EQUATIONS FOR THE
MICHIGAN BROWN-DEARDORFF-STERN CGE TRADE MODEL

(All Sectors Tradable)

COUNTRY EQUATIONS (i=1,…,m; j=1,…,n)

A. Final Demand

(1) $\hat{C}_{ij} = \hat{E}_i - \hat{P}_{ij}$

B. Total Demand

(2) $\hat{D}_{ij} = v_{ijo}\hat{C}_{ij} + \sum_{k=1}^{n} v_{ijk}\hat{S}_{ik}$

C. Product Demand

(3) $\hat{D}_{ij}^{M} = \hat{D}_{ij} + \sigma\theta_{ij}^{i}(\hat{P}_{ij}^{i} - \hat{P}_{ij}^{M}) - \frac{\sigma(1+\mu\sigma)}{\sigma-1}\theta_{ij}^{i}\hat{n}_{ij}$

(4*a*) $\hat{D}_{ij}^{i} = \hat{D}_{ij} + \sigma\theta_{ij}^{M}(\hat{P}_{ij}^{M} - \hat{P}_{ij}^{i}) - \frac{\sigma(1+\mu\sigma)}{\sigma-1}\theta_{ij}^{i}\hat{n}_{ij} + \mu\sigma\hat{n}_{ij}$

(4*b*) $\hat{D}_{ij}^{r} = \hat{D}_{ij}^{M} + \sigma(\hat{P}_{ij}^{M} - \hat{P}_{ij}^{r}) + \mu\sigma\hat{n}_{rj}$ $\quad r \neq i; r = 1,...,m+1$

D. Prices

(5) $\hat{P}_{ij} = \theta_{ij}^{i}\hat{P}_{ij}^{i} + \theta_{ij}^{M}\hat{P}_{ij}^{M} - \frac{(1+\mu\sigma)}{\sigma-1}\theta_{ij}^{i}\hat{n}_{ij}$

(6) $\hat{P}_{ij}^{M} = \sum_{r\neq i}^{m+1}\theta_{ij}^{rM}\hat{P}_{ij}^{r} - \sum_{r\neq i}^{m+1}\frac{(1+\mu\sigma)}{\sigma-1}\theta_{ij}^{rM}\hat{n}_{rj}$

(7) $\hat{P}_{ij}^{r} = \hat{P}_{wj}^{r} + \hat{t}_{ij}^{rMeq}$ $\quad r = 1,...,m+1$

(8) $$\hat{P}^i_{wj} = \theta^{MC}_{ij} \hat{MC}_{ij} + \theta^{FC}_{ij} (\hat{P}^V_{ij} + \hat{n}_{ij} - \hat{S}_{ij})$$

(9) $$\hat{P}^i_{wj} = \hat{MC}_{ij} + \frac{\hat{\eta}_{ij}}{1+\eta_{ij}}$$

(10) $$\hat{P}^V_{ij} = \theta^L_{ij} \hat{w}_i + \theta^K_{ij} \hat{r}_i$$

E. Marginal Cost

(11) $$\hat{MC}_{ij} = \frac{\theta^{VK}_{ij}}{\theta^{MC}_{ij}} b_{ijo} \hat{P}^V_{ij} + \sum_{k=1}^{n} \frac{b_{ikj}}{\theta^{MC}_{ij}} \hat{P}_{ik}$$

F. Demand for Primary Inputs

(12) $$\hat{L}_{ij} = \hat{n}_{ij} + \theta^{VK}_{ij} \hat{V}_{ij} + \bar{\sigma} \theta^K_{ij} (\hat{r}_i - \hat{w}_i)$$

(13) $$\hat{K}_{ij} = \hat{n}_{ij} + \theta^{VK}_{ij} \hat{V}_{ij} - \bar{\sigma} \theta^L_{ij} (\hat{r}_i - \hat{w}_i)$$

G. Demand Elasticities

(14) $$\hat{\eta}^r_{ij} = \frac{(\sigma - 1)\theta^r_{ij}}{\eta^r_{ij} n_{rj}} (\hat{P}^r_{ij} + \hat{D}^r_{ij} - \hat{P}_{ij} - \hat{D}_{ij}) \qquad r = 1,...,m+1$$

(15) $$\hat{\eta}_{ij} = \sum_{r=1}^{m+1} \frac{\delta^r_{ij} \eta^i_{rj}}{\eta_{ij}} \hat{\eta}^i_{rj}$$

H. Primary Factors Market Equilibrium

(16) $$\sum_{j=1}^{n} K_{ij} \hat{K}_{ij} = 0$$

(17) $$\sum_{j=1}^{n} L_{ij} \hat{L}_{ij} = 0$$

I. Nontariff Barriers

$$(18) \qquad \hat{t}_{ij}^{rMeq} = \hat{t}_{ij}^{r} - \frac{\theta_{ij}^{rQ}}{(1-\theta_{ij}^{rQ})\sigma}(\hat{Q}_{ij}^{r} - \hat{D}_{ij}^{r} - \hat{n}_{rj}) \qquad r \neq i;\ r = 1,...,m+1$$

WORLD EQUATIONS

J. Trade Balance and Income Determination

$$(19) \qquad 0 = dB_i^T = \sum_{j=1}^{n}[X_{ij}\hat{P}_{wj}^{i} + \sum_{r\neq i}^{m} X_{ij}^{r}(\hat{D}_{rj}^{i} + \hat{n}_{ij}) + X_{ij}^{m+1}L_E] - \sum_{j=1}^{n}\sum_{r\neq i}^{m+1} M_{ij}^{r}(\hat{n}_{rj} + \hat{P}_{wj}^{r} + \hat{D}_{ij}^{r}) \qquad i = 1,..m$$

K. Goods Market Equilibrium

$$(20) \qquad S_{ij}\hat{S}_{ij} = n_{ij}D_{ROWj}^{i}L_E + \sum_{r=1}^{m} n_{ij}D_{rj}^{i}(\hat{D}_{rj}^{i} + \hat{n}_{ij}) \qquad j = 1,...,n;\ i = 1,...,m$$

L. ROW Goods Market Equilibrium

$$(21) \qquad dS_j^{ROW} = \sum_{i=1}^{m} D_{ij}^{ROW}\hat{D}_{ij}^{ROW} \qquad j = 1,...,n$$

$$(22) \qquad dS_j^{ROW} = S_j^{ROW}\varepsilon_j^{ROW}\hat{P}_{wj}^{ROW} \qquad j = 1,...,n$$

M. ROW Import Licensing

$$(23) \qquad \sum_{j=1}^{n}(dS_j^{ROW} + S_j^{ROW}\hat{P}_{wj}^{ROW}) = L_E M_{ROW} + \sum_{i=1}^{m} M_{ROWj}^{i}\hat{P}_{wj}^{i}$$

Variables

C_{ij} Final demand for good j in contry i.

E_i Household income in country i.

E_{ij} Household income spent on good j in country i.

E_{ij}^M Household income spent on imports of good j in country i.

Z_{ijk} Intermediate demand for good j by industry k in country i.

P_{ij} Price index of good j in country i.

P_{ij}^r Price of good j produced in country r paid by consumers in country i.

S_{ij} Total production of good j in country i.

D_{ij} Final plus intermediate demand for good j in country i.

D_{ij}^M Demand for imports of good j in country i.

P_{ij}^M Import price index of good j in country i.

n_{ij} Number of firms in sector j in country i.

D_{ij}^r Demand in country i for the output of a representative firm in country r producing good j.

t_{ij}^{rMeq} Tariff equivalent applied to import by country i of good j from country r.

P_{wj}^i World price of good j produced in country i.

MC_{ij} Marginal cost for a firm producing good j in country i.

P_{ij}^V Price index in for a unit of the primary input aggregate in industry j in country i.

η_{ij} Elasticity of demand perceived by a firm producing good j in country i.

η_{rj}^i Elasticity of demand in country r for good j produced by a representative firm in country i.

w_i Wage paid to labor in country i.

r_i Return to capital in country i.

V_{ij} Primary input aggregate in industry j in country i.

L_{ij} Labor demand in industry j in country i.

K_{ij} Capital demand in industry j in country i.

B_i^T Trade balance of country i.

Q_{ij}^r Import quota imposed by country i on imports of good j from country r.

L_E Import licensing variable for ROW.

S_j^{ROW} Supply of good j by ROW.

Parameters

α_{ij} Budget share of good j in country i.

v_{ijo} Final demand share of total demand for good j in country i.

v_{ijk} Intermediate demand by industry k share of total demand for good j in country i.

σ Elasticity of substitution among different varieties of each good.

μ Parameter measuring substitutability among varieties produced by each firm.

θ_{ij}^{M} Fraction of expenditure on good j in country i devoted to imports.

θ_{ij}^{r} Fraction of expenditure on good j in country i devoted to goods produced in country r.

θ_{ij}^{rM} Fraction of country i's imports of good j purchased from country r.

θ_{ij}^{MC} Variable cost share of total cost in industry j in country i.

θ_{ij}^{FC} Fixed cost share of total in industry j in country i.

θ_{ij}^{L} Labor's share of expenditure on primary inputs in industry j in country i.

θ_{ij}^{K} Capital's share of expenditure on primary inputs in industry j in country i.

θ_{ij}^{VK} Variable capital's share of total capital employed in industry j in country i.

b_{ijo} Primary inputs share of total cost in industry j in country i.

b_{ikj} Intermediate input k's share of total cost in industry j in country i.

$\overline{\sigma}$ Elasticity of substitution between capital and labor.

δ_{ij}^{r} Fraction of production of good j in country i that is sold to country r.

t_{ij}^{r} Tariff rate applied to imports of good j by country i from country r.

θ_{ij}^{rQ} Fraction of imports of good j by country i from country r subject to an import quota.

X_{ij} Exports of good j by country i.

X_{ij}^{r} Exports of good j by country i to country r.

M_{ij}^{r} Imports of good j by country i from country r.

M_{ROW} ROW total imports.

ε_{j}^{ROW} Elasticity of supply of good j by ROW.