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THE VALUE OF SINGLE FAMILY RESIDENTIAL PROPERTY:
AN EMPIRICAL INVESTIGATION

by
Douglas Oliver Stewart

A dissertation submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
(Economics)
in The University of Michigan
1974

Doctoral Committee:

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Chapter 1

INTRODUCTION

A. PURPOSE AND MOTIVATION

The purpose of this paper is to present a theoretical model for an empirical investigation of the value of single family residential properties and report the results of such an investigation. The results of the investigation indicate the relative importance to consumers of a property's characteristics. These empirical results of a residential housing market study are of interest for two reasons. First, they are useful for supporting or rejecting the hypotheses developed in various theoretical works.¹ The empirical results of the model are valuable too because "models of this kind would be of enormous value to appraisers in estimating the market value of real estate for tax and other purposes."² "Other purposes" includes measuring the impact on property values of public investment projects, zoning and land-use changes, and the effect of changes in the socio-economic characteristics of a neighborhood's residents. It is

¹Ralph Turvey, The Economics of Real Property: An Analysis of Property Values and Patterns of Use (London: George Allen and Unwin Limited, 1957); William Alonso, Location and Land Use; Toward a General Theory of Land Rent (Cambridge, Massachusetts: Harvard University Press, 1964); Richard F. Muth, "Urban Residential Land and Housing Markets," Issues in Urban Economics, ed. H. S. Perloff and L. Wingo, Jr. (Baltimore, Maryland: The Johns Hopkins Press, 1968), pp. 285-334.

²John F. Kain and John M. Quigley, "Measuring the Value of Housing Quality," Journal of the American Statistical Association, (1970), LXV, p. 533.

important to know how the windfall gains and losses resulting from these changes are distributed.

It is clear from the available data that current assessment practices leave room for improvement. The coefficient of dispersion from the median ratio of assessed value to sales value is often used to indicate the accuracy of assessment. Values of this coefficient were reported for nonfarm houses in selected local areas in the 1967 Census of Governments. Some of the findings were as follows: in Michigan the coefficient ranged from a low of 7.0 percent for the City of Livonia to a high of 22.8 percent for the City of Pontiac, with the City of Ann Arbor having a coefficient of 9.4 percent; in California the coefficient ranged from a low of 7.2 percent for the City of Anaheim to a high of 28.9 percent for the San Francisco area; and in New York it ranged from a low of 10.6 percent for the Tompkins area to a high of 69.2 percent for the County of Albany.³ The failure of assessment practices to correctly determine the relative market value of properties results from failing to consider some of the factors which affect market values. Even if all factors were considered, it is quite likely that their importance would be over- or under-estimated by assessors.

The empirical results of housing market investigations may permit marginal changes in the current methods which would increase the adequacy of assessment practices. Secondly, it may be possible to

³U.S. Bureau of the Census, Census of Governments, 1967, Vol. 2: Taxable Property Values (Washington, D.C.: Government Printing Office, 1968), California pp. 125, 127, Michigan p. 133 and New York p. 137. The coefficient of dispersion is the percentage by which the various individual sales items differ, on the average, from the median assessment ratio.

develop radically different techniques for assessing property values. For example, California has been experimenting for some time with computer-assisted appraisals.⁴ These appraisals consist essentially of using data on characteristics of exchanged houses to estimate the coefficients of a multiple regression equation. These coefficients are applied to the characteristics of an unsold house to estimate its expected market value. The legal assessment sales ratio is applied to the expected market value to determine the house's assessed value. For this technique to yield accurate results and be financially feasible, two conditions must obtain. First, a property's sale value must vary in a systematic way with the values of various measurable characteristics. Secondly, the coefficients must remain fixed over relatively long periods of time, or at least change in a way which permits inexpensive updating of the regression results. The empirical results of a market value study are useful in evaluating experiments such as the ones being conducted in California.

⁴Ronald B. Welch states in a letter to Harvey E. Brazier on March 2, 1971 made available to me, "Bob Gustafson is currently working with representatives of some 12 to 15 California counties which are either experimenting with computer-assisted appraisals or actually implementing a countywide program. There are perhaps a half-dozen counties in the State where part of the 1971 assessment rolls will be based on computer-assisted appraisals, and it is alleged that all single family assessments in Orange County are so based."

Previous empirical investigations⁵ of single family property values suffer from one or more deficiencies among which the most common are poor specification of the dependent and/or independent variables⁶ and the use of highly aggregated data. These deficiencies limit the extent to which the results of previous studies can be used to improve assessment practices. For assessment purposes the dependent variable, the variable to be predicted, must be the market value of individual properties. Few empirical studies have concerned themselves with this variable. And where the dependent variable has been specified in accordance with assessors' needs, the investigator usually has been satisfied by including a limited number of independent variables which explained only a small part of the variation in the dependent variable.

Motivation for yet another study of the market value of single family property lay in the belief that it is possible to develop a

⁵A partial list of such studies, those most closely related to the present study, includes the following: Richard F. Muth, "Urban Residential Land and Housing Markets," op. cit.; John F. Kain and John M. Quigley, "Measuring the Value of Housing Quality," op. cit., pp. 532-548; Harold Brodsky, "Residential Land and Improvement Values in a Central City," Land Economics (1970), XLVI, pp. 229-247; Warren R. Seyfried, "Location and the Centrality of Urban Land Values," Land Economics (1970), XLVI, pp. 329-333; R. Harris, G. Tolley, C. Harrell, "The Residence Site Choice," Review of Economics and Statistics (1968), L, pp. 241-247; J. P. Crecine, O. Davis, J. Jackson, "Urban Property Markets: Some Empirical Results and Their Implications for Municipal Zoning," The Journal of Law and Economics (1967), pp. 79-99; Wallace E. Oates, "The Effects of Property Taxes and Local Public Spending on Property Values: An Empirical Study of Tax Capitalization and the Tiebout Hypothesis," Journal of Political Economy (1969), pp. 957-971; and Eugene F. Brigham, "The Determinants of Residential Land Values," Land Economics (1965), XLI, pp. 325-334.

⁶Poor specification of variables may result from failure to develop a firm theoretical foundation for the empirical work or from inadequate data.

theoretical foundation which suggests the proper specification of important explanatory variables. Furthermore, the availability of data with a low level of aggregation suggested the possibility of a study with results more widely applicable than those of previous studies. The objective of the present study was to obtain results which will be useful in solving the problems which up to this time have seemed to be inherent in the appraisal of real property.

The remainder of this chapter consists of three parts. Section B1 contains a review of some of the major theoretical works in the economics of real property, beginning with David Ricardo's efforts to explain land values and ending with those of William Alonso. Section B2 contains reviews of four previous empirical studies. These reviews pay particular attention to the deficiencies of the investigations as they related to property appraisal. Finally, Section C contains an outline of the contents of the remaining chapters of this paper.

B. REVIEW OF THE LITERATURE

1. The Theoretical Literature

Although the entire literature on market determination of prices is applicable to the particular case of the residential housing market, much has been written about this specific market. The locational elements in urban land values were of primary interest to the early writers in this area. This resulted directly from the attention previously given to the determinants of agricultural land values. It is a short, albeit rather difficult, step from analyzing agricultural land values to a more general analysis of land values which

would include urban land values. However, the foundation for the present-day theory of the determinants of urban land value can be traced to David Ricardo's theory of land rent in his Principles of Political Economy and Taxation, 1817. Although giving most of his attention to fertility differentials, Ricardo recognized both locational and fertility differentials as bases for agricultural land rent. The rent on more productive land depends on its advantage over less productive land. Competition among farmers insures that landlords receive all land rent.

Whereas earlier writers restricted their analyses to agricultural land values, Alfred Marshall discussed urban land values as well in his Principles of Economics. However, in the chapter devoted to urban land values, Marshall dealt almost exclusively with non-agricultural firms, largely neglecting the value of residential land.⁷ He defined "situation value" as the sum of money values placed on the differential advantages which a site confers upon a firm. His main emphasis was on locational advantages, and as an example he mentioned access to specialized labor. He concluded that the price which a piece of urban land yields if cleared of buildings and sold in a free market, its bare "site value," is the sum of its agricultural value and its situation value. Another improvement in the theory of land value made by Marshall was his discussion of the size of site in relation to the height of the building upon it. Concerning this issue Marshall stated, "If land is cheap he (the developer) will take much of it; if it is dear he will take less and build high."⁸ Until this time the

⁷Ninth (Variorum) Edition, Volume I, The Macmillan Company, New York, 1961. Book V, Chapter XI.

⁸Ibid., p. 448.

effect of the value per unit of land on the type of improvements placed on a site had not been considered.

Those persons advocating the replacement of a tax on both improvements and land by a tax on site value believe that it is theoretically and practically possible to separate property value into its components. Marshall indicated general agreement with this possibility. His position was as follows:

The rent of a house (or other building) is a composite rent, of which one part belongs to the site and the other to the buildings themselves. . . . The corresponding rents can always be distinguished analytically, and sometimes they can be separated commercially.⁹

The analysis of urban land values was extended in 1903 when Richard M. Hurd published his Principles of City Land Values. Hurd outlined a theory of urban land values which, according to William Alonso,¹⁰ closely resembles that of von Thunen's theory of agricultural land values:

As a city grows, more remote and hence inferior locations must be utilized and the difference in desirability between the two grades produces economic rent in locations of the first grade, but not in those of the second. As land of a still more remote and inferior grade comes into use, ground rent is forced still higher in land of the first grade, arises in land of the second grade, but not in the third grade, and so on. Any utility may compete for any location within a city and all land goes to the highest bidder....¹¹

⁹ Ibid., p. 453.

¹⁰ Op. cit., p. 5.

¹¹ Richard M. Hurd, Principles of City Land Values, The Record and Guide, New York, 1903, p. 11.

Hurd made it very clear by the following statement that he recognizes the importance of accessibility as a factor determining land values: "Since value depends on economic rent, and rent on location, and location on convenience, and convenience on nearness, we may eliminate the intermediate steps and say that value depends on nearness. . ."¹²

Although he considered the basis of residence values to be social and not economic, Hurd discussed at greater length than Marshall those factors which cause variations in the value of residential land. For example, he mentioned nearness to parks, favorable transportation facilities, and the absence of nuisances.¹³ Presumably all of these are scarce factors and have implicit prices which could be discovered using the appropriate statistical methods. To our knowledge Hurd's study is the first to present actual data supporting the hypothesis that in urban areas accessibility is sufficiently important for land values to decline as inaccessibility or distance from the center of the city is increased. The data are presented in the form of maps which show land values for both the business and residential sections of several American cities.¹⁴

¹²Ibid., p. 13.

¹³Ibid., p. 78.

¹⁴In his review of Hurd's study, Alonso states that Hurd does not consider the size of the site, op. cit., p. 6. However, this is not strictly true. Hurd demonstrates in at least one place that he recognizes the importance of considering site size; in his map illustrating land values he uses as units of value "dollars per front foot for a lot of average width and depth."

Beginning in the 1920's at the time of rapidly increasing urbanization of the American population and continuing through the 1960's, there has been growing interest in the urban land market as evidenced by the volume of work on the subject. The article by Robert M. Haig is typical of what was written between 1920 and 1955.¹⁵ Haig emphasized the savings in transportation costs resulting from locating near the center of an urban area as the key factor in firms' location decisions and the consequent pattern of land rents. Haig recognized that

. . . activities differ in the degree to which they can make effective use of the quality of physical proximity or accessibility possessed by the center--that is, in the degree that they can turn accessibility into profits. An order of precedence of activities is worked out by competitive bidding, the relative size of the bids being determined fundamentally by the degree to which the various activities can profitably utilize sites¹⁶

Haig's view of differential advantage or varying accessibility as the source of land rents is substantially the same as Hurd's and Marshall's view. However, unlike these previous authors, Haig completely neglected consideration of the size of site.

Residential location is the one phase of the land rent-determination question in which Haig has a more well-developed discussion than Marshall or Hurd. He made explicit in the following statement the similarity between a firm's location decision and a family's location decision:

¹⁵"Toward an Understanding of the Metropolis," Quarterly Journal of Economics (1926), Vol. XL, pp. 402-434. Other authors prominent during this period include R. T. Ely, G. S. Wehrwein, Richard U. Ratcliff, and Edward W. Morehouse.

¹⁶Haig, op. cit., p. 420.

In choosing a residence purely as a consumption proposition, one buys accessibility precisely as one buys clothes or food. He considers how much he wants the contacts furnished by the central location, weighing the 'costs of friction' involved--the various possible combinations of site rent, time value, and transportation costs; he compares this want with his other desires and his resources, and he fits it into his scale of consumption, and buys. . .¹⁷

More recently Ralph Turvey has presented an analysis of property values and has attempted to explain the pattern of property uses.¹⁸ Turvey's theoretical analysis is couched in terms of property markets in general. However, since this study is interested in the market for single-family properties, discussion of his analysis will be put in terms of that narrowly-defined market.

Turvey asserts that there are three factors which determine the relative market values of single family properties: (1) the physical condition of the building, (2) its "general accessibility" which Turvey defines as the property's "nearness in terms of travel cost rather than distance per se to all other buildings of the town," and (3) its "special accessibility," defined as "its nearness to particular complementary facilities."¹⁹ The first factor includes such attributes as the number of bathrooms, square feet of living space, degree of deterioration, and other on-site characteristics. The second refers to the accessibility to economic or social activity in general, while the third factor might be interpreted for residential properties as the accessibility to specific facilities such as schools and shopping areas.

¹⁷Ibid., p. 423.

¹⁸Turvey, op. cit.

¹⁹Ibid., p. 48.

Although Turvey argues that at any point in time the property market is not in long-run equilibrium and that rents, hence property values, "are to be explained less in terms of equilibrium analysis than with reference to the absence of equilibrium,"²⁰ he does not present much discussion as to what happens in the absence of equilibrium. Turvey attempts to justify this failure when he makes the following statement:

In the short run, however, an equilibrium analysis in general terms can contribute some understanding. Given the existing stock of buildings, the pattern of rents can be analyzed by examining demand conditions, as can the pattern of land use insofar as it is flexible--i.e. to the extent that some buildings are capable of more than one use. . . Thus as a first approximation, the way in which existing buildings are used and the pattern of market values may be discussed in equilibrium terms. . . .²¹

Turvey does suggest some factors operating in the housing market to slow the tendency to equilibrium, including imperfect knowledge of the market by buyers and sellers, the long life of structures, and legal complexities and taxation rules which prevent the typical owner from making the most advantageous decisions.²² The argument that the housing market is not in long-run equilibrium at any point in time is of great importance for an empirical study of the type reported in this paper. If this theoretical argument seems to apply and is

²⁰Ibid., p. 28.

²¹Ibid., p. 48. Further discussion of the question of equilibrium analysis versus disequilibrium analysis appears in Chapter II of the dissertation.

²²Alonso criticizes Turvey and others who discuss these market imperfections since there is no explicit model of the perfect market to which the imperfections would apply. op. cit., p. 13.

disregarded in the analysis, it calls into question the meaning of the coefficients of the explanatory variables in a cross-section context.

Turvey also discusses the possibility of separating property value into its land and improvements components. He states, "This supposed 'physical' division of value into site and building value has no analytical value, and is meaningless except in long-run stationary equilibrium."²³ However, he does mention two methods of estimating the value of a given site, the "comparative" and the "residual." In applying the first method, the site value "is formed on the basis of information about the price fetched by bare sites of comparable nature."²⁴ The second is one in which "the cost of building and developer's profit are subtracted from the value which it is estimated would be possessed by the building most suitable to the site."²⁵ Turvey implies that the division is impractical in most instances due to the vast amount of judgment and uncertainty inherent in these methods.

Others who, in the context of a discussion of replacing a general property tax with a tax on land, have considered the possibility of dividing property value into improvement and site components are more optimistic than Turvey about the accurate estimation of site value. In his review article on implementing site-value taxation, Daniel Holland writes,

²³Ibid., p. 23

²⁴Ibid., p. 82.

²⁵Ibid.

. . . Methods are available despite the lack of direct sales evidence for vacant sites to permit reasonably good assessment of land values. These methods are standard, have always been available to assessors, but have not always been used by them.

In this connection Kenneth Back stresses particularly the Allocation Method. . . . Back considers that 'Potentially, however, it is the key to accurate and uniform land evaluation' which would permit the derivation of a meaningful allocation of the value of land and buildings for 'every parcel of improved real property. . . if the assessor subjects a modicum of information about a number of properties having similar use and located in an homogeneous area to some rather elementary statistical procedures.' "26

However, these statements are unsupported assertions, and an extensive review of the literature reveals that, in general, those supporting site-value taxation for reasons other than ease of implementation state that site value can be accurately estimated, and those who do not support site-value taxation point to the difficulty or impossibility of making accurate estimates of site value.

Within the last decade there have been attempts to generalize further the theory of land rent. William Alonso has succeeded to the extent that he has allowed two choice variables for the consumer to consider when seeking property, size of the site and its location vis-a-vis the center of the city, rather than the one, location, allowed previously. In addition, he has structured his theory in such a way as to include all land uses, namely, business, agricultural, and residential.

In formulating his model, Alonso makes the following assumptions:
(1) the city is located on a featureless plain, (2) all land is of

²⁶ Daniel M. Holland, "Implementing Site Value Taxation," Explorations in Urban Land Economics, ed. John J. Sullivan (Hartford, Connecticut: University of Hartford, 1970), pp. 18, 19.

equal quality and ready for use without further improvement, (3) land is freely bought and sold, and (4) the assumptions of perfect competition are fulfilled.²⁷ Applying the tools of indifference analysis to the problems of where the individual household should locate and what quantity of land it should purchase, Alonso concludes that a household will locate where the savings from cheaper land are at least as great as the increased commuting costs. Using similar techniques, Alonso derives the conditions for equilibrium location by an urban business and by an agricultural business. He then derives bid price curves²⁸ for all potential types of users and combines these to find the market equilibrium solution in terms of location and size of site.

Alonso applies his model by considering the effects on locations and densities of changes in income, population, technology, and zoning regulations.²⁹ For example, a reduction in marginal commuting costs to the center of the city from any given location leads the household to increase its land holdings and move farther from the center of the city. By these applications, he is able to demonstrate the complexity of the land market and resulting difficulty in drawing general conclusions about the relative effects of alternative public policies.

However, even though Alonso has developed a theory whose workings are rather complex, his model is far too simple to permit an adequate analysis of public policy decisions. He has neglected entirely the fact that sites have varying amenity levels. Differential amenity

²⁷ Alonso, op. cit., pp. 15, 16.

²⁸ "A bid price curve is a set of combinations of land prices and distances among which the individual is indifferent." Ibid., p. 71.

²⁹ Ibid., chapter 6.

levels might be the most important factor in the explanation of land values, particularly residential land values. Furthermore, Alonso's treatment does not capture the complex nature of the desire for accessibility.

This review of the literature has not included any discussion of a theory of the amount and types of improvements appearing on residential sites. The explanation for this omission is that there has been little theoretical discussion concerning the combinations of improvements and land which one should expect to find in various residential areas. That work which has been done in the area of demand for housing consists primarily of aggregate stock-adjustment models.³⁰ These bear almost no relation to the type of model needed as a foundation for the empirical work in this study. These models are useful for investigating the income and price elasticities of demand for housing, but, as a result of the high level of aggregation, they are of little help in explaining the value to a consumer of various housing characteristics.

Some of the hypotheses suggested in the theoretical works reviewed in this section have been subjected to empirical investigation. A review of the methods and results of four of these investigations is contained in Section B2.

2. Review of Some Earlier Empirical Studies

Previous empirical studies of urban land and residential housing markets may be grouped into two almost mutually exclusive categories:

³⁰Models of this type include H. S. Houthakker and Lester D. Taylor, Consumer Demand in the United States (2nd. ed.; Cambridge, Massachusetts: Harvard University Press, 1970) and Richard F. Muth, "The Demand for Non-Farm Housing," The Demand for Durable Goods, ed. Arnold C. Harberger (Chicago: The University of Chicago Press, 1960), pp. 29-96.

(1) studies which attempt to test the hypotheses of location theory and (2) those which investigate the effect of various neighborhood and on-site characteristics on residential property values.³¹ Eugene Brigham's study³² is typical of those in the first category. For this study he arbitrarily selected three rays running from the Civic Center to the Los Angeles County boundaries. The average appraised value per square foot of each block through which one of these rays passed was treated as an observation of the dependent variable. This dependent variable was regressed on an accessibility potential variable, a variable for amenity, one for variations in topography, and a variable to take account of historical accidents. In this study amenity referred to all those characteristics associated with a given site with the exception of the site's accessibility and non-optimal surrounding land-use characteristics. For example, amenity value included the values of dwelling-unit crowding and nonwhite proportion of the population. The historical factor is the use of land in a way which is no longer suitable. Among the independent variables the accessibility potential variable received the most careful specification. For any given site i , the value of this variable P_i depends on total employment E_j in the j -th workplace and the distance D_{ij} between i and j . The variable takes the form

$$P_i = \sum_{j=1}^N \frac{E_j}{a + bD_{ij}^c},$$

³¹The studies to be discussed in this review do not include those whose purpose was to determine the price and income elasticities of the aggregate demand for housing. As mentioned earlier, studies of that type bear little relation to the present study.

³²"The Determinants of Residential Land Values," op. cit.

where a, b, and c are parameters. Census tract data were used to find the values taken on by P_i . Very little effort was made to control for variations in amenity, topography, and historical factors, simple proxies being used.³³

Although Brigham identified a clear relationship between average appraised value per square foot and accessibility to urban activities, more careful specification of all the variables, independent and dependent, would lead to more useful results. Brigham does recognize most of the shortcomings of his study and suggests some possible improvements.

A recent study by John Kain and John Quigley seems to be the best available study falling in the second category, studies which investigate the effect of various neighborhood and on-site characteristics on residential property values.³⁴ They estimated regression coefficients for two equations. The dependent variable in one equation was the market price of owner-occupied dwelling units, and the dependent variable in the other equation was the market price of renter-occupied dwelling units. Kain and Quigley regressed these dependent variables on the following

³³Brigham used three variables as proxies for amenities: (1) to control for neighborhood effects, he used median family income for ray 1 and the percentage of dwelling units with more than 1.01 persons per room for ray 2 and (2) to control for on-site improvements, he used the average value of the dwelling-unit structures in a given block. He used a dummy variable for topographic variation. It took on the value of 1 in the Santa Monica Mountains along ray 1 and for hills about one-third of the way out ray 2 and a value of zero everywhere else. In the regression results he did not report on a proxy for historical factors. Ibid., p. 331.

³⁴"Measuring the Value of Housing Quality," op. cit.

... Measures of the qualitative and quantitative dimensions of the housing bundle. The measures of residential quality associated with individual dwelling units were obtained by using factor analysis to aggregate some 39 indexes of the quality of narrowly defined aspects of the dwelling units, structures, parcels, and micro-neighborhoods.³⁵

With the exception of data on variables such as the incidence of crime which were obtained from public records, the information necessary to complete the study was obtained through home interviews and from a survey conducted by St. Louis city building inspectors. With this data available, Kain and Quigley were able to investigate thoroughly the effect of their quality and quantity variables on the dependent variables. They summarized their findings in the following statement: "The quality of a bundle of residential services has at least as much effect on its price as such quantitative aspects as number of rooms, number of bathrooms, and lot size."³⁶ It should be recalled that Brigham lacked the necessary information for a thorough investigation of the effect of different quality on appraised value.

Kain and Quigley did not deal as thoroughly with the locational aspects of dwelling market value as Brigham did. The only accessibility variable on which they report, miles from the CBD, proved to be statistically insignificant in explaining the variance in property market values. A possibility which they recognized is that the insignificance might have been the result of improper specification. In a footnote they mention that an employment-accessibility index was also used, but the results obtained were no better. In any case these

³⁵Ibid., p. 532.

³⁶Op. cit., p. 546.

results are at variance with Brigham's conclusion that there is a definite relationship between land values and accessibility to urban activity.

A third empirical study bears mentioning because it includes a model in which the misspecification discussed in Section A occurs. This striking example of poor specification is the model used by Crecine, et. al. Whereas they used as the dependent variable "the market price of the i -th property (single family dwelling) divided by the number of square feet in the site," the dependent variable should have been "the value per square foot of the i -th site."³⁷ This belief is based on a hypothesis, as yet unconfirmed by empirical work, that if zoning ordinances are motivated by a desire to minimize negative externalities, it is the value of the site rather than the value of the improvements which is affected.³⁸ Further, they failed to include variables reflecting variations in the quality and quantity of improvements. This omission means that their study can make only a minimal contribution to the effort required to improve assessment practices.

Crecine, Davis, and Jackson were not attempting to formulate a model which could be used to improve assessment practices. Their model was derived for the purpose of testing the hypothesis that negative externalities exist in urban property markets. They regressed

³⁷"Urban Property Markets: Some Empirical Results and Their Implications for Municipal Zoning," op. cit., p. 84.

³⁸This statement concerning the burden of externalities implies certain relative elasticities of supply of and demand for sites and supply of and demand for improvements. In addition, with total property value consisting partially of site values, variations in improvement values for Crecine's cross-section data might be such as to wash out any effect of externalities on value. This could yield the conclusion reached by Crecine, et. al., that significant externalities do not exist in the urban housing market.

the aforementioned dependent variable on several neighborhood variables³⁹ which they expected would exert a negative influence on single family property values. The evidence of Crecine, et. al., suggests that independence rather than interdependence prevails in urban property markets.⁴⁰

In a 1967 article,⁴¹ Harris, Tolley, and Harrell summarized the results of a study of the amenity and travel savings components of land value in Raleigh, North Carolina. This study is reviewed here because of the rather unique method used. In contrast to Brigham's use of census tract data for his accessibility potential variable, Harris, et. al., interviewed households to determine the destinations of trips by family members.⁴² They calculated the yearly costs of these trips for the present location of the household and also for the lowest travel cost location at the low rent margin of the city, the difference being the cost savings from an inframarginal location. These travel savings were put on a square foot basis.

Harris, et. al., assumed that total land values are equal to the sum of travel cost savings or locational values plus the value of amenities, where amenities are the externalities imposed on a particular site by the characteristics of surrounding sites. An appraisal of land values for tax purposes served as the basis for the data on total land

³⁹Op. cit., p. 88.

⁴⁰Op. cit., pp. 92-93 and pp. 97-99.

⁴¹"The Residence Site Choice," op. cit.

⁴²The method for selecting households is described in the article on pages 242 and 243.

value per square foot. For each cluster⁴³ of households the average value of amenities per square foot was estimated by subtracting the travel cost savings per square foot and the land value per square foot at the low rent margin from average land value.

Average amenity value per square foot multiplied by lot size was regressed on demand variables to test the hypothesis that households have similar tastes for amenities. Harris, et. al., found that income was the only statistically significant variable. With an income elasticity of between 2.5 and 4.0 for expenditures on amenities, they concluded, ". . . amenity is one of the main commodities people wish to acquire when their income rise."⁴⁴ Their hypothesis concerning similarity of tastes for amenities is not rejected.

None of the studies reviewed in this section had as its objective the formulation of a model which would be useful in improving property appraisal practices. Then it naturally follows that each is limited in its usefulness for the improvement of appraisal practices. The limitations of the Crecine, et. al., study have already been indicated. The failure of the Brigham study to make a significant contribution results from both the high level of aggregation of his data and his relative preoccupation with the accessibility potential variable. The results of the Kain and Quigley investigation are the most useful in terms of their potential contribution to improving appraisal practices. However, their relative neglect of the importance of accessibility is a major drawback. This neglect is probably a major factor in the explanatory

⁴³" . . . 20 blocks were randomly selected and clusters of three to five blocks were obtained by combining each of the 20 blocks with contiguous blocks." Ibid.

⁴⁴Op. cit., p. 246.

power of their independent variables. Their R^2 's range between .72 and .77. The investigation of Harris, et. al., had a very limited objective and could not be expected to make more than a minor contribution toward improvements in appraisal practices. Finally, both the Kain and Quigley and the Harris, et. al., studies employed an interview method for gathering at least part of the data which is necessary for making assessments. The interview method would be a relatively expensive method of gathering data for appraisal purposes.

This concludes the discussion of previous theoretical and empirical investigation of land and housing values. The format for the remainder of this paper is presented in Section C of this chapter.

C. DESCRIPTION OF THE CONTENT OF THE REMAINING CHAPTERS

The general theoretical foundation for the analysis of sale prices for single family residential properties is laid in Chapter II. The discussion in Chapter II includes the following: justification for the application of Lancaster's activity analysis approach to consumer behavior, development of the housing characteristics budget equation, justification for the assumption of perfectly inelastic supply curves for housing characteristics, and theoretical determination of the market-day equilibrium price of a housing characteristic.

The hypotheses to be tested are developed in Chapter III. The earlier sections of the chapter contain discussion of the way in which several variables affect the price per unit of urban land. The later sections show how expenditures on improvements are affected by variations in several variables once expenditures on land have been subtracted from the sale price of a property.

Chapters IV and V contain the empirical investigation of single family residential property values. The data used in this investigation are described in Chapter IV, and the method employed in and empirical results of the investigation are reported in Chapter V.

CHAPTER II

THE GENERAL FRAMEWORK FOR A THEORETICAL ANALYSIS OF HOUSING PRICES

A. Assumptions Concerning the Types of Services Yielded by Housing

When the United States was a rural agricultural society with little large-scale manufacturing, housing location generally was dictated by business considerations. For example, the location of a family's farmland determined within a few hundred feet where its house would be located. Even in the cities and villages shopkeepers and workers tended to locate their houses at or very near their places of business and employment.

In these earlier times, housing yielded primarily the following two flows of production inputs: (1) transportation inputs in that housing gave a family a particular location vis-a-vis economic activity and (2) inputs for the maintenance of human capital by serving as a place for eating and sleeping.¹ The places where economic activity was carried on consisted of employment centers and places to purchase goods and services. At the same time the sale of most goods and services tended to be more concentrated at the point of production than is the case today. Location of housing very close to the center of economic activity doubtlessly resulted from two factors. The first is that

¹Even at this early date, housing provided extensive consumption inputs for higher income groups. But a house which yielded consumption inputs was more the exception to the rule than it is today.

transportation was quite time consuming prior to the development of the automobile and other modern means of conveyance. Secondly, production was relatively labor intensive, and long hours at work were required to eke out a meager existence for one's self and family. With long working hours it was not feasible to spend much time in traveling to and from work at the expense of leisure. Further, long working hours and the lack of relatively rapid transport facilities limited social contacts, making accessibility to economic activity the most important determinant of an individual's housing location. In his history of the growth of Boston during the last half of the nineteenth century, Sam Bass Jr. makes the following statement about the intermingling of housing and economic activity:

Throughout the tiny metropolitan region [Boston] of 1850, streets of the well-to-do lay hard by workers' barracks and tenements of the poor; many artisans kept shop and home in the same building or suite; and factories, wharves, and offices were but a few blocks from middle class homes...²

Alternatives to the horse-and-buggy, streetcar, and walking as means of transportation have been developing in modern society. The increased capital intensity of production has permitted most Americans to have both higher incomes and more leisure time. Being able to travel faster and having more time in which to travel, i.e., more leisure time, Americans have chosen to increase the quantity of social contacts demanded. This increased quantity has reduced the importance of the location of production and sale of goods and services as a determinant of

²Streetcar Suburbs, Harvard University Press, Cambridge, Massachusetts, 1962, p. 19. In addition to the evidence cited by Bass, the structure of other American cities prior to 1900 suggests that accessibility to employment was probably the most important determinant of an individual's housing location.

housing location.³ In addition, with rising per capita income, many families have chosen to own more than one car. The increasing mobility of family members, particularly women and teenagers, has further complicated the location decision.

As previously mentioned housing provides production inputs other than transportation inputs. The other production inputs are those necessary for maintaining human capital and consist of a place for eating and sleeping. Until recent decades housing was designed to serve only these needs, and the range of housing characteristics available to all but the highest income families was limited. This was due to both the low level of income and the small amount of leisure time which most families had. There just was no time for other home-centered activities.

Although housing continues to provide the same two basic flows of production inputs, the consumer's choice with respect to housing has become more complex. This increased complexity results from the increasingly difficult location decision and from the wider range of housing characteristics which have been made available. The potential diversity of the activities to which housing contributes flows of

³ Not only the price elasticity of demand for social contacts is high, but also the income elasticity is high. This suggests that urban sprawl, although probably speeded by a public policy of providing increasingly easy access to employment centers from large distances, would have occurred in any case as a result of the high income elasticity of the demand for social contacts. The possibility of avoiding certain types of social contacts, e.g., whites with blacks, is also enhanced by higher incomes. The high income elasticity has meant that the availability of social contacts has, with rising per capita income, grown in relative importance when compared to the importance of accessibility to employment as a determinant of housing location.

consumption services has given rise to the wide range. Included among these diverse activities are entertaining, recreation, eating, and reading.⁴

If the fact that there is a wide range of housing characteristics in which a consumer is interested is combined with the assumption that it is the characteristics of the property which are desired rather than the property qua property, it is natural to apply Kelvin Lancaster's adaptation of activity analysis⁵ to an analysis of market demand for residential property. Lancaster's contribution to the theory of consumer behavior is the market demand curve for a characteristic in which consumers are interested rather than the market demand curve for a commodity. The market demand for a characteristic can be derived by a summation of the demand by individuals where the demand by an individual takes the form

$$P_1 = f(c_1, P_0, c_0, Y) \quad (1)$$

where c_1 is the quantity of the first characteristic, P_1 is the price of the first characteristic, P_0 is the price of other characteristics,

⁴ The line between those activities to which housing contributes consumption inputs and those to which it contributes production inputs is drawn arbitrarily. Housing probably contributes flows of both types of inputs to most activities. Eating seems to be an activity for which the size of the flows is approximately equal. In fact, for a gourmet cook the size of the consumption inputs from eating probably exceeds the size of production inputs.

⁵ "A New Approach to Consumer Theory," Journal of Political Economy, LXXIV (1966), pp. 132-157. The assumption that individuals are interested in the characteristics of property rather than property qua property does not rule out purchase for speculative purposes.

c_0 is the quantity of other characteristics, and Y is the consumer's income. When written in this form, the demand relation is interpreted as indicating the willingness-to-pay, P_1 , for various amounts of c_1 .

B. Application of Lancaster's Analysis to the Demand Side of the Single Family Property Market

It is not yet possible to proceed directly from the general market demand function for a housing characteristic to the estimating equation used in this study. First, we introduce the notion of a strongly separable or additive utility function.⁶ The general form of the additive utility function is (2):

$$U = U_a(c_{a1}, \dots, c_{a\alpha}) + U_b(c_{b1}, \dots, c_{b\beta}) + \dots + U_m(c_{m1}, \dots, c_{m\gamma}) \quad (2)$$

where $\alpha + \beta + \gamma$ is the total number of characteristics. This is a form of the utility tree suggested by Professor Robert Strotz.⁷ A utility function of this type is justified by the argument that the first step in the budgeting process is to allocate among broad groups of characteristics--each broad characteristics group associated with a different term in the utility function.⁸ The proportion of the budget allocated to the various branches depends on the relative price indices

⁶ This concept is introduced to insulate the housing market from changes which might occur in other markets.

⁷ R. H. Strotz, "The Empirical Implications of a Utility Tree," *Econometrica*, 1957 and "The Utility Tree--A Correction and Further Appraisal," *Econometrica*, 1959.

⁸ The utility tree is usually couched in terms of broad groups of commodities. However, no apparent damage to the Strotz analysis results from assuming that characteristics are the arguments in branch utility functions.

associated with the branches. Once the allocation among branches has been determined by the consumer, the total expenditure allocated to each branch may be treated as a budget constraint for that branch. Let one of the branches of the utility function, say the first, $U_a(c_{a1}, \dots, c_{aK})$, be that associated with the characteristics of housing, and let $\underline{E}$ be the amount allocated to expenditure on this branch. The second step in the budget process is to allocate $\underline{E}$ among c_a , i.e., solve the program

$$\begin{array}{ll} \text{Maximize} & U_a(c_{a1}, \dots, c_{aK}) \\ \text{Subject to} & E \geq Px \end{array} \quad (3)$$

where the x 's are goods having characteristics $c_{a1}, \dots, c_{aK}$.

The budgeting process implied by an additive utility function seems to approximate the way in which consumers make house-purchasing decisions. Simply put the process is as follows: An individual consumer decides that, given the trade-off between the characteristics associated with housing and the characteristics associated with other activities, he will allocate a portion of his budget to housing characteristics, yielding an absolute dollar figure to be spent. This dollar amount is the present value of a desired flow of expenditures. Then the consumer looks at the available housing market alternatives having a present value of costs no greater than the amount he is willing to spend. Alternative possible choices will yield different combinations of housing characteristics. Realizing that he is constrained by his budget, the consumer weighs the value of the various characteristics and chooses the property possessing a combination of characteristics which maximizes his benefits.

With an additive utility function, the Lancaster analysis lends itself very nicely to an investigation of characteristics prices in the housing market. As a beginning assume that the individual consumer has already allocated his budget among the broad groups of characteristics and is now ready to decide on the bundle of housing characteristics he will purchase. The set of housing characteristics which maximizes the housing branch utility function $U_a(c_{a1}, \dots, c_{a\alpha})$ and simultaneously satisfies the branch budget constraint in goods space

$$E \geq Px \tag{4}$$

will also satisfy the budget constraint in characteristics space

$$E \geq pc \tag{5}$$

where the p 's in this expression are implicit characteristics' prices. Determination of the vector $\underline{p}$ of implicit characteristics' prices is the objective of the empirical work in this study.

An individual's demand curve for a housing characteristic can be derived by finding the price-consumption locus for that characteristic and plotting the combination of $\underline{p}$ and $\underline{c}$ in the appropriate space. The demand curve which results from this derivation is one for which money expenditure on housing is held constant. Real expenditure on housing does vary along the individual's characteristic demand curve. This variation in real expenditure introduces a difficulty into the analysis. With invariant price indices and money expenditures for the branches other than housing, a change in real expenditure on the housing branch implies that total real expenditure or income has changed. One would expect a change in the budget proportion allocated to each branch and a consequent change from the initial level of money expenditures on each branch along with a change in real income.

However, if one assumes that the money expenditure on each characteristic is a small proportion of total money expenditure, then little harm is done by disregarding the feedback effect on a branch budget constraint of a change in the price of a housing characteristic.

C. THE STOCK OF A HOUSING CHARACTERISTIC

1. Importance of Time Period to the Analysis

The time period of the analysis is an important aspect of any discussion of available stock in a market. The length of the time period under consideration has implications for the possibility of varying the stock of a characteristic. In most markets, the longer the time period, the greater is the possibility of variations in stock. Basically there are three time periods: market day, short run and long run. These periods are defined in terms of the possibility of varying the amount of inputs applied to producing the characteristic in question, rather than in terms of calendar time.

The market day is defined as the time period in which variation in the stock of a characteristic is not possible. Consequently, in the market day the supply of a characteristic is perfectly price inelastic.⁹ The short run is defined as the time period in which it is possible to vary the amounts of some, but not all, inputs, allowing variation in the stock of the characteristic. It is true by definition that in the short run the price elasticity of supply is greater than

⁹If a commodity's supply price-elasticity is zero and its demand price-elasticity is greater than zero, then the market price may be said to be demand-determined. A demand-determined price is one for which the price elasticity of supply is zero and the price elasticity of demand is greater than zero.

zero.¹⁰ Finally, the long run is defined as the time period in which it is possible to vary the amounts of all factors of production. Then for any given characteristic, it follows that the supply price-elasticity is at least as great in the long run as it is in the short run.

2. Characteristic Stock Elasticity Assumptions

The expected price-elasticity of the stock of a housing characteristic bears heavily on the realism of any equilibrium analysis of price determination in the housing market.¹¹ The reader should recall from Chapter I Ralph Turvey's discussion of the efficacy of equilibrium analysis in the housing market. It is Turvey's opinion that whereas short run equilibrium analysis is useful, an assumption of long run equilibrium would yield results not closely approximating the actual layout of towns and the pattern of buildings. In his words,

If the determinants of the equilibrium constellation of prices and resources-use changed infrequently or slowly, while adjustments to such changes took place relatively rapidly and without much friction, the actual pattern of prices and resource allocation would usually correspond fairly closely to the equilibrium pattern. It would thus be possible to analyze the existing state of affairs in terms of an equilibrium construction. Now so far as the long run is concerned, this is not generally the case with urban property, because the great durability of buildings makes urban change a very slow process and one that is never completed. . .

In the short run, however, an equilibrium analysis in general terms can contribute some understanding. Given the existing standing stock of buildings, the pattern of rents can be analyzed by examining demand conditions, as can the pattern of land use insofar as it is flexible--i.e. to the extent that some buildings are capable of more than one use.

¹⁰ In the short run, market price is not demand-determined.

¹¹ In keeping with analysis of the demand side of the market, we are discussing the supply of a housing characteristic rather than the supply of a commodity called "housing".

The point is that the actual rent and use pattern of a given stock of buildings will approach the equilibrium pattern within a period sufficiently short for changes in that stock (by reconstruction and new building) to be negligibly small. . .¹²

Although his discussion is couched in terms of a stock of buildings and the present analysis is in terms of housing characteristics, Turvey's discussion is very suggestive of the stock price-elasticity assumption which should be made in the present discussion.

There are three general classes of residential property characteristics: (1) public services levels, (2) externalities, and (3) on-site characteristics. Each class is made available under different institutional arrangements. Decisions about the level of public services offered are generally political in nature and as such require deliberation on the part of public officials and often on the part of the electorate as well. It is an oft-made assertion that in the political process a great deal of time passes between the point at which a desired change surfaces and the point at which it is ratified by public officials and by the electorate.

The possibility of varying within short time periods the quantities of those characteristics which might be thought of as externalities is also remote in our present institutional setting. By the definition of externalities, these characteristics are a by-product occurring as a result of the production of other characteristics. Further the producer of these externalities does not take account of the value placed on them by individuals in the community, i.e. there is not a functional relationship between a price

¹²Ralph Turvey, op. cit., pp. 47-48.

and the available stock. This is a case of perfectly price-inelastic stock, and the value of the characteristic is demand-determined.

Only the last of the three general classes of property characteristics would seem to contain elements of which the stock is variable within the one year period of this analysis. Even within this class, there are probably only a limited number of characteristics whose stock is variable within such a short time period. The stock of characteristics of the buildings themselves, e.g. shower or tub in the bathroom, existence of air conditioning, may be changed within the period. However, variations in the stock of other typical on-site improvements such as mature trees and other landscaping features, and total reconstruction of the buildings probably require more than one year from the time of a variation in demand.

In accordance with the preceding discussion, the assumption of a perfectly price inelastic one-year characteristic stock curve is appropriate. The assumption seems to be a good approximation even for those characteristics which one would expect to have a stock price elasticity greater than zero within the period of the analysis. An implication of the perfectly inelastic stock assumption is that, at least within the twelve month period used for this analysis, characteristics' prices are demand-determined.

The demand and stock sides of the market, dealt with separately to this point, must now be related to each other for a discussion of the determination of characteristics' prices. This discussion is the subject matter of the next section of this chapter.

D. Market Determination of a Characteristic's Price

To illustrate the workings of the market for an individual housing characteristic $\underline{c}$, let us assume that the market is in equilibrium at the beginning of a market-day period, i.e. at the existing characteristic price, the available quantity of the characteristic is exactly equal to the quantity demanded. This situation is illustrated in Figure II-1 where S_{MD} or C_1 is the available quantity

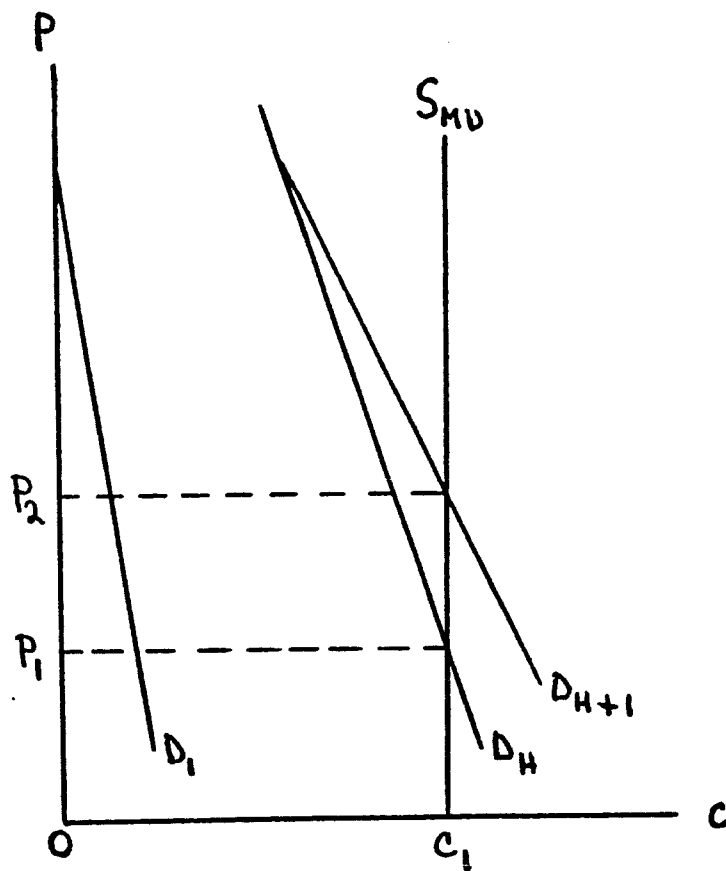


Figure II-1

of the characteristic, D_H is demand, and P_1 is the existing equilibrium price. Now assume that at the beginning of the market day period a person from out of town interviews for a job which subsequently is offered to and accepted by him. Let his demand for $\underline{c}$ be represented

by D_1 in Figure II-1. The total demand for the characteristic is now D_{H+1} . As a result of this individual's demand for c there is excess demand for the characteristic at price P_1 . The mechanics of the process by which this excess demand is registered in the market and affects a new equilibrium price is a matter for conjecture. There are several methods by which the individual might indicate a demand for the characteristic: (1) he could contract with a builder to construct an amount of the characteristic, the amount built dependent upon the construction costs, (2) the individual could make contact either personally or through real estate brokers with current holders of the characteristic to let these holders know of his desire to obtain some of the characteristic, and (3) he could place advertisements in periodicals, probably local newspapers, to inform current holders of his interest in obtaining the characteristic. The individual could use more than one of these methods at the same time.

Although the effect of these methods of registering increased demand on the characteristic's price in later periods is uncertain, the effect in the current market day period is to increase the price of the characteristic. If the new individual elects to contract for construction of some of the characteristic but finds it necessary to obtain some of the characteristic before construction is completed, i.e., before the end of the current market day period, he must offer to current holders a dollar amount which is higher than the existing price P_1 . The price will rise to the new equilibrium price P_2 in one or more transactions depending upon the ease with which market information can be obtained. Information about the availability of characteristics and their prices is readily obtained in the housing market, and

housing expenditures constitute a large portion of most families' budgets. Consequently one would expect most characteristics' transactions to take place at or very near the new equilibrium price, P_2 .¹³ In subsequent periods when changes in stock are allowed, the relationship between the first equilibrium price P_1 and new equilibrium price depends on the elasticity of the short run and long run supply curves. However, the long run equilibrium price would not exceed the cost of constructing additional amounts of a housing characteristic.

The discussion of the methods through which an increase in demand is registered during the market day period raises the issue of the constancy of market demand in Ann Arbor during the period of this analysis, calendar year 1970. If there are variations in demand during the year, some attempt to account for this variation must be made. It is well-known that most people move during the summer months of the year. Alone this does not indicate that demand for a housing characteristic is higher or lower during these months than at other times during the year. For if the rate at which families are moving out of a community equals the rate at which families are moving into a community, the tastes and incomes of the families involved may be such that demand remains constant. However, if the community is growing, we would anticipate housing characteristics' prices to be higher during the summer months than during other months of the year. Then for

¹³ A change in the rate at which a city is gaining or losing population and/or a change in the rate at which demand for a characteristic is increasing or decreasing can lead to transactions which occur at prices not near the equilibrium price. Transactions at prices not near the equilibrium price result from changes in price expectations lagging behind changes in the rate of change of characteristic demand.

growing communities, including Ann Arbor, the relation between month of sale and the price of characteristic is as illustrated in Figure II-2. The technique used to account for this seasonal variation in characteristics' price is discussed in a subsequent chapter.

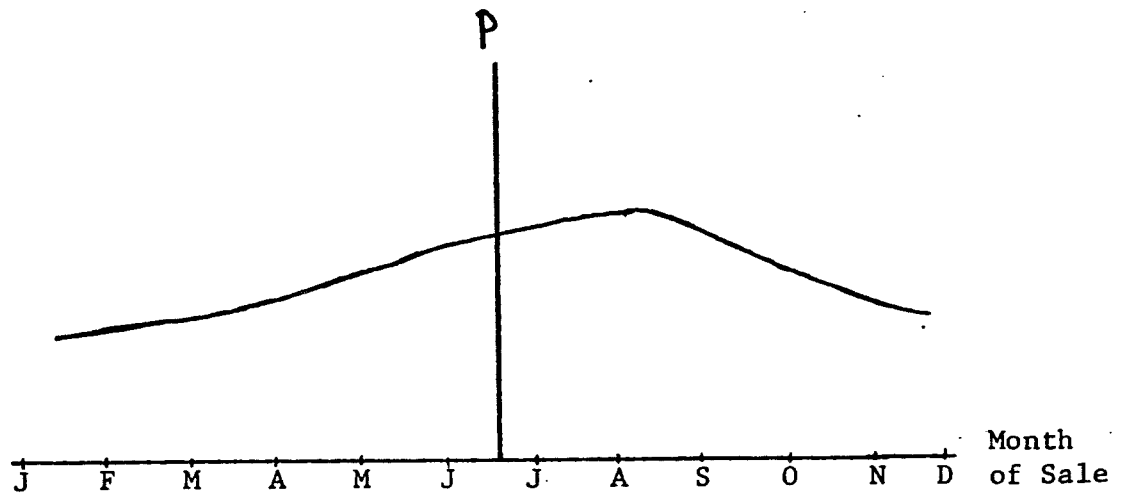


Figure II-2

This concludes the introductory discussion of the market determination of a characteristic's price. The next chapter of this paper contains a development of the specific hypotheses tested in the study.

CHAPTER III

DEVELOPMENT OF SPECIFIC TESTABLE HYPOTHESES

A. Land Price Determination Hypotheses

So far no specific testable hypotheses have been developed in this paper. The previous chapter merely provided a general framework which can be used for analyzing the effects of any of several variables on the price an individual will pay for various quantities of any particular housing characteristic. The development of specific housing market hypotheses is the purpose of this chapter.

First let us consider which characteristics from the set of all possible characteristics might enter as arguments in an individual's housing branch utility function, $U = U_a(c_{a1}, \dots, c_{a\alpha})$. We believe that the set of housing characteristics which enter the utility function are capable of being summarized at a high level of aggregation by two characteristics' vectors $\underline{I}$ and $\underline{L}$ such that $U = U(I, L)$ where $\underline{L}$ is a vector of characteristics associated with land, and $\underline{I}$ is a vector of improvement characteristics.

There are several elements of the land-characteristics vector $\underline{L}$ which enter as arguments in an individual's characteristics utility function. The i th individual's utility function may be written as:

$$U_i = U_i(D, s, n, z, t, r, c, a, I)$$

where

$D \equiv$ accessibility,

$s \equiv$ public services,

$n \equiv$ neighborhood characteristics,

$z \equiv$ zoning classification,

$t \equiv$ number of square feet in plot,

$r \equiv$ shape of the plot,

$c \equiv$ whether or not corner lot,

$a \equiv$ property taxes, and

$I \equiv$ quantity of improvements. The amount of improvements I is not a choice variable at this time.

Now let us investigate the process by which land prices are determined. In Figure III-1 below we have depicted an individual holding a fixed amount of improvements who is choosing among different quantities

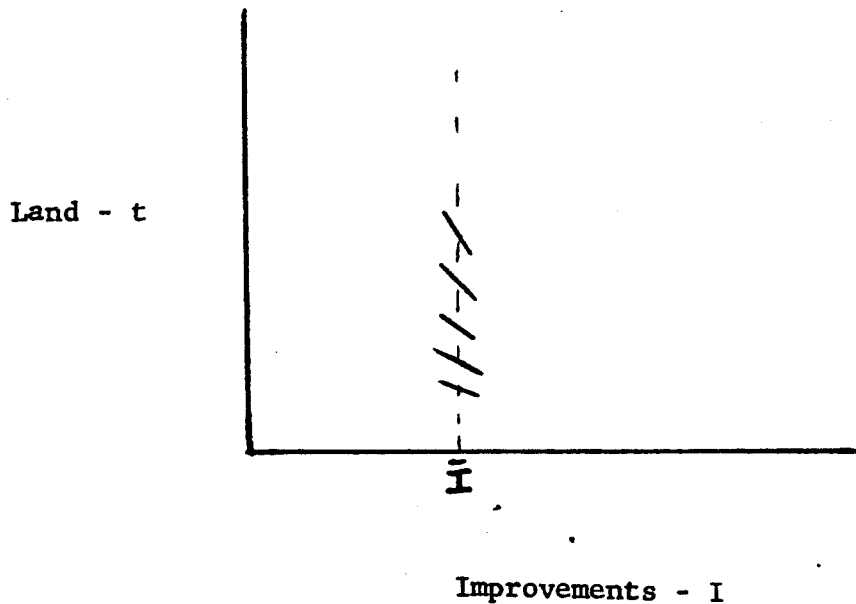


Figure III-1

of equi-quality land.¹ If $\frac{\partial U}{\partial t} > 0$, $\frac{\partial^2 U}{\partial t^2} < 0$, $\frac{\partial U}{\partial I} > 0$, and $\frac{\partial^2 U}{\partial I^2} < 0$, then an individual will choose greater t as P_t/P_I decreases. For an individual with I the relationship depicted in Figure III-2 holds. If the price of improvements, P_I , is treated as the numeraire, then

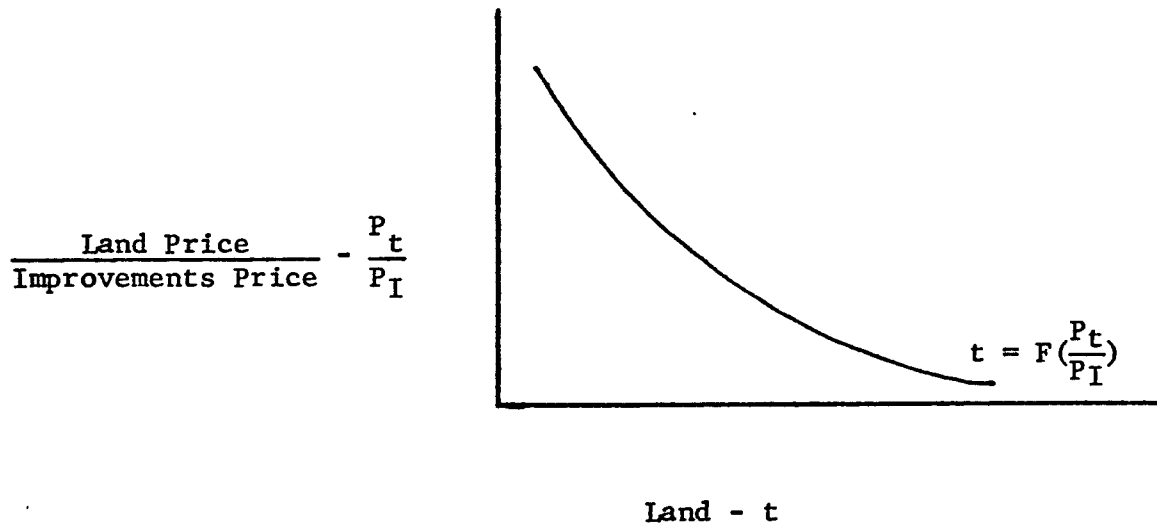


Figure III-2

Figure III-2 is a graphical illustration of $t_i = F_i(P_t)$, the i -th individual's demand for land. Since we are interested in the individual's willingness-to-pay, we could write $P_{ti} = g_i(t)$ where $g_i(t) = f_i^{-1}(P_t)$. The aggregate willingness-to-pay curve or market demand curve is found by a horizontal summation of the individual willingness-to-pay curves. Let us look at the aggregate willingness-to-pay when the supply of land S is perfectly inelastic. This is depicted in Figure III-3. Panel (a) is the first individual's willingness-to-pay curve, (b) is the second's, and (c) is their aggregate willingness-to-pay. The market price for t units of land will be established at the

¹ Equi-quality land means that the value of other land-associated characteristics is fixed.

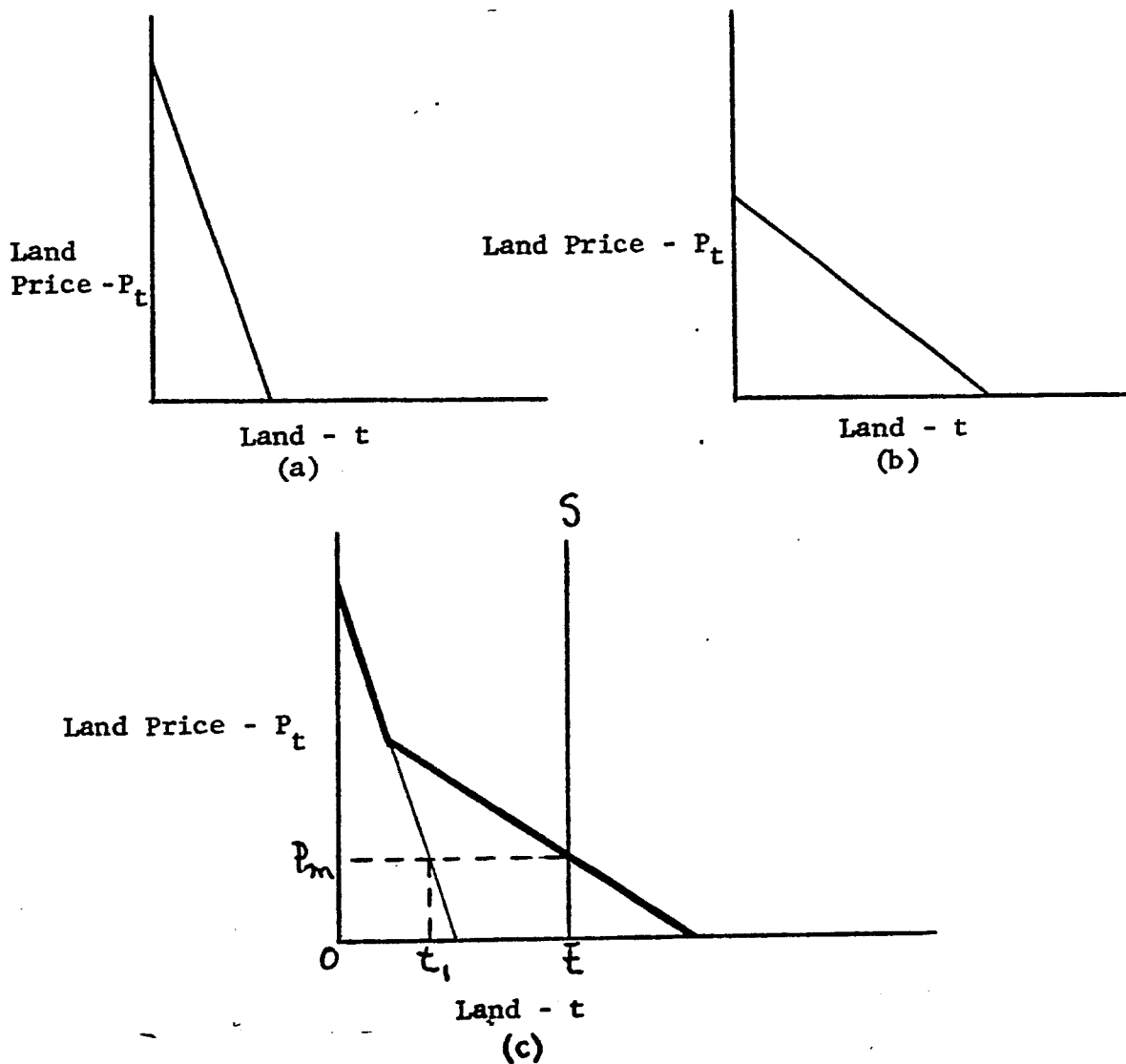


Figure III-3

level P_m , with the first individual purchasing t_1 units of land and the second purchasing $(t-t_1)$ units. The willingness-to-pay for the last unit of land purchased is the same P_m for each individual. Now that we know how much consumers are willing to pay for the t -th unit of land, we can consider the effect of varying amounts of other characteristics associated with land on the aggregate willingness-to-pay for land.

1. The Effect on Land Prices of Varying Accessibility to Economic Activity

The first land-associated characteristic that we might allow to vary is the accessibility to economic activity. Accessibility or nearness varies with the distance or time from economic activity and is a decreasing function of the distance or time required to get from a given place in a city to a given center of economic activity in a city. In Ann Arbor traffic flows at a fairly constant miles per hour rate throughout the city so that time from a center and distance from a center are highly correlated. Either of these variables is obtainable, but distance is less costly to obtain. Thus we used distance as the measure of accessibility in this study.

Let us look at the effect of varying the distance from economic activity on an individual's willingness-to-pay for a particular piece of land. Consider the individual whose willingness-to-pay for equi-quality land is shown in Figure III-4. The price P_1 is what he would pay for the t_1 -th unit at a distance d_1 from the center of economic activity. If $d_1 = 0$, the price

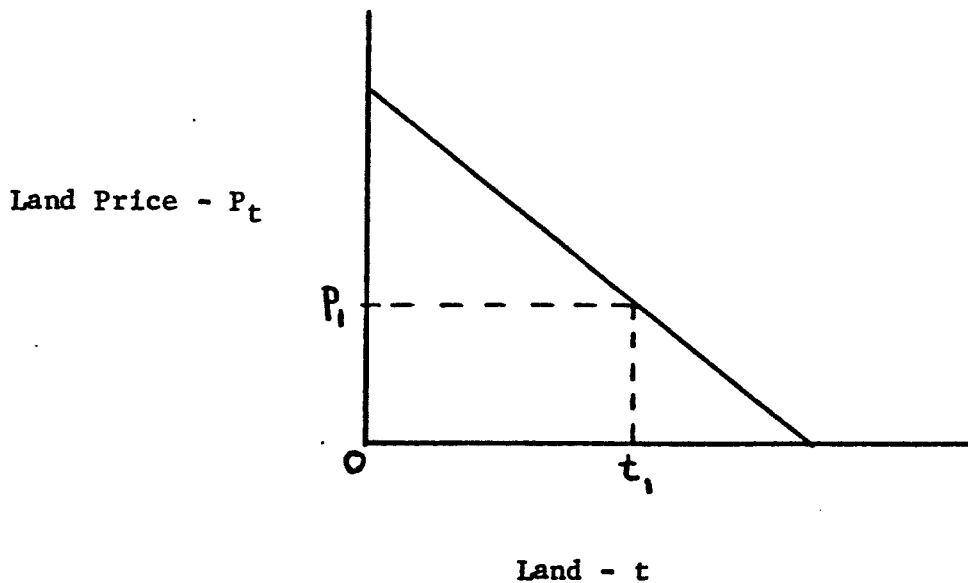


Figure III-4

P_1 is the vertical intercept of $P(d)$ in the p - d plane. This is illustrated in Figure III-5. If $\frac{\partial U}{\partial d} < 0 \Big|_{t=t_1}$ and $\frac{\partial^2 U}{\partial d^2} < 0 \Big|_{t=t_1}$, and $\frac{\partial U}{\partial t} > 0$ and $\frac{\partial^2 U}{\partial t^2} < 0$, then $\frac{\partial P(d)}{\partial d} < 0$. These mathematical relation-

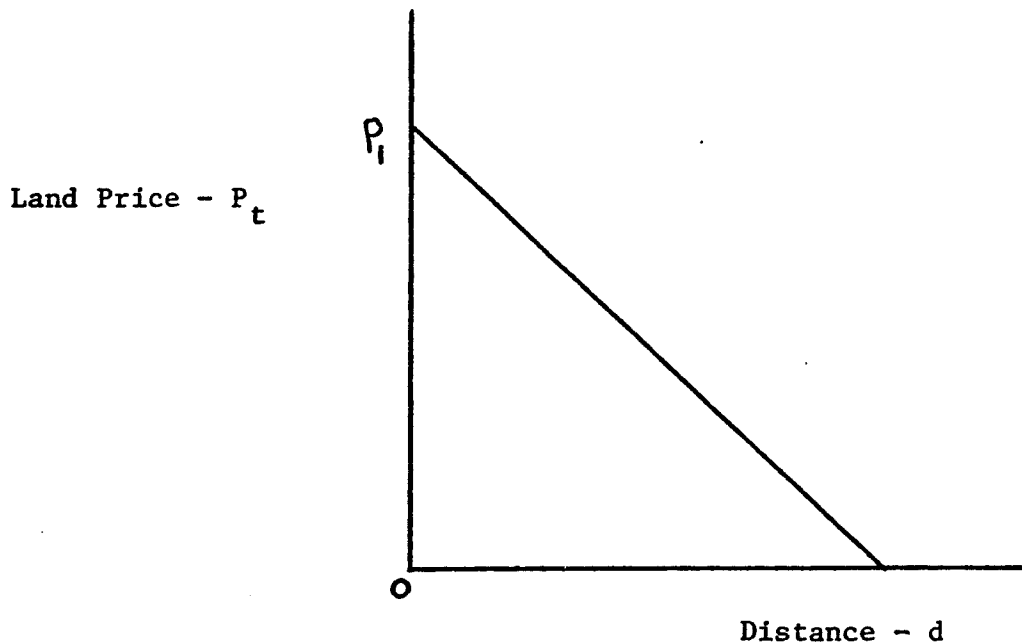


Figure III-5

ships state that as the $\underline{t}_1$ unit is moved such that accessibility diminishes (distance from economic activity increases), an individual's willingness-to-pay for the $\underline{t}_1$ unit of land diminishes from $\underline{P}_1$. This relationship is shown in Figure III-5. Figure III-6 shows the relationship for one individual between $\underline{P}_t$ and $\underline{t}$ and $\underline{P}_t$ and $\underline{d}$ in the P_t - t - d space. One of these surfaces exists for each individual. As in the case where we were determining the aggregate willingness-to-pay for land, we can find the aggregate willingness-to-pay for land at varying distances by summing the individual quantities demanded at various prices for given distances from the center of economic activity. To illustrate this summation, consider the individuals

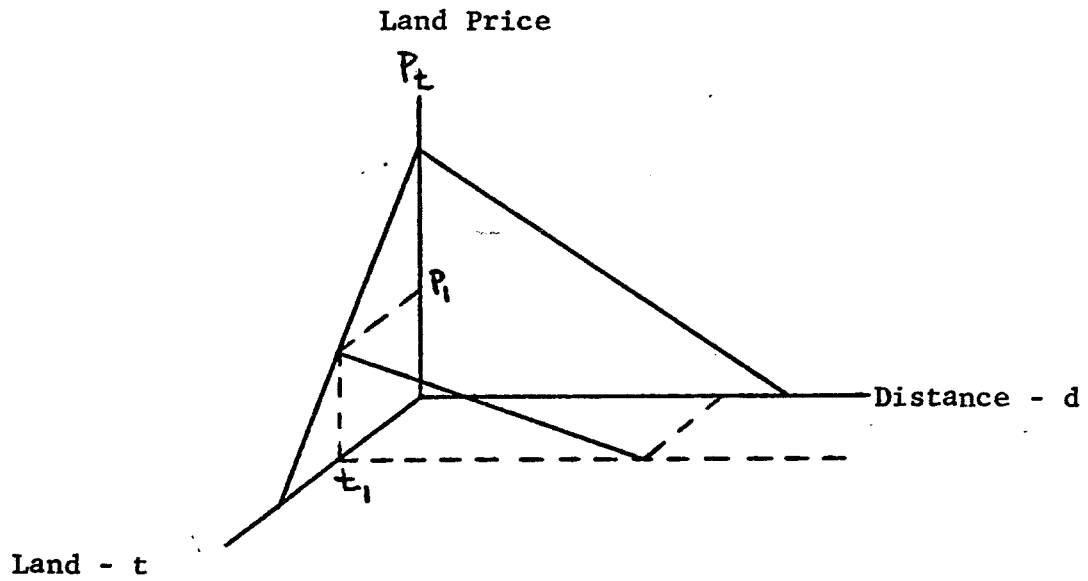


Figure III-6

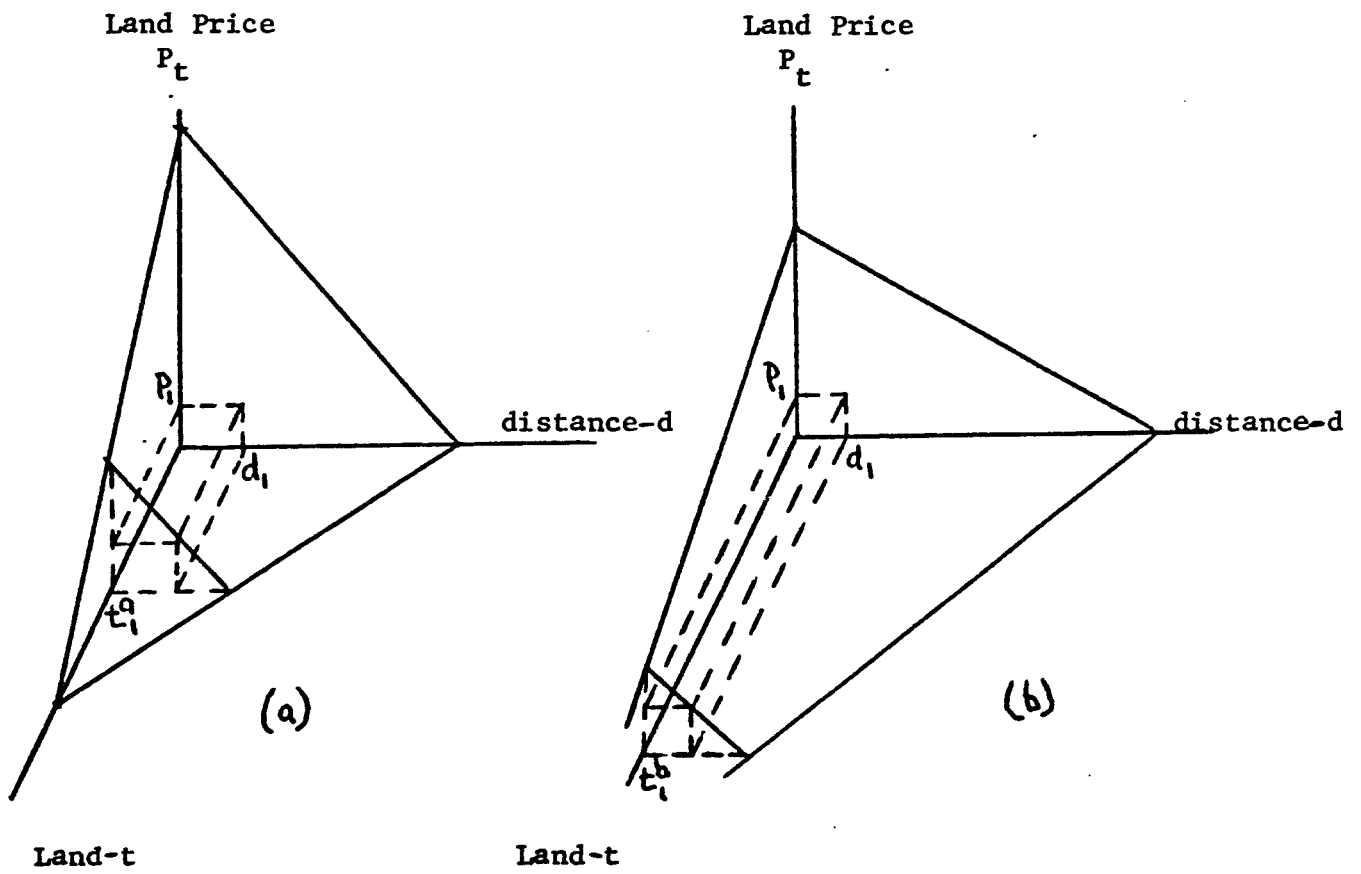


Figure III-7

with willingness-to-pay surfaces shown in Figure III-7 (a) and III-7 (b). At price P_1 and distance d_1 , individual $\underline{a}$ is willing to purchase t_1^a units of land and $\underline{b}$ is willing to purchase t_1^b units of land. Thus one point on the aggregate willingness-to-pay curve is $(d_1, P_1, t_1^a + t_1^b)$ shown in Figure III-8. The aggregate willingness-to-pay surface resulting from this method of summation is drawn in Figure III-9.

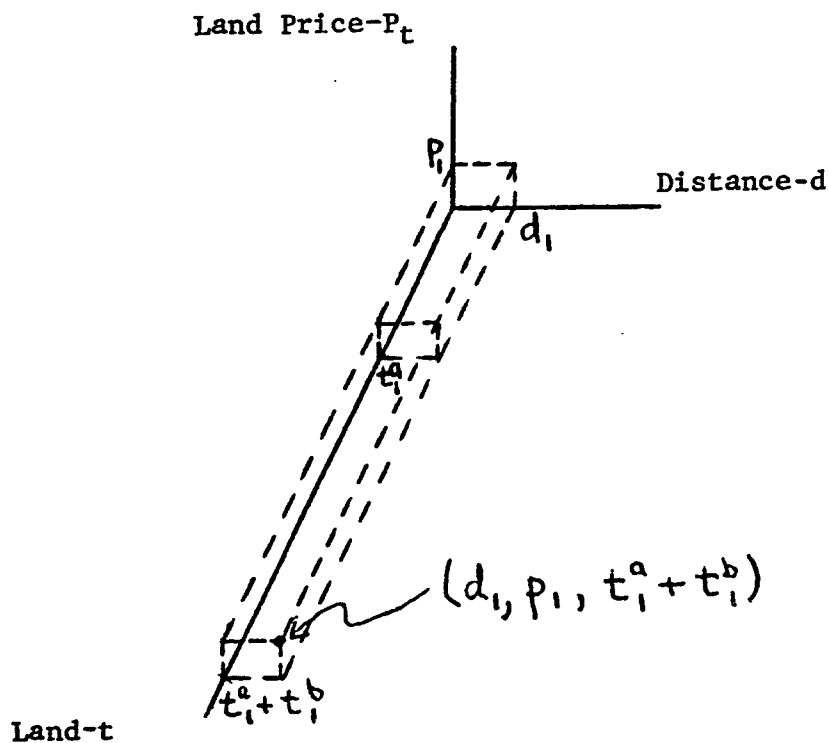


Figure III-8

If the city in question were circular with one center of economic activity at the geographic center and no holes in it, then the total amount of land available in a city with $\underline{d}$ radius is T , where

$$T = \pi d^2. \quad (1)$$

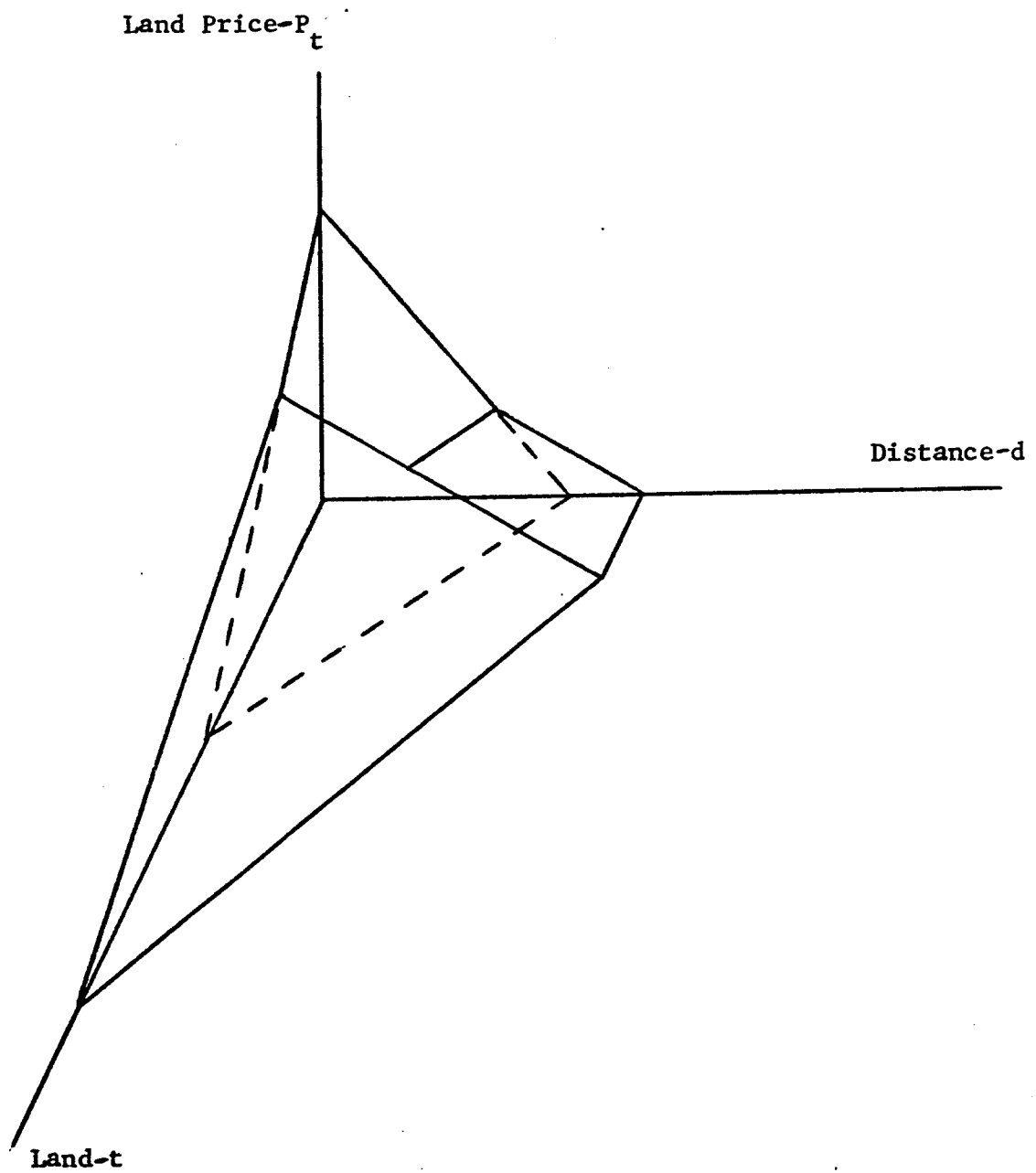


Figure III-9

The amount of land available at a given distance from the center of economic activity is

$$t = 2\pi d \quad (2)$$

where each plot of land has a width equal to the width of a line. The last expression, equation (2), is the circumference of a circle with a radius d . This expression indicates the supply of land available at a given distance from the center of a city, and is shown graphically in Figure III-10. Equation (2) is an equation for a straight line with a

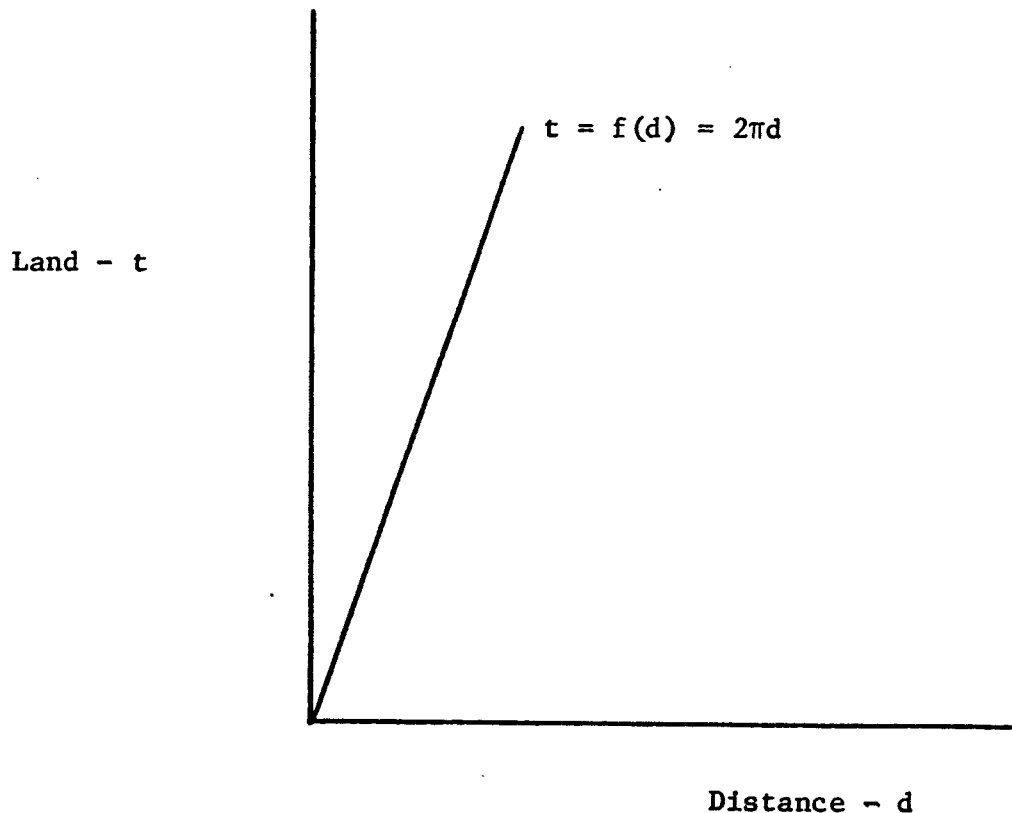


Figure III-10

vertical intercept at $t=0$ and a slope equal to 2π . In Pdt -space the supply of land is represented as a plane as illustrated in Figure III-11. The supply of land at any given distance from the center of a city is invariant

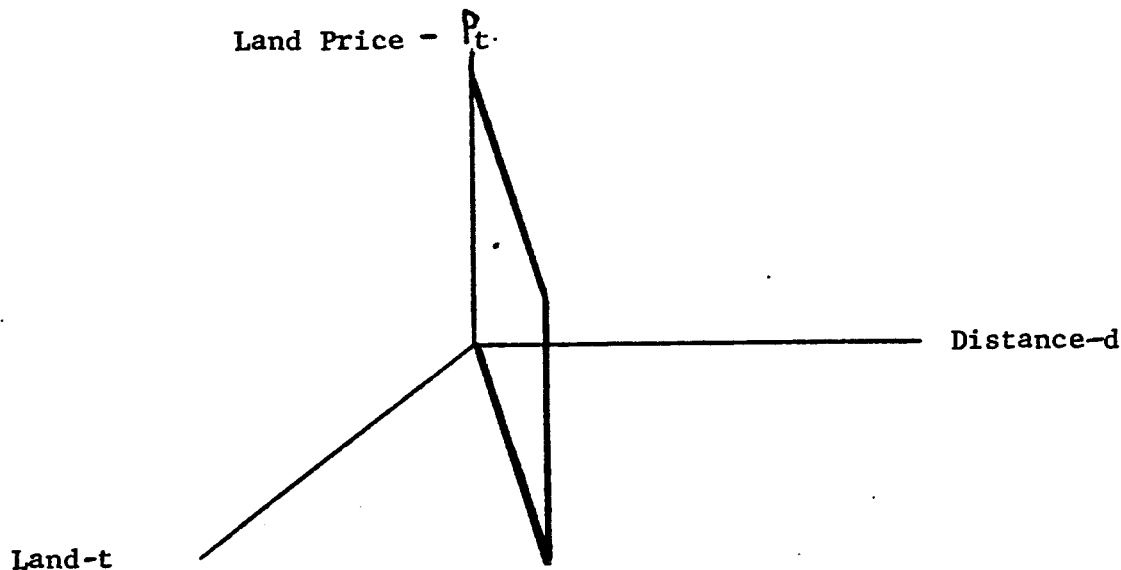


Figure III-11

with respect to the price of land since the supply is physically determined. The equilibrium price structure for land is determined by finding, for a given $\underline{d}$, the price at which quantity supplied is equal to quantity demanded. The land price structure for the willingness-to-pay surface in Figure III-9 and the supply plane in Figure III-11 is illustrated in Figure III-12.

The functional form of the relationship between the price of land and the quantity and distance from the center of a city is a question which can only be answered by empirical investigation. However, some characteristics of the function have been derived from basic utility analysis assumptions. Since the quantity of land supplied does not vary with price, the equilibrium (p, t, d) combinations are demand determined, and our attention need

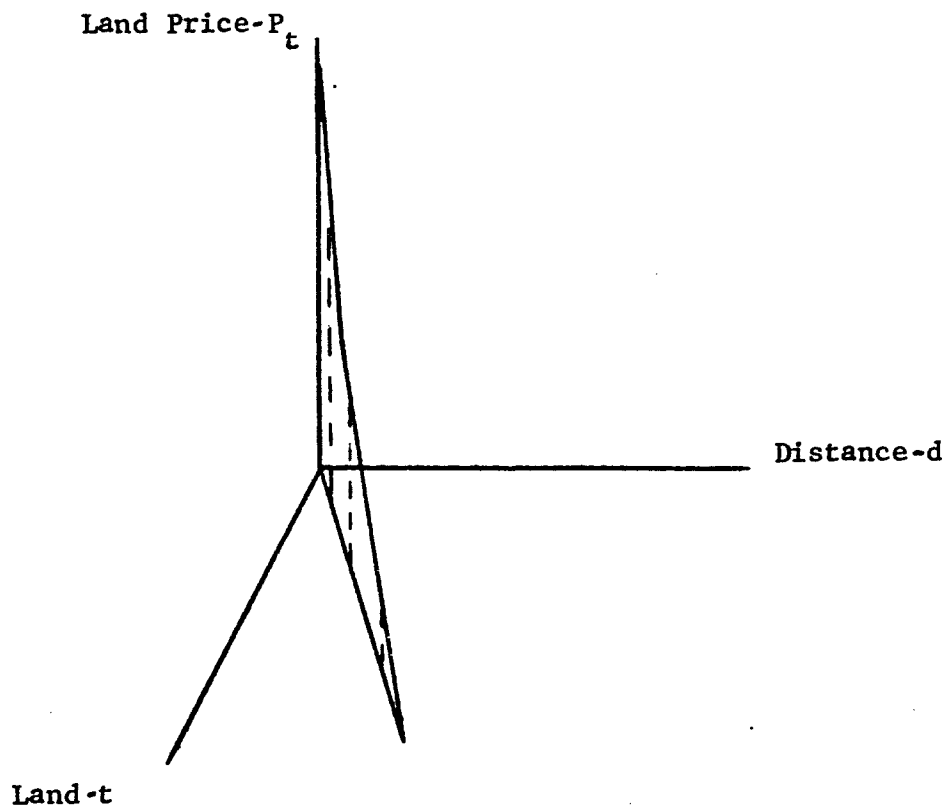


Figure III-12

be directed only at the characteristics of the demand function. We know from previous discussion that

$$p = F(d, t). \quad (3)$$

Let us examine what we know about the function

$$p = F(d, \bar{t}) \quad (4)$$

where $\bar{t}$ is a constant quantity of land. We know that

$$\frac{\partial F}{\partial d} < 0,$$

i.e., the willingness-to-pay for a given amount of land $\bar{t}$ decreases as d increases. We do not know a priori at what rate p changes as d varies.

However, p probably does not decrease at an increasing rate as $\underline{d}$ increases. Most people depict the relationship between p and $\underline{d}$ so that

$$\frac{\partial^2 F}{\partial d^2} > 0,$$

indicating that p decreases at a decreasing rate as $\underline{d}$ increases. These values for the partial derivatives added to the assumption that $\underline{t}$ and $\underline{d}$ act independently in the determination of willingness-to-pay for land yield the following relationship between p and $\underline{d}$

$$p = a_1 - b_1^1 d + b_2^1 d^2 \quad (5)$$

where $|-b_1^1| > 2b_2^1 d$. If for $d = \bar{d}$ such that

$$p = F(\bar{d}, t),$$

$$\frac{\partial F}{\partial t} < 0, \text{ and}$$

$$\frac{\partial^2 F}{\partial t^2} > 0, \text{ then}$$

the relationship between p and $\underline{t}$ may take the following form

$$p = a_2 - b_1^2 t + b_2^2 t^2 \quad (6)$$

where $|-b_1^2| > 2b_2^2 t$. If equations (5) and (6) accurately describe the relationship between p and $\underline{d}$ and between p and $\underline{t}$, and $\underline{d}$ is independent from $\underline{t}$ and vice versa, then

$$F(d, t) = a_1 - b_1^1 d + b_2^1 d^2 + a_2 - b_1^2 t + b_2^2 t^2. \quad (7)$$

If the effect of $\underline{d}$ on p is not independent of the value of $\underline{t}$, and the effect of $\underline{t}$ on p is not independent of the value of $\underline{d}$, then the function relating $\underline{d}$ and $\underline{t}$ to p must include an interaction term so that the functional form is as follows

$$p = F(d, t) = a_1 - b_1^1 d + b_2^1 d^2 + a_2 - b_1^2 t + b_2^2 t^2 - b_3 dt, \quad (8)$$

where $b_3 > 0$. The effect on the relationship between $\underline{d}$ and $\underline{p}$ as $\underline{t}$ varies is illustrated in Figure III-13. The negative sign on b_3 indicates

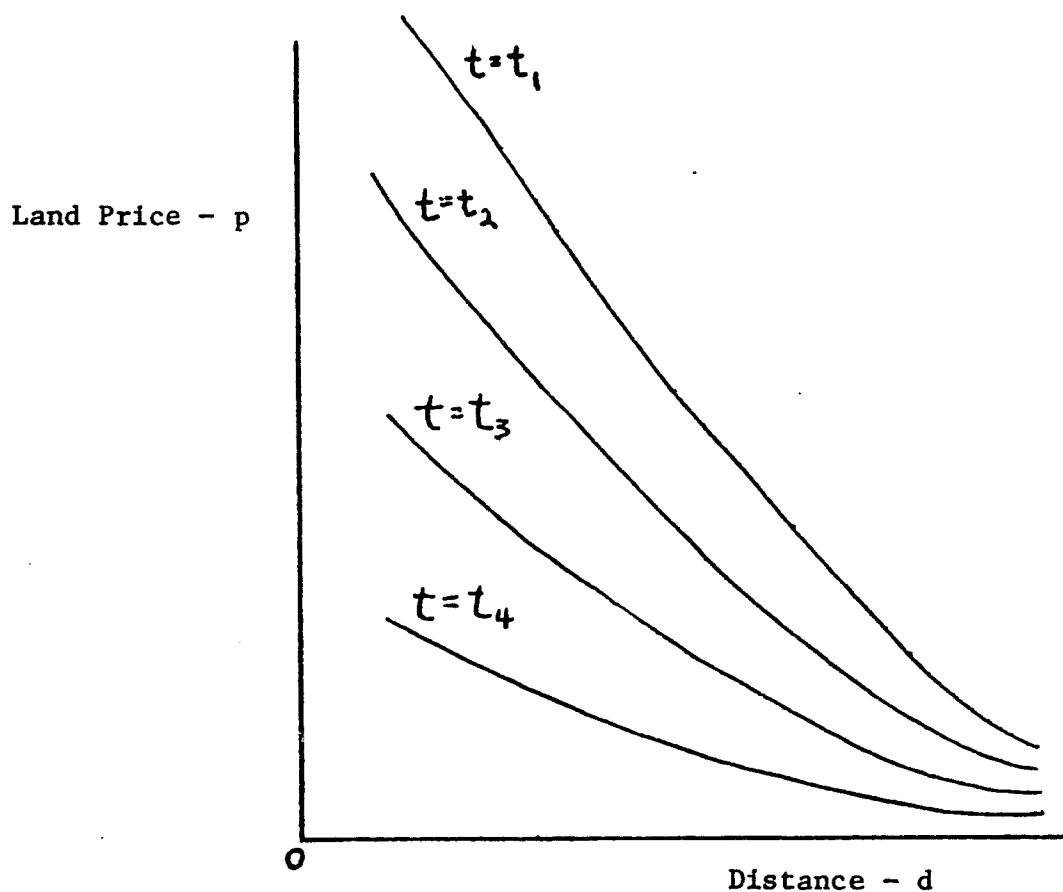


Figure III-13

that for any $\underline{d}$, $\frac{\partial F(\underline{d}, \underline{t})}{\partial \underline{d}}$ becomes less negative as $\underline{t}$ increases. In this figure, $t_4 > t_3 > t_2 > t_1$. Similarly, for any $\underline{t}$, $\frac{\partial F(\underline{d}, \underline{t})}{\partial \underline{t}}$ becomes less negative as $\underline{d}$ increases. This interaction term would follow from characteristics' indifference curves for individuals in $\underline{d}$, $\underline{t}$ under the following conditions:

If $U = U(D, t)$, then on the average

$$\frac{\partial U}{\partial d} < 0, \frac{\partial U}{\partial t} > 0, \frac{\partial^2 U}{\partial d^2} > 0, \frac{\partial^2 U}{\partial t^2} < 0, \frac{\partial^3 U}{\partial d^3} < 0, \text{ and } \frac{\partial^3 U}{\partial t^3} > 0.$$

The hypothesis concerning the effect on land prices of a change in accessibility to the center of a city which we have been discussing has been altered to allow for more than one center. Alonso discusses some possible complications of the generalized hypothesis.² One of the complications considered is the case of competing centers in the sense of complementary centers. In this case there is more than one center, each of which provides a different combination of activities. For example, the competing centers may include one or more retail centers, a financial center, an educational center, and a manufacturing center.³ The willingness-to-pay surface for land in a city with complementary centers will have several peaks, one for each of the centers. This hypothesis for multiple competing centers is applicable to the empirical investigation of the Ann Arbor housing market. According to this hypothesis, there will be an equation for each center with the same terms that appear in equation (7) or equation (8), depending upon whether or not interaction between distance from each center and number of units of land is assumed. If

$$P = F(d_1, d_2, \dots, d_n, t)$$

and no interaction is assumed, then

$$p = a + \sum_{i=1}^n \left[-b_1^i d_i + b_2^i d_i^2 \right] - b_3 t + b_4 t^2. \quad (9)$$

² Op. cit., pp. 134-142.

³ Ibid., p. 137. It is not necessary that a given activity be undertaken at only one center.

If interaction is assumed, the function relating the price of land to the distance from various competing centers and the quantity of land is the following equation

$$p = a + \sum_{i=1}^n \left[-b_1^i d_i + b_2^i (d_i)^2 - b_3^i d_i t \right] - b_4 t + b_5 t^2. \quad (10)$$

Both of these specifications were utilized in the empirical portion of this study. The results are reported in Chapter V.

2. The Effect on Land Prices of Varying Neighborhood Characteristics

Next we consider the effect of variations in neighborhood characteristics on land prices. The first step in analyzing the effects of these variations is to define what is meant by the term "neighborhood". In Webster's New World Dictionary of the American Language, neighborhood is defined as "... a community, district, or area, esp. with regard to some characteristic or point of reference ..."⁴ The point of reference which we use in this paper is an individual, single family property, and we define "neighborhood" as a geographical area consisting of all city blocks which face any given house. Thus, for the purposes of this paper, a neighborhood consists of an area two, three or four square blocks. The situation giving rise to each of these cases is illustrated in Figure III-14 where point A is the point of reference, i.e. a given house. Panel (a) is the two-block neighborhood, panel (b) is the three-block neighborhood, and panel (c) is the four-block neighborhood. A "neighborhood" has been defined in this way

⁴ Second College Edition, The World Publishing Company, Englewood Cliffs, N.J., 1970.

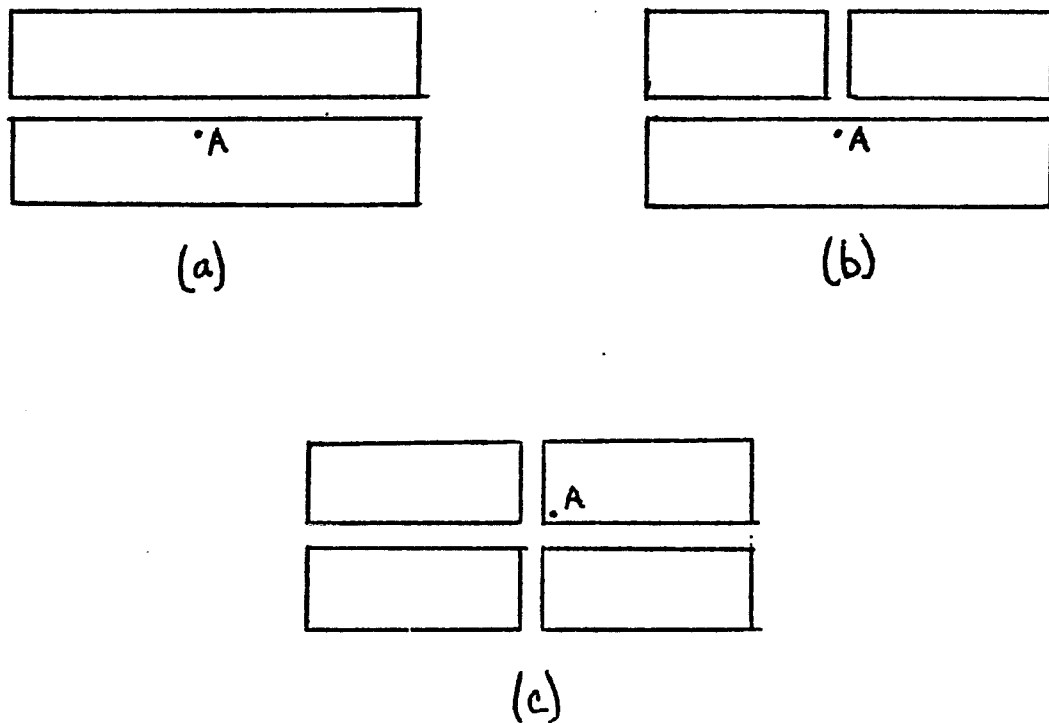


Figure III-14

to allow use of census block data in the empirical investigation of property values.⁵

The land-associated neighborhood characteristic which we will consider in this paper is the racial composition.⁶ Let us begin our

⁵ If the empirical results of this study indicate that the characteristics of a neighborhood as here defined affect the value of an individual property, further research will be necessary to determine if a neighborhood actually encompasses a greater area.

⁶ The analysis of the effect on land prices of varying racial composition is not intended as an explanation of an historical sequence of land prices within a given neighborhood as the proportion of blacks in the neighborhood varies, nor is it intended to be an explanation of why the racial composition of some neighborhoods changes over time and the composition of others does not change. The intention is merely to suggest the relationship between land prices and neighborhood racial composition which we expect if we observe at a point in time the various combinations of land price and racial composition for different neighborhoods within a city.

discussion of the effect on land prices of varying neighborhood racial composition with the market willingness-to-pay or demand relation developed earlier in this chapter. This relation is depicted in Figure III-15. The only housing characteristic allowed to vary along

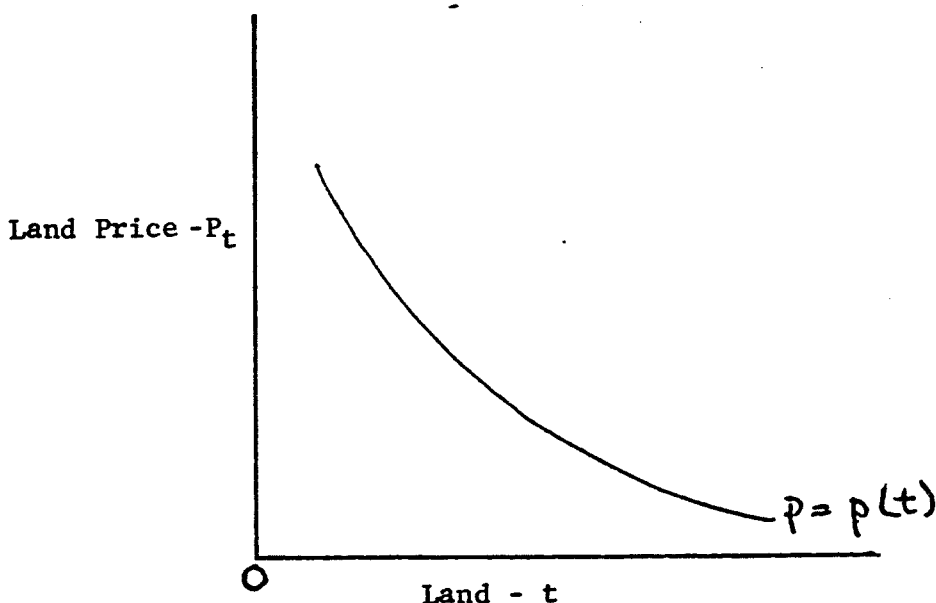


Figure III-15

this curve is the quantity of land. The values of all other housing characteristics are held constant. The relation depicted in Figure III-15 shows that the quantity of land demanded is a decreasing function of the price of land.

Now let us consider what happens to the willingness-to-pay for land in the white segment of the market as we vary the percentage of a neighborhood's residents who are black. If we assume that the relation depicted in Figure III-15 is drawn for a neighborhood which is 100 percent white and that whites prefer to live among other whites, then the white's willingness-to-pay for land decreases as we increase the proportion of residents who are black. This effect is illustrated

in p - t space in Figure III-16, where $\underline{R}$, the percent black, rises from R_1 to R_4 . The same effect is illustrated in Figure III-17 in p - t - R space. The line AB in Figure III-17 is an equi-willingness-to-pay line which indicates combinations of percentage black and quantity of land for which whites would be willing to pay the same amount per unit of land.

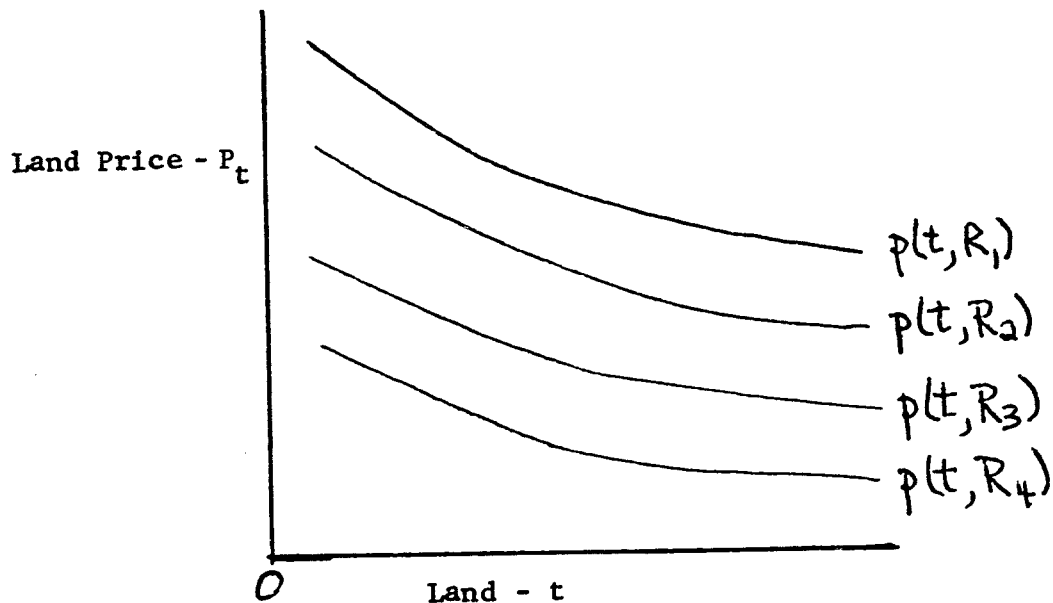


Figure III-16

Similarly, if blacks prefer to live among other blacks, their willingness-to-pay for land increases as the percent black increases. This variation in willingness-to-pay is illustrated in p - t space in Figure III-18 where $\underline{R}$ rises from R_1 to R_4 and in p - t - R space in Figure III-19. The line AB in Figure III-19 is an equi-willingness-to-pay line. It indicates the various combinations of percentage black and quantity of land for which blacks would be willing to pay the same amount per unit of land.

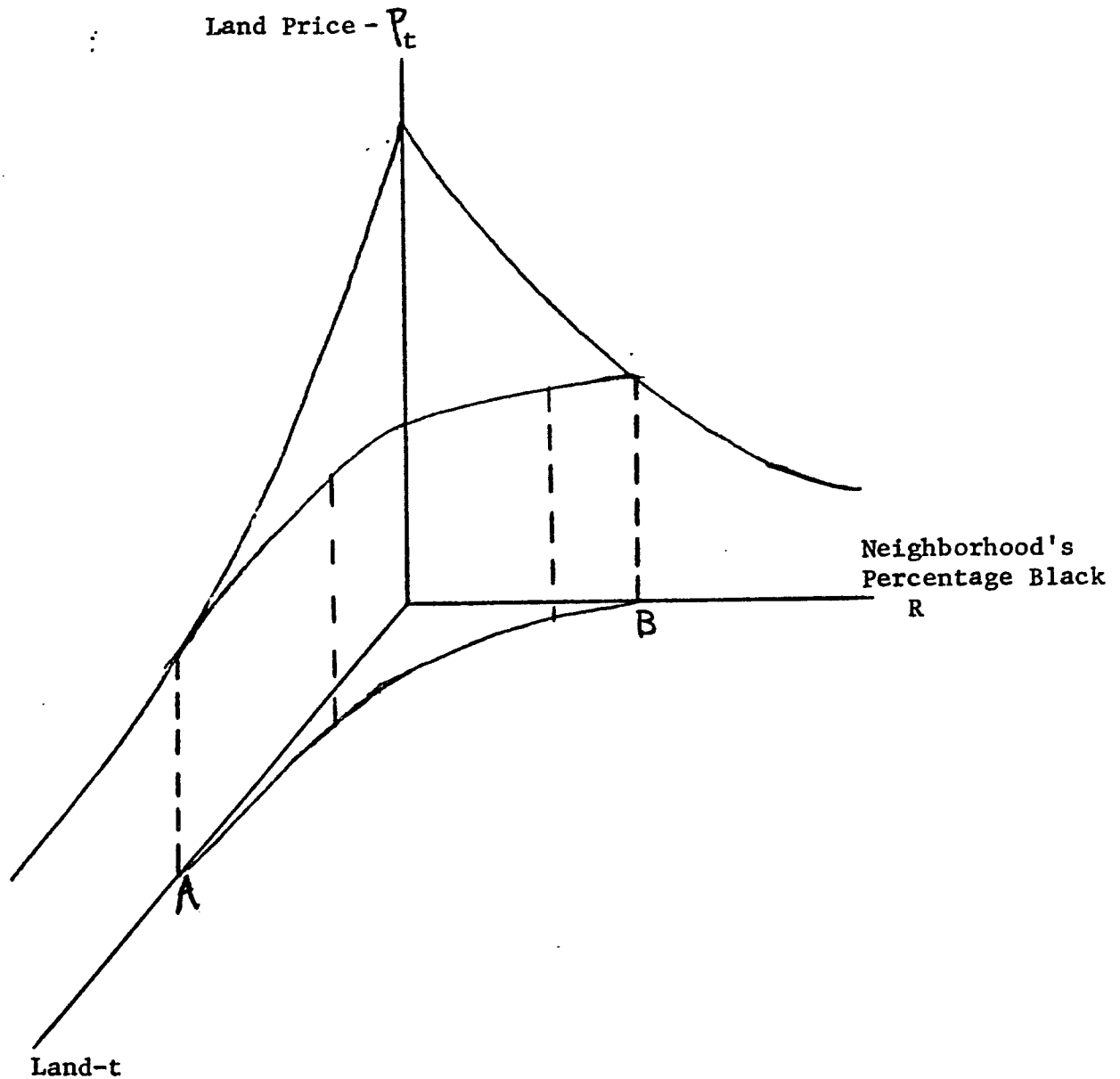


Figure III-17

If we place in the same space the white's willingness-to-pay surface from Figure III-17 and the black's willingness-to-pay surface from Figure III-19 and project the intersection of the resulting surface and a plane for which $t = \bar{t}$ into p-R space, the result is a curve such as $p(R)$ in Figure III-20. This relationship indicates that, for some given quantity of land, the market willingness-to-pay for

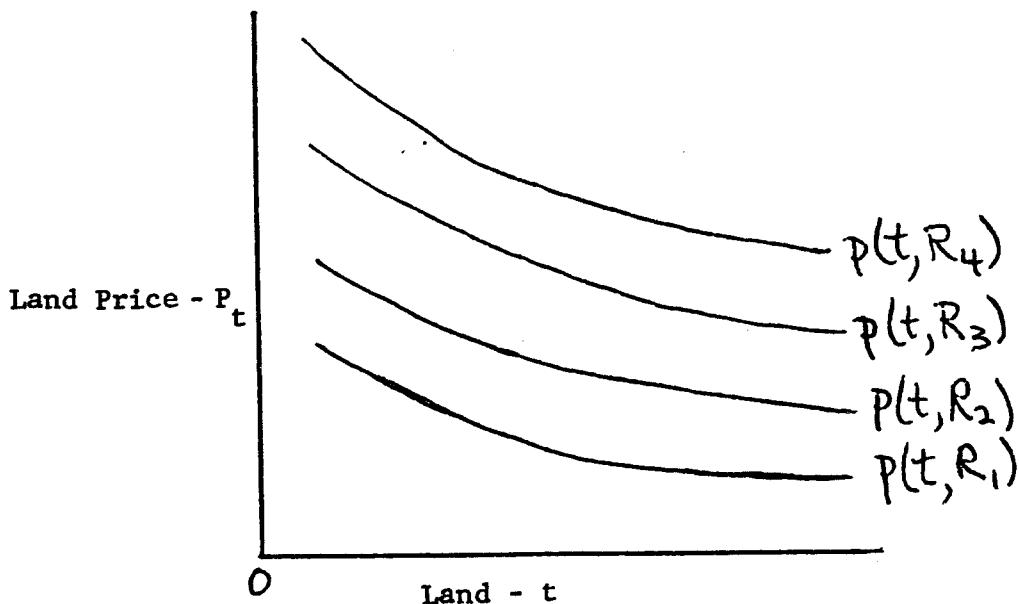


Figure III-18

land decreases as $\underline{R}$, the percentage of black residents in a neighborhood, increases towards R_m , reaches a minimum at $R = R_m$, and increases beyond R_m .⁷ The relationship shown graphically in Figure III-20 is described mathematically by the following equation:

$$p = -b_1 R + b_2 R^2 + b_3 \quad (14)$$

where b_1 , b_2 , and b_3 are constants. The particular value of $\underline{R}$ which minimizes the value of $\underline{p}$ can be solved mathematically when the constant values are known. However, in general, for $\underline{p}$ to be a minimum

$$\frac{dp}{dR} = 0 \text{ and } \frac{d^2p}{dR^2} > 0. \text{ For the expression in equation (14)}$$

$$\frac{dp}{dR} = b_1 + 2b_2 R = 0, \text{ and if}$$

⁷ The value of $\underline{R}$ for which $p(R)$ is a minimum might be interpreted as the percentage of black residents which make a neighborhood a "black" neighborhood rather than a "white" neighborhood.

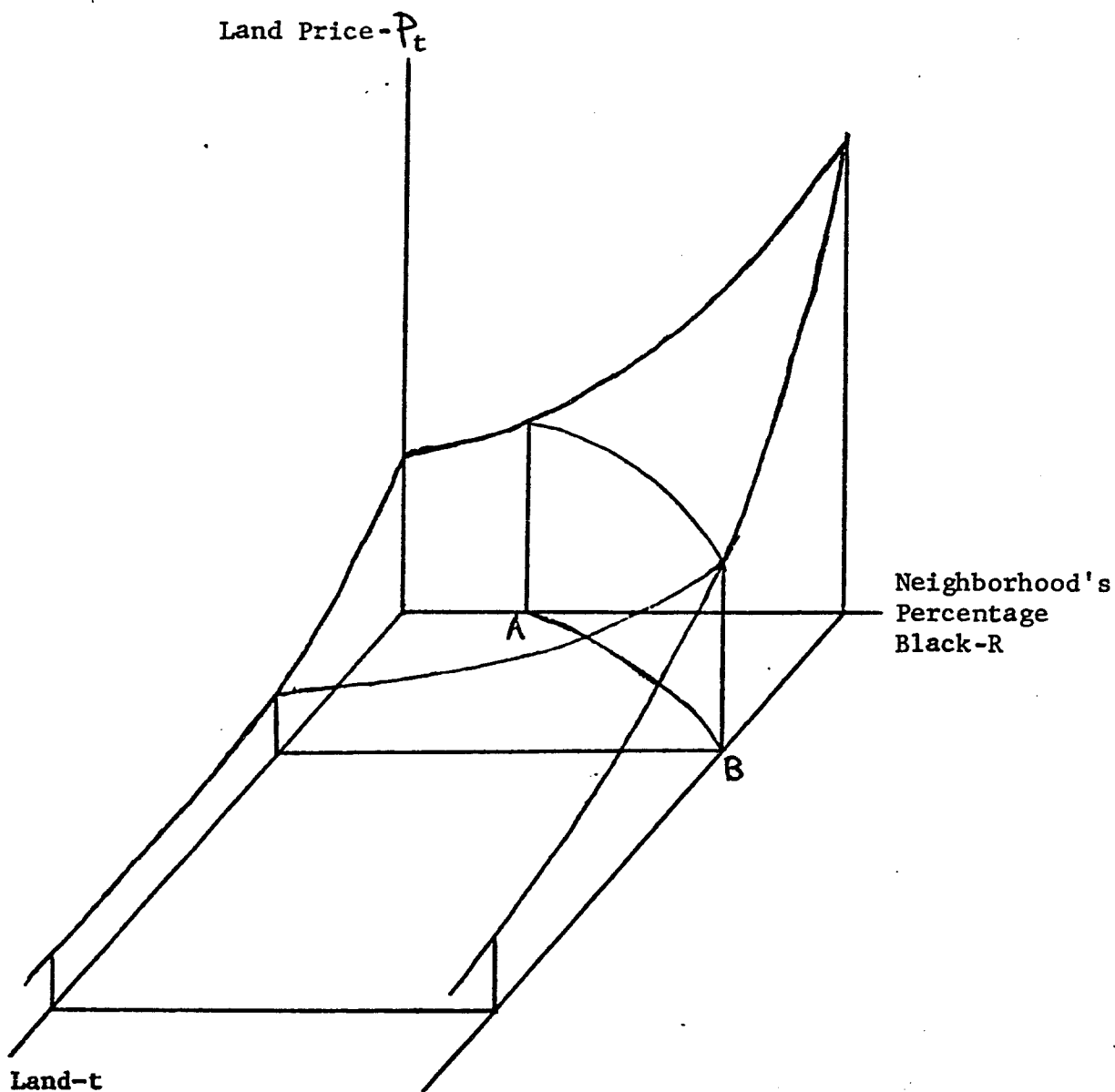


Figure III-19

$\frac{d^2 p}{dR^2} = 2b_2 > 0$, $R = \frac{b_1}{2b_2}$ is the value of $\underline{R}$ which yields the minimum value of $\underline{p}$.

We believe that the relationship between $\underline{p}$ and $\underline{R}$ shown in Figure III-20 and stated in equation (14) is probably not the best functional

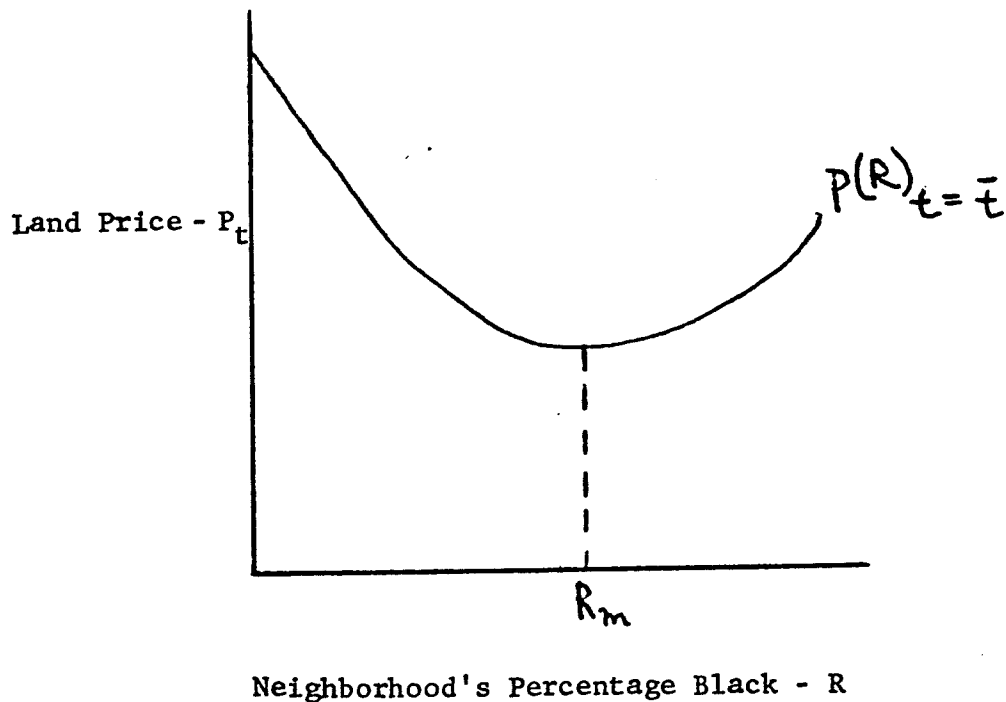


Figure III-20

form for describing the relationship which exists between land prices and a neighborhood's racial composition. It is likely that a few blacks might not be seen by the whites as a threat to the "whiteness" of the neighborhood. Similarly, a few whites might not be seen as a threat to the "blackness" of the neighborhood. The implication of these statements is that land prices in neighborhoods with a small proportion white or a small proportion black may not be much lower than in all white or all black neighborhoods. However, if the proportion of blacks in a white neighborhood or whites in a black neighborhood is increased by a couple of percentage points, then land prices may be much lower. The following figure, III-21, contains a graphical representation of this last relationship between land prices and neighborhood racial composition. This relationship is described mathematically by

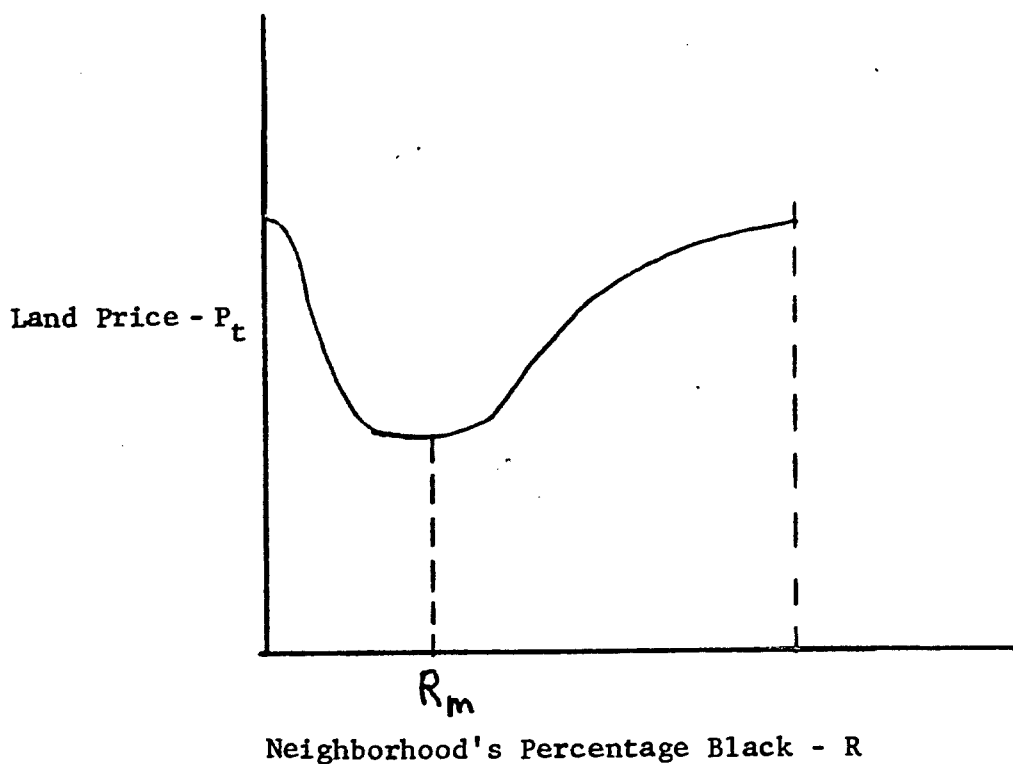


Figure III-21

the following equation:

$$P = -b_1 R + b_2 R^2 - b_3 R^3 + b_4 R^4 + b_5. \quad (15)$$

The price of land $\underline{p}$ takes on its minimum value where

$$\frac{dp}{dR} = -b_1 + 2b_2 R - 3b_3 R^2 + 4b_4 R^3 = 0 \quad (16)$$

and

$$\frac{d^2p}{dR^2} = 2b_2 - 6b_3 R + 12b_4 R^2 > 0. \quad (17)$$

Both specifications, equations (14) and (15), were utilized in the empirical work, the results of which are reported in Chapter V.

3. The Effect on Land Prices of Varying Assessment Ratios

A study by Wallace Oates showed that high levels of expenditures on education tend to offset the effects on property values of the high

tax rates necessary to finance these expenditures.⁸ As a corollary to this, the Tiebout hypothesis,⁹ one would expect that within a city where expenditures and millage rates are constant, land prices would vary inversely with the assessment ratio applied to land, i.e.,

$$p = bT + F(t, d, R) \quad (11)$$

where $\underline{T}$ is the assessment ratio and $\underline{b}$ is a negative number. Under this relationship $\underline{p}$ decreases as $\underline{T}$ increases, other determinants of $\underline{p}$ held constant. However it seems neither logically nor statistically correct to use this model to test the corollary of the Tiebout hypothesis. Let us first consider the logical defect of this formulation. Noting that $T = A/pt$, where $\underline{A}$ is the assessed valuation, we are able to write

$$p = (bA/pt) + F(t, d, R). \quad (12)$$

Hence the assessment ratio, $\underline{T}$, and the price of land, $\underline{p}$, are simultaneously determined at the time a land sale is concluded. The "high" price of land can be attributed to a "low" assessment ratio, but the "low" assessment ratio can be attributed to the "high" price of land.

If $p = F(t, D, R)$ as we have hypothesized, then including $\underline{I}$, i.e., (bA/pt) , as an "independent" variable in our estimating equation yields explanatory variables which are highly correlated. The multicollinearity introduced by including the variable $\underline{T}$ in the estimating equation is undoubtedly so severe that we do not want to contaminate the empirical results by including $\underline{T}$.

⁸ "The Effect of Property Taxes and Local Public Spending on Property Values," Journal of Political Economy, (1969), LXXVI, pp. 957-971.

⁹ Charles M. Tiebout, "A Pure Theory of Public Expenditures," Journal of Political Economy, (1956), LXIV, pp. 416-24.

4. The Effect on Land Prices of Varying Zoning Classification

Two types of zoning are in use in Ann Arbor, land-use zoning and density-intensity zoning. Land-use zoning regulations prevent certain uses on many parcels of land. For example, a parcel which is zoned residential, R, cannot be used for industrial or commercial activities. At the same time within each land use, density-intensity zoning regulations are applied. These regulations require various minimum lot sizes per family. For example, within the single family residence land use, there are four zoning classifications; R1A, R1B, R1C, and R1D, each of which requires a different minimum lot size. Zoning classification R1A requires the largest minimum lot size in the single-family-residence land-use, and classification R1D, the smallest.

A thorough theoretical analysis of the effect of varying zoning classification on land prices is not within the purview of this study. However, a simple hypothesis based upon the work of Alonso can be tested.¹⁰ Alonso states the following concerning the effect of land-use zoning on land price:

The effects of a binding land use zoning regulation are simple. On the site directly affected, since the highest bid is disallowed, a lower bid takes precedence, and the price of land on that site will be lower than under free-market conditions. For the zoned-out land use, since its free-market equilibrium bid price function is not permitted, it will be forced to a higher one, resulting in lower profits or utility, ...¹¹

Following is the hypothesis tested in this study: The more restrictions which are placed upon the use of a plot of land, the lower will be its

¹⁰ Op. cit., pp. 117-123.

¹¹ Op. cit., pp. 117.

price per unit. This hypothesis does not follow directly from Alonso's statement. However, it does seem that the land which faces the least use restrictions would be most likely to sell at the free market price and be used in the best use in terms of the free market solution, and that the land which faces the greatest use restrictions would be least likely to sell at the free market price. Hence the preceding hypothesis.

5. The Tested Land Price-Determination Hypotheses

Several hypotheses concerning the relationship between land prices and the values of several quantifiable variables resulted from the previous discussion. These include the following hypotheses:

$$(1) \quad p = a + \sum_{i=1}^n \left[-b_1^i d_i + b_2^i (d_i)^2 \right] - b_3 t + b_4 t^2 - b_5 R + b_6 R^2 + b_7 Z,$$

$$(2) \quad p = a + \sum_{i=1}^n \left[-b_1^i d_i + b_2^i (d_i)^2 \right] - b_3 t + b_4 t^2 - b_5 R + b_6 R^2 - b_7 R^3 \\ + b_8 R^4 + b_9 Z,$$

$$(3) \quad p = a + \sum_{i=1}^n \left[-b_1^i d_i + b_2^i (d_i)^2 \right] - b_3^i d_i t \left[-b_4 t + b_5 t^2 - b_6 R \right. \\ \left. + b_7 R^2 - b_8 Z, \text{ and} \right]$$

$$(4) \quad p = a + \sum_{i=1}^n \left[-b_1^i d_i + b_2^i (d_i)^2 - b_3^i d_i t \right] - b_4 t + b_5 t^2 - b_6 R \\ + b_7 R^2 - b_8 R^3 + b_9 R^4 + b_{10} Z.$$

If we have found all of the characteristics which affect the market price of land, and if one of the hypothesized relationships among the variables is the true relationship, then multiplying the price of a particular plot of land by the total number of units in that plot yields the total value of land, i.e.,

$$\text{Value of Land} = P_L(L),$$

where $L \equiv$ quantity of land. If improvements is the only other thing purchased when an individual buys a single family property, we can determine the value of those improvements by subtracting the value of land from the purchase price of the property. The expenditure function for improvements can be derived from the housing branch budget constraint in characteristics space. In its general form, this housing budget constraint is

$$E = P_I(I) + P_L(L), \quad (13)$$

where

P_I = a vector of improvement characteristics prices, and

I = a vector of improvement characteristics.

Thus $P_I(I)$ is the total value of improvements. Once the correct form for the land price function has been determined empirically, we can find, for any particular single family residential property, the total value of land. Subtracting $P_L(L)$, value of land, from both sides of equation (13) yields the total value of improvements:

$$P_I(I) = E - P_L(L).$$

In this manner we are able to separate the sale value of a property into two elements: (1) value of the land and (2) value of the improvements. This division of total sale price into the two components is consistent with current property assessment practices in Ann Arbor. Determining the expected form of the improvement-value function is the next step in our investigation of single family property values.

B. The Improvement-Value Function

In the last section we derived the general form of the improvement-value function from the housing branch budget constraint. In this section we discuss the arguments of this function and arrive at

the form to be used in the empirical portion of the study.

Most of the arguments of the improvement-value function can be determined by casual observation of the improvement characteristics in which people are interested when considering a single family residential property for possible purchase and by the properties which they do purchase. People have revealed by their actions that the arguments of the improvement-value function include the following characteristics: (1) number of square feet of living space; (2) kitchen built-ins, e.g., range, refrigerator, dishwasher; (3) types of building materials from which the improvements are constructed; (4) types and number of bathroom facilities; and (5) area for housing motor vehicles. All of the above are on-site characteristics. There may also be characteristics of the neighborhood which affect the market value of the improvements contained in the property. The neighborhood characteristic, racial composition, considered in our discussion of land-price determination is one neighborhood characteristic likely to affect the market value of improvements. Let us first consider the effect on the value of improvements of variations in the on-site characteristics of the property.

1. The Effect on Improvement Value of Varying On-Site Characteristics

The complete list of the on-site characteristics whose variation we will consider includes the following items: (1) portion of house insulated, (2) foundation material, (3) type of interior trim, (4) type of kitchen built-ins, (5) type of flooring, (6) type of roofing, (7) type of exterior wall materials, (8) type of interior finish, (9) size of central air conditioner, (10) types and number of bathroom facilities, (11) number of fireplaces, (12) square feet of living area, (13) square

feet of garage space, and (14) dwelling age. We do not choose to consider here the effect on improvement value of variations in each separate on-site characteristic. That would result in unnecessary repetition. Rather let us consider in a general way the effect of variations in these characteristics.

If each on-site characteristic independently affects the total value of improvements, the improvement-value function may be written in the following form:

$$E - P_L L = E_I = P_I I \quad (15)$$

where E_I is defined as the total value of improvements and the other symbols are as previously defined. The fourteen on-site characteristics mentioned above are elements of the vector, $\underline{I}$. Each of the on-site characteristics variables can be defined so that its value increases as the characteristic moves from a less to a more desirable type and whose partial effect on total improvement value is positive. Consider the characteristic, roofing type, as an example. There are several different materials which can be used in constructing a roof. If these materials are ranked from least to most desirable in terms of durability and appearance, then we have a variable for which an increased value will yield an increased total improvement value.

We have attempted to apply this procedure to all but one of the on-site characteristics. The exception is dwelling age. Dwelling age is used as a proxy for depreciation, and we expect total improvement-value to diminish as dwelling age increases, ceteris paribus. Let us now turn our attention to the effect on improvement value of varying neighborhood racial composition.

2. The Effect on Improvement Value of Varying Neighborhood Racial Composition¹²

Let us begin by assuming that a neighborhood's racial composition is an independent factor in the determination of the value of improvements of single family residential properties. If this is the case then we can consider the effect of varying neighborhood racial composition alone on improvement value while neglecting the amount of other improvement characteristics contained in a particular property. Further, the relationship between racial composition and improvement value may be written as follows:

$$E_I = E(R) + P_I I \quad (16)$$

where $E(R)$ is the functional relationship between improvement value and racial composition, R . We are now ready to analyze the functional form of this relationship.

If whites prefer to live among other whites, then, ceteris paribus, the white's willingness-to-pay for improvements decreases as we increase the proportion of residents who are black. This effect is

¹²For the purpose of this discussion a neighborhood is defined as in the discussion of the effect on land prices of varying neighborhood characteristics. Further, the analysis of the effect on improvement value of varying neighborhood racial composition is not intended as an explanation of an historical sequence of improvement values within a given neighborhood as the proportion of blacks in the neighborhood varies, nor is it intended to be an explanation of why racial composition of some neighborhoods changes over time and the composition of others does not change.

illustrated in $E_I - R$ space in Figure III-22. $[E(R)]_w$ is the

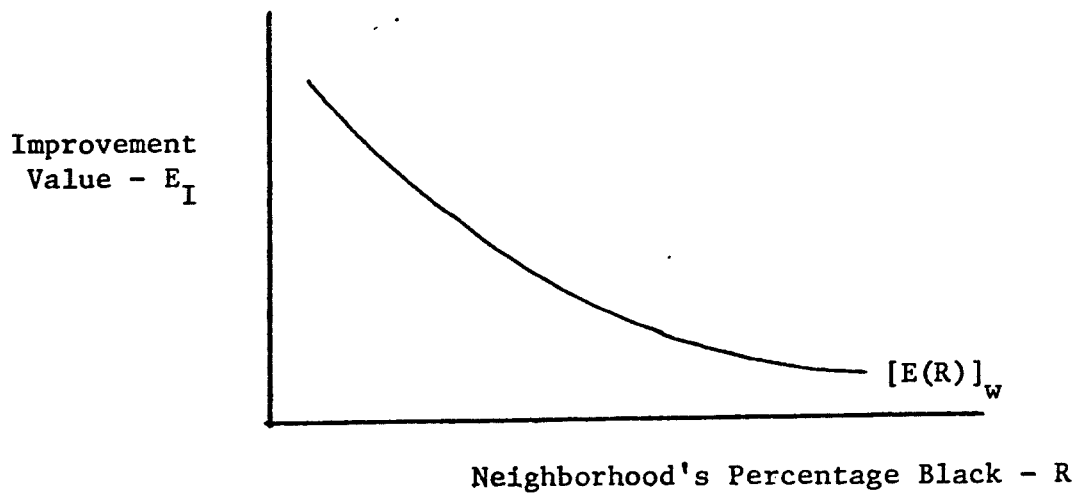


Figure III-22

relationship between improvement value and racial composition for whites. Similarly, if blacks prefer to live among other blacks, their willingness-to-pay for improvements increases as the percent black increases. This variation in willingness-to-pay is illustrated in $E_I - R$ space in Figure III-23, where $[E(R)]_B$ is the relationship between improvement value and racial composition for blacks.

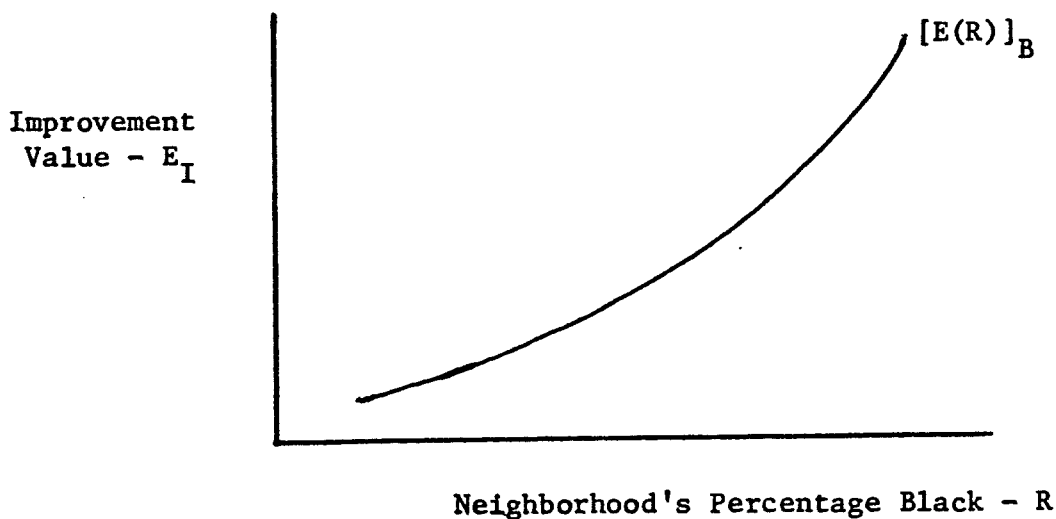


Figure III-23

If we place in the same $E_I - R$ space the relations $E(R)_w$ and $E(R)_b$, the result is the relation $E(R)$ in Figure III-24. This relationship indicates that, for some given combination of improvements, the market value of those improvements decreases as R increases toward R_m , reaches a minimum at $R = R_m$, and increases beyond R_m . The relationship shown graphically in Figure III-24 is described mathematically by the equation:

$$E_I = -b_1 R + b_2 R^2 + b_3 \quad (17)$$

where b_1 , b_2 , and b_3 are constants.

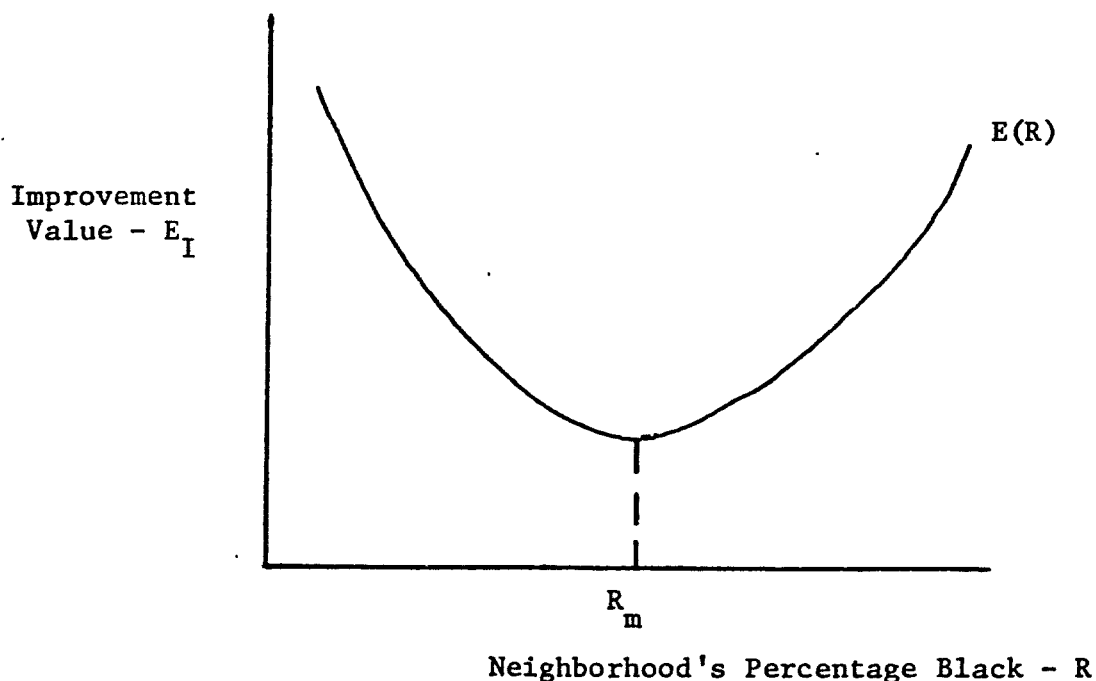


Figure III-24

As in the case of the effect of racial composition on land price, there is an alternative hypothesis for the situation in which a few blacks are not seen by whites as a threat to the "whiteness" of the neighborhood, and a few whites are not seen as a threat to the "blackness" of the neighborhood. This alternative hypothesis is summarized

by the following equation:

$$E_I = -b_1 R + b_2 R^2 - b_3 R^3 + b_4 R^4 + b. \quad (18)$$

Both specifications, equations (17) and (18), were utilized in the empirical work.

This completes our development of the specific testable hypotheses. In the next chapter we describe the data which were used in testing these hypotheses. The final chapter contains the results of this testing and our conclusion concerning the efficacy of using the computer in assessing single family residential property value for property tax purposes.

CHAPTER IV

DESCRIPTION OF THE DATA USED FOR EMPIRICAL INVESTIGATION

The geographical area chosen for the empirical investigation of single family residential property values was Ann Arbor, Michigan. Ann Arbor was chosen for the following reasons: (1) the University of Michigan is located in Ann Arbor and the expense of collecting the data was low since we did not have to travel to another city; (2) the assessor's office in Ann Arbor has fairly accurate records of the characteristics which are possessed by the single family residences within the City of Ann Arbor; (3) the assessor's office in Ann Arbor has accurate records of the dates of sale and sale prices for single family residences in Ann Arbor;¹ (4) all of the sample properties are within the City of Ann Arbor so that public services and property tax rates can be assumed constant; and (5) all of the properties are within the same assessor's jurisdiction, and he does not assess outside of the area from which the sample is taken.

The data used in the study are of the following four types: (1) the sale prices of the 951 single family residential properties and 69 vacant lots which were sold in 1970; (2) the physical characteristics of the same 951 single family residential properties and 69 vacant lots; (3)

¹The accuracy of the Ann Arbor assessor's records of dates of sale and sale prices for single family residences was checked by a survey of some of the current property owners. We asked the owners of thirty sample properties when they purchased their property and how much they paid for it. Fifteen responded. All had purchased in the month recorded by the assessor, and none had paid more than one percent more or less than the amount recorded by the assessor.

the locational characteristics of the 1020 properties in the sample; and (4) the neighborhood characteristics of the 1020 properties in the sample. Each of these types is described in detail in a separate section of this chapter; the sale prices in Section A; physical characteristics in Section B; location characteristics in Section C; and neighborhood characteristics in Section D.

A. Data on Sale Prices

The source of the sale prices data was the Ann Arbor Assessor's file of field data cards. We decided to use as our sample those single family residences and vacant lots which were sold in 1970.² The assessor notes on the field assessment cards the date of sale and price for properties sold. Thus it was possible to leaf through the files and find cards for the properties sold in 1970. It turned out that 951 single family residential properties and 69 vacant lots were sold during the year. We then took from the field cards the sale price of each property. One of these cards with the type of information collected is shown in Exhibit IV-1.³

The 1970 sale price for the 1020 properties in the sample show considerable variation. The lowest sale price was \$200, the highest was \$90,000 and the mean 1970 sale price was \$28,910. In Figure IV-1 is a frequency distribution showing the number of properties sold in 1970 in various price ranges. It is this variation in sale price which we have attempted to explain in our theoretical and empirical investi-

²The year 1970 was chosen because of the availability of census data collected during that year.

³We also took from the field cards the physical characteristics data for each property. An "X" in a space in Exhibit IV-1 indicates that the corresponding information was recorded and is used in the empirical investigation.

ASSESSMENT RECORD

[illegible]

[illegible]

Exhibit IV-1 continued

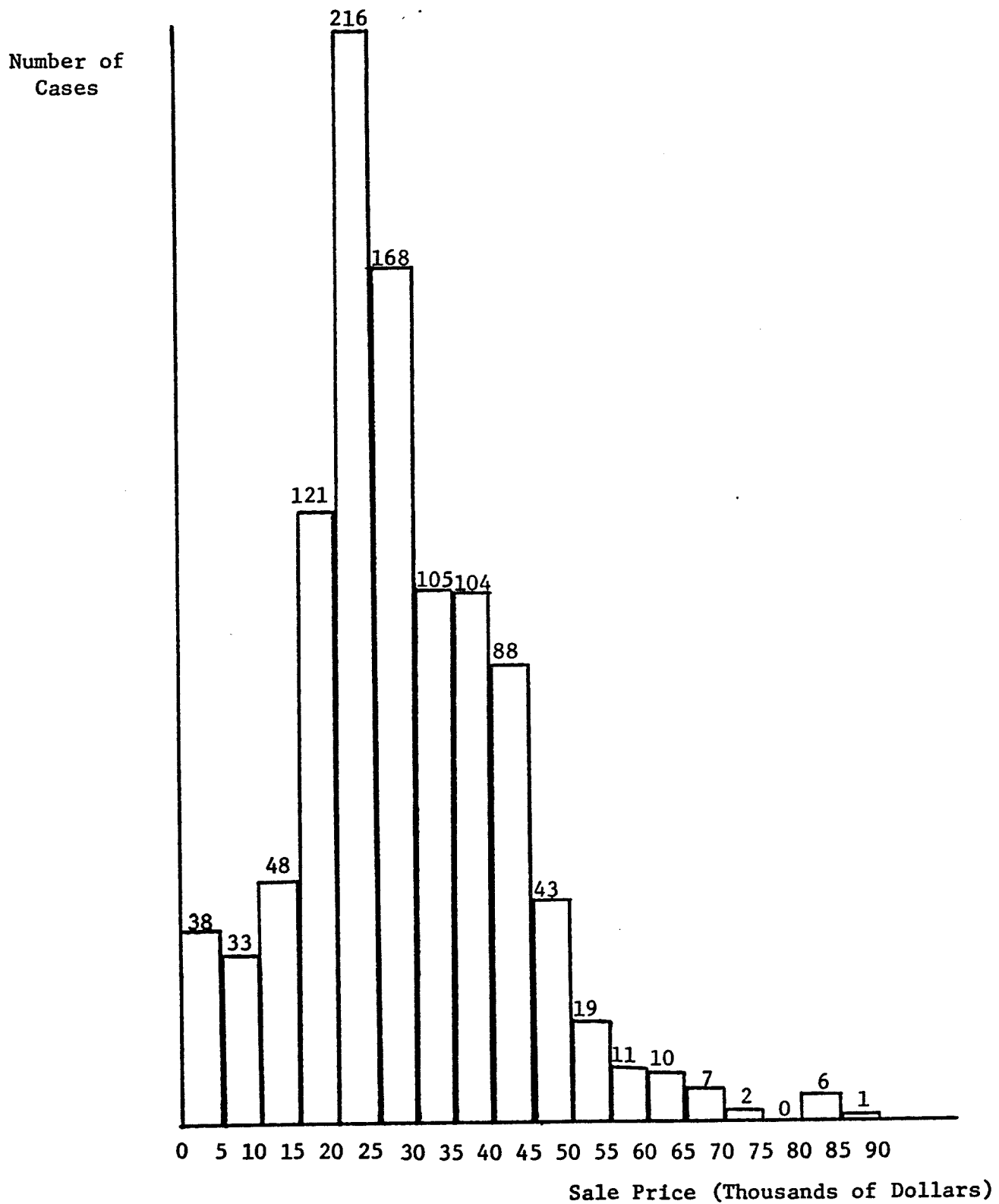


Figure IV-1

gation.

B. Physical Characteristics Data

Exhibit IV-1 suggests that data were collected on many physical characteristics. For the 69 vacant lots the physical characteristics data consisted of the zoning classification, the depth of the lot, and its frontage. In many instances the given information for those 951 properties consisting of a plot of land and a single family residence was whether or not the residence contained a given piece of equipment. For example, under the general category "built-ins," many items are listed: disposal, dishwasher, oven and range, and several others. We decided that a variable called "built-ins" should be included in our variable list rather than including each piece of equipment as a dichotomous variable.⁴

Table IV-2 contains a list of the variables which are composites of related physical characteristics, their components, and range of values.⁵ The zoning variable has the value of 1,2,...,10 for the zoning classification R1A, R1B,...,R4D, respectively. The other composite variables were formed by ranking the components in terms of construction costs and then assigning a number from within the given range to each cost class. Where the range contains more integers than there are components, properties contained combinations of the compo-

⁴If each piece of equipment is a dichotomous variable, the variable takes on a value of zero if the equipment is not present and one if it is present. Although the decision was not originally based on limitations imposed by the use of the computer in our regression analysis, it turned out that, even after diminishing the number of variables by eliminating these dichotomous variables, we were barely within computer core limitations.

⁵Appendix I contains a list of the names and descriptions of all variables used in the study for which regression coefficients are reported in Chapter V.

TABLE IV-2

COMPOSITE VARIABLES, THEIR COMPONENTS,
AND THEIR RANGE OF VALUES

Composite Variables	Components	Range
Zoning	R1A, R1B, R1C, R1D, R2A, R2B, R4A, R4B, R4C, R4D	1-10
Insulation	none, roof, walls, both	1-4
Foundation	piers, brick, concrete, block, poured	1-4
Trim	none, pine, hardwood	1-3
Built-in	disposal, dishwasher, oven and range, oven and range with broiler, refrigerator, tile wains, inter- com, attic fan, incinerator, kitchen fan-hood	1-32
Flooring	earth, cement, pine, hardwood, plywood and carpeting, tile, slate	1-14
Roofing	none, metal, roll, tile, asbestos, asphalt, tar and gravel, wood, slate	1-9

nents, and these combinations were included in the rankings.

We found in the sample properties thirteen types of exterior wall materials. These materials appeared both singly and in combination. Since it was not economical to determine the amount of each material used in construction, we assumed that for a single property the fraction of the exterior wall constructed of a particular material was equal to one divided by the number of materials used. This fraction is the value entered for each exterior wall material used. Zero is the value entered for each exterior wall material not used for a particular property. Hence we have thirteen exterior wall materials variables with values ranging from zero to one. We recognize that this method of varying exterior wall type yields approximations to the actual variation in exterior wall type, but we were unable to find an economical method which yielded better approximation. There is one additional drawback to the selected method: for each property, the value entered for one exterior wall materials variable is not independent of the values entered for the other exterior wall materials variables.

Five types of interior finish appeared in the sample properties: plaster, drywall, paneling, accoustical and unfinished. These finishes appeared singly and in combination. We used the same method of determining the values for each of these variables as was used for determining the values of the exterior wall materials variables. The reservations in this application are the same as those mentioned when we explained the possible values for the exterior wall materials variables.

All other physical characteristics variables merely have the numerical value recorded on the field assessment card. There are

eight groups of these variables, including (1) land depth in feet, (2) land frontage in feet, (3) central air conditioning in tons, (4) plumbing, (5) fireplaces, (6) living area in square feet, (7) garage space in square feet, and (8) dwelling age in years.

C. Location Characteristics Data

The third type of data is the location characteristics of the 1020 properties in the sample. The first step in determining the location characteristics was to find the centers of economic activity from which distance was to be measured. From traffic patterns and employment opportunities, we decided that there are nine centers of economic activity in Ann Arbor. These centers are shown on the map in Exhibit IV-2. Then we measured the distance to the nearest 300 feet between each of these centers and each of the sample properties. As is indicated in Chapter V, variations in location as small as 300 feet do not account for variations in the sale prices of the sample properties. Consequently we combined the 300 feet distance intervals into three alternatives, one-third mile, one-half mile, and one mile intervals. We experimented with these four interval measures and found that the one-half mile interval yielded the most satisfactory results. Therefore, we used distance between each of the centers and each of the sample properties, measured to the nearest one-half mile, in the investigations reported in Chapter V.

D. Neighborhood Characteristics Data

The fourth type of data is the neighborhood characteristics of the 1020 properties in the sample. We believe that there are four neighborhood characteristics which are likely to affect the value of a property. They are the following: (1) attractiveness of a neighborhood; (2) the

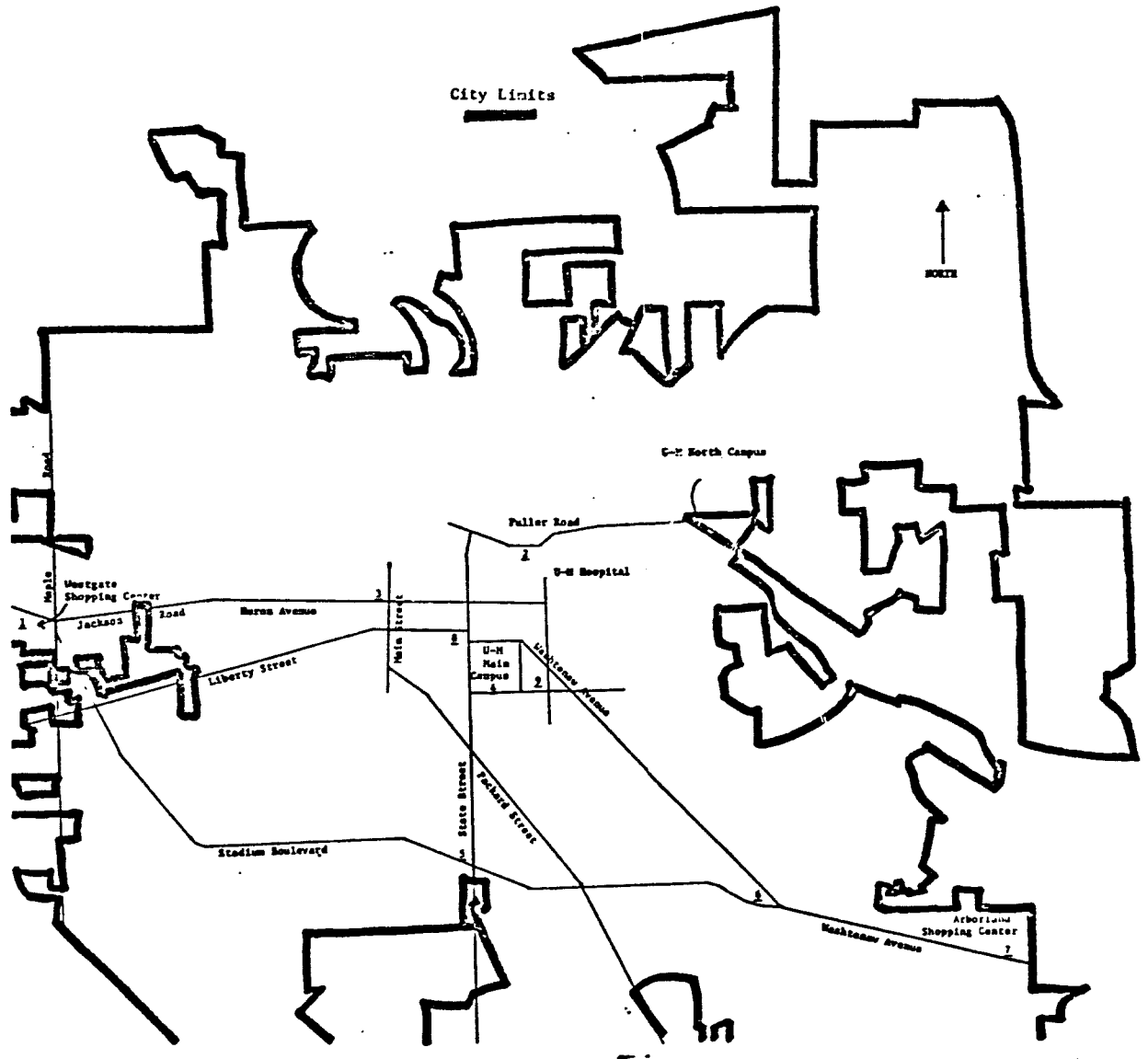


Exhibit IV-2
Map Showing 9 Centers of Economic Activity

income level of a neighborhood's residents; (3) the educational level attained by residents; and (4) the neighborhood's racial composition.

We did not attempt to construct an index of the attractiveness, and were not able to find data by census block on income and education. However, we were able to determine the racial composition of the neighborhood for each property sold. The measure of racial composition which we wanted to use was the percentage of a neighborhood's population which was black. The 1970 Census of Population, Block Statistics for Ann Arbor, Michigan gives for each census block the total population and the percentage of this population which is black. This information enabled us to calculate the measure of racial composition which we wanted to use in our empirical investigation of property values.

This concludes the description of the data which were used for the empirical investigation. Chapter V contains a description of the statistical methods applied in and the empirical results of the investigation. It also contains the conclusion regarding the implications of the results for possible improvement in assessment practices.

CHAPTER V

EMPIRICAL RESULTS AND CONCLUSIONS

While deriving the specific hypotheses to be tested in this study, we discovered that the theories of land values and of housing values are not advanced sufficiently to allow more than the testing of provisional hypotheses and the inference of additional hypotheses from empirical results. Thus, the results which we report in this chapter should be viewed as laying a foundation for further study of the value of single family residential properties. In addition to containing the results of our investigations when the data described in Chapter IV are used,¹ this chapter contains our conclusion regarding the possibility of applying regression analysis in assessing property values for property tax purposes. Section A contains the regression results for our alternative land price equations.

A. Empirical Land Price Equations

Four alternative specifications of the land price equation were examined for their power to predict the price per square foot for land.² The data used for the independent variables in this examination consisted of the physical characteristics, location characteristics, and

¹The data for two of the dependent variables in the regressions are transformations of the saleprices of our sample properties. These dependent variables are described in this chapter.

²See page 65 for comparison of the four specifications or hypotheses.

neighborhood characteristics of the 69 vacant lots which were sold in 1970. The dependent variable, price per square foot, was computed by dividing the saleprice of each vacant lot by its area in square feet. Table V-1 contains lists of the terms included in each specification, and the degrees of freedom, the F statistic, the significance level, and the R^2 for each equation.

Land price specification IV-A has the most predictive power. For this regression equation we have displayed the coefficients and their standard errors in Table V-2. We have indicated which variables have coefficients at least as large as their standard errors by placing an "X" in the last column of Table V-2. Nearly all of the coefficients are at least as large as their standard errors and eleven are at least twice as large as their standard errors although not all of these have the expected sign. Those variables whose coefficients are both significant at the 5 percent level and have the expected sign include the following: (1) ZONING, (2) DIS412M, (3) DIS412MA, (4) DIS412M2, and (5) DIS912M2.

. The sign of the zoning coefficient supports the hypothesis that a plot of land of a given size with fewer lot restrictions tends to sell for a higher price than one of the same size with more use restrictions placed upon it. Although the evidence which we have found supports the hypothesis, this result should be viewed with caution since, as we indicated in Chapter III, the hypothesis is not based on a strong theoretical foundation. Much theoretical and empirical work remains to be done in this area.

The variables DIS412M, DIS412MA, and DIS412M2 are all related to the distance from the center of economic activity which most people

TABLE V-1

INCLUDED TERMS, F-STATISTIC, SIGNIFICANCE LEVEL, AND R² FOR
ALTERNATIVE LAND PRICE EQUATIONS

Term	Equations			
	I	II	III	IV-A
ZONING	X	X	X	X
DISTANCES	X	X	X	X
DISTANCES2	X	X	X	X
DISTANCES X AREA			X	X
AREA	X	X	X	X
AREA2DIV	X	X	X	X
PARBLAK	X	X	X	X
PARBLAK2	X	X	X	X
PARBLAK3		X		X
PARBLAK4		X		X
CONSTANT	X	X	X	X
Degrees of Freedom:				
Regression	23	25	32	34
Residual	45	43	36	34
<u>F</u> -Statistic	4.60418	4.20483	3.33516	4.28393
Significance Level	0.01000	0.01000	0.01000	0.01000
<u>R</u> ²	0.67503	0.70970	0.74777	0.81075

TABLE V-2

INCLUDED TERMS, COEFFICIENTS, AND STANDARD
ERRORS FOR SPECIFICATION IV-A

Variable	B-Coefficient	Standard Error of B	"X" if $\frac{B}{SE_B} \geq 1$
ZONING	0.36095	0.11308	X
AREA	-0.00013	0.00014	
AREA2DIV	0.00012	0.00019	
PARBLAK	-2.69106	5.12983	
PARBLAK2	-38.17641	36.22122	X
PARBLAK3	145.90818	78.46354	X
PARBLAK4	-118.57889	51.98538	X
DIS112M	-0.67194	0.39577	X
DIS112MA	0.00003	0.00002	X
DIS112M2	0.05486	0.03633	X
DIS212M	2.37137	0.99471	X
DIS212MA	-0.00015	0.00005	X
DIS212M2	-0.03914	0.09236	
DIS312M	-0.78405	0.71761	X
DIS312MA	-0.00001	0.00003	
DIS312M2	0.06022	0.04956	X
DIS412M	-3.89838	0.95615	X
DIS412MA	0.00011	0.00004	X
DIS412M2	0.27744	0.09993	X
DIS512M	0.84364	0.60790	X
DIS512MA	-0.00002	0.00003	
DIS512M2	-0.10893	0.05565	X
DIS612M	0.10113	0.50060	X
DIS612MA	0.00001	0.00002	
DIS612M2	0.06052	0.02963	X
DIS712M	0.03680	0.62351	
DIS712MA	0.00001	0.00002	
DIS712M2	-0.01536	0.02850	
DIS812M	3.28809	0.74444	X
DIS812MA	0.00007	0.00004	X
DIS812M2	-0.40298	0.08998	X
DIS912M	-1.20117	0.69687	X
DIS912MA	-0.00003	0.00004	
DIS912M2	0.10923	0.05378	X
(CONSTANT)	-1.04795	0.44128	X

familiar with Ann Arbor would expect to have the strongest impact on land prices, the University of Michigan main campus.³ According to our results, land price per square foot decreases at a decreasing rate as the distance from the Michigan campus increases. Table V-3 shows the decline in land price as the distance from the University of Michigan campus increases.

TABLE V-3

RELATIONSHIP BETWEEN LAND PRICE AND DISTANCE FROM
UNIVERSITY OF MICHIGAN CAMPUS

Distance from U-M Campus (miles)	Change in land price from U-M Campus (\$)
0.5	-1.88
1.0	-3.62
1.5	-5.22
2.0	-6.88
2.5	-8.01
3.0	-9.20
3.5	-10.25

The other distance variables in specification IV-A either do not have coefficients which are significant at the 5 percent level or do not have the expected sign.⁴ However, some of the distance variables do have the expected sign and have coefficients which are at least as great as

³As mentioned in Chapter IV, initially we measured distance to the nearest 300 feet from each plot of land to each center of economic activity. However, when we first attempted to find the relationship between land price and distance from the centers, we found no statistically significant relationship. We experimented with distance to the nearest one-third mile, one-half mile, and one mile. Distance measured to the nearest one-half mile yielded the best results. Thus, that measure was the one used for the results reported here.

⁴The exception is DIS912M2. It is mentioned in this paragraph with its corresponding distance variable, DIS912M.

their standard errors. These include the following groups of distance variables: (1) DIS112M, DIS112MA, DIS112M2; (2) DIS312M, DIS312M2; and (3) DIS912M, DIS912M2. These results could be explained by any combination of three factors: (1) there is multicollinearity, (2) the distance is not measured from a "center" of economic activity, or (3) the distance variable is correlated with an excluded variable which has a strong positive effect on land price.

Now let us consider the implication of our regression results for the effect on land price of variations in neighborhood racial composition. First, it should be noted that, while only PARBLAK4, (percentage black residents)⁴, has a coefficient which is significant at the 5 percent level, PARBLAK is the only neighborhood racial composition variable whose coefficient is not at least as great as its standard error. But, aside from the significance of the coefficients, what do our regression results indicate about the relationship between land price and neighborhood racial composition? The indicated relationship is depicted in Figure V-1. The depicted relationship is basically the one which was

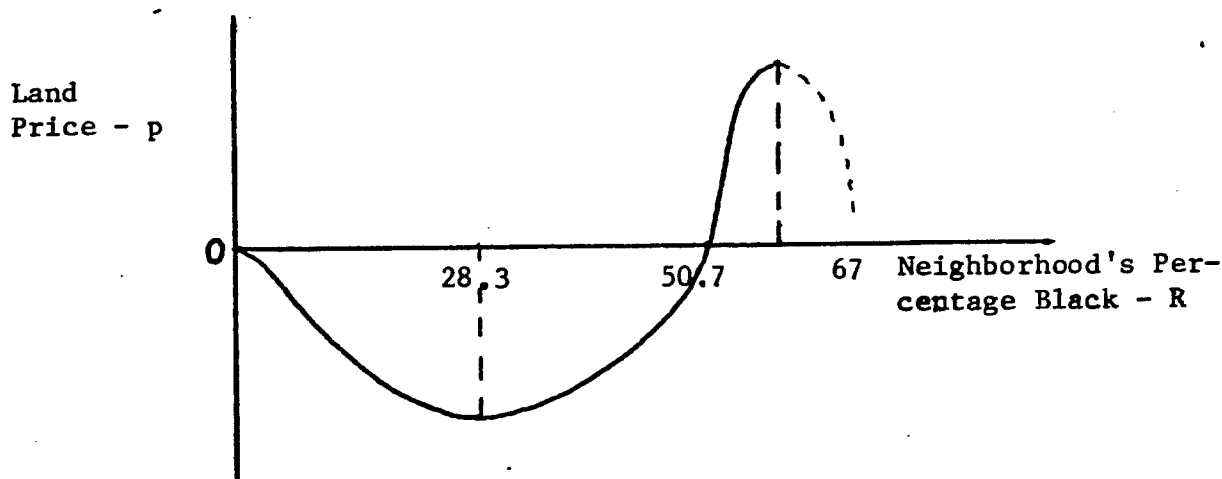


Figure V-1

hypothesized: as the percentage of a neighborhood's population which is black increases, land price first falls, and then increases. Our results suggest that land prices are actually highest, ceteris paribus, in a neighborhood whose black residents constitute 67 percent of the total population.⁵ Further, our results indicate that, ceteris paribus, land prices are lowest where blacks constitute 28.3 percent of the neighborhood's population.

After considering the regression results for specifications I, II, III, and IV-A, we decided to eliminate those variables from specification IV-A which did not have coefficients at least as large as their standard errors. The procedure was to use a stepwise regression program which brought into the regression only those variables whose coefficients are at least as large as their standard errors. The resulting regression equation was designated specification IV-B. Table V-4 contains for specification IV-B a list of the entering variables, their coefficients, and standard errors; and the degrees of freedom, the F statistic, the significance level, and the R².⁶ Although the variables in specification IV-B have coefficients with values not substantially different from the values of the coefficients of the same variables in specification IV-A,

⁵In our sample no neighborhood had a black population which constituted more than 78 percent of the total. Thus, no special significance should be attached to 67 percent.

⁶The variables DIS312MA and DIS912MA would have had coefficients greater than their standard errors, had they entered the regression equation. However, they did not enter the equation since they would have added only a negligible amount to the R². These variables did not have coefficients which were as large as their standard errors in specification IV-A.

Two variables, AREA and DIS512MA, entering specification IV-B, did not have coefficients as large as their standard errors in specification IV-A, and one variable not entering specification IV-B, DIS612M did have a coefficient as large as its standard error in specification IV-A.

TABLE V-4

INCLUDED TERMS, COEFFICIENTS, AND STANDARD
ERRORS FOR SPECIFICATION IV-B

Variable	B-Coefficient	Standard Error of B
ZONING	0.36961	0.06994
PARBLAK2	-46.18974	11.41303
PARBLAK3	151.59964	35.29431
PARBLAK4	-118.73140	27.55677
DIS112M	-0.63467	0.26130
DIS112MA	0.00004	0.00001
DIS212M	1.91988	0.51484
DIS212MA	-0.00014	0.00003
DIS412M	-3.10791	0.65210
DIS412MA	0.00009	0.00003
DIS412M2	0.21072	0.05688
DIS512M	0.70388	0.32444
DIS512MA	-0.00002	0.00001
DIS512M2	-0.08325	0.02977
DIS612M2	0.04888	0.01119
DIS812M	3.12042	0.56486
DIS812MA	0.00005	0.00002
DIS812M2	-0.36115	0.06731
DIS912M	-1.51727	0.39530
DIS912M2	0.10216	0.04051
AREA	-0.00013	0.00003
DIS112M2	0.04289	0.01879
DIS312M	-0.85203	0.43766
DIS312M2	0.06529	0.03938
(CONSTANT)	-0.14520	0.41251
Degrees of Freedom:		
Regression	24	
Residual	44	
F-Statistic	6.73286	
Significance Level	0.01000	
R^2	0.78598	

two important points should be noted when comparing specification IV-A and IV-B. First, the coefficient of every variable in Table V-4 is larger relative to its standard error in specification IV-B than it is in specification IV-A. Second, the R^2 decreases from 0.81075 in specification IV-A to 0.78598 in specification IV-B, a decrease of only 0.02477. Both of these facts indicate substantial multicollinearity among the variables in specification IV-A. By diminishing the number of explanatory variables from 34 in specification IV-A to 24 variables in specification IV-B, the explanatory power of the equation is not substantially reduced but predictions of the values for land price per square foot is obtained more economically.

As an indication of the size of the change in predicted land price per square foot caused by the variation of the coefficients between specification IV-A and specification IV-B, we calculated the difference between land price at the center of the University of Michigan campus and land price at various distances from the University of Michigan campus using the coefficients of DIS412M and DIS412M2 in specification IV-B. The results are displayed in Table V-5.⁷ We found that the coefficients of the DIS412M and DIS412M2 from specification IV-B yield smaller values for the change in land value than the coefficients from specification IV-A yield.

As a measure of the accuracy of forecasts made when applying the regression results and to compare these forecasts with the assessments made by the Ann Arbor assessor's office, we calculated the coefficient of dispersion from the mean of the quantity, estimated land value divided by actual saleprice, and the coefficient of dispersion from the mean of the quantity, appraised value divided by actual saleprice.

⁷ These results should be compared with the results in Table V-3.

TABLE V-5

RELATIONSHIP BETWEEN LAND PRICE AND DISTANCE FROM
UNIVERSITY OF MICHIGAN CAMPUS

Distance from U-M Campus (miles)	Change in Land Price From U-M Campus (\$)
0.5	-1.50
1.0	-2.89
1.5	-4.19
2.0	-5.37
2.5	-6.45
3.0	-7.43
3.5	-8.29

The general formula for these coefficients of dispersion is given by the following equation:

$$CD = \frac{\sum_{i=1}^n |R_i - \bar{R}|}{n}, \text{ where}$$

CD is the coefficient of dispersion, R_i is the ratio estimated land value divided by actual saleprice for the i-th plot of land, $\bar{R}$ is the mean of the ratios of estimated land value divided by actual saleprice, and n is the number of properties used to calculate the coefficient.⁸ The coefficient of dispersion from the mean was chosen as our measure of forecasting accuracy because a similar coefficient, the coefficient of dispersion from the median, is reported for metropolitan areas in the Census of Governments. Thus, calculation of the coefficient of dispersion allows comparison of the accuracy of our forecasts not only with the accuracy of the Ann Arbor assessments but with the accuracy of assessments in many metropolitan areas.

The coefficient of dispersion for the forecast made when applying the specification IV-A regression coefficients is .903. This coefficient of dispersion indicates that the average percentage difference between the estimated land price divided by actual saleprice and the mean of that quantity is 90.3 percent. The coefficient of dispersion for the forecast made when applying the specification IV-B regression coefficients is .623. The coefficient of dispersion from the mean of the quantity appraised value divided by actual saleprice is 0.33.⁹ Thus, the estimates of the Ann Arbor assessor are, on the average, much closer to

⁸The value of n is 69 for the coefficients of dispersion reported in this section.

⁹Appraised value for each property in the sample was taken from the Ann Arbor assessor's field cards.

the mean of the distribution than are our estimates. We must conclude that the forecasting equation should not be used to predict land values for assessment for property tax purposes.

Let us turn now to a discussion of our empirical attempt to separate the value of property into the value of land and the value of improvements. The results of this attempt are presented in Section B.

B. Separation of Land Value and Improvements Value

In Chapter III we suggested the possibility of separating property value into its components, land value and improvements value. To make the separation we first needed an estimate of the value of land for each of the 951 properties containing both land and improvements. We obtained two estimates of the value of land. The first was obtained by applying the regression coefficients displayed in Table V-2 to the value of the corresponding variable for each of the 951 sample properties having improvements. This procedure yielded a predicted value for the price per square foot of the land contained in each property. Then we obtained the predicted total land value for each of the 951 properties by multiplying predicted land price per square foot by the actual land area. The second estimate of the price per square foot of land was obtained by applying the regression coefficients displayed in Table V-4 to the value of the corresponding variable for each of the 951 sample properties having improvements. By multiplying the second estimate of the price per square foot by the actual land area we obtained a second predicted total land value for each of the 951 properties.

If separating property value into its land value component and improvement value component is possible and the regression coefficients in Table V-2 yield reasonably accurate estimates of land value, then by

subtracting land value from property value¹⁰ we obtained one set of values for the dependent variable used in the regression analysis, the results of which are reported in this section. That dependent variable was improvements value of the 951 single family residential properties containing both land and improvements. A second set of values for the dependent variable, improvements value, was obtained by subtracting from the saleprice for each of the 951 sample properties the land value estimates derived by applying the regression coefficients in Table V-4.

Initially we ran four regressions in which improvements value appeared as the dependent variable, including the following regressions: (1) the set of values for improvements derived from applying the coefficients in Table V-2 with those variables expected to explain variations in property value but not including those variables which were used in land price specification IV-A,¹¹ (2) the set of values for improvements derived from applying the coefficients in Table V-2 with those variables expected to explain variations in property value including those variables which were used in land price specification IV-A, (3) the set of values for improvements derived from applying the coefficients in Table V-4 with those variables expected to explain variations in property value but not including those variables which were used in land price specification IV-B, and (4) the set of values for improvements derived from applying the coefficients in Table V-4 with those variables expected to explain variations in property value including those variables which were used in land price specification IV-B.

¹⁰The property value is the saleprice obtained from the Ann Arbor assessor's field cards for each of the 951 sample properties containing both land and improvements.

¹¹See Section C of this chapter for a discussion of how the group of independent variables for this regression were chosen from the entire 120 variables discussed in Chapter IV.

The results of the first two regressions, improvements value equations I-A and I-B, are reported in Table V-6. The results of the last two regressions, improvements value equations II-A and II-B, are reported in Table V-7.

Generally, the results of the two sets of regressions reported in Tables V-6 and V-7 were similar in nature, and most of the statements which apply to the results shown in Table V-6 also apply to the results reported in Table V-7. Hence, in our discussion let us consider only the results reported in Table V-6.

First, in nearly all cases, the coefficients of those variables which entered both regressions shown in Table V-6 do not differ significantly at the .05 level from one regression to another.¹² This may be an indication that the land price equation variables, excluded from equation I-A, are independent of the remaining variables in their effect on the value of improvements. However, the upward bias in the size of the standard errors resulting from multicollinearity among the variables which entered both regressions prevents us from concluding that the coefficients are not significantly different from one regression equation to the other.

Second, the land price variables from specification IV-A added a substantial amount to the explained proportion of variation in improvements value. The $\underline{R}^2$ rises from 0.46203 to 0.90353 when the land price variables were entered in the regression equation. This large increase in the $\underline{R}^2$ suggests that there is not one set of variables explaining land price and a distinct set explaining improvements value but rather

¹²This is indicated by the fact that in most cases the interval of two standard errors about the coefficient of a variable in one regression overlaps the interval of two standard errors about the coefficient of the same variable in the other regression.

TABLE V-6

VARIABLES LISTS, COEFFICIENTS, AND STANDARD ERRORS
FOR IMPROVEMENTS VALUE EQUATIONS I-A AND I-B

Variable	Equation I-A		Equation I-B	
	B-Coefficient	Standard Error of B	B-Coefficient	Standard Error of B
INSULAT1	-1194.30114	716.02561	225.09153	332.06027
EXT3	729.25631	1749.84176	-721.83981	776.18536
EXT4	455.05797	2391.48095	-2495.97297	1070.58462
CENTAIR	245.85371	610.72123	824.14152	271.57700
INTE3F	4816.44654	2961.86685	-662.78490	1327.61881
BATILFLO	1042.05205	1045.34032	1272.96933	466.54747
TWOPCLAV	29.70724	1291.03209	2174.58554	576.89655
LAVTILFL	2982.97799	1660.98327	595.25559	735.37325
LAVSTASH	3502.49709	1682.41699	1103.29703	743.14495
EXWASBWL	-1983.77642	1499.66017	1334.67191	692.51096
SINGFIRE	3200.61209	1199.83391	1917.82537	543.75949
SQFT2SNB	9.86049	3.98612	5.19262	1.80335
SQFTCCP	3.60476	8.44904	4.86357	3.75215
SQFT2S04	-205.75469	169.11806	-89.93714	75.67524
ATTACHGA	14.26576	4.00577	2.45361	1.82179
DETACHGA	2.69874	3.74416	1.04409	1.67109
SQFTPATI	3.66340	6.27645	6.56168	2.80155
SQFTWPP	11.98656	16.30162	17.32944	7.27230
BUILTIN	76.29754	106.87002	171.71043	47.38741
FLOORS	461.06434	306.10190	216.13003	135.76267
EXT8	21684.30812	12450.01856	7114.53135	5732.25274
EXT13	-3930.43112	13150.68615	-4596.05273	5729.45641
INTE1S	-1989.39546	2361.49191	-546.42236	1076.87826
INTE3S	-3596.85070	5737.30481	-3265.75022	2530.26987
INTE5S	-4550.12107	4140.36798	-2992.36493	1821.83761
SQFT2SB	12.02958	3.12587	7.26670	1.44543
SQFT1SB	13.37007	2.62438	6.46592	1.20577
CARPORT	8.34624	8.33293	4.30550	3.68009
SQFTWGEP	11.36468	19.37438	2.27096	8.50139
TRIMS	-2107.61226	3180.27630	-12.46410	1393.33222
EXT7	-1659.66587	8551.49724	-9855.64332	3795.91534
FLOORF	135.62786	259.89548	54.04580	116.40803
BATHS	1048.91521	2372.12172	2305.74258	1035.38520
ROOFING	3240.72351	1243.15316	1185.36707	562.87430
FOUNDAT	-3.80116	1131.36830	952.56084	509.61495
MONHSAL4	21.05499	6.26801	4.07070	2.74611
MONTHSAL	-12857.46140	5120.70813	-1913.35268	2239.13273
MONHSAL2	4460.33009	1483.37621	795.79506	649.43164
MONHSAL3	-539.19602	165.58467	-101.70223	72.53660
SQFTCPP	12.01398	10.00943	17.60418	4.41687
INTE2B	351.00340	3717.68164	732.11718	1642.52288
TRIMB	2989.60558	2533.05987	1405.40631	1120.02872
FLOORB	-411.53020	589.17655	-84.06292	259.72104

TABLE V-6 (con't)

Variable	Equation I-A		Equation I-B	
	B-Coefficient	Standard Error of B	B-Coefficient	Standard Error of B
EXT11	-532.15405	7883.92099	2677.51819	3473.20570
SQFTCGEP	-39.83731	17.33617	-11.86541	7.65078
SQFT4SB	14.87871	4.96272	12.87417	2.29292
DWELAGE	-172.83260	51.41726	-89.91642	25.73798
SQFT1SNB	13.07404	2.40814	4.83887	1.12523
INTE4F	-25038.43183	19247.92664	-22062.41623	9135.97794
EXT6	11400.90152	3828.81767	4764.08288	1698.18552
INTE1B	1266.08380	6572.98412	3483.78858	2871.76339
INTE1F	-634.48019	1803.28108	-795.15577	824.43515
BATILWAN	2225.32990	1448.90691	585.37344	658.61268
LAVTILWA	-2104.90316	2584.29539	-230.44665	1174.67459
SQFTCSEP	4.11465	15.58721	-4.74732	6.85722
DBLFIRE	8583.80626	1752.27197	2183.55773	804.12471
INTE5B	-956.27875	2378.53461	-1018.58247	1057.17249
ZONING			-2740.96038	265.36871
DIS112MA			-0.29487	0.04056
DIS212MA			2.35675	0.12190
DIS312MA			0.77760	0.12579
DIS412MA			-2.56086	0.14464
DIS512MA			0.70895	0.08945
DIS612MA			-0.29926	0.08300
DIS712MA			-0.06090	0.05048
DIS812MA			-2.12304	0.15274
DIS912MA			1.92767	0.12402
DIS112M			5960.06615	869.95292
DIS212M			-22119.28540	1532.56020
DIS312M			3686.21087	1801.14634
DIS412M			35254.97791	2321.03996
DIS512M			-10314.58137	1199.28961
DIS612M			1299.12975	1228.41155
DIS712M			790.94127	1023.82478
DIS812M			-18263.31209	2000.97626
DIS912M			-2254.59316	2079.46919
DIS112M2			-559.54690	81.00887
DIS212M2			-299.58256	135.56062
DIS312M2			-807.93878	172.13119
DIS412M2			-1215.64773	242.49447
DIS512M2			917.44156	113.13718
DIS612M2			-497.80192	86.60325
DIS712M2			19.96746	64.99427
DIS812M2			3796.35670	189.87661
DIS912M2			-1140.45866	199.88492
PARBLAK			-1275.99119	15146.14519
PARBLAK2			362910.24519	104181.03029
PARBLAK3			-1162743.39100	226589.70124
PARBLAK4			874015.06677	147849.35124
(CONSTANT)	7715.08998	15678.87996	28519.95328	6761.71731

TABLE V-6 (con't)

Degrees of Freedom:		
Regression	57	89
Residual	893	861
<u>F</u> -Statistic	13.45506	90.60670
Significance Level	0.01000	0.01000
<u>R</u> ²	0.46203	0.90353

TABLE V-7

VARIABLES LISTS, COEFFICIENTS, AND STANDARD ERRORS
FOR IMPROVEMENTS VALUE EQUATIONS II-A AND II-B

Variable	Equation II-A		Equation II-B	
	B-Coefficient	Standard Error of B	B-Coefficient	Standard Error of B
INSULAT1	-779.81459	495.63502	90.85736	340.65414
EXT3	307.03374	1211.24559	77.33532	801.17063
EXT4	-270.77881	1655.39012	-2189.48159	1110.36952
CENTAIR	597.11213	422.74303	910.48301	279.94956
INTE3F	-1589.92114	2050.21291	-2105.70940	1358.31628
BATIFLO	645.45905	723.58363	1223.47454	479.78111
TWOFCLAV	2118.70243	893.65619	2554.24577	591.60996
LAVTILFL	1960.44679	1149.73748	-75.35855	756.51177
LAVSTASH	1578.14074	1164.57397	1217.80225	768.96631
EXWASBWL	-784.50794	1038.06916	1239.50161	709.16060
SINGFIRE	2114.89553	830.52855	2112.35361	556.22093
SQFT2SNB	8.60729	2.75920	4.85935	1.84480
SQFTCCP	5.02844	5.84845	5.02312	3.88552
SQFT2SOH	-72.09233	116.37181	-69.19280	77.87062
ATTACHGA	2.84188	2.77280	5.25724	1.87541
DETACHGA	0.69682	2.59172	1.35477	1.72854
SQFTPATI	7.78823	4.34458	4.68958	2.87677
SQFTWPP	10.91323	11.28403	8.63696	7.41391
BUILTIN	128.62581	73.97574	160.41724	48.93793
FLOORS	292.12387	211.88463	263.07752	140.08819
EXT8	17119.41247	8617.93931	10242.47552	5857.21198
EXT13	-1950.77193	9102.94347	-6356.43417	5924.82150
INTE1S	129.77096	1634.63161	-1082.77800	1101.64713
INTE3S	-1181.79077	3971.37919	-4603.43269	2618.17160
INTE5S	-4775.76151	2865.97485	-3687.62184	1883.94049
SQFT2SB	8.48318	2.16373	7.59860	1.47957
SQFT1SB	9.29544	1.81660	7.97255	1.24036
CARPORT	3.87504	5.76808	7.94413	3.78874
SQFTWGEP	8.17892	13.41100	3.73097	8.80674
TRIMS	-1161.38093	2201.39657	-12.43293	1443.09183
EXT7	-4974.72618	5919.37144	-6577.90148	3927.07057
FLOORF	-65.94365	179.90041	-12.68767	119.54142
BATHS	2338.04825	1641.98960	2434.92409	1072.17805
ROOFING	2303.98802	860.51426	1717.71140	574.24883
FOUNDAT	331.76794	783.13645	905.21972	524.55106
MONHSAL4	11.79307	4.33873	3.11590	2.84759
MONTHSAL	-8028.76645	3544.56917	-916.50547	2318.28678
MONHSAL2	2651.40664	1026.79736	499.35007	672.84552
MONHSAL3	-310.11387	114.61819	-72.32248	75.19074
SQFTCPP	13.46024	6.92856	14.84953	4.57161
INTE2B	-1996.07509	2573.39012	257.30804	1687.13765
TRIMB	2141.52266	1753.39146	2115.78527	1154.29013
FLOORB	68.00702	407.82974	1.54148	268.72573
EXT11	269.47171	5457.27320	1580.24733	3575.20435

TABLE V-7 (con't)

Variable	Equation II-A		Equation II-B	
	B-Coefficient	Standard Error of B	B-Coefficient	Standard Error of B
SQFTCGEP	-26.66333	12.00015	-12.70713	7.90989
SQFT4SB	10.80718	3.43521	15.20227	2.33704
DWELAGE	-215.21101	35.59118	-102.95394	26.24608
SQFT1SNB	5.61109	1.66692	6.08695	1.15328
INTE4F	-36991.74800	13323.47120	-11279.45582	8928.12677
EXT6	8512.40161	2650.31881	5416.35778	1743.02509
INTE1B	-1678.50916	4549.83886	2919.97397	2974.65966
INTE1F	-420.60250	1248.23645	-150.01956	849.49738
BATILWAN	1856.65792	1002.93761	1197.39663	675.09792
LAVTILWA	-1457.25370	1788.85683	666.83522	1195.21260
SQFTCSEP	3.42343	10.78951	-4.11061	7.10508
DBLFIRE	3150.73787	1212.92779	2372.01273	820.56667
INTE5B	-2071.73877	1646.42862	-655.13758	1091.61175
ZONING			-2889.03027	267.51102
DIS112MA			-0.59747	0.03792
DIS212MA			2.47374	0.11306
DIS412MA			-1.32115	0.12742
DIS512MA			0.60069	0.05360
DIS812MA			-1.07699	0.11875
DIS112M			9124.47919	834.65423
DIS212M			-24919.36723	1210.41997
DIS312M			10594.75689	1664.84203
DIS412M			25480.35275	2265.29878
DIS512M			-7724.08112	1091.08299
DIS812M			-19722.61198	1852.18206
DIS912M			10036.92061	1764.91179
DIS112M2			-595.66835	73.42230
DIS312M2			-907.20856	170.53564
DIS412M2			-1419.99578	230.83210
DIS512M2			654.60444	108.51190
DIS612M2			-528.88488	44.11897
DIS812M2			2902.12706	181.47040
DIS912M2			-507.87275	192.08739
PARBLAK2			258287.22263	42170.20665
PARBLAK3			-868749.29754	132400.62585
PARBLAK4			674973.31483	101354.77751
(CONSTANT)			9292.10277	7027.84816
Degrees of Freedom:				
Regression		57		80
Residual		893		870
F-Statistic		16.97610		44.59064
Significance Level		0.01000		0.01000
R^2		0.52006		0.80393

some variables affect just land value, some just improvements value, and some variables affect both land value and improvements value. Further, it is possible that some land price variables are correlated with some excluded variables upon which improvements value depends, and that these land price variables served as proxies for the improvements value variables.

In comparing the regression results for all four regressions in Tables V-6 and V-7 we found that improvements value equation I-B had the highest $\underline{R}^2$. It is also true that the values for the dependent variable for this equation were derived using the coefficients from land price specification IV-A, the land price specification which had the highest $\underline{R}^2$. On the basis of the $\underline{R}^2$, the best prediction of a property's value would be obtained by applying the regression coefficients in Table V-2 in determining land value, applying the regression coefficients for improvements value equation I-B in determining improvements value, and summing the land value and improvements value to find total property value.

After considering the regression results reported in Table V-6 and V-7, we ran two more improvements value regressions. In one of these we regressed the dependent variable improvements value, using the values used in equations I-A and I-B, on the list of variables in equation I-B. However, for this regression we used a stepwise regression program which brought into the regression only those variables whose coefficients are at least as large as their standard errors. The resulting regression equation was designated improvements value equation I-C. The results for this regression are presented in Table V-8.

The last of the improvements value regressions was run using the same stepwise regression program. However, for this regression we used the

TABLE V-8

VARIABLES LISTS, COEFFICIENTS, AND STANDARD ERRORS
FOR IMPROVEMENTS VALUE EQUATIONS I-C AND II-C

Variable	Equation I-C		Equation II-C	
	B-Coefficient	Standard Error of B	B-Coefficient	Standard Error of B
DIS212MA	2.35386	0.12188	2.46913	0.11157
BUILTIN	167.61860	46.82793	161.18635	47.17894
DIS212M	-21319.60277	1524.99195	-24961.16632	1192.32080
ZONING	-2797.06072	260.88179	-2927.16643	259.79693
TWOPCLAV	2631.73875	549.44435	2802.18084	550.51152
DIS712MA	-0.06156	0.04929		
DIS912MA	1.90378	0.12227		
DIS812MA	-2.12315	0.15216	-1.07728	0.11666
DIS812M2	3724.47554	190.01505	2872.30572	178.56803
DIS612MA	-0.28017	0.08141		
BATILFLO	1357.92257	426.48640	1179.32113	467.02031
DIS912M2	-1218.61080	199.70372	-461.24853	188.90843
DIS712M	933.22424	666.31119		
DIS412MA	-2.55077	0.14375	-1.32104	0.12457
DIS412M	34232.07076	2310.55132	25903.32064	2232.95547
DIS912M	-1406.97235	2070.84315	9704.64940	1733.10659
DWELAGE	-114.61549	22.63977	-105.31554	24.40142
TRIMS	334.49065	1384.30916	-147.58328	1402.84931
SQFTPATI	6.51302	2.74604	5.17237	2.81223
DIS812M	-17808.88906	1998.61485	-19460.63296	1824.89886
DIS212M2	-375.91836	134.49633		
DIS512M2	967.16471	111.73980	679.14788	105.66970
DIS312M	3242.04420	1787.50378	10255.70378	1626.64874
DIS512M	-10659.98498	1184.11808	-7939.42869	1065.61069
DIS612M2	-486.12819	76.78901	-540.87219	42.99100
DIS112M2	-543.76711	80.55504	-585.32394	72.03403
DIS412M2	-1132.85210	240.55190	-1461.07458	227.74964
DIS112M	5691.90552	846.15255	8933.46254	816.77404
DIS312M2	-734.15120	171.42390	-879.53988	167.34651
DIS312MA	0.77965	0.12531		
DIS512MA	0.70068	0.08792	0.60407	0.05231
DIS112MA	-0.28463	0.04004	-0.59517	0.03740
BATHS	2689.16048	1033.85723	2693.12210	1055.75541
EXWASBWL	1371.56328	673.57082	1335.52999	694.50194
EXT6	4711.72039	1656.98641	5079.11762	1683.96057
SQFTCPP	17.70191	4.35921	15.36628	4.47416
SINGFIRE	2028.31265	529.97967	2101.73267	547.05781
DBLFIRE	2268.07784	779.94358	2413.77104	794.35168
SQFTWPP	17.21511	7.27020	9.85295	7.32355
CENTAIR	951.34777	268.90945	909.21665	274.61836
SQFT4SB	12.81220	2.26996	15.46206	2.28409
INTE4F	-20551.57295	9078.31319	-11061.49962	8818.00650
EXT7	-9705.98099	3806.42721	-6515.68689	3875.37722
ROOFING	1231.72773	548.86886	1680.42924	555.53809

TABLE V-8 (con't)

Variable	Equation I-C		Equation II-C	
	B-Coefficient	Standard Error of B	B-Coefficient	Standard Error of B
EXT4	-2510.33370	1069.64135	-2101.38065	1076.60446
TRIMB	1589.58533	1046.91241	2380.58561	1036.93114
SQFT2SB	7.42397	1.42593	7.55159	1.45134
SQFT1SB	6.70338	1.18455	7.92963	1.21306
SQFT1SNB	4.99748	1.10392	6.09424	1.13324
SQFT2SNB	5.46258	1.80041	4.83782	1.81874
PARBLAK	62732.99515	8407.41974		
PARBLAK2	-155492.02669	23495.58733	257023.23201	41467.29373
PARBLAK4	121818.34133	23076.16520	672620.79250	99963.87478
LAVSTASH	1277.85845	720.44295	1110.50315	733.02152
FLOORS	189.17253	131.81741	264.47411	134.28912
SQFT2SOH	-115.82297	75.06962	-78.63627	76.88852
SQFTCGEP	-13.03267	7.54193	-12.42776	7.73174
EXT8	6474.84021	5743.93447	10203.06181	5790.95932
FOUNDAT	789.79376	497.85291	826.28013	501.11316
SQFTCCP	4.80304	3.68468	4.80989	3.79544
INTE5B	-1290.88831	811.40617		
MONTHSAL	627.21322	308.13386		
MONHSAL2	-41.37326	22.70536		
INTE5S	-2349.61831	1764.25556	-3751.93422	1833.39239
DIS612M	1215.62158	1121.86736		
INTE3S	-2524.39643	2440.73725	-4726.67268	2570.21501
EXT3	-802.56875	769.43653		
EXT13	-5986.12722	5763.40889		
ATTACHGA			4.40036	1.63659
PARBLAK3			-865590.98990	130489.50459
CARPORT			7.58230	3.60757
BATILWAN			1320.29262	640.47682
INTE3F			-2182.93351	1303.03409
INTE1B			3025.26034	2751.55771
INTE1S			-1167.14209	1044.94825
(CONSTANT)			11231.43912	6986.74328
Degrees of Freedom:				
Regression		68		62
Residual		882		888
F-Statistic		115.90369		58.09058
Significance Level		0.01000		0.01000
R^2		0.89935		0.80221

values for the dependent variable which were used in equations II-A and II-B. We regressed improvements value on the list of variables in II-B. The resulting equation was designated improvements value equation II-C. The results for this regression are also presented in Table V-8.

The usefulness of a stepwise regression program such as the one which we used is that the number of independent variables can be easily reduced. With multicollinearity, reduction of the number of variables entering the equation does not result in a substantial reduction in the R^2 as can be seen by comparing the results for equations I-B and II-B with I-C and II-C, respectively. For example, the number of explanatory variables in equation II-B is 80 and the R^2 is 0.80393, while the number of variables in equation II-C is 62 and the R^2 is 0.80221. The explanatory power of the equation is not reduced substantially, but prediction of the values for improvements value is obtained more economically.

We calculated the coefficients of dispersion from the means of two estimates of property value¹³ divided by actual saleprice and the coefficient from the mean of appraised property value divided by saleprice. The coefficient of dispersion for the first estimate of property value is .198, for the second estimate .338, and for appraised property value 0.16. The relative size of these coefficients of dispersion implies that the forecasting technique applied by the Ann Arbor assessor is superior to the forecasting technique which we have applied.

¹³The first property value estimate was obtained by applying the regression coefficients in Table V-2 to obtain land value, applying the coefficients for improvements equation I-C to obtain improvements value, and summing land value and improvements value to obtain property value. The second estimate was obtained using the same procedure. However, the coefficients in Table V-4 were applied to obtain land value, and the coefficients for improvements equation II-C were applied to obtain improvements value.

Overall, we believe that the evidence indicates that our attempt to separate property value into two components, land value and improvements value, was relatively successful. The belief is based not only on the values of the coefficients of dispersion but also on the proportion of variation in both improvements value and land value which we were able to explain, although our improvements value results were better than our land value results. Of course, as stated earlier, our results should be viewed as laying a foundation for further study of the problem of predicting the value of single family residential properties, and we believe that further study will result in predicting equations which are even better than the ones which we have developed, particularly with respect to land value.

This completes our discussion of the attempt to separate property value into land value and improvements value. The next two sections contain a discussion of the results of regressing saleprice on several variables.

C. Analysis of Property Value Factors

There are fourteen factors resulting from a principle component analysis which account for 64 percent of the total variance of the original data matrix consisting of 120 variables. Thirteen of these factors, along with the variables loading most heavily, are presented in Appendix II.¹⁴ When the variable saleprice was regressed on the separate groups of variables loading most heavily on these factors, the results were as shown in Appendix III.¹⁵ Those variables which had a coefficient at least as great as its standard error were deemed to be significant in explaining variations in saleprice, and they were used in the improvements value regressions, the results of which were reported in Section B of this

¹⁴No variables loaded most heavily on one of the factors.

¹⁵Data for each of our 1020 sample properties were included in these regressions.

chapter and in the property value regressions, the results of which are reported in Section D of this chapter. In addition, we used any variable from the land price equation whether or not the variable had a coefficient as large as its standard error. Using this procedure we reduced the number of variables under consideration to a total of 93 variables. Section D and E contain the results of regressing saleprice on these variables and our conclusion regarding the possibility of applying regression analysis in assessing property values for property tax purposes.

D. Empirical Property Value Equations

Although, as indicated in Section B of this chapter, the results of our attempts to separate property value into its components, land value and improvements value, were relatively successful, we thought that it would be useful to include in one regression all variables which affect land value, improvements value, or both land and improvements value. This section contains the results of the regression analysis.

The first regression has as its values for the dependent variable the saleprices, as reported on the assessor's field cards, of all 1020 single family properties in the sample. The set of independent variables for this regression, property value equation I, consists of the 93 variables listed in Table V-9. Table V-9 also contains the regression coefficients and standard errors; and the degrees of freedom, the F statistic, the significance level, and the R^2 . The last column of Table V-9 contains an "X" if the variable has a coefficient at least as large as its standard error.

As was true of the regression results reported in Section A and B of this chapter, one's attention is drawn to the fact that although the variables in this regression significantly explain a high proportion of the variation in the saleprice of single family residential properties,

TABLE V-9

INCLUDED TERMS, COEFFICIENTS, AND STANDARD ERRORS
FOR PROPERTY VALUE EQUATION I

Variable	B-Coefficient	Standard Error of B	"X" if $\frac{B}{SE_B} \geq 1$
DIS612MA	-0.01836	0.04826	
INTE3F	626.28812	915.06605	
BATILFLO	1930.77613	321.41292	X
TWOPCLAV	2034.41859	397.40811	X
LAVTILFL	1920.95358	507.40954	X
LAVSTASH	1042.15045	512.97590	X
SINGFIRE	1671.19246	372.44889	X
ATTACHGA	3.98155	1.25886	X
DETACHGA	2.18743	1.15515	X
SQFTPATI	4.70465	1.93062	X
SQFTWPP	14.75644	5.00450	X
BUILTIN	136.37771	32.64057	X
DIS112MA	0.03904	0.02828	X
DIS212MA	0.12797	0.07595	X
DIS312MA	0.10382	0.07466	X
DIS412MA	-0.19016	0.08806	X
DIS512MA	0.11012	0.05169	X
DIS712MA	0.02170	0.03397	
DIS812MA	-0.11408	0.09201	X
FLOORS	234.88413	93.75590	X
EXT8	-3933.34040	3938.98135	
INTE5S	-2808.27694	1259.36159	X
SQFT2SB	6.80759	0.98721	X
SQFT1SB	6.25875	0.82633	X
CARPORT	3.55800	2.55077	X
DIS712M2	-91.05031	43.08248	X
EXT7	-5633.50879	2626.85989	X
DIS612M	-2043.20731	796.18340	X
DIS612M2	188.27450	57.09051	X
FLOORF	116.15333	78.93412	X
BATHS	719.20043	707.99007	X
ROOFING	1854.07288	388.42984	X
PARBLAK	-26140.15981	9879.59097	X
PARBLAK2	101377.68526	67206.67276	X
SQFTCPP	8.99350	3.06692	X
TRIMB	1843.14771	777.02132	X
SQFT4SB	14.64804	1.56480	X
DIS212M2	-133.48639	91.00302	X
DIS312M2	-17.24545	111.07161	
DIS412M	-4099.08448	1534.29527	X
DIS412M2	628.97645	160.23982	X
DIS512M2	-42.45775	75.68747	
DIS812M	1040.93027	1311.67381	

TABLE V-9 (con't)

Variable	B-Coefficient	Standard Error of B	"X" if $\frac{B}{SE_B} \geq 1$
DIS812M2	-112.12848	127.73187	
ZONING	-481.52530	177.07323	X
DWELAGE	-110.95223	17.33224	X
RDEPHFRT	-780.99609	571.84907	X
LANDEPTH	12.46620	13.48100	
INTE1B	7252.32510	1989.37063	X
LAVTILWA	-979.58623	810.20561	X
SQFTCSEP	5.06222	4.74539	X
DBLFIRE	1994.11705	549.99585	X
INTE5B	-237.64752	733.21584	
INSULAT1	106.19539	229.11818	
EXT3	-242.28736	537.92690	
SQFT2SOH	-111.60204	52.19086	X
LANFRONT	12.55582	17.17134	
EXT13	-2614.85298	3966.81283	
INTE1S	75.59817	742.93464	
INTE3S	472.65121	1749.39654	
SQFTWGEP	2.60330	5.89888	
TRIMS	141.36854	964.86554	
UNDEVEL	1193.81681	2672.12678	
FOUNDAT	481.78221	351.52554	X
MONHSAL4	-0.10685	1.83715	
MONTHSAL	934.73017	1509.20204	
MONHSAL2	-119.29742	437.01168	
MONHSAL3	5.98860	48.68546	
INTE2B	-627.94827	1138.04507	
FLOORB	112.07161	179.47665	
SQFTCGEP	7.45429	5.30351	X
INTE4F	-5021.85813	6232.29279	
INTE1F	-251.32068	566.92041	
BATILWAN	84.42530	455.27592	
EXT4	-1114.71966	740.78049	X
CENTAIR	679.75160	187.04197	X
EXWASBWL	1134.95231	475.85167	X
SQFT2SNB	5.53136	1.23839	X
SQFTCCP	3.93488	2.60652	X
EXT11	-1323.79215	2408.06845	
SQFT1SNB	4.98116	0.77308	X
EXT6	929.50843	1172.06377	
DIS912MA	-0.06517	0.07819	
DIS112M	267.38551	555.12736	
DIS712M	1166.44822	688.48169	X
DIS112M2	-32.28330	54.22099	
PARBLAK4	68419.27503	92474.02823	
PARBLAK3	-147518.54448	143224.00535	X

TABLE V-9 (con't)

Variable	B-Coefficient	Standard Error of B	"X" if $\frac{B}{SE_B} \geq 1$
DIS212M	-334.67142	1019.82911	
DIS312M	86.83014	1173.88747	
DIS512M	-255.82818	793.28049	
DIS912M	-178.70211	1380.36043	
DIS912M2	71.49552	132.12441	
(CONSTANT)	7431.14273	4686.98391	X

Degrees of Freedom:

Regression	93
Residual	926
<u>F</u> Statistic	80.17395
Significance Level	0.01000
<u>R</u> ²	0.88953

relatively few of the variables have coefficients which are large when compared to their standard errors. The fact that few of the variables have coefficients which are large when compared to their standard errors suggests the existence of a high degree of multicollinearity. While the existence of a high degree of multicollinearity does not prevent us from using all of these variables together for the purpose of predicting the market value of a single family residential property, its existence does lead to imprecise estimates of the regression coefficients. The multicollinearity further limits the possibility of testing hypotheses concerning the signs and sizes of the coefficients of the variables.¹⁶

Although most of the regression coefficients are not large relative to their standard errors, they have, with few exceptions, the expected sign. The coefficients of the included exterior wall material variables, EXT3, EXT4, EXT7, EXT8, EXT11, EXT13, are all less than zero. The materials represented by these variables are wood shingles, composition shingles, brickcrete, stone veneer, concrete cinder block, and stucco brick. The sign of the coefficients of these variables indicates that they are inferior materials for exterior wall construction. The only other exterior wall material variable in property value equation I is EXT6, brick.¹⁷

The negative signs on the coefficients of the following interior finish variables indicate the use of inferior materials: INTELF,

¹⁶We mentioned earlier that the theory of housing values is not advanced sufficiently to allow more than the testing of provisional hypotheses and the inference of additional hypotheses from the empirical results.

¹⁷The other six exterior wall material variables do not have coefficients as large as their standard error in the property value factor regressions. See Appendix III.

plastered first floor walls; INTE2B, drywall basement walls; INTE4F, acoustical tile on first floor; INTE5S, unfinished second floor walls; and INTE5B, unfinished basement walls. The other interior finish variables in property value equation I are the following: INTE1B, plastered basement walls; INTE1S, plastered second floor walls; INTE3F, paneled first floor walls; and INTE3S, paneled second floor walls.

There are fifteen variables in property value equation I which represent the area in square feet of various parts of the dwelling, for example, ATTACHGA, square feet in the attached garage, and SQFT1SB, square feet of one story house with basement. Of the fifteen variables, only SQFT2SOH, number of square feet of second story overhang, has a coefficient with a negative sign. We expected all of these variables to have positive signs, but perhaps a second story overhang is associated with an architectural style which is not currently appealing to most home buyers.

Of the sets of variables related to distance from the centers of economic activity, only one set has coefficients for all three variables greater than two times their standard errors. That set, DIS412M, DIS412M2, DIS412MA, is the one related to the distance from the center of the University of Michigan campus. The signs of these coefficients indicate that property values decline at a declining rate as the distance from the campus increases. The coefficients of the variables measuring the distances from centers six and nine also indicate property values declining at a declining rate as the distance from these centers increases. Thus property values are highest along a ray extending from the center of the University of Michigan main campus toward the southeast edge of the city and decline in all directions from

the ray.

After considering the regression results for property value equation I, we decided to eliminate those variables which did not have coefficients at least as large as their standard errors. We used the same procedure as we used to eliminate variables from the land price and improvements value equations. We used a stepwise regression program which brought into the regression only those variables whose coefficients are at least as large as their standard errors. The resulting regression equation was designated property value equation II. Table V-10 contains a list of the entering variables, their coefficients, and standard errors; and the degrees of freedom, the F statistic, the significance level, and the $\underline{R}^2$ ¹⁸.

Comparison of the two property value equations reveals the following two facts: (1) the coefficient of nearly every variable in Table V-10 is larger relative to its standard error in equation II than it is in equation I; (2) the $\underline{R}^2$ decreases from 0.88953 in equation I to 0.88412 in equation II, a decrease on only 0.00541. Both of these facts indicate substantial multicollinearity among the variables in property value equation I. By diminishing the number of explanatory variables from 93 in equation I to 54 variables in equation II, the explanatory power of the equation is not reduced substantially but prediction of the values for property value is obtained more economically.

¹⁸Two of the variables listed in Table V-10, LAVTILWA and FOUNDAT, do not have coefficients larger than their standard errors. Each of these variables entered with coefficients greater than their standard errors but the coefficients were reduced to less than the standard errors on subsequent steps when other variables entered the equation.

TABLE V-10

INCLUDED TERMS, COEFFICIENTS, AND STANDARD
ERRORS FOR PROPERTY VALUE EQUATION II

Variable	B-Coefficient	Standard Error of B
PARBLAK3	-45313.87446	22798.41053
BATILFLO	1982.84546	292.96549
TWOPCLAV	2058.68865	375.04561
LAVTILFL	1782.19886	495.64587
LAVSTASH	1025.12639	500.87073
SINGFIRE	1853.46094	360.14025
ATTACHGA	3.64817	1.19620
DETACHGA	1.88507	1.11298
SQFTPATI	4.64624	1.88046
SQFTWPP	16.21873	4.92388
BULTIN	148.10709	31.14677
DIS112MA	0.04801	0.01778
DIS212MA	0.13427	0.04700
DIS312MA	0.11009	0.02956
DIS412MA	-0.27331	0.05725
DIS512MA	0.14690	0.02471
DIS812MA	-0.12682	0.03953
FLOORS	228.99169	92.13804
INTE5S	-2932.89388	836.69638
SQFT2SB	6.79824	0.94894
SQFT1SB	5.95365	0.76863
CARPORT	4.40834	2.47575
DIS712M2	-127.31970	36.59636
EXT7	-4992.32672	2566.49128
DIS612M	-2675.91301	423.28384
DIS612M2	215.69319	45.80494
FLOORF	121.38398	76.22477
BATHS	814.32337	676.86945
ROOFING	1712.83388	244.63381
PARBLAK	-23551.43442	6114.33364
PARBLAK2	59725.62788	24355.22604
SQFTCPP	8.98535	2.99306
TRIMB	1866.84018	695.15278
SQFT4SB	15.87707	1.48140
DIS212M2	-161.27796	53.46619
DIS512M2	-82.35525	26.24160
DIS412M2	238.64483	68.53341
ZONING	-193.66873	140.47458
DWELAGE	-102.81169	14.06275
RDEPHFRT	-552.66070	249.98172
INTE1B	8363.25466	1830.89251
LAVTILWA	-494.95856	769.25215
SQFTCSEP	6.23421	4.65684

TABLE V-10 (con't)

Variable	B-Coefficient	Standard Error of B
DBLFIRE	1944.16285	518.37629
SQFT2SOH	-108.14913	50.84260
FOUNDAT	283.08829	298.15366
SQFTCGEP	9.28720	5.11086
EXT4	-1081.87931	710.74636
CENTAIR	690.69408	183.19873
EXWASBWL	1152.42239	452.92778
SQFT2SNB	5.30415	1.20810
SQFTCCP	3.55538	2.53532
SQFT1SNB	4.66688	0.72708
DIS712M	2049.94172	472.88536
(CONSTANT)	3761.67191	4702.31458
Degrees of Freedom:		
Regression	54	
Residual	965	
<u>F</u> -Statistic	136.34465	
Significance Level	0.01000	
<u>R</u> ²	0.88412	

Again we calculated the coefficient of dispersion from the mean of the quantity, estimated property value divided by actual saleprice, and the coefficient of dispersion from the mean of the quantity, appraised value divided by actual saleprice. The coefficient of dispersion for the prediction made when applying the equation I regression coefficients is .219. This coefficient of dispersion indicates that the average percentage difference between the estimated property value divided by actual saleprice and the mean of that quantity is 21.9 percent. The coefficient of dispersion for the prediction made when applying the equation II regression coefficients is .188. The coefficient of dispersion from the mean of the quantity, appraised value divided by actual saleprice, is 0.163.¹⁹ This is a smaller difference between the coefficients of dispersion than the differences between the coefficients reported in Sections A and B of this chapter. The conclusion to be drawn on the basis of our results is that if regression analysis is to be used for appraisal, a single equation including variables expected to account for variations in improvements and/or land values should be used rather than one equation for land values and one equation for improvements values.

The last section of this chapter contains our conclusions concerning the use of regression analysis in assessing property values for property tax purposes and mention of the areas in which we believe more research would be worthwhile.

¹⁹The coefficient of dispersion from the median of the ratio appraised value to actual saleprice for a sample of properties sold in Ann Arbor during a six month period in 1971 was reported in the 1972 Census of Governments to be .207.

E. Conclusion and Areas for Further Research

The conclusion of this empirical investigation into the determinants of the value of single family residential properties is that regression analysis can be used to generate a prediction of the value of a single family residential property which is accurate enough for the purpose of residential property taxation. We have demonstrated with coefficients of dispersion that regression analysis can predict saleprices in Ann Arbor nearly as well as can the Ann Arbor assessor. With improvements in measurement of some variables and further research into determinants of property values, we are confident that regression techniques can yield estimates that are superior to those yielded by current assessment techniques.²⁰ We believe that, at a minimum, the Ann Arbor assessor should consider our regression coefficients when evaluating the weight placed on various housing characteristics in his appraisal of property.²¹

The primary advantage of the application of regression analysis is that, once the data base is established, reassessments can be made annually with a minimum of effort and at a relatively low cost. The Ann Arbor assessor's office currently employs three appraisers of residential property, two appraisers of commercial and industrial property, one appraiser of personal property, and has an operating

²⁰It should be kept in mind that we have judged the accuracy of our predictions against the accuracy of the predictions of an assessor who does well when compared to many assessors. See page 2 of this paper.

²¹We mention the Ann Arbor assessor specifically because it is doubtful that the regression coefficients which we obtained would apply in other cities.

budget of over \$125,000.²² We believe that, using regression analysis, single family residential properties in Ann Arbor could be re-appraised every year with the same expenditure which is currently made to re-appraise every third year without substantially decreasing the quality of appraisal. When property values are rising and falling unequally across the taxing district, infrequent reassessments may be the prime cause of inequitable relative property tax bills.

However, we do think that effort should be made to improve the measurement of some variables which enter our regression equation. Those variables whose measurement need improvement are the interior and exterior construction materials variables. As indicated in Chapter 4, the values which we have used are only approximations to the values which an assessor should impute to these variables. We believe that an assessor can arrive at better approximations of the value of these variables much less expensively than we could.

Finally, the areas into which further research would seem worthwhile include the relationship between zoning and property values; and the definition of a center of economic activity, the relationship between accessibility to centers of economic activity, and the interaction between this relationship and other determinants of property values. Our results clearly indicate that much about these relationships is not understood.

²²He had an operating budget of \$133,610 for calendar year 1973.

APPENDIX I

VARIABLE NAMES USED IN REGRESSION AND DESCRIPTION OF VARIABLE

Variable Name	Description
ATTACHGA	Number of square feet of attached garage.
BATHS	Number of bathrooms.
BATILFLO	Whether or not bathroom has tiled floor.
BATILTUB	Whether or not bathroom has tiled tub area.
BATILWAN	Whether or not bathroom has tiled wains.
BUILTIN	Built-in appliances: 1, none; 2, kitchen fan-hood; 3, incinerator; 4, disposal; 5, intercom system; 6, dishwasher or incinerator and fan-hood; 7, oven and range; 8, disposal and fan-hood or disposal and attic fan; 9, disposal and incinerator; 10, dishwasher and incinerator; 11, oven and range and fan-hood or disposal and dishwasher; 12, disposal and oven and range; 13, disposal and dishwasher and fan-hood; 14, disposal and dishwasher and incinerator; 15, disposal and oven and range and fan-hood or disposal and dishwasher and incinerator and fan-hood or disposal and dishwasher and intercom system and fan-hood or disposal and oven and range and incinerator; 16, disposal and oven and range with broiler and fan-hood; 17, disposal and oven and range and incinerator and fan-hood; 18, dishwasher and oven and range and fan-hood; 19, disposal and dishwasher and oven and range; 20, disposal and dishwasher and oven and range and tile wains in kitchen; 21, disposal and dishwasher and oven and range and fan-hood or disposal and dishwasher and oven and range with broiler; 22, dishwasher and oven and range and intercom system and fan-hood or dishwasher and oven and range and incinerator and fan-hood; 23, disposal and dishwasher and oven and range and incinerator or disposal and dishwasher and oven and range with broiler and fan-hood or disposal and dishwasher and intercom system and attic fan and incinerator and fan-hood; 24, disposal and dishwasher and oven and range and incinerator and fan-hood or disposal and dishwasher and oven and range and tile wains in kitchen and fan-hood; 25, disposal and dishwasher and oven and range and attic fan and fan-hood; 26, disposal and dishwasher and oven and range and intercom system and fan-hood; 27, disposal and dishwasher and oven and range with broiler and intercom system and fan-hood; 28, disposal and dishwasher and oven and range and incinerator and fan-hood;

Variable Name	Description
	29, disposal and dishwasher and oven and range and refrigerator; 30, dishwasher and oven and range and refrigerator and intersom system; 31, disposal and dishwasher and oven and range and refrigerator and fan-hood or disposal and dishwasher and oven and range and intercom system and attic fan and fan-hood; 32, disposal and dishwasher and oven and range and refrigerator and intercom system and incinerator and fan-hood.
CARPORT	Number of square feet of carport.
CENTAIR	Size of central air conditioning unit in tons.
CORNLOT	Whether or not the property is at the corner of a block.
DBLFIRE	Number of double fireplaces.
DETACHGA	Number of square feet of detached garage.
DIS112M, DIS212M, ..., DIS912M	Distances from centers of economic activity 1, 2, ..., 9, respectively, measured to the nearest one-half mile.
DIS112M2, DIS212M2, ..., DIS912M2	Square of distances from centers of economic activity 1, 2, ..., 9, respectively.
DIS112MA, DIS212MA, ..., DIS912MA	Area of lot multiplied by distances from centers of economic activity 1, 2, ..., 9, respectively.
DWELAGE	Age of dwelling in years.
EXT1	Proportion of exterior wall material which is wood siding.
EXT2	Proportion of exterior wall material which is aluminum siding.
EXT3	Proportion of exterior wall material which is wood shingles.
EXT4	Proportion of exterior wall material which is composition shingles.
EXT5	Proportion of exterior wall material which is brick veneer.
EXT6	Proportion of exterior wall material which is brick.

Variable Name	Description
EXT7	Proportion of exterior wall material which is brick-crete.
EXT8	Proportion of exterior wall material which is stone veneer.
EXT9	Proportion of exterior wall material which is stone.
EXT10	Proportion of exterior wall material which is artificial stone.
EXT11	Proportion of exterior wall material which is concrete cinder block.
EXT12	Proportion of exterior wall material which is stucco block.
EXT13	Proportion of exterior wall material which is stucco brick.
EXTRSTOL	Number of extra stools.
EXWASBWL	Number of extra washbowls.
FLOORB, FLOORF, FLOORS, FLOORT	Types of flooring material in basement, on first floor, on second floor, on third floor, respectively: 1, none; 2, cement; 3, pine; 4, cement and tile or tile or pine and tile or cement and tile and slate; 5, cement and hardwood and tile or hardwood and tile; 6, hardwood and tile and slate; 7, pine and hardwood or cement and hardwood or hardwood; 8, cement and carpeting and tile or plywood and carpeting and tile or hardwood and slate; 9, hardwood and carpeting and tile or cement and hardwood and carpeting and tile; 10, cement and hardwood and carpeting or pine and hardwood and carpeting; 11, pine and carpeting or cement and carpeting; 12, hardwood and carpeting and tile and slate or hardwood and carpeting or carpeting and tile and slate; 13, carpeting or hardwood and carpeting and slate; 14, carpeting and slate.
FOUNDAT	Type of foundation: 1, piers; 2, brick; 3, concrete block; 4, poured concrete.
INTE1B, INTE2B, ..., INTE5B	Proportion of the basement which is finished with plaster, drywall, paneling, acoustical tile, or unfinished, respectively.
INTE1F, INTE2F, ..., INTE5F	Proportion of the first floor which is finished with plaster, drywall, paneling, acoustical tile, or unfinished, respectively.

Variable Name	Description
INTE1S, INTE2S, ..., INTE5S	Proportion of the second floor which is finished with plaster, drywall, paneling, acoustical tile, or unfinished, respectively.
INTE1T, INTE2T, ..., INTE5T	Proportion of the third floor which is finished with plaster, drywall, paneling, acoustical tile, or unfinished, respectively.
INSULAT1	No insulation, insulation in walls or roof, or insulation in both walls and roof.
LANDEPTH	Depth of lot in feet.
LANFRONT	Frontage of lot in feet.
LAVSTASH	Number of stall showers.
LAVTILFL	Whether or not lavatory has tiled floor.
LAVTILWA	Whether or not lavatory has tiled wains.
MONHSAL2	Square of month in 1970 in which property was sold.
MONHSAL3	Cube of month in 1970 in which property was sold.
MONHSAL4	Fourth power of month in 1970 in which property was sold.
MONTHSAL	Month in 1970 in which property was sold.
PARBLAK	Proportion of a neighborhood's residents who are black.
PARBLAK2	Square of the proportion of a neighborhood's residents who are black.
PARBLAK3	Cube of the proportion of a neighborhood's residents who are black.
PARBLAK4	Fourth power of the proportion of a neighborhood's residents who are black.
RDEPHFRT	Ratio of land depth to land frontage.
ROOFING	Type of roofing material: 1, none; 2, metal; 3, roll; 4, tile; 5, asbestos; 6, asphalt; 7, tar and gravel; 8, wood; 9, slate.
SINGFIRE	Number of single fireplaces.
SQFT3SB	Number of square feet of second story overhang.
SQFTCGEP	Number of square feet of concrete and glass enclosed porch.

Variable Name	Description
SQFTCPP	Number of square feet of concrete platform porch.
SQFTCSEP	Number of square feet of concrete and screen enclosed porch.
SQFTEB	Number of square feet of enclosed breeze-way.
SQFTOB	Number of square feet of open breeze-way.
SQFTPATI	Number of square feet of patio.
SQFTWCP	Number of square feet of wood covered porch.
SQFTWGEP	Number of square feet of wood and glass enclosed porch.
SQFTWPP	Number of square feet of wood platform porch.
SQFT1SB	Number of square feet of one story house with basement.
SQFT2SB	Number of square feet of two story house with basement.
SQFT3SB	Number of square feet of one and one-half story house with basement.
SQFT4SB	Number of square feet of two and one-half story house with basement.
SQFT1SNB	Number of square feet of one story house without basement.
SQFT2SNB	Number of square feet of two story house without basement.
SQFT1SOH	Number of square feet of first story overhang.
SQFT2SOH	Number of square feet of second story overhang.
TRIMB, TRIMF, TRIMS, TRIMT	Type of trim in basement, on first floor, on second floor, on third floor, respectively: 1, none; 2, pine; 3, hardwood.
TWOPCLAV	Number of two piece lavatories.
UNDEVEL	Whether or not the property is a vacant lot.
ZONING	Zoning classification of property.

APPENDIX II

FACTORS AND MOST HEAVILY LOADED
PROPERTY CHARACTERISTICS

Factor	Variable Name	Loading
1	EXT2	-0.313
	EXT3	0.274
	EXT4	0.159
	EXT5	-0.411
	CENTAIR	-0.376
	INTE3F	-0.510
	INTE2S	-0.679
	BAT1LFLO	-0.506
	BAT1LTUB	-0.599
	TWOPCLAV	-0.782
	LAVTILFL	-0.690
	LAVSTASH	-0.641
	EXWASBWL	-0.612
	SINGFIRE	-0.464
	SQFT2SNB	-0.224
	SQFT1SOH	-0.548
	SQFTCCP	-0.260
	SQFT2SOH	-0.402
	ATTACHGA	-0.723
	DETACHGA	0.379
	SQFT3SB	0.049
	SQFTPATI	-0.207
	SQFTWPP	-0.169
	BUILTIN	-0.723
	INSULAT1	-0.468
2	LANFRONT	0.821
	EXT9	0.109
	DIS112MA	0.898
	DIS212MA	0.947
	DIS312MA	0.962
	DIS412MA	0.968
	DIS512MA	0.869
	DIS612MA	0.880
	DIS712MA	0.904
	DIS812MA	0.966
	DIS912MA	0.970
3	EXT8	-0.091
	EXT12	-0.169
	EXT13	-0.073
	INTE2F	0.425

APPENDIX II (con't)

Factor	Variable Name	Loading
3	INTE3S	-0.081
	INTE5S	0.823
	SQFT2SB	-0.712
	SQFT1SB	0.658
	INTE1S	-0.628
	CARPORT	0.179
	SQFTWGEP	-0.167
	TRIMS	-0.803
	FLOORS	-0.681
4	EXT7	-0.075
	SQFTEB	-0.042
	DIS112M	-0.740
	DIS612M	0.939
	DIS712M	0.965
	DIS112M2	-0.690
	DIS612M2	0.942
	DIS712M2	0.977
5	INTE5F	0.965
	BATHS	-0.720
	UNDEVEL	0.968
	ROOFING	-0.925
	FOUNDAT	-0.710
	TRIMF	-0.664
	FLOORF	-0.534
6	MONTHSAL	0.924
	MONTHSAL 2	0.993
	MONTHSAL 3	0.995
	MONTHSAL 4	0.956
7	PARBLAK	-0.834
	PARBLAK 2	-0.992
	PARBLAK 3	-0.987
	PARBLAK 4	-0.940
8	INTE2B	0.375
	INTE3B	0.607
	INTE4B	0.214
	TRIMB	0.316
	FLOORB	0.607
	EXT11	-0.058
	SQFTCGEP	0.145

APPENDIX II (con't)

Factor	Variable Name	Loading
8	SQFTCPP	0.151
9	INTE1T	0.678
	INTE2T	0.494
	INTE3T	0.210
	INTE5T	-0.985
	TRIMT	0.888
	FLOORT	0.672
	SQFT4SB	0.720
10	DIS212M	-0.816
	DIS212M2	-0.822
	DIS312M	-0.847
	DIS312M2	-0.847
	DIS412M	-0.930
	DIS412M2	-0.943
	DIS512M	-0.665
	DIS512M2	-0.644
	DIS812M	-0.911
	DIS812M2	-0.911
	DIS912M	-0.899
	DIS912M2	-0.915
	EXT1	0.262
	ZONING	0.460
	EXTRSTOL	0.174
	SQFT1SNB	-0.249
	SQFTWCP	0.321
	DWELAGE	0.506
11	INTE4F	-0.941
	INTE4S	-0.600
12	RDEPHFRT	0.688
	LANDEPTH	0.682
	CORNLOT	-0.204
13	EXT6	-0.190
	EXT10	-0.094
	INTE1B	-0.280
	INTE1F	-0.451
	BATILWAN	-0.574
	LAVTILWA	-0.423
	DBLFIRE	-0.380
	INTE5B	0.113
	SQFTCSEP	-0.099
	SQFTTOB	-0.056

APPENDIX III

INCLUDED TERMS, COEFFICIENTS, AND STANDARD ERRORS
FOR REGRESSIONS OF SALEPRICE ON EACH GROUP OF
VARIABLES LOADING MOST HEAVILY ON A FACTOR

Factor	Variable	B-Coefficient	Standard Error of B	Used in Other Regressions
1	INSULAT1	953.89706	269.60064	Yes
	EXT2	-543.86147	919.22077	No
	EXT3	1928.96102	763.24217	Yes
	EXT4	-1324.36195	1041.44650	Yes
	EXT5	357.77704	869.19513	No
	CENTAIR	352.66819	265.00905	Yes
	INTE3F	1945.73782	1253.96759	Yes
	INTE2S	105.19940	765.76919	No
	BATILFLO	5168.96445	381.86628	Yes
	BATILTUB	20.61717	621.58474	No
	TWOPCLAV	2851.48657	533.20631	Yes
	LAVTILFL	2384.97537	676.59296	Yes
	LAVSTASH	3245.95047	709.98480	Yes
	EXWASBWL	826.94574	642.42917	Yes
	SINGFIRE	3483.37050	442.58874	Yes
	SQFT2SNB	-1.47580	1.42776	Yes
	SQFT1SOH	6.71770	9.54775	No
	SQFTCCP	4.94344	3.58352	Yes
	SQFT2SOH	-160.76980	73.36550	Yes
	ATTACHGA	9.04358	1.64461	Yes
	DETACHGA	7.64440	1.48260	Yes
	SQFT3SB	2.78827	4.96607	No
	SQFTPATI	4.32185	2.66932	Yes
	SQFTWPP	27.79471	7.03684	Yes
	BUILTIN	216.53174	44.91242	Yes
2	DIS912MA	0.07621	0.08815	Yes ^a
	LANFRONT	151.69532	23.76443	Yes
	EXT9	-6080.71798	9498.13080	No
	DIS112MA	-0.07011	0.02895	Yes
	DIS212MA	0.51760	0.08550	Yes
	DIS312MA	-0.12557	0.07173	Yes
	DIS412MA	-0.24552	0.10082	Yes
	DIS512MA	0.09884	0.05055	Yes
	DIS612MA	-0.24802	0.05091	Yes
	DIS712MA	0.16688	0.03417	Yes

APPENDIX III (con't)

Factor	Variable	B-Coefficient	Standard Error of B	Used in Other Regressions
2	DIS812MA	-0.20269	0.09410	Yes
3	FLOORS	955.92564	187.62823	Yes
	EXT8	15977.23767	7973.88659	Yes
	EXT12	1669.76376	3180.67538	No
	EXT13	-15051.03528	8637.67740	Yes
	INTE2F	793.22738	876.46361	No
	INTE1S	-9992.59533	1130.35038	Yes
	INTE3S	-10003.33503	3631.65789	Yes
	INTE5S	-9278.25612	2468.81786	Yes
	SQFT2SB	6.25375	1.38792	Yes
	SQFT1SB	14.47089	0.83492	Yes
	CARPORT	9.33306	5.02967	Yes
	SQFTWGEP	17.26915	11.92817	Yes
	TRIMS	4772.18318	1972.85499	Yes
4	DIS712M2	-312.60264	85.30160	Yes
	EXT7	-9821.32906	6735.88690	Yes
	SQFTEB	-27.58732	67.23179	No
	DIS112M	-127.44579	909.12780	Yes ^a
	DIS612M	-8202.35162	1074.19830	Yes
	DIS712M	6975.34852	1062.64668	Yes
	DIS112M2	181.54773	76.17004	Yes
	DIS612M2	575.23639	109.12619	Yes
5	FLOORF	847.03492	172.42474	Yes
	INTE5F	2954.84248	10606.71762	No
	BATHS	5395.48779	1616.27104	Yes
	UNDEVEL	15549.58326	10635.59868	Yes
	ROOFING	4196.94332	908.47091	Yes
	FOUNDAT	3650.20073	656.75577	Yes
	TRIMF	598.91581	1351.06208	No
6	MONHSAL 4	12.20034	5.01979	Yes
	MONTHSAL	-5522.06919	4125.44430	Yes
	MONHSAL 2	2274.77187	1193.78916	Yes
	MONHSAL 3	-299.68880	132.98208	Yes
7	PARBLAK 4	459422.50800	235430.98631	Yes
	PARBLAK	-111297.36477	23367.59918	Yes
	PARBLAK 2	432092.60722	166370.25591	Yes
	PARBLAK 3	-778061.30944	360312.19965	Yes

APPENDIX III (con't)

Factor	Variable	B-Coefficient	Standard Error of B	Used in Other Regressions
8	SQFTCPP	25.03682	7.73439	Yes
	INTE2B	-4131.67759	2789.69106	Yes
	INTE3B	-1773.80365	1899.69106	No
	INTE4B	4268.38770	6261.41740	No
	TRIMB	5160.91939	1915.33187	Yes
	FLOORB	2803.24590	421.44732	Yes
	EXT11	-8763.23293	6264.10024	Yes
	SQFTCGEP	19.79191	13.63655	Yes
9	SQFT4SB	18.05819	4.32564	Yes
	INTE1T	80277.71265	*****	No
	INTE2T	80409.39078	*****	No
	INTE3T	77453.95474	*****	No
	INTE5T	84183.77902	*****	No
	TRIMT	-827.15782	8566.15782	No
	FLOORT	733.05414	824.63385	No
10	DWELAGE	-70.88233	30.03414	Yes
	DIS212M	5252.29293	1730.61720	Yes
	DIS212M2	-171.00072	177.29071	Yes ^a
	DIS312M	10225.17376	2112.10624	Yes
	DIS312M2	-1020.71606	227.80037	Yes
	DIS412M	-353.85095	3311.13229	Yes ^a
	DIS412M2	-450.60936	367.85705	Yes
	DIS512M	1481.42034	1408.63154	Yes
	DIS512M2	-76.10156	140.80602	Yes ^a
	DIS812M	-1850.12646	2770.75224	Yes ^a
	DIS812M2	185.35386	290.38880	Yes ^a
	DIS912M	-8918.35711	2746.91995	Yes
	DIS912M2	948.66048	296.32206	Yes
	EXT1	45.93759	1082.30094	No
	ZONING	-2191.97745	356.11426	Yes
	EXTRSTOL	894.42963	1541.09861	No
	SQFT1SNB	1.66347	1.27987	Yes
	SQFTWCP	7.96471	8.24299	No
11	INTE4S	-29000.00000	32880.49099	No
	INTE4F	41272.92429	19002.21739	Yes
12	CORNLOT	796.46323	1089.24899	No
	RDEPHFRT	-7742.17276	656.24091	Yes
	LANDEPTH	125.45552	11.41152	Yes

APPENDIX III (con't)

Factor	Variable	B-Coefficient	Standard Error of B	Used in Other Regressions
<hr/>				
13	SQFTOB	2.79959	22.19003	No
	EXT6	3718.42579	2869.69533	Yes
	EXT10	-10982.44748	11055.86662	No
	INTE1B	20322.78307	4442.88274	Yes
	INTE1F	-5180.19906	826.60864	Yes
	BATILWAN	4266.95067	913.52635	Yes
	LAVTILWA	7985.96147	1768.55956	Yes
	SQFTCSEP	38.54249	11.52642	Yes
	DBLFIRE	6132.25762	1188.48251	Yes
	INTE5B	-4543.97503	1267.48908	Yes

a: denotes $B/S.E. < 1$.

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