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# New Goods and Rising Skill Premium:

# A Theoretical Investigation

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**Abstract**

This paper examines the effects of new goods on the relative wages of skilled-labor and trade patterns in a two-cone Heckscher-Ohlin model and shows that: *(i)* new goods can be a valid theoretical explanation for the rising skill premium in the U.S. *(ii)* new goods have both domestic and international factor market effects, and their interplay determines the outcome and gives rise to surprising results; *(iii)* new goods that are "friendly" to the abundant (scarce) factors move the relative factor prices in the direction of convergence (divergence). The setup is general in the goods dimension so that the introduction of new goods is completely unrestricted, and the results apply to any one or any combination of the relative demand shocks for skilled labor. The results also apply when non-tradable goods are present.

**Key Words**: new goods; rising skill premium; international and domestic factor market effects; lens condition for factor price equalization; production set

**JEL Classification:** F11; J31; O30

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**Section 1. Introduction**

It is well-documented that the wages of skilled workers relative to unskilled workers increased steadily in the U.S. in the late 1970s and 1980s (e.g. Bound and Johnson 1992, Katz and Murphy 1992). Meanwhile, many new products also emerged in this period (e.g. fiber optic cables, Windows series software, VCRs and soft contact lenses…).[2] Is there a causal link? This paper examines the effects of new goods on the relative wages of skilled labor (i.e. the skill premia) and the pattern of trade in a global economy with both developed (Home) and developing (Foreign) countries (i.e. with two diversification cones[3]). The Home country is relatively abundant in skilled labor, and the Foreign country is relatively abundant in unskilled labor.

New goods could be a valid theoretical explanation for the rising skill premium[4] if on average, they use skilled labor more intensively than old goods. This is because following the creation of the new goods, demand shifts away from the old goods towards them so that the production of the old goods contracts, releasing both skilled and unskilled labor. Since the new goods are more skilled-labor intensive on average, they demand a higher proportion of skilled labor compared with the factors released by the old sectors, creating excess relative demand for skilled labor and pushing up its relative wage. I call this effect the "domestic factor market effect".

Investigating the empirical validity of this explanation is interesting because new goods provide a direct measure of technology,[5] the leading explanation for rising skill premium in the

[2] See Xiang (2002) for more examples. Even more anecdotes can be found in, for instance, Gray (1992), Zeisset and Wallace (1998), and various case studies by the now defunct Office of Technology Assessment.
[3] A diversification cone is a subset of the factor space. Countries that are identical except for their factor endowments achieve factor price equalization if their endowments are in the same cone.
[4] Wage inequality has two components: (1) skill premium, or the wage difference between workers with different skills and (2) residual wage inequality, or the wage difference between workers with similar skills (see Katz and Autor 1998).
[5] Technology has two effects on the relative demand for skilled labor: the direct effect is to create new goods, and the indirect effect is to change the production techniques of the old goods. The literature on technology and skill premia has focused on the indirect effect (e.g. Krueger 1993, Autor, Katz and Krueger 1997), and been unable to find a satisfactory measure for it (e.g. DiNardo and Pischke 1997; see also Berman, Bound and Machin 1998). See Xiang (2002) for more details.

literature.[6] In Xiang (2002), I find that in the U.S. manufacturing sector, new goods are on average over 40% more skilled-labor intensive than old goods, and could account for about 30% of the increase in the relative demand for skilled labor. Thus new goods appear to be a valid empirical explanation for the rising skill premium.

However, in an open economy with two diversification cones, new goods also exert their influence through what I call the "international factor market effect". To see how this effect works, suppose that before new goods appear, Home produces only radios, Foreign produces only shoes, and radios are more skilled-labor intensive than shoes. Now suppose that a new good, the computer, is created in Home and it is more skilled-labor intensive than both the radio and the shoes. Then the consumption shares of the radio and the shoes decline, and the aggregate consumption share of Foreign products, the shoes, falls relative to the aggregate consumption share of Home products, the radio and the computer. In other words, world demand shifts in favor of Home products. Because trade in goods can be thought of as trade in the factor services embedded in these goods, the demand for Home factor services increases so that Home factors become more expensive, other things equal. Then the radio becomes more expensive to produce in Home, and so its production switches to Foreign. Thus Home produces only computers. Because Home produces only radios before computers appear and computers are more skilled-labor intensive than radios, the average skilled-labor intensity has increased for Home, pushing up the relative wage of skilled labor in this country.

Therefore the first theme of this paper is the interplay between the domestic and international factor market effects. The domestic factor market effect is related to the change of the average factor-usage intensity, and new goods are said to be "friendly" to a factor in a country if they increase the average intensity of this factor's usage in this country. In the previous example, the new goods, computers, are skilled-labor friendly in Home because they increase the average skilled-labor intensity in this country. On the other hand, the international factor market effect is

[6] See Bound and Johnson (1992), Berman, Bound and Griliches (1994), Feenstra and Hanson (1999).

related to the change of the aggregate demand for one country's products relative to the other country's products, and new goods are said to be "friendly" to a country if they increase the relative demand for this country's products. In the previous example, the computers are Home friendly because they shift world demand in favor of the Home country's products. The factor-friendliness and country-friendliness of the new goods are key parameters to the results.

The interplay between the domestic and international factor market effects also gives rise to surprising results. Suppose new goods appear only in Home and they are unskilled-labor friendly. Would the Home country's skill premium necessarily decline? The answer is "no" because of the international factor market effect. New goods are Home friendly and so they increase the relative demand for Home products and Home factor services. Then Home factors become more expensive and Home produces a narrower range of products so that its average skilled-labor intensity increases. Thus the relative wage of skilled labor could increase in Home, as shown in Section 5. Now suppose that new goods appear only in Home and they are skilled-labor friendly. Could Home expand its production into unskilled-labor intensive sectors? The answer is "yes" because of the domestic factor market effect. New goods are skilled-labor friendly and so they tend to increase the relative wage of skilled labor in Home. The other side of the coin is that unskilled labor becomes relatively cheap so that the marginal costs of the unskilled-labor intensive sectors decline. Thus Home could expand its production into these sectors, as shown in Section 5.

The second theme of this paper is that the analytical framework is general in the goods dimension. First, there is no restriction on the new goods and the old goods regarding their numbers, consumption shares, productivity parameters, or factor usage intensities. There can be either a finite number or a continuum of goods. An individual new good can be more, or less, skilled-labor intensive than every old good, or have an intermediate skilled-labor intensity. The new goods could appear in both Home and Foreign. They might account for a tiny fraction of the aggregate consumption expenditure, or the bulk of it, and their average skilled-labor intensity can

be identical to or drastically different from the old goods, whose productivity parameters may vary across sectors. Second, the exogenous change of introducing new goods is modeled as a shock to the production set, the set of all the goods that are produced by the world economy. Because this shock could also result from changes in consumer tastes, production techniques, or productivity parameters of the old goods, the model treats all these relative demand shocks for skilled labor in a unified framework, and the results apply to any one, or to any combination of them, as shown in Section 4. The results also apply when non-tradable goods are present, as shown in Section 6. Finally, the results hold for large changes. This is useful for the related empirical work, because if new goods really matter for the rising skill premium, they would have to either capture a significant portion of the consumption expenditure, or employ a considerably higher skilled-unskilled mix.

This general framework also allows us to think about the effects of new goods from the perspective of the lens condition for factor price equalization (FPE)[7]. In an open economy with two diversification cones, FPE is not achieved because the difference between sector factor usages is too small compared with the difference in national factor endowments. If new goods are friendly to the abundant (scarce) factors in both countries, they tend to increase (decrease) the difference in sector factor usages and move the economy closer to (farther away from) achieving FPE; thus factor prices move in the direction of convergence (divergence).

This paper adopts a Heckscher-Ohlin model under CES preferences with two diversification cones. A multi-cone model is not only suitable when both developed and developing countries are present (e.g. Deardorff 1998), but also consistent with a few empirical studies (e.g. Debaere and Demiroglu 1998; Schott 1997). In contrast, a small-open-economy model has factor prices pinned down by exogenous commodity prices, and so fails to fully consider the general equilibrium effects of new goods, and a Heckscher-Ohlin model with one diversification cone and FPE

[7] See, for example, Deardorff (1994) and Xiang (2001).

behaves like a closed economy, and can be regarded as a special case of the two-cone model because the international factor market effect is absent.

A number of studies examine the effects of factor-biased and/or sector-biased technological changes on relative wages, and all of them use a one-cone model with FPE or Hicks-neutral differences in technology.[8] Zhu (2001) is closest in spirit to this paper, but she mainly focuses on what happens to the relative wage of skilled labor in the developing country under Cobb-Douglas preferences when every new good appears in the developed country and is more skilled-labor intensive than every old good.

This paper is organized as follows. Section 2 sets up the model and discusses the key properties of the equilibrium. Section 3 presents the main results and the intuition behind their proof, and then illustrates the intuition based on the lens condition for FPE. Section 4 discusses the applications of these results to various relative demand shocks for skilled labor. Section 5 shows some surprising results graphically using a simple case, and Section 6 discusses the case of many countries with two diversification cones and the case of non-tradable goods. Finally, Section 7 concludes.

## Section 2. Setup and Equilibrium

### 2.1. Setup

Consider a Heckscher-Ohlin model with two diversification cones. The universe of goods that the world can produce is $K$, and the subset of goods that it actually produces is $T$. Let $T \subseteq K \subseteq$ °. If $T$ has a continuum of elements (e.g. $T = [0,1]$), the model becomes the continuum Heckscher-Ohlin model *a la* Dornbusch, Fischer and Samuelson (1980). If $T$ has a finite number

[8] See, for example, Leamer (1996), Krugman (2000) and Xu (2001). This literature is relevant because technological changes, or process innovation, are sometimes difficult to distinguish from new goods, or product innovation (e.g. is a titanium bike process innovation or product innovation?). In this paper, new goods refer to well-defined final (consumer) products.

of elements (e.g. $\mathsf{T} = \{1, 2, 3, 4, 5, 6\}$), the model becomes the finite-good Heckscher-Ohlin model as used in Deardorff (1979).

Index the goods in $\mathsf{K}$ by $z$. Production exhibits constant returns to scale and uses two factors, skilled labor and unskilled labor. The factor endowments are $L_s$ and $L_u$; let $s \equiv L_s/L_u$. Factor prices are $w_s$ and $w_u$, and let $\omega \equiv w_u/w_s$ be the relative wage of *unskilled* labor. For a good $z$, its price is $P(z)$, and its unit factor requirements are $a_s(z)$ and $a_u(z)$. Let $s(z) \equiv a_s(z)/a_u(z)$ be the skilled-labor intensity of sector $z$. Assume that factor intensity reversal is absent. Then the goods can be ranked in an ascending order by their skilled-labor intensities (i.e. the goods with larger index numbers are more skilled-labor intensive). Also, let the factor income shares of sector $z$ be $\theta_s(z)$ and $\theta_u(z)$. Notice that in general, $s(z)$, $\theta_s(z)$ and $\theta_u(z)$ all depend on factor prices.

On the demand side, the representative consumer has the following CES preferences:

$$U = [\int_K c(z)^{1-\gamma} x(z)^{\gamma}]^{1/\gamma}; \quad \gamma \equiv (\sigma - 1)/\sigma$$

The elasticity of substitution is $\sigma$ ($\sigma \geq 1$), and $c(z)$'s are parameters showing how much the consumer favors good $z$, whose quantity consumed is $x(z)$ and whose share in consumption expenditure is:

$$b(z) = c(z)P(z)^{1-\sigma}/P; \; P \equiv \int_K c(z)P(z)^{1-\sigma} dz$$

Notice that the preferences are defined over $\mathsf{K}$. Before new goods appear, their costs are infinite so that their prices are infinite and their consumption shares are 0. As technological progress decreases the costs of these new goods to some finite number, their prices drop, and they are consumed.[9] Also notice that $\forall z \in T$, $b(z)$ can either be infinitesimal[10] (e.g. $\mathsf{T} = [0,1]$) or a positive number between 0 and 1 (e.g. $\mathsf{T} = \{1, 2, 3, 4, 5, 6\}$), and they add up to 1 (i.e. $\int_T b(z)dz = 1$). Let $b$ denote a distribution of $b(z)$ across $z$.

[9] Notice that (1) when $\sigma = 1$, the preferences become Cobb-Douglas with $b(z)$'s as constants. Thus the preferences should be defined over $\mathsf{T}$ (otherwise new goods would have positive consumption shares even before they appear), and so the emergence of new goods necessarily involves a change in preferences; (2) "technological progress" is to be broadly interpreted: a clever idea (e.g. opening a fast food restaurant) is "technological progress", and so is automation.

[10] That is, $b(z)dz$ is the consumption share of goods in $[z, z+dz]$.

There are two countries, Home and Foreign, identical except for factor endowments. Home is relatively abundant in skilled labor, and the superscript "*" denotes Foreign variables. Without loss of generality, assume $L_u = L_u^*$. Furthermore, all the goods markets are perfectly competitive and there is no barrier to international trade. Factors are perfectly mobile domestically, but immobile internationally.

**2.2. Equilibrium**

For a good $z \in \mathsf{T}$, its price equals its marginal cost, *MC(z)*:

(1) $$P(z) = MC(z) = w_s A_s(z);\ A_s(z) \equiv a_u(z)[\omega + s(z)] \quad \forall z \in \mathsf{T}$$

Equation (1) decomposes *MC(z)* into two parts: the wage of skilled labor ($w_s$) and the unit factor requirement for a fictional factor "equivalent skilled labor" ($A_s(z)$)[11] that always has the same price as skilled labor. The importance of thinking about equivalent skilled labor will become clear soon.

Second, Home and Foreign specialize and "carve up" the production set $\mathsf{T}$. Let $\mathsf{T}_H$ and $\mathsf{T}_F$ denote the Home and Foreign production sets and let both be non-empty. As skilled labor is more abundant in Home, every good in $\mathsf{T}_H$ is more skilled-labor intensive than every good in $\mathsf{T}_F$ except for the common good, when it exists, that both countries produce; i.e., $\mathsf{T}_H$ and $\mathsf{T}_F$ may have at most one good in common. Let $\bar{g}(X)$ ($\underline{g}(X)$) be the most (least) skilled-labor intensive good in a set $X$.[12] Then the pattern of trade and specialization can be summarized as:

(2) $$\mathsf{T}_H \cup \mathsf{T}_F = \mathsf{T},\ \bar{g}(\mathsf{T}_F) \leq \underline{g}(\mathsf{T}_H)$$

[11] To see why, skilled labor is converted into "equivalent skilled labor" one-for-one, but one unit of unskilled labor is converted into $\omega$ units of equivalent skilled labor; thus a unit of good $z$ uses $a_s(z) + a_u(z)\omega = a_u(z)[\omega + s(z)] = A_s(z)$ units of equivalent skilled labor.

[12] Assume that the closure of $\mathsf{T}$ is a subset of $\mathsf{K}$ and is compact. Then $\bar{g}(\mathsf{T}_F)$ and $\underline{g}(\mathsf{T}_H)$ always exist, although they might not be in the set $\mathsf{T}$, in which case $\exists z \in \mathsf{T}_F(\mathsf{T}_H)$ arbitrarily close to $\bar{g}(\mathsf{T}_F)$ ($\underline{g}(\mathsf{T}_H)$).

For (2) to be consistent with the marginal costs determined by the Home and Foreign factor prices, a Foreign good must be no cheaper to produce in Home and a Home good no cheaper to produce in Foreign. Let *h(z)* be the relative marginal cost (of Home production) for good *z*:

(3) $$h(z) \geq 0 \ \forall z \in T_F, \ h(z) \leq 0 \ \forall z \in T_H; \ h(z) \equiv ln[MC(z)/MC^*(z)]$$

Thirdly, when all domestic markets clear in Home:[13]

(4) $$1/\omega = \mu(\omega; T_H, b)/s; \ \mu(\omega; T_H, b) \equiv \int_{P_H} b(z)\,\theta_s(z)dz / \int_{P_H} b(z)\,\theta_u(z)dz$$

The term $\mu(.)$[14] is the ratio of the average income share of skilled labor to the average income share of unskilled labor, both weighted by sector shares in consumption. It can be thought of as the average skilled-labor intensity[15] across all sectors of the Home economy. Clearly, the average skilled-labor intensity is the relative demand for skilled labor. On the other hand, the relative supply of skilled labor is the ratio of skilled-labor endowment to unskilled-labor endowment (*s*). Thus by (4), the relative wage of skilled labor ($1/\omega$) is determined by its relative demand ($\mu(.)$) and relative supply (*s*).

Similarly for Foreign:

(5) $$1/\omega^* = \mu(\omega^*; T_F, b)/s^*; \ \mu(\omega^*; T_F, b) \equiv \int_{P_F} b(z)\,\theta_s^*(z)dz / \int_{P_F} b(z)\,\theta_u^*(z)dz$$

Finally, international payments must be balanced. Since international trade in goods can be viewed as the exchange of factor services, this balance-of-payments condition can be written as the equilibrium condition of an international factor market for the service of equivalent skilled labor (see Appendix 1 for the derivation of (6)):

(6) $$\Lambda(T_H, T_F, b; \omega, \omega^*) = \left(\frac{w_s^*}{w_s}\right)^{\sigma} \frac{\omega^* + s^*}{\omega + s}; \ \Lambda(T_H, T_F, b; \omega, \omega^*) \equiv \frac{\int_{P_F} c(z) A_s^{*1-\sigma}(z)dz}{\int_{P_H} c(z) A_s^{1-\sigma}(z)dz}$$

[13] See Appendix 1 for the derivation of (4). Notice that it holds under any preference specification consistent with the existence of a representative consumer and/or when there are more than two factors.

[14] At equilibrium, $\mu(.)$ depends on $\omega$ only because: 1. $\theta_s(z)$, $\theta_u(z)$ depend on $\omega$ only; 2. the prices of Home goods depend on $\omega$ only by (1); 3. the price index *P*, in the definition of *b*, appears in both the numerator and the denominator of $\mu(.)$ and cancels out.

[15] The "intensity" here is based on factor shares, not unit factor requirements as in Section 2.1; these two measures yield the same ranking under the same factor prices, and will be used interchangeably henceforth.

The term $\Lambda(.)$ is proportional to the relative demand for Foreign goods and can be interpreted as the relative demand for the service of Foreign equivalent skilled labor. Its relative supply is $(\omega^*+s^*)/(\omega + s)$ because for every unit of Foreign (Home) unskilled labor there are $\omega^*+s^*$ $(\omega+s)$ units of Foreign (Home) equivalent skilled labor.[16] Finally, the relative price of Foreign equivalent skilled labor is simply $w_s^*/w_s$. Therefore, other things equal, an increase in $\Lambda(.)$ tends to increase $w_s^*/w_s$, and an increase in $(\omega^* + s^*)/(\omega + s)$ tends to decrease it.

The equilibrium (2)~(6) is built on two important properties:

**Property 1 (downward-sloping relative demand curve):** $\omega\mu(\omega; T_H, b)$ strictly increases in $\omega$, and $\omega^*\mu(\omega^*; T_F, b)$ strictly increases in $\omega^*$.

**Property 2 (comparative advantage):** $h(z)$ decreases in $z$.

**Proof:** See Appendix 2.

Since the relative wages of skilled labor are $1/\omega$ and $1/\omega^*$, Property 1 says that the relative demand curves for skilled labor are strictly decreasing. This property ensures that the domestic factor market effect functions properly. Suppose new goods are skilled-labor friendly; then the relative demand for skilled labor increases. Property 1 guarantees that when this happens, the relative wage of skilled labor tends to increase. On the other hand, Property 2 says that the relative marginal cost (of Home production) decreases (not necessarily strictly) with the goods index; i.e. Home has a comparative advantage in producing skilled-labor intensive goods. This property ensures that the international factor market effect functions properly. Suppose new goods are Home friendly; then the relative demand increases for Home factor services. As a result, Home factors become more expensive, and Home's production set contracts. Property 2 guarantees that when this happens, the production of the most unskilled-labor intensive Home goods switches to Foreign so that the average skilled-labor intensity increases in Home.

[16] Recall that one unit of skilled (unskilled) labor is converted to one ($\omega$) unit(s) of equivalent skilled labor.

## Section 3. Main Results

### 3.1. Definitions and Propositions

Model new goods as an exogenous shock to the production set that takes 3 steps to complete. First, a new production set $T_1$ replaces the original one $T_0$ and a new allocation of consumption shares $b'$ replaces the original allocation $b$.[17] Second, all the goods in $T_1$ are ranked by their skilled-labor intensities at the original factor prices in an ascending order. Finally, factor prices change and the economy adjusts to a new equilibrium. Let the subscript "0" denote the variables at the original equilibrium, "1" those associated with the new production set and the new consumption-share allocation but the original factor prices, and "$n$" those at the new equilibrium. For example, the Home production set is $T_{H,0}$ at the original equilibrium ($T_0$, $b$ and $\omega_0$), becomes $T_{H,1}$ under $T_1$, $b'$ and $\omega_0$, and changes to $T_{H,n}$ at the new equilibrium ($T_1$, $b'$ and $\omega_n$). The factor-friendliness and country-friendliness of the shock then determine the outcome.

**Definition 1. Factor Friendliness (Home):** Let $\psi_H \equiv \mu(\omega_0; T_{H,1}, b') - \mu(\omega_0; T_{H,0}, b)$; i.e. $\psi_H$ is the change in the Home average skilled-labor intensity at the pre-shock factor prices. Then a shock is friendly to skilled (unskilled) labor if $\psi_H > 0$ ($< 0$), and factor-neutral if $\psi_H = 0$.

**Definition 2. Factor Friendliness (Foreign):** Let $\psi_F \equiv \mu(\omega_0^*; T_{F,1}, b') - \mu(\omega_0^*; T_{F,0}, b)$; i.e. $\psi_F$ is the change in the Foreign average skilled-labor intensity at the pre-shock factor prices. Then a shock is friendly to skilled (unskilled) labor if $\psi_F > 0$ ($< 0$), and factor-neutral if $\psi_F = 0$.

**Definition 3. Country Friendliness:** Let $\Gamma \equiv \int_{P_{H,1}} b'(z)dz / \int_{P_{F,1}} b'(z)dz - \int_{P_{H,0}} b(z)dz / \int_{P_{F,0}} b(z)dz$; i.e. $\Gamma$ is the change in the relative demand for Home goods (at the pre-shock factor prices). A shock is friendly to Home (Foreign) if $\Gamma > 0$ ($< 0$), and country-neutral if $\Gamma = 0$.

**Definition 4. Neutral Shock:** A neutral shock is factor-neutral in both countries and country-neutral (i.e. $\psi_H = \psi_F = \Gamma = 0$).

[17] Notice that the preference parameters $c(z)$'s might change as part of the shock.

The main results are:

**Proposition 1. (ω)** The relative wage of skilled labor strictly rises (falls) in Home if the shock is friendly to Home (Foreign) or country-neutral and friendly also to the abundant (scarce) factors or factor-neutral in both countries; i.e., $\omega_n < \omega_0$ ($\omega_n > \omega_0$) if $\Gamma \geq 0$ ($\leq 0$), $\psi_H \geq 0$ ($\leq 0$), $\psi_F \leq 0$ ($\geq 0$), and at least one inequality is strict.

**Proposition 2. ($\omega^*$)** The relative wage of skilled labor strictly rises (falls) in Foreign if the shock is friendly to Home (Foreign) or country-neutral and friendly also to the scarce (abundant) factors or factor-neutral in both countries; i.e., $\omega_n^* < \omega_0^*$ ($\omega_n^* > \omega_0^*$) if $\Gamma \geq 0$ ($\leq 0$), $\psi_H \leq 0$ ($\geq 0$), $\psi_F \geq 0$ ($\leq 0$), and at least one inequality is strict.

**Proposition 3. (Production sets)** The Foreign (Home) production set contracts if the shock is friendly to Foreign (Home) or country-neutral, and friendly also to skilled (unskilled) labor or factor-neutral in both countries; i.e., $T_{F,n} \subseteq T_{F,1}$ ($T_{H,n} \subseteq T_{H,1}$) if $\Gamma \leq 0$ ($\geq 0$), $\psi_H \geq 0$ ($\leq 0$), $\psi_F \geq 0$ ($\leq 0$), and at least one inequality is strict[18].

**Proposition 4. (Neutral shock)** A neutral shock has no effect on $\omega_0$, $\omega_0^*$, $T_{H,1}$ and $T_{F,1}$.

**Proof**: See Appendix 3.

Because moving from one equilibrium to another with identical exogenous variables is a neutral shock and $\omega$, $\omega^*$, $T_H$ and $T_F$ determine all the other endogenous variables, Proposition 4 implies that:

**Corollary 1. (Uniqueness)** The equilibrium is unique when it exists.

Propositions 1~4 are general because they hold for large changes and apply to any one or any combination of the following relative demand shocks for skilled labor: new goods, preference

[18] Notice that (1) $T_H$ contracts (expands) if and only if $T_F$ expands (contracts); (2) a contraction in $T_H$ ($T_F$) might not involve a change in $\underline{g}(T_H)$ ($\overline{g}(T_F)$) because when the common good exists and has a large consumption share, the contraction could show up as a change in Home's (Foreign's) share in the production of this common good; (3) The contraction is not strict because when $\overline{g}(T_F) < \underline{g}(T_H)$, the gap could be so large that Home is completely insulated from what happens in Foreign.

changes for the old goods, and changes in production techniques and sector-specific and country-specific productivity parameters, as shown in Section 4. On the other hand, Propositions 1~4 also accommodate surprising results (some of which are mentioned in the Introduction), as shown in Section 5.

**3.2. Proof: Intuition**

Proving Propositions 1~4 is not trivial. First, a shock to the production set is difficult to parameterize because it can result from different relative demand shocks that require different parameterizations (e.g. a change in preferences versus a change in production techniques). Second, even if a parameterization is feasible, differentiation is difficult because the production sets might be discrete and there might not exist a common good that both countries produce. To get around these problems, I prove Propositions 1~4 by contradiction.

Take Proposition 1 as an example, and consider a shock that is friendly to Home and friendly also to the abundant factors in both countries (i.e. $\psi_H > 0$, $\psi_F < 0$ and $\Gamma > 0$). We want to show that the relative wage of skilled labor strictly increases in Home (i.e. $\omega_n < \omega_0$). Suppose at the new equilibrium, this relative wage decreases (i.e. $\omega_n \geq \omega_0$). First, the relative demand for skilled labor is higher in Home at the pre-shock factor prices (i.e. $\omega_0$) because the shock is skilled-labor friendly, and an increase in $\omega$ raises this relative demand still further. The only way to absorb the excess relative demand is for Home to expand its production into unskilled-labor intensive sectors. Then the Foreign production set must contract, lowering the relative demand for skilled labor in this country, *ceteris paribus*; since the shock is unskilled-labor friendly in Foreign, the relative wage of skilled labor must decline in this country (i.e. $\omega_n^* > \omega_0^*$). Thirdly, the relative demand for Foreign equivalent skilled labor ($\Lambda(.)$) decreases because the shock is Home friendly and on top of this, Home has expanded its production set. Now consider the good $\underline{g}_n \equiv \underline{g}(\mathsf{T}_{H,n})$, and let $h_n \equiv h(\underline{g}_n)$ denote its relative marginal cost (of Home production). As shown in Appendix 3, the changes in $\omega$, $\omega^*$ and $\Lambda(.)$ imply that $h_n > 0$ at the new equilibrium. In other words, the

hypothetical increase in $\omega$ necessarily implies that it is cheaper to produce a Home good, $\underline{g}_n$, in the Foreign country. This contradicts (3), and so $\omega$ must strictly decrease.

**3.3. Illustration**

Figures 3.1 ~ 3.3 summarize the effects of Home-friendly, Foreign-friendly and country-neutral shocks respectively. In each graph, $\psi_H$ is on the vertical axis and $\psi_F$ on the horizontal axis so that each point identifies the factor friendliness and country friendliness of the shock (e.g. a point in the 4th quadrant of Figure 3.3 has $\psi_H < 0$, $\psi_F > 0$ and $\Gamma = 0$). Thus Propositions 1~4 can be applied to the quadrants, axes and origin of each graph. First, the results in the quadrants are explicitly shown. For example, in the 2nd quadrant of Figure 3.3, the shock is skilled-labor friendly in Home, unskilled-labor friendly in Foreign, and country-neutral, and so the relative wage of skilled labor increases in Home by Proposition 1 (i.e. $\omega \downarrow$) and decreases in Foreign by Proposition 2 (i.e. $\omega^* \uparrow$). Second, the results on the axes and origins contain those of all the quadrants they border except for the origin of Figure 3.3. For instance, at the origin of Figure 3.2, both $\omega^*$ and $\omega$ increase and $T_F$ contracts since the origin borders all the four quadrants so that Propositions 1~3 all apply. Thirdly, a small circle represents the origin of Figure 3.3, which corresponds to a neutral shock that has no effect, by Proposition 4. Finally, when the label of an endogenous variable does not appear in a quadrant, this variable may either increase or decrease as illustrated in Section 5.[19] For example, Section 5.1 shows that $\omega$ could either rise or fall if the shock is unskilled-labor friendly in Home, factor-neutral in Foreign, and Home friendly. Thus the label of $\omega$ does not appear in the 3rd and 4th quadrants of Figure 3.1.

**3.4. More Intuition: the Lens Condition for FPE**

[19] In other words, more information than the signs of $\psi_F$, $\psi_H$ and $\Gamma$ is needed to determine how the variable changes. However, to gather such information, a simpler setup with more structure (e.g. Section 5) is more suitable.

The intuition based on the domestic and international factor market effects provides a "micro" view of the results: it helps the most when the results are examined case by case. A "macro" view of Propositions 1~2 can be obtained by thinking about the lens condition for FPE.

Stated in the context of the integrated world economy (IWE),[20] a hypothetical world in which factors are perfectly mobile across national borders, the lens condition formalizes the intuition that to achieve FPE, the differences in factor endowments across countries have to be less, in some sense, than the differences in factor-usage intensities across goods. As the vectors of factor endowments form the "endowment lens" and those of sector factor usages at the IWE factor prices form the "goods lens", the lens condition states that if FPE is achieved, then the endowment lens must lie inside the goods lens, and the converse is also true with 2 countries or 2 factors.[21] Since countries specialize in this paper and FPE is not achieved, the lens condition fails; i.e. the difference in sector factor usages is not large enough. If this difference is increased (decreased) by the shock, the economy moves closer to (farther away from) achieving FPE, and it is intuitive for factor prices to converge (diverge) in some sense.

As Home is relatively abundant in skilled labor, its relative wage is lower in this country than in Foreign at the original equilibrium (i.e. $\omega_0 > \omega_0^*$). Thus an increase (decrease) in the relative wage of skilled labor in Home and/or a decrease (increase) in it in Foreign is a movement in the direction of convergence (divergence). On the other hand, a shock that is friendly to the abundant (scarce) factors in both countries increases (decreases) the difference in sector factor usages. Reorganizing Propositions 1 and 2:

**Corollary 2. Abundant-factor friendly shocks lead to convergence:** If a shock is skilled-labor friendly or factor-neutral in Home but unskilled-labor friendly or factor-neutral in Foreign, the relative wage of skilled labor strictly increases (decreases) in Home (Foreign) if the shock is

[20] See, for example, Dixit and Norman (1980).
[21] The converse might not be true with more than 2 factors. See Demiroglu and Yun (1999).

friendly to Home (Foreign) or country-neutral; i.e. $\omega_n < \omega_0$ ($\omega_n^* > \omega_0^*$) if $\psi_H \geq 0$, $\psi_F \leq 0$, $\Gamma \geq 0$ ($\leq 0$), and at least one inequality is strict.

**Corollary 3. Scarce-factor friendly shocks lead to divergence:** If a shock is unskilled-labor friendly or factor-neutral in Home but skilled-labor friendly or factor-neutral in Foreign, the relative wage of skilled labor strictly decreases (increases) in Home (Foreign) if the shock is friendly to Foreign (Home) or country-neutral; i.e. $\omega_n > \omega_0$ ($\omega_n^* < \omega_0^*$) if $\psi_H \leq 0$, $\psi_F \geq 0$, $\Gamma \leq 0$ ($\geq 0$), and at least one inequality is strict.

Figures 3.4 ~ 3.6 illustrate Corollaries 2 and 3 using new goods as an example[22]. Figure 3.4 shows the factor endowment box of an IWE producing 3 goods, 1~3, whose factor usage vectors are labeled as such. The goods lens is the polygon with these vectors as its sides. Since it is symmetric about the diagonal, Figure 3.4 shows only its upper half. Point E represents the distribution of factor endowments between Home and Foreign; since it is outside the goods lens, the lens condition does not hold, and FPE is not achieved.

Look at Corollary 2 first. As the Home new goods are skilled-labor friendly, they congregate at the lower corner of the goods lens. They tend to use skilled labor more intensively, and so tend to make the lower portion of the goods lens bigger. Similarly, the Foreign new goods are unskilled-labor friendly and so they congregate at the upper corner of the goods lens and tend to make its upper portion bigger. When both conditions hold, the goods lens becomes bigger, in some sense, and the IWE is pushed in the direction of FPE. Then it is intuitive for factor prices to move in the direction of convergence. In Figure 3.5, a Home new good appears in the economy described by Figure 3.4, its skilled-labor intensity is higher than good 3, and its factor usage vector is labeled "*n*". The average skilled-labor intensity of the IWE increases and $\omega_{IWE}$ decreases (notice that the slopes of the factor usage vectors of the old goods, 1~3, are flatter), and the new

[22] Notice that this intuition is loose. The lens condition has not been defined for a continuum of goods, and Corollaries 2 and 3 do not say how $\omega/\omega^*$ will change unless the shock is country-neutral ($\Gamma = 0$). In that case, $\omega\downarrow$ and $\omega^*\uparrow$ if the shock is friendly to the abundant factors in both countries (by Corollary 2), and $\omega\uparrow$ and $\omega^*\downarrow$ if the shock is friendly to the scarce factors (by Corollary 3).

goods lens has the old one inside it, and gets "closer" to covering point E. Convergence occurs in that the relative wage of skilled-labor increases in Home (i.e. $\omega$ falls).

Similarly, in the case of Corollary 3, the new goods congregate in the middle portion of the goods lens, and tend to make the lens smaller. As the IWE is pushed away from achieving FPE, it is intuitive for factor prices to move in the direction of divergence. In Figure 3.6, a Foreign new good, *n*, appears in the economy of Figure 3.4, and its skilled-labor intensity is lower than good 2 but higher than good 1. At unchanged IWE factor prices the new goods lens lies inside the old one and gets "farther away" from covering point E. Divergence occurs in that the relative wage of skilled labor decreases in Home (i.e. $\omega$ rises).

**Section 4. Applications**

Since there is no restriction on the relation between $T_0$ and $T_1$ or between $b$ and $b'$ in Propositions 1~4, these propositions are powerful results applicable to different relative demand shocks for skilled labor, each of which corresponds to a restriction on $T_1$, $T_0$, $b'$ and $b$. First, let the sets of new goods and old goods be $N$ and $O$, and $N \cap O = \varnothing$. Propositions 1~4 can then be applied to new goods by having $T_0 = O_0$, $T_1 = O_1 \cup N$, and $O_1 \subseteq O_0$.[23] Second, the shift of consumer tastes among the old goods (a pure taste change) corresponds to $T_1 = O_1 \subseteq T_0 = O_0$. Finally, because a change in the production techniques[24] of an old good $z$ can be thought of as the creation of a new good with the new production techniques and consumption share $b(z)$ plus the demise of good $z$, it is a special case of new goods with the additional requirements that $\forall z_0 \in T_0$, $\exists z_1 \in T_1$ such that $b'(z_1) = b(z_0)$, and $z_1 \neq z_1'$ if $z_0 \neq z_0'$.

Propositions 1~4 also apply to changes in Hicks-neutral productivity parameters. First, let $A(z) \geq 1$ be the productivity parameter of sector $z$ so that an increase in $A(z)$ reflects an increase in

[23] The distinction between $O_0$ and $O_1$ is necessary because following the arrival of the new goods some old goods may cease to be consumed.

[24] This refers to an exogenous change in $\theta_s$ and $\theta_u$ with no change in productivity (thus the word "techniques" rather than "technologies").

the productivity of sector $z$. Let $b_0(z)$ denote the consumption share of good $z$ when $A(z) = 1\ \forall z$. Then (see Appendix 4 for their derivations):

$$\mu(\omega; X, b) = [\int_X b_0(z)A^{\sigma-1}(z)\theta_s(z)dz][\int_X b_0(z)A^{\sigma-1}(z)\theta_u(z)dz]^{-1};\ X = T_H,\ T_F,$$
$$\Lambda(.) = (w_s^*/w_s)^{1-\sigma}[\int_{P_F} b_0(z)A^{\sigma-1}(z)dz][\int_{P_H} b_0(z)A^{\sigma-1}(z)dz]^{-1}$$

Thus when $\sigma = 1$, the change in the $A(z)$'s has no effect, and when $\sigma > 1$, it has the same effect as a pure taste change. Next, let $A_H \geq 1$ be the Home-specific productivity parameter and suppose $A_H$ increases; i.e. the productivity of both factors increases by the same proportion in every sector in Home. Then $\mu(., T_H)$ and $\mu(., T_F)$ are unchanged and the prices of Home goods tend to fall; i.e. the increase in $A_H$ has no domestic factor market effect, and its international factor market effect is to expand the Home production set. Therefore the increase in $A_H$ can be modeled as a Foreign-friendly shock that is factor-neutral in both countries (this is shown rigorously in Appendix 4).

Furthermore, Propositions 1~4 apply to any combination of the above-mentioned shocks, and these shocks can be large changes. In other words, there can be any number of new goods with any allocation of consumption shares, and the new goods can have their own sector-specific productivity parameters, their arrival accompanied by changes in taste and production techniques of the old goods in a completely unrestricted way.

The case of new goods merits more discussion because their creation might change the preferences of the old goods (i.e. $c(z)$'s might change for the old goods) so that the consumption shares of some old goods might decline by (proportionately) more than the others. I call the preference changes of old goods caused by the creation of new goods "induced preference changes". For example, following the creation of personal computers, the (proportional) decline in the typewriter's consumption share is likely to exceed the (proportional) decline in the consumption share of, say, the beef. This is not a problem for theory because of the generality of Propositions 1~4, but could be a problem for the empirical work that tries to identify the effects of new goods because in the real world, new goods and pure taste changes might occur side by side, and the induced preference changes are hard to distinguish from the pure taste changes.

Intuitively, the induced preference changes will not affect the outcome if the "complementarities" between the old goods and the new goods are "evenly" distributed among the old goods; then the above-mentioned problem is solved. To formalize this intuition, suppose new goods are the only exogenous change; then at pre-shock factor prices, the aggregate consumption share of the old goods declines from 1 to $\rho \equiv 1 - \int_N b'(z)dz \in (0,1)$. Let $\varepsilon(z) \equiv b'(z) - \rho b(z)$; then for an old good $z$, $\varepsilon(z)$ is the deviation of its consumption share from the benchmark $\rho b(z)$ and represents its complementarity with the new goods. In other words, the old good, $z$, is a "complement" ("substitute") of the new goods if $\varepsilon(z) > 0$ ($< 0$) so that its consumption share falls by less (more) than the average (of all the old goods). Then for $\varepsilon(z)$'s to be "evenly" distributed among the old goods:

(7.1) $\int_X \varepsilon(z)\theta_s(z)dz = 0$ and $\int_X \varepsilon(z)\theta_u(z)dz = 0$; $X = O_{H,1}, O_{F,1}$

(7.2) $\int_X \varepsilon(z)dz = 0$, $X = O_{F,1}$, $O_{H,1}$

In other words, the complementarities between the old goods and the new goods are uncorrelated with the factor income shares of the old goods ((7.1)) and unrelated to where the old goods are produced ((7.2)). Thus the induced preference changes can be neutralized by imposing (7.1)~(7.2)[25].

Under (7.1) and (7.2), we can also highlight the role of new goods by having the following alternative definitions of their factor-friendliness and country-friendliness:

(8.1) $\psi_H^n \equiv \mu(\omega_0; N_H, b') - \mu(\omega_0; O_{H,0}, b)$; $\psi_F^n \equiv \mu(\omega_0^*; N_F, b') - \mu(\omega_0^*; O_{F,0}, b)$

(8.2) $\Gamma^n \equiv \int_{N_H} b'(z)dz / \int_{O_{H,0}} b(z)dz - \int_{N_F} b'(z)dz / \int_{O_{F,0}} b(z)dz$

[25] When the induced preference changes are absent (i.e. $\sigma > 1$ and $c(z)$'s do not change), (7.1) ~ (7.2) also hold because $b'(z)/b(z) = P/P'$ ($P = \int_O c(z)P(z)^{1-\sigma}dz$; $P' = \int_{N\cup O} c(z)P(z)^{1-\sigma}dz$) so that $\rho = P/P'$ and $\varepsilon(z) = 0$ $\forall z$. Thus imposing (7.1) and (7.2) amounts to saying that either the induced preference changes are absent or they are present but do not affect the outcome.

because $\psi_H^n$, $\psi_F^n$ and $\Gamma^n$ have the same signs as $\psi_F$, $\psi_H$ and $\Gamma$ in Definitions 1~4.[26] In other words, new goods are skilled-labor friendly in Home (Foreign) ($\psi_H > 0$ or $\psi_F > 0$) if and only if their average skilled-labor intensity exceeds the old goods ($\psi_H^n > 0$ or $\psi_F^n > 0$), and new goods are friendly to Home (Foreign) ($\Gamma > 0$ or $\Gamma < 0$) if and only if in this country, their consumption share is higher relative to the old goods and so, in some sense, more of them have appeared ($\Gamma^n > 0$ or $\Gamma^n < 0$).

Finally, in a closed economy or an open economy with one diversification cone and FPE, the international factor market effect is absent and the outcome depends solely on the domestic factor market effect. Apply Proposition 1 with $\psi_F = \Gamma = 0$:

**Corollary 4.** The relative wage of skilled labor strictly increases (decreases) if and only if the shock is skilled (unskilled)-labor friendly.

## Section 5. Surprising Results

To illustrate some of the surprising results contained in Propositions 1~4, consider the simple case in which there is a continuum of goods and the preferences are Cobb-Douglas (i.e. $\mathcal{T} = [0,1]$ and $\sigma = 1$). Then there exists a common good, $g$, that both countries produce. To accommodate new goods, dig holes in the continuum $[0, 1]$; as new goods appear, these holes are filled up. The Home new good is $z_n$, and it fills the hole $[z_n, z_n+dz_n]$, and in this example there is no Foreign new good.[27] Finally, assume that all the goods have identical consumption shares (i.e. $b(z) = b(z')$ $\forall z, z'$), and the consumption shares of the old goods all decrease by the same proportion (i.e. $b'(z) = \rho\, b(z)$ $\forall z$; $0 < \rho < 1$) following the creation of $z_n$.

### 5.1. The Effect on the (Home) Relative Wage of Skilled Labor

[26] See Appendix 4 for the rigorous proof. To see the distinction between $\psi_H^n$, $\psi_F^n$, $\Gamma^n$ and $\psi_F$, $\psi_H$, $\Gamma$, notice that, for example, $\psi_H = \mu(\omega_0; N_H \cup O_{H,1}, b') - \mu(\omega_0; O_{H,0}, b)$ in the case of new goods.

[27] The effects of Foreign new goods have a similar intuition to Home new goods. See Appendix 5 for the treatment of Foreign new goods in this setup.

In this example, the relative wage of skilled labor could increase even if the new good is unskilled-labor friendly, according to Proposition 1. As shown in Appendix 5:

(9.1) $$sgn(d(w_s/w_u)) = -sgn(d\omega) = sgn(b_\psi + c_1 b_\Gamma);$$
$$c_1 > 0,\ b_\Gamma = dz_n/(1-g),\ b_\psi = [\theta_s(z_n)/\int_g^1 \theta_s(z)dz - \theta_u(z_n)/\int_g^1 \theta_u(z)dz]dz_n$$

In other words, the change in the relative wage of skilled labor is determined by two terms, $b_\psi$ and $c_1 b_\Gamma$. First, $b_\psi$ is the marginal increase in the Home average skilled-labor intensity following the creation of the new good, $z_n$, and so $b_\psi > 0\ (< 0)$ if $z_n$ is friendly to skilled (unskilled) labor. Thus $b_\psi$ measures the domestic factor market effect. Notice that $b_\psi > 0\ (< 0)$ if and only if $s(z_n) > s\ (< s)$[28], and that $b_\psi$ increases with $s(z_n)$; i.e. the magnitude of the domestic factor market effect is larger the more $z_n$'s skilled-labor intensity differs from the Home average. On the other hand, $b_\Gamma$ measures the marginal increase in the relative demand for Home equivalent skilled labor following the introduction of $z_n$, and so $b_\Gamma > 0$ as $z_n$ is Home friendly in this example. Thus $c_1 b_\Gamma$ measures the international factor market effect. Notice that $c_1 b_\Gamma$ does not depend on $s(z_n)$; i.e. the magnitude of the international factor market effect does not depend on the skilled-labor intensity of the new good, $z_n$.

Figure 5.1 graphs the two components of $d(w_s/w_u)$, $b_\psi$ and $c_1 b_\Gamma$, against the skilled-labor intensity of the new good, $s(z_n)$. The first component, $b_\psi$, shows up as the dashed upward-sloping line DD that intersects the horizontal axis at $s(z_n) = s$, and this line represents the domestic factor market effect. The other component, $c_1 b_\Gamma$, shows up as the dashed horizontal line II above the horizontal axis, and this line represents the international factor market effect. To obtain the total effect, add up the domestic and international factor market effects by shifting DD up to the solid line TT by the distance between II and the horizontal axis in Figure 5.1. If TT is above (below) the horizontal axis, the creation of $z_n$ leads to an increase (decrease) in $w_s/w_u$.

[28] $s(z_n) > (<)\ s \Leftrightarrow s(z_n)/\omega > (<)\ s/\omega = \mu(\omega,g) \Leftrightarrow \theta_s(z_n)/\theta_u(z_n) > (<)\ \mu(\omega,g) \Leftrightarrow b_\psi > (<)\ 0.$

Suppose $z_n$ is slightly unskilled-labor friendly (i.e. $s(z_n)$ is slightly to the left of $s$ on the horizontal axis). Then its domestic factor market effect is but slightly different from 0 (the DD line is slightly below the horizontal axis), but its international factor market effect is sizeable (the II line is considerably above the horizontal axis) because this effect does not depend on $z_n$'s skilled-labor intensity. Thus the international factor market effect dominates and $d(w_s/w_u) > 0$ (the TT line is above the horizontal axis). In other words, the relative wage of skilled labor has *increased* in Home even though *ceteris paribus*, the new good *reduces* the relative demand for skilled labor. To drive home the point that this surprising result is due to the international factor market effect, consider a closed economy producing the goods *[g, 1]*. The international factor market effect is then shut off, and in Figure 5.1, the sign of $d(w_s/w_u)$ is determined by the DD line alone. Clearly, the relative wage of skilled labor rises (falls) if and only if $z_n$ is friendly to skilled (unskilled) labor.

Finally, notice that when $z_n$ is skilled-labor friendly ($s(z_n)$ is to the right of $s$ on the horizontal axis*),* the relative wage of skilled labor increases because both the domestic and international factor market effects work in the same direction (both the II and DD lines are above the horizontal axis). Also notice that the international factor market effect depends on not only $b_I$, but also other coefficients of the model (these coefficients are grouped into $c_1$). For example, let $a_{21}$ denote the marginal increase of the average skilled-labor intensity in Home following a marginal increase of the common-good index, *g.* An increase in the value of $a_{21}$ implies a larger $c_1$ and so a stronger international factor market effect. In Figure 5.2, this increase shows up as an upward shift of the TT line to T'T'. In other words, when the new good is unskilled-labor friendly, the relative wage of skilled labor is more likely to increase the larger is the value of $a_{21}$.

**5.2 The Effect on the Pattern of Trade**

Another surprising result in this example is that Home could expand its production into unskilled-labor intensive sectors (i.e. the index of the common good, $g$, decreases) even if the new good is skilled-labor friendly, according to Proposition 3. As shown in Appendix 5:

(9.2) $sgn(dg) = sgn(b_\Gamma + a_{12}b_\psi/a_{22})$

$a_{12} = 1/(\omega+s) - 1/(\omega+s(g)) < 0,\ a_{22} = \partial[ln(\omega\mu(.))]/\partial\omega > 0$

As in Section 5.1, $b_\Gamma$ measures the international factor market effect, and $b_\Gamma > 0$ because the new good, $z_n$, is Home friendly. Figures 5.3 and 5.4 graph the logs of Home and Foreign marginal costs against the goods index. The (solid) Home marginal cost line HH is flatter than the (solid) Foreign marginal cost line FF[29] due to Property 2. The intersection of HH and FF determines the common-good index, $g$, and Home (Foreign) produces the goods to the right (left) of $g$ for which HH lies below (above) FF. Because the creation of $z_n$ makes Home factors more expensive and so increases the Home marginal costs of all the goods, the international factor market effect shows up in Figures 5.3 and 5.4 as an upward shift of HH to the dashed line II. The intersection of II and FF is to the right of $g$, showing that the international factor market effect tends to contract Home's production set.

On the other hand, the domestic factor market effect is $a_{12}b_\psi/a_{22}$. Suppose $z_n$ is skilled-labor friendly ($b_\psi > 0$); then the relative wage of skilled labor tends to increase (by $b_\psi/a_{22}$). The other side of the coin is that unskilled labor becomes relatively cheap, and so the Home marginal cost of good $g$, which has a lower-than-average skilled-labor intensity ($s(g) < s$), declines (by $|a_{12}b_\psi/a_{22}|$). However, for a good $z$ with a higher-than-average skilled-labor intensity ($s(z) > s$), exactly the opposite happens: its Home marginal cost goes up (by $|a_{12}(z)b_\psi/a_{22}|$; $a_{12}(z) = 1/(\omega+s) - 1/(\omega+s(z)) > 0$). In other words, the skilled-labor friendly new good, $z_n$, reduces (increases) the Home marginal costs of unskilled (skilled)-labor intensive goods through the domestic factor market effect. Therefore this effect shows up in Figure 5.3 as a counter clockwise tilting of HH to the dashed line DD at $z_0$ such that $s(z_0) = s$. Notice that the intersection of DD and FF (not explicitly shown) is to the left of $g$, showing that the domestic factor market effect tends to expand Home's production set.

[29]For illustration, both HH and FF are drawn as linear and upward sloping.

To obtain the total effect, add up the domestic and international factor market effects by shifting DD up by the distance between II and HH in Figure 5.3 to get the thick solid line TT. If TT intersects FF to the right (left) of $g$, the creation of $z_n$ leads to a contraction (expansion) of Home's production set. If $z_n$ is very skilled-labor friendly, the domestic factor market effect is strong and so HH tilts a lot, as in Figure 5.3. Then TT intersects FF to the left of $g$. In other words, even though the new good appears in Home and is more skilled-labor intensive than average, Home could expand its production into unskilled-labor intensive sectors, and the more skilled-labor intensive is the new good, the more is this scenario likely to happen. To drive home the point that this surprising result is due to the domestic factor market effect, consider a continuum Ricardian model (*a la* Dornbusch, Fischer and Samuelson (1977)) with only one factor of production, skilled labor. Then the domestic factor market effect is shut off, and in Figure 5.3, the sign of $dg$ is determined by the II line alone. Clearly, Home's production set contracts because the new good, $z_n$, is Home friendly (see Appendix 6 for the rigorous proof).

Notice that when $z_n$ is unskilled-labor friendly ($b_\psi < 0$), the HH line tilts clockwise as in Figure 5.4 so that the TT line intersects FF to the right of $g$: Home's production set has contracted because the international and domestic factor market effects work in the same direction. Notice also that the domestic factor market effect depends on the coefficient $a_{22}$, which is negatively related to the slope of the relative demand curve for Home skilled labor. A decrease in the value of $a_{22}$ makes this relative demand curve steeper so that a larger change in the relative wage of skilled labor is needed to restore equilibrium in the Home factor markets following an exogenous shock; i.e. the domestic factor market effect is stronger. This is illustrated in Figure 5.5, where the flatter $T_1T_1$ (steeper $T_0T_0$) line corresponds to a high (low) value of $a_{22}$ and the new good, $z_n$, is skilled-labor friendly ($b_\psi > 0$). The $T_1T_1$ ($T_0T_0$) line intersects FF to the right (left) of $g$, showing

that Home's production set contracts (expands)[30]. In other words, when the new good is skilled-labor friendly, Home's production set is more likely to expand the steeper is the relative demand curve for Home skilled labor (i.e. the lower is $a_{22}$).

**Section 6. More Applications**

First, Propositions 1~4 readily apply to many countries with two diversification cones because countries in the same cone have identical factor prices and marginal costs and can be aggregated into a single country bloc.

Second, to apply Propositions 1~4 when non-tradable goods are present, one more assumption is needed. Let the superscript "T" denote sets of tradable goods:

(10) $$s(\omega; \underline{g}(T_H^T)) \leq \mu(\omega; T_H, b); s(\omega^*; \overline{g}(T_F^T)) \geq \mu(\omega^*; T_F, b)$$

In other words, the least (most) skilled-labor intensive Home (Foreign) tradable good is less (more) skilled-labor intensive than the national average (including both tradable and non-tradable goods). To see why (10) is necessary, suppose Home new goods are unskilled-labor friendly and all non-tradable, and $s(\omega; \underline{g}(T_H^T)) > \mu(\omega; T_H, b)$. Then the relative wage of skilled labor tends to decrease in Home and the (Home) marginal cost of the good $\underline{g}(T_H^T)$ tends to decrease because this good has a higher-than-average skilled-labor intensity. Since $\underline{g}(T_H^T)$ is also the least skilled-labor intensive tradable good in Home, this country's production set tends to *expand*. In other words, the domestic factor market effect on $T_H$ is exactly opposite to Sections 2~5, in which non-tradable goods are absent (see, in particular, Section 5.2). However, once (10) is imposed, such problems do not arise, and so Propositions 1~4 apply with one minor modification: the country-friendliness of the shock is determined by the tradable goods alone.

[30] Appendix 6 shows two concrete examples, in both of which $z_n = 1$: (1) the production technology is Leontief (i.e. $\forall z$, the elasticity of substitution in production, $\eta(z)$, is 0) with $a_u(z)$ being a step function with 3 discrete values, and $g$ decreases; (2) the production technology has $\eta(z) \geq 1$ $\forall z$, and $g$ increases. The first (second) example corresponds to a low (high) value of $a_{22}$.

Thirdly, factor-biased technological changes can also be analyzed, although doing so in a general way is beyond the scope of this paper. For example, for a closed economy (or an open economy with FPE) producing goods 1 and 2 with $\sigma > 1$, let $A_{s,2} \geq 1$ be the productivity parameter of skilled labor in sector 2, and $\eta(.)$ the elasticity of substitution in production. Then an increase in $A_{s,2}$ can be treated as a decrease in $w_s$ for sector 2 by the same proportion and: (1) the income share of skilled labor in sector 2 ($\theta_s(2)$) increases (decreases) if $\eta(2) > 1$ ($< 1$) and does not change if $\eta(2) = 1$; (2) the marginal cost of sector 2 decreases, and so the price of good 2 drops and its consumption share ($b(2)$) increases since $\sigma > 1$.[31] Because sector 1 is unaffected by the increase in $A_{s,2}$, how $\mu(.)$ changes depends on sector 2's elasticity of substitution ($\eta(2)$): if $\eta(2) \geq 1$, $\mu(.)$ increases so that $\omega$ decreases; but if $\eta(2) < 1$, the changes of $\mu(.)$ (and so $\omega$) are ambiguous. Therefore, an increase in skilled labor's productivity might not be skilled-labor friendly.

Finally, the intuition based on the domestic and international factor market effects also works when other exogenous variables change. For example, suppose $L_s$ increases (i.e. $s$ increases). The domestic factor market effect tends to increase $\omega$ because the relative supply of skilled labor has increased. This effect also increases the Home marginal costs of unskilled-labor intensive goods and so tends to contract Home's production set. On the other hand, the international factor market effect tends to expand Home's production set because the relative supply of Home's equivalent skilled labor has increased and so Home factors have become cheaper. This effect also decreases the average skilled-labor intensity in Home, and so tends to increase $\omega$. Thus $\omega$ increases but Home's production set could either expand or contract.

**Section 7. Conclusion and Discussion**

This paper analyzes the effects of new goods on factor prices and the pattern of trade in a Heckscher-Ohlin setup with CES preferences and two diversification cones, and both the setup

[31] The former corresponds to the "direct impact effect" and the latter corresponds to the "indirect price effect" in the literature (see, for example, Xu 2001).

and the results are general in the goods dimension. New goods (as well as other exogenous changes of the relative demand for skilled labor) are modeled as a shock to the production set so that new goods could appear in a completely unrestricted way and the results apply to any one or any combination of the relative demand shocks for skilled labor. The results also apply when non-tradable goods are present. Finally, the results are proved by contradiction and they hold for large changes.

In this setup, new goods exert their influence through the domestic and international factor market effects. The former (latter) depends on the factor (country) friendliness of the new goods, and works by changing the relative demand for skilled labor (factor services) in the domestic (international) factor markets. The interplay between these two effects gives rise to surprising results. For example, Home new goods that are unskilled-labor friendly could increase the relative wage of skilled labor in this country due to the international factor market effect, and Home new goods that are skilled-labor friendly could expand this country's production into unskilled-labor intensive sectors due to the domestic factor market effect. However, skilled-labor friendly new goods tend to increase the relative wage of skilled labor via the domestic factor market effect, and so new goods are a valid theoretical explanation for the rising skill premium in the U.S.

While the domestic and international factor market effects provide a "micro" intuition for the results, the lens condition for FPE provides a "macro" one. A shock that is friendly to the abundant (scarce) factors in both countries tends to increase (decrease) the difference in sector factor usages and make the goods lens bigger (smaller); thus it pushes the IWE in the direction of (away from) FPE and moves the factor prices in the direction of converge (divergence) in the sense that the relative wage of skilled labor either rises (falls) in Home, or falls (rises) in Foreign, or both, depending on the country friendliness of the shock.

Finally, the theoretical framework can be implemented empirically. In particular, (1) the finding that the relative demand for skilled labor is the average skilled-labor intensity holds in a

multi-good and multi-factor setting, and is robust to the specifications of preferences and production technologies; (2) the results are robust to different relative demand shocks for skilled labor and hold for large changes; (3) the average skilled-labor intensity is straightforward to compute. Therefore, once we have identified the new goods, we can implement the theoretical framework to measure their contribution to the rising skill premium in the U.S., as in Xiang (2002).

## Appendix 1: The Derivations of (4) and (6)

### A1.1 The derivation of (4) with two factors

**Proof:** Let $A$ be a set of goods that are produced in Home, $y$ the income of all the consumers who are buying from this country, $Q(z)$ the output of good $z$, and $l_s(z)$, $l_u(z)$ the demand for skilled and unskilled labor by sector $z$. Since $b(z) \equiv P(z)Q(z)/y$, $\theta_x(z) \equiv [l_x(z)w_x][P(z)Q(z)]^{-1}$ $(x=s, u)$, $\mu(\omega; A, b) = [\int_A b(z)\theta_s(z)dz][\int_A b(z)\theta_u(z)dz]^{-1} = (w_s/w_u)[\int_A l_s(z)dz][\int_A l_u(z)dz]^{-1}$. When the factor markets clear, $\int_A l_s(z)dz = L_s$, $\int_A l_u(z)dz = L_u$. Since $s \equiv L_s/L_u$, $\omega\mu(\omega; A, b) = s$. Θ

### A1.2 The derivation of (4) with multiple factors

Suppose there are a finite number of factors indexed by $i = 1, 2...J$. The factor endowments are $E_1$, $E_2...E_J$, and let $s_i \equiv E_i/E_1 \ \forall i$. Denote the unit factor requirements of the $J$ factors in sector $z$ by $a_i(z)$, and let $s_i(z) \equiv a_i(z)/a_1(z) \ \forall i$ (for factor 1, $s_1(z) = 1$). The factor prices are $w_i$, $i = 1, 2...J$, and let $\omega_i \equiv w_1/w_i$, $\forall i>1$. Define the income share of factor $i$ in sector $z$ as $\theta_i(z) \equiv w_i\, a_i(z)/P(z)$. Then $\Sigma_{i=1}^{J} \theta_i(z) = 1 \ \forall z$, $w_i = P(z)\partial Q^s(z)/\partial l_i \ \forall i, z$ and $L_i^d(z) = [y/w_i]b(z)\theta_i(z) \ \forall i, z$. Call factor 1 "unskilled labor" and factor 2 "skilled labor", and a similar manipulation as that in Appendix 1.1 yields $1/\omega_2 = \mu(.)/s_2$, which is similar to (4) with $\omega_2$ and $s_2$ replacing $\omega$ and $s$. This exercise can be done for any pair of factors.

### A1.3 The derivation of (6)

To balance international payments, $(w_s L_s + w_u L_u) \int_{P_F} b(z)dz = (w_s^* L_s^* + w_u^* L_u^*) \int_{P_H} b(z)dz$; i.e. Home imports equal its exports. Thus $\int_{P_F} b(z)dz / \int_{P_H} b(z)dz = [w_s^*(\omega^* + s^*)] / [w_s(\omega + s)]$. Plugging the definition of $b(z)$ and (1) into this condition yields (6).

## Appendix 2: Properties 1 and 2

### A2.1 Property 1

**Lemma 1.** $\partial \ln A_s(z)/\partial \ln \omega = \omega/(\omega+s(z)) = \theta_u(z)$.

**Proof:** By definition, $MC(z) = w_s A_s(z)$, and so $\partial MC(z)/\partial w_u = \partial A_s(z)/\partial \omega$. Since $MC(z)$ is the cost function of sector $z$ when the output is 1 physical unit, $\partial MC(z)/\partial w_u = a_u(z)$. $\therefore \partial A_s(z)/\partial \omega = a_u(z)$, and $\partial lnA_s(z)/\partial ln\,\omega = a_u(z)\omega/A_s(z) = \omega/(\omega+s(z)) = \theta_u(z)$. Θ

**Property 1. $\omega\mu(.)$ strictly increases in $\omega$.**

**Proof:** Let $A$ be a set of goods. Then:

$$\omega\mu(\omega; A, b) = \frac{w_u}{w_s}\frac{\int_A b(z)\theta_s(z)dz}{\int_A b(z)\theta_u(z)dz} = \frac{\frac{1}{P}\int_A c(z)P(z)^{1-\sigma}\frac{w_s a_s(z)}{P(z)}\frac{1}{w_s}dz}{\frac{1}{P}\int_A c(z)P(z)^{1-\sigma}\frac{w_u a_u(z)}{P(z)}\frac{1}{w_u}dz} = \frac{\int_A c(z)P(z)^{-\sigma}a_s(z)dz}{\int_A c(z)P(z)^{-\sigma}a_u(z)dz}$$

At equilibrium, $a_s(z)$, $a_u(z)$ and $P(z)$ all depend on $\omega$ only, and so $\mu(.)$ and $\omega\mu(.)$ both depend on $\omega$ only. Let $D_s \equiv \int_A c(z)a_s(z)P(z)^{-\sigma}dz$ and $D_u \equiv \int_A c(z)a_u(z)P(z)^{-\sigma}dz$. Then $ln[\omega\mu(.)] = lnD_s - lnD_u$ and:

$$\partial ln[\omega\mu(.)]/\partial\omega = \left[\frac{1}{D_s}\int_A c(z)\frac{\partial a_s(z)}{\partial\omega}P(z)^{-\sigma}dz - \int_A c(z)\frac{\partial a_u(z)}{\partial\omega}P(z)^{-\sigma}dz\frac{1}{D_u}\right]$$

$$-\sigma\left[\frac{1}{D_s}\int_A c(z)\frac{\partial \ln P(z)}{\partial\omega}a_s(z)P(z)^{-\sigma}dz - \int_A c(z)\frac{\partial \ln P(z)}{\partial\omega}a_u(z)P(z)^{-\sigma}dz\frac{1}{D_u}\right]$$

The first square-bracketed term is non-negative since $\partial a_s(z)/\partial\omega \geq 0$ and $\partial a_u(z)/\partial\omega \leq 0$. The second square-bracketed term is negative, as explained below. Thus $\partial ln[\omega\mu(.)]/\partial\omega > 0$.

Define $x_k(z) \equiv c(z)a_k(z)P(z)^{-\sigma}$ $(k = s, u)$, and $B \equiv \{\int_A[\partial lnP(z)/\partial\omega]x_s(z)dz\}\{\int_A[\partial lnP(z)/\partial\omega]x_u(z)dz\}^{-1}$. Then $x_s(z)>0$, $x_u(z)>0$ $\forall z$, and $x_s(z)/x_u(z) = s(z)$. Furthermore, $D_s/D_u = (\int_A x_s(z)dz)/(\int_A x_u(z)dz)^{-1}$.

The sign of the second square-bracketed term depends on the sign of $[B - D_s/D_u]$ and $\partial lnP(z)/\partial\omega$. Let $w_s = 1$. Then $A_s(z) = P(z)$ and by **Lemma 1**, $\partial lnP(z)/\partial\omega > 0$ and decreases with $z$. Thus compared with $D_s/D_u$, $B$ has more weight put on low-indexed goods whose skilled-labor intensities are low so that $B < D_s/D_u$ (a rigorous proof is available from the author upon request). Therefore the second bracketed term is negative. Θ

**A2.2 Property 2: $h(z_l; \omega,\omega^*) \geq h(z_h; \omega,\omega^*)$ if $z_l \leq z_h$ $\forall \omega, \omega^*$ s.t. $\omega > \omega^*$.**

**Proof:** Plug (1) and (6) into the definition of $h(.)$:

$$h(z; \omega,\omega^*) = lnA_s(z) - ln(\omega+s)/\sigma + ln(\omega^*+s^*)/\sigma - lnA_s^*(z) - lnΛ(T_H, T_F, b; \omega, \omega^*)/\sigma$$

Thus $h(z_l; \omega,\omega^*) - h(z_h; \omega,\omega^*) = [lnA_s(\omega, z_l) - lnA_s(\omega, z_h)] - [lnA_s(\omega^*, z_l) - lnA_s(\omega^*, z_h)]$. By **Lemma 1**, $\partial[lnA_s(\omega, z_l) - lnA_s(\omega, z_h)]/\partial ln\,\omega = [\theta_u(z_l) - \theta_u(z_h)] \geq 0$ $\forall\omega$ as $z_l \leq z_h$. Since $\omega > \omega^*$, $h(z_l; \omega,\omega^*) - h(z_h; \omega,\omega^*) = (\omega - \omega^*)\{\partial[lnA_s(\omega_j, z_l) - lnA_s(\omega_j, z_h)]/\partial\omega\} \geq 0$ by the mean value thoerem, where $\omega_j \in [\omega^*,\omega]$. Θ

## Appendix 3: Propositions 1~4

This appendix proves that when $\psi_H \geq 0$, $\psi_F \leq 0$, $\Gamma \geq 0$, and at least one inequality is strict, $\omega$ strictly falls. The proofs of the other cases in Propositions 1~4 are similar.

**A3.1. Preparation**

Define:

$$f(z;\ T_H, T_F,\ b;\ \omega,\omega^*) \equiv lnA_s(z) - ln(\omega+s)/\sigma + ln(\omega^*+s^*)/\sigma - lnA_s^*(z) - ln\Lambda(T_H, T_F,\ b;\ \omega,\ \omega^*)/\sigma$$

The difference between *f(.)* and *h(.)* is that *h(.)* is the value of *f(.)* at the equilibrium factor prices and production sets, and that *f(.)* could take any factor prices and production sets as its arguments.

**Lemma 2**. $\partial f(z;\ T_H, T_F,\ b;\ \omega,\omega^*)/\partial\omega = (\omega+s(z))^{-1} - [\sigma(\omega+s)]^{-1} - [(\sigma-1)/\sigma]\ [\omega+\omega\mu(\omega;T_H,\ b)]^{-1}$

$$\partial f(z;\ T_H, T_F,\ b;\ \omega,\omega^*)/\partial\omega^* = [(\sigma-1)/\sigma]\ [\omega^*+\omega^*\mu(\omega^*;T_F,\ b)]^{-1} + [\sigma(\omega^*+s^*)]^{-1} - (\omega^*+s^*(z))^{-1}$$

**Proof:** Define $D_H \equiv \int_{P_H} c\,(z)A_s^{1-\sigma}(z)dz$. Then $\partial ln\Lambda(.)/\partial\omega = (\sigma-1)\frac{1}{D_H}\int_{P_H} c\,(z)A_s^{-\sigma}(z)\frac{\partial A_s(\omega)}{\partial\omega}dz = (\sigma-1)$

$\frac{1}{D_H}\int_{P_H} c\,(z)A_s^{1-\sigma}(z)\ \ \frac{1}{\omega+s(z)}dz$ (by **Lemma 1**). Now $\omega\mu(\omega;\ T_H,\ b) = \frac{\int_{P_H} c(z)P(z)^{-\sigma}a_s(z)dz}{\int_{P_H} c(z)P(z)^{-\sigma}a_u(z)dz} =$

$\frac{\int_{P_H} c(z)A_s(z)^{1-\sigma}s(z)[\omega+s(z)]^{-1}dz}{\int_{P_H} c(z)A_s(z)^{1-\sigma}[\omega+s(z)]^{-1}dz}$ (by Appendix 2 and (1)). Thus $\omega+\omega\mu(\omega;\ T_H,\ b) =$

$\frac{\int_{P_H} c(z)A_s(z)^{1-\sigma}dz}{\int_{P_H} c(z)A_s(z)^{1-\sigma}[\omega+s(z)]^{-1}dz} = \frac{D_H}{\int_{P_H} c(z)A_s(z)^{1-\sigma}[\omega+s(z)]^{-1}dz}$ and so $\partial ln\Lambda(.)/\partial\omega = (\sigma-1)[\omega+\omega\mu(\omega;$

$T_H,\ b)]^{-1}$. Thus $\partial f(z;T_H, T_F,\ b;\omega,\omega^*)/\partial\omega = \partial lnA_s(z)/\partial\omega - [\partial ln(\omega+s)/\partial\omega]/\sigma - [\partial\ln\Lambda(.)/\partial\omega]/\sigma = (\omega+s(z))^{-1} - [\sigma(\omega+s)]^{-1} - [(\sigma-1)/\sigma][\omega+\omega\mu(\omega;T_H,\ b)]^{-1}$. The proof for $\partial f(z;\ T_H, T_F,\ b;\ \omega,\omega^*)/\partial\omega^*$ is analogous. Θ

**A3.2 The Proof**

Let $\underline{g}_n \equiv \underline{g}\,(T_{H,n})$, $\underline{g}_0 \equiv \underline{g}\,(T_{H,0})$, $\underline{g}_1 \equiv \underline{g}\,(T_{H,1})$, $\overline{g}_n \equiv \overline{g}\,(T_{F,n})$, $\overline{g}_0 \equiv \overline{g}\,(T_{F,0})$ and $\overline{g}_1 \equiv \overline{g}\,(T_{F,1})$.

(See Section 3 for the exact meanings of the subscripts "0", "1" and "n".)

**(i).** Suppose ω rises; i.e. $\omega_n \geq \omega_0$. Then $\underline{g}_n \leq \underline{g}_1$

Suppose $\underline{g}_n > \underline{g}_1$. (A3.1)

Then: $\omega_n\mu(\omega_n;\ T_{H,n},\ b') - \omega_0\mu(\omega_0;\ T_{H,0},\ b)$

$= [\omega_n\mu(\omega_n;\ T_{H,n},\ b') - \omega_0\mu(\omega_0;\ T_{H,n},\ b')]$ *($\geq 0$* by **Property 1***)*

$+ [\omega_0\mu(\omega_0;T_{H,n},\ b') - \omega_0\mu(\omega_0;\ T_{H,1},\ b')]$ *($> 0$* by (A3.1)*)*

$+ [\omega_0\mu(\omega_0;T_{H,1},\ b') - \omega_0\mu(\omega_0;\ T_{H,0},\ b)]$ *($\geq 0$* since $\psi_H \geq 0$*)*

$\therefore\ \omega_n\mu(\omega_n;\ T_{H,n},\ b') - \omega_0\mu(\omega_0;\ T_{H,0},\ b) > 0.$

This contradicts (4), which implies that $\omega_n\mu(\omega_n;\ T_{H,n},\ b') - \omega_0\mu(\omega_0;\ T_{H,0},\ b) = s - s = 0.$ $\therefore\ \underline{g}_n \leq \underline{g}_1$

**(ii).** Since $T_{H,1} \cup T_{F,1} = T_{H,n} \cup T_{F,n} = T_1$, $\underline{g}_n \leq \underline{g}_1 \Rightarrow \overline{g}_n \leq \overline{g}_1$ and $\underline{g}_n \leq \overline{g}_1$ (A3.2)

**(iii)**. $\overline{g}_n \leq \overline{g}_1$ implies that $\omega_n^* \geq \omega_0^*$

Suppose $\omega_n^* < \omega_0^*$; then:

$\omega_n^*\mu(\omega_n^*;\ T_{F,n},\ b') - \omega_0^*\mu(\omega_0^*;\ T_{F,0},\ b)$

$= [\omega_n^*\mu(\omega_n^*;\ T_{F,n},\ b') - \omega_0^*\mu(\omega_0^*;\ T_{F,n},\ b')]$ *($< 0$* by **Property 1***)*

$+ [\omega_0^*\mu(\omega_0^*;T_{F,n},\ b') - \omega_0^*\mu(\omega_0^*;\ T_{F,1},\ b')]$ *($\leq 0$* by (A3.2)*)*

$+ [\omega_0^*\mu(\omega_0^*;T_{F,1},\ b') - \omega_0^*\mu(\omega_0^*;\ T_{F,0},\ b)]$ *($\leq 0$* since $\psi_F \leq 0$*)*

$\therefore\ \omega_n^*\mu(\omega_n^*;\ T_{F,n},\ b') - \omega_0^*\mu(\omega_0^*;\ T_{F,0},\ b) < 0.$

This contradicts (5), which implies that $\omega_n^* \mu(\omega_n^*;\ T_{F,n},\ b') - \omega_0^* \mu(\omega_0^*;\ T_{F,0},\ b) = s^* - s^* = 0.\ \therefore \omega_n^* \geq \omega_0^*$.

**(iv)**. $\Lambda_n(b') \leq \Lambda_0(b)$ where $\Lambda_n(b') \equiv \Lambda(T_{H,n},\ T_{F,n},\ b';\ \omega_0,\ \omega_0^*);\ \Lambda_0(b) \equiv \Lambda(T_{H,0},\ T_{F,0},\ b;\ \omega_0,\ \omega_0^*)$.

Denote $\int_A b(z)dz$ by $M(A;\ b)$ where $A$ is any set of goods ($\omega_0$ and $\omega_0^*$ are not shown in the brackets because they are the same for $\Lambda_n(b')$ and $\Lambda_0(b)$ ). Then $\Gamma = M(T_{H,1};\ b')/M(T_{F,1};\ b') - M(T_{H,0};\ b)/M(T_{F,0};\ b)$.

$$\begin{aligned}
&\therefore \Lambda_0(b) = [M(T_{F,0};\ b)/M(T_{H,0};\ b)](w_{s,0}/w_{s,0}^*)^{1-\sigma} \\
&\geq [M(T_{F,1};\ b')/M(T_{H,1};\ b')](w_{s,0}/w_{s,0}^*)^{1-\sigma} && \text{(since } \Gamma \geq 0\text{)} \\
&\geq [M(T_{F,n};\ b')/M(T_{H,n};\ b')](w_{s,0}/w_{s,0}^*)^{1-\sigma} = \Lambda_n(b') && \text{(by \textbf{(i)} and \textbf{(ii)})}
\end{aligned}$$

**(v)**. Suppose $\bar{g}_1 \leq \bar{g}_0$. Then by (A3.2), $\underline{g}_n \leq \bar{g}_0$ (A3.2')

**(v1).** By **(iv)**, $f(\bar{g}_0;\ T_{H,n}, T_{F,n}, b';\ \omega_0, \omega_0^*) - f(\bar{g}_0;\ T_{H,0}, T_{F,0}, b;\ \omega_0, \omega_0^*) = -(\ln\Lambda_n(b') - \ln\Lambda_0(b))/\sigma \geq 0$ (A3.3)

**(v2).** $f(\underline{g}_n;\ T_{H,n}, T_{F,n},\ b';\ \omega_0, \omega_n^*) \geq f(\bar{g}_0;\ T_{H,n}, T_{F,n},\ b';\ \omega_0, \omega_n^*)$ by **Property 2** and (A3.2'); $f(\bar{g}_0;\ T_{H,n}, T_{F,n}, b';\ \omega_0, \omega_n^*) - f(\bar{g}_0;\ T_{H,n}, T_{F,n},\ b';\ \omega_0, \omega_0^*) = (\omega_n^* - \omega_0^*)[\partial f(.;\ \omega_j^*, \bar{g}_0)/\partial\omega^*]$ by the mean value theorem (**MVT**) with $\omega_j^* \in [\omega_0^*, \omega_n^*]$. Now $\partial f(.;\ \omega_j^*, \bar{g}_0)/\partial\omega^* \equiv \partial f(\bar{g}_0;\ T_{H,n}, T_{F,n},\ b';\ \omega_0, \omega_j^*)/\partial\omega^* = (1/\sigma)[s(\omega_j^*; \bar{g}_0) - s^*]/D_1 + [(\sigma-1)/\sigma][s(\omega_j^*; \bar{g}_0) - \omega_j^* \mu(\omega_j^*; T_{F,n}, b')]/D_2$ with $D_1 \equiv (\omega_j^* + s^*)(\omega_j^* + s(\omega_j^*; \bar{g}_0)) > 0$ and $D_2 \equiv (\omega_j^* + s^*(\omega_j^*; \bar{g}_0))[\omega_j^* + \omega_j^* \mu(\omega_j^*;\ T_{F,n},\ b')] > 0$ by **Lemma 2**. Since $s(\omega_j^*;\ \bar{g}_0) \geq s(\omega_0^*;\ \bar{g}_0) \geq s^* = \omega_n^* \mu(\omega_n^*;\ T_{F,n},\ b') \geq \omega_j^* \mu(\omega_j^*;\ T_{F,n},\ b')$, $\partial f(.;\ \omega_j^*, \bar{g}_0)/\partial\omega^* \geq 0$. Since $\omega_n^* \geq \omega_0^*$ by **(iii)**, $f(\bar{g}_0;\ T_{H,n}, T_{F,n},\ b';\ \omega_0, \omega_n^*) - f(\bar{g}_0;\ T_{H,n}, T_{F,n},\ b';\ \omega_0, \omega_0^*) \geq 0$.

$$\therefore f(\underline{g}_n;\ T_{H,n}, T_{F,n},\ b';\ \omega_0, \omega_n^*) - f(\bar{g}_0;\ T_{H,n}, T_{F,n},\ b';\ \omega_0, \omega_0^*) \geq 0 \qquad \text{(A3.4)}$$

**(v3).** $f(\underline{g}_n;\ T_{H,n}, T_{F,n},\ b';\ \omega_n, \omega_n^*) - f(\underline{g}_n;\ T_{H,n}, T_{F,n},\ b';\ \omega_0, \omega_n^*) = (\omega_n - \omega_0)[\partial f(.;\ \omega_j, \underline{g}_n)/\partial\omega]$ by **MVT** with $\omega_j \in [\omega_0,\ \omega_n]$. Now $\partial f(.;\ \omega_j, \underline{g}_n)/\partial\omega \equiv \partial f(\underline{g}_n;\ T_{H,n}, T_{F,n},\ b';\ \omega_j, \omega_n^*)/\partial\omega = (1/\sigma)\ [\ s - s(\omega_j;\ \underline{g}_n)]/D_1 + [(\sigma-1)/\sigma]\ [\ \omega_j \mu(\omega_j;\ T_{H,n}, b') - s(\omega_j;\ \underline{g}_n)]/D_2$ with $D_1 \equiv (\omega_j + s)(\omega_j + s(\omega_j;\ \underline{g}_n)) > 0$ and $D_2 \equiv (\omega_j + s(\omega_j;\ \underline{g}_n))[\omega_j + \omega_j \mu(\omega_j;\ T_{H,n}, b')] > 0$ by **Lemma 2**. Since $s(\omega_j;\ \underline{g}_n) \leq s(\omega_n;\ \underline{g}_n) \leq s$ and $\omega_j \mu(\omega_j;\ T_{H,n},\ b') - s(\omega_j;\ \underline{g}_n) \geq 0$ ($\underline{g}_n$ is the least skilled-labor intensive good in $T_{H,n}$), $\partial f(.;\ \omega_j, \underline{g}_n)/\partial\omega \geq 0$. Since $\omega_n \geq \omega_0$ by **(i)**,

$$\therefore f(\underline{g}_n;\ T_{H,n}, T_{F,n},\ b';\ \omega_n, \omega_n^*) - f(\underline{g}_n;\ T_{H,n}, T_{F,n},\ b';\ \omega_0, \omega_n^*) \geq 0 \qquad \text{(A3.5)}$$

**(v4).** By A(3.3) ~ (A3.5) (at least one inequality is strict), $h(\underline{g}_n;\ \omega_n,\ \omega_n^*) = f(\underline{g}_n;\ T_{H,n}, T_{F,n},\ b';\ \omega_n, \omega_n^*) > f(\bar{g}_0;\ T_{H,0}, T_{F,0},\ b;\ \omega_0, \omega_0^*) = h(\bar{g}_0;\ \omega_0,\ \omega_0^*) \geq 0$ (the last inequality is because $\bar{g}_0 \in T_{F,0}$). This contradicts Equation (3), which implies that $h(\underline{g}_n;\ \omega_n,\ \omega_n^*) \leq 0$ since $\underline{g}_n \in T_{H,n}$.

**(v5)**. Now suppose $\bar{g}_1 > \bar{g}_0$. Replace $\bar{g}_0$ in **(v1)** ~ **(v4)** with $\bar{g}_1$, and the proof goes through. In particular, in **(v2)**, $s(\omega_j^*; \bar{g}_1) \geq s(\omega_0^*; \bar{g}_1) \geq s(\omega_0^*; \bar{g}_0) \geq s^*$ so that (A3.4) holds; in **(v4)**, $h(\bar{g}_1; \omega_0, \omega_0^*) \geq 0$ because $\bar{g}_1 \in \mathsf{T}_{F,1}$. Θ

## Appendix 4

### A4.1 Sector-specific productivity parameter

Let these parameters be $A(z)$; without loss of generality, let $A(z) \geq 1\ \forall z$. Let the subscript "0" denote the case when $A(z) = 1\ \forall z$. Let $B$ be a set of goods, $P_0 \equiv \int_B c(z)P_0(z)^{1-\sigma}dz$ and $P_1 \equiv \int_B c(z)P(z)^{1-\sigma}dz$. Then an increase in $A(z)$ decreases $MC(z)$ by the same proportion, and since price equals marginal cost, $P(z) = P_0(z)A(z)^{-1}$. Thus $P_1 = \int_B c(z)P_0(z)^{1-\sigma}A(z)^{\sigma-1}dz$, and $b(z) = c(z)P(z)^{1-\sigma}/P_1 = (P_0/P_1)A(z)^{\sigma-1}[c(z)P_0(z)^{1-\sigma}/P_0] = (P_0/P_1)A(z)^{\sigma-1}b_0(z)$. On the other hand, the changes in $A(z)$ do not affect $\theta_s(z), \theta_u(z)$, $s(z)$ and $h(z)$. Thus:

$$\mu(.) = [\int_B b(z)\theta_s(z)dz]\,[\int_B b(z)\theta_u(z)dz]^{-1} = [\int_B b_0(z)A(z)^{\sigma-1}\theta_s(z)dz]\,[\int_B b_0(z)A(z)^{\sigma-1}\theta_u(z)dz]^{-1}$$

$$\Lambda(.) = (w_s^*/w_s)^{1-\sigma}[\int_{P_F} b(z)dz]\,[\int_{P_H} b(z)dz]^{-1} = (w_s^*/w_s)^{1-\sigma}[\int_{P_F} b_0(z)A(z)^{\sigma-1}dz]\,[\int_{P_H} b_0(z)A(z)^{\sigma-1}dz]^{-1}$$

### A4.2 Country-specific productivity parameter

Denote the country-specific productivity parameters by $A_H$ and $A_F$. Without loss of generality, let $A_F = 1$ and $A_H \geq 1$. Then $\mu(\omega; \mathsf{T}_H, b)$ and $\mu(\omega^*; \mathsf{T}_F, b)$ are unaffected, and Equations (6) and (3) become:

$$A_H^{1-\sigma}\Lambda(T_H, T_F, b; \omega, \omega^*) = (w_s^*/w_s)^{\sigma}[(\omega^*+s^*)/(\omega+s)] \quad \text{(A4.1)}$$

$$MC^*(\bar{g}(P_F)) \leq MC(\bar{g}(P_F))/A_H;\ MC^*(\underline{g}(P_H))) \geq MC(\underline{g}(P_H))/A_H \quad \text{(A4.2)}$$

Let $w_s' \equiv w_s/A_H$ and $w_u' = w_u/A_H$; then $\omega$ and $\omega^*$ are unchanged, and (A4.1) and (A4.2) become:

$$A_H\Lambda(T_H, T_F, b; \omega, \omega^*) = (w_s^*/w_s')^{\sigma}[(\omega^*+s^*)/(\omega+s)] \quad \text{(A4.3)}$$

$$MC^*(\bar{g}(P_F)) \leq w_s'A_s(\bar{g}(P_F));\ MC^*(\underline{g}(P_H))) \geq w_s'A_s(\underline{g}(P_H)) \quad \text{(A4.4)}$$

Compare (A4.3) with (6) and (A4.4) with (3), $A_H$ augments the left-hand side of (6) and $w_s'$ replaces $w_s$. Thus an increase in $A_H$ has the same effects on $\omega$, $\omega^*$, $\mathsf{T}_H$ and $\mathsf{T}_F$ as a shock that is Foreign-friendly and factor-neutral in both countries (i.e. $\Gamma < 0$, $\psi_H = \psi_F = 0$).

### A4.3 $\psi_H^n$, $\psi_F^n$ and $\Gamma^n$ have the same signs as $\psi_H$, $\psi_F$ and $\Gamma$ under (7.1) and (7.2)

It is sufficient to show that $\psi_H^n > 0 \Leftrightarrow \psi_H > 0$. The proofs for the other cases are similar.

**Proof**: Notice that $\mu(\omega_0; O_{H,1}, b') = \int_{O_{H,1}} b'(z)\theta_s(z)dz \,/\, \int_{O_{H,1}} b'(z)\theta_u(z)dz$

$$= \int_{O_{H,0}} b'(z)\theta_s(z)dz \,/\, \int_{O_{H,0}} b'(z)\theta_u(z)dz \qquad (b'(z) = 0\ \forall z \in O_{H,0} \text{ but } \notin O_{H,1})$$

$$= (\int_{O_{H,0}} \rho\, b(z)\theta_s(z)dz + \int_{O_{H,0}} \varepsilon(z)\theta_s(z)dz)\,(\int_{O_{H,0}} \rho\, b(z)\theta_u(z)dz + \int_{O_{H,0}} \varepsilon(z)\theta_u(z)dz)^{-1}$$

$$= \int_{O_{H,0}} b(z)\theta_s(z)dz \,/\, \int_{O_{H,0}} b(z)\theta_u(z)dz \qquad \text{(by } \mathbf{(7.1)}\text{)}$$

$$= \mu(\omega_0; O_{H,0}, b)$$

Thus $\psi_H^n > 0 \Leftrightarrow \mu(\omega_0; N_H, b') > \mu(\omega_0; O_{H,0}, b) \Leftrightarrow \mu(\omega_0; N_H \cup O_{H,1}, b') > \mu(\omega_0; O_{H,0}, b) \Leftrightarrow \psi_H > 0$. Θ

## Appendix 5: Simple Case

It is straightforward to show that in the simple case considered in Section 5, (2) ~ (6) become:

$$h(g) = 0;\ h(g) \equiv ln[MC(g)/MC^*(g)] = ln(w_s/w_s^*) + ln[A_s(g)/A_s^*(g)] \quad (A5.1)$$

$$1/\omega = \mu(\omega,g)/s;\ \mu(\omega,g) \equiv \int_g^1 \theta_s(z)dz/\int_g^1 \theta_u(z)dz \quad (A5.2)$$

$$1/\omega^* = \mu^*(\omega^*,g)/s^*;\ \mu^*(\omega^*,g) \equiv \int_0^g \theta_s^*(z)dz/\int_0^g \theta_u^*(z)dz \quad (A5.3)$$

$$g/(1-g) = [w_s^*(\omega^* + s^*)] / [w_s(\omega + s)] \quad (A5.4)$$

Since the hole $[z_n,\ z_n+dz_n]$ has infinitesimal length, the equilibrium is well approximated by (A5.1) ~ (A5.4). Substitute (A5.4) into (A5.1) and differentiate (A5.1) and the logs (A5.2) and (A5.3):

$$\begin{bmatrix} a_{11}(+) & a_{12}(-) & a_{13}(-) \\ a_{21}(+) & a_{22}(+) & 0 \\ a_{31}(+) & 0 & a_{33}(+) \end{bmatrix} \begin{bmatrix} dg \\ d\omega \\ d\omega^* \end{bmatrix} = \begin{bmatrix} b_\Gamma(+) \\ -b_\psi \\ 0 \end{bmatrix} \quad (A5.5)$$

$$b_\Gamma = dz_n/(1-g),\ b_\psi = [\theta_s(z_n)/\int_g^1 \theta_s(z)dz - \theta_u(z_n)/\int_g^1 \theta_u(z)dz]dz_n$$

$$a_{11} = 1/[g(1-g)] - \partial h(g)/\partial g,\ a_{12} = 1/(\omega+s) - 1/(\omega+s(g)),\ a_{13} = 1/(\omega^*+s^*(g)) - 1/(\omega^*+s^*),$$

$$a_{21} = \theta_u(g)/\int_g^1 \theta_u(z)dz - \theta_s(g)/\int_g^1 \theta_s(z)dz,\ a_{22} = \partial[ln(\omega\mu(.))]/\partial\omega,$$

$$a_{31} = -\theta_u(g)/\int_0^g \theta_u(z)dz + \theta_s(g)/\int_0^g \theta_s(z)dz,\ a_{33} = \partial[ln(\omega^*\mu^*(.))]/\partial\omega^*$$

1. $a_{11}$ measures (1). the effect of a small change in $g$ on the demand for Foreign goods relative to Home goods ($\Lambda(.)$) (the first term); as $g$ goes up, $\Lambda(.)$ goes up. (2). The effect of a small change in $g$ on the relative marginal cost (the second term); as $g$ goes up, $h(g)$ goes down (by Property 2). 2. $-a_{12}$ measures the effect of a small change in $\omega$ on $MC(g)$; as $\omega$ goes up, $MC(g)$ goes up. 3. $a_{13}$ measures the effect of a small change in $\omega^*$ on $MC^*(g)$; as $\omega^*$ goes up, $MC^*(.)$ goes down. 4. $a_{21}$ measures the effect of a small change in $g$ on $\mu(.)$, the average skilled-labor intensity in Home; as $g$ goes up, $\mu(.)$ increases. 5. $a_{22}$ is negatively related to the slope of the relative demand curve for skilled labor in Home; $a_{22} > 0$ by Property 1. 6. $a_{31}$ is analogous to $a_{21}$, and $a_{33}$ analogous to $a_{22}$.

By Cramer's Rule, $d\omega = [(-a_{11}a_{33} + a_{13}a_{31})b_\psi - a_{21}a_{33}b_\Gamma]/D$ and $dg = (a_{12}a_{33}b_\psi + a_{22}a_{33}b_\Gamma)/D;\ D \equiv a_{11}a_{22}a_{33} - a_{13}a_{31}a_{22} - a_{12}a_{21}a_{33} > 0$. Let $c_1 = -a_{21}a_{33}/(a_{13}a_{31} - a_{11}a_{33})$, we have (9.1), and (9.2) is straightforward to show.

When there is a Foreign new good $z_n^*$ filling up the hole $[z_n^*,\ z_n^*+dz_n^*]$, (A5.5) becomes:

$$\begin{bmatrix} a_{11}(+) & a_{12}(-) & a_{13}(-) \\ a_{21}(+) & a_{22}(+) & 0 \\ a_{31}(+) & 0 & a_{33}(+) \end{bmatrix} \begin{bmatrix} dg \\ d\omega \\ d\omega^* \end{bmatrix} = \begin{bmatrix} b_\Gamma \\ -b_\psi \\ -b_\psi^* \end{bmatrix}$$

$$b_\Gamma = dz_n/(1-g) - dz_n^*/g;\ b_\psi^* = [\theta_s(z_n^*)/\int_0^g \theta_s(z)dz - \theta_u(z_n^*)/\int_0^g \theta_u(z)dz]dz_n^*$$

The other coefficients are the same as in (A5.5). Applying Cramer's rule: $d\omega = [(-a_{11}a_{33} + a_{13}a_{31})b_\psi - a_{13}a_{21}b_\psi^* - a_{21}a_{33}b_\Gamma]/D$, $d\omega^* = [-a_{12}a_{31}b_\psi + (-a_{11}a_{22} + a_{21}a_{12})b_\psi^* - a_{22}a_{31}b_\Gamma]/D$ and $dg = (a_{12}a_{33}b_\psi + a_{13}a_{22}b_\psi^* + a_{22}a_{33}b_\Gamma)/D;\ D = a_{11}a_{22}a_{33} - a_{13}a_{31}a_{22} - a_{12}a_{21}a_{33} > 0$.

## Appendix 6: Concrete Examples

### A6.1. Preparation

When $\mathsf{T} = [0, 1\text{-}dz)$, $N_H = [1\text{-}dz, 1]$ and $N_F = \varnothing$, denote the common good by $g_c$ rather than $g$. By Appendix 3 of Dornbusch, Fischer and Samuelson (1980):

$$\int_{g_c}^{1} b(z)dz \,/\, \int_{0}^{g_c} b(z)dz = [(\omega+s)/(\omega^*+s^*)][A_s^*(g_c)/A_s(g)] \tag{A6.1}$$

$$\int_{g_c}^{1} b(z)(s-s(z))[\omega+s(z)]^{-1}dz = 0 \tag{A6.2}$$

$$\int_{0}^{g_c} b(z)(s^*-s^*(z))[\omega^*+s^*(z)]^{-1}dz = 0 \tag{A6.3}$$

Differentiating the equilibrium conditions yields:

$$\begin{bmatrix} a_{11}(+) & a_{12}(-) & a_{13}(-) \\ a_{21}(+) & a_{22}(+) & 0 \\ a_{31}(+) & 0 & a_{33}(+) \end{bmatrix} \begin{bmatrix} dg_c \\ d\omega \\ d\omega^* \end{bmatrix} = \begin{bmatrix} b_{11}(+) \\ b_{21}(-) \\ 0 \end{bmatrix}$$

$a_{11} = b(g_c)/\int_0^{g_c} b(z)dz + b(g_c)/\int_{g_c}^{1} b(z)dz - \partial h(g)/\partial g$, $a_{12} = 1/(\omega+s) - 1/(\omega+s(g_c))$, $a_{13} = 1/(\omega^*+s^*(g_c)) - 1/(\omega^*+s^*)$, $b_{11} = b(1)/\int_{g_c}^{1} b(z)dz$, $a_{21} = [b(g_c)/(\omega+s(g_c))]\,(s - s(g_c))$, $a_{22} = \int_{g_c}^{1} [b(z)/(\omega+s(z))^2]\,[(\partial s(z)/\partial\omega)\,(\omega+s) + (s - s(z))]dz$, $b_{21} = [b(1)/(\omega+s(1))]\,(s - s(1))$, $a_{31} = -[b(g_c)/(\omega^*+s^*(g_c))]\,(s^* - s^*(g_c))$, $a_{33} = \int_0^{g_c} [b(z)/(\omega^*+s^*(z))^2]\,[(\partial s^*(z)/\partial\omega^*)\,(\omega^*+s^*) + (s^* - s^*(z))]dz$.

By Cramer's Rule, $dg_c = a_{33}\,(b_{11}a_{22} - a_{12}b_{21})/D$ with $D = a_{11}a_{22}\,a_{33} - a_{13}a_{31}a_{22} - a_{12}a_{21}a_{33} > 0$; thus $sgn(dg_c) = sgn(b_{11}a_{22} - a_{12}b_{21})$. Let $\mu_S \equiv \int_{g_c}^{1} \theta_s(z)\,b(z)dz$, $\mu_U \equiv \int_{g_c}^{1} \theta_u(z)\,b(z)dz$, and $E_H \equiv \int_{g_c}^{1} b(z)\,dz$. Then $\mu_S + \mu_U = E_H$, $s = \omega\mu_S/\mu_U$ (by (4)) and $s(z) = \omega\theta_s(z)/\theta_u(z)\ \forall z$. Thus $b_{11} = b(1)/E_H$, $a_{12} = [\mu_U - E_H\theta_u(g_c)]/[\omega E_H]$ and $b_{21} = b(1)[\theta_u(1)E_H - \mu_U]/\mu_U$.

**A6.2 If $\eta(z) \geq 1\ \forall z$, $dg_c > 0$.**

**Proof**: Let $v(z) \equiv \partial s(z)/\partial\omega - \theta_s(z)/\theta_u(z)$; then $v(z) \geq 0$ if $\eta(z) \geq 1$ since $\eta(z) = \partial ln s(z)/\partial ln\,\omega$. Thus:

$$a_{22} = \int_{g_c}^{1} \frac{b(z)}{[\omega+s(z)]^2}\left(\frac{\partial s(z)}{\partial\omega}\,\omega - s(z)\right)dz + \int_{g_c}^{1} \frac{b(z)}{[\omega+s(z)]^2}\left(\frac{\partial s(z)}{\partial\omega} + 1\right)s\,dz$$

$$= \int_{g_c}^{1} \frac{b(z)s(z)}{[\omega+s(z)]^2}\,(\eta(z) - 1)dz + \int_{g_c}^{1} \frac{b(z)sv(z)}{[\omega+s(z)]^2}\,dz + \int_{g_c}^{1} \frac{b(z)s}{[\omega+s(z)]^2}\left(\frac{\theta_s(z)}{\theta_u(z)} + 1\right)dz$$

$$= A_{22} + \mu_U s/\omega^2 \left(A_{22} \equiv \int_{g_c}^{1} \frac{b(z)}{[\omega+s(z)]^2}\,[s(z)(\eta(z)-1) + v(z)s]dz > 0 \text{ since } \eta(z) \geq 1\right) = A_{22} + \mu_S/\omega$$

$$\therefore\ b_{11}a_{22} - a_{12}b_{21} = [b(1)/E_H]A_{22} + b(1)\mu_S[E_H\omega]^{-1}\left[1 - \frac{(\mu_U - \theta_u(g_c)E_H)}{\mu_S\mu_U}\,(E_H\theta_u(1) - \mu_U)\right]$$

Since $\mu_S + \mu_U = E_H$ and $\theta_u(1) < 1$, $\therefore\ 0 < E_H\theta_u(1) - \mu_U < \mu_S$ and $0 < \mu_U - E_H\theta_u(g_c) < \mu_U$,

$\therefore [1-\frac{(\mu_U - \theta_u(g_c)E_H)}{\mu_S \mu_U}(E_H\theta_u(1) - \mu_U)] > 0. \therefore b_{11}a_{22} - a_{12}b_{21} > 0$ and $dg_c > 0.$ Θ

**A6.3. If $\eta(z) = 0 \ \forall z$, $dg_c < 0$ if $\theta_u(z) = \beta_1$ for $z \in [g_c, x_1)$, $\beta_2$ for $z \in [x_1, x_2)$, and $\beta_3$ for $z \in [x_2, t]$, where $\beta$'s depend on $\omega$ and $\beta_1 > \beta_2 > \beta_3$.**

**Proof:** Since $\eta(z) = 0 \ \forall z$, $\partial s(z)/\partial\omega = 0$, $a_{22} = \int_{g_c}^{1} \frac{b(z)}{[\omega + s(z)]^2}(s\text{-}s(z))dz = \int_{g_c}^{1} \frac{b(z)\theta_u^2(z)}{\omega}(\frac{E_H}{\mu_U} - \frac{1}{\theta_u(z)})dz$

$= \int_{g_c}^{1} \frac{b(z)\theta_u(z)}{\omega\mu_U}[E_H\theta_u(z) - \mu_U]dz$. Thus $b_{11}a_{22} - a_{12}b_{21} = \frac{b(1)}{E_H\omega\mu_U}\{\int_{g_c}^{1} \theta_u(z)b(z)[E_H\theta_u(z) - \mu_U]dz + [\theta_u(1)E_H$

$- \mu_U][E_H\theta_u(g_c) - \mu_U]\}$. By (A4.2), $\int_{g_c}^{1} b(z)[\theta_u(z)E_H - \mu_U]dz = 0$. Using the assumption about $\theta_u(z)$, $b_{11}a_{22} -$

$a_{12}b_{21} = \frac{b(1)}{E_H\omega\mu_U}[(\beta_1 - \beta_3)(E_H\beta_1 - \mu_U)\int_{g_c}^{x_1} b(z)dz + (\beta_2 - \beta_3)(E_H\beta_2 - \mu_U)\int_{x_1}^{x_2} b(z)dz + (E_H\beta_3 - \mu_U)(E_H\beta_1 - \mu_U)]$.

Since $E_H\beta_3 - \mu_U = (\beta_3 - \beta_1)\int_{g_c}^{x_1} b(z)dz + (\beta_3 - \beta_2)\int_{x_1}^{x_2} b(z)dz$,

$\therefore b_{11}a_{22} - a_{12}b_{21} = \frac{b(1)}{E_H\omega\mu_U}(\beta_2 - \beta_3)(\int_{x_1}^{x_2} b(z)dz)E_H(\beta_2 - \beta_1) < 0 \therefore dg_c < 0$ Θ

**A6.4 The case of a continuum Ricardian model**

In this case, the equilibrium conditions collapse to:

$\int_0^{g_c} b(z)dz / \int_{g_c}^{1} b(z)dz = (w_s^* L_s^*)/(w_s L_s)$; $w_s/w_s^* = a(g_c)$; $a(z) \equiv a_s^*(z)/a_s(z)$ and increases in $z$

Let $w_s^* = 1$; totally differentiating these equations yields: $c_{11}dg_c + dlnw_s = c_{12}$ and $dlnw_s = c_{13}dg_c$ where $c_{11}$ $= b(g_c)[1/\int_0^{g_c} b(z)dz + 1/\int_{g_c}^{1} b(z)dz] > 0$, $c_{12} = b(1)/\int_{g_c}^{1} b(z)dz > 0$, and $c_{13} = \partial lna(g_c)/\partial g_c > 0$. $\therefore dg_c =$ $c_{12}/(c_{11} + c_{13}) > 0$, $\therefore dlnw_s > 0$, and $dMC(g_c) = a_s(g_c)\Delta w_s > 0$.

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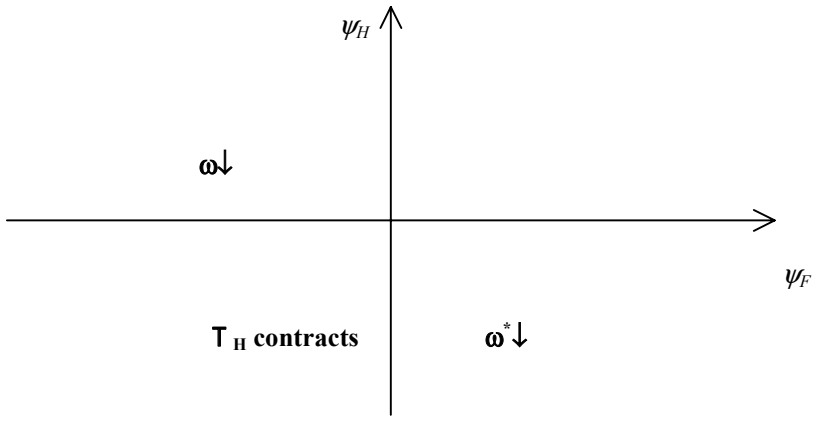



**Figure 3.1. Home-friendly shock (Γ>0)**

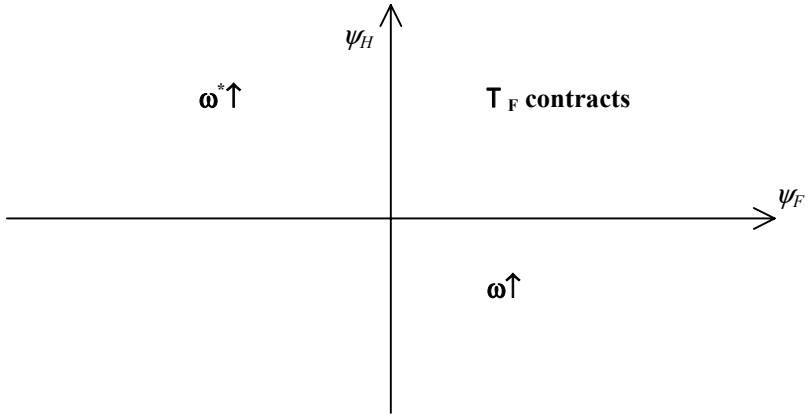



**Figure 3.2. Foreign-friendly shock (Γ<0)**

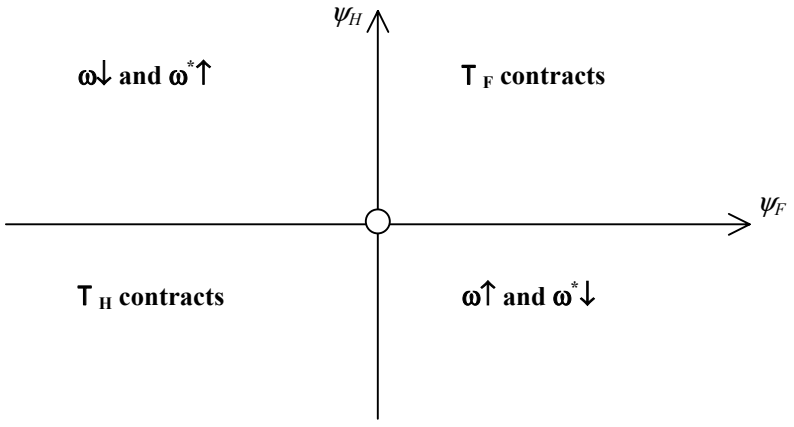



**Figure 3.3. Country-neutral shock (Γ=0)**

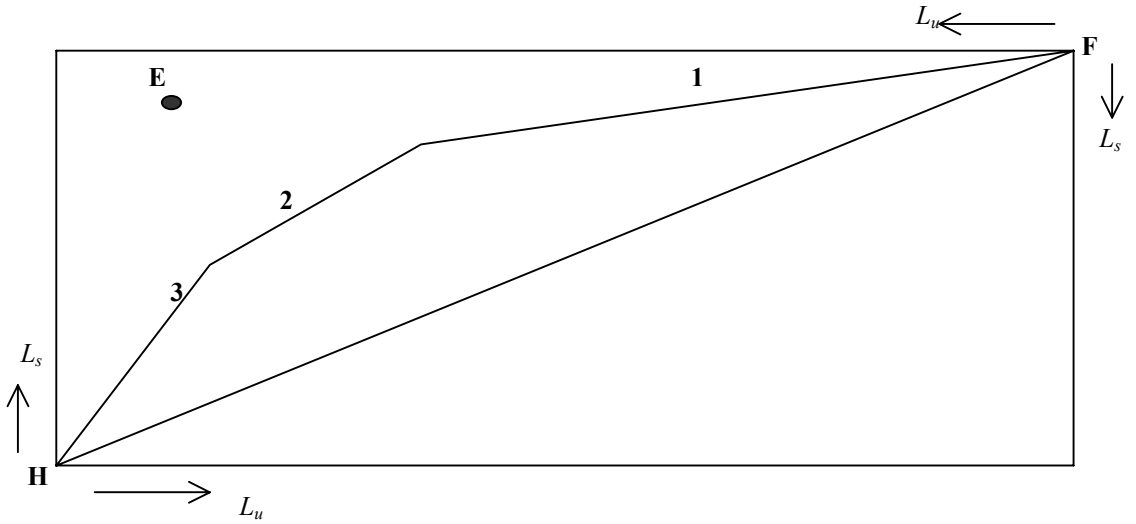



**Figure 3.4. The Integrated World Economy (IWE)**

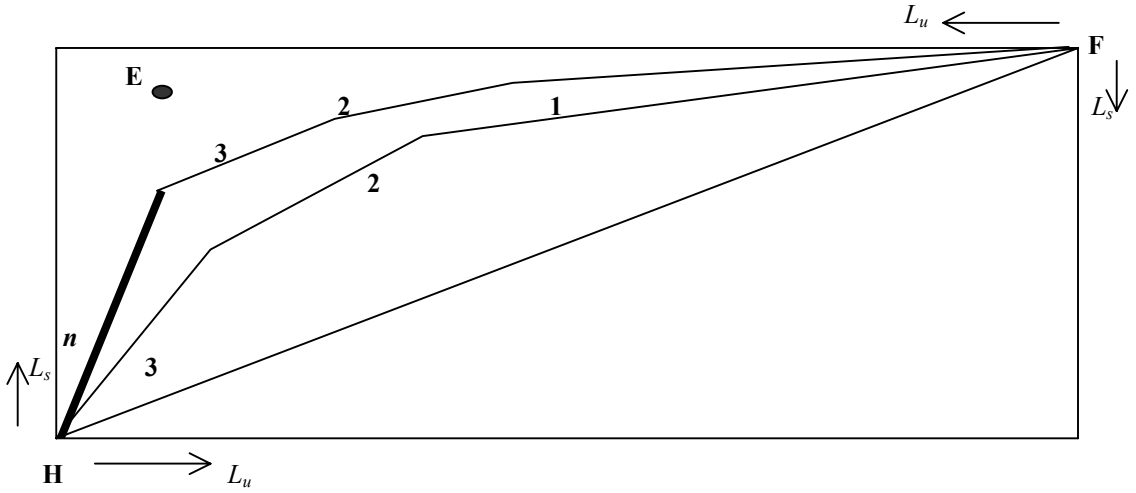



**Figure 3.5. Convergence ($\omega_{IWE}\downarrow$)**

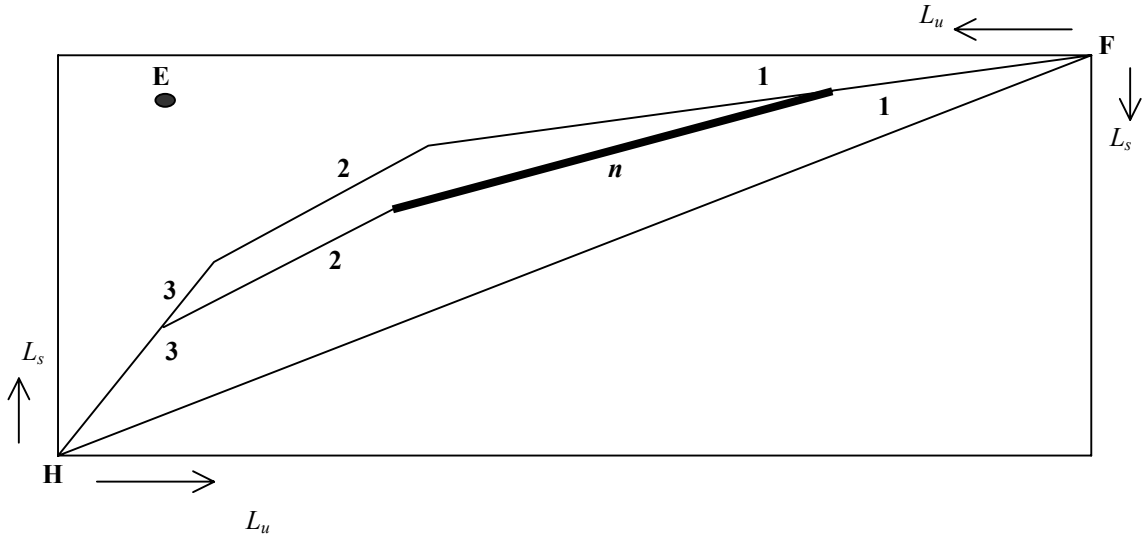



**Figure 3.6. Divergence ($\omega_{IWE}$ unchanged)**

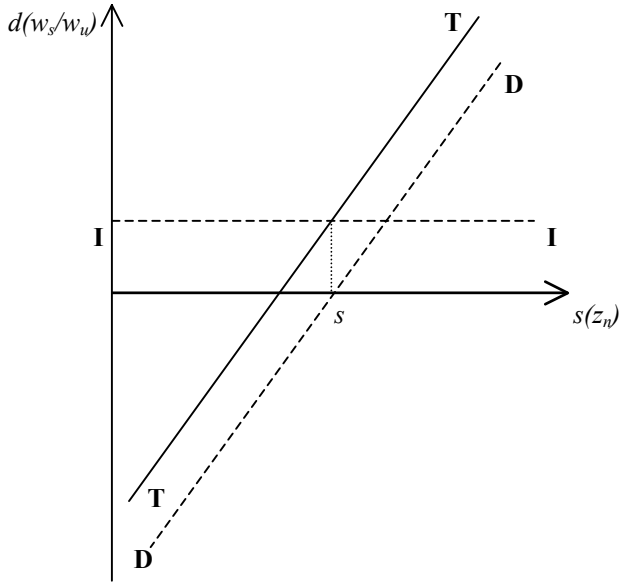



**Figure 5.1 The effect on $w_s/w_u$**

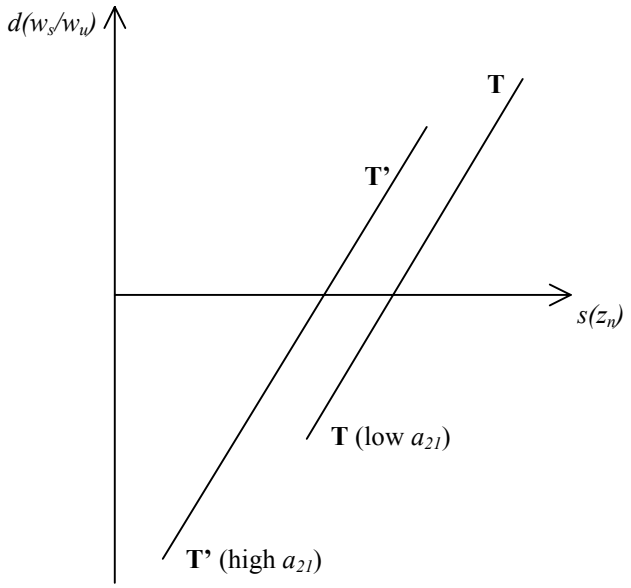



**Figure 5.2 The role of $a_{21}$**

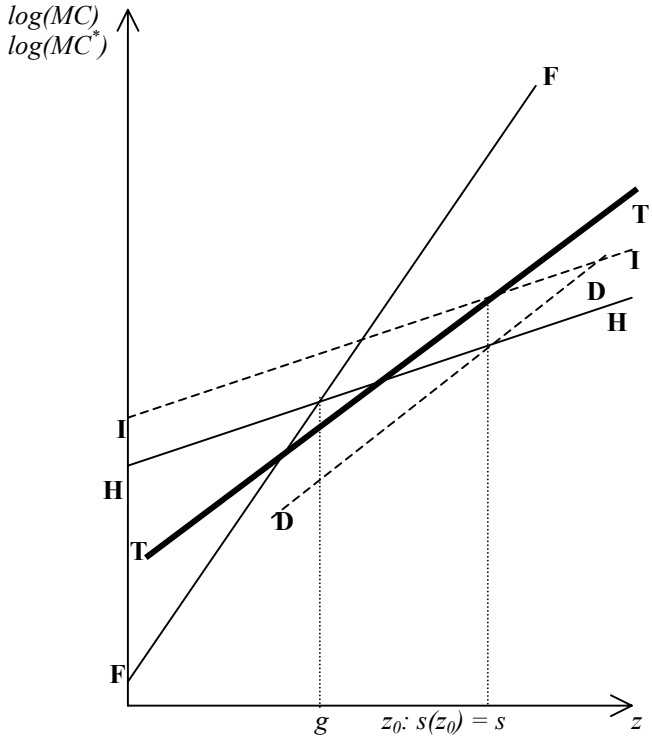



**Figure 5.3 The effect on $g$: $b_\psi > 0$**

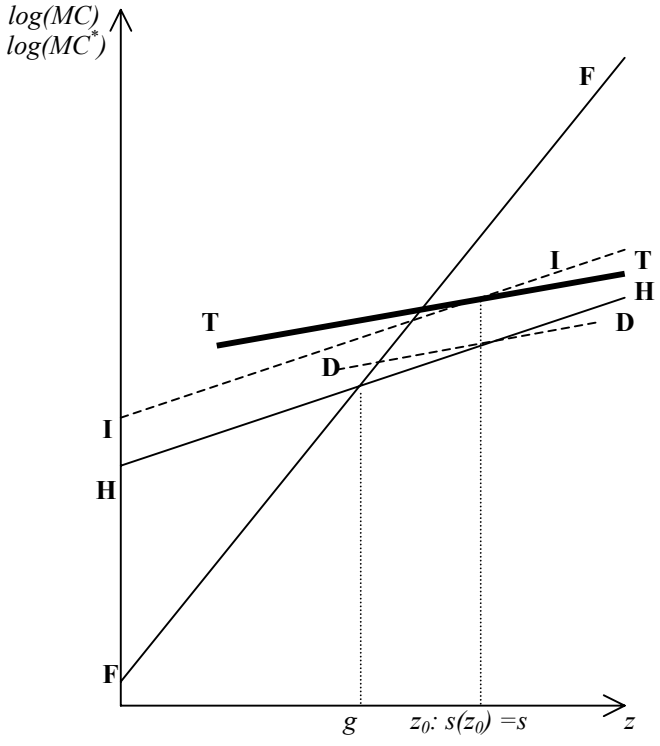



**Figure 5.4 The effect on $g$: $b_\psi < 0$**

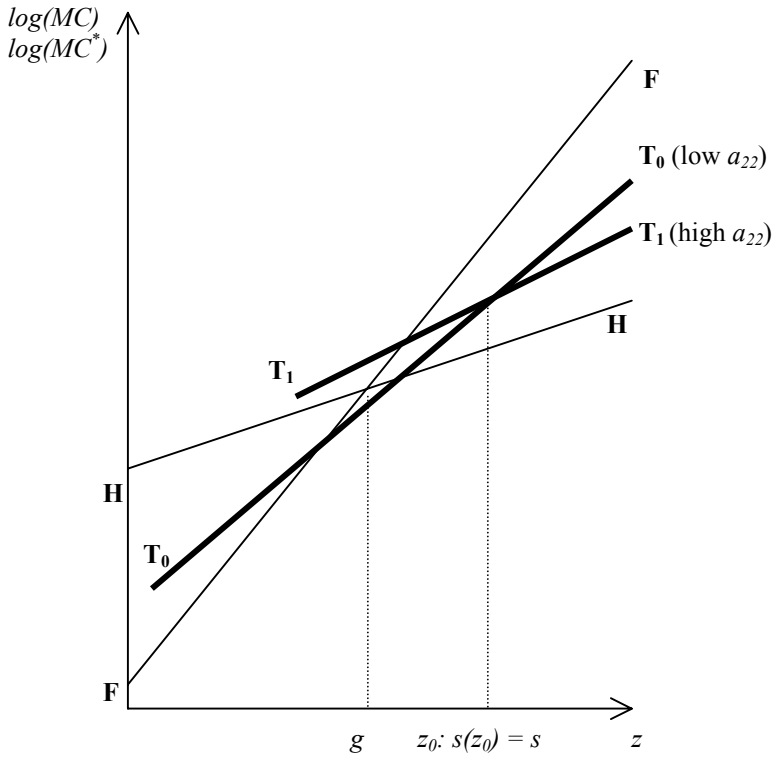



**Figure 5.5 The role of $a_{22}$ : $b_\psi > 0$**