IMPACT OF GOVERNMENT GUARANTEES ON DEBT CAPACITY AND OPTIMAL CAPITAL STRUCTURE OF PRIVATELY-FINANCED INFRASTRUCTURE PROJECTS

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ABSTRACT

Governments use concession agreements and guarantees to induce the development and operation of many types of infrastructure projects by the private sector in which private promoters form a separate owning company that finances, builds, and operates the facility. Key decisions for promoters include the project's financial feasibility and its debt/equity capital structure. The formation of a separate owning company allows off-balance-sheet financing where the project must repay its debt through its own net operating income without other collateral. This paper investigates the effect of government guarantees on the amount of debt that a project can accommodate under the possibility of bankruptcy (its debt capacity), as well as on its optimal capital structure (debt and equity). An important finding is that investors should not borrow as much as debt capacity but should instead seek to maximize the project's net present value or alternatively the investors' return on equity. These concepts are illustrated with an example that determines and compares debt

capacity and optimal capital structure for a project that has no guarantees, a production guarantee, or a minimum-revenue guarantee.

INTRODUCTION

As existing infrastructure ages and demand for new facilities increases due to population growth and technological advancement, governments worldwide no longer have funds in place, nor the required bonding capacity to finance all the public facilities, public services, and infrastructure that they would like to provide (World Bank 2017). In addition, there is increased understanding among governments that they should not own and/or operate certain types of facilities and infrastructure because of their less effective utilization of resources when compared with the more flexible and cost-conscious private sector. Private enterprise can benefit from these opportunities by providing its financial resources as well as its technical and managerial skills to increase its share of the infrastructure market. In 2023, for example, infrastructure investments with private participation amounted to $86 billion, representing 0.2 percent of the GDP of all low- and middle-income countries (World Bank 2023).

The use of public–private partnerships (PPP) to develop infrastructure projects through concession agreements that leverage private financing is not new and dates back to the early 19th century (Irwin 2007). Since then, PPP agreements have also often included government guarantees that aim to attract the interest of private project promoters by enhancing the project's financial viability and mitigating the promoters' risk exposure. In doing so, it is understood that an equitable risk-sharing principle is to structure concessions and guarantees so that project risks are allocated to the parties that can best control and manage those risks.

This paper examines the effect of government guarantees on the financial structure of privately-financed infrastructure projects and in particular on their debt capacity and optimum financial structure under the risk of bankruptcy. Debt capacity represents the maximum amount of debt that can be raised to finance a project. The optimal financial structure is that mixture of debt and equity that maximizes a financial objective such as the equityholders' return on investment, or the project's net present value. A project's financial structure is of paramount importance to the promoting team

and in particular to construction contractors with limited funds because it constrains their ability to go ahead with the project.

An important consideration in the evaluation of risky debt is to include explicitly both the possibility of bankruptcy and the effect of taxes as these are the two main factors that influence debt policy (Brealey et al. 2022). The explicit consideration of bankruptcy costs is of particular importance because privately-financed projects often employ off-balance-sheet financing and provide no collateral to debtholders.

A market equilibrium approach is used to determine the value of the project and its financial components (debt and equity) and to show that the presence of guarantees increases the debt capacity of the owning company as well as the value of its equity as the project becomes less risky. As a result, guarantees increase the project's net present value and the investors' return on investment. The determination of the market value of debt and equity in the presence of a production or a minimum revenue guarantee are similar to that for a project without a guarantee (Dias and Ioannou 1995). Due to space limitations, only the formulation for a project with a production guarantee is presented. The analyses for a minimum-revenue guarantee or no guarantee are similar. A detailed example determines and compares the debt and equity results for a project under all three cases (no guarantee, production guarantee, minimum-revenue guarantee).

Key findings include the conditions under which debt capacity exists, its value, and that excessive borrowing can make the promoting team worse off. The promoting team should instead strive to achieve the optimum debt-to-equity ratio, and if it does not have the necessary equity, it should search for additional investors until enough resources become available. Alternatively, it should negotiate with the sponsor government for more favorable guarantees to reduce the risk to debtholders and thus increase the optimum amount of debt financing.

The nature of this important topic requires a relatively complex mathematical treatment. The most important analytical results are presented and explained so that they can be verified and serve as a starting point for further investigation. Care has been taken to explain most of the results using common-sense concepts so that the assumptions, results, and especially the conclusions can be

understood by a wide audience.

CONCESSION AGREEMENTS

A concession contract is one possible arrangement that can be used by governments to raise the necessary funds to finance revenue-generating infrastructure projects when their access to traditional sources of capital is constrained or undesirable. In Build-Operate-Transfer (BOT) projects, the sponsor government provides a concession that permits a promoting team to build a facility and to operate it for a specific amount of time. Project promoters use the revenues produced during the concession period to pay back lenders, other shareholders, and to get a return on their investment. When the concession period elapses, the operation of the facility and its revenues are transferred to the sponsor government that may have infused very few monetary resources into its construction. A well known BOT project, for example, is the Channel Tunnel linking France and the UK by rail.

Similarly, Build-Operate-Own (BOO) projects also produce revenues from their cash flows to cover debt, operation and maintenance costs and return profit gains to the promoter companies. In BOO arrangements, however, private promoters have an unlimited amount of time to operate the facility as well as full ownership of the underlying assets. Examples of such projects are power plants (constructed and operated by private utility companies) and public office buildings. This process can be used not only for financing but also for the privatization of public services.

Concession-financed projects are funded through a combination of debt and equity. Debt is provided by lending institutions (*e.g.*, banks) while equity is provided by companies that have a direct interest in the project (*e.g.*, contractors and operators) and by entities that view the project as an investment opportunity (*e.g.*, pension funds). The use of debt is essential because promoters rarely have the financial resources needed for large projects. The use of equity is also essential to show the support of promoters and provide financial flexibility. Whereas debt instruments present rigid amounts and payment dates, and normally do not offer large grace periods, equity is more flexible as dividends are paid based on the availability of funds.

Once a project concession is granted by the sponsor, the promoting team creates a company,

referred to as the *owning company*, which is responsible for the financing, construction, and operation of the facility and which retains ownership during the concession period. The creation of an owning company as a separate entity is of great benefit to the promoting companies because it allows them to use *off-balance-sheet financing* and raise debt without providing a portion of their own assets as collateral. In this arrangement, the debt is repaid solely by the revenues of the project and is not secured directly by the promoters. Hence, the debt does not appear on their balance sheets, but only on the balance sheet of the owning company. If the project does not produce enough revenues to fully repay the debt, the lenders receive only partial payment and do not have recourse towards the remainder from the promoters. And this makes government guarantees valuable to all parties.

GOVERNMENT GUARANTEES

Project guarantees are formal assurances provided by the host government with the objective of reducing the number of project risks and limiting their impact on the participants of a privately-promoted infrastructure project. The government, by furnishing a guarantee, expresses a commitment towards the project and assumes responsibility to fulfill its conditions. Examples of guarantees include:

- Production guarantee — protection against uncertainty where the government ensures a minimum demand for the service provided by the facility (Shi et al. 2020).
- Revenue guarantee — covenant that guarantees a minimum operating income or a minimum demand volume to the owning company (Zapata 2019).
- Convertibility guarantee — mechanism that assures foreign investors that they will be authorized to convert local currency earnings into foreign currency (Irwin 2007).
- Foreign exchange guarantee — government assurance that the owning company will be protected against project revenue losses due to currency rate fluctuations (Demacopoulos 1989).
- No-second-facility guarantee — government assurance that it will not construct, nor allow

others to construct and to explore competing facilities (Liu 2014).

An example of a project with a production guarantee is the BOT power-plant developed in Shajiao, China, where the government agreed to purchase a minimum quantity of electricity over a 10-year concession period. The Malaysian North-South Expressway offers a more complex example of a minimum-revenue guarantee where the government assured a certain level of revenue and hence, in case the facility is not able to reach this level, provided the necessary funds. In this case the Malaysian government agreed to provide a loan facility if the traffic volume fell below an assured minimum level in the first 17 years of the concession period. Numerous other examples appear in several investigations that examined the various types and extents of government guarantees and provide advice to governments about the structure and effectiveness of project guarantees (Almassi et al. 2013; Chiara and Kokkaew 2013; He at al. 2022; Irwin 2007; Sant'Anna et al. 2022; Tokiwa and Queiroz 2017; Vassallo and Soliño 2006; Werneck and Vieira 2017; Wibowo and Kochendoerfer 2011; Wibowo et al. 2012). In addition, other investigations examined the impact of guarantees from the view of the private sector and provide guidelines to project promoters (Ashuri et al. 2012; Deep at al. 2024).

BANKRUPTCY COSTS

A privately-financed project should produce sufficient net operating income to repay its debt. Otherwise, the owning company does not meet its debt obligation, enters a state of financial distress and becomes bankrupt. In this case, debtholders take ownership of the company and pay the costs of bankruptcy before they receive any payment. Details of this process are described in (Martin and Scott 1976; Hong and Rappaport 1978; Kim 1978).

In this investigation, the adopted model for bankruptcy costs is that suggested by Kim (1978):

$$\tilde{B} = b_f + b_v \tilde{X} \quad (0 \leq \tilde{B} \leq \tilde{X}) \tag{1}$$

In Eq. (1), $\tilde{B}$ represents the uncertain cost of bankruptcy and $\tilde{X}$ is the company's uncertain net operating income (random variables are indicated by a tilde over their symbol). The cost coefficient

b_f represents the expected value of the fixed components of bankruptcy costs (expressed in monetary units) that are independent of $\tilde{X}$ (*i.e.*, those costs that do not depend on the size of the owning company). The cost coefficient b_v relates the variable cost of bankruptcy $\tilde{B}$ to the net operating income $\tilde{X}$ and can assume values from -1 to 1 (if $\tilde{B}$ is independent of $\tilde{X}$, then b_v is zero).

It is assumed that once in bankruptcy the owning company is liquidated and its proceedings get distributed according to the Bankruptcy Reform Act of 1978. Thus, administrative expenses associated with liquidating the project (*i.e.*, bankruptcy costs), such as fees and other compensation paid to trustees, attorneys, accountants, etc., are paid before the debtholders claims on the project assets. Empirical studies on bankruptcies show that administrative expenses range from 4 to 20% of a company's assets depending on the type of firm analyzed and other characteristics (Van Horne 2001).

VALUATION BASED ON PROJECT REVENUES

The basic variable for the analysis of debt capacity and optimal capital structure in the absence of guarantees is the project's net operating income, $\tilde{X}$. The following derivations, however, start at the more basic level of project revenues, $\widetilde{Rev}$, because some forms of guarantees (*e.g.*, minimum-revenue) secure levels of revenue rather than net operating income. Nonetheless, project revenue by itself is not sufficient to determine a project's debt capacity and optimal capital structure because it does not provide the essential information of how much money there is at the end of a period to repay the debt and to be distributed to investors. To determine the project's net operating income $\tilde{X}$ it is also necessary to consider project expenses, which for the purposes of this study are assumed to be a function of revenues:

$$\widetilde{Exp} = e_f + e_v \widetilde{Rev} \tag{2}$$

In Eq. (2), $\widetilde{Exp}$ represents the uncertain value of expenses at the end of the period, e_f is the fixed amount of expenses expressed in monetary units and e_v is the variable component expressed as a fraction of the project revenues $\widetilde{Rev}$. Thus, net operating income can be defined as:

$$\tilde{X} = (1 - e_v)\widetilde{Rev} - e_f \tag{3}$$

In addition, the following are key variables for the formulation of revenue and net operating income when a project has a production or a minimum-revenue guarantee:

- $\widetilde{Rev}_g$ is the maximum of two values: project revenues, $\widetilde{Rev}$, and the project guarantee, g:

$$\widetilde{Rev}_g = \begin{cases} \widetilde{Rev} & \text{if } \widetilde{Rev} \geq g \\ g & \text{if } \widetilde{Rev} < g \end{cases} \tag{4}$$

- b'' is the maximum amount of $\widetilde{Rev}_g$ that does not provide any return to debtholders (*i.e.*, all the money generated by the project is used to pay bankruptcy costs):

$$b'' = \frac{b_f + e_f(1 - b_v)}{(1 - b_v)(1 - e_v)} \tag{5}$$

- a'' is the minimum amount of $\widetilde{Rev}_g$ that avoids bankruptcy (*i.e.*, the money generated by the project is sufficient to pay the full debt repayment amount d_1 as well as all project expenses):

$$a'' = \frac{d_1 + e_f}{1 - e_v} \tag{6}$$

The actual market values of debt and equity for a privately-financed infrastructure project secured by a production or a minimum-revenue guarantee depend on the value of the guarantee g relative to the variables b'' and a'' and are given by different expressions in each of three regions:

- For $g < b''$, the guarantee is too small and does not secure any repayment to the debtholders.
- For $b'' \leq g < a''$, the guarantee is large enough to secure part of the project's debt obligations (making the debt less risky) but does not secure any return to the project promoters.
- For $a'' \leq g$, the guarantee secures the entire debt repayment amount (*i.e.*, the debt becomes riskless) as well as some level of return to equityholders.

NET OPERATING INCOME AND GUARANTEES

A production guarantee assures a minimum demand for the service provided by a privately promoted project and thus protects both revenue and expenses. The resulting net operating income

is shown as $\tilde{X}_p$. A minimum-revenue guarantee protects revenue even in the absence of expenses and the resulting net operating income is shown as $\tilde{X}_m$. The guaranteed project revenue, $\widetilde{Rev}_g$, and the corresponding net operating income with guarantees are:

$$\widetilde{Rev}_g = \delta_k \widetilde{Rev} + g(1 - \delta_k) \tag{7}$$

$$\tilde{X}_p = (1 - e_v)[\delta_k \widetilde{Rev} + g(1 - \delta_k)] - e_f \tag{8}$$

$$\tilde{X}_m = \delta_k \widetilde{Rev} - e_v \widetilde{Rev} + g(1 - \delta_k) - e_f \tag{9}$$

Here, the binary variable δ_k is defined as follows to show results succinctly:

$$\delta_k = \begin{cases} 0 & \text{if } \widetilde{Rev} < g \\ 1 & \text{if } \widetilde{Rev} \geq g \end{cases} \tag{10}$$

PRESENT VALUE OF DEBT

For a one-period privately-financed project, the amount D that the owning company can borrow when the nominal interest rate charged by the lenders is *Int* depends on the risk characteristics of the amount d_1 it promises to repay at the end of the period.

$$d_1 = D(1 + Int) \tag{11}$$

Because the loan is risky, the amount $E[\tilde{D}_1]$ that the lenders expect to receive, however, is less than the full promised amount d_1. Thus, the expected return on the debt $E[\tilde{r}_D]$ is less than *Int*:

$$E[\tilde{D}_1] = D(1 + E[\tilde{r}_D]) = DE[\tilde{R}_D] \tag{12}$$

(For brevity of notation, variables that represent one-plus-a-rate-of-return are shown with the corresponding capital letter, *e.g.*, $\tilde{R}_D = 1 + \tilde{r}_D$).)

As the promised debt repayment amount d_1 increases, so does the risk faced by the debtholders and so does the nominal interest rate *Int* they demand. When d_1 exceeds a certain level, the required nominal interest *Int* becomes so large that the debt amount D can actually decrease (even though

the owning company promises to pay more). This is true even when the project is backed up by a production or a minimum-revenue guarantee (albeit not a large one).

To illustrate this behavior in a mathematically tractable manner, and without loss of generality, the remaining discussion focuses on the evaluation of debt and equity for a one-period project backed by a production guarantee. The analysis for multiperiod projects is similar, but is best undertaken using numerical methods such as those suggested in (Ashuri et al. 2012).

The loan amount D_p for a project with a production guarantee (*i.e.*, the present value of a project's debt as determined by the market) can be computed according to the CAPM, as suggested by Hamada (1971):

$$D_p = \frac{E[\tilde{D}_{1,p}] - \lambda \operatorname{Cov}(\tilde{D}_{1,p}, \tilde{R}_m)}{R_f} \tag{13}$$

Here, $E[\tilde{D}_{1,p}]$ is the expected value of the debt at time 1; $\operatorname{Cov}(\tilde{D}_{1,p}, \tilde{R}_m)$ is the covariance between the value of debt at time 1 and one-plus-the-rate-of-return-on-the-market ; $\lambda = (E[\tilde{R}_m] - R_f)/\sigma_m^2$ is the market price per unit of risk; and $R_f = 1 + r_f$ is one-plus-the-risk-free-rate.

The end-of-period value of debt, $\tilde{D}_{1,p}$, depends on the end-of-period project net operating income, $\tilde{X}_p$, and can be expressed as:

$$\tilde{D}_{1,p} = \begin{cases} d_1 & \text{if } \tilde{X}_p \geq d_1 \\ \tilde{X}_p - \tilde{B} & \text{if } \tilde{B} \leq \tilde{X}_p < d_1 \\ 0 & \text{if } \tilde{X}_p < \tilde{B} \end{cases} \tag{14}$$

Or equivalently,

$$\tilde{D}_{1,p} = \begin{cases} d_1 & \text{if } \widetilde{Rev}_g \geq a'' \\ (1 - b_v)[(1 - e_v)\widetilde{Rev}_g - e_f] - b_f & \text{if } b'' \leq \widetilde{Rev}_g < a'' \\ 0 & \text{if } \widetilde{Rev}_g < b'' \end{cases} \tag{15}$$

Thus, if the net cash flow at the end of the period is greater than the promised amount d_1, the debtholders receive the full debt payment. Otherwise, they receive the net cash flow minus the bankruptcy costs, provided that this difference is positive, and nothing if the difference is negative

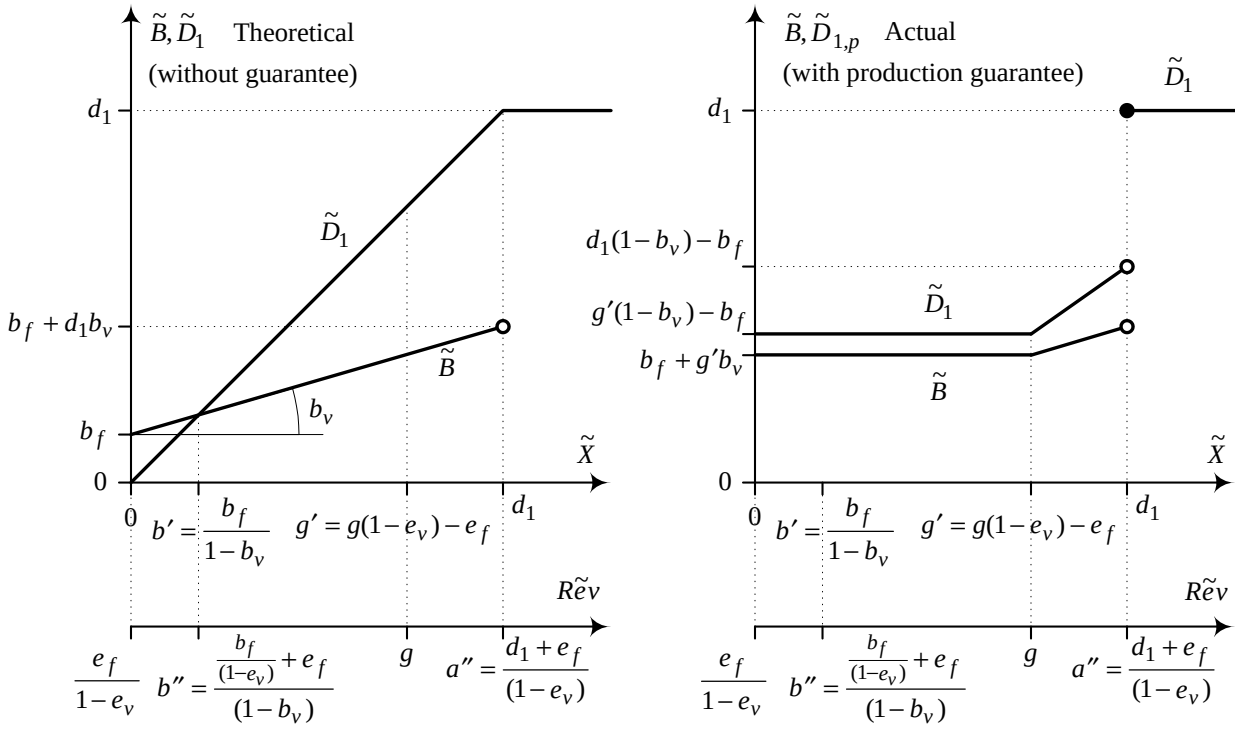


Fig. 1. Value of debt ($\tilde{D}_1$) and bankruptcy costs ($\tilde{B}$) as a function of $\tilde{X}$ and $\widetilde{Rev}$

(the entire net operating income is consumed by bankruptcy costs). Figure 1 shows the value of $\tilde{D}_{1,p}$ as a function of $\tilde{X}$ and $\widetilde{Rev}$ for a production guarantee in the range $b'' \leq g < a''$. The graph on the left is for a project without a guarantee and shows $\tilde{B}$ and $\tilde{D}_1$ (when bankruptcy costs are not considered). The graph on the right illustrates the actual values of $\tilde{D}_{1,p}$ given by Eq. (15) when the production guarantee and bankruptcy costs are considered.

$\tilde{D}_{1,p}$ can also be given by the following single expression as an alternative to Eq. (15):

$$\tilde{D}_{1,p} = d_1(1-\delta_b)\delta_q + \delta_b\delta_q\tilde{X}_p - \delta_b\delta_q(b_f + b_v\tilde{X}_p) \tag{16}$$

Here δ_b and δ_q are binary variables defined as follows:

$$\delta_b = \begin{cases} 0 & \text{if } \widetilde{Rev}_g \geq a'' = \frac{d_1+e_f}{1-e_v} \\ 1 & \text{if } \widetilde{Rev}_g < a'' \end{cases} \tag{17}$$

$$\delta_q = \begin{cases} 0 & \text{if } \widetilde{Rev}_g < b'' = \frac{b_f + e_f(1-b_v)}{(1-b_v)(1-e_v)} \\ 1 & \text{if } \widetilde{Rev}_g \geq b'' \end{cases} \tag{18}$$

Substituting Eq. (8) into Eq. (16) gives $\tilde{D}_{1,p}$ as a function of $\widetilde{Rev}$:

$$\begin{aligned} \tilde{D}_{1,p} &= d_1(1-\delta_b)\delta_q + (1-e_v)(1-b_v)[\delta_b\delta_k\delta_q\widetilde{Rev} + g(\delta_b\delta_q - \delta_b\delta_k\delta_q)] - \\ &\quad -[e_f(1-b_v) + b_f]\delta_b\delta_q \end{aligned} \tag{19}$$

In the following sections, the future value $\tilde{D}_{1,p}$ given by Eq. (19) is substituted into Eq. (13) to evaluate the present value of debt D_p for a production guarantee g that falls in each of the three regions: $g < b''$; $b'' \leq g < a''$; and $a'' \leq g$.

Present value of debt for a guarantee that does not assure any debt repayment ($0 \leq g < b''$)

The present value of debt for a guarantee that is not large enough to cover any debt repayment, under the assumption that $\widetilde{Rev}$ follows a Normal distribution, is found by entering into Eq. (19) the appropriate $(0, 1)$ values for δ_b, δ_k and δ_q that correspond to $g < b''$, and substituting the result into Eq. (13):

$$\begin{aligned} D_p &= \frac{1}{R_f}\Big\{d_1\left(1 - F_{Rev}(a'')\right) + (1-e_v)(1-b_v)\Big[E[\widetilde{Rev}]\left(F_{Rev}(a'') - F_{Rev}(b'')\right) + \\ &\quad +\sigma^2_{Rev}\left(f_{Rev}(b'') - f_{Rev}(a'')\right)\Big] - [e_f(1-b_v) + b_f]\left(F_{Rev}(a'') - F_{Rev}(b'')\right) - \\ &\quad -\lambda\,\mathrm{Cov}(\widetilde{Rev}, \tilde{R}_m)\Big[(1-e_v)(1-b_v)\left[F_{Rev}(a'') - F_{Rev}(b'')\right] + \\ &\quad +\left[b_f + b_v d_1\right] f_{Rev}(a'')\Big]\Big\} \end{aligned} \tag{20}$$

It should be noted that the present value of debt given by Eq. (20) is not a function of g and thus is the same as for a project without a guarantee. *I.e.*, a guarantee smaller than b'' covers only bankruptcy costs and does not assure any repayment to the debtholders. Thus, the guarantee has no value to the debtholders and does not affect the present value of the debt.

Present value of debt for a guarantee that assures partial debt repayment ($b'' \leq g < a''$)

The market value of debt for a guarantee that covers only part of the debt obligations (*i.e.*, $b'' \leq g < a''$) is given by Eq. (13) when the appropriate values for the binary variables δ_b, δ_k, and δ_q are substituted into Eq. (19):

$$\begin{aligned} D_p \;=\; & \frac{1}{R_f}\Big\{ d_1\left(1 - F_{Rev}(a'')\right) + (1-e_v)(1-b_v)\Big[E[\widetilde{Rev}]\left(F_{Rev}(a'') - F_{Rev}(g)\right) + \\ & + \sigma^2_{Rev}\left(f_{Rev}(g) - f_{Rev}(a'')\right) + g F_{Rev}(g)\Big] - [e_f(1-b_v) + b_f] F_{Rev}(a'') - \\ & - \lambda\, \mathrm{Cov}(\widetilde{Rev}, \tilde{R}_m)\Big[(1-e_v)(1-b_v)\left[F_{Rev}(a'') - F_{Rev}(g)\right] + \\ & + \left[b_f + b_v d_1\right] f_{Rev}(a'')\Big]\Big\} \end{aligned} \tag{21}$$

Eq. (21) reflects the fact that the project's debt has no systematic relationship with the market when debtholders exercise their production guarantee. This is because the guarantee offered by the government does not depend on any market conditions.

Present value of debt for a guarantee that assures full debt repayment ($a'' \leq g$)

For a production guarantee that assures full debt repayment ($a'' \leq g$), Eq. (19) gives:

$$\tilde{D}_{1,p} = d_1(1 - \delta_b) \tag{22}$$

The expected value of $\tilde{D}_{1,p}$ is $E[\tilde{D}_{1,p}] = d_1(1 - E[\delta_b]) = d_1$ (*i.e.*, the full repayment amount) and the covariance between the project debt and the market return, $\mathrm{Cov}(\tilde{D}_{1,p}, \tilde{R}_m)$, is zero ($\beta_{D_p} = 0$). Thus, the market value of debt, D_p, given by Eq. (13) is:

$$D_p = \frac{d_1}{R_f} \tag{23}$$

This shows that the project's debt does not have any systematic relationship with the market. In this region, the guarantee covers all project expenses and the promised debt repayment amount, and the owning company would always be able to repay its debt in full, regardless of how the market behaves. In this situation, the promised debt amount is discounted by the risk-free rate because the loan is riskless (full debt repayment is assured).

PRESENT VALUE OF EQUITY

The actual market value of the equity in a project can be expressed in a form similar to Eq. (13) for the present value of the debt:

$$S_p = \frac{E[\tilde{S}_{1,p}] - \lambda \, \mathrm{Cov}(\tilde{S}_{1,p}, \tilde{R}_m)}{R_f} \tag{24}$$

In Eq. (24), $E[\tilde{S}_{1,p}]$ is the expected value of the end-of-period value of equity when the project is backed by a production guarantee and $\mathrm{Cov}(\tilde{S}_{1,p}, \tilde{R}_m)$ is the covariance between the end-of-period value of equity and the return on the market.

The end-of-period value of equity, $\tilde{S}_{1,p}$, represents the value of the company after debt obligations are paid to debtholders and taxes are paid to the government. Thus, it is given by:

$$\tilde{S}_{1,p} = \begin{cases} (1-T)(\tilde{X}_p - d_1) + T(A-D) & \text{if } \tilde{X}_p \geq d_1 \\ 0 & \text{if } \tilde{X}_p < d_1 \end{cases} \tag{25}$$

Alternatively, $\tilde{S}_{1,p}$ can be expressed in the following equation form:

$$\tilde{S}_{1,p} = (1-T)(\tilde{X}_p - d_1)(1-\delta_b)\delta_q + T(A-D)(1-\delta_b)\delta_q \tag{26}$$

Substituting $\tilde{X}_p = (1-e_v)[\delta_k \widetilde{Rev} + g(1-\delta_k)] - e_f$ into Eq. (26) gives:

$$\begin{aligned} \tilde{S}_{1,p} \;=\; & (1-T)(1-e_v)\left[(\widetilde{Rev} - g)(\delta_k\delta_q - \delta_b\delta_k\delta_q)\right] + \\ & +(1-T)\left(g(1-e_v) - e_f - d_1\right)(\delta_q - \delta_b\delta_q) + T(A-D)(\delta_q - \delta_b\delta_q) \end{aligned} \tag{27}$$

In the following two sections, the future value $\tilde{S}_{1,p}$ given by Eq. (27) is substituted into Eq. (24) to evaluate the present value of equity S_p depending on whether the guarantee does ($a'' \leq g$) or does not ($0 \leq g < a''$) assure any return to the investors.

Present value of equity for a guarantee that does not assure any equity returns ($0 \leq g < a''$)

The market value of equity for a guarantee that does not assure any return to the equityholders ($0 \leq g < a''$) is given by Eq. (24) after substituting into Eq. (27) the appropriate values for the

binary variables δ_b, δ_k, and δ_q:

$$\begin{aligned} S_p &= \frac{1}{R_f}\left\{\left[(1-T)\left((1-e_v)E[\widetilde{Rev}]-d_1-e_f\right)\right](1-F_{Rev}(a''))+\right. \\ &\quad +(1-T)(1-e_v)\sigma^2_{Rev}f_{Rev}(a'')+T(A-D)\,(1-F_{Rev}(a''))- \\ &\quad -\lambda\,\mathrm{Cov}(\widetilde{Rev},\tilde{R}_m)\left\{(1-T)(1-e_v)\,(1-F_{Rev}(a''))+\right. \\ &\quad \left.\left.+T(A-D)f_{Rev}(a'')\right\}\right\} \end{aligned} \tag{28}$$

The present value of equity given by Eq. (28) is not a function of g and is the same as if the guarantee does not exist. This is similar to the present value of debt when $g < b''$, where the entire amount secured by the guarantee will pay the liquidators in case of bankruptcy. When $b'' \leq g < a''$, however, the guarantee does provide indirect value to equityholders. That is, while the guarantee does not secure any money to investors but only to debtholders, this secured debt repayment, though partial, reduces the risk faced by the lenders. Thus, other things being equal, the owning company can increase the amount borrowed while promising to pay the same repayment amount d_1 as in the case of no project guarantees (or guarantees smaller than b'').

Present value of equity for a guarantee that assures some return on equity ($a'' \leq g$)

The market value of equity for a guarantee that assures some level of return to the equityholders ($a'' \leq g$) is given by Eq. (24) when the appropriate values for the binary variables δ_b, δ_k, and δ_q are substituted into Eq. (27):

$$\begin{aligned} S_p &= \frac{1}{R_f}\left\{(1-T)\left\{(1-e_v)\left[E[\widetilde{Rev}](1-F_{Rev}(g))+gF_{Rev}(g)+\sigma^2_{Rev}f_{Rev}(g)\right]-d_1-\right.\right. \\ &\quad \left.\left.-e_f\right\}+T(A-D)-\lambda\,\mathrm{Cov}(\widetilde{Rev},\tilde{R}_m)\left\{(1-T)(1-e_v)[1-F_{Rev}(g)]\right\}\right\} \end{aligned} \tag{29}$$

Eq. (29) for the present value of equity S_p is an increasing function of the production guarantee g. When the guarantee becomes so large as to make $F_{Rev}(g) = 1$ and $f_{Rev}(g) = 0$, then $\delta_q = 1$ and $\delta_k = \delta_b = 0$, and Eq. (29) simplifies to Eq. (27) discounted by the risk-free rate .

PROJECT DEBT CAPACITY

Project debt capacity, shown as D^c, is the maximum amount an owning company can borrow to finance a project. Promising to pay more than the corresponding repayment amount d_1^c increases the debt risk and thus the required interest rate and results in a smaller loan. For debt capacity to be meaningful, there must exist a repayment amount that maximizes debt, and this maximum debt value $D_{\max}$ must be smaller than the initial cost of the project A. That is, if $D_{\max} < A$, then $D^c = D_{\max}$. Otherwise, 100% debt financing is possible and $D^c = A$.

Debt capacity for a guarantee that does not assure any debt repayment ($g < b''$)

For a production guarantee that is too small to assure any repayment to debtholders (*i.e.*, $g < b''$), d_1^c exists when $\partial D/\partial d_1 = 0$ and $\partial^2 D/\partial d_1^2 < 0$.

The second derivative of D given by Eq. (20) with respect to d_1 is negative when the expected value of project revenue can cover the promised repayment amount d_1 (*i.e.*, when $a'' < E[\widetilde{Rev}]$), a condition that any project worth financing should satisfy. Thus, a promised amount d_1^c that satisfies $\partial D/\partial d_1 = 0$, corresponds to a maximum. Since for low values of d_1 the slope $\partial D/\partial d_1$ is positive, higher values of d_1 that satisfy $\partial D/\partial d_1 < 0$ assure that $\partial D/\partial d_1 = 0$ at some intermediate debt repayment level and that debt capacity exists.

The first derivative is negative, $\partial D/\partial d_1 < 0$, when:

$$b_f + b_v d_1 \;>\; \frac{1 - F_{Rev}(a'') - \lambda \operatorname{Cov}(\widetilde{Rev}, \tilde{R}_m) f_{Rev}(a'')}{\frac{f_{Rev}(a'')}{1-e_v}\left[1 - (\frac{a'' - E[\widetilde{Rev}]}{\sigma^2_{Rev}})\lambda \operatorname{Cov}(\widetilde{Rev}, \tilde{R}_m)\right]} \tag{30}$$

Therefore, if the above condition is satisfied (*i.e.*, if the cost of bankruptcy is large enough), there is a finite limit on the owning company's ability to raise debt. This maximum debt amount, $D_{\max}$, is calculated by substituting the promised amount d_1^c that satisfies $\partial D/\partial d_1 = 0$ into Eq. (20).

For the case $g < b''$, where a guarantee is too small relative to the cost of bankruptcy, the results for a production guarantee or a minimum-revenue guarantee are exactly the same as those for a project without a guarantee. Therefore, a project with such guarantees has the same d_1^c and the same debt capacity D^c as a project without any guarantees.

Debt capacity for a guarantee that assures full debt repayment ($a'' \leq g$)

When $a'' \leq g$, the guarantee is large enough to cover all project expenses as well as the full repayment amount d_1. Hence, debtholders face no repayment risks, the project has no possibility of going bankrupt, and the loan is considered riskless. In this region, the loan amount D is given by Eq. (23) and increases linearly with the repayment amount d_1. The maximum amount of riskless debt, $D^g_{\max}$, that can be raised due to the presence of a guarantee is:

$$D^g_{\max} = [g(1 - e_v) - e_f]/R_f \tag{31}$$

This occurs when d_1 assumes its maximum fully-secured value, *i.e.*, $d^g_1 = g(1 - e_v) - e_f$.

From a project cost perspective, the guarantee necessary to secure 100% debt financing is:

$$g_{100} = \frac{AR_f + e_f}{1 - e_v} \tag{32}$$

A project with a guarantee g_{100} can be 100% debt financed and does not have debt capacity as an operational concept. In this case, $g_{100} = a''$, $D^g_{\max} = A$, and $d_1 = g_{100}(1 - e_v) - e_f = AR_f$. A guarantee greater than g_{100} does not provide any extra value to debtholders; the extra amount over g_{100} benefits the equityholders and the government (in the form of taxes).

Debt capacity for a guarantee that assures partial debt repayment ($b'' \leq g \leq a''$)

When $b'' \leq g < a''$, the promised repayment amount d_1 can increase beyond the level covered by the guarantee (*i.e.*, $d_1 > d^g_1$), and the loan becomes risky. In this region, the maximum debt that can be raised for a production guarantee is shown as $D^s_{\max}$ and is given by Eq. (21). The corresponding promised debt repayment amount is d^s_1.

Notice that even though the market value of debt D_p given by Eq. (21) is a function of g, its first derivative is not, and thus neither is d^s_1. This leads to the somewhat surprising result that the unsecured repayment amount d^s_1 that maximizes debt capacity in the region $d_1 > d^g_1$ (*i.e.*, when $b'' \leq g < a''$) is the same no matter whether the project has a production guarantee, a minimum revenue guarantee, or no guarantee at all. Although the optimum d^s_1 is the same for all three cases,

the corresponding maximum debt amounts $D^s_{\max}$ are not. $D^s_{\max}$ is largest for a project with a minimum revenue guarantee and smallest for a project without any guarantees.

Possible debt capacity values

From the above, it follows that there two repayment amounts, d^g_1 and d^s_1 (where $d^g_1 < d^s_1$), that correspond to two local maxima of debt, $D^g_{\max}$ and $D^s_{\max}$. The value $D^g_{\max}$ is the maximum amount of riskless debt ($a'' \leq g$) and occurs at d^g_1, and $D^s_{\max}$ is the maximum amount of risky debt ($b'' \leq g < a''$) and occurs at d^s_1. The global maximum debt $D_{\max}$ that can be raised to fund the project is the larger of the two values:

$$D_{\max} = \max(D^g_{\max}, D^s_{\max}) \tag{33}$$

For a given guarantee g, increasing the promised debt repayment amount d_1 increases the minimum revenue required for full debt repayment a'' (which is a linear function of d_1 given by Eq. (6)), while g remains constant. Thus, small promised debt repayment amounts (up to d^g_1) ensure that the debt is riskless (*i.e.*, until $a'' = g$). For values d_1 slightly larger than d^g_1, the debt becomes risky, and the possibility of bankruptcy causes the corresponding debt amount D to drop below $D^g_{\max}$, forming a vertical discontinuity. As d_1 continues to increase, so does the corresponding risky debt amount D until it reaches a local maximum $D^s_{\max}$ at $d_1 = d^s_1$. Whether the global maximum debt amount $D_{\max}$ occurs at a relatively small d^g_1 that results in riskless debt $D^g_{\max}$, or at a higher d^s_1 that results in risky debt $D^s_{\max}$ must be investigated on a case-by-case basis. Project debt capacity exists (*i.e.*, $D^c = D_{\max}$) if $D_{\max}$ is less than the project cost A. Otherwise, 100% debt financing is possible.

Table 1 illustrates how to determine the debt capacity of a project $D_{\max}$ given different possible values of g and d_1. An important observation in Table 1 is that for risky debt (*i.e.*, when $b'' < g < a''$), the optimum d^s_1 remains constant while debt capacity does not. Thus, an increase in the guarantee amount g (provided $b'' < g < a''$) results in an increase in the debt capacity of the project (D^c) but does not change the optimal promised debt amount d^s_1. This is because an increase in the guarantee decreases the risk faced by debtholders and hence, decreases the nominal interest *Int* they

TABLE 1. Project debt capacity as a function of g and d_1

Riskless Debt (*i.e.*, $d_1 \leq d_1^g \Leftrightarrow a'' \leq g$) (1)	Risky Debt (*i.e.*, $d_1 > d_1^g \Leftrightarrow g < a''$) (2)	Project Debt Capacity D^c (3)
if $g < g_{100}$ then $D^g_{\max} = \frac{1}{R_f}[g(1-e_v) - e_f]$	if $g < b''$ then $D^s_{\max}$ given by Eq. (20) if $b'' \leq g$ then $D^s_{\max}$ given by Eq. (21)	$D^c = \max(D^g_{max}, D^s_{max})$ (if $D^s_{\max} \geq A$ then 100% debt fin. possible)
if $g \geq g_{100}$ then $D^g_{\max} = A$		100% debt fin. possible

charge on the loan even though the promised debt amount d_1^s remains the same. When $g \geq a''$, the guarantee provides full collateral for the debt, therefore, if d_1 remains constant, then an increase in the guarantee beyond a'' does not increase the debt capacity of the project. This implies that as g increases, the promised debt repayment amount d_1 should also increase so that $g = a''$. Thus, $d_1^c = d_1^g = g(1-e_v) - e_f$ and $D_c = D^g_{\max} = d_1^g/R_f$. In this case, increasing g increases both d_1^c and D_c.

The above concepts are clearly illustrated in the example application.

OPTIMAL CAPITAL STRUCTURE

This section examines how the optimal financial structure for a privately financed project is influenced by production and minimum-revenue guarantees. Two financial objectives are considered, return on equity (*ROE*) and net present value (*NPV*).

The *ROE* objective leads to the maximization of:

$$E[\widetilde{ROE}] = \frac{E[\tilde{S}_1]}{A - D} \tag{34}$$

The *NPV* objective leads to the maximization of the difference between the market value of equity S and the amount $A - D$ that is invested as equity in a project that costs A and has a debt D:

$$NPV = S - (A - D) = S + D - A = V - A \tag{35}$$

The determination of the optimal financial structure requires separate formulations depending

on whether the guarantee does or does not assure any debt repayment and on whether the assured debt repayment is partial or full.

Case I: Guarantee does not assure any debt repayment ($0 < g < b''$)

For a project with a production or minimum-revenue guarantee that does not assure any debt repayment ($0 < g < b''$), maximizing the equityholders' return on investment *ROE*, or the project's *NPV*, leads to the same results as for a project without a guarantee.

Differentiating Eq. (34) with respect to d_1 shows that when $\partial E[\widetilde{ROE}]/\partial d_1 = 0$ we always have $\partial D/\partial d_1 > 0$. This means that the optimal capital structure for a concession project always occurs before its debt capacity is reached (as is the case for no guarantees). That is, $d_1^{ROE} < d_1^c$, where d_1^{ROE} is the value of d_1 that corresponds to $\partial E[\widetilde{ROE}]/\partial d_1 = 0$. Similarly, maximizing *NPV* leads to an optimum debt that is even less than that allowed by the maximization of the return on investment (*i.e.*, $d_1^{NPV} < d_1^{ROE} < d_1^c$).

This leads to the important conclusion that an owning company that has a privately financed infrastructure project with a guarantee smaller than b'' and with $D^c < A$ should not borrow at debt capacity but only at the amount sufficient to maximize its return on investment *ROE*, or the project's net present value *NPV*. If the project debt capacity allows 100% debt financing (*i.e.*, $D^c = A$), then obviously the optimal capital structure occurs at 100% debt financing.

The optimal debt amounts d_1^{ROE} and D^{ROE}, as well as d_1^{NPV} and D^{NPV}, for a project with a guarantee smaller than b'' are equal to those for a project without a guarantee. This is yet another indication that a guarantee (either production or minimum-revenue) smaller than b'' has no value to investors as it does not guarantee any repayment either to debtholders or to equityholders.

Case II: Guarantee assures full or partial debt repayment ($b'' \leq g$)

For a production or minimum-revenue guarantee that is large enough to cover full or partial debt repayment ($b'' \leq g$), the objective of maximizing *NPV* occurs when the debt repayment amount d_1 equals d_1^g (*i.e.*, the owning company should borrow the maximum level of debt that is fully secured). The maximum expected return to project investors, $E[\widetilde{ROE}]_{\max}$, however, may occur at d_1^g (riskless debt) or at a higher repayment amount $d_1 > d_1^g$ (risky debt). This makes it necessary

to evaluate and compare the maximum expected returns for both cases.

Case IIa: Riskless debt ($0 < d_1 \leq d_1^g \quad \Leftrightarrow \quad a'' \leq g$)

In the interval $0 < d_1 \leq d_1^g$, the guarantee assures full debt repayment (*i.e.*, $a'' \leq g$) and the expected return on investment for a production guarantee is given by:

$$E[\widetilde{ROE}] = \frac{1}{A - \frac{d_1}{R_f}}(1-T)\left\{(1-e_v)\left[E[\widetilde{Rev}](1-F_{Rev}(g)) + gF_{Rev}(g) + \sigma^2_{Rev}f_{Rev}(g)\right] - \right.$$
$$\left. -d_1 - e_f\right\} + T \tag{36}$$

Here, $E[\widetilde{ROE}]$ increases as d_1 increases and thus $E[\widetilde{ROE}]^g_{\max}$ always occurs at the boundary $d_1 = d_1^g = g(1-e_v) - e_f$ (*i.e.*, when $a'' = g$).

Case IIb: Risky debt ($d_1^g < d_1 \quad \Leftrightarrow \quad b'' \leq g < a''$)

When the debt repayment amount d_1 exceeds d_1^g, the guarantee assures only partial debt repayment (*i.e.*, $b'' \leq g < a''$). The expected return on investment for a production guarantee is given by:

$$E[\widetilde{ROE}] = \frac{1}{A-D}\left\{(1-T)\left[(1-e_v)E[\widetilde{Rev}] - d_1 - e_f\right](1-F_{Rev}(a'')) + \right.$$
$$\left. + (1-T)(1-e_v)\sigma^2_{Rev}f_{Rev}(a'')\right\} + T(1-F_{Rev}(a'')) \tag{37}$$

The maximum $E[\widetilde{ROE}]^s_{\max}$ is determined by setting the derivative of Eq. (37) with respect to d_1 equal to zero.

The capital structure that maximizes the investors' return is the largest of the two $E[\widetilde{ROE}]_{\max}$ values:

$$E[\widetilde{ROE}]_{\max} = \max(E[\widetilde{ROE}]^g_{\max}, E[\widetilde{ROE}]^s_{\max}) \tag{38}$$

Clearly, when $E[\widetilde{ROE}]_{\max} = E[\widetilde{ROE}]^g_{\max}$ (*i.e.*, when the maximum expected return on investment occurs at d_1^g), then the objective of maximizing the equityholders' wealth (*i.e.*, maximizing *NPV*) provides the same optimal capital structure as the objective of maximizing their returns. Otherwise,

TABLE 2. Input parameters for the example project (all $ values are in millions)

Project		Market	
Variable	Value	Variable	Value
(1)	(2)	(3)	(4)
A	$ 2,200	$E[\tilde{R}_m]$	1.12
$E[\widetilde{Rev}]$	$ 3,800	σ_m	0.20
σ_{Rev}	$ 1,500	T	0.35
ρ_{Rev,R_m}	0.70	R_f	1.06
e_f	$ 500		
e_v	0.20		
b_f	$ 100		
b_v	0.30		

when $E[\widetilde{ROE}]_{\max} = E[\widetilde{ROE}]^s_{\max}$, the optimum debt that maximizes returns is larger than that which maximizes wealth.

Investors prefer larger guarantee amounts because an increase in g increases $E[\widetilde{ROE}]_{\max}$. The reason for this increase depends on whether the debt is risky or riskless. When the debt is risky (*i.e.*, $b'' \leq g < a''$), it can be seen from Eq. (37) that as g increases, everything remains constant except D (which increases); hence $E[\widetilde{ROE}]$ also increases. When the debt is riskless (*i.e.*, $a'' \leq g$), $D = d_1/R_f$ does not change by an increase in g, but $E[\tilde{S}_1]$ increases, therefore $E[\widetilde{ROE}]$, given by Eq. (36), also increases. The same results hold true for minimum-revenue guarantees.

VALUE OF GUARANTEES

A guarantee has value for both debtholders and equityholders. The value of a guarantee to debtholders at a given d_1 can be calculated by subtracting the market value of debt without a guarantee, D, from the market value of debt with a guarantee, D_p (for a production guarantee) or D_m (for a minimum revenue guarantee). Similarly, the value of a guarantee to equityholders at a given d_1 is the difference between the market value of equity with a guarantee, S_p or S_m, and the market value of equity without any guarantee, S.

A guarantee can be viewed as a put option; that is, at the end of the period, if the revenues are larger than the guarantee, then the option expires worthless. However, if revenues are smaller than

TABLE 3. Example results for the case of no guarantees (all $ in millions)

Promised debt amount d_1 (1)	Market value of debt D (2)	Market value of equity S (3)	Market value of project V (4)	Net Present Value NPV (5)	Debt to cost ratio D/A (6)	Effective return on debt $E[\tilde{r}_D]$ (7)	Nominal interest on debt Int (8)	Required return on equity $E[\tilde{r}_S]$ (9)	Return on equity invstmnt $E[\widetilde{Roe}]$ (10)	∂D $/\partial d_1$ (11)	$\partial E[\widetilde{ROE}]$ $/\partial d_1$ (12)
0	0.00	2,117.70	2,117.70	-82.30	0.000	0.060	0.060	0.139	0.097	0.914	9.91E-06
171	155.78	1,959.00	2,114.78	-85.22	0.071	0.073	0.100	0.146	0.098	0.900	6.17E-06
343	308.34	1,801.70	2,110.03	-89.97	0.140	0.076	0.111	0.154	0.099	0.881	1.37E-07
514	457.11	1,646.40	2,103.51	-96.49	0.208	0.080	0.124	0.162	0.098	0.855	-8.92E-06
685	600.99	1,494.03	2,095.02	-104.98	0.273	0.084	0.140	0.172	0.095	0.823	-2.20E-05
857	738.77	1,345.65	2,084.42	-115.58	0.336	0.089	0.159	0.184	0.090	0.784	-4.04E-05
1,028	869.11	1,202.43	2,071.54	-128.46	0.395	0.094	0.183	0.197	0.081	0.737	-6.55E-05
1,199	990.65	1,065.61	2,056.26	-143.74	0.450	0.100	0.210	0.211	0.067	0.681	-9.88E-05
1,370	1,102.09	936.34	2,038.43	-161.57	0.501	0.107	0.243	0.227	0.047	0.619	-1.42E-04
1,542	1,202.22	815.69	2,017.91	-182.09	0.546	0.114	0.282	0.245	0.018	0.550	-1.95E-04
1,580	1,222.97	789.98	2,012.94	-187.06	0.556	0.116	0.292	0.250	0.010	0.533	-2.08E-04
1,581	1,223.50	789.31	2,012.81	-187.19	0.556	0.116	0.292	0.250	0.010	0.533	-2.08E-04
1,713	1,290.10	704.46	1,994.57	-205.43	0.586	0.122	0.328	0.265	-0.020	0.476	-2.57E-04
1,884	1,365.07	603.21	1,968.27	-231.73	0.620	0.130	0.380	0.287	-0.070	0.399	-3.25E-04
2,056	1,426.82	512.14	1,938.96	-261.04	0.649	0.138	0.441	0.311	-0.132	0.322	-3.94E-04
2,227	1,475.49	431.19	1,906.68	-293.32	0.671	0.147	0.509	0.337	-0.204	0.247	-4.54E-04
2,398	1,511.63	359.98	1,871.61	-328.39	0.687	0.155	0.586	0.365	-0.286	0.176	-4.99E-04
2,570	1,536.17	297.94	1,834.12	-365.88	0.698	0.163	0.673	0.395	-0.374	0.112	-5.20E-04
2,741	1,550.41	244.37	1,794.78	-405.22	0.705	0.171	0.768	0.427	-0.463	0.056	-5.15E-04
2,912	1,555.86	198.49	1,754.35	-445.65	0.707	0.178	0.872	0.463	-0.549	0.009	-4.88E-04
3,083	1,554.21	159.52	1,713.74	-486.26	0.706	0.184	0.984	0.501	-0.629	-0.027	-4.43E-04
3,255	1,547.19	126.71	1,673.90	-526.10	0.703	0.189	1.104	0.543	-0.701	-0.053	-3.89E-04
3,426	1,536.44	99.37	1,635.80	-564.20	0.698	0.193	1.230	0.589	-0.762	-0.071	-3.31E-04

the guarantees, then the option is exercised and debtholders receive the guarantee amount from equityholders. Obviously, guarantees enhance the market value of a project because they reduce the risks associated with the project's revenue.

EXAMPLE PROJECT

This section presents a comprehensive example to illustrate and clarify the above concepts. Table 2 shows the input parameters and their values that are used to determine the debt capacity and optimal capital structure for a privately-financed project without a guarantee, with a production guarantee, and with a minimum-revenue guarantee.

The numerical results for a project with no guarantees are shown in Table 3. Table 4 shows

TABLE 4. Example results for the case of a $\$2,600M$ production guarantee (all \$ in millions)

Promised debt amount d_1 (1)	Market value of debt D (2)	Market value of equity S (3)	Market value of project V (4)	Net Present Value NPV (5)	Debt to cost ratio D/A (6)	Effective return on debt $E[\tilde{r}_D]$ (7)	Nominal interest on debt Int (8)	Required return on equity $E[\tilde{r}_S]$ (9)	Return on equity invstmnt $E[\widetilde{Roe}]$ (10)	$\partial D/\partial d_1$ (11)	$\partial E[\widetilde{ROE}]/\partial d_1$ (12)
0	0.00	2,250.63	2,250.63	50.63	0.000	0.060	0.060	0.117	0.143	0.943	4.46E-05
171	161.60	2,092.22	2,253.83	53.83	0.073	0.060	0.060	0.122	0.151	0.943	5.20E-05
343	323.21	1,933.82	2,257.03	57.03	0.147	0.060	0.060	0.127	0.161	0.943	6.13E-05
514	484.81	1,775.42	2,260.23	60.23	0.220	0.060	0.060	0.133	0.172	0.943	7.34E-05
685	646.42	1,617.02	2,263.43	63.43	0.294	0.060	0.060	0.140	0.186	0.943	8.95E-05
857	808.02	1,458.62	2,266.63	66.63	0.367	0.060	0.060	0.149	0.203	0.943	1.11E-04
1,028	969.62	1,300.21	2,269.84	69.84	0.441	0.060	0.060	0.159	0.225	0.943	1.43E-04
1,199	1,131.23	1,141.81	2,273.04	73.04	0.514	0.060	0.060	0.173	0.253	0.943	1.89E-04
1,370	1,292.83	983.41	2,276.24	76.24	0.588	0.060	0.060	0.191	0.291	0.943	2.62E-04
1,542	1,454.43	825.01	2,279.44	79.44	0.661	0.060	0.060	0.216	0.346	0.943	3.89E-04
1,580	1,490.57	789.59	2,280.16	80.16	0.678	0.060	0.060	0.224	0.362	0.943	4.29E-04
1,581	1,343.43	760.52	2,103.96	-96.04	0.611	0.086	0.177	0.254	0.113	0.533	-1.61E-04
1,713	1,410.04	677.19	2,087.23	-112.77	0.641	0.093	0.215	0.270	0.088	0.476	-2.16E-04
1,884	1,485.00	578.04	2,063.04	-136.96	0.675	0.102	0.269	0.292	0.044	0.399	-2.99E-04
2,056	1,546.76	489.19	2,035.95	-164.05	0.703	0.111	0.329	0.316	-0.014	0.322	-3.88E-04
2,227	1,595.43	410.52	2,005.95	-194.05	0.725	0.119	0.396	0.342	-0.089	0.247	-4.75E-04
2,398	1,631.57	341.61	1,973.18	-226.82	0.742	0.128	0.470	0.371	-0.176	0.176	-5.45E-04
2,570	1,656.11	281.85	1,937.96	-262.04	0.753	0.136	0.552	0.401	-0.274	0.112	-5.88E-04
2,741	1,670.34	230.47	1,900.82	-299.18	0.759	0.143	0.641	0.435	-0.376	0.056	-5.97E-04
2,912	1,675.80	186.67	1,862.47	-337.53	0.762	0.150	0.738	0.470	-0.476	0.009	-5.73E-04
3,083	1,674.15	149.63	1,823.78	-376.22	0.761	0.155	0.842	0.509	-0.571	-0.027	-5.23E-04
3,255	1,667.12	118.57	1,785.70	-414.30	0.758	0.160	0.952	0.551	-0.655	-0.053	-4.59E-04
3,426	1,656.37	92.78	1,749.15	-450.85	0.753	0.163	1.068	0.597	-0.727	-0.071	-3.89E-04

the results for a project with a $\$2,600M$ production guarantee and Table 5 shows the results for a project with a $\$2,600M$ minimum-revenue guarantee.

The columns in these tables show the values of D, S, V, NPV, D/A, $E[\tilde{r}_D]$, Int, $E[\tilde{r}_S]$, $E[\widetilde{Roe}]$, $\partial D/\partial d_1$, and $\partial E[\widetilde{Roe}]/\partial d_1$ for different d_1 values. Before examining these results in more detail, it is expedient to show that the production guarantee g_{100} needed to secure 100% debt financing for this project is larger than the available guarantee of \$2,600M and equals:

$$g_{100} = \frac{AR_f + e_f}{1 - e_v} = \$3,540M \tag{39}$$

Also, the values of b'' (the minimum $\widetilde{Rev}_g$ that provides some return to debtholders) and d_1^g

TABLE 5. Example results for the case of a $\$2,600M$ minimum-revenue guarantee (all \$ in millions)

Promised debt amount d_1 (1)	Market value of debt D (2)	Market value of equity S (3)	Market value of project V (4)	Net Present Value NPV (5)	Debt to cost ratio D/A (6)	Effective return on debt $E[\tilde{r}_D]$ (7)	Nominal interest on debt Int (8)	Required return on equity $E[\tilde{r}_S]$ (9)	Return on equity invstmnt $E[\widetilde{Roe}]$ (10)	∂D $/\partial d_1$ (11)	$\partial E[\widetilde{ROE}]$ $/\partial d_1$ (12)
0	0.00	2,280.92	2,280.92	80.92	0.000	0.060	0.060	0.113	0.154	0.943	4.92E-05
171	161.60	2,122.52	2,284.13	84.13	0.073	0.060	0.060	0.117	0.163	0.943	5.73E-05
343	323.21	1,964.12	2,287.33	87.33	0.147	0.060	0.060	0.121	0.173	0.943	6.76E-05
514	484.81	1,805.72	2,290.53	90.53	0.220	0.060	0.060	0.127	0.186	0.943	8.09E-05
685	646.42	1,647.32	2,293.73	93.73	0.294	0.060	0.060	0.133	0.201	0.943	9.87E-05
857	808.02	1,488.91	2,296.93	96.93	0.367	0.060	0.060	0.141	0.220	0.943	1.23E-04
1,028	969.62	1,330.51	2,300.13	100.13	0.441	0.060	0.060	0.151	0.244	0.943	1.57E-04
1,199	1,131.23	1,172.11	2,303.34	103.34	0.514	0.060	0.060	0.163	0.275	0.943	2.08E-04
1,370	1,292.83	1,013.71	2,306.54	106.54	0.588	0.060	0.060	0.179	0.317	0.943	2.89E-04
1,542	1,454.43	855.30	2,309.74	109.74	0.661	0.060	0.060	0.201	0.378	0.943	4.28E-04
1,580	1,490.57	819.89	2,310.45	110.45	0.678	0.060	0.060	0.207	0.395	0.943	4.73E-04
1,581	1,379.84	751.78	2,131.62	-68.38	0.627	0.076	0.146	0.255	0.150	0.533	-1.41E-04
1,713	1,446.45	668.92	2,115.36	-84.64	0.657	0.083	0.184	0.271	0.128	0.476	-1.97E-04
1,884	1,521.41	570.40	2,091.81	-108.19	0.692	0.092	0.239	0.293	0.087	0.399	-2.84E-04
2,056	1,583.16	482.23	2,065.39	-134.61	0.720	0.101	0.298	0.318	0.030	0.322	-3.82E-04
2,227	1,631.83	404.24	2,036.08	-163.92	0.742	0.110	0.365	0.344	-0.044	0.247	-4.78E-04
2,398	1,667.97	336.04	2,004.01	-195.99	0.758	0.118	0.438	0.373	-0.133	0.176	-5.60E-04
2,570	1,692.52	276.96	1,969.48	-230.52	0.769	0.126	0.518	0.404	-0.234	0.112	-6.13E-04
2,741	1,706.75	226.26	1,933.00	-267.00	0.776	0.133	0.606	0.437	-0.341	0.056	-6.29E-04
2,912	1,712.20	183.09	1,895.29	-304.71	0.778	0.140	0.701	0.473	-0.447	0.009	-6.07E-04
3,083	1,710.56	146.63	1,857.19	-342.81	0.778	0.145	0.803	0.512	-0.547	-0.027	-5.56E-04
3,255	1,703.53	116.10	1,819.63	-380.37	0.774	0.150	0.911	0.554	-0.637	-0.053	-4.87E-04
3,426	1,692.78	90.78	1,783.55	-416.45	0.769	0.153	1.024	0.600	-0.714	-0.071	-4.12E-04

(the maximum debt repayment amount d_1 that is fully covered by the guarantee) are:

$$b'' = \frac{b_f + e_f(1-b_v)}{(1-b_v)(1-e_v)} = \frac{100 + 500(1-0.3)}{(1-0.3)(1-0.2)} = \$804M \tag{40}$$

$$d_1^g = g(1-e_v) - e_f = 2,600(1-0.2) - 500 = \$1,580M \tag{41}$$

In contrast, a'' (the minimum $\widetilde{Rev}_g$ that ensures full payment of d_1 to debtholders) is not a constant but a linear function of the promised repayment amount d_1 (in $\$M$):

$$a'' = \frac{d_1 + e_f}{1-e_v} = \frac{d_1 + 500}{1-0.2} = 1.25d_1 + 625 \tag{42}$$

At $d_1 = d_1^g = \$1,580M$, $a'' = g = \$2,600M$. Depending on the relative magnitude of d_1, the production guarantee for this example is large enough to cover full/partial debt repayment (*i.e.*, $g = \$2,600M > b'' = \$804M$). For values $d_1 < \$1,580M$, the debt is riskless (*i.e.*, $a'' < g$), and the market value of debt, D_p, is calculated from Eq. (23), while the market value of equity, S_p, is given by Eq. (29). Similarly, for $d_1 > \$1,580M$, the debt is risky (*i.e.*, $a'' > g$), and the market value of debt, D_p, is calculated from Eq. (21), while the market value of equity, S_p, is given by Eq. (28).

The market value of the project, V, is the sum of D and S (assuming $S \geq 0$). The net present value, NPV, is equal to the market value of the project, V, minus the initial project costs, A. The debt to cost ratio, D/A, is the fraction of the initial cost of the project that is provided by the debtholders.

The effective return on debt, $E[\tilde{r}_D] = \{E[\tilde{D}_1]/D\} - 1$, is the expected return to the debtholders, where $E[\tilde{D}_1]$ is the repayment amount debtholders expect to receive at the end of the period. The promised return on debt, $Int = (d_1/D) - 1$, is the nominal interest rate debtholders would charge the owning company in order to lend them D.

The required return on equity, $E[\tilde{r}_S]$, is the return investors would expect to receive if they had invested in an openly-traded asset that presents the same degree of risk as the concession-financed project (*i.e.*, $\beta_{\text{asset}} = \beta_{\text{project}}$). The expected return on equity investment, $E[\widetilde{Roe}]$, is the ratio between the expected end-of-period value of the project after all obligations have been paid and the amount infused by investors at the beginning of the period, minus one.

Figure 2 displays D, S, and V, for the example project as functions of d_1 for the three cases of no guarantee, a production guarantee equal to \$2,600, and a minimum-revenue guarantee equal to \$2,600.

The market value of the debt D is given by Eq. (13) and is the amount debtholders expect to receive at the end of the period minus the systematic risk premium on the project's debt (*i.e.*, the amount lenders charge to buy part of the project's systematic operating risk premium from the owning company), divided by R_f.

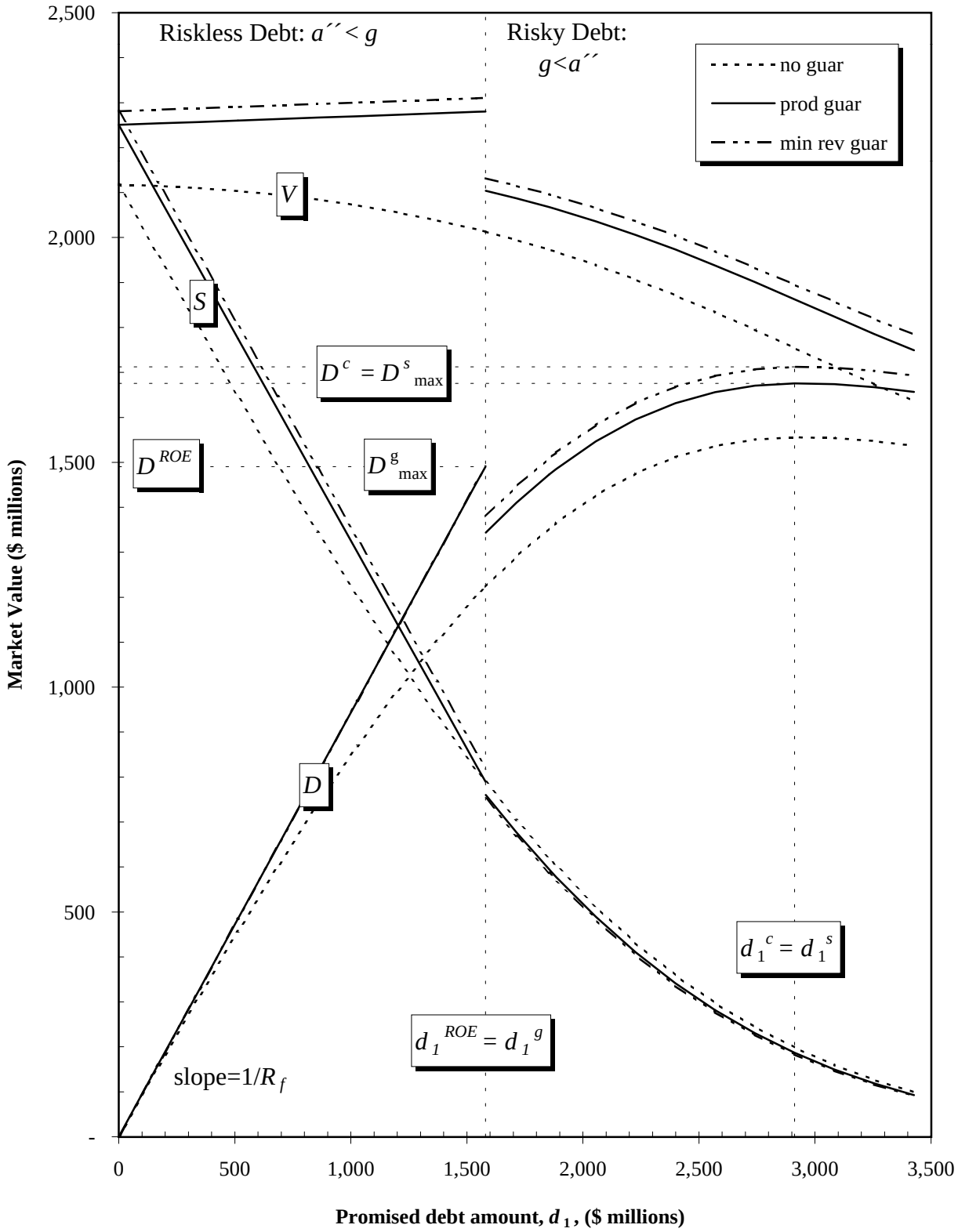


Fig. 2. Present (market) values of V, D, and S as functions of d_1

As shown in Figure 2, the value of the debt D in all cases tends to increase with the promised repayment amount d_1. For a production or a minimum-revenue guarantee, D has a discontinuity and a local maximum to the left of d_1^g. The global maximum with and without a guarantee occurs at d_1^c which for this example occurs at d_1^s.

At $d_1 = d_1^c = d_1^s$, the market value of the debt reaches $D^c = D^s$, which is the maximum amount of money the owning company can borrow. This is the debt capacity of the project. This maximum amount depends on the type as well as the magnitude of the project guarantee. As shown in Fig. 2, a project without a guarantee has the smallest debt capacity, while a project with a minimum-revenue guarantee has the largest. Thus, the type of guarantee can also increase debt capacity and enable promoters to borrow more money while promising to pay the same amount d_1^s.

The D and S curves for a project with guarantees in Figure 2 are discontinuous when d_1 crosses $d_1^g = \$1,580M$. At that point, $a'' = 1.25 \times 1,580 + 625 = \$2,600M$. For $d_1 < d_1^g = \$1,580M$, the repayment amount d_1 is small enough to be completely covered by the guarantee (*i.e.*, $a'' < g = \$2,600M$) and the debt is riskless. For $d_1 > d_1^g = \$1,580M$, the repayment amount d_1 is larger than what the guarantee can secure (*i.e.*, $a'' > g = \$2,600M$) and the debt is risky. Therefore, it is necessary to use two functions to express the market values of debt and equity for a project with either a production or a minimum-revenue guarantee. Determining the project debt capacity requires the comparison of the maximum point of both of these functions. For this example, a $\$2,600M$ production guarantee gives: $D^g_{\max} = \$1,491M$ and $D^s_{\max} = \$1,676M < A = \$2,200M$; therefore, the debt capacity of the project is $D^c = D^s_{\max} = \$1,676M$. Similarly, a minimum-revenue guarantee gives: $D^g_{\max} = \$1,491M$, $D^s_{\max} = \$1,712M < A = \$2,200M$; therefore, the project debt capacity is $D^c = D^s_{\max} = \$1,712M$. For this example, debt capacity occurs when the debt is risky at a debt repayment amount $d_1 = d_1^c = d_1^s = \$1,912M$.

Figure 3 displays $E[\widetilde{Roe}]$ as a function of d_1 for a project without a guarantee, as well as for a project with either a production guarantee or a minimum revenue guarantee equal to $\$2,600M$. In part, guarantees increase $E[\widetilde{Roe}]$ because they enable promoters to borrow more money from lenders while promising to pay a fixed amount, and hence require a lower equity investment.

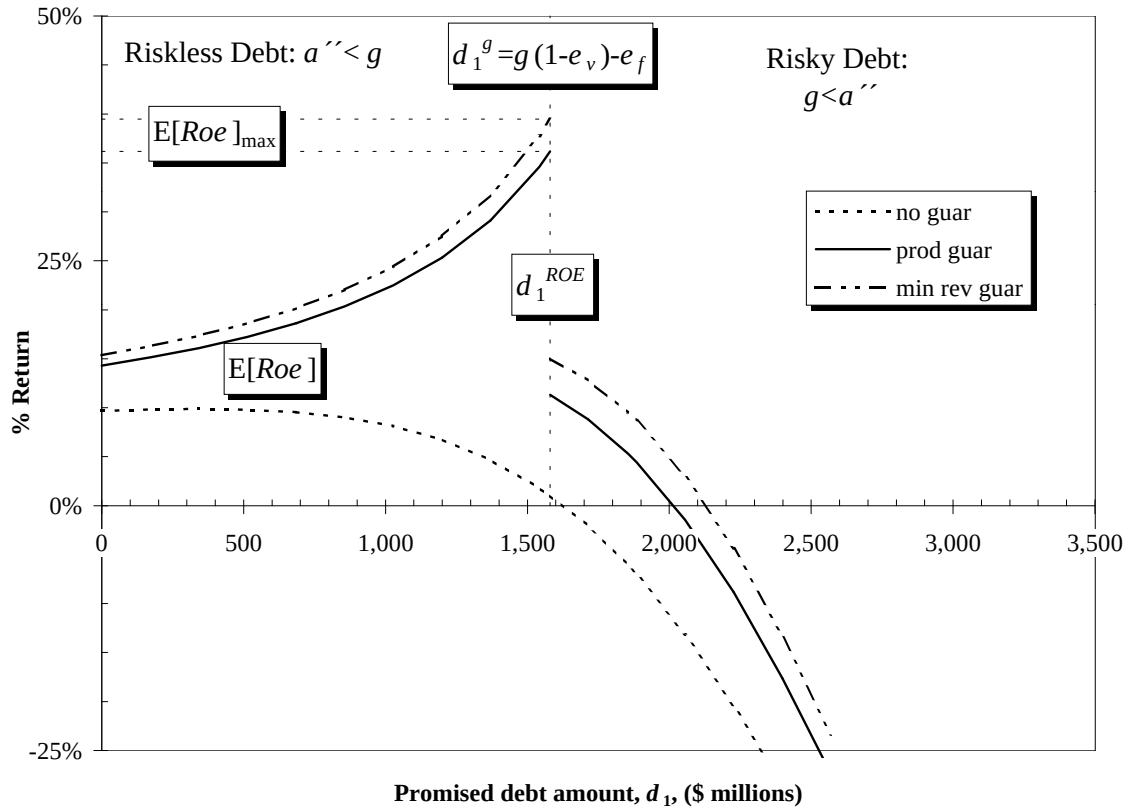


Fig. 3. Expected value of the return to investors, $E[\widetilde{Roe}]$, as a function of d_1

The curves for projects with guarantees in Figure 3 are discontinuous at $d_1^g = \$1,580M$ because at that point the debt changes from being riskless to being risky (a'' changes from $a'' < g$ to $a'' > g$). Therefore, in order to find $E[\widetilde{Roe}]_{\max}$ and determine the optimal capital structure for the project, it is necessary to use a procedure similar to that used to determine project debt capacity. Figure 3 shows that for a $\$2,600M$ production guarantee, the maximum return on investment is $E[\widetilde{Roe}]^g_{\max} = 0.362$ and occurs when $d_1 = d_1^g = \$1,580M$.

Thus, for this example, the objective of maximizing the project's *NPV* and maximizing the equityholders's returns provide the same optimal capital structure. Both occur at $d_1 = d_1^g = \$1,580M$, i.e., when the debt is still riskless (even though debt capacity occurs when the debt is risky, at $d_1 = d_1^s = d_1^c = \$1,912M$). The optimum debt is $D^{ROE} = D^{NPV} = 1,490M$.

Moreover, this example shows the value of guarantees. Table 3 shows that without a guarantee

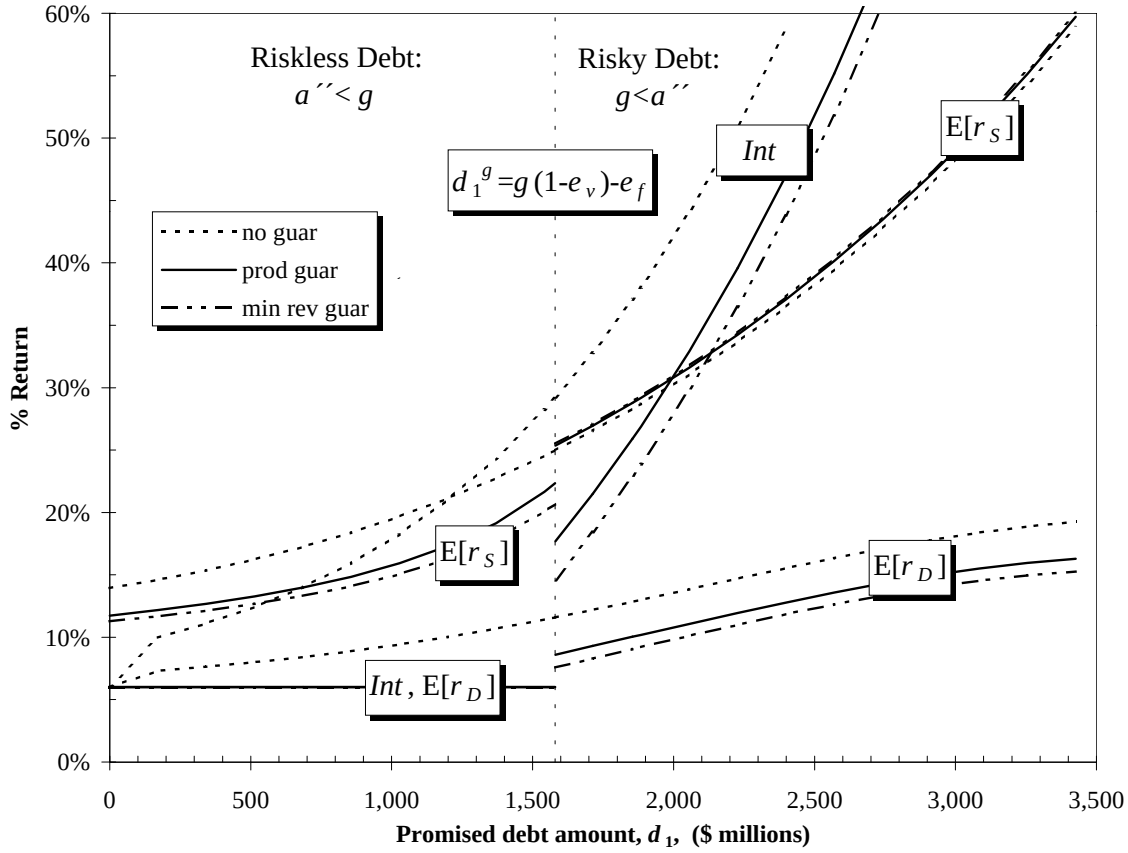


Fig. 4. $E[\tilde{r}_D]$, *Int*, and $E[\tilde{r}_S]$, as functions of d_1

this project has at best a negative $NPV = -\$82.30M$. A $\$2,600M$ guarantee and a loan of $D^{ROE} = D^{NPV} = 1,490M$ can boost this project's *NPV* to being positive, *i.e.*, $NPV = \$80.16M$ for a production guarantee and $NPV = \$110.45M$ for a minimum-revenue guarantee.

Figure 4 shows $E[\tilde{r}_D]$, *Int*, and $E[\tilde{r}_S]$ as functions of d_1 for the same three guarantee cases. Because projects with guarantees are less risky than projects without guarantees, lenders can charge lower interest rates and also expect a lower return on debt. As in Figures 2 and 3, Figure 4 also shows that the curves related to guarantees are discontinuous. Note that $E[\tilde{r}_D] = Int = R_f$ for promised debt amounts to the left of the discontinuity. This is because at borrowing levels to the left of d_1^g the guarantee is sufficient to provide full debt repayment, and thus, the debt is riskless.

Figure 5 shows the impact of guarantees on a project's debt capacity. As expected, it shows that higher levels of guarantees g increase D^c/A, the project's debt capacity as a percentage of the initial

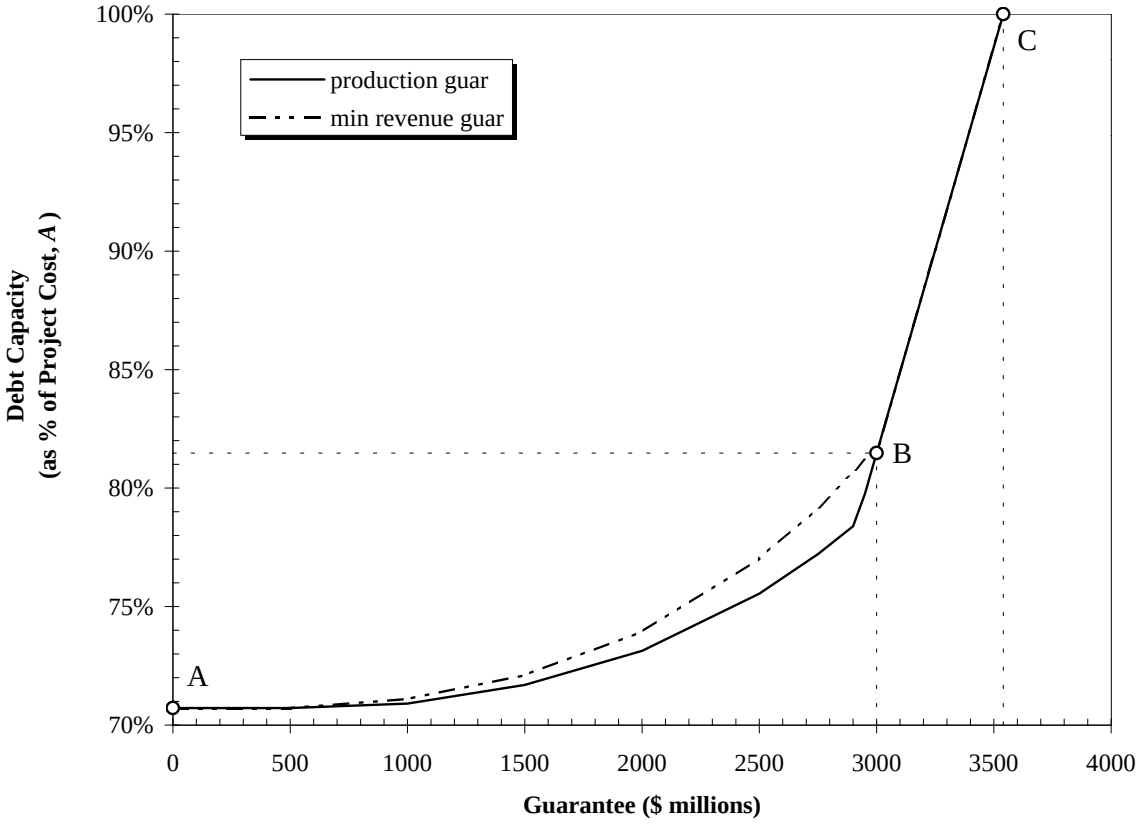


Fig. 5. Effect of guarantees on project debt capacity

project cost. It also shows that debt capacity presents two different behaviors. In the region (A, B), debt capacity occurs when the debt is risky, *i.e.*, when $g < a''$ (as was the case for this example) and $D^c = D^s_{\max}$. In the region (B, C), debt capacity occurs when the debt is riskless, *i.e.*, when $a'' \leq g$, and $D^c = D^g_{\max}$. In this case, debt capacity $D^c = D^g_{\max}$ occurs when the company promises to repay the maximum amount d^g_1 that is fully covered by the guarantee g. Point A represents the case of no guarantees, point B is the case where $D^c = D^s_{\max} = D^g_{\max}$, and point C represents the case where the guarantee is sufficient to secure 100% debt financing (*i.e.*, $g = g_{100}$).

CONCLUSION

Privately financed infrastructure projects provide alternatives for developing and implementing many types of projects that range from civil infrastructure works to industrial facilities. Private-sector involvement facilitates access to new sources of finance, provides gains in efficiency arising

from better business expertise (including innovation, marketing and management skills), and greater incentives for the control of construction, operating, and maintenance costs. A detailed discussion of the benefits of concession-financed projects appears in (World Bank 2017, 2023; Irwin 2007).

Project development by a purpose-formed single-project owning company is attractive to private-sector corporate equityholders because of the limited liability afforded by off-balance-sheet financing. This situation creates risky and expensive debt because debtholders increase their risk premiums to account for the probability that the owning company defaults. As a result, there is a limit to what owning companies can borrow to finance a project (debt capacity). The presence of a guarantee that secures more than just the cost of bankruptcy (*i.e.*, $b'' < g$) increases the debt capacity of a project. This is because the project becomes less risky and lenders are willing to finance it at a lower interest rate than they would if there were no guarantees. For the same promised amount d_1, project promoters should be able to raise more debt financing, D, as the level of guarantees increases. If the guarantee is large enough to cover the full repayment amount, then the debt is riskless and the possibility of bankruptcy does not even exist. When the guarantee exceeds g_{100}, then the project can be 100% debt financed risk-free.

The presence of guarantees increases the expected return on investment as well as the project's net present value. An increase in a guarantee that does not cover the full debt obligations increases expected return because it decreases the required equity capital (*i.e.*, the amount that can be borrowed increases) while the expected value of equity at the end of the period remains constant. Increasing a guarantee that already assures full debt repayment increases expected return because the expected value of equity at the end of the period increases while the amount that can be borrowed remains constant (*i.e.*, $D = d_1/R_f$). If the maximum expected return on investment occurs at the maximum fully secured debt level ($g = a''$), the objective of maximizing the project's net present value provides the same optimal capital structure as the objective of maximizing the equityholders' returns. If it also happens that the corresponding $D^g_{\max}$ is equal to D^c, the maximum expected return on the equityholders' investment as well as the maximum project's *NPV* occur at the project's debt capacity; otherwise, the optimal capital structure always requires less borrowing than the available

debt capacity.

Comparing the objective of maximizing equity returns with the more traditional objective of maximizing equityholders' wealth (*i.e.*, net present value) shows that the capital structure used to maximize returns allows more (or equal) debt financing than the more traditional objective of maximizing wealth and *NPV*.

A project's net present value *NPV* and the equityholders' expected return on investment $E[\widetilde{ROE}]$ can be quite sensitive to the selected debt-equity ratio and can decline rapidly as the owning company borrows more than the optimal amount. Thus, excessive borrowing to achieve debt capacity can make the promoting team worse off. Instead, the promoting team should strive to achieve the optimum debt-to-equity ratio that maximizes net present value or the expected return on investment, and if it does not have the necessary equity, it should invite additional investors until enough funds become available. Alternatively, it can negotiate with the sponsor government for a more favorable guarantee in an effort to reduce the interest required by the debtholders and thus to increase the amount of debt and reduce the required equity at the optimum. Clearly, the issue of optimal capital structure merits significant attention at the project planning phase.

A one-period project was considered in order to keep the mathematical analysis as simple as possible. The same general conclusions, however, are also valid for multi-period projects. The analysis, in this case, is best undertaken using numerical methods that apply the basic mathematical results presented here over many periods.

APPENDIX I. NOTATION

The following symbols are used in this paper. Random variables are indicated by a tilde (~) over their symbols.

A = Total project cost;

a'' = the minimum amount of $\widetilde{Rev}_g$ that avoids bankruptcy (*i.e.*, the money generated by the project is sufficient to pay project expenses plus debt expenses);

$\tilde{B}$ = Total cost of bankruptcy;

$b' = b_f/(1 - b_v)$, net operating income ($\tilde{X}$) level at which the lenders are repaid nothing;

b'' = the maximum amount of $\widetilde{Rev}_g$ that does not provide any return to debtholders (*i.e.*, all the money generated by the project is used to pay bankruptcy costs);

b_f = Fixed cost of bankruptcy;

b_v = Variable cost of bankruptcy;

D = Amount borrowed by owning company (project debt) while promising to repay d_1 at the end of period 1;

$\tilde{D}_1$ = Uncertain market value of a project's debt at the end of period 1;

d_1 = $D(1 + Int)$, amount promised to be paid to lenders at the end of period 1, (*i.e.*, principal + interest);

d_1^g = the maximum promised repayment amount d_1 that is fully covered by the guarantee g;

d_1^c = Amount of d_1 that maximizes the debt amount for a project (*i.e.*, the project reaches its debt capacity, D^c);

d_1^{ROE} = Amount of d_1 that maximizes the investors' return on equity;

d_1^V = Amount of d_1 that maximizes the project's value, NPV;

D^c = Debt capacity;

D_{max}^g = Maximum riskless debt that can be secured;

D_{max}^s = Maximum risky debt that can be secured;

$E[\tilde{U}]$ = Expected value of the random variable U;

$\widetilde{Exp}$ = Project expenses;

e_f = Fixed portion of project expenses;

e_v = Coefficient for the variable portion of project expenses;

$F_U(u)$ = Cumulative distribution function (CDF) of random variable U evaluated at u;

$f_U(u)$ = Probability density function (PDF) of random variable U evaluated at u;

g = Guarantee;

g_{100} = Guarantee level that provides riskless 100% debt-financing;

Int = Nominal interest rate charged by the lenders which creates a repayment obligation d_1 (at the end of period 1) on a loan D;

$\tilde{R} = 1 + \tilde{r}$, one plus the rate of return on a single-period project;

$R_f = 1 + r_f$, one plus the risk-free rate of interest;

$\tilde{R}_m = 1 + \tilde{r}_m$, one plus the rate of return on the market;

$\widetilde{Rev}$ = Project revenues;

$\widetilde{Rev}_g$ = Project revenues (protected by guarantee);

$\widetilde{ROE} = 1 + \widetilde{Roe}$, one plus the investors' return on equity;

$\tilde{r}_D$ = The actual rate of return on the debt D (depends on how much of the promised amount d_1 is actually paid to the lenders at the end of period 1);

$\tilde{r}_S$ = The actual rate of return on equity S;

S = Present market value of a project's equity when the market is in equilibrium;

$\tilde{S}_1$ = Uncertain market value of a project's equity at the end of period 1;

T = Corporate income tax rate;

V = Present (actual) market value of a project when the market is in equilibrium;

$\tilde{V}_1$ = Uncertain market value of a project at the end of period 1;

$\tilde{X}$ = Net Operating Income (NOI) generated by the project at the end of period 1;

β = Asset beta as defined by the capital asset pricing model (CAPM);

δ_i = Several binary (0,1) auxiliary variables defined when introduced in the text;

λ = Market price of a unit of risk (the expected excess return that must be given up by an asset to reduce a unit of its risk);

σ_U^2 = Variance of the random variable U.

DATA AVAILABILITY STATEMENT

All data, models, and code generated or used during the study appear in the submitted article.

ACKNOWLEDGMENTS

The authors would like to express their gratitude to the College of Engineering at the University of Michigan and to CAPES - Coordenação de Aperfeiçoamento de Pessoal de Nível Superior for their financial support of this research.

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