ENGINEERING RESEARCH INSTITUTE
UNIVERSITY OF MICHIGAN
ANN ARBOR

Status Report

THEORY OF CERTAIN ENERGY SURFACES AND BRILLOUIN ZONES

June 1, to October 31, 1954

ERNST KATZ

Project 2158

DEPARTMENT OF THE AIR FORCE, AIR RESEARCH AND DEVELOPMENT COMMAND
CONTRACT NO. AF 18(600)-750
E. O. NO. 670-729-BR-1, PROJECT NO. R-355-40-10

December, 1954

THEORY OF CERTAIN ENERGY SURFACES AND BRILLOUIN ZONES

PART I

BRILLOUIN ZONES

1. Introduction

The general purpose of this project is the investigation of the relations between the shapes of energy surfaces and of Brillouin zones in solids. The reasons why this investigation is important, and the proposed methods for carrying out the investigation are discussed in detail in the introduction to Quarterly Progress Report No. 1,* hereafter referred to as Q.P.R. No. 1.

2. Applications of the Nearly-Free-Electron Approximations to Metals and Alloys

The nearly-free-electron approximation is described in Q.P.R. No. 1, and some problems in the theory of metals and alloys which can be investigated by this method are discussed in Q.P.R. No. 4, June, 1954. It appears that many complex phenomena which occur in three-dimensional problems occur also in two-dimensional problems, which are simpler to analyze. Consequently, we are investigating in detail the two-dimensional square lattice. Some of the phenomena of interest occur in the limiting cases when only one or two of the coefficients of the Fourier expansions of the periodic potential are nonzero, that is, for potentials of the form

$$V(F) = 2V_{20}(\cos 4\pi x + \cos 4\pi y)$$

*Air Research and Development Command Contract AF 18(600)-750, Project No. R-355-40-10, September, 1953. This report was erroneously labeled No. V instead of No. 1.

or

$$V(F) = 2V_{10}(\cos 2\pi x + \cos 2\pi y) + 2V_{20}(\cos 4\pi x + \cos 4\pi y),$$

where x and y go from 0 to 1 in the unit cell. The Schrödinger equation with either of these potentials is separable and yields Mathieu's equation for the first potential and Hill's equation for the second potential. Thus, it is possible to obtain exact solutions for these problems, with which to compare the nearly-free-electron approximations.

The curve showing the energy E as a function of the wave number k has been calculated for the Mathieu equation, for a particular value of V_{20}, by a method due to Whittaker.* The next steps will be to plot the two-dimensional energy contours and to compute the density of states. Ince has extended Whittaker's method to the Hill equation.** However, for our problem we need several high-order terms not given in the formulas of Ince. These are being derived at present and then the energy curve and density of states will be computed. Certain problems in the zone theory of metals and alloys can then be discussed.

PART II

THE PHOTOGRAPHIC LATENT IMAGE

1. Introduction

In order to acquaint the reader of the reports with this new branch of the work done under this project, a summary will here be given of its general aims and motivation.

The motivation is based on two theoretical papers by the undersigned in which it was made plausible that certain effects should exist and their study yield useful information. The measurements required to study these effects involve new techniques and new instruments, the development of which has taken much time and effort but considerable progress has been made. The effects referred to above are:

*See for example, N. W. McLachlan, Theory and Application of Mathieu Functions, Clarendon Press, Oxford, 1947, Sec 4, 30.

**Ibid, Sec 6, 30.

a. A dependence of the slope of reciprocity failure graphs as low light intensity, on the grain size, other factors being equal.

b. An inhibiting effect of infrared radiation, of wavelengths longer than those producing the ordinary "Hershel" effect, on latent image formation at low (actinic) light intensities. This inhibiting effect should manifest itself when the infrared and the actinic light strike the plate simultaneously.

c. A number of relations between parameters of reciprocity failure graphs and sequence loops.

d. A study of the sequence loops with a view of providing a quantitative method for determining the minimum number of silver atoms constituting a just developable speck under various circumstances.

It was hoped that a program of study along these four lines would throw more light on important parts of the process of latent image formation. The progress made along these four lines up to the present is as follows:

a.1. Equipment has been built and techniques have been perfected for the measurement of low-intensity reciprocity failure diagrams. The present state of our reciprocimetry, although susceptible to minor improvements, is believed to be very accurate and efficient.

a.2. Considerable effort was devoted to fractionating emulsions into components of different grain size without affecting their photographic properties. This effort was partly successful, but it became clear that good results could only be obtained with a considerable extension of the size of this contract so the problem was therefore shelved. A substitute procedure was adopted as next best, namely to work with emulsions, made for this project especially by the Kodak research laboratories, which differed in grain size but were as much alike as possible in all other respects.

b.1. A certain amount of progress was made in performing preliminary experiments on the inhibition effect. The requirements of purity of the infrared radiation turned out to be so stringent that we are still in the preliminary stage with this branch of the program.

c.1 and d.1. A sequence loop meter is being developed on the basis of experience that we have gained and are still gaining with

pilot models. The preliminary results indicate that this branch will progress rapidly in the near future. In conjunction with the reciprocimeter results this should enable us soon to reach relevant conclusions.

2. Reciprocimetry

An improved reciprocimeter, based on experience with a previously constructed model, was constructed, and after about two months of preliminary adjustments and checking, the instrument was found to permit very satisfactory control of several factors which had been uncontrolled before, in addition to its having greater speed and extended intensity range. In particular:

a. The light intensity was found to be uniform to within 2 percent over the entire area of the plate.

b. Stray light, if present, was undetectably weak.

c. The temperature is controlled to $\pm$ 0.5°F.

d. The humidity is controlled to 30 $\pm$ 5 percent.

A program was set up for measuring reciprocity failure curves of a group of five pure AgBr emulsions, kindly furnished for this purpose by the research laboratories of the Eastman Kodak Company. These emulsions differ markedly in average grain size but are otherwise as closely alike as possible. With the improved apparatus it was found that a major source of errors lies in the nonuniform coating thickness of the plates; methods for correcting this difficulty are now under study.

3. The Infrared Effect

It was found previously that the requirements for the infrared beam were very stringent. It should be both of high intensity in the near infrared (1-2μ) and free from actinic light to better than 1 part in 10^9. Attempts to overcome this difficulty are being made along three lines.

a. By controlling the temperature of the plate, conditions may be found which permit relaxing the factor of 10^9 somewhat.

b. By using Lippman emulsions, the factor 10^9 can be relaxed because of the very fine grains.

c. By developing the use of Si or Ge as both filter and lense, the intensity and purity of the infrared beam will be increased.

4. The Sequence Experiment

It is recalled that the purpose of these experiments is to obtain information on the latent image formation by a study of the exposure, required to produce a given density (after development), if the exposure is administered partly at high intensity and partly at low intensity. The result depends on the ratio of the high and low intensities used, on the exposure ratio p of the high- and low-intensity parts of the exposure given, and on the order or sequence in which the two exposure parts are admitted. Consequently a plot of the total exposure required to produce a given density, as a function of p, will have the form of a loop. The raw data are obtained as isoexposure densities, from which the theoretically more significant isodense loops are to be derived.

The old reciprocimeter was converted into an instrument yielding isoexposure loops. Mainly, this instrument has been used to study the design of a second instrument, which should enable us to photograph on a single 4" x 10" plate all data required for isoexposure and isodense loops. A small-scale working model of the new instrument is nearing completion and will soon provide further checks on some features to be built into the final instrument.

Results obtained with the first isoexposure sequence meter show that the illumination of the plate area is satisfactory but that sources of error arising from nonuniformity in plate thickness, development, etc. have to be controlled with as much care as was found necessary for the reciprocimeter. In particular, in comparing development with a simple rocking machine, with brushing, and with a commercial developing machine, it was found that if there exists any difference between these three methods the simple rocking machine gives the most uniform development.

5. Outlook

During the next interval Mr. Martin will work on some further problems in reciprocimetry and grain size studies, while Dr. Enns will work on developing the sequence loop meter.