column.

A mixing parameter may be defined in terms of a simplified Prandtl mixing length, postulating mixing occurs only by back mixing. Assuming that every differential element in a column, perpendicular to the liquid flow, is of uniform concentration, within the element; and that the concentration in the element is a function of the concentration in the previous element and the concentration in some element Δw downstream, the following mixing model may be set up.

A material balance around the element $dV/V = dw$ (see Fig. 45) gives the following difference-differential equation:

$$(y_n - y_{n-1}) \frac{dV}{V} = \frac{1}{V} d[Lx_n(w) + Kx_n(w + \Delta w)] \quad (22)$$

where K is the mass of fluid moving from position $w + \Delta w$ to position w .

Equation (22) can be approximately linearized if $x_n(w + \Delta w) = (1 - k)x_n(w)$. This assumption does not presume a linear concentration gradient, but instead a variable mixing length Δw which allows the approximation of the difference in concentration by a constant.

Now assuming:

- (a) the point efficiency (E_{OG}), the liquid rate (L), and the vapor rate (V), are constant at all point in the plate;
- (b) the equilibrium relation can be represented by $y_n^* = mx_n + b$; and
- (c) the inlet gas is perfectly mixed (i.e., $y_{n-1} = \text{constant}$);

then:

$$(y_n - y_{n-1}) = \left(\frac{1}{\lambda} + \frac{\beta}{mV}\right) \frac{dy_n^*}{dw} \quad (22a)$$

where $\lambda = \frac{mV}{L}$ and $\beta = K(1 - k)$.

$$\text{If } \frac{dy_{n-1}}{dw} = 0 \quad \text{and} \quad E_{OG} = \frac{y_n - y_{n-1}}{y_n^* - y_{n-1}}$$

then

$$\frac{dy_n}{dw} - E_{OG} \mu y_n = -E_{OG} \mu y_{n-1} \quad (40)$$

where $\frac{1}{\mu} = \frac{1}{\lambda} + \frac{\beta}{mV}$.

The solution to equation (40) is:

$$y_n = y_{n-1} + A \exp(EOG \mu w)$$

and the appropriate boundary condition is:

$$y_n = (y_n)_0 \text{ at } w = 0 .$$

This boundary condition is permissible in this case as the basic differential equation contains the mixing factor and is therefore capable of predicting the concentration discontinuity at $w = 1$. If $[(y_n)_{ave} = \int_0^1 y_n dw]$ is substituted into the definition of the Murphree plate efficiency E_{MV} , the following relation results:

$$\left. \begin{aligned} E_{MV} &= \left[\frac{1}{\lambda} + \frac{\beta}{mV} \right] \left[\exp \frac{EOG}{1/\lambda + \beta/mV} - 1 \right] \\ \text{or} \quad EOG &= \left[\frac{1}{\lambda} + \frac{\beta}{mV} \right] \ln \left[\frac{E_{MV}}{1/\lambda + \beta/mV} + 1 \right] \end{aligned} \right\} . \quad (23)$$

For no back mixing, $\beta = 0$, and the equation is identical to equation (6) for plug flow.

To study the mixing characteristics of a bubble-cap column, a permanganate solution was injected at a steady rate into the inlet downcomer of a typical plate, and the concentration measured, by colorimetric methods, as a function of time for the dye injection period and also after cut-off of the dye. Application to this case of the proposed mixing theory follows.

Dye Injection

Consider the differential element of unit depth for the column shown schematically in Fig. 45.

Input to element	$LU(\theta, w) + KU(\theta, w + \Delta w)$
Out of element	$LU(\theta, w) + KU(\theta, w + dw) + \frac{\partial}{\partial w} [LU(\theta, w) + KU(\theta, w + \Delta w)]$
Accumulation	$A_c \frac{\partial U}{\partial \theta}$

where

U - dimensionless concentration

w - dimensionless distance downplate

L - liquid feed rate to plate

K - mass of back mixing eddy

θ - elapsed time

A_c - clear liquid cross-section on plate.

The above material balance would give a partial differential-difference equation; to linearize this equation the following assumption is made:

$$U(\theta, w + \Delta w) = \begin{cases} 0 & \text{for } 0 < \theta < \frac{A_c(w + \Delta w)}{L + K} \\ (1 - k) U(\theta, w) & \text{for } \theta > \frac{A_c}{L + K} (w + \Delta w) \end{cases}$$

and

$$U(\theta, w + dw) = \begin{cases} U(\theta, w) & \text{for } 0 < \theta < \frac{A_c}{L + K} (w + \Delta w) \\ (1 - k) U(\theta, w) & \text{for } \theta > \frac{A_c}{L + K} (w + \Delta w) \end{cases}$$

This discontinuity in the formulation of the equation is necessary because of the assumption that only back mixing is occurring; i.e., the concentration at a point "w" is zero until the dye front reaches it in time $\frac{A_c}{L + K} w$, and therefore there is only dilution occurring at a point "w" till $\theta = \frac{A_c(w + \Delta w)}{L + K}$.

Case I: for $0 < \theta < \frac{A_c(w + \Delta w)}{L + K}$ the differential equation is:

$$-KU(\theta, w) - (L + K) \frac{\partial U(\theta, w)}{\partial w} = A_c \frac{\partial U(\theta, w)}{\partial \theta}$$

The Laplace transform yields the ordinary differential equation

$$\frac{du(s, w)}{dw} + \left(\frac{A_c s}{L + K} + \frac{K}{L + K} \right) u(s, w) = U(0, w) \left(\frac{A_c}{L + K} \right)$$

where

$$u(s, w) = L [U(\theta, w)]$$

Initially there is no dye on the plate, so $U(0,w) = 0$ and the solution to the ordinary differential equation is:

$$u(s,w) = A \exp \left[- \frac{A_c w}{L + K} \left(s + \frac{K}{A_c} \right) \right] .$$

During dye injection the liquid concentration at the inlet is constant, i.e., $U(\theta,0) = 1$ or $u(s,0) = \frac{1}{s}$, which fixes the value of the constant "A". The solution to the partial differential equation is the inverse Laplace transform of the solution of the ordinary differential equation:

$$U(\theta,w) = S_K(\theta) \exp \left(- \frac{K_w}{L + K} \right) \quad (41)$$

where

$$S_K(\theta) = \begin{cases} 0 & \text{for } 0 < \theta < \frac{A_c w}{L + K} \\ 1 & \text{for } \theta > \frac{A_c w}{L + K} \end{cases} .$$

Physically, the initial dye concentration at time θ drops exponentially with distance from the inlet downcomer, till the dye front is reached; then it drops discontinuously to zero.

Case II: For $\theta > \frac{A_c(w + \Delta w)}{L + K}$ the partial differential equation, using a new time variable $t = \theta - \frac{A_c(w + \Delta w)}{L + K}$, is:

$$\frac{\partial U(t,w)}{\partial w} = - \frac{A_c}{L + \beta} \frac{\partial U(t,w)}{\partial t} .$$

The Laplace transform of this equation is:

$$\frac{du(s,w)}{dw} + \frac{A_c s u}{L + \beta} = \frac{A_c}{L + \beta} U(0,w) .$$

The solution for $\theta < \frac{A_c(w + \Delta w)}{L + K}$ is $U(0,w) = \exp(-\frac{K_w}{L + K})$ for $t = 0$, and the solution to the ordinary differential equation is:

$$u(s,w) = \frac{\exp \left(- \frac{K_w}{L + K} \right)}{\left[s - \frac{K(L + \beta)}{A_c(L + K)} \right]} + C \exp \left(- \frac{A_c w}{L + \beta} \right) .$$

With the boundary condition $U(t,0) = 1$ or $u(s,0) = \frac{1}{s}$, the inverse transform is:

$$U(t,w) = \exp\left[\frac{K(L+\beta)t}{A_c(L+K)}\right] \exp\left[-\frac{K_w}{L+K}\right] +$$

$$S_K(t) - \exp\left[\frac{K(L+\beta)t}{A_c(L+K)}\right] \exp\left[-\frac{K_w}{L+K}\right] S_K(t) \quad (42)$$

where

$$S_K(t) = \begin{cases} 0 & \text{for } 0 < t < \frac{A_c w}{L+\beta} \\ 1 & \text{for } t > \frac{A_c w}{L+\beta} \end{cases}.$$

I.e., the solution for $\frac{A_c w}{L+\beta} < \theta < \frac{2 A_c w}{L+\beta}$ is

$$U(\theta,w) = \exp\left[-\frac{K_w}{L+K}\right] \exp\left[\frac{K(L+\beta)t}{A_c(L+K)}\right]. \quad (30a)$$

Dye Decay

The material balance for the differential element in Fig. 45 is identical for dye decay as for dye injection, with a slightly different definition of the mixing stream concentration.

$$U(\theta, w + \Delta w) = \begin{cases} 1 & \text{for } 0 < \theta < \frac{A_c(w + \Delta w)}{L+K} \\ (1-k) U(\theta, w) & \text{for } \theta > \frac{A_c(w + \Delta w)}{L+K} \end{cases}$$

and

$$U(\theta, w + dw) = \begin{cases} U(\theta, w) & \text{for } 0 < \theta < \frac{A_c(w + \Delta w)}{L+K} \\ (1-k) U(\theta, w) & \text{for } \theta > \frac{A_c(w + \Delta w)}{L+K} \end{cases}$$

Case I: The partial differential equation for $0 < \theta < \frac{A_c(w + \Delta w)}{L+K}$ is:

$$\frac{\partial U(\theta, w)}{\partial w} = \frac{A_c}{L+K} \frac{\partial U(\theta, w)}{\partial \theta} - \frac{K}{L+K} U(\theta, w) + \frac{K}{L+K}.$$

The Laplace transform of this equation, when $U(0,w) = 1$, is:

$$\frac{du(s,w)}{dw} + \left(\frac{Acs}{L+K} + \frac{K}{L+K} \right) u(s,w) = \frac{Ac}{L+K} \frac{K}{(L+K)s}$$

whose solution is:

$$u(s,w) = \frac{1}{s} + C \exp \left[- \frac{Acs}{L+K} + \frac{K}{L+K} \right] w$$

and $U(0,0) = 0$ or $u(s,0) = 0$ and $C = -\frac{1}{s}$.

The inverse is then the solution to the partial differential equation:

$$U(\theta, w) = 1 - \exp \left(- \frac{Kw}{L+K} \right) S_K(\theta) \quad (43)$$

where

$$S_K(\theta) = \begin{cases} 0 & \text{for } 0 < \theta < \frac{Acw}{L+K} \\ 1 & \text{for } \theta > \frac{Acw}{L+K} \end{cases}.$$

That is, at time θ the dye concentration increases exponentially across the plate till position $w = \frac{\theta(L+K)}{Ac}$, at which point it jumps discontinuously to 1.

Case II: For $\theta > \frac{Ac(w+\Delta w)}{L+K}$, the partial differential equation using a new time variable $t = \theta - \frac{Ac(w+\Delta w)}{L+K}$, is:

$$\frac{\partial U(t,w)}{\partial w} = - \frac{Ac}{L+\beta} \frac{\partial U(t,w)}{\partial t}$$

the Laplace transform of which is:

$$\frac{du(s,w)}{dw} + \frac{Acsu(s,w)}{L+\beta} = \frac{Ac}{L+\beta} \left[1 - \exp \left(- \frac{Kw}{L+K} \right) \right]$$

for $U(0,w) = 1 - \exp \left(- \frac{Kw}{L+K} \right)$, (which is the solution for Case I). The solution of the ordinary differential equation is

$$u(s,w) = \frac{1}{s} - \frac{\exp \left(- \frac{Kw}{L+K} \right)}{s - \frac{K(L+\beta)}{Ac(L+K)}} + C \exp \left(- \frac{Acsw}{L+\beta} \right)$$

but $U(t,0) = 0$ or $u(s,0) = 0$ and therefore

$$C = \frac{1}{s - \frac{K(L + \beta)}{A_c(L + K)}} - \frac{1}{s}$$

And the solution to the partial differential equation is:

$$U(t,w) = 1 - \exp\left(-\frac{K_w}{L + K}\right) \exp\left(\frac{K(L + \beta)t}{A_c(L + K)}\right) (1 - S_K(t)) - S_K(t) \quad (44)$$

where

$$S_K(t) = \begin{cases} 0 & \text{for } 0 < t < \frac{A_c w}{L + \beta} \\ 1 & \text{for } t > \frac{A_c w}{L + \beta} \end{cases}$$

That is, for $t < \frac{A_c w}{L + K}$ or $\frac{A_c w}{L + K} < \theta < \frac{2A_c w}{L + K}$,

$$U(t,w) = 1 - \exp\left(-\left(\frac{K_w}{L + K}\right) \exp\left(\frac{K(L + \beta)t}{A_c(L + K)}\right)\right) \quad (45)$$

and

$$U(t,w) = 0 \text{ for } t > \frac{A_c w}{L + K}$$

As can be seen from a plot of $(1 - U)$ versus θ (Fig. 52), equation (45) does not fit the data. It is also curious that the solution is not symmetrical for dye decay and for dye injection. It appears that the fault lies in too rigorous initial and boundary conditions for the case of dye decay, i.e., $U(\theta,0) \neq 0$, because of back mixing. An approximate solution can be obtained by separation of variables using the solution for Case I in dye decay as a guide to choose the boundary conditions.

The partial differential equation for Case II is:

$$\frac{\partial U(\theta,w)}{\partial w} = -\frac{A_c}{L + \beta} \frac{\partial U(\theta,w)}{\partial \theta}$$

Now if the solution is of the form $U = W(w)\Theta(\theta)$, then:

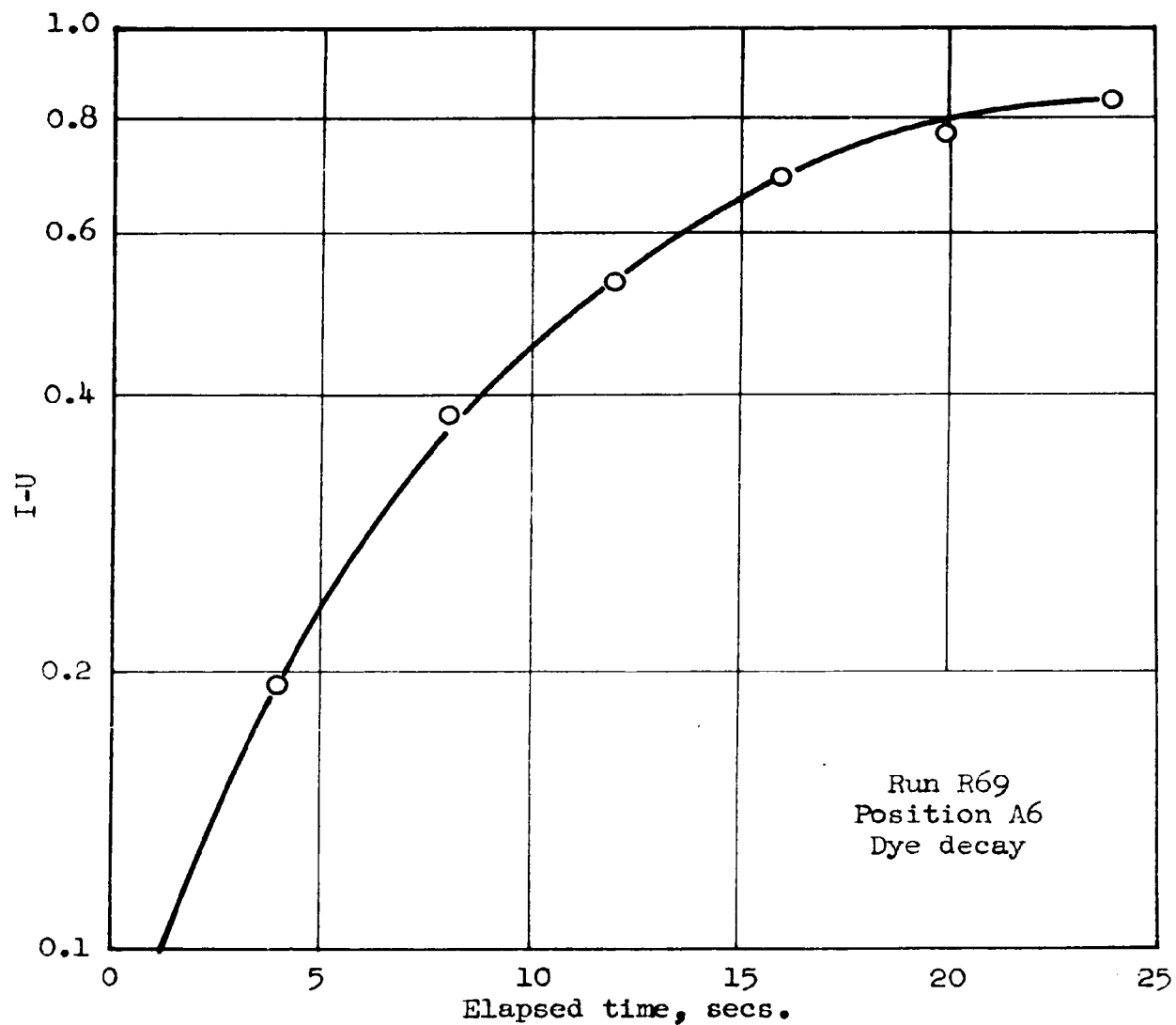


Fig. 52. Concentration vs elapsed time, dye decay Run R69.

$$\frac{W'}{W} = - \frac{A_c}{L + \beta} \frac{\theta'}{\theta} = \delta^2$$

or

$$U(\theta, w) = A \exp(\delta^2 w) \exp\left[- \frac{\delta^2(L + \beta)}{A_c} \theta\right] .$$

When $U(0,0) = 1$, then $A = 1$, and $\frac{\partial U(\theta,0)}{\partial w} = + \frac{K}{L + K}$

or

$$U(\theta, w) = \exp\left(\frac{K_w}{L + K}\right) \exp\left(- \frac{K(L + \beta)}{A_c(L + K)} \theta\right) . \quad (31a)$$

This equation is valid only for $\theta > \frac{A_c w}{L + \beta}$ and may be written more rigorously:

$$U(\theta, w) = S_K(t) \left[\exp\left(\frac{K_w}{L + K}\right) \exp\left(- \frac{K(L + \beta)\theta}{A_c(L + K)}\right) - 1 \right] + 1 \quad (46)$$

where

$$S_K(t) = \begin{cases} 0 & \text{for } 0 < \theta < \frac{A_c w}{L + \beta} \\ 1 & \text{for } \theta > \frac{A_c w}{L + \beta} \end{cases} .$$

Essentially, the only difference in the derivation of equation (46) is the choice of the boundary condition at $w = 0$; it is probably more reasonable, in dye decay, to expect the boundary to reduce its concentration, with time, at the same rate as the rest of the plate. If at $w = 0$, $U(\theta,0) = \exp[- K(L + \beta)\theta/A_c(L + K)]$ is taken as the boundary condition in the Laplace transform method, equation (46) is obtained. Apparently in dye injection $U(\theta,0) = 1$ is a reasonable boundary condition, as both methods of solution give the same results.

SAMPLE CALCULATIONS

Baffle clearance - 3-1/2-inches

TABLE VII

Trial and Error Calculation of Froth Density
Profile and Comparison with MIDAC Calculation

					MIDAC*		
Assumed $c \times 10^6$					3.8	4.5	4.33
$\int_0^{Z_f} \rho_f dZ$					3.77	3.65	---
Comparison with measured $Z_c = 3.62$ inches					high	check	check
Height above tray, inches	Run 26a I, mv	Run 26b I, mv	Average I, mv	$I/I_1 \times 10^{-3}$	ρ_f gm/cc	ρ_f gm/cc	ρ_f gm/cc
0.5	0.11	0.11	0.11	0.106	0.976	0.970	0.96
1.0	0.25	0.23	0.24	0.254	0.896	0.880	0.87
1.5	0.41	0.40	0.40	0.423	0.774	0.745	0.75
2.0	0.65	0.63	0.64	0.677	0.605	0.572	0.56
2.5	0.90	0.91	0.90	0.952	0.475	0.445	0.44
3.0	1.11	1.20	1.16	1.228	0.386	0.359	0.35
3.5	1.33	1.40	1.37	1.450	0.333	0.309	0.30
4.0	1.55	1.60	1.58	1.672	0.292	0.271	0.26
4.5	1.70	1.77	1.74	1.841	0.268	0.248	0.25
5.0	1.77	1.92	1.85	1.958	0.254	0.234	0.23
5.5	1.80	2.00	1.90	2.011	0.247	0.228	0.22
6.0	1.80	2.01	1.90	2.011	0.247	0.228	0.22
6.5	1.80	2.01	1.90	2.011	0.247	0.228	0.22

* MIDAC was programmed to stop calculating when Z_c checked within 0.05 inches.

$$\text{Superficial velocity} = \frac{G}{A_a} = V_s = \frac{128.0}{(0.615)(60)} = 3.47 \text{ ft/sec}$$

$$F\text{-factor} = V_s \sqrt{\rho_G} = (3.47)(0.274) = 0.936$$

$$\text{Liquid rotameter scale reading} = 50.0$$

$$\text{Calibrated liquid rate} = 23.5 \text{ gal/min}$$

$$\text{Liquid rate} = \frac{23.5}{7.37} = 3.19 \text{ gal/min - inch of weir}$$

$$\text{Average clear liquid height, } Z_c = 2.90 \text{ inches}$$

$$\text{Average froth height, } Z_f = 7.5 \text{ inches}$$

Mixing factor calculations:

Table VIII shows the original fractional light extinction as a function of time for four positions in the froth.

The slope, θ_R , of the curve θ versus logarithm of $\frac{\ln I_0/I_f}{\ln I_0/I}$ is $\theta_R = \frac{Z_c d}{\beta}$. From this slope the mixing parameter, γ , is calculated by:

$$\gamma = \beta/L = \frac{Z_c d}{\theta_R L}$$

(see Fig. 47).

TABLE VIII

Calculation of γ Mixing Parameter from
Fractional Light Extinction for Mixing

Run R-51

Position	I_0/I				$\frac{\ln I_0/I_f}{\ln I_0/I}$			
	A2	A4	B2	B4	A2	A4	B2	B4
Time, secs.								
0	3.67	4.20	2.92	2.93	1.00	1.00	1.00	1.00
1	3.00	3.53	2.63	2.53	1.18	1.14	1.11	1.17
2	2.45	2.40	1.90	2.21	1.45	1.63	1.67	1.40
3	1.95	1.91	1.55	1.94	1.96	2.22	2.45	1.83
4	1.69	1.58	1.34	1.64	2.48	3.15	3.72	2.53
5	1.50	1.38	1.21	1.47	3.24	4.49	6.10	3.35
6	1.35	1.24	1.12	1.31	4.52	6.93	9.45	5.00
7	1.25	1.15	1.08	1.21	6.26	10.21	13.10	7.51
8	1.18	1.09	1.04	1.15	7.29	16.50	20.00	13.32
9	1.12	1.05	1.03	1.11	11.75	38.30	38.80	22.00
10	1.08	1.03	1.01	1.05	17.15			47.40
11	1.05	1.02	1.00	1.03	32.35			
12	1.02	1.00		1.01	44.00			
Slope θ_R					3.28	2.72	2.88	2.88
β					0.046	0.055	0.053	0.053
γ					0.88	1.06	1.01	1.01

APPENDIX C
ORIGINAL DATA

TABLE IX

TEST OF LIGHT TRANSMISSION RELATION (EQUATION 15) IN SPECIAL POROUS CELL

Run No.	Air Rate, CFM	Liquid Height, in.	Froth Height, in.	Froth Density, ρ_f	$I/I_i \times 10^{-3}$	$(I/I_i)^2 \times 10^{-6}$	$(1/\rho_f)^2$	$c \times 10^6$
C-1	34	1.0	4.0	0.25	8.6	74.0	16.0	0.203
C-2	34	1.5	4.5	0.33	7.1	50.2	9.0	0.160
C-3	34	2.0	5.5	0.36	6.4	41.0	7.6	0.160
C-5	34	3.0	8.5	0.35	6.5	42.1	8.0	0.166
C-27	34	2.0	5.2	0.38	3.1	38.4	6.9	0.154
C-26	30	1.2	4.2	0.29	3.3	43.5	11.6	0.244
C-28	30	2.7	6.5	0.42	3.2	41.0	5.6	0.112
C-29	30	3.5	8.2	0.42	3.2	41.0	5.6	0.112
C-30	30	4.1	9.2	0.44	3.0	36.0	5.1	0.114
C-31	30	4.5	10.0	0.45	2.9	33.6	4.9	0.117
C-32	30	5.0	11.0	0.46	3.0	36.0	4.8	0.106
C-17	27	1.7	4.5	0.39	4.0	64.0	6.6	0.09
C-18	27	2.5	5.7	0.44	4.2	70.5	5.3	0.06
C-19	27	3.5	7.2	0.48	3.8	57.6	4.3	0.06
C-20	27	4.0	8.5	0.47	3.9	59.1	4.5	0.06
C-21	27	5.0	9.5	0.53	3.6	51.9	3.6	0.05

Plexiglas cell — 4 x 4 x 20 inches.

Gas diffuser — 1/2-inch furnace brick.

TABLE X
SUMMARIZED POINT DENSITY DATA
Bubble-cap tray, 3-inch weir, exit baffle 1/2 inches above tray floor.

Run No.	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	24	25	26	27	28	29	30	31	32 ⁽⁴⁾	33	34	35	36
Liquid Rate, gal/min	8.0	15.9	23.9	31.9	31.9	31.9	23.9	31.9	31.9	31.9	23.9	31.9	31.9	31.9	31.9	31.9	31.9	31.9	31.9	31.9	31.9	31.9	31.9	31.9	31.9	31.9	31.9	31.9	31.9
Gas Rate, CFM at 60°F	159.0	160.0	160.0	159.0	116.0	115.0	117.0	117.0	117.0	117.0	117.0	117.0	117.0	117.0	117.0	117.0	117.0	117.0	117.0	117.0	117.0	117.0	117.0	117.0	117.0	117.0	117.0	117.0	117.0
Superficial Velocity, ft/sec ⁽¹⁾	4.3	4.3	4.3	4.3	3.2	3.1	3.2	3.2	3.2	3.2	3.2	3.2	3.2	3.2	3.2	3.2	3.2	3.2	3.2	3.2	3.2	3.2	3.2	3.2	3.2	3.2	3.2	3.2	3.2
$F = V_g \sqrt{\rho_g}$	1.17	1.17	1.17	1.17	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.86
Avg Gas Temp, °F	80.4	88.0	88.1	88.4	81.3	83.2	86.8	87.2	87.2	87.2	87.2	87.2	87.2	87.2	87.2	87.2	87.2	87.2	87.2	87.2	87.2	87.2	87.2	87.2	87.2	87.2	87.2	87.2	87.2
Avg Liquid Temp, °F	57.0	59.3	59.4	59.8	60.4	60.8	60.0	60.6	60.6	60.6	60.6	60.6	60.6	60.6	60.6	60.6	60.6	60.6	60.6	60.6	60.6	60.6	60.6	60.6	60.6	60.6	60.6	60.6	60.6
Avg Froth Height, in.	7.5	11.0	15.0	18.5	6.0	9.0	12.0	16.5	5.0	5.8	6.3	8.0	5.8	8.0	10.0	14.0	16.5	11.0	8.8	6.1	7.1	10.8	13.8	18.3	6.0	8.0	11.3	20.5	11.0
Clear Liquid Height, position ⁽²⁾	0	2.60	3.55	4.50	6.20	3.60	4.55	6.30	2.90	3.40	3.95	4.85	2.70	3.25	4.35	6.15	6.50	4.31	3.69	2.60	2.55	3.65	4.35	6.40	2.20	2.50	3.80	7.10	3.50
Bulk Froth Density, ρ/ρ_f	0.35	0.32	0.30	0.34	0.40	0.40	0.38	0.38	0.38	0.38	0.38	0.38	0.38	0.38	0.38	0.38	0.38	0.38	0.38	0.38	0.38	0.38	0.38	0.38	0.38	0.38	0.38	0.38	0.38
Light Transmission Constant, $c \times 10^6$	6.0	3.3	2.7	1.7	3.1	2.5	1.7	2.7	1.4	0.8	0.5	2.9	2.0	0.9	0.6	1.5	2.7	2.7	4.3	7.1	8.2	4.1	3.6	2.0	10.9	8.4	2.8	1.7	3.8
Point Froth Density vs Height above Tray, in. ⁽⁵⁾	3.71	0.72	0.75	0.89	0.61	0.61	0.61	0.61	0.61	0.61	0.61	0.61	0.61	0.61	0.61	0.61	0.61	0.61	0.61	0.61	0.61	0.61	0.61	0.61	0.61	0.61	0.61	0.61	0.61
	2.0	0.47	0.55	0.67	0.51	0.51	0.51	0.51	0.51	0.51	0.51	0.51	0.51	0.51	0.51	0.51	0.51	0.51	0.51	0.51	0.51	0.51	0.51	0.51	0.51	0.51	0.51	0.51	0.51
	3.0	0.29	0.37	0.42	0.32	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31
	4.0	0.21	0.28	0.36	0.37	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23
	5.0	0.17	0.22	0.25	0.25	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20
	6.0	0.17	0.19	0.21	0.21	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16
	7.0	0.18	0.19	0.21	0.21	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16
	8.0	0.18	0.19	0.21	0.21	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16
	9.0	0.18	0.19	0.21	0.21	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16
	10.0	0.18	0.19	0.21	0.21	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16
	11.0	0.18	0.19	0.21	0.21	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16
	12.0	0.18	0.19	0.21	0.21	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16

(1) Superficial velocity based on area between inlet downcomer and outlet baffle, 0.61 sq ft.
(2) Positions: 0 - downstream calding section; B - between first and second row of caps; C - upstream calding section.
(3) Point density is constant up to given froth height after repeated number.
(4) Run 32 through 35 without exit baffle.

TABLE XI
SUMMARIZED POINT DENSITY DATA
Bubble-cap tray, 2-inch weir, exit baffle 2-1/2 inches above tray floor.

Run No.	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68
Liquid Rate, gal/min	7.8	15.9	23.9	31.7	8.0	15.9	23.9	31.6	8.0	15.9	23.9	31.6	8.0	15.8	23.8	31.7
Gas Rate, CFM at 60°F	108.0	113.5	116.0	117.0	168.0	166.5	166.5	164.5	52.5	47.0	42.5	37.0	84.0	86.0	86.0	86.0
Superficial Velocity, ft/sec (1)	2.9	3.1	3.2	3.2	4.6	4.5	4.5	4.5	1.4	1.3	1.2	1.0	2.3	2.3	2.3	2.3
$F = V_g \sqrt{\rho_g}$	0.81	0.84	0.86	0.87	1.24	1.23	1.22	1.21	0.39	0.35	0.32	0.28	0.62	0.64	0.64	0.64
Avg Gas Temp, °F	69.0	72.1	73.1	74.0	78.0	78.7	80.4	80.3	76.1	73.6	72.9	71.0	71.6	71.8	72.5	72.3
Avg Liquid Temp, °F	57.7	56.4	56.9	55.8	56.2	55.8	55.7	54.8	55.9	55.4	55.4	54.4	56.3	56.1	56.2	56.0
Avg Froth Height, in.	4.5	6.0	8.1	12.3	5.0	7.5	10.0	14.0	3.5	4.3	5.0	5.8	3.9	5.0	6.5	9.0
Clear Liquid Height, position (2)	0	1.55	2.20	3.00	4.50	1.60	2.20	4.65	1.75	2.15	2.55	3.00	1.65	2.10	2.70	3.75
	B	1.50	2.05	2.80	4.40	1.50	2.10	4.60	1.70	2.05	2.45	2.95	1.60	2.00	2.60	3.65
	C	1.60	2.20	3.00	4.55	1.60	2.20	4.75	1.75	2.15	2.55	3.05	1.70	2.10	2.70	3.85
Bulk Froth Density, Z_c/η_f	0.34	0.37	0.36	0.37	0.32	0.32	0.29	0.33	0.50	0.51	0.51	0.52	0.42	0.42	0.42	0.42
Light Transmission Constant, $c \times 10^6$	20.0	14.0	5.0	2.2	15.0	9.0	5.0	1.1	12.0	7.5	4.7	1.5	18.0	10.0	5.0	3.0
Point Froth Density vs Height above Tray, in. (3)	0.5	0.90	0.93	0.93	0.98	0.92	0.87	0.88	0.87	0.94	0.82	0.77	0.86	0.91	0.92	0.99
	1.0	0.82	0.81	0.80	0.93	0.66	0.62	0.66	0.62	0.76	0.59	0.52	0.65	0.74	0.77	0.91
	1.5	0.50	0.51	0.63	0.78	0.44	0.50	0.50	0.50	0.63	0.48	0.42	0.50	0.59	0.57	0.61
	2.0	0.28	0.33	0.45	0.61	0.26	0.35	0.40	0.39	0.53	0.45	0.39	0.43	0.41	0.46	0.47
	2.5	0.19	0.23	0.35	0.48	0.17	0.25	0.31	0.34	0.39	0.41	0.39	0.43	0.27	0.33	0.37
	3.0	0.12	0.18	0.27	0.40	0.12	0.17	0.26	0.31	0.29	0.41		0.18	0.26	0.31	0.43
	3.5	0.08	0.15	0.23	0.34	0.09	0.13	0.23	0.28	0.24		0.13	0.22	0.27	0.30	0.39
	4.0	0.06	0.13	0.20	0.30	0.08	0.12	0.20	0.26			0.11	0.18	0.26	0.34	0.40
	4.5		0.11	0.18	0.27	0.08	0.11	0.18	0.25			0.16	0.25	0.30	0.36	0.43
	5.0		0.10	0.17	0.25		0.11	0.16	0.25				0.15	0.25	0.30	0.36
	5.5		0.10	0.17	0.25			0.15					0.15	0.25	0.30	0.36
	6.0		0.09					0.15					0.15	0.25	0.30	0.36
	6.5							0.15					0.15	0.25	0.30	0.36
	7.0							0.20					0.15	0.25	0.30	0.36
	7.5							0.20					0.15	0.25	0.30	0.36

(1) Superficial velocity based on area between inlet downcomer and outlet baffle, 0.615 sq ft.
(2) Positions: 0 - downstream calming section; B - between first and second row of caps; C - upstream calming section.
(3) Point density is constant up to given froth height after repeated number.

TABLE XII

SUMMARIZED POINT DENSITY DATA

Bubble cap tray, 1-inch weir, exit baffle 1-1/2 inches above tray floor.

Run No.	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84
Liquid Rate, gal/min	8.0	15.9	23.9	31.6	31.6	15.8	23.6	31.6	8.0	15.9	23.8	31.6	8.0	15.8	23.9	31.6
Gas Rate, CFM at 60°F	163.5	164.5	165.5	164.5	119.0	129.0	128.0	128.0	95.0	95.0	96.0	91.5	42.5	37.0	36.0	37.0
Superficial Velocity, ft/sec ⁽¹⁾	4.4	4.5	4.5	4.5	3.2	3.5	3.5	3.5	2.6	2.6	2.6	2.5	1.2	1.0	1.0	1.0
$F = V_s \sqrt{\rho_G}$	1.22	1.22	1.22	1.21	0.88	0.95	0.94	0.94	0.70	0.70	0.71	0.68	0.32	0.27	0.27	0.27
Avg Gas Temp, °F	70.4	75.4	76.3	78.5	76.6	77.8	78.6	78.3	75.4	74.3	74.5	73.3	69.8	70.3	67.0	-----
Avg Liquid Temp, °F	51.9	49.3	48.8	50.0	49.3	49.7	49.6	48.6	49.2	48.4	48.7	47.1	48.0	48.8	48.1	-----
Avg Froth Height, in.	4.5	6.0	7.8	13.5	3.0	4.5	6.0	10.5	2.5	3.5	4.5	7.0	2.3	3.0	3.9	4.5
Clear Liquid Height, position ⁽²⁾	0 B C	1.10	1.60	2.30	4.15	1.05	1.50	2.25	3.80	1.25	1.55	2.00	2.80	1.30	1.55	1.85
		1.05	1.55	2.20	4.15	1.00	1.45	2.20	3.70	1.25	1.45	1.90	2.70	1.30	1.55	1.85
		1.15	1.65	2.35	4.35	1.10	1.55	2.35	3.90	1.30	1.60	2.00	2.90	1.35	1.60	1.90
Bulk Froth Density, Z_c/Z_f	0.24	0.27	0.30	0.32	0.35	0.34	0.37	0.36	0.50	0.44	0.45	0.40	0.60	0.52	0.49	0.53
Light Transmission Constant, $c \times 10^6$	22.8	20.0	10.0	5.0	23.3	17.9	12.5	5.4	17.3	17.0	10.4	5.1	13.9	11.6	9.0	3.3
Point Froth Density vs Height above Tray, in. ⁽³⁾	0.5	0.95	0.66	0.75	0.94	0.97	0.98	0.98	0.96	0.97	0.97	0.95	0.91	0.92	0.95	0.69
	1.0	0.77	0.60	0.69	0.85	0.90	0.96	0.91	0.92	0.89	0.92	0.93	0.90	0.76	0.78	0.43
	1.5	0.39	0.37	0.55	0.66	0.57	0.56	0.63	0.75	0.51	0.67	0.67	0.78	0.61	0.65	0.40
	2.0	0.19	0.22	0.40	0.52	0.20	0.28	0.35	0.53	0.19	0.33	0.41	0.52	0.18	0.36	0.51
	2.5	0.10	0.14	0.29	0.37	0.12	0.15	0.23	0.38	0.11	0.19	0.29	0.38	0.14	0.18	0.31
	3.0	0.05	0.10	0.21	0.29	0.09	0.11	0.17	0.29	0.13	0.13	0.20	0.30	0.12	0.12	0.21
	3.5	0.03	0.08	0.16	0.26		0.11	0.15	0.23	0.13	0.15	0.15	0.25			
	4.0	0.03	0.08	0.13	0.22		0.13	0.13	0.21	0.13	0.12	0.12	0.23			
	4.5			0.12	0.20		0.13	0.19	0.19	0.12	0.12	0.22				
	5.0			0.12	0.19			0.17	0.17							
	5.5				0.18											
	6.0				0.17											
	6.5				0.17											

(1) Superficial velocity based on area between inlet downcomer and outlet baffle, 0.615 sq ft.

(2) Positions: 0 - downstream calming section; B - between first and second row of caps; C - upstream calming section.

(3) Point density is constant up to given froth height after repeated number.

TABLE XIII

SUMMARYED POINT DENSITY DATA

Sieve tray, 3-inch weir, exit baffle 3-1/2 inches above tray floor.

Run No.	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52
Liquid Rate, gal/min	8.0	15.9	23.9	31.7	8.0	15.8	23.9	31.6	8.0	15.9	23.8	31.7	8.0	15.9	23.6	31.6
Gas Rate, CFM at 60°F	156.5	160.0	160.0	160.0	96.0	98.0	98.0	98.0	77.5	78.5	80.5	80.5	36.0	35.0	35.0	33.0
Superficial Velocity, ft/sec ⁽¹⁾	4.3	4.3	4.3	4.3	2.6	2.7	2.7	2.7	2.1	2.1	2.2	2.2	0.98	0.95	0.95	0.90
$F = V_g \sqrt{\rho_g}$	1.15	1.16	1.16	1.16	0.70	0.72	0.72	0.72	0.57	0.58	0.60	0.60	0.27	0.26	0.26	0.26
Avg Gas Temp, °F	86.7	87.8	91.0	-----	87.5	87.5	87.5	87.0	76.7	77.2	79.5	79.5	77.5	76.8	76.8	75.8
Avg Liquid Temp, °F	57.5	56.0	56.3	-----	56.8	56.7	55.4	54.4	57.5	56.1	56.3	55.4	57.5	55.9	55.8	54.5
Avg Froth Height, in.	7.3	10.5	13.0	18.0	6.0	9.0	10.8	16.0	6.0	8.4	9.8	12.0	5.2	6.7	8.0	8.8
Clear Liquid Height, position ⁽²⁾	0	2.30	3.00	3.70	6.00	2.35	3.15	3.60	2.40	3.30	3.80	5.20	2.50	3.40	4.05	4.30
	B	2.35	3.10	4.00	6.30	2.40	3.25	3.80	2.40	3.40	3.90	5.40	2.55	3.50	4.15	4.40
	C	2.35	3.00	3.90	6.10	2.40	3.20	3.70	2.40	3.40	3.90	5.30	2.50	3.50	4.10	4.40
Bulk Froth Density, Z_c/Z_f	0.32	0.29	0.30	0.34	0.40	0.36	0.35	0.38	0.40	0.41	0.40	0.44	0.49	0.52	0.51	0.53
Light Transmission Constant, $c \times 10^6$	2.0	1.6	1.2	0.6	2.2	1.0	0.9	0.4	2.0	0.9	0.8	0.5	1.5	0.7	0.6	0.3
Point Froth Density vs Height above Tray, in. ⁽³⁾	1.0	0.50	0.41	0.42	0.54	0.55	0.54	0.48	0.53	0.53	0.49	0.51	0.56	0.60	0.54	0.52
	2.0	0.30	0.28	0.27	0.33	0.34	0.36	0.32	0.32	0.34	0.36	0.44	0.39	0.43	0.43	0.52
	3.0	0.25	0.22	0.26	0.33	0.27	0.32	0.32	0.29	0.33	0.36	0.44	0.38	0.43	0.43	
	4.0	0.21	0.21	0.26	0.25	0.25	0.30	0.41	0.29	0.33	0.36	0.44	0.38	0.43	0.43	
	5.0	0.18	0.21		0.25	0.29			0.29	0.33			0.38			
	6.0	0.18			0.27											
	7.0				0.25											
	8.0				0.25											

(1) Superficial velocity based on area between inlet downcomer and outlet baffle, 0.615 sq ft.

(2) Positions: 0 - downstream calming section; B - between first and second row of caps; C - upstream calming section.

(3) Point density is constant up to given froth height after repeated number.

TABLE XIV
SUMMARIZED DATA MIXING RUNS
Bubble-cap tray, 5-1/2-inch weir, exit baffle 4 inches above tray floor.

Run No.	R61	R62	R63	R64	R65	R66	R67	R68	R69	R70	R71	R72	R73	R74	R75	R76	R77	R78	R79 ⁽⁴⁾	R80	R81	R82
Liquid Rate, gal/min	7.9	15.7	23.5	31.0	7.9	15.7	23.5	31.0	7.9	15.7	23.5	31.0	7.9	15.7	23.5	31.0	3.2	27.5	6.3	15.7	23.5	31.0
Gas Rate, CFM at 60°F	89.0	92.0	90.7	88.8	116.0	119.0	120.5	120.5	167.0	165.5	165.0	163.5	55.5	51.0	46.0	37.6	112.5	118.0	124.5	127.0	128.0	127.0
Superficial Velocity, ft/sec ⁽¹⁾	2.4	2.5	2.5	2.4	3.2	3.2	3.3	3.3	4.5	4.5	4.5	4.4	1.5	1.4	1.3	0.28	0.84	0.88	0.93	0.95	0.96	0.95
$F = V_g \sqrt{D_C}$	0.66	0.69	0.68	0.66	0.87	0.89	0.90	0.90	1.25	1.24	1.23	1.22	0.41	0.38	0.34	91.2	79.6	82.2	82.2	83.0	83.8	83.8
Avg Gas Temp, °F	79.6	81.0	80.2	88.5	91.2	93.1	94.4	95.7	93.3	96.9	97.5	98.3	93.5	92.8	91.4	11.5	5.0	16.0	5.5	6.5	7.5	18.0
Avg Liquid Temp, °F	52.6	52.8	52.8	54.7	56.7	55.0	54.3	54.3	56.8	55.1	54.0	54.4	55.3	54.4	55.4	5.8	3.1	6.5	2.0	2.3	2.7	5.3
Avg Froth Height, in.	6.0	9.5	12.5	17.0	6.5	11.0	14.5	18.5	9.5	14.0	16.5	20.0	5.5	6.5	9.0	61	370	100	474.0	1450	706	138
Avg Clear Liquid Height, in. ⁽²⁾	3.5	4.4	5.2	6.9	3.5	4.5	5.5	7.2	3.5	4.5	5.6	7.3	3.6	4.2	4.9							
Mixing Factor, γ ⁽³⁾	0.95	0.99	1.07	0.86	0.87	0.93	0.89	0.72	0.82	0.86	0.86	0.91	1.00	0.95	0.84							
$\sqrt{V_g/z_c^2} L, \sqrt{ft/sec}/ft^{1/2}/sec$	104.0	344	160	69	1190	373	168	73	1460	430	190	82	796	274	129							

(1) Superficial velocity based on area between inlet downcomer and exit baffle, 0.615 sq ft.
(2) Clear liquid height interpolated from measurements of L. A. Warzel (75).
(3) Average of 8 measurements.
(4) Runs R79 through R82 without exit baffle.

TABLE XV

SUMMARIZED DATA MIXING RUNS

Bubble-cap tray, 3-1/2-inch weir, exit baffle 4 inches above tray floor.

Run No.	R15 ⁽⁴⁾	R16	R18	R19	R20	R21	R22	R23	R24	R25	R26	R27	R28	R29	R30	R31	R32	R33
Liquid Rate, gal/min	23.5	31.5	7.9	15.7	23.5	31.0	7.9	15.7	23.5	31.0	7.9	15.7	23.5	31.0	26.6	26.6	26.6	26.6
Gas Rate, CFM at 60°F	170	164.5	120.5	124.5	124.5	120.5	75.5	75.5	81.0	72.0	48.0	49.0	56.7	58.5	106.0	164.5	75.0	38.0
Superficial Velocity, ft/sec ⁽¹⁾	4.6	4.5	3.3	3.4	3.4	3.3	2.1	2.1	2.2	2.0	1.3	1.3	1.5	1.6	2.9	4.5	2.0	1.0
$F = V_g \sqrt{\rho_G}$	1.25	1.21	0.90	0.92	0.92	0.89	0.56	0.56	0.60	0.53	0.36	0.36	0.42	0.45	0.79	1.22	0.56	0.28
Avg Gas Temp, °F	83.6	84.6	70.0	72.3	74.5	75.9	69.7	71.0	70.8	71.4	69.0	70.0	71.0	71.3	66.0	69.2	69.9	69.0
Avg Liquid Temp, °F	43.1	43.5	47.4	46.7	46.2	45.4	47.2	46.8	46.3	45.9	48.9	48.0	46.4	46.2	47.8	46.5	47.5	48.6
Avg Froth Height, in.	10.5	14.0	3.3	7.0	9.0	15.0	2.0	4.5	6.5	11.0	2.0	4.5	6.0	10.0	11.0	14.0	7.5	5.5
Avg Clear Liquid Height, in. ⁽²⁾	3.2	5.2	1.3	2.7	3.4	5.9	1.2	2.4	3.0	5.5	1.2	2.6	3.2	4.9	4.4	4.6	3.5	3.3
Mixing Factor, γ ⁽³⁾	0.83	1.00	1.10	1.10	0.87	0.95	1.06	1.10	0.95	0.94	1.06	1.03	1.04	1.05	0.95	0.77	1.05	0.92
$\sqrt{V_g/2} L, \sqrt{\text{ft}^3/\text{sec}}/\text{ft}^2/\text{sec}$	582	163	8800	985	445	110	8150	1020	456	96	6480	702	335	110	214	243	284	227

(1) Superficial velocity based on area between inlet downcomer and outlet baffle, 0.615 sq ft.

(2) Clear liquid height calculated from measured froth height and correlated average froth density.

(3) Average of 8 measurements.

(4) Exit weir leaked in runs R15 through R33.

TABLE XVI
SUMMARIZED DATA MIXING RUNS
Bubble-cap-tray, 2-inch weir, exit baffle 2-1/2 inches above tray floor.

Run No.	R44	R45	R46	R47	R48	R49	R50	R51	R52	R53	R54	R55	R56	R57	R58	R59	R60
Liquid Rate, gal/min	7.9	7.9	15.7	23.5	31.0	7.9	15.7	23.5	31.0	7.9	15.7	23.5	31.0	7.9	15.7	23.5	31.0
Gas Rate, CFM at 60°F	116.0	174.5	177.0	177.0	179.5	120.5	123.5	128.0	129.5	98.0	98.5	97.0	97.0	37.5	36.5	35.5	34.8
Superficial Velocity, ft/sec ⁽¹⁾	3.2	4.7	4.8	4.8	4.9	3.3	3.4	3.5	3.5	2.7	2.7	2.6	2.6	1.0	1.0	1.0	0.9
$F = V_g \sqrt{\rho_G}$	0.86	1.29	1.30	1.30	1.31	0.88	0.91	0.94	0.95	0.72	0.72	0.71	0.71	0.28	0.27	0.26	0.26
Avg Gas Temp, °F	75.0	79.1	84.8	86.5	87.6	86.7	86.2	86.2	86.6	85.4	85.0	84.4	84.0	80.0	79.5	79.5	79.4
Avg Liquid Temp, °F	52.1	52.0	51.2	50.7	49.9	50.7	51.0	50.5	50.3	50.8	50.8	50.2	49.9	51.7	50.8	50.3	50.0
Avg Froth Height, in.	3.5	4.5	7.5	9.0	14.0	3.5	5.5	7.5	12.5	2.3	4.2	6.0	8.5	2.0	3.8	4.5	5.5
Avg Clear Liquid Height, in. ⁽²⁾	1.6	1.6	2.2	3.1	4.7	1.6	2.2	2.9	4.5	1.7	2.2	2.8	4.1	1.7	2.2	2.5	3.0
Mixing Factor, $\gamma^{(3)}$	1.44	1.34	0.98	1.01	1.06	1.47	1.13	0.98	0.93	1.38	1.06	1.05	1.03	1.25	1.15	0.83	0.96
$\sqrt{V_g}/z_c^2 L, \sqrt{\text{ft}}/\text{sec}/\text{ft}^5/\text{sec}$	5680	7430	1950	650	213	5790	1630	623	194	4900	1460	605	206	2860	890	453	236

(1) Superficial velocity based on area between inlet downcomer and exit baffle, 0.615 sq ft.
 (2) Clear liquid height interpolated from previous measurements.
 (3) Average of 8 measurements.

APPENDIX D
CALIBRATION DATA

Two identical liquid rotameters were used in this investigation. Fisher and Porter Flowrator W70-4024/2 was calibrated gravimetrically by Ashby (5), and his calibration was used in this study. The duplicate liquid rotameter was calibrated by weighing the water discharged into a 55-gallon drum on an industrial scale. The calibration is given in Table XVII.

The gas rotameter used in this study was calibrated by Thatcher (66), using an oxygen tracer technique.

The iron-constantan thermocouple readings were converted using U. S. Bureau of Standards' tables. One thermocouple was checked against a calibrated mercury thermometer, and agreed with the tabulated voltages to ± 0.25 degrees Fahrenheit.

TABLE XVII
Calibration of Rotameter 5601D1038B1
Water at 60°F

Scale Reading	Water rate, gal/min
13.5	4.31
19.0	6.20
24.5	7.50
34.5	11.03
48.0	15.50
60.0	18.95
63.5	19.75
75.0	23.45
90.0	28.40
94.0	29.40
96.5	30.00

NOMENCLATURE

- a - surface area for transfer of mass, per unit volume, ft^2/ft^3 .
- A_a - tray active area, based on area between the inlet downcomer and the outlet baffle, ft^2 .
- $A_c = Z_c d$ - clear liquid cross section, ft^2 .
- b - constant.
- c - light transmission constant in equation (15).
- $C = \frac{x_o - x_i}{x_o - x_e}$ - Warzel (73) mixing parameter.
- d - differential operator.
- d - tray width, ft.
- D - molecular diffusivity.
- D_E - eddy diffusivity.
- $\exp(x) = e^x$.
- E_{ML} - Murphree liquid efficiency for a plate.
- E_{MV} - Murphree vapor efficiency for a plate.
- E_o - overall column efficiency, ideal plates/actual plates.
- E_{OG} - point vapor efficiency.
- $F = V_s \sqrt{\rho_G}$, F-factor.
- $F = \frac{x_i - x_e}{x_i - x_o}$ - Oliver and Watson (51) mixing factor.
- G - superficial molar mass velocity of vapor, lb. moles/hr-ft², based on area between downcomer and baffle (0.615 ft²).
- h - liquid submergence, inches, taken as the distance from the middle of the slots to the liquid level above the weir.
- H - Henry's law constant, inches Hg/mole fraction.
- I - transmitted light from froth.
- I_f - transmitted light by froth at steady state dye concentration.
- I_i - incident light to froth.
- I_o - transmitted light by clear froth.

- k - concentration difference between points "w" and "Δw" on tray.
- k_G - individual gas phase mass transfer coefficient.
- k_L - individual liquid phase mass transfer coefficient.
- K - back mixing eddy mass.
- K_G - overall mass transfer coefficient based on gas phase.
- K_L - overall mass transfer coefficient based on liquid phase.
- ln - natural logarithm, base e.
- L - molar liquid rate, lb. moles/hour.
- m - slope of equilibrium curve, dy/dx.
- $M = \frac{S L}{D_E Z_c d M_L}$ - Anderson (4) mixing parameter.
- MIK - methyl-isobutyl-ketone.
- M_L - liquid molar density, lb. moles/ft³.
- n - number of pools in Gautreaux and O'Connell (27) pool theory.
- N_G - number of transfer units, gas phase.
- N_L - number of transfer units, liquid phase.
- p - partial pressure of transferring component, inches of Hg.
- P - total pressure, inches of Hg or atms.
- r - mean bubble radius.
- R - universal gas constant.
- s - Laplace transform of time.
- S - distance down plate or total length, ft.
- t - time of contact of gas and liquid on tray, secs.
- T - absolute temperature, °R.
- u - Laplace transform of dye concentration.
- U - dimensionless dye concentration.
- v_s - bubble rise velocity.
- V - molar vapor rate, lb. mole/hr.

- V_s - linear vapor velocity, ft/sec, based on area between downcomer and splash baffle (0.615 ft^2).
- w - fractional distance down tray ($w = 0$ at overflow weir).
- x - mole fraction solute in liquid stream.
- x^* - mole fraction solute in liquid in equilibrium with gas.
- x_e - mole fraction solute on downstream side of inlet weir.
- y - mole fraction solute in gas stream.
- y^* - mole fraction solute in gas in equilibrium with liquid.
- Z_c - clear liquid height above tray floor, inches.
- Z_f - froth height above tray floor, inches.
- Z_w - weir height above tray floor, inches.
- $\beta = K(1 - k)$.
- γ - proposed mixing factor.
- δ - integration constant.
- ϵ - liquid hold-up on tray, ft^3 .
- θ - elapsed time, secs.
- $\lambda = mV/L$, slope of equilibrium line/slope of operating line.
- μ - liquid viscosity, centipoise.
- ρ_f - true froth density at a point - gms/cc.
- ρ_F - froth layer density.
- ρ_G - gas density.
- ρ_L - liquid density - gms/cc.
- ϕ - average froth specific gravity, Z_c/Z_f .

Subscripts

- G - gas phase.
- i - inlet.
- L - liquid phase.
- n - nth plate of column.
- o - exit.

BIBLIOGRAPHY

1. American Institute of Chemical Engineers Research Committee, Tray Efficiency Project, Third Annual Report (1955).
2. American Institute of Chemical Engineers Research Committee, Tray Efficiency Project, Special Progress Report, University of Michigan, May 14, 1956.
3. American Institute of Chemical Engineers Research Committee, Tray Efficiency Project, Fourth Annual Report (1956).
4. Anderson, J. E., Sc. D. Thesis, Chem. Eng., Massachusetts Institute of Technology, 1954.
5. Ashby, B. B., Ph. D. Thesis, Chem. Eng., University of Michigan, 1955.
6. Atkins, G. T., Chem. Eng. Prog., 50, 116 (1954).
7. Atteridge, P. T., Lemieux, E. J., Schreiner, W. C., and Sundback, R. A., A. I. Ch. E. Journal, 2, 3 (1956).
8. Bakowski, S., Chem. Eng. Science, 1, 266 (1952).
9. Brown, J. W., B. S. Thesis, Chem. Eng., Massachusetts Institute of Technology, 1954.
10. Bryan, J. C., and Garber, H. J., Paper presented at the Am. Inst. Chem. Engrs., Detroit Meeting, November 27-30, 1955.
11. Byfield, A., S. M. Thesis, Chem. Eng., Massachusetts Institute of Technology, 1939.
12. Cauley, J. E., Moore, G. E., and Van Winkle, M., Chem. Eng. Prog., 49, 553 (1953).
13. Clark, N. O., Trans. Faraday Soc., 44, 13 (1948).
14. Clay, H. A., Hutson, T., Jr., and Kleiss, L. D., Chem. Eng. Prog., 50, 517 (1954).
15. Colburn, A. P., Ind. Eng. Chem., 33, 459 (1941).
16. Coppock, P. D., and Meiklejohn, G. T., Trans. Instn. Chem. Engrs. (London), 29, 75 (1951).
17. Danckwerts, P. V., Appl. Sci. Res., Section A, 3, 279 (1952).
18. Danckwerts, P. V., and Sellers, E. S., Industrial Chemist, 27, 395 (1951).
19. Datta, R. L., Napier, D. H., and Newitt, D. M., Trans. Instn. Chem. Engrs. (London), 28, 14 (1950).

20. Drickamer, H. G., and Bradford, J. R., Trans. Am. Inst. Chem. Engrs., 39, 319 (1943).
21. Edminster, W. C., Pet. Engr., 21, C-45 (January, 1949).
22. Eduljee, H. E., Trans. Instn. Chem. Engrs. (London), 24, 128 (1946).
23. Fossett, H., Trans. Instn. Chem. Engrs. (London), 29, 322 (1951).
24. Freshwater, D. C., Ph. D. Thesis, Chem. Eng., University of Birmingham, 1954.
25. Garner, F. H., and Freshwater, D. C., Trans. Instn. Chem. Engrs. (London), 33, 280 (1955).
26. Garner, F. H., and Hammerton, D., Chem. Eng. Science, 3, 1 (1954).
27. Gautreaux, M. F., and O'Connell, H. E., Chem. Eng. Prog., 51, 232 (1955).
28. Geddes, R. L., Trans. Am. Inst. Chem. Engrs., 42, 79 (1946).
29. Gerster, J. A., Bonnet, W. E., and Hess, I., Chem. Eng. Prog., 47, 523 (1951).
30. Gerster, J. A., Bonnet, W. E., and Hess, I., Chem. Eng. Prog., 47, 621 (1951).
31. Gerster, J. A., Colburn, A. P., Bonnet, W. E., and Carmody, T. W., Chem. Eng. Prog., 45, 716 (1949).
32. Gerster, J. A., Koffolt, J. H., and Withrow, J. R., Trans. Am. Inst. Chem. Engrs., 39, 37 (1943).
33. Gerster, J. A., Koffolt, J. H., and Withrow, J. R., Trans. Am. Inst. Chem. Engrs., 41, 393 (1945).
34. Gerster, J. A., Mizushina, T., Marks, T. N., and Catanach, A. W., A. I. Ch. E. Journal, 1, 536 (1955).
35. Guyer, A., and Peterhans, E., Helv. Chem. Acta, 26, 1099 (1946).
36. Higbie, R., Trans. Am. Inst. Chem. Engrs., 31, 365 (1935).
37. Hughes, R. D., Handlos, A. E., Evans, H. D., and Maycock, R. L., Chem. Eng. Prog., 51, 557 (1955).
38. Hunt, C. d'A., Hanson, D. N., and Wilke, C. R., A. I. Ch. E. Journal, 1, 441 (1955).
39. Hutchinson, M. H., Buron, A. G., and Miller, B. P., Paper presented at the Los Angeles Regional Meeting of the Am. Inst. Chem. Engrs., May, 1949.

40. Johnson, A. I., Robinson, D. G., and Michellepis, C. P., Paper presented at the Am. Inst. Chem. Engrs. Annual Meeting, Detroit, November 27-30, 1955.
41. Kirschbaum, E., "Distillation and Rectification," p. 278, Chemical Publishing Co. Brooklyn, N. Y. (1948).
42. Kirschbaum, E., Dechema Monographien, 6, 19, Berlin (1933).
43. Klein, J. H., Sc. D. Thesis, Chem. Eng., Massachusetts Institute of Technology, 1950.
- 44a. Lewis, W. K., Jr., Ind. Eng. Chem., 28, 399 (1936).
- 44b. Lewis, W. K., and Whitman, W. G., Ind. Eng. Chem., 16, 1215 (1924).
45. Marshall, W. R., Jr., and Pigford, R. L., "The Application of Differential Equations to Chemical Engineering Problems," University of Delaware, Newark, Delaware (1947).
46. Miller, O., 15 me. Congr. Chim. Ind. (Bruxelles), p. 352 (1936).
47. Mitkevich, G. P., Zhur. Priklad. Khim., 22, 846 (1949).
48. Nord, M., Ind. Eng. Chem., 38, 657 (1946).
49. Nord, M., Trans. Am. Inst. Chem. Engrs., 42, 863 (1946).
50. O'Connell, H. E., Trans. Am. Inst. Chem. Engrs., 42, 741 (1946).
51. Oliver, E. D., and Watson, C. C., A. I. Ch. E. Journal, 2, 18 (1956).
52. Peavy, C. C., and Baker, E. M., Ind. Eng. Chem., 29, 1056 (1937).
53. Polich, W. F., Ph. D. Thesis, Chem. Eng., Carnegie Institute of Technology, 1953.
54. Ross, S., J. Phys. Chem., 47, 266 (1943).
55. Ross, S., and Cutillas, M. J., J. Phys. Chem., 59, 68 (1955).
56. Schubauer, G. B., Natl. Adv. Comm. Aeronaut. Tech. Rept., 524 (1935).
57. Sherwood, T. K., and Woertz, B. B., Ind. Eng. Chem., 31, 1034 (1939).
58. Simkin, D. J., Strand, C. P., and Olney, R. B., Chem. Eng. Prog., 50, 565 (1954).
59. Souders, M., Jr., Huntington, R. L., Cornell, H. G., and Emert, F. L., Ind. Eng. Chem., 30, 86 (1938).
60. Spells, K. E., and Bakowski, S., Trans. Instn. Chem. Engrs. (London), 28, 38 (1950).

61. Spells, K. E., and Bakowski, S., Trans. Instn. Chem. Engrs. (London), 30, 189 (1952).
62. Stone, H. L., Sc. D. Thesis, Chem. Eng., Massachusetts Institute of Technology, 1953.
63. Taylor, G. I., Proc. London Math. Soc., A 151, 421 (1935).
64. Taylor, G. I., Proc. London Math. Soc., 20, 196 (1921).
65. Taylor, G. I., Chilton, T. H., and Handforth, S. L., Ind. Eng. Chem., 23, 860 (1931).
66. Thatcher, C. M., Ph. D. Thesis, Chem. Eng., University of Michigan, 1954.
67. Towle, W. L., and Sherwood, T. K., Ind. Eng. Chem., 31, 457 (1939).
68. Towle, W. L., and Seder, L. A., Ind. Eng. Chem., 31, 462 (1939).
69. Trice, V. G., Jr., and Rodger, W. A., A. I. Ch. E. Journal, 2, 205 (1956).
70. van Krevelen, D. W., and Hoftijzer, P. J., Chem. Eng. Prog., 46, 29 (1950).
71. Vermeulen, T., Williams, G. M., and Langlois, G. E., Chem. Eng. Prog., 51, 85F (1955).
72. Walter, J. F., and Sherwood, T. K., Ind. Eng. Chem., 33, 493 (1941).
73. Warzel, L. A., Ph. D. Thesis, Chem. Eng., University of Michigan, 1955.
74. West, F. B., Gilbert, W. D., and Shimizu, T., Ind. Eng. Chem., 44, 2470 (1952).
75. Wharton, L., B. S. Thesis, Chem. Eng., Massachusetts Institute of Technology, 1955.
76. Whitman, W. G., Chem. Met. Engr., 29, 146 (1923).
77. Winslow, C. E., Ph. D. Thesis, Chem. Eng., North Carolina State College, 1955.
78. Zenz, F. A., Pet. Ref., 33, No. 2, 99 (1954).