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LARGE-SIGNAL CHARACTERISTICS OF AVALANCHE-DIODE OSCILLATORS

by

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## A B S T R A C T

### LARGE-SIGNAL CHARACTERISTICS OF AVALANCHE-DIODE OSCILLATORS

by

Paul Theodore Greiling

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The purpose of this study is to investigate the large-signal operating characteristics of IMPATT diodes. Large-signal analyses for two types of models are presented and experimental results are obtained and compared with the theoretical ones.

A large-signal analysis of a Read diode is presented. The analysis incorporates the assumptions associated with a Read diode. The admittance is obtained as a function of frequency, RF voltage, dc bias current and transit angle. The limitations on the RF voltage with respect to maintaining field-independent velocities are also investigated.

The derivation and results of a large-signal analysis for studying the dc, single-frequency and multifrequency characteristics of realistic IMPATT diodes are presented. The analysis allows for carrier multiplication by impact ionization to occur at every point in the diode and therefore allows for a wide range of doping profiles to be investigated. The admittance, power output, efficiency and bias voltage are obtained as a function of the frequency, RF voltage and dc bias current for different profiles. Several important results are obtained which indicate that an optimum frequency exists for maximum negative conductance, the doping

density is an important parameter in determining the admittance and the negative conductance can be enhanced or degraded by the relative phase angles of the RF voltages in multifrequency operation.

The operating characteristics are measured experimentally. An equivalent circuit for the diode package and mount is obtained and verified by C-V measurements. Also from C-V data the doping density of two different diodes is determined. The admittance is measured by both a reflection-type circuit and an oscillator circuit as a function of RF voltage, dc bias current and frequency. The experimental results are correlated with the theoretical results.

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## CHAPTER I. INTRODUCTION

### 1.1 Historical Background

1.1.1 Transit-Time Negative Resistance in Semiconductors. The concept of negative-resistance devices, due to transit-time effects in semiconductors, was first demonstrated by Shockley<sup>1</sup> in 1954. He showed that the negative resistance of the device was due to a phase lag of the externally induced current with respect to the voltage caused by the diffusion time across the base region of a transistor-type structure. The device proposed by Shockley consisted of a pnp structure in which carriers are injected at one of the p-n junctions and diffuse through the n-region to the other junction. The device impedance has a negative real part when the external current induced by the diffusing carriers lags the voltage by more than 90 and less than 270 degrees.

In 1958 Read<sup>2</sup> proposed a diode structure in which the phase lag between the current in the device and the voltage across it was obtained by both transit-time effects of carriers and their generation by impact ionization. The structure proposed by Read was an  $n^+p_i p^+$  diode in which the injection of the carriers into the intrinsic or drift region was accomplished by avalanche multiplication of the carriers at the  $n^+p$  junction. Read showed theoretically, after several simplifying assumptions, that such a device was capable of producing high-power microwave oscillations with efficiencies of 30 percent. Several problems associated with the semiconductor technology had to be overcome before

this particular structure could be realized. It was not until 1965 that Lee et al.<sup>3</sup> solved the technological problems and obtained oscillations from a Read structure at 180 MHz. However, around the same time, Johnston et al.<sup>4</sup> obtained microwave oscillations at 10 GHz from a simple, avalanching p-n junction. These results indicated that it was not necessary to fabricate the Read structure in order to obtain negative-resistance devices at microwave frequencies but that a broad class of structures existed which exhibited negative resistance. All of these structures, including the Read diode, are now generally referred to as IMPATT diodes for impact ionization avalanche transit time.

Misawa<sup>5</sup> showed that a small-signal negative resistance of the transit-time type exists in junction diodes with a wide range of doping profiles. Misawa's computer solutions for the small-signal impedance were compared to those obtained by Read and were shown to be qualitatively similar for a wide range of doping profiles. Gilden and Hines<sup>6</sup> derived a simple equivalent circuit for the Read diode using a closed-form expression for the small-signal impedance and assuming a short transit angle. The electronic tuning effects due to the dc current could be clearly seen from their equivalent circuit. Fisher<sup>7</sup> developed a small-signal analysis in which the avalanche region of the diode, although still assumed to be narrow as in the Read diode, could be located any place in the space-charge region of the diode. Also several of the assumptions used in other analyses,<sup>2,6</sup> such as equal saturated drift velocities and ionization rates of the carriers, were removed. His closed-form solutions for the impedance were shown to be in agreement with Misawa's results. Gummel and Scharfetter<sup>8</sup> extended Gilden and Hines' small-signal analysis to

include diodes in which the avalanche region is not necessarily narrow. They obtained small-signal admittance plots for a Read diode and for more realistic diode structures in which the avalanche region occupies some fraction of the total depletion region. They found that optimum performance is achieved when the avalanche region is approximately one-third of the drift region.

Small-signal results predict the range of current and frequency in which an IMPATT diode will oscillate if placed in the proper resonant circuit. These results also indicate the magnitude of the maximum negative conductance which can be generated by the diode. However, no data on power output, efficiency and other important effects such as electronic tuning can be obtained since IMPATT diodes are nonlinear devices and the space-charge distribution under large-signal conditions is much different from the small-signal case.

The first large-signal analysis which was employed to predict the large-signal operating characteristics was performed by Evans and Haddad.<sup>9</sup> They obtained a closed-form solution of the nonlinear equations of a Read model by assuming a short transit time in the drift region. Several important large-signal characteristics were predicted from this simple model. The restriction of a short transit time was removed from the analysis by Greiling and Haddad.<sup>10</sup> Scharfetter and Gummel<sup>11</sup> developed a computer program for obtaining a numerical solution to the basic equations which govern the operation of the IMPATT diode. These include the continuity equations for holes and electrons and Poisson's equation. They have attempted to make their model as exact as possible by including carrier generation and recombination due to impact ionization and defects,

respectively, and the variation of mobility as a function of electric field and doping density. They have also assumed that these effects are not equal for both electrons and holes and have included the effect of the microwave circuit on the diode in which it is placed. Their program is presumed to give the most accurate and detailed solution for the large-signal operating characteristics of IMPATT diodes. However, the amount of computer time necessary for its execution is prohibitive for any detailed study of the effect of different diode parameters on its operating characteristics. A large-signal analysis was developed by Greiling et al.<sup>12</sup> in which IMPATT diodes with any doping profile could be analyzed. Though not all of the physical parameters included by Scharfetter and Gummel were included in this analysis, it does give good physical results for the large-signal operating characteristics of IMPATT diodes. The amount of computer time required for execution is such that it is also possible to use the analysis to study the effect of different diode parameters on the operating characteristics.

Because of the broadband negative resistance and the nonlinearities, IMPATT diodes operate under multifrequency conditions as well as single-frequency conditions. Although the analyses of Scharfetter and Gummel and Greiling et al. can be employed to investigate multifrequency operation, the amount of computer time necessary for execution becomes too costly for a detailed study. Blue<sup>13</sup> employed a Read-type model analysis and obtained some results concerning multifrequency operation. Schroeder et al.<sup>14</sup> have also used this type of analysis and have performed a more extensive study of the multifrequency large-signal operating characteristics of Read diodes. These results were also compared to more realistic diode

structures and good agreement was obtained. These results indicate that under the proper operating conditions the output power, efficiency and frequency range could be somewhat improved. Also these theoretical results were found to be in excellent agreement with experimental results reported by Swan.<sup>15</sup>

1.1.2 High-Efficiency Mode. In 1967 Prager et al.<sup>16</sup> observed experimentally high-efficiency operation of IMPATT diodes under pulse conditions at low frequencies. Since the negative resistance of IMPATT diodes is caused by a phase shift in the current, which is due in part to the transit time of the carriers, there exists a low-frequency cutoff below which the phase shift is not sufficient for negative resistance to exist. The frequency of operation observed by Prager et al. was well below this cutoff frequency and thus the negative resistance was not due to transit-time effects. This has been referred to as the "anomalous mode." The low frequency was found to be a subharmonic of a high-frequency oscillation which was trapped in the diode cavity and was in the normal transit-time range. Johnston et al.<sup>17</sup> obtained oscillations from Ge diodes operating pulsed with efficiencies exceeding 40 percent at frequencies of 2 to 3 GHz. Also the second harmonic of the signal was found to be present at the diode. Using these experimental voltage waveforms as input for their computer program, Scharfetter et al.<sup>18</sup> discovered the cause for the oscillations at the low frequencies. They found that under certain conditions it was possible to form an "avalanche shock front"<sup>19</sup> which propagates through the diode at a velocity which is much greater than the saturated velocities of the carriers. The front leaves behind it a region of very high carrier concentration and low field. The carriers are trapped

in this low-field region but the electric field is still large enough to move the carriers. A low-frequency recovery transient then occurs which brings the diode back to the initial state and the process is repeated. Thus the diode switches from a low-voltage, high-current state after the avalanche shock front has formed to a high-voltage, low-current state after the trapped carriers have been extracted from the diode. This current and voltage waveform produces a high-efficiency diode at the low frequency. The name assigned to this mode by Scharfetter et al.<sup>18</sup> was the "TRAPATT" mode for trapped plasma avalanche triggered transit.

Hoefflinger<sup>20</sup> presented an explanation for the high-efficiency, low-frequency operation which did not require the existence of the trapped plasma.<sup>18</sup> By superposition of several harmonically related voltages across the diode, his analysis predicted high-efficiency operation without trapped plasma formation. However, some doubt has been raised concerning this analysis due to recent work by Schroeder and Haddad.<sup>21</sup> Clorfeine et al.<sup>22</sup> have presented a simplified explanation of the high-efficiency mode based on the existence of an avalanche shock front in which they obtained a closed-form solution for the operating characteristics of the device. A similar explanation of the high-efficiency mode was also presented by Scharfetter.<sup>23</sup>

This study will be concerned with the transit-time mode of operation of IMPATT diodes and the high-efficiency mode will not be considered.

## 1.2 State of the Art

IMPATT diodes have become practical microwave devices. Since their inception a great deal of work has been performed on them and very

significant results concerning power output, efficiency and frequency range have been obtained. These diodes have been operated as oscillators and amplifiers from 500 MHz to 340 GHz. Operation has been achieved from a single diode chip, several diode chips in one package, several diodes mounted in parallel and series and in single-frequency, multifrequency and high-efficiency modes.

IMPATT diode performance has been greatly enhanced due to technological developments. First, by providing a better heat sink, the output power has been greatly increased in X-band. This was obtained by mounting the diode "upside down" with the junction bonded to the heat sink (flip chip) rather than the substrate. Swan<sup>24</sup> has shown that by using a diamond instead of a copper heat sink the performance could be improved. He obtained 2.7 W CW in X-band with a single diode and 4.7 W CW with two diodes mounted on the same heat sink. The efficiency obtained was 8 percent. Recently work has been reported by Kim and Armstrong<sup>25</sup> in which 11 percent efficiency was obtained at X-band from a GaAs Schottky-barrier IMPATT diode. More recently they and others<sup>26,27</sup> have reported efficiencies between 10 and 15 percent in X-band from GaAs and Ge Schottky-barrier IMPATT diodes.

A second improvement has been the reduction of the skin-effect losses in the substrate at high frequencies. This has been shown to contribute a significant loss to the diode at high frequencies by DeLoach.<sup>28</sup> Edwards et al.<sup>29</sup> have reduced this loss by making diodes with very thin substrates ( $\approx 5 \mu\text{m}$ ). They obtained 10 percent efficiency at 55 GHz with 650 mW CW output power and 3.2 percent efficiency at 110 GHz with 75 mW CW output power.

At the low-frequency end of the IMPATT diode spectrum the diodes are operated in the high-efficiency mode rather than the transit-time mode. Some of the best CW results in this mode have been obtained by Iglesias and Evans<sup>30</sup> with 5.5 W CW from a single diode operating at 450 MHz and with 43 percent efficiency. Liu and Risko<sup>31</sup> have operated five diode chips in a single package in the high-efficiency mode and obtained 1.2 kW of pulsed power at 1.1 GHz with 25.6 percent efficiency. A summary of some of the significant experimental results are presented in Table 1.1 and Fig. 1.1. These indicate the state of the art at this time.

### 1.3 Dynamic Characteristics of IMPATT Diodes

1.3.1 Introduction. Before discussing the physical operation of an IMPATT diode, it is helpful to derive the maximum output power and efficiency for certain voltage and current waveforms. In the next section it will be shown how currents and voltages of the form shown in Fig. 1.2 are generated in IMPATT diodes.

The average power dissipated for the waveforms shown in Fig. 1.2 is

$$P = \frac{1}{2\pi} \int_{\pi}^{x\pi} I_{\max} V_{RF} \sin \omega t \, d\omega t, \quad (1.1)$$

where  $I_{\max}$ ,  $V_{RF}$  and  $x$  are illustrated in the figure. Performing the integration, the following result is obtained:

$$P = - \frac{V_{RF} I_{\max}}{2\pi} (1 + \cos x\pi) . \quad (1.2)$$

The negative sign means that the power is delivered rather than dissipated.

Table 1.1  
Status of IMPATT Diodes \*

| Frequency<br>(GHz) | Power<br>Output | Efficiency<br>(Percent) | Mode of<br>Operation  | Material | Authors                          | Number of<br>Diodes |
|--------------------|-----------------|-------------------------|-----------------------|----------|----------------------------------|---------------------|
| 9.9                | 540 mW          | 15.3                    | IMPATT, CW            | GaAs     | de Nobel <sup>27</sup>           | 1                   |
| 11.1               | 500 mW          | 11.5                    | IMPATT, CW            | GaAs     | Kim and Armstrong <sup>25</sup>  | 1                   |
| 13.3               | 4.7 W           | 8                       | IMPATT, CW            | Si       | Swan <sup>24</sup>               | 2                   |
| 55                 | 650 mW          | 10                      | IMPATT, CW            | Si       | Edwards et al. <sup>29</sup>     | 1                   |
| 110                | 75 mW           | 3.2                     | IMPATT, CW            | Si       | Edwards et al. <sup>29</sup>     | 1                   |
| 8                  | 30 W            | 4-5                     | IMPATT, pulsed        | Si       | Gilden and Moroney <sup>32</sup> | 1                   |
| 21                 | 8.8 W           | 4-5                     | IMPATT, pulsed        | Si       | Gilden and Moroney <sup>32</sup> | 1                   |
| 341                | 1 mW            | too small               | IMPATT, pulsed        | Si       | Bowman and Burris <sup>33</sup>  | 1                   |
| 0.45               | 5.3 W           | 43                      | High- $\eta$ , CW     | Ge       | Iglesias and Evans <sup>30</sup> | 1                   |
| 1.1                | 1.2 kW          | 25.6                    | High- $\eta$ , pulsed | Si       | Liu and Risko <sup>31</sup>      | 5                   |
| 3                  | 20 W            | 30                      | High- $\eta$ , pulsed | Si       | Grace and Gibbons <sup>34</sup>  | 1                   |
| 6.5                | 3 W             | 17                      | High- $\eta$ , pulsed | Si       | Ying et al. <sup>35</sup>        | 1                   |

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\* This data was compiled by M. S. Gupta, Electron Physics Laboratory, The University of Michigan, Ann Arbor.

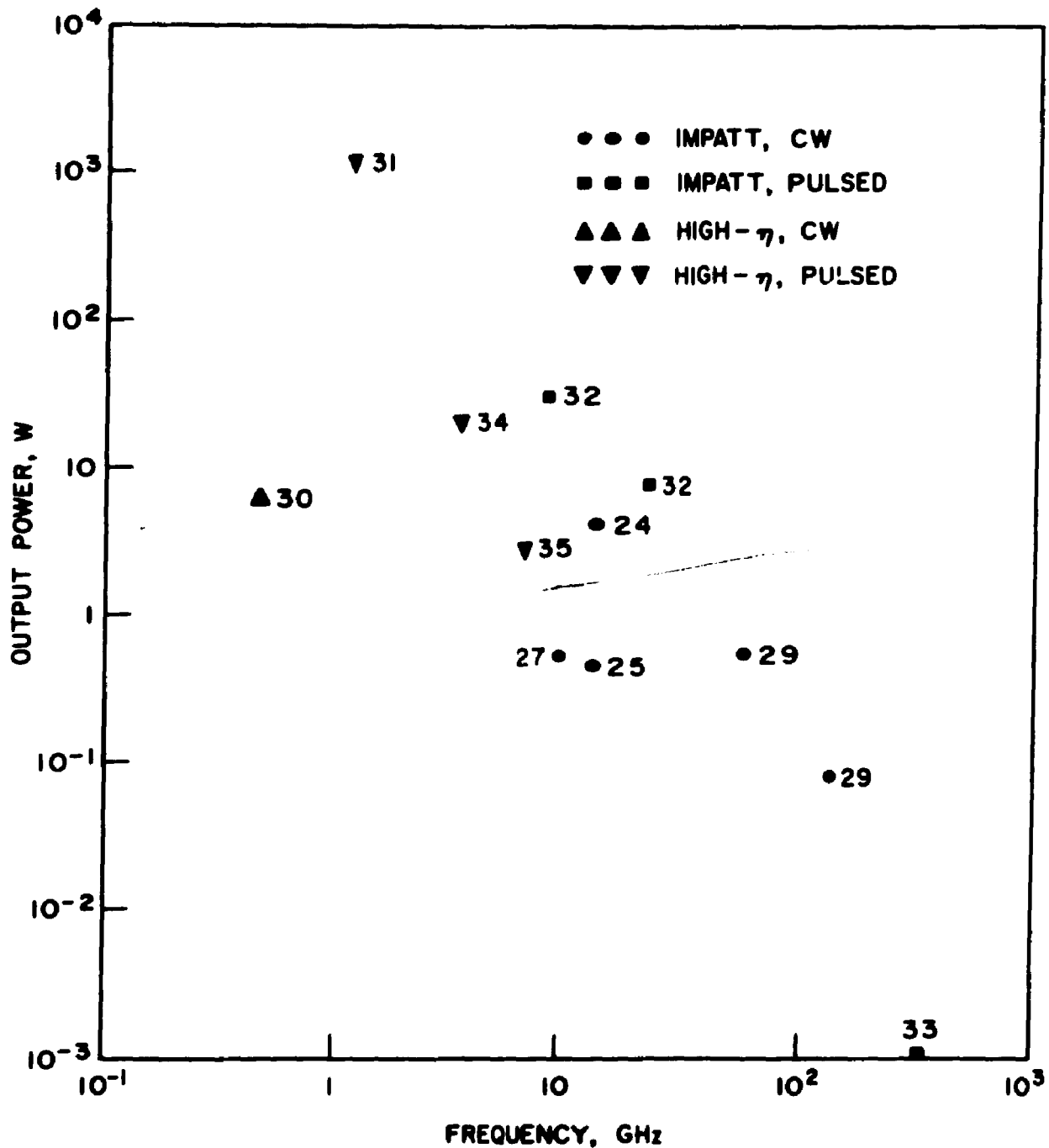


FIG. 1.1 STATUS OF IMPATT DIODES. (THIS DATA WAS COMPILED BY M. S. GUPTA, ELECTRON PHYSICS LABORATORY, THE UNIVERSITY OF MICHIGAN, ANN ARBOR. THE NUMBERS NEXT TO THE DIODE POINTS REFER TO THE REFERENCE NUMBERS)

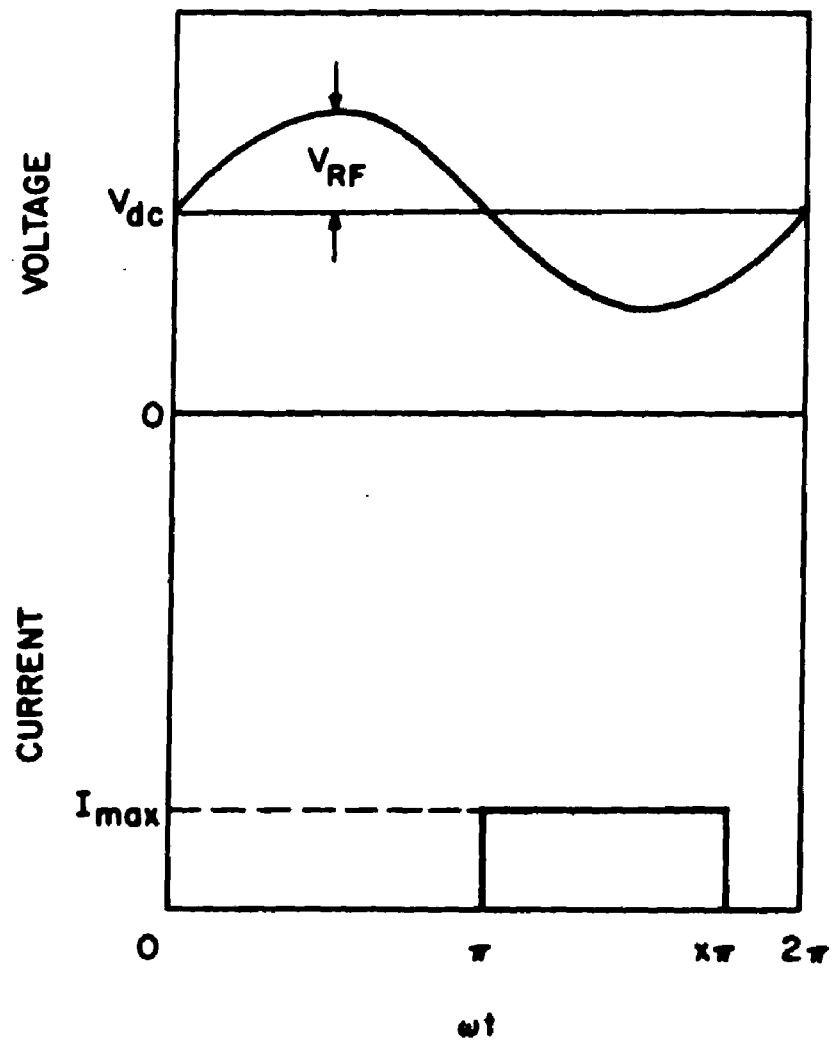


FIG. 1.2 VOLTAGE AND CURRENT WAVEFORMS.

The maximum current,  $I_{\max}$ , is related to the dc current by

$$I_{dc} = \frac{1}{2\pi} \int_{\pi}^{x\pi} I_{\max} d\omega t \quad (1.3)$$

which reduces to

$$I_{dc} = I_{\max} \left( \frac{x - 1}{2} \right) \quad (1.4)$$

Therefore

$$I_{\max} = \frac{2I_{dc}}{x - 1} \quad (1.5)$$

which when substituted into Eq. 1.2 gives

$$P = - \frac{V_{RF} I_{dc}}{\pi} \left( \frac{1 + \cos x\pi}{x - 1} \right) \quad (1.6)$$

The maximum output power can be found by taking the derivative of the output power with respect to  $x$  and finding the value of  $x$  which makes the derivative zero. However, the resulting equation for the derivative is difficult to solve explicitly for  $x$  and thus an easier method is to plot the output power as a function of  $x$ . This is done in Fig. 1.3 where the output power is normalized by setting

$$\frac{V_{RF} I_{dc}}{\pi} = 1 \quad (1.7)$$

The maximum output power occurs for  $x$  approximately equal to 1.742.

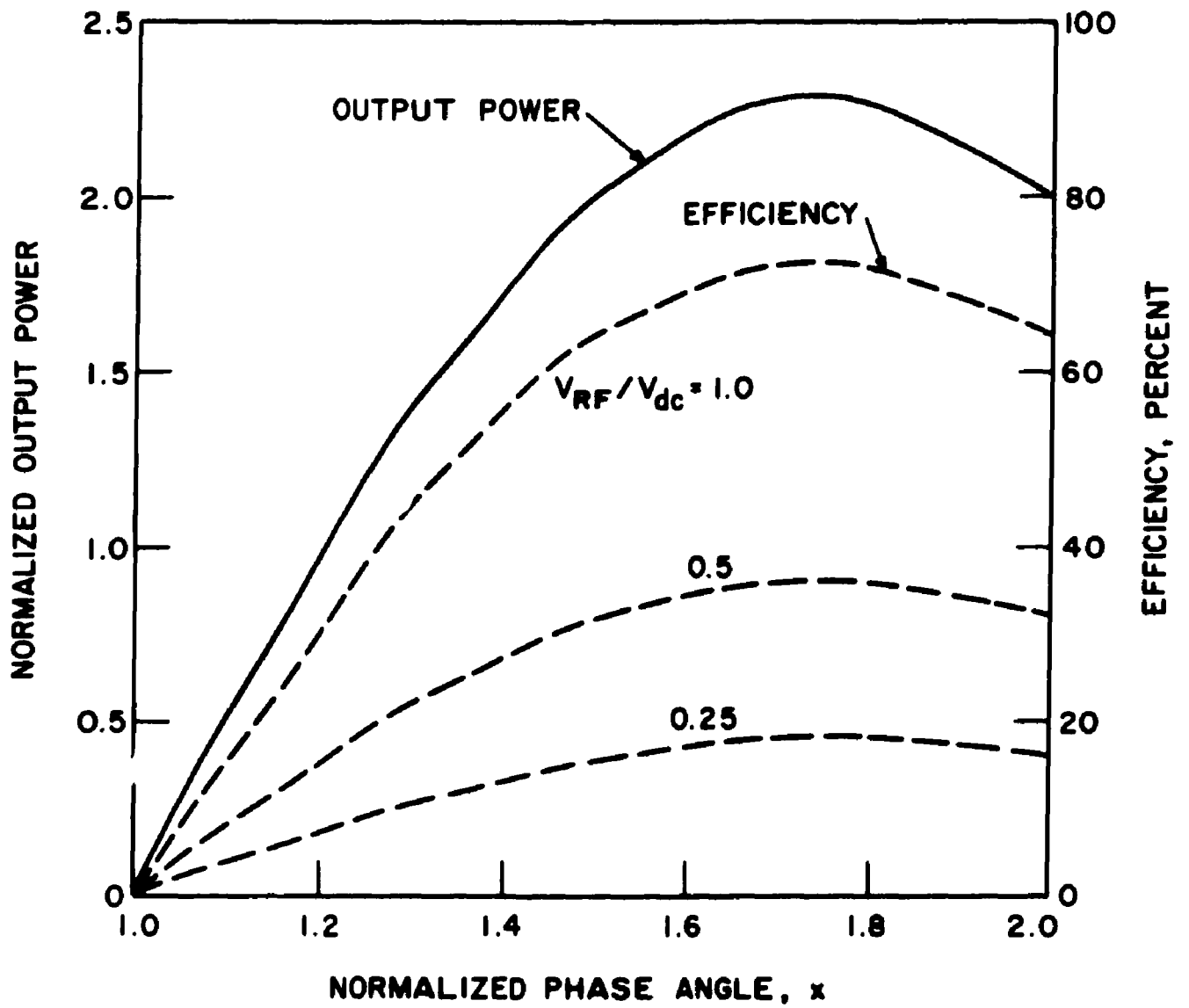


FIG. 1.3 NORMALIZED OUTPUT POWER AND EFFICIENCY VS. NORMALIZED PHASE ANGLE  $x$ .

The efficiency  $\eta$  of a device with these waveforms is given by

$$\eta = P / (V_{dc} I_{dc}) \quad (1.8)$$

which yields for this case

$$\eta = \frac{V_{RF}}{V_{dc}} \pi \left( \frac{1 + \cos x\pi}{x - 1} \right) . \quad (1.9)$$

The efficiency for different values of  $V_{RF}/V_{dc}$  is plotted as a function of  $x$  in Fig. 1.3 and also has a maximum for  $x$  equal to approximately 1.742.

1.3.2 The Read Diode. An understanding of the dynamic operating characteristics of IMPATT diodes can best be obtained by considering the operation of a Read diode. The structure and field distribution for a reverse-biased Read diode are shown in Fig. 1.4. The avalanche region (p-region) is assumed to be narrow and doped such that at low values of reverse bias all of the voltage drop occurs across the  $n^+p$  junction. As the bias is increased the depletion region extends through the intrinsic region to the  $ip^+$  junction. The bias is increased until avalanche breakdown occurs. At this point the field distribution is such that carrier multiplication by impact ionization occurs in the avalanche region, carriers drift at saturated or field-independent velocities in the drift or i-region and no impact ionization occurs in the drift region. Under these conditions the current in the diode is primarily due to the holes generated in the avalanche region and drifting through the drift region. The electrons which are generated in the avalanche region are quickly swept out into the  $n^+$ -region.

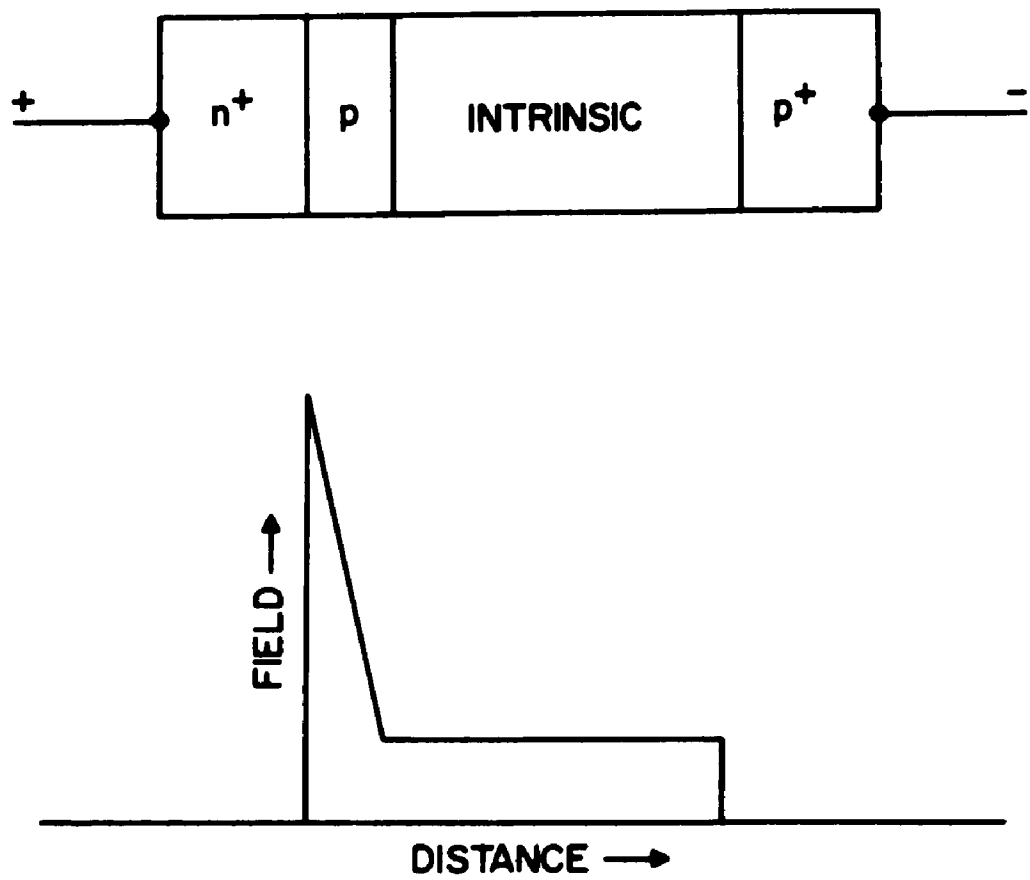


FIG. 1.4 STRUCTURE AND FIELD DISTRIBUTION OF A REVERSE-BIASED READ DIODE.

The negative resistance or conductance of a Read diode is attributed to a phase shift between the current through the diode and the voltage across it. This phase shift consists of two components. There is a phase delay of the current caused by the avalanche multiplication process and by the finite transit time of the holes drifting through the drift region. The phase delay caused by the avalanche process results from the condition that the rate of generation of electron-hole pairs in the avalanche region is proportional to both the electric field and the density of pairs that are already present. If a small RF voltage of sufficiently high frequency is assumed to be superimposed along with a dc voltage near breakdown across the diode, the rate of generation of electron-hole pairs will exceed the rate at which the pairs leave the avalanche region. Thus both the density of carriers and the current will grow exponentially with time as long as the RF and dc voltages add to give a total field above breakdown. When the RF voltage changes sign and subtracts from the dc voltage causing the field to fall below breakdown, the generation rate starts decreasing, causing the current to decrease. Thus the current generated by the avalanche process will have its maximum when the RF field goes through zero. Thus, as shown in Fig. 1.5, the avalanche process contributes a 90 degree inductive phase lag to the current generated in the avalanche region. This current is then injected into the drift region of the diode. The current induced in the external circuit by this charge drifting through the drift region at a saturated drift velocity is shown at the bottom of Fig. 1.5. Thus the fundamental component of the external current is more than 90 degrees out of phase with the RF voltage causing the resistance or conductance of the diode

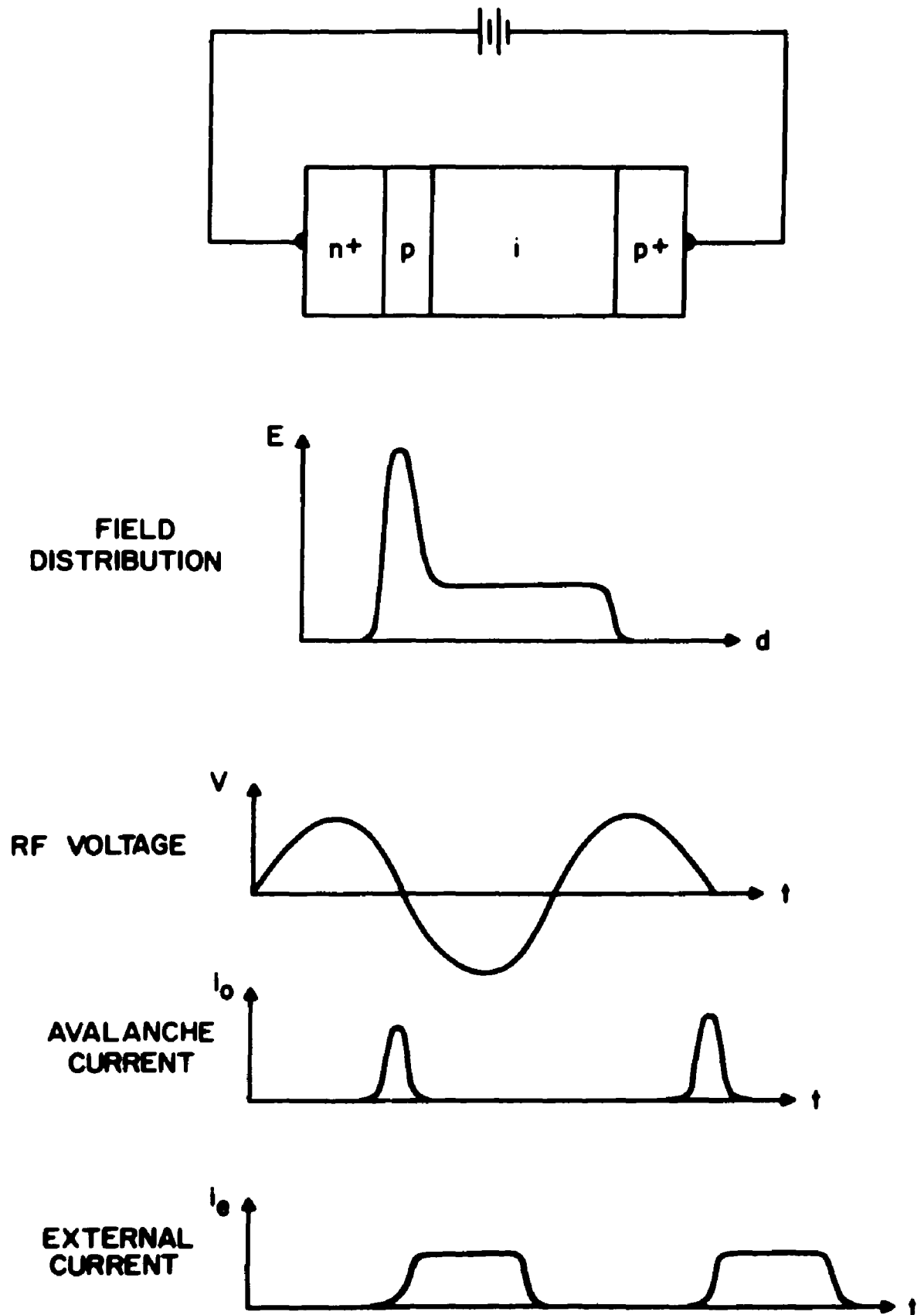


FIG. 1.5 READ DIODE WAVEFORMS.

to be negative. In order for the diode to operate as a stable oscillator, the negative conductance of the diode must decrease with increasing RF voltage. The RF voltage across the diode will grow until the admittance of the diode is balanced by the admittance of the diode mount and microwave circuit. The diode will oscillate at a frequency and RF voltage level where the diode admittance is equal and of opposite sign to the circuit admittance.

There are several important effects which must be included in this explanation. First is the effect of the space charge of the carriers. As mentioned previously, the current in the diode consists primarily of holes which will cause the electric field to decrease behind them and to increase in front of them as they drift across the drift region. As the holes are generated in the avalanche region and injected into the drift region they cause the field in the avalanche region to be depressed. In Fig. 1.6 the effect of the space charge on the field in the avalanche region is illustrated. The dashed line corresponds to the field and current without space charge while the solid line represents the field and current when space charge is taken into account. Thus the effect of the space charge of the holes is to decrease the field and cause it to drop below the breakdown level before the RF voltage does. Therefore the phase lag of the current due to the avalanche process is decreased. If the RF voltage is held constant and the dc current is increased, which means the space charge in the diode is increased, the field in the avalanche region will drop below the breakdown level earlier in the cycle. Thus the phase lag of the current will decrease with increasing dc current and constant RF voltage as shown in Fig. 1.7. Conversely, if the dc current is held constant but the RF voltage is increased the phase lag

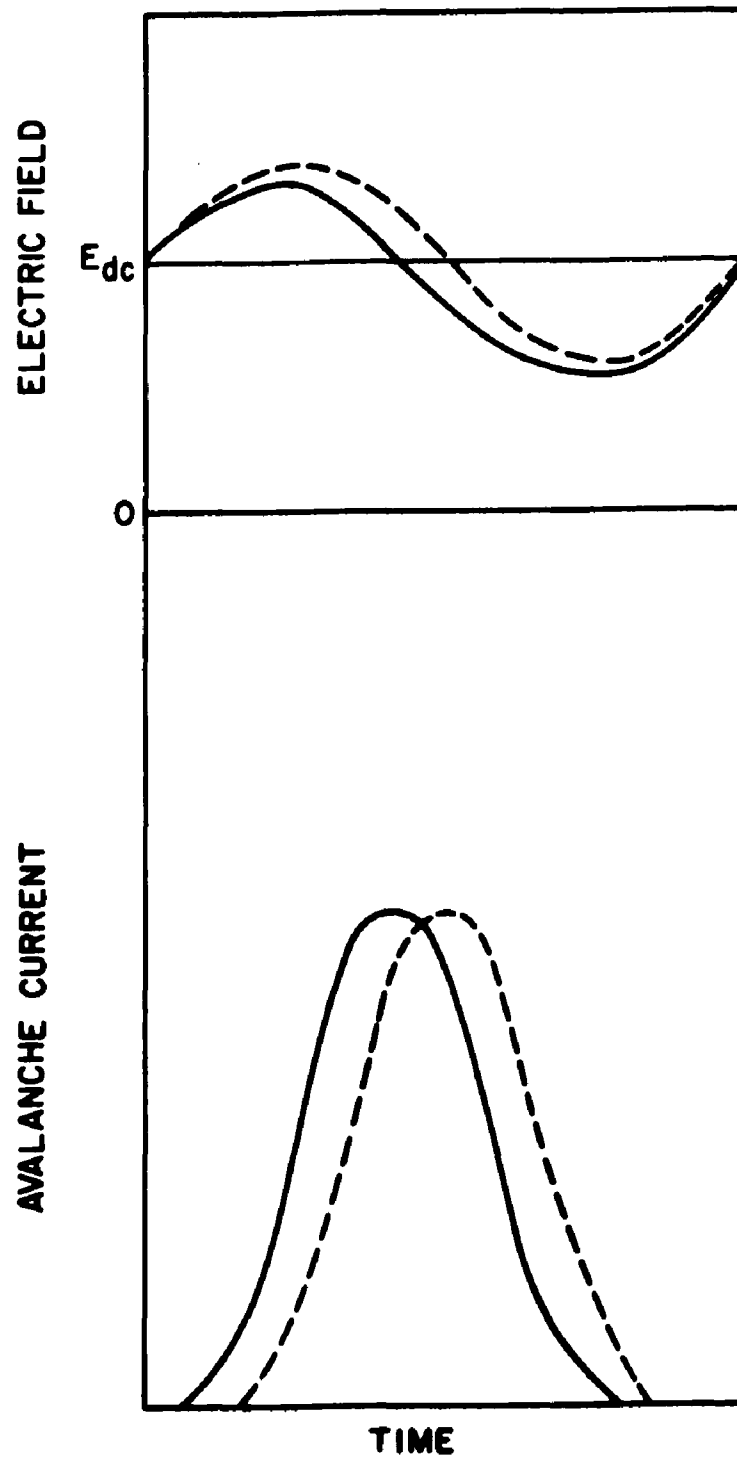


FIG. 1.6 ELECTRIC FIELD AND CURRENT IN THE AVALANCHE REGION AS A FUNCTION OF TIME.

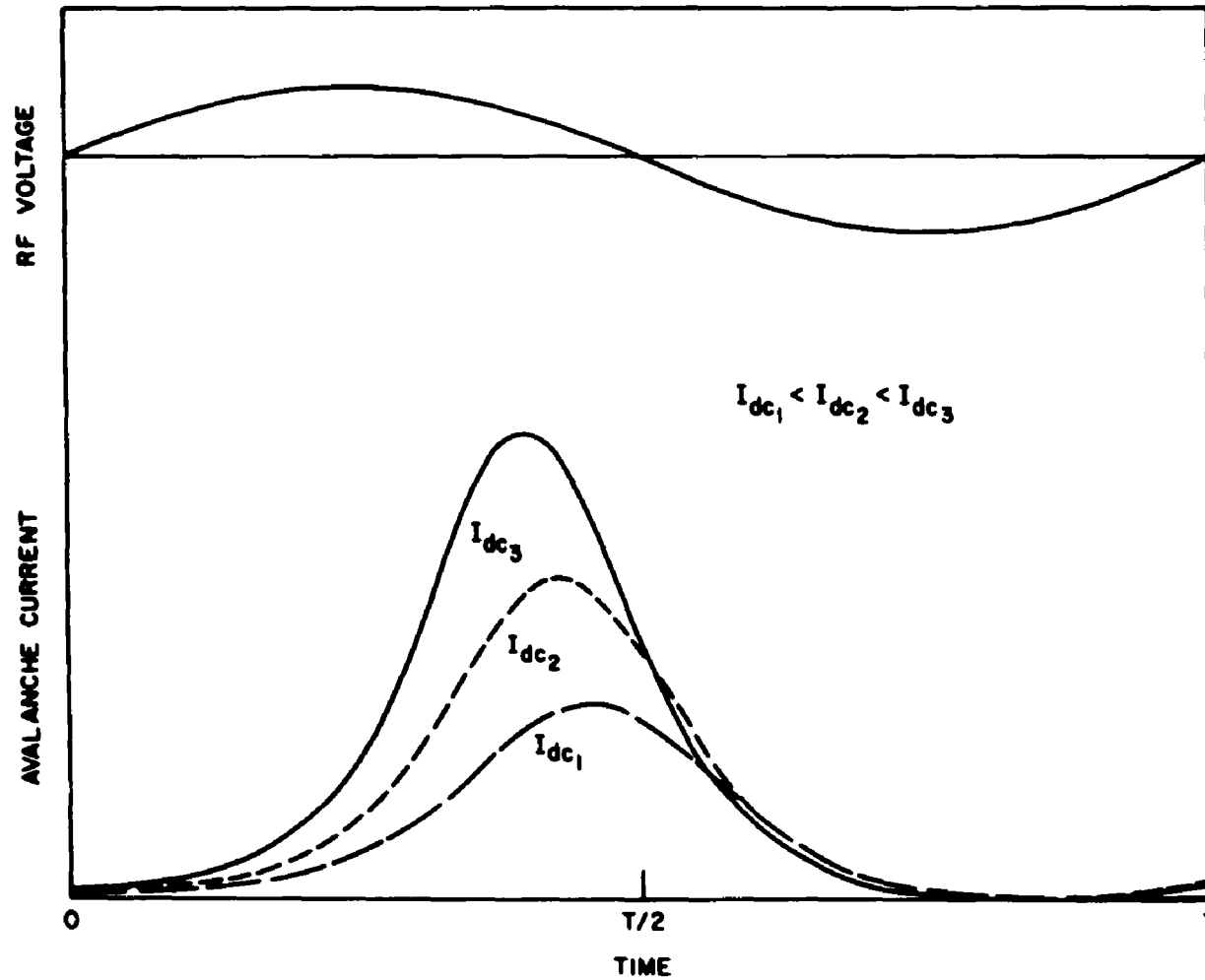


FIG. 1.7 AVALANCHE CURRENT AS A FUNCTION OF THE PHASE OF THE RF VOLTAGE FOR DIFFERENT dc CURRENT VALUES (CONSTANT RF VOLTAGE).

of the current is increased. For constant dc current the amount of space charge is constant and thus the field is decreased by a set value. But as the RF voltage and field increase in magnitude this set value of depression of the field causes the field to drop below the breakdown level later in the cycle. This effect is illustrated in Fig. 1.8. As the holes drift across the diode and are collected on the opposite side, the space charge in the diode decreases and the RF field decays toward the no-space-charge field as shown in Fig. 1.6.

The additional phase delay needed for negative conductance is caused by the transit time of the holes. The effect of the transit time on the frequency range of operation of the diode can be easily understood by assuming that the current generated in the avalanche region is generated in a sharp pulse. The exact time in the cycle that this pulse is generated will be a function of the dc current and the RF voltage as discussed above. Making this assumption, the field, avalanche current and external current are shown in Fig. 1.9. The phase delay caused by the transit time is proportional to  $\omega\tau$ , where  $\tau$  is the transit time in seconds. For a given diode the transit time  $\tau$  remains constant but the phase delay decreases with decreasing frequency. At a sufficiently low frequency the phase delay of the transit time plus the phase lag of the avalanche process are not sufficient to have the fundamental component of the external current lag the RF voltage by 90 degrees or more. Therefore below a certain cutoff frequency the conductance of the diode is positive. As can be seen from the effect of the RF voltage on the phase lag in the avalanche process, the conductance will be negative at lower frequencies for larger RF voltages. However, for these frequencies the diode will not be a self-starting oscillator since the small-signal conductance is positive.

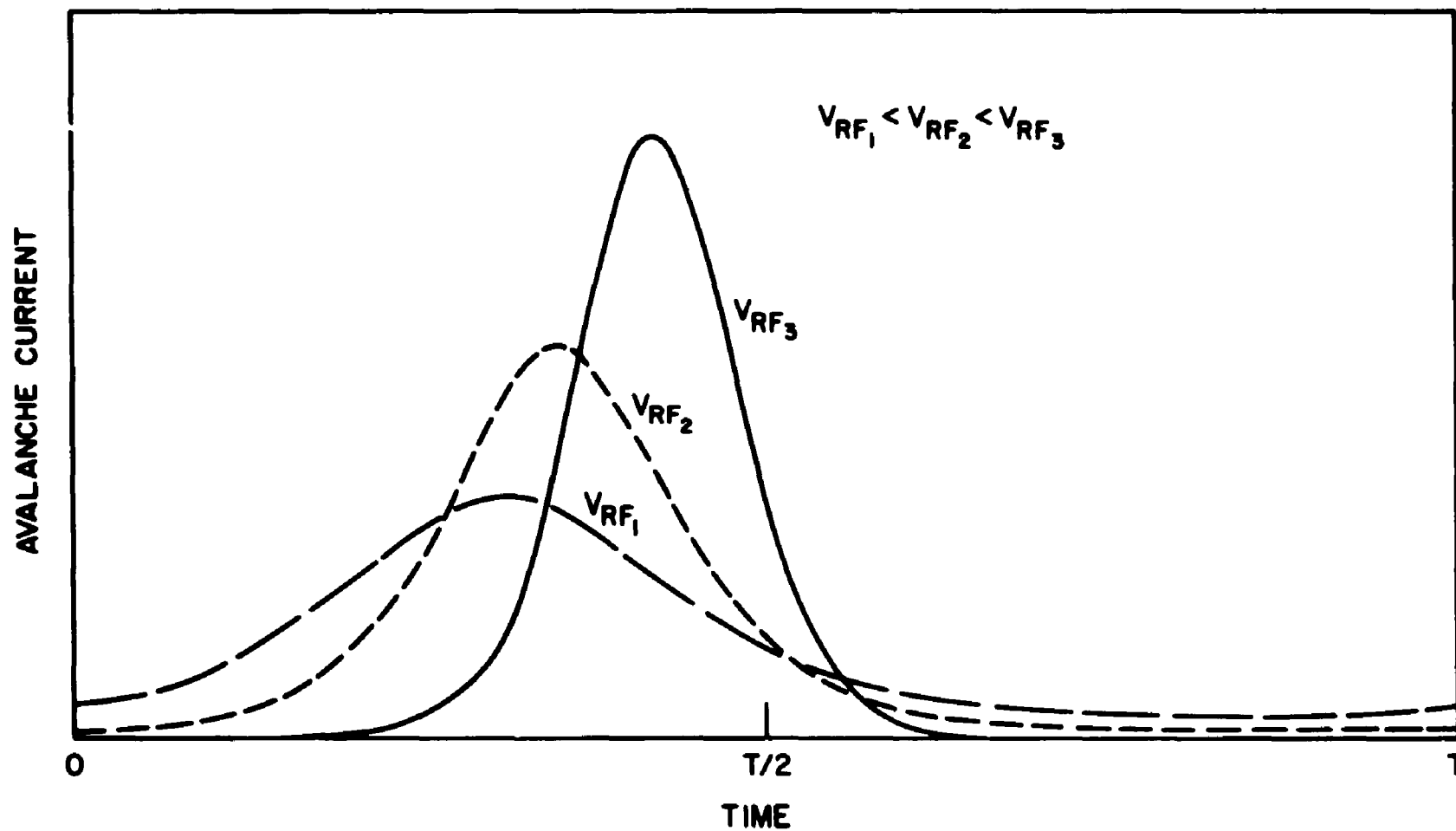


FIG. 1.8 AVALANCHE CURRENT AS A FUNCTION OF THE PHASE OF THE RF VOLTAGE FOR DIFFERENT MAGNITUDES OF RF VOLTAGE (CONSTANT dc CURRENT).

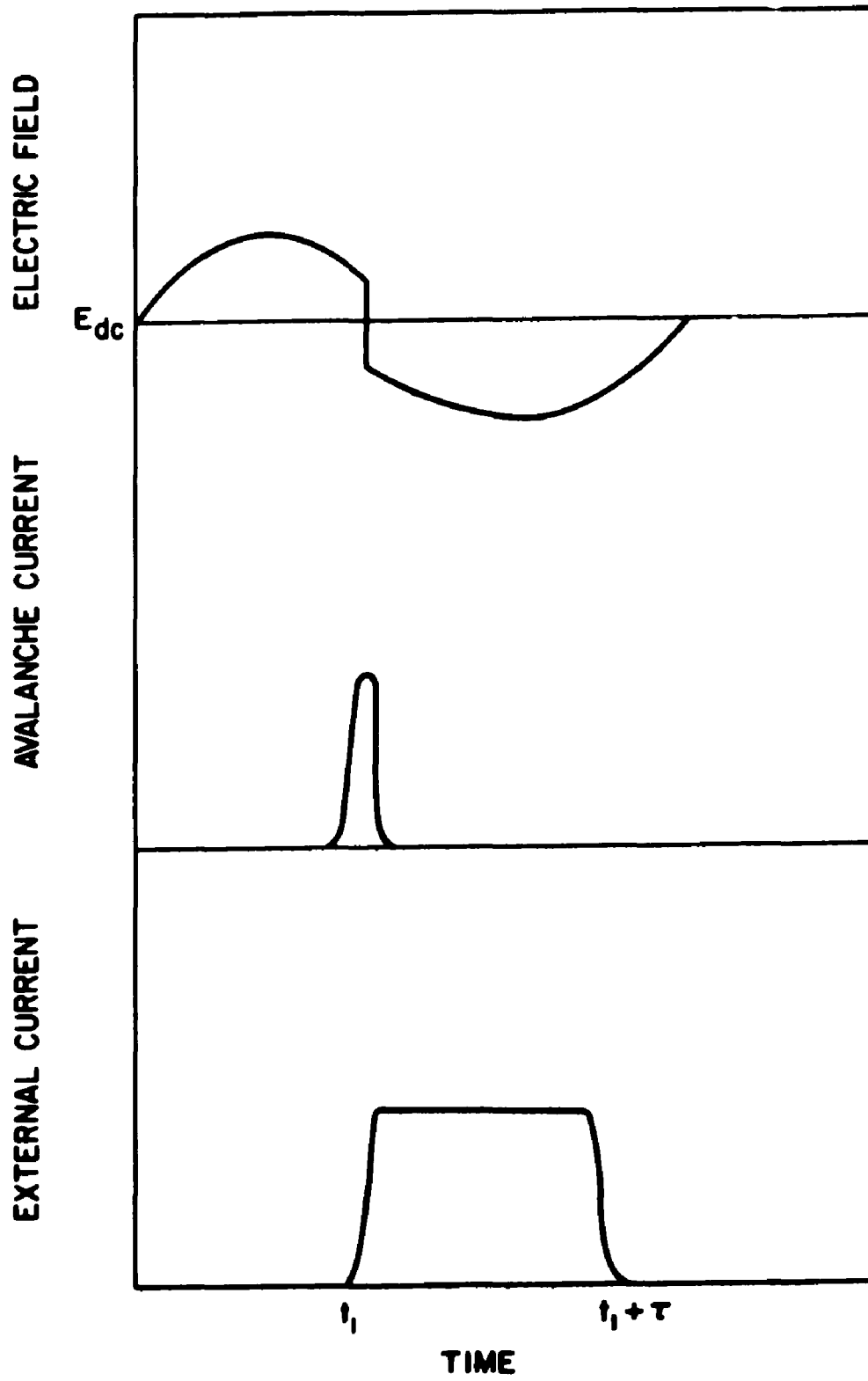


FIG. 1.9 CURRENT AND ELECTRIC FIELD WAVEFORMS FOR SHOWING TRANSIT-TIME EFFECTS.

As the dc current is increased the cutoff frequency is increased due to the effect of the current on the phase lag in the avalanche process. Gilden and Hines<sup>6</sup> showed that the small-signal cutoff frequency increased as the square root of the dc current.

1.3.3 IMPATT Diodes. The Read diode is a convenient structure for analytical purposes and for obtaining a physical understanding of IMPATT diodes, but it is difficult to construct. The p-n junction is easier to fabricate but much more difficult to analyze. In the p-n junction the avalanche region and drift region are not separated but rather carriers are generated while they are drifting through an appreciable portion of the diode. Also both electrons and holes take part in the conduction process rather than just one type of carrier. Thus it is not possible to separate the two phase-delay components as in the Read diode. However the mechanisms causing the phase delay between the voltage and the current in IMPATT diodes are still the transit time and the avalanche process. Another difference in IMPATT diodes is the cutoff frequency. Misawa<sup>5</sup> has shown that the cutoff frequency for a diode with a uniform field in the depletion region (a pin diode) goes to dc. This is due to space-charge effects on the field distribution which are comparable to those observed in a gas discharge rather than to transit-time effects. Hoefflinger<sup>38</sup> has shown that for high current densities (large space charge) the cutoff frequency for a Read structure goes to dc due to avalanching in the drift region caused by the large space charge present.

#### 1.4 Outline of the Present Study

In this study large-signal models of a Read and an IMPATT diode are obtained and analyzed. Experimental results are also presented and compared with the theoretical ones. In Chapter II a large-signal model for a Read diode is presented for which any transit angle can be investigated. The analysis removes the restriction of a short transit angle used by Evans and Haddad.<sup>9</sup> The results show that there exists an optimum transit angle for maximum negative conductance as discussed previously. Limits on the maximum RF voltage which can exist across the diode without causing the carriers in the drift region to drop out of saturated velocities are investigated as a function of the transit angle. These results show that there exists an optimum transit angle for maximum efficiency which differs from that for maximum negative conductance. Also the effects of nonsaturated drift velocities in the drift region on the properties of the diode are discussed.

In Chapter III a large-signal model for an IMPATT diode with any doping profile is derived. Starting with the continuity equations for holes and electrons, Poisson's equation and an equation for the field dependence of the carrier generation rate due to impact ionization, a nonlinear integro-differential equation for the total current density is obtained as a function of the electric field. For a given electric field the equation has a periodic solution. The total current density is then separated into the corresponding hole, electron and displacement current densities. Using the hole and electron current densities the electric field is modified to give the proper space charge and then this new field is used to find the total current density again. The iterations are repeated

until the admittance of the diode changes by less than a prescribed percentage. The significant contribution of this analysis is that realistic diodes with different doping profiles can be compared. The analysis is not limited to having a narrow avalanche region and a long drift region but can handle doping profiles such as the pin diode where avalanching by impact ionization occurs throughout the entire diode. Thus the analysis can be employed to compare realistic diode structures with different doping profiles with respect to admittance, frequency range, power output, efficiency and other operating characteristics. A comparison of this type is presented in Chapter III.

Chapter IV describes the experimental method for measuring the large-signal admittance of an IMPATT diode as a function of frequency, RF voltage and dc current. An equivalent circuit for the diode package and mount is developed which is used in measuring the admittance of the diode by two different methods. In the first method the diode is used as a reflection-type amplifier and in the second, the admittance seen by the oscillating diode is measured. The results of both methods are compared and these experimental results are compared with the theoretical results of Chapter III.

## CHAPTER II. LARGE-SIGNAL ANALYSIS OF A READ DIODE

### 2.1 Introduction

In this chapter a mathematical model is presented which describes the large-signal operating characteristics of a Read diode. Following the analysis of Evans and Haddad,<sup>9</sup> a nonlinear integro-differential equation is obtained which describes the time dependence of the current generated in the avalanche region on the electric field. The solution of the equation was obtained by an iterative technique on an IBM 7090 computer. From these results, the large-signal operating characteristics are obtained for different diode parameters and biasing conditions. The dc breakdown voltage and the effect of the RF voltage on the field in the drift region are investigated. The maximum RF voltage which can exist without having the diode forward biased or having the carriers drift at nonsaturated velocities is found. Finally, the effect of nonsaturated drift velocities of the carriers on the negative conductance of the diode is discussed.

### 2.2 Large-Signal Analysis

2.2.1 Basic Equations and Assumptions. The continuity equations for holes and electrons and Poisson's equation relate the current and the electric field in the diode. The one-dimensional continuity equations with terms included to account for carrier generation by impact ionization are given by

$$\frac{\partial p}{\partial t} = -\frac{1}{q} \frac{\partial J_p}{\partial x} + \alpha_p p v_p + \alpha_n n v_n \quad (2.1)$$

and

$$\frac{\partial n}{\partial t} = \frac{1}{q} \frac{\partial J_n}{\partial x} + \alpha_p p v_p + \alpha_n n v_n, \quad (2.2)$$

where  $p$  = the hole density ( $\text{cm}^{-3}$ ),

$n$  = the electron density ( $\text{cm}^{-3}$ ),

$J_p$  = the hole current density ( $\text{A}/\text{cm}^2$ ),

$J_n$  = the electron current density ( $\text{A}/\text{cm}^2$ ),

$q$  = the electronic charge (C),

$\alpha_p$  = the ionization rate for holes ( $\text{cm}^{-1}$ ),

$\alpha_n$  = the ionization rate for electrons ( $\text{cm}^{-1}$ ),

$v_p$  = the hole velocity ( $\text{cm}/\text{s}$ ) and

$v_n$  = the electron velocity ( $\text{cm}/\text{s}$ ).

The ionization rates  $\alpha_p$  and  $\alpha_n$  are dependent on the electric field  $E$ .

An approximate relation for the ionization rate which can be applied to several semiconductors may be expressed as

$$\alpha = A' \exp \left[ - \left( \frac{b'}{E} \right)^{m'} \right], \quad (2.3)$$

where  $A'$ ,  $b'$  and  $m'$  are determined experimentally. Values for these constants, which depend on the type of carrier and material, may be found in the literature.<sup>37,38</sup>

Poisson's equation relates the field and carrier densities as

$$\frac{\partial E}{\partial x} = \frac{q}{\epsilon} (N_D - N_A + p - n) , \quad (2.4)$$

where  $N_D$  = the donor density ( $\text{cm}^{-3}$ ),  
 $N_A$  = the acceptor density ( $\text{cm}^{-3}$ ) and  
 $\epsilon$  = the dielectric constant (F/cm).

In order to determine the characteristics of IMPATT diodes it is necessary to solve Eqs. 2.1 through 2.4. Misawa<sup>5</sup> has obtained computer solutions to these equations assuming the usual small-signal assumptions. Evans and Haddad<sup>9</sup> made several simplifying assumptions to these equations and obtained a closed-form solution under large-signal conditions. The analysis presented in this chapter is an extension of their analysis and therefore the derivation of the equations relating the field and current which are solved on the computer will not be presented. The derivation can be obtained from Evans.<sup>39</sup> However the assumptions which have been employed in this analysis will be discussed so that the results can be interpreted correctly and the analysis can be compared with the more general analysis presented in Chapter III.

The first assumption is that the Read diode is divided into two regions: a narrow end region in which the field is assumed high enough to generate carriers by impact ionization, and a second, larger region in which the carriers injected from the avalanche region drift at saturated or field-independent velocities. Thus the avalanche and drift regions are assumed constant in width and independent of the RF voltage and current density.

A second assumption is  $\alpha_p = \alpha_n = \alpha$ . In GaAs the ionization rates are equal<sup>37</sup> and in Ge they are approximately equal<sup>38</sup> whereas in Si the

rates<sup>37</sup> differ by about a factor of ten. However, for the Read diode the avalanche region injects only one type of carrier into the drift region and therefore the contribution of the other carrier to the total current is small.

The electron and hole velocities are assumed to be equal and saturated. In Si, the velocities are approximately equal.<sup>40</sup> In Ge and GaAs, the amount of data in the literature on the saturated velocity of the carriers is quite limited. Paige<sup>41</sup> has measured the velocity of electrons and holes in Ge as a function of the electric field for different temperatures. His results give a saturation velocity for electrons but do not show any clear saturation of the hole velocity before impact ionization occurs. Results by Butcher and Fawcett<sup>42</sup> show a saturation velocity for electrons in GaAs. However, the only results for the hole velocity have been measured by an indirect method of inferring the velocity from the small-signal impedance of an avalanche diode which was done by Kim.<sup>43</sup> Thus the exact number to be used for the velocity and whether or not the electron and hole velocities can be assumed equal have not been resolved yet. However, since in the Read diode the current consists primarily of one type of carrier the main effect of the magnitude of the saturated drift velocity is on the frequency range of operation. The phase delay of the transit time of the carriers is proportional to the transit angle. The transit angle is

$$\omega\tau = (2\pi f) \cdot (x_D/v_s) , \quad (2.5)$$

where  $f$  = the operating frequency (c/s),

$x_D$  = the length of the drift region (cm) and

$v_s$  = the saturated drift velocity (cm/s).

Thus the phase delay is inversely proportional to the saturated velocity. Therefore in order to obtain a given transit angle the frequency will be a function of the magnitude of the saturated velocity, but the behavior of the admittance, power output and efficiency will not be directly affected by the velocity other than changing the frequency of operation.

The next assumption is that the total carrier current  $J_o(t)$  in the avalanche region is independent of  $x$ , i.e.,

$$J_o(t) = J_p(x,t) + J_n(x,t) . \quad (2.6)$$

This means that the displacement current in the avalanche region is also independent of  $x$  as can be seen from Maxwell's equation for  $\nabla \times \vec{E}$ :

$$\nabla \times \vec{H} = J_o + \epsilon \frac{\partial \vec{E}}{\partial t} \quad (2.7)$$

and

$$\nabla \cdot \nabla \times \vec{H} \equiv 0 . \quad (2.8)$$

Therefore for the one-dimensional case under consideration here,

$$\frac{\partial}{\partial x} \left( J_o + \epsilon \frac{\partial E}{\partial t} \right) = 0 . \quad (2.9)$$

If

$$\frac{\partial}{\partial x} J_o = 0 , \quad (2.10)$$

then

$$\frac{\partial}{\partial x} \left( \epsilon \frac{\partial E}{\partial t} \right) = 0 . \quad (2.11)$$

For the case where the electric field in the avalanche region is assumed to be only a function of time (constant with respect to  $x$ ) then the assumption is valid. However, a field which is independent of  $x$  in the avalanche region does not include the space-charge effects of the carriers in that region. If the avalanche region is narrow and the transit time of the carriers through this region is short compared to the period of oscillation then this will be a reasonable assumption.<sup>2,44</sup> Gummel and Blue<sup>45</sup> have shown that the assumption of a constant total carrier current in the avalanche region produces an error in the calculation of the small-signal cutoff or avalanche frequency. In a Read diode the small-signal cutoff frequency as derived by Gilden and Hines<sup>6</sup> is

$$f_a = \frac{1}{2\pi} \left( \frac{2\bar{\alpha}' v_s J_{o_{dc}}}{\epsilon} \right)^{1/2} , \quad (2.12)$$

where  $\bar{\alpha}' = d\bar{\alpha}/dE$  ,

$\bar{\alpha}$  = the average value of the ionization rate ( $\text{cm}^{-1}$ ) and

$J_{o_{dc}}$  = the dc carrier-current density in the avalanche region ( $\text{A/cm}^2$ ).

This frequency corresponds to the point where both the real and imaginary parts of the admittance of the Read diode go to zero. It was shown<sup>45</sup> that this frequency is in error by approximately 20 percent. However, in this analysis the avalanche frequency is taken as an independent parameter and therefore the results will not be affected greatly.

Another important assumption concerns the expansion of the ionization rate given in Eq. 2.3. As derived by McKay,<sup>46</sup> avalanche breakdown occurs when the integral of the ionization rate over the avalanche region equals unity. The field in the avalanche region is assumed to be independent of  $x$ . Thus the integral can be evaluated easily to give the critical field for breakdown. The ionization rate is expanded in a Taylor series about this critical field and only the terms through the first order are retained. By neglecting the higher-order terms, the effect of decreasing the average value of the critical field (and thus the dc voltage) with increasing RF voltage will not appear.

The remaining two assumptions are that diffusion effects and reverse saturation currents are negligible. Diffusion effects would not become important until large RF voltages existed across the diode causing the avalanche current to be generated in a sharp pulse. Diffusion would spread the pulse out as it drifted through the diode. The reverse saturation current determines the minimum current which can flow at any instant. Thus this would affect the amount of avalanche current modulation caused by the RF voltage. For reasonable values of reverse saturation current this effect is negligible.<sup>47</sup>

Making the above assumptions, the following equations relating the electric field in the avalanche region  $E(t)$  and the total carrier-current density  $J_o(t)$  can be derived:<sup>39</sup>

$$\frac{\tau_a}{2} \frac{d}{dt} J_o(t) = \frac{m J_o(t) E(t)}{E_c} , \quad (2.13)$$

where  $m \triangleq E_c \alpha'(E_c)/\alpha(E_c)$ ,

$E_c$  = the critical field for avalanche breakdown (V/cm) and

$\tau_a$  = the transit time through the avalanche region;

$$E(t) = E_b - \frac{V(t)}{x_D} - \frac{1}{\epsilon\tau} \int_{t-\tau}^t (\tau - t + \gamma) J_o(\gamma) d\gamma, \quad (2.14)$$

where  $E_b \triangleq E_{po} - E_c + (V_p - V_d)/x_D$ ,

$E_{po}$  = the field at punch-through at  $x = 0$  (V/cm),

$V_d$  = the dc bias voltage (V),

$V_p$  = the dc voltage drop across the avalanche region at punch-through (V) and

$V(t)$  = the RF voltage across the diode (V).

The external current  $J_e$  induced by the carriers drifting through the diode is

$$J_e(t) = \frac{1}{\tau} \int_{t-\tau}^t J_o(\gamma) d\gamma. \quad (2.15)$$

### 2.2.2 Outline of the Method of Solution of the Equations. By

substituting Eq. 2.14 into Eq. 2.13 the following differential equation is obtained:

$$\frac{dJ_o(t)}{dt} = J_o(t) \left( B + C \sin(\omega t) - D \int_{t-\tau}^t (\tau - t + \gamma) J_o(\gamma) d\gamma \right), \quad (2.16)$$

where

$$B = \frac{2mE_b}{\tau_a E_c},$$

$$C = \frac{2mV_{RF}}{\tau_a E_c x_D}$$

and

$$D = \frac{2m}{\epsilon \tau_a \tau E_c} .$$

Also single-frequency operation has been assumed so that

$$V(t) = -V_{RF} \sin(\omega t) . \quad (2.17)$$

In order for the solution to Eq. 2.16 to be periodic the following condition must be satisfied:

$$E_b = \frac{J_{dc} \tau}{2\epsilon} . \quad (2.18)$$

A proof of this constraint is given in Appendix A.

Equation 2.16 was solved by using the IBM 7090 digital computer. The input parameters to the computer program are listed in Table 2.1.

Table 2.1

Input Parameters for Read-Diode Analysis

|            |   |                                                          |
|------------|---|----------------------------------------------------------|
| $f_a$      | = | the avalanche frequency (Hz)                             |
| $I_{dc}$   | = | the dc current (A)                                       |
| $A$        | = | the cross-sectional area of the diode (cm <sup>2</sup> ) |
| $V_{RF}$   | = | the magnitude of the RF voltage (V)                      |
| $\theta_a$ | = | the transit angle at the avalanche frequency (rad)       |
| $v_s$      | = | the saturated drift velocity (cm/s)                      |
| $\epsilon$ | = | the dielectric constant (F/cm)                           |

The constants in Eq. 2.16 are obtained from the input parameters as follows:

$$\omega_a = 2\pi f_a , \quad (2.19)$$

$$\tau = \theta_a / (2\pi f_a) , \quad (2.20)$$

$$B = \omega_a^2 \tau / 2 , \quad (2.21)$$

$$C = \omega_a^2 V_{RF} \epsilon / (I_{dc} \tau v_s) \quad (2.22)$$

and

$$D = \omega_a^2 / (I_{dc} \tau) . \quad (2.23)$$

The solution of Eq. 2.16 is obtained by an iteration technique. An initial conjecture for the time dependence of  $J_o(t)$  for one period is obtained from either the closed-form solution<sup>39</sup> for short-transit angles or by Read's intermediate amplitude solution.<sup>2</sup> The better the initial conjecture for  $J_o(t)$  the fewer iterations are needed to converge on the correct  $J_o(t)$ . Using this initial conjecture for  $J_o(t)$  the electric field in the avalanche region can be calculated from Eq. 2.14. This gives the coefficient of  $J_o(t)$  on the right-hand side of Eq. 2.16. If it is assumed that this coefficient is only a function of time and is independent of  $J_o(t)$  then the solution to Eq. 2.16 is

$$J_o(t) = J_o(0) \exp \left( \int_0^t F(\gamma) d\gamma \right) , \quad (2.24)$$

where

$$F(\gamma) = B + C \sin(\omega\gamma) - D \int_{\gamma-\tau}^{\gamma} (\tau - \gamma + \beta) J_o(\beta) d\beta .$$

The initial value of the current  $J_o(0)$  is determined by the dc current in the following manner:

$$J_{dc} = \frac{1}{T} \int_0^T J_o(t) dt , \quad (2.25)$$

where  $T$  = the period of oscillation (s). Therefore substituting Eq. 2.24 into Eq. 2.25 and evaluating yields

$$J_o(0) = \frac{J_{dc}}{\left[ \frac{1}{T} \int_0^T \exp \left( \int_0^t F(\gamma) d\gamma \right) dt \right]} . \quad (2.26)$$

By Fourier analysis the fundamental component of  $J_o(t)$  is obtained and can be represented by

$$J_{o_1}(t) = R_1 \sin(\omega t + \theta_1) , \quad (2.27)$$

where  $R_1$  = the magnitude of the fundamental component (A/cm<sup>2</sup>) and

$\theta_1$  = the phase angle of the fundamental component (rad).

The fundamental component of the external current is found by substituting Eq. 2.27 into Eq. 2.15 to give

$$J_{e_1}(t) = \frac{1}{\tau} \int_{t-\tau}^t R_1 \sin(\omega \gamma + \theta_1) d\gamma . \quad (2.28)$$

The admittance of the diode, which is found by the phasor division of the fundamental component of the external current by the RF voltage, is

$$Y = G + jB , \quad (2.29)$$

where

$$G = \frac{AR}{\omega\tau V_{RF}} \left( [1 - \cos(\omega\tau)]\sin(\theta_1) + \cos(\theta_1)\sin(\omega\tau) \right)$$

and

$$B = \frac{AR}{\omega\tau V_{RF}} \left( [\cos(\omega\tau) - 1]\cos(\theta_1) + \sin(\theta_1)\sin(\omega\tau) \right) .$$

The conductance and susceptance as calculated by Eq. 2.29 are compared with the values obtained in the previous solution for  $J_o(t)$ . If the change is greater than several percent then the value for  $J_o(t)$  as found in Eq. 2.24 is used as the initial conjecture to solve for a new electric field. All of the steps are then repeated to find a new admittance. These iterations are repeated until the change in the admittance is less than 1 percent at which point the solution to Eq. 2.16 is obtained. This condition for convergence will be discussed later.

### 2.3 Results of the Large-Signal Analysis

The above analysis was used to study the large-signal operating characteristics of Read diodes. Three diodes with different transit angles at the avalanche frequency were studied. The avalanche frequency was assumed to be 10 GHz at 100 mA dc bias current and the three transit angles were  $0.25\pi$ ,  $0.50\pi$  and  $0.75\pi$ .

In Figs. 2.1 and 2.2 the current generated in the avalanche region,  $J_o(t)$ , is plotted as a function of the phase of the RF voltage. In Fig. 2.1 the dc current equals 100 mA and is held constant. The RF voltage is increased from 15 V to 150 V. Since the dc current is constant the average amount of space charge in the diode is constant, therefore, as the RF voltage increases the RF field in the avalanche region becomes

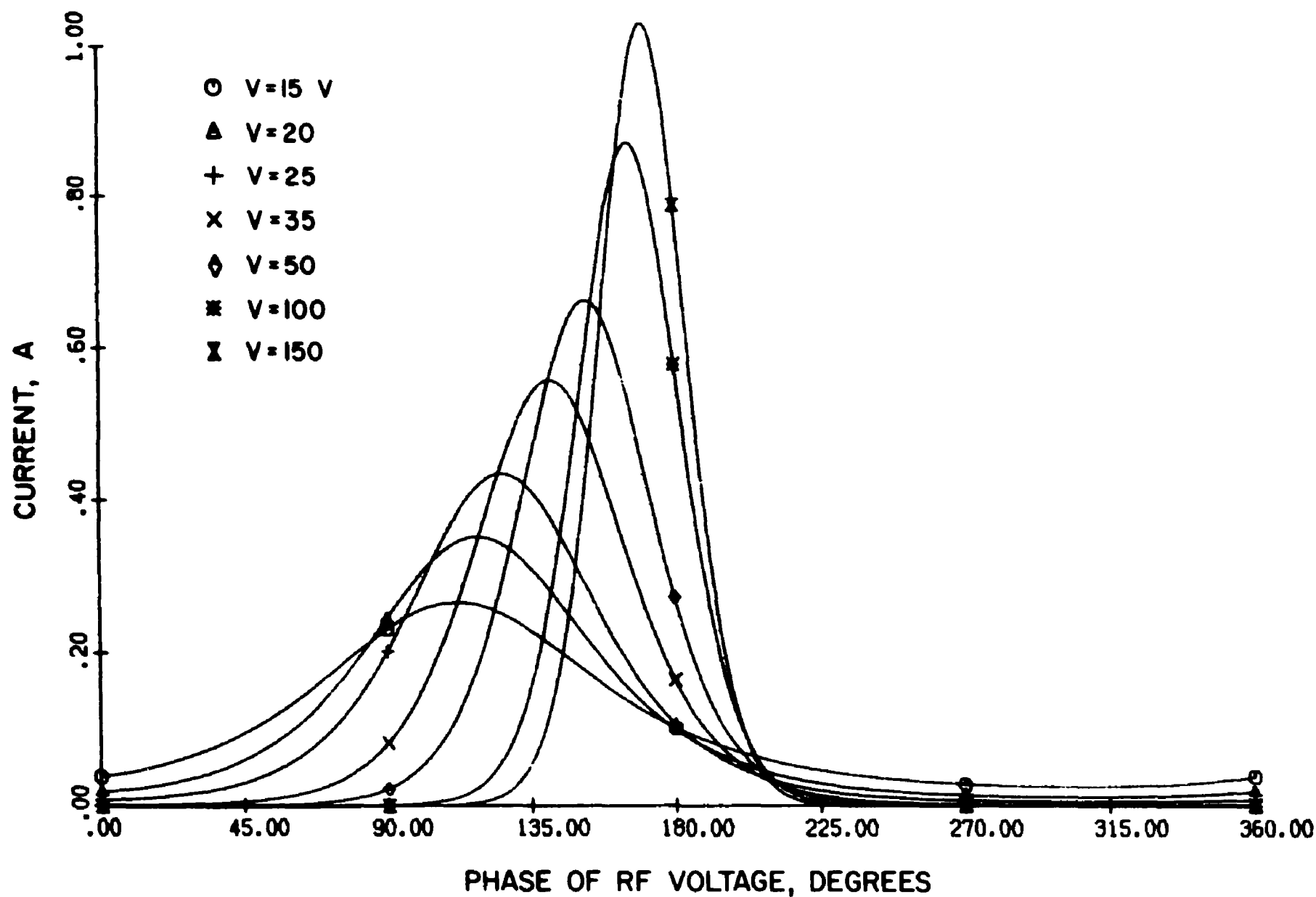
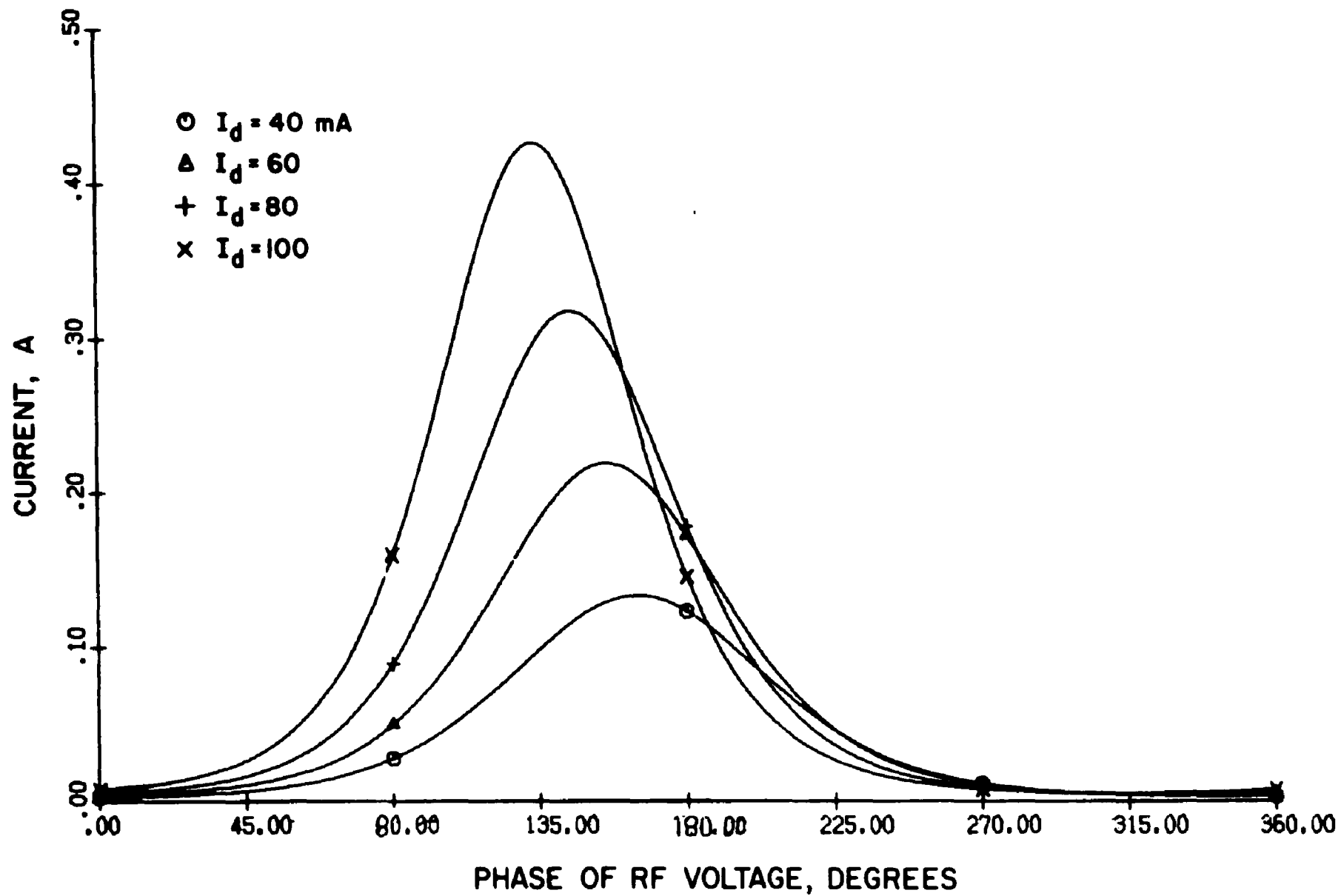


FIG. 2.1 AVALANCHE CURRENT VS. PHASE OF RF VOLTAGE. ( $f_a = 10$  GHz AT 100 mA,  $f = 10$  GHz,  $I_{dc} = 100$  mA AND  $\omega_a \tau = 0.75\pi$ )



-04-

FIG. 2.2 AVALANCHE CURRENT VS. PHASE OF RF VOLTAGE. ( $f_a = 10$  GHz AT 100 mA,  $f = 10$  GHz,  $V_{RF} = 25$  V  
AND  $\omega_a \tau = 0.75\pi$ )

negative at a later time in the cycle. This can be seen from Eqs. 2.14 and 2.17. Thus the avalanche current reaches its maximum value at a later time in the RF cycle causing an increase in the amount of phase lag of the current with respect to the RF voltage. In Fig. 2.2 the RF voltage has a constant magnitude of 25 V and the dc current is increased from 40 to 100 mA. As the dc current is increased, the average amount of space charge increases and the RF field becomes negative earlier in the cycle. Thus the avalanche current maximizes earlier in the cycle as the dc current is increased. This decreases the phase lag of the current with respect to the RF voltage. It is therefore seen that the dc current and RF voltage have opposing effects on the phase delay of the current in the avalanche region.

In Figs. 2.3 through 2.5 the conductance vs. RF voltage is shown for different operating frequencies. In Fig. 2.3 for the  $0.25\pi$  transit-angle diode, the negative conductance increases as the frequency increases. The increase in operating frequency effectively increases the transit angle of the diode. If for  $f = 10$  GHz the transit angle  $\omega_a \tau = 0.25\pi$ , then for  $f = 20$  GHz the transit angle  $\omega_{20} \tau = 0.5\pi$ . In Figs. 2.4 and 2.5 the negative conductance increases for increasing frequency until a maximum negative conductance is obtained and then any increase in frequency causes a decrease in the negative conductance. In Fig. 2.4 the maximum negative conductance occurs for a frequency of 16 GHz which corresponds to a transit angle of  $0.8\pi$ . For the  $0.75\pi$  diode the maximum negative conductance occurs for a frequency of 12 GHz which corresponds to a transit angle of  $0.9\pi$ . If for this last case the operating frequency were 11 GHz, the negative

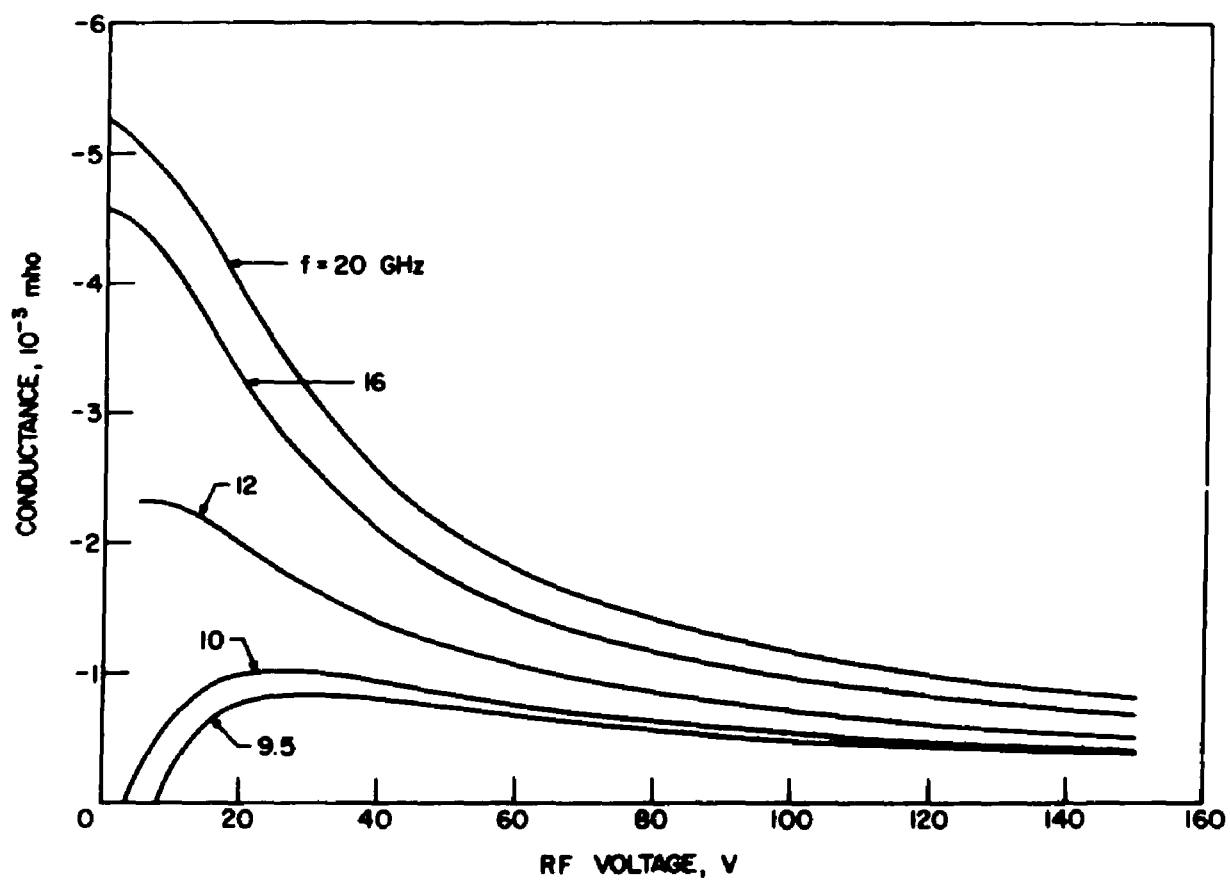


FIG. 2.3 CONDUCTANCE VS. RF VOLTAGE. ( $f_a = 10$  GHz AT 100 mA,  $I_{dc} = 100$  mA AND  $\omega_a \tau = 0.25\pi$ )

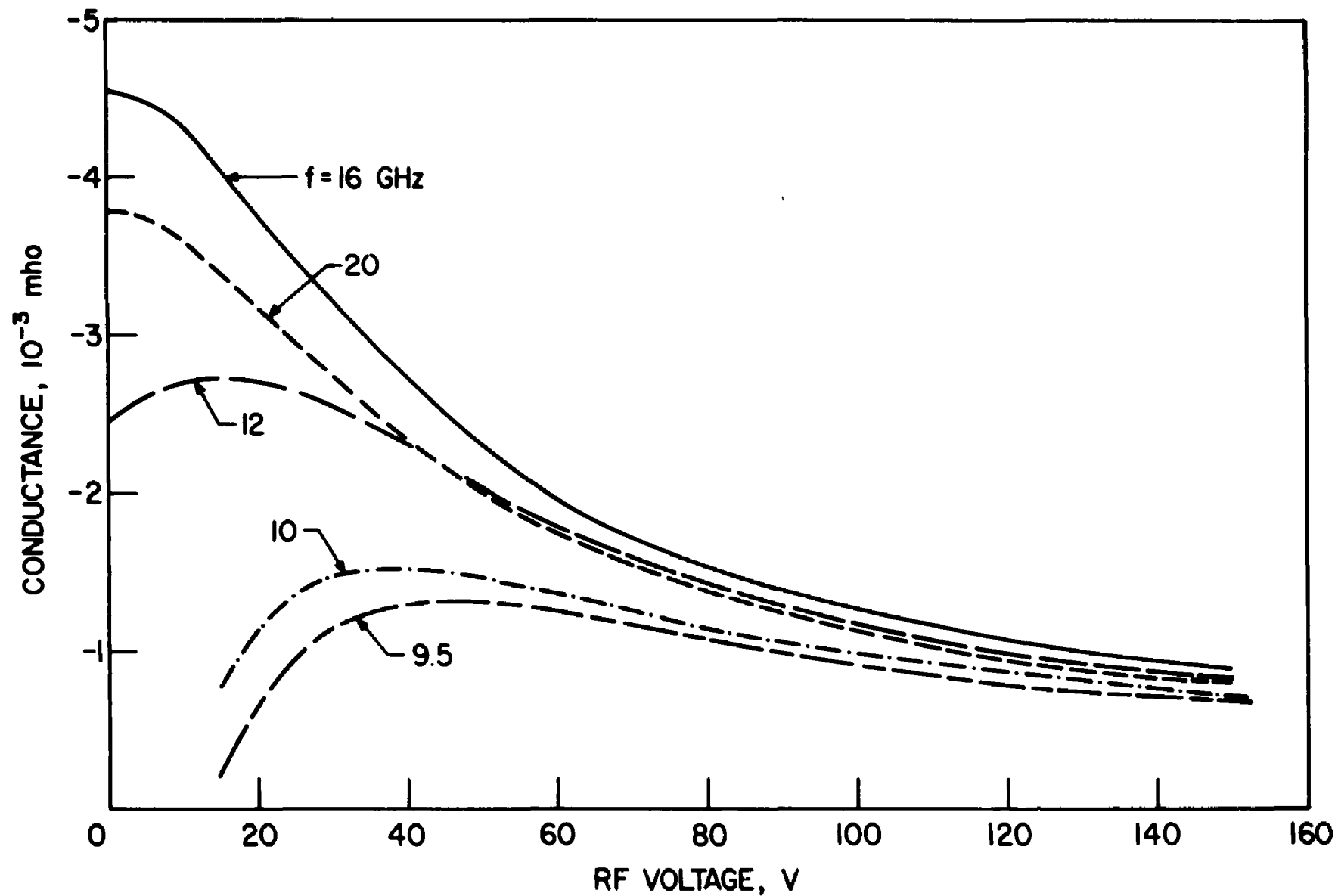


FIG. 2.4 CONDUCTANCE VS. RF VOLTAGE. ( $f_a = 10$  GHz AT 100 mA,  $I_{dc} = 100$  mA AND  $\omega_a \tau = 0.5\pi$ )

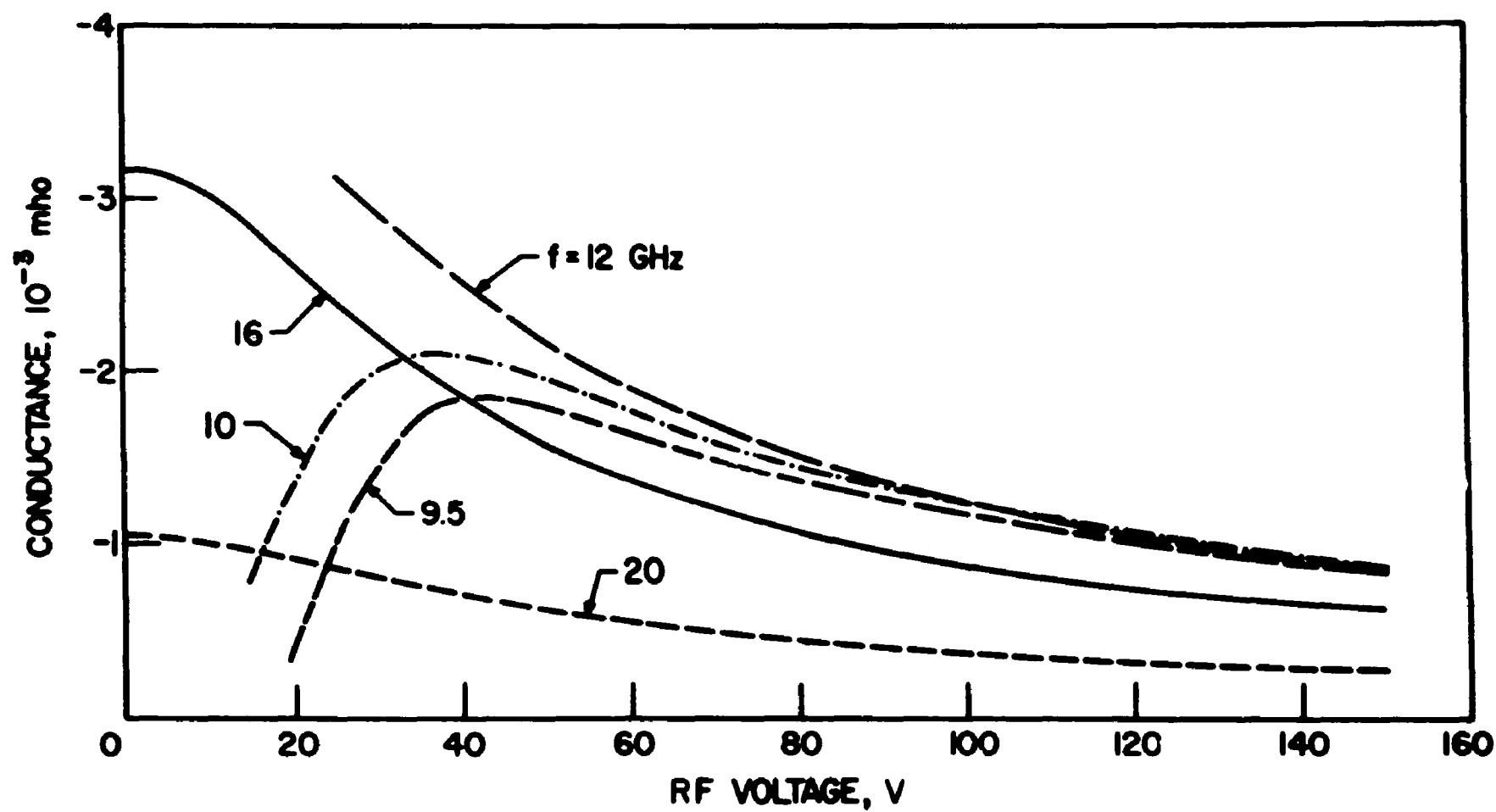


FIG. 2.5 CONDUCTANCE VS. RF VOLTAGE. ( $f_a = 10$  GHz AT 100 mA,  $I_{dc} = 100$  mA AND  $\omega_a \tau = 0.75\pi$ )

conductance would be larger and this would give a transit angle of  $0.825\pi$ . Gummel and Scharfetter<sup>8,11</sup> have shown that for a Read diode, the maximum small-signal conductance occurs for an operating frequency which corresponds to an effective transit angle of  $0.833\pi$ . In Appendix B a derivation of the maximum negative conductance for the large-signal waveform in Read diodes shows the optimum transit angle to be between approximately  $0.7\pi$  and  $1.0\pi$  depending on the RF voltage, dc current and frequency. Note that in the small-signal case (small RF voltages) the analysis shows a positive conductance for frequencies below the avalanche frequency and as the RF voltage is increased the conductance becomes negative.

In Figs. 2.6 through 2.8 the conductance vs. RF voltage for different dc bias currents is given. The diode is operated at a frequency of 10 GHz so that as the dc bias current is increased from 40 mA to 120 mA the avalanche frequency increases from 6.4 GHz to 11 GHz. Again, for the small-signal case the conductance is positive for an operating frequency below the avalanche frequency and becomes negative for larger RF voltages.

Figures 2.9 through 2.11 show the effect of frequency on power output for different loads,  $G_L$ . For the  $0.25\pi$  case the power increases for increasing frequency in the same manner as the negative conductance. The  $0.5\pi$  and  $0.75\pi$  cases show an optimum frequency of operation for maximum output power which corresponds to the same frequency for maximum negative conductance.

The susceptance as calculated by this analysis does not include the depletion layer capacitance of the diode. This capacitance,  $C_d$ , is independent of the operating conditions and is only a function of the

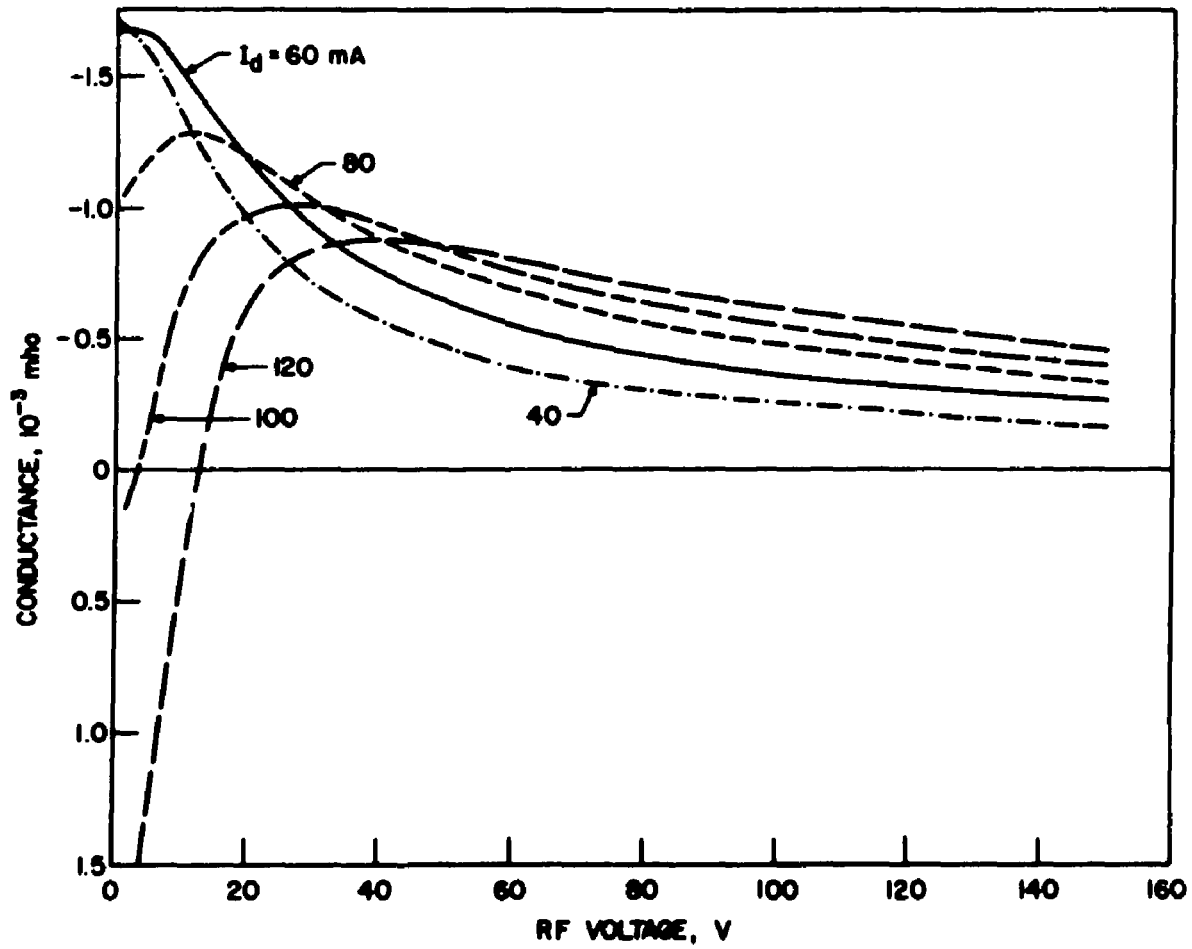


FIG. 2.6 CONDUCTANCE VS. RF VOLTAGE. ( $f_a = 10$  GHz AT 100 mA,  $f = 10$  GHz  
AND  $\omega_a \tau = 0.25\pi$ )

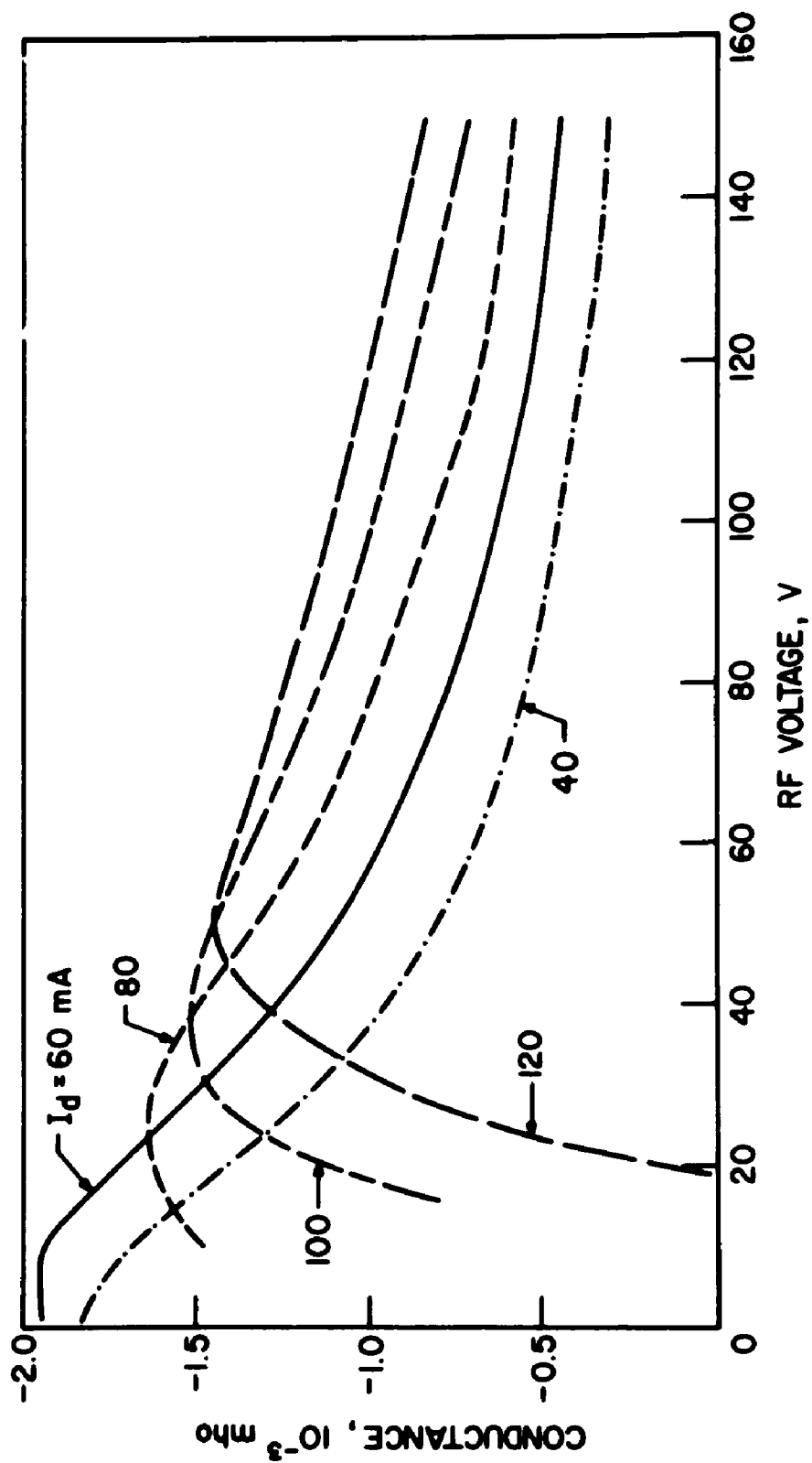


FIG. 2.7 CONDUCTANCE VS. RF VOLTAGE. ( $f_a = 10$  GHz AT 100 mA,  $f = 10$  GHz AND  $\omega \tau = 0.5\pi$ )

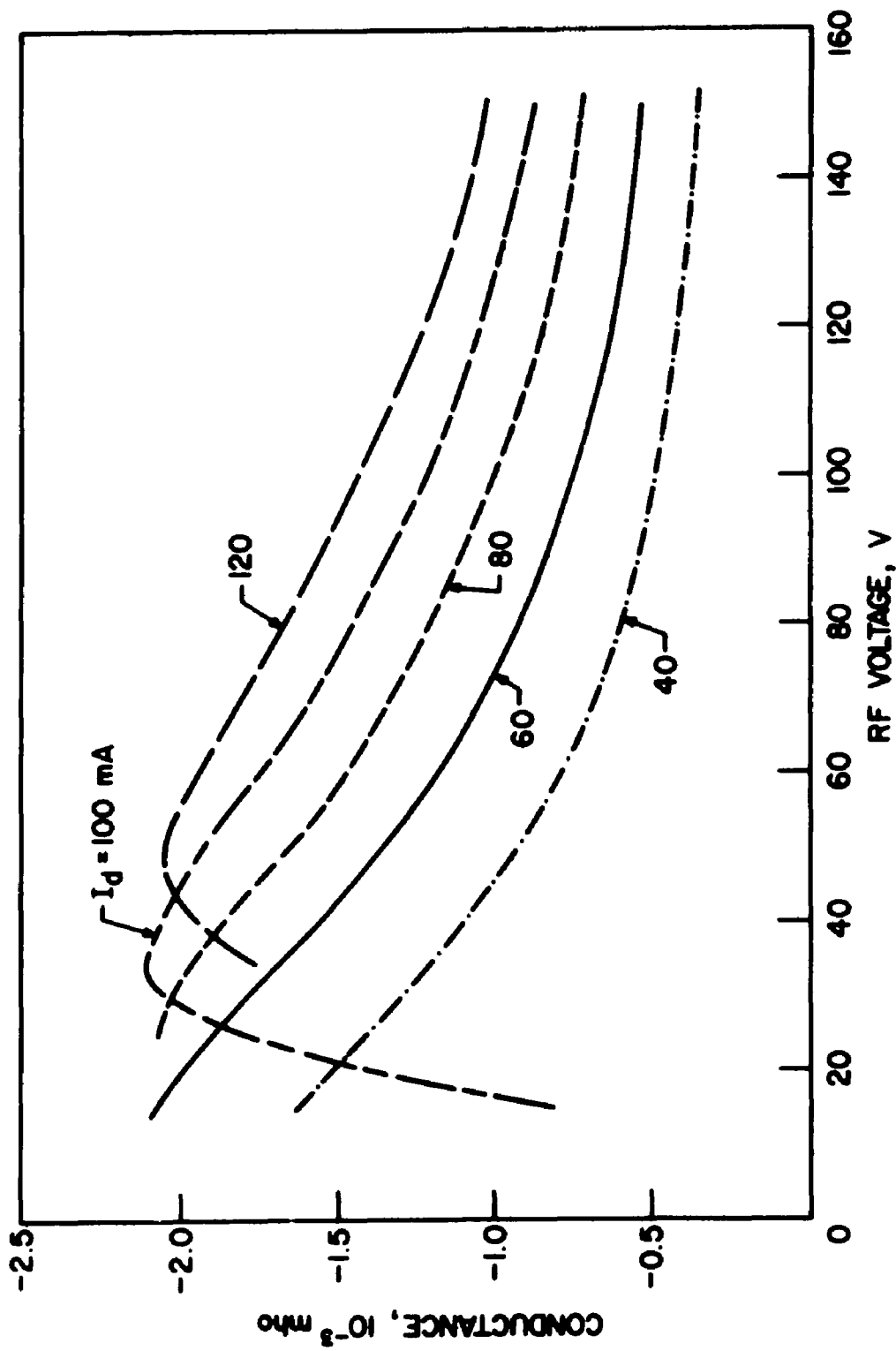


FIG. 2.8 CONDUCTANCE VS. RF VOLTAGE. ( $f_a = 10 \text{ GHz}$  AT  $100 \text{ mA}$ ,  $f = 10 \text{ GHz}$  AND  $\omega_a \tau = 0.75\pi$ )

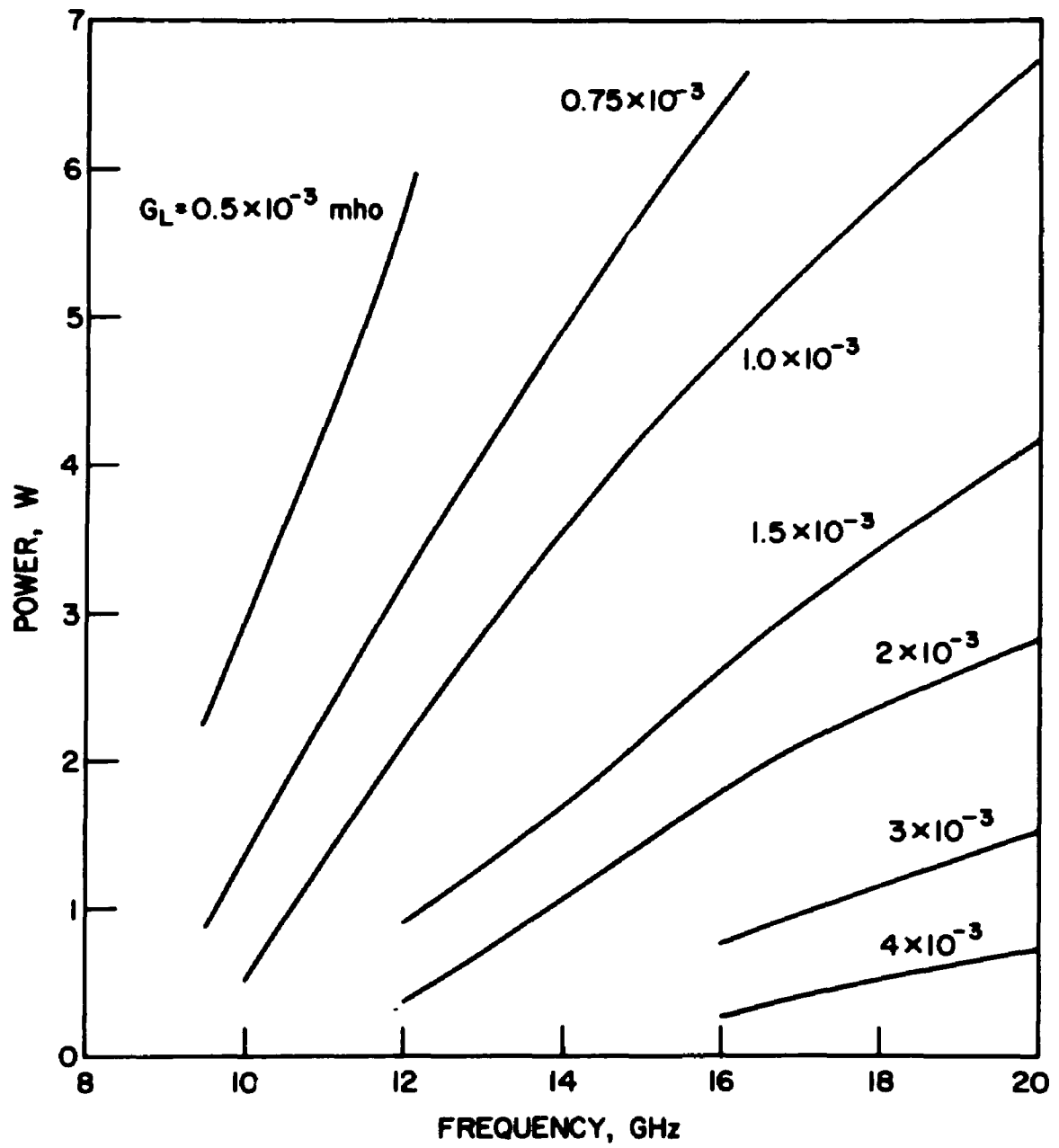


FIG. 2.9 POWER VS. FREQUENCY. ( $f_a = 10$  GHz AT 100 mA,  $I_{dc} = 100$  mA AND  $\omega_a \tau = 0.25\pi$ )

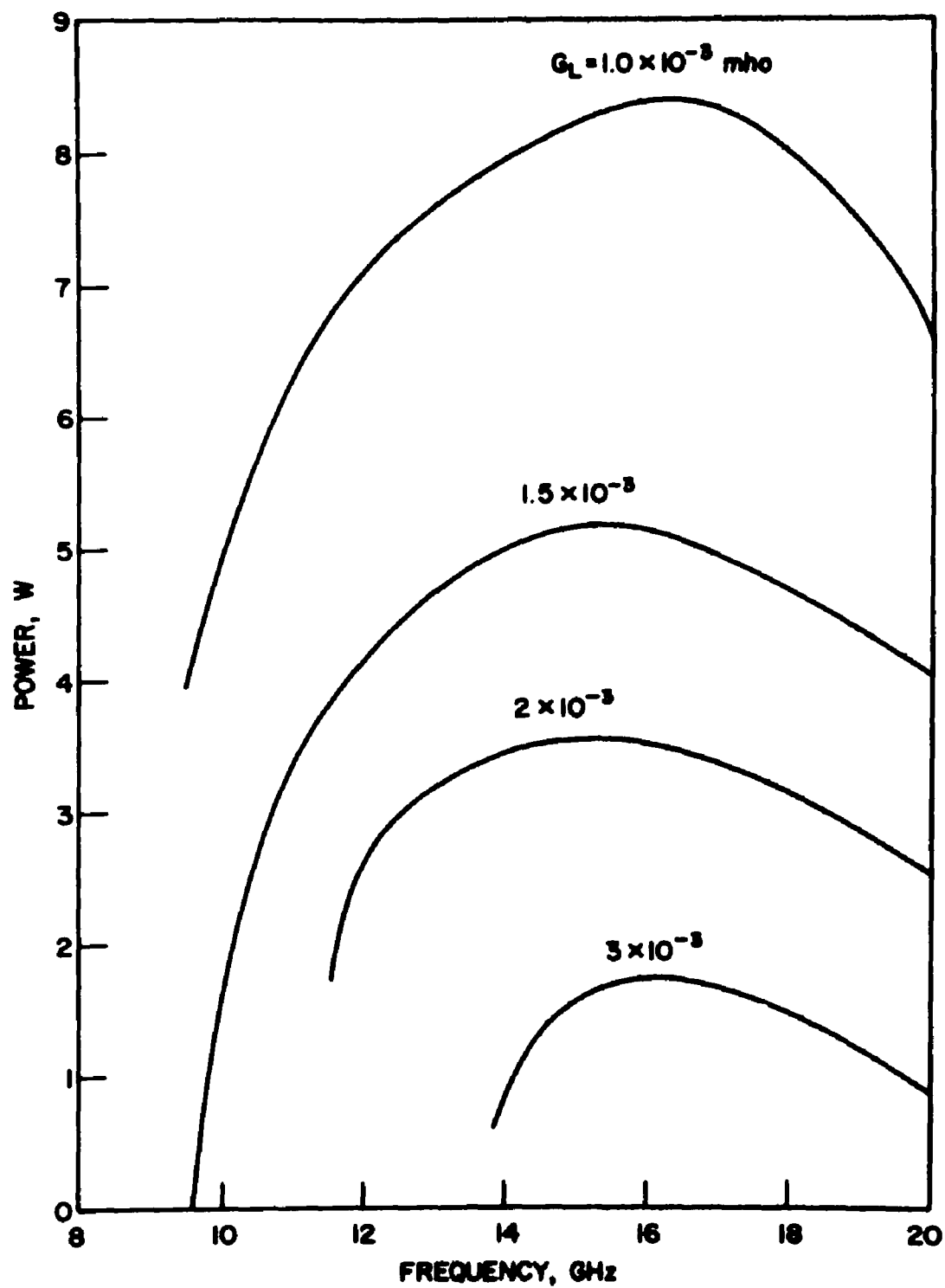


FIG. 2.10 POWER VS. FREQUENCY. ( $f_a = 10 \text{ GHz}$  AT  $100 \text{ mA}$ ,  $I_{dc} = 100 \text{ mA}$  AND  $\omega_a \tau = 0.5\pi$ )

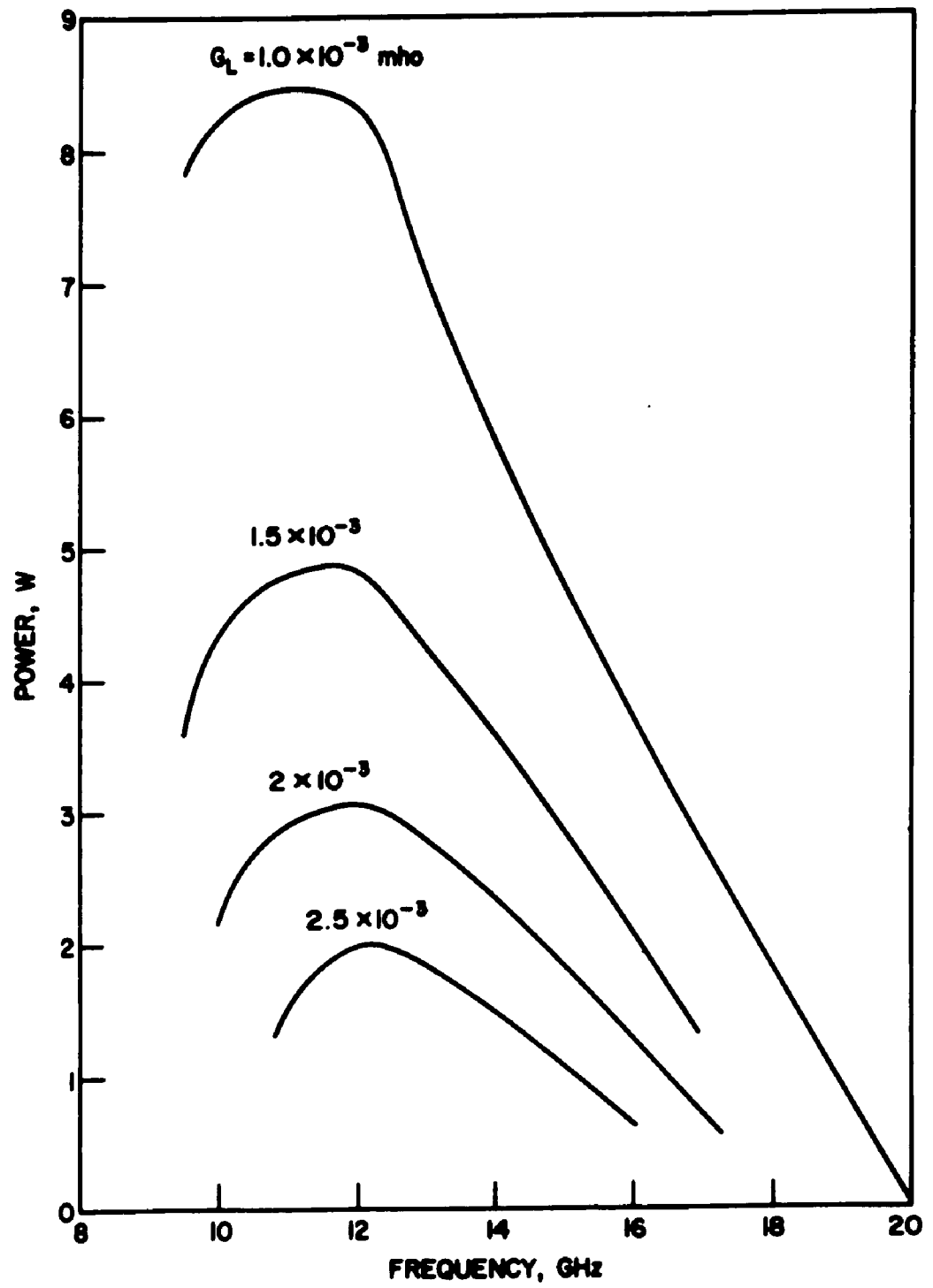


FIG. 2.11 POWER VS. FREQUENCY. ( $f_a = 10$  GHz AT 100 mA,  $I_{dc} = 100$  mA AND  $\omega_a \tau = 0.75\pi$ )

area and width of the diode. Thus for a given area and width (proportional to the transit time) the total susceptance of the diode would be the susceptance presented here plus  $\omega C_d$ . Figures 2.12 through 2.14 show the calculated conductance vs. susceptance for different operating frequencies as a function of the RF voltage. As the effective transit angle is increased, either by increasing the frequency or by going from the  $0.25\pi$  to  $0.75\pi$  transit-angle diode, the negative susceptance decreases. This can be explained by considering Fig. 2.15. The fundamental component of the current generated in the avalanche region  $J_{o1}$  lags the RF voltage by approximately 90 degrees. The phase delay  $\theta$  caused by the transit time is proportional to the frequency. The fundamental component of the external current  $J_{e1}$  lags the RF voltage by the sum of these two phase delays. The conductance and susceptance of the diode are proportional to the projection of the external current on the imaginary and real axes, respectively. Thus as the frequency increases,  $\theta$  increases until B goes to zero. For the  $0.75\pi$  diode with an operating frequency of 20 GHz, the susceptance becomes positive since the total phase lag is greater than 180 degrees.

A comparison of the admittance calculated by this analysis and that obtained from the short-transit angle, closed-form solution<sup>9</sup> are shown in Figs. 2.16 and 2.17. For frequencies near the avalanche frequency where the transit angle is small, the two analyses are in good agreement. However, as the frequency increases (larger transit angles) the analyses show some differences as would be expected due to the short-transit angle assumption.<sup>9</sup>

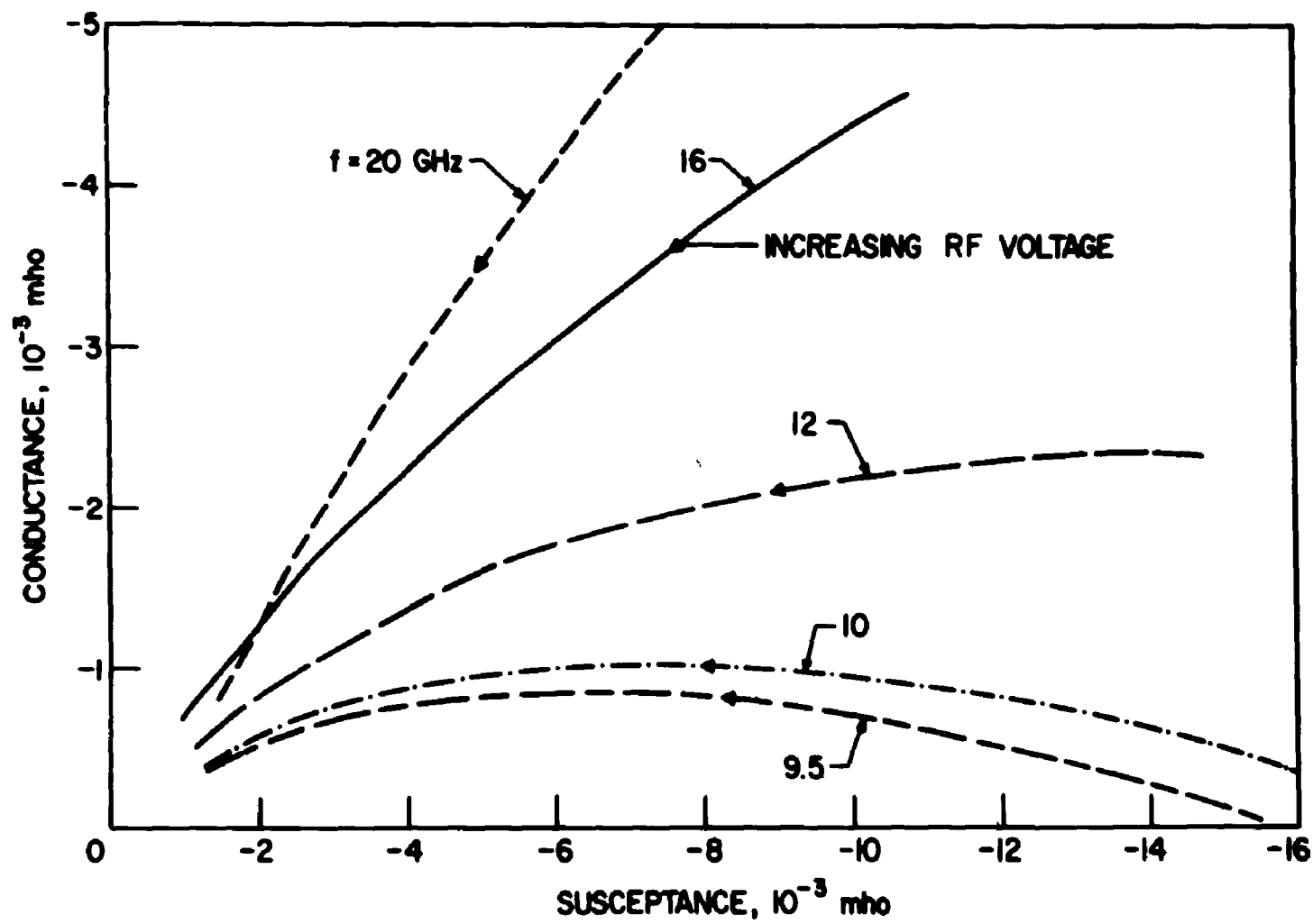


FIG. 2.12 CONDUCTANCE VS. SUSCEPTANCE. ( $f_a = 10$  GHz AT 100 mA,  $I_{dc} = 100$  mA AND  $\omega_a \tau = 0.25\pi$ )

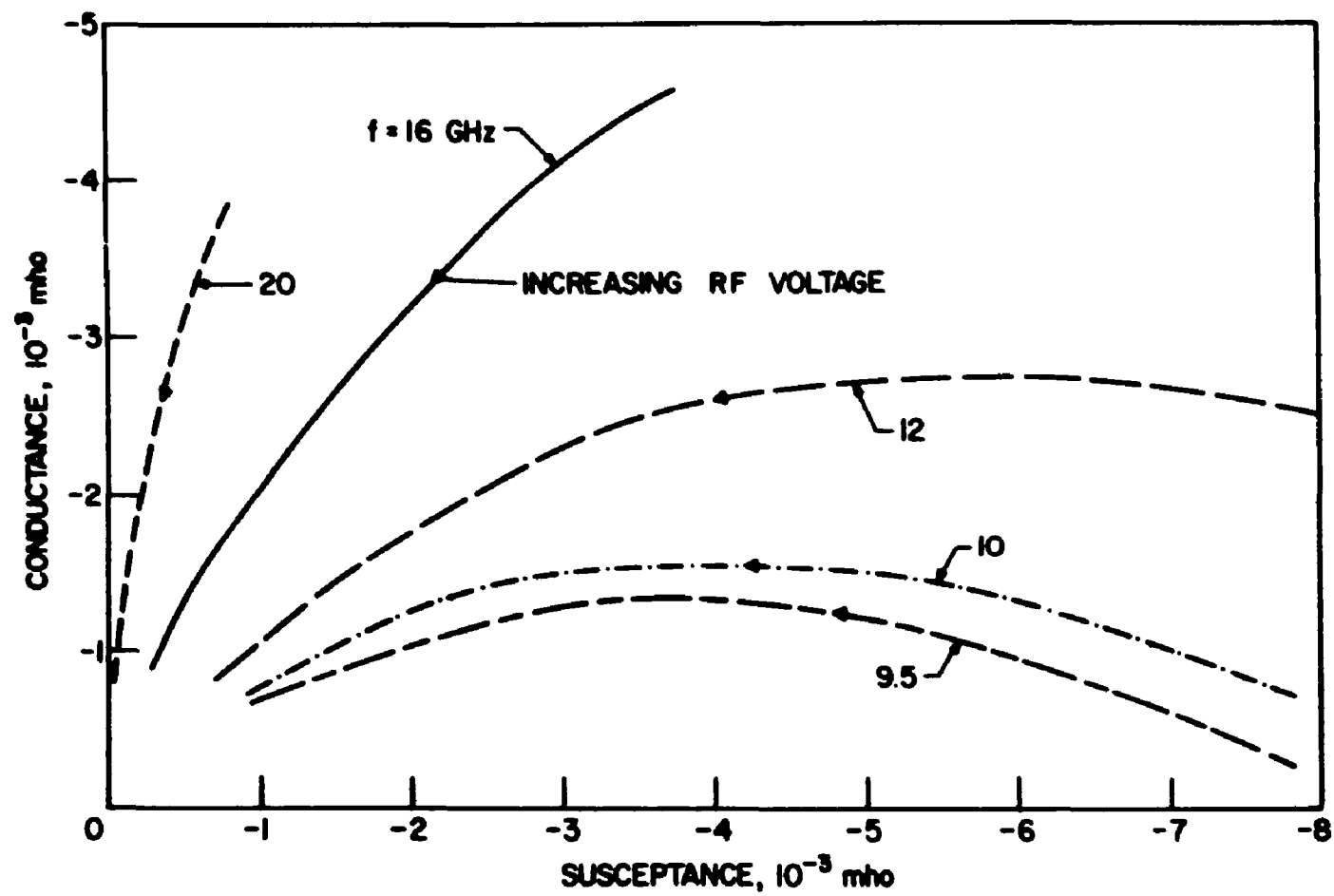


FIG. 2.13 CONDUCTANCE VS. SUSCEPTANCE. ( $f_a = 10 \text{ GHz}$  AT  $100 \text{ mA}$ ,  $I_{dc} = 100 \text{ mA}$  AND  $\omega_a \tau = 0.5\pi$ )

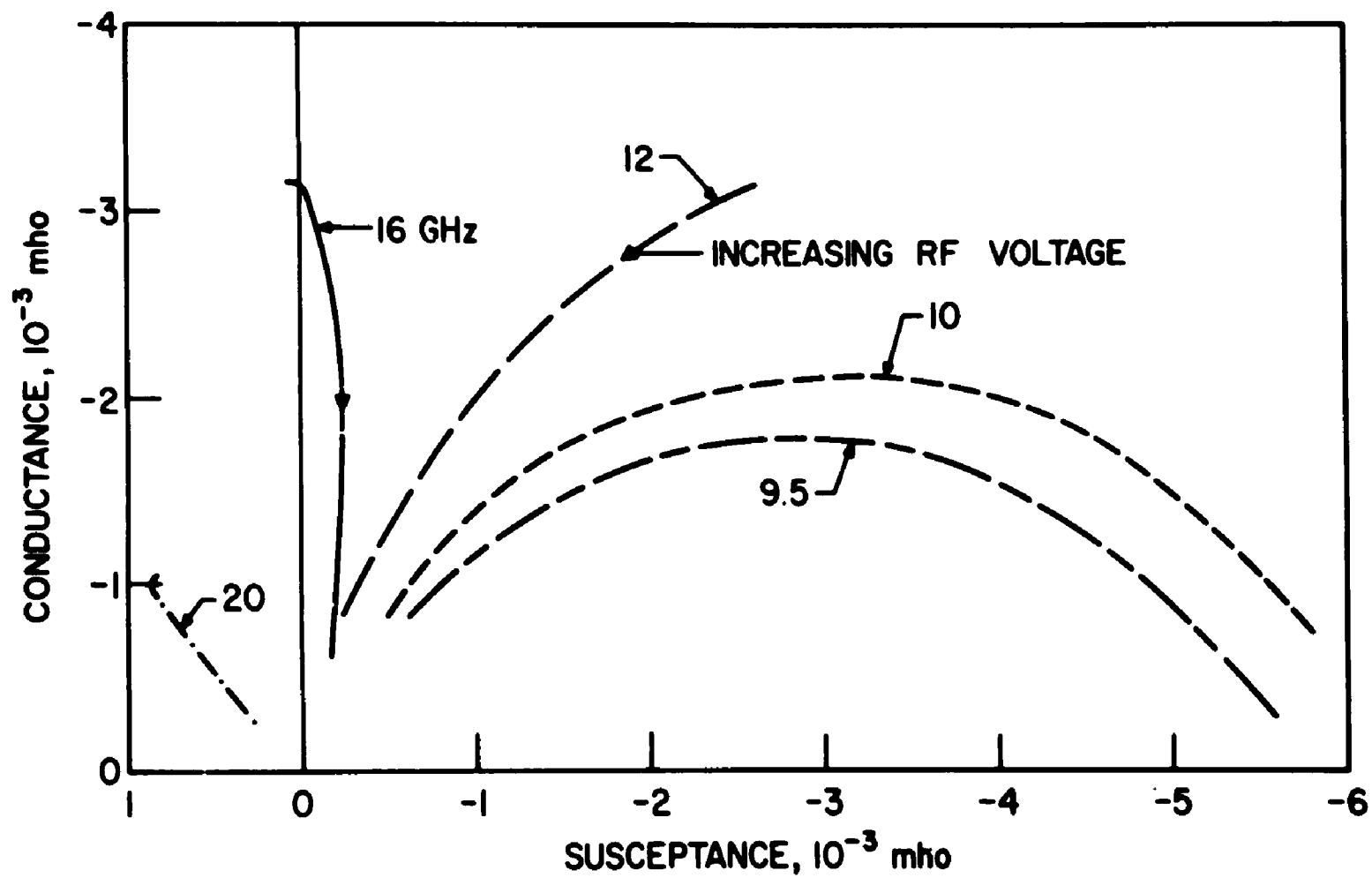


FIG. 2.14 CONDUCTANCE VS. SUSCEPTANCE. ( $f_a = 10$  GHz AT 100 mA,  $I_{dc} = 100$  mA AND  $\omega_a \tau = 0.75\pi$ )

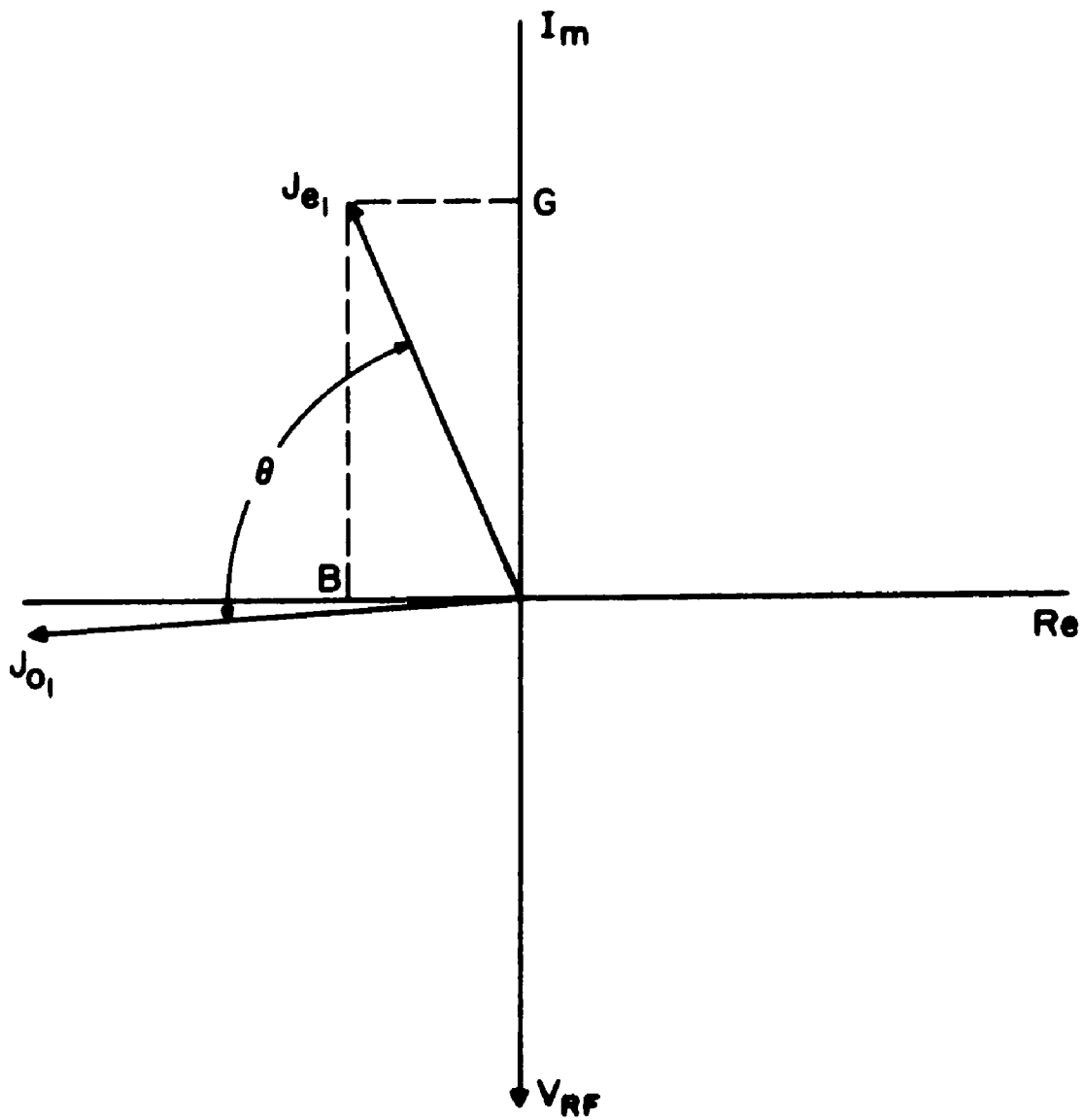


FIG. 2.15 PHASOR DIAGRAM FOR A READ DIODE.

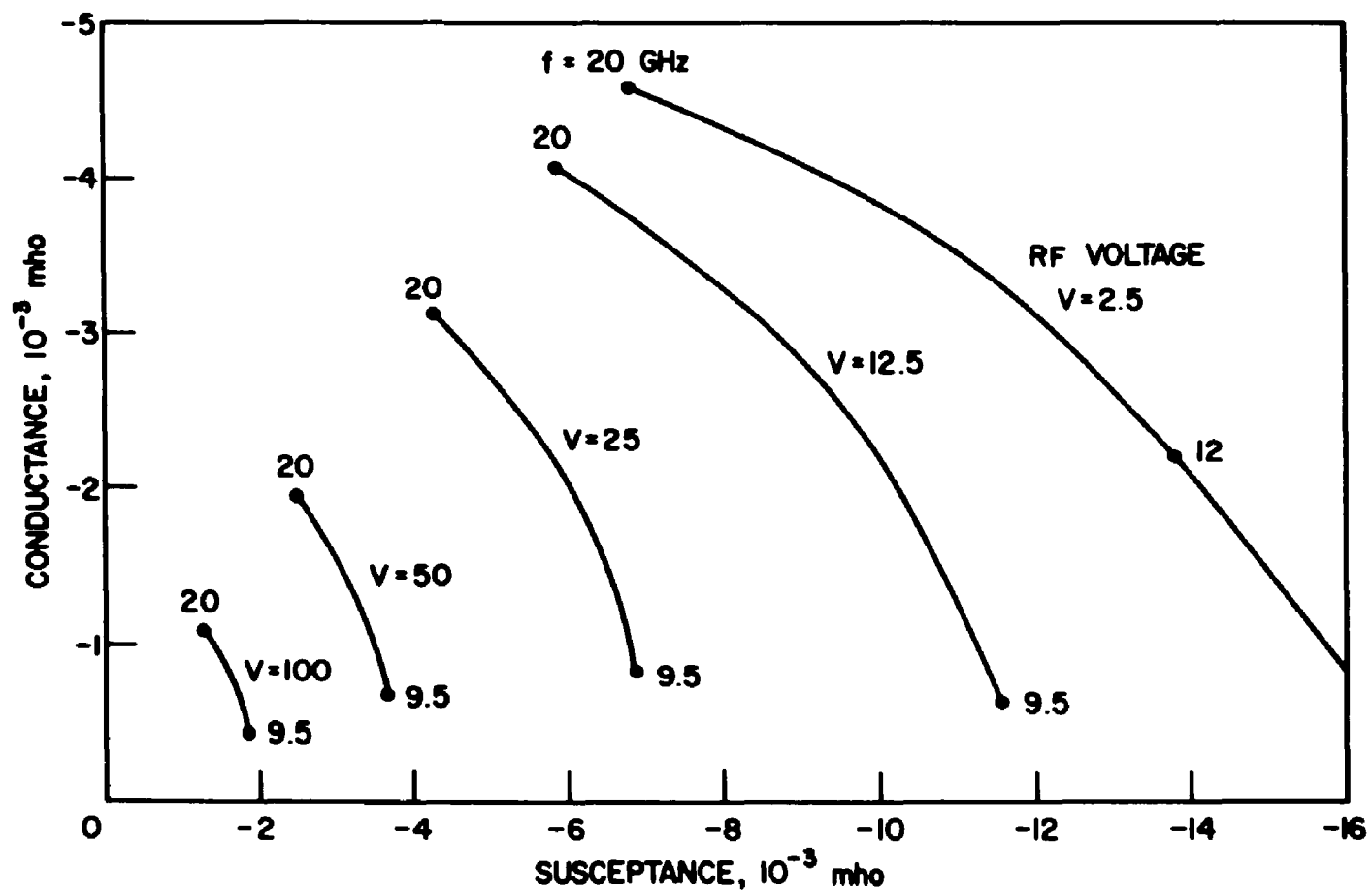


FIG. 2.16 CONDUCTANCE VS. SUSCEPTANCE. ( $f_a = 10$  GHz AT 100 mA,  $I_{dc} = 100$  mA AND  $\omega_a \tau = 0.25\pi$ )  
(EVANS<sup>39</sup>)

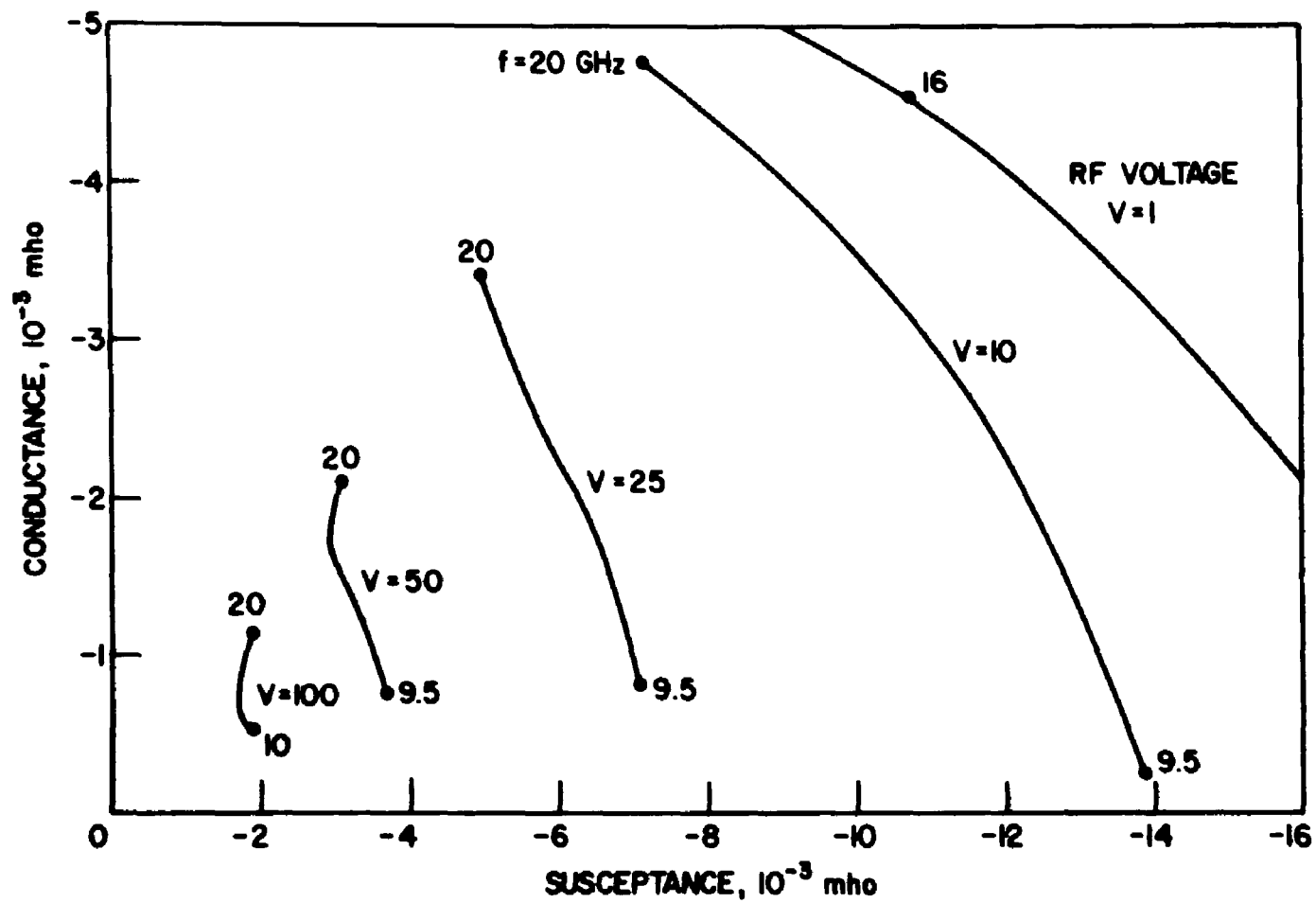


FIG. 2.17 CONDUCTANCE VS. SUSCEPTANCE ( $f_a = 10$  GHz AT 100 mA,  $I_{dc} = 100$  mA AND  $\omega_a \tau = 0.25\pi$ )

In Fig. 2.18 the efficiency as a function of frequency is plotted for the three transit-angle diodes. The results are for a constant RF voltage of 50 V and a breakdown voltage of 80 V. Thus the input power is 8 W.

An important limitation of this analysis which must be noted is that there is no constraint on the magnitude of the RF voltage. The analysis does not calculate the breakdown voltage of the diode and obviously the RF voltage must be less than this voltage or the diode would be forward biased during a portion of the RF cycle. Even more important is the fact that if during the negative portion of the RF cycle (when carriers are drifting through the diode) the RF voltage is too large, then the field in the drift region may drop below the value necessary to satisfy the assumption of saturated drift velocities of the carriers. As will be shown in the next section, the RF voltage will be constrained to values somewhat less than those used in this section.

## 2.4 Maximum Allowable RF Voltage

2.4.1 Limitation Due to Breakdown Voltage. The breakdown voltage of Read diodes is a function of the dimension and assumed doping profile. The length of the drift region in the three transit-angle diodes considered in the previous section can be obtained from

$$\omega_a \tau = \theta_a \quad (2.30)$$

and

$$\tau = x_D / v_s \quad (2.31)$$

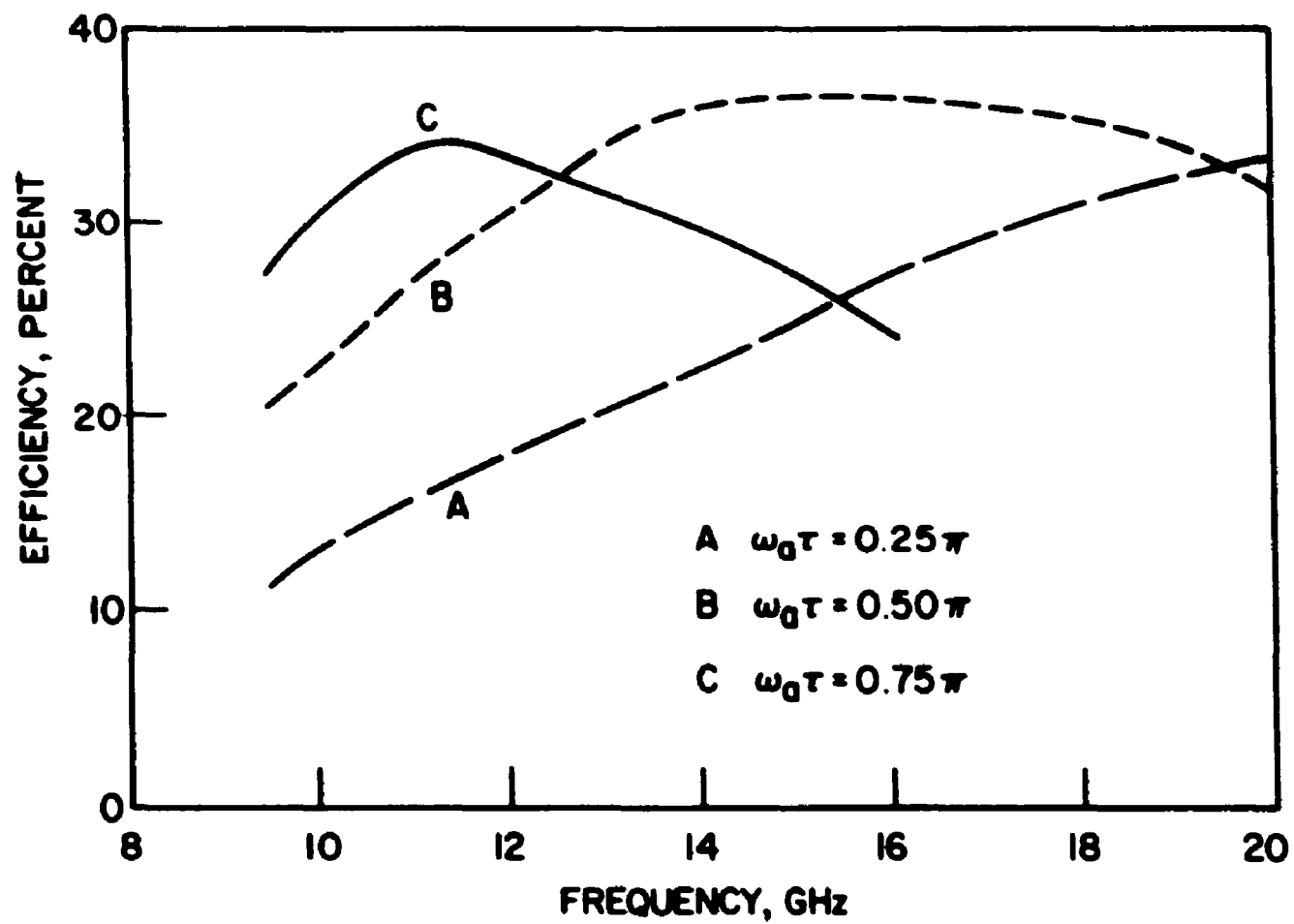


FIG. 2.18 EFFICIENCY VS. FREQUENCY. ( $f_a = 10$  GHz AT 100 mA,  $I_{dc} = 100$  mA, RF VOLTAGE = 50 V, INPUT POWER = 8 W AND  $V_B = 80$  V)

Therefore the width of the drift region is

$$x_D = \frac{\theta_a v_s}{\omega_a} . \quad (2.32)$$

For the diodes considered previously the avalanche frequency was 10 GHz and the saturated drift velocity was assumed to be  $1 \times 10^7$  cm/s; the width of the avalanche region  $x_a$  was taken as one-tenth of the drift region width. The dimensions of the diodes are given in Table 2.2.

Table 2.2

| Diode Dimensions       |                                             |                                         |
|------------------------|---------------------------------------------|-----------------------------------------|
| Transit Angle<br>(rad) | Avalanche Region<br>Width ( $\mu\text{m}$ ) | Drift Region<br>Width ( $\mu\text{m}$ ) |
| $0.25\pi$              | 0.125                                       | 1.25                                    |
| $0.50\pi$              | 0.250                                       | 2.50                                    |
| $0.75\pi$              | 0.375                                       | 3.75                                    |

The doping profile in the Read diode is such that, when the diode is reverse biased, the electric field is high enough for substantial avalanching to occur in the narrow avalanche region and for the carriers to drift at saturated velocities throughout the diode. The condition<sup>46</sup> for dc breakdown due to impact ionization for equal ionization rates is

$$\int_0^{x_L} \alpha[E(x)] dx = 1 , \quad (2.33)$$

where  $x_L$  = the total length of the diode (cm). The constants for the ionization rate for the calculations in this section are taken for holes in Si.<sup>37</sup>

A doping profile which can be evaluated easily is one which produces a constant field in the avalanche region. If it is also assumed that all of the ionization occurs in the avalanche region, then Eq. 2.33 reduces to

$$\int_0^{x_A} \alpha(E_c) dx = 1, \quad (2.34)$$

where  $E_c$  = the field in the avalanche region necessary for breakdown. Substituting the expression for the ionization rate into this equation and integrating, the following result is obtained:

$$A' \exp \left[ - \left( \frac{b'}{E_c} \right) \right] x_A = 1. \quad (2.35)$$

This equation can easily be solved for the critical field for breakdown. The voltage across the avalanche region of the diode is then

$$V_a = E_c x_A. \quad (2.36)$$

Results of these calculations for the different transit-angle diodes are given in Table 2.3. The field in the drift region is assumed constant and equal to  $2 \times 10^5$  V/cm which is a reasonable value for a saturated drift velocity and no impact ionization. Thus the total voltage across the diode at breakdown is also calculated and given in Table 2.3.

Table 2.3

Parameters for Avalanche Breakdown in Read Diodes

| <u>Transit Angle<br/>(rad)</u> | <u>Breakdown<br/>Field (V/cm)</u> | <u>Voltage Across<br/>Avalanche Region (V)</u> | <u>Breakdown<br/>Voltage (V)</u> |
|--------------------------------|-----------------------------------|------------------------------------------------|----------------------------------|
| $0.25\pi$                      | $5.78 \times 10^5$                | 7.23                                           | 32.23                            |
| $0.50\pi$                      | $5.14 \times 10^5$                | 12.84                                          | 62.84                            |
| $0.75\pi$                      | $4.84 \times 10^5$                | 18.14                                          | 93.14                            |

The breakdown voltage can also be calculated for a more realistic doping profile for a Read diode as shown in Fig. 2.19. The doping density in the  $p^+$  and  $n^+$  regions are assumed to be large so that the field does not penetrate into these regions. The doping density in the p-region is allowed to vary while the doping density in the "intrinsic" region is set at  $10^{12} \text{ cm}^{-3}$ . The value of the electric field  $E_m$ , which gives avalanche breakdown, is calculated for the different transit-angle diodes by Eq. 2.33. Again the ionization rates for holes and electrons are assumed to be equal. Once the maximum electric field  $E_m$  is determined then the field as a function of  $x$  is known throughout the diode. The avalanche width,<sup>48</sup> the width for which the ionization integral in Eq. 2.33 is equal to 0.95, is calculated from the known field. The breakdown voltage for the diode is also calculated from the known field.

Figure 2.20 shows the maximum electric field  $E_m$  as a function of the doping density in the p-region for three different-length diodes. For a large doping density, all three diodes behave like a one-sided abrupt p-n junction since all of the impact ionization occurs in the p-region and only a negligible amount occurs in the intrinsic region because of the low field in this region. As the doping density in the p-region

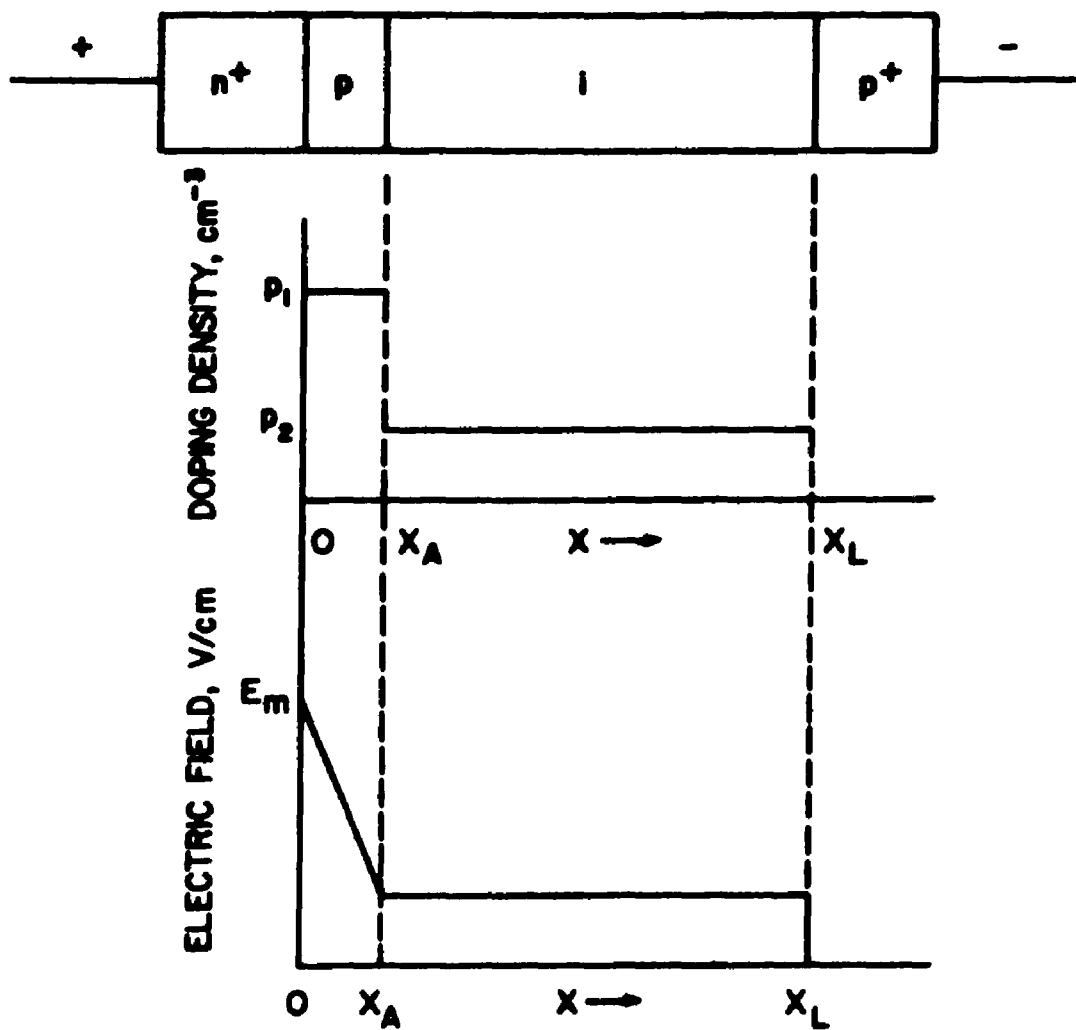


FIG. 2.19 READ-TYPE DIODE WITH ASSUMED DOPING DENSITY AND ELECTRIC FIELD. ( $x_L = x_D + x_A$ )

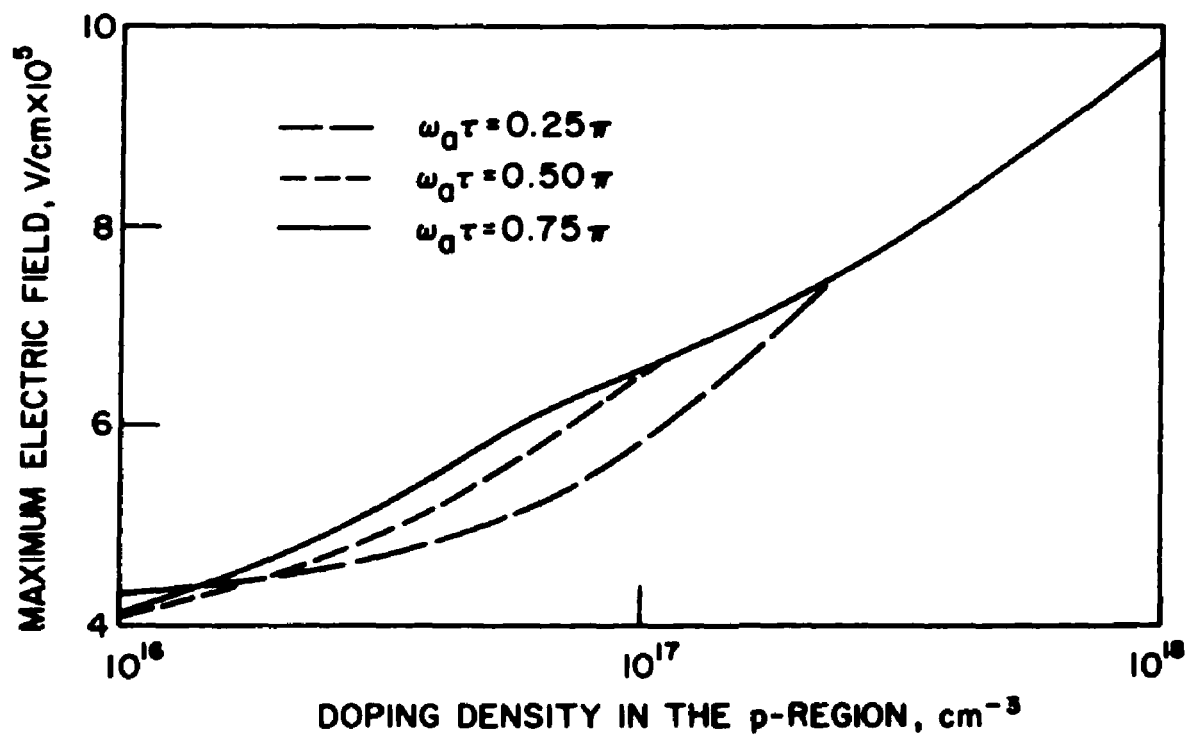


FIG. 2.20 MAXIMUM ELECTRIC FIELD VS. DOPING DENSITY IN THE p-REGION.

decreases, the field in the intrinsic region increases, thereby causing impact ionization to occur throughout the diode. The shorter the distance over which impact ionization occurs, the higher the field must be for breakdown to occur. Thus for the shorter diode at a low doping density, the maximum field is larger than that for the longer diodes. Figure 2.21 shows the avalanche width as a function of the doping density in the p-region for all three diodes. Again for a large doping density the avalanche width is the same for all diodes. When the doping density decreases so that the field in the intrinsic region is large enough for impact ionization to occur, the avalanche width for the three diodes is no longer equal. For a low doping density, the avalanche width extends nearly across the diode to the value of  $x_L$  as can be seen in Fig. 2.21. The breakdown voltage for the three diodes is plotted in Fig. 2.22. For a large doping density the breakdown voltages of the three diodes are equal since the field in the intrinsic region is so low that the contribution to the voltage is negligible. For a small doping density where the field in the intrinsic region is much larger, the breakdown voltage of the three diodes is proportional to the width of the diode.

In order to determine an approximate magnitude of the breakdown voltage for the different transit-angle diodes, the doping density which gives an avalanche width equal to the width of the avalanche region as defined in Table 2.2 is selected from Fig. 2.21. For this doping density the maximum field  $E_m$  and the breakdown voltage can be determined. The parameters for breakdown under these conditions are shown in Table 2.4.

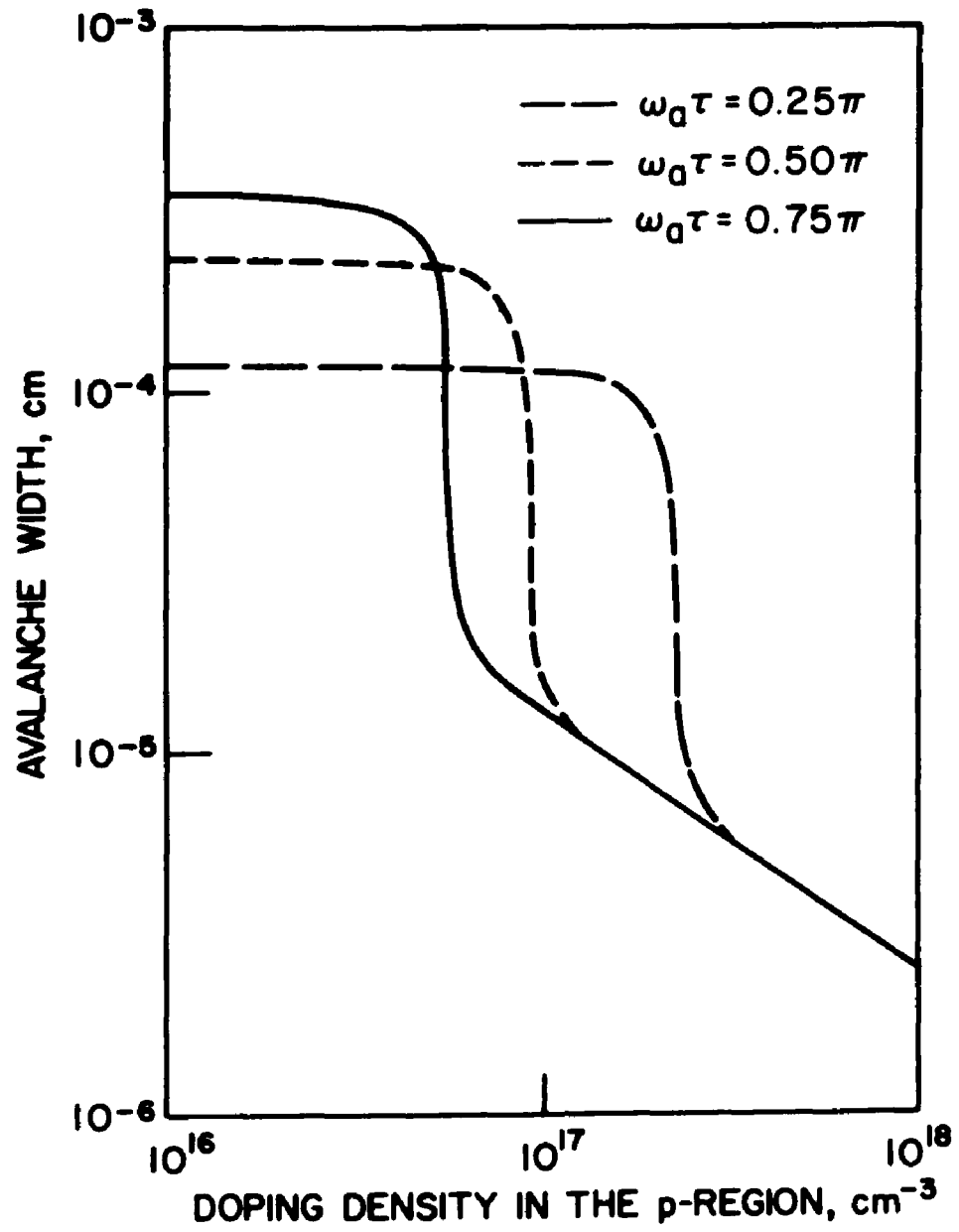


FIG. 2.21 AVALANCHE WIDTH VS. DOPING DENSITY IN THE p-REGION.

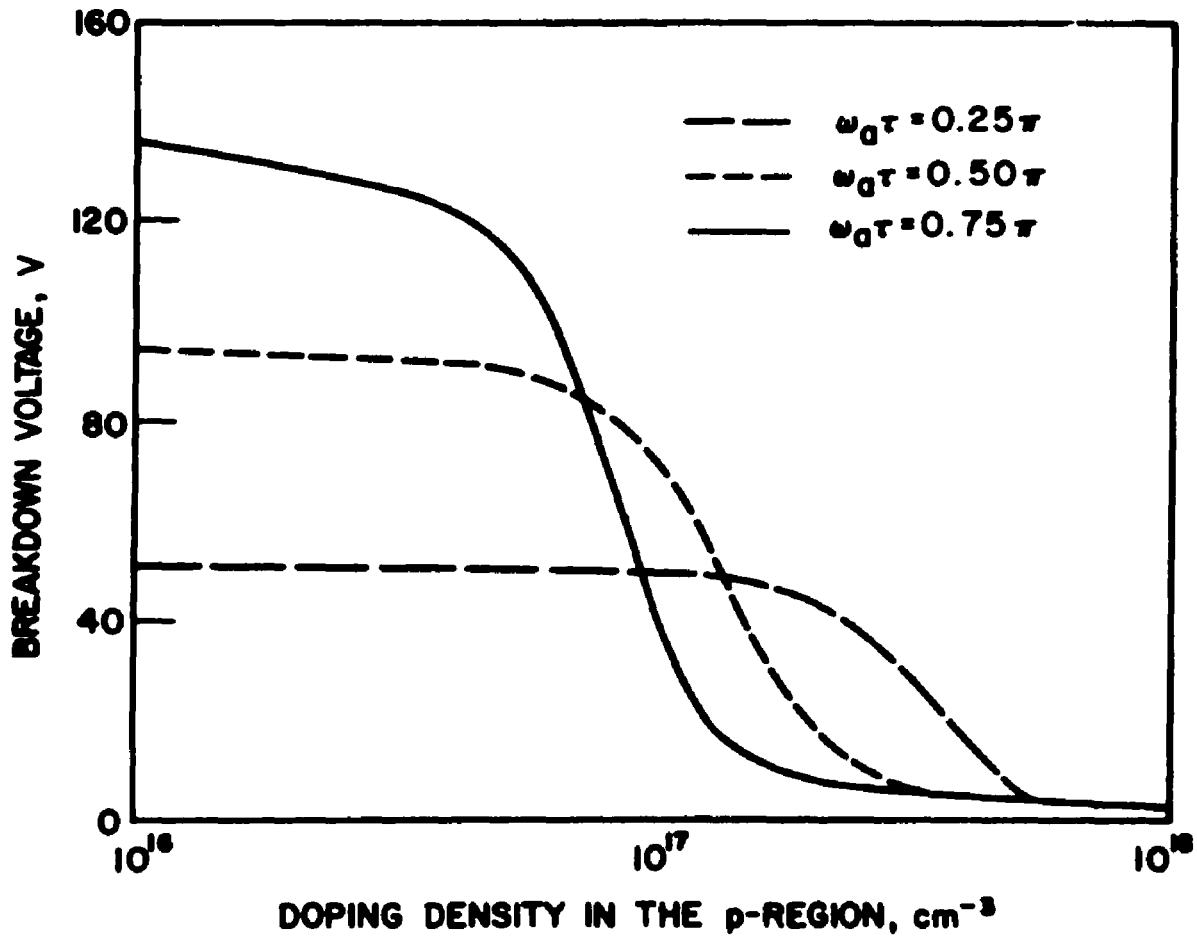


FIG. 2.22 BREAKDOWN VOLTAGE VS. DOPING DENSITY IN THE p-REGION.

Table 2.4

Parameters for Avalanche Breakdown in Realistic Read Diodes

| <u>Avalanche Width (<math>\mu\text{m}</math>)</u> | <u>Doping Density (<math>\text{cm}^{-3}</math>)</u> | <u>Maximum Electric Field (<math>\text{V/cm}</math>)</u> | <u>Breakdown Voltage (V)</u> |
|---------------------------------------------------|-----------------------------------------------------|----------------------------------------------------------|------------------------------|
| 0.125                                             | $2.3 \times 10^{17}$                                | $7.40 \times 10^5$                                       | 41                           |
| 0.250                                             | $9.5 \times 10^{16}$                                | $6.45 \times 10^5$                                       | 75                           |
| 0.375                                             | $5.6 \times 10^{16}$                                | $5.95 \times 10^5$                                       | 110                          |

From the results presented in Tables 2.3 and 2.4 an approximate value of the breakdown voltage for Read diodes can be deduced. Therefore the maximum RF voltage which could exist across the diode is obtained. These values are less than the maximum RF voltages assumed in the previous section and should be used as an upper limit on the magnitude of the RF voltage in those results.

2.4.2 Limitation Due to a Nonsaturated Drift Velocity. A second limitation on the magnitude of the RF voltage is its effect on the field in the drift region. In this case the field in the drift region may become too low for a saturated drift velocity of the carriers to exist during a portion of the RF cycle. The maximum allowable swing of the RF voltage is calculated for which a saturated drift velocity of the carriers exists in the drift region. The data from the large-signal analysis along with several simplifying assumptions are used. The avalanche current as calculated by the large-signal analysis is assumed to be injected at  $x = 0$  in the diode as shown in Fig. 2.23. The charge, which this current injects, drifts through the drift region to  $x_D$  in a transit time  $\tau$ . At any instant of time  $t$  the charge which is injected at  $x = 0$  is given by

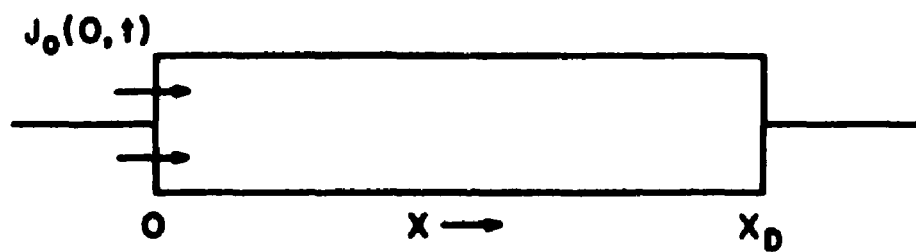


FIG. 2.23 DIODE STRUCTURE ASSUMED FOR CALCULATION OF THE EFFECT OF THE RF VOLTAGE SWING.

$$\rho(o,t) = \frac{J_o(o,t)}{v_s} . \quad (2.37)$$

The avalanche current is assumed to be constant between each increment of time  $\delta t$ , where  $\delta t$  is taken as  $1/360$  of the period of the RF voltage. Thus, charge which is injected at time  $t$  takes  $\tau$  seconds to drift across the diode, where

$$\tau = x_D / v_s \cong n \delta t , \quad (2.38)$$

in which  $n$  is an integer. Therefore, at some instant of time  $t$  the charge density as a function of  $x$  in the drift region is given by

$$\rho(x,t) = \frac{J_o(o, t - x/v_s)}{v_s} \quad (2.39)$$

and the field due to this space charge as a function of  $x$  is

$$E_{sc}(x,t) = \int_0^x \frac{\rho(x,t)}{\epsilon} dx . \quad (2.40)$$

The voltage drop across the drift region, due to the space charge, is given by

$$V_{sc}(t) = - \int_0^{x_D} E_{sc}(x,t) dx . \quad (2.41)$$

At any instant of time  $t$  the voltage across the diode is  $V(t)$ , where  $V(t)$  is assumed to be of the form

$$V(t) = V_{bias} + V_{RF} \sin(\omega t) . \quad (2.42)$$

As shown in Fig. 2.24, the voltage  $V(t)$  across the diode can be related to the field in the diode by adding the voltage drop  $V_{sc}(t)$  due to the space charge and voltage drop  $E_{min}(t) \cdot x_D$ . Thus the minimum field which occurs at  $x = 0$  is related to the space-charge voltage by

$$V(t) = V_{bias} + V_{RF} \sin(\omega t) = E_{min}(t) \cdot x_D + V_{sc}(t) . \quad (2.43)$$

Solving for  $E_{min}(t)$ , the following result is obtained,

$$E_{min}(t) = \frac{V_{bias} + V_{RF} \sin(\omega t) - V_{sc}(t)}{x_D} . \quad (2.44)$$

The minimum electric field  $E_{min}$ , which occurs at the point  $x = 0$  of the drift region, is plotted as a function of the phase of the RF voltage for the three different diodes in Figs. 2.25 through 2.27. The bias voltage for these diodes was selected so that the field in the drift region with no RF or space-charge voltage would be  $2 \times 10^5$  V/cm. This value of field assumes a saturated drift velocity of the carriers and is low enough that no appreciable impact ionization occurs.

For a diode with a transit angle of  $0.25\pi$  the avalanche current has transversed the drift region before the RF voltage approaches its minimum. Since the avalanche current is in the drift region for such a short time the amount of space charge is small and has little effect on the minimum field in the diode. Also, since the avalanche current is not in the drift region when the RF voltage goes through its minimum value, the RF voltage can approach the breakdown voltage in magnitude without causing the carriers to have a nonsaturated drift velocity. In

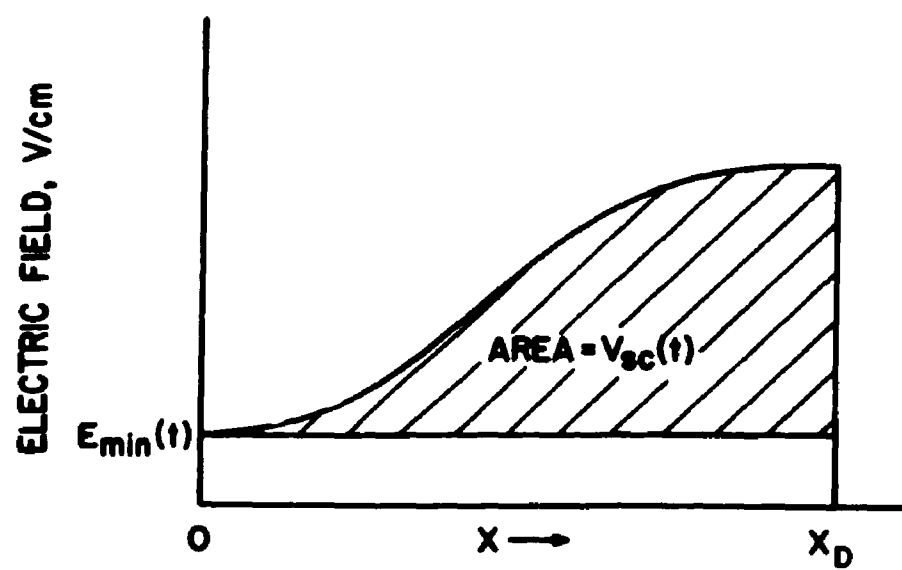


FIG. 2.24 ELECTRIC FIELD IN THE DRIFT REGION.

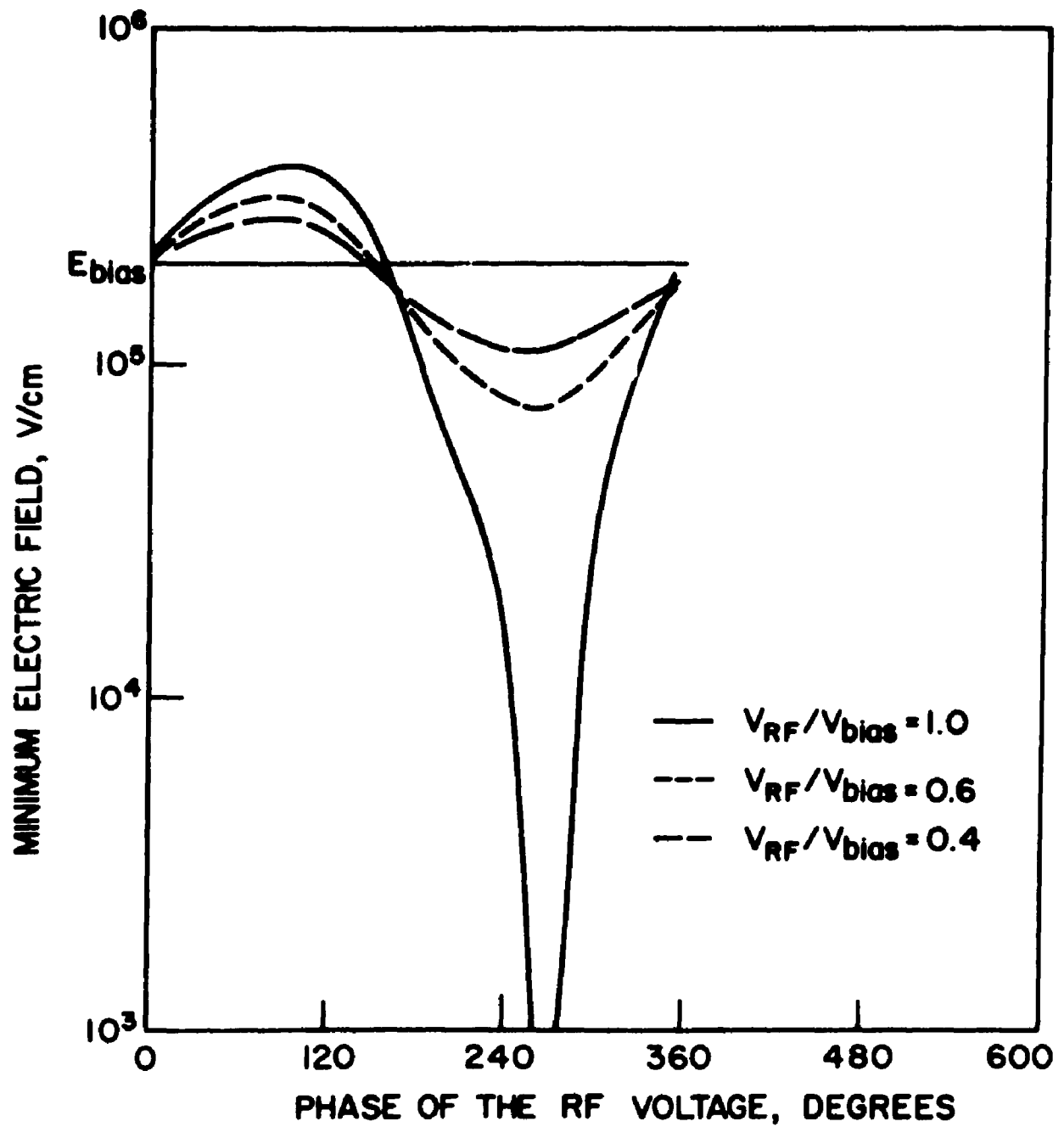


FIG. 2.25 MINIMUM ELECTRIC FIELD VS. PHASE OF THE RF VOLTAGE. ( $\omega_a \tau = 0.25\pi$ ,  $f = 10$  GHz,  $V_{bias} = 25$  V AND  $J_{dc} = 3000$  A/cm<sup>2</sup>)

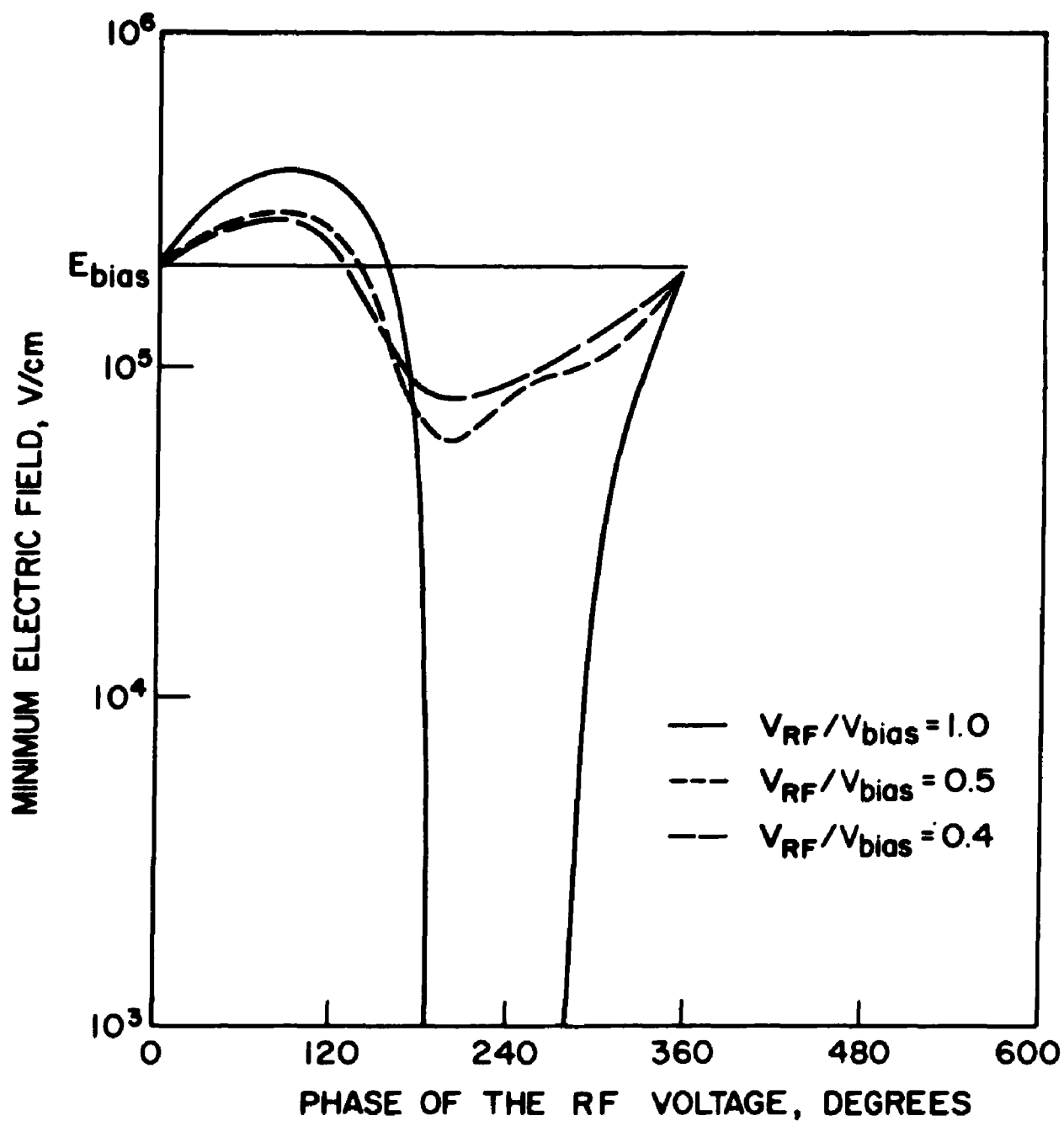


FIG. 2.26 MINIMUM ELECTRIC FIELD VS. PHASE OF THE RF VOLTAGE. ( $\omega_a \tau = 0.5\pi$ ,  
 $f = 10$  GHz,  $V_{\text{bias}} = 50$  V AND  $J_{\text{dc}} = 3000$  A/cm<sup>2</sup>)

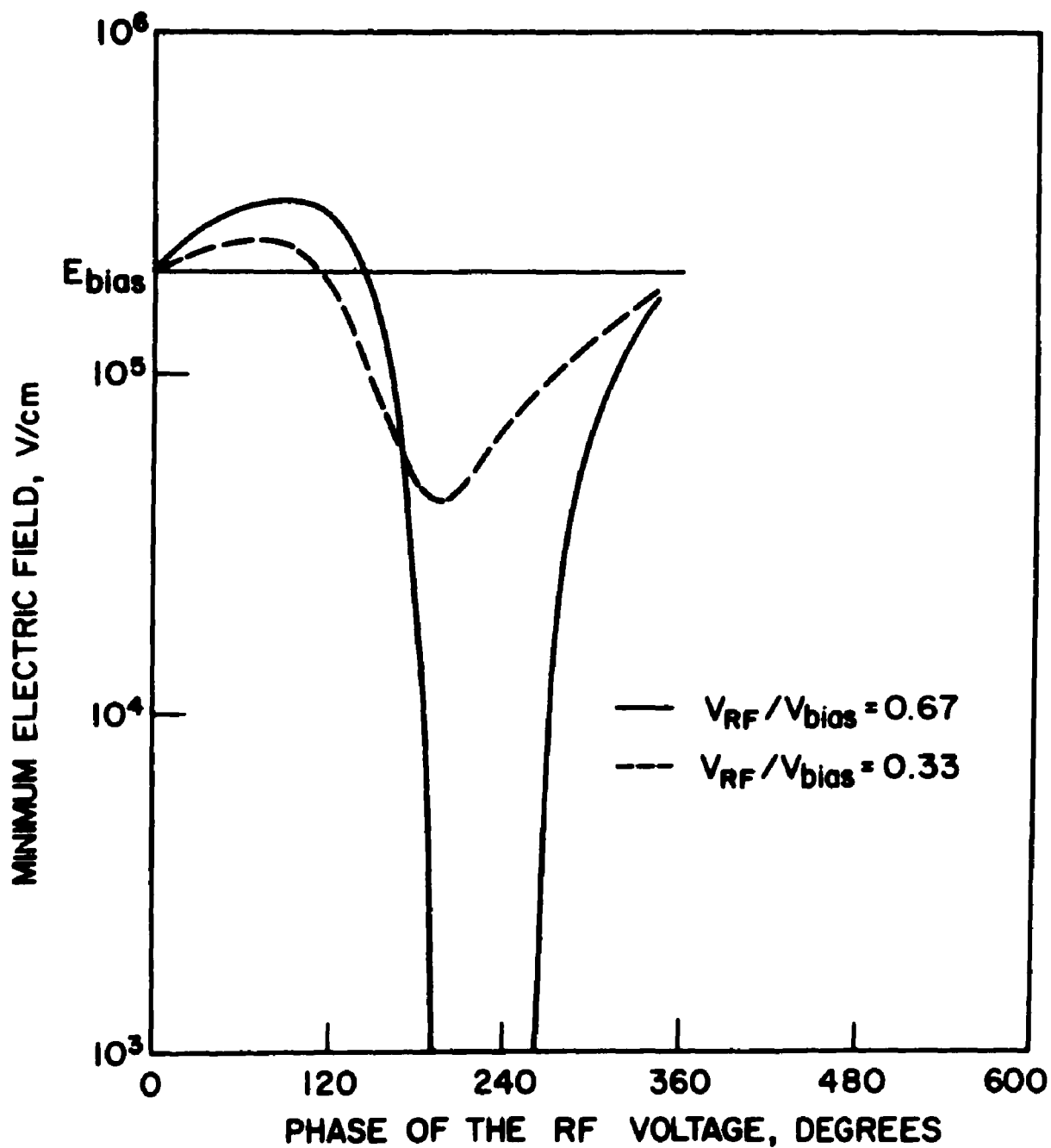


FIG. 2.27 MINIMUM ELECTRIC FIELD VS. PHASE OF THE RF VOLTAGE. ( $\omega_a \tau = 0.75\pi$ ,  
 $f = 10$  GHz,  $V_{bias} = 75$  V AND  $J_{dc} = 3000$  A/cm<sup>2</sup>)

the  $0.5\pi$  diode the avalanche current is in the drift region for a longer period of time and therefore there is enough space charge present to affect the minimum field. Nonsaturated drift velocity of electrons and holes occurs for fields less than approximately  $4 \times 10^4$  V/cm in silicon.<sup>40</sup> Thus for the  $0.5\pi$  diode the RF voltage is limited to some value less than the breakdown voltage since the avalanche current is still in the drift region when the minimum field drops below the value for saturated drift velocity. The diode with a  $0.75\pi$  transit angle shows even more effect of the space charge since the charge is in the drift region for most of the negative portions of the RF voltage. Here the RF voltage is limited to a still smaller fraction of the breakdown voltage. In Fig. 2.28 the minimum field which occurs during the cycle is plotted as a function of the normalized RF voltage for the three diodes. It is worth noting that the major portion of the charge which is drifting through the drift region does not see this minimum field which occurs at the beginning of the drift region. This charge sees some slightly larger field because of space-charge effects. For this analysis for saturated drift velocity the RF voltage can be increased until the minimum field during the transit time of the charge is slightly below the field necessary for saturated drift velocity.

Large RF voltages are necessary for maximum efficiencies. Thus, in the  $0.25\pi$  diode where the RF voltage can approach the breakdown voltage of the diode the efficiency would be larger than in the  $0.75\pi$  diode where the RF voltage is limited to less than one-half of the breakdown voltage. This is illustrated in Fig. 2.29. It must be noted though that the negative conductance is larger for the  $0.75\pi$  diode than for the  $0.25\pi$  diode and thus this effect works in the opposite manner. The only transit angles

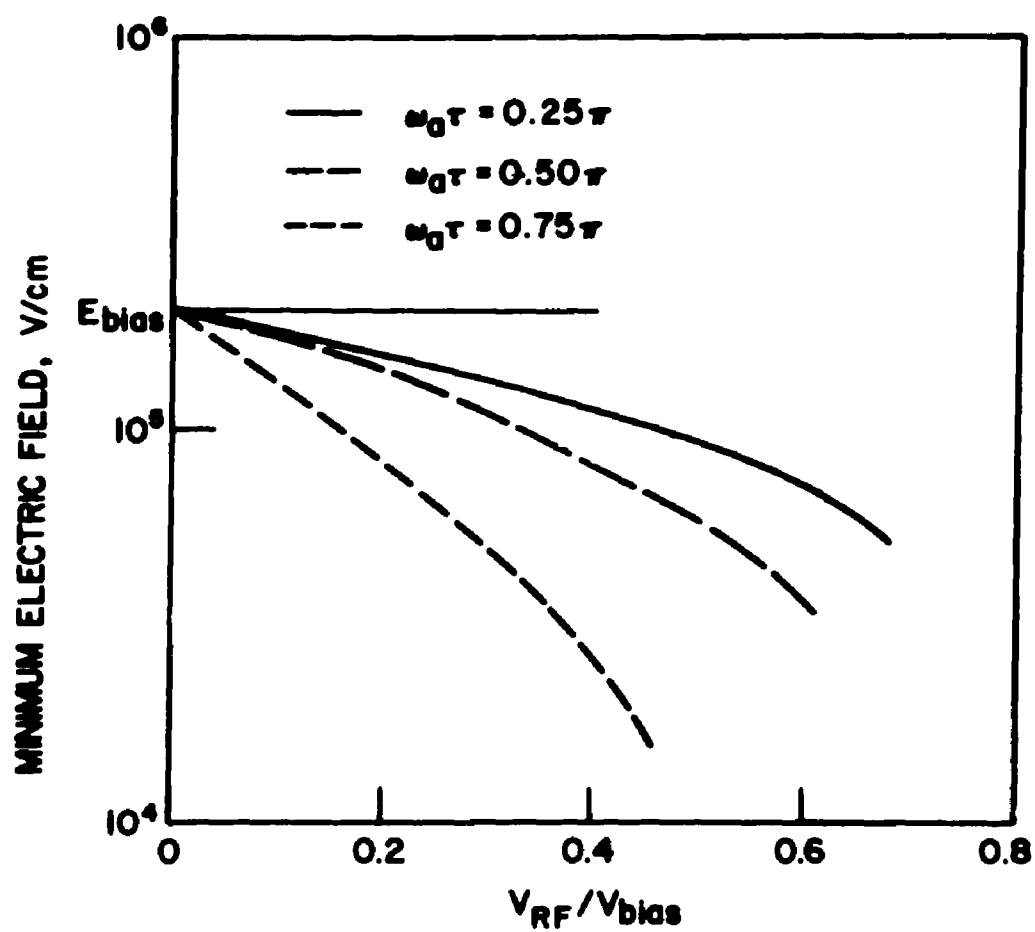


FIG. 2.28 MINIMUM ELECTRIC FIELD VS.  $V_{RF}/V_{bias}$ . ( $V_{bias} = 2 \times 10^5 \cdot x_D$  AND  $f = 10$  GHz)

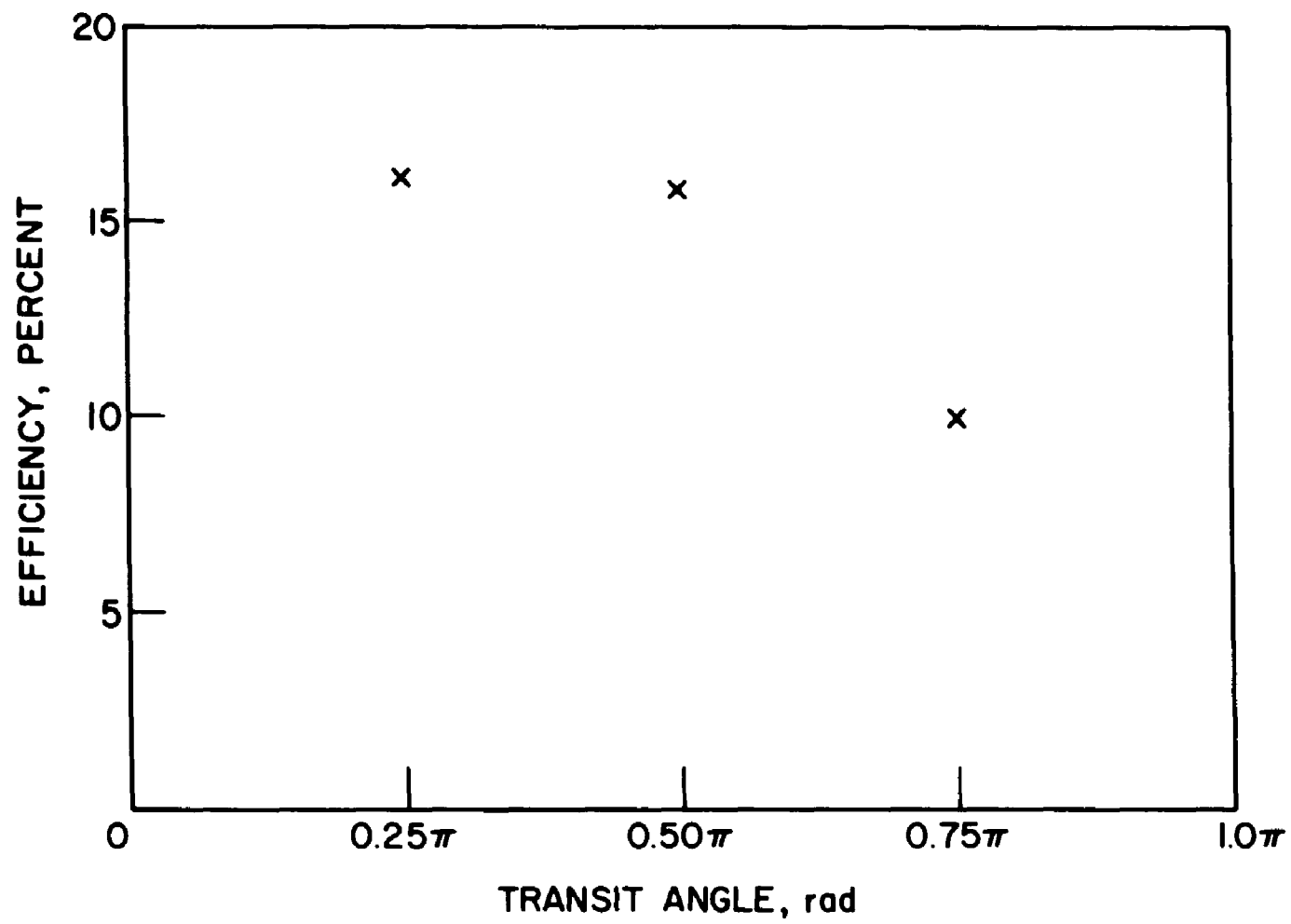


FIG. 2.29 EFFICIENCY VS. TRANSIT ANGLE. ( $f_a = 10$  GHz AT  $I_{dc} = 100$  mA)

which have been considered are these three. It is apparent that there is some optimum transit angle less than  $0.8\pi$  for maximum efficiency if a saturated velocity has to be maintained. This can be seen from the fact that for small-transit angles the RF voltage can approach the breakdown voltage but the negative conductance is decreasing with decreasing transit angle. As the transit angle approaches the optimum  $0.8\pi$  the negative conductance is maximized but the RF voltage is limited to a small fraction of the breakdown voltage. Thus the efficiency falls off at both extremes of transit angles. It is worth noting that these conclusions are based on the assumption of maintaining a saturated drift velocity which is not necessary in an actual diode.

## 2.5 Effect of a Nonsaturated Drift Velocity on the Operating Characteristics

It was shown in the previous section that as the RF voltage increases, the field in the drift region can become low enough during a portion of the RF period for the carriers to drift at nonsaturated velocities. In this case the effect of a nonsaturated drift velocity can be qualitatively understood. When the field becomes low enough for the carriers to drift at nonsaturated velocities, the effective transit angle of the carriers is increased due to the longer time necessary for the carriers to traverse the drift region. As the RF voltage increases, the transit angle of the carriers increases due to the lower field. If a diode is operating at a frequency where its transit angle is less than the optimum transit angle,  $0.8\pi$ , then as the RF voltage approaches the breakdown voltage of the diode, the field in the drift region will become low enough for the carriers to drift at nonsaturated velocities and the effective transit angle of the diode is increased. For the RF voltage which

corresponds to the  $0.8\pi$  transit angle of the carriers, the conductance will be maximum and the RF voltage will be closer to the breakdown voltage. Thus it is possible to have large RF voltages with respect to the breakdown voltage and maximum negative conductance. Therefore the output power and efficiency will be improved.<sup>49</sup> However, if the effective transit angle becomes larger than the optimum for maximum negative conductance due either to an operating frequency which is too high or to a nonsaturated velocity which is too low, the output power will be degraded. The amount of improvement in output power and efficiency would have to be determined by a detailed analysis. However, it is apparent that the effect of nonsaturated drift velocities could improve the performance of the diode.

## 2.6 Conclusions

In this chapter a mathematical model for studying the large-signal operating characteristics of a Read diode has been presented. The model has no limitation on the transit angle. It was demonstrated that the results agree with the short-transit angle analysis<sup>9</sup> for transit angles less than  $0.25\pi$  and that the optimum transit angle for maximum negative conductance is approximately  $0.8\pi$ . In Section 2.4 the limitations on the magnitude of the RF voltage with respect to the breakdown voltage and to maintaining saturated drift velocity throughout the diode at every instant of time in the cycle were investigated. These limitations showed that an optimum transit angle for maximum efficiency existed which is less than the angle for maximum negative conductance. Finally, in Section 2.5 a qualitative explanation was presented to show that improved output

power and efficiency could be obtained by having the RF voltage large enough to cause the carriers to drift at field-dependent velocities. In Chapter III many of the limitations of this analysis will be removed by a more general large-signal analysis of IMPATT diodes.

## CHAPTER III. LARGE-SIGNAL ANALYSIS OF IMPATT DIODES

### 3.1 Introduction

In this chapter a mathematical model is developed which describes the large-signal operating characteristics of IMPATT diodes. A wide range of doping profiles and materials can be analyzed since the model allows for electron and hole multiplication by impact ionization to occur at every point in the diode. Given the particular doping profile and dimensions for a diode, its operating characteristics can be determined. These include admittance, power output, efficiency and bias voltage as a function of the operating frequency, dc current density and RF voltage. This analysis is used to study the dc, single-frequency and multifrequency operating characteristics of IMPATT diodes for several different doping profiles. A physical understanding of the differences in the characteristics of the different doping profiles will also be discussed.

### 3.2 Derivation of the Large-Signal Model

3.2.1 Introduction. Before deriving the large-signal model it is helpful to outline the essential features of the analysis. Most large-signal analyses of Read-type diodes neglect the divergence of the displacement current in the avalanche region and assume that the period of the oscillation is much larger than the transit time of the carriers through the avalanche region. Large-signal analyses of Read-type structures that make this assumption in the narrow avalanche region give good qualitative results but predict the wrong cutoff or avalanche frequency<sup>45</sup>

and efficiency<sup>11</sup> for the diode. As the width of the avalanche region increases the effect of this assumption on the results of the analysis becomes appreciable. For this case the divergence of the displacement current cannot be neglected since it takes the conduction current a finite time to cross the avalanche region and during this transit time the field changes, thus giving rise to an appreciable change in the displacement current. Therefore, the conduction current is not independent of  $x$  in the avalanche region. For this reason, large-signal analyses of diode structures in which the avalanching occurs over an appreciable portion of the diode cannot neglect the divergence of the displacement current.

This large-signal analysis does not neglect the divergence of the displacement current in the avalanche region. A differential equation is obtained relating the total current and the field in the diode. The equation is solved on an IBM 360-67 computer. For a given doping density and RF voltage, a solution for the total current is found which has the prescribed dc value. From the total current, the space-charge field is obtained by separating the conduction current into the proper hole and electron currents and solving Poisson's equations for the field. The space-charge field is used to modify the original field and a new solution for the total current is obtained. This procedure is repeated for a given dc current and RF voltage until the admittance of the diode converges. Since the analysis solves for the total current density without neglecting the change in the displacement current at any point, avalanching by impact ionization can occur throughout the diode and is not assumed to be confined to a narrow region.

3.2.2 Assumptions and Derivation. The basic equations governing the operation of an IMPATT diode were presented in Chapter II. These equations, Eqs. 2.1 through 2.4, are repeated here for the sake of clarity.

Continuity Equations

$$\frac{\partial p}{\partial t} = -\frac{1}{q} \frac{\partial J_p}{\partial x} + \alpha_p v_p p + \alpha_n v_n n \quad (2.1)$$

and

$$\frac{\partial n}{\partial t} = \frac{1}{q} \frac{\partial J_n}{\partial x} + \alpha_p v_p p + \alpha_n v_n n \quad (2.2)$$

Ionization Rate Equation

$$\alpha = A' \exp \left[ - \left( \frac{b'}{E} \right)^{m'} \right] \quad (2.3)$$

Poisson's Equation

$$\frac{\partial E}{\partial x} = \frac{q}{\epsilon} (N_D - N_A + p - n) \quad (2.4)$$

In order to solve these equations several simplifying assumptions are made. The velocity of holes and electrons are assumed equal and field-independent, i.e.,  $v_p = v_n = v$ . In Chapter II the validity of this assumption was discussed. The ionization rates for holes and electrons are also assumed equal. The value for the ionization rate is taken to be the average or effective rate of the two carriers which is obtained by solving for the dc breakdown voltage with unequal rates and then finding a rate equal for both carriers which gives the same breakdown voltage. A discussion of this method and its results are presented in Appendix C.

Making use of these two simplifications and the fact that

$$J_p = qvp \quad , \quad (3.1)$$

$$J_n = qvn \quad (3.2)$$

and

$$J_c = J_p + J_n \quad , \quad (3.3)$$

a differential equation involving the conduction current  $J_c$  and the hole and electron currents can be obtained by combining the two continuity equations. Thus

$$\frac{1}{v} \frac{\partial J_c}{\partial t} = \frac{\partial}{\partial x} (J_n - J_p) + 2\alpha J_c \quad . \quad (3.4)$$

The total current  $J_T$  which is independent of  $x$  due to continuity of current, consists of the conduction and displacement currents and is given by

$$J_T(t) = J_c(x,t) + \epsilon \frac{\partial E}{\partial t}(x,t) \quad . \quad (3.5)$$

Solving for the conduction current from this equation and substituting it into Eq. 3.4, the following equation is obtained:

$$\frac{1}{v} \frac{\partial J_T}{\partial t} - \frac{\epsilon}{v} \frac{\partial^2 E}{\partial t^2} = \frac{\partial}{\partial x} (J_n - J_p) + 2\alpha J_T - 2\alpha \epsilon \frac{\partial E}{\partial t} \quad . \quad (3.6)$$

The dependence on  $x$  in this equation can be eliminated by integrating the equation over the length of the diode. Thus

$$\begin{aligned} \frac{1}{v} \int_0^{x_L} \frac{\partial J_T}{\partial t} dx - \frac{\epsilon}{v} \int_0^{x_L} \frac{\partial^2 E}{\partial t^2} dx &= (J_n - J_p) \Big|_0^{x_L} + 2 \int_0^{x_L} \alpha J_T dx \\ &- 2\epsilon \int_0^{x_L} \alpha \frac{\partial E}{\partial t} dx \quad . \quad (3.7) \end{aligned}$$

Note that this equation is integrated over the entire length of the diode rather than just the avalanche region. Therefore impact ionization can occur anywhere in the diode. Since the total current is independent of  $x$ , Eq. 3.7 reduces to

$$\begin{aligned} \tau \frac{d}{dt} J_T - \frac{\epsilon}{v} \int_0^{x_L} \frac{\partial^2 E}{\partial t^2} dx &= (J_n - J_p) \Big|_0^{x_L} + 2J_T \int_0^{x_L} \alpha dx \\ &- 2\epsilon \int_0^{x_L} \alpha \frac{\partial E}{\partial t} dx \quad , \quad (3.8) \end{aligned}$$

where  $\tau = x_L/v$ . Since  $x$  and  $t$  are independent, the order of integration and differentiation can be interchanged in the second term. This term then reduces to

$$- \frac{\epsilon}{v} \int_0^{x_L} \frac{\partial^2 E}{\partial t^2} dx = - \frac{\epsilon}{v} \frac{d^2}{dt^2} \int_0^{x_L} E dx = - \frac{\epsilon}{v} \frac{d^2}{dt^2} V(t) \quad , \quad (3.9)$$

where  $V(t)$  is the voltage across the diode. Note that the sign convention which has been assumed is

$$V(t) = \int_0^{x_L} E dx \quad . \quad (3.10)$$

The boundary conditions for the diode are shown in Fig. 3.1. These boundary conditions are as follows:

at  $x = 0$ ,

$$J_T(t) = J_{ps} + J_n(0,t) + \epsilon \frac{\partial E}{\partial t}(0,t) \quad (3.11)$$

and at  $x = x_L$ ,

$$J_T(t) = J_p(x_L,t) + J_{ns} + \epsilon \frac{\partial E}{\partial t}(x_L,t) \quad (3.12)$$

Since

$$(J_n - J_p) \Big|_0^{x_L} = J_n(x_L,t) - J_p(x_L,t) - J_n(0,t) + J_p(0,t) \quad , \quad (3.13)$$

each of these terms can be evaluated from the boundary conditions to give

$$\begin{aligned} (J_n - J_p) \Big|_0^{x_L} &= J_{ns} - \left( J_T(t) - J_{ns} - \epsilon \frac{\partial E}{\partial t}(x_L,t) \right) \\ &\quad - \left( J_T(t) - J_{ps} - \epsilon \frac{\partial E}{\partial t}(0,t) \right) + J_{ps} \quad . \quad (3.14) \end{aligned}$$

This equation reduces to

$$(J_n - J_p) \Big|_0^{x_L} = 2(J_{ns} + J_{ps}) - 2J_T(t) + \epsilon \frac{\partial E}{\partial t}(E(0,t) + E(x_L,t)) \quad . \quad (3.15)$$

Substituting this expression into Eq. 3.8 gives

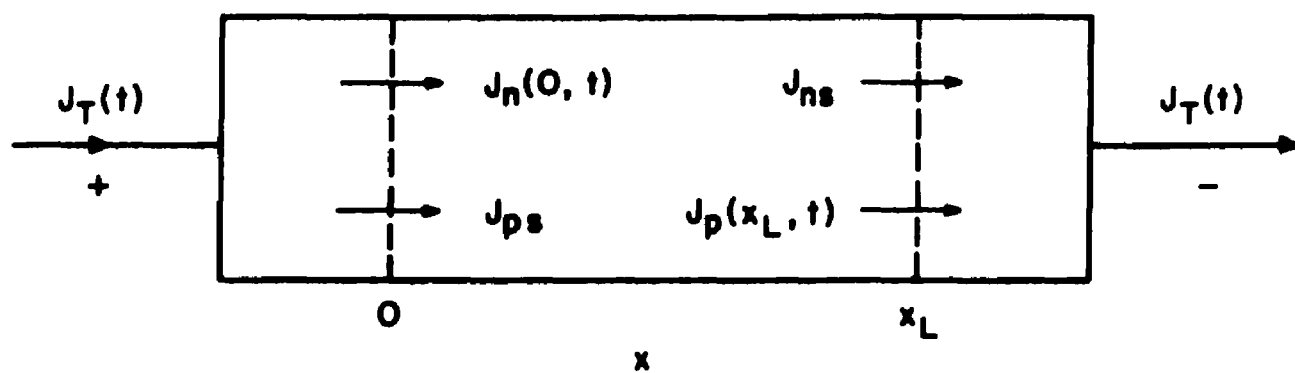


FIG. 3.1 BOUNDARY CONDITIONS FOR AN IMPATT DIODE.

$$\tau \frac{d}{dt} J_T(t) - \frac{\epsilon}{v} \frac{d^2}{dt^2} V(t) = 2J_{sat} - 2J_T(t) + \epsilon \frac{d}{dt} \left( E(0,t) + E(x_L,t) \right) + 2J_T(t) \int_0^{x_L} \alpha [E(x,t)] dx - 2\epsilon \int_0^{x_L} \alpha [E(x,t)] \frac{\partial E}{\partial t} (x,t) dx, \quad (3.16)$$

where  $J_{sat} = J_{ns} + J_{ps}$ .

The voltage across the diode can be defined as

$$V(t) = V_B + \sum_{n=1}^l V_n \sin(n\omega t + \theta_n), \quad (3.17)$$

where  $V_B$  = the bias voltage (V),

$n$  = the harmonic number of the operating frequency,

$l$  = the number of harmonics present,

$V_n$  = the magnitude of the  $n$ th harmonic voltage (V) and

$\theta_n$  = the phase of the  $n$ th harmonic voltage (rad).

The phases of the harmonics are the relative phases among the harmonic voltages at  $t = 0$  and thus for single-frequency operation the phase can be set equal to zero. Using this expression for the voltage the second term in Eq. 3.16 reduces to

$$- \frac{\epsilon}{v} \frac{d^2}{dt^2} V(t) = \sum_{n=1}^l \frac{n^2 \omega^2 \epsilon}{v} V_n \sin(n\omega t + \theta_n). \quad (3.18)$$

By combining terms, Eq. 3.16 can be written in the following form:

$$\frac{d}{dt} J_T(t) + A(t)J_T(t) = Y(t), \quad (3.19)$$

where

$$A(t) = \frac{2}{\tau} \left( 1 - \int_0^{x_L} \alpha \, dx \right)$$

and

$$Y(t) = \frac{2J_{\text{sat}}}{\tau} + \frac{\epsilon}{\tau} \frac{d}{dt} \left( E(0,t) + E(x_L,t) \right) - \frac{2\epsilon}{\tau} \int_0^{x_L} \alpha \frac{\partial E}{\partial t} \, dx \\ - \sum_{n=1}^l \frac{n^2 \omega^2 \epsilon}{x_L} V_n \sin(n\omega t + \theta_n) \quad .$$

This differential equation can be solved on the computer in the following manner. An initial conjecture for the electric field is assumed. For this field which is assumed to be independent of the total current, the solution<sup>50</sup> of the differential equation is

$$J_T(t) = \exp \left( - \int_0^t A(\gamma) d\gamma \right) \left[ J_T(0) + \int_0^t Y(\gamma) \exp \left( \int_0^\gamma A(\beta) d\beta \right) d\gamma \right] \quad . \quad (3.20)$$

A periodic solution for  $J_T$  is obtained when the initial value,  $J_T(0)$ , is selected properly. The proper initial value for a periodic solution is

$$J_T(0) = \frac{\int_0^T Y(\gamma) \exp \left( \int_0^\gamma A(\beta) d\beta \right) d\gamma}{\exp \left( \int_0^T A(\gamma) d\gamma \right) - 1} \quad , \quad (3.21)$$

where  $T$  is the period of the fundamental frequency. A proof of the periodicity of  $J_T$  is given in Appendix D.

The dc current is obtained from the total current by

$$J_{dc} = \frac{1}{T} \int_0^T J_T(t) dt \quad . \quad (3.22)$$

The conductance and susceptance at each frequency are obtained by Fourier analysis of  $J_T$  and phasor division by the RF voltage, thus

$$G(f_n) = \frac{\frac{1}{\pi} \int_0^{2\pi} J_T \sin(n\omega t + \theta_n) d\omega t}{V_n} \quad (3.23)$$

and

$$B(f_n) = \frac{\frac{1}{\pi} \int_0^{2\pi} J_T \cos(n\omega t + \theta_n) d\omega t}{V_n} \quad . \quad (3.24)$$

Equation 3.20 is the correct solution for the assumed electric field. The electric field must be modified to include the effect of the space charge of the carriers at each point and every instant in time. First the conduction current  $J_c$  is calculated from the total current and the assumed electric field by

$$J_c(x,t) = J_T(t) - \epsilon \frac{\partial E}{\partial t}(x,t) \quad . \quad (3.25)$$

Once the conduction current is known for all  $x$  and  $t$  then the hole and electron currents can be obtained from the continuity equations as follows:

$$\frac{\partial p}{\partial t} = -\frac{1}{q} \frac{\partial J_p}{\partial x} + \frac{\alpha J_c}{q} \quad (3.26)$$

and

$$\frac{\partial n}{\partial t} = \frac{1}{q} \frac{\partial J_n}{\partial x} + \frac{\alpha J_c}{q} \quad . \quad (3.27)$$

These equations can be rewritten as

$$\frac{1}{v} \frac{\partial J_p}{\partial t} + \frac{\partial J_p}{\partial x} = \alpha J_c \quad (3.28)$$

and

$$\frac{1}{v} \frac{\partial J_n}{\partial t} - \frac{\partial J_n}{\partial x} = \alpha J_c . \quad (3.29)$$

Let

$$\xi = x - vt \quad (3.30)$$

and

$$\eta = x + vt , \quad (3.31)$$

then

$$t = \frac{\eta - \xi}{2v} \quad (3.32)$$

and

$$x = \frac{\eta + \xi}{2} . \quad (3.33)$$

Therefore

$$\left. \frac{\partial t}{\partial \xi} \right|_{\eta=\text{const}} = -\frac{1}{2v} , \quad (3.34)$$

$$\left. \frac{\partial t}{\partial \eta} \right|_{\xi=\text{const}} = \frac{1}{2v} , \quad (3.35)$$

$$\left. \frac{\partial x}{\partial \xi} \right|_{\eta=\text{const}} = \frac{1}{2} \quad (3.36)$$

and

$$\left. \frac{\partial x}{\partial \eta} \right|_{\xi=\text{const}} = \frac{1}{2} . \quad (3.37)$$

Using the above relationships and Eqs. 3.28 and 3.29 it is easily shown that

$$\frac{\partial J_p}{\partial \eta} \triangleq \frac{\partial J_p}{\partial t} \frac{\partial t}{\partial \eta} + \frac{\partial J_p}{\partial x} \frac{\partial x}{\partial \eta} = \frac{1}{2v} \frac{\partial J_p}{\partial t} + \frac{1}{2} \frac{\partial J_p}{\partial x} = \frac{\alpha J_c}{2} \quad (3.38)$$

and

$$\frac{\partial J_n}{\partial \xi} \triangleq \frac{\partial J_n}{\partial t} \frac{\partial t}{\partial \xi} + \frac{\partial J_n}{\partial x} \frac{\partial x}{\partial \xi} = -\frac{1}{2v} \frac{\partial J_n}{\partial t} + \frac{1}{2} \frac{\partial J_n}{\partial x} = -\frac{\alpha J_c}{2} . \quad (3.39)$$

These two differential equations can be solved in the following manner.

Equation 3.38 can be integrated with respect to  $\eta$  with the constraint that

$$d\xi = 0 \quad (3.40)$$

which implies that

$$dx - v dt = 0 \quad (3.41)$$

or

$$dx = v dt . \quad (3.42)$$

Thus, since

$$d\eta = dx + v dt \quad (3.43)$$

then

$$d\eta = 2 dx . \quad (3.44)$$

Therefore Eq. 3.38 yields

$$J_p(x,t) - J_p(0,t) = \int_0^x \alpha J_c(x,t) dx \Big|_{dx=v dt} . \quad (3.45)$$

A similar argument can be applied to Eq. 3.39 to yield

$$J_n(x,t) - J_n(x_L,t) = - \int_{x_L}^x \alpha J_c(x,t) dx \Big|_{dx=-v dt} . \quad (3.46)$$

Applying the boundary conditions for the diode, the following equations are obtained:

$$J_p(x,t) = J_{ps} + \int_0^x \alpha J_c(x,t) dx \Big|_{dx=v dt} \quad (3.47)$$

and

$$J_n(x,t) = J_{ns} + \int_x^{x_L} \alpha J_c(x,t) dx \Big|_{dx=-v dt} . \quad (3.48)$$

These equations have a simple physical explanation. The value of the hole current at  $x = 0$  is the reverse saturation current  $J_{ps}$ , and similarly for the electron current at  $x = x_L$ . The integrals are performed along the trajectory of the particles since for each  $\delta x$  the time is changed by  $\delta t = \delta x/v$ .

Once the hole and electron currents are determined, an electric field is obtained which includes the space charge of the carriers. Poisson's equation is

$$\frac{\partial E}{\partial x} = \frac{q}{\epsilon} (N_D - N_A + p - n) , \quad (3.49)$$

which when divided into two parts as

$$\frac{\partial E}{\partial x} = \frac{q}{\epsilon} (N_D - N_A) + \frac{qv}{\epsilon v} (p - n) \quad (3.50)$$

and incorporating Eqs. 3.1 and 3.2 yields

$$\frac{\partial E(x,t)}{\partial x} = \frac{q}{\epsilon} \left( N_D(x) - N_A(x) \right) + \frac{J_p(x,t) - J_n(x,t)}{\epsilon v} . \quad (3.51)$$

This equation is integrated to yield

$$E(x,t) = E_D(x) + \int_0^x \frac{J_p(\gamma,t) - J_n(\gamma,t)}{\epsilon v} d\gamma + E(0,t) , \quad (3.52)$$

where  $E_D(x)$  = the electric field due to the doping profile and is independent of time and

$E(0,t)$  = the electric field at  $x = 0$  and is an arbitrary constant for each  $t$ .

The voltage across the diode is

$$V(t) = \int_0^{x_L} E(x,t) dx \quad (3.53)$$

in which the same sign convention as Eq. 3.10 has been assumed. Substituting for the field from Eq. 3.52 and for the voltage from Eq. 3.17, the following equation is obtained:

$$V_B + \sum_{n=1}^l V_n \sin(n\omega t + \theta_n) = \int_0^{x_L} E_D(x) dx + \int_0^{x_L} \left( \int_0^x \frac{J_p(\gamma,t) - J_n(\gamma,t)}{\epsilon v} d\gamma \right) dx + E(0,t) \cdot x_L . \quad (3.54)$$

Therefore

$$E(0,t) = \frac{1}{x_L} \left[ V_B + \sum_{n=1}^L V_n \sin(n\omega t + \theta_n) - \int_0^{x_L} E_D(x) dx - \int_0^{x_L} \left( \int_0^x \frac{J_p(\gamma,t) - J_n(\gamma,t)}{\epsilon v} d\gamma \right) dx \right]. \quad (3.55)$$

The only unknown in this equation is  $V_B$ . This voltage is selected such that the dc current obtained in Eq. 3.22 is the specified value. For the first iteration the space-charge term is set equal to zero in Eqs. 3.52 and 3.55 in order to obtain the field. The voltage  $V_B$  is adjusted until the specified dc value of total current is found. This will be the correct solution for this field. The conductance and susceptance are calculated from the total current and then the electric field is modified to include the space charge of the carriers. This field is used to calculate a new current which has the specified dc value due to the adjustment of  $V_B$  again. After each iteration the admittance is calculated and compared with the previous value. When the change is less than 1-1/2 percent the iterations are stopped. This condition for convergence is discussed in Appendix E. The dc bias voltage is then obtained by integrating the field as

$$V_{dc} = \frac{1}{T} \int_0^T \left( \int_0^{x_L} E(x,t) dx \right) dt. \quad (3.56)$$

The output power is

$$P_o(f_n) = \frac{1}{2} V_n^2 G(f_n) \quad (3.57)$$

and the efficiency is

$$\eta = \frac{P_o(f_n)}{V_{dc} \cdot J_{dc}}. \quad (3.58)$$

3.2.3 Dc Analysis. The dc equations can be obtained easily from the ac analysis by setting all the time derivatives to zero. Therefore Eq. 3.16 yields

$$0 = 2J_{\text{sat}} - 2J_T + 2J_T \int_0^{x_L} \alpha [E(x)] dx \quad (3.59)$$

which reduces to

$$J_T = \frac{J_{\text{sat}}}{1 - \int_0^{x_L} \alpha dx} \quad (3.60)$$

Since there is no displacement current, the total current equals the conduction current. The derivation of the hole and electron currents from the conduction current yields

$$J_p(x) = J_{ps} + J_c \int_0^x \alpha dx \quad (3.61)$$

and

$$J_n(x) = J_{ns} + J_c \int_x^{x_L} \alpha dx \quad (3.62)$$

The electric field can be obtained from the hole and electron currents.

The arbitrary constant, the field at  $x = 0$ , is then given by

$$E(0) = \frac{1}{x_L} \left[ V_B - \int_0^{x_L} E_D(x) dx - \int_0^{x_L} \left( \int_0^x \frac{J_p(\gamma) - J_n(\gamma)}{\epsilon v} d\gamma \right) dx \right] \quad (3.63)$$

Therefore the same procedure can be followed in the dc case as in the ac case.

### 3.3 Large-Signal Operating Characteristics

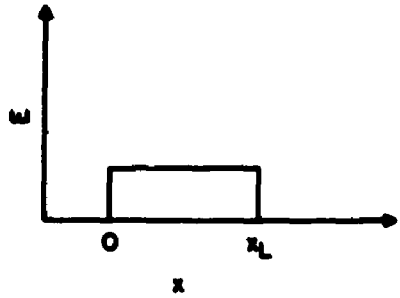
3.3.1 Introduction. The large-signal analysis presented in the previous section has been used to compare the operating characteristics of IMPATT diodes with different doping profiles and materials. The different diode structures and their respective electric fields which have been analyzed are shown in Fig. 3.2. Several assumptions have been made for the diodes shown. First, the doping in the  $p^+$  and  $n^+$  regions is large so that the field does not penetrate these regions but goes to zero at the boundary. Second, the width of all the active regions is  $5\text{ }\mu\text{m}$ . Finally, the doping in each region is assumed constant so that the field in each region is given by

$$E_D(x) = E(x_0) + \frac{q}{\epsilon} (N_D - N_A)(x - x_0) \quad , \quad (3.64)$$

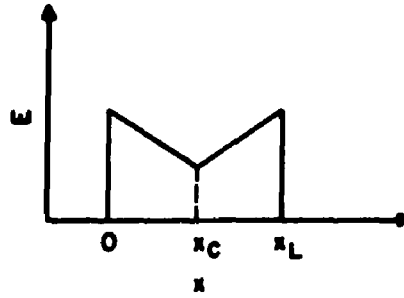
where  $E(x_0)$  = the field at the beginning of the region (V/cm),  
 $x_0$  = the beginning of a region of constant doping (cm),  
 $N_D$  = the donor concentration ( $\text{cm}^{-3}$ ) and  
 $N_A$  = the acceptor concentration ( $\text{cm}^{-3}$ ).

For the pin diode three different materials were compared; namely, Si, GaAs and Ge. For the double injection and p-n junction diodes the doping density in the p- and n-regions was  $1 \times 10^{15} \text{ cm}^{-3}$ . Also point  $x_0$  was in the middle of the diode. For the one-sided abrupt junction the doping density was varied from  $1 \times 10^{15} \text{ cm}^{-3}$  to  $5 \times 10^{15} \text{ cm}^{-3}$ . The Read diode had a doping density in the assumed  $1\text{-}\mu\text{m}$  wide avalanche region of  $1 \times 10^{16} \text{ cm}^{-3}$ .

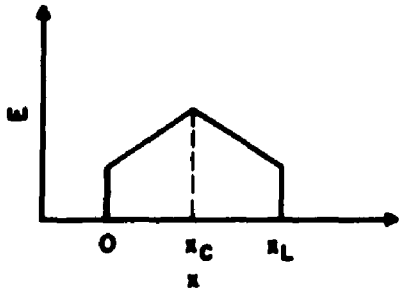
The final assumption which was made was that the total reverse saturation current for all of the structures was  $0.01 \text{ A/cm}^2$ . Also the reverse hole and electron saturation currents were assumed to be equal.



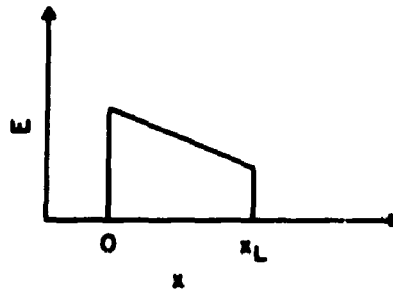
(a) pin DIODE



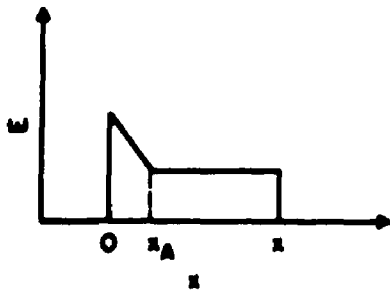
(b) DOUBLE-INJECTION DIODE



(c) p-n JUNCTION



(d) ONE-SIDED ABRUPT JUNCTION



(e) READ DIODE

FIG. 3.2 DIODE STRUCTURES AND FIELD PROFILES.

First, the dc results for several of the doping profiles are presented and explained. Then the single-frequency ac results for admittance and efficiency are discussed. Finally, a few multifrequency admittance plots are shown and discussed.

3.3.2 Dc Characteristics. The static J-V characteristics for several of the different diode structures and materials are compared in Fig. 3.3. The solid curves are for the three pin diodes of different material and the dashed curves are for the double-injection, p-n junction and abrupt-junction Si diodes with a p- and n-region doping density of  $1 \times 10^{15} \text{ cm}^{-3}$ . The three pin diodes have the same breakdown characteristics except for the magnitude of the breakdown voltage which is due to the different ionization rates for the different materials. The constants for the ionization rates which have been used are given in Table 3.1.

Table 3.1

Effective Constants for Ionization Rates of Si, GaAs and Ge

| <u>Material</u> | <u><math>A' (\text{cm}^{-1})</math></u> | <u><math>b' (\text{V/cm})</math></u> | <u><math>m'</math></u> |
|-----------------|-----------------------------------------|--------------------------------------|------------------------|
| Si              | $13.15 \times 10^6$                     | $2.46 \times 10^6$                   | 1                      |
| GaAs            | $3.5 \times 10^5$                       | $6.85 \times 10^5$                   | 2                      |
| Ge              | $3.595 \times 10^5$                     | $7.528 \times 10^5$                  | 1                      |

The corresponding ionization rates are plotted in Fig. 3.4. An approximate value of the breakdown voltage with no space-charge effects for the pin diode can be found by assuming a constant field and solving for the ionization rate which makes the ionization integral equal to unity. Thus

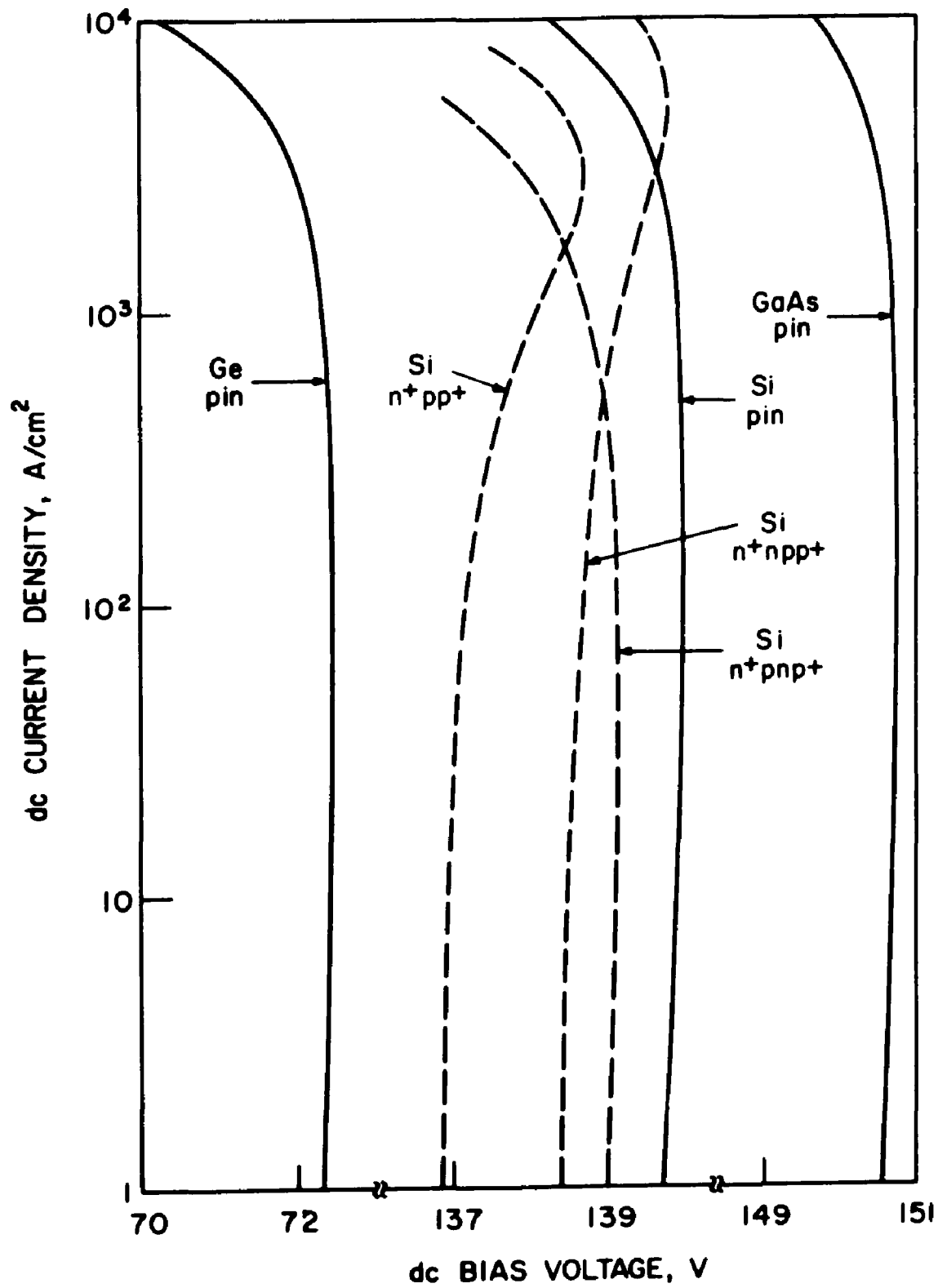


FIG. 3.3 STATIC J-V CHARACTERISTICS.

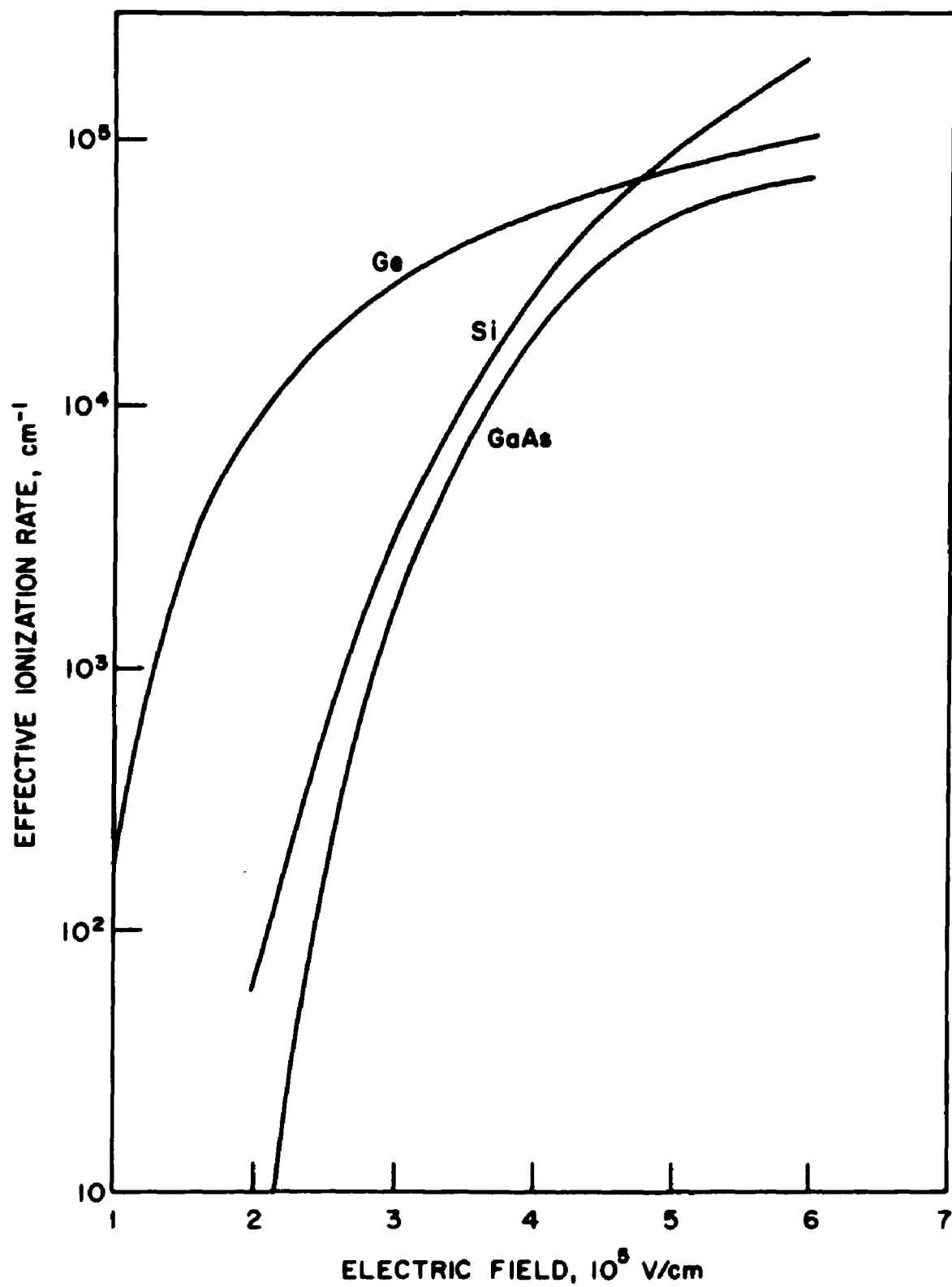


FIG. 3.4 EFFECTIVE IONIZATION RATES VS. ELECTRIC FIELD FOR Si, GaAs AND Ge.

$$\alpha = \frac{1}{x_L} \quad (3.65)$$

which for these diodes is

$$\alpha = 2 \times 10^3 \text{ cm}^{-1} . \quad (3.66)$$

The field which satisfies this equation is

$$E = \frac{b'}{\left[ \ln \left( \frac{A'}{2 \times 10^3} \right) \right]} \quad (3.67)$$

and the corresponding voltage is

$$V = E \cdot 5 \times 10^{-4} . \quad (3.68)$$

The appropriate breakdown fields and voltages are given in Table 3.2. The difference in the breakdown voltage is important when considering the amount of heat generated in the diode for a given current density. The input power to the Ge pin diode is approximately one half that for the Si and GaAs diodes. However, for the same efficiency the Si and GaAs diodes are generating twice the output power.

Table 3.2

Approximate Breakdown Fields and Voltages

| <u>Material</u> | <u>Breakdown Field (V/cm)</u> | <u>Breakdown Voltage (V)</u> |
|-----------------|-------------------------------|------------------------------|
| Si              | $2.8 \times 10^5$             | 140.0                        |
| GaAs            | $3.02 \times 10^5$            | 151.0                        |
| Ge              | $1.45 \times 10^5$            | 72.5                         |

The corresponding electric fields for different current densities for the different Si diode structures are shown in Figs. 3.5 through 3.8. In Fig. 3.9 the static J-V characteristics for the one-sided abrupt junction with different doping densities are shown. The corresponding electric field for the case where the doping density is  $3 \times 10^{15} \text{ cm}^{-3}$  is shown in Fig. 3.10. All of these results are in agreement with the results of Bowers.<sup>51</sup>

Above a certain current density these diodes exhibit a negative dc differential conductance, i.e., the bias voltage decreases for increasing current. In the double-injection diode this occurs above about  $25 \text{ A/cm}^2$  while in the pin diode this occurs above about  $100 \text{ A/cm}^2$ . In the abrupt junction the dc differential conductance becomes negative above about  $3000 \text{ A/cm}^2$  for a doping density of  $1 \times 10^{15} \text{ cm}^{-3}$  and as the doping is increased the value of current density for which the conductance becomes negative increases. Hoefflinger<sup>36</sup> showed that Read diodes with large current densities exhibited negative dc differential conductance when a second avalanche was present at the end of the drift region. This is also the case for the diodes considered here. As the current density is increased the amount of space charge in the diode increases. Even though the total current is independent of  $x$  the hole current density increases from left to right and the electron current density from right to left. Thus at each end of the diode the current density consists of one type of carrier while in the middle for the symmetrical diodes the current density is divided equally. Therefore the space charge at the ends of the diode is large and at the center it is small. The field at the ends will in turn be increased above the field caused by the background doping density due to the space charge. The voltage across the diode is the area under the

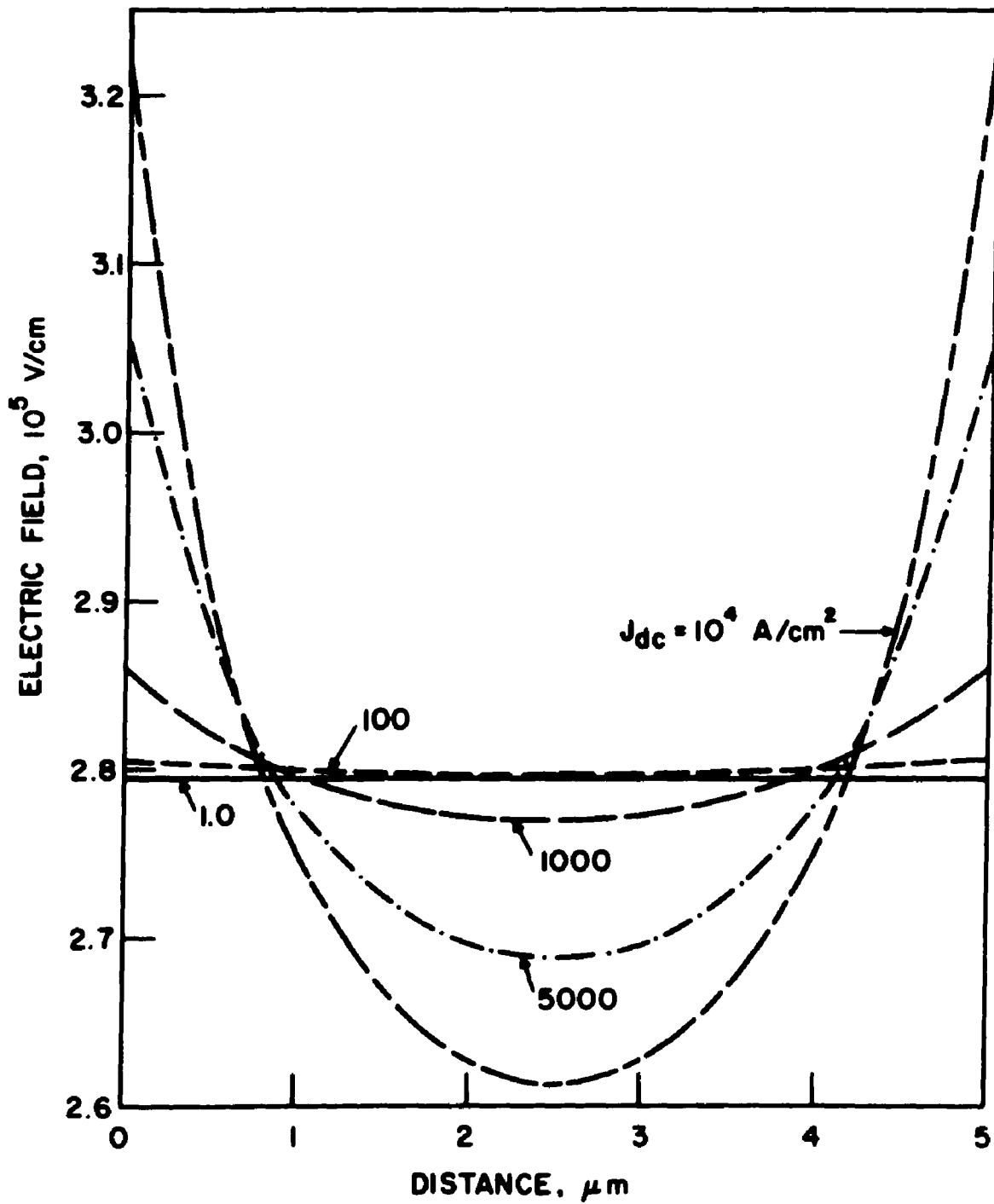


FIG. 3.5 ELECTRIC FIELD VS. DISTANCE FOR A Si pin DIODE.

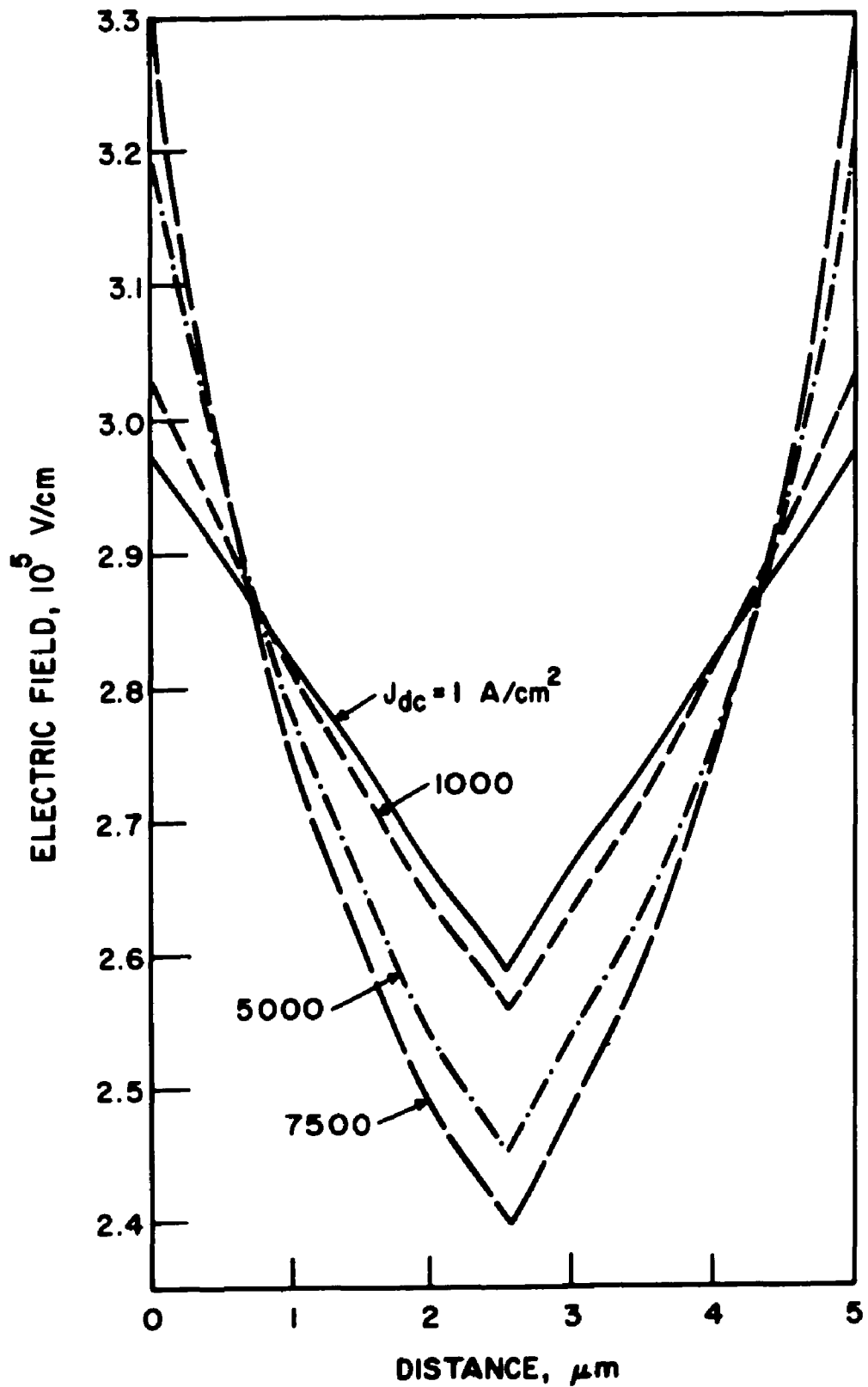


FIG. 3.6 ELECTRIC FIELD VS. DISTANCE FOR A SI DOUBLE-INJECTION DIODE.

( $p = 1 \times 10^{15} \text{ cm}^{-3}$  AND  $n = 1 \times 10^{15} \text{ cm}^{-3}$ )

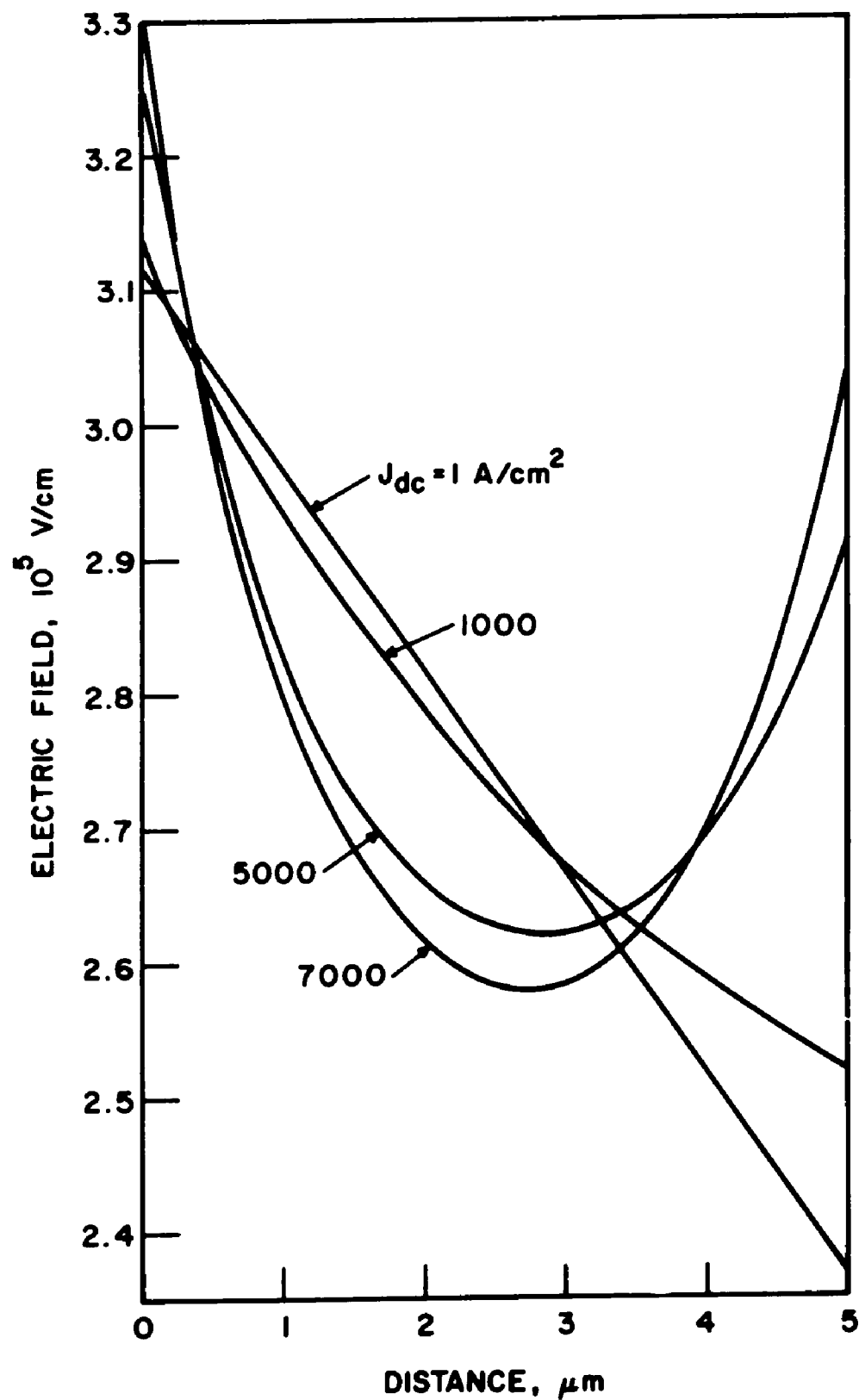


FIG. 3.7 ELECTRIC FIELD VS. DISTANCE FOR A Si ABRUPT-JUNCTION DIODE.  
 ( $p = 1 \times 10^{15} \text{ cm}^{-3}$ )

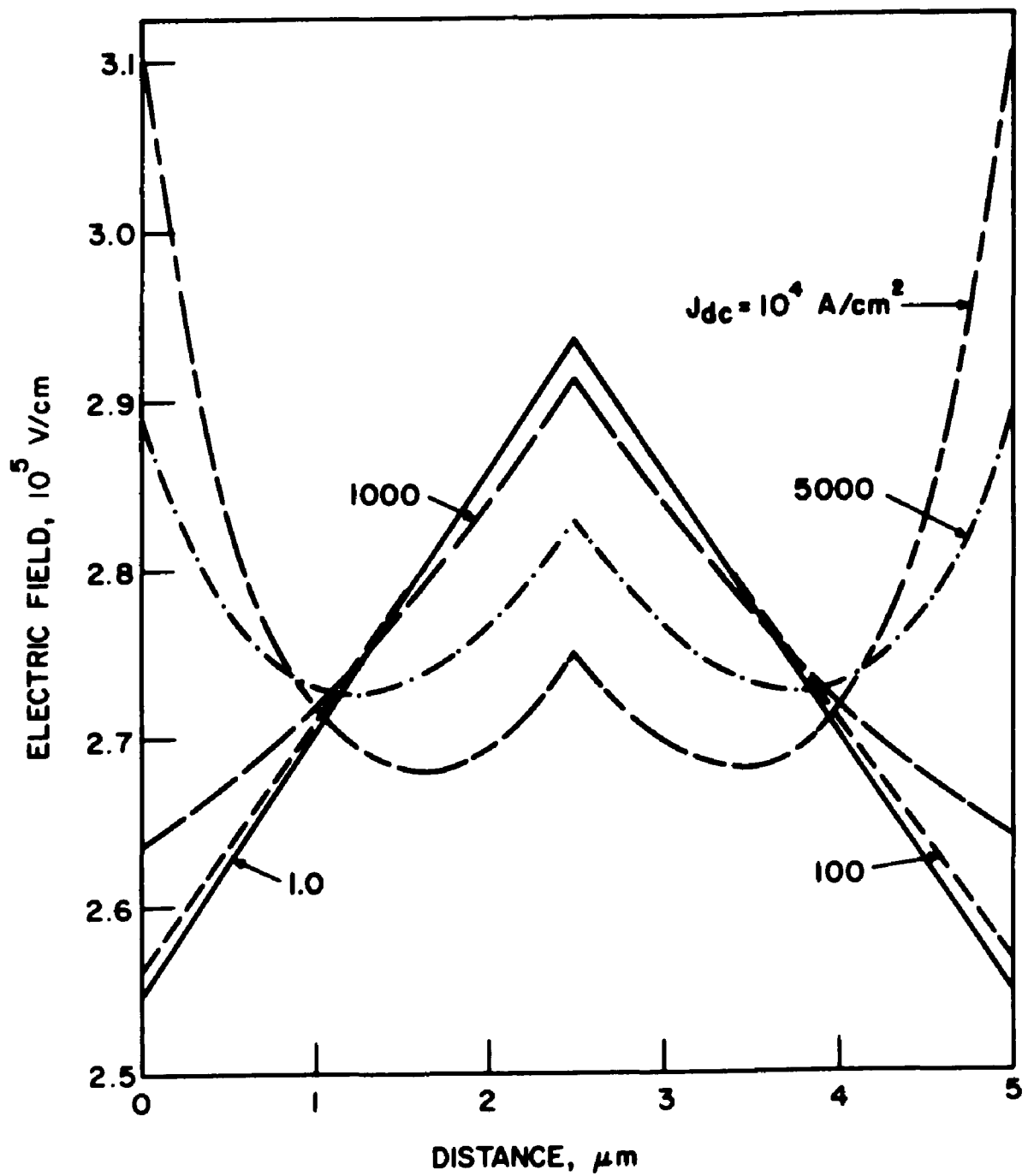


FIG. 3.8 ELECTRIC FIELD VS. DISTANCE FOR A Si p-n JUNCTION DIODE.

( $n = 1 \times 10^{15} \text{ cm}^{-3}$  AND  $p = 1 \times 10^{15} \text{ cm}^{-3}$ )

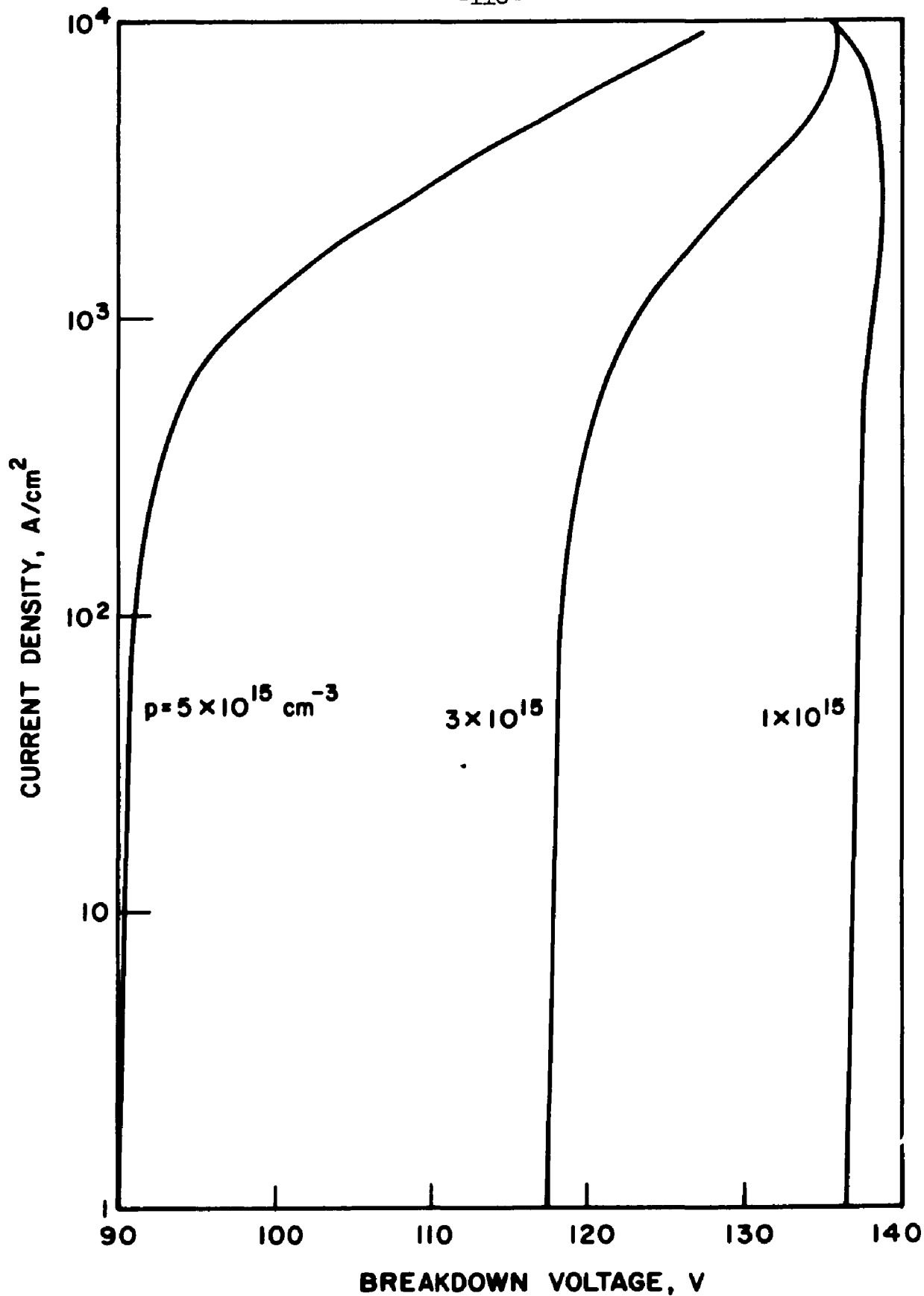


FIG. 3.9 STATIC J-V CHARACTERISTICS FOR A Si ONE-SIDED ABRUPT-JUNCTION DIODE.

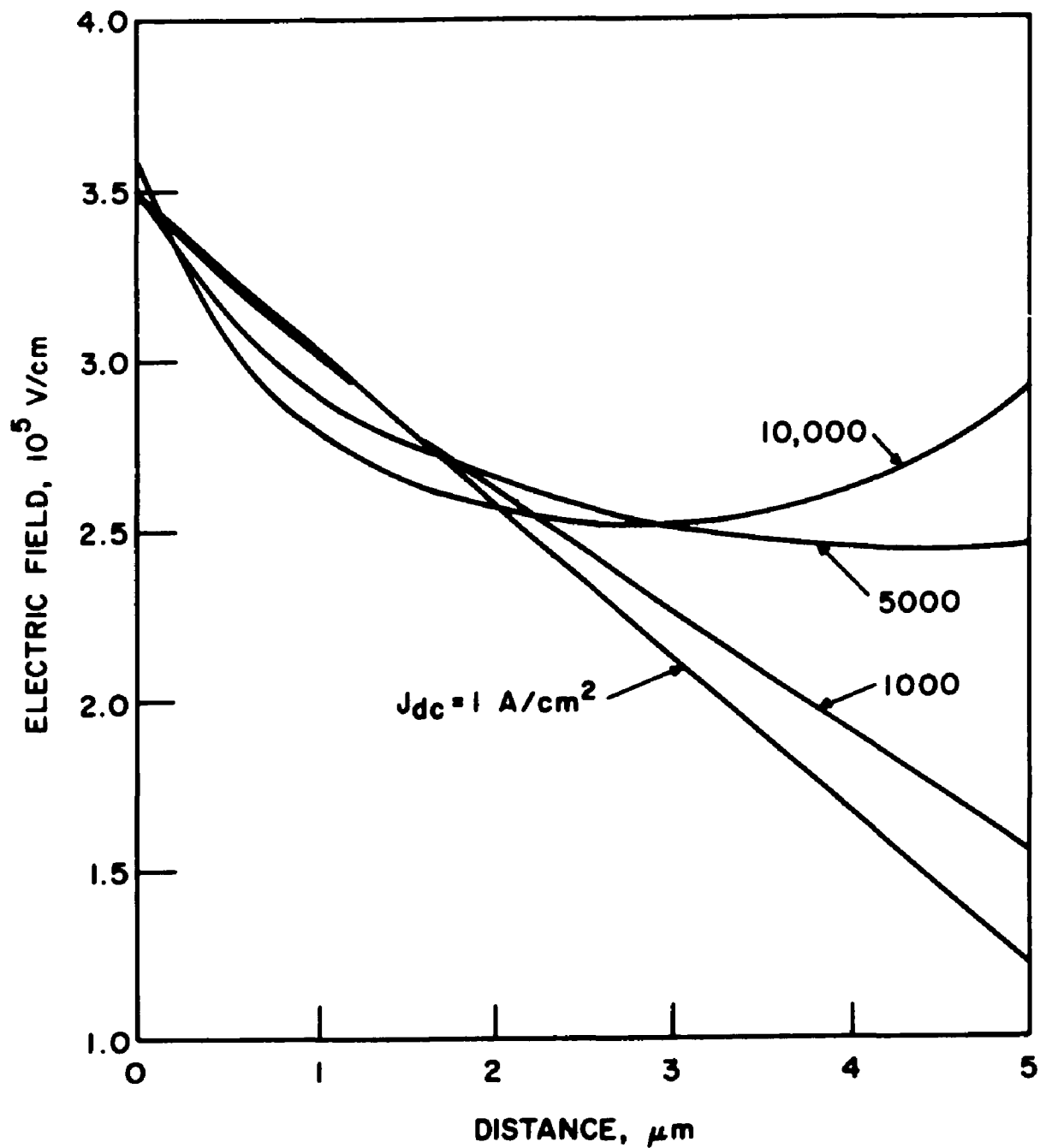


FIG. 3.10 ELECTRIC FIELD VS. DISTANCE FOR A Si ABRUPT-JUNCTION DIODE.  
( $p = 3 \times 10^{15} \text{ cm}^{-3}$ )

curve for the field. This voltage adjusts itself in such a way that the correct dc current density is generated. Due to the nonlinearity of the ionization rates a slight increase in the field at the ends generates a large amount of current and thus above a certain current density the voltage decreases for increasing current density. The current density for which the dc differential conductance becomes negative is the value for which the space charge of the carriers increases the field at both ends to the point where significant avalanching is occurring. By comparing the abrupt junction with different doping densities this effect can be shown. In Fig. 3.7 the electric field for the doping density of  $1 \times 10^{15} \text{ cm}^{-3}$  was shown. In Fig. 3.10 the field for the doping density of  $3 \times 10^{15} \text{ cm}^{-3}$  was shown. In Figs. 3.11 and 3.12 the corresponding hole and electron current densities are plotted. From Fig. 3.9 the dc differential conductance becomes negative between  $10^3 \text{ A/cm}^2$  and  $5 \times 10^3 \text{ A/cm}^2$  for the doping density of  $1 \times 10^{15} \text{ cm}^{-3}$ . In Fig. 3.11 the hole and electron current densities for  $10^3 \text{ A/cm}^2$  increase smoothly through the diode. However for  $5 \times 10^3 \text{ A/cm}^2$  the current densities have an inflection point on the right-hand side due to a larger ionization rate at  $x = x_L$  than toward the middle of the diode. For the abrupt junction with a doping density of  $3 \times 10^{15} \text{ cm}^{-3}$  the dc differential conductance becomes negative between  $5 \times 10^3 \text{ A/cm}^2$  and  $10^4 \text{ A/cm}^2$ . Figure 3.12 shows the same condition of an inflection point in the growth of the electron and hole current densities for  $10^4 \text{ A/cm}^2$ . This inflection point in the electron and hole current densities corresponds to more avalanching occurring at the end than in the middle. The dependence of the dc differential conductance on the doping density is now easily shown. The double-injection diode has a negative dc differential conductance

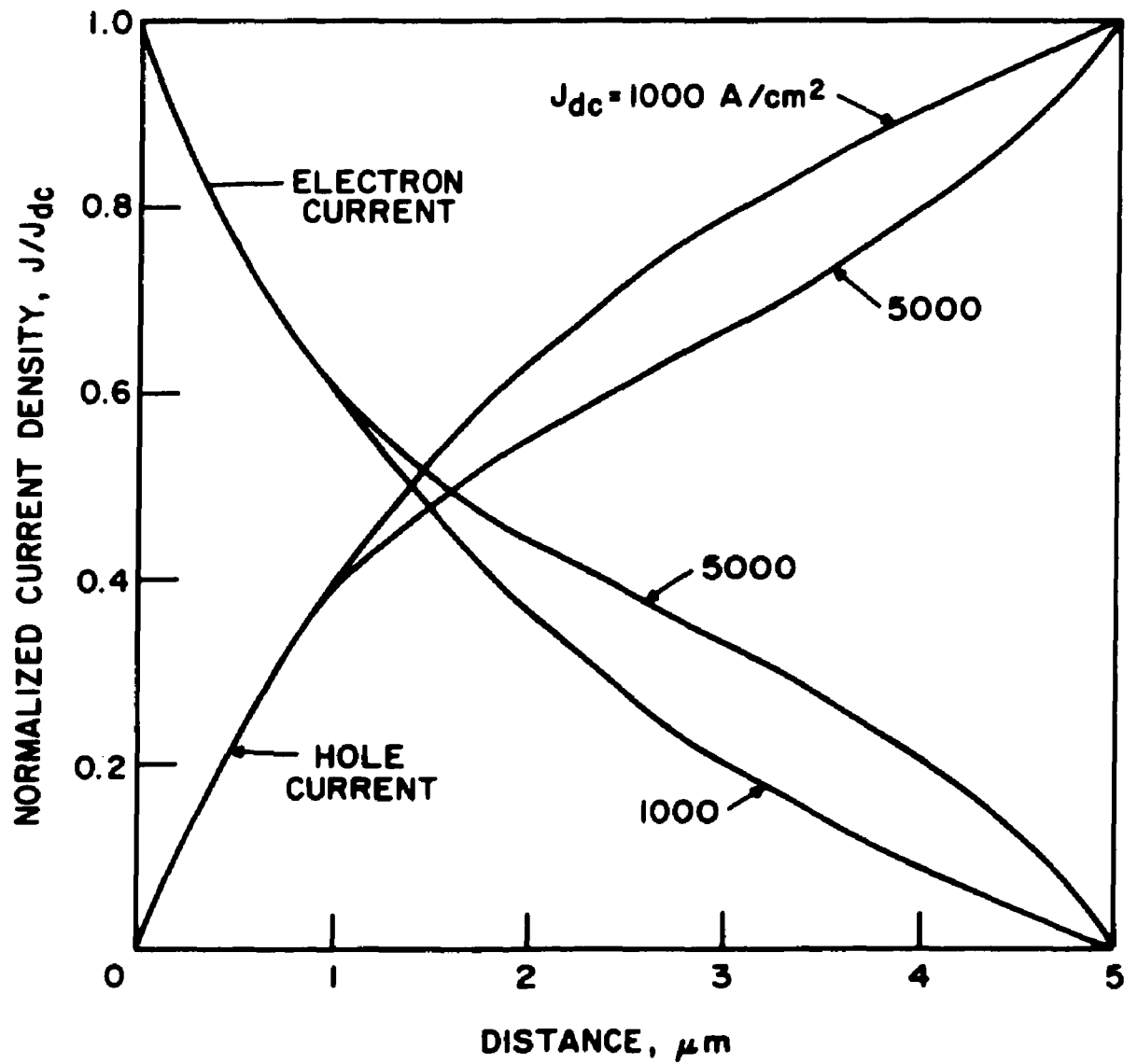


FIG. 3.11 HOLE AND ELECTRON CURRENT DENSITIES VS. DISTANCE FOR A Si ABRUPT-JUNCTION DIODE. ( $p = 1 \times 10^{15} \text{ cm}^{-3}$ )

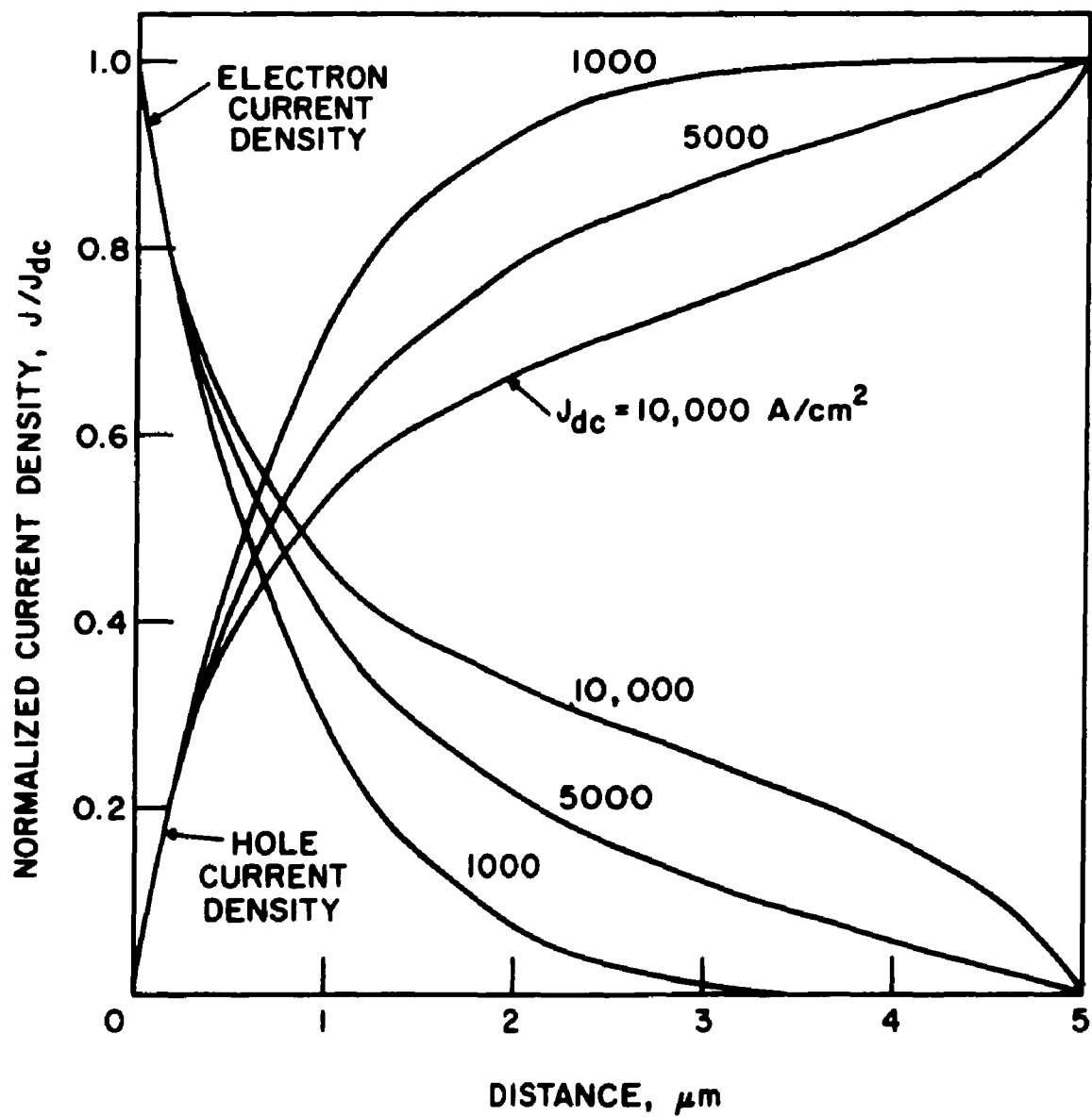


FIG. 3.12 HOLE AND ELECTRON CURRENT DENSITIES VS. DISTANCE FOR A Si ABRUPT-JUNCTION DIODE. ( $p = 3 \times 10^{15} \text{ cm}^{-3}$ )

at very low current densities due to its doping profile. The pin diode has the next lowest value of current density for which the conductance is negative since the amount of space charge necessary to increase the fields at the ends of the diode is small. As the doping density is increased in the abrupt junction it takes an increasing amount of space charge to raise the field to a high enough value to cause avalanching at both ends of the diode.

3.3.3 Single-Frequency Characteristics. The large-signal admittance of the diode structures shown in Fig. 3.2 is calculated from the above analysis for single-frequency operation. Assuming a single-frequency RF voltage across the diode and obtaining the admittance of the diode is the same as solving the problem with the diode in a microwave circuit of this admittance at the operating frequency under steady-state conditions. Inherent in this single-frequency assumption is that all the harmonics of the voltage are shorted by the microwave circuit. This assumption will be discussed and justified in detail in Chapter IV where an equivalent circuit for the diode and package are derived.

The admittance for the three pin diodes of Si, GaAs and Ge are shown in Figs. 3.13 through 3.15. For the current density of  $500 \text{ A/cm}^2$  the dc differential conductance is negative as shown in Fig. 3.3 and thus for the small-signal case (1 V RF), the conductance remains negative as the frequency is decreased. For larger RF voltages the conductance becomes positive as the frequency is decreased. Therefore the small-signal cutoff frequency for the pin diode at this current density goes to zero. Note though that the negative conductance at the low frequencies decreases rapidly with increasing RF voltage. The admittance plots for the three diodes are similar except for the following points. The small-signal

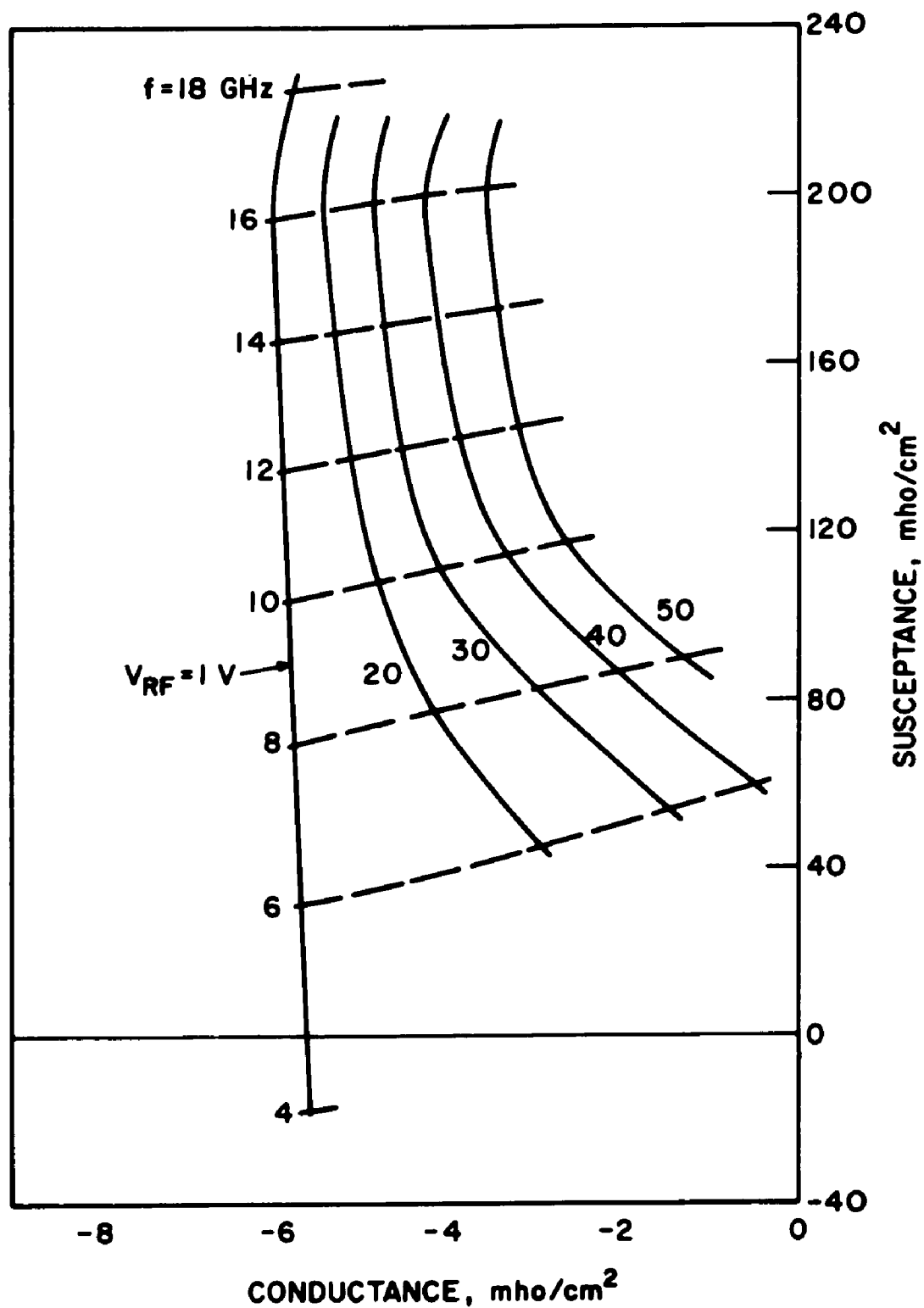


FIG. 3.13 CONDUCTANCE VS. SUSCEPTANCE FOR A Si pin DIODE. ( $J_{dc} = 500$  A/cm<sup>2</sup>)

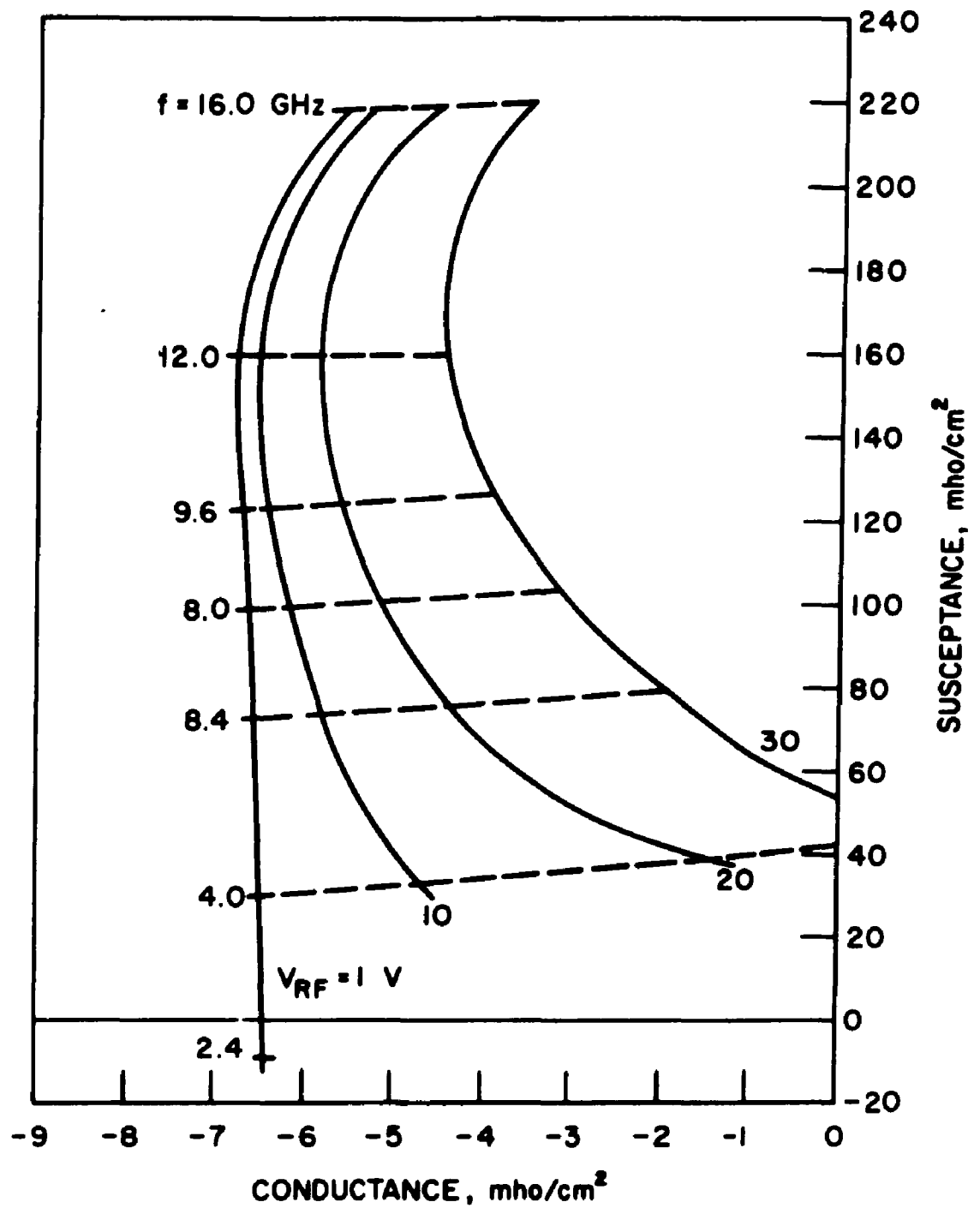


FIG. 3.14 CONDUCTANCE VS. SUSCEPTANCE FOR A GaAs pin DIODE. ( $J_{dc} = 500$  A/cm²)

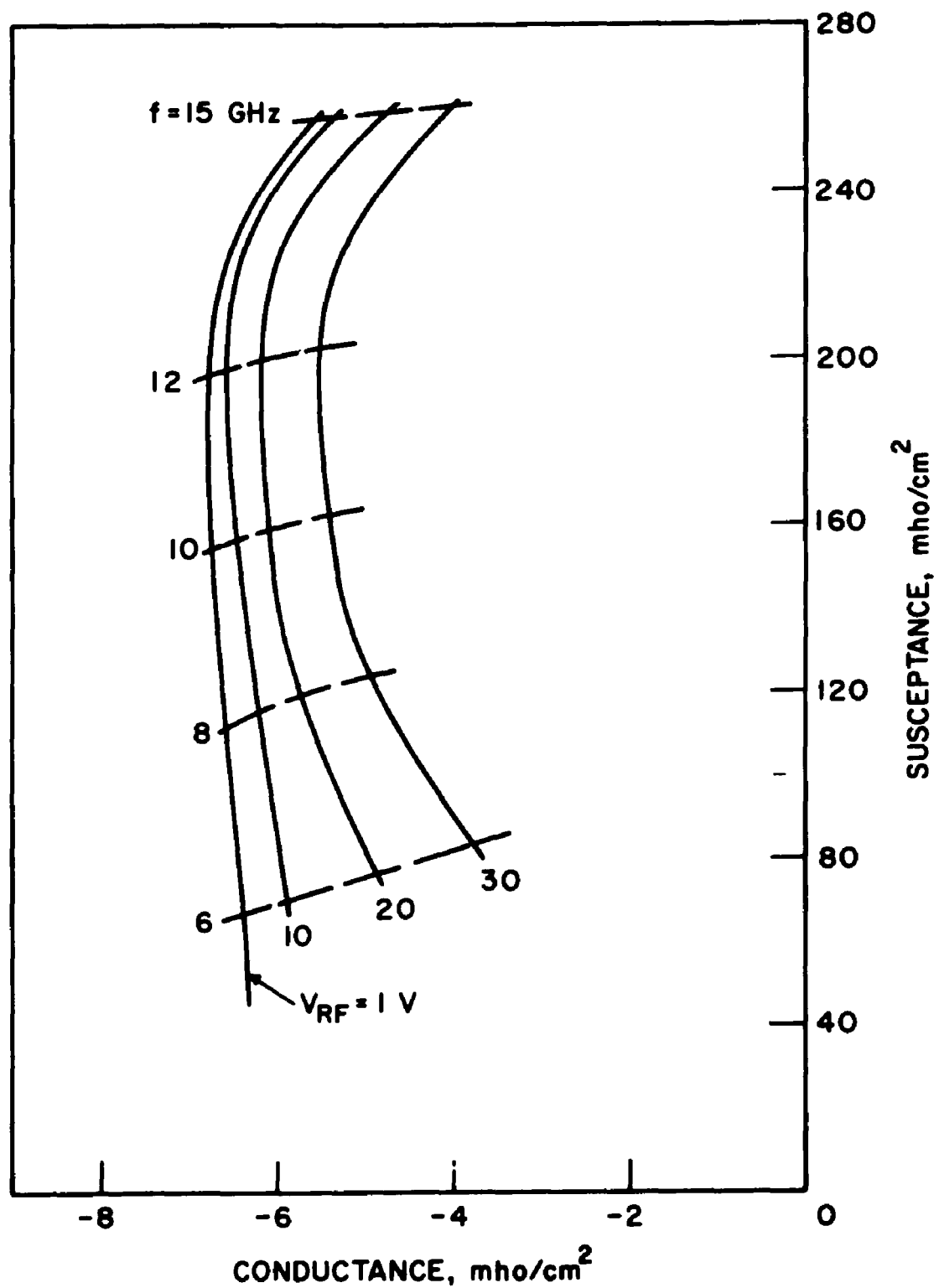


FIG. 3.15 CONDUCTANCE VS. SUSCEPTANCE FOR A Ge pin DIODE. ( $J_{dc} = 500$  A/cm<sup>2</sup>)

negative conductance of the GaAs and Ge diodes is slightly larger than for the Si diode. The susceptance of the three diodes is different for the same operating frequency. Finally, the optimum operating frequency (maximum negative conductance) is slightly different for each diode. It would be expected that the admittances are similar since the only assumed differences in the diodes are the dielectric constant of the material, the ionization rate, and the saturated drift velocity. There appears to be no simple physical explanation for the slightly smaller negative conductance in the Si pin diode. However the shift in the susceptance for a given operating frequency is due to the different dielectric constants of the material. The biggest contribution to the susceptance is the depletion layer capacitance which is directly proportional to the dielectric constant. Thus for a given frequency, 12 GHz, the susceptance is largest for the Ge diode ( $\epsilon_r = 16.5$ ) and smallest for the Si diode ( $\epsilon_r = 11.8$ ). The GaAs diode ( $\epsilon_r = 12.5$ ) has a slightly larger susceptance than the Si diode. The final point, the difference in the optimum operating frequency, can be explained by the difference in the saturated drift velocities which have been assumed for the carriers. As shown in Chapter II the negative conductance is maximum for some given phase delay. This phase delay is inversely proportional to the drift velocity. Thus in order to obtain the same optimum phase delay in the three materials the Si diode must have a higher operating frequency due to its higher drift velocity ( $v = 1 \times 10^7$  cm/s) than the GaAs ( $v = 0.80 \times 10^7$  cm/s) and Ge ( $v = 0.75 \times 10^7$  cm/s) diodes. The maximum negative conductance for the Si diode occurs at approximately 16 GHz, and for the GaAs and Ge diodes, at approximately 12 GHz. A more detailed discussion of the phase delay will be presented later in this

section. In Fig. 3.16 the efficiency of the three pin diodes is compared for the optimum operating frequency as a function of the normalized RF voltage. This RF voltage is the assumed RF voltage divided by the actual dc operating voltage. Due to the nonlinearity of the ionization rates with respect to the field, as the RF voltage increases in magnitude the dc voltage across the diode must decrease in order to maintain the dc current constant. This effect is shown in Fig. 3.17 for the Si pin diode operating at 10 GHz. The GaAs diode has the largest efficiency due to two factors. Since the efficiency is plotted against normalized RF voltage, the RF voltage is much larger in the GaAs diode than in the Ge diode for a given normalized RF voltage. Even though the input power in the GaAs diode is twice that of the Ge diode the output power is proportional to the square of the RF voltage and therefore it is much larger for a given normalized RF voltage. The difference in the efficiencies of the Si and GaAs diodes is due to the above-mentioned effect which is small for these two diodes and is also due to the difference in the magnitude of the negative conductance. The efficiency for the Si diode at 8 GHz is plotted to show the effect of a nonoptimum operating frequency.

The admittance for the double-injection diode is plotted in Fig. 3.18. For the current density of  $500 \text{ A/cm}^2$  the dc differential conductance is negative and the conductance becomes more negative for decreasing frequency. However this low-frequency negative conductance decreases rapidly with increasing RF voltage. The optimum operating frequency for this diode is approximately 10 to 12 GHz. In Fig. 3.19 the effect of increasing the current density on the small-signal conductance is presented. As the current density is decreased the space-charge effects decrease and the characteristics are similar to a pin diode.

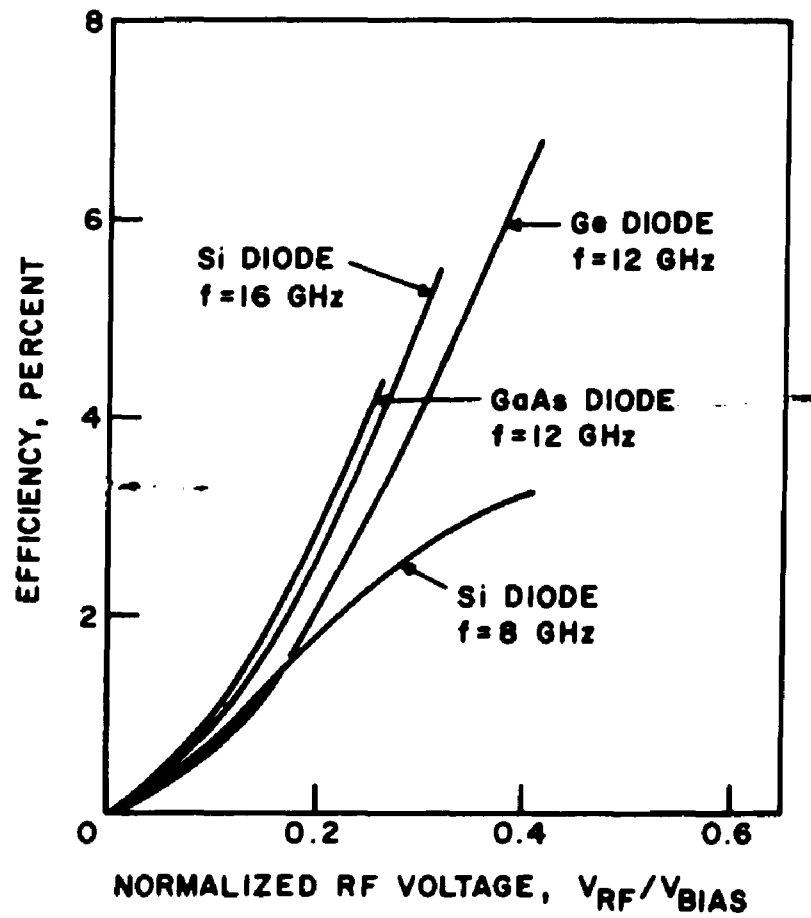


FIG. 3.16 EFFICIENCY VS. NORMALIZED RF VOLTAGE FOR pin DIODES.

( $J_{dc} = 500 \text{ A/cm}^2$ )

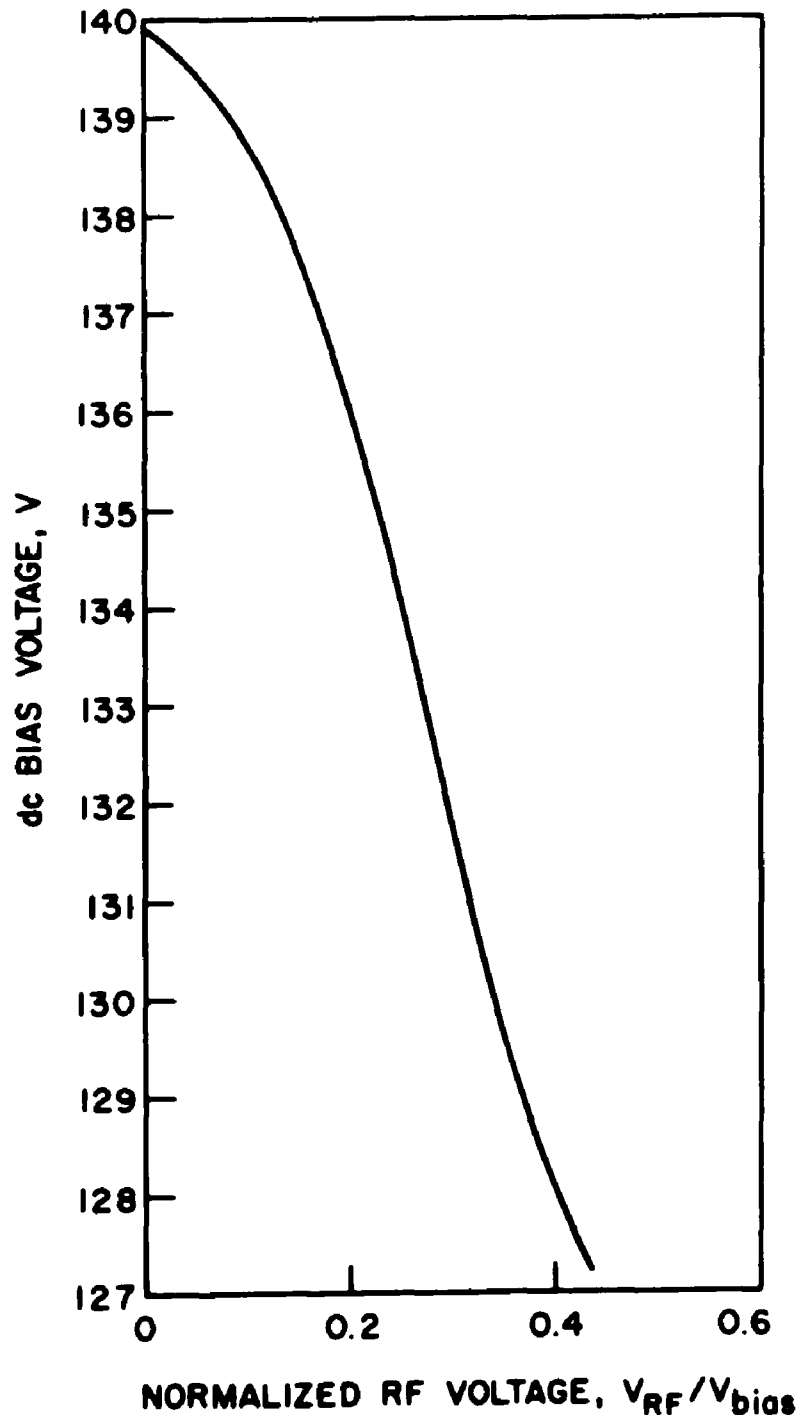


FIG. 3.17 Dc BIAS VOLTAGE VS. NORMALIZED RF VOLTAGE FOR A Si pin DIODE.

( $J_{dc} = 500 \text{ A/cm}^2$  AND  $f = 10 \text{ GHz}$ )

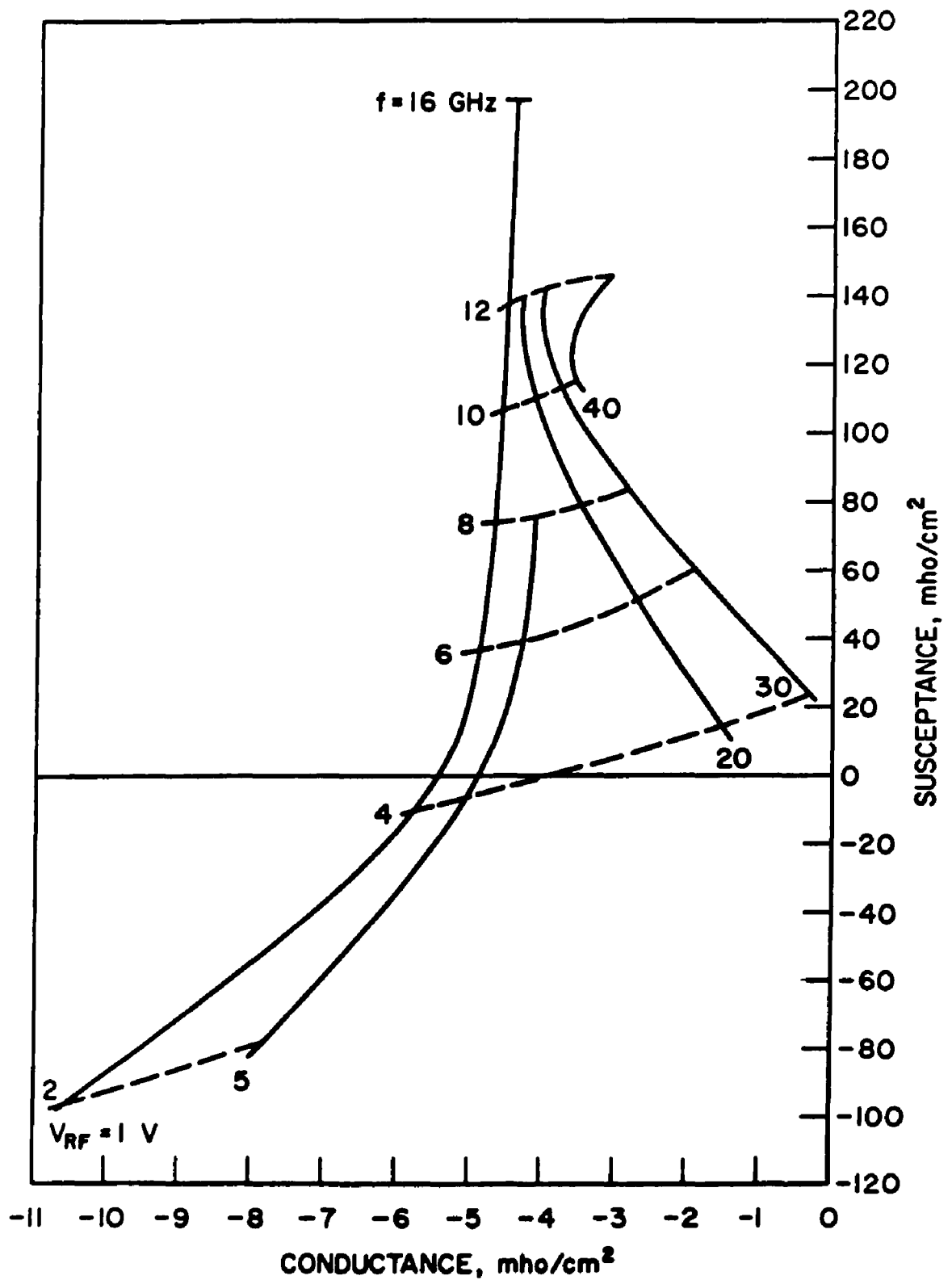


FIG. 3.18 CONDUCTANCE VS. SUSCEPTANCE FOR A Si  $n^+pnp^+$  DIODE. ( $J_{dc} = 500$  A/cm<sup>2</sup> AND  $n = p = 1 \times 10^{15}$  cm<sup>-3</sup>)

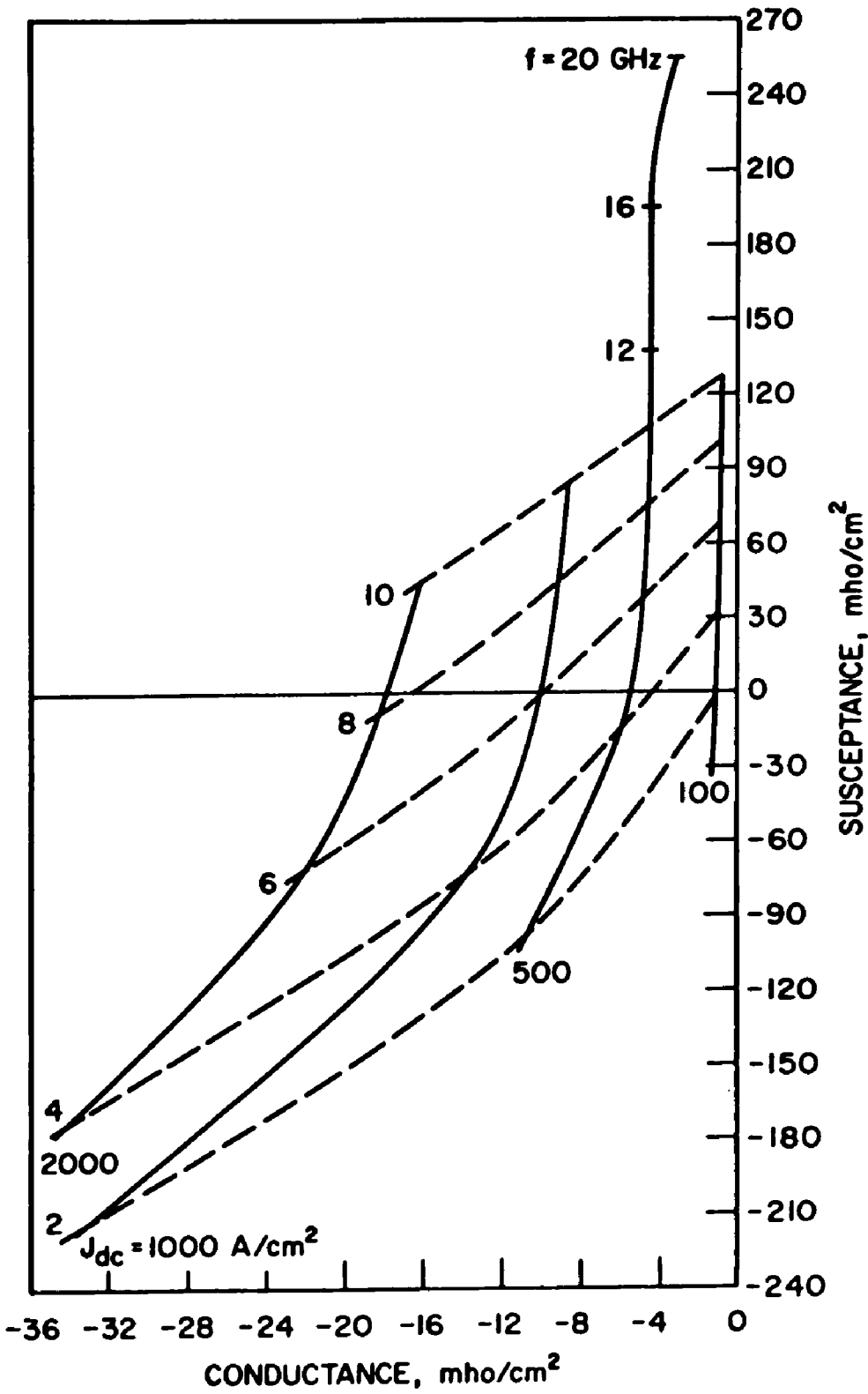


FIG. 3.19 CONDUCTANCE VS. SUSCEPTANCE FOR A Si  $n^+pnp^+$  DIODE. ( $V_{RF} = 1$  V AND  $n = p = 1 \times 10^{15} \text{ cm}^{-3}$ )

The next structure for which the admittance is calculated is the p-n junction. This admittance is plotted in Fig. 3.20 for a current density of  $500 \text{ A/cm}^2$ . For this current density the dc differential conductance is positive for this structure as shown in Fig. 3.3. Therefore as the frequency is decreased the conductance becomes positive even for the small-signal case. The optimum operating frequency is approximately 16 GHz as in the pin diode. Also the negative conductance is larger for this diode than for the previous two diode structures.

The admittance for the one-sided abrupt junction with different doping densities is plotted in Figs. 3.21 through 3.23. These characteristics are for a current density of  $500 \text{ A/cm}^2$  and therefore the conductance becomes positive for decreasing frequency. As the doping density is increased the negative conductance of the diode increases and the optimum operating frequencies decrease slightly.

Abrupt junctions with higher doping densities were not analyzed since the field goes to zero in the depletion layer of the diode and thus violates the assumption of saturated drift velocities for the carriers. Higher doping densities in the p-n junction, considered previously, were not investigated since the negative conductance would increase in the same manner as for the abrupt junction. However, two points should be made: the optimum operating frequency would be higher and the heat dissipation could be a problem at high power levels since the avalanche region is located in the center of the diode.

Finally, the admittance for a Read diode is shown in Fig. 3.24 for a current density of  $500 \text{ A/cm}^2$ . The magnitude of the small-signal negative conductance is slightly less than that of the abrupt junction with

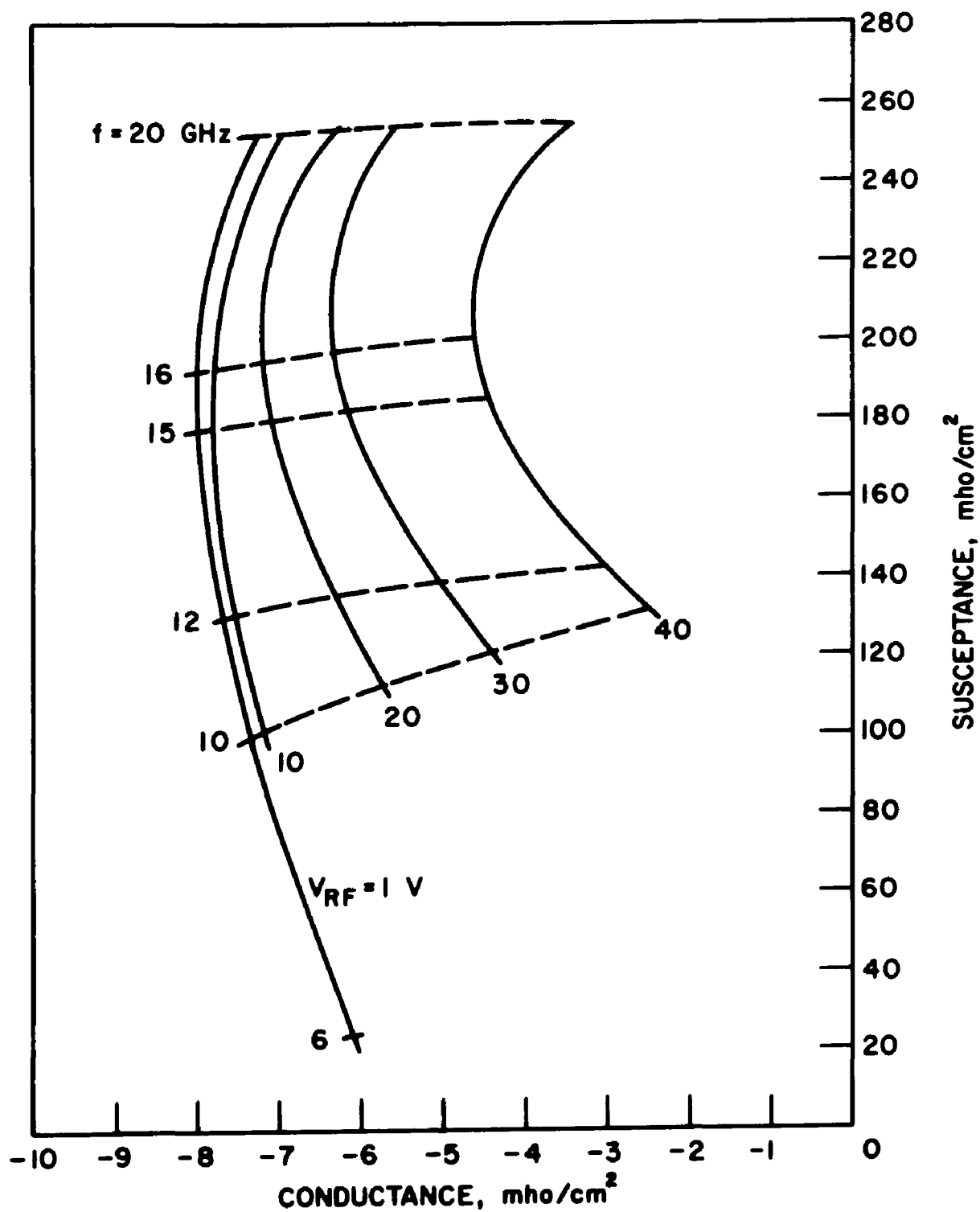


FIG. 3.20 CONDUCTANCE VS. SUSCEPTANCE FOR A Si  $n^+npp^+$  DIODE. ( $J_{dc} = 500$  A/cm<sup>2</sup> AND  $n = p = 1 \times 10^{15}$  cm<sup>-3</sup>)

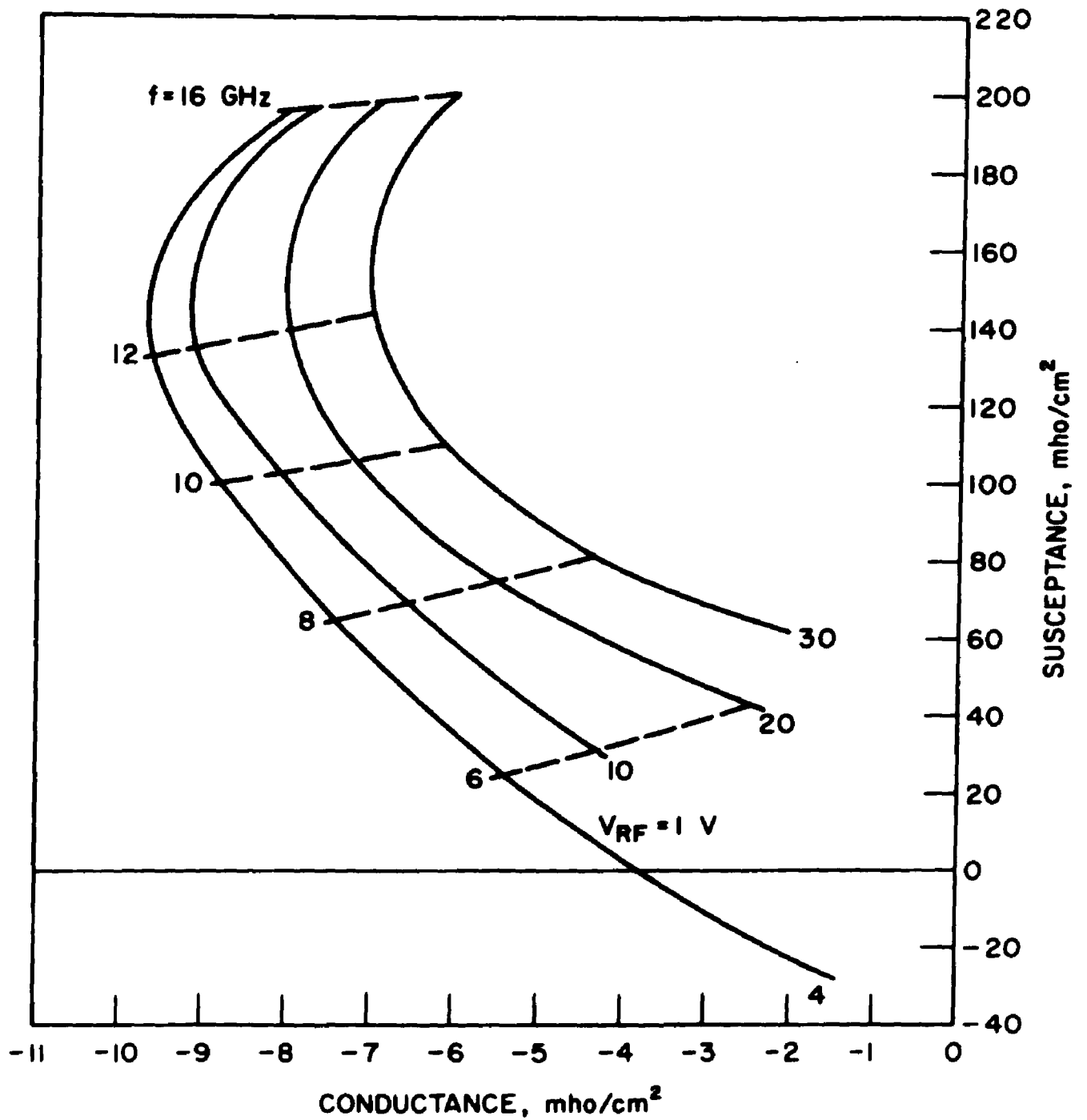


FIG. 3.21 CONDUCTANCE VS. SUSCEPTANCE FOR A Si  $n^+pp^+$  DIODE. ( $J_{dc} = 500$  A/cm<sup>2</sup> AND  $p = 1 \times 10^{15}$  cm<sup>-3</sup>)

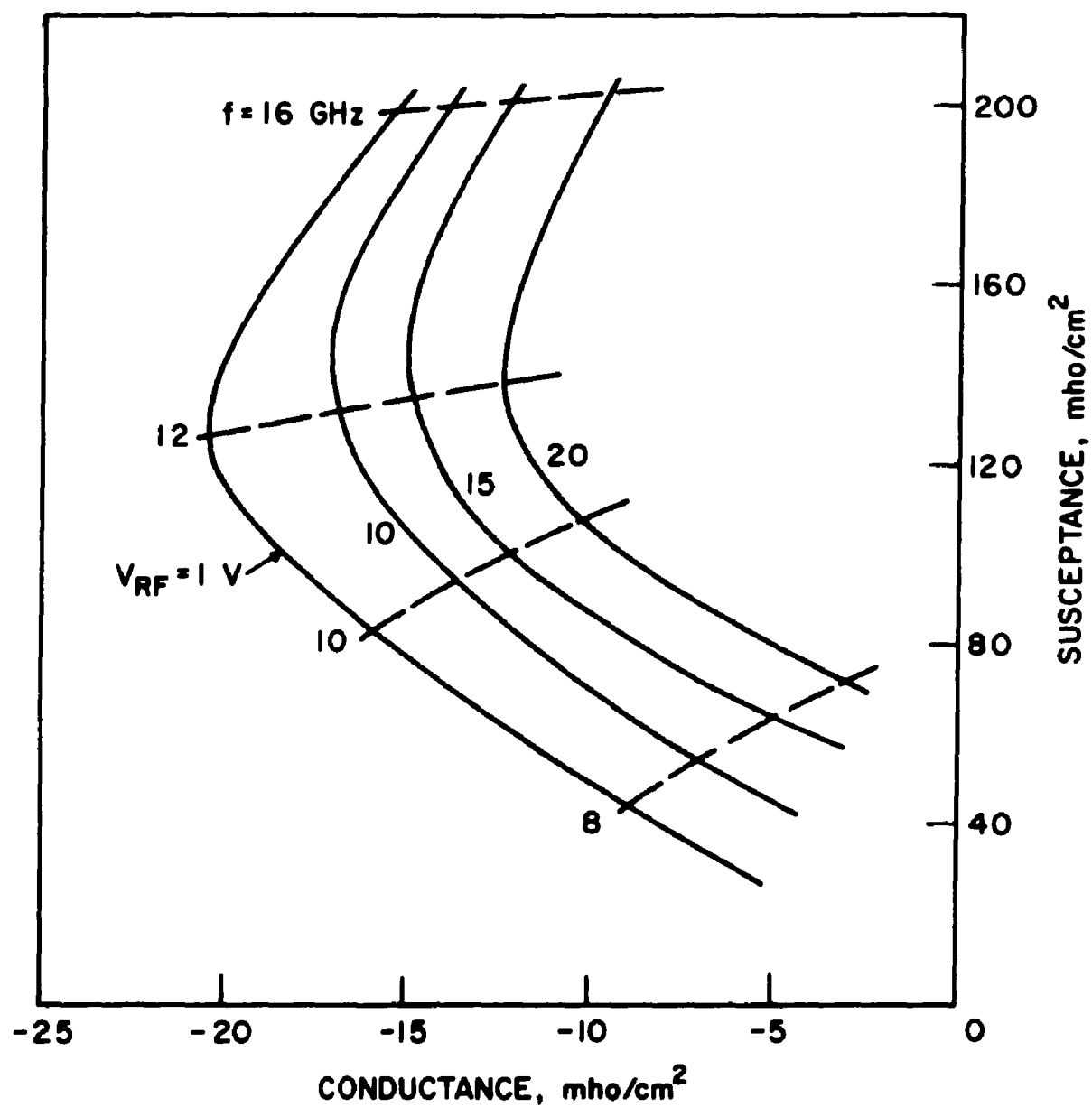


FIG. 3.22 CONDUCTANCE VS. SUSCEPTANCE FOR A Si  $n^+pp^+$  DIODE. ( $J_{dc} = 500 \text{ A/cm}^2$  AND  $p = 3 \times 10^{15} \text{ cm}^{-3}$ )

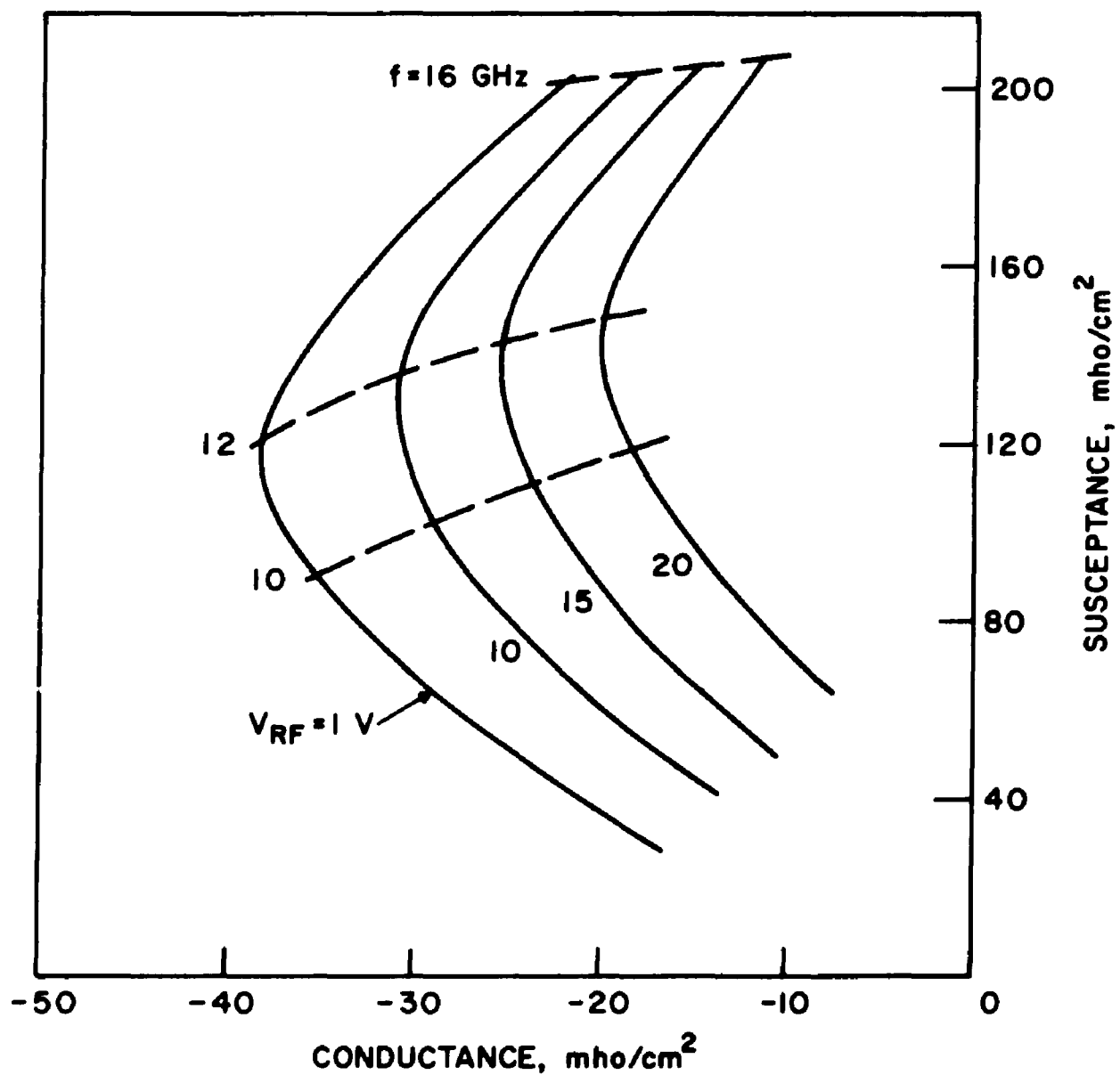


FIG. 3.23 CONDUCTANCE VS. SUSCEPTANCE FOR A Si  $n^+pp^+$  DIODE. ( $J_{dc} = 500 \text{ A/cm}^2$  AND  $p = 5 \times 10^{15} \text{ cm}^{-3}$ )

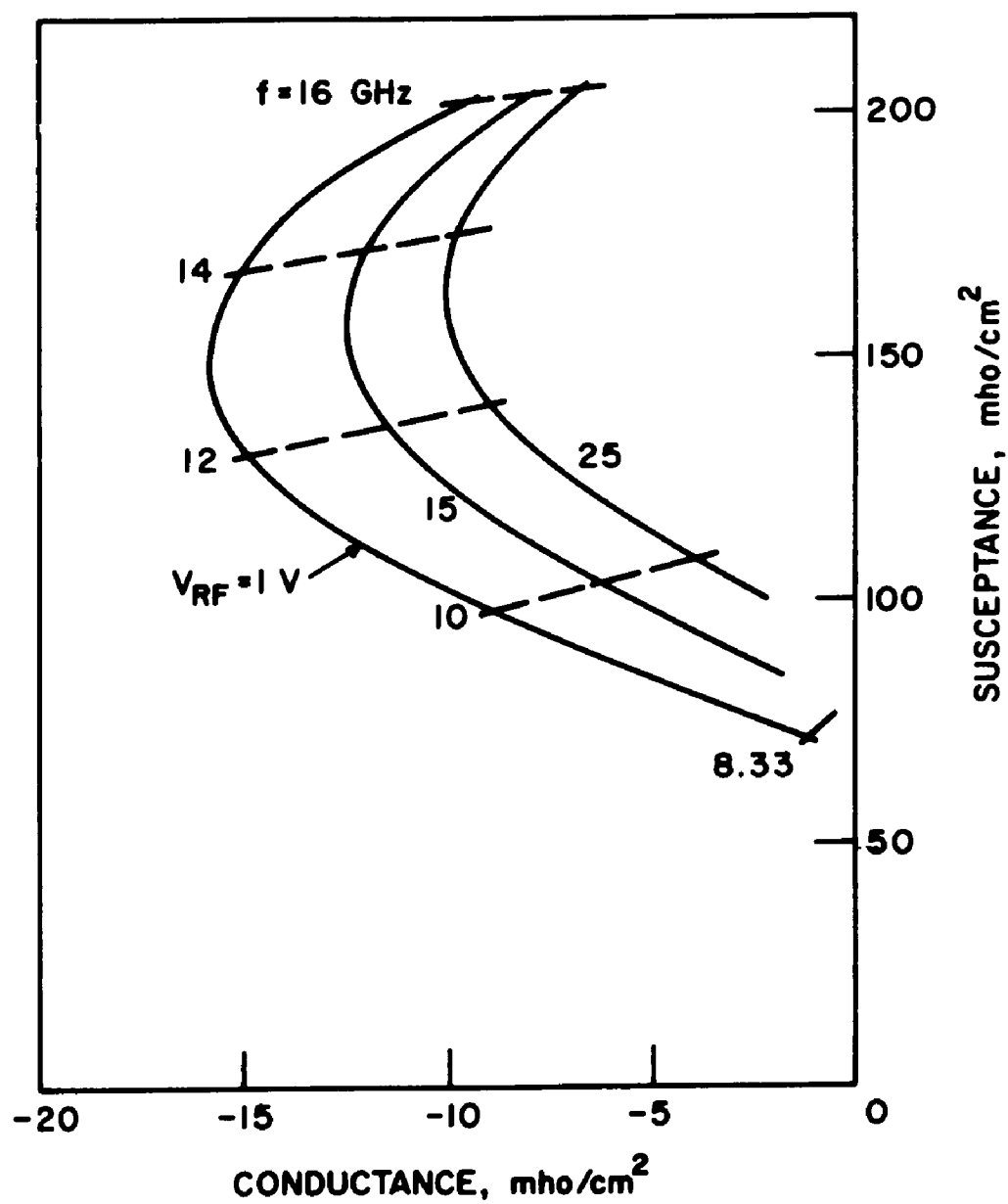


FIG. 3.24 CONDUCTANCE VS. SUSCEPTANCE FOR A Si READ DIODE. ( $J_{dc} = 500 \text{ A/cm}^2$  AND  $p_A = 1 \times 10^{16} \text{ cm}^{-3}$ )

a doping density of  $3 \times 10^{15} \text{ cm}^{-3}$ . Also the optimum frequency is approximately 13 GHz.

In Fig. 3.25 the conductance at the optimum operating frequency is plotted as a function of the normalized RF voltage for the different diode structures. The ideal Read diode results are for an analysis as described in Chapter II. This is included for reference. The first point to be considered is the reason for the difference in magnitude of the small-signal conductance for different doping profiles. This can be understood by considering the conduction current density in the diode as it is generated. In Figs. 3.26 through 3.32 the conduction current density is plotted as a function of distance for different phases of the RF voltage cycle. For the double-injection diode the conduction current density is generated over a wide region of the diode as can be seen by the relatively flat conduction-current curve. The pin diode generates the current over a slightly narrower but still large region. As the doping density in the abrupt junction is increased the current modulation increases. This is shown in Fig. 3.33 where the conduction current density at a phase angle of approximately 180 degrees is plotted for several of the diodes. In the pin diode due to the large region over which carriers are generated, the transit angle of the carriers varies over a wide range of angles and only a few of them experience the same phase delay. As the doping density is increased in the abrupt junction the avalanche region becomes narrower and the conduction current is generated in a sharper pulse. Thus more of the carriers experience the same transit angle. For the proper frequency, more of the carriers can be made to experience the optimum transit angle as the doping density is increased. Thus the negative conductance of the

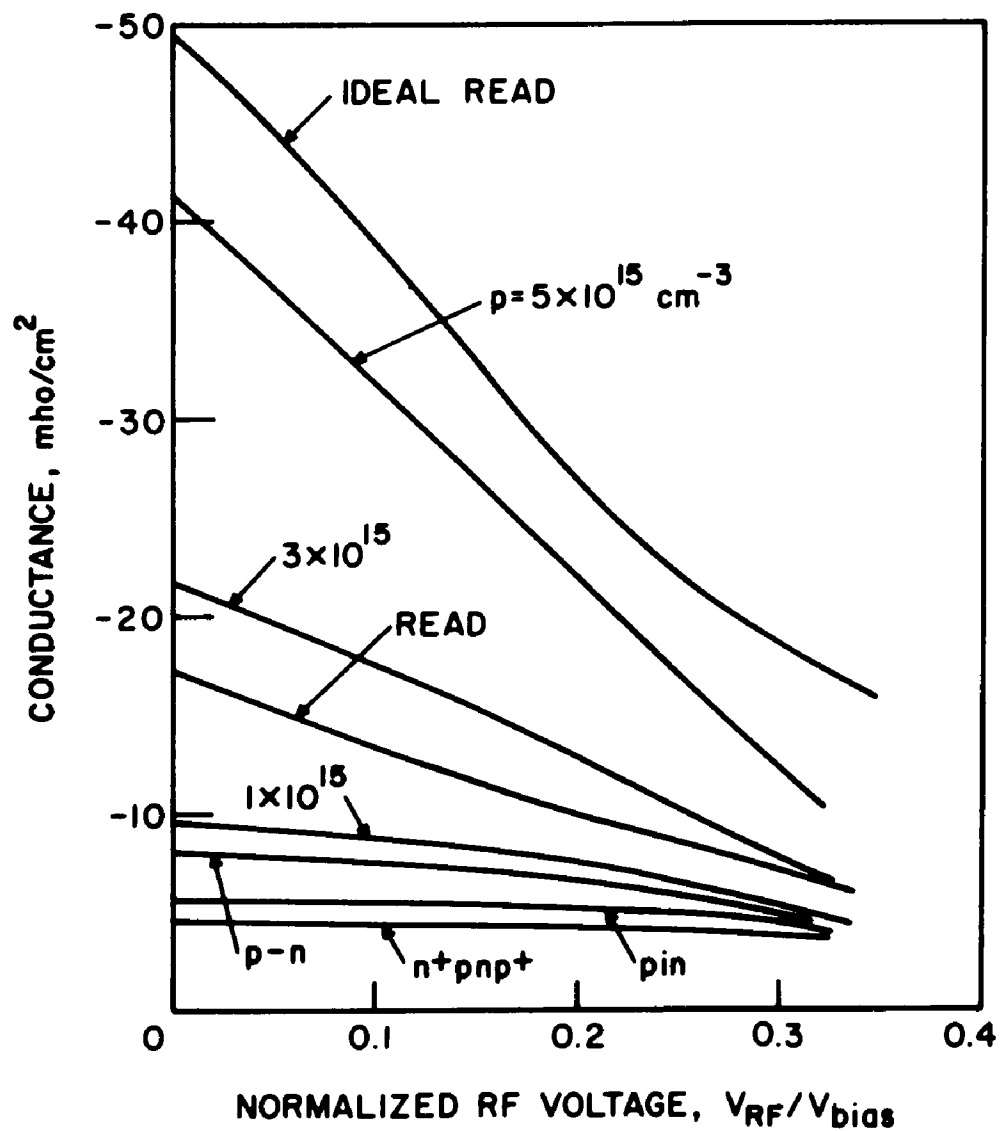
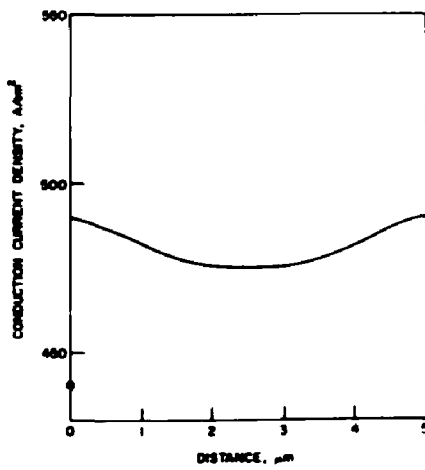


FIG. 3.25 CONDUCTANCE VS. NORMALIZED RF VOLTAGE FOR DIFFERENT Si DIODE STRUCTURES AT THE OPTIMUM FREQUENCY. ( $J_{dc} = 500 \text{ A/cm}^2$ )

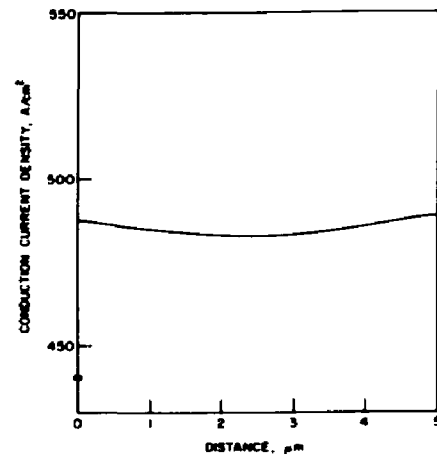
diode is larger when the distribution of the transit angles of the carriers is concentrated around the optimum transit angle rather than when the distribution is spread out over a wide range but has an average equal to the optimum.

A second point to consider is the optimum frequency of operation for each structure. This frequency is determined by the total phase delay of the current due to both the avalanching process and the transit time. The phase delay caused by the generation of carriers in the avalanche process can be determined as was done in Chapter II. The avalanche process is stopped when the RF field in the avalanche region goes through zero. For the diodes investigated in this chapter, the avalanche region will be defined as the region where the conduction current density is maximum during the avalanche process. The electric field in the avalanche region is normalized to the value of the field at  $\omega t = 0$  and plotted in Fig. 3.34 for three of the diode structures. In the pin diode the avalanche region is spread over a large region of the diode but as shown in Fig. 3.27 the conduction current density peaks in the middle of the diode. Therefore the middle of the diode is considered as the avalanche region and the carriers drift approximately one half the length of the diode. In the one-sided abrupt junction with a doping density of  $5 \times 10^{15} \text{ cm}^{-3}$ , Fig. 3.32 shows that the avalanche region is approximately  $0.5 \mu\text{m}$  from the left of the diode and the drift region is  $4.5 \mu\text{m}$  long.

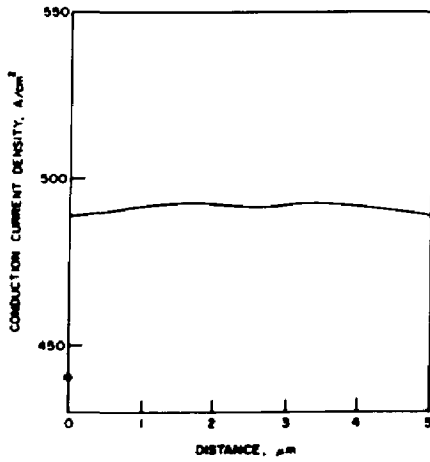
The Read diode has an avalanche region approximately  $0.35 \mu\text{m}$  from the left end of the diode as shown in Fig. 3.29. The instant of time at which the avalanche process is turned off and the conduction current density stops growing is when the RF field in the avalanche region goes through zero



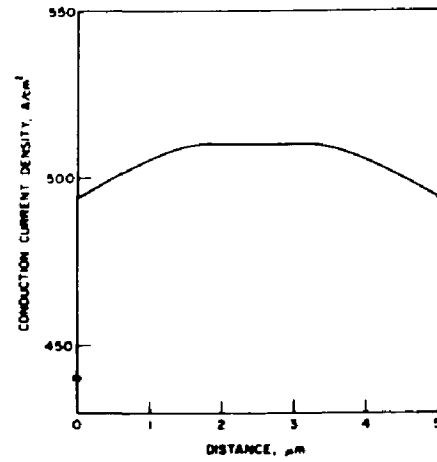
(a)  $\omega t = 7$  DEGREES



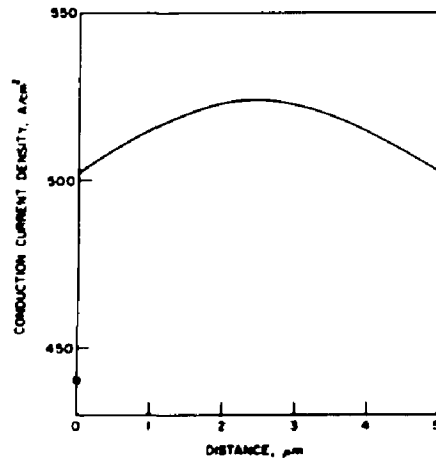
(b)  $\omega t = 43$  DEGREES



(c)  $\omega t = 79$  DEGREES

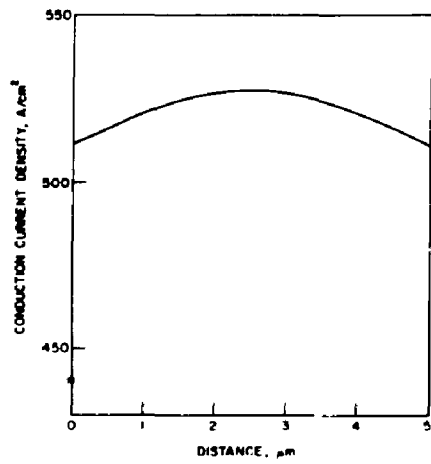


(d)  $\omega t = 115$  DEGREES

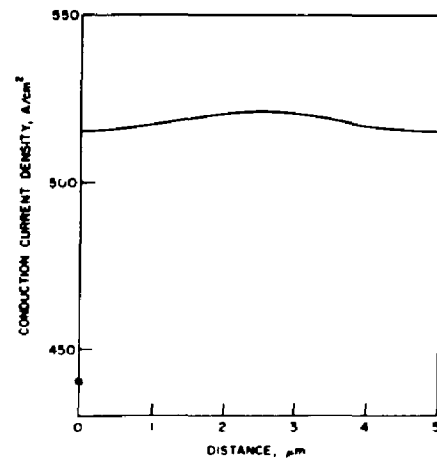


(e)  $\omega t = 151$  DEGREES

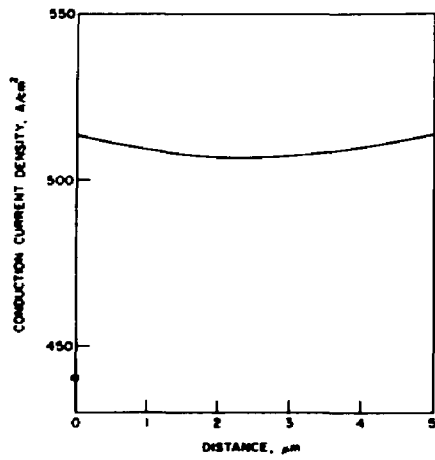
FIG. 3.26 CONDUCTION CURRENT DENSITY VS. DISTANCE FOR A Si DOUBLE-INJECTION DIODE AT DIFFERENT PHASES OF THE RF VOLTAGE. ( $J_{dc} = 500 \text{ A/cm}^2$ ,  $V_{RF} = 1 \text{ V}$ ,  $f = 12 \text{ GHz}$  AND  $n = p = 1 \times 10^{15} \text{ cm}^{-3}$ )



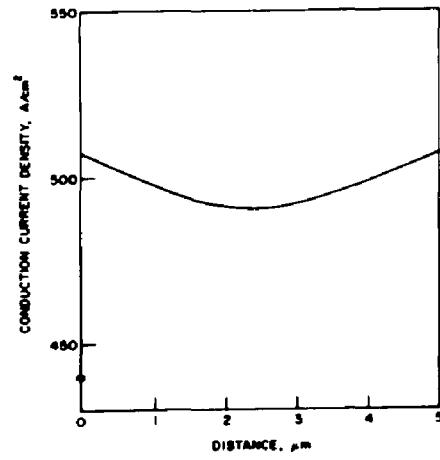
(f)  $\omega t = 187$  DEGREES



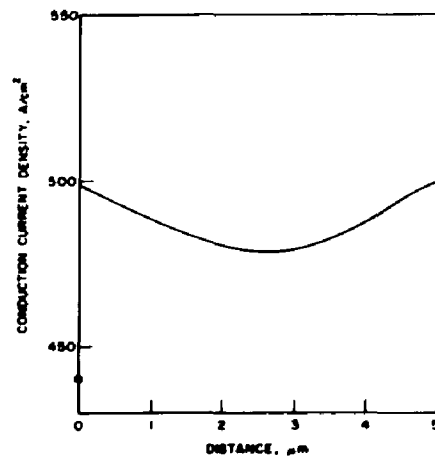
(g)  $\omega t = 223$  DEGREES



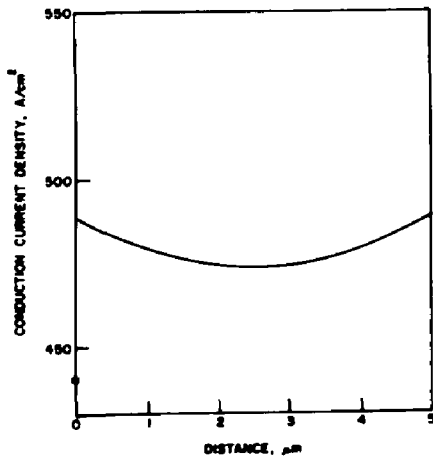
(h)  $\omega t = 259$  DEGREES



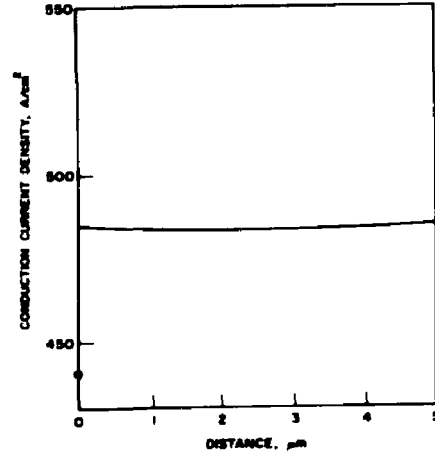
(i)  $\omega t = 295$  DEGREES



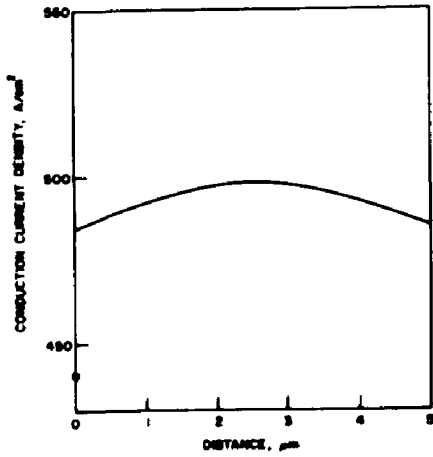
(j)  $\omega t = 331$  DEGREES



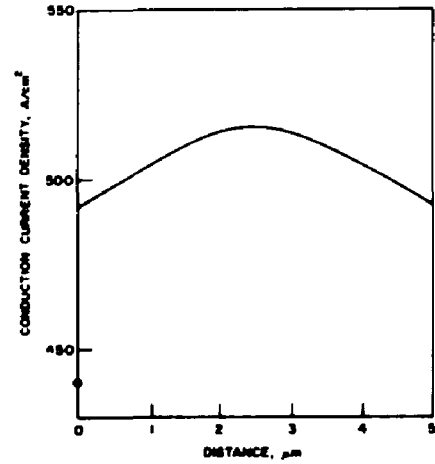
(a)  $\omega t = 7$  DEGREES



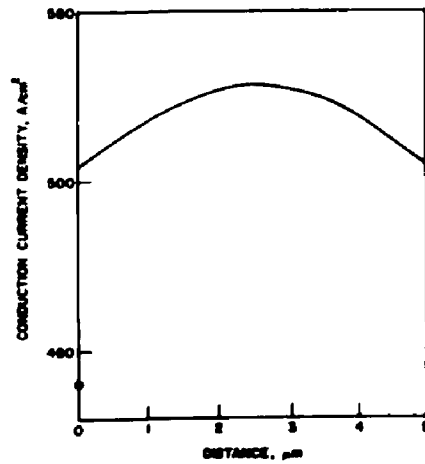
(b)  $\omega t = 43$  DEGREES



(c)  $\omega t = 79$  DEGREES

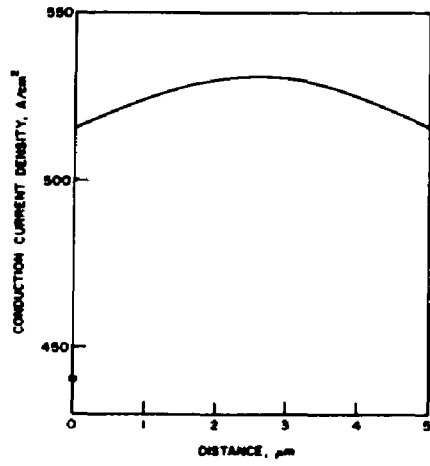


(d)  $\omega t = 115$  DEGREES

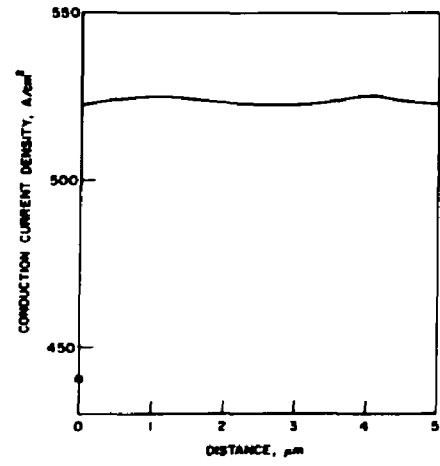


(e)  $\omega t = 151$  DEGREES

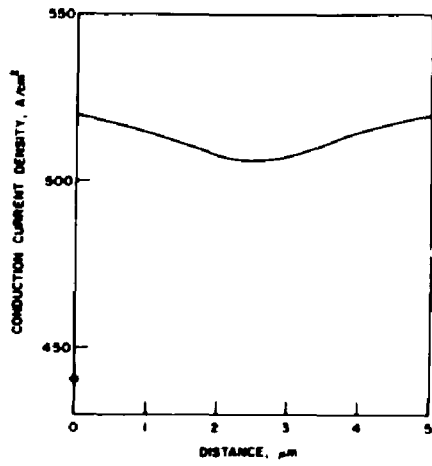
FIG. 3.27 CONDUCTION CURRENT DENSITY VS. DISTANCE FOR A Si pin DIODE  
AT DIFFERENT PHASES OF THE RF VOLTAGE. ( $f = 12$  GHz,  $V_{RF} = 1$  V  
AND  $J_{dc} = 500$  A/cm<sup>2</sup>)



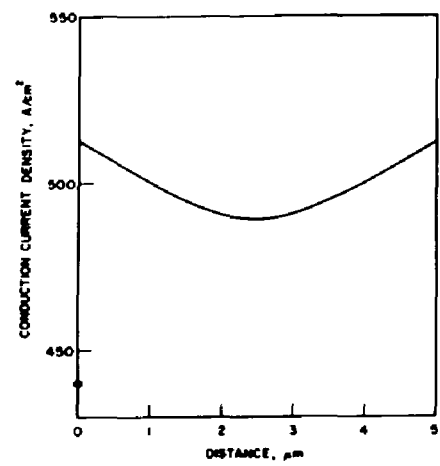
(f)  $\omega t = 187$  DEGREES



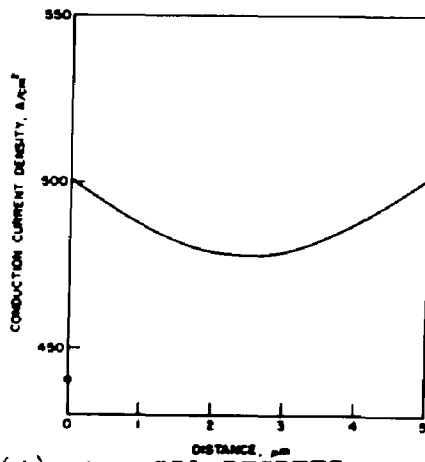
(g)  $\omega t = 223$  DEGREES



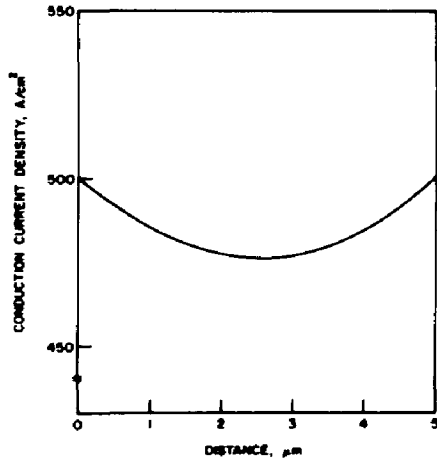
(h)  $\omega t = 259$  DEGREES



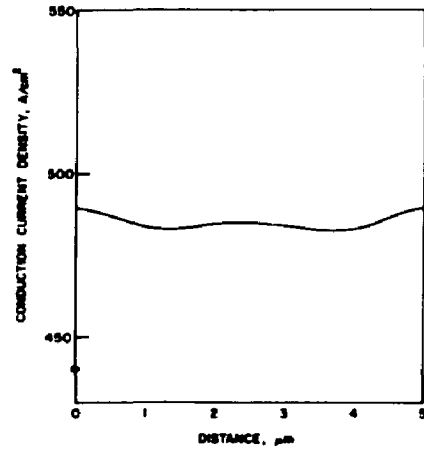
(i)  $\omega t = 295$  DEGREES



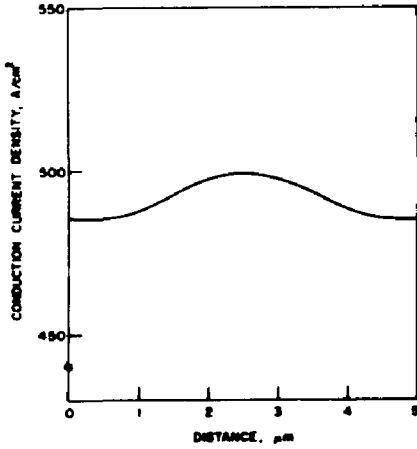
(j)  $\omega t = 331$  DEGREES



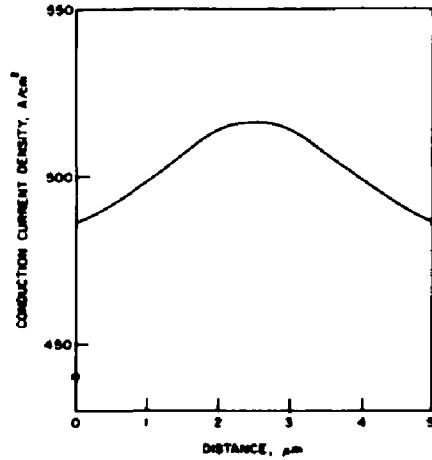
(a)  $\omega t = 7$  DEGREES



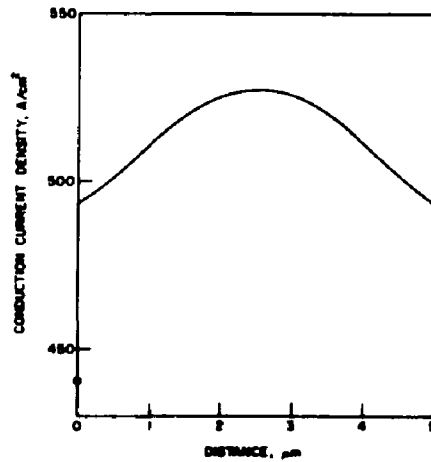
(b)  $\omega t = 43$  DEGREES



(c)  $\omega t = 79$  DEGREES

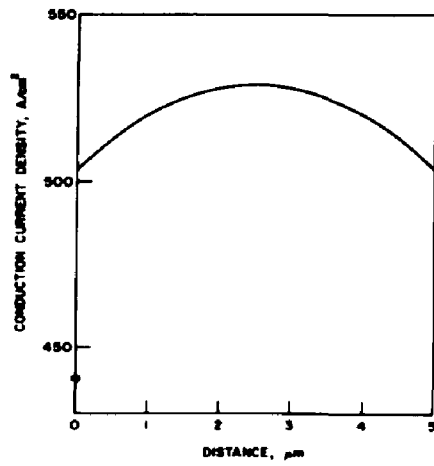


(d)  $\omega t = 115$  DEGREES

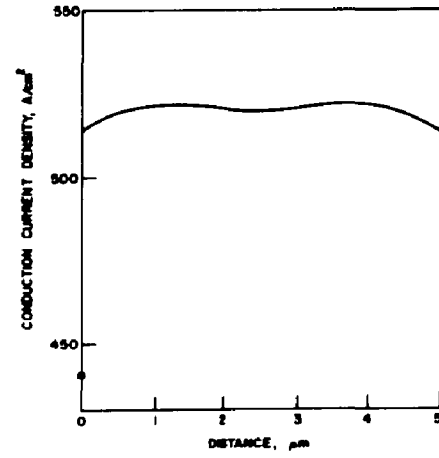


(e)  $\omega t = 151$  DEGREES

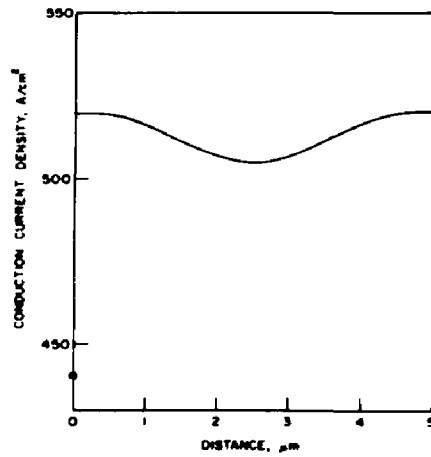
FIG. 3.28 CONDUCTION CURRENT DENSITY VS. DISTANCE FOR A Si p-n JUNCTION DIODE AT DIFFERENT PHASES OF THE RF VOLTAGE. ( $J_{dc} = 500 \text{ A/cm}^2$ ,  $V_{RF} = 1 \text{ V}$ ,  $f = 16 \text{ GHz}$  AND  $n = p = 1 \times 10^{15} \text{ cm}^{-3}$ )



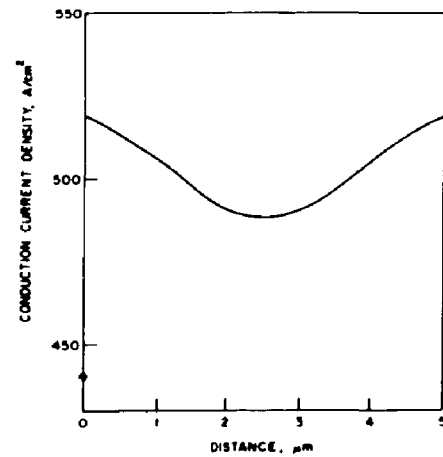
(f)  $\omega t = 187$  DEGREES



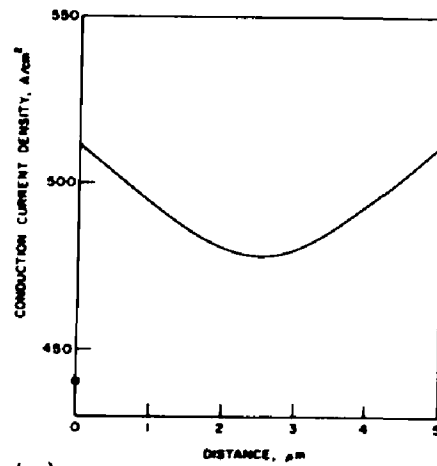
(g)  $\omega t = 223$  DEGREES



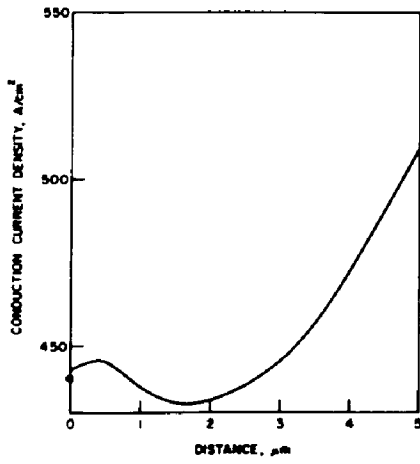
(h)  $\omega t = 259$  DEGREES



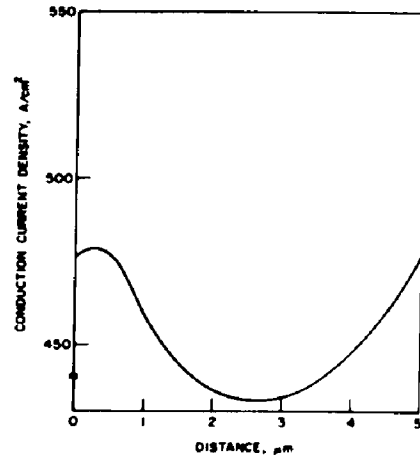
(i)  $\omega t = 295$  DEGREES



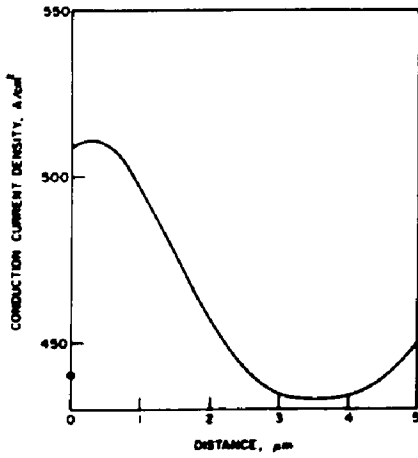
(j)  $\omega t = 331$  DEGREES



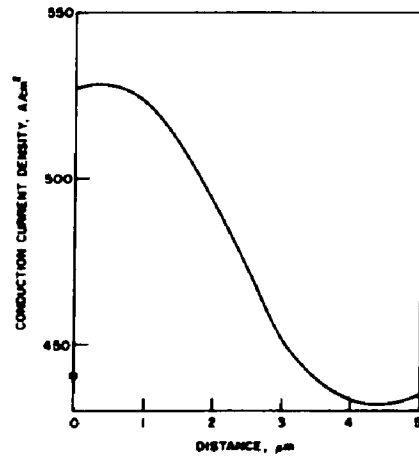
(a)  $\omega t = 7$  DEGREES



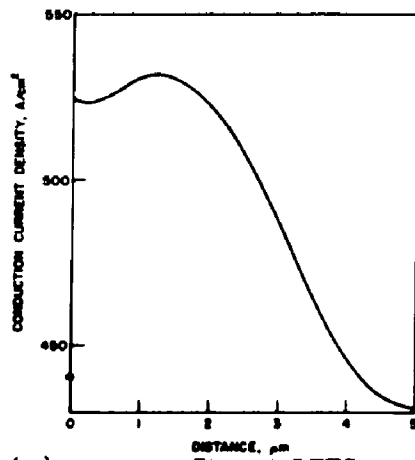
(b)  $\omega t = 43$  DEGREES



(c)  $\omega t = 79$  DEGREES

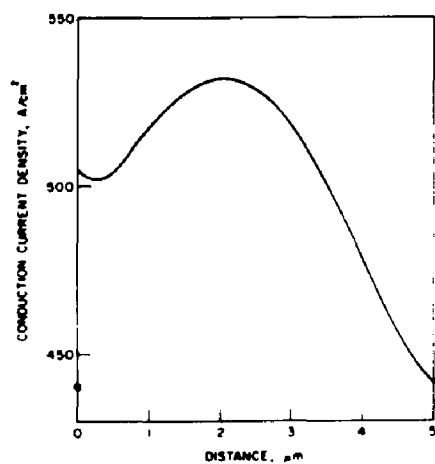


(d)  $\omega t = 115$  DEGREES

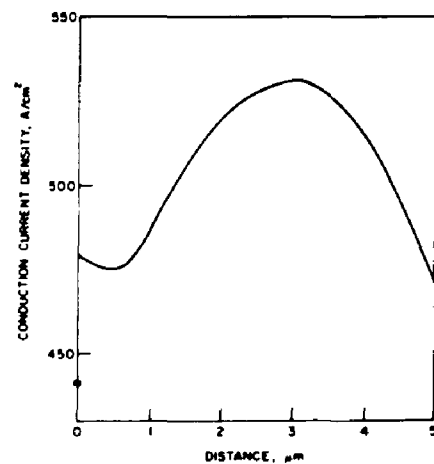


(e)  $\omega t = 151$  DEGREES

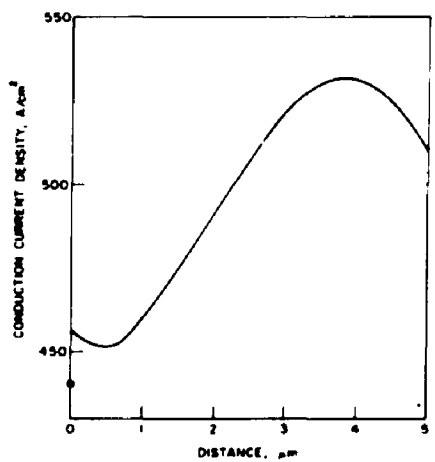
FIG. 3.29 CONDUCTION CURRENT DENSITY VS. DISTANCE FOR A Si READ DIODE AT DIFFERENT PHASES OF THE RF VOLTAGE. ( $f = 12$  GHz,  $J_{dc} = 500$  A/cm<sup>2</sup>,  $V_{RF} = 1$  V,  $p_A = 1 \times 10^{16}$  FOR  $1 \mu\text{m}$  AND  $p = 1 \times 10^{13}$  FOR  $4 \mu\text{m}$ )



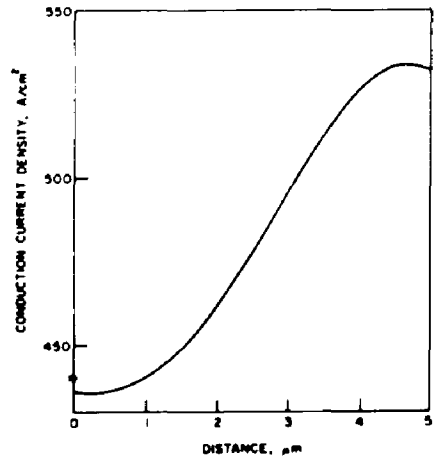
(f)  $\omega t = 187$  DEGREES



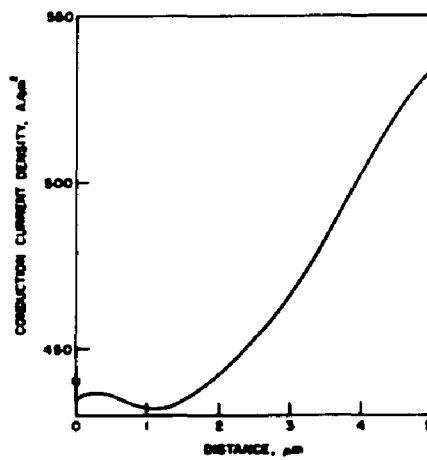
(g)  $\omega t = 223$  DEGREES



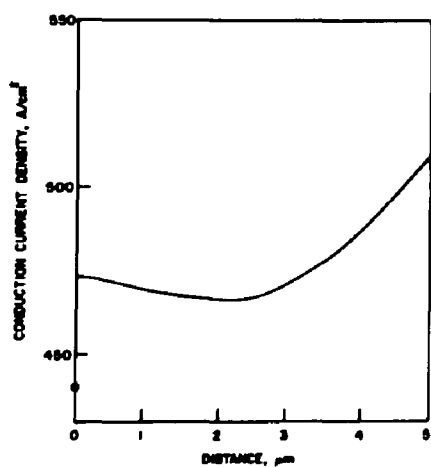
(h)  $\omega t = 259$  DEGREES



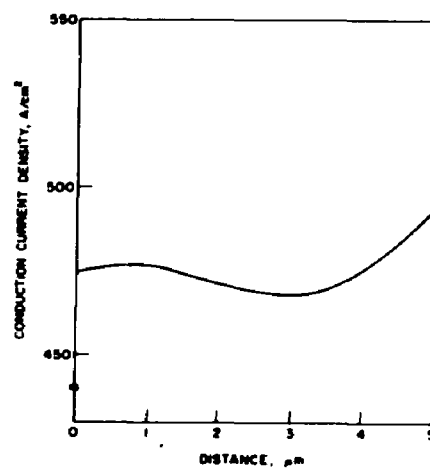
(i)  $\omega t = 295$  DEGREES



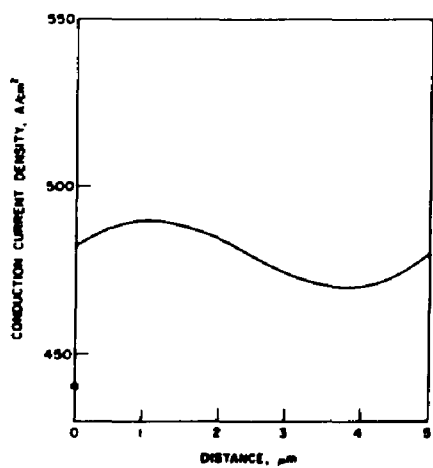
(j)  $\omega t = 331$  DEGREES



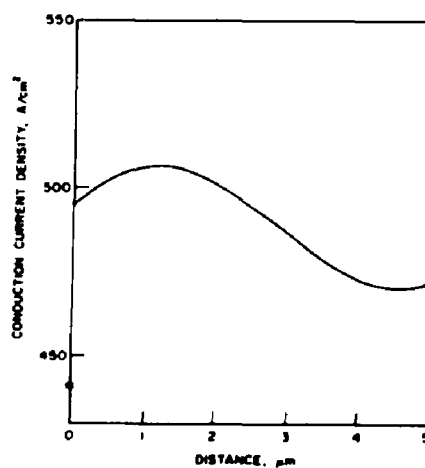
(a)  $\omega t = 0$  DEGREES



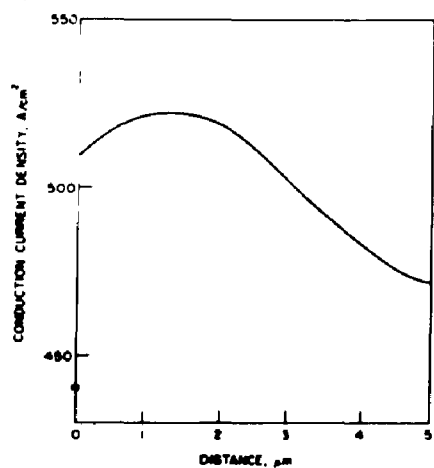
(b)  $\omega t = 30$  DEGREES



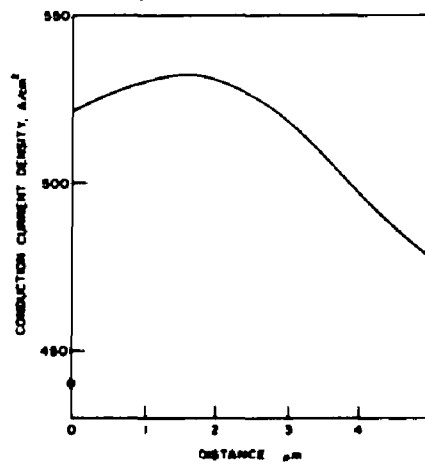
(c)  $\omega t = 60$  DEGREES



(d)  $\omega t = 90$  DEGREES

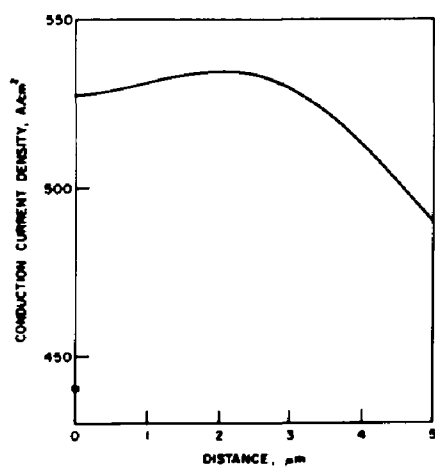


(e)  $\omega t = 120$  DEGREES

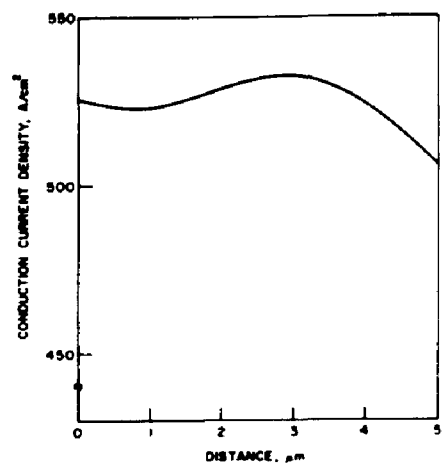


(f)  $\omega t = 150$  DEGREES

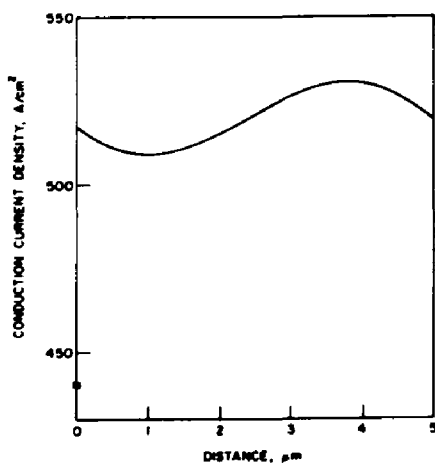
FIG. 3.30 CONDUCTION CURRENT DENSITY VS. DISTANCE FOR A Si ABRUPT-JUNCTION DIODE AT DIFFERENT PHASES OF THE RF VOLTAGE. ( $p = 1 \times 10^{15}$ ,  $J_{dc} = 500 \text{ A/cm}^2$ ,  $f = 12 \text{ GHz}$  AND  $V_{RF} = 1 \text{ V}$ )



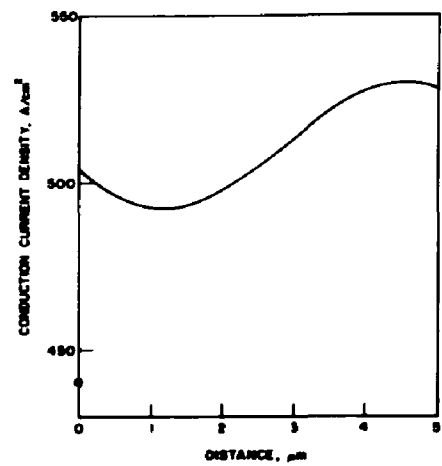
(g)  $\omega t = 180$  DEGREES



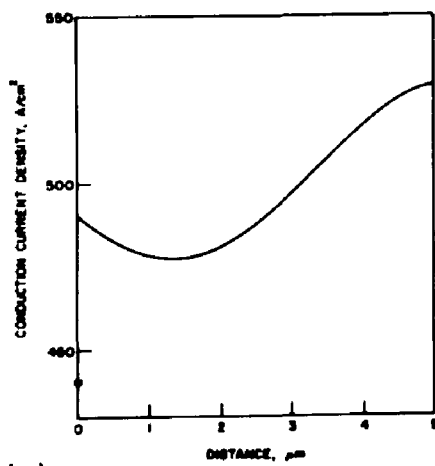
(h)  $\omega t = 210$  DEGREES



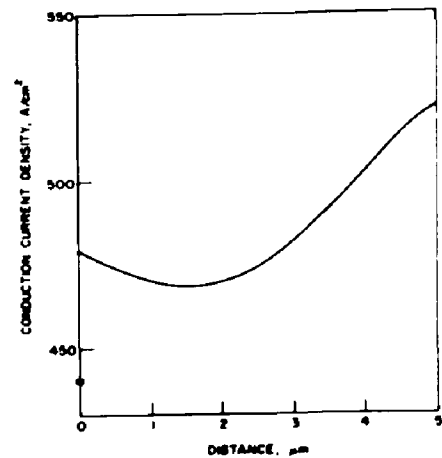
(i)  $\omega t = 240$  DEGREES



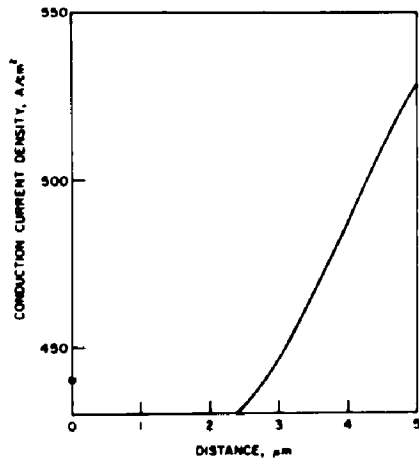
(j)  $\omega t = 270$  DEGREES



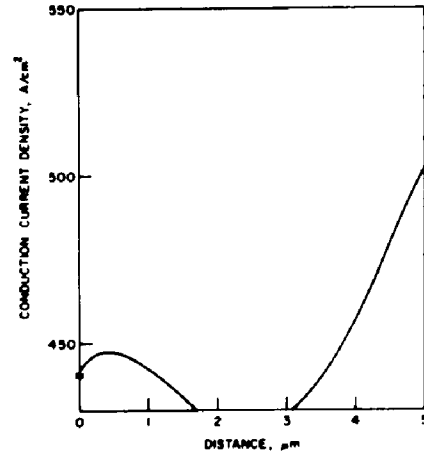
(k)  $\omega t = 300$  DEGREES



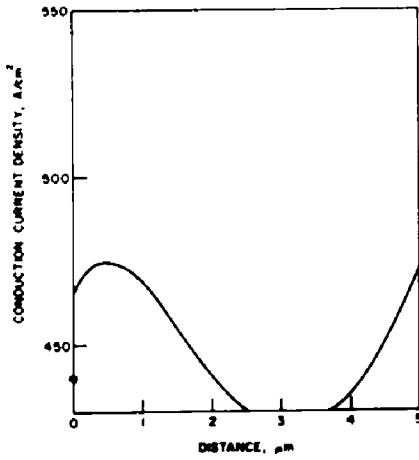
(l)  $\omega t = 330$  DEGREES



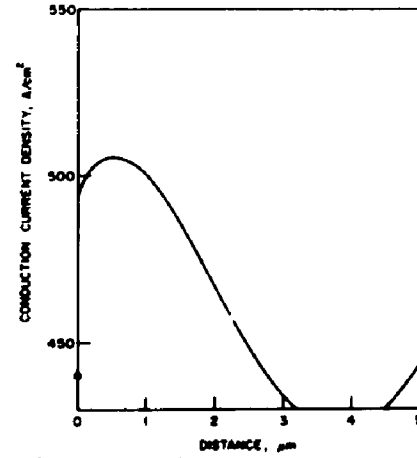
(a)  $\omega t = 6$  DEGREES



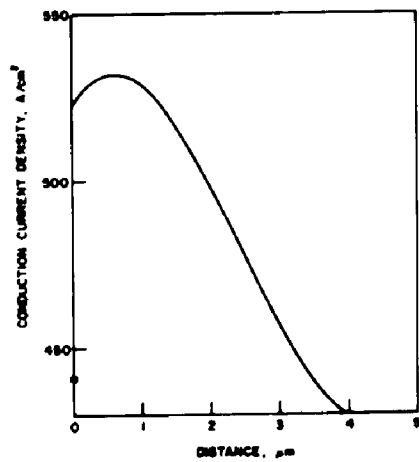
(b)  $\omega t = 36$  DEGREES



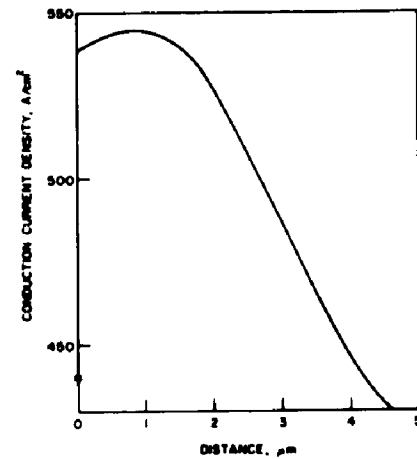
(c)  $\omega t = 66$  DEGREES



(d)  $\omega t = 96$  DEGREES



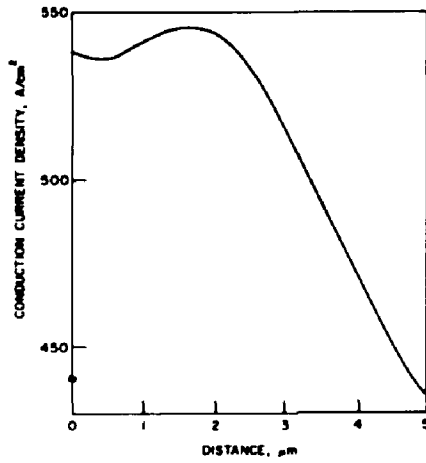
(e)  $\omega t = 126$  DEGREES



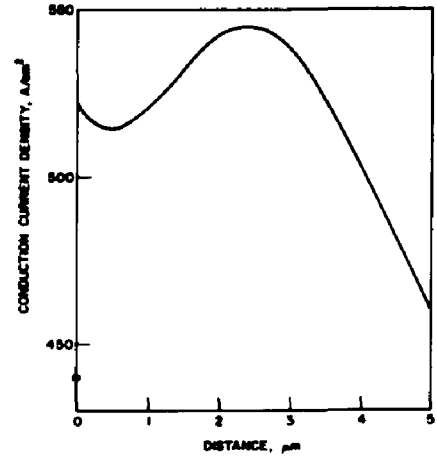
(f)  $\omega t = 156$  DEGREES

FIG. 3.31 CONDUCTION CURRENT DENSITY VS. DISTANCE FOR A Si ABRUPT-JUNCTION DIODE AT DIFFERENT PHASES OF THE RF VOLTAGE.

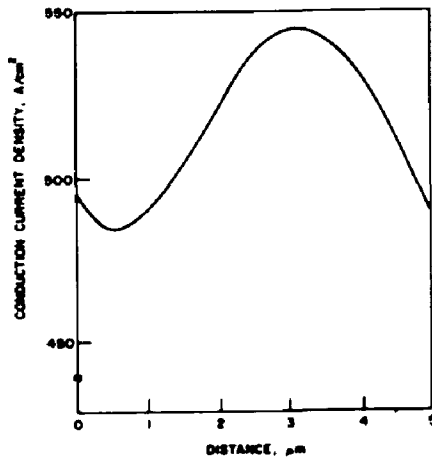
( $p = 3 \times 10^{15}$ ,  $f = 12$  GHz,  $V_{RF} = 1$  V AND  $J_{dc} = 500$  A/cm<sup>2</sup>)



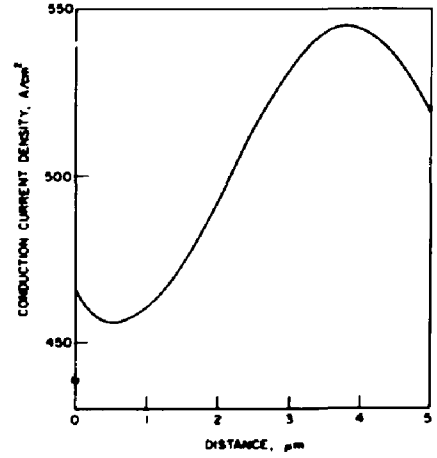
(g)  $\omega t = 186$  DEGREES



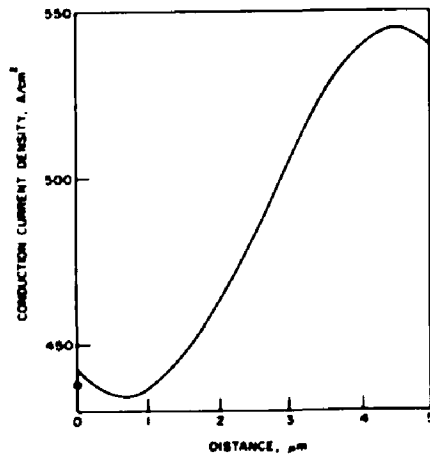
(h)  $\omega t = 216$  DEGREES



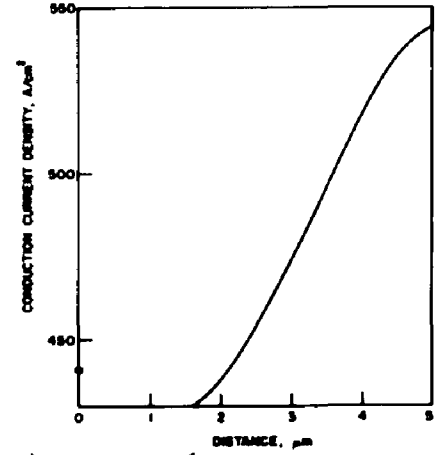
(i)  $\omega t = 246$  DEGREES



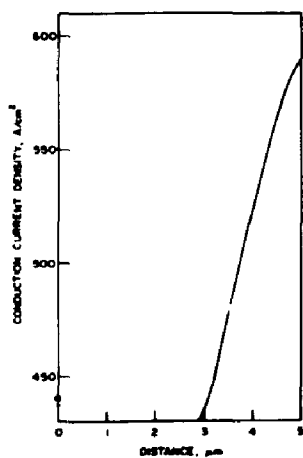
(j)  $\omega t = 276$  DEGREES



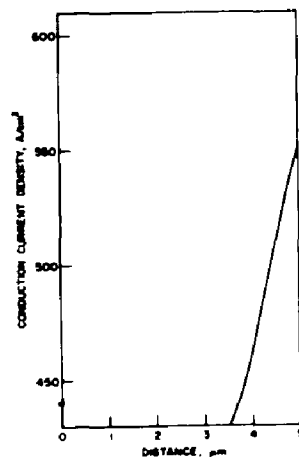
(k)  $\omega t = 306$  DEGREES



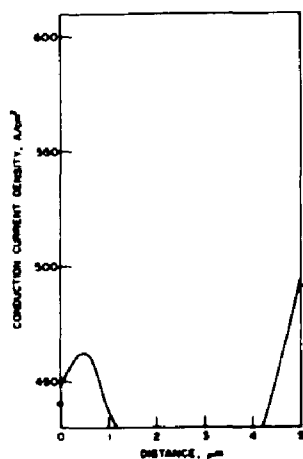
(l)  $\omega t = 336$  DEGREES



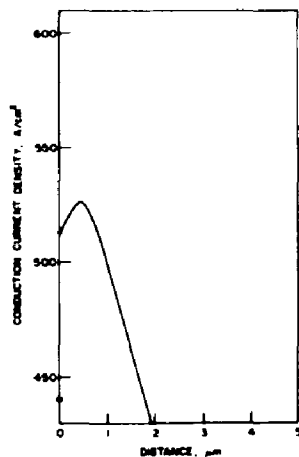
(a)  $\omega t = 6$  DEGREES



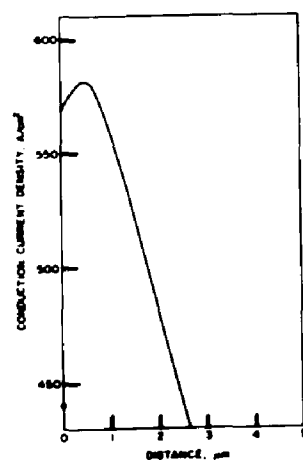
(b)  $\omega t = 36$  DEGREES



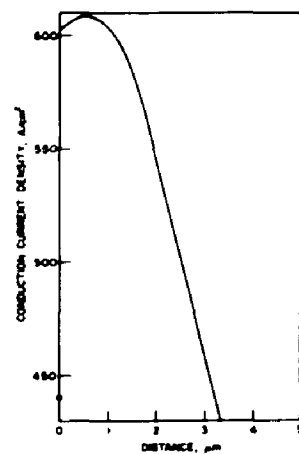
(c)  $\omega t = 66$  DEGREES



(d)  $\omega t = 96$  DEGREES

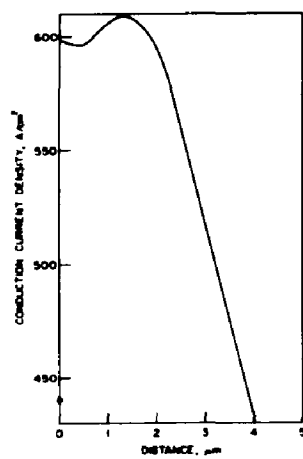


(e)  $\omega t = 126$  DEGREES

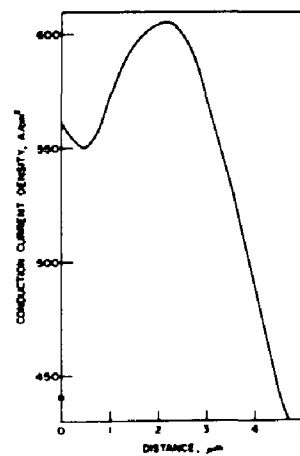


(f)  $\omega t = 156$  DEGREES

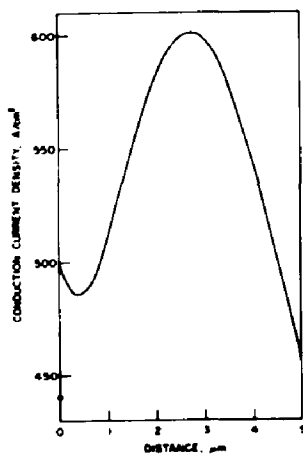
FIG. 3.32 CONDUCTION CURRENT DENSITY VS. DISTANCE FOR A Si ABRUPT-JUNCTION DIODE AT DIFFERENT PHASES OF THE RF VOLTAGE. ( $p = 5 \times 10^{15}$ ,  $V_{\text{RF}} = 1$  V,  $f = 12$  GHz AND  $J_{\text{dc}} = 500$  A/cm<sup>2</sup>)



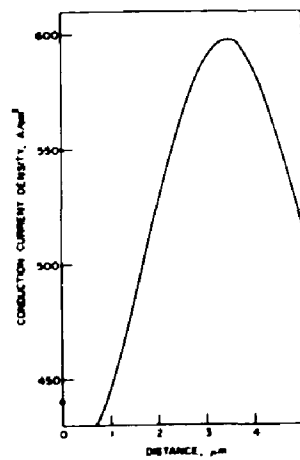
(g)  $\omega t = 186$  DEGREES



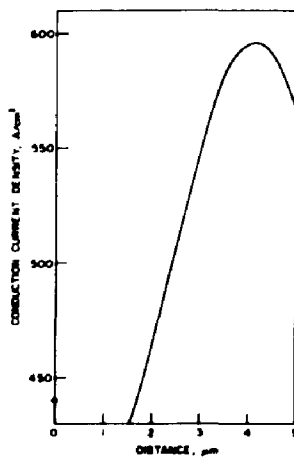
(h)  $\omega t = 216$  DEGREES



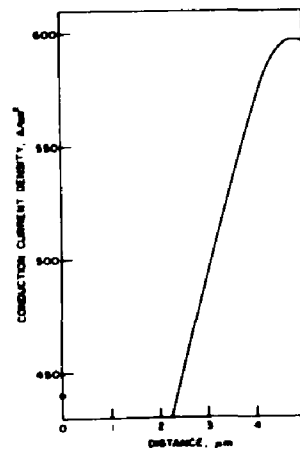
(i)  $\omega t = 246$  DEGREES



(j)  $\omega t = 276$  DEGREES



(k)  $\omega t = 306$  DEGREES



(l)  $\omega t = 336$  DEGREES

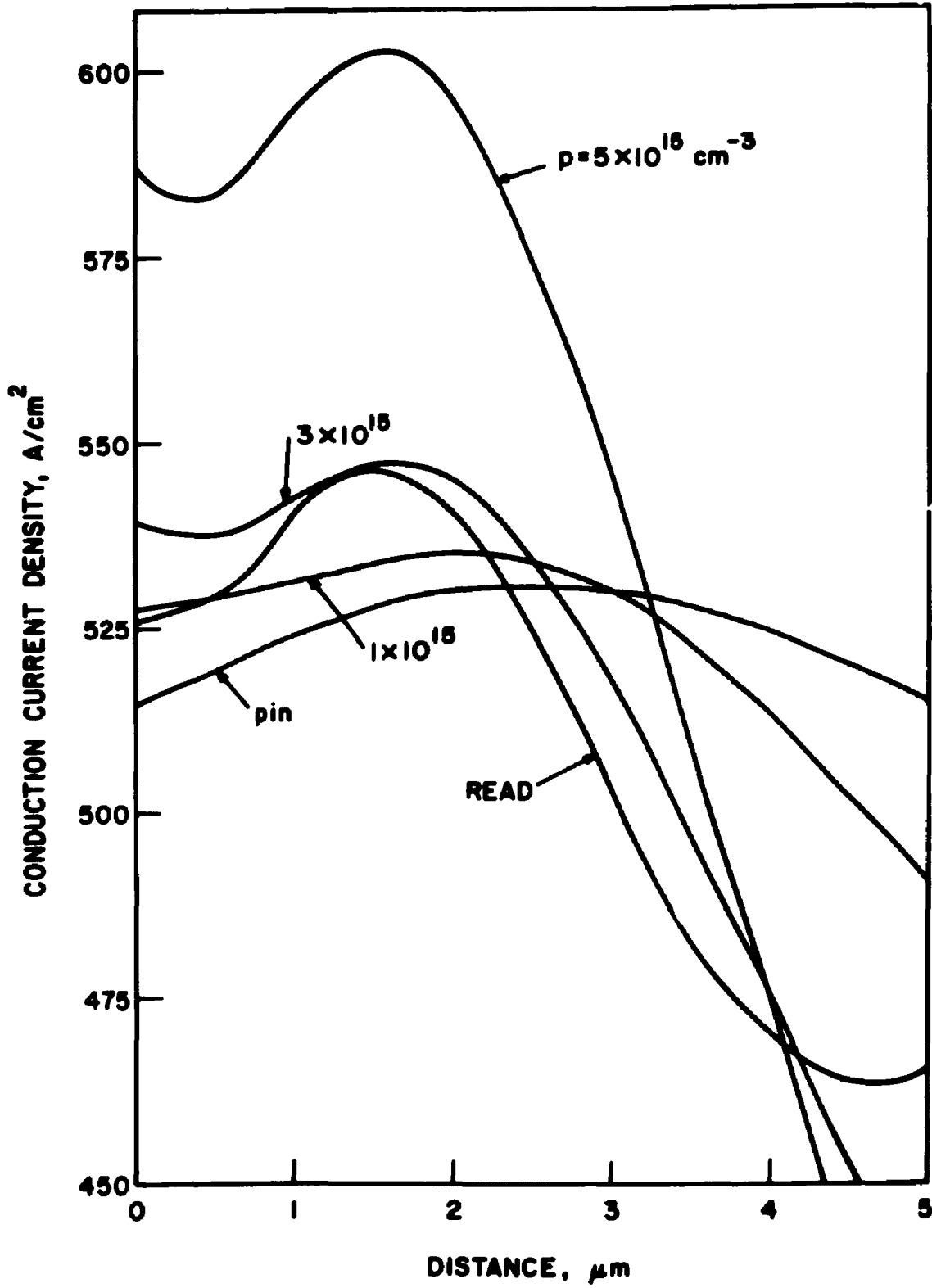


FIG. 3.33 CONDUCTION CURRENT DENSITY VS. DISTANCE FOR DIFFERENT DOPING DENSITIES. ( $J_{dc} = 500 \text{ A/cm}^2$ ,  $f = \text{OPTIMUM FREQUENCY AND } \omega t \approx 180 \text{ DEGREES}$ )

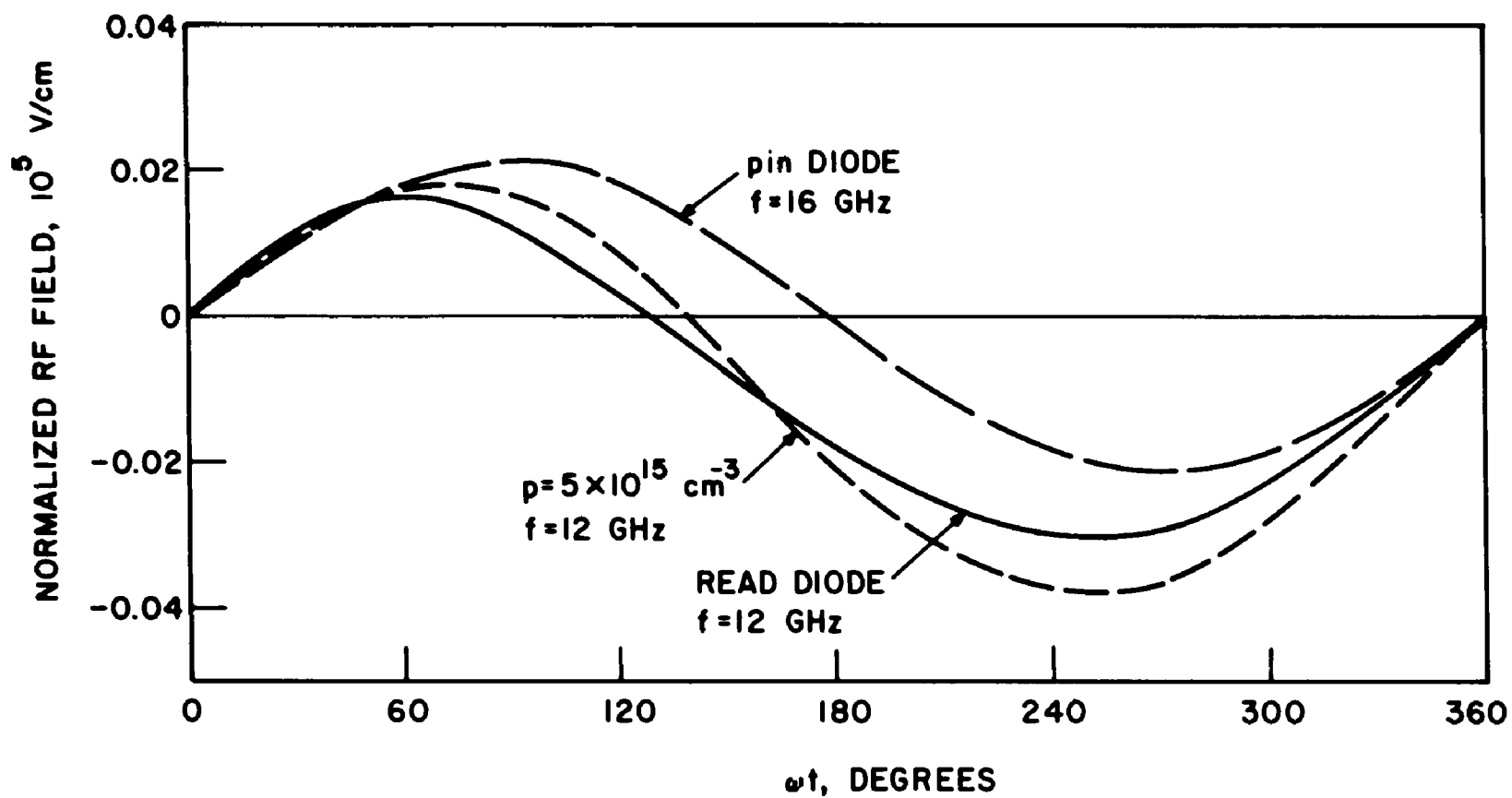


FIG. 3.34 NORMALIZED RF FIELD VS.  $\omega t$  FOR DIFFERENT DIODES. ( $J_{dc} = 500 \text{ A/cm}^2$  AND  $V_{RF} = 1 \text{ V}$ )

and becomes negative. For the rest of the RF cycle the conduction current drifts through the diode without increasing in magnitude. Thus the two phase delay components can be separated much the same as for the ideal Read diodes discussed in Chapter II.

The total phase delay for the pin diode can be approximated as follows. The phase delay due to the avalanche process from Fig. 3.34 is 88 degrees. The phase delay due to transit time is

$$\omega\tau = 2\pi \cdot f \cdot x_D' / v_s , \quad (3.69)$$

where  $x_D'$  = the effective drift length for the pin diode which is approximately 2.5  $\mu\text{m}$ . Thus

$$\omega\tau = 144 \text{ degrees} . \quad (3.70)$$

Therefore the total phase delay is 232 degrees. For the abrupt junction the avalanche process contributes 48 degrees of phase delay and the transit time another 195 degrees for a total phase delay of 243 degrees at a frequency of 12 GHz. For the Read diode the avalanche process contributes only 39 degrees and the transit time 218 degrees for a total phase delay of 257 degrees at 13 GHz. The same field behavior as shown for 12 GHz is assumed for 13 GHz in the Read diode. As the phase delay of the avalanche process is decreased the transit-time phase delay must increase to compensate for this decrease. Also since the transit time of the carriers during the positive portion of the RF voltage contributes a loss (current in phase with the voltage) the total phase delay must be larger for the diodes with the shorter phase delay in the avalanche process.

The reason the RF field goes through zero at different times in the cycle is due to the space charge in the diode. The time when the RF field becomes negative is proportional to the amount of space charge in the diode. As the space charge increases the RF field becomes negative sooner and thus the phase delay between the maximum in the RF voltage and the instant when the conduction current maximizes and starts to drift decreases. In Fig. 3.35 the electron and hole current densities at an instant in time are plotted for the pin diode and the abrupt junction. This is for the same instant in the cycle and the same dc current density. Since the space charge at any point is proportional to the difference in the hole and electron current densities, the abrupt junction has much more space charge present at this instant. Thus the phase delay due to the avalanche process for the abrupt junction is less than for the pin diode.

Special attention must be given the Read diode since, even though the doping density in the avalanche region is higher than any of the other diodes, the negative conductance is smaller than several of the diodes. This can be explained by considering Fig. 3.36 in which the hole and electron current densities and the electric field are plotted for the Read diode at  $\omega t = 150$  degrees. This corresponds to the instant when the avalanche process has turned off as defined previously. As can be seen for the avalanche region doping density considered ( $p_A = 1 \times 10^{16} \text{ cm}^{-3}$ ), the field at breakdown in the drift or intrinsic region is large enough for impact ionization to occur. Thus a small but appreciable number of holes and electrons are generated in the drift region. The amount of generation is clearly shown in Fig. 3.37 in which the two current densities are plotted for  $\omega t = 260$  degrees. Even though the avalanching in the avalanche region

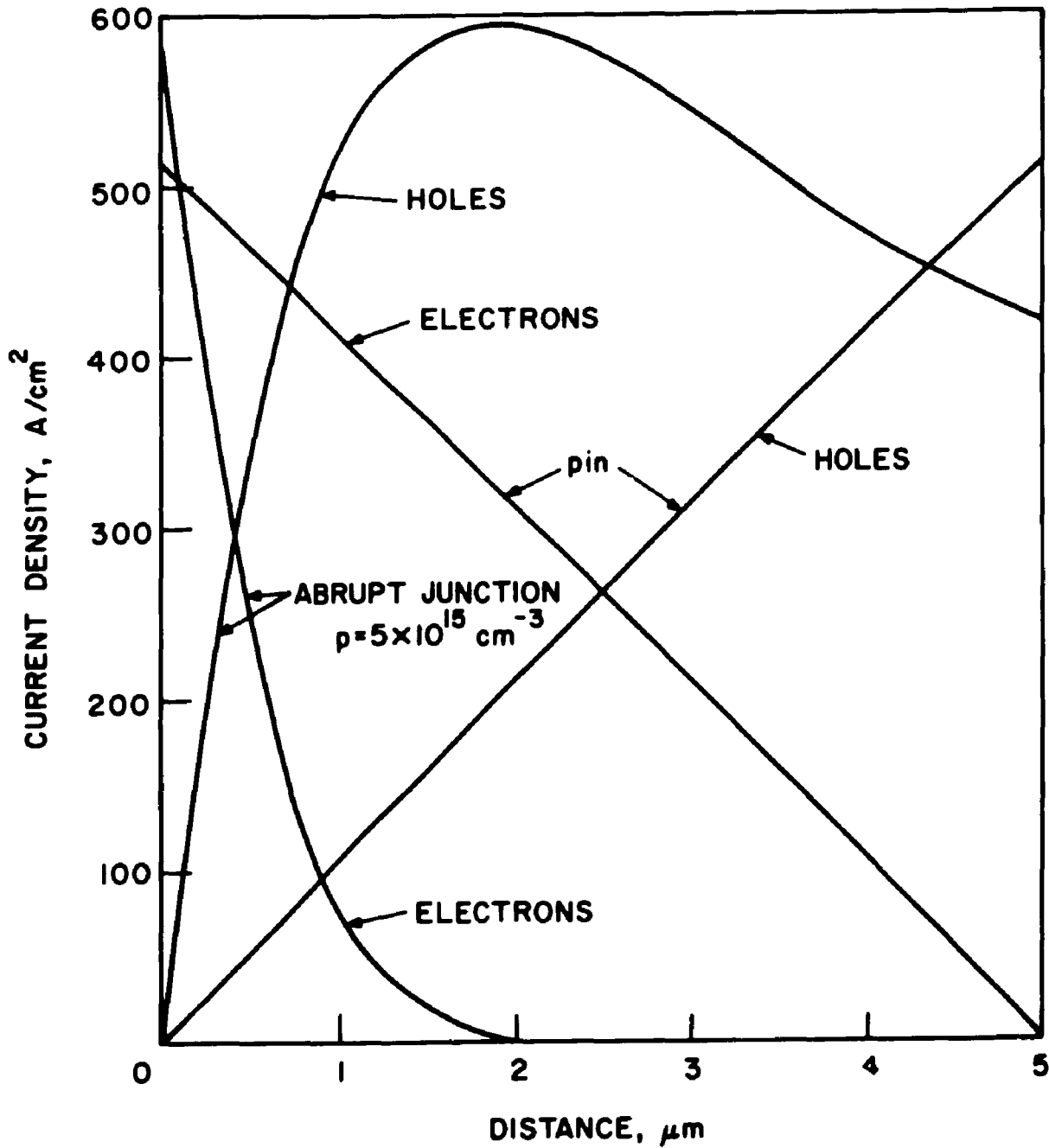


FIG. 3.35 ELECTRON AND HOLE CURRENT DENSITIES VS. DISTANCE FOR pin AND ABRUPT-JUNCTION DIODES. ( $J_{dc} = 500 A/cm^2$  AND  $\omega t = 150$  DEGREES)

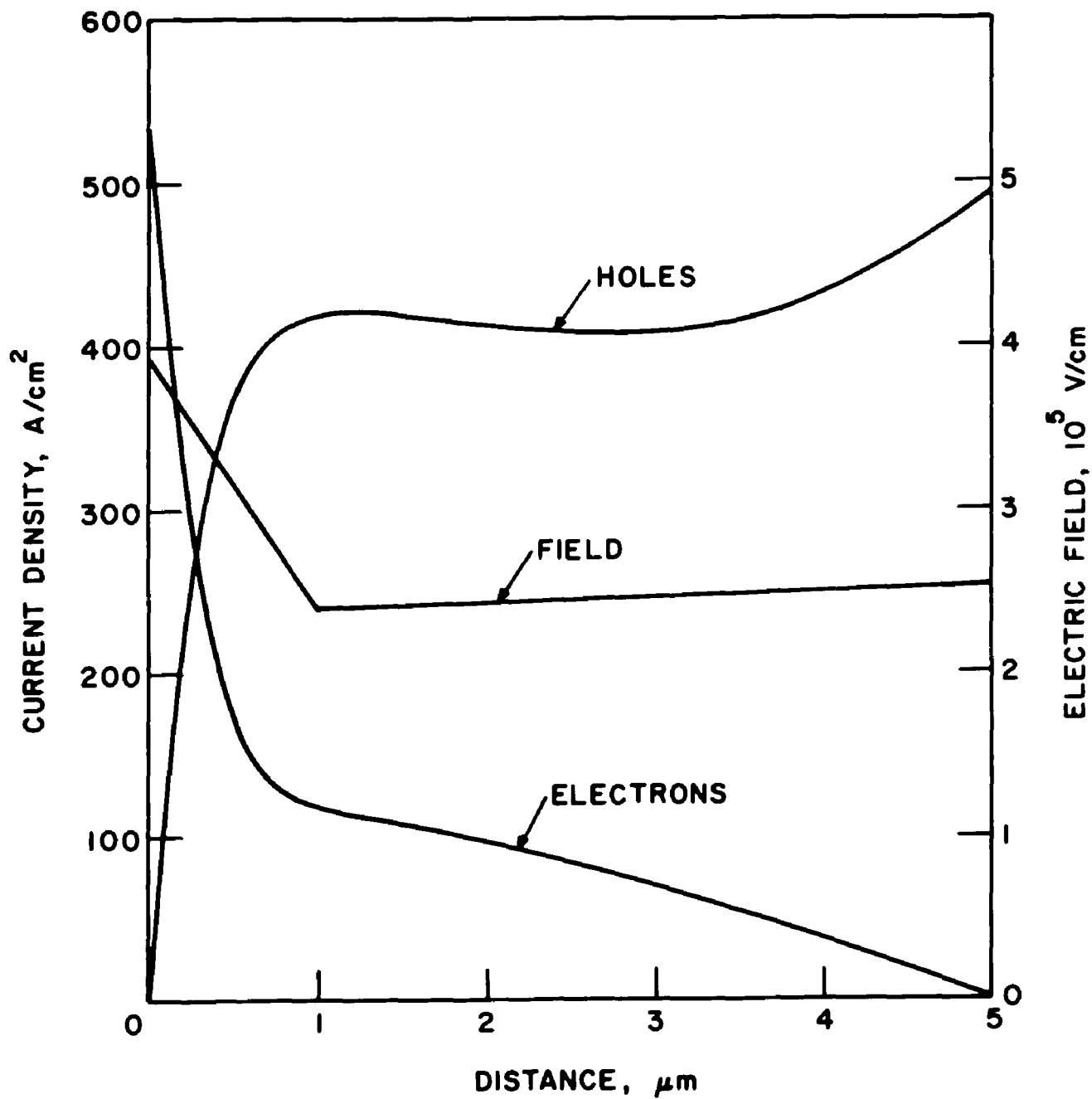


FIG. 3.36 ELECTRON AND HOLE CURRENT DENSITIES AND ELECTRIC FIELD VS. DISTANCE FOR A READ DIODE. ( $J_{dc} = 500 \text{ A/cm}^2$ ,  $p_A = 1 \times 10^{16} \text{ cm}^{-3}$  AND  $\omega t = 130 \text{ DEGREES}$ )

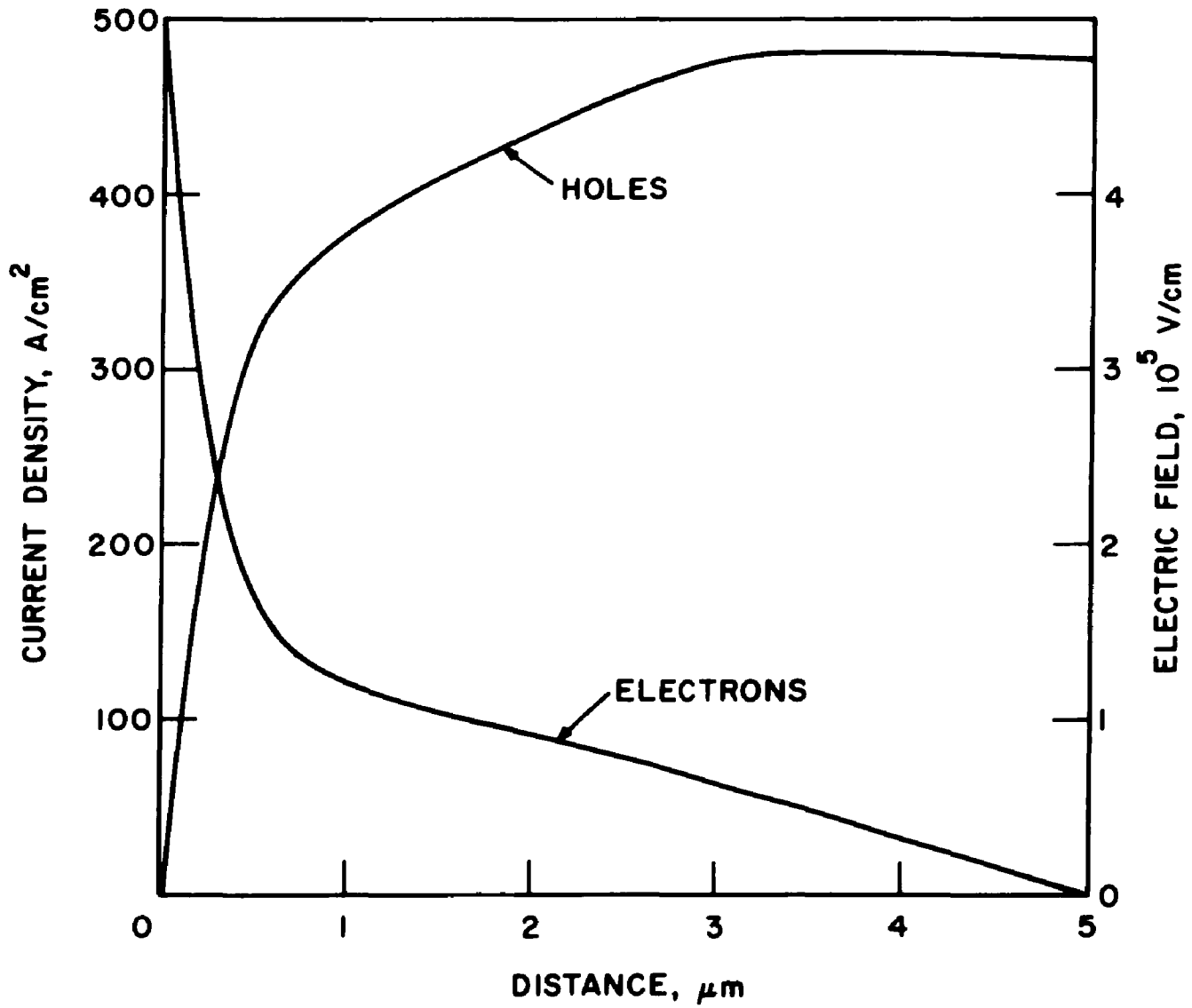


FIG. 3.37 ELECTRON AND HOLE CURRENT DENSITIES VS. DISTANCE FOR A READ DIODE. ( $J_{dc} = 500 \text{ A}/\text{cm}^2$ ,  $p_A = 1 \times 10^{16} \text{ cm}^{-3}$  AND  $\omega t = 260$  DEGREES)

has stopped, the hole current increases in magnitude as it drifts across the diode. Thus the transit angle of the carriers is spread over a wide range of values. The avalanche region causes a large number of the carriers to be generated in a pulse but the field in the drift region is high enough to generate enough more carriers to degrade the negative conductance of the diode. The normalized field plotted in Fig. 3.34 for the Read diode is for the field in the avalanche region. The phase delay calculations are for the carriers generated in the avalanche region but not for the carriers generated in the drift region. This Read diode behaves like a combination of an abrupt-junction and pin diode. As the doping density in the avalanche region is increased the field in the drift region decreases and therefore fewer carriers are generated and the negative conductance of the diode increases. This is shown in Fig. 3.38 in which the small-signal admittance for a Read diode with a doping density of  $2 \times 10^{16} \text{ cm}^{-3}$  in the avalanche region is plotted. Figures 3.39 through 3.41 show the hole and electron current densities as well as the field at three times in the RF cycle. For this diode the field in the drift region is low enough for no impact ionization to occur and all of the avalanching occurs in the avalanche region. Figure 3.39 shows the currents at the instant the peak in the conduction current starts to drift. Figure 3.40 shows that as the hole current leaves the avalanche region 36 degrees later than Fig. 3.39 it has grown slightly in magnitude. Figure 3.41 shows that the hole current does not grow in magnitude after it leaves the avalanche region. Thus this Read diode looks like a highly doped abrupt junction with a drift region in which the field is high enough for saturated drift velocities, but low enough for no impact ionization to occur and long enough for approximately X-band operation.

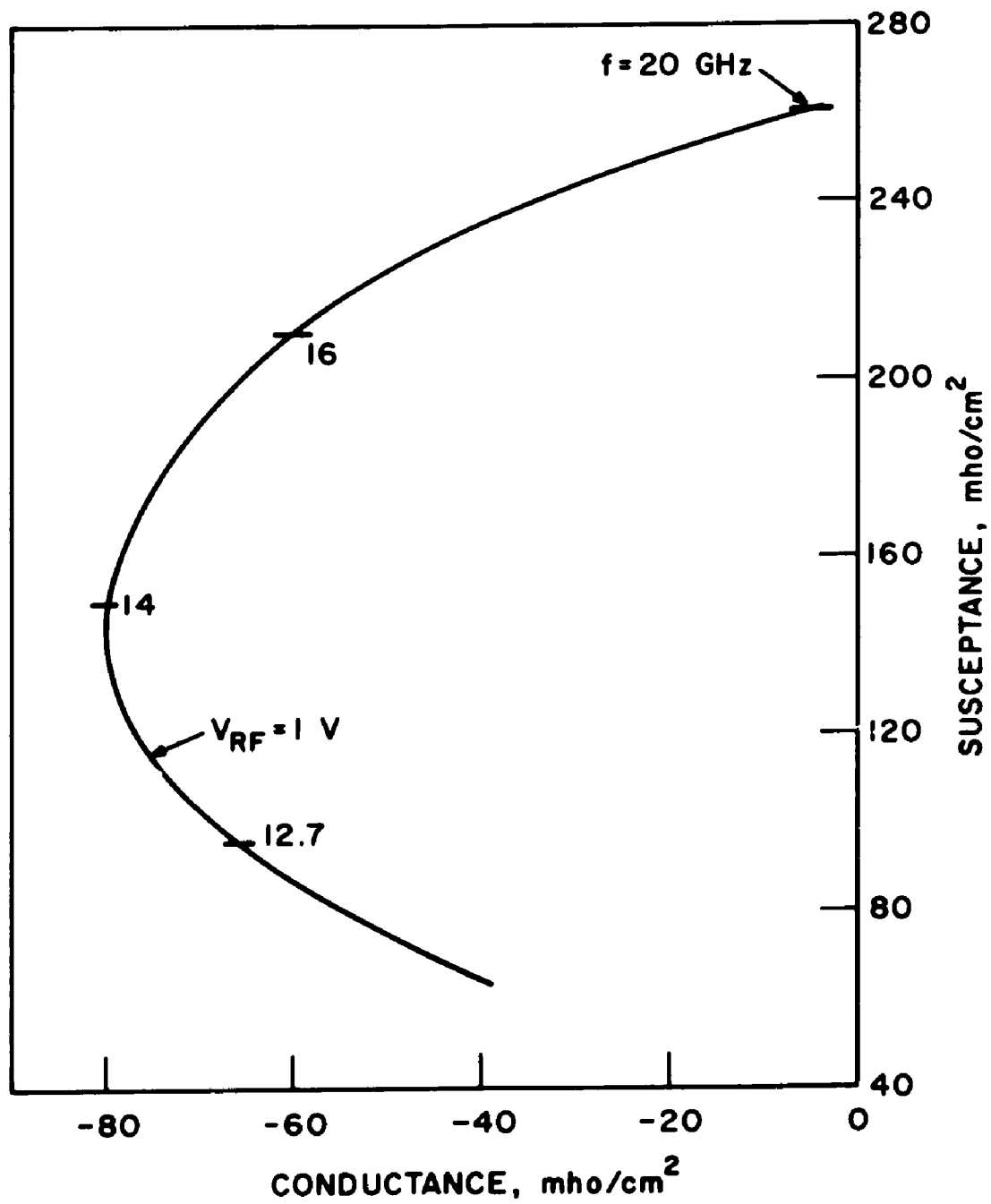


FIG. 3.38 SMALL-SIGNAL ADMITTANCE FOR A READ DIODE. ( $J_{dc} = 500 \text{ A/cm}^2$   
AND  $p_A = 2 \times 10^{18} \text{ cm}^{-3}$ )

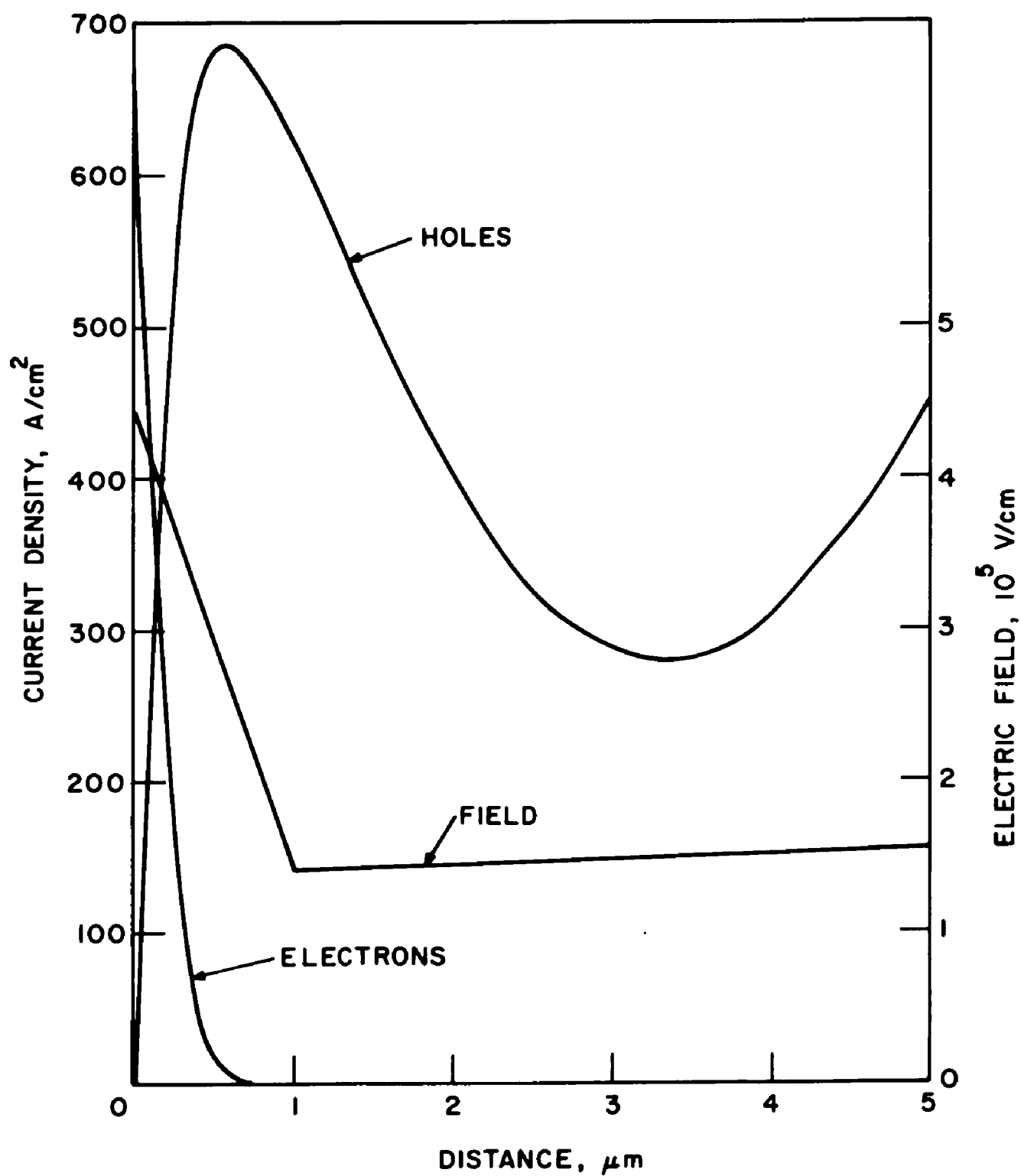


FIG. 3.39 HOLE AND ELECTRON CURRENT DENSITIES AND ELECTRIC FIELD VS. DISTANCE FOR A READ DIODE. ( $J_{dc} = 500 \text{ A}/\text{cm}^2$ ,  $p_A = 2 \times 10^{18} \text{ cm}^{-3}$ ,  $f = 16 \text{ GHz}$  AND  $\omega t = 144 \text{ DEGREES}$ )

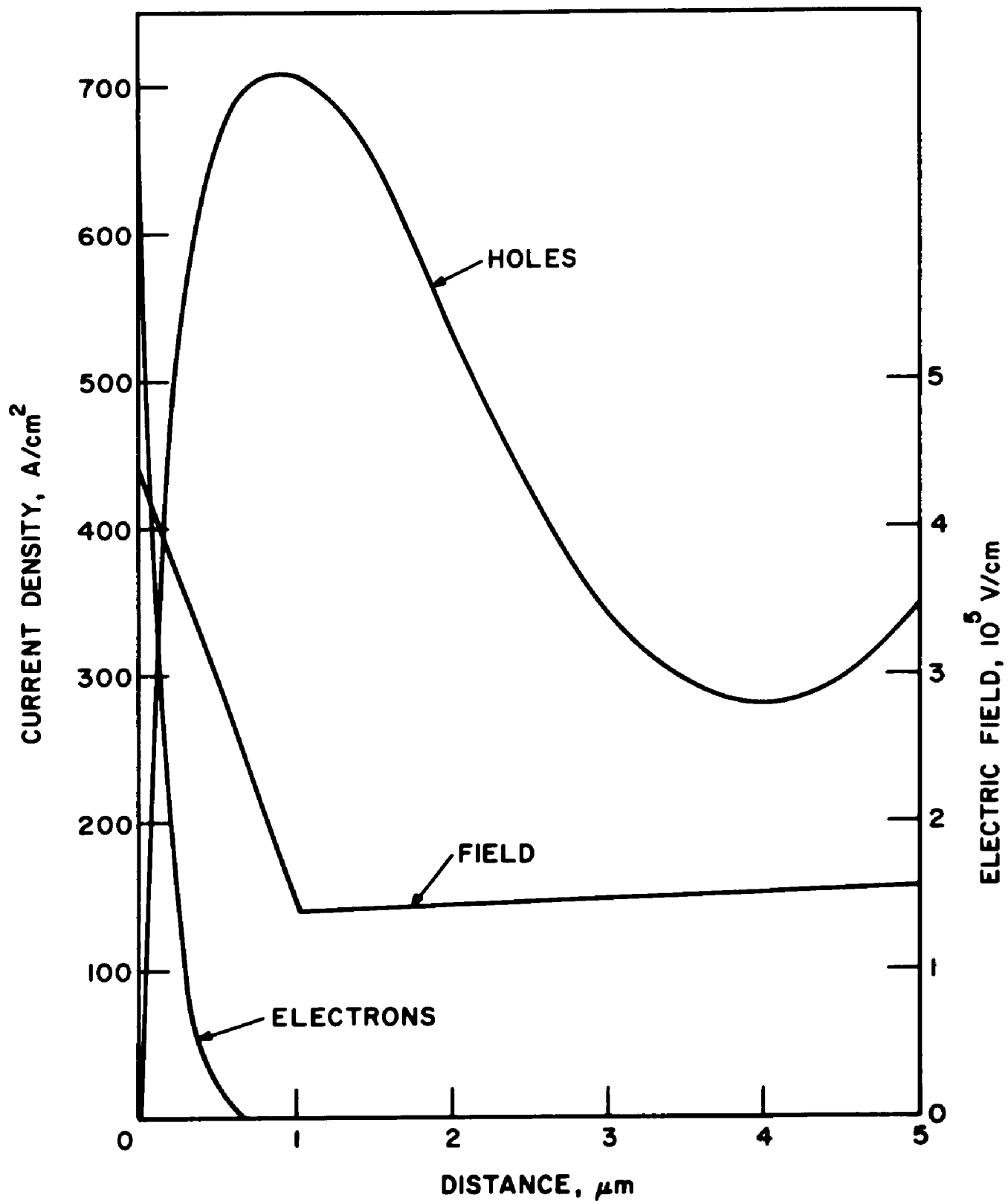


FIG. 3.40 HOLE AND ELECTRON CURRENT DENSITIES AND ELECTRIC FIELD VS. DISTANCE FOR A READ DIODE. ( $J_{\text{dc}} = 500 \text{ A}/\text{cm}^2$ ,  $p_A = 2 \times 10^{18} \text{ cm}^{-3}$ ,  $f = 16 \text{ GHz}$  AND  $\omega t = 180 \text{ DEGREES}$ )

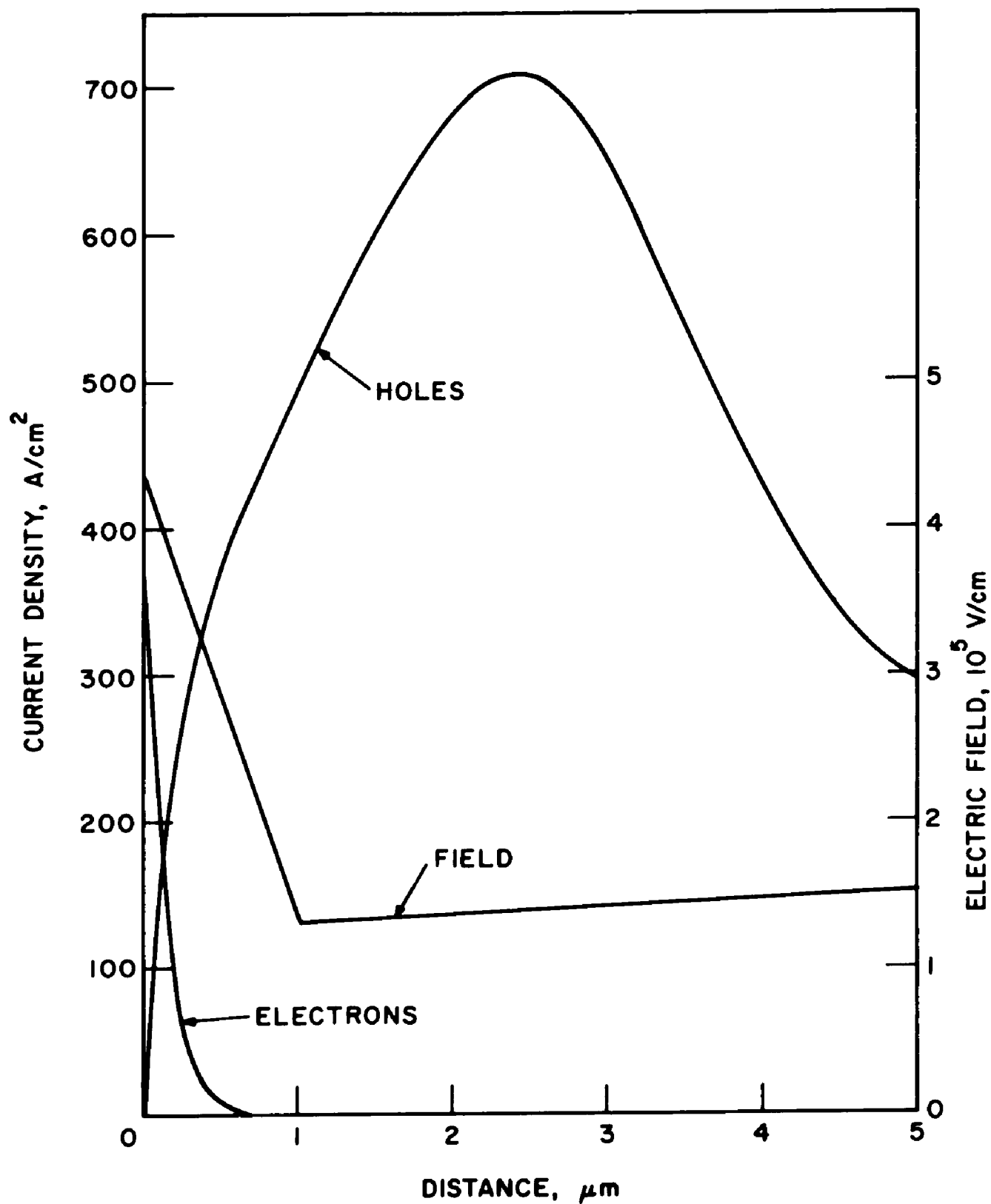


FIG. 3.41 HOLE AND ELECTRON CURRENT DENSITIES AND ELECTRIC FIELD VS. DISTANCE FOR A READ DIODE. ( $J_{dc} = 500 \text{ A}/\text{cm}^2$ ,  $p_A = 2 \times 10^{18} \text{ cm}^{-3}$ ,  $f = 16 \text{ GHz}$  AND  $\omega t = 266 \text{ DEGREES}$ )

The last point which requires elaboration is the difference in the rate of decrease of the negative conductance for the different diodes. This can be understood by looking at the electric field in the diode as shown in Fig. 3.42 for a given instant in time. The straight line is for the electric field with no space charge and is shown for reference. As the RF voltage is increased the modulation of the conduction current increases and thus when the carriers are being generated there is more space charge in the diode than for the lower RF voltage. Therefore, as the RF voltage increases the field behaves as shown and the avalanche region is widened. Thus carriers are generated over a wider region of the diode and the transit angle of the carriers is spread over a wider range. This avalanche widening effect causes the diode to look more like a pin diode for larger RF voltages. For the Read diode the avalanche widening effect occurs in the avalanche region causing a decrease in the negative conductance but an opposite effect occurs in the drift region. As the RF voltage increases the field in the drift region becomes smaller while the carriers are drifting and thus there is less generation of carriers in the drift region. Therefore the negative conductance does not decrease at the same rate as the other diode structures for increasing RF voltage.

This effect of the different rates of decrease in the negative conductance is apparent in the efficiency of the Read, pin and abrupt-junction diodes which is plotted in Fig. 3.43. As the avalanche region is spread out the diodes' behavior approaches that of a pin diode.

3.3.4 Multifrequency Characteristics. The analysis has been used to study the large-signal operating characteristics of IMPATT diodes when oscillating at two frequencies which are harmonically related. Because of

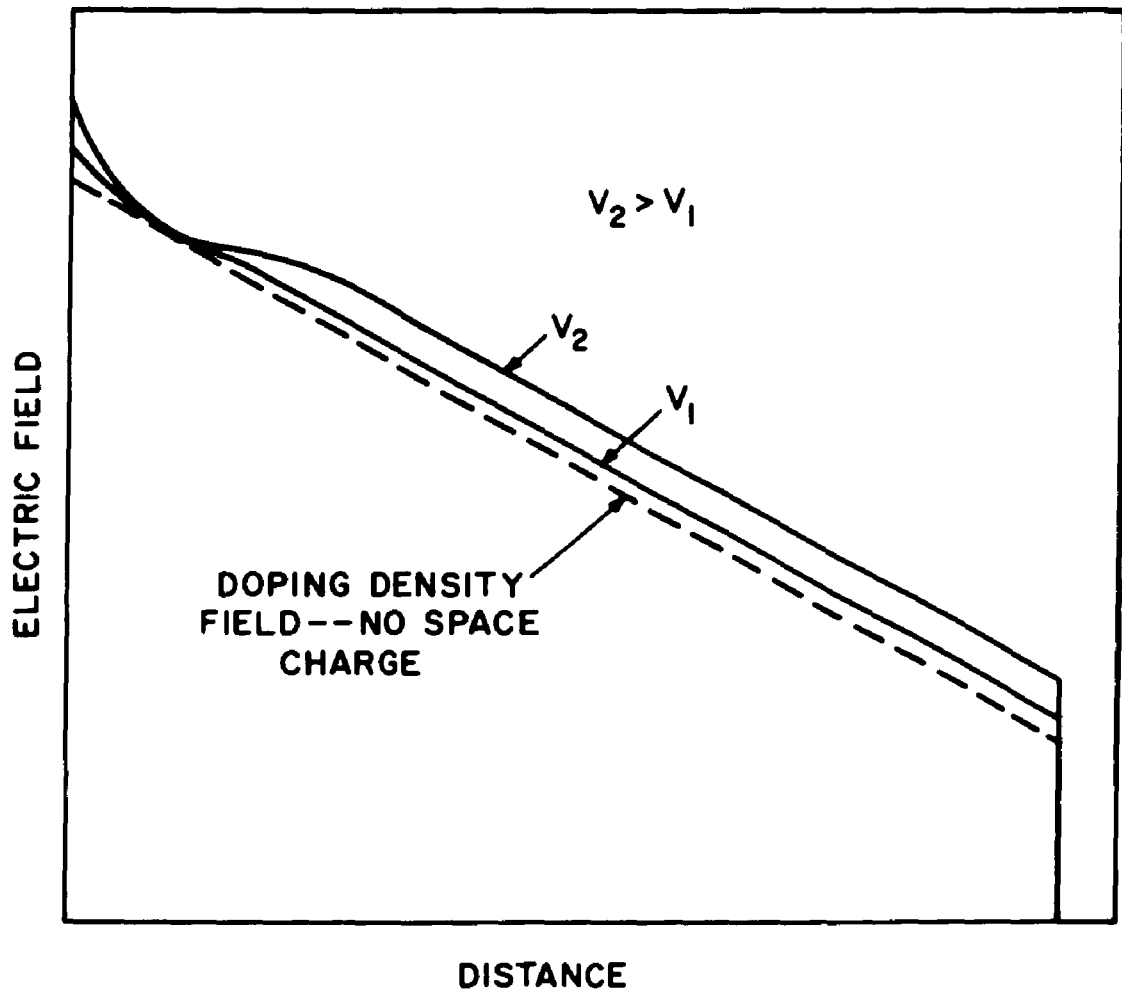


FIG. 3.42 ELECTRIC FIELD VS. DISTANCE FOR DIFFERENT RF VOLTAGES.

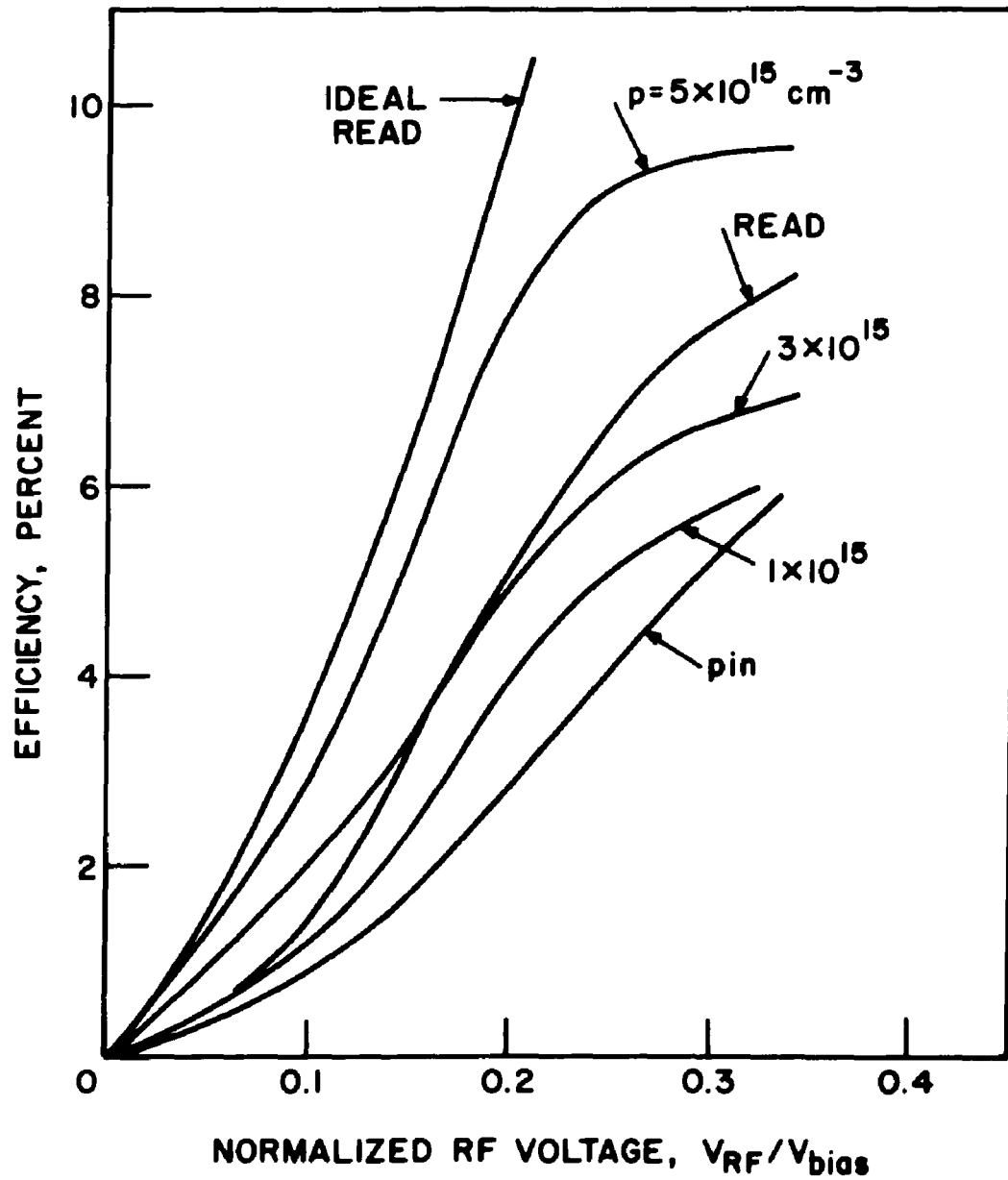


FIG. 3.43 EFFICIENCY VS. NORMALIZED RF VOLTAGE FOR DIFFERENT Si DIODES AT THE OPTIMUM OPERATING FREQUENCY. ( $J_{dc} = 500 \text{ A/cm}^2$ )

the number of parameters which can be adjusted and the amount of computer time necessary for execution, the analysis has been used only for comparison with a simplified multifrequency analysis.<sup>14</sup>

As shown by Schroeder et al.,<sup>14</sup> the negative conductance of a Read diode can be enhanced by having harmonics or subharmonics of the signal frequency present. This enhancement of the negative conductance is a function of the phase between the two frequencies, the RF voltage amplitudes and the relative value of the frequencies with respect to the optimum operating frequency (frequency which produces maximum negative conductance). It was shown that for a Read diode with a cutoff frequency around 7 GHz (small-signal conductance is positive below this frequency) and an optimum operating frequency of 11 GHz, the diode could oscillate with significant output power at 5 GHz if a properly phased oscillation at 10 GHz were present. This is due to waveshaping of the RF voltage across the diode which causes an additional phase delay in the generation of the current in the avalanche process. Under single-frequency operation at 5 GHz the total phase delay of the avalanche process and the transit-time is not enough for the fundamental current to be more than 90 degrees out of phase with the RF voltage. However, by adding the second harmonic of the signal frequency with the proper phase it is possible to generate a waveform which causes the RF field in the avalanche region to go through zero at a much later time in the cycle and thus increase the phase delay of the avalanche process. This additional phase delay is enough for the conductance to become negative. Therefore the phase angle between the fundamental and harmonic RF voltages is an important parameter in determining the negative conductance of the diode. It was also shown that

it is possible to degrade the performance of the diode with certain phase angles.

The large-signal analysis developed in this chapter was used to show that realistic IMPATT diodes behave in the same manner as the ideal Read diode. The pin diode is an extreme case for comparison with the ideal Read diode since carriers are generated over the entire length of the diode at the same time that they are drifting through it. However, the admittance for the pin diode as shown in Fig. 3.44 has the same dependence on phase angle and the negative conductance is maximum for the same angle of 135 degrees as predicted by the simplified theory.<sup>14</sup> Figures 3.45 and 3.46 show the admittance for the abrupt junction with a doping density of  $3 \times 10^{15} \text{ cm}^{-3}$  operating at two frequencies. Note that in Fig. 3.45 the diode would not oscillate at a single frequency of 5 GHz. When the second harmonic is present there exists a substantial negative conductance at 5 GHz. Also note for this case that the conductance is not negative at both frequencies simultaneously for these RF voltages and therefore power would have to be supplied at that frequency with a positive conductance. Finally, as can be seen from the figures, certain phase angles can diminish the negative conductance and cause it to become positive.

These few results do show that the simplified analysis for the Read diode can be used to study the multifrequency operation of diodes and the results obtained will be qualitatively correct.

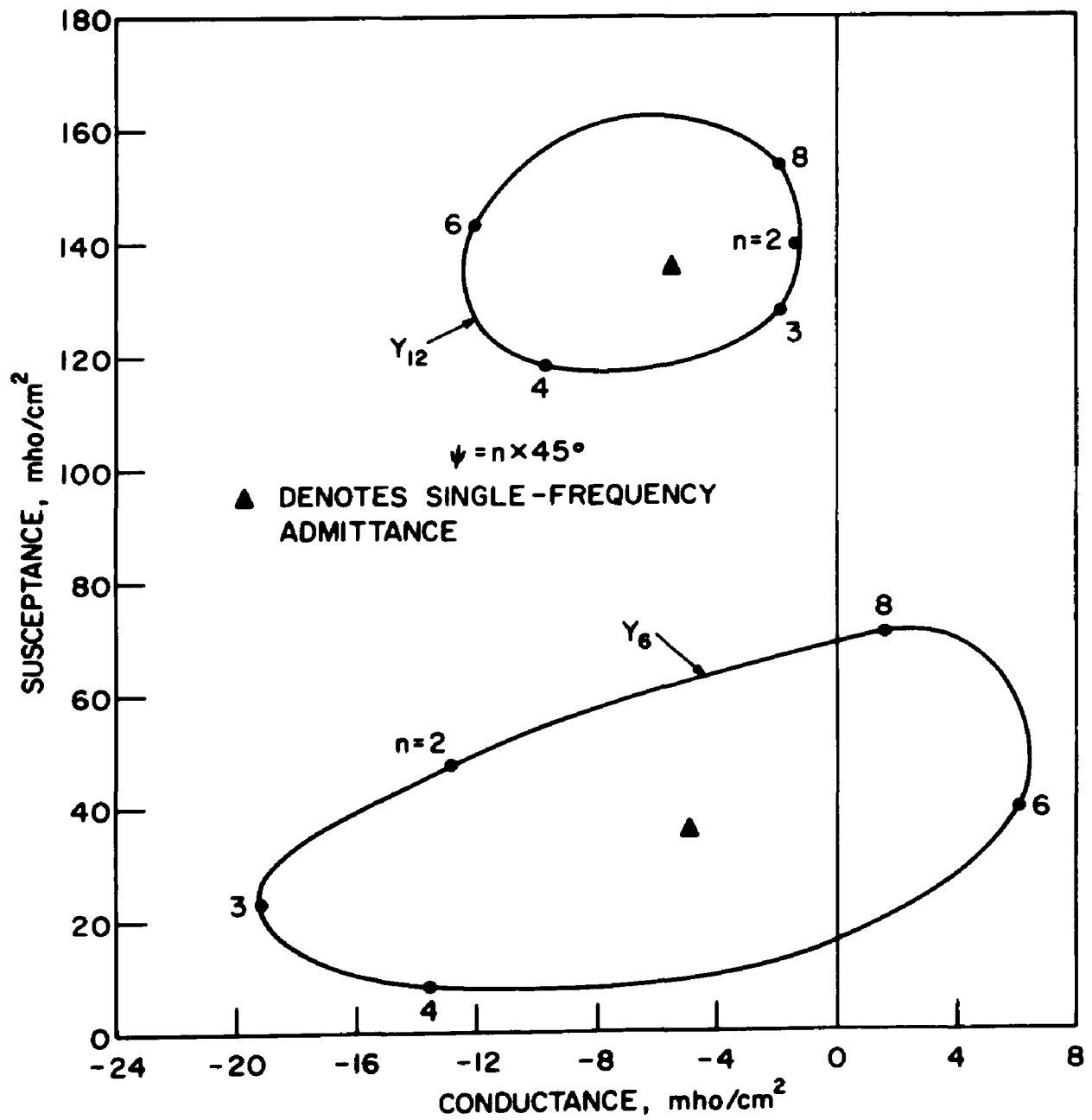


FIG. 3.44 ADMITTANCE FOR TWO-FREQUENCY OPERATION OF A pin DIODE.

$$[V_{RF} = 10 \sin(\omega t) + 10 \sin(2\omega t + \psi), f = 6 \text{ GHz AND}$$

$$J_{dc} = 500 \text{ A/cm}^2]$$

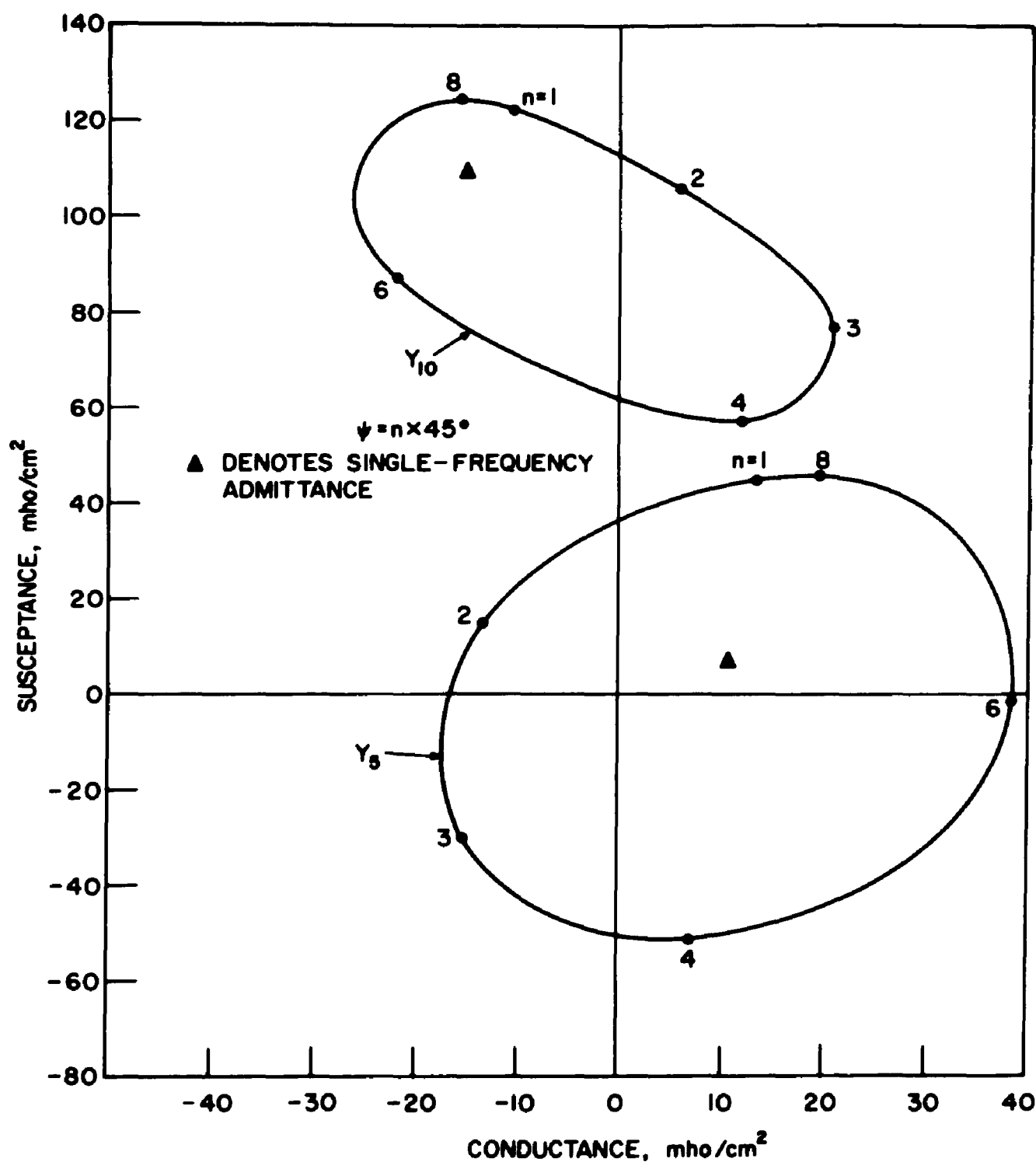


FIG. 3.45 ADMITTANCE FOR TWO-FREQUENCY OPERATION OF AN  $n^+pp^+$  DIODE.

$[V_{RF} = 10 \sin(\omega t) + 10 \sin(2\omega t + \psi), f = 5 \text{ GHz}, J_{dc} = 500 \text{ A/cm}^2 \text{ AND } p = 3 \times 10^{15} \text{ cm}^{-3}]$

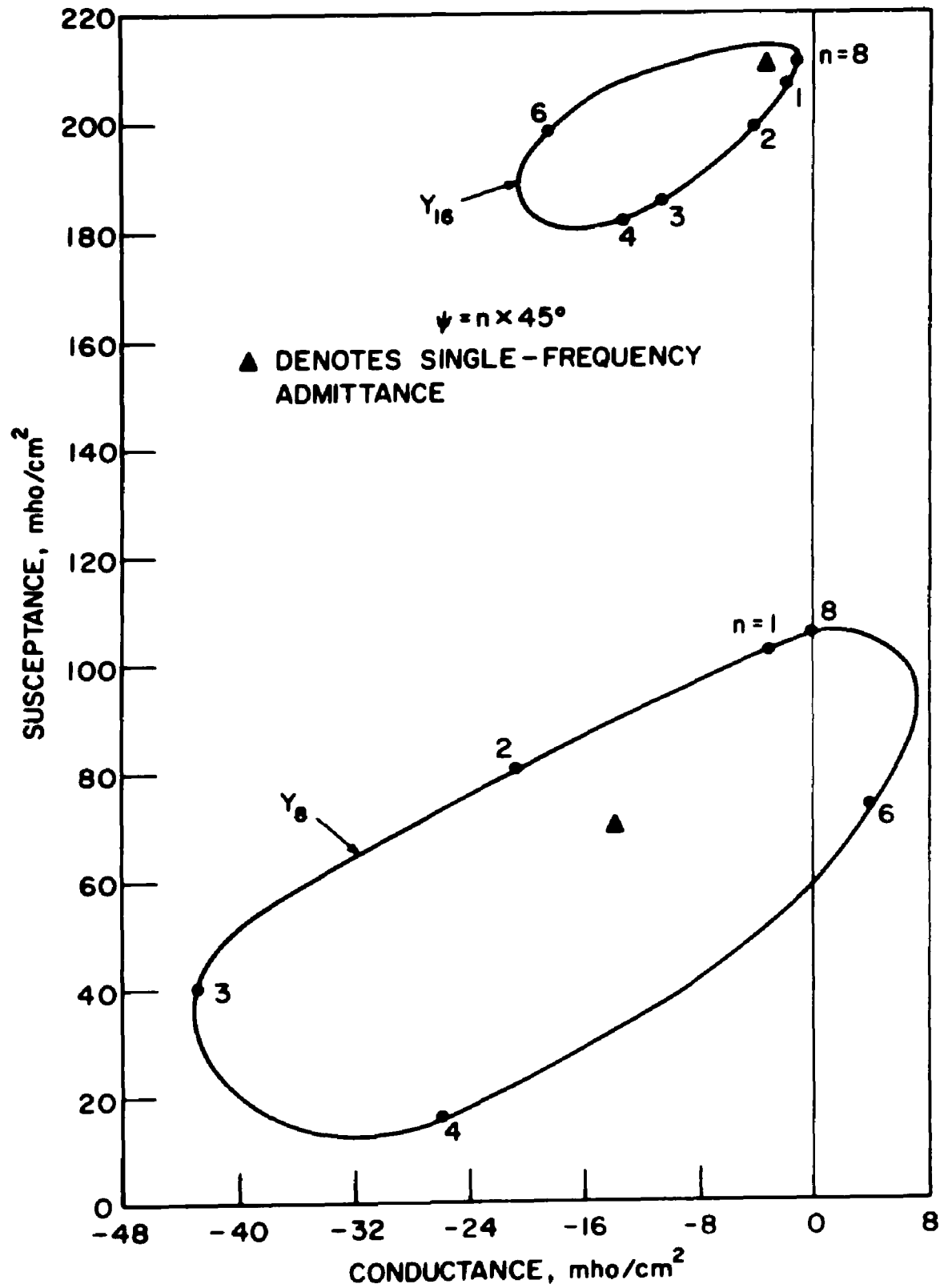


FIG. 3.46 ADMITTANCE FOR TWO-FREQUENCY OPERATION OF AN  $n^+pp^+$  DIODE.

$[V_{RF} = 10 \sin(\omega t) + 10 \sin(2\omega t + \psi), f = 8 \text{ GHz}, J_{dc} = 500 \text{ A/cm}^2 \text{ AND } p = 3 \times 10^{15} \text{ cm}^{-3}]$

### 3.4 Conclusions

In this chapter a mathematical model has been developed which describes the large-signal operating characteristics of realistic IMPATT diodes. The model allows for impact ionization to occur at every point in the diode thereby allowing for a wide range of doping profiles to be analyzed. The model predicts the admittance, output power, efficiency and dc bias voltage as a function of the RF voltage, frequency and dc current density. In Section 3.3 dc and ac operating characteristics were presented for several different doping profiles and materials. Both single-frequency and multifrequency operation were studied. The single-frequency results showed that the doping profile is an important parameter in determining the admittance and efficiency of the diodes. It was shown why the different diodes have a different magnitude of negative conductance and how, as the RF voltage is increased, the negative conductance of the diodes approaches that of the pin diode. Finally, a few multifrequency results were presented along with a discussion of the reasons for the enhancement of the performance of the diode by multifrequency operation.

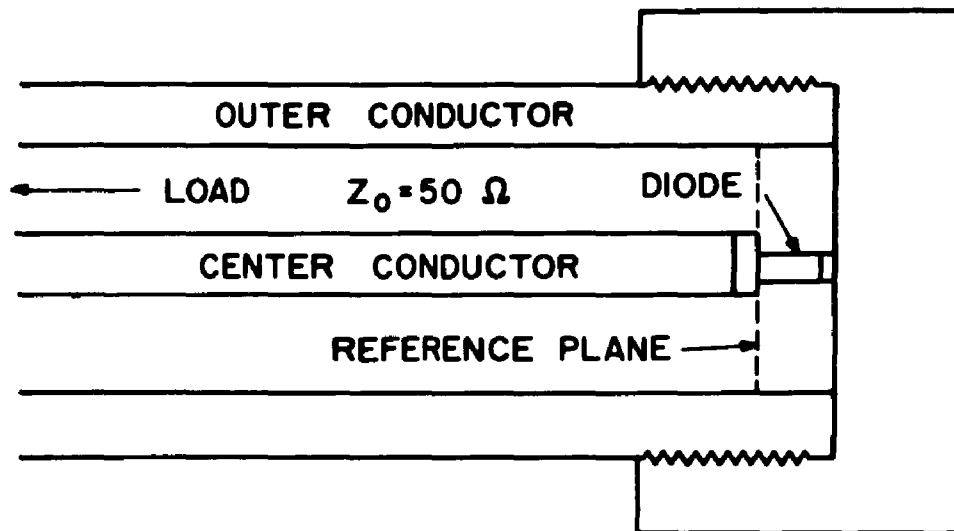
## CHAPTER IV. EXPERIMENTAL INVESTIGATION OF THE OPERATING CHARACTERISTICS OF IMPATT DIODES

### 4.1 Introduction

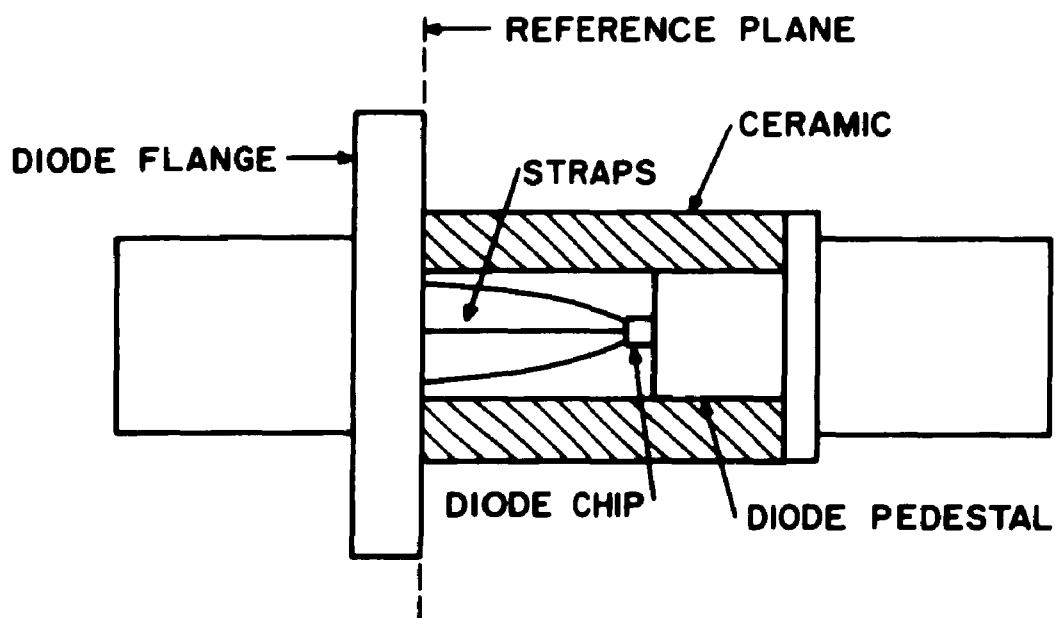
In Chapters II and III the large-signal operating characteristics of IMPATT diodes were investigated theoretically. In this chapter a discussion of the experimental work which was performed to measure the operating characteristics is presented along with the experimental results. In Section 4.2 the equivalent circuit of the diode package and mount is obtained. In Section 4.3 the capacitance of the diode is measured as a function of the reverse bias voltage to determine both the validity of the equivalent circuit and the doping density of the diode. In the next section the small-signal admittance of the diode is measured in a reflection-type amplifier circuit. Also the large-signal admittance is obtained by measuring the admittance of the microwave circuit seen by the oscillating diode. The results of both of these measurements are compared with the theoretical results obtained in Chapters II and III. Finally, a discussion of single-frequency operation of IMPATT diodes in series and parallel external circuits is presented.

### 4.2 Determination of the Equivalent Circuit of the Diode Package and Mount

4.2.1 Introduction. The coaxial circuit and package in which the IMPATT diode is mounted is shown in Fig. 4.1. The equivalent circuit which has been assumed for this mount and package is shown in Fig. 4.2.



(a) COAXIAL CIRCUIT



(b) DIODE PACKAGE

FIG. 4.1 MICROWAVE CIRCUIT AND DIODE PACKAGE.

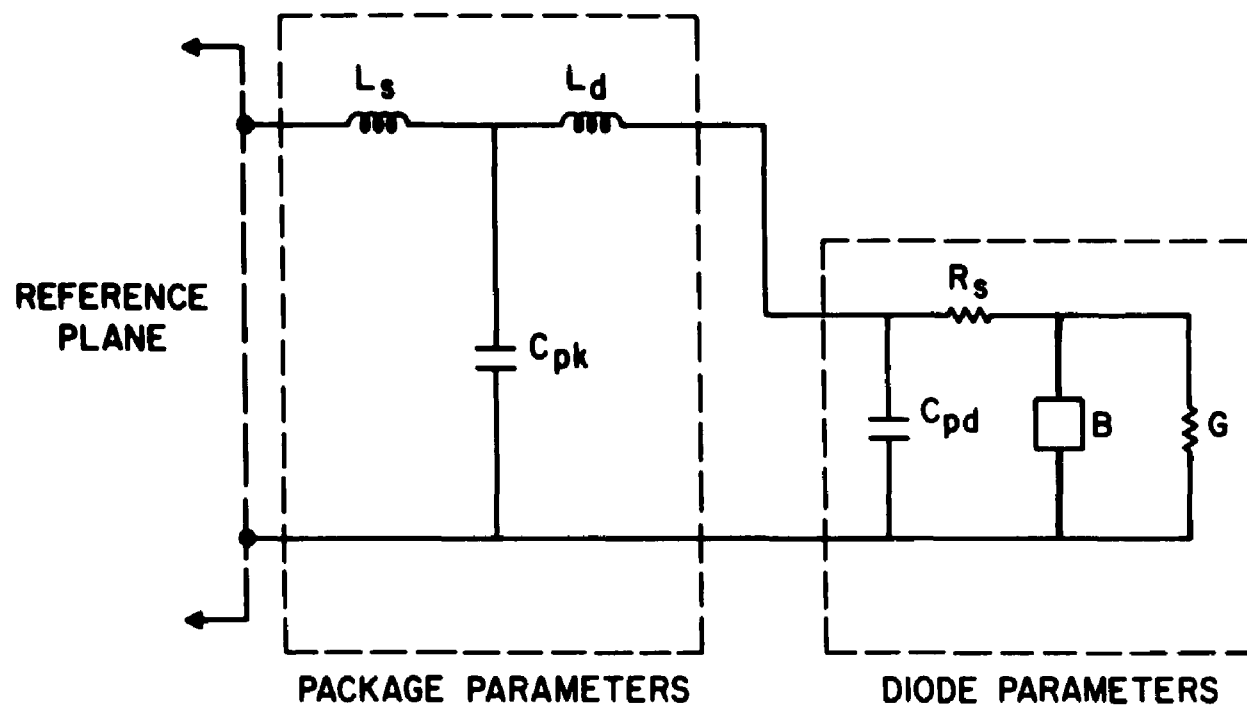


FIG. 4.2 EQUIVALENT CIRCUIT OF THE DIODE PACKAGE AND THE DIODE.

The lumped elements of the equivalent circuit are as follows. The conductance of the diode is represented by  $G$  and the susceptance by  $B$ . This susceptance can be either capacitive or inductive. The series resistance of the diode due to contact losses, the resistance of the substrate and the undepleted region of the diode is represented by  $R_s$ . The pad capacitance  $C_{pd}$  is due to the capacitance between the metal contact on top of the diode and the pedestal on which the diode is mounted. This capacitance which is a function of the diode geometry is negligible for the diodes investigated.<sup>52</sup> The lead inductance  $L_d$  is the inductance due to the four wire straps between the flange and the top of the diode. The capacitance of the package and the fringing fields at the end of the coaxial line is represented by  $C_{pk}$ . The series inductance  $L_s$  is the inductance caused by the high-impedance, shorted-transmission line of the pedestal<sup>53</sup> and the magnetic energy storage in the cylindrical volume around the diode.<sup>54</sup>

The series inductance and package capacitance are determined from the measured impedance of an open-circuited package. The lead inductance is determined by two methods. First from the impedance of a short-circuited package and then from the impedance of a forward-biased diode. These impedance measurements are made in the test circuit shown in Fig. 4.3. A shorting washer is placed at the end of the center conductor in place of the diode in order to establish a reference plane. The location of the minimum of the standing-wave ratio is measured as a function of frequency with the shorting washer at the reference plane. This minimum location is then shifted by the thickness of the diode flange

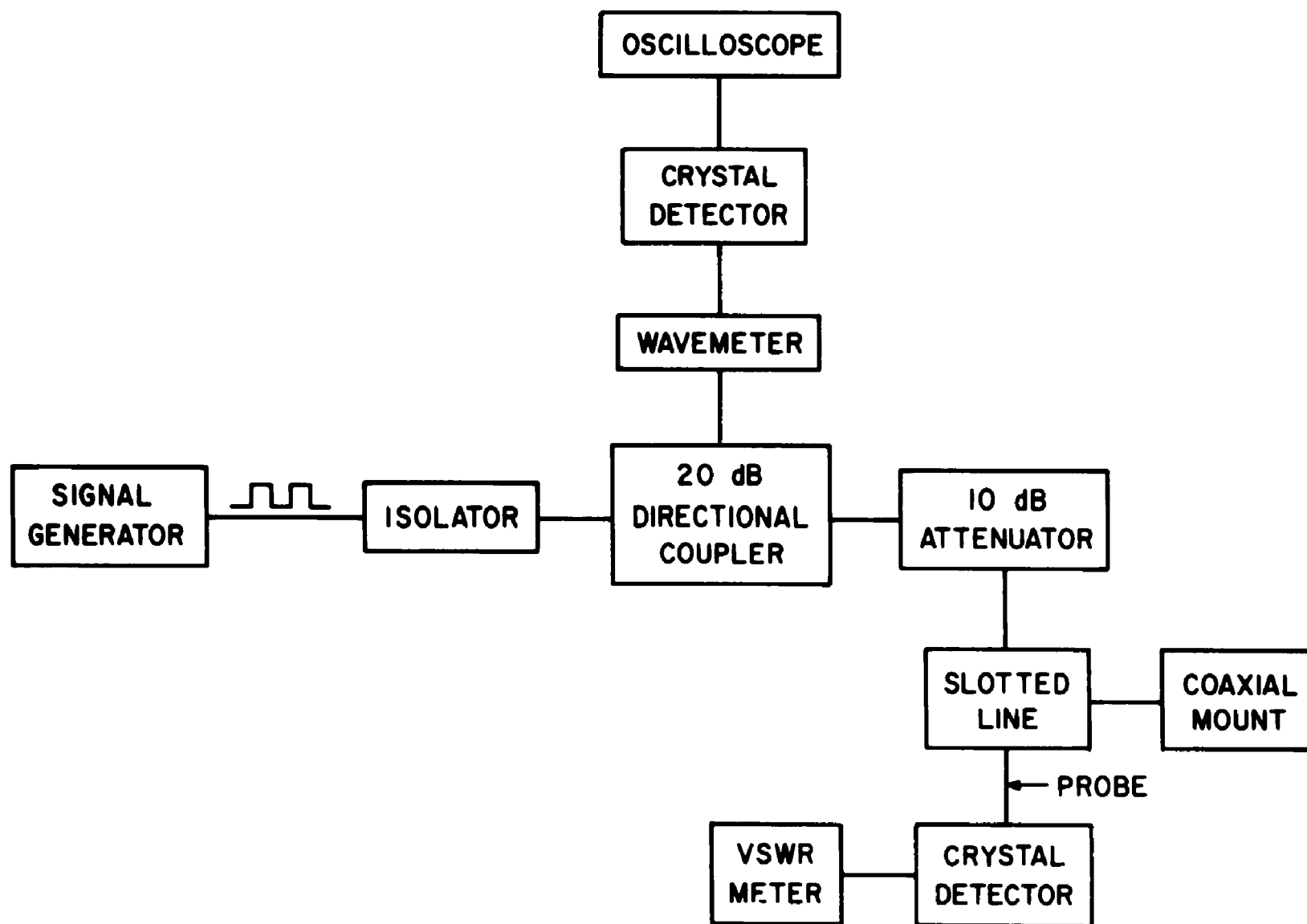


FIG. 4.3 EXPERIMENTAL TEST CIRCUIT FOR OPEN- AND SHORT-CIRCUITED PACKAGE MEASUREMENTS.

so that it corresponds to a reference plane located between the flange and the ceramic of the diode.

4.2.2 Open-Circuited Package Measurement. The impedance of the open-circuited package was measured as a function of frequency and is shown in Fig. 4.4. The package is assumed lossless so only the reactance is plotted. The experimental values are plotted for each frequency and a least-squares fit of the data is also shown. The method of least squares<sup>55</sup> is applied by assuming an equivalent circuit for the open-circuited package as shown in Fig. 4.5. In Appendix F the derivation of  $L_s$  and  $C_{pk}$  from a least-squares fit of the data is presented.

From the data plotted in Fig. 4.4 the values for the series inductance and the package capacitance are

$$L_s = 0.22 \text{ nH} \quad (4.1)$$

and

$$C_{pk} = 0.32 \text{ pF} \quad (4.2)$$

From the geometry of the package and mount, approximate values for these two parameters can be obtained. For the series inductance the pedestal looks like a short transmission line with a short on the end. Because of the ceramic a TEM mode does not exist and determination of the characteristic impedance of the line is not a simple problem.<sup>56</sup> However an approximate value can be obtained by assuming a characteristic impedance between that for an air line and for a dielectric ( $\epsilon_r = 9.6$ ) filled coaxial line. Since the ceramic is small the characteristic impedance is approximately equal to the air-line impedance and is assumed to be  $100 \Omega$ . Because of the short length of the transmission line the reactance of the line  $j\omega L_s$  is given by

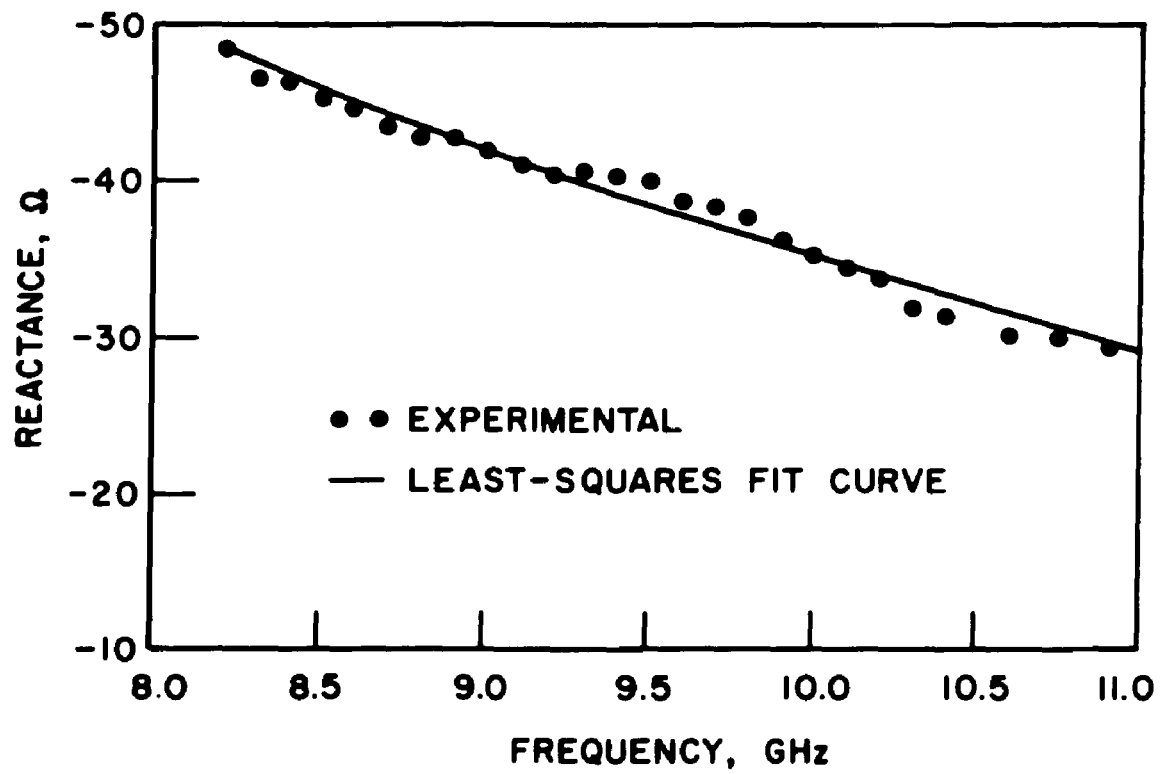


FIG. 4.4 REACTANCE VS. FREQUENCY FOR AN OPEN-CIRCUITED PACKAGE.

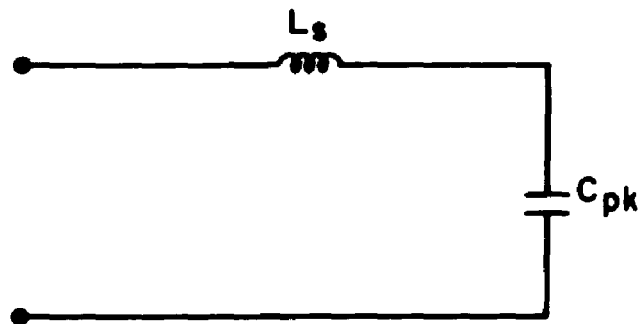


FIG. 4.5 EQUIVALENT CIRCUIT OF AN OPEN-CIRCUITED PACKAGE.

$$j\omega L_s \cong jZ_o \frac{\omega l}{c} , \quad (4.3)$$

where  $c$  = the velocity of the wave on the line (cm/s) and

$l$  = the length of the pedestal (cm).

Thus  $L_s$  can be easily found and is given by

$$L_s = 0.2286 \text{ nH} . \quad (4.4)$$

This gives remarkable agreement with Eq. 4.1.

The package capacitance can be approximated by considering three capacitors in parallel: a capacitance between the pedestal and the flange with an air dielectric, a capacitance between the pedestal and the flange with a ceramic dielectric and one due to the fringing between the flange and the outer conductor. The first two capacitors can be calculated from

$$C = \epsilon A/d , \quad (4.5)$$

where  $A$  = the area ( $\text{cm}^2$ ) and

$d$  = the spacing between the flange and the pedestal (cm).

For the first two capacitors the values are approximately 0.015 pF and 0.27 pF, respectively. The third capacitance due to the fringing is approximated by the capacitance due to a change in the center conductor in a coaxial line. This capacitance can be found in the literature<sup>57</sup> for the given dimensions and is approximately 0.015 pF. Thus the total capacitance due to the package and mount is approximately 0.3 pF. This also agrees with the measured value given in Eq. 4.2.

4.2.3 Short-Circuited Package Measurement. The same test circuit as shown in Fig. 4.3 was used to measure the impedance of the short-circuited package. The equivalent circuit for the short-circuited package which was assumed is shown in Fig. 4.6. The series inductance as obtained above is assumed to be the same value. However the package capacitance changes due to the effect on the electric field of the four metal straps which short the flange to the pedestal. Experimentally measured values for the reactance of the shorted package are shown in Fig. 4.7. Again the package is assumed lossless. Two curves have been fitted to the data by the method of least squares. The solid curve is for the case where both the package capacitance and the lead inductance are adjusted to give the best fit. The dashed curve is for the case where the package capacitance is set at the value determined by the open-circuited package and the lead inductance is adjusted to give the best fit. It is seen that better agreement is obtained when the package capacitance is assumed to be different from the open-circuited value. The amount of change in the package capacitance due to the shorting straps appears rather large but will be verified in later sections.

4.2.4 Forward-Biased Diode Measurement. Another method of measuring the lead inductance is by measuring the impedance of a forward-biased diode.<sup>58</sup> The equivalent circuit<sup>59</sup> for a forward-biased diode is shown in Fig. 4.8. The series inductance and package capacitance are the values obtained above. The pad capacitance has been assumed negligible for this diode structure. The depletion-layer capacitance  $C_d$  is smaller than the diffusion capacitance<sup>59</sup>  $C_{df}$  when the diode is forward biased. The diffusion capacitance is given by

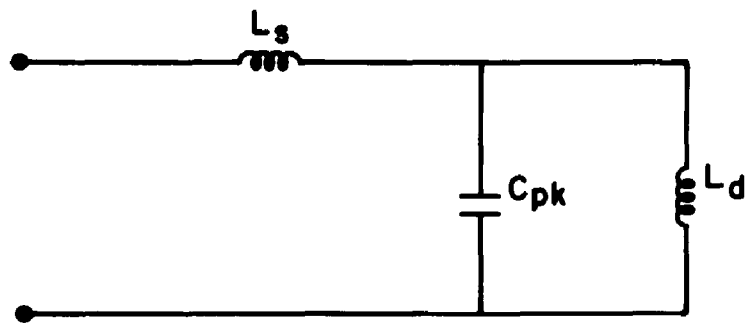


FIG. 4.6 EQUIVALENT CIRCUIT OF A SHORT-CIRCUITED PACKAGE.

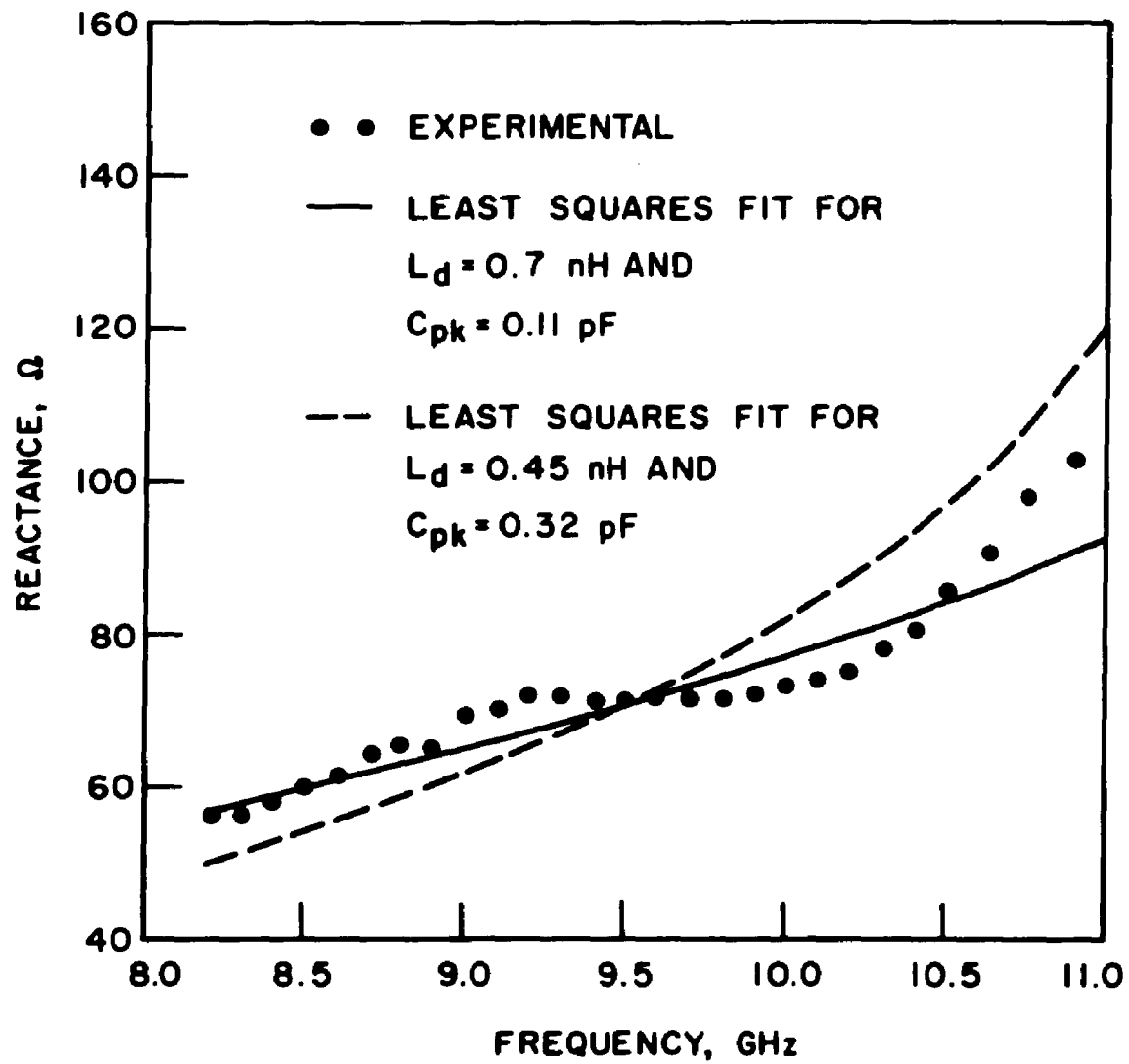
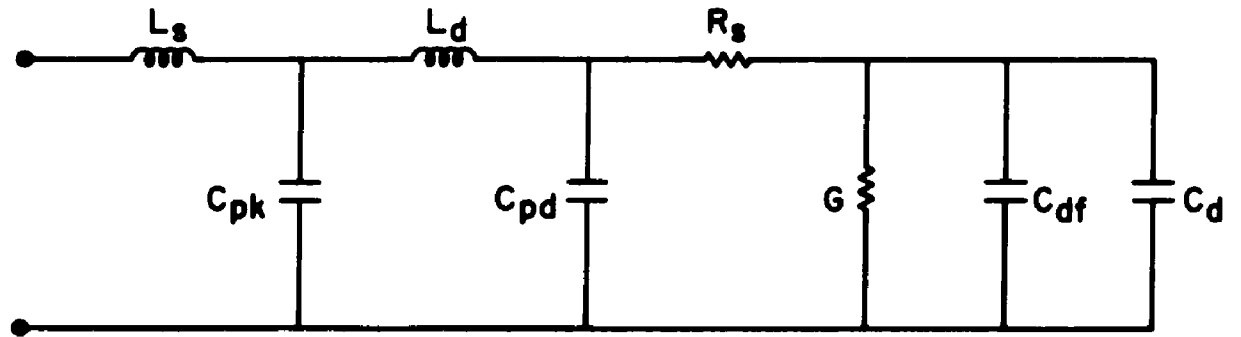
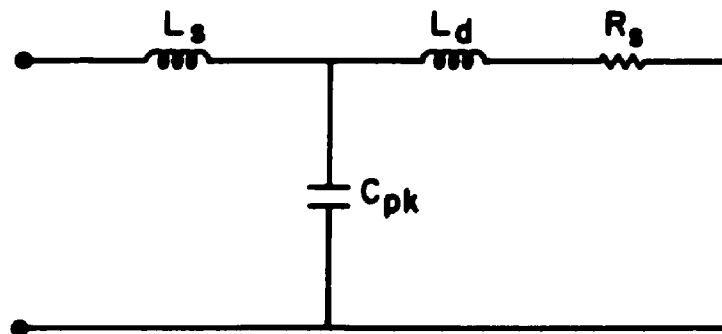


FIG. 4.7 REACTANCE VS. FREQUENCY FOR A SHORT-CIRCUITED PACKAGE.



(a) COMPLETE EQUIVALENT CIRCUIT



(b) ASSUMED EQUIVALENT CIRCUIT

FIG. 4.8 EQUIVALENT CIRCUIT OF A FORWARD-BIASED DIODE.

$$C_{df} = \frac{qI_{dc} W_{sc}^2}{3kTD_p} \quad (4.6)$$

where  $W_{sc}$  = the width of the undepleted junction (cm),

$k$  = the Boltzmann constant (J/°K),

$T$  = the absolute temperature (°K) and

$D_p$  = the diffusion constant for holes (cm<sup>2</sup>/s).

Typical values<sup>60</sup> for these constants for the diodes under consideration are  $W_{sc} = 3.5 \times 10^{-4}$  cm and  $D_p = 10$  cm<sup>2</sup>/s. The dc current was 10 mA and thus,

$$C_{df} = 16.33 \times 10^{-10} \text{ F} \quad (4.7)$$

The impedance of the diffusion capacitance at 10 GHz is therefore

$$X = \frac{1}{\omega C_{df}} \cong 10^{-2} \Omega \quad (4.8)$$

Thus the effect of the diffusion and depletion-layer capacitors is negligible. The assumed equivalent circuit for the forward-biased diode is shown in Fig. 4.8. From a measurement of the impedance of the forward-biased diode the lead inductance and the series resistance can be calculated. These experimental results are shown in Fig. 4.9 as a function of frequency. The average values of the lead inductance and the series resistance are  $L_d = 0.793$  nH and  $R_s = 3.03 \Omega$ . The lead inductance agrees fairly well with the value obtained for the short-circuited package. The series resistance measured by this method is not needed for the determination of the package equivalent circuit. The series resistance as shown in Fig. 4.2 accounts for the losses in the mount, package and

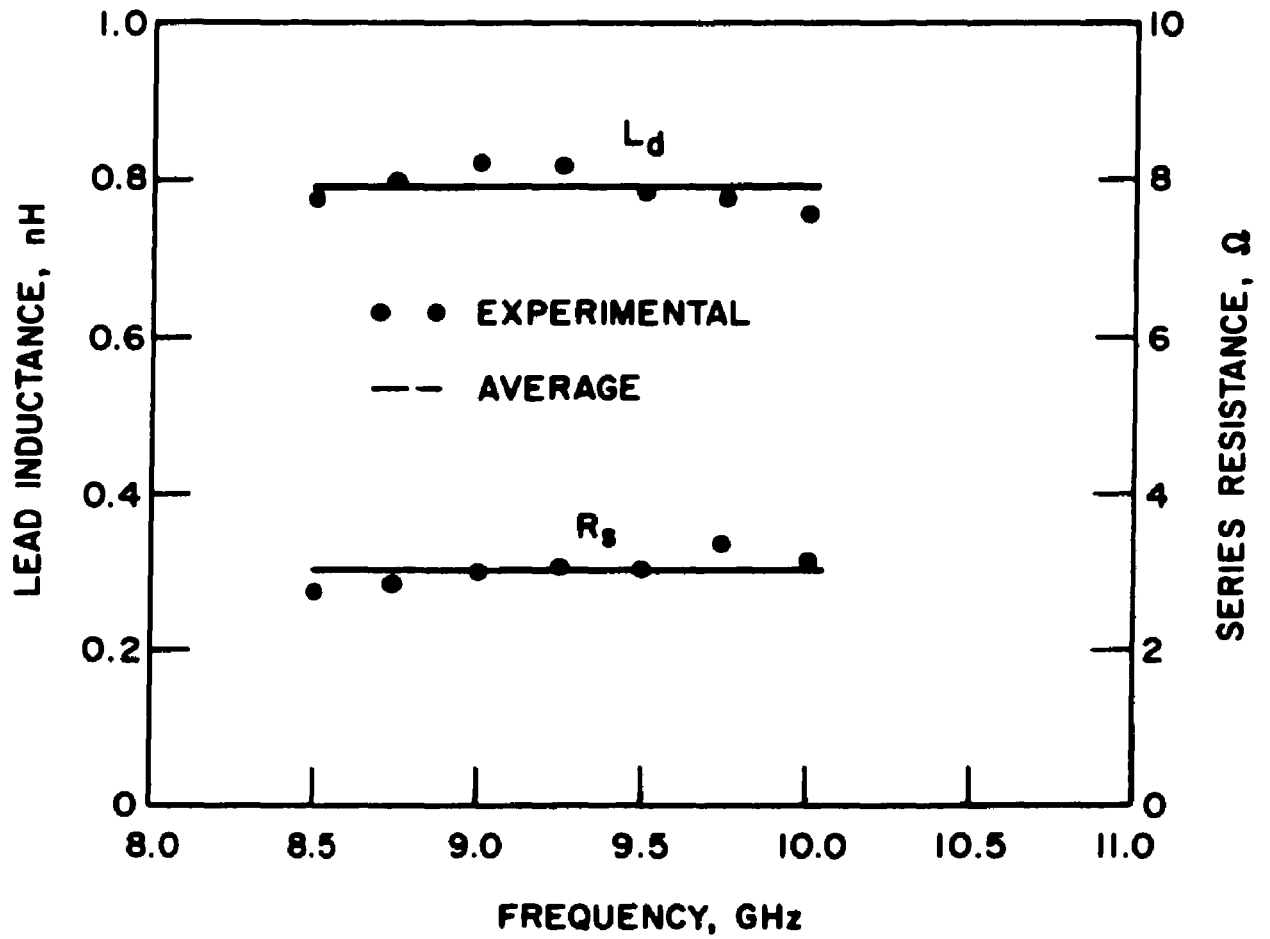


FIG. 4.9 LEAD INDUCTANCE AND SERIES RESISTANCE VS. FREQUENCY FOR A FORWARD-BIASED DIODE. ( $I_{dc} = 10 \text{ mA}$ )

diode when the diode is biased into the avalanche region. This resistance will be measured by another method discussed in the next section.

#### 4.3 Capacitance vs. Voltage Measurements

4.3.1 Verification of Equivalent Circuit. The equivalent circuit for the diode package and mount obtained in the previous section transforms the diode's conductance and susceptance from the package pedestal to the reference plane in the coaxial line. Verification that the lumped elements obtained for the equivalent circuit give the correct transformation over the frequency range considered was obtained by measuring the depletion-layer capacitance as a function of the reverse voltage at 1 MHz and at X-band. At 1 MHz, the elements in the equivalent circuit are negligible and the depletion-layer capacitance which is the susceptance of the diode below breakdown is obtained directly. The test circuit which was used to make this measurement is shown in Fig. 4.10. Both the reverse voltage and the depletion-layer capacitance were measured on digital voltmeters. The capacitance meter is first nulled with an open-circuited package in the coaxial mount; then the diode is placed in the mount and the capacitance is measured as a function of the reverse voltage from 0 V to breakdown.

The depletion-layer capacitance was also measured at X-band as a function of the reverse voltage. The test circuit is the same as that in Fig. 4.3 except for the addition of a bias tee between the 10 dB attenuator and the 20 dB directional coupler. A power supply and a digital voltmeter were used to provide the reverse voltage via the bias tee. The impedance of the diode was measured at frequencies of

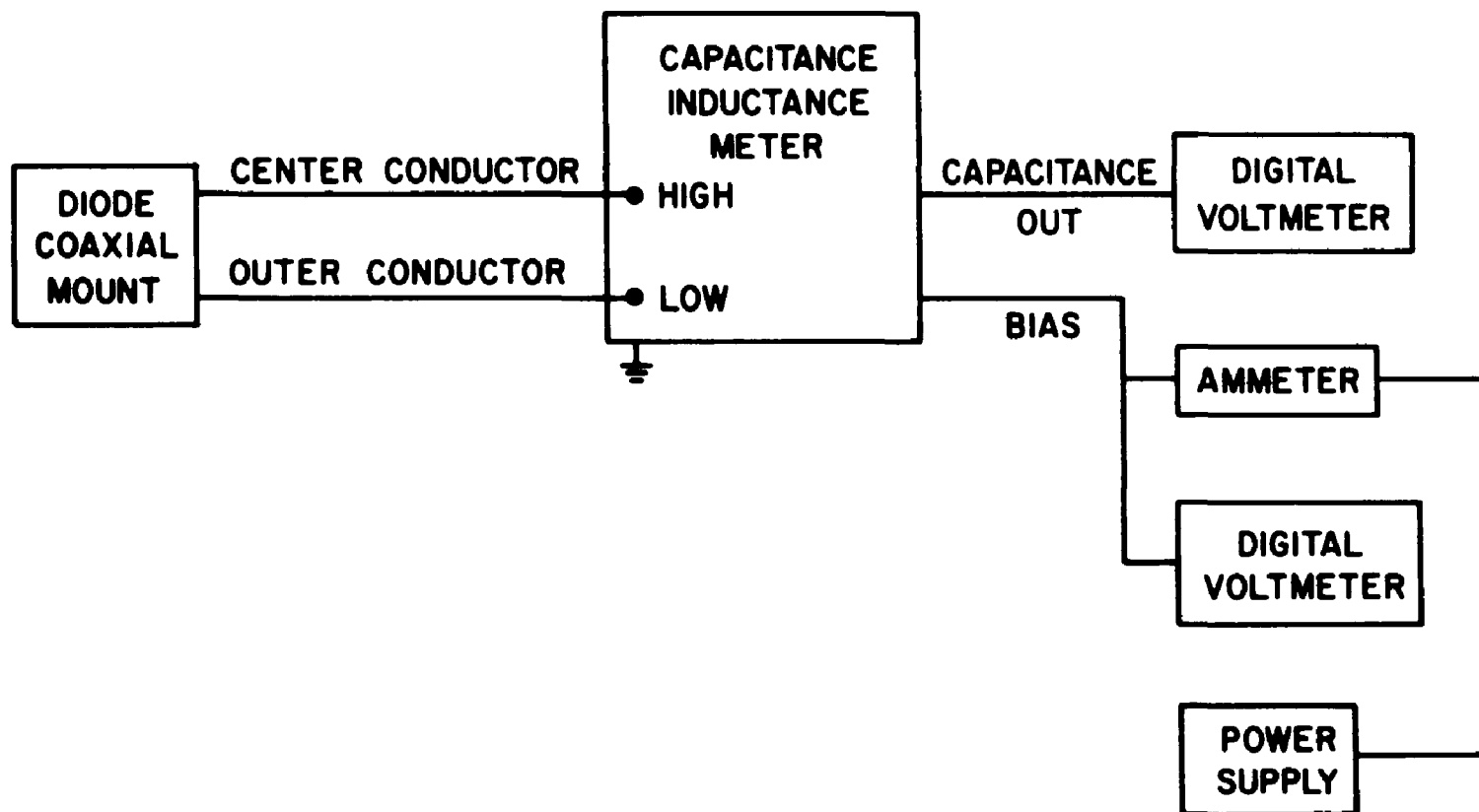


FIG. 4.10 EXPERIMENTAL CIRCUIT FOR CAPACITANCE VS. REVERSE VOLTAGE MEASUREMENT AT 1 MHz.

8.5, 8.75, 9.0, 9.25, 9.5, 9.75 and 10.0 GHz as a function of the reverse voltage. From this impedance the depletion-layer capacitance and series resistance of the diode were obtained by transforming the impedance through the equivalent circuit.

In Fig. 4.11, the two methods of measuring the depletion-layer capacitance are compared. The solid curve is for the 1 MHz measurement. Case A shows the capacitance obtained at X-band using the equivalent circuit derived in the previous section. Case B is for the capacitance obtained at X-band using the values obtained for the open-circuited package capacitance and the resulting lead inductance. The spread in the data shows the minimum and maximum capacitance which was obtained at all the frequencies employed. If the depletion-layer capacitance is assumed to be independent of frequency,<sup>52</sup> then these results show that the transformation performed by the equivalent circuit element values given in Table 4.1 is correct. The series resistance of the diode is taken as the resistance<sup>52</sup> at breakdown with  $I_{dc} = 1$  mA. This resistance is obtained from the impedance measurement at X-band. The resistance is plotted as a function of the reverse voltage in Fig. 4.12. Again the minimum and maximum values obtained over the frequencies considered are shown. At breakdown (78.8 V) the average resistance is approximately 1.1  $\Omega$ .

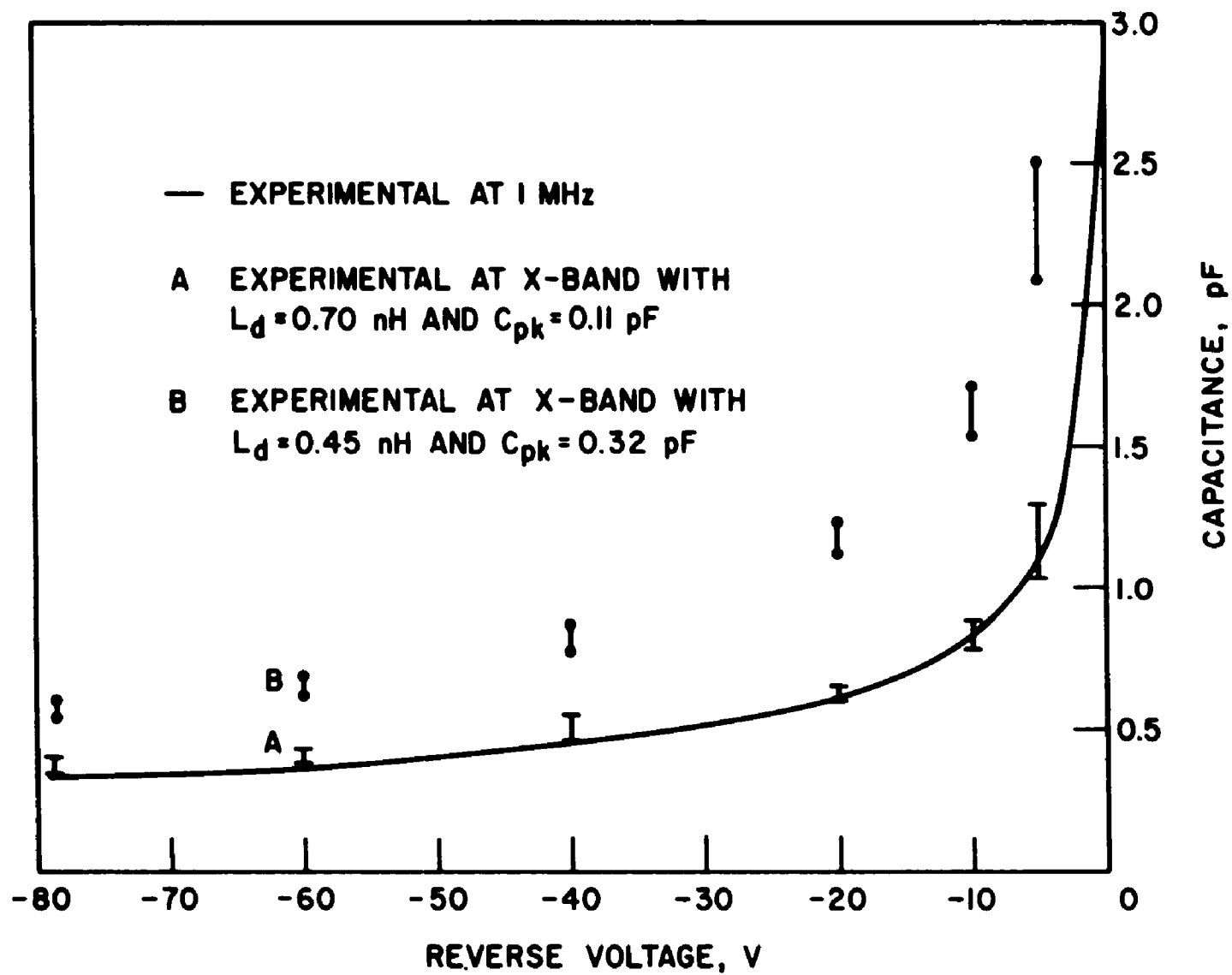


FIG. 4.11 CAPACITANCE VS. REVERSE VOLTAGE FOR DIODE NO. TI 40.

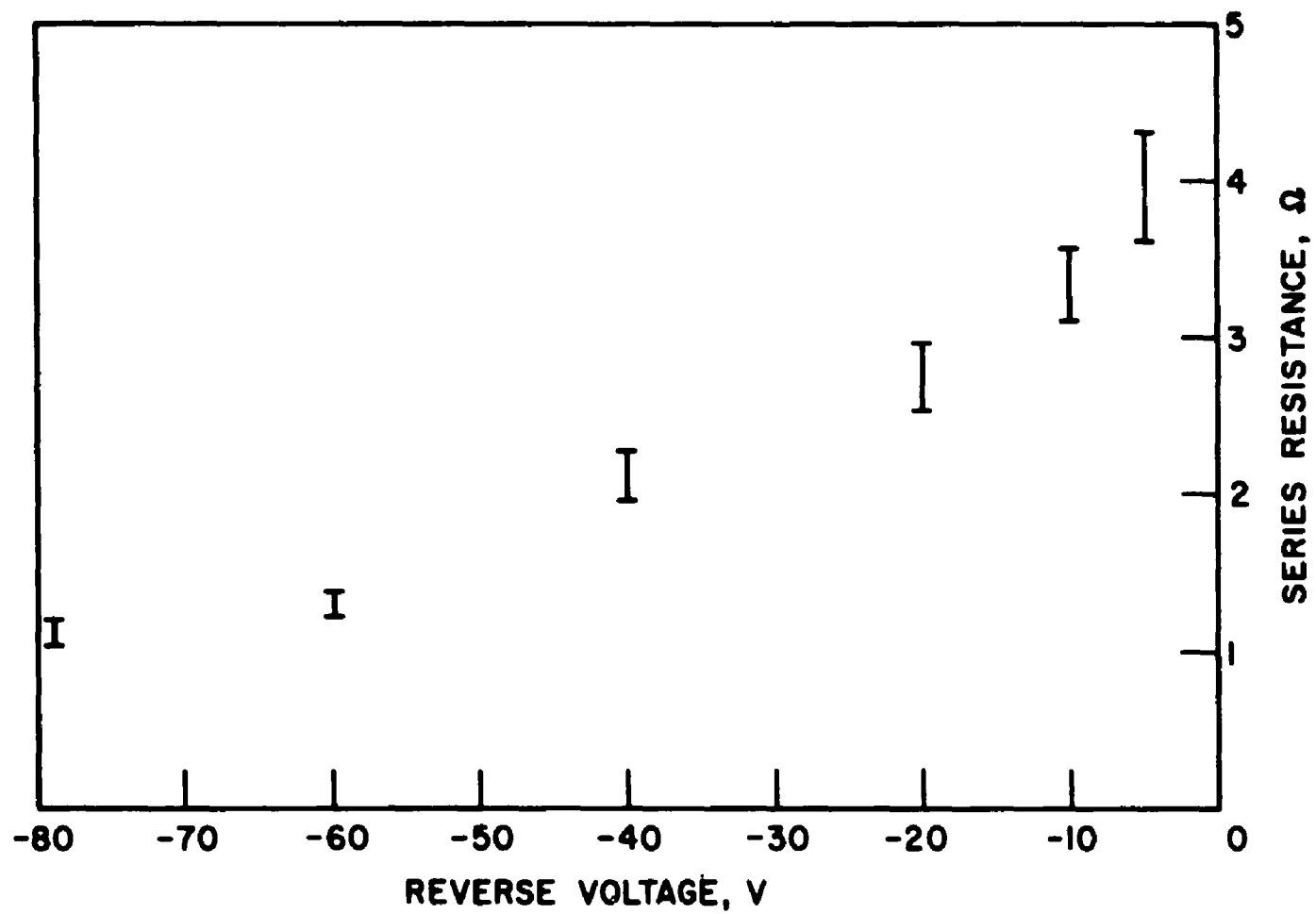


FIG. 4.12 SERIES RESISTANCE VS. REVERSE VOLTAGE FOR DIODE NO. TI 40.

Table 4.1

Element Values for Equivalent Circuit

$$\begin{aligned}L_s &= 0.22 \text{ nH} \\C_{pk} &= 0.11 \text{ pF} \\L_d &= 0.70 \text{ nH} \\C_{pd} &= 0 \text{ pF} \\R_s &= 1.1 \Omega\end{aligned}$$

The important property of the individual element values in Table 4.1 is that they transform the conductance and susceptance of the diode properly to the reference plane. This circuit could be represented by just a reactance and resistance measured at each frequency which yield the proper transformation. The results in Fig. 4.11 show that the transformation is correct. The series resistance however is not verified by this method but the value obtained is quite reasonable.

4.3.2 Determination of Doping Density. The doping density in a one-sided abrupt junction can be determined<sup>61</sup> from the C-V data obtained above. Assuming a one-sided abrupt junction, the doping density  $N(x)$  is related to the capacitance and the reverse voltage of a diode by<sup>62</sup>

$$N(x) = \frac{K C^3}{D^4 \cdot \left( -\frac{dC}{dV} \right)} \quad (4.9)$$

and the depth  $x$  in microns from the junction is given by

$$x = K_2 D^2 / C , \quad (4.10)$$

where

$$K_1 = \frac{16}{q\epsilon\pi^2} \frac{10^{-8}}{(25.4)^4} ,$$

$$K_2 = \frac{\epsilon\pi}{4} (25.4)^2 \times 10^8 ,$$

C = the capacitance (pF),

D = the junction diameter (mils) and

V = the reverse voltage (V).

These equations have assumed a one-sided abrupt junction which is one-dimensional and of constant area. For a diffused junction, the one-sided approximation is reasonable. Also for the diode structures investigated, the junction area is approximately constant with respect to x. Thus the doping profiles obtained from the C-V data are approximately correct. The area or diameter of the junction could not be obtained without destroying the diode, therefore the nominal value was assumed.

One of the problems which exists when taking the C-V data is that the capacitance increases with heating. Starting at 0 V bias the capacitance was measured at 100 values of reverse voltage up to breakdown. When the reverse voltage was brought back to zero the zero-voltage capacitance was slightly larger. After several minutes the capacitance decayed back to the original zero-voltage value. The C-V data were then taken for ascending and descending values of voltage and the capacitance was averaged.

The C-V data are reduced by a computer program and plots of the capacitance as a function of voltage and the doping density as a function of  $x$  are obtained. Figures 4.13 and 4.14 show the Calcomp plots for the capacitance vs. voltage and doping density vs.  $x$  for diode No. TI 40. The nominal parameters for this diode were a doping density of  $6 \times 10^{15} \text{ cm}^{-3}$  and a diameter of 0.005 inch. In Figs. 4.15 and 4.16 similar plots for diode No. TI 40B are shown. Again the nominal parameters were a doping density of  $3 \times 10^{15} \text{ cm}^{-3}$  and a diameter of 0.005 inch. Diode No. TI 40B shows that the electric field at breakdown has punched through to the substrate as can be seen by the doping density beyond  $3.5 \text{ } \mu\text{m}$ . This is also shown by the nearly zero slope of the capacitance for large reverse voltages.

#### 4.4 Diode Admittance Measurements

4.4.1 Reflection-Type Amplifier Measurement. The admittance of the IMPATT diode was measured in the reflection-type amplifier test circuit shown in Fig. 4.17. The power levels of the incident signal from the klystron and the reflected signal from the diode are compared. The admittance of the diode is measured with the slotted line as a function of frequency and dc bias current. It was necessary to compare incident and reflected power levels as part of the admittance measurement in order to determine the sign of the VSWR. As the bias current through the diode is increased the negative conductance of the diode increases. For a large enough dc bias current the negative resistance is larger than the series resistance and thus the reflected power is larger than the incident power. The VSWR therefore increases to

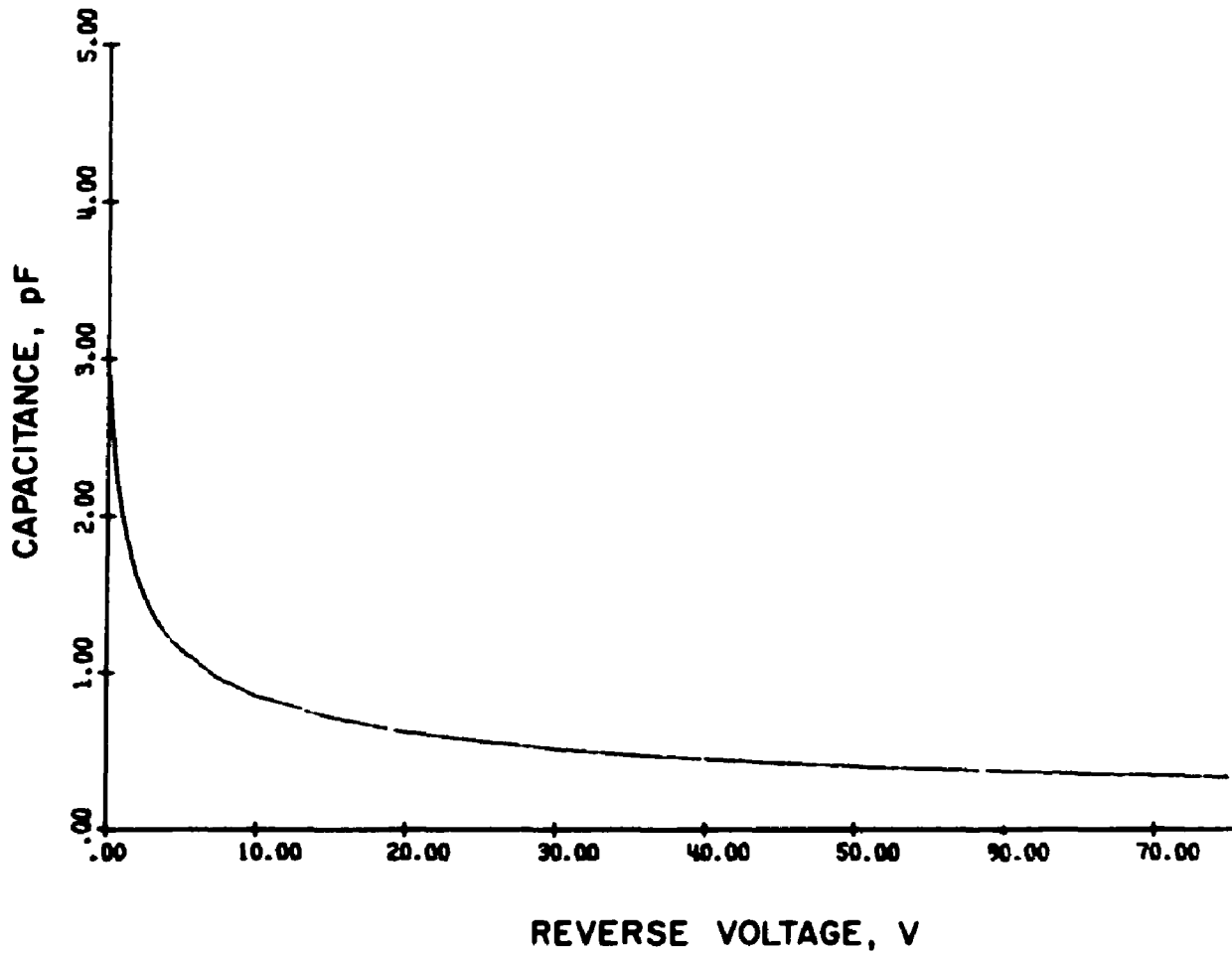


FIG. 4.13 CAPACITANCE VS. REVERSE VOLTAGE. (DIODE NO. TI 40,  
D = 0.005 INCH)

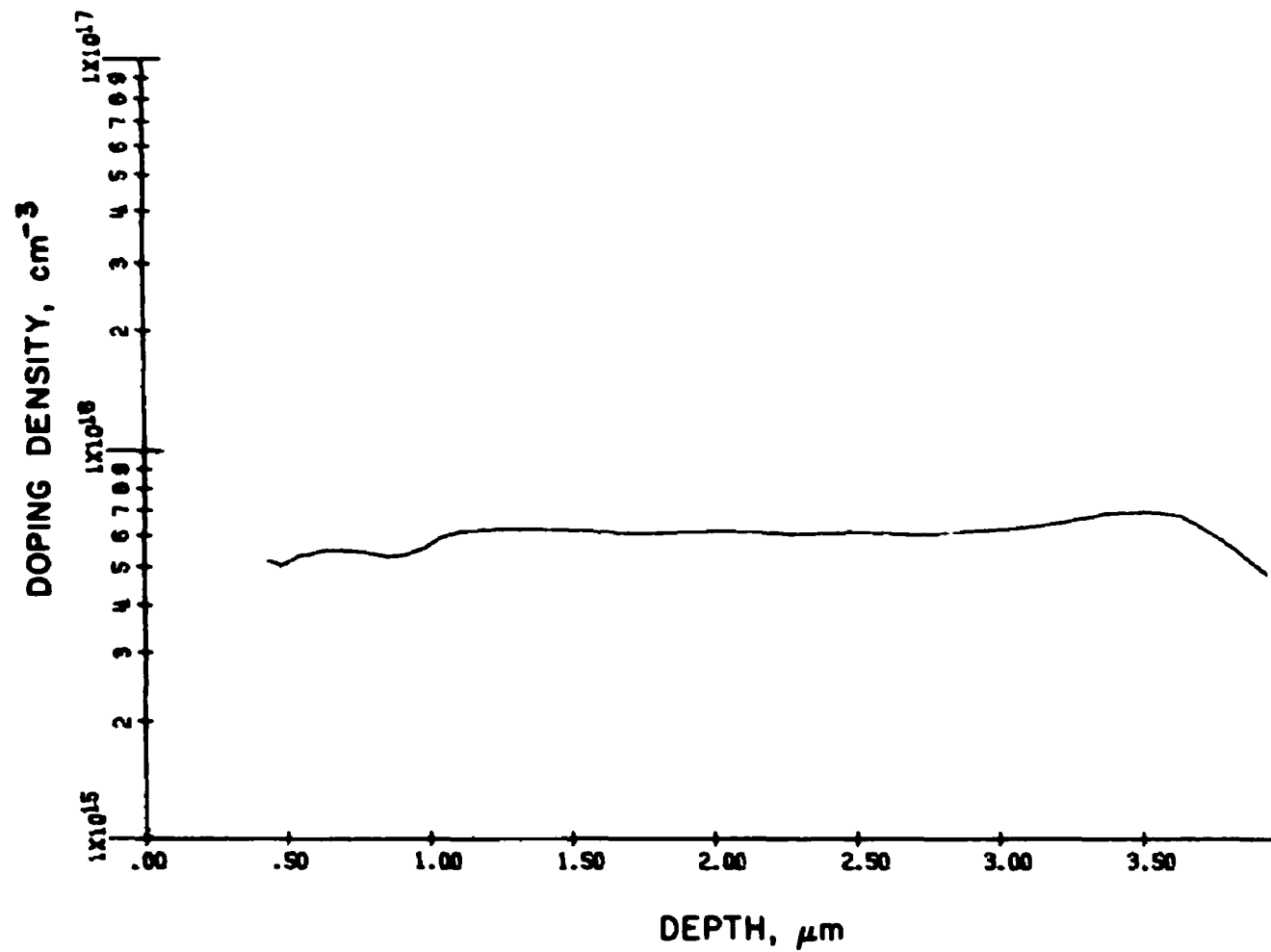


FIG. 4.14 DOPING DENSITY VS. DEPTH OF DEPLETION REGION. (DIODE NO. TI 40,  $D = 0.005$  INCH)

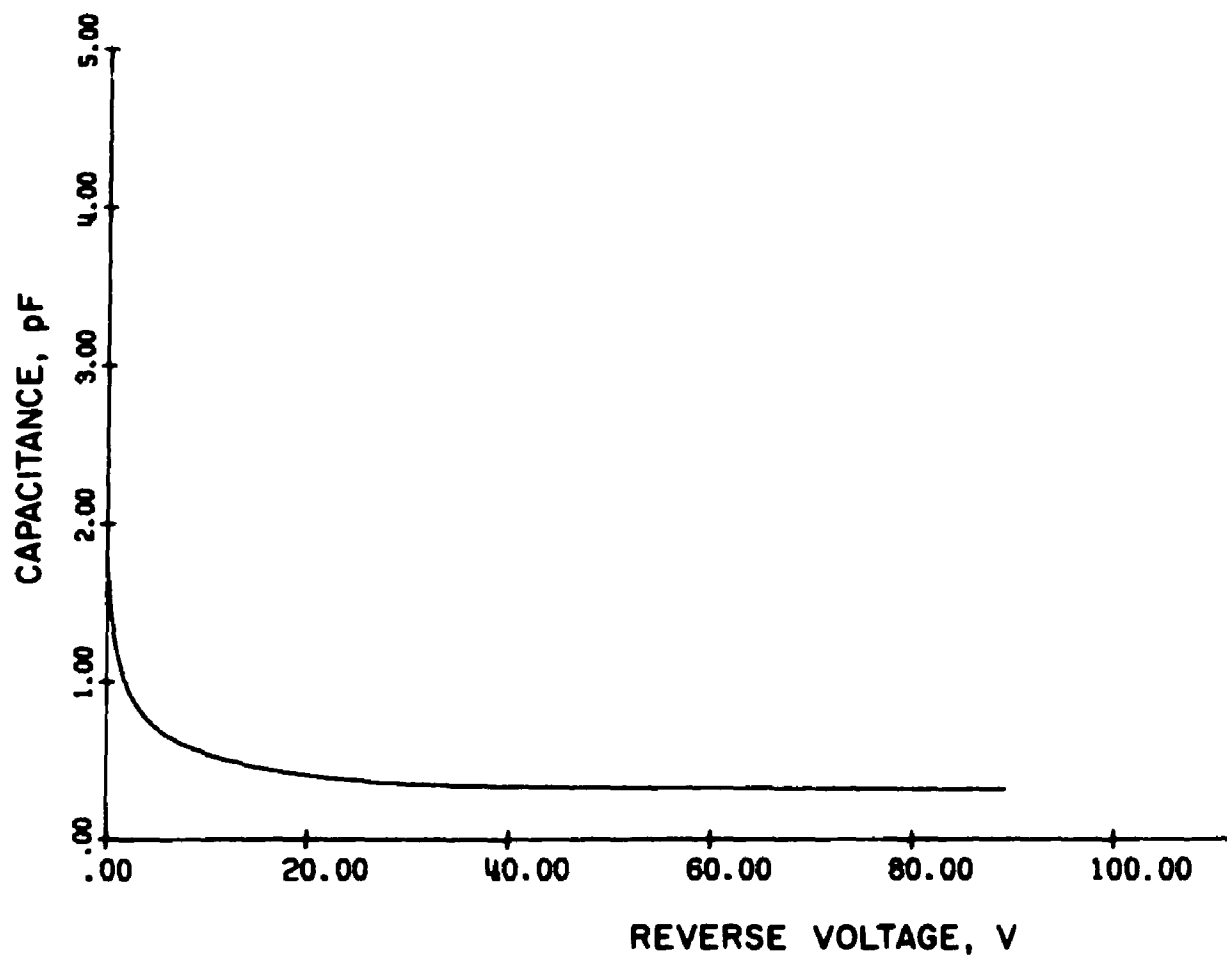


FIG. 4.15 CAPACITANCE VS. REVERSE VOLTAGE. (DIODE NO. TI 40B,  
D = 0.005 INCH)

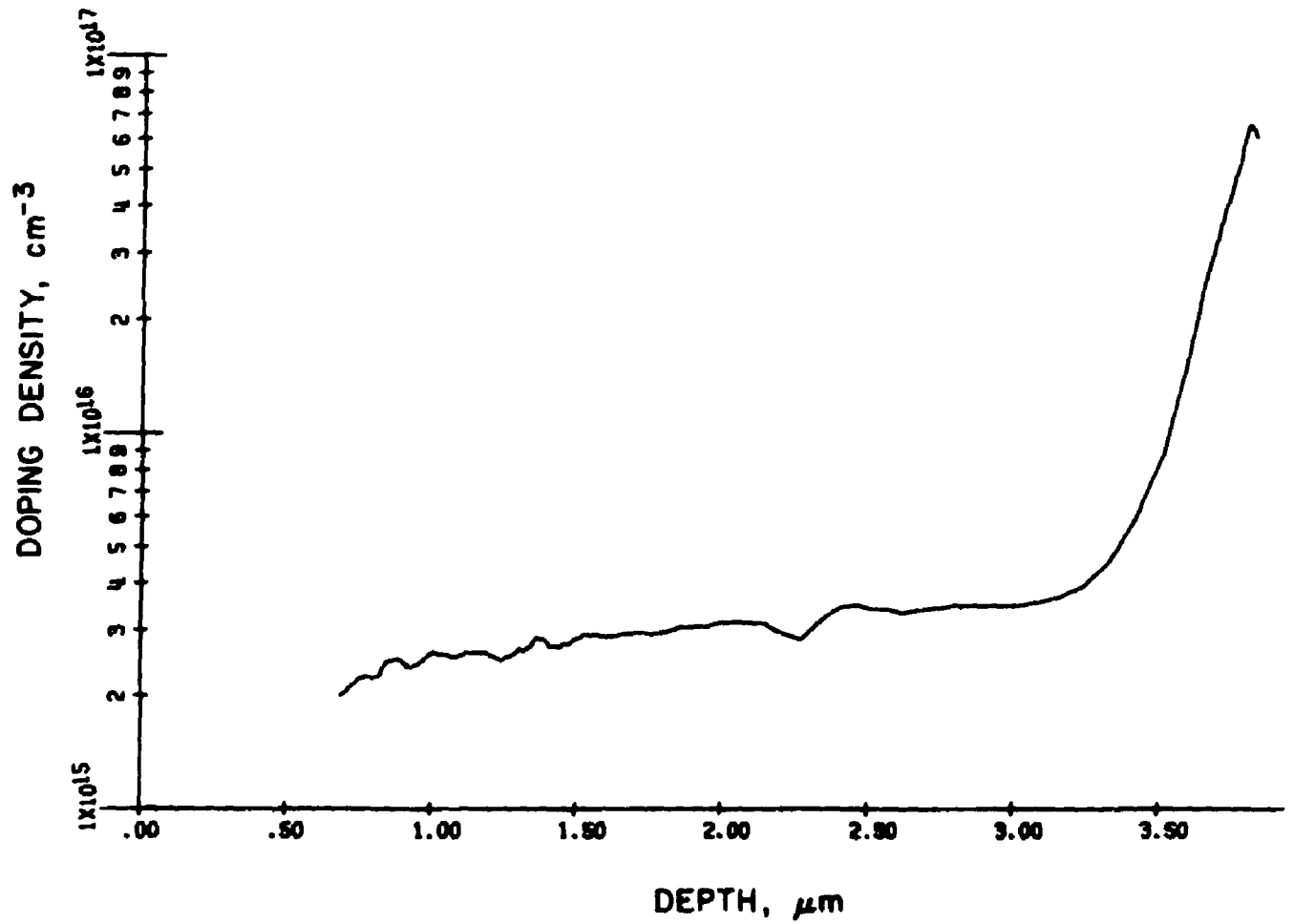


FIG. 4.16 DOPING DENSITY VS. DEPTH OF DEPLETION REGION. (DIODE NO. TI 40B, D = 0.005 INCH)

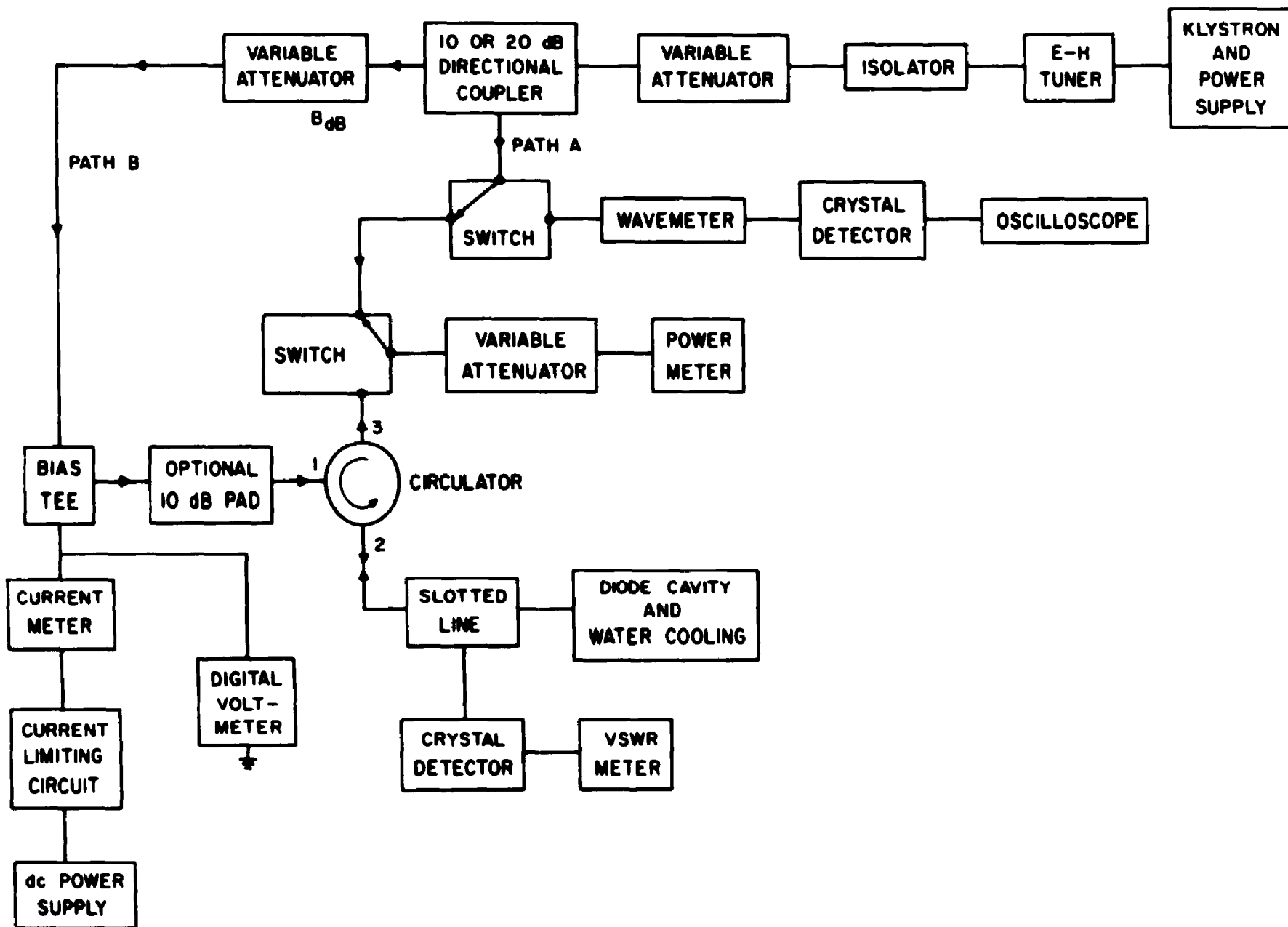


FIG. 4.17 EXPERIMENTAL TEST CIRCUIT FOR REFLECTION-TYPE AMPLIFIER MEASUREMENTS.

a large positive value as the bias current is increased. At the value of current where the reflected power is larger than the incident power the VSWR changes sign and decreases in magnitude. This effect is shown in Fig. 4.18 for a single-frequency measurement. As a check, the diode was mounted in a tunable microwave circuit and was shown to oscillate at these dc current levels.

Because of the high VSWR's which had to be measured it was necessary to use the double-minimum method<sup>63</sup> of measuring a high VSWR. A micrometer which measured the relative position of the probe in the slotted line to within 0.001 inch was used to determine the position of the two 3 dB points of the standing wave. In order to determine when the reflected power was greater than the incident power the following procedure was used. An adjustable microwave short was substituted for the diode cavity and tuned for maximum reflected power. The variable attenuator  $B_{dB}$  was adjusted to give equal incident power through path A and reflected power through path B. It was necessary to use an adjustable microwave short because of the small reflections due to connectors between the circulator and diode cavity. The variation in reflected power was small as the short was moved. However by adjusting the short properly the point where the reflected power equaled the incident power corresponded exactly with a VSWR which was too large to be measured.

The reduction of the slotted-line measurements to give the conductance and susceptance of the diode is done in the following manner. The load impedance  $Z_L$  measured by the VSWR and the voltage minimum is given by<sup>63</sup>

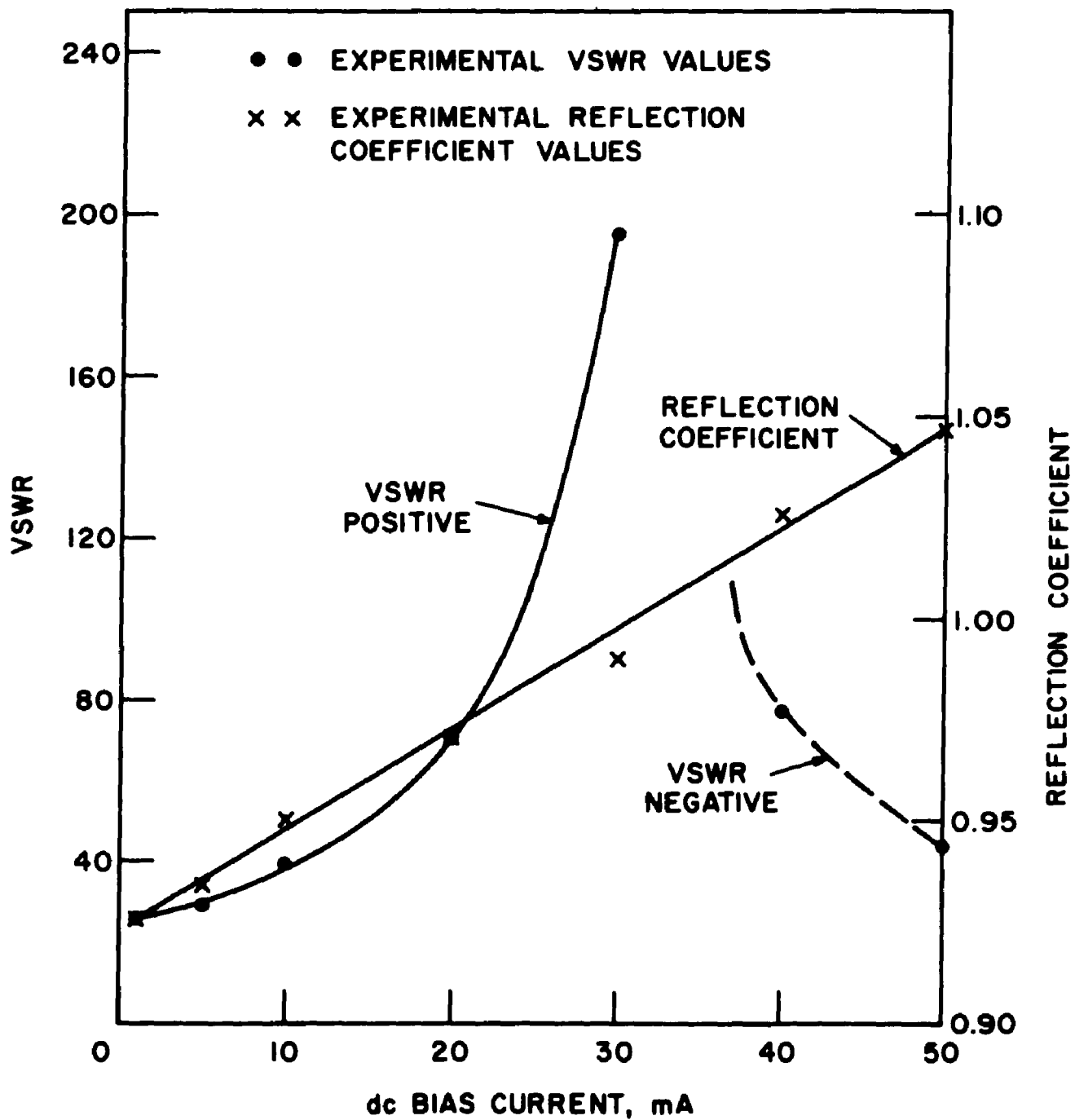


FIG. 4.18 VSWR AND REFLECTION COEFFICIENT VS. dc BIAS CURRENT FOR DIODE NO. TI 40B. ( $f = 8$  GHz)

$$Z_L = Z_0 \left( \frac{1 - jr \tan \theta}{r - j \tan \theta} \right) , \quad (4.11)$$

where  $Z_0$  = the characteristic impedance of the coaxial line ( $\Omega$ ),

$r$  = VSWR,

$\theta$  =  $2\pi(x_1 - x_0)/\lambda$  (rad),

$\lambda$  = the wavelength of the signal frequency (mils),

$x_1$  = the position of the voltage minimum with the load (mils) and

$x_0$  = the position of the voltage minimum with the short (mils).

The impedance is divided into real and imaginary parts as

$$\frac{Z_L}{Z_0} = R + jX \quad (4.12)$$

in which

$$R = \frac{r}{r^2 \cos^2 \theta + \sin^2 \theta} \quad (4.13)$$

and

$$X = \frac{(1 - r^2) \sin 2\theta}{2 \cdot (r^2 \cos^2 \theta + \sin^2 \theta)} . \quad (4.14)$$

Referring to the equivalent circuit in Fig. 4.2 the measured impedance of the diode in the package is related to the conductance and susceptance of the diode by

$$R + jX = j\omega L_s + \frac{1}{j\omega C_{pk} + \frac{1}{j\omega L_d + \frac{1}{j\omega C_{pd} + \frac{1}{R_s + \frac{1}{G + jB}}}}} \quad (4.15)$$

By rearranging the equation in the following way, it is found that

$$G + jB = \frac{1}{\frac{1}{\frac{1}{\frac{1}{R + j(X - \omega L_s)} - j\omega C_{pk}} - j\omega L_d} - j\omega C_{pd}} - R_s \quad (4.16)$$

Defining,

$$AA \triangleq 1 + \omega(X - \omega L_s)(C_{pk} + C_{pd}) - \omega^2 L_d C_{pd} [1 + \omega C_{pk} \cdot (X - \omega L_s)] \quad , \quad (4.17)$$

$$BB \triangleq \omega R(C_{pk} + C_{pd}) - \omega^3 L_d C_{pk} C_{pd} R \quad , \quad (4.18)$$

$$CC \triangleq R(1 - \omega^2 L_d C_{pk}) - R_s \cdot AA \quad (4.19)$$

and

$$DD \triangleq (X - \omega L_s) - \omega L_d [1 + \omega C_{pk} (X - \omega L_s)] + R_s \cdot BB \quad , \quad (4.20)$$

the conductance and susceptance can be expressed as

$$G = \left( \frac{AA \cdot CC - BB \cdot DD}{(CC)^2 + (DD)^2} \right) \quad , \quad (4.21)$$

and

$$B = - \left( \frac{BB \cdot CC + AA \cdot DD}{(CC)^2 + (DD)^2} \right) \quad . \quad (4.22)$$

In addition to the conductance and susceptance of the diode it is necessary to determine the RF voltage across this admittance. In order to do this it is first necessary to determine the incident power on the diode. By referring to the simplified representation of the test circuit illustrated in Fig. 4.19 the incident power can be obtained. It is assumed that all of the losses in paths A and B, mentioned previously, can be attributed to the directional coupler. The known parameters are the measured power levels  $P_{out}$  and  $P_3$ , and the variable precision attenuator settings  $A_{dB}$  and  $B_{dB}$ . Therefore referring to Fig. 4.19,

$$B_{dB} = 10 \log_{10} (P_2 / P_{out}) \quad (4.23)$$

which yields

$$P_2 = P_{out} \cdot 10^{(B_{dB}/10)} \quad (4.24)$$

Also

$$(DC)_{dB} = 10 \log_{10} (P_1 / P_3) \quad , \quad (4.25)$$

where  $(DC)_{dB}$  accounts for the difference in power levels in dB due to the directional coupler and all losses except the variable attenuator  $B_{dB}$ . Since

$$P_1 = P_2 + P_3 \quad , \quad (4.26)$$

then

$$(DC)_{dB} = 10 \log_{10} \left( \frac{P_2 + P_3}{P_3} \right) \quad (4.27)$$

and from Eq. 4.24 this reduces to

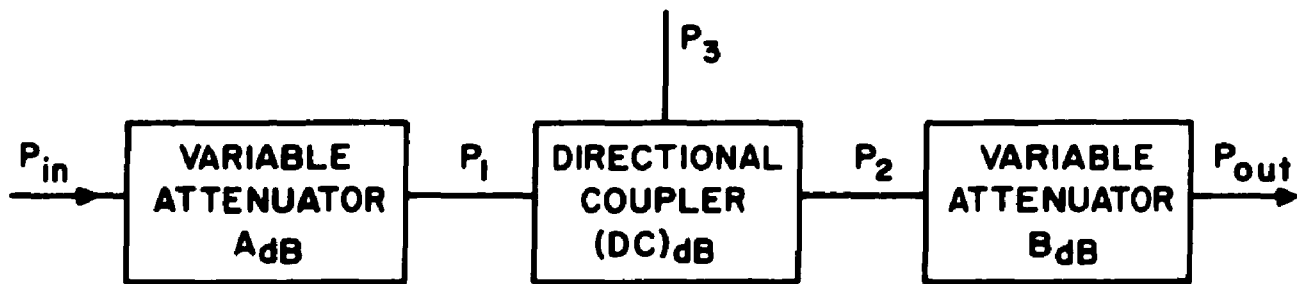


FIG. 4.19 SIMPLIFIED REPRESENTATION OF THE TEST CIRCUIT.

$$(DC)_{dB} = 10 \log_{10} \left( 1 + \frac{P_{out}}{P_3} \cdot 10^{(B_{dB}/10)} \right) . \quad (4.28)$$

The input power  $P_{in}$  from the klystron is obtained by

$$A_{dB} = 10 \log_{10} \left( \frac{P_{in}}{P_1} \right) \quad (4.29)$$

which yields

$$P_{in} = P_1 \cdot 10^{(A_{dB}/10)} \quad (4.30)$$

and from Eqs. 4.24 and 4.26 which give

$$P_{in} = \left( P_3 + P_{out} \cdot 10^{(B_{dB}/10)} \right) \cdot 10^{(A_{dB}/10)} . \quad (4.31)$$

The input power is assumed to remain constant during the measurements at a given frequency. However the attenuator settings are changed to adjust the power level incident on the diode. The incident power which is  $P_{out}$  in Fig. 4.19 is related to the input power by the following set of equations:

$$P_{inc} = P_2 \cdot 10^{-(B_{dB}/10)} , \quad (4.32)$$

$$P_1 = P_2 + P_3 , \quad (4.33)$$

$$P_2 = P_3 \cdot \left( 10^{[(DC)_{dB}/10]} - 1 \right) \quad (4.34)$$

and

$$P_1 = P_{in} \cdot 10^{-(A_{dB}/10)} . \quad (4.35)$$

These equations are solved for the incident power as a function of the input power and the attenuator settings. The solution is

$$P_{inc} = \frac{P_{in} \cdot \left( 10^{[(DC)_{dB}/10]} - 1 \right)}{10 \left( \frac{A_{dB} + B_{dB} + (DC)_{dB}}{10} \right)} \quad (4.36)$$

In order to determine the RF voltage across the admittance of the diode the transmitted power is calculated from the VSWR and incident power by

$$P_T = P_{inc} \cdot \frac{4r}{(r + 1)^2} \quad (4.37)$$

For a negative VSWR the transmitted power is negative which corresponds to the diode delivering power rather than dissipating power. The only circuit elements in the equivalent circuit of the diode shown in Fig. 4.2 which could either dissipate or deliver power are the series resistance and the conductance of the diode. Thus for the calculation of the RF voltage the equivalent circuit can be reduced to the circuit shown in Fig. 4.20. The admittance of this circuit is

$$Y = \frac{G + R_s(G^2 + B^2)}{(1 + R_s G)^2 + R_s^2 B^2} + j \frac{B}{(1 + R_s G)^2 + (R_s B)^2} \quad (4.38)$$

which can be represented by

$$Y = G' + jB' \quad (4.39)$$

The transmitted power is related to the equivalent conductance  $G'$  by

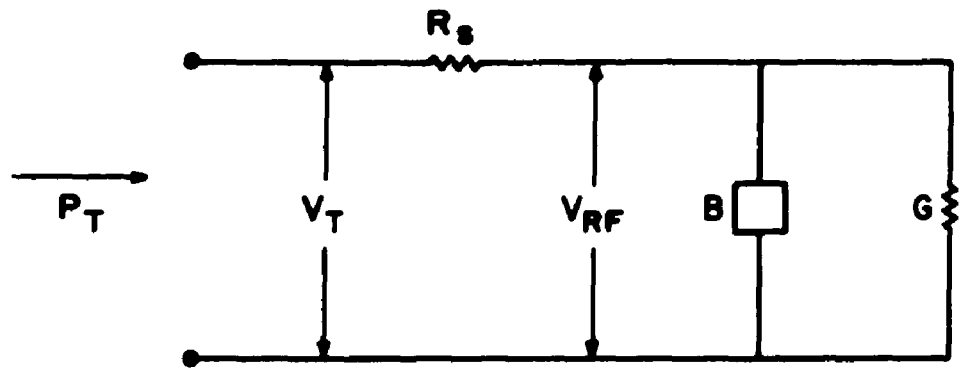


FIG. 4.20 EQUIVALENT CIRCUIT FOR CALCULATION OF RF VOLTAGE.

$$P_T = \frac{1}{2} V_T^2 G' , \quad (4.40)$$

where  $V_T$  is the RF voltage across the input terminals. Thus

$$V_T = \left( \frac{2P_T}{G'} \right)^{1/2} . \quad (4.41)$$

The RF voltage across the admittance of the diode is

$$V_{RF} = \frac{V_T}{[(1 + GR_s)^2 + (R_s B)^2]^{1/2}} . \quad (4.42)$$

Therefore combining Eqs. 4.41 and 4.42 the RF voltage is

$$V_{RF} = \left( \frac{2P_T}{G + R_s(G^2 + B^2)} \right)^{1/2} . \quad (4.43)$$

In Figs. 4.21 and 4.22 the experimentally measured admittances for the two diodes, Nos. TI 40 and TI 40B, are plotted as a function of dc bias current and frequency. For all of the points shown the RF voltage across the diode as calculated above was 1 V or less. Thus these admittance plots are for the small-signal admittance. As the incident power was increased, the VSWR and voltage minimum remained constant until the RF voltage was slightly larger than 1 V. Then the measurements showed the presence of other frequencies which were large enough in magnitude to affect the VSWR and minimum position measurements. The microwave test circuit shown in Fig. 4.17 is assumed to present a 50- $\Omega$  match to the diode at all frequencies. But in reality the connectors and circulators are not perfect and the diode is not loaded with a 50- $\Omega$

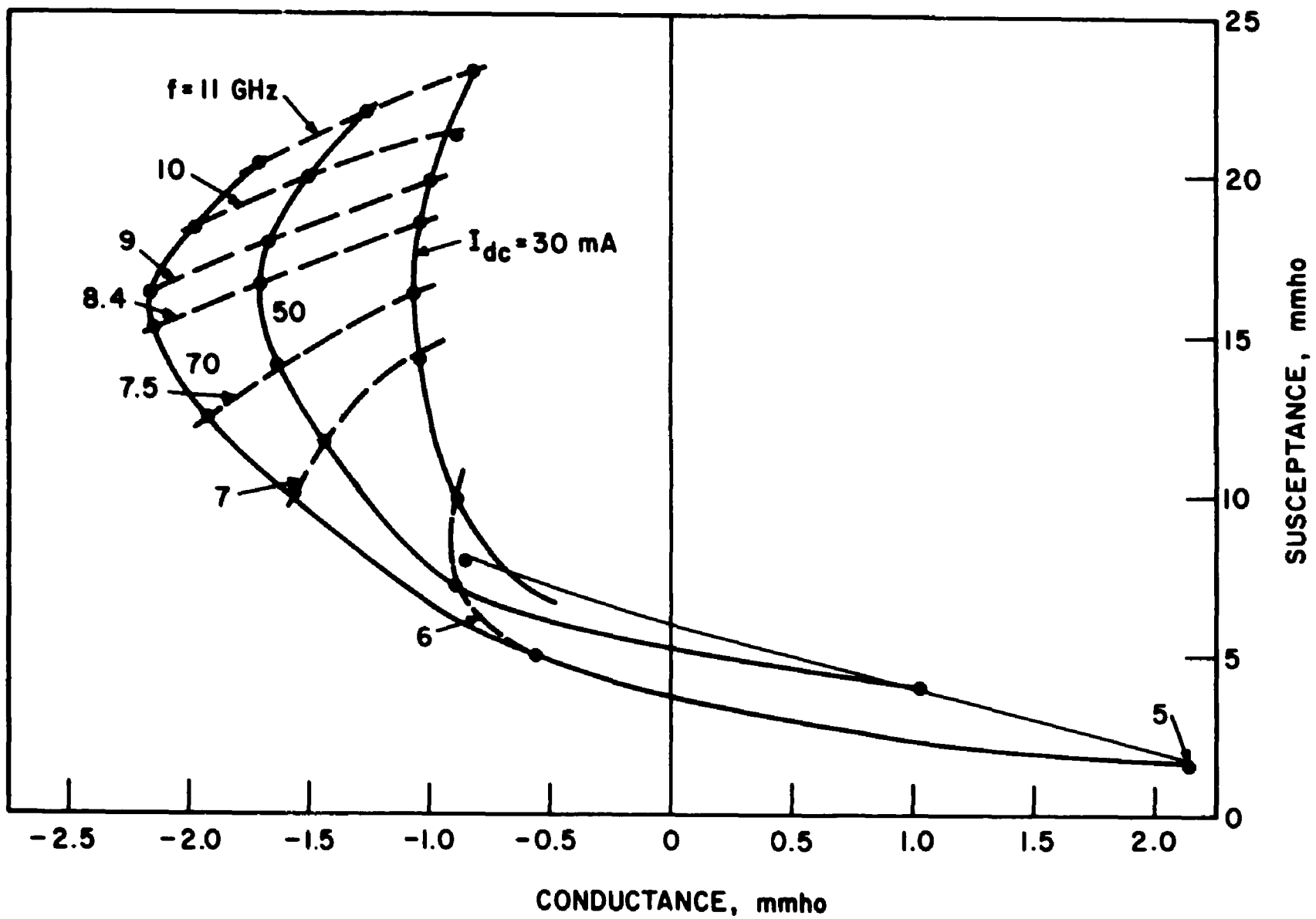


FIG. 4.21 ADMITTANCE FOR DIODE NO. TI 40. ( $V_{RF} \leq 1$  V)

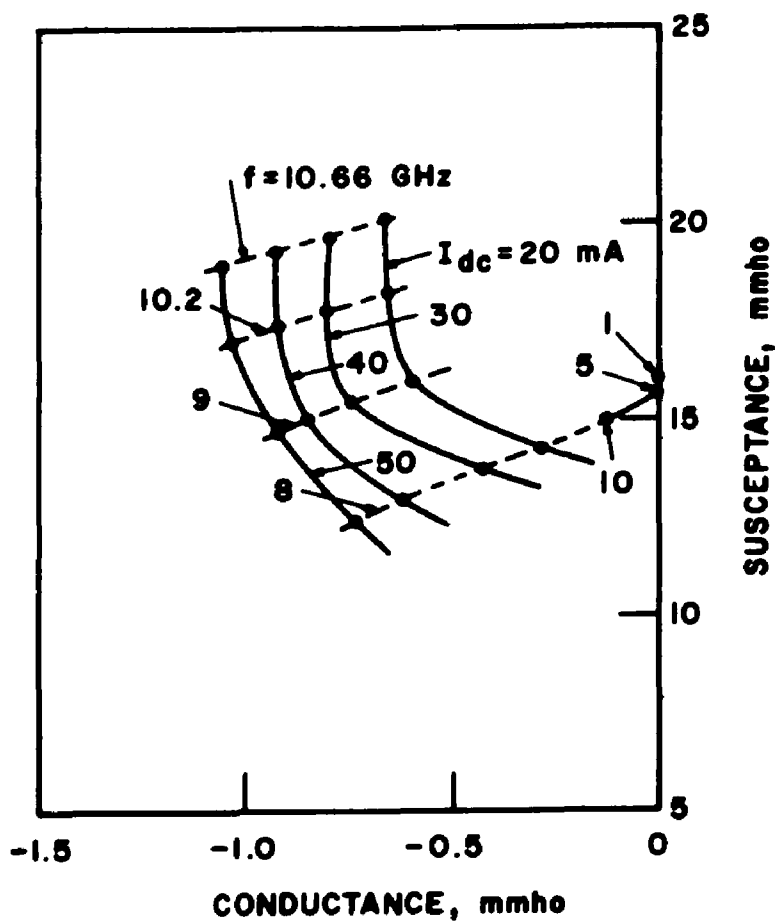


FIG. 4.22 ADMITTANCE FOR DIODE NO. TI 40B. ( $V_{RF} \leq 1 \text{ V}$ )

load at all frequencies. For the measurements plotted in Fig. 4.21 two circulators had to be used, one designed for 4 to 8 GHz operation and the other for 8 to 12 GHz. Circulators act as a reactance rather than a load out of their designed frequency band and thus the diodes are not necessarily loaded with 50  $\Omega$  at all frequencies. Due to the broadband negative resistance of IMPATT diodes and its nonlinear behavior the diode can generate power at frequencies other than the impressed frequency when the impressed signal becomes large enough in magnitude and the load at the other frequencies is small. Thus it was necessary in these measurements to keep the RF voltage small. By using better wideband, low VSWR connectors and a combination of broadband directional couplers which are lossy out of band rather than the circulator, this problem might be eliminated.

The results plotted in Figs. 4.21 and 4.22 do show qualitative agreement with the predicted theoretical trends presented in Chapters II and III. The negative conductance increases with increasing dc bias current and the susceptance decreases slightly. Also the cutoff frequency, the frequency below which the conductance is positive, decreases with decreasing dc bias current (Fig. 4.21). Finally, the negative conductance for the abrupt junction with the higher doping density is larger than for the lower doping density. However, in order to compare the admittances of the two diodes at a given bias current, the error involved in the determination of the diodes' areas should be considered. As shown previously for diode No. TI 40 the doping density for a 0.005-inch diameter was  $6 \times 10^{15} \text{ cm}^{-3}$ . Assuming a variation in the doping density of  $\pm 10$  percent which is within the range specified by the manufacturer, this yields an error in the determination of the area of  $\pm 5$  percent. For diode TI 40B the doping density was  $3 \times 10^{15} \text{ cm}^{-3}$  for a 0.005-inch diameter. Assuming the same variation in doping density a  $\pm 5$  percent error is also possible. With this tolerance value it is seen that in the

worst case the negative conductance for the diode with the higher doping density is still larger than the lower doping density one.

Diode No. TI 40B was not operated above 50 mA since it was found that punch-through diodes burn out more easily than nonpunch-through diodes. This is believed to be caused by the fact that nonuniform avalanching causes filaments to form which, when the diode is operating from a constant current source, will cause the current density in this small filament to increase and burn out the diode. For nonpunch-through diodes however if a filament forms, the undepleted region of the diode acts as a series resistance for each small filament. As the current increases in the filament the voltage drop across the corresponding undepleted region increases causing a decrease in voltage across the filament and decreasing the avalanching.

4.4.2 Oscillator Measurements. The second method for measuring the admittance of an IMPATT diode is an indirect method of measuring the admittance of the microwave circuit in which the diode is oscillating. The admittance of the diode package and mount must be the negative of the admittance of the microwave circuit. The measured admittance is then transformed through the equivalent circuit to calculate the admittance of the diode.

The microwave test circuit used for the oscillator measurements is shown in Fig. 4.23. The diode is tuned with a slug in the coaxial cavity to oscillate at a single frequency for a given dc bias current. Single-frequency operation is verified by inserting a probe in the diode cavity above the diode and monitoring the output on the spectrum analyzer. The diode is tuned in such a way that all frequencies except the operating frequency are down in power by at least 30 dB. The frequency is measured

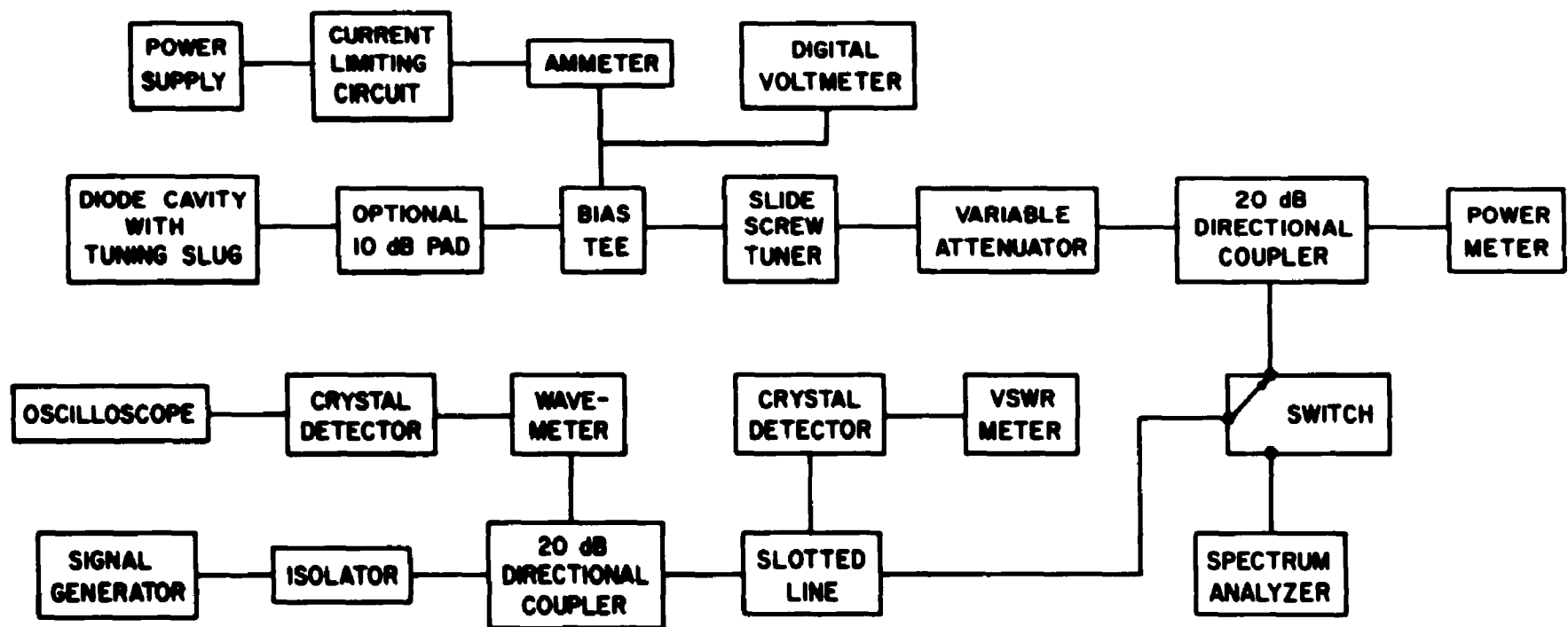


FIG. 4.23 EXPERIMENTAL TEST CIRCUIT FOR OSCILLATOR MEASUREMENTS.

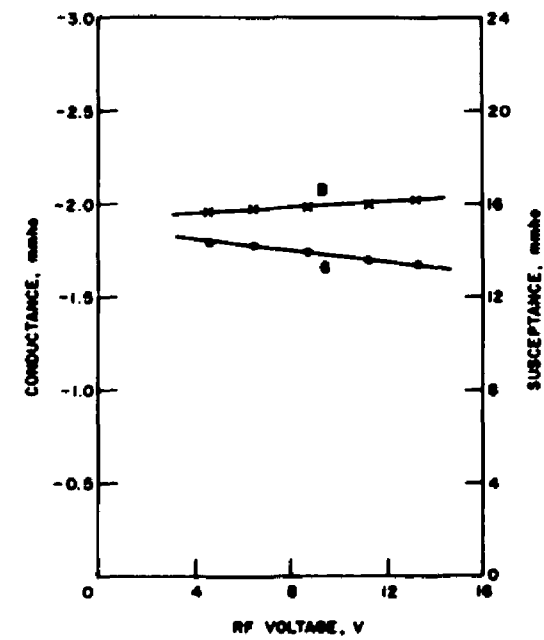
by matching the frequency from the signal generator and the oscillating diode on the spectrum analyzer with a 1 MHz/cm spectrum width and then measuring the signal-generator frequency with the wavemeter. The power output or RF voltage level of the diode is varied by changing the insertion of the slide screw tuner, which is measured with a micrometer. The power level is measured for each insertion of the slide screw tuner. The next step is to remove the diode from the diode cavity and connect a 50- $\Omega$  coaxial-line adapter and the slotted line in place of the diode. The admittance of the microwave circuit is measured for each insertion of the slide screw tuner and at the frequency at which the diode was oscillating. The admittance of the microwave circuit is measured with respect to the reference plane located at the flange of the diode and then transformed through the package to the diode. Finally, the input power from the signal generator is measured along with the power which is transmitted through the circuit to the power meter at the end of the circuit.

The calculation of the admittance of the diode from the experimental VSWR and voltage minimum position is the same as for the reflection-type amplifier measurement. The RF voltage across the diode is found by calculating the loss in the circuit between the diode and the power meter. This is done by calculating the transmitted power from the signal generator to the circuit from the input power and the measured VSWR as shown in Eq. 4.37. The loss is found from this power and from the measured power which is transmitted through the circuit. From the loss in the circuit and the power measured when the diode was oscillating the power delivered by the diode is obtained. Finally, knowing the

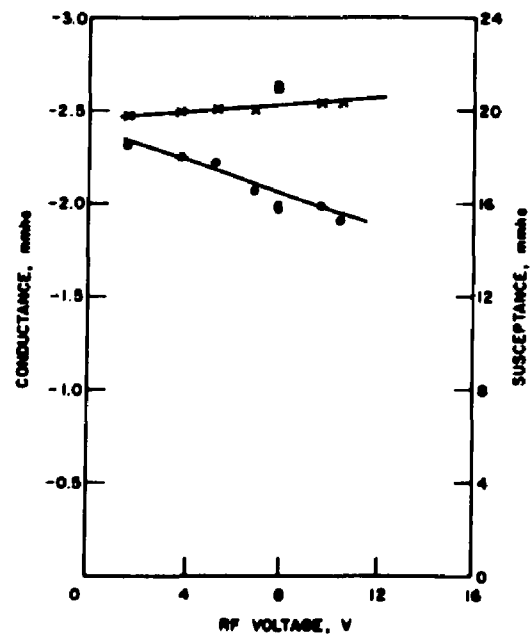
power delivered and the admittance of the diode, the RF voltage is calculated as was done previously.

The conductance and susceptance for diode No. TI 40 are plotted in Fig. 4.24 for several different frequencies and a bias current of 50 mA. The results show that the conductance decreases and the susceptance increases for increasing RF voltage as predicted in Chapter III (Fig. 3.23). The decrease in the negative conductance for a change in RF voltage from 1 V to 10 V is approximately 20 percent at 9.9 GHz whereas the decrease predicted in Chapter III for an abrupt junction with a doping density of  $5 \times 10^{15} \text{ cm}^{-3}$  at the optimum frequency is approximately 23 percent. Also the decrease in negative conductance for the lower frequency (8.75 GHz) is less than at the higher frequencies, however the susceptance does not change as much as predicted by the results of Chapter III. The magnitude of the negative conductance is somewhat less than predicted for a comparable doping density. It is again difficult to compare the magnitude of the negative conductances for a given current density since the area of the junction is not known exactly for the experimental diode, however the errors involved in this case have been discussed previously.

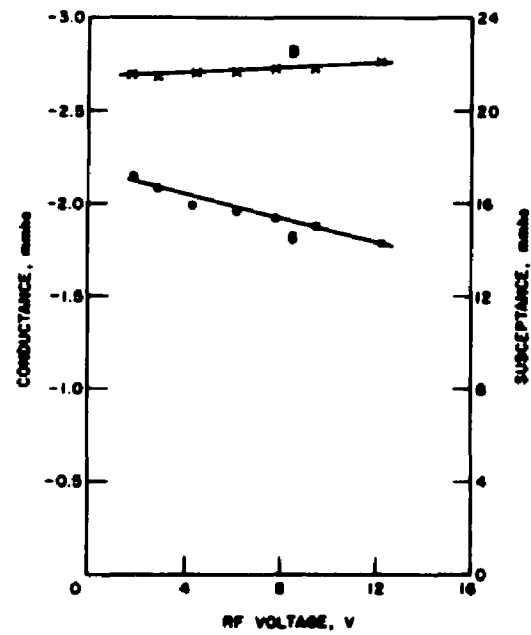
Some elaboration must be given to several points about this measurement. The microwave circuit was kept as simple as possible and thus it was not always possible to change the RF voltage with the slide screw tuner while holding the diode in a constant, single-frequency mode. This is illustrated by the results in Fig. 4.24d in which the RF voltage remained relatively constant for many different insertions of the slide screw tuner. A second point is that as the slide screw tuner was inserted the spectrum width of the oscillations would change slightly. Also a point was reached for which the diode would not



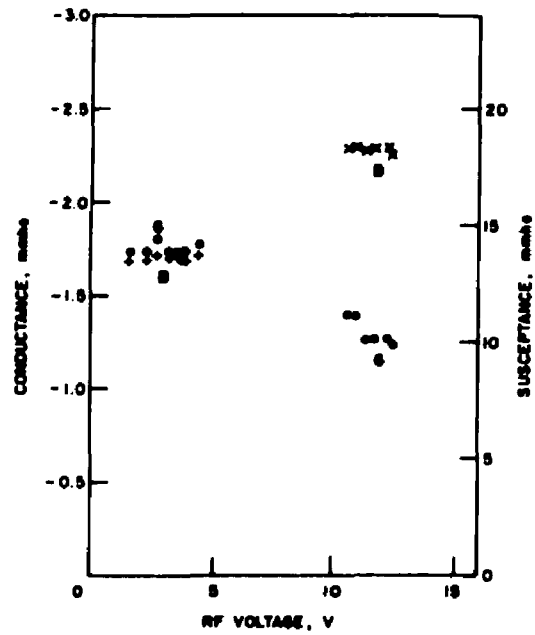
(a)  $f = 8.75$  GHz



(b)  $f = 9.9$  GHz



(c)  $f = 10.75$  GHz



(d)  $\bullet + f = 7.46$  GHz

$\circ \times f = 9.09$  GHz

FIG. 4.24 CONDUCTANCE AND SUSCEPTANCE VS. RF VOLTAGE FOR DIODE

NO. TI 40. ( $I_{dc} = 50$  mA)

maintain single-frequency operation. Either several other frequencies would appear or a very noisy spectrum would result. As the insertion was decreased, it was not possible to continuously decrease the RF voltage without having the diode stop oscillating before the voltage reached zero. This is illustrated in the figures since the admittance for 1 V or less was not obtained. Finally, the frequency of the oscillating diode changed slightly as the slide screw tuner was inserted. In Fig. 4.25 the change in frequency is plotted as a function of the RF voltage which corresponds to a change in the insertion. As is shown, the change is less than 0.25 percent; however, it was accounted for when the admittance measurements were made. Since the frequency decreases slightly for increasing RF voltage the susceptance should increase a little faster than measured.

In Fig. 4.26 the conductance and susceptance are plotted for the same diode with a dc bias current of 30 mA. The conductance is less for the lower bias current as predicted. With the microwave circuit used it was difficult to obtain single-frequency operation at 70 mA bias current due to the large, broadband negative conductance.

The errors which can be introduced into the measurement of the admittance of the diode come from several sources. One source is due to the mechanical stress and torque on the circuit. When the diode is removed from the circuit and the 50- $\Omega$  coaxial adapter is connected, mechanical stress or torque on the connectors and adapters in the circuit can give rise to errors in the measured circuit admittance. Another source of errors is in the measurement of the voltage minimum position. Even though the probe is inserted as far as possible to detect the minimum position, the location cannot be determined exactly due to the high VSWR and noise in the system. The effect of an error in the minimum position on the negative conductance is shown in Fig. 4.27. The conductance is

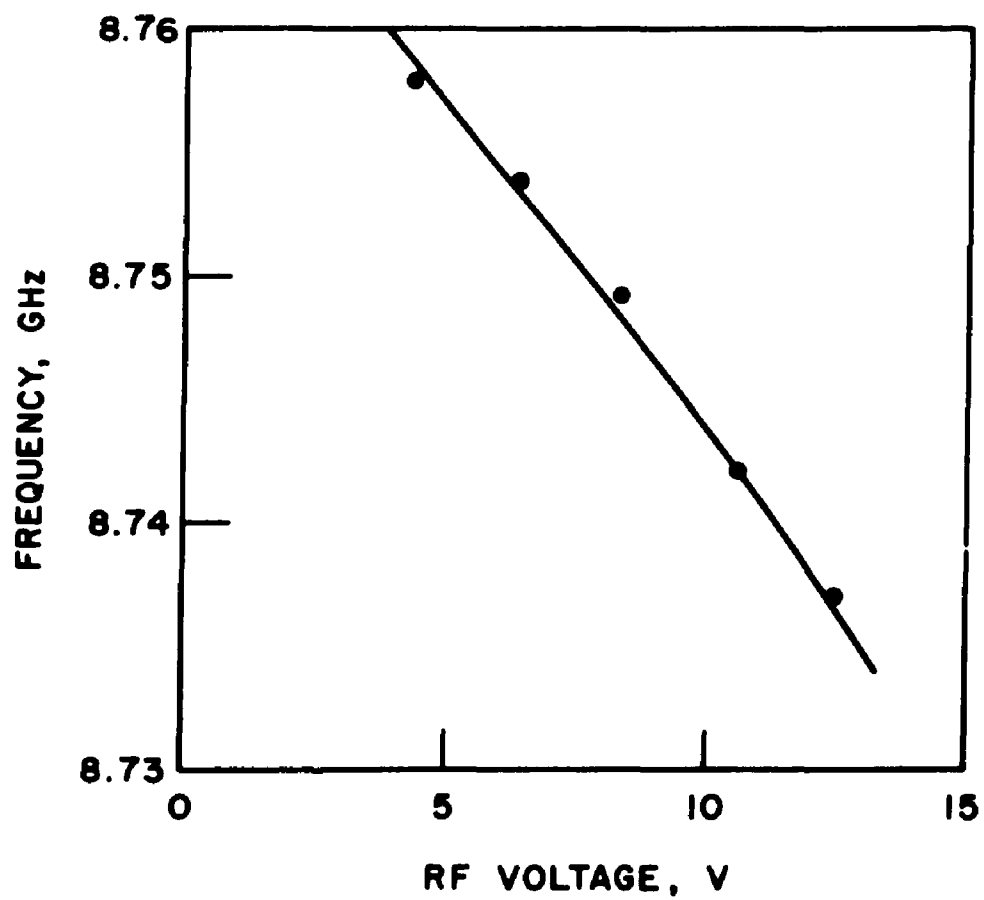
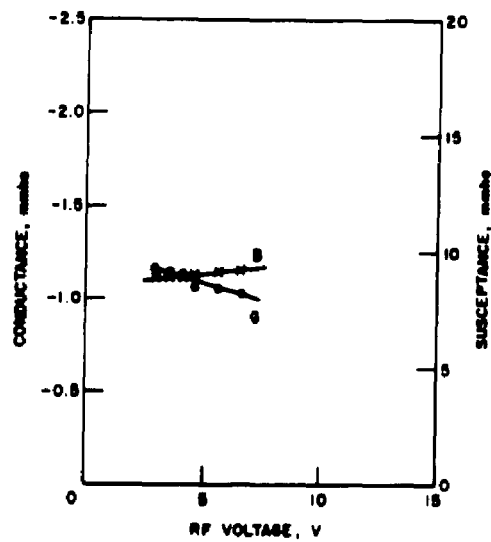
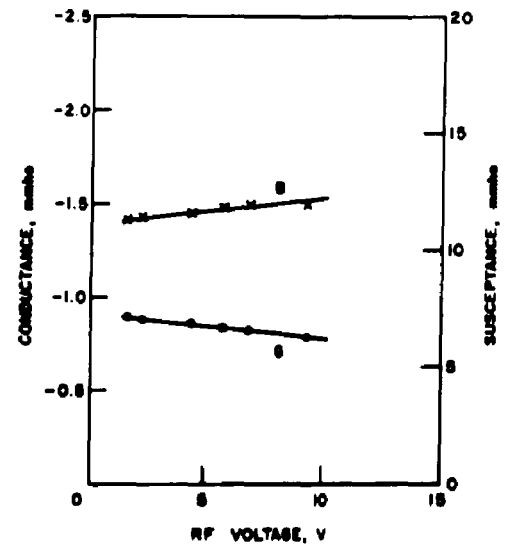


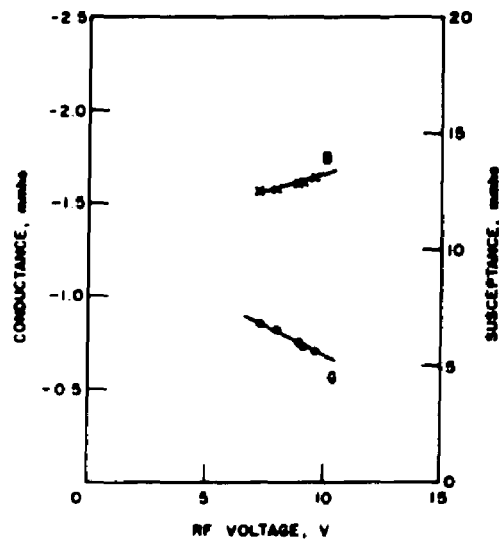
FIG. 4.25 FREQUENCY VS. RF VOLTAGE FOR DIODE NO. TI 40. ( $I_{dc} = 50$  mA)



(a)  $f = 7.22$  GHz



(b)  $f = 8.60$  GHz



(c)  $f = 9.99$  GHz

FIG. 4.26 CONDUCTANCE AND SUSCEPTANCE VS. RF VOLTAGE FOR DIODE

NO. TI 40. ( $I_{dc} = 30$  mA)

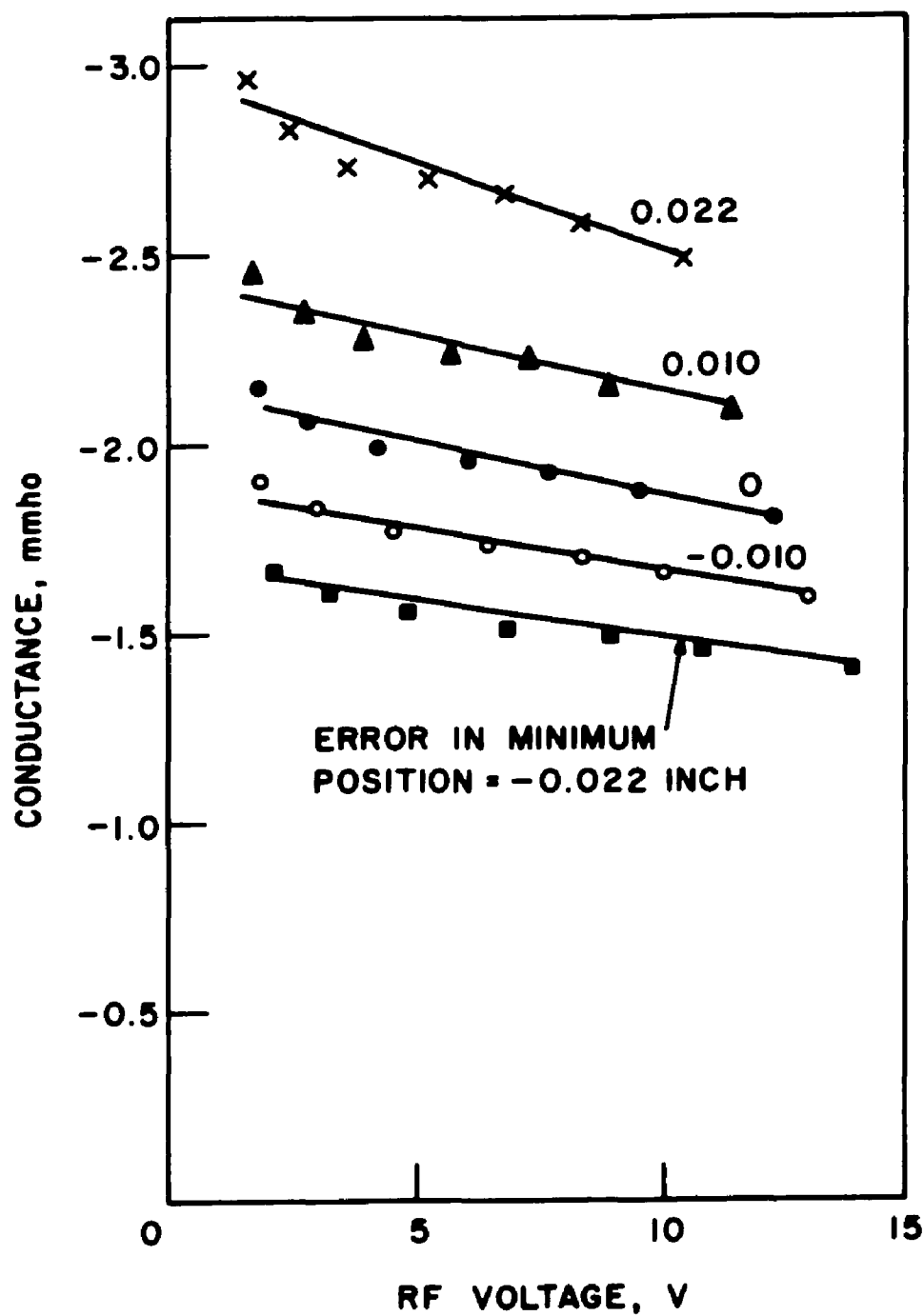


FIG. 4.27 CONDUCTANCE VS. RF VOLTAGE FOR DIODE NO. TI 40. ( $I_{dc} = 50$  mA  
AND  $f = 10.75$  GHz)

plotted as a function of the RF voltage for different values of error in the minimum position. At this frequency, 10.75 GHz, the effect of the error is larger than at lower frequencies due to the shorter wavelength. The error in the measurement of the minimum position could be as large as  $\pm 0.010$  inch since the minimum position must be measured for both the reference plane and the admittance. Another source of error is in the measurement of the distance used for the double-minimum method for high VSWR. The measurement of the two positions where the standing wave is 3 dB above the minimum is not as difficult as the minimum position measurement since the noise does not affect the signal appreciably at this power level. The VSWR is related to the distance between the two 3-dB points by<sup>63</sup>

$$\text{VSWR} = \frac{\lambda}{\pi x_0} , \quad (4.44)$$

where  $\lambda$  = the wavelength in the coaxial line (mils) and

$x_0$  = the distance between the two 3-dB points of the VSWR (mils).

The effect of an error in the measurement of  $x_0$  on the negative conductance is shown in Fig. 4.28. The error is largest at the high frequency as before. The uncertainty in the measurement of  $x_0$  was less than  $\pm 0.003$  inch. This error has less effect on the conductance than the error in the minimum position. The final source of error to be discussed is the measurement of the power transmitted through the circuit to the power meter from the signal generator. Since the signal generator is low power and the reflection coefficient of the circuit is large, the power transmitted is small and difficult to measure accurately. However

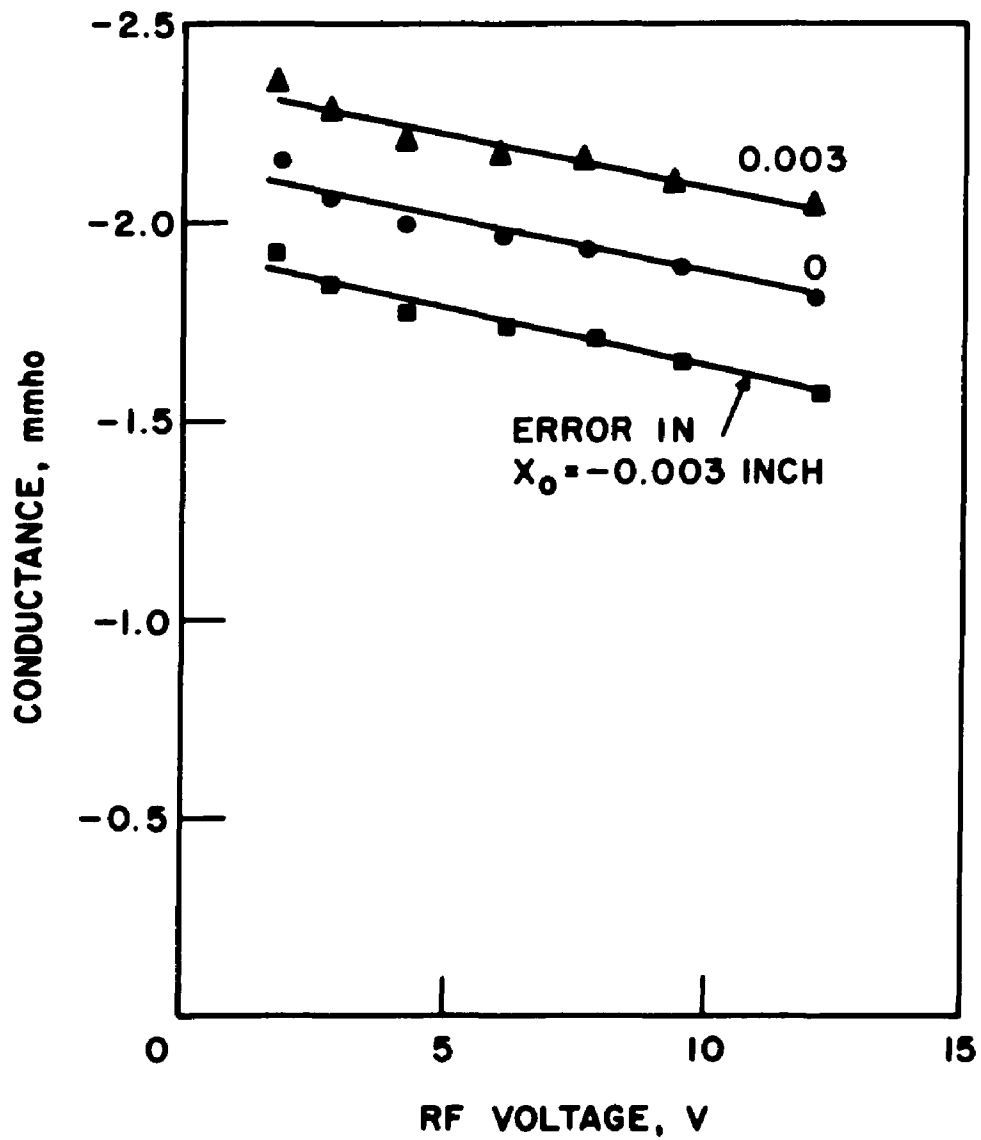


FIG. 4.28 CONDUCTANCE VS. RF VOLTAGE FOR DIODE NO. TI 40. ( $I_{dc} = 50$  mA AND  $f = 10.75$  GHz)

this measurement affects only the magnitude of the RF voltage rather than the magnitude of the negative conductance.

In Fig. 4.29 a comparison of the two methods of measuring the admittance is shown. The negative conductances and susceptances for the oscillator test are taken as approximate values from Fig. 4.24. The dc bias current is 50 mA. The susceptance values are in fairly good agreement. At the lower frequencies where the errors mentioned previously are less, the conductances agree fairly well whereas for the higher frequencies there is more discrepancy. However considering the errors just mentioned the agreement is not as bad as indicated. The susceptance is much easier to measure accurately since it is large whereas the conductance is more difficult to measure since it is only one tenth that of the susceptance. Another point must be made in that the two methods of measuring the admittance do not present the same equivalent circuit to the diode. For the oscillator measurement the circuit is tuned to obtain single-frequency operation which means the harmonics of the current through the diode are shorted. Whereas for the reflection-type amplifier measurement the circuit is not tuned and the impedance seen by the harmonics of the current might be large as will be discussed in the next section. Thus it is not apparent that the two methods should yield the same result for all frequencies.

#### 4.5 Single-Frequency Operation

The two large-signal analyses presented in Chapters II and III assume that the diode oscillates with a single-frequency RF voltage across it. This assumes that all the harmonics of the voltage are

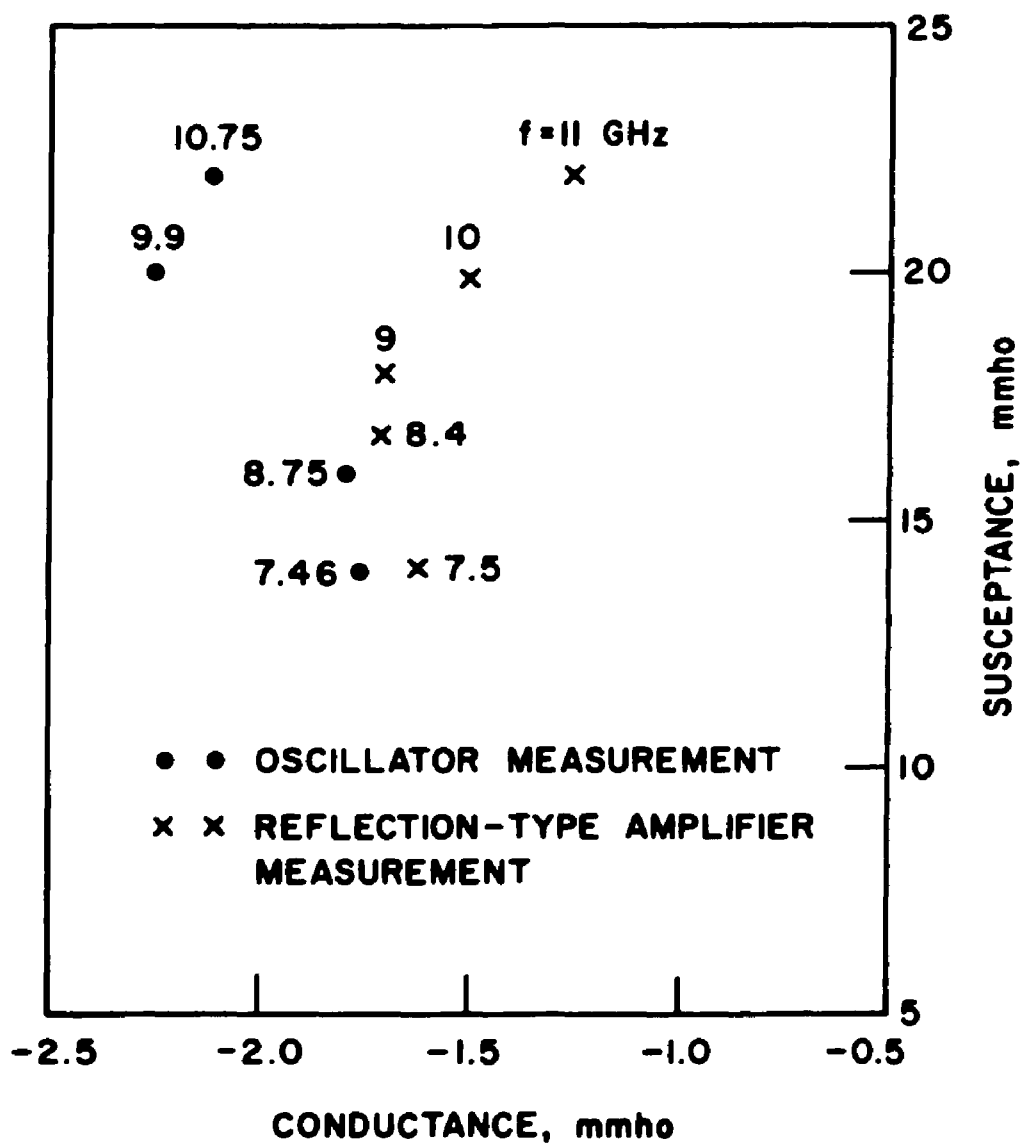


FIG. 4.29 ADMITTANCE OF DIODE NO. TI 40. ( $I_{dc} = 50 \text{ mA}$  AND  $V_{RF} \leq 1 \text{ V}$ )

shorted by the package and microwave circuit. The experimental results presented in this chapter were obtained with the diode oscillating essentially in a single-frequency mode with all other frequencies at least 30 dB below the fundamental oscillating frequency. In order to show how the diode could oscillate in a single-frequency mode, the impedance seen by the diode was adjusted to match the impedance of the diode at the oscillation frequency; it was then calculated as a function of frequency to investigate its behavior at the harmonics. In order for single-frequency operation to exist, the load impedance must match the impedance of the diode at the oscillation frequency and must be small at the harmonics.

In Fig. 4.30 the equivalent circuit of the diode, diode package and microwave circuit is shown. The conductance of the diode is  $-G$  and the susceptance is separated into an inductance  $L_a$  and a depletion-layer capacitance  $C_d$ . The parameters  $R_s$ ,  $L_d$ ,  $C_{pk}$  and  $L_s$  have been defined previously. The microwave circuit is represented by  $R_L$  and  $X_L$ .

For a given admittance of the diode at a given frequency,  $R_L$  and  $X_L$  must have the value such that the admittance seen by the diode is the negative of the diode admittance. The diode admittance is

$$Y = -G - \frac{j}{\omega L_a} \quad (4.45)$$

and therefore the circuit admittance at the oscillation frequency must be

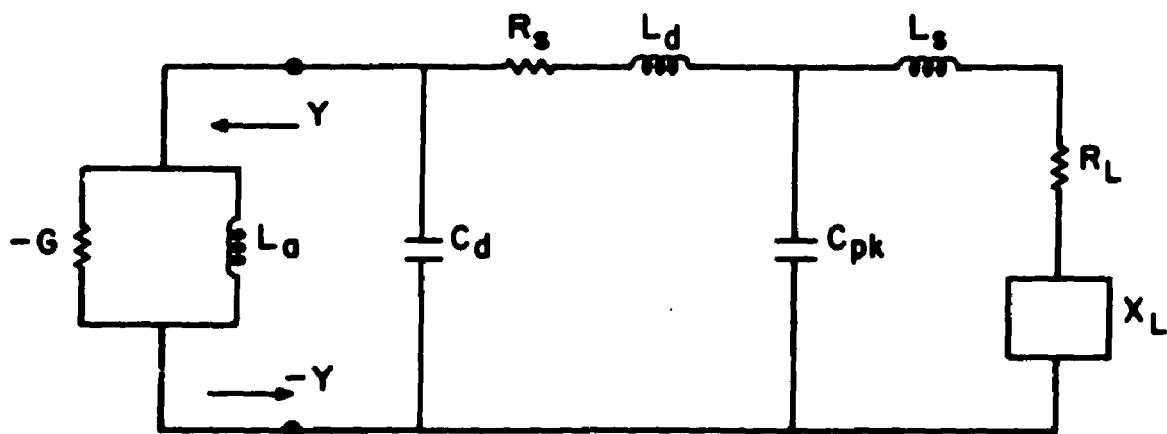


FIG. 4.30 EQUIVALENT CIRCUIT OF DIODE, DIODE PACKAGE AND MICROWAVE CIRCUIT.

$$G + \frac{j}{\omega L_a} = j\omega C_d + \frac{1}{R_s + j\omega L_d + \frac{1}{j\omega C_{pk} + \frac{1}{R_L + j(\omega L_s + X_L)}}} \quad (4.46)$$

The values of  $R_L$  and  $X_L$  are obtained from this equation at the oscillation frequency. Defining,

$$AA \triangleq 1 - R_s G + \omega L_d \left( \frac{1}{\omega L_a} - \omega C_d \right), \quad (4.47)$$

$$BB \triangleq R_s \left( \frac{1}{\omega L_a} - \omega C_d \right) + \omega L_d G, \quad (4.48)$$

$$CC \triangleq G - \omega C_{pk} BB \quad (4.49)$$

and

$$DD \triangleq \frac{1}{\omega L_a} - \omega C_d - \omega C_{pk} AA, \quad (4.50)$$

the load resistance which satisfies Eq. 4.46 at the oscillation frequency is

$$R_L = \frac{AA \cdot CC - BB \cdot DD}{(CC)^2 + (DD)^2} \quad (4.51)$$

and the load reactance is

$$X_L = -\omega L_s - \left( \frac{BB \cdot CC + AA \cdot DD}{(CC)^2 + (DD)^2} \right). \quad (4.52)$$

For this load,  $R_L$  and  $X_L$ , the diode will oscillate at the given frequency.

The load resistance and reactance may be represented by either a series or a parallel load. In the series load the resistance is assumed independent of frequency and is given by Eq. 4.51. The series reactance is represented by a sliding short which has a frequency dependence given by

$$X_L = Z_o \tan \theta_f . \quad (4.53)$$

For the operating frequency

$$\theta_{f_o} = \arctan \left( \frac{X_L}{Z_o} \right) , \quad (4.54)$$

where  $X_L$  is calculated from Eq. 4.52. For other frequencies the angle is given by

$$\theta_f = \theta_{f_o} \cdot \frac{f}{f_o} . \quad (4.55)$$

The circuit for the parallel load is shown in Fig. 4.31. For the parallel load the resistance  $R_p$  is assumed independent of frequency while the reactance has the same dependence as Eq. 4.53. The impedance of this circuit is

$$Z_L = \frac{R_p}{1 + \left( \frac{R_p}{Z_o \tan \theta_f} \right)^2} + j \frac{Z_o \tan \theta_f}{1 + \left( \frac{Z_o \tan \theta_f}{R_p} \right)^2} . \quad (4.56)$$

In order for the admittance of the circuit to satisfy the condition in Eq. 4.46,

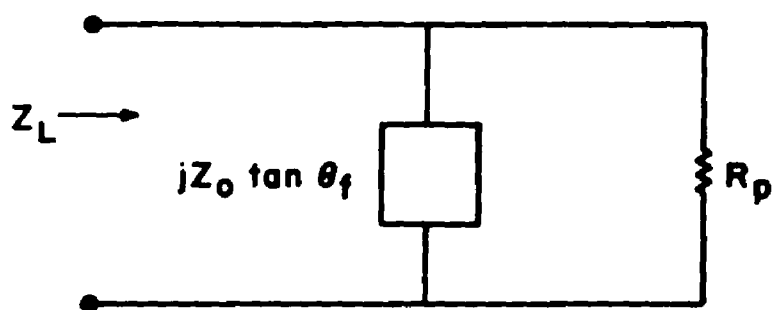


FIG. 4.31 MICROWAVE CIRCUIT FOR THE PARALLEL LOAD.

$$R_L = \frac{R_p}{1 + \left( \frac{R_p}{Z_o \tan \theta_{f_o}} \right)^2} \quad (4.57)$$

and

$$X_L = \frac{Z_o \tan \theta_{f_o}}{1 + \left( \frac{Z_o \tan \theta_{f_o}}{R_p} \right)^2} \quad (4.58)$$

Solving these two equations for  $R_p$  and  $\theta_{f_o}$ , the following results are obtained:

$$R_p = \frac{X_L^2 + R_L^2}{R_L} \quad (4.59)$$

and

$$\theta_{f_o} = \arctan \left( \frac{X_L^2 + R_L^2}{Z_o X_L} \right) \quad (4.60)$$

The frequency dependence of  $R_L$  and  $X_L$  for the parallel load is given by Eqs. 4.57 and 4.58 where  $\theta_f$  is calculated by Eq. 4.55.

The admittance seen by the diode as shown in Fig. 4.30 is given by the right-hand side of Eq. 4.46. Defining,

$$EE \triangleq R_s [1 - \omega C_{pk} (\omega L_s + X_L)] + R_L (1 - \omega^2 L_d C_{pk}) \quad , \quad (4.61)$$

$$FF \triangleq R_s R_L \omega C_{pk} - \omega^2 L_d C_{pk} (\omega L_s + X_L) + \omega (L_d + L_s) + X_L \quad , \quad (4.62)$$

$$GG \triangleq 1 - \omega C_{pk} (\omega L_s + X_L) - \omega C_d \cdot FF \quad (4.63)$$

and

$$HH \triangleq \omega C_{pk} R_L + \omega C_d \cdot EE , \quad (4.64)$$

where  $R_L$  and  $X_L$  are calculated by either Eqs. 4.51 and 4.53 or by Eqs. 4.57 and 4.58 for each type of load and frequency. The admittance is then given by

$$Y = \frac{GG + jHH}{EE + jFF} \quad (4.65)$$

or the impedance is

$$Z = 1/Y . \quad (4.66)$$

Therefore

$$|Z| = \left( \frac{(EE)^2 + (FF)^2}{(GG)^2 + (HH)^2} \right)^{1/2} \quad (4.67)$$

and

$$\angle Z = \arctan \left( \frac{FF}{EE} \right) - \arctan \left( \frac{HH}{GG} \right) . \quad (4.68)$$

The magnitude of the impedance given by Eq. 4.67 as seen by the diode is plotted in Figs. 4.32 through 4.35 for different oscillation frequencies. The negative conductance of the diode has been assumed to be -1.5 mmho. The depletion layer capacitance is calculated by

$$C_d = \frac{\epsilon A}{d} , \quad (4.69)$$

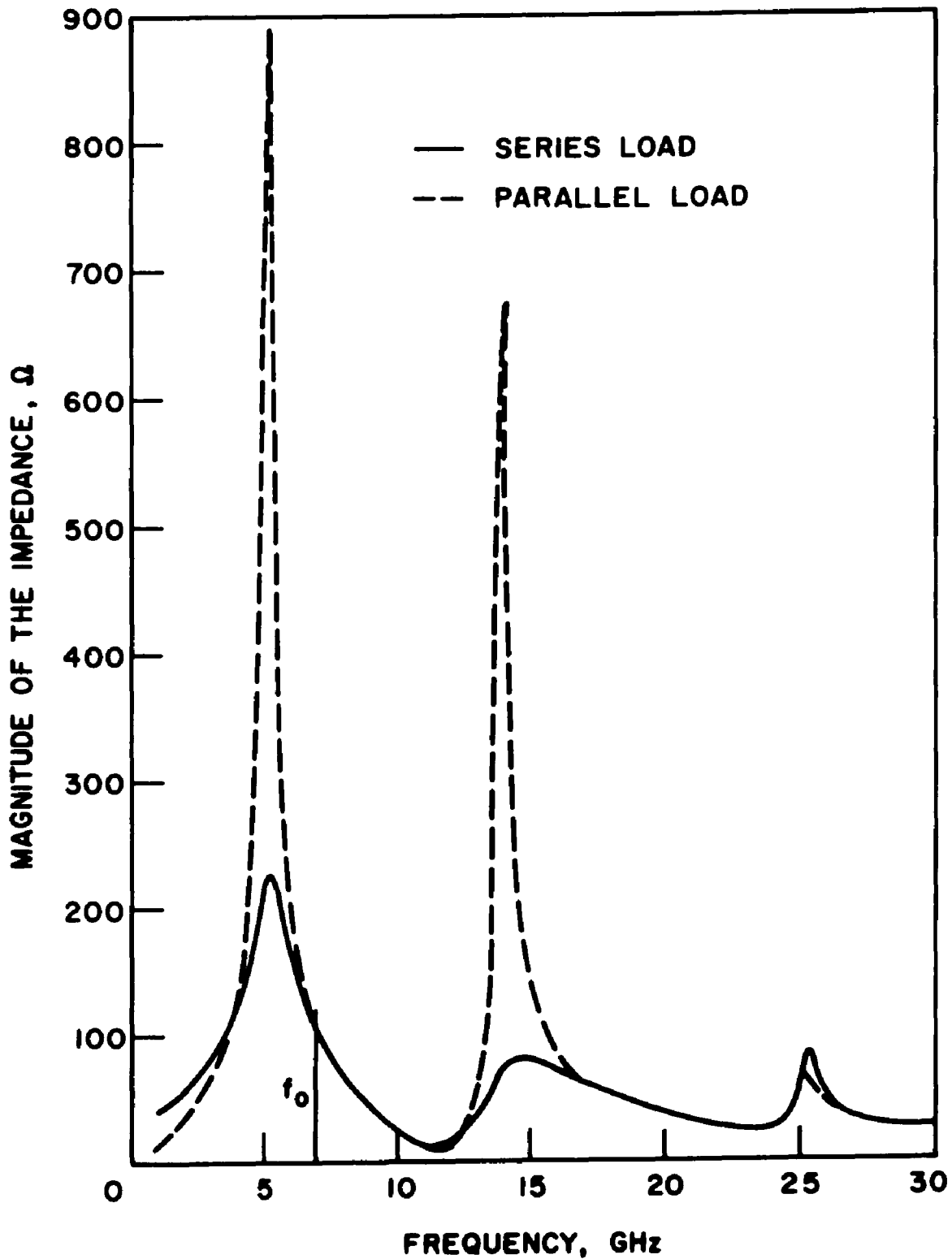


FIG. 4.32 MAGNITUDE OF THE IMPEDANCE VS. FREQUENCY. ( $G = -1.5 \text{ mmho}$ ,  
 $L_a = 2.4 \text{ nH}$  AND  $f_o = 7 \text{ GHz}$ )

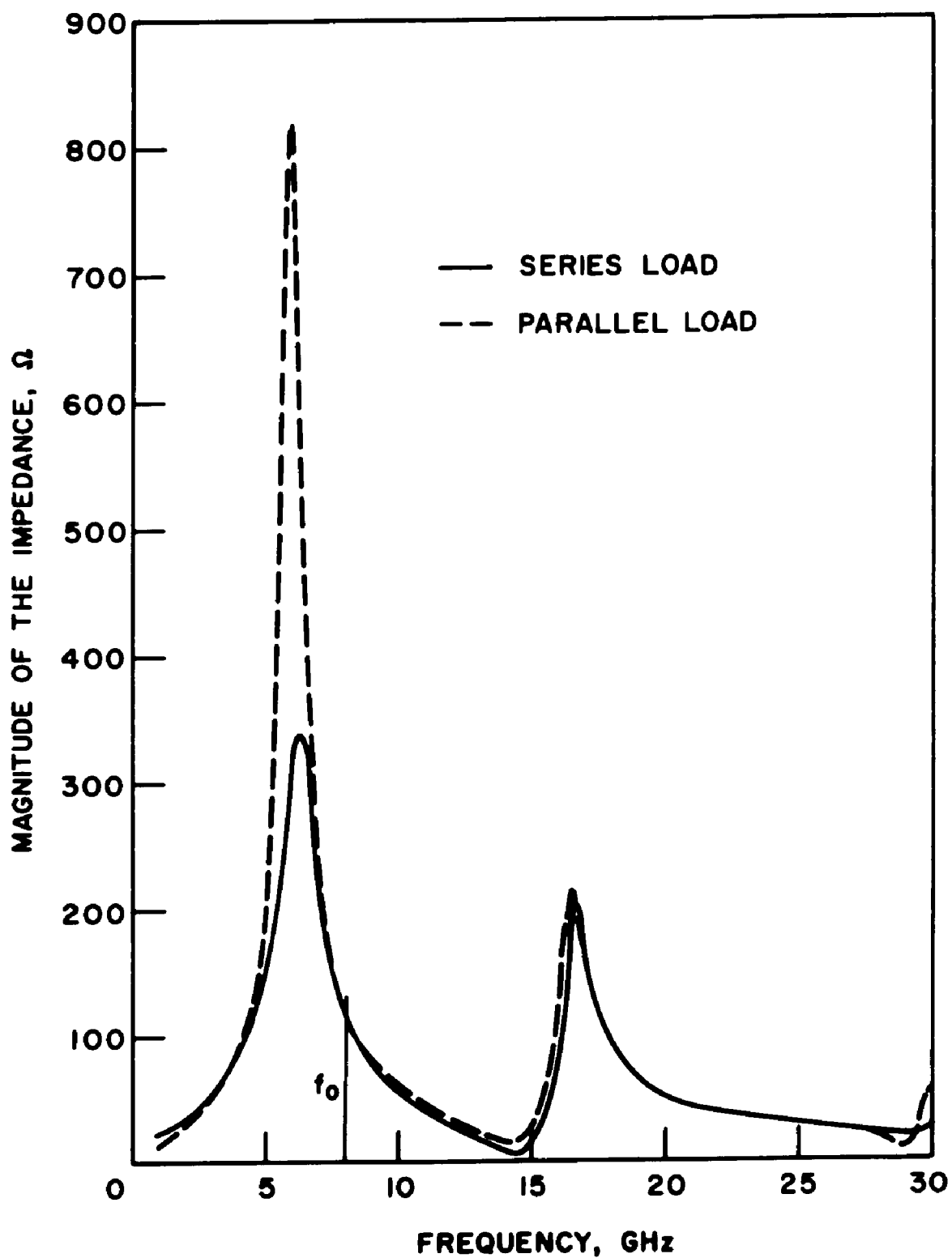


FIG. 4.33 MAGNITUDE OF IMPEDANCE VS. FREQUENCY. ( $G = -1.5 \text{ mmho}$ ,  
 $L_a = 2.4 \text{ nH}$  AND  $f_o = 8 \text{ GHz}$ )

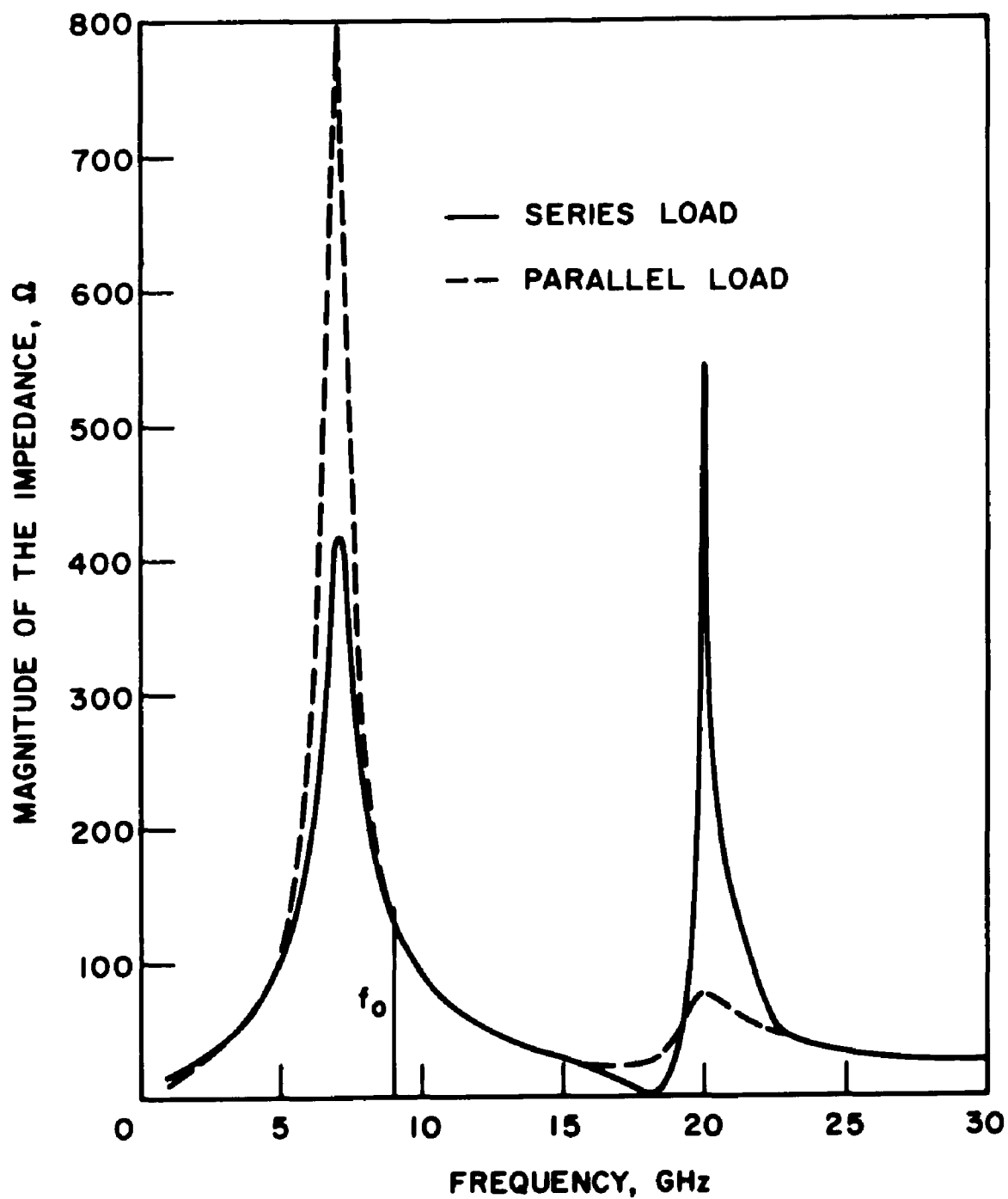


FIG. 4.34 MAGNITUDE OF THE IMPEDANCE VS. FREQUENCY. ( $G = -1.5$  mmho,  
 $L_a = 2.4$  nH AND  $f_0 = 9$  GHz)

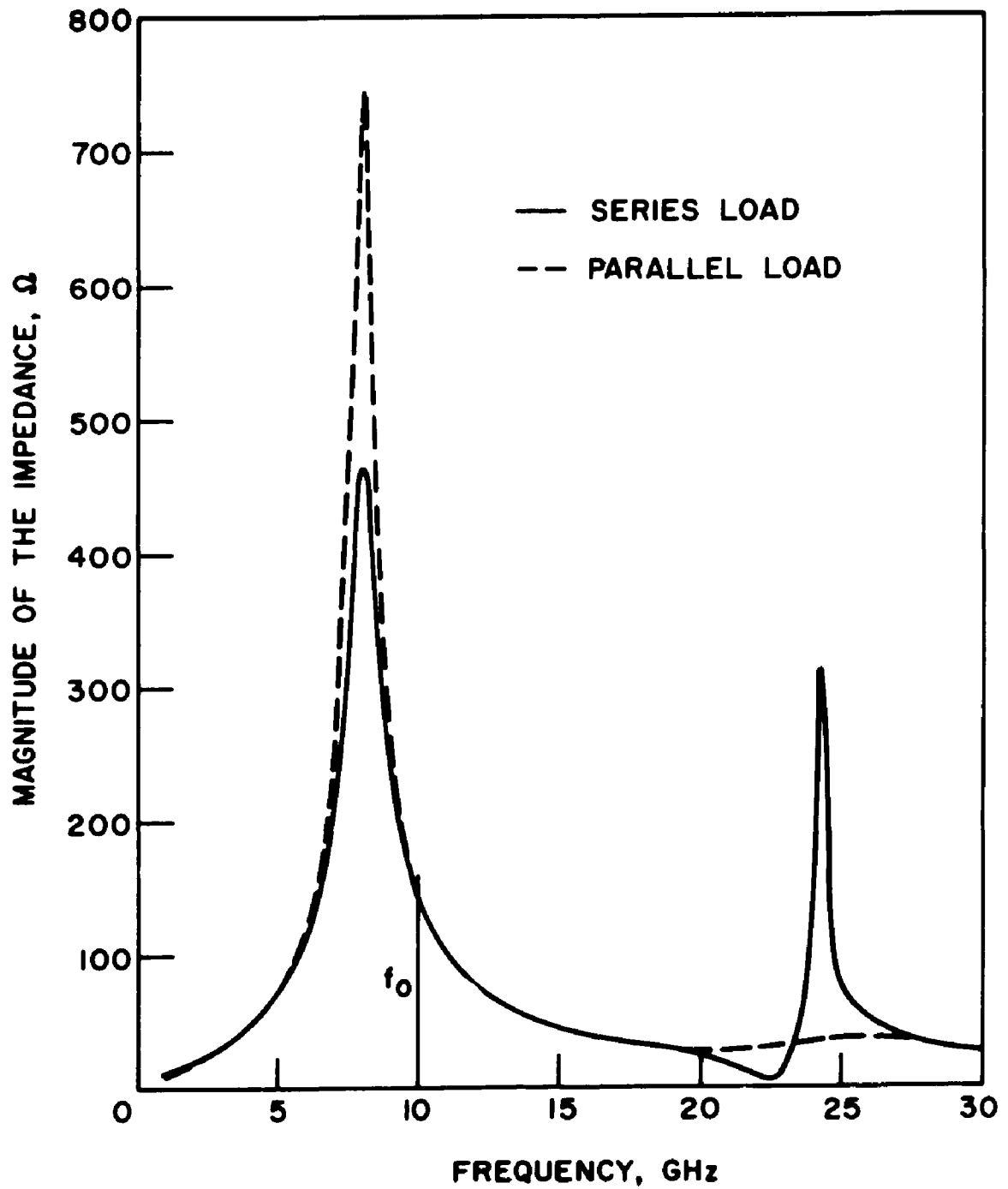


FIG. 4.35 MAGNITUDE OF THE IMPEDANCE VS. FREQUENCY. ( $G = -1.5$  mmho,  
 $L_a = 2.4$  nH AND  $f_o = 10$  GHz)

where  $\epsilon = 10^{-12}$  F/cm,

$A = 10^{-4}$  cm<sup>2</sup> and

$d = 4 \times 10^{-4}$  cm.

Thus  $C_d$  is 0.25 pF and the inductance due to the avalanche process is obtained from the small-signal cutoff or avalanche frequency<sup>6</sup> in which

$$\omega_a^2 = \frac{1}{L_a C_d}, \quad (4.70)$$

where the avalanche frequency is taken as 6.5 GHz. Substituting these values into Eq. 4.70, the inductance is 2.4 nH. The circuit elements for the package are the values measured experimentally and presented in this chapter.

Even though the impedance of the microwave circuit has been greatly simplified, the results obtained show several important effects. First, for the case where the diode is oscillating at 9 GHz (Fig. 4.34) the impedance at the oscillation frequency is 133  $\Omega$ . The RF voltage at this frequency is equal to this impedance times the magnitude of the current component at this frequency. At the second harmonic, 18 GHz, the magnitude of the impedance seen by the diode for the series load is 1.26  $\Omega$ . The magnitude of the second-harmonic current component will be at most one fifth of the fundamental current component and thus the RF voltage at the second harmonic is at most 1/500 of the RF voltage at the fundamental. This corresponds to essentially shorting the harmonic and having the diode oscillate in a single-frequency mode. The second effect is that for other tuning conditions as shown by the case where the oscillation frequency is 7 GHz (Fig. 4.32) the opposite

condition can exist. The magnitude of the impedance at 7 GHz is  $10^4 \Omega$  while at 14 GHz the impedance is  $670 \Omega$  for the parallel load. Using a similar approximation as before, the RF voltage at the second harmonic would be larger than at the fundamental. Thus the diode would not oscillate in a single-frequency mode. The final effect is the comparison of the parallel and series loads. As can be seen the frequency response of the impedance seen by the diode is different for the two loads, however, either type of load can be used to make the diode oscillate in a single-frequency mode with the proper choice of circuit parameters. This is due to the fact that the microwave circuit load is only one part of the impedance seen by the diode and that the package parameters which transform this load are the other important part of the impedance.

#### 4.6 Conclusions

Experimental results of the operating characteristics of IMPATT diodes have been presented in this chapter. An equivalent circuit for the diode package and mount was obtained. The validity of the transformation of the diode admittance through the equivalent circuit to the microwave circuit was verified by comparing C-V data obtained at 1 MHz and at X-band. The C-V data were also used to calculate the doping density of two different diodes as a function of the depletion layer width. In Section 4.4 the small-signal admittance of these two diodes was measured as a function of dc bias current with a reflection-type circuit. The results showed that the negative conductance of the diode increases with bias current and doping density as predicted by the theoretical results. The large-signal admittance was obtained from a

measurement of the admittance of the microwave circuit seen by the diode when oscillating. The RF voltage across the diode is also obtained from the power measurement. These large-signal results show the same decrease in negative conductance with increasing RF voltage as predicted by the theoretical results. Finally, the assumption of single-frequency operation is discussed with results showing that the harmonic voltage can be any magnitude from essentially zero to larger than the fundamental depending upon the circuit parameters. Also the results show that it is possible to use both parallel and series microwave circuits for single-frequency operation.

## CHAPTER V. SUMMARY, CONCLUSIONS AND SUGGESTIONS FOR FURTHER STUDY

### 5.1 Summary and Conclusions

The purpose of this study has been to investigate the large-signal operating characteristics of IMPATT diodes. Two large-signal analyses were derived for studying the operating characteristics of the idealized Read structure and more realistic diode structures as a function of the diode parameters and bias conditions. The operating characteristics were measured experimentally and correlated with the theoretical results.

A large-signal analysis for studying the operating characteristics of a Read diode was presented. The analysis was an extension of the short-transit-angle analysis<sup>9</sup> but was not limited to small-transit-angle diodes. A discussion of the assumptions incorporated in analyzing a Read diode was given. The iterative technique used for solving the integro-differential equation relating the current generated in the avalanche region and the RF voltage across the diode was discussed. The admittance of a Read diode was obtained as a function of the operating frequency, RF voltage, dc bias current and transit angle of the diode. The results showed that the negative conductance increases with increasing dc bias current and that the cutoff frequency, the frequency below which the conductance is positive, increases with bias current. Both of these effects were verified by experimental results. An optimum transit angle for maximum negative conductance was shown to be  $0.8\pi$ . However the limitations on the magnitude of the RF voltage with respect to the breakdown voltage and to maintaining saturated drift velocity of the carriers throughout the

diode at every instant of time in the RF cycle showed that an optimum transit angle for maximum efficiency was less than the optimum transit angle for maximum negative conductance. This was due to the fact that for larger transit angles the amount of space charge in the diode is larger since the carriers are traversing the diode for a longer time. The space charge of the carriers suppresses the field in the drift region causing the carriers to drop out of saturated drift velocities. Finally a discussion was given on the effect of allowing carriers to drift at nonsaturated velocities. This effect increases the output power due to two factors. The transit angle of the diode could be improved to increase the negative conductance by the longer drift time of the carriers and at the same time the RF voltage across the diode would be larger.

The derivation and results of a large-signal analysis for studying the dc, single-frequency and multifrequency operating characteristics of realistic IMPATT diodes were presented. The analysis allowed for electron and hole multiplication by impact ionization to occur at every point in the diode and did not assume that the avalanche region was narrow so that the divergence of the displacement current was neglected in this region. Therefore, it was possible to investigate a wide range of doping profiles and materials. The analysis did assume equal saturated drift velocities and ionization rates for the carriers which therefore did not allow for nonpunch-through diode structures to be considered. A differential equation relating the total current density (conduction and displacement) through the diode to the RF voltage across it was derived. The equation was solved by an iteration technique on a computer. The electric field, hole current density and electron current density were

obtained as a function of the phase of the RF voltage and position in the diode. The dc current density was plotted as a function of the bias voltage for the different diode structures. The current density above which the dc differential conductance was negative was shown to be related to an electric field profile in which the majority of the avalanching occurred at both ends of the diode rather than in the middle. The admittance, power output, efficiency and bias voltage of the diode were obtained as a function of the operating frequency, RF voltage and dc bias current density for the different diode structures. For the three pin diodes of Si, GaAs and Ge which were compared, the GaAs diode showed a slightly larger efficiency as a function of the RF voltage at the optimum operating frequency. All of the diodes considered showed an optimum operating frequency for maximum negative conductance which was related to the total phase delay between the RF voltage and current of the diode. An important effect predicted by this analysis is the effect of the doping density on the negative conductance of the diode. It was shown that as the doping density increased the avalanche region (the region where the carriers were generated) became narrower.

This caused more of the carriers which were generated to experience the same total phase delay and thus the negative conductance was larger for the proper operating frequency. This effect was verified experimentally. As the RF voltage was increased this narrow avalanche region became wider due to space-charge effects and the negative conductance approached that of a pin diode where the carriers were generated throughout most of the diode. Finally, the multifrequency results showed that the phase angle between the two harmonically related RF voltages was an important

parameter. With the proper phase angle the RF voltage waveform was such that the current generated in the avalanche process was delayed enough for operation below the cutoff frequency. However with another phase angle the conductance of the diode was degraded with respect to the single-frequency conductance.

The operating characteristics of IMPATT diodes were evaluated experimentally. An equivalent circuit for the diode package and mount was obtained. The transformation of the diode admittance which this circuit performed was verified by comparing the depletion layer capacitance of the diode as a function of the reverse voltage measured at 1 MHz where the transformation is unity and at X-band. The doping density for two different abrupt-junction diodes was obtained from this C-V measurement. The admittance of these two diodes was measured as a function of the dc bias current and frequency by a reflection-type circuit. Since the circuit was not a perfect 50- $\Omega$  match at all frequencies due to reflections in the connectors and circulator the measurement was made under small-signal conditions. The experimental results verify qualitatively several of the theoretical ones. These include the effects of bias current, frequency and doping density on the small-signal admittance. The increase in the cutoff frequency with bias current was also verified experimentally. The admittance of the diode was obtained from the admittance of the microwave circuit seen by the diode when oscillating at a single frequency. These large-signal results showed agreement between theory and experiment concerning the decrease in the magnitude of the negative conductance with increasing RF voltage. However, the magnitude of the conductance measured by both of these methods was somewhat less than the value predicted by the theory.

Also the change in susceptance with RF voltage which was measured was less than predicted. Due to the errors in the area of the diode the magnitude of the conductance and susceptance per square centimeter could be closer to the theoretical values than shown. Finally, a discussion of single-frequency operation was presented. By assuming a simplified microwave circuit it was shown that the impedance seen by the diode at the harmonics of the oscillation frequency could be very small or very large depending upon the operating conditions and the circuit. Thus the harmonics of the RF voltage could be very small in magnitude or of the same magnitude as the fundamental oscillation RF voltage.

## 5.2 Suggestions for Further Study

Although the large-signal analyses developed in this investigation for studying the operating characteristics of realistic IMPATT diodes provide good physical results and insight into the operation of an oscillating diode, there are a number of interesting questions which remain to be answered. The first effect to investigate would be the effect of higher current densities on the operating characteristics in order to determine the current saturation effects. Also it would be informative to investigate more diode structures and doping densities. The restriction of saturated drift velocities of the electrons and holes would have to be removed along with the capability of handling an undepleted region in the diode. Diodes with higher doping densities could then be analyzed to determine whether an optimum doping density for maximum negative conductance exists. The effect of unequal drift velocities and ionization rates would also be of interest. It appears that both of these effects would degrade the performance of the diode since this

would spread the total phase delay of the carriers over a wider range of values. Finally, the effect of diffusion should be studied to determine if and when it is important. Again diffusion could degrade the performance of the diode since the sharp pulse of carriers which is generated in highly doped diodes with large RF voltages would be spread out.

Experimentally there are several measurements which should be made. The diode should be operated both in the single-frequency and multifrequency modes. The admittance as seen by the diode at the fundamental and harmonics should be measured and compared to determine if the harmonics are shorted for single-frequency operation. A circuit should be designed for tuning both the fundamental and the second harmonic frequencies independently. Also it would be convenient to design a circuit in which the conductance and susceptance seen by the diode could be tuned somewhat independently. Characterization of these circuits should be done in such a way that the tuning of the diodes would take less trial and error.

These are a few of the experimental measurements which should be made as related to the present investigation. A great deal of other experimental work remains to be done relating to designing circuits for higher output power and efficiency, lower noise figure and lower thermal resistance.

## APPENDIX A. CONDITIONS FOR A PERIODIC SOLUTION IN READ-DIODE ANALYSIS

In Chapter II the differential equation involving the avalanche current was found to be

$$\frac{dJ_o(t)}{dt} = \left( B + C \sin \omega t - D \int_{t-\tau}^t (\tau - t + t') J_o(t') dt' \right) J_o(t) .$$

This equation reduces to

$$\frac{d}{dt} \ln J_o(t) = B + C \sin \omega t - D \int_{t-\tau}^t (\tau - t + t') J_o(t') dt' . \quad (A.1)$$

If  $J_o(t)$  is a periodic function with some positive average value then the logarithm must also be periodic. Also the derivative of a periodic function is periodic with a zero average value, thus

$$\left\langle \frac{d}{dt} \ln J_o(t) \right\rangle = 0 . \quad (A.2)$$

This implies that

$$\left\langle B + C \sin \omega t - D \int_{t-\tau}^t (\tau - t + t') J_o(t') dt' \right\rangle = 0 \quad (A.3)$$

or that

$$B = D \left\langle \int_{t-\tau}^t (\tau - t + t') J_o(t') dt' \right\rangle . \quad (A.4)$$

The current which is periodic can be represented by a Fourier series as

$$J_o(t) = J_{dc} + \sum_{-\infty}^{\infty} J_n e^{jn\omega t} . \quad (A.5)$$

If this relation is substituted into Eq. A.4 and the integration performed, the following result is obtained:

$$B = D \left\langle J_{dc} \frac{\tau^2}{2} + \sum_{-\infty}^{\infty} J_n e^{jn\omega t} \left( \frac{\tau}{jn\omega} + \frac{1}{n^2\omega^2} (1 - e^{-jn\omega\tau}) \right) \right\rangle . \quad (A.6)$$

Because of the exponential in the summation the average value of the sum is zero and Eq. A.6 reduces to

$$B = \frac{DJ_{dc}\tau^2}{2} . \quad (A.7)$$

Substituting in the relations for B and D as given in Eq. 2.16, the equation reduces to

$$E_b = \frac{J_{dc}\tau}{2\epsilon} . \quad (A.8)$$

Thus this condition is necessary for the solution of the differential equation, Eq. 2.16, to be periodic but it is not sufficient.

APPENDIX B. OPTIMUM TRANSIT ANGLE IN A READ-TYPE DIODE WITH SPACE-CHARGE  
EFFECTS INCLUDED

Under large-signal conditions the waveforms for the RF voltage, RF field, avalanche current and external current in a Read diode can be approximated by the waveforms shown in Fig. B.1. For these waveforms the optimum transit angle for maximum output power and efficiency will be derived.

The jump in the RF field when a sharp pulse of current is generated in the avalanche region is a function of the amount of charge in the pulse which in turn is proportional to the dc current by

$$Q = I_{dc}/f , \quad (B.1)$$

where  $f$  is the frequency of oscillation. Thus

$$\Delta E = \frac{I_{dc}}{f\epsilon A} . \quad (B.2)$$

If it is assumed that the current pulse is symmetrical then the RF field goes to zero when one-half of the charge is generated. This instant of time will be a function of the magnitude of the RF field as

$$E_{RF} \sin y\pi = \frac{I_{dc}}{2f\epsilon A} , \quad (B.3)$$

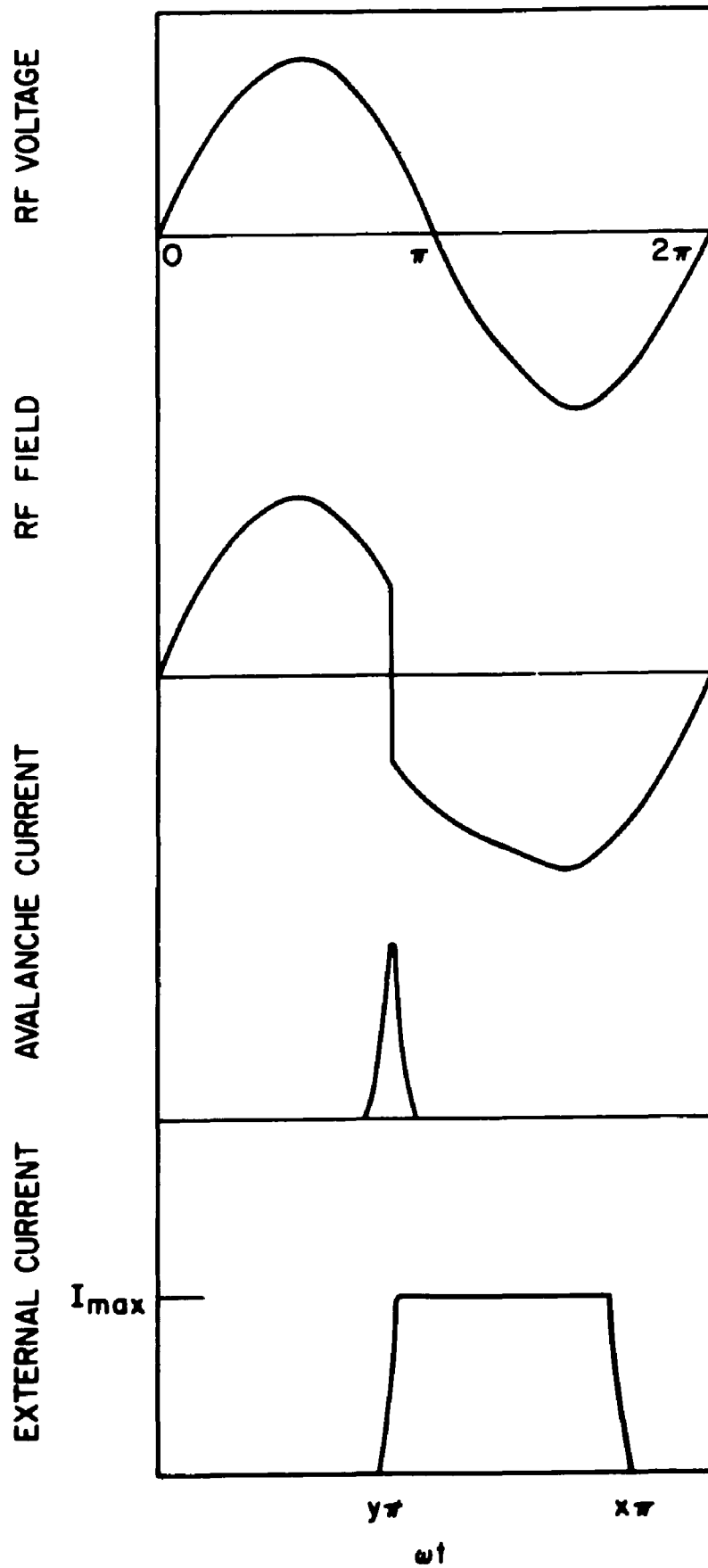


FIG. B.1 VOLTAGE, FIELD AND CURRENT WAVEFORMS IN A READ DIODE FOR A LARGE RF VOLTAGE.

where  $y$  is defined in Fig. B.1. Assume that the voltage is related to this field by

$$V_{RF} = E_{RF} \cdot x_L, \quad (B.4)$$

where  $x_L$  = the width of the diode. Thus

$$\sin y\pi = \frac{I_{dc} x_L}{2f\epsilon AV_{RF}}, \quad (B.5)$$

where  $0.5 \leq y \leq 1$ . The width of the diode can be obtained from the transit time which is a function of  $x$  and  $y$  by

$$x_L = v_s \cdot (x - y)\pi \cdot \frac{1}{2\pi f}, \quad (B.6)$$

where  $v_s$  = the saturated drift velocity. Thus

$$x_L = \frac{v_s (x - y)}{2f}. \quad (B.7)$$

Substituting this into Eq. B.5 the following relation is obtained:

$$\sin y\pi = \frac{I_{dc} v_s (x - y)}{4f^2 \epsilon AV_{RF}}. \quad (B.8)$$

This equation imposes a constraint on the parameters since the sine function is always less than or equal to unity. The equation for the average power dissipated by this waveform is given by

$$P = \frac{1}{2\pi} \int_{y\pi}^{x\pi} I_{max} V_{RF} \sin \omega t \, d\omega t. \quad (B.9)$$

This integral can be evaluated to give

$$P = \frac{I_{\max} V_{RF}}{2\pi} (\cos y\pi - \cos x\pi) . \quad (B.10)$$

The dc current is related to the maximum current by

$$I_{dc} = \frac{1}{2\pi} \int_{y\pi}^{x\pi} I_{\max} d\omega t \quad (B.11)$$

which gives

$$I_{dc} = I_{\max} \left( \frac{x - y}{2} \right) . \quad (B.12)$$

Therefore

$$I_{\max} = \frac{2I_{dc}}{x - y} \quad (B.13)$$

and substituting this into Eq. B.10 gives

$$P = \frac{I_{dc} V_{RF}}{\pi} \left( \frac{\cos y\pi - \cos x\pi}{x - y} \right) . \quad (B.14)$$

The efficiency of the device with these waveforms is

$$\eta = P / (V_B I_{dc}) , \quad (B.15)$$

where  $V_B$  is the dc bias voltage. The bias voltage is proportional to the length of the diode which is a function of the transit angle. A typical value for bias voltage is 100 V for a  $0.75\pi$  transit angle at 10 GHz.

Thus the bias voltage can be expressed as

$$V_B = 100 f(\tau) , \quad (B.16)$$

where  $f(\tau)$  is the dependence on the transit time. This function is given by

$$f(\tau) = x_L(\tau) / x_L(\tau_{0.75\pi}) , \quad (\text{B.17})$$

where

$$x_L(\tau) = v_s \tau = v_s (x - y) \pi / (2\pi f)$$

and

$$x_L(\tau_{0.75\pi}) = v_s \tau_{0.75\pi} = v_s (0.75\pi) / (2\pi \cdot 10^{10}) .$$

Therefore

$$f(\tau) = \frac{(x - y) \cdot 10^{10}}{0.75f} . \quad (\text{B.18})$$

The bias voltage is

$$V_B = 10^{12} \cdot \frac{(x - y)}{0.75f} \quad (\text{B.19})$$

thus the efficiency is

$$\eta = \frac{V_{RF}}{\pi} \left( \frac{0.75f}{10^{12}} \right) \left( \frac{\cos y\pi - \cos x\pi}{(x - y)^2} \right) . \quad (\text{B.20})$$

In order to determine the maximum output power and efficiency as a function of the transit angle, a computer program was written to evaluate the output power and efficiency. The diode parameters  $v_s$ ,  $\epsilon$  and  $A$  were taken as follows:

$$\begin{aligned} v_s &= 1 \times 10^7 \text{ cm/s}, \\ \epsilon &= 10^{-12} \text{ F/cm and} \\ A &= 10^{-4} \text{ cm}^2. \end{aligned}$$

Different values for  $I_{dc}$ ,  $f$  and transit angle  $(x - y)$  were specified. The breakdown voltage was then calculated and the different values of RF voltage

were determined relative to the breakdown voltage. For each RF voltage the output power and efficiency were calculated. These results are plotted in Figs. B.2 through B.4. These figures show the essential features of the Read diode. An optimum transit angle exists which depends on the RF voltage, dc current and frequency of operation. As the RF voltage is increased for constant dc current the optimum transit angle decreases due to the additional phase delay in the avalanche process (Fig. B.2). The optimum transit angle is smaller for maximum efficiency than for maximum output power due to the effect of transit angle on the input and output power (Fig. B.3). As the dc current is increased for a constant normalized RF voltage ( $V_{RF}/V_B$ ) the optimum transit angle increases due to the decrease in phase delay in the avalanche process (Fig. B.4).

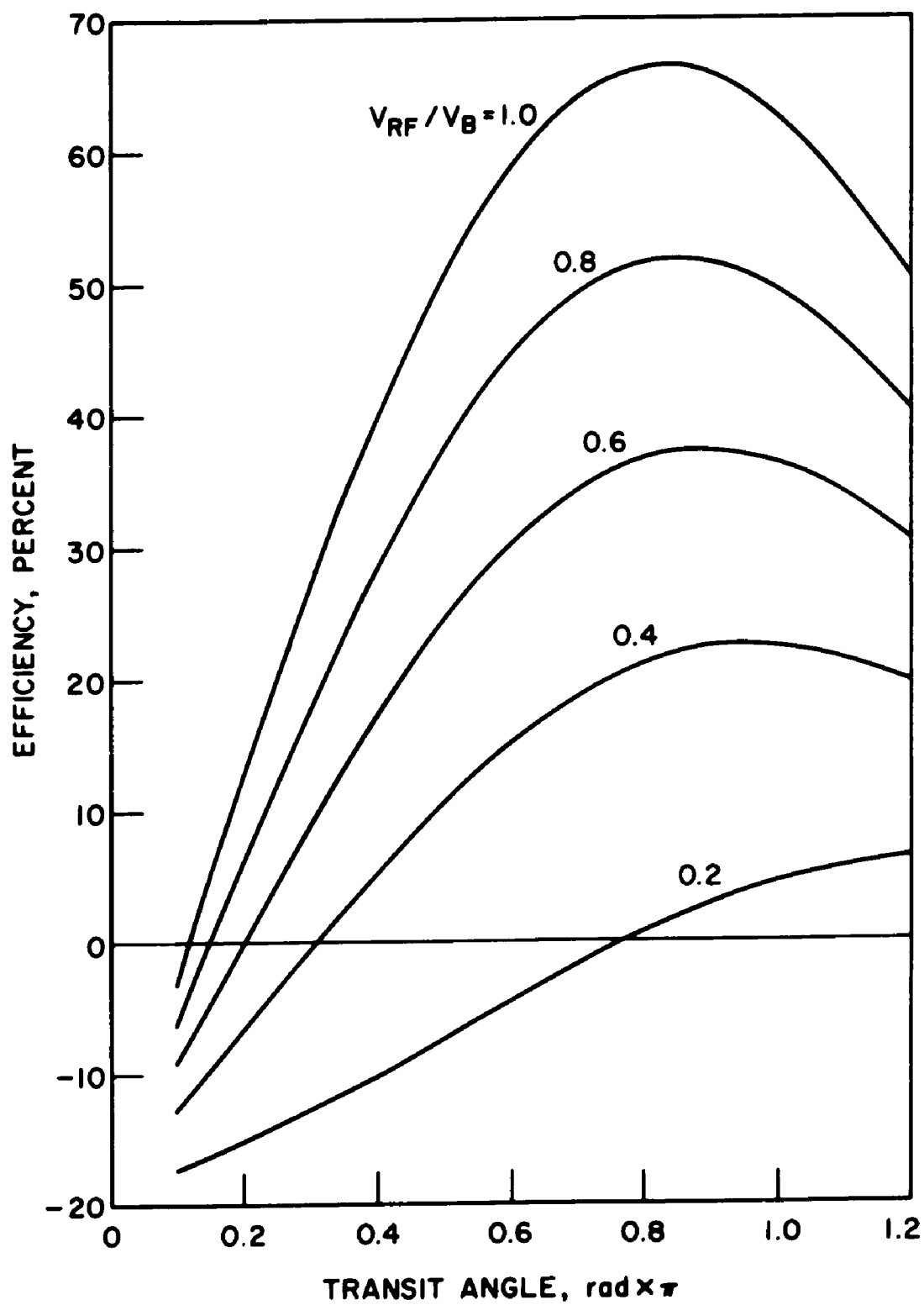


FIG. B.2 EFFICIENCY VS. TRANSIT ANGLE. ( $I_{dc} = 100 \text{ mA}$  AND  $f = 10 \text{ GHz}$ )

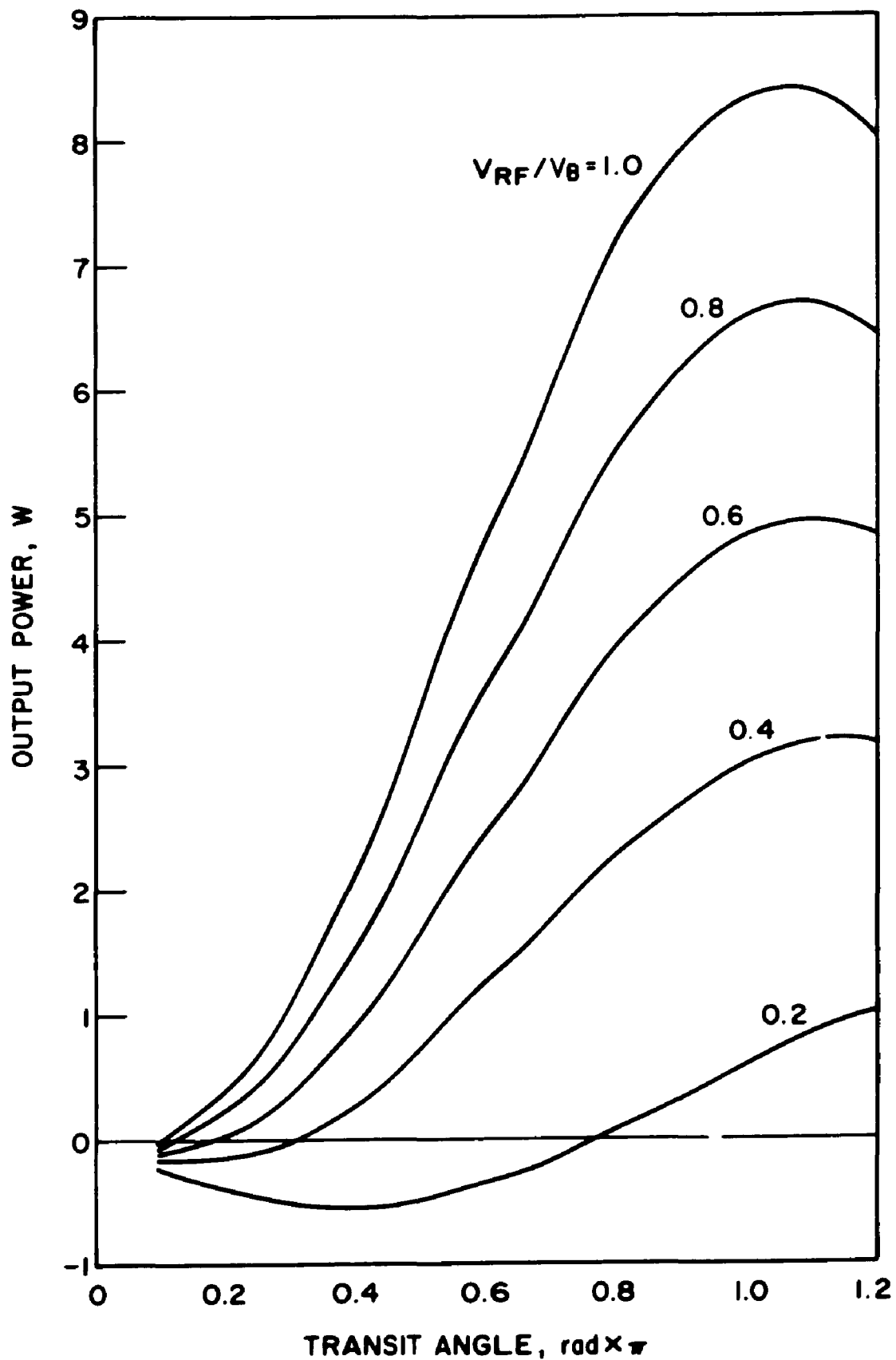


FIG. B.3 OUTPUT POWER VS. TRANSIST ANGLE. ( $I_{dc} = 100$  mA AND  $f = 10$  GHz)

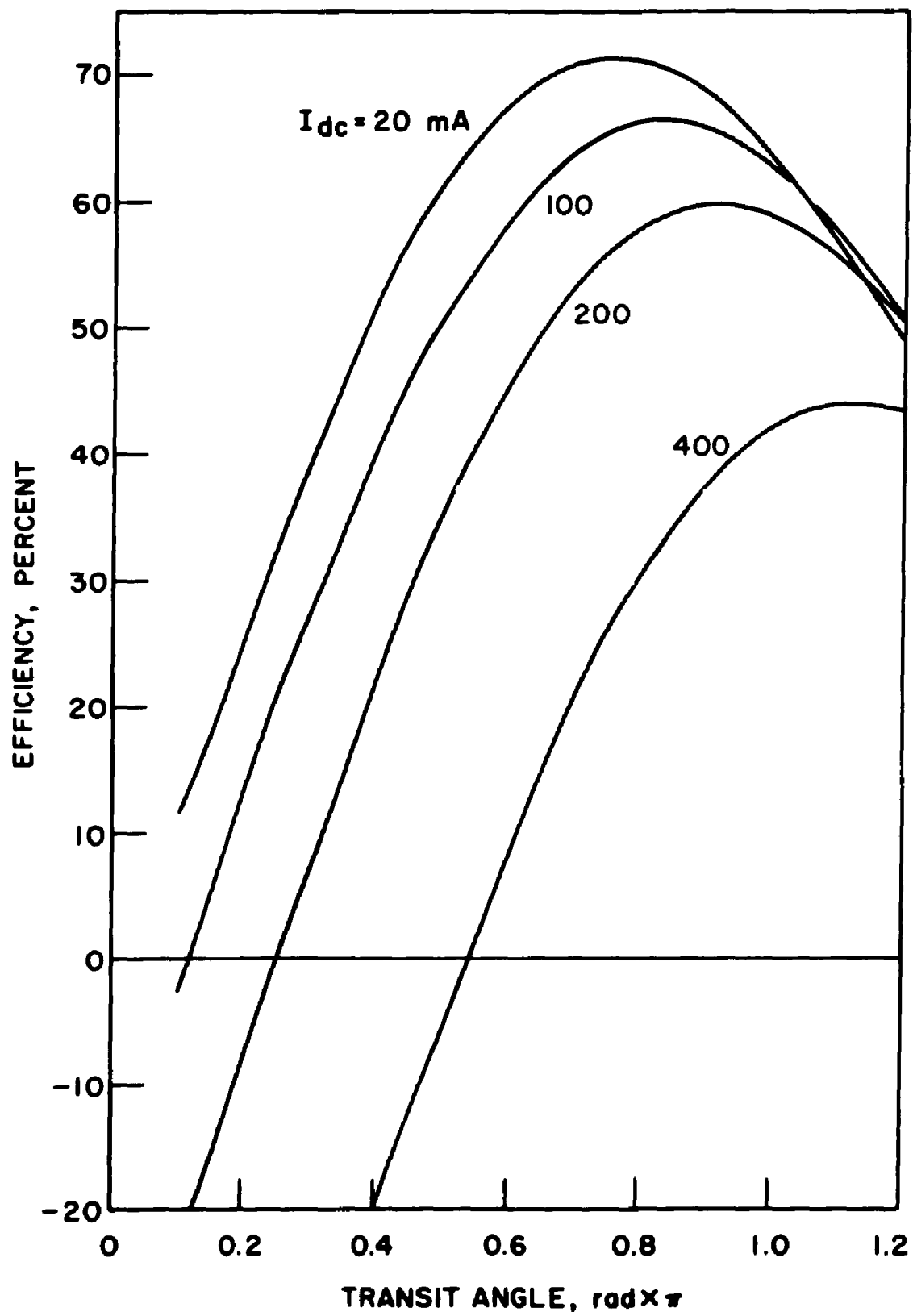


FIG. B.4 EFFICIENCY VS. TRANSIT ANGLE. ( $V_{RF}/V_B = 1$  AND  $f = 10 \text{ GHz}$ )

## APPENDIX C. EXPLANATION OF THE EFFECTIVE IONIZATION RATE

The effective or average ionization rate which has been used in the large-signal analysis is obtained from the experimental rates for Si, GaAs and Ge. The dc breakdown voltage is determined for the diode by solving for the field which satisfies the condition for breakdown with unequal ionization rates. This condition<sup>64</sup> is

$$\int_0^{x_L} \alpha_p \exp \left( - \int_0^x (\alpha_p - \alpha_n) d\gamma \right) dx = 1 . \quad (C.1)$$

The electric field in the diode is determined by the doping profile from Poisson's equation by

$$E(x) = E(0) + \int_0^x [N_D(\gamma) - N_A(\gamma)] d\gamma , \quad (C.2)$$

where  $E(0)$  is an arbitrary value of the field at  $x = 0$ . Space-charge effects have been neglected in Eq. C.2. The field  $E(0)$  is determined in such a way that Eq. C.1 is satisfied. Then the voltage at breakdown is

$$V_B = \int_0^{x_L} E(\gamma) d\gamma . \quad (C.3)$$

The effective ionization rate is obtained by finding the constants  $A'$  and  $b'$  of the ionization rate which satisfy the breakdown condition for equal ionization rates,

$$\int_0^{x_L} \alpha \, dx = 1 \quad , \quad (C.4)$$

and give the same value of breakdown voltage. These constants are approximately the mean value of the constants for the unequal rates. These equations were solved by an iteration technique on the computer. The effective ionization rate for Si is plotted in Fig. C.1 along with the experimental rates for comparison. For GaAs there is no need for an effective rate since the rates for electrons and holes are equal. The effective rate for Ge is plotted in Fig. C.2 and is compared with the experimental values. The constants A' and b' which have been calculated for the effective ionization rates are given in Table 3.1.

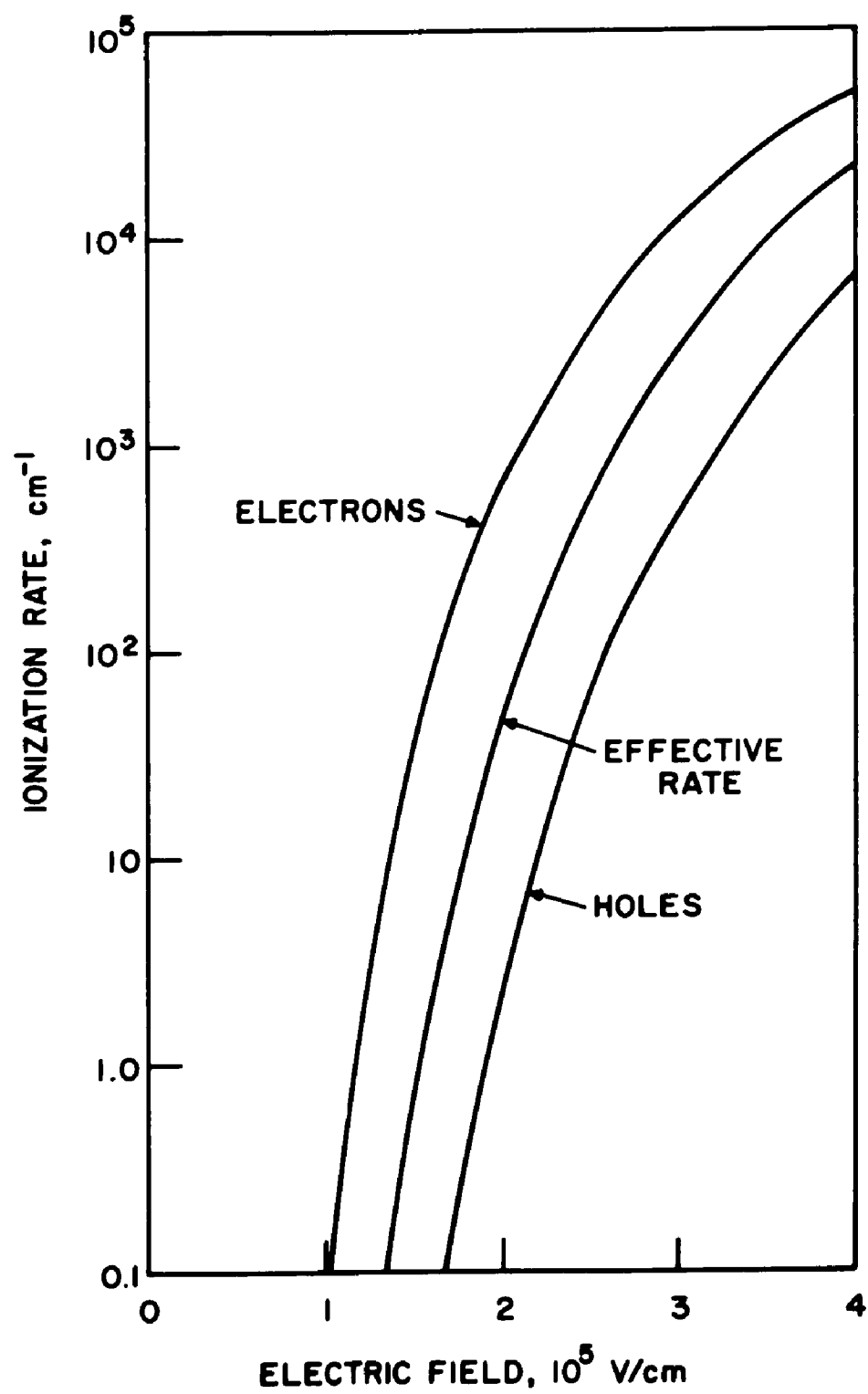


FIG. C.1 IONIZATION RATES VS. ELECTRIC FIELD IN Si.

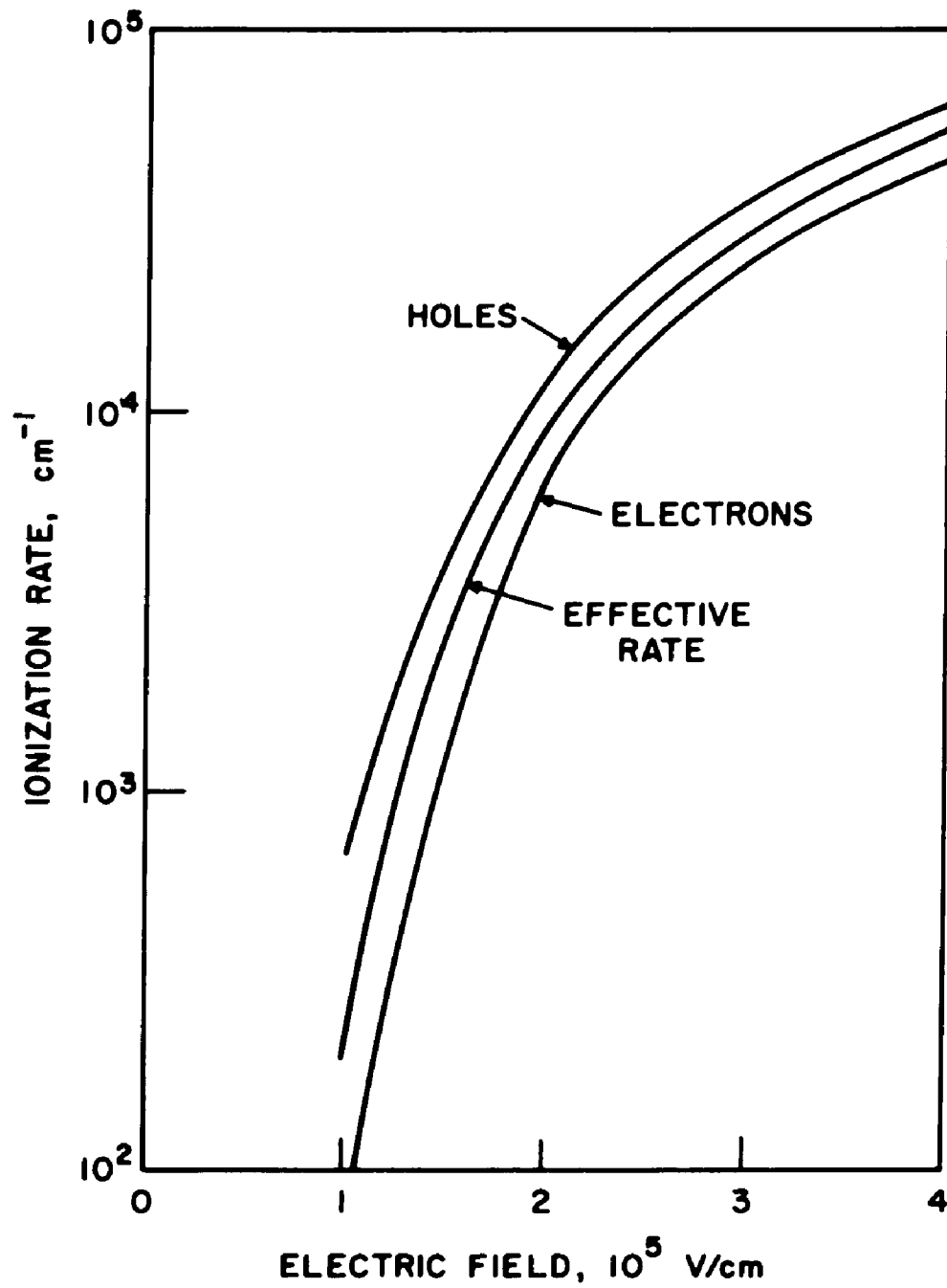


FIG. C.2 IONIZATION RATES VS. ELECTRIC FIELD IN Ge.

# APPENDIX D. PERIODIC SOLUTIONS OF LINEAR DIFFERENTIAL EQUATIONS WITH PERIODIC COEFFICIENTS

The following theorem<sup>65</sup> proves that the solution to Eq. 3.19 is periodic.

## Theorem

Let the solution to Eq. 3.19,

$$\frac{d}{dt} J_T(t) + A(t)J_T(t) = Y(t) ,$$

be

$$J_T(t) = \exp \left( - \int_0^t A(\gamma) d\gamma \right) \left[ J_T(0) + \int_0^t Y(\gamma) \exp \left( \int_0^\gamma A(\beta) d\beta \right) d\gamma \right] , \quad (D.1)$$

where  $A(t)$  and  $Y(t)$  are periodic functions of  $t$  with period  $T$ ; then a necessary and sufficient condition for  $J_T(t)$  to be periodic with a period  $T$  is

$$J_T(0) \left[ 1 - \exp \left( \int_0^T A(\gamma) d\gamma \right) \right] + \int_0^T Y(\gamma) \exp \left( \int_0^\gamma A(\beta) d\beta \right) d\gamma = 0 . \quad (D.2)$$

## Proof of Theorem

It is obvious that for a periodic RF voltage the coefficients  $A(t)$  and  $Y(t)$  defined in Eq. 3.19 are also periodic.

a. Necessity: If  $J_T(t)$  is periodic with period  $T$ , then  $J_T(T) = J_T(0)$ .

But

$$J_T(T) = \exp \left( - \int_0^T A(\gamma) d\gamma \right) \left[ J_T(0) + \int_0^T Y(\gamma) \exp \left( \int_0^\gamma A(\beta) d\beta \right) d\gamma \right] . \quad (D.3)$$

Hence by substituting  $J_T(T) = J_T(0)$ , Eq. D.2 is obtained.

b. Sufficiency: It must be shown that if the condition in Eq. D.2 exists, the solution is periodic or that

$$J_T(t + T) = J_T(t) \quad (D.4)$$

for any  $t$ . Thus

$$J_T(t + T) = \exp \left( - \int_0^{t+T} A(\gamma) d\gamma \right) \left[ J_T(0) + \int_0^{t+T} Y(\gamma) \exp \left( \int_0^\gamma A(\beta) d\beta \right) d\gamma \right] \quad (D.5)$$

which can be written as

$$J_T(t + T) = \exp \left( - \int_0^T A(\gamma) d\gamma \right) \exp \left( - \int_T^{t+T} A(\gamma) d\gamma \right) \cdot \left[ J_T(0) + \int_0^T Y(\gamma) \exp \left( \int_0^\gamma A(\beta) d\beta \right) d\gamma + \int_T^{t+T} Y(\gamma) \exp \left( \int_0^\gamma A(\beta) d\beta \right) d\gamma \right] . \quad (D.6)$$

Since  $A(t)$  and  $Y(t)$  are periodic the integrals from  $T$  to  $t + T$  can be shifted by making the following substitution:

$$u = \gamma - T \quad (D.7)$$

and

$$du = d\gamma \quad (D.8)$$

Therefore

$$\begin{aligned} J_T(t + T) &= \exp \left( - \int_0^T A(\gamma) d\gamma \right) \exp \left( - \int_0^t A(u) du \right) \\ &\cdot \left[ J_T(0) + \int_0^T Y(\gamma) \exp \left( \int_0^\gamma A(\beta) d\beta \right) d\gamma \right. \\ &\quad \left. + \int_0^t Y(u) \exp \left( \int_0^{u+T} A(\beta) d\beta \right) du \right] \quad (D.9) \end{aligned}$$

The last term in Eq. D.9 can be rewritten as

$$\exp \left( \int_0^{u+T} A(\beta) d\beta \right) = \exp \left( \int_0^T A(\beta) d\beta \right) \exp \left( \int_T^{u+T} A(\beta) d\beta \right) \quad (D.10)$$

but

$$\int_T^{u+T} A(\beta) d\beta = \int_0^u A(\beta) d\beta \quad (D.11)$$

since  $A(t)$  is periodic. Therefore

$$\exp \left( \int_0^{u+T} A(\beta) d\beta \right) = \exp \left( \int_0^T A(\beta) d\beta \right) \exp \left( \int_0^u A(\beta) d\beta \right) \quad (D.12)$$

Equation D.9 can be reduced to

$$\begin{aligned}
 J_T(t + T) &= \exp \left( - \int_0^T A(\gamma) d\gamma \right) \exp \left( - \int_0^t A(u) du \right) \\
 &\cdot \left[ J_T(0) + \int_0^T Y(\gamma) \exp \left( \int_0^\gamma A(\beta) d\beta \right) d\gamma + \exp \left( \int_0^T A(\beta) d\beta \right) \right. \\
 &\quad \left. \cdot \int_0^t Y(u) \exp \left( \int_0^u A(\beta) d\beta \right) du \right] \quad . \quad (D.13)
 \end{aligned}$$

Using the definition of  $J_T(t)$ , this equation can be written in the following form:

$$\begin{aligned}
 J_T(t + T) &= J_T(t) - J_T(0) \exp \left( - \int_0^t A(\gamma) d\gamma \right) + \exp \left( - \int_0^T A(\gamma) d\gamma \right) \\
 &\cdot \exp \left( - \int_0^t A(u) du \right) \left[ \int_0^T Y(\gamma) \exp \left( \int_0^\gamma A(\beta) d\beta \right) d\gamma + J_T(0) \right] \quad (D.14)
 \end{aligned}$$

which can be rearranged to yield

$$\begin{aligned}
 J_T(t + T) &= J_T(t) + \exp \left( - \int_0^T A(\gamma) d\gamma \right) \exp \left( - \int_0^t A(u) du \right) \\
 &\cdot \left\{ J_T(0) \left[ 1 - \exp \left( \int_0^T A(\gamma) d\gamma \right) \right] + \int_0^T Y(\gamma) \exp \left( \int_0^\gamma A(\beta) d\beta \right) d\gamma \right\} \quad . \\
 &\hspace{25em} (D.15)
 \end{aligned}$$

By applying the condition of the theorem the last term is then zero and

$$J_T(t + T) = J_T(t) \quad . \quad (D.16)$$

Thus the theorem is proved and the solution is periodic.

One point must be noted in the solution of Eq. 3.19; the integral of the coefficient  $A(t)$  over a period must not be zero. Mathematically if this is the case, then  $J_T(0)$  cannot be evaluated as defined in Eq. 3.21. Physically this will never happen as can be seen by considering the dc case. The coefficient  $A$  is related to the multiplication factor  $M$  such that in the dc case if  $A$  goes to zero then  $M$  goes to infinity. The coefficient will be close to zero but never exactly zero.

## APPENDIX E. EFFECT OF CONVERGENCE CONDITIONS ON CURRENT WAVEFORMS

The solutions to the differential equations obtained in the large-signal analyses of Chapters II and III were obtained by an iteration technique. The solutions were assumed to have converged when the conductance and susceptance at the fundamental frequency changed by less than a prescribed percent. Since it may be possible that many different waveforms could have the same fundamental components, the effect of this convergence condition on the total current waveform or solution of the large-signal analyses must be investigated.

In Chapter II the solution of the differential equation gave the avalanche current generated in the avalanche region as a function of the phase of the RF voltage. In Table E.1 values of this current are listed at different phase angles of the RF voltage as a function of the iteration number. Two points should be made. First, as discussed in Appendix A, the condition obtained for a periodic solution was necessary but not sufficient, but these results show that the solution is periodic. Second, after six iterations which correspond to convergence in the fundamental admittance the current waveform also converges.

In Chapter III the solution to the differential equation derived for the large-signal analysis gave the total current density as a function of the phase of the RF voltage. In Fig. E.1 this current density

Table E.1

Avalanche Current vs. Phase of the RF Voltage for Different Iteration Numbers

( $I_{dc} = 100 \text{ mA}$ ,  $f = 9.5 \text{ GHz}$ ,  $f_a = 10 \text{ GHz}$  at  $100 \text{ mA}$  and  $V_{RF} = 20 \text{ V}$ )

| Iteration<br>Number | Phase of the RF Voltage, Degrees |       |       |       |       |       |
|---------------------|----------------------------------|-------|-------|-------|-------|-------|
|                     | 0                                | 90    | 160   | 180   | 270   | 360   |
| 1                   | 5.508                            | 121.3 | 321.0 | 183.6 | 12.99 | 5.507 |
| 2                   | 5.488                            | 120.9 | 322.0 | 184.5 | 12.95 | 5.487 |
| 3                   | 5.473                            | 120.6 | 322.6 | 184.7 | 12.91 | 5.472 |
| 4                   | 5.466                            | 120.5 | 322.8 | 184.8 | 12.89 | 5.465 |
| 5                   | 5.465                            | 120.5 | 322.9 | 184.7 | 12.88 | 5.463 |
| 6                   | 5.466                            | 120.5 | 322.8 | 184.6 | 12.88 | 5.464 |
| 7                   | 5.466                            | 120.5 | 322.8 | 184.6 | 12.89 | 5.465 |
| 8                   | 5.467                            | 120.5 | 322.8 | 184.6 | 12.89 | 5.466 |
| 9                   | 5.467                            | 120.5 | 322.8 | 184.6 | 12.89 | 5.466 |

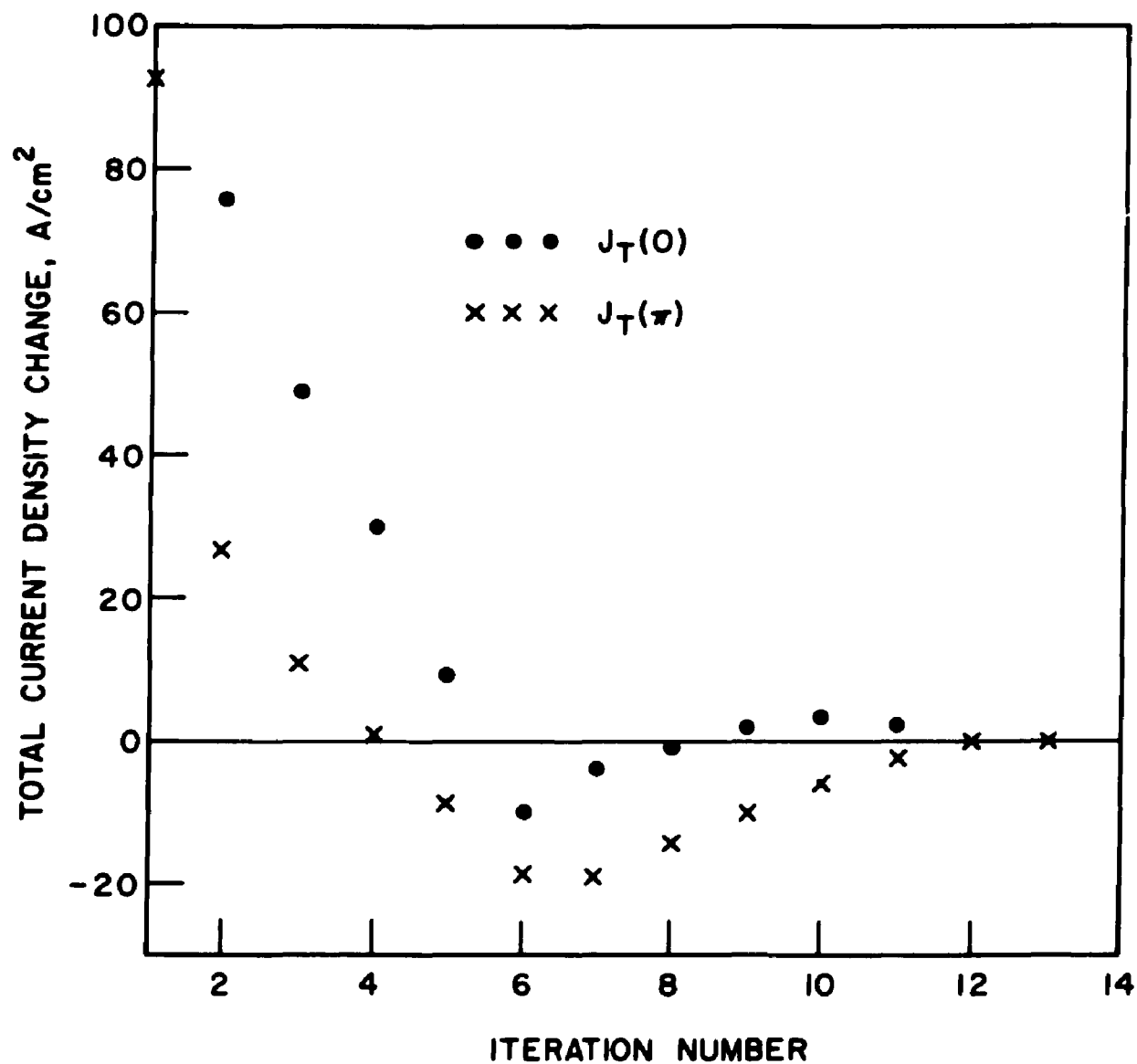


FIG. E.1 TOTAL CURRENT DENSITY CHANGE VS. ITERATION NUMBER FOR A Si READ DIODE. ( $f = 12$  GHz,  $J_{dc} = 500$  A/cm<sup>2</sup> AND  $V_{RF} = 20$  V. FINAL VALUE OF CURRENT IS  $J_T(0) = 3310$  A/cm<sup>2</sup> AND  $J_T(\pi) = -2305$  A/cm<sup>2</sup>)

is plotted for both 0 and  $\pi$  rad as a function of the iteration number. In Appendix D the condition imposed to obtain a periodic solution to this equation was shown to be necessary and sufficient. For this case the conductance and susceptance converged on the thirteenth iteration which corresponds to the iteration number when the waveform for the total current converged.

As shown by these two results, the convergence conditions imposed in the large-signal analyses produce a convergent current waveform.

## APPENDIX F. DERIVATION OF LEAST-SQUARES FIT OF EXPERIMENTAL DATA

The lumped circuit elements in the equivalent circuit for the diode package and mount are obtained from the experimental data by a least-squares fit of the theoretical impedance of the equivalent circuit to the data. The following derivation is for the open-circuited package which has the equivalent circuit shown in Fig. 4.5. The derivation for the short-circuited package follows in a similar manner and is not included.

The reactance of the open-circuited package is

$$X = \omega L_s - \frac{1}{\omega C_{pk}} \quad (F.1)$$

which can be written as

$$\omega X = \omega^2 L_s - \frac{1}{C_{pk}} \quad (F.2)$$

The curve to which the data is to be matched is of the form

$$y = ax^2 - b \quad , \quad (F.3)$$

where a and b are constants. Following the method of least squares, the difference between the measured values and the calculated values from Eq. F.3 is  $\delta$ . Thus for the  $i$ th value

$$y_i - (ax_i^2 - b) = \delta_i \quad (F.4)$$

The sum of the squares of the differences at each point is

$$E^2 \triangleq \sum_{i=1}^n \delta_i^2 = (y_1 - ax_1^2 + b)^2 + (y_2 - ax_2^2 + b)^2 + \cdots + (y_n - ax_n^2 + b)^2 , \quad (F.5)$$

where n equals the number of data points.

The least-squares criterion is that the constants a and b should be chosen so as to make the sum of the squares of the deviations  $E^2$  as small as possible. This is done by applying the conditions for minimizing a function of two variables which is to equate to zero the first two partial derivatives, thus

$$\begin{aligned} \frac{\partial E^2}{\partial a} &= 2(y_1 - ax_1^2 + b)(-x_1^2) + 2(y_2 - ax_2^2 + b)(-x_2^2) \\ &\quad + \cdots + 2(y_n - ax_n^2 + b)(-x_n^2) = 0 \end{aligned} \quad (F.6)$$

and

$$\frac{\partial E^2}{\partial b} = 2(y_1 - ax_1^2 + b) + 2(y_2 - ax_2^2 + b) + \cdots + (y_n - ax_n^2 + b) = 0 . \quad (F.7)$$

Collecting terms of the constants a and b, the following equations are obtained:

$$-2 \sum_{i=1}^n y_i x_i^2 + 2a \sum_{i=1}^n x_i^4 - 2b \sum_{i=1}^n x_i^2 = 0 \quad (F.8)$$

and

$$2 \sum_{i=1}^n y_i - 2a \sum_{i=1}^n x_i^2 + 2nb = 0 \quad . \quad (F.9)$$

From Eq. F.9 a relation for b is obtained,

$$b = \frac{1}{n} \left( a \sum_{i=1}^n x_i^2 - \sum_{i=1}^n y_i \right) \quad (F.10)$$

and using this relation in Eq. F.8 yields

$$a \sum_{i=1}^n x_i^4 - \sum_{i=1}^n x_i^2 \left[ \frac{1}{n} \left( a \sum_{i=1}^n x_i^2 - \sum_{i=1}^n y_i \right) \right] = \sum_{i=1}^n y_i x_i^2 \quad . \quad (F.11)$$

Solving for a from this equation results in

$$a = \frac{\left( n \sum_{i=1}^n y_i x_i^2 - \sum_{i=1}^n x_i^2 \sum_{i=1}^n y_i \right)}{n \sum_{i=1}^n x_i^4 - \left( \sum_{i=1}^n x_i^2 \right)^2} \quad . \quad (F.12)$$

The parameters y, x, a and b are related to X,  $\omega$ ,  $L_s$  and  $C_{pk}$  as can be seen from Eqs. F.2 and F.3. Using these relations one obtains

$$L_s = \frac{\left( n \sum_{i=1}^n \omega_i^3 X_i - \sum_{i=1}^n \omega_i^2 \sum_{i=1}^n \omega_i X_i \right)}{n \sum_{i=1}^n \omega_i^4 - \left( \sum_{i=1}^n \omega_i^2 \right)^2} \quad (F.13)$$

and

$$C_{pk} = \frac{n}{L_s \sum_{i=1}^n \omega_i^2 - \sum_{i=1}^n \omega_i X_i} . \quad (F.14)$$

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## LIST OF SYMBOLS

|                             |                                                                 |
|-----------------------------|-----------------------------------------------------------------|
| $A$                         | Diode area.                                                     |
| $A', b', m'$                | Parameters appearing in the expression for the ionization rate. |
| $A_{dB}, B_{dB}, (DC)_{dB}$ | Attenuation.                                                    |
| $A(t), Y(t)$                | Parameters defined by Eq. 3.19.                                 |
| $B$                         | Diode susceptance.                                              |
| $B, C, D$                   | Parameters defined by Eq. 2.16.                                 |
| $C$                         | Capacitance.                                                    |
| $C_d$                       | Depletion layer capacitance.                                    |
| $C_{df}$                    | Diffusion capacitance.                                          |
| $C_{pd}$                    | Pad capacitance.                                                |
| $C_{pk}$                    | Package capacitance of equivalent circuit.                      |
| $D$                         | Junction diameter.                                              |
| $E$                         | Electric field.                                                 |
| $E_b$                       | Dc field due to the dc current flow in the diode.               |
| $E_c$                       | Critical field for avalanche breakdown.                         |
| $E_D$                       | Electric field due to the doping profile.                       |
| $E_m$                       | Maximum electric field for avalanche breakdown.                 |
| $E_{min}$                   | Minimum field in the drift region.                              |
| $E_{po}$                    | Field at the edge of the avalanche region at punch through.     |
| $E_{RF}$                    | RF electric field.                                              |

|              |                                                                             |
|--------------|-----------------------------------------------------------------------------|
| $E_{sc}$     | Electric field due to space charge.                                         |
| $f$          | Frequency.                                                                  |
| $f_a$        | Small-signal cutoff frequency defined in Eq. 2.12.                          |
| $G$          | Diode conductance.                                                          |
| $G_L$        | Load conductance.                                                           |
| $I_{dc}$     | Dc current.                                                                 |
| $I_{max}$    | Maximum value of the current.                                               |
| $i, n$       | Index integers.                                                             |
| $J_o, J_c$   | Conduction current density.                                                 |
| $J_{o_1}$    | Fundamental component of the conduction current density.                    |
| $J_{o_{dc}}$ | Dc conduction current density.                                              |
| $J_{dc}$     | Dc current density.                                                         |
| $J_e$        | Current density induced in the external circuit.                            |
| $J_{e_1}$    | Fundamental component of the externally induced conduction current density. |
| $J_n$        | Electron current density.                                                   |
| $J_{ns}$     | Electron reverse-saturation current density.                                |
| $J_p$        | Hole current density.                                                       |
| $J_{ps}$     | Hole reverse-saturation current density.                                    |
| $J_{sat}$    | Total reverse-saturation current density.                                   |
| $J_T$        | Total current density.                                                      |
| $j$          | $\sqrt{-1}$ .                                                               |
| $k_1$        | Parameter defined in Eq. 4.9.                                               |
| $k_2$        | Parameter defined in Eq. 4.10.                                              |

|                 |                                                             |
|-----------------|-------------------------------------------------------------|
| $L_a$           | Inductance of the diode.                                    |
| $L_d$           | Lead inductance of the equivalent circuit.                  |
| $L_s$           | Series inductance of the equivalent circuit.                |
| $l$             | Number of harmonic RF voltages across the diode.            |
| $m$             | Parameter defined by $m = E_c [\alpha'(E_c)/\alpha(E_c)]$ . |
| $N_A$           | Number of acceptor atoms/cm <sup>3</sup> .                  |
| $N_D$           | Number of donor atoms/cm <sup>3</sup> .                     |
| $N(x)$          | Doping density as a function of distance.                   |
| $n$             | Harmonic number of the operating frequency.                 |
| $n$             | Number of conduction electrons/cm <sup>3</sup> .            |
| $P$             | Average power.                                              |
| $P_1, P_2, P_3$ | Power levels defined by Fig. 4.16.                          |
| $P_{in}$        | Input power to the test circuit.                            |
| $P_{inc}$       | Incident power on the diode coaxial cavity.                 |
| $P_o$           | Power delivered by the diode.                               |
| $P_{out}$       | Power measured at the diode coaxial cavity.                 |
| $P_T$           | Transmitted power to the diode.                             |
| $p$             | Number of conduction holes/cm <sup>3</sup> .                |
| $Q$             | Total charge in the pulse of the conduction current.        |
| $q$             | Electronic charge.                                          |
| $R$             | Resistance.                                                 |
| $R_L, R_p$      | Load resistance.                                            |
| $R_s$           | Series resistance.                                          |
| $r$             | Voltage standing-wave ratio.                                |

|            |                                                      |
|------------|------------------------------------------------------|
| $T$        | Period of oscillation.                               |
| $t$        | Time.                                                |
| $V$        | RF voltage across the diode.                         |
| $V$        | Total voltage across the diode.                      |
| $V_B$      | Bias voltage.                                        |
| $V_{bias}$ | Average voltage across the diode.                    |
| $V_d$      | Dc voltage drop across the diode.                    |
| $V_{dc}$   | Dc bias voltage.                                     |
| $V_n$      | Magnitude of the nth harmonic voltage.               |
| $V_p$      | Voltage drop at punch through.                       |
| $V_{RF}$   | Magnitude of the RF voltage.                         |
| $V_{sc}$   | Voltage across the drift region due to space charge. |
| $V_T$      | RF voltage across the input terminals of the diode.  |
| $v, v_s$   | Saturated drift velocity.                            |
| $v_n$      | Electron drift velocity.                             |
| $v_p$      | Hole drift velocity.                                 |
| $X$        | Reactance.                                           |
| $X_L$      | Load reactance.                                      |
| $x$        | Distance.                                            |
| $x$        | Parameter defined in Fig. 1.2.                       |
| $x_A$      | Length of the avalanche region.                      |
| $x_D$      | Length of the drift region.                          |
| $x'_D$     | Effective drift length.                              |
| $x_L$      | Total length of the depleted region.                 |

|                 |                                                                                                             |
|-----------------|-------------------------------------------------------------------------------------------------------------|
| $Y$             | Admittance.                                                                                                 |
| $y$             | Parameter defined in Fig. B.1.                                                                              |
| $Z$             | Impedance.                                                                                                  |
| $Z_o$           | Characteristic impedance of the coaxial line (50 $\Omega$ ).                                                |
| $Z_L$           | Load impedance.                                                                                             |
| $\alpha$        | Ionization rate.                                                                                            |
| $\bar{\alpha}$  | Average ionization rate.                                                                                    |
| $\bar{\alpha}'$ | Derivative of average ionization rate defined by $\bar{\alpha}' = d\bar{\alpha}/dE$ .                       |
| $\alpha_n$      | Ionization rate for electrons.                                                                              |
| $\alpha_p$      | Ionization rate for holes.                                                                                  |
| $\Delta E$      | Field variation due to space charge.                                                                        |
| $\epsilon$      | Dielectric constant.                                                                                        |
| $\epsilon_r$    | Relative dielectric constant.                                                                               |
| $\eta$          | Efficiency.                                                                                                 |
| $\theta$        | Phase angle obtained for impedance measurement and defined by Eq. 4.11.                                     |
| $\theta$        | Transit-time phase delay.                                                                                   |
| $\theta_a$      | $\omega_a \tau$ .                                                                                           |
| $\theta_f$      | Phase angle for the subscripted frequency for the reactance of the short at the end of a transmission line. |
| $\theta_n$      | Phase angle of the nth harmonic voltage.                                                                    |
| $\rho$          | Charge density.                                                                                             |
| $\tau$          | Transit time through the diode.                                                                             |
| $\tau$          | Transit time through the drift region.                                                                      |

|          |                                                                                                     |
|----------|-----------------------------------------------------------------------------------------------------|
| $\tau_a$ | Transit time through the avalanche region.                                                          |
| $\psi$   | Phase angle between the two RF voltages.                                                            |
| $\omega$ | $2\pi f$ (all subscripted $\omega$ 's are related to the corresponding frequency by this relation). |