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Abstract

AN ANALYSIS OF SYSTEM RECOVERY VOLTAGES AND METHODS FOR REDUCING THEIR SEVERITY

by Roger C. Quisenberry

The purpose of this analysis is to set up a mathematical procedure for predetermination of the recovery voltage waveforms which appear across the terminals of an arc interrupting device immediately following arc extinction. The analysis is developed with a three-phase power system in mind and the results are checked at each point by means of an experimental miniature power system. Besides establishing a procedure for analysis, the effects of various impedance networks shunted across the arc interrupter are investigated, and two effective means of reducing recovery voltage severity are described and analyzed.

In Section I, the mathematical analysis is developed using the Laplace Transform and the principle of superposition.

In Section II of the study, a mathematical analysis is made of recovery voltages in nine different cases involving various fault locations, different arrangements of system elements, and several types of arc-interrupter shunting impedances.

The Appendix contains the experimental analysis which offers a convenient check on the mathematically computed

waveforms. The same nine cases are studied and experimental waveforms of recovery voltages are photographed directly from a cathode ray oscilloscope. This recording of waveforms is made possible by means of a multiple pole synchronous switch, carefully synchronized and phased with the supply voltage. With the switch simulating faults and arc interrupters, the transient recovery voltage is repeated many times per second and results in a steady picture on the face of the oscilloscope.

Both the experimental and mathematical analyses involve the same assumptions: (1) Arc voltage drop prior to arc interruption is neglected, (2) Conduction current following current-zero is neglected, (3) Fault impedance is neglected, (4) An infinite-capacity power bus is assumed.

For the most part, the results of the experimental and mathematical analyses checked quite well. Two of the several cases involving impedance networks shunted across the arc interrupter resulted in effective reduction of recovery voltage severity: (1) Series resistance and capacitance connected in shunt with the arc interrupter, and (2) A pure resistance shunt adjusted for critical damping or slight overdamping.

The writer wishes to express his appreciation to the members of his committee for their very capable criticism and for their considerable patience and painstaking care exerted in improving and perfecting this paper.

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LIST OF SYMBOLS

$v(t)$ = system recovery voltage developed across circuit breaker terminals.

$\mathcal{L}$ = an operator representing the Unilateral Laplace Transform of a function.

$\mathcal{L}^{-1}$ = an operator representing the inverse of the Unilateral Laplace Transform of a function.

S = the complex variable, $x + iy$.

$i(t)$ = the steady state fault current flowing through the circuit breaker prior to arc interruption time.

$V(S) = \mathcal{L}[v(t)]$

$I(S) = \mathcal{L}[i(t)]$

$Z(S)$ = complex operational network impedance.

ω = fundamental power frequency in radians per second.

$\alpha, \beta, \gamma, \delta$ = constants, real as specified.

E_m = peak value of fundamental steady state system voltage from line to neutral.

L, C, R = circuit parameters, assumed constant.

$\cong$ = "very nearly equal to", within slide-rule accuracy, 2% error.

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INTRODUCTION

As the circuit breaker terminals separate in an attempted short-circuit current interruption, a column of hot ionized gas called an arc is formed between the contacts, thus prolonging the life of the short-circuit current beyond the time of contact separation. Normally the arc is extinguished within a few fundamental-frequency cycles at a zero point of the alternating current wave. The interruption of the arc is made easier due to the fact that there are two normal zero points in each cycle of the alternating-current wave.

Just prior to arc interruption the voltage across circuit breaker contacts equals the arc voltage, and at the instant of arc extinction the voltage attempts to recover to the generated value. This transient recovery voltage rises at a rate determined by the ability of the circuit to charge the system capacitance existing between the circuit breaker terminals and ground.

Once the arc has been interrupted, the rate of rise of the system voltage which builds up across the separating contacts becomes very important. For satisfactory interruption, it is essential that the dielectric strength of the space between the contacts increases at a more rapid rate than the buildup of this voltage. Otherwise all the complicating factors of restriking arcs will be added to the interruption problem.

The voltage buildup across separating terminals consists of the superposition of voltages of normal power system frequency, along with damped transient voltages of various natural frequencies, usually much higher than the generated frequency. The composition of this complex waveform depends upon a number of factors such as fault location, arrangement and size of the power system units (transformers, current limiting reactors, lines, rotating machines, etc.), and upon the capacitances of the various units to ground.

The time-zero point in recovery voltage studies throughout this paper will be taken at the current zero point corresponding to the instant of arc extinction. With the exception of one specialized study in the Appendix, the arc voltage immediately prior to arc extinction will be taken as zero. The error introduced by this approximation appears mainly in fixing the exact point on the fundamental voltage wave at which arc extinction occurs. The chief obstacle in the way of including arc voltage in the analysis is the fact that the arc represents a time-varying element and in general the arc voltage drop is an unknown function of time.

A few words should perhaps be said about the actual process of arc extinction. This phenomenon has been very well covered in references (2) and (3). Briefly, arc extinction normally occurs in two stages. In the first stage, arc resistance is increased to a high value by rapid

lengthening and deionization of the path. Second, the dielectric strength of the arc space is raised so high by deionizing processes inside the breaker that the arc will not normally be reignited by the system recovery voltage. Dielectric strength is finite [9] at all times and depends on the amount of ionization remaining and the rate at which the circuit breaker reduces the ionization. This strength increases toward the end of a half cycle and may reach high values just before a current zero. Unless at some time the recovery voltage exceeds the dielectric strength, the latter continues to increase after current zero, and the transition to an insulating state is completed. Normally, in modern high speed circuit breakers approximately three cycles are required between trip time and arc extinction [2]. Regardless of the point on the current cycle of the faulted phase at which tripping occurs, final arc extinction is delayed until a time very near to that of a normal current zero. The number of cycles that intervene between the trip time and final arc extinction depends of course upon the circuit breaker characteristics (Fig. 1a).

In this paper, recovery voltage waveforms under various operating conditions are first determined analytically and then checked experimentally. In the experimental analysis, a specially constructed multipolar switch, synchronized and properly phased with the system voltage, is used to simulate fault application and circuit breaker operation. The

Fig. 1a-- Typical Sequence of Events in High Speed Circuit Breaker Fault Interruption.

Fig. 1b-- Sequence of Events in Synchronous Switch Analysis of Recovery Voltages.

FIGURE 1a

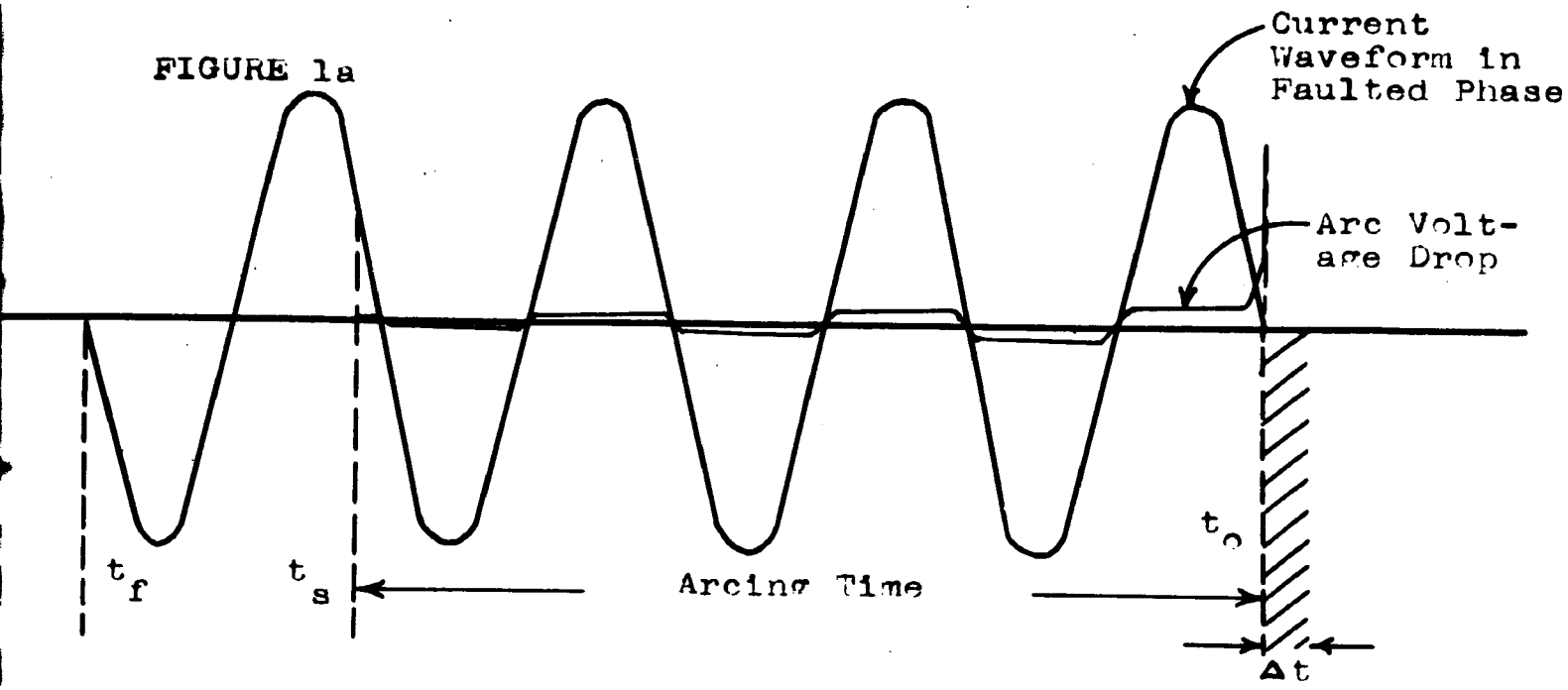
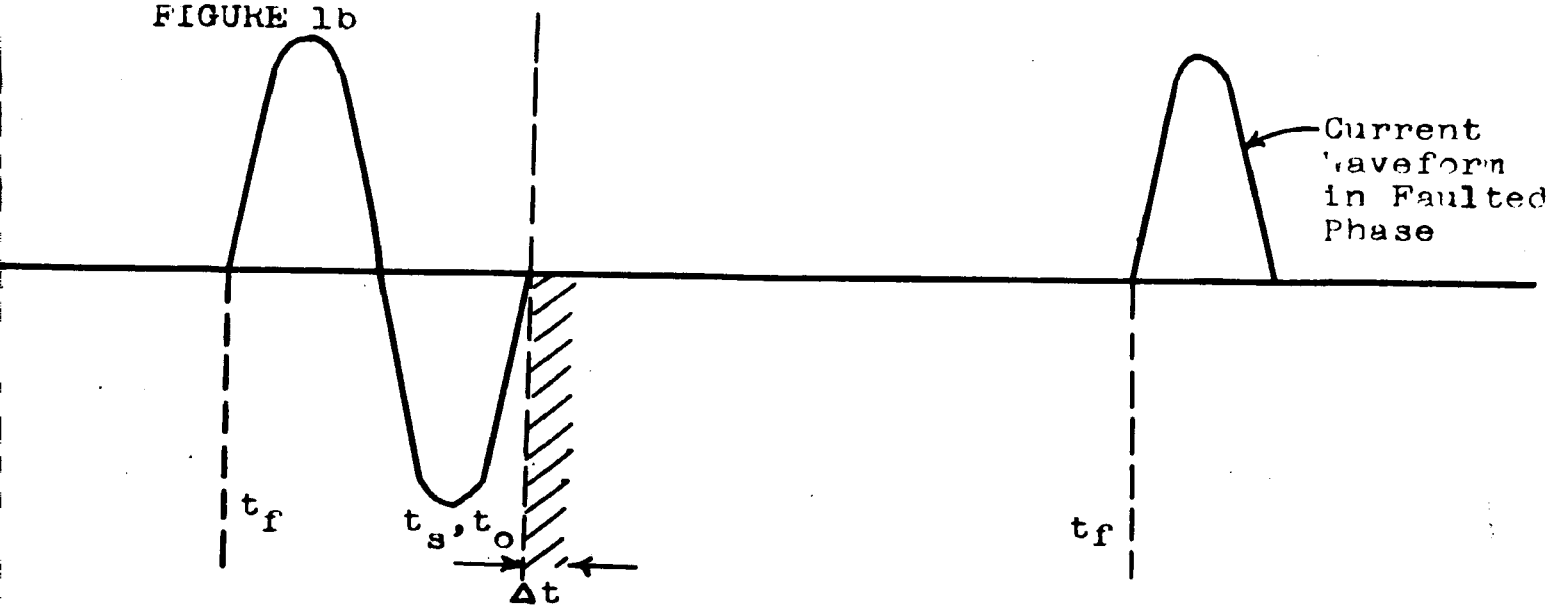


FIGURE 1b



t_f---- Fault Occurrence Time

t_s---- Contact Separation Time

t_o---- Arc Interruption Time (Time Zero)

Δt--- Recovery Voltage Time Interval.

phasing is carefully adjusted so that the "circuit breaker" switch opens precisely at the point on the current cycle where the steady state current in the faulted phase is going through a normal zero. The "fault" switch is closed a full cycle ahead of this instant to allow an abundance of time for any switching transients to die out before the inspection period immediately following current interruption begins (Fig. 1b). A cathode ray oscilloscope is used to inspect the repeated transient of recovery voltage which develops across the "circuit breaker" switch terminals. This transient is repeated twenty times per second and results in a picture that can readily be photographed.

The same assumptions are adhered to in both the mathematical and experimental analyses. The arc voltage drop within the interrupting device prior to arc extinction is neglected, and the conduction current after arc extinction is assumed to be zero. All faults are assumed to be one-line-to-ground and of zero impedance, although a method is described in the Appendix for including the effect of a constant impedance fault. An infinite capacity bus is assumed throughout the paper primarily so that the generator internal impedance, made negligible in size by the infinite bus assumption, may be treated as a constant instead of a flux-decaying transient.

In making the mathematical analysis, the Laplace Transform method of operational mathematics simplifies the solution of the fourth, fifth, and sixth degree

differential equations which frequently appear in deriving expressions for recovery voltage. Mathematical tables [6] are used to translate functions of the complex variable S back to the real t-plane.

SECTION I - Mathematical Method used in the Analysis of Recovery Voltage

Since the analysis of recovery voltage is clearly a study of transient quantities, the Laplace Transform method is used in obtaining solutions to the differential equations which arise. Constant circuit parameters are assumed throughout, which allows the superposition of currents or voltages throughout each problem. If we assume that the sudden application of a fault can be represented analytically by a switch closing, while the extinction of an arc in a circuit breaker at an alternating current zero point can be represented by a switch opening [6], the problems involved in recovery voltage computations are closely related to those involving switching transients in alternating current networks. In solving the latter type of problem by the Laplace Transform method, the initial values of currents and voltages and their derivatives may be introduced into the equations in the process of the transformation. But frequently the determination of these initial values is quite difficult and may be avoided as described below:

In an electrical network suddenly modified by the opening of a switch, the principle of superposition may be applied. The voltage existing across the switch terminals after the switching operation may be considered to be composed of two components:

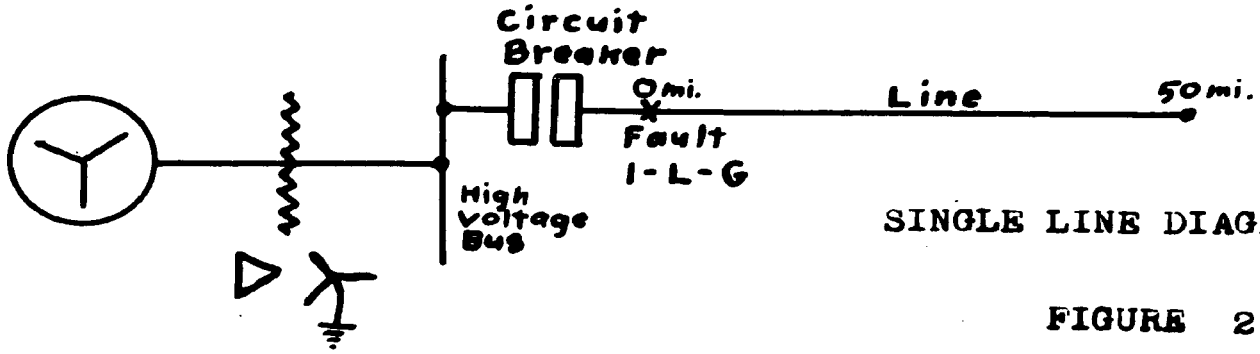
First, the branch voltage that would exist if no change had taken place. This component is clearly zero since the voltage existing across a closed switch is zero. Second, the branch voltage that would exist if the passive (no stored energy) network were suddenly excited by a current source connected into the network at the switching point and of polarity opposite to that of the current interrupted by the opening of the switch. Therefore, if V(S) represents the Laplace Transform of voltage developed across the switch after it opens, and if I(S) is the Laplace Transform of the steady state fault current interrupted by the switch opening, and finally if Z(S) represents the transform impedance of the network as viewed from the switch terminals, then

$$V(S) = I(S) \cdot Z(S)$$

and the system recovery voltage v(t) can be obtained by taking the inverse Laplace Transform of V(S). A knowledge of symmetrical component theory is of course essential in determining the steady-state fault current. The method is illustrated in the simple case of recovery voltage determination which follows.

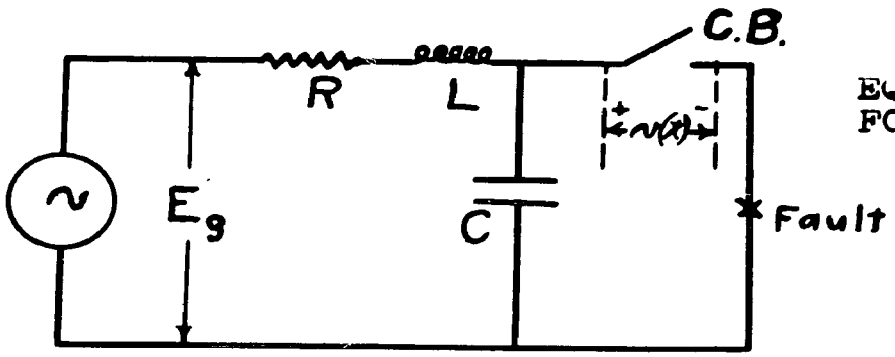
SECTION II- RECOVERY VOLTAGE STUDIES

CASE A- Fault at Input End of Transmission Line. No Corrective Measures Used.



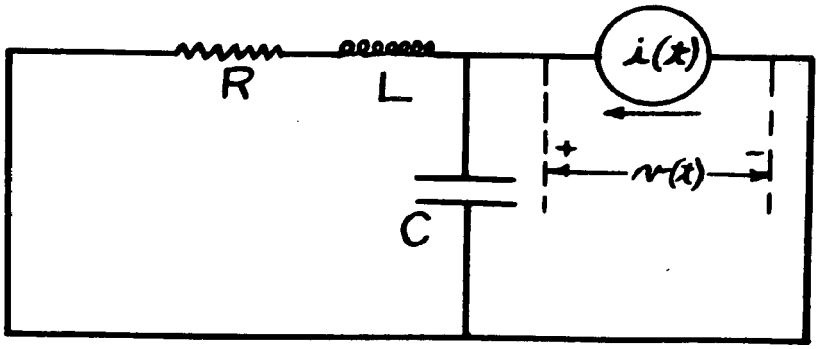
SINGLE LINE DIAGRAM

FIGURE 2a



EQUIVALENT CIRCUIT FOR FAULTED PHASE

FIGURE 2b



EQUIVALENT TRANSIENT CIRCUIT

FIGURE 2c

CASE A - Fault at Input End of Transmission Line.
No Corrective Measures Used.

In the preceding figure, R and L represent the impedance of the voltage source (generators and transformers), E_g is source phase voltage, C represents the stray capacitance of bus and transformer to ground, approximated by a lumped C. CB represents the circuit breaker, originally closed, but assumed to open on a normal zero point of the fault current through F. Then v(t) represents recovery voltage. In order to conform to values actually measured on a miniature 3-phase power system set up for experimentally verifying analytical results, the following parameters will be used in the computations:

R = 6 ohms (transformer)

L = 31.2 mh (transformer)

C = .02 uf (bus and transformer stray capacitance to ground)

E_g = 10 volts (maximum), assumed to be taken from an infinite bus

I_{max} = .758 amp

Let $i(t) = I_m \sin \omega t$

$$I(S) = \mathcal{L}\left[i(t)\right] = \frac{I_m \omega}{S^2 + \omega^2}$$

$$Z(S) = \frac{(S + \frac{R}{L})}{C(S^2 + \frac{RS}{L} + \frac{1}{LC})}$$

$$V(S) = I(S) \cdot Z(S)$$

$$= \frac{I_m \omega (S + \frac{R}{L})}{C(S^2 + \omega^2)\ (S^2 + \frac{RS}{L} + \frac{1}{LC})}$$

$$v(t) = \mathcal{L}^{-1}\left[V(S)\right]$$

$$= \frac{I_m}{C}\left[\frac{(\frac{R}{L})^2 + \omega^2}{(\frac{1}{LC} - \omega^2)^2 + (\frac{R\omega}{L})^2}\right]^{\frac{1}{2}} \sin(\omega t + \psi_1)$$

$$+ \frac{I_m \omega}{C\sqrt{1 - \frac{R^2C}{4L}}} \cdot \frac{1}{\left[(\frac{1}{LC} - \omega^2)^2 + (\frac{R\omega}{L})^2\right]^{\frac{1}{2}}} \cdot e^{-\frac{R}{2L}t} \cdot$$

$$\cdot \sin\left[\sqrt{\frac{1}{LC} - (\frac{R}{2L})^2} \cdot t - \psi_2\right]$$

where $\psi_1 = \tan^{-1}\frac{\omega L}{R} - \tan^{-1}\frac{\omega R}{\frac{1}{C} - \omega^2 L}$

and $\psi_2 = \tan^{-1}\sqrt{\frac{4L}{R^2C} - 1}$

$$- \tan^{-1}\frac{\sqrt{\frac{1}{LC} - (\frac{R}{2L})^2}}{\frac{1}{RC} - \frac{R}{2L} - \frac{\omega^2 L}{R}}$$

Substitution of

$$I_m = \frac{E_m}{\sqrt{R^2 + \omega^2 L^2}} \text{ , gives}$$

$$v(t) = \frac{E_m \sin(\omega t + \psi_1)}{\left[1 + \omega^4 L^2 C^2 - 2\omega^2 LC + R^2\omega^2 C^2\right]^{\frac{1}{2}}}$$

$$+ \frac{E_m \omega e^{-\frac{R}{2L} t} \sin\left[\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2} \cdot t - \psi_2\right]}{C\left[\left(1 - \frac{R^2 C}{4L}\right)(R^2 + \omega^2 L^2)\right]^{\frac{1}{2}}\left[\left(\frac{1}{LC} - \omega^2\right)^2 + \left(\frac{R\omega}{L}\right)^2\right]^{\frac{1}{2}}}$$

where $\psi_1 = \tan^{-1}\frac{\omega L}{R} - \tan^{-1}\frac{\omega R}{\frac{1}{C} - \omega^2 L}$

and $\psi_2 = \tan^{-1}\sqrt{\frac{4L}{R^2 C} - 1} - \tan^{-1}\frac{\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}}{\frac{1}{RC} - \frac{R}{2L} - \frac{\omega^2 L}{R}}$

The above expression may be considerably simplified by several approximations which give results within slide rule accuracy if reasonable values for circuit parameters are used.

$$v(t) \cong E_m \sin(\omega t + \psi_1)$$

$$+ \frac{E_m \omega L}{\sqrt{R^2 + (\omega L)^2}} e^{-\frac{R}{2L} t} \sin\left[\frac{1}{\sqrt{LC}} t - \psi_2\right]$$

where $\psi_1 \cong \tan^{-1}\frac{\omega L}{R}$

and $\psi_2 \cong \tan^{-1}\frac{2}{R}\sqrt{\frac{L}{C}} \cong 90°$

If we use experimental values from the Appendix,

$$\omega = 377 \text{ rads./sec.}$$
$$R = 6\ \Omega$$
$$L = 31.2 \text{ mh}$$
$$C = .02 \text{ uf}$$
$$E_m = 10 \text{ volts}$$

$$v(t) \cong 10 \sin (377t + 63°5') + 8.95\, e^{-96t} \sin (40{,}000t - 90°).$$

This relationship is plotted in the following Figure 3. An important check which holds for all recovery voltage functions in cases where arc voltage drop is neglected is that

$$v(0^+) = 0$$

As a first approximation, assume R = 0.

Then $$V(S) = \frac{I_m \omega}{C(S^2 + \omega^2)} \cdot \frac{S}{\left[S^2 + \frac{1}{LC}\right]}$$

and $$v(t) = \mathcal{L}^{-1}\left[V(S)\right]$$

$$= \left[\frac{I_m}{\frac{1}{\omega L} - C\omega}\right] \left(\cos \omega t - \cos \frac{1}{\sqrt{LC}}\, t\right)$$

again note that the important check holds,

$$v(0^+) = 0$$

This check is an example of the well known physical

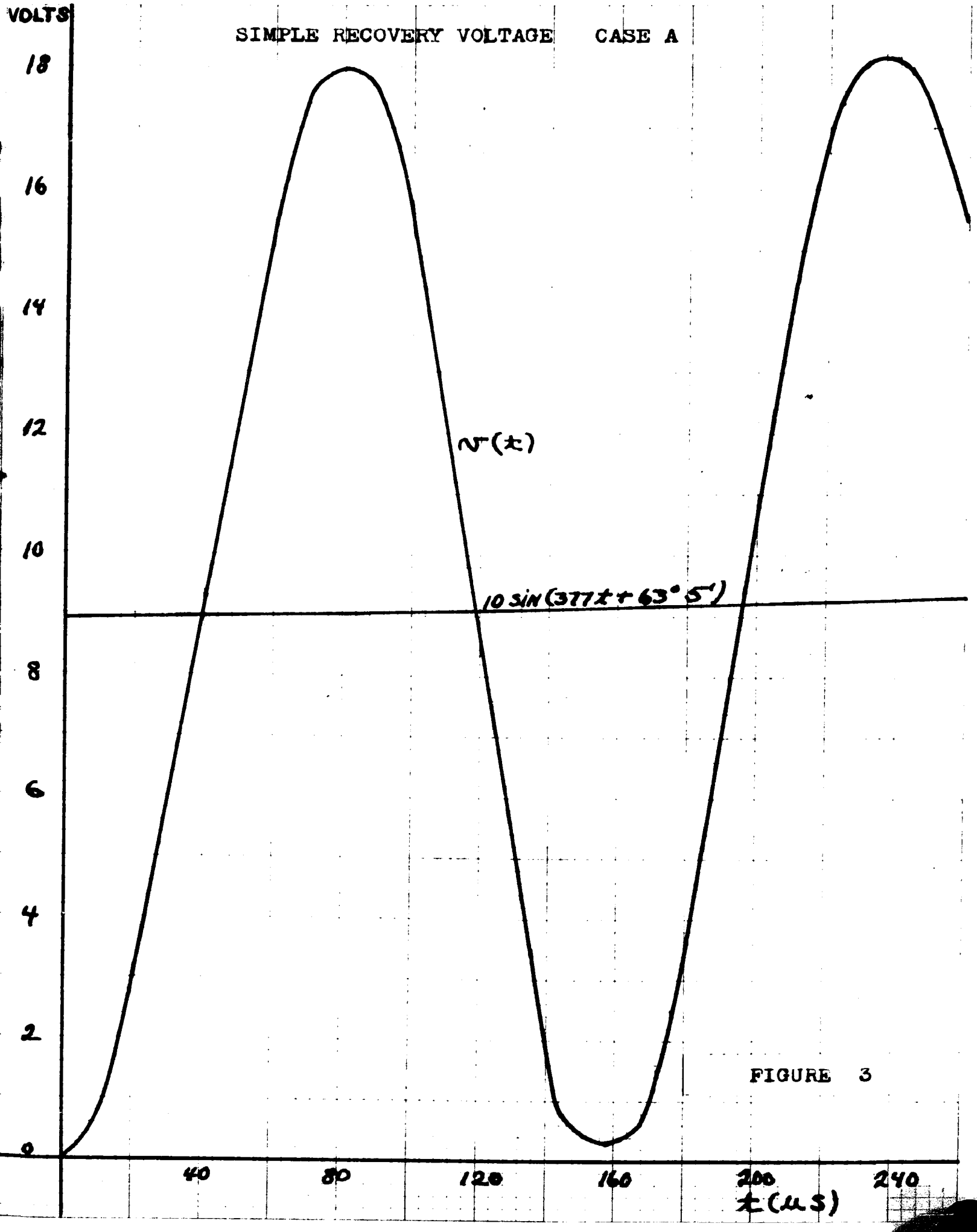

SIMPLE RECOVERY VOLTAGE CASE A
VOLTS
18
16
14
12
10
8
6
4
2
0
v(t)
10 sin(377t + 63° 5')
40
80
120
160
200
240
t (μs)

FIGURE 3

fact that it is not possible to place a finite voltage other than zero upon a condenser in zero time with a finite current as the charging source. At zero time, (see Fig. 2b) the condenser is shorted out by the closed circuit breaker and the current is then going through a normal zero. Therefore at the instant of switch opening, the recovery voltage must start from zero value.

Find the Maximum Value of Recovery Voltage

Maximum recovery voltage in this simple case occurs 180 degrees (measured in terms of the high frequency harmonic) after zero time. That is when

$$t = \pi\sqrt{LC} = 78.5 \text{ uS}$$

also, $$\frac{1}{\sqrt{LC}} = 4.02 \times 10^4 = 2\pi f$$

$$f = 6{,}400 \text{ cycles/sec.}$$

From the well-known symmetrical component theory for a one-line-to-ground fault,

$$(I_{fault})_{max} = 3\left[\frac{E_m}{3\omega L}\right] = \frac{E_m}{\omega L}$$

$$\left[v(t)\right]_m = \frac{\frac{E_m}{\omega L}}{\frac{1}{\omega L} - \omega C}\left(\cos \pi\omega\sqrt{LC} - \cos\pi\right)$$

$$= \frac{E_m}{1 - LC\omega^2}\left(\cos 377\sqrt{LC}\,\pi + 1\right)$$

and $377\pi\sqrt{LC}$ radians = 1.69°

Therefore, $$\left[v(t)\right]_m = \frac{E_m}{1 - LC\omega^2} (\cos 1.69^\circ + 1)$$

$$= \frac{E_m (2 - .000435)}{1 - .000088}$$

$$\cong 2E_m$$

For a first approximation, consider the lossless case, R = 0. The recovery voltage

$$v(t) = \frac{I_m}{\frac{1}{\omega L} - \omega C} (\cos \omega t - \cos \frac{1}{\sqrt{LC}} t)$$

and

$$v'(t) = \frac{I_m}{\frac{1}{\omega L} - \omega C} (-\omega \sin \omega t + \frac{1}{\sqrt{LC}} \sin \frac{t}{\sqrt{LC}})$$

an inspection of Figure 3 shows that the greatest rate of rise will take place when

$t = \frac{1}{4}$ cycle measured on the 6.4 KC harmonic component of recovery voltage.

Or, in seconds, $t = \frac{1}{4} \times \frac{1}{f} = \frac{\pi}{2}\sqrt{LC}$.

Therefore, $$\left[v'(t)\right]_{max} = \frac{I_m}{\frac{1}{\omega L} - \omega C} \sqrt{\frac{1}{LC}}$$

$$= \frac{E_m}{\sqrt{LC}} = 4.02 \times 10^4 E_m \frac{\text{volts}}{\text{sec}}$$

when $t = \frac{\pi}{2}\sqrt{LC} = 39.2$ micro-sec.

SECTION II- RECOVERY VOLTAGE STUDIES

CASE B- Recovery Voltage with Two-Frequency Transient. No Corrective Measures Used.

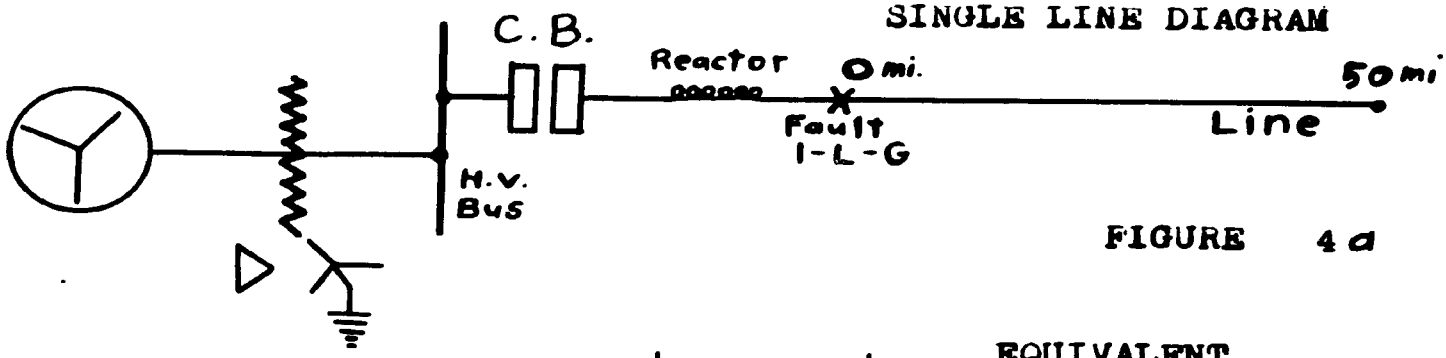


FIGURE 4a

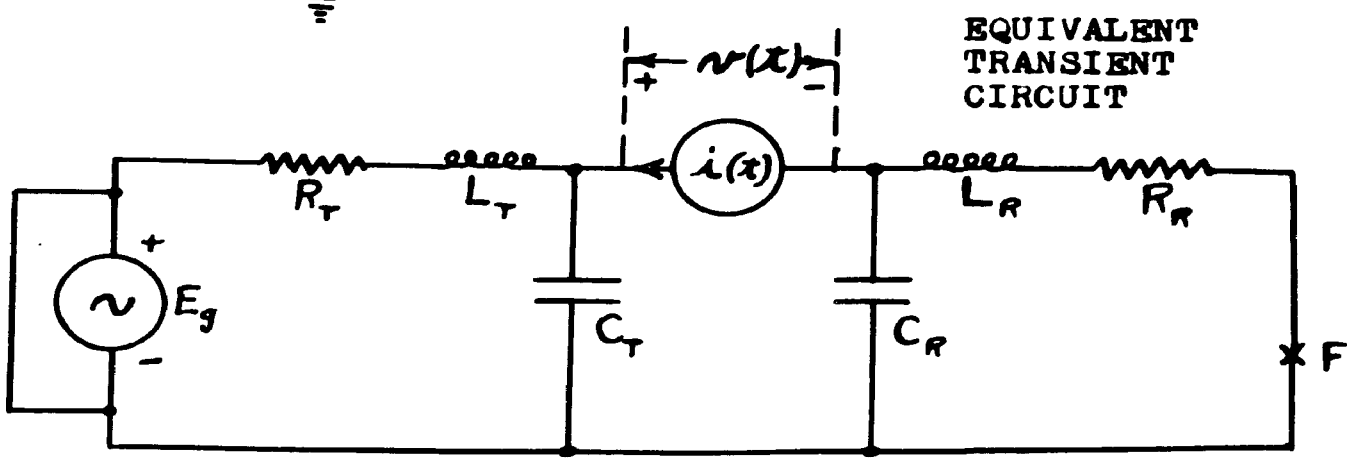


FIGURE 4b

E_g-GENERATED VOLTAGE

R_T-TRANSFORMER R

L_T-TRANSFORMER LEAKAGE INDUCTANCE

C_T-LUMPED TRANSFORMER & BUS C TO GND.

L_R-REACTOR INDUCTANCE

R_R-REACTOR RESISTANCE

C_R-LUMPED REACTOR CAPACITANCE TO GROUND

$i(t)$-CURRENT GENERATOR USED IN TRANSIENT STUDY.

CASE B - Recovery Voltage With Two-Frequency Transient

The second case of recovery voltage to be studied is one which involves a two-frequency transient of recovery voltage superimposed on fundamental voltage. The single line diagram is shown in Fig. 4a.

Symmetrical component theory must be used to find the steady state fault current for a single line to ground fault in this case.

Since at fundamental frequency, transformer and reactor capacitance to ground are negligible,

fault current $I_f = I_1 + I_2 + I_o$

$$I_1 = \frac{E}{3\left[R_T + jX_T + R_R + jX_R\right]}, \text{ and } I_F = 3I_1,$$

$$(I_F)_{max} = \frac{E_m}{\left[(R_T + R_R)^2 + \omega^2(L_T + L_R)^2\right]^{\frac{1}{2}}}$$

where I_1 = positive-sequence fault current

I_2 = negative-sequence fault current

I_o = zero-sequence fault current.

Approximate equivalent L-circuits were chosen to represent the transformer and reactor. This simplification neglects the comparatively small harmonics of natural frequency of these two power elements. The equivalent transient circuit becomes Figure 4b.

$$Z(S) = \frac{\left[R_T + SL_T\right]\frac{1}{SC_T}}{R_T + SL_T + \frac{1}{SC_T}} + \frac{\left[R_R + SL_R\right]\frac{1}{SC_R}}{R_R + SL_R + \frac{1}{SC_T}}$$

$$= \frac{\frac{R_T}{L_T} + S}{C_T\left[S^2 + S\frac{R_T}{L_T} + \frac{1}{L_T C_T}\right]} + \frac{\frac{R_R}{L_T} + S}{C_R\left[S^2 + S\frac{R_R}{L_R} + \frac{1}{L_R C_R}\right]}$$

$$V(S) = I(S) \cdot Z(S)$$

$$= \frac{\left[S + \frac{R_T}{L_T}\right] I_m \omega}{C_T\left[S^2 + S\frac{R_T}{L_T} + \frac{1}{L_T C_T}\right](S^2+\omega^2)} +$$

$$+ \frac{\left[S + \frac{R_R}{L_R}\right] I_m \omega}{C_R\left[S^2 + S\frac{R_R}{L_R} + \frac{1}{L_R C_R}\right](S^2+\omega^2)}$$

$$V(S) = \frac{I_m \omega}{C_T} \frac{\left[S + \frac{R_T}{L_T}\right]}{\left[S^2+\omega^2\right]\left[\left(S + \frac{R_T}{2L_T}\right)^2 + \left(\frac{1}{L_T C_T} - \frac{R_T^2}{4L_T^2}\right)\right]}$$

$$+ \frac{I_m \omega}{C_R} \frac{\left(S + \frac{R_R}{L_R}\right)}{(S^2+\omega^2)\left[\left(S + \frac{R_R}{2L_R}\right)^2 + \left(\frac{1}{L_R C_R} - \frac{R_R^2}{4L_R^2}\right)\right]}$$

$$v(t) = \mathcal{L}^{-1}\left[V(S)\right]$$

$$= \frac{I_m}{C_T}\left[\frac{\left(\frac{R_T}{L_T}\right)^2 + \omega^2}{\left(\frac{1}{L_T C_T} - \omega^2\right)^2 + \frac{R_T^2}{L_T^2}\omega^2}\right]^{\frac{1}{2}} \sin(\omega t + \psi_1)$$

$$+ \frac{1}{\left[\frac{1}{L_T C_T} - \frac{R_T^2}{4L_T^2}\right]^{\frac{1}{2}}}\left[\frac{\frac{1}{L_T C_T}}{\left(\frac{1}{L_T C_T} - \omega^2\right)^2 + \frac{R_T^2}{L_T^2}\omega^2}\right]^{\frac{1}{2}} \cdot \frac{I_m \omega}{C_T}$$

$$\cdot e^{\frac{-R_T t}{2L_T}} \sin\left[\left(\frac{1}{LC} - \left(\frac{R}{2L}\right)^2\right)^{\frac{1}{2}} t - \psi_2\right]$$

$$+ \left[\text{two identical terms with subscripts changed to R}\right].$$

where $\psi_1 = \tan^{-1}\frac{\omega L}{R} - \tan^{-1}\frac{\frac{R\omega}{L}}{\frac{1}{LC} - \omega^2}$

and $\psi_2 = \tan^{-1}\frac{\left[\frac{1}{LC} - \frac{R^2}{4L^2}\right]^{\frac{1}{2}}}{\frac{R}{2L}} - \tan^{-1}\frac{\left[-\frac{R}{L}\left(\frac{1}{LC} - \frac{R^2}{4L^2}\right)^{\frac{1}{2}}\right]}{\frac{R^2}{2L^2} - \frac{1}{LC} + \omega^2}$

and $I_m = \frac{E_m}{\left[(R_T + R_R)^2 + \omega^2(L_T + L_R)^2\right]^{\frac{1}{2}}}$

The following circuit quantities were measured in the experimental setup described in the Appendix:

R_R = 3.77 ohms

L_R = 10 mh

C_R = 0.008 μf

R_T = 6 ohms

L_T = 21.2 mh

C_T = 0.02 uf (distributed C of transformer and H. V. bus to ground)

$$I_m = \frac{E_m}{\left[(R_T + R_R)^2 + \omega^2(L_T + L_R)^2\right]^{\frac{1}{2}}}$$

$$= 0.418 - j\,0.503 = 0.655 \angle -50°20'$$

$$v(t) = 6.54 \sin (377t + 53.1°) + 3.49 \sin (377t + 45°)$$
$$+ 5.22\, e^{-141.5t} \sin (48{,}600t - 90°)$$
$$+ 2.47\, e^{-189t} \sin (112{,}000t - 90°)$$

again, the important check holds,

$$v(0^+) = 0$$

Like terms can be combined to give

$$v(t) = 10 \sin (377t + 50°14')$$
$$+ 5.22\, e^{-141.5t} \sin (48{,}600t - 90°)$$
$$+ 2.47\, e^{-189t} \sin (112{,}000t - 90°)$$

Considerable simplification can be obtained in plotting v(t) if all losses are neglected. Let $R_R = 0 = R_T$. Then

$$Z(S) = \frac{S}{C_T\left(S^2 + \frac{1}{L_T C_T}\right)} + \frac{S}{C_R\left(S^2 + \frac{1}{L_R C_R}\right)}$$

$$V(S) = I(S) \cdot Z(S)$$

$$= I_m \omega \left[\frac{S}{C_T\left(S^2 + \frac{1}{L_T C_T}\right)\left(S^2 + \omega^2\right)} + \frac{S}{C_R\left(S^2 + \frac{1}{L_R C_R}\right)\left(S^2 + \omega^2\right)} \right]$$

$$v(t) = \mathcal{L}^{-1}\left[V(S)\right] = I_m \omega \left[\frac{\cos \frac{1}{\sqrt{L_T C_T}} t - \cos \omega t}{C_T\left(\omega^2 - \frac{1}{L_T C_T}\right)} + \frac{\cos \frac{1}{\sqrt{L_R C_R}} t - \cos \omega t}{C_R\left(\omega^2 - \frac{1}{L_R C_R}\right)} \right]$$

where $I_m = \frac{E_m}{\omega (L_T + L_R)}$

$$v(t) = \frac{E_m}{(L_T + L_R)} \left[\frac{\cos \frac{1}{\sqrt{L_T C_T}} t - \cos \omega t}{C_T\left(\omega^2 - \frac{1}{L_T C_T}\right)} + \frac{\cos \frac{1}{\sqrt{L_R C_R}} t - \cos \omega t}{C_R\left(\omega^2 - \frac{1}{L_R C_R}\right)} \right]$$

In both terms, so far as practical values are concerned,

$$\frac{1}{LC} >> \omega^2$$

(example: $23{,}800 \times 10^5 >> 1.41 \times 10^5$)

Therefore neglecting ω^2,

$$v(t) = E_m \cos \omega t - \frac{E_m}{(L_T + L_R)} \left[L_T \cos \frac{1}{\sqrt{L_T C_T}} t + L_R \cos \frac{1}{\sqrt{L_R C_R}} t \right]$$

If experimental values from the Appendix are substituted into the preceding expression,

$$v(t) = 10 \cos 377t - 6.8 \cos 48{,}600t - 3.2 \cos 112{,}000t$$

The usual check applies,

$$v(0^+) = 0$$

The expression for v(t) is plotted on the following figure.

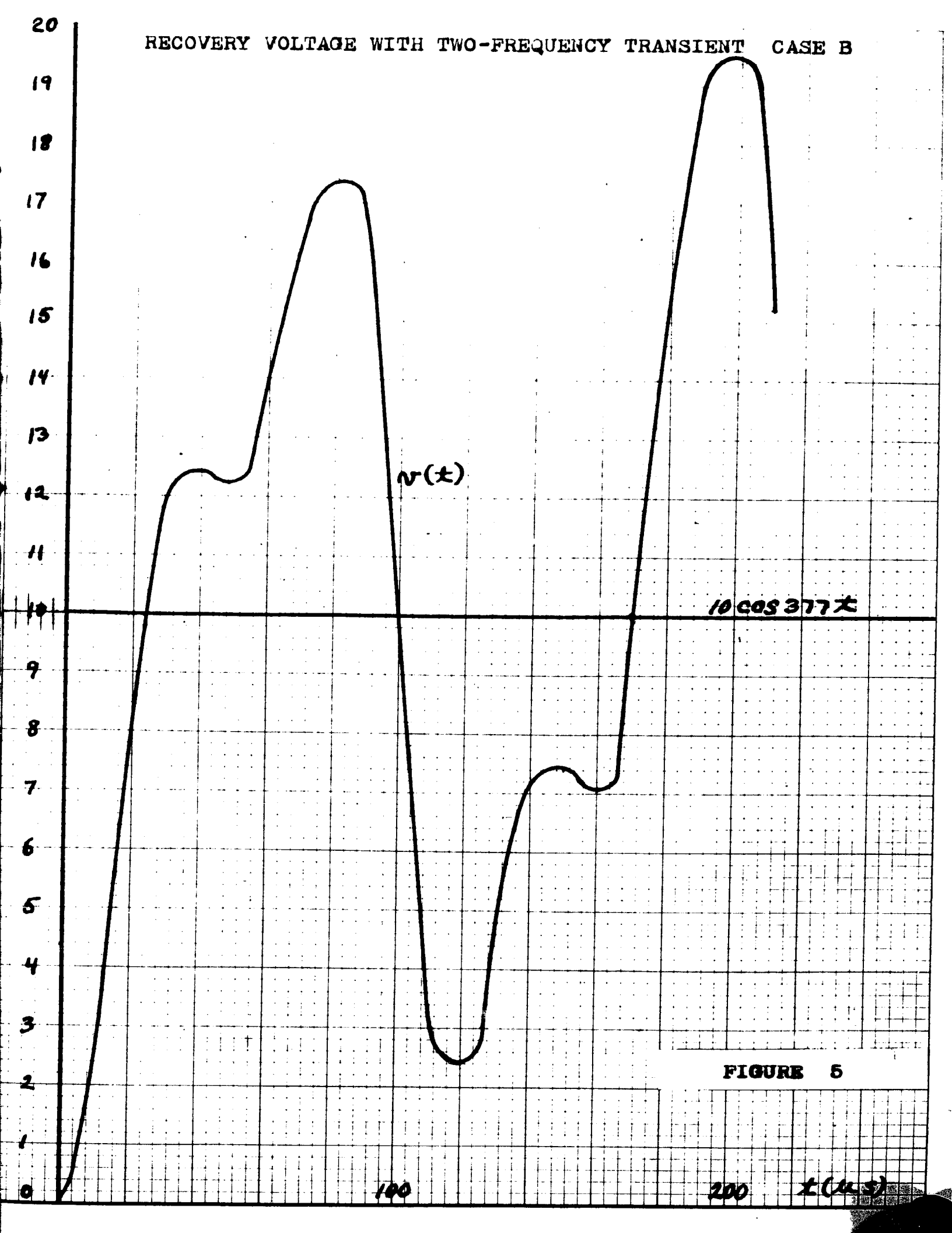

RECOVERY VOLTAGE WITH TWO-FREQUENCY TRANSIENT CASE B
20
19
18
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0
v(t)
10 cos 377t
100
200
t (μs)

FIGURE 5

SECTION II- RECOVERY VOLTAGE STUDIES

CASE C-RECOVERY VOLTAGE WITH RESISTANCE SHUNTED ACROSS CIRCUIT BREAKER. FIRST CORRECTIVE MEASURE.

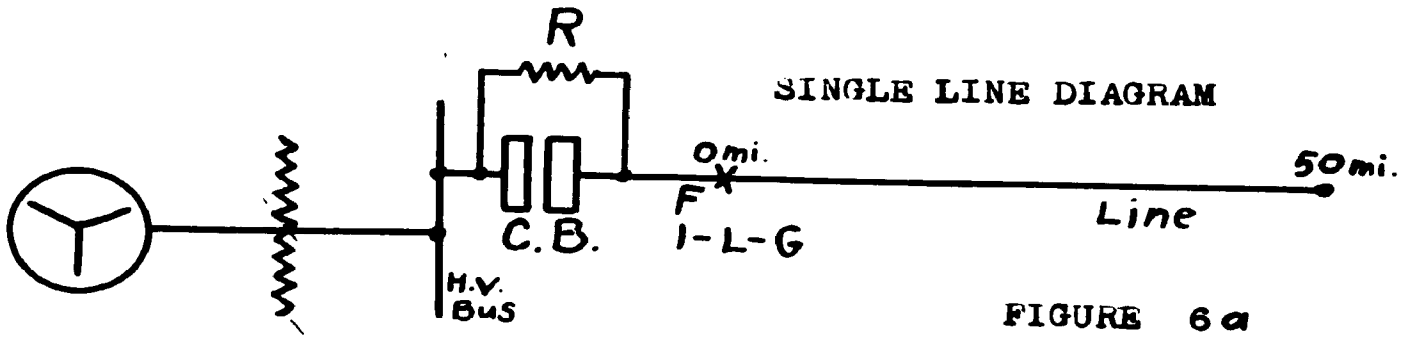


FIGURE 6a

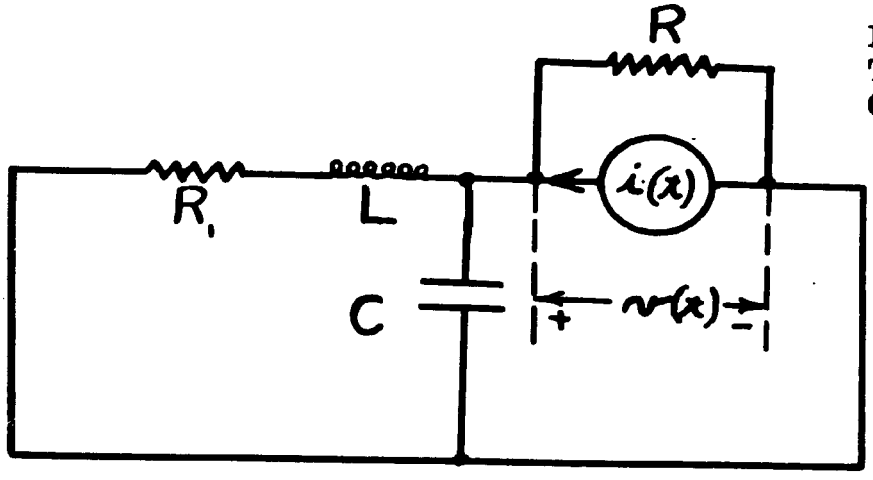


FIGURE 6 b

L-TRANSFORMER LEAKAGE INDUCTANCE

R_1-TRANSFORMER EFFECTIVE RESISTANCE

C-LUMPED TRANSFORMER & BUS CAPACITANCE TO GND.

R-RESISTANCE SHUNTED ACROSS CIRCUIT BREAKER.

CASE C - First Corrective Case, Resistance Shunted across Breaker

$$i(t) = I_m \sin \omega t$$

(For values used in Appendix, gives:

$$i(t) = .758 \sin 377t)$$

$$I(S) = \mathcal{L}\left[i(t)\right] = \frac{I_m \omega}{S^2 + \omega^2}$$

$$\frac{1}{Z(S)} = \frac{1}{R_1 + SL} + SC + \frac{1}{R}$$

$$V(S) = I(S) \cdot Z(S)$$

$$= \frac{I_m \omega}{C} \frac{(S + \frac{R_1}{L})}{(S^2 + \omega^2)\left[S^2 + \left(\frac{R_1 RC + L}{LCR}\right)S + \frac{R + R_1}{LCR}\right]}$$

As a first simplification, assume the lossless case,

$$R_1 = 0$$

V(S) becomes,

$$V(S) = \frac{I_m \omega}{C} \frac{S}{(S^2 + \omega^2)\left[S^2 + \frac{1}{CR}S + \frac{1}{LC}\right]}$$

where $I_m = \frac{E_m}{[\omega L]}$

$$v(t) = \mathcal{L}^{-1}\left[V(S)\right]$$

$$v(t) = \frac{E_m}{LC} \frac{1}{\left[\left(\frac{1}{LC} - \omega^2\right)^2 + \left(\frac{\omega}{CR}\right)^2\right]^{\frac{1}{2}}} \sin(\omega t + \psi_1)$$

$$+ \frac{E_m}{LC} \left\{ \frac{\frac{1}{LC}}{\left[\left(\frac{1}{LC} - \omega^2\right)^2 + \left(\frac{\omega}{CR}\right)^2\right]\left[\frac{1}{LC} - \left(\frac{1}{2RC}\right)^2\right]} \right\}^{\frac{1}{2}} \cdot$$

$$\cdot e^{-\frac{t}{2RC}} \sin(\beta t + \psi_2)$$

where $\beta = \left[\frac{1}{LC} - \left(\frac{1}{2RC}\right)^2\right]^{\frac{1}{2}}$, $\left[R \text{ assumed large; } \frac{1}{LC} > \left(\frac{1}{2RC}\right)^2\right]$

and $\psi_1 = 90° - \tan^{-1} \frac{\omega}{\frac{R}{L} - \omega^2 RC}$

and $\psi_2 = \tan^{-1}(-2RC)\left[\frac{1}{LC} - \left(\frac{1}{2RC}\right)^2\right]^{\frac{1}{2}}$

$$-\tan^{-1} \frac{\frac{1}{RC}\left[\frac{1}{LC} - \left(\frac{1}{2RC}\right)^2\right]^{\frac{1}{2}}}{-\omega^2 + \frac{1}{LC} - \frac{1}{2R^2C^2}}$$

If we examine the exact expression for v(t) in the lossless case, we can see that so far as fundamental amplitude is concerned, and for practical values of circuit parameters,

$$\frac{1}{LC} \gg \omega^2,$$

therefore neglecting ω^2,

$$v(t) = \frac{E_m}{\left[1 + \left(\frac{\omega L}{R}\right)^2\right]^{\frac{1}{2}}} \sin(\omega t + \psi_1)$$

$$+ \left[\text{harmonic term}\right]$$

Normally the shunting resistance of optimum value for reducing the recovery voltage "kick" will be around 1,000 ohms. For example, for values used in the Appendix,

$$\left(\frac{\omega L}{R}\right)^2 = .000372$$

Consequently, the maximum value of fundamental recovery voltage will be only very slightly reduced from steady-state line-to-ground system voltage. Now by application of the same very close approximation to the harmonic,

$$v(t) \cong \frac{E_m}{\left[1 + \left(\frac{\omega L}{R}\right)^2\right]^{\frac{1}{2}}} \sin(\omega t + \psi_1)$$

$$+ \frac{E_m}{\left\{\left[1 + \left(\frac{\omega L}{R}\right)^2\right]\left[1 - \frac{L}{4R^2C}\right]\right\}^{\frac{1}{2}}} e^{-\frac{t}{2RC}} \sin(\beta t + \psi_2)$$

where $\beta = \left[\frac{1}{LC} - \left(\frac{1}{2RC}\right)^2\right]^{\frac{1}{2}}$

and $\psi_1 = 90° - \tan^{-1} \frac{\omega}{\frac{R}{L} - \omega^2 RC}$

$$\cong 90° - \tan^{-1} \frac{\omega L}{R} \cong 90°$$

$$\text{and} \quad \psi_2 = -\tan^{-1}\left[\frac{4R^2C}{L} - 1\right]^{\frac{1}{2}}$$

$$-\tan^{-1}\frac{\left[\frac{1}{LC} - \left(\frac{1}{2RC}\right)^2\right]^{\frac{1}{2}}}{RC\left[-\omega^2 + \frac{1}{LC} - \frac{1}{2R^2C^2}\right]}$$

From the preceding expression for v(t), it can be seen that for usable values of shunting resistance, the fundamental of recovery voltage will be essentially a cosine wave equal in magnitude to normal system phase voltage. On examination of the harmonic equation it is obvious that variation in R will have a vast effect on amplitude, phase, and damping of the harmonic. This will be examined in detail.

If we go back to the general case (system losses included), the effects of varying the shunting resistance can be studied.

$$V(S) = \frac{I_m\omega}{C} \cdot \frac{\left(S + \frac{R_1}{L}\right)}{(S^2 + \omega^2)\left[S^2 + \left(\frac{R_1RC + L}{LCR}\right)S + \frac{R_1 + R}{LCR}\right]}$$

The quadratic factor in the denominator determines the nature of the harmonic component of recovery voltage. This factor may be rewritten as follows:

$$\left[\left(S + \frac{R_1RC + L}{2LCR}\right)^2 + \frac{R_1 + R}{LCR} - \left(\frac{R_1RC + L}{2LCR}\right)^2\right]$$

It is easily seen that in order to obtain repeated roots for

S, (critical damping), the last two terms must give zero.

$$\frac{R_1 + R}{LCR} = \left(\frac{R_1RC + L}{2LCR}\right)^2$$

If we solve for critical R,

$$R_c = \frac{-R_1 + 2\sqrt{\frac{L}{C}}}{4 - \frac{R_1^2C}{L}}$$

$\left(\text{for practical values, } \frac{R_1^2C}{L} \text{ may usually be neglected}\right)$.

For the lossless case this reduces to

$$R_c = \frac{1}{2}\sqrt{\frac{L}{C}}$$

If the shunting resistance is made equal to R_c, then

$$V(S) = \frac{I_m\omega\left(S + \frac{R_1}{L}\right)}{C(S^2 + \omega^2)\left(S + \frac{R_1R_cC + L}{2LCR_c}\right)^2}$$

$$v(t) = \mathcal{L}^{-1}\left[V(S)\right]$$

$$= \frac{I_m\sqrt{R_1^2 + \omega^2L^2}}{LC\left[\omega^2 + \left(\frac{R_1R_cC + L}{2LCR_c}\right)^2\right]} \sin(\omega t + \psi)$$

$$+ \frac{I_m\omega}{C}\left\{\frac{\frac{R_1}{L} - \frac{R_1R_cC + L}{2LCR_c}}{\omega^2 + \left(\frac{R_1R_cC + L}{2LCR_c}\right)^2}\right\} \cdot t$$

$$+ \frac{\frac{2R_1}{L}\left(\frac{R_1R_cC + L}{2LCR_c}\right) + \omega^2 - \left(\frac{R_1R_cC + L}{2LCR_c}\right)^2}{\left[\omega^2 + \left(\frac{R_1R_cC + L}{2LCR_c}\right)^2\right]^2} \Bigg\} \cdot e^{-\frac{R_1R_cC + L}{2LCR_c} \cdot t}$$

where $\psi = \tan^{-1}\frac{\omega L}{R_1} - 2\tan^{-1}\frac{2\omega LCR_c}{R_1R_cC + L}$

If we substitute

$$I_m = \frac{E_m}{\left(R_1^2 + \omega^2L^2\right)^{\frac{1}{2}}}$$

and $\left(\frac{R_1R_cC + L}{2LCR_c}\right)^2 = \frac{R_1 + R_c}{LCR_c}$

Thus the preceding expression for v(t) simplifies to

$$v(t) = \frac{E_m}{\omega^2CL + \frac{R_1 + R_c}{R_c}} \sin(\omega t + \psi) +$$

$$+ \frac{E_m\,\omega}{C\left(R_1^2 + \omega^2L^2\right)^{\frac{1}{2}}} \left[\frac{\frac{R_1}{L} - \alpha}{\omega^2 - \alpha^2}\, t + \frac{\frac{2R_1}{L}\alpha + \omega^2 - \alpha^2}{(\omega^2 + \alpha^2)^2}\right] e^{-\alpha t}$$

where $\alpha = \left(\frac{R_1 + R_c}{LCR_c}\right)^{\frac{1}{2}}$

and $\psi = \tan^{-1}\frac{\omega L}{R_1} - 2\tan^{-1}\frac{\omega}{\alpha}$

Now if we make use of the experimental values from the Appendix,

$$R_1 = 6\,\Omega$$

$$L = 31.2 \text{ mh}$$

$$C = 0.02 \ \mu\text{f}$$

$$E_m = 10 \text{ volts}$$

an expression for v(t) can be developed corresponding to critical damping.

First, if we solve for critical damping resistance,

$$\left(\frac{R_1 R_c C + L}{2LCR_c}\right)^2 = \frac{R_1 + R_c}{LCR}$$

$$R_c = \frac{2\sqrt{\frac{L}{C}} - R_1}{4 - \frac{R_1^2 C}{L}} = 621 \text{ ohms.}$$

$$\alpha = \sqrt{\frac{R_1 + R_c}{LCR_c}} = 40{,}000 \text{ radians/sec}$$

$$\psi = \tan^{-1}\frac{\omega L}{R_1} - 2\tan^{-1}\frac{\omega}{\alpha}$$

$$= 62°50' - 2(0°32') = 61°46'$$

Therefore,

$$v(t) = 10 \sin (377t + 61°46') - 3.56 \times 10^5 \, t \, e^{-40{,}000t} - 8.8 \, e^{-40{,}000t}$$

again the check holds, that

$$v(0^+) = 0$$

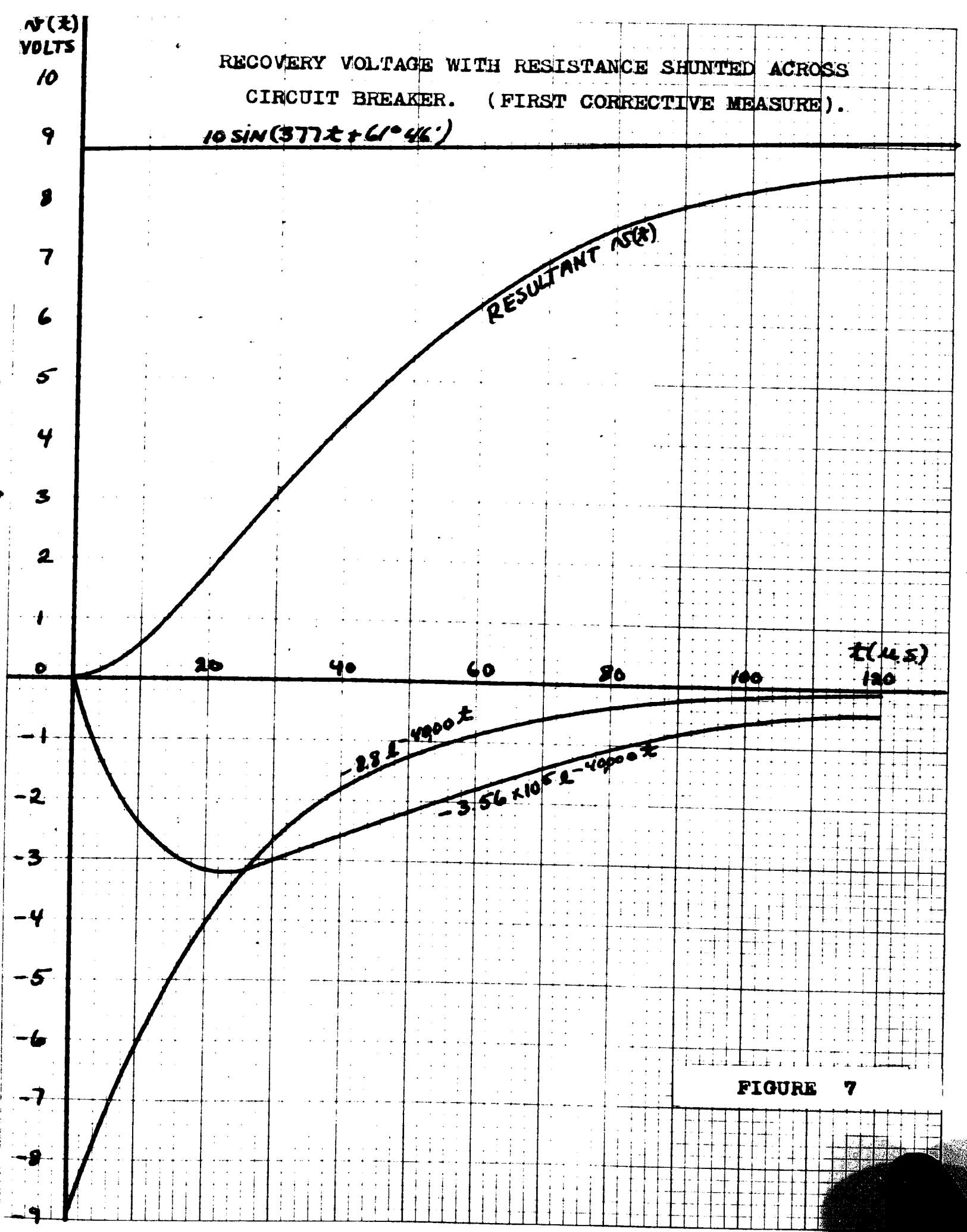

v(t) VOLTS
RECOVERY VOLTAGE WITH RESISTANCE SHUNTED ACROSS CIRCUIT BREAKER. (FIRST CORRECTIVE MEASURE).
10 sin(377t + 61° 46')
RESULTANT v(t)
t(μs)
-2.8 ε^-4900t
-3.56 x 10^5 ε^-4900t

FIGURE 7

Now we shall return to the general case of shunting resistance, giving special attention to rate of rise of recovery voltage:

$$V(S) = \frac{I_m \omega}{C} \frac{\left(S + \frac{R_1}{L}\right)}{(S^2 + \omega^2)\left[S^2 + \frac{R_1 RC + L}{LCR} S + \frac{R + R_1}{LCR}\right]}$$

If $f(t) = 0$ when $t = 0^+$,

then $\mathcal{L}^{-1}[SV(S)] = f'(t)$

The above boundary condition is satisfied since the recovery voltage in the case considered must build up across line-to-ground capacitance of transformer and high voltage bus. This capacitance is shorted by the circuit breaker prior to opening, consequently if arc voltage is neglected, the recovery voltage starts at voltage zero.

Therefore, the rate of rise $f'(t)$ is given by

$$f'(t) = \frac{E_m}{ZC}\left[\frac{\omega^4 + a_1\omega^2}{(\beta_o^2 - \omega^2)^2 + 4\alpha^2\omega^2}\right]^{\frac{1}{2}} \sin(\omega t + \psi_1)$$

$$+ \frac{\omega E_m}{\beta ZC}\left[\frac{(\alpha^2 - \beta^2 - a_1\alpha)^2 + (a_1 - 2\alpha)^2}{(\beta_o^2 - \omega^2) + 4\alpha^2\omega^2}\right]^{\frac{1}{2}} e^{-\alpha t} \sin(\beta t - \psi_2)$$

where $\psi_1 = \tan^{-1}\frac{a_1}{-\omega} - \tan^{-1}\frac{2\alpha\omega}{\beta_o^2\omega^2}$

$$\psi_2 = \tan^{-1}\frac{\beta(a_1 - 2\alpha)}{\alpha^2 - \beta^2 - a_1\alpha} - \tan^{-1}\frac{(-2\alpha\beta)}{\alpha^2 - \beta^2 + \omega^2}$$

and $$a_1 = \frac{R_1}{L}$$

$$\alpha = \frac{R_1RC + L}{2LCR}$$

$$\beta^2 = \frac{R + R_1}{LCR} - \left(\frac{R_1RC + L}{2LCR}\right)^2$$

$$\beta_o^2 = \frac{R + R_1}{LCR}$$

$$Z = \sqrt{R_1^2 + \omega^2L^2}$$

To simplify the analysis, assume the lossless case, $R_1 = 0$

Then $$a_1 = 0$$

$$\alpha = \frac{1}{2RC}$$

$$\beta^2 = \frac{1}{LC} - \left(\frac{1}{2RC}\right)^2$$

$$\beta_o^2 = \frac{1}{LC}$$

$$Z = \omega L$$

Then, $$f'(t) = \frac{\omega E_m}{\left[(1 - \omega^2LC)^2 + \left(\frac{\omega L}{R}\right)^2\right]^{\frac{1}{2}}} \sin(\omega t + \psi_1)$$

$$+ \frac{E_m \cdot e^{-\frac{t}{2RC}} \sin\left(\sqrt{\frac{1}{LC} - \frac{1}{4R^2C^2}}\, t - \psi_2\right)}{\left\{\left[LC - \left(\frac{L}{2R}\right)^2\right]\left[\left(1 - \omega^2LC\right)^2 + \left(\frac{\omega L}{R}\right)^2\right]\right\}^{\frac{1}{2}}}$$

where $$\psi_1 = -\tan^{-1}\frac{\omega}{\frac{R}{L} - \omega^2CR}$$

$$\psi_2 = \tan^{-1} \frac{\beta}{RC\left(\frac{1}{LC} - \frac{1}{2R^2C^2}\right)} - \tan^{-1} \frac{\beta}{RC\left(\frac{1}{LC} - \frac{1}{2R^2C^2} - \omega^2\right)}$$

To find the initial rate of rise, set t = 0

$$f'(o) = \frac{\omega E_m}{\left(1 - \omega^2 LC\right)^2 + \left(\frac{\omega L}{R}\right)^2} \left[\frac{\omega L}{R} - \frac{1}{\sqrt{\left(\frac{R}{\omega L}\right)^2 + 1 + (\omega RC)^2 - \frac{2R^2C}{L}}} \right]$$

Using experimental values from Appendix,

$E_m = 10$ v.

$L = 31.2$ mh

$C = .02 \times 10^{-6}$ mf

$R = R_c = 621$ Ω

$f'(o) = 0$ (within slide-rule accuracy)

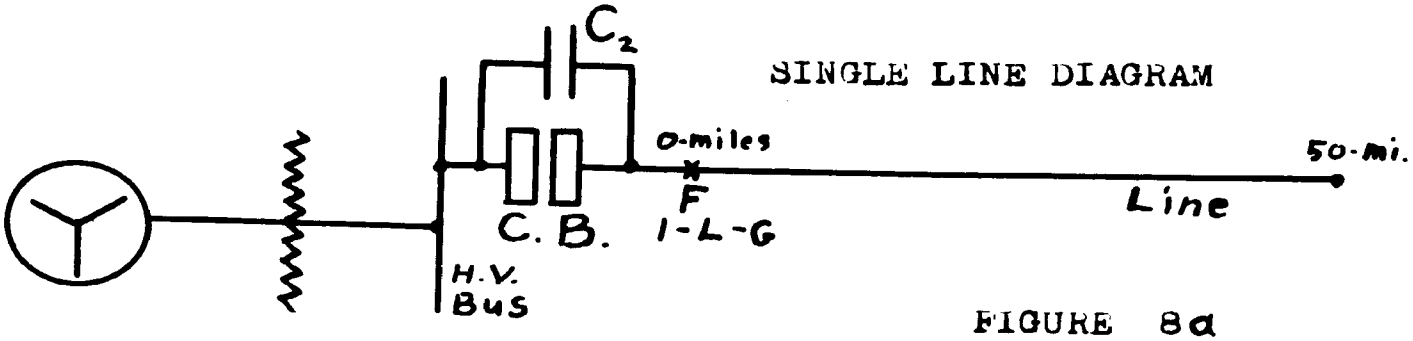
(Compare with preceding curve Fig. 7)

Likewise, maximum recovery rates could be computed and their time of occurrence be determined by well known maximum-minimum technique.

Thus, set $f''(t) = 0$ and solve for t.

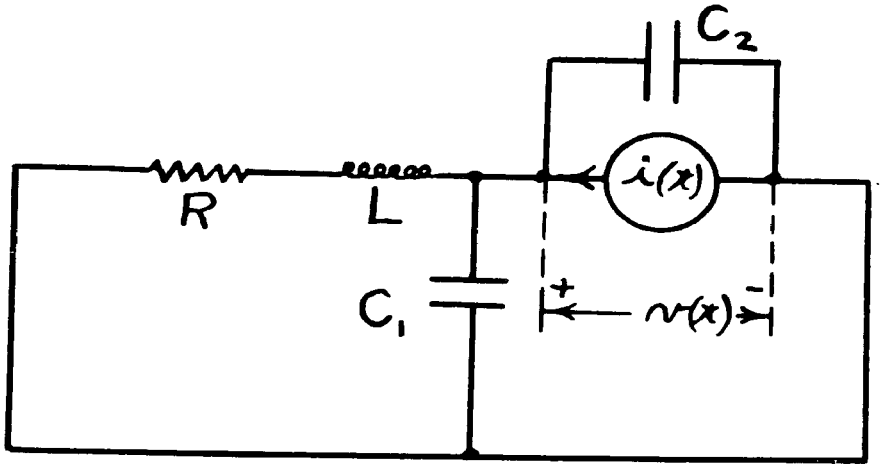
SECTION II- RECOVERY VOLTAGE STUDIES

CASE D- RECOVERY VOLTAGE WITH CAPACITANCE SHUNTED ACROSS CIRCUIT BREAKER. SECOND CORRECTIVE MEASURE.



SINGLE LINE DIAGRAM

FIGURE 8a



EQUIVALENT TRANSIENT CIRCUIT

FIGURE 8 b

L-TRANSFORMER LEAKAGE INDUCTANCE

R-TRANSFORMER EFFECTIVE RESISTANCE

C_1-LUMPED TRANSFORMER & BUS CAPACITANCE TO GND.

C_2-CAPACITANCE SHUNTED ACROSS CIRCUIT BREAKER.

CASE D - Second Corrective Measure.

Capacitance in Shunt with Circuit Breaker.

So far as recovery voltage is concerned, the effect of increased capacitance is primarily a large increase in the period of harmonic oscillation, and a slight shift in phase of this harmonic from near 90° (small C) to a calculated value of 86°55' for an extremely large C of 10 uf.

Enlarged capacitance has little or no effect on recovery voltage peak, but does have some effect on (1) initial rate of rise (due to phase shift in harmonic), and (2) on maximum rate of rise (due to decreased frequency of harmonic). These qualitative conclusions will be verified in the analysis which follows, and also in the experimental data of the Appendix.

$$\text{Let} \quad C = C_1 + C_2$$

$$V(S) = \frac{I_m \omega}{C} \frac{\left(S + \frac{R}{L}\right)}{(S^2 + \omega^2)\left(S^2 + \frac{R}{L}S + \frac{1}{LC}\right)}$$

$$v(t) = \mathcal{L}^{-1}[V(S)]$$

$$= \frac{E_m \sin(\omega t + \psi_1)}{\left[1 + \omega^4 L^2 C^2 - 2\omega^2 LC + R^2\omega^2 C^2\right]^{\frac{1}{2}}}$$

$$+ \frac{E_m \omega \left[\frac{1}{LC}\right]^{\frac{1}{2}} e^{-\frac{R}{2L}t} \sin\left[\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}\, t - \psi_2\right]}{C\left[(R^2 + \omega^2 L^2)\left(\frac{1}{LC} - \frac{R^2}{4L^2}\right)\right]^{\frac{1}{2}}\left[\left(\frac{1}{LC} - \omega^2\right)^2 + \left(\frac{R\omega}{L}\right)^2\right]^{\frac{1}{2}}}$$

where

$$\psi_1 = \tan^{-1}\frac{\omega L}{R} - \tan^{-1}\frac{R\omega}{\frac{1}{C} - \omega^2 L}$$

$$\psi_2 = \tan^{-1}\frac{\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}}{\frac{R}{2L}} - \tan^{-1}\frac{\left(-\frac{R}{L}\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}\right)}{\frac{R^2}{2L^2} - \frac{1}{LC} + \omega^2}$$

The above expression is of course identical with that given in Case A except that the capacitance term has been increased by the amount of the shunting capacitance, C_2.

Within slide-rule accuracy, the preceding expression can be greatly simplified:

$$v(t) \cong E_m \sin(\omega t + \psi_1)$$

$$+ E_m \cdot \frac{\omega L}{\sqrt{R^2 + \omega^2 L^2}}\, e^{-\frac{R}{2L}t} \sin\left(\frac{1}{\sqrt{LC}}t - \psi_2\right)$$

where $\psi_1 \cong \tan^{-1}\frac{\omega L}{R}$

$$\psi_2 \cong \tan^{-1}\frac{2}{R}\sqrt{\frac{L}{C}}$$

From the above, it is clear that the magnitude and phase of the fundamental are practically independent of C. (This is true within slide-rule accuracy for all C < 1 uf). The same is true for harmonic amplitude. Harmonic frequency (for all C < 1 uf) varies inversely as the square root of C, and clearly the harmonic phase angle Ψ_2 decreases from 90° for C = 0, to slightly smaller values for large C.

C	Ψ_2
0	90°
1 uf	89°
10 uf	86°55'

So far as harmonic damping is concerned, varying the capacitance would at first appear to have no effect. However, it was noted experimentally (see Appendix Fig. 43 - 46) that increasing C did reduce harmonic damping considerably. This was due to change in effective resistance caused by a wide change in harmonic frequency. This change in effective R as a function of frequency will be investigated in a separate section of the Appendix, Fig. 47.

Initial and Maximum Rates of Rise of Recovery Voltage for Capacitance-Shunted Breakers

In systems where inductive reactance is large compared to resistance, fundamental voltage is maximum and of zero slope at time zero. So that for the first several cycles of harmonic voltage, the fundamental can be considered as

a line of zero slope. For small values of capacitance consistent with magnitudes actually found in power systems, harmonic voltage has been shown to be of form

$$\sin(\beta t - 90^\circ)$$

Thus, harmonic voltage also has zero slope at time zero. If C is greatly increased, the harmonic phase angle decreases slightly from 90° and initial rate of rise increases slightly from zero. If the capacitance were made very large, initial rate of rise would approach maximum rate of rise. Within slide rule accuracy, the maximum rate of rise equals the slope of the harmonic at its first inflection point. This can be found by differentiating the expression for the harmonic of v(t) with respect to t, as follows:

$$\max v'(t) = \frac{E_m \omega \left(\frac{1}{LC}\right)^{\frac{1}{2}}}{C\sqrt{R^2 + \omega^2 L^2}\left[\left(\frac{1}{LC} - \omega^2\right)^2 + \left(\frac{R\omega}{L}\right)^2\right]^{\frac{1}{2}}} \cong \frac{E_m}{\sqrt{LC}}$$

where $C < 1$ uf, $R \ll \omega L$

and will occur at points where

$$\left[\frac{1}{LC} - \left(\frac{R}{2L}\right)^2\right]^{\frac{1}{2}} \cdot t - \psi_2 = 0, \pi, 2\pi, \text{ etc.}$$

where $$\psi_2 = \tan^{-1}\sqrt{\frac{4L}{R^2C} - 1} - \tan^{-1}\frac{\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}}{\frac{1}{CR} - \frac{R}{2L} - \frac{\omega^2 L}{R}}$$

The general equation for recovery rate will now be developed:

$$\text{Since } \mathcal{L}^{-1}\left[S\,V(S)\right] = f'(t) \quad \left[\text{where } f(0^+) = 0\right]$$

$$v'(t) = \mathcal{L}^{-1}\left[\frac{E_m \omega \cdot S\left(S + \frac{R}{L}\right)}{C\sqrt{R^2 + \omega^2 L^2}\,(S^2 + \omega^2)\left(S^2 + \frac{R}{L}S + \frac{1}{LC}\right)}\right]$$

$$v'(t) = \frac{E_m \omega \sin(\omega t + \psi_1)}{LC\left[\left(\frac{1}{LC} - \omega^2\right)^2 + \left(\frac{R\omega}{L}\right)^2\right]^{\frac{1}{2}}}$$

$$+ \frac{E_m \omega e^{-\frac{R}{2L}t} \sin\left[\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2} \cdot t - \psi_2\right]}{LC^2\left[R^2 + \omega^2 L^2\right]^{\frac{1}{2}}\left[\frac{1}{LC} - \left(\frac{R}{2L}\right)^2\right]^{\frac{1}{2}}\left[\left(\frac{1}{LC} - \omega^2\right)^2 + \left(\frac{R\omega}{L}\right)^2\right]^{\frac{1}{2}}}$$

Where $\psi_1 = \tan^{-1}\left(\frac{-R}{\omega L}\right) - \tan^{-1}\frac{\omega R}{\frac{1}{C} - L\omega^2}$

$$\psi_2 = \tan^{-1}\frac{\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}}{\frac{\omega^2 L}{R} + \frac{R}{2L} - \frac{1}{RC}}$$

From the above it can easily be seen that, for

$$R = 0, \qquad v'(0) = 0$$

Also it was shown that, for $R \ll \omega L$ and for $C < 1$ uf, to a close degree of approximation, max. $v'(t) \propto \frac{1}{\sqrt{LC}}$. This max. rise occurs at a time very nearly equal to a quarter cycle of harmonic voltage.

SECTION II- RECOVERY VOLTAGE STUDIES

CASE E-1- RECOVERY VOLTAGE WITH FAULT AT FAR END OF TRANSMISSION LINE. NO CORRECTIVE MEASURE USED.

SINGLE LINE DIAGRAM

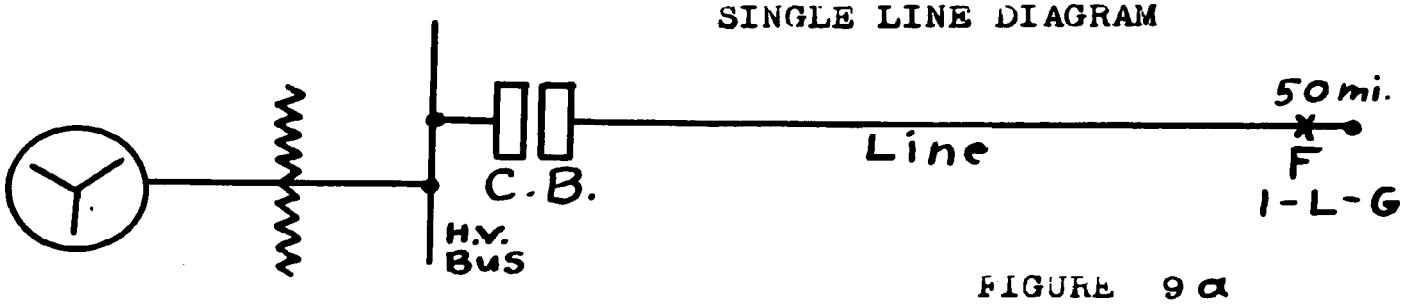


FIGURE 9a

EQUIVALENT CIRCUIT OF FAULTED PHASE

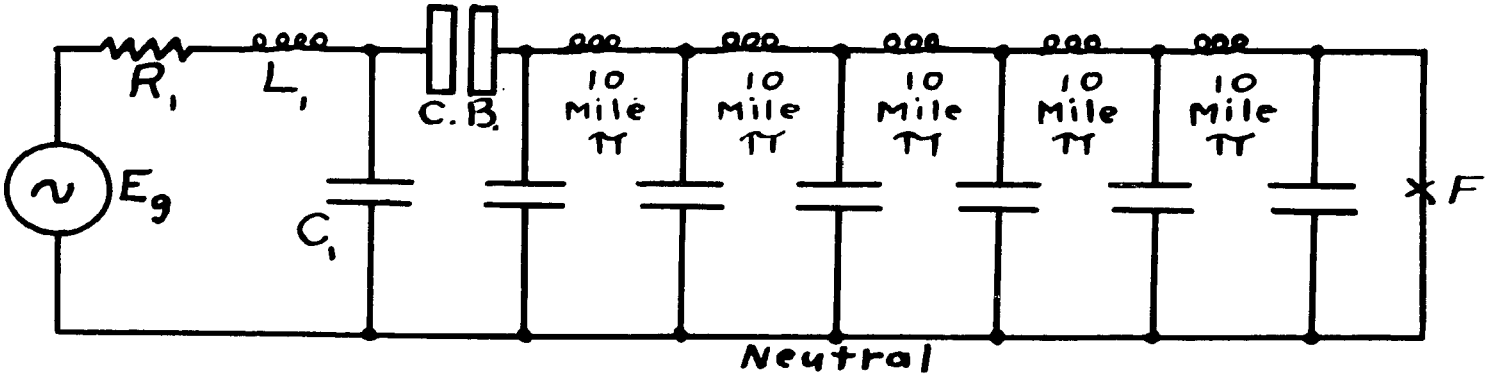
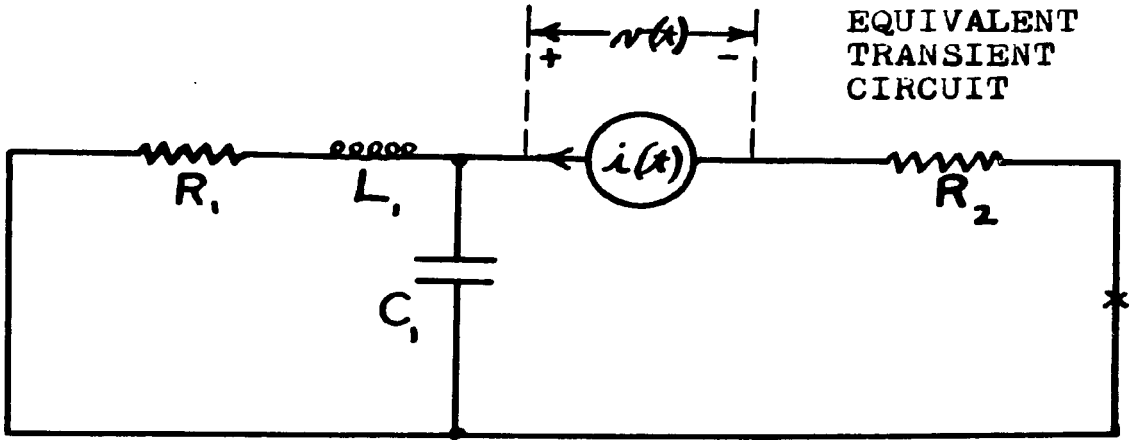


FIGURE 9b



EQUIVALENT TRANSIENT CIRCUIT

R_2-LINE SURGE IMPEDANCE

FIGURE 9c

CASE E1 - Recovery Voltage with Fault at Far End of Transmission Line

This analysis is concerned with the effect on system recovery voltage of a transmission line located between the circuit breaker and a one line to ground fault.

Overhead lines contain distributed parameters and are capable of supporting traveling surges. In such surges, it is found that [1] a given magnitude of current is always associated with a proportionate voltage, so that from the input standpoint, before reflections appear, the circuit appears to be a resistance. Surge or characteristic impedance is defined as the ratio of surge voltage to surge current. In overhead lines the surge impedance commonly ranges from about 300 to 400 ohms [1].

At the shorted end of the line, the reflected voltage is reversed and the reflected current has the same polarity as the approaching wave. The rate of propagation of waves on overhead lines is very close to the speed of light, about 1,000 feet per micro-second [1].

The experimental setup contained in the Appendix Fig. 23 makes use of an artificial 5-pi-section, 50 mile line, with parameters of value comparable to an actual overhead line. Surge impedance on a line which is essentially lossless, [1], [14], is given by

$$Z_s = \sqrt{\frac{L}{C}} = 280 \text{ ohms}$$

This value will be used in making the transient analysis so that experimental and theoretical results can be compared. From the equivalent transient circuit,

$$Z(S) = R_2 + \frac{\left(S + \frac{R_1}{L_1}\right)}{C_1\left(S^2 + S\frac{R_1}{L_1} + \frac{1}{L_1C_1}\right)}$$

$$V(S) = I(S) \cdot Z(S)$$

$$= \frac{I_m \omega R_2}{S^2 + \omega^2} + \frac{I_m \omega \left(S + \frac{R_1}{L_1}\right)}{C_1\left(S^2 + \omega^2\right)\left(S^2 + S\frac{R_1}{L_1} + \frac{1}{L_1C_1}\right)}$$

and $v(t) = \mathcal{L}^{-1}\left[V(S)\right]$

Therefore for the first 528 u-secs (pulse travel time on 50-mile line) following arc extinction, the recovery voltage is given by:

$$v(t) = \frac{E_m R_2}{\sqrt{R_o^2 + (\omega L_o)^2}} \sin \omega t$$

$$+ \frac{E_m}{L_1C_1\sqrt{R_o^2 + (\omega L_o)^2}} \left[\frac{R_1^2 + (\omega L_1)^2}{\left(\frac{1}{L_1C_1} - \omega^2\right)^2 + \left(\frac{R_1\omega}{L_1}\right)^2}\right]^{\frac{1}{2}} \sin(\omega t + \psi_1)$$

$$+ \frac{E_m \omega \left(\frac{1}{L_1C_1}\right)^{\frac{1}{2}} e^{-\frac{R_1}{2L_1}t} \sin\left(\sqrt{\frac{1}{L_1C_1} - \left(\frac{R_1}{2L_1}\right)^2}\, t - \psi_2\right)}{C_1\left[\left(R_o^2 + \omega^2 L_o^2\right)\left(\frac{1}{L_1C_1} - \frac{R_1^2}{4L_1^2}\right)\right]^{\frac{1}{2}}\left[\left(\frac{1}{L_1C_1} - \omega^2\right)^2 + \left(\frac{R_1\omega}{L_1}\right)^2\right]^{\frac{1}{2}}}$$

where $$\psi_1 = \tan^{-1} \frac{\omega L_1}{R_1} - \tan^{-1} \frac{R_1 \omega}{\frac{1}{C_1} - \omega^2 L_1}$$

$$\psi_2 = \tan^{-1} \frac{2L_1}{R_1} \sqrt{\frac{1}{L_1 C_1} - \left(\frac{R_1}{2L_1}\right)^2} - \tan^{-1} \frac{\sqrt{\frac{1}{L_1 C_1} - \left(\frac{R_1}{2L_1}\right)^2}}{\frac{1}{R_1 C_1} - \frac{R_1}{2L_1} - \frac{\omega^2 L_1}{R_1}}$$

Experimental values from the Appendix are as follows:

E_m = 10 volts

R_1 = 6 ohms

L_1 = 31.2 mh

C_1 = .02 uf

R_2 = 280 ohms

$R_o = R_1 = 6$ (assume lossless line)

L_o = 200 mh

Note: R_o and L_o are each one-third the sum of the positive, negative, and zero sequence system resistance and inductance respectively taken at fundamental frequency. Capacitance can be neglected at this frequency. If we apply the above parameters to the expression for v(t), an approximate and greatly simplified expression may be derived.

$$v(t) \cong \frac{E_m R_2}{\omega L_o} \sin \omega t + \frac{E_m \sqrt{R_1^2 + (\omega L_1)^2}}{\omega L_o} \sin (\omega t + \psi_1)$$

$$+ \frac{L_1 E_m}{L_o} \cdot e^{-\frac{R_1}{2L_1} t} \sin \left(\frac{1}{\sqrt{L_1 C_1}} t - \psi_2 \right)$$

where $$\psi_2 \cong \tan^{-1} \frac{2}{R_1} \sqrt{\frac{L_1}{C_1}} - \tan^{-1} \frac{R_1}{\sqrt{\frac{L_1}{C_1}}} \cong 90°$$

and $$\psi_1 = \tan^{-1} \frac{\omega L_1}{R_1}$$

Numerically,

$$v(t) \cong 37.1 \sin 377t + 1.75 \sin (377t + 63°)$$
$$+ 1.56 e^{-96t} \sin (40{,}000t - 90°)$$
$$\left(\text{For } t < 528 \text{ uS}\right)$$

Again the check holds, that

$$v(0^+) = 0$$

An examination of the numerical relation just developed for v(t) reveals that for t = ∞, v(t) does not coincide with steady-state fundamental system voltage ($E_m = 10$). This apparent discrepancy is due to the fact that the relation for v(t) holds only until the transient pulse, initiated by a switch opening, travels to the far end of the 50 mile line, is reflected at the shorted end, and returns to the input end; a total time of 528 u-seconds. In the equivalent transient circuit, a pure resistance was

substituted for the pi-sections of pure L and C representing the line. However, the transient solution for v(t) requires that I_m, the steady-state fault current before the breaker opens, be known. This quantity I_m is concerned only with the fundamental frequency and is obtained from the steady-state equivalent circuit of the faulted phase, with capacitance neglected. Thus, the apparent discrepancy is explained and the results validated.

The function v(t) is plotted on the following figure (Fig. 10).

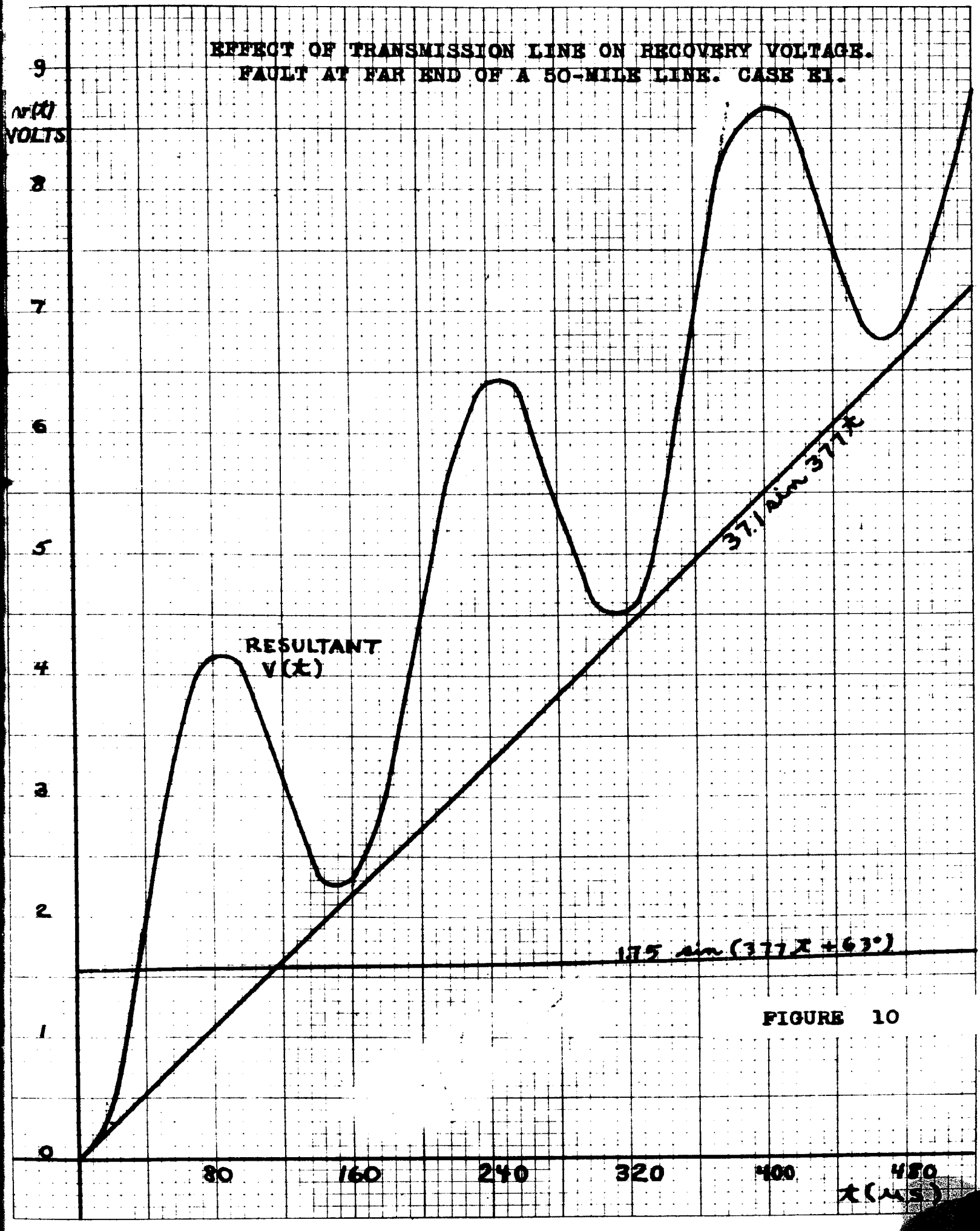
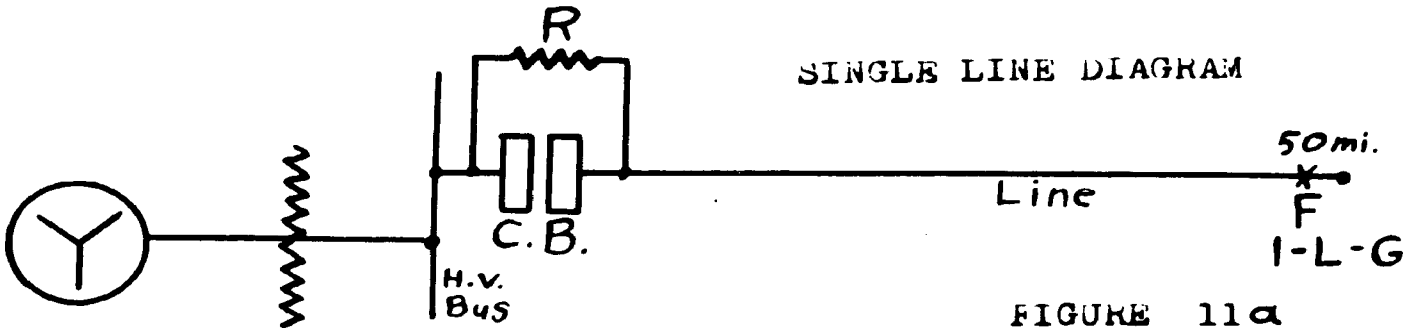

EFFECT OF TRANSMISSION LINE ON RECOVERY VOLTAGE.
FAULT AT FAR END OF A 50-MILE LINE. CASE E1.
v(t) VOLTS
9
8
7
6
5
4
3
2
1
0
RESULTANT
V(t)
37.1 sin 377t
175 sin (377t + 63°)
80
160
240
320
400
480
t (μs)

FIGURE 10

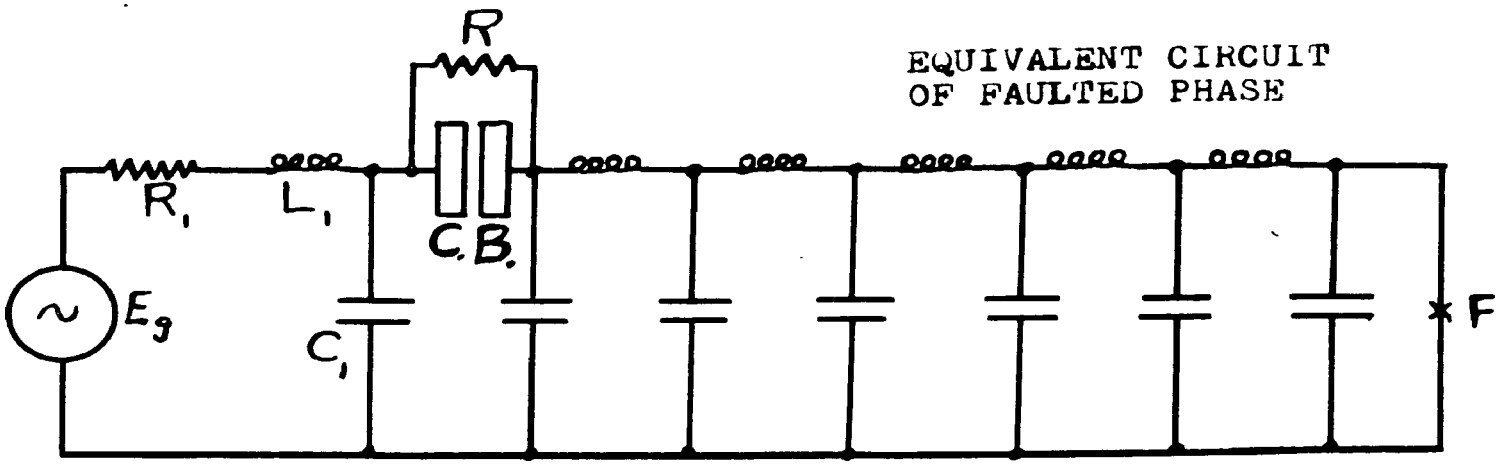
SECTION II- RECOVERY VOLTAGE STUDIES

CASE E- 2- RECOVERY VOLTAGE WITH FAULT AT FAR END OF TRANSMISSION LINE. RESISTANCE SHUNTED ACROSS C.B.



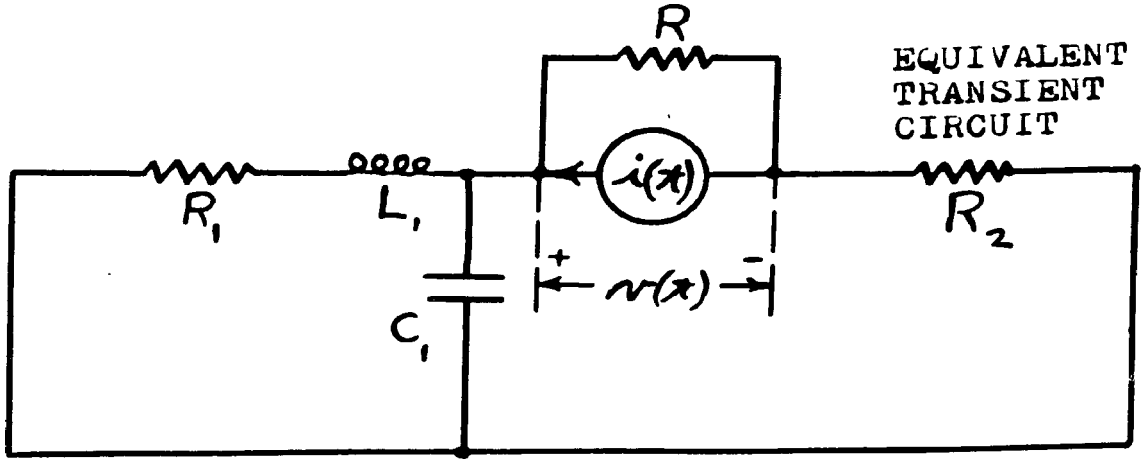
SINGLE LINE DIAGRAM

FIGURE 11a



EQUIVALENT CIRCUIT OF FAULTED PHASE

FIGURE 11b



EQUIVALENT TRANSIENT CIRCUIT

FIGURE 11c

R_2-LINE SURGE IMPEDANCE

R-SHUNTING RESISTANCE

CASE E-2

This analysis will be concerned with the effect of a transmission line between circuit breaker and fault on system recovery voltage, with the breaker shunted with pure resistance.

First the general expression for recovery voltage under these conditions will be developed. From the single line diagram, Figure 11a, and the equivalent transient circuit, Figure 11c,

$$Z(S) = \frac{R\left[R_2 + \dfrac{\frac{1}{SC_1}(R_1 + SL_1)}{R_1 + SL_1 + \frac{1}{SC_1}}\right]}{R + R_2 + \dfrac{(R_1 + SL_1)}{1 + SC_1R_1 + S^2C_1L_1}}$$

$$Z(S) = \frac{R_2R}{R + R_2} \cdot \frac{\left[S^2 + S\left(\frac{1}{R_2C_1} + \frac{R_1}{L_1}\right) + \frac{R_1 + R_2}{R_2L_1C_1}\right]}{S^2 + S\left[\frac{R_1}{L_1} + \frac{1}{(R + R_2)C_1}\right] + \left[\frac{1}{L_1C_1} + \frac{R_1}{L_1C_1(R+R_2)}\right]}$$

$$V(S) = I(S)\cdot Z(S) = \frac{I_m \omega}{S^2 + \omega^2} \cdot Z(S)$$

V(S) is of the form:

$$V(S) = A_1\left\{\frac{S + A_1S + A_o}{(S^2 + \omega^2)\left[(S + \alpha)^2 + \beta^2\right]}\right\}$$

where $A_1 = I_m \omega \left(\frac{R_2 R}{R + R_2} \right)$

$$a_1 = \frac{1}{R_2 C_1} + \frac{R_1}{L_1}$$

$$a_o = \frac{R_1 + R_2}{R_2 L_1 C_1}$$

$$\alpha = \frac{R_1}{2L_1} + \frac{1}{2C_1 (R + R_2)}$$

$$\beta = \left[\frac{1}{L_1 C_1} + \frac{R_1}{2L_1 C_1 (R + R_2)} - \left(\frac{R_1}{2L_1} \right)^2 - \left(\frac{1}{2C_1 (R + R_2)} \right)^2 \right]^{\frac{1}{2}}$$

$$\beta_o^2 = \frac{1}{L_1 C_1} + \frac{R_1}{L_1 C_1 (R + R_2)}$$

$$v(t) = \mathcal{L}^{-1} [V(s)]$$

$$v(t) = \frac{A_1}{\omega} \left[\frac{(a_o - \omega^2)^2 + a_1^2 \omega^2}{(\beta_o^2 - \omega^2)^2 + 4\alpha^2 \omega^2} \right]^{\frac{1}{2}} \sin (\omega t + \psi_1)$$

$$+ \frac{1}{\beta} \left[\frac{(\alpha^2 - \beta^2 - a_1 \alpha + a_o)^2 + \beta^2 (a_1 - 2\alpha)^2}{(\beta_o^2 - \omega^2)^2 + 4\alpha^2 \omega^2} \right]^{\frac{1}{2}} \cdot$$

$$\cdot e^{-\alpha t} \sin (\beta t - \psi_2)$$

Where $\psi_1 = \tan^{-1} \frac{a_1 \omega}{a_o - \omega^2} - \tan^{-1} \frac{2\alpha \omega}{\beta_o^2 - \omega^2}$

$$\psi_2 = \tan^{-1} \frac{\beta (a_1 - 2\alpha)}{\alpha^2 - \beta^2 - a_1 \alpha + a_o} - \tan^{-1} \frac{2\alpha\beta}{\beta^2 - \alpha^2 - \omega^2}$$

This solution for v(t), with R unrestricted, is quite cumbersome and will not be simplified further. It is of the same general form as most recovery voltage solutions: a fundamental term added to a damped harmonic oscillation. Again, the same restriction applies as in the preceding case, namely,

$$0 \leq t < 528 \text{ } \mu\text{-second}$$

for the case considered in the Appendix, or in general, the relation for v(t) is valid only during the time elapsed between arc interruption and the arrival of the reflected pulse back at the generator-end of the transmission line. The purpose of the shunting resistance is of course to control the size and form of the harmonic component of recovery voltage. A very interesting value of resistance is that which causes critical damping of the harmonic. That is, that value of R which makes $\beta = 0$

Set $\beta = 0$,

$$\frac{1}{L_1 C_1} + \frac{R_1}{2L_1 C_1 (R + R_2)} - \frac{R_1^2}{4L_1^2} - \frac{1}{4C_1^2 (R + R_2)} = 0$$

Solving for R,

$$\text{Critical } R = R_c = \frac{R_1 - 2\sqrt{\frac{L_1}{C_1}}}{\frac{R_1^2 C_1}{L_1} - 4} - R_2$$

Using values from the Appendix,

$$R_1 = 6\ \Omega$$
$$L_1 = 31.2\ \text{mh}$$
$$C_1 = .02\ \mu\text{f}$$
$$R_2 = 280\ \Omega$$

Gives $R_c = 343\ \Omega$

With critical resistance shunted across the circuit breaker, the operational solution becomes,

$$V(S) = \frac{I_m \omega R_2 R_c}{(R_c + R_2)} \frac{\left[S^2 + S\left(\frac{1}{R_2 C_1} + \frac{R_1}{L_1}\right) + \frac{R_1 + R_2}{R_2 L_1 C_1}\right]}{(S^2 + \omega^2)\left[S + \left(\frac{R_1}{2L_1} + \frac{1}{2C_1}(R_c + R_2)\right)\right]^2}$$

$$v(t) = \mathcal{L}^{-1}\left[V(S)\right]$$

$$= \mathcal{L}^{-1}\left[A_1 \frac{(S^2 + a_1 S + a_o)}{(S^2 + \omega^2)(S + \alpha)^2}\right]$$

$$= A_1 \frac{\left[(a_o - \omega^2)^2 + a_1^2 \omega^2\right]^{\frac{1}{2}}}{\omega(\alpha^2 + \omega^2)} \sin(\omega t + \psi)$$

$$+ A_1 \left[\frac{\alpha^2 - a_1 \alpha + a_o}{\alpha^2 + \omega^2} t + \frac{a_1(\omega^2 - \alpha^2) + 2\alpha(a_o - \omega^2)}{(\alpha^2 + \omega^2)^2}\right] e^{-\alpha t}$$

where $\psi = \tan^{-1} \frac{a_1 \omega}{a_o - \omega^2} - 2\tan^{-1} \frac{\omega}{\alpha} = 2°24' - 1°5' = 1°19'$

and A_1, a_1, a_o, α are as defined:

$$A_1 = I_m \omega \left(\frac{R_2 R_c}{R_c + R_2} \right) = 7,620$$

$$a_o = \frac{R_1 + R_2}{R_2 L_1 C_1} = 1.64 \times 10^9$$

$$a_1 = \frac{1}{R_2 C_1} + \frac{R_1}{L_1} = 1.79 \times 10^5$$

$$\alpha = \frac{R_1}{2L_1} + \frac{1}{2C_1(R_c + R_2)} = 4 \times 10^4$$

Therefore, for this case,

$$v(t) = 20.7 \sin (377t + 1°19') - (18,700t + .462)\, e^{-40,000t}$$

This relationship is plotted on the following Figure 12.

EFFECT OF TRANSMISSION LINE ON RECOVERY VOLTAGE.
CIRCUIT BREAKER SHUNTED WITH CRITICAL DAMPING RESISTANCE.
CASE E-2

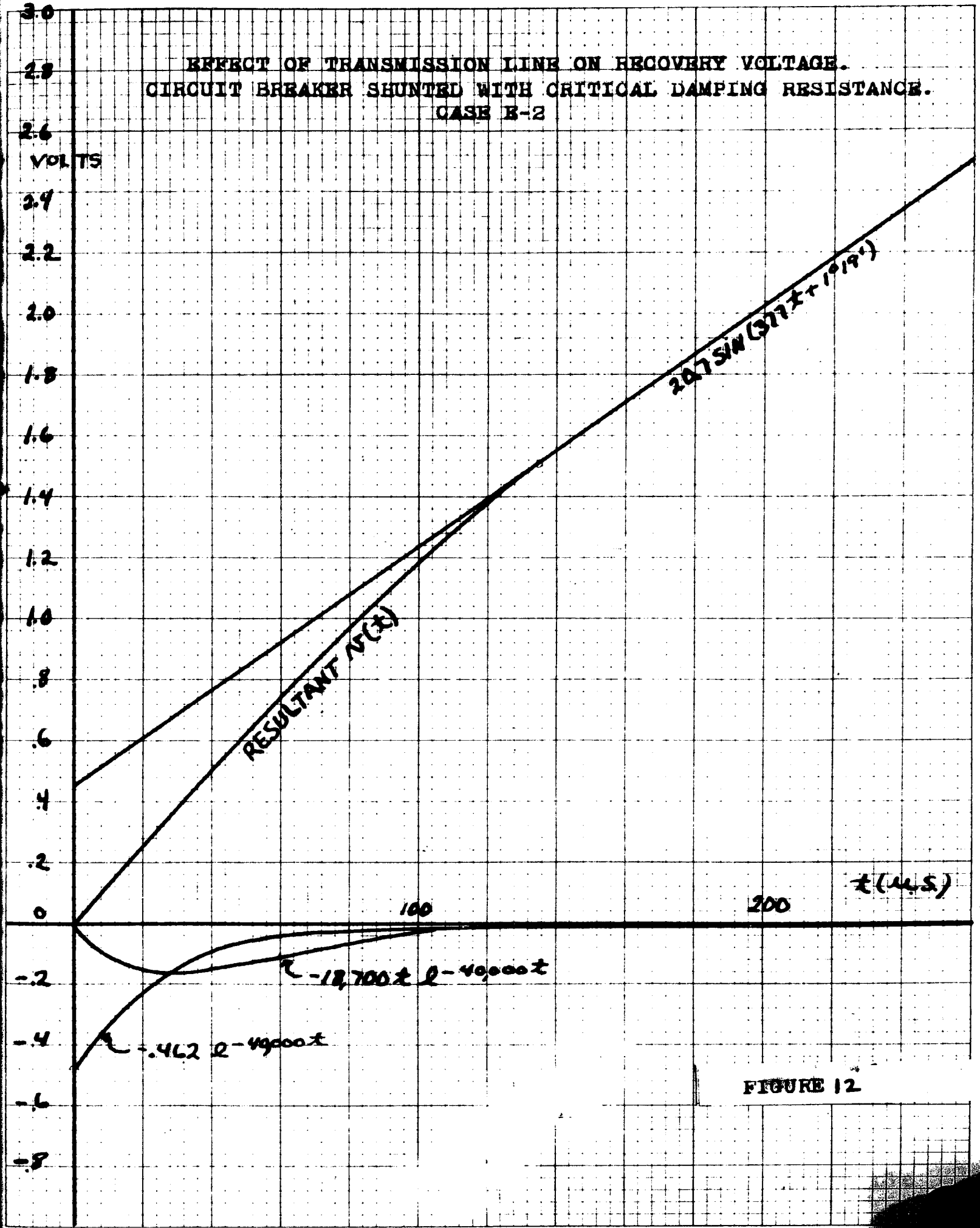


FIGURE 12

SECTION II- RECOVERY VOLTAGE STUDIES

CASE F-1,2,3,4- RECOVERY VOLTAGES WITH VARIOUS IMPEDANCE COMBINATIONS SHUNTED ACROSS CIRCUIT BREAKER.

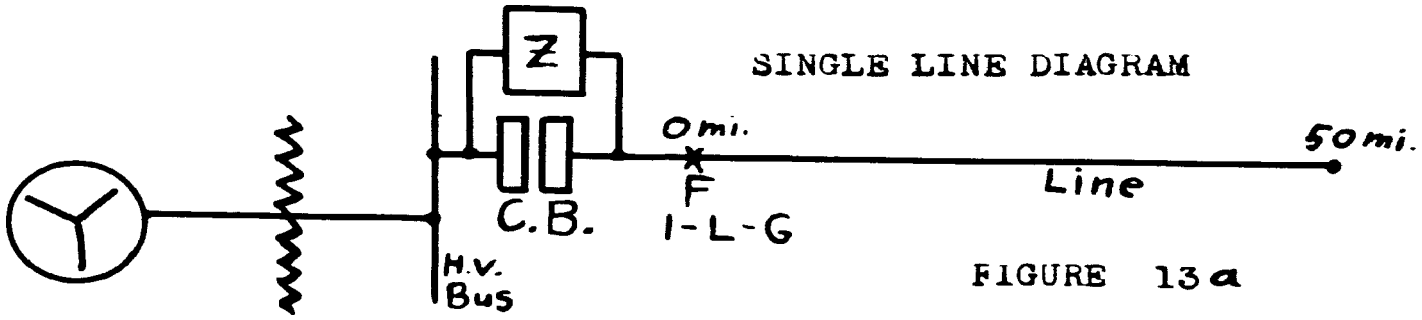
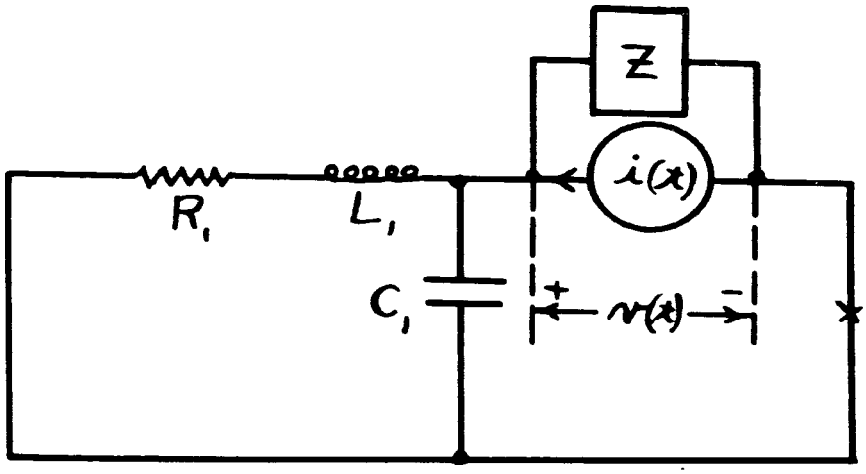


FIGURE 13a



EQUIVALENT TRANSIENT CIRCUIT

FIGURE 13b

FOUR DIFFERENT SHUNT Z COMBINATIONS STUDIED:

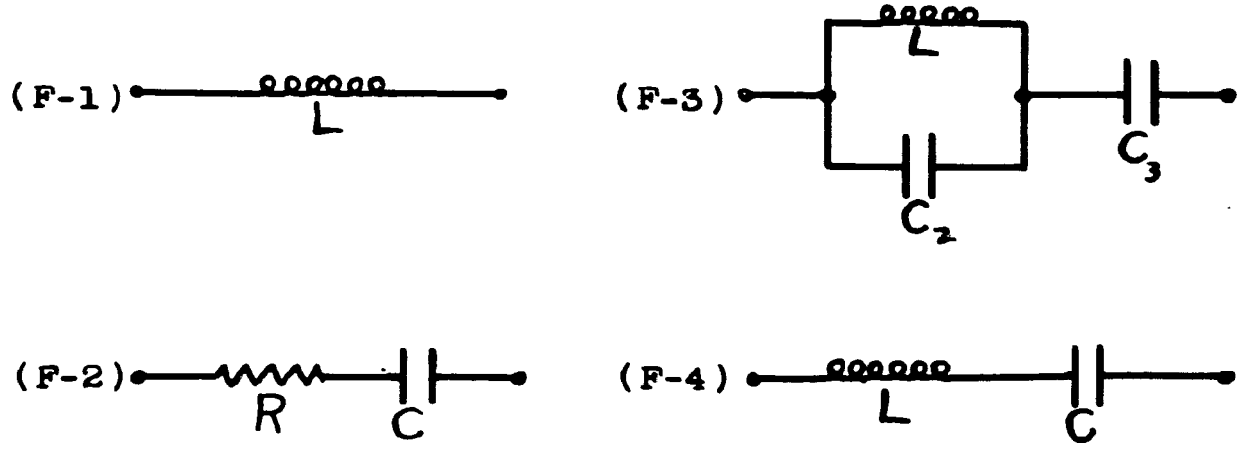


FIGURE 13c

F(1)-Inductance Paralleled With Breaker

Since, in the equivalent transient circuit, a shunting inductance is also in parallel with transformer and generator inductance, its effect is to lessen the effective inductance which determines harmonic frequency: $f = \frac{1}{2\pi\sqrt{LC}}$.

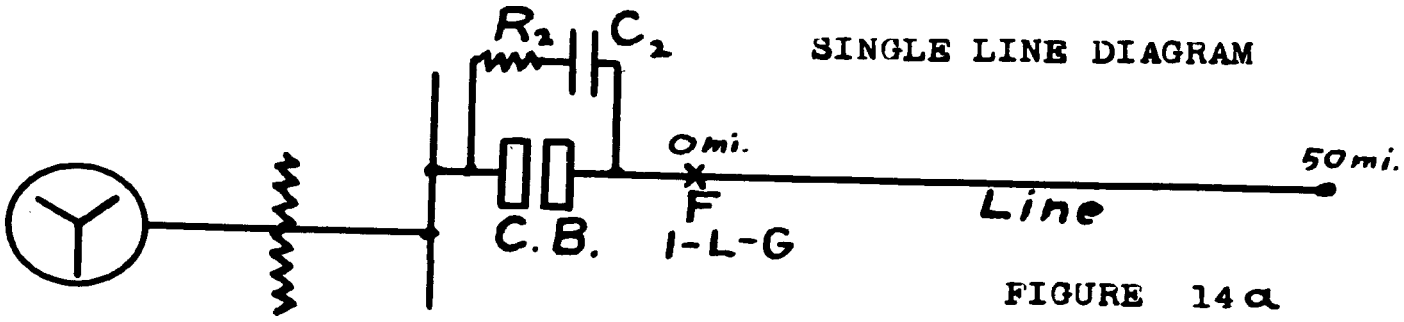
Obviously, this results in raising harmonic frequency and thus increasing the rate of rise of recovery voltage. Its effect on fundamental or harmonic amplitude is very slight. Also, using practical values for system parameters, its effect on fundamental and harmonic phase angles is very slight. In order to avoid excess breaker shunting currents, the shunt inductance must be much larger than that of generator and transformer leakage inductance; consequently, as a parallel impedance it has small effect in any case, and that small effect is the undesirable one of increasing the rate of rise of recovery voltage. Therefore, this case is impractical and will not be considered further.

F(3) Series-Parallel L + C. (Fig. 13c - F3)

This series-parallel combination can also be disposed of rather quickly. One might consider the following possibility: design L and C_1 to be parallel-resonant at fundamental frequency, then utilize C_2 to decrease the frequency of harmonic oscillation and thus decrease the rate of rise. However, using practical-sized L and C_1, at harmonic frequency L would be essentially an open circuit and this case would reduce to one already considered, that is, the shunt capacitor case. Nothing very important is to be gained from this case so it will not be considered further.

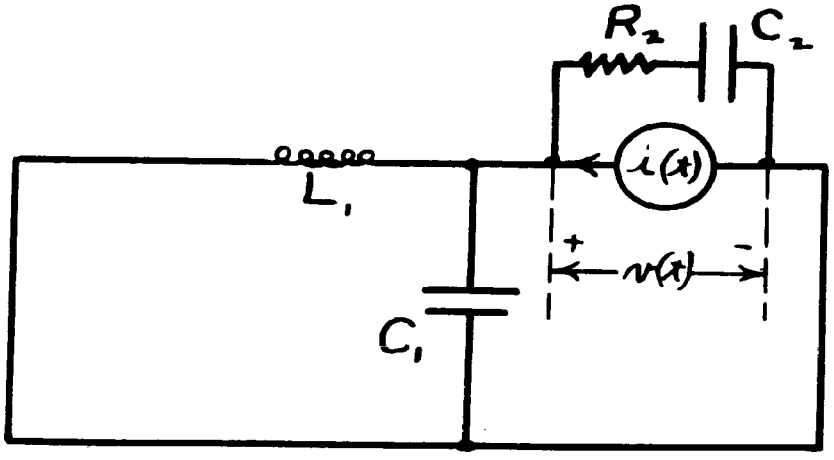
SECTION II- RECOVERY VOLTAGE STUDIES

CASE F-2- RECOVERY VOLTAGE WITH RESISTANCE AND CAPACITANCE SHUNTED ACROSS CIRCUIT BREAKER.



SINGLE LINE DIAGRAM

FIGURE 14a



EQUIVALENT TRANSIENT CIRCUIT

FIGURE 14b

CASE F-2

System losses are neglected.

$$Z(S) = \frac{\frac{SL_1}{SC_1}\left(R_2 + \frac{1}{SC_2}\right)}{\frac{SL_1}{SC_1} + \frac{1}{SC_2} + R_2}$$

$$= \frac{S}{C_1} \frac{\left(S + \frac{1}{R_2 C_2}\right)}{\left(S^3 + S^2 \frac{C_1 + C_2}{C_1 C_2 R_2} + S\frac{1}{L_1 C_1} + \frac{1}{L_1 C_1 C_2 R_2}\right)}$$

$$V(S) = I(S) \cdot Z(S)$$

$$= Z(S) \cdot \frac{I_m \omega}{S^2 + \omega^2}$$

In form, $V(S) = \frac{I_m \omega}{C_1} \frac{S\left(S + \frac{1}{R_2 C_2}\right)}{\left(S^2 + \omega^2\right)\left(S + \gamma\right)\left[(S + \alpha)^2 + \beta^2\right]}$

The roots of the following cubic must be found:

$$\left(S^3 + S^2 \frac{C_1 + C_2}{C_2 C_1 R_2} + S \frac{1}{L_1 C_1} + \frac{1}{L_1 C_1 C_2 R_2}\right)$$

Using Weisner's Notation [15],

$$a_o = 1$$

$$a_1 = \frac{C_1 + C_2}{3C_1C_2R_2}$$

$$a_2 = \frac{1}{3L_1C_1}$$

$$a_3 = \frac{1}{L_1C_1C_2R_2}$$

$$\frac{G}{2} = \frac{1}{2C_1C_2R_2}\left[\frac{1}{L_1} - \frac{C_1 + C_2}{3L_1C_1} + \frac{2}{27}\frac{(C_1 + C_2)^3}{(C_1C_2R_2)^2}\right]$$

$$H = \frac{1}{3L_1C_1} - \frac{(C_1 + C_2)^2}{9(C_1C_2R_2)^2}$$

$$G^2 = \frac{1}{(C_1C_2R_2)^2}\left[\frac{1}{L_1^2} - \frac{2(C_1 + C_2)}{3L_1^2C_1} + \frac{(C_1 + C_2)^2}{9L_1^2C_1^2}\right.$$

$$\left. + \frac{4}{729}\frac{(C_1 + C_2)^6}{(C_1C_2R_2)^4} + \frac{4}{27}\frac{(C_1 + C_2)}{L_1(C_1C_2R_2)^2} - \frac{4}{81}\frac{(C_1 + C_2)^4}{L_1C_1(C_1C_2R_2)^2}\right]$$

$$4H^3 = \frac{4}{27L_1^3C_1^3} - \frac{4}{27}\frac{(C_1 + C_2)^2}{L_1^2C_1^4C_2^2R_2^2} + \frac{4}{81}\frac{(C_1 + C_2)^4}{L_1C_1^5C_2^4R_2^4}$$

$$- \frac{4}{729}\frac{(C_1 + C_2)}{(C_1C_2R_2)^6}$$

Let $A = \sqrt[3]{\frac{G}{2} + \frac{1}{2}\sqrt{G^2 + 4H^3}}$, $B = \sqrt[3]{\frac{G}{2} - \frac{1}{2}\sqrt{G^2 + 4H^3}}$

Then

$$A = \left[\frac{1}{2C_1C_2R_2}\left\{\frac{1}{L_1} - \frac{C_1 + C_2}{3L_1C_1} + \frac{2}{27}\frac{(C_1 + C_2)^3}{(C_1C_2R_2)^2}\right\} + \right.$$

$$\left. + \frac{1}{2}\cdot\sqrt{\frac{1}{(C_1C_2R_2)^2}\left\{\frac{1}{L_1^2} - \frac{2}{3}\frac{(C_1 + C_2)}{L_1^2C_1} + \frac{(C_1 + C_2)^2}{9L_1^2C_1^2} + \frac{4}{27}\frac{(C_1 + C_2)^3}{L_1(C_1C_2R_2)^2}\right\} + \frac{4}{27L_1^3C_1^3} - \frac{4}{27}\frac{(C_1 + C_2)^2}{L_1^2C_1^4C_2^2R_2^2}}\right]^{\frac{1}{3}}$$

$$B = \left[\frac{1}{2C_1C_2R_2}\left\{\ \right\} - \frac{1}{2}\cdot\sqrt{\ \ }\right]^{\frac{1}{3}}$$

Then in terms of the above defined quantities, the three roots are:

$$X_1 = -a_1 - (A + B)$$

$$X_2 = \left[-a_1 + \frac{1}{2}(A + B)\right] - j\left[\frac{\sqrt{3}}{2}(A - B)\right]$$

$$X_3 = \left[-a_1 + \frac{1}{2}(A + B)\right] + j\left[\frac{\sqrt{3}}{2}(A - B)\right]$$

where $a_1 = \frac{C_1 + C_2}{3C_1C_2R_2}$ as defined above.

Therefore $\alpha = a_1 - \frac{1}{2}(A + B)$

$\beta = \pm \frac{\sqrt{3}}{2}(A - B)$

$\gamma = a_1 + (A + B)$

Now find $\mathcal{L}^{-1}[V(S)]$

$V(S) = I(S) \cdot Z(S)$

$$= \frac{I_m \omega}{C_1} \frac{S\left(S + \frac{1}{R_2 C_2}\right)}{(S^2 + \omega^2)(S + \gamma)[(S + \alpha)^2 + \beta^2]}$$

$\mathcal{L}^{-1}[V(S)] = \frac{d}{dt}\left[\mathcal{L}^{-1}\frac{V(S)}{S}\right] = v(t)$, if

$v(0^+) = 0$, which is true here.

$$\mathcal{L}^{-1}\left[\frac{V(S)}{S}\right] = \frac{I_m \omega}{C_1}\left[A e^{-\gamma t} + B \sin(\omega t + \psi_1) + C e^{-\alpha t} \sin(\beta t - \psi_2)\right]$$

$$v(t) = \frac{d}{dt}\left\{\mathcal{L}^{-1}\left[\frac{V(S)}{S}\right]\right\}$$

$$= \frac{I_m \omega}{C_1}\left[-\gamma A_o e^{-\gamma t} + \omega B_o \cos(\omega t + \psi_1) + C_o e^{-\alpha t} \beta \cos(\beta t - \psi_2) - \alpha e^{-\alpha t} C \sin(\beta t - \psi_2)\right]$$

where $A_o = \dfrac{\frac{1}{R_2C_2} - \gamma}{(\omega^2 - \gamma^2)\left[(\alpha - \gamma)^2 + \beta^2\right]}$

$$B_o = \frac{1}{\omega}\left[\frac{\left(\frac{1}{R_2C_2}\right)^2 + \omega^2}{(\gamma^2 + \omega^2)\left[(\alpha^2 + \beta^2 - \omega^2)^2 + 4\alpha^2\omega^2\right]}\right]^{\frac{1}{2}}$$

$$C_o = \frac{1}{\beta}\left\{\frac{\left(\frac{1}{R_2C_2} - \alpha\right)^2 + \beta^2}{\left[(\gamma - \alpha)^2 + \beta^2\right]\left[(\alpha^2 + \beta^2 - \omega^2)^2 + 4\alpha^2\omega^2\right]}\right\}^{\frac{1}{2}}$$

and $\psi_1 = \tan^{-1}\omega R_2C_2 - \tan^{-1}\frac{\omega}{\gamma} - \tan^{-1}\frac{2\alpha\omega}{\alpha^2 + \beta^2 - \omega^2}$

$$\psi_2 = \tan^{-1}\frac{\beta}{\frac{1}{R_2C_2} - \alpha} - \tan^{-1}\frac{\beta}{\gamma - \alpha} - \tan^{-1}\frac{2\alpha\beta}{\beta^2 - \alpha^2 - \omega^2}$$

The above expression for v(t) is rather complex in form. However, a special case of great importance is that in which R_2 and C_2 are adjusted to give critical damping. Clearly we have two independent variables R_2 and C_2, and with these, two degrees of freedom are obtained in controlling recovery voltage waveform and rates of rise.

In order to obtain critical damping, set $\beta = 0$

$$\beta = 0 = \frac{\sqrt{3}}{2}(A - B)$$

Therefore $A = B$, and from the definition of A and B given on page **63**, A can equal B only if

$$\sqrt{\quad} = 0$$

This gives the following equation:

$$R_2^4\left(4C_1C_2^4\right) + R_2^2\left(27L_1C_1^2C_2^2 - 18\,C_oL_1C_1C_2^2\right) + \\ - C_o^2L_1C_2$$

$$+\left(4C_o^3L_1^2\right) = 0$$

where $C_o = \left(C_1 + C_2\right)$

If we let $X = R_2^2$, this fourth degree equation in the unknown R_2 is of the form:

$$b_oX^2 + b_1X + b_2 = 0$$

where, $b_o = 4C_1C_2^4$

$$b_1 = 27L_1C_1^2C_2^2 - 18C_oL_1C_1C_2^2 - C_o^2L_1C_2^2$$

$$b_2 = 4C_o^3L_1^2$$

Therefore $$X = \frac{-b_1 \pm \sqrt{b_1^2 - 4b_ob_2}}{2b_o}$$

and $$R_2 = \sqrt{X}$$

Critical damping R_2 will now be computed from the equations above, assuming an arbitrary value of 0.5 uf for C_2.

From the Appendix,

$$L_1 = 31.2 \text{ mh}$$
$$C_1 = .02 \ \mu f$$
$$C_o = C_1 + C_2 = .52 \ \mu f$$

Calculated values:

$$b_o = 0.5 \times 10^{-32}$$
$$b_1 = -34.9 \times 10^{-28}$$
$$b_2 = 5.45 \times 10^{-22}$$
$$X = 45.96 \times 10^4, \text{ or } 23.76 \times 10^4$$
R_2 = 678 ohms or 487 ohms for critical damping in series with
$$C_2 = 0.5 \ \mu f$$

Analysis of critically-damped case

Set $\beta = 0$

$$V(S) = \frac{I_m \omega}{C_1} \frac{S\left(S + \frac{1}{R_2 C_2}\right)}{\left(S^2 + \omega^2\right)(S + \gamma)(S + \alpha)^2}$$

$$v(t) = \mathcal{L}^{-1}\left[V(S)\right]$$

$$v(t) = \frac{E_m}{LC_1}\left[\frac{\gamma^2 - \frac{\gamma}{R_2C_2}}{(\gamma^2+\omega^2)(\alpha-\gamma)^2}\, e^{-\gamma t} + \right.$$

$$+ \frac{\left(\omega^2 + \frac{1}{R_2^2C_2^2}\right)^{\frac{1}{2}}}{(\gamma^2+\omega^2)^{\frac{1}{2}}(\alpha^2+\omega^2)} \sin(\omega t - \psi) + \frac{\alpha^2 - \frac{\alpha}{R_2C_2}}{(\gamma-\alpha)(\alpha^2+\omega^2)} te^{-\alpha t}$$

$$\left. + \frac{(\gamma-\alpha)(\alpha^2+\omega^2)\left(\frac{1}{R_2C_2} - 2\alpha\right) - \left(\alpha^2 - \frac{\alpha}{R_2C_2}\right)\left(3\alpha^2+\omega^2-2\alpha\gamma\right)}{(\gamma-\alpha)^2(\alpha^2+\omega^2)^2} e^{-\alpha t}\right]$$

where $\psi = \tan^{-1}\frac{1}{-\omega R_2C_2} - \tan^{-1}\frac{\omega}{\gamma} - 2\tan^{-1}\frac{\omega}{\alpha}$

If we use experimental values from the Appendix, an actual expression for critically-damped v(t) can be developed:

Use the smaller value of critical R_2:

$$R_2 = 487\ \Omega$$
$$C_1 = .02\text{uf}$$
$$C_2 = .5\ \text{uf}$$
$$L_1 = 31.2\ \text{mh}$$

$$A = B = \left[10.25 \times 10^{10}\{32.1 - 277.7 + 438\}\right]^{\frac{1}{3}}$$
$$= 27{,}000$$

$$\alpha = \frac{C_1 + C_2}{3C_1C_2R_2} - A = 8{,}590$$

$$\gamma = \frac{C_1 + C_2}{3C_1C_2R_2} + 2A = 89{,}590$$

(Check: $\alpha^2\gamma = \frac{1}{L_1C_1C_2R_2}$

$66 \times 10^{11} = 66 \times 10^{11}$).

If the larger value of critical R_2 is used: $R_2 = 678$

$A = B = -11,380$

$\alpha = 36,945$

$\gamma = 2,805$

Note: since both α and γ are positive, the system is stable, and both values of R_2 are therefore usable. The smaller value will be used in the following analysis.

If we substitute into the equation just developed for critically damped v(t),

$$v(t) = 2.33\, e^{-89,590t} + 10 \sin (377t + 89°58') + 10.36 \times 10^4\, t\, e^{-8,590t} - 12.3\, e^{-8,590t}$$

Again the check holds, that

$$v(0^+) = 0$$

The above expression is plotted on the following figure (Fig.15) for comparison with previous cases.

By comparing this curve with the curve for pure resistance shunted across the breaker (Fig. 7), we can see that rate of rise is very close to that of the pure resistance case. The peak voltage rises to 116% of normal phase voltage 210 micro-seconds after current interruption.

In the experimental section of the Appendix, a large

number of resistance-capacitance combinations are investigated. A curve is plotted Fig. 54, giving the locus of optimum values of R and C to be used in shunt with the circuit breaker. Thus a vast amount of mathematical computation is avoided. It is found that a combination of C_2 = 4.5 uf and an R_2 = 200 ohms gives a more desirable recovery voltage waveform (Fig. 50) than the case of critical damping just analyzed.

The other critically damped case, using the larger value of R_2 = 678 ohms, gives a waveform very similar to the one just analyzed. The experimental curves for both cases are virtually identical (Figs. 48-49) and check quite well in general form with the computed curve (Fig. 15).

RECOVERY VOLTAGE: CRITICALLY DAMPED CASE

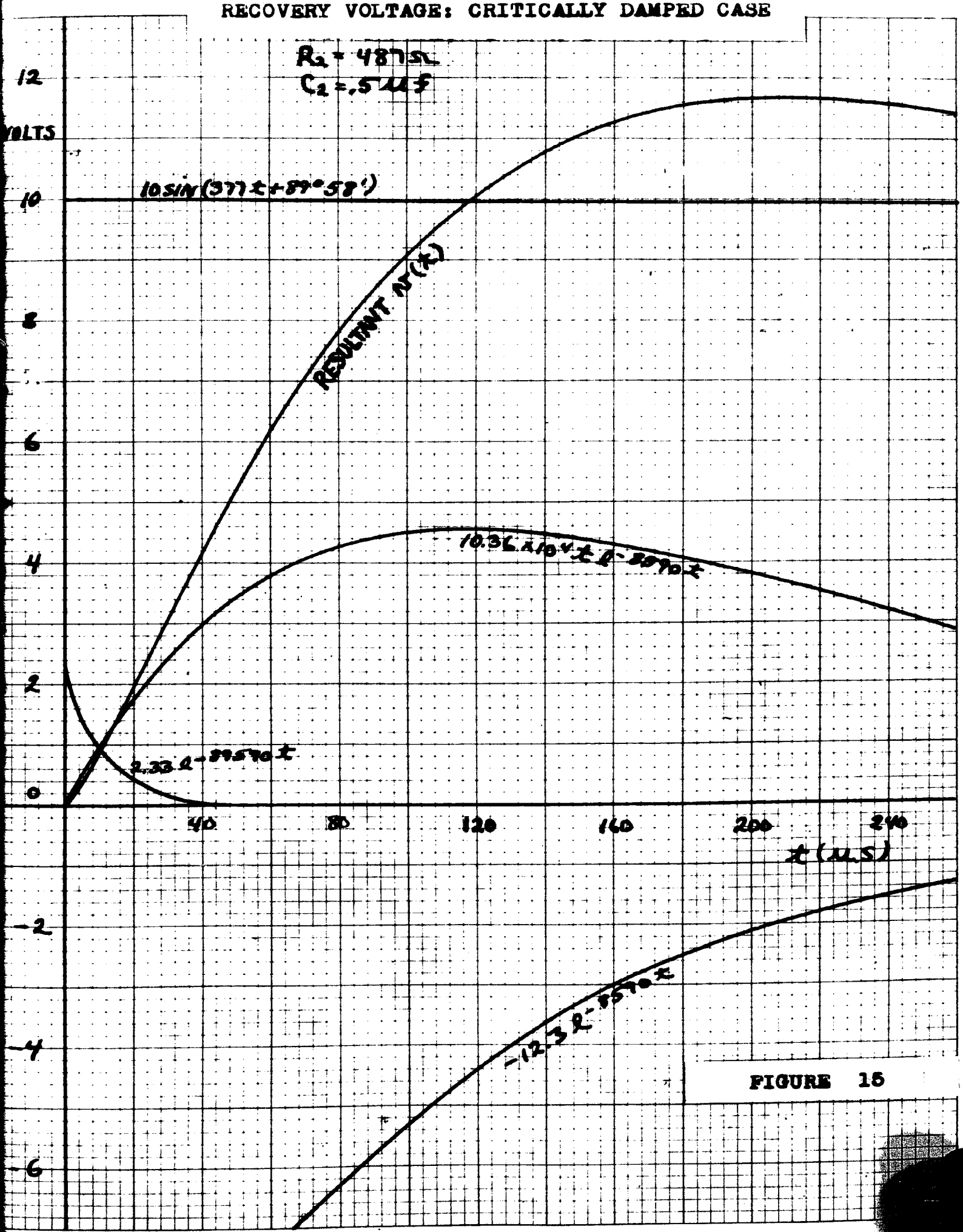


FIGURE 15

Since it was found experimentally (see Appendix Fig. 50) that the overdamped case of resistance and capacitance shunted across the circuit breaker gave the least severe recovery voltage waveform of all corrective measures attempted, this case will now be analyzed.

The form for V(S) is:

$$V(S) = \frac{E_m}{L_1 C_1} \frac{S\left(S + \frac{1}{R_2 C_2}\right)}{\left(S^2 + \omega^2\right)(S+\gamma)(S+\alpha)(S+\delta)}$$

Since only the first 250 micro-seconds are of interest, the approximation can be made that

$$I_m \sin \omega t \cong I_m \omega t$$

So that the expression for V(S) becomes,

$$V(S) = \frac{E_m}{L_1 C_1} \frac{\left(S + \frac{1}{R_2 C_2}\right)}{S(S+\alpha)(S+\gamma)(S+\delta)}$$

In order to obtain v(t), an extension of the transform tables [6] is required.

$$\text{If } \mathcal{L}^{-1}\left[F(S)\right] = f(t)$$

$$\text{Then } \mathcal{L}^{-1}\left[\frac{1}{S} F(S)\right] = \int_0^t f(t)\, dt$$

Defining $$F(S) = \frac{S + a_o}{(S + \alpha)(S + \gamma)(S + \delta)}$$

Then $$f(t) = \frac{a_o - \alpha}{(\gamma - \alpha)(\delta - \alpha)} e^{-\alpha t}$$

$$+ \frac{a_o - \gamma}{(\alpha - \gamma)(\delta - \gamma)} e^{-\gamma t} + \frac{a_o - \delta}{(\alpha - \delta)(\gamma - \delta)} e^{-\delta t}$$

and

$$\int_o^t f(t)\,dt = \frac{a_o - \alpha}{\alpha(\gamma - \alpha)(\delta - \alpha)} \left[1 - e^{-\alpha t}\right]$$

$$+ \frac{a_o - \gamma}{\gamma(\alpha - \gamma)(\delta - \gamma)} \left[1 - e^{-\gamma t}\right]$$

$$+ \frac{a_o - \delta}{\delta(\alpha - \delta)(\gamma - \delta)} \left[1 - e^{-\delta t}\right]$$

where

$$a_o = \frac{1}{R_2 C_2}$$

$$\alpha = -a_1 - (A + B)$$

$$\gamma = \left[-a_1 + \tfrac{1}{2}(A + B)\right] - j\left[\tfrac{\sqrt{3}}{2}(A - B)\right]$$

$$\delta = \left[-a_1 + \tfrac{1}{2}(A + B)\right] + j\left[\tfrac{\sqrt{3}}{2}(A - B)\right]$$

(a_1, A, and B are defined on page 63)

In order to obtain an actual waveform, numerical values for shunting impedance must be chosen. So that this case may be compared with the pure resistance shunting case, the values of R_2 and C_2 are chosen so that the combined series impedance to fundamental frequency currents is equal in magnitude to critical resistance (621 ohms).

Assume $C_2 = 4.5\ \mu f$

Then $X_{C_2} = 590$ ohms ($\omega = 377$)

And $R_2 = 200$ ohms

Using these values, the constants defined on p.63 have the following values:

$$a_1 = 83{,}600$$

$$A = \left[.518 + j\ .0193\right]^{\frac{1}{3}} \times 10^5 = .802 \times 10^5 \underline{\lfloor -43'}$$

$$B = \left[.518 - j\ .0193\right]^{\frac{1}{3}} \times 10^5 = .802 \times 10^5 \underline{\lfloor -43'}$$

$$X_1 = -83{,}600 - 2 \times 80{,}200 \cos 43'$$
$$= -244{,}000 = -\alpha$$

$$X_2 = -83{,}600 + 80{,}200 \times 2 \cos (60° - 43')$$
$$= -1{,}600 = -\gamma$$

$$X_3 = -83{,}600 + 80{,}200 \times 2 \cos (60° + 43')$$
$$= -5{,}100 = -\beta$$

Then,

$$v(t) = \frac{E_m}{L_1 C_1}\left[\frac{\frac{1}{R_2C_2} - \alpha}{\alpha(\gamma - \alpha)(\delta - \alpha)}\left(1 - e^{-\alpha t}\right)\right.$$

$$\left. + \frac{\frac{1}{R_2C_2} - \gamma}{\gamma(\alpha-\gamma)(\delta-\gamma)}\left(1 - e^{-\gamma t}\right) + \frac{\frac{1}{R_2C_2} - \delta}{\delta(\alpha-\delta)(\gamma-\delta)}\left(1 - e^{-\delta t}\right)\right]$$

$$v(t) = -.275\left(1 - e^{-244,000t}\right) - 4.9\left(1 - e^{-1,600t}\right)$$

$$+ 15\left(1 - e^{-5,100t}\right)$$

again the check holds, that

$$v(0^+) = 0\text{, also, } \quad v(\infty) \tilde{=} 10$$

The above equation for v(t) is plotted on the following figure.

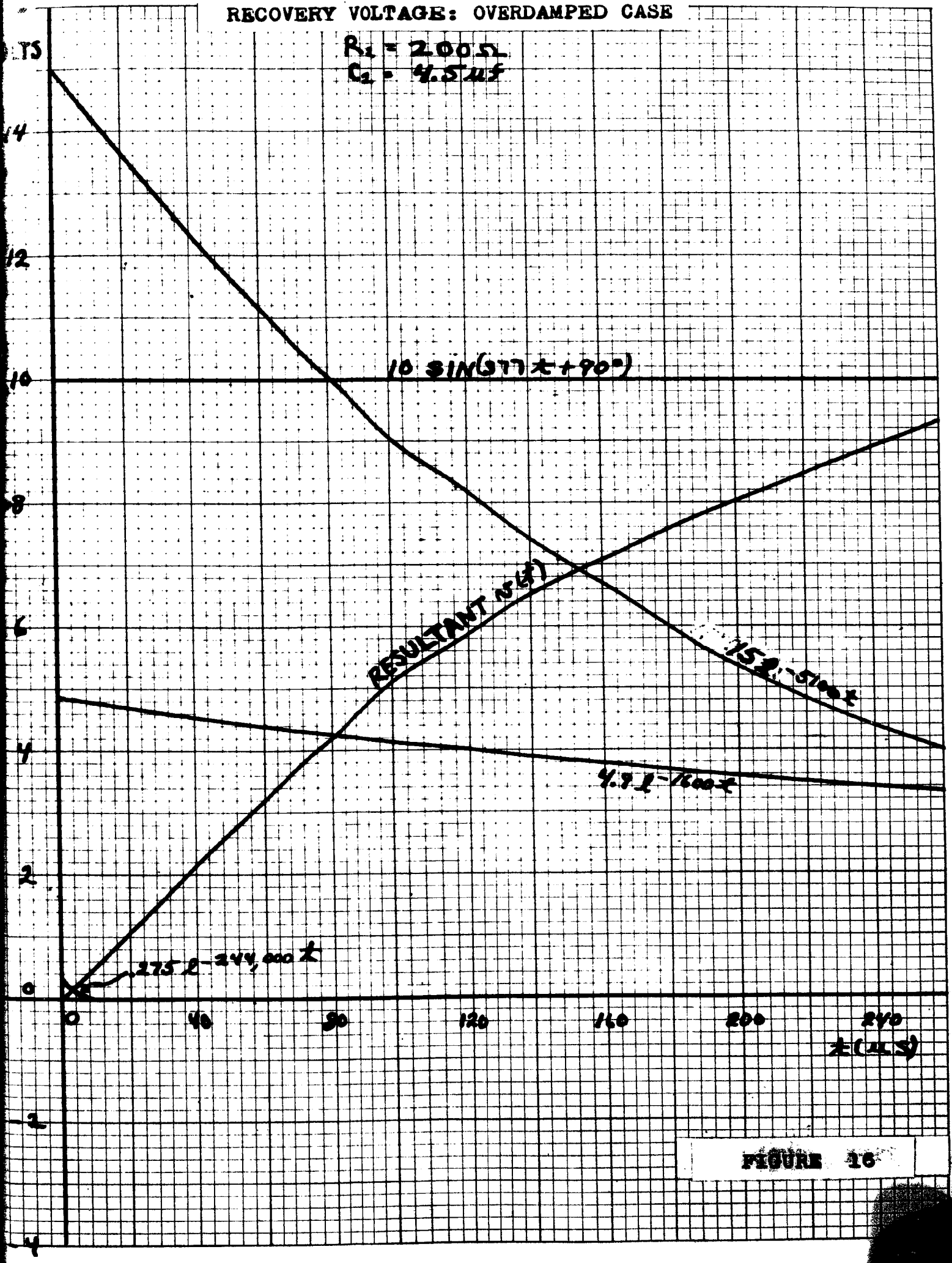
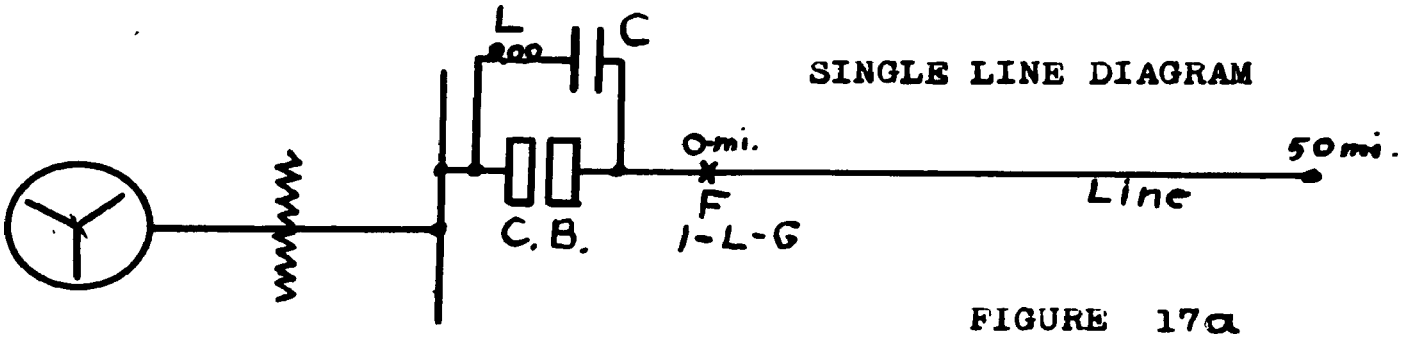

RECOVERY VOLTAGE: OVERDAMPED CASE
$R_1 = 200\,\Omega$
$C_2 = 4.5\,\mu f$
$10\,SIN(377t + 90°)$
RESULTANT $v_2(t)$
$15.2\,e^{-510t}$
$4.7\,e^{-100t}$
$.275\,e^{-244,000t}$
16
14
12
10
8
6
4
2
0
-2
-4
0
40
80
120
160
200
240
$t(\mu s)$

FIGURE 16

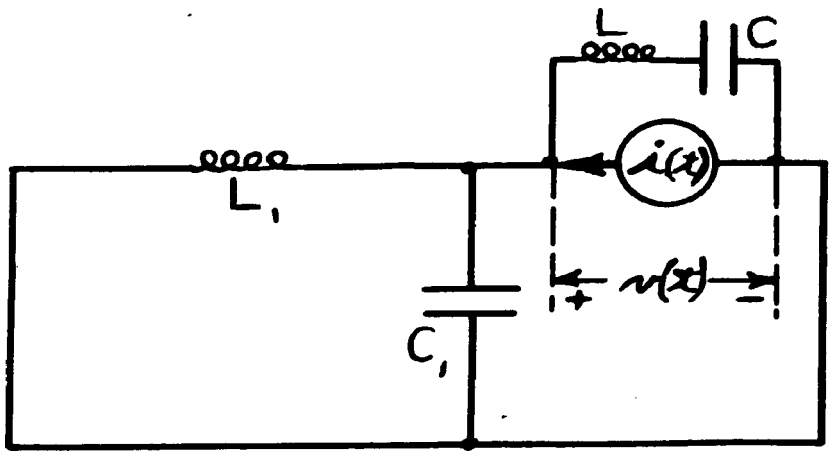
SECTION II- RECOVERY VOLTAGE STUDIES

CASE F-4- RECOVERY VOLTAGE WITH INDUCTANCE. AND CAPACITANCE SHUNTED ACROSS CIRCUIT BREAKER.



SINGLE LINE DIAGRAM

FIGURE 17a



EQUIVALENT TRANSIENT CIRCUIT

FIGURE 17b

CASE F-4 Inductance and Capacitance in Series Shunted Across Circuit Breaker

From the Equivalent Transient Circuit, Figure 17b,

$$Z(S) = \frac{S\left(S^2 + \frac{1}{LC}\right)}{C_1\left[S^4 + S^2\left(\frac{1}{L_1C_1} + \frac{1}{LC} + \frac{1}{LC_1}\right) + \frac{1}{L_1LC_1C}\right]}$$

Since we are primarily interested in the first 200 micro-seconds of recovery voltage,

$$i = I_m \sin \omega t \cong I_m \omega t$$

(for $t < 200$ uS)

and $$\mathcal{L}[i(t)] \cong \frac{I_m \omega}{S^2} = I(S)$$

$$V(S) = I(S) \cdot Z(S)$$

$$= \frac{I_m \omega}{C_1} \cdot \frac{\left(S^2 + \frac{1}{LC}\right)}{S\left[S^4 + S^2\left(\frac{1}{L_1C_1} + \frac{1}{LC} + \frac{1}{LC_1}\right) + \frac{1}{L_1LC_1C}\right]}$$

Define $$\lambda^2 = \frac{-}{+} \sqrt{\left(\frac{1}{L_1C_1} + \frac{1}{LC} + \frac{1}{LC_1}\right)^2 - \frac{4}{L_1LC_1C}}$$

$$2$$

Define $$\beta^2 = \frac{\left(\frac{1}{L_1C_1} + \frac{1}{LC} + \frac{1}{LC_1}\right) \pm \sqrt{\left(\frac{1}{L_1C_1} + \frac{1}{LC} + \frac{1}{LC_1}\right)^2 - \frac{4}{L_1LC_1C}}}{2}$$

$$\text{Then} \quad V(S) = \frac{I_m \omega}{C_1} \frac{\left(S^2 + \frac{1}{LC}\right)}{S\left[S^4 + S^2(\lambda^2 + \beta^2) + \lambda^2\beta^2\right]}$$

$$v(t) = \mathcal{L}^{-1}\left[V(S)\right]$$

$$= \frac{I_m \omega}{(\lambda^2 - \beta^2)C_1}\left[\left(1 + \frac{1}{\beta^2}\right)\cos \beta t + \left(\frac{1}{\lambda^2} - 1\right)\cos \lambda t - \left(\frac{1}{\beta^2} + \frac{1}{\lambda^2}\right)\right]$$

Experimentally, L and C of the shunt branch are adjusted to be series resonant at system (L_1 and C_1) harmonic frequency, and at the same time to offer sufficiently high impedance at fundamental frequency so that the shunt current is not excessive. Since it is the system harmonic (L_1 and C_1) which causes the high peaks and steep rates of rise of recovery voltage, the series-resonant shunt branch seems to offer possibilities: however, in ridding the recovery voltage of one harmonic, two new harmonics are introduced as verified by the above equation. This result is borne out experimentally also (Figs. 57 and 58). Nothing is gained, therefore, by this corrective attempt.

SECTION II- RECOVERY VOLTAGE STUDIES

CASE G- RECOVERY VOLTAGE WITH FAULT AT FAR END OF TRANSMISSION LINE. CAPACITANCE SHUNTED ACROSS C.B.

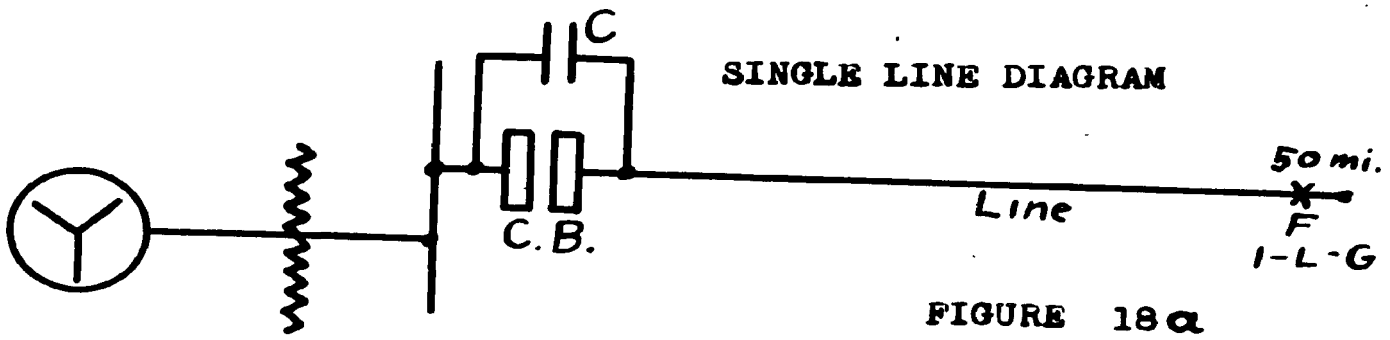


FIGURE 18a

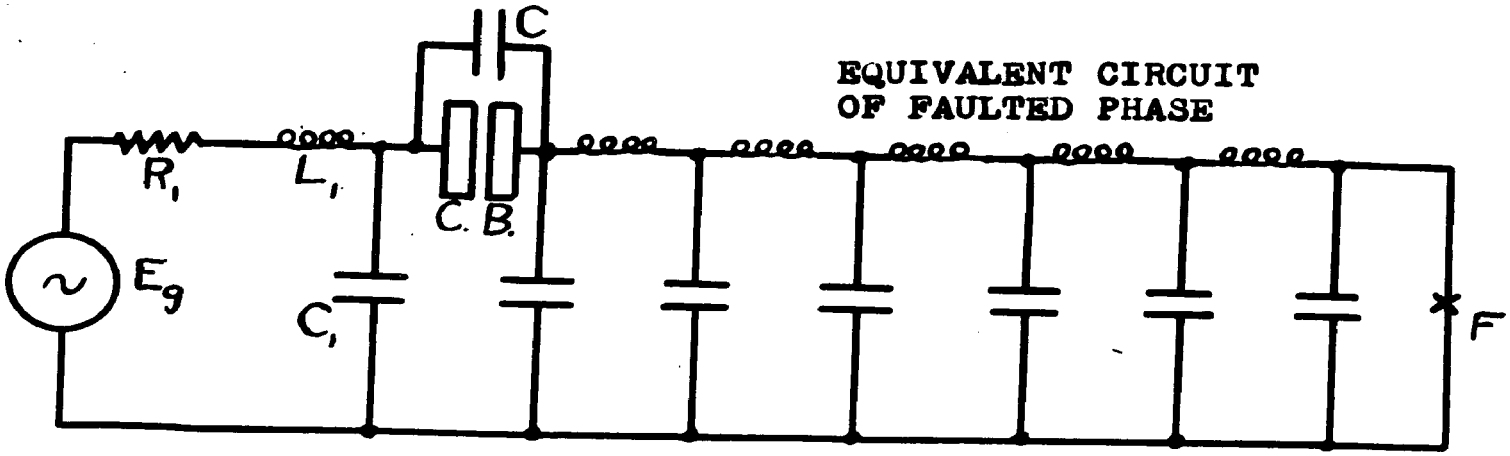


FIGURE 18b

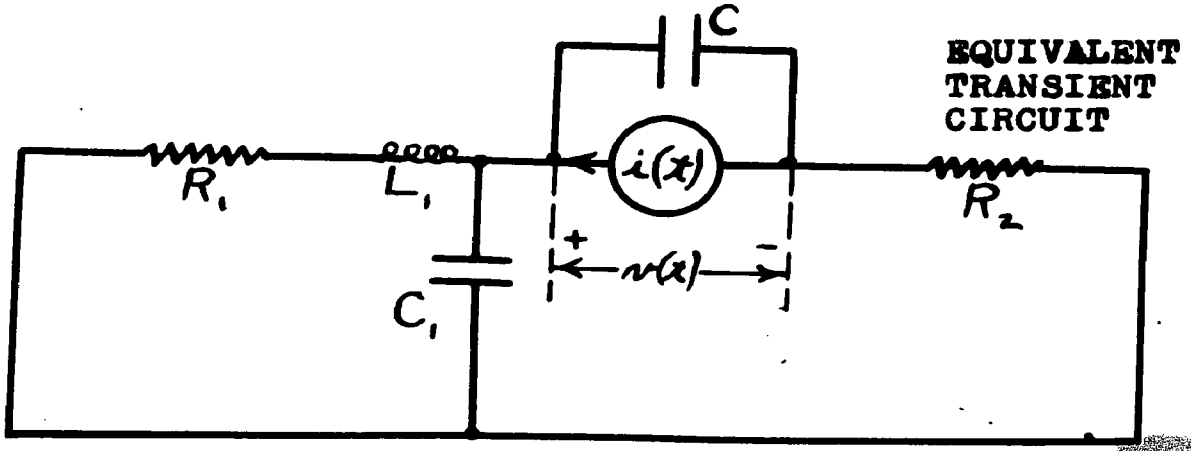


FIGURE 18c

R_2-LINE SURGE IMPEDANCE

C-SHUNTING CAPACITANCE

CASE G - Recovery Voltage with Fault at Far End of Transmission Line. Capacitance Shunted Across Circuit Breaker.

From the Equivalent Transient Circuit, Fig. 18c,

$$Z(S) = \frac{\left[S^2 + S\left(\frac{C_1R_1R_2 + L_1}{L_1C_1R_2}\right) + \frac{R_1 + R_2}{L_1C_1R_2}\right]}{C\left[S^3 + S^2\left(\frac{R_1}{L_1} + \frac{1}{CR_2} + \frac{1}{C_1R_2}\right) + S\left(\frac{R_1C + R_1C_1 + CR_2}{L_1C_1R_2C}\right) + \frac{1}{L_1C_1R_2C}\right]}$$

$$V(S) = \frac{I_m \omega}{S^2 + \omega^2} \cdot Z(S) = \frac{A\left(S^2 + a_1S + a_o\right)}{\left(S^2 + \omega^2\right)\left[(S + \alpha)^2 + \beta^2\right](S + \gamma)}$$

This particular case is very tedious of solution. However, the form of solution is easily determined. The $(S^2 + \omega^2)$ term of course yields the 60-cycle fundamental, the $(S + \gamma)$ term is a damping exponential, and the $\left[(S + \alpha)^2 + \beta^2\right]$ term is a damped harmonic of angular frequency β and with a time constant of $\frac{1}{\alpha}$.

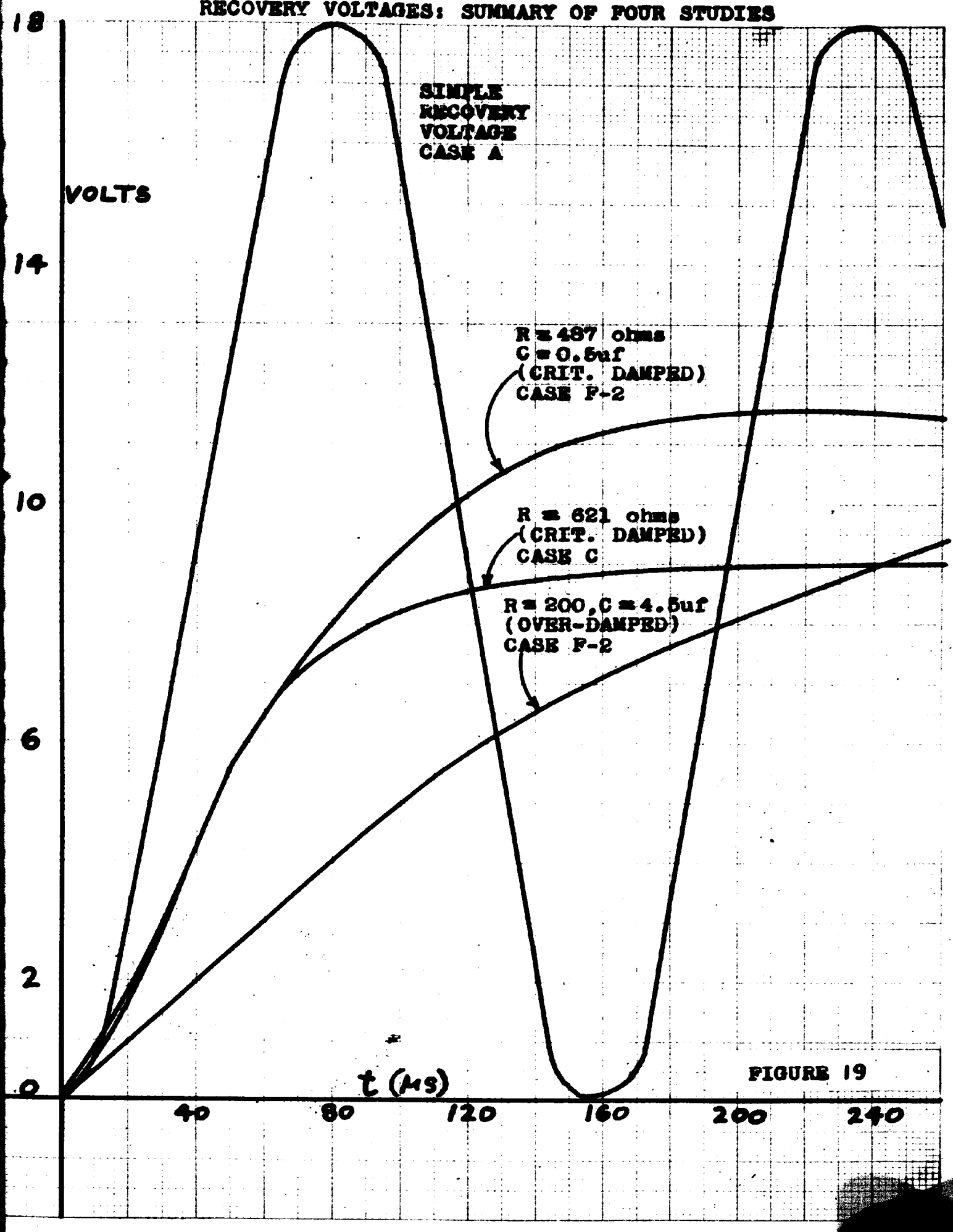

RECOVERY VOLTAGES: SUMMARY OF FOUR STUDIES
18
14
10
6
2
0
VOLTS
SIMPLE RECOVERY VOLTAGE CASE A
R = 487 ohms
C = 0.5uf
(CRIT. DAMPED)
CASE F-2
R = 621 ohms
(CRIT. DAMPED)
CASE C
R = 200, C = 4.5uf
(OVER-DAMPED)
CASE F-2
t (ms)
40
80
120
160
200
240

FIGURE 19

APPENDIX

Experimental Study of Recovery Voltage

Introduction

Because of the large amount of time involved in mathematical methods of analysis, and also to serve as a check on the results obtained through the Laplace Transform method, a miniature three-phase power system was set up and transient solutions obtained for various fault locations with the aid of a specially constructed synchronous switch and a cathode-ray oscilloscope. The schematic diagram of equipment used is shown below (Fig. 20).

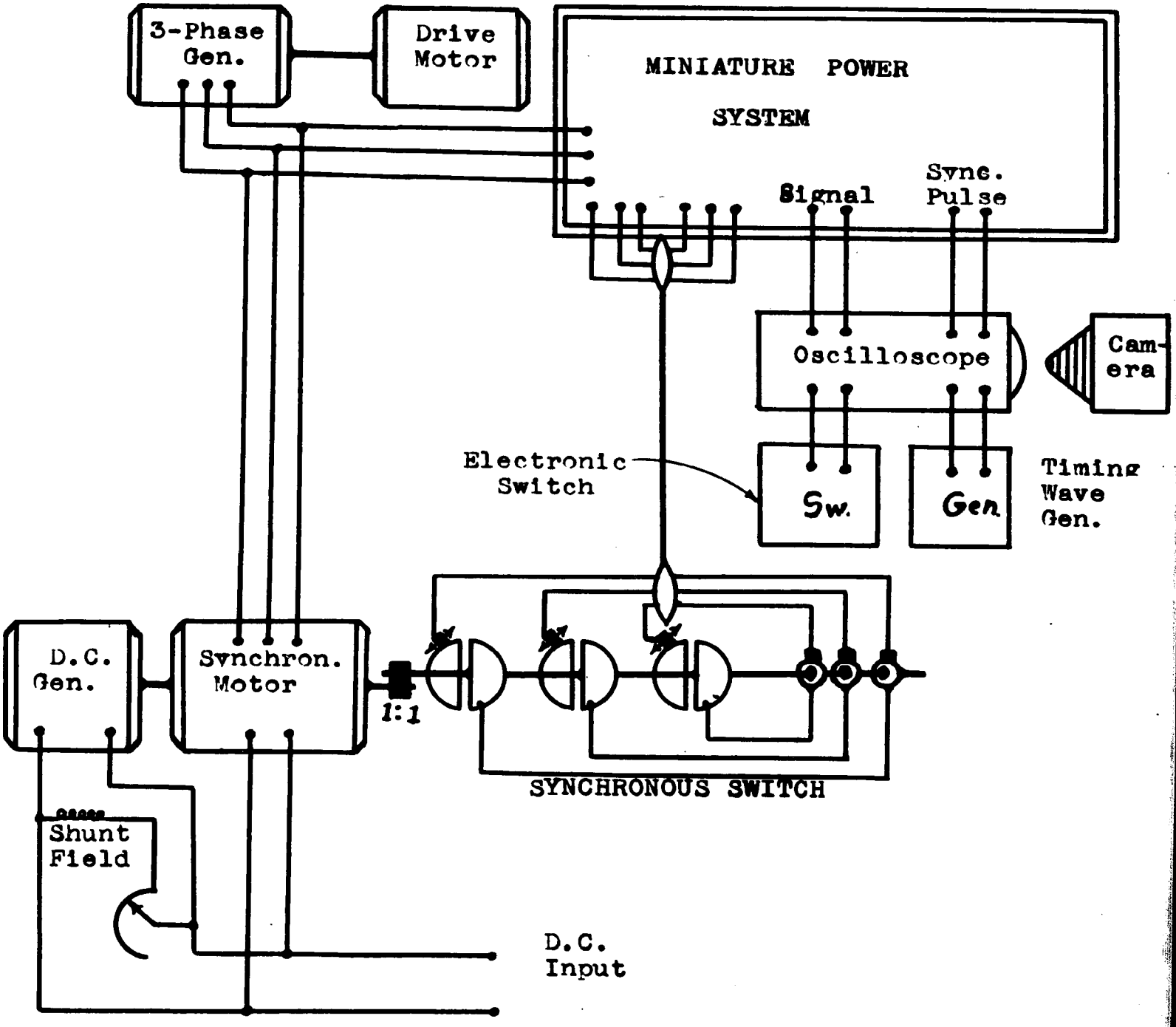
In all recovery voltage waveforms shown, the ordinate and abscissa scales are as follows:

Peak voltage on all waveforms is 10 volts.

Horizontal time scale is marked off by the timing wave oscillator which has a frequency of 5,000 cycles per second and therefore a period of 200 microseconds.

SYSTEM DIAGRAM

FIGURE 20



MINIATURE THREE PHASE POWER SYSTEM

SINGLE LINE DIAGRAM

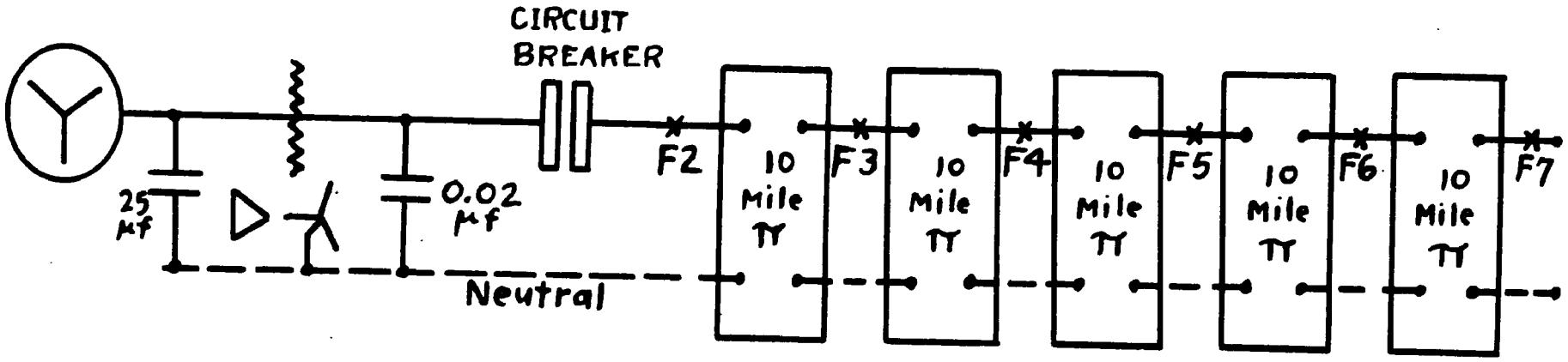


FIGURE 21

SYNCHRONIZING PULSE GENERATING CIRCUIT

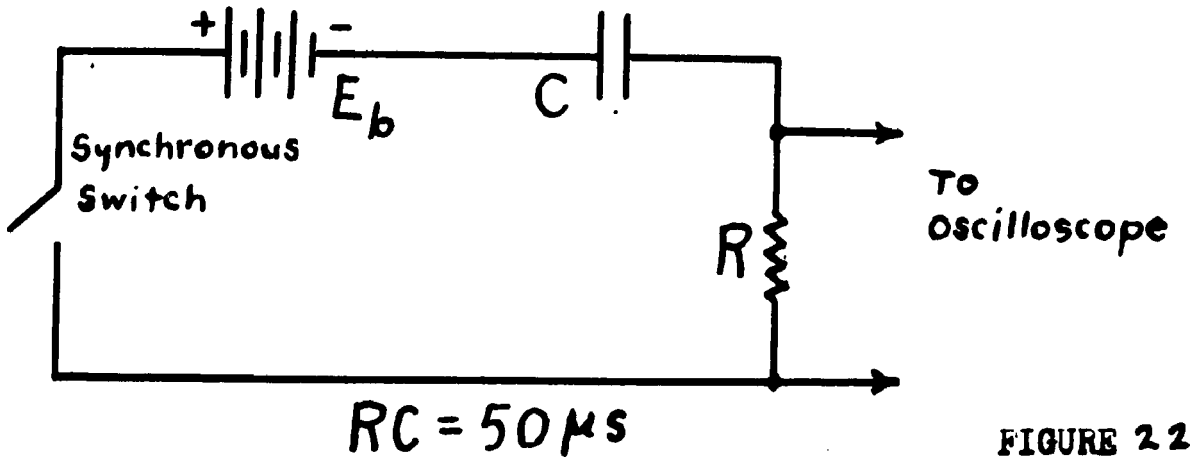


FIGURE 22

CAMERA DATA

Camera Number 1 - 4" x 5" Linhof

Lens - 6 3/8" Wollensak Raptar

Film - Ansco Triple-S Pan 4 x 5

Exposures - f/4.5 - from 1/10 to 1/2 sec.

Distance - 2 feet from scope face

Steady sweep - 1/2 sec.

Jerky sweep - 1/10 sec.

Camera Number 2 - Rolliflex

Lens - Zeiss Tessar 3.5

Distance - 2 1/2 feet from scope face

Exposure - f/3.5

Steady sweep - 1/5 sec.

Jerky sweep - 1/25 sec.

Film - Super XX and Plus X



Throughout this paper, a large-capacity supply system is assumed. This infinite bus assumption rules out the necessity of considering synchronous machine reactance variations. Three large 25 uf condensers provide a virtual short-circuit bypass path to neutral for the high frequency components of recovery voltage, in conformance with the infinite bus assumption. Three .02 uf condensers are added from the high-tension transformer side to neutral to simulate reasonable values of transformer and high voltage bus capacitance to ground. Three current-limiting reactors of common size are inserted with a corresponding capacitance to ground in order to add another component of high frequency recovery voltage (Case B only). The reactors connect to a five-pi-section simulated medium voltage line. Each pi represents a ten-mile section of line. Calculations for line constants are based on the following assumptions:

A typical 66 KV single-circuit line, transposed regularly, equivalent spacing approximately 10.4 ft., $X_0 = 3.5\ X_1$. Computed values of delta and wye capacitances, and wye reactances are given in the circuit diagram of the line, Fig. 23.

ARTIFICIAL THREE PHASE TRANSMISSION LINE

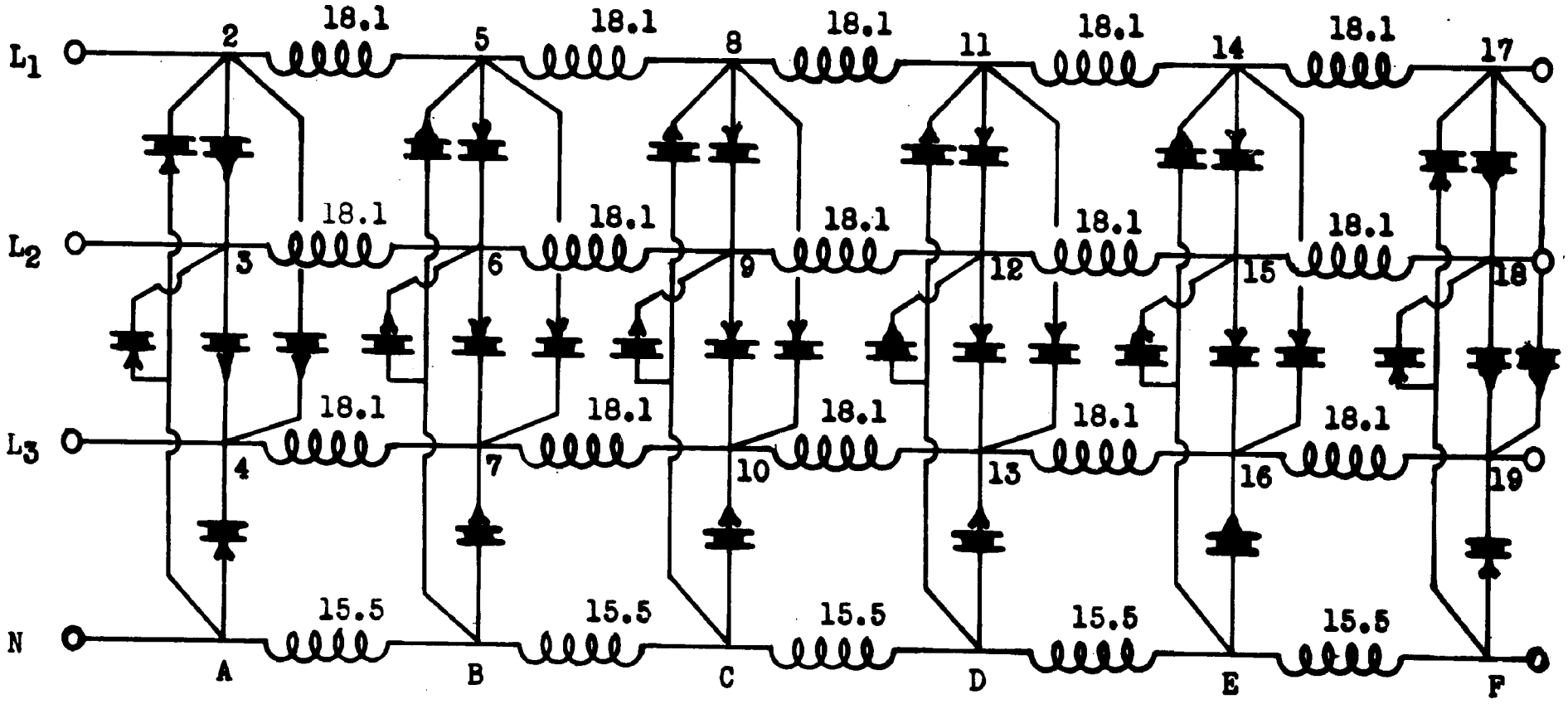


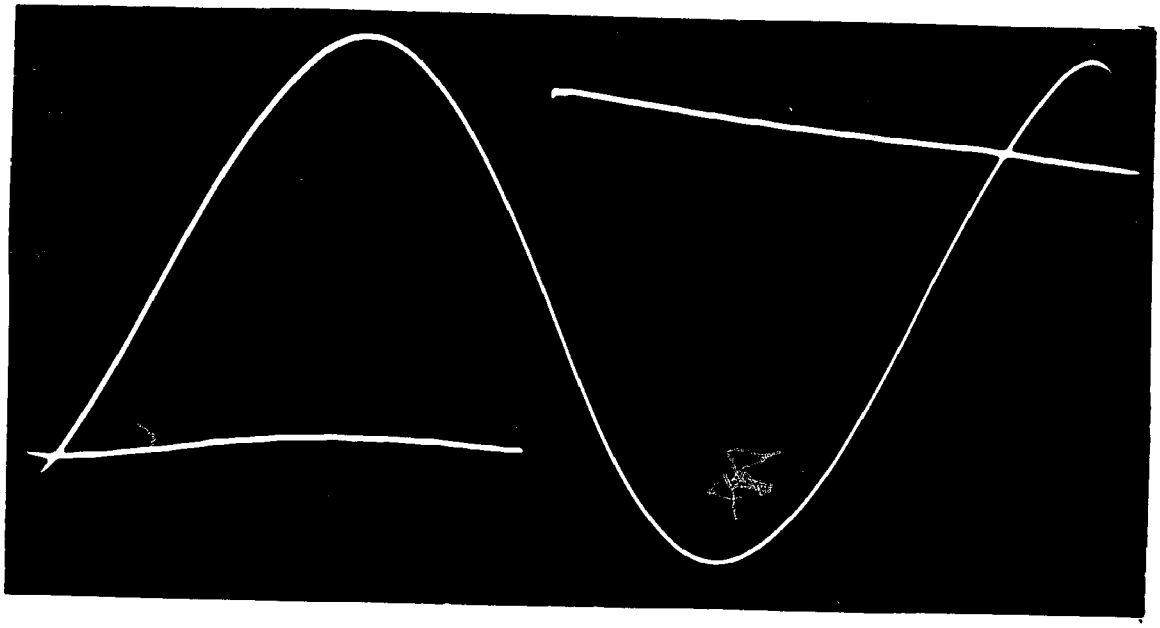
FIGURE 23

NOTE: ALL REACTANCE VALUES IN MILLIHENRYS

Both faults and circuit breakers are represented by the various synchronous switch contacts, properly phased with the 60-cycle power frequency. This phasing is carried out by use of an oscilloscope in conjunction with an electronic switch so that the switching operation, appearing as a square wave (Fig. 24), can be properly phased with the 60 cycle current or voltage waveform concerned. Coarse adjustments of phasing are carried out by loosening the wing nuts on the synchronous switch brush mounting and shifting the brushes. Fine adjustments (one or two electrical degrees) are made by adjusting the shunt field current of the d-c machine which is coupled to the synchronous drive motor, and thus power flow between the d-c and three-phase a-c supplies is controlled. The torque angle of the synchronous machine can thus be adjusted both leading or lagging, and hence the phasing of the direct-connected synchronous switch is adjusted. [See Figure 24: the sine wave represents steady-state fault current, phase A; the square wave corresponds to circuit breaker switching]. Throughout the main part of this paper, the arc in the circuit breaker is not considered and the short-circuit current in the faulted phase is interrupted at a normal zero. Consequently the circuit breaker square-wave in waveform /1 crosses the sine wave of steady-state fault current at a normal zero. The arc is not considered since it represents a time-varying network element



PHASING THE SYNCHRONOUS SWITCH WITH SYSTEM VOLTAGE



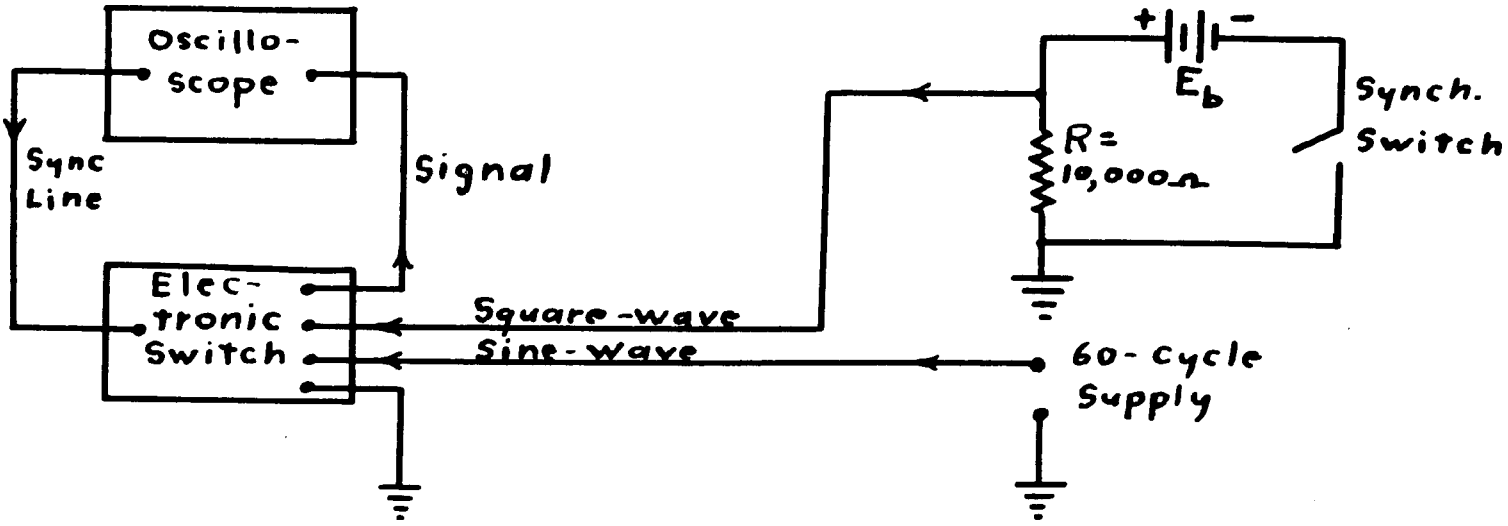


FIGURE 24

whose voltage drop is in general unknown. If the arc voltage drop were known, the arc could be considered to be a virtual voltage source that acts upon the fixed part of the network along with the actual sources, and the superposition of the currents produced by these sources would give the actual current being interrupted. (See Appendix, p. 132).

The oscilloscope used is a Dumont, synchronized by a differentiated square wave (see Fig. 25) which is generated by one of the synchronous switch contacts, properly phased and acting on a battery and RC circuit of proper time constant. The time constant of the sync pulse shown is 50 micro-seconds. The phasing of the sync pulse is set so that the oscilloscope sweep is started right before the portion of the recovery voltage of greatest interest. Therefore, using a high speed sweep, the high-frequency component of recovery voltage occurring immediately after circuit-breaker current zero can be expanded enough to check peaks and frequencies.

Synchronous Switch Details

The device used for performing switching operations, for applying faults, and for synchronizing the scope with the supply frequency is constructed as follows: four one-inch-thick clear bakelite disks are mounted on a three-fourths inch ball-bearing supported shaft, and to each of these disks a split steel ring is bolted. On the end of

SYNCHRONIZING PULSE

(DIFFERENTIATED SQUARE WAVE)

50 MICROSECOND TIME CONSTANT

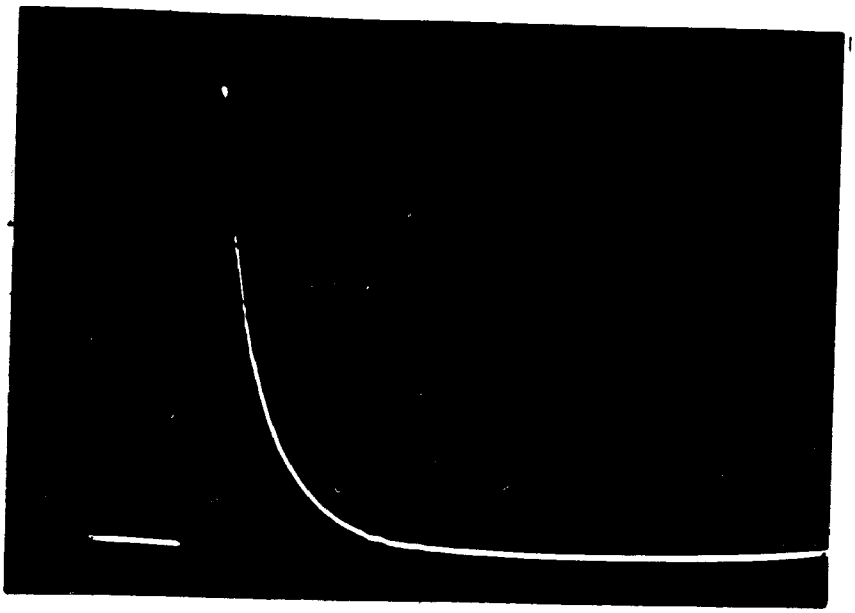


FIGURE 25

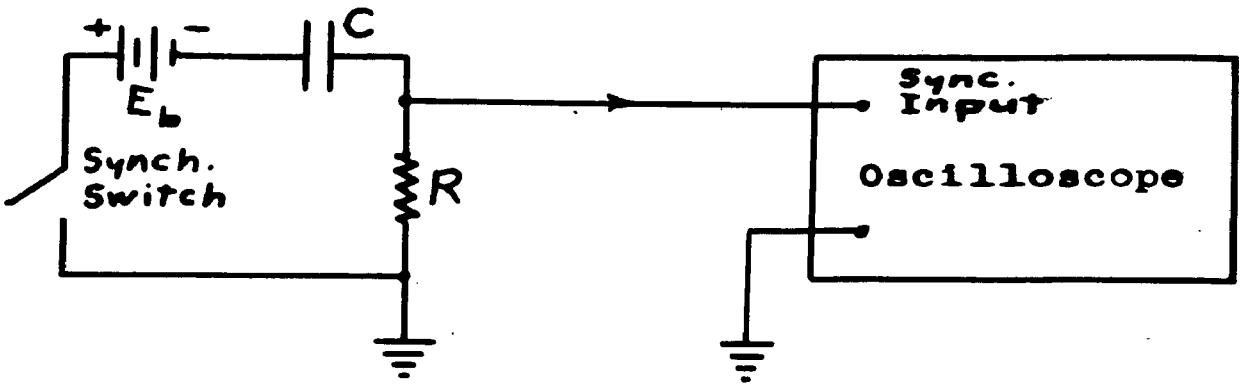


FIGURE 26

Identification of Equipment in the Following Figures (27,28)

1 . . . . . Timing Wave Generator

2 . . . . . Electronic Switch for Simultaneous Waveform Display

3 . . . . . Oscilloscope

4 . . . . . Power Supply Synchronizing Switch

5 . . . . . Synchronous Switch

6 . . . . . Synchronous Drive Motor

7 . . . . . Pulse Generating Circuit for Synchronizing the Oscilloscope

8 . . . . . Brass Slip Rings

9 . . . . . Bakelite Spacers and Steel Ring Supports

10. . . . . Split Steel Rings

11. . . . . Lumped System Parameters

12. . . . . High Frequency Bypass Condensers

13. . . . . Artificial Transmission Line

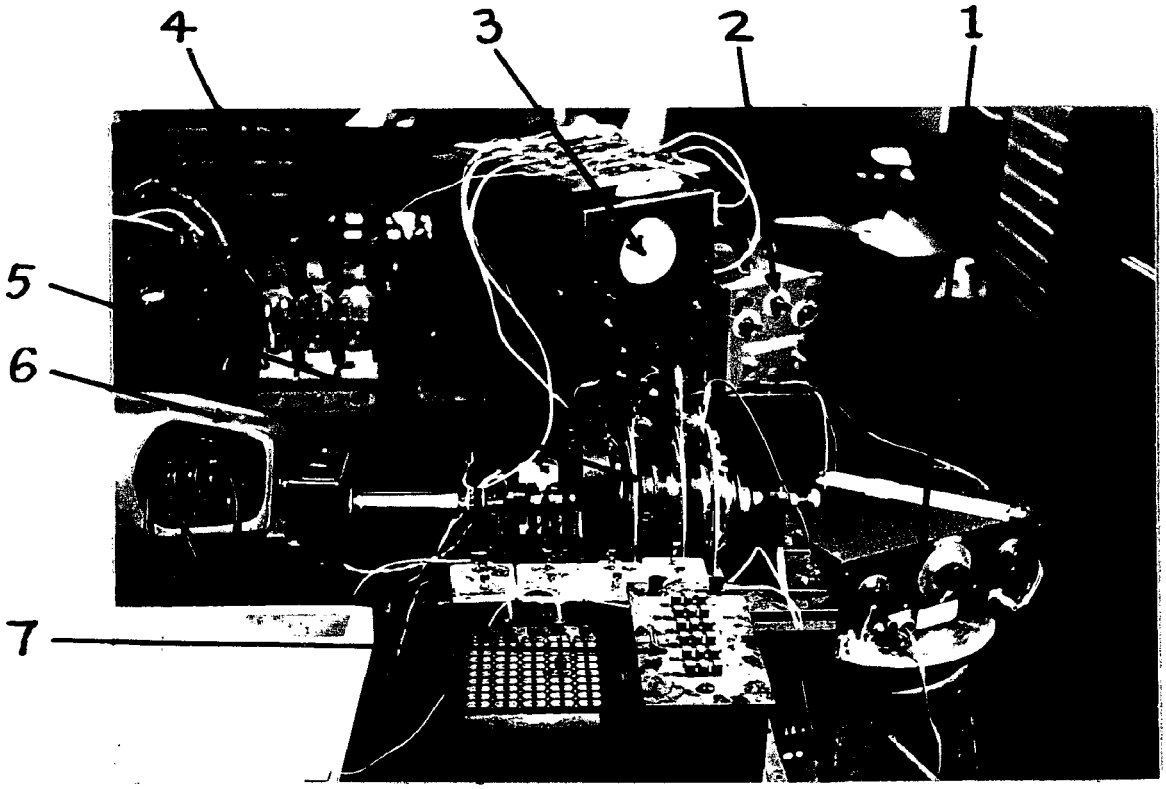
14. . . . . Direct Current Generator

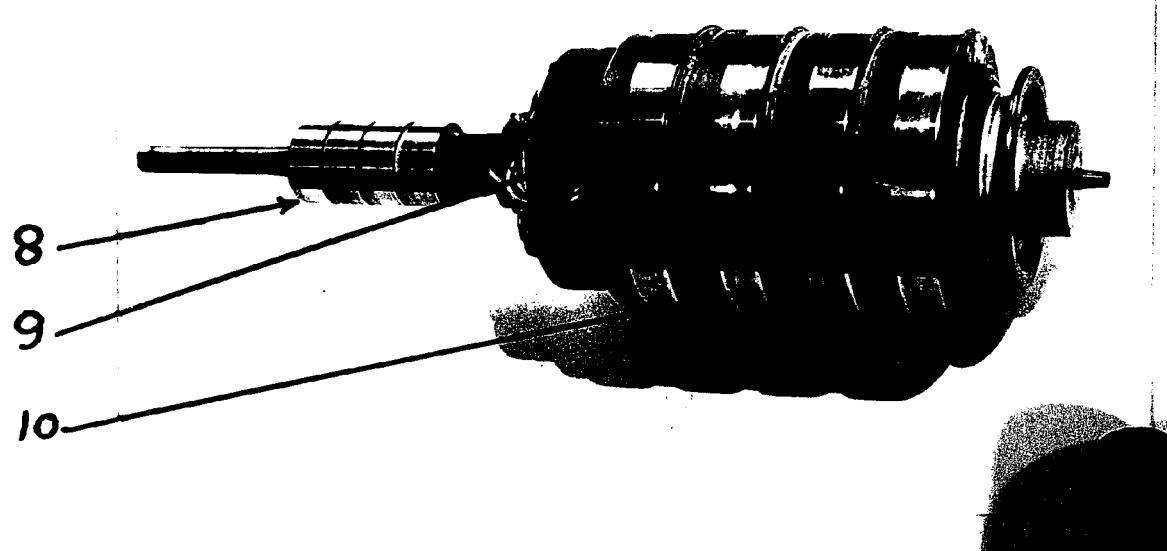
15. . . . . Three Phase Power Transformer

16. . . . . Circuit Breaker Shunting Impedances

EQUIPMENT LAYOUT

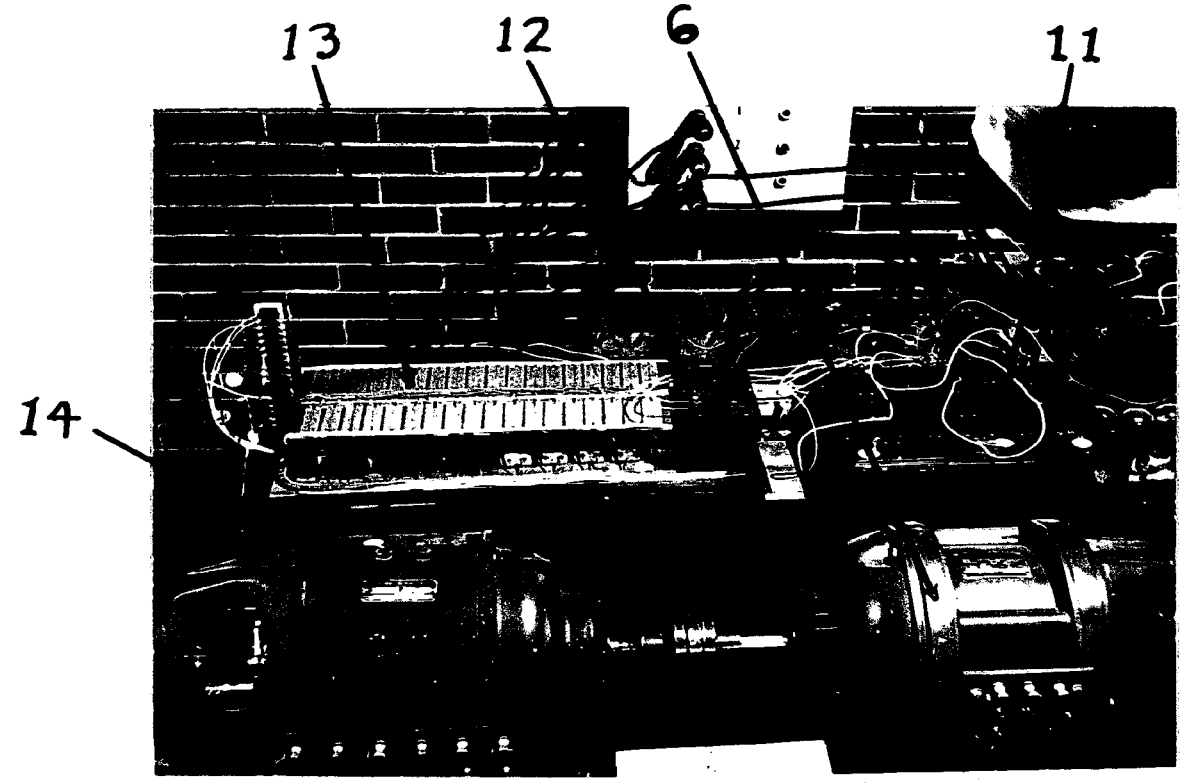
FIGURE

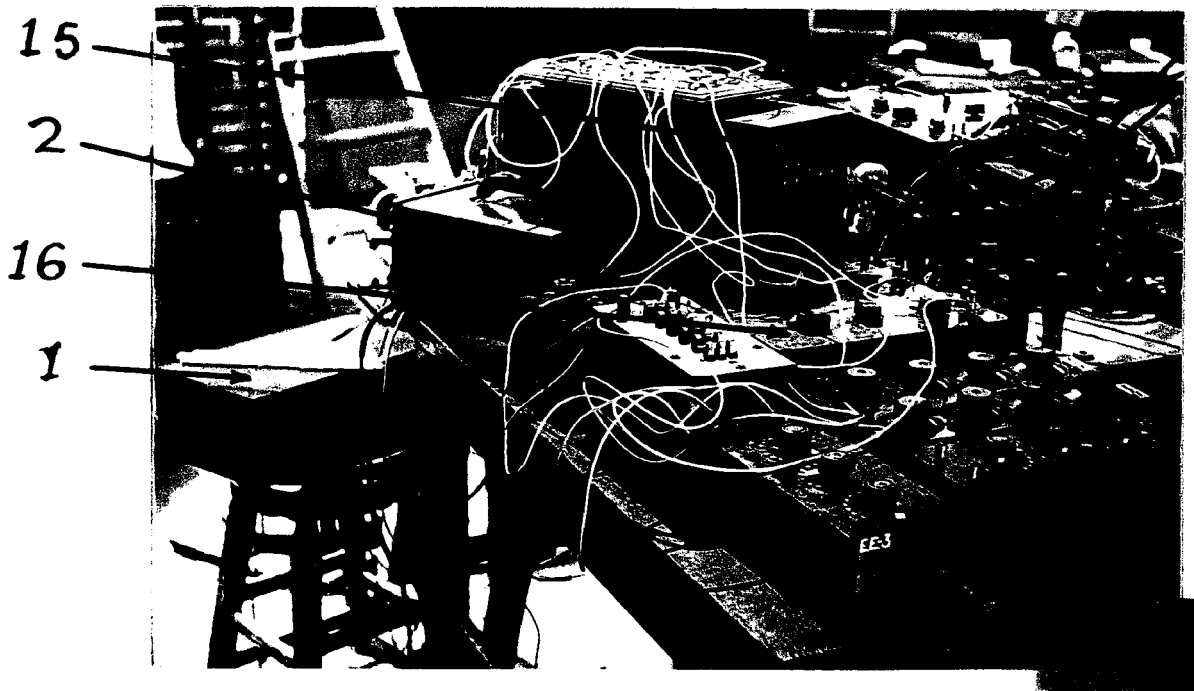




EQUIPMENT LAYOUT

FIGURE





the shaft are mounted four separately insulated brass rings, each ring electrically connected to one of the steel ring halves. Fixed brushes are mounted on the brass rings, and circumferentially adjustable brushes are mounted on the steel rings. Therefore by pairing one brass ring with one steel ring, a phase-adjustable switch results which is closed during one-half of each revolution of the synchronous machine, and open one-half revolution. Since the synchronous machine has six poles, the switch is open for 1 1/2 cycles, and closed for 1 1/2 cycles. Since one cycle of power frequency corresponds to 16,700 micro-seconds, and since in most practical cases the portion of recovery voltage of most vital concern is over within less than 2,000 micro-seconds after current zero, the three cycles elapsing between consecutive faulting and switching operations are more than adequate to reach a condition of stability between operations. Thus, the synchronous switch, by repeating periodically the same transient, allows the display of this transient on the face of a cathode-ray oscilloscope, as contrasted to the one-shot oscillograph and film method. By use of the oscilloscope, much faster waveforms can be presented, and the effect of a change in circuit parameters on the transient waveform can be quickly and easily studied. Then when suitable conditions are attained, the waveforms can be photographed from the face of the oscilloscope as was done in this paper. Careful planning and construction were required in

building the switch to keep vibration down to an absolute minimum, and to insure excellent and continuous electrical contact at high peripheral velocities.

Recovery Voltage Study - Case A

This is the case of a one-line-to-ground fault occurring at the input end of a 50 mile transmission line. On the corresponding mathematical analysis, this was found to be the most severe case of recovery voltage, both from the standpoint of rate of rise and of highest peak (see Fig. 3). If damping effects were not present, the initial peak would reach twice normal line-to-ground voltage. The presence of appreciable effective resistance at fundamental frequency displaces the fault current from its lossless 90° lag to its actual lag of 63° behind phase voltage. From the mathematical analysis, the initial peak amounts to 178% of the normal 60 cycle peak. Part of the reduction from the theoretical 200% is due to the forementioned phase shifting effect of resistance, and part is due to reduction in the amplitude of the harmonic by resistance. In this case the harmonic is reduced to 89% of its undamped value. The waveform (Fig. 29) is rather simple, consisting of the 60 cycle fundamental, and the damped 6,370 cycle harmonic which is the resonant frequency of the transformer leakage reactance acting in conjunction with the lumped high voltage bus and transformer capacitance to ground.

Discussion of Figures 29 - 34 Inclusive

All the waveforms which follow were photographed directly from the face of a Dumont oscilloscope. An electronic switch was used to present both the waveform under study and the timing wave generator output simultaneously.

The 60 cycle waveform which is superimposed on all the recovery voltage waveforms which follow is actually normal, steady-state, pre-fault phase voltage existing on the secondary of the transformer. After the recovery voltage transient dies out (see Figure 1b), there is a full 60-cycle period of time available during which the "circuit breaker" switch is open and the "fault" switch is closed. During this interval, therefore, a normal pre-fault voltage appears across the breaker and can be observed on the oscilloscope. By a suitable adjustment of sweep speed, this normal voltage can be made to appear on top of the recovery voltage transient. Thus, by using a little ingenuity, a very useful voltage reference is presented. This is the voltage to which recovery voltage is attempting to recover.

These experimental waveforms (Figs. 29-34) illustrate the several important effects that various lengths of transmission line, located between circuit breaker and fault point, have on the resultant recovery voltage waveform which appears across the circuit breaker. Figure 29 portrays the most severe fault case, with no transmission

line between fault and breaker to ease fault severity. The first peak of recovery voltage approximates twice normal system voltage, with a very steep rate of rise. Figure 29 should be compared with Figure 3 in the analytical section.

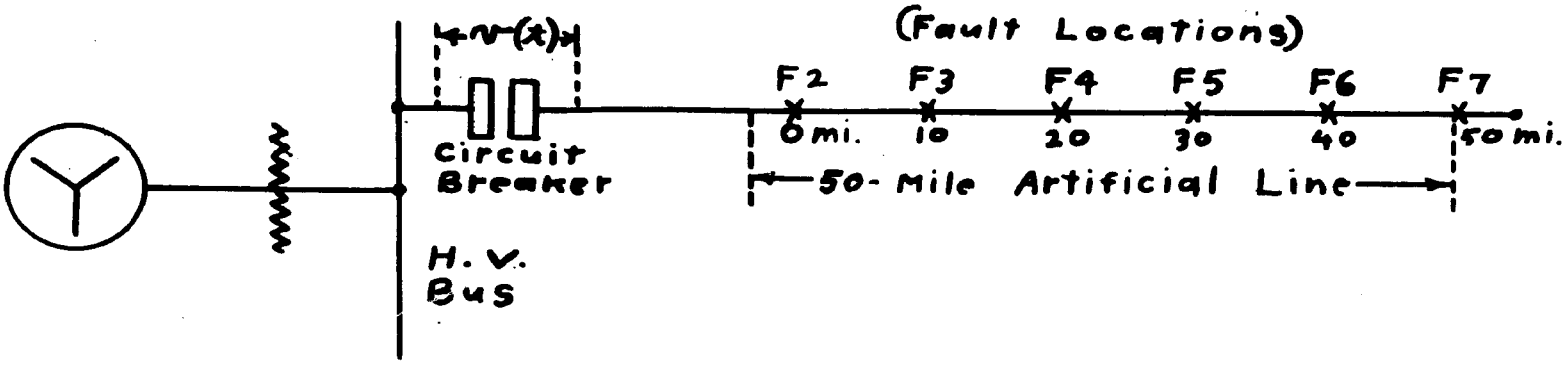
Figure 30 shows the effect of a ten mile line section between fault and breaker. Recovery voltage severity is greatly reduced. Also, in addition to the 60-cycle fundamental and 6,370 cycle transformer harmonic, we notice what appears to be a new harmonic somewhere between 60 and 6,370 cycles. This component is caused by successive reflections within the ten mile line section of the transient initiated by current interruptions.

Figures 31, 32, and 33 illustrate the effect of successively longer sections of line between fault and breaker, with a correspondingly greater reduction of recovery voltage severity, both in initial rate of rise and initial peak voltage. In addition, the harmonic due to line reflections has a successively longer period as the length of line is increased.

Figure 34 was taken with the entire 50 mile section of line between breaker and fault. The 6,370 cycle component appears larger than the corresponding component in the preceding figure. This is due to the failure of the "circuit breaker" switch to open precisely at a normal zero of fault current. Even a fraction of a degree error in synchronous switch timing results in appreciable distortion of harmonic

size. Even with direct coupling of the synchronous switch to the synchronous motor, some slight hunting takes place (in spite of flywheel and heavy dampers), and the experimental difficulty arises in photographing the waveform at the precise instant when the switch is opening exactly at a current zero. At that instant the harmonic shrinks to a minimum. Of the five attempts made, the Figure 34 waveform came nearest to the ideal. Figure 34 should be compared with Figure 10 in the analytical section.

TRANSMISSION LINE FAULT SERIES



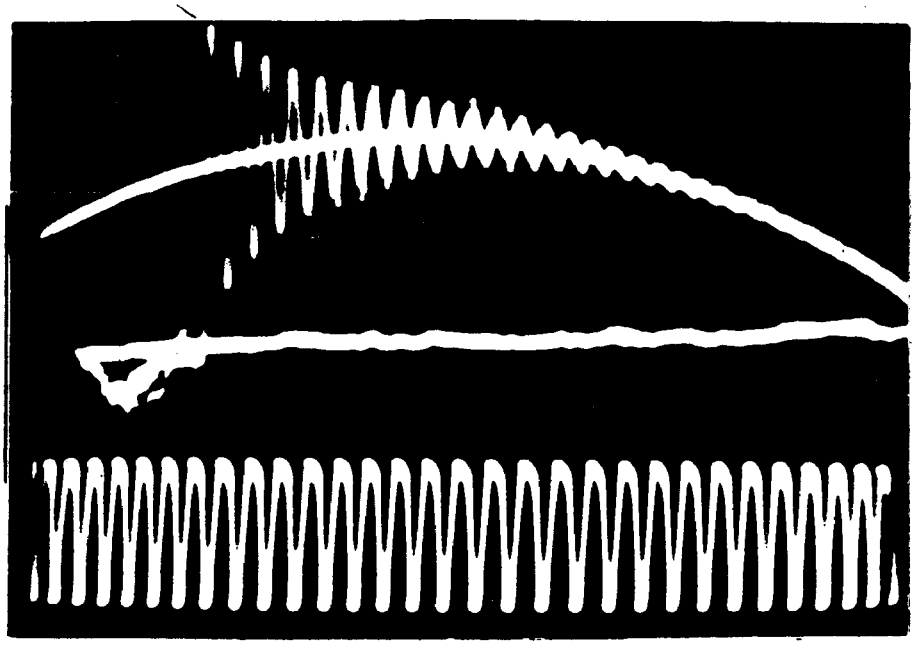


FIGURE 29

FAULT AT F2

f = 5 kc

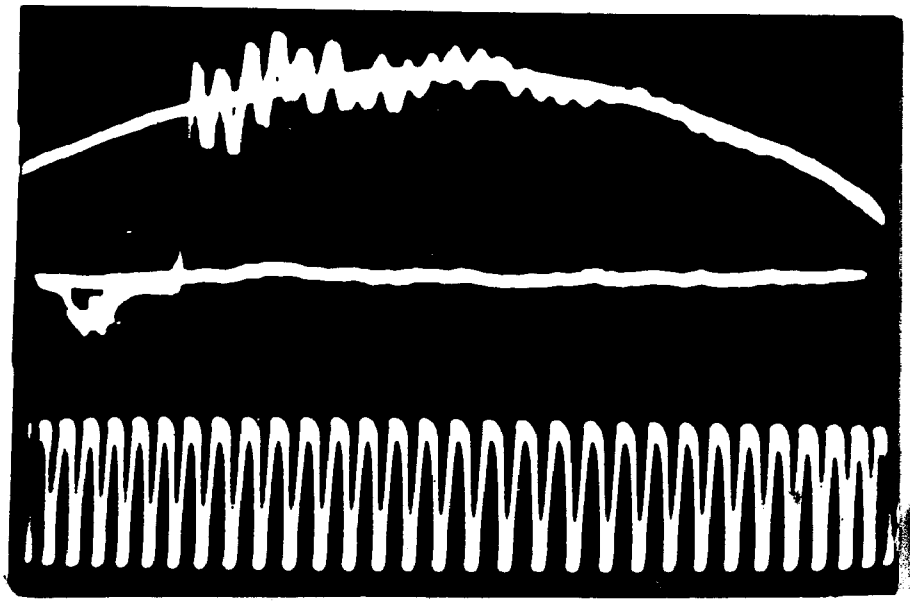


FIGURE 30

FAULT AT F3

f = 5 kc

TRANSMISSION LINE FAULT SERIES (CONTINUED)

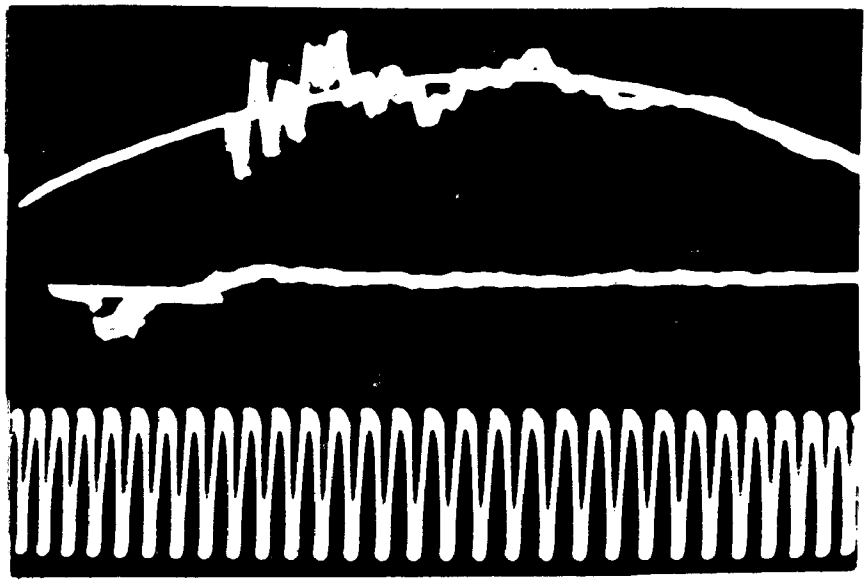


FIGURE 31

FAULT AT F4

f = 5 KC

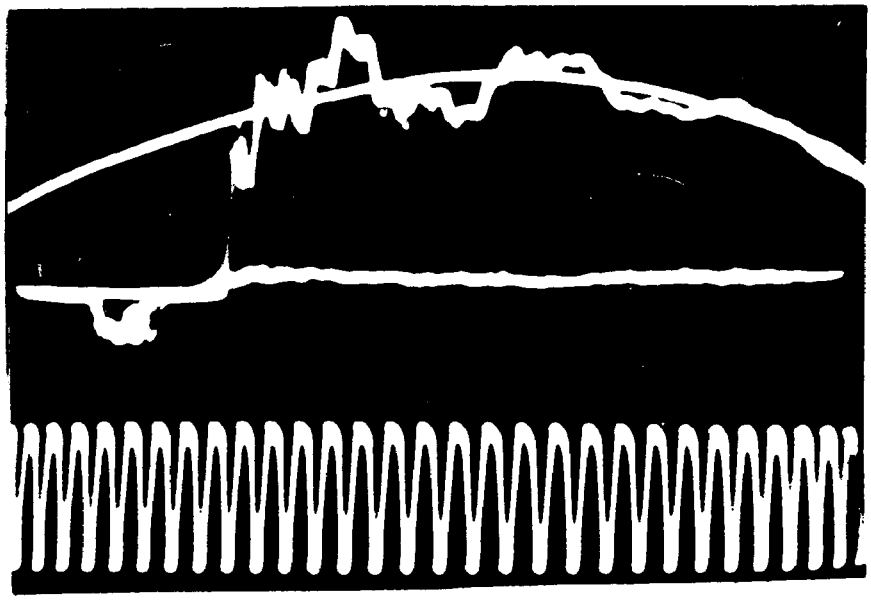


FIGURE 32

FAULT AT F5

f = 5 KC

TRANSMISSION LINE FAULT SERIES (CONCLUDED)

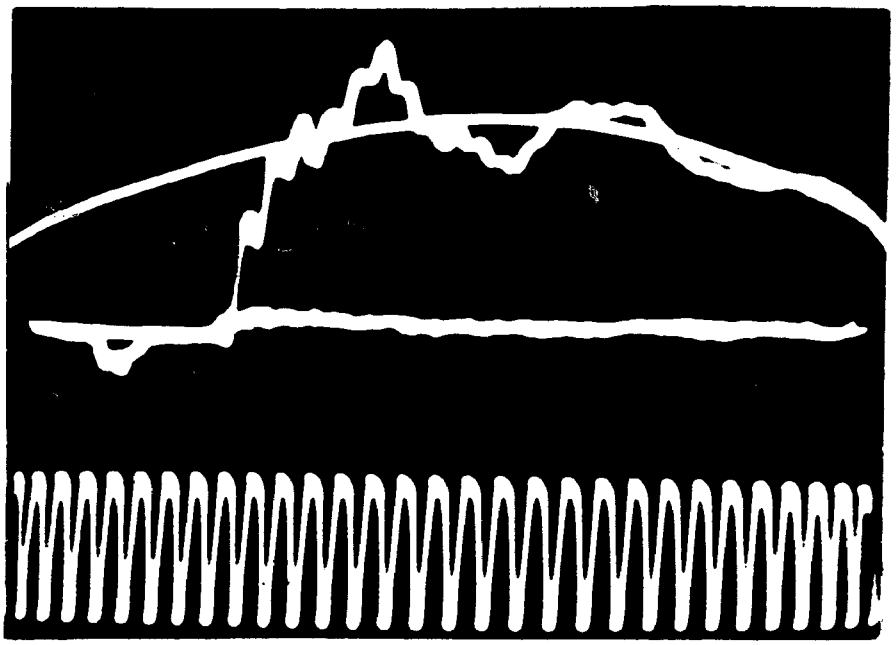


FIGURE 33

FAULT AT F6

f = 5 kc

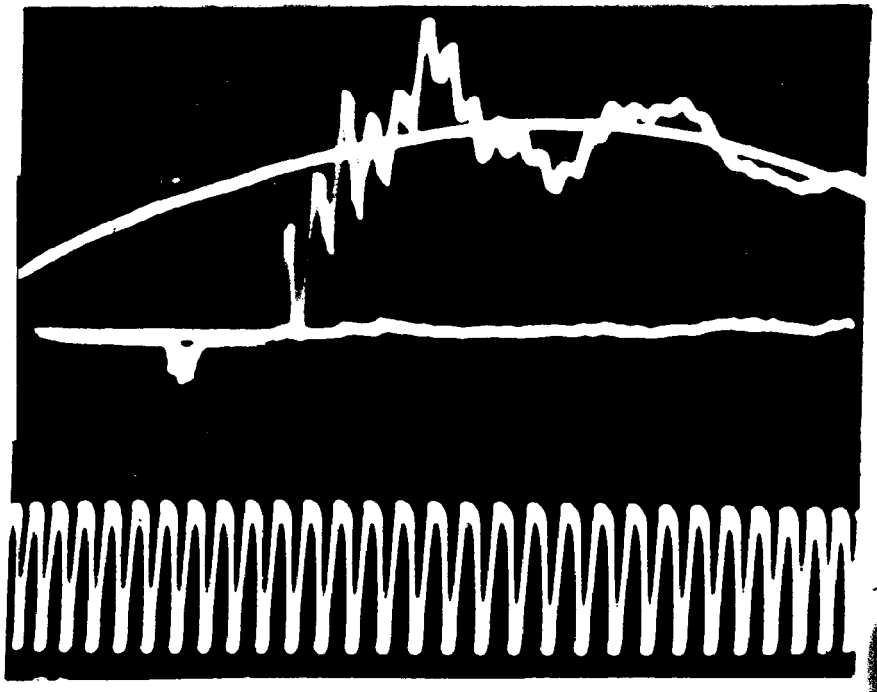


FIGURE 34

FAULT AT F 7

f = 5 kc

Discussion of Figures 35 and 36

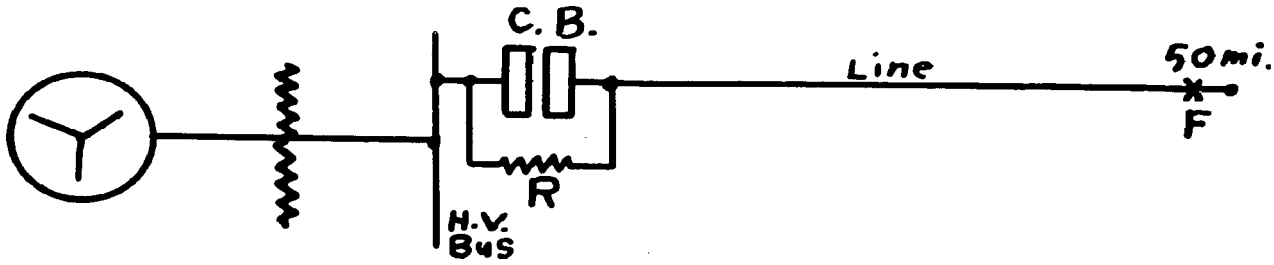
These Figures illustrate the damping effect of shunting pure resistance across a circuit breaker which is interrupting a fault at the far end of a 50 mile line. Neither waveform exceeds normal peak phase voltage, and initial rates of rise are quite low. From the theory developed in the analytical section under Case E-2, critical damping resistance was found to be 343 ohms. A close similarity can be verified between Figures 35 and 22. Figure 36 is an under-damped case of shunting resistance. Its initial rate of rise is appreciably less. However, too small a shunting impedance would be undesirable in a practical application.

The "hash" that appears on the base line beneath the recovery transient is caused by scope pickup from the line following "circuit breaker" opening.

RECOVERY VOLTAGE WITH SHUNT RESISTANCE

EFFECT OF TRANSMISSION LINE

CASE E2



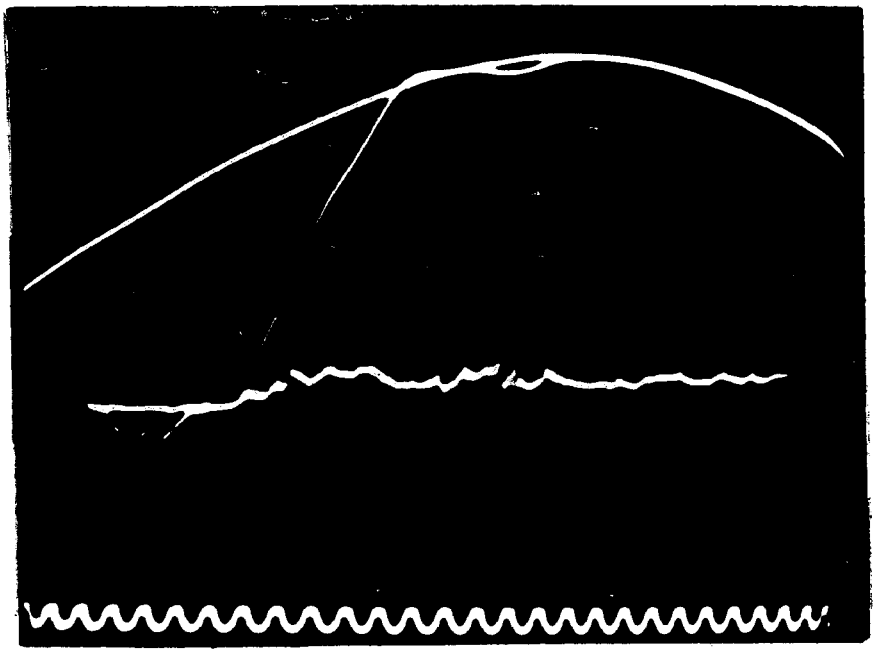


FIGURE 35

R = 343 ohms
(Critical R)

f = 5 kc

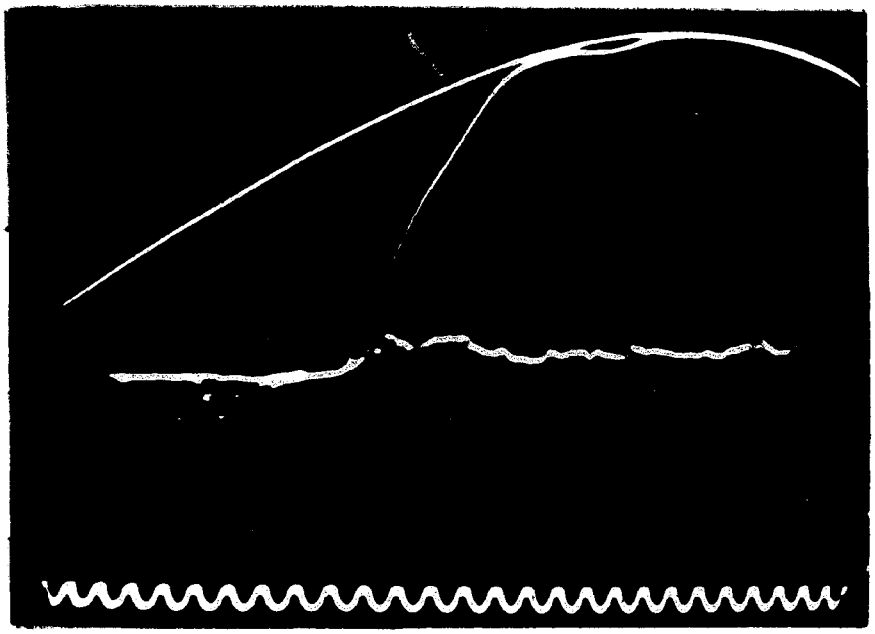


FIGURE 36

R = 281 ohms

f = 5 kc

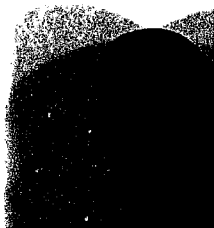
Discussion of Figures 37 to 42 Inclusive

These Figures represent recovery voltage corrective measures in the form of pure resistance shunted across circuit breaker terminals and applied to the case of greatest severity so far as recovery transients are concerned. The fault is located at the input end of the line. Figure 37 illustrates the uncorrected case (Case **A**, Figure 29), and is inserted here purely for comparison purposes.

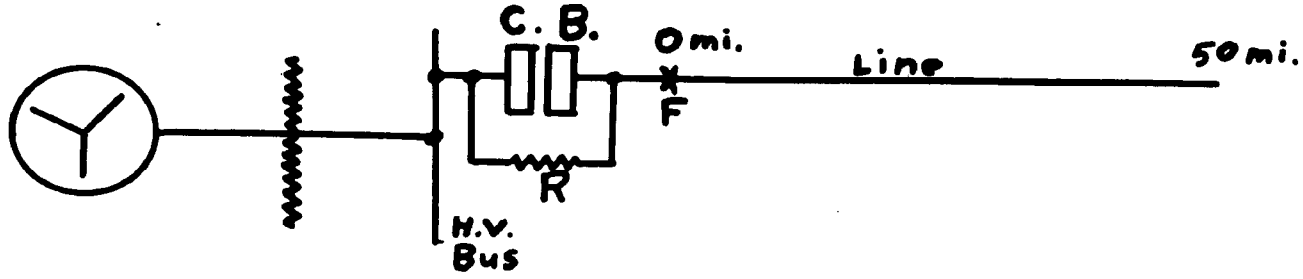
Figure 38 very graphically illustrates the effectiveness of a shunting resistance of critical value in removing the severe recovery transient shown in Figure 37. An expression for critical resistance was developed in the analytical section, and yielded a value of 621 ohms. Compare Figure 38 with Figure 7.

Figures 39 and 40 portray the underdamped case. The tendency towards oscillation is apparent, especially in the larger resistance case, Figure 40.

Figures 41 and 42 illustrate the overdamped case. The only appreciable difference is that the rate of rise is decreased as the shunting resistance is decreased. The "hash" appearing in the base line after time-zero is noise pickup from the switch.



RECOVERY VOLTAGE WITH SHUNT RESISTANCE -CASE C



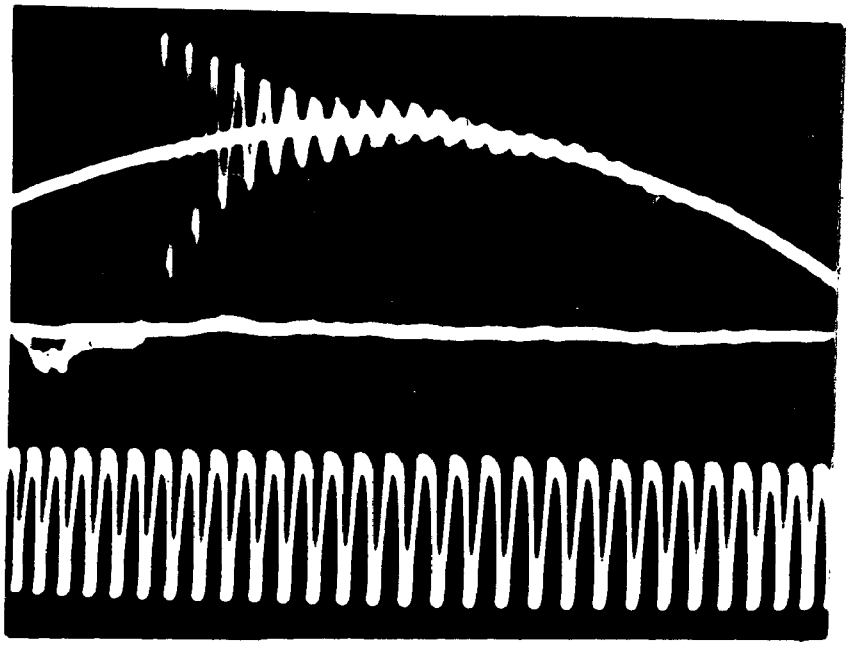


FIGURE 37

$R = \infty$

f = 5 KC

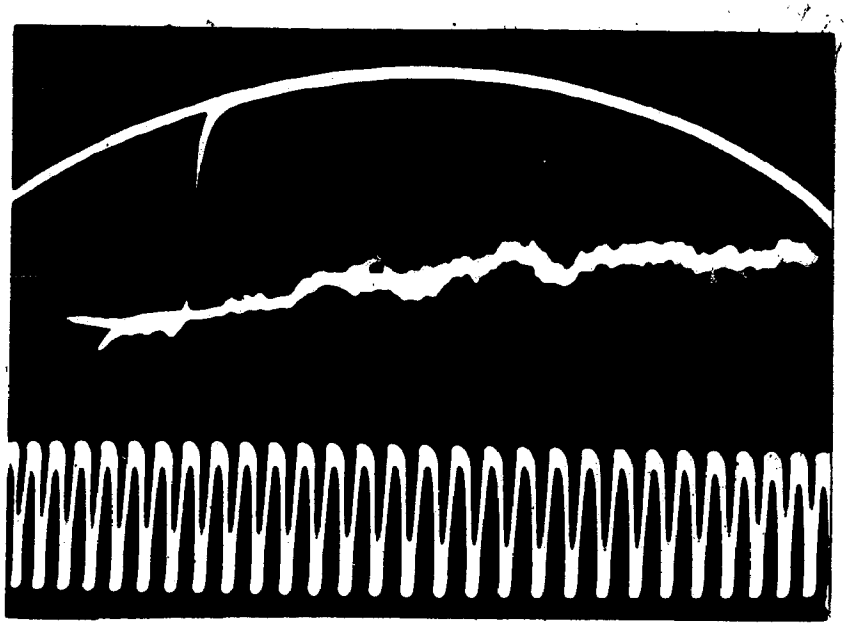


FIGURE 38

$R = 621\ \Omega = R_{crit.}$

f = 5 KC

RECOVERY VOLTAGE WITH SHUNT RESISTANCE (CONTINUED)

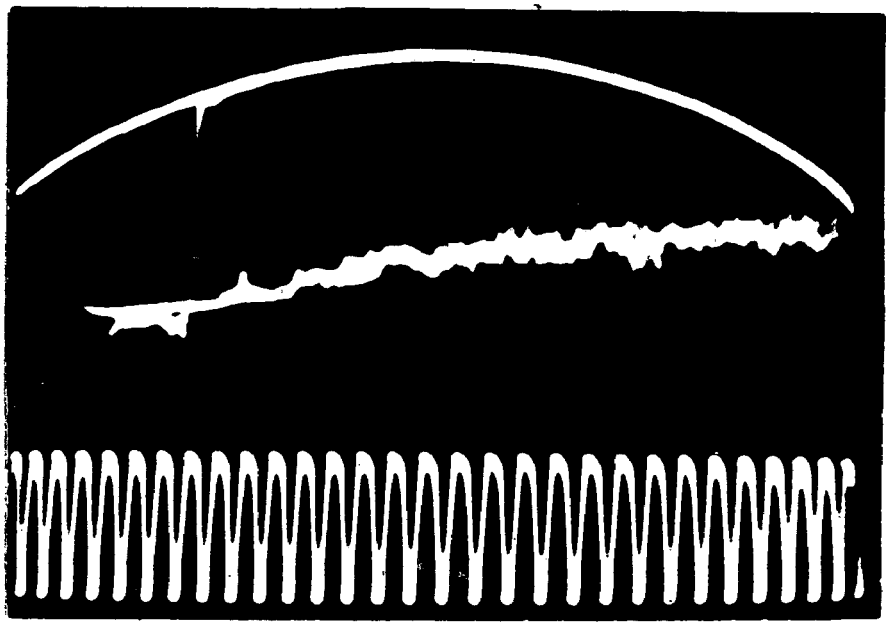


FIGURE 39

R = 1,000 Ω

f = 5 KC

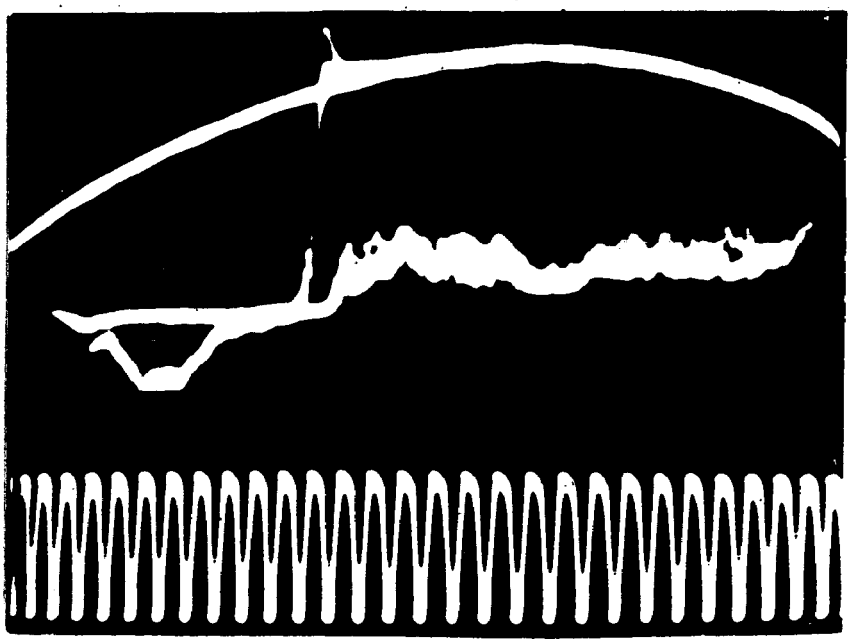


FIGURE 40

R = 1,500 Ω

f = 5 KC

RECOVERY VOLTAGE WITH SHUNT RESISTANCE (CONCLUDED)

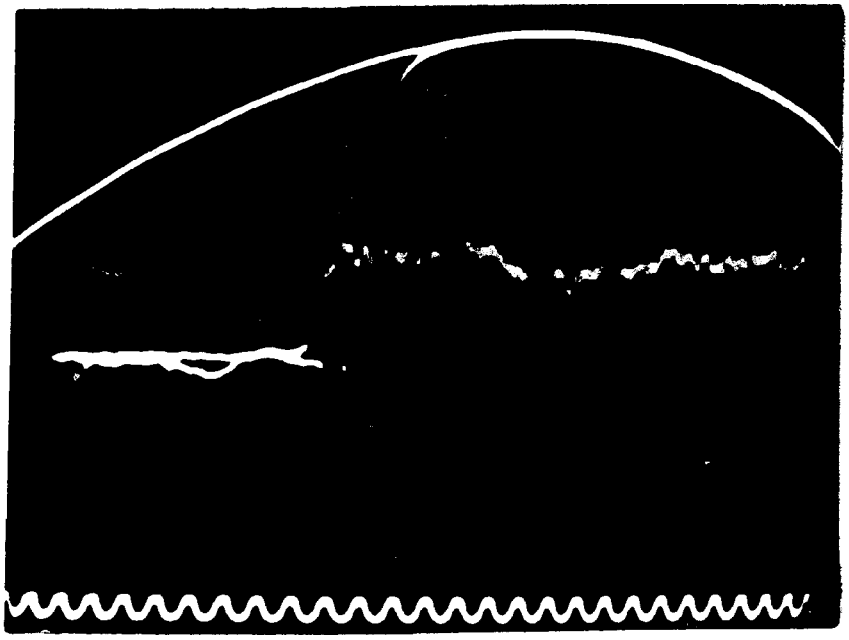


FIGURE 41

$R = 400\ \Omega$

$f = 5$ KC

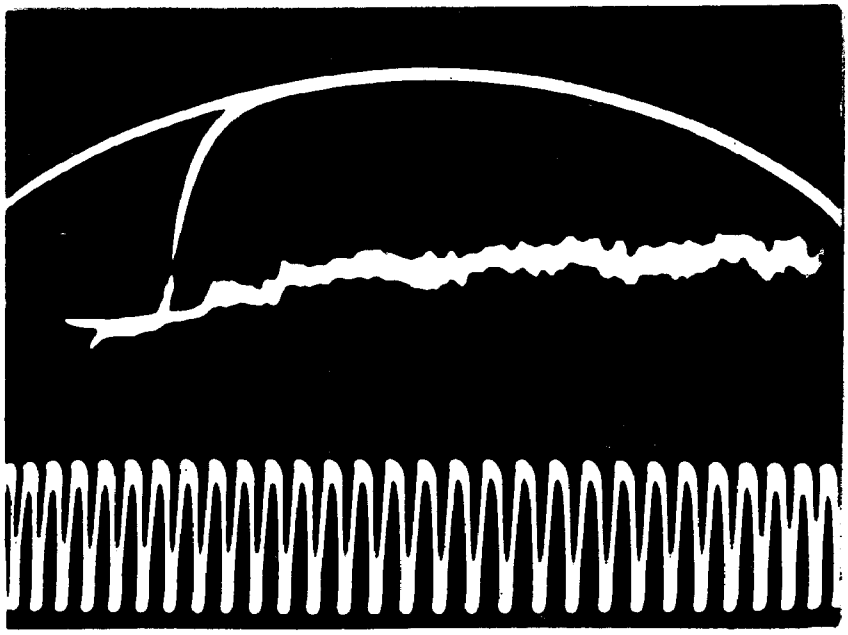


FIGURE 42

$R = 200\ \Omega$

$f = 5$ KC

Case D - Effect of Shunt Capacitance on Recovery Voltage

From the qualitative standpoint, a reasonably large capacitance in shunt with the breaker acts as a very low impedance to the high frequency harmonics of recovery voltage generated by system elements such as transformers and current limiters. Therefore these harmonics which cause the severe case of high peaks and rapid rates of rise of voltage do not appear in the waveforms; however, the presence of fairly large shunting capacitance introduces new harmonics of lower frequency (since $f \propto \frac{1}{\sqrt{LC}}$). From the mathematical analysis it was found that the shunt capacitor had negligible effect on the size of the harmonic it introduces, also negligible effect (for capacitance $<$ 1 uf) on the phase between the fundamental voltage and the steady-state fault current; hence the peak of recovery voltage is not appreciably affected by shunt capacitance. This is borne out experimentally in Fig. 46 where it is found that adding a shunt C = .46 uf has no noticeable effect on maximum recovery voltage magnitude. The harmonic has a frequency of 1300 cycles.

Owing to the lower frequency of the harmonic, the rate of rise of recovery voltage is slightly less severe than without the shunt capacitance. This is the only apparent advantage. Another effect to be noted is that the system damping so far as the harmonic is concerned is considerably less for the 1300 cycle harmonic (Fig. 46) than in the

case of the 6 K C harmonic (Fig. 29) with no shunt capacitance. From the mathematical analysis, it is seen that shunt capacitance has no direct effect on damping, since the damping exponential remains the same $e^{\frac{-Rt}{2L}}$; however, the great reduction in frequency due to shunt capacitance apparently has considerable effect on the effective value of R (see Fig. 47). This is of course no advantage from an application standpoint.

Case F-2 - Effect on Recovery Voltage of R and C in Series Shunted Across Circuit Breaker (Figs. 48-53).

Qualitatively speaking, this case is rather similar to Case C where pure R was shunted across the breaker terminals. The presence of C, however, has the additional advantage of being frequency selective in that it offers a much higher impedance to the fundamental of shunt current than to the harmonic of recovery voltage. Also, the presence of C, as in the shunt capacitance case, has the effect of lowering the harmonic frequency and of shifting the phase of this harmonic, which gives control on rate of rise of recovery voltage.

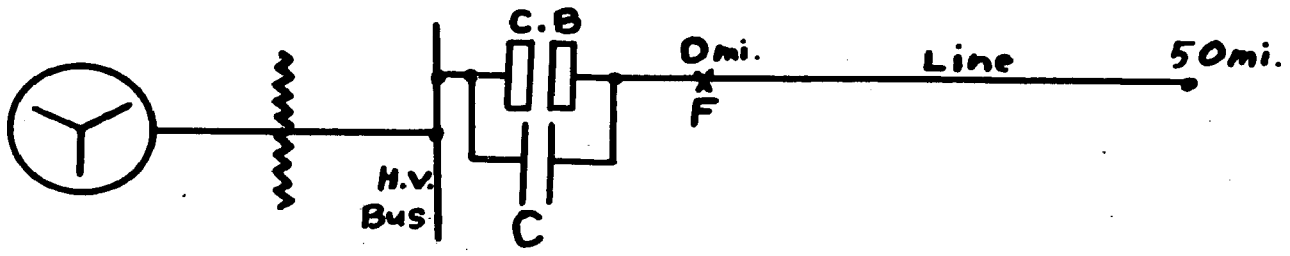
Discussion of Figures 43 to 46 Inclusive

These figures illustrate the effect of shunting various amounts of capacitance across a circuit breaker with the fault located at the input end of the line. An examination of these four figures reveals that the recovery voltage peak is virtually unaffected by shunt C. Increasing C merely increases the period of the harmonic, and thus decreases somewhat the initial rate of rise. A side effect is the reduction of damping rate with reduction in harmonic frequency. This effect is due to the increase of effective resistance with frequency increase and will be investigated below.

The wavy lines and multiple trace which appear to have replaced the reference voltage trace are due to the greatly decreased damping rate of the harmonic component of recovery voltage. These oscillations persist even a full cycle later than time-zero. These greatly reduced oscillations are very difficult to keep in synchronism with the initial transient, consequently a multiple trace is seen on the photograph. If the center line of the wavy trace is taken on Figures 45 and 46, a sufficiently accurate voltage reference is obtained.

RECOVERY VOLTAGE WITH SHUNT CAPACITANCE - CASE D

[EFFECT OF FREQUENCY ON EFFECTIVE RESISTANCE]



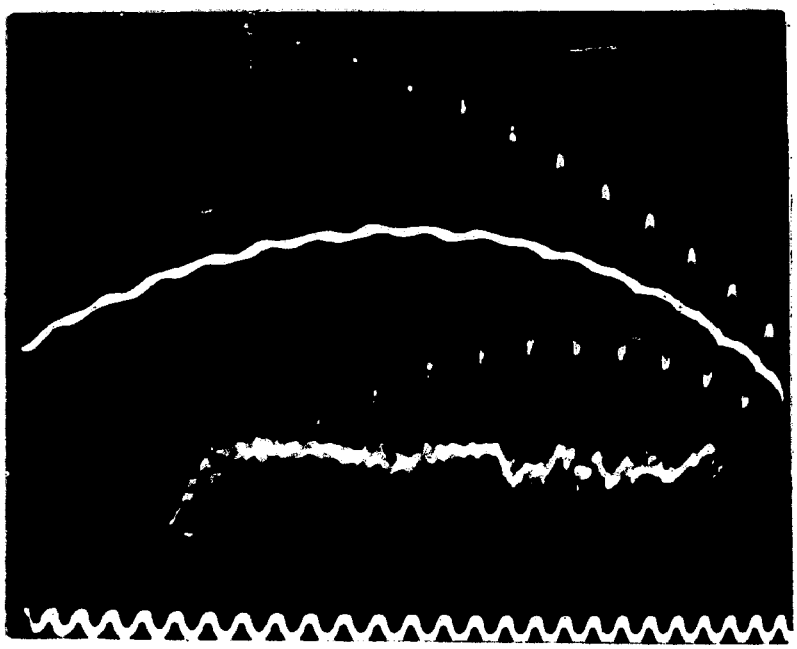


FIGURE 43

C = .08 μf

f = 5 KC

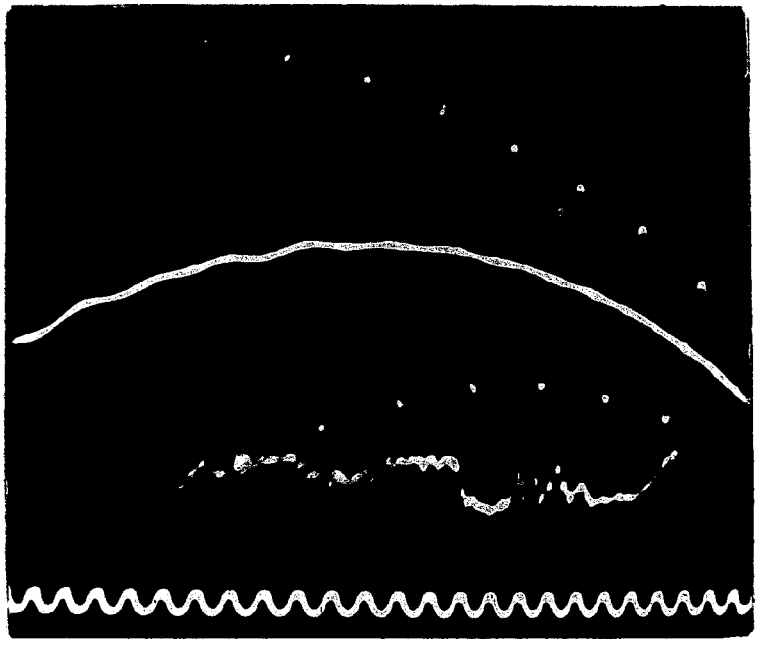


FIGURE 44

C = .18 μf

f = 5 KC

RECOVERY VOLTAGE WITH SHUNT CAPACITANCE - CASE D

[EFFECT OF FREQUENCY ON EFFECTIVE RESISTANCE]

FIGURE 45

C = .38 μf

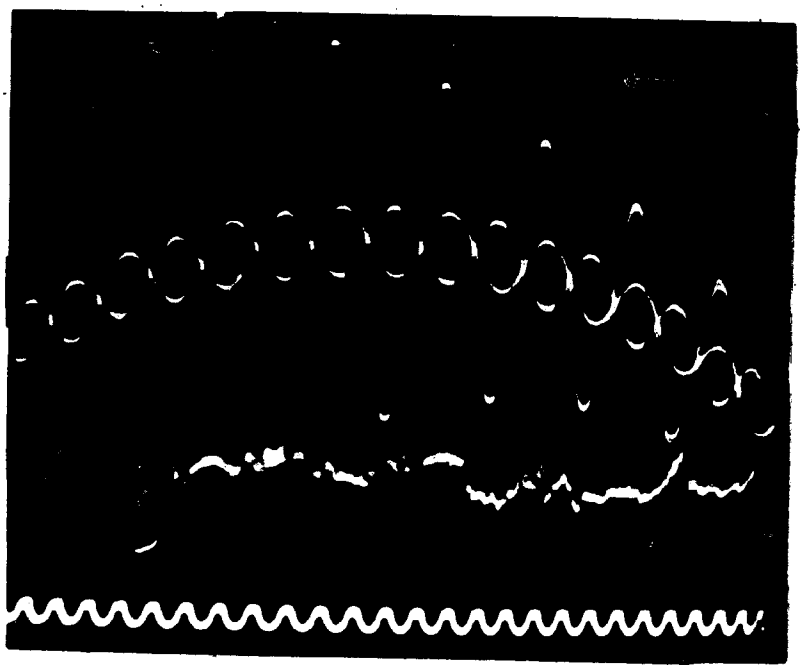
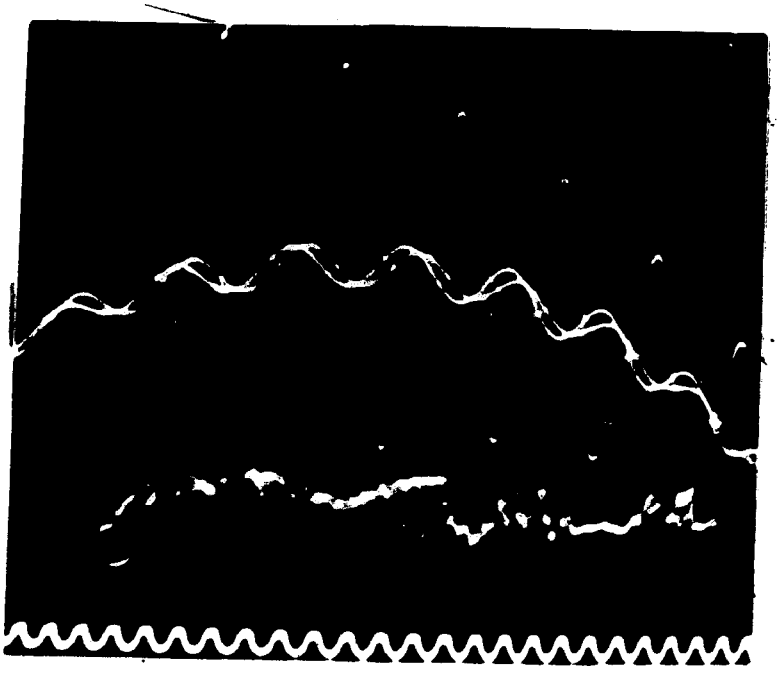


FIGURE 46

C = .46 μf.



Effect on Recovery Voltage Harmonic Frequency on Transformer Resistance

In making the experimental analysis of Case D, with capacitance shunted across the circuit breaker, it was noticed that variations of harmonic frequency had a very marked effect on exponential decay of the harmonic of recovery voltage introduced by transformer leakage reactance and system capacitance to ground. This exponential envelope is of the form,

$$f(t) = A\, e^{\frac{-Rt}{2L}}$$

This relation was developed in Case A. The time required for each respective exponential to decay to 36.8% of its initial value can be measured from the exponential waveforms and equated to $\frac{2L}{R}$. On the assumption that inductance variation over limited frequency variation is small, effective values of resistance can thus be obtained as a function of frequency. The corresponding data and curve follow.

EFFECT OF HARMONIC FREQUENCY ON TRANSFORMER EFFECTIVE RESISTANCE

Figure	Page	L (mh)	C (uf)	Measured Time Constant (us)	Computed Harmonic Frequency (KC)	Effective Resistance (ohms)
29	101	31.2	.02	1,000	6.37	62.4
43	113	31.2	.1	2,550	2.85	24.5
44	113	31.2	.2	3,450	2.01	18.1
45	114	31.2	.4	3,800	1.42	16.4
46	114	31.2	.48	4,200	1.30	14.8
--	---	----	---	∞	0	6.0

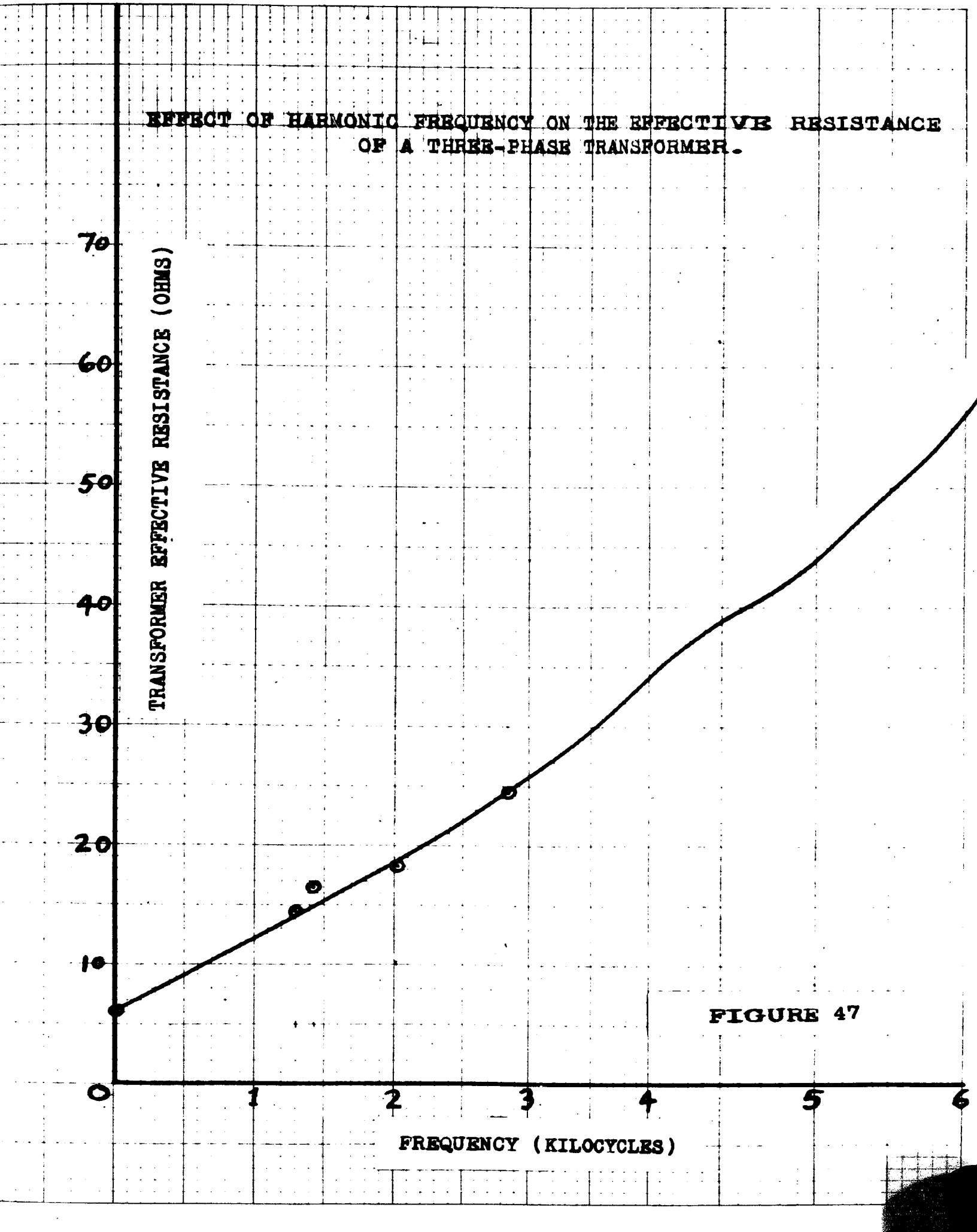

EFFECT OF HARMONIC FREQUENCY ON THE EFFECTIVE RESISTANCE OF A THREE-PHASE TRANSFORMER.
TRANSFORMER EFFECTIVE RESISTANCE (OHMS)
70
60
50
40
30
20
10
0
1
2
3
4
5
6
FREQUENCY (KILOCYCLES)

FIGURE 47

Discussion of Figures 48 to 53 Inclusive

These figures illustrate the effect of shunting various combinations of series resistance and capacitance across a circuit breaker with the fault located at the input end of the line. This combination seems to offer the best possibility of commercial application for reduction of recovery voltage severity. From the analytical section of this paper which involved the general solution of a cubic for each value of capacitance assumed, the solution gave two critical values of resistance. Experimentally it was found that all values of R between these two values yielded non-oscillating recovery transients.

Figures 48 and 49 are experimental waveforms using an assumed C = 0.5 uf and critical resistances of 487 ohms and 678 ohms respectively. Compare Figure 48 with Figure 15 in the analytical section. A large number of resistances and capacitances were experimentally tested and the results are plotted in Figure 54, which gives the locus of non-oscillatory combinations. Figure 51 is located to the left of this locus while Figures 52 and 53 are located to the right. Figures 48 and 49 are borderline cases (critical values).

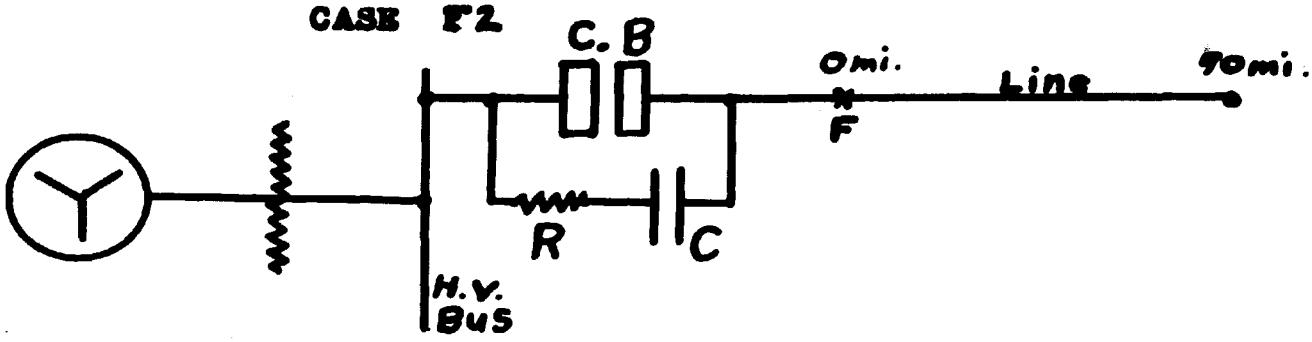
The characteristic "hump" by which transient recovery voltage initially exceeds normal phase voltage is very clearly evident in the analytical waveform, Figure 15, as well as in the experimental waveforms, Figures 48 and 49. The

large amount of "hash" present in the baselines to the right of time-zero in Figures 48 to 53 is due to noise pickup on the line and "circuit breaker" terminal which are isolated from ground by the condenser during the open-switch period.

Practically speaking, Figure 50 probably illustrates the most desirable recovery transient since it is non-oscillatory, has a low initial rate of rise, and nowhere exceeds normal phase voltage. The shunt impedance offered by this combination is not excessively small, and comparing this case with the pure resistance shunt (Case C), the capacitor in the RC combination would make this combination much easier to design from the high voltage standpoint. Compare Figure 50 with Figure 16.

RECOVERY VOLTAGE WITH SERIES R AND C SHUNTED ACROSS CIRCUIT BREAKER

CASE F2



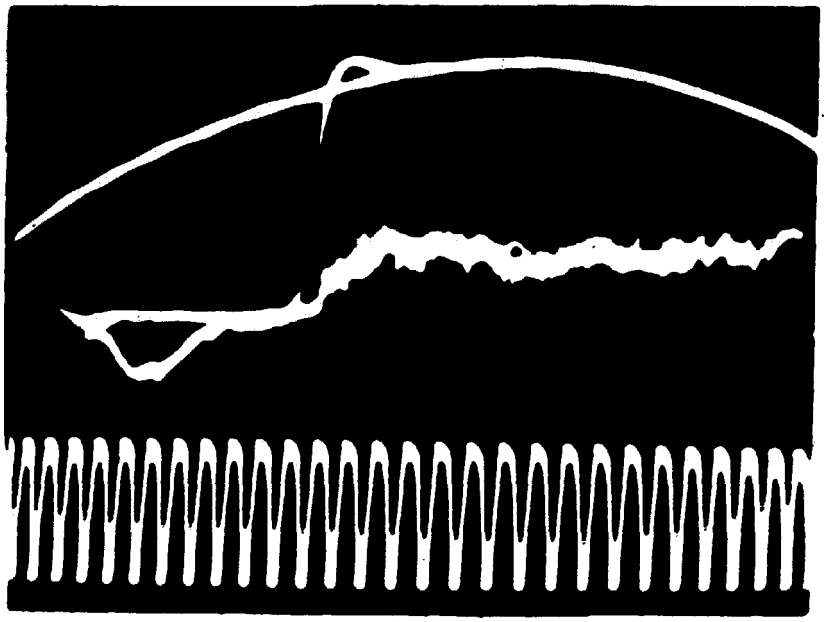


FIGURE 48

R = 487 ohms

C = 0.5 uf

(Crit. Damp.)

f = 5 KC

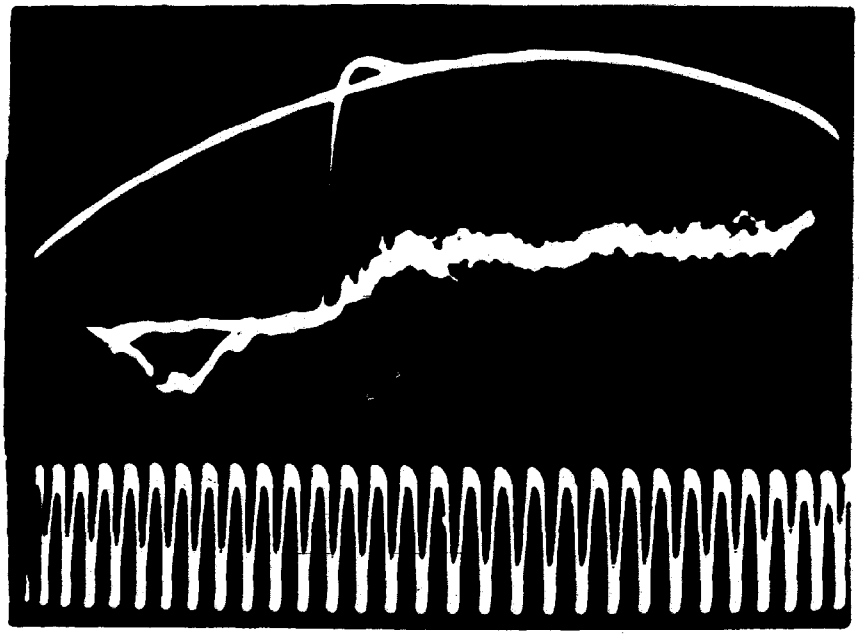


FIGURE 49

R = 678 ohms

C = 0.5 uf

(Crit. Damp.)

f = 5 KC

SERIES R AND C CONTINUED

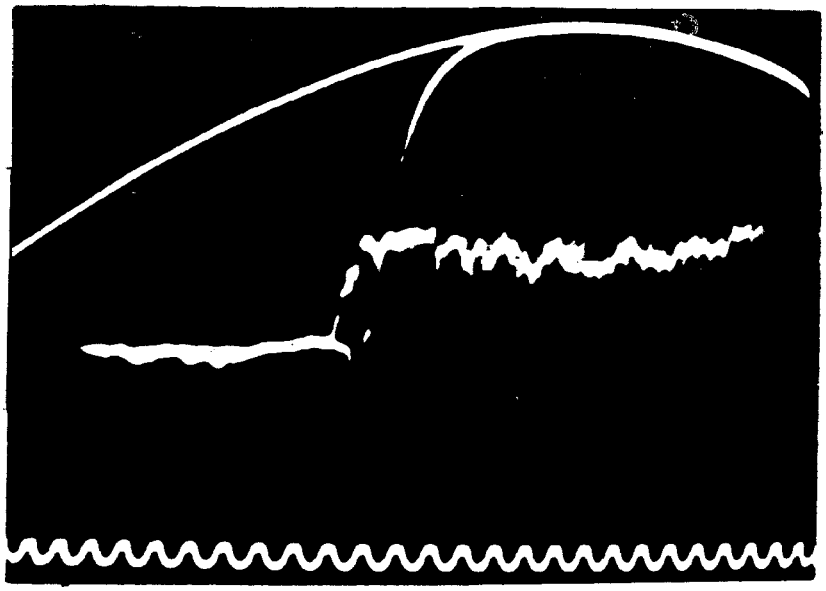


FIGURE 50

R = 200 ohms

C = 4.5 uf

f = 5 KC

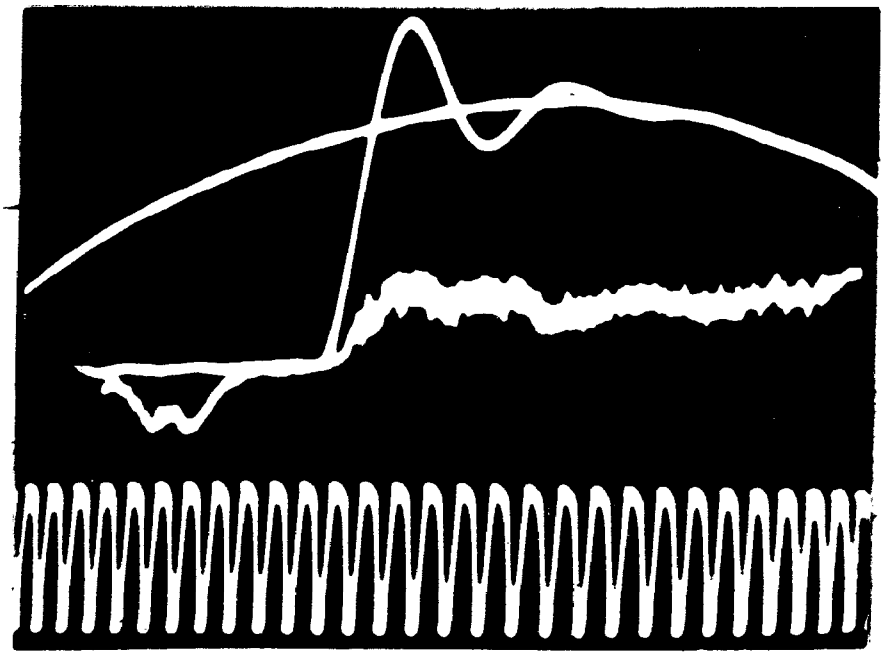


FIGURE 51

R = 100 ohms

C = 0.8 uf

f = 5 KC

SERIES R AND C CONCLUDED

FIGURE 52

R = 1,000 ohms

C = 0.5 uf

f = 5 KC

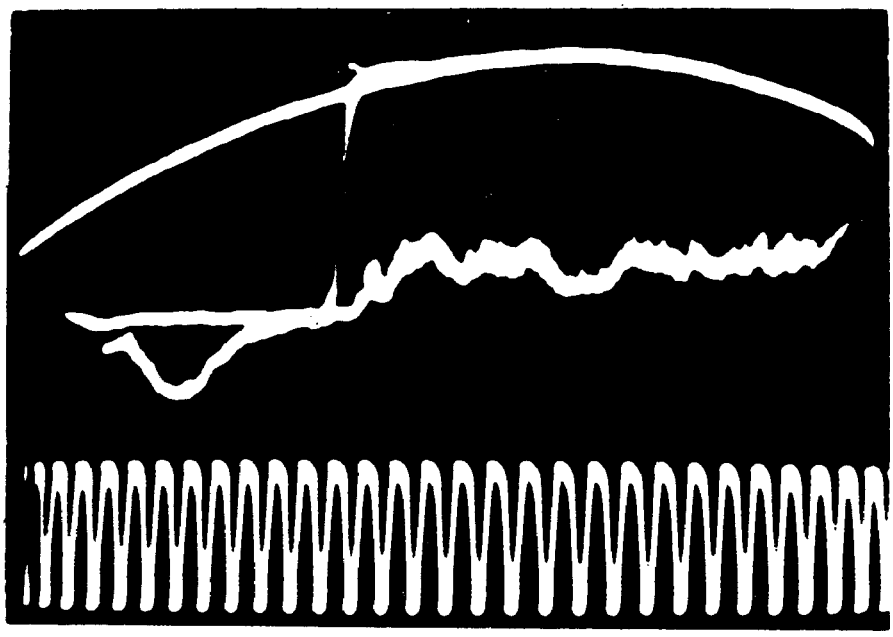
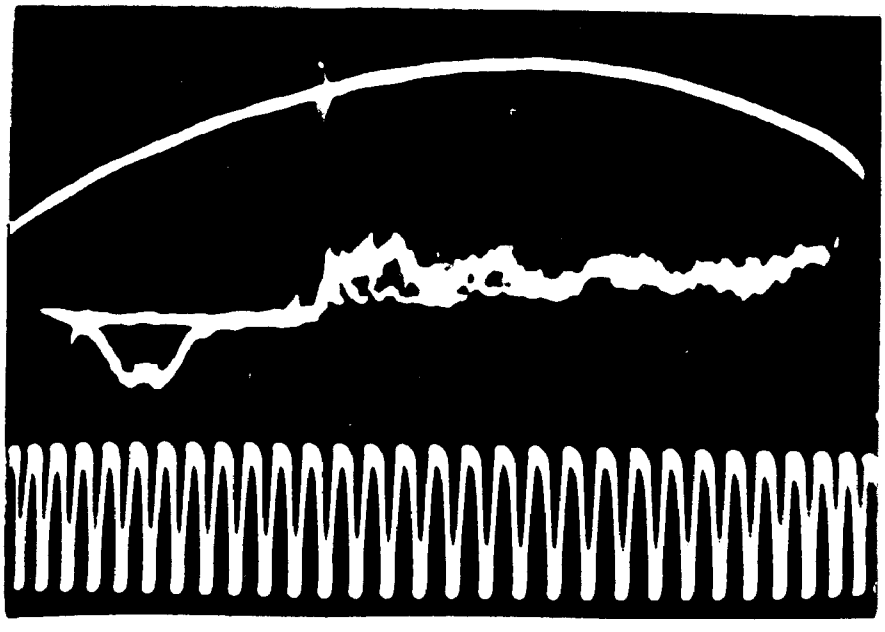


FIGURE 53

R = 1,000 ohms

C = 0.8 uf

f = 5 KC



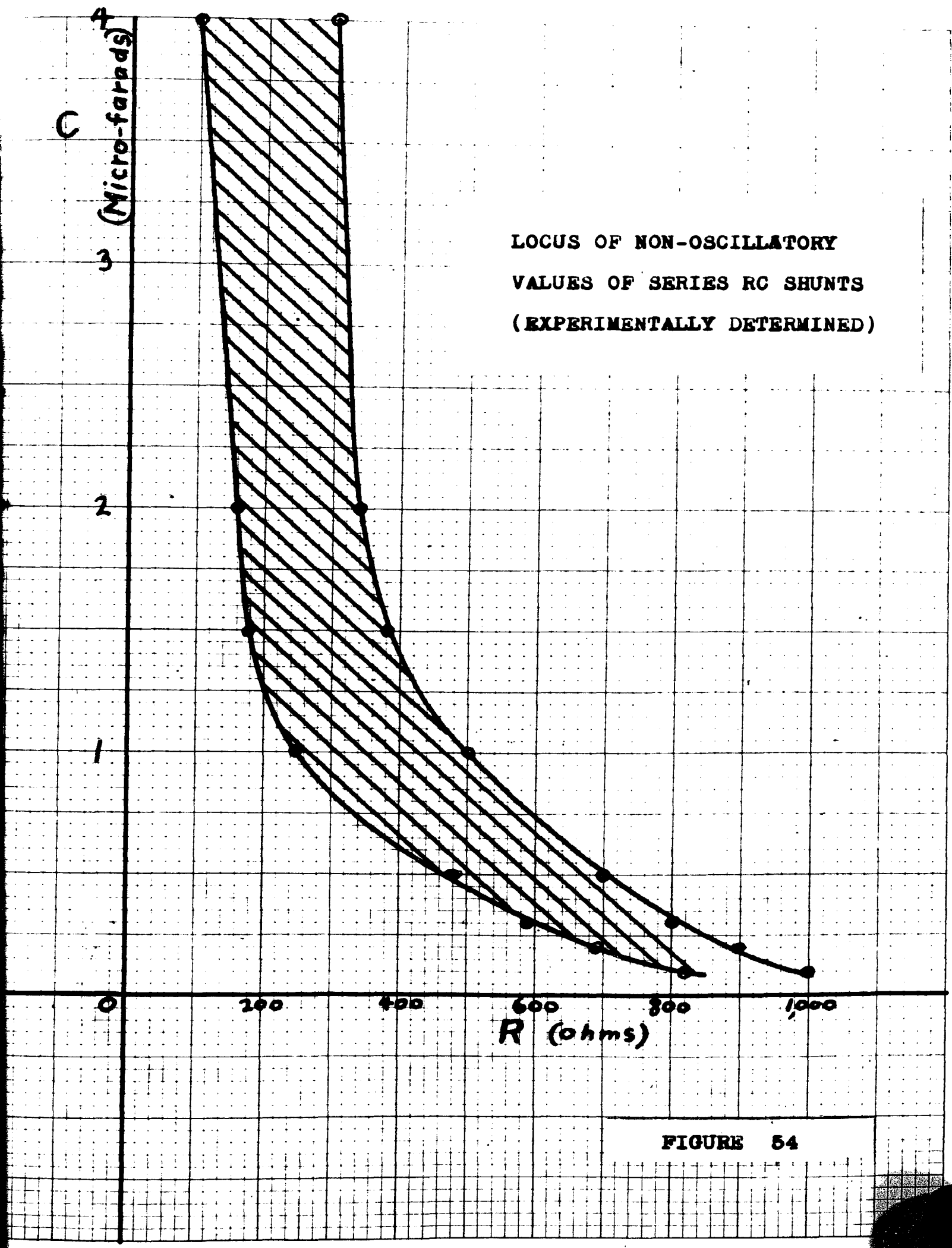

C (Micro-farads)
4
3
2
1
0
200
400
600
800
1,000
R (ohms)
LOCUS OF NON-OSCILLATORY
VALUES OF SERIES RC SHUNTS
(EXPERIMENTALLY DETERMINED)

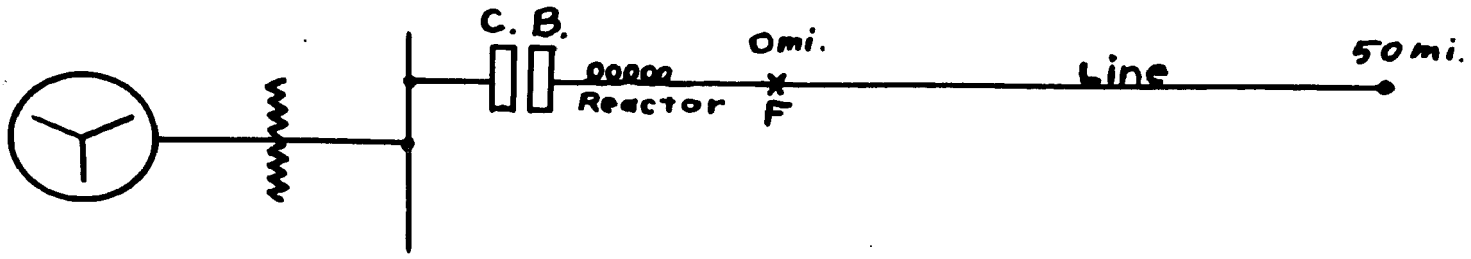
FIGURE 54

Discussion of Figures 55 and 56

These figures illustrate a two-harmonic type of recovery voltage. In addition to the 60-cycle fundamental, the transient contains a 7.74 KC harmonic introduced by transformer leakage inductance and capacitance to ground, and also a 17.8 KC harmonic introduced by current-limiting-reactor inductance and capacitance to ground. Figure 55 should be compared with Figure 5. Experimentally this was a very difficult waveform to photograph satisfactorily. Even a very slight hunting on the part of the synchronous switch caused considerable changes in waveform. The trace was very fast and faint to begin with, so that after sharing the sweep time with the timing wave only a ghost of a trace remains as illustrated in Figure 56. Therefore, Figure 55 was photographed with the transient occupying 100% of the sweep time, and the horizontal scale considerably enlarged. The time scale was obtained by calibrating the oscilloscope face with the timing wave and leaving the horizontal gain fixed.

RECOVERY VOLTAGE, TWO-HARMONIC--CASE B

HARMONICS:7.74 KC & 17.8 KC



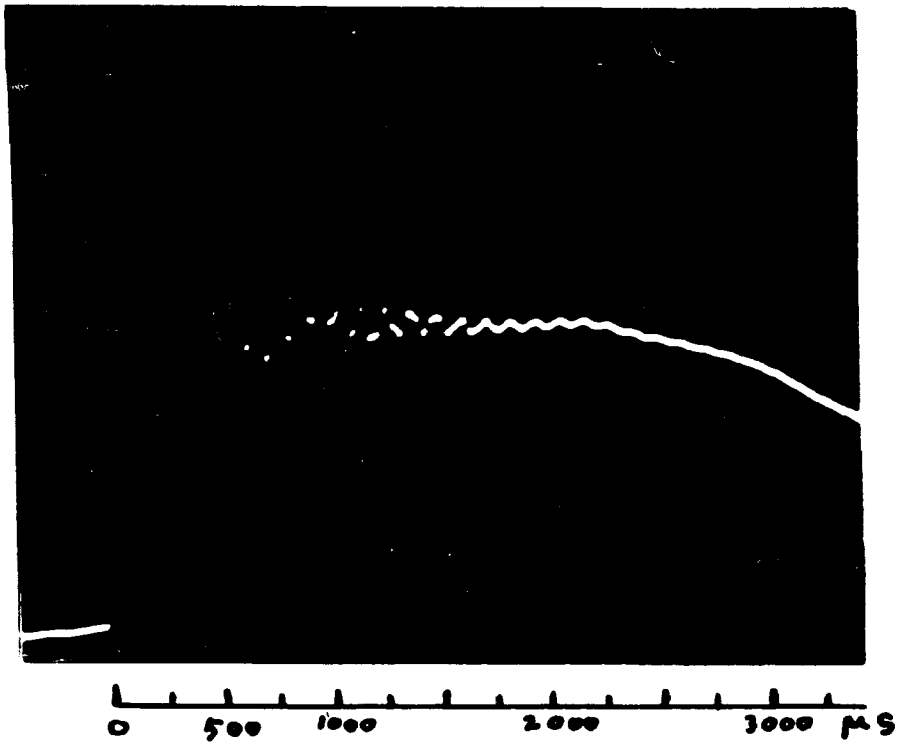


FIGURE 55

Transformer:
R = 6 ohms
L = 20 mh
C = 0.02 uf
Reactor:
R = 3.77 ohms
L = 10 mh
C = 0.008 uf

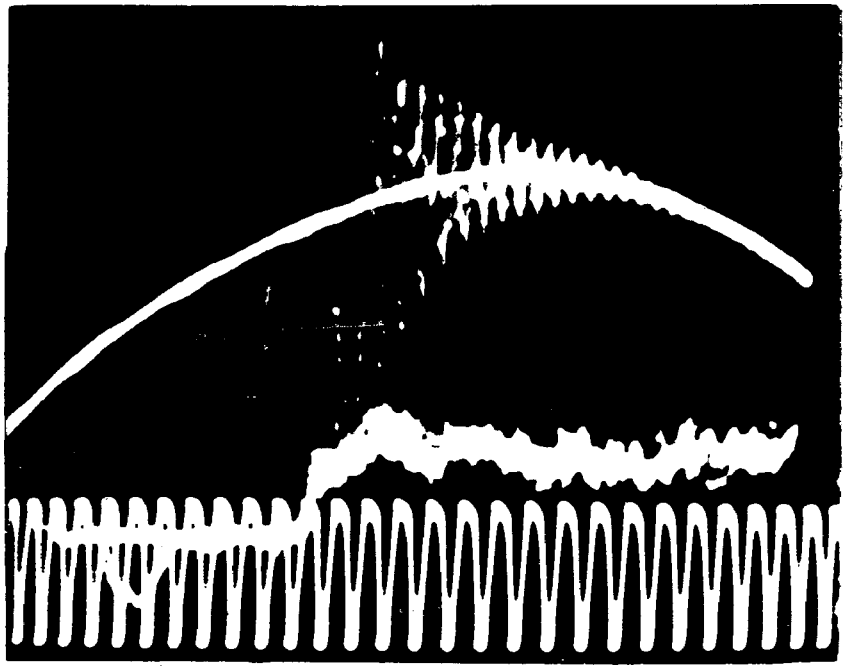


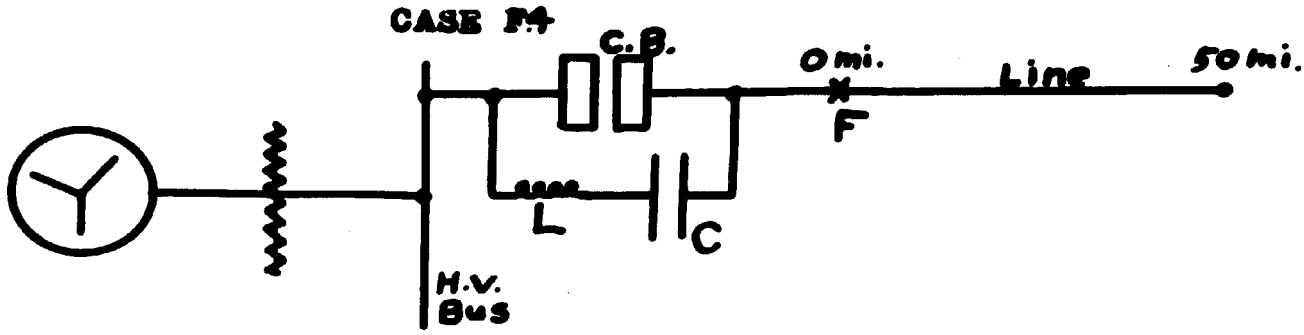
FIGURE 56

(Same constants as Fig. 55)

Discussion of Figures 57 and 58

These two figures illustrate a case of academic interest only, as was explained in the analytical discussion of this case. Adding inductance in series with capacitance shunting a circuit breaker merely adds a high frequency harmonic to the results obtained in Case D (shunt capacitance). This harmonic raises the peak value of the transient to an even higher value than would be obtained in Case D.

RECOVERY VOLTAGE WITH SERIES L AND C SHUNTED ACROSS CIRCUIT BREAKER



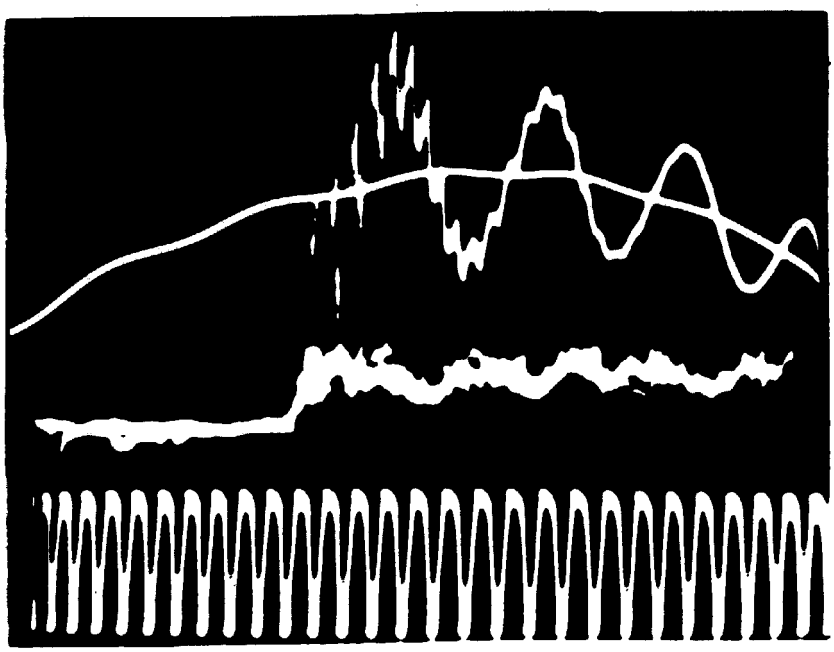


FIGURE 57

L = 50 mh

C = 0.2 uf

f = 5 kc

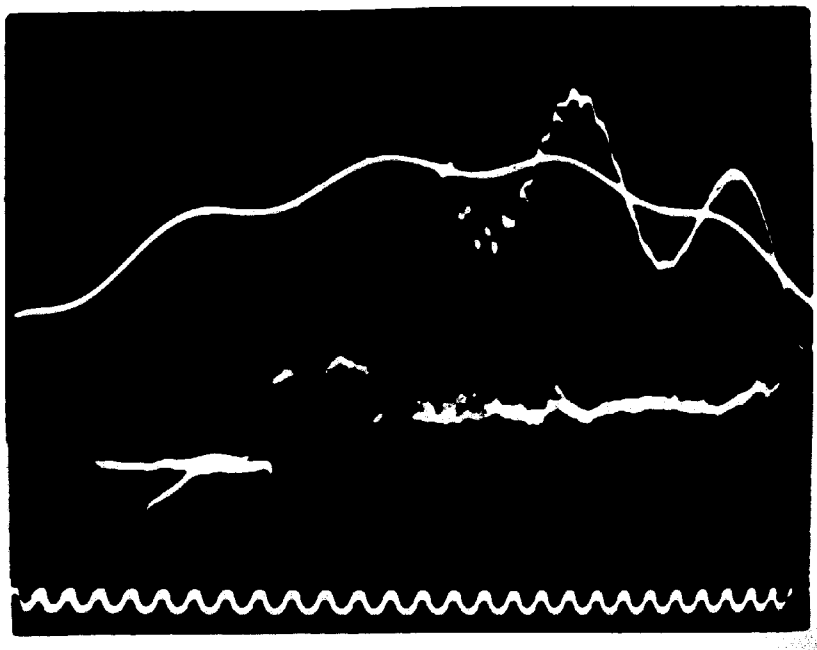


FIGURE 58

L = 50 mh

C = 0.5 uf

f = 5 kc

Effect of Arc Voltage Drop on Recovery Voltage

The main body of this paper dealt with circuit transient recovery voltage characteristics and did not include any effect the circuit breaker itself might have. The following factors were not considered:

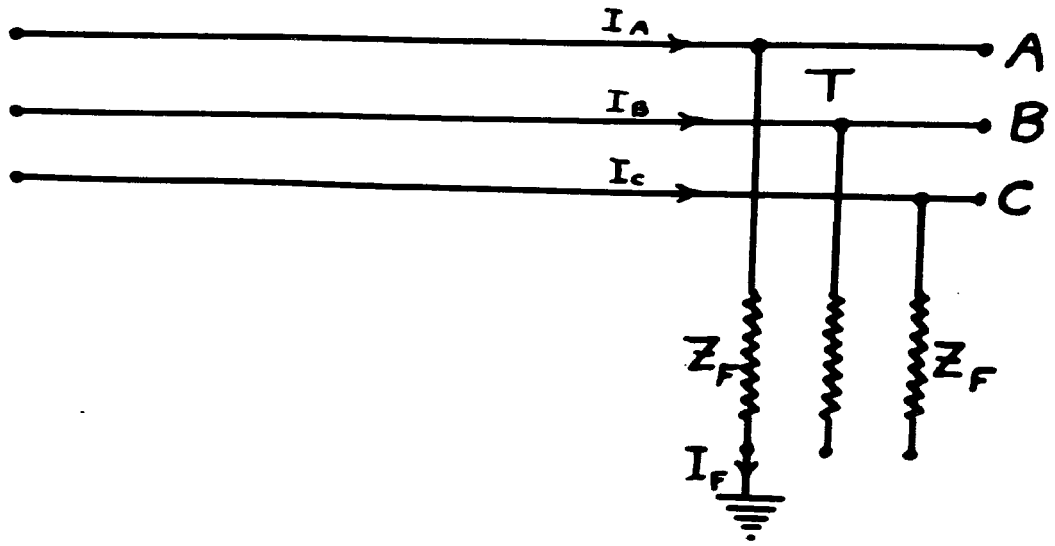
1. Arc voltage drop in circuit breaker.
2. Conduction current in circuit breaker after current zero.
3. Arc voltage drop in fault.
4. Rotating machine flux-transients subsequent to faulting.

Factor 4 was neglected by assuming an infinite bus power supply. Therefore generator impedance is negligible and the transient effect of flux decay need not be considered.

Factor 3 can be taken care of quite easily if the assumption of constant fault impedance is made. By well-known symmetrical component theory, constant fault impedance can be entered into the positive, negative, and zero sequence breadboards used in computing steady-state fault current as follows:

VOLTAGE DROP ACROSS FAULT IMPEDANCE

FIGURE 59



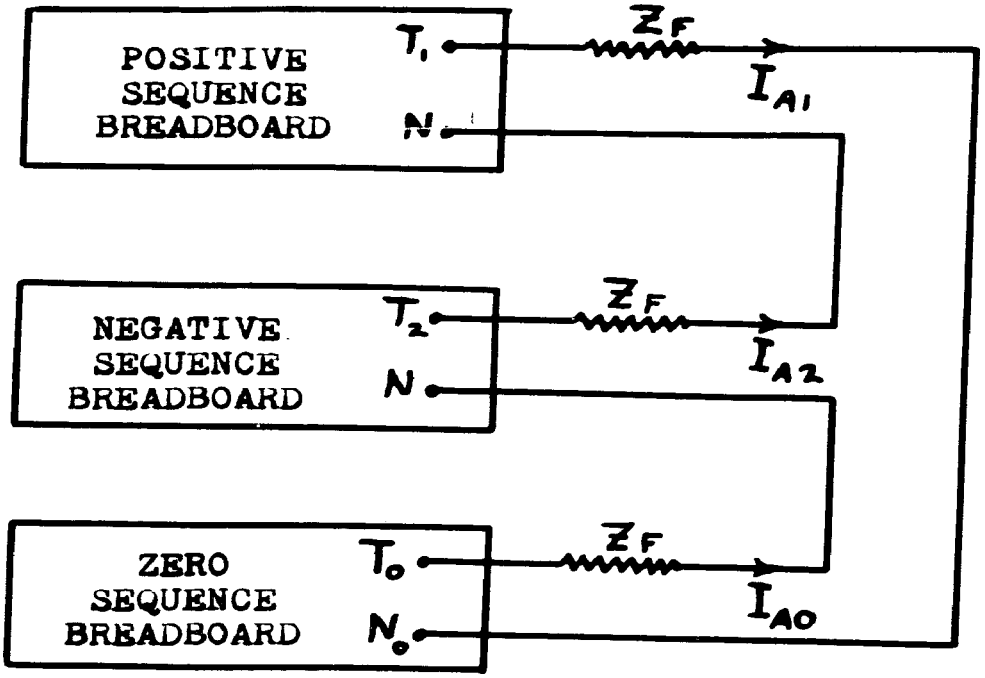


FIGURE 60

$$I_m = (I_F)_{Max} = \frac{3(E_1)_{Max}}{Z_1 + Z_2 + Z_0 + 3Z_F}$$

Factor 2 is generally more difficult to consider in recovery voltage calculations since residual ionization, immediately following arc interruption, is equivalent to a time-varying resistance connected across the circuit-breaker terminals. This function would depend upon the nature of the fault and upon the type of arc-interrupting device. Normally this conduction current is negligible compared to arc current [9], and cannot even be detected on fault current oscillograms. Its main importance lies in its effect on recovery voltage waveform. When capacitance adjacent to the circuit breaker is very small, resulting in a very high natural frequency of oscillation and also resulting in a very small capacitor current comparable in size with conduction current after arc extinction, then conduction current effect is closely comparable with the case of shunting resistance. The previously illustrated damping effect of resistance on recovery voltage takes place. In the example which follows conduction resistance is considered, along with arc voltage, but assumed to remain constant at 10,000 ohms during the short recovery voltage interval considered.

Factor 1 will be introduced in the analysis as explained in the following problem.

APPENDIX- SPECIAL PROBLEM

Effect of Arc Voltage Prior to Arc Extinction on Recovery Voltage.

SINGLE LINE DIAGRAM

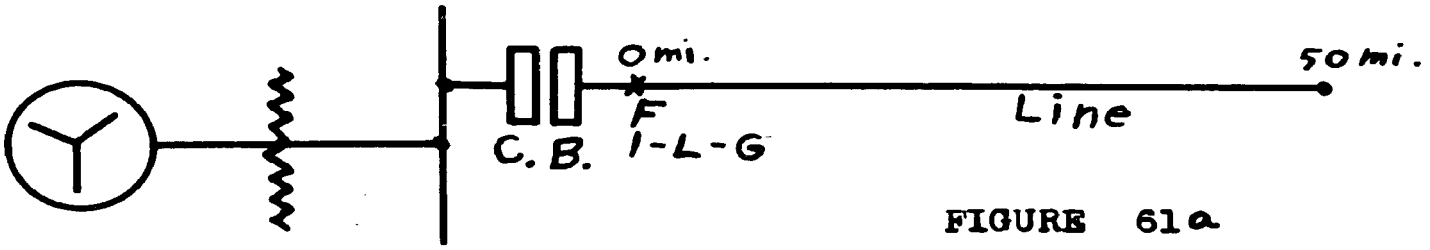


FIGURE 61a

EQUIVALENT CIRCUIT FOR PHASE A PRIOR TO ARC EXTINCTION

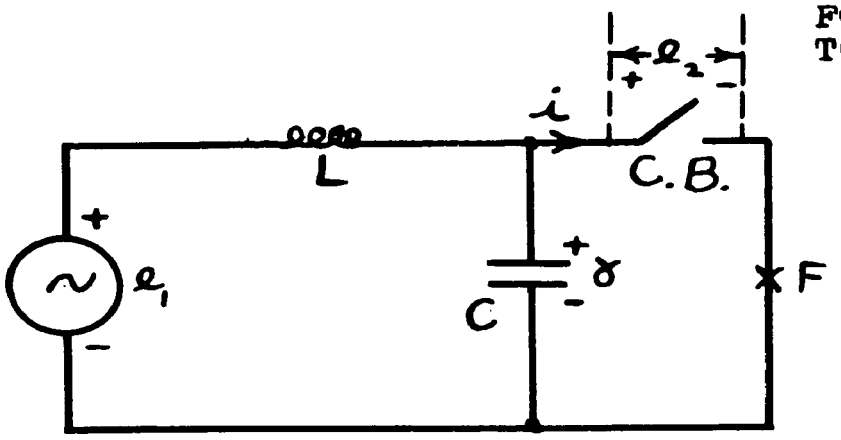


FIGURE 61b

L = Transformer Leakage Inductance (31.2 mh)

C = Transformer & H.V. Bus Capacitance to Ground (0.02 μf)

γ = Condenser Voltage at Time t_2

EQUIVALENT TRANSIENT CIRCUIT

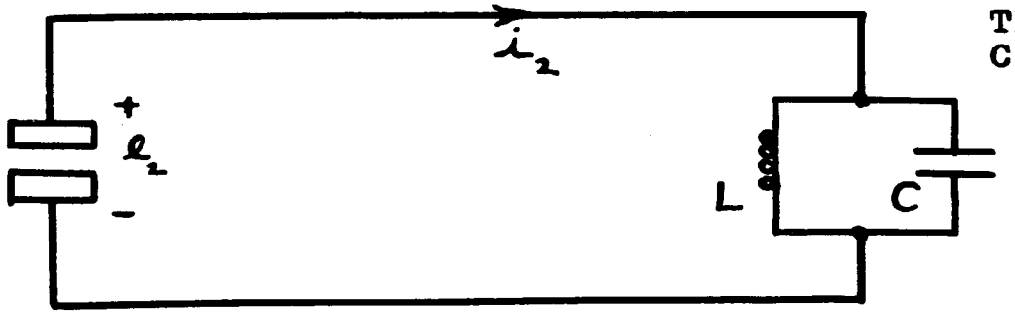


FIGURE 62

Special Problem - Effect of Arc Voltage Prior to Arc Extinction on Recovery Voltage

Reasoning: The actual current being interrupted can be considered to be the superimposed resultant of two currents [6]:
(1) the steady-state sinusoidal fault-current i_1 which flows prior to current interruption.
(2) the transient current i_2 that flows in the network caused by a virtual voltage source representing the time-varying voltage rise (e_2) across the circuit-breaker arc. This transient component of current must be derived from known circuit parameters and from a predetermined time variation of arc voltage, $e_2(t)$. This voltage depends of course upon the type of circuit breaker used. It starts out at zero and rises to the value of system voltage, e_1, which exists at the time of arc extinction. (See Figure 64). For the purpose of analysis, let us assume that the voltage rise across the circuit breaker arc can be represented by a function of time, $e_2(t)$.

Referring to Figure 62, we can see that

$$I_2(S) = \frac{E_2(S)}{Z(S)}$$

$$Z(S) = \frac{S}{C(S^2 + \frac{1}{LC})}$$

$$i_2(t) = \mathcal{L}^{-1}\left\{\frac{\mathcal{L}[e_2(t)]}{\frac{S}{C(S^2 + \frac{1}{LC})}}\right\}$$

Therefore the transient component of current, $i_2(t)$, due to arc voltage can be determined. By adding steady-state fault current $i_1(t)$ prior to contact separation to this transient component of current, the actual arc current waveform can be determined. (See Figure 64). The arc interruption time, t_1, occurs when the sinusoidal component, $i_1(t)$, is equal and opposite to the unidirectional transient component, $i_2(t)$.

At the instant of arc extinction, the system voltage, $e_1(t)$, has a value of γ volts, and a charge corresponding to this voltage will remain on the condenser after arc interruption is accomplished.

Now if we assume that there are no restrikes and that the conduction gases left in the path of the arc provide a finite leakage path, we can represent the system following arc interruption by Figure 63a. In this figure, R represents leakage resistance left in the path of the arc which is assumed to be constant during the recovery voltage time interval. Since this interval is very brief compared to the period of normal system voltage, let us assume that the latter remains constant at a value V_1 which of course is equal in magnitude to the condenser voltage γ.

APPENDIX- SPECIAL PROBLEM (CONTINUED)

Effect of Arc Voltage Prior to Arc Extinction on Recovery Voltage.

EQUIVALENT CIRCUIT FOR FAULTED PHASE ($t \geq t_1$)

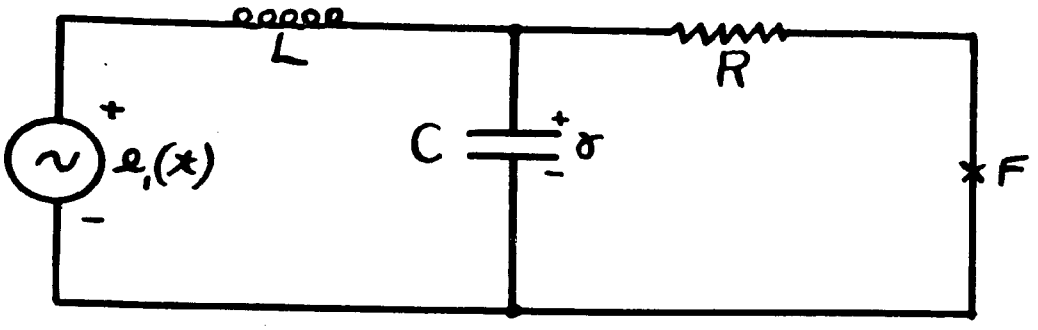


FIGURE 63a

EQUIVALENT TRANSIENT CIRCUIT ($t \geq t_1$)

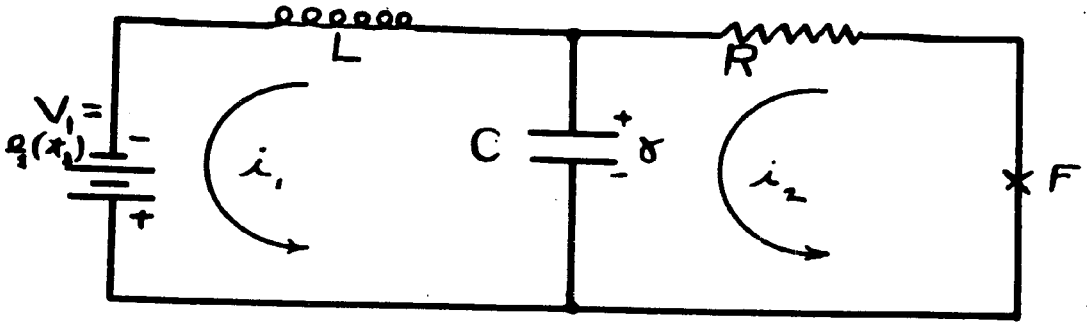


FIGURE 63b

δ -VOLTAGE ON CONDENSER AT t_1

V_1-SYSTEM VOLTAGE AT t_1

R-LEAKAGE RESISTANCE LEFT IN ARC PATH

C- TRANSFORMER & H.V.BUS LUMPED C TO GND.

t_1-TIME OF ARC EXTINCTION

L-TRANSFORMER LEAKAGE INDUCTANCE.

We now proceed to the calculation of recovery voltage, using the following constants:

$$L = 31.2 \text{ mh}$$

$$C = 0.02 \text{ uf}$$

$$R = 10{,}000 \text{ ohms}$$

$$\gamma = \text{constant}$$

$$e_1(t) = V_1 = \text{constant} = \gamma$$

Let us refer now to Figure 63b which represents the system immediately following arc extinction and for the duration of the recovery voltage time interval. From this figure, we can derive the transient loop equations:

$$(1) \quad \frac{1}{C} \int_o^t (i_1 - i_2)\, dt - V_1 + L \frac{di_1}{dt} - \gamma = 0$$

$$(2) \quad i_2 R + \frac{1}{C} \int_o^t (i_2 - i_1)\, dt + \gamma = 0$$

Transforming both equations gives:

$$(1) \quad \frac{1}{CS} \cdot I_1(S) - \frac{1}{CS} \cdot I_2(S) - \frac{V_1}{S} + LS \cdot I_1(S) = \frac{\gamma}{S}$$

$$(2) \quad R \cdot I_2(S) + \frac{1}{CS} \cdot I_2(S) - \frac{1}{CS} \cdot I_1(S) = \frac{-\gamma}{S}$$

Solving for $I_2(S)$,

$$I_2(S) = \frac{\begin{vmatrix} \frac{1}{CS} + LS & \frac{1}{S}(V_1 + \gamma) \\ -\frac{1}{CS} & -\frac{\gamma}{S} \end{vmatrix}}{\begin{vmatrix} \frac{1}{CS} + LS & -\frac{1}{CS} \\ -\frac{1}{CS} & R + \frac{1}{CS} \end{vmatrix}}$$

$$I_2(S) = \frac{-\gamma\left(S^2 - \frac{V_1}{\gamma LC}\right)}{RS\left(S^2 + \frac{S}{RC} + \frac{1}{LC}\right)}$$

$$V_2(S) = I_2(S) \cdot R = \frac{-\gamma\left(S^2 - \frac{V_1}{\gamma LC}\right)}{S\left[\left(S + \frac{1}{2RC}\right)^2 + \left(\frac{1}{LC} - \frac{1}{4R^2C^2}\right)\right]}$$

Therefore, the recovery voltage is given by

$$v_2(t) = \mathcal{L}^{-1}\left[V_2(S)\right]$$

$$v_2(t) = V_1 - \frac{\gamma LC}{\left(1 - \frac{L}{4R^2C}\right)^{\frac{1}{2}}}\left\{\left[\frac{1}{2R^2C^2} - \frac{1}{LC}\left(1 + \frac{V_1}{\gamma}\right)\right]^2 +\right.$$

$$\left. + \left[\frac{1}{LC} - \frac{1}{4R^2C^2}\right]\frac{1}{R^2C^2}\right\}^{\frac{1}{2}} e^{-\frac{t}{2RC}} \sin\left[\left(\frac{1}{LC} - \frac{1}{4^2R^2C}\right)^{\frac{1}{2}} t + \psi\right]$$

where

$$\psi = \tan^{-1} \frac{\left[\frac{1}{LC} - \frac{1}{4R^2C^2}\right]^{\frac{1}{2}} \left(-\frac{1}{RC}\right)}{\frac{1}{2R^2C^2} - \frac{1}{LC}\left(1 + \frac{V_1}{\gamma}\right)}$$

$$+ \tan^{-1} \left[2RC \left(\frac{1}{LC} - \frac{1}{4R^2C^2}\right)^{\frac{1}{2}}\right]$$

The preceding equation can be greatly simplified and still give results within slidrule accuracy.

$$v_2(t) \cong V_1 - 2 V_1 e^{-\frac{t}{2RC}} \sin \left[\left(\frac{1}{LC} - \frac{1}{4R^2C^2}\right)^{\frac{1}{2}} t + \psi\right]$$

$$\psi \cong \tan^{-1} 2RC \left(\frac{1}{LC} - \frac{1}{4R^2C^2}\right)^{\frac{1}{2}} \cong 90°$$

If we let $t = 0$ (time of arc extinction),

then $v_2(t) = -V_1$

Therefore the recovery voltage transient begins at a negative value equal to arc voltage at the instant of extinction. Park and Skeats [1] verify these results from actual waveforms. The above equation would give a waveform very similar to Figure 3 except that it would begin at a negative value and would be damped out more rapidly. The time constant of the waveform shown in Figure 3 is 0.0104 seconds, while the time constant of the above waveform is 0.0004 seconds.

Therefore it has been shown that the effect of arc

voltage just prior to arc interruption is threefold:

1. The high frequency oscillation of recovery voltage begins at a negative value corresponding to arc voltage immediately before interruption.
2. The exponential decay of the recovery voltage waveform occurs faster than in the other cases considered.
3. The arc voltage prior to interruption shifts the current-zero point (time of arc extinction) to the left on the time axis. The amount of shift depends of course upon the time-function of arc voltage. This effect can be verified by an examination of Figure 64.

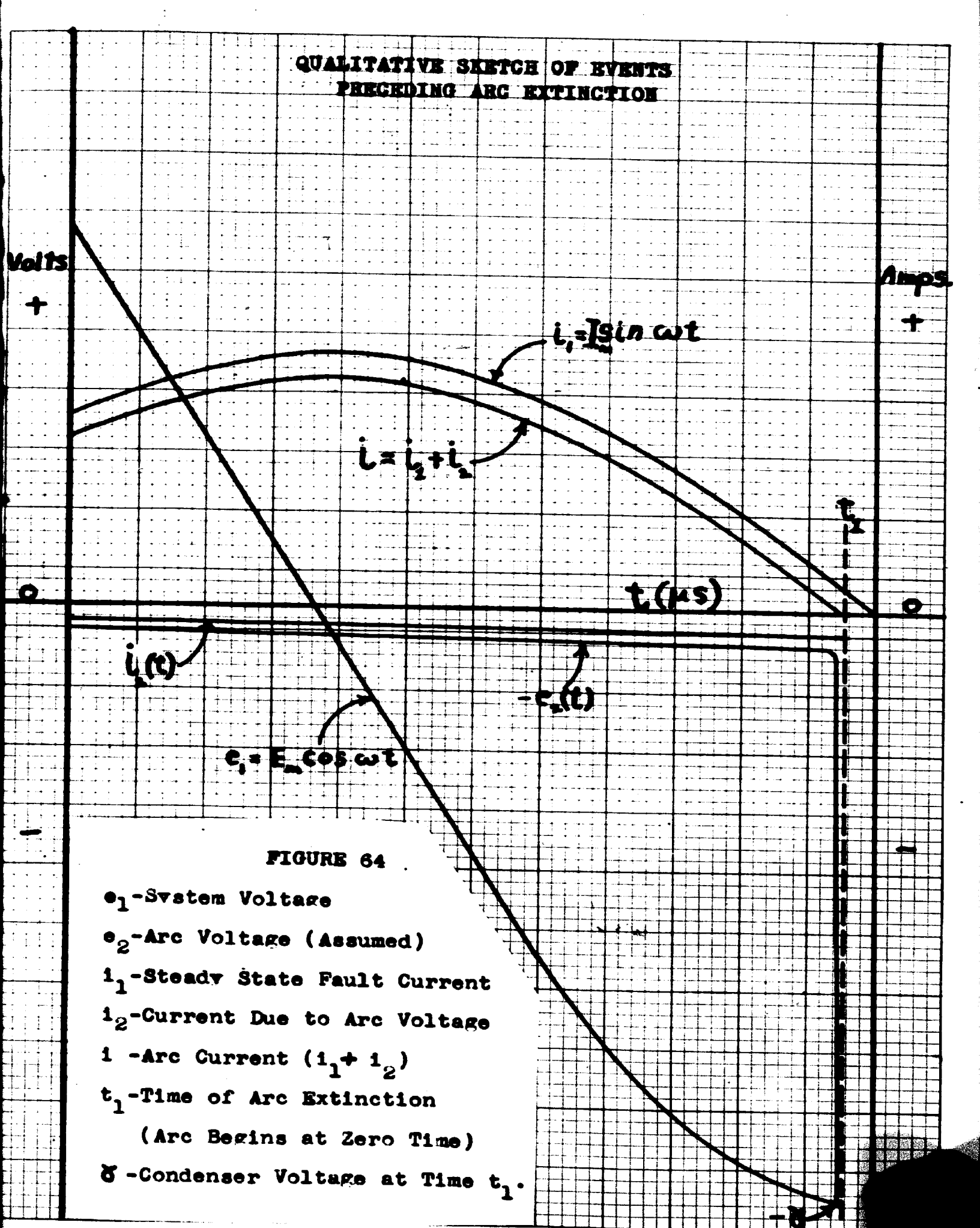


FIGURE 64

e_1-System Voltage

e_2-Arc Voltage (Assumed)

i_1-Steady State Fault Current

i_2-Current Due to Arc Voltage

i -Arc Current ($i_1 + i_2$)

t_1-Time of Arc Extinction

(Arc Begins at Zero Time)

σ -Condenser Voltage at Time t_1.

CONCLUSIONS

The important results of this study are threefold:

(1) An experimental method has been demonstrated which, within the limitations of the four previously mentioned assumptions, makes possible the presentation of recovery voltage waveforms quite similar to those actually obtained in practice [1], [9]. The presentation of the repeated transient waveforms are stable enough to be photographed. In addition, by using a miniature power system with lumped parameters, a great variety of fault interruption problems can be investigated and their corresponding recovery voltage waveforms can be recorded with a minimum of time and effort.

(2) A mathematical method has been established for predetermination of recovery voltage waveforms. By comparison of the experimental waveforms contained in the Appendix with the corresponding computed waveforms in the first sections of this paper, we can verify the close check between the mathematical and experimental methods.

If a particular investigation warrants a very close inspection, the mathematical method would of course give more precise results. The greater part of the investigation was made under the same four limiting assumptions as the experimental study. One section of the Appendix describes methods for overcoming these limitations.

(3) Two effective means of reducing recovery voltage

severity have been described and analyzed, both mathematically and experimentally.

The first method involves a series resistance-capacitance combination shunted across the circuit breaker terminals (Case F-2, Figures 15, 16, 48-53 inclusive). This method greatly reduces recovery voltage peaks and initial rates of rise. For a specified minimum shunting impedance this method is most effective in reducing transient severity. An experimentally determined locus is presented in Figure 54 which allows a rapid determination of effective resistance-capacitance combinations for the particular system under study.

The second method consists of a pure resistance shunted across the circuit breaker (Case C, Figures 7, 37-42 inclusive). For a specified minimum shunting impedance, this method is not quite as effective as the first; however, a considerable reduction in both recovery voltage peaks and in rates of rise can be accomplished.

Park and Skeats [1] describe the practical use of a shunting resistance to reduce recovery voltage severity. Their conclusions were quite favorable. An arbitrary value of 1,000 ohms was used, but no methods were developed for predetermining an optimum value of resistance. The resistor was somewhat costly since it had to withstand rather high voltages. From the practical application standpoint, the resistance-capacitance combination would be easier and cheaper to construct.



Numerical Comparison of Experimental and Analytical Waveforms

Figure 3 (p. 14) and Figure 29 (p. 101)
- Figure 3 - period = 157 us
  - First peak = 18 volts
- Figure 29 - period = 170 us
  - First peak = 16.6 volts

Figure 7 (p. 33) and Figure 38 (p. 107)
- Figure 7 - time constant = 54 us
- Figure 38 - time constant $\cong$ 60 us

Figure 10 (p. 49) and Figure 34 (p. 103)
- Figure 10 - time to reach 3rd peak = 400 us
  - Voltage, 1st peak = 4.2
  - Voltage, 2nd peak = 6.4
  - Voltage, 3rd peak = 8.7
- Figure 34 - time to reach 3rd peak = 420 us
  - Voltage, 1st peak = 5.4
  - Voltage, 2nd peak = 8.1
  - Voltage, 3rd peak = 12

Figure 15 (p. 71) and Figure 48 (p. 120)
- Figure 15 - time constant of exponential rise = 68 us
  - Time to reach peak = 210 us
  - Voltage of peak = 11.7
- Figure 48 - time constant $\cong$ 80 us
  - Time to reach peak $\cong$ 280 us
  - Voltage of peak = 10

Figure 16 (p. 76) and Figure 50 (p. 121)
- Figure 16 - Time constant = 136 us
- Figure 50 - Time constant $\cong$ 150 us

Figure 12 (p. 56) and Figure 35 (p. 105)
- Figure 12 - rate of rise (for $t < 120$us) = 12,500 v/sec.
  - Rate of rise (for $120 < t < 528$) = 7,800 v/sec.
- Figure 35 - rate of rise (for $t < 120$us) = slightly greater than 10,000 v/sec.
  - Rate of rise (for $120 < t < 528$) $\cong$ 10,000 v/sec.

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