DOCTORAL DISSERTATION SERIES

PUBLICATION: 5679

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University of Michigan

TITLE: THE THERMISTOR AS A DEVICE FOR THE MEASUREMENT OF VELOCITY IN FLOWING WATER

University Microfilms, Ann Arbor, Michigan, 1953

THE THERMISTOR AS A DEVICE FOR THE MEASUREMENT OF VELOCITY IN FLOWING WATER

by
Edwin E. Henry, Jr.

A dissertation submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy in the
University of Michigan
1953

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-Abstract-

THE THERMISTOR AS A DEVICE FOR THE MEASUREMENT OF VELOCITY IN FLOWING WATER

by
Edwin E. Henry, Jr.

The thermistor is a semiconductor whose electrical resistance changes markedly with a small variation in its temperature. When used to measure water velocity, heat is introduced electrically and is transferred from the thermistor to the water. The thermistor temperature depends on the amount of heat introduced, the size and shape of the thermistor, and the water temperature and velocity. It is the purpose of this thesis to study, by a heat transfer approach, the operation of the thermistor as a flow measuring device. Particular attention is given to the various calibration curves that are obtained when the thermistor is connected to different basic electrical measurement circuits. Values for physical quantities given are for the specific thermistors tested; however, the concepts shown should be generally valid.

In Chapter I, a relationship expressing the heat transfer from the thermistor to the water is established. Physical quantities are measured by conventional means with the exception of the thermistor temperature. The temperature of the thermistor is found by comparing

its electrical pulse impedance when it is heated by d c power and subjected to flow with its pulse impedance when it is immersed in static water at known temperatures. The pulses used are of short duration and occur at a frequency low enough so that they cannot cause the thermistor temperature to rise above the water temperature. The relationship which expresses the heat transfer from the thermistor to the water is then formed by correlating the data through the use of dimensionless ratio equations.

In Chapter II, this relationship is used to construct a graphical model of the thermistor velocity meter. From this model, various calibration curves are formed. The errors to be expected with each are discussed.

Chapter III describes the measurement of point-by-point velocities of a flow situation in a hydraulic flume.

The first appendix is a manual for those who wish to use thermistors to measure flow, without major concern as to the origin of the response curves. The second appendix reviews other methods of flow measurement.

This study has had three significant results. The first is the acquirement of an overall understanding of the thermistor as a flow measuring device. The second is a description of basic measurement circuits and their manner of use. The third is a suggestion of a method of study of other thermal indicators for the measurement of velocity in various fluids.

PREFACE

The project of finding an instrument capable of measuring fluid velocities at a set position in the flow was first suggested to the author by Dr. W.W. Hagerty of the Engineering Mechanics Department at the College of Engineering, University of Michigan. The thermistor was later decided upon. It is hoped that a small contribution has been made where there was a need for such a device.

There are several existing methods for the measurement of fluid velocities. Among these are:(1) the pitot tube, (2) the hot wire anemometer, (3) the electromagnetic method, and (4) photographic techniques. Fluid viscosity and capillary action have made very small pitot tubes impractical. The hot-wire anemometer, while successful in gases, has not been used widely with water, since it burns out when surrounded by an air bubble or foreign matter. The electromagnetic method has met with some success, but there are situations where it cannot be used. Photographic techniques may be cumbersome.

The thermistor is an electrical resistor with a high thermal coefficient of resistivity. Its use as a velocity meter is based on the same principle as the hot-wire anemometer. The purpose of this thesis is to gain a better understanding of it when it is used to measure fluid velocities. For this work, all thermistors must be calibrated, and the values given for physical quantities are for only the specific

ones tested. However, the relationships shown should hold true for all.

The author wishes to acknowledge the aid of all who have assisted him during the course of this investigation; in particular Dr. Hagerty for suggesting the problem, Professor W.G. Dow for suggesting the pulse method and his many hours of guidance, Mr. G. Hok for his helpful discussions, Mr. Milo Kaufman for help in constructing apparatus and for photographic assistance, Mr. Carl Bonnewell for use of electrical equipment, and last, but not least, his wife Mrs. Myra Henry, who has spent many hours reading manuscript.

TABLE OF CONTENTS

	Page
PREFACE	ii
LIST OF TABLES	vi
LIST OF ILLUSTRATIONS	vii
INTRODUCTORY OUTLINE	1
Chapter	
I. EXPERIMENTAL TECHNIQUE AND DATA	7
Preparation of Thermistor	7
Determination of Water Velocity	10
Determination of Thermistor Temperature (Electrical Pulse Technique)	11
Response of the Oscilloscope	21
Over-all Accuracy	23
The Testing of a Thermistor Probe Under Flow Conditions	28
Calculation of Data	30
Development of Dimensionless Ratios for the Analysis of Data	34
Correlation of Experimental Data through the Use of Dimensionless Parameters	38
An Analysis of Errors	52
II. A MATHEMATICAL MODEL OF THE THERMISTOR IN FLUID FLOW	65
The Relationships Between the Current Flowing through the Thermistor and the Velocity at Different Water Temperatures When the Voltage is Held Constant	80
The Relationships between the Voltage across the Thermistor and the Velocity at Different Water Temperature When the Current is Constant	86
The Relationships between the Power Input to the Thermistor and the Velocity and Temperature of the Water When the Thermistor Temperature is Held Constant	92
A Wheatstone Bridge with Compensation By Another Thermistor for Change in Water Temperature	103
III. VELOCITY MEASUREMENTS	110
IV. SUMMARY OF THESIS	120

TABLE OF CONTENTS, cont.

	Page
APPENDIX A. A SHORT MANUAL OF OPERATION	136
APPENDIX B. A LITERATURE REVIEW OF OTHER METHODS . . .	153
BIBLIOGRAPHY	168

LIST OF TABLES

Table	Description	Page
I.	Determination of Film Coefficient for Thermistor No. 1	125
II.	Determination of Film Coefficient for Thermistor No. 2	127
III.	Evaluation of Dimensionless Ratios for Thermistor No. 1	128
IV.	Evaluation of Dimensionless Ratios for Thermistor No. 2	130
V.	Determination of Constant, k''' for Thermistor No. 1	131
VI.	Determination of Constant, k''' for Thermistor No. 2	132
VII.	Determination of Constant, k''''' for Thermistor No. 1	133
VIII.	Determination of Constant, k''''' for Thermistor No. 2	134
IX.	Velocity Traverse of Hydraulic Flume . . .	135

LIST OF ILLUSTRATIONS

Figure No.	Description	Page No.
1.	Measuring Probe	8
2.	Preparation of Western Electric Thermistors and Cutaway Assembly	9
3.	Cutaway Section of Calibrating Apparatus .	12
4.	Circuit Diagram of Pulsing Unit	14
5.	Dumont Type 279 Cathode Ray Oscilloscope .	15
6.	Narrow Width Pulse	16
7.	Medium Width Pulse	17
8.	Wide Pulse	18
9.	Comparator Microscope	20
10.	Relative Amplification of Voltages of Different Frequencies as compared to a Voltage of 1 Kilocycle by the Dumont Type 279 Oscilloscope	22
11.	Voltage-Current Curve for 100,000 ohm Resistor by Pulse Method	24
12.	Voltage-Current Curve for Thermistor in 34.5°, 69.5°, and 106.8°F. Still Water.	26
13.	Impedance of Thermistor by Pulse Method vs. Temperature	27
14.	Circuit Used for Measuring Thermistor Temperature	29
15a.	Voltage-Current Curves of Thermistor No. 2 in flow	31
15b.	Ibid.	32
16.	Equipment Used to Determine Thermistor Temperature	33
17.	Distribution of Observations for Plotting hD/k vs. $EI/\rho v^3 D^2$	41

Figure No.	Description	Page No.
18.	Plot of hD/k vs. $EI/\rho v^3D^2$ for Thermistor No. 1	42
19.	Distribution of Observations for Plotting hD/k vs. $\rho vD/\mu$	44
20.	Plot of hD/k vs. $\rho vD/\mu$ for Thermistor No. 1	45
21.	Percentage of Observations Within Percentage Deviation from Mean Value of k'''	47
22.	Plot of $h/\rho vC_p$ vs. $EI/\rho v^3D^2$	49
23.	Percentage of Observations Within Percentage Deviation from Mean Value of k''''' . . .	51
24.	Plot of Kinematic Viscosity vs. Temperature	54
25.	Plot of Thermal Conductivity vs. Temperature of Water	55
26.	Two Dimensional Flow Patterns	61
27.	Two Dimensional Flow Patterns	62
28a.	Power Diagram For Thermistor in Fluid Flow-Water Temperature 45°F.	67
28b.	Ibid.	68
28c.	Ibid.	69
29a.	Power Diagram for Thermistor in Fluid Flow-Water Temperature 52°F.	70
29b.	Ibid.	71
29c.	Ibid.	72
30a.	Power Diagram for Thermistor in Fluid Flow-Water Temperature 60°F.	73
30b.	Ibid.	74
31.	Direct Current Resistance of Thermistor No. 2	76
32.	Voltage-Current Relationships for a Thermistor under a Flow of 3 ft./sec. . .	78

Figure No.	Description	Page No.
33.	Voltage-Current Relationships for a Thermistor under a Flow of 9 ft./sec. .	79
34.	Current Flowing in Thermistor with 10 Volts Applied	81
35.	Current Flowing in Thermistor with 25 Volts Applied	82
36.	Ibid.	83
37.	Relationships between Velocity and Water Temperature with Constant Power	84
38.	Ibid.	85
39.	Voltage Across Thermistor with Constant Current	87
40.	Ibid.	89
41.	Relationships between Velocity and Water Temperature with Constant Power	90
42.	Power Required to Maintain Thermistor Temperature with Water of 45°F.	93
43.	Ibid., 52°F.	94
44.	Ibid., 60°F.	95
45.	Power Required to Maintain Temperature Difference Between Thermistor and Water of 3° F.	97
46.	Ibid., 4° F.	98
47.	Relationships Between Velocity and Water Temperature with Temperature Difference Between Thermistor and Water of 3° F. .	99
48.	Relationships Between Velocity and Water Temperature with Temperature Difference Between Thermistor and Water of 4° F. .	101
49.	Power Required to Maintain Thermistor at 60° F.	102
50.	Value of Variable Resistor to Balance Wheatstone Bridge	105
51.	Water Temperature-Velocity Relationships that Produce a Constant Value of Variable Resistor	106

Figure No.	Description	Page No.
52.	Resistance-Temperature Relationship of a Thermistor Calculated to compensate for Water Temperature Change at a Velocity of 1 ft./sec.	108
53.	Value of a Variable Resistor to Balance Wheatstone Bridge	109
54.	Hydraulic Flume Used For Velocity Measurements	111
55.	Series DC Circuit Used With Thermistors. .	112
56.	Calibration Curve for a No. 14A Thermistor	113
57.	The Probe Used to Determine the Velocity Traverse in Hydraulic Flume	114
58.	Velocity Distribution of a Flow Situation in a Hydraulic Flume as Determined by a No. 14A Thermistor	116
59.	Current Flowing in an Iron Oxide Thermistor with 30^{V} Applied	122
60.	Velocity Distribution of a Flow Situation in Hydraulic Flume as Determined by an Iron Oxide Thermistor	123
A-1.	Power Diagram for a Thermistor in Fluid Flow	137
A-2.	Current Flowing in an Iron Oxide Thermistor	139
A-3.	Cutaway Section of Calibrating Apparatus .	142
A-4.	Power Required to Maintain Thermistor Temperature	143
A-5.	Power Required to Maintain Thermistor Temperature of 60° F.	144
A-6.	Voltage Across Thermistor with Constant Current	146
A-7.	Relationships Between Velocity and Water Temperature with Constant Power	147
A-8.	Value of Variable Resistor to Balance Wheatstone Bridge	148

Figure No.	Description	Page No.
A- 9.	Water Temperature-Velocity Relationships that Produce a Constant Value of Variable Resistor in Wheatstone Bridge	149
A-10.	Resistance-Temperature Relationship of a Thermistor Calculated to Compensate for Water Temperature Change at a Velocity of 1 ft./sec.	151
A-11.	Value of Variable Resistor to Balance Wheatstone Bridge	152
B- 1.	The Attachment at the Head of the Experimental Assembly with a Tube Through the Center to Inject Dye into the Water for Photographic Studies	163

INTRODUCTORY OUTLINE

A THERMISTOR AS A DEVICE FOR THE MEASUREMENT OF FLUID VELOCITIES

Thermistors [1,2,3,4] are electrical semiconductors and are usually composed of metallic oxides. Their unique property is that they exhibit a high temperature coefficient of resistivity, usually negative, which means that their electrical resistance changes considerably with a small change in temperature. If the resistance decreases as the temperature increases, the coefficient is negative.

Thermistors are formed from sintered powders and may be made in any desired shape, which for this problem is an advantage. Likewise they may be made the size of a pin head or possibly smaller. This is an advantage since any device introduced into a flow system must not disturb the system; otherwise the measurements, no matter how accurate, would be valueless. They can also be used as a portion of a surface, such as a ship model, where they will not disturb the flow stream.

The thermistor, like the hot-wire anemometer, operates on the principle that heat is imparted to the device electrically, and in turn, is transferred to the surrounding fluid. The temperature to which the thermistor must rise is a function of the water temperature and velocity, and the amount of

heat imparted. Also, the thermistor's resistance depends on its temperature. Thus, it is possible to eliminate the thermistor temperature, and find a correlation between the resistance of the thermistor and the velocity and temperature of the fluid. There are two possible sources of power (heat): (1) the electrical power, and (2) friction between the fluid and the thermistor surface. The latter is considered to be small for incompressible fluids, and will be neglected.

This thesis describes the various response or calibration curves that are obtained when the thermistor is used with various electrical measurement circuits. These curves are obtained from a power diagram, which shows the amount of power transferred in relation to the thermistor temperature when the water velocity and temperature are held constant, or are considered as parameters. The power input to the thermistor, which is a function of the applied voltage and the thermistor resistance, is also shown. Thus, this diagram becomes a mathematical model of the thermistor velocity meter. The presentation of the power diagram and the response curves is found in Chapter II.

Before this diagram can be constructed, it is necessary to determine the mathematical relationship that expresses the heat transfer from the thermistor to the water. This is found in Chapter I.

The basic heat transfer equation[5] that is used is

$$Q = h\,A(\Delta T)$$

where Q = the amount of heat transferred (BTU/hr.)

h = the film coefficient or overall coefficient of heat transfer from the thermistor to the water. This varies with different flow conditions and amounts of power transferred. (BTU/sq. ft./hr./°F.)

A = the area of the thermistor that is in contact with the water. (sq. ft.)

ΔT = the temperature difference between the surface of the thermistor and the main body of water that is flowing around the thermistor. (°F.)

In order to use this equation, it is necessary to determine the area and the temperature of the thermistor, the water velocity and temperature, and the amount of power applied. The thermistor temperature is found by comparing its electrical impedance when it is subjected to known conditions of flow and a known amount of power, with its impedance when it is surrounded with static water of different temperatures. The impedance is found through a voltage-current curve when an electrical pulse is applied to the thermistor. This pulse is of sufficiently short duration and is applied at a frequency low enough so that it, by itself, cannot cause the thermistor temperature to rise above the temperature of the surrounding water. The voltage-current curve is determined through a calibrated photograph of the pulse when it is displayed on a cathode ray oscilloscope.

Before testing a thermistor in flow, two conditions must be satisfied. First, the method and the equipment must accurately determine the impedance of the thermistor. The accuracy is found by substituting a known carbon-type resistor.

Secondly, the actual pulse applied must not cause the thermistor to rise above the temperature of the water. This condition is satisfied when the voltage-current curve is a straight line.

After these conditions are satisfied, the pulsing circuit may be connected in parallel with a dc battery circuit that is supplying the heating power. Data, sufficient to determine the film coefficient (h) with different flow conditions and different amounts of applied power, can then be collected.

The next task is to determine from the data the mathematical relationship that expresses the heat transfer from the thermistor to the water; that is, to determine how the film coefficient (h) varies. To determine this relationship rigorously, would involve, in addition to the heat transfer problem, an expression for the flow pattern of the water around the thermistor probe. A set of differential equations,[6] which when solved simultaneously present this flow pattern, exists. These equations have been solved in certain cases where simplifying assumptions have been made. However, no general solution exists.

Therefore another approach, known as the Principle of Similarity,[7] is used. This approach uses equations composed of dimensionless ratios. These ratios are not merely numbers, but can be expressed physically and are derived for each situation. More than one equation can usually be found with the same numbers or ratios, and in this particular study two equations are found. The data must be correlated with each equation in order to determine which represents the data more correctly.

In such a correlation procedure, the effect of one ratio upon the heat transfer coefficient (h), or the ratio containing this term, is studied. Other ratios should then be kept constant. This was not possible experimentally. The data are grouped, however, so that the other ratios vary only throughout a specified range, and the mathematical relationship which expresses the transfer of heat from the thermistor to the water, is shown.

The final section of Chapter I presents an analysis of the effect of an error in measuring the physical quantities on the mathematical expression as presented. Each portion of this outline is discussed in detail.

A technician, wishing to use this method, will proceed immediately to either one or a combination of the calibration curves which appear in Chapter II and are the result of this study. As this method depends on heat transfer from the thermistor to the flowing fluid around it, the calibration curves are studied by means of a heat transfer approach, in order that they may be fully understood, and their comparative merits explored. Also, a technique for future similar heat transfer studies is shown.

Chapter III describes the measurement of an actual flow situation. A velocity traverse was obtained of a situation in a flume, located in the Hydraulics Laboratory at the University of Michigan. Simultaneously, the flow was measured with a calibrated orifice, which was located in the feeder pipe to the flume. The average velocity, as determined by the thermistor, was compared with the average velocity as determined by the

orifice. The velocity traverse obtained is shown in graphical form in Fig. 58, page 116.

Chapter IV is a short summary, suggesting the type of electrical measurement circuit which should be used under various hydraulic conditions. Recommendations for future research are also given.

The Appendix is divided into two parts: (1) a brief manual of operation, and (2) a summary of the methods for the measurement of flow other than by the thermistor. The manual of operation is intended for those who wish to use the thermistor, but are not interested in the development of the various response or calibration curves.

In summary, this thesis presents the thermistor as a device by means of which research personnel may determine fluid velocities at any fixed position. Through a study of the heat transfer from the thermistor to the fluid, there are presented the different electrical responses when thermistors are used as portions of different electrical measurement systems. The hydraulic conditions, under which each measurement system is advantageous, are discussed. Finally, an actual measurement of a hydraulic flow situation is described.

CHAPTER I.

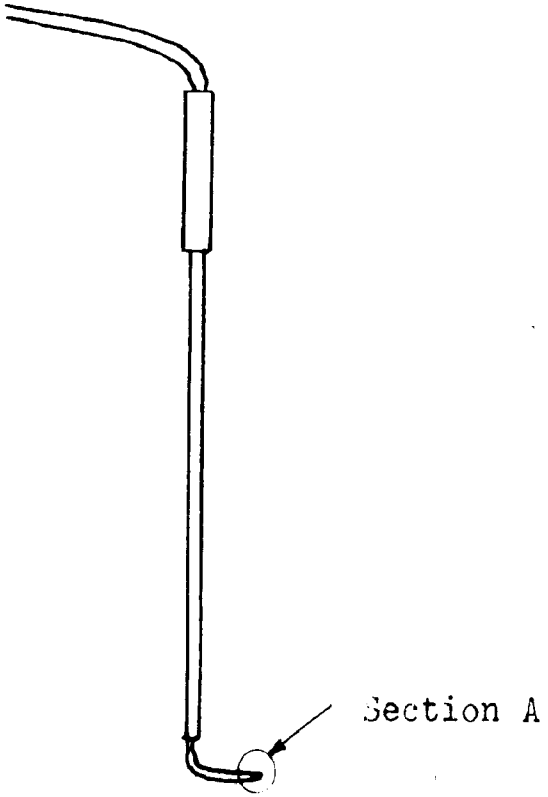
EXPERIMENTAL TECHNIQUE AND DATA.

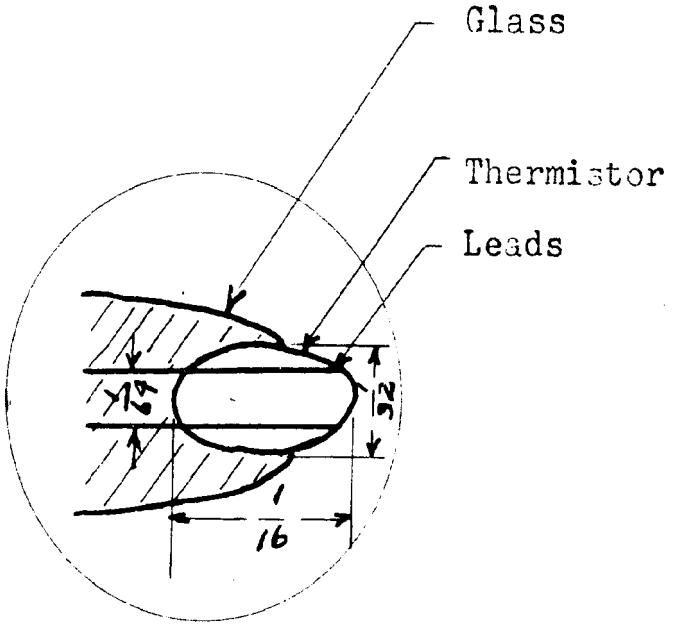
The thermistor chosen for this work was a Western Electric Co. No. 14A thermistor which is composed of a binary combination of nickel and manganese oxides. When received, it was sintered in a sheath of pyrex glass about 2 inches long and 1/16 inch in diameter. It was exposed by dipping the sheath into a solution of hydrofluoric acid and etching the glass away. Small pieces of lead wires, extending from the exposed bead, were then carefully removed and the wire ends lacquered to prevent any electrical conduction through the water. The remainder of the exposed thermistor had to be free from lacquer. The glass was softened about 1 inch to the rear of the thermistor and bent to form an "L" shaped probe. The resulting probe is shown in Figs. 1 and 2.

As mentioned in the Introductory Outline, the water temperature and velocity, the thermistor temperature, and the electrical power applied to the thermistor must be measured in order to determine the heat transfer from the thermistor to the water. The electrical quantities were measured by conventional means. The water temperature was measured by a mercury thermometer. The determination of the water velocity and the thermistor temperature are more complex and will be discussed.

Fig. 1.

Measuring Probe





Section A

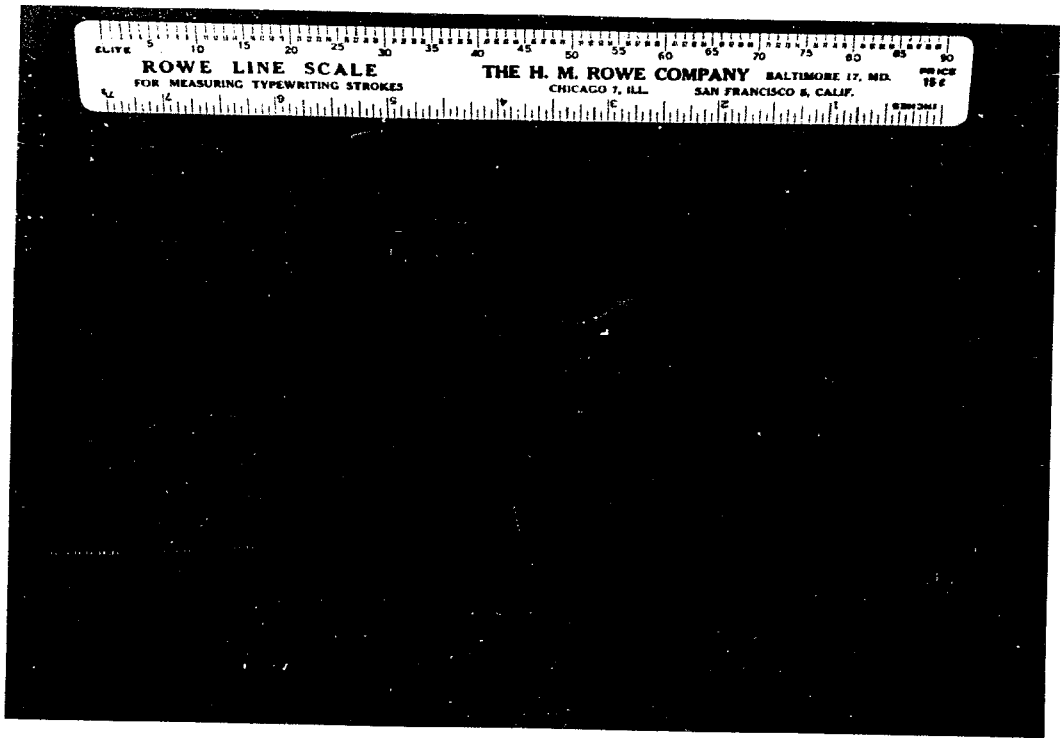


Fig. 2. This Figure shows the Preparation of Western Electric Thermistors and a Cutaway Assembly.

Determination of Water Velocity.

The hydraulic calibration apparatus was built around the observation that at a well rounded constriction, the velocity profile across the pipe of smaller diameter is nearly uniform.[8] By determining the quantity of fluid that has flowed during a unit of time, and the area of the constriction, the velocity was found by taking the quotient of the two. This is considered to be one of the most accurate means of calibration.

The basis for this method of calibration lies in the theoretical work of Boussinesq and Prandtl, and the experimental work of J. Nikuradse. Nikuradse[9] found that for a well rounded entrance the assumption of a uniform flow was valid until x/rR was equal to approximately 0.04. Here x equals the distance downstream from the entrance, r equals the radius of the pipe and R equals Reynolds' number. This applies to laminar flow. For turbulent flow, Nikuradse found that when x/d was 2.5 the maximum velocity exceeded the average velocity by approximately 10%. Here, x is as previously mentioned and d is the diameter of the pipe.

The advantage of this calibration method is that it applies not only for laminar flow, but also for turbulent flow.

Nikuradse's work was based on a large reservoir of fluid supplying the pipe with the well rounded entrance, and he considered that the fluid was motionless in the reservoir. The diameter of the supply pipe was approximately twice as large as that of the constriction, so that the average velocity of flow in the feeder pipe was approximately one-fourth of

the velocity in the constriction. Thus, it was felt that the conditions of Nikuradse's experiment would apply in this case.

The actual piece containing the constriction was made of Lucite and the edges were well rounded and polished. It is shown in a cutaway photograph in Fig. 2 and in a line diagram in Fig. 3.

Determination of the Thermistor Temperature.

To determine the thermistor temperature, its impedance when it was subjected to flow was compared with its impedance when it was subjected to static water of various temperatures. This impedance was found by means of a voltage-current curve. Thus, for measurement purposes, a voltage had to be applied to the thermistor, without applying enough power to raise its temperature above the temperature of the surrounding water. A means of doing this is an electrical pulse. In order to satisfy this requirement the pulse voltage-current curve must be a straight line. The next step was to find the duration and frequency of a satisfactory pulse.

A pulsing circuit,[10] as in Fig. 4, was set up. It consisted of a multivibrator, producing a negative square pulse. Its output was connected through a differentiating network to a unishot vibrator, which in turn produced at each triggering a single square pulse of variable width. This pulse output drove a cathode follower stage through a variable grid resistance which permitted the pulse voltage to be adjusted.

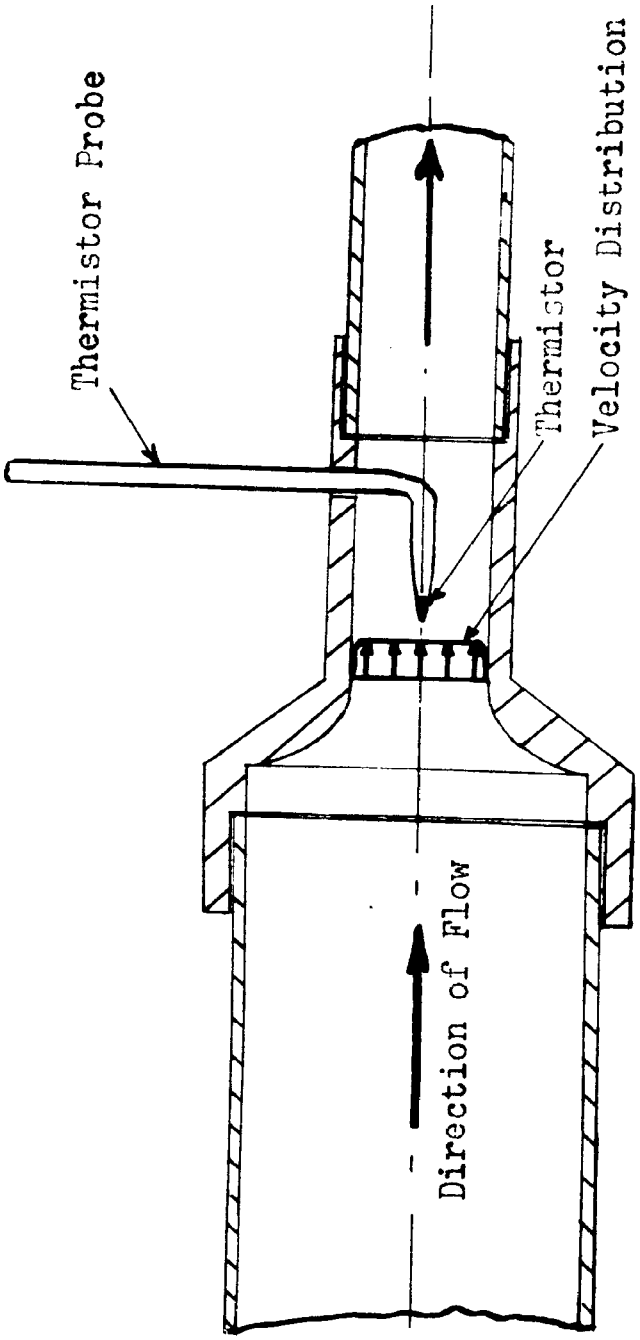


Fig. 3.

Cutaway Section of Calibrating Apparatus
(Not drawn to scale)

The negative "spike" from the differentiating network triggered the sweep circuit of a Dumont dual beam cathode ray oscilloscope type 279, while the positive "spike" activated the unishot vibrator. The visual pulse was centered on the oscilloscope screen by varying the width of the multivibrator pulse. Two independent pulses could be displayed simultaneously.

The thermistor was connected in series with a resistor of known value, and this combination was connected to the output terminals of the pulsing unit. The total pulse unit voltage drove one of the vertical amplifiers of the oscilloscope, while the voltage across the known series resistor drove the other. The voltage across the thermistor was the algebraic difference between these two voltages. A direct voltage calibration unit was provided integral with the oscilloscope.

A thermistor probe was set in a vacuum bottle, filled with water at various temperatures, and the pulse voltage was varied from zero to 90 volts (approximately) for three different pulse widths. Photographic pictures of the pulse images were taken for various voltages at each pulse width and at each temperature of the water with a Dumont cathode ray oscilloscope camera. Calibration pictures were taken between each set. Sample pictures of pulse voltages are shown in Figs. 6, 7, and 8. The upper image is the total voltage pulse while the lower is the voltage across the series resistor.

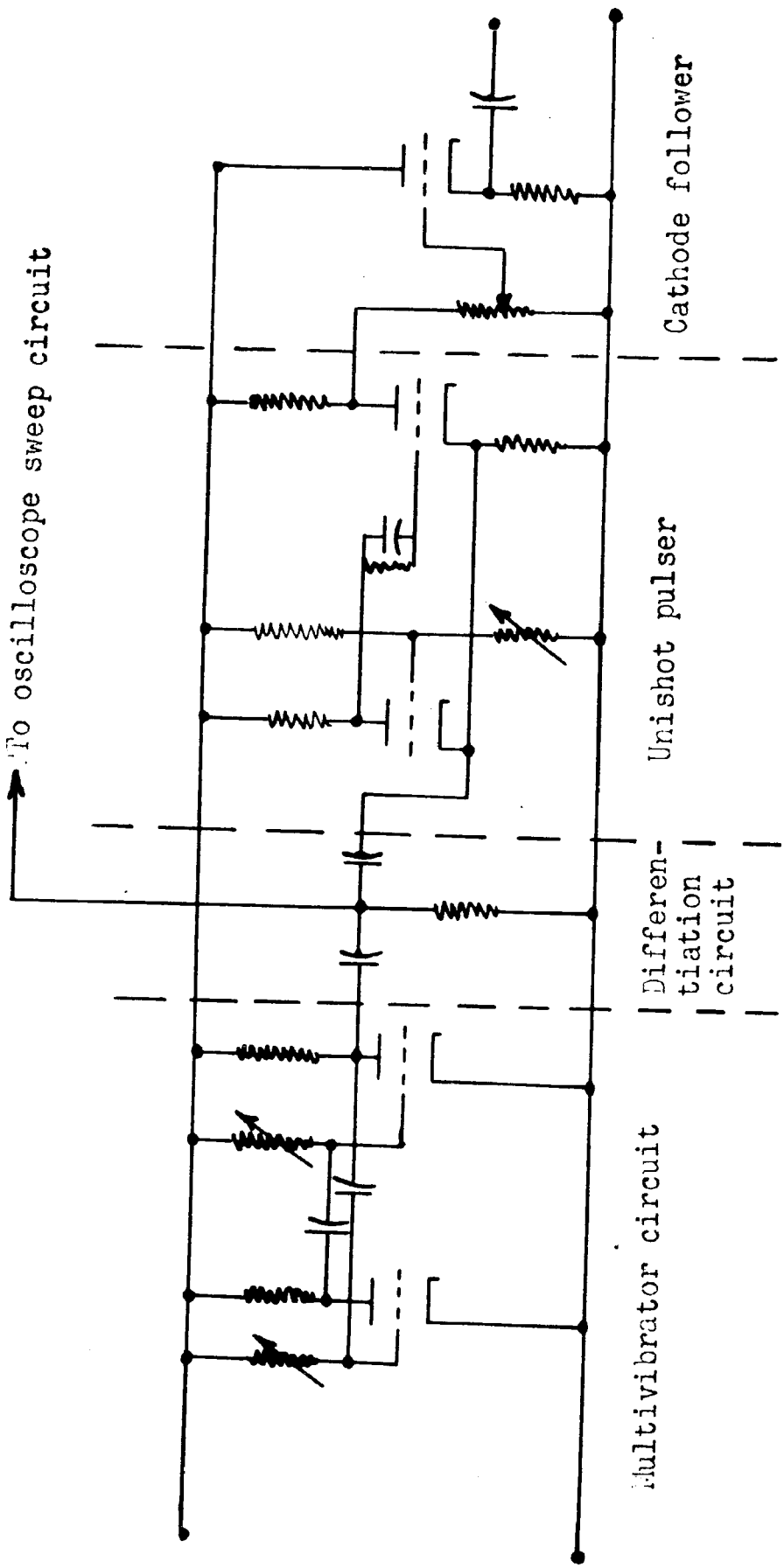


Fig. 4.

Circuit Diagram
of
Pulsing Unit

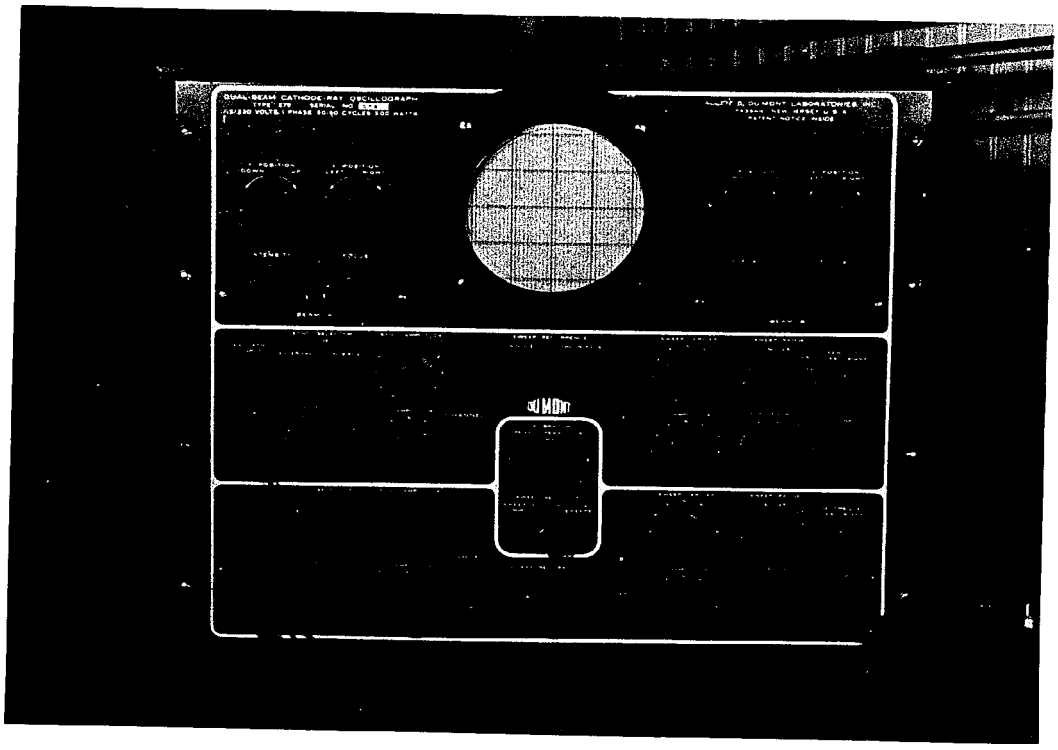


Fig. 5. Dumont Type 279 Cathode Ray Oscilloscope.

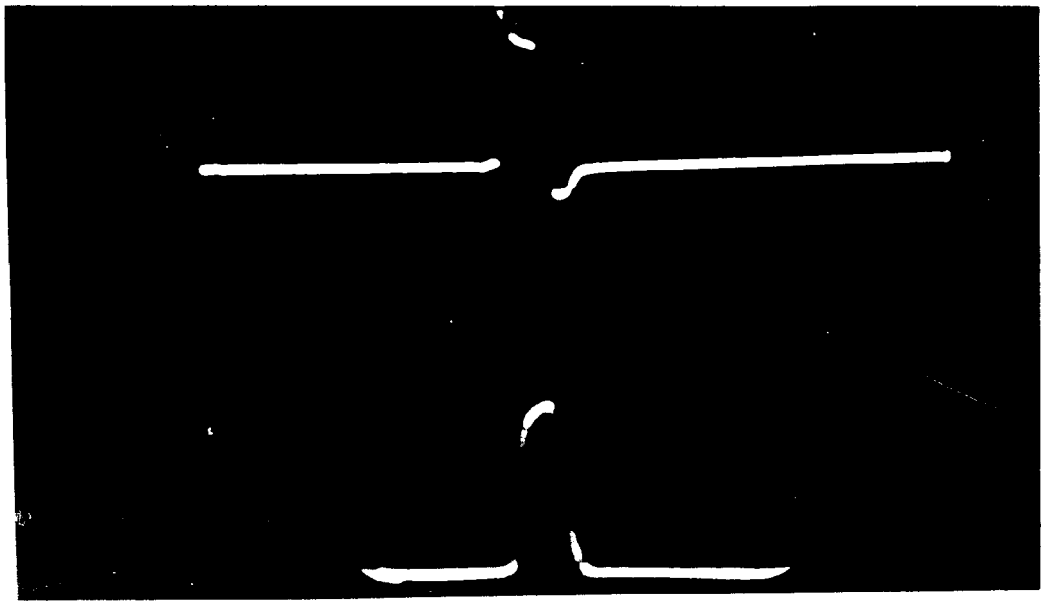


Fig. 6. Narrow Pulse

15.6 Microseconds Duration

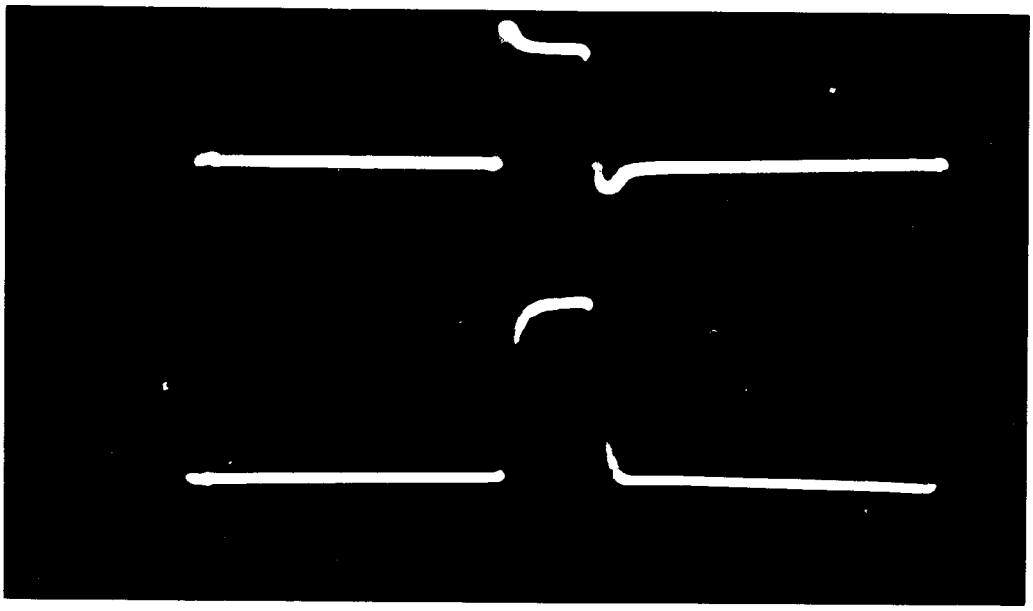


Fig. 7. Medium Pulse

22 Microseconds Duration

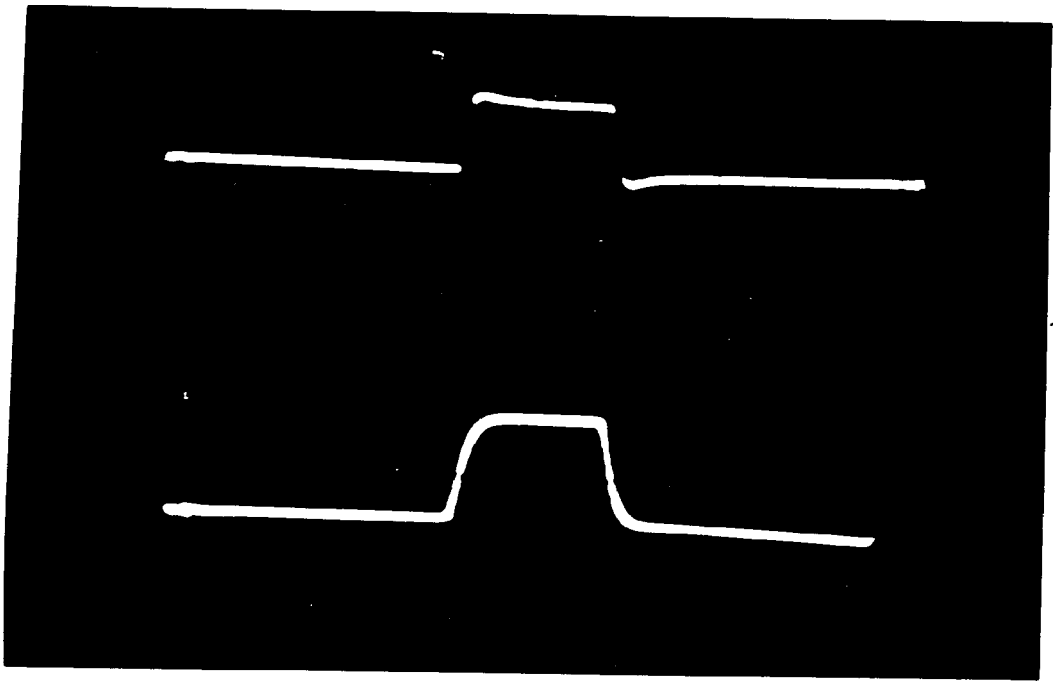


Fig. 8. Wide Pulse

28 Microseconds Duration

The pulse images on the film were measured by a comparator microscope, which was graduated to read to 0.001 mm.. This was mounted above the film track of a photographic contact printer. The film was held flat to the glass surface.

A sample pulse is shown below.



Total Pulse Current Pulse

The thermistor may have been capacitative, and some capacitance between the leads and at the lacquered tips undoubtedly was present. The current pulse should contain some current used to charge these capacitances. For this reason, the point B was used as a reading point.

When the current dropped from some value to zero at the trailing edge of the pulse, there was a dip below the reference line on the scope due to the discharging of the capacitances. See Figs. 6, 7, and 8. For this reason the point A was taken as a measuring point for the zero value.

The duration of the pulse was calibrated by comparing the width of the pulse with the width or period of a wave of known frequency from a Hewlett-Packard oscillator (20kc.). The three pulse widths used were 15.6, 21.8, and 28.0 microseconds. The frequency was about 20 cps..

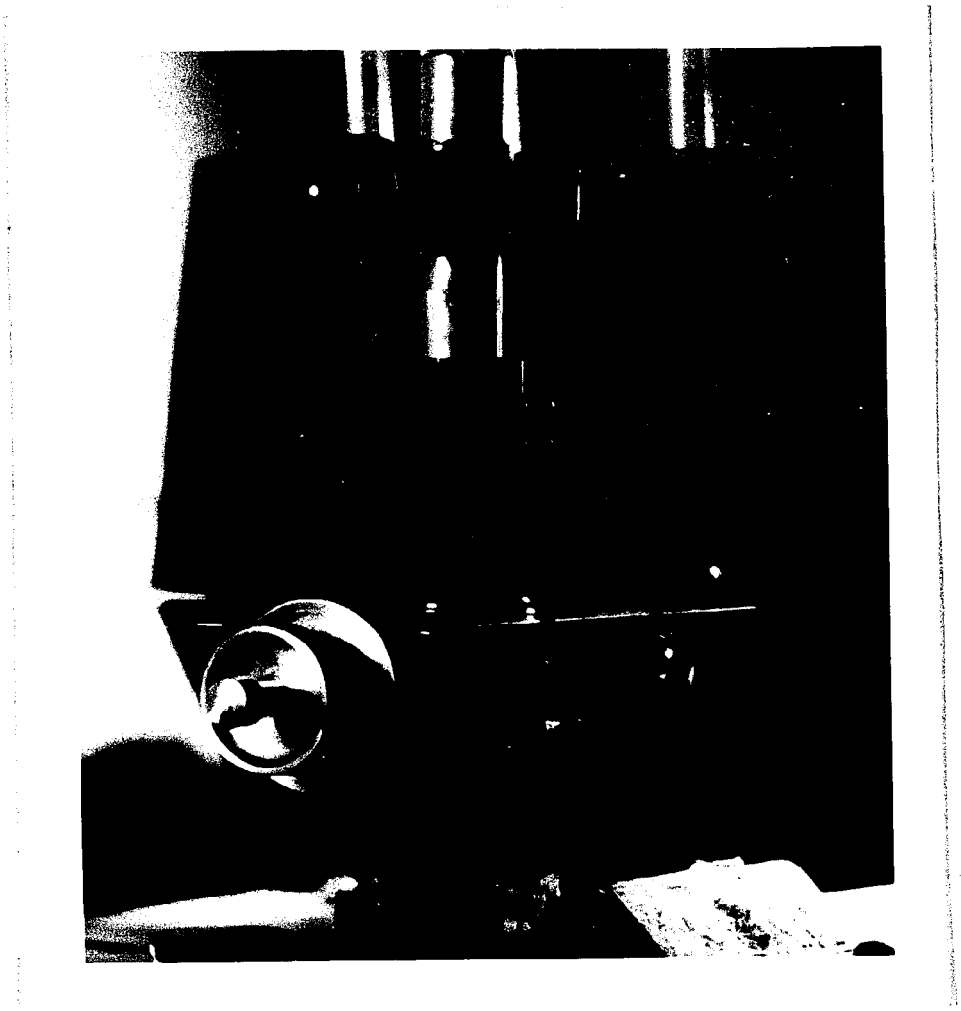


Fig. 9. This Figure Shows The Microscope Used To Measure Pulses From The Film Record.

Response of the Oscilloscope.

Consideration should be given to the response of the oscilloscope to a pulse, since a pulse is composed of many frequencies with varying voltages. Inasmuch as the impedance of inductors and capacitators varies with frequency, voltages of different frequencies will not be amplified alike by the amplifiers in the oscilloscope.

Goldman[11] has examined mathematically the transmission of a pulse through a pulse amplifier with uniform transmission between the frequency of zero to $w_s/2$ and no transmission beyond this frequency. He assumed that the phase shift of the amplifier varied with the frequency. For a single pulse Goldman found that optimum transmission was approached when

$$F_s = \frac{3}{2(T_2 - T_1)}$$

where F_s = the cut-off frequency of the amplifier (cps.)
$(T_2 - T_1)$ = the pulse width (sec.)

The frequency response diagram for the Dumont oscilloscope[12] used is shown in Fig. 10. Nothing is known, however, about the phase characteristics.

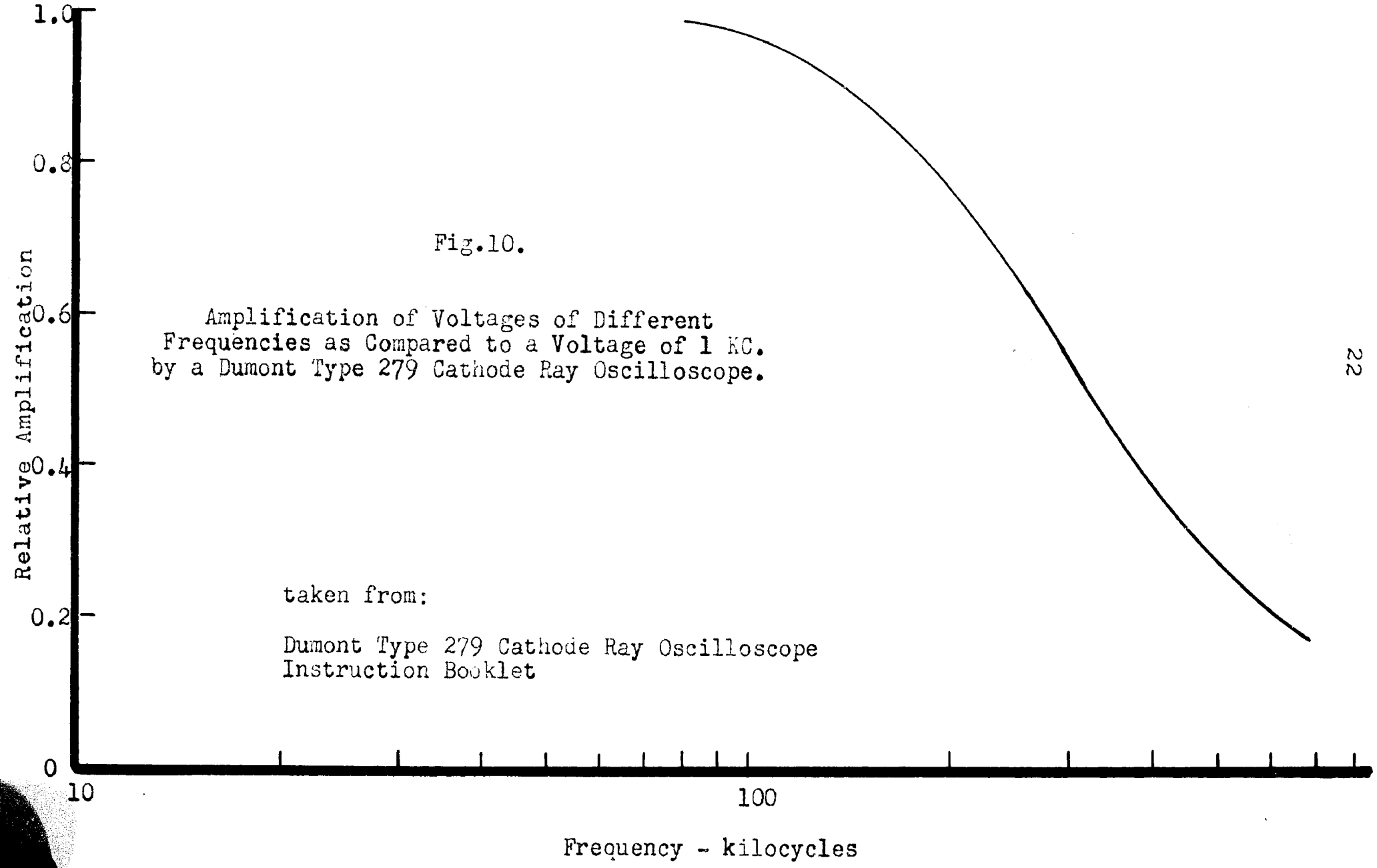


Fig.10.

Amplification of Voltages of Different Frequencies as Compared to a Voltage of 1 KC. by a Dumont Type 279 Cathode Ray Oscilloscope.

taken from:

Dumont Type 279 Cathode Ray Oscilloscope Instruction Booklet

The expression was rearranged to read

$$F_s = k \frac{1}{(T_2 - T_1)}$$

and was solved for each pulse width. For the 15.6 microsecond pulse, the value of k was 1.24; for the 21.8 microsecond pulse, 1.74; and for the 28.0 microsecond pulse, 2.24. A value of 80,000 cps. was used as the cut off frequency (F_s). The medium pulse was chosen for this study since the value of 1.74 is the closest to the optimum value of 1.5.

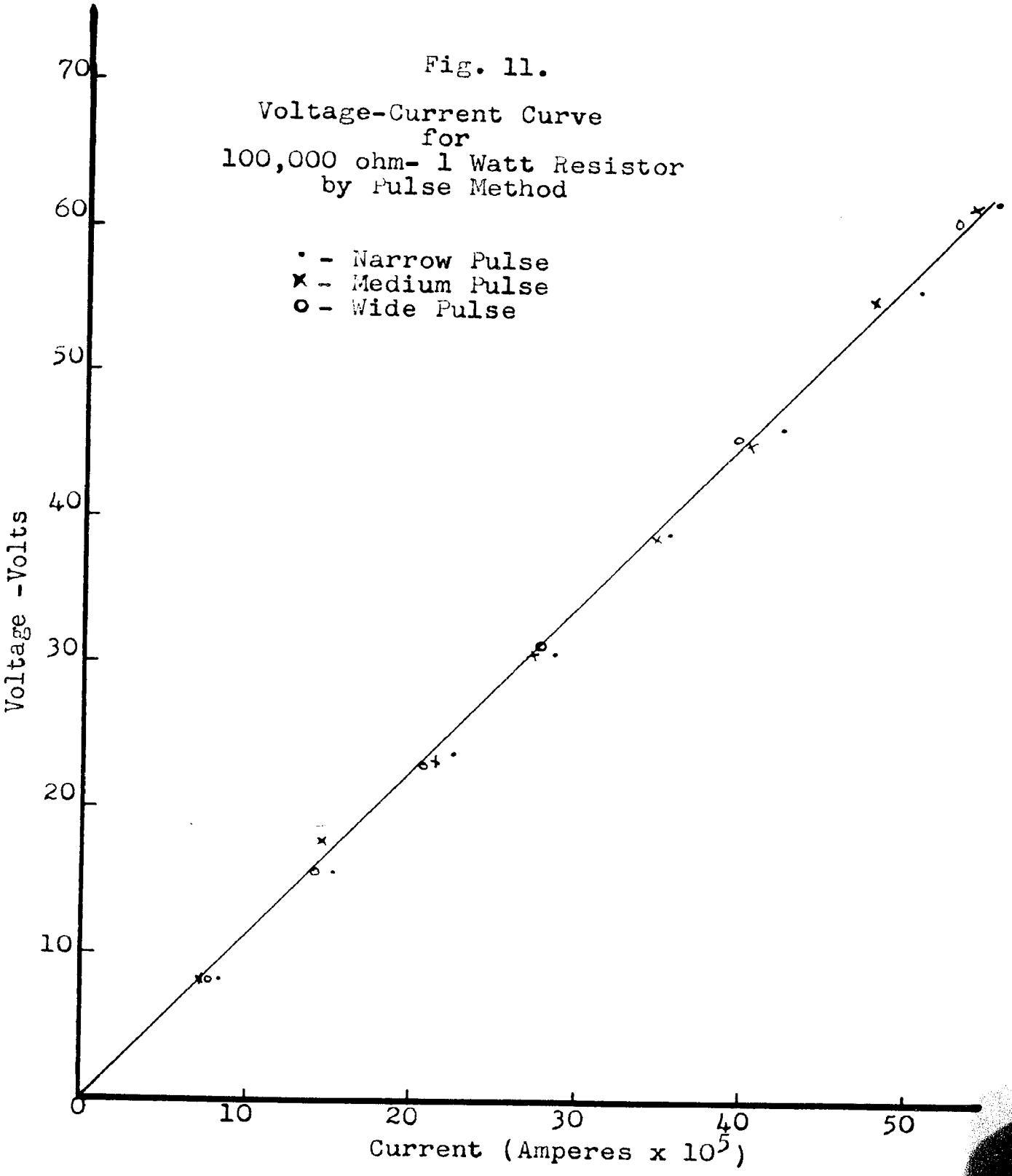
Overall Accuracy.

The overall accuracy of the measurement method was found by substituting a 100,000 ohm - 1 watt carbon resistor for the thermistor. This resistor was calibrated on a subsequent day in another laboratory at the University of Michigan with an L and N Wheatstone bridge (Serial No. 729864) and was found to be 109,875 ohms (abs. units). This was the average of the two readings of 108,600 ohms and 111,150 ohms. When measured by the pulse method, this same resistor was 114,200 ohms. This figure indicates an error of 4,325 ohms or 3.94%. The voltage-current curve for this resistor is shown in Fig. 11.

There are many sources of error. The pulse method consists of the comparison of the voltage across the thermistor with the voltage across the series resistor, which was measured on the same L and N Wheatstone bridge. Any error

Fig. 11.

Voltage-Current Curve
for
100,000 ohm- 1 Watt Resistor
by Pulse Method

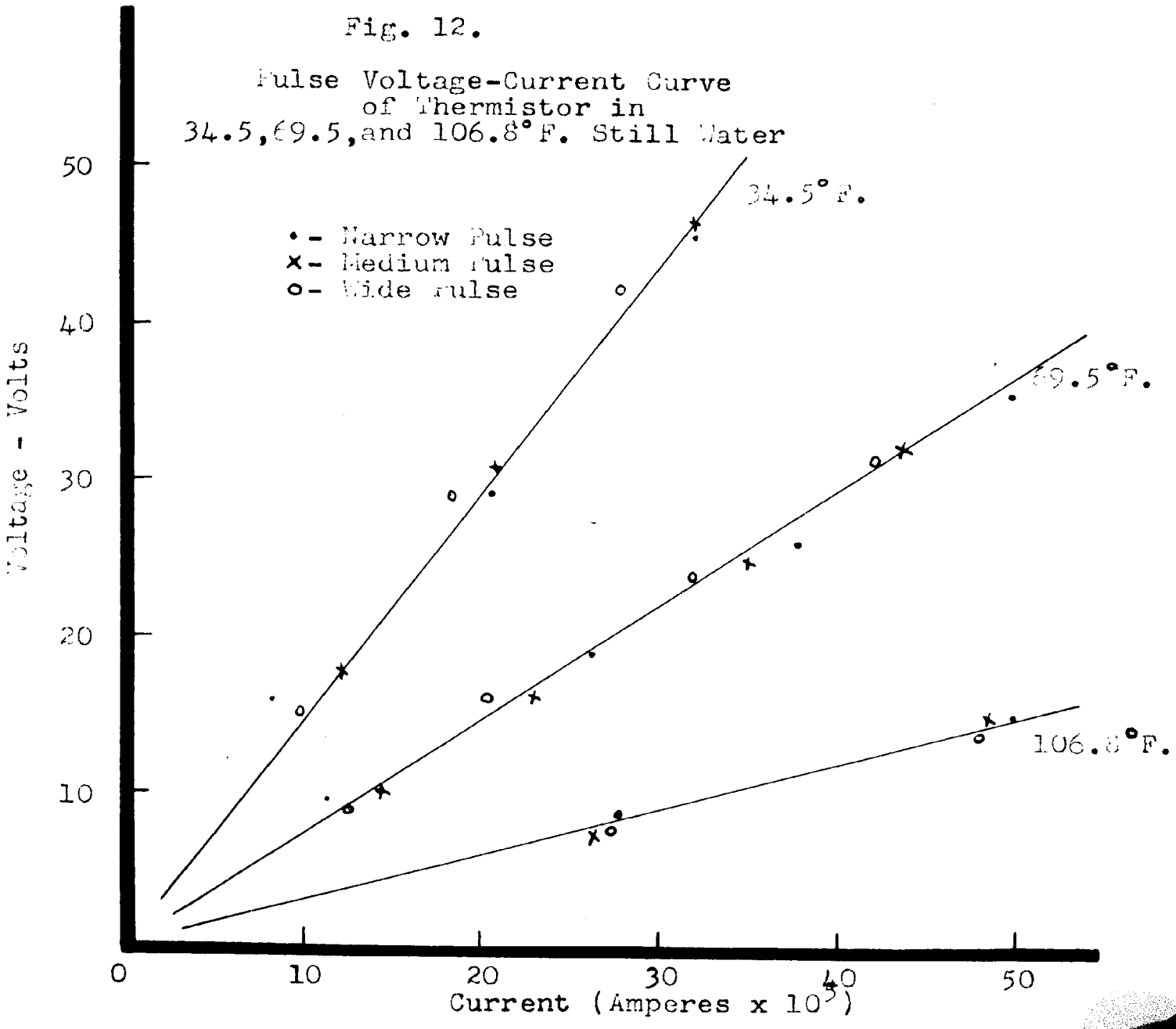


in the measurement of the series resistor will result in an error of the 100,000 ohm resistor. The camera was positioned by eye and it is possible that it was rotated a few degrees from the vertical. This error could be corrected by rotating the photographic printer holding the developed film. The width of the photographed traces depended upon the intensity of the oscilloscope beam. The beam was adjusted independently for each pulse image and each calibration image, thereby possibly giving different widths during different tests and calibrations. There were also errors associated with the use of the microscope, although any backlash of the activating screw was removed.

The thermistor probe was tested in still water with water temperatures of 32.0°F., 34.5°F., 44.0°F., 57.0°F., 65.7°F., 69.5°F., 89.0°F., and 106.8°F.. The three pulse widths were used at each temperature. Sample curves are shown in Fig. 12. The pulse impedance of the thermistor as a function of temperature is shown in Fig. 13. All voltage-current curves were straight lines; that is, there was no indication in any of the curves that the pulse was applying enough power to heat the thermistor above the temperature of the water. For later work, however, only the 21.8 microsecond pulse was used.

Fig. 12.

Pulse Voltage-Current Curve
of Thermistor in
34.5, 69.5, and 106.8° F. Still Water



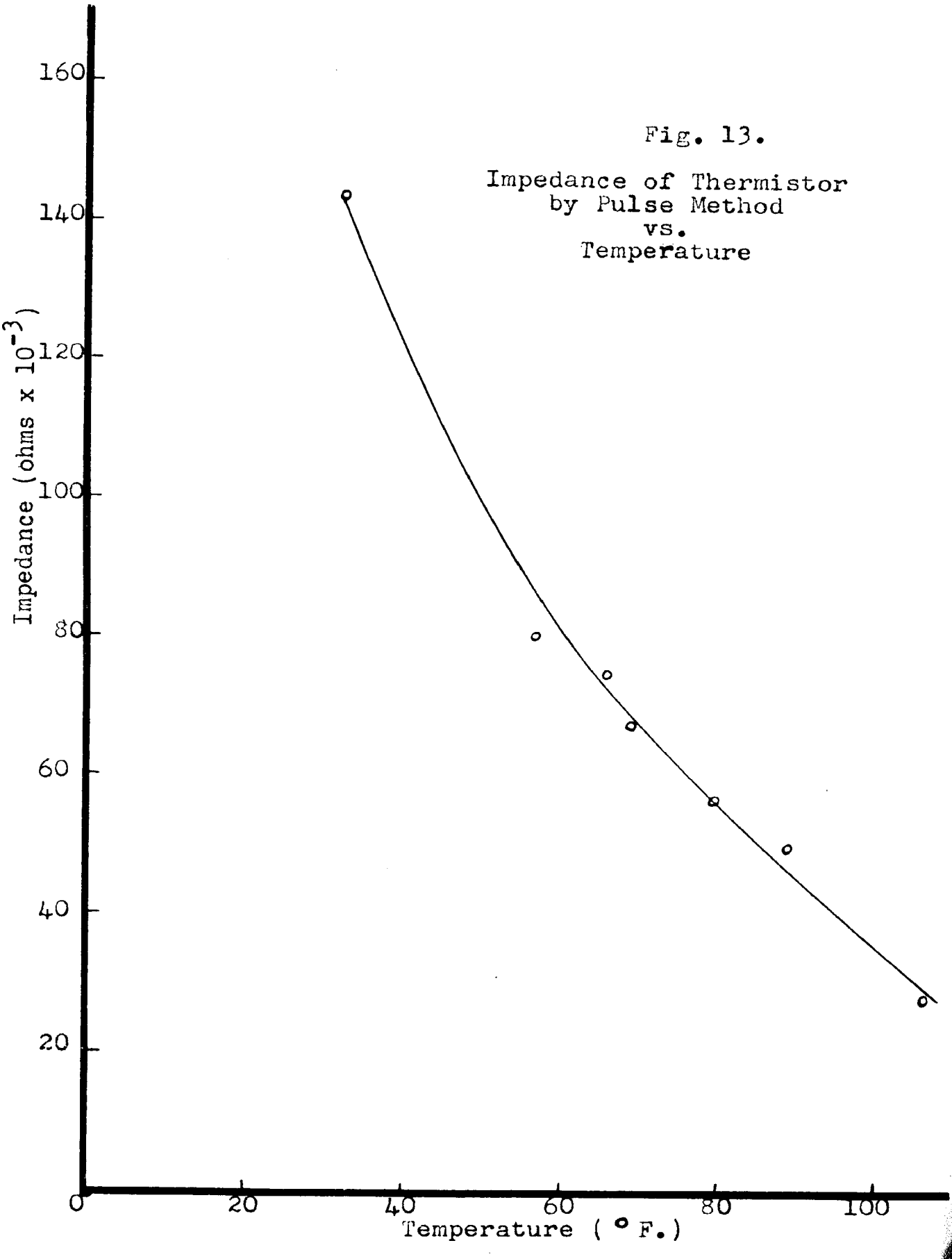


Fig. 13.

Impedance of Thermistor
by Pulse Method
vs.
Temperature

The Testing of a Thermistor Probe Under Flow Conditions.

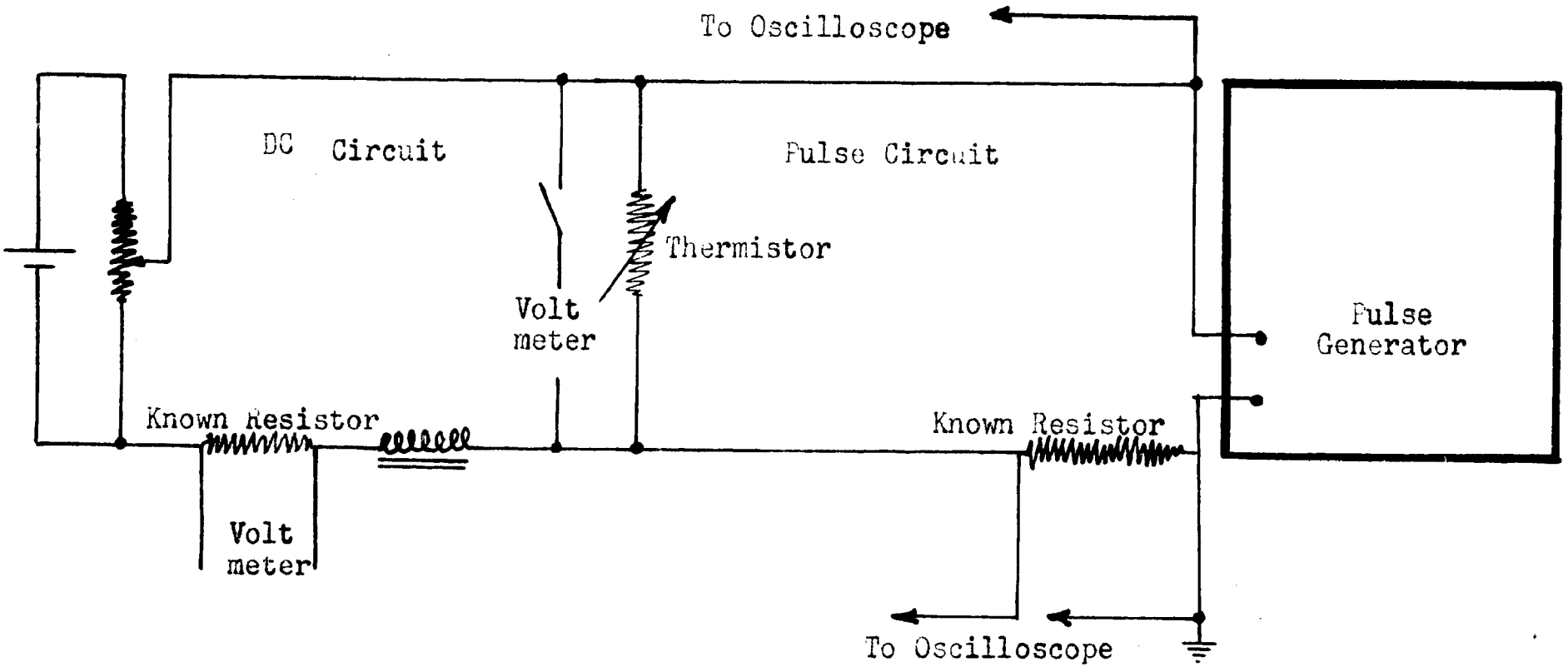
The next step was to connect the pulse measurement system to the flow measurement system, and determine the temperature of the thermistor under actual flow conditions. The thermistor was inserted into the flow tube and the pulse system was connected in parallel with a dc circuit as shown in Fig. 14. The cathode follower stage of the pulsing unit had a capacitator at its output to block the dc from that circuit. Inductance was added to the dc circuit to block the pulse from it.

The vacuum- tube voltmeter (Hickok Type 110-B Voltmeter), used to measure the dc voltage across the thermistor, has a dc input impedance of 11 megohms. However, its input impedance to the pulse was much less. Therefore, the voltmeter was disconnected when the pulse photographs were taken.

The water velocity was varied from 1 to 9 ft./sec. in five steps. As the temperature of the water varied somewhat with the velocity, time was allowed after each change for the water temperature to reach equilibrium. Two thermistors were tested. The first (No. 1) was tested in July when the water was relatively warm (60-70°F.), and the second (No. 2) was tested in January when the water was relatively cold (40-50°F.). When thermistor No. 1 was tested, the dc voltage was varied from zero to 30 volts in 10 volt steps. For thermistor No. 2, the dc voltage was varied from zero to 25 volts in 5 volt steps. Determinations of the water velocity and temperature, dc voltage and current were made at each test condition. Approximately

Fig. 14.

Circuit Used for Measuring Thermistor Temperatures



4 photographs of variable pulse voltages were taken at each test condition. From these photographs a voltage-current curve was drawn and from this curve, the pulse impedance of the thermistor was found.

After each of the tests, the thermistor was removed and placed, as previously described, in a vacuum bottle containing water at different temperatures. Photographs of variable voltage and current pulses were taken for each water temperature, and a curve similar to Fig. 13 was constructed.

By comparing the pulse impedance of the thermistor, heated by dc power and subjected to flowing water, with its pulse impedance in static water of various temperatures, the temperature of the thermistor was found for each condition. Typical pulse voltage-current curves are shown in Fig. 15.

CALCULATION OF DATA

From the data, the coefficient of heat transfer was calculated for each condition using the equation

$$Q = hA(T_t - T_w)$$

or

$$h = \frac{Q}{A(T_t - T_w)}$$

where Q = the heat transferred by thermistor (BTU/hr.)

h = the coefficient of heat transfer (BTU/sq.ft./hr./°F.)

A = the area of thermistor (sq.ft.)

T_t = the thermistor temperature (°F.)

T_w = the water temperature (°F.)

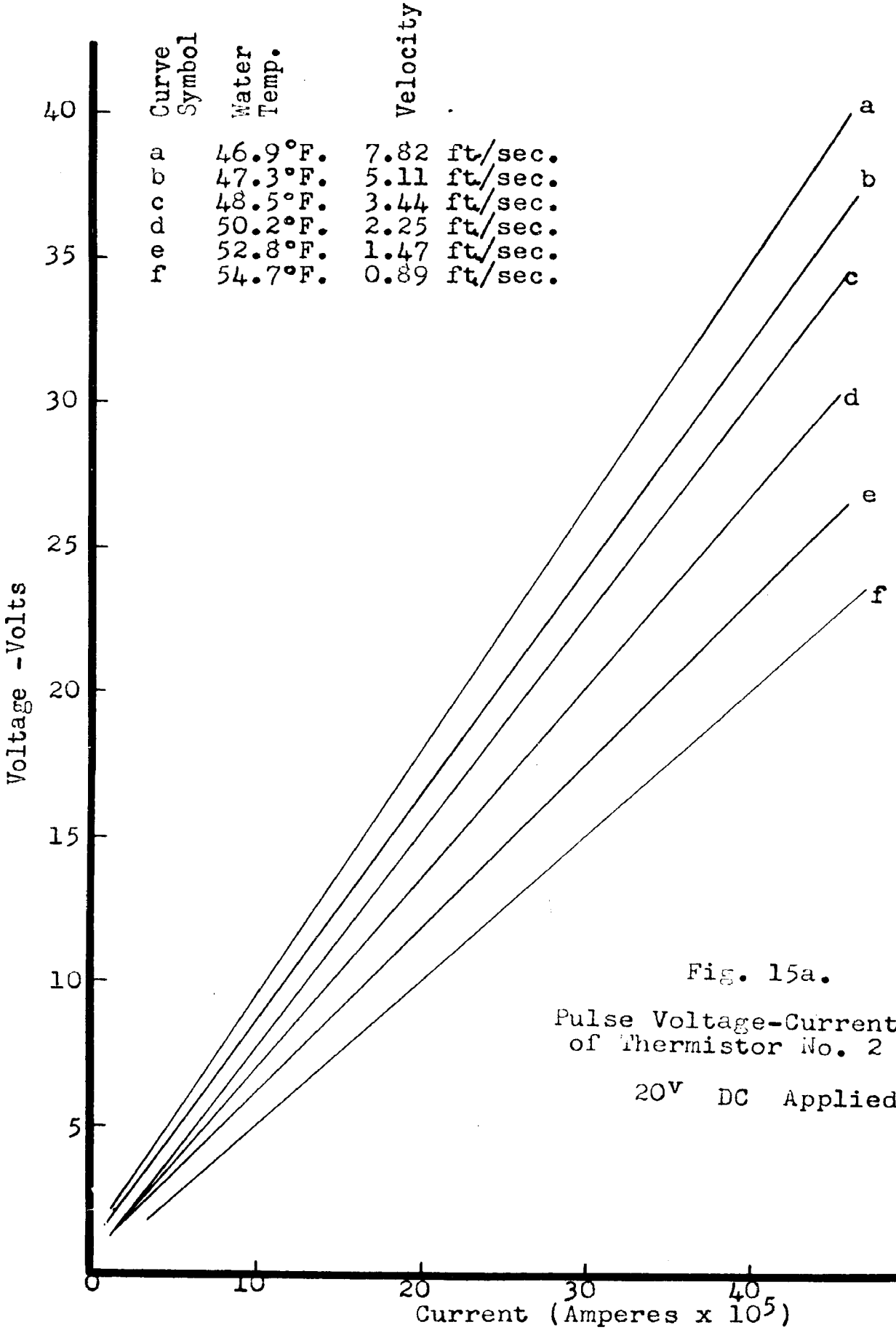


Fig. 15a.

Pulse Voltage-Current Curves of Thermistor No. 2 in Flow

20^{V} DC Applied

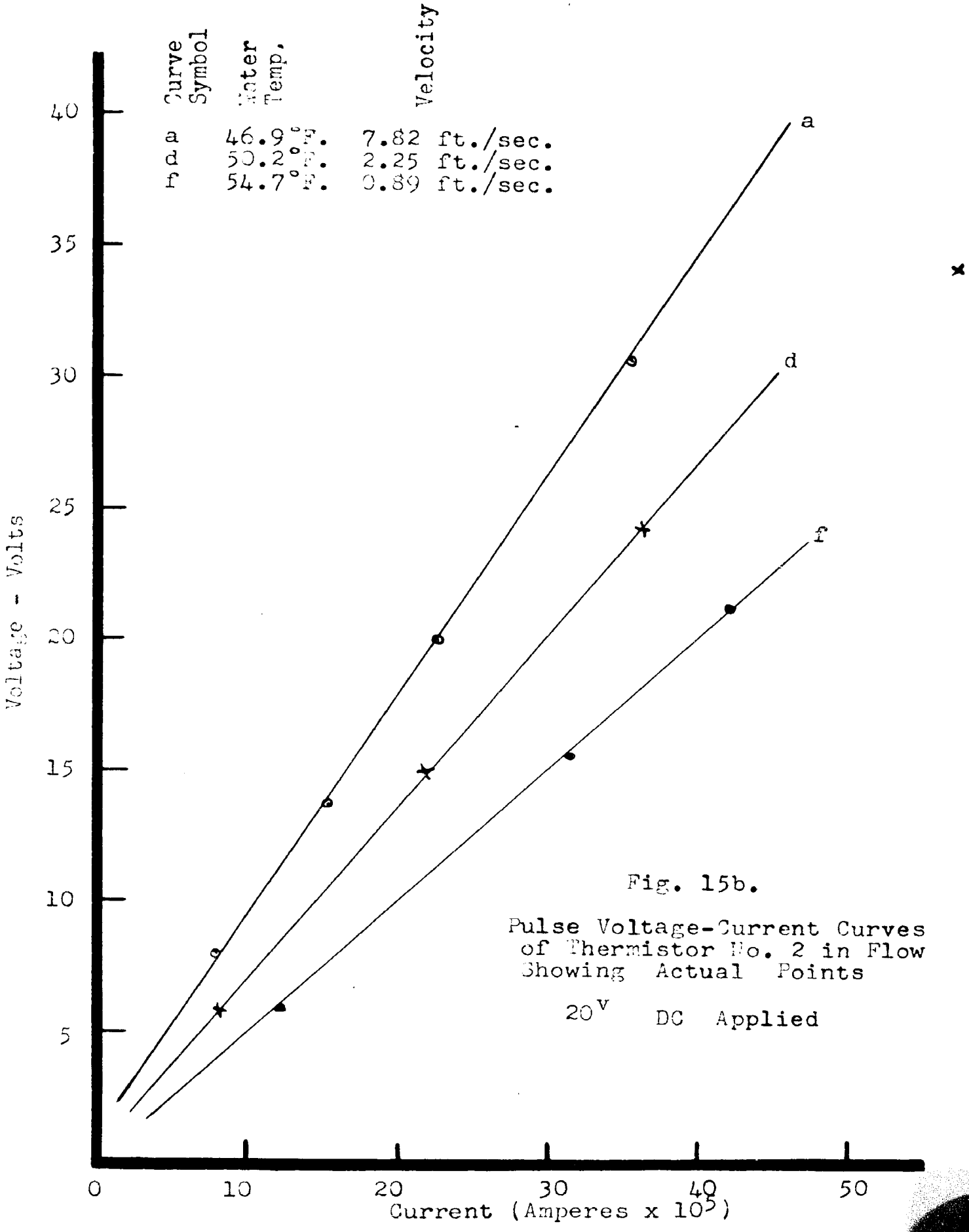


Fig. 15b.

Pulse Voltage-Current Curves of Thermistor No. 2 in Flow Showing Actual Points

20^{V} DC Applied

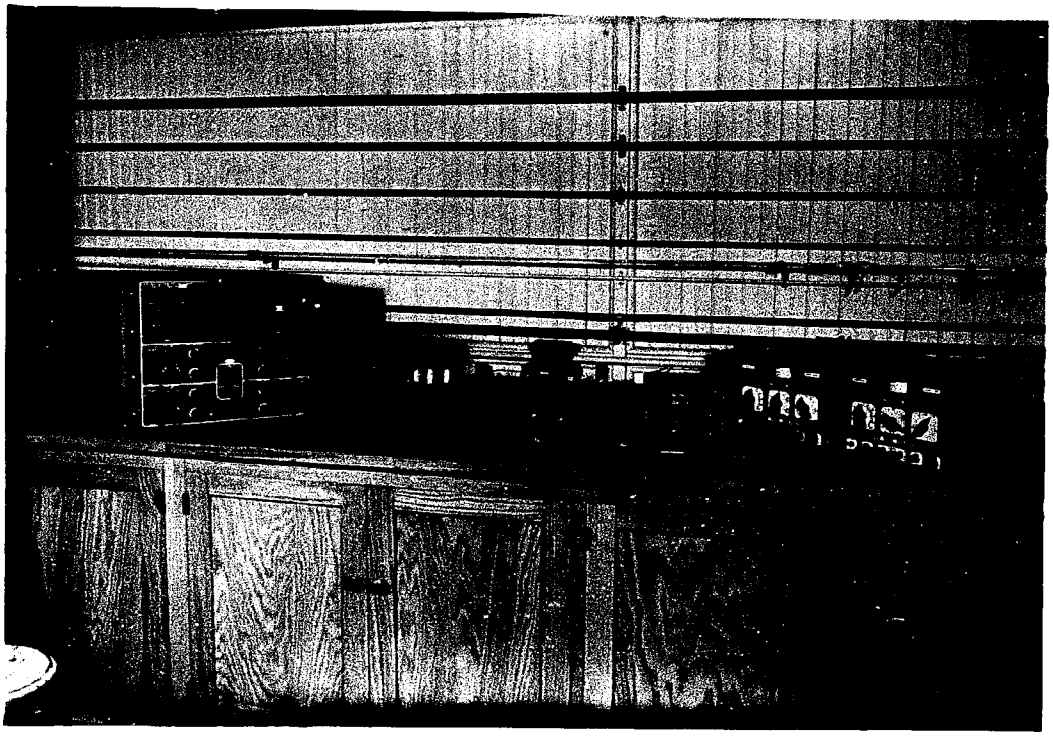


Fig. 16. This Figure Shows the Equipment Used to Determine the Temperatures of the Thermistor under Use.

The area of the thermistor was found from the expression[13]

$$A = (\pi a/2)(\sqrt{b^2 + a^2})$$

Where A = the area of the ellipsoid (sq.ft.)

b = the length of the ellipsoid (ft.)

a = the radius of the ellipsoid (ft.)

The areas were 0.00194 sq.in. (0.0000134 sq.ft.) for thermistor No. 1, and 0.00133 sq.in. (0.0000092 sq.ft.) for thermistor No. 2. The exposed length of thermistor No. 1 was 0.054 in. and the radius was 0.016 in.. The same dimensions of thermistor No. 2 were 0.032 in. (length) and 0.018 in. (radius). These measurements were made with a micrometer.

The data, as computed, are shown in Tables I and II.

DEVELOPMENT OF DIMENSIONLESS RATIOS FOR THE ANALYSIS OF DATA[14]

In order to analyze the data of Tables I and II, it is necessary to develop dimensionless parameters. This allows the use of the Principle of Similarity.

The coefficient of heat transfer (h) may be expressed as a function of the following independent variables:

$$h = f(D, v, \rho, \mu, L, k, C_p, E, I)$$

where h = the coefficient of heat transfer (BTU/sq.ft./hr./^{o}F.)

D = the diameter of the probe (ft.)

v = the velocity of the water (ft./sec.)

ρ = the density of the fluid (lb./cu.ft.)

μ = the absolute viscosity of the fluid (lb./hr./ft.)

L = the length of the probe (ft.)

k = the thermal conductivity of the fluid (BTU/ft./hr./°F.)

C_p = the specific heat of the fluid at constant pressure (BTU/lb./°F.)

E = the voltage applied to the thermistor (volts)

I = the current flowing in the thermistor (amperes)

These variables have the following basic dimensions in terms of length (L), time (T), mass (M), heat (Q), temperature (θ), and electrical charge (C).

$$Q/L^2T = f\left[(L),(L/T),(M/L^3),(M/LT),(L),(Q/LT\theta),(Q/M\theta),(ML^2/T^2C),(C/T)\right].$$

There are 10 variables, and since $\frac{Q}{\theta}$ always occurs as a ratio, there are 5 fundamental dimensions. From these, through the use of a theorem known as the π theorem, 5 dimensionless parameters or ratios may be set up. These ratios are known as π terms and may be set up in the following manner:

$\pi_1 = D^a v^b \rho^c k^d E^e h$

$\pi_2 =$ " " " " " μ

$\pi_3 =$ " " " " " L

$\pi_4 =$ " " " " " C_p

$\pi_5 =$ " " " " " I

$$\pi_1 - M^oL^oT^o\left(\frac{Q}{\theta}\right)^oC^o = L^a\left(\frac{L}{T}\right)^b\left(\frac{M}{L^3}\right)^c\left(\frac{Q}{LT\theta}\right)^d\left(\frac{ML^2}{T^2C}\right)^e\left(\frac{Q}{L^2T\theta}\right)^1$$

$M^o: 0 = c + e$

$L^o: 0 = a + b - 3c - d + 2e - 2$

$T^o: 0 = -b - d - 2e - 1$

$\left(\frac{Q}{\theta}\right)^o: 0 = d + 1$

$C^o: 0 = -e$

from which $a = 1,\ b = 0,\ c = 0,\ d = -1,\ e = 0$

$$\pi_1 = \frac{hD}{k} \quad \text{(Nusselt's number)}$$

$$\pi_2 - M^oL^oT^o\left(\frac{Q}{\theta}\right)^oC^o = L^a\left(\frac{L}{T}\right)^b\left(\frac{M}{L3}\right)^c\left(\frac{Q}{LT\theta}\right)^d\left(\frac{ML^2}{T^2C}\right)^e\left(\frac{M}{LT}\right)^1$$

$$M^o{:}0 = c + e + 1$$

$$L^o{:}0 = a + b - 3c - d + 2e - 1$$

$$T^o{:}0 = -b - d - 2e - 1$$

$$\left(\frac{Q}{\theta}\right)^o{:}0 = d$$

$$C^o{:}0 = -e$$

from which $a = -1,\ b = -1,\ c = -1,\ d = e = 0$

$$\pi_2 = \frac{\mu}{\rho v D} = \frac{\rho v D}{\mu} \quad \text{(Reynolds' number).}$$

$$\pi_3 - M^oL^oT^o\left(\frac{Q}{\theta}\right)^oC^o = L^a\left(\frac{L}{T}\right)^b\left(\frac{M}{L3}\right)^c\left(\frac{Q}{LT\theta}\right)^d\left(\frac{ML^2}{T^2C}\right)^e L^1$$

$$M^o{:}0 = c + e$$

$$L^o{:}0 = a + b - 3c - d + 2e + 1$$

$$T^o{:}0 = -b - d - 2e$$

$$\left(\frac{Q}{\theta}\right)^o{:}0 = d$$

$$C^o{:}0 = -e$$

from which $a = -1,\ b = c = d = e = 0$

$$\pi_3 = \frac{L}{D} \quad \text{(aspect ratio)}$$

$$\pi_4 - M^oL^oT^o\left(\frac{Q}{\theta}\right)^oC^o = L^a\left(\frac{L}{T}\right)^b\left(\frac{M}{L3}\right)^c\left(\frac{Q}{LT\theta}\right)^d\left(\frac{ML^2}{T^2C}\right)^e\left(\frac{Q}{M\theta}\right)^1$$

$$M^o{:}0 = c + e - 1$$

$$L^o{:}0 = a + b - 3c - d + 2e$$

$$T^o{:}0 = -b - d - 2e$$

$$\left(\frac{Q}{\Theta}\right)^{o}: 0 = d + 1$$

$$C^{o}: 0 = -e$$

from which $a = 1,\ b = 1,\ c = 1,\ d = -1,\ e = 0$

$$\pi_4 = \frac{C_p \rho v D}{k} \qquad \text{(Peclet's number)}$$

$$\pi_5 - M^{o}L^{o}T^{o}\left(\frac{Q}{\Theta}\right)^{o}C^{o} = L^{a}\left(\frac{L}{T}\right)^{b}\left(\frac{M}{L^3}\right)^{c}\left(\frac{Q}{LT\Theta}\right)^{d}\left(\frac{ML^2}{T^2C}\right)^{e}\left(\frac{C}{T}\right)^{1}$$

$$M^{o}: 0 = c + e$$

$$L^{o}: 0 = a + b - 3c - d + 2e$$

$$T^{o}: 0 = -b - d - 2e - 1$$

$$\left(\frac{Q}{\Theta}\right)^{o}: 0 = d$$

$$C^{o}: 0 = -e + 1$$

from which $a = -2,\ b = -3,\ c = -1,\ d = 0,\ e = 1$

$$\pi_5 = \frac{E\ I}{\rho v^3 D^2} \qquad \text{(power ratio)}$$

These π terms are

(1) hD/k - Nusselt's number - Nu.

(2) $\frac{C_p \rho v D}{k}$ - Peclet's number - Pe.

(3) $\frac{\rho v D}{\mu}$ - Reynolds' number - Re.

(4) $\frac{L}{D}$ - aspect ratio

(5) $\frac{EI}{\rho v^3 D^2}$ - power ratio

also (6) $\frac{C_p \mu}{K}$ - Prandtl's number - Pr.

Peclet's number is the product of Reynolds' number and Prandtl's number (Pr.).

Another number that is of importance is Stanton's number (St.). It is formed as follows:

$$St = \frac{Nu}{Re\ Pr} = \frac{h\ D/k}{(\rho v\ D/\mu)(C_p\mu/k)} = \frac{h}{\rho v\ C_p}$$

All of these dimensionless ratios may be expressed physically and are not mere numbers. Using these dimensionless parameters that have been developed, the coefficient of heat transfer (h) may be expressed in the following manners:

$$\frac{hD}{k} = k''\ (\frac{L}{D})^a(\frac{C_p\mu}{k})^b(\frac{\rho vD}{\mu})^c\ (\frac{E\ I}{\rho v^3D^2})^d$$

and/or $$\frac{h}{\rho vC_p} = k'''\ (\frac{L}{D})^e(\frac{E\ I}{\rho v^3D^2})^f$$

k", k"", a, b, c, d, e, and f have to be determined from the experimental data.

CORRELATION OF EXPERIMENTAL DATA USING THE DIMENSIONLESS PARAMETERS.

Two dimensionless equations have been developed for the correlation of the heat transfer coefficient with the variables of the problem. For each thermistor, the $\frac{L}{D}$ ratio is a constant. Therefore, another constant k''' and k"", will be set up as follows:

$$k''' = k''\ (\frac{L}{D})^a$$

and

$$k'''' = k''''(\frac{L}{D})^e$$

The equations can then be expressed

$$\frac{hD}{k} = k''' \left(\frac{C_p \mu}{k}\right)^b \left(\frac{\rho v D}{\mu}\right)^c \left(\frac{E\ I}{\rho v^3 D^2}\right)^d$$

and

$$\frac{h}{\rho v\ C_p} = k'''' \left(\frac{E\ I}{\rho v^3 D^2}\right)^f$$

The first step in the correlation of the experimental data is to calculate the values of each of the dimensionless ratios for each experimental observation. These are shown in Tables III and IV.

When studying the influence of either Prandtl's number ($C_p \mu/k$), Reynolds' number ($\rho vD/\mu$), or $EI/\rho v^3 D^2$ on Nusselt's number (hD/k), the remaining numbers should be kept constant. With the equipment used, this was not possible. The water velocity and the electrical voltage could be controlled. The glass tube, which contained the constriction and the thermistor, was located along the laboratory wall. As the laboratory temperature was above that of the water, the water acquired some heat, the amount of which depended on the velocity of the water. Thus, as the velocity was changed, the water temperature changed. Therefore, not all of the dimensionless ratios could be kept constant, which would be necessary if each ratio was to be studied independently.

Because the variables could not all be controlled, the data were grouped in such a way that the dimensionless ratios which were not being studied were allowed to vary only through a restricted range. Figs. 17 and 19 show this grouping for thermistor No. 1. The numbers in these figures are the observation numbers (See Table III) and the notation for each square correspond to the notation on the curves of Figs. 18 and 20.

An example may clarify this grouping and plotting. Fig. 18 shows the relationship between hD/k (Nusselt's number) and $EI/\rho v^3D^2$. To obtain these curves, Reynolds' number, which varied from zero to 2500, was divided into 500-unit divisions (See Fig. 17). Likewise, Prandtl's number ($Cp\mu/k$) varied from 6.6 to 7.2 and was divided into 6 units with a unit width of 0.1. Curve e-3 of Fig. 18 shows the relationship between hD/k and $EI/\rho v^3D^2$ for all the experimental points where Reynolds' number ($\rho vD/\mu$) varied from 500 to 1000, and $Cp\mu/k$ (Prandtl's number) varied from 6.8 to 6.9. The observations for thermistor No. 1, falling within these specifications, are observations 8, 17, 18, 19, and 35. These points are plotted on log-log graph paper. The other curves in Fig. 18 were obtained in the same manner.

The equation was stated:

$$\frac{hD}{k} = k^{VI} \left(\frac{EI}{\rho v^3 D^2}\right)^d$$

Fig. 17.

Distribution of Observations
for plotting hD/k vs. $EI/\rho v^2 D^2$

Thermistor No. 1

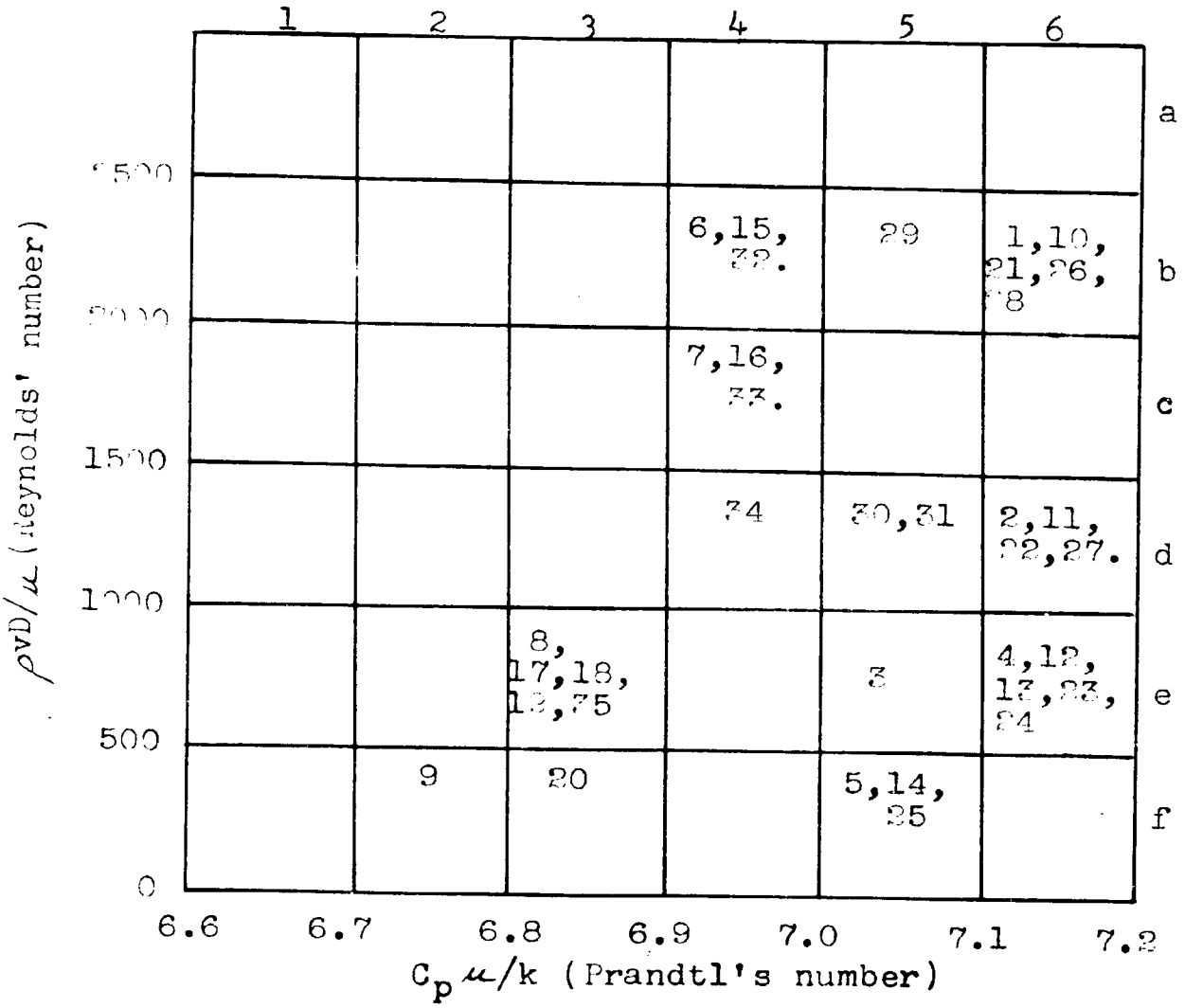
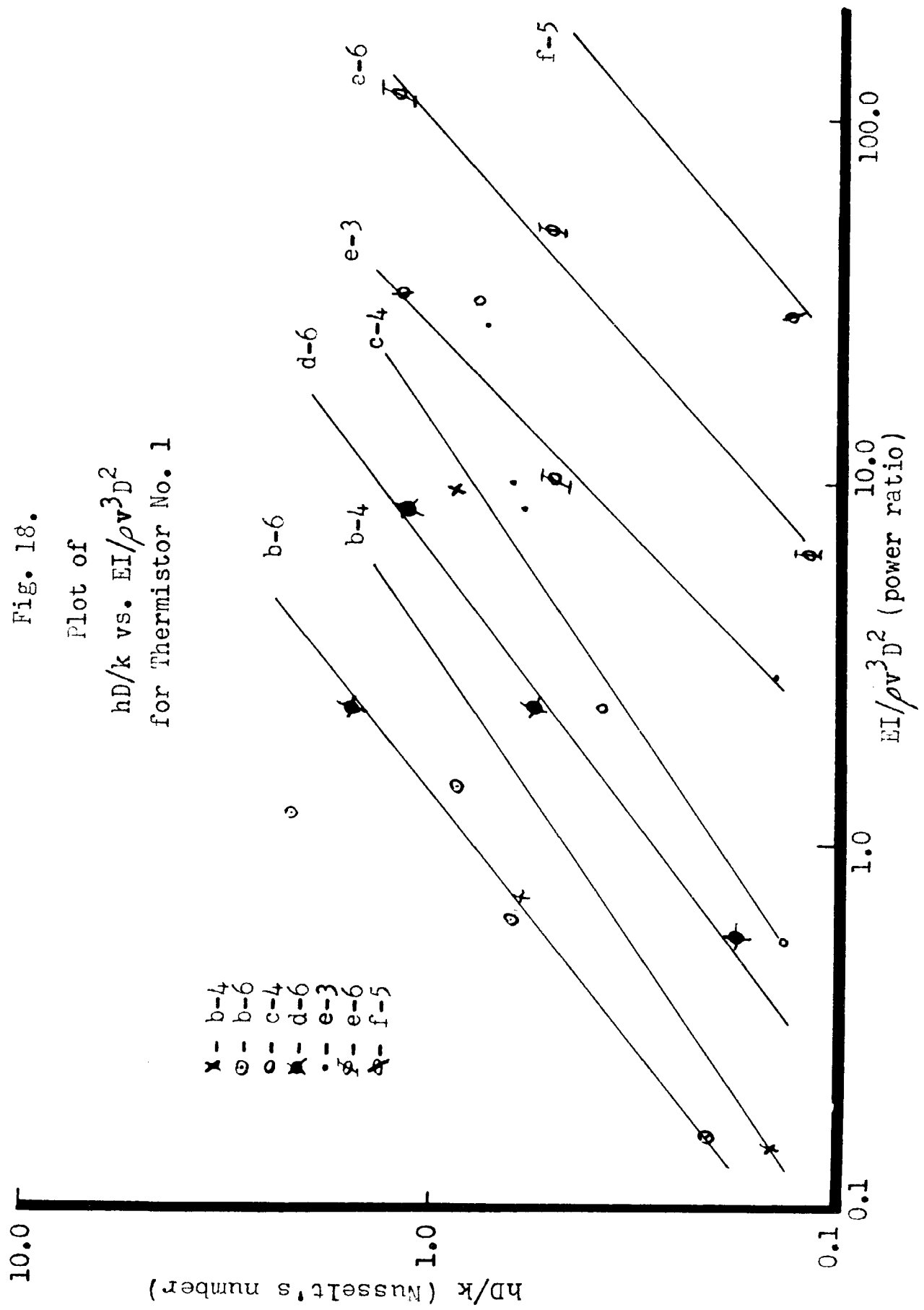


Fig. 18.

Plot of
hD/k vs. $EI/\rho v^3 D^2$
for Thermistor No. 1



Taking the logarithm of each side

$$\log \frac{hD}{k} = \log k^{VI} + d \log \left(\frac{EI}{\rho v^3 D^2}\right)$$

and the value of "d" is equal to the slope of the curves of Fig. 18. The average slope of these individual curves for thermistor No. 1 was 0.74. For thermistor No. 2, the average value of "d" was 0.89, and the over-all average was 0.81.

The variation of $\frac{hD}{k}$ (Nusselt's number) with $\rho vD/\mu$ (Reynolds' number) was studied similarly. The division of the experimental observations is shown in Fig. 19, and the variation shown in Fig. 20. For thermistor No. 1, the value of"c"was found to be 2.46; and for thermistor No. 2, 3.05. The overall average for "c" was then 2.75.

Within the range tested, $\frac{hD}{k}$ (Nusselt's number) was found to be independent of Prandtl's number ($Cp\mu/k$). When $\rho vD/\mu$ (Reynolds' number) and $EI/\rho v^3D^2$ (power ratio) were subdivided, and hD/k plotted against $Cp\mu/k$ on log-log graph paper, the result was a series of short nearly vertical curves. This series indicates a nearly independent relationship between hD/k and $Cp\mu/k$. The variation of $Cp\mu/k$ was very small, however.

In the equation, as determined to this point

$$\frac{hD}{k} = k''' \left(\frac{Cp\mu}{k}\right)^b \left(\frac{\rho vD}{\mu}\right)^{2.75} \left(\frac{EI}{\rho v^3 D^2}\right)^{0.81}$$

only k''' and "b" have yet to be determined. Because of the feeling that this plotting method was insufficient, a series of

Fig. 19.

Distribution of Observations for plotting hD/k vs. $\rho vD/\mu$

Thermistor No. 1

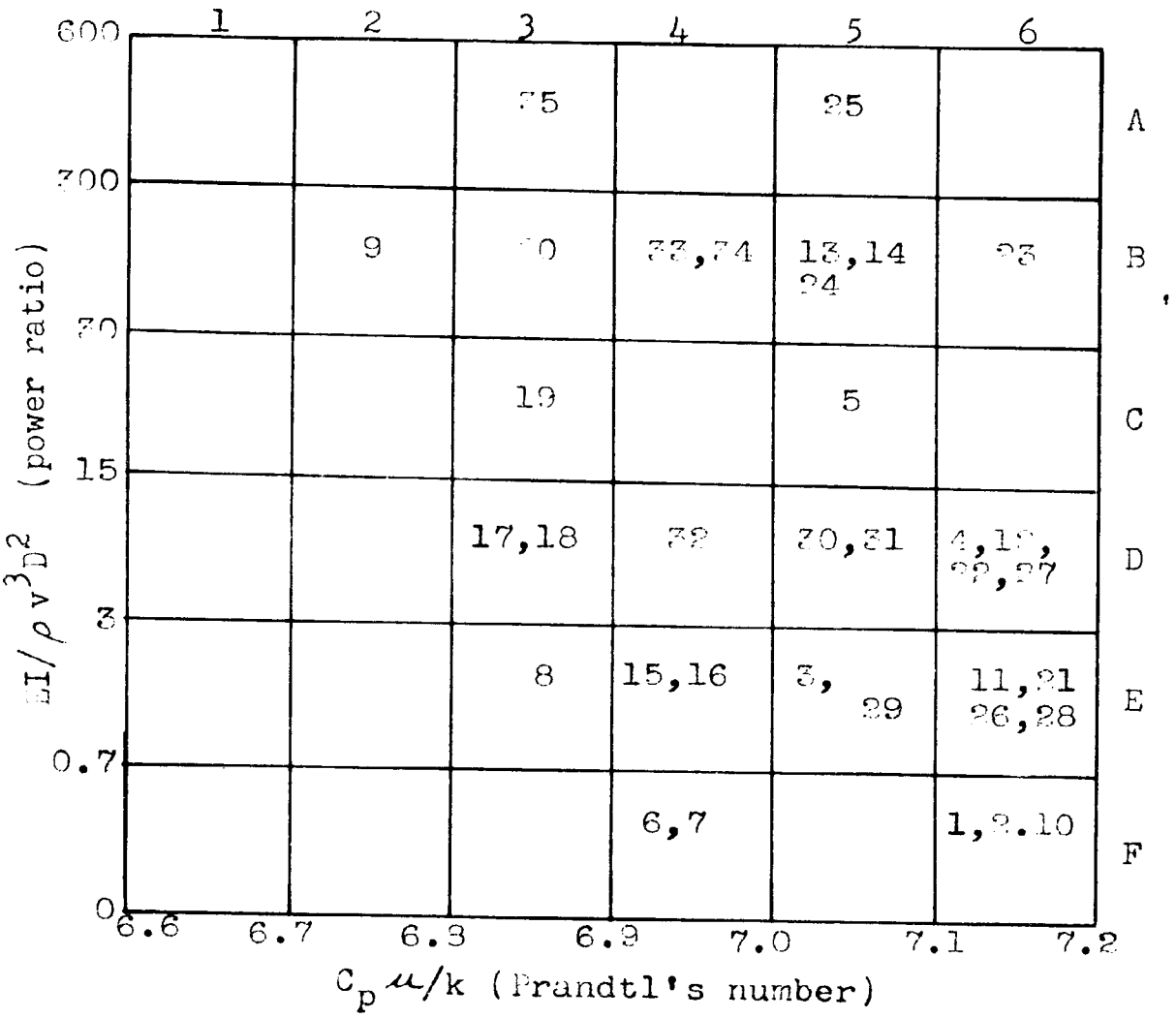
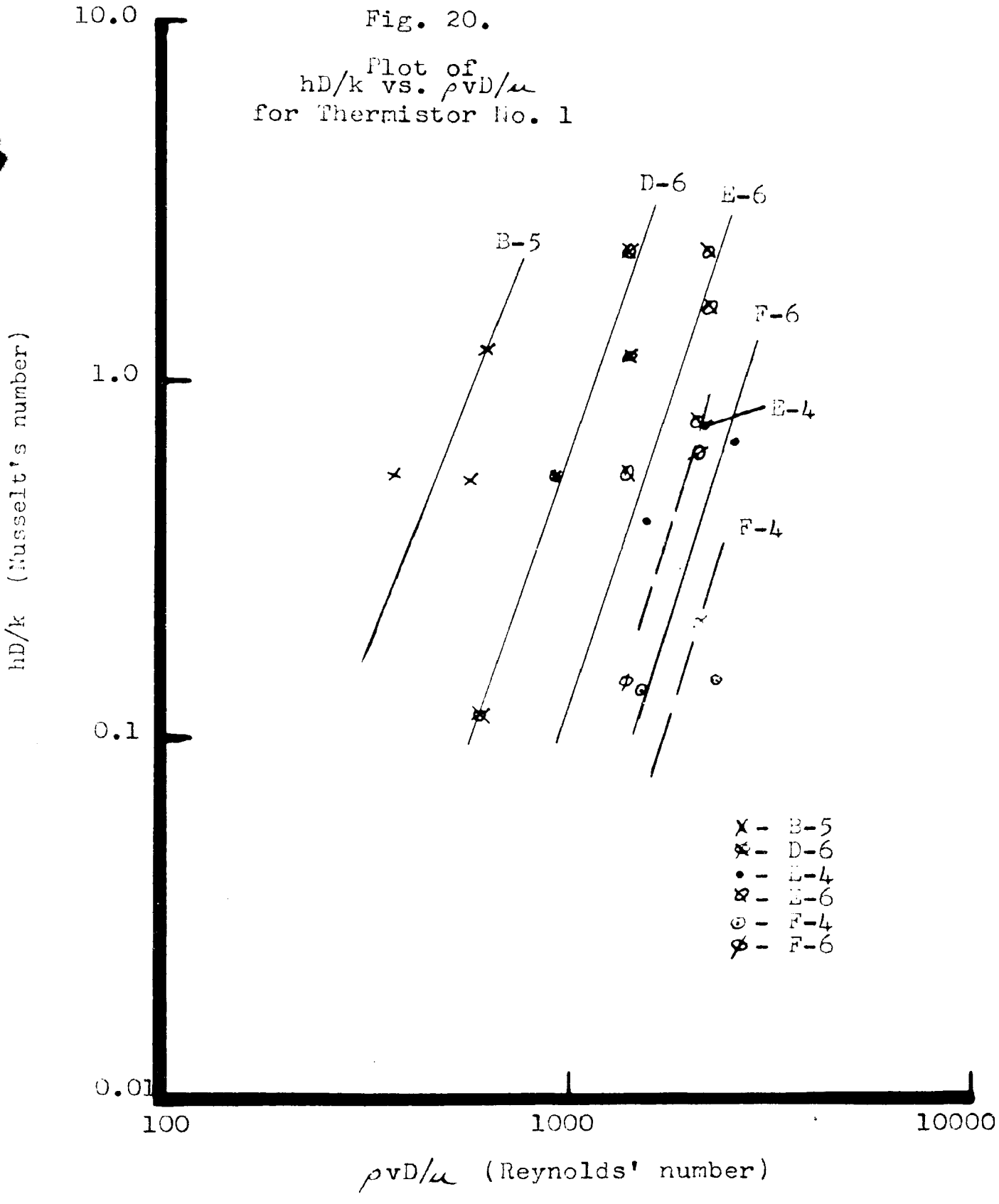


Fig. 20.

Plot of
hD/k vs. $\rho vD/\mu$
for Thermistor No. 1



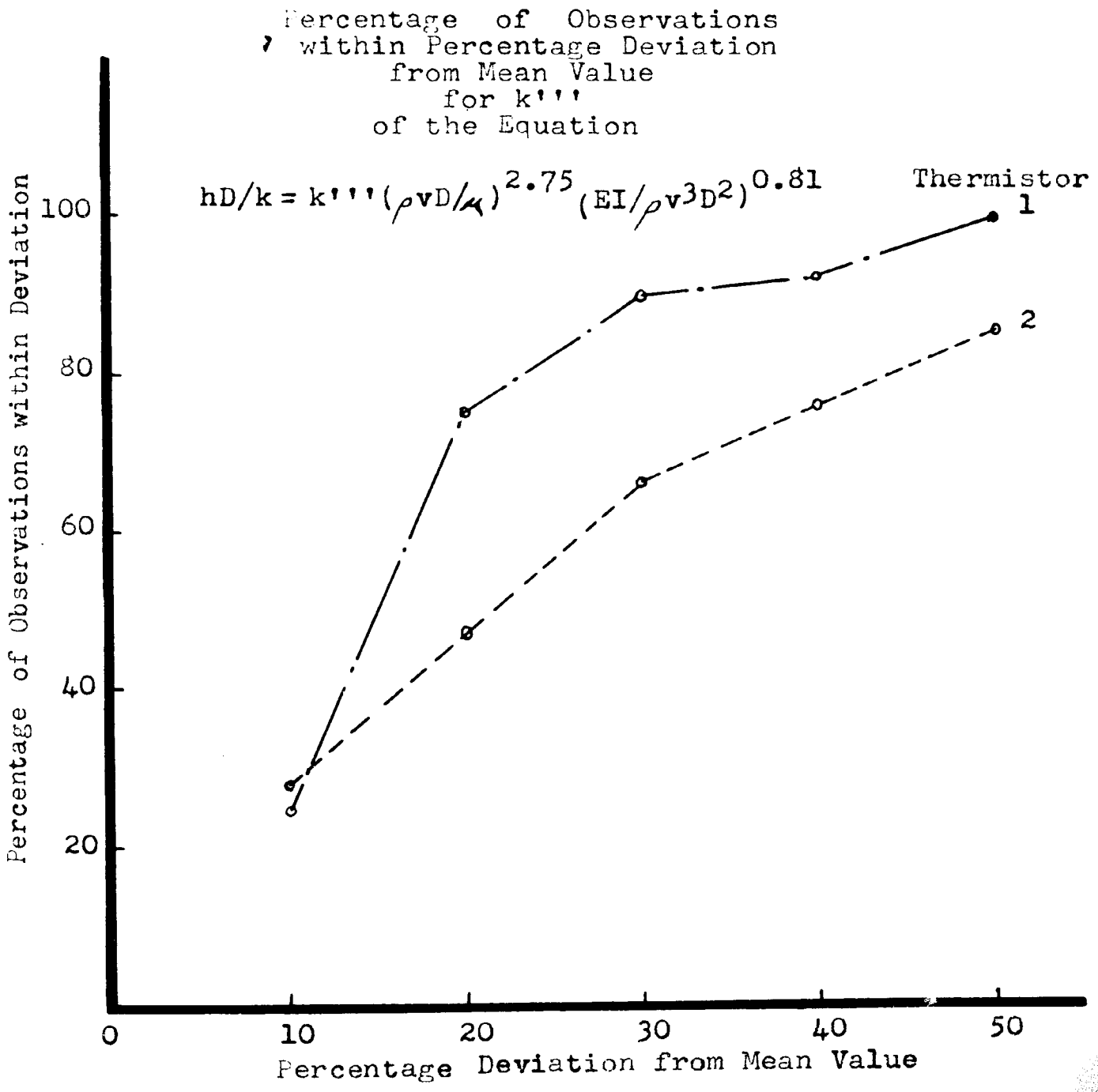
simultaneous solutions to determine "b" from different sets of two observations was undertaken. The value of "b", determined by this method, was just as inconclusive. The values ranged from high negative to high positive values. Again, the difficulty was the very small variation of Prandtl's number ($C_p\mu/k$), so that any errors or scattering of data were highly magnified. Thus, it has been concluded that <u>for the range tested</u>, hD/k did not vary with $Cp\mu/k$.

The equation showing the relationship between the film coefficient (h) and the other variables has been determined to be:

$$\frac{hD}{k} = k''' \left(\frac{\rho vD}{\mu}\right)^{2.75} \left(\frac{EI}{\rho v^3 D^2}\right)^{0.81}$$

Only k''' has yet to be determined. This is done for all experimental points in Table V for thermistor No. 1, and in Table VI for thermistor No. 2. The mean value of k''' was 0.551×10^{-9} for thermistor No. 1, and 8.134×10^{-9} for thermistor No. 2. Values of k''' that were extremely out of range were rejected before averaging. Fig. 21 shows the percentage of the observations with various percentage deviations from the mean value of k'''.

Fig. 21.


Percentage of Observations within Percentage Deviation from Mean Value for k''' of the Equation
$hD/k = k'''(\rho vD/\mu)^{2.75}(EI/\rho v^3D^2)^{0.81}$
Thermistor
1
2
Percentage of Observations within Deviation
100
80
60
40
20
0
10
20
30
40
50
Percentage Deviation from Mean Value

As shown

$$k''' = k'' (L/D)^a$$

Thermistor No. 1 had an L/D ratio equal to 1.69 and thermistor No. 2 had an L/D ratio equal to 0.89. Thus k" has a value of 4.95×10^{-9} and "a" has a value of -4.18. These values were determined from only two thermistors. The completed equation is as follows:

$$\frac{hD}{k} = 4.95 \times 10^{-9} \left(\frac{L}{D}\right)^{-4.18} \left(\frac{\rho vD}{\mu}\right)^{2.75} \left(\frac{EI}{\rho v^3 D^2}\right)^{0.81}$$

The other equation previously shown on page 38 was

$$\frac{h}{\rho v c_p} = k'''' \left(\frac{L}{D}\right)^e \left(\frac{EI}{\rho v^3 D^2}\right)^f$$

and for any particular thermistor

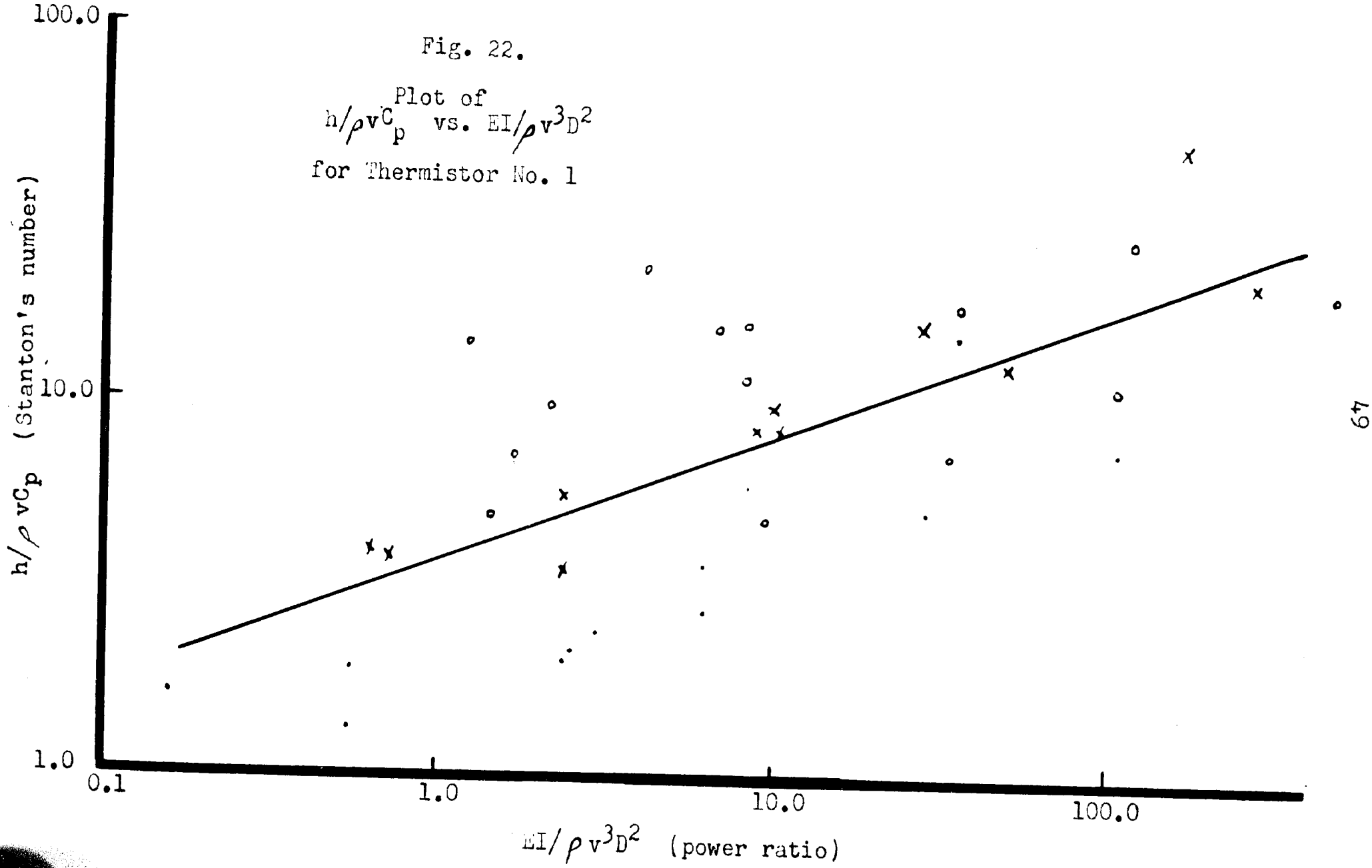
$$\frac{h}{\rho v c_p} = k''''' \left(\frac{EI}{\rho v^3 D^2}\right)^f$$

where $k''''' = k'''' \left(\frac{L}{D}\right)^e$

The values for each dimensionless ratio were plotted ($h/\rho v c_p$ vs $EI/\rho v^3 D^2$) for the experimental observations of both thermistor No. 1 and No. 2. A regression line was determined by the method of least squares,[15] using the factor $EI/\rho v^3 D^2$ as the independent variable. Fig. 22 shows this plot for thermistor No. 1. The value of "f" was 0.331 for the first thermistor and 0.325 for the second thermistor. The average of these values is 0.328. The value of k""' was then

Fig. 22.

Plot of $h/\rho v C_p$ vs. $EI/\rho v^3 D^2$ for Thermistor No. 1



calculated for each experimental observation. These values are shown in Tables VII and VIII. The average value of k"'' was 3.80×10^{-5} for thermistor No. 1 and 16.5×10^{-5} for thermistor No. 2. Again the deviation of the individual values from the mean values was found. The percentage of the observations within a designated percentage deviation from the mean value is shown in Fig. 23. From the equation

$$k'''' = k'''' (L/D)^e$$

and the values of the L/D ratio, as shown on page 48, the values of k"" and "e" were determined to be 1.26×10^{-4} and -2.18 respectively.

A comparison of Fig. 21 with Fig. 23 shows that the equation

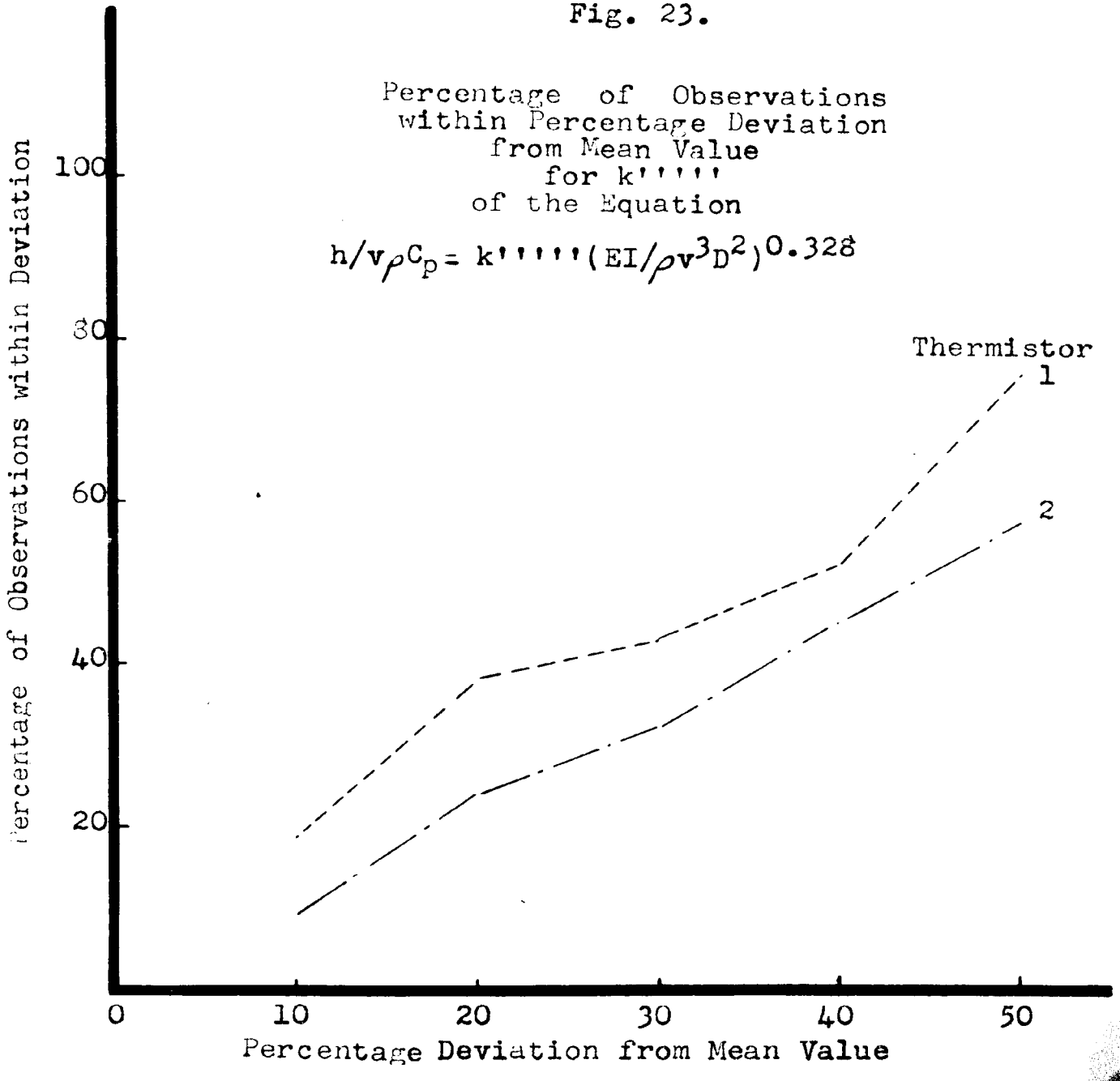
$$\frac{h}{\rho v C_p} = k'''' \left(\frac{EI}{\rho v^3 D^2}\right)^{0.328}$$

does not follow the experimental data as well as the equation

$$\frac{hD}{k} = k'''' \left(\frac{\rho vD}{\mu}\right)^{2.75} \left(\frac{EI}{\rho v^3 D^2}\right)^{0.81}$$

Therefore, the latter equation will be used as the expression relating the heat transfer from the thermistor to the water.

Fig. 23.


Percentage of Observations within Percentage Deviation from Mean Value for k''''' of the Equation
$h/v\rho C_p = k'''''(EI/\rho v^3 D^2)^{0.328}$
Thermistor
1
2
100
80
60
40
20
0
10
20
30
40
50
Percentage of Observations within Deviation
Percentage Deviation from Mean Value

AN ANALYSIS OF ERRORS

The closest correlation of the data has been made by using the expression

$$\frac{hD}{k} = k''' \left(\frac{\rho v D}{\mu}\right)^{2.75} \left(\frac{EI}{\rho v^3 D^2}\right)^{0.81}$$

An examination of some of the errors to be expected now follows.

$$k''' = \frac{\left(\frac{3.4115}{3600} EID\right)\left(\frac{\pi D}{4\sqrt{2}}\left(\sqrt{(2L)^2 + D^2/4}\right)(T_t - T_w)(k/3600)\right)}{\left((\rho vD/\mu)^c (EI/\rho v^3 D^2)^d\right)}$$

where the 3600 term is used to reduce the "time dimension" to seconds. Now

$$k''' = f(E, I, v, T_t, T_w, L, D)$$

which are the measured variables. See pages 30, 34 and 35. A total error in k"' will be noted as $\Delta k'''$. It is the sum of the errors caused by an erroneous measurement of any of the variables, and is found in the following manner:

$$\Delta k''' = \frac{\partial f}{\partial E}\Delta E + \frac{\partial f}{\partial I}\Delta I + \frac{\partial f}{\partial T_t}\Delta T_t + \frac{\partial f}{\partial T_w}\Delta T_w + \frac{\partial f}{\partial v}\Delta v + \frac{\partial f}{\partial L}\Delta L + \frac{\partial f}{\partial D}\Delta D$$

where

$$f(E, I, v, T_t, T_w, L, D) = \frac{hD/k}{\left(\frac{\rho vD}{\mu}\right)^c \left(\frac{EI}{\rho v^3 D^2}\right)^d}$$

and, for example,

$\frac{\partial f}{\partial E}$ represents the rate of change of this function with respect to a change in the voltage (dc) applied

ΔE represents a change or an error in the voltage (dc) applied.

The partial differentials ($\partial f/\partial$) are now found.

1. $$\frac{\partial f}{\partial E} = \frac{hD}{Ek} \cdot \frac{1 - d}{\left(\frac{vD}{\nu}\right)^c \left(\frac{EI}{\rho v^3 D^2}\right)^d} = k''' \frac{1-d}{E}$$

where ν = kinematic viscosity = μ/ρ

2. $$\frac{\partial f}{\partial I} = \frac{hD}{Ik} \cdot \frac{1 - d}{\left(\frac{vD}{\nu}\right)^c \left(\frac{EI}{\rho v^3 D^2}\right)^d} = k''' (1-d)/I$$

3. $$\frac{\partial f}{\partial T_t} = -k''' \left(1/(T_t - T_w)\right)$$

The three partial derivatives shown have been obtained in a straightforward manner. The next, $\partial f/\partial T_w$, is more complex, for k (thermal conductivity) and ν (kinematic viscosity) are a function of the temperature of the water.[7] These relationships are shown as a function of the water temperature in Figs. 24 and 25.

4. $$\frac{\partial f}{\partial T_w} = k''' \left(\frac{c}{\nu} \frac{\partial \nu}{\partial T_w} - \frac{\partial k/\partial T_w}{k} + 1/(T_t - T_w)\right)$$

5. $$\frac{\partial f}{\partial v} = k''' (3\, d/v - c/v)$$

6. $$\frac{\partial f}{\partial L} = -k''' \frac{4L}{(2L)^2 + \frac{D^2}{4}}$$

7. $$\frac{\partial f}{\partial D} = k''' \left(2\, d/D - c/D - \frac{D}{4\left[(2L)^2 + D^2/4\right]}\right)$$

Fig. 24.

Kinematic Viscosity of Water
vs. Temperature

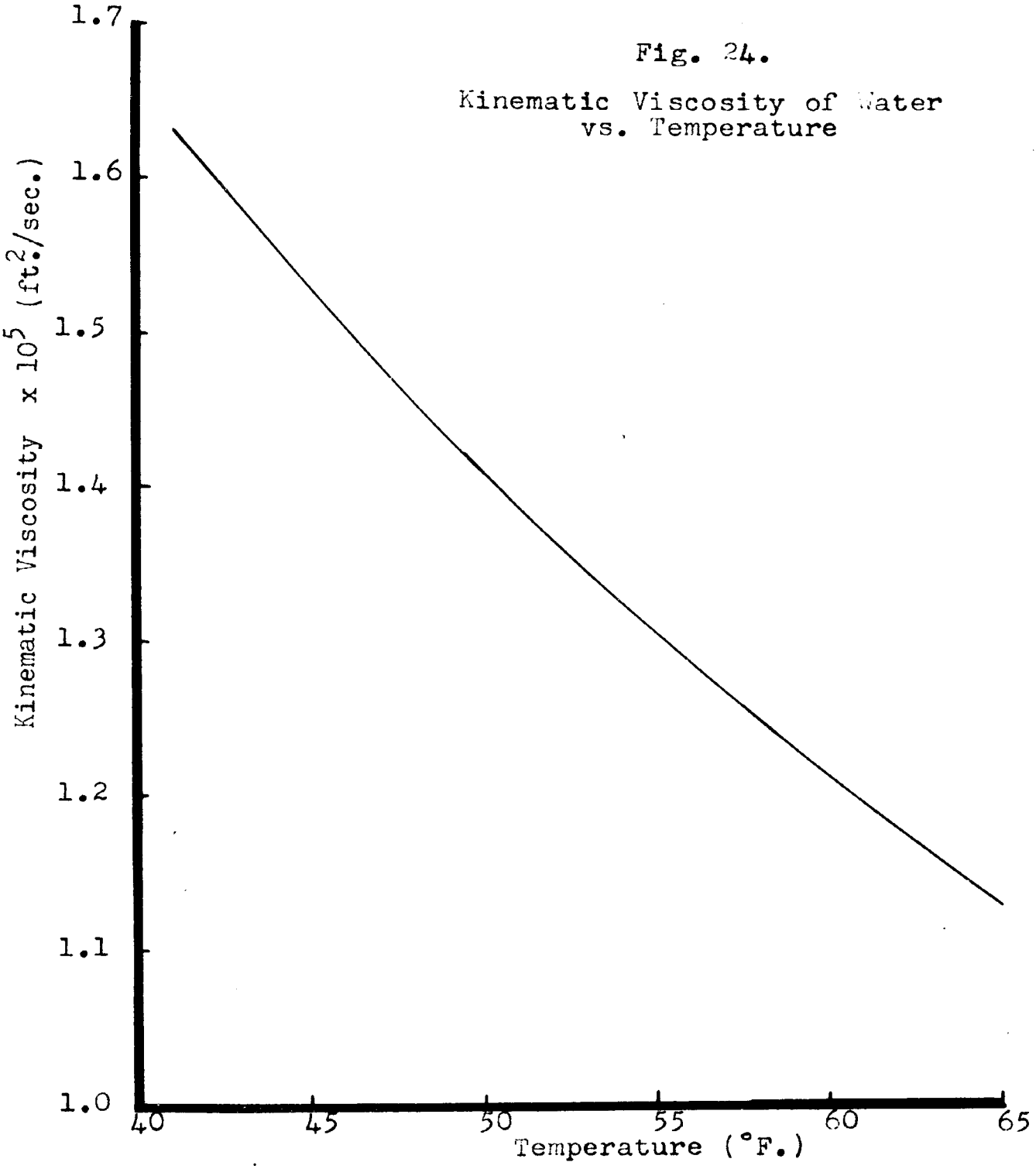
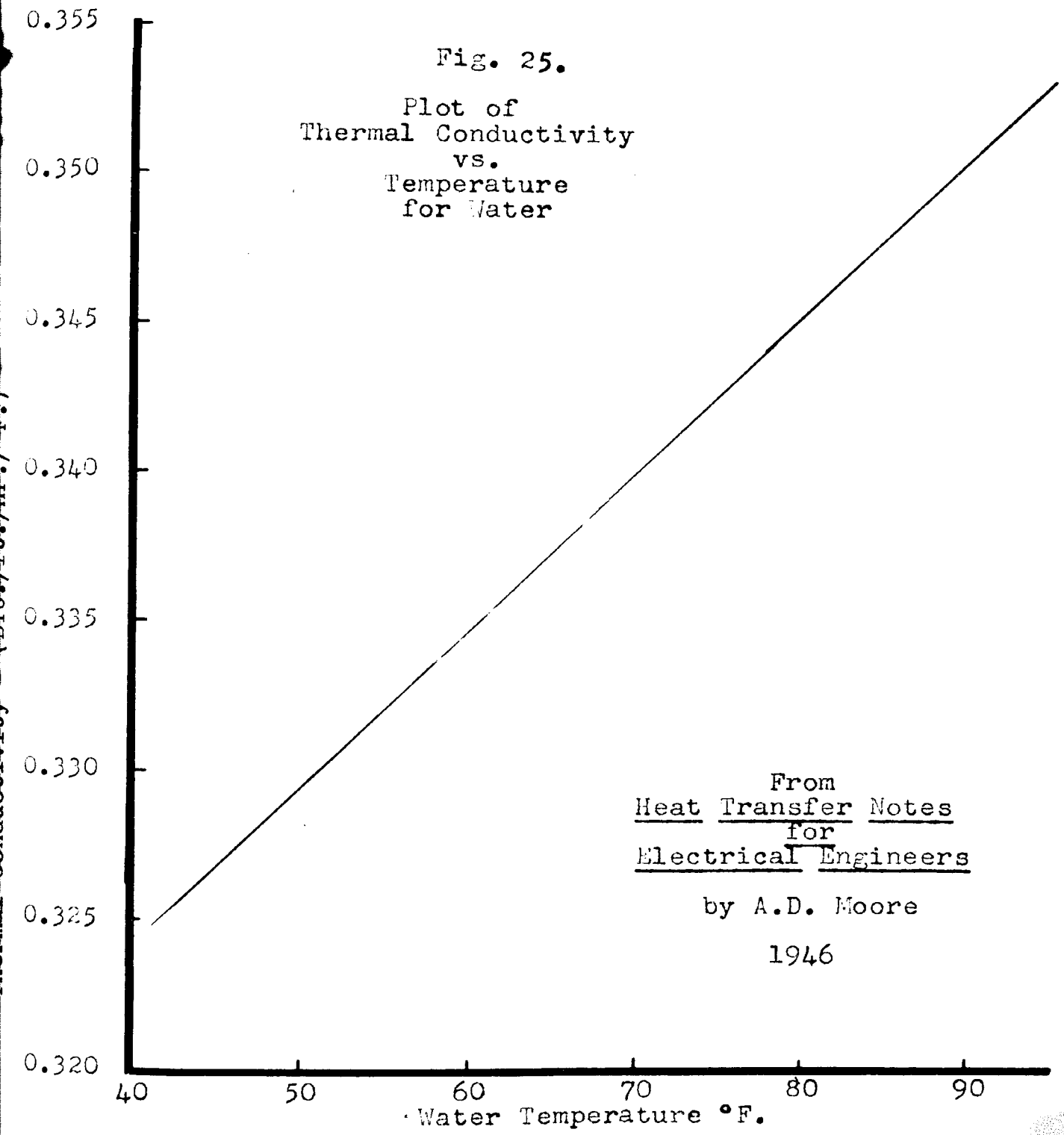


Fig. 25.

Plot of
Thermal Conductivity
vs.
Temperature
for Water



From
Heat Transfer Notes
for
Electrical Engineers

by A.D. Moore

1946

Now $\partial f/\partial E = k''' (1 - d)/E$ and $k'''_E = k''' \left[(1 - d)/E\right] \Delta E$

where $\Delta k'''_E$ means the error in k"' due to an error in the value of E. The voltage was measured by a Hickok vacuum-tube voltmeter (Type 110B) and the zero reading which could be adjusted by a panel control was rather sensitive. However, the maximum error should not have been greater than 1/2 volt in 25 volts

$$\Delta k'''_E = (1 - .81)(.02)k''' = 0.0038\ k'''$$

or about 0.38% error.

The error with respect to the current is

$$\Delta k'''_I = \left[(1 - d)/I\right] \Delta I$$

This quantity was read from the same type of meter, as a voltage across a 10,000 ohm resistor (nominal value). An average reading was about 1.5 volts, denoting a current of 1.5×10^{-4} amperes. A maximum error would be in the order of 0.02 volts, or $\Delta k'''_I$ equals 0.25% k"'.

The error due to an error in the measurement of the thermistor temperature may be expressed as

$$\frac{\partial f}{\partial T_t} = -k''' \frac{1}{T_t - T_w}$$

and

$$\Delta k'''_{T_t} = -k''' \frac{1}{T_t - T_w} \Delta T_t$$

Under certain circumstances an error of 1°F. in measuring T_t may result in an error, which is a multiple of k"'. If $(T_t - T_w)$ is large, i.e., at high power and/or low velocity, the error will be relatively small. Since the relationship

between the pulse impedance and the thermistor temperature is used to determine this temperature, any shift of this relationship, or a measurement error would result in an erroneous measurement. An estimate of an error of 1 degree in a temperature difference of 5 degrees, or a maximum error of 20% is considered reasonably correct.

An error due to an erroneous measurement of the water temperature can be found in the following manner:

$$\frac{\partial f}{\partial T_W} = k''' \left(c \frac{\partial \upsilon / \partial T_W}{\upsilon} - \frac{\partial k / \partial T_W}{k} + \frac{1}{T_t - T_W} \right)$$

and

$$\Delta k'''_{T_W} = k''' \left(c \frac{\partial \upsilon / \partial T_W}{\upsilon} - \frac{\partial k / \partial T_W}{k} + \frac{1}{T_t - T_W} \right) \Delta T_W \quad .$$

k, (shown in Fig. 24) may be approximated by the expression:

$$k = 0.324 + 0.00505 \ (T_W - 40) \ \text{BTU/ft./hr./}^{\circ}\text{F.},$$

and k at 52° F. is equal to 0.3305.

Kinematic viscosity may be approximated by the expression:

$$\upsilon = 3.04 \ e^{-.015 \, T_W} \ (\text{X } 10\text{-}5) \ \text{ft.}^2/\text{sec.}$$

A likely value for $(T_t - T_W)$ was considered to be approximately 5° when determining the error due to an erroneous measurement of the thermistor temperature, and thus

$$\Delta k'''_{T_W} = k''' \left[2.75(-0.0150) - (0.005/0.330) + 1/5 \right] \Delta T_W$$

$$= 0.144 \ k''' \Delta T_W .$$

The water temperature was measured with a mercury thermometer resting in the water that had passed through the tube. The thermometer was readable to approximately 0.1 of a degree. Thus an error $\Delta k'''_{T_W} = 1.44\% \ k'''$ could be expected.

The error due to an error in measuring velocity should be relatively small. Now

$$\partial f/\partial v = k''' (3d - c)/v$$

and

$$\Delta k'''_v = k''' (3d - c) \Delta v/v$$

since

$$d = 0.81 \text{ and } c = 2.75, (3d - c) = -0.32$$

and

$$\Delta k''' = -0.32 \Delta v/v \quad k'''$$

This measurement was made by determining the time necessary to collect a quantity of water (2000 cc.) flowing past the thermistor. A stop watch with a sensitivity of 0.2 of a second was used. The expression $v = 97.6$ ft./time (sec.) was used to calculate the velocity. The constant (97.6) was calculated from the diameter of the receptacle holding the thermistor and the quantity of water collected. The expression $\Delta k'''_v = -0.32\, k''' \Delta T/T$ would also be valid where T is the time measured. Let us assume an error of 0.2 of a second in a time of 50 seconds. Thus

$$\Delta k'''_v = -0.32\, k''' (0.2/50) = -0.32\, (.004)\, k''' = -0.128\%\, k'''$$

The maximum would be about 0.2 of a second in 11 seconds or

$$\Delta k'''_v = -0.58\% k'''$$

This value takes into account the operator's reaction time when collecting the water and any small variation of the actual velocity profile in the constriction from the theoretical profile.

The error due to an error in the length measurement of the probe is as follows:

$$\frac{\partial f}{\partial L} = -k''' \frac{4L}{(2L)^2 + D^2/4}$$

and

$$\Delta k'''_L = -k''' \frac{4L}{(2L)^2 + D^2/4} \Delta L$$

The most extreme condition would occur when $L \gg D$

$$\Delta k'''_L \simeq -k''' \frac{\Delta L}{L}$$

If the diameter (D) and the length (L) are equal, the term in the denominator due to the value of L is 16 times as great as that due to D. Thus the approximation is still valid.

Let us assume an error of 0.005 of an inch in a length of 0.050 of an inch.

$$\Delta k'''_L = -10\% \; k'''$$

This again is a maximum. This error will be a part of the total error, but since it is constant for all observations, it will not be a portion of any correlation error.

The error due to an error in the measurement of the diameter (D) is expressed by

$$\Delta k'''_D = k''' \left[(2d - c)/D - D/(D^2 + 16 L^2)\right] \Delta D$$

$$= -k''' \left[(1.13/D) + D/(D^2 + 16 L^2)\right] \Delta D$$

The last term is small compared with the others (unless L is equal to a small fraction of D) and therefore it will be neglected.

$$\Delta k''_D{}' = -1.13 \frac{\Delta D}{D}$$

An accurate assessment of this ratio is difficult to make. The value of D is given in the expression for k"' in three places: (1) vD/ν , (2) $EI/\rho v^3 D^2$, and (3) the area of the thermistor exposed. The major portion of this error is due to Reynolds' number (vD/ν), and the ratio $EI/\rho v^3 D^2$. Reynolds' number is related to the flow of water around the probe, and the factor $EI/\rho v^3 D^2$ is the ratio of the electrical power to the power of the water impinging on the probe. A consideration to be made is whether the diameter at the base of the thermistor bead or the diameter of the host probe should be used.

If the glass is etched away so that the slope from the bead diameter to the host probe diameter is gradual, one effective diameter may be used, but if the change from one diameter to the other is abrupt, another value may be needed. This is shown in Figs. 26 and 27 by two dimensional flow patterns, in which the dimensions representing the bead diameter and the host probe diameter are the same.

For these figures, the representation was designed so as to be geometrically similar to the thermistor probe in the calibration tube. The velocity of the fluid in the channel was varied so as to compare favorably to the flow in the tube with respect to Reynolds' number.

Fig. 26 shows that with a gradual change from the thermistor to the probe, the water follows the change rather closely, even at rather high velocities. When the change

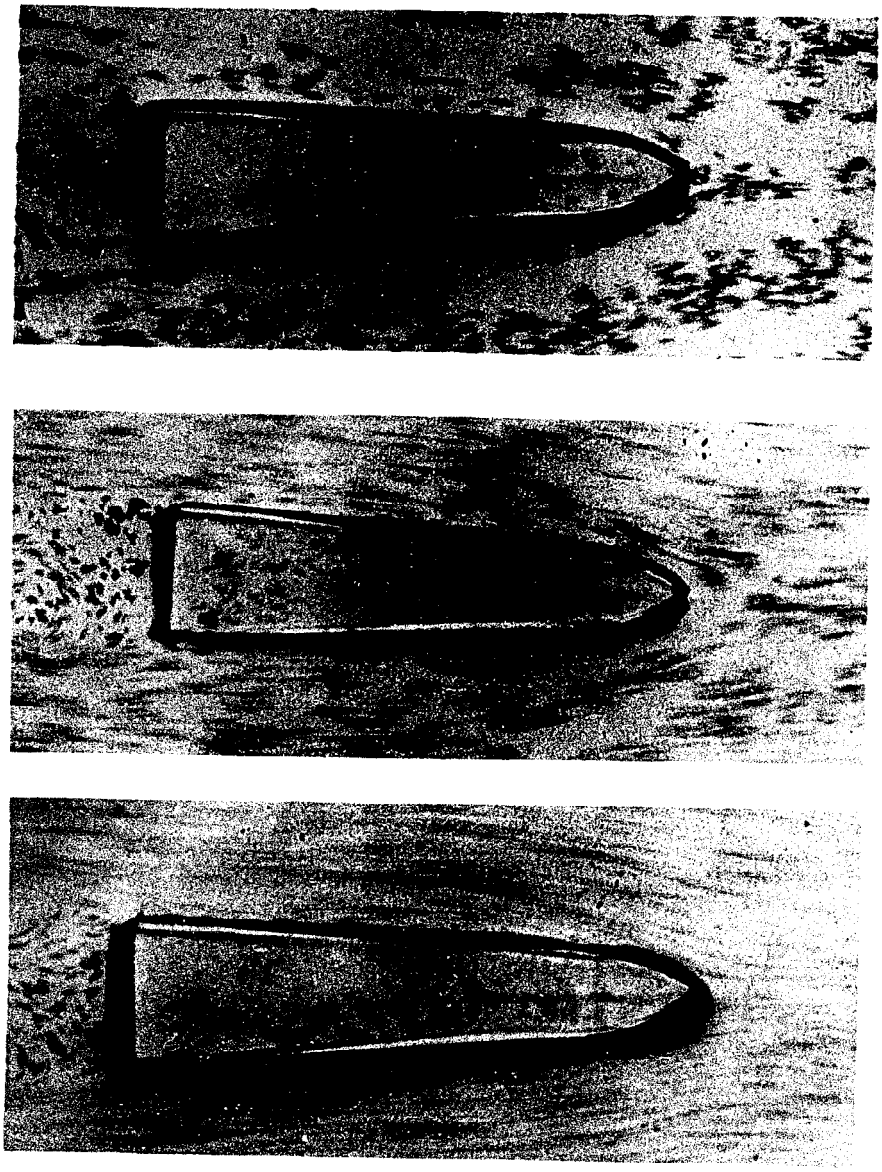


Fig. 26. Two Dimensional Flow Patterns.

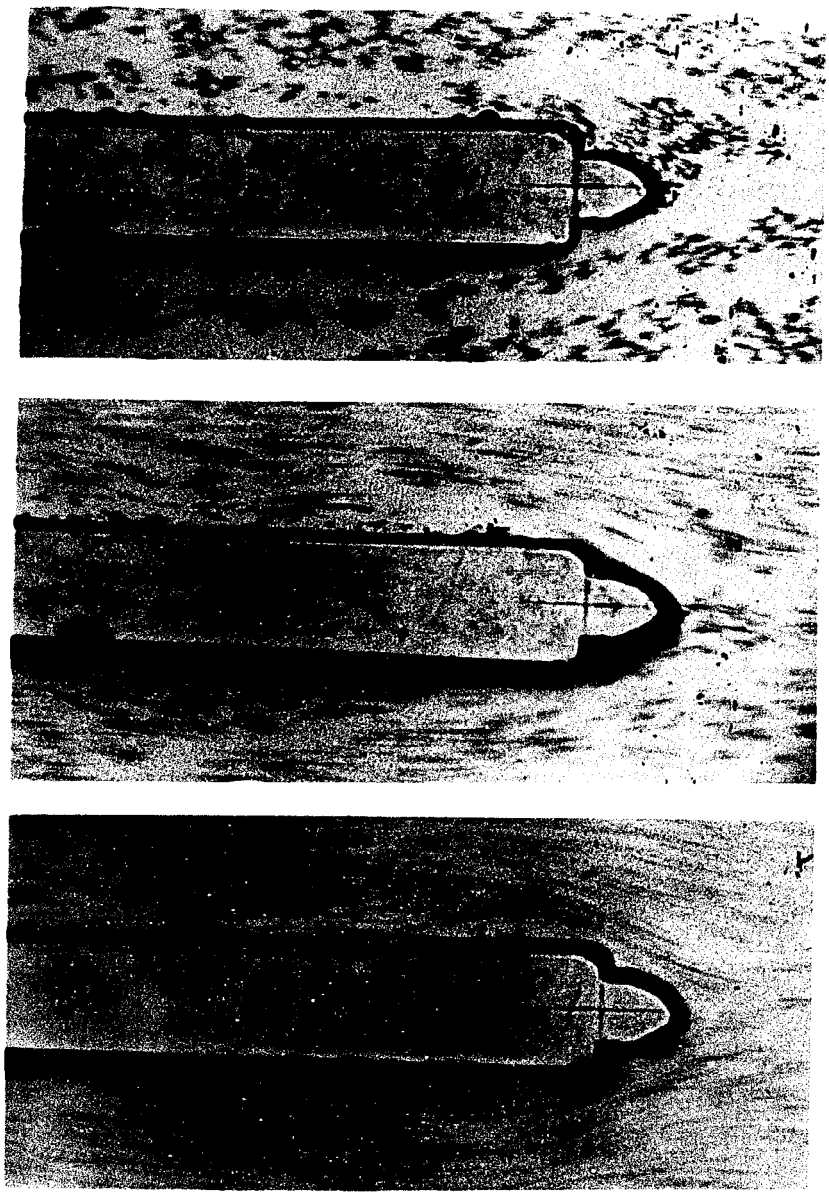


Fig. 27. Two Dimensional Flow Patterns.

from the probe to the thermistor bead is abrupt, some cavitation is formed along the host probe as shown in Fig. 27. This cavitation increases as the velocity increases. For the Principle of Similarity to be applicable, this cavitation must be considered when an effective diameter is assigned. The distance of maximum cavitation from the tip of the probe must vary with the velocity, in order to apply the Principle of Similarity.

The dimension at the base of the thermistor bead has been taken as the diameter. The change from the diameter of the thermistor to that of the host probe was kept gradual, probably somewhere between the limits shown by the two sets of photographs. An error of 0.002 of an inch will be assigned with a diameter 0.035 of an inch.

This results in an error of $-1.13 \times 0.002/0.035\ k''' $ or 5.4% k'''. This again was considered as a portion of the total error and constant for all observations.

The most probable error[16] for k''' is considered to be the square root of the sum of the squares of the individual errors due to a deviation from the true value of the various variables of the problem. Therefore, the most probable $\Delta k'''$ $(m.p.\ \Delta k)'''$ is found as follows:

$$m.p.\ \Delta k''' = \sqrt{(\Delta k'''_E)^2 + (\Delta k'''_I)^2 + (\Delta k'''_{T_t})^2 + (\Delta k'''_{T_w})^2 + (\Delta k'''_v)^2}$$

$$= 20.04\%\ k'''$$

The dominant error in the determination of k''', as indicated in Fig. 21, is the misrepresentation of the temperature of the thermistor when operated under flow. An accurate determination of the most probable error is nearly dependent on an accurate estimation of an error in this temperature. An estimation has been made that there may be a 1 degree error in a temperature difference of 5 degrees. The actual temperature differences have ranged from approximately 2° to 10° for thermistor No. 2 (in colder water and transferring a small amount of power) and from 10° to 40° for thermistor No. 1 (in warmer water and transferring considerably more power than thermistor No. 2). The error in temperature measurement includes a shift of the pulse impedance-temperature relationship as well as errors in actual measurement. After careful study, the 1° in 5° error figure was considered to be reasonable.

CHAPTER II.

A MATHEMATICAL MODEL OF THE THERMISTOR IN FLUID FLOW.

The preceding work found a means of expressing the various relationships between the electrical power being transferred and the velocity and temperature of the water. From this expression, the various types of measurement circuits may be examined to find under what conditions each may be superior. It should be emphasized that the figures given in this section are for a particular thermistor, and in this case thermistor No. 2.

This work has been based on the equation for heat transfer through forced convection.

$$Q = hA\Delta T$$

This equation may be rearranged as

$$\Delta T = Q/hA$$

Now

$$hD/k = k''' (vD/\nu)^{2.75} (EI/\rho v^3 D^2)^{0.81}$$

and

$$h = k''' (vD/\nu)^{2.75} (EI/\rho v^3 D^2)^{0.81} (k/D)$$

By substitution,

$$\Delta T = QD/Ak''' k (vD/\nu)^{2.75} (EI/\rho v^3 D^2)^{0.81}$$

By substituting the values of k''', A, D, for thermistor No. 2, and noting that Q = 3.4115 EI, then

$$\Delta T = \frac{3.4115D}{k''A} \frac{1}{\left(\frac{vD}{\nu}\right)^{2.75}} k \frac{EI}{\left(\frac{EI}{\rho v^3 D^2}\right)^{0.81}}$$

$$= 1.370 \times 10'' \frac{1}{\left(\frac{vD}{\nu}\right)^{2.75}} k \frac{EI}{\left(\frac{EI}{\rho v^3 D^2}\right)^{0.81}}$$

This equation shows the difference in temperature between the thermistor and the water necessary to transfer a certain quantity of power under specified conditions of water temperature and velocity. It may be represented graphically by plotting the power (EI) as the ordinate and the thermistor temperature (T_t) as the abscissa, with the water velocity as a parameter on different diagrams; the water temperature (T_w) was specified for each diagram. Graphical figures for these curves with water temperatures of 45°F., 52°F., and 60°F. are shown in Figs. 28 through 30. The curves which were calculated from this equation are labeled "power output curves". These curves relate the temperature difference necessary to transfer a given amount of power with the velocity and temperature of the water not only from this particular thermistor, but also from any solid of the same dimensions, with the same surface conditions, and held in the same manner. A method of introducing power, in the form of heat or otherwise, would have to be found. If the power were kept constant as the velocity of the water was varied, a line parallel to the base line and at the set power value could be introduced. This curve would intersect the "power output curves" and show the temperature difference necessary to transfer that amount of power. It

Fig. 28a.

Power Diagram for Thermistor
in Fluid Flow
Water Temperature 45 °F.

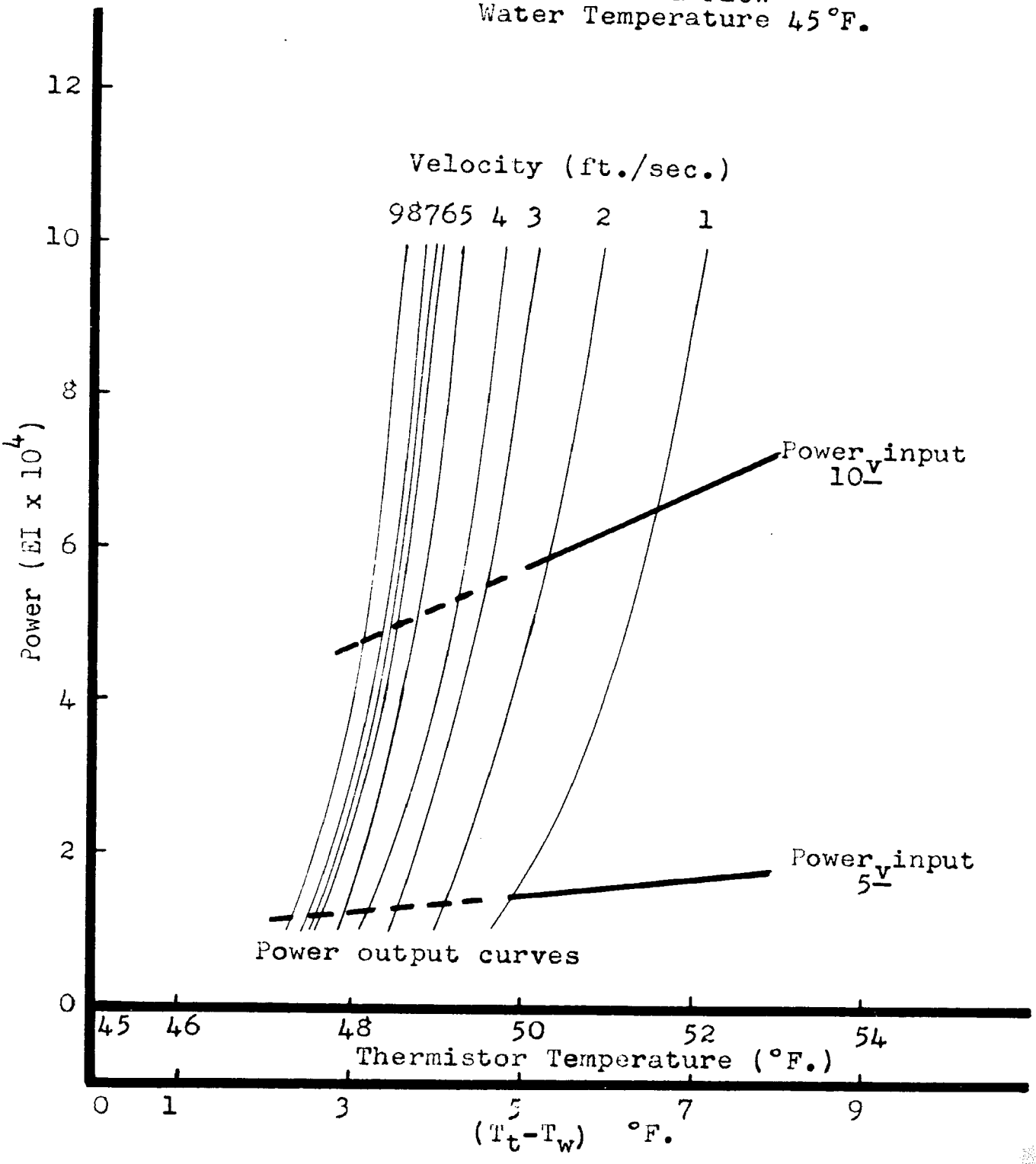


Fig. 28b.

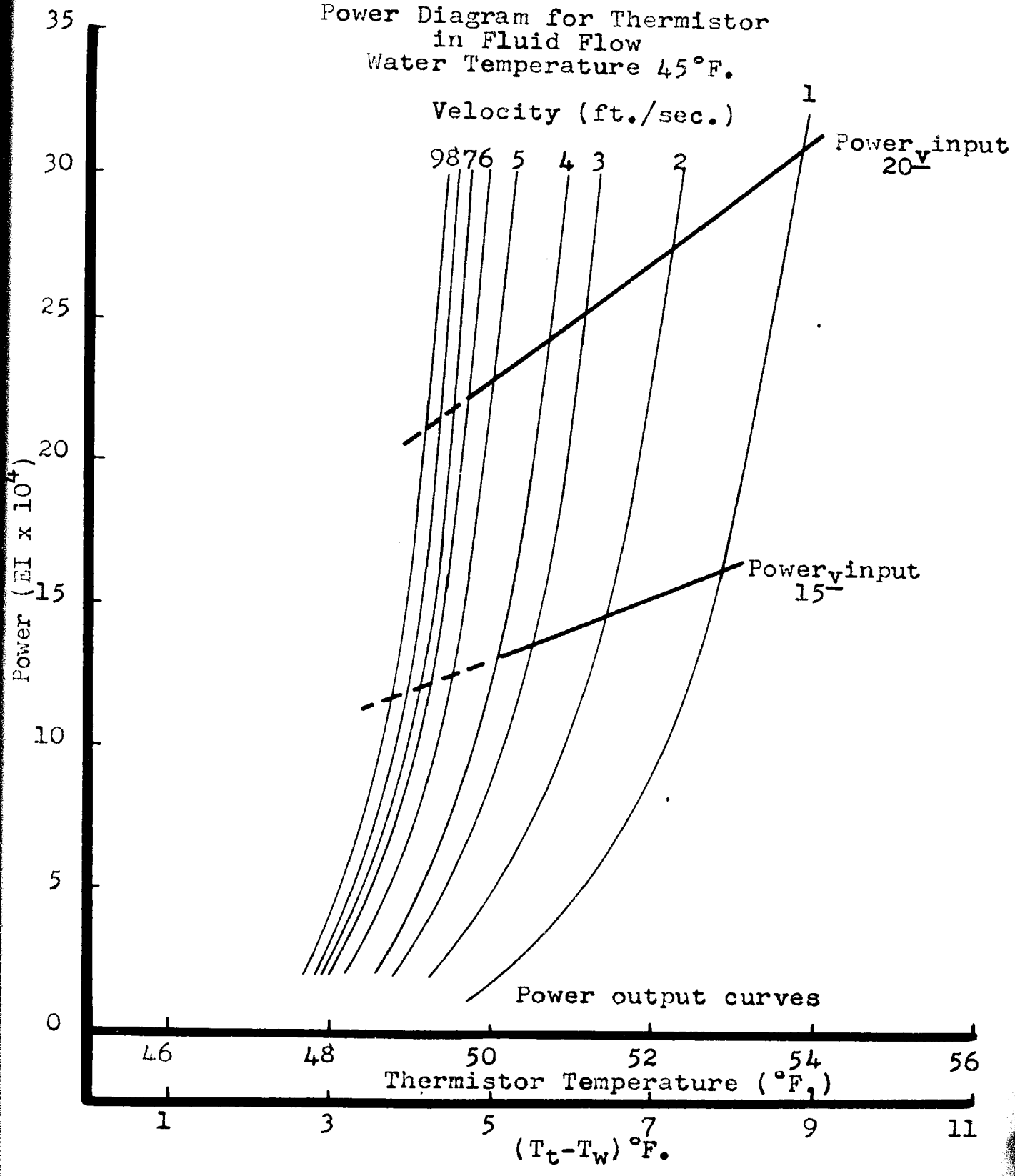

Power Diagram for Thermistor
in Fluid Flow
Water Temperature 45°F.
Velocity (ft./sec.)
1
9 8 7 6 5 4 3 2
Power input
20 V
Power input
15 V
Power output curves
Power (EI x 10^4)
35
30
25
20
15
10
5
0
46
48
50
52
54
56
Thermistor Temperature (°F.)
1
3
5
7
9
11
(T_t - T_w) °F.

Fig. 28c.

Power Diagram for Thermistor
in Fluid Flow
Water Temperature 45°F.

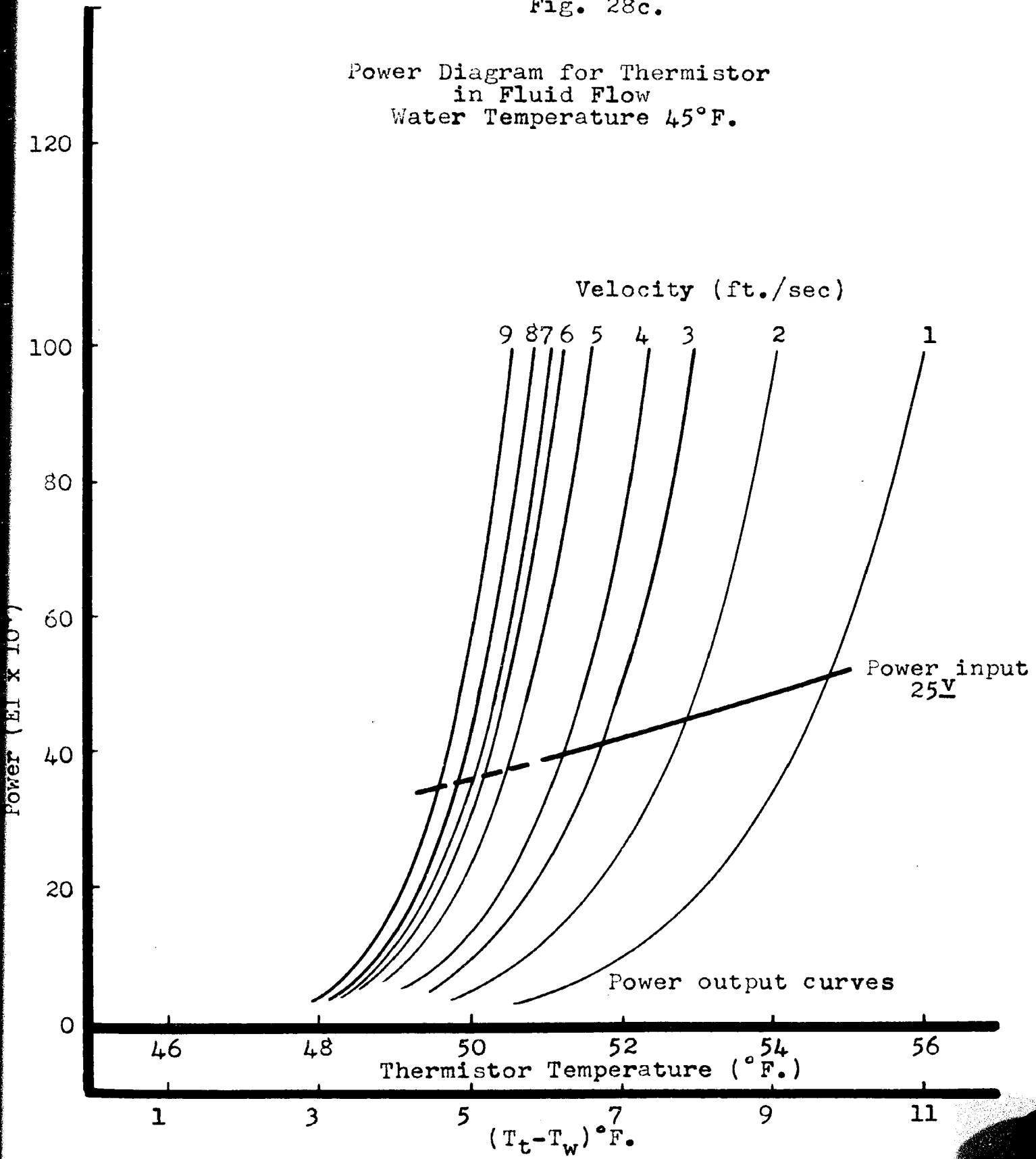
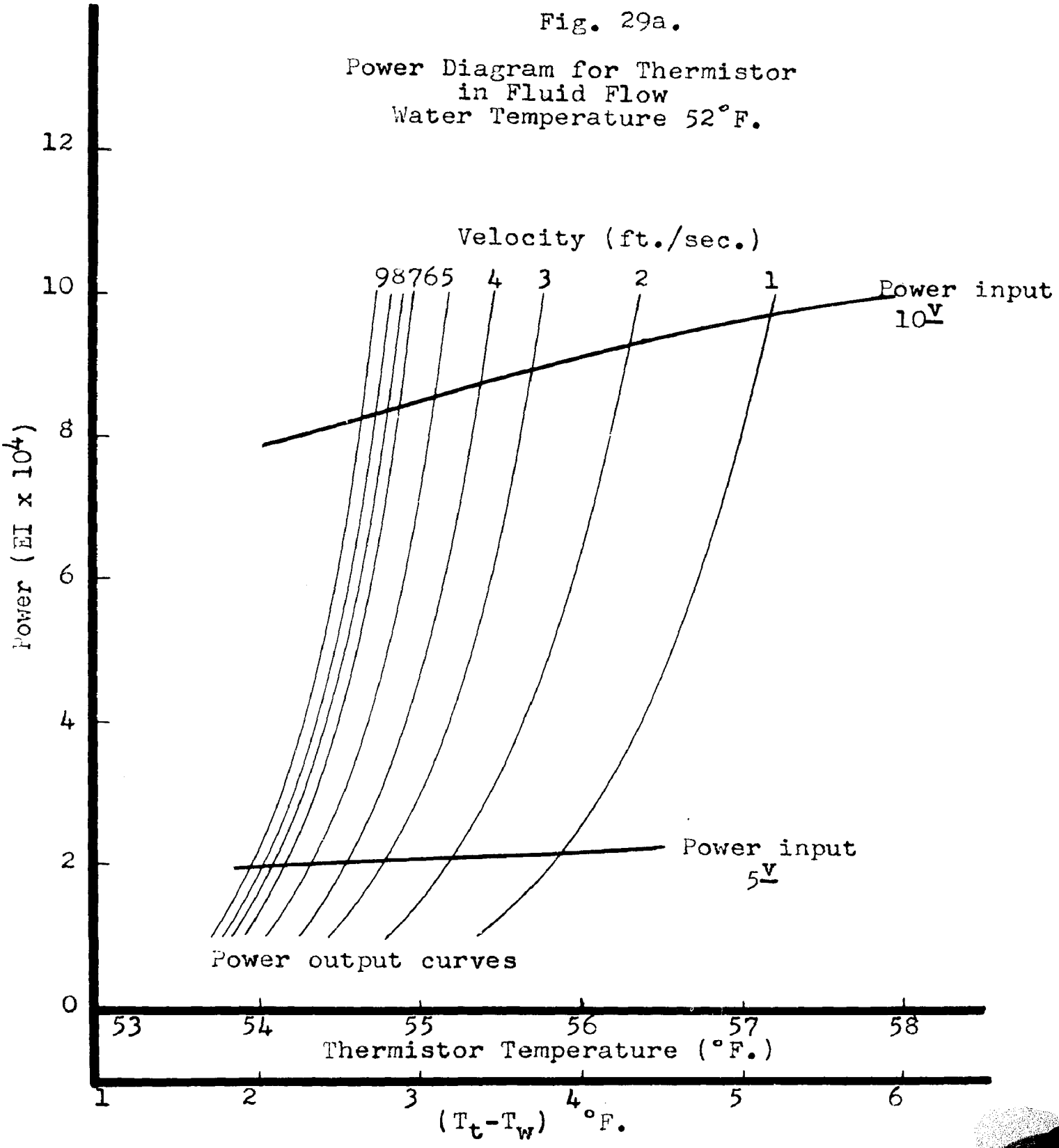


Fig. 29a.

Power Diagram for Thermistor
in Fluid Flow
Water Temperature 52°F.



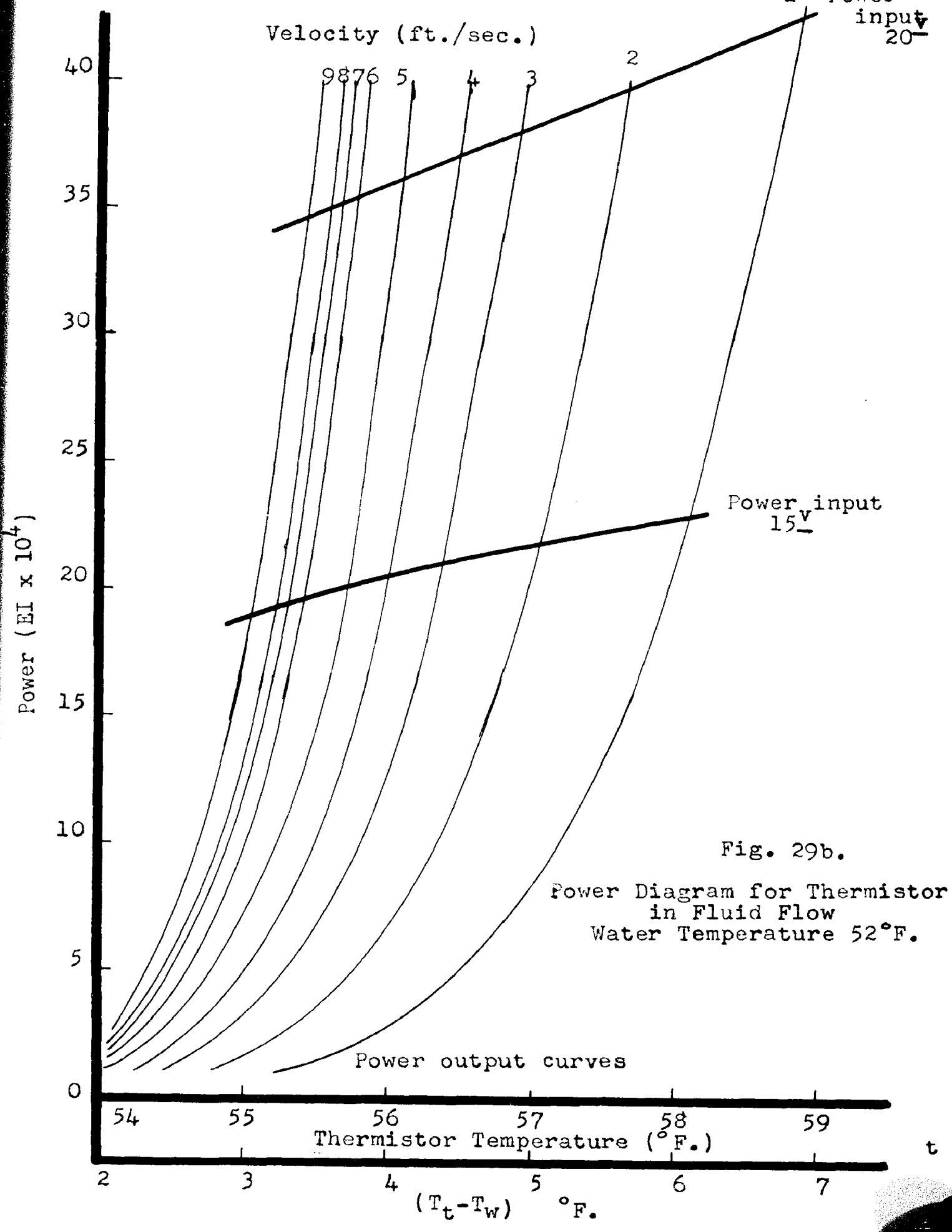
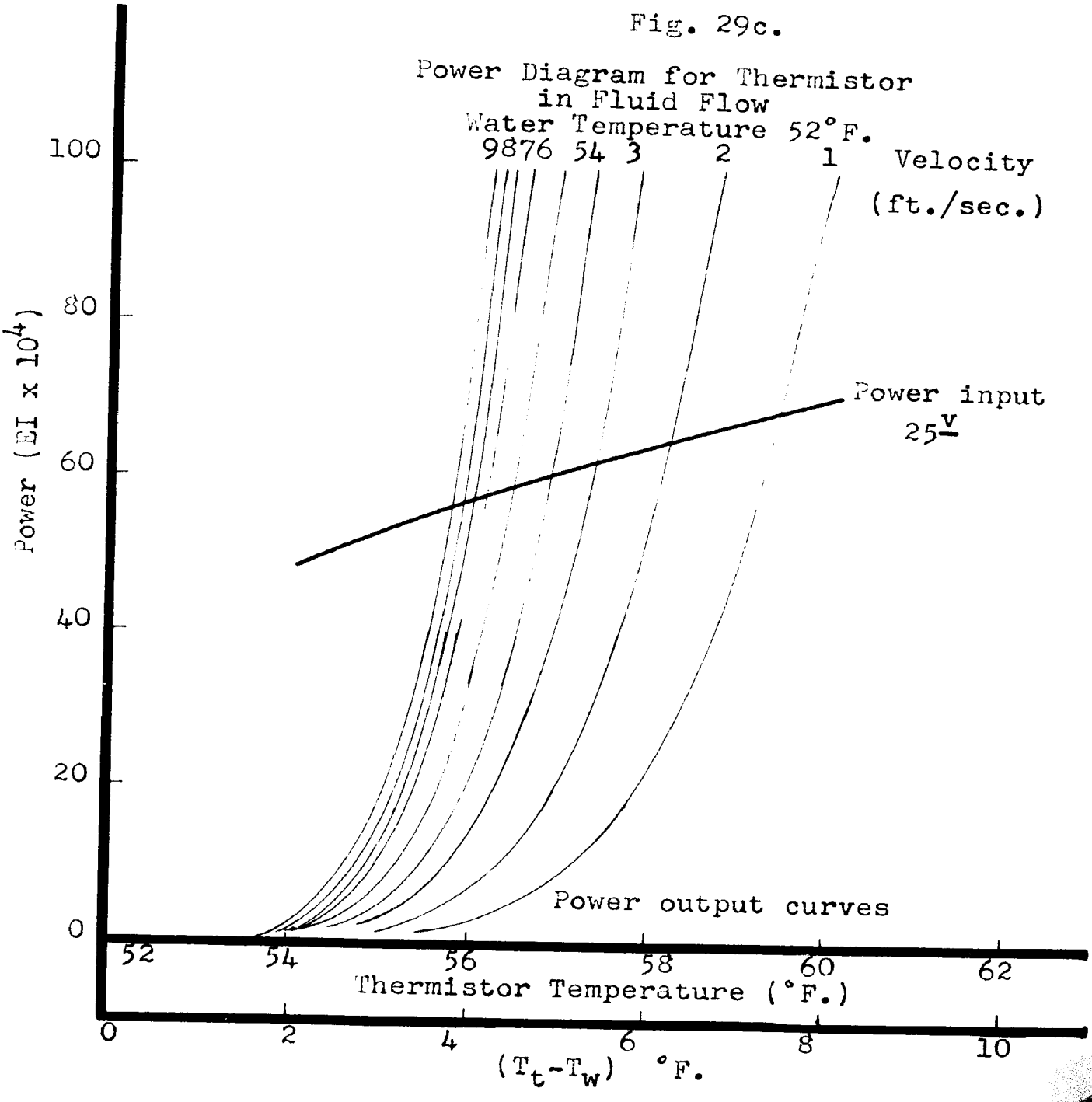


Fig. 29b.

Power Diagram for Thermistor in Fluid Flow
Water Temperature 52°F.

Fig. 29c.


Power Diagram for Thermistor
in Fluid Flow
Water Temperature 52°F.
9 8 7 6 5 4 3 2 1 Velocity
(ft./sec.)
Power input
25 V
Power output curves
Power (EI x 10⁴)
100
80
60
40
20
0
52
54
56
58
60
62
Thermistor Temperature (°F.)
0
2
4
6
8
10
(T_t - T_w) °F.

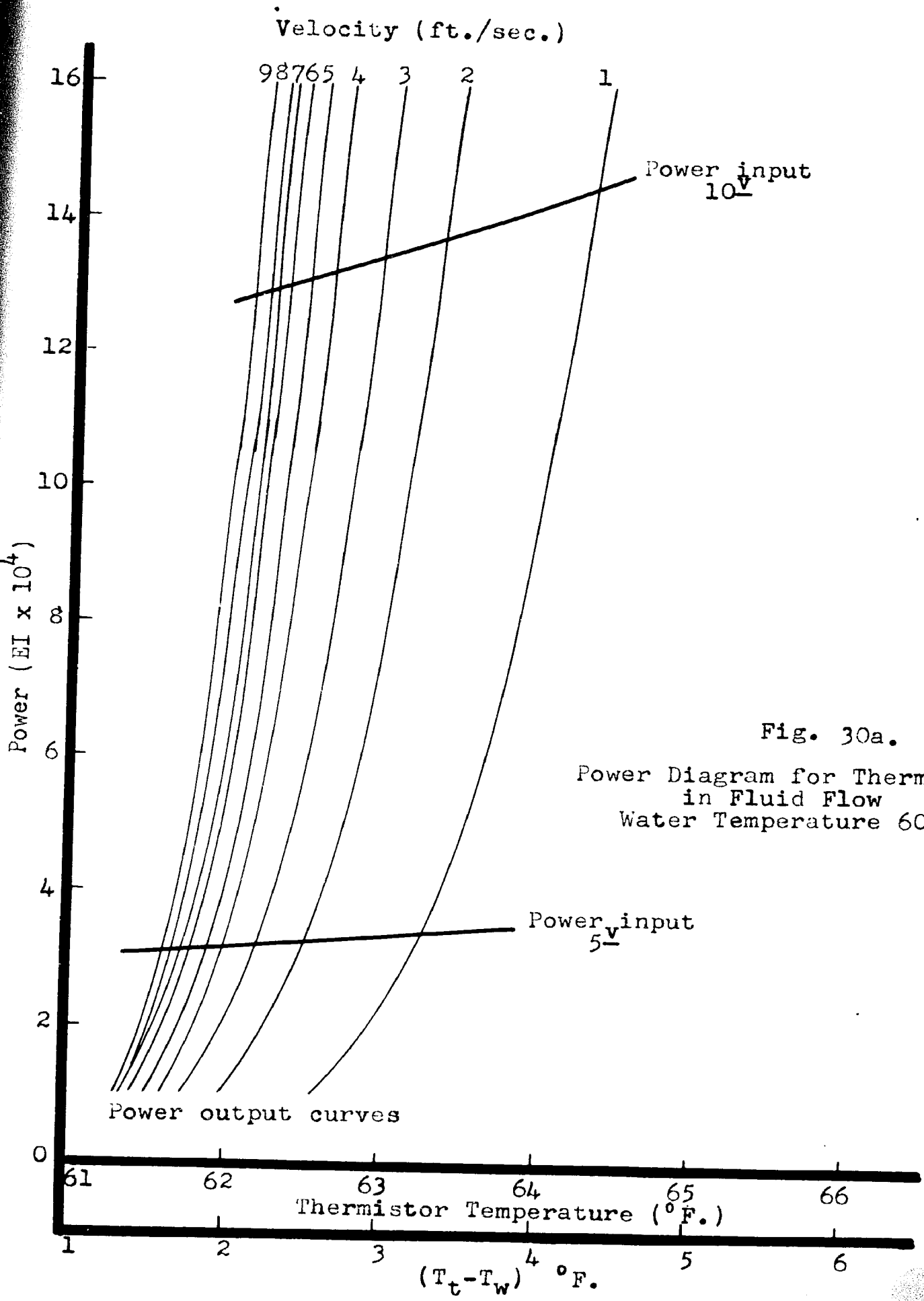
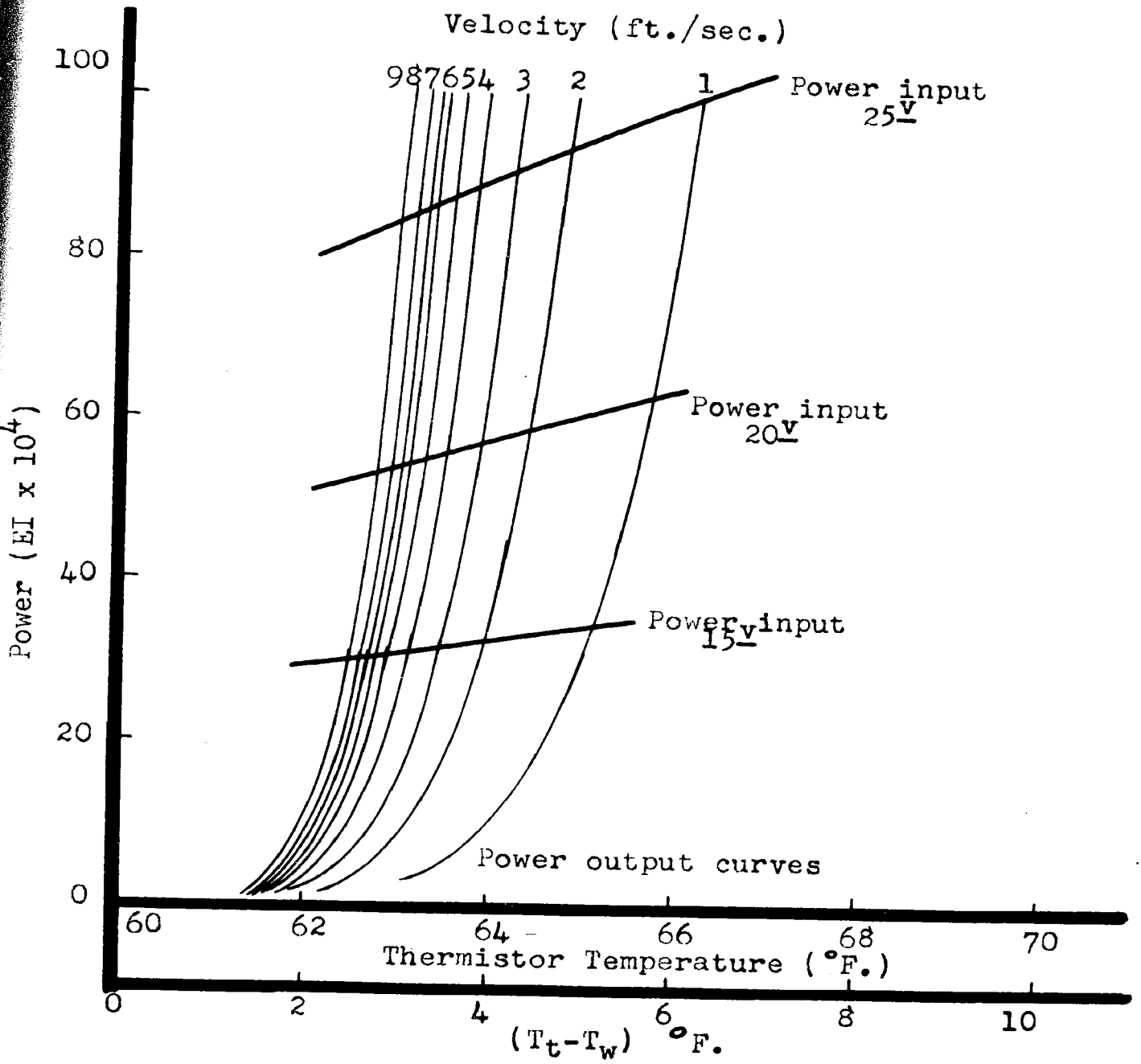


Fig. 30a.

Power Diagram for Thermistor in Fluid Flow
Water Temperature 60°F.

Fig. 30b.

Power Diagram for Thermistor
in Fluid Flow
Water Temperature 60°F.



would also show how this temperature difference varies as the velocity varies. Thus, any means introduced into the solid to record the temperature difference would fulfill the requirements of a successful velocity meter.

For thermistor No. 2, the "heat input" curves are shown when 5^V, 10^V, 15^V, 20^V, and 25^V (dc) are applied to the thermistor under flow conditions. The power is calculated from the equation

$$P = E^2/R_t$$

where P = power (watts)

E = voltage applied (volts)

R_t = resistance of the thermistor (ohms)

The relationship between the resistance of the thermistor and the thermistor temperature is shown in Fig. 31, which was constructed from data taken from Table II, page 127.

Certain general conclusions can be drawn from an examination of these figures. First, the thermistor is able to transfer considerable heat to the surrounding water without raising its own temperature markedly. Secondly, a large amount of power is not necessary to obtain a temperature difference sufficient to give sensitivity to the instrument. Experience has shown that if the thermistor is not subjected to high amounts of power, it is more stable. Thirdly, the temperature difference between the thermistor and the water increases as the power applied increases. Fourthly, this temperature difference also increases as the velocity decreases. Fifthly, under the

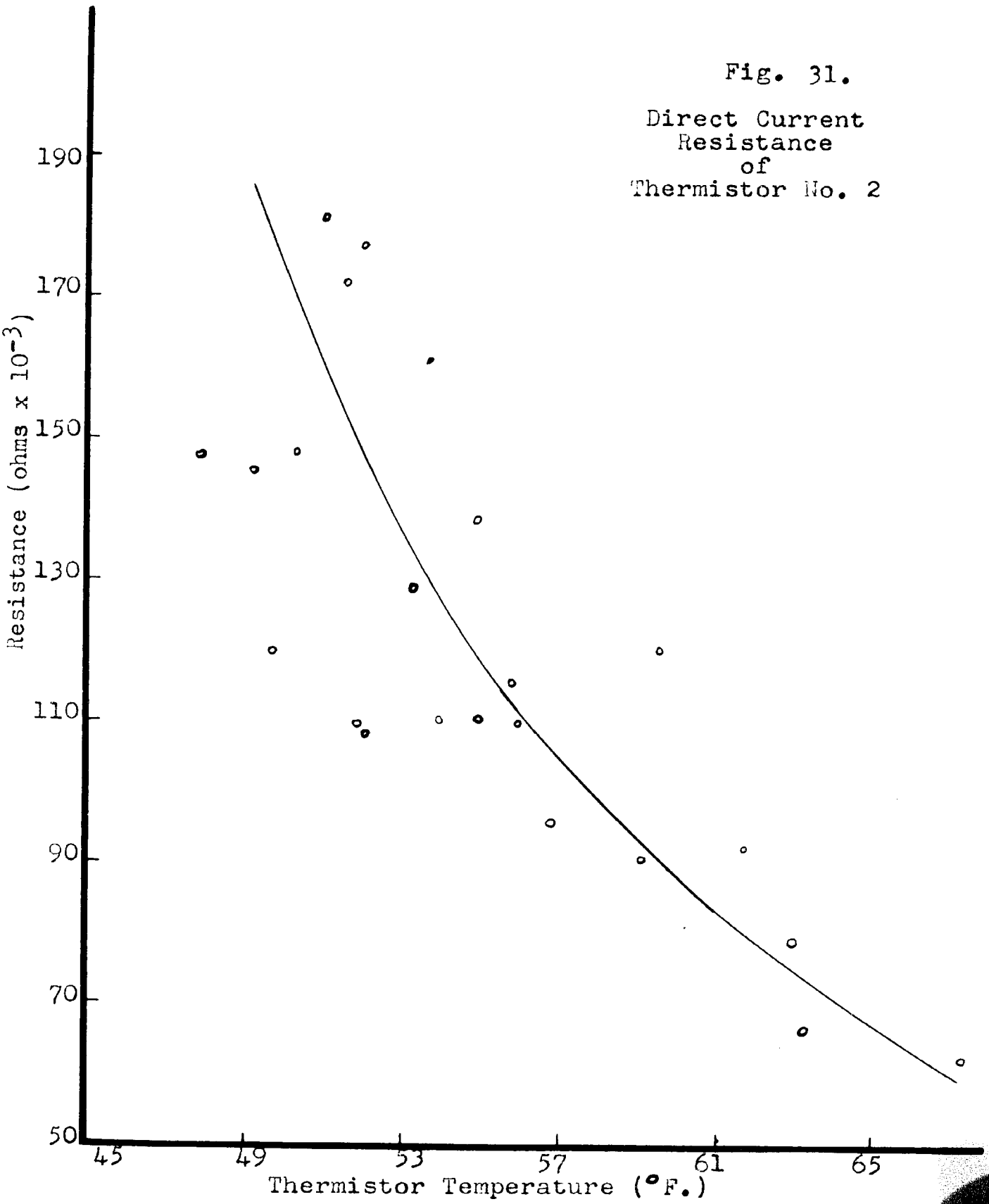


Fig. 31.

Direct Current Resistance of Thermistor No. 2

same conditions of power and velocity, it appears that the difference between the thermistor temperature and water temperature decreases as the water becomes warmer due to the decrease in viscosity and the increase in thermal conductivity. There are two ways of increasing the sensitivity of the instrument: (1) increasing the power to increase the temperature difference, and/or(2) increasing the thermal coefficient of resistivity with respect to the temperature of the thermistor. Both ways present some difficulties. Another method, however, is the amplification of the response by external amplifiers.

From diagrams similar to Figs. 28 through 30, which were drawn for different water temperatures from 45°F. to 60°F., the dc voltage-current relationships for the thermistor were obtained. Figs. 32 and 33 are representative of these relationships. The water velocity was considered constant for each figure and the water temperature was used as a parameter for each curve in the figure. The velocity was varied graphically from 1 to 9 ft./sec..

An examination of these curves shows that a non-linear relationship between the voltage and current exists. As the velocity increases, the entire set of curves tends to "squeeze" toward lower current values.

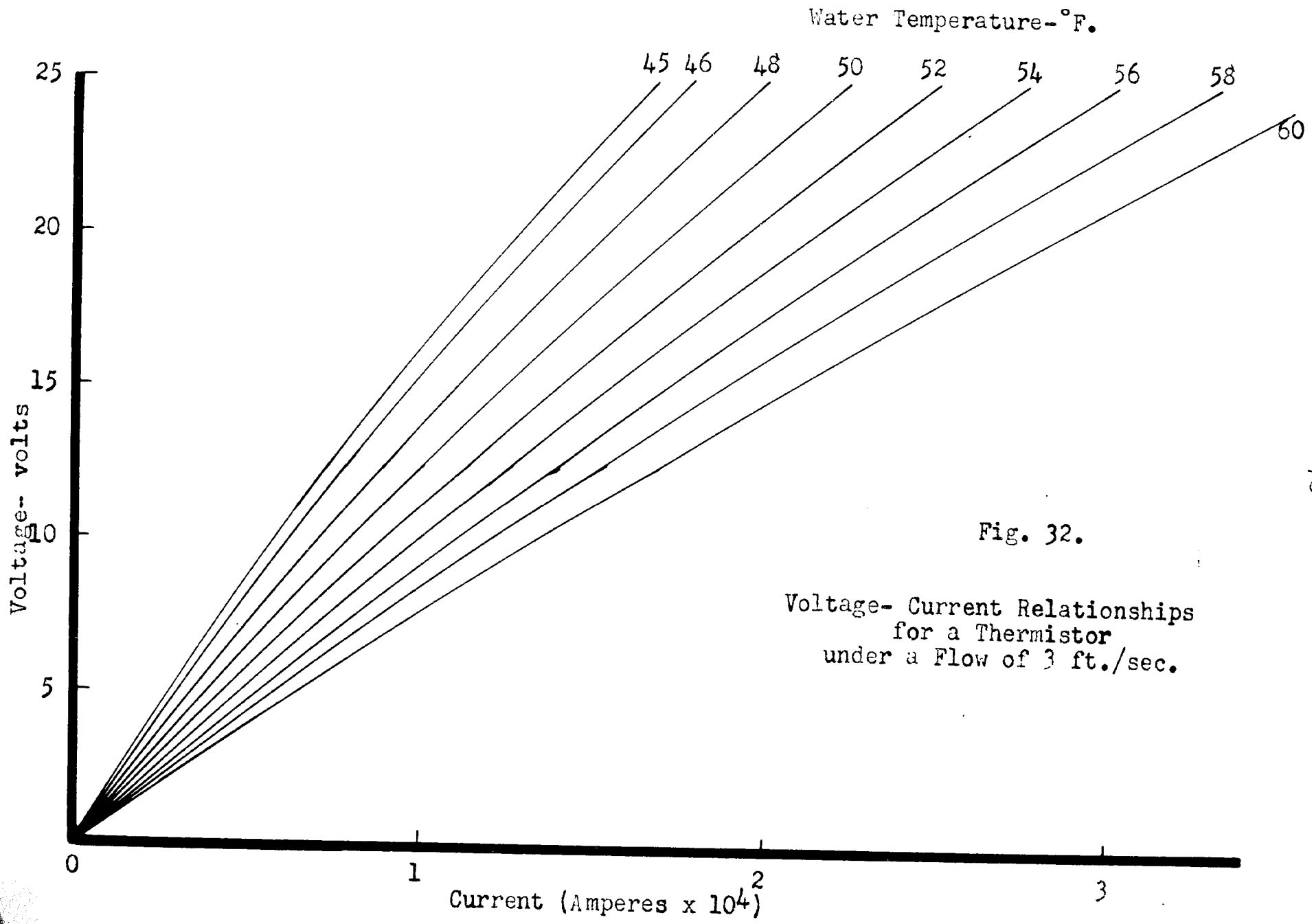


Fig. 32.

Voltage- Current Relationships
for a Thermistor
under a Flow of 3 ft./sec.

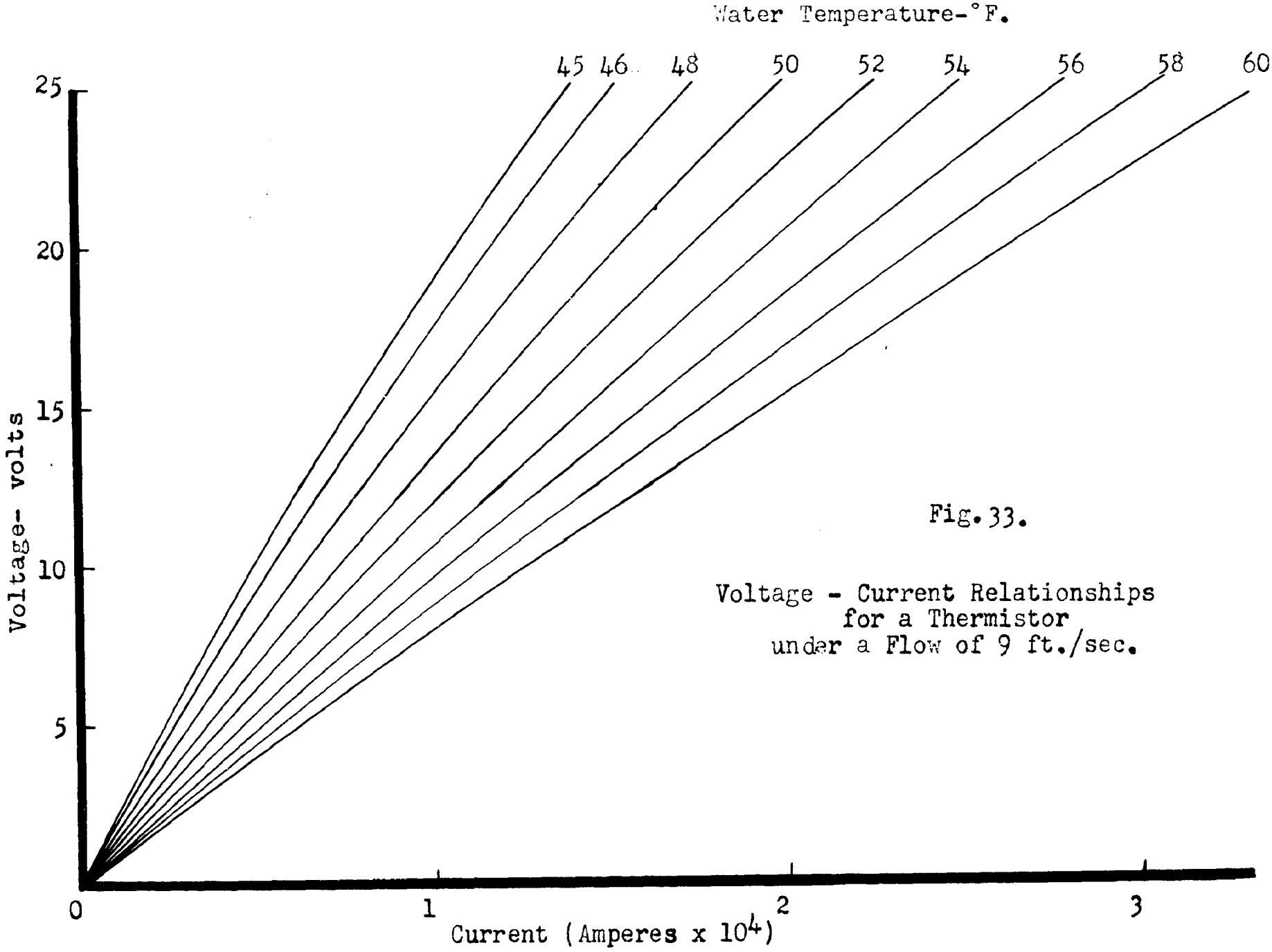


Fig. 33.

Voltage - Current Relationships
for a Thermistor
under a Flow of 9 ft./sec.

RELATIONSHIPS BETWEEN THE CURRENT FLOWING THROUGH THE THERMISTOR AND THE VELOCITY AT DIFFERENT WATER TEMPERATURES WHEN THE VOLTAGE IS HELD CONSTANT.

Using these voltage-current curves and voltages of 5^V, 10^V, 15^V, 20^V, and 25^V, graphical relationships were obtained which show the current flowing through the thermistor as a function of velocity with the water temperature as a parameter. Figs. 34 and 35 are representative of this set. These curves show the weakness of this type of calibration. The difference in the amount of current flowing through the thermistor is as great when the water temperature varies 2 degrees as when the velocity is varied through the entire range of 1 to 9 ft./sec.. The sensitivity at the higher velocities is rather low and decreases as the water becomes warmer. This type of calibration lends itself to a situation where the velocity can be held constant for a period of time, but during this period the water temperature varies and this variation cannot be controlled by the operator. This type of calibration curve is shown in Fig. 36. Figs. 37 and 38 show the relationship between water temperature and velocity under constant power conditions. They also show the error that would occur with this type of measurement when the temperature of the water is varied. If the temperature varies a degree or so, the error can be appreciable. For example, with 15^V and $1.25 \times 10^{-4}A$ current, the velocity recorded would be 1 ft./sec. with a water temperature of 48^oF; 1.65 ft./sec. with a water temperature of

Fig. 34.

Current Flowing
in Thermistor with
10 Volts Applied

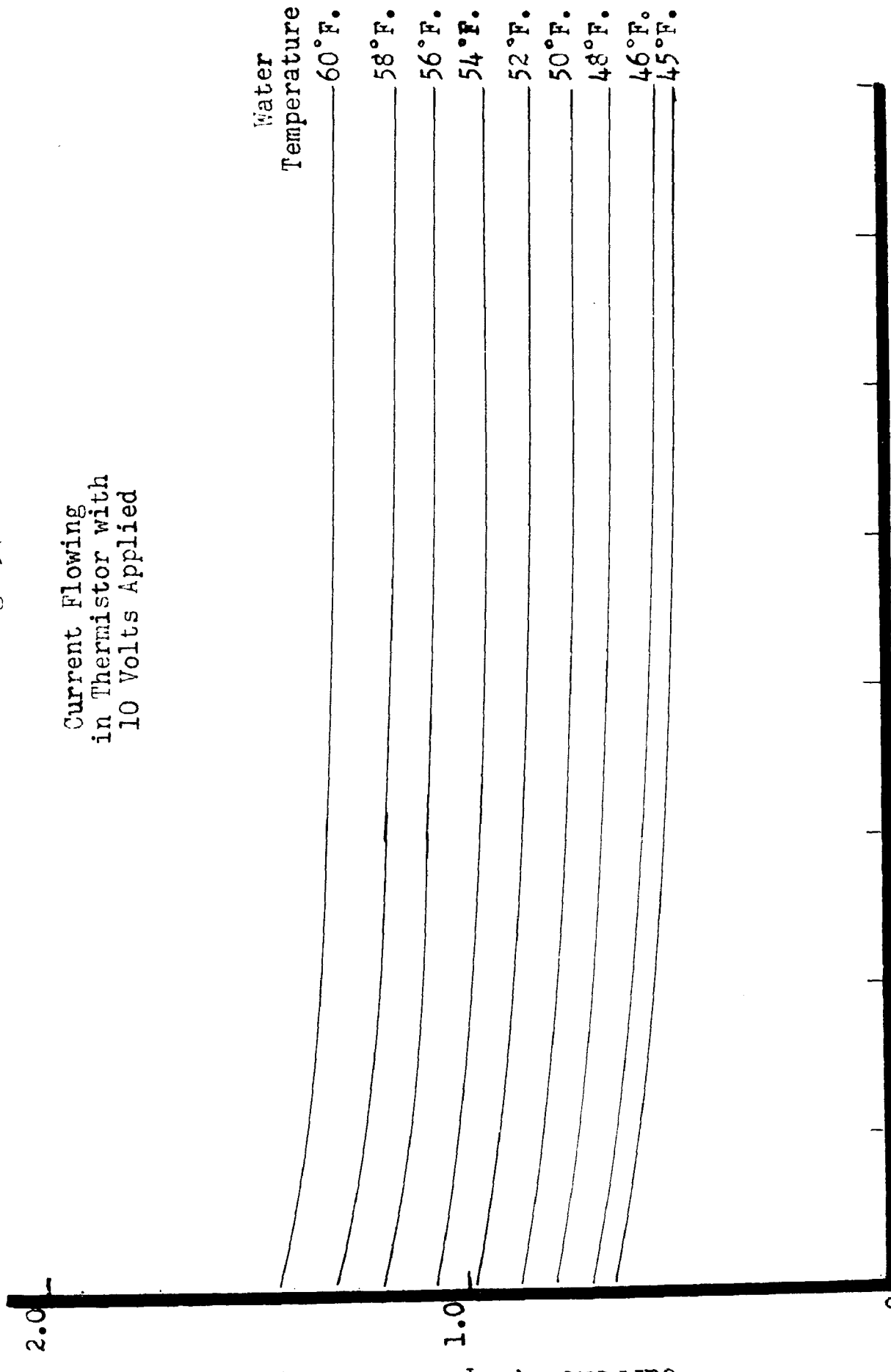
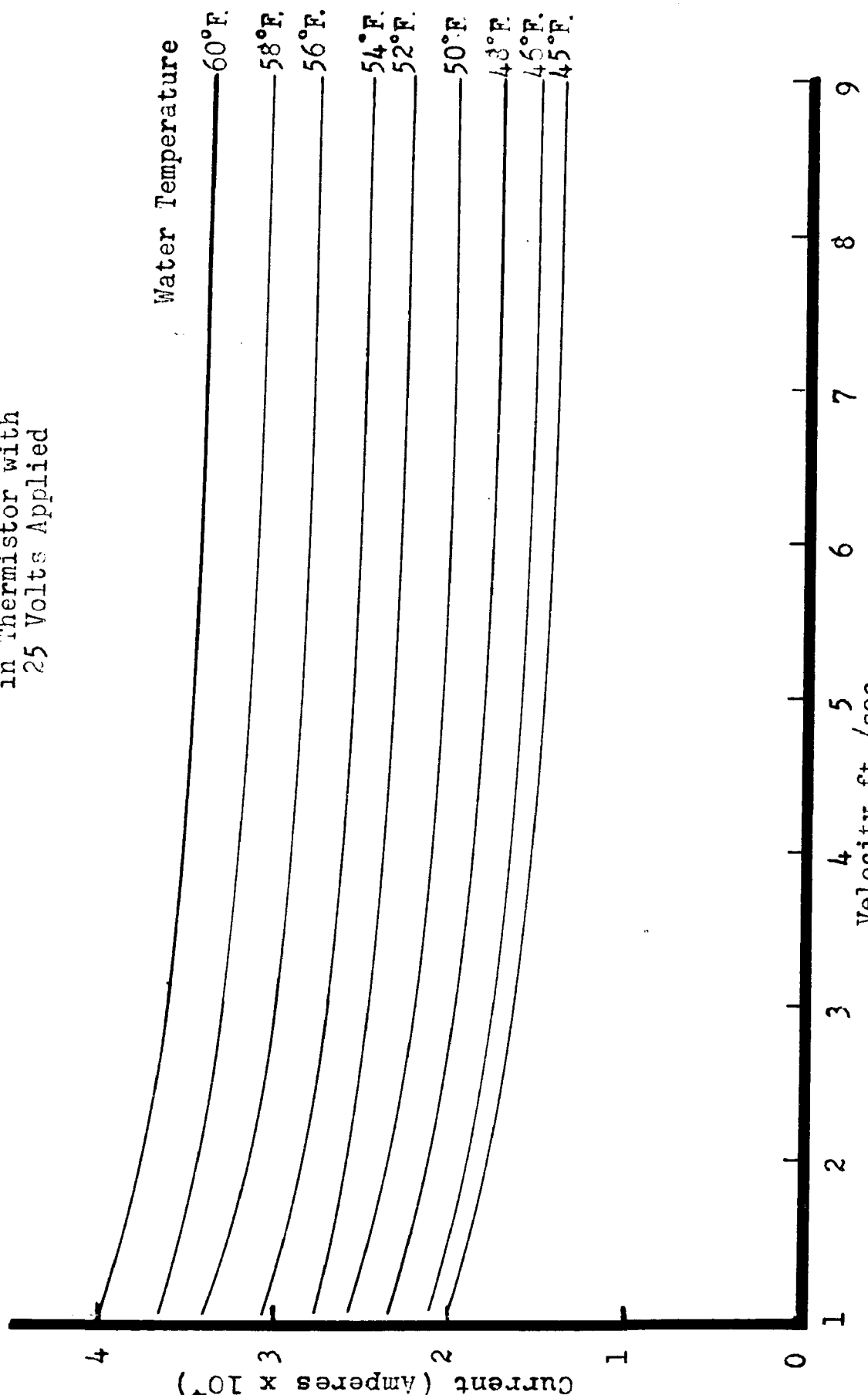


Fig. 35.

Current Flowing
in Thermistor with
25 Volts Applied



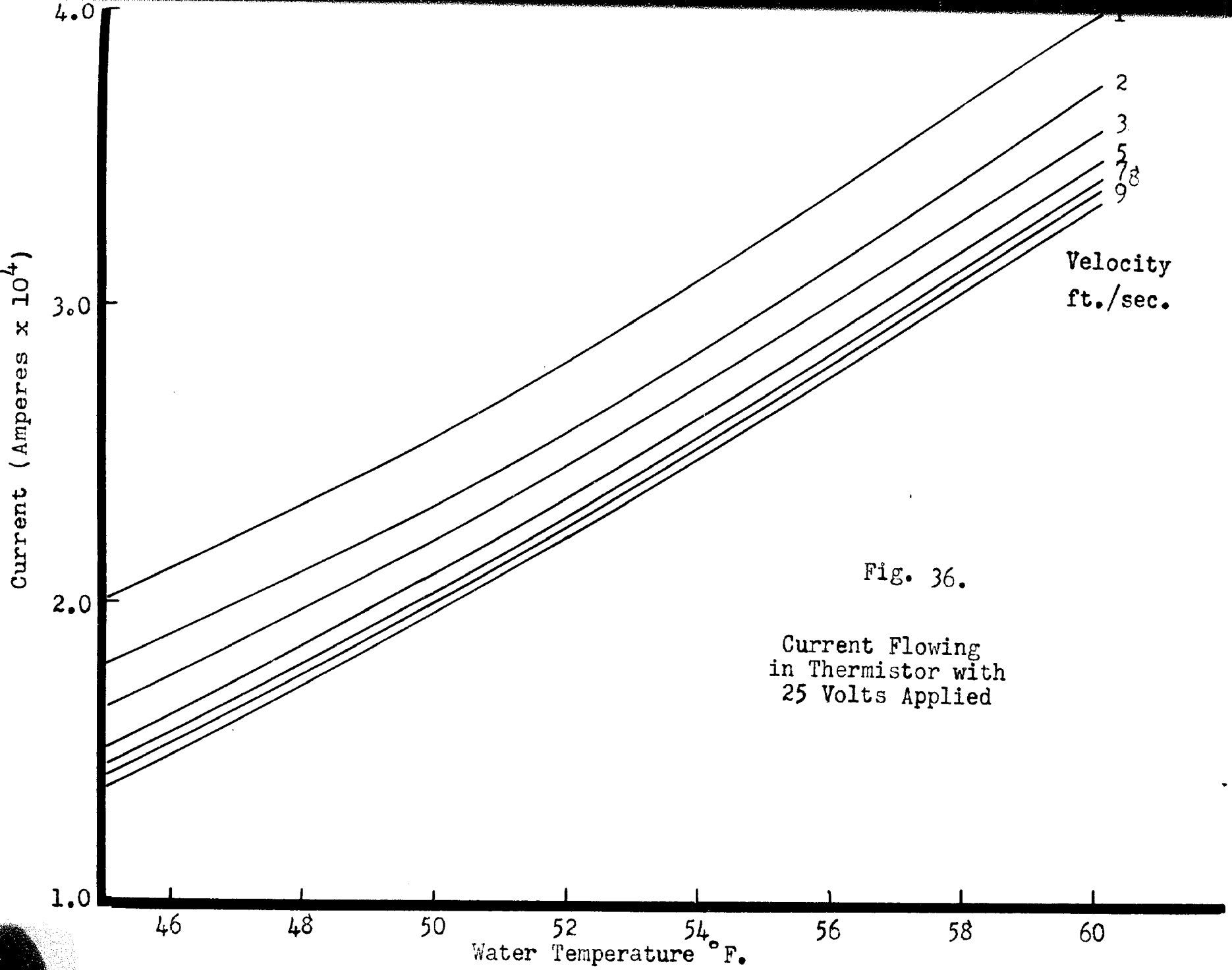


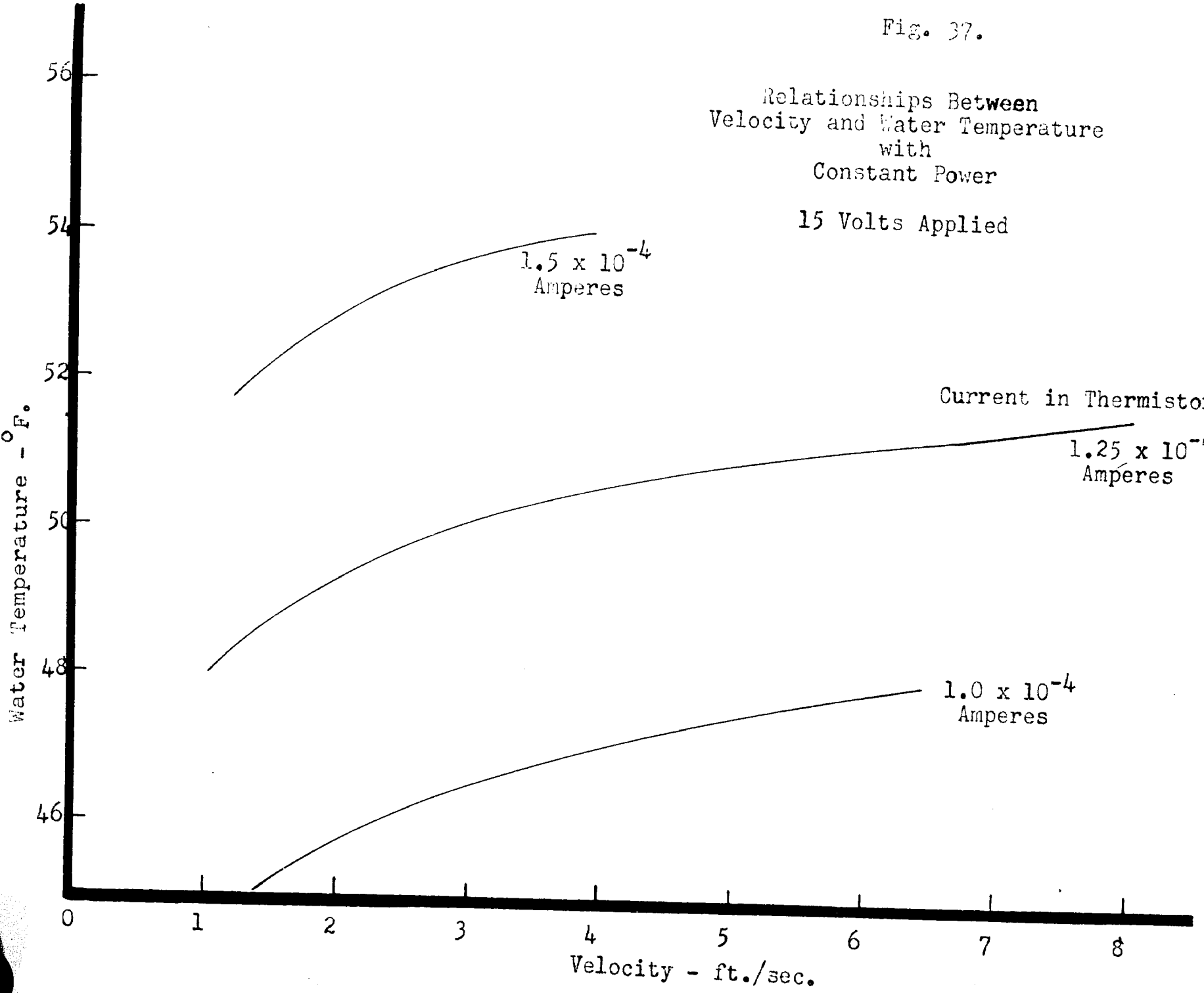
Fig. 36.

Current Flowing
in Thermistor with
25 Volts Applied

Fig. 37.

Relationships Between
Velocity and Water Temperature
with
Constant Power

15 Volts Applied



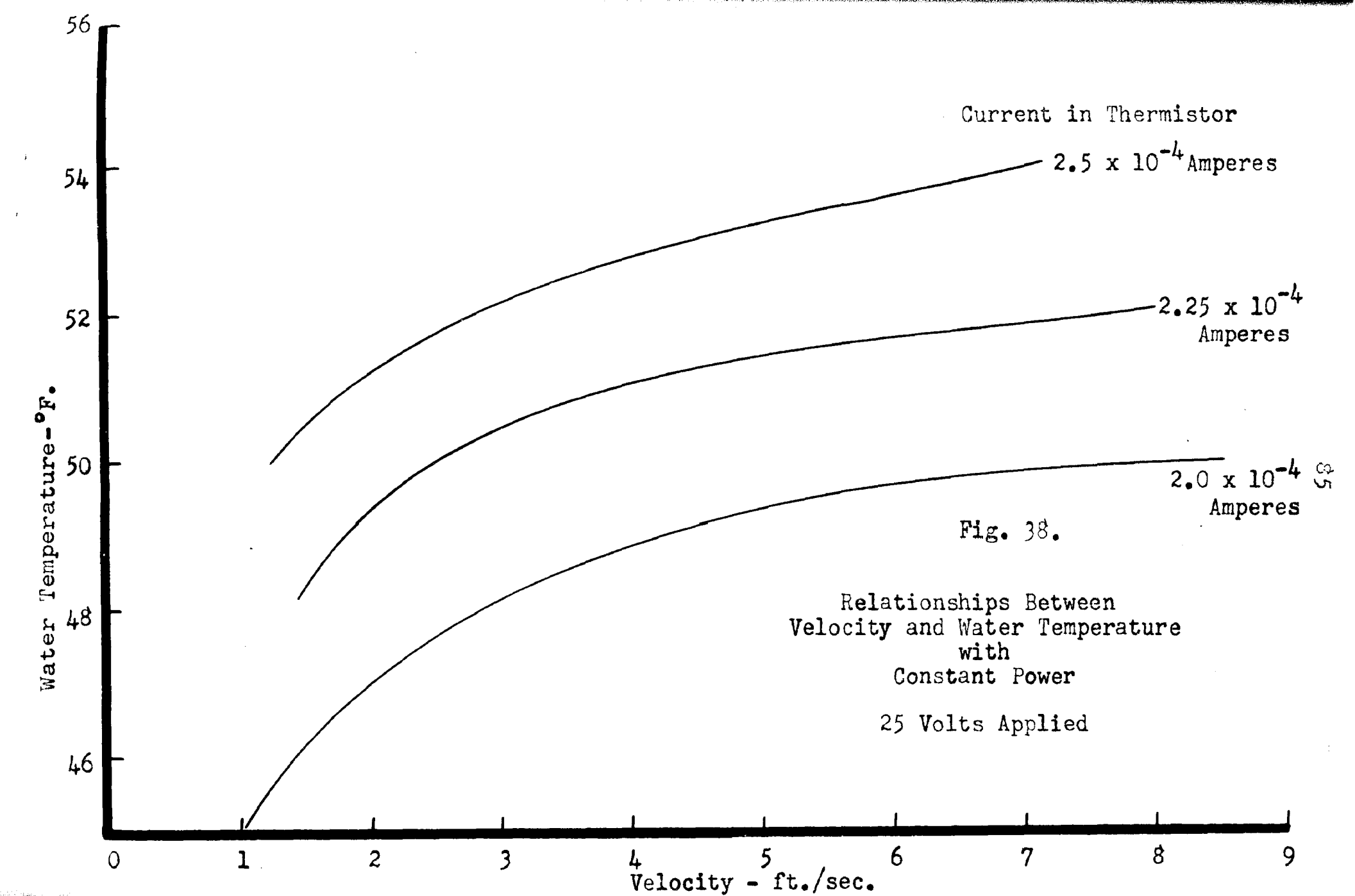


Fig. 38.

Relationships Between
Velocity and Water Temperature
with
Constant Power

25 Volts Applied

49°F; 2.80 ft./sec. with a water temperature of 50°F.; 5.5 ft./sec. with a water temperature of 51°F; and close to 10 ft./sec. with a water temperature of 52°F.. If the temperature error is small, for example, 0.2 of a degree at 48°F. with a velocity of 1.6 ft./sec., the error will be in the order of 0.1 ft./sec.. At 51°F. and a velocity of 5.5 ft./sec. the error would be approximately 0.9 ft./sec.,or close to 20%, for the same error in water temperature.

Since, by appearance, the other curves in these figures are parallel to and similar to the curve discussed, similar errors should occur under the conditions to which each curve applies.

THE RELATIONSHIP BETWEEN THE VOLTAGE ACROSS THE THERMISTOR AND THE VELOCITY AND TEMPERATURE OF THE WATER FLOWING PAST THE THERMISTOR WHEN THE CURRENT IS KEPT CONSTANT.

From the graphical figures of which Figs. 32 and 33 are representative, and with the current constant at $1x10^{-4}$, $1.5x10^{-4}$, and $2x10^{-4}$ amperes, the voltage across the thermistor was obtained as a function of the water velocity, with the water temperature as a parameter. Fig. 39 is representative of this set.

Again the sensitivity handicap is apparent. The sensitivity increases with an increase of current and a decrease in water temperature. However, at low current values and with relatively warm water, this method is relatively insensitive. With 60°F. water and a current of $1.5x10^{-4}$ amperes flowing

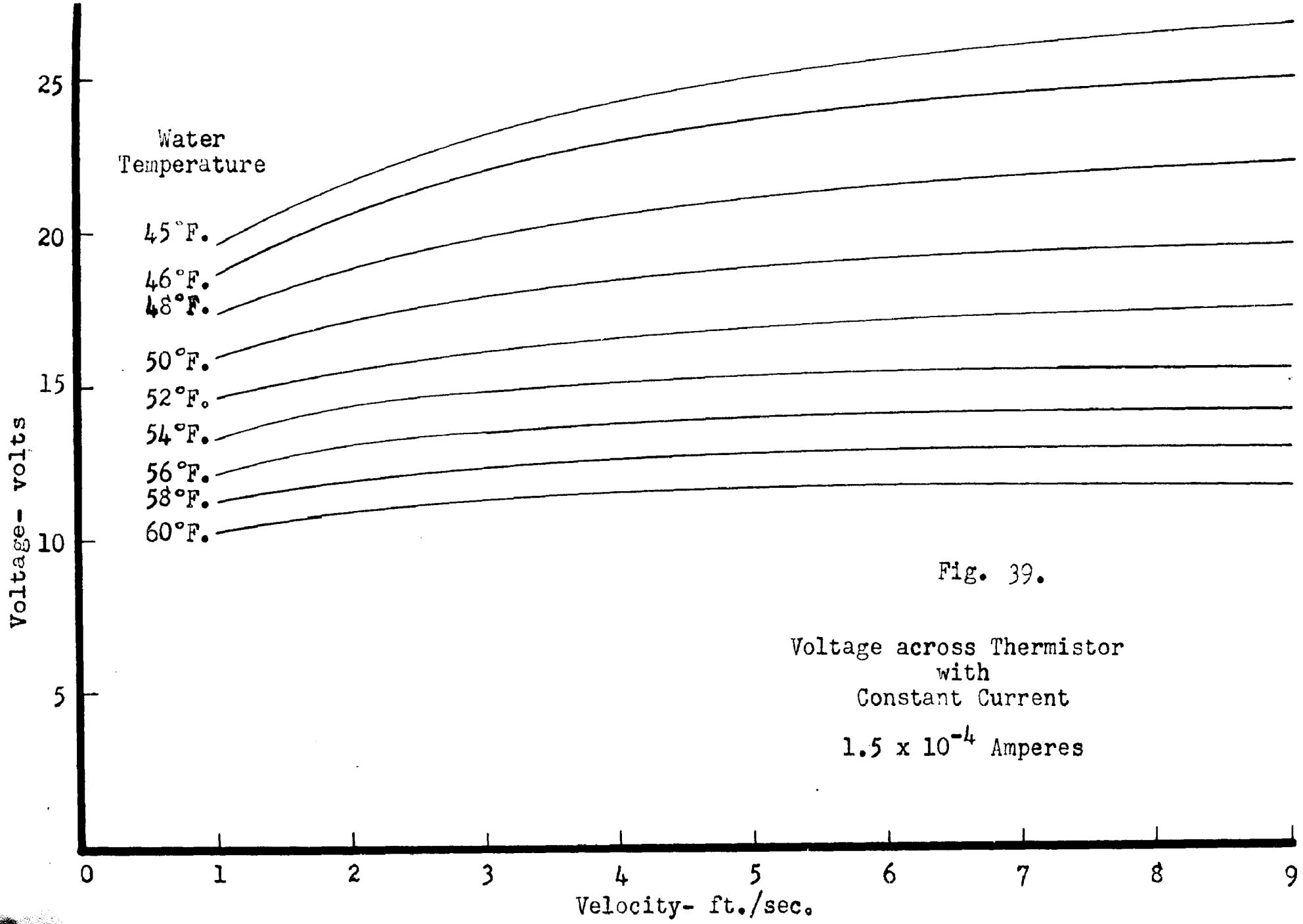


Fig. 39.

Voltage across Thermistor
with
Constant Current

1.5×10^{-4} Amperes

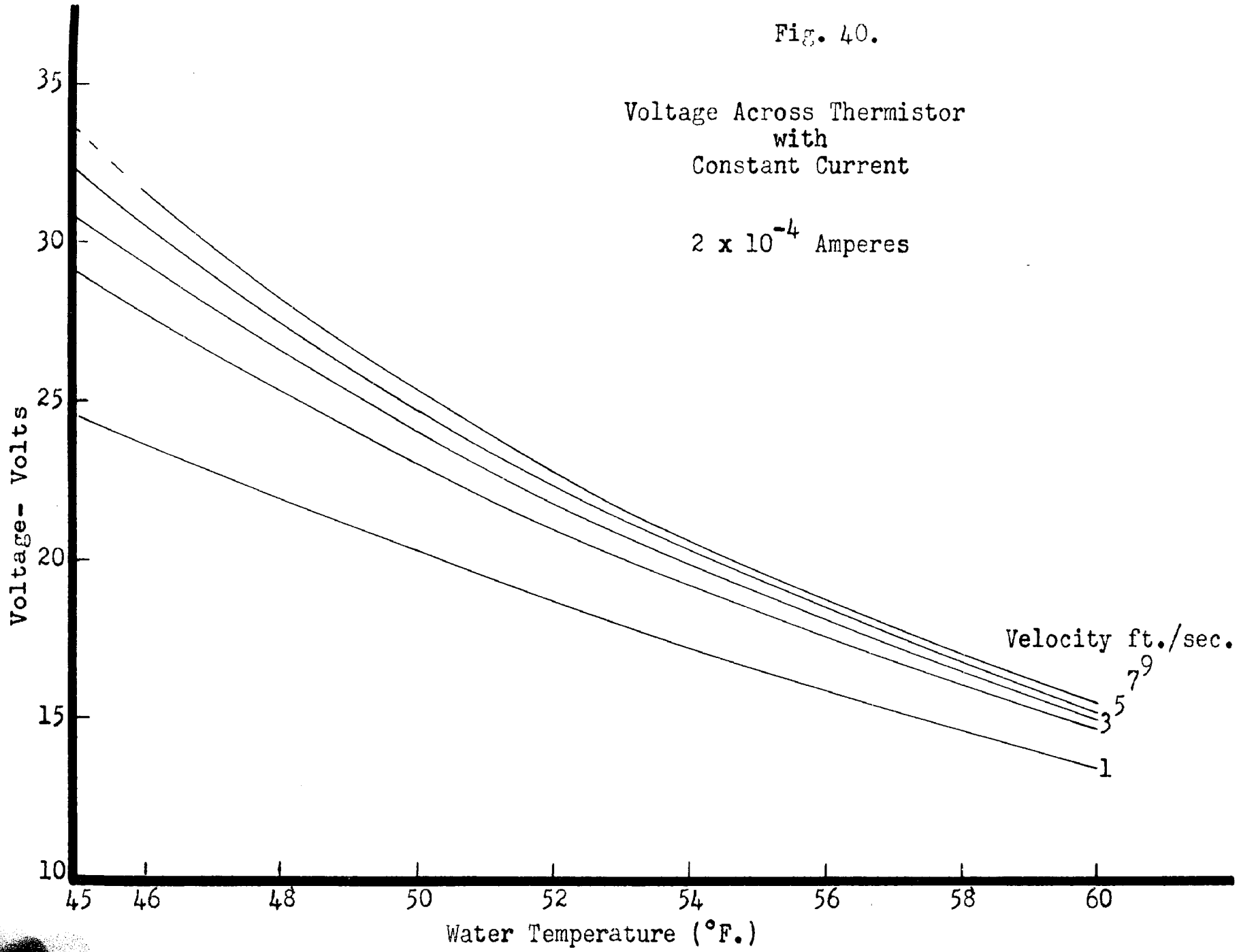
through the thermistor, Fig. 39 shows a voltage change from 11.3 to 11.6 volts when the velocity of the water is changed from 4 to 9 ft./sec.. This is a change of 0.3 of a volt, or 2.8% This is caused by a thermistor temperature change of a major fraction of a degree F.. This is also shown in Fig. 30. By a relatively small amount of amplification, this sensitivity may be increased markedly.

This type of calibration may also be applied in a situation where the temperature of the water cannot be controlled by the operator. A typical calibration curve is shown in Fig. 40.

Again the velocity error will be appreciable with an error in water temperature. Considering the curve in Fig. 41 where the voltage across the thermistor is 17.5^{v} and the current flowing in the thermistor is 1.5×10^{-4} amperes and water at 48°F., the velocity would be 1.1 ft./sec.; with 49°F. water, the velocity would be 1.6 ft./sec.; with 50°F. water, the velocity would be 2.3 ft./sec.; with 51°F. water, 3.9 ft./sec.; and with 52°F. water, 8.6 ft./sec.. Thus an error of a degree or so in the water temperature would result in a rather large error in the measured fluid velocity.

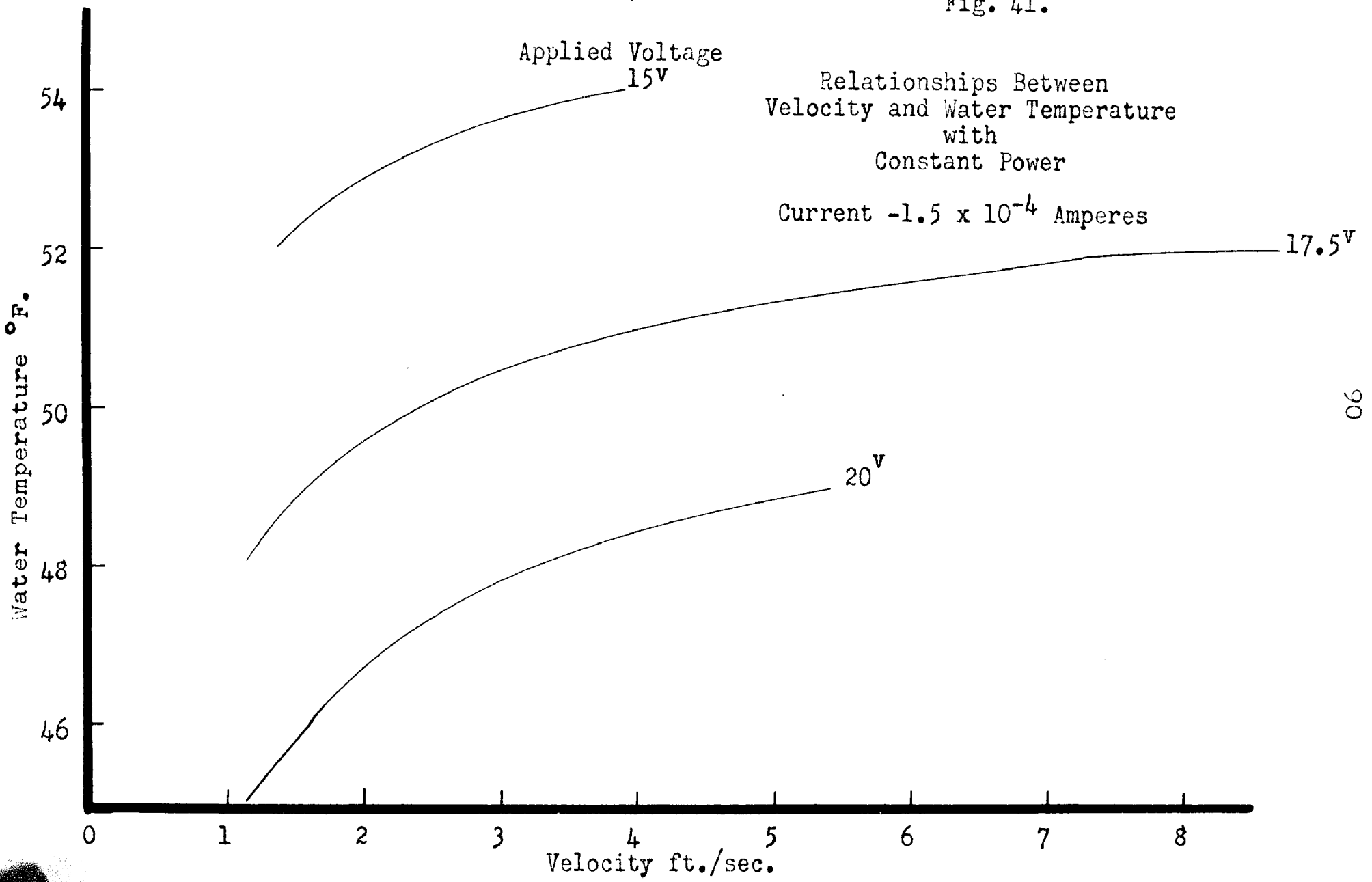
An error of 0.2 of a degree with water at 48°F. would produce a velocity error of approximately 0.1 ft./sec.; with 49°F. water, 0.1 ft./sec.; with 50°F. water, 0.2 ft./sec.; with 51°F. water, 0.5 ft./sec.; with 52°F. water, 1.8 ft./sec. In percentages this error would be 8% with 48°F. water, 6% with 49°F. water, 8% with 50°F. water, 12% with 51°F. water,

Fig. 40.


Voltage Across Thermistor
with
Constant Current
2 x 10^-4 Amperes
Voltage- Volts
35
30
25
20
15
10
45
46
48
50
52
54
56
58
60
Water Temperature (°F.)
Velocity ft./sec.
7.9
3.5
1

V

Fig. 41.



Relationships Between
Velocity and Water Temperature
with
Constant Power

Current -1.5×10^{-4} Amperes

and 21% with 52 F. water. Thus again, with an increase in velocity and in water temperature, the error increases.

Both the constant voltage and constant current methods of utilizing the thermistor use the same basic circuit. However, there is a fundamental difference between the two methods. The power is given by the formula

$$P = E^2/R_t = I^2R_t$$

where E and R_t are as previously mentioned and I is the current flowing through the thermistor. With the constant-voltage type of circuit, the power applied is proportional to $1/R_t$. As the velocity increases and/or the water becomes cooler, the resistance of the thermistor tends to increase and, as a consequence, less power is applied. This, in turn, increases the resistance of the thermistor and decreases the power. Theoretically, this could lead to an unstable situation producing an erroneous answer.

The constant-current circuit applies power proportional to R_t . When the thermistor is under flow conditions that can absorb more power, more power is applied. When the flow is at a lower velocity and the fluid is warmer, less power is applied. The power varies with the ability of the water to absorb it. This indicates a stable measurement technique.

RELATIONSHIPS BETWEEN THE POWER INPUT TO THE THERMISTOR AND THE VELOCITY AND TEMPERATURE OF THE WATER WHEN THE THERMISTOR TEMPERATURE IS HELD CONSTANT.

This is a method which may be used when the water temperature can be controlled. The physical arrangement would take the form of a Wheatstone bridge where the resistance of the thermistor has been set previously by the adjustment of an adjacent arm of the bridge to correspond to a temperature of the thermistor. This resistance-temperature relationship would have been determined previously by a separate Wheatstone bridge arrangement with not more than 1 volt energizing the bridge and a sensitive galvanometer.

As the different flow conditions of operation and calibration are imposed upon the thermistor, the input power to the experimental bridge could be raised or lowered until a null indication is obtained.

Returning to the main construction figures (Figs. 28 through 30) for the mathematical model of the thermistor velocity meter and plotting the $EIx10^4$ that would be required to maintain the thermistor temperature selected, Figs. 42 through 44 are produced. The $EIx10^4$ is the product of the voltage across the thermistor and the current in the thermistor.

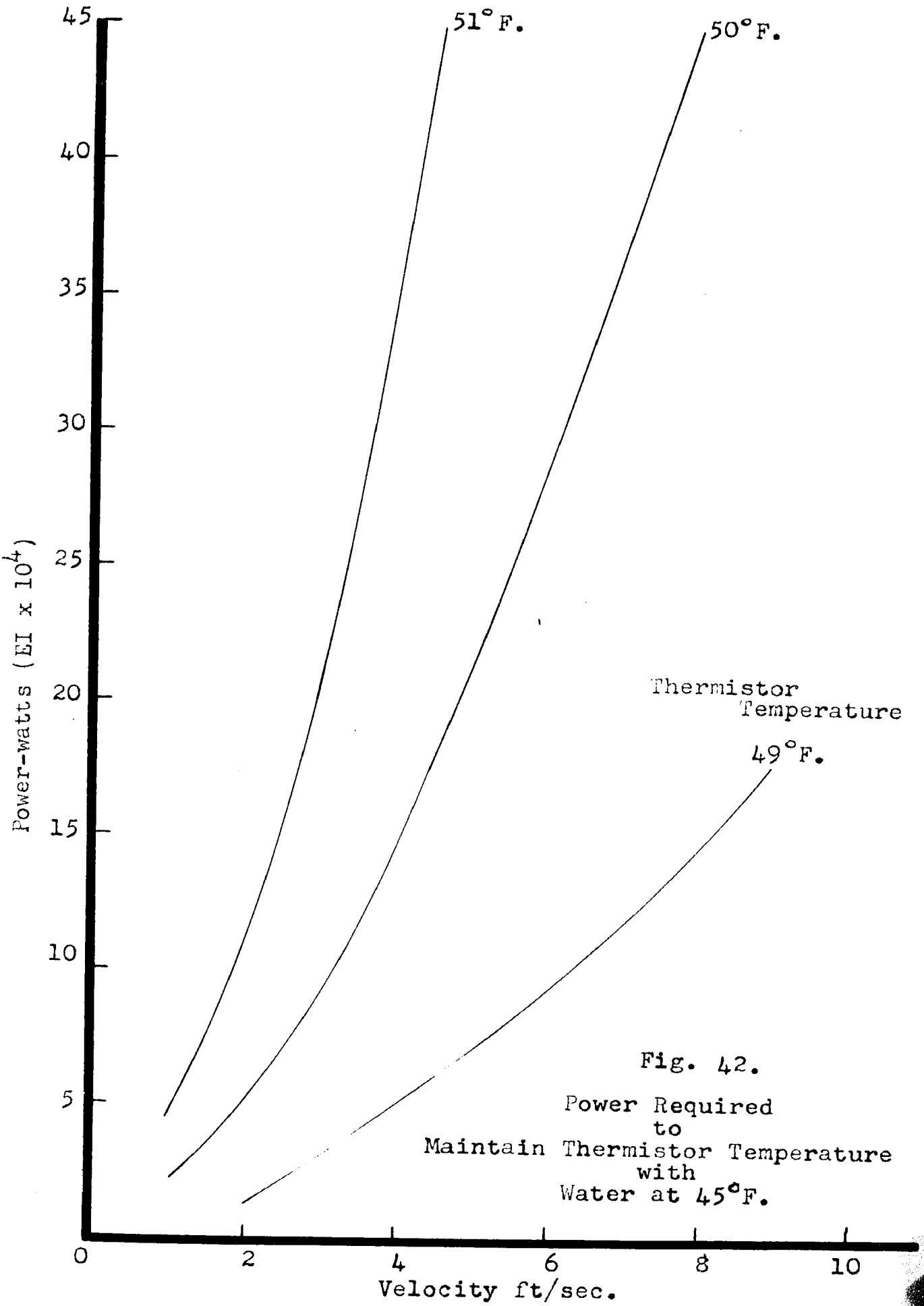


Fig. 42.

Power Required to Maintain Thermistor Temperature with Water at 45°F.

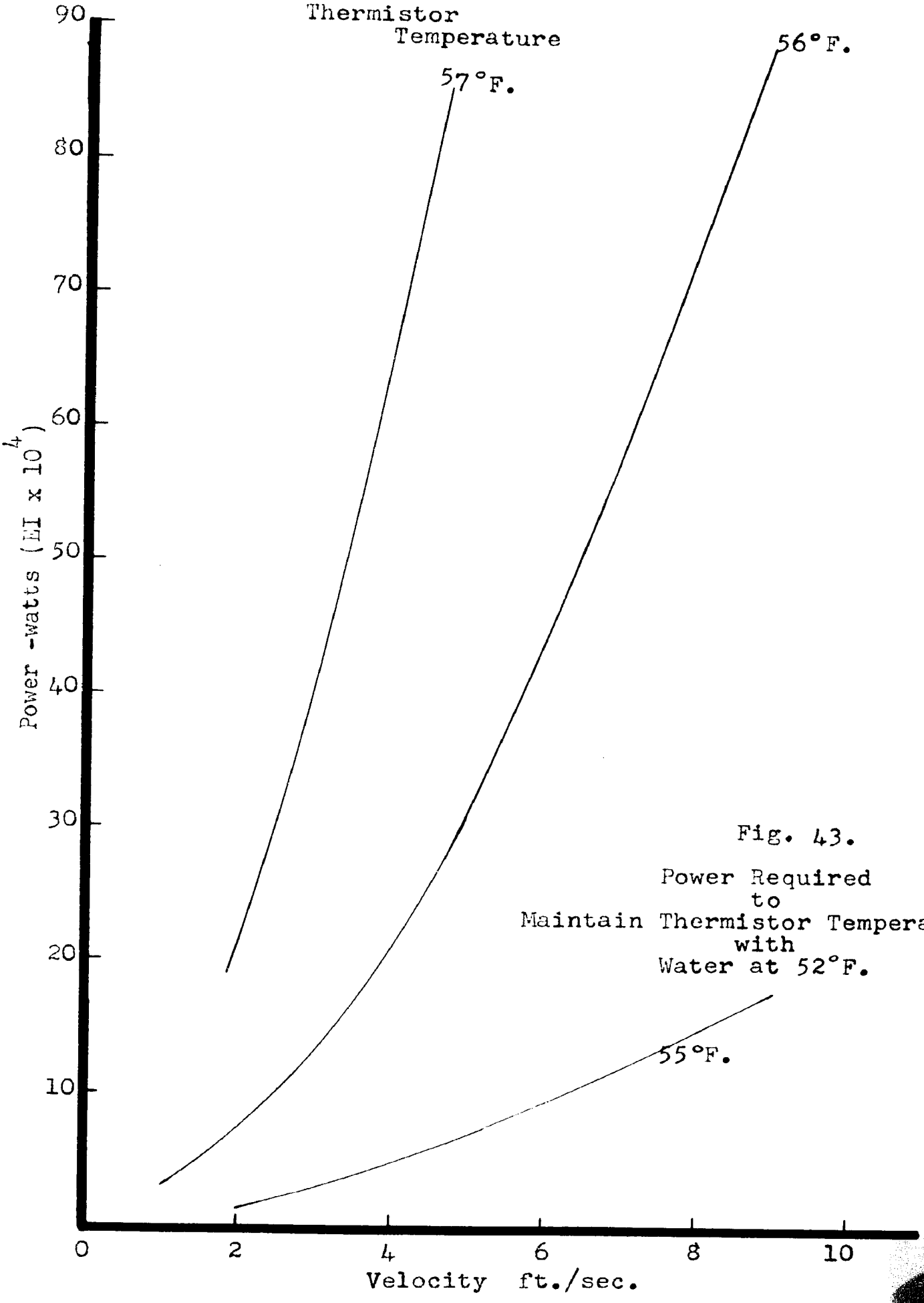


Fig. 43.

Power Required
to
Maintain Thermistor Temperature
with
Water at 52°F.

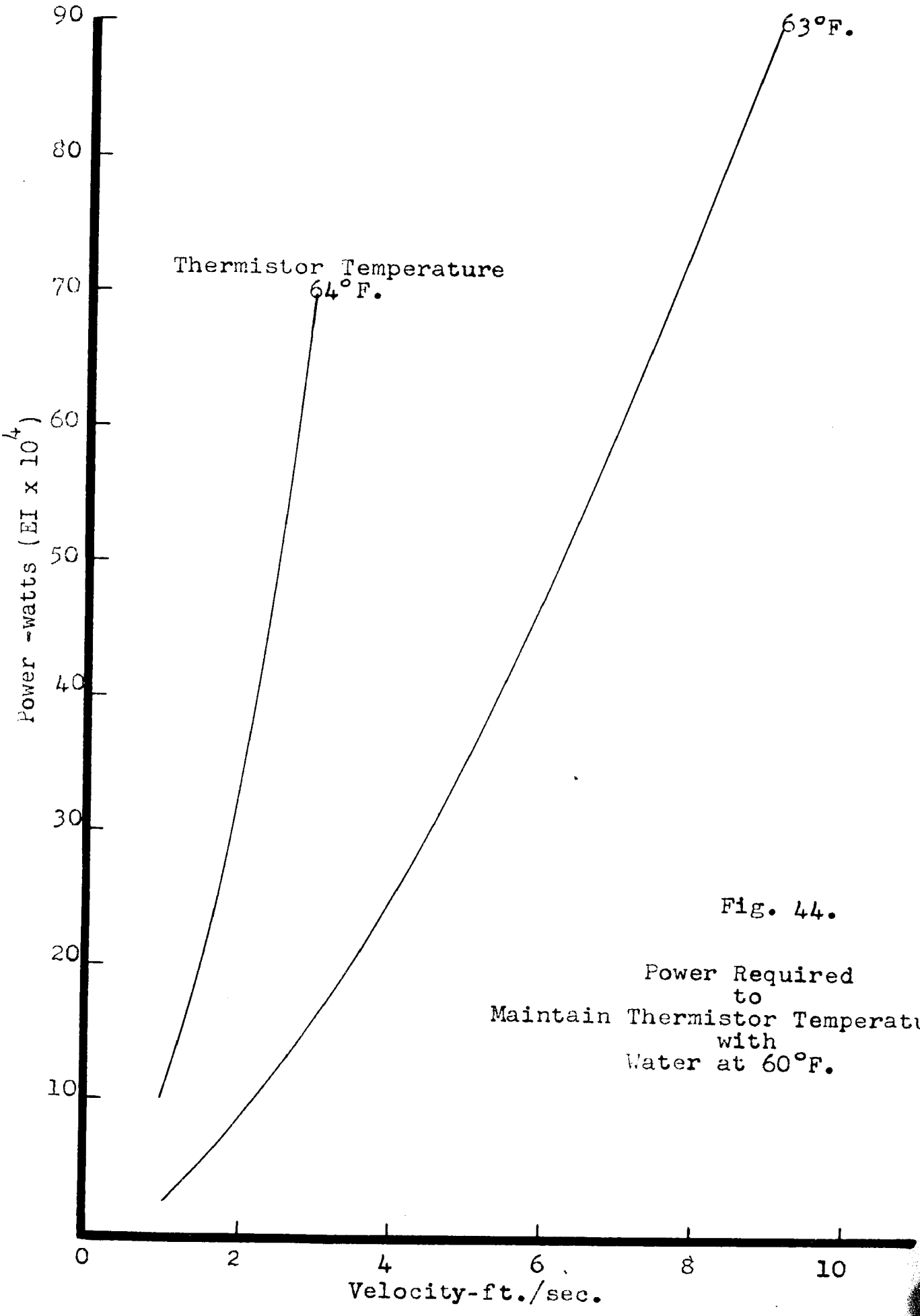


Fig. 44.

Power Required
to
Maintain Thermistor Temperature
with
Water at 60°F.

The most striking note of this method, according to the observation of these figures, is the sensitivity of the thermistor to the velocity when used in this manner. By adjusting the temperature of the thermistor, the sensitivity of the instrument may be adjusted in the velocity range in question.

If these data are reworked so that the temperature difference between the thermistor and water is held constant, Figs 45 and 46 are obtained. Fig. 45 shows the power necessary to maintain a temperature difference of 3 degrees under the various conditions of operation; Fig. 46, a 4 degree temperature difference. These figures show that this method is more sensitive in warm water than in cold water.

An insight into the errors of this method is shown in Figs. 47 and 48 for low values of power (watts).

With a temperature difference of 3 degrees and an observed power reading of 15×10^{-4} watts from Fig. 45, it was found that at a water temperature of 60° F. there will be a velocity of 3 ft./sec.; at 58°F., 4 ft./sec.; at 56°F., 5.05 ft./sec.; at 54°F., 6.3 ft./sec.; and at 52°F., 8 ft./sec..

With an error in water temperature of 0.2 of a degree, the error in velocity with water at 60°F. will be 0.1 ft./sec. or 3.3%; with 58°F. Water, 0.1 ft./sec. or 2.5%; with 56°F. water, 0.1 ft./sec. or 2%; with 54°F. water, 0.15 ft./sec. or 2.34%; and with 52°F. water, 0.2 ft./sec. or 2.5%.

Fig. 46 shows the same data only with a temperature difference of 4 degrees. With the same observed power reading of 10×10^{-4} watts, the velocity would be 2.1 ft./sec. with

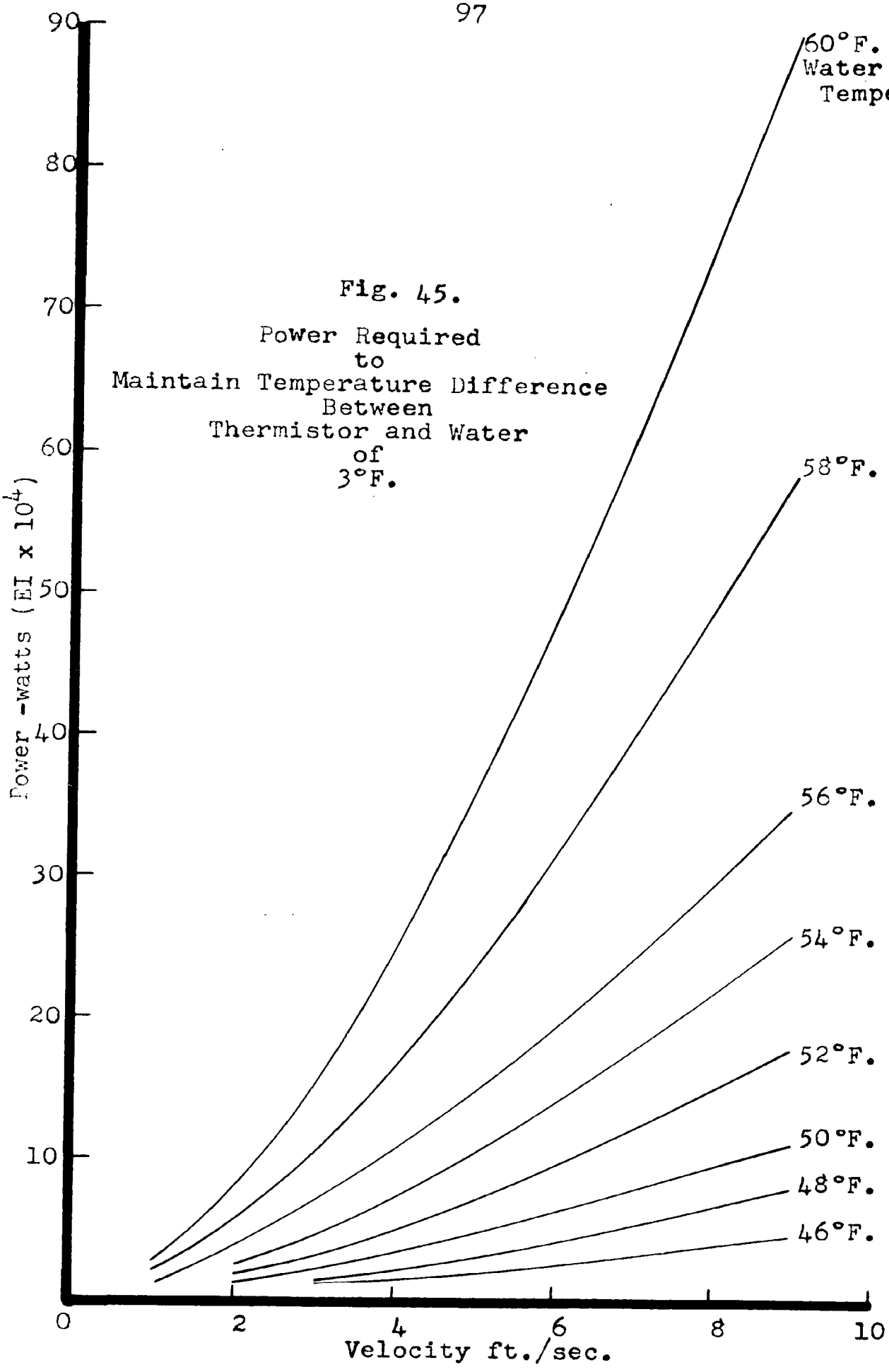


Fig. 45.

Power Required
to
Maintain Temperature Difference
Between
Thermistor and Water
of
3°F.

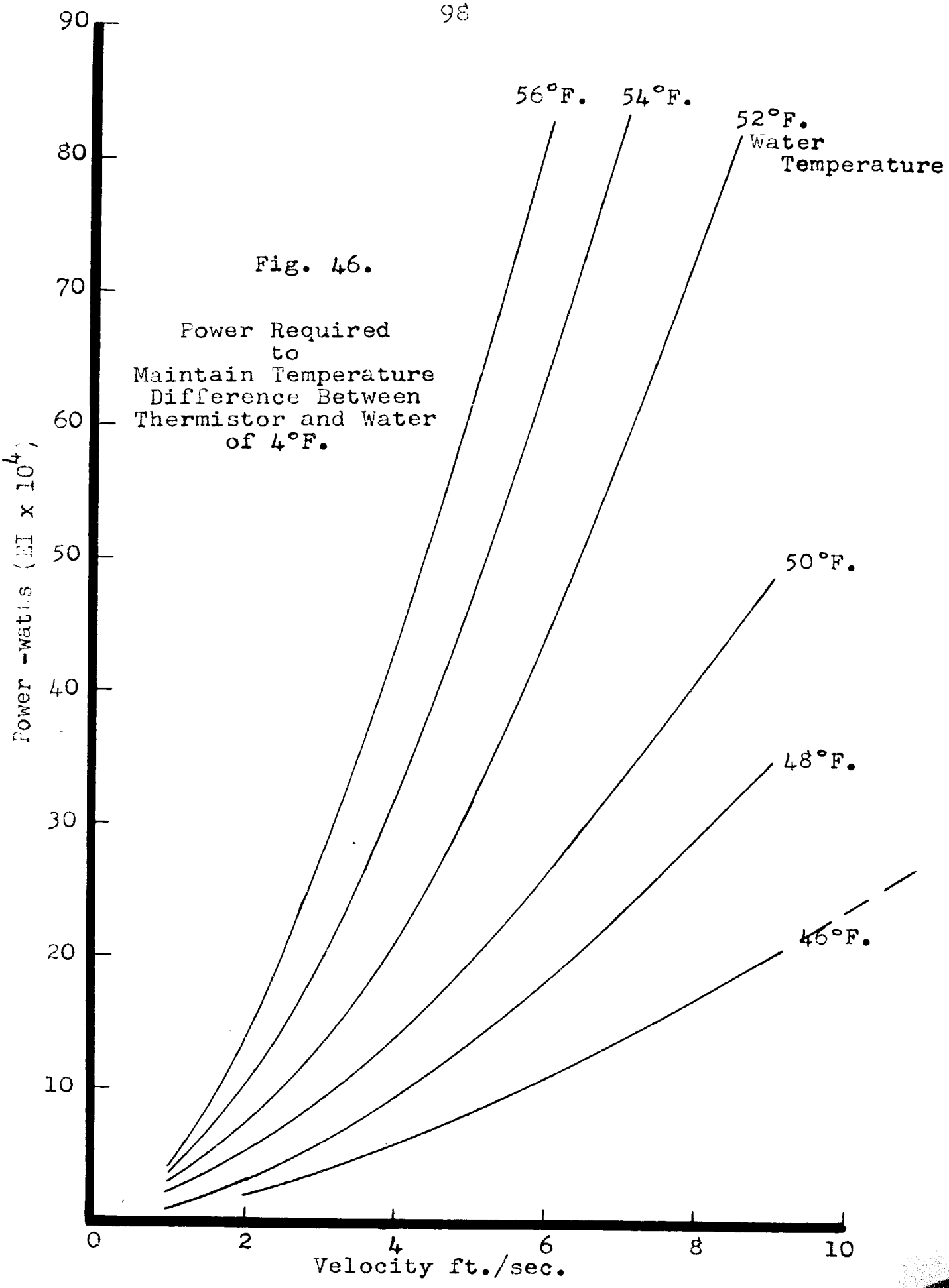
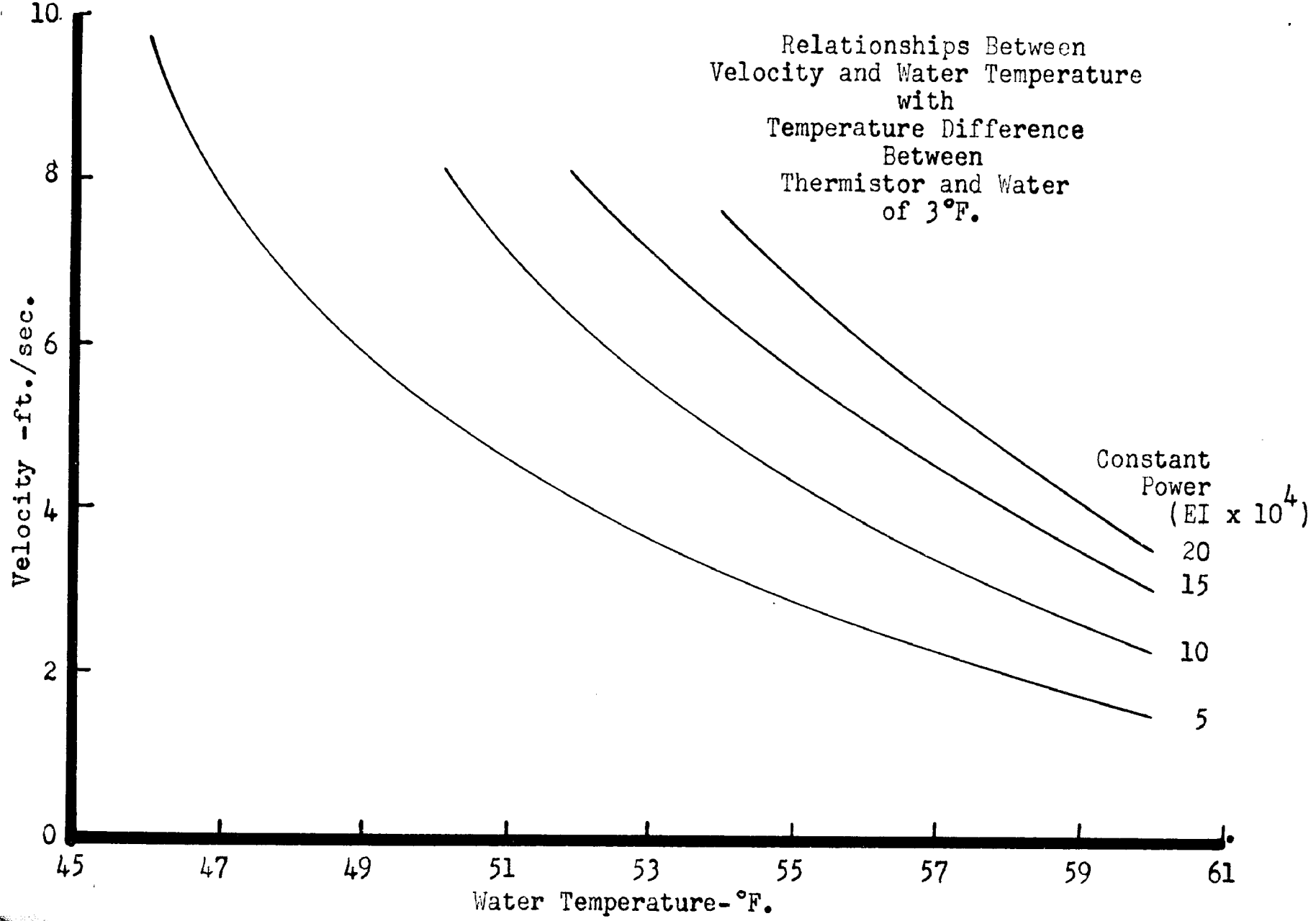


Fig. 46.

Power Required to Maintain Temperature Difference Between Thermistor and Water of 4°F.

Fig. 47.


Relationships Between
Velocity and Water Temperature
with
Temperature Difference
Between
Thermistor and Water
of 3°F.
Constant
Power
(EI x 10^4)
20
15
10
5
Velocity -ft./sec.
0
2
4
6
8
10
45
47
49
51
53
55
57
59
61
Water Temperature-°F.

water at 56°F.; 2.6 ft./sec. at 54°F.; 3.25 ft./sec. at 52°F.; 4.15 ft./sec. at 50°F.; 5.45 ft./sec. at 48°F.; 7.4 ft./sec. at 46°F..

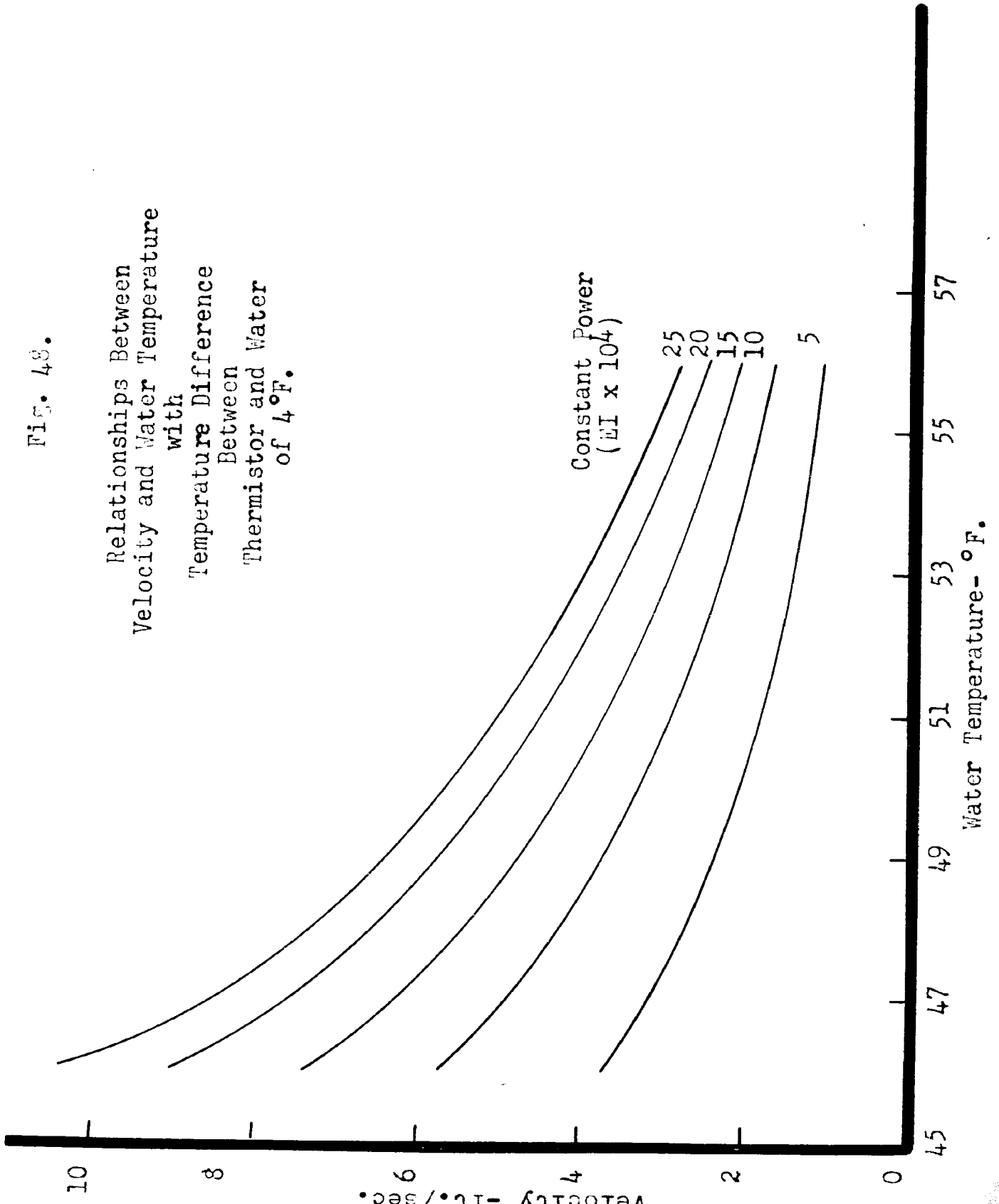
With an error in the water temperature of 0.2 of a degree at 56°F., the velocity error would be approximately 0.1 ft./sec. or 4.8%; at 54°F., 0.1 ft./sec. or 3.15%; at 52°F., 0.1 or 3.08%; at 50°F., 0.15 or 3.62%; at 48°F., 0.2 ft./sec. or 3.67%; at 46°F., 0.3 ft./sec., or 4.06%.

Another way of showing errors with this type of measurement circuit is to keep the thermistor temperature constant and plot the power required to maintain that thermistor temperature as the water temperature and velocity are varied. This is done in Fig. 49 for a thermistor temperature of 60°F. and a water temperature of 54°F., 56°F., and 58°F.. Thus, an increase in the temperature difference increases the sensitivity, but also increases the possibility of error.

This method requires that the experimenter must have a closed calibration system in which the temperature of the water can be ordered within moderately close limits. However, a method may be devised for use with uncontrolled water temperatures, as long as they can be measured and will stay relatively constant for a period of time. This would be beneficial for naval tanks that are located in buildings where the temperature does not vary appreciably from day to day.

Fig. 48.

Relationships Between
Velocity and Water Temperature
with
Temperature Difference
Between
Thermistor and Water
of 4°F.



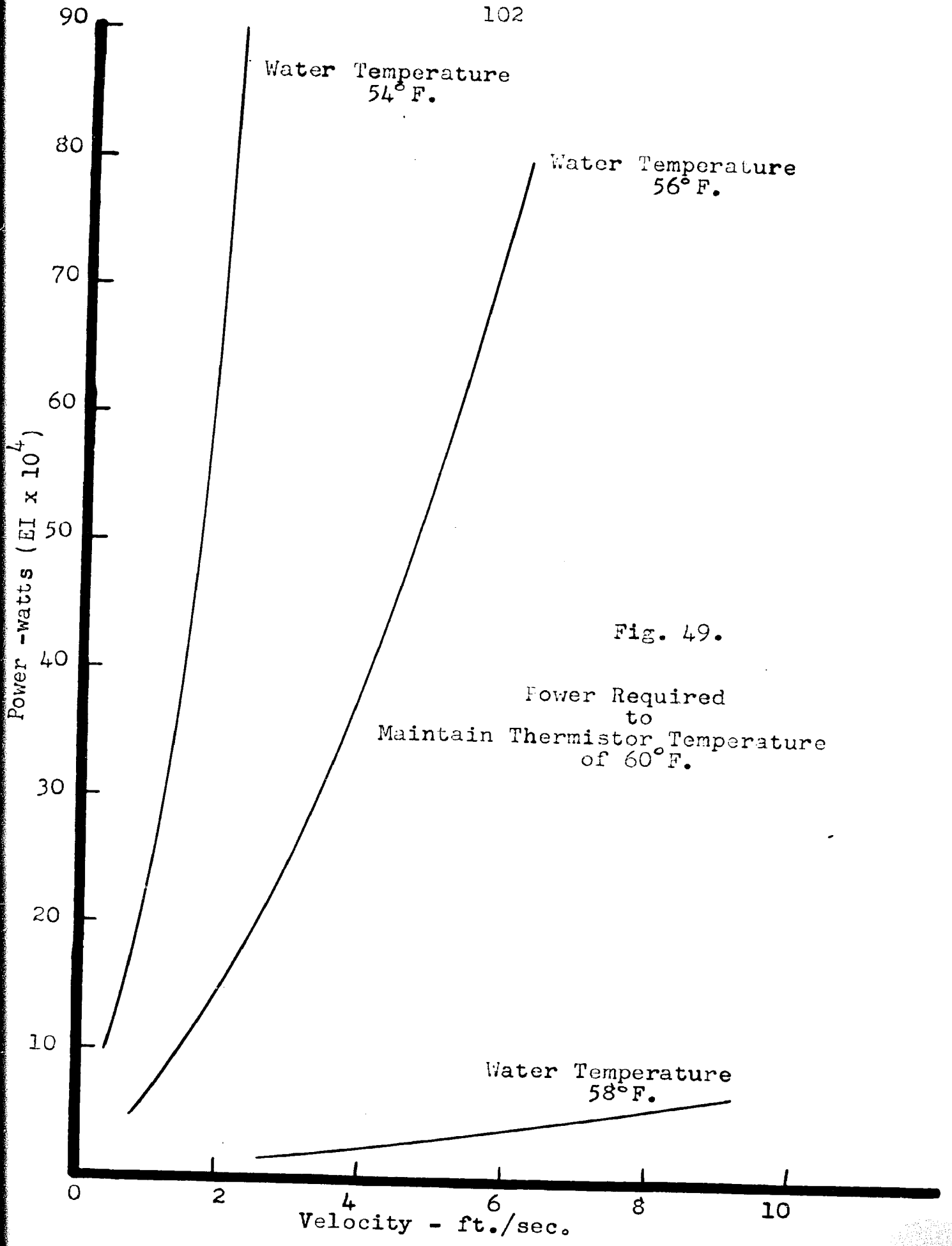
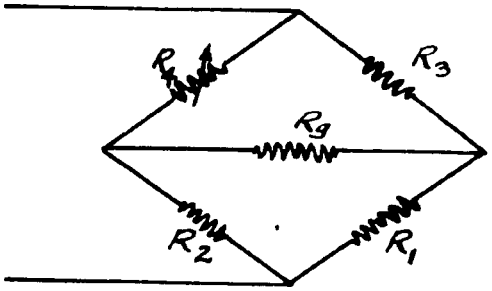


Fig. 49.

Power Required
to
Maintain Thermistor Temperature
of 60° F.

THE WHEATSTONE BRIDGE WITH COMPENSATION BY ANOTHER THERMISTOR FOR CHANGE IN WATER TEMPERATURE

A Wheatstone Bridge is shown in its simplest form below.



Consider as R_1, thermistor No. 2 which was about 190,000 ohms at 49°F., and 70,000 ohms at 64°F.. Because of this large change, a temperature compensating thermistor which is identical to R_1 will be used as R_2. Thermistors may be purchased in pairs which are identical both electrically and in size. However, what is needed is a thermistor which possesses an electrical resistance similar to that of the velocity meter element, but which is much larger in mass. A compensating thermistor which is the size of the meter element would be too small, for the current would undoubtedly heat it. It is possible that such a large thermistor with essentially the same resistance-temperature characteristics could be procured. This resistor would have a surface large enough so that the passage of the small amount of current would not be great enough to raise its temperature above the temperature of the water.

The resistor R_3 (opposite R_2) is arbitrarily set at 100,000 ohms and the resistor R_4 is variable. Now when the bridge is balanced, $R_4 = R_2R_3/R_1$.

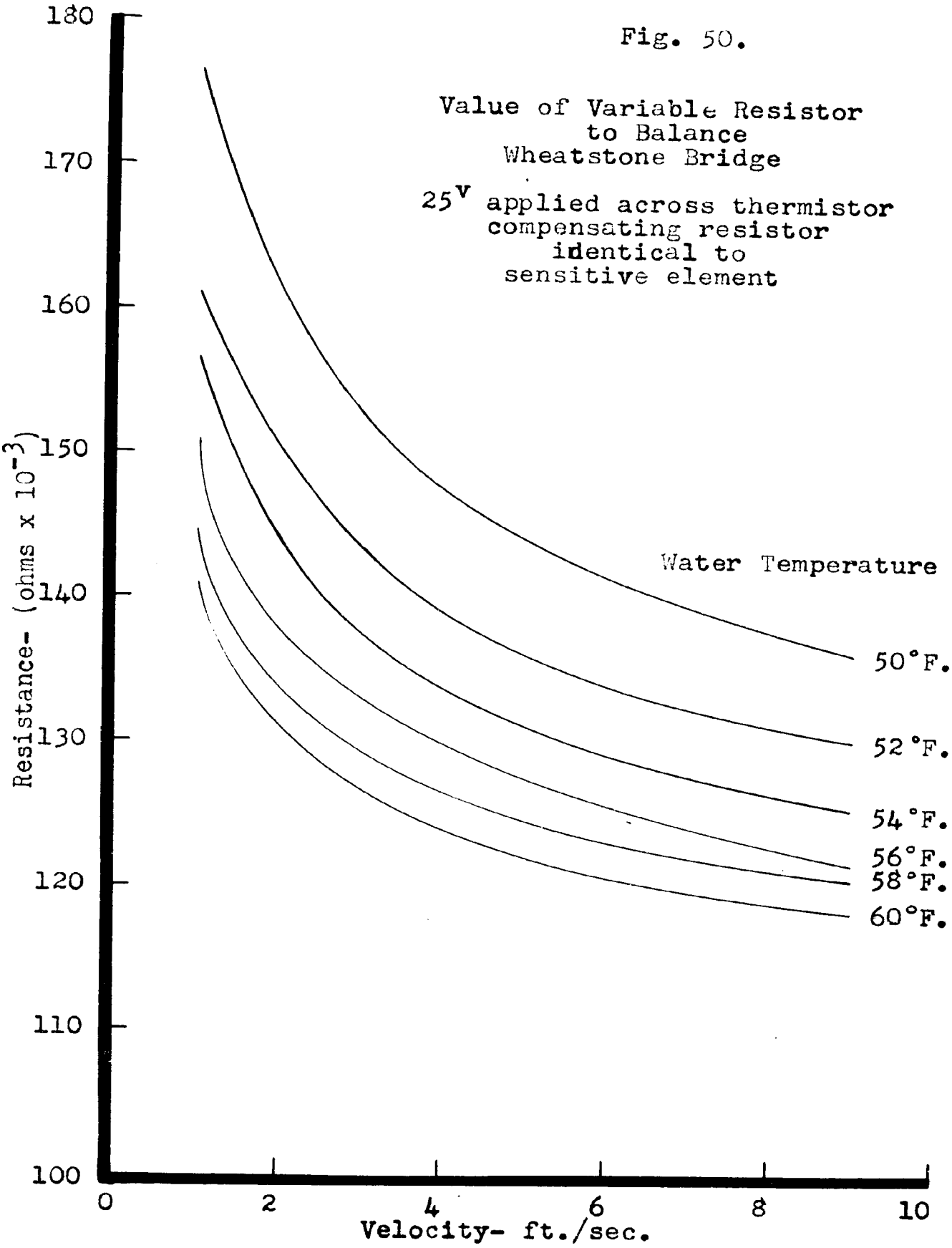
The values of R_4 under flow conditions may be found by multiplying the resistance of the compensating resistor, determined from the temperature of the water and Fig. 31, by 100,000 ohms and then dividing this product by the resistance of the sensitive element. This resistance is determined from the temperature of the sensitive thermistor under actual flow conditions and Fig. 31. This has been done in Fig. 50.

If the compensating thermistor were to compensate for the change in water temperature, each of the curves in Fig 50 would coincide with each other curve. This is not the case, however, as an examination of Fig. 50 will show. As the water temperature decreases, because of differences in water properties, and the thermistor resistance changes caused by this water temperature change, the difference or "spread" between the curves increases. As the water temperature increases the sensitivity of the instrument decreases.

Fig. 51 shows the various combinations of velocity and water temperature for a constant value of the variable resistor. If the compensation were ideal, the balance resistor would be constant for each value of velocity (vertical in the figure shown). The device shows large error.

If the value of the variable resistor were to be arbitrarily set at 140,000 ohms for a velocity of 1 ft./sec. at all water temperatures, and the resistance-temperature relationship of the compensating resistor (R_2) calculated, this relationship would be as shown in Fig. 52. If the balance resistor were to be calculated for other velocities and water

Fig. 50.


Value of Variable Resistor
to Balance
Wheatstone Bridge
25V applied across thermistor
compensating resistor
identical to
sensitive element
Water Temperature
50°F.
52°F.
54°F.
56°F.
58°F.
60°F.
Resistance- (ohms x 10-3)
180
170
160
150
140
130
120
110
100
0
2
4
6
8
10
Velocity- ft./sec.

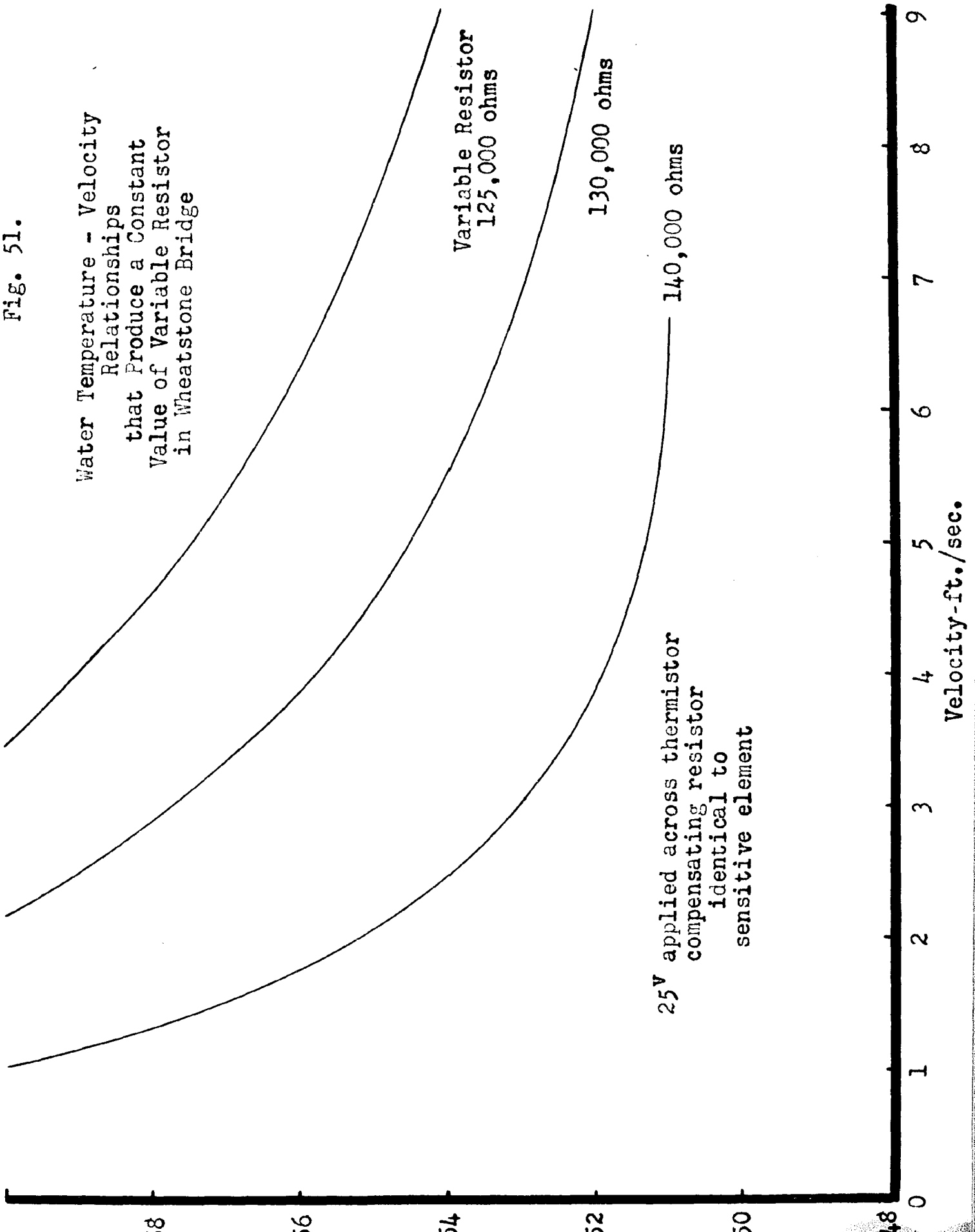

Water Temperature - Velocity Relationships that Produce a Constant Value of Variable Resistor in Wheatstone Bridge
Variable Resistor 125,000 ohms
130,000 ohms
140,000 ohms
25V applied across thermistor compensating resistor identical to sensitive element
Water Temperature- °F.
Velocity-ft./sec.
48
50
52
54
56
58
0
1
2
3
4
5
6
7
8
9

Fig. 51.

temperatures, the values would appear much as shown in Fig. 53. This figure shows that it is possible to compensate for a change in water temperature when the velocity range is limited, but beyond this range some errors must be expected. This, undoubtedly, is due to changes in thermal properties of the fluid itself as the temperature of the fluid changes.

This has been a discussion of the basic circuits. One investigator[17] has used a combination of these circuits. It is a feedback bridge circuit, using a dc amplifier to maintain the temperature of the thermistor at a constant value, and also incorporating a compensating thermistor in an arm adjacent to the active thermistor. The output of the bridge was correlated with the velocity of the fluid, which in this case was kerosene. In order to maintain the stability of the circuit, the resistance values of the three arms (the two arms containing fixed resistors and the arm containing the compensating resistor) were much greater than the active thermistor. The compensation was effective. The flow was limited, however, to velocities of 0-4 in./sec..

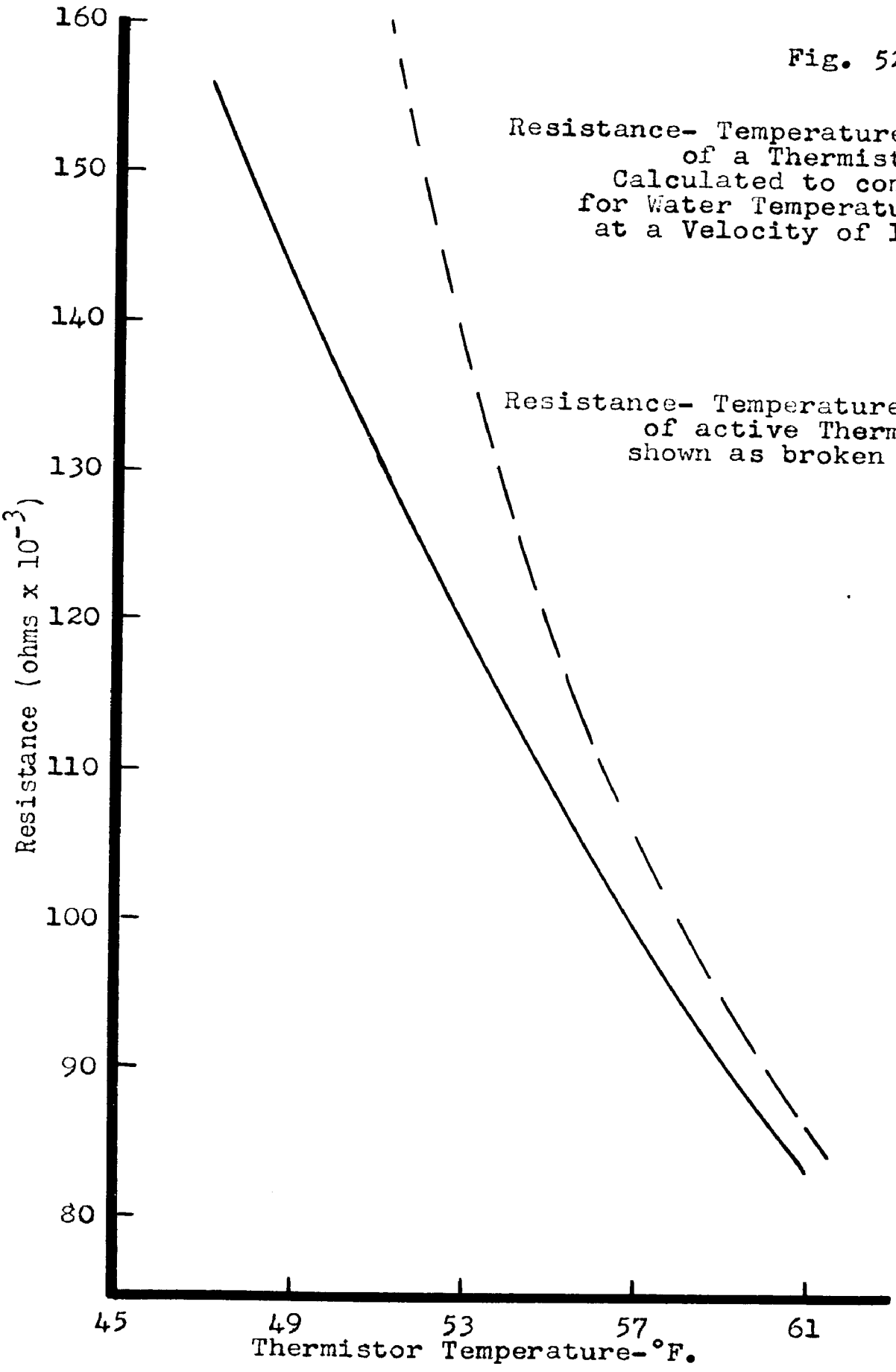


Fig. 52.

Resistance- Temperature Relationship
of a Thermistor
Calculated to compensate
for Water Temperature Change
at a Velocity of 1 ft./sec.

Resistance- Temperature Relationship
of active Thermistor
shown as broken curve

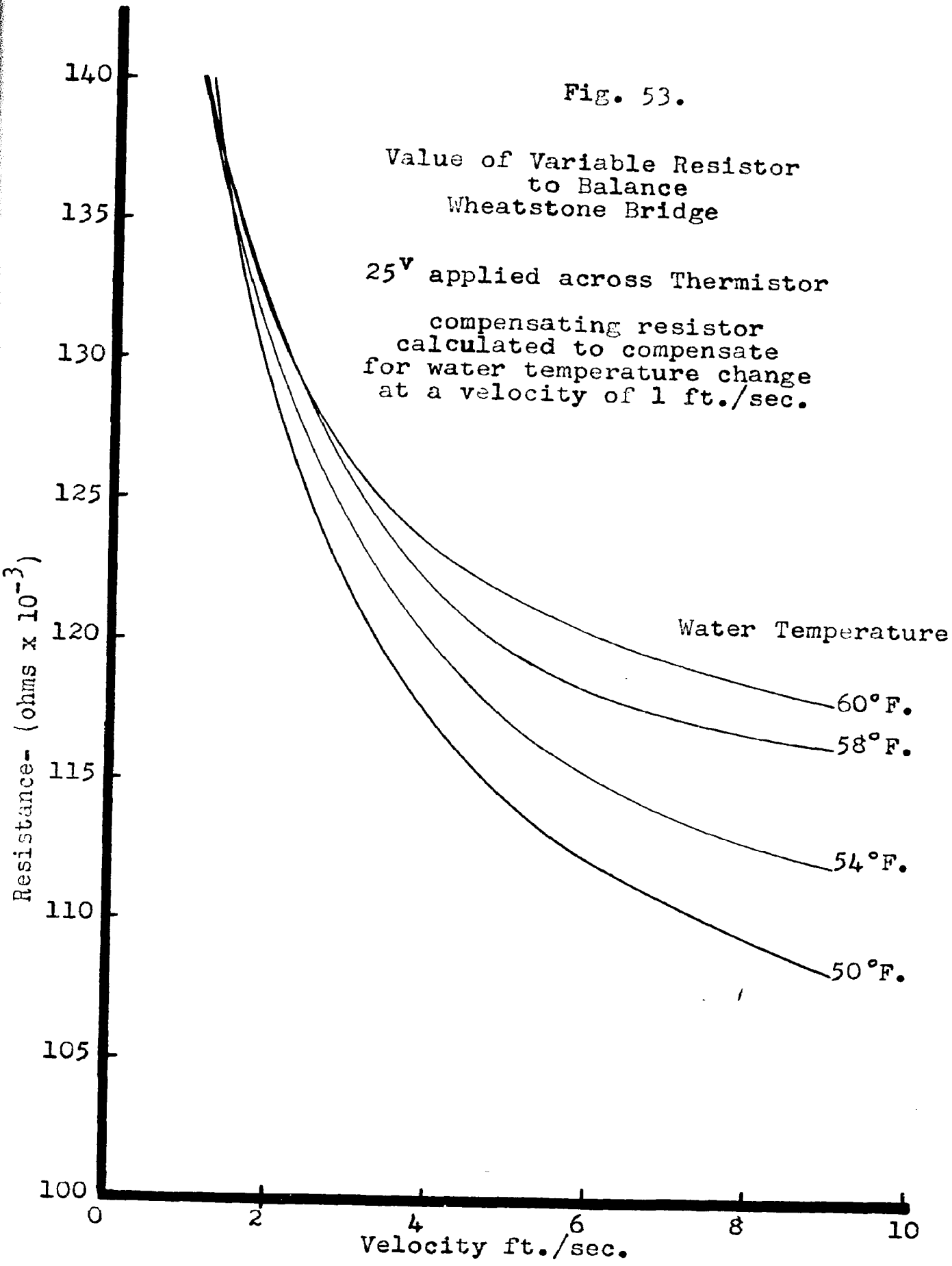

Value of Variable Resistor
to Balance
Wheatstone Bridge
25V applied across Thermistor
compensating resistor
calculated to compensate
for water temperature change
at a velocity of 1 ft./sec.
Water Temperature
60°F.
58°F.
54°F.
50°F.
Resistance- (ohms x 10-3)
140
135
130
125
120
115
110
105
100
0
2
4
6
8
10
Velocity ft./sec.

Fig. 53.

CHAPTER III.

VELOCITY MEASUREMENTS.

The thermistor was used to determine a velocity traverse of a flow situation in a flume, shown in Fig. 54. The average velocity of the water was found in the following manner. A No. 14A thermistor was prepared as previously described and mounted in the calibration tube. Inasmuch as the average velocity was desired, the constant-voltage type measurement circuit was used. The voltage selected was 25^{V} dc.

The velocity in the calibration tube was varied in four steps from approximately 1 ft./sec. to 8 ft./sec.. The current flowing in the thermistor was plotted as a function of the water temperature for each of the four velocities. The current was obtained as the voltage across a 10,000 ohm resistor, in series with the thermistor, as shown in Fig. 55. The actual value of the series resistor was not found, since the same meters and resistor were used for the velocity measurements in the flume. The calibration curve is shown in Fig. 56.

The thermistor was removed from the calibration tube and inserted into a Lucite rod to form a probe. This probe is shown in Fig. 57. No attempt was made to make the probe rigid enough to eliminate vibration. This would be necessary if turbulence were being measured.

A traverse of the velocities in the flume was then made.



Fig. 54. This Figure Shows the Hydraulic Flume Used for Velocity Measurements.

Fig. 55.

Series DC Circuit used with Thermistors

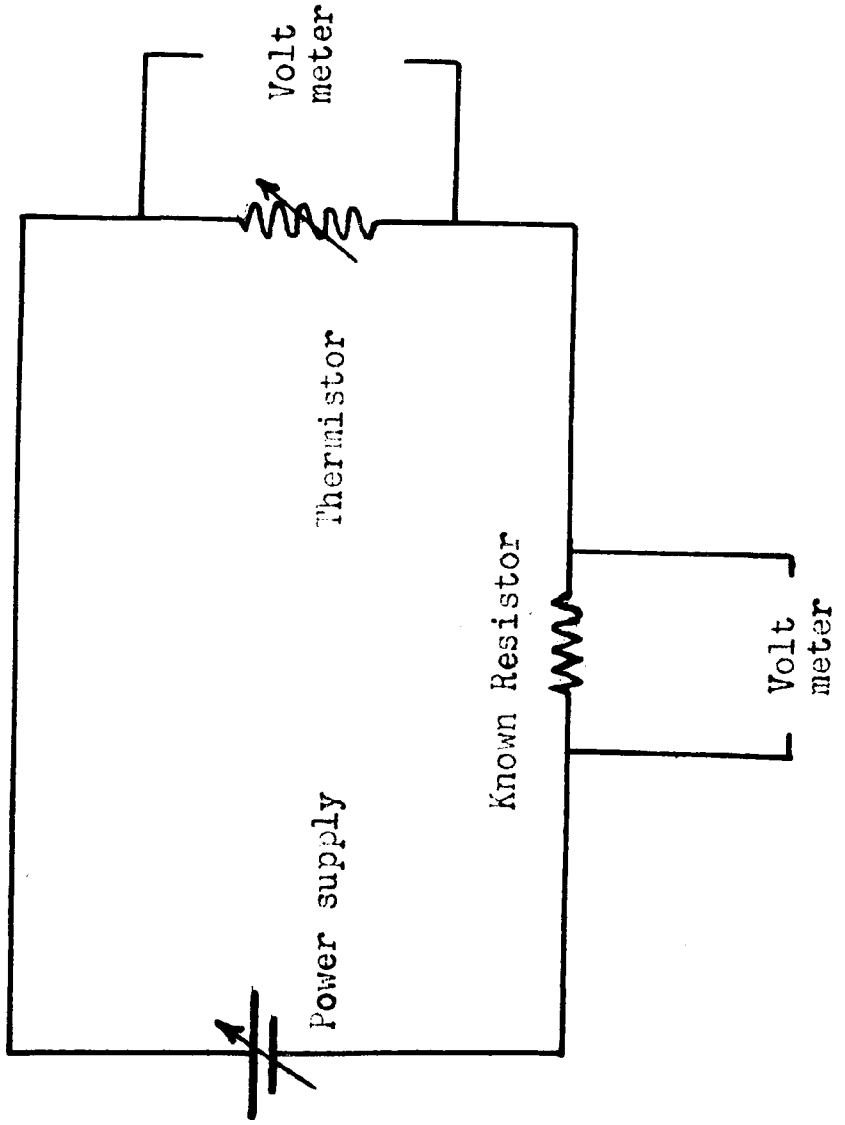
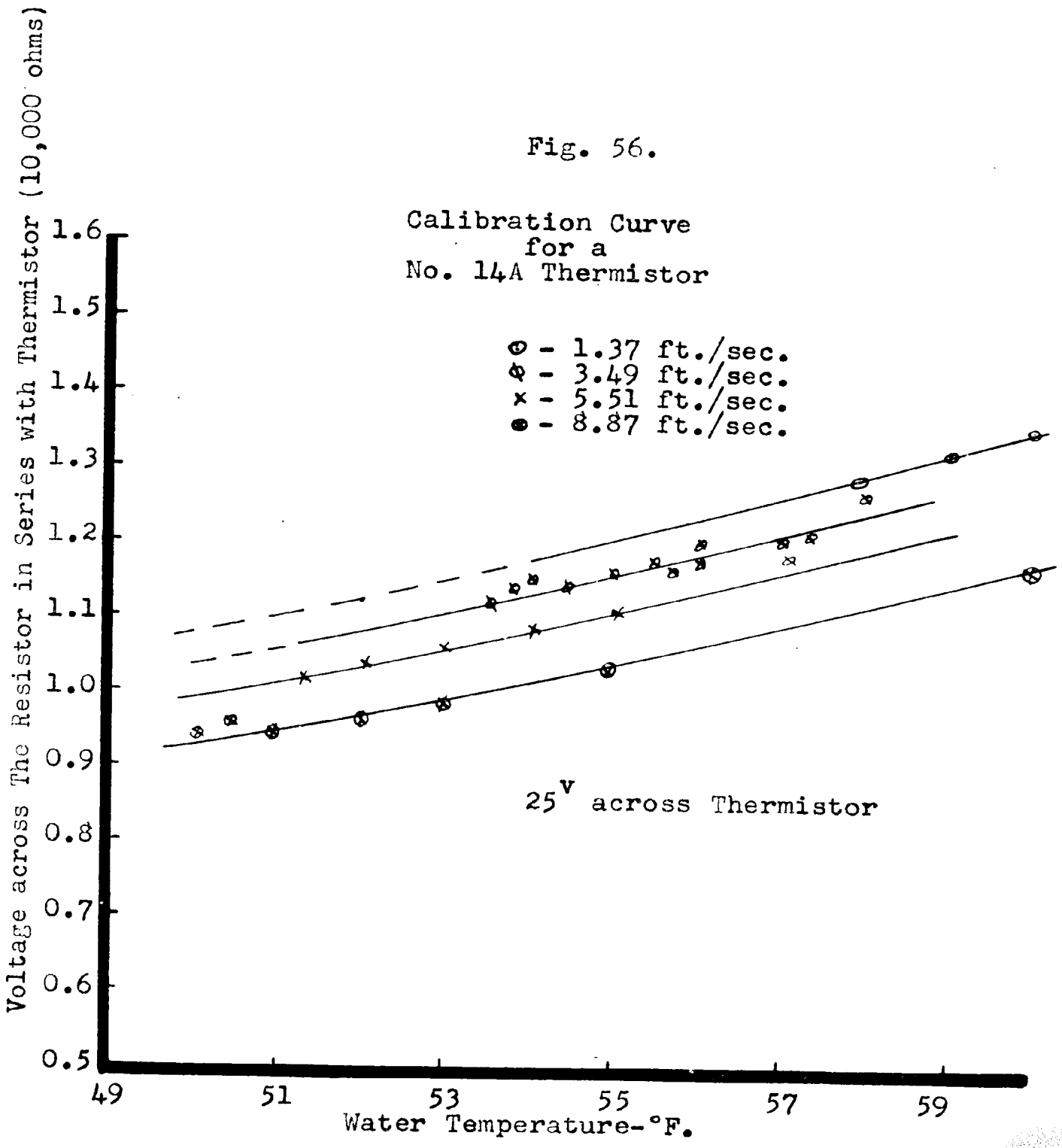


Fig. 56.


Calibration Curve
for a
No. 14A Thermistor
⊙ - 1.37 ft./sec.
⊗ - 3.49 ft./sec.
× - 5.51 ft./sec.
● - 8.87 ft./sec.
25V across Thermistor
Voltage across The Resistor in Series with Thermistor (10,000 ohms)
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
49
51
53
55
57
59
Water Temperature-°F.

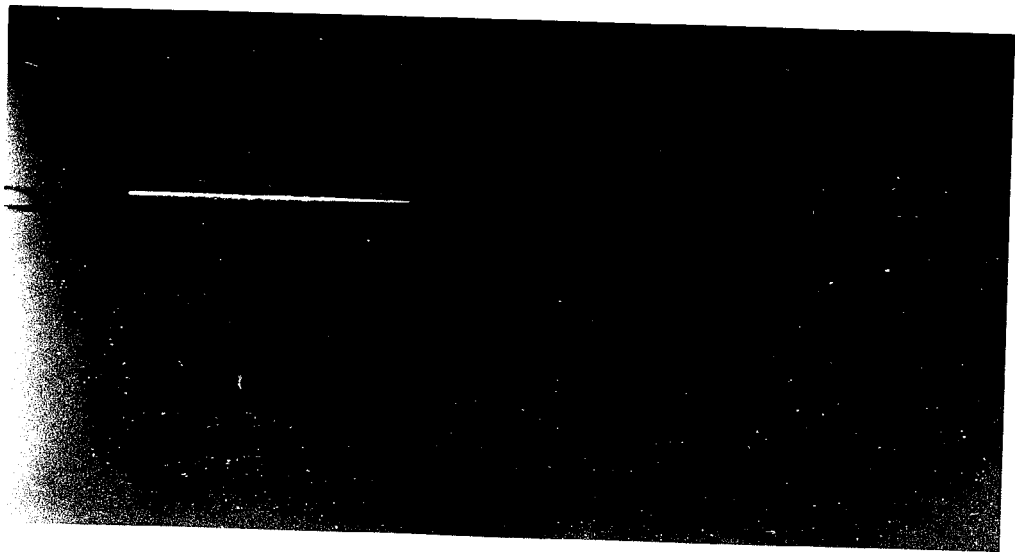


Fig. 57. This Figure Shows The Probe Used to Determine the Velocity Traverse in the Hydraulic Flume.

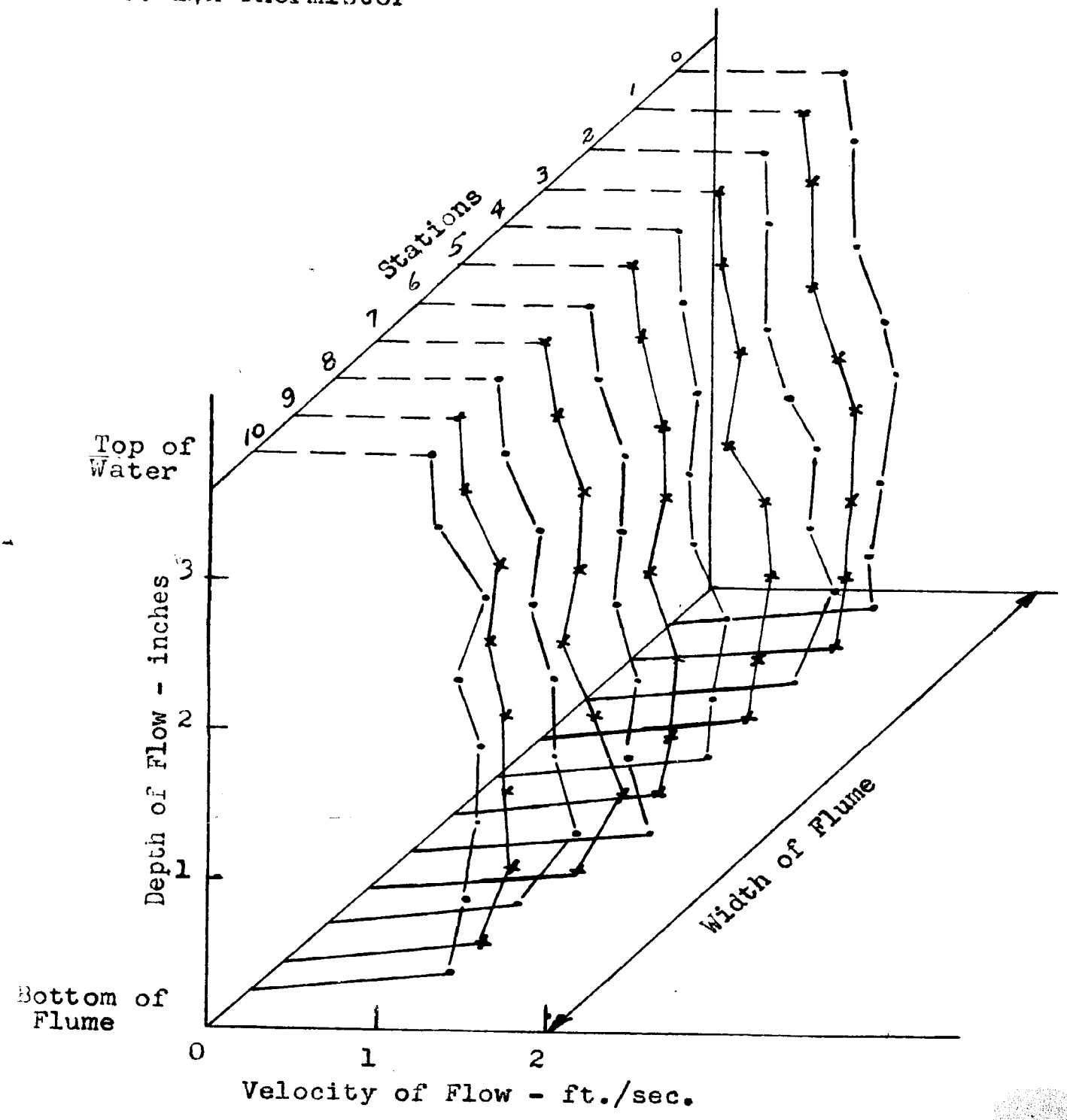
The probe was swept across the flume at depths of 1/8, 5/8, 1-1/8, 1-5/8, 2-1/8, 2-5/8, 3-1/8, and 3-5/8 inches. Eleven readings were taken at different stations at each depth. The width of the flume was 11-3/4 inches and the nearest position of the probe was 1 inch from each side wall. There was 1 inch between each of the readings except between stations 10 and 11 where the distance was 3/4 of an inch. The readings and velocities which they represent are shown in Table IX. A plot of the velocity traverse is shown in Fig. 58. From this traverse, the average velocity was 1.19 ft./sec.. This was found by determining the average of the mean velocity at each of the eleven stations.

The arithmetic mean value of the velocity in the flume as found from the eighty-eight values, equally weighted, was 1.18 ft./sec.. The mean square error or the standard deviation was 0.02 ft./sec., and the standard error of the mean value was 0.002 ft./sec..

The amount of water flowing through the flume was measured by an orifice located in the feeder pipe. Pressure tubes were connected from the orifice to a mercury manometer, as shown in Fig. 54. The difference in pressure reading on the manometer board was 1.91 inches of mercury (2.166 ft. of water). This orifice was calibrated on November 7, 1946. From the calibration chart for the orifice and this reading, it was found that 0.320 cu. ft./sec. of water was flowing. The water depth, as measured by a steel scale, was 3.85 in. on one side and 3.7 in. on the other side of the flume, or an average of 3.77 in. (0.314 ft.). The cross-sectional area of the flow was 0.307 sq.ft.

Fig. 58.

Velocity Distribution of a Flow Situation in a Hydraulic Flume as determined by a No. 14A Thermistor



Thus the average velocity was 0.320/0.307 or 1.042 ft./sec.. The average velocity as determined by the use of the probe was 1.19 ft./sec., or an indicated error of 0.15 ft./sec..

Since the orifice was calibrated in 1946, this calibration was questioned. It was decided to recalibrate the orifice at this one rate of flow. This was done on two occasions by actually weighing the amount of water flowing through the orifice. The average flow was 0.26 cu.ft./sec.. This indicates a large error between the actual flow and the flow indicated by the calibration curve. This error was 0.06 cu.ft./sec.. The reason for this error was an accumulation of foreign matter (rust) which decreased the area of the orifice over a period of time.

The cross-section of the flow was 0.307 sq.ft.. The actual average velocity of water flowing in the flume was 0.26/0.307 ft./sec., or 0.85 ft./sec.. This indicates an error in the average velocity as determined by the thermistor probe of 0.34 ft./sec., which represents a rather large fraction of the actual velocity.

The reasons for this error are numerous. First, the velocity at the side walls is zero. This was not considered in determining the average velocity with the probe, since the closest position of the probe to the side wall was 1 inch. However, the amount of this error should be small. The velocity at the bottom surface is also zero. However, at a depth of 1/8 of an inch above this surface, the velocity was 1.2 ft./sec..

Secondly, a portion of this error was due to the measurement of the water temperature. A mercury thermometer was used to measure the water temperature and the reading was estimated by eye. Theory indicates that a small error in temperature measurement may result in a rather large velocity error. It is suggested that if this method is used to study a flow situation, a more sensitive means of temperature measurement be employed both at the time of the measurement and at the time of probe calibration.

Thirdly, a portion of the error arose from the drift of the thermistor, due both to the actual drift of the temperature-resistance relationship of the thermistor and to the type of measurement circuit used. The constant-voltage type of circuit is susceptible to a drift to a higher value of resistance, which would indicate an apparent velocity greater than the actual velocity. The indicated error is in this direction. A constant-current type circuit is recommended for all future measurements.

Fourthly, the average velocity was calculated from only one reading at each of the eighty-eight different positions. If a detailed study were to be presented, it would be necessary to take several readings at each position as the probe was swept across the flume and a like number of readings when the probe was moved to different depths at each station. The data would then be treated statistically.

This error analysis has been concerned with errors in

velocity measurement at each point in the hydraulic flume. A fifth source of error would undoubtedly arise during the calibration process. These errors would also be analyzed by statistical means.

A sixth source of error lies in the possibility of turbulent flow during the flume measurement with the probe being calibrated in laminar flow. This would indicate a velocity greater than the actual velocity, which is the observed effect. A calculation of Reynolds' number for the flow in the flume indicates the possibility of turbulent flow. Visual observation of small particles during the measurement led the author to conclude that the flow was laminar, or nearly so.

Other sources of error undoubtedly exist. In the mind of the author, the source of the large error experienced lies in the drift of the resistance-temperature relationship of the thermistor, and the use of the constant-voltage type circuit. The error lies in the direction that a consideration of the theoretical power relationship would indicate with the use of this type of circuit.

This discussion has been concerned with an error in the absolute measurement of the average velocity in the flume. Fig. 58 shows that the relative point-by-point velocity measurements are rather uniform, which seems to be reasonable. Thus the error appears to be a consistent one which can be elimihated in a detailed study of a flow situation.

CHAPTER IV.

SUMMARY OF THESIS.

This thesis has presented the thermistor as a device for the measurement of velocity at any particular point in the flow. This ability is a result of a transfer of heat from the thermistor to the water. As the velocity and temperature of the water vary, the temperature and electrical resistance of the thermistor change. This heat is supplied in the form of electrical power, which, in turn, varies as the flow conditions change. This thesis presents these changes in the form of calibration curves when different types of electrical measurement circuits are used with the thermistor. The calibration curves will vary with each individual thermistor, but the concepts shown are still valid.

The type of thermistor used for this study was the Western Electric Co. No. 14A thermistor. Other types of thermistors may be used, and the shapes and forms of calibration curves may be derived for these thermistors by making appropriate changes in the "power input curves" of Figs. 28-30, pages 67-74. These changes are made in accordance with the resistance-temperature relationship of the thermistor.

It is felt, however, that thermistors which will produce single-valued calibration curves should be selected. In the beginning of this study, thermistors, composed mainly of iron oxide, were used. Under certain conditions of operation, the current flowing in the thermistor reached a minimum

near the center of the calibration range. Such a calibration curve is shown in Fig. 59. These thermistors may still be used provided that the operator knows the range of velocities in which the thermistor is operating. A velocity traverse, obtained by this type of thermistor, is shown in Fig. 60.

It is necessary that the thermistor and leads be electrically insulated from the water to insure a thermal response. Even a small amount of conduction through the water will completely obscure the results. A simple test consists of determining the current flowing when the thermistor, with a constant voltage applied, is in air and is in water. The thermistor current should be less when the thermistor is in water. The water temperature should be equal to, or less than that of the air.

Various types of electrical measurement circuits may be used. The main criterion is whether or not the operator can control the temperature of the water during the test. If the water temperature can be controlled, a Wheatstone bridge without compensation for a change in water temperature may be used. If the water temperature cannot be controlled, and the range of velocity changes to be studied is small, a Wheatstone bridge, utilizing another thermistor as the compensating resistor, may be used. If the water temperature cannot be controlled and the range of velocity changes is large, a circuit supplying the thermistor with a constant current should be used. A circuit supplying a constant voltage across the thermistor is not recommended.

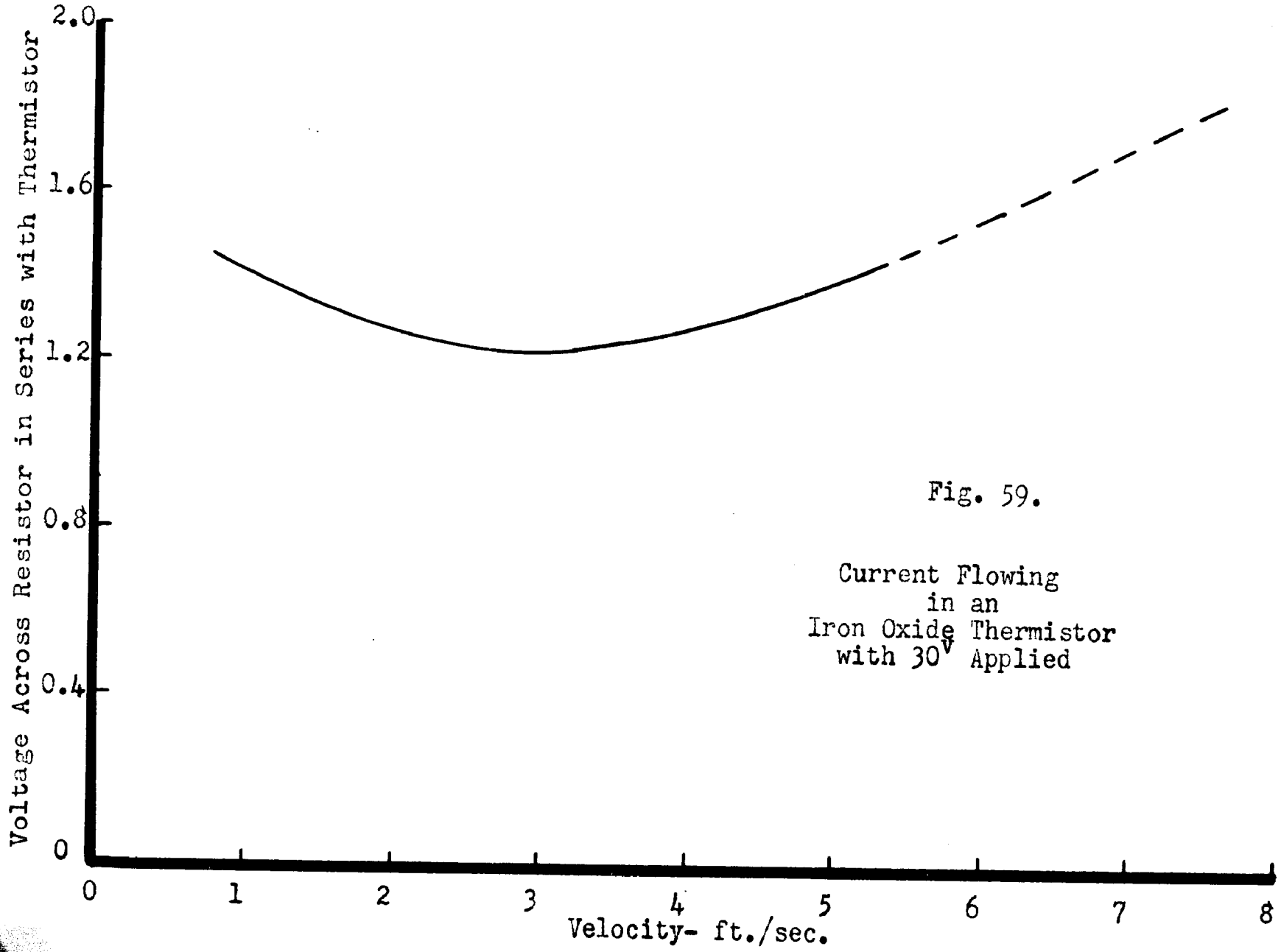


Fig. 59.

Current Flowing
in an
Iron Oxide Thermistor
with 30^{V} Applied

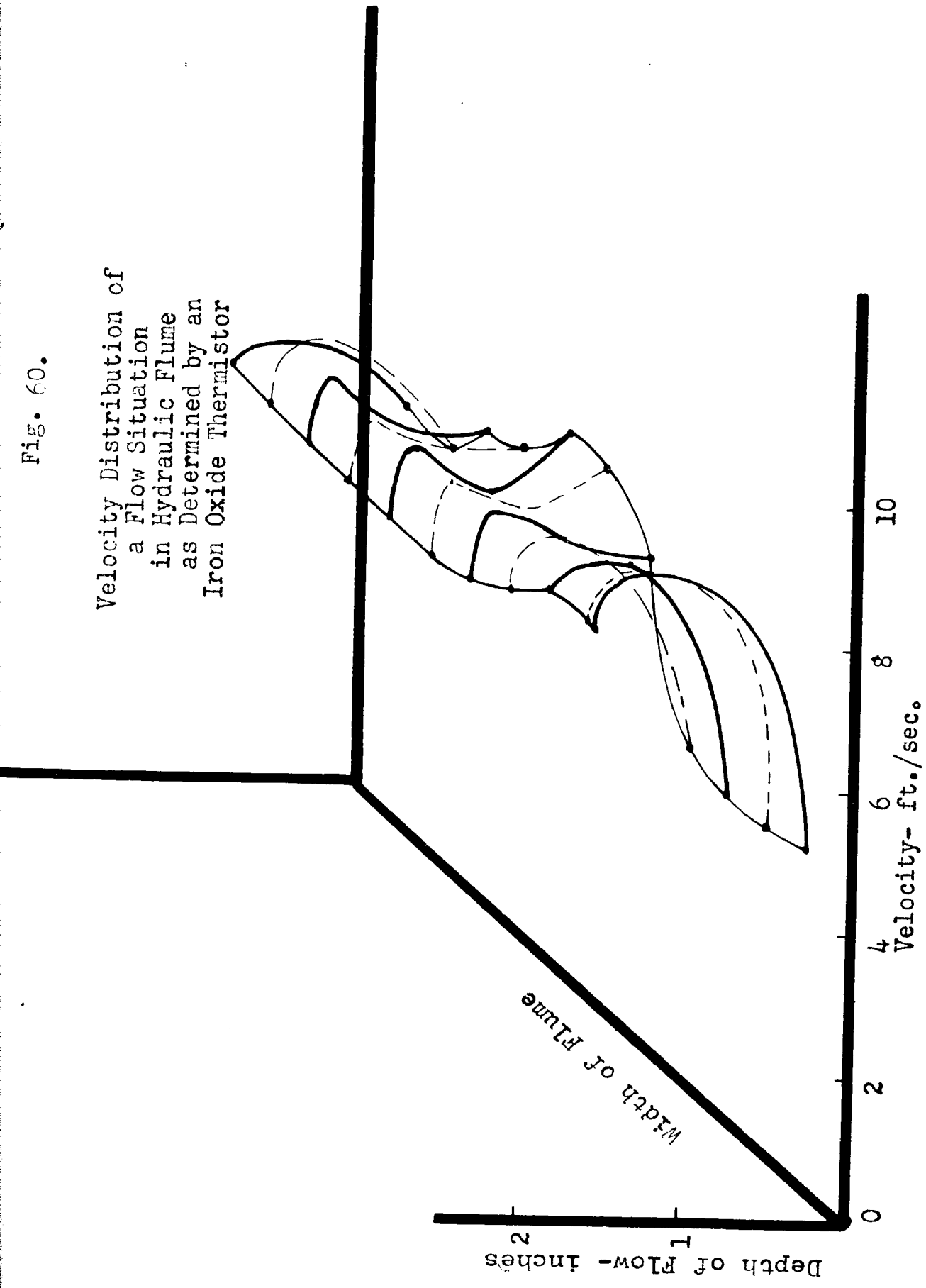


Fig. 60.

Velocity Distribution of a Flow Situation in Hydraulic Flume as Determined by an Iron Oxide Thermistor

The fundamental concepts of a very interesting and useful instrument are shown. Further research may proceed in two directions. The first is the measurement of velocities in turbulent flow. The thermistor, activated by a circuit supplying a constant amount of current, should be adequate for this type of study. Experimentation, however, is necessary to determine the limitations of the instrument. The second is the determination of the direction as well as the magnitude of a velocity at any point in the flow. This could be accomplished by placing more than one thermistor on the same probe, and noting the asymmetry of the response of the different thermistors. In this study there would be two problems. The first would be the uniqueness of the individual thermistors, which arises from the variation in the size, the amount of surface exposed, and the resistance-temperature relationship of the various thermistors. The second would be a means of calibration, which might be the more difficult of the two.

TABLE I

DETERMINATION OF FILM COEFFICIENT, h

Thermistor No. 1

Area = 1.34×10^{-5} sq.ft.

Observation	Velocity ft./sec.	T_w* °F.	E Volts	10^{-4} I Amps.	Q BTU./hr.	T_t* °F	$T_t - T_w$ °F	h BTU/hr./sq.ft./°F.
1	8.7	68.1	10	1.35	0.004599	81	12.9	26.6
2	5.57	68.4	10	1.32	0.004499	83.5	15.1	22.2
3	3.56	69.0	10	1.34	0.004568	86	17.	20.0
4	2.4	68.6	10	1.16	0.003957	88.5	19.9	14.8
5	1.51	69.2	10	1.37	0.004670	89.7	20.5	16.9
6	9.03	70.7	10	1.46	0.00497	91	20.3	18.2
7	5.88	70.7	10	1.49	0.005076	92.7	22.0	17.2
8	3.41	71.3	10	1.59	0.005414	93.5	22.2	18.2
9	1.04	72.6	10	1.64	0.005581	97.0	24.4	17.1
10	8.78	67.5	20	2.75	0.01877	85.5	18	77.7
11	5.71	67.7	20	3.0	0.02063	89.6	21.9	70.2
12	3.67	68.0	20	3.35	0.02335	93.5	25.5	69.1
13	2.35	68.5	20	4.34	0.02963	102	33.5	66.1
14	1.47	69.2	20	6.14	0.04096	113.6	44.4	68.7
15	8.87	70.3	20	3.57	0.02435	94.5	24.2	75.1
16	6.1	70.5	20	3.67	0.02504	109	38.5	48.6
17	3.82	70.8	20	3.17	0.02165	92	21.4	75.6
18	3.82	71.1	20	3.74	0.02551	94.5	23.4	81.3
19	2.64	71.3	20	3.42	0.02335	89.7	18.4	94.7
20	1.61	71.5	20	4.61	0.03146	85	13.5	172.5

*T_w = Water Temperature

*T_t = Thermistor Temperature

TABLE I (Cont.)

Observation	Velocity ft./sec.	T_w* °F.	E Volts	I 10^{-4} Amps.	Q BTU/hr.	T_t* °F.	$T_t - T_w$ °F.	h BTU/hr./sq.ft./°F.
21	8.78	67.4	30	4.13	0.04242	100	32.6	97
22	5.81	67.6	30	7.34	0.07511	105.5	37.9	147.8
23	3.75	67.9	30	7.88	0.08073	107.5	39.6	152.1
24	2.57	68.5	30	8.87	0.09084	112.5	44	154
25	1.61	68.9	30	10.07	0.10302	115	46.1	167.1
26	9.11	68.4	30	4.16	0.04263	79.5	11.1	286
27	5.92	68.4	30	3.81	0.03908	78.5	10.1	289
28	9.03	68.5	30	7.68	0.07866	98	29.5	198.5
29	9.2	69.4	30	5.95	0.06090	100	30.6	148.2
30	5.7	69.4	30	6.96	0.07130	95.5	26.1	204
31	5.88	69.4	30	6.22	0.06597	93	23.6	208
32	9.29	70.	30	3.37	0.03451	96	26	99
33	6.25	70.4	30	3.52	0.03603	97	26.6	101
34	4.17	70.6	30	3.47	0.03552	96.5	25.9	102.2
35	2.47	71.0	30	3.17	0.03248	93	22	110

TABLE II

DETERMINATION OF FILM COEFFICIENT, h

Thermistor No. 2

Area = 0.92×10^{-5} sq.ft.

Observation	Velocity ft./sec.	T_w* °F.	E Volts	I 10^{-4} Amps.	Q BTU/hr.	T_t* °F.	$T_t - T_w$ °F.	h BTU/hr./sq.ft./°F.
1	8.06	48.1	10	0.5523	0.00187	51.0	2.9	70.3
2	5.42	48.6	10	0.5623	0.00192	52.0	3.4	61.4
3	3.67	48.8	10	0.5824	0.001985	51.7	2.9	73.1
4	2.49	50.3	10	0.6225	0.002122	53.7	3.4	67.8
5	1.59	51.8	10	0.6828	0.002327	57.3	5.5	46.0
6	1.06	53.5	10	0.8334	0.002842	58.5	5	61.8
7	7.62	47.3	15	1.014	0.005188	50.3	3.0	188.0
8	5.24	47.3	15	1.0845	0.005547	53.9	6.6	91.2
9	3.33	48.8	15	1.1648	0.005960	53.3	4.5	144.0
10	2.25	50.1	15	1.2953	0.006625	55.7	5.6	127.4
11	1.42	51.7	15	1.3656	0.006907	56.0	4.3	176.7
12	1.05	53.7	15	1.6267	0.008320	61.7	8.0	112.3
13	7.82	46.9	20	1.355	0.009245	47.7	0.8	1256.0
14	5.11	47.3	20	1.376	0.009385	48.2	1.9	523.0
15	3.44	48.5	20	1.677	0.011438	49.7	1.2	994.0
16	2.25	50.2	20	1.807	0.012332	54.0	3.8	353.0
17	1.47	52.8	20	2.209	0.015072	59.2	6.4	256.0
18	.88	54.7	20	2.531	0.017262	62.0	7.3	257.0
19	7.8	47.2	25	2.279	0.01944	51.7	4.5	464.0
20	5.38	47.4	25	2.309	0.01969	52.0	4.6	465.0
21	3.63	48.3	25	2.410	0.02055	52.2	3.9	566.0
22	2.45	49.8	25	2.611	0.02226	56.7	6.9	348.0
23	1.71	51.8	25	3.765	0.03211	63.2	11.4	305.0
24	1.13	53.3	25	4.017	0.03435	67.2	13.9	268.0

*T_w = Water Temperature

*T_t = Thermistor Temperature

TABLE III

EVALUATION OF DIMENSIONLESS RATIOS

Thermistor No. 1

L/D = 1.69

Observation	ρ Mass/ ft.^{3}	ν Kinematic Viscosity	hD/K Nusselt's Number	$C_p \rho vD/k$ Peclets Number	$\rho vD/\mu$ Reynolds' Number	$C_p \mu/k$ Prandtl's Number	$EI/\rho v^3 D^2$ Power Ratio	$h/v\rho C_p$ Stanton's Number
1	1.9367	1.0822	.2100	15,350	2147	7.14	0.157	1.62
2	1.9366	1.078	.1740	9,870	1378	7.10	0.554	1.79
3.	1.9365	1.069	.1575	6,300	889	7.06	2.135	2.52
4	1.9366	1.075	.1165	4,240	596	7.10	6.230	2.78
5	1.9364	1.066	.1338	2,667	378	7.04	28.300	5.05
6	1.936	1.045	.1430	15,960	2310	6.90	0.145	0.91
7	1.936	1.045	.1345	10,340	1505	6.90	0.533	1.31
8	1.9359	1.037	.1425	6,000	878	6.85	2.900	2.40
9	1.9356	1.020	.1338	1,824	272	6.75	106.800	7.40
10	1.9368	1.091	0.613	15,570	2150	7.18	9.622	3.98
11	1.9368	1.088	0.552	10,060	1400	7.15	2.360	5.53
12	1.9367	1.0836	0.540	6,480	903	7.14	10.100	8.47
13	1.9366	1.076	0.52	4,160	583	7.10	48.700	12.65
14	1.9364	1.066	0.539	2,595	368	7.04	276.0	21.00
15	1.9361	1.051	0.591	15,670	2252	6.95	.701	3.80
16	1.9361	1.048	0.380	10,795	1552	6.93	2.34	3.58
17	1.9360	1.044	0.591	6,745	977	6.89	8.33	8.91
18	1.9360	1.040	0.637	6,745	977	6.88	9.80	9.58
19	1.9359	1.037	0.742	4,645	679	6.85	27.15	16.15
20	1.9359	1.045	1.355	2,850	411	6.85	161.0	48.20
21	1.9368	1.09	0.767	15,590	2150	7.17	1.42	4.97
22	1.9368	1.09	1.17	10,270	1421	7.16	8.15	11.48
23	1.9367	1.085	1.197	6,635	922	7.14	32.5	18.23

TABLE III (Cont.)

Observation	ρ Mass/ft.^{3}	ν Kinematic Viscosity	hD/K Nusselt's Number	$C_p \rho vD/k$ Peclet's Number	$\rho vD/\mu$ Reynolds' Number	$C_p \mu/k$ Prandtl's Number	$EI/\rho v^3 D^2$ Power Ratio	$h/v\rho C_p$ Stanton's Number
24	1.9366	1.076	1.215	4,550	638	7.10	113.8	27.
25	1.9365	1.073	1.312	2,840	401	7.06	525.	46.7
26	1.9366	1.078	2.25	16,100	2255	7.10	1.203	14.15
27	1.9366	1.078	2.27	10,440	1466	7.10	4.03	22.7
28	1.9366	1.076	1.57	15,960	2247	7.10	2.36	9.9
29	1.9364	1.063	1.165	16,200	2310	7.03	1.66	7.24
30	1.9364	1.063	1.601	10,090	1432	7.03	8.2	16.1
31	1.9364	1.063	1.639	10,400	1478	7.03	6.92	15.86
32	1.9362	1.055	0.778	16,380	2350	6.97	9.22	4.88
33	1.9361	1.049	0.791	11,020	1590	6.94	31.4	7.27
34	1.9361	1.047	0.803	7,350	1063	6.92	106.3	11.07
35	1.9360	1.041	0.866	4,355	635	6.89	462.	20.

TABLE IV

EVALUATION OF DIMENSIONLESS RATIOS

Thermistor No. 2 L/D = 0.89

Observation	ρ (Mass/ ft.^{3})	ν Kinematic Viscosity	hD/K Nusselt's Number	$C_p \rho vD/k$ Peclet's Number	$\rho vD/\mu$ Reynolds' Number	$C_p \mu/k$ Prandtl's Number	$EI/\rho v^3 D^2$ Power Ratio	$h/v\rho C_p$ Stanton's Number
1	1.939	1.462	.642	16,570	1650	9.3	.06	3.9
2	1.939	1.44	.56	11,110	1129	9.23	.20	5.1
3	1.939	1.429	.667	7,550	770	9.20	.67	8.9
4	1.939	1.409	.616	5,110	530	9.02	2.33	12.2
5	1.939	1.385	.418	3,260	345	8.82	9.77	13.0
6	1.939	1.336	.560	2,160	238	8.63	40.2	26.2
7	1.939	1.459	1.718	15,660	1567	9.40	.19	11.1
8	1.939	1.459	.836	10,800	1078	9.40	.64	7.9
9	1.939	1.430	1.312	6,835	698	9.20	2.72	19.4
10	1.939	1.408	1.160	4,620	479	9.05	9.86	25.4
11	1.939	1.385	1.608	2,910	329	8.82	41.1	56.
12	1.939	1.334	1.017	2,140	236	8.60	120.5	48.2
13	1.939	1.476	11.50	16,100	1587	9.45	0.29	72.2
14	1.939	1.459	4.80	10,500	1042	9.40	1.18	46.1
15	1.939	1.44	9.1	7,100	716	9.24	4.72	130
16	1.939	1.409	3.215	4,620	482	9.04	18.17	70.6
17	1.939		2.325	3,000	402	8.70	80.0	78.3
18	1.939	1.311	2.33	1,805	203	8.48	427.0	130
19	1.939	1.460	4.25	15,980	1592	9.42	0.69	26.8
20	1.939	1.458	4.24	11,090	1100	9.40	2.12	38.8
21	1.939		5.17	7,480	742	9.25	7.22	70.2
22	1.939	1.410	3.17	5,030	522	9.07	25.5	63.9
23	1.939	1.382	2.77	3,500	372	8.80	107.5	80.3
24	1.939	1.340	2.415	2,307	253	8.65	400.0	106.7

TABLE V

DETERMINATION OF CONSTANT, k'''

istor No. 1

$k''' = \text{Units} \times 10^{-9}$

n	$\frac{hD}{k}$	$\frac{\rho v D^{2.75}}{u}$	$\frac{EI}{\rho v^3 D^2}^{0.81}$	k'''	Dev. of k'''	% Dev.
	.21	1.45×10^9	.222	.65	.099	18.0
	.174	4.37×10^8	.62	.642	.091	16.5
	.157	1.29×10^8	1.85	.658	.107	19.4
	.1165	4.36×10^7	4.41	.606	.055	10.0
	.1338	1.23×10^7	15.0	.725	.174	31.6
	.143	1.74×10^9	.21	.393	-.158	28.6
	.1345	5.5×10^8	.602	.407	-.144	26.1
	.1425	1.26×10^8	2.37	.478	-.073	13.2
	.1338	5.75×10^6	44.	.529	-.022	4.0
	.613	1.46×10^9	.680	.619	.068	12.3
	.552	4.47×10^8	2.01	.614	.063	11.4
	.54	1.32×10^8	6.47	.633	.082	14.8
	.52	3.98×10^7	23.2	.562	.011	2.0
	.539	1.12×10^7	95.	.507	-.044	8.0
	.591	1.62×10^9	.75	.487	-.064	11.6
	.380	5.88×10^8	1.99	.324	-.227	41.2
	.591	1.7×10^8	5.57	.623	.072	13.1
	.637	1.7×10^8	6.34	.591	.040	7.2
	.742	6.31×10^7	14.5	.811	.260	47.1
	.767	1.35×10^9	1.328	.427	-.124	22.5
	1.17	4.68×10^8	5.50	.453	-.098	17.8
	1.197	1.41×10^8	16.7	.507	-.044	8.0
	1.215	5.9×10^7	46.2	.447	-.104	18.9
	1.312	1.44×10^7	160.	.57	.019	3.4
	1.57	1.61×10^9	2.01	.486	-.065	11.8
	1.165	1.78×10^9	1.507	.435	-.116	21.0
	1.601	4.78×10^8	5.52	.607	.056	10.1
	1.639	5.25×10^8	4.80	.65	.099	18.0
			Mean Value	.551		

The value of k''' for the following differed appreciably from the mean value.

1.355	1.58×10^7	61.5	1.39
2.25	1.62×10^9	1.16	1.19
2.27	5.01×10^8	3.10	1.46
0.778	1.86×10^9	6.04	.069
0.791	7.25×10^8	16.4	.066
0.803	2.04×10^8	43.5	.090
0.866	5.02×10^7	144.	.12

TABLE V

DETERMINATION OF CONSTANT, k'''

Thermistor No. 1 $k''' = \text{Units} \times 10^{-9}$

Observation	$\frac{hD}{k}$	$\frac{\rho v D^{2.75}}{u}$	$\frac{EI}{\rho v^3 D^2}^{0.81}$	k'''	Dev. of k'''	% Dev.
1	.21	1.45×10^9	.222	.65	.099	18.0
2	.174	4.37×10^8	.62	.642	.091	16.5
3	.157	1.29×10^8	1.85	.658	.107	19.4
4	.1165	4.36×10^7	4.41	.606	.055	10.0
5	.1338	1.23×10^7	15.0	.725	.174	31.6
6	.143	1.74×10^9	.21	.393	-.158	28.6
7	.1345	5.5×10^8	.602	.407	-.144	26.1
8	.1425	1.26×10^8	2.37	.478	-.073	13.2
9	.1338	5.75×10^6	44.	.529	-.022	4.0
10	.613	1.46×10^9	.680	.619	.068	12.3
11	.552	4.47×10^8	2.01	.614	.063	11.4
12	.54	1.32×10^8	6.47	.633	.082	14.8
13	.52	3.98×10^7	23.2	.562	.011	2.0
14	.539	1.12×10^7	95.	.507	-.044	8.0
15	.591	1.62×10^9	.75	.487	-.064	11.6
16	.380	5.88×10^8	1.99	.324	-.227	41.2
17	.591	1.7×10^8	5.57	.623	.072	13.1
18	.637	1.7×10^8	6.34	.591	.040	7.2
19	.742	6.31×10^7	14.5	.811	.260	47.1
21	.767	1.35×10^9	1.328	.427	-.124	22.5
22	1.17	4.68×10^8	5.50	.453	-.098	17.8
23	1.197	1.41×10^8	16.7	.507	-.044	8.0
24	1.215	5.9×10^7	46.2	.447	-.104	18.9
25	1.312	1.44×10^7	160.	.57	.019	3.4
28	1.57	1.61×10^9	2.01	.486	-.065	11.8
29	1.165	1.78×10^9	1.507	.435	-.116	21.0
30	1.601	4.78×10^8	5.52	.607	.056	10.1
31	1.639	5.25×10^8	4.80	.65	.099	18.0
			Mean Value	.551		

The value of k''' for the following differed appreciably from the mean value.

20	1.355	1.58×10^7	61.5	1.39
26	2.25	1.62×10^9	1.16	1.19
27	2.27	5.01×10^8	3.10	1.46
32	0.778	1.86×10^9	6.04	.069
33	0.791	7.25×10^8	16.4	.066
34	0.803	2.04×10^8	43.5	.090
35	0.866	5.02×10^7	144.	.12

TABLE VI

DETERMINATION OF CONSTANT, k'''

hermistor No. 2 $k''' = \text{Units} \times 10^{-9}$

bser-ation	$\frac{hD}{k}$	$\frac{\rho v D^{2.75}}{\mu}$	$\frac{EI^{0.81}}{\rho v^3 D^2}$	k'''	Dev. of k'''	% Dev.
1	.642	7.07×10^8	.103	8.82	.69	8.5
2	.56	6.92×10^8	.275	2.95	-5.18	63.6
3	.667	8.5×10^7	.726	10.80	2.67	32.8
4	.616	3.16×10^7	1.97	9.89	1.56	19.7
5	.418	9.55×10^6	6.32	6.96	-1.17	14.2
6	.560	3.47×10^6	20.	8.07	- .06	0.7
7	1.718	6.3×10^8	.27	10.10	1.97	24.2
8	.836	2.04×10^8	.702	5.84	-2.29	28.2
9	1.312	6.61×10^7	2.25	8.82	.69	8.5
10	1.160	2.34×10^7	6.38	7.77	- .36	4.4
11	1.1608	8.31×10^6	20.2	9.58	1.45	17.8
12	1.017	3.47×10^6	48.2	6.08	-2.05	25.2
16	3.215	2.4×10^7	10.6	12.6	4.47	54.8
17	2.325	1.44×10^7	35.0	4.59	3.54	43.5
18	2.33	2.24×10^6	135.	7.71	- .42	5.2
19	4.25	6.31×10^8	.74	9.11	.98	12.1
20	4.24	2.29×10^8	1.837	10.10	-1.97	24.2
21	5.17	7.75×10^7	4.97	13.42	5.29	65.
22	3.17	3.02×10^7	13.8	7.6	0.53	6.5
23	2.77	1.17×10^7	44.0	5.38	2.75	33.8
24	2.415	4.07×10^6	128.0	4.65	3.48	42.8
			Mean Value	8.13		

The value of k''' for the following differed appreciably from the mean value.

13	11.50	6.31×10^8	.368	49.6
14	4.80	1.99×10^8	1.18	20.4
15	9.1	7.07×10^7	3.52	36.6

TABLE VII

DETERMINATION OF CONSTANT, k'''''

Thermistor No. 1

$k''''' = \text{Units} \times 10^{-5}$

Observation	$\frac{EI}{\rho v^3 D^2}^{0.328}$	$\frac{h}{\rho v C_p}$	k'''''	Deviation of k''''' from mean value	% Dev.
1	.544	1.62	2.98	- .82	21.6
2	.824	1.79	2.17	-1.63	43.
3	1.282	2.52	1.97	-1.83	48.2
4	1.82	2.78	1.53	-2.27	59.8
5	3.0	5.05	1.68	-2.12	56.8
6	0.53	.91	1.72	-2.08	54.7
7	0.813	1.31	1.61	-2.19	57.8
8	1.417	2.40	1.69	-2.11	55.5
9	4.62	7.40	1.60	-2.20	58.
10	.856	3.98	4.64	.84	22.1
11	1.325	5.53	4.17	.37	9.8
12	2.133	8.47	3.97	.17	4.5
13	3.58	12.65	3.54	- .26	6.8
14	6.60	21.	3.18	- .62	16.3
15	.890	3.8	4.27	.47	12.4
16	1.321	3.50	2.71	-1.09	28.7
17	2.00	8.91	4.45	.65	17.1
18	2.115	9.58	4.53	.73	19.2
19	2.96	16.15	5.45	1.65	43.4
21	1.122	4.97	4.41	.61	16.1
22	1.99	11.48	5.78	1.98	51.2
23	3.13	18.23	5.82	2.04	53.7
24	4.73	27.	5.70	1.90	50.
25	7.8	46.7	5.99	2.19	57.7
28	1.326	9.9	7.45	3.65	96.
29	1.181	7.24	6.12	2.42	63.7
30	1.997	16.1	8.02	4.20	111.
31	1.888	15.86	8.40	4.60	121.
32	2.07	4.88	2.36	-1.44	37.9
33	3.10	7.27	2.34	-1.46	38.4
34	4.61	11.07	2.40	-1.40	36.9
35	7.48	20.0	2.68	-1.12	29.5
		Mean Value	3.8		

The value of k''''' for the following differed appreciably from the mean value.

20	5.28	48.2	9.13
26	1.062	14.15	13.35
27	1.58	22.7	14.37

TABLE VIII

DETERMINATION OF CONSTANT, k'''''

Thermistor No. 2 $\qquad k''''' = \text{Units} \times 10^{-5}$

Observation	$\frac{EI}{\rho v^3 D^2}^{0.328}$	$\frac{h}{\rho v C_p}$	k'''''	Deviation of k''''' from mean value	% Dev.
1	.397	3.93	9.88	-6.62	40.0
2	.592	5.11	8.64	-7.86	47.6
3	.987	8.95	9.07	-7.43	45.0
4	1.32	12.25	9.28	-7.22	43.7
5	2.11	13.00	6.17	-9.83	59.5
6	3.36	26.2	7.80	-8.70	52.8
7	.587	11.1	18.9	2.4	14.5
8	.868	7.96	9.17	-7.33	44.5
9	1.388	19.4	14.0	-2.5	15.1
10	2.11	25.25	11.96	-4.54	27.6
11	3.39	56.	16.50	-	0
12	4.8	48.2	10.0	-6.5	39.4
16	2.59	70.6	27.2	10.7	65.
17	4.2	78.3	18.6	2.1	12.7
18	7.3	130.0	17.8	1.3	7.9
19	.885	26.8	30.3	13.8	83.7
20	1.279	38.8	30.3	13.8	83.7
21	1.92	70.2	36.6	20.1	122.
22	2.90	63.9	22.	5.5	33.4
23	4.65	80.3	17.3	0.8	4.8
24	7.15	106.7	14.9	-1.6	9.7
		Mean Value	16.5		

The value of k''''' for the following differed appreciably from the mean value.

13	.668	72.2	108.
14	1.057	46.1	43.7
15	1.665	130.	78.1

TABLE IX

VELOCITY TRAVERSE OF HYDRAULIC FLUME

Distance From Bottom (in.)	1/8		5/8		1-1/8		1-5/8		2-1/8		2-5/8		3-1/8		3-5/8	
Water Temp. (^{o}F.)	51.4		51.5		51.6		51.6		51.7		51.8		51.8		51.9	
Position	Rdg.	Vel.	Rdg.	Vel.	Rdg.	Vel.	Rdg.	Vel.	Rdg.	Vel.	Rdg.	Vel.	Rdg.	Vel.	Rdg.	Vel.
0	1.12	1.2	1.125	1.15	1.125	1.25	1.12	1.32	1.13	1.2	1.14	1.05	1.14	1.05	1.15	0.9
1	1.12	1.2	1.12	1.25	1.12	1.32	1.12	1.32	1.13	1.2	1.14	1.05	1.14	1.05	1.145	1.0
2	1.12	1.2	1.11	1.5	1.12	1.32	1.12	1.32	1.13	1.1	1.14	1.05	1.14	1.05	1.145	1.0
3	1.12	1.2	1.12	1.25	1.12	1.32	1.12	1.32	1.14	1.0	1.14	1.15	1.14	1.05	1.145	1.0
4	1.12	1.2	1.12	1.25	1.12	1.32	1.13	1.1	1.135	1.1	1.14	1.15	1.14	1.05	1.145	1.0
5	1.12	1.2	1.12	1.25	1.12	1.32	1.13	1.1	1.13	1.2	1.14	1.15	1.14	1.05	1.145	1.0
6	1.11	1.4	1.12	1.25	1.12	1.32	1.12	1.32	1.13	1.2	1.14	1.25	1.14	1.05	1.145	1.0
7	1.12	1.2	1.11	1.5	1.12	1.32	1.13	1.1	1.13	1.2	1.13	1.25	1.14	1.05	1.145	1.0
8	1.12	1.2	1.11	1.5	1.12	1.32	1.12	1.32	1.13	1.2	1.13	1.25	1.14	1.05	1.145	1.0
9	1.12	1.2	1.12	1.25	1.12	1.32	1.12	1.32	1.13	1.2	1.13	1.25	1.14	1.05	1.145	1.0
10	1.12	1.2	1.12	1.25	1.12	1.32	1.12	1.32	1.13	1.2	1.12	1.35	1.14	1.05	1.145	1.0

Depth at station 0 - 3.85 inches = .321 ft.
10 3.7 inches .308 ft. Av. = 0.314

Orifice Rdg's. 14.07 - 12.16 inches Hg.
= 1.91 inches Hg.
1 in. Hg ≡ 1.134 ft. H_2O (62^{o}F.).

APPENDIX A.

A SHORT MANUAL OF OPERATION FOR THE USE OF A THERMISTOR TO MEASURE FLUID FLOW.

This section is intended for the technician who wishes to use the thermistor, but does not wish to read the previous work.

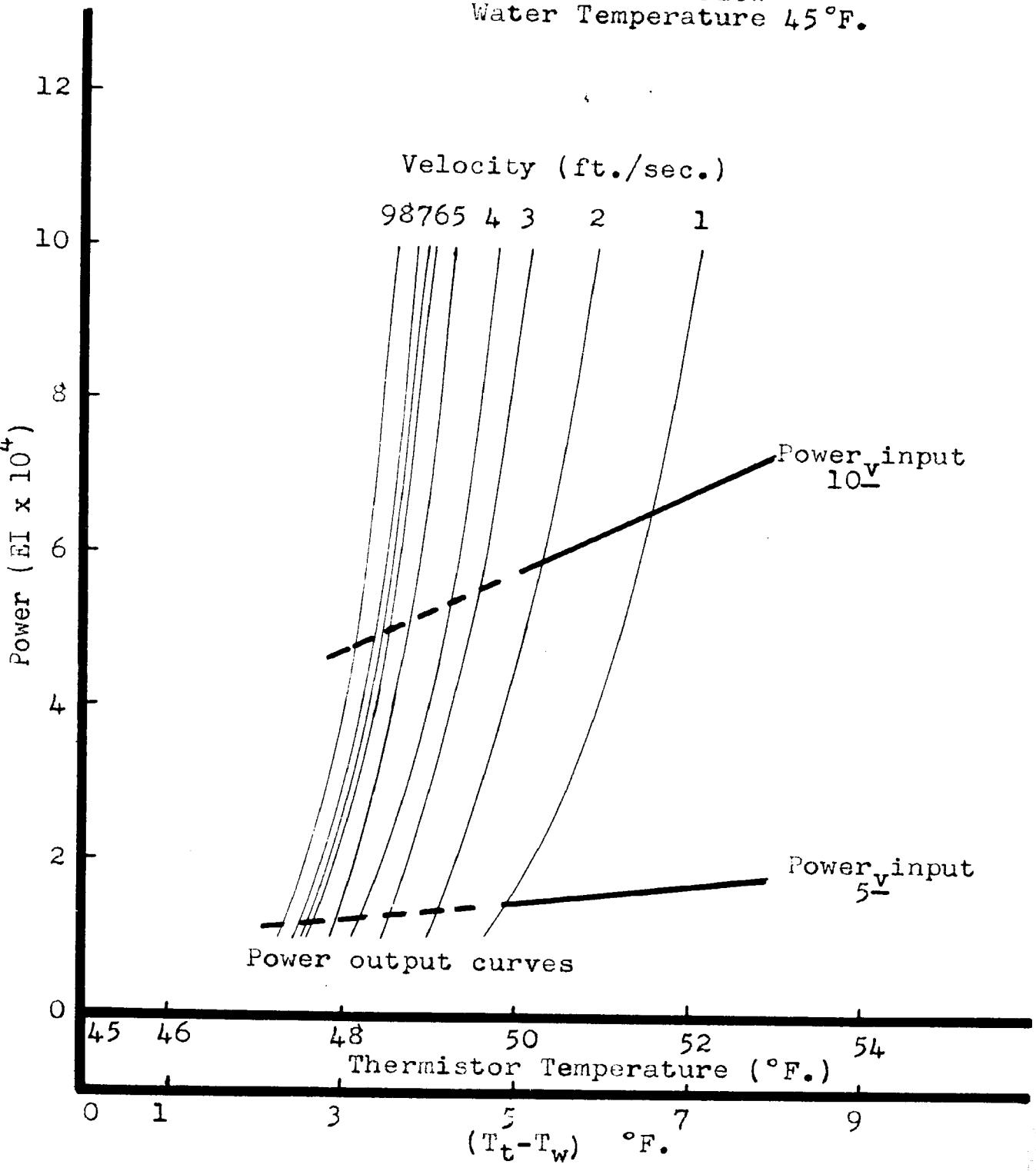
The thermistor operates on the principle that heat is imparted to it electrically, and is then transferred to the fluid flowing past it. The temperature to which the thermistor must rise depends upon the amount of power imparted, the thermistor specifications, and the velocity and temperature of the water. This temperature is measured by the electrical resistance of the thermistor through the electrical voltage and current. An insight into the power relationships for the Western Electric Co. No. 14A thermistor that was used for this study is shown in Fig. A-1. One of these diagrams can be constructed for each water temperature.

Selection, Preparation, And Mounting of Thermistors

Thermistors are made in many shapes which depend on the specifications of any particular situation. If rapid response to small and sudden velocity changes is necessary, a thermistor that has a large surface-to-volume ratio is preferable. The exposed area should not be too great for the amount of power applied, which is limited by the size of the thermistor and is usually specified by the manufacturer. The exposed areas for the thermistors tested were approximately one square millimeter.

Fig. A-1.

Power Diagram for Thermistor
in Fluid Flow
Water Temperature 45°F.



With about 0.005 watt of electrical power, the difference between the water and the thermistor temperature varied from about 2 to 10°F., which was sufficient. Either a high or a low resistance thermistor may be selected, as long as the thermal coefficient of resistivity is the same. Quantitatively, the response of the lower resistance thermistor will be less, and additional amplification will be needed. The thermal noise should be less, however.

Thermistors are made of sintered oxides of various materials. Prior to the selection of the No. 14A thermistor, which is composed of nickel and manganese oxides, considerable work was done with thermistors composed primarily of iron oxides. It was found that under certain conditions of operation a response curve was obtained that had a minimum value near the center of the calibration range. Such a calibration curve is shown in Fig. A-2. This type of thermistor may be used if the operator is sure of the velocity range. In general, however, it is not recommended.

The preparation depends on the condition in which the thermistor is received from the manufacturer. The No. 14A thermistor was received in a pyrex glass sheath which was sintered to the thermistor. This sheath was removed by etching the glass from the thermistor with hydrofluoric acid.

In general, the following conditions must be met in any particular preparation:

(1) The thermistor should be exposed to the flow.

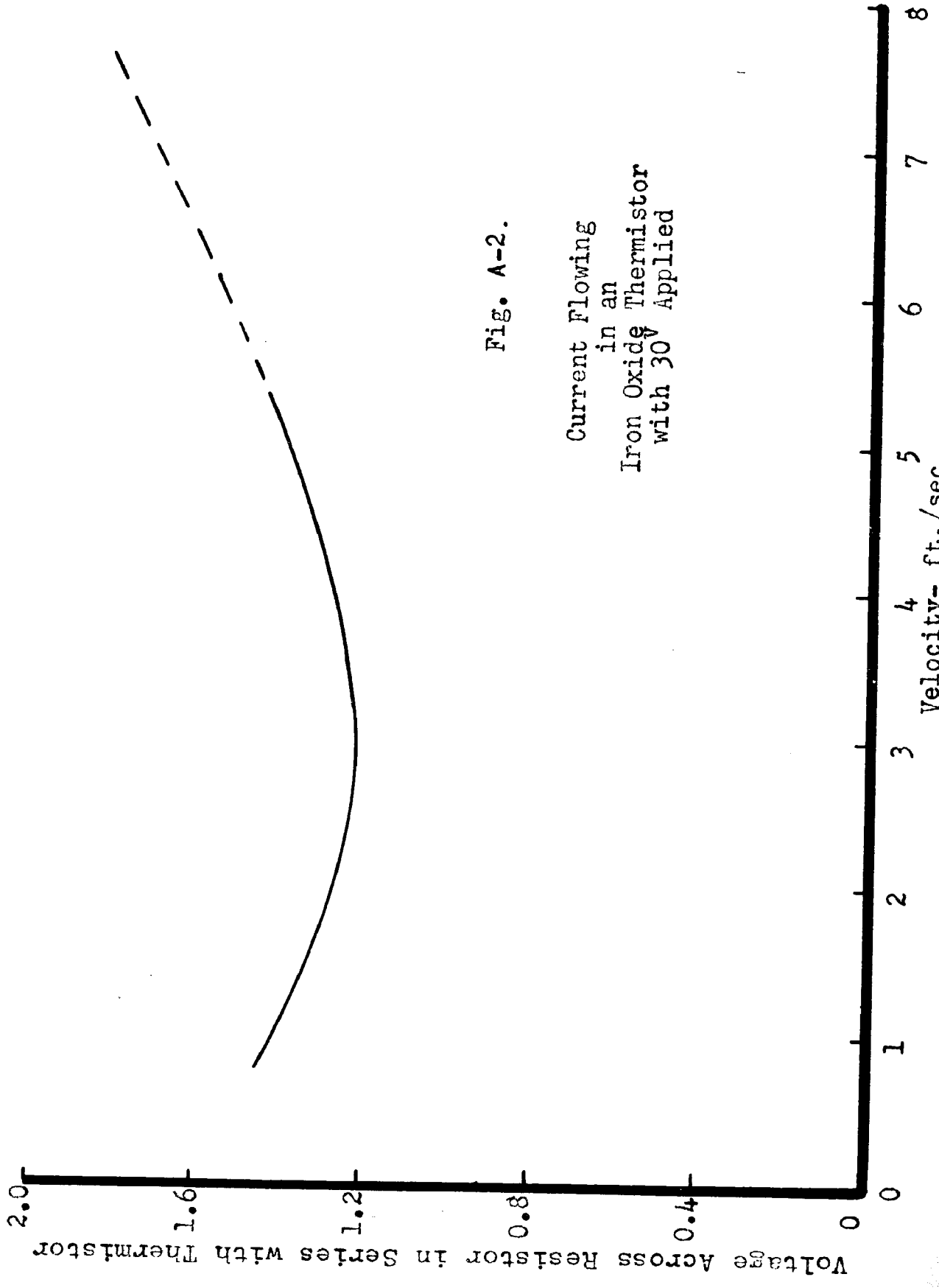


Fig. A-2.

Current Flowing
in an
Iron Oxide Thermistor
with 30 V Applied

(2) The amount of exposed area should not be too great for the amount of allowable power that can be applied to the thermistor.

(3) The electrical leads and connections must be perfectly insulated from the fluid flow. Any conduction of electricity through the water usually obscures the response due to thermal effects.

A simple test for this condition is to determine the thermistor response when it is exposed to both air and water. If the current in the thermistor is held constant by external circuitry, the voltage across the thermistor should be greater when the thermistor is exposed to water. The water should be slightly colder than the air for this test.

A thermistor should be mounted rigidly in a durable material. Of the number of thermistors used for this thesis, the main cause of failure was due to the glass mounting. The mount should be designed so that the material next to the thermistor is a thermal insulator. This will allow all heat to be transferred directly to the fluid. The seal between this material and the thermistor should be waterproof so that the water cannot penetrate into the mount and short the thermistor leads. The thermistor should be mounted so that it may be easily removed from the test situation for calibration. Since thermistors "drift" from their original calibration, a means of recalibration is necessary.

Calibration

Known flow velocities are necessary in order to calibrate the thermistor. There are several ways by which calibration velocities may be found. The particular method used is shown in principle in Fig. A-3. The velocity profile just downstream from a constriction in a round, smooth pipe has been shown to be nearly uniform, when the velocity in the larger of the two pipes is considerably less than that in the smaller pipe. This method applies when the flow is both laminar and turbulent. For this thesis, the constriction used was made of plastic which could be easily lathe-turned and polished.

Choice of Circuits

The choice of circuits depends mainly on whether or not the water temperature can be controlled. If it can, a Wheatstone bridge can be set up, where the thermistor temperature may be fixed and the power necessary to maintain this temperature may be measured. This temperature is maintained by keeping the value of the thermistor resistance constant by varying the power input to the bridge, either automatically or manually. A typical calibration curve is shown in Fig. A-4. An insight into the errors to be expected with this type of circuit is shown in Fig. A-5. This circuit should provide a very sensitive instrument, where the sensitivity may be adjusted in the desired velocity range by varying the thermistor temperature.

When the water temperature cannot be controlled, two basically different circuits may be used. The first is a potentiometric circuit utilizing a constant current in the thermistor.

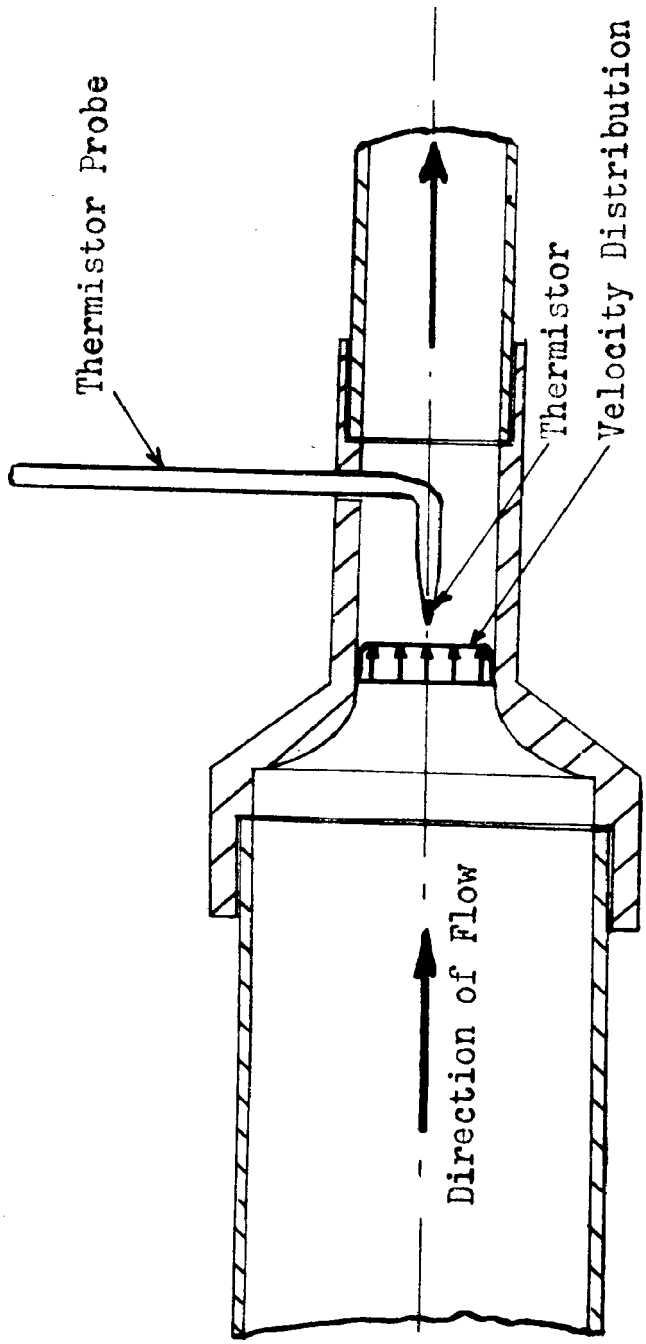


Fig. A-3.

Cutaway Section of Calibrating Apparatus
(Not drawn to scale)

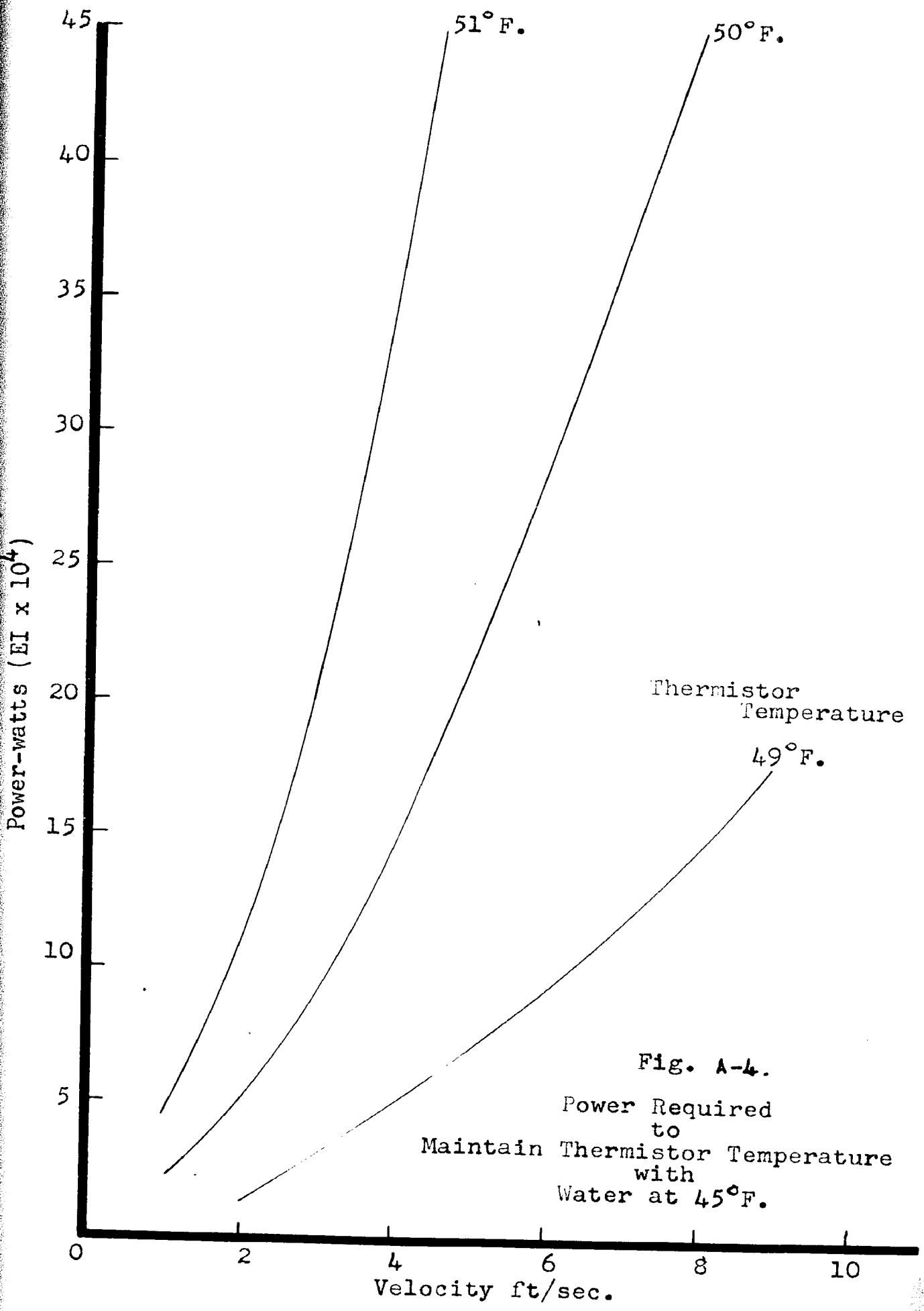


Fig. A-4.

Power Required to Maintain Thermistor Temperature with Water at 45°F.

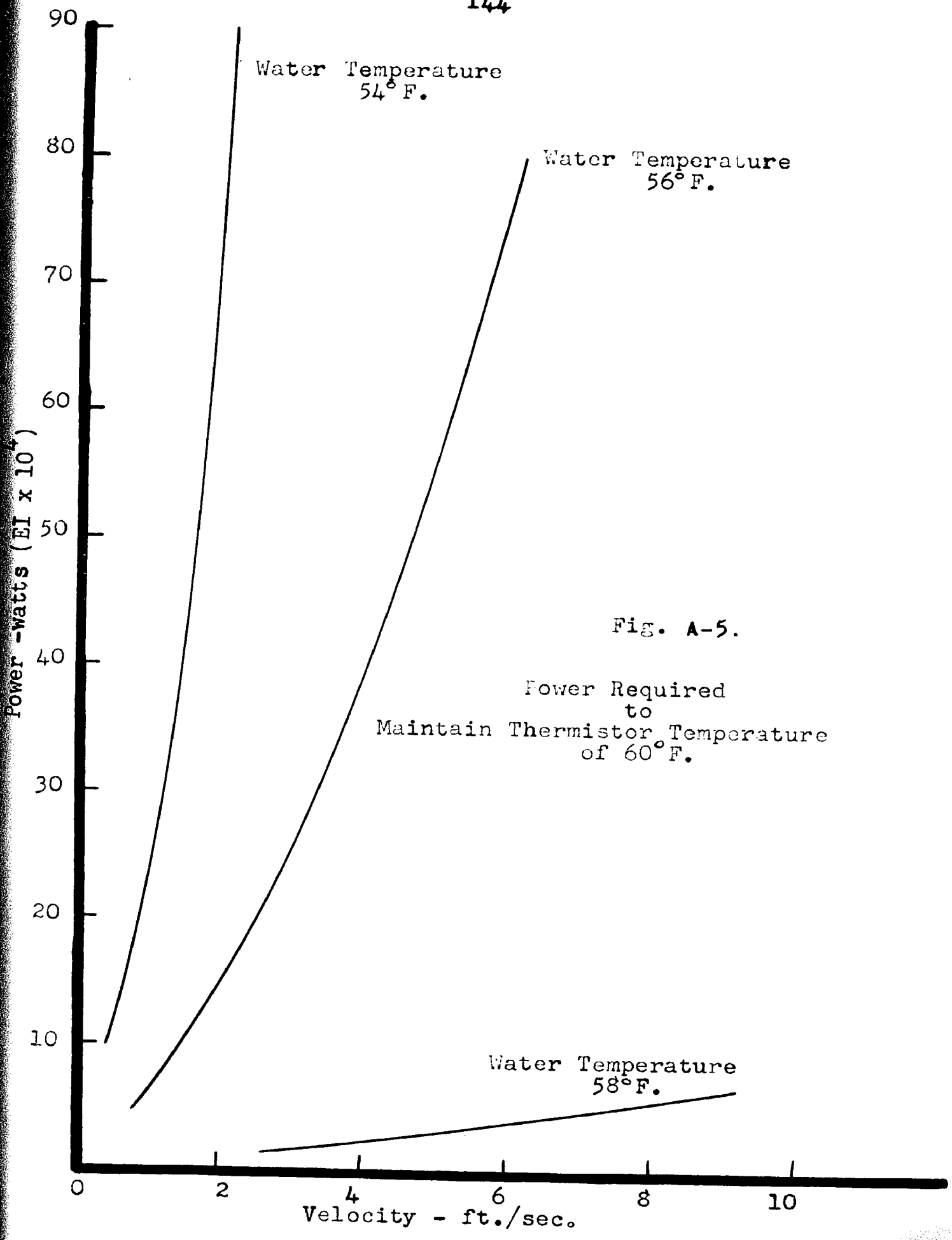


Fig. A-5.

Power Required
to
Maintain Thermistor Temperature
of 60° F.

The voltage across the thermistor varies with the velocity and the temperature of the water. The current may be kept constant through the use of a vacuum tube, or a "swamping resistor". Such a resistor is connected in series with the thermistor and has a resistance much larger than that of the thermistor, so that it controls the current. A typical calibration curve is shown in Fig. A-6. Fig. A-7 shows the water temperature-velocity relationships for the thermistor studied that would occur at constant power. In effect, it shows the errors to be expected with this circuit and demonstrates that careful operation is necessary.

Another circuit that is similar to this one maintains a constant voltage across the thermistor. The current varies, then, with the water temperature and velocity. This circuit is not recommended since there is a tendency for the response to drift toward higher current values. These values indicate apparent velocities that are greater than the actual ones. This drift can be theoretically predicted.

The second method is a Wheatstone bridge with another thermistor to compensate for a water temperature change. This thermistor is connected into an arm of the bridge adjacent to the active, velocity-measuring thermistor. The values of the variable resistor to balance a bridge that was composed of two particular No. 14A thermistors and a fixed resistance of 100,000 ohms is shown in Fig. A-8. The various combinations of water temperature and velocity that result in a constant value of this balancing resistor are shown in Fig. A-9. This

Fig. A-6.

Voltage Across Thermistor
with
Constant Current

2×10^{-4} Amperes

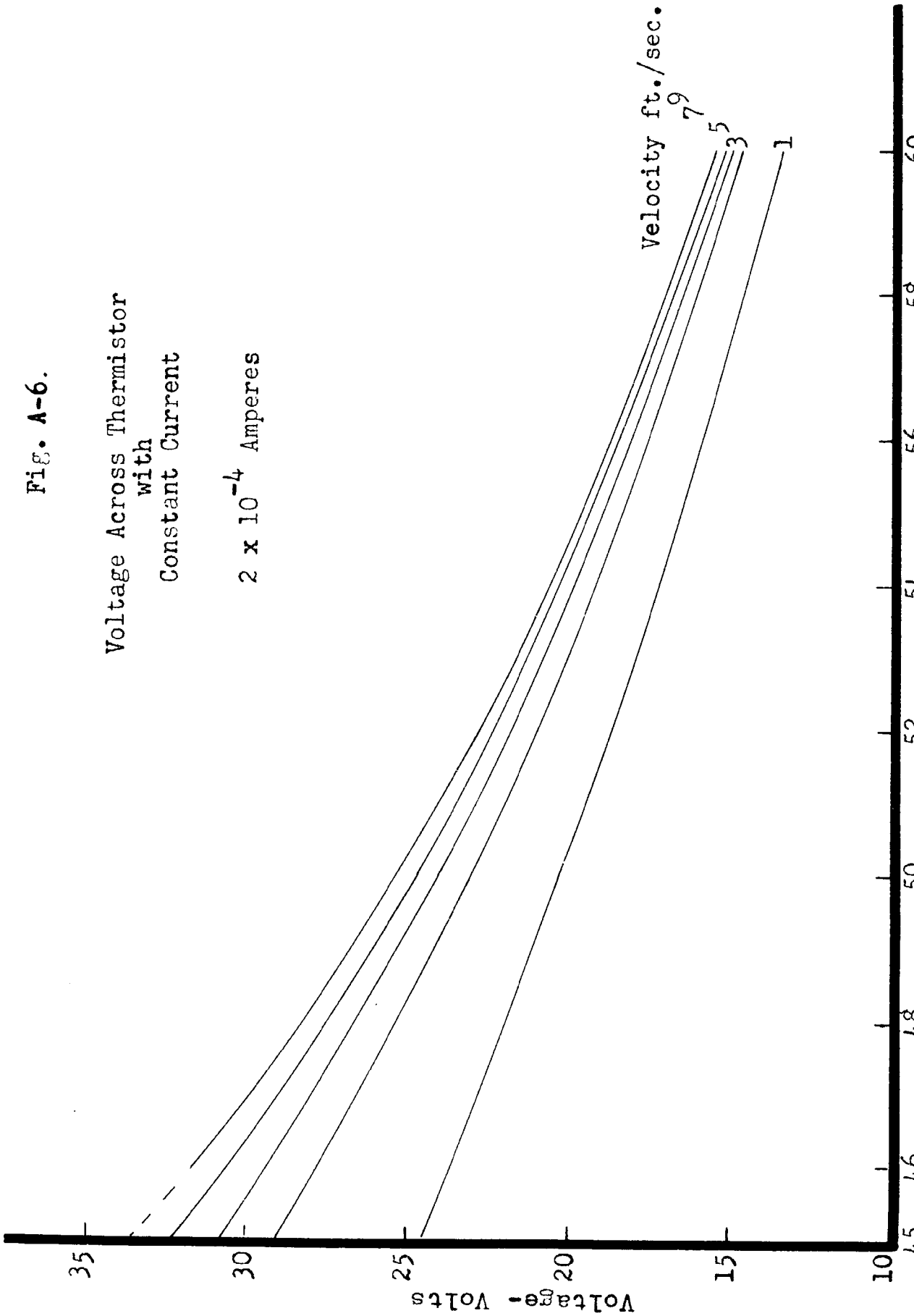


Fig. A-7.

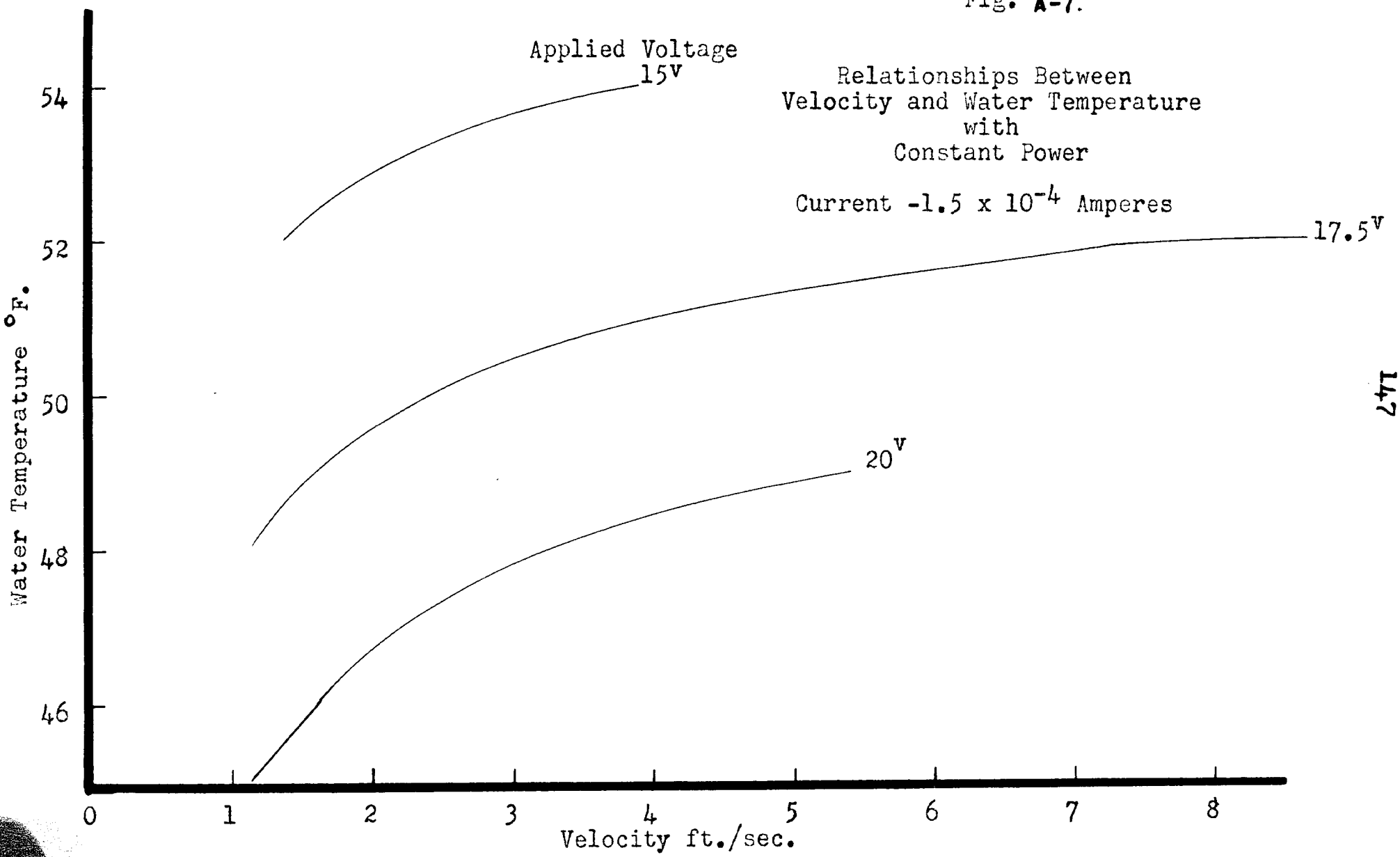

V
Applied Voltage
15V
17.5V
20V
Relationships Between
Velocity and Water Temperature
with
Constant Power
Current -1.5 x 10-4 Amperes
Water Temperature °F.
54
52
50
48
46
0
1
2
3
4
5
6
7
8
Velocity ft./sec.

Fig. A-8.

Value of Variable Resistor
to Balance
Wheatstone Bridge

25^{V} applied across thermistor
compensating resistor
identical to
sensitive element

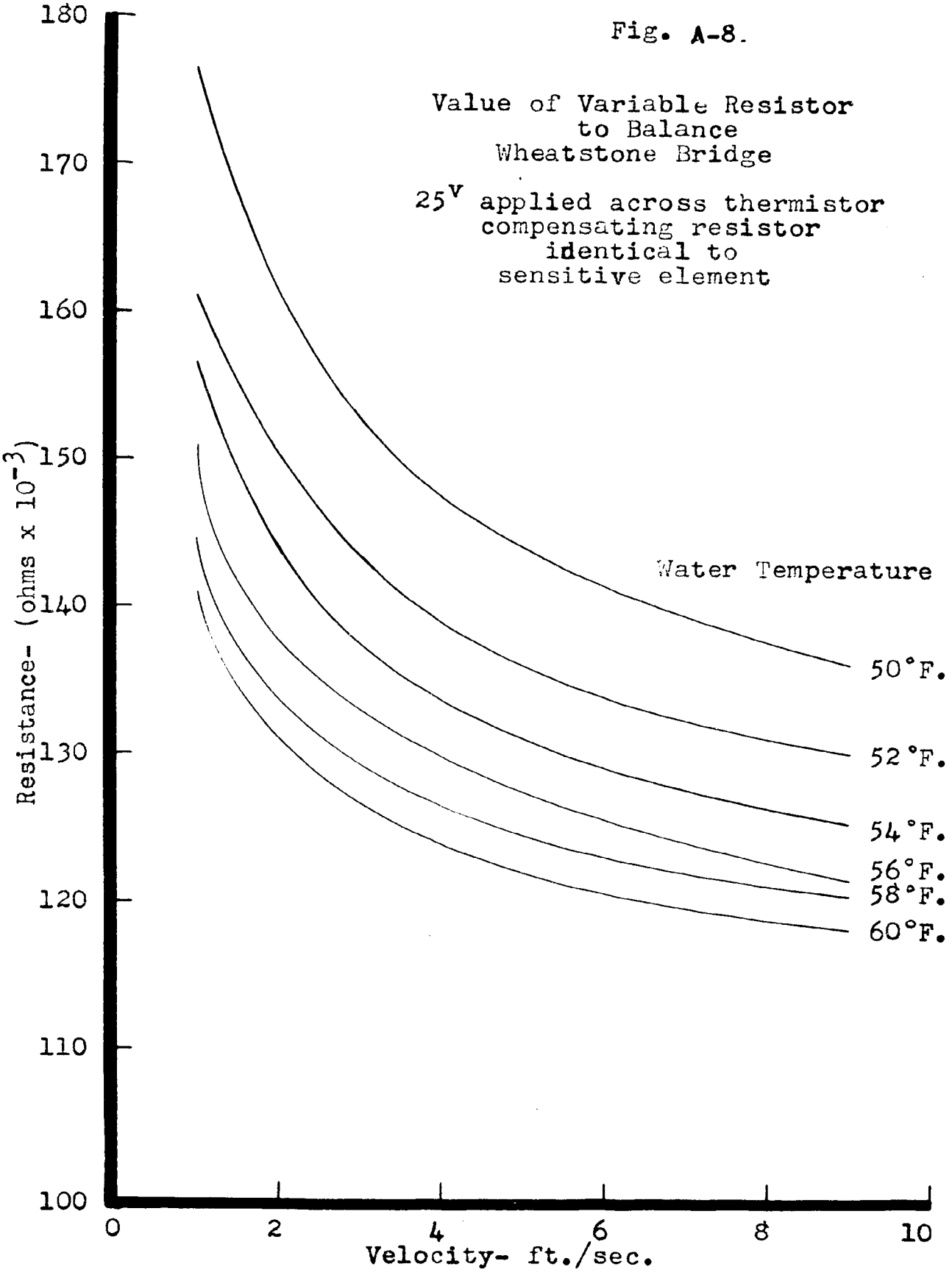
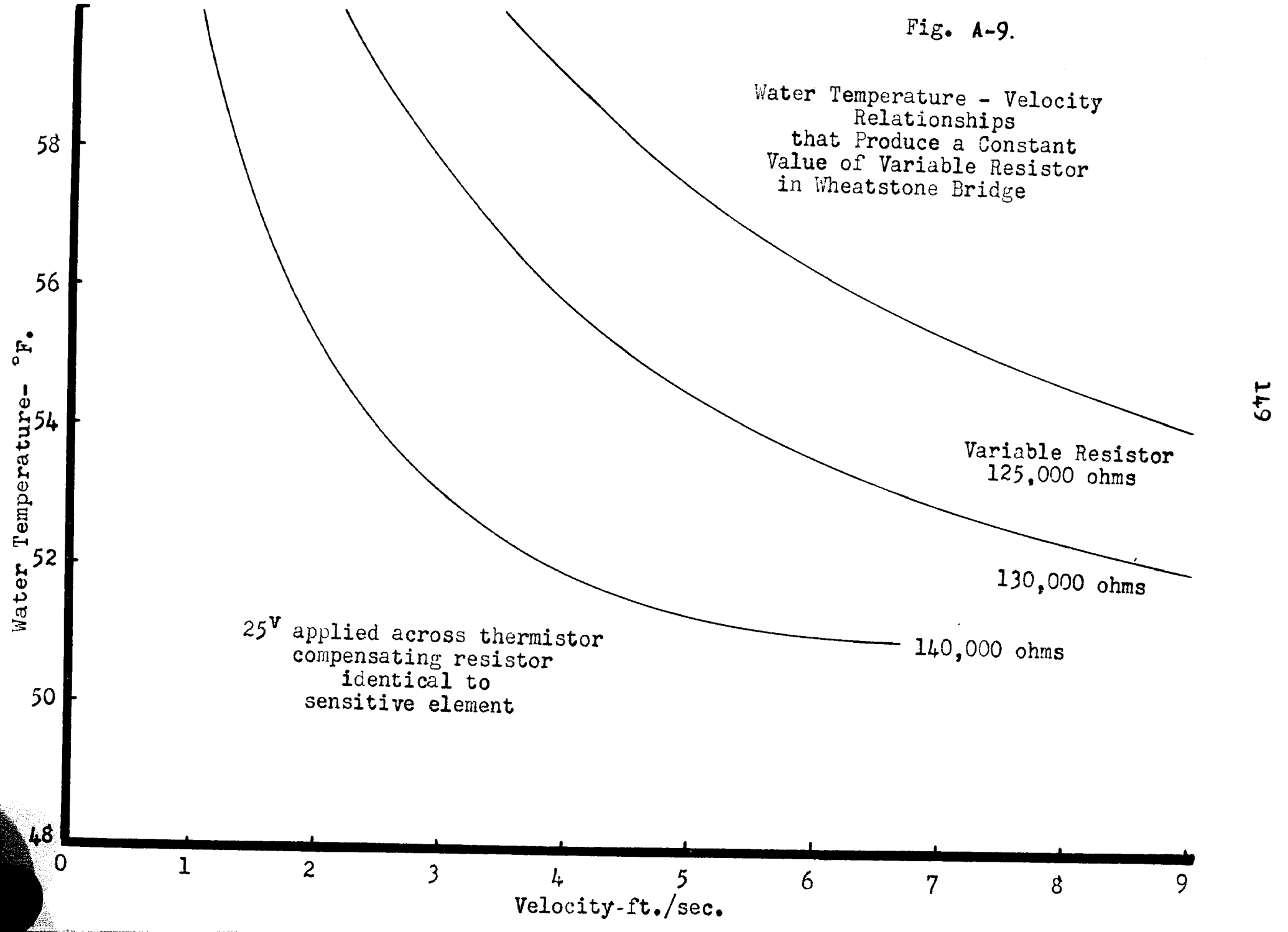


Fig. A-9.


Water Temperature - Velocity
Relationships
that Produce a Constant
Value of Variable Resistor
in Wheatstone Bridge
Variable Resistor
125,000 ohms
130,000 ohms
140,000 ohms
25V applied across thermistor
compensating resistor
identical to
sensitive element
Water Temperature- °F.
58
56
54
52
50
48
0
1
2
3
4
5
6
7
8
9
Velocity-ft./sec.

shows the errors to be expected and again indicates the necessity of careful operation.

The resistance-temperature relationship of a thermistor that would be required in this bridge in order to fully compensate for a water temperature change at a velocity of 1 ft/sec. is shown in Fig. A-10. With this "thermistor", the values of the variable resistor which balance a Wheatstone bridge are shown in Fig. A-11. This figure shows that the bridge may be balanced for any particular velocity by using an adequate compensating thermistor, but that at other velocities errors are to be expected.

This has been a discussion of the basic circuits. One investigator[17] has used a combination of these circuits. It is a feedback bridge circuit, using a dc amplifier to maintain the temperature of the thermistor at a constant value, and also incorporating a compensating thermistor in an arm adjacent to the active thermistor. The output of the bridge was correlated with the velocity of the fluid, which in this case was kerosene. In order to maintain the stability of the circuit, the resistance values of the three arms (the two arms containing fixed resistors and the arm containing the compensating resistor) were much greater than the active thermistor. The compensation was effective. The flow was limited, however, to velocities of 0-4 in./sec..

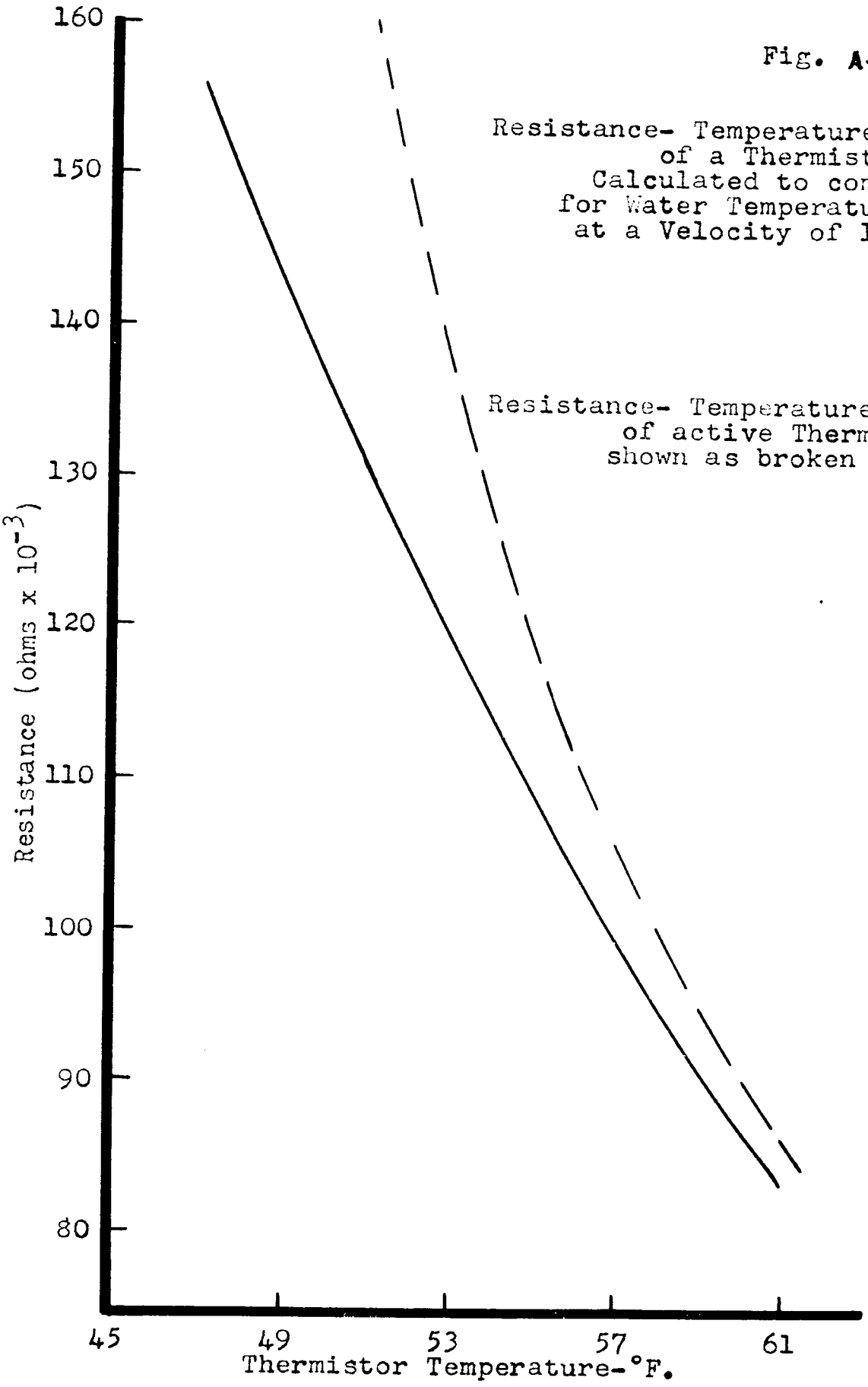
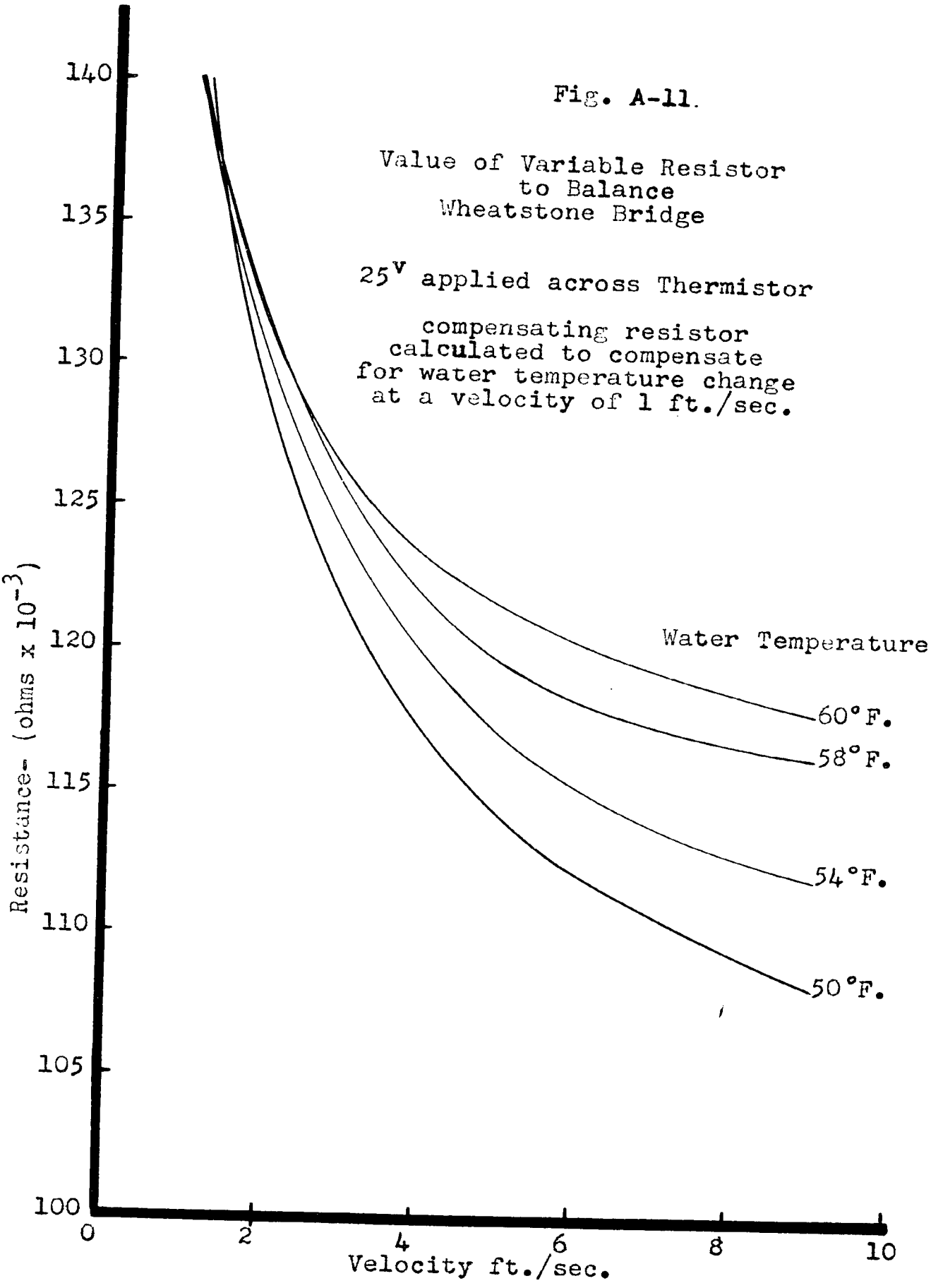


Fig. A-10.

Resistance- Temperature Relationship
of a Thermistor
Calculated to compensate
for Water Temperature Change
at a Velocity of 1 ft./sec.

Resistance- Temperature Relationship
of active Thermistor
shown as broken curve


Fig. A-11.
Value of Variable Resistor
to Balance
Wheatstone Bridge
25v applied across Thermistor
compensating resistor
calculated to compensate
for water temperature change
at a velocity of 1 ft./sec.
Water Temperature
60°F.
58°F.
54°F.
50°F.
Resistance- (ohms x 10-3)
140
135
130
125
120
115
110
105
100
0
2
4
6
8
10
Velocity ft./sec.

APPENDIX B.

A REVIEW OF LITERATURE.

A field, closely allied to the field of this problem, is the determination of the flow of fluids, where the purpose is to determine the total quantity of a fluid that has flowed and also the rate of flow. A. Linford has published a volume entitled, Flow Measurement and Meters,[18] in which the problem of metering flow is divided into general categories: (1) meters to be used with closed conduits; and (2) meters to be used with open conduits, in the following manner:

1. Meters for measuring the flow in closed conduits
   - (a) Mechanical flow meters
     - (1) positive meters
     - (2) semi-positive meters
     - (3) inferential meters
     - (4) combination meters
   - (b) Differential pressure flow meters
     - (1) venturi tubes
     - (2) orifice plate
     - (3) nozzles
     - (4) pitot tubes
   - (c) Shunt type flow meters (that combine certain characteristics of the two preceding groups)
   - (d) Variable aperture flow meters
     - (1) cone and float type meters
2. Meters for measuring the flow through open conduits
   - (a) Weir flow meters
   - (b) Venturi flume or standing wave flow meters
   - (c) Meters for the gauging of open streams
     - (1) velocity methods
     - (2) chemical methods

The mechanical meters possess mechanical movements which are activated by the flow being measured. A piston type meter, actually a "pump" operated in reverse, is said to be positive if it is capable of measuring flow down to a zero flow, and a semi-positive meter if it is only accurate to some minimum flow. The inferential meters are constructed mainly with fans, rotors, and turbine impellers as the activated members. The rate and quantity of flow are determined from the speed of rotation and the total number of revolutions of the impeller. Combination meters are constructed with more than one conduit. Different types of meters are placed in each conduit, and a means is provided so that the flow may be divided from one conduit to another so that each meter may be operated at the point of optimum characteristics. All mechanical meters must be calibrated and usually some means of calibration adjustment is provided.

The Venturi tube, the orifice, and the nozzle are differential pressure generating devices. These devices consist, in the main, of a constriction in the conduit where some of the pressure, or potential energy of the flow is converted into kinetic energy. If the pressures at certain selected positions and the area of the constriction are known, the quantity and rate of flow can be determined. The nozzle has an advantage over the others in that when the ratio of the pressure below the constriction to the pressure above the constriction, is below a critical value, the velocity of flow is constant. The discharge is then only a function of the upstream pressure and a pressure

reading gauge may be calibrated directly into the rate of flow.

Shunt type flow meters combine certain characteristics of the mechanical meters and differential pressure meters.

The variable aperture meters consist mainly of three different types: (1) cone and float type, (2) cone and disc type, and (3) hinged-gate type. The cone and float type meter and the cone and disc type meters operate on the same principle. The former may be constructed for all liquids, while the latter is used primarily for cold water. They consist of a tapered glass cone, mounted with its axis vertical and the smaller diameter at the bottom. This cone contains a float, the largest diameter of which is slightly smaller than the vertical glass tube in which it rests. The flow is upwards through the tube and the float rises to the position where the upward thrust on the float is equal to the weight of the float. The hinged-gate type of meter consists of a chamber, containing a weighted hinged gate that covers the opening from the pipe line. When there is flow the gate is deflected. Its effective weight is then balanced by the differential pressure that is formed by the orifice consisting of the gate and its seat.

Open channel flow meters are divided into two classes: (1) the weir, and (2) the Venturi flume. The weir is the oldest of the flow meters and consists of a dam built across the stream. The liquid flow is forced to rise and go over the top of the dam. The depth of flow over the dam is a measure of the flow rate. The Venturi flume is similar to the Venturi tube, except that the top is open. It consists of a contrac-

tion in the channel cross-section. There are many different types of flumes.

The last section of this volume discusses river gauging. Inasmuch as a river is usually too large for the construction of flumes and dams, just for metering purposes, an approximation of the flow may be determined through a velocity profile. Some methods for finding the velocity profile are as follows: (1) floats, (2) current meters, or more correctly, generators, (3) pitot tubes, and (4) chemical means.

Floats may take a variety of shapes and forms. They may be surface floats, subsurface floats, twin float assemblies, float rods, and captive floats. They all float downstream at a rate approximately 85% of the stream velocity. This rate is always calibrated. They should be sufficiently submerged that they are uneffected by wind.

Current meters and generators utilize propellers or cup-type vanes as the activating element. These devices operate an electrical make-and-break contact system, or a small electrical generator. The speed of rotation of the propeller type is greater than that of the cup vane type for the same velocity conditions. However, the cup form operates at a lower velocity.

Open streams may also be gauged by chemical methods. In recent years this has been highly perfected and has been used in acceptance tests in connection with hydroelectric plants. The apparatus consists of a source of power, a recording milliammeter, and two pairs of insulated electrodes. The electrodes are mounted in the channel, a convenient distance apart. Salt

solution is injected into the stream above the electrodes. Electrical conductivity curves are obtained from the latter. From the time necessary for the salt solution to traverse the measured distance between the electrodes and the contour of the stream, the quantity of water flowing may be determined. The channel should be at least 60 feet long and have a cross-section that is sufficiently uniform to allow some degree of accuracy in the computation of the flow. The method is more accurate if longer channels are used.

In the same manner, Thomas and Archibald[19] have used radio-active isotopes and have found that a much lower concentration is needed. They do not cause observable density currents, while salt brine does. Apparatus for making counts is simple and precise. In small flow systems the passage of a radio-active tracer cloud may be detected by radiation through conduit walls, which eliminates sampling the flow. Chemical, biochemical and temperature changes do not affect the rate of disintegration. One disadvantage is the danger to personnel when over-exposed to radiation. The isotope used was radio-active Iodine(I-131).

Linear Resistance Flow Meter[20]

A linear resistance flow meter is a plug of cotton, steel wool, or glass wool inserted in a closed conduit, carrying gas. The difference in pressure, measured by a manometer at each side of the plug, is a measure of the flow. The calibration is linear from zero flow to a certain maximum value.

Beyond this value it is not linear. The calibration varies with the specific weight and the length of the plug.

Thus far, this review has dealt with those methods which are used to determine the quantity and rate of flow. The following section will deal with those methods by which a velocity traverse, or the velocity of flow at any particular point, may be found.

The Pitot Tube

The pitot tube is a differential pressure generating device, used for the measurement of velocities in both fluids and gases. Its simplest form consists of a small tube with a hole drilled in the wall near a sealed end or tip. When this tube is inserted in the fluid being measured, with the hole pointing upstream, the line of flow directly in front of the tube is stopped. The kinetic energy of the fluid at this point is converted by impact into potential energy, which, in turn, is converted into pressure and is measured through the use of a differential manometer. A differential manometer is a piece of glass tubing bent in the form of a U. One leg of the U is connected to the pitot tube and the other to a device which will determine the static pressure of the fluid at the location of the pitot tube. It should be mentioned that the pressure from the pitot tube is the total pressure, i.e., the sum of the static pressure of the fluid and the pressure due to the velocity. The pressure recorded from the pressure tap, if it is constructed accurately, registers only the static pressure of the fluid. The velocity of the fluid may be found from the

difference between these two pressures.

A device which measures only the static pressure of the fluid is known as the static tube. This tube is designed with the criterion that it must not disturb the flow of the fluid.

The pitot-static tube has been devised to do the work of the pitot tube and the static tube simultaneously. It is similar to the static tube, except that a small hole is drilled in the tip facing the stream. The total pressure is measured through this hole, and the side hole measures the static pressure of the fluid.

Pitot tubes have been made in various sizes and shapes. Their design becomes very complex when special properties of the flow are measured. An interesting flow meter[21] has been constructed by using pitot tubes and static tubes located at the three-quarter radius of the conduit, and placed alternately at 45 degree angles. Any asymmetry of flow can be detected by examining the readings from the individual tubes, while the mean pitot tube reading, minus the mean static pressure reading, can be related to the quantity of flow. The three-quarter radius was chosen since the calibration factor should be unity. However, the device is calibrated by a traverse with another pitot-static tube. This device has been used in air and is suitable for high speed flow.

The Electromagnetic Flow Meter

The electromagnetic induction method of measuring the velocity of flow is based on Faraday's law of electromagnetic induction which states that an electromagnetic force is induced in a conductor moving normal to a magnetic field. The fluid acts as a group of conductors and if a magnetic field is set up, a voltage proportional to the field strength and the fluid velocity is induced in it.

Many[22] have used this method for the determination of velocities. Morris and Chadwick[23] used it to measure the flow to and from two tanks, located on each side of a ship model, in a study to produce a method to counteract the roll of a destroyer. They stated their requirements as follows: (1) the instrument must measure fluid velocity linearly in the velocity range of zero to 18 ft./sec.; (2) the data obtained should be recorded simultaneously with other data; (3) the instrument should be simple, inexpensive, readily available, easy to operate, and reliable; (4) the instrument should not appreciably disturb or distort the flow; (5) the instrument should respond only to fluid velocity changes, and be insensitive to static or kinetic pressure forces; (6) the instrument must have an instantaneous time response without lag.

This method has some disadvantages. Morris and Chadwick observed ac noise, erratic disturbances, and drift of the calibration which were due to polarization at the electrodes.

Grossman[24] was interested in a study of turbulence and velocity fluctuations rather than the mean or average velocity.

He used this method for the determination of the rms. velocity components parallel to and normal to the mean flow of the fluid as a function of the radial position and the flow rate. Grossman used a Lucite tube with 2.0 in. o.d. and 1.732 in. i.d.. The magnetic pole faces were 3 in. square, separated by 2 in., and with battery operation, produced 2600 gauss. The probe was 0.034 in. in diameter, made with two insulated copper sheathed thermocouple wires spaced 0.032 of an inch apart. They were treated with colloidal graphite which formed a layer at the points of contact with the fluid. The two probe conductors were connected to the primary winding of a transformer whose function was to prevent the grounding of either side of the probe. The secondary winding was connected to a voltage amplifier. This amplifier was battery operated, resulting in a hum level of about 3 microvolts. The output was led to the parallel arrangement of a cathode ray oscilloscope, a vacuum tube voltmeter, and a milliammeter.

Grossman listed certain disadvantages. The method involves the measurement of potentials of the order of 10^{-5} volts which may be masked by spurious potentials and the thermal noise level. Another difficulty is that the potential gradient, due solely to the induction effect will be disturbed by the presence of current flow within the fluid field, at the flow boundaries, or in the measuring circuit. These are eliminated by the use of non-conducting pipe walls and high impedance detectors. "Short circuiting" currents between points in the fluid where the velocity gradients are large, are possible. These

induced currents may also have the secondary effect of modifying the original flow due to Amperian forces. However, this is negligible. It was found that the probe required a very rigid support. Any breakdown in the electrical insulation, due to liquid leaks, would completely erase any results. The measurement equipment had to be carefully shielded from stray voltages (60 cps.). It was found that the piping system itself acted as an antenna. This was eliminated by wrapping the system with copper strip.

Bordon[25] has also tried this method to measure turbulence and found it possible, though difficult. Arnold[26] has used the same principle, using two sets of electrodes that balance each other to study transient flow.

The Photographic Method

A method, used to measure velocity distributions, utilizes the technique of photographing oil droplets in the flow. This has been called the "Oil Injection method".[27] Dyed oil, having the same density as the fluid, is injected considerably upstream from the section being studied. It forms droplets and flows interspersed through the fluid. The section under study is photographed in two orthogonal planes. This section is made of Lucite and can be surrounded by a water-filled transparent manifold which helps to eliminate certain lighting problems. The flow pattern is then determined from an analysis of the photographs. In the same manner, dye has been injected into a stream to study turbulence.

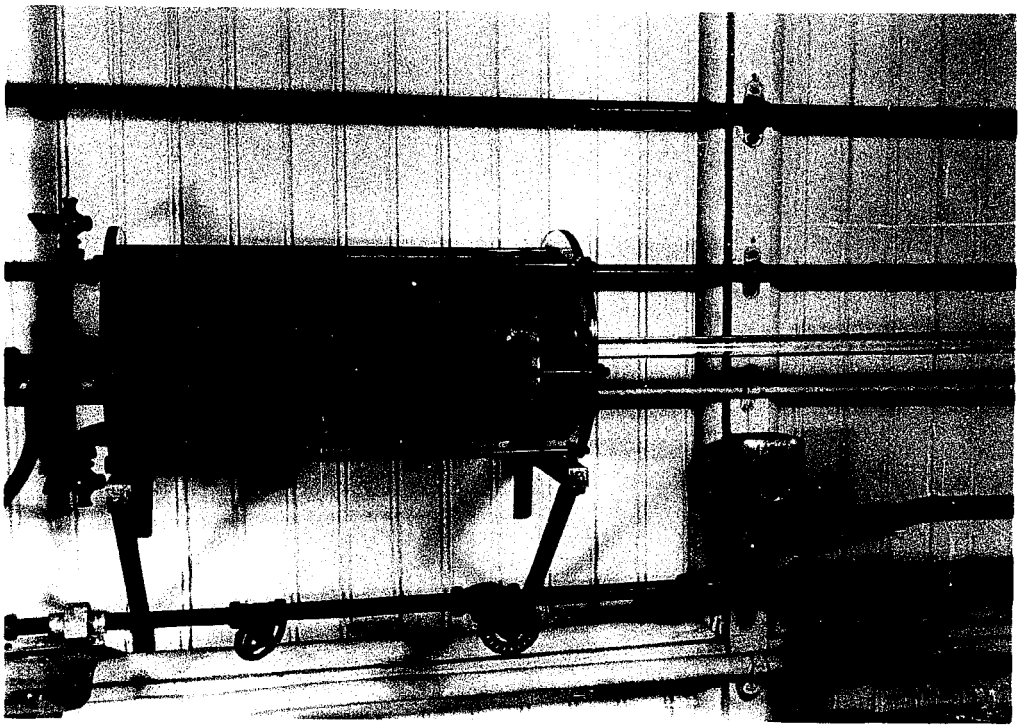


Fig. B-1. This Figure Shows the Attachment at the Head of the Experimental Assembly with a Central Tube to Inject Dye into the Water for Photographic Studies.

Hagerty and Yagle[28] have observed an effect in glycerine which they call photo-viscosity. They do not understand the physical or chemical reasons for this phenomenon, except that the material is in a state of shear. When the glycerine passes from a storage tank through a channel containing a flow situation, lines appear which show how the flow is taking place. When the flow is suddenly stopped, these lines persist. The residual pattern remains visible for 30 to 45 minutes. Photographs should be taken, however, during the first 15 minutes as the lines become "fuzzy" in appearance after this time. The glycerine, used in the examples, was not pure, but contained about 4% of water.

Use of Doubly Refracting Solutions

Rosenberg[29] has introduced a process that is similiar to the technique of photoelasticity for solids. In photoelasticity, a plane polarized light is passed through a section of bakelite, Fosterite, etc., which is in a stressed condition. The stresses are shown as colored bands and can be quantitatively calibrated. Rosenberg has set up two dimensional flow systems and has used colloidal solutions of Bentonite ($Al_2O_3 \cdot 4\ SiO_2\ x\ H_2O$) as the fluid. Through the use of an optical system, similar to that used for photoelasticity, he has been able to show flow patterns. The results are affected by: (1) particle size, (2) concentration of the Bentonite, (3) temperature, and (4) acidity of the solution.

The Hot Wire Anemometer

The hot wire anemometer is a device used in aeronautical research to determine the velocity of air in wind tunnels both for laminar and turbulent flow. This device operates on the same thermal principle as the thermistor velocity meter. An excellent review is the report ACA-19[30] (Australian Committee on Aeronautics Report No. 19, published in 1945) which reviews a considerable number of both American and foreign articles and is a summary of the theory and methods for the measurement of turbulence characteristics.

The wire is tungsten and is very small (0.001 of an inch diameter), which enables the measurements to be made without disturbing the air flow. It is mounted in the air stream and is usually connected as one arm of a Wheatstone bridge. In this bridge, the resistance of the wire or the current may be kept constant. The resistance and the current may also be measured potentiometrically. The main disadvantage is the loss of response at high speeds and at the higher frequencies of air speed fluctuations.

The direction of air flow may be measured by the use of two or more wires. If three wires are grouped to form the apex edges of a symmetrically triangular or square pyramid, and the assembly is capable of rotation, a position will be found where the heat loss will be identical for each wire. The apex will point toward the direction of the air flow. Another type of direction meter that has been used successfully at the California Institute of Technology consists of

a copper-constantan thermocouple placed behind the wire. If the meter is placed in the direction of flow, the thermocouple will be warmed by air that is heated by the wire.

Macovsky,[27] at the Taylor Model Basin has used the hot wire method for the measurement of turbulence and steady velocities of fluids flowing past ship models. He has found that a wire having a time constant of 0.002 of a second in air would have a time constant of approximately 50 to 100 microseconds in water. The principal difficulty in water was dirt accumulation. Consistent results with a constant calibration can be obtained only by observing data immediately after the wire has been cleaned.

A Method for the Measurement of Small Liquid Velocities

Dr. I. E. Eugen,[31] at the Max Planck Institute for Hydraulic Research, has developed a method using a cooling principle for the measurement of fluid velocities of approximately 1 cm./sec..

His instrument is similar to that developed at the California Institute of Technology. Eugen's instrument measured the water temperature directly by a thermocouple instead of measuring the wire resistance. Eugen constructed a two-part cylindrical probe. The head is electrically and thermally insulated from the rear portion. The head contains the heating element and a place for soldering the cold junction of the thermocouple, which compensates for any change in water temperature. The rear portion of the probe contains the hot

junction of the thermocouple and is at the temperature of the surrounding heated fluid. The thermocouple voltage is proportional to the square root of the velocity of the fluid flowing past the probe. This proportionality exists as long as heat transfer by natural convection is not important. The probe, however, must always be calibrated.

From experience, it has been found that an accuracy of about 0.2 mm./sec. could be obtained with a velocity of 1 cm./sec..

A Comparison of the Thermistor Method for Measuring Velocities with Other Methods.

A thermistor may become a portion of a surface, such as a ship model. This is a distinct advantage over the other methods. Since the thermistor is composed of metallic oxides sintered together, it should not burn out. It is affected, however, by conditions which disturb the heat transfer from the thermistor to the fluid. It is not as cumbersome to use as are photographic techniques. The response of the electromagnetic method is linear with velocity, while the response of the thermistor is not. However, the response of the thermistor can be made considerably greater, requiring less amplification. A small thermistor is superior to a large one. This is an advantage over the pitot tube. The thermistor can also measure relatively small velocities.

The main disadvantage of the thermistor lies in its calibration drift. For this reason it appears to be a laboratory instrument for the measurement of velocity distributions.

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