

direction is

$$\begin{aligned}\Gamma_2 &= \omega_1(\cos \alpha_2/\cos \alpha_1) (jh/dn)dn \\ &= \omega_1(\cos \alpha_2/\cos \alpha_1) jh\end{aligned}$$

which gives

$$\Gamma_2 = (dU_1/dz) \left\{ \int (L_l/q_l) - \int (L_u/q_u) \right\} U_1 \quad (4)$$

The circulation of this is a right-handed screw facing downstream.

The vorticity due to the shed circulation from the blades is also present downstream. The circulation associated with this is

$$\Gamma_3 = (dU_1/dz)s \cos \alpha_1 [\tan \alpha_1 - \tan \alpha_2] \quad (5)$$

The direction of Γ_3 is the same as Γ_2 .

The total streamline circulation is therefore

$$\begin{aligned}\Gamma &= \Gamma_1 - \Gamma_2 - \Gamma_3 \\ &= s(dU_1/dz) \left[\left\{ (\sin 2\alpha_1 - \sin 2\alpha_2)/2 \cos \alpha_1 \right\} - \right. \\ &\quad \left. \cos \alpha_1 (\tan \alpha_1 - \tan \alpha_2) \right] \quad (6)\end{aligned}$$

when $\alpha_1 = -\alpha_2$ as in an impulse cascade

$$\Gamma = 0 \quad (7)$$

The above method may be applied to an accelerating cascade when the following results are obtained. These results are the same as those obtained by Hawthorne² with a different method.

$$\Gamma_1 = (dU_1/dz) \left[(s/2) \left\{ (\sin 2\alpha_2 - \sin 2\alpha_1)/\cos \alpha_1 \right\} + \left\{ \int (L_l/q_l) - \int (L_u/q_u) \right\} U_1 \right]$$

$$\Gamma_2 = (dU_1/dz) \left[\int (L_l/q_l) - \int (L_u/q_u) \right] U_1$$

$$\Gamma_3 = (dU_1/dz)s \cos \alpha_1 (\tan \alpha_2 - \tan \alpha_1)$$

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On Ménard Inserts in Supersonic Nozzles†

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INTRODUCTION

Intentional changes of the design Mach Number in a nozzle with fixed contours by means of movable sidewall inserts were first reported by Ménard.¹ His work formed the starting point for studies in the Aerodynamics Laboratory of The University of Michigan into the potential usefulness of such inserts at low and high Mach Numbers. The study was motivated by the possibility of using Ménard inserts as a simple and extremely economical technique of changing the Mach Number of a set of fixed nozzle blocks.

DISCUSSION

Consider a given two-dimensional symmetric nozzle with a uniform exit flow of Mach Number M_0 ; add to the plane walls of this nozzle properly shaped and located half bodies such that a new uniform flow of Mach Number $M_1 = M_0 + \Delta M$ results. The determination of the contours of these inserts and of their location relative to the basic nozzle cannot be carried out by any of the usual design procedures for two-dimensional or axisymmetric nozzles. Only the simple one-dimensional area ratio

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theory or the cumbersome technique of characteristics for general three-dimensional supersonic flows appear applicable. The specific design procedure employed is given in references 2-5.

RESULTS

Low Mach Number Inserts

The technique described above was used to design a set of inserts for the 19 in. $\times$ 27.5 in. wind tunnel of the Ordnance Aerophysics Laboratory so that their Mach 1.5 nozzle would give Mach 1.6 flow of acceptable uniformity.

Inserts of the recommended design were manufactured and tested by the Ordnance Aerophysics Laboratory. The results were highly satisfactory and are reported in reference 2. Briefly, the desired Mach Number of 1.6 was achieved with the inserts in design position, and the flow uniformity was no worse than that of the basic Mach 1.5 nozzle. Off-design positions of the inserts, 6 in. upstream and downstream of the design, resulted in usable flows of Mach 1.57 and 1.63, respectively, with the most uniform flow being obtained in the latter position (Fig. 1). The cost of these inserts and their installation amounted to approximately 13 per cent of the expected cost of a new set of Mach 1.63 nozzle blocks.

High Mach Number Inserts

After this success at low Mach Numbers, the program of designing inserts for the OAL facility became quite ambitious. The first high Mach Number inserts were to boost the OAL Mach 2.23 nozzle to Mach 3.0. It was found that no inserts could be designed which would have a reasonable chance of giving acceptable flow at the Mach 3.0 level.³

Next, a series of inserts (Fig. 2) were studied to bring the OAL Mach 2.75 nozzle to a Mach Number of 3.0. Exploratory tests with models of the proposed inserts were conducted in the Mach 2.84 nozzle of the 8 in. $\times$ 13 in. wind tunnel of the University. The first series of tests were made of so-called "short" inserts, shown in the upper portion of Fig. 2. The negative results of these tests are given in reference 4. The unsatisfactory flow of all three short inserts was primarily due to a strong lateral shock believed to originate in local regions of rapid expansion and cross flow just downstream of the effective throat of the nozzle-insert combination.

In order to alleviate the local expansion, a long cylindrical insert was tested⁵ as shown in the lower half of Fig. 2. The main result is given in Fig. 3 and shows that a flow of reasonable uniformity was obtained, and that the Mach 3.0 level of the basic nozzle was increased by an increment of 0.20. The lateral shocks

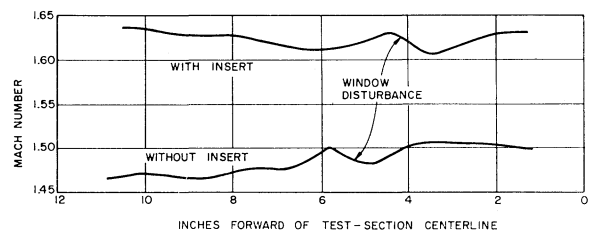


FIG. 1. Centerline Mach Number, OAL Mach 1.50 nozzle.

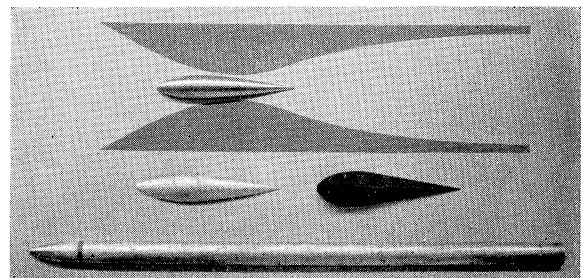


FIG. 2. Models of Ménard sidewall bodies designed to give M-3 flow in OAL M-2.75 nozzle. Tested in Univ. of Mich. M-2.84 nozzle.

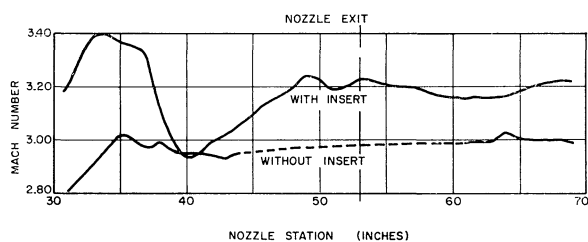


FIG. 3. Effect of cylindrical sidewall body on the centerline Mach Number (Univ. of Mich. Mach 2.84 nozzle).

mentioned before were now sufficiently weak and were quickly attenuated.

From these tests one can predict a usable Mach Number 3.0 flow for the OAL Mach 2.75 nozzle with cylindrical inserts. However, in case one wants to terminate these cylindrical inserts in a smooth trailing point upstream of the test section, additional model tests are strongly recommended to assure that the flow uniformity remains acceptable.

CONCLUSIONS

The work with Ménard inserts to date has indicated the following:

- (1) Ménard inserts are an effective and economical means of increasing the Mach Number of an existing two-dimensional nozzle.
- (2) At low Mach Numbers (around 1.5), increments of up to 10 per cent of the basic Mach Number can be achieved without loss in flow uniformity.
- (3) The design of inserts for the low Mach Number range can be carried out by a modified area-ratio theory. In most cases no model tests will be needed.
- (4) At high Mach Numbers (above 2) increments in Mach number due to inserts will be less than 10 per cent and some deterioration of the uniformity of the flow must be anticipated.
- (5) The design of inserts for the high Mach Number regime should be substantiated by model tests, especially in the case of large increment inserts in short nozzles.

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Physical Mechanism of Vortex-Ring Cascade

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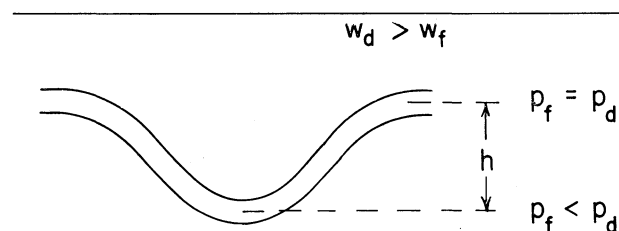
IN A RECENT LETTER,¹ C. S. Hsu reports the observation of a vortex-ring cascade phenomenon. The writer suggests that the phenomenon may be explained by the following physical mechanism.

Consider a dye drop entering a quiet body of fluid and forming a vortex ring. Assume the dye to be denser than the surrounding fluid. The vortex ring would be a perfect torus only if the dye drop were perfectly symmetric and the fluid absolutely

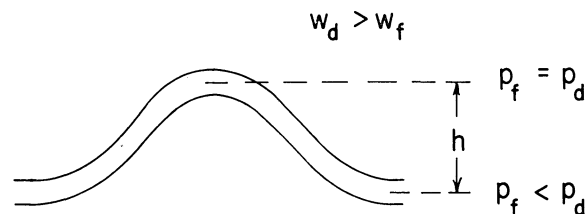
quiet. Such a ring would be in unstable equilibrium and so would not initiate a cascade as long as the motion and conditions remained ideal. Practically speaking, however, the ring will have small distortions. For a downward dip [Fig. 1(a)], the pressure at the level of the undistorted part of the ring is the same in the fluid and within the dye filament, but along the dip, this equality does not hold. Since the filament is continuous, the pressure within it at the bottom of the dip is $w_d h$ (dye density $\times$ head), just as it would be in a mechanical tube. The pressure in the fluid at the h level is $w_f h$ (fluid density $\times$ head), which is less than that in the dip since $w_f < w_d$. Therefore, the dye in the lower part of the dip tends to push away the fluid immediately surrounding it. Since there is no restraining tube (other than the effect of surface tension), the dye will flow from ring level down into the dip thus expanding the dip [Fig. 2(b, c)].

Meanwhile, the entire vortex ring is moving downward. The drag force created by this motion causes a flattening at the bottom of the dip such that the dip takes on an approximately ellipsoidal shape [Fig. 2(c)]. The asymmetry, which is bound to be present in the ellipsoid, will allow the entering filaments to take on a configuration that produces a rotational flow about a vertical axis [shown by the arrows in Fig. 2(c)]. By centrifugal action, the ellipsoid changes to a torus. The central part of the ellipsoid gets thinner, becomes a membrane, and breaks [Fig. 2(d, e)]. The vortex ring, which has been supplying the dye for the small *pro forma* rings, has emptied; the near-vertical filaments break [Fig. 2(e)], leaving a torus. While the rotational flow has been growing, the *pro forma* ring continues to descend. The relative upward velocity of the fluid past the *pro forma* ring [Fig. 2(b, d, e)] imparts a vortex motion so that after the membrane has broken a vortex ring results [Fig. 2(f)].

Returning to the original vortex ring, what happens if a distortion is upward [Fig. 1(b)]? At the top of the distortion, the pressures in the fluid and the dye are equal. At the level of the undistorted ring, the pressure within the filament is more than that in the fluid because the dye density is greater than the fluid density. Therefore, the dye at the ring level tends to push away the surrounding fluid, thus permitting the dye in the distorted portion to flow down to ring level and so reduce the amplitude of the distortion. It can be seen that the ring is stable to upward disturbances but will produce rings if given a downward distortion.



a. DOWNWARD DISTORTION



b. UPWARD DISTORTION

FIG. 1. Pressure conditions for distorted vortex ring when dye is denser than surrounding fluid.