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A COMPARISON OF TWO SEQUENCES FOR TEACHING PERCENT

by
Bruce Robert Maxim

A dissertation submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
(Education)
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1982

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For My Loving Wife Norma Jean

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CHAPTER I

PROBLEM INTRODUCTION

Background

People use percent in a great number of daily situations at home and on the job, yet it has been regarded as a difficult school topic by both students and teachers. With this recognized difficulty, very little research has been conducted to see what makes percent so difficult to learn. The research on percent which has been reported to date has tended to focus on the frequencies of student errors or on differences in mean group performance following exposure to established methods of teaching percent. There have been few attempts made at trying to ascertain what might be done to increase the effectiveness of the materials used to teach percent.

Understanding percent, like understanding fractions and decimals, requires students to develop a number of interpretive and analytic skills. The problem areas in learning percent seem to be different from those problems found in the application of decimal fractions. Percent requires an understanding of a technical vocabulary and the ability to interchange different, but equivalent, forms with assurance (Butler and Wren, 1960).

Recent published test results (Carpenter, 1980) from the National Assessment of Educational Progress (NAEP)

indicate that a majority of the subjects tested had little understanding of the meaning of decimals or percents. Test subjects frequently seemed to apply rules in an arbitrary manner, with no means of verifying the correctness of their responses. Results from the Michigan Educational Assessment Program (MEAP) suggest that students have problems grasping the notion of equivalence, between decimals and fractions and between percents and decimals (Porter). MEAP test results provide additional evidence for the hypothesis that students are applying rules in an arbicrary manner, without checking the reasonableness of their answers.

A casual review of middle school textbooks published during the last thirty years reveals a general decrease in the number of pages devoted to teaching percent. One widely used seventh grade text (Dolciani, 1970) only had five lessons on percent topics. This reduced interest on the part of textbook publishers, coupled with poor student test performance, suggests a need to reevaluate the teaching of percent.

Some lessons about teaching rational number topics may be learned from researchers at the University of Michigan (Payne, 1975). These researchers have been quite successful at teaching initial fraction concepts to young children. Their first step in designing instructional materials was to analyze the teaching of fraction concepts into it component parts. Following this analysis logical sequences of instruction were developed. The materials developed during

this research made good use of concrete diagrams and careful use of language.

The sequencing of instructional materials may be facilitated if one is able to identify the interrelationships between the component parts. One type of logical structure which can aid in this process is called a learning hierarchy. Novillis (1973) was able to demonstrate the validity of a learning hierarchy for the fraction concept. She did not however attempt to develop learning materials based on her analysis.

There is a strong need to examine the nature of the difficulties students have with the understanding and application of percent concepts. This requires the analysis of the teaching of percent into its component parts and the organization of a learning hierarchy which delineates their interrelationships to one another. The purpose of this analysis is to aid in the development of a meaningful sequence for percent instruction.

Problem Statement

The primary purpose of this study was to develop and evaluate the effectiveness of percent materials based on the notion of extending children's understanding of fractions to include percent. A related goal of this study was to evaluate the relative effectiveness of two approaches to finding a percent of a number. The investigator also examined the nature of student difficulties in understanding and applying percent concepts.

Research questions investigated by the author included the following:

1. Does the approach used to teach the solution of percent problems affect student test performance?
2. Does student performance on selected test questions vary over time?
3. Is student performance on fraction or decimal test questions related to their performance on percent test questions?
4. Is the validity of a percent learning hierarchy independent of the method of instruction?

Operational Definitions

The term Learning Hierarchy refers to a set of behavioral objectives which contains one or more ordered sequences of student behaviors. In a well ordered learning hierarchy it is assumed that a student will not have attained a terminal behavior without first attaining all prerequisite or subordinate behaviors. Figure 1 contains a graphical depiction of the percent hierarchy postulated for this study.

Case 1 Percent Problems refer to problems in which the student is given a rate, expressed as a percent, and a base (or whole unit) and is expected to compute a percentage (or fractional part).

Case 2 Percent Problems are problems in which the student is given a base and a percentage and is asked to compute a rate, expressed as a percent.

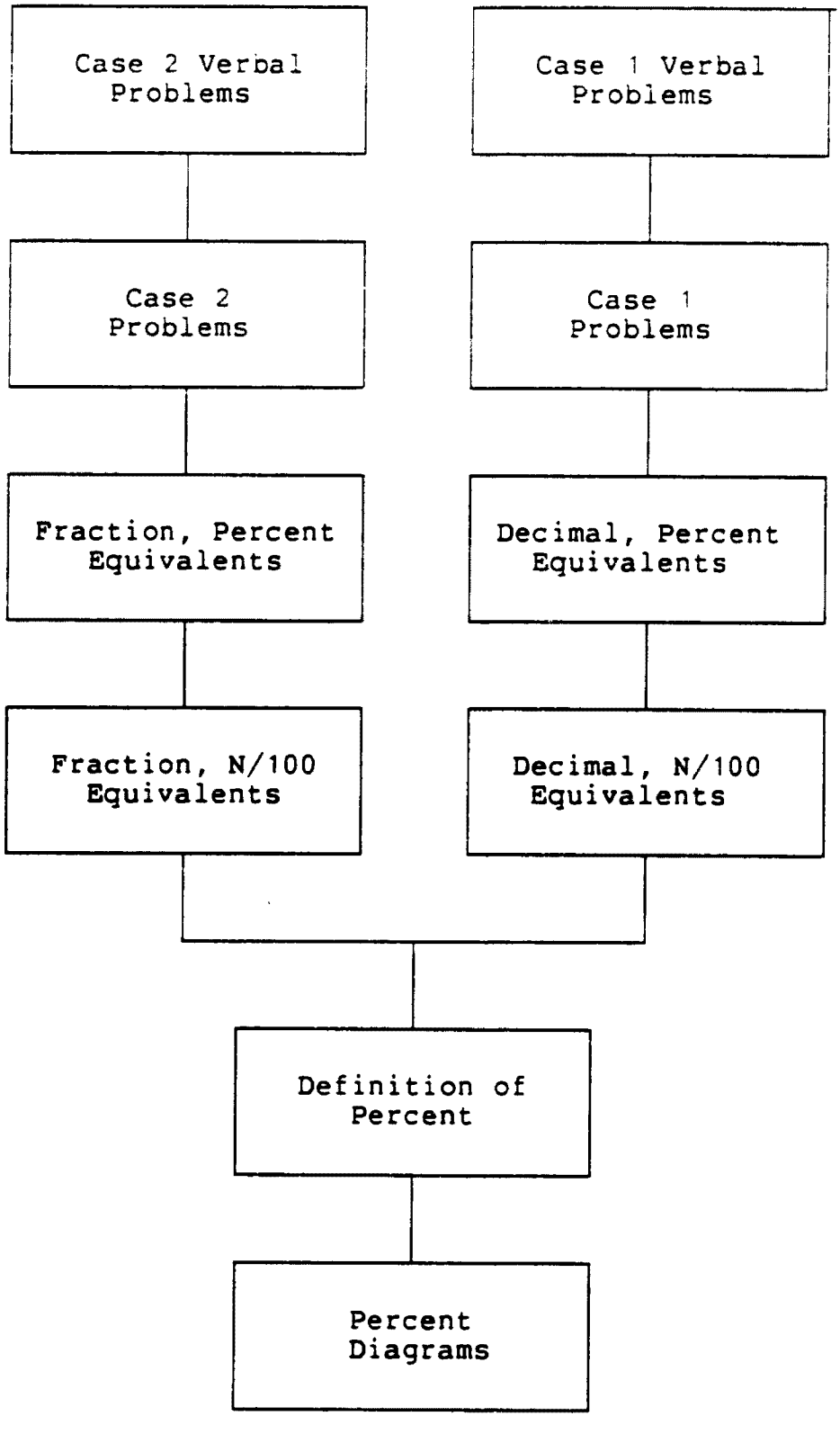


Fig. 1 Percent Learning Hierarchy.

In Case 3 Percent Problems the student is given a percentage and a rate, expressed as a percent and is expected to find the base.

The Proportion Method for solving percent problems teaches the student to use the information given in a percent problem to set up a proportion of the form a/n = c/100. The student then solves for the missing quantity, typically using the cross product. To find 10% of 40 the student would set up the equation a/40 = 10/100 and then solve for "a".

The Factor Product Method (sometimes called the decimal multiplication method) for solving percent problems teaches the student to replace unknown percents with their fraction or decimal equivalents. The next step for the student is to set up and evaluate an appropriate multiplication or division expression to find the unknown quantity. To find 10% of 40 the student would replace 10% with an equivalent fraction or decimal and then evaluate a multiplication expression (0.1 x 40 or 1/10 x 40).

The author uses the term Tracking Questions to refer to a set of fourteen test questions on which student performance was monitored during the course of the study. These questions were administered to students at least three times during the study (on the pretest, post test, and retention test). Parallel forms of some of these questions were administered to the students on quizzes following major units of instruction as well.

Cross-lagged panel correlation analysis is a quasi-experimental design which can be used to examine two variables, measured at two points in time, for causality. In general the $Time_1$ variable with the larger cross-lagged correlation, as indicated in Figure 2, is inferred to be causally predominant. In other words, a variable is considered to be causally predominant if the correlation between A_1 and B_2 (r_1) is greater than the correlation between B_1 and A_2 (r_2). This study design allows the investigator to examine relationships among variables without having to arbitrarily classify them as being independent or dependent.

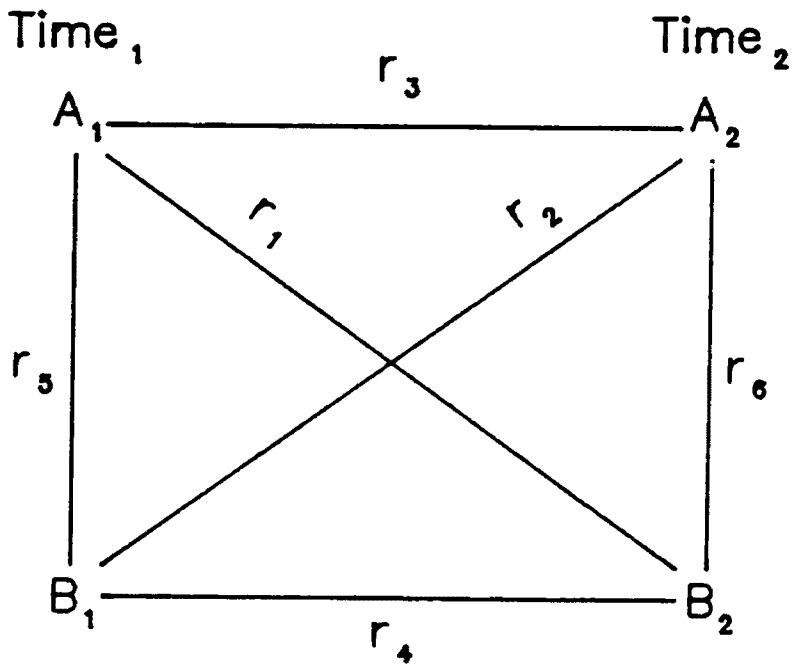


Fig. 2 Cross-Lagged Panel Correlation Design

To rule out the possibility that the difference between the cross-lagged correlation coefficients is not caused by a third variable, the assumptions of synchronicity and stationarity must reasonably be met (Kenny, 1975). By measuring the two variables during the same test administration one is reasonably sure of meeting the assumption of synchronicity. The assumption of stationarity may be statistically tested by examining the equality of the synchronous correlations, the correlation between A_1 and B_1 (r_5) and the correlation between A_2 and B_2 (r_6). If the assumption of stationarity is rejected, then the synchronous correlations must be adjusted for attenuation and the cross-lagged correlations must be adjusted for reliability. The extent to which the results of cross-lagged panel correlation analyses results can be relied upon is dependent on several factors. Large changes in the reliabilities of the individual constructs measured at times 1 and 2 make it very hard to meet the assumption of stationarity. Small sample sizes (less than 150) seldom yield significant differences between cross-lagged correlations and causal relationships may be hidden from the investigator. Uncontrolled background variables may often times affect the results, separate analyses should be conducted for each type of subject whenever the sample sizes remain large enough (Calsyn, 1976).

The Pearson-Filon z test is used to test the equality of the synchronous correlations and to test the equality of

the cross-lagged correlations (Steiger, 1980). The equations needed to compute the test statistic to test the equality of correlations r_1 and r_2 are given below:

$$z = \frac{N^{.5}(r_1 - r_2)}{((1 - r_1^2)^2 + (1 - r_2^2)^2 - k)^{.5}}$$

$$k = (r_5 - r_4 r_1)(r_6 - r_4 r_2) + (r_3 - r_5 r_2)(r_4 - r_5 r_1) + (r_5 - r_3 r_2)(r_6 - r_3 r_1) + (r_3 - r_1 r_6)(r_4 - r_6 r_2)$$

Summary

The topic of percent has wide spread applicability and utility to the student's everyday activities. Yet, test data indicate that students are not gaining a clear understanding of percent from their educational experiences. Very little research has been done concerning how instruction may be modified to improve students' learning of percent concepts.

This study was conducted to examine the feasibility of using a percent learning hierarchy as a basis for designing instructional sequences for percent. The author compared the relative strengths of two popular approaches to teaching the solution of case 1 percent problems. The effectiveness of teaching the solution of case 2 percent problems before case 1 was examined. Student performance across time was evaluated on selected test questions.

CHAPTER II

REVIEW OF RELATED LITERATURE

Prior to the development of any instructional materials, or the formulation of the study design, the author conducted a review of the related literature. To better understand what was currently known about teaching percent, the literature on percent error analyses and methods of teaching percent was examined. An understanding of the notion of fraction appeared to be crucial to the student's acquiring an understanding of percent so relevant literature on the teaching of fraction and decimal concepts was examined. Research on human information processing was examined to gain insight into the roles that language and models play in facilitating concept attainment. To determine the utility of learning hierarchies to curricular sequencing the author reviewed the literature on learning hierarchies and hierarchy validation techniques in mathematics instruction.

Percent Error Analysis

Early studies seemed to concentrate on analyzing student errors on percent test questions. Many of these error studies were conducted during the twenties and thirties. A study by Edwards (1930) was typical of these. He gave eight groups of seventh grade students (N=215) a

standardized percent test, and then attempted to tabulate and classify the student errors by type. The test content and error analysis are summarized in Table 1.

Table 1

Average Number of Student Errors in Edwards' Study

Question Type	No. of Items	Errors (%)
Changing fractions to percents	15	21
Changing percents to fractions	15	27
Changing percents to decimals	15	23
Changing decimals to percents	12	28
Associating percents less than 100 with region diagrams	10	36
Associating percents greater than 100 with region diagrams	3	66
Selecting correct problem solution from five given	15	55
Solving percent problems by inspection	15	52
Solving percent problems requiring written work	13	66

Edwards wrote that students' errors were not caused by doing a problem wrongly, but by choosing to do the wrong thing to solve the problem. In support of this observation, he found a number of instances where students blindly followed rules without regard to the meaning of percent. An interesting phenomenon reported by Edwards was the tendency

of students to regress back to an easier, but better known, solution if there was some uncertainty in the type of answer a question required. He also reported correlations between student percent scores and other ability test scores: Terman Group Intelligence Test (r=0.52), Stanford Reading Test (r=0.36) and Advanced Compass Arithmetic Test (r=0.7).

Edwards concluded that students were not being taught enough about percent to be able to determine the reasonableness of their answers. This observation is based on the fact that both bright and dull students made the same errors, only the frequency of the errors differed. He recommended that researchers needed to explore the hierarchy of connections between various problem types and student responses. He also suggested that teachers should make use of counter examples and mixed exercise sets to aid students in developing problem discrimination skills.

Brueckner (1930) stated that knowledge of ratio and equivalent decimals was prerequisite to work on the application of percent. Brueckner analyzed work with percent into seven required skills and devised a sixty-three item test on percent, based on this analysis. He administered this test to a group of Minneapolis seventh graders (N=405). Student errors were analyzed by examining the written work on the test papers. The errors were classified as those involving percent, or those having other causes. The test composition and error analysis are summarized in Table 2.

Table 2

Average number of Student Errors
in Brueckner's study

Content Area	No. of Items	Errors (%)
Changing decimals to percent	10	46
Changing percent to decimals	10	28
Changing fractions to percent	12	15
Changing fractions to hundredths and percent	4	79
Finding a percent of a number (Case 1)	9	52
Finding what percent one number is of another (Case 2)	9	75
Finding number when percent is known (Case 3)	6	87

The least number of student errors occurred while changing simple fractions to percents. In changing decimals to percents, and conversely, students had the most trouble with numbers involving percents greater than 100 or less than 1. Students also made many errors, while writing percents as decimals, when decimal percents were present (ie. 16.5%). Based on student errors, Brueckner concludes that Case 3 problems are more difficult than Case 2 problems, and Case 2 problems are more difficult than Case 1 problems.

Brueckner felt that most of the student errors were either caused by a failure to manipulate decimal numbers properly, or by a lack of understanding of the process

required by the problem. As a partial remedy to these difficulties, he suggested that students should receive extensive practice on problems similar to those on which students exhibited large numbers of errors. Brueckner also suggests that children make many of their errors because they are not taught to check their work.

Guiller administered an arithmetic computation test to ninth graders and college freshpersons. Ten questions on this test dealt with five types of percent problems. His ninth grade sample (N=931) was drawn from five public school systems. His college sample (N=925) was drawn from three successive freshperson education classes. Guiller examined the written work of the high school (Guiller, June 1946) and college (Guiller, October 1946) students separately. Student errors were distributed as shown in Table 3.

The errors made by the two groups were remarkably similar, both in type and relative frequency. Guiller writes that the greatest single cause of student errors seemed to result from a failure of students to understand the meaning of percent. Students exhibited a lack of ability to apply the notion that percent means hundredths. Students often had difficulty in selecting a procedure appropriate to the information present in the problem. His data suggest that they also had difficulty in generating percent equivalents for fractions and decimals. Computational difficulties, principally troubles with multiplication and division, accounted for the second

Table 3

Average Number of Student Errors in Guiller's Study

Ability Measured	No. of Items	Grade 9 (%)	College (%)
Finding percent of number (Case 1)	2	52	51
Finding what percent one number is of another (Case 2)	2	48	54
Finding a number when percent is known (Case 3)	2	94	88
Finding result of percent increase or decrease on a number	2	72	50
Finding percent increase or decrease of one number on another	2	83	75

largest number of student errors. The third greatest cause of student errors was misplacing or omitting the decimal point in the answer.

Guiller's work is important because it reveals the prevalence of specific student difficulties with percent. Even though Guiller used a small number of test items, one is struck by the large number of errors occurring in both age groups. His data suggest that a widespread misunderstanding of percent existed at the time this test was given. Based on his work, it would seem that not much happens during the high school years to help students who had difficulties with percent prior to entering high school.

A study by Montgomery (1958) was conducted to gain insight into the nature of incorrect responses associated with percent questions. His approach was to test several groups of seventh grade students (N=553). No percent instruction was attempted in this study. He examined the effects of three test variables (percent case, percent magnitude and mode of problem presentation) on student performance. Montgomery used four presentation modes for his percent problems (regions, line segments, number problems and verbal problems). His analyses tended to focus on student difficulties as they related to the three cases of percent problems in each of the four modes of presentation.

Montgomery found that students had difficulty applying their knowledge of related topics of arithmetic (ie. common fractions and decimals). Students often selected the wrong arithmetic operation, dividing rather than multiplying or vice versa. Based on student errors, case 3 problems were identified as the most difficult and case 1 problems the least difficult. His data suggest that student difficulties with percent may be related to the magnitude of the percent used in a given problem. Students in his study tended to treat the graphical problems (those involving the interpretation of region or line segment diagrams) as novel problems. Hence, student performance on the graphical problems was poorer than on either the numerical or verbal percent problems. Montgomery reported large numbers of

students failing to answer questions or guessing at answers. He also reported the tendency of students to regress to easy, one step, arithmetic solutions when confronted by difficult problems. Students would often just count off two decimal places or simply divide the larger number by the smaller number, without regard to the nature of the problem.

Montgomery concluded that a better understanding of fractions and decimals would have improved student scores. He concluded that students don't have equal abilities on all four problem presentation modes. This writer would attribute these student differences to the fact that students, in general, have differing exposures to the four presentation modes, rather than to differing student abilities to deal with them. In 1958 when Montgomery did his study, the use of visual models in percent instruction was not as wide spread as instruction based on rule processing.

Percent Teaching Methods

There have been a number of studies in which two or more methods of teaching percent were compared. These studies tended to concentrate on teaching the computational aspects of percent problems, with little emphasis on developing an understanding of the meaning of percent or the analysis of student errors. The five studies reported here compare treatment groups on the basis of mean differences, as measured by investigator developed tests.

A study by McMahon (1959) compared the effectiveness of teaching percent to seventh graders (N=245) by the ratio and the conventional methods. The conventional method focused on the solution of percent problems according to their case type. The algorithmic method of attack in this method seemed to be a rule oriented decimal method. Students in the ratio method were expected to solve percent problems by setting up and solving proportions. McMahon found no significant differences between the two teaching methods with respect to developing student ability to interpret percent statements. There was no significant difference between the post test total scores obtained by students in the two treatments. Significant differences favoring the ratio method were observed on the computational subsection of the post test and for overall performance on the six week retention test. Neither treatment was successful with low ability students.

A study by May (1965) compared the effectiveness of teaching percent to seventh graders (N=156) using the traditional, the ratio and the discovery methods. The traditional method was described as one in which the teacher presents a rule, the students memorize the rule and then work problems using the rule. The rules presented are very specific to the type of problem studied. One percent problem type is presented at a time. The ratio method was an expository development of the solution of percent problems by setting up and solving proportions. The

discovery method, which used the same text as the traditional method, was an inductive development of the solution of percent problems. The students in the discovery treatment would be given a problem to solve and the teacher would aid them in developing generalizations to solve the problem. These students were encouraged to test their generalizations and figure out why they worked. All of her treatments focused on teaching percent as a special case of the fraction concept. May's principle finding was that the traditional method is not the most effective method. Significant differences favoring both the ratio and discovery methods were discovered on both the post test and the six week retention test. There were no treatment interactions with either sex or mathematical ability as measured by the School College Aptitude Test.

A study by Wynn (1965) compared the unitary analysis, the formula and the decimal methods of teaching percent. His subjects were twenty-seven seventh grade classes, taught by nine teachers. In the unitary analysis treatment, students are taught to find the value of 1% of the whole first, then use that information to solve the problem. The formula method was a percent case based method, in which the students learned to classify percent problems by case and the make use of the variations of the factor-product equation (Percentage = Rate X Base). In the decimal method, students were taught to use the decimal form of the percent and then decide whether to multiply or divide, based on

problem percent case type. Wynn performed an analysis of covariance, using IQ score as the covariate, and found no significant differences between treatment groups on the post test. A second analysis of covariance, using the post test score as the covariate, was performed on the six week retention test score and again no significant differences were observed between treatment groups. Wynn concluded that none of the treatments was superior to the other two.

Mason (1975) conducted a three day study with college students (N=71). In this study he compared the effectiveness of the time function approach to teaching percent, with that of the formula and the proportion methods of teaching percent. The time function approach is a variation on the proportion method. If no percent of increase or decrease is involved, the students are taught to set up a proportion and solve for the missing term. If percent of change is involved, students are taught to classify all problem elements, based on their time functional relationships to one another. Emphasis is placed on correctly identifying the whole unit or base (in time functional terms the base is the element which occurs first in time) and then identify the relation (increase or decrease) needed to arrive at the problem's final state. To find how many percent less than 50 the number 40 is using this method, the student would identify 50 as the element occurring first in time and then complete the table that follows:

	First	Decrease	Last
M	50	b	40
P%	100	y	z

The student would then set up and solve the proportion 50/100 = 40/z. Since 40 is less than 50, this is a percent decrease problem. In a decrease problem Decrease = First - Last, so that the percent decrease is 100 - 80 = 20. The formula method of instruction has the students classify the problems by case type, and then make use of variations on the factor-product equation (Percentage = Rate X Base). In the proportion method the students are taught to solve percent problems by setting up and solving proportions.

Mason wrote a programmed study module for each of the three treatments. An analysis of variance performed on the pretest scores showed no significant differences existed between the groups prior to beginning the study. An analysis of covariance, using the pretest score as a covariate, was performed for the post test scores and indicated a significant difference favoring the students in the time function treatment. In addition t tests for related scores were performed on the differences between the pre and post test scores. The only significantly different score was for the time function treatment. Mason concluded that, given the success and apparent ease of teaching, the time function approach should be considered for use in

teaching high school students. This writer feels that the three day instructional treatment was too brief to allow for an adequate comparison of the three treatments.

Jamski (1976) investigated the effects of using calculators to teach fraction-decimal-percent conversion algorithms to seventh graders (N=136). This study is a fairly typical calculator experimental group and non-calculator control group comparison. Both groups of students received the same instructional materials, the treatment variable was the presence or absence of the calculator.

Jamski compared the student abilities, in the experimental and control classes, to perform the six conversions formed by taking the permutations of fraction, decimal and percent taken two at a time. Of the six conversions, the only significant difference favoring the calculator treatment was on the conversion of fractions to decimals on the post test. No significant differences between treatment groups were found for any other conversions on the post test or on any of the six conversions on the four week retention test. He found no evidence of treatment-ability interactions, implying that use or non-use of the calculator is neither detrimental or helpful to low or high ability students.

Fractions

Several studies at the University of Michigan dealt with the learning of initial fraction concepts. The research summarized by Payne (1976) seems to have several recurring themes. Quality instruction must be preceded by a careful analysis of the concept to be taught. Concept formation seems to be aided by making good use of appropriate visual models. Explicit work linking the verbal name, written symbol and the model of the concept is necessary for the student to internalize fraction concepts. The work describing the development of the initial fraction sequence is of particular relevance to the study of the percent concept.

Muangnapoe's (1975) dissertation showed that fraction concepts can be mastered by students as early as grade 3. Muangnapoe described the development and effectiveness of a sequence to teach initial fraction concepts to third and fourth graders. Investigators at The University of Michigan developed an initial fraction sequence consisting of nine lessons. The focus of the learning sequence was on developing an understanding of the meaning of fractions. The importance of identifying the unit and the necessity of equal size partitioning of the unit was stressed. Classroom work was introduced using concrete models and students learned the meaning of verbal names for fractions prior to learning the meaning of the written symbols. Based on his pilot studies, Muangnapoe choose to exclude set models from

the final sequence. He did this to reduce the amount of potential learning interference from using dissimilar models and to emphasize the measurement aspects of the linear and region fraction models.

The fraction sequence was taught to students (N=91) in four intact classes, two third grade and two fourth grade, taught by their regular teachers. Muagnapoe taught the fraction sequence to five small groups of third and fourth graders (N=15). The students were pretested prior to beginning the study and were given a post test, a transfer test and a two week retention test following the nine days of instruction. The test scores were analyzed using both analysis of variance and analysis of covariance, using the pretest score as the covariate. No statistically significant differences were found between student post test scores, for student in the small groups or intact classes. Statistically significant differences favoring students in the small group instruction treatment were found on both mean transfer and retention test scores. Statistically significant differences were found favoring the third and fourth grade students, in this study, over seventh grade performance levels reported for selected NLSMA (National Longitudinal Study on Mathematics Achievement) test questions.

Ellerbruch's research was concerned with extending the initial fraction sequence to the addition of fractions. He and Coxford (1977) wrote that the region model seemed to be

the most natural fraction model for children to understand and the easiest for them to use. His initial fraction work relied on paper folding activities to develop an understanding of the verbal names for fractions and to develop the equivalent fraction generality (a/b=an/bn). Emphasis is placed on helping the student connect the oral name and written symbol with the model, before trying to connect the oral name and the written symbol. Ellerbruch and Payne (1978) analyzed the process of naming fractions into four parts: recognizing the unit, recognizing the presence of equal size partitioning of the unit, naming the number of pieces to be considered and naming the number of pieces in the unit. The part whole comparison, implied by a measurement model of a fraction, is essential to this analysis.

At the University of Texas, Novillis (1973) demonstrated the feasibility of formulating, and testing, a subconcept hierarchy for the fraction concept. Novillis defines a subconcept as a concept whose attainment is prerequisite to a second concept. Her research focused on seeing whether or not her proposed hierarchy was supported by test data and whether or not the hierarchy varied with student age.

Novillis developed a fraction concept test consisting of sixty-eight items, constructed to measure student attainment of sixteen selected concepts. The test was administered to students (N=279) in grades four, five and

six. No classroom instruction on fractions was included in her study prior to administering this test. Validation of her learning hierarchy was conducted using the so called Walbesser ratios, computed from her test data. Her test data supported eighteen, of twenty-three, postulated subordinate concept dependencies. She found no differences in the hierarchical dependencies across grade levels four, five or six. It should be noted that eleven of the dependencies could not be examined using all three groups of students because the younger children tended to omit the more difficult questions. Novillis found that associating fractions with part-whole and part-group models was prerequisite to associating a fraction with a number line point or using a fraction in a comparison situation. She also found that being able to associate a fraction with a model whose members were congruent was prerequisite to being able to associate a fraction with a model whose members were not congruent. She criticized her own work because it involved evaluating student understanding of the fraction concept without teaching them about fractions.

Decimals

Research on learning decimal fractions has tended to focus on the effects of teaching strategies on student computational skills. There has been some work done which is relevant to the teaching of percent concepts. O'Brien (1968) wrote that decimal instruction should be based on the relation of decimals to both fraction and numeration

concepts. Galloway (1976) was successful in teaching decimal concepts to third graders following instruction on initial fraction concepts. Payne (1975) felt that Galloway's success was due, in part, to the good understanding of fractions that the children acquired during the first part of her treatment. A similar success was reported, at the middle school level, in an unpublished master's thesis. The thesis evaluated the effectiveness of teaching decimals as a logical extension of the initial fraction sequence reported by Ellerbruch and Payne (1978). It is interesting to note that Wilson (1972) found that student achievement was not affected by the order in which fractions and decimals were introduced to the students.

Concept Attainment

Van Engen (1953) writes that mathematics teachers are concerned with teaching two types of meanings for symbols, syntactic and semantic. He feels that teachers spend too much time on the former and neglect the latter. Van Engen's feeling is that a person must have a referent for understanding a symbol by itself (semantic meaning), before a person is able to use the symbol in relation to other symbols (syntactic meaning). He writes that a combination of abstract presentation and concrete experience will yield greater fundamental efficiency in concept development than either alone. He also writes that concepts learned logically are more quickly remembered and retained longer.

Greeno's (1978) work on procedural knowledge suggests that the student's ability to represent quantities spatially is essential to understanding mathematical concepts. His work also reaffirms the importance of the role language plays in developing quantitative concepts. Greeno writes that children who over-learn the component parts of a concept are more likely to be able to transfer their knowledge of the concept in novel situations. He also warns that over learning the component parts of a concept may make it more difficult for the student to integrate them into one whole.

Sowder (1980) writes that concept attainment is demonstrated when a common response is given to dissimilar stimuli. He describes five levels of responses that a person may give to indicate his understanding of the concept represented by the concept stimuli. Responses may range between being able to recognize a previously shown example of a concept and being able to distinguish examples from non-examples at the lower levels. Higher level responses would include being able to provide an example of a concept and stating a formal definition of the concept (in one's own words). Sowder's feeling is that teachers do not make adequate use of non-examples in teaching concepts. The role of language in concept development is often not appreciated by teachers. Sowder writes that the laboratory study of artificial concepts is not as relevant to educators as classroom like studies on the attainment of real concepts.

Hierarchy Validation

For a learning hierarchy to be of any use in the sequencing of instructional materials it must be ordered so that when a student has exhibited a higher level behavior one is certain that the student has exhibited all prerequisite behaviors. To ensure that a learning hierarchy is properly ordered it is necessary to perform a test of its validity. A review of the literature revealed a number of methods for the validation of postulated learning hierarchies. Several studies comparing the effectiveness of these methods were examined.

Phillips (1971) conducted a study to develop and evaluate indirect procedures for validating learning hierarchies from test data. He was interested in seeing whether or not the sequence of objectives within a set of instructional materials would affect student achievement or their time to complete the materials. He used seven methods of ordering the objectives in his learning hierarchy for the addition of fractions: logical sequence, item difficulty, Guttman scalogram (Cox and Graham, 1966; Guttman, 1944), ϕ correlation analysis (Capie and Jones, 1968), Walbesser ratios (Eisenberg and Walbesser, 1971), textbook and random. His logical sequence ordering of objectives was based on a task analysis and confirmed by pattern analysis (Rimoldi and Grib, 1960). The relative number of student errors

determined the sequence of objectives in the item difficulty ordering. He used a standard textbook ordering of the objectives, and a completely random ordering as well.

To conduct his study, Phillips first postulated a hierarchy of eleven objectives for the addition of fractions with like denominators. This hierarchy was evaluated by testing (N=163) elementary school students in grades four through six. Based on this test data, he modified his original ordering of objectives to arrive at the seven orderings described previously. Phillips developed seven sets of programmed study materials, one set for each ordering of objectives. Two pretests were administered to a group of fourth grade students, who were then randomly assigned to treatment groups. The purpose of the pretests was to remove from the study population those students who were missing the prerequisite skills required by the study or who had already mastered the study materials. Following the completion of the programmed materials the students (N=142) were given a post test and a transfer test. The transfer test dealt with the subtraction of fractions having like denominators. Two weeks later the students were given a retention test.

Phillips performed several analyses of variance and found no significant differences among treatment groups on the post, transfer or retention tests. He found no significant differences among groups on mean completion time for the lesson materials. Post hoc comparisons, using the

Duncan multiple range test did reveal some pairwise differences between certain treatment groups. Significant differences between group mean retention scores were found favoring the logical sequence and Walbesser ratios treatments over the textbook treatment. Differences in mean retention test scores favored the Walbesser ratios treatment over the item difficulty treatment. Significant differences in mean completion time, favoring the logical sequence treatment over the correlational analysis treatment, were also found.

Phillips concluded from his study that the overall efficiency of a set of instructional materials can be affected by the sequence of objectives within the materials. He also observed that retention seems to be more susceptible to sequence affects than are students' achievement or transfer abilities. One must be careful in the interpretation of Phillips results, as he reported two confounding factors which may have affected his results. In scoring his tests, Phillips required students to reduce all fractions completely to get full credit. When Phillips relaxed this requirement his initial results changed radically. A second confounding factor was that the teachers in this study often did more than merely answer questions when the students encountered difficulties in the programmed materials. The teachers were often observed to be teaching needed skills out of sequence to alleviate students' frustrations with the materials.

A study by MacGregor (1976) compared the White and Clark C test (White, 1974) and the Walbesser methods of validating learning hierarchies. He developed a learning hierarchy for the addition of fractions, along with the appropriate instructional materials. The programmed instructional materials contained ten, 30 minute lessons. A test was included within the materials used for each lesson.

MacGregor administered two pretests to four eighth grade classes. The pretests were used to eliminate students from the study, because they did not have the necessary prerequisite knowledge or because they had already mastered the study material. He evaluated the test results using both the C test and the Walbesser ratios. The C test supported four of the postulated hierarchical relationships, while the Walbesser ratios supported none of them. MacGregor concluded that the two methods were not equivalent.

A study by Mauk (1981) also compared the White and Clark C test with the Walbesser ratios, as methods for validating learning hierarchies. Specifically, she wanted to see if the two methods identified the same dependencies between skills in a hierarchy and whether or not they made the same suggestions for modification of the hierarchy. She was also interested in seeing whether or not the assumptions made by the two techniques were valid for her hierarchy of objectives for the addition and subtraction of integers.

Mauk developed a test, which had two test questions per hierarchical objective. The test was administered to several college algebra classes. Students (N=255) volunteered to allow their test papers to be used in her study. She found no pattern of agreement between the two methods. The Walbesser ratios seemed to be the more stringent test. It should be noted that Mauk made no effort to control for the effects of instruction in her study. She found that the C test made few suggestions for modifying the hierarchy, while the Walbesser ratios have established criteria for making modifications. She concluded that the assumptions made by the C test were almost impossible to guarantee in an education study. The assumptions made by the Walbesser ratios depend almost entirely on the arbitrarily set criteria for determining mastery of each objective. Her recommendation is to use the Walbesser ratios, if the hierarchy has a small number of objectives, of slightly increasing difficulty.

The investigator, after examining the literature cited previously, concluded that the Walbesser technique was the most appropriate validation technique to use in his study on percent. The Walbesser ratios used by this technique were developed to answer criticism's of Gagne's proportion of positive transfer (Gagne and Paradise, 1961). The Walbesser ratios provide a means for hierarchy validation and provide cues for modifying the hierarchy. The Walbesser ratios seem to be a very demanding criteria for establishing

hierarchical relationships. The ratios often call for the rejection of postulated relationships, when other techniques support them. This technique requires the calculation of three ratios to measure the consistency, adequacy and completeness of the hierarchy. The quantities used in the calculation of these ratios are derived from contingency tables like Figure 3. In Figure 3, f(0,1) represents the number of students mastering the first objective and not mastering the next objective in the hierarchy.

Subordinate Objective	Terminal Objective	
	Fail	Pass
Fail	f(0,0)	f(1,0)
Pass	f(0,1)	f(1,1)

$$\text{consistency ratio} = \frac{f(1,1)}{f(1,1) + f(1,0)}$$

$$\text{adequacy ratio} = \frac{f(1,1)}{f(1,1) + f(0,1)}$$

$$\text{completeness ratio} = \frac{f(1,1)}{f(1,1) + f(0,0)}$$

Fig. 3 Pass-fail contingency table.

The consistency ratio tests the hypothesis that the acquisition of the terminal behavior implies the acquisition of all subordinate behaviors. A hierarchy which is not consistent has objectives which are not sequenced properly. The adequacy ratio tests the hypothesis that, given appropriate instruction, students possessing the subordinate behaviors will acquire the terminal behavior. A hierarchy which is not adequate is presumed to have too great a jump between objectives and needs intermediate objectives added to it. The completeness ratio estimates the number of students capable of traversing the hierarchy. A hierarchy which is not complete is valid for only a portion of the population.

The literature reports two criteria for determining whether or not Walbesser ratios support a hierarchical relationship. Walbesser's own criteria calls for each ratio to be 0.85 or greater. Mauk (1981) reports the use of a second criteria which requires the consistency ratio to exceed 0.85, the adequacy ratio to exceed 0.7 and the completeness ratio ratio to exceed 0.5.

Summary

The studies on percent error analyses reported large numbers of student errors, and a tendency of students to apply rules without reasons. Students often selected incorrect arithmetic operations or performed them incorrectly.

Some investigators stated that knowledge of fraction concepts and equivalent decimals was prerequisite to the study of percent. Research on the teaching of fraction and decimal concepts suggests the importance of using visual models and the effectiveness of using precise language in the instructional process. The initial fraction sequence, developed by researchers at the University of Michigan, provides a solid basis for instruction on rational numbers.

Learning hierarchies may provide a valuable tool in sequencing instructional materials. The Walbesser ratios seem to provide a sufficiently stringent criteria for validating hierarchical relationships.

CHAPTER III

METHOD AND INSTRUCTIONAL MATERIALS

Instructional Materials

All materials and procedures were field tested thoroughly in four pilot studies. The purpose of the pilot studies was to gain insight into the composition and sequencing of topics within an initial percent unit at the middle school level. The students involved in the pilot studies were enrolled in Michigan schools, in grades six through eight. The teachers used in the pilot studies were recruited by the investigator with the help of local mathematics coordinators.

First Pilot Study

The first pilot study was conducted in February, 1981. The students involved in this pilot study made up the entire seventh grade class (N=46) of a small, private secondary school located in Ann Arbor, Michigan. The students were homogeneously grouped, by mathematics ability, into four classes by the school's faculty. Class sizes ranged from four to fifteen students.

The instructional materials used in this study consisted of the first portion of the author's proposed percent materials. The sequence of topics within these materials was as follows:

Lesson 1. Meaning of percent, using region diagrams.

Lesson 2. Equivalent ratios.

Lesson 3. Fractions and percent.

Lesson 4. Decimals and percent.

Lesson 5. Fraction-decimal-percent equivalents.

Lesson 6. Finding 1% and 10% of numbers.

Lesson 7. Finding percents of numbers.

Lesson 8. Ways to state percent problems.

During the week prior to instruction the students were given a diagnostic pretest. The students received eight, fifty minute periods of instruction on percent. They were taught by their regular mathematics teacher. The investigator observed a number of class sessions and assisted students as they worked with the materials. During the week following instruction the students were given a post test and a percent transfer test. Students were randomly selected from each class and interviewed by the investigator to gain further insight into how they solved percent problems.

Using test and interview data, supplemented by the author's classroom observations, a number of student learning difficulties were identified. Students seemed to have some difficulty interpreting the wording of percent problem statements. This suggested that more practice on interpreting percent statements needed to be included in the next revision of the percent sequence.

Some students were observed to have problems in understanding the relation between fractions and decimals. These students tended to read decimal numbers a single digit at a time, and were unable to write 0.53 as a fraction. The author observed that some students had difficulty in understanding place value as it relates to percents expressed using decimal notation. At times, it seemed as if these students did not understand the meaning of the thousandths place in decimal notation. There was a tendency among some students to simply add, or delete, the percent symbol to change forms between decimals and percents.

On the transfer test the greatest difficulty observed was that students seemed to have trouble selecting the correct value for the whole unit involved in the problem. In addition, on many of the transfer test problems students attempted to solve the problems by the application of a single arithmetic operation when most problems required the use of two.

Second Pilot Study

The second pilot study was conducted in May, 1981. The students involved in this study were enrolled in two intact sixth grade classes (N=55), at a public elementary school in Waterford, Michigan. One of the classes was regularly managed as a completely individualized, open classroom, the other was managed in a more traditional manner.

The instructional materials used in this study were based on the author's earlier work and incorporated revisions suggested by the results of the first pilot study. The sequence of lessons within these materials was as follows:

Lesson 1. Review of initial fraction concepts.

Lesson 2. Equivalent fraction generalities.

Lesson 3. Equivalent ratios.

Lesson 4. Meaning of percent, using region diagrams.

Lesson 5. Fractions and percent.

Lesson 6. Discount and mark up.

Lesson 7. Ways to state case 2 percent problems.

Lesson 8. Decimal representation of tenths and hundredths.

Lesson 9. Equivalent decimals.

Lesson 10. Decimals and percent.

Lesson 11. Finding fractional parts of numbers.

Lesson 12. Finding percents of numbers.

The students were given a diagnostic pretest prior to beginning instruction. The students were then given twelve hours of instruction, based on the author's fraction multiplication percent sequence. The students were taught by their regular classroom teachers. Approximately one-half of the class sessions were observed by the author. The students received short quizzes covering the previous day's work at the first class period following lessons: 3, 5, 7

and 10. Following the instructional treatment the students were given a post test. Students from each class were randomly selected and interviewed by the investigator.

Based on student test results and interview data, several observations on student performance in this pilot study may be made. Students made a number of errors which seemed to imply that they were interpreting decimal place value incorrectly. They seemed to have little difficulty in associating two digit decimals (tenths and hundredths) with percent. Yet, they were observed to have a great deal of difficulty interpreting the meaning of single digit decimals and percents less than 10%. The author hypothesizes that this may be caused by the students not having a good understanding of the meaning of the thousandths place in decimal notation. By not understanding the relationship between thousandths and percent, the students had no valid interpretation of a number like 12.5%. This implied to the author that more work on equivalent decimals to thousandths needed to be included in the next revision of the percent sequence.

Associating easy fractions, those whose denominators are divisors of one hundred, with percents seemed to be easy for most students. In writing percents as fractions, the problem demonstrated by most students was not in writing an equivalent fraction for the percent. Rather, the problem seemed to be that the students failed to reduce the fractions completely or did so incorrectly. This problem

seemed to be a potentially serious confounding factor to the investigator. It was decided in the future to have students record both an equivalent fraction and the reduced equivalent fraction separately, so that the source of student errors would be easier to identify.

The students were observed to do quite well on fractions dealing with basic percent concepts, like equating the whole with 100%. The students did not perform as well on the post test as they did on comparable questions on the quiz following lesson 8. These questions required students to interpret and solve case 2 percent problem statements. The typical error seemed to be multiplying the numbers in the problem regardless of their context. This implied to the author that work on discriminating between case 1 and case 2 problem statements should be included in later revisions of the materials. Students were observed to have a great deal of difficulty interpreting the wording of percent problems, apparently because of the similarities in the language used in case 1 and case 2 problem statements. A lesson on discriminating between the two types of statements needed to included in the next revision of the sequence.

Third Pilot Study

The third pilot study was conducted during September, 1981. The students involved in this study made up the entire seventh grade class (N=44) of the same private secondary school used in the first pilot study. The school

was located in Ann Arbor, Michigan. The students were enrolled in three intact heterogeneously grouped, mathematics classes. The teacher had previously used the author's percent materials in the first pilot study.

The instructional materials used in this study were based on the author's earlier work and represented the most recent draft of the author's proportion sequence. The sequence of lessons for these materials was as follows:

Lesson 1. Basic fraction concept review.

Lesson 2. Equivalent fraction generalities.

Lesson 3. Equivalent ratios.

Lesson 4. Cross products.

Lesson 5. Meaning of percent, using region diagrams.

Lesson 6. Fractions and percent.

Lesson 7. Percent discount and mark up.

Lesson 8. Ways to state case 2 percent problems.

Lesson 9. Decimal representations of tenths and hundredths.

Lesson 10. Equivalent decimals.

Lesson 11. Decimal representation of thousandths.

Lesson 12. Decimals and percent.

Lesson 13. Finding fractional parts of numbers.

Lesson 14. Estimating percents of numbers.

Lesson 15. Finding percents of numbers.

Lesson 16. Percents of discounts and percents of mark ups.

Lesson 17. Percent of increase and percent of decrease.

Lesson 18. Discrimination between case 1 and case 2 problem statements.

The students were given a diagnostic pretest prior to beginning instruction. Students received eighteen fifty-minute periods of instruction on percent. Two of the classes were taught by their regular mathematics teacher. The third class was taught by the investigator's faculty advisor and a visiting professor of mathematics from The University of Michigan. The investigator observed several periods of instruction. Students received daily quizzes covering the previous day's instruction. Following the instructional period the students were given a post test.

There were some common student errors which indicated sequence revisions were required. There was evidence of some confusion over correct decimal point placement, when the decimal representation of a percent required the use of the thousandths place. Students had trouble with decimal and percent equivalents for thirds and eighths. Some problems in reducing fractions correctly were also observed.

Students had some difficulty in selecting the appropriate denominator, when setting up a proportion to solve case 1 percent problems. Students exhibited difficulties in solving percent verbal problems correctly. There were large numbers of student errors made while solving percent increase and decrease problems. In this writer's opinion, percent increase and percent decrease

problems are so difficult for students at this level that they should only be introduced with expectation that students will not attain mastery of them.

Fourth Pilot Study

The fourth pilot study was conducted during October, 1981. The students involved in this study were enrolled in an intact eighth grade mathematics class (N=23). The students attended a public secondary school, north of Petosky, Michigan.

The instructional materials were based on revisions of the author's work and represented the author's latest draft of his factor product sequence. The sequence of the lessons within these materials was the same as that used in the third pilot study. Only the algorithmic method of attack for solving case 1 percent problems distinguished the materials used in this pilot study from those used in the third pilot study.

The students were given a pretest prior to beginning instruction. The students were taught eighteen lessons on percent. The students were not able to keep up the one lesson per day pace suggested by the author. Their teacher, who provided all the instruction, felt that the students' home lives were such they they could not complete the lessons outside of class time. Otherwise the lesson plans were adhered to as closely as possible. Students received

daily quizzes covering the previous day's instruction. Following the instructional treatment the students were given a post test.

Verbal problems seemed very difficult for the students in this pilot study. Their teacher observed that students often did not even attempt the verbal problems included the lesson work sheets. They may have had reading difficulties or the content of the verbal problems may have been too far removed from their life experiences.

These students were observed to have problems interpreting decimal percents. There also seemed to be some confusion as to when the percent symbol should be included as a part of the problem's answer. The students had some difficulty in reducing fractions correctly, although writing equivalent fractions of higher terms did not seem difficult for them. There were a number of instances where students attempted to solve case 1 percent problems by division, rather than multiplication. Some students seemed to have difficulty in identifying the whole unit in percent increase or decrease problems.

Summary of Findings

In the course of conducting these pilot studies, the investigator became aware of several difficulties which seemed to affect student performance on percent questions. Students seemed to have difficulty in applying the relation between fraction and decimal representation of numbers. Some students had a tendency to read decimal numbers a

single digit at a time, seemingly without thinking about the place value meaning of each digit. They had problems understanding how decimal place value relates to percent notation. There was a tendency among some students to do decimal-percent conversions by simply adding or deleting the percent symbol, without regard to the place value meaning of the numbers. Students had difficulty representing the meaning of single digit percents correctly using diagrams (ie. 0.7 and 7% were likely to be represented using the same diagram).

In this writer's opinion, the ability to move easily between fraction, decimal and percent representation of the same quantity is essential to mastery of percent topics. Writing decimal and percent equivalents for thirds and eighths is hard for most students and requires special attention during instruction. The most likely student errors in fraction-percent conversions occur when the students attempt to reduce the equivalent fraction.

Students seemed to have difficulty interpreting the wording of percent statements. Admittedly, the language of percent is at times awkward and needs to be taught carefully. It seemed to be difficult for the students to distinguish between the wording of case 1 and case 2 percent statements. Students seem uncertain as to when to include the percent symbol as a part of the answer. Students had some difficulty identifying the whole unit in problem statements, a skill essential to mastery of the proportion

method of solving percent problems. Students had some difficulty in selecting the appropriate arithmetic operation (ie. multiplication, not division).

Verbal problems involving percent seem very difficult to the students. Part of this may be due to the fact that students generally have trouble with verbal problems. Another part of this is that the students are not yet comfortable with the basic concepts of percent. There was an observed tendency, on the part of some students, to try to solve verbal problems by applying a single arithmetic operation without regard to the nature of the quantities in the problem.

Percent Lessons

The investigator's extensive pilot work was used in the design of the instructional materials used in this study. The investigator developed a first unit on percent which consisted of seventeen fifty-minute lessons. Two approaches to teaching the solution of case 1 percent problems, the proportion method (P) and the factor product method (F), were conceptualized. Appropriate materials were written for each of the two methods.

A copy of the student materials and teacher lesson plans appear in the appendices to this paper. A summary of the content and sequence of percent lessons follows:

Lesson 1. Meaning of Fractions and Equivalent Fractions. The connections between oral names, written symbols and diagramatic representations of fractions were developed.

Lesson 2. Equivalent Fraction Generalizations. Strategies for generating sets of equivalent fractions were developed. Students were taught how generate equivalent fractions having 100 as the denominator.

Lesson 3. Equivalent Fractions and Cross Products. The term proportion was defined. Students were taught how to use cross products as a means to find equivalent fractions. Opportunities were provided for the students to decide whether to use equivalent fraction generalities or cross products to find equivalent fractions.

Lesson 4. Meaning of Percent. The notion of percent meaning hundredths was introduced by having students solve percent problems using a hundreds grid. Students were taught to associate one hundred percent with the whole unit, and given practice on estimating parts of units using percent terminology.

Lesson 5. Fractions and Percents. The work on equivalent fractions was extended to include percents using region diagrams. The emphasis

was on thinking of percents as alternate ways to express fractions with 100 as the denominator.

Lesson 6. Finding Percents. Students were exposed to a variety of equivalent ways to state percent problems. Students were taught to identify the part and whole in each problem, prior to attempting to solve it.

Lesson 7. Finding Discounts and Mark Ups. Students were taught the meaning of the terms discount and markup. Students were taught how to express the the quantity of change in a number as a percent. Emphasis was on the notion of treating the percent change as a part-whole comparison.

Lesson 8. Decimals for Tenths and Hundredths. The meaning of decimals was presented, using region diagrams, as an alternate way of representing fractions. Students received practice work on associating the oral names, written symbols, and region diagrams for decimals. The meaning of place value was stressed.

Lesson 9. Equivalent Decimals. The equivalence of decimal numbers was developed by comparing region diagrams for numbers represented by

both tenths and hundredths. Students were taught to write tenths as hundredths and hundredths as mixed tenths and hundredths.

Lesson 10. Decimals to Thousandths. The previous lesson's work was extended to include thousandths. Students were taught how to represent tenths and hundredths as thousandths.

Lesson 11. Decimals and Percents. Diagram work with decimals was compared to that used to represent fractions and percents. Given a fraction, decimal or percent, students were asked to write the missing equivalents.

Lesson 12. Fractional Parts of Numbers. Students found fractional parts of numbers using region diagrams.

F: Students were taught to rewrite problems as decimal or fraction multiplications.

P: Students were taught to set up and solve proportions.

Lesson 13. Estimating Percents Using 1%, 10%, 50%, 100%.

F and P: Using appropriate method, students were given practice on computing 1% and 10% of numbers.

Lesson 14. Finding Percents of Numbers.

F: Students were taught to solve case 1 percent problems by rewriting problems as fraction/ decimal multiplications.

P: Students were taught to solve case 1 percent problems by setting up and solving proportions.

Lesson 15. Percent Mark Up and Percent Discount.

F and P: Using appropriate method, students were taught how to compute discounts or markups, given the percent difference and the original price. This work was compared to students' earlier work with discount and markup.

Lesson 16. Finding Percents and Percents of Numbers.

F and P: Using the appropriate method, students were given practice on the solution of percent verbal problems. The emphasis of the instruction was on trying to determine the presence of both the part and the whole in the problem statement.

Lesson 17. Review.

Lesson 18. Test.

Study Design

An author designed pretest was administered to eleven seventh grade mathematics classes during the first week of March, 1982. The students then received about eighteen days of mathematics instruction on percent topics. The first eleven days of instruction were the same for all students. This initial period of instruction was be directed toward using the students' understanding of fractions as a basis for learning to solve case 2 percent problems. During the

remaining days of instruction the students were divided into two groups. Five classes studied the solution of case 1 percent problems using the proportion method. The remaining six classes studied the solution of case 1 percent problems using the factor product or decimal method. Figure 4 graphically depicts the chronology of the study.

The students' week long spring vacation fell during the course of the study. For the Rural students it came after the completion of lesson 8 and in the City school following lesson 17. An unusual April snow fall delayed the administration of the post test until after the City students had returned from their vacation. In both schools, the students were given a day of review following their vacation, before continuing with the program.

Students in both treatments received daily quizzes to evaluate the day-to-day results of the instructional sequences. Quizzes were given at the start of the period on the day following instruction. Following the treatment period all classes were given a test which measured their mastery of percent concepts and a three week retention test. The author interviewed randomly selected individuals from several of the classes after the post test was administered using Lankford's (1972) think aloud technique.

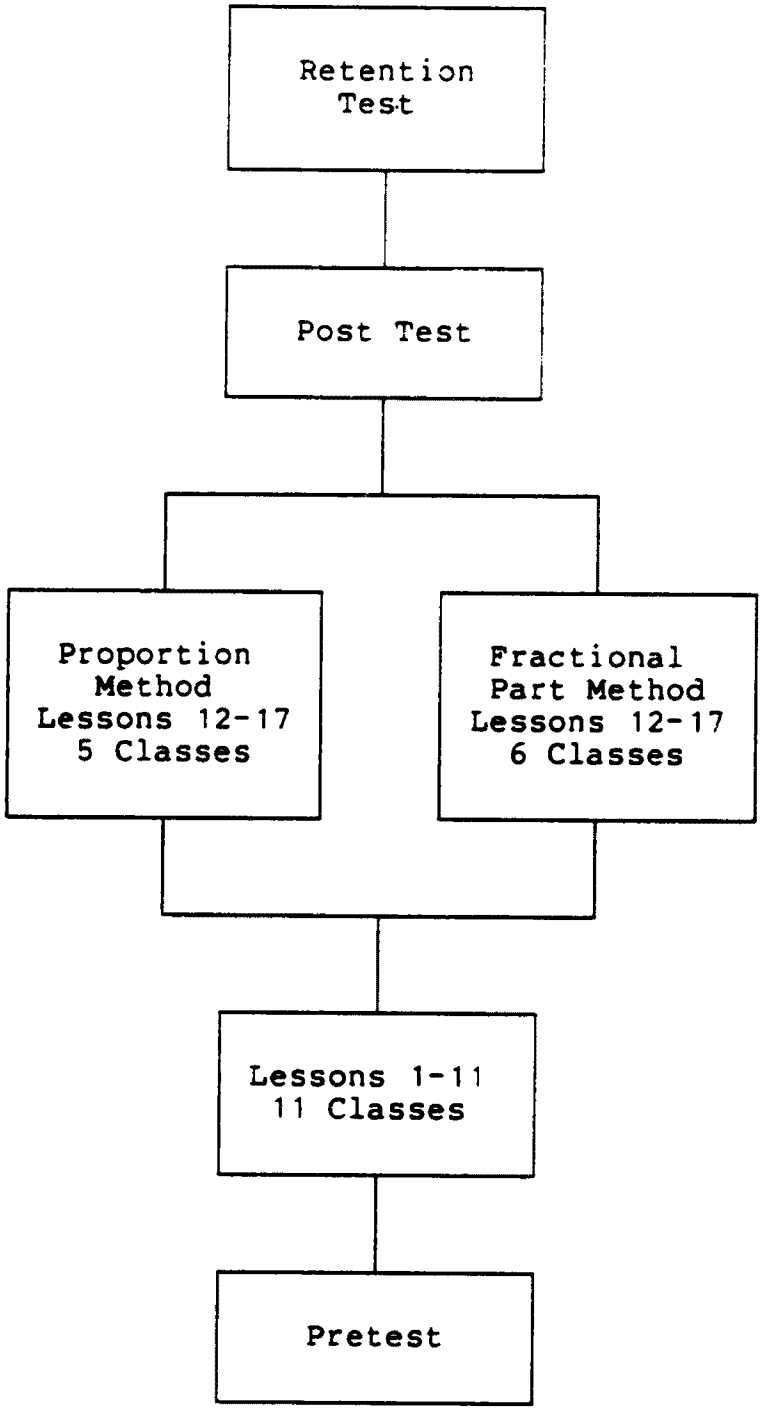


Fig. 4 Study Chronology

The investigator developed a set of fourteen tracking questions which were administered to students at several points in time during the study. The purpose of these test items was to examine changes in student learning over time. A complete set of these questions and where they appeared in the test materials are discussed in the next chapter. These test questions appeared on each of the major tests and on selected quizzes during the study. The same form of the test question was used on each major test and on the first quiz following instruction. On subsequent quizzes parallel items were used.

Instruments

The author used three major tests (a pretest, a post test and a retention test) and sixteen daily quizzes during the course of this study. All test instruments were written by the author and field tested during the author's pilot studies. Copies of the test instruments appear in the appendices.

The pretest was administered to all subjects prior to beginning the instructional treatment. The pretest was designed to be used as a covariate in analyses of covariance, not as a parallel form of the post test. The pretest was designed to measure initial student understanding of fraction, decimal and percent concepts. The pretest was the first point in time at which the tracking questions were administered.

The percent post test was administered to the students at the conclusion of the instructional treatment. The post test had one subscale for each objective in the percent hierarchy. The post test contained five questions taken from two standardized mathematics tests. Three questions were taken from the NEAP (National Assessment for Educational Progress) test. Two questions were taken from the seventh grade assessment test given by the Minnesota State Department of Education. The purpose of including these questions was to get a better indication of the success of the instructional treatments compared to national results. The post test represented the second point in time at which the entire set of tracking questions, in their original form, were administered to the students.

The retention test was a parallel form of the post test and was administered three weeks after the post test. The retention test represented the final point in time at which the tracking questions were administered to the students.

Daily criterion referenced quizzes were administered to students in both sequences. The quizzes covered the previous day's instruction, plus any relevant tracking questions. In general tracking questions appeared only on quizzes following their inclusion in the instructional treatment. Tracking questions appeared on quizzes following lessons: 4, 8 and 11.

Subjects

The students used in this study were enrolled in seventh grade mathematics classes at either a Detroit suburban middle school (City) or a secondary school in northern Michigan (Rural). All seventh graders (N=240) at the City school, except those enrolled in the school's advanced placement mathematics program or those not participating in regular class activities, were included in the study. All seventh graders (N=38) in the Rural school, except for two not regularly working with the rest of the class, were included in the study. The three teachers involved with this study were recruited by the investigator from candidates suggested by local mathematics coordinators. One of the teachers had used some of the investigators's materials previously in one of the pilot studies.

Each of the eleven classes was randomly assigned to either the factor product treatment (F) or the proportion treatment (P). All instruction was performed by the regular class teacher. Treatment assignments were made so that each teacher was teaching approximately the same number of sections of each treatment. Mean MEAP (Michigan Educational Assessment Program) core objective scores are reported for each class in Table 4. There are twenty-eight core objectives on the MEAP seventh grade test. The average number of objectives attained for all Michigan seventh graders (N=130173) was 21.54 (MEAP State Summary 1982). The

students used in the author's study scored slightly better than the state average, this was expected since no remedial mathematics classes were included in his study.

Table 4

Mean MEAP Core Objective Scores
by School, Teacher and Class

Class	Treatment	N	Mean	S.D.
Teacher A (Rural)				
Class 1	F	21	22.5	5.9
Class 2	P	12	21.2	3.7
Teacher B (City)				
Class 1	P	27	23.3	3.3
Class 2	F	28	23.4	3.3
Class 3	F	27	24.2	3.0
Class 4	P	29	23.4	4.0
Teacher C (City)				
Class 1	P	26	23.8	4.1
Class 2	P	28	23.9	3.5
Class 3	F	26	23.8	3.4
Class 4	F	23	24.8	2.0
Class 5	F	14	24.7	1.6

CHAPTER IV

DISCUSSION OF RESULTS

The data analysis for this study had several facets. Student test performance in each of the two treatments was compared. Student errors or percent questions in this study were compared to those reported in the literature by other investigators. Student test performance on selected subscales was examined for causal relationships. The author's postulated learning hierarchy was tested for validity using student performance data from this study.

Total Score Comparisons

Kuder-Richardson formula 20 test reliability coefficients, along with mean test performance by treatment, are reported in Table 5. The test reliabilities tended to vary directly with the number of items on each test. The major tests (Pre, Post and Retention) had reliabilities which were high and generally consistent (about .94). Analyses of variance were conducted and indicated no significant differences between total mean test performance of students assigned to the two treatment groups, on any of the tests or quizzes. Analyses of covariance, using the students' pretest and MEAP scores as covariates, also yielded no significant differences between student Post or Retention test performances in the two treatments. The adjusted mean scores for the post and retention tests are

reported in Table 6. These results were not altogether unexpected, since the students used many of the same instructional materials during the study.

Table 5

Mean Test Scores by Treatment Group

Test			Treatment Group					
			Factor Product			Proportion		
Name	No.of Items	K-R_{20}	N	Mean	S.D.	N	Mean	S.D.
MEAP	28	...	139	23.9	3.6	122	23.4	3.7
Pretest	69	.95	141	23.2	14.0	117	20.5	12.9
Quiz 1	6	.29	133	5.8	.5	118	5.7	.6
Quiz 2	6	.38	134	5.2	.8	120	5.1	1.1
Quiz 3	6	.66	132	4.8	1.4	116	4.7	1.5
Quiz 4	7	.27	131	6.3	.9	116	6.4	.7
Quiz 5	12	.77	131	9.6	2.4	115	9.3	2.5
Quiz 6	6	.74	128	4.9	1.6	114	4.9	1.4
Quiz 7	5	.67	127	3.3	1.5	115	3.4	1.4
Quiz 8	14	.69	127	11.1	2.2	110	11.1	2.2
Quiz 9	8	.67	126	6.5	1.6	110	6.1	1.8
Quiz10	8	.69	128	6.8	1.6	115	6.9	1.5
Quiz11	10	.69	129	8.2	1.8	118	8.0	2.0
Quiz12	23	.84	130	16.0	4.6	118	15.8	4.7
Quiz13	13	.79	128	11.0	2.6	114	10.8	2.3
Quiz14	6	.70	128	4.5	1.6	110	4.6	1.6
Quiz15	5	.45	126	3.4	1.1	111	3.4	1.2
Quiz16	8	.70	130	5.5	2.0	115	5.4	2.2
Post	65	.93	141	50.2	10.6	123	47.9	10.9
Retention	65	.94	140	49.8	10.7	121	48.9	11.9

Table 6

Adjusted Mean Test Scores by Treatment Group

Test	Covariate	Factor Product N	Factor Product Mean	Factor Product Adj. Mean	Proportion N	Proportion Mean	Proportion Adj. Mean
Post	Pretest	138	50.4	49.8	116	47.9	48.6
Post	Pretest,MEAP	133	50.3	49.5	112	47.9	48.8
Retention	Pretest	137	50.0	49.5	111	49.1	49.7
Retention	Pretest,MEAP	132	50.1	49.4	108	48.9	49.8

Subscore Comparisons

To identify specific differences in student learning, between treatments, items on the major tests were grouped into subscales and student scores on these subscales were examined. One subscale was defined for each of the sixteen pretest objectives:

1. Associating fractions with region diagrams (F.Diag)
2. Associating percents with region diagrams (%Diag)
3. Associating decimals with region diagrams (D.Diag)
4. Writing decimals as hundredths (D..N/100)
5. Writing fractions as hundredths (F..N/100)
6. Writing percents as hundredths (%..N/100)
7. Writing fractions as decimals (F..D)
8. Writing percents as decimals (%..D)
9. Writing decimals as fractions (D..F)

10. Writing percents as fractions (%..F)
11. Equivalent fractions (Equiv.F)
12. Writing fractions as percents (F..%)
13. Writing decimals as percents (D..%)
14. Interpreting case 1 percent statements (Case1%)
15. Interpreting case 2 percent statements (Case2%)
16. Percent verbal problems (%Verbal)

Montgomery (1958) reported that the magnitude of the percent used and the case type of the problem affect student performance. This investigator further classified the percent items by case type (1 or 2) and magnitude (easy or hard). Easy percent items were those items involving halves, quarters, fifths, tenths or single digit hundredths without requiring the use of equivalent fractions. A subscale consisting of all verbal problem items, regardless of content, was also defined. The items making up each subscale are reported in Table 7.

A similar classification of test items was conducted for the post and retention tests. One subscale was defined for each of the fourteen objectives on the post and retention tests:

1. Associating percents with region diagrams (%Diag)
2. Simple verbal problems requiring the application of the definition of percent (%Def)
3. Writing fractions as hundredths (F..N/100)
4. Writing decimals as hundredths (D..N/100)
5. Writing percents as decimals (%..D)

Table 7

Pretest Items Included in Each Subscale

Subscale	Items
F.Diag	1,2
%Diag	3,4,7,8
D.Diag	5,6
D..N/100	9-11
F..N/100	12-14
%..N/100	15-17
F..D	18-21,50,52,54
%..D	22-25
D..F	26,28,30,32
%..F	34,36,38,40
Equiv.F	27,29,31,33,35,37,39,41
F..%	42-45,51,53,55
D..%	46-49
Case1%	59-61
Case2%	62-64
%Verbal	67,69
Fraction	1,2,12-14,27,29,31,33,35,37,39,41,56,65,66
Decimal	5,6,9-11,18-21,26,28,30,32,50,52,54,57,58
Percent	3,4,7,8,15-17,22-25,34,36,38,40,42-49,51, 53,55,59-64,67-69
Easy%	3,4,8,15-17,22,23,34,36,38,40,42-47,49,51, 59,61,62,67,68
Hard%	7,24,25,48,53,55,60,63,64,69
Case1	59-61,67
Case2	62-64,69
Verbal	65-69

6. Writing percents as fractions (%..F)
7. Equivalent fractions (Equiv.F)
8. Writing fractions as percents (F..%)
9. Writing decimals as percents (D..%)
10. Writing fractions as decimals (F..D)
11. Interpreting case 1 percent statements (Case1%)
12. Interpreting case 2 percent statements (Case2%)
13. Case 1 verbal problems (Case1.V)
14. Case 2 verbal problems (Case2.V)

The author defined subscales consisting of the: fraction, decimal, percent, case1, case2, verbal, easy% and hard% items. The author defined two subscales as well, consisting of those items on which 80% or more of the students had correct answers (GE.80) and those items on which fewer than 80% of the students had correct (LT.80). The items making up each post (P) and retention (R) test subscale are reported in Table 8.

The mean pretest subscores and subscale reliabilities are reported in Table 9. Analyses of variance were conducted and yielded no significant differences in mean student performance between treatments for any subscales except one. Significant differences ($p<.05$) favoring the students in the factor product treatment were found for the fraction subscale. This further confirmed the comparability of the two populations of students at the beginning of the study.

Table 8

Post and Retention Test Items Included in Each Subscale

Subscale	Test	Items
%Diag	P,R	1-3
%Def	P,R	4-7
F..N/100	P,R	8-10
D..N/100	P,R	11-13
%..D	P,R	14-18
%..F	P,R	19,21,23,25,27
Equiv.F	P,R	20,22,24,26,28
F..%	P,R	29-33,40,42,44
D..%	P,R	34-38
F..D	P,R	39,41,43
Case1%	P,R	45-49
Case1.V	P,R	62-65
Case2.V	P,R	50,57-59
GE.80	P	1-17,19,21,23,25,27,29-31,34-36,40,51,55
	R	1-17,19-21,23-25,27,29-31,34-36,39,40,51
LT.80	P	18,20,22,24,26,28,32,33,37-30,41-50, 52-54,56-65
	R	18,22,26,28,32,33,37,38,41-50,52-65
Fraction	P,R	8-10,20,24,26,28,56
Decimal	P,R	11-13,39,41,43
Percent	P,R	1-7,14-19,21,25,27,29-28,40,42,44-55, 57-65
Easy%	P	1,2,4,5,7,14-16,19,21,23,25,29-31,34-36, 38,40,45-47,49,51,62-65
	R	1,2,4,5,7,14-16,19,21,23,25,29-31,34-36, 38,40,45-48,51,62-65
Hard%	P	3,6,17,18,27,32,33,37,42,44,48,50,52-55, 57-61
	R	3,6,17,18,27,32,33,37,42,44,49,50,52-55, 57-61
Case1	P,R	45-49,62-65
Case2	P,R	5,6,50-55,57-61
Verbal	P,R	4-7,50,56-65

Table 9

Mean Pretest Subscores by Treatment Group

Subscale			Treatment Group			
			Factor Product (N=141)		Proportion (N=117)	
Name	No.of Items	K-R_{20}	Mean	S.D.	Mean	S.D.
F.Diag	2	.63	1.9	.4	1.9	.5
%Diag	4	.58	1.8	1.3	1.8	1.2
D.Diag	2	.04	1.0	.6	.9	.7
D..N/100	3	.75	1.3	1.3	1.1	1.2
F..N/100	3	.79	1.0	1.2	.8	1.2
%..N/100	3	.88	1.7	1.4	1.4	1.4
F..D	7	.81	2.0	2.2	1.7	1.9
%..D	4	.70	1.0	1.2	1.0	1.3
D..F	4	.90	1.6	1.8	1.2	1.7
%..F	4	.92	1.6	1.8	1.2	1.6
Equiv.F	8	.86	1.9	2.5	1.3	2.1
F..%	7	.74	2.0	1.9	1.8	1.7
D..%	4	.64	1.1	1.2	.9	1.1
Case1%	3	.41	.2	.5	.2	.5
Case2%	3	.40	.3	.6	.3	.6
%Verbal	2	.11	.5	.6	.4	.6
Fraction *	16	.85	6.3	3.9	5.3	3.4
Decimal	18	.87	6.3	4.6	5.6	4.2
Percent	35	.90	10.6	7.0	9.6	6.7
Easy%	25	.89	9.7	6.2	8.8	5.9
Hard%	10	.63	.9	1.4	.9	1.4
Case1	4	.36	.6	.8	.6	.8
Case2	4	.44	.4	.7	.4	.8
Verbal	5	.56	1.9	1.4	1.7	1.4

* $p<.05$

Mean post subscores and subscale reliabilities are reported in Table 10. Significant differences ($p<.05$) favoring students in the factor product treatment were found on four subscales: %..D, F..%, D..% and Hard%. The difference between mean scores on the F..% subscale is likely to have been the result of the slightly superior initial fraction skills (as measured by the pretest) of the factor product students. A possible explanation for the differences on the other subscales is the fact that the factor product students received slightly more practice on decimal percent conversions, by the nature of that method. A nearly significant result ($p=.0503$) favoring the factor product students was found on the percent subscale. This is also likely to be related to initial differences in fraction skills.

Mean retention test subscores and subscale reliabilities are reported in Table 11. The only significant difference maintained from the post test was on the %..D subscale ($p<.05$). This difference and the significant difference ($p<.05$) favoring the factor product treatment on the decimal subscale may be related to the additional work with decimals that was inherent in the factor product approach.

Item Analyses

Item analyses performed by treatment group. Factor Product (F) or Proportion (P), showing the percent of students getting each pretest item correct are contained in

Table 10

Mean Post Test Subscores by Treatment Group

Subscale				Treatment Group			
				Factor Product (N=141)		Proportion (N=123)	
Name		No.of Items	K-R_{20}	Mean	S.D.	Mean	S.D.
%Diag		3	.51	2.7	.6	2.7	.7
%Def		4	.45	3.5	.9	3.5	.7
F..N/100		3	.48	2.6	.7	2.7	.7
D..N/100		3	.32	2.8	.6	2.7	.5
%..D	*	5	.57	4.6	.7	4.2	1.1
%..F		5	.07	4.8	.4	4.7	.5
Equiv.F		5	.65	3.4	1.4	3.2	1.5
F..%	*	8	.74	6.6	1.7	6.1	2.0
D..%	*	5	.64	4.2	1.1	3.9	1.3
F..D		3	.66	2.0	1.1	1.8	1.2
Case1%		5	.74	3.3	1.6	3.2	1.8
Case2%		5	.68	3.6	1.5	3.6	1.5
Case1.V		4	.47	2.4	1.3	2.2	1.3
Case2.V		4	.52	2.2	1.3	2.1	1.2
GE.80		31	.84	28.5	3.5	27.9	3.7
LT.80		34	.91	21.7	7.8	20.0	7.9
Fraction		9	.71	6.7	2.0	6.6	2.1
Decimal		6	.66	4.7	1.4	4.6	1.4
Percent		50	.91	38.8	8.1	36.7	8.5
Easy%		29	.83	24.7	3.8	23.9	4.2
Hard%	*	21	.86	14.1	4.8	12.9	4.7
Case1		9	.76	5.7	2.6	5.4	2.6
Case2		13	.80	8.4	3.1	7.9	3.0
Verbal		15	.76	9.6	3.3	9.0	3.9

* $p<.05$

Table 11

Mean Retention Test Subscores by Treatment Group

Subscale				Treatment Group			
				Factor Product (N=140)		Proportion (N=121)	
Name		No.of Items	K-R_{20}	Mean	S.D.	Mean	S.D.
%Diag		3	.52	2.8	.5	2.8	.6
%Def		4	.64	3.6	.8	3.5	1.0
F..N/100		3	.62	2.7	.8	2.6	.8
D..N/100		3	.36	2.8	.5	2.7	.6
%..D	*	5	.57	4.6	.8	4.2	1.1
%..F		5	.48	4.8	.6	4.7	.7
Equiv.F		5	.64	3.8	1.3	3.9	1.4
F..%		8	.73	6.5	1.6	6.2	2.0
D..%		5	.71	4.0	1.3	4.0	1.3
F..D		3	.47	2.1	1.0	1.9	1.0
Case1%		5	.81	3.0	1.9	2.7	2.0
Case2%		5	.80	3.0	1.9	3.4	1.7
Case1.V		4	.54	2.5	1.3	2.3	1.3
Case2.V		4	.59	1.9	1.3	2.0	1.4
GE.80		33	.86	30.2	3.6	29.7	4.5
LT.80		32	.92	19.6	8.0	19.3	8.3
Fraction		9	.71	7.2	1.9	7.2	2.0
Decimal	*	6	.52	4.9	1.1	4.6	1.4
Percent		50	.93	37.7	8.5	37.1	9.4
Easy%		29	.86	24.3	4.0	23.8	4.7
Hard%		21	.88	13.4	5.0	13.3	5.2
Case1		9	.82	5.5	2.7	5.1	3.0
Case2		13	.86	7.6	3.7	8.3	3.5
Verbal		15	.80	9.6	3.3	9.7	3.6

* $p<.05$

Table 12. Substantially different student performances were found on six of the items. Performance differences favoring the factor product students were found for items: 15, 17, 20, 33, 34. These items dealt with associating easy percents with common fraction hundredths. Performance on item 58 tended to favor students in the proportion treatment. Item 58 was a straight forward decimal multiplication problem.

On only two pretest items (1 and 2) did 80% of the students in either treatment have correct answers. These items dealt with associating fractions with region diagrams, a relatively simple task for seventh graders. The pretest items on which the fewest number of students gave correct answer dealt with writing percents for thirds (item 53, 4% and 3%) or eighths (item 55, 3% both groups) or interpreting percent statements (item 60, 1% and 4%; item 61, 1% and 3%; item 63, 3% both groups). This item analysis provides good evidence for the students lack of familiarity with the instructional materials used in this study.

An item analysis for the post test appears in Tables 13 and 14. Student performance on each test item was examined to identify large differences between treatments. One difference favoring the proportion group was found for item 7. This item was solvable by subtraction from 100% and student in the factor product group made more computational errors. Differences favoring the factor product group were found for items: 15, 17, 27, 31, 37, 38, 42, 60, 64. Four

Table 12

Pretest Percent of Correct Responses by Treatment Group

Item	F		P		Item	F		P	
	N	%	N	%		N	%	N	%
1	131	93	108	92	36	57	40	38	32
2	133	94	110	94	37	48	34	29	25
3	70	50	63	54	38	62	44	44	38
4	44	31	39	33	39	37	26	21	18
5	104	74	82	70	40	58	41	38	32
6	37	26	28	24	41	24	17	16	14
7	28	20	17	14	42	58	41	46	39
8	105	74	91	78	43	80	57	64	55
9	44	31	29	25	44	30	21	25	21
10	67	48	47	40	45	56	40	40	34
11	76	54	56	48	46	73	52	50	43
12	53	38	32	27	47	33	23	19	16
13	44	31	27	23	48	15	11	6	5
14	47	33	35	30	49	41	29	27	23
15	74	52	48	41	50	37	26	28	24
16	82	58	60	51	51	43	30	34	29
17	86	61	56	48	52	15	11	10	8
18	77	55	60	51	53	6	4	4	3
19	53	38	40	34	54	16	11	8	7
20	53	38	30	26	55	4	3	3	3
21	37	26	28	24	56	77	55	56	48
22	63	45	51	44	57	6	4	8	7
23	33	23	25	21	58	51	36	57	49
24	28	20	29	25	59	21	15	17	14
25	15	11	16	14	60	2	1	5	4
26	56	40	38	32	61	2	1	3	3
27	37	26	24	20	62	29	21	24	20
28	59	42	42	36	63	4	3	4	3
29	29	21	17	14	64	8	6	9	8
30	48	34	30	26	65	87	62	60	51
31	40	28	25	21	66	46	33	34	29
32	57	40	35	30	67	53	38	40	34
33	29	21	14	12	68	70	50	59	50
34	42	30	23	20	69	16	11	8	7
35	21	15	9	8					

Table 13

Post Test Items 1 to 33 Percent of Correct Responses by Treatment Group

Item	F		P	
	N	%	N	%
1. Diagram 70%	139	99	116	94
2. Diagram 25%	126	89	113	92
3. Diagram 7%	115	82	102	83
4. Easy % verbal	129	92	106	86
5. Easy % verbal	127	90	108	88
6. Easy % verbal	123	87	103	84
7. Easy % verbal	120	85	119	97
8. 2/5=N/100	132	94	113	92
9. 3/4=N/100	126	89	116	94
10. 3/3=N/100	110	78	102	83
11. 0.3=N/100	121	86	110	89
12. 0.07=N/100	129	92	108	88
13. 0.97=N/100	138	98	120	98
14. 47% as decimal	141	100	120	98
15. 9% as decimal	127	90	97	79
16. 50% as decimal	140	99	121	98
17. 23.7% as decimal	130	92	95	77
18. (62 1/2)% as decimal	108	77	89	72
19. 59% as fraction	140	99	121	98
20. Reduce answer 19	116	82	94	76
21. 6% as fraction	132	94	118	96
22. Reduce answer 21	106	75	92	75
23. 40% as fraction	141	100	123	100
24. Reduce answer 23	106	75	83	68
25. 75% as fraction	141	100	123	100
26. Reduce answer 25	104	74	93	76
27. (66 2/3)% as fraction	126	89	94	76
28. Reduce answer 27	41	29	32	26
29. 1/5 as %	134	95	112	91
30. 7/10 as %	135	96	112	91
31. 7/100 as %	138	98	114	93
32. 3/8 as %	93	66	72	58
33. 2/3 as %	93	66	78	63

Table 14

Post Test Items 34 to 65 Percent of Correct Responses by Treatment Group

Item	F		P	
	N	%	N	%
34. 0.13 as %	135	96	118	96
35. 0.6 as %	125	89	103	84
36. 0.03 as %	129	92	110	89
37. 0.837 as %	108	77	77	63
38. 1.0 as %	98	70	66	54
39. 1/4 as decimal	113	80	93	76
40. 1/4 as %	124	88	101	82
41. 1/8 as decimal	82	58	63	51
42. 1/8 as %	106	75	76	62
43. 1/3 as decimal	85	60	71	58
44. 1/3 as %	108	77	88	72
45. 1% of 630	106	75	93	76
46. 12% of 60	101	72	78	63
47. 4% of 86	87	62	75	61
48. (33 1/3)% of 90	74	52	57	46
49. 20% of 80	101	72	90	73
50. Case 2 verbal	91	64	83	68
51. ?% of 10 is 6	125	89	106	86
52. ?% of 300 is 9	101	72	93	76
53. ?% of 32 is 8	100	71	82	67
54. ?% of 80 is 50	71	50	56	46
55. 15 is ?% of 20	114	81	10	83
56. Fraction verbal	104	74	82	67
57. Case 2 verbal	95	67	83	68
58. Case 2 verbal	29	21	19	15
59. Case 2 verbal	92	65	68	55
60. % markup	50	36	24	20
61. % discount	60	43	43	35
62. Case 1 verbal	97	69	90	73
63. Case 1 verbal	77	55	59	48
64. Case 1 verbal	96	68	70	57
65. Case 1 verbal	67	48	54	44

of these items (15, 17, 31, 37) dealt with associating decimals and percents, the common errors in both treatments were incorrectly placing the decimal point and omitting the percent symbol. Item 27 dealt with writing a fraction for (66 2/3)% while item 42 requested students to write 1/8 as a percent. Item 60 was a percent markup question. These items were difficult for the students and no systematic errors were apparent.

The items on which fewer than 80% of the students showed correct solutions dealt with equivalent fractions, writing percents for thirds and eighths or discriminating between case 1 or 2 percent problems. Perhaps, if more work on equivalent fractions were done prior to beginning the study of percent, student performance on these items would be improved. Responses during the student interviews indicated that students are not comfortable with writing division problem remainders as fractions, this may be part of the reason that thirds and eighths give them so many problems. During the student interviews it seemed that to be able to solve a case 1 or 2 percent problem, the student had to both set up the correct equations needed to solve the problem and ascertain whether a percent symbol needed to be included with the answer.

For both treatments the most difficult items, based on the number of student errors, were: item 28, 29% and 26%; item 58, 21% and 15%; item 60, 36% and 20%. The most common errors on item 28 were not answering the question or

simplifying incompletely (writing 33.33/50 rather than 2/3). On item 58 the students were to find the percent of the whole based on a fractional part of the whole which was not given explicitly in the problem. The word "not" may have caused some confusion, as the most common wrong answers were generated by taking the part given in the problem, rather than the part not given in the problem. Similar errors were observed during the student interviews when students were asked to find the percent correct, given the total number of items and the number missed, the typical wrong answer was the percent missed. Item 60 as mentioned earlier was a percent markup question with no systematic errors. Students seem to want to solve percent increase problems by applying a simple arithmetic operation to whatever numbers happen to be present in the problem.

The item analysis for the retention test appears in Table 15. Substantial treatment differences in student performance were found for twelve items. Differences favoring the proportion students were found for items: 51, 52, 60. Items 51 and 52 dealt with the solution of case 2 percent problems. These differences seem to have arisen out of drops in student performance for the factor product students. The students in the factor product treatment seemed more likely to have failed to identify the correct whole to use as the denominator in solving these items. The students in the proportion treatment, by the nature of the method, received more practice using cross products than the

Table 15

Retention Test Percent of Correct Responses by Treatment Group

Item	F		P		Item	F		P	
	N	%	N	%		N	%	N	%
1	138	99	118	98	34	133	95	117	97
2	132	94	115	95	35	122	87	103	85
3	128	91	107	88	36	124	89	112	93
4	128	91	107	88	37	95	68	78	64
5	121	86	107	88	38	91	65	78	64
6	125	89	105	87	39	112	80	99	82
7	126	90	108	89	40	127	91	107	88
8	131	94	105	87	41	84	60	67	55
9	125	89	110	91	42	100	71	80	66
10	115	82	103	85	43	100	71	67	55
11	123	88	97	80	44	115	82	86	71
12	130	93	111	92	45	90	64	83	69
13	139	99	116	96	46	94	67	68	56
14	138	99	116	96	47	89	64	65	54
15	132	94	104	86	48	89	64	73	60
16	137	98	115	95	49	64	46	43	36
17	126	90	92	76	50	78	56	80	66
18	110	79	80	66	51	108	77	106	88
19	139	99	119	98	52	79	56	87	72
20	128	91	107	88	53	82	59	78	64
21	125	89	113	93	54	65	46	59	49
22	105	75	96	79	55	92	66	83	69
23	139	99	119	98	56	102	73	88	73
24	115	82	97	80	57	36	26	41	34
25	138	99	119	98	58	58	41	53	44
26	106	76	99	82	59	99	71	71	59
27	127	91	104	86	60	61	44	70	58
28	80	57	67	55	61	61	44	62	51
29	131	94	107	88	62	104	74	86	71
30	133	95	113	93	63	63	45	51	42
31	134	96	111	92	64	95	68	66	54
32	83	59	75	62	65	83	59	77	64
33	85	61	76	63					

students in the factor product treatment and one would expect them to be better at solving case 2 problems. This was not true on the post test, so it may be the case that the proportion students had better retention of this skill than the factor product students. Item 60 was a percent discount problem, again the factor product students seemed to have more difficulties in identifying the correct whole than did the students in the proportion treatment. Differences favoring the factor product group were found for items: 8, 15, 17, 18, 43, 44, 46, 59, 64. Most of these items dealt with associating fractions or decimals with percents. This finding is consistent with the other findings reported for this study. The differences on initial fraction skills and extra decimal work may have had long term effects on student performance. Items 46 and 64 dealt with the solution of case 1 percent problems. The common errors made by the factor product students seemed to arise from computational errors, while no systematic errors could be detected for the proportion students. Item 59 was a case 2 problem in which the whole was divided into three unequal sets, the proportion students did not always correctly compute (or identify) the whole to be used as the denominator.

Many of the items on which less than 80% of the students had correct answers received student responses which indicated that students could have made better use of their estimation skills. These items were reported as being

difficult on the post test earlier in this paper. The most difficult items for students in the proportion treatment were: item 49 (36%), item 57 (34%), item 58 (44%), item 63 (42%). The most difficult items for students in the factor product treatment were: item 57 (26%), item 58 (41%), item 60 (44%), item 61 (44%). Items 57 and 58 dealt with the solution of case 2 verbal problems. There seemed to be some confusion for students in both treatments with respect to identifying the fractional part to be considered. It should be noted that on the post test item 58 contained the word "not" while on the retention test item 57 contained the word "not". All other aspects of these two problems was the same on the two tests. Student errors on these problems may have arisen from the students recording their post test answer without reading the retention question carefully. As mentioned earlier, student interview data suggested that students often ignored the logic implied by the word "not" in formulating a problem's solution. Item 49 was concerned with the solution of a case 1 percent problem, the common error seemed to have been omitting the problem or misplacing the decimal point. Items 60 and 61 dealt with percent discount and markup. The most common error on these items was a failure to identify the whole and attempting to solve the problem using a single arithmetic operation. Item 63 was a case 1 verbal problem and the most common wrong answers was incorrect multiplication and decimal point misplacement (writing 252, rather than 2520).

Tracking Questions

Table 16 displays the test and quiz items which made up the author's set of tracking questions. The original form of each question appeared on the three major tests and on the first quiz on which the students saw the question. Parallel forms of the question were used on subsequent quizzes.

Table 16

Tracking Question Item Placement

Question	Test Item Number					
	Pretest	Quiz4	Quiz8	Quiz12	Post	Retention
1	66	7	13	8	56	56
2	7		7	10	3	3
3	68		14	9	7	7
4a	40		8	14	23	23
4b	41		9	15	24	24
5a	34		10	16	21	21
5b	35		11	17	22	22
6	63		12	7	53	53
7	47			12	35	35
8	48			13	37	37
9	25			11	17	17
10a	50			18	39	39
10b	51			19	40	40
11a	52			20	41	41
11b	53			21	42	42
12a	54			22	43	43
12b	55			23	44	44
13	60				46	46
14	67				65	65

Figure 5 depicts the typical pattern of student performance across time on the tracking questions. The students tended to exhibit steady growth on most questions. Performance was similar for students in the two treatments.

Table 17 shows the percent of students giving correct answers to each question by treatment group, Factor Product (F) and Proportion (P). In general the slight advantage displayed by the factor product students on the pretest was continued throughout the study, but there were a few reversals. In most cases the difference between treatments is 5 percentage points or less. No tests of significance were attempted. The placement and generally small magnitude of the declines in student performance can probably be explained by the recency of student practice. The small decline in student performance between the post and retention tests was most likely due to normal forgetting patterns.

There were exceptions to the performance pattern described above. The peak performance for question 1 occurs on quiz 12. Question 1 was a fraction verbal problem, and practice work on these types of problems would have been discontinued about that time in the percent lessons. Question 3 shows a decline between quizzes 8 and 12. This may be a reflection of the fact that student performance on quiz 12 was generally lower than on the other quizzes. One reason for this may have been that the students were growing tired of the daily quiz routine. A second possibility is

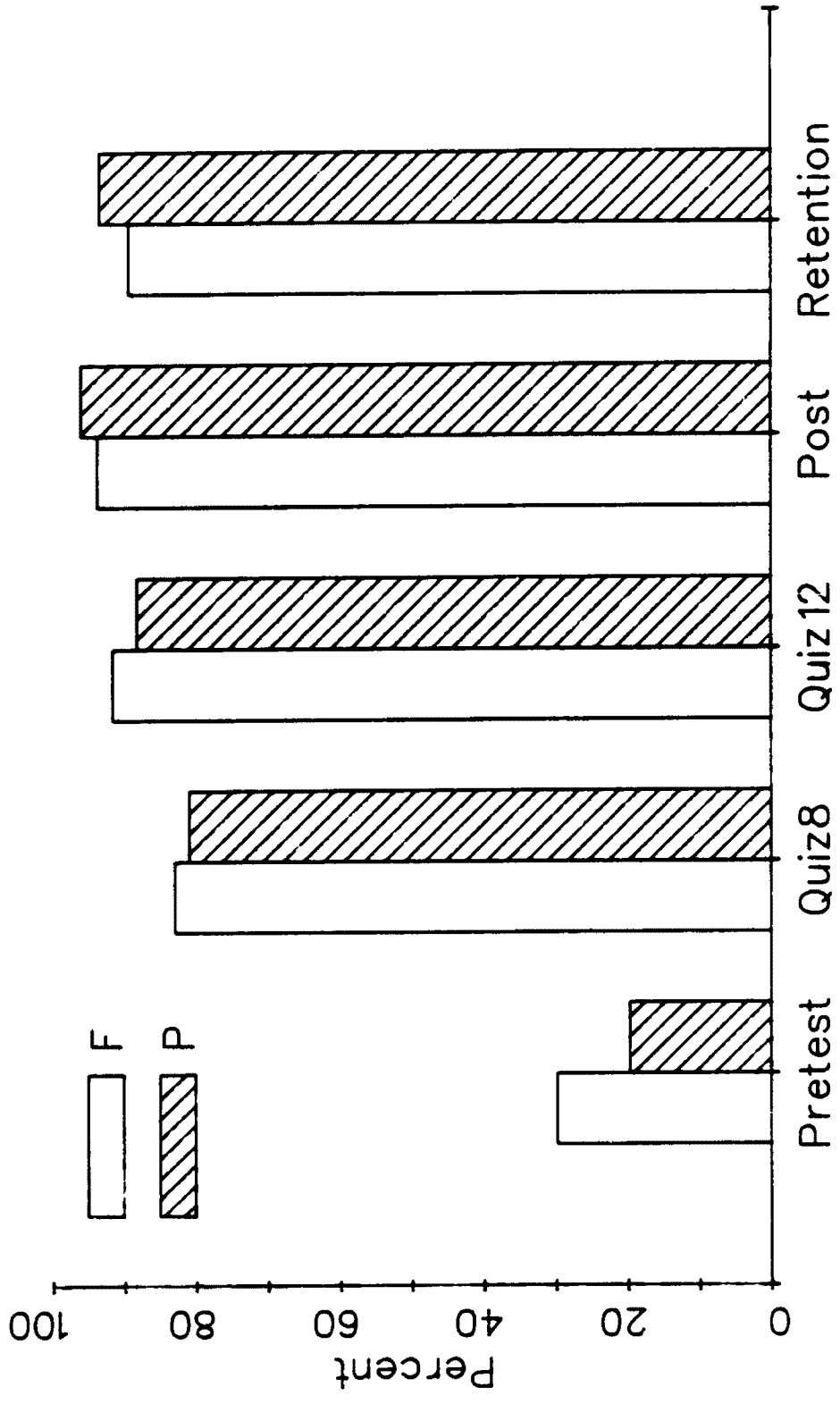


Fig. 5 Student Performance on Tracking Question 5a

Table 17

Percent of Correct Responses to Tracking Questions by Treatment Group

Question	Question Administration											
	Pretest		Quiz4		Quiz8		Quiz12		Post		Retention	
N=	F 141	P 117	F 131	P 116	F 127	P 110	F 130	P 118	F 141	P 123	F 140	P 121
1	32.6	29.1	61.1	56.6	73.2	80.0	84.6	87.3	73.8	66.7	72.9	72.9
2	19.9	14.5			23.6	22.7	76.9	81.4	81.6	82.9	91.4	88.4
3	49.6	50.4			85.0	85.5	77.7	71.2	85.1	96.7	90.0	89.3
4a	41.1	32.5			94.5	97.3	97.7	96.6	100.0	100.0	99.3	98.3
4b	17.0	13.7			57.5	60.9	64.6	72.0	75.2	67.5	82.1	80.2
5a	29.8	19.7			82.9	80.9	91.5	88.1	93.6	95.9	89.3	93.4
5b	14.9	7.7			55.9	60.0	66.9	60.2	75.2	74.8	75.0	79.3
6	2.8	3.4			63.0	57.3	32.2	39.8	70.9	66.7	58.6	64.5
7	23.4	16.2					81.5	74.6	88.7	83.7	87.1	85.1
8	10.6	5.1					65.4	61.0	76.6	62.6	67.9	64.5
9	10.6	13.7					73.8	63.6	92.2	77.2	90.0	76.0
10a	26.2	23.9					70.8	76.3	80.1	75.6	80.0	81.8
10b	30.5	29.1					83.8	83.1	87.9	82.1	90.7	88.4
11a	10.6	8.5					40.0	28.0	58.2	51.2	60.0	55.4
11b	4.3	3.4					70.8	71.2	75.2	61.8	71.4	66.1
12a	11.3	6.8					35.4	16.1	60.3	57.7	71.4	55.4
12b	2.8	2.6					75.4	72.9	76.6	71.5	82.1	71.1
13	1.4	4.3							71.6	63.4	67.1	56.2
14	37.6	34.2							47.5	43.9	59.3	63.6

that the decimal work covered in the intervening lessons may have caused some interference. Question 4b requested the students to write a fraction in simplest terms for 40%. The nature of the proportional method doesn't place much emphasis on reducing fractions, so a slight decline for this treatment group is not surprising. Question 6 showed the largest drop in student performance between quizzes 8 and 12. Lesson 12 is the first lesson in which case 1 percent problems were presented to the students. Question 6 dealt with the solution of a case 2 problem. It has been the author's experience that discrimination between case 1 and case 2 problems is quite hard for students. This case 1 interference coupled with the generally low performance on quiz 12 might account for this decline. Question 11b asked students to write a percent for the fraction 1/8.

Comparisons to Other Studies

The post test contained three items which were included in the NEAP thirteen year old test battery and two items used in the Minnesota seventh grade mathematics assessment test. Student performance on the items is summarized in Table 18. Inspection of this Table reveals that students in both treatment groups tended to perform substantially higher than those in either of the original test populations. No tests of significance were attempted.

Table 19 contains student performance data, by treatment group, for each of the objectives tested by the major tests. Chi-square analyses revealed few statistically

Table 18

Percent Correct on Selected NEAP and Minnesota State Assessment Test Questions by Test Sample

Post Test Item	Test Population			
	Minn Grade 7	NEAP Age 13	Factor Product	Proportion
29		41.0	95.0	91.1
49	35.7		71.6	73.2
55	64.3		80.9	82.9
56		20.0	73.8	66.7
63		10.0	54.6	48.0

significant differences between groups, with regard to the number of students attaining the criterion level on each objective. Those differences which were found are potentially explainable by the greater initial knowledge of equivalent fractions possessed by and the greater practice on decimals received by the factor product group.

An examination of Table 19 reveals that student performance increased quite dramatically following instruction. Student performance on the %Diag, %..F, %..D and D..% objectives was a bit higher than that reported by Edwards (1930) and Brueckner (1930), while student performance on F..% is a bit lower. This may be explained by the fact that students received instruction immediately prior to the author's test, where no instruction was given

Table 19

Percent of Students Attaining Criterion Level on Each of the Test Objectives by Treatment Group

Objective	Pretest					Post Test					Retention Test				
	Criterion		F		P	Criterion		F		P	Criterion		F		P
			N	%	N %			N	%	N %			N	%	N %
F.Diag	1.00		127	90	108 92										
D.Diag	1.00		27	19	22 19										
%Diag	.75		44	31	37 32	.67		135	96	115 94	.67		135	96	116 96
%Def						.75		127	90	110 89	.75		129	92	107 88
F..N/100	.67		47	33	31 27	.67		128	91	116 94	.67		129	92	112 93
D..N/100	.67		68	48	50 43	.67		135	96	121 98	.67		137	98	115 95
%..N/100	.67	p=.0379	83	59	55 47										
Equiv.F	.75		18	13	9 8	.80		86	61	65 53	.80		98	70	89 74
F..D	.71		24	17	13 11	.67		92	65	74 60	.67		105	75	80 66
D..F	.75		49	35	36 31										
%..F	.75		56	40	36 31	.80		141	100	120 98	.80		138	99	117 97
F..%	.71		13	9	11 9	.75		110	78	88 72	.75		105	75	88 73
D..%	.75		24	17	12 10	.80	p=.0064	114	81	82 67	.80		100	74	84 69
%..D	.75		25	18	20 17	.80	p=.0014	127	90	93 76	.80	p=.0003	129	92	92 76
Case1%	.67		2	1	5 4	.80		83	59	68 55	.80		76	54	59 49
Case2%	.67		8	6	7 6	.80		89	63	74 60	.80		73	52	70 58
%Verbal	.67		9	6	4 3										
Case1.V						.75		73	52	59 48	.75		77	55	59 49
Case2.V						.75		65	46	45 37	.75		51	36	48 40

as a part of any error studies discussed in the author's literature review chapter. This is tempered, however, by the fact during the 20's a great deal of time was spent on percent as part of the middle school curriculum.

The author's data contradicts the often heard claim that case 1 problems are easier for students than case 2 problems (Brueckner, 1930 and Montgomery, 1958). This is especially significant because the author's case 2 questions were likely to contain more difficult percents than his case 1 problems. It seems likely that students will learn best what they practice the most. Traditionally the solution of case 1 problems is taught first. In the author's study case 2 problems were taught first and received greater emphasis due to the nature of his approach.

The author's data, both test (%Def) and interview, did not support Guiller's (1946) finding of a general lack of ability to apply the notion that percent means hundredths. Students in the author's study would solve a problem like item 5 on the post test by reasoning that 4 (the part) goes into 100% (the whole) 25 times so that 1/4 is equivalent to 25%. Montgomery's (1958) finding that students had more difficulty with diagrams than interpreting fraction-percent equations was not supported by the author's data. This is probably due to the fact that students in the author's study were provided with early and substantial practice on associating percents with region diagrams, so that the problems were not novel situations to the students as they

were in Montgomery's study. Analyses of student errors indicated that the magnitude of the percent has some effect on student performance, as Montgomery reported. As discussed earlier, thirds and eighths are difficult as are fractional percents.

The students in the author's study seemed to have some of the problems in manipulating the decimal point that were reported by Guiller (1946) and Brueckner (1930). More work on the meaning of decimal numbers and decimal operations may need to be done prior to studying percent to avoid these difficulties. The tendency of students in the author's study to simply multiply or divide numbers given in verbal problem was similar to that reported by Montgomery (1958) and Edwards (1930). More work on identifying the whole and more practice on discrimination between case 1 and case 2 percent problems may be needed to reduce this arbitrary behavior. This blind application of rules seemed most likely to occur on problems involving percent increase or decrease. Problems of this type probably should only be introduced at the seventh grade level, without expecting mastery.

Cross-Lagged Correlation Analysis

Cross-lagged panel correlation analysis was used to identify causal relationships among the fraction (F), decimal (D) and percent (%) subscales on the major tests used in this study. Because each subscale was derived from a single test administration the assumption of synchronicity

appears reasonable. To test the assumption of stationarity the synchronous correlations were tested using Pearson-Filon z tests for independent correlations (Steiger, 1980). Because the differences in synchronous correlations were much greater for decimal and fraction subscales, pairwise rather than simultaneous tests for equality were conducted. All synchronous correlations which were not statistically equal were corrected for attenuation and the corresponding cross-lagged correlations were corrected for reliability (Kenny, 1975). The correlation coefficients among the various subscales are reported in Table 20.

Examining the cross-lagged correlations it appears that student's knowledge of fractions and decimals, as measured by the pretest, is more likely to affect their performance on the post and retention test percent subscales, than conversely. This effect is reversed for the analysis of the post and retention test subscales, although the decimal-percent cross-lagged difference is not statistically significant. This suggests the existence of a positive feedback loop, in which the students' knowledge of fractions and decimals assists their learning of percent skills which in turn aids their learning of fraction topics. It may be desirable to teach fraction, decimal and percent topics in a spiral manner. The results of these analyses also support Wilson's (1972) finding that the presentation order within the curriculum, of fractions and decimals, has no effect on student achievement.

Table 20

Cross-Lagged Correlations Among the Fraction, Decimal and Percent Subscores

N=245			Correlation Coefficients						z	Corrected z
			Cross-Lagged		Stability		Synchronous			
$Time_1$	$Time_2$	A B	A_1B_2	B_1A_2	A_1A_2	B_1B_2	A_1B_1	A_2B_2		
Pre	Post	F %	.55	.39	.45	.50	.75	.67	2.99 **	
		D %	.48	.34	.37	.50	.67	.63	2.33 *	
		F D	.38	.38	.45	.37	.77	.54	.00	-.32
Pre	Ret	F %	.46	.31	.38	.44	.75	.68	2.64 **	
		D %	.43	.30	.32	.44	.67	.63	2.10 *	
		F D	.36	.26	.38	.32	.77	.51	1.53	1.36
Post	Ret	F %	.56	.68	.65	.83	.67	.68	-2.71 **	
		D %	.51	.58	.53	.83	.63	.63	-1.35	
		F D	.45	.45	.65	.53	.54	.51	.00	

* $p<.05$

** $p<.01$

Separate analyses were not conducted for each treatment group since sample sizes less than 150 are not considered large enough to use in cross-lagged panel analysis. One needs to be cautious in generalizing the results of these analysis, because the subscale reliabilities involved should be less variable than those reported in this study to accurately assess the assumption of equal synchronous correlations. To have greater confidence in the statistical results of these analyses they should be replicated with larger samples, multiple sample groups and more reliable scores.

Hierarchy Analyses

Results of the author's hierarchical analyses for students in both treatment groups are reported in Table 21 and are depicted graphically in Figure 6. The test data from the post test supported eight of the author's thirteen postulated relationships, while the retention test data supported seven of the relationships. In all cases the consistency ratios were high, indicating that the relative placement of the objectives within the sequence was correct with regard to item difficulty. For all dependencies, except Case2% -> Case2.V the completeness ratios indicated that a majority of the students were capable of traversing the hierarchy. Where a relationship failed to be supported, the cause seemed to be a low adequacy ratio. A low adequacy ratio indicates that more skills are required to make the transition between the subordinate and terminal objective.

Table 21

Walbesser Ratios for Post and Retention Test Dependencies Both Treatment Groups Combined

Objective		Test					
		Post (N=264)			Retention (N=261)		
Subordinate	Terminal	Consistency	Adequacy	Completeness	Consistency	Adequacy	Completeness
%Diag	%Def	.96	.91	.98 **	.97	.91	.99 **
%Def	F..N/100	.93	.96	.96 **	.94	.96	.96 **
%Def	D..N/100	.91	.98	.99 **	.91	.97	.99 **
F..N/100	%..F	.93	1.00	.99 **	.93	.98	.99 **
F..N/100	F..%	.98	.80	.92 **	.98	.78	.92 *
D..N/100	%..D	.98	.84	.98 *	.98	.86	.98 **
D..N/100	D..%	1.00	.77	.96 *	.99	.73	.96 *
%..F	Case2%	.99	.62	.99	.99	.56	.97
F..%	Case2%	.88	.72	.76 *	.89	.66	.71
%..D	Case1%	.91	.63	.82	.92	.56	.81
D..%	Case1%	.82	.63	.75	.84	.60	.68
Case2%	Case2.V	.87	.59	.52	.89	.62	.45
Case1%	Case1.V	.80	.70	.55	.71	.71	.53

* valid at lower level (.85, .70, .50)

** valid at higher level (.85, .85, .85)

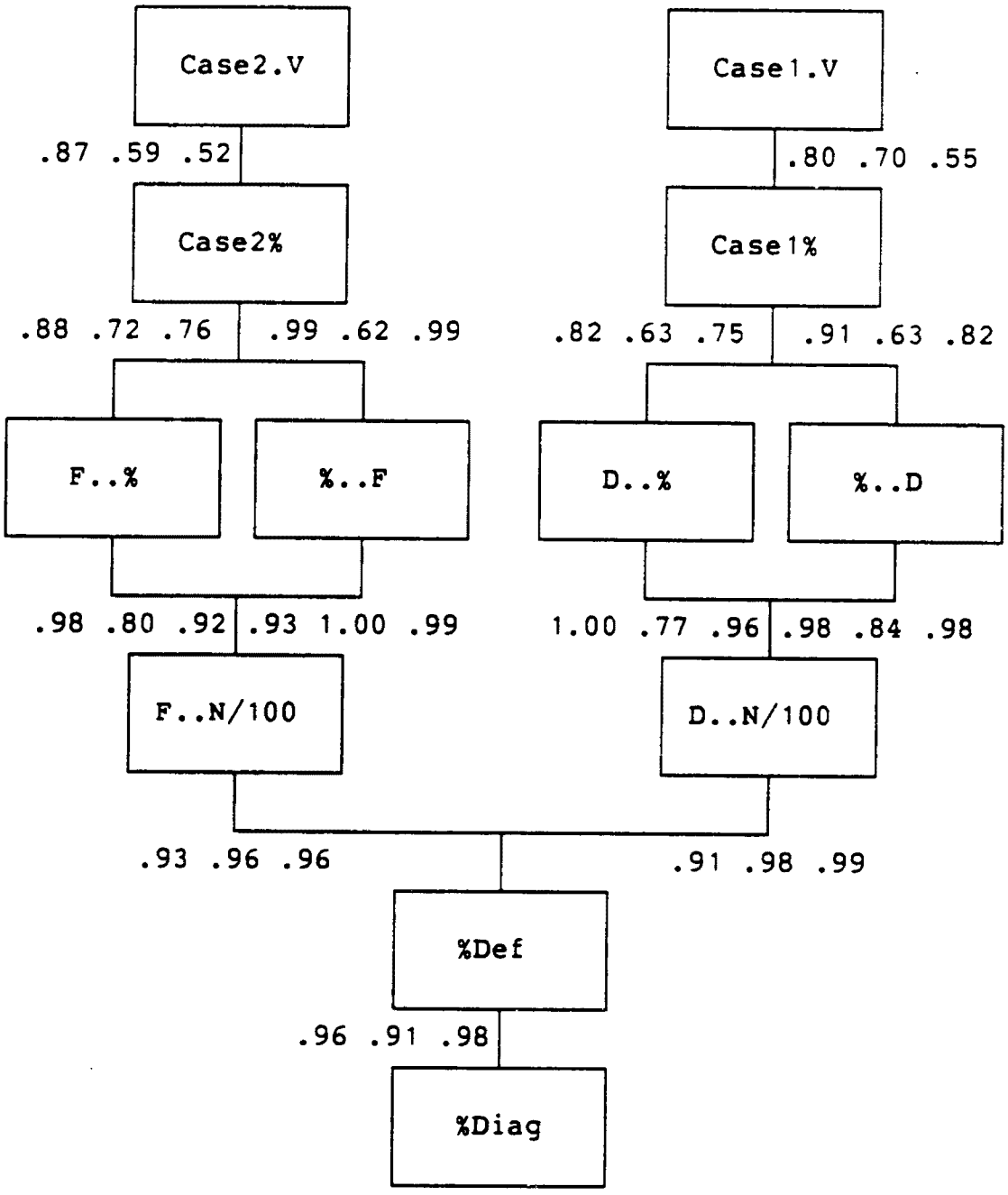


Fig. 6 Post Test Walbesser Ratios.

On the case 2 side of the hierarchy all dependencies were supported except %..F -> Case2% and Case2% -> Case2.V. The first failure may be due to the fact that %..F may not really be necessary to solving case 2 percent problems. In case 2 percent problems the typical task is to correctly form the part-whole fraction and write it as a percent. Verbal problems are always difficult for students, so it is not surprising that computational facility with percents is not the only skill needed to solve case 2 verbal problems. Given the author's general approach to teaching percent, by extending the students' part-whole fraction skills to include percent, it was gratifying to see this portion of the sequence substantiated. The dependency between F..% and Case2% was not preserved from the post to the retention test.

On the case 1 branch of the learning hierarchy, it would appear that more than facility with the equivalence of decimals and percents is required. Perhaps work on finding fractional parts of numbers and rational number multiplication should be part of a revised learning sequence.

Table 22 reports the post test analyses and Table 23 reports the retention test analyses conducted separately for each treatment group. On the post test the same dependencies were supported in both treatment groups, with the exception of D..N/100 -> D..% which was not supported by data from the proportion treatment. On the retention test

Table 22

Walbesser Ratios for Post Test Dependencies by Treatment Groups

Objective		Factor Product (N=141)			Proportion (N=123)		
Subordinate	Terminal	Consistency	Adequacy	Completeness	Consistency	Adequacy	Completeness
%Diag	%Def	.98	.92	.98 **	.94	.90	.99 **
%Def	F..N/100	.95	.96	.94 **	.91	.95	.98 **
%Def	D..N/100	.91	.97	.98 **	.90	.99	.99 **
F..N/100	%..F	.91	1.00	1.00 **	.96	.99	.98 **
F..N/100	F..%	.97	.84	.91 *	.99	.75	.94 *
D..N/100	%..D	.98	.92	.98 **	.99	.76	.99 *
D..N/100	D..%	1.00	.84	.95 *	1.00	.68	.98
%..F	Case2%	1.00	.63	1.00	.99	.61	.97
F..%	Case2%	.90	.73	.78 *	.85	.72	.72 *
%..D	Case1%	.95	.62	.89	.87	.63	.74
D..%	Case1%	.87	.63	.82	.76	.63	.68
Case2%	Case2.V	.83	.61	.57	.93	.57	.48
Case1%	Case1.V	.84	.73	.57	.75	.65	.52

* valid at lower level (.85, .70, .50)

** valid at higher level (.85, .85, .85)

Table 23

Walbesser Ratios for Retention Test Dependencies by Treatment Groups

Objective		Factor Product (N=140)			Proportion (N=121)		
Subordinate	Terminal	Consistency	Adequacy	Completeness	Consistency	Adequacy	Completeness
%Diag	%Def	.98	.93	.98 **	.96	.89	.99 **
%Def	F..N/100	.95	.95	.97 **	.93	.97	.95 **
%Def	D..N/100	.93	.99	.98 **	.89	.95	.99 **
F..N/100	%..F	.92	.98	1.00 **	.94	.98	.98 **
F..N/100	F..%	.96	.78	.94 *	1.00	.79	.91 *
D..N/100	%..D	.98	.93	.99 **	.97	.78	.96 *
D..N/100	D..%	.98	.74	.99 *	1.00	.73	.93
%..F	Case2%	1.00	.53	.97	.99	.59	.96
F..%	Case2%	.89	.62	.71	.89	.70	.71 *
%..D	Case1%	.97	.57	.89	.85	.54	.71
D..%	Case1%	.86	.63	.71	.81	.57	.65
Case2%	Case2.V	.88	.62	.42	.90	.61	.48
Case1%	Case1.V	.69	.70	.57	.73	.73	.48

* valid at lower level (.85, .70, .50)

** valid at higher level (.85, .85, .85)

the same pattern of dependency was present for both treatment groups, with the exception of F..% -> Case2% which was not supported by data from the factor product treatment. In fact it appears that the decrease in the adequacy ratio for this group resulted in this relationship not being supported in the combined group analysis described previously. This may suggest that students in the proportion group are more likely to maintain their case 2 problem solving skills if they can deal with the numbers used in the problem.

Summary

Comparisons of total scores indicated no significant differences between students in the two treatment groups. There was, however, a non-significant trend which tended to favor the factor product students on the total test scores. Comparisons of subscores on the major tests indicated some significant differences favoring students in the factor product treatment. Students in the factor product treatment tended to score higher than students in the proportion treatment on certain subscales relating percents to fractions and decimals. The cause of these differences may have been related to the somewhat superior initial fraction knowledge of the factor product students or to greater amount of practice on decimal numbers inherent in the factor product method.

Student performance on the author's tracking questions increased steadily during the study period. Student performance in the author's study was superior to that reported for selected NEAP and Minnesota assessment questions. Even though the students in the author's study seemed to have learned enough about percent to outperform the subjects on these tests, their performance still did not approach mastery on all topics. The implication seems to be that four weeks of study may not be long enough to be sure of mastery.

The author's analyses of his fraction, decimal and percent subscales suggest that the students' knowledge of fractions and decimals is likely to affect their learning of percent. The discovery of a positive feedback loop among these topics suggests that student knowledge on percent is likely to affect their later performance on fraction items. The author's study data tend to support his learning hierarchy for case 2 percent problems, but not case 1 percent problems. With case 2 problems it seems that the most significant skills required to insure mastery are the ability to form a part whole fraction and write it as a percent. With case 1 problems skills other than the ability to write percents as decimals seemed to be important prerequisites to attaining mastery.

CHAPTER V

SUMMARY AND CONCLUSIONS

Summary of Study

The main purpose of this study was the development and evaluation of two sequences for teaching percent. The two sequences differed in their treatment of the solution of case 1 percent problems, one used the factor product method and the other the proportion method. Student test performance was examined for error patterns. The interrelationships among the students' knowledge of fractions, decimals and percents was examined. The author's model for the learning of percent was evaluated.

Based on the author's pilot studies, instructional materials were developed and a learning hierarchy, based on the objectives included in these materials, was postulated. The materials were taught in seventh grade public school classrooms by the students' regular mathematics teachers. Test instruments were developed to examine the effectiveness of the materials and the relationships among the students' rational number skills. Students were given a pretest, daily quizzes, a post test and a retention test.

Analyses of variance and covariance were conducted and differences between students' test scores and subscores in the two treatment groups were examined. Item analyses were conducted for the major tests and student error patterns

between treatment groups were examined. Cross-lagged panel correlation analyses were conducted to examine the students' knowledge of fractions, decimals and percents for indications of causality. Walbesser ratios were computed and used to examine the author's postulated learning hierarchy.

Study Limitations

The generalizability of the results of this study was moderated by several factors. Although the total number of subjects used in this study was reasonably large, they were taken from just two schools and taught by three experienced teachers. In addition the extremely capable mathematics students from the City school were excluded from the study, because enrollment in the advanced placement mathematics program begins in the seventh grade. The students' week long spring vacation fell during the instructional phase of the study and this may have had some impact on the post test scores. Due to limitations on the amount of class time available to the author, some of his test subscales had a small number of items and correspondingly lower reliability than would have been optimal. Lastly, the MEAP test score was the only standard test score available to the author. Because the MEAP is a minimal performance test, it is not able to make fine discriminations between student abilities.

Findings and Conclusions

The author's data analyses revealed a number of interesting findings, relevant to the teaching of percent. These findings and the author's conclusions were presumed to be valid only within the context of the study limitations set forth earlier in this paper.

Conclusion 1.

There were minimal differences in student performance between the factor product and proportion treatments, but there was a definite trend favoring the factor product treatment.

Factors other than the method of teaching the solution of case 1 percent problems seem to have a greater impact on student test performance. Some of these factors seem to be the students' knowledge of fractions, the magnitude of the percent used in the problem and the question's wording. The factor product students seem to have a better understanding of how fractions, decimals and percents interrelate. This may be due to the fact that the factor product students received more work with decimals. A second explanation is that it is easier for them to select the correct method of algorithmic attack, because the solution of case 1 percent problems is distinctly different from the solution of case 2 problems unlike the proportion method.

Conclusion 2.

After four weeks of study students were able to exceed results reported in other studies, but did not attain mastery on all topics.

The fact that students performed quite well on the topics taught early in the sequence, but not the later topics, suggests that more time should be spent on teaching percent. It was apparent to the author, as he interviewed students, that one additional week should be spent on discrimination between case 1 and case 2 percent problems. To include more material than that which the author included in his sequences would require adding still more time to that which is spent on teaching percent.

Conclusion 3.

There was a positive feedback loop among the fraction, decimal and percent subscales on the major tests.

Work on decimal and fraction skills should precede and follow the work on percent. It is the investigator's opinion that the student's ability to understand and apply the notions of equivalent fractions is one of the keys to being able to understand and manipulate percents. Initial work on percent should occur early during the seventh grade year, to be followed up by additional work on fractions, decimals and percent after the student develops good equivalent fraction skills.

Conclusion 4.

The student test data supports the author's learning hierarchy for case 2 percent problems.

The student's ability to interpret percent as hundredths and to write percents for given hundredths seem to have been the primary skills needed to solve case 2 percent problems. Case 2 problems seem only to require the use of equivalent fraction and translation skills. Some additional skills seem to have been required to ensure that students were able to solve case 2 verbal problems. This investigator would suggest that certain styles of stating percent verbal problems may make it difficult to ascertain that a percent is the expected problem outcome.

Conclusion 5.

Student test data did not support the author's learning hierarchy for case 1 percent problems.

Skills other than the students facility with associating decimals and percents seem to have been more important to the solution of case 1 percent problems. Some of these skills are likely to include the ability to find fractional parts of numbers and the ability to multiply decimal numbers. An additional required skill is likely to be the ability to discriminate between case 1 and case 2 problem statements. This may simply be a question of spending more time in the sequence having the students identify whether the part and the whole are both given in a problem statement.

Conclusion 6.

There was comparable student performance on case 1 and case 2 percent problems.

Case 1 problems are not always easier than case 2 problems for the students, as often reported in the literature. Student performance will likely be higher on the problem type on which the students receive the most practice. Given the author's approach of teaching solution of case 2 problems first one would expect that students would be able to handle case 2 percents quite well, and this did seem to be true. Even though the magnitudes of the percents used in the case 2 problems were more difficult than those used in the case 1 problems there were no discernable differences in the number of student errors related to problem type.

Conclusion 7.

Students had a very difficult time generating percent equivalents for certain fractions (ie. thirds and eighths).

Students may not have received varied enough practice on generating equivalent fractions. It is possible that a different approach to equivalent fractions may be needed for denominators which are not divisors of, or multiples of, 100. A second possible explanation was revealed during the student interviews, students seemed to have trouble dealing with fractional remainders during long division. This may be because the students have gotten used to using decimal

division to solve percent problems and the concept of infinitely repeating decimal fractions is too foreign for them to compute the quotient properly.

Recommendations

Based on results reported in this document the author would make several recommendations for further study. To increase the generalizability of this study it should be repeated with more students, involving more schools and teachers. Replication of this study using subscales having more test questions and greater reliability would reduce the likelihood that these results are caused by sampling errors. The use of weekly, rather than daily quizzes, might increase quiz reliabilities and decrease potential measurement errors caused by student fatigue. A longer time lag between the post and retention tests (six months or more) would allow a more meaningful investigation of the retention of the percent knowledge taught. The study should be conducted earlier in the school year, so that the students' spring vacations don't interfere with the instructional phase of the study.

The results of this study point out the need for research to be undertaken to be able to identify and order the component skills needed to solve case 1 percent problems. Since the instructional method does not seem to be the major determinant of student percent performance, more significant factors need to be identified and their effects on student performance substantiated. Predominant

among these other factors is likely to be the magnitude of the percent. Research should be conducted on how the students' knowledge of fractions, decimals and percent is integrated to form a general understanding of rational numbers. Research should be conducted to examine the effects of question wording on student percent performance.

The author's work suggests that some changes in his percent sequences are necessary. Additional and more varied work on equivalent fractions should precede the teaching of percent. A different approach for teaching students to generate percent equivalents for thirds and eighths may be needed. It should be noted that, even though completing the author's four week percent sequence enabled students to out perform national test results, many percent topics had to be omitted due to lack of time. These omitted topics included: percents greater than 100, case 3 percent problems and percent of change. To include these topics in an introductory percent unit may not be possible, unless one is prepared to add three more weeks of instructional time. The author's feeling is that even without these topics, one could spend one additional week teaching students to discriminate between case 1 and case 2 percent problems, and trying to improve student abilities to solve percent verbal problems.

It is the author's belief that percent has become a forgotten topic in the mathematics curriculum. Textbook publishers have seemed content to ignore it in recent years,

perhaps because it is hard to teach or is not glamorous. This trend is beginning to reverse itself as student difficulties with percent in the business world come to light. The author believes that percent can be taught well and should be introduced at the seventh grade level. Special lessons will need to be developed for difficult topics. Mathematics educators must realize that, if percent is to be mastered, more time will need to be spent on it in the curriculum.

APPENDICES

APPENDIX A

TEACHER INSTRUCTIONS

GENERAL INSTRUCTIONS

The purpose of this study is to gain a better understanding of student difficulties in learning percent. In this study I will be comparing the effectiveness of two methods of teaching percent. This is an educational experiment and extra care must be taken to follow all procedures exactly. If questions about procedures arise, you should get in touch with me immediately.

Since different teaching methods are being compared, it is important that they receive equal consideration to ensure a fair experiment. The first eleven lessons in both treatments are identical. It will be important to keep the teaching methods separate from one another during the remaining seven lessons of the study. Separate worksheets are used in each treatment for lessons 12 and 14. No additional practice exercises are to be assigned, beyond those mentioned in the lesson plans. Worksheet problems marked with an "*" are optional and may be assigned to your better students. The daily routine for each class session should be as follows:

1. Quiz on previous day's work (5-10 minutes)
2. Go over quiz questions (5-10 minutes)
3. Introduce new material (10-15 minutes)
4. Class practice on exercises (10-20 minutes)
5. Correction of work sheets (5-10 minutes)

Note: Some quizzes will take more than 10 minutes, on these days the time spent introducing new material will need to be reduced.

Since I won't be meeting with you every day, I will need to have you do some record keeping for me. In addition to checking the tests and quizzes, please keep careful track of student

absences. If a student is absent on the day of instruction or on the day of a quiz, write the student's name and the word absent on a blank quiz paper. Keep test and quiz papers separated by treatment and class. Return test and quiz papers to me on a weekly basis. I will need to have you provide me with class lists, so that I may better keep track of the student test papers. No list of student names will be published in any written account of this study.

Verbal Explanation of Study

The purpose of this study is to examine the comparative effectiveness of two methods of teaching percent, the factor product method and the proportion method. I believe both teaching methods are educationally sound and students should gain comparable, though slightly different, amounts of knowledge about percent from either instructional treatment. There is no deception involved in this study. Individual students or teachers will not be identified or compared in this study. My primary interest is in identifying the respective strengths of the two teaching methods involved, and in testing the appropriateness of the sequencing of the topics within each treatment.

If you agree to participate in this study, you will be asked to spend eighteen days teaching one of the two percent sequences to your seventh grade mathematics class according to my instructions. Prior to beginning instruction you will be asked to administer a diagnostic pretest to your students. During the course of the study you will be asked to give daily five minute quizzes to your students, covering the previous day's instruction. At the completion of the instructional period you will be asked to give the students a posttest covering the instructional material. Three weeks following the posttest, you will be asked to give the posttest a second time to your students, this is to see how much of the material they are able to recall. All student lesson materials, tests, quizzes and lesson plans will be

written by me and furnished to you in classroom quantities. Following the posttest, you will be asked to make selected students available to me to interview. These students will be asked to work percent problems similar to those covered in the instructional treatments, and to explain their methods of solution as they do so.

Please read over the informed consent document that I have given you. When you feel I have answered all your questions concerning this study, please sign the form and date your signature in the blanks provided on the form.

INFORMED CONSENT FORM

The research project to be conducted by Bruce R. Maxim, a doctoral candidate, of the University of Michigan's School of Education as a dissertation study has been explained to me. I understand that my participation is completely voluntary. Further, I understand that if I agree to participate, I will:

Teach the student materials provided by the investigator, according to the methods prescribed by him.

Administer all test instruments according to the investigator's instructions.

Provide the investigator with the opportunity to interview selected students at the completion of the study.

I further understand that:

All information is confidential and my identity, and the identities of my students, will not be revealed.

I am free to withdraw my consent and to discontinue my participation in the project at any time.

Any questions I have about the project will be answered.

On the basis of the above statements, I agree to participate in this project.

Participant's signature

Date

Investigator's signature

Bruce R. Maxim
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Ann Arbor, MI 48103
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LESSON 1

Meaning of Fractions and Equivalent Fractions

Objectives:

1. To represent fractions using circles and/or rectangles as models, and to interpret meanings of fraction models
2. To use fraction models to write the missing numerators for equivalent fractions

Methods:

1. -Emphasize whole unit size

   -Draw two candy bars 

   -Ask students which bar they would choose and why

   -Discuss the importance of knowing bar size before choice

2. -Emphasize equal size pieces

   -Tell students you have 2 candy bars cut in 3 pieces

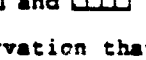
   -Ask students if it makes any difference which bar they choose

   -Show them  and , make sure the students see that one piece from the second could be larger than any from the first

3. -Emphasize number of pieces in the whole

   -Tell students that you have two bars, the same size cut into equal size pieces

   -Ask students if they need any more information to choose the largest piece of candy

   -Show them  and 

   -Make the observation that the greater the number of pieces (in the whole) the smaller each is

4. -Review naming of fractions

   -Emphasize the order of the numbers

   -When naming a fraction (ie. two thirds), the second number tells how many parts the whole is divided into and the first tells how many parts are to be considered

5. -Discuss the models on page 2 of the lesson 1 worksheet

   -Show students the twice as many pieces relation that holds between pictures on the left side of the page

   -Show the twice as many pieces relation to go from fifths to tenths, and the ten times as many pieces to go from tenths to hundredths

   -Make sure that students see that only the number of pieces is increasing, not the size of the whole

6. -Have students do lesson 1 worksheet

Note:

-Stress the terms whole, part and fraction name for parts through out the lesson

-Read all fractions, such as 3/4 as three fourths, not as 3 over 4

LESSON 2

Equivalent Fraction Generalizations

Objectives:

1. To identify the part (or parts) and the whole in verbal statements or written ratios
2. To use equivalent fractions to find the missing numerator in a proportion
3. To write equivalent ratios having 100 as the denominator

Method:

1. -Answer questions on lesson 1, give quiz 1
2. -Draw 1 whole unit on the board (use a rectangular model)

   -Divide the unit into thirds, shade one piece

   -Have students name the fraction

   -Divide unit into 6 pieces

   -Ask students what happened to the shaded piece (did it change in size?)

   -Ask students if 1/3=2/6?, Why? (Both diagrams show the same amount)

3. -Have students generate sequences of equivalent fractions by doubling (1/2=2/3=4/8 = ... or 1/3=2/6=4/12 = ...)

   -Write sequences on the board

4. -Have students generate sequences of equivalent fractions by tripling (1/2=3/6=9/18= ...)

5. -Work some problems like: $\frac{1}{2} = \frac{\quad}{10}$ (×5 above, ×5 below)

   -Ask students what must be done to the denominator to get 10? (Tell them to do the same to the numerator)

6. -Work on simplifying fractions (this will be hard)

   -Go back to $\frac{1}{2} \overset{\times 2}{=} \frac{2}{4}$ and point out that $\frac{2}{4} \overset{\div 2}{=} \frac{1}{2}$

   -Ask students what must be done to the denominator to get 2? (Tell them to do the same to the numerator)

7. -Work some examples dividing numerator and denominator by 2

$$\frac{6 \div 2}{14 \div 2} = ?$$

8. -Work some examples dividing numerator and denominator by the same number

9. -Work some problems like:

$$\frac{2}{8} = \frac{\ }{4} \qquad \frac{5}{15} = \frac{\ }{5} \qquad \frac{30}{100} = \frac{\ }{10}$$

10. -Practice $\frac{3}{4} = \frac{\ }{8}$ vs $\frac{2}{8} = \frac{\ }{4}$

-Do some oral work like: What do you do to 4 to get 12?
What do you do to 8 to get 4?

11. -Have students do lesson 2 worksheet

Decision point:

-Kids must learn to look at the denominators first and to decide whether to multiply or divide

-Divide to go from a larger denominator to a smaller one

-Multiply to go from a smaller one to a larger one

LESSON 3

Equivalent Fractions and Cross Products

Objectives:

1. To use cross products to solve proportions with one unknown (N) and to find equivalent ratios
2. To determine whether the equivalent fraction or cross product method is more appropriate to find equivalent ratios
3. To interpret verbal problems involving ratios, and to compute equivalent ratios having 100 as the denominator

Method:

1. -Answer questions on lesson 2, give quiz 2
2. -Begin with $\frac{1}{2} = \frac{3}{6}$ note: $1 \times 6 = 2 \times 3$

   $\frac{3}{10} = \frac{30}{100}$ $3 \times 100 = 10 \times 30$

3. -Relate the forms $\frac{2}{3} = \frac{\quad}{12}$ to $\frac{2}{3} = \frac{N}{12}$

   -Find number represented by N, using cross products one gets

   $2 \times 12 = 3 \times N$

   -Tell students to guess answer, then test their guess
4. -Tell students that some problems are more difficult and may require the use of division to solve for N

   $\frac{2}{3} = \frac{N}{7}$ => $2 \times 7 = 3 \times N$

   -Divide each side by 3 to isolate N

   -Caution: Must emphasize getting N by itself, some students will want to divide by 2 or 7
5. -Give students some practice in deciding whether to use equivalent fractions or cross products
6. -Have students do lesson 3 worksheet

Decision Step:

1. In deciding whether or not to use equivalent fractions look for fractions which have denominators like: 2, 4, 5, 10, 25, 50 and perhaps 3

2. If you decide to use cross products check to see if the fractions can be simplified before cross multiplying

LESSON 4

Meaning of Percent

Objectives:

1. To associate one whole with 100%, and to interpret N% as as N/100 using models.
2. To compare two percents and write > , < or =

Main Idea:

Percent means hundredths (put a sign on the bulletin board or write it on the chalk board every day)

Method:

1. -Answer questions on lesson 3, give quiz 3
2. -Emphasize that the whole unit (either circle or square) is divided into 100 pieces, and each piece is 1/100 or 1% of the whole
3. -Draw 4 squares on the board, have kids explain how to shade each to show: 40%, 1%, (1/2)%, (40 1/2)%

   -Students will want to show (1/2)% as 50 hundredths

   -It is important to emphasize that (1/2)% < 1% < 100%

   -You may also have to point out that

   $$(40\ 1/2)\% = (40 + 1/2)\%$$

4. -Ask students how they would show (50 1/2)% using both a square and a circle as the unit

   -Make sure that kids see that the unit (and all of its pieces) is 100%

5. -Ask students which is greater (1/4)% or 4%?

   -Get students to describe pictures of each

6. -Have students do lesson 4 worksheet

LESSON 5

Fractions and Percents

Objectives:

1. To write fractions as percents using the cross product or equivalent fraction method, and to decide which is easier to use (for a given problem)
2. To compare fractions and percents, and to determine which relation holds: $=$, $<$, $>$

Main Idea:

Percents are alternative ways of writing fractions which have 100 as the denominator

Method:

1. -Answer questions on lesson 4, give quiz 4
2. -Put this table on the board:

	Votes	
Carey	37	
Jim		23%
Beth	40	40%
Total	100	100%

-Have students complete the table

-Jim got 23% of the votes, how many votes did he get? Why? (23% means 23 per 100)

3. -Equivalent fraction method

-Write $\frac{1}{10} = \frac{?}{100}$

-How do you get from 10 to 100? (multiply by 10)

-So that 1/10 = 10/100 or 10%

4. -Work some problems using the equivalent fractions

-Write as percents: $\frac{1}{4}$, $\frac{1}{20}$, $\frac{50}{200}$

5. -Review cross products

$\frac{1}{4} = \frac{N}{20}$ $\quad 1 \times 20 = 4 \times N$ $\quad N = 5$

6. -Cross product method

$\frac{1}{3} = \frac{N}{100}$ $\quad 1 \times 100 = 3 \times N$ $\quad N = \frac{100}{3}$

$\frac{1}{3} = (33\ 1/3)\%$

-Caution: some students will want to write 1/3 = 33 1/3, they forget that finding N isn't the end of the problem

7. -Work some problems which require the use of cross products

-Write as percents: $\frac{2}{3}$, $\frac{1}{8}$, $\frac{7}{35}$

8. -Have students do the lesson 5 worksheet

Decision Step:

-When is it easier to use equivalent fractions? (when the denominator is a multiple or divisor of 100)

LESSON 6

Finding Percents

Objectives:

1. To interpret different forms of percent statements, and compute the correct percent
2. To solve verbal problems involving percent, using either the equivalent fraction or cross product method

Main Idea:

Identify the part and whole, write a fraction and then write a percent

Method:

1. -Answer questions on lesson 5, give quiz 5
2. -Verbally review:

   Write as fractions (12 1/2)% , 25% , (33 1/3)%

   Write as percents $\frac{3}{4}$, $\frac{7}{7}$, $\frac{1}{8}$

   -Note: Students can write 1/8 as a percent, but may have trouble writing (12 1/2)% as a fraction

3. -Write these examples on the board:

   $\frac{3}{10} = ___\%$

   3 out of 10 is what % ?

   3 is what % of 10?

   What % of 10 is 3?

   Compare 3 to 10 and write as a %

   -Have students identify the part and whole in each

   -Make sure students write percent sign if required

4. -Review cross product and equivalent fraction methods
5. -Have students do lesson 6 worksheet

LESSON 7

Finding Discounts and Mark Ups

Objectives:

1. To compute sales price, fractional discount and percent discount from original price and discount.
2. To compute selling price, fractional mark up and percent markup from purchase price and mark up.
3. To solve verbal problems involving percent discount and percent mark up.

Main Idea:

Original Purchase Price ---- - Discount (Decrease) / + Mark up (Increase) ----> Sale Selling Price

Method:

1. -Answer questions on lesson 6, give quiz 6
2. -Review some fraction-percent equivalents orally

   -Use fractions like: 1/3 , 1/8 , 1/2 , 1/4
3. -Put this table on the board:

Original Price (Whole)	Amount of Discount (Part)	Fraction Discount	Percent Discount
$4	$1		
$10	$2		
$16	$6		

   -Have students help complete the table, one column at a time
4. -Have students work problems 1 through 9 on lesson 7 worksheet

   -Note: Some students will get the fraction discount and not know how to get the percent discount (may need to review equivalent fraction and cross product methods)

5. -Put this table on the board:

Purchase Price (Whole)	Mark up Price (Part)	Selling Price	Fraction Mark up	Percent Mark up
$10	$2	$12	$\frac{2}{10}$	
$20		$4		

-Have students help complete the table

-Emphasize the difference between sale price and selling price, but point out the similarity of part/whole ideas in both tables

6. -Have students do remainder of lesson 7 worksheet

Teaching Tip:

It might be helpful to put some newspaper ads, showing sales and discounts, on the bulletin board

LESSON 8

Decimals for Tenths and Hundredths

Objectives:

1. To relate models for tenths and hundredths to decimal notation
2. To relate fractions, and mixed numbers, with denominators of 10 or 100 to decimal notation
3. To count by tenths

Method:

1. -Answer questions on lesson 7, give quiz 7
2. -Draw a rectangle on the board, call it one whole unit

   -Divide the unit into 10 equal parts

   -Ask students how they would show: 3 tenths? 8 tenths? 10 tenths?

   -Shade 7 pieces and ask for the oral name for the shaded part of the rectangle
3. -Show students this table:

Tens	Ones	Tenths
	.	

   -Ask students about the role of the decimal point (to identify the ones place)
4. -Write 0.4 on the board

   -Read the number (four tenths)

   -Ask students how 0.4 differs from 40.0 and 4.0

Tens	Ones	Tenths
	0.	4
4	0.	0
	4.	0

5. -Ask students how to write 0.4 as a fraction (capitalize on identical verbal names)

   -Draw a model for 0.4 (use rectangle for unit)

6. -Extend place value table to include hundredths

Hundreds	Tens	Ones	Tenths	Hundredths

-Discuss the table's symmetry about the one's place

-Note: Make a point of emphasizing the "th" in tenths and hundredths, do not let students read 0.4 as zero point four, insist on them reading 0.4 as four tenths

7. -Put 0.04 on the board

-Ask students how 0.04 compares to 40.0 , 4.0 and 0.4

Tens	Ones	Tenths	Hundredths
	0.	0	4
	0.	4	
	4.	0	
4	0.		

-Ask students how they would write 0.04 as a fraction

-Ask students how they would represent 0.04 using a rectangular model

-Ask students how they would represent 0.05 and 0.09 using rectangular models

8. -Have students do lesson 8 worksheet

LESSON 9

Equivalent Decimals

Objectives:

1. To use models and word names to write equivalent decimals for tenths and hundredths
2. To compare decimals (to hundredths) and write > , < or =

Method:

1. -Answer questions on lesson 8, give quiz 8
2. -Draw 2 rectangles on the board (no divisions)

   -Ask students how they would show 0.7

   -Ask students how they would show 0.07

   -Ask students how the two diagrams compare
3. -Have students look at question 7, lesson 9 worksheet

   -Have students shade 20 hundredths of A and shade 2 tenths of B

   -Ask students what they observe about the shaded parts of A and B (equal areas)
4. -Ask students how to write 0.7 as a fraction

   -Ask students to complete: 7 tenths = ___ hundredths

   -Make sure that students write 70 as the answer, not $\frac{70}{100}$
5. -Ask students how they would write 0.07 as a fraction

   -Ask students how to write 0.07 in words (seventy hundredths)

   -Ask students how they would complete:

   70 hundredths = $\underset{(7)}{___}$ tenths $\underset{(0)}{___}$ hundredths

   -Note: In comparing 7.6 to 7.06, make sure that students compare the largest places, not the number of digits
6. -Have students do lesson 9 worksheet

LESSON 10

Decimals to Thousandths

Objectives:

1. To relate models and fractions for thousandths to decimal notation
2. To use models and word names to write equivalent decimals to thousandths
3. To compare decimals (to thousandths) and write > , < or =

Methods:

1. -Answer questions on lesson 9, give quiz 9
2. -Draw a 100 grid on the board

   -Divide one of the small squares into 10 pieces

   -Ask students how many pieces there would be in the whole if each of the small squares were divided into 10 parts (1000)

   -Ask students what part of the whole each piece is $\frac{1}{1000}$

   -Ask students how many pieces would be in one row (100)

   -Ask students what part of the whole it is $\frac{100}{1000} = \frac{1}{10}$
3. -Put this table on the board:

Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths

   -Write 0.4 , 0.04 , 0.004 in the table

   -Ask students how to write 0.4 , 0.04 and 0.004 as fractions

   -Ask students how to draw models for each

   -Note: Remind students to look at place value when comparing decimals
4. -Have students do lesson 10 worksheet

LESSON 11

Decimals and Percents

Objectives:

1. To write decimals (to thousandths) as percents and conversely
2. To write fraction-decimal-percent equivalents for easy fractions
3. To compare percents and decimals, and to decide which relation holds: > , < or =

Methods:

1. -Answer questions on lesson 10, give quiz 10
2. -Have students take dictation; read fractions, decimals and percents out loud and have the students write them down

   -For example: Write 3 tenths as a decimal

   Write 43 hundredths as a fraction
3. -Review, orally, writing fractions as percents

   -Ask students to solve questions like:

   What are 8 hits out of 10, written as a percent?

   Write 12 out of 24 as a percent

   Write 16 out of 24 as a percent

   -Emphasize the importance of thinking hundredths
4. -Ask students how to write 0.62 as a percent

   $0.62 = 62 \text{ hundredths} = \frac{62}{100} = 62\%$

   -Have students complete: $0.3 = \frac{\quad}{100} = \quad\%$
5. -Ask students how to write 0.001 as a percent

   $0.001 = \frac{1}{1000} = \frac{0.1}{100} = 0.1\ \%$

   -Ask students to complete: $0.278 = \frac{\quad}{100} = \quad\%$

6. -Ask students how to write 38% as a decimal

$$38\% = \frac{38}{100} = 0.38$$

-Ask students how to write 12.5% as a decimal

$$12.5\% = \frac{12.5}{100} = \frac{125}{1000} = 0.125$$

7. -Tell students they should try to memorize the fraction-decimal-percent equivalents for:

$$\frac{1}{2} \quad \frac{1}{4} \quad \frac{1}{5} \quad \frac{1}{10} \quad \frac{1}{8} \quad \frac{1}{3}$$

-Note: Students should be reminded that cross products can be used for tough fractions like $\frac{1}{6}$

8. Have students do lesson 11 worksheet

LESSON 12F

Fractional Parts of Numbers

Objectives:

To find fractional parts of numbers and percents of numbers, by drawing pictures and by multiplication

Methods:

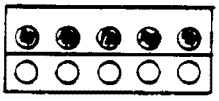
1. -Answer questions on lesson 11, give quiz 11
2. -The first problem set will be tough for students, they will have trouble identifying the whole and dividing it into equal parts

   -Put a rectangle on the board

   -Ask students how to represent $\frac{1}{2}$ of 1 
3. -Ask students how to show $\frac{1}{2}$ of 2

   -Ask students if it will require more than 1 unit (yes)

   -Draw 2 rectangles and shade 1 

   -Ask students what $\frac{1}{2}$ of 2 equals (1)
4. -Ask students how to show $\frac{1}{2}$ of 10 

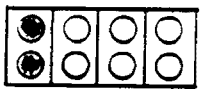
   -Stress that 10 is the whole

   -Ask students how to show 0.1 of 10
5. -Review easy fraction-decimal-percent equivalents for:

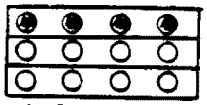
   $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{5}$ $\frac{1}{10}$ $\frac{1}{8}$ $\frac{1}{3}$
6. -Ask students how to show:

   50% of 2 

25% of 8



(33 1/3)% of 12



-Be sure to label parts and wholes

7. -Ask students what "of" seems to indicate (division by a whole number, or multiplication by a fraction)

-Discuss the process of rewriting "of" statements as multiplication problems

$$\frac{1}{4} \text{ of } 20 = \frac{1}{4} \times 20 = \frac{1}{4} \times \frac{20}{1} = \frac{1 \times 20}{4 \times 1} = \frac{20}{4} = 5$$

-Note: Remind students to simplify when changing decimals and percents to fractions

8. -Have students do lesson 12F worksheets

LESSON 12P

Fractional Parts of Numbers

Objectives:

1. To solve proportions written in terms of fractions, decimals and percents for the unknown quantity N
2. To set up proportions to solve statements involving "of" (ie. 20% of 50)

Methods:

1. -Answer questions on lesson 11, give quiz 11
2. -Review solving proportions using cross products
3. -Think of 1 of 20 as:

   1 out of 4 N out of 20

   -Then write 

   $\frac{1}{4} = \frac{N}{20}$

   -Generalize to 

   -Label $\frac{\text{part}}{\text{whole}} = \frac{1}{4} = \frac{N}{20} = \frac{\text{part}}{\text{whole}}$

4. -Point out that to do problems like 0.25 of 20 or 25% of 20, rewrite the decimal or percent as a fraction, set up a proportion and then solve

   -Note: Remind students to simplify, if possible, when changing decimals and percents to fractions

5. -Have students do lesson 12P worksheet

LESSON 13

Estimating Percents Using 1%, 10%, 50%, 100%

Objectives:

1. To find 100%, 50%, 10% and 1% of whole numbers
2. To estimate percents of numbers using 1%, 10%, 50% and 100% of numbers

Methods:

1. -Answer questions on lesson 12, give quiz 12
2. -Work questions 1 through 3, lesson 13 worksheet, together as a class
3. -Find 1% of 3600

   -Rewrite problem as:

   $$\frac{1}{100} \times 3600 = 36 \quad \text{or} \quad 0.01 \times 3600 = 36$$

4. -Find 10% of 3600

   -Rewrite problem as:

   $$\frac{1}{10} \times 3600 = 360 \quad \text{or} \quad 0.1 \times 3600 = 360$$

5. -Find 100%, 10% and 1% of 700

   -Get students to make an observation about the decimal point

   -For 1% of N, things get smaller, decimal point moves two places to the left

   -For 10% of N, things get smaller, decimal point moves one place to the left

   -Note: Remind students not to depend on the rule, if they forget the rule they should solve the problem the long way

6. -Work through several problems like questions 16-38, lesson 13 worksheet

   -Have students explain the logic of their choices

7. -Have students do lesson 13 worksheet

LESSON 14F

Finding Percents of Numbers

Objectives:

1. To find N% of a number by multiplication, equivalent fractions or mentally
2. To determine when paper and pencil or mental calculations are more efficient to solve percent problems

Methods:

1. -Answer questions on lesson 13, give quiz 13
2. -Ask students how to find 0.3 of 10
   (Rewrite as a multiplication problem)

   -Remind students that they can get the answer without using fractions (3 tenths x 10 = 30 tenths or 3.0)

   -Ask students: What is 0.7 of 2?

   What is 0.3 of 8?
3. -Ask students what is 15% of 10 (0.15 x 10 = 1.5)

   -Note: Remind students to be careful with decimal point placement in the answer, and to not include a % sign when writing the answer

   -Ask students what 10% of 630 is

   $0.1 \times 630 = 63$ or $\frac{1}{10} \times 630 = \frac{630}{10} = 63$

   -Ask students: What is 34% of 635? (215.9)

   What is 5% of 40? (2)
4. -Have students do lesson 14F worksheet

LESSON 14P

Finding Percents of Numbers

Objectives:

1. To find N% of numbers using cross products, equivalent fractions or mentally
2. To determine when paper and pencil or mental calculations are more efficient to solve percent problems

Method:

1. -Answer questions on lesson 13, give quiz 13
2. -Ask students what 10% of 70 is

   -Rewrite problem as:

   10 out of 100 = N out of 70

   $$\frac{\text{part}}{\text{whole}} = \frac{10}{100} = \frac{N}{70} = \frac{\text{part}}{\text{whole}}$$

   -Solve using cross products: $700 = 100N \Rightarrow N = 7$
3. -Ask students what 15% of 10 is

   -Rewrite as:

   15 out of 100 = N out of 10

   $$\frac{\text{part}}{\text{whole}} = \frac{15}{100} = \frac{N}{10} = \frac{\text{part}}{\text{whole}}$$

   $$150 = 100N \Rightarrow N = 1.5$$

   -Ask students: What is 25% of 630? (157.5)

   What is 5% of 40? (2)

   -Note: Remind students to estimate answers before solving, and to not write a % sign as part of their answer
4. -Have students do lesson 14P worksheet

LESSON 15

Percent Mark Up and Percent Discount

Objectives:

1. To calculate the sale price from the original price and percent discount
2. To calculate the selling price from the purchase price and the percent mark up
3. To solve verbal problems involving percent discount and percent mark up

Method:

1. -Answer questions on lesson 14, give quiz 14
2. -Note: Relate lesson 15 to lesson 7, in lesson 7 the task was to find a percent, in lesson 15 task is to find a fractional part of a number (or a percent of a number)
3. -Stress the part-whole comparison

   -Point out the similarity between the discount and mark up problems, explain how the diagram on the lesson 15 worksheet relates to the problems
4. -Students need to see that these are two step problems:

   a) Find the part discount or mark up

   b) Compute the final price by addition or subtraction
5. -Caution students to identify the part and whole in each verbal problem, before attempting any calculations
6. -Have students do lesson 15 worksheet

Decision Point:

Students need to see that discount or decrease problems require the use of subtraction, while mark up or increase problems require the use of addition

LESSON 16

Finding Percents and Percents of Numbers

Objectives:

1. To distinguish between finding percents and finding percents of numbers
2. To provide practice on solving percent verbal problems

Main Idea:

Students need to know when to write the percent symbol in the answer and when not to

Method:

1. -Answer questions on lesson 15, give quiz 15
2. -Work through problems 1-10, lesson 15 worksheet, together as a class

   -The emphasis is on how the problems are to be solved, not on getting the right answer
3. -Have students complete lesson 16 worksheet

Decision Point:

-Students must learn to ask if they are finding a percent

If so, write a $\frac{\text{part}}{\text{whole}}$ fraction, then write it as a %

If they are finding a percent of a number instead, then they should either write a multiplication sentence (Factor Product Treatment) or set up and solve a proportion (Proportion Treatment)

LESSON 17

Review

1. -Answer questions on lesson 17, give quiz 17

2. -Provide general review of material as needed

   -Stress correct decimal point placement

   -Stress the importance of knowing when to write the % symbol in the answer

   -Review fraction-decimal-percent equivalents for:

   $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{5}$ $\frac{1}{8}$ $\frac{1}{10}$

   -Review simplifying fractions

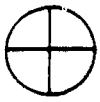
3. Have students do lesson 18 worksheet

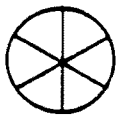
APPENDIX B

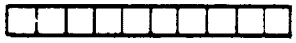
STUDENT MATERIALS

LESSON 1

Shade each figure to show:

1. 
3 fourths

2. 
1 sixth

3. 
3 tenths

Draw lines to make equal size pieces.

4. 
halves

5. 
thirds

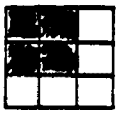
6. 
fourths

7. 
fifths

8. 
eighths

9. 
tenths

Write a fraction for the shaded part.

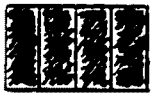
10. 

11. 

12. 

13. 

14. 

15. 

Use a rectangle for the unit. Draw diagrams for each fraction.

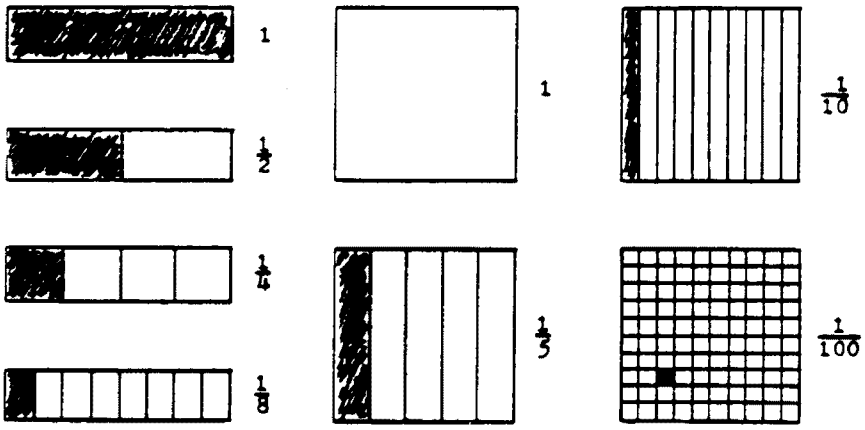
16. $\frac{1}{5}$ 17. $\frac{2}{3}$ 18. $\frac{7}{10}$ 19. $\frac{4}{4}$

Write the missing numerator.

20. $1 = \frac{\quad}{2}$ 21. $1 = \frac{\quad}{4}$ 22. $1 = \frac{\quad}{8}$ 24. $1 = \frac{\quad}{5}$

24. $1 = \frac{\quad}{10}$ 25. $1 = \frac{\quad}{20}$ 26. $1 = \frac{\quad}{3}$ 27. $1 = \frac{\quad}{6}$

Use the pictures to find the missing numerator in questions 28-47.



28. $\frac{1}{2} = \frac{\quad}{4}$ 29. $\frac{1}{2} = \frac{\quad}{8}$ 30. $\frac{3}{4} = \frac{\quad}{8}$ 31. $\frac{1}{5} = \frac{\quad}{10}$

32. $\frac{2}{5} = \frac{\quad}{10}$ 33. $\frac{3}{5} = \frac{\quad}{10}$ 34. $\frac{1}{10} = \frac{\quad}{100}$ 35. $\frac{2}{10} = \frac{\quad}{100}$

36. $\frac{5}{10} = \frac{\quad}{100}$ 37. $\frac{8}{10} = \frac{\quad}{100}$ 38. $\frac{9}{10} = \frac{\quad}{100}$ 39. $\frac{10}{10} = \frac{\quad}{100}$

40. $\frac{1}{5} = \frac{\quad}{100}$ 41. $\frac{3}{5} = \frac{\quad}{100}$ 42. $\frac{5}{5} = \frac{\quad}{100}$ 43. $1 = \frac{\quad}{100}$

44. $\frac{20}{100} = \frac{\quad}{5}$ 45. $\frac{60}{100} = \frac{\quad}{5}$ 46. $\frac{80}{100} = \frac{\quad}{5}$ 47. $\frac{100}{100} = \frac{\quad}{5}$

Write the missing numerator.

48. $\frac{1}{4} = \frac{\quad}{8}$ 49. $\frac{3}{4} = \frac{\quad}{8}$ 50. $\frac{4}{8} = \frac{\quad}{2}$ 51. $\frac{1}{5} = \frac{\quad}{10}$

52. $\frac{2}{5} = \frac{\quad}{10}$ 53. $\frac{6}{10} = \frac{\quad}{5}$ 54. $\frac{8}{10} = \frac{\quad}{5}$ 55. $1 = \frac{\quad}{100}$

Lesson 2

In each problem below, circle each number and label it P for part and W for whole. If the whole is not given find it. Then for each write a fraction part/whole.

1. Sue made 8 out of 20 free throws she attempted.

2. Gibson got 347 hits for each 1000 times at bat.

3. 10 minutes out of every 60 minutes are spent traveling to class.

4. 12 parts out of 24 were gold.

5. Mac made 10 free throws in 10 attempts.

6. The original price of a shirt was $9, the discount is $2.

7. Mark saves $.10 out of every dollar he earns.

8. A team won 8 out of it last 10 games.

9. There are 8 boys and 7 girls in a class.

   whole=students=____

10. There are 5 students and 2 teachers on a committee.

    whole=people=_____

For each problem decide whether you need to multiply or divide, then write the missing numerator.

11. $\frac{1}{3} = \frac{\quad}{6}$

12. $\frac{1}{2} = \frac{\quad}{6}$

13. $\frac{2}{3} = \frac{\quad}{9}$

14. $\frac{2}{5} = \frac{\quad}{15}$

15. $\frac{1}{10} = \frac{\quad}{20}$

16. $\frac{5}{8} = \frac{\quad}{24}$

17. $\frac{1}{2} = \frac{\quad}{10}$

18. $\frac{3}{4} = \frac{\quad}{20}$

19. $\frac{2}{6} = \frac{\quad}{3}$

20. $\frac{4}{10} = \frac{\quad}{5}$

21. $\frac{9}{12} = \frac{\quad}{4}$

22. $\frac{24}{30} = \frac{\quad}{10}$

23. $\frac{0}{100} = \frac{\quad}{5}$

24. $\frac{1}{4} = \frac{\quad}{100}$

25. $\frac{3}{4} = \frac{\quad}{100}$

26. $\frac{2}{4} = \frac{\quad}{100}$

27. $\frac{4}{4} = \frac{\quad}{100}$ 28. $\frac{75}{100} = \frac{\quad}{4}$ 29. $\frac{100}{100} = \frac{\quad}{5}$ 30. $\frac{50}{100} = \frac{\quad}{2}$

Write as equivalent ratios having 100 as the denominator.

31. 4 out of 20 32. 2 out of 5 33. 6 out of 25

34. 5 for 20 35. 7 for 50 36. 3 for 10

37. 5 per 50 38. 2 per 25 39. 8 per 20

40. $\frac{1}{4}$ 41. $\frac{2}{5}$ 42. $\frac{3}{4}$

43. $\frac{4}{200}$ 44. $\frac{20}{25}$ 45. $\frac{1}{10}$

*46. $\frac{4}{40}$ *47. $\frac{8}{40}$ *48. $\frac{35}{70}$

Find each part/whole ratio, then write each as a fraction N/100

49. John made 10 out of 25 free throws.

50. Jenny got 4 hits out of 10 times at bat.

*51. 18 carat gold is 18 parts gold, out of 24 parts.

52. There are 7 boys and 3 girls in a class.
whole=students=____

53. There are 2 teachers and 18 students in a group.
whole=people=____

Lesson 3

Use cross products, to find the number which should replace N to make each a true sentence.

1. $\frac{1}{2} = \frac{N}{10}$
2. $\frac{1}{3} = \frac{N}{24}$
3. $\frac{N}{4} = \frac{3}{6}$
4. $\frac{2}{7} = \frac{N}{21}$
5. $\frac{N}{10} = \frac{18}{5}$
6. $\frac{N}{12} = \frac{6}{8}$
7. $\frac{6}{10} = \frac{N}{25}$
8. $\frac{N}{3} = \frac{5}{10}$
9. $\frac{4}{6} = \frac{N}{9}$
10. $\frac{N}{14} = \frac{12}{21}$
11. $\frac{75}{100} = \frac{N}{30}$
12. $\frac{5}{8} = \frac{N}{100}$
13. $\frac{4}{15} = \frac{N}{60}$
14. $\frac{N}{6} = \frac{3}{4}$
15. $\frac{5}{12} = \frac{N}{5}$

Using equivalent fractions, write each ratio as an equivalent ratio having 100 as the denominator.

16. $\frac{1}{2}$
17. $\frac{2}{5}$
18. $\frac{3}{4}$
19. $\frac{1}{10}$
20. $\frac{3}{50}$
21. $\frac{7}{20}$
22. $\frac{13}{25}$
*23. $\frac{3}{6}$

Using either equivalent fractions, or cross products, write each ratio as an equivalent ratio having 100 as the denominator.

24. $\frac{1}{3}$ 25. $\frac{1}{4}$ 26. $\frac{1}{8}$ 27. $\frac{7}{10}$

28. $\frac{4}{4}$ 29. $\frac{3}{40}$ 30. $\frac{3}{20}$ 31. $\frac{3}{3}$

Solve the following using proportions.

32. Jim made 3 free throws out of 5 attempts. How many free throws is he likely to make in 100 attempts?

33. Sandy paid $1 sales tax on a $25 purchase. How much sales tax would she pay on a $100 purchase?

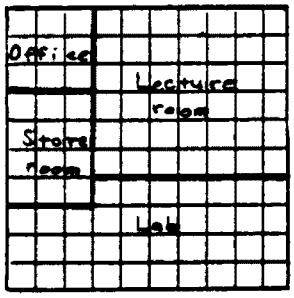
34. Jenny got 4 hits in 10 times at bat. How many hits is she likely to get in 100 times at bat?

35. Sam got 3 heads when he tossed 8 coins. How many heads should he expect if he tosses 100 coins?

36. 3 out of every 10 bolts are defective. How many defective light bulbs would be expected out of every 100?

LESSON 4

1. Mr. Owen designed a new science area for the school. Notice that it is divided into 100 parts. The office takes up 9 hundredths of the total space. This can be written as nine percent.

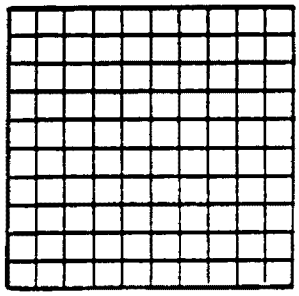


a) What space is occupied by each of the other rooms?

b) Put a 3 percent closet inside the lecture room.

c) Four separate tables take up ten percent of the total science area. Place these 4 tables in the lab section. (The tables don't have to be the same size.)

2. This grid represents the basement of a home.



a) Divide up the space so that:

15 percent is a bathroom

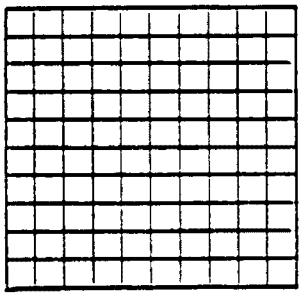
20 percent is a laundry room

21 percent is a store room

the remaining space is a rec room

b) What percent of the basement is the rec room?

3. This 100 grid represents the first floor of an office building.



68 percent of the space is used for 4 separate offices (they don't have to be the same size)

20 percent of the space is used for hallways

12 percent of the space is used for 2 separate bathrooms

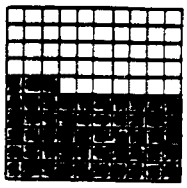
4. How many small squares are in A?

5. ___ of A is shaded?
   100

6. ___% of A is shaded?

7. ___ of A is not shaded?
   100

8. ___% of A is not shaded?



A

9. ___% of A shaded + ___% of A not shaded = ___% A altogether

10. Write the following in the small squares of B, putting one letter in each square.

    a) 20 x's b) 46 w's

    c) 4 y's d) 5 o's

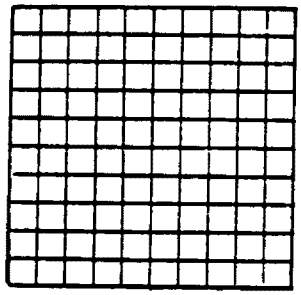
11. ___% of the squares in B have x's

12. ___% of the squares in B have w's

13. ___% of the squares in B have y's

14. ___% of the squares in B have o's

15. ___% of the squares in B are blank

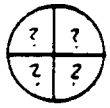


B

Write the percents that are missing to make the whole.

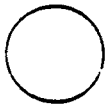
16. 

17. 

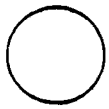
18. 

Draw lines which would approximately divide each circle according to the percents given.

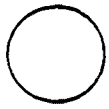
19. 50% 40% 10%



20. 60% 20% 20%



21. 85% 10% 5%



Solve the following.

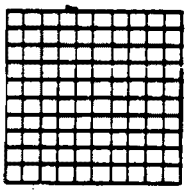
22. Sue missed 10% of the questions on a test. What percent of the test questions did she get correct?

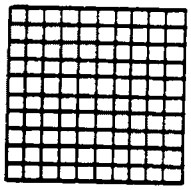
23. If two friends contribute equally to buy a car, what percent of the money does each contribute?

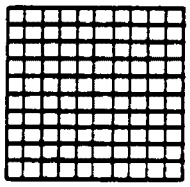
24. If 25% of the cars made are red and 20% are blue, what percent of the cars made are other colors?

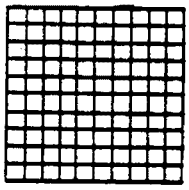
25. Five men buy a house. If each contributes the same amount, what percent of the purchase price does each contribute?

Shade each grid to show the percent.

26.  50%

27.  (1/2)%

28.  (80 1/2)%

29.  (2 3/4)%

Replace ___ with > , < or = (Try to visualize the percent grid representation of each.)

30. $\frac{1}{2}$ % ___ 50%

31. $\frac{1}{5}$ % ___ 5%

32. 23% ___ $\frac{2}{3}$ %

33. 13% ___ (12 1/2)%

34. 20% ___ $\frac{2}{10}$ %

35. $\frac{3}{3}$ % ___ 100%

LESSON 5

Write as percents.

1. $\frac{24}{100}$ 2. $\frac{17}{100}$ 3. $\frac{70}{100}$ 4. $\frac{7}{100}$

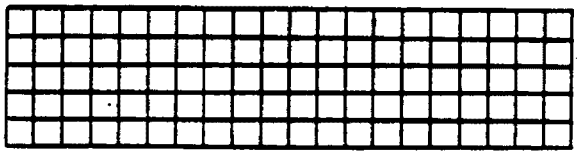
5. $\frac{(33\ 1/3)}{100}$ 6. $\frac{3}{100}$ 7. $\frac{1}{100}$ 8. $\frac{100}{100}$

Write as fractions with denominators of 100.

9. 41% 10. 36% 11. 25% 12. 99%

13. 100% 14. 5% 15. 1% 16. 0%

Use diagram A for questions 17-21.



A

17. Put ▨ on 1/2 of the small squares of A.
18. Each small square is what % of the rectangle?
19. What percent of A has ▨ on it?
20. Put ▨ on 1/4 of the rectangle.
21. What percent of A has ▨ on it?

Use diagram B for questions 22-25.



B

22. Put ▨ on 50% of rectangle B.
23. Put ▨ on 10% of rectangle B.
24. Put ▨ on 5% of rectangle B.
25. Shade 1% of rectangle B.

Use equivalent fractions. Write each fraction as a percent.

26. $\frac{3}{4}$ 27. $\frac{1}{5}$ 28. $\frac{3}{5}$ 29. $\frac{1}{10}$

30. $\frac{8}{10}$ 31. $\frac{10}{10}$ 32. $\frac{7}{10}$ 33. $\frac{3}{50}$

Using cross products, write each fraction as a percent.

34. $\frac{1}{4}$ 35. $\frac{1}{3}$ 36. $\frac{2}{3}$ 37. $\frac{1}{8}$

38. $\frac{3}{8}$ 39. $\frac{5}{8}$ 40. $\frac{7}{8}$ 41. $\frac{7}{7}$

Using either equivalent fractions or cross products, write each fraction as a percent.

42. $\frac{50}{500}$ 43. $\frac{13}{40}$ 44. $\frac{7}{25}$ 45. $\frac{13}{200}$

Identify the part and whole in each question. Write the fraction part/whole and its percent equivalent.

46. On a test with 50 questions, you get 40 correct.

47. The soccer team won 15 out of 20 games played.

48. A 1000 liter tank contains 540 liters of gas.

49. Sam made 6 hits in 20 times at bat.

Replace ___ with > , < or = (Try to visualize diagrams for each.)

50. 45% ___ $\frac{4}{5}$ 51. $\frac{8}{10}$ ___ 8% 52. $\frac{1}{4}$ ___ 4%

53. $\frac{1}{2}$ ___ 50% 54. $\frac{1}{5}$ ___ 20% *55. $\frac{1}{100}$ ___ $\frac{3}{3}$ %

LESSON 6

Solve the following. Circle the whole, box the part and write the percent. (If the whole is not given find it.)

1. $\frac{20}{100}$ = ___%
2. 30 is ___% of 100
3. ___% of 100 = 50
4. 3 out of 100 = ___%
5. $\frac{3}{10}$ = ___%
6. 1 out of 4 = ___%
7. 4 per 100 = ___%
8. $\frac{5}{8}$ is ___%
9. 5 compared to 15 is ___%
10. 16 compared to 24 is ___%
11. 1 compared to 8 is ___%
12. What % of 100 is 9?
13. What percent of 50 is 5?
14. What percent of 200 is 4?
15. What % of 400 is 16?
16. What percent of 300 is 9?
17. What percent of 9 is 3?
18. 4 is what % of 8?
19. 12 is what % of 48?
20. 7 is what percent of 21?
21. Andy won 15 games of the 25 he pitched. What percent of the games did he win?
22. In a class of 30 students, 12 had perfect papers on a test. What percent of the class had perfect papers?
23. Stacy has 17 out of 20 problems correct. What percent of the problems did she have correct?

24. In a class election, John got 35 votes and Mary got 15. What percent of the total votes did each get?

25. There were three candidates running for office. Mark got 17 votes, Chris got 18 votes and Joan got 15. Find the percent of the total vote each got.

26. In a class there were 15 boys and 10 girls. Find the percent of boys and the percent of girls in the class.

27. Fourteen of the 20 people in the science club have paid their dues. What percent have paid their dues? What percent have not paid their dues?

28. A committee is composed of 2 teachers and 8 students. Find the percent of the committee members who are students and the percent who are teachers.

29. A team has won 8 games and lost 4. Find the percent of games won and the percent of games lost.

*30. Mr. Jones paid $4000 for his car. It decreased in value $800 the first year he owned it. What percent of his car's value was lost the first year?

*31. A woman buys radios for $100 and marks them up $40 and sells them. What part of the radios' cost is added to the price before they are sold.

LESSON 7

Find the missing numbers.

	Original Price (whole)	Amount of Discount (part)	Sale Price	Fraction Discount	Percent Discount
1.	$10	$1			
2.	$20	$4			
3.	$15		$10		
4.	$16		$12		
5.		$8	$16		
6.		$20	$30		
7.	$100		$60		
8.	$8	$3			
9.	$12	$4			

Solve the following. Circle the whole, box the part and write the percent requested. Find the part if it is not given.

10. The price of a liter of milk was $.50. If it is decreased by $.05, what is the percent decrease in the price of milk?

11. During a sale, a hat originally priced at $10 was reduced to $8. What was the amount of decrease? What was the percent decrease in price?

12. Joyce bought a shirt for $14. If the original price was $20, what was the percent discount on the shirt price?

*13. Records are sold for $4, after a $2 discount. What is the percent discount?

You own a store. You buy merchandise for the amount listed under purchase price and add the markup to get the selling price.

Find the missing numbers.

	Purchase Price (whole)	Mark up (part)	Selling Price	Fraction Mark up	Percent Markup
14.	$10		$11		
15.	$20		$25		
16.	$30	$10			
17.	$12	$3			
18.	$100		$125		
19.	$60	$20			
20.		$40	$100		
21.		$50	$150		

Solve the following. Circle the whole, box the part and write the percent requested. Find the part if it is not given.

22. An increase from $5 to $7 is how much of an increase? What percent increase?

23. Ties are sold for $4 each. They cost $3 to make. What is the percent mark up?

24. A store owner buys stereos for $500 and sells them with a $50 mark up. What is the percent mark up on the stereos?

*25. A woman bought a used car and sold it with an $800 mark up for $3200. What was the percent mark up on the car?

LESSON 8

Draw pictures to show.

1. 0.2 2. 0.9 3. 0.6

Divide the rectangle into tenths.

4. Put  on 0.1
5. Put  on 0.2
6. Put  on 0.3
7. How much of the rectangle is left unshaded?



Write as you count by tenths.

8. 0.1, ___, ___, ___, ___, ___, ___, ___, ___, ___, ___
9. 3.4, ___, ___, ___, ___, ___, ___, ___, ___, ___, ___
10. 120.9, ___, ___, ___, ___, ___, ___, ___, ___, ___, ___

Write a decimal number to show.

11. 3 tenths 12. 8 tenths 13. 10 tenths 14. 4 tenths

15. $\frac{9}{10}$ 16. $\frac{6}{10}$ 17. $\frac{1}{10}$ 18. $3\frac{7}{10}$

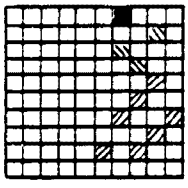
19. $2\frac{3}{10}$ 20. $8\frac{3}{10}$ 21. 1 22. $5\frac{0}{10}$

Write as fractions or mixed numbers.

23. 0.3 24. 0.1 25. 0.4 26. 1.0

27. 2.3 28. 10.8 29. 12.1 30. 37.0

Use the picture at the right to answer questions 31-37.



31. Put x's on 0.03

32. Put y's on 0.07 more

33. Put w's on $\frac{4}{100}$ more

34. Put o's on 8 hundredths more

35. Write a decimal for the part that is shaded like:

 ____  ____  ____

36. How much would you shade to show 100 hundredths?

37. 1 vertical row = ___ hundredths

1 horizontal row = ___ hundredths

2 rows = ___ hundredths 5 rows = ___ hundredths

7 rows = ___ hundredths 10 rows = ___ hundredths

Write a decimal to show:

38. one hundredth 39. 5 hundredths 40. 100 hundredths

41. $\frac{9}{100}$ 42. $\frac{1}{100}$ 43. $\frac{8}{100}$

44. $3\ \frac{4}{100}$ 45. $8\ \frac{7}{100}$ 46. $7\ \frac{0}{100}$

Write as fractions or mixed numbers.

47. 0.03 48. 0.01 49. 0.09 50. 0.1

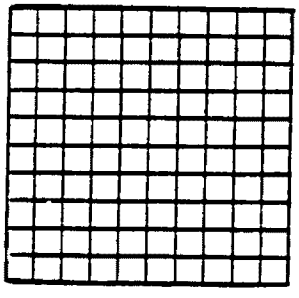
51. 4.01 52. 11.02 53. 6.00 54. 14.2

LESSON 9

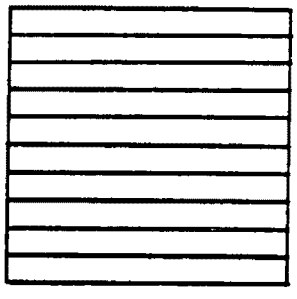
Complete the following with whole numbers.

1. 2 rows of A = ___ hundredths
2. 2 rows of B = ___ tenths
3. 5 rows of A = ___ hundredths
4. 5 rows of B = ___ tenths
5. 10 rows of A = ___ hundredths
6. 10 rows of B = ___ tenths

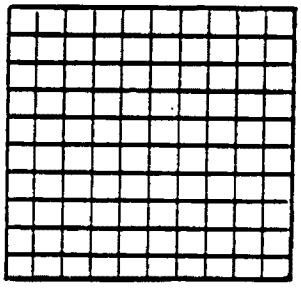
A



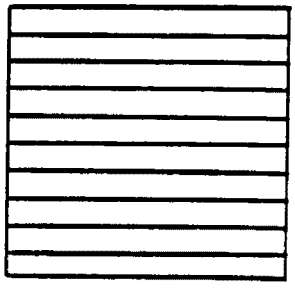
B



C



D



Shade horizontal rows to show each.



7. (A) 20 hundredths
   (B) 2 tenths



8. (A) 40 hundredths more
   (B) 4 tenths more



9. (C) 10 hundredths
   (D) 1 tenth



10. (C) 20 hundredths more
    (D) 2 tenths more

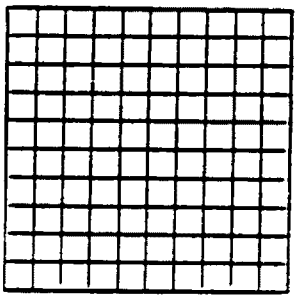
11. There are ___ hundredths left unshaded in A
12. There are ___ tenths left unshaded in B
13. There are ___ hundredths left unshaded in C
14. There are ___ tenths left unshaded in D

Complete the following with whole numbers.

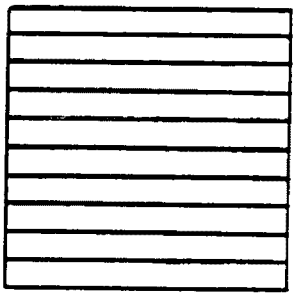
15. 1 tenth = ___ hundredths
16. 4 tenths = ___ hundredths
17. 10 tenths = ___ hundredths
18. 0.2 = ___ hundredths
19. 0.1 = ___ hundredths
20. 1.2 = one and ___ hundredths

Shade horizontal rows to show each:

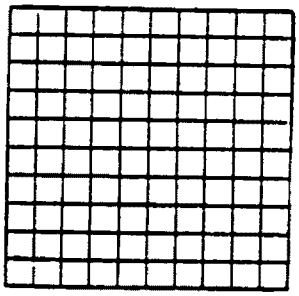
21. (A) 27 hundredths
(B) same amount
22. (A) 32 hundredths
(B) same amount
23. (B) 2 tenths
(A) same amount
24. (B) 1 tenth
(A) same amount
25. (C) 13 hundredths
(D) same amount
26. (C) 8 hundredths
(D) same amount
27. (C) 21 hundredths
(D) same amount



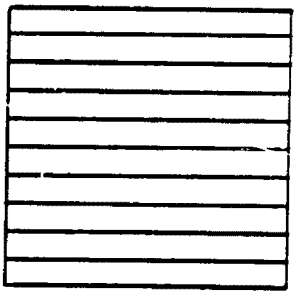
A



B



C



D

Complete the following with whole numbers.

28. 42 hundredths = 4 tenths ___ hundredths

29. 13 hundredths = ___ tenths ___ hundredths

30. 7 hundredths = ___ tenths ___ hundredths

31. 78 hundredths = ___ tenths ___ hundredths

32. 40 hundredths = ___ tenths ___ hundredths

33. 100 hundredths = ___ tenths

Write a decimal for each fraction.

34. $\frac{19}{100}$ 35. $\frac{60}{100}$ 36. $1\ \frac{14}{100}$ 37. $\frac{97}{100}$

38. $\frac{3}{4}$ 39. $\frac{14}{20}$ 40. $\frac{2}{5}$ 41. $1\ \frac{1}{2}$

Replace ___ with > , < or =

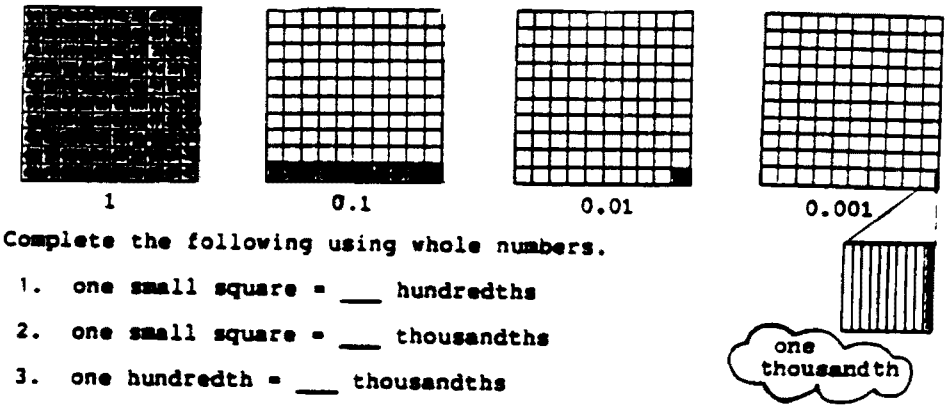
42. 0.4 ___ 0.08 43. 0.67 ___ 0.7 44. 0.06 ___ 0.07

45. 0.06 ___ 0.6 46. 0.9 ___ 0.5 47. 6.7 ___ 5.72

48. 7 ___ 5.98 49. 0.2 ___ 0.17 50. 0.3 ___ 0.30

LESSON 10

Use these diagrams to answer questions 1-6.



Complete the following using whole numbers.

1. one small square = ___ hundredths
2. one small square = ___ thousandths
3. one hundredth = ___ thousandths
4. one row = ___ thousandths
5. one tenth = ___ thousandths
6. one whole = ___ thousandths

Write decimals for the following.

7. 5 thousandths
8. 23 thousandths
9. 45 thousandths
10. 6 tenths
11. six and 2 thousandths
12. forty-two thousandths
13. 18 and 2 thousandths
14. 94 and 6 thousandths
15. 2 tenths 3 hundredths 4 thousandths
16. one hundred three thousandths

Complete the following with whole numbers.

17. 432 thousandths = 4 tenths 3 hundredths ___ thousandths
18. 13 thousandths = ___ tenths ___ hundredths ___ thousandths
19. 3 thousandths = ___ tenths ___ hundredths ___ thousandths
20. 780 thousandths = ___ tenths ___ hundredths ___ thousandths
21. 20 thousandths = ___ tenths ___ hundredths ___ thousandths
22. 300 thousandths = ___ tenths ___ hundredths ___ thousandths

Complete the following with whole numbers.

23. 2 tenths = ___ hundredths
24. 2 tenths = ___ thousandths
25. 4 tenths = ___ thousandths
26. 0.04 = ___ thousandths
27. 0.12 = ___ thousandths
28. 0.005 = ___ thousandths
29. 0.5 = ___ thousandths
30. 0.150 = ___ thousandths
31. 0.600 = ___ thousandths
32. 0.60 = ___ thousandths
33. 1 = ___ thousandths
34. 5.93 = 5 and ___ thousandths

Write a decimal for each fraction.

34. $\frac{125}{1000}$
35. $\frac{60}{1000}$
36. $\frac{200}{1000}$
37. $\frac{3}{1000}$

38. $\frac{3}{5}$
39. $\frac{1}{4}$
40. $\frac{37}{100}$
*41. $\frac{7}{8}$

Write a fraction for each decimal, then write the fraction in simplest terms.

42. 0.013
43. 0.75
44. 0.520
45. 0.375

Replace ___ with > , < or =

46. 0.2 ___ 0.175
47. 0.07 ___ 0.578
48. 0.060 ___ 0.07

49. 7 ___ 6.95
50. 6.700 ___ 6.7
51. 0.125 ___ 0.126

LESSON 11



1. Shade $\frac{60}{100}$ of A
2. Write a decimal to show how much of A is shaded.
3. Shade $\frac{6}{10}$ of B
4. Write a decimal to show how much of B is shaded.
5. Shade $\frac{3}{5}$ of C
6. Write a decimal to show how much of C is shaded.
7. ___% of A is shaded
8. ___% of B is shaded
9. ___% of C is shaded

Complete the following with whole numbers.

10. 0.75 = $\frac{\quad}{100}$ = ___%
11. 0.38 = $\frac{\quad}{100}$ = ___%
11. 0.07 = $\frac{\quad}{100}$ = ___%
13. 0.10 = $\frac{\quad}{100}$ = ___%

Write as percents.

14. 0.4	15. 0.04	16. 0.004	17. 0.9
18. 0.5	19. 0.01	20. 0.375	21. 0.700
22. 0.055	23. 0.333	24. 0.005	25. 0.632

Write as decimals.

26.	24%	27.	5%	28.	10%	29.	1%
30.	13%	31.	86%	32.	3%	33.	9%
34.	14.5%	35.	12.9%	36.	8.6%	37.	100%
38.	98.4%	39.	62.5%	40.	0.1%	41.	(1/10)%

Replace ___ with > , < or = (Try to think of the percent equivalent for each number before deciding.)

42.	20% ___ 0.2	43.	25% ___ 0.3	44.	100% ___ 0.100
45.	0.6 __ 6%	46.	17% ___ 1.7	47.	4.5 ___ 4.5%
48.	2% ___ 0.25	49.	1% ___ 1	50.	30 ___ .30%

Complete the table.

	Fraction (Reduced)	Fraction (N/100)	Decimal	Percent
51.	$\frac{1}{2}$	$\frac{50}{100}$	0.50	
52.		$\frac{25}{100}$		
53.			0.2	
54.				10%
55.			0.04	
56.				45%

LESSON 12F

Draw a picture to solve.

1. $\frac{1}{2}$ of 6
2. $\frac{1}{4}$ of 12
3. $\frac{1}{3}$ of 1
4. 0.1 of 20
5. 0.25 of 2
6. 0.2 of 1
7. 25% of 8
8. (33 1/3)% of 6
9. 50% of 9
10. (12 1/2)% of 16
11. 10% of 1
12. 100% of 6.5

Rewrite each problem as a multiplication problem, then solve.

13. $\frac{1}{4}$ of 20
14. $\frac{1}{3}$ of 15
15. $\frac{1}{10}$ of 30
16. $\frac{1}{8}$ of 24
17. $\frac{1}{10}$ of 95
18. $\frac{1}{5}$ of 40
19. 0.2 of 6
20. 0.5 of 70
21. 0.8 of 30
22. 0.25 of 60
23. 0.24 of 100
24. 0.09 of 100

25. 10% of 20 26. (33 1/3)% of 12 27. (12 1/2)% of 16

28. 50% of 70 29. 20% of 50 30. 100% of 8.9

Solve the following.

31. (33 1/3)% of 300 32. 50% of 65 33. 25% of 9

34. 20% of 500 35. 10% of 800 36. 50% of 7

37. (12 1/2)% of 160 38. (12 1/2)% of 8 39. 20% of 200

40. 10% of 60 41. 25% of 4000 42. (33 1/3)% of 60

43. 50% of 37 44. (33 1/3)% of 120 45. 25% of 30

LESSON 12P

Solve the following for N.

1. $\frac{1}{2} = \frac{N}{6}$
2. $\frac{1}{4} = \frac{N}{12}$
3. $\frac{1}{3} = \frac{N}{1}$
4. $0.1 = \frac{N}{20}$
5. $0.25 = \frac{N}{2}$
6. $0.2 = \frac{N}{1}$
7. $25\% = \frac{N}{8}$
8. $(33\ 1/3)\% = \frac{N}{6}$
9. $50\% = \frac{N}{9}$
10. $10\% = \frac{N}{1}$
11. $100\% = \frac{N}{.5}$
12. $(12\ 1/2)\% = \frac{N}{16}$

Rewrite each problem as a proportion, then solve.

13. $\frac{1}{4}$ of 20
14. $\frac{1}{3}$ of 15
15. $\frac{1}{10}$ of 30
16. $\frac{1}{8}$ of 24
17. $\frac{1}{10}$ of 95
18. $\frac{1}{5}$ of 40
19. 0.2 of 6
20. 0.5 of 70
21. 0.8 of 30
22. 0.25 of 60
23. 0.24 of 100
24. 0.09 of 100

25. 10% of 20 26. (33 1/3)% of 12 27. (12 1/2)% of 16

28. 50% of 70 29. 20% of 50 30. 100% of 8.9

Solve the following.

31. (33 1/3)% of 300 32. 50% of 65 33. 25% of 9

34. 20% of 500 35. 10% of 800 36. 50% of 7

37. (12 1/2)% of 160 38. (12 1/2)% of 8 39. 20% of 200

40. 10% of 60 41. 25% of 4000 42. (33 1/3)% of 60

43. 50% of 37 44. (33 1/3)% of 120 45. 25% of 30

LESSON 13

Find N.

1. 1 out of 100 is N out of 400
2. 1 out of 100 is N out of 700
3. 1 out of 100 is N out of 500

Write only the answer, complete the table.

	100%	50%	10%	1%
4.	800			
5.	400			
6.	200			
7.	750			
8.	420			
9.	280			
10.	683			
11.	897			
12.	621			
13.	50			
14.	40			
15.	70			

Circle the correct number. Use you knowledge of 100%, 50%, 10% and 1% to estimate the answer.

16. 10% of 500 is 5 50 500
17. 20% of 500 is 10 100 1000
18. 30% of 500 is 1.5 15 150
19. 40% of 500 is 200 2000 20000
20. 80% of 500 is 400 4000 40000
21. 40% of 60 is 24 240 2400
22. 60% of 30 is 1.8 18 180
23. 70% of 800 is 56 560 5600
24. 13% of 3000 is 39 390 3900

25. 56% of 200 is 112 1120 11200

26. 12% of 80 is .96 9.6 96

27. 8% of 520 is 4.16 41.6 416

28. 31% of 10000 is 31 310 3100

29. 76% of 800 is 608 6080 60800

30. 49% of 50 is 2.45 24.5 245

31. 6% of 830 is 49.8 498 4980

32. 35% of 6 is 2.1 21 210

33. 45% of 110 is 49.5 495 4950

34. 81% of 4000 is 324 3240 32400

35. 5% of 60 is .3 3 30

36. 11% of 400 is 4.4 44 440

37. 26% of 2000 is 52 520 5200

*38. 130% of 300 is 3.9 39 390

*39. 200% of 60 is 12 120 1200

Replace ___ with > , < or =

40. 10% of 50 ___ 10% of 60

41. 10% of 20 ___ 100% of 20

42. 1% of 10 ___ 10% of 1

43. 10% of 0.25 ___ 10% of 0.3

*44. 1% of $\frac{1}{2}$ ___ 1% of $\frac{1}{3}$

*45. 1% of $\frac{1}{10}$ ___ 10% of $\frac{1}{10}$

LESSON 14F

Rewrite each problem as a decimal multiplication, then solve.

1. 15% of 80
2. 33% of 90
3. 60% of 100
4. 80% of 50
5. 75% of 120
6. 9% of 30
7. 2% of 100
8. 7% of 90
9. 86% of 840
10. 97% of 650
11. 37.5% of 960
12. 70% of 10

Solve each problem in your head.

13. 10% of 60
14. 10% of 600
15. 50% of 640
16. 10% of 1680
17. 1% of 840
18. 1% of 6740

Solve the following in your head, or use multiplication.

19. 25% of 400
20. 10% of 850
21. 12.5% of 16
22. (33 1/3)% of 600
23. 43% of 98
24. 60% of 120
25. 1% of 650
26. 50% of 320
27. 90% of 200
28. 4% of 830
29. 7.6% of 900
30. 20% of 350

31. 30% of the cherry crop was lost due to frost. If the full crop was estimated to be 650 tons, how much of the crop was lost?

32. Mr. Black made a 7% commission when he sold a house for $80,000. How much was his commission?

33. A car is sold for $850. Sales tax is 4%. What is the amount of sales tax paid on the car?

34. What is the amount of sales tax you pay, in Michigan, on a $7.50 purchase?

35. There are 50 students in a choir. 40% of the choir members are boys? How many boys are in the choir? How many girls are in the choir?

36. A woman makes a 10% down payment on a $5,500 car. How much money did she put down?

LESSON 14P

Rewrite each problem as a proportion using N for the missing term, then solve each problem.

1. 15% of 80
2. 33% of 90
3. 55% of 100
4. 75% of 50
5. 75% of 120
6. 9% of 30
7. 2% of 100
8. 7% of 90
9. 86% of 840
10. 97% of 650
11. 37.5% of 960
12. 70% of 10

Solve each problem in your head.

13. 10% of 60
14. 10% of 600
15. 50% of 640
16. 10% of 1680
17. 1% of 840
18. 1% of 6740

Solve the following in your head, or set up proportions and use cross products.

19. 25% of 400
20. 10% of 850
21. 12.5% of 16
22. (33 1/3)% of 600
23. 43% of 98
24. 60% of 120
25. 1% of 650
26. 50% of 320
27. 90% of 200
28. 4% of 830
29. 7.6% of 900
30. 20% of 350

31. 30% of the cherry crop was lost due to frost. If the full crop was estimated to be 650 tons, how much of the crop was lost?

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35. There are 50 students in a choir. 40% of the choir members are boys? How many boys are in the choir? How many girls are in the choir?

36. A woman makes a 10% down payment on a $5,500 car. How much money did she put down?

LESSON 15

Starting price ---(+ mark up / - discount)---> Ending price

Complete the table.

	Original Price (whole)	Percent Discount	Amount of Discount (part)	Sale Price
1.	$40	25%		
2.	$60	(33 1/3)%		
3.	$70	50%		
4.	$15	10%		
5.	$800	30%		
6.	$9	25%		
7.	$65	20%		
8.	$3	40%		
9.	$40	(37 1/2)%		

Complete the table.

	Purchase Price (whole)	Percent Mark up	Amount of Mark up (part)	Selling Price
10.	$650	8%		
11.	$400	12%		
12.	$842	10%		
13.	$100	5.5%		
14.	$128	5%		
15.	$65	50%		
16.	$80	15%		
17.	$12	2%		

Solve the following.

18. The original price on a bike is $100. If the bike is sold with a 25% discount, how much money does the buyer save?

19. Mr. Brown bought $3,000 worth of merchandise. If he received a 2% discount, how much money did he save?

20. Jim received a 10% increase in pay. If his old pay was $5/hour, how large was his raise?

21. A store marks up electronics equipment 20% before it's sold. How large is the mark up on a television which was purchased for $210?

22. A farmer is using a new type of fertilizer, which promises a 15% increase in the yield from his crops. If his previous yield was 300 metric tons, how much of an increase should he expect with th new fertilizer?

23. A store owner gets a 12.5% discount when buying large lots of goods. How much does she save on $1600 of goods?

*24. A store sells day old baked goods at 20% off. How much will a person pay for a $1.35 cake?

*25. What percent mark up is being added to an item which is purchased for $40 and sold for $45?

LESSON 16

How would you solve the problem? Set up, but DO NOT SOLVE problems 1-8.

1. $\frac{1}{4}$ of 40
2. 25% of 200
3. What % of 10 is 5?
4. 0.2 of 70
5. 65.2% of 432
6. 0.72 of 682
7. 4.8% of 1672
8. 2 compared to 3 is what %?
9. What % of 40 is 30?
10. 10% of 680

Solve the following.

11. 1% of 600 =
12. 10% of 800 =
13. Find 10% of 70
14. 1% of 150 =
15. 10% of 85 =
16. $\frac{30}{100}$ = %
17. 20 is ___% of 100
18. ___% of 100 = 40
19. 3 out of 100 = ___ %
20. 3 out of 4 = ___%
21. 6 per 100 = ___%
22. 3 compared to 9 is ___%
23. What % of 100 is 7?
24. 3 compared to 8 is ___%
25. What % of 50 is 5?
26. What % of 200 is 10?
27. 6 is what % of 12?
28. 9 is what % of 45?

29. Find the amount of discount on a dress marked $25, but on sale with a 20% discount.

30. John's allowance is $4.50 a week. If he saves $1.50 each week, what percent of his allowance does he save?

31. Ruth won 6 sets of tennis and her brother won 10. What percent of the total sets played did Ruth win?

32. Tom put 75 eggs in an incubator and 60 hatched. What percent of the eggs hatched?

33. A sofa marked $120, is sold with a 5% discount for paying cash. How much money was saved? What did the sofa sell for?

34. There were 42 students in the eighth grade class. (33 1/3)% of the students are boys. How many students are boys?

35. Compute the 4% sales tax on a $36 purchase.

*36. A company was able to reduce its losses, due to waste, from $540 to $162. What was the percent decrease in its losses?

LESSON 17

Write as percents.

1. $\frac{24}{100}$ 2. $\frac{5}{100}$ 3. $\frac{1}{100}$ 4. 0.75

5. 0.4 6. 0.03 7. 0.125 8. 0.005

9. $\frac{1}{3}$ 10. $\frac{3}{8}$ 11. $\frac{1}{5}$ 12. $\frac{3}{4}$

Write as fractions with 100 as the denominator.

13. 41% 14. 0% 15. 7% 16. 40%

17. 0.1 18. 0.35 19. 0.03 20. 0.70

21. $\frac{9}{20}$ 22. $\frac{1}{4}$ 23. $\frac{3}{5}$ 24. $\frac{1}{10}$

Write as decimals.

25. 23% 26. 3% 27. 60% 28. 62.5%

29. $\frac{3}{10}$ 30. $\frac{80}{100}$ 31. $\frac{25}{100}$ 32. $\frac{125}{1000}$

Solve the following.

33. What % of 5 is 3?

34. 10% of 16 =

35. 15% of 70 =

36. What % of 80 is 30?

37. What % of 400 is 300?

38. Find 23% of 45

39. 3% of 150 =

40. 4 out of 12 = ___%

41. Find 10% more than 7

42. Find 5% less than 20

43. Find 25% less than 60

44. Find 100% more than 8.5

45. What is the percent increase in price on a radio purchased for $60 and sold for $75?

46. What is the sale price of a $30 pair of jeans sold with a 15% discount?

47. Who has the better free throw average, Bob who is shooting 14 for 16 or Jody who is shooting 20 for 25? (Compute the percent of free throws each makes.)

48. On an opinion poll 30 people favored a proposition, 15 people were opposed and 5 people had no opinion. What percent of the people are in each category?

49. Compute 4% sales tax on a $72 purchase.

APPENDIX C

TEST INSTRUMENTS

Name _____

Pretest 1/82

Write your answers in the blanks. Please show all your work in the space provided on your test paper.

Write a fraction for the shaded part of each figure.

1. _____ 

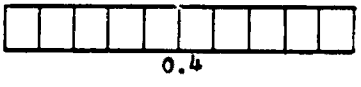
2. _____ 

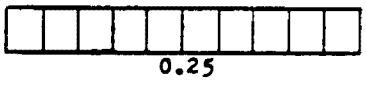
Write a percent for the shaded part of each figure.

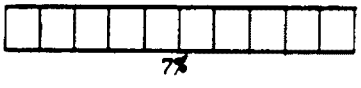
3. _____ 

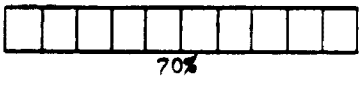
4. _____ 

Shade each drawing to show the decimal or percent.

5. 

6. 

7. 

8. 

Write a fraction with a denominator of 100 for each.

9. _____ 0.5

10. _____ 0.07

11. _____ 0.13

12. _____ $\frac{3}{4}$

13. _____ $\frac{2}{5}$

14. _____ $\frac{3}{3}$

15. _____ 8%

16. _____ 49%

17. _____ 90%

Write a decimal equivalent to each.

18. _____ $\frac{1}{2}$

19. _____ $\frac{3}{4}$

20. _____ $\frac{4}{100}$

21. _____ $\frac{3}{5}$

22. _____ 17%

23. _____ 4%

24. _____ 125%

25. _____ 23.7%

Write a fraction equivalent to each. Then write each fraction in simplest terms.

	Fraction		Simplest Terms
26.	_____	27. _____	0.4
28.	_____	29. _____	0.84
30.	_____	31. _____	0.007
32.	_____	33. _____	0.370
34.	_____	35. _____	6%
36.	_____	37. _____	23%
38.	_____	39. _____	75%
40.	_____	41. _____	40%

Write as percents.

42. _____ $\frac{5}{100}$

43. _____ $\frac{69}{100}$

44. _____ $\frac{2}{5}$

45. _____ $\frac{9}{10}$

46. _____ 0.98

47. _____ 0.6

48. _____ 0.837

49. _____ 1.0

Write an equivalent decimal (to thousandths) and a percent for each.

Decimal	Percent
50. _____	51. _____ $\frac{1}{4}$
52. _____	53. _____ $\frac{1}{8}$
54. _____	55. _____ $\frac{1}{3}$

Solve the following. Show all your work.

56. _____ What is $\frac{1}{4}$ of 80 ?

57. _____ What is 0.40 of 80 ?

58. _____ 0.27 x 320 =

59. _____ Find 1% of 600

60. _____ Find 12% of 60

61. _____ Find 30% of 25

62. _____ What percent of 10 is 6 ?

63. _____ What percent of 32 is 8 ?

64. _____ What percent of 20 is 1 ?

65. _____ A candy bar is broken into 3 pieces of the same size. Each piece is what part of the candy bar?

66. _____ There are 13 boys and 15 girls in a group. What fractional part of the group is boys?

67. _____ 50% of 50 is ____ times as much as 25% of 50

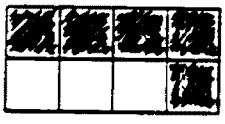
68. _____ 10% of the trees in a park are elm, 28% are maple and 32% are spruce. What percent of the trees are other kinds?

69. _____ In a school election with 3 candidates, Joe received 120 votes, Mary received 50 votes and George received 30 votes. What percent of the total votes did Joe receive?

Name _____

Quiz 1

1. Write a fraction that names the shaded part of the whole figure.



2. Draw a diagram which shows $\frac{3}{4}$

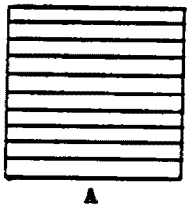
Write the missing numerator.

3. $\frac{1}{4} = \frac{\quad}{8}$ 4. $\frac{2}{4} = \frac{\quad}{2}$ 5. $\frac{2}{5} = \frac{\quad}{10}$ 6. $1 = \frac{\quad}{100}$

Name _____

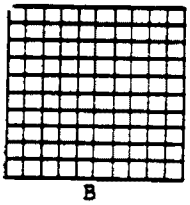
Quiz 2

1. Shade 25 hundredths of A



A

2. Shade 3 tenths of B



B

Write the missing numerator.

3. $\frac{7}{8} = \frac{\quad}{24}$ 4. $\frac{7}{20} = \frac{\quad}{100}$ 5. $\frac{6}{14} = \frac{\quad}{7}$ 6. $\frac{60}{100} = \frac{\quad}{5}$

Name _____

Quiz 3

Find the number which sould replace X to make each a true sentence. Show all your work.

1. ___ $\frac{X}{8} = \frac{5}{10}$ 2. ___ $\frac{X}{10} = \frac{12}{5}$ 3. ___ $\frac{5}{8} = \frac{X}{100}$ 4. ___ $\frac{4}{10} = \frac{X}{15}$

Solve the following using proportions.

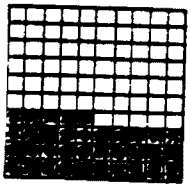
5. _____ Jenny got 6 hits in 10 times at bat. How many hits is she likely to get in 100 times at bat?

6. _____ 12 computer chips out of every 25 are defective. How many defective chips would you find out of every 100?

Name _____

Quiz 4

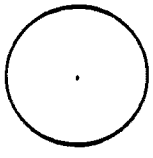
1. What percent of the diagram is shaded?



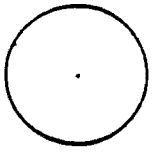
2. Put ▨ on 23% of the diagram.

Draw lines which would approximately divide each circle according to the percents given.

3. 50% 25% 25%



4. 70% 20% 10%



Solve the following. Show all your work.

5. _____ If a student gets 80% of the test questions right, what percent did she get wrong?

6. _____ Two friends pool their money to buy a car. If they are equal partners, what percent of the money does each contribute?

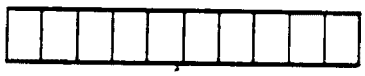
7. _____ There are 13 boys and 15 girls in a group. What fractional part of the group is boys?

Name _____

Quiz 5

1. Put ▨ on 20% of the diagram.

2. Put ■ on 5% more.



Write equivalent percents for each.

3. _____ $\frac{39}{100}$ 4. _____ $\frac{4}{5}$ 5. _____ $\frac{4}{200}$ 6. _____ $\frac{1}{3}$

First write a fraction for each percent. Then write each fraction in simplest terms.

		23%	5%	60%
Fraction	7.	_____	9. _____	11. _____
Simplest Terms	8.	_____	10. _____	12. _____

Name _____

Quiz 6

Solve the following. Show all your work.

1. _____ 3 per 100 = ____%

2. _____ What percent of 8 is 2 ?

3. _____ 30 is what percent of 50 ?

4. _____ 6 is what percent of 300 ?

5. _____ 4 out of 12 = ____%

6. _____ Jim made 19 out of 20 free throws. What percent of his free throws did Jim make?

Name _____

Quiz 7

Solve the following. Show all your work.

1. _____ A $3 discount on a $10 shirt is what fraction discount?

2. _____ A $3 discount on a $10 shirt is what percent discount?

3. _____ The sale price is $40. The original price was $50. What is the percent discount?

4. _____ An increase in price from $12 to $15 is what percent mark up?

5. _____ A man buys a product for $25, marks it up $5 and resells it. What is the percent mark up?

Name _____

Quiz 8

Write decimals equivalent to each.

1. _____ 4 tenths 2. _____ $\frac{7}{100}$ 3. _____ $\frac{1}{10}$

Write fractions equivalent to each.

4. _____ 0.9 5. _____ 0.03 6. _____ 57 hundredths

7. Shade the diagram to show 7%



First write a fraction equivalent to each. Then write the fraction in simplest terms.

	Fraction	Simplest Terms
8.	_____	
9.	_____	40%
10.	_____	
11.	_____	6%

Solve the following. Show all your work.

12. _____ What percent of 32 is 8 ?

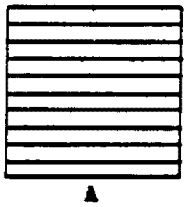
13. _____ There are 12 boys and 13 girls in a group. What fractional part of the group is girls?

14. _____ 10% of the trees in a park are elm, 28% are maple and 32% are spruce. What percent of the trees are other kinds?

Name _____

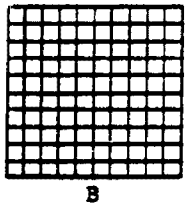
Quiz 9

1. Shade 0.35 of A



A

2. Shade 0.5 of B



B

Complete the following.

3. 0.7 = ___ hundredths

4. 0.57 = ___ hundredths

5. 0.03 = ___ hundredths

Write a decimal equivalent to each fraction.

6. _____ $\frac{63}{100}$

7. _____ $\frac{1}{4}$

8. _____ $\frac{7}{10}$

Name _____

Quiz 10

Write decimals equivalent to each.

1. _____ 42 thousandths

2. _____ four and 3 thousandths

3. _____ $\frac{80}{1000}$

Complete the following with whole numbers.

4. 0.4 = ___ thousandths

5. 0.12 = ___ thousandths

6. 0.04 = ___ thousandths

7. 15 thousandths = ___ tenths ___ hundredths ___ thousandths

8. 562 thousandths = ___ tenths ___ hundredths ___ thousandths

Name _____

Quiz 11

Write decimals equivalent to each.

1. _____ 25% 2. _____ 3% 3. _____ 40%

4. _____ 7.8% 5. _____ 41.6% 6. _____ (62 1/2)%

Write percents equivalent to each.

7. _____ 0.63 8. _____ 0.3

9. _____ 0.07 10. _____ 0.423

Name _____

Quiz 12

Solve the following. Show all your work.

1. _____ $\frac{1}{4}$ of 80 2. _____ $\frac{3}{5}$ of 15 3. _____ $\frac{1}{10}$ of 25

4. _____ 0.03 of 20 5. _____ 0.4 of 80 6. _____ 0.66 of 50

7. _____ What percent of 36 is 9 ?

8. _____ There are 11 boys and 14 girls in a group. What fractional part of the group is girls?

9. _____ 20% of the trees in a park are spruce, 35% are birch and 14% are oak. What percent are other kinds?

10. Shade the drawing to show 9% 

11. _____ Write 23.7% as a decimal.

Write equivalent percents for each.

12. _____ 0.6 13. _____ 0.837

First write equivalent fractions for each. Then write each fraction in simplest terms.

	80%	4%
Fraction	14. _____	16. _____
Simplest Terms	15. _____	17. _____

Write an equivalent decimal (to nearest thousandth) and an equivalent percent for each.

	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{3}$
Decimal	18. _____	20. _____	22. _____
Percent	19. _____	21. _____	23. _____

Name _____

Quiz 13

Find 1%, 10% and 100% of each number.

	300	430	70
1%	1. _____	4. _____	7. _____
10%	2. _____	5. _____	8. _____
100%	3. _____	6. _____	9. _____

Select the best answer.

10. 12% of 70 is about 0.84 8.4 84 840

11. 24% of 30 is about 7 30 110 60

12. 95% of 600 is about 95 450 550 300

13. 15% of 15 is about 3 2.25 1.50 1

Name _____

Quiz 14

Solve the following. Show all your work.

1. _____ Find 30% of 50

2. _____ Find 4% of 80

3. _____ Find 23% of 40

4. _____ Find 7.4% of 200

5. _____ A man makes a 10% down payment on a $7000 car. How much money does he put down?

6. _____ 40% of the students in a school are girls. If there are 250 students in the school, how many are girls?

Name _____

Quiz 15

Solve the following. Show all your work.

1. _____ Bruce bought a suit with an original price of $70. If there was a 10% discount, how much did Bruce save?

2. _____ Janie received a 2% discount for paying cash. How much money does she save when she buys a $200 TV set?

3. _____ A store owner buys records for $8 and marks them up 25% How much does she mark up each record?

4. _____ A filling station owner buys gas for $.25 per liter and marks it up 20%. How much does he mark up each liter?

5. _____ A $30 pair of jeans is marked down in price to $27. What is the percent discount on the original price?

Name _____

Quiz 16

Solve the following. Show all your work.

1. _____ Find 10% of 30

2. _____ Find 25% of 12

3. _____ What % of 50 is 5 ?

4. _____ What % of 200 is 80 ?

5. _____ Find 2% of 30

6. _____ What % of 30 is 10 ?

7. _____ There are 9 boys and 11 girls in a choir. What percent of the choir members are boys?

8. _____ 15% of the students were absent from class last Tuesday. If there are 20 students in the class, how many were absent?

Name _____

Posttest 1/82

Write your answers in the blanks. Please show all your work in the space provided on your test paper.

Shade each drawing to show the percent.

1. 70%



2. 25%



3. 7%



Solve the following. Show all your work.

4. _____ Fred missed 30% of the problems on this test. What percent did he get right?

5. _____ Four women bought a house, each paying an equal part of the cost. What percent of the cost did each pay?

6. _____ Six boys live in the same house. All six play soccer. What percent of the boys play soccer?

7. _____ 10% of the trees in a park are elm, 28% are maple and 32% are spruce. What percent of the trees are other kinds?

Write a fraction with a denominator of 100 for each.

8. _____ $\frac{2}{5}$

9. _____ $\frac{3}{4}$

10. _____ $\frac{3}{3}$

11. _____ 0.3

12. _____ 0.07

13. _____ 0.97

Write as decimals.

14. _____ 47%

15. _____ 9%

16. _____ 50%

17. _____ 23.7%

18. _____ (62 1/2)%

Write a fraction for each. Then write each fraction in simplest terms.

	Fraction		Simplest Terms
19.	_____	20. _____	59%
21.	_____	22. _____	6%
23.	_____	24. _____	40%
25.	_____	26. _____	75%
27.	_____	28. _____	(66 2/3)%

Write as percents.

29. _____ $\frac{1}{5}$

30. _____ $\frac{7}{10}$

31. _____ $\frac{7}{100}$

32. _____ $\frac{3}{8}$

33. _____ $\frac{2}{3}$

34. _____ 0.13

35. _____ 0.6

36. _____ 0.03

37. _____ 0.837

38. _____ 1.0

Write a decimal equivalent (to nearest thousandth) and a percent for each.

	Decimal		Percent
39.	_____	40.	_____ $\frac{1}{4}$
41.	_____	42.	_____ $\frac{1}{8}$
43.	_____	44.	_____ $\frac{1}{3}$

Solve the following. Show all your work.

45. _____ Find 1% of 630

46. _____ Find 12% of 60

47. _____ Find 4% of 86

48. _____ Find (33 1/3)% of 90

49. _____ What is 20% of 80 ?

A. 4 B. 16 C. 20 D. 80 E. Don't Know

50. _____ A school is attended by 300 students. On any one day 9 students are usually absent. What percent of the students are usually absent?

51. _____ What percent of 10 is 6 ?

52. _____ What percent of 300 is 9 ?

53. _____ What percent of 32 is 8 ?

54. _____ What percent of 80 is 50 ?

55. _____ 15 is ____ % of 20 ?

A. 65% B. 70% C. 75% D. 80% E. Don't Know

56. _____ There are 13 boys and 15 girls in a group. What fractional part of the group is boys?

57. _____ Six members of a thirty member club paid their dues. What percent of the members have paid?

58. _____ Of 64 programs in a computer library, 8 are games. What percent of the computer programs are NOT games?

59. _____ In a school election Joe received 24 votes, Karen received 14 and Judy received 12. What percent of the total votes did Karen get?

60. _____ A radio purchased for $32, is marked up and sold for $40. What is the percent markup?

61. _____ A $75 suit is sold with a $15 discount. What is the percent discount on the original price?

62. _____ A woman made a 10% down payment on a $5800 car. How much money did she put down?

63. _____ Candidate A received 70 percent of the votes cast in an election. If 4200 votes are cast in the election, how many votes did he receive?

64. _____ If Michigan sales tax is 4%, how much tax is paid on a $26 purchase?

65. _____ 50% of 50 is ____ times as much as 25% of 50

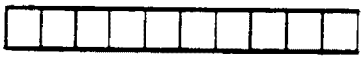
Name _____

Retention Test 1/82

Write your answers in the blanks. Please show all your work in the space provided on your test paper.

Shade each drawing to show the percent.

1. 70% 

2. 75% 

3. 7% 

Solve the following. Show all your work.

4. _____ Sue missed 20% of the problems on this test. What percent did she get right?

5. _____ Five men bought a farm, each paying an equal part of the cost. What percent of the cost did each pay?

6. _____ Six boys live in the same house. All six play soccer. What percent of the boys play soccer?

7. _____ 10% of the trees in a park are elm, 28% are maple and 32% are spruce. What percent of the trees are other kinds?

Write a fraction with a denominator of 100 for each.

8. _____ $\frac{1}{5}$

9. _____ $\frac{11}{20}$

10. _____ $\frac{8}{8}$

11. _____ 0.7

12. _____ 0.03

13. _____ 0.93

Write as decimals.

14. _____ 53%

15. _____ 5%

16. _____ 60%

17. _____ 23.7%

18. _____ (37 1/2)%

Write a fraction for each. Then write each fraction in simplest terms.

	Fraction	Simplest Terms
19.	_____	
20.	_____	67%
21.	_____	
22.	_____	6%
23.	_____	
24.	_____	40%
25.	_____	
26.	_____	75%
27.	_____	
28.	_____	(33 1/3)%

Write as percents.

29. _____ $\frac{2}{5}$

30. _____ $\frac{9}{10}$

31. _____ $\frac{3}{100}$

32. _____ $\frac{5}{8}$

33. _____ $\frac{2}{3}$

34. _____ 0.27

35. _____ 0.6

36. _____ 0.02

37. _____ 0.837

38. _____ 1.0

Write a decimal equivalent (to nearest thousandth) and a percent for each.

	Decimal	Percent
39.	_____	40. _____ $\frac{1}{4}$
41.	_____	42. _____ $\frac{1}{8}$
43.	_____	44. _____ $\frac{1}{3}$

Solve the following. Show all your work.

45. _____ Find 1% of 540

46. _____ Find 12% of 60

47. _____ Find 4% of 38

48. _____ Find 30% of 60

49. _____ Find (12 1/2)% of 40

50. _____ A school is attended by 300 students. On any one day 6 students are usually absent. What percent of the students are usually absent?

51. _____ What percent of 10 is 4 ?

52. _____ What percent of 300 is 6 ?

53. _____ What percent of 32 is 8 ?

54. _____ What percent of 80 is 70 ?

55. _____ What percent of 20 is 15 ?

56. _____ There are 13 boys and 15 girls in a group. What fractional part of the group is boys?

57. _____ Six members of a thirty member club paid their dues. What percent of the members have NOT paid?

58. _____ Of 64 programs in a computer library, 8 are games. What percent of the computer programs are games?

59. _____ In a school election Joe received 24 votes, Karen received 14 and Judy received 12. What percent of the total votes did Judy get?

60. _____ A $40 radio is sold with an $8 discount. What is the percent discount on the original price?

61. _____ A suit is purchased for $60, marked up $15 and sold. What is the percent mark up on the purchase price?

62. _____ A man made a 10% down payment on a $6400 car. How much money did he put down?

63. _____ Candidate A received 60 percent of the votes cast in an election. If 4200 votes are cast in the election, how many votes did he receive?

64. _____ If Michigan sales tax is 4%, how much tax is paid on a $52 purchase?

65. _____ 50% of 50 is ____ times as much as 25% of 50

Name _____

Interview
4/82

1. Write $\frac{1}{2}$ as a decimal and as a percent.

2. Write 0.53 as a fraction.

3. Write 38.6% as a decimal.

4. Which is bigger $\frac{2}{5}$ or 25%? Why?

5. Jim has 15 red marbles and 14 blue marbles. What fractional part of his marbles are blue?

6. A pie is divided in 5 equal pieces. What percent of the whole is each piece?

7. On a 100 item test Jenny missed 13 items. What percent of her test did she get right?

8. What is 25% of 40?

9. What percent of 15 is 10?

* 10. What number is 10% more than 50?

APPENDIX D

QUIZ ITEM ANALYSES

Table 24

Percent of Correct Responses on Quiz 1
Items by Treatment Group

Item	Factor Product		Proportion	
	N	%	N	%
1	129	97	115	98
2	129	97	115	98
3	129	97	111	94
4	124	93	107	91
5	123	92	105	89
6	132	99	117	99

Table 25

Percent of Correct Responses on Quiz 2
Items by Treatment Group

Item	Factor Product		Proportion	
	N	%	N	%
1	78	58	78	65
2	102	76	92	77
3	133	99	116	97
4	130	97	116	97
5	125	93	114	95
6	127	95	101	84

Table 26

Percent of Correct Responses on Quiz 3 Items by Treatment Group

Item	Factor Product		Proportion	
	N	%	N	%
1	116	88	100	86
2	110	83	87	75
3	72	54	74	64
4	109	83	91	78
5	119	90	101	87
6	109	83	88	76

Table 27

Percent of Correct Responses on Quiz 4 Items by Treatment Group

Item	Factor Product		Proportion	
	N	%	N	%
1	127	97	111	96
2	128	98	116	100
3	128	98	114	98
4	112	86	101	87
5	131	100	114	98
6	124	95	115	99
7	80	61	66	57

Table 28

Percent of Correct Responses on Quiz 5
Items by Treatment Group

Item	Factor Product		Proportion	
	N	%	N	%
1	121	92	107	93
2	105	80	89	77
3	124	95	109	95
4	108	82	82	71
5	94	72	73	64
6	80	61	68	59
7	128	98	108	94
8	105	80	88	76
9	125	95	108	94
10	76	58	62	54
11	128	98	110	96
12	64	49	66	57

Table 29

Percent of Correct Responses on Quiz 6
Items by Treatment Group

Item	Factor Product		Proportion	
	N	%	N	%
1	114	89	105	92
2	104	81	84	74
3	113	88	105	92
4	100	78	94	82
5	80	62	69	60
6	111	87	103	90

Table 30

Percent of Correct Responses on Quiz 7
Items by Treatment Group

Item	Factor Product		Proportion	
	N	%	N	%
1	105	83	96	84
2	105	83	98	85
3	58	46	62	54
4	47	37	48	42
5	105	83	92	80

Table 31

Percent of Correct Responses on Quiz 8
Items by Treatment Group

Item	Factor Product		Proportion	
	N	%	N	%
1	121	95	106	96
2	115	91	98	89
3	118	93	105	96
4	122	96	105	96
5	122	96	103	94
6	126	99	105	96
7	30	24	25	23
8	120	94	107	97
9	73	58	67	61
10	105	83	89	81
11	71	56	66	60
12	80	63	63	57
13	93	73	88	80
14	108	85	94	86

Table 32

Percent of Correct Responses on Quiz 9 Items by Treatment Group

Item	Factor Product		Proportion	
	N	%	N	%
1	111	88	101	92
2	63	50	51	46
3	92	73	70	64
4	99	79	87	79
5	108	86	89	81
6	123	98	100	91
7	100	79	73	66
8	117	93	103	94

Table 33

Percent of Correct Responses on Quiz 10 Items by Treatment Group

Item	Factor Product		Proportion	
	N	%	N	%
1	121	94	104	90
2	121	94	107	93
3	117	91	99	86
4	89	70	84	73
5	100	78	91	79
6	90	70	89	77
7	112	88	107	93
8	122	95	114	99

Table 34

Percent of Correct Responses on Quiz 11 Items by Treatment Group

Item	Factor Product		Proportion	
	N	%	N	%
1	126	98	116	98
2	113	88	102	86
3	123	95	111	94
4	91	70	75	64
5	97	75	91	77
6	67	52	61	52
7	124	96	113	96
8	109	84	94	80
9	116	90	104	88
10	90	70	77	65

Table 35

Percent of Correct Responses on Quiz
12 Items by Treatment Group

Item	Factor Product		Proportion	
	N	%	N	%
1	124	95	107	91
2	96	74	94	80
3	102	78	98	83
4	61	47	61	52
5	78	60	79	67
6	79	61	78	66
7	42	32	47	40
8	110	85	103	87
9	101	78	84	71
10	100	77	96	81
11	96	74	75	64
12	106	82	88	75
13	85	65	72	61
14	127	98	114	97
15	84	65	85	72
16	119	92	104	88
17	87	67	71	60
18	92	71	90	76
19	109	84	98	83
20	52	40	33	28
21	92	71	84	71
22	46	35	19	16
23	98	75	86	73

Table 36

Percent of Correct Responses on Quiz
13 Items by Treatment Group

Item	Factor Product		Proportion	
	N	%	N	%
1	115	90	104	91
2	119	93	105	92
3	114	89	105	92
4	108	84	90	79
5	115	90	101	89
6	111	87	102	90
7	100	78	95	83
8	106	83	91	80
9	109	85	99	87
10	117	91	95	83
11	110	86	92	81
12	111	87	95	83
13	69	54	55	48

Table 37

Percent of Correct Responses on Quiz
14 Items by Treatment Group

Item	Factor Product		Proportion	
	N	%	N	%
1	110	86	92	84
2	92	72	86	78
3	107	84	92	84
4	58	45	65	59
5	105	82	93	84
6	98	77	83	76

Table 38

Percent of Correct Responses on Quiz
15 Items by Treatment Group

Item	Factor Product		Proportion	
	N	%	N	%
1	112	89	97	87
2	101	80	96	86
3	109	86	92	83
4	65	52	53	48
5	38	30	38	34

Table 39

Percent of Correct Responses on Quiz
16 Items by Treatment Group

Item	Factor Product		Proportion	
	N	%	N	%
1	105	81	94	82
2	97	75	79	69
3	112	86	94	82
4	90	69	76	66
5	65	50	65	57
6	64	49	68	59
7	89	68	60	52
8	89	68	86	75

APPENDIX E

STUDENT INTERVIEW WRONG ANSWERS

Wrong Answers For Factor Product Students (N=16) and Proportion Students (N=13)

1. Write 1/2 as a decimal and as a percent.

Factor Product Student Answers

.05 5%	1/2 is .05 which equals 5% (1 student)
.5 1/2%	1/2 is .5, written as a percent is 1/2% (1 student)
No Answer	I don't know how to do this (1 student)

Proportion Student Answers

.5 50/100	Percent means out of 100, so 50 out of 100 (1 student)
.01 505	1/2 equals .01 because it's one hundredth (1 student)
.5 20%	Must multiply .5 and 20 to get 100, so 1/2 equals 20% (1 student)
2.5 50%	1/2 is 2 point 5 (1 student)
.5	No reason given (1 student)

2. Write 0.53 as a fraction.

Factor Product Student Answers

0.53/100	It's in the hundredths place so write it over 100 (1 student)

Proportion Student Answers

53%	It's in the hundredths place, move it two places over (1 student)

3. Write 38.6% as a decimal.

Factor	Product Student Answers
3.86	Just take away the percent sign (1 student)
38.06	I think it's 38.06 (1 student)

Proportion	Student Answers
3.86%	Just move the decimal point one place to make it one hundredth (1 student)
38.6	Just write 38.6, I'm not sure (1 student)

4. Which is bigger 2/5 or 25%? Why?

Factor	Product Student Answers
25%	25% equals 1/4 which is greater than 2/5 (1 student)
25%	Because it's bigger (1 student)
25%	I don't know why (1 student)
2/5	2/5 equals 33 1/3 % which is greater than 25% (1 student)
2/5	2/5 is only 10 (1 student)

Proportion	Student Answers
25%	Because it's equal to 3/4 (1 student)
25%	25% = 1/4 out of 100 and 1/4 > 2/5 (1 student)
25%	25% is just more (1 student)
25%	I don't know why (1 student)
2/5	2 point 5 is 50% and 50% > 25% (1 student)
2/5	(2/5) x (20/20) = 42/100 (1 student)

5. Jim has 15 red marbles and 14 blue marbles. What fractional part of his marbles are blue?

Factor Product	Student Answers
14/100	I'm not too good at story problems (1 student)
3/20	15/100 equals 3/20 (1 student)
15/29	Find the denominator, 14 plus 15, the top would be 15 (1 student)
14/15	No reason given (2 students)

Proportion	Student Answers
15/29	No reason given (1 student)

6. A pie is divided in 5 equal pieces. What percent of the whole is each piece?

Factor Product	Student Answers
1%	Percent means over 100, 5 pieces is 5%, so one piece is 1% (1 student)
95%	100 plus 5 equals 105 and 100 minus 105 is 95 (1 student)
No answer	Not sure skip it (2 students)

Proportion	Student Answers
70%	I'm not sure why (1 student)
5/100	No reason given (1 student)

7. On a 100 item test Jenny missed 13 items. What percent of her test did she get right?

Factor	Product Student Answers
13%	13/100 is 13% (1 student)
77%	100 plus 13 equals 123 and 100 minus 123 equals 77 (1 student)
14	100-13 = 87, (13/87) x 100 = 1300/87 = 14 (1 student)

Proportion	Student Answers
13	The part is 13, the whole is 100, 13/100 = 13% right (3 students)
11 31/100	13/100 = N/87, 100N/100 = 1311/100, N = 11 31/100 (1 student)

8. What is 25% of 40?

Factor	Product Student Answers
10%	40 x .25 = 10 (1 student)
16%	Multiply (25/100) by 40 and get 4000, then divide by 25 and get 16 (1 student)
62.5%	25/40 = N/100, N = 2500/40 = 62.5 (1 student)

Proportion	Student Answers
10%	25% of 40 is 10% or 40 divided by 4 equals 10% (2 students)
15	40 minus 15 is 25 (1 student)
160	25/200 = 40/N, 1/4 = 40/N, N = 160 (1 student)
8/40	25/100 = N/40, 100N = 800, 8/40 (1 student)

9. What percent of 15 is 10?

Factor Product	Student Answers
1.5	Multiply 15 by .1 (1 student)
60.6	10/15 = N/100, 1000 divided by 15 equals 60.6 (1 student)
16 10/15 %	10/15 = N/100, 1000 divided by 15 equals 60 10/15 (1 student)
150	Multiply 15 by 10 (1 student)
15%	15/10 = 15/(10 x 10) = 15% (1 student)
60%	10 divided by 15 equals .606 (1 student)
66%	10 divided by 15 equals .66 (2 students)

Proportion	Student Answers
1.5	15/100 = N/10, 150/100 = (100/100) x N, N = 1.5 (1 student)
73 1/5	N/100 = 10/15, 1000 divided by 15 equals 73 1/5 (1 student)
5%	15 minus 10 is 5 (2 students)
66 1/3	10/5 = N/100, 1000 divided by 15 is 66 10/15 which is 66 1/3 (1 student)
66 1/6 %	N/100 = 10/15, 15N = 1000, 1000 divided by 15 is 66 1/6 (1 student)

10. What number is 10% more than 50?

Factor Product Student Answers	
60%	10% means 10 out of 100, 50 out of 100 plus 10 out of 100 is 60 (7 students)
No answer	Don't know how to do this (4 students)
5	10/100 = 50/1 = 500/100 = 5 (2 students)
70	10/50 = 20/100, 50 plus 20 is 70 (1 student)
20	No reason given (1 student)

Proportion Student Answers	
5/50	10/100 = N/50, (10/2) divided by (100/2) is 5/50 (1 student)
70	I guess, 10% of 50 is 5, 5 is 10% more, so put 50 on top, 10/100 = 50/N (1 student)
5	10/100 = N/50, N=5 (4 students)
60	50/100 plus 10/100 is 60/100 (1 student)
5%	I don't know (1 student)
500	10/100 = 50/N, 1/10 = 50/N, N/500 (2 students)
80	I guess (1 student)
No answer	(2 students)

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