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by
Tung H. Lin

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by

Tung H. Lin

The basic equation in Newtonian mechanics states that force is proportional to mass times acceleration. Accurate measurement of acceleration is very important in the study of mechanics. The spring-mass accelerometer commonly used gives fairly reliable readings for motions with frequencies below 70 percent of the natural frequency of the instrument, but is unreliable for transients like shock or impact with components of motions which have frequencies near to or exceeding the natural frequency.

In structural and machine design, the maximum force on the parts usually gives critical design conditions, so the designer is most interested in the peak acceleration. For high frequency and transient motions, contact peak accelerometers designed to measure the maximum acceleration occurring during the motion, have been used. These contact accelerometers are very unreliable and their theory has not been worked out. This investigation is made to study, theoretically and experimentally, the response of a spring-mass contact accelerometer and to explore the means of

improving its design so as to give reliable responses.

The responses of a spring-mass accelerometer with damping, to acceleration of simple harmonic motions of different frequencies, including those far beyond the natural frequency of the system, are shown with distributed mass of the spring considered.

Then the responses of a contact-break accelerometer without damping, under both steady state and transient motions, are given. The factor of proportionality and time lag vary appreciably as the natural frequency is approached. A method has also been derived for calculating the responses under the steady state motion after the contact breaks.

An analysis of the contact accelerometer responses with distributed viscous damping in addition to the distributed mass of the spring is given. With this damping, the variation of the factor of proportionality and the phase angle lag with frequency is reduced. This variation can be further reduced by increasing the ratio of the top mass to the spring mass, but the complete elimination of this variation would require zero spring mass.

For checking the method derived, a contact accelerometer has been built and tested under simple harmonic motions generated by an MB electromagnetic vibrator. Experimental

results agree fairly well with calculated values. It was also tested under transient motions of free damped vibrations. The theoretical and experimental results agree within 10 percent.

Two new types of contact accelerometers are here proposed. The first is a balanced type, in which four identical springs, with viscous damping, are used to balance the mass effect upon each other under vertical, lateral and rotational motions, enabling the responses to be independent of the frequency of the springs. The other is a magnetic type, in which the equivalent spring load is adjusted by regulating the current. The mass effect of a mechanical spring is completely eliminated. Hence, the factor of proportionality is constant and the phase angle lag vanishes. Tests have been made on the magnetic type and the results are good. These two proposed types increase, to a very great extent, the reliability of responses to both transient and steady state motions.

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NOTATIONS

a	velocity of wave propagation in elastic medium
A	cross-section area of elastic rod
A_n	Fourier constant
$\bar{A}$	amplitude of gap, a constant
A'	a constant
B_n	Fourier constant
B'	a constant
$\bar{B}$	a constant
C_n	Fourier constant
C	dashpot damping coefficient
c	distributed viscous damping per unit length
C'	a constant
E	Young's modulus of elasticity
e	compressive strain
e_o	preset compressive strain
f	a function, f' and f" are first and second derivatives of "f" with respect to its argument
F	a constant, a function, F' and F" are first and second derivatives of "F" with respect to its argument
g	gravitational acceleration
G	a constant
i	an integral number, square root of minus one

K spring constant

L length of spring or elastic rod

L_1 a length = a/ω

m an integral number

M a constant

M_1 top mass

M_2 spring mass

n an integral number

N a constant

p angular velocity

p_n angular velocity

$q_n = \xi_n + i\eta_n$

Q amplitude of a vibrating force

r_1 a non-dimensional constant

r_2 a non-dimensional constant

r^2 a non-dimensional constant

R ratio of the preset compressive force to the product of the top mass times acceleration at the break of the contact

S force

t time

t' time measured from an arbitrary instant

u displacement

V a non-dimensional constant

x distance from the lower end of spring

α ratio of spring mass to top mass, damping coefficient of damped free vibration

β a number, phase angle

γ specific weight of elastic rod

η a constant with the dimension of one over length

λ a non-dimensional ratio

ξ a constant with the dimension of one over length

ρ density of elastic rod

Σ summation

φ a non-dimensional ratio

χ function of x only

ψ a ratio $= -\left(\frac{\partial^2 u}{\partial t^2}\right)_{x=L} / A_0$

ω angular velocity

Dots indicate differentiation with respect to time. The equations are denoted by two numbers separated by a dash. The number preceeding the dash denotes the chapter in which the equation first appears and the number following the dash denotes the number of the equation in the chapter.

INTRODUCTION

The basic equation of Newtonian mechanics states that force is proportional to mass times acceleration. Therefore an accelerometer, which is an instrument for measuring such accelerations, is important in the study of mechanics. The fundamental requirement of such an accelerometer is to produce an output or response which reliably indicates the variation of acceleration, which usually consists of either a periodic or a transient acceleration.

The most commonly used accelerometer is a spring-mass, damped system (Reference 1). This type of accelerometer consists of a case containing a mass which is suspended from the case by springs. The case of the instrument is rigidly attached to the vibrating body and moves with the body. This excites the suspended mass into motion. The relative motion between the vibrating case and the mass is related to the acceleration being measured in a definite way.

Since the differential equation for the spring-mass system is linear, the principle of superposition can be applied. All motions can be represented by a Fourier series. The requirements of an accelerometer can be summarized as follows: (1) The response must be proportional

to the acceleration of a simple harmonic motion with the factor of proportionality the same for all frequencies present in the motion, (2) the phase angle lag of any particular frequency must be proportional to that frequency so as to give constant time lag for motions of all frequencies (Reference 2), (3) the response must be damped out sufficiently fast so the response at one instant is free from the effect of the preceding motions. Usually in the design of an ordinary accelerometer a natural frequency is selected which is twice that of the highest frequency to be measured, with damping between sixty to seventy per cent of the critical damping. For impact or shock, the frequency is likely to be high and it is quite likely to have harmonic terms near to or exceeding the natural frequency of the accelerometer. It is impossible to increase the natural frequency without reducing the sensitivity.

The peak acceleration usually gives critical design conditions, consequently it is of utmost interest to the designer, and is therefore of practical importance in design. There are many types of peak accelerometers such as the crusher gage, the indenter gage, the contact-break and the contact-make gage, etc., but none of them seem to be able to give reliable responses under various types of

motions (References 3 and 4). Since the crusher or indenter type involves plastic deformation which varies with the dynamic conditions of loading, the contact type which is free os such deformation would have fewer sources of error. Comparing the contact-make and contact-break types with the same spring and mass, the former has a stationary mass with respect to the case and has a higher natural frequency. For this reason the contact-break type has been chosen for the present investigation.

The contact accelerometer consists of a mass on a spring attached to a case. The case is fastened to the body whose acceleration is being measured. The mass is pushed down by a setscrew preloading the spring. It has been commonly assumed that when the product of the mass and the acceleration equals the preset spring load, the contact between the screw and the top mass is broken. The theory of these contact accelerometers has not been worked out in detail. In practice they are very unreliable (Reference 3), so the following investigation was made to study, theoretically and experimentally, the response of a spring-mass contact accelerometer and to explore the means of improving its design so that it will give more reliable response.

CHAPTER I

RESPONSE OF A SPRING MASS ACCELEROMETER WITH DISTRIBUTED MASS OF SPRING CONSIDERED

The responses of a spring-mass accelerometer to simple harmonic and transient motions have been calculated by Draper (Reference 5), Levy (Reference 6) and others for the condition where the mass of the spring has been neglected.

In this chapter, an accelerometer, shown in Figure 1, with distributed mass of spring and unconstrained top mass is analyzed covering frequencies far beyond the natural frequency of the system. This serves as a preliminary study of the characteristics of a contact accelerometer in which only the spring mass has relative motion with respect to the case.

The wave equation is solved in Fourier's series and the end motions are used to determine the Fourier constants. The solution of the differential equation with viscous damping is shown and the case without damping can be obtained by letting the damping coefficient "C" be zero.

With the spring replaced by an elastic rod of area "A" and modulus of elasticity "E", force on any cross section of the rod, $S = EA \frac{\partial u}{\partial x}$, (Cf. page viii for symbols.)

$$dS = EA \frac{\partial^2 u}{\partial x^2} dx = \frac{\gamma A}{g} dx \frac{\partial^2 u}{\partial t^2}$$

$a^2 = Eg/\gamma$ "a" is velocity of sound in the rod.

$$a^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} \qquad 1\text{-}1$$

$u = f(x-at) + F(x+at)$, (References 7 to 9) 1-2

"f" and "F" are functions.

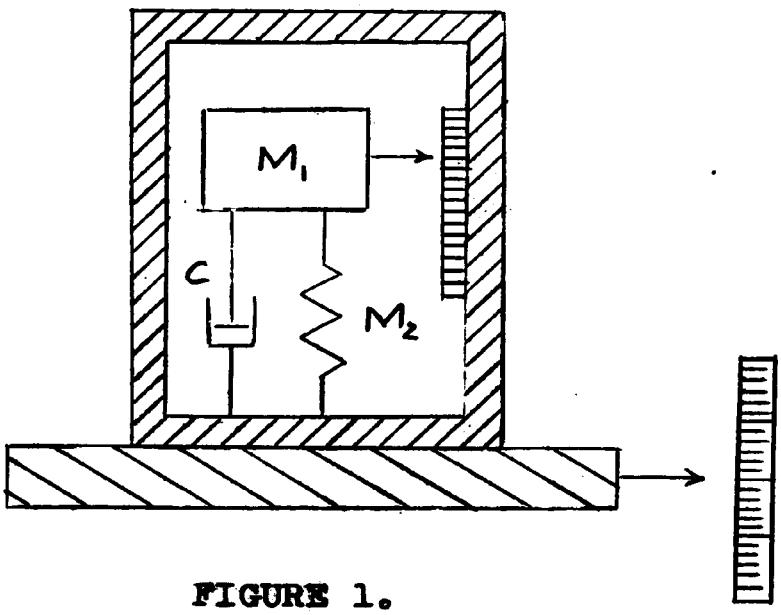


FIGURE 1.

SPRING-MASS ACCELEROMETER

End conditions: $u_{x=0}$ is the motion to be measured.

$$x = L, \qquad S = -M_1\left(\frac{\partial^2 u}{\partial t^2}\right) - C\,\frac{\partial(u_{x=L} - u_{x=0})}{\partial t} = EA\left(\frac{\partial u}{\partial x}\right)_{x=L} \qquad 1\text{-}3$$

From Equations 1-2 and 1-3,

$$u_{x=0} = f(-at) + F(at) \qquad 1\text{-}4$$

$$EA\,(f'+F')_{x=L} = -M_1 a^2 (f''+F'')_{x=L} - C\left[(-f''+F'')_{x=L} - (-f''+F'')_{x=0}\right] \qquad 1\text{-}5$$

where the single primes denote the first derivatives and double primes denote second derivatives. With the functions "f" and "F" expressed in trigonometric series,

$$u = \sum_{n=1}^{\infty}\left[A_n \sin\frac{n(x-at)}{L_1} + B_n \cos\frac{n(x-at)}{L_1} + C_n \sin\frac{n(x+at)}{L_1} + D_n \cos\frac{n(x+at)}{L_1}\right] \qquad 1\text{-}6$$

where L_1 is the length to be determined later, (References 10-11). Substituting Equation 1-6 into Equation 1-3,

$$EA\sum_{n=1}^{\infty}\left[\frac{A_n n}{L_1}\cos\frac{n(L-at)}{L_1} - \frac{B_n n}{L_1}\sin\frac{n(L-at)}{L_1} + \frac{C_n n}{L_1}\cos\frac{n(L+at)}{L_1} - \frac{D_n n}{L_1}\sin\frac{n(L+at)}{L_1}\right] = M_1\frac{a^2}{L_1^2}\sum_{n=1}^{\infty}\left[A_n n^2 \sin\frac{n(L-at)}{L_1} + B_n n^2\cos\frac{n(L-at)}{L_1} + C_n n^2 \sin\frac{n(L+at)}{L_1} + D_n n^2\cos\frac{n(L+at)}{L_1}\right]$$

$$+ \frac{Ca}{L_1}\sum_{n=1}^{\infty}\left[A_n n\left(\cos\frac{n(L-at)}{L_1} - \cos\frac{nat}{L_1}\right) - B_n n\left(\sin\frac{n(L-at)}{L_1} - \sin\frac{nat}{L_1}\right) - C_n n\left(\cos\frac{n(L+at)}{L_1} - \cos\frac{nat}{L_1}\right) + D_n n\left(\sin\frac{n(L+at)}{L_1} - \sin\frac{nat}{L_1}\right)\right] \qquad 1\text{-}7$$

$$\frac{EA}{L_1}\sum_1^\infty\Big[A_n n\Big(\cos\frac{n\ell}{L_1}\cos\frac{n\alpha t}{L_1}+\sin\frac{n\ell}{L_1}\sin\frac{n\alpha t}{L_1}\Big)$$

$$-B_n n\Big(\sin\frac{n\ell}{L_1}\cos\frac{n\alpha t}{L_1}-\cos\frac{n\ell}{L_1}\sin\frac{n\alpha t}{L_1}\Big)+C_n n\Big(\cos\frac{n\ell}{L_1}\cos\frac{n\alpha t}{L_1}-\sin\frac{n\ell}{L_1}\sin\frac{n\alpha t}{L_1}\Big)$$

$$-D_n n\Big(\sin\frac{n\ell}{L_1}\cos\frac{n\alpha t}{L_1}+\cos\frac{n\ell}{L_1}\sin\frac{n\alpha t}{L_1}\Big)\Big]$$

$$=\frac{M_1\alpha^2}{L_1^2}\sum_1^\infty\Big[A_n n^2\Big(\sin\frac{n\ell}{L_1}\cos\frac{n\alpha t}{L_1}-\cos\frac{n\ell}{L_1}\sin\frac{n\alpha t}{L_1}\Big)$$

$$+B_n n^2\Big(\cos\frac{n\ell}{L_1}\cos\frac{n\alpha t}{L_1}+\sin\frac{n\ell}{L_1}\sin\frac{n\alpha t}{L_1}\Big)$$

$$+C_n n^2\Big(\sin\frac{n\ell}{L_1}\cos\frac{n\alpha t}{L_1}+\cos\frac{n\ell}{L_1}\sin\frac{n\alpha t}{L_1}\Big)$$

$$+D_n n^2\Big(\cos\frac{n\ell}{L_1}\cos\frac{n\alpha t}{L_1}-\sin\frac{n\ell}{L_1}\sin\frac{n\alpha t}{L_1}\Big)\Big]$$

$$+\frac{C\alpha}{L_1}\sum_1^\infty\Big[A_n n\Big(\cos\frac{n\ell}{L_1}\cos\frac{n\alpha t}{L_1}+\sin\frac{n\ell}{L_1}\sin\frac{n\alpha t}{L_1}-\cos\frac{n\alpha t}{L_1}\Big)$$

$$-B_n n\Big(\sin\frac{n\ell}{L_1}\cos\frac{n\alpha t}{L_1}-\cos\frac{n\ell}{L_1}\sin\frac{n\alpha t}{L_1}+\sin\frac{n\alpha t}{L_1}\Big)$$

$$-C_n n\Big(\cos\frac{n\ell}{L_1}\cos\frac{n\alpha t}{L_1}-\sin\frac{n\ell}{L_1}\sin\frac{n\alpha t}{L_1}-\cos\frac{n\alpha t}{L_1}\Big)$$

$$+D_n n\Big(\sin\frac{n\ell}{L_1}\cos\frac{n\alpha t}{L_1}+\cos\frac{n\ell}{L_1}\sin\frac{n\alpha t}{L_1}-\sin\frac{n\alpha t}{L_1}\Big)\Big] \quad 1\text{-}8$$

Equating the coefficients of Cos (nat) and Sin (nat).gives

$$\frac{EA}{L_1}\Big(A_n n\cos\frac{n\ell}{L_1}-B_n n\sin\frac{n\ell}{L_1}+C_n n\cos\frac{n\ell}{L_1}-D_n n\sin\frac{n\ell}{L_1}\Big)$$

$$=\frac{M_1\alpha^2}{L_1^2}\Big(A_n n^2\sin\frac{n\ell}{L_1}+B_n n^2\cos\frac{n\ell}{L_1}+C_n n^2\sin\frac{n\ell}{L_1}+D_n n^2\cos\frac{n\ell}{L_1}\Big)$$

$$+\frac{C\alpha}{L_1}\Big[A_n n\Big(\cos\frac{n\ell}{L_1}-1\Big)-B_n n\sin\frac{n\ell}{L_1}-C_n n\Big(\cos\frac{n\ell}{L_1}-1\Big)+D_n n\sin\frac{n\ell}{L_1}\Big]$$

1-9

$$\frac{EA}{L_1}\left(A_n n \sin\frac{nL}{L_1} + B_n n \cos\frac{nL}{L_1} - C_n n \sin\frac{nL}{L_1} - D_n n \cos\frac{nL}{L_1}\right)$$

$$= \frac{M_1 a^2}{L_1^2}\left(-A_n n^2 \cos\frac{nL}{L_1} + B_n n^2 \sin\frac{nL}{L_1} + C_n n^2 \cos\frac{nL}{L_1} - D_n n^2 \sin\frac{nL}{L_1}\right)$$

$$+ \frac{Ca}{L_1}\left[A_n n \sin\frac{nL}{L_1} + B_n\left(\cos\frac{nL}{L_1} - 1\right) + C_n n \sin\frac{nL}{L_1} + D_n n\left(\cos\frac{nL}{L_1} - 1\right)\right]$$

1-10

For steady state simple harmonic motion, and from Equation 1-6,

$$u(t)_{x=0} = A_0 \sin \omega t, \qquad \frac{a}{L_1} = \omega$$

$$L/L_1 = \lambda, \qquad A_0 = C_1 - A_1, \qquad B_1 + D_1 = 0.$$

$$A_{n\neq 1} = B_{n\neq 1} = C_{n\neq 1} = D_{n\neq 1} = 0$$

Substituting the above in Equations 1-9 and 1-10,

$$\frac{EA}{L_1}(A_1 + C_1)\cos\lambda = \frac{M_1 a^2}{L_1^2}(A_1 + C_1)\sin\lambda$$

$$+ \frac{Ca}{L_1}\left[-A_0(\cos\lambda - 1) - (B_1 - D_1)\sin\lambda\right] \qquad 1\text{-}11$$

$$\frac{EA}{L_1}\left[-A_0 \sin\lambda + (B_1 - D_1)\cos\lambda\right] = \frac{M_1 a^2}{L_1^2}\left[A_0 \cos\lambda\right.$$

$$\left. + (B_1 - D_1)\sin\lambda\right] + \frac{Ca}{L_1}(A_1 + C_1)\sin\lambda \qquad 1\text{-}12$$

$$r_1 = \frac{M_1 a^2}{EA L_1} = \frac{M_1}{M_2}\lambda \qquad 1\text{-}13$$

$$r_2 = \frac{Ca}{EA} \qquad 1\text{-}14$$

From Equations 1-11 and 1-12,

$$(A_1+C_1) = \frac{A_0[r_2(\cos\lambda-1)(r_1\sin\lambda-\cos\lambda)+r_2(-\sin\lambda-r_1\cos\lambda)\sin\lambda]}{(r_1\sin\lambda-\cos\lambda)^2+r_2^2\sin^2\lambda}$$

$$= \frac{A_0[-r_2-r_1r_2\sin\lambda+r_2\cos\lambda]}{(r_1\sin\lambda-\cos\lambda)^2+r_2^2\sin^2\lambda} \qquad 1\text{-}15$$

$$(B_1-D_1) = \frac{A_0[-(r_1\sin\lambda-\cos\lambda)(\sin\lambda+r_1\cos\lambda)-r_2^2\sin\lambda(\cos\lambda-1)]}{(r_1\sin\lambda-\cos\lambda)^2+r_2^2\sin\lambda} \qquad 1\text{-}16$$

Let $\Delta g = u_{x=0} - u_{x=L}$

$$\Delta g = \sum_1^\infty \left[A_n\left(\sin\frac{-n\alpha t}{L_1} - \sin\frac{n(L-\alpha t)}{L_1}\right)\right.$$
$$+B_n\left(\cos\frac{-n\alpha t}{L_1} - \cos\frac{n(L-\alpha t)}{L_1}\right)+C_n\left(\sin\frac{n\alpha t}{L_1} - \sin\frac{n(L+\alpha t)}{L_1}\right)$$
$$\left.+D_n\left(\cos\frac{n\alpha t}{L_1} - \cos\frac{n(L+\alpha t)}{L_1}\right)\right] \qquad 1\text{-}17$$

For simple harmonic motion,

$$\Delta g = (C_1-A_1)\sin\frac{\alpha t}{L_1} - A_1\sin\frac{(L-\alpha t)}{L_1} - C_1\sin\frac{(L+\alpha t)}{L_1}$$
$$+(B_1+D_1)\cos\frac{\alpha t}{L_1} - B_1\cos\frac{(L-\alpha t)}{L_1} - D_1\cos\frac{(L+\alpha t)}{L_1}$$
$$= A_0(1-\cos\lambda)\sin\frac{\alpha t}{L_1} - (A_1+C_1)\sin\lambda\cos\frac{\alpha t}{L_1}$$
$$-(B_1-D_1)\sin\lambda\sin\frac{\alpha t}{L_1} \qquad 1\text{-}18$$

Let $\Delta g = \bar{A}\sin\left(\frac{\alpha t}{L_1} - \beta\right)$, where A is the amplitude and β is the phase angle.

$$\frac{A}{A_0} = \left\{\left[1-\cos\lambda + \frac{(r_1\sin\lambda-\cos\lambda)(\sin\lambda+r_1\cos\lambda)+r_2^2\sin\lambda(\cos\lambda-1)}{(r_1\sin\lambda-\cos\lambda)^2+r_2^2\sin^2\lambda}\sin\lambda\right]^2 + \left[\frac{r_2(1+r_1\sin\lambda-\cos\lambda)\sin\lambda}{(r_1\sin\lambda-\cos\lambda)^2+r_2^2\sin^2\lambda}\right]^2\right\}^{\frac{1}{2}} \qquad 1\text{-}19$$

$$\tan\beta = \frac{r_2(1+r_1\sin\lambda-\cos\lambda)\sin\lambda}{(1-\cos\lambda)\left[(r_1\sin\lambda-\cos\lambda)^2+r_2^2\sin^2\lambda\right]+(r_1\sin\lambda-\cos\lambda)(\sin\lambda+r_1\cos\lambda)\sin\lambda+r_2^2(\cos\lambda-1)\sin^2\lambda} \qquad 1\text{-}20$$

For the case with no damping, $r_2 = 0$, $\beta = 0, \pi$.

$$\frac{A}{A_0} = 1-\cos\lambda + \frac{\sin^2\lambda + r_1\cos\lambda\sin\lambda}{r_1\sin\lambda-\cos\lambda}$$

$$= 1 - \frac{1}{\cos\lambda - r_1\sin\lambda} \qquad 1\text{-}21$$

The ratio of the static force required to cause the relative displacement of the top mass with respect to the case, to the product of the top mass times the maximum acceleration, for the case with top mass equal to spring mass can be calculated as follows:

$$R = \frac{EA\Delta g/L}{M_1 A_0 \omega^2} = \frac{EA\Delta g/L}{M_2 A_0 \omega^2}$$

$$= \frac{\Delta g}{A_0}\frac{g^2}{L^2\omega^2} = \frac{\Delta g}{A_0\lambda^2} \qquad 1\text{-}22$$

This ratio "R" and the phase angle "β" for different frequencies, are shown in Table I and Figure 2.

TABLE I

CALCULATION OF RESPONSE OF A SPRING-MASS ACCELEROMETER WITH DISTRIBUTED MASS OF SPRING CONSIDERED.

(1) λ in degrees	10	14.3	30	45	60	75	90
λ in radians	.1745	.25	.524	.785	1.047	1.31	1.57
(2) Cos λ	.984	.968	.866	.707	.50	.258	0
(3) Sin λ	.1736	.247	.50	.707	.866	.966	1
(4) Tan λ	.176	.255	.577	1	1.732	3.74	∞
(5) $\text{Tan}^2\lambda$	.0311	.065	.333	1	3	14.0	∞
(6) Tanλ + 1	.350	.505	1.103	1.785	2.78	5.05	
(7) λ tanλ -1	-.97	-.936	-.698	-.215	.813	3.9	
(8) (6)x(7)	-.34	-.473	-.768	-.384	2.26	19.7	
(9) 1-Cos λ	.016	.032	.134	.293	.500	.742	1
(10) $(7)^2$	.94	.876	.486	.046	.660	15.2	
(11) (10)+r_2^2(5)	.97	.941	.819	1.046	3.66	29.2	
(12) l_2Secλ	-.017	-.033	-.154	-.41	-1.0	-2.88	$-\infty$
(13) $r_2$tan λ(12)	-.003	-.008	-.089	-.41	-1.732	-10.77	$-\infty$
(14) [(8)+(13)](3)/(11)	-.0614	-.126	-.523	-.536	.125	.296	
(15) (9)-(14)	-.0454	-.094	-.389	-.243	.625	1.038	
(16) (15)	.00206	.0088	.151	.059	.39	1.08	
(17) Secλ +(7)	.048	.097	.456	1.20	2.81	7.8	
(18) (17)r_2(4)/(11)	.0087	.0263	.321	1.15	1.33	1.00	
(19) $(18)^2$	.00008	.00069	.103	1.32	1.77	1.00	
(20) (16)+(19)	.00213	.00953	.254	1.38	2.16	2.08	
(21) $\sqrt{(20)} = \bar{A}/A_o$	.0461	.0976	.604	1.17	1.47	1.44	1.11
(22) Tan β = -(18)/(15)	.191	.28	.825	4.73	-2.13	-.914	-.655
(23) β in degrees	10.8	15.6	39.5	78	115	134	147
(24) R = (21)/λ^2	1.51	1.56	1.81	1.90	1.34	.84	.453

For the Case with No Damping

(25) $\bar{A}/A_o$ = (9)+(6)(3)/(7)	.66	5.58	3.46	1.99	1.636	
(26) R = (26)/λ^2	2.4	9.05	3.16	1.16		

Damping coefficient $C = EA/a$

Mass of spring equals top mass, $M_1 = M_2$

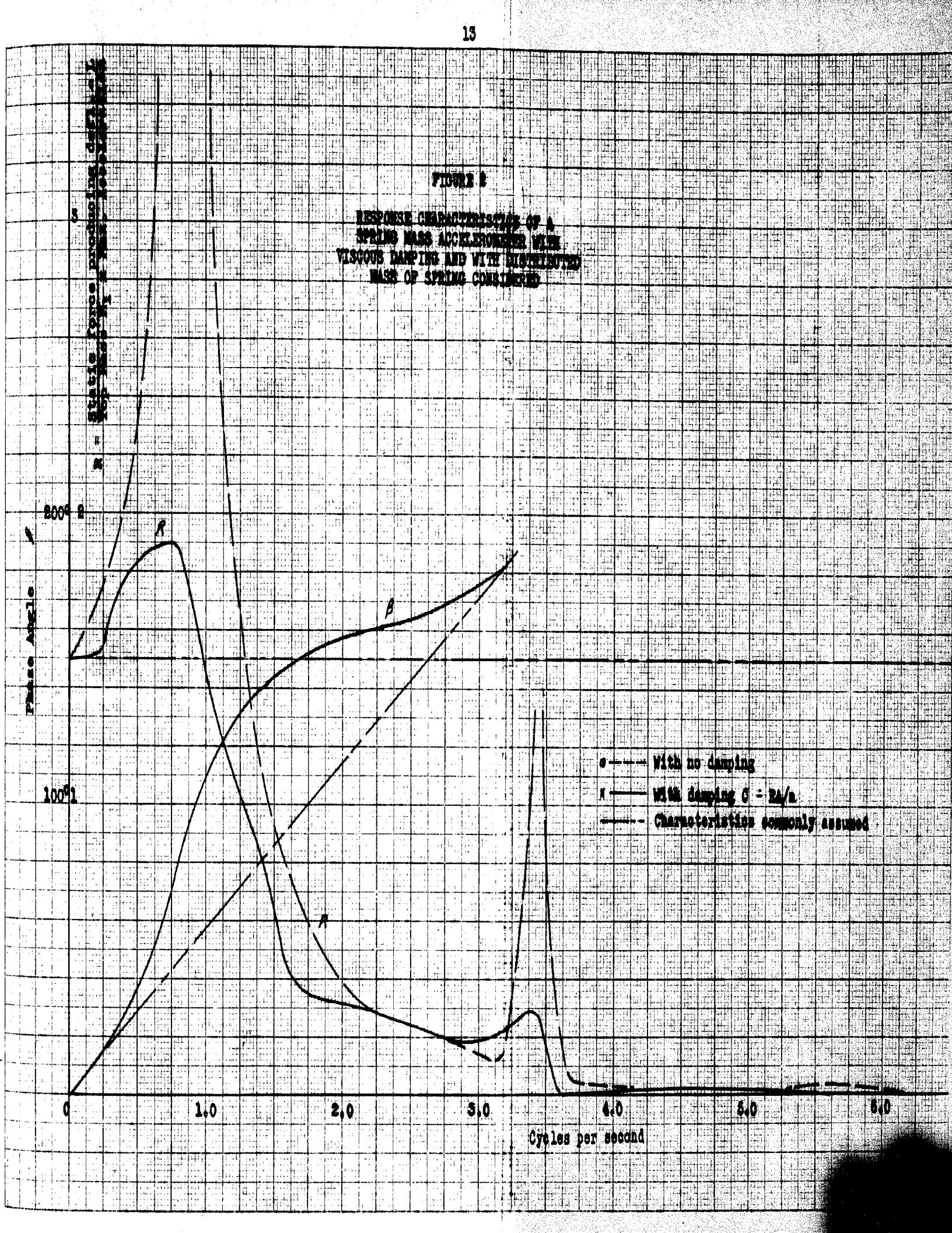
$r_1 = \lambda$ $\qquad$ $r_2 = Ca/EA = 1$

TABLE I -- Continued

120	150	180	195	200	215	270	315	360
2.094	2.62	3.14	3.405	3.6	3.75	4.72	5.50	6.28
-.50	-.866	-1	-.966	-.940	-.819	0	.707	1
.866	.500	0	-.258	-.342	-.573	-1	-.707	0
-1.732	-.577	.0	.268	.364	.700	. . .	1	0
3	.333	0	.072	.132	.490		1	0
.362	2.04	3.14	3.48	3.54	4.45		4.50	6.28
-.462	-2.51	-1	-.089	.27	1.63		-6.50	-1
-1.67	-5.10	-3.14	-.31	.956	7.66		-29.0	-6.28
1.50	1.866	2.0	1.966	1.94	1.82	1	.293	0
21.3	6.30	1	.0079	.073	2.66		42.2	1
24.3	6.63	1	.08	.205	3.15		43.2	1
3	2.15	2	2.03	2.06	2.22		-.41	0
-5.2	-1.24	0	.544	.75	1.55		.41	0
-.245	-.479	0	-.726	2.85	1.68		-.469	0
1.25	1.387	2	1.24	-.91	0.14		.762	0
1.67	1.92	4	1.54	.827	.0196		.58	0
-6.62	-3.66	-2	-.946	-.79	.41		-5.09	0
.472	.318	0	-3.17	-1.40	.091		.118	0
.223	0.10	0	10.1	1.96	.0083		.014	0
1.89	2.02	4	11.6	2.79	.028		.59	0
1.37	1.42	2	3.40	1.67	.167	.635	.77	0
-.378	-.230	0	2.55	-1.54	-.65	-.212	-.155	0
159	167	180	68.5	123	147	168	171	180
.312	.207	.202	.293	.136	.012	.0285	.0312	0
1.44	1.46	1.14	12.1	.	.	.788	.78	0
.328	.213	.116	1.04			.035	.026	0

FIGURE 2

RESPONSE CHARACTERISTICS OF A SPRING MASS ACCELEROMETER WITH VISCOUS DAMPING AND WITH DISTRIBUTED MASS OF SPRING CONSIDERED



As shown in Table I and Figure 2, the factor of proportionality varies appreciably, especially when the motion is near the natural frequency or $(2n+1)$ times the natural frequency. Besides, the phase angle "β" is not proportional to the frequency, so distortion of response will be present in complex harmonic motions. This may explain why this type of accelerometer is only adaptable to motions with frequencies less than about 70% of the natural frequency of the system.

The characteristics of the contact-make type of accelerometer will be the same as shown above, before the contact is made. Consequently, this also explains the inadequacy of the contact-make type for measuring the acceleration of high frequency or shock motion.

CHAPTER II

RESPONSE CHARACTERISTICS OF A CONTACT-BREAK ACCELEROMETER WITH NO DAMPING

In this chapter, the analysis of the response characteristics of a contact-break type accelerometer, shown in Figure 3, under a steady state and a transient motion of the type $A_0(1 - \cos\omega t)$, is shown. The response after the contact breaks, and the response with distributed viscous damping applied between the spring and the case, are shown in Chapters III and IV, and hence are not considered here.

Under Steady State Motion

Under the steady state motion with $u_{x=0} = A_0(1-\cos\omega t)$, the contact opens when

$$-M_1\left(\frac{\partial^2 u}{\partial t^2}\right) = EA\left(\frac{\partial u}{\partial x}\right)_{x=L} - EA\,e_0 . \qquad 2\text{-}1$$

From Equation 1-6,

$$u = A_0 + A_1 \sin\frac{x-at}{L_1} + B_1\cos\frac{x-at}{L_1} + C_1\sin\frac{x+at}{L_1}$$

$$+ D_1\cos\frac{x+at}{L_1} = \left[(C_1-A_1)\cos\frac{x}{L_1} + (B_1-D_1)\sin\frac{x}{L_1}\right]\sin\frac{at}{L_1}$$

$$+ \left[(A_1+C_1)\sin\frac{x}{L_1} + (B_1+D_1)\cos\frac{x}{L_1}\right]\cos\frac{at}{L_1} + A_0 \qquad 2\text{-}2$$

With $(C_1-A_1) = 0, \qquad B_1+D_1 = -A_0, \qquad \frac{a}{L_1} = \omega$

$$u = A_0 + (B_1-D_1)\sin\frac{x}{L_1}\sin\frac{at}{L_1} + \left[(A_1+C_1)\sin\frac{x}{L_1} - A_0\cos\frac{x}{L_1}\right]\cos\frac{at}{L_1} \qquad 2\text{-}3$$

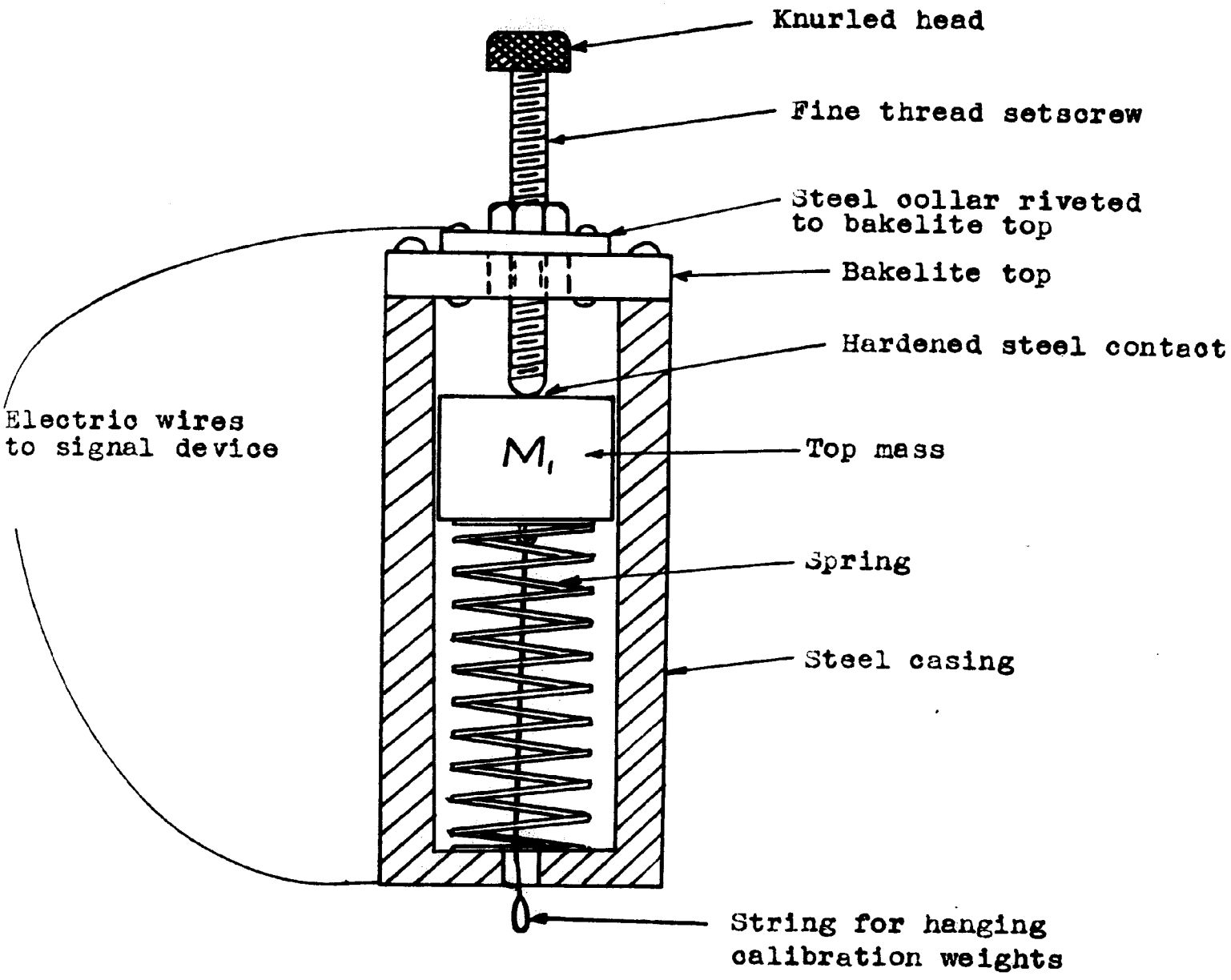


FIGURE 3

A CONTACT-BREAK ACCELEROMETER

Before the contact breaks, $u_{x=L} = u_{x=0}$. Therefore

$$u_{x=L} = A_0 + (B_1 - D_1)\sin\lambda \sin\frac{at}{L_1} + [(A_1 + C_1)\sin\lambda - A_0\cos\lambda]\cos\frac{at}{L_1}$$

$$= A_0(1 - \cos\omega t) \qquad 2\text{-}4$$

$$(B_1 - D_1) = 0, \qquad (A_1 + C_1)\sin\lambda - A_0\cos\lambda = -A_0$$

$$A_1 + C_1 = \frac{A_0(-1+\cos\lambda)}{\sin\lambda} \qquad 2\text{-}5$$

$$u = A_0\left[1 - \left(\frac{1-\cos\lambda}{\sin\lambda}\sin\frac{x}{L_1} + \cos\frac{x}{L_1}\right)\cos\frac{at}{L_1}\right] \qquad 2\text{-}6$$

$$\dot{u} = A_0\frac{a}{L_1}\left[\frac{1-\cos\lambda}{\sin\lambda}\sin\frac{x}{L_1} + \cos\frac{x}{L_1}\right]\sin\frac{at}{L_1} \qquad 2\text{-}7$$

$$\ddot{u} = A_0\frac{a^2}{L_1^2}\left[\frac{1-\cos\lambda}{\sin\lambda}\sin\frac{x}{L_1} + \cos\frac{x}{L_1}\right]\cos\frac{at}{L_1} \qquad 2\text{-}8$$

$$\frac{\partial u}{\partial x} = \frac{A_0}{L_1}\left(-\frac{1-\cos\lambda}{\sin\lambda}\cos\frac{x}{L_1} + \sin\frac{x}{L_1}\right)\cos\frac{at}{L_1} \qquad 2\text{-}9$$

Substituting the above in Equation 2-1,

$$EA\left[\epsilon_0 - \frac{A_0}{L_1}\left(\frac{-(1-\cos\lambda)\cos\lambda}{\sin\lambda} + \sin\lambda\right)\cos\frac{at}{L_1}\right]$$

$$= M_1 A_0\frac{a^2}{L_1^2}\left(\frac{1-\cos\lambda}{\sin\lambda}\sin\lambda + \cos\lambda\right)\cos\frac{at}{L_1}$$

$$EA\epsilon_0 = \left[A_0\frac{EA}{L_1}\frac{1-\cos\lambda}{\sin\lambda} + M_1 A_0\frac{a^2}{L_1^2}\right]\cos\frac{at}{L_1} \qquad 2\text{-}10$$

At $t = 0$, $\cos\frac{at}{L_1} = 1$. The contact opens if

$$EA\epsilon_0 \geq A_0\left(\frac{EA}{L_1}\frac{1-\cos\lambda}{\sin\lambda} + M_1\frac{a^2}{L_1^2}\right) \qquad 2\text{-}11$$

$$R = \frac{EAe_0}{M_1 A_0 \omega^2} \leq \frac{\frac{EA}{L_1}\frac{1-\cos\lambda}{\sin\lambda} + M_1\frac{g^2}{L_1^2}}{\frac{M_1}{M_2}\frac{\gamma}{g}LA\frac{g^2}{L_1^2}}$$

$$\leq \frac{M_2}{M_1}\left(\frac{1-\cos\lambda}{\lambda\sin\lambda}\right)+1 = \frac{M_2}{M_1}\frac{1}{\lambda}\tan\frac{\lambda}{2}+1 \qquad 2\text{-}12$$

The above ratio of the preset force to the product of the top mass times the imposed acceleration, shows the acceleration required to open the contact. The greater the ratio of masses M_1/M_2, the less will be the spring effect.

Under Transient Motion

Under the transient motion shown in Figure 4, $u_{x=0} = A_0 (1 - \cos\omega t')$ for $t' < 0$, and $u_{x=0} = 0$ for $t' > 0$. For $t' < 0$, Equation 2-6 is valid. For $t' > 0$, the equation of motion is derived as follows:

At $t' = 0$, $\cos\omega t' = 1$.

$$u = A_0\left[1 - \frac{1-\cos\lambda}{\sin\lambda}\sin\frac{x}{L_1} + \cos\frac{x}{L_1}\right] \qquad 2\text{-}13$$

$$\dot{u} = 0, \qquad u_{x=0} = 0. \qquad 2\text{-}14$$

The series satisfying the simple wave differential equation and Equation 2-14, is given as

$$\sum_{n=1}^{\infty} A_n \sin\frac{\omega_n x}{a}\cos\omega_n t'$$

$$u_{x=L} = 0, \qquad \frac{\omega_n L}{a} = n\pi, \qquad \omega_n = \frac{n\pi a}{L}$$

$$u = \sum_{n=1}^{\infty} A_n \sin\frac{n\pi x}{L} \cos\frac{n\pi a t}{L} . \qquad \text{2-15}$$

In order to satisfy Equation 2-13 at t' = 0,

$$\sum_{n=1}^{\infty} A_n \sin\frac{n\pi x}{L} = A_0\left[1 - \left(\frac{1-\cos\lambda}{\sin\lambda}\sin\frac{x}{L_1} + \cos\frac{x}{L_1}\right)\right]. \qquad \text{2-16}$$

Let $A_0\left[\frac{1-\cos\lambda}{\sin\lambda}\sin\frac{x}{L_1} + \cos\frac{x}{L_1}\right] = M\cos\left(\frac{x}{L_1}+\beta\right)$

$$= M\left(\cos\frac{x}{L_1}\cos\beta - \sin\frac{x}{L_1}\sin\beta\right)$$

$$A_0 - M\cos\left(\frac{x}{L_1}+\beta\right) = \sum_{n=1}^{\infty} A_n \sin\frac{n\pi x}{L}$$

Multiplying the above equation by (Sin $m\pi x/L$) dx, and integrating from x = 0 to x = L,

$$\int_0^L A_0 \sin\frac{m\pi x}{L}\,dx = A_0\frac{L}{m\pi}\left[1-(-1)^m\right]$$

$$\int_0^L M\cos\left(\frac{x}{L_1}+\beta\right)\sin\frac{m\pi x}{L}\,dx$$

$$= \frac{M}{2}\int_0^L\left[\sin\left(\frac{m\pi x}{L}+\frac{x}{L_1}+\beta\right)+\sin\left(\frac{m\pi x}{L}-\frac{x}{L_1}-\beta\right)\right]dx$$

$$= \frac{ML}{2}\left[-\frac{\cos(m\pi+\lambda+\beta)+\cos\beta}{m\pi+\lambda} + \frac{-\cos(m\pi-\lambda-\beta)+\cos\beta}{m\pi-\lambda}\right]$$

$$= \frac{ML}{2}\left[-\cos(\lambda+\beta)\left(\frac{1}{m\pi+\lambda}+\frac{1}{m\pi-\lambda}\right)(-1)^m + \cos\beta\left(\frac{1}{m\pi+\lambda}+\frac{1}{m\pi-\lambda}\right)\right]$$

$$= \frac{ML\,m\pi}{(m\pi)^2-\lambda^2}\left[(-1)^{m+1}\cos(\lambda+\beta)+\cos\beta\right]$$

$$\int_0^L \sum_{n=1}^{\infty} A_n \sin\frac{n\pi x}{L}\sin\frac{m\pi x}{L}\,dx = A_m\frac{L}{2}$$

$$A_m\frac{L}{2} = A_0\frac{L}{m\pi}\left[1-(-1)^m\right] - \frac{ML\,m\pi}{(m\pi)^2-\lambda^2}\left[(-1)^{m-1}\cos(\lambda+\beta)+\cos\beta\right].$$

Expanding Cos $(\lambda+\beta)$ and substituting the expressions for M Cos β and M Sin β, into the above equation,

$$A_n = \frac{2A_0}{n\pi}\left[1-(-1)^n\right] - \frac{2n\pi}{(n\pi)^2-\lambda^2}\left[(-1)^{n-1}(M\cos\lambda\cos\beta - M\sin\lambda\sin\beta) + M\cos\beta\right]$$

$$= \frac{2A_0}{n\pi}\left[1-(-1)^n\right] - \frac{2A_0\,n\pi}{(n\pi)^2-\lambda^2}\left[(-1)^{n+1}+1\right] \qquad 2\text{-}17$$

$$u = \sum_{n=1}^{\infty} A_n \sin\frac{n\pi x}{L}\cos\frac{n\pi a t'}{L}$$

$$= \sum_{n=1}^{\infty} 2A_0\left[\frac{1-(-1)^n}{n\pi} + \frac{\left[(-1)^n-1\right]n\pi}{(n\pi)^2-\lambda^2}\right]\sin\frac{n\pi x}{L}\cos\frac{n\pi a t'}{L}$$

$$u = 2A_0 \sum_{n=1}^{\infty} [1-(-1)^n] \frac{-\lambda^2}{n\pi(n^2\pi^2-\lambda^2)} \sin\frac{n\pi x}{L} \cos\frac{n\pi a t'}{L} \qquad 2\text{-}18$$

$$\frac{\partial u}{\partial x} = -\frac{2A_0\lambda^2}{L} \sum_{n=1}^{\infty} \frac{1-(-1)^n}{(n\pi)^2-\lambda^2} \cos\frac{n\pi x}{L} \cos\frac{n\pi a t'}{L} \qquad 2\text{-}19$$

$$\left(\frac{\partial u}{\partial x}\right)_{x=L} = \frac{-2A_0\lambda^2}{L} \sum_{n=1}^{\infty} \frac{[1-(-1)^n](-1)^n}{n^2\pi^2-\lambda^2} \cos\frac{n\pi a t'}{L} \qquad 2\text{-}20$$

$$\frac{\partial^2 u}{\partial t^2} = \frac{2A_0\lambda^2 a^2}{L^2} \sum_{n=1}^{\infty} \frac{n\pi[1-(-1)^n]}{(n\pi)^2-\lambda^2} \sin\frac{n\pi x}{L} \cos\frac{n\pi a t'}{L} \qquad 2\text{-}21$$

$x = 0 \quad L, \quad \frac{\partial^2 u}{\partial t^2} = 0.$

Substituting the above into Equation 2-1,

$$e_0 = \frac{2A_0\lambda^2}{L} \sum_{n=1}^{\infty} \frac{[1-(-1)^n](-1)^{n+1}}{n^2\pi^2-\lambda^2} \cos\frac{n\pi a t'}{L} \qquad 2\text{-}22$$

With t' = 0, cos ($n\pi$ at'/L) = 1. When "n" is even, the right hand term vanishes. Retaining only the odd terms, Equation 2-22 becomes

$$e_0 = \frac{4A_0\lambda^2}{L\pi^2} \sum_{1,3,5\cdots} \frac{1}{n^2-\left(\frac{\lambda}{\pi}\right)^2} \cos\frac{n\pi a t'}{L}$$

$$\lambda^2 = \frac{L^2\omega^2}{a^2} = \frac{\omega^2 L M_2}{EA},$$

$$\frac{n\pi a t'}{L} = \frac{n\pi a L_1 t'}{L_1 L} = \frac{n\pi}{\lambda}\omega t'$$

$$R = \frac{EAe_0}{M_1\omega^2 A_0} = \frac{M_2}{M_1}\frac{4}{\pi^2} \sum_{1,3,5\cdots} \frac{1}{n^2-\left(\frac{\lambda}{\pi}\right)^2} \cos\frac{\pi}{\lambda} n\omega t' \qquad 2\text{-}23$$

This ratio "R" is calculated as shown in Table III and plotted in Figure 4. The shaded area in the figure shows the region where the contact does not open under steady state motion, but does open under the motion $u_{x=0} = A_o (1 - \cos\omega t)$ for $t < 0$, and $u_{x=0} = 0$ for $t > 0$.

For a single wave transient of the type $A_o (1 - \cos\omega t)$ for $0 < t < 2\pi/\omega$ and with zero displacement outside of this interval, the response can be obtained by superposing the response of the motion $u_2 = A_o (1 - \cos\omega t)$ for $t < 2\pi/\omega$, and $u_2 = 0$ for $t > 2\pi/\omega$, on the response of the motion $u_1 = -A_o (1 - \cos\omega t)$ for $t < 0$, and equals zero for $t > 0$. Hence

$$u_{1x=0} = -A_o(1-\cos\omega t), \qquad t < 0$$

$$u_{1x=0} = 0 \qquad t > 0$$

$$u_{2x=0} = A_o(1-\cos\omega t) \qquad t < \frac{2\pi}{\omega}$$

$$u_{2x=0} = 0 \qquad t > \frac{2\pi}{\omega}$$

$$u = u_1 + u_2$$

$$\left(\frac{\partial^2 u_2}{\partial t^2}\right)_{x=L} = A_o\omega^2\cos\omega t, \qquad t < \frac{2\pi}{\omega}$$

From Equation 2-9,

$$\left(\frac{\partial u_2}{\partial x}\right)_{x=L} = \frac{A_o}{L_1}\tan\frac{\lambda}{2}\cos\omega t, \qquad t < \frac{2\pi}{\omega}$$

$$\left(\frac{\partial^2 u_2}{\partial t^2}\right)_{x=L} = 0, \qquad t > \frac{2\pi}{\omega}$$

From Equation 2-21,

$$\left(\frac{\partial u_2}{\partial x}\right)_{x=L} = \frac{4A_0\lambda^2}{L} \sum_{1,3,5\ldots} \frac{1}{(n\pi)^2 - \lambda^2} \cos\frac{n\pi a\left(t-\frac{2\pi}{\omega}\right)}{L}, \qquad t > \frac{2\pi}{\omega}$$

$$\left(\frac{\partial^2 u_1}{\partial t^2}\right)_{x=L} = -A_0\omega^2 \cos\omega t, \qquad t < 0$$

$$\left(\frac{\partial u_1}{\partial x}\right)_{x=L} = -\frac{A_0}{L_1}\tan\frac{\lambda}{2}\cos\omega t, \qquad t < 0$$

$$\left(\frac{\partial^2 u_1}{\partial t^2}\right)_{x=L} = 0, \qquad t > 0$$

$$\left(\frac{\partial u_1}{\partial x}\right)_{x=L} = -\frac{4A_0\lambda^2}{L} \sum_{1,3,5\ldots} \frac{1}{n^2\pi^2 - \lambda^2} \cos\frac{n\pi a t}{L}, \qquad t > 0$$

$t < 0, \quad u = u_1 + u_2 = 0, \quad \frac{\partial^2 u}{\partial t^2} = 0, \quad \frac{\partial u}{\partial x} = 0$

$0 < t < \frac{2\pi}{\omega}$,

$$\left(\frac{\partial u}{\partial x}\right)_{x=L} = \frac{A_0}{L}\tan\frac{\lambda}{2}\cos\omega t - \frac{4A_0\lambda^2}{L} \sum_{1,3,5\ldots} \frac{1}{n^2\pi^2 - \lambda^2} \cos\frac{n\pi a t}{L}$$

$$\left(\frac{\partial^2 u}{\partial t^2}\right)_{x=L} = A_0\omega^2\cos\omega t$$

$t > \frac{2\pi}{\omega}, \qquad \left(\frac{\partial^2 u}{\partial t^2}\right)_{x=L} = 0$

$$\left(\frac{\partial u}{\partial x}\right)_{x=L} = \frac{4A_0\lambda^2}{L}\sum_{1,3,5..}\frac{1}{n^2\pi^2-\lambda^2}\left[\cos\frac{n\pi a(t-\frac{2\pi}{\omega})}{L} - \cos\frac{n\pi a t}{L}\right]$$

From Equation 2-1, $0 < t < \frac{2\pi}{\omega}$,

$$-M_1 A_0 \omega^2 \cos\omega t = EA\frac{A_0}{L}\left[\tan\frac{\lambda}{2}\cos\omega t - 4\lambda^2\sum_{1,3,5..}\frac{1}{n^2\pi^2-\lambda^2}\cos\frac{n\pi a t}{L}\right] - EAe_0$$

2-24

$$\frac{EAe_0}{M_1A_0\omega^2} = \frac{EA}{M_1\omega^2 L}\left[\tan\frac{\lambda}{2}\cos\omega t - 4\lambda^2\sum_{1,3,5..}\frac{1}{n^2\pi^2-\lambda^2}\cos\frac{n\pi a t}{L}\right] + \cos\omega t$$

$$= \left(\frac{EA}{M_1\omega^2 L_1}\tan\frac{\lambda}{2} + 1\right)\cos\omega t - \frac{4EA\lambda^2}{M_1\omega^2 L_1}\sum_{1,3,5..}\frac{1}{n^2\pi^2-\lambda^2}\cos\frac{n\pi a t}{L}$$

2-25

$$\frac{EA}{M_1\omega^2 L_1} = \frac{EAM_2}{M_1 L_1\omega^2\rho A L} = \frac{M_2}{M_1}\frac{a^2}{\omega^2 L_1 L} = \frac{M_2}{M_1}\frac{L_1}{L} = \frac{M_2}{M_1}\frac{1}{\lambda}$$

The contact opens when

$$\frac{EAe_0}{M_1A_0\omega^2} \leq \left(\frac{M_2}{M_1}\frac{1}{\lambda}\tan\frac{\lambda}{2} + 1\right)\cos\omega t - \frac{M_2}{M_1}\frac{4\lambda}{\pi^2}\sum_{1,3,5..}\frac{1}{n^2-(\frac{\lambda}{\pi})^2}\cos\frac{n\pi a t}{L}$$

2-26

For $t > \frac{2\pi}{\omega}$, from Equation 2-24,

$$0 = EA\frac{4A_0\lambda^2}{L}\sum_{1,3,5..}\frac{1}{(n\pi)^2-\lambda^2}\left[\cos\frac{n\pi a(t-\frac{2\pi}{\omega})}{L} - \cos\frac{n\pi a t}{L}\right] - EAe_0$$

$$\frac{EAe_0}{M_1A_0\omega^2} \leq \frac{M_2}{M_1}\frac{4\lambda}{\pi^2}\sum_{1,3,5...}\frac{1}{n^2-(\frac{\lambda}{\pi})^2}\left[\cos\frac{n\pi a(t-\frac{2\pi}{\omega})}{L} - \cos\frac{n\pi a t}{L}\right]$$

$$\frac{EAe_0}{M_1 A_0 \omega^2} = \frac{M_2}{M_1}\frac{4\lambda}{\pi^2}\sum_{1,3,5\ldots}\frac{1}{n^2-\left(\frac{\lambda}{\pi}\right)^2}\left[\left(\cos\frac{2n\pi^2}{\lambda}-1\right)\cos\frac{n\pi at}{L}\right.$$

$$\left.+\sin\frac{2n\pi^2}{\lambda}\sin\frac{n\pi at}{L}\right] \qquad 2\text{-}27$$

From this derivation, the responses of both the steady state and the transient motions of the type A_0 (1 - cosωt) can be calculated.

TABLE II

CALCULATION OF BREAK OF CONTACT OF CONTACT ACCELEROMETER UNDER SIMPLE HARMONIC MOTIONS

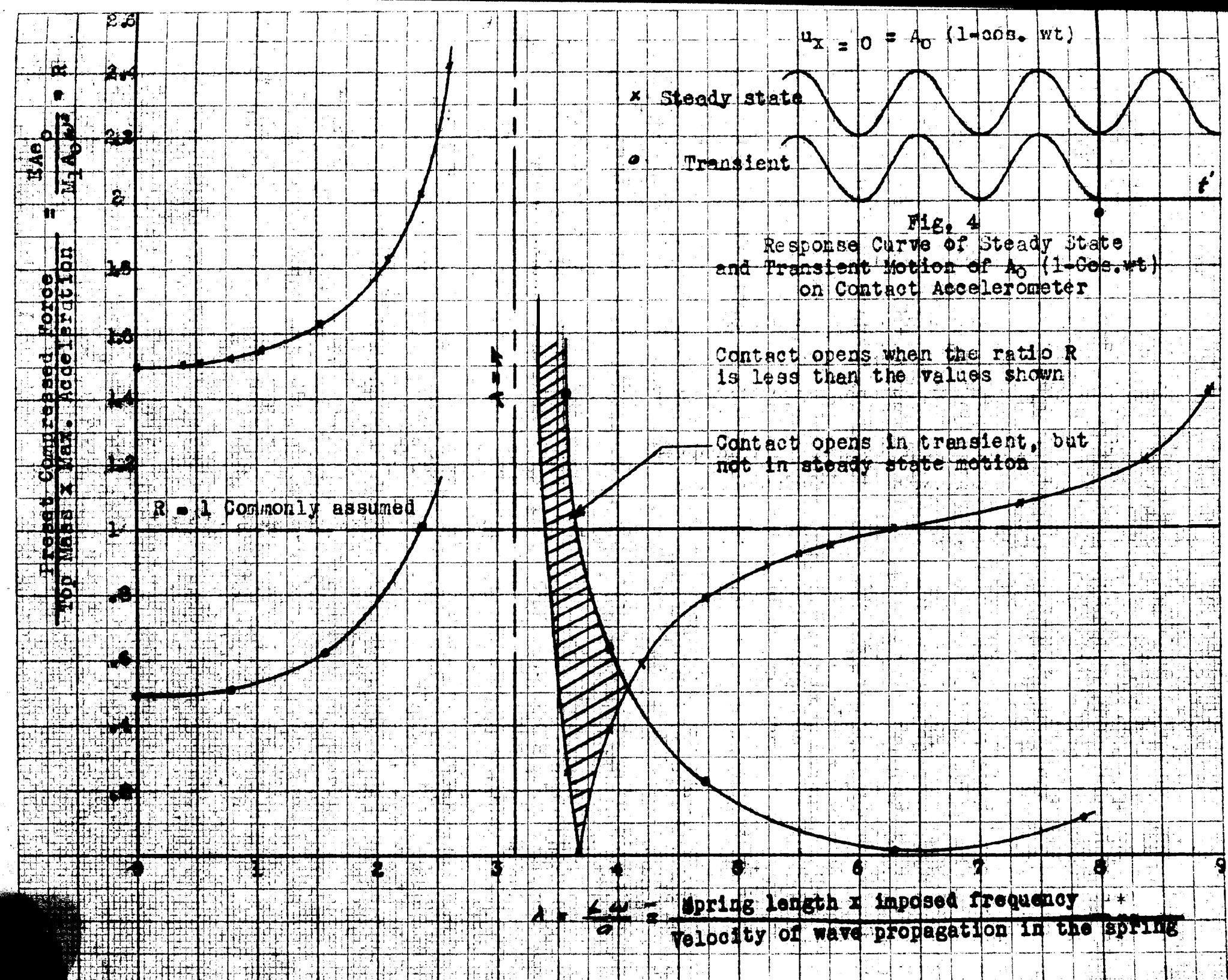
λ		$\tan\frac{\lambda}{2}$	$\frac{1}{\lambda}\tan\frac{\lambda}{2}$	$\frac{M_1}{M_2}\frac{1}{\lambda}\tan\frac{\lambda}{2}+1$
in degrees	in radians			
0	0	0	.5	1.5
22.5	.393	.199	.505	1.505
30	.524	.268	.511	1.511
45	.786	.414	.526	1.526
60	1.047	.577	.552	1.552
90	1.57	1	.637	1.637
120	2.094	1.732	.828	1.828
135	2.36	2.42	1.026	2.026
150	2.62	3.73	1.42	2.42
180	3.14	∞	∞	∞
205	3.58	-4.52	-1.26	-.26
210	3.67	-3.73	-1.02	-.02
225	3.93	-2.42	- .616	.384
240	4.19	-1.732	- .413	.587
270	4.72	-1	- .212	.788
300	5.24	- .577	- .110	.890
315	5.50	- .414	- .0753	.925
330	5.76	- .268	- .0465	.953
360	6.28	0	0	1
420	7.33	.577	.079	1.079
480	8.38	1.732	.206	1.206
510	8.91	3.74	.420	1.420
540	9.44	∞	∞	∞

$\frac{M_1}{M_2} = 1$

TABLE III

CALCULATION OF CONTACT ACCELEROMETER RESPONSE UNDER TRANSIENT MOTIONS OF A_o (1-COS ω t)

$-\frac{1}{n^2-(\frac{\Delta}{\pi})^2}\cos\frac{n\pi\omega t}{\lambda}$							$R=-\frac{4}{\pi}\Sigma\frac{1}{n^2(\frac{\Delta}{\pi})^2}\cos\frac{n\pi\omega t}{\lambda}$
n=1	n=3	n=5	n=7	n=9	n=11	n=13	
$\lambda = 0$, $n\pi\omega t/\lambda = n\pi$							
1	.11	.04	.02	.012	.008	.006	.486
$\lambda = \pi/8$, $n\pi\omega t/\lambda = n\pi$							
1.02	1.1	.04	.02	.01	.008	.006	.49
$\lambda = \pi/4$, $n\pi\omega t/\lambda = n\pi$							
1.07	.112	.04	.02	.01	.008	.006	.508
$\lambda = \pi/2$, $n\pi\omega t/\lambda = n\pi$							
1.33	.114	.04	.02	.01	.008	.006	.622
$\lambda = 3/4\,\pi$, $n\pi\omega t/\lambda = n\pi$							
2.29	.118	.04	.02	.01	.008	.006	1.01
$\lambda = 9/8\,\pi$, $n\pi\omega t/\lambda = 0$							
3.70	-.13	-.042	-.02	-.01	-.008	-.006	1.41
$\lambda = 5/4\,\pi$, $n\pi\omega t/\lambda = 0$							
1.738	-.13	-.043	-.02	-.017	-.008	-.006	.630
$\lambda = 3/2\,\pi$, $n\pi\omega t/\lambda = 7/6\,n\pi$							
.80	-.148	-.044	-.02	-.013	-.008	-.006	.298
$\lambda = 2\pi$, $n\pi\omega t/\lambda = n\pi$							
.333	-.20	-.052	-.022	-.013	-.008	-.006	.162


$u_x = 0 = A_0 (1-\cos. \omega t)$
x Steady state
o Transient
t'
Fig. 4
Response Curve of Steady State and Transient Motion of $A_0 (1-\cos.\omega t)$ on Contact Accelerometer
Contact opens when the ratio R is less than the values shown
Contact opens in transient, but not in steady state motion
R = 1 Commonly assumed
Thrust Compressed Force / Top Mass x Max. Acceleration = $\frac{kA_0}{mA_0\omega^2} = R$
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
.8
.6
.4
.2
0
1
2
3
4
5
6
7
8
9
$\lambda = \frac{2\omega}{a} = \frac{\text{Spring length x imposed frequency}}{\text{Velocity of wave propagation in the spring}}$

CHAPTER III

RESPONSE CHARACTERISTICS OF A CONTACT-BREAK ACCELEROMETER AFTER CONTACT BREAKS

For the contact-break accelerometer, the break of contact is transmitted to a signal device. The amount of gap and the length of the time interval of the break required to operate the signal, depend on the type of signal. With an electronic bulb indicator and a contact of a hardened steel setscrew on an inserted tungsten surface, it has been found that a gap of the order of 10^{-5} inches is sufficient to operate the signal (Reference 12). The response characteristics after the contact breaks are important in the operation of signals, especially for high frequency motions. In this chapter, the response after the contact breaks and the maximum gap obtainable in a given motion are derived.

Let the motion be $u_{x=0} = A_0(1-\cos\omega t)$. Before the contact breaks, the displacement at any point in the rod is given by Equation 2-6 and is denoted by u_1 here. Therefore

$$u_1 = A_0\left[1-\left(\frac{1-\cos\lambda}{\sin\lambda}\sin\frac{x}{L_1}+\cos\frac{x}{L_1}\right)\cos\omega t\right]. \qquad 3\text{-}1$$

Assume that the contact breaks at "t" = "t_1". Hence at that instant, the force between the top mass and the setscrew is zero. From equation 2-10,

$$EAe_o = A_o\left[\frac{EA}{L_1}\frac{1-\cos\lambda}{\sin\lambda} + M_1\frac{a^2}{L_1^2}\right]\cos\frac{at_1}{L_1} \qquad 3\text{-}2$$

$$u_1 = A_o\left[1-\left(\frac{1-\cos\lambda}{\sin\lambda}\sin\frac{x}{L_1} + \cos\frac{x}{L_1}\right)\cos\frac{at_1}{L_1}\right] \qquad 3\text{-}3$$

$$\dot{u}_1 = A_o\left[\frac{1-\cos\lambda}{\sin\lambda}\sin\frac{x}{L_1} + \cos\frac{x}{L_1}\right]\frac{a}{L_1}\sin\frac{at_1}{L_1} \qquad 3\text{-}4$$

After the contact breaks, the simple wave differential equation remains the same. This wave equation consists only of terms of second derivatives of "u" with respect to "t" and "x", so with the addition of terms of the first power of "x" and "t", the solution still satisfies the differential equation. Adding $e_o x$ to Equation 2-3,

$$u = A_o + (B_1-D_1)\sin\frac{x}{L_1}\sin\frac{at}{L_1} + \left[(A_1+C_1)\sin\frac{x}{L_1} - A_0\cos\frac{x}{L_1}\right]\cos\frac{at}{L_1} + e_o x \qquad 3\text{-}5$$

This equation satisfies the lower end condition. Differenting gives

$$\frac{\partial u}{\partial x} = \frac{(B_1-D_1)}{L_1}\cos\frac{x}{L_1}\sin\frac{at}{L_1} + \left(\frac{A_1+C_1}{L_1}\cos\frac{x}{L_1} + \frac{A_0}{L_1}\sin\frac{x}{L_1}\right)\cos\frac{at}{L_1} + e_o \qquad 3\text{-}6$$

$$\left(\frac{\partial u}{\partial x}\right)_{x=L} = \frac{(B_1-D_1)}{L_1}\cos\lambda\sin\frac{at}{L_1} + \left(\frac{A_1+C_1}{L_1}\cos\frac{x}{L_1} + \frac{A_0}{L_1}\sin\lambda\right)\cos\frac{at}{L_1} + e_o \qquad 3\text{-}7$$

$$\frac{\partial^2 u}{\partial t^2} = -\frac{a^2}{L_1^2}(B_1 - D_1)\sin\frac{x}{L_1}\sin\frac{at}{L_1} - \frac{a^2}{L_1^2}\left[(A_1 + C_1)\sin\frac{x}{L_1} - A_0\cos\frac{x}{L_1}\right]\cos\frac{at}{L_1}$$

3-8

After the contact breaks, from the equilibrium condition at x = L,

$$-M_1\frac{a^2}{L_1^2}\left\{(B_1 - D_1)\sin\lambda\sin\frac{at}{L_1} + \left[(A_1 + C_1)\sin\lambda - A_0\cos\lambda\right]\cos\frac{at}{L_1}\right\}$$

$$= -\frac{EA}{L_1}\left\{(B_1 - D_1)\cos\lambda\sin\frac{at}{L_1} + \left[(A_1 + C_1)\cos\lambda + A_0\sin\lambda\right]\cos\frac{at}{L_1}\right.$$

$$-EAc_0 + EAc_0 \qquad 3\text{-}9$$

Equating the coefficients of $\sin\frac{at}{L_1}$ and $\cos\frac{at}{L_1}$,

$$M_1\frac{a^2}{L_1^2}(B_1 - D_1)\sin\lambda = \frac{EA}{L_1}(B_1 - D_1)\cos\lambda$$

$$B_1 - D_1 = 0 \qquad 3\text{-}10$$

$$(A_1 + C_1)\left[\frac{EA}{L_1}\cos\lambda - M_1\frac{a^2}{L_1^2}\sin\lambda\right] = -A_0\left[\frac{EA}{L_1}\sin\lambda + M_1\frac{a^2}{L_1^2}\cos\lambda\right]$$

$$r^2 = \frac{M_1 a^2}{EA L_1} \qquad 3\text{-}11$$

$$(A_1 + C_1) = \frac{-A_0(r^2 + \tan\lambda)}{1 - r^2\tan\lambda} \qquad 3\text{-}12$$

The displacement "u" of the rod after contact breaks, is represented by two parts: one is obtained directly from Equation 3-5 and is denoted by "u_2"; the other, from the natural vibration of the rod is denoted by "u_3". Hence

$$u_2 = A_0\left[1-\left(\frac{r^2+\tan\lambda}{1-r^2\tan\lambda}\sin\frac{x}{L_1}+\cos\frac{x}{L_1}\right)\cos\frac{at}{L_1}\right]+e_0 x. \qquad 3\text{-}13$$

The end conditions of the natural vibration of a prismatic bar with mass at the top are $u_{3_{x=0}}$, and $AE\left(\frac{\partial u_3}{\partial x}\right)_{x=L} = -M_1\left(\frac{\partial^2 u_3}{\partial t^2}\right)_{x=L}$

$$u_3 = X(A'\cos pt + B'\sin pt), \qquad \text{(Reference 7).} \qquad 3\text{-}14$$

$$X = C'\cos\frac{px}{a} + D'\sin\frac{px}{a} \qquad 3\text{-}15$$

where A', B', C' and D' are constants. The end condition

$$u_{3_{x=0}} = 0 \qquad \text{gives} \qquad C' = 0 \qquad 3\text{-}16$$

In order to satisfy the other end condition

$$AE\left(\frac{\partial u_3}{\partial x}\right)_{x=L} = -M_1\left(\frac{\partial^2 u_3}{\partial t^2}\right)_{x=L},$$

$$AE\frac{p}{a}\cos\frac{pL}{a} = M_1 p^2\sin\frac{pL}{a}$$

Let $\beta = \frac{pL}{a}$, $\qquad p = \frac{\beta a}{L}$.

$$\tan\beta = \frac{AE}{M_1 p a} \qquad 3\text{-}17$$

$$\beta\tan\beta = \frac{AE\rho L}{M_1 p a^2} = \frac{M_2}{M_1} = \alpha \qquad 3\text{-}18$$

where α denotes the ratio of the spring mass to the top mass.

$$u_3 = \sum_{i=1}^{i=\infty} \sin\frac{\beta_i x}{L}\left(A_i' \cos\frac{\beta_i a t}{L} + B_i' \sin\frac{\beta_i a t}{L}\right) \qquad 3\text{-}19$$

After the contact breaks, the displacement is equal to

$$u = u_2 + u_3 = A_0\left[1 - \left(\frac{r^2+\tan\lambda}{1-r^2\tan\lambda}\sin\frac{x}{L_1} + \cos\frac{x}{L_1}\right)\cos\frac{at}{L_1}\right]$$

$$+ e_0 x + \sum_{i=1}^{i=\infty} \sin\frac{\beta_i x}{L}\left(A' \cos\frac{\beta_i a t}{L} + B' \sin\frac{\beta_i a t}{L}\right) \qquad 3\text{-}20$$

The above satisfies the differential equation and the end conditions.

$$\dot{u} = \dot{u}_2 + \dot{u}_3 = A_0\frac{a}{L_1}\left[\frac{r^2+\tan\lambda}{1-r^2\tan\lambda}\sin\frac{x}{L_1} + \cos\frac{x}{L_1}\right]\sin\frac{at}{L_1}$$

$$+ \sum_{i=1}^{i=\infty} \frac{\beta_i a}{L} \sin\frac{\beta_i x}{L}\left(-A' \sin\frac{\beta_i a t}{L} + B' \cos\frac{\beta_i a t}{L}\right)$$

3-21

At $t = t_1$, the values of u and $\dot{u}$, before and after the contact breaks, must be the same.

$$A_0\left[1-\left(\frac{1-\cos\lambda}{\sin\lambda}\sin\frac{x}{L_1}+\cos\frac{x}{L_1}\right)\cos\frac{at_1}{L_1}\right]$$

$$=A_0\left[1-\left(\frac{r^2+\tan\lambda}{1-r^2\tan\lambda}\sin\frac{x}{L_1}+\cos\frac{x}{L_1}\right)\cos\frac{at_1}{L_1}\right]+e_0x$$

$$+\sum_{i=1}^{i=\infty}\sin\frac{\beta_i x}{L}\left(A_i'\cos\frac{\beta_i a t_1}{L}+B_i'\sin\frac{\beta_i a t}{L}\right) \qquad 3\text{-}22$$

$$A_0\frac{a}{L_1}\left(\frac{1-\cos\lambda}{\sin\lambda}\sin\frac{x}{L_1}+\cos\frac{x}{L_1}\right)\sin\frac{at_1}{L_1}$$

$$=A_0\frac{a}{L_1}\left(\frac{r^2+\tan\lambda}{1-r^2\tan\lambda}\sin\frac{x}{L_1}+\cos\frac{x}{L_1}\right)\sin\frac{at_1}{L_1}$$

$$+\sum_{i=1}^{\infty}\frac{\beta_i a}{L}\sin\frac{\beta_i x}{L}\left(-A_i'\sin\frac{\beta_i a t_1}{L}+B_i'\cos\frac{\beta_i a t_1}{L}\right) \qquad 3\text{-}23$$

Let $$F_i=\left(A_i'\cos\frac{\beta_i a t_1}{L}+B_i'\sin\frac{\beta_i a t}{L}\right) \qquad 3\text{-}24$$

$$G_i=\left(-A_i'\sin\frac{\beta_i a t_1}{L}+B_i'\cos\frac{\beta_i a t_1}{L}\right) \qquad 3\text{-}25$$

$$f(\lambda)=\left[\frac{r^2+\tan\lambda}{1-r^2\tan\lambda}-\frac{1-\cos\lambda}{\sin\lambda}\right] \qquad 3\text{-}26$$

$$A_0 f(\lambda)\sin\frac{x}{L_1}\cos\frac{at_1}{L_1}-e_0x-\sum_{i=1}^{\infty}F_i\sin\frac{\beta_i x}{L}=0 \qquad 3\text{-}27$$

$$A_0\lambda f(\lambda)\sin\frac{x}{L_1}\sin\frac{at_1}{L_1}+\sum_{i=1}^{\infty}G_i\beta_i\sin\frac{\beta_i x}{L}=0 \qquad 3\text{-}28$$

Multiplying the above two equations by $\sin\frac{\beta_i x}{L}dx$ and integrating from x = 0 to x = L.

$$\int_0^L \sin\frac{\beta_i x}{L}\sin\frac{x}{L_1}\,dx = \frac{L}{2}\left[\frac{\sin(\lambda-\beta_i)}{\lambda-\beta_i} - \frac{\sin(\lambda+\beta_i)}{\lambda+\beta_i}\right]$$

$$\int_0^L x\sin\frac{\beta_i x}{L}\,dx = -\frac{L^2}{\beta_i}\cos\beta_i + \frac{L^2}{\beta_i^2}\sin\beta_i$$

$$\int_0^L \sin^2\frac{\beta_i x}{L}\,dx = \frac{L}{2}\left(1 - \frac{\sin 2\beta_i}{2\beta_i}\right)$$

$m \neq i \qquad \alpha = \beta_i \tan\beta_i = \beta_m \tan\beta_m$

$$\int_0^L \sin\frac{\beta_i x}{L}\sin\frac{\beta_m x}{L}\,dx = -\frac{L}{\beta_m}\sin\beta_i\cos\beta_m + \int_0^L \frac{\beta_i}{\beta_m}\cos\frac{\beta_m x}{L}\cos\frac{\beta_i x}{L}\,dx$$

$$= -\frac{L}{\beta_m}\sin\beta_i\cos\beta_m + \frac{\beta_i L}{\beta_m^2}\sin\beta_m\cos\beta_i + \int_0^L \frac{\beta_i^2}{\beta_m^2}\sin\frac{\beta_m x}{L}\sin\frac{\beta_i x}{L}\,dx$$

$$\left(1-\frac{\beta_i^2}{\beta_m^2}\right)\int_0^L \sin\frac{\beta_m x}{L}\sin\frac{\beta_i x}{L}\,dx$$

$$= \left(1-\frac{\beta_i^2}{\beta_m^2}\right)\frac{L}{\alpha}\sin\beta_i\sin\beta_m$$

$$\int_0^L \sin\frac{\beta_m x}{L}\sin\frac{\beta_i x}{L}\,dx = -\frac{L}{\alpha}\sin\beta_i\sin\beta_m \qquad 3\text{-}29$$

$$\int_0^L \sin\frac{\beta_i x}{L}\sum_{i=1}^{\infty} F_i \sin\frac{\beta_i x}{L}\,dx = \frac{L}{2}\left(1-\frac{\sin 2\beta_i}{2\beta_i}\right) - \frac{L}{\alpha}\sin\beta_i \sum_{m=1,2,3\neq(i-1),(i+1)\ldots} F_m \sin\beta_m \qquad 3\text{-}30$$

$$\sum_{i=1}^{i=\infty} F_i \sin\beta_i = A_0 f(\lambda) \sin\lambda \cos\frac{at_i}{L_i} - c_0 L \qquad 3\text{-}31$$

$$\sum_{i=1}^{i=\infty} G_i \beta_i \sin\beta_i = -A_0 \lambda f(\lambda) \sin\lambda \sin\frac{at_i}{L_i} \qquad 3\text{-}32$$

$$\sum_{m=1,2\cdots(i-1)(i+1)\cdots} F_m \sin\beta_m = A_0 f(\lambda) \sin\lambda \cos\frac{at_i}{L_i} - c_0 L - F_i \sin\beta_i \qquad 3\text{-}33$$

$$\sum_{m=1,2\cdots(i-1)(i+1)\cdots} G_m \beta_m \sin\beta_m = -A_0 \lambda f(\lambda) \sin\lambda \sin\frac{at_i}{L_i} - G_i \beta_i \sin\beta_i \qquad 3\text{-}34$$

From Equations 3-27 and 3-28,

$$A_0 f(\lambda) \cos\frac{at_i}{L_i} \frac{L}{2}\left[\frac{\sin(\lambda-\beta_i)}{\lambda-\beta_i} - \frac{\sin(\lambda+\beta_i)}{\lambda+\beta_i}\right]$$

$$- c_0\left[-\frac{L^2}{\beta_i}\cos\beta_i + \frac{L^2}{\beta_i^2}\sin\beta_i\right] - F_i\frac{L}{2}\left(1 - \frac{\sin 2\beta_i}{2\beta_i}\right)$$

$$+ \frac{L}{\alpha}\sin\beta_i\left[A_0 f(\lambda)\sin\lambda\cos\frac{at_i}{L_i} - c_0 L - F_i\sin\beta_i\right] = 0$$

$$A_0 \lambda f(\lambda)\sin\frac{at_i}{L_i}\frac{L}{2}\left[\frac{\sin(\lambda-\beta_i)}{\lambda-\beta_i} - \frac{\sin(\lambda+\beta_i)}{\lambda+\beta_i}\right]$$

$$+ G_i\beta_i\frac{L}{2}\left(1 - \frac{\sin 2\beta_i}{2\beta_i}\right) - \frac{L}{\alpha}\sin\beta_i\left[-A_0\lambda f(\lambda)\sin\lambda\sin\frac{at_i}{L_i}\right.$$

$$\left. - G_i\beta_i\sin\beta_i\right] = 0$$

$$A_0 f(\lambda)\cos\frac{a t_i}{L_i}\left[\sin\lambda\cos\beta_i\frac{\beta_i}{\lambda^2-\beta_i^2}-\cos\lambda\sin\beta_i\frac{\lambda}{\lambda^2-\beta_i^2}\right]$$

$$-e_0\left[-\frac{L}{\beta_i}\cos\beta_i+\frac{L}{\beta_i^2}\sin\beta_i\right]-F_i\left[\frac{1}{2}\left(1-\frac{\sin 2\beta_i}{2\beta_i}\right)+\frac{\sin^2\beta_i}{\alpha}\right]$$

$$+\frac{\cos\beta_i}{\beta_i}\left[A_0 f(\lambda)\sin\lambda\cos\frac{a t_i}{L_i}-e_0 L\right]=0$$

$$A_0 f(\lambda)\cos\frac{a t_i}{L_i}\left[\sin\lambda\cos\beta_i\left(\frac{\beta_i}{\lambda^2-\beta_i^2}+\frac{1}{\beta_i}\right)-\cos\lambda\sin\beta_i\frac{\lambda}{\lambda^2-\beta_i^2}\right]$$

$$-e_0\frac{L}{\beta_i^2}\sin\beta_i-\frac{F_i}{2}\left(1+\frac{\sin^2\beta_i}{\alpha}\right)=0 \qquad 3\text{-}35$$

$$A_0\lambda f(\lambda)\sin\frac{a t_i}{L_i}\left[\sin\lambda\cos\beta_i\frac{\beta_i}{\lambda^2-\beta_i^2}-\cos\lambda\sin\beta_i\frac{\lambda}{\lambda^2-\beta_i^2}\right]$$

$$+\frac{G_i\beta_i}{2}\left(1-\frac{\sin 2\beta_i}{2\beta_i}\right)-\frac{\sin\beta_i}{\alpha}\left[-A_0\lambda f(\lambda)\sin\lambda\sin\frac{a t_i}{L_i}\right.$$

$$\left.-G_i\beta_i\sin\beta_i\right]=0$$

$$\frac{\sin\beta_i}{\alpha}=\frac{\cos\beta_i}{\beta_i},$$

$$A_0\lambda f(\lambda)\sin\frac{a t_i}{L_i}\left[\sin\lambda\cos\beta_i\left(\frac{\beta_i}{\lambda^2-\beta_i^2}+\frac{1}{\beta_i}\right)-\cos\lambda\sin\beta_i\frac{\lambda}{\lambda^2-\beta_i^2}\right]$$

$$+G_i\left[\frac{\beta_i}{2}\left(1-\frac{\sin 2\beta_i}{2\beta_i}\right)+\sin\beta_i\cos\beta_i\right]=0$$

$$A_0\lambda f(\lambda)\sin\frac{a t_i}{L_i}\left[\sin\lambda\cos\beta_i\frac{\lambda^2}{(\lambda^2-\beta_i^2)\beta_i}-\cos\lambda\sin\beta_i\frac{\lambda}{\lambda^2-\beta_i^2}\right]$$

$$+G_i\left[\frac{\beta_i}{2}\left(1-\frac{\sin\beta_i\cos\beta_i}{\beta_i}\right)+\sin\beta_i\cos\beta_i\right]=0 \qquad 3\text{-}36$$

From Equations 3-35 and 3-36, F_i and G_i can be found. Then from Equations 3-24 and 3-25, A_i' and B_i' are known. Substituting these values in Equation 3-20 gives the equation of motion after the contact breaks. The maximum gap and the time interval during which the contact remains open, can be obtained. The above equations are valid until the top mass hits the screw. Then the end conditions will be the same as those before the break of contact and the equation of motion can be obtained in the same way as shown above.

CHAPTER IV

RESPONSE OF A CONTACT ACCELEROMETER WITH DISTRIBUTED VISCOUS DAMPING ON SPRING

For a contact-break type accelerometer, there is no relative motion between the top mass and the case, so the dashpot damping between these two has no effect on the break of contact. However, the different parts of the spring do have relative motion with respect to the casing. Damping applied between the spring and the case will modify the response. In this chapter, response of the contact accelerometer with distributed viscous damping on the spring (Reference 13), under simple harmonic motions, is analyzed.

Due to the distributed viscous damping, the differential Equation 1-1 is modified as follows:

$$EA\frac{\partial^2 u}{\partial x^2} = \frac{\gamma A}{g}\frac{\partial^2 u}{\partial t^2} + c\left(\frac{\partial u}{\partial t} - \frac{\partial u_{x=0}}{\partial t}\right)$$

$$\frac{\gamma}{Eg}\frac{\partial^2 u}{\partial t^2} + \frac{c}{EA}\frac{\partial u}{\partial t} - \frac{c}{EA}\frac{\partial u_{x=0}}{\partial t} = \frac{\partial^2 u}{\partial x^2} \qquad 4\text{-}1$$

Under a simple harmonic motion,

$$u_{x=0} = A_0 \sin \omega t, \qquad \frac{\partial u_{x=0}}{\partial t} = A_0 \omega \cos \omega t$$

Equation 4-1 is solved by first finding the general solution "u_1" by letting $\frac{\partial u_{x=0}}{\partial t} = 0$, and then the particular solution "u_2" with $\frac{\partial u_{x=0}}{\partial t} = A_0 \omega \cos \omega t$.

$$u_1 = \sum_{n=1}^{\infty} \bar{A} e^{i p_n t \pm q_n x} \qquad \text{(Reference 12).} \qquad 4\text{-}2$$

$$q_n = \xi_n + i\eta_n, \quad \text{where } \xi_n \text{ and } \eta_n \text{ are real numbers,} \qquad 4\text{-}3$$

Substituting Equation 4-2 in Equation 4-1, with $\frac{\partial u_{x=0}}{\partial t} = 0$ for the general solution and equating the coefficients of $e^{i p_n t \pm q_n x}$,

$$-\frac{\gamma}{Eg} p_n^2 + \frac{c}{EA} i p_n = q_n^2 = 2i\xi_n \eta_n - \eta_n^2 + \xi_n^2$$

$$\xi_n^2 - \eta_n^2 = -\frac{\gamma}{Eg} p_n^2, \qquad 2\xi_n \eta_n = \frac{c}{EA} p_n \qquad 4\text{-}4$$

For the particular solution,

$$\frac{\gamma}{Eg} \frac{\partial^2 u_2}{\partial t^2} + \frac{c}{EA} \frac{\partial u_2}{\partial t} = \frac{\partial^2 u_2}{\partial x^2} + c \frac{A_0}{EA} \omega \cos \omega t \qquad 4\text{-}5$$

Let $u_2 = F \sin \omega t + G \cos \omega t$, where F and G are constants.

$$\frac{\gamma}{Eg}\left[-F\omega^2 \sin \omega t - G\omega^2 \cos \omega t\right] + \frac{c}{EA}\left[F\omega \cos \omega t - G\omega \sin \omega t\right]$$

$$= \frac{c}{EA} A_0 \omega \cos \omega t. \qquad 4\text{-}6$$

Equating the coefficients of $\sin \omega t$ and $\cos \omega t$,

$$-\frac{\gamma}{Eg} F\omega^2 - \frac{c}{EA} G\omega = 0$$

$$-\frac{\gamma}{Eg} G\omega^2 + \frac{c}{EA} F\omega = \frac{c}{EA} A_0 \omega$$

$$F = \frac{A_0}{1 + \left(\frac{A\gamma\omega}{cg}\right)^2} \qquad 4\text{-}7$$

$$G = \frac{-A_0 \frac{A\gamma\omega}{cg}}{1 + \left(\frac{A\gamma\omega}{cg}\right)^2} \qquad 4\text{-}8$$

Let $\frac{A\gamma g}{cg} = V$

$$u = u_1 + u_2 = \sum_{n=1}^{\infty} \bar{A}_n e^{i p_n t \pm q_n x} + \frac{A_0}{1+V^2} \sin\omega t$$

$$- \frac{A_0 V}{1+V^2} \cos\omega t \qquad 4\text{-}9$$

Before the contact breaks, $u_{x=0} = u_{x=L}$. To satisfy this condition, "u" is expressed as follows,

$$u = \sum_{n=1}^{\infty} \bar{A}_n \left(e^{i p_n t \pm q_n x} + e^{i p_n t \pm q_n x'}\right)$$

$$+ \frac{A_0}{1+V^2} \sin\omega t - \frac{A_0 V}{1+V^2} \cos\omega t \qquad 4\text{-}10$$

where $x' = L - x$ and $\bar{A}_n$s are constants.

Expressing "u" in trigonometric series,

$$u = \sum_{n=1}^{\infty} \bar{A}_n \left[e^{-\xi_n x} \sin(p_n t - \eta_n x) + e^{-\xi_n x'} \sin(p_n t - \eta_n x') \right]$$

$$+ \sum_{n=1}^{\infty} \bar{B}_n \left[e^{-\xi_n x} \cos(p_n t - \eta_n x) + e^{-\xi_n x'} \cos(p_n t - \eta_n x') \right]$$

$$+ \frac{A_e}{1+V^2} \sin \omega t - \frac{A_e V}{1+V^2} \cos \omega t, \qquad 4\text{-}11$$

where A_n and B_n are constants.

At $x = 0$,

$$\sum_{n=1}^{\infty} \bar{A}_n \left[\sin p_n t + e^{-\xi_n L} \sin(p_n t - \eta_n L) \right]$$

$$+ \sum_{n=1}^{\infty} \bar{B}_n \left[\cos p_n t + e^{-\xi_n L} \cos(p_n t - \eta_n L) \right]$$

$$+ \frac{A_e}{1+V^2} \sin \omega t - \frac{A_e V}{1+V^2} \cos \omega t = A_e \sin \omega t \qquad 4\text{-}12$$

$$\sum_{n=1}^{\infty} \bar{A}_n \left[\sin p_n t + e^{-\xi_n L} (\sin p_n t \cos \eta_n L - \cos p_n t \sin \eta_n L) \right]$$

$$+ \sum_{n=1}^{\infty} \bar{B}_n \left[\cos p_n t + e^{-\xi_n L} (\cos p_n t \cos \eta_n L + \sin p_n t \sin \eta_n L) \right]$$

$$+ \frac{A_e}{1+V^2} \sin \omega t - \frac{A_e V}{1+V^2} \cos \omega t = A_e \sin \omega t. \qquad 4\text{-}13$$

Retaining only terms with $n = 1$, letting $p_1 = \omega$ and equating the coefficients of $\sin p_1 t$ and $\cos p_1 t$,

$$\bar{A}_1(1+e^{-\xi_1 L}\cos\eta_1 L) + \bar{B}_1 e^{-\xi_1 L}\sin\eta_1 L + \frac{A_0}{1+V^2} = A_0$$

$$-\bar{A}_1 e^{-\xi_1 L}\sin\eta_1 L + \bar{B}_1(1+e^{-\xi_1 L}\cos\eta_1 L) - \frac{A_0 V}{1+V^2} = 0$$

$$\bar{A}_1 = \frac{A_0 V}{1+V^2}\,\frac{V(1+e^{-\xi_1 L}\cos\eta_1 L) - e^{-\xi_1 L}\sin\eta_1 L}{(1+e^{-\xi_1 L}\cos\eta_1 L)^2 + e^{-2\xi_1 L}\sin^2\eta_1 L} \qquad 4\text{-}14$$

$$\bar{B}_1 = \frac{A_0 V}{1+V^2}\,\frac{(1+e^{-\xi_1 L}\cos\eta_1 L) + V e^{-\xi_1 L}\sin\eta_1 L}{(1+e^{-\xi_1 L}\cos\eta_1 L)^2 + e^{-2\xi_1 L}\sin^2\eta_1 L} \qquad 4\text{-}15$$

For "n" ≠ 1,

$$\bar{A}_n(1+e^{-\xi_n L}\cos\eta_n L) + \bar{B}_n e^{-\xi_n L}\sin\eta_n L = 0$$

$$-\bar{A}_n e^{-\xi_n L}\sin\eta_n L + \bar{B}_n(1+e^{-\xi_n L}\cos\eta_n L) = 0$$

$$\bar{A}_{n\neq 1} = 0, \qquad \bar{B}_{n\neq 1} = 0 \qquad 4\text{-}16$$

Only the term "n" = 1 does not vanish, so hereafter the subscript "1" is dropped. Therefore

$$u = \bar{A}\left[e^{-\xi x}\sin(\omega t - \eta x) + e^{-\xi x'}\sin(\omega t - \eta x')\right]$$

$$+ \bar{B}\left[e^{-\xi x}\cos(\omega t - \eta x) + e^{-\xi x'}\cos(\omega t - \eta x')\right]$$

$$+ \frac{A_0}{1+V^2}\sin\omega t - \frac{A_0 V}{1+V^2}\cos\omega t \qquad 4\text{-}17$$

From Equation 4-4,

$$\xi^2 - \eta^2 = -\frac{\gamma}{Eg}\omega^2 \qquad 4\text{-}18$$

$$2\xi\eta = \frac{c}{EA}\omega \qquad 4\text{-}19$$

$$\eta^2 = \frac{c^2\omega^2}{4E^2A^2\eta^2} + \frac{\gamma}{Eg}\omega^2$$

$$\eta^2 = \frac{\gamma}{2Eg}\omega^2 \pm \left(\frac{\gamma^2}{4E^2g^2}\omega^4 + \frac{c^2\omega^2}{4E^2A^2}\right)^{\frac{1}{2}}$$

$$\eta = \left(\frac{\gamma}{2Eg}\right)^{\frac{1}{2}}\omega\left[1+\left(1+\frac{c^2g^2}{\omega^2A^2\gamma^2}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}}$$

$$= \left(\frac{1}{2q^2}\right)^{\frac{1}{2}}\omega\left[1+\left(1+\frac{1}{V^2}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}} \qquad 4\text{-}20$$

$$\xi = \left(\frac{1}{2q^2}\right)^{\frac{1}{2}}\frac{\omega}{V}\left[1+\left(1+\frac{1}{V^2}\right)^{\frac{1}{2}}\right]^{-\frac{1}{2}} \qquad 4\text{-}21$$

From Equation 4-17,

$$\frac{\partial u}{\partial x} = \bar{A}\left[-\xi e^{-\xi x}\sin(\omega t-\eta x) - e^{-\xi x}\eta\cos(\omega t-\eta x)\right.$$

$$\left. + \xi e^{-\xi x'}\sin(\omega t-\eta x') + e^{-\xi x'}\eta\cos(\omega t-\eta x')\right]$$

$$+ \bar{B}\left[-\xi e^{-\xi x}\cos(\omega t-\eta x) + e^{-\xi x}\eta\sin(\omega t-\eta x)\right.$$

$$\left. + \xi e^{-\xi x'}\cos(\omega t-\eta x') - e^{-\xi x'}\eta\sin(\omega t-\eta x')\right]$$

$$\left(\frac{\partial u}{\partial x}\right)_{x=L} = \bar{A}\left[-\xi e^{-\xi L}\sin(\omega t-\eta L) - e^{-\xi L}\eta\cos(\omega t-\eta L)\right.$$

$$\left. + \xi\sin\omega t + \eta\cos\omega t\right] + \bar{B}\left[-\xi e^{-\xi L}\cos(\omega t-\eta L)\right.$$

$$\left. + e^{-\xi L}\eta\sin(\omega t-\eta L) + \xi\cos\omega t - \eta\sin\omega t\right]$$

$$= \left[-\bar{A}(\xi e^{-\xi L}\cos\eta L + \eta e^{-\xi L}\sin\eta L - \xi)\right.$$

$$\left. - \bar{B}(\xi e^{-\xi L}\sin\eta L - \eta e^{-\xi L}\cos\eta L + \eta)\right]\sin\omega t$$

$$+ \left[\bar{A}(\xi e^{-\xi L}\sin\eta L - \eta e^{-\xi L}\cos\eta L + \eta)\right.$$

$$\left. + \bar{B}(-\xi e^{-\xi L}\cos\eta L - \eta e^{-\xi L}\sin\eta L + \xi)\right]\cos\omega t \qquad 4\text{-}22$$

Let the above coefficients of $\sin\omega t$ and $\cos\omega t$ be denoted by "M" and "N".

$$\left(\frac{\partial u}{\partial x}\right)_{x=L} = M\sin\omega t + N\cos\omega t \qquad 4\text{-}23$$

The contact breaks, when

$$EA\left[\epsilon_0 - \left(\frac{\partial u}{\partial x}\right)_{x=L}\right] \leq M_1\left(\frac{\partial^2 u}{\partial t^2}\right)_{x=L} \qquad 4\text{-}24$$

Before the contact breaks,

$$u_{x=0} = u_{x=L} = A_0 \sin \omega t$$

$$\left(\frac{\partial^2 u}{\partial t^2}\right)_{x=L} = \left(\frac{\partial^2 u}{\partial t^2}\right)_{x=0} = -A_0 \omega^2 \sin \omega t$$

$$EA\left[c_0 - \left(\frac{\partial u}{\partial x}\right)_{x=L}\right] = -M_1 A_0 \omega^2 \sin \omega t$$

$$EAc_0 = EA(M \sin \omega t + N \cos \omega t) - M_1 A_0 \omega^2 \sin \omega t$$

The above equation gives the condition required to break the contact. With the terms of the equation rearranged,

$$EAc_0 = (EAM - M_1 A_0 \omega^2) \sin \omega t + EAN \cos \omega t \qquad 4\text{-}25$$

Let $(EAM - M_1 A_0 \omega^2) \sin \omega t + EAN \cos \omega t$

$$= Q \sin(\omega t - \beta) = Q(\cos\beta \sin \omega t - \sin\beta \cos \omega t) \qquad 4\text{-}26$$

$$EAc_0 = Q \sin(\omega t - \beta)$$

$$Q \sin\beta = -EAN$$

$$Q \cos\beta = (EAM - M_1 A_0 \omega^2)$$

$$\tan\beta = -\frac{EAN}{EAM - A_0 \omega^2 M_1} \qquad 4\text{-}27$$

$$Q = M_1 A_0 \omega^2 \left[\left(\frac{EAN}{M_1 A_0 \omega^2} \right)^2 + \left(\frac{EAM}{M_1 A_0 \omega^2} - 1 \right)^2 \right]^{\frac{1}{2}} \qquad 4\text{-}28$$

For the case with no damping, $c = 0$

$$\eta = \sqrt{\frac{\gamma}{Eg}}\,\omega, \qquad \xi = 0, \qquad V = \infty$$

$$\bar{A} = \frac{A_0(1+\cos\eta L)}{(1+\cos\eta L)^2 + \sin^2\eta L} = \frac{A_0}{2}$$

$$\bar{B} = \frac{A_0 \sin\eta L}{2(1+\cos\eta L)}$$

$$M = -\frac{A_0}{2}\eta \sin\eta L \left(1 + \frac{1-\cos\eta L}{1+\cos\eta L}\right)$$

$$= -\frac{A_0 \eta \sin\eta L}{1+\cos\eta L}$$

$$N = \frac{A_0}{2}\eta(1-\cos\eta L) - \frac{A_0}{2}\frac{\eta \sin^2\eta L}{1+\cos\eta L} = 0$$

$$\tan\beta = 0, \qquad \beta = 0.$$

$$Q = EAM - A_0 \omega^2 M_1$$

$$= -EA\left(A_0 \frac{\eta \sin\eta L}{1+\cos\eta L}\right) - A_0 \omega^2 M_1$$

$$= -EA\left(A_0 \eta \frac{1-\cos\eta L}{\sin\eta L}\right) - A_0 \omega^2 M_1$$

$$\eta L_1 = \lambda, \qquad \eta = \frac{1}{L_1}$$

$$Q = -EA\left(\frac{A_0}{L_1}\frac{1-\cos\lambda}{\sin\lambda}\right) - A_0\omega^2 M_1$$

From Equation 4-25 and Equation 4-26,

$$EAe_0 = Q\sin\omega t$$

$$EA\left(e_0 + \frac{A_0}{L_1}\frac{1-\cos\lambda}{\sin\lambda}\sin\omega t\right) = -M_1 A_0\omega^2\sin\omega t$$

At $\sin\omega t = -1$,

$$EA\left(e_0 - \frac{A_0}{L_1}\frac{1-\cos\lambda}{\sin\lambda}\right) = M_1 A_0\omega^2 \qquad 4\text{-}29$$

The above is the condition required for the contact to break without damping. It checks with Equation 2-11.

An illustrative example with distributed viscous damping is calculated as follows: The mass of the spring is taken as equal to the top mass. The distributed damping for the whole spring length is taken to be equal to the critical dashpot damping with one third of the spring mass included in the top mass in a single degree of freedom system (Reference 7).

$$M_2 = M_1 = \frac{A\gamma L}{g}$$

$$c = \frac{4}{L}\sqrt{\frac{M_1 EA}{3L}}$$

$$n = \frac{cg}{A\gamma} = \frac{4}{L}\sqrt{\frac{M_1 EA}{3L}}\frac{g}{A\gamma} = \frac{4}{\sqrt{3}}\frac{g}{L}$$

$$\lambda = \frac{\omega L}{a} = \frac{4}{\sqrt{3}}\frac{\omega}{n}, \qquad V = \frac{\sqrt{3}}{4}\lambda$$

$$\xi L = 1.63\left[1+\left(1+\frac{16}{3\lambda^2}\right)^{\frac{1}{2}}\right]^{-\frac{1}{2}}$$

$$\eta L = .707\lambda\left[1+\left(1+\frac{16}{3\lambda^2}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}}$$

$$\bar{A} = \frac{A_0\frac{\sqrt{3}}{4}\lambda}{1+\frac{3}{16}\lambda^2}\,\frac{\frac{\sqrt{3}}{4}\lambda\left(1+e^{-\xi L}\cos\eta L - e^{-\xi L}\sin\eta L\right)}{\left(1+e^{-\xi L}\cos\eta L\right)^2+e^{-2\xi L}\sin^2\eta L}$$

$$\bar{B} = \frac{A_0\frac{\sqrt{3}}{4}\lambda}{1+\frac{3}{16}\lambda^2}\,\frac{\left(1+e^{-\xi L}\cos\eta L\right)+\frac{\sqrt{3}}{4}\lambda e^{-\xi L}\sin\eta L}{\left(1+e^{-\xi L}\cos\eta L\right)^2+e^{-2\xi L}\sin^2\eta L}$$

From Equation 4-22 and Equation 4-23,

$$ML = -\bar{A}[\xi L(e^{-\xi L}\cos\eta L - 1) + \eta L e^{-\xi L}\sin\eta L]$$

$$-\bar{B}[\xi L e^{-\xi L}\sin\eta L + \eta L(1 - e^{-\xi L}\cos\eta L)]$$

$$NL = \bar{A}[\xi L e^{-\xi L}\sin\eta L + \eta L(1 - e^{-\xi L}\cos\eta L)]$$

$$+\bar{B}[\xi L(1 - e^{-\xi L}\cos\eta L) - \eta L e^{-\xi L}\sin\eta L]$$

From Equation 4-25,

$$R = \frac{EAe_0}{M_1 A_0 \omega^2} = \left[\left(\frac{ML}{A_0\lambda^2\left(\frac{M_1}{M_2}\right)} - 1\right)\sin\omega t + \frac{NL}{A_0\lambda^2\left(\frac{M_1}{M_2}\right)}\cos\omega t\right]$$

$$= \frac{Q\sin(\omega t - \beta)}{M_1 A_0 \omega^2}$$

$$\frac{Q}{M_1 A_0 \omega^2} = \left[\left(\frac{ML}{A_0\lambda^2\left(\frac{M_1}{M_2}\right)} - 1\right)^2 + \left(\frac{NL}{A_0\lambda^2\left(\frac{M_1}{M_2}\right)}\right)^2\right]^{\frac{1}{2}}$$

$$\tan\beta = \frac{\frac{NL}{A_0\lambda^2}\left(\frac{M_2}{M_1}\right)}{\frac{ML}{A_0\lambda^2}\left(\frac{M_1}{M_2}\right) - 1}$$

The calculations are shown in Table IV and plotted in Figure 5.

TABLE IV

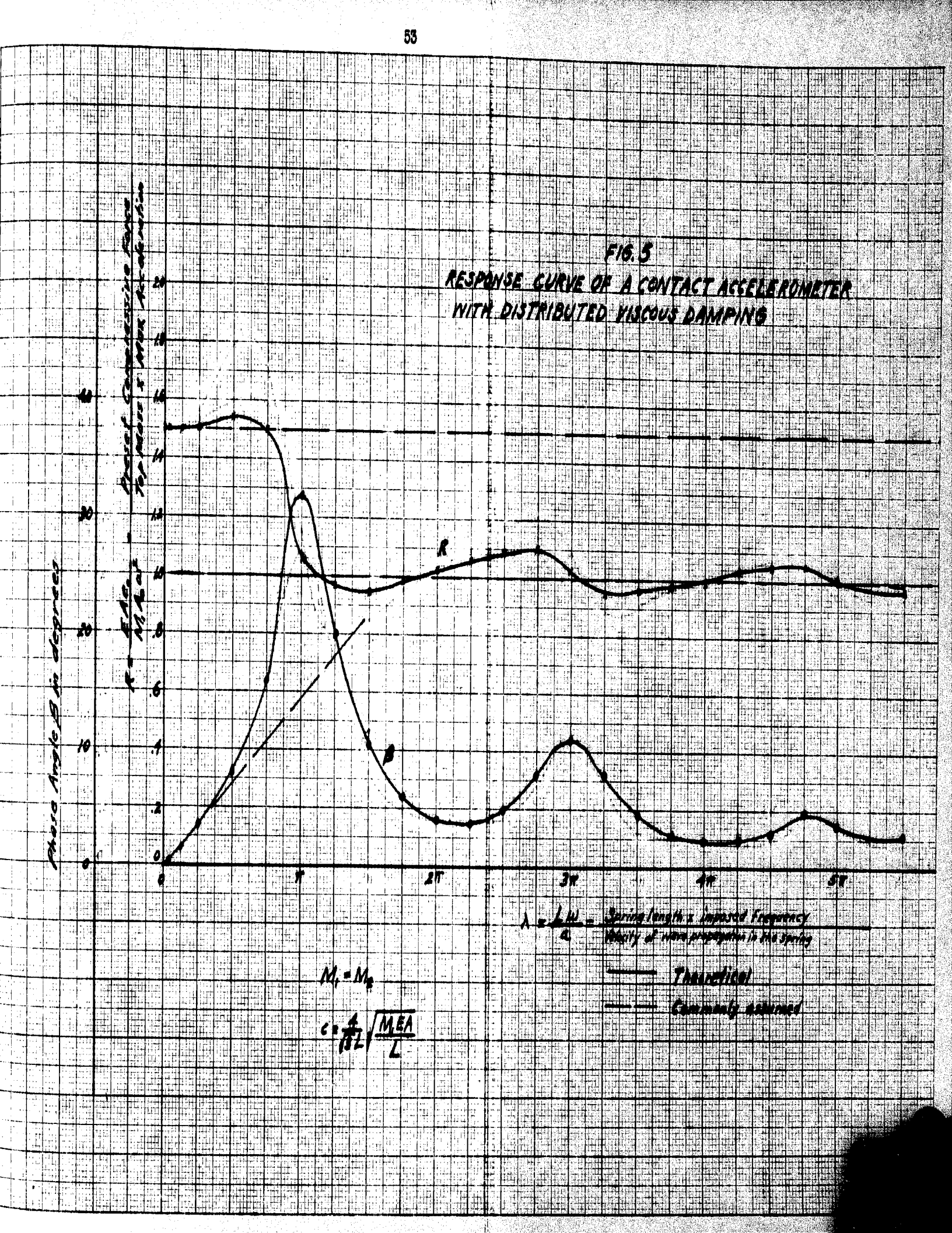
CALCULATION OF RESPONSE OF CONTACT ACCELEROMETER WITH DISTRIBUTED VISCOUS DAMPING

(1)	λ	1/4	1/2	3/4	1	1 1/4	2	2 1/4
(2)	λ^2	.618	2.46	5.58	9.87	22.2	39.4	61.6
(3)	$\frac{16}{3\lambda^2}$	8.63	2.16	.956	.54	.240	.135	.0865
(4)	$\frac{\sqrt{3}\lambda}{4}$	.339	.678	.943	1.36	2.04	2.71	3.38
(5)	$(1+\frac{16}{3\lambda^2})^{\frac{1}{2}}$	3.1	1.78	1.40	1.24	1.11	1.06	1.04
(6)	$[1+(5)]^{\frac{1}{2}}$	2.02	1.67	1.55	1.50	1.45	1.43	1.43
(7)	$\xi L = 1.63/(6)$	.806	.975	1.05	1.48	1.12	1.14	1.14
(8)	$\eta L = .707(1)(6)$	1.12	1.85	2.58	3.33	4.83	6.35	7.92
(9)	$e^{-\xi L}$	.446	.378	.350	.340	.326	.320	.320
(10)	$\sin \eta L$	.898	.96	.528	-.19	-.99	.07	.998
(11)	$\cos \eta L$	.436	-.276	-.848	-.982	.122	1	-.07
(12)	$e^{-\xi L}\sin \eta L$	.401	.363	.185	-.065	-.322	.022	.320
(13)	$e^{-\xi L}\cos \eta L$	.195	-.104	-.297	-.334	.040	.320	-.022
(14)	$1+(13)$	1.195	.896	.703	.666	1.04	1.32	.978
(15)	$(14)^2$	1.43	.800	.495	.433	1.08	1.74	.958
(16)	$(12)^2$	.160	.132	.034	.004	.104	.00048	.102
(17)	$(4)(14)-(12)$	.004	.244	.477	.971	2.44	3.56	3.00
(18)	$(15)+(16)$	1.59	.932	.529	.437	1.18	1.74	1.06
(19)	$\frac{(4)}{1+(4)^2}$	.304	.465	.500	.466	.394	.324	.271
(20)	$\bar{A}/A_0 = (17)(19)/(18)$	.00077	.122	.451	1.033	.815	.644	.767
(21)	$(4)(12)+(14)$	1.331	1.142	.877	.576	.38	1.38	2.06
(22)	$\bar{B}/A_0 = (2)(19)/(18)$	.255	.570	.829	.614	.127	.257	.526
(23)	$1-(13)$	.805	1.10	1.30	1.33	.96	.680	1.022
(24)	$(7)(23)-(8)(12)$	.200	.400	.883	1.66	2.63	.635	-1.37
(25)	$(20)(26)$	.00015	.049	.398	1.72	2.14	.410	-1.05
(26)	$(7)(12)+(8)(23)$	1.225	2.39	3.54	4.36	4.28	4.34	8.47
(27)	$(25)(22)$	.312	1.36	2.94	2.68	.544	1.115	4.45
(28)	$ML/A_0 = (25)-(27)$	-.312	-1.31	-2.54	.96	1.60	-.705	-5.50
(29)	$(20)/(26)$	.00094	.292	1.59	4.50	3.48	2.80	6.49
(30)	$(22)(24)$	.051	.228	.732	1.02	.344	.163	-.720
(31)	$NL/A_0 = (29)+(30)$	.052	.520	2.32	5.52	3.81	2.96	5.77
(32)	$ML/A\lambda^2 = (28)(2)$	-.505	-.533	-.455	.097	.072	-.018	-.089
(33)	$(32)-1$	-1.505	-1.533	-1.455	-.903	-.93	-1.018	-1.089
(34)	$(33)^2$	2.26	2.36	2.12	.816	.866	1.04	1.19
(35)	$NL/A_0\lambda^2 = (31)/(2)$	.084	.212	.416	.559	.172	.075	.0935
(36)	$(35)^2$	.0071	.045	.173	.313	.0295	.0056	.0088
(37)	$R = \sqrt{(34)+(36)}$	1.505	1.548	1.51	1.06	.946	1.02	1.09
(38)	$\tan\beta = \frac{(35)}{(33)}$	.056	.138	.286	.62	.185	.0737	.086
(39)	β in degrees	3.2	7.9	16	32	10.5	4.2	5

TABLE IV---Continued

2.75	3	3.25	3.50	4	4.50	4.75	5	5.50
74.8	88.8	104	121	158	200	222	247	298
.0712	.060	.0573	.0441	.0338	.0266	.0240	.0216	.0179
3.74	4.06	4.41	4.76	5.43	6.10	6.44	6.79	7.46
1.034	1.03	1.028	1.022	1.017	1.013	1.012	1.011	1.009
1.426	1.426	1.424	1.420	1.42	1.418	1.418	1.418	1.418
1.14	1.14	1.14	1.15	1.15	1.15	1.15	1.15	1.15
8.70	9.51	10.26	11.06	12.62	14.16	14.92	15.72	17.30
.320	.320	.320	.316	.316	.316	.316	.316	.316
.669	-.087	-.744	-.997	.005	.998	.694	-.034	-.948
-.744	-.998	-.669	.078	1.00	-.034	-.719	.998	.034
.214	-.028	-.248	-.316	.0016	.316	.220	-.0107	-.300
-.238	-.320	-.214	.025	.316	-.011	-.227	-.316	-.011
.762	.680	.786	1.025	1.316	.99	.773	.684	.99
.580	.463	.618	1.05	1.73	.98	.60	.468	.98
.046	.0008	.062	.099	.000002	.099	.048	.0001	.090
2.64	2.79	3.72	5.19	7.15	5.72	4.75	4.65	7.70
.626	.471	.680	1.15	1.73	1.08	.650	.468	1.07
.249	.232	.215	.201	.177	.159	.151	.143	.123
1.05	1.37	1.18	.908	.732	.842	1.10	1.42	.885
1.56	.566	-.304	-.480	1.32	2.92	2.19	.611	-1.24
.62	.279	-.0096	-.084	.135	.430	.509	.187	-.143
1.24	1.325	1.21	.975	.684	1.01	1.247	.684	.99
-.45	1.77	3.92	4.62	.766	-.3132	-1.87	.953	6.33
-.473	2.42	4.62	4.19	.561	-2.79	-2.06	1.35	5.61
11.04	12.5	12.1	10.4	8.65	14.7	18.6	10.8	16.8
6.84	3.48	-1.16	-.87	1.17	6.33	9.46	2.02	-2.40
-7.31	-1.06	5.78	5.06	.61	-9.12	-11.52	-.67	8.01
11.6	17.1	14.3	9.45	6.33	12.4	20.5	15.3	14.9
-.28	.493	-.376	-.388	.103	-1.43	-.951	.178	-.905
11.3	17.6	14.0	9.06	6.43	11.0	19.5	15.5	14.0
-.098	-.0119	.056	.042	.004	-.046	-.052	-.0027	.027
-1.098	-1.012	-.944	-.958	-.996	-1.046	-1.052	-1.003	-.973
1.20	1.026	.891	.918	.992	1.092	1.108	1.006	.946
.151	.198	.135	.075	.041	.055	.088	.065	.047
.023	.039	.018	.006	.0017	.0030	.0078	.0042	.0022
1.104	1.03	.953	.962	.997	1.047	1.053	1.005	.973
.137	.196	.143	.078	.041	.053	.084	.065	.048
8°	11°	8°	4.5°	24°	3°	4.8°	3.7°	28°

FIG. 5
RESPONSE CURVE OF A CONTACT ACCELEROMETER WITH DISTRIBUTED VISCOUS DAMPING



It is seen from the figure, that the factor of proportionality "R" remains fairly constant up to "λ" = 0.7π and the phase angle "β" curve is nearly a straight line within this range. So for this particular ratio of the top mass to the spring mass, the accelerometer will give readings of reasonable accuracy on periodic motions within the range of "λ" = $.7\pi$.

It is commonly assumed that the contact opens when the product of the top mass times the maximum acceleration is equal to, or greater than, the preset compressive force on the spring. This implies (1) "R" is equal to unity, (2) the phase angle "β" of every harmonic in the motion is proportional to its frequency, and (3) the response of the motion at one instant is completely damped out in the response of the succeeding instant. As shown in Figure 5, beyond the frequency "λ" = $.7\pi$, the constant of proportionality "R" drops and the phase angle "β" deviates appreciably from a straight line, so this particular instrument will give distorted or unreliable responses to the periodic and transient motions with frequencies exceeding the value "λ" = $.7\pi$.

As seen from Chapter II, there are motions in different parts of the spring, after the imposed motion on the accelerometer ceases. These motions of the different parts of the spring do produce tensile strain at the top of the

spring and influence the break of contact. The combined effect of this tensile strain and the tensile strain caused by the subsequent motion on the accelerometer actually determine the opening of the contact. In order that the response denotes the acceleration at a particular instant only, the response due to the preceeding motion must be completely damped out. Since the motions in the spring are not completely damped out instantly, these motions will introduce errors in the response of the instrument to the subsequent acceleration.

However, with increasing ratio of the top mass to the spring mass, the curve "R" will have less deviation from unity and the phase angle "β" will approach zero. The greater the damping, the faster the response will be damped out. Consequently, this type of contact accelerometer will give reliable responses only when the ratio of masses approaches infinity i.e. with spring mass being zero and when the damping is of infinite magnitude. These ideal conditions are not fulfilled in the instruments commonly used. Hence, unreliable readings are obtained. Under a particular motion, the magnitude of the error in the breaking of the contact depends on the amount of deviation from these ideal conditions.

CHAPTER V

EXPERIMENTAL STUDY OF A CONTACT ACCELEROMETER

In order to check experimentally, the equations derived in the previous chapters, a contact accelerometer with top mass soldered to a spring was built. Since the spring mass is the main cause of the variation of response for motions of different frequencies, the spring mass was made approximately equal to the top mass. The spring is made of 1/16" piano wire with the outside diameter 1-3/8". The pitch is 1/4". Total number of coils is 21.75. The spring weighs .085 lb. Since the top and bottom coils are inactive, the number of active coils is 19.75. The mass of the active coils is .085 x 19.75/21.75 = .0771 lb. The top mass and spring weigh .156 lb. (71 gm.), so the top mass is .156 - .0771 - .085/21.75 = .074 lb. $M_1/M_2 = .961$. The spring constant is .421 lb/in. The resonant frequency is $\frac{1}{2}\sqrt{K/M_2} = \frac{1}{2}\sqrt{.421 \times 386 / .0771} = 22.9$ cycles per second. The accelerometer was tested with both steady state and transient motions.

TESTS WITH STEADY STATE MOTION

This accelerometer was mounted on an MB electromagnetic vibrator, which gives simple harmonic force of

varying frequencies from sixteen to twenty thousand cycles per second. . On the accelerometer, a Statham accelerometer with natural frequency of 150 cycles per second was mounted and was electrically connected to a Brush electro-magnetic recorder, thus recording the actual acceleration. A Hewett-Packet Oscillator was connected in series with the contact accelerometer. While the contact was closed, high frequency wave from the oscillator was showing on a DuMont 308 Oscilloscope. The break of the contact was shown by the break of the oscillator wave on the oscilloscope. The precompressed force was adjusted by the setscrew and measured by hanging weights on a pan attached through a string to the top mass until the contact broke as shown on the oscilloscope. This setup is shown in Figure 6.

Based on the spring data and the top mass, the response curve without damping was calculated and is shown in Figure 7. Then the damping between the spring and the case is assumed to be a distributed viscous damping "c". The magnitude of this damping was obtained by letting the calculated response be equal to the experimental value at the second resonant frequency ($\lambda = 3\pi$). Then the theoretical response curve with this distributed damping was calculated for all frequencies. The calculated values and the experimental

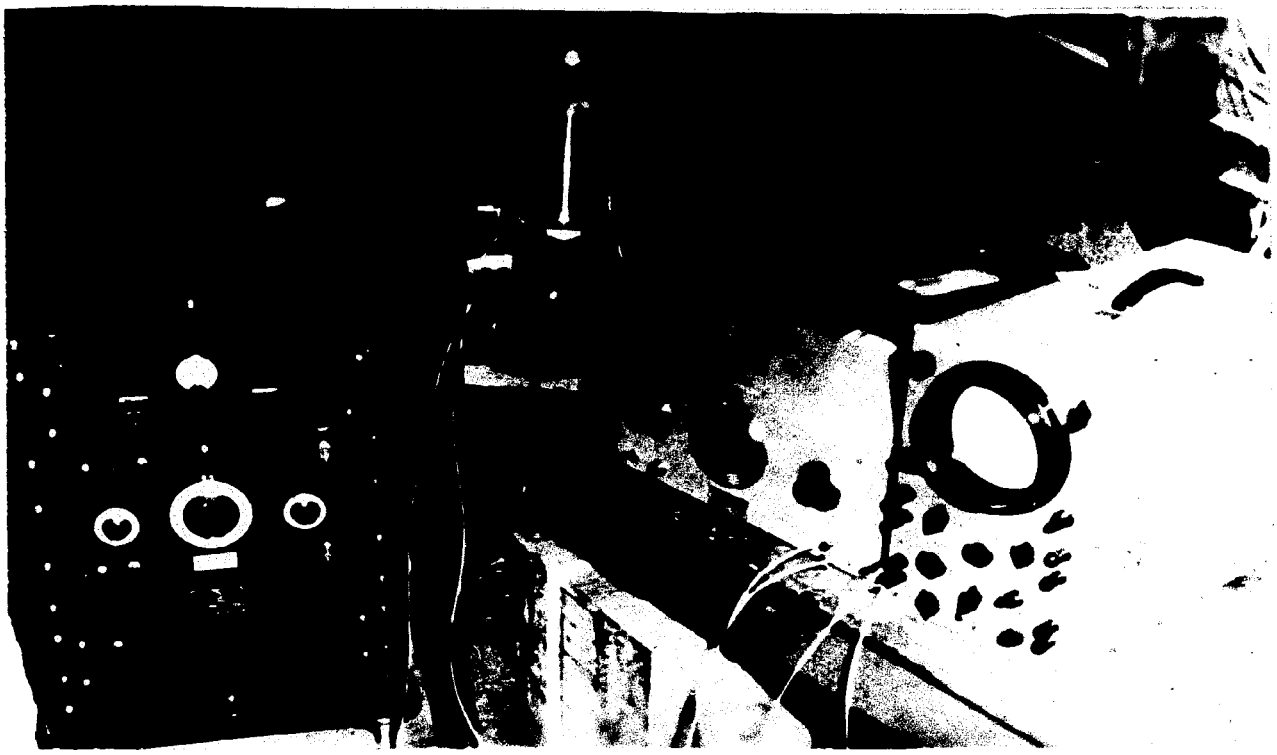


FIGURE 6

TEST SETUP OF A CONTACT ACCELEROMETER
UNDER SIMPLE HARMONIC MOTIONS

data are plotted in Figure 7. The agreement seems to be fairly good with the consideration of some non-uniformity of the spring and the deviation from viscous damping as assumed.

The distributed damping was found by the way explained above.

$$c = \frac{0.12}{L}\sqrt{\frac{M_1 EA}{L}}$$

$$\frac{M_2}{M_1} = 1.04$$

$$n = \frac{cg}{A\gamma} = \frac{.12}{L}\sqrt{\frac{M_1 EA}{L}}\frac{g}{A\gamma} = .12\sqrt{\frac{M_1}{M_2}}\sqrt{\frac{EA}{LM_2}}$$

$$= .12\sqrt{\frac{1}{1.04}}\sqrt{\frac{EAg}{LA\gamma L}} = .12 \times .981\frac{g}{L}$$

$$= .119\frac{g}{L}$$

$$\lambda = \frac{\omega L}{a} = .119\frac{\omega}{n}$$

$$V = \frac{A\gamma\omega}{cg} = \frac{\omega}{n} = \frac{\lambda}{.119} = 8.42\lambda$$

$$\xi L = \left(\frac{1}{2g^2}\right)^{\frac{1}{2}}\frac{\omega L}{V}\left[1+\left(1+\frac{1}{V^2}\right)^{\frac{1}{2}}\right]^{-\frac{1}{2}}$$

$$= \frac{1}{\sqrt{2}g}\frac{\omega L}{8.42\lambda}\left[1+\left(1+\frac{1}{V^2}\right)^{\frac{1}{2}}\right]^{-\frac{1}{2}}$$

$$\xi L = \frac{1}{1.414 \times 8.42}\left[1+\left(1+\frac{1}{V^2}\right)^{\frac{1}{2}}\right]^{-\frac{1}{2}}$$

$$\eta L = .707\,\lambda\left[1+\left(1+\frac{1}{V^2}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}}$$

The values of $\bar{A}$ and $\bar{B}$ were obtained from Equation 4-14 and Equation 4-15. Equation 4-22 and Equation 4-23 were used to obtain the values of M and N. The acceleration required to cause the break of contact was calculated from Equation 4-25 as shown in Table VI and plotted in Figure 7.

TABLE V

EXPERIMENTAL RESPONSE OF CONTACT ACCELEROMETER UNDER SIMPLE HARMONIC MOTIONS

Cycles per second. (1)	Acceleration* in mm. shown on Brush Recorder. (2)	$R = EAe_o/M_1A_o\omega^2$ (3)
16	4.7	1.63
20	2.7	2.84
22.3	0.7	11.0
25	2.6	2.95
29	4.4	1.74
30	8.5	.903
32	7.8	.984
34	8.8	.872
40	8.6	.893
45	8.8	.872
50	7.6	1.01
55	8.2	.93
60	8.6	.893
65	2.7	2.84
67.5	2.0	3.84
70	13.2	.582
75	11.5	.667
80	10.4	.738
85	9.0	.853

$EAe_o = .154$ lbs.

*$2g = 3.6$ mm.

$$R = \frac{.154 \times 3.6}{.074 \times (2)} = \frac{7.48}{(2)}$$

Simple harmonic motions were generated by an MB vibrator.

TABLE VI

CALCULATION OF THEORETICAL RESPONSE OF THE CONTACT ACCELEROMETER TESTED (WITH DISTRIBUTED VISCOUS DAMPING)

(1)	λ/π	.1	1/8	1/4	1/2	3/4	1	5/4
(2)	C.P.S.	.73	2.86	5.73	11.5	17.2	22.9	28.6
(3)	λ^2	.01	.154	.614	2.46	5.56	9.87	15.4
(4)	$V = 8.42\lambda$	.842	3.30	6.60	13.2	19.9	26.4	33.0
(5)	$\left[1+\left(1+\frac{1}{(4)^2}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}}$	1.596	1.43	1.414	1.414	1.414	1.414	1.414
(6)	$\xi L = \frac{1}{1.41\times 8.42(5)}$	.0522	.0582	.0588	.0588	.0588	.0588	.0588
(7)	$\eta L = .707\lambda(5)$	.113	.397	.785	1.57	2.36	3.14	3.92
(8)	$e^{-\xi L}\sin\eta L$	.1067	.363	.667	.943	.667	0	-.667
(9)	$e^{-\xi L}\cos\eta L$	.942	.871	.667	0	-.667	-.943	-.667
(10)	1+(9)	1.942	1.871	1.667	1	.333	.057	.333
(11)	(4)(10)-(8)	1.527	5.80	10.3	12.3	5.96	1.50	11.7
(12)	$(10)^2+(8)^2$	3.79	3.64	3.22	1.89	.557	.00324	.556
(13)	$\frac{M_1}{1+(4)^2}$	.492	.277	.148	.075	.050	.0378	.030
(14)	$A/A_0 = (11)(13)/(10)$	.198	.441	.469	.488	.535	17.5	.631
(15)	(4)(8)+(10)	2.03	3.07	6.07	13.4	13.8	0	-.21.7
(16)	$B/A_0 = (15)(13)/(10)$	.264	.234	.279	.531	1.24	.44	-1.17
(17)	1-(9)	.058	.129	.333	1	1.67	1.943	1.667
(18)	(6)(17)-(7)(8)	-.0091	-.136	-.504	-1.42	1.47	.114	2.71
(19)	(14)(18)	-.0018	-.060	-.236	-.694	-.786	2.00	1.71
(20)	(6)(8)+(7)(17)	.0122	.0725	.300	1.625	3.97	6.10	6.50
(21)	(20)(16)	.00322	.0172	.086	.863	4.92	2.68	-7.61
(22)	$ML/A_0 = (19)-(21)$	-.0050	-.0774	-.323	-1.56	-5.71	-.68	9.32
(23)	(14)(20)	.00242	.032	.141	.794	2.13	106.8	4.10
(24)	(16)(18)	-.0024	-.0324	-.141	-.755	-1.82	.050	-3.17
(25)	$NL/A_0 = (23)+(24)$	.00003	-.0004	-.004	-.039	.31	106.8	.93
(26)	$\frac{ML}{A_0\lambda^2}\left(\frac{M_2}{M_1}\right)$	-.520	-.521	-.546	-.634	-1.03	-.072	.630
(27)	(26)-1	-1.52	-1.52	-1.546	-1.06	-2.07	-1.07	-.370
(28)	$(27)^2$	2.32	2.32	2.39	2.76	4.29	1.148	.137
(29)	$\frac{NL}{A_0\lambda}\left(\frac{M_2}{M_1}\right)$	.0031	-.0027	-.0068	-.0166	.058	11.2	.062
(30)	$(29)^2$					.0031	126	.0038
(31)	$R = \sqrt{(28)+(30)}$	1.52	1.52	1.55	1.67	2.08	11.3	.375

$$M_1/M_2 = .961$$

$$c = \frac{.12}{L}\sqrt{\frac{M_1 EA}{L}}$$

TABLE VI---Continued

3/2	7/4	2	9/4	5/2	11/4	2.95	2.98	3	3.25
34.4	40.0	45.8	51.5	57.3	63	67.5	68.3	68.7	74.5
22.2	30.2	39.4	50.1	61.6	74.8	85.9	87.6	88.8	104.6
39.6	46.2	52.9	59.5	66.2	76.7	78.0	78.8	79.4	86.0
1.414	1.414	1.414	1.414	1.414	1.414	1.414	1.414	1.414	1.414
.0588	.0588	.0588	.0588	.0588	.0588	.0588	.0588	.0588	.0588
4.71	5.49	6.28	7.07	7.86	9.11	9.26	9.36	9.43	10.22
-.943	-.667	0	.667	.943	.667	.147	.0575	0	-.667
0	.667	.943	.667	0	-.667	-.929	-.940	-.943	-.667
1	1.667	1.943	1.667	1	.333	.071	.060	.057	.333
40.5	77.7	.03	98.4	65.3	24.8	5.40	4.67	4.52	29.3
1.89	3.24	3.78	3.24	1.89	.556	.0266	.0069	.00326	.556
.0251	.0216	.0189	.0168	.0151	.0131	.0128	.0127	.0126	.0116
.538	.518	.515	.510	.522	.600	2.67	8.60	7.47	.612
-36.4	-29.1	1.943	41.4	63.4	51.5	11.8	5.09	.057	57.1
-.483	-.191	.0097	.215	.506	1.21	5.67	8.60	.220	-1.21
1	.333	.057	.333	1	1.667	1.929	1.94	1.943	1.667
4.50	3.78	.00335	-.469	-7.35	-5.96	-1.25	-4.24	.114	6.91
2.42	1.95	.0017	-2.37	-3.83	-3.58	-3.34	-3.64	1.99	4.22
4.65	1.79	.358	2.40	7.92	15.2	17.87	18.16	18.3	16.96
-22.4	-.347	.0035	.516	4.00	18.4	101.0	156.1	4.02	-20.5
4.66	2.30	-.0018	-2.91	-7.80	-22.0	-109.3	-159.7	-2.03	24.7
2.50	.930	.184	1.22	4.13	9.12	47.6	.56	320	10.4
-.217	-.734	0	-1.06	-3.72	-7.20	-7.09	-3.65	.025	-8.36
.33	.196	.184	.21	.41	1.92	39.7	152	-320	2.00
.218	.079	0	-.061	-.132	-.275	-1.26	-1.89	-.024	.245
-.782	-.921	-1	-1.06	-1.13	1.276	-2.26	-2.89	-1.024	-.755
.611	.850	1	1.12	1.28	1.63	5.10	8.35	1.05	.570
.0155	.0068	.0049	.0044	.0069	.024	.48	1.80	3.74	.0199
.00024	.00005	.00002	.00002	.00005	.0005	.231	3.24	14.0	.0004
.78	.922	1.00	1.06	1.13	1.28	2.31	3.40	3.88	.754

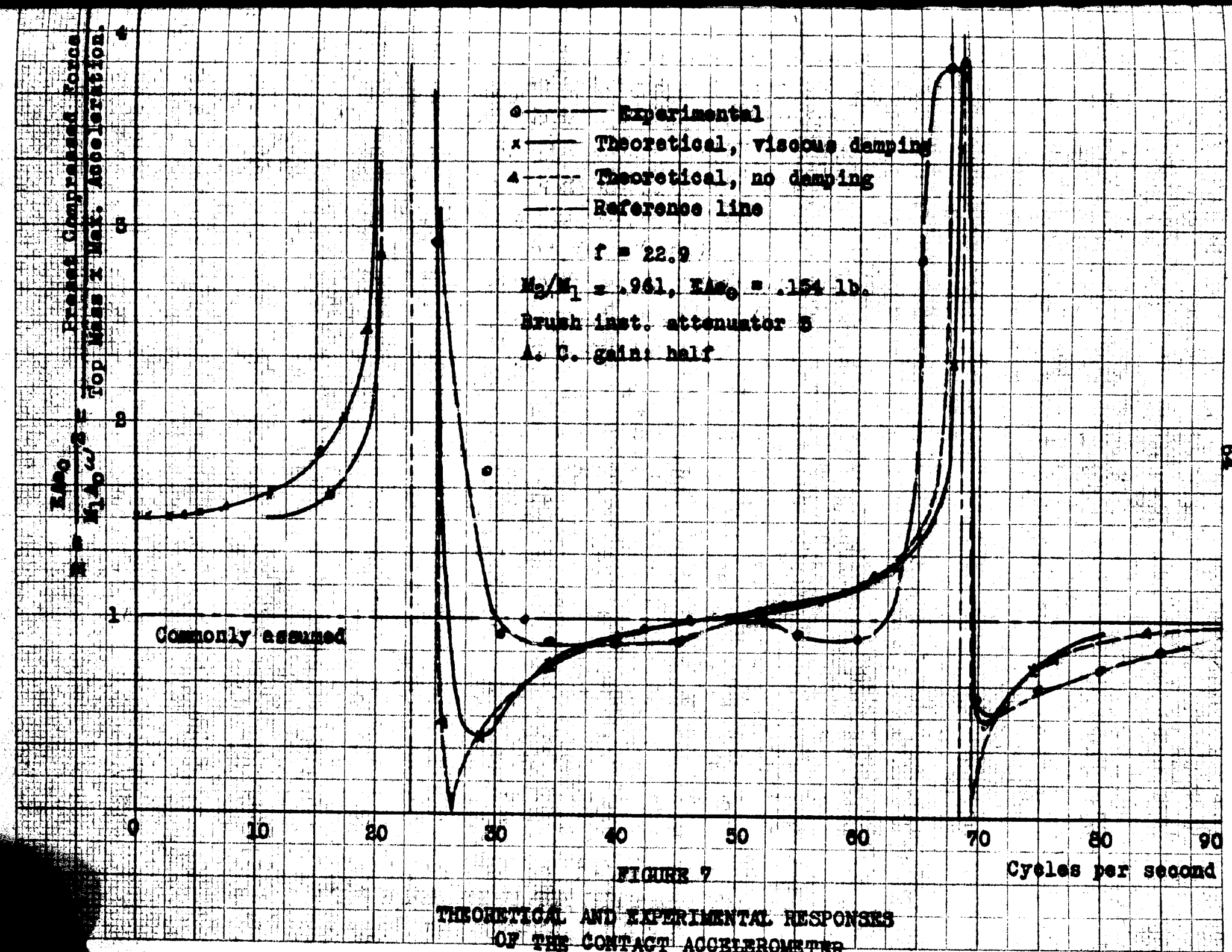


FIGURE 7

THEORETICAL AND EXPERIMENTAL RESPONSES
OF THE CONTACT ACCELEROMETER

TESTS UNDER TRANSIENT MOTION

A damped free vibration was chosen for the transient motion. A 17ST aluminum alloy cantilever beam 3/4" x 3/4" x 11'-0", was used with the contact accelerometer mounted near the free end. The acceleration was measured by a Statham accelerometer and recorded with the break of contact on a two channel Brush Recorder. The free end was raised and then released. The single break of contact, where the contact breaks only in the first cycle, was obtained by adjusting the distance that the free end was raised. For multiple breaks of the contact, the free end was raised further. The arrangement is shown in Figure 8.

The theoretical response to this motion is calculated as follows: Air damping on the beam is assumed to act as a viscous damping (Reference 13), giving a damped free vibration $A_0 e^{-\alpha t} \cos \omega t$, where A_0 is the initial amplitude, "α", the viscous damping coefficient obtained empirically from the logarithmic decrement of the vibration curve, "ω" is the angular velocity of the vibration measured from the test curve.

From Equation 1-2 with $x = 0$,

$$u_{x=0} = f(-at) + F(at)$$

$$= A_0 e^{-\alpha t} \cos \omega t$$

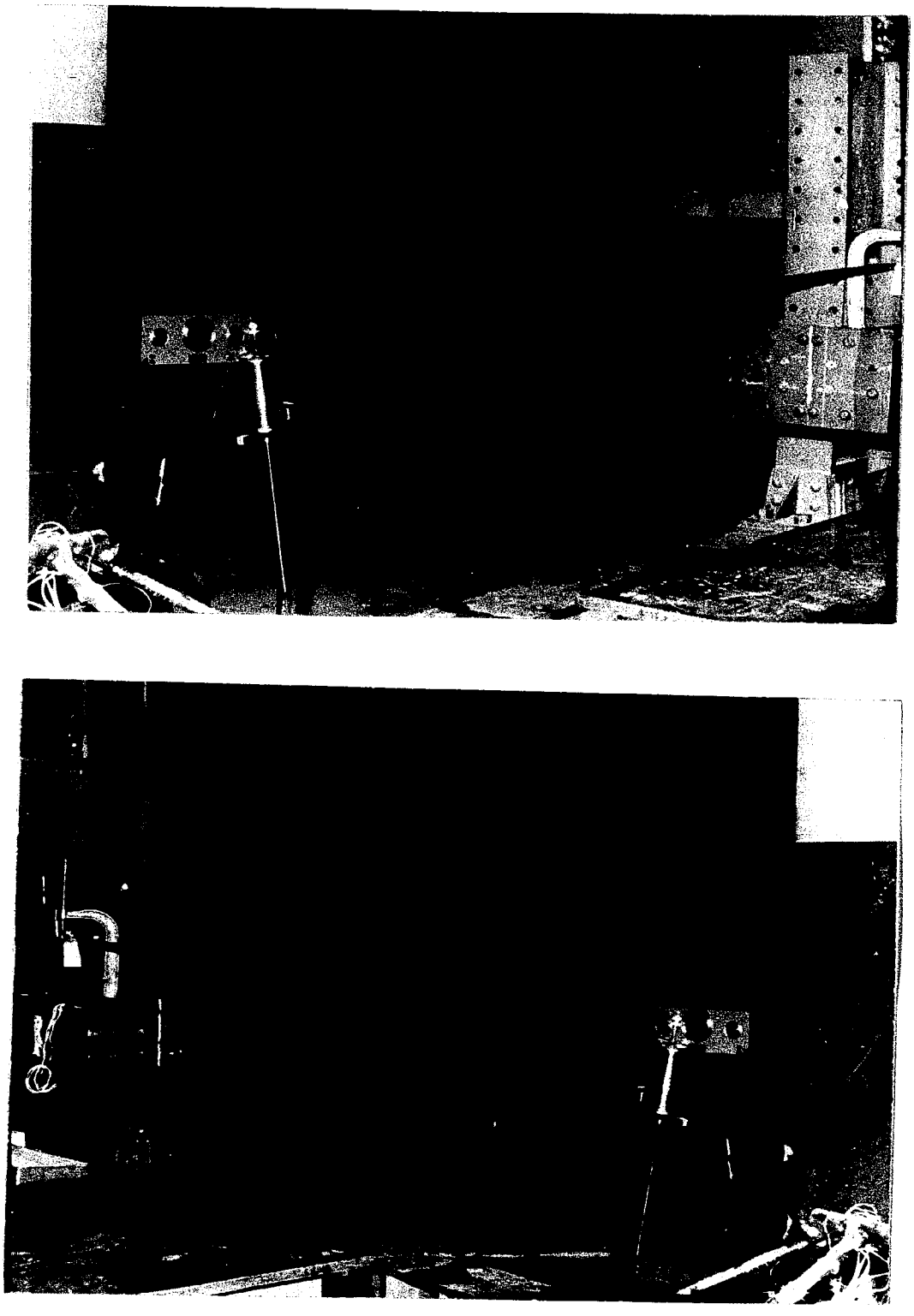


FIGURE 8

TEST SETUP OF A CONTACT ACCELEROMETER
UNDER DAMPED FREE VIBRATIONS

From Equation 1-6,

$$u = \sum_{n=1}^{\infty} \left[A_n \sin \frac{n(x-at)}{L_1} + B_n \cos \frac{n(x-at)}{L_1} + C_n \sin \frac{n(x+at)}{L_1} + D_n \cos \frac{n(x+at)}{L_1} \right].$$

Before the contact breaks,

$$\sum_{n=1}^{\infty} \left[-A_n \sin \frac{nat}{L_1} + B_n \cos \frac{nat}{L_1} + C_n \sin \frac{nat}{L_1} + D_n \cos \frac{nat}{L_1} \right]$$

$$= \sum_{n=1}^{\infty} \left[A_n \sin \frac{n(L-at)}{L_1} + B_n \cos \frac{n(L-at)}{L_1} + C_n \sin \frac{n(L+at)}{L_1} + D_n \cos \frac{n(L+at)}{L_1} \right].$$

Equating the coefficients of $\sin \frac{nat}{L_1}$ and $\cos \frac{nat}{L_1}$,

$$(A_n + C_n) = (B_n + D_n) \frac{1 - \cos n\lambda}{\sin n\lambda} = (B_n + D_n) \tan \frac{n\lambda}{2} \qquad 5\text{-}1$$

$$(-A_n + C_n) = (B_n - D_n) \frac{\sin n\lambda}{1 - \cos n\lambda} = (B_n - D_n) \cot \frac{n\lambda}{2} \qquad 5\text{-}2$$

From Equation 1-6 with x = 0,

$$\sum_{n=1}^{\infty} \left[(-A_n + C_n) \sin \frac{nat}{L_1} + (B_n + D_n) \cos \frac{nat}{L_1} \right]$$

$$= \sum_{n=1}^{\infty} \left[(B_n - D_n) \cot \frac{n\lambda}{2} \sin \frac{nat}{L_1} + (B_n + D_n) \cos \frac{nat}{L_1} \right]$$

$$= A_0 e^{-\alpha t} \cos \omega t \qquad 5\text{-}3$$

The acceleration at x = 0 is obtained by differentiating the above equation with respect to "t" twice,

$$-\sum_{n=1}^{\infty}\left[(B_n - D_n)\cos\frac{n}{2}A(n\omega)^2\sin n\omega t + (B_n + D_n)(n\omega)^2\cos n\omega t\right.$$

$$= A_0 e^{-\alpha t}\left[(\alpha^2 - \omega^2)\cos\omega t + 2\alpha\omega\sin\omega t\right]$$

To find the coefficients $(B_n - D_n)$ and $(B_n + D_n)$, multiply the above equation by Sin $m\omega t\, dt$ and integrate from $t = 0$ to $t = 2\pi/\omega$, and then multiply the above equation by Cos $m\omega t\, dt$ and integrate with the same limits. By the orthogonal relationship of the trigonometric series, the coefficients are obtained as follows:

$$\int_0^{\frac{2\pi}{\omega}} e^{-\alpha t}\cos\omega t\sin m\omega t\, dt$$

$$= \frac{(1 - e^{-\alpha\frac{2\pi}{\omega}})}{2}\left[\frac{(m+1)\omega}{\alpha^2 + (m+1)^2\omega^2} + \frac{(m-1)\omega}{\alpha^2 + (m-1)^2\omega^2}\right] \qquad 5\text{-}4$$

$$\int_0^{\frac{2\pi}{\omega}} e^{-\alpha t}\cos\omega t\cos m\omega t\, dt$$

$$= \frac{(1 - e^{-\alpha\frac{2\pi}{\omega}})}{2}\left[\frac{\alpha}{\alpha^2 + (m-1)^2\omega^2} + \frac{\alpha}{\alpha^2 + (m+1)^2\omega^2}\right] \qquad 5\text{-}5$$

$$\int_0^{\frac{2\pi}{\omega}} e^{-\alpha t} \sin \omega t \sin m\omega t$$

$$= \frac{1-e^{-\alpha\frac{2\pi}{\omega}}}{2}\left[\frac{\alpha}{\alpha^2+(m-1)^2\omega^2} - \frac{\alpha}{\alpha^2+(m+1)^2\omega^2}\right] \quad 5\text{-}6$$

$$(B_n - D_n)\cot\frac{n\lambda}{2}(n\omega)^2\frac{\pi}{\omega}$$

$$= \int_0^{\frac{2\pi}{\omega}} A_0 e^{-\alpha t}\left[(\omega^2-\alpha^2)\cos\omega t - 2\alpha\omega\sin\omega t\right]\sin n\omega t\, dt$$

$$= A_0(\omega^2-\alpha^2)\frac{1-e^{-\alpha\frac{2\pi}{\omega}}}{2}\left[\frac{(n+1)\omega}{\alpha^2+(n+1)^2\omega^2} + \frac{(n-1)\omega}{\alpha^2+(n-1)^2\omega^2}\right]$$

$$-2A_0\alpha\omega\frac{1-e^{-\alpha\frac{2\pi}{\omega}}}{2}\left[-\frac{\alpha}{\alpha^2+(n+1)^2\omega^2} + \frac{\alpha}{\alpha^2+(n-1)^2\omega^2}\right]$$

$$(B_n - D_n) = \frac{A_0(1-e^{-\alpha\frac{2\pi}{\omega}})\omega}{2\pi}\frac{1}{(n\omega)^2}\tan\frac{n\lambda}{2}$$

$$\times\left\{(\omega^2-\alpha^2)\left[\frac{(n+1)\omega}{\alpha^2+(n+1)^2\omega^2} + \frac{(n-1)\omega}{\alpha^2+(n-1)^2\omega^2}\right]\right.$$

$$\left.-2\alpha^2\omega\left[\frac{1}{\alpha^2+(n-1)^2\omega^2} - \frac{1}{\alpha^2+(n+1)^2\omega^2}\right]\right. \quad 5\text{-}7$$

$$(B_n + D_n)(n\omega)^2 \frac{T}{2} = \int_0^{\frac{2\pi}{\omega}} A_0 e^{-\alpha t}\left[(\omega^2-\alpha^2)\cos\omega t - 2\alpha\omega \sin\omega t\right]\cos n\omega t\, dt$$

$$= A_0(\omega^2-\alpha^2)\frac{1-e^{-\alpha\frac{2\pi}{\omega}}}{2}\left[\frac{\alpha}{\alpha^2+(n-1)^2\omega^2} - \frac{\alpha}{\alpha^2+(n+1)^2\omega^2}\right]$$

$$- A_0 2\alpha\omega \frac{1-e^{-\alpha\frac{2\pi}{\omega}}}{2}\left[\frac{(n+1)\omega}{\alpha^2+(n+1)^2\omega^2} + \frac{(n-1)\omega}{\alpha^2+(n-1)^2\omega^2}\right]$$

$$(B_n + D_n) = A_0 \frac{(1-e^{-\alpha\frac{2\pi}{\omega}})\omega}{2\pi} \frac{1}{(n\omega)^2}\left\{(\omega^2-\alpha^2)\left[\frac{\alpha}{\alpha^2+(n-1)^2\omega^2} + \frac{\alpha}{\alpha^2+(n+1)^2\omega^2}\right] - 2\alpha\omega\left[\frac{(n+1)\omega}{\alpha^2+(n+1)^2\omega^2} + \frac{(n-1)\omega}{\alpha^2+(n-1)^2\omega^2}\right]\right\} \quad 5\text{-}8$$

From Equation 1-6,

$$u = \sum_{n=1}^{\infty}\left[(A_n+C_n)\sin\frac{n\pi}{L_1}\cos\frac{n\alpha t}{L_1} + (-A_n+C_n)\cos\frac{n\pi}{L_1}\sin\frac{n\alpha t}{L_1} + (B_n+D_n)\cos\frac{n\pi}{L_1}\cos\frac{n\alpha t}{L_1} + (B_n-D_n)\sin\frac{n\pi}{L_1}\sin\frac{n\alpha t}{L_1}\right]$$

$$\left(\frac{\partial u}{\partial x}\right)_{x=L} = \sum_{n=1}^{\infty}\frac{n}{L_1}\left[(A_n+C_n)\cos n\lambda - (B_n+D_n)\sin n\lambda\right]\cos\frac{n\alpha t}{L_1} + \sum_{n=1}^{\infty}\frac{n}{L_1}\left[(A_n-C_n)\sin n\lambda + (B_n-D_n)\cos n\lambda\right]\sin\frac{n\alpha t}{L_1}$$

Substituting Equation 5-1 and Equation 5-2 in the above,

$$\left(\frac{\partial w}{\partial x}\right)_{x=L} = \sum_{n=1}^{\infty} \frac{n}{L_1}\left[(B_n + D_n)\tan\frac{n\lambda}{2}\cos n\lambda - (B_n + D_n)\sin n\lambda\right]\cos\frac{n\omega t}{L_1}$$

$$+ \sum_{n=1}^{\infty} \frac{n}{L_1}\left[-(B_n - D_n)\cot\frac{n\lambda}{2}\sin n\lambda + (B_n - D_n)\cos n\lambda\right]\sin\frac{n\omega t}{L_1}$$

$$= -\sum_{n=1}^{\infty} \frac{n}{L_1}(B_n + D_n)\tan\frac{n\lambda}{2}\cos\frac{n\omega t}{L_1} - \sum_{n=1}^{\infty} \frac{n}{L_1}(B_n - D_n)\sin\frac{n\omega t}{L_1}$$

$$= -\sum_{n=1}^{\infty} \frac{n}{L_1} A_0 \tan\frac{n\lambda}{2}\cos n\omega t \frac{(1 - e^{-\alpha\frac{25}{25}})\omega}{2\pi}\frac{1}{(n\omega)^2}$$

$$\times\left[(\omega^2 - \alpha^2)\left(\frac{\alpha}{\alpha^2 + (n-1)^2\omega^2} + \frac{\alpha}{\alpha^2 + (n+1)^2\omega^2}\right)\right.$$

$$\left. -2\alpha\omega\left(\frac{(n+1)\omega}{\alpha^2 + (n+1)^2\omega^2} + \frac{(n-1)\omega}{\alpha^2 + (n-1)^2\omega^2}\right)\right]$$

$$-\sum_{n=1}^{\infty} \frac{n}{L_1} A_0 \frac{(1 - e^{-\alpha\frac{25}{25}})}{2\pi}\omega\tan\frac{n\lambda}{2}\frac{1}{(n\omega)^2}\sin n\omega t$$

$$\times\left[(\omega^2 - \alpha^2)\left(\frac{(n+1)\omega}{\alpha^2 + (n+1)^2\omega^2} + \frac{(n-1)\omega}{\alpha^2 + (n-1)^2\omega^2}\right)\right.$$

$$\left. -2\alpha\omega\left(\frac{1}{\alpha^2 + (n-1)^2\omega^2} - \frac{1}{\alpha^2 + (n+1)^2\omega^2}\right)\right]$$

$$\left(\frac{\partial u}{\partial x}\right)_{x=L} = \frac{-A_o}{L_1}\frac{(1-e^{-\alpha\frac{2\pi}{\omega}})}{2\pi}\sum_{n=1}^{\infty}\tan\frac{n\lambda}{2}$$

$$\times\left\{\left[\frac{\alpha[-(2n+1)\omega^2-\alpha^2]}{[\alpha^2+(n+1)^2\omega^2]n\omega}+\frac{\alpha[(2n-1)\omega^2-\alpha^2]}{[\alpha^2+(n-1)^2\omega^2]n\omega}\right]\cos n\omega t\right.$$

$$\left.+\left[\frac{(n+1)\omega^2-(n-1)\alpha^2}{[\alpha^2+(n+1)^2\omega^2]n}+\frac{(n-1)\omega^2-(n+1)\alpha^2}{[\alpha^2+(n-1)^2\omega^2]n}\right]\sin n\omega t\right\} \quad 5\text{-}9$$

When $\alpha \to 0$, terms with $n \neq 1$ vanish.

$$\left(\frac{\partial u}{\partial x}\right)_{\substack{x=L\\ \alpha\to 0}} = \frac{A_o}{L_1}\frac{1-e^{-\alpha\frac{2\pi}{\omega}}}{2\pi}\frac{\alpha\omega^2}{\alpha^2\omega}\tan\frac{\lambda}{2}\cos\omega t$$

$$= \frac{A_o}{L_1}\tan\frac{\lambda}{2}\cos\omega t$$

This checks with Equation 2-9.

For the case with no damping, with the substitution of the above equation in Equation 2-1, the condition for breaking the contact is obtained. The calculation is shown in Table VIIa and Table VIIb.

From the experimental data, the amplitude of vibration was reduced by the ratio 5.6/10.8 or 1/1.93 in seven cycles. From the logarithmic decrement, the damping coefficient is found as follows:

Ln 1.93 = 7 x 1.06, Period T = 1.06 sec.

α = .0875 lbs per ft./sec., ω = 5.93 rad./sec.

M_1 = .074 lbs., M_2 = .0771 lbs.

L = 5".

$\sqrt{K/M_2}$ = 45.8

$a = \sqrt{K/M_2} \times L = 45.8 \times 5/12 = 19.1$ ft./sec.

$L_1 = a/\omega = 19.1/5.93 = 3.22$ ft.

$\lambda = L\omega/a = L/L_1 = .129$

$$\left(\frac{\partial u}{\partial x}\right)_{x=L} = -\frac{A_o}{L_1}\frac{1-e^{-.0875\times1.06}}{2\pi}\sum_1^\infty \tan .0645n$$

$$\times\left\{\left[\frac{.0875[(2n+1)5.93^2-.0875^2}{[.0875^2+(n+1)^2 5.93^2]n5.93}+\frac{.0875[(2n-1)5.93^2-.0875^2]}{[.0875^2+(n-1)^2 5.93^2]n5.93}\right]\cos n\omega t\right.$$

$$\left.+\left[\frac{(n+1)5.93^2-(n-1).0875^2}{[.0875^2+(n+1)^2 5.93^2]n}+\frac{(n-1)5.93^2-(n+1).0875^2}{[.0875^2+(n-1)^2 5.93^2]n}\right]\sin n\omega t\right\}$$

$.0875^2$ is negligible as compared to 5.93^2.

$$\left(\frac{\partial u}{\partial x}\right)_{x=L} = -\frac{A_o}{L_1}\frac{.088}{6.28}\sum_{n=1}^\infty\left\{\left[\frac{.0875(2n+1)}{n(n+1)^2 5.93}+\frac{.0875(2n-1)5.93}{[.0875^2+(n-1)^2 5.93^2]n}\right]\cos n\omega t\right.$$

$$\left.+\left[\frac{1}{n(n+1)}+\frac{(n-1)5.93^2-(n+1).0875^2}{[.0875^2+(n-1)^2 5.93^2]n}\right]\sin n\omega t\right\}\tan .0645n.$$

The above has been calculated as shown in Table VIIa and Table VIIb.

With e_o = 1/5,

K = .421 lb/in. The equivalent EA = 2.105 lbs per unit strain.

$$EA\left[e_0 - \left(\frac{\partial u}{\partial x}\right)_{x=L}\right] - M_1\left(\frac{\partial^2 u}{\partial t^2}\right)_{x=L} = 0$$

$$EA = E\frac{M_2 g}{\gamma L} = M_2\frac{a^2}{L}$$

$$EAe_0 = M_2\frac{a^2}{L}\left(\frac{\partial u}{\partial x}\right)_{x=L} + M_1\left(\frac{\partial^2 u}{\partial t^2}\right)_{x=L}$$

Let $\left(\frac{\partial u}{\partial x}\right)_{x=L} = -\frac{A_0}{L_1}\varphi, \qquad \left(\frac{\partial^2 u}{\partial t^2}\right)_{x=L} = -A_0\psi$

$$EAe_0 = M_2\frac{a^2}{L}\left(-\frac{A_0}{L_1}\varphi\right) - M_1 A_0\psi$$

$$\frac{EAe_0}{M_1 A_0 \omega^2} = -\frac{M_2}{M_1}\frac{a^2}{\omega^2 L L_1}\varphi - \frac{\psi}{\omega^2}$$

$$= -\frac{M_2}{M_1}\frac{1}{\lambda}\varphi - \frac{\psi}{\omega^2}$$

$$\frac{EAe_0}{M_1\left(\frac{\partial^2 u}{\partial t^2}\right)_{x=0}} = -\frac{M_2}{M_1}\frac{a^2}{L L_1}\frac{\varphi}{\psi} - 1$$

$$\frac{EAe_0}{M_1\left(\frac{\partial^2 u}{\partial t^2}\right)_{x=0}} = \frac{M_2}{M_1}\frac{\omega^2}{\lambda}\frac{\varphi}{\psi} + 1$$

$$= \left(\frac{.0771}{.074}\frac{5.93^2}{.129}\frac{\varphi}{\psi} + 1\right)$$

The above gives the condition required to open the contact.

From Table VIb,

$$\frac{\varphi}{\psi} = \frac{.0612}{34}$$

$$\frac{EAC_0}{M_1\left(-\frac{\partial^2 u}{\partial t^2}\right)_{x=L}} \leq 1.51$$

The test value is 1.66 as shown in Table VIII, so the discrepancy is ten percent. This may be due to some non-uniformity of the spring, which was wound by hand, and due to the error in the assumption of viscous damping in the system.

TABLE VIIa

FOURIER CONSTANTS FOR $\left(\frac{\partial u}{\partial x}\right)_{x=L}$ OF CONTACT ACCELEROMETER UNDER DAMPED FREE VIBRATIONS

n	$\overline{A_n}$	$\overline{B_n}$
1	.0612	-.00136
2	.000032	.00122
3	.000010	.00062
4	.000006	.00049
5	.000004	.00039
6	.000003	.00033
7	.000002	.00028
8	.000001	.00025
9	.000001	.00023
10	.000001	.00021

$$\left(\frac{\partial u}{\partial x}\right)_{x=L} = -\frac{L_1}{A_0}\sum_{1}^{\infty}(A_n \cos n\omega t + B_n \sin \omega t)$$

$$\overline{A_n} = \frac{.088}{6.28}\left[\frac{.0875(2n+1)}{n(n+1)^2 5.93} + \frac{.0875(2n-1)5.93}{[.0875^2+(n-1)^2 5.93^2]n}\right]$$

$$\overline{B_n} = \frac{.088}{6.28}\left[\frac{1}{n(n+1)} + \frac{(n-1)5.93^2-(n+1).0875^2}{[.0875^2+(n-1)^2 5.93^2]n}\right]$$

TABLE VIIb

CALCULATION OF $\left(\frac{\partial^2 u}{\partial t^2}\right)_{x=L}$ AND $\left(\frac{\partial u}{\partial x}\right)_{x=L}$
OF CONTACT ACCELEROMETER UNDER
DAMPED FREE VIBRATIONS

(1) ωt in degrees	0	30	60
(2) ωt in radians	0	.524	1.047
(3) t	0	.0844	.1766
(4) αt	0	.00773	.0155
(5) $e^{-\alpha t}$	1	1	.985
(6) $\cos \omega t$	1	.866	.500
(7) $\sin \omega t$	0	.500	.866
(8) $35.3 \cos \omega t$	35.3	30.6	17.7
(9) $1.038 \sin \omega t$	0	.52	.90
(10) (8)+(9)	35.3	31.12	18.6
(11) φ = (5)(10)	35.3	31.12	18.3
(12) $A_1 \cos \omega t$	.0612	.053	.0306
(13) $A_2 \cos 2\omega t$	.00003	.000015	-.000015
(14) $A_3 \cos 3\omega t$	.00001	0	-.00001
(15) $A_4 \cos 4\omega t$	.000006	-.000003	-.000003
(16) $A_5 \cos 5\omega t$	.000004	-.0000034	.000002
(17) $A_6 \cos 6\omega t$	.000003	-.000003	.000003
(18) $A_7 \cos 7\omega t$	.000002	-.0000017	.000001
(19) $B_1 \sin \omega t$	0	-.00068	-.00118
(20) $B_2 \sin 2\omega t$	0	.00106	0
(21) $B_3 \sin 3\omega t$	0	.00062	-.00062
(22) $B_4 \sin 4\omega t$	0	.00042	-.00042
(23) $B_5 \sin 5\omega t$	0	.00020	-.00032
(24) $B_6 \sin 6\omega t$	0	0	0
(25) $B_7 \sin 7\omega t$	0	-.00014	.00024
(26) $\varphi = \Sigma A_n \cos n\omega t + \Sigma B_n \sin n\omega t$	.06126	.0546	.02969

$$\left(\frac{\partial^2 u}{\partial t^2}\right)_{x=L} = -A_0 \varphi$$

$$\left(\frac{\partial u}{\partial x}\right)_{x=L} = -\frac{A_e}{L_1} \varphi$$

TABLE VIIb-Continued

90	120	150	180
1.57	2.096	2.62	3.14
.265	.354	.425	.530
.0232	.0310	.0372	.0464
.976	.970	.964	.954
0	-.50	-.866	-1
1	.866	.50	0
0	-17.7	-30.6	-35.6
1.04	.90	.52	0
1.04	-16.8	-30.1	-35.6
1.015	-16.3	-29.0	-34.0
0	-.0306	-.053	-.0612
-.00003	-.000015	.000015	.00003
0	.00001	0	-.00001
.000006	-.000003	-.000003	.000006
0	-.000002	.000003	-.000004
-.000003	.000003	-.000003	.000003
0	-.000001	.0000017	-.000002
-.00136	-.00118	.00118	0
0	-.00116	-.00106	0
-.00062	0	.00062	0
0	.00042	-.00042	0
.00039	-.00034	-.00034	.00020
0	0	0	0
-.00014	.00024	-.00014	0
-.00161	-.03277	-.05246	-.0612

TABLE VIII

EXPERIMENTAL RESPONSE OF CONTACT ACCELEROMETER - UNDER DAMPED FREE VIBRATIONS

Preset Force in lbs. EA_o (1)	Acceleration in mm. on Brush Recorder (2)	Acceleration* / g (3)	Top Mass x Acceleration (4)	$R = \frac{(1)}{(4)}$ (5)
Single Break of Contact				
.0135	.85	.121	.00895	1.51
.0286	1.50	.214	.01584	1.81
.0436	2.30	.329	.0244	1.78
.0586	3.35	.479	.0354	1.65
.0736	4.40	.629	.0466	1.58
			Average R	1.66
Multiple Breaks of Contact				
.015	.90	.1284	.0095	1.58
.030	1.65	.236	.0175	1.71
.045	2.40	.343	.0254	1.77
.060	3.40	.486	.0360	1.67
.0751	4.40	.629	.0465	1.61
.090	5.00	.715	.0529	1.70
.105	5.80	.830	.0614	1.71
			Average R	1.68

*2g = 14 mm. Theoretical R = 1.51. Discrepancy 10%.

CHAPTER VI

TWO PROPOSED TYPES OF CONTACT ACCELEROMETERS

For measuring accelerations of transient and steady state motions, the instrument is required to give (1) constant factor of proportionality for all frequencies in the motion, (2) constant time lag and (3) instantaneous damping out of responses. To meet these three requirements especially in high frequency transient motions, the spring mass effect must be completely eliminated. Based on this principle of eliminating the spring mass effect, the following two types of contact accelerometer are proposed.

Balanced Type Contact Accelerometer

In this type of accelerometer, four identical springs are used to balance the spring mass effect upon each other. The main parts of the instrument are shown in Figure 9. The four springs A, B, C and D are attached to the bar TU, which is fixed to the case, so the lower ends of the springs A and B and the upper ends of the springs C and D have the same motion as the case. The two levers RS and VW are hinged by needle bearings at points 2 and 5 respectively to the case. The four bell cranks FGH, IJK, LMN and OPQ of equal dimensions are hinged to the case at points 7, 8, 9 and 10 with the center of gravity of each bell crank on the

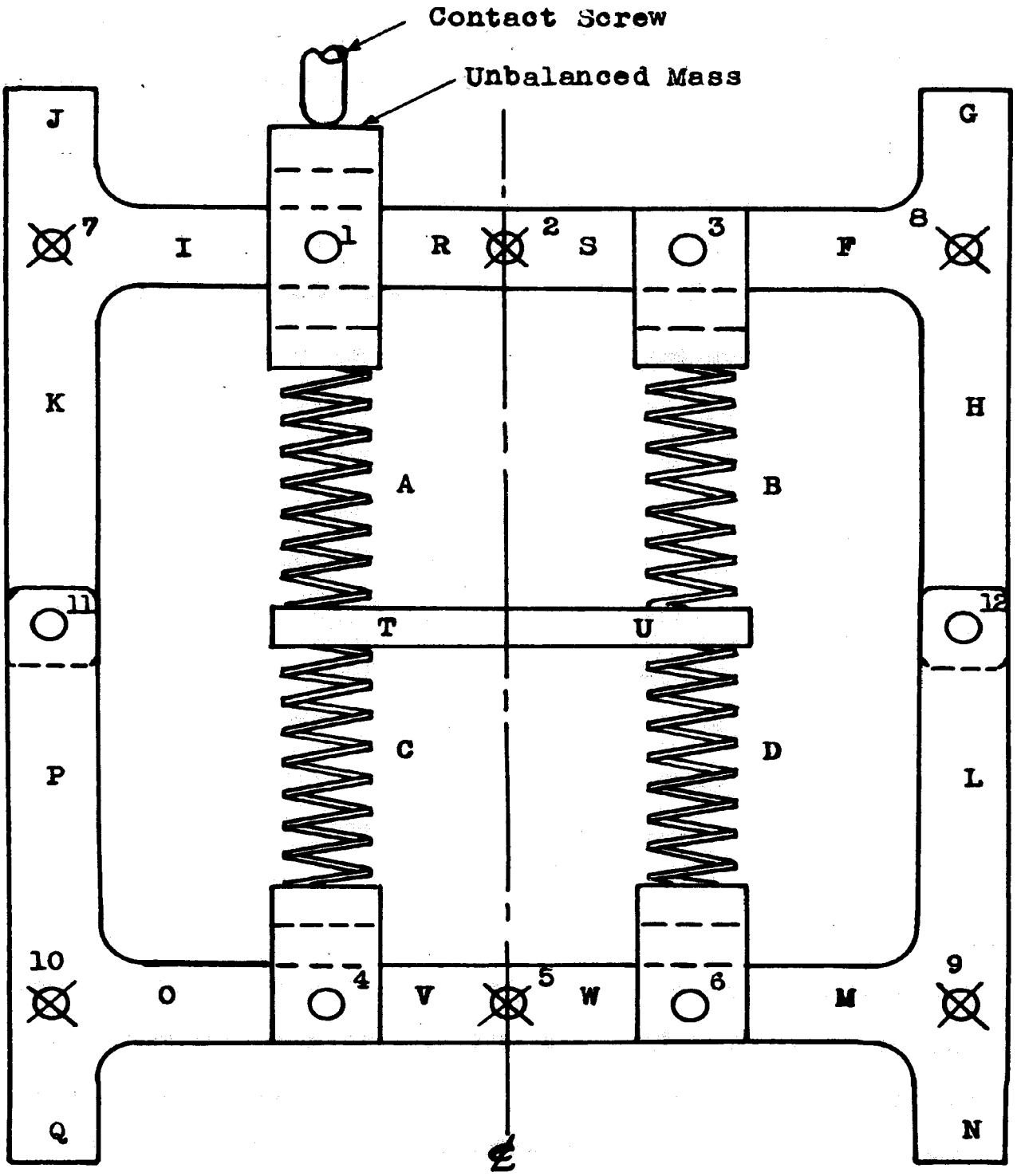


FIGURE 9

A BALANCED TYPE CONTACT ACCELEROMETER

⊗ Fixed hinges ○ Movable hinges

horizontal line passing through the respective hinge point. Hinge 1 connects the horizontal arm of the bell crank IJK, the left end of the lever RS and the unbalanced mass. Similarly, hinges 3, 4 and 6 connect the fittings of the springs, bell cranks and levers as shown. The vertical arms of the bell cranks are connected by hinges 11 and 12. The mass attached to spring A is greater than that attached to spring B. When this excess or unbalanced mass times acceleration overcomes the preset compressive force by the setscrew, the contact opens.

The springs and linkages are all in one plane, so this instrument is only sensitve to motions in this plane. This plane motion is resolved into a vertical motion, a horizontal motion, and a rotation about the center. For horizontal acceleration, each bell crank is balanced, i.e. J balances K, G balances H, etc. Therefore, horizontal acceleration produces no effect on the break of contact.

For an angular acceleration, if it is clockwise about the center, the inertia force will be counterclockwise. Vertical arms of bell cranks FGH and LMN tend to move in opposite directions at hinge 12, so the equal and opposite forces acting at hinge 12 balance each other. The lever RS tends to rotate counterclockwise about joint 2 causing the vertical arm of bell crank FGH to move to the left, but the lower lever VW also tends to rotate counterclockwise about joint 5 causing the vertical arm L of bell crank

LMN to move to the right. The horizontal force to the right from bell crank LMN balances the horizontal force to the left from the bell crank FGH. Due to this angular acceleration, the spring A tends to be compressed and the spring C tends to be under tension. Hinges 1 and 4 tend to move down, but the downward movement of hinge 1 tends to move joint 11 to the left and that of hinge 4 tends to move the joint to the right. So the horizontal forces at joint 11 from the springs A and C balance each other. Similarly, the mass effects of springs B and C balance each other. Therefore, angular acceleration produces no effect on the break of contact.

For vertical acceleration, since the bell cranks, levers, springs and fittings are symmetrical about the vertical center line, their mass effects balance out. The only unbalanced mass is that under the setscrew. The contact will open when this unbalanced mass times the acceleration overcomes the preset compression. This preset compression can be measured by adding weights to the unbalanced mass until the contact breaks. The lower ends of the springs A and B are subjected to the same motion, and these two springs are identical, so the strains at the top ends due to vertical acceleration of any motion are the same. This also holds for the two lower springs. Therefore, this instrument is free from the spring mass effect for all motions up to the instant of the break of the contact. It satisfies the three

requirements listed above and, at the same time, is only sensitive to the vertical acceleration.

Because of the manufacturing difficulties in making exactly identical springs, viscous damping is used to decrease the effects due to the small differences among the springs. The balance effect discussed above, has been tested with two identical springs as shown in Figure 10 with the test results shown in Figure 11.

Magnetic Type Contact Accelerometer

In this type of accelerometer, an electromagnet replaces the mechanical spring. The electromagnetic force replaces the preset compression. The electromagnetic force can be regulated by adjusting the current in the winding. This arrangement is shown in Figure 12. This is equivalent to a mass-less spring, hence the mass effect is completely eliminated and the three requirements are fulfilled.

For operating the indicating device, the gap at the contact must be finite. With a neon bulb, the gap required has been found to be of the order of 10^{-5} inch (Reference 13). To produce this gap, the mass must have an acceleration relative to the case. For the balanced type, the acceleration of the top mass relative to the case is accompanied by angular accelerations of the bell cranks, levers and fittings. This increases the amount of inertia force of the unbalanced

mass to give that gap, especially for high frequency motions. Hence the masses of the bell cranks, levers and fittings should be minimized in the detail design. In the magnetic type, the only moving mass relative to the case is the mass giving the inertia force in acceleration. The mechanical linkage does not exist, so no extra force is required to accelerate linkages. Besides, as soon as the contact breaks, the magnetic force on the mass is reduced instead of being increased as is the case with the mechanical spring. This reduces the extra inertia force required to get the finite gap after the contact breaks. Tests have been run with this magnetic type contact accelerometer and the results are favorable as shown in Figure 13.

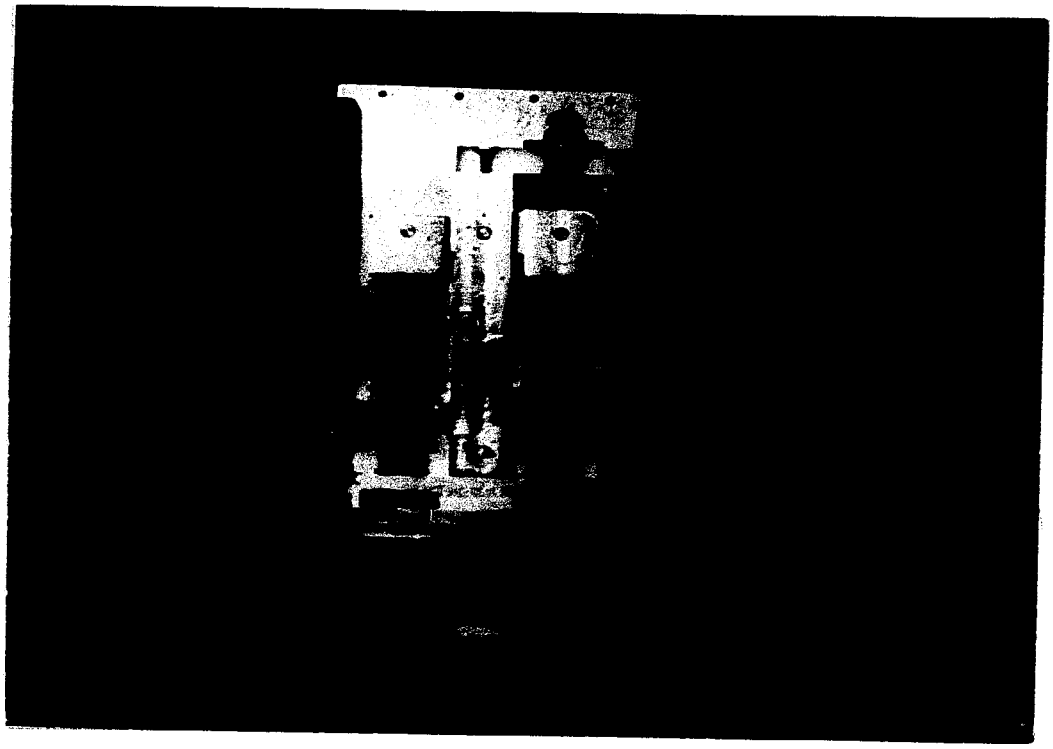


FIGURE 9

A CONTACT ACCELEROMETER WITH SPRING MASSES
BALANCED UNDER VERTICAL MOTIONS

FIGURE 11

EXPERIMENTAL RESPONSES OF A BALANCED TYPE CONTACT ACCELEROMETER WITH TWO SPRINGS

K_{ac} = 46 gm. Attenuator [illegible]: No. 3

With oil damping. Viscosity: 1.24 x 10^{-4} slug/ft. sec. @ 100° F.

Test data made 8/4/52 Theoretical

Temperature at test 80° F.

Note: Spring mass effects balance each other only for natural acceleration but not for angular acceleration.

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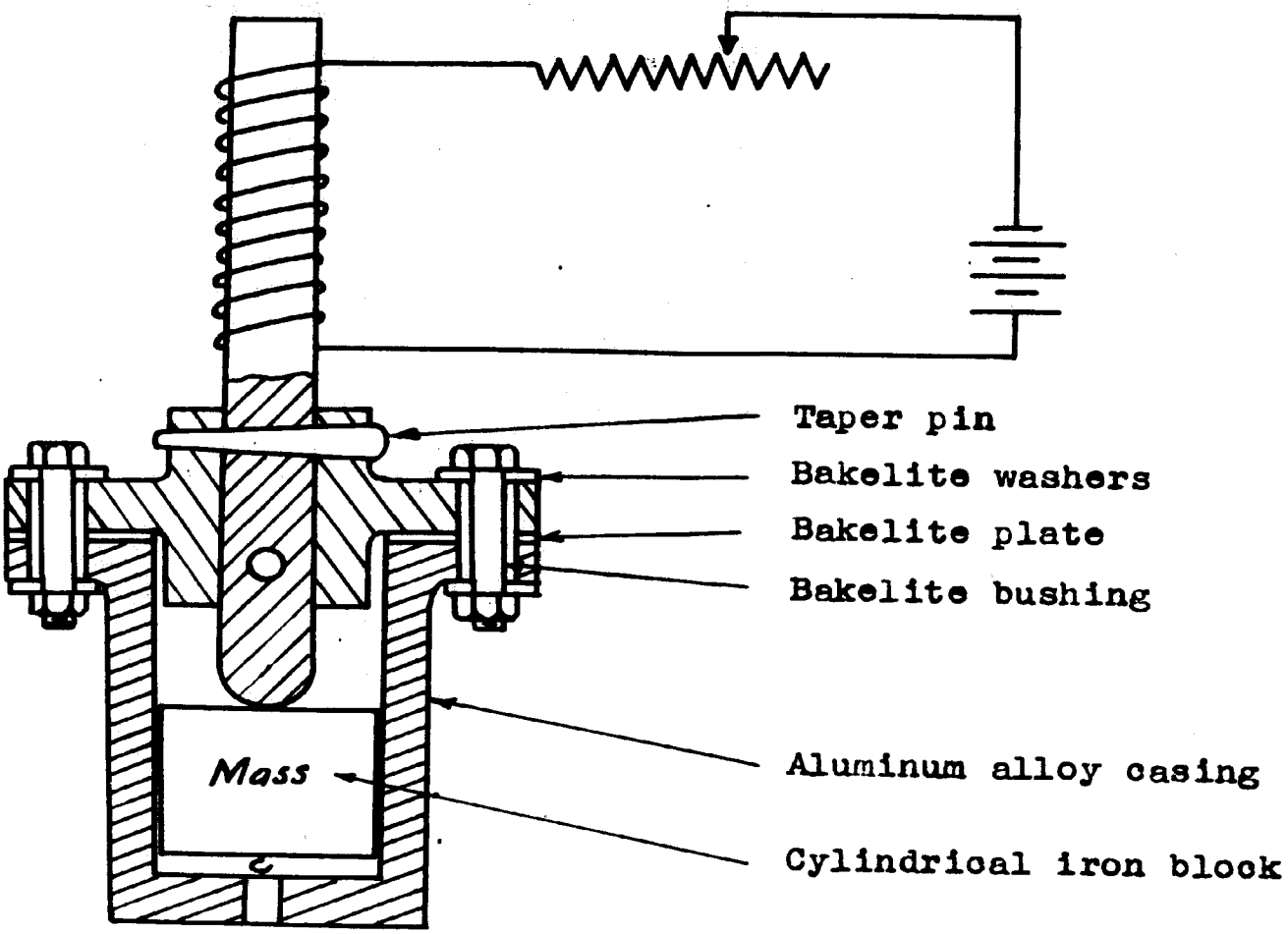
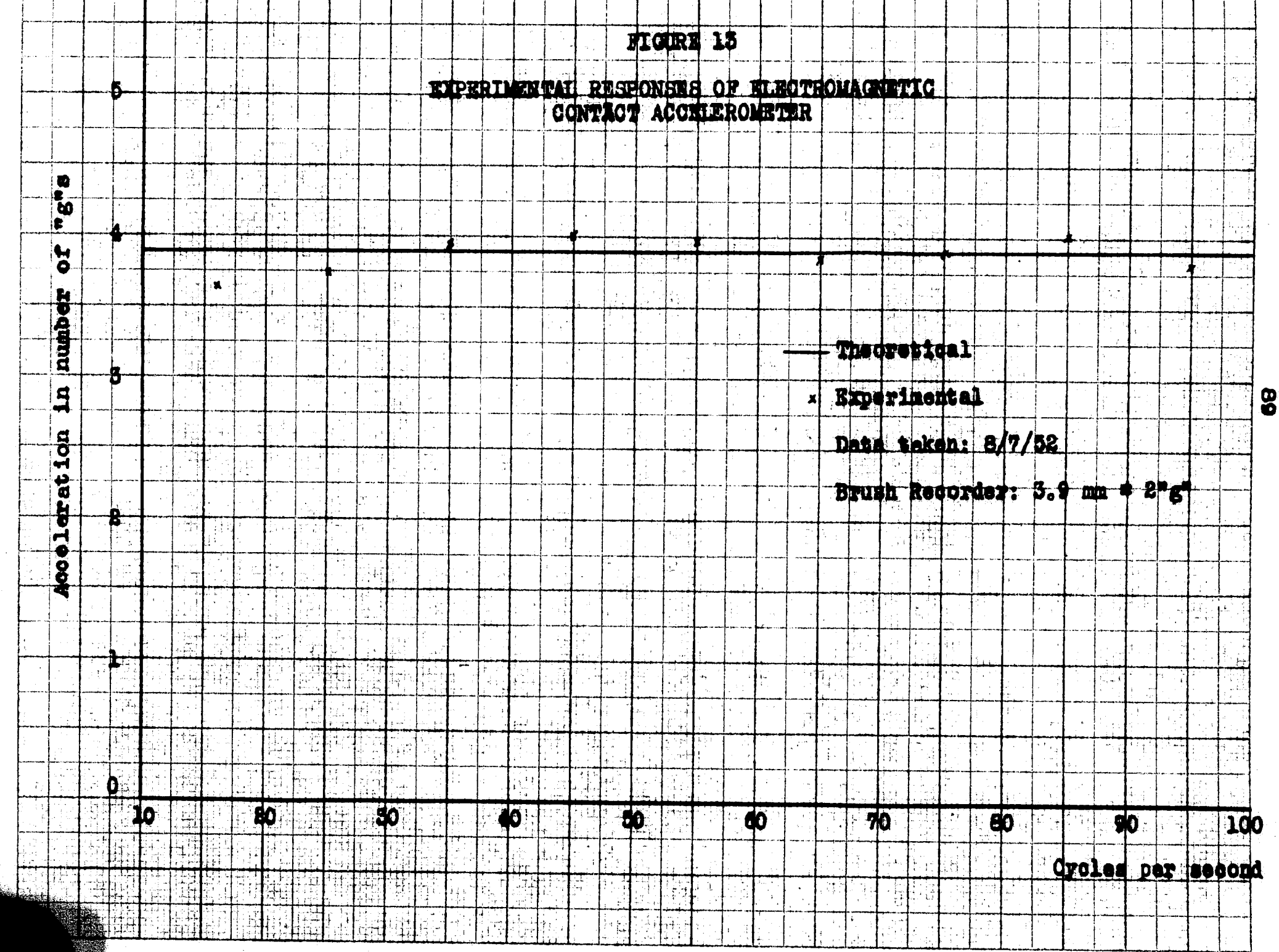


FIGURE 12

MAGNETIC TYPE CONTACT ACCELEROMETER

FIGURE 13

EXPERIMENTAL RESPONSES OF ELECTROMAGNETIC CONTACT ACCELEROMETER



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CHAPTER VII

GENERAL CONCLUSIONS AND SUGGESTIONS FOR FURTHER STUDY

General Conclusions

From the equations, which are derived in Chapters I to IV, and checked experimentally as shown in Chapter V, the unreliable responses of the break of contact of the conventional spring-mass contact accelerometer, with and without viscous damping, are showed to be caused by the following two main sources of errors:

(1) As the resonant frequency of the spring is approached or exceeded, the constant of proportionality "R" between the response and the measured acceleration varies appreciably, and the phase angle "β" deviates greatly from being proportional to the frequencies giving different time lags which distort the responses. The variation in "R" would give misleading responses for all motions of different frequencies including simple harmonic motions. The different time lags would distort responses to periodic motions composed of motions of different frequencies.

(2) The response to acceleration at one instant is not damped out instantly. Therefore, the motions of the different parts of the spring, caused by preceeding motions on the accelerometer, will introduce error to the response

of the acceleration of the succeeding motions as seen in the case of transient motions shown in Chapter II.

Near or exceeding the natural frequency of the spring, the variations of "R" and time lag "β", as shown in Figures 4, 5 and 7, are large. For transient motions such as shock or impact, the response caused by the previous motion also introduces error by being superposed on the response due to the succeeding motions. This explains mainly why the conventional spring-mass contact accelerometer is unreliable for high frequency motions.

As seen from the equations derived, the errors mentioned are due to the dynamic effects of the mass of the spring. In order to eliminate these errors, two new types of contact accelerometers, free of spring mass effects are proposed. One is a Balanced Type, in which four identical springs are used to balance out the mass effect (Figure 9). The other is a Magnetic Type, in which the magnetic force is used in place of spring force as shown in Chapter VI, so a mass-less spring is obtained. Both types are free from the two errors. The magnetic type has the advantage of having less movable parts. At the same time, after the contact breaks, the magnetic force is reduced instead of being increased as is the case of a mechanical spring force. This fact will help to increase the gap to the amount required for the operation

of the signal.

Suggestions for Further Study

A method of calculating response after contact breaks, has been derived for the spring-mass contact accelerometer as shown in Chapter III. As a finite gap is required to operate the signal, it is suggested that further study be made on the variation of the magnetic force with the increase of gap and on the calculation of the acceleration needed for obtaining the required gap for both steady and transient motions for the two types of contact accelerometers proposed.

A group of peak accelerometers with cantilever beams as springs with different preset forces has been used to indicate acceleration at a particular instant (Reference 12). An instrument with a number of the magnetic type units with different electromagnetic forces, will indicate the acceleration at one instant. If a number of these instruments are electrically controlled to operate in turn at different instants, the acceleration versus the time curve can be obtained. The break of contact of the new type of contact accelerometer is free from the spring mass effect, so an accelerometer built with these units would give much more reliable acceleration versus time curve for transient

and steady state motions. It is suggested that this accelerometer be designed, built and tested. This would be valuable in the study of high frequency steady state and transient motions including shocks and impacts.

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