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**McALOON, Sister Mary deLourdes, 1923-
AN EXPLORATORY STUDY ON TEACHING
UNITS IN LOGIC TO GRADES THREE AND
SIX.**

**The University of Michigan, Ph.D., 1969
Education, theory and practice**

University Microfilms, Inc., Ann Arbor, Michigan

*AN EXPLORATORY STUDY ON TEACHING UNITS
IN LOGIC TO GRADES THREE AND SIX*

by
Sister Mary deLourdes McAloon

A dissertation submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy in
The University of Michigan
1969

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ABSTRACT

*AN EXPLORATORY STUDY ON TEACHING UNITS
IN LOGIC TO GRADES THREE AND SIX*

Sr. Mary de Lourdes McAloon

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PURPOSE

This study was designed to investigate the role of logic in mathematics in grades 3 and 6 and to examine the teaching of various principles of logical thinking and their relation to learning mathematics.

PROCEDURE

Twenty-five teachers, 1100 students from middle class socio-economic backgrounds in grades three and six from 16 schools in the Archdiocese of Detroit participated in the project for eight weeks in the 1967-68 school year. The 16 hours of logic instruction were taken solely during the mathematics class periods. Four treatments in each grade were designed: LM, in which logic was taught interwoven with mathematics; L, in which logic was taught separately from mathematics; I, in which teachers of these

students attended in-service classes in mathematics; C, the control group in which students were not taught logic and whose teachers had no in-service classes during this study. Teachers of both LM and L groups attended in-service classes in logic during the project.

Units in logic included topics such as: sets, relation of sets, statements, truth-falsity, class logic such as meaning of all, some, none, disjunction, conjunction, negations and implications, basic ideas of validity and invalidity.

Instruments used in the study were: adapted forms of Cornell Class and Conditional Reasoning Tests, Wisconsin Contemporary Elementary Mathematics Test, given to all groups as pretest and posttest. Groups LM and L took a Logic Final and Logic Retention Test based on logic material taught. An attitude test of seven items was given to all groups at the end of the project. Students were of average and above average intelligence as indicated by school records and I. Q. as measured by the California Test of Mental Maturity.

FINDINGS

1. The group taught logic scored significantly higher, $p \leq .01$, on the mathematics achievement test than the group not taught logic.

2. Although the group taught logic with mathematics scored higher in mathematics than the group taught logic

separated from mathematics, these differences were not significant in either grade.

3. The groups taught logic scored significantly higher, $p \leq .01$, than groups not taught logic on the reasoning tests in both grades.

4. There was no significant difference in scores on the class and conditional reasoning tests between those taught logic interwoven with mathematics and those taught logic separated from mathematics. In the Logic Final, the LM group scored significantly higher, $p \leq .05$, in grade 3 and the L group in grade 6 scored significantly higher, $p \leq .01$, than LM in both the Logic Final and Logic Retention Test.

5. There was a significant difference, $p \leq .01$, between the scores of students in grade 3 and those in grade 6 in achievement on the reasoning and logical thinking tests.

6. There was no significant difference in the logic or mathematics achievement between boys and girls at either third or sixth grade level.

7. Symbolism and the principles of transitivity in both class and conditional reasoning were easy for both grades. Generally class reasoning was easier than conditional reasoning. In class reasoning, the principle that

class exclusion is symmetric and in conditional reasoning, modus ponens, were in the highest rankings in the percent of correct responses.

8. Fallacy principles and the principles of contraposition and converse, while more difficult to teach at both levels, were more difficult to teach in grade 3 than in grade 6.

9. Tests were designed to have Yes, No, and Maybe answers. Items with a correct answer of Maybe were answered the least well of any group on both class and conditional reasoning tests.

10. Answers to principles of transitivity, modus ponens, and class exclusion being symmetric indicated that about 70% of the students knew these logical ideas prior to instruction.

11. Children in grade 3 had more trouble with items which had content not familiar or specific to them than those in grade 6.

DISCUSSION

This study, in answering some questions about the relation of logical thinking and mathematics achievement, has raised additional questions. Taking time from mathematics did not decrease mathematics achievement, but, in fact, helped it. However, optimum amount of time to be devoted to logic is yet to be determined. More work is needed with small groups to make it easier to assess the thinking processes of the child.

ACKNOWLEDGMENTS

I wish to express my appreciation:

To Dr. Joseph N. Payne, my advisor and Chairman of my Doctoral Committee for his perceptive and constructive criticism, encouragement and advice.

To members of my Doctoral Committee for their helpful suggestions.

To Dr. Robert Ennis, Cornell University, for his generosity in sharing his research and for his encouragement during this study.

To the teachers and students participating in this project for their genuine enthusiasm and time given to the study of logic and mathematics.

To my professional colleagues for their assistance and encouragement during this study.

To my secretary, Mrs. Sarina Krupp, for her generous assistance in many aspects of this doctoral study.

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CHAPTER I

STATEMENT OF THE PROBLEM

In the early years of the contemporary revolution in mathematics education, much emphasis was given to the content. In the elementary school, the stress seemed to be placed on the relatively new topics such as sets, numeration systems, certain topics in geometry not previously part of the elementary curriculum, or to the change in grade placement of topics. Recently there has been a revival of interest in methodology and in the behavioral aspect of learning and teaching including a growing awareness of and alertness to the logical and psychological aspects of mathematics.

The Committee on the Undergraduate Program in Mathematics of the Mathematics Association of America (76), among other groups, has recommended that students acquire an understanding of the language and nature of deductive reasoning. While it is not intended that they be introduced to formal logic, they should understand what it means to prove something. In explaining these recommendations, the report suggests the inclusion of topics from logic such as the meaning of all, none, inclusion, implication, if-and-only-if. This suggests, at least, that elementary teachers are expected to teach such topics directly or to use them as

background for teaching other mathematical topics.

Piaget's monumental studies on the thought processes of children have gained much attention in pedagogical circles but there has been a paucity of research regarding the role of experience in the development of logical thinking. Questions such as the following are heard often.

What degree of rigor or mathematical precision in language and logic is appropriate at various grade levels? Can we structure experience to accelerate this cognitive growth which Piaget describes as developmental stages?
(80:2)

There has been an obvious need, from an educational standpoint, to investigate the relationship between the development of cognitive abilities described by Piaget and the intellectual components of school mathematics. Furthermore, there is ". . . a need, both from a theoretical and empirical point of view, to build a bridge of knowledge about children's thinking and what is or might be done in school to cultivate children's thinking." (1:1)

Consistency of thought, "straight thinking," critical thinking, logical thinking--all have been a major objective in elementary and secondary mathematics. Piaget, in his work on the development of cognitive processes in children, did not indicate the specific role of teachers in this developmental process.

THE PROBLEM

This study, designed to investigate the role of logic in elementary school mathematics, is concerned with examining the teaching of various principles of logical thinking and their relation to learning mathematics. It is not as much concerned about whether the stages can be accelerated, as Piaget called this the "great American question," (31) as it is to ferret out specifically those aspects of logical thinking which will aid students in learning mathematics, and perhaps, because of their general applicability, save him valuable learning time in his school career.

Certain teaching strategies and lessons were offered to children to give them the opportunities of manipulating logical ideas such as class inclusion, class exclusion, implication, conjunction and disjunction in a way that is coordinate with and cognizant of their maturity, experience, and level of mathematical achievement.

Four treatment groups were used: LM, with logic taught and interwoven with mathematical ideas; L, logic taught but not related to mathematics; I, in-service training for teachers in mathematics; and C, a control group. Teachers in treatments LM and L received in-service training in logic while teachers in I, a control group for in-service training, received in-service in mathematics on the content they were teaching.

Children in grade 3 and 6 were selected for the study. In an effort to coordinate some of the logical ideas with mathematics, it was necessary to study closely the mathematical sequence, some of the major ideas of logic, and interweave the two in a sequentially realistic way.

QUESTIONS ASKED IN THE STUDY

The major questions asked in the study were:

1. Can the teaching of logic interwoven within mathematics significantly affect the mathematics achievement of third and sixth grade students?
2. Can the teaching of logic significantly affect the development of logical thinking in third and sixth grade children?
3. Is there a difference in the attainment of logical thinking ability in third and sixth grade children? If so, in what way is there a difference?
4. How much logic can be taught to third graders? to sixth graders?
5. What are the trouble areas for third graders in logic? for sixth graders?
6. What are the easy areas of logic for third graders? for sixth graders?
7. What did the third graders seem to have learned in logic before their logic instruction? sixth graders?
8. What specific mathematical topics seem to be aided by instruction in logic for third graders? for sixth graders?

9. What are some of the similarities and differences in the logical thinking ability of third and sixth graders?

Certain facets of the problem posed in question form are stated in null hypothesis form. Questions 1-3 give rise to six hypotheses stated below.

Hypothesis 1. There is no significant difference in the mathematics achievement of students in grade 3 and grade 6 between those who have been taught logic and those who have not been taught logic.

Hypothesis 2. There is no significant difference in mathematics achievement of students in grade 3 and grade 6 between those who have been taught logic interwoven with mathematics and those who have been taught logic separated from and not linked with mathematics instruction.

Hypothesis 3. The teaching of logic does not significantly affect the development of logical thinking abilities in grade 3 and grade 6.

Hypothesis 4. There is no significant difference in logic achievement in grade 3 and grade 6 between those who have been taught logic interwoven with mathematics and those who have been taught logic separated from and not linked with mathematics instruction.

Hypothesis 5. There is no significant difference in the attainment of logical thinking abilities between third and sixth grade students.

Hypothesis 6. There is no significant difference in the logic achievement between boys and girls in grade 3 and in grade 6.

Answers to the other questions rely more on analysis of individual items on tests, percents of correct responses and written reports of teachers. It was difficult to state these questions in null hypothesis form.

In this chapter, the writer has presented the problem, defined the purpose of the study. Chapter II offers a review of the literature relating to the problem; Chapter III presents the research design with detailed description of the instruments. Chapter IV offers the analysis of data, with Chapter V presenting a restatement of the problem, conclusions, and suggestions for further research.

CHAPTER II

RELATED LITERATURE

It would be very helpful to define precisely what logical thinking is. However, it is far too broad an area to expect anything but what one could call an "operational description," something that, while it is there, is nonetheless a bit nebulous to put into a single, neat category.

Characterizations of logical thinking have been made by many people, including such notables as Dewey and Russell. On the current educational scene, Hill made the definition ". . .for present purposes, 'logical reasoning' is defined as, and limited to, the ability to derive correct conclusions from sets of given premises." (51:3)

Dewey made a distinction between proximate and ultimate subject matter of logic as follows:

Contemporary logical theory is marked by an apparent paradox. There is general agreement as to its proximate subject-matter. With respect to this proximate subject-matter no period shows a more confident advance. Its ultimate subject matter, on the other hand, is involved in controversies

which show little sign of abating. Proximate subject-matter is the domain of the relations of propositions to one another, such as affirmation-negation, inclusion-exclusion, particular-general, etc. No one doubts that the relations expressed by such words as is, is-not, if-then, only (none but), and, or, some-all, belong to the subject matter of logic in a way so distinctive as to mark off a special field. (25:1)

Dewey was concerned with the psychological, the process of thinking, the logical method as opposed to unchanging logical form. He warned that subject matter of formal logic is strictly impersonal and yet again he states, "It does not follow, however, that formal treatment lacks all value for education. It has its value, provided it is put and kept in its place." (24:72-73)

The problem of definition is complex because there are many facets. Russell wrote:

It is possible to make a logical distinction between the materials of thinking (such as perceptions), the motives in thinking (such as attitudes influencing it), the processes of thinking (such as classifying), and the ability in thinking which produces ease and precision in problem situations. (107:27)

Recognizing the difficulty of precise definition, a somewhat global view of logical thinking was taken in this study. The intent of the view of logical thinking is demonstrated in Chapter III in the outline of the logic material taught.

If logic were to be taken in an even broader sense to mean a set of actions with logic-like rules, such as transitivity, then children's behavior at all age levels manifests some form of logic.

Since logic is related to intelligence, as are all cognitive processes, some basic ideas about intelligence and thinking must be made. Three assumptions about intelligence are made, derived from the works of Piaget and others:

1) Intelligence is not a unitary trait but the variety of thinking skills by which a person processes, organizes and manipulates information from the environment.

2) Children learn to be intelligent. By interaction with environment, a child gradually develops the skills and understanding of logical thought that enable him to deal with the world rationally and creatively. This assumption does not deny any hereditary influence in the development of the intellect but focuses attention on the importance of providing appropriate learning experiences for children.

3) Intellectual growth proceeds through a definite sequence of stages or steps. (32) (100) (104) (35) (118)

BASIC VOCABULARY IN PIAGET

Since some of this study is related to the stages of cognitive development as identified by Piaget, it is necessary to be familiar with some basic ideas which pervade all of his works on cognitive development in children. The

descriptions are given as operational and, in no way, complete.

An Operation is a type of action; it can be carried out either directly, in the manipulation of objects, or internally, when it is categories or (in case of formal logic) propositions which are manipulated. . Roughly, an operation is a means for mentally transforming data about the real world so that they can be organized and used selectively in the solution of problems. An operation differs from simple action or goal-directed behavior in that it is internalized and reversible. (60:xiii)

Piaget describes a wide variety such as infralogical operations which are cognitive actions bearing on position and distance relationships apropos of concrete spatiotemporal objects of configurations, and logical operations in which considerations of spatial and temporal positions, proximity are essentially irrelevant. (38:166) (59:292)

Equilibration is a process whereby there is a consistent and constant integration and reintegration of knowledge--new pieces of information are acquired and made parts of larger wholes. . It is the mechanism of transition, a process operating continuously in all exchanges between the growing subject and his environment, a propellant for change and transition. This continuous process gives rise to successive, essentially discontinuous equilibrium states, that is, organized systems of actions whose attributes as systems are describable in equilibrium terms. (38:238)

Adaptation is the cognitive striving of the organism--the thinking person--to find an equilibrium between himself and his environment and it depends upon two interrelated processes, according to Piaget, accommodation and assimilation.

Assimilation involves a person's adaptation of the environment to himself and represents the individual's use of his environment as he would conceive it.... Experiences are taken in or absorbed only as far as the individual himself can preserve and consolidate them in terms of his own subjective experience.

Accommodation represents the impact of the actual environment. To accommodate is to conceive and to incorporate the environmental experience as it truly is.... (104:68) (75:86) Both accommodation and assimilation always act together. They are interlocked; assimilation is always balanced by the force of accommodation while accommodation is possible only with the function of assimilation.

Reversibility is the permanent possibility of returning to the starting point of the operation in question. From a structural standpoint, it can appear in either of two distinct and complementary forms, as indicated by Piaget:

First, one can return to the starting point by canceling an operation which has already been performed--i.e., by inversion or negation. In this case, the product of the direction operation and its inverse is the null or identical operation.

Second, one can return to the starting point by compensating a difference (in the logical sense of the term)--i.e., by reciprocity. In this case, the product of the two reciprocal operations is not a null operation but an equivalence. (60:272)

Inversion and reciprocity are necessary conditions of equilibrium for the most elementary behavior as well as for the most highly organized. Thus, with variations in form, they are found at all developmental stages.

Conservation is the cognition that certain properties (quantity, number, length, etc.) remain invariant (are conserved) in the face of certain transformations (displacing objects or object parts in space, sectioning an object into pieces, changing its shape, etc.)

PIAGET'S STAGES IN COGNITIVE DEVELOPMENT

In order to understand more fully some of the aspects of this study, it is necessary to review briefly the stages of development in cognitive growth which Piaget has formulated. Although Piaget believes that the order in which stages appear holds true for all children, he also believes as Elkind points out, "that the ages at which the stages evolve will depend upon the native endowment of the children and upon the quality of the physical and social environment in which he is reared." (31:59) In a very real sense, then, Piaget's is both a nature and nurture theory.

These stages can be separated into three phases which can be further subdivided:

1. Period of sensory-motor intelligence
(0-2 years)
2. Period of preparation for and organization
of concrete operations (2-11 years)
3. Period of formal operations (11-15)

Ages are approximate and the development is a gradual, growing sort of process. As Piaget says, "Mental development is a continuous construction comparable to the erection of a

vast building that becomes more solid with each addition." (100:4) It is almost axiomatic of Piaget's approach to development that a given period can be properly understood only in the context of earlier ones from which it springs. (38:202)

Perhaps it would be wise to add a note here about these stages of development; they are listed as three in some books, four in others, even six. All really are describing the same stages of growth. In one of his most recent publications, Piaget writes:

For greater clarity, we shall distinguish six stages or periods of development which mark the appearance of these successively constructed structures:

1) The reflex or hereditary stage, at which the first instinctual nutritional drives and the first emotions appear.

2) The stage of the first motor habits and of the first organized percepts, as well as of the first differentiated emotions.

3) The stage of sensorimotor or practical intelligence (prior to language), of elementary affective organizations, and of the first external affective fixations. These first three stages constitute the infancy period--from birth till the age of one and a half to two years--i.e., the period prior to the development of language and thought as such.

4) The stage of intuitive intelligence, of spontaneous interpersonal feelings, and of social relationships in which the child is subordinate to the adult (ages two to seven years or 'early childhood').

5) The stages of concrete intellectual operations (the beginning of logic) and of moral and social feelings of cooperation (ages seven to eleven or twelve or "middle childhood.")

6) The stage of abstract intellectual operations, of the formation of the personality and of affective and intellectual entry into the society of adults (adolescence).
(100:5-6)

This study was done with two levels of age, namely of children in third grade with average age of 8.7 years and children in sixth grade with average age of 11.6 years. In order to examine more closely the concrete operational and formal operational periods, distinctions between the two stages are made in terms of definite logical principles and activities.

CHARACTERISTICS OF PRE-OPERATIONAL THOUGHT

The following description gives the major characteristics of pre-operational thought:

- . "representation formation;" internalization of actions into thoughts;
- . actions occurring in the past and elsewhere added to actions occurring in the child's immediate spatial environment;
- . mental division of objects and collections into parts;
- . absence of concepts of conservations on any level higher than sensory-motor;

- . absence of reversible operations;
- . child's judgments of quantity lacking systematic transitivity.

CHARACTERISTICS OF CONCRETE OPERATIONS

Major characteristics of concrete operations are:

- . attainment of a state of "mobile" equilibrium, that is, the acquisition of the character of reversability (of being able to return to their original state or starting point);
- . operations carried out only on objects themselves;
- . operations belonging to the logic of classes and relations but not taking into account the totality of possible transformations of classes or relations (i.e., their combinatorial possibilities);
- . classification or inclusion of classes under each other other;
- . seriation or the linking of assymmetrical transitive relations into a system;
- . acquisition of logic techniques in this concrete operational period restricted in two essential respects:
 - a) operations at this level have an insufficiently formal character; not yet completely disassociated from concrete

data to which they apply; operations develop separately field by field, resulting in a progressive structuralization of these fields, without complete generality being attained;

- b) Operational systems are fragmentary. One can, with the aid of concrete operations, classify, order serially, form equalities or set up correspondence between objects without these operations being combined into a single structural whole;

- . operations not yet completely achieving equilibrium.

CHARACTERISTICS OF PROPOSITIONAL OR FORMAL OPERATIONS

Major characteristics of propositional or formal operations are:

- . new feature, reasoning by hypothesis.
Hypothetico-deductive reasoning is characterized by possibility of accepting any sort of data as purely hypothetical and reasoning correctly from them;
- . thought no longer proceeding from actual to theoretical but starting from theory so as to establish or verify actual relationships between things;

- . . . not merely coordination of facts about the actual world but hypothetical-deductive reasoning drawing out the implications of possible statements and thus, giving rise to a unique synthesis of the possible and necessary;
- . subjects' logic now concerned with propositions as well as with objects;
- . new kind of operations: propositional operations which are more than just a simple setting up of concrete correspondences;
- . . . discovery of certain kinds of invariants which fall outside the range of empirical verifications;
- . most interesting psychological problem connected with appearance of a new group of operations or "operational schemata," apparently unrelated to the logic of propositions:
 - i) deals with combinatorial operations in general (combinations, permutations, aggregations)
 - ii) proportions
 - iii) mechanical equilibrium, involving equality between action and reaction;

- . There are other operational schemata such as schemata relating to probabilities, correlations, multiplicative compensations, etc.

In summary, then, two important acquisitions occur in the propositional or formal operations period:

- 1) the logic of propositions, which is both a formal structure holding independently of content and a general structure coordinating the various logical operations into a single system;
- 2) a series of operational schemata which have no apparent connection with each other nor with the logic of propositions.

Certain of the characteristics indicated in concrete operational thinking and formal operational thinking are pertinent to the study of logical thinking in this project as will be shown in Chapter V.

While there has been a paucity of research of certain aspects of logical thinking, there has been research on the broad aspects of critical thinking at all educational levels. Those which have some relevance to this present study are here reviewed.

LOGIC ABOVE GRADE 6

Hale (46) sought to produce a text in formal logic for secondary school students and measure their ability to learn from this. Students in grades 8-12 were separated into groups on the basis of grade level, type of mathematical background and type of intelligence tests furnished by teachers. Students in the project were being taught mathematics from textbooks produced by the University of Illinois Committee on School Mathematics. An interesting finding, perhaps not the major one of this study, was that achievement scores of the student were affected by the type of mathematical background he had as well as by the knowledge he had of logical topics. The text was found to be suitable for all students in grades 9-12 and for those in grade 8 who had begun the study of algebra.

In this study of 165 junior high school students who had not had formal logic, Howell (53) found that there is a slight growth in inferential reasoning ability without formal instruction in logic. Less than two-thirds of the students in each grade demonstrated an understanding of valid inference patterns. All of the students participating in this study failed to recognize the invalidity of both the converse and inverse pattern.

Hrabi (54) used grade 7 and grade 9 students in a study of a unit on logic and proof strategy to determine if it contributed to the development of critical thinking ability and relative achievement. He found no significant differences between the grade 7 experimental and grade 7 control groups, but significant differences between grade 9 experimental and grade 9 control in favor of grade 9 experimental. Grade 9 achieved significantly superior mean

scores in logic achievement than did grade 7 when mean scores were adjusted for differences in intelligence. When mean scores were adjusted for differences in both intelligence and critical thinking ability, the superiority of grade 9 students was not so pronounced. This experimenter, in the conclusions of his study, recommended the inclusion of this unit on logic and proof strategy for grade 9 algebra classes and bright 7th grade classes but did not recommend that it be included in the mathematics curriculum of average and below average grade 7 students. He claims that the study of such content contributes to both the mathematical, literary, and critical thinking ability of these students.

In his study on inferential questioning, Drum (29) discovered that among the 7th and 8th grade students that the following items proved most difficult: i) implied deduction of a given term; ii) identification of the most suitable generalization; iii) statement of the most suitable generalization about a designated term and the composite item category, and iv) selection and writing of given information to support a conclusion.

Greif (45) investigated the effect of distinct emphasis on creative and critical thinking in student teaching in several education methods courses. He found that emphasis on creative and critical thinking effected highly significant gains in ability to implement such teaching which encourages children to think, in the student teacher's ability to think and in their general teaching ability. Placing a distinct emphasis on one aspect of teaching brought attendant improvement in other aspects of teaching, according to the findings of this study. Teachers were given precise help in critical thinking.

STUDIES OF THINKING IN CHILDREN

In determining what kinds of logical operations would be appropriate for a study of this kind, it was necessary to examine carefully research done in strategies of thinking and teaching of inquiry techniques. Larson (71), in comparing theories of Piaget and Bruner in order to find dimensions for studying strategies of inquiry, indicated that both identified the following intellectual components of strategy: content, organization, and proof. Bruner included components associative with values; Piaget did not. Bruner identified focusing and scanning strategies in adolescence which were differentiated according to whether subjects searched for alternatives or eliminated hypotheses from a system.

Piaget had used similar criteria for differentiating between concrete operations in middle childhood and formal operations in adolescence. If scanning and focusing strategies were identified in middle childhood, then Larson (71) maintains that it would be an indication that some variable other than structure, as defined by Piaget, would be needed to describe their inquiring behaviors. Significant differences in strategies were not obtained between Piaget's and Bruner's problem situations but children's strategies were significantly more systematic on the problems that entailed a limited and understandable set of alternatives and unambiguous information. Their strategies were significantly less systematic on problems that entailed an unlimited or nonunderstandable set of alternatives and ambiguous information. This seems related to Piaget's idea of concrete operations where there is not a systematic organization of thought processes such as classification and seriation. There were no significant differences between

boys and girls, a fact that seems to pervade most of the studies in thinking.

Several educators have proposed the idea that cognitive progress may be prompted by conflict-producing events. Dewey observed that the origin of thinking is some perplexity, confusion, or doubt.

. . . Reflective thinking involves
 1) a state of doubt, hesitation, or perplexity, mental difficulty, in which thinking originates, and 2) an act of searching, hunting, inquiring to find material that will resolve the doubt, settle and dispose of the perplexity.
 (24:12)

. . . the origin of thinking is some perplexity, confusion or doubt. Thinking is not a case of spontaneous combustion; it does not occur just on 'general principles.' There is something specific which occasions or evokes it. General appeals to a child (or to a grown-up) to think, irrespective of the existence of his own experience of some difficulty that troubles him and disturbs his equilibrium, are as futile as advice to lift himself by his bootstraps. (24:15)

Palmer (87) enlarged on the work of Berlyne by employing three conflict-producing procedures in first grade children. First of these was what he called the "set effect," an attempt to elicit two incompatible conclusions simultaneously. The other two procedures consisted of exposing nonconservers of number to the corrective influence of others. In one of the treatments the researcher gave a verbal expression of surprise when a conclusion indicating nonconservation was given by a child, and in the other, nonconservers were exposed

to the contradictory conclusions of their peers who were already conservers.

Among many interesting results of this study are two relevant to this project: the gains in conservation were significantly greater in the treatments in which there were conflict-producing situations and the boys outperformed the girls in solving cognitive conflicts constructively.

Smedslund (111) investigated concrete reasoning in children from four to 11 years of age. He used the terms inference and reasoning interchangeably while defining an inference as involving the combination of two or more initial states (premises) into a final state (conclusion). An inference is concrete, if and only if:

- i) It involves symbolic processes, i.e., processes which represents events and
- ii) the events represented by the initial stages have been perceived by the subject and
- iii) the structure of the inference is isomorphic with a logical structure, that is, if the conclusion follows necessarily from the premises.

Although his findings supported many of Piaget's ideas, Smedslund indicated that in experiments of the kind which he did, clearer distinctions must be made between percept, goal object, and inference patterns.

Stern (113) explored the possibilities of teaching young children strategies which would improve their efficiency in solving problems. She believed that children generally adopt

a random approach to learning, failing to profit from much of the information which is available to them. After instructing third graders in two different strategies for information processing, she found overwhelming evidence that instruction in problem-solving strategies involving the discovery of a rule was superior to the policy of providing children with an opportunity to practice and to formulate for themselves procedures without help.

In this experiment in which a single hypothesis and a multiple hypothesis strategy were used, it was found that more capable children profited more from the multiple hypothesis than the less capable children. In her conclusions, Stern recommends that primary school curriculum include what might be considered as a scientific attitude toward information getting and processing and that teachers systematically encourage the young child to ask questions and pose hypotheses on his own.

While Hilda Taba (118) (119) used social studies as the subject area in which to experiment with cognitive skills, her findings are relevant to mathematics. She used students in grades two through six and concluded that the transformation of concrete operations into formal ones begins in a small measure in the second grade, increases slowly through the third and fourth grades, until in the fifth and sixth grades formal thought represented approximately one-sixth of all thought units offered. This suggests an earlier beginning of formal thought than has been postulated. There was, however, no comparative data from untrained classes available.

In her recommendations for further study, Taba indicated that more research is needed to determine the nature and the

amount of concrete operations required to create a maximum opportunity for making the transition into formal thought, and to ascertain the difference in the nature and duration of this preparation according to ability, age, and grade level. Her data suggests that it is more fruitful to consider the development of thought as a continuous stream than as an additive accretion of specific skills.

Her findings indicated that the most marked single influence on cognitive performance was that the nature of the questions teachers ask is especially influential in either limiting or enhancing the capacity of students to think. Finally, Taba indicated that to study thinking, adequately it is necessary to extend the testing procedures to tap not only the products of students' thinking but also their processes, for the ways of arriving at answers is as important as the quality of the answers. (119: 179-180)

It is necessary, as several experimenters have pointed out (24:17) (50:80) (10:4), that the approach to the teaching of logical thinking in children be from a dual role, namely, that while trying to improve the logical, we be mindful of the psychological.

Berlyne observes:

. . . Insufficient attention has been paid to motivational factors in thinking. This is largely because of the ease with which artificial motives can be induced in human subjects with sufficient strength for purposes of an experiment. To present a human subject with a problem and to ask him to attempt a solution is often all that is required, although

the procedure may be more effective if praise or heightened self-esteem or some token of merit is made to depend on a successful outcome.

A high proportion of everyday thinking is, no doubt, prompted by comparable social pressures. But we need to know what factors determine when thinking will occur spontaneously and at what targets it will be aimed. Little or no research has been devoted directly and intensively to these factors, but some hints may be found among the data and theoretical ideas that come out of work on human and animal exploratory behavior. (10:4)

We try in our own formulation of thoughts and the teaching of mathematical ideas to reflect consistency, but do admit in general transactions that not everyone engages precisely in this kind of thinking. Perhaps one of the deficiencies of some previous research projects has been inattention to this factor, that people use words differently. Perhaps there has been a deceptive regard for language and its role in cognitive development.

Indirectly, but relevant to this idea of the concern for the psychological is the atmosphere of the premises. Woodward and Sells (126) guessed that judgment of the conclusion of a syllogism depends on the impression or atmosphere created by the premises as well as by the logical relations of the premises. Negative premises set up negative attitudes. Affirmative premises set up an affirmative atmosphere. And conclusion in agreement with the atmosphere thus engendered are likely to be accepted. There are complications, but specified predictions can be

made from the atmosphere of the premises. Sells presented syllogisms with a large percentage of invalid conclusions to a group of educated adults and found that of these invalid conclusions, those in agreement with his predictions were accepted much more often than the others. . . (126)

Henry and Brown (50) attempted to find the logic pervading all the disciplines and maintained:

In sum, all the devices generally associated with logical or critical thinking or problem solving are generally taught separately from the regular subject matter or given more emphasis than usual. It is generally a short course or "unit" in logic or creativity or reasoning, a shot in the arm supposedly to carry the pupil through the rest of the year. . . (50:29)

They were concerned with finding the actual process the mind must go through when one is learning to understand the most deductively necessary relations among concepts and the creation of a structure for handling new concepts--"what is the relation between the order of propositions (the logic of subject matter) and the order of experience in learning that logic of propositions?" (50:29)

What is perhaps more significant about the study is the extensive examination of theory and rationale of teaching logic. The project attempted to set up certain strategies of learning and researchers maintained:

- 1) that in all disciplines idea runs ahead of "stubborn facts" (information, data, instances);

2) that the mobilizing of facts (instances) into categories is psychologically the same process in all disciplines;

3) that structuring concepts (whatever kind they are: tragedy, energy, imperialism, light, atonality) into a pattern is logically the same operational process.
(50:98)

Henry and Brown emphasized throughout their report that a pupil's making an organization of his own by means of his knowledge of logical form does help him to reason. Further, they supplied some evidence to the need of integrating logic with subject matter. In fact, their three major hypotheses were built on this idea: 1) That a common logic (a similar set of strategies in thinking) runs through all disciplines; 2) That pupils would think better if every discipline were deliberately to emphasize the common strategies identified within this logic; 3) That a transfer of training can be based on this common logic, if pupils were taught these strategies under the same method of concept development.

Students of high school age were used in the project and those who were taught strategies of relation using two different disciplines, mathematics and English, improved significantly in the eight weeks period. They did much better than those taught only the strategies in one discipline, but all improved except the control group.

HILL'S AND OSBORNE'S STUDIES

In order to measure logical ability in children of six, seven, and eight, Hill (51) devised a 100-item test of three broad categories in elementary logic: sentential

logic, classical syllogism and logic of quantification. Responses were "Yes" if the conclusion was valid and "No" if it were not valid. All items consisted of two premises, a question, and two possible responses. In every item, the child was required to discriminate between a necessary conclusion and the negation of a necessary conclusion.

Mean scores of 71.18, 79.54 and 85.58, much higher than the 50.00 chance score, were obtained and Hill concluded that the investigation's results "are counter to Piaget's theories," (51:76), that children, therefore, are capable of hypothetico-deductive reasoning.

Osborne (85) asks the following questions:

1. Does success with the Hill test of logical abilities show the existence of a system or organization of logical operations or does it show the existence of ability in component, and as yet unrelated, facets of an organization of logical abilities?
2. Do the children deal with the problem hypothetically, considering the possibilities on a non-actual level of thought instead of the child in fact treating the arguments in terms of the particulars? Does the child think in terms of the reality of instantiation being a subset of the possible hypotheses?
3. Does the child in reasoning escape from the phenomenological present? If the child is given the conclusion in terms of the particulars of the argument and does not have to cast around for the solution, does he have to indulge in hypothetical thinking in a true sense?

In order to reject the Piagetian theory, it is necessary to answer each of the three questions in the affirmative. (85:18)

Osborne seriously questions where the child thinks in terms of the organizational model, whether the child is using hypothetical thought. Hill's test indicates that children definitely have logical abilities at an early age but does not consider the interrelations of these of those abilities or the "nature of the child's treatment of reality in terms of the arguments." (85:19)

O'BRIEN AND SHAPIRO STUDY

O'Brien and Shapiro (83), in a paper delivered at the American Educational Research Association's convention in February, 1968, on the "Development of Logical Thinking in Children," felt that:

. . . behavioral manifestation of hypothetical-deductive thinking chosen by Hill was but one of several possible behaviors. The problem was that in each of the test items a necessary conclusion followed from the premises and the task of the student in each case was simply to discern whether the third statement given was the conclusion or the negation of the conclusion. In no case was the student called upon to test the logical necessity of the conclusion, a behavior which the present investigators felt was vital to an adequate consideration of hypothetical-deductive reasoning. Therefore, the present study was undertaken to investigate this second behavior and to determine the differences, if any, between the ability of young children to discriminate between logically necessary conclusions and their ability to test the logical necessity of a conclusion. (83:3)

Two measuring instruments were used. Test A was the original Test of Logical Abilities used by Hill while Test B "opened up" 33 of the original 100 test items so that no necessary conclusion followed from the premises and for every item in test B a "Not Enough Clues" option was added to the "Yes" and "No" responses of Test A.

Test B, like Test A, contained content that was familiar but not suggestive and vocabulary that was appropriate for first, second, and third graders. Findings on Test A confirmed Hill's original findings with the exception that while there were statistically significant differences between age levels, the present data showed these differences to be significant only between the means of the six-year olds and those of the seven and eight-year olds. Little growth was detected between age 7 and age 8.

With Test B, the scores were significantly lower than test A scores, because of the 33 altered items. O'Brien and Shapiro concluded that children "peak out" early in their ability of recognizing logically necessary conclusions. They experience great difficulty in testing the logical necessity of a conclusion and show slow growth in this ability.

That these two behavioral manifestations of hypothetical-deductive thinking occur at such different levels among children of the same age seems to bring into question the challenge that the original research gave to Piaget's theory regarding the growth of this kind of logical thinking in children.

It seems clear that for one reason or another, children avoided the "not enough clues" response, thus indicating an avoidance of "open" situations. What

the reasons for this behavior might be at present is unknown to the investigators. What is known is that the existing primary school curriculum seems to have had little effect (at least immediately) on children's tolerance of or ability to recognize "open" situations. (83:11)

O'Brien and Shapiro, with Osborne, agree that caution is needed in interpreting the findings of Hill. In fact, the authors suggest that more questions are raised by their study than answers. Some of these are:

1. In what ways does the children's ability to test for logical necessity develop at later ages? At what age does it begin to peak out?
2. What is the relation between a child's performance on a paper-and-pencil logic test and his performance of logical operations in everyday life situations? Further, in the light of current interest in children's ability to deal with symbolic logic, what is the relation between the children's ability in this regard and the two logical abilities previously mentioned?
3. What systematic influences, if any, can be brought to bear to increase children's ability to test for logical necessity and to derive conclusions in hypothetical situations?

CORNELL CRITICAL THINKING STUDY

In the long range project in Critical Thinking Readiness in Grades 1-12 under the direction of Robert Ennis at Cornell University, the first phase was concerned with deductive reasoning in adolescence. Ennis and associates (35), while giving operational definitions of critical thinking and in defining their problem indicated that "logic is that part of critical thinking which deals with whether a conclusion follows necessarily from the premises that are offered in support of it." (35:II-2 and II-3)

Two of the most common types of logic were used, conditional and class logic, with basic elementary principles, twelve in number, used for each. The principles were grouped according to conversion, negation of antecedent, contraposition, transitivity, and "only-if" principles.

Subjects of the study were 803 students in grades 4-12 with approximately one class in each grade taught logic. Grades 5-7-9-11 worked with conditional logic while grades 4-6-8-10-12 had class logic. More detailed explanations of the type of questions are given in Chapter III which describes the types of items used in the current study. The class and conditional reasoning tests were adapted, with the permission of the director of the project, Dr. Robert Ennis, from this phase of the Cornell Critical Thinking study for this investigation.

Ennis found that there is a fairly gradual improvement in knowledge of logic from age 10-18 among students who were not given any special training in logic. While the improvement held for logical fallacies and transitivity, it did not hold for contraposition. Basic principles of class and

conditional logic were not generally mastered by students aged 11-12 who had not received special instruction in logic. At age 17, students not receiving instruction in logic definitely had not mastered the basic principles of conditional reasoning but were much closer to general mastery of the basic principles of class reasoning.

Class logic appears to be much easier all along the line than conditional logic. An increase in mastery of contraposition occurred under instruction while an increase in mastery of transitivity did not. No relationship was found between sex and knowledge of logic and fairly small correlations were found between socio-economic status and knowledge of logic. Correlations between knowledge and chronological age were significant and medium in children of age six.

Ennis claims that Piaget's studies did suggest certain questions:

1. Is there actually a development of logical ability as children grow older?
2. Does this development (if there is any) come in stages?
3. Is conditional logic mastered by age 11-12?
4. Is class logic mastered by 11-12?
5. Is the truth-validity characteristic (essentially the mastery of the suggestive component) achieved by age 11-12?
6. Within each type of logic is there a development of one sort of thing before another such that some things are more difficult than others at a given level?

Answers to the first five of these questions are suggested or indicated in the literature and are fairly obvious and routine for anyone who has worked with children and thought about the questions. The answers are as follows:

1. Yes
2. That depends on what you count as a stage, but gross plateaus extending over a period of years were not evident.
3. Partly
4. Partly
5. Partly

The sixth question is an extension of the third, fourth, and fifth in that it is concerned with the extent to which various parts are mastered. It is the sixth question which is the interesting one to our minds and is the one toward which this study has made contribution, if it has made any contribution in the area of the natural-cultural development of logic. Part of the contribution lies in the formulation of the question and part lies in the empirical findings. The latter part is inevitably quite modest because of the size of the group and the fact that it is composed of students from only one small part of the world. (35:VIII-16)

Ennis indicated in his summary that the basic principles of neither class nor conditional logic are mastered by age 11-12 among students not receiving instruction in logic; that students at 17 not receiving instruction in logic definitely had not mastered the basic principles of conditional reasoning.

This crucial finding has definite implication for the researcher of this study in attempting to determine just what principles were learned from the instruction in logic and which ones, at different age levels, were either not mastered or had already been learned in some other way than formal instruction.

CHAPTER III

RESEARCH DESIGN

This study was designed to ascertain the effects of teaching logic to children in grades three and six. A primary question to be answered was whether logic interwoven with mathematics or logic taught separately produced the best results on a logic test and on mathematics achievement. Since the teachers teaching the logic needed in-service training, it was necessary to establish two control groups, one with in-service only and one with no in-service.

Twenty-five teachers from 16 schools in the Archdiocese of Detroit were randomly assigned to one of four treatments. There were 511 students in 13 classes in grade 3 and 589 students in 13 classes in grade 6.

TREATMENTS

Four treatments were designed:

Treatment LM: Logic was taught and interwoven into the regular mathematics instruction for grades three and six for approximately eight weeks. Time for teaching logic was taken only during the mathematics period and in this treatment, it was interwoven and integrated with the mathematics instruction.

Treatment L: Logic was taught to students in grades three and six but not integrated with mathematics instruction. The only time taken for the teaching of logic was during mathematics period, but in this treatment, this instruction was not integrated with mathematics.

Teachers in both treatments LM and L attended a two-hour in-service training class in logic each week for nine weeks, beginning one week before the project started and continuing during the project. Total teaching time for logic was 16 hours of instruction.

Treatment I: No logic was taught but teachers attended in-service classes for two hours during a nine-week period. Content of these sessions was determined, for the most part, by the specific topics which the teachers were covering at the time of the in-service training. This included, for grade three, additional work in multiplication and division, non-metric geometry and metric geometry such as measurement of length, area, liquid, and weight; additional work in place value and subtraction, with consistent attention throughout on problem-solving techniques.

In grade six, topics considered at in-service classes were negative numbers, geometry, measurement, sets, place value, decimals and fractions, numeration systems, inequalities, and again during these classes, techniques of problem-solving. One in-service class was devoted to enrichment topics.

Treatment C: This is the control group. No logic was taught and the teachers did not have in-service training of any kind either in mathematics or logic.

In all groups, the following tests were administered, both as pretests in December, 1967, and as posttests in March, 1968.

1. Cornell Class Reasoning Test. (adapted)
2. Cornell Conditional Reasoning Test. (adapted)
3. Wisconsin Contemporary Elementary Mathematics Test, Form A, grades 3-4 for grade 3; Form A, grades 5-6, for grade 6.

Treatments LM and L were given a final logic test during the week of March 11, 1968, and the same test as a retention test during the week of May 6, 1968.

POPULATION

Teachers and students participating in the project were from selected schools in the school system of the Archdiocese of Detroit. There were 25 teachers from 16 schools located in middle class neighborhoods. Notices were sent to schools requesting volunteers for the project and asking that i) students be either average or above average; ii) teachers have had no previous experience in logic and iii) teachers not be beginning teachers. No attempt was made to control the socio-economic backgrounds of either the teachers or the students. Classes were assigned randomly to teachers for the different treatments.

Students were in the third and sixth grades with average or above-average intelligence as indicated by school records, teacher reports, and scores on the California Test of Mental Maturity.

CONTENT OF MATERIAL TAUGHT IN TREATMENTS LM AND L

In outline form, the following topics were included in the eight weeks of classes given to the students. Total time was approximately 16 hours of instruction. A rigid schedule of a given number of minutes per day was not followed because of difference in grade level and local situations,

Rather than determining that a specific amount of logic content should be taught each day, it was decided that it would be better not to be too rigid about this because of differences in locale, school programs, holidays, etc. However, since each teacher in treatments LM and L attended classes each week, there was a strong tendency, within a week's time, for all to be about in the the same place in the sequential logic units. Experiences were shared and much opportunity was given for exchanging ideas and discussing student reactions and teacher observations at these two-hour weekly sessions. Also, the director of the project was on call for each teacher in the project and there was added "instruction" over the phone when specific incidents had occurred during the project time.

OUTLINE OF CONTENT

I. Class Logic

A. Categorization

1. Sets, set membership, set operations
2. Relations between sets: inclusion, exclusion, empty set
3. Simple inferences to be made from set ideas

- B. Relation of sets such as universal sets, set inclusion with universal statements such as:
All A's are B's; No A's are B's
- C. Necessary Symbolism

II. SENTENCES, STATEMENTS, TRUTH-FALSITY

- A. Ideas of sentences, statements, truth-falsity
- B. Negation of simple statements and sentences
- C. Definitions and "reversible" statements
such as: No A's are B's and no B's are A's.
- D. Connection between reversability and the
idea of converse.

III. IMPLICATION

- A. Informal approach to the if-then statements
 - 1. What they tell
 - 2. What they don't tell
- B. Inferences made from if-then statements such as:
modus ponens, modus tollens
- C. Relation of implication to class logic:
informal approach including the use of
Venn diagrams
- D. Converse and intuitive use of validity;
with third grade, use of "O. K." and "Not O. K."
to determine if ideas followed validly from
initial statements.

IV. COMBINATION OF STATEMENTS: CONJUNCTIONS AND DISJUNCTIONS

- A. Connective and: conjunction
- B. Truth and falsity of conjunctions with stress on meaning of and
- C. Disjunctive or: intuitive idea of exclusion and inclusion
- D. Truth and falsity of disjunctions with stress on meaning of or

V. CHOICE MAKING OR DECISION MAKING

- A. Relating of ideas in I-IV by means of diagrams, Euler circles
- B. Inferential statements based on major ideas in preceding sections
- C. Recognition of probable inference--that there is not always an absolute, a "right" answer
- D. Relation of logical concepts to some basic notions in mathematics such as inequalities.

With the LM group, there was a consistent attempt to bring in mathematical ideas with the teaching of logical concepts. This was avoided in the L treatment. A deliberate attempt was made to include logic vocabulary as precisely as the maturity level of the students warranted, but at times, this created a problem. For example, it was discovered more meaningful to use "O. K." to indicate that conclusions followed necessarily from the premises with third graders. This was not necessary in the sixth grade.

A concerted effort was made to be careful of the language used, distinguishing vocabulary mainly from three aspects: ordinary and/or colloquial usage; mathematical usage which is technical, including the logical; mathematical colloquialism.

CORNELL CLASS AND CONDITIONAL REASONING TESTS

In 1961, Dr. Shirley Hill devised a test to measure the degree of knowledge of logic among six, seven, and eight-year olds. Her test, satisfactory for group administration, was based on an attempt at a comprehensive coverage of basic principles of valid arguments in class and sentence logic. However, it was considered not satisfactory for purposes of this study.

All of the proposed conclusions on the Hill test either followed necessarily or contradicted the premises, not allowing for probable inference. An example of this is the following where students were asked to answer "Yes" or "No":

If Jack is in the race then Joe will not win.

Jack is in the race.

Will Joe win?

There are various classes of invalid arguments, and among them, those in which the conclusion contradicts the premises and those in which the conclusion which, while it does not follow the premises, yet does not contradict them. This involves the situation in which, under certain conditions, the conclusion could be true.

In logic and mathematics, one cannot assume the truth of the converse but there are specific instances where the converse is true. For example, in mathematics, two lines which are in the same plane and do not intersect are parallel. Its converse is true also: Parallel lines are those which are in the same plane and do not intersect. It seemed that this important distinction was missing on Dr. Hill's test.

To secure separate measures of some of the basic principles of logic, the Cornell Deduction Tests, in an adapted form were used. Dr. Robert Ennis, director of the Cornell Critical Thinking Project at Cornell University, gave permission for the use of the tests. Only certain principles were used on the adapted form. There were two separate tests, class reasoning and conditional reasoning. They were administered as a pretest in December, 1967, and as a posttest in March, 1968, to students in all treatments. In these tests, there was provision for the maybe situation, in which the conclusion, under certain conditions, could be valid. The "maybe" situation occurred when there was not enough information given.

LOGICAL FORM OF ITEMS IN THE CLASS AND CONDITIONAL REASONING TESTS

The original Cornell test was constructed in such a manner that it tested for knowledge of twelve principles or combinations of principles. Six items were assigned to each principle or combination of principles. The adapted test, used in this study, consisted of items testing seven principles with three items assigned to each principle. There

were 21 items in class reasoning and 21 items in conditional reasoning. In tables 3.1 and 3.2 is summarized the basic principles used in the adapted test. The complete tests are in Appendix A, table A.1 and A.2.

THREE CONTENT COMPONENTS

Wilkins (1928) specified four types of content: concrete familiar, symbolic, unfamiliar, and suggestive--and attempted to see which kind made syllogisms more difficult. The Cornell tests included three of the four but excluded unfamiliar. The four types are described as follows:

a. Concrete familiar. The content mentioned is concrete articles and qualities with which the subject has been associated. However, no statements are made which the subject has reason to believe to be true or false because these specific objects might not be known by the subject. For example, instead of saying, "All cats are black," a statement with which the subject is likely to disagree, a statement might be "Mary's cats are black." The latter is less likely to meet with agreement or disagreement, although there is still the possibility of disagreement, if the subject identifies the Mary of the statement with some Mary he knows.

b. Symbolic. Symbols like 'x', 'y', 'A' and 'B' are used in key places, instead of terms that refer to particular objects. An example of a sentence using symbolic content is "All A's are B's."

c. Unfamiliar. The basic terms are scientific terms with which the subject is expected to be unfamiliar or

TABLE 3.1

BASIC PRINCIPLES OF CLASS LOGIC IN ADAPTED CORNELL TEST

PRINCIPLE	SYMBOLIZED ARGUMENTS	CONCRETE ARGUMENTS
1. Whatever is a member of a class is not a non-member of that class and vice versa.	All A's are B's. Therefore it is false that at least some A's are not B's. <u>Valid</u>	All of Mickey's shirts are white. Therefore it is false that at least some of Mickey's shirts are not white.
2. Whatever is a member of a class is also a member of a class in which the first is included. Therefore inclusion that class inclusion is transitive.	All A's are B's. All B's are C's. Therefore all A's are C's. <u>Valid</u>	All of the bikes in the garage are Mike's. All Mike's bikes are Speedways. Therefore all of the bikes in the garage are Speedways.
3. Whatever is a member of a class is not (as a result of that relationship) necessarily a member of a class included in that class.	All A's are B's. Therefore all B's are A's. <u>Invalid</u>	All Joe's pens are red. All of the pens on the table are red. At least some of the pens on the table are Joe's.
4. Class exclusion is symmetric.	No A's are B's (and there are B's). Therefore no B's are A's. <u>Valid</u>	None of Joan's dolls have hats. Therefore none of the dolls that have hats are Joan's.
5. Whatever is a member of a class is not a member of a class excluded from the first.	All A's are B's. No C's are B's. Therefore no A's are C's. <u>Invalid</u>	All Frank's homework is due today. None of the homework due today is in history. Therefore none of Frank's homework is in history.

6. Whatever is not a member of a class is not a member of that relation-ship) necessarily a member of (nor a non-member of) another class, which is excluded from the first.

No A's are B's.
 No C's are B's.
 Therefore at least some A's are C's.
Invalid.

None of Jim's shirts are wool.
 None of the shirts hanging up in the closet are wool.
 Therefore at least some of Jim's shirts are hanging up in the closet.

7. Whatever is not a member of a class is also not a member of any class included in the first.

All A's are B's.
 No C's are B's.
 Therefore no C's are A's.
Valid.

All of Joan's homework is due today. None of the math homework is due today.
 Therefore none of the math homework is for Joan.

TABLE 3.2

BASIC PRINCIPLES OF CONDITIONAL LOGIC IN ADAPTED CORNELL TEST

<u>PRINCIPLE</u>	<u>SYMBOLIZED ARGUMENT</u>	<u>CONCRETE ARGUMENT</u>
1. Given an if-then sentence, the affirmation of the if-part implies the affirmation of the then-part.	If p, then q. Therefore q. <u>Valid.</u>	If the hat on the table is blue, then it belongs to Joan. The hat on the table is blue. Therefore the hat on the table belongs to Joan.
2. Given an if-then sentence, the denial of the if-part does not by itself (as a result of its being an if-part) imply the denial of the then-part.	If p, then q. Not p. Therefore not q. <u>Invalid.</u>	If Tom lives in the white house, then his last name is Smith. Tom does not live in the white house. Therefore Tom's last name is not Smith.
3. Given an if-then sentence, the affirmation of the then-part does not by itself (as a result of its being a then-part) imply the affirmation of the if-part.	If p, then q. Therefore p. <u>Invalid.</u>	If Mary lives in the white house, then her last name is Brown. Mary's last name is Brown. Therefore Mary lives in the white house.
4. Given an if-then sentence, the denial of the then-part implies the denial of the if-part.	If p, then q. Not q. Therefore not p. <u>Valid.</u>	If the car in the parking lot is Mr. Smith's, then it is blue. The car in the parking lot is not blue. Therefore the car in the parking lot is not Mr. Smith's.

5. The if-then relationship is transitive.

If p, then q.
If q, then r.
Therefore, if p, then r.
Valid.

6. The if-then sentence implies its contrapositive.

If p, then q.
Therefore, if not q,
then not p.
Valid

7. The if-then relation is non-symmetric.

If p, then q.
Therefore, if q, then p.
Invalid.

If Sam misses the bus, he will walk to school. If Sam walks to school, he will cross the bridge. Therefore, if Sam misses the bus, he will cross the bridge.

If Mrs. Smith entered the flower show, then she entered her roses. . . . Therefore, if Mrs. Smith didn't enter her roses, then she didn't enter the flower show.

If the chair is green, then the table is black. Therefore, if the table is black, then the chair is green.

"nonsense words invented to sound like scientific terms." For example, "All hentras are globiculous" is a sentence with unfamiliar content. The Cornell researchers did not think that this unfamiliar content is likely to be encountered in situations that make a difference. When a person meets arguments with such content he will generally not know what to do with the conclusion anyway. The investigator in this study agreed with them and did not feel that this was a major portion of logical thinking of interest at this time.

d. Suggestive. The content is familiar but the truth status of this content is known by the subject. Furthermore, the truth or falsity of these statements (conclusions) is at variance with their validity. In other words, the truth status of the conclusion was different from the validity status of the argument. A person might be tempted to make an argument invalid because he disagrees with it. In the adapted form used in this study, there was only one suggestive item on the class reasoning and only one on the conditional reasoning.

In tables 3.3 and 3.4 is a summary of these content components of the adapted Cornell test.

In the original Cornell Deduction tests, four of the items in each item group had concrete familiar content, a fifth had symbolic content and a sixth had suggestive content. There were some formal variations. Basic logical symbolic form was assigned to each group. (See tables 3.1 and 3.2). This basic form was used for two of the concrete familiar items, shown under CF 1 and CF 2; the symbolic, SY; and suggestive, SU. The third and fourth concrete familiar items,

TABLE 3.5

LOGICAL FORM OF AND ANSWERS TO ITEMS IN THE ADAPTED CORNELL CLASS REASONING TEST

Prin- ciple	Basic form, which was used for first two concrete fami- liar items (CF1 and CF2), the sym- bolic item (SY) and the suggestive item (SU)	Answer for Basic Form	Form of CF3 (Same answer as basic form)	Form of CF4 (Same answer as basic form)	ITEM NUMBER					
					CF1	CF2	CF3	CF4	SY	SU
1	All A's are B's. : At least some A's are not B's.	No	X is an A. : X is not an A.	At least some A's are B's. : No A's are B's.	10	20				24
2	All A's are B's. All B's are C's. : All A's are C's.	Yes	All A's are B's. X is an A. : X is a B.	At least some A's are B's. All B's are C's. : At least some A's are C's.	21	13	15			
3	All A's are B's. All C's are B's. : At least some C's are A's.	Maybe	All A's are B's. X is a B. : X is an A.	At least some A's are B's. All C's are B's. : At least some A's are C's.	12	16	8			

TABLE 3.3 Continued

		CF1	CF2	CF3	CF4	SY	SU
4	No A's are B's. : no B's are A's.	Yes	No A's are B's. X is an A. : X is not a B.	No A's are B's. and there are B's. : At least some B's are not A's.	9	11	7
5.	All A's are B's. No C's are B's. : At least some A's are C's.	No	All A's are B's. X is not a B. : X is an A.	At least some A's are B's. No C's are B's. : All A's are C's.	18	14	27
6.	No A's are B's. No C's are B's. : At least some A's are C's.	May- be	No A's are B's. X is not a B. : X is an A.	At least some A's are not B's. At least some C's are not B's. : At least some A's are C's.	22	26	19
7.	All A's are B's. : all non-B's are also non A's.	Yes	All A's are B's. X is a non-B. : X is a non-A., B's.	All A's are B's, and there are non- non-B's are non-A's.	15	17	23

NOTE: The symbol ':' is used to introduce the proposed statement. It symbolizes "therefore."

TABLE 3.4

LOGICAL FORM OF AND ANSWERS TO ITEMS IN CORNELL CONDITIONAL REASONING TEST

Principle	Basic Form, which was used for the first two concrete familiar items (CF1 and CF2), the sum-bolic item (SY) and the suggestive items (SU).	Answer for Basic Form	Form of CF3	Answer to CF3	Form of CF4 (same answer as basic form)	CF1	CF2	CF3	CF4	SY	SU
1	If p, then q. p. : q.	Yes	If p, then q. : not q.	No	Not p. If not p, then q. : q.	18	14	10			
2.	If p, then q. Not p. : not q.	Maybe	Same as basic form	Maybe	p. If not p, then q. : not q.	13	19	25			
3	If p, then q. q. : p.	Maybe	Same as basic form.	Maybe	q. If not p, then q. : not p.	17	9	21			
4	If p, then q. Not q. : p.	No	If p, then q. : not p.	Yes	Not q. If not p, then q. : not p.	8	26	16			

TABLE 3.4 Continued

		CF1	CF2	CF3	CF4	SY	SU
5	If p, then q. If q, then r. : if p, then r.	Yes	Same as basic form	Yes	If q, then r. If not p, then q. : if not p, then r.	7 15 22	
6	If p, then q. : if not q, then not p.	Yes	Same as basic form	Yes	If not p, then q. : if not q, then p.	12 24	20
7	If p, then q. : if q, then p.	May- be	Same as basic form	May- be	If not p, then q. If q, then not p.	11 23	27

NOTE: The symbol ':' is used to introduce the proposed statement. It symbolizes therefore.

CF 3 and CF 4, involved slight formal change in order to provide greater variety in the representatives of each principle of combination. Because class and conditional reasoning differ in their basic structure, these variations differed.

In the conditional reasoning test, the third concrete familiar item negated the conclusion unless there were good reasons not to do so. That is, if the conclusion called for by the basic symbolic form is positive, the one provided is negative; and if the conclusion called for is negative, then the one provided is positive. Reasons for not doing this are two:

i) If the basic form is invalid, then denying the conclusion reduces the temptation to judge it valid, making the item considerably easier. So for those four item groups or principles in which the basic form is invalid, the third concrete familiar item conformed to the basic form.

ii) If the conclusion itself is a conditional statement, a denial of it becomes awkward to state. Note that the denial of "If Sam misses the bus, then he will walk to school" is not "If Sam misses the bus, he will not walk to school" which is not awkward to state. Instead, the denial is "It is not the case that if Sam misses the bus he will walk to school" which is awkward.

The fourth concrete familiar item in conditional reasoning varied from the basic form by having one of the basic sentences negated throughout and by having premises in different order, when there are two or more premises. In class reasoning, the third concrete familiar item contained

at least one class membership statement in the premises instead of all class inclusion and exclusion. . . . A class membership statement asserts the membership or nonmembership of one thing in a class whereas the class inclusion or exclusion statement speaks about the relation between classes. . . . The fourth concrete familiar item in class reasoning contained at least one statement of partial inclusion.

Item Format

The Cornell reasoning test sought an item format that would meet the following criteria:

- a. Provide a way for a subject to show that he knows that a proposed conclusion does not follow, even though it does not contradict the premises.
- b. Avoid technical language, yet ask whether or not an argument is valid.
- c. Provide more than two alternatives in multiple-choice form.
- d. Provide fairly equal numbers of each type of answer.
- e. Allow for separation of different items of the same logical form.
- f. Be understandable to fourth graders without insulting twelfth graders.

- g. Avoid confusing truth and validity.
- h. Not require elaborate directions.
- i. Not require elaborate scoring procedure.
- j. Minimize irrelevant errors.

After much experimentation and discussion, the Cornell researchers settled upon a format which asks the subjects to suppose certain premises and to decide further, on the basis of the supposition, a further thing would be true. They called the further thing the "proposed statement" and the original supposition the "supposed statement." There are three possible answers:

"Yes" which indicates that the subject thinks the proposed statement follows necessarily;

"No" signifies that the proposed statement contradicts what has gone before.

"Maybe" means that the proposed statement neither follows necessarily nor contradicts--that its truth is not necessarily determined by the establishment of the truth of the premises.

The "No" response was introduced partly in order to make possible three choices instead of the two suggested by the distinction between the validity and invalidity. The Cornell researcher wanted three choices in order to lessen the attractiveness of wild guessing and to lessen the chances

of its success. The "No" response was introduced as a correct answer by taking conclusions to valid arguments and then denying them. For the "No" response, the proposed statement is then a denial of a conclusion that follows necessarily from the supposed statement. The second reason for having a "No" response is that by doing so, it enabled the testers to avoid a preponderance of "Yes" answers over "Maybe" answers.

The "No" response involves a risk--namely, there is an additional principle involved in each of these items; that the denial of a statement which follows necessarily from supposed statements cannot be accepted, if the supposition is accepted. This principle is elementary but it must be invoked. Failure to invoke it, as contrasted with failure to know the basic principles, could explain errors that are made. Reasons for failure on these items could be difficult to determine precisely.

WISCONSIN CONTEMPORARY ELEMENTARY MATHEMATICS TESTS

The Wisconsin Contemporary Test of Elementary Mathematics (WCEM) is a new test. The content focuses on those mathematical facts and concepts which have been introduced into elementary mathematics curriculum and tests primarily since 1962. It is designed to measure modern mathematics achievement at the end of grade 3; all of grades 4, 5, 6; and the beginning of grade 7.

Although the test is divided into two parts, facts and concepts, total scores were used. It was felt that distinction between facts and concepts are made with a great deal of thought to the instructional experiences of the

child at the time that he was taking the test. There appeared to be no significant differences between the types of items in the two parts for the purposes for which it was to be used. Therefore, no distinction was made between these two parts.

The Wisconsin Contemporary Test of Elementary Mathematics (WCEM) is a broad-spectrum contemporary test built to provide up-to-date measurement of the skills and content common to many mathematics programs now in use. The content of the test represents the middle part of the continuum which begins with extremely traditional mathematics and goes to extremely modern range. The Wisconsin test authors maintain that they have selected the middle range of this continuum.

ITEM INFORMATION

Items on the WCEM can be put into three broad categories:

1. Fundamentals
2. Operations
3. Measurement-Geometry

Each of these has from 8-10 items. In the fundamentals are included numeration and sets; in operations are included addition, subtraction, multiplication, division of integers and rational numbers. In the measurement-geometry section are included all of the items related to metric and non-metric geometry. Items selected for final inclusion in the test had the following general characteristics:

1. Difficulty from 5% to 98% with mean difficulty approximately 50%.
2. Correlation with total test score ranges from .45 to .90 with mean correlation approximately .60.

There were 52 items in the total test for grades 3-4 with 26 in the facts section and 26 in the concepts section. In the grade 5-6 WCEM were 30 items on facts and 30 on concepts.

The Wisconsin Contemporary Test of Elementary Mathematics was administered to all students in every treatment as the pretest in the latter part of December, 1967, and given as a posttest in mid-March, 1968.

An additional analysis of sub-categories in mathematics and logically-related items is made in Chapter IV.

LOGICAL THINKING TEST

In an attempt to measure the kinds of logical thinking principles taught to the students in treatments LM and L, an instrument of 47 items was devised by the investigator for administration at the end of the teaching projects in mid-March, 1968. Eight weeks later, in the first part of May, the same test was administered as a retention test. In contrast to the previous instruments, there were questions which involved students actually supplying conclusions to syllogisms as well as statements of negation and class reasoning. A copy of this instrument is in Appendix A, Table A.3.

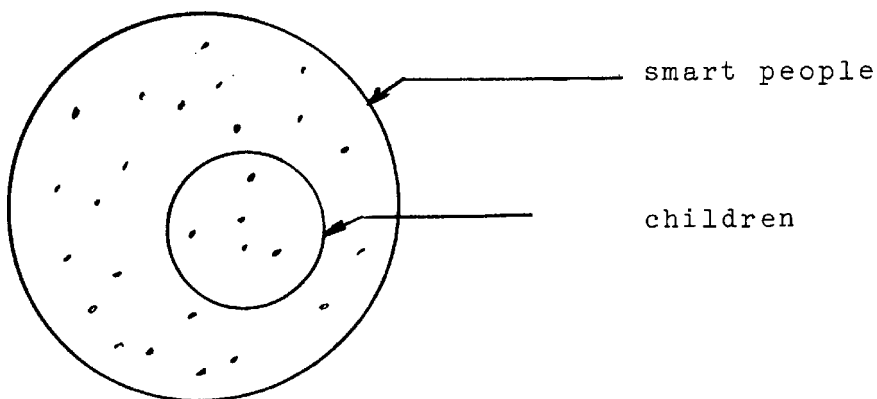
This test was directly related to the content of the logic taught in the project. Items on the test were placed into several different categories, not all of which were disjoint. These categories are: i) negations; ii) mathematical; iii) non-mathematical; iv) ideas contrary to regular or ordinary experiences; v) ideas contrary to mathematical experience; vi) symbolic; vii) class and conditional reasoning (similar to those on the Cornell reasoning tests). Also, items were separated into "Yes", "No", and "Maybe" categories. These categorizations do not involve a partitioning of the set of items but were classified in order to identify areas of strength and weakness.

DESCRIPTION OF TYPES OF ITEMS ON LOGICAL THINKING TEST

Under the non-mathematical category were items on Venn diagrams which had nothing to do with mathematics per se, the writing of negations, completion statements where students were required to write their own conclusions; judgment of validity of conclusions and determining "Yes," "No," or "Maybe" to questions asked concerning the validity of certain arguments. Some examples are given here.

NON-MATHEMATICAL EXAMPLES

1. Use the diagram and write one true sentence:



4. Write the negation for: All cats have tails.
23. Study the first two statements in each and then complete the sentence:

Suppose that all girls like to play with dolls and
Tillie is a girl, then _____.

29. Check O. K. if the conclusion follows from premises;
Not O. K. if it does not.

If Mike is a dog, then he is an animal.

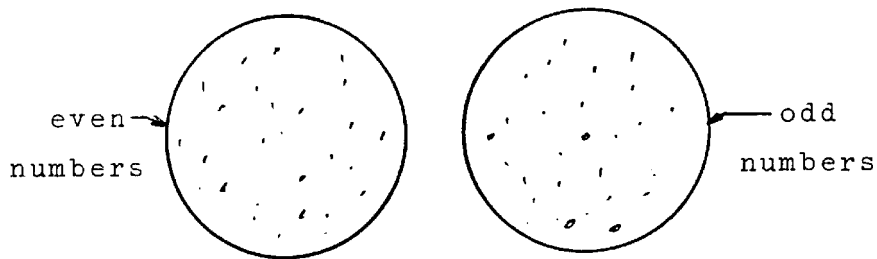
Mike is an animal.

Therefore, Mike is a dog.

O. K. ☐ Not O. K. ☐

MATHEMATICAL EXAMPLES:

3. Use the diagram and write one true sentence.



8. Write the negation for: All squares are rectangles.
9. Study the first two statements and then complete the statement:
- If all triangles are polygons and ABC is a triangle,
then _____.

25. Show whether the conclusion is O. K. or not O. K., that is, whether it follows from the given sentences or not:

All numbers divisible by 6 are whole numbers.
Therefore, all whole numbers are divisible by 6.

O. K. ☐ Not O. K. ☐

39. In answering the question below, use only what you are told in each exercise. There are three possible answers.

Yes. It must be true.
No. It cannot be true.
Maybe. You can't be certain. It
may be true or it may not
be true. You are not told
enough information to be
certain it is Yes or No.

If the figure is a circle, then it is a closed curve.

If the figure is a closed curve, is it a circle?

Yes ☐ No ☐ Maybe ☐

EXAMPLES OF NEGATION

Examples 4 and 7 given in the mathematical and non-mathematical examples illustrate negations as do the following:

10. Suppose if the figures are polygons, either they are triangles or they are rectangles.

The figures are polygons.

The figures are not triangles.

Are they rectangles?

36. If it snows, then Lulu does not go to school.

Lulu does not go to school.

Therefore, it is snowing.

O. K. ☐ Not O. K. ☐

Note that in the negations, it was difficult to make a judgment on whether the "atmosphere" was negative or not. Therefore, it was determined that any item which had a negative expression such as "not" or "it is false that" would be considered under the category "Negation." Note that except for mathematical and non-mathematical, the categories are not necessarily disjoint.

EXAMPLES OF SYMBOLISM

15. In each of the following show whether the sentence is true or false. Mark T for true; F for false.

$$\begin{array}{c} S \wedge J \\ \hline T \quad T \\ \hline \end{array}$$

40. All A's are B's.

All C's are B's.

Are some A's also C's?

Yes ☐ No ☐ Maybe ☐

EXAMPLES OF ITEMS INDICATING IDEAS CONTRARY TO ORDINARY EXPERIENCES

25. Suppose that lemons are bigger than pineapples and pineapples are bigger than basketballs, then

_____.

38. If all third graders are dogs, then they can meow.

All third graders are dogs.

Can they meow?

Yes ☐ No ☐ Maybe ☐

EXAMPLES OF ITEMS INDICATING IDEAS CONTRARY TO MATHEMATICAL EXPERIENCE

26. Suppose that all numbers which are divisible by 3 are even and 10 is divisible by 3, then

_____.

37. If two numbers are even, then their sum is odd.

These two numbers are even.

Therefore, their sum is odd.

O. K. ☐ Not O. K. ☐

There are separate categories indicated for "Yes," "No," and "Maybe" correct answers as indicated in the analyses in Chapter IV. Included in this summary are sections where "O. K." was a correct answer and "Not O. K." was a correct answer.

With exception of the Wisconsin Mathematics tests, all of the instruments of evaluation contained much logic, most of it, however, not directly related to mathematics per se. Only in the logical thinking test administered to treatments LM and L were comparable items included which emphasized the same logical principle: one of a mathematical nature and one of a non-mathematical nature.

ATTITUDE TEST

A five-point scale of attitudes toward subjects and topics in the student's school curriculum was administered at the end of the project to all participants in the project. Faces were drawn, with expressions ranging from the most miserable to the most delighted, indicating on a five-point scale, how the student felt about the subject or topic. A box in between the happy face and the middle face, for example, shows a medium liking. If the child liked the subject very much, he would put a 5. If he disliked it very much, he would put a 1.

Topics included on the attitude scale were: school, science, reading, mathematics, addition, subtraction, and logic.

CHAPTER FOUR

ANALYSIS OF DATA

In the initial section of this chapter, analysis is made of the age and intelligence of the sample populations. The next section presents the analysis of the results from the Class and Conditional Reasoning tests and the Wisconsin Contemporary Elementary Mathematics tests. A major portion of this section is devoted to analyzing strengths and weaknesses at each grade level of the extent to which basic logic principles were learned.

Following this is the analysis of the scores on the Logical Thinking Test administered as a final and retention test only to those students in treatments LM and L. Concluding this is a brief analysis of the attitude test results administered to all students.

DESCRIPTIVE DATA

INTELLIGENCE

The intelligence quotients were taken from the California Test of Mental Maturity, administered at the schools for appropriate levels for both grade three and grade six. Analyses of variance of the I.Q. and age in months are reported in Tables 4.1 and 4.3. The differences in the I.Q. in the third grade were significant, $p \leq .05$, only in the pairwise comparison of LM + L with I + C as shown in Table 4.2. There were no significant differences in third grade in the pairwise comparison as shown by the Scheffé test reported in Table 4.2.

TABLE 4.1

ANALYSIS OF VARIANCE OF I.Q. FOR GRADES 3 AND 6

GRADE 3

<u>GROUP</u>	<u>MEAN</u>	<u>EFFECT</u>	<u>STANDARD DEVIATION</u>	<u>N</u>
LM	111.03	1.12	10.35	205
L	110.82	0.91	7.13	111
I	108.60	-1.32	9.26	114
C	107.69	-2.22	7.93	81

<u>SOURCE OF VARIATION</u>	<u>DF</u>	<u>SUM OF SQUARES</u>	<u>MEAN SQUARE</u>	<u>F RATIO</u>	<u>SIGN. LEVEL</u>
Between groups	3	946.33	315.44	3.79	.05
Within groups	507	42179.88	83.20		
TOTAL	510	43126.21			

GRADE 6

<u>GROUP</u>	<u>MEAN</u>	<u>EFFECT</u>	<u>STANDARD DEVIATION</u>	<u>N</u>
LM	110.88	-4.23	13.02	119
L	117.11	1.99	13.60	202
I	116.72	1.60	15.16	113
C	114.61	-0.51	11.76	155

<u>SOURCE OF VARIATION</u>	<u>DF</u>	<u>SUM OF SQUARES</u>	<u>MEAN SQUARE</u>	<u>F RATIO</u>	<u>SIGN. LEVEL</u>
Between groups	3	3265.03	1088.34	6.11	.01
Within groups	585	104249.89	178.20		
TOTAL	588	107514.92			

*F = 2.62 significant at .05

*F = 3.83 significant at .01

In grade six, there was a significant difference, $p \leq .01$, for the following pairwise comparisons as shown in Table 4.2. L was higher than LM, $p \leq .01$; I over LM at $p \leq .05$; but all other pairwise comparisons, including LM + L and I + C, were not significant. Since there were differences in I.Q., convariance analyses were necessary.

AGE

Average chronological age of third graders was 8.68 years or 104.13 months and of sixth graders 11.57 years or 138.87 months. Analyses of variance produced no evidence of significant differences in ages among the groups.

TABLE 4.2
SCHEFFE TEST FOR DIFFERENCES IN I.Q.

<u>PAIRWISE GROUPS</u>	<u>SIGN. LEVEL GR. 3</u>	<u>HIGHER MEAN GR. 3</u>	<u>SIGN. LEVEL GR. 6</u>	<u>HIGHER MEAN GR. 6</u>
LM-L	NS	LM	.01	L
LM-I	NS	LM	.05	I
LM-C	NS	LM	NS	C
L-I	NS	L	NS	L
L-C	NS	L	NS	L
I-C	NS	I	NS	I
(LM+L)-(I+C)	.05	LM+L	NS	I+C

TABLE 4.3

ANALYSIS OF VARIANCE OF AGE IN MONTHS

GRADE 3

<u>GROUP</u>	<u>MEAN</u>	<u>EFFECT</u>	<u>STANDARD DEVIATION</u>	<u>N</u>	
LM	103.53	-0.44	5.17	205	
L	103.82	-0.15	5.03	111	
I	104.23	0.26	4.81	114	
C	104.94	0.97	5.26	81	
<u>SOURCE OF VARIATION</u>	<u>DF</u>	<u>SUM OF SQUARES</u>	<u>MEAN SQUARE</u>	<u>F RATIO</u>	<u>SIGN. LEVEL</u>
Between Groups	3	125.41	41.80	1.62	NS
Within Groups	507	13058.20	25.76		
TOTAL	510	13183.62			

GRADE 6

<u>GROUP</u>	<u>MEAN</u>	<u>EFFECT</u>	<u>STANDARD DEVIATION</u>	<u>N</u>	
LM	138.70	-0.17	4.44	119	
L	138.83	-0.04	5.12	202	
I	138.94	0.07	4.83	113	
C	138.99	0.13	4.13	155	
<u>SOURCE OF VARIATION</u>	<u>DF</u>	<u>SUM OF SQUARES</u>	<u>MEAN SQUARE</u>	<u>F RATIO</u>	<u>SIGN. LEVEL</u>
Between groups	3	6.72	2.24	0.10	NS
Within groups	585	12812.95	21.90		
TOTAL	588	12819.67			

CORNELL REASONING PRETEST

The class and conditional reasoning tests as described in Chapter III were taken by all students in all treatments in both grades. All of the third graders had their tests read to them. Analysis of variance of pretest data is reported in Tables 4.4 and 4.5. There were significant differences in Grade 6 conditional reasoning and Grade 3 class reasoning.

Pairwise comparison using the Scheffé test (Table 4.6) showed in grade 3 that there was a significant difference at the .05 level between treatments LM and I, and LM and C with LM having the higher mean in both cases. There was a significant difference, $p \leq .01$, between the LM + L groups as contrasted with the I + C treatments in grade 3 also with LM + L having the higher mean.

In the sixth grade, pairwise comparison showed a .05 level of significance between LM and I with LM having the higher mean and a .05 level of significance between L and I with L having the higher mean. There was no significant difference in either grade in conditional reasoning pretest and class reasoning pretest between boys and girls.

CORNELL REASONING POSTTEST

Since there were differences in the Cornell Reasoning pretest among the groups, covariance was used in analyzing posttests. Covariates used were I.Q. and Cornell pretests.

Results of the analyses of covariance are reported in Tables 4.7 to 4.10. For all analyses of covariance, $p \leq .01$ for class reasoning and conditional reasoning in grades 3 and 6.

TABLE 4.4

ANALYSIS OF VARIANCE OF CLASS REASONING PRETEST

<u>GRADE 3</u>					
<u>GROUP</u>	<u>MEAN</u>	<u>EFFECT</u>	<u>STAND. DEVIATION</u>	<u>N</u>	
LM	12.79	0.50	3.24	205	
L	12.59	0.31	3.26	111	
I	11.70	-0.59	3.21	114	
C	11.43	-0.86	3.17	81	
<u>SOURCE OF VARIATION</u>	<u>DF</u>	<u>SUM OF SQUARES</u>	<u>MEAN SQUARE</u>	<u>F RATIO</u>	<u>SIGN.</u>
Between groups	3	159.66	53.22	5.11	.01
Within groups	507	5281.05	10.42	10.42	
TOTAL	510	5440.71			
<u>GRADE 6</u>					
<u>GROUP</u>	<u>MEAN</u>	<u>EFFECT</u>	<u>STAND. DEVIATION</u>	<u>N</u>	
LM	14.57	-0.25	2.82	119	
L	15.02	0.20	2.97	202	
I	14.39	-0.43	2.82	113	
C	15.08	0.25	2.92	155	
<u>SOURCE OF VARIATION</u>	<u>DF</u>	<u>SUM OF SQUARES</u>	<u>MEAN SQUARE</u>	<u>F RATIO</u>	<u>SIGN.</u>
Between groups	3	46.63	15.54	1.85	NS
Within groups	585	4915.00	8.40		
TOTAL	588	4961.64			

TABLE 4.5

ANALYSIS OF VARIANCE OF CONDITIONAL REASONING PRETEST

<u>GRADE 3</u>					
<u>GROUP</u>	<u>MEAN</u>	<u>EFFECT</u>	<u>STAND. DEVIATION</u>	<u>N</u>	
LM	9.65	0.03	2.30	205	
L	10.07	0.45	2.62	111	
I	9.42	-0.20	2.24	114	
C	9.22	-0.40	2.37	81	
<u>SOURCE OF VARIATION</u>	<u>DF</u>	<u>SUM OF SQUARES</u>	<u>MEAN SQUARE</u>	<u>F RATIO</u>	<u>SIGN.</u>
Between groups	3	40.24	13.41	2.38	NS
Within groups	507	2853.62	5.63		
TOTAL	510	2893.86			
<u>GRADE 6</u>					
<u>GROUP</u>	<u>MEAN</u>	<u>EFFECT</u>	<u>STAND. DEVIATION</u>	<u>N</u>	
LM	10.92	0.36	2.65	119	
L	10.71	0.15	3.18	202	
I	9.74	-0.81	2.62	113	
C	10.66	0.11	3.01	155	
<u>SOURCE OF VARIATION</u>	<u>DF</u>	<u>SUM OF SQUARES</u>	<u>MEAN SQUARE</u>	<u>F RATIO</u>	<u>SIGN.</u>
Between groups	3	96.53	32.18	3.74	.05
Within groups	585	5031.04	8.60		
TOTAL	588	5127.57			

TABLE 4.6

SCHEFFE TEST FOR DIFFERENCES IN CLASS AND
CONDITIONAL REASONING PRETESTSClass Reasoning Pretest

<u>PAIRWISE GROUPS</u>	<u>SIGN. LEVEL GR. 3</u>	<u>HIGHER MEAN GR. 3</u>	<u>SIGN. LEVEL GR. 6</u>	<u>HIGHER MEAN GR. 6</u>
LM-L	NS	LM	NS	L
LM-I	.05	LM	NS	LM
LM-C	.05	LM	NS	C
L-I	NS	L	NS	L
L-C	NS	L	NS	C
I-C	NS	I	NS	C
(LM+L)-(I+C)	.01	LM+L	NS	LM+L

Conditional Reasoning Pretest

<u>PAIRWISE GROUPS</u>	<u>SIGN. LEVEL GR. 3</u>	<u>HIGHER MEAN GR. 3</u>	<u>SIGN. LEVEL GR. 6</u>	<u>HIGHER MEAN GR. 6</u>
LM-L	NS	L	NS	LM
LM-I	NS	LM	.05	LM
LM-C	NS	LM	NS	LM
L-I	NS	L	.05	L
L-C	NS	L	NS	L
I-C	NS	I	NS	C
(LM+L)-(I+C)	NS	LM+L	NS	LM+L

In grade 3, the adjusted mean was highest for LM and lowest for I with L and C about the same. Almost the same kind of pattern can be seen on the conditional reasoning; LM is the highest with I and C the lowest. Using I.Q. as covariates, LM and L have an obvious advantage over I and C. The absolute difference favoring LM and/or L is in the neighborhood of one point. Overall, there was a decided advantage in grade 3 on this reasoning test for students taught logic interwoven with mathematics.

Using Scheffé test for pairwise comparison with Class Reasoning pretest as covariate, scores of students in the LM group differed significantly, $p \leq .01$, over I but that was the only significant difference in the Class Reasoning posttest. With I.Q. as covariate, LM differed significantly from I at the .01 level and the group taught logic, namely, LM and L, differed significantly, $p \leq .01$, from the group not taught logic, I and C. (Table 4.11).

In the conditional reasoning in third grade, with I.Q. as covariate and also with conditional reasoning pretest as covariate, there was a significant difference, $p \leq .01$ of LM over I and LM over C and also of the logic groups over groups not taught logic. (Table 4.11).

In the sixth grade in all cases, the L and LM groups were substantially higher than I and C, with a one to one-and-a-half point advantage. But the L group was consistently better than the LM group, contrary to grade 3. Thus it appears that logic taught alone produced better results in grade 6. One can only conjecture that this indicates that the students in grade 6 are better able to work with logical ideas divorced from the subject matter content at this level than might occur at grade 3.

TABLE 4.7

ANALYSIS OF COVARIANCE OF CLASS REASONING POSTTEST
SCORES WITH CLASS REASONING PRETEST AS COVARIATE

GRADE 3

Correlation = .463

<u>GROUP</u>		<u>MEAN</u>		<u>ADJUSTED MEAN</u>	
LM		14.59		14.30	
L		13.35		13.17	
I		12.35		12.69	
C		12.74		13.24	

<u>SOURCE OF VARIATION</u>	<u>DF</u>	<u>ADJUSTED SUM SQUARE</u>	<u>MEAN SQUARE</u>	<u>F RATIO</u>	<u>SIGN.</u>
Between groups	3	218.54	72.85	5.21	.01
Within groups	506	7068.65	13.97		
TOTAL	509	72.87.18			

GRADE 6

Correlation = .484

<u>GROUP</u>		<u>MEAN</u>		<u>ADJUSTED MEAN</u>	
LM		15.67		15.79	
L		16.38		16.29	
I		14.66		14.86	
C		15.52		15.40	

<u>SOURCE OF VARIATION</u>	<u>DF</u>	<u>ADJUSTED SUM SQUARE</u>	<u>MEAN SQUARE</u>	<u>F RATIO</u>	<u>SIGN.</u>
Between groups	3	163.28	54.43	9.55	.01
Within groups	584	3327.24	5.70		
TOTAL	587	3490.52			

TABLE 4.8

ANALYSIS OF COVARIANCE OF CONDITIONAL REASONING POSTTEST
SCORES WITH CONDITIONAL REASONING PRETEST AS COVARIATE

GRADE 3

Correlation = .418

CORRELATION = .7410					
<u>GROUP</u>		<u>MEANS</u>		<u>ADJUSTED</u> <u>MEAN</u>	
LM		11.26		11.25	
L		10.89		10.70	
I		10.07		10.16	
C		9.97		9.96	
<u>SOURCE OF</u> <u>VARIATION</u>	<u>DF</u>	<u>ADJUSTED SUM</u> <u>SQUARE</u>	<u>MEAN</u> <u>SQUARE</u>	<u>F RATIO</u>	<u>SIGN.</u>
Between groups	3	139.28	46.43	9.32	.01
Within groups	506	2519.50	4.98		
TOTAL	509	2658.78			

GRADE 6

Correlation = .453

CORRELATION = .7455			ADJUSTED		
<u>GROUP</u>		<u>MEANS</u>		<u>MEAN</u>	
LM		12.43		12.28	
L		12.90		12.84	
I		10.88		11.21	
C		11.65		11.61	
<u>SOURCE OF VARIATION</u>	<u>DF</u>	<u>ADJUSTED SUM SQUARE</u>	<u>MEAN SQUARE</u>	<u>F RATIO</u>	<u>SIGN.</u>
Between groups	3	237.84	79.28	13.05	.01
Within groups	584	3547.95	6.08		
TOTAL	587	3785.79			

TABLE 4.9

ANALYSIS OF COVARIANCE OF CLASS REASONING POSTTEST
SCORES WITH I.Q. AS COVARIATE

GRADE 3

Correlation = .258

<u>GROUP</u>		<u>MEAN</u>		<u>ADJUSTED MEAN</u>	
LM		14.59		14.46	
L		13.35		13.25	
I		12.35		12.50	
C		12.74		12.98	

<u>SOURCE OF VARIATION</u>	<u>DF</u>	<u>ADJUSTED SUM SQUARE</u>	<u>MEAN SQUARE</u>	<u>F RATIO</u>	<u>SIGN.</u>
Between groups	3	328.72	109.57	6.65	.01
Within groups	506	8331.73	16.47		
TOTAL	509	8660.45			

GRADE 6

Correlation = .291

<u>GROUP</u>		<u>MEAN</u>		<u>ADJUSTED MEAN</u>	
LM		15.67		15.93	
L		16.38		16.26	
I		14.66		14.57	
C		15.52		15.55	

<u>SOURCE OF VARIATION</u>	<u>DF</u>	<u>ADJUSTED SUM SQUARE</u>	<u>MEAN SQUARE</u>	<u>F RATIO</u>	<u>SIGN.</u>
Between groups	3	217.03	72.34	10.69	.01
Within groups	584	3952.61	6.77		
TOTAL	587	4169.63			

TABLE 4.10

ANALYSIS OF COVARIANCE OF CONDITIONAL REASONING
POSTTEST SCORES WITH I.Q. AS COVARIATE

GRADE 3

Correlation - .336

<u>GROUP</u>		<u>MEAN</u>	<u>ADJUSTED MEAN</u>
LM		11.26	11.17
L		10.89	10.82
I		10.07	10.18
C		9.79	9.98

<u>SOURCE OF VARIATION</u>	<u>DF</u>	<u>ADJUSTED SUM SQUARE</u>	<u>MEAN SQUARE</u>	<u>F RATIO</u>	<u>SIGN.</u>
Between groups	3	116.87	38.96	7.20	.01
Within groups	506	2739.23	5.41		
TOTAL	509	2856.09			

GRADE 6

Correlation = .291

<u>GROUP</u>		<u>MEAN</u>	<u>ADJUSTED MEAN</u>
LM		12.43	12.68
L		12.90	12.78
I		10.88	10.78
C		11.65	11.68

<u>SOURCE OF VARIATION</u>	<u>DF</u>	<u>ADJUSTED SUM SQUARE</u>	<u>MEAN SQUARE</u>	<u>F RATIO</u>	<u>SIGN.</u>
Between groups	3	357.31	119.10	17.16	.01
Within groups	584	4054.48	6.94		
TOTAL	587	4411.80			

TABLE 4.11

SCHEFFE TEST FOR DIFFERENCES IN REASONING POSTTESTS

Class Reasoning Posttest with I.Q. as Covariate

<u>PAIRWISE GROUPS</u>	<u>SIGN. LEVEL GR. 3</u>	<u>HIGHER MEAN GR. 3</u>	<u>SIGN. LEVEL GR. 6</u>	<u>HIGHER MEAN GR. 6</u>
LM-L	NS	LM	NS	L
LM-I	.01	LM	.01	LM
LM-C	NS	LM	NS	LM
L-I	NS	L	.01	L
L-C	NS	L	NS	L
I-C	NS	C	.05	C
(LM+L)-(I+C)	.01	LM+L	.01	LM+L

Class Reasoning Posttest with Class Reasoning
Pretest as Covariate

<u>PAIRWISE GROUPS</u>	<u>SIGN. LEVEL GR. 3</u>	<u>HIGHER MEAN GR. 3</u>	<u>SIGN. LEVEL GR. 6</u>	<u>HIGHER MEAN GR. 6</u>
LM-L	NS	LM	NS	L
LM-I	.01	LM	.05	LM
LM-C	NS	LM	NS	LM
L-I	NS	L	.01	L
L-C	NS	C	.01	L
I-C	NS	C	NS	C
(LM+L)-(I+C)	NS	LM+L	.01	LM+L

TABLE 4.11 (continued)

SCHEFFE TEST FOR DIFFERENCES IN REASONING POSTTESTS

Conditional Reasoning Posttest with I.Q. as Covariate

<u>PAIRWISE GROUPS</u>	<u>SIGN. LEVEL GR. 3</u>	<u>HIGHER MEAN GR. 3</u>	<u>SIGN. LEVEL GR. 6</u>	<u>HIGHER MEAN GR. 6</u>
LM-L	NS	LM	NS	L
LM-I	.01	LM	.01	LM
LM-C	.01	LM	.05	LM
L-I	NS	L	.01	L
L-C	NS	L	.01	L
I-C	NS	I	NS	C
(LM+L)-(I+C)	.01	LM+L	.01	LM+L

Conditional Reasoning Posttest with Conditional
Reasoning Pretest as Covariate

<u>PAIRWISE GROUPS</u>	<u>SIGN LEVEL GR. 3</u>	<u>HIGHER MEAN GR. 3</u>	<u>SIGN. LEVEL GR. 6</u>	<u>HIGHER MEAN GR. 6</u>
LM-L	NS	LM	NS	L
LM-I	.01	LM	.01	LM
LM-C	.01	LM	NS	LM
L-I	NS	L	.01	L
L-C	NS	L	.01	L
I-C	NS	I	NS	C
(LM+L)-(I+C)	.01	LM+L	.01	LM+L

In grade 6 in class reasoning, there was a significant difference, $p \leq .01$, of LM over I; L over I and the group taught logic over the group not taught logic. There was also a significant difference, using I.Q. as covariate, $p \leq .05$, of the C group over the I group. (Table 4.11).

With class reasoning pretest as covariate, there was a significant difference in grade 6, $p \leq .01$, of the L group over I and C, and the group taught logic over the group not taught logic. (Table 4.11). At the .05 level, there was a significant difference of the scores of the LM group over the I group.

In conditional reasoning in grade 6, using the pretest as covariate, there was a significant difference, $p \leq .01$, of LM over I, L over I and C, and the group taught logic over the group not taught logic. With I.Q. as covariate, there was a significant difference, $p \leq .01$, of LM over I, L over I and C, and the group taught logic over the group not taught logic. At the .05 level, there was a significant difference of LM over C also.

Difficult to explain is the superiority of C over I that occurred consistently in grade 6. It happened that in the I treatment, there was one sixth grade teacher for the three classes in the group. This teacher, while not dogmatic or authoritarian, was very systematic as evidenced in her classroom teaching. Whether this style of teaching is influential as evidenced by the scores not only in the reasoning but in the mathematics tests cannot be determined. It has been indicated in other projects that not all teachers come from in-service training with the same ideas and certainly there is a variety of impact on these people.

There was no significant difference between boys and girls even at the .05 level in either class or conditional reasoning pretests or

posttests either at the third or sixth grade level. These results are in agreement with Piaget, Hill, and O'Brien, that sex is not a factor in the differences in scores.

ITEM ANALYSES ON CLASS AND CONDITIONAL REASONING

Hill (51) found that primary children were, on the whole, able to do the validity principles but she did not test for fallacy principles. Shapiro and O'Brien (82) did test for the fallacy principles and found that primary children did poorly with them. Both researchers used paper and pencil tests. Findings in this study agree with Shapiro in this regard. Both in the pretest of class and conditional reasoning, third and sixth graders did poorly on those principles involving fallacies. In class reasoning, these were principles 3 and 6; in conditional reasoning, principles 2,3, and 7.

Tables 4.12 and 4.13 show the percent of correct responses for the total population in grade 3 and in grade 6 on both pretest and posttest of the class reasoning and conditional reasoning.

In the pretest in grade 3, 44.58% answered principle 3 correctly in class reasoning, while in the posttest, 39.54% answered it. Fallacy principle 6 was answered by only 35.05% of the third graders in class reasoning pretest correctly with 45.80% in the posttest. Fallacy principle 6 in class reasoning was answered correctly by 57.40% of the sixth graders in the pretest and 64.12% in the posttest.

In conditional reasoning, only about 23% of the third graders were able to answer fallacy principles 2,3, and 7 correctly with about 30% of the sixth graders answering these correctly.

TABLE 4.12

PERCENT OF CORRECT RESPONSES IN RANK ORDER
ON CLASS REASONING PRINCIPLES

GRADE 3

<u>PRINCIPLE</u>	<u>GRADE 3 PRETEST</u>	<u>PRINCIPLE</u>	<u>GRADE 3 POSTTEST</u>	<u>PRINCIPLE</u>	<u>TOTAL GR. 3</u>
2	78.16	2	81.73	2	79.95
1	70.99	4	79.85	1	72.74
5	69.72	1	74.49	4	70.07
4	60.29	5	70.32	5	70.02
7	52.32	7	58.16	7	55.24
3	44.58	6	45.80	3	42.06
6	35.05	3	39.54	6	40.43

GRADE 6

<u>PRINCIPLE</u>	<u>PRETEST</u>	<u>PRINCIPLE</u>	<u>POSTTEST</u>	<u>PRINCIPLE</u>	<u>TOTAL</u>
1	90.90	1	91.99	1	91.45
2	84.07	2	90.38	2	87.23
4	79.34	4	88.04	4	83.69
5	67.66	5	71.45	5	69.56
7	57.76	7	67.66	7	62.71
6	57.40	6	64.12	6	60.76
3	57.10	3	55.61	3	56.36

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TABLE 4.13

PERCENT OF CORRECT RESPONSES IN RANK ORDER
ON CONDITIONAL REASONING PRINCIPLES

GRADE 3

<u>PRINCIPLE</u>	<u>PRETEST</u>	<u>PRINCIPLE</u>	<u>POSTTEST</u>	<u>PRINCIPLE</u>	<u>TOTAL</u>
1	78.74	5	74.02	1	75.99
5	66.63	1	73.23	5	70.33
4	60.64	4	67.93	4	64.29
6	45.72	6	57.24	6	51.48
2	23.61	3	28.67	3	25.95
3	23.23	2	23.90	2	23.76
7	20.83	7	21.76	7	21.30

GRADE 6

<u>PRINCIPLE</u>	<u>PRETEST</u>	<u>PRINCIPLE</u>	<u>POSTTEST</u>	<u>PRINCIPLE</u>	<u>TOTAL</u>
1	75.26	1	81.77	1	78.52
4	71.60	5	79.33	5	73.84
5	68.34	4	75.22	4	73.41
6	45.75	6	57.68	6	51.50
3	37.57	3	43.30	3	40.44
2	28.68	2	30.15	2	29.42
7	21.55	7	25.04	7	23.30

TABLE 4.13 (continued)

PERCENT OF CORRECT RESPONSES IN RANK ORDER
ON CONDITIONAL REASONING PRINCIPLES

<u>PRINCIPLE</u>	<u>TOTAL PRETEST GR. 3 AND 6</u>	<u>PRINCIPLE</u>	<u>TOTAL POSTTEST GR. 3 AND 6</u>
1	77.00	1	77.50
5	67.49	5	76.68
4	66.12	4	71.58
6	45.74	6	57.48
3	30.41	3	35.99
2	26.15	2	27.02
7	21.19	7	23.41

The percents of change in correct responses to the various principles from pretest to posttest are reported in Tables 4.14 to 4.17. Included also in these tables are the percent of correct responses with Yes, No, or Maybe answers as well as other types such as concrete familiar, symbolic, and suggestive items. There was an increase in the number of students getting correct answers in fallacy principles from pretest to posttest, in general.

In conditional reasoning in both grades 3 and 6, greater changes were made, on the whole, in the contrapositive principles than on fallacy principles. Grade 3 in class reasoning made greater increases in fallacy than in contrapositive while grade 6 did the opposite, increase contrapositive score 12.53% over a 4.36% increase in fallacy. (See Tables 4.14 to 4.17.)

In the third grade class reasoning from pretest to posttest, the greatest increase in percent of students who answered a principle correctly occurred in principle 4 which says that class exclusion is symmetric, that is, no A's are B's means that no B's are A's. The second greatest increase was in principle 6 which is a fallacy principle: whatever is not a member of a class is not (as a result of that relationship) necessarily a member of (nor a non-member of) another class which is excluded from the first. (See Tables 4.14 and 4.15)

There was the greatest increase in treatments LM and L over I and C in these two principles. In the third grade also there was the third greatest increase in principle 7, the contrapositive principle: whatever is not a member of a class is also not a member of any class included in the first. In the rank order of principles, principle 7 was fifth and principle 6 was last; principle 4 was third.

TABLE 4.14

PERCENT OF CHANGE FROM PRETEST TO POSTTEST
IN PRINCIPLES OF CLASS REASONING

GRADE 3

<u>PRINCIPLE</u>	<u>TREATMENT LM</u>	<u>TREATMENT L</u>	<u>TREATMENT I</u>	<u>TREATMENT C</u>
1	9.7%	10.9%	-1.2%	.5%
2	1.6	5.0	6.6	9.0
3	1.0	-43.0	6.6	5.0
4	33.0	34.7	27.6	34.6
5	3.9	.3	-6.6	-1.0
6	32.4	30.2	24.3	35.4
7	10.7	17.4	14.2	2.1
Total for Principles	11.4	6.8	8.1	10.2
<u>YES ANSWERS</u>	12.4	17.7	-1.0	15.1
<u>NO ANSWERS</u>	6.8	5.1	-1.0	-.3
<u>MAYBE ANSWERS</u>	15.9	-13.5	11.7	18.7
<u>Components</u>				
CF1	5.0	-7.9	21.9	15.9
CF2	12.0	7.7	13.8	26.1
CF3	.4	20.5	7.8	19.4
CF4	7.6	~~4.9~~	2.5	.02
SY	26.8	19.6	2.7	-6.5
SU	12.6	18.5	0	6.9

TABLE 4.15
PERCENT OF CHANGE FROM PRETEST TO POSTTEST
IN PRINCIPLES OF CLASS REASONING

GRADE 6

<u>PRINCIPLE</u>	<u>TREATMENT LM</u>	<u>TREATMENT L</u>	<u>TREATMENT I</u>	<u>TREATMENT C</u>
1	-1.9	3.6	2.4	1.0
2	7.1	17.9	4.0	1.7
3	.7	2.2	14.2	.8
4	12.6	14.2	13.5	3.8
5	18.0	10.4	.6	-5.9
6	19.6	24.3	.5	1.0
7	53.8	11.5	3.9	7.9
Total for Principles	13.0	11.8	2.4	1.2
<u>YES ANSWERS</u>	19.7	14.8	7.3	4.1
<u>NO ANSWERS</u>	6.1	6.5	1.6	-2.0
<u>MAYBE ANSWERS</u>	10.3	13.3	-7.5	-.1
<u>Components</u>				
CF1	12.7	15.0	4.6	-4.1
CF2	3.7	12.3	-1.6	3.0
CF3	18.6	5.8	2.3	6.2
CF4	8.4	9.6	2.4	-.6
SY	34.3	30.1	-25.5	-5.0
SU	-2.6	9.4	9.3	1.0

TABLE 4.16

PERCENT OF CHANGE FROM PRETEST TO POSTTEST
IN PRINCIPLES OF CONDITIONAL REASONING

GRADE 3

<u>PRINCIPLE</u>	<u>TREATMENT LM</u>	<u>TREATMENT L</u>	<u>TREATMENT I</u>	<u>TREATMENT C</u>
1	-10.1	1.7	-5.1	-11.1
2	18.9	-3.6	3.4	17.3
3	84.7	8.4	12.4	-25.8
4	25.9	8.3	5.6	8.6
5	10.4	21.6	2.4	11.3
6	30.7	24.8	19.8	25.9
7	12.5	-5.2	60.0	-29.5
Total for Principles	17.6	8.9	4.5	.7
YES ANSWERS	7.6	11.2	4.2	4.4
NO ANSWERS	32.0	14.3	1.6	17.3
MAYBE ANSWERS	37.9	.3	19.8	-24.0
<u>Components</u>				
CF1	32.9	3.5	.8	10.2
CF2	10.0	15.6	9.2	5.6
CF3	49.1	9.1	3.3	.2
CF4	-7.7	.2	5.1	-2.5
SY	26.2	17.4	6.9	28.2
SU	3.9	-11.0	87.3	-26.5

TABLE 4.17

PERCENT OF CHANGE FROM PRETEST TO POSTTEST
IN PRINCIPLES OF CONDITIONAL REASONING

GRADE 6

<u>PRINCIPLE</u>	<u>TREATMENT LM</u>	<u>TREATMENT L</u>	<u>TREATMENT I</u>	<u>TREATMENT C</u>
1	9.9	7.9	5.9	10.9
2	7.6	34.9	-17.0	-10.5
3	-2.6	60.0	-10.9	10.8
4	14.3	2.6	3.5	6.1
5	19.8	21.3	9.3	13.9
6	54.4	11.1	22.2	19.2
7	24.7	15.3	27.9	.1
Total for Principles	17.6	18.3	4.2	9.1
YES ANSWERS	22.4	11.8	9.3	11.8
NO ANSWERS	15.5	3.6	-5.2	12.2
MAYBE ANSWERS	7.5	39.4	-5.9	1.1
CF1	9.0	21.0	-18.5	.1
CF2	21.8	14.1	9.1	13.6
CF3	17.4	29.1	1.7	3.5
CF4	15.5	12.2	8.9	6.6
SY	22.7	17.8	10.7	14.9
SU	11.5	7.8	25.8	19.1

A caution should be made about using such tables as 4.14 - 4.17. If students score high on certain principles there can't be much change from pretest to posttest. Similarly, it could quite possibly be that they score very low on both pretest and posttest and change would be minimal. Therefore, these tables must be used in conjunction with other tables which give total percents of correct answers. In other words, if 90% of a group answers a principle correctly, there might not be much change in scores from pretest to posttest because the students already indicate a knowledge of the principle. Also, if 10% of a group might answer an item correctly, only 5% more need to score higher in the posttest to make a 50% increase. It is imperative for careful interpretation of these tables that rank order of principles be taken into consideration.

The pattern of increase in grade six was more complex. For treatments I and C, where no logic was taught, there were the greatest increases in principles 7, 2, and 4, although these increases were considerably less than those in treatments LM and L. Fallacy principle 3 showed a small gain in treatment L from pretest to posttest while there was a negative difference in treatments LM, I, and C. (See Tables 4.16 and 4.17).

In contrast, fallacy principle 6 showed substantial increase in treatments LM and L, a decrease in treatment I and a slight gain in treatment C. Note that principle 6 showed substantial gains in all treatments in grade 3.

THE "MAYBE" FACTOR

O'Brien and Shapiro (82) agreed that in Hill's test of 100 items, the students were not called upon to test the logical necessity of the conclusion, a behavior which they felt was vital to an adequate understanding of hypothetical-deductive reasoning. They "opened up" 33 of the original 100 items so that no necessary conclusion followed from the premises and another option was added to the existing "Yes" and "No" on Hill's test, namely, "Not enough clues."

In a discussion of their results, O'Brien and Shapiro report :

In considering the meaning of these results, it would seem that as Hill suggested, children of ages six, seven and eight have considerable success in recognizing logically necessary conclusions. They "peak out" high and early in this ability. Children of the same age, however, experience great difficulty in testing the logical necessity of a conclusion and they show slow growth in this ability. That these two behavioral manifestations of hypothetical-deductive thinking occur at such different levels among children of the same age seems to bring into question the challenge that the original research gave to Piaget's theory regarding the growth of this kind of logical thinking in children.

One of the peculiar findings of the present research was that the children's performance of the 33 "open" items in test B was, as is indicated by the negative corrected scores for these Test B items, below the chance level. That is, the children could have done better on these items by simple random guessing. It seems clear that for one reason or another, the children avoid "Not enough clues" response, thus indicating an avoidance of "open" situations. What the reasons for this behavior might be is at present unknown to the investigators. What is known is that the existing primary school curriculum seems to have had little effect (at least immediately) on children's tolerance of or ability to recognize "open" situations. (82)

This study showed that the students in both third and sixth grades did not do as well in either pretests or posttests on those items which had "Maybe" as a correct answer. This is shown in the tables on the rank

order of items, 4.18 - 4.22, which indicate the differences not only in the items but also in the totals of correct answers for "Yes", "No", and "Maybe".

In the conditional reasoning in grade three, the nine "Maybe" items were at the bottom nine in the rank order in treatments L, I, and C, both pretests and posttests. In treatment LM, one of the items went from 18th place in the pretest to 9th place in the posttest. All others were in the bottom third of the test. With few variations, almost this same pattern occurred in grade 6 in conditional reasoning. There were occasional differences in the ranking in the LM and L treatments as indicated in Tables 4.20 - 4.22.

For class reasoning, the "Maybe" items were, for the most part, in the bottom third of the ranking in grade three. There was little change in placement in grade three, but an increase in percent of students who answered "Maybe" items correctly except in treatment L, an unexpected finding. This decrease was almost exclusively in principle 3 of class reasoning. (See Table 4.18).

The sixth grade students varied their patterns of "Maybe" answers in treatments LM and L where they were scattered in the bottom two-thirds of the items. Treatments I and C in class reasoning followed the conditional reasoning pattern in that they had most of their "Maybe" answers in the bottom third of the item ranking as shown in Table 4.19.

The relative position of many items remained more nearly the same in pretests and posttests with regard to "Maybe" answers than items with correct answers of "Yes" and "No." (See Tables 4.18 - 4.21). A random check of the students' answers indicated that when they answered

TABLE 4.18

RANK ORDER OF ITEMS IN CLASS REASONING

GRADE 3Groups in Treatments LM and L

RANK	<u>LM PRETEST</u>			<u>LM POSTTEST</u>			<u>L PRETEST</u>			<u>L POSTTEST</u>		
	PRIN.ANS. %			PRIN.ANS. %			PRIN.ANS. %			PRIN.ANS. %		
1	21	Y	87.20	7	Y	88.24	14	N	87.07	13	Y	88.60
2	13	Y	86.26	25	Y	85.78	21	Y	81.90	11	Y	87.72
3	20	N	78.67	11	Y	83.33	13	Y	81.03	7	Y	85.96
4	25	Y	76.30	18	N	82.84	25	Y	77.59	14	N	84.21
5	14	N	76.30	13	Y	81.57	10	N	75.86	18	N	83.33
6	18	N	76.30	14	N	81.37	20	N	72.44	25	Y	82.46
7	10	N	67.77	20	N	80.39	27	N	69.82	21	Y	81.58
8	7	Y	67.77	21	Y	78.43	18	N	68.97	10	N	80.70
9	11	Y	65.40	10	N	78.43	15	Y	64.66	20	N	78.95
10	24	N	64.45	9	Y	77.45	24	N	62.93	15	Y	78.95
11	15	Y	61.13	15	Y	74.51	8	M	62.93	24	N	74.56
12	17	Y	54.98	24	N	72.55	11	Y	62.93	9	Y	70.18
13	27	N	54.50	19	M	64.22	9	Y	62.07	27	N	57.72
14	9	Y	54.03	8	M	82.84	12	M	58.62	19	M	55.26
15	26	M	54.03	17	Y	54.90	7	Y	56.03	17	Y	55.26
16	12	M	49.76	26	M	54.90	26	M	50.86	23	Y	52.63
17	8	M	47.39	27	N	50.98	23	Y	49.66	26	M	48.25
18	23	Y	45.50	22	M	49.02	17	Y	44.83	22	M	41.23
19	22	M	45.02	23	Y	47.06	16	M	43.10	12	M	36.84
20	16	M	46.60	12	M	44.01	22	M	41.38	16	M	29.82
21	19	M	27.96	16	M	40.20	19	M	18.97	8	M	27.19

Y = Yes

N = No

M = Maybe

% = percent of students who answered item correctly.

TABLE 4.18 (continued)
RANK ORDER OF ITEMS IN CLASS REASONING
GRADE 3

RANK	<u>I PRETEST</u>			<u>I POSTTEST</u>			<u>C PRETEST</u>			<u>C POSTTEST</u>		
	PRIN.	ANS.	%	PRIN.	ANS.	%	PRIN.	ANS.	%	PRIN.	ANS.	%
1	21	Y	81.51	13	Y	90.76	20	N	81.52	7	Y	88.04
2	13	Y	78.15	9	Y	85.81	21	Y	80.43	13	Y	84.76
3	10	N	76.47	21	Y	85.71	14	N	78.26	21	Y	82.61
4	18	N	76.47	10	N	78.15	13	Y	77.17	10	N	81.52
5	20	N	75.63	7	Y	75.63	10	N	71.74	14	N	73.91
6	25	Y	70.59	11	Y	75.63	24	N	63.04	18	N	71.74
7	14	N	69.75	20	N	71.43	18	N	63.04	11	Y	71.74
8	7	Y	69.75	14	N	70.59	15	Y	60.87	25	Y	69.57
9	11	Y	63.87	15	Y	69.75	25	Y	59.78	20	N	68.48
10	24	N	61.34	25	Y	68.91	11	Y	59.78	15	Y	68.48
11	27	N	56.30	18	N	67.23	27	N	59.78	9	Y	68.48
12	15	Y	52.94	24	N	61.34	7	Y	57.61	24	N	67.39
13	23	Y	52.94	17	Y	57.98	23	Y	54.35	17	Y	54.34
14	9	Y	52.10	27	N	51.26	9	Y	52.17	27	N	53.26
15	17	Y	47.90	23	Y	47.90	8	M	42.39	22	M	42.39
16	12	M	43.70	12	M	44.54	26	M	42.39	12	M	39.13
17	8	M	42.86	22	M	43.70	17	Y	38.04	19	M	39.13
18	22	M	40.34	8	M	40.34	16	M	34.78	26	M	39.13
19	16	M	35.29	16	M	39.50	12	M	31.52	16	M	38.04
20	26	M	32.77	26	M	38.66	22	M	26.09	8	M	36.96
21	19	M	20.17	19	M	33.61	19	M	20.65	23	Y	33.70

Y = Yes

N = No

M = Maybe

% = percent of students who answered item correctly

TABLE 4.19
RANK ORDER OF ITEMS IN CLASS REASONING
GRADE 6

GROUPS IN TREATMENTS LM AND L

RANK	<u>LM PRETEST</u>			<u>LM POSTEST</u>			<u>L PRETEST</u>			<u>L POSTTEST</u>		
	PRIN.ANS. %			PRIN.ANS. %			PRIN.ANS. %			PRIN.ANS. %		
1	10	N	95.80	13	Y	95.00	10	N	96.14	10	N	97.13
2	20	N	94.96	10	N	94.17	20	N	95.65	13	Y	96.95
3	24	N	92.44	20	N	93.33	13	Y	92.75	20	N	96.95
4	13	Y	89.08	21	Y	91.67	7	Y	89.37	25	Y	94.74
5	14	N	89.08	24	N	90.00	24	N	83.57	11	Y	94.74
6	7	Y	87.39	7	Y	90.00	11	Y	82.61	7	Y	93.30
7	21	Y	84.87	15	Y	80.17	14	N	82.61	21	Y	91.87
8	11	Y	81.51	14	N	88.33	21	Y	79.23	24	N	91.39
9	25	Y	77.31	11	Y	87.50	18	N	75.36	9	Y	89.95
10	26	M	67.23	25	Y	82.50	9	Y	71.50	14	N	86.51
11	22	M	66.39	9	Y	80.83	15	Y	69.08	18	N	80.86
12	12	M	64.71	18	N	79.17	26	M	69.08	26	M	78.95
13	9	Y	60.50	26	M	74.17	25	Y	68.60	15	Y	78.95
14	18	N	55.46	22	M	70.83	12	M	66.67	19	M	76.56
15	16	M	54.62	23	Y	70.00	22	M	66.67	22	M	75.12
16	23	Y	54.62	19	M	65.00	8	M	64.73	23	Y	69.81
17	27	N	50.42	12	M	64.17	23	Y	60.87	12	M	68.11
18	15	Y	49.58	27	N	62.50	17	Y	57.97	8	M	63.16
19	8	M	49.58	17	Y	56.67	16	M	53.62	17	Y	60.77
20	19	M	42.02	8	M	54.17	19	M	49.76	27	N	59.81
21	17	Y	36.13	16	M	51.67	27	M	47.83	16	M	57.89

Y = Yes

N = No

M = Maybe

% = percent of students who answered item correctly

TABLE 4.19 (continued)

RANK ORDER OF ITEMS IN CLASS REASONING

GRADE 6GROUPS IN TREATMENTS I AND C

RANK	<u>I PRETEST</u>			<u>I POSTTEST</u>			<u>C PRETEST</u>			<u>C POSTTEST</u>		
	PRIN.ANS. %			PRIN.ANS. %			PRIN.ANS. %			PRIN.ANS. %		
1	10	N	94.23	10	N	92.90	10	N	94.51	10	N	95.06
2	20	N	91.03	20	N	91.61	13	Y	90.85	13	Y	93.83
3	13	Y	91.03	7	Y	83.97	20	N	90.24	7	Y	92.59
4	25	Y	85.90	25	Y	90.97	24	N	87.20	20	N	91.98
5	7	Y	83.97	13	Y	90.76	14	N	86.59	24	N	87.65
6	21	Y	82.05	11	Y	90.32	7	Y	85.37	25	Y	85.80
7	11	Y	80.13	21	Y	87.74	21	Y	84.15	11	Y	84.57
8	14	N	80.13	9	Y	85.81	25	Y	82.93	14	N	83.33
9	24	N	75.00	14	N	82.58	11	Y	82.32	21	Y	82.72
10	18	N	75.00	24	N	81.94	9	Y	76.22	15	Y	78.40
11	9	Y	71.15	18	N	75.77	18	N	71.95	18	N	76.54
12	15	Y	67.31	15	Y	69.75	26	M	68.90	9	Y	75.93
13	23	Y	62.82	23	Y	63.87	15	Y	68.29	8	M	69.75
14	12	M	59.62	26	M	55.16	8	M	67.07	26	M	67.77
15	26	M	57.05	17	Y	52.26	22	M	62.80	23	Y	65.43
16	27	N	49.36	8	M	49.03	17	Y	62.80	22	M	63.46
17	8	M	48.72	12	M	49.03	12	M	58.54	17	Y	56.79
18	17	Y	48.72	27	N	46.45	23	Y	54.88	16	M	56.17
19	22	M	45.51	22	M	43.87	19	M	53.05	19	M	55.31
20	16	M	44.23	19	M	43.23	16	M	53.05	12	M	51.23
21	19	M	40.38	16	M	32.90	27	N	47.17	27	N	34.57

Y = Yes

N = No

M = Maybe

% = percent of students who answered item correctly

TABLE 4.20

RANK ORDER OF ITEMS IN CONDITIONAL REASONING

GRADE 3GROUPS IN TREATMENTS LM AND L

RANK	<u>LM PRETEST</u>			<u>LM POSTTEST</u>			<u>L PRETEST</u>			<u>L POSTTEST</u>		
	PRIN.ANS. %			PRIN.ANS. %			PRIN.ANS. %			PRIN.ANS. %		
1	14	Y	82.78	8	N	89.27	10	Y	83.62	10	Y	87.72
2	10	Y	79.43	7	Y	82.90	14	Y	81.90	8	N	82.46
3	22	Y	78.47	10	Y	81.46	8	N	80.17	7	Y	82.46
4	7	Y	77.51	22	Y	78.54	26	Y	74.14	18	Y	74.56
5	18	Y	68.90	16	Y	77.56	22	Y	67.24	22	Y	73.68
6	8	N	67.46	18	Y	70.24	18	Y	62.93	26	Y	72.81
7	26	Y	66.99	20	Y	67.32	7	Y	62.93	20	Y	68.42
8	24	Y	46.41	15	Y	60.98	20	Y	56.03	15	Y	64.04
9	15	Y	45.45	21	M	60.00	15	Y	50.86	14	Y	62.28
10	16	N	43.06	16	N	56.59	12	Y	46.55	16	N	61.40
11	12	Y	42.58	14	Y	56.10	16	N	45.69	12	Y	59.65
12	20	Y	40.19	12	Y	52.68	24	Y	45.69	24	Y	57.02
13	23	M	36.84	24	Y	48.78	9	M	39.66	9	M	42.11
14	9	M	33.01	9	M	47.32	23	M	36.21	21	M	39.47
15	25	M	29.19	25	M	35.12	13	M	33.62	13	M	30.70
16	13	M	28.71	13	M	31.70	21	M	32.76	25	M	28.07
17	27	M	24.88	11	M	30.73	25	M	25.86	23	M	27.19
18	21	M	21.05	19	M	29.76	19	M	22.41	11	M	26.32
19	19	M	19.62	17	M	28.78	27	M	20.69	19	M	20.18
20	17	M	19.62	13	M	27.32	17	M	19.83	27	M	18.42
21	11	M	16.75	27	M	25.85	11	M	18.97	17	M	18.42

Y = Yes

N = No

M = Maybe

% = percent of students who answered item correctly

TABLE 4.20 (continued)

RANK ORDER OF ITEMS IN CONDITIONAL REASONING

GRADE 3GROUPS IN TREATMENTS I AND C

RANK	<u>I PRETEST</u>			<u>I POSTTEST</u>			<u>C PRETEST</u>			<u>C POSTTEST</u>		
	PRIN.ANS. %			PRIN.ANS. %			PRIN.ANS. %			PRIN.ANS. %		
1	14	Y	86.18	7	Y	83.61	10	Y	85.71	10	Y	88.89
2	7	Y	85.25	14	Y	81.97	7	Y	81.32	7	Y	83.33
3	10	Y	84.55	22	Y	81.97	14	Y	75.82	22	Y	75.56
4	18	Y	77.23	10	Y	77.05	18	Y	75.82	18	Y	72.22
5	8	N	67.48	18	Y	76.23	26	Y	73.63	26	Y	71.11
6	22	Y	65.04	26	Y	70.49	8	N	71.43	15	Y	71.11
7	26	Y	62.60	8	N	59.84	22	Y	64.84	8	N	68.89
8	15	Y	60.16	20	Y	59.02	15	Y	60.44	24	Y	63.33
9	24	Y	58.54	24	Y	54.10	24	Y	50.55	20	Y	58.89
10	20	Y	48.78	12	Y	52.46	12	Y	43.96	16	N	52.22
11	16	N	43.09	16	N	52.46	20	Y	38.46	14	Y	50.00
12	12	Y	30.89	15	Y	50.00	16	N	31.87	12	Y	45.16
13	9	M	28.46	21	M	30.33	25	M	25.27	25	M	22.22
14	19	M	21.95	25	M	25.41	27	M	24.18	27	M	17.78
15	25	M	21.95	9	M	25.41	9	M	20.88	9	M	16.67
16	21	M	20.33	27	M	21.31	13	M	19.78	13	M	14.44
17	13	M	19.51	13	M	20.49	23	M	19.78	19	M	13.33
18	17	M	14.63	19	M	19.67	21	M	16.48	11	M	12.22
19	23	M	13.82	23	M	19.23	19	M	15.38	23	M	11.11
20	11	M	12.20	11	M	18.85	11	M	14.29	17	M	10.00
21	27	M	11.38	17	M	15.57	17	M	12.09	21	M	10.00

Y = Yes

N = No

M = Maybe

% = percent of students who answered item correctly

TABLE 4.21

RANK ORDER OF ITEMS IN CONDITIONAL REASONING

GRADE 6GROUPS IN TREATMENTS LM AND L

RANK	<u>LM PRETEST</u>			<u>LM POSTTEST</u>			<u>L PRETEST</u>			<u>L POSTTEST</u>		
	PRIN.ANS. %			PRIN.ANS. %			PRIN.ANS. %			PRIN.ANS. %		
1	10	Y	81.82	7	Y	96.69	26	Y	86.47	10	Y	98.08
2	8	N	81.82	10	Y	95.04	10	Y	84.06	26	Y	87.50
3	26	Y	80.17	26	Y	90.08	7	Y	76.33	7	Y	87.02
4	7	Y	76.86	8	N	87.60	8	N	74.88	22	Y	86.06
5	22	Y	76.03	22	Y	85.12	22	Y	74.88	21	M	78.85
6	18	Y	72.72	18	Y	81.81	18	Y	71.50	18	Y	78.37
7	14	Y	71.90	15	Y	73.55	14	Y	68.60	15	Y	70.67
8	15	Y	60.33	16	N	72.72	20	Y	57.49	16	N	69.23
9	16	N	57.02	24	Y	72.72	16	N	56.04	8	N	66.35
10	21	M	56.20	14	Y	71.90	21	M	53.14	9	M	65.87
11	20	Y	50.41	12	Y	64.46	24	Y	50.72	20	Y	65.38
12	24	Y	45.45	20	Y	64.46	15	Y	49.76	14	Y	65.38
13	9	M	43.80	21	M	56.20	12	Y	40.58	24	Y	64.90
14	13	M	36.36	9	M	40.50	9	M	40.10	19	M	48.56
15	12	Y	34.71	11	M	37.19	13	M	35.26	13	M	47.12
16	23	M	29.75	19	M	33.06	19	M	31.88	17	M	42.31
17	25	M	25.62	23	M	30.58	23	M	31.40	11	M	39.42
18	17	M	25.62	13	M	30.58	25	M	29.47	12	Y	35.10
19	11	M	22.31	17	M	25.62	11	M	27.54	25	M	34.61
20	27	M	21.49	27	M	23.97	17	M	23.67	23	M	30.29
21	19	M	18.66	25	M	23.14	27	M	23.19	27	M	25.00

Y = Yes

N = No

M = Maybe

% = percent of students who answered item correctly

TABLE 4.21 (continued)

RANK ORDER OF ITEMS IN CONDITIONAL REASONING

GRADE 6GROUPS IN TREATMENT I AND C

RANK	<u>I PRETEST</u>			<u>I POSTTEST</u>			<u>C PRETEST</u>			<u>C POSTTEST</u>		
	PRIN.ANS. %			PRIN.ANS. %			PRIN.ANS. %			PRIN.ANS. %		
1	7	Y	83.97	7	Y	83.87	26	Y	85.28	10	Y	92.64
2	26	Y	81.41	22	Y	81.94	10	Y	82.82	7	Y	87.73
3	14	Y	79.49	26	Y	80.65	8	N	79.14	26	Y	82.21
4	10	Y	77.56	14	Y	80.65	18	Y	74.23	14	Y	82.21
5	8	N	73.71	18	Y	78.06	22	Y	73.01	8	N	80.98
6	22	Y	73.08	10	Y	76.77	7	Y	73.00	18	Y	80.37
7	18	Y	65.38	15	Y	61.29	14	Y	73.00	22	Y	74.23
8	15	Y	50.64	8	N	59.35	20	Y	58.90	16	N	71.16
9	16	N	46.79	16	N	54.84	16	N	56.44	15	Y	63.80
10	24	Y	44.87	20	Y	52.90	21	M	56.44	20	Y	63.80
11	20	Y	42.31	24	Y	51.61	15	Y	52.15	21	M	59.51
12	9	M	37.82	12	Y	45.16	24	Y	47.24	12	Y	57.06
13	12	Y	35.26	21	M	35.48	12	Y	41.10	24	Y	54.60
14	21	M	33.97	9	M	33.55	9	M	38.65	9	M	41.72
15	13	M	26.92	25	M	23.23	13	M	31.90	13	M	28.83
16	19	M	25.64	13	M	20.00	19	M	31.90	19	M	28.22
17	17	M	23.08	19	M	17.42	23	M	30.68	25	M	26.99
18	25	M	20.51	23	M	17.42	25	M	30.06	23	M	25.77
19	23	M	14.10	27	M	16.13	11	M	21.47	17	M	24.54
20	27	M	12.82	17	M	15.48	17	M	18.40	11	M	24.54
21	11	M	10.90	11	M	14.84	27	M	12.88	27	M	15.34

Y = Yes

N = No

M = Maybe

% = percent of students who answered item correctly

TABLE 4.22

DIFFERENCES IN PERCENTS BETWEEN PRETESTS AND
POSTTESTS WITH "YES," "NO," AND "MAYBE"
AS CORRECT ANSWERS

CLASS REASONINGGRADE 3

<u>ANSWERS</u>	<u>LM</u>			<u>L</u>			<u>I</u>			<u>C</u>		
	<u>PRE</u>	<u>POST</u>	<u>POST-PR</u>	<u>PRE</u>	<u>POST</u>	<u>POST-PR</u>	<u>PRE</u>	<u>POST</u>	<u>POST-PR</u>	<u>PRE</u>	<u>POST</u>	<u>POST-PR</u>
Yes	66.51	74.86	<u>8.25</u>	64.52	75.93	<u>11.41</u>	63.31	73.12	<u>9.81</u>	60.02	69.08	<u>9.06</u>
No	69.67	74.43	<u>4.76</u>	72.85	76.58	<u>3.73</u>	69.33	68.67	<u>-.66</u>	69.56	69.38	<u>-.18</u>
Maybe	44.63	51.72	<u>7.09</u>	45.98	39.77	<u>-6.21</u>	35.86	40.06	<u>4.20</u>	32.97	39.13	<u>6.16</u>

CLASS REASONINGGRADE 6

Yes	69.00	82.59	<u>13.59</u>	74.66	85.68	<u>11.02</u>	74.79	80.27	<u>5.48</u>	76.42	79.56	<u>3.14</u>
No	79.79	84.58	<u>4.89</u>	80.19	85.39	<u>5.20</u>	77.47	78.71	<u>1.24</u>	79.78	78.20	<u>-1.58</u>
Maybe	57.43	63.34	<u>5.91</u>	61.76	69.97	<u>8.21</u>	49.25	45.54	<u>-3.71</u>	60.57	60.62	<u>-.05</u>

CONDITIONAL REASONINGGRADE 3

Yes	62.87	67.66	<u>4.79</u>	63.19	70.26	<u>7.07</u>	65.92	68.69	<u>2.77</u>	65.06	67.95	<u>2.89</u>
No	55.26	72.93	<u>17.67</u>	62.93	71.93	<u>9.00</u>	55.29	56.15	<u>.86</u>	51.65	60.56	<u>8.91</u>
Maybe	25.52	35.18	<u>9.66</u>	27.78	27.88	<u>.10</u>	18.25	21.86	<u>3.61</u>	18.68	14.19	<u>-4.49</u>

CONDITIONAL REASONINGGRADE 6

Yes	65.04	79.58	<u>14.54</u>	66.04	73.85	<u>7.81</u>	63.40	69.29	<u>5.89</u>	66.07	73.87	<u>7.82</u>
No	69.42	80.16	<u>10.74</u>	65.46	67.79	<u>2.33</u>	60.25	57.10	<u>-3.15</u>	67.79	76.07	<u>8.28</u>
Maybe	31.09	33.43	<u>2.34</u>	32.85	45.78	<u>12.93</u>	22.86	21.51	<u>-1.35</u>	30.27	30.60	<u>.33</u>

incorrectly the "Yes" and "No" items; they very seldom used "Maybe." When they failed to answer correctly the "Maybe" items, they tended to use "No." In oral work, at least in the third grade, by contrast, when discussing these "Maybe" answers, the children would often respond, "Yes, but maybe."

The Cornell project worked extensively with the idea of concrete familiar, symbolic and suggestive. An explanation of these terms, as well as indications of categories of items is in Chapter Three. Students in this study followed the same pattern in regard to the concrete familiar. Since there was only one item that could be categorized as suggestive in both class and conditional reasoning, there are not enough data to make any definite statement. (See tables 4.14 - 4.17). However, it should be noted that the suggestive items are usually not as high in placement as the others, yet in the class reasoning, approximately 65% of the students answered it correctly. It has a "No" answer and by the very item itself, one wonders if it is really typical. "All dogs are brown." Then would this be true? "At least some dogs are not brown." In brief discussion with students after the posttest, it seemed that they were answering the question, "No, not all dogs are brown." This needs further investigation.

In contrast to this was the symbolic item on the conditional reasoning test. "Suppose you know that 'If a dog can stand on its front legs, then it is a puppy.' Then would this be true? 'If a dog is a puppy, then it can stand on its front legs.'"

The answer is, of course, "Maybe." Only about 20% of the students answered this question correctly.

A careful re-examination of differences between pretests and posttests as shown in Tables 4.14 - 4.17 indicates very little change or growth in

certain logical principles in grade 3. Except for treatment LM, there was a decrease in the number of students who answered principle 5 correctly in class reasoning, principle 1 in conditional reasoning in all treatments. In grade 6, treatments I and C, class reasoning principle 3 indicated a marked decrease while conditional principle 2 went down in those two treatments also.

WISCONSIN MATHEMATICS TESTS

To obtain a measure of mathematical achievement for all treatment groups, the Wisconsin Contemporary Test of Elementary Mathematics (WCEM) was used both as a pretest and a posttest. Differences between groups on the Wisconsin pretest were significant at the .05 level for grade 3 and at the .01 level for grade 6 as shown in Table 4.23. In grade 3, in a pairwise comparison using the Scheffé test, there was only a significant difference at the .01 level between groups LM and C, favoring LM, as shown in Table 4.24.

There was a significant difference, pairwise, in the Wisconsin pretest in grade 6 between groups LM and I, favoring LM; between L and I, favoring L; and between I and C, favoring C, $p \leq .01$.

Since there was a significant difference, $p \leq .01$, between the treatments in both grades 3 and 6 as shown in the analyses of covariance in Tables 4.25 and 4.26, it was necessary to make pairwise comparisons of the groups using the Scheffé test. In grade three, LM group scored higher than the other groups and significantly higher, $p \leq .01$ than the C group. There was a significant difference, using the Wisconsin pretest as

TABLE 4.23

ANALYSIS OF VARIANCE OF WISCONSIN
CONTEMPORARY ELEMENTARY MATHEMATICS PRETEST

GRADE 3

<u>GROUP</u>	<u>MEAN</u>	<u>EFFECT</u>	<u>STAND. DEVIATION</u>	<u>N</u>	
LM	28.55	0.83	7.15	205	
L	28.01	0.29	6.17	111	
I	27.23	-0.49	6.40	114	
C	25.91	-1.81	6.30	81	
<u>SOURCE OF VARIATION</u>	<u>DF</u>	<u>SUM OF SQUARES</u>	<u>MEAN SQUARE</u>	<u>F RATIO</u>	<u>SIGN. LEVEL</u>
Between groups	3	442.81	147.60	3.34	.05
Within groups	507	22418.17	44.22		
TOTAL	510	22860.98			

GRADE 6

<u>GROUP</u>	<u>MEAN</u>	<u>EFFECT</u>	<u>STAND. DEVIATION</u>	<u>N</u>	
LM	34.92	-0.35	7.40	119	
L	36.27	1.00	7.41	202	
I	31.40	-3.88	8.23	113	
C	37.06	1.79	8.30	155	
<u>SOURCE OF VARIATION</u>	<u>DF</u>	<u>SUM OF SQUARES</u>	<u>MEAN SQUARE</u>	<u>F RATIO</u>	<u>SIGN. LEVEL</u>
Between groups	3	2410.21	803.40	13.17	.01
Within groups	585	35688.78	61.01		
TOTAL	588	38098.99			

TABLE 4.24

SCHEFFE TEST FOR DIFFERENCES ON WISCONSIN PRETEST

<u>PAIRWISE GROUPING</u>	<u>SIGN. LEVEL GR.3</u>	<u>HIGHER MEAN GR.3</u>	<u>SIGN. LEVEL GR.6</u>	<u>HIGHER MEAN GR.6</u>
LM-L	NS	LM	NS	L
LM-I	NS	LM	.01	LM
LM-C	.01	LM	NS	C
L-I	NS	L	.01	L
L-C	NS	L	NS	C
I-C	NS	I	.01	C
(LM+L and I+C)	NS	LM+L	NS	LM+L

TABLE 4.25

ANALYSIS OF COVARIANCE OF WISCONSIN POSTTEST
WITH WISCONSIN PRETEST AS COVARIATE

GRADE 3

Correlation = .736

<u>GROUP</u>		<u>MEAN</u>	<u>ADJUSTED MEAN</u>		
LM		32.04	31.43		
L		31.18	30.97		
I		30.51	30.87		
C		26.56	27.90		
<u>SOURCE OF VARIATION</u>	<u>DF</u>	<u>ADJUSTED SUM SQUARE</u>	<u>MEAN SQUARE</u>	<u>F RATIO</u>	<u>SIGN.</u>
Between groups	3	741.11	247.04	11.74	.01
Within groups	506	10647.94	31.04		
TOTAL	509	11389.05			

GRADE 6

Correlation = .793

<u>GROUP</u>		<u>MEAN</u>	<u>ADJUSTED MEAN</u>		
LM		39.71	39.99		
L		39.14	38.34		
I		34.15	37.15		
C		39.49	38.06		
<u>SOURCE OF VARIATION</u>	<u>DF</u>	<u>ADJUSTED SUM SQUARE</u>	<u>MEAN SQUARE</u>	<u>F RATIO</u>	<u>SIGN.</u>
Between groups	3	493.95	164.65	6.77	.01
Within groups	584	14208.30	24.33		
TOTAL	587	14702.25			

TABLE 4.26

ANALYSIS OF COVARIANCE OF WISCONSIN POSTTEST
WITH I.Q. AS COVARIATE

GRADE 3

Correlation = .386

<u>GROUP</u>		<u>MEAN</u>	<u>ADJUSTED MEAN</u>		
LM		32.04	31.74		
L		31.18	30.93		
I		30.51	30.87		
C		26.56	27.16		
<u>SOURCE OF VARIATION</u>	<u>DF</u>	<u>ADJUSTED SUM SQUARE</u>	<u>MEAN SQUARE</u>	<u>F RATIO</u>	<u>SIGN.</u>
Between groups	3	1225.59	408.53	10.39	.01
Within groups	506	19905.24	39.34		
TOTAL	509	21130.83			

GRADE 6

Correlation = .511

<u>GROUP</u>		<u>MEAN</u>	<u>ADJUSTED MEAN</u>		
LM		39.71	41.12		
L		39.14	38.48		
I		34.05	33.52		
C		39.49	39.66		
<u>SOURCE OF VARIATION</u>	<u>DF</u>	<u>ADJUSTED SUM SQUARE</u>	<u>MEAN SQUARE</u>	<u>F RATIO</u>	<u>SIGN.</u>
Between groups	3	3762.99	1254.33	28.73	.01
Within groups	584	25498.68	43.66		
TOTAL	587	29261.68			

TABLE 4.27

SCHEFFE TEST FOR DIFFERENCES ON WISCONSIN POSTTEST
WISCONSIN POSTTEST WITH WISCONSIN PRETEST AS COVARIATE

<u>PAIRWISE GROUP</u>	<u>SIGN. LEVEL GR. 3</u>	<u>HIGHER MEAN GR. 3</u>	<u>SIGN. LEVEL GR. 6</u>	<u>HIGHER MEAN GR. 6</u>
LM-L	NS	LM	.05	LM
LM-I	NS	LM	.01	LM
LM-C	.01	LM	.05	LM
L-I	NS	L	NS	L
L-C	.01	L	NS	L
I-C	.01	I	NS	C
(LM+L)-(I+C)	.01	LM+L	.01	LM+L

WISCONSIN POSTTEST WITH I.Q. AS COVARIATE

LM-L	NS	LM	.01	LM
LM-I	NS	LM	.01	LM
LM-C	.01	LM	NS	LM
L-I	NS	L	.01	L
L-C	.01	L	NS	C
I-C	.01	I	.01	C
(LM+L)-(I+C)	.01	LM+L	.01	LM+L

covariate also, $p \leq .01$, of L over C and I over C, with the logic group, LM + L over I + C as shown in Table 4.27.

In grade 6, using the Wisconsin pretest as covariate, LM scored significantly higher, $p \leq .05$, than both the L and C groups, and $p \leq .01$, over the I group. The group taught logic scored significantly higher, $p \leq .01$, than the group not taught logic.

With I.Q. as covariate, there was a significant difference, $p \leq .01$, of LM over C; L over C and I over C, and the logic group over the group not taught logic in the third grade Wisconsin posttest. In grade six, there was a significant difference, $p \leq .01$, of LM over L and I, L over I, and C over I, with the logic group scoring significantly higher than the group not taught logic.

It is possible to make several analyses of the scores on the mathematics achievement but there are two particularly relevant to this study: 1) to look at mean scores and percent of correct answers for each treatment; 2) to examine percent of change.

Total percent correct for the four treatments in the third grade Wisconsin mathematics test are shown as follows:

<u>LM Pretest</u>	<u>LM Posttest</u>	<u>L Pretest</u>	<u>L Posttest</u>
58.22	63.58	56.48	62.14
<u>I Pretest</u>	<u>I Posttest</u>	<u>C Pretest</u>	<u>C Posttest</u>
54.76	62.77	51.70	54.57

Percent of change in total Wisconsin Gr. 3 score:

<u>LM</u>	<u>L</u>	<u>I</u>	<u>C</u>	<u>Total</u>	<u>LM + L</u>	<u>I + C</u>
9.2%	10.0%	14.6%	5.7%	10.4%	9.6%	10.2%

Total percent correct for Grade 6 Wisconsin test:

<u>LM Pretest</u>	<u>LM Posttest</u>	<u>L Pretest</u>	<u>L Posttest</u>
62.18	68.42	62.98	66.70
<u>I Pretest</u>	<u>I Posttest</u>	<u>C Pretest</u>	<u>C Posttest</u>
52.91	56.33	61.45	65.76

Percent of change in total Wisconsin Gr. 6 score:

<u>LM</u>	<u>L</u>	<u>I</u>	<u>C</u>	<u>Total</u>	<u>LM + L</u>	<u>I + C</u>
10.0%	5.9%	6.5%	7.0%	8.6%	8.0%	6.8%

Thus it appears that taking 16 hours of class time to teach logic was more helpful in improving scores on mathematics tests than the same amount of instruction on regular mathematical content, because scores on the Wisconsin posttests at both grades 3 and 6, with pretest and I.Q. as covariates indicate a significant highly score, $p \leq .01$, of the group taught logic over the group not taught logic.

CATEGORIES IN GRADE 3 WISCONSIN

In order to determine certain aspects of mathematics which might be significantly affected by the teaching of logic to the experimental group, the Wisconsin tests were analyzed using two broad categories: mathematical category and logically-related category.

The items in the mathematics category were analyzed using the following sub-categories, with the number of items on each shown after the abbreviation:

Fractions (FR), 9

Addition (A), 5

Subtraction (SU), 4

Multiplication (MU), 6

Division (D), 2

Place Value (PV), 3

Geometry (G), 8

Measurement of money (ME), 2

Measurement of time (MT), 3

Measurement of length, liquid, weight (LLW), 3

Fundamentals (FU), 7

(this included such items as ordinals, odds and evens, sequences, distributive property, Roman numerals and the idea that numbers have many names)

Categorization into mathematical topics agreed with the description which the authors of the test submitted for their test plans.

In the 17 items which could be categorized logically-related, the form of the items was that of a number sentence where a student was required to substitute to make the sentence true. Typical of these was the following:

1. Which number makes the statement true?

$$\square + 4 = 12$$

- a) 16 b) 9 c) 8 d) 3

The percent of correct responses for each category arranged in rank order by treatment for grade 3 is shown in Table 4.28. In the pretest, group LM had the greatest percent of correct answers in the following categories: fractions, multiplication, division, place value, measurement of money, and measurement of length, liquid, and weight. Treatment L had

TABLE 4.28

RANK ORDER OF TOPICS ON WISCONSIN MATHEMATICS TEST

GRADE 3

RANK	<u>LM PRE</u> TOPIC %		<u>LM POST</u> TOPIC %		<u>L PRE</u> TOPIC %		<u>L POST</u> TOPIC %	
1	ME	82.46	A	81.64	ME	79.74	A	80.35
2	A	80.96	ME	79.96	A	72.93	ME	76.52
3	PV	69.71	PV	76.33	PV	68.96	PV	70.73
4	SU	64.43	D	75.85	SU	65.95	D	69.57
5	D	62.50	SU	75.85	FU	55.05	SU	66.96
6	FU	54.74	FU	62.32	G	54.31	FU	60.37
7	G	52.23	MU	61.35	MT	53.74	G	60.11
8	MT	51.28	G	58.76	D	51.29	MU	55.22
9	MU	43.43	MT	51.05	MU	42.68	MT	54.20
10	LLW	40.39	FR	44.07	LLW	39.94	FR	45.70
11	FR	38.25	LLW	41.87	FR	36.74	LLW	43.77

ABBREVIATIONS USED:

A = Addition	ME = Measurement: money
D = Division	MT = Measurement: time
FR = Fractions	MU = Multiplication
FU = Fundamentals	PV = Place Value
G = Geometry	SU = Subtraction
LLW = Length, liquid weight measurement	

% = percent of students who answered the topics correctly

TABLE 4.28 (continued)

RANK ORDER OF TOPICS ON WISCONSIN MATHEMATICS TEST
GRADE 3

RANK	<u>I PRE</u> TOPIC %		<u>I POST</u> TOPIC %		<u>I PRE</u> TOPIC %		<u>I POST</u> TOPIC %		<u>TOTAL PRE</u> Topic %		<u>TOTAL POST</u> Topic %	
1	A	82.35	A	83.39	A	76.22	A	75.38	ME	78.18	A	80.19
2	ME	76.05	D	81.46	ME	74.45	ME	71.43	A	78.12	ME	74.12
3	PV	67.23	PV	76.88	PV	69.78	PV	69.96	PV	68.92	D	73.62
4	SU	59.88	ME	68.55	SU	59.45	D	67.58	SU	62.43	PV	73.48
5	FU	55.24	SU	65.12	D	54.45	SU	54.12	D	54.25	SU	63.10
6	G	51.67	FU	60.94	FU	40.00	MU	49.27	FU	53.76	FU	58.04
7	D	48.74	MU	58.33	G	46.53	MT	49.08	G	51.19	G	51.85
8	MT	48.63	G	57.56	FR	37.04	FU	48.51	MT	47.11	MU	56.05
9	MU	37.96	MT	52.42	MU	35.11	G	47.21	MU	40.04	MT	53.61
10	FR	37.63	LLW	44.62	MT	34.81	FR	39.93	FR	37.41	FR	42.69
11	LLW	36.98	FR	41.17	LLW	31.85	LLW	27.84	LLW	37.29	LLW	39.52

the greatest percent of correct answers in addition and fundamentals. Treatment C in the pretest in no category scored the highest percent but had the second highest in division.

Recall that for the third grade students, there was no significant difference in I.Q. for the various treatment groups but there was a significant difference, $p \leq .05$, between LM + L and I + C. In the pretests the LM group maintained second highest percent of correct answers in addition, subtraction, measurement of time and geometry. L group was second highest in multiplication, place value, fundamentals, measurement of money and measurement of length, liquid, and weight. I group had the second highest percent of correct answers in fractions in the pretest.

In the posttests LM group had the highest percent of correct answers in multiplication, fundamentals, measurement of money. This group had the second highest percent of correct answers in fractions, addition, subtraction, division, place value and geometry. The L group had the greatest percent of correct answers in fractions, subtraction, measurement of time and geometry, with second highest percent of correct answers in measurement of money and measurement of length, liquid, and weight.

The I group, in contrast to two first places in pretests, had four first places in percent of correct answers in addition, division, place value and the measurement of length, liquid, and weight. It also had second highest percent of correct answers in multiplication, fundamentals and measurement of time. In posttests, the C group had no highest or second highest percent of correct answers in any mathematical topic.

According to the lesson plans of the teachers in the project, those in I and C both taught more topics during the time of the project than the teachers in LM and L. Topics which were emphasized during this time were: division, extension of addition and subtraction to more difficult problems; place value such as expanded notation in terms of ones, tens and hundreds, measurement, Roman numerals and ordinals with certain topics in geometry.

Despite this progression of topics, it is apparent from the tables that groups LM and L maintained their achievement in the mathematics.

In terms of the second point of view, namely, percent of change, students in classes which were not taught logic "progressed" in more topics than those who took time for logic during their mathematics classes. For example, in the lesson plans submitted during this study, approximately 50% more time was spent in classes in treatment I and C on multiplication and division than in classes in LM and L groups.

Table 4.29 shows the percent of change on the various categories from pretest to posttest. Note that despite this factor of time and natural progression through the mathematical topics, that treatments LM and L while not gaining as much of an increase in multiplication and division as did I and C, nevertheless did have a substantial increase in these topics.

In the overall picture of change from pretest to posttest, out of the 11 categories, LM + L had the greater percent of change than I + C. From Table 4.29, note that in the separate treatments, LM had the greatest increase in three topics: fundamentals, geometry, measurement of money; L in two, fractions and addition; I in five, namely, division,

TABLE 4.29

PERCENT OF CHANGE FROM WISCONSIN PRETEST TO POSTTEST

GRADE 3

TOPIC	<u>LM</u> % CHANGE	<u>L</u> % CHANGE	<u>I</u> % CHANGE	<u>C</u> % CHANGE	<u>TOTAL</u> % CHANGE
Addition	.8	10.2	1.3	-1.1	2.6
Division	21.4	26.5	67.1	19.4	28.3
Fractions	15.2	24.4	9.4	7.8	14.1
Fundamentals	13.7	9.7	10.3	-3.0	8.0
Geometry	12.5	10.8	11.4	1.5	11.1
Length, Liquid, Weight: Meas.	3.72	9.6	20.7	-12.6	6.0
Measurement: Money	-3.0	-4.0	-9.9	-4.1	-5.2
Measurement: Time	- .4	1.0	7.8	41.0	13.8
Multiplica- tion	41.3	29.4	53.7	36.4	40.0
Place Value	9.5	2.6	14.4	3.1	70.0
Subtraction	2.7	1.5	8.8	-9.0	1.1

TOPIC	<u>LM+L</u> % CHANGE	<u>I+C</u> % CHANGE
Addition	5.3	.1
Division	27.8	44.4
Fractions	19.7	8.6
Fundamentals	11.7	4.0
Geometry	11.6	6.7
Length, Liquid, Weight Measurement	6.6	5.3
Measurement: Money	-3.5	-7.0
Measurement: Time	.2	21.6
Multiplication	35.4	45.2
Place Value	6.0	8.8
Subtraction	2.1	.1

measurement of length, liquid, and weight, place value, multiplication, and subtraction. C ranked highest in improvement in measurement of time only.

I was the control group that was not instructed in logic but the teachers attended in-service sessions each week during the study. The lessons taught at these sessions were, for the most part, self-directed. That is, the topics taught were chosen the previous week by the teachers themselves and this was the main thrust of the meetings. Several minutes each session were concerned with discussing the week's work of the students. During this time with the third grade teachers, measurement was the main topic of two of the sessions, with a rather thorough examination of those elements related to the primary level. Also included in these sessions were several attempts to clarify the problem-solving procedures, including the translation of ordinary situations into mathematical language. For the most part, this involved getting the idea into a number sentence. This might be helpful in interpreting the results of the changes in treatment I. A deliberate attempt was made to avoid anything relating to logic but it was impossible in dealing with the types of problems in third grade not to bring in the idea of number sentences.

LOGIC-RELATED CATEGORY

The logic-related items were divided into four categories: those which were concerned with 1) addition; 2) subtraction; 3) multiplication and 4) division. There was only one item which involved a number sentence which was not concerned with operations: namely a sentence that involved almost exclusively memory:

What number makes the statement true?

32 ounces = pounds

- a) 1 b) 2 c) 3 d) 4

Since this involved memory, it is not included as a logic-related item. One could conceivably memorize this information, substitute it in the correct box without doing any operation normally described as logical.

It was difficult to isolate the logic factor in some of the categories for the following reasons. All of the addition items involve number sentences. There are no addition problems not involving the replacement of a variable to make the statement true. This was not the case in multiplication, where only four out of the six items were concerned with number sentences.

Only one item, really subtraction of fractions, was in the logically-related category of subtraction that was not in the regular subtraction category. There were only two items on division in the entire test and both of these were in the number sentence form. Therefore, it is difficult to isolate the logic factor in these items.

Tables 4.30 and 4.31 show the percent of correct responses on logic-related items as well as the percent of change from pretest to posttest. In the percent of change in the logic-related item groups, I came out on top as far as improvement was concerned, having a higher percent of increase in subtraction and division. In subtraction the difference was not great from LM and L. LM + L had the highest percent of increase in addition, subtraction, and the memory item, while I + C maintained the highest percent of improvement in multiplication and division.

TABLE 4.30

PERCENT OF CORRECT RESPONSES ON LOGIC-RELATED
ITEMS ON WISCONSIN TEST

GRADE 3

ITEM TYPE	<u>LM</u>		<u>L</u>		<u>I</u>		<u>C</u>	
	PRE %	POST %	PRE %	POST %	PRE %	POST %	PRE %	POST %
<u>Logic: addition</u> (1-1,1-13,1-20, 2-20,2-25)	70.58	73.14	68.20	74.09	70.08	72.10	70.44	66.59
Logic: subtraction (1-2,1-8, 1-19,2-1,2-13)	58.75	62.03	59.48	62.61	53.61	59.48	54.67	50.11
Logic: Multiplication (1-7,1-14, 2-2,2-8)	52.41	72.85	52.37	67.39	48.70	69.56	44.72	63.74
Logic: Division (1-25, 1-26)	62.50	75.85	51.29	69.57	48.74	81.46	54.45	67.58
Logic: Memory (2-9)	24.52	32.85	35.34	30.43	30.25	35.48	23.33	10.99
Total on logic-related items including Logic: Memory	53.75	63.34	53.34	60.82	50.28	63.24	49.52	51.80
Total on logic-related items without Logic: Memory	61.06	70.96	57.84	68.42	55.28	70.18	56.07	62.01

TABLE 4.30 (continued)

PERCENT OF CORRECT RESPONSES ON LOGIC-RELATED
ITEMS ON WISCONSIN TESTGRADE 3

ITEM TYPE	<u>LM + L</u>		<u>I + C</u>		<u>TOTAL</u>	<u>TOTAL</u>
	PRE %	POST %	PRE %	POST %	PRE %	POST %
<u>Logic: addition</u> (1-1,1-13,1-20, 2-20,2-25)	69.39	73.62	70.26	69.30	69.83	71.46
<u>Logic: subtraction</u> (1-2,1-8, 1-19,2-1,2-13)	59.12	62.32	54.14	53.85	56.63	58.09
<u>Logic: multiplication</u> (1-7, 1-14,2-2,2-8)	52.39	70.12	46.71	66.65	49.55	68.39
<u>Logic: division</u> 1-25,1-26	56.90	72.71	51.60	74.52	54.25	73.62
<u>Logic: memory</u> (2-9)	29.93	31.64	26.79	23.24	28.36	27.44
Total on Logic- related items including Logic Memory	53.55	62.08	49.90	57.52	51.73	59.80
Total on Logic- related items with- out Logic: Memory	59.45	69.69	55.68	66.10	57.57	67.90

TABLE 4.31

PERCENT ON CHANGE ON LOGIC-RELATED ITEMS
ON WISCONSIN TESTS

GRADE 3

ITEM GROUPS IN LOGIC	<u>LM</u> %	<u>L</u> %	<u>I</u> %	<u>C</u> %	<u>LM+L</u> %	<u>I+C</u> %	<u>TOTAL</u> %
Addition	3.6	8.6	2.9	-5.5	6.1	-1.4	2.3
Subtraction	5.6	5.3	7.4	-8.3	5.4	- .5	2.6
Multiplication	39.0	28.7	42.8	42.5	33.8	42.7	38.0
Division	21.4	35.6	67.1	24.1	27.8	44.4	35.7
Memory	34.0	-13.9	17.3	-52.9	5.7	-13.3	-3.2
Total with memory item	17.8	14.0	25.8	4.6	15.9	15.3	15.6
Total without memory item	16.2	18.3	27.0	10.6	17.2	18.7	17.9

% = percent of change from pretest to posttest on logic-related items
on Grade 3, Wisconsin Contemporary Elementary Mathematics Test

WISCONSIN GRADE 6 ITEM ANALYSIS

Items on the 6th grade Wisconsin Mathematics test were categorized into the following 13 mathematical sub-categories:

Addition (A), 6
 Division (D), 4
 Fractions (FR), 6
 Fundamentals (FU), 3
 Geometry (G), 6
 Geometry: angles (GA), 4
 Measurement: area (MA), 5
 Measurement: in general (ME), 2
 Measurement: length (ML), 3
 Multiplication (MU), 6
 Place Value (PV), 5
 Sets (S), 8
 Subtraction (SU), 2

The logic-related items on the test were of two kinds predominately:

1) the regular number sentence items such as 1-5:

Which sign makes the statement true?

$$350 - 50 \bigcirc 196$$

a) > b) - c) < d) =

and 2) if-then statements either with or without placeholders.

Examples of this type are:

2-13 If $\square + \triangle = \diamond$, then $\triangle + \square$ must equal:

a) $\diamond$ b) $\triangle$ c) $\square$ d) $\square$

2-7. If $77 + p = 107$, and $107 - p = \square$, then $\square$ is equal to:

- a) 77 b) p c) 30 d) 10

Students were required to think of truth and falsity in the first type and there were several aspects of logic concerned in the second type. There was on the 6th grade test, in contrast to the third grade form, greater variety in the topics under mathematical categories. For example, not all of the items in division involved number sentences as they did in grade three. This was a factor in providing more variety for the students in addition to complicating the categorization by the researcher. However, it was viewed as a minor difficulty.

There were different kinds of mathematical operations involved in the number sentences in the logic-related items, although there was a strong tendency to concentrate on properties of operations such as commutativity, associativity, and the distributive law of multiplication over addition. These were involved in five of the twelve logic-related items.

The rank order of mathematical topics according to percent of students with correct answers is in Table 4.32. However, this table should be used with Tables 4.33 and 4.34 in order to determine some of the aspects of change present in all treatment groups. From Table 4.32, eight of the greatest percent correct were obtained in LM posttest scores; three in L posttest scores; 2 in C, one in pretest and one in posttest scores. What this actually indicates, one can't be sure, except that it is fair to conjecture that teaching logic during mathematics did, in no way, penalize the students as far as mathematics achievement is concerned and might even have assisted it.

TABLE 4.32

RANK ORDER OF TOPICS ON WISCONSIN
MATHEMATICS TESTGRADE 6

RANK	<u>LM PRE</u> TOPIC %		<u>LM POST</u> TOPIC %		<u>L PRE</u> TOPIC %		<u>L POST</u> TOPIC %	
1	ME	80.42	ME	88.33	ME	84.54	ME	89.00
2	FU	74.17	SU	80.00	FU	78.10	FU	80.22
3	SU	74.17	FU	77.50	S	76.75	S	79.84
4	A	70.70	A	76.81	SU	72.22	SU	79.19
5	S	69.58	S	73.75	A	68.60	PV	71.96
6	PV	67.33	MU	71.25	PV	65.99	A	70.57
7	ML	64.44	PV	67.33	MU	64.24	MU	65.23
8	MU	60.69	ML	66.95	ML	63.29	ML	62.52
9	GA	60.21	MA	61.67	MA	53.62	MA	57.80
10	D	54.79	GA	60.21	D	51.93	D	57.65
11	MA	50.00	D	58.75	GA	50.12	GA	53.59
12	FR	45.28	FR	55.28	G	46.22	FR	50.16
13	G	36.53	G	51.67	FR	42.83	G	49.36

ABBREVIATIONS

A = Addition

D = Division

FR = Fractions

FU = Fundamentals

G = Geometry

GA = Geometry: angles

MA = Measurement: area

ME = Measurement: in general

ML = Measurement: length

MU = Multiplication

PV = Place Value

S = Sets

SU = Subtraction

% = percent of students who answered the topics correctly

TABLE 4.32 (continued)

RANK ORDER OF TOPICS ON WISCONSIN
MATHEMATICS TEST

GRADE 6

RANK	<u>I PRE</u> TOPIC %		<u>I POST</u> TOPIC %		<u>C PRE</u> TOPIC %		<u>C POST</u> TOPIC %		<u>TOTAL PRE</u> TOPIC %		<u>TOTAL POST</u> TOPIC %	
1	ME	80.77	ME	82.59	FU	79.76	ME	84.78	ME	79.38	ME	86.17
2	S	62.50	S	67.74	S	76.15	FU	78.65	S	71.25	FU	74.98
3	SU	61.54	SU	65.16	ME	71.78	S	78.34	SU	69.01	S	74.92
4	ML	60.08	FU	63.55	MU	69.12	MU	72.55	FU	68.91	MU	74.90
5	A	57.48	PV	58.45	SU	68.10	A	71.53	A	65.32	SU	71.93
6	PV	55.77	A	58.39	PV	66.87	ML	69.57	PV	63.99	A	69.33
7	MU	52.56	MU	57.96	A	64.52	PV	69.56	ML	62.45	PV	66.83
8	FU	48.93	ML	51.40	ML	61.96	SU	63.36	MU	61.64	ML	62.61
9	MA	47.68	MA	49.81	MA	60.74	MA	60.37	MA	53.01	MA	57.41
10	FR	44.77	GA	45.81	D	52.61	D	56.21	D	49.17	D	54.36
11	G	40.49	D	44.84	FR	44.38	FR	56.01	GA	47.82	GA	52.25
12	GA	37.98	FR	43.76	GA	42.95	GA	49.38	FR	44.31	FR	51.30
13	D	37.34	G	42.80	G	39.88	G	44.62	G	40.78	G	47.11

% = percent of students who answered the topics correctly

In Table 4.33, where percent of change from pretest to posttest is listed, LM had the greatest percent in three areas; L in two; I in four; and C in four. A study of lesson plans and topics taught during that interval of the project indicates that, for example, in addition where C had the greatest increase, fractions, measurement -- all were topics taught during this interval. This was true with certain topics in which I had the greatest increase, namely, fundamentals (most of which were concerned with ideas of "greater than" and "less than"), geometry of angles and sets.

There was no indication, in the lesson plans of teachers, in treatments LM and L, that topics in which these groups had the greatest percent of correct answers were formally taken in class during that time, although it is obvious that some of them must have been. However, what is interesting about treatments LM and L, that while geometry was not a formal part of any of the plans during this period of the study, it had a significant increase in treatment LM. In relating logic and mathematics, it was found that geometric topics lend themselves very well to this, particularly in the class reasoning as well as in disjunction and conjunction.

There was no conscious attempt to teach any new geometric ideas during this period except to assure that students could use geometric examples meaningfully and without being distracted from the main thrust of the logic. Also, there is some relationship with these topics as shown in group LM as seen from Table 4.33. There was a great increase in measurement of area and while nothing was taught on the other topics in measurement, there was an increase. This was not the case in treatment

TABLE 4.33

PERCENT OF CHANGE FROM WISCONSIN PRETEST TO POSTTEST

GRADE 6

<u>TOPIC</u>	<u>LM</u>	<u>L</u>	<u>I</u>	<u>C</u>	<u>TOTAL</u>	<u>LM+L</u>	<u>I+C</u>
Addition	8.6	2.9	1.8	10.9	6.1	5.8	6.5
Division	7.2	11.0	20.1	6.8	11.3	9.1	12.3
Fractions	22.1	17.1	2.3	26.2	16.9	19.7	11.9
Fundamentals	4.5	2.7	29.9	4.9	8.8	3.6	10.5
Geometry	41.4	6.8	6.2	11.9	15.5	22.1	8.8
Geometry: angles	0	6.9	20.6	15.0	9.3	3.1	17.6
Measurement: area	23.3	7.8	4.5	-0.1	8.3	15.3	1.6
Measurement: in general	9.8	5.3	2.3	18.1	8.6	7.5	9.7
Measurement: length	3.9	-1.2	-14.4	12.3	0.2	1.4	-1.8
Multiplica- tion	17.4	1.5	10.3	5.0	9.3	9.2	7.3
Place Value	0	9.0	4.8	4.0	4.4	4.5	4.4
Sets	6.0	4.0	8.4	2.9	5.2	5.0	5.4
Subtraction	7.9	9.7	5.9	-7.0	4.2	8.7	-1.8

LOGIC-RELATED ITEMS ON GR. 6 WISCONSIN

Logic: Number Sentences	20.1	7.0	-1.9	12.1	9.9	13.4	6.0
If-Then Statements	12.0	11.4	8.3	10.0	10.4	12.1	9.2
Logic:total	16.1	9.2	3.7	11.1	10.2	12.8	7.7
Entire Total of All Items on Test	10.0	5.9	6.5	7.0	8.6	8.0	6.8

TABLE 4.34

PERCENT OF CORRECT RESPONSES ON LOGIC-RELATED
ITEMS ON WISCONSIN TEST

GRADE 6

NUMBER SENTENCES	<u>LM</u>		<u>L</u>		<u>I</u>		<u>C</u>		<u>TOTAL</u>	
	PRE	POST	PRE	POST	PRE	POST	PRE	POST	PRE	POST
(1-5,1-7, 1-8,2-14, 2-19,2-25)	62.78	75.42	65.30	69.85	53.95	53.44	61.96	69.44	61.00	67.04
<u>IF-THEN STATEMENTS</u>										
(1-14,1-19, 1-28,2-1, 2-7,2-13)	68.20	76.39	65.54	73.04	54.49	59.03	66.26	72.88	63.62	70.34
TOTAL	65.49	75.91	65.42	71.45	54.22	56.24	64.14	71.16	62.32	68.69

	<u>LM + L</u>		<u>I + C</u>	
	PRE	POST	PRE	POST
Number Sentences	64.04	72.64	57.96	61.44
If-Then State- ments	66.87	74.97	60.38	65.96
TOTAL	65.46	73.81	59.17	63.70

I or C. For example, in treatment C, there was the greatest percent correct in measurement in general and measurement of length but a negative difference in measurement of area. In I, there was a small increase in measurement of area and measurement in general, but a noticeable decrease in measurement of length. In treatment LM, there was an increase in all aspects of measurement and while no particular new topic of measurement was taught, nevertheless there could have been a transfer. Measurement presents several challenges but the idea of measurement, regardless of what particular emphasis, is universal: namely, finding a suitable unit, using it to count (i.e., to measure) the object whose measure is desired.

It is interesting to note that group I had the greatest increase in sets. Sets, per se, were not taught in the logic group but were used constantly throughout the project. The items on sets in the Wisconsin test were fairly routine. Three items were on equivalence of sets, union and intersection, subset and disjoint sets. It is doubtful whether these items are meaningful as far as mathematics achievement is concerned--in the routine, almost memory-like approach used on this test. Sets were considered in this project, like the logic itself, as a tool for gaining greater mastery in mathematics. Therefore, students in the logic project "used" sets but could quite possibly have been trapped by over-verbalization on some of the set items.

The results for logic-related items are in Table 4.34. Treatment LM showed the greatest increase in both the number sentences and if-then sentences in the logic-related items on the Wisconsin test. In the number sentences, both L and C showed substantial increases also. The greatest increases in these items were on those where treatment C had

been taught particular topics such as negative numbers, division, and number properties. This same held true for if-then statement increases. It is difficult, as it was with the third grade, to isolate precisely the mathematical from the logical.

Note that in regard to both mathematical topics and logic-related topics, LM + L had the greatest gain. In the entire total of the test, as indicated in Table 4.33, the greatest percent of gain was obtained by treatment LM with 10.0%, then C with 7%, then I with 6.5% and L with 5.9%. LM + L had 8.0% of change with I + C, 6.8%. In the logic-related items, the relative placement in regard to percent of change was about the same. There was a 16.1% increase in treatment LM; 11.1% increase in C; 9.2% increase in L, with a 3.7% increase in I. There was not a great difference between L and C as shown by Tables 4.33 and 4.34.

Two things should be noted with regard to treatment C. 1) Three of the four teachers in treatment C had strong mathematical backgrounds and all had received extensive in-service training on contemporary mathematics. In fact, one of these teachers was currently attending a rather advanced course in contemporary mathematics. Although we had asked these teachers not to include anything on logic during the time of the project, it is quite conceivable that logically-related items were part of the program. Every contemporary program includes much on number sentences, some work on if-then, with perhaps not a conscious effort to examine these very closely.

2) Treatment C teachers had covered in their classes several more topics than either of the other three treatments, even more than I where there was the in-service class.

LOGIC FINAL AND LOGIC RETENTION

In grade 6 logic final there was a significant difference at the .01 level between scores of treatment LM and L, favoring L as shown in Table 4.36. This was true also for the logic retention. Note in Table 4.1 that the mean I.Q. for group L is 117.11, while it is 110.88 for students in group LM. Since I.Q. is an important factor, the question arises whether this difference can be attributed to I.Q. or not.

There was a significant difference at the .05 level in scores on the logic final between treatments LM and L in grade 3, favoring the LM group, but no significant difference in the retention test as shown in Table 4.35. I.Q.'s for both treatments LM and L were not significantly different with mean I.Q. for LM students = 111.03 and for L = 110.82, a difference of .21. (See Table 4.1). This was in contrast to the .01 level of significance between I.Q.'s of grade 6 treatments LM and L as indicated in the Scheffé Table 4.2.

In grade 3, group LM had a substantially higher percent of correct answers on the mathematical items on the logic final and logic retention as seen in Tables 4.37 and 4.38. On the non-mathematical items, treatment L had a higher percent of correct answers in the logic final, but in the logic retention, LM was higher.

For the mathematical items on the logic final, grade 6 group LM had a higher percent than treatment L, but in the retention, group L had higher than LM. Group L had a higher percent on the non-mathematical items in both the logic final and retention. Note that L has been substantially higher in many aspects throughout the entire project.

TABLE 4.35

ANALYSIS OF VARIANCE FOR LOGIC FINAL AND RETENTION
FOR GROUPS LM AND L

GRADE 3

<u>GROUP</u>		<u>MEAN</u>		<u>EFFECT</u>	
LM		33.92		0.46	
L		32.61		-0.85	

<u>SOURCE OF VARIATION</u>	<u>DF</u>	<u>SUM OF SQUARES</u>	<u>MEAN SQUARE</u>	<u>F RATIO</u>	<u>SIGN.</u>
Between groups	1	122.53	122.53	4.04	.05
Within groups	314	9533.93	30.36		
TOTAL	315	9656.47			

COMPARE GROUPS BY PAIRS FOR LOGIC FINAL

LM	N= 205	MEAN= 33.92	SIGMA= 5.90	T RATIO = 2.01	P ≤ .05
L	N= 111	MEAN= 32.61	SIGMA= 4.69	F RATIO = 1.58	

LOGIC RETENTION

<u>GROUP</u>		<u>MEAN</u>		<u>EFFECT</u>	
LM		32.39		0.38	
L		31.30		-0.71	

<u>SOURCE OF VARIATION</u>	<u>DF</u>	<u>SUM OF SQUARES</u>	<u>MEAN SQUARE</u>	<u>F RATIO</u>	<u>SIGN.</u>
Between groups	1	85.25	85.25	2.15	NS
Within groups	314	12441.75	39.62		
TOTAL	315	12527.00			

COMPARE GROUPS BY PAIRS FOR LOGIC RETENTION

LM	N= 205	MEAN = 32.39	SIGMA = 6.80	T RATIO = 1.47	NS
L	N= 111	MEAN = 31.30	SIGMA = 5.22	F RATIO = 1.70	

TABLE 4.36

ANALYSIS OF VARIANCE FOR LOGIC FINAL AND RETENTION
FOR GROUPS LM AND LGRADE 6LOGIC FINAL

<u>GROUP</u>	<u>MEAN</u>	<u>EFFECT</u>
LM	34.72	-2.23
L	38.27	1.31

<u>SOURCE OF VARIATION</u>	<u>DF</u>	<u>SUM OF SQUARES</u>	<u>MEAN SQUARE</u>	<u>F RATIO</u>	<u>SIGN.</u>
Between groups	1	940.89	940.89	42.34	.01
Within groups	319	7089.41	22.22		
TOTAL	320	8030.30			

COMPARE GROUPS BY PAIRS FOR LOGIC FINAL

LM	N = 119	MEAN = 34.72	SIGMA = 4.63	T RATIO = -6.51	P ≤ .01
L	N = 202	MEAN = 38.27	SIGMA = 4.76	F RATIO = 1.06	

LOGIC RETENTION

<u>GROUP</u>	<u>MEAN</u>	<u>EFFECT</u>
LM	35.56	-2.62
L	39.73	1.54

<u>SOURCE OF VARIATION</u>	<u>DF</u>	<u>SUM OF SQUARES</u>	<u>MEAN SQUARE</u>	<u>F RATIO</u>	<u>SIGN.</u>
Between groups	1	1298.85	1298.85	63.03	.01
Within groups	319	6573.30	30.61		
TOTAL	320	7872.16			

COMPARE GROUPS BY PAIRS FOR LOGIC RETENTION

LM	N = 119	MEAN = 35.56	SIGMA = 4.51	T RATIO = -7.94	p ≤ .01
L	N = 202	MEAN = 39.73	SIGMA = 4.56	F RATIO = 1.02	

TABLE 4.37

PERCENT OF CORRECT RESPONSES ON MATHEMATICAL
ITEMS ON LOGIC FINAL AND LOGIC RETENTION TESTS

ITEMS	3LM		3L		6LM		6L	
	FINAL	RETEN.	FINAL	RETEN.	FINAL	RETEN.	FINAL	RETEN.
3	92.75	92.23	92.24	84.07	99.13	97.44	99.05	98.54
11	67.15	82.52	75.00	78.76	87.83	87.18	57.62	91.71
12	14.98	33.01	18.03	17.24	23.48	44.44	26.19	62.44
13	91.78	84.47	91.38	83.19	97.39	88.03	92.38	87.32
14	86.96	92.23	91.38	85.84	94.78	94.02	97.62	97.56
27	33.33	30.58	32.76	26.55	66.96	63.25	57.62	60.49
TOTAL	64.49	69.17	66.67	61.65	78.26	79.06	71.75	83.01
6	66.18	52.43	62.07	56.64	64.35	67.52	61.40	62.93
8	71.01	43.69	49.14	31.86	42.61	69.23	60.00	65.85
9	88.41	86.89	63.79	84.07	93.04	99.15	97.14	98.05
10	62.80	60.68	62.93	57.52	83.48	82.05	77.62	83.90
22	51.21	65.08	50.00	57.52	87.83	90.60	84.76	90.73
TOTAL	67.92	61.75	57.59	57.52	74.26	81.71	76.28	80.29
24	82.13	83.98	76.72	80.53	96.52	95.73	92.86	99.02
26	77.78	74.27	70.69	68.14	93.04	95.73	96.19	97.56
TOTAL	79.96	79.13	73.71	74.34	94.78	95.73	94.53	98.29
32	50.24	55.34	45.69	61.06	70.43	73.50	65.71	70.73
34	43.00	42.23	39.66	30.97	59.13	51.28	59.52	70.24
37	79.23	80.58	78.45	71.68	95.65	93.16	92.86	92.68
TOTAL	57.49	59.38	54.60	54.57	75.07	72.65	72.70	77.88
39	56.52	49.03	55.17	47.79	70.43	77.78	63.33	75.12
41	60.87	59.71	59.48	58.41	76.52	74.36	70.00	75.61
42	72.95	70.39	59.48	66.37	91.30	81.20	89.52	88.29
43	65.70	62.14	63.79	51.33	74.78	73.50	69.52	67.80
44	47.83	46.12	46.55	42.48	55.65	51.28	47.14	53.66
TOTAL	60.77	57.49	56.89	53.28	73.74	71.62	67.90	72.10
TOTAL OF MATHEMA- TICAL ITEMS	66.13	65.38	61.89	60.27	79.22	80.15	76.63	82.31

TABLE 4.38

PERCENT OF CORRECT RESPONSES ON NON-MATHEMATICAL
ITEMS ON LOGIC FINAL AND LOGIC RETENTION TESTS

ITEMS	3LM		3L		6LM		6L	
	FINAL	RETEN.	FINAL	RETEN.	FINAL	RETEN.	FINAL	RETEN.
1	68.67	70.39	65.82	62.83	91.30	88.03	89.04	95.12
2	85.99	88.35	93.10	87.61	95.65	97.44	98.57	99.02
TOTAL	77.33	79.37	79.46	75.22	93.48	92.74	93.81	97.07
4	71.98	74.76	66.38	67.26	39.13	67.52	59.05	79.51
5	84.54	70.39	79.31	61.95	67.83	71.79	78.57	80.49
7	66.18	93.20	62.07	88.50	64.35	91.45	61.90	96.10
TOTAL	74.23	79.45	69.25	72.57	57.10	76.92	61.55	85.37
23	97.10	94.66	94.83	92.04	96.52	98.29	92.86	99.51
25	48.79	54.37	54.31	55.75	76.52	87.18	80.48	81.95
TOTAL	72.95	74.52	74.57	73.90	86.52	92.74	86.67	90.73
28	91.30	90.78	95.69	92.92	98.26	95.73	95.24	96.59
29	54.11	54.37	50.00	42.48	36.52	67.52	66.19	86.34
30	81.64	76.21	73.28	73.45	91.30	88.89	90.00	90.24
31	89.86	87.86	94.83	88.50	97.39	94.02	95.71	94.15
33	64.25	76.69	75.00	62.83	77.39	66.67	76.19	84.39
35	78.26	82.04	81.90	78.76	94.78	95.73	99.52	98.05
36	66.18	66.50	50.43	52.21	57.76	59.83	74.29	85.85
TOTAL	75.09	76.35	74.45	70.16	79.06	81.20	85.31	90.80
38	75.85	75.24	74.14	75.22	88.70	93.16	92.86	97.56
46	73.91	76.70	83.62	75.22	92.17	94.87	92.38	92.20
47	85.02	86.89	93.10	86.73	86.09	91.45	92.86	90.73
TOTAL	78.26	79.61	83.62	79.06	88.99	93.16	92.70	93.50
TOTAL OF NON-MATH- EMATICAL ITEMS	75.57	77.86	76.27	74.18	81.03	87.35	85.00	91.49

Whether this can be attributed to a higher I.Q. is debatable. Two of the teachers in treatment L had extensive mathematical background but detailed comparisons were not made on the teacher's background in all treatments. Number of hours of mathematics studied is not sufficient to gain information about this contribution because courses and instructors vary so greatly.

For items that had some negation in them, group LM in grade 3 had a higher percent correct than treatment L in both the logic final and retention as shown in Table 4.39. The reverse was true in grade 6, with group L having a higher percent correct in both tests. Interesting enough, in grade 6, each treatment shared eight items which had highest percent correct. There were 16 items with negation in them and despite the fact that the total favored treatment L, they divided equally items with highest percent of correct answers. Therefore, it is important to search through the items and examine in detail what each entailed. Generally, treatment LM did better on those sections concerned with validity, with answers of O.K. or Not O.K. and Yes, No, Maybe in negation.

Symbolism was in a class by itself, being neither mathematical nor non-mathematical. Except for numbers 40 and 45, the items were concerned with recognition of symbols used for disjunction, conjunction, and implication to determine truth values. The teaching of these symbols presented no difficulties on either 3rd or 6th grade level because all symbols were used throughout the project. Table 4.40 indicates this, particularly in items from 15-21. Number 40 had a Maybe answer, a determining factor which seems to pervade all aspects of the logic. Other than that item, symbolic items rated as high as any on the test.

TABLE 4.39

PERCENT OF CORRECT RESPONSES ON ITEMS WITH
NEGATION ON LOGIC FINAL AND LOGIC RETENTION TESTS

ITEMS	3LM		3L		6LM		6L	
	FINAL	RETEN.	FINAL	RETEN.	FINAL	RETEN.	FINAL	RETEN.
4	71.98	74.76	66.38	67.26	39.13	67.52	59.05	79.51
5	84.54	70.39	79.31	61.95	67.83	71.79	78.57	80.49
6	66.18	52.43	62.07	56.64	64.35	67.52	61.90	62.93
7	86.96	93.20	95.69	88.50	92.17	91.45	96.19	96.10
8	71.01	43.69	49.14	31.86	42.61	69.23	60.00	65.85
TOTAL	76.13	66.89	70.52	61.24	61.22	73.50	71.14	76.98
10	62.80	60.68	62.93	57.52	83.48	82.05	77.62	83.90
22	51.21	65.05	50.00	57.52	87.83	90.60	84.76	90.73
TOTAL	57.01	62.87	56.47	57.52	85.66	86.33	81.19	87.32
30	81.64	76.21	73.28	73.45	91.30	88.89	90.00	90.24
31	89.86	87.86	94.83	88.50	97.39	94.02	95.71	94.15
32	50.24	55.34	45.69	61.06	70.43	73.50	65.71	70.73
36	66.18	66.50	50.43	52.21	57.76	59.83	74.29	85.85
TOTAL	71.98	71.48	66.06	68.81	79.22	79.06	81.43	85.24
41	60.87	59.71	59.48	58.41	76.52	74.36	70.00	75.61
42	72.95	70.39	59.48	66.37	91.30	81.20	89.52	88.29
43	65.70	62.14	63.79	51.33	74.78	73.50	69.52	67.80
45	62.80	61.65	55.17	53.98	73.91	67.52	72.86	74.15
47	85.02	86.89	93.10	86.73	86.09	91.45	92.86	90.73
TOTAL	69.47	68.16	66.20	63.36	80.52	77.71	78.95	79.32
TOTAL ON ITEMS WITH NEGATION	68.65	67.35	64.81	62.73	76.66	79.15	78.18	82.22

TABLE 4.40

PERCENT OF CORRECT RESPONSES ON ITEMS WITH
SYMBOLISM ON LOGIC FINAL AND LOGIC RETENTION

ITEMS	3LM		3L		6LM		6L	
	FINAL	RETEN.	FINAL	RETEN.	FINAL	RETEN.	FINAL	RETEN.
15	99.52	99.51	97.41	99.12	95.65	93.16	99.05	100.00
16	97.58	96.12	97.41	96.46	88.70	91.45	99.05	99.51
17	83.09	71.84	76.72	77.88	82.61	74.36	88.57	84.39
18	98.55	97.57	96.55	99.12	92.17	90.60	100.00	99.51
19	68.60	58.25	49.14	53.98	62.61	49.57	73.33	72.20
20	98.55	97.57	96.55	98.23	93.04	92.31	97.62	100.00
21	99.52	99.03	97.41	97.35	85.22	94.02	100.00	99.02
TOTAL	92.20	88.56	87.31	88.88	85.71	83.64	93.93	93.52
40	32.37	31.55	52.59	38.94	26.09	32.48	39.05	41.95
45	62.80	61.65	55.17	53.98	73.91	67.52	72.86	74.15
TOTAL	47.59	46.60	53.88	46.46	50.00	50.00	55.96	58.05
TOTAL ON ITEMS WITH SYMBOLISM	69.90	67.58	70.60	67.67	67.86	66.82	74.96	75.79

In grade 3, group L did better on the final and group LM only slightly better than L on the retention. Group L had a higher percent of correct responses on both the final and retention tests when two scores were averaged together.

It was difficult to classify items into class reasoning and conditional reasoning because there were various types of each on the test. However, in order better to compare it with the Cornell tests, just the items concerned with validity and invalidity were categorized. The first three items, for example, as well as numbers 4-8, were all concerned with class reasoning. It is necessary to keep in mind the criteria used in subdividing the items.

In grade 3 in class reasoning, treatment L did better than LM on the final but not as well on retention as shown in Table 4.41. In grade 6, group LM did better than group L on the final, but not as well on retention. In conditional reasoning, treatment LM in grade 3 had a higher percent of correct responses in both the logic final and retention. The reverse was true in grade 6, with group L doing better than LM in both.

In contrast to the findings on the Cornell study with the two tests of reasoning, the students overall did better in all treatment and all grades in conditional reasoning than in class reasoning. It is difficult to determine the reason for this, because two of the three Maybe answers are in conditional reasoning, and all the "contrary to ordinary experience" and "contrary to mathematical experience" items are in conditional reasoning.

TABLE 4.41

PERCENT OF CORRECT RESPONSES ON CLASS AND
CONDITIONAL REASONING ITEMS ON LOGIC FINAL AND RETENTION TESTS

<u>CLASS REASONING</u>								
ITEM	<u>3LM</u>		<u>3L</u>		<u>6LM</u>		<u>6L</u>	
	FINAL	RETEN.	FINAL	RETEN.	FINAL	RETEN.	FINAL	RETEN.
28	91.30	90.78	95.69	92.92	98.26	95.73	95.24	96.59
34	43.00	42.23	39.66	30.97	59.13	51.28	59.52	70.24
40	32.37	31.55	52.59	38.94	26.09	32.48	39.05	41.95
41	60.87	59.71	59.48	58.41	76.52	74.36	70.00	75.61
42	72.95	70.39	59.48	66.37	91.30	81.20	89.52	88.29
44	47.83	46.12	46.55	42.48	55.65	51.28	47.14	53.66
45	62.80	61.65	55.17	53.98	73.91	67.52	72.86	74.15
46	73.91	76.70	83.62	75.22	92.17	94.87	92.38	92.20
47	85.02	86.89	93.10	86.73	86.09	91.45	92.86	90.73
TOTAL	63.34	62.89	65.04	60.67	73.24	71.13	73.17	75.94

<u>CONDITIONAL REASONING</u>								
ITEM	<u>3LM</u>		<u>3L</u>		<u>6LM</u>		<u>6L</u>	
	FINAL	RETEN.	FINAL	RETEN.	FINAL	RETEN.	FINAL	RETEN.
29	54.11	54.37	50.00	42.48	36.52	67.52	66.19	86.34
30	81.64	76.21	73.28	73.45	91.30	88.89	90.00	90.24
31	89.86	87.86	94.83	88.50	97.39	94.02	95.71	94.15
32	50.24	55.34	45.69	61.06	70.43	73.50	65.71	70.73
33	64.25	76.69	75.00	62.83	77.39	66.67	76.19	84.39
35	78.26	82.04	81.90	78.76	94.78	95.73	99.52	98.05
36	66.18	66.50	50.43	52.21	57.76	59.83	74.29	85.85
37	79.23	80.58	78.45	71.68	95.65	93.16	92.86	92.68
38	75.85	75.24	74.14	75.22	88.70	93.16	92.86	97.56
39	56.52	49.03	55.17	47.79	70.43	77.78	63.33	75.12
43	65.70	62.14	63.79	51.33	74.78	73.50	69.52	67.80
TOTAL	69.26	69.64	67.52	64.12	77.74	80.34	80.56	85.72
TOTAL CLASS AND CON- DITIONAL REASONING	66.30	66.27	66.28	62.40	75.49	75.74	76.87	80.83

Items 34, 40, and 47 seem to be the lowest in class reasoning. Example 34 involves the validity or invalidity of the converse, a principle low in percent of correct responses on both class and conditional reasoning in the Cornell tests. Maybe is the answer to 40. However, on the conditional reasoning, 33 involves the converse and students in both grades did considerably better on it. One wonders if the familiar item, the policeman, has something to do with this. Certainly policemen are more interesting to most youngsters than whole numbers divisible by 6. This examination of familiar and more interesting object needs to be considered in analyzing this data. It turned up again and again in oral presentation. Some familiar examples used to teach logical principles were helpful; others were not. It was conjectured that teachers must know, at least in a broad sense, the experiences of the children at the level at which he is teaching. On the rank order of items on both the logic final and retention, numbers 34 and 40 were in the lowest fourth.

A factor in the logic final test was the idea of determining validity when the things talked about were contrary to experience, as for example, lemons being bigger than pineapples and pineapples bigger than basketballs. Also some items were contrary to mathematical experience such as all numbers being divisible by three being even. Table 4.42 shows the percent of correct responses for such items.

In the "contrary to non-mathematical experiences" in grade 3, treatment LM did better than treatment L in both the final and retention while in grade 6, treatment L did better than LM in the final and only slightly better than LM in the retention. In the "contrary to mathematical experiences" in both grades, group LM did better than L in both the logic final and logic retention.

TABLE 4.42

PERCENT OF CORRECT RESPONSES ON ITEMS WITH IDEAS
CONTRARY TO EXPERIENCE (MATHEMATICAL AND NON-MATHEMATICAL)

CONTRARY TO REGULAR OR NON-MATHEMATICAL EXPERIENCE

ITEM	3LM		3L		6LM		6L	
	FINAL	RETEN.	FINAL	RETEN.	FINAL	RETEN.	FINAL	RETEN.
25	48.79	54.37	54.31	55.75	76.52	87.18	80.48	81.95
30	81.64	76.21	73.28	73.45	91.30	88.89	90.00	90.24
35	78.26	82.04	81.90	78.76	94.78	95.73	99.52	98.05
38	75.85	75.24	74.14	75.22	88.70	93.16	92.86	97.56
TOTAL	71.14	71.97	70.91	70.80	87.83	91.24	90.72	91.95

CONTRARY TO MATHEMATICAL EXPERIENCE

ITEM	3LM		3L		6LM		6L	
	FINAL	RETEN.	FINAL	RETEN.	FINAL	RETEN.	FINAL	RETEN.
26	77.78	74.27	70.69	68.14	93.04	95.73	96.19	97.56
32	50.24	55.34	45.69	61.06	70.43	73.50	65.71	70.73
37	79.23	80.58	78.45	71.68	95.65	93.16	92.86	92.68
43	65.70	62.14	63.79	51.33	74.78	73.50	69.52	67.80
TOTAL	68.24	68.08	64.66	63.05	83.48	83.97	81.07	82.19
TOTAL OF ITEMS CONTRARY TO EXPERIENCE	69.69	70.03	67.79	66.93	85.66	87.61	85.90	87.07

During the teaching of items which seemed to contradict what the children were familiar with, in the third grade there was a kind of reluctance to suppose things that were against ordinary, classroom and perhaps, family experience. For example, if the third graders were to "suppose that their teacher who was tall, was short," "that their Dad was bald (and he was not)," there was great difficulty in working with logical principles. On the other hand, they found it quite easy, and in fact, were often eager, to accept that "cows could smoke", "airplane could talk", "horses had their hair done", "cats made good quarterbacks," probably a kind of Walt Disney syndrome. This rigidity almost, to the mundane, to the daily experience is evident in some of the observations of experiments which Piaget did. There seemed to be little problem along these lines with the Sixth graders as it was not reported by any teacher in the discussion of the above-mentioned aspect.

In order to determine what effect kinds of answers such as O.K., Not O.K., Yes, No, and Maybe had on correct responses, Table 4.43 clearly shows that items with a Maybe answer had the least percent of correct responses. In fact, it seems that this Maybe answer phenomenon pervades several analyses. Items 28-37 were concerned with validity and children were requested merely to indicate O.K. if the conclusion followed necessarily from the premises and Not O.K., if it did not. The Maybe factor did not enter into this section.

In the next section, validity was a key point, that is, while the form of the items was different from those of the previous section, nevertheless validity was the idea being tested. If one eliminates the Maybe answers as shown in Table 4.43, there was no apparent or substantial

TABLE 4.43

PERCENT OF CORRECT RESPONSES ON SELECTED ITEMS
ON LOGIC FINAL AND LOGIC RETENTION TESTS

ITEM	3LM		3L		6LM		6L	
	FINAL	RETEN.	FINAL	RETEN.	FINAL	RETEN.	FINAL	RETEN.
<u>ANSWERS</u>								
YES	74.24	74.11	72.41	72.47	90.72	89.74	91.59	92.68
NO	68.60	67.60	67.89	62.61	77.83	76.71	76.31	77.07
MAYBE	45.57	42.23	51.44	43.07	50.72	53.85	49.84	56.91
<u>ITEMS</u>								
28-37	69.81	71.26	68.49	65.49	77.86	78.63	81.52	86.93
<u>VALIDITY</u>								
<u>ITEMS</u>								
38-47	63.38	61.94	64.31	59.64	73.56	73.75	72.95	75.71
<u>VALIDITY, INVALIDITY AND MAYBE</u>								
<u>ITEMS</u>								
38-47	71.01	70.39	69.83	66.75	83.35	82.29	82.86	83.76
<u>WITHOUT MAYBE ITEMS</u>								
<u>ITEMS</u>								
28, 30, 31, 35, 37 WITH O.K. AS ANSWERS	84.06	83.49	84.83	81.06	95.48	93.51	94.67	94.34
<u>ITEMS 29, 32, 33, 34, 36 WITH NOT O.K. AS ANSWERS</u>								
29, 32, 33, 34, 36	55.56	59.03	52.16	49.91	60.25	63.76	68.38	79.51

difference between the forms of the items. In the first section, items 28-37, the conclusion was stated in a sentence form. In the second section, items 38-47, a question was asked about the conclusion. Once again, this seems in line with the findings of O'Brien and Shapiro (82), and would seem to firm up the caution about interpreting the findings of Hill in regard to this Maybe factor.

In section 28-37, with two choices of answers, O.K. and Not O.K., Table 4.43 indicates that there is a substantial difference in percent of correct responses for these answers. In all treatments in both grades, the O.K. items were answered substantially better than the Not O.K. answers. Except for one of the O.K. answers, the items were all conditional reasoning and except for one also, all non-mathematical.

To make it more difficult, the Not O.K. items, except for one, were conditional reasoning also, and except for two, were also non-mathematical. Perhaps this follows the same pattern with regard to the Yes, No answers. While there is not as much difference between them as there is between these and the Maybe answers, in this study, the Yes answers consistently have a higher percent of students answering them correctly.

CORRELATION

In the correlation Tables, 4.44 and 4.45, the greatest relation appears to be between pretests and posttests of the same test, as would be expected. There was a significant correlation between the logic final and logic retention in both the LM and L treatments. In treatment LM, the correlation between the logic final and Cornell reasoning was higher than L treatment on the pretest and lower than L on the posttest. The

TABLE 4.44

INTERCORRELATIONS OF DESCRIPTIVE AND FINAL
VARIABLES BY TREATMENT GROUPS LM AND L

GRADES 3 AND 6

VARI- ABLE	GROUP	CLASS PRE	COND. PRE	WIS. PRE	CLASS POST	COND. POST	WIS. POST	LOGIC FINAL	LOGIC RET.	ATTI- TUDE	I.Q.	AGE
Class	LM	1.00										
Pre	L	1.00										
Cond.	LM	.36	1.00									
Pre	L	.34	1.00									
Wis.	LM	.53	.38	1.00								
Pre	L	.51	.36	1.00								
Class	LM	.42	.20	.34	1.00							
Post	L	.58	.29	.55	1.00							
Cond.	LM	.37	.46	.40	.26	1.00						
Post	L	.51	.38	.49	.53	1.00						
Wis.	LM	.50	.35	.81	.34	.38	1.00					
Post	L	.49	.30	.80	.52	.48	1.00					
Logic	LM	.54	.31	.52	.40	.45	.51	1.00				
Final	L	.52	.28	.66	.59	.52	.66	1.00				
Logic	LM	.54	.35	.45	.36	.44	.45	.73	1.00			
Ret.	L	.52	.29	.59	.63	.51	.61	.77	1.00			
Atti-	LM	.12	.06	.11	.03	.09	.13	.15	.11	1.00		
tude	L	.18	.06	.19	.15	.15	.20	.16	.18	1.00		
I.Q.	LM	.37	.25	.38	.19	.29	.40	.41	.36	.14	1.00	
	L	.36	.27	.54	.34	.31	.58	.45	.41	.12	1.00	
Age	LM	.22	.19	.34	.06	.17	.41	.02	.18	-.06	-.10	1.00
	L	.33	.07	.46	.42	.29	.42	.44	.58	.27	.16	1.00

TABLE 4.45

INTERCORRELATIONS OF DESCRIPTIVE AND FINAL
VARIABLES BY TREATMENT GROUPS I AND CGRADES 3 AND 6

VARI- ABLE	GROUP	CLASS PRE	COND. PRE	WIS. PRE	CLASS POST	COND. POST	WIS. POST	ATTI- TUDE	I.Q.	AGE
Class	I	1.00								
Pre	C	1.00								
Cond.	I	.36	1.00							
Pre	C	.44	1.00							
Wis.	I	.47	.32	1.00						
Pre	C	.67	.45	1.00						
Class	I	.55	.25	.39	1.00					
Post	C	.67	.41	.64	1.00					
Cond.	I	.41	.51	.35	.40	1.00				
Post	C	.53	.54	.50	.58	1.00				
Wis.	I	.39	.26	.79	.37	.35	1.00			
Post	C	.66	.36	.86	.65	.44	1.00			
Atti- tude	I	.01	.02	.17	-.04	.04	.16	1.00		
	C	.05	.13	.21	.15	.11	.20	1.00		
I.Q.	I	.41	.29	.57	.40	.34	.57	.04	1.00	
	C	.50	.34	.58	.49	.47	.55	.27	1.00	
Age	I	.38	.02	.22	.36	.14	.18	.16	.21	1.00
(in months)	C	.52	.28	.57	.42	.35	.57	.17	.43	1.00

correlation between the logic final and Wisconsin pretest and posttest was about .14 lower in treatment LM than in L. It is uncertain what this could indicate.

Except for treatment LM, the correlation between the pretest and posttest of class reasoning was higher than for the correlation between pretest and posttest of conditional reasoning. I.Q. correlated highest with the Wisconsin mathematics test generally in all treatments and particularly in treatments LM and L, it correlated significantly with the logic final and logic retention. After this, I.Q. scored next highest with class reasoning rather than conditional reasoning.

In all treatments, the conditional pretest correlated highest with conditional posttest; Wisconsin pretests with Wisconsin posttest and logic final with logic retention. The class reasoning pretest correlated highest with class reasoning posttest in treatments L, I, and C, but not in LM where it correlated highest with logic final and logic retention.

Class posttest correlated highest with class pretest in treatments LM, I, and C, but not in L where it correlated highest with logic retention and logic final. An interesting note occurs in conditional posttest where it correlated highest with conditional pretest only in treatments LM and I. In treatments L and C, it correlated highest with class reasoning posttests.

Correlations regarding attitude were not high but both Wisconsin pretest and posttest ranked among the highest correlations to this variable. I.Q. correlated more highly with both Wisconsin tests in general than any other score, but in treatments LM and L, there was a high correlation between I.Q. and logic final. High correlations

regarding age were varied but Wisconsin pretest ranked among the highest in each treatment. Logic final and logic retention were highly correlated with age in treatment L, but not in treatment LM. Class pretest was highly correlated in treatments I and C but not in treatments LM and L.

In treatments LM and L, the logic final and logic retention were highly correlated with both class reasoning pretest and posttest, Wisconsin pretest and posttest, with conditional posttest but not with conditional pretest.

ATTITUDE TEST

There was a significant difference among the four treatments in scores on the attitude test with $p \leq .01$ for both grades 3 and 6. See Tables 4.46 and 4.47.

In both grades neither one of the logic groups scored highest in attitude. Treatment group C scored highest in grade 3 and group I in grade 6.

The higher the score, the more positive the attitude as the scale on the test was: from 5 for like very much to 1 for dislike very much. Since there were seven items on the attitude instrument: school, science, reading, mathematics, addition, subtraction and logic, 35 was the maximum score.

Even with the means on the attitude test adjusted with both I.Q. and age, the level of significance, $p \leq .01$, remained. The overall picture on the attitude tests indicates that in rank order of preference, the following is true:

TABLE 4.46

ANALYSIS OF VARIANCE OF ATTITUDE TEST

GRADE 3

<u>GROUP</u>	<u>MEAN</u>	<u>EFFECT</u>	<u>STAND. DEVIATION</u>	<u>N</u>
LM	26.14	0.17	4.97	205
L	24.39	-1.78	4.41	111
I	26.60	0.43	5.10	114
C	27.58	1.41	5.20	81

<u>SOURCE OF VARIATION</u>	<u>DF</u>	<u>SUM OF SQUARES</u>	<u>MEAN SQUARE</u>	<u>F RATIO</u>	<u>SIGN.</u>
Between groups	3	540.24	180.08	7.43	.01
Within groups	507	12285.28	24.23		
TOTAL	510	12825.53			

GRADE 6

<u>GROUP</u>	<u>MEAN</u>	<u>EFFECT</u>	<u>STAND. DEVIATION</u>	<u>N</u>
LM	25.92	-1.22	4.50	119
L	26.82	-0.32	4.19	202
I	28.16	1.01	3.62	113
C	27.77	0.62	4.32	155

<u>SOURCE OF VARIATION</u>	<u>DF</u>	<u>SUM OF SQUARES</u>	<u>MEAN SQUARE</u>	<u>F RATIO</u>	<u>SIGN.</u>
Between groups	3	374.77	124.92	7.13	.01
Within groups	585	10250.67	17.52		
TOTAL	588	10625.44			

TABLE 4.47

SCHEFFE TEST FOR DIFFERENCES IN ATTITUDE TEST

<u>PAIRWISE GROUPS</u>	<u>SIGN. LEVEL GR. 3</u>	<u>HIGHER MEAN GR. 3</u>	<u>SIGN. LEVEL GR. 6</u>	<u>HIGHER MEAN GR. 6</u>
LM-L	.05	LM	NS	L
LM-I	NS	I	.01	I
LM-C	NS	C	.01	C
L-I	.05	I	NS	I
L-C	.01	C	NS	C
I-C	NS	C	NS	I
(LM+L)-(I+C)	.01	I+C	.01	I+C

<u>Grade 3</u>		<u>Grade 6</u>	
<u>Rank Order</u>	<u>Topic</u>	<u>Rank Order</u>	<u>Topic</u>
1	Mathematics (4.20)	1	Mathematics (4.56)
2	Science (4.10)	2	Reading (4.10)
3	School (4.08)	3	Addition (3.96)
4	Subtraction (3.76)	4	Science (3.90)
5	Reading (3.31)	5	Subtraction (3.56)
6	Addition (3.38)	6	School (3.50)
7	Logic (2.17)	7	Logic (2.29)

About the only commonality existing between the two grade levels is the fact that they liked mathematics the most and logic the least. Test scores regarding the attitude towards logic seemed contradictory to statements made by teachers about the students' enjoying logic where it was taught in treatments LM and L. It is conjectured that perhaps attitude on tests of this kind reflect that students have a greater feeling of success or achievement in certain areas where they have not yet felt this towards the logic regardless of how they might have enjoyed classroom experiences.

Boys apparently liked school more than girls as shown in every treatment group in Table 4.48. Except for the C group in grade 3, girls favored science more than the boys and in every group, girls liked mathematics more than the boys.

While logic was low not only for the I and C groups, but for the LM and L groups also, yet not enough time was allowed for any kind of mastery or even "student satisfaction" with lessons in logic. Perhaps

too many principles were included in the study whereas fewer principles and more in-depth teaching might have indicated otherwise. No explanation can be offered for girls liking science and mathematics more than the boys.

An interesting result in the comparison of the attitude total scores and mean scores on the pretests and posttests in class reasoning, conditional reasoning, and Wisconsin contemporary mathematics, indicates in both grade 3 and grade 6 that the groups with highest attitude scores had lowest scores in achievement. For example, in grade 3, C boys had highest attitude scores and lowest in six tests; C girls had second highest in attitude and second lowest in reasoning and mathematics. (See Appendix B, Tables B.3 and B.4).

In grade 6, I boys had highest score on the attitude test and lowest scores on the reasoning and mathematics test; I girls had second highest in attitude and second lowest in the six tests mentioned previously. Looking at the picture from the highest ranking in achievement: in grade 3, LM boys ranked highest but fourth in attitude; LM girls ranked second highest in achievement but fifth in attitude. (See Appendix B, Tables B.3 and B.4).

LM girls ranked highest in achievement in grade 6 but fifth in attitude while L girls ranked second highest in achievement and tied for fifth in attitude. It might be that there was still enough uncertainty in both LM and L groups to influence the attitude towards logic. Certainly with I and C groups, where the only contact with logic was the taking of pretests and posttests in class and conditional reasoning, not much can be inferred. It is interesting, however, in the overall

TABLE 4.48
MEAN SCORES ON ATTITUDE TEST

<u>GRADE 3</u>								
GROUP	SCHOOL	SCIENCE	READ- ING	MATHE- MATICS	ADDI- TION	SUBTRAC- TION	LOGIC	TOTAL SCORE
LM BOYS	4.16	3.83	3.70	4.26	3.60	3.43	2.13	26.16
LM GIRLS	3.56	4.30	3.62	4.32	3.65	3.39	2.23	26.49
L BOYS	4.25	3.76	2.61	3.63	2.68	3.39	1.80	23.10
L GIRLS	3.63	4.37	3.04	4.40	3.12	3.63	2.17	25.85
I BOYS	4.32	3.86	3.27	4.08	3.31	3.86	2.03	25.80
I GIRLS	3.87	4.29	3.55	4.33	3.51	4.16	2.33	27.45
C BOYS	4.64	4.23	3.62	4.10	3.62	3.87	2.38	27.49
C GIRLS	4.24	4.19	3.07	4.45	3.57	4.33	2.26	27.67
MEAN	4.08	4.10	3.31	4.20	3.38	3.76	2.17	

<u>GRADE 6</u>								
GROUP	SCHOOL	SCIENCE	READ- ING	MATHE- MATICS	ADDI- TION	SUBTRAC- TION	LOGIC	TOTAL SCORE
LM BOYS	3.37	3.64	3.63	4.22	3.47	3.19	2.00	24.54
LM GIRLS	3.33	4.21	3.97	4.54	3.84	3.59	2.26	27.13
L BOYS	3.65	3.83	3.90	4.40	3.87	3.23	2.20	26.26
L GIRLS	3.16	3.93	4.05	4.60	4.17	3.57	2.28	27.29
I BOYS	4.04	3.63	4.33	4.69	4.27	3.96	2.44	28.35
I GIRLS	3.05	4.15	4.41	4.79	4.16	3.87	2.39	28.00
C BOYS	4.06	3.73	4.28	4.48	3.99	3.65	2.41	27.92
C GIRLS	3.36	4.04	4.24	4.73	3.93	3.42	2.30	27.64
MEAN	3.50	3.90	4.10	4.56	3.96	3.56	2.29	

picture in both grades, that the I and C groups tended to have a higher score in their attitude towards logic. Maybe it could be conjectured that a "little knowledge" makes it interesting whereas the "imbalance" engendered by some attention, perhaps not enough, inspires students to realize their ignorance and not like it particularly. Obviously, this needs added research.

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

In the final chapter there is a brief restatement of the problem, followed by findings about the hypotheses, answers to questions posed and suggestions for further research in the area of logical thinking and mathematical learning.

RESTATEMENT OF THE PROBLEM

This study was designed to investigate the role of logic in mathematics in grades 3 and 6 and to examine the teaching of various principles of logical thinking and their relation to learning mathematics.

Four treatment groups were used: LM, in which logic was taught interwoven with mathematics; L, in which logic was taught separated from any mathematical application; I, in which teachers of this group received in-service instruction in mathematics; and C, a control group which received no instruction in logic. Teachers in LM and L received in-service instruction in logic. Treatment I was designed to help isolate the factor of in-service training. Teachers in C received no in-service training and students in this group were taught no logic.

Major questions asked in the study were:

1. Can the teaching of logic interwoven within mathematics significantly affect the mathematics achievement of third and sixth grade students?
2. Can the teaching of logic significantly affect the development of logical thinking in third and sixth grade children?
3. Is there a difference in the attainment of logical thinking ability in third and sixth grade children? If so, in what way is there a difference?
4. How much logic can be taught to third graders? to sixth graders?
5. What are the trouble areas for third graders in logic? for sixth graders?
6. What are the easy areas of logic for third graders? for sixth graders?
7. What did the third graders seem to have learned in logic before their logic instruction? sixth graders?
8. What specific mathematical topics seem to be aided by instruction in logic for third graders? for sixth graders?

9. What are some of the similarities and differences in the logical thinking ability of third and sixth graders?

Questions 1-3 are restated as six null hypotheses.

FINDINGS AND CONCLUSIONS

Hypothesis 1. There is no significant difference in the mathematics achievement of students in grade 3 and grade 6 between those who have been taught logic and those who have not been taught logic.

Findings: The hypothesis is rejected. The two groups, LM and L, taught logic achieved significantly higher, $p \leq .01$, than the two groups, I and C, in both grades 3 and 6 on the posttests in mathematics achievement using I. Q. or mathematics pretest as covariates.

Hypothesis 2. There is no significant difference in mathematics achievement of students in grade 3 and grade 6 between those who have been taught logic interwoven with mathematics and those who have been taught logic separated from and not linked with mathematics instruction.

Findings: The hypothesis is accepted. While the group taught logic interwoven with mathematics scored higher in both grade 3 and grade 6 than the group taught logic without mathematics, these differences were not significant.

In the third grade pretest in mathematics, the LM group scored about one-half a point higher than the L group, while in grade 6, the L group scored about one and one-half points higher. In the posttests, the LM group in grade 3 scored about nine-tenths of a point higher than L; while in grade 6, the LM group scored about seven-tenths of a point higher.

Hypothesis 3. The teaching of logic does not significantly affect the development of logical thinking abilities in grade 3 and grade 6.

Findings: The hypothesis is rejected. The groups taught logic scored significantly higher, $p \leq .01$, than the groups not taught logic on both the class and conditional reasoning tests, using the corresponding pretests or I. Q. as covariates.

Hypothesis 4. There is no significant difference in logic achievement in grade 3 and grade 6 between those who have been taught logic interwoven with mathematics and those who have been taught logic separated from and not linked with mathematics instruction.

Findings: The hypothesis is accepted on the basis of the class and conditional reasoning tests. Although LM scored higher than group L in third grade and L scored higher than LM in grade six, in both reasoning tests, these differences were not significant, using corresponding pretests or I. Q. as covariates.

In the third grade, the hypothesis is rejected on the basis of the logic final where group LM scored significantly higher, $p \leq .05$, than L. While LM scored higher than L in the logic retention, this difference was not significant. In the sixth grade, the hypothesis is rejected on the basis of both the logic final and logic retention as group L scored significantly higher, $p \leq .01$, than the LM group.

Hypothesis 5. There is no significant difference in the attainment of logical thinking abilities between third and sixth grade students.

Findings: The hypothesis is rejected. There was a significant difference, $p \leq .01$, between the scores of students in grade 3 and grade 6 on both class reasoning pretest and posttest as well as conditional reasoning pretest and posttest. Scores on the logic final and logic retention administered only to groups LM and L, showed that sixth graders scored substantially higher on this test than did the third graders.

Hypothesis 6. There is no significant difference in the logic achievement of boys and girls in grade 3 and grade 6.

Findings: The hypothesis is accepted. There was no significant difference in any area of the study, either in mathematics achievement or logic achievement, between boys and girls in either grade 3 or grade 6.

Several of the questions asked were not able to be translated into statements of null hypotheses to be answered precisely statistically. These are discussed below.

Question 4. How much logic can be taught to third graders? to sixth graders?

Findings: The added or hidden question of "What should be taught?" as opposed to "What can be taught?" enters the analyses here. Certain principles in class reasoning in grade 3, such as transitivity, the fact that class exclusion is symmetric and the idea that a thing cannot be both a member of the class and a non-member of that same class, seemed to be easy for the third graders as well as the sixth graders. There was a definite division of principles in the conditional reasoning in both grade 3 and grade 6 which did not exist in class reasoning. In other words, principles 1, 4, and 5 in both levels were in top brackets and were learned much more readily by both groups, as scores indicated, than the rest of the principles. In class reasoning the spread of difference was more gradual and there was not a pronounced division. Class reasoning, as previously mentioned, was easier for both groups than conditional reasoning. Students were able, in both groups, to learn symbolism with a minimum of difficulty as evidenced by the fact that about 72% of the students in both grades were able to do the symbolic items correctly.

The only symbolism which was taught was used extensively. It was not an artificial lesson to determine whether children could learn but an efficient tool, for example, to write

$\wedge$ for "and" and $\vee$ for "or," $\longrightarrow$ for "implies." The fact that these were used consistently throughout the instruction perhaps was the greatest single factor in the mastery of them. The use of symbolism was of particular advantage to the third graders whose motor coordination is somewhat less than speedy. This meant that they would not have to write as extensively. Motivation for symbolism, it was felt by the researcher, was built into the activities.

An analysis of the tables indicates, for example, that there was only a slight change in the third graders' scores on the contrapositive, this, despite more emphasis on this principle in the instruction than towards others. Previous research had indicated that children do not do well on this. The third grade and sixth grade both did better than anticipated, but third graders did not seem to improve as much as the instruction period might have warranted. Furthermore, they were better able to do some kinds of contraposition, namely, ones closely associated with their own familiar experiences, than those concerned with incidents which were either contrary to their own experience or entirely divorced from anything they might have known.

Question 5. What are the trouble areas for third graders in logic? for sixth graders?

There is, of necessity, some overlapping in this question with question 4 because in explaining what can be taught, one necessarily has to point up some difficulties.

One of the difficulties in analyzing the data, other than the fact that there is so much of it, is that students were pretested and posttested on the Cornell class and conditional reasoning tests. Yet certain of these principles

were not emphasized that much on the regular instruction, particularly in third grade. However, on the logic final test as administered to only LM and L groups, there was a consistency of results in those topics and principles which matched the items on the Cornell reasoning tests.

For example, children in third grade did poorly on items of fallacy concerning the converse. The converse is not always true; it may be. There are two general ways of checking on the form of this principle. The first one seemed to be easier for the students, namely, to ask the following:

1) All A are B.

Is it true that all B are A?

Yes ☐ No ☐ Maybe ☐

Secondly, to ask the same thing in terms of determining whether it is valid or invalid, where the "Maybe" factor does not enter in. This could be as follows:

2) If all red candies on the dish are licorices,
then all licorices are red candies on the dish.

O. K. ☐ Not O. K. ☐

When the children in third grade were not "accosted" with a "Maybe" choice, they did better on the converse.

They also did better when the things they were talking about were familiar to them or at the other extreme, fantastic. However, in regard to "supposing things" that were contrary to situations "close to home," they did not do well. This phenomenon did not occur in grade 6. An example would serve to illustrate this better.

Third graders had no trouble, it seemed, supposing that cows could smoke, cats are good quarterbacks, but were reluctant to suppose that the teacher was tall if she were short, that their fathers were bald if they had a lot of hair. This might be in line with what Piaget says about children at this age: they cannot seem to gather in all possibilities and hypotheses of a situation, to reason hypothetical-deductively in every illustration of a principle. This was true with third grade and more obvious in oral work than in written work. What apparently seemed like mastery of a principle, even in transitivity, where the third grade scores were high, was not as solid when examples were selected that contradicted their observations. It is difficult to explain this other than perhaps calling it a kind of "Walt Disney" syndrome and conjecture that animated cartoons and TV programs had made them more attuned to the fantastic but less eager for the hypothetical.

In sixth grade, class reasoning was much easier than conditional reasoning generally. While there was a small change in the learning of the converse fallacy, when demonstrated in class reasoning, it seemed easier than when shown in conditional reasoning. In other words, students were

better able to see that if someone said that All A are B, it was not necessarily true that all B are A. When it was in the form of a conditional: If p implies q, then does it follow that a q implies p, the principle became much more difficult for the students in both grades, as evidenced by their scores on the tests. However, more changes were made with grade 6 in this regard.

Question 6. What are the easy areas of logic for grade 3? for grade 6?

Findings: Transitivity was easy for both grade levels and there was not much change evidenced. This can be interpreted to mean that the student already knew this and so not much change could be made. Although this study did not determine, by way of pretest, the students' mastery of statements with negations in them, nevertheless on the logic final, 66% of grade 3 and 78% of grade 6 answered items with negatives in them correctly. Much work has been done in the teaching of negatives, and once the students became adept at it, they did not seem to lose it as evidenced not only by the logic final, but the logic retention. Symbolism, as mentioned previously, was easy.

Conjunction was much easier to teach than disjunction and once students became aware of differences between form and content, certain logical principles seemed more readily mastered. More detailed work was necessary in these areas in grade 3 than in grade 6, because language had to be built carefully. This is perhaps a key observation of this project, the gradual development of a working language that had precise meaning in logic and mathematics and yet could

and often would not be interpreted correctly in ordinary conversation.

It seemed almost impossible to teach certain topics in conditional reasoning to grade 3 children and not much time was spent after the initial attempt. For example, with implications, third graders were only briefly taught what the if-then statement meant, what it did not mean, and some familiar usages. In grade 6, more was studied, and it apparently seemed to be easier for these students.

Question 7. What did the third graders seem to have learned in logic before their logic instruction? the sixth graders?

Findings: Scores on the pretest of class reasoning indicate that both third and sixth graders scored highly on the second principle which is transitivity and both seemed to have a fairly good idea that whatever is a member of a class cannot be a non-member of it. This last principle seems to be one fairly associated with common sense and children reflected this. Sixth graders scored substantially higher in both of these principles than the third graders.

In conditional reasoning, both levels scored well on the pretest in transitivity. One can say with a fair degree of certainty that children by the third grade level have a good understanding of the principle of transitivity, with the sixth graders having a more firm hold on it, of course. Both grade 3 and grade 6 did well on the pretest on the first principle which is that of detachment, or modus ponens. The remaining scores on the conditional for third graders would

suggest that this is about all that one can determine that they have very much knowledge of. With the sixth graders in conditional reasoning, 75% answered correctly the contrapositive, principle 4, but did not maintain as much consistency as one would hope from students who seem to have an understanding of a principle.

Question 8. What specific mathematical topics seem to be aided by instruction in logic for third graders? for sixth graders?

Findings: There were substantial changes in the group taught logic in the third grade in class reasoning in class exclusion being symmetric, and the fallacy principle: No A's are B's. No C's are B's. Therefore at least some A's are C's, which is invalid. There was not much change in the results as far as transitivity because third graders had a fair understanding of this as indicated in the pretest scores. There was only slight improvement in the idea of the invalidity of the converse and also in some aspects of the contrapositive. Both LM and L improved in principle 7, but made only a slight gain in principle 5, both of which are contrapositives. There was substantial change in the knowledge of symbolism as shown on the Cornell tests and as exhibited on the logic final and logic retention in third grade.

In conditional reasoning, LM made gains in all principles except the first one, but they seemed to have had a better understanding of this than of the other principles of conditional reasoning. L group increased in all principles except 2 and 7, but their gains, for the most part,

were not as great as the LM group.

Generally, almost all principles seemed to have been aided by the logical instruction but the fact that overall in grade three, conditional reasoning while showing a certain percent of improvement, nevertheless, was not learned as well as class reasoning. Basic ideas of conditional reasoning, the meaning of the if-then relationship, certainly, were improved.

In grade 6, instruction did not appear to change students' scores on transitivity and the fact that an object could not be both a member and a non-member of a set at the same time. The most improvement in grade 6 in class reasoning was in principle 7, a form of the contrapositive which says: All A's are B's. No C's are B's. Therefore no C's are A's. It appears that more progress was made in grade 6 in those principles which seemed to have been very difficult for grade 3 and in which, while there was some improvement, nevertheless the total scores on these were not high.

Grade 6, like grade 3, had a higher percent of change in conditional reasoning than in class reasoning, but in grade 6, the percent of correct responses was much higher in conditional reasoning. The idea of fallacy, contrapositive, and converse seemed to have taken hold much more by aid of instruction in grade 6 than in grade 3.

Question 9. What are some of the similarities and differences in the logical thinking ability of third and sixth graders?

Findings: Both groups seemed to have a good understanding of transitivity. While the third grade maintained lower scores on these principles, nevertheless they, along with the sixth graders, seemed to have a fair amount of knowledge of the basic idea that whatever is a member of a class is not a non-member of that class and also that class exclusion was symmetric. After those three principles, the other four seemed to be much different as far as initial understanding was concerned. Both groups had no difficulty with symbolism.

In conditional reasoning, both groups listed among the top three in scoring, modus ponens, the contrapositive, and the transitivity of the if-then relationship. The other principles were far below these in percent of correct responses. This pattern occurred both in grade 3 and grade 6. In class reasoning, there was a gradual spread of the principles in contrast to this. Both found the converse more difficult in conditional reasoning than in class reasoning.

Third graders could not, it seemed, understand certain principles, either in class or conditional reasoning if they did not have familiar content. Furthermore, they had more trouble in working with expressions and statements which included words for which there is no meaning, such as "flexums", "glorbs" and so on. Because the LM group did better in logic in grade 3 than the L group and the reverse is true in grade 6, it would seem that the younger children need more blending of logical principles in the subject matter content than do the sixth graders.

DISCUSSION AND RECOMMENDATIONS

While this study has answered some questions about the relation of logical thinking and mathematics achievement, it has probably raised more questions than it has answered. Because students can learn certain logical principles and apparently have used them in working out new examples as the tests in this project indicated, does not indicate what they will do with these principles in other subject matter content.

Some questions which need to be asked and have further research are:

1. Would the same performance be obtained by students in lower socio-economic backgrounds than obtained by students in this project who were, for the most part, of a higher social background?
2. What would be the results if test items were open-ended and not multiple-choice? Although some open-ended questions were asked on the logic final, there were not enough to make a comparison with the multiple-choice type.
3. What would be the result if the logic were taught in grade four and five, seven and eight?
4. How many basic logical ideas can be incorporated into the teaching of mathematics at the first and second grade levels?

5. A next step could be the teaching of logic to small groups so that the researcher would have more than a paper-pencil type of instrument to determine the logical thought processes of the child. How would this knowledge affect the kind of material that would be useful?

6. What are some of the fringe benefits of teaching logic to elementary school students? While not able to be statistically validated, there has been evidence that students taught logic are more careful in their reading, ask more questions, and are more eager to attempt new ideas and problems.

7. What are some of the basic logical principles pervading elementary mathematics? How can these be interwoven more closely?

It would seem, at this stage, that we have found that students can learn logic and have little difficulties with certain principles. What needs to be determined is the amount of logic that can be taught at various stages in collaboration with the subject-matter, what are the best ways of determining this, and learning how best to teach teachers basic principles of logical thinking.

APPENDIX A

APPENDIX A

Table A.1. Class Reasoning Test

Table A.2. Conditional Reasoning Test

Table A.3. Logic Final Test

Table A.4. Attitude Test

TABLE A.1

CLASS REASONING TEST

(adapted from the Cornell Critical Thinking Project tests, Cornell University, Ithaca, New York: reprinted with permission of Dr. Robert Ennis, project director.)

GENERAL DIRECTIONS

This is a test to see how well you do a particular kind of thinking. We call it CLASS REASONING. You will see that you already do some of this kind of thinking. The sample questions make clear what is expected.

DO NOT GUESS WILDLY.

There is a scoring penalty for guessing wrong. If you think you have the answer, but are not sure, mark that answer. But if you have no idea, then SKIP the question.

There are six sample questions, then 21 others. You should work as quickly as you can, but do not rush. This is not a speed test. Once you do the samples, you will be able to move right along.

ANSWERING THE QUESTIONS

In answering each question, use only what you are told in that question. In order to do this, you should imagine that your mind is blank, because some of the things you are told are obviously false. Even so, you should suppose that they are true--for that question only.

You will be given one or more sentences with which to think. You will then be given another sentence, about which you must decide, using only what you are told.

There are three possible answers. This is what they mean.

- Y. Yes. It must be true.
- N. No. It can't be true.
- M. Maybe. You can't be certain. It may be true or it may not

be true. You aren't told enough information to be certain that it is YES or NO.

The meaning of the possible answers is given at the top of each page to help you remember. Each question has only one answer.

Mark your answers on your answer sheet by drawing a line through the right answer.

SAMPLE Y N M

REMEMBER: If you have no idea what the answer is, SKIP the question and go on to the next. Do not guess wildly. If you think you know, then answer the question.

SAMPLE QUESTIONS

Read the first question and see how it is marked.

1. Suppose you know that
Bill is next to Sam.

Then would this be true?

Sam is next to Bill.

1. Yes	No	Maybe
Y	N	M

The correct answer is Y, YES. If Bill is next to Sam, then Sam must be next to Bill. It must be true, so a mark is drawn through Y which represents YES.

Here is another sample. On your answer sheet, you mark the answer.

2. Suppose you know that
The sparrow is over the hawk

Then would this be true?

The hawk is over the
sparrow.

You should have marked N, NO. If the sparrow is over the hawk, then the hawk can't be over the sparrow. It can't be true.

Mark the answer to this next sample. Be careful.

3. Suppose you know that
Jane is standing near Betsy.
Then would this be true?
Betsy is standing near Jane.

The correct answer is M, MAYBE. Even if Jane is standing near Betsy, Betsy might be sitting. Betsy might be standing near Jane, but she might be sitting near Jane or something else. You were not told enough to be CERTAIN about it, so MAYBE is the answer.

Mark the answer to this next sample question on your answer sheet. Remember that your mind is supposed to be blank at the beginning of each question.

4. Suppose you know that
 California is near
 New York
 Then would this be true?
 New York is near
 California.

The correct answer is Y, YES, even though New York and California are not really near to each other. If California were near to New York, then New York WOULD BE near to California. It would have to be true.

Remember: You should suppose that what you are told is true--for the question you are answering.

So far in the sample questions you were told only one thing. In this one you are told two things. Mark your answer on the sample question section of your answer sheet.

5. Suppose you know that
 The pit is inside the
 mouth of the fox.
 The cherry is inside
 the mouth of the fox.
 Then would this be true?
 The pit is inside the
 cherry.

The correct answer is M, MAYBE. All you are told is that the pit and the cherry are both in the mouth of the fox. There is no way to be certain whether the pit is in the cherry or not.

Here is the last sample question. This time the letters "A" and "B" are used. They can stand for anything you like. Mark your answer on your answer sheet.

6. Suppose you know that
 A is next to B.
 Then would this be true?
 B is next to A.

The correct answer is Y, YES, no matter what A and B stand for. If A is next to B, then B must be next to A.

Now that you have done the practice questions, you probably understand what is expected. If you have any questions, ask them now.

7. Suppose you know that
None of the fifth grade boys are on the football team.
John is a fifth grade boy.
Then would this be true?
John is not on the football team.

8. Suppose you know that
At least some of Kate's pencils are blue.
All the pencils in the box are blue.
Then would this be true?
At least some of Kate's pencils are in the box.

9. Suppose you know that
None of Jane's dolls have hats.
Then would this be true?
None of the dolls that have hats are Jane's.

10. Suppose you know that
Eileen is one of the children on the playground.
Then would this be true?
Eileen is not one of the children on the playground.

11. Suppose you know that
None of Sue's books are about animals.
Then would this be true?
None of the books about animals are Sue's.

12. Suppose you know that
All the books about sailing are Bill's.
All the green books are Bill's.
Then would this be true?
At least some of the green books are about sailing.

13. Suppose you know that
All the pets of the Greens' won some prize in the pet show.
Fido is one of the Greens' pets.
Then would this be true?
Fido won a prize in the pet show.

14. Suppose you know that
At least some of the books on the table are about stars.
None of Bob's books are about stars.
Then would this be true?
All of the books on the table are Bob's.

15. Suppose you know that
All of the red pencils are broken.

Emil's pencils are not broken.
Then would this be true?
Emil's pencil is not red.

16. Suppose you know that
All Jean's pencils are red.
All the pencils on the table are red.

Then would this be true?
At least some of the pencils on the table are Jean's.

17. Suppose you know that
All of the boys are running, but not everyone is running.

Then would this be true?
At least some of the people not running are not boys.

18. Suppose you know that
All members of the football team weigh over 150 pounds.
Henry does not weigh over 150 pounds.

Then would this be true?
Henry is on the football team.

19. Suppose you know that
No Z's are Y's
No X's are Y's.
Then would this be true?
At least some Z's are X's.

21. Suppose you know that
All the cars in the garage are Mr. Smith's.
All Mr. Smith's cars are Fords.

Then would this be true?
All of the cars in the garage are Fords.

22. Suppose you know that
None of Tom's books are on the shelf.
No science books are on the shelf.

Then would this be true?
At least some of Tom's books are science books.

23. Suppose you know that
All X's are Y's.
Then would this be true?
All things that are not Y's are also not X's.

24. Suppose you know that
All dogs are brown.
Then would this be true?
At least some dogs are not brown.

25. Suppose you know that
At least some of the children in the Martin family take out books from the library.

All people who take out
books from the library
have library cards.

Then would this be true?

At least some of the
children in the Martin
family have library
cards.

26. Suppose you know that

At least some of Mrs.
Brown's flowers are not
roses.

At least some of the
flowers in the flower
show are not roses.

Then would this be true?

At least some of Mrs.
Brown's flowers are in
the flower show.

27. Suppose you know that

All X's are Y's.

No Z's are Y's.

Then would this be true?

At least some X's are
Z's.

TABLE A.2
CONDITIONAL REASONING TEST

- | | |
|---|--|
| <p>7. Suppose you know that
 If Sam misses the bus,
 he will walk to school.
 If Sam walks to school,
 he will cross the bridge.
 Then would this be true?
 If Sam misses the bus,
 he will cross the bridge.</p> <hr/> <p>8. Suppose you know that
 If the car in the parking
 lot is Mr. Smith's, then
 it is blue.
 The car in the parking
 lot is not blue.
 Then would this be true?
 The car in the parking
 lot is Mr. Smith's.</p> <hr/> <p>9. Suppose you know that
 If Bill lives on a farm,
 then he has a pet dog.
 Then would this be true?
 Bill lives on a farm,</p> <hr/> <p>10. Suppose you know that
 If there is an X, then
 there is a Y.
 There is an X.
 Then would this be true?
 There is a Y.</p> <hr/> <p>11. Suppose you know that
 If the chair is green,
 then the table is black.
 Then would this be true?
 If the table is black,
 then the chair is green.</p> <hr/> | <p>12. Suppose you know that
 If Nancy bought a new dress,
 then she went to the shop
 on Main Street.
 Then would this be true?
 If Nancy didn't go to the
 shop on Main Street,
 then she didn't buy a new
 dress.</p> <hr/> <p>13. Suppose you know that
 If Tom lives in the white
 house, then his last name
 is Smith.
 Tom does not live in the
 white house.
 Then would this be true?
 Tom's last name is not Smith.</p> <hr/> <p>14. Suppose you know that
 Joe's bicycle is not working
 today.
 If Joe's bicycle is not work-
 ing, then he has to walk to
 school.
 Then would this be true?
 Joe has to walk to school.</p> <hr/> <p>15. Suppose you know that
 If Kate is in Mrs. Jones'
 class, then she is out on
 the playground.
 If Kate is out on the play-
 ground, then she is jumping
 rope.
 Then would this be true?
 If Kate is in Mrs. Jones'
 class, then she is jumping
 rope.</p> <hr/> |
|---|--|

16. Suppose you know that
If there is an X, then
there is a Y.

There is not a Y.

Then would this be true?

There is an X.

17. Suppose you know that
If Mary lives in the
white house, then her last
name is Brown.

Mary's last name is Brown.

Then would this be true?

Mary lives in the white
house.

18. Suppose you know that
If Jean goes to the park,
she will see her friend
Pat.

Today, Jean is going to
the park.

Then would this be true?

Today, Jean will see her
friend Pat.

19. Suppose you know that
If Mary lives in the
green house, then her
last name is Jones.

Mary doesn't live in the
green house.

Then would this be true?

Mary's last name is
not Jones.

20. Suppose you know that
If there is an X, then
there is a Y.

Then would this be true?

If there is not a Y,
then there is not an X.

21. Suppose you know that
If Jane fell off her
horse, then she hurt
herself badly.

Jane hurt herself badly.

Then would this be true?

Jane fell off her horse.

22. Suppose you know that
If the school team loses
this game, Brighton High
will win the league
pennant.

If Joe does not hit a
homer on this pitch, the
school team will lose
this game.

Then would this be true?

If Joe does not hit a
homer on this pitch,
Brighton High will win
the league pennant.

23. Suppose you know that
If Bill has an apple in
his lunchbox, then Sally
has a cracker in her
lunchbox,

Then would this be true?

If Sally has a cracker
in her lunchbox, then
Bill has an apple in his
lunchbox.

24. Suppose you know that

If Bob did not buy a
new baseball glove,
then he played basket-
ball today.

Then would this be true?

If Bob did not play
basketball today,
then he did buy a new
baseball glove.

25. Suppose you know that

Fred went to a movie
last night.

If Fred does not go to
a movie, he feels bad
the next day.

Then would this be true?

Fred does not feel bad
today.

26. Suppose you know that

If the bicycle in the
garage is Bob's, then
it is red.

The bicycle in the
garage is not red.

Then would this be true?

The bicycle in the
garage is not Bob's.

27. Suppose you know that

If a dog can stand on
its front legs, then
it is a puppy.

Then would this be true?

If a dog is a puppy,
then it can stand on
its front legs.

TABLE A.3

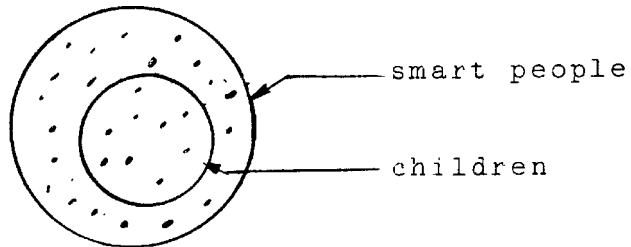
LOGICAL THINKING TEST

NAME _____

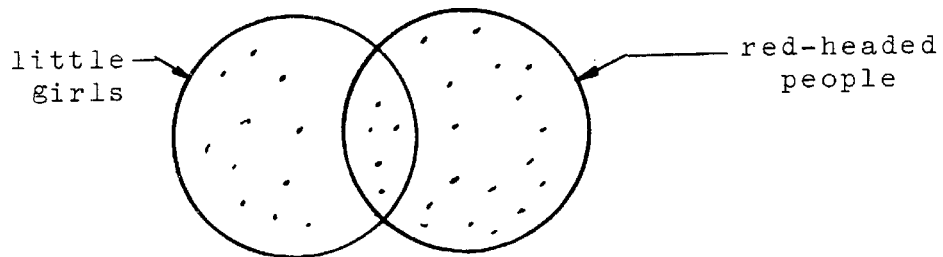
SCHOOL _____ GRADE _____

(Write all answers on this test paper, please.)

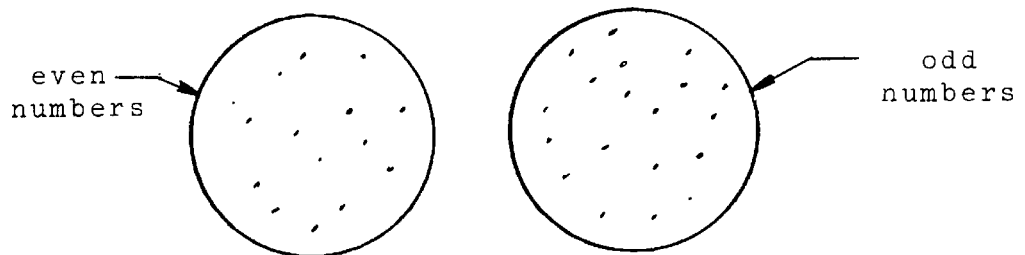
1-3. Use the diagrams and write one true sentence for each.



1. _____



2. _____



3. _____

4-8. Write the negation of the following:

4. All cats have tails.

5. No boys like dolls.

6. Some numbers are greater than seven.

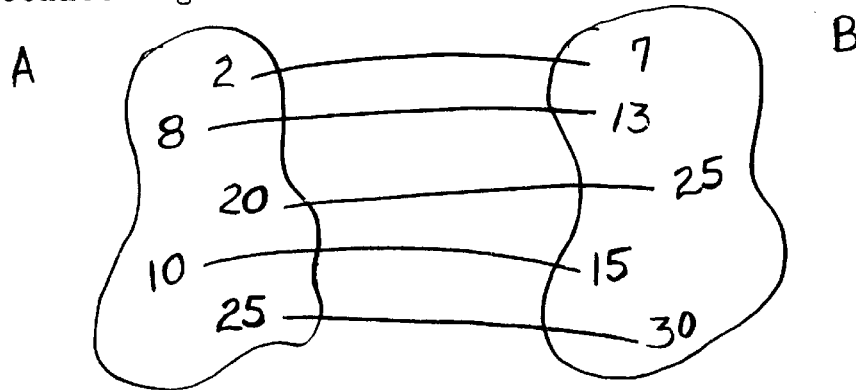
7. Jane did not lose her money.

8. All squares are rectangles.

9. Suppose you know that two numbers are unequal and that A is greater than B. Is it true that B is greater than A?

10. Suppose if the figures are polygons, either they are triangles or they are rectangles. The figures are polygons. The figures are not triangles. Are they rectangles?

Suppose you have two sets of numbers matched as shown by connecting lines:



Show whether the following are true or false.

11. All numbers in set A are even.

True ☐

False ☐

12. All numbers in set B are five greater than those in set A.

True ☐

False ☐

13. For each number in A there is a number in B that is five greater.

True ☐

False ☐

14. Some numbers in B are odd numbers.

True ☐

False ☐

15-21 In each of the following, show whether the sentence is true or false. Mark T for true; F for false.

15. $\frac{S \wedge J}{T \quad T}$

—

17. $\frac{P \wedge Q}{F \quad T}$

—

19. $\frac{R \vee K}{T \quad F}$

—

21. $\frac{C \wedge D}{F \quad F}$

—

16. $\frac{R \vee T}{T \quad T}$

—

18. $\frac{M \rightarrow N}{T \quad T}$

—

20. $\frac{H \vee K}{F \quad F}$

—

22. Suppose you are given a number from set $A = \{53, 24, 71, 48, 29, 36\}$ and the number you choose is less than 50 and is not a multiple of 4. This number must be _____.

In 23-26, study the first two statements in each and then complete the statement.

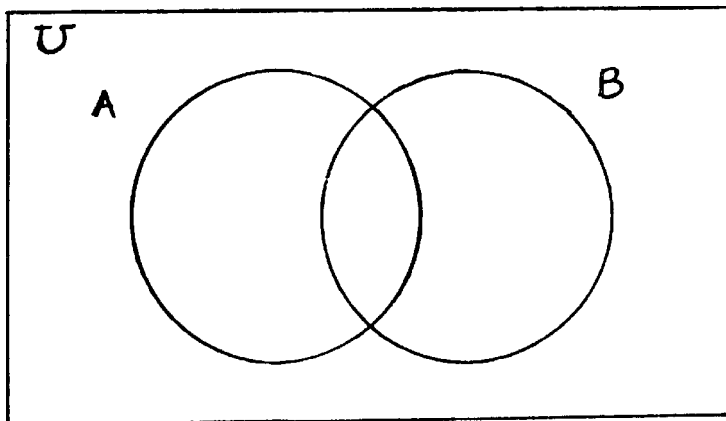
23. Suppose that all girls like to play with dolls and Tillie is a girl, then
- _____.

24. If all triangles are polygons and ABC is a triangle, then
- _____.

25. Suppose that lemons are bigger than pineapples and pineapples are bigger than basketballs, then
- _____.

26. Suppose that all numbers which are divisible by 3 are even and 10 is divisible by 3, then
- _____.

27. You are given two sets for which diagrams are shown.
A is the set of all even numbers
B is the set of all whole numbers greater than 8
U = the set of all whole numbers.



Given the set

$$\{13, 4, 9, 2, 6, 11, 16, 12\}$$

Write these numbers in correct region in the diagram.

In answering each of the questions, use only what you are told in that exercise. In 28-37, show whether the conclusion is OK or not OK, that is, whether it follows from the sentence given.

28. All the bikes in the garage are the Bumbleys.

All the Bumbley's bikes are red.

Therefore, all the bikes in the garage are red.

OK ☐ Not OK ☐

29. If Mike is a dog, then he is an animal.

Mike is an animal.

Therefore, Mike is a dog.

OK ☐ Not OK ☐

30. If a penny has the same value as a dime, then two pennies have the same value as twenty cents.

Two pennies do not have the same value as twenty cents.

Therefore, a penny does not have the same value as a dime.

OK ☐ Not OK ☐

31. If the weather gets colder, then the birds will not come back.

The weather gets colder.

Therefore, the birds will not come back.

OK ☐ Not OK ☐

32. If some numbers are greater than 8, then there are some numbers less than 8.

No numbers are less than 8.

Therefore, some numbers are greater than 8.

OK ☐ Not OK ☐

33. If that man is a policeman, then he is brave.
He is brave.
Therefore, that man is a policeman.

OK ☐ Not OK ☐

34. All numbers divisible by 6 are whole numbers.
Therefore, all whole numbers are divisible by 6.

OK ☐ Not OK ☐

35. If a nickel has the same value as a dollar, then 2
nickels equal 2 dollars.
A nickel has the same value as a dollar.
Therefore, 2 nickels equal 2 dollars.

OK ☐ Not OK ☐

36. If it snows, Lulu does not go to school.
Lulu does not go to school.
Therefore, it is snowing.

OK ☐ Not OK ☐

37. If two numbers are even, then their sum is odd.
These two numbers are even.
Therefore, their sum is odd.

OK ☐ Not OK ☐

In answering each of the questions below, use only what you are told in each exercise.

There are three possible answers.

YES. It must be true.

NO. It cannot be true.

MAYBE. You can't be certain. It may be true or it may not be true. You are not told enough information to be certain it is YES or NO.

38. If all third graders are dogs, then they can meow.
All third graders are dogs.

Can they meow?

YES ☐ NO ☐ MAYBE ☐

39. If the figure is a circle, then it is a closed curve.
If the figure is a closed curve, is it a circle?

YES ☐ NO ☐ MAYBE ☐

40. All A's are B's.
All C's are B's.
Are some A's also C's?

YES ☐ NO ☐ MAYBE ☐

41. All of the numbers in set A are in set B and no numbers in set C are in set B.

Are some of the numbers in set A in set C?

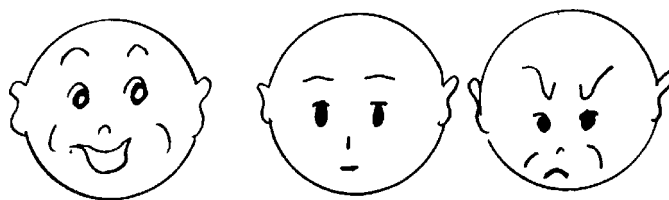
YES ☐ NO ☐ MAYBE ☐

42. No numbers in set A are even.
Is it true that no even numbers are in set A?
YES ☐ NO ☐ MAYBE ☐
43. If two numbers are even, then their sum is even.
If the sum is even, then the product is not even.
If the two numbers are even, is the product even?
YES ☐ NO ☐ MAYBE ☐
44. Some of the numbers in set G are divisible by four.
All of the numbers in set K are divisible by four.
Are some of the numbers in set G in set K?
YES ☐ NO ☐ MAYBE ☐
45. All M are N.
S is not an N.
Is S an M?
YES ☐ NO ☐ MAYBE ☐
46. All glorbs are flexums.
George is a glorb.
Is George a flexum?
YES ☐ NO ☐ MAYBE ☐
47. All of the candies in the dish are red.
No one in 6th grade wants a red candy because they all like licorice.
Does anyone in the 6th grade want candy from the dish?
YES ☐ NO ☐ MAYBE ☐

TABLE A. 4
ATTITUDE TEST

NAME _____ SCHOOL _____

TEACHER _____ GRADE _____ DATE _____



	<u>5</u>	<u>4</u>	<u>3</u>	<u>2</u>	<u>1</u>
1. School	☐	☐	☐	☐	☐
2. Science	☐	☐	☐	☐	☐
3. Reading	☐	☐	☐	☐	☐
4. Mathematics	☐	☐	☐	☐	☐
5. Addition	☐	☐	☐	☐	☐
6. Subtraction	☐	☐	☐	☐	☐
7. Logic	☐	☐	☐	☐	☐

APPENDIX B

APPENDIX B

- Table B.1. Percent of Correct Answers on the Wisconsin
Contemporary Elementary Mathematics Test,
Grade 3
- Table B.2. Percent of Correct Answers on the Wisconsin
Contemporary Elementary Mathematics Test,
Grade 6
- Table B.3. Summary of Mean Scores of Boys and Girls
on Reasoning and Mathematics Tests
- Table B.4. Summary of Mean Scores on Attitude, I. Q.,
Logic Final and Retention of Boys and
Girls

TABLE B.1
PERCENT OF CORRECT ANSWERS ON THE WISCONSIN CONTEMPORARY
ELEMENTARY MATHEMATICS TEST, GRADE 3

Treatments LM and L

<u>Topic and Item No.</u>	<u>LM Pre- test</u>	<u>LM Post- test</u>	<u>L Pre- test</u>	<u>L Post- test</u>
<u>Fractions</u>				
1-11	53.85	59.90	46.55	60.87
1-12	57.69	60.87	43.97	80.00
1-13	60.58	70.05	64.66	67.83
1-24	58.65	61.84	58.62	64.35
2-11	11.06	11.11	8.62	8.70
2-12	18.27	38.16	12.07	26.96
2-13	36.06	45.41	33.62	45.22
2-17	13.46	12.56	18.10	22.61
2-20	34.62	36.71	44.44	34.78
<u>Addition</u>				
1-1	90.87	83.57	57.76	88.70
1-20	87.98	88.89	90.52	92.17
2-7	70.19	69.08	51.72	52.17
2-19	76.92	80.19	81.03	81.74
2-25	78.85	86.47	83.62	86.96
<u>Subtraction</u>				
1-2	91.83	90.34	91.38	86.96
1-8	47.60	53.62	49.14	56.52
1-19	73.08	77.29	77.59	77.39
2-1	45.19	43.48	45.69	46.96
<u>Multiplication</u>				
1-7	78.85	89.86	77.59	81.74
1-14	36.06	60.87	40.52	41.74
2-2	31.25	56.52	23.28	66.09
2-8	63.46	84.06	68.10	80.00
2-14	26.92	51.21	18.10	34.78
2-26	24.04	25.60	28.45	26.96

Table B.1 Continued

<u>Topic and Item No.</u>	<u>LM Pre- test</u>	<u>LM Post- test</u>	<u>L Pre- test</u>	<u>L Post- test</u>
<u>Division</u>				
1-25	59.62	68.12	51.72	64.35
1-26	65.38	83.57	50.86	74.78
<u>Place Value</u>				
2-6	61.54	71.50	43.10	64.35
2-18	76.92	72.46	83.62	60.87
2-24	70.67	85.02	80.17	86.96
<u>Fundamentals</u>				
1-5	91.83	97.58	93.97	95.65
1-6	65.38	88.41	58.62	89.57
1-17	54.81	58.45	55.17	51.30
1-18	91.83	92.75	91.38	98.26
1-23	27.40	39.61	31.03	27.83
2-5	35.10	43.96	38.79	45.22
2-23	16.83	15.46	16.38	14.78
<u>Measurement:</u>				
<u>Money</u>				
1-3	92.79	93.24	91.38	93.91
2-4	72.12	66.67	68.10	59.13
<u>Measurement:</u>				
<u>Time</u>				
1-4	55.29	50.72	58.62	44.35
2-3	33.65	28.99	44.83	56.52
2-21	64.90	73.43	57.76	61.74

Table B.1 Continued

<u>Topic and Item No.</u>	<u>LM Pre- test</u>	<u>LM Post- test</u>	<u>L Pre- test</u>	<u>L Post- test</u>
<u>Measurement:</u>				
length, liquid, and weight				
1-9	23.56	17.87	19.83	31.30
1-16	73.08	74.88	64.66	69.57
2-9	24.52	32.85	35.34	30.43
<u>Geometry</u>				
1-10	75.00	88.89	81.90	90.43
1-15	93.27	97.58	93.10	95.65
1-21	75.00	80.19	78.45	86.96
1-22	17.79	19.81	15.52	15.65
2-10	42.79	46.38	60.34	60.00
2-15	78.37	66.18	71.55	82.61
2-16	18.27	40.58	25.86	34.78
2-22	17.31	30.43	7.76	14.78
<u>Total on Test</u>				
	58.22	63.58	56.48	62.14

Table B.1 (continued)

<u>Topic and Item No.</u>	<u>Treatments I and C</u>			
	<u>I Pre- test</u>	<u>I Post- test</u>	<u>C Pre- test</u>	<u>C Post- test</u>
<u>Fractions</u>				
1-11	47.90	58.87	66.67	76.92
1-12	56.30	63.71	51.11	64.84
1013	62.18	70.97	52.22	67.03
1-24	55.46	55.65	43.33	54.95
2-11	20.17	7.26	6.67	7.69
2-12	13.45	44.35	25.56	14.29
2-13	28.57	27.42	35.56	34.07
2-17	21.85	10.48	8.89	12.09
2-20	32.77	30.65	43.33	27.47
<u>Addition</u>				
1-1	85.71	83.06	93.33	79.12
1-20	94.12	91.13	82.22	78.02
2-7	68.07	80.65	51.11	69.23
2-19	88.24	77.42	73.33	69.23
2-25	76.63	84.68	81.11	81.32
<u>Subtraction</u>				
1-2	93.28	87.10	88.89	76.92
1-8	45.38	41.13	45.56	43.96
1-19	66.39	79.84	70.00	64.84
2-1	34.45	52.42	33.33	30.77
<u>Multiplication</u>				
1-7	70.59	95.16	68.89	82.42
1-14	52.94	47.58	31.11	48.35
2-2	19.33	66.13	20.00	43.96
2-8	52.94	69.35	58.87	80.22
2-14	12.61	52.42	17.78	24.18
2-26	19.33	19.35	20.00	16.48

Table B.1 Continued

<u>Topic and Item No.</u>	<u>I Pre- test</u>	<u>I Post- test</u>	<u>C Pre- test</u>	<u>C Post- test</u>
<u>Division</u>				
1-25	49.58	70.97	52.22	59.34
1-26	47.90	91.94	56.67	75.82
<u>Place Value</u>				
2-6	42.86	65.32	48.89	53.85
2-18	71.43	78.23	70.00	78.02
2-24	87.39	87.10	84.44	78.02
<u>Fundamentals</u>				
1-5	89.92	97.58	86.67	92.31
1-6	58.82	72.58	46.67	73.63
1-17	55.56	54.84	56.67	41.76
1-18	89.92	95.16	87.78	90.11
1-23	31.93	42.74	18.89	12.09
2-5	33.61	47.58	30.00	19.78
2-23	26.89	16.13	23.33	9.89
<u>Measurement:</u>				
<u>Money</u>				
1-3	93.28	91.94	86.67	91.21
2-4	58.82	45.16	62.22	51.65
<u>Measurement:</u>				
<u>Time</u>				
1-4	55.96	57.26	47.78	57.14
2-3	26.05	31.45	18.87	25.27
2-21	63.87	68.55	37.78	64.84

Table B.1 Continued

<u>Topic and Item No.</u>	<u>I Pre- test</u>	<u>I Post- test</u>	<u>C Pre- test</u>	<u>C Post- test</u>
<u>Measurement:</u>				
length, liquid, and weight				
1-9	13.45	31.45	17.78	17.58
1-16	67.23	66.94	54.44	54.95
2-9	30.25	35.48	23.33	10.99
<u>Geometry</u>				
1-10	77.31	82.26	57.78	70.33
1-15	94.11	94.35	94.44	87.91
1-21	52.94	77.42	60.00	69.23
1-22	10.92	12.90	6.67	19.78
2-10	36.97	62.10	45.56	52.75
2-15	83.17	83.06	70.00	71.43
2-16	32.74	33.06	18.89	23.08
2-22	25.21	15.32	18.89	13.19
<u>Total</u>				
for				
<u>Test</u>				
	54.76	62.77	51.70	54.57

TABLE B.2

PERCENT OF CORRECT ANSWERS ON THE WISCONSIN CONTEMPORARY
ELEMENTARY MATHEMATICS TEST, GRADE 6

Treatments LM and L

<u>Topic and Item No.</u>	<u>LM Pre- test</u>	<u>LM Post- test</u>	<u>L Pre- test</u>	<u>L Post- test</u>
<u>Fractions</u>				
1-23	52.50	85.83	65.22	83.73
1-29	48.33	40.00	32.85	40.67
2-12	51.67	61.67	56.04	44.98
2-19	48.33	79.17	39.61	62.20
2-26	43.33	42.50	39.61	34.45
2-30	27.50	22.50	23.67	34.93
<u>Addition</u>				
1-19	49.17	64.17	33.33	39.23
1-20	70.83	86.67	82.13	76.56
2-1	80.83	90.00	82.61	92.34
2-7	65.00	50.83	52.17	57.89
2-8	69.17	76.67	74.40	66.51
<u>Subtraction</u>				
1-1	77.50	72.50	71.01	77.99
1-8	70.83	87.50	73.43	80.38
<u>Multiplication</u>				
1-2	21.67	58.33	21.67	37.80
1-7	70.83	85.00	79.23	79.90
1-26	75.83	81.67	85.51	84.21
2-2	90.00	96.67	98.07	93.27
2-14	45.83	55.00	52.66	47.37
2-20	60.00	50.83	48.31	48.80

Table B.2 Continued

<u>Topic and Item No.</u>	<u>LM Pre- test</u>	<u>LM Post- test</u>	<u>L Pre- test</u>	<u>L Post- test</u>
<u>Division</u>				
1-13	60.00	66.67	45.41	45.50
1-14	65.83	79.17	65.70	75.60
1-25	41.67	31.67	39.61	40.67
2-25	51.67	57.50	57.00	58.81
<u>Place Value</u>				
1-6	80.00	80.83	83.09	84.21
2-5	97.50	99.17	94.69	95.69
2-6	83.33	77.50	84.06	88.04
2-23	32.50	40.83	29.95	39.23
2-29	43.33	38.33	38.16	52.63
<u>Fundamentals</u>				
1-5	89.17	88.33	89.86	90.43
1-12	80.83	87.50	91.79	93.30
1-18	52.50	56.67	52.66	56.94
<u>Measurement</u>				
1-3	84.17	85.83	81.16	84.21
2-15	76.67	90.83	87.92	93.78
<u>Length</u>				
1-9	83.33	86.67	85.51	88.52
1-27	34.17	35.00	32.37	27.27
2-9	75.83	79.17	71.98	71.77
<u>Area</u>				
1-10	50.83	59.17	44.44	53.11
1-28	59.17	81.67	72.46	82.30
2-3	64.17	73.33	76.33	70.33
2-16	28.33	35.00	21.74	22.01
2-28	47.50	59.17	53.14	61.24

Table B.2 Continued

<u>Topic and Item No.</u>	<u>LM Pre- test</u>	<u>LM Post- test</u>	<u>L Pre- test</u>	<u>L Post- test</u>
<u>Geometry:</u>				
<u>Angles</u>				
1-4	67.50	74.17	59.42	59.81
1-15	56.67	62.50	42.51	48.33
1-21	53.33	50.83	54.59	55.50
1-22	63.33	53.33	43.96	50.72
<u>Geometry</u>				
1-16	58.33	61.67	56.52	51.20
2-4	31.67	50.00	44.44	55.98
2-10	41.67	63.33	61.35	69.38
2-21	58.33	70.83	65.22	70.81
2-22	21.67	42.50	42.51	41.15
2-27	7.50	21.67	7.25	7.66
<u>Sets</u>				
1-11	85.00	87.50	81.16	91.87
1-17	92.50	96.67	94.69	94.26
1-24	48.33	45.83	54.11	57.89
1-30	50.83	65.00	67.63	77.03
2-11	94.17	92.50	94.20	93.30
2-17	87.50	85.83	85.02	85.17
2-18	80.83	88.33	84.54	86.12
2-24	17.50	28.33	52.66	53.11
<u>Total on</u>				
<u>Test</u>	62.18	68.42	62.96	66.70

TABLE B.2 (continued)
Treatments I and C

<u>Topic and Item No.</u>	<u>I Pre- test</u>	<u>I Post- test</u>	<u>C Pre- test</u>	<u>C Post- test</u>
<u>Fractions</u>				
1-23	76.92	72.90	65.64	89.44
1-29	30.13	23.23	36.81	37.27
2-12	50.00	48.39	66.26	65.84
2-19	66.67	45.81	47.24	78.88
2-26	25.64	35.48	28.83	38.51
2-30	19.23	36.77	21.47	26.09
<u>Addition</u>				
1-19	25.00	35.48	41.10	44.10
1-20	69.23	73.55	79.14	80.12
2-1	74.36	81.94	65.03	90.06
2-7	44.87	34.84	52.76	51.55
2-8	56.41	46.45	65.03	72.67
2-13	75.00	78.06	84.05	90.68
<u>Subtraction</u>				
1-1	66.03	79.35	71.17	53.42
1-8	57.05	50.97	65.03	73.24
<u>Multiplication</u>				
1-2	21.79	62.58	22.70	45.96
1-7	58.97	76.13	74.85	78.16
1-26	63.46	52.26	88.96	81.37
2-2	81.41	89.68	92.64	92.55
2-14	41.67	39.35	55.21	54.04
2-20	48.08	27.74	80.37	83.23

Table B.2 Continued

<u>Topic and Item No.</u>	<u>I Pre- test</u>	<u>I Post- test</u>	<u>C Pre- test</u>	<u>C Post- test</u>
<u>Division</u>				
1-13	35.90	61.94	56.44	56.52
1-14	43.59	54.19	69.33	71.43
1-25	34.62	23.87	36.20	55.90
2-25	35.26	39.35	48.47	40.99
<u>Place Value</u>				
1-6	57.69	72.26	78.53	79.50
2-5	87.18	85.16	92.02	94.41
2-6	75.00	71.61	83.44	83.83
2-23	14.74	36.77	36.81	39.75
2-29	44.23	26.45	43.56	50.31
<u>Fundamentals</u>				
1-5	64.10	69.03	80.98	91.30
1-12	66.02	76.13	87.12	87.58
1-18	16.67	45.48	71.17	57.08
<u>Measurement</u>				
1-3	74.36	80.65	54.60	80.12
2-15	87.18	84.52	88.96	89.44
<u>Length</u>				
1-9	76.92	61.29	81.60	86.34
1-27	20.51	31.61	33.74	45.34
2-99	82.82	61.29	70.55	77.02
<u>Area</u>				
1-10	58.33	53.55	52.15	51.55
1-28	64.10	69.68	85.28	89.44
2-33	56.41	56.77	68.71	75.16
2-16	9.62	21.29	38.04	36.65
2-28	49.95	47.74	59.51	49.07

Table B. 2 Continued

<u>Topic and Item No.</u>	<u>I Pre- test</u>	<u>I Post- test</u>	<u>C Pre- test</u>	<u>C Post- test</u>
<u>Geometry</u>				
<u>Angles</u>				
1-4	43.59	52.90	52.76	62.11
1-15	28.85	45.16	43.56	47.83
1-21	50.64	47.74	32.52	40.37
1-22	28.85	37.42	42.94	47.20
<u>Geometry</u>				
1-16	40.38	34.19	52.15	49.07
2-4	38.46	44.52	44.79	41.61
2-10	75.64	80.65	43.56	60.25
2-21	44.87	49.03	57.67	59.63
2-22	34.62	38.71	33.13	45.34
2-27	8.98	9.68	7.98	11.80
<u>Sets</u>				
1-11	80.13	85.16	87.12	92.55
1-17	85.26	93.55	92.02	93.17
1-24	23.72	25.81	53.37	44.72
1-30	57.05	50.32	65.03	75.16
2-11	84.62	88.39	92.02	92.55
2-17	77.56	86.45	84.66	88.82
2-18	73.72	76.13	79.75	91.30
2-24	17.95	36.13	55.21	48.45
Total on <u>Test</u>	61.45	56.33	59.77	65.76

TABLE B.3
SUMMARY OF MEAN SCORES OF BOYS AND GIRLS ON
REASONING AND MATHEMATICS TESTS

GRADE 3							
<u>Group</u>	<u>Pre- Class</u>	<u>Pre- Cond.</u>	<u>Pre- Wisc.</u>	<u>Post Class</u>	<u>Post Cond.</u>	<u>Post Wisc.</u>	<u>Mean Total</u>
LM Boys	12.98	9.47	28.81	15.32	11.20	31.84	18.27
LM Girls	12.62	9.81	28.33	13.96	11.32	32.22	18.04
L Boys	12.63	9.85	28.95	13.61	10.80	31.98	17.97
L Girls	12.56	10.33	26.94	13.06	11.00	30.27	17.36
I Boys	11.36	9.22	27.03	12.15	9.83	29.41	16.50
I Girls	12.07	9.64	27.44	12.56	10.33	31.69	17.12
C Boys	11.33	9.33	25.91	12.54	9.56	25.67	15.73
C Girls	11.52	9.12	25.86	12.93	10.00	27.38	16.14
GRADE 6							
LM Boys	13.81	10.73	33.71	14.98	12.25	38.24	20.62
LM Girls	15.31	11.11	36.25	16.41	12.62	41.16	22.14
L Boys	15.28	10.12	36.13	16.48	12.94	38.95	21.65
L Girls	14.84	11.10	36.30	16.28	12.87	39.25	21.75
I Boys	13.75	9.25	29.87	14.40	10.48	32.62	18.40
I Girls	14.93	10.16	32.70	14.89	11.21	35.28	19.86
C Boys	15.11	10.65	37.14	15.18	11.89	38.92	21.48
C Girls	15.05	10.68	37.00	15.80	11.45	39.98	21.66

TABLE B.4
SUMMARY OF MEAN SCORES OF BOYS AND GIRLS ON
ATTITUDE, I. Q., LOGIC FINAL AND RETENTION
GRADE 3

<u>Group</u>	<u>Attitude</u>	<u>I. Q.</u>	<u>Logic Final</u>	<u>Logic Retention</u>
LM Boys	26.16	111.89	33.84	32.82
LM Girls	26.49	110.31	33.98	32.02
L Boys	23.10	111.25	32.46	30.53
L Girls	25.85	110.33	32.79	32.17
I Boys	25.80	108.37		
I Girls	27.45	108.34		
C Boys	27.49	109.18		
C Girls	27.67	106.31		

GRADE 6

LM Boys	24.54	108.86	33.61	34.56
LM Girls	27.13	113.15	35.93	36.70
L Boys	26.26	115.91	37.94	39.11
L Girls	27.29	117.82	38.45	40.10
I Boys	28.35	113.00		
I Girls	28.00	119.39		
C Boys	27.92	113.42		
C Girls	27.64	115.62		

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