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VISUAL PATTERN ANALYSIS

A LOOK AT MAPS

by

Phillip Charles Muehrcke

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Doctoral Committee:

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A B S T R A C T

VISUAL PATTERN ANALYSIS
A LOOK AT MAPS

by
Phillip Charles Muehrcke

Chairman: Waldo R. Tobler

The utility of thematic maps in scientific argument is considered in this paper. Maps are viewed as graphic models, with particular attention on the interaction between statistics and graphics. The quality of visual information retrieval from two-dimensional displays is discussed in the context of this map-model formulation. It is concluded that cartographic processing is valuable in inductive hypothesis-generation and model refinement, but not well suited to deductive model testing. Map generalization and map comparison are identified in this evaluation as important but poorly understood cartographic procedures. Although both processes can be pursued in the context of the philosophical constructs mentioned above, map comparison is chosen as focus for further discussion. An experiment is conducted in which subjects are instructed to rank-order sets of dot, choropleth, and profile maps of various complexities (i.e., linear, quadratic, cubic) in terms of their degree of correlation with a referent. Results are evaluated by means of a statistical model of association known as the coefficient of determination, r^2. Subject response indicates that there is a highly significant graphic-complexity interaction. The graphic main effect is highly significant (choropleth maps were compared most effectively, dot maps least effectively). The complexity main effect is not significant. From this it is inferred that the distributions tested were too simple and/or the range in surface complexity too small to evoke significant differences in subject response trends. Errors are distributed from graphic to graphic as a function of the degree of correlation between map and format. The range in ($\pm$) r^2 values tested, where ($\pm$) refers to the sign of the original r value, is: (+).90, .80, .70, .60, .50, .40, .20, 0, (-).20, .40, .50, .60, .70, .80, .90, 1.00. Noting the larger increment in r^2 between

map pairs as values approached 0, experimental results can be summarized as follows: Choropleth maps produced a bimodal function (most errors with medium ($\pm$) r^2 values); dot maps produced a bimodal function (most errors with high ($\pm$) r^2 values); and profile maps produced a unimodal function (most errors with medium to low ($\pm$) r^2 values. Results from subject responses are interpreted to mean that, given the types of graphics and distributions employed, choropleth maps were far better information-transmitting media than either dot or profile maps. It was also noted that, although profile maps generated fewer errors than dot maps, these errors were larger. Results suggested that dot maps could be improved by increasing the number of dots in the high density zones (i.e., apparent value versus absolute value scaling). Profile map effectiveness might similarly be improved by employing a larger vertical exaggeration factor and by increasing the number of data points (which in effect would smooth the surface). The experimental design is evaluated and future topics of research are outlined.

TABLE OF CONTENTS

INTRODUCTION 1

PART I

Section

I. MAPS AND MODELS 7

II. MAPS AND GRAPHICS 14

III. THE SCIENTIFIC USE OF MAPS 22

Maps in Research
Pattern Comparison Situations
Visual Pattern Comparison

PART II

IV. EXAMPLE EXPERIMENT 34

Problem
Data
Map Compilation
Experimental Method

V. DISCUSSION OF THE RESULTS 45

Statistical Evaluation
Graphic/Complexity Interaction
Graphics
Complexity
Subject Consistency
Interpretation of the Results

VI. FUTURE RESEARCH 71

Graphics
Complexity
Test Design
Increment
Data
Statistical Models

Learning
Image Processing

VII. IN RETROSPECT 82

APPENDICES 84

Map Compilation Procedure
Data Listing
Test Map Sets
Confusion Matrices
Corrected Map Ordering
Tables (Miscellaneous)

BIBLIOGRAPHY 169

LIST OF ILLUSTRATIONS

TABLES

Table	Title	Page
5.1	Analysis of Variance for Factorial Design	48
5.2	Tabular Summary of Experimental Results	60
6.1	Variable Matrix From Factorial Design	72

FIGURES

Figure	Title	Page
1	Cartographic System	2
1.1	Map Utility Function	8
4.1	The Function r^2	36
4.2	Trend Surfaces	40
5.1	Mean Tau Statistics with Standard Deviations	46
5.2	Total Ranking Displacement and Total Ranking Errors	50
5.3	Constant Increments in r^2	51
5.4	Total Ranking Displacement	54
5.5	Total Ranking Errors	55
5.6	Range in Total Errors Between Three Surface Complexities	57
5.7	Mean Standard Deviation in Ranking Displacement	59
5.8	Generalized Confusion Pattern	61
5.9	Generalized Utility Functions for Map Graphics	63
5.10	Apparent Intensity Transformations	65
5.11	Learning Function	67

PREFACE

I would like to make special acknowledgement of the many sources of financial support that made my graduate studies program possible: NDEA Title IV Fellowship in Cartography, Rotary Foundation Fellowship for International Understanding, Rackham Prize Fellowship (Ford Foundation funds), Graduate Dissertation Research Grant, Robert G. Buzzard Graduate Scholarship Award, and Assistantships in the Department of Geography, The University of Michigan. I am especially indebted to my advisor, Professor Waldo R. Tobler, Department of Geography, University of Michigan, for his suggestions, criticisms, and continued patience. Special thanks are extended to Associate Professor Daniel J. Weintraub, Department of Psychology, University of Michigan, for his encouragement, advice in formalizing the experimental design, and many constructive comments. Associate Professor Gunnar Olsson and Professor John D. Nystuen are thanked for their suggested revisions of the thesis draft. I am grateful to Martin Weinrich, University of Michigan Statistical Laboratory, for his enthusiastic help in statistical reduction of subject response data. The experimental section could not have been executed without the use of the Data Simulation Program written by Clemens Johnson, Consultant at the University of Michigan Computing Center. Computer programming assistance was provided by Stephen Gale, Ph.D. candidate in the Department of Geography, The University of Michigan. The myriad computations and plots would not have been possible without the facilities of the University of Michigan Computing Center. I am indeed grateful to the many subjects that consented so willingly to participate in the experiment. And finally, I thank my parents for their unquestioned support, endless encouragement, and depth of understanding. To Nancy I owe the chance to finish--the motivation was the love of Juliana.

INTRODUCTION

Analysis of two-dimensional patterns is common. Micrographs, medical thermographs, bubble-chamber photographs, aerial photographs and imagery, radar images, X-rays, fingerprints, and maps are a few examples from the wide range of pictorial displays employed in scientific inquiry. The existence and variation in pattern properties (i.e., shapes, sizes, intensities, textures, positions, associations, etc.) depicted by these two-dimensional displays are interpreted in light of the theory characterictic of the respective fields of interest. Results of such analysis form the basis of inference to spatial structure and process.

The special case of cartographic representation is the focus of this paper. Maps have proven to be a convenient means of portraying and analyzing certain types of information.[1] For some disciplines they constitute a major form of communication and serve as a means of facilitating the solution of diverse problems. In particular, scientists who study the two-dimensional distributions of phenomena on or near the surface of the earth (i.e., ecologists, geographers, geologists, meteorologists, and so on) have traditionally been interested in map analysis. Since the map has provided a model for pattern analysis, it figures significantly in the theoretical basis of each discipline.

In recent decades these fields have undergone a major transformation in technique, often described as the "quantitative revolution." Explicit in this change in emphasis from a qualitative to a quantitative approach is the assumption that subjective map analysis of spatial distributions and spatial associations (central research themes in the fields mentioned) is not sufficiently rigorous to provide the foundation for a scientific discipline. The results obtained from visual pattern comparison are asserted to be imprecise, or inconsistent, and often more apparent than real.[2] The nature of the map as a graphic model, on one hand, and the inherent and learned limitations of the human perceptual mechanism on the other, are cited in support of this assertion.

The "quantifiers" responded by borrowing a host of statistical procedures from other fields for "objective"analysis of pattern association. Two advantages of the quantitative approach over visual estimation are that the former allows: (a) the detection and appreciation of smaller differences, and (b) provides a sounder basis of judgment of the significance of these differences.[3] It is also postulated that use of the cartographic technique seriously limits the number of variables that can be incorporated into an analysis; while quantitative techniques are well suited to inclusion of large numbers of variables.[4] With the range of quantitative techniques now available and easily applicable with high-speed, electronic computers, more sophisticated and rigorous pattern comparison is asserted to be within our grasp. The map, so long held in unquestioned status as a major tool of these sciences, requires re-examination and critical evaluation in light of the growing emphasis on precise quantitative analysis.

During the present transitional stage it is not clear when the use of a quantitative technique or the use of a qualitative technique is the more appropriate, or what the most judicious mix of the two approaches should be. The information required to clarify this little understood interaction between statistics and graphics would appear to be forthcoming only after rigorous assessment of the practical potentialities of the various methods. The quality of information retrieval from cartographic display becomes a major focus of interest in this research. The problem is more clearly defined when viewed in the theoretical context of cartographic processing. It is convenient to characterize the cartographic process as a series of transformations (see Figure 1).[5]

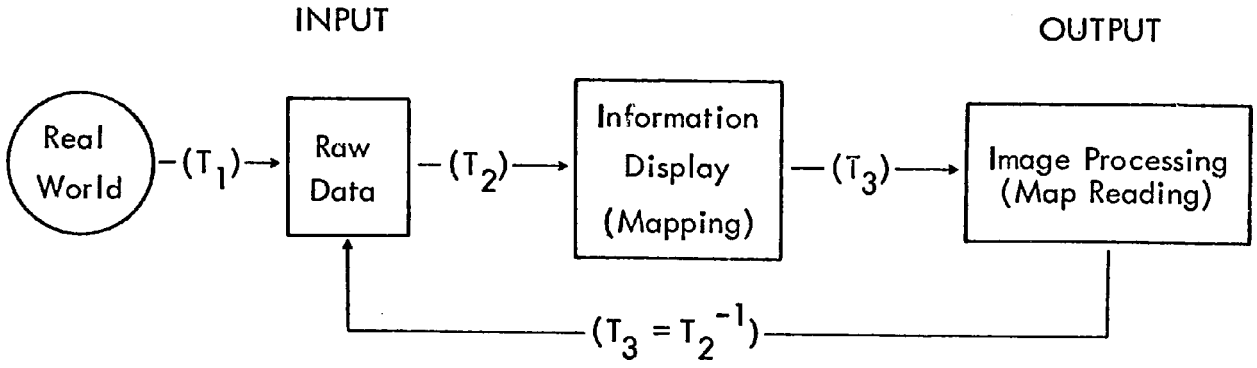


FIGURE 1

Data are selected from the real world (T_1). The cartographer transforms (i.e., encodes) these data into a map (T_2). Information is retrieved (i.e., decoded) from the map through an interpretative reading process (T_3). Cartographic research is devoted to the task of finding a transformation, T_2, such that output from T_3 is equal to input to T_2, that is, $T_3 = T_2^{-1}$. This conceptual simplification should not be interpreted to mean that the transformations themselves are simple. Yet thinking from this point of view could lead to useful results. The main emphasis of previous cartographic research has been with T_2. Although T_3 is equally, if not more, important than T_2, it has been essentially neglected.[6] This failure to recognize cartographic communication as a human problem has resulted in a system that is often improperly tailored to those who use it.[7] A great deal of this oversight can be attributed to a failure on the part of cartographers to conceive cartography as a feedback system. To the contrary, there has been a tendency to consider the completion of map drafting or printing to be the final stage of the cartographic process.

Since little philosophical attention has been devoted to this problem area, the preparation of maps for scientific purposes is vaguely defined and too often represents a misdirected effort. It is inefficient to make more maps than is necessary, to make too few maps, to make the wrong map for a given purpose, or to use maps in exclusion of a more appropriate technique.[8] It is the author's opinion that there are two major sources of this inefficiency: (a) researchers have a poor understanding of the intrinsic nature of maps as graphic models, and (b) researchers have a poor understanding of the relative effectiveness of various cartographic techniques as information transmitting media. The rigorous attempt to attain positional and magnitudinal accuracy, so characteristic of cartographic efforts, constitutes a major expense in cartographic processing. Striving for the resulting gain in detail and complexity is questionable for several reasons: (a) it may not be appreciated due to limitations in map interpreters' information retrieval mechanism; and (b) it may actually distract from graphic communication of the essential map theme by introducing "noise" into the system. From the point of view of interpretability, it may be that up to a point increased generalization would more than offset consequent planimetric displacement. Finding the appropriate balance

between these factors could result in considerable map compilation economies.

Confusion surrounding the cartographic process warrants urgent attention for several reasons: (a) the high cost of map production distracts significantly from their useability; (b) cartographic processing permits the flow (transmission) of huge amounts of information at a useful rate; therefore, properly designed maps could provide a partial solution to the growing problem of information overload in the environmental sciences; and (c) the increasing challenge of the scientific utility of maps threatens their continued use in scientific argument. Developments in computer graphics (e.g., interactive computer consoles) create further need to resolve the uncertainty encompassing this issue.[9]

The author concludes that cartographers have not dealt adequately with their own essential subject matter--symbolic representation. Research cartographers in particular demand revitalizing philosophical insight. There is need for generative ideas of the sort that will transform the whole study of cartography from "conventional wisdom" into modern science. This only seems attainable through the reconception of cartographic knowledge under new abstractive principles, in new intellectual perspectives. Part I of this dissertation outlines one such philosophical perspective. The formulation is based on the notion of maps as graphic models; that is, maps are viewed in light of the role of models and graphics in scientific argument. Discussion draws on the literature of cartography, graphic communication, and scientific model building. The distinction between the inductive and the deductive use of maps is emphasized. Cartography is treated as an information handling process, with emphasis placed on the map user as the key element in this system. The problem becomes one of expediting the transfer of information from producer (source) to destination (user).

This discussion is intended to expose critical research topics in the cartographic processing system and serve as an introduction for a formal statement of visual pattern analysis. It is hoped that this initial attempt at formulation and logical explanation of concepts will promote or facilitate continuing research in this vein, or suggest fresh approaches to basic problems.

Two major research areas in experimental cartography can be identified and considered within the philosophical constructs discussed above: (a) map generalization, and (b) map analysis. Of foremost concern is the need for better understanding of the role, mechanics, and implications of map generalization. This is especially critical to the continued development of automated cartography. Map analysis probably consists of the more basic operations of pattern description, recognition, comparison, and so on. The approach toward analysis can be either qualitative (e.g., visual) or objective (e.g., mathematical), or both. The trend has been toward increased emphasis on quantitative techniques of pattern (map) analysis under the conjecture that visual analysis is not sufficiently rigorous for scientific purposes. Of course it is true that prior to recent developments in digital computer capability which made many powerful quantitative spatial analyses available, researchers concerned with the spatial character of their data had to rely on cartographic processing.

Although map generalization and map analysis are interrelated concepts (i.e., not mutually exclusive), for purposes of research it is often convenient to treat them as separate operations as is done above. Continuing along this line, the scope of the experimental phase (Part II) of this dissertation is limited to a focus on visual pattern analysis. In particular, several aspects of visual map comparison are evaluated by means of a statistical model known as the coefficient of determination, r^2. Variables considered include: type of graphic (dots, choropleths, profiles); surface complexity (linear, quadratic, cubic); and degree of association with a referent (range from $\pm$ 1). Information on the relative effectiveness of various cartographic techniques could prove useful in establishing guidelines to facilitate selection of optimal graphics or graphic combinations for map comparison purposes. The proposed research should lead to a better understanding of visual pattern comparison, which may in turn suggest ideas for the design of more effective map display graphics, that is, ones that take into account the strengths and weaknesses of the human perceptual mechanism.

LIST OF REFERENCES

1. Throughout this dissertation the term "map" refers to the class of special purpose (thematic) maps commonly called "statistical maps," as opposed to the more general category of topographic maps. Since these two broad types of maps differ widely in character and function, they will not be considered together in the following discussion. Indeed, many arguments put forward in this thesis are erroneous when viewed in the context of topographic mapping.

2. Harold H. McCarty and Neil E. Salisbury, Visual Comparison of Isopleth Maps as a Means of Determining Correlations Between Spatially Distributed Phenomena, State University of Iowa Studies in Geography, No. 3 (Iowa City: State University of Iowa, 1961), pp. 78-80.

3. Peter Greig-Smith, Quantitative Plant Ecology (2d ed.; Washington: Butterworths, 1964), p. 212.

4. Kevin R. Cox, "Regional Anomalies in the Voting Behavior of the Population of England and Wales: 1921-1951" (unpublished Ph.D. dissertation, University of Illinois, 1967), p. 21.

5. Waldo R. Tobler, "Transformations," in The Philosophy of Maps, Michigan Inter-University Community of Mathematical Geographers, No. 12 (1968), pp. 3-4; and Joel L. Morrison, "Towards a Schematization of the Cartographic Process," Department of Geography, University of Wisconsin (Madison, 1968), 12 p. (Mimeographed.)

6. This is not to say that the map user has not been implicitly considered by cartographers. The entire set of map choices in, for example, drafting as to line weight, letter size and style, symbol combinations, and so on, are based on assumptions as to the response of map users. What the author criticizes is that this "conventional wisdom" has not been evaluated other than by feelings or impressions of what "looks" to be right.

7. E. E. Ahnell, Information Retrieval and Digital Display, Proceedings of the Third Annual Conference on Urban Planning Information Systems and Programs (Chicago, 1965), p. 42.

8. O. M. Miller and Robert J. Voskuil, "Thematic Map Generalization," The Geographical Review, 54, 1 (1964), 13-19.

9. R. A. Siders, et al., Computer Graphics: A Revolution in Design (New York: American Management Association, 1966).

I. MAPS AND MODELS

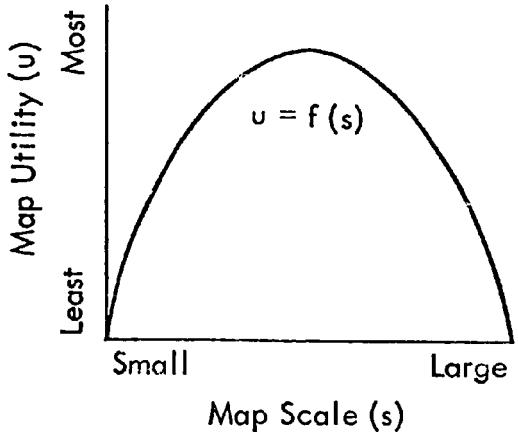
Models play an important role in scholarship.[1] This role derives in large part from their economizing, stimulating and suggestive nature. The highest levels of intellectual achievement often result through successful model building. The abstractive character of models is what gives them their scientific value. The detail and complexity of the real world is partially overcome by creating an idealized (simplified and apparently rational) representation demonstrating only certain of its properties. Although information is lost and "noise" is introduced in the process of symbolism, the abstraction gained allows the model to become more general and useful than reality itself for certain purposes. In particular, the model is generally much easier to manipulate than its referent(s).

There are numerous ways to classify models. In one of the simpler and more illustrative schemes, Ackoff and others suggest a progressively more abstract three-stage breakdown into <u>iconic</u> (represents properties at different scales), <u>analogue</u> (represents one property by another), and <u>symbolic</u> (represents properties by symbols) models.[2] It is recognized that some properties of maps are common to all models.[3] Slight paraphrasing of a section of Pierce's work on communication theory introduces the map-model notion: ". . . many of the most general and powerful discoveries of [the biological, physical and earth sciences] have arisen, not through the study of phenomena as they occur in nature, but, rather, through the study of phenomena [depicted cartographically]. This is because the phenomena [displayed on maps] are simplified and ordered in comparison with those occurring naturally, and it is these simplified phenomena that man understands most easily."[4] The disparity between the flat, schematized map and every day reality leads naturally to a consideration of the merits of generalization versus replication. In fact, the gradient between abstraction and reality is central to any discussion or theory concerning the nature of knowledge. To explore this gradient from a cartographic point of view it may be illustrative to begin with Lewis Carroll:

"'What do you consider the largest map that would be really useful?'
'About six inches to the mile.'
'Only six inches!' exclaimed Mein Herr. 'We very soon got to six yards to the mile. Then we tried a hundred yards to the mile. And then came the grandest idea of all! We actually made a map of the country, on the scale of a mile to the mile!'
'Have you used it much?' I enquired.
'It has never been spread out yet', said Mein Herr: 'The farmers objected: they said it would cover the whole country, and shut out the sunlight! So we now use the country itself, as its own map, and I assure you it does nearly as well.'"[5]

Brambrough, a philosopher, satirizes Mein Herr, saying he could have done even better and made the country do just as well as the map by constructing a full-scale relief map utilizing working models in authentic materials of every detail of the landscape.[6] He succinctly summarizes Carroll's note by concluding: "The ideal limiting case of representation is reduplication, and a duplicate is too true to be useful . . . . And this goes not only for maps, but also for descriptions, pictures, portraits and theories."[7] Polanyi defines this requirement of symbols as the Law of Manageability: "A map is the more accurate the nearer its scale approaches unity, but if it were to reach unity and represent the features of the landscape in their natural size, it would become useless, since it would be as difficult to find one's way on the map as in the region represented by it."[8] This is interesting because it suggests that map utility is a U-shaped function of map scale (see Figure 1.1).

FIGURE 1.1



In fact, we may conclude that an element of schematism or symbolism is common to all representation. ". . . a map is a representation but a highly simplified one. We do not expect to see actual rivers and mountains drawn upon it in order to enable us to read it. A map does reproduce some of the features of the country which it represents, but some others are entirely omitted and all are put down in a symbolic, not literal, fashion. This interplay between the literal and the symbolic is normal in all forms of communication."[9]

As models, maps cannot be both revealing and complete. The detail and complexity of the real world would appear to preclude a completely faithful pictorial portrayal (representation) of it even if this was an ideal.[10] Nature is always transformed, interpreted, distorted in representation.[11] "We cannot know reality; we can only have an abstract picture of it.[12] As was seen above, this is not altogether undesirable. The abstract picture we can have represents a highly simplified and subjective approximation to only selective (but supposedly significant) features and relationships. Distortion in scale, content, form, and relation is acceptable in various proportions in order to help reduce confusion by incidental detail and clarify the fundamental picture,[13] simply because the alternative to a map possessing the distorting effects of abstraction is no map at all (i.e., it can only be the real object).[14] Thus it is absurd to criticize maps on the ground that they have distorted the phenomena represented. From this point of view it is strange and unfortunate that cartographic distortion has been conventionally considered as a distraction rather than an asset. The positive approach is refreshing and would appear more revealing.[15]

Map generalization is a process of evaluation, selection, and emphasis that leads to simplification or elimination of unessential detail.[16] This raises the question of how much abstraction is essential and/or tolerable in a given situation. It is a choice not between perfection and imperfection but rather between essential and non-essential. In this context "ideal" must be defined as essential or what ought to be. Gombrich, who seems to have grasp of this notion, states: "But what matters to us is the correct portrait, like the useful map, is an end product on a long road through schema and correction. It is not a faithful record

of a visual experience but the faithful construction of a relational model."[17] Like a good model, the map suggests ". . . further questions, taking us beyond the phenomena from which it began, and tempts us to formulate hypotheses which turn out to be experimentally fertile."[18]

In a somewhat different perspective on maps as models, Morrison suggests that it may be illustrative to view maps as probability statements.[19] Suppose we draw finite samples from urn I and urn II and use these data in construction of maps I and II. The data set portrayed on each map has a certain probability of arriving from its parent distribution. In practice, however, the composition of either urn is not known and the probability statement is not specified. Intuitively, it seems as though it would be necessary to make a special probability statement within each model framework. For example, the reliability, comparability, and significance of mapped data might be thought of as functions of: (1) the inherent characteristics of the data (scale, discrete-continuous, tangible-intangible, static-dynamic, etc.); (2) the sampling scheme (location and number of sample points or the size and shape of enumeration areas); (3) the observational, sampling, and bias error; (4) the choice of class interval in data collection and presentation; (5) the degree of approximation; (6) the method of interpolation between control points; and so on. In the absence of a probability statement we do not know how well a map replicates its parent distribution(s), nor do we know the magnitude of introduced noise.[20] This is especially critical if maps are to be compared.

Unfortunately, because the map is so neat and sometimes appears so simple as compared to the spatial organization of phenomena in their natural state (i.e., at a 1 to 1 scale with reality), the graphic is relied upon more and more, often to the point where the symbol is taken for the then forgotten reality. In other words, ". . . just as superstitious people may think of the statue of a saint as the saint himself, we think of the symbol as that for which it stands."[21] Geographers and non-geographers alike have frequently treated the map as visually equivalent to geographic reality. In consequence of this traditional reliance on the map as a "given", it has evolved almost as an article of faith.

Some of its users, particularly in geography, have often forgotten that it is but a model -- one of many possible models. The problem arises when the model (map) is misinterpreted as the reality -- that is, when the disparity between "model world" and "real world" goes unrecognized. Map users accustomed to thinking of a map as a scaled replica of the original region are prone to overlook the fact that maps are more properly viewed as generalizations created from selective information that is available concerning the original situation.[22]

The above discussion sheds some light on the danger of employing a single or several maps as the sole test for accepting or rejecting a hypothesis. It is likewise apparent that "blanket" rejection of the cartographic technique -- as some researchers, namely quantifiers, have done -- because a single or several graphic versions of a distribution are judged to be inadequate, is probably unjustified. In order to appreciate fully the ramifications of this discussion of maps as models it is necessary to consider briefly the role that graphics play in scientific argument. The distinction between graphics and models is of course not mutually exclusive.

I. MAPS AND MODELS

1. Richard Chorley and Peter Haggett, eds., Models in Geography (London: Methuen & Co., Ltd., 1967).

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3. C. Board, "Maps as Models," in Models in Geography, ed. by R. Chorley and Peter Haggett (London: Methuen, 1967), pp. 671-725.

4. John R. Pierce, Symbols, Signals and Noise: The Nature and Process of Communication, Harper Torchbooks (New York: Harper & Row, Publishers, 1961), p. 19.

5. Lewis Carroll [C. Dodgson], Sylvie and Bruno Concluded (London: Macmillan, 1893), p. 169.

6. Renford Brambrough, "Principia Metaphysica," Philosophy, 39 (1964), p. 97.

7. Brambrough, "Principia Melaphysica," p. 98.

8. Michael Polanyi, Personal Knowledge: Towards a Post-Critical Philosophy (London: Routledge & Kegan Paul, 1958), p. 81.

9. George Boas and Harold Holmes Wrenn, What is a Picture? (New York: Schocken Books, 1966), p. 67.

10. Ernst H. Gombrich, Art and Illusion: A Study in the Psychology of Pictorial Representation (New York: Pantheon Books, 1962), p. 78.

11. John Hospers, Meaning and Truth in the Arts (Chapel Hill: University of North Carolina Press, 1946), p. 20.

12. Leslie Curry, "Climatic Change as a Random Series," Annals of the Association of American Geographers, 52 (1962), 21.

13. Presumably maps, as models, preserve some important characteristics of the original or else they (maps) would not be recognizable. If only the preserved aspect of the model is used there is presumably least distortion. There can be different models of the same referent object, each preserving different characteristics of the original (e.g., witness the numerous

equally "correct" map projections). The point is that the focus of this thesis (on statistical maps) relates to some quite different characteristics of the referent than those of the topographer.

14. Stephen Toulmin, The Philosophy of Science (New York: Harper & Row, 1960), p. 127.

15. There are several advocates of this point of view. For example, see Waldo R. Tobler, "Geographical Area and Map Projections," The Geographical Review, 53 (1963), 59-78.

16. Miller and Voskuil, "Thematic-Map Generalization," p. 13.

17. Gombrich, Art and Illusion, p. 78.

18. Toulmin, The Philosophy of Science, p. 38.

19. Joel L. Morrison, personal communication, Ann Arbor, 1968.

20. Some research is being done on this aspect of cartographic processing. See especially George Jenks, "The Data Model Concept in Statistical Mapping," A paper presented at the third International Conference on Cartography of the International Cartographic Association, Amsterdam, 1967, 10 p. (Mimeographed.)

21. Boas and Wrenn, What is a Picture?, p. 67.

22. James E. Esler and Floyd W. Preston, Fortran IV Program for the GE 625 to Compute the Power Spectrum of Geological Surfaces, Computer Contribution No. 16 (Lawrence: State Geological Survey, University of Kansas, 1967), p. 2.

II. MAPS AND GRAPHICS

Graphics play an important role in scientific research. "By far the most frequent visualizations of physical happenings are representations in terms of spatial relations that completely replace direct pictures."[1] Scientific illustrations running all the way from the most meticulous literal reproductions or pictures of objects to highly schematized diagramatic devices are common.[2] Visualization is made possible by arbitrary graphical concepts, called symbols, which are operationally defined to stand for or signify particular natural phenomena.[3] Indirect readings -- that is, readings from these representational forms -- take the place of genuine observation. For, in the long run, two-dimensional representations often appear to be the best way we have of understanding and communicating to others the spatial character of things. "The sense-data on which the propositions of modern science rest are, for the most part, little photographic spots and blurs, or inky curved lines on paper."[4] Scientific premises derived from interpretation of these representations of "physical fact" are subject to two types of errors, however. Not all features of symbols correspond to characteristics of their referents. For example, maps which have colors may actually be only representative by the sizes and shapes depicted. The failure to make this distinction between the endogenous and exogenous properties of symbols has been called the error of "map reading."[5] A second problem arising in symbolization is the frequent inability to read from the symbol all that has gone into it. Leaving out of a symbol reading something which should be in it could be called the error of oversimplification. This is commonly referred to as visual "smoothing," "filtering," or "generalization." Both of the above mentioned map reading errors are identified with the T_3 phase of the cartographic process. In particular, either the map reader may add information (I) to that used in map compilation (i.e., $(I)T_3 > (I)T_2^{-1}$), or the map reader may fail to extract as much infor-

mation from a map as was used in its compilation (i.e., $(I)T_3 < (I)T_2^{-1}$). It is most probable that $T_3 \neq T_2^{-1}$. (The above notation was defined on pages 3 and 4 of the Introduction). The point is that the sensory qualities of map symbols are extremely important to their interpretation.

Maps constitute a special case of graphics. This realization is illustrative. Spatial reality is reduced to a skeletal form in the flat map image. Therefore, the map is an instrument of illusion.[6] Lines, points, areas, and so on, are intended to transcend their physical identity to become symbols. These symbols are not unique and specific to particular phenomena, but rather are explicitly defined from map to map situation.[7] The coordination is arbitrary. Just as a number of phenomena can be mapped in a single graph, a single phenomenon can be mapped in a large number of graphics. The choice of phenomena to be coordinated to specific symbols is left to the cartographer.[8] The concern of the "cartographic researcher" should be the possibility of symbolizing mappable phenomena and their relationships in the most effective manner. Thus, development in cartographic processing should in large part be the history of the gradual accumulation and elaboration of an appropriate set of graphic symbols. But a given map can only employ a limited number of symbols from this set. If the cartographer contributes a particular skill to the cartographic system then the mental selection of ensembles of map symbols would be expected to differ significantly from the probability that the same series of symbols was selected by chance. In any case, interpretation of these symbols forms the basis of map reading. We must consider a double reality. The map (symbol) itself is a physical object seen as such, but it should also evoke quite other objects, namely, the phenomena symbolized. It is the cartographer's task to make us reject the first reality, while conveying the second, so that we see the referents of symbols and not mere points, lines, or patches of color on a flat surface.[9] That is, if you shift your attention from the meaning of the cartographic symbols to the symbols as objects viewed in themselves, you destroy their meaning.[10]

The problem of graphical representation has seldom been recognized in its full scope.[11] In spite of the widespread use of graphical representation in scientific literature, the role and potential of graphics in scientific argument is vaguely defined and poorly understood. As

Tobler states: ". . . very little of substance is known scientifically concerning visual information processing, except that it seems effective."[12] Because graphical capability remains largely unexplored, researchers have fallen far short of realizing the full power of graphics in communicating information.[13] Yet it is apparent that humans rely heavily on the visual sense. Most of our thinking is done in terms of visual models.[14] "Sheer observation is hard to contradict, for sense data have an inalienable semblance of fact."[15] The prevalence of platitudes dealing with sight (e.g., "seeing is believing," "The eyes of the Lord preserve knowledge and he over-throweth the words of the transgressor (Proverbs 22:12)," "a picture is worth a thousand words," etc.) are ample testimony to its power in our lives. Multi-million dollar advertising schemes take full advantage of this trust in our eyes.

Why is visualization so important? Reichenbach conjectures that graphical representation is more than a matter of convenience and that it rests on a basic necessity of human thinking. People cannot think abstractly about relations without some method of symbolic representation which supplies a concrete model.[16] Whatever the explanation, visual observation of actual things or graphical representations of them is widely thought to yield to the brain a true, vivid, and lasting impression. "People have found that visualized information is faster, more direct, more dramatic, and easier to remember."[17] Or, as Reichenbach asserts, the reason every physical or technical text is filled with drawings or curves is that the clarifying effect of graphical representation enables us to understand the most complicated phenomena.[18]

Map display, as compared with the corresponding tabular, textual or functional presentation, makes use of asserted visual attributes to extract facts from figures. Since the spatial character of alphabetically or magnitudinally arranged geographic data is largely obscured, the purpose of statistical mapping would seem to be to provide better comprehension of spatial series data and more penetrating analysis than is otherwise possible. Indeed, maps can often show immediately information that is not easily deduced from ordinary tables and thus may reveal functional relationships quite unsuspected from our knowledge of the

original figures.[19] A belief in this utility of maps is well documented in the literature. The consensus is that "maps are often indispensible in locating problems, testing hypotheses, analyzing data, and discovering hidden facts and relationships."[20] Cartographic presentation is asserted in a greater or lesser degree to enhance pattern comparability, clarity, speed of interpretation, effect or impact, image permanence, flexibility with respect to data selection and apportionment of emphasis, and detection of errors and omissions in statistical compilations and reckonings.

It is likewise recognized (though generally not by those who make assertions in the above mentioned vein) that the map exhibits several equally definite limitations, as compared with the corresponding text, formulas or tables. In fact, it would be quite hard to agree with more than a few of the above assertions without reservations. Dogmatic statements of the type so characteristic of the literature are in most cases unproven. The principal disadvantages of maps are: (a) that the data may not be read from the map with such fine distinction, and (b) that the map is generally more costly to produce and reproduce. The author suggests three broad explanations for the inability to derive precision information from map display: (a) the subjective nature of map compilation, (b) the inherent characteristics of graphic portrayal, and (c) the subjective nature of visual analysis (see next section).[21]

The subjective nature of map compilation was discussed previously in the context of model building. The conclusion was that one can construct innumerable models of reality, each possessing a substantial amount of artificial noise. Further introduced noise occurs in choosing the type of graphic representation from the many techniques and types applicable in any given instance. The cartographer can present the map interpreter with only one of a multitude of different graphic versions of a mathematical model or statistical distribution.[22] Each version produces a somewhat different pattern effect and modifies the visual impact created by the map accordingly. In addition, the limitations imposed by the media and tools of graphics and methods of reproduction considerably influence the level of precision attainable.[23]

For the above mentioned reasons, a map is never a substitute for actual figures if detailed accuracy is required, and should not be expected to display the accuracy of tabular data.[24] One of the map's functions is to stimulate the desire to go to the tabular data for more complete information once a summary view is obtained. "Accuracy in graphics is more a matter of portraying a clear, reliable picture than reiterating exact values."[25] Butsch provides a clear statement of this notion: "In many instances. . . it is more important that the reader see clearly the contrast between two or more quantities, or recognize the trend of the information, than that he know precisely, to last decimal place, how large a particular difference may be."[26] These advantages and limitations of map display are put into proper perspective by way of analogy to the slide rule. "Charts [maps] have a great deal in common with slide rules. Both are suited for providing quick answers accurate for many purposes, yet neither will provide the exact answer in larger figures which is sometimes needed."[27]

There are statements in the geographical literature which say the opposite -- which goes to show that in some cases the role of graphics is poorly understood by those most involved with their use. Maps should not ordinarily displace tables, text, or functions as a means of communication, but instead should supplement them -- each filling a particular need. Again, a look at the several geographical and geological journals indicates that this policy is not rigorously followed. On the contrary, preoccupation with the accuracy of mapped data is common.[28] In deciding whether or not to present a statistical map in a given situation, it seems that the researcher should follow two guidelines: (a) if the analysis deals with general values, trends, and relationships, present the map; and (b) if the analysis deals with precise quantities, omit the map.[29] This conclusion is in general agreement with the contentions of the "quantifiers" mentioned in an earlier section.

Presumably the next steps are to evaluate the assertions regarding map utility considered in this chapter (a) in terms of their relevance vis a vis the discussion of models and maps, or (b) to decide which assertions can and need be tested scientifically. Available evidence tends to suggest that there are serious limitations on the ability to read data from maps. It is unfortunate that most of this research does not include statistical maps in addition to topographic maps.[30]

II. LIST OF REFERENCES

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(3) Refer to the section on "Representation" (Chapter II) in John Hospers, Meaning and Truth in the Arts (Chapel Hill: University of North Carolina Press, 1946), pp. 29-61.

(4) Susanne K. Langer, Philosophy in a New Key: A Study in the Symbolism of Reason, Rite, and Art, Mentor Books (2nd ed.; New York: The New American Library, 1951), p. 29.

(5) Abraham Kaplan, The Conduct of Inquiry: Methodology for Behavioral Science (San Francisco: Chandler Publishing Company, 1964), pp. 284-285.

(6) Edward Hill, The Language of Drawing, A Spectrum Book (Englewood Cliffs, N. J.: Prentice Hall, Inc., 1966), p. 43.

(7) Hospers (Meaning and Truth in the Arts, p. 29) is referring to this type of representation when he states: "Perhaps the most common type of symbol is that which is generally called a conventional or arbitrary symbol, which stands for its referent only by a common convention resulting from an explicit stipulation of the meaning to be attached to it." The map key or legend is used to define these symbols in uses where their meaning is not obvious through cartographic convention.

(8) There are, of course, certain symbols called pictographs which actually are designed to look somewhat like the thing they stand for. In this special case, coordination is not arbitrary.

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(12) Waldo Tobler, review of Semiologie Graphique: Les Diagrammes - Les Reseau - Les Cartes, by Jacques Bertin, in The American Statistical Association Journal, March 1969, pp. 391-392.

(13) Jeremy Anderson, "Putting the Graph Back in Geography: The Role of Graphs in Geographic Analysis," The Professional Geographer, XXI, 1 (1969), 23-27.

(14) Marshall McLuhan, Understanding Media: The Extensions of Man, Signet Book (New York: The New American Library, Inc., 1964)

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(16) Reichenbach, The Philosophy of Space and Time, p. 107.

(17) Rudolf Modley and Dyno Lowenstein, Pictographs and Graphs: How to Make and Use Them (New York: Harper & Brothers, 1952), p. 3.

(18) Reichenbach, The Philosophy of Space and Time, pp. 101-102.

(19) Polanyi, Personal Knowledge, p. 83.

(20) Calvin F. Schmid, Handbook of Graphic Presentation (New York: The Ronald Press, 1954), p. 184.

(21) It may be important to point out that: (a) many federal topographic mapping agencies consider that the only feasible way of making all of the data available is in graphic form, and measurements can be made on the topographic maps; and (b) some people refer to the data, not the graphic, as the map.

(22) George Jenks, "Generalization and Statistical Mapping," Annals of the Association of American Geographers, 53 (1963), p. 26.

(23) For an extended review of research on cartometric measurements see D. H. Maling, "How Long is a Piece of String?", The Cartographic Journal, 5, 2 (1968), 147-156.

(24) Miller and Voskuil, "Thematic-Map Generalization," pp. 13-19.

(25) Anna C. Rogers, Graphic Charts Handbook (Washington: Public Affairs Press, 1961), p. 10.

(26) Russell L. C. Butsch, How to Read Statistics (Milwaukee: The Bruce Publishing Company, 1946), p. 29.

(27) Rogers, Graphic Charts Handbook, p. 6.

(28) What is unfortunate is that reliance on map measurements has reached the proportion that several researchers have felt the need to clarify the subject through extended testing. See Hans Boesch and Haruko Kishimoto, Accuracy of Cartometric Data (Zurich: Geographisches Institut Der Universitat Zurich, 1966); A. I. Shabanova, "On the Accuracy of Small-Scale Maps," Geodesiya i Kartografiya, 10 (1957), 36-44; D. H. Maling, "How Long is a Piece of String?", The Cartographic Journal, 5, 2 (1968), 147-156.

(29) L. Edwin Smart and Sam Arnold, Practical Rules for Graphic Presentation (Columbus: Bureau of Business Research, Ohio State University, 1947), p. 3.

(30) Refer to the references cited in footnote 28 above.

III. THE SCIENTIFIC USE OF MAPS

(A) Maps in Research

Since maps are not probability statements[1] they are less than adequate for rigorous hypothesis testing, but map portrayal preserves the spatial pattern of a set of data and allows communication of this pattern at a glance. This trait is useful in the theory-formulation and model-refinement stages of research on spatial structure or spatial process. For example, it is widely accepted that some form of qualitative investigation naturally precedes quantitative analysis. Greig-Smith suggests that ". . . in examining a particular problem it is essential to grasp the broad aspects before starting quantitative investigations. . . . quantitative observations should not be made without a definite objective, . . ."[2] Lack of precision and rigor is less important in this case. The broader picture that the map affords often suffices. Thus, the potential of maps in speculative (inductive) hypothesis-generation is asserted to exceed their utility as deductive, model-testing devices.[3] "It would appear that while map comparison is an excellent tool for suggesting possible solutions to a geographical problem, it has a low degree of utility in determining or demonstrating the sufficiency of these solutions."[4] McCarty has suggested a research system involving hypothesis formulation, testing, and reformulation in which a balanced integration of statistical techniques and the statistical map is achieved.[5]

A difficulty in using maps in research arises from the fact that a single map is just one of many possible graphic interpretations of descriptions of a spatial data set. Each version yields a special pattern which in turn modifies the visual impact the map evokes.[6] This could be valuable; science thrives on an overabundance of hypothesis. But the complementary nature of maps as models must be recognized.[7] Unfortunately, little research has been devoted to a consideration of the capabilities of the various cartographic expressions in terms of the flow or exchange of information.[8] The general form of this input-output relationship might be written in symbolic notation as $R = f(S)$,

where the response-dependent variable, R, is a function of the stimulus-independent variable, S.[9] Following somewhat Johnson's discussion of the "stimulus pattern - perceptual pattern" disparity, information retrieval (R) is asserted to be a function of the stimulus information (S) multiplied by some empirically derived constant (k) plus and/or minus information distortion (N) introduced by both the human system and the cartographic process.[10] Written in functional notation as $R = f(k \cdot S \pm N)$, this essentially means that map readers deal with fallible information in making inferences from maps.

In the context of information theory, the problem, then, is what sort of graphic (signal) to use (send) so as best to convey messages of a given type over the particular sort of noisy circuit known as the cartographic system.[11] This problem might be visualized as the consequences of encoding.

It seems intuitive that the efficiency of transmitting information cartographically would increase as the display became more abstract and less complex, and that map comparison performance should increase accordingly. The problem is to find the proper balance between planimetric and magnitudinal accuracy on one hand, and conceptual simplicity on the other. The relationship can be experimentally studied by varying the stimulus class (i.e., map graphics) and observing subjects' perceptual responses. This procedure is consistent with the notion that ". . . the map is a graphic presentation of spatial knowledge and its evaluation will depend upon the effectiveness of transmitting this knowledge."[12]

Morrison suggests that both the quality and the quantity of information retrieved from map display would appear to be subject to probability analysis.[13] After extensive differential threshold (i.e., just noticeable difference) testing, the cartographer should be able to say that, given a pair of maps of specified distributional and graphic characteristics, at a specified level of significance at least X percent of map readers will perceive the degree of association within $\pm$ Y of the calculated measure of correlation, all things being equal. From a statistical point of view the actual measure of the association may not be as important as its probability of being significant. The idea is at least suggestive.

On the other hand, at the present time map users generally have little notion how accurate the answers provided by maps need to be. Considering the level of sophistication of contemporary theory, it may be that qualitative visual judgments suffice and will do so for some time! In the meantime the need for extensive research is evident.

A second useful function of maps in research is their role as so-called spatial worksheets. Although data coded with locational coordinates are most flexible for spatial analyses, in many instances data are only available in tabular form. In these cases researchers have several options. First, data can be laboriously coded with locational coordinates and then subjected to data processing. This coding operation usually involves either: (a) looking up coordinates of data units on the appropriate map; or (b) plotting data on a map and then, as another step, recording coordinates of each data unit. As an alternative, volume of data or expediency may suggest that data be grouped into areal subdivisions. In this case tabular data are typically plotted on a map, summarized through an areal aggregating or cell counting procedure, and then subjected to additional data processing. Although in both procedures (i.e., recording coordinates and cell counts) maps serve an intermediate step in data analyses, they are not analyzed directly. Since the worksheet function differs significantly from the active part maps play in hypothesis generation or model refinement, this use of maps will not be considered further.

(B) Pattern Comparison Situations

This paper focuses on visual analysis of pattern association in the context of cartography. Two map comparison situations come to mind. (1) Map(s) 1 . . . n can be compared with a referent (anchor stimulus) in search of a best fit. The inductive use of maps in theory-formulation seems to involve this technique of searching for association. The task is to search for "most similar" distributions. A rank-ordering results; that is, measurement is on an ordinal scale. (2) Map A may be compared with map B and the degree of fit or association estimated or calculated, yielding some coefficient of association between the two distributions. The deductive use of maps in model testing seems to involve this technique of estimating the degree of association. The task

is to ascertain how much of the variation in the first distribution can be explained by the second. Measurement is at least on an interval scale.

Each of these two hypothetical pattern comparison situations can be generalized to include at least three quite different task situations.

(1) Observed versus reality -- The map may be tested against reality or vice versa. The process of investigation may start with a map-generated hunch or speculation and lead to reality. Scale discrepancy varies with the point of perspective. It is greatest with persons on the ground, least with persons in space. But the map scale is always small compared to what is visually perceived on the earth's surface. Field research and verification of the nature common to the environmental sciences are typical examples of operations falling into this category. The problem is analogous to comparing "typical" textbook examples of a wide range of phenomena with observations of the actual thing. A "perfect" match would be extremely rare due to the generalized nature of the example and the natural variations of the observed case. Traditional map evaluation is included in this task situation, especially in the military intelligence sense, since it is generally based upon empirical measures of performance on such map-reading tasks as position location, direction location, road map use and object identification.[14]

The map-reality comparison has yet to be formalized, largely because it appears to be extremely difficult, if not impossible, to develop objective statistical tests of the relationship between the model and that selective (spatial and topical) portion of the real world under consideration. If the way people who use other types of models handle this situation can provide insight, then a potential partial solution would be to test the map against less abstract models, such as aerial photographs or orthophoto maps.

(2) Observed-vs-Observed -- Comparison (cross-correlation) is often made between several maps. Several variations exist. (a) Maps covering the same geographic area representing two or more sections in time may be compared in search of similarities, change or trends. This technique has been utilized by Imhof[15] and Tobler[16] to estimate the rate of cartographic

progress in terms of the history of positional accuracy on maps. Similarly, it is also possible to use regression analysis of stated positions on maps to help trace the ancestry and projection (where unspecified) of a given map. More commonly, the technique is used to identify change in the cultural or physical landscape over time, or to establish growth trends that can be used in historical interpolation or prediction. (b) Maps covering different geographical areas may be compared with respect to one or more phenomena in search of similarities between the distributional geometries of disparate regions. (c) Maps of the same geographic area depicting two or more variables may be compared in search of areal correspondence or similarities in distribution geometries of diverse phenomena, and this association can be treated over time.

(3) Observed-vs-Theoretical -- It is common to compare maps of observed areal distributions with idealized distributions with which they are theorized to have a high degree of correspondence. The use of the normal probability surface for description, or analysis of a wide variety of geographic distributions characterized by an exponential decay function are two examples.[17]

In a somewhat different procedure, the map of an observed phenomenon may be compared (tested) against a theoretically derived pattern (model) of the same phenomenon. Comparison between simulated and empirical data in map form would generally be employed in an attempt to test the validity of the theory; but could be employed to test the degree of real world deviation from optimal theory.

The general technique was suggested above in the case where the map distribution is compared with a range of theoretical distributions generated by various spatial processes, e.g., stochastic, regular, or contagious. In the extensive literature on the simulation of spatial distributions the theoretical models are invariably compared with the appropriate empirical distributions in an attempt to varify and modify the theory.[18]

A somewhat different procedure than the above involves comparing a map of an observed phenomenon exposed to a subject at time t with the subject's mental recreation of the map at $t + 1$, $t + 2$, . . ., $t + n$. This technique could be used in an investigation of mental pattern generalization processes. It is speculated that such research would prove

a revealing aid towards solution of the important problems of abstraction and generalization in map design.

Another application of model-map testing is found in studies of a group's functional interpretation (distortion) of the psychological image of their physical environment. Many examples of the discongruity between geographical image and reality may be cited. In several instances maps have been constructed which caricature the geographic perspective of people in different regions. In any case, since attitudes and decisions are based upon psychological conceptions (often based on erroneous notions) of geographical reality, discongruity between this image and environmental reality is a valid topic for research. The problem is to test the degree of similarity or difference between the map recreation of a group's (defined by region or social class) functional space and a more conventional (but less relevant) Euclidean description of it. It may be further possible and desirable to group these "mental maps" into clusters asserted to represent different types of individuals within a given region or cultural complex.[19]

In conclusion, comparison of a generated, theoretical pattern with a map of the actual distribution introduces one more potential source of artificial noise into the cartographic system. Neither the map pattern nor the theoretical pattern portrays the only "real" one. A statement of probability for the theoretical distribution would appear to be even harder to come by than is that for the map itself.

(C) Visual Pattern Comparison

We see then that the search for association between several distributions is a common map-reading task of the "fishing expedition" type of science.[20] Visual analysis of pattern association constitutes the traditional method. In addition to random and systematic noise introduced into the cartographic system by the map maker through his available tools and materials, the map user, by virtue of inherent or acquired limitations, exhibits an inability to read the depiction without further distorting the flow of information.

The accessibility of cartographically displayed information has not yet been measured satisfactorily. Several variables are involved. First, maps and other symbols are intended to communicate something.

They are to be read and not merely stared at.[21] Even if maps could be made in forms which best induced an understanding of their messages, comprehension of these messages would still always rest on the ability of the map reader.[22] In part because maps are extremely complicated symbols, one has to learn how to read them. Unfortunately, instruction in map interpretation too often begins and ends with training the interpreter in awareness of the key (legend). The truth is that little is known of proper (i.e., most efficient) map-reading technique. The result is probably a wide range in map interpretation abilities even with so-called trained professionals.

Secondly, the capacity of the map reader as a data processing system is finite and asserted to be relatively small.[23] An overload of information input in the form of mapped data could easily strain this limited capacity and lead to poorer quality retrieval. At this time we know neither the human capability nor how to measure "bits" of information displayed on a map. We may assume, however, that if the amount of data that can be transmitted effectively at a glance is finite and small, then the logical goal in cartographic design is to maximize the signal-to-noise ratio. The problem of efficient encoding, then, is how to treat a signal plus noise so as to get a best estimate of the signal.[24] One approach is to eliminate as much distracting detail (high frequency components)[25] as possible, as if the mind's eye does in fact function as a high-pass filter.[26] It seems as though, as far as the map reader is concerned, mapped data that are not used or are not even recognized as information, for most practical purposes do not even exist. If the map reader consistently "filters out" certain types of graphically presented information, then there would seem to be little reason for its inclusion in the first place. In the same vein, if it is found that certain graphic forms are consistently over-weighted by the map user, then in the interest of maximizing the signal-to-noise ratio these features might better be avoided. Unless, that is, such graphics can be employed to positive ends.

Thirdly, the map reader views a map with a high degree of personal bias and is easily influenced by the character of the display and the nature of the subject. Even the most accurate maps in terms of positional or magnitudinal variables may evoke "impressions" as well as reveal en-

coded information. Although this ability to evoke impressions may be useful, it can also introduce undesirable noise. The map is thus a visual as well as a factual implement and must be treated as a "psychological force" as well as a form of scientific illustration and analysis. Special purpose cartography is largely founded, albeit implicitly, upon this notion. It is a simple task to create maps to support theories impressionistically. The type, size, color, design, and positioning of cartographic symbols can all be used to advantage. The range of equally "correct" projections provides an additional alternative. Propaganda maps--that is, maps intended to convince--provide an extreme example of special purpose cartography.[27] In this case an explicit attempt is made to create a desired impression, even if this involves deception. Propaganda mapping, particularly from the point of view of successful advertising, is important enough to warrant further attention. In particular, a better understanding may be gained of the impact or impression on the reader created by different symbols, design features, and so forth. Information of this nature would seem to be of value in the design of more effective maps for presentation purposes. Again, a look at maps in advertising may prove revealing.

And, lastly, visual pattern comparison yields subjective assessments, regardless of the power of the perception. These subjective conclusions (descriptive terms) are generally of the nature "highly correlated," "poor association," and so on rather than "I am stimulated to go to the tabular data." It is apparent that these estimates are neither precise nor consistent. In spite of the ease of use and the speed of visual estimates, many situations demand more sensitive and/or objective (repeatable) measures for describing the degree of association between spatial distributions. By turning to quantitative techniques, researchers claim to gain the objectivity and comparability lacking in subjective visual assessments. More often than not, however, the derivation and use of elegant quantitative descriptions of spatial associations has posed far less a problem than has the difficulty in interpreting the results.[28]

The conclusion to be drawn from the discussion in Chapter III is that "eye ball" methods of map comparison are probably unsatisfactory for certain purposes. This is to be expected of a "tool" designed for a specific task and is no real cause for alarm. The map cannot be criticized

for its misuse on the part of the unaware researcher. What is needed is a clear and definite statement of the utility and limitations of maps in scientific argument. Knowing which numerical measures of association most suitably replicate the visual notion of pattern association would allow the researcher to roughly interpret his intuitive judgment in light of a quantitative model.

In the absence of extensive research into the various aspects of visual pattern comparison, important questions remain unanswered. Has the fundamental dependence on visual comparison of maps been an efficient and sound scientific technique? Can the intuitive, qualitative processes generating percepts in the experimental observer in a pattern comparison situation be put into quantitative notation? Would psycho-physical testing of visual map evaluation yield results that were meaningful and applicable in setting future cartographic standards? Quantitative techniques have been advanced as being more efficient and objective, but pose formidable problems in interpretation. When are what specific techniques applicable? When are the added costs in time and labor associated with their use warranted? In what situations, if any, is visual analysis sufficient or actually more powerful? And finally, using the above information, can a set of rejection levels be established to guide the researcher in his decision of whether or not to abandon the visual technique for more objective quantitative measures in a given situation? The next section is intended to outline one possible approach toward gathering the sorts of information required to answer some of these questions.

III. LIST OF REFERENCES

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18. David Harvey, "Geographical Processes and the Analysis of Point Patterns," Transactions and Papers of the British Institute of Geographers, 40 (1966), 81-95.

19. Peter Gould, On Mental Maps, Michigan Inter-University Community of Mathematical Geographers, No. 9 (1966); and David Stea, "On the Measurement of "Mental Maps"," Department of Psychology, Clark University (Worcester, 1968). (Typescript.)

20. John P. Cole and Cuchlaine A. M. King, Quantitative Geography: Techniques and Theories in Geography (London: John Wiley & Sons, 1968), pp. 227-335.

21. Making measurements on maps is purposefully excluded from consideration in the belief that it should at most play a minor role and therefore should be de-emphasized in retrieving information from (decoding) statistical maps.

22. Michael Polanyi, The Study of Man, pp. 21-22.

23. James Miller, "The Individual as an Information Processing System," in Information Storage and Neutral Control, ed. by William Fields and Walter Abbott (Springfield, Illinois: Charles Thomas, Publisher, 1963).

24. Pierce, Symbols, Signals and Noise, pp. 19-44.

25. Detail is of course not necessarily noise. It could be fine grained information that would become the signal at another level of analysis. This sort of detail is to be distinguished from the random or systematic noise generated in cartographic processing.

26. J. Leith Halloway, "Smoothing and Filtering of Time Series and Space Fields," Advances in Geophysics, 4 (1958), p. 386.

27. Louis Quam, "Maps in Propaganda," Journal of Geography, 42 (1943), 21-32; and Hans Spier, "Magic Geography," Social Research, 8 (1941), 310-330.

28. Leslie Curry, "Quantative Geography, 1967," The Canadian Geographer, 11 (1967), 265-279.

IV. EXAMPLE EXPERIMENT

(A) Problem

Man's perceptual information-processing mechanism is intricate and inaccessible. "What little is already known about its capabilities. . . appears to rest largely upon inferences from observed correlations between externally imposed inputs (or stimuli) and the ensuing outputs (or responses)."[1] The evaluation of human capabilities of visual pattern comparison is one example of this sort of research. An appropriate experimental procedure is available and well documented in the literature of intuitive statistics. In particular, ". . . a number of investigators have used formal statistical theory as a point of reference for the study of human inferences. For many uncertain situations, statistical theory provides models for making optimal inferences. The psychological research consists of examining the relation between inferences made by man and corresponding optimal inferences as would be made by "statistical man".[2] Thus, numerical techniques for detecting non-randomness between distributions provide a convenient standard against which visual estimates might be evaluated. In spite of the obvious application of this technique as a means to further understanding visual map comparison, the dearth of references on the topic is outstanding.

A simple experimental procedure is to test if subjects are capable of judging whether map-like patterns are identical, similar, or otherwise related. Psychologists have explored this concept in research on "visual sameness."[3] Decision time is the common standard of evaluation. A complex but more interesting procedure is to test the ability of subjects to discriminate the degree of "sameness" between map-like displays. The classic work in the field of map comparison is the monograph by McCarty and Salisbury.[4] Groups of subjects (Ss) with different levels of "training" were given a series of tests in which map images

were displayed in flash card fashion or projected from Kodachrome slides. Subjects were asked to (a) estimate the degree of similarity between pairs of isopleth maps in terms of a simple product-moment correlation coefficient (r), or (b) select the proper map (e.g., most or least correlated) from several choices. Results of their experiment tend to indicate that an optimum combination of variables for purposes of map comparison include: few classes or intervals (quartile or quintile), low order surface complexity, gradient system of color (either gray-scale or chromatic), high positive correlations, subjects with considerable geographic training, and selection of most or least correlated maps from several choices. McCarty and Salisbury suggested that ". . . a principal defect in visual comparison lies in respondent's inabilities to estimate differences accurately as well as in their inabilities to detect the direction of the associations that are involved."[5] They conclude by stating:

> In brief, the findings do not support the contention that visual comparisons of isopleth maps provide an effective means of determining or demonstrating the degree of association that exists between sets of spatially distributed phenomena. Only in cases in which the degree of association is very high does the process produce results which approach the standards of accuracy generally demanded in present-day research and teaching. . . .
>
> On the other hand, visual methods produce generally satisfactory results, even among untrained students, when the sole object is to select pairs of maps whose distributions are most highly correlated when several alternative choices are available; and in this role of guiding the progress of a geographic investigation by suggesting new independent variables, map comparison seems to perform effectively.[6]

In review of the McCarty-Salisbury project, it should be mentioned that there is little if any intuitive reason to use r as a statistical model against which to evaluate visual pattern comparison. On the contrary, r is an abstract concept measuring the degree of association on a scale ranging between $\pm$ 1. In order to obtain the percentage or amount of variance in one distribution explained by a second, r must be squared. The relationship between r and r^2 is shown in Figure 4.1.

Study of Figure 4.1 reveals one reason why it is so difficult for subjects to estimate r in the zone of medium correlation. Regardless

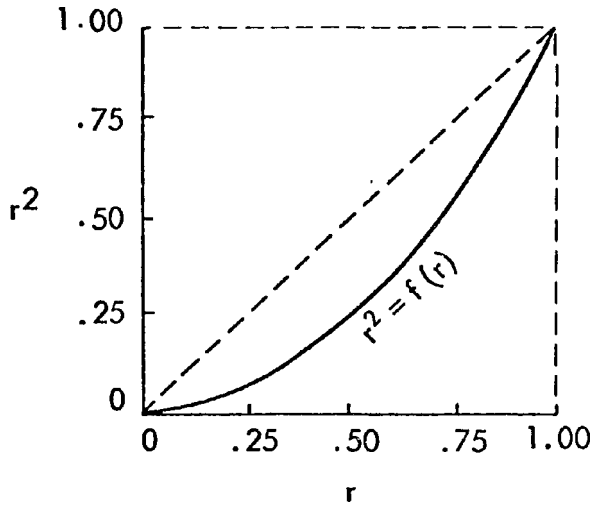


FIGURE 4.1

of the relation between r and r^2, requiring subjects to estimate a numerical coefficient of correlation of any kind appears to be unrealistic. For it is highly unlikely that this procedure is followed in actual map comparison situations.

In spite of its lack of conceptual or methodological sophistication, Dent's work with the census tracts of Canberra, Australia, is interesting.[7] He used five maps, each divided into four classes, and colored in a gradient brown sequence. Both "geographers" (13) and "non-geographers" (11) were asked to estimate the simple product-moment coefficients of correlation, r, between variables from the map patterns by indicating their relative positions on a line scaled from +1 to -1. Comparing results over 10 trials, each with the relevant correlation matrix, it was concluded that the difference in performance (accuracy) between the two groups was not significant. Dent's research was undoubtedly biased by the use of variable sized statistical units (census tracts), an arbitrary color scheme, not unlike that found on many maps in use, and small samples. Regardless of these criticisms, however, his observation that subjects made more and larger errors with negatively correlated items is consistent with the findings of McCarty and Salisbury.

The works of Dent and McCarty-Salisbury dealt directly with perceived correlations from maps versus Pearsonian product-moment correlations between data sets employed in map compilation. Olson reasoned that original data are considerably transformed or distorted in the process of map generalization.[8] Therefore, she dealt with perceived correlations from choropleth maps versus rank-correlations (Kendall's tau) measured between class intervals on the same maps. Comparisons

were made on distances between stimuli as all possible combinations of three maps were displayed in flash card fashion. Stimuli A, B, and C were presented and the subjects asked to judge whether B or C were more like A. Taking each variable by itself, at the 5 percent level, one-way analysis of variance showed that: (1) training produced no significant difference in results; (2) a significant increase in errors resulted from increasing map complexity; (3) there was a significant increase in errors with decreasing differences in correlation (Δ tau) between map choices; and (4) there was a significant increase in errors with decreasing correlation between map choices. Discrimination between map choices was most difficult with medium correlations (tau = 0.4, 0.6). Olson also concluded that map pattern must have influenced results in some way not measured by the test procedure.

The high level of agreement between Olson's findings and previous work suggests that distortions resulting from cartographic processing were limited in the case of the choropleth maps utilized. At least distortion was distributed so as not to upset the probable monotonic relationship between Kendall's tau and Pearson's r, when measured between stimuli maps and original data sets, respectively.

Consideration of the McCarty-Salisbury, Dent, and Olson studies exhausts known extant map comparison research. The idea is to use these studies in the context of Part I of this dissertation to design a more effective experimental procedure. The role of maps in scientific research was discussed in depth in the section on "Information Retrieval." The conclusion was that maps serve best in the initial hypothesis generation phase of research, as opposed to the latter hypothesis testing phase. The inductive use of maps in theory-formulation seems to involve either: (a) choosing (identifying) the maps of independent variables from which others (dependent variables) are derivatives; or (b) in the more sophisticated sort of research, the comparison of map(s) 1 . . . n with a base map (anchor stimulus) in search of a "best fit." A rank ordering naturally results; that is, measurement is in an ordinal scale. In the section of "Maps and Graphics" it was emphasized that a single map is just one of many possible graphic interpretations of a single spatial data set. Each version

yields a slightly different pattern which in turn would seem to modify the visual impact evoked by the map. Although little is known of the capabilities of the various cartographic expressions in terms of the exchange of information, one might surmise that the diverse techniques of portraying statistical data are not equally effective in attaining communication objectives. Since information transmission is fundamental to pattern analysis, it follows that the different graphics are not equally efficient for map comparison purposes. In spite of the inclusion of a pattern complexity variable in several of the experimental designs mentioned above (i.e., the research of McCarty-Salisbury and Olson), little is known of the pattern complexity problem. However, evidence does suggest that there are rather severe limits on the complexity of patterns that subjects can compare effectively.[9] On the other hand, it seems that pattern complexity in the computational sense may not necessarily imply complexity in the visual sense, and vice versa. The complexity variable does warrant further consideration.

An experiment is now proposed which includes three independent variables: type of graphic, pattern complexity, and degree of association. In the proposed experiment the amount of information that goes into the visual presentations of the graphical form of the distributions will be held constant. The portrayal graphic will be varied.[10] The problem is not psychophysical but rather refers to the relative effectiveness of a range of graphics when they are employed for a particular purpose, in this case visual pattern comparison. Map complexity will be minimized in the proposed experiment by considering only three conceptually and computationally simple distributions.[11] The experiment will utilize an incomplete cartwheel design based on the method of rank order in which subjects are presented with all n stimuli and asked to rank order n-1 of them against a referent.[12]

The specific problem can be formulated in the following way:

(a) How is the performance level in visually estimating the degree of association between map pairs influenced by (1) the type of cartographic depiction, (2) the complexity of the map distribution, and (3) the degree of correlation between maps?

(b) Does the coefficient of determination, r^2, a common numerical measure of association, serve as a satisfactory substitute for

(or means of evaluating) visual estimates?

(B) Data

Hypothetical data having specified correlations were generated with a computer program.[13] All variables consist of 256 observations, with a mean $\simeq$ 50 and a standard deviation $\simeq$ 18. Variable pairs exhibit a range of Pearsonian product-moment correlation values, r, between $\pm$ 1 such that the intervals (Δr) are constant at .10 for r^2 (coefficient of determination). For both theoretical and practical reasons, prior to map compilation the number of variable pairs was reduced from 21 to 17 by eliminating the $\pm$ 30 and $\pm$ 10 r^2 values. (See Appendix A). Data then consisted of a single independent variable of 256 non-dimensional observations and 16 dependent variables of the same size.

(C) Map Compilation

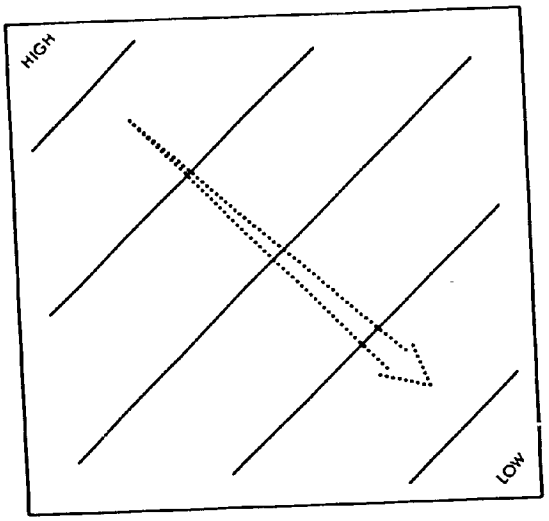
These data could be organized to create a large number of trend surfaces (e.g., linear, quadratic, cubic, etc.) by specifying different spatial formats. Three relatively simple trend surfaces were chosen for this experiment (Figure 4.2). The first distribution is linear, progressing from one high value zone in the upper left corner to one low value zone in the lower right corner. The second distribution is quadratic, progressing from one high value zone in the center of the display to low value zones in each of the four corners. The third distribution is labeled cubic, progressing from two high zones in the centers of diagonal quadrants to two low value zones in the outside corners of the other two quadrants. Note that the more complex the trend surface, the "rougher" it must be because values of the same magnitude are spread over a wider area.

It was believed that a person's ability to extract information from a map is a function of the type of graphic utilized in the display. To test this hypothesis, it was necessary to select a testable (small) but representative sample from the wide range of available graphics. Three commonly used techniques were chosen: dots (point symbols), choropleths (areal symbols), and profiles (line symbols). Each data set (representing one of the three distribution types) was mapped in each of these graphic forms. (See Appendix B).

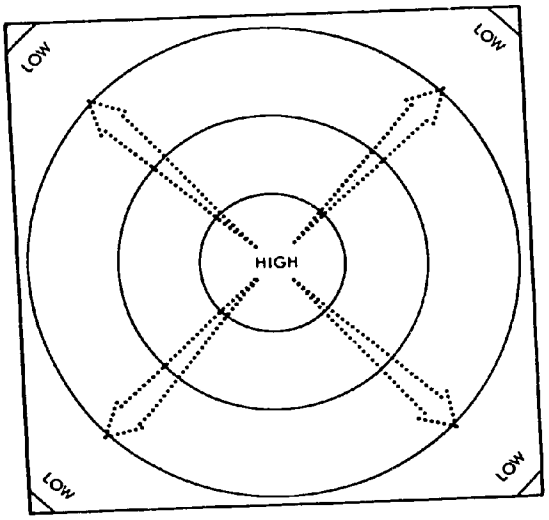
FIGURE 4.2

TREND SURFACES

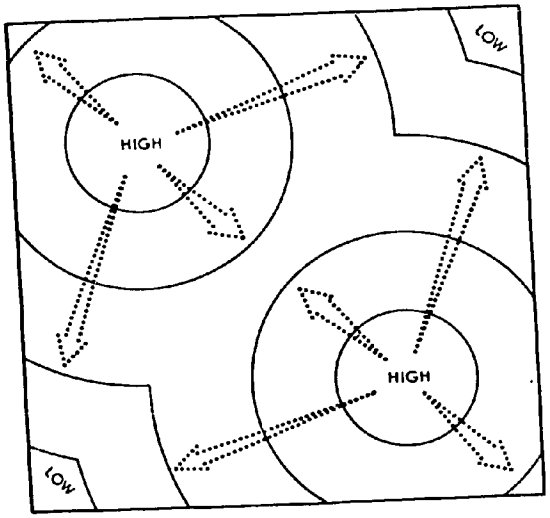
LINEAR



QUADRATIC



CUBIC



(D) Experimental Method

(1) Subjects

Subjects were chosen from the students and faculty at the University of Michigan. The sample (consisting of: Physical Geography class (62 Ss), Geography graduate students (18 Ss), Introductory Cartography class (22 Ss), and Psychology and Geography staff members (9 Ss)) was selected on the basis of: (a) map familiarity, (b) availability in sufficient numbers to yield significant results, and (c) interest and willingness to participate in the experiment. Faculty (volunteers) and graduate students (paid--$1.50/session--volunteers) were run individually. Undergraduate classes were run in groups of up to 12 individuals, with a $5 prize for the best test return within each group.

(2) Materials

Each subject was presented with a large Manila envelope containing the following materials:

(a) a page-size sheet displaying a linear surface map in each of three graphics (i.e., dots, choropleths, profiles);

(b) a letter-size brown envelope labeled Sample Tests containing three sets of four maps (Format + 3 displays) representing the quadratic dot, quadratic choropleth, and cubic profile cases;

(c) nine randomly ordered test packets in letter-size white envelopes representing possible test combinations (i.e., linear dot, quadratic dot, cubic dot, linear choropleth, quadratic choropleth, cubic choropleth, linear profile, quadratic profile, cubic profile), each containing seventeen maps (Format + 16 randomly ordered displays).

(3) Procedure and Design

Approximately 15 minutes were taken to explain informally to subjects the nature and design of the experiment. The following topics were covered: (a) the contemporary interest in visual pattern analysis in diverse fields of research, (b) the role of pattern comparison in map analysis, (c) the criteria used in selecting subjects, (d) the importance of high motivation and the incentive provided, and (e) the timed nature of the experiment (9 five-minute tests). Subjects were told that no deception was involved and questions arising throughout

the experiment would be openly answered.

Subjects were then instructed to empty the large envelope found in front of them, being careful not to alter the order (arrangement) of its contents. The experimenter, referring to the sheet containing the three displays, gave a brief explanation of each map. This included principles of construction and the nature of the various graphics. Subjects were told that displays were properly oriented only when the marginal black spot was at the top. After all questions were answered and when it seemed apparent that subjects understood that all three maps depicted the same linear distribution, they were instructed to put the display sheet aside. Subjects were then informed that the task of the following experiment would be to study a base map (labeled Format) and then rank-order 16 other maps in terms of their degree of likeness or association with the referent display. The 16 maps were to be arranged in a continuum such that the map most highly correlated with the Format positively fell at one end, and the one most highly correlated with the Format negatively fell at the other end. The map with the least degree of likeness to the Format should be placed midway between these extremes. The result should be a uniform progression in correlation from the high positive to the high negative end maps. To make this clearer, subjects were instructed to open the envelope labeled Sample Tests. The three sets of four maps found within were used to familiarize subjects with the experimental procedure. Taking the choropleth set first, subjects were told to look at the Format and then rank-order the remaining three displays as instructed above. After all subjects had successfully completed and seemed to understand this test, the procedure was repeated with the dot and profile samples.

When the sample tests were completed, subjects were informed that the remainder of the experiment would be timed. They had 4 1/2 minutes to rank-order each test packet, which thereafter consisted of 16, not 3, displays to be ranked. After 4 minutes they would be informed that there were 30 seconds left in which to make final adjustments. Subjects were asked not to make noises of delight, frustration, etc.; not to talk; and not to begin or pick up a test series before notified to do so. The importance of picking up displays so as to preserve their

order was emphasized. Maps were to be stacked with the Format on top and the most negatively associated display on the bottom. This could be accomplished by picking the Format up first, the most positively correlated display next, and so on, such that the last display to be picked up would be the most negatively associated map. When this was accomplished, the pile of maps was to be placed back into the envelope from which it came and the subject should wait for instructions to continue on to the next test packet.

The testing procedure was repeated until all nine test envelopes had been completed. Upon completion of the tests subjects were asked to fill out a personal data sheet. When this was done they were told to put all materials (i.e., display sheet, sample tests packet, nine test packets, personal data sheet) back into the large envelope. This terminated the experiment.

IV. LIST OF REFERENCES

1. Roger N. Shepard, "Analysis of Proximities as a Technique for the Study of Information Processing in Man," Human Factors, 5 (1963), p. 33.

2. Camron Peterson and L. Roy Beach, "Man as an Intuitive Statistician," Psychological Bulletin, 68 (1967), p. 29.

3. Robert W. Sekuler and Michael Abrams, "Visual Sameness: A Choice Time Analysis of Pattern Recognition Processes," Journal of Experimental Psychology, 77, 2 (1968), 232-238.

4. McCarty and Salisbury, Visual Comparison of Isopleth Maps.

5. Ibid., p. 63.

6. Ibid., p. 78.

7. Owen Dent, "Map Comparison Survey," Australian National University (Canberra, 1965). (Mimeographed.)

8. Judy Olson, "Visual Comparison of Maps," Seminar Paper, Department of Geography, University of Wisconsin (Madison, 1968). (Mimeographed.)

9. Sekuler and Abrams, "Visual Sameness," 232-238.

10. A major controversy in psychophysics is whether the sensation intensity (ψ) increases as the logrithm of the stimulus intensity (ϕ), that is, $\psi = k \cdot \log \phi$ (Weber-Fechner Law), or whether the relation between a stimulus magnitude and the corresponding sensation magnitude can be adequately described by the power function $\psi = k \cdot \phi^n$ (S. Stevens, "The Psychophysics of Sensory Function," American Scientist, 48 (1960), 226-253). Ratio data are required to resolve this dilemma. But stimuli vary in kind as well as amount. It is equally interesting to study perception in an ordinal scale.

11. Complexity will be roughly defined as the order of the term that can be used to explain the greatest amount of variation in the surface.

12. Clyde H. Coombs, The Theory of Data (New York: John Wiley and Sons, 1964), pp. 32-45.

13. Clemens M. Johnson, "Data Simulation Program: MAD-7090," University of Michigan Computing Center (Ann Arbor, 1967). This program can generate a maximum of seven variables with the following parameters specified by the user: number of observations/variable, means, standard deviations, and cross-correlations.

V. DISCUSSION OF THE RESULTS

(A) Statistical Evaluation

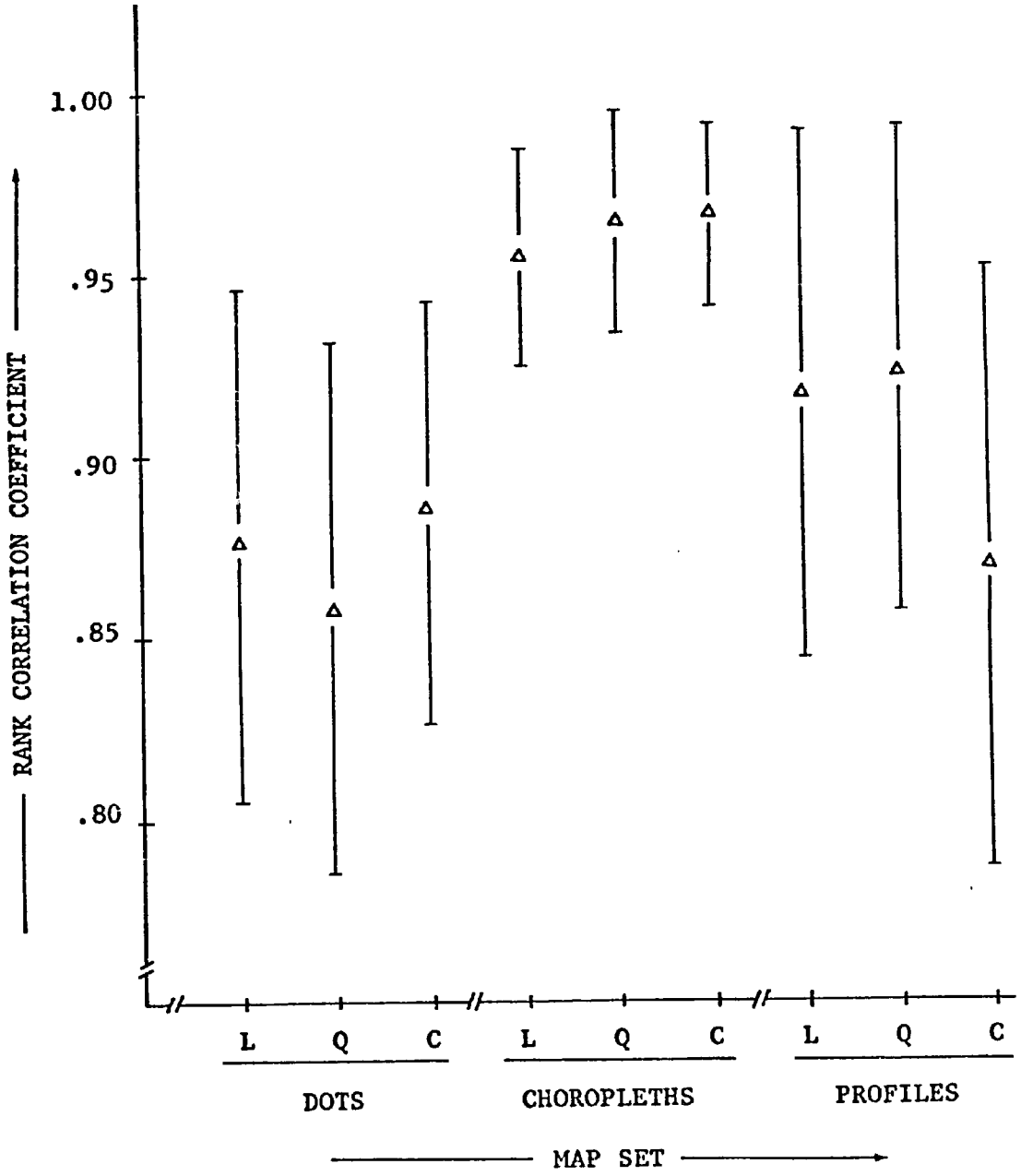
A large number of nonparametric statistical tests, sometimes referred to as "order statistics" or "ranking statistics", are appropriate for testing hypotheses with data measured in an ordinal scale.[1] Data from the present experiment consist of judged rankings versus statistical rankings on n = 16 items (maps). A measure of the degree of correspondence between these two rankings can be thought of as a measure of the intensity of their rank correlation. That is, if the estimated rank of each map on each of the nine data sets is denoted as $X_1, X_2, \ldots, X_n$, and the actual rank in each case is represented by $Y_1, Y_2, \ldots, Y_n$, we may use a measure of rank correlation to determine the relation between the X's and the Y's. The various differences, $d_i = X_i - Y_i$, hereafter called ranking displacement, can be thought of as an indication of the disparity between the two sets of rankings.[2] Thus, rank correlation is a test of the efficiency or a measure of the ability to appraise or detect something by ranking. In this case, it is a measure of subjects' success at the task of visual map comparison.

A rank correlation coefficient known as Kendall's τ (tau) was computed. Tau values for the nine map sets averaged for the 100 subjects are plotted in Figure 5.1. The lowest τ scored by any of the subjects on the 900 map tests was .5833. Since this "low" coefficient is highly significant ($p < .01$), the author comes to two conclusions. First, each subject's judged ranks are significantly related to actual ranks (i.e., each subject can do the task). Second, since 100 percent of the sample performed significantly above chance there is little doubt that, in the population of which this sample is representative, virtually everyone can do the job.

The idea is to investigate whether one combination of map variables was more efficient in transmitting mapped information than another. If τ values were normally distributed then the difference between τ scores for various combinations of the nine map cases could be compared, approximately, by an ordinary t-test. But in this instance τ scores were

FIGURE 5.1

MEAN TAU STATISTICS WITH STANDARD DEVIATIONS
(n = 100)



LEGEND

L = linear; Q = quadratic; C = cubic

Δ = mean tau

I = standard deviation

highly skewed and all values were concentrated in the zone of high positive correlation. For this reason, it was decided that analysis of variance using ranking displacement data would be more appropriate and easier to interpret than further analysis of r scores.

Each stimulus (map) was ranked the same number of times (100), providing arithmetic convenience. Since rankings were made in terms of a limited number of categories, the observed frequencies may be conveniently arranged in n x m matrices such as those found in Tables 1 . . . 9 (Appendix D). To aid in the interpretation of these tables (confusion matrices) data were both statistically and graphically analyzed.

Study of the nine confusion matrices indicated that several pairs of maps were consistently inverted in subject rankings. Scrutiny of the anomalous map pairs suggested that in each case subjects grossly misweighted unnatural or randomly occurring map details (see Appendix E for further discussion). Although these inversions could provide an informative means of looking into the perceptual mechanism involved in this map comparison situation, this was not the principal goal of the experiment. Since these non-conforming mis-rankings introduce "noise" into the general response trend, the author elected to re-invert the affected map pairs for purposes of visualization (see the following graphs). The original data consisting of ranking displacements were left intact for statistical analyses.

(B) Graphic-Complexity Interaction

The idea is to use analysis of variance to examine the factorial design and observe whether variations of the independent variables (map attributes) significantly affect the dependent variable (subject response). Two-way analysis of variance using ranking displacement data indicates that there is a highly significant (.005 level) interaction between the graphic and complexity variables (Table 5.1). Thus it is interesting to look at various combinations of the map sets to see whether there is a simple, direct relationship between these variables.

(C) Graphics

When total ranking displacements are averaged across the complexity variable there is a highly significant graphic main effect (Table 5.1). The results of pooling errors over the complexity variable graphic by

TABLE 5.1

ANALYSIS OF VARIANCE FOR
FACTORIAL DESIGN

Analysis of Variance	Source of Variation	Degrees of Freedom	Sums of Squares	Mean Squares	F	Significance Level
Two-Way (3 x 3)	Graphic	2	12668.7	6334.3	204	.005
	Complexity	2	147.9	74.0	2.4	.10
	Graphic/ Complexity	4	1866.9	466.7	15	.005
	Within Replicates	891	27459.0	30.8		
	Total	899	42142.6			

graphic are illustrated in Figure 5.2a and 5.2b. The difference between the areas under the 3 curves in Figure 5.2a implies that the discrepancy in total ranking displacement between pooled dot and profile map sets is less than the discrepancy in total ranking displacement between grouped dot or profile map sets and choropleth map sets. These observations are confirmed by Tables 1 and 2 in Appendix E.

Figure 5.2b illustrates the frequency of ranking errors for items 1. . . 16, as opposed to the size of errors (rank displacement) which is plotted in Figure 5.2a. Although this has the effect of abstracting the size of errors from consideration, the comparative relations of the 3 curves in Figure 5.2b remain the same as shown in Figure 5.2a. It is evident that the mean size and number of errors vary from graphic to graphic. This is important when contemplating the use of maps in research. The choice is between using a map type that minimizes the chance of making (a) more but smaller errors or (b) fewer but larger errors.

The distribution of ranking displacement and ranking errors in relation to items 1. . . 16 is also revealing (Figures 5.2a and 5.2b). When interpreting these and following graphs it is important to note that the increment (Δ) in r^2 values between ranks 6 (+.40) through 10 (-.40) is 20 percent, not 10 percent, as is the case with the remaining ranks. Thus, it is expected that relatively fewer errors would be made in this zone. With the exception of profile maps, this appears to be the case. The roughness or higher incidence of hidden profiles in plots as r^2 approaches 0 appears to have obscured the percentage of volume under the surface described by the profiles. Since Δr^2 was greater in the medium to low r^2 zone, it is reasonable to assume that map sets of constant Δr^2 would emphasize the error pattern even more. The assertion is supported by evidence given in Figure 5.3a, which illustrates the results of presenting map sets of constant $\Delta r^2 = .20$ to 11 Cartography students. Although use of a Δr^2 as large as .20 made map comparisons too easy to yield powerful results, it is interesting to note that the profile map sets now exhibited the largest number of total errors and these errors occurred primarily in the medium to low correlation zone.

It may also be interesting to compare responses between the two groups of cartography students. Data from the section run according to the standard testing design might be transformed into a form comparable

FIGURE 5.2a

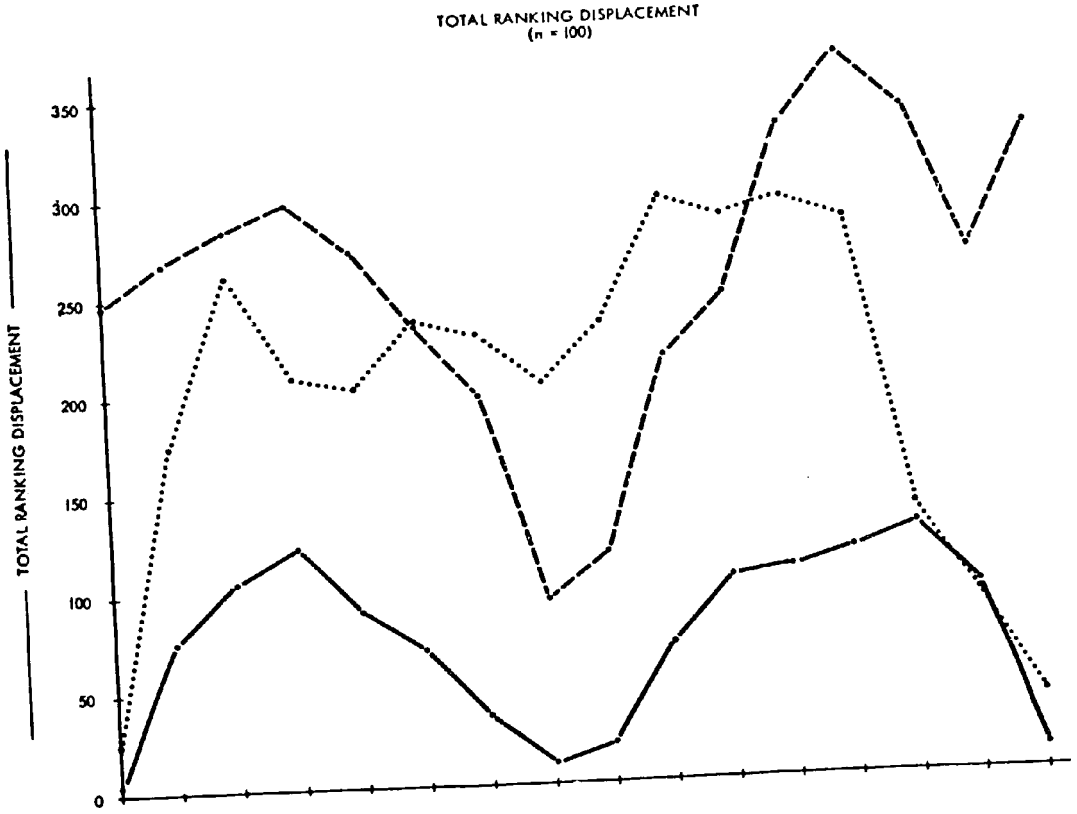
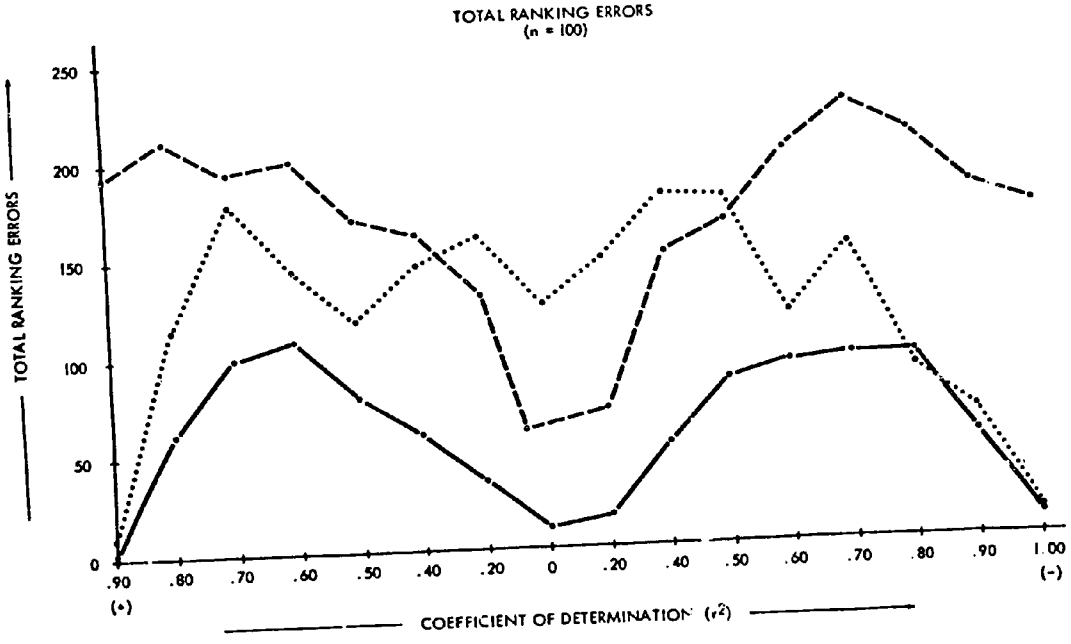


FIGURE 5.2b



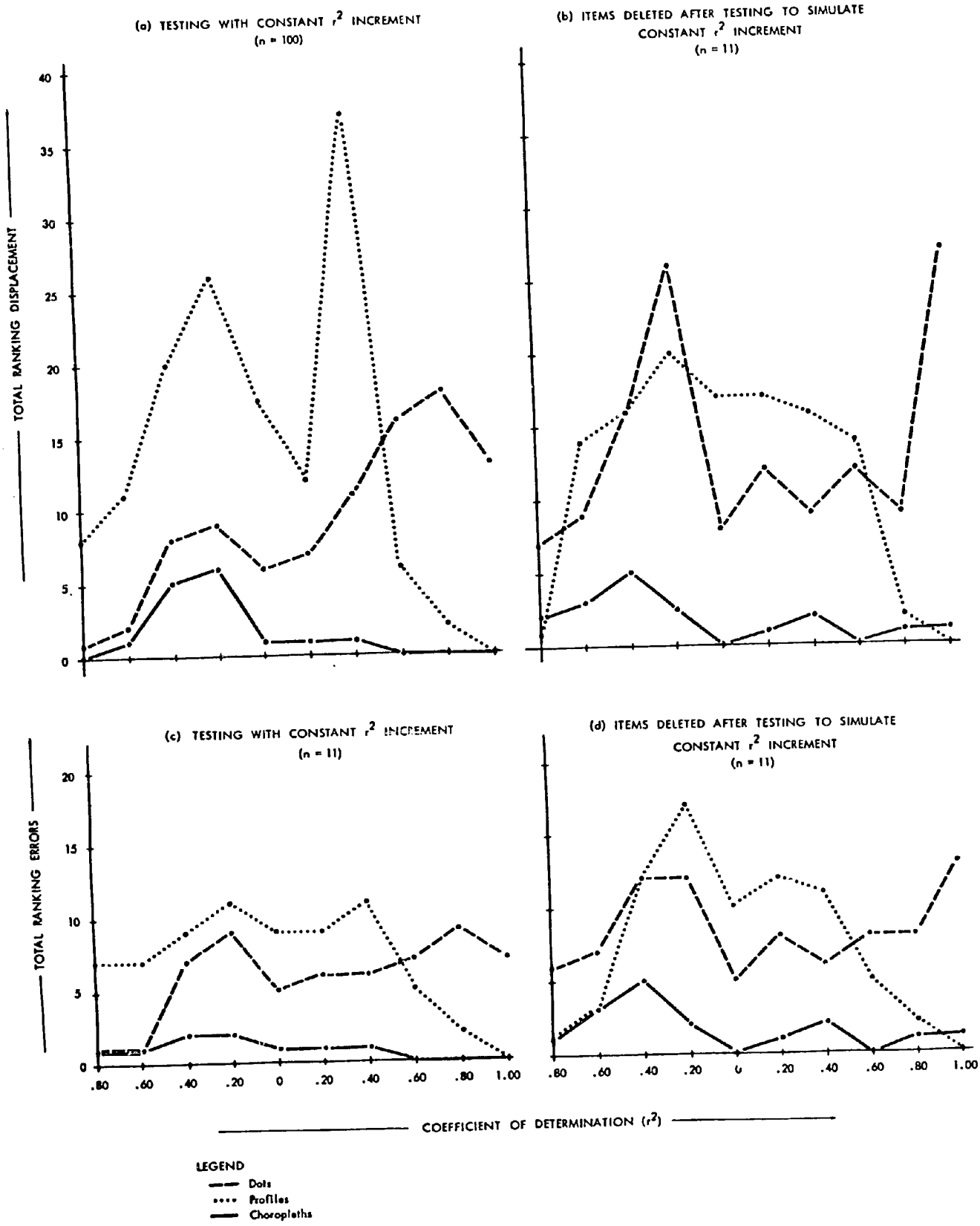
LEGEND

— — Dots

···· Profiles

——— Choropleths

FIGURE 5.3


(a) TESTING WITH CONSTANT r^2 INCREMENT
(n = 100)
(b) ITEMS DELETED AFTER TESTING TO SIMULATE CONSTANT r^2 INCREMENT
(n = 11)
(c) TESTING WITH CONSTANT r^2 INCREMENT
(n = 11)
(d) ITEMS DELETED AFTER TESTING TO SIMULATE CONSTANT r^2 INCREMENT
(n = 11)
TOTAL RANKING DISPLACEMENT
TOTAL RANKING ERRORS
0
5
10
15
20
25
30
35
40
.80
.60
.40
.20
0
.20
.40
.60
.80
1.00
COEFFICIENT OF DETERMINATION (r^2)
LEGEND
Dots
Profiles
Choropleths

with the special section by deleting ranking displacements between $\pm$ .40 to +1.00 and -.40 to -1.00 if they are < 2, taking into consideration the direction of mis-rankings with items $\pm$ .40 (r^2). Figures 5.3b and 5.3d show the results of this transformation. Since there appears to be no substantial difference between Figures 5.3a and 5.3b, and Figures 5.3c and 5.3d, here is further evidence of the similarity between the psychological distance between maps and their proximity in a statisitical sense. One might also tentatively (the small sample size does not seem to justify rigorous assessments) conclude that the above transformation was founded on tenable assumptions.

Intuitively it seems that relatively smaller and fewer errors should be made as r^2 values closely approached ($\pm$) 1, since errors can be made in only one direction from either end of the ranking. Comparison of the curves in Figures 5.2a and 5.2b indicates that this edge effect does exist. The trend is most apparent in the choropleth and profile map plots, which appear to differ only in magnitude of errors. Dot maps do not exhibit such a strong pattern. On the contrary, the largest number of errors with dot maps were made with the highest r^2 values. One interpretation of why subjects had most difficulty ordering highly correlated dot maps is that the density gradient was not strong enough to provide sufficient visual cues to allow perception of map differences. Mackay's research, indicating that dot density tends to be underestimated in high density areas,[5] and Birch's convincing demonstration that localized dot density appears to the eye to vary inversely with that of the surrounding area,[6] tend to support this finding. The distortion of the visual field noted by Birch is similar to the findings of psychologists on field effects in two-dimensional form perception.[7] Since dot density, or, for that matter, choropleth tone categories, can be considered to be a montage of symbols (forms), the notion of field effects may yield useful insight.

Upon comparison, response plots for choropleth maps (combining errors across the complexity variable) appear nearly identical in Figures 5.2a and 5.2b. This is also true for the profile maps. But comparison of the plots of errors for pooled dot maps reveals a deviation in the high r^2 value zones. This means that in these cases a large number of small errors were made, thereby diminishing the plot of ranking errors

relative to the medium r^2 value zones where errors were about the same in number but larger in size. Several possible explanations have been mentioned: (a) limiting the errors to one direction has the effect of diminishing their number and size, and (b) the eye appears to "smooth" and diminish dot intensity.

Discussion of Figures 5.2a and 5.2b is concluded with the observation that there appears to be little difference between the total number of errors made on positive and negative items once the several "mis-weightings" are corrected. Since this observation contradicts McCarty and Salisbury's conclusion that more errors are made with negatively correlated items, here is more evidence to indicate that subjects often failed to record minus signs with estimated r values in their experiments. Why Dent found more errors with negative items is less clear. The reason why this conclusion contradicts a number of observations in psychological literature that subjects show greater accuracy in estimating positive correlations is even more obscure.[8] The author speculates that these conflicting conclusions provide further evidence that subjects do in fact have a working notion of graphic correlation.

(D) Complexity

When total ranking displacements are averaged across graphics there is no significant complexity main effect (Table 5.1). But there is an interesting relationship between surface complexities, graphic by graphic (Figures 5.4 and 5.5).

Subject response to choropleth map complexity is shown in Figures 5.4c and 5.5c. Only slight differences exist between the three curves. They are entwined throughout. There is a slight improvement in subjects' ability to compare choropleth maps as surface complexity increased.

Subject response to dot map complexity is illustrated in Figures 5.4b and 5.5b. Although the curves exhibit less braiding than the choropleth set, the pattern is essentially the same. The most complex surface evoked the least error, but in this instance the simplest surface did not exhibit the fewest errors. Thus, it cannot be said that there is a general trend in the pattern of errors commensurate with increasing surface complexity, as is the case with choropleth maps. It is possible,

FIGURE 5.4

TOTAL RANKING DISPLACEMENT
(n = 100)

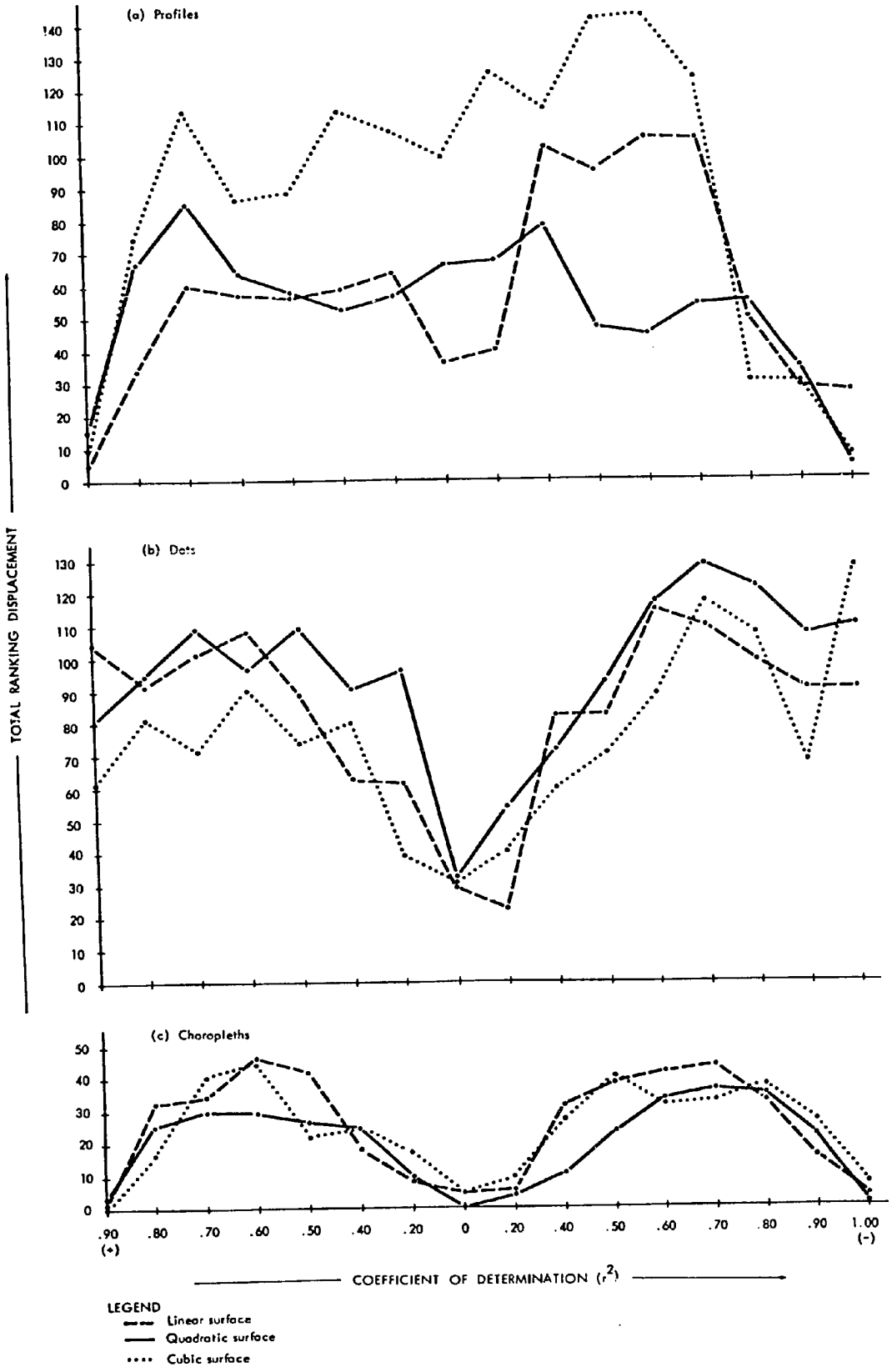
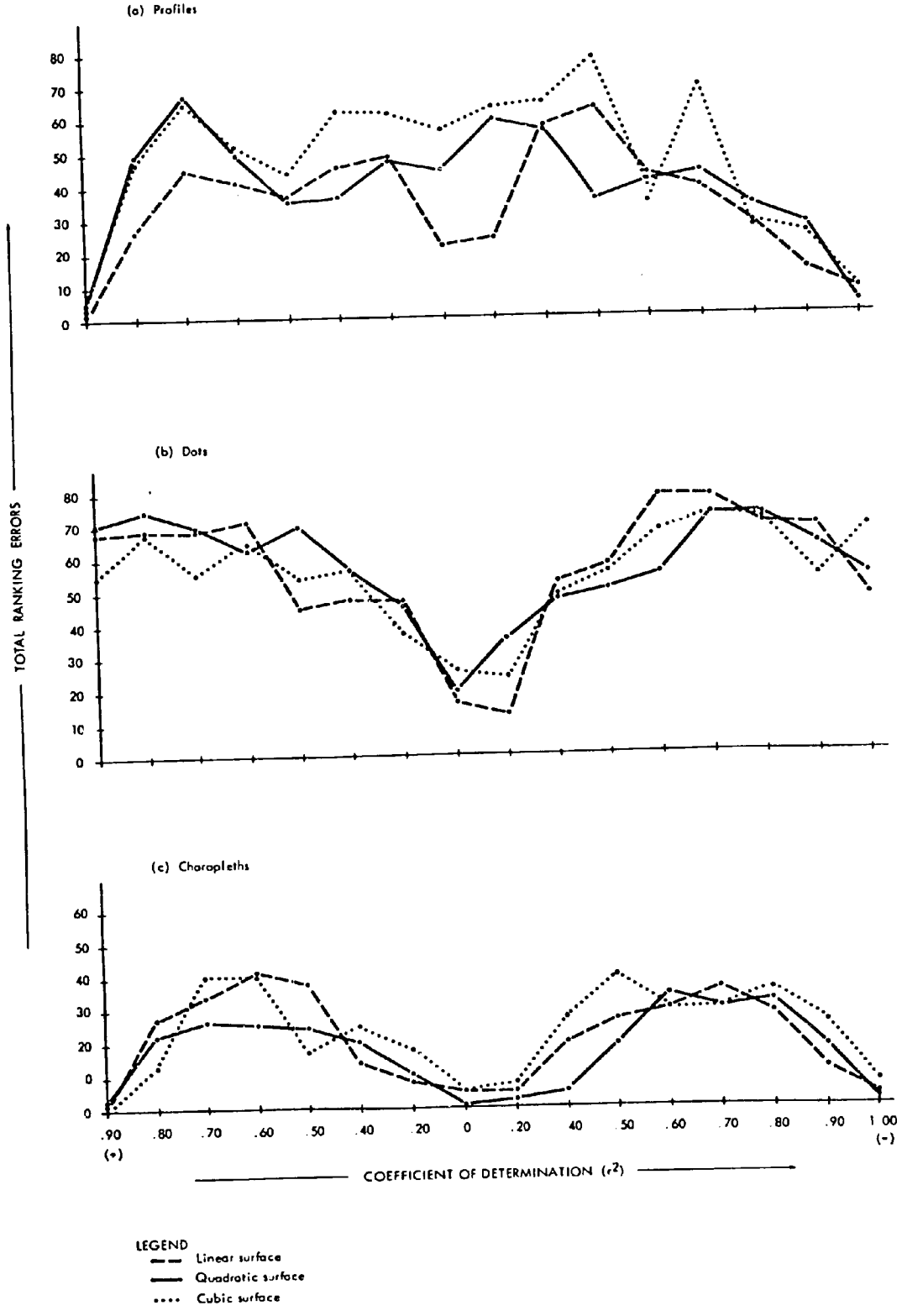


FIGURE 5.5

TOTAL RANKING ERRORS
(n = 100)



however, that such a trend exists but was obscured in this experiment by displays having cartographic, photographic, or printing imperfections. This notion is supported by the pattern of errors in the two most highly positively correlated items. Although the most highly negatively correlated items do not exhibit the same pattern, it was already mentioned that a special weighting mechanism possibly involved in this instance would negate general error trends.

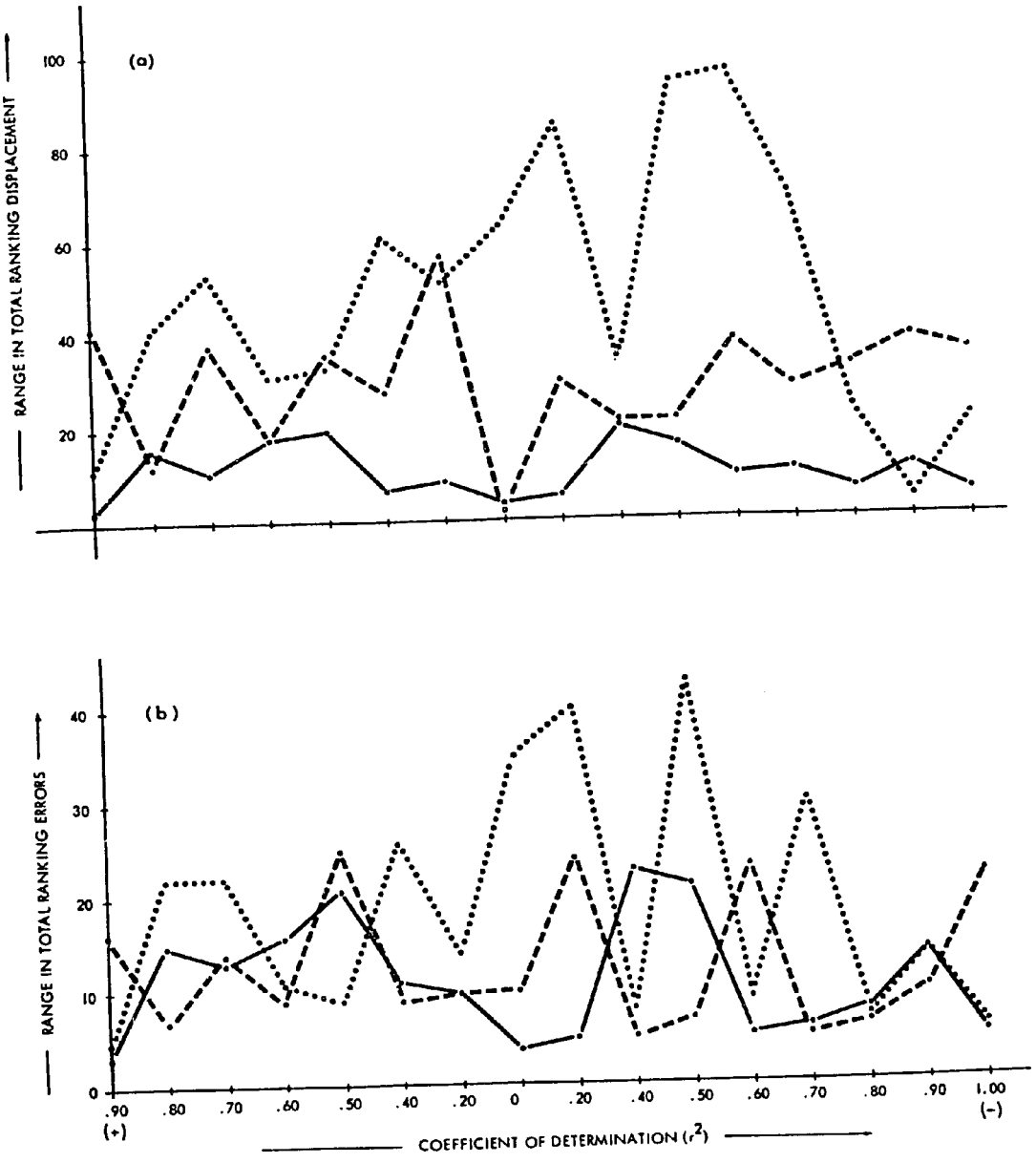
Subject response to profile map complexity is shown in Figures 5.4a and 5.5a. The curves exhibit less similarity and interlacing than either of the previous examples. More specifically, the linear and quadratic sets exhibit the most braiding, and all sets are more highly entwined in the zones of high correlation. The linear case is bi-modal, with more errors made with negatively correlated items. In the quadratic case, errors become more evenly distributed throughout the r^2 value range. The cubic case distinctly exhibits the most errors in the medium to low r^2 zones, with more errors made with negatively associated items.

The similarity in the pattern of ranking errors or ranking displacement between graphics can be studied in previous figures, but it is more easily visualized by plotting the range in errors between pooled surfaces for each graphic (Figures 5.6a and 5.6b). Considering the 3 ranking error curves together, it is apparent that subjects perceived choropleth maps to be most similar to each other and profile maps to be least similar to each other, with dot maps falling between these two. In other words, subjects were least influenced by the range of complexity in choropleth map distributions and most influenced by the range in complexity in profile map distributions. Considering the distribution of errors in terms of r^2 values, it is evident that the perceived differences in profile map complexity was generally greatest in the low to medium r^2 zone. Dot and choropleth maps show no obvious trend.

The difference between the curves on the two graphs (Figures 5.6a and 5.6b) indicates that the size and distribution of errors varied considerably from surface to surface and graphic to graphic. The greater the discrepancy between the two figures the larger the size of the average error made on that particular item. The large ranking errors in the low to medium r^2 zones with profile maps, positively correlated dot maps, and high negatively correlated dot maps, is notable.

FIGURE 5.6

RANGE IN TOTAL ERRORS BETWEEN THREE SURFACE COMPLEXITIES*



LEGEND

* Note scale difference between graphs.

—— Choropleths

••• Profiles

– – Dots

(E) Subject Consistency

Grouped subject consistency is indicated in Figures 5.7a, 5.7b, and 5.7c (unlike previous graphs, these plots are based on the original ranking displacement data). Subject response to the 3 choropleth map complexities was almost equally consistent (Figure 5.7c). The plots of standard deviations are bi-modal, with lowest values occurring in high positive, high negative, and low correlations. Slightly more larger deviations were made with negative correlations, but this was largely due to several anomalous mis-rankings (discussed above).

Subject response to the 3 dot map cases was less consistent (i.e., there were larger discrepancies between the 3 curves) than was the case with choropleth map sets (Figure 5.7). The linear and quadratic sets are definitely bi-modal, with smallest deviations in the high positive and low correlation zones. The cubic case shows smallest deviations with high positive, high negative, and low correlations, but indicates an increasing size in standard deviations from the medium positive through the low to the medium negative r^2 zones. All 3 cases (complexities) show higher deviations with high negative than with high positive correlations. As was asserted above, this can be explained largely by the several map inversions that resulted from visually over-weighting empty cells.

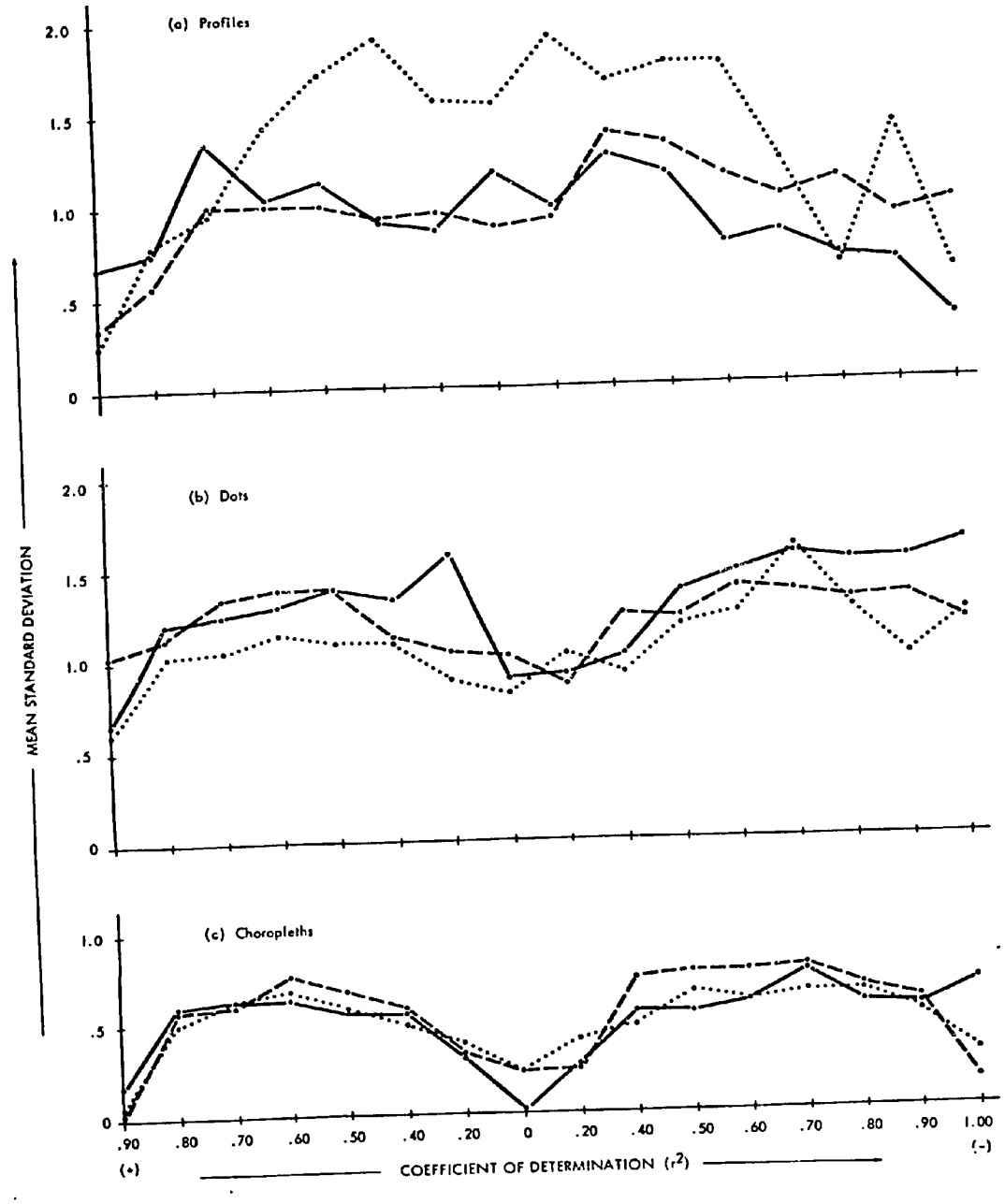
Subject response to the 3 profile map complexities was the least consistent of the 3 graphics considered (Figure 5.7a). The linear case exhibits larger deviations with negative than with positive correlations, but smallest deviations are seen in the highest correlation zones. The deviations trend with quadratic maps is not distinct, except that smallest deviations were made with high negative correlations. The cubic case represents a major break from the linear or quadratic pattern. Largest deviations were made in the medium to low correlation zones.

(F) Interpretation of the Results

A tabular summary of the experimental results is presented in Table 5.2. This is self-explanatory. The same data are depicted graphically in Figure 5.8, which is a generalized illustration of the pattern of error dispersion on the nine confusion matrices (Appendix D). Recall

FIGURE 5.7

MEAN STANDARD DEVIATION IN RANKING DISPLACEMENT
(n = 100)



LEGEND
- Linear surface
- Quadratic surface
- Cubic surface

TABLE 5.2

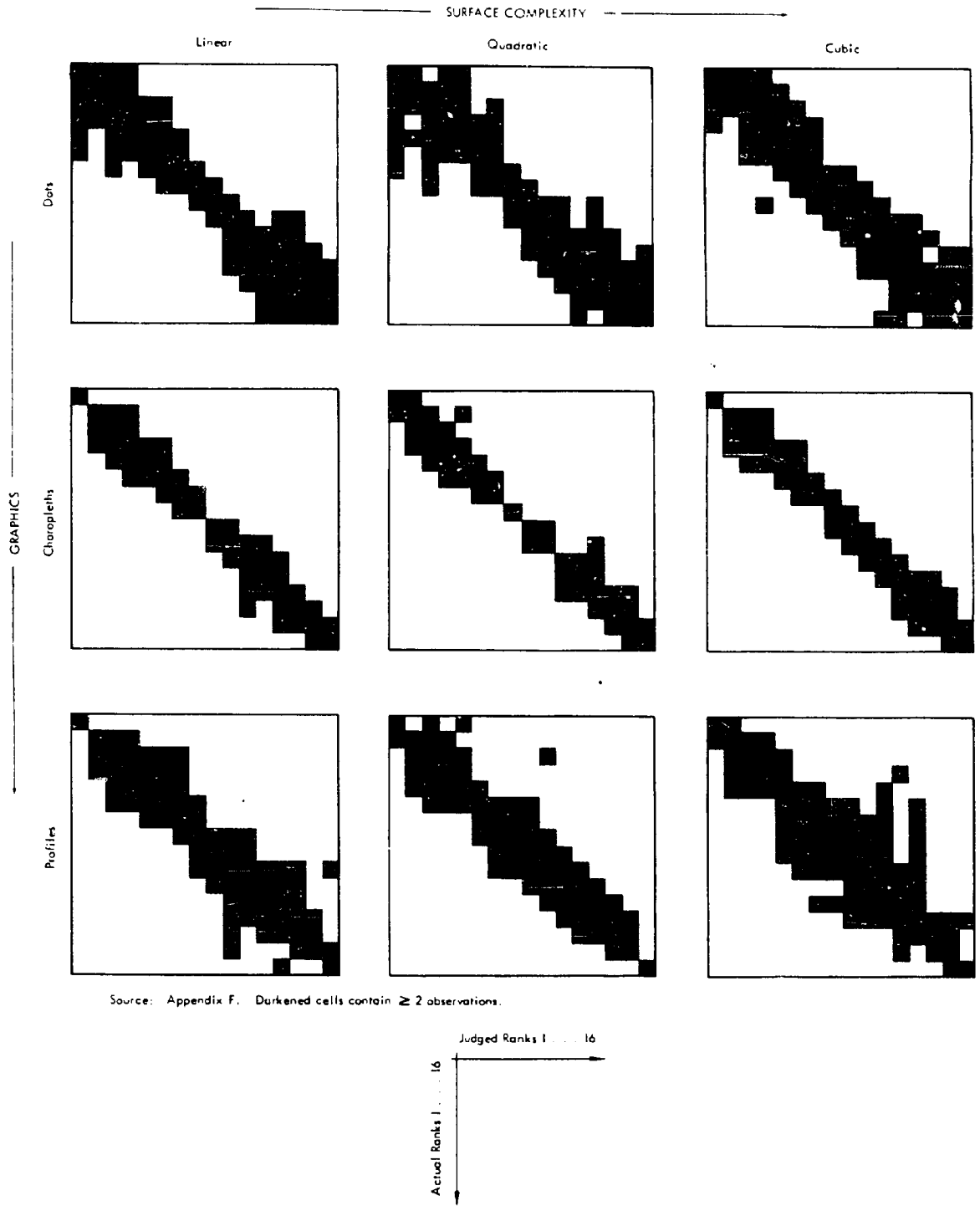
TABULAR SUMMARY OF THE EXPERIMENTAL DESIGN

HYPOTHESES	RESULTS	SIGNIFICANCE	REFER TO	COMMENTS
1. Can subjects visually rank sets of maps in terms of their degree of association with a referent?	Yes	$p < .01$	Figure 5.1	A constant increment in r^2 values would have generated more errors, but probably not enough to make the ranking non-significant.
2. Is the performance level in visually ranking sets of maps in terms of their degree of association with a referent influenced by:				
(a) The type of cartographic depiction?	Yes	$p < .005$	Figures 5.2, 5.3	Dot, choropleth, and profile graphics were tested. Choropleths were ranked most effectively, dot maps least effectively.
(b) Map Complexity?	No	$p < .10$	Figures 5.4, 5.5	Simple linear, quadratic and cubic surfaces were tested. Increasing complexity evoked more errors with profile maps and fewer errors with dot maps. Choropleth maps show no distinct trend.
(c) Degree of correlation?	Yes	*	Figures 5.2, 5.5	Given the larger increment between maps as r^2 approached 0, the most errors were made in the zone of: high $(\pm)$ r^2 values with dot maps; medium $(\pm)$ r^2 values with choropleth maps; and medium to low $(\pm)$ r^2 values with profile maps.

* Not tested statistically.

FIGURE 5.8

GENERALIZED CONFUSION PATTERN
(n = 100)



Source: Appendix F. Darkened cells contain ≥ 2 observations.

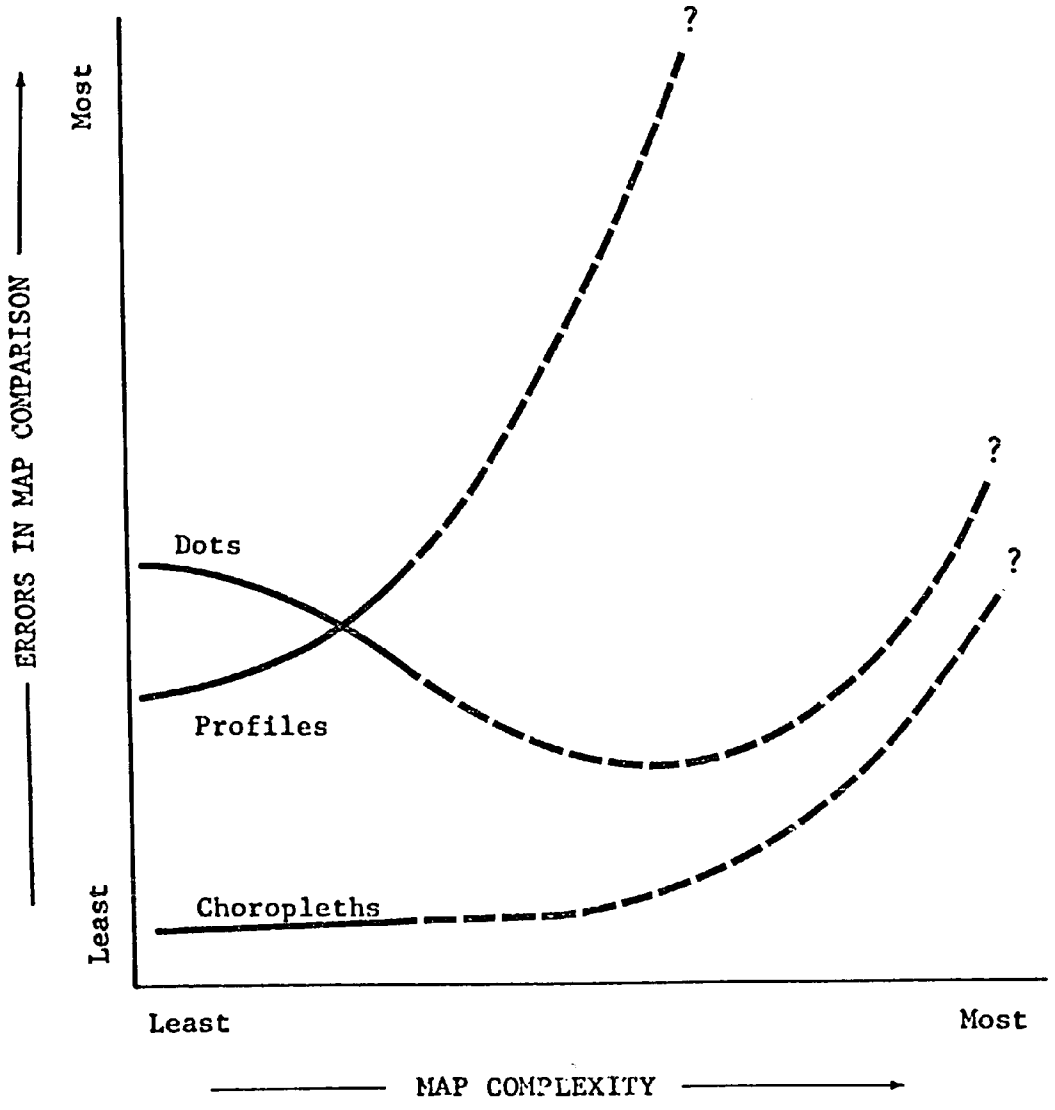
that perfect orderings would result in a straight line along the principal diagonal. The dot map graphs illustrate the most flare at the extremes, but this flare lessens as complexity increases. Choropleth graphs illustrate a bi-modal flare, which also decreases with increasing complexity. Profile maps illustrate a range in the pattern of flare from distinctly bi-modal (linear) to slightly bi-modal (quadratic) to strongly uni-modal (cubic). Zones of r^2 and degrees of complexity where each graphic is strongest or weakest in conveying mapped information is readily apparent. In general, these schematized confusion matrices should be interpreted with caution in light of variable r^2.

The utility of various graphics is summarized in Figure 5.9. Solid lines depict the generalized function: errors = f (complexity). Broken lines are possible extrapolations of these curves. Choropleth maps were compared well by all subjects and no distinct trend is apparent. The implication is that the choropleth maps used were much more effective information communicating devices than either the dot or profile maps. This suggests that choropleth maps could be extended well into higher degrees of complexity before errors would reach the proportion found with the dot and profile maps used in this experiment.

The trend depicted in Figure 5.9 suggests that the efficiency of profile maps for pattern comparison breaks down quickly with increasing surface complexity. Both the number and size of errors increased directly with the complexity of surfaces. This is intuitively sound, since the more random the distribution, the rougher the profile depiction becomes (i.e., the greater the number of hidden lines). It should be remembered, however, that comparison of profile maps is quite effective with high correlations and these are generally of most interest to researchers using maps. The conclusion that profile maps are of questionable utility would seem to contradict the prevailing belief in the superior visual characteristics of three-dimensional or perspective representations, as opposed to the conventional two-dimensional map format.[9] The visual attributes of profile maps seem to be inversely related to map complexity. Yet there is reason to mistrust such generalization. In particular, the 16 x 16 matrix (= 256 data points) used to generate maps in the present experiment results in a surface of quite different appearance than is generated by, for example, an 80 x 80 matrix (=6400

FIGURE 5.9

GENERALIZED UTILITY FUNCTIONS FOR
MAP GRAPHICS



data points).

Dot maps exhibit fewer errors with increasing surface complexity, but this trend must reverse itself eventually. How soon this reversal takes place and how few errors are involved with greater surface complexities are intriguing questions left unanswered. In spite of the larger number of errors with simpler surfaces, it should be remembered that the most errors were made with highest correlations. This undesirability from a research standpoint is somewhat lessened by the fact that the average error was small in this zone and therefore would not seem to hamper picking highly correlated items from a large group of maps. Finer distinction within this subset would be of dubious reliability.

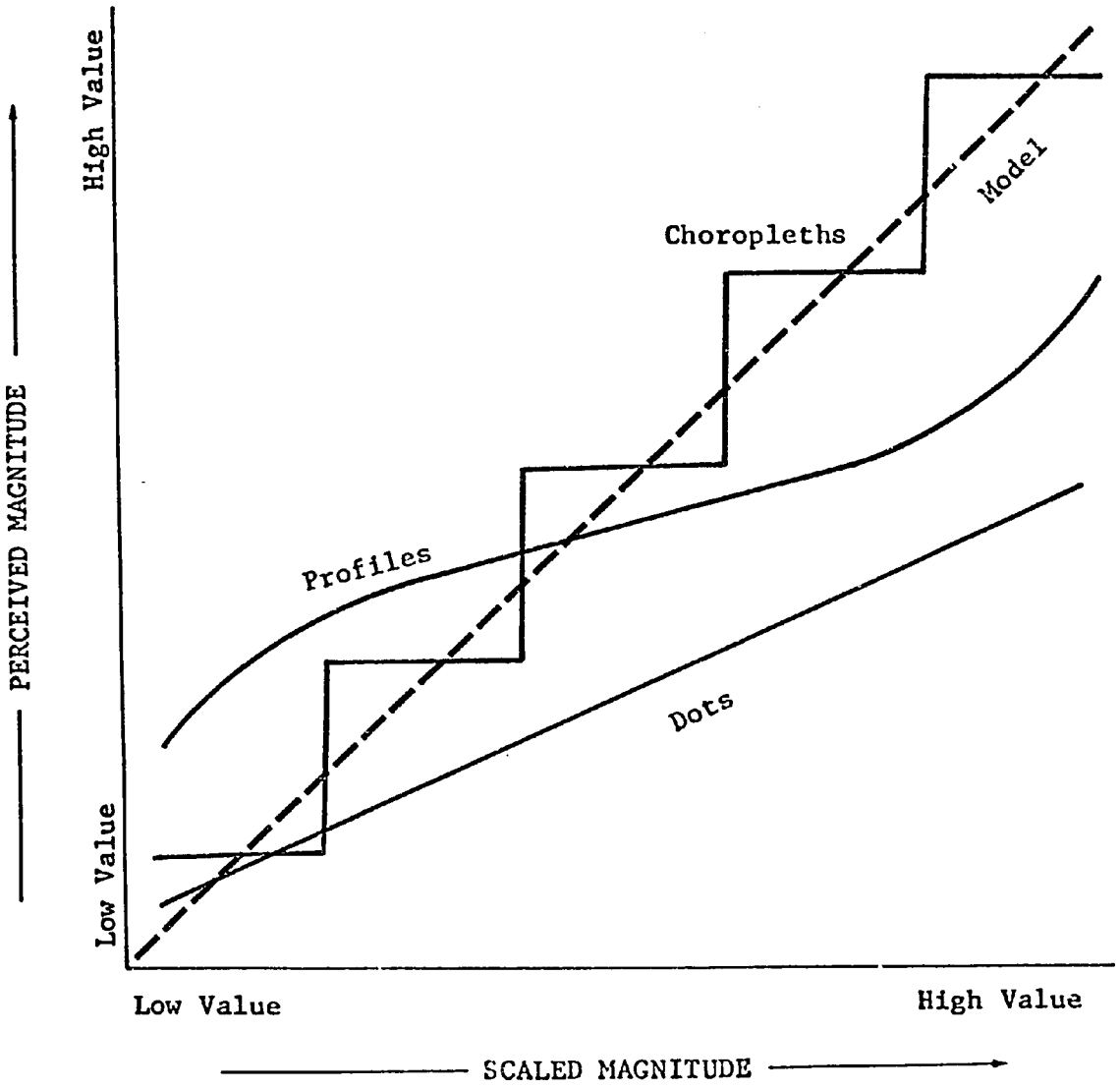
The results of this experiment are reviewed in Figure 5.10. Plots represent conceptual notions, not true data. Perceived or apparent intensity (magnitude) of the range of map distribution values is graphed as a function of the scaled or actual distribution values. If subject perception conformed perfectly to absolute scaling, a straight line relationship (model) would accrue. That is, when a map cell contained value 5, it would be perceived as 5, not higher or lower. The goal of cartographic research is to design graphic symbol schemes that approximate as closely as possible this straight line model.

Choropleth symbols (shadings) were based on the findings of perceived versus actual gray scale intensities. Thus, they closely resemble the model. Increasing the number of categories would lead to a better approximation, but would introduce counter-balancing confusion from increased map detail and complexity. The quintile interval with equal appearing shadings has been found to be the best compromise between generalization and visualization.[10] The results of this experiment suggest that the steps in shading were effective in producing maps that lend themselves well to comparison.

Dot-map scaling was not based on findings of previous research which suggest that the number of dots in the high density zone must be increased by 1/3 to provide a true sensory impression of the data mapped.[11] Researchers have been reluctant to adopt apparent value scaling on dot maps in spite of this information. The author chose absolute

FIGURE 5.10

APPARENT INTENSITY TRANSFORMATIONS



scaling and thus has rechecked Mackay's findings and established a basis for future work along these lines. It does appear that subjects had difficulty distinguishing between maps, especially in the zone of highest correlation. Subjects also show greater ease of comparing more complex map sets with their more rapid and widespread variation in pattern, for this made it more difficult for the eye to smooth local variation. This suggests that increasing dot density in high value zones might be more effective than absolute scaling, that is, it might reduce the discrepancy between the derived and theoretical curves on the graph.

Response to profile maps is more difficult to interpret. It does seem as though an increase in vertical exaggeration might produce different results. The author tends to feel that there was not sufficient range between high and low values on the maps. In any case, variation in values between these zones is extremely hard to discern. Although an increase in vertical exaggeration may enhance perception of the intensity gradient, middle values would probably always remain hardest to discriminate.

It is illustrative to view all of these suggested modifications as attempts to transform the 3 curves into ones more closely approximating the model. In particular, available information implies that an increase in dot density in high value zones or more vertical exaggeration with profile maps might improve results. Of course, additional variables must be considered simultaneously so that a proper balance results.

The author has found that visual map comparison and rank correlation are highly associated. That is, one is a good mirror or model for the other. This could indicate that there is a high degree of similarity between map comparison and a statistical ranking procedure. This conclusion is interesting in that review of previous literature revealed that, within the given experimental designs, subjects were generally poor at map comparison. Present findings directly contradict Smedslund's conclusion that subjects with no statistical training apparently have no adequate concept of correlation.[12] Since the present experiment employed graphical, as opposed to tabular or sequential, stimuli, we might conclude from the divergent results that subjects do in fact possess a cognitive structure isomorphic with the notion of graphical correlation in spite of the seeming absence of correlational reasoning in a statistical sense.

The personal data sheets provide another sort of information. Subjects were encouraged to comment freely on the test design and on their approach to ranking the various map sets. Test instructions and test design (i.e., time, number of maps/set, negative versus positive correlation, and general procedure) seem to have caused no serious problems with subjects. In fact, most participants rated their performance as good. Subjects in general said that they found profile maps to be most difficult to order, while stating that choropleth maps were easiest to arrange. This is interesting considering that the poorest performance resulted in ordering dot maps, if total ranking displacement is used as the evaluating criteria. It is true, however, that larger (but fewer) ranking displacements were made with profile maps.

It was observed that subjects spent less time ranking map sets as they gained experience at the task, which suggests a function similar to that depicted in Figure 5.11. Even though 4 1/2 minutes were given on every test, after a few sets most subjects were able to complete the ordering within 2 to 2 1/2 minutes. This progressive, but decreasing, reduction in task time suggests that "learning" played a role in initial attempts at map comparison.

It is significant that subjects exhibited some tendency to rely on "gimmicks" as a means of making the map comparison task easier. Although test results were undoubtedly influenced by such procedures,

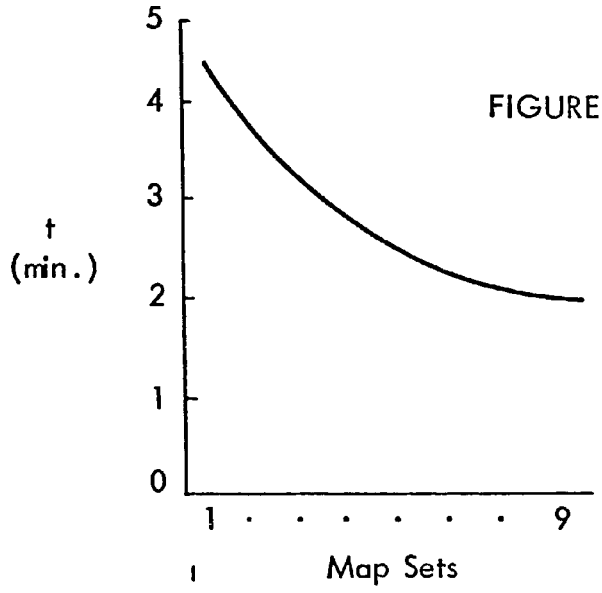


FIGURE 5.11

there is no evidence to support their general validity. On the contrary, subjects were likely to be led astray by discovering a so-called "easy" solution to the task. Most notable examples were: (a) to compare only several small sample sections over the map set, or (b) to compare the height of a corner from one profile map to the next. A more effective comparison technique discovered by many subjects was to look first at the overall pattern (macro-pattern). This technique generally sufficed to get the maps roughly ordered. Next, working in from the ends (high $\pm$ correlations), the degree of "roughness" or "smoothness" (micro-pattern) was used in making pair-wise comparisons. In most cases fine judgments could be made in this manner. The final step was to stand back, squint, take off glasses, etc., in an attempt to blur or smooth the pattern so that items out of place in the general trend (progression) could be detected.

Subjects generally indicated that they looked for correspondence between highest value zones first, lowest value zones next, and in-between value zones last. Olson confirms this observation.[13] It is of interest to note that subjects seem to have searched explicitly for correspondence, while, at most, implicitly for the absence of relationship. This observation is supported by several psychological studies in which the authors concluded that people do not deal consciously nor efficiently with negative evidence.[14]

V. LIST OF REFERENCES

1. Sydney Siegel, Nonparametric Statistics for the Behavioral Sciences (New York: McGraw-Hill Book Company, 1956).

2. Ranks are denoted by ordinal numbers, 1, 2, 3, ..., n where n is the number of items. Thus the item that comes fifth in the ranking has the rank 5. If the estimated rank for a particular item was 5, and the actual rank was 8, then ranking displacement = 8-5, or +3.

3. Maurice G. Kendall, Rank Correlation Methods (London: Charles Griffen & Company Limited, 1948), p. 20.

4. The qualitative classification of sets of hypothetical data according to degree and direction of association is as follows: r^2 = 0, .20 (low); .40, .50 (medium); .60, .70 (medium-high); .80, 1.00 (high). Unfortunately, this scale differs significantly from McCarty and Salisbury's scheme and therefore cannot be directly compared.

5. J. Ross Mackay, "Dotting the Dot Map: An Analysis of Dot Size, Number, and Visual Tone Density," Surveying and Mapping, 9, 1 (1949), 3-10.

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9. George Jenks and Dwight Brown, "Three-Dimensional Map Construction," Science, 154 (1966), 857-863.

10. McCarty and Salsibury, Visual Comparison of Isopleth Maps, p.

11. Mackay, "Dotting the Dot Map," 9; and Andrew C. O'Dell, "Population Maps: A Discussion ... opened by Professor C. B. Fawcett," Geographical Journal, 85 (1935), 151-152.

12. Jan Smedslund, "The Concept of Correlation in Adults," Scandinavian Journal of Psychology, 4 (1963), 165-173.

13. Olson, "Visual Comparison of Maps," pp. 36-37.

14. Smedslund, "The Concept of Correlation," 165-173; P. C. Watson, "The Processing of Positive and Negative Information," Quarterly Journal of Experimental Psychology, 11 (1959), 92-107; _____, "On the Failure to Eliminate Hypotheses in a Conceptual Task," Quarterly Journal of Experimental Psychology, 12 (1960), 129-140; H. M. Jenkins and W. C. Ward, "Judgement of Contingency Between Responses and Outcomes," Psychological Monographs, 79 (1965); Erlick and Mills, "Perceptual Quantification," 9-14.

VI. FUTURE RESEARCH

Figure 6.1 illustrates the variables that were tested in relation to possible combinations of test variables. The lack of completeness is conspicuous. A randomized selection may have been more appropriate. On the other hand, the present experiment was designed to provide a foundation for future research. To answer all questions raised is of course desirable, but it is equally interesting to have raised additional questions. A number of suggested research topics come to mind.

(A) Graphics

Dot, choropleth, and profile graphics were involved in this study. Within each graphic only one of many possible combinations of variables was presented. For example, dot size and unit-value/dot could be varied to produce different graphical effects. Different choropleth intervals and shadings should lead to different subject response. Altering the vertical exaggeration and incorporating vertical profiles would change the appearance of profile maps. Changing overall map dimensions might yield interesting results. The point is that before sweeping generalizations can be made about the effectiveness of these graphics in transmitting mapped information, a more thorough testing of these map variables will be required.

A much wider range of graphics is also available for testing. Although dots, choropleths, and profiles are representative of point, areal, and line symbols, respectively, they are but a few of the graphics available to the cartographer. The sort of map evaluation performed in this dissertation should eventually be extended to include topographic maps. The evaluation procedure would have to be modified considerably to take into account how people use topographic maps in practice. That is, measurement on topographic maps (e.g., elevations, areas, distances, etc.) seems a somewhat different use of maps than visual pattern analysis of thematic maps.

TABLE 6.1

VARIABLE MATRIX FROM
FACTORIAL DESIGN

GRAPHIC/ COMPLEXITY	Dot Linear	Dot Quadratic	Dot Cubic	Choropleth Linear	Choropleth Quadratic	Choropleth Cubic	Profile Linear	Profile Quadratic	Profile Cubic
Dot Linear	■								
Dot Quadratic		■							
Dot Cubic			■						
Choropleth Linear				■					
Choropleth Quadratic					■				
Choropleth Cubic						■			
Profile Linear							■		
Profile Quadratic								■	
Profile Cubic									■

■ Tested graphic/complexity combinations

Subjects should also be tested with mixed (i.e., unlike) graphics. Either the displays to be compared to a referent may be of a different but common graphic, or a large number of graphics could be represented in a single set of maps to be compared. Such information might provide useful guidelines in those common situations encountered in research in which maps to be considered are not available in a single graphic.

The concept of apparent value scaling warrants more attention.[1] it is possible that maps can be scaled in such a manner that perceived map information is nearly identical with the original data that were to be displayed. If this could be attained it would represent a major step forward in graphic communication.

(B) Complexity

Three degrees of surface complexity were evaluated in the present experiment. There are a myriad other surfaces of a linear, quadratic, or cubic nature. A test of only one of each type does not warrant making broad generalizations about subject response. The degree of complexity can also be extended to include far more complex surfaces, that is, those approximating more closely to a random distribution. It is expected that subjects' ability to work with increasing complexity is limited by the type of graphic utilized as well as by the inherent nature of the perceptual mechanism.[2]

It would be of interest to compare maps of simple, yet mixed complexities. For example, what is the effect of comparing a linear to a quadratic surface, a cubic to a quadratic, and so on? The main point is that the distributions tested were artificial "degradations" of a base map. An actual map comparison situation would probably utilize quite different displays. Thus, it is imperative that more "real looking" surfaces be tested as well.

(C) Test Design

A rank ordering procedure was used in this study. Several drawbacks inherent in rank data deserve mention: (a) all judgments are not independent due to a progressive reduction in alternatives; (b) errors must occur in pairs; and (c) errors are propagated throughout. There are several advantages of a ranking procedure that help compensate for its deficiencies. It closely approximates the speculated map comparison task;

it requires a minimum of testing effort and cost; and it generates data that can be analyzed with little difficulty. Previous researchers have utilized: (a) numerical estimates of r from paired comparisons, (b) scaled estimates of r on a $\pm$ 1 continuum from paired comparisons, and (c) a triadic ranking design. The experimental designs of McCarty-Salisbury[3] and Dent[4] (i.e., a and b above) were felt to be inadequate in previous discussions. Olson's cartwheel design deserves further consideration.[5] It is in fact far more powerful than a full ranking procedure in that it possesses maximum error potential. In addition, a pair-wise comparison may well be the basis of a full ranking procedure even though it is not explicit. On the other hand, the triadic design requires a great deal of extra testing and analysis effort for a questionable increase in information content. In conclusion, the procedure of pair-wise comparison is worth further attention.

In any event, the goal of the researcher should be a test design that most closely approximates the actual map interpretation process. Since the proper technique of map comparison is not known, one can only speculate on what the best experimental design might be. A rank ordering was chosen for this experiment. Several additional procedures are worth mentioning. For example, if a standard plus n stimuli were displayed in loose-leaf fashion, subjects could be instructed to select and remove the 1/n stimuli judged to be most like the standard. With no replacement, the subject would then remove the 1/n-1 display most like the referent, and so on. As another example, it might be interesting to present a large number of maps and ask subjects to pick the n displays that "add up to" the referent map. This procedure is based on the notion that a complex surface can be broken down into a number of lower degree (i.e., less complex) distributions.[6] Instead of thinking in terms of linear, quadratic, cubic, and other components of a continuous distribution, the design might be founded on the assumption that we can think of a pattern of discrete points as consisting of random, regular, and clustered component distributions in varying relative proportions. Can subjects choose regular, random, and clustered map distribution components such that they "add up" to a referent display? Or, in another test altogether, can subjects make the distinction between ran-

dom, regular, and clustered point distributions?[7] Visual judgments could be evaluated against cell counts and nearest neighbor analysis. In this same context it would be interesting to attempt to discern the characteristics of the "cartographic filter" operating in the mind's eye. In general, what speculations can be made regarding visual or mental filtering of map pattern information? The object of research might be the study of correlation among numerous spatially distributed phenomena, among which no variables can be specified as independent or dependent. In this case, partial correlation methods are appropriate. The idea is to measure that part of the correlation between variables X and Y that is not simply a reflection of their relations with variable Z. Cox asserts that "while graphical co-variation has offered a cartographic equivalent to the statistical coefficient of correlation, there is no such cartographic equivalent to the coefficient of partial correlation."[8] Thus, regardless of the fact that high partial correlations might be expected even with comparatively low coefficients of correlation, variables have often been rejected in cartographic analysis if no obvious co-variation with the dependent variable is evident. This criticism of cartographic analysis should be investigated. As another example, researchers might seek to understand the nature and strength of the relationship between two sets of variables (i.e., maps). Given sets of maps $y_1 \ldots y_n$ and $x_1 \ldots x_n$, the question is what combination, x_i, of maps $x_1 \ldots x_n$ best predicts the common trend, y_i, of the set of maps $y_1 \ldots y_n$. The techniques of canonical correlation and canonical trend-surface analysis provides an appropriate statistical model in this situation.[9]

The method in which maps are presented may influence test results. Map comparison studies have utilized flash cards of map pairs, map pairs projected from slides, loose-leaf map pairs and map sets, and a triadic (A, B, C) flash card design. Review of the literature indicates that map patterns are often superimposed under the assumption that this technique aids detection of correspondence. Maps to be compared are often presented in a number of positions; that is, side by side, or one above the other, on a single or on separate pages. The scale and orientation of maps presented for comparison are in many cases inconsistent within

a single article. These situations undoubtedly influence in various degrees the ability to compare the maps. The various procedures could be easily replicated in experimental designs. Sequential presentation of maps to be compared might yield interesting results in terms of "pattern memory" or recall of pattern characteristics. Mental pattern generalization or smoothing would seem to be involved. Along somewhat different lines but still related to the notion of "pattern memory" is the interesting research with "mental maps."[10] This approach may provide valuable insight toward further understanding of pattern memory.

(D) Increment

The increment (Δr^2) between maps to be compared obviously has a great influence on subjects' response. It would be of interest to test in greater detail the relationship between Δr^2, map graphic, and map complexity. It seems likely that these three variables have a balancing effect on each other.

(E) Data

Present data were artificially generated and contained several statistical attributes that "upset" subject response trends. In particular, several empty cells and several randomly misplaced high value cells were involved. These perturbations provide insight into the perceptual weighing mechanism, thus indicating a technique in which this type of research could be pursued. Small, controlled tamperings with the data might evoke the desired perceptual results yet not significantly alter the overall statistical attributes of the data. On the other hand, these "upsets" are undesirable when looking for overall effects. It would be an equally simple task to "clean up" the data, since the several alterations would be small and not have a significant effect on statistical parameters of the data.[11]

(F) Statistical Models

In the present and previous studies researchers have sought a statistical model that approximates closely the response pattern in visual pattern comparison. The coefficients of correlation r and τ, and, in this study, the coefficient of determination, r^2, have been utilized. Yet the range of statistical measures of association is nearly unlimited. Since

the actual choice of a model in large part determines the meaningfulness of results, the difficulty is to make the "most appropriate" choice from the multitude of available measures.[12] Important as they might be in the evaluation of the adequacy of models, mathematical simplicity and computational convenience are not substitutes for theoretical justification.[13] The main point is that there are many so-called objective (i.e., repeatable) measures for comparing maps, each based on certain assumptions and satisfying specific requirements. Any one of the numerical models might be compared with a visual estimate of the correlation of the same pair of patterns. The correspondence between the calculated and the estimated statistic would serve as a measure of perceptual accuracy. An interesting point is that almost all attempts to formulate quantitative measures of pattern comparison use visual judgments in evaluating whether a measure yields intuitively plausible results.[14] In any case, the problem is to ascertain which ones are the most suitable in a given situation. Is one technique more applicable than the others, or is it even reasonable to liken the process carried on in the human mind to a series of arithmetic operations? From the outset one might speculate that a model based on the assumption of spatial dependence (i.e., cross-spectral correlation) would be more meaningful than one with a spatial independence assumption (e.g., simple cross-correlation). Since the coefficients r, Υ, and r^2 are all founded on the assumption of spatial independence, an important research area has been neglected. It is possible that optical computers could aid this inquiry with tremendous savings in time and cost.[15]

In any case, because subject judgment does not conform to a particular statistical model should not be grounds for dismissing visual pattern comparison altogether, as some people have done using the McCarty-Salisbury results.[16] On the contrary, such information should be used in the search for a more appropriate statistic. The goal is to discover an underlying scale, starting with as few pre-conceived notions as possible. From this point of view the analysis of proximities approach documented in multi-dimensional scaling literature might be useful.[17]

Olson's use of Υ represents the sole attempt to work with apparent pattern, as opposed to the statistical attributes of original data. Re-

search of this nature should be continued and expanded. Olson is presently continuing work along these lines.[18]

(G) Learning

On an a priori basis it is expected that training and experience exert a direct influence on subjects' ability to visually compare maps. The problem arises in trying to define training and experience in a relevant manner. This author interprets these terms to mean: (a) knowledge of the statistical notion of correlation, (b) background in methods of cartographic processing of statistical information, and (c) familiarity with techniques applicable to the analysis of pattern association between pairs of maps. Previous map comparison studies have utilized a much more general notion, "years of geographic experience," as an index. This author concludes that their results must be considered inconclusive for several reasons: (a) in light of most geographic training programs there is little theoretical basis for using "years of geographic experience" as an index of pattern recognition experience, and (b) it is extremely difficult to distinguish between training and task motivation, even if training could be operationally defined. The degree to which lack of either phenomenon is responsible for errors is unknown. Experiments incorporating levels of control far beyond those achieved to date will be required to obtain this information.

There appear to be two approaches toward research of the notion of "learning" in the context of map reading. First, groups of subjects representing varying backgrounds could be tested in an attempt to determine which sort of training is most valuable. All of the problems outlined above must be considered. An alternative approach is to attempt to train a group of subjects in the task of map reading. The program could draw on the literature of cartography, perception psychology, learning theory, statistics, and other fields believed to be helpful. Observations from the present research might be of value. Progress could then be observed by repeated testing throughout the training period. Whatever the approach, if it can be established that knowledge of certain subjects or techniques substantially improves map interpretation, great potential for cartographic training will have been exposed.

(H) Image Processing

Cartographic display is a technique of making certain types of information visible. Since there are still no operational automatic pattern recognition systems capable of map interpretation,[19] visualization is usually done for the benefit of human users, who are particularly well equipped to assimilate and appreciate spatial patterns.[20] Yet the human image processing system is not infallible. For example, it was asserted that pattern characteristics had a significant influence on perceived correlations in the present research. Authors of previous map comparison studies came to a similar conclusion. In particular, interaction appears to have existed between different elements on the perceptual surface. Dot density, tone density, blank areas, and so on, were mentioned above as possible sources of weighting errors. If this is the case, the next step logically is to try to determine the characteristics of this interaction. It may be that the most intriguing part of the present research is not the quality of information retrieved from maps under different conditions, but rather how the map reader went about his search for the required information. Erlick and Mills ". . . concluded that S's estimates appear to correspond to a simple summation-of-discrepancies. . . rather than a squared discrepancy model underlying statistical correlation."[21] This could be tested further. Sekuler and Abrams demonstrated that subjects adopt distinctive modes of pattern recognition as a function of pattern complexity. In particular, patterns tend to be processed as wholes when the complexity of the pattern permits, that is, when it is simple enough.[22] Comments from subjects in the present experiment are suggestive as well. Research on pattern recognition and eye movement parameters may provide an interesting approach to study of visual image processing.[22] A large area of potentially fruitful research has been largely ignored by cartographers.

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VII. IN RETROSPECT

Two objectives were pursued in this dissertation. First, the role of maps in scientific argument was discussed, paying particular attention to the nature of models and graphics. It was concluded that inherent characteristics of the map, in combination with perceptual distortion on the part of the map reader, make "complete accuracy" an unlikely result of the cartographic process. This contention tends to support the quantifiers' claim that reliance on cartographic analysis has been detrimental to the "scientification" of the disciplines involved. Yet it was emphasized that there are a multitude of equally correct cartographic interpretations of a single data set; thus the inadequacy of one or several graphic techniques is not sufficient cause for general rejection of maps. The conclusion was that maps are valuable in inductive hypothesis formulation and model-refinement, but are not sufficient deductive, model-testing devices.

This being the case, it is suggested that a logical extension of the computer storage of spatial data, as opposed to map storage of such information, would seem to be machine analysis of distributional characteristics and relations with precise results presented in textual notation. Maps would then be used for more schematic, less complicated, and more easily interpreted presentations. The schematized design format would orient the map toward producing a forceful impression. The map would serve in a hypothesis-generation capacity and would be used primarily to illustrate and convince the reader that quantitative conclusions enumerated in manuscript form were indeed credible. A significant reduction in map construction costs and an equally significant increase in information flow would probably occur. The map generalization problem is directly related to this theme, for increasing the level of generalization may result in an equally significant increase in map comparability. The main point is that special purpose maps must sacrifice application neutrality in order to gain optimal

application effectiveness. Researchers have been reluctant to adopt this concept.

Secondly, an attempt was made to evaluate experimentally several cartographic forms to determine whether information was in fact more accessible to human viewers in one than in another. This research was pursued with the motivation that understanding of visual map interpretation will make it possible to assign quantitative values to the merits of the visual method, thereby confirming or dismissing intuitive and qualitative evaluations. More questions were raised than were answered. But it was observed that distribution complexity, type of graphic, and degree of association all interact to evoke diverse trends in subject response patterns. Although the questions asked are not identical, the present conclusions definitely refute the widely quoted McCarty-Salisbury results. Present findings seem to agree with the data obtained by Olson in spite of differences in methods and conditions. In all cases, research was designed to be repeatable by anyone who cares to acquire the maps and do the testing. That is, results can be easily checked or referred to for comparative studies.

In conclusion, it is appropriate to ask whether the results of the present research could be used to increase significantly the transmission of information in visual pattern comparison situations. Would this increase be sufficient to satisfy present and future data processing demands? This consideration would not be complete without mention of the potentially powerful and promising developments in automated systems of pattern recognition and analysis utilizing electronic or optic scanners and computers. If the many conceptual and mechanical problems can be solved, then the speed, accuracy, and flexibility of an automated system of pattern analysis would appear to provide one answer to contemporary and future demands. However, automatic processing is not expected to replace human viewing. The day is not foreseeable when man will entirely yield map interpretation to the machine. But it is certain that together they would make a good team; with machines handling computational problems and humans performing evaluation processes. Automatic processing would be used for highly accurate information demands and human viewing for quick reference. Thus, regardless of developments in automation, we still must investigate the problem of pattern comparison from the human point of view.

LIST OF APPENDICES

		Page
A.	Map Compilation Procedure	85
B.	Data	96
C.	Map Test Sets	115
D.	Confusion Matrices	152
E.	Corrected Map Ordering	162
F.	Tables (Miscellaneous)	166

APPENDIX A

MAP COMPILATION PROCEDURE

Data -- Since the Data Simulation Program could only generate six variable pairs (i.e., one independent and up to six dependent variables) in a single run, and since 21 such pairs were needed to make up the complete range of desired r values, data were generated in four complete runs. The four data sets were combined only after the author was satisfied that the statistical attributes of the four independent variables were sufficiently similar to justify such action. At this point several reasons suggested that economy in variables was desirable: (a) correlation values approaching 0 were not especially interesting in the context of the role of maps in scientific research; (b) fewer displays per test set was thought to contribute to a more effective testing procedure; and (c) fewer displays would result in a saving in the cost and labor of map compilation. For these reasons the number of variables was reduced from 21 to 17 by eliminating the ($\pm$) .30 and ($\pm$) .10 r^2 values (map pairs).

Data were punched on computer cards, a listing of which can be found in Appendix B. One row of observations, consisting of an independent plus six dependent variables, was recorded on each card.

Map compilation -- The same technique was used to create each distribution format. First, the elements of the number sequence 1 . . . 256 were assigned to a 16 x 16 cell matrix in the manner shown in Tables 1, 2, and 3. The idea was to produce a progressive increase in values with distance from the center(s) where lowest values were plotted. Next, the values of the independent variables were rank ordered and assigned to the cells of the square grid according to each pre-determined spatial format. For example, the rank # 1 value was positioned in cell # 1, the value with rank # 2 was placed in cell # 2, and so on. The plotting procedure was executed by numbering the rank ordered cards sequentially from 1 to 256.

TABLE 1

MAP FORMAT A

1	3	6	10	15	21	28	36	45	55	66	78	91	105	120	136
2	4	8	13	19	26	34	43	53	64	76	89	103	118	134	151
5	7	11	17	24	32	41	51	62	74	87	101	116	132	149	165
9	12	16	22	30	39	49	60	72	85	99	114	130	147	163	178
14	18	23	29	37	47	58	70	83	97	112	128	145	161	176	190
20	25	31	38	46	56	68	81	95	110	126	143	159	174	188	201
27	33	40	48	57	67	79	93	108	124	141	157	172	186	199	211
35	42	50	59	69	80	92	106	122	139	155	170	184	197	209	220
44	52	61	71	82	94	107	121	137	153	168	182	195	207	218	228
54	63	73	84	96	109	123	138	152	166	180	193	205	216	226	235
65	75	86	98	111	125	140	154	167	179	191	203	214	224	233	241
77	88	100	113	127	142	156	169	181	192	202	212	222	231	239	246
90	102	115	129	144	158	171	183	194	204	213	221	229	237	244	250
104	117	131	146	160	173	185	196	206	215	223	230	236	242	248	253
119	133	148	162	175	187	198	208	217	225	232	238	243	247	251	255
135	150	164	177	189	200	210	219	227	234	240	245	249	252	254	256

TABLE 2

MAP FORMAT B

253	251	245	231	207	194	187	152	156	193	199	208	230	234	248	255
247	226	218	201	176	144	129	125	137	149	170	178	180	214	222	242
240	216	172	164	133	117	109	101	105	113	121	135	162	174	219	236
233	192	160	97	89	81	73	65	69	77	85	93	99	166	203	228
223	183	140	95	61	57	45	37	41	49	53	63	92	142	190	211
212	168	123	87	55	33	27	21	19	25	35	58	83	119	185	210
205	139	115	79	51	29	13	11	7	15	30	47	76	112	146	209
158	131	107	67	39	17	5	1	4	6	24	44	72	104	128	154
155	127	103	71	43	23	9	3	2	10	18	40	68	108	132	159
195	147	111	75	48	31	16	8	12	14	32	52	80	116	138	196
197	167	120	84	60	36	20	26	22	28	34	56	88	124	169	202
206	186	151	91	64	54	50	42	38	46	59	62	96	148	184	237
227	191	175	100	94	86	78	70	66	74	82	90	98	173	215	239
241	213	181	171	143	122	114	106	102	110	118	141	165	182	225	246
250	220	217	198	163	150	136	130	126	134	145	161	177	221	238	252
254	244	232	224	200	188	179	157	153	189	204	229	235	243	249	256

TABLE 3

MAP FORMAT C

170	144	128	103	89	102	123	149	166	189	209	237	240	248	254	256
154	111	84	59	43	73	78	116	164	185	208	230	233	245	250	252
109	96	51	37	19	35	55	75	141	176	202	226	229	232	243	246
131	69	27	11	3	15	31	65	132	173	197	212	217	220	239	242
92	49	23	7	1	9	25	47	99	183	195	198	203	213	219	236
112	63	33	17	5	13	29	71	147	187	192	177	171	179	191	204
125	94	57	41	21	39	53	97	136	157	162	153	117	140	160	184
143	104	80	67	45	61	83	110	133	145	120	90	100	106	139	152
167	159	115	119	88	124	135	134	121	108	68	48	62	77	113	150
175	174	148	142	163	158	146	126	85	54	32	22	40	58	72	127
201	186	182	181	188	196	161	129	66	28	14	6	18	36	64	107
216	207	194	190	200	199	172	101	46	26	10	2	8	24	44	91
234	222	225	206	210	205	178	118	70	42	16	4	12	30	76	122
214	228	223	221	224	211	180	151	95	56	38	20	34	52	82	137
251	249	244	227	215	218	193	165	105	81	74	50	60	79	86	156
255	253	247	238	235	231	214	168	155	114	93	87	98	130	138	169

Using each spatial format, the author then began with the first column in the first row and proceeded to the last column in the first row, picking up the approximately ranked cards. This procedure was repeated for rows two through sixteen.

In light of the relationship between the independent and dependent variables, it should be clear that once the independent variable is plotted, the formats of the dependent variables are specified. Data matrices were constructed next by arranging seven square grids (16 x 16) labeled variable 1 through variable 7 on a table, and then, starting with the first card (column 1, row one), recording the 7 variable values in the appropriate cells on their respective grids. This was repeated with cards 2 through 256, at which time the cells of the seven matrices were filled. The procedure was repeated with each of the four data sets.

At this point the data were numerically mapped in matrix fashion. For identification purposes the 4 data sets were numbered as follows: set 1 (Format, +.80, +.60, +.40, +.20, 0); set 2 (+.90, +.70, +.50); set 3 (-.90, -.70, -.50); and set 4 (-1.00, -.80, -.60, -.40, -.20). The ± numbers in ( ) indicates the r^2 values included from each set. The ± signs are valid only as indicators of the sign of the original r values. Random numbers from 0 to 9 were assigned to each set of r^2 values according to the following scheme: +.90 (r^2) = 6 (random #), +.80 = 1, +.70 = 9, +.60 = 3, +.50 = 8, +.40 = 0, +.20 = 9, 0 = 6, -.20 = 6, -.40 = 4, -.50 = 2, -.60 = 5, -.70 = 1, -.80 = 3, -.90 = 8, and -1.00 = 0. The linear, quadratic and cubic distributions were labeled A, B, and C, respectively. The proper alpha-numeric code of the form A12 was placed at the top of each display. In this example (A13), the code specifies, from left to right, the distribution type (A), the data set (1), and the random number (3) assigned to a particular r^2 value. This identification device was useful in tabulating test results in a latter phase of research. To insure that the code would not influence subject judgments it was covered with removable black Prestape.

Three graphic techniques were employed in construction of stimuli displays (maps): dots, choropleths, and profiles.

Dot maps -- The most commonly used point symbol is the dot. Distribution maps utilizing dots of uniform size, each having a specific value, are simple and useful. Their wide popularity is attributed to: (1) ease of construction; (2) the confidence felt in the method, due primarily to the simple, obvious underlying construction principle; (3) their ability to show distributional variation well; (4) the visually impressive and informative result; and (5) the ease of comparison with other distributional maps, particularly those of the isotherm group.[2] It should be noted, however, that the last three attributes mentioned above, namely (3), (4), and (5) have yet to be substantiated through rigorous testing.

The success of dot maps is dependent upon several variables: (1) the value represented by each dot, (2) the size of the dots, (3) the size and shape of the statistical units, (4) the placing of the dots, and (5) the statistical nature of the distribution. Most standard cartography texts[1] and a number of journal articles[2] have dealt with these problems. Suffice it to say that, in spite of conflicting statements in the literature, it is generally accepted as desirable to maximize variation of the ratio of "black" dots to "white" background. This involves manipulating the interdependent variables (1), (2), (3) and (5), mentioned above. In practice, where compilation effort is a principal concern, the "theoretically optimum" display is seldom attained. In the present study the large number (51) of maps required and the limited time available for compilation set practical constraints on statistical unit size, dot size and value/dot. An 8 cm x 8 cm square array of 256 cells was used (Tables 1, 2, and 3). The absolute value of each cell was divided by an empirically derived constant (3) to determine the number of dots to be placed in each cell. Using a #3 Leroy pen, dots were visually placed as uniformly (but not regularly) as possible within each cell. This procedure was perfectly reasonable since the data were artificially generated and located only as to cell.

Completed displays were photographically reduced by 1/2 (= 1/4 area) and reproduced by offset lithography (Appendix C).

Choropleth maps -- Quantitative areal maps are widely used.[3] This map class has been divided into those that indicate quantities by lines of equal value (isograms) and those that depict average values per unit area (chorograms).[4] A subset of the chorogram group, known as choropleth maps, was chosen to represent quantitative areal maps in this experiment. In this technique areal symbols (shadings) are applied to existing statistical units to make up density categories. A review of the several variables which contribute to the effectiveness of choropleth maps should clarify the choice. These include: (1) the geometric nature of the statistical units, (2) the statistical nature of the distribution, (3) the number and scaling of density categories, and (4) the tone density of category shadings. Although extensively dealt with in the literature,[5] solutions to the problems surrounding these variables are by no means definitive. Yet the present study should be evaluated with the above-mentioned variables in mind. The uniform shapes and sizes of the statistical units (i.e., grid cells) tend not to yield natural, as opposed to artificial, patterns on density maps and thus produce quite effective choropleth maps. The smoothness of the format displays (i.e., the small range of local variation) contributes to a minimum degree of overall generalization of the distribution in map form. In addition, as maps become less correlated with their format, the pattern depicted becomes progressively fragmented. Choropleth mapping preserves this character well. Five density categories were chosen on the basis of desired map detail and in light of knowledge of asserted map reader capabilities.[6] Category boundary scaling was derived by summing the values of the 256 grid cells for each map and dividing the total by five. Beginning with the lowest value, the rank-ordered cell values were then progressively resummed until the boundary values were reached. This procedure was only approximate since the calculated interval boundary invariably fell between the accumulated total X_i and the accumulated total X_{i+1}, where $X_{1...n}$ represent cell values. In practice, this meant that the proper interval value was exceeded by some portion of the last considered cell value, or, in the case of

multiple equal-valued cells, by some portion of the summed value of these cells. In no instance was this discrepancy considered significant, however. Once the individual map density intervals were established, a range of tone densities was chosen on the basis of research on the grey-scale. Information suggests that approximately equal apparent density value shading increments are best suited for effective map reading.[7] But the five class system and available materials complicated selection of equal density intervals, especially since the writer chose, for intuitive reasons, to avoid both black and white categories. In compromise, Zip-a-Tone Coordinated Dot Screens with percentage area inked of 10 (32 1/2 line ordering system), 20 (27 1/2 line ordering system), 40 (27 1/2 line ordering system), 60 (27 1/2 line ordering system), and 80 (27 1/2 line ordering system), seemed to provide an acceptable approximation to suggested screen patterns.

In spite of the fact that category boundaries are generally accentuated by inking on choropleth maps, the author decided against the practice. The belief was that boundaries would tend to fragment apparent gradient visualization, particularly as map patterns became more random.

Completed maps (16 cm x 16 cm) were photographically reduced by 1/2 and copies produced by offset lithography (Appendix C).

Profile maps -- Development and use of 3-D cartographic models (displays) have long excited researchers who work with maps.[8] In addition to becoming a source of considerable realism and interest, the illusion of depth also has important applications in the visualization of scientific data.[9] The perspective case of planar projection appears most effectual and is a common method of representing a function of the form $z = f(x,y)$.[10] Such 3-D depictions are especially valuable in cases where precise planimetric measurements are not required. In this instance the distortion in original map planimetry is acceptable in order to gain better qualitative areal description.

The mathematical function $z = f(x,y)$ completely describes a statistical surface. In perspective drawing the idea is to make a selection of points that will conveniently and effectively represent the infinity of points on the surface (representing the function).

A 2-D rectangular array is most convenient for this purpose. The row and column values yield two families of orthogonal generating curves (profiles) which generally depict the function in a well defined and aesthetically pleasing perspective view. The profiles tend to portray the "look" or "feel" of the surface to an extent unattained with many other cartographic techniques.

An efficient method of plotting an oblique projection of a 3-D surface was available in the form of a computer plotting program.[11] The method consists of: (a) computing points of the surface along equally spaced parallel cuts (rows and/or columns), each displaced to establish the desired projection; and (b) drawing oblique profiles (consisting of straight line segments between adjacent points of each successive cut) along the rows using the University of Michigan Plotting System Library (CALCOMP PLOTTER). Although angular perspective projection provides a more realistic (less distorted) depiction of a surface and is therefore more pleasing to the eye than oblique perspective, the latter projection is the simplest representation and possesses the desirable feature of having the near face undistorted.[12]

Data for the present study were appropriately organized in a 16 x 16 square array. Row values of this matrix were used to construct oblique horizontal profiles. Hidden portions of profiles were deleted to aid interpretation. Knowledge from previous study of aesthetically pleasing plots was utilized in setting the vertical exaggeration and perspective angle.

The CALCOMP plots were photographically reduced by 1/5 and copies made through offset lithography (Appendix C).

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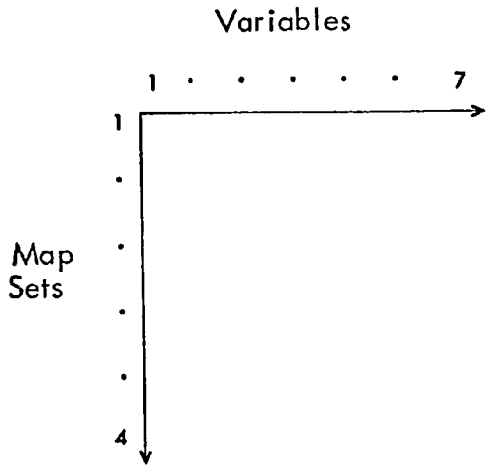
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APPENDIX B

DATA LISTING



Note: Variable 1 of each set is the independent variable for that set.

R	3.602	9.964	7.865	24.662	21.358	40.726	93.244
T	5.149	8.234	4.712	28.869	16.687	40.709	83.317
U	5.513	17.323	14.705	8.000	38.879	30.334	92.422
V	7.180	20.160	23.461	5.690	43.464	44.159	53.019
B	9.567	8.028	-9.467	35.040	11.684	23.953	59.523
K	9.594	20.584	11.108	34.769	56.138	57.587	74.191
AA	10.264	12.807	25.430	22.293	13.462	42.202	81.885
G	12.890	10.381	20.859	36.154	51.589	27.099	67.493
O	14.254	15.210	15.423	30.130	19.112	37.452	44.044
S	15.120	8.037	17.177	32.391	4.540	32.161	50.589
C	15.248	10.688	34.063	25.872	50.569	65.273	51.874
S	15.533	15.135	31.395	16.606	24.721	79.975	62.277
I	15.857	12.214	13.501	22.021	43.471	41.430	56.646
CC	15.932	5.523	30.938	42.902	27.786	33.931	64.961
CC	17.474	17.967	5.071	24.945	54.282	69.869	89.611
I	17.594	18.977	68.021	28.605	41.395	72.428	57.729
U	17.802	26.011	13.329	18.135	4.722	76.619	74.605
A	17.940	24.016	42.985	40.644	33.415	71.203	72.410
E	18.361	21.694	9.802	30.601	32.174	60.139	77.371
F	18.572	32.702	19.375	26.377	46.464	42.898	66.031
J	19.797	31.880	16.953	33.572	30.452	64.038	57.115
DD	19.767	39.034	10.243	42.861	66.743	63.010	45.860
U	20.504	20.583	10.405	27.687	21.643	27.038	63.599
T	21.083	22.538	20.568	25.386	25.181	94.686	79.976
E	21.662	17.801	27.260	14.146	35.901	81.622	78.485
C	21.896	30.733	42.275	27.930	33.331	41.923	73.459
A	22.791	10.048	22.504	20.463	38.222	43.041	81.499
K	22.979	32.820	32.385	43.923	48.859	60.206	81.291
W	23.034	31.971	39.926	55.959	31.580	67.770	34.593
W	24.077	39.963	44.490	42.663	33.559	51.745	97.412
Y	25.226	33.089	22.811	54.658	31.265	47.553	54.009
J	25.769	18.154	37.201	55.899	37.802	54.362	85.557
X	25.981	22.787	15.478	29.912	18.237	40.059	51.707
P	26.414	33.308	46.051	13.375	59.126	50.194	64.322
FF	26.912	43.553	26.605	13.298	43.446	40.149	77.084
S	27.030	20.683	33.780	39.990	44.974	22.390	60.807
Z	27.346	27.114	41.639	37.428	39.387	56.531	90.828
Z	27.516	28.341	51.200	32.043	54.128	22.967	67.237
BB	27.884	13.437	43.902	42.557	46.427	15.190	44.438
P	28.006	20.111	32.997	45.919	42.102	73.399	23.086
N	28.068	31.776	42.625	29.157	53.417	78.113	61.129
M	28.070	33.007	25.854	27.017	27.116	53.143	57.451
X	28.175	22.044	25.888	16.295	26.950	28.500	41.976
BB	28.408	30.834	41.232	34.103	11.011	60.311	55.452
G	28.674	26.688	24.591	43.098	28.780	87.373	22.725
J	28.742	31.726	28.614	30.235	19.968	90.588	71.959
Y	29.374	38.160	14.745	42.314	41.519	37.110	82.454
AA	29.623	43.267	19.290	64.577	46.171	67.075	70.738
E	29.775	30.705	20.731	41.080	47.043	34.165	61.220
V	30.160	28.865	44.439	18.183	46.878	51.979	62.418
I	30.574	35.468	39.232	22.752	26.112	13.568	63.223
C	31.451	23.587	29.566	58.636	32.499	5.252	85.336
O	31.439	42.328	39.291	29.389	46.675	59.349	60.826
D	31.677	15.503	29.145	31.763	43.814	20.696	23.425
E	31.854	26.653	30.678	35.712	45.689	22.033	70.606
Y	32.437	32.982	35.198	41.257	32.353	53.044	33.773
FF	32.965	32.816	45.112	31.648	42.356	70.429	38.947
N	33.762	30.730	34.812	35.485	40.162	31.745	23.590
Z	33.744	32.567	36.522	46.679	36.280	42.985	90.484
K	33.944	45.395	39.686	43.124	4.690	82.766	43.738

S	34.255	36.744	44.623	57.193	18.689	75.952	54.160
V	34.345	37.664	61.035	11.940	79.434	32.836	36.210
BB	34.393	40.433	21.539	40.235	48.366	42.472	44.014
O	34.973	31.906	41.438	11.579	49.760	70.370	51.556
M	35.010	20.653	38.555	34.247	66.530	40.637	40.283
O	35.328	41.400	40.509	38.575	65.796	51.001	52.567
D	35.574	39.439	36.014	41.708	41.644	100.249	47.942
F	35.671	35.356	21.187	14.837	79.482	14.320	37.614
W	35.944	47.903	31.103	45.708	11.803	57.242	51.569
AA	36.057	36.456	29.701	36.694	79.065	57.303	41.290
R	36.026	38.165	37.542	50.317	32.767	57.101	86.634
J	36.105	36.324	34.340	39.187	14.576	61.705	52.171
A	36.319	38.124	50.871	18.226	32.073	53.597	58.828
H	36.648	38.928	44.514	45.855	41.995	72.244	68.221
H	36.658	40.432	18.522	51.591	62.377	58.701	68.822
D	37.245	20.637	40.342	24.994	50.029	34.304	68.301
D	37.246	40.034	35.101	48.527	45.832	39.150	81.577
E	37.370	34.257	43.129	50.098	46.046	20.878	48.644
W	37.377	36.191	48.341	55.062	45.294	51.389	54.998
C	37.792	38.220	37.517	16.016	49.578	63.415	60.715
H	37.860	46.814	48.884	33.900	29.627	57.286	40.345
V	38.142	27.841	40.777	33.922	54.454	59.535	35.479
E	38.454	46.121	51.832	16.724	45.600	57.581	71.870
N	38.800	37.468	34.084	25.951	29.295	36.645	31.402
D	38.805	41.511	37.739	62.966	46.036	40.851	68.210
L	38.973	38.747	58.475	37.695	39.640	58.231	44.631
EE	39.067	39.386	41.960	13.041	86.378	78.388	34.482
G	39.212	39.665	50.226	58.049	64.487	18.543	58.900
EE	39.387	34.051	32.461	36.942	30.597	62.031	52.638
H	39.528	38.725	54.817	43.588	77.909	29.455	53.988
K	39.992	62.052	53.524	22.134	36.982	69.292	48.318
I	40.057	47.608	47.057	20.775	52.584	58.972	48.037
Z	40.221	32.159	66.351	63.382	36.519	18.849	34.422
N	40.444	33.428	47.595	32.821	41.325	43.180	83.578
I	40.753	45.237	32.320	62.467	63.648	35.920	41.431
T	40.753	47.069	47.982	67.527	71.796	59.823	46.514
BB	40.923	31.595	28.808	44.738	31.513	53.260	42.564
K	41.330	36.404	19.293	53.317	4.475	57.694	54.434
AA	41.869	26.513	39.464	48.872	38.902	47.105	64.609
R	42.063	50.048	25.200	57.032	58.204	14.877	36.828
A	42.470	55.484	37.795	40.659	36.541	39.770	41.477
C	42.545	48.674	42.324	68.484	31.738	56.423	54.596
Z	42.727	56.345	38.927	44.703	27.696	44.219	73.401
X	43.066	42.787	25.174	76.783	67.030	72.211	43.551
K	43.143	37.579	28.145	40.448	43.903	42.211	61.102
N	43.522	37.962	36.708	45.605	66.336	70.762	46.579
M	43.718	36.384	47.496	26.711	29.359	61.901	78.760
F	43.785	42.425	48.307	16.074	50.766	54.756	74.115
Z	43.770	45.441	31.818	43.087	50.188	51.214	60.973
FF	43.832	43.461	43.935	54.545	64.901	80.195	42.362
F	44.289	45.808	50.165	57.758	29.802	49.271	52.537
W	45.058	34.230	60.979	32.264	17.216	47.342	65.869
F	45.088	48.213	50.742	41.304	63.223	47.962	51.558
G	45.172	41.019	37.389	52.754	36.099	31.012	55.504
R	45.893	40.112	70.292	65.890	9.541	58.627	49.910
D	45.910	43.862	56.587	47.542	50.561	51.030	72.357
CC	45.996	44.777	48.948	62.285	40.695	51.126	80.406
DD	46.192	45.782	41.358	29.392	22.314	13.916	65.575
Y	46.449	55.916	40.989	32.563	72.462	52.008	38.690
CC	46.831	39.873	42.850	52.356	40.337	37.117	56.282

V	47.185	31.491	46.297	70.270	55.116	42.031	60.124
H	47.198	63.113	47.694	33.621	38.238	62.520	46.293
O	47.289	46.389	49.431	24.846	49.384	58.265	46.003
X	47.729	42.308	47.162	33.054	65.135	51.523	56.685
CC	48.103	36.094	53.861	41.072	54.395	65.243	59.501
EE	48.395	44.485	27.779	44.064	61.105	61.084	49.025
I	48.387	61.867	54.220	32.777	54.991	20.776	42.387
V	48.668	46.094	42.999	59.290	86.301	51.841	51.756
X	48.793	36.948	24.702	63.100	59.265	76.918	62.203
G	49.003	45.206	69.744	46.314	60.572	40.813	57.969
P	49.275	43.560	29.244	54.555	41.812	61.470	38.265
O	49.552	47.390	57.599	32.017	67.820	25.362	47.404
A	49.626	31.299	61.771	13.536	59.322	25.460	67.963
E	49.879	48.599	35.725	57.122	39.169	63.582	53.394
O	50.013	66.709	58.114	58.468	54.893	61.021	58.673
W	50.148	57.247	27.638	46.977	58.668	49.189	55.559
CC	50.255	40.273	67.562	61.629	38.260	23.749	51.232
X	50.687	58.677	61.730	37.280	30.879	39.217	23.725
T	51.140	31.613	50.868	59.861	53.490	70.928	79.163
T	51.176	37.771	53.770	53.570	43.308	55.962	59.970
O	51.190	40.037	43.655	44.731	70.648	54.342	39.593
W	51.216	53.147	64.540	60.549	35.214	71.374	48.012
M	51.571	53.020	47.799	57.752	44.434	66.041	39.563
O	51.704	43.105	60.340	51.352	53.614	35.779	18.085
A	51.766	55.159	41.845	65.842	66.195	23.562	53.521
S	52.087	63.177	30.914	53.026	56.792	46.096	70.760
A	52.310	39.836	47.688	54.519	54.492	74.343	45.666
OO	52.303	48.923	46.079	67.766	59.483	46.105	65.859
K	52.326	53.118	44.833	58.110	57.253	65.786	40.876
Z	52.656	64.559	53.836	58.351	41.171	53.754	30.147
Y	52.758	41.847	43.191	56.159	40.438	58.199	28.713
U	52.893	51.028	72.610	52.394	55.795	46.257	56.041
R	53.075	55.084	31.786	40.467	50.782	41.072	57.570
T	53.278	70.490	30.406	27.897	29.096	43.698	55.268
FF	54.100	60.327	64.512	41.539	52.188	46.752	52.766
O	54.550	54.237	60.479	49.448	16.086	49.577	52.149
V	54.616	51.244	48.697	70.082	40.895	63.332	38.907
S	55.094	53.622	37.648	42.798	45.421	39.595	35.835
H	55.354	51.565	59.033	61.820	66.642	64.681	14.056
N	55.691	61.567	32.533	56.651	44.025	61.572	37.382
R	55.719	48.920	55.388	69.903	41.370	54.514	64.502
X	55.774	48.029	61.653	46.321	55.341	68.990	27.972
S	55.751	55.485	35.615	53.561	54.169	54.130	21.884
M	56.049	64.105	61.375	43.696	60.662	64.051	45.447
E	56.125	56.288	54.300	49.620	54.411	60.151	35.058
C	56.104	59.542	49.959	61.057	53.040	74.921	31.869
T	56.274	45.005	60.874	60.265	43.191	26.296	59.470
R	56.300	41.074	59.642	37.172	57.364	37.525	41.421
H	56.440	66.457	52.997	46.231	24.868	37.986	26.863
P	56.570	46.975	60.021	51.305	44.393	61.655	75.332
W	56.883	56.753	60.499	42.620	33.970	65.456	49.114
R	57.039	52.077	46.702	26.067	49.695	78.182	58.617
F	57.003	63.870	50.632	29.877	56.681	31.882	48.554
P	57.286	57.117	71.482	78.595	86.776	53.380	48.948
Z	57.643	42.171	39.634	41.028	40.531	36.566	20.341
D	57.738	50.435	28.219	44.019	48.752	25.476	29.229
I	58.019	56.776	43.674	38.329	50.039	79.850	81.513
BB	58.316	58.514	49.697	52.028	76.131	57.110	48.248
M	58.664	63.353	60.490	23.576	53.698	77.643	38.634
Y	58.746	45.316	70.477	48.207	36.473	42.851	36.284

BB	59.111	58.271	63.074	60.031	47.991	42.835	32.090
J	59.262	51.040	51.652	68.070	18.487	74.349	54.173
A	59.250	54.651	59.556	52.117	35.882	64.058	52.771
M	59.212	61.960	70.522	55.399	26.782	48.278	47.738
CC	59.395	58.769	58.595	73.319	54.038	34.152	65.547
J	59.706	68.699	40.270	48.031	24.746	53.376	53.298
T	59.805	58.821	55.158	47.628	35.032	42.996	66.442
T	59.852	66.919	66.134	40.109	49.658	64.201	10.386
P	59.992	40.988	59.340	65.122	55.415	24.035	60.833
T	59.968	55.308	48.439	65.409	68.686	15.945	65.220
R	60.198	70.650	58.732	63.998	36.736	38.875	44.356
EE	60.263	77.821	46.528	41.841	52.963	57.890	18.321
P	60.325	76.690	61.319	63.651	44.331	42.071	55.128
D	60.478	51.597	78.477	55.010	29.780	77.144	55.059
P	60.940	58.692	50.889	56.983	75.241	67.587	58.228
Y	60.914	63.370	40.595	64.457	84.463	50.538	53.834
G	61.116	49.118	58.282	40.897	54.027	33.500	81.947
D	61.210	62.269	44.517	45.344	39.316	52.267	24.232
N	61.359	41.363	56.278	83.445	55.565	32.997	30.619
EE	61.924	73.516	61.978	61.520	53.353	74.587	48.262
A	62.202	67.729	44.461	52.767	80.411	39.220	77.063
R	62.421	62.070	50.571	52.936	53.628	73.887	76.412
DD	62.811	56.190	40.992	69.612	46.152	76.176	51.245
EE	62.853	60.724	42.084	61.789	68.545	43.887	40.690
U	63.178	36.979	52.238	49.800	49.162	43.760	42.372
H	63.362	51.296	62.393	49.999	51.996	52.248	33.787
DD	63.468	61.640	46.016	59.648	58.203	59.800	34.374
K	63.744	48.779	26.463	59.158	42.484	56.423	28.914
P	63.796	51.356	52.998	43.148	57.990	29.541	34.888
G	63.942	63.231	54.615	62.911	33.308	57.471	50.656
EE	64.004	66.410	60.696	63.379	41.263	39.398	73.341
D	64.270	65.824	65.394	36.869	68.697	45.997	47.021
EE	64.434	64.853	50.712	40.495	68.983	44.445	55.065
C	64.635	79.966	63.431	71.586	18.114	62.866	17.547
U	65.133	58.624	58.933	76.780	61.784	16.639	18.577
EE	65.538	71.327	65.402	69.708	48.872	48.034	71.320
X	65.929	62.674	85.190	90.861	36.612	67.598	18.136
BB	66.163	69.966	56.335	54.519	43.859	57.589	26.046
EE	66.556	61.509	69.977	55.691	78.834	50.191	16.723
I	66.670	67.093	77.050	71.629	49.655	60.654	36.844
R	66.934	52.939	49.469	55.457	57.007	52.640	21.873
F	67.589	63.128	64.849	66.020	56.705	72.665	16.716
FF	67.643	63.523	47.849	75.103	68.311	57.332	30.095
M	68.366	53.088	32.356	64.723	69.012	53.140	27.758
I	68.578	64.999	61.696	54.226	45.950	54.652	68.043
J	68.643	69.499	64.416	67.245	79.274	66.381	41.486
Y	70.317	62.972	53.966	58.804	46.957	68.009	64.094
T	70.722	76.467	56.769	80.993	59.801	42.068	64.225
I	70.913	73.218	74.689	51.049	53.659	45.595	44.620
P	71.248	61.674	66.423	73.913	43.364	42.374	61.342
DD	71.711	62.822	57.863	83.861	71.428	64.884	63.980
N	71.770	63.482	67.340	79.170	44.404	54.417	40.901
T	71.808	57.088	70.601	70.731	35.947	68.724	52.059
O	72.310	64.887	48.332	79.367	60.572	55.824	51.971
C	72.489	63.030	50.795	48.927	58.057	56.694	41.517
I	72.472	71.605	42.978	56.982	50.577	53.188	13.573
U	72.659	72.102	49.459	70.853	80.202	45.553	49.086
AA	73.111	77.237	68.343	62.891	59.973	59.319	24.760
EE	73.598	69.177	77.416	42.199	53.390	74.133	73.971
F	73.819	65.526	78.473	63.782	69.186	41.995	44.569

BB	74.416	61.145	58.699	86.936	61.949	53.158	61.589
DD	74.730	72.398	69.146	54.081	85.163	67.366	49.043
CC	75.085	68.785	69.329	47.900	40.906	48.540	45.057
S	75.004	69.512	68.229	50.278	77.516	55.090	62.834
C	75.402	74.312	60.098	56.685	45.148	17.370	63.620
D	77.436	76.510	68.101	68.004	34.746	36.245	31.165
AA	78.704	67.618	56.836	57.654	31.296	52.056	44.922
DD	79.140	71.878	62.180	50.920	45.523	42.219	45.809
W	79.240	90.085	65.732	62.069	53.625	57.917	65.134
J	82.800	78.915	80.089	43.954	101.016	61.534	52.060
D	83.949	96.277	87.278	73.475	61.862	4.262	58.752
P	84.078	64.190	96.208	64.324	67.283	42.738	60.969
AA	84.375	76.609	86.438	87.575	68.962	38.587	23.933
H	88.801	93.518	74.421	51.768	68.508	80.520	25.883
R	94.758	84.600	100.671	77.765	69.906	71.047	41.239
AA	102.819	102.487	84.316	76.876	87.284	75.219	33.270
T	7.519	4.357	20.950	10.003	3.075	24.334	24.666
I	8.722	8.966	15.152	-4.767	23.829	24.644	32.418
K	8.925	8.705	17.897	22.136	26.156	19.239	-6.731
EE	11.166	16.620	10.325	14.764	21.589	14.955	18.896
I	16.181	17.554	30.419	11.619	29.475	3.163	15.232
R	17.644	18.956	18.395	48.282	12.688	31.829	15.139
H	18.106	19.980	17.184	28.904	28.456	37.452	14.832
E	18.201	10.999	25.909	32.314	33.715	13.053	14.346
M	19.353	15.477	18.944	30.112	28.671	45.423	41.383
H	19.561	22.018	44.223	15.692	26.417	19.956	21.870
D	19.814	21.695	23.354	38.109	44.978	5.754	36.715
CC	19.985	38.325	20.678	34.805	19.606	37.372	64.871
I	21.552	25.437	24.580	41.543	32.094	23.483	49.393
EE	22.549	31.184	24.029	35.718	34.672	1.062	27.206
A	22.757	33.525	31.099	47.691	37.492	35.201	34.436
DD	24.386	17.307	28.021	34.551	26.158	19.038	39.327
X	25.260	30.135	22.014	24.012	35.661	12.720	18.075
E	25.238	30.823	26.472	16.798	33.696	38.163	18.843
BB	25.885	29.489	14.953	20.048	32.702	57.487	12.101
C	25.883	24.551	18.571	37.145	24.238	29.045	43.884
N	25.847	18.181	26.126	28.605	20.145	28.471	51.075
CC	26.549	25.774	19.449	27.341	35.360	26.678	31.253
J	26.634	35.212	17.773	30.499	10.791	31.366	54.642
T	26.779	33.676	44.372	53.551	44.134	36.233	31.028
K	26.943	31.792	38.833	31.199	33.068	46.733	41.962
W	27.044	35.382	30.184	41.415	47.407	42.709	24.684
D	27.791	30.000	27.475	40.280	20.608	33.116	40.680
AA	28.081	17.751	43.826	36.039	26.709	25.388	41.382
Y	28.116	24.350	26.706	30.561	33.255	10.847	18.613
K	28.159	34.287	27.914	31.091	19.051	36.538	34.839
E	28.763	26.692	32.437	39.806	17.311	36.196	31.316
J	29.471	21.521	24.675	40.136	33.946	45.454	21.828
V	29.857	21.852	34.963	25.409	21.328	42.299	34.168
J	30.484	32.053	24.362	18.298	49.657	52.827	37.718
DD	30.658	31.054	37.037	23.251	50.452	41.184	35.392
CC	31.399	29.519	32.450	49.697	40.511	20.081	38.148
I	31.335	30.120	34.762	40.791	44.692	34.297	47.684
H	31.474	31.717	26.593	39.624	41.110	34.756	42.726
J	31.975	27.418	21.337	35.195	44.555	19.675	57.084
R	32.275	29.870	19.603	19.026	55.107	27.664	56.631
K	32.698	20.126	28.049	58.702	29.722	21.765	21.153
CC	32.676	27.604	46.055	34.483	34.297	21.430	31.811
D	32.684	34.730	22.544	33.647	56.144	37.122	54.496
F	32.721	45.816	39.557	22.412	39.213	61.907	61.943

	BB	33.106	32.303	45.094	31.361	26.534	19.767	40.642
	B	33.110	33.986	34.005	21.991	20.863	30.283	43.200
	EE	33.419	35.505	28.699	37.906	30.476	24.082	37.209
	M	34.220	30.949	47.421	38.583	47.254	34.461	48.763
	Z	34.705	29.841	45.062	39.519	38.605	22.848	41.700
	P	35.220	37.864	43.320	44.251	44.649	28.715	55.907
	S	35.216	34.049	23.394	49.056	55.224	41.322	59.193
	DD	35.993	28.131	29.530	47.478	37.647	32.809	23.506
	P	36.050	36.507	36.261	30.082	31.678	28.296	48.882
	P	36.557	38.907	33.509	29.877	46.264	45.616	37.714
	FF	36.611	35.335	40.196	33.091	38.872	47.879	53.350
	M	37.103	40.143	33.009	6.047	41.979	40.198	50.024
	FF	37.437	41.850	32.508	33.453	27.269	65.648	19.498
	T	37.508	34.291	29.992	50.000	35.542	44.902	23.588
	H	37.791	39.820	43.133	36.735	50.584	53.579	22.531
	O	38.116	31.683	48.648	40.422	41.405	50.163	34.419
	T	38.526	36.899	49.089	45.561	45.594	41.729	22.172
	AA	38.989	33.371	41.218	39.232	30.603	21.723	48.976
	R	39.551	40.284	35.288	30.009	35.794	20.967	48.999
	C	39.535	29.790	34.677	55.575	29.597	51.717	51.076
	G	39.892	45.039	48.756	39.488	50.375	49.578	71.666
	C	40.338	42.247	46.790	37.214	40.695	54.019	37.663
	X	40.324	39.567	48.970	46.061	40.472	44.995	71.666
	F	40.497	43.085	44.376	32.673	52.803	46.659	59.819
	P	40.728	40.832	50.007	41.801	38.056	64.746	38.771
	H	40.733	34.594	40.419	48.667	46.777	42.250	43.208
	A	40.971	38.454	40.576	37.777	17.360	23.264	46.492
	H	41.152	43.257	34.034	39.860	34.078	26.392	41.323
	P	41.283	35.496	57.317	31.877	26.263	60.731	42.681
	J	41.441	45.932	39.721	43.249	49.396	38.077	47.825
	M	41.440	45.723	41.940	29.129	44.273	42.521	52.381
	AA	41.504	34.412	45.192	44.852	37.235	34.870	45.583
	E	41.683	47.955	52.572	40.187	56.966	48.027	39.885
	R	41.673	27.523	53.985	43.481	33.136	49.751	41.022
	X	41.618	40.587	46.282	33.098	44.754	34.708	70.157
	BB	41.833	43.483	58.118	54.408	58.091	42.490	57.761
	X	42.190	45.172	48.042	42.400	34.184	51.275	59.323
	U	42.211	40.479	46.127	35.549	45.961	44.326	33.049
	U	43.413	46.119	39.898	38.848	31.928	55.764	24.289
	I	43.419	42.636	35.122	37.938	40.635	37.223	40.322
	U	43.434	36.359	48.211	53.501	57.644	35.364	57.189
	E	43.549	40.553	43.485	49.167	62.243	46.186	52.161
	G	43.687	44.236	54.466	40.902	28.984	67.939	61.990
	V	43.786	30.025	45.528	56.024	35.536	58.953	43.197
	Z	43.813	46.570	57.889	29.979	49.082	46.860	43.288
	O	43.836	39.478	34.616	48.273	35.611	38.118	67.538
	V	44.219	42.835	49.358	50.161	37.768	7.445	49.161
	R	44.512	53.926	45.919	44.909	48.257	58.341	44.316
	H	44.875	30.972	49.103	45.870	65.516	64.269	40.981
	FF	44.966	41.199	51.872	69.520	26.416	49.474	44.325
	BB	44.974	34.582	42.123	61.448	28.753	52.492	51.848
12	FF	45.456	43.628	43.853	48.048	45.206	46.678	33.651
11	A	45.520	39.497	48.648	54.814	35.551	38.145	38.870
10	A	45.621	38.719	54.566	53.661	56.620	45.612	34.972
9	O	45.872	52.328	50.850	64.177	31.744	38.868	22.453
8	K	45.875	36.284	53.536	44.457	52.804	49.600	37.465
7	M	46.061	40.353	38.910	43.516	59.980	42.822	64.897
6	M	46.273	42.430	41.267	54.985	49.277	70.484	35.141
5	FF	46.381	41.979	53.984	37.992	50.687	44.367	34.707
4	G	46.720	48.124	66.713	58.830	31.892	59.603	47.003
3								

	F	47.181	41.680	43.966	52.539	28.536	51.848	49.561
	F	47.209	53.154	39.513	45.979	56.044	12.013	33.805
	M	47.409	51.015	44.622	43.900	55.780	28.270	50.208
	V	47.911	37.966	51.046	53.074	42.683	74.972	40.080
	A	48.064	43.772	60.346	47.253	47.714	47.659	30.680
	D	48.129	59.446	34.666	42.200	77.617	44.149	57.024
	K	48.224	51.154	41.118	46.694	49.754	54.098	46.436
	S	48.244	48.681	59.528	68.385	49.211	65.190	49.437
	D	48.373	39.309	40.421	41.131	38.292	40.977	51.091
	T	48.931	43.833	44.318	59.920	62.893	46.251	44.778
	T	49.262	47.386	48.936	50.127	49.154	67.460	44.841
	A	49.366	48.432	46.699	54.530	59.046	46.447	30.482
	J	49.667	53.865	50.268	46.900	49.083	74.944	32.972
	D	49.736	50.769	51.960	66.903	53.414	18.318	51.010
	M	49.886	63.270	51.211	55.022	50.909	43.156	44.790
	AA	49.883	48.196	45.010	61.365	39.769	38.819	58.088
	V	49.920	57.796	45.010	47.499	58.264	36.290	46.284
	I	50.318	50.758	49.711	52.103	44.675	63.270	34.205
	H	50.440	49.782	50.383	56.439	59.512	62.640	63.647
	V	50.553	46.444	40.513	50.846	52.150	51.249	54.516
	P	50.729	46.832	51.636	60.619	51.235	35.468	47.130
	D	50.770	49.491	66.433	52.206	49.161	44.103	69.366
	F	51.298	51.133	43.113	54.974	45.725	54.959	51.552
	O	51.276	48.171	41.335	46.930	65.877	70.598	64.772
	R	51.323	54.999	37.545	40.864	49.613	61.748	26.705
	7	51.339	48.292	41.148	48.463	41.091	67.799	27.458
	R	51.346	46.188	41.874	55.095	55.066	67.827	48.317
	N	51.384	48.691	62.284	39.619	58.717	57.975	61.093
	DD	51.438	49.219	57.038	70.644	53.871	78.924	59.039
	W	51.635	50.223	59.890	46.850	47.813	52.905	37.504
	N	51.967	55.317	38.814	64.097	41.048	39.669	61.613
	AA	52.292	56.035	44.490	48.190	54.970	44.473	62.588
	O	52.425	47.495	62.846	42.757	53.374	46.228	37.668
	N	52.436	52.017	55.000	55.063	48.809	51.043	52.845
	R	52.512	57.592	56.242	54.073	50.429	54.337	22.340
	Y	52.634	50.599	47.096	56.140	70.451	57.486	47.643
	D	52.664	55.538	52.521	46.949	61.670	31.907	56.974
	T	52.873	57.027	40.599	55.674	53.263	56.774	69.944
	DD	53.079	59.732	68.432	49.277	36.724	55.263	63.326
	7	53.236	56.940	53.796	52.752	44.643	50.055	47.677
	I	53.420	47.289	68.358	36.344	43.728	61.142	33.485
		53.646	52.283	49.702	48.362	39.954	41.512	55.033
	O	53.857	61.235	55.135	48.606	51.283	65.280	64.406
	V	53.972	55.860	48.598	54.700	51.899	61.024	55.100
	D	54.265	50.673	70.403	59.288	40.382	63.576	50.250
	BB	54.324	55.796	52.586	45.647	49.227	46.872	30.056
	R	54.470	52.097	52.648	54.263	56.663	75.243	44.149
	S	54.470	50.225	54.433	53.044	54.655	51.646	53.531
	H	54.515	51.287	59.644	39.169	37.301	33.482	79.142
	M	54.674	54.859	48.851	52.522	50.652	39.747	76.521
	P	54.802	56.948	40.201	61.566	68.452	40.562	64.213
12	S	55.106	53.547	57.302	51.655	55.448	64.045	53.769
11	G	55.222	50.871	45.700	50.768	30.257	47.288	50.304
10	P	55.393	53.879	62.894	71.440	38.430	74.782	40.706
9	K	55.400	57.017	50.919	36.072	55.644	47.056	44.201
8	T	55.678	52.268	46.696	60.311	57.419	45.029	67.911
7	W	55.727	48.549	72.317	58.219	46.313	51.184	59.851
6	S	55.724	55.935	56.068	53.810	51.260	64.049	80.576
5	J	55.990	50.930	59.370	46.743	63.526	36.170	46.238
4	F	56.054	57.939	59.069	57.114	48.699	36.354	41.124
3								

	W	56.141	51.616	46.068	64.589	74.078	23.932	51.387
	FF	56.443	48.869	73.721	56.412	64.772	52.629	38.274
	W	56.438	52.334	50.005	65.319	58.215	57.718	58.868
	H	56.814	57.604	52.924	72.228	56.501	49.581	55.815
	BB	57.235	53.246	58.267	53.249	64.517	45.466	40.697
	FF	57.718	52.448	53.366	56.171	45.731	46.233	49.541
	S	57.774	59.643	63.967	59.860	73.156	41.169	52.905
	C	57.904	55.848	54.171	55.575	49.008	52.570	44.774
	Y	58.274	55.923	68.007	49.634	54.786	66.926	72.063
	M	58.565	60.120	51.497	76.302	38.598	44.584	65.405
	F	58.668	53.608	61.939	49.605	29.582	48.029	29.861
	C	58.679	59.354	56.772	53.861	62.191	57.396	63.750
	FF	58.870	53.673	55.970	32.855	77.080	48.103	58.197
	CC	58.923	60.256	48.311	72.461	60.398	41.319	64.205
	AA	59.019	57.521	62.310	53.264	49.540	51.354	70.935
	Z	59.321	71.276	62.067	58.379	53.872	63.049	65.048
	B	60.182	63.037	43.287	58.300	61.736	57.324	30.472
	T	60.207	59.084	57.278	70.537	46.056	69.849	48.814
	DD	60.265	56.765	60.075	62.408	38.518	54.183	58.248
	X	60.239	59.327	47.063	57.964	46.029	64.034	60.073
	O	60.304	66.535	57.922	46.460	56.263	57.222	38.529
	G	60.394	51.859	55.325	37.807	38.433	31.488	51.990
	R	60.684	65.867	62.134	45.268	49.820	42.172	40.355
	C	60.896	63.469	50.166	62.997	60.767	62.804	42.327
	FF	61.069	56.898	56.850	66.276	55.846	50.996	58.523
	B	61.189	60.228	77.469	59.161	42.012	53.563	55.755
	W	61.332	58.323	61.689	69.470	62.076	58.575	67.261
	D	61.313	56.569	72.664	63.191	61.597	71.984	76.450
	T	61.445	65.933	67.390	53.412	60.124	44.788	55.099
	E	61.411	64.055	62.891	52.240	53.381	55.843	63.519
	P	61.619	64.380	67.809	51.829	37.879	50.360	51.471
	H	62.117	65.884	66.221	70.855	76.974	79.950	53.620
	C	63.398	54.181	64.139	66.482	63.547	65.840	25.229
	W	63.556	62.806	56.703	46.730	74.418	55.448	46.687
	D	63.963	57.052	66.362	49.250	62.146	41.466	56.007
	O	64.550	68.483	72.617	61.613	43.452	59.267	61.073
	CC	64.640	63.578	58.133	55.902	64.757	74.618	61.348
	Y	64.761	53.549	64.514	62.309	56.439	65.658	50.602
	H	64.945	65.355	69.056	58.192	56.839	75.027	82.514
	N	65.030	64.760	57.656	65.492	54.156	55.285	65.705
	J	65.286	68.489	53.043	66.488	59.084	60.310	34.396
	P	65.278	70.025	72.841	72.492	62.839	53.553	66.959
	EE	65.448	68.135	66.789	60.124	86.371	52.880	64.266
	F	65.790	66.305	54.194	69.909	63.951	77.256	53.361
	FF	66.258	71.154	62.366	70.960	65.306	68.141	76.552
	AA	66.272	65.706	59.803	60.218	63.932	73.455	78.890
	D	66.224	58.745	70.981	76.089	66.962	47.541	45.157
	T	66.375	76.115	57.710	58.496	80.324	84.341	44.204
	M	66.783	61.626	70.361	73.170	58.862	38.026	58.792
	O	66.704	66.016	55.279	65.691	55.979	71.660	66.482
	E	66.806	69.878	59.305	47.824	70.826	86.192	78.620
12	E	66.971	70.250	64.238	81.023	70.232	69.276	69.459
11	I	67.259	69.157	67.613	58.237	76.904	63.271	40.754
10	I	67.319	58.779	58.643	59.986	87.947	61.993	35.500
9	S	67.359	68.587	76.353	66.520	54.340	34.972	54.128
8	Y	67.382	63.968	68.789	68.204	62.418	74.758	57.541
7	CC	67.536	73.342	78.618	65.251	55.499	72.014	38.644
6	V	67.768	63.729	66.449	49.688	72.380	59.154	62.331
5	H	67.936	62.539	67.942	69.973	72.452	64.458	66.737
4	C	68.104	64.112	71.681	64.217	64.228	85.560	66.291
3								

I	68.329	71.316	49.767	72.029	54.757	74.031	58.862
AA	69.275	65.495	69.401	62.073	49.952	60.084	75.425
W	69.491	67.664	66.181	54.725	55.031	55.879	63.672
A	70.378	69.603	77.508	76.472	64.099	67.262	58.297
EE	70.560	62.692	57.186	54.199	55.455	70.245	48.868
BB	70.767	70.996	65.863	65.455	66.009	70.176	76.600
A	70.941	62.239	64.339	83.727	88.452	48.210	77.117
Y	71.343	72.427	67.640	65.892	33.802	64.193	26.305
P	71.411	69.816	64.123	68.887	52.549	75.625	62.244
T	71.522	69.949	66.620	64.422	56.663	52.916	63.565
S	71.883	66.111	76.806	75.201	55.556	69.924	43.004
C	72.049	74.206	76.195	79.653	66.923	85.865	66.247
Z	72.805	64.143	77.140	70.062	71.680	86.613	65.612
C	72.805	75.458	56.715	67.747	53.564	83.764	90.488
G	73.458	72.707	75.054	68.159	60.488	47.447	82.912
DD	74.725	72.401	66.311	65.867	64.141	75.020	57.529
FF	74.882	74.255	73.245	81.417	62.468	83.755	54.810
X	74.932	71.623	68.730	84.668	84.662	68.764	71.595
T	75.125	70.418	74.505	68.094	84.972	60.997	68.254
K	75.157	71.652	69.409	72.870	81.283	72.089	72.379
CC	76.701	68.247	75.313	80.528	85.533	65.883	79.295
X	76.826	76.623	75.725	96.151	73.423	80.083	71.407
G	77.776	77.482	70.196	68.930	80.578	45.542	54.769
N	77.911	69.882	73.130	77.765	81.161	76.817	54.559
DD	80.326	77.722	71.706	69.599	72.247	70.565	77.165
T	84.046	80.107	75.923	92.783	66.778	69.038	46.470
X	84.330	85.323	74.566	79.357	63.969	66.523	81.753
Z	84.391	88.931	68.834	67.400	69.168	71.383	105.387
Z	87.429	92.692	87.160	66.782	62.526	65.231	74.703
H	88.515	92.237	76.489	92.242	85.360	104.528	110.419
Y	88.748	76.465	89.969	78.907	85.461	79.773	66.750
BB	96.540	89.955	83.613	97.432	105.887	83.148	60.662
D	8.064	93.099	89.445	86.789	89.997	88.441	89.916
O	9.023	90.316	91.373	80.531	72.741	68.745	89.090
O	10.069	89.026	90.347	86.107	73.018	100.732	73.238
V	13.801	85.103	85.156	99.572	77.556	56.037	65.378
V	16.467	85.170	72.200	90.024	82.779	73.896	79.944
BB	18.400	79.645	69.820	78.916	56.226	69.309	62.224
F	18.627	76.655	82.116	77.246	85.375	67.057	92.572
H	18.709	78.418	77.657	84.503	83.667	64.224	106.518
N	20.533	80.740	79.583	88.928	74.109	62.001	98.978
O	21.859	80.199	69.671	85.844	72.974	54.226	77.120
A	22.018	79.307	73.385	77.178	72.402	75.954	66.499
H	24.487	75.264	73.795	61.808	72.076	73.268	54.779
F	24.534	71.142	72.199	64.063	65.073	60.292	77.468
P	26.275	72.461	83.224	69.183	69.708	67.811	65.353
I	26.593	74.656	72.201	60.738	81.560	64.310	61.822
O	27.125	70.040	65.849	79.759	63.904	62.224	75.232
J	27.406	72.132	75.648	86.009	68.591	73.503	59.809
J	27.660	70.293	68.606	76.625	74.050	71.621	74.609
W	28.236	71.953	73.960	59.565	64.778	62.153	75.802
Z	28.456	75.085	61.504	57.356	69.425	56.093	74.282
J	28.558	75.048	78.279	56.315	63.513	63.704	72.879
X	28.776	70.560	71.897	70.399	73.592	80.679	63.622
T	28.957	68.953	75.897	77.855	68.496	69.847	52.992
K	29.056	72.617	64.856	71.506	68.931	75.212	62.033
U	29.157	70.802	73.714	72.850	72.705	68.817	56.968
P	29.570	69.717	74.103	64.617	55.429	63.275	72.192
A	30.335	73.186	89.443	72.145	78.673	68.824	64.932
I	30.601	67.570	69.555	67.164	60.180	73.651	65.188

Z	30.832	68.207	70.765	66.988	71.746	63.752	57.799
FF	30.846	64.233	64.649	62.218	61.039	72.634	66.504
D	31.071	67.553	70.607	76.054	55.157	48.010	81.205
R	31.321	64.926	65.261	74.047	79.772	51.995	66.716
M	31.430	73.685	68.645	64.707	69.952	48.726	39.392
J	31.991	69.188	61.815	58.465	70.077	63.410	44.596
EE	32.736	66.581	76.963	68.300	63.480	71.010	58.244
EE	32.835	65.376	69.187	52.304	60.232	49.891	39.479
P	32.877	64.697	64.535	80.209	68.712	49.844	67.123
BB	33.059	70.341	62.355	61.404	45.793	68.597	39.470
T	33.253	65.968	69.863	77.833	50.049	60.134	52.279
R	33.272	66.130	68.342	68.346	69.238	66.879	63.062
A	33.484	71.027	64.840	62.906	77.717	55.903	44.768
K	34.373	68.859	71.067	56.077	92.551	61.381	77.457
N	34.546	66.656	55.925	60.436	55.429	66.627	54.339
C	34.850	63.572	70.206	59.007	53.913	81.237	71.706
P	35.758	64.110	72.412	60.792	67.257	66.846	59.078
I	36.246	66.735	60.134	74.552	62.346	52.592	69.721
Y	36.793	60.858	67.341	70.554	49.480	65.948	68.307
S	36.701	67.606	61.621	59.578	59.439	41.402	73.639
K	36.803	62.070	57.255	60.258	66.507	60.864	53.774
H	36.942	59.682	66.510	59.473	59.436	36.794	77.255
P	37.558	61.418	59.401	57.823	77.368	46.086	57.188
F	37.759	66.018	61.767	51.278	64.840	58.051	34.008
P	37.985	63.142	55.338	65.705	51.835	40.965	60.522
V	38.166	62.796	64.356	62.268	68.121	60.764	52.591
D	38.178	63.111	57.790	76.297	51.903	68.191	55.166
H	38.183	60.366	59.771	75.453	54.534	54.524	62.578
CC	38.263	57.555	54.505	53.592	46.797	58.248	53.115
FF	38.542	60.415	65.147	53.306	65.654	53.733	43.064
H	38.592	65.879	61.193	63.214	51.975	66.613	53.011
E	39.956	58.638	53.465	52.527	57.841	69.684	60.542
B	40.022	56.344	53.436	78.137	48.981	65.443	62.045
N	40.230	59.959	64.904	70.335	61.513	36.291	44.662
D	40.259	60.285	57.121	67.786	70.277	58.655	55.361
A	40.483	58.837	53.722	63.318	37.027	51.215	72.756
AA	40.708	55.513	64.158	61.952	57.997	47.282	43.768
T	40.842	56.929	54.638	51.699	53.466	49.684	92.085
AA	41.396	58.434	63.654	57.201	50.831	51.795	65.506
T	41.473	57.186	54.801	51.121	59.581	41.620	64.042
O	41.766	60.425	55.422	67.617	62.497	68.437	56.922
V	42.099	56.875	55.761	58.868	53.429	64.590	54.518
J	42.299	59.505	50.207	58.609	61.025	71.242	30.780
BB	42.309	59.958	65.539	54.523	61.589	79.507	58.321
CC	42.720	55.102	52.056	62.190	54.765	61.942	53.087
G	43.372	55.924	57.161	53.796	54.579	58.642	61.874
C	43.575	60.278	59.456	57.897	65.801	70.473	45.303
N	43.583	54.613	49.491	54.321	66.352	50.158	48.460
E	44.353	53.444	57.782	42.999	68.086	69.671	58.713
BB	44.451	55.266	66.225	50.559	46.276	41.306	53.658
EE	44.685	56.316	62.296	52.086	57.052	39.365	44.983
AA	44.726	53.003	53.226	55.108	53.777	67.947	48.293
E	44.886	56.452	43.310	45.303	46.068	50.028	33.417
S	45.064	55.821	57.540	47.049	47.636	75.922	56.134
A	45.607	58.602	48.545	48.153	59.980	35.389	57.134
Y	45.830	56.794	62.697	52.574	57.953	48.425	54.730
BB	45.952	55.609	55.496	54.863	49.751	58.145	50.526
T	46.298	52.060	57.742	47.524	43.985	58.390	31.283
O	46.279	51.568	44.950	53.991	49.889	53.816	58.286
E	46.507	52.842	57.470	52.051	44.059	61.157	40.572

I	46.777	52.849	60.064	52.364	62.707	60.861	41.926
D	46.761	54.359	50.344	48.802	48.624	55.220	50.041
I	46.781	53.664	49.376	53.366	52.350	71.853	70.414
C	46.817	51.941	42.944	67.068	57.148	43.036	39.704
H	46.960	53.350	54.178	60.813	36.161	30.514	76.955
FF	46.923	51.107	50.632	47.373	42.210	61.715	57.810
V	47.024	54.638	60.654	54.257	48.973	68.722	56.367
X	47.549	47.965	52.446	48.913	55.721	40.071	70.338
O	47.720	53.922	64.812	42.803	44.386	64.562	44.921
E	47.945	52.690	44.548	49.810	46.268	48.341	45.052
G	48.050	52.732	50.104	61.301	52.852	41.500	57.202
A	48.217	51.477	55.339	55.453	49.740	71.269	60.713
C	48.344	53.626	35.301	49.600	56.715	51.451	45.770
CC	48.574	50.807	48.833	63.496	47.962	52.026	37.670
Y	48.603	54.708	55.502	52.215	44.957	44.647	40.116
D	48.687	49.525	41.738	60.449	62.375	46.987	52.746
K	49.153	49.113	61.352	44.310	43.372	65.884	62.792
Y	49.337	50.443	58.681	53.460	47.123	23.546	51.828
DD	49.682	47.688	47.650	37.867	45.813	66.938	42.219
Z	49.804	43.243	55.921	43.033	52.649	60.182	64.579
Z	49.756	53.629	51.354	45.574	41.028	58.960	41.209
CC	49.711	49.234	61.066	53.744	47.552	52.847	58.874
Z	50.011	50.836	49.329	53.326	60.456	55.983	57.791
K	50.140	51.561	42.351	57.918	28.873	61.089	64.455
FF	50.293	48.226	46.939	43.535	43.251	60.102	48.690
Z	50.287	47.042	47.540	63.403	39.337	56.765	63.651
G	50.531	50.232	50.141	47.053	44.694	35.581	62.041
R	50.606	46.210	57.332	44.331	50.936	44.887	65.107
T	50.602	52.001	52.738	51.765	60.622	56.447	67.365
AA	50.795	47.970	46.137	38.130	43.869	43.559	37.274
S	50.879	50.847	54.929	44.423	49.208	57.834	48.914
N	50.977	51.410	45.399	42.815	59.679	51.117	54.140
AA	51.282	51.204	46.382	54.175	46.097	43.335	55.772
EE	51.336	46.360	46.164	54.655	30.640	48.959	28.718
D	51.487	51.313	42.992	48.656	51.393	41.009	54.383
I	51.746	49.572	38.917	50.250	56.422	44.824	44.971
J	51.788	45.210	46.940	43.148	35.805	50.784	56.544
I	51.714	46.326	44.121	59.151	62.837	45.978	65.157
O	51.928	49.576	47.530	36.136	44.061	17.081	40.650
U	52.040	46.000	42.805	41.867	57.114	68.197	38.526
I	52.073	48.922	54.317	39.333	40.392	43.820	38.810
O	52.060	51.380	45.394	59.082	35.668	34.421	46.647
H	52.365	48.609	46.625	45.133	34.730	51.392	26.386
K	52.346	45.388	45.800	47.511	59.818	47.220	65.084
CC	52.711	42.929	51.231	47.482	47.097	54.016	35.143
N	52.761	47.447	52.635	57.095	46.088	53.109	49.156
EE	52.783	49.046	56.405	34.397	47.575	47.612	57.787
CC	52.701	44.138	49.447	29.950	56.184	33.770	61.722
C	52.809	45.357	42.916	48.841	39.585	50.443	23.703
B	52.891	46.286	46.910	43.079	48.550	44.035	59.212
E	53.167	44.223	38.839	43.741	41.356	50.578	43.671
X	53.693	48.420	53.636	57.082	44.113	49.813	49.558
AA	53.720	48.524	45.369	47.580	50.749	59.392	47.489
G	53.820	42.098	43.808	43.901	40.096	47.444	53.207
A	54.144	49.279	45.243	30.641	46.354	57.888	33.830
E	54.166	47.807	47.477	42.705	44.046	31.556	37.675
G	54.146	48.686	49.431	44.915	53.230	56.214	49.871
O	54.461	44.995	46.703	34.416	31.820	43.531	43.551
FF	55.120	43.284	43.376	50.658	29.915	44.346	46.826
DD	55.221	41.416	41.840	52.942	50.421	57.166	63.793

H	55.770	42.257	45.905	57.764	42.809	61.302	47.791
T	55.737	44.306	46.881	52.626	59.381	44.073	66.955
W	55.961	48.356	45.082	52.550	30.881	52.687	42.418
O	55.972	46.117	42.104	48.306	29.299	59.077	43.177
CC	56.220	43.741	50.194	61.105	49.241	42.087	45.508
X	56.268	42.723	31.741	52.326	56.726	54.802	58.458
M	56.503	46.353	39.230	47.426	53.015	28.845	39.246
P	56.736	43.177	37.586	43.657	34.179	46.521	15.892
I	57.036	42.592	48.379	46.276	30.084	49.802	45.116
O	57.334	42.504	41.219	40.232	68.443	51.812	41.339
C	57.441	44.101	45.788	37.536	40.363	40.550	28.535
H	57.914	41.392	50.536	51.876	52.078	43.815	43.145
C	58.406	42.227	45.201	46.677	35.390	38.406	32.233
V	58.437	41.434	40.245	53.586	40.972	42.163	44.478
DD	58.550	41.786	48.314	46.200	41.344	42.240	20.815
V	58.604	42.373	40.169	29.012	44.925	27.534	42.610
N	58.613	44.289	51.978	22.396	35.824	31.690	39.303
S	58.720	44.225	35.325	38.818	40.954	43.693	45.877
Y	58.764	41.294	32.730	40.413	52.728	35.529	57.051
FF	58.826	38.975	44.974	44.674	53.218	50.223	80.208
AA	59.078	42.443	42.651	34.866	44.638	25.940	52.243
Z	59.390	43.868	32.939	42.804	48.309	59.885	52.284
P	59.780	42.276	41.272	34.836	42.100	60.130	18.305
M	59.808	40.231	43.564	28.444	58.241	26.081	30.656
R	59.820	41.708	43.074	30.323	51.518	28.756	46.768
C	60.204	38.788	44.327	49.906	41.052	54.490	32.048
DD	60.290	39.568	38.034	43.933	41.167	44.942	36.628
X	60.262	41.895	38.803	50.411	32.871	46.581	45.256
A	60.267	43.422	38.016	40.800	52.434	52.213	55.925
H	60.353	39.617	42.391	33.413	48.068	48.501	63.390
C	60.419	44.872	42.311	32.334	52.204	42.040	32.685
DD	60.790	40.191	44.091	42.874	48.583	45.325	20.060
P	60.896	40.479	45.144	37.004	39.877	48.110	33.654
H	61.012	39.070	41.289	41.818	30.613	63.013	55.950
RR	61.147	40.502	37.368	35.608	38.505	45.685	29.981
DD	61.358	40.107	45.444	47.728	48.157	43.826	38.694
E	61.440	42.098	35.470	35.797	48.425	41.604	60.326
S	61.512	41.324	39.375	30.666	39.263	50.223	26.210
P	61.748	37.233	50.430	33.076	54.380	62.667	33.805
M	61.833	40.343	42.736	44.777	33.871	35.550	53.522
RR	61.991	38.205	34.643	46.076	54.187	35.073	40.274
D	61.995	35.488	36.597	42.459	21.991	41.351	54.899
E	62.049	36.228	48.773	43.175	43.662	41.436	34.702
T	62.058	37.693	41.482	31.170	30.439	31.740	41.939
O	62.028	34.047	32.745	43.808	28.903	46.671	48.315
I	62.246	38.672	40.575	35.614	27.461	39.440	20.661
M	62.519	36.345	33.552	36.575	38.359	32.919	33.071
T	62.661	37.806	37.721	32.854	30.149	48.522	35.233
FF	62.745	40.551	40.235	38.081	31.702	40.515	38.282
H	62.866	40.355	37.627	54.701	46.727	33.931	23.466
DD	63.126	38.761	49.906	36.507	43.882	18.225	36.512
E	63.139	31.686	39.100	27.796	45.470	42.769	45.408
O	63.342	36.933	34.188	40.505	39.550	23.811	69.904
E	63.582	39.756	29.520	37.051	42.151	25.905	45.035
V	63.558	38.461	33.068	24.791	37.751	50.781	56.374
R	63.816	33.765	32.380	25.963	29.145	29.048	41.191
X	64.573	32.151	27.834	28.841	22.819	41.911	38.791
H	64.962	37.091	27.162	44.686	38.196	49.008	48.550
RR	65.021	35.477	36.187	39.695	29.787	31.949	44.358
R	65.011	35.242	32.613	44.735	51.151	46.859	68.084

DD	65.139	33.557	29.774	28.742	44.762	40.606	19.825
U	65.656	35.531	34.862	31.113	31.395	35.635	42.297
Y	65.765	30.419	20.007	42.578	49.846	23.431	36.015
EE	65.969	33.087	28.897	42.948	47.480	45.673	50.114
J	66.240	31.260	31.684	27.415	32.585	36.399	25.853
L	66.388	35.071	34.080	32.109	29.111	30.818	27.352
V	66.358	31.757	32.306	48.889	22.407	36.975	43.621
W	66.460	36.521	35.352	44.252	33.870	33.479	24.562
F	66.544	34.742	37.078	39.294	51.630	46.683	43.426
I	66.603	34.671	30.998	35.781	36.708	51.456	14.006
U	66.668	32.053	24.072	27.639	28.023	32.233	34.372
C	66.940	28.514	36.284	28.147	34.633	43.349	39.084
M	67.087	36.136	45.195	32.579	45.004	26.695	48.052
R	67.906	32.454	35.372	32.543	30.494	40.072	45.817
W	67.952	36.680	40.118	27.178	50.345	21.069	48.415
S	68.516	32.156	32.391	15.703	31.734	31.215	40.337
U	68.645	30.490	30.803	41.867	13.402	45.679	47.792
CC	68.840	35.000	26.959	33.865	39.574	39.126	36.034
U	68.913	31.872	26.064	24.241	45.825	27.900	40.762
D	69.269	34.799	31.258	30.545	23.129	35.116	34.562
Y	69.332	30.473	29.291	26.896	42.103	53.686	42.838
J	69.940	28.462	29.396	36.562	37.652	33.366	30.745
N	70.598	32.987	28.402	33.825	37.263	28.756	34.524
T	70.629	27.119	29.137	35.193	29.588	28.003	34.982
N	70.816	30.231	34.742	35.471	23.029	29.392	22.828
X	70.845	33.637	26.745	18.712	28.961	37.014	48.050
Y	71.081	31.176	24.196	36.273	49.438	50.959	34.201
Z	73.158	29.214	34.941	22.410	16.102	25.320	21.697
FF	74.104	27.479	20.730	34.150	28.332	26.954	47.136
S	74.484	24.293	20.802	32.228	18.060	45.262	37.438
EE	74.516	23.167	22.628	35.795	39.709	24.445	28.552
K	74.895	24.299	21.030	25.118	24.664	24.878	14.284
W	75.003	26.455	29.735	32.723	18.696	35.540	30.524
F	75.393	24.258	20.377	17.158	21.099	33.108	27.404
T	77.208	25.058	28.674	26.442	33.770	35.115	23.504
G	78.358	18.920	28.068	27.194	30.793	18.210	27.776
T	78.759	19.827	10.261	21.041	22.910	31.961	12.606
O	79.639	18.321	24.957	28.201	19.449	28.643	34.581
Y	80.815	17.620	24.095	27.724	44.134	33.653	45.906
P	81.049	23.207	16.471	9.128	26.148	43.190	30.369
I	81.425	22.332	25.954	22.417	37.765	32.362	29.082
R	82.305	17.338	24.484	21.241	18.320	21.578	9.500
FF	82.804	20.357	19.351	10.108	26.444	6.466	10.115
AA	82.812	17.155	13.035	21.094	22.115	39.514	14.040
M	83.395	17.194	9.404	16.383	15.385	26.963	30.599
R	91.353	6.310	13.599	3.230	13.670	11.522	11.098
N	92.578	7.038	4.906	22.376	28.808	22.250	30.100
S	98.193	4.992	.224	3.631	10.744	26.351	27.328
N	7.513	91.368	90.881	86.944	79.775	60.793	61.577
I	8.947	92.436	88.999	105.600	64.656	60.094	44.915
T	13.072	87.959	83.263	66.612	63.654	56.683	57.211
FF	14.956	90.156	96.611	66.286	56.583	67.049	47.613
P	16.677	79.848	81.786	94.875	83.935	60.629	54.900
P	18.663	81.262	84.641	77.673	81.798	57.027	56.759
S	19.530	83.519	81.687	79.958	61.735	74.379	74.372
I	19.991	79.646	87.482	45.835	64.988	57.252	57.735
E	21.496	77.567	66.636	78.256	55.610	51.432	39.825
S	22.258	76.369	75.434	65.706	62.479	43.276	67.886
U	22.368	79.073	84.695	73.193	79.390	40.836	50.646
E	23.021	79.581	68.382	61.598	57.117	54.612	49.029

W	23.642	72.733	80.626	58.200	46.569	55.188	26.919	
EE	23.657	74.830	69.544	66.531	55.665	45.395	68.924	
H	24.227	75.150	68.799	64.082	64.410	62.633	55.064	
DD	24.630	78.958	68.072	70.121	64.260	58.502	45.198	
G	24.940	76.563	84.809	71.949	46.416	82.601	51.807	
M	24.916	80.013	69.116	80.522	53.799	36.496	47.852	
D	25.666	70.919	80.371	63.747	69.885	92.020	10.525	
X	26.018	74.739	66.475	71.446	76.711	64.996	40.762	
AA	26.073	79.635	69.911	66.958	56.866	44.707	35.091	
C	26.468	66.426	57.229	62.013	43.685	56.053	62.580	
I	27.365	72.136	76.019	66.356	76.122	19.858	53.860	
X	27.584	66.884	64.362	76.233	88.478	68.195	30.403	
U	27.685	69.798	70.640	82.936	51.894	53.746	34.637	
S	29.090	71.702	77.479	67.266	62.473	66.316	42.476	
U	29.619	69.935	75.391	50.933	47.133	47.337	73.062	
J	29.767	72.446	64.025	53.129	59.843	58.528	38.529	
S	30.483	73.056	77.279	76.592	64.658	81.385	38.610	
S	30.690	69.187	70.240	70.349	68.574	51.783	49.821	
J	31.036	67.981	65.492	45.649	65.401	64.668	63.529	
BB	31.373	66.135	66.079	85.223	60.656	66.518	74.708	
U	31.772	68.220	80.901	57.249	58.712	42.444	53.985	
R	31.823	68.714	77.421	67.524	65.313	62.784	47.291	
BB	32.698	64.851	60.179	67.955	39.516	69.009	60.854	
T	32.649	65.824	68.668	69.019	60.188	74.193	70.197	
DD	32.608	67.336	62.308	71.348	64.502	57.238	.904	
W	32.727	63.834	63.069	85.775	68.915	107.290	54.304	
M	32.794	65.106	63.080	59.396	40.019	74.142	70.496	
J	32.888	68.635	69.625	70.680	37.401	53.962	40.561	
F	33.096	67.539	56.273	47.503	51.225	49.112	9.110	
T	33.430	64.531	61.931	66.119	66.758	56.634	40.692	
V	33.447	65.704	60.000	65.356	62.102	66.039	30.854	
T	34.097	66.019	73.553	56.442	51.113	71.809	35.490	
X	34.182	71.119	64.438	67.008	60.814	50.347	69.866	
C	34.576	63.192	83.376	75.158	57.931	37.404	74.755	
K	34.644	59.678	62.829	67.312	75.281	44.160	65.188	
I	34.665	66.111	57.882	59.685	56.705	45.650	56.310	
Z	34.777	71.844	68.069	58.489	76.209	50.404	76.857	
V	35.052	61.765	60.166	65.120	51.112	59.350	41.938	
M	35.968	66.587	64.389	46.524	77.382	49.003	71.461	
T	36.116	60.906	66.656	69.931	76.766	57.968	36.484	
K	36.598	61.581	67.866	72.449	70.088	49.467	36.146	
Y	37.044	64.612	62.905	80.273	78.082	75.628	62.682	
EE	37.162	66.769	56.870	50.994	60.618	54.224	48.706	
X	37.631	63.740	70.982	68.293	52.312	60.987	54.250	
EE	37.770	63.906	65.701	50.678	75.174	55.098	52.902	
Z	38.028	59.195	51.085	53.447	62.205	45.928	57.583	
A	38.355	61.368	64.105	58.448	62.445	72.773	43.064	
C	38.495	58.058	57.630	46.166	75.535	41.401	67.723	
O	38.444	62.052	60.335	45.464	69.714	45.102	52.006	
X	38.642	61.985	55.291	56.160	47.157	54.110	67.923	
U	38.695	62.766	66.112	52.068	60.209	47.598	40.681	
R	38.661	63.542	65.890	51.667	74.662	40.919	48.099	
V	39.076	54.062	41.619	86.888	72.799	44.083	57.740	
H	39.008	58.549	60.083	53.178	51.677	49.642	63.688	
F	39.802	62.855	54.024	44.853	79.574	82.876	56.497	
DD	39.949	59.021	62.159	67.641	57.666	33.964	73.305	
M	40.061	56.378	56.815	62.734	71.073	59.489	52.849	
G	40.895	57.113	51.812	73.519	83.311	43.916	42.375	
P	40.950	57.282	60.884	60.336	46.319	45.317	55.615	
Z	40.900	60.932	65.049	51.004	60.091	17.601	73.409	

	DD	41.094	57.351	58.817	66.188	55.016	51.003	70.236
	A	41.163	60.529	60.719	52.949	60.084	60.713	17.770
	CC	41.290	56.521	44.533	64.524	35.889	44.654	56.386
	EE	41.311	61.464	59.968	55.943	57.784	57.893	36.596
	FF	41.869	54.490	63.233	55.298	32.379	94.385	16.369
	P	41.882	58.142	58.317	53.899	41.494	35.834	75.651
	T	41.995	57.880	52.135	44.863	37.728	44.629	68.899
	P	42.323	60.030	62.795	68.268	44.644	48.742	55.790
	K	42.760	57.629	53.976	77.617	71.480	61.216	10.870
	W	42.760	58.550	59.453	78.672	46.941	30.920	12.460
	BB	43.083	53.684	72.182	60.595	75.275	31.310	51.575
	V	43.008	56.908	49.409	67.119	59.218	56.214	40.186
	N	43.110	58.100	58.772	52.267	57.147	48.960	37.734
	J	43.646	56.577	50.908	56.253	48.105	47.967	56.064
	CC	43.712	58.101	50.093	40.190	47.204	63.018	57.857
	AA	44.126	55.370	75.182	47.002	45.981	59.205	46.196
	O	44.374	57.475	44.250	47.732	56.654	75.684	10.392
	O	44.893	53.341	47.521	53.569	69.435	50.198	31.245
	DD	45.181	48.043	59.913	41.877	75.900	53.665	74.004
	Y	45.351	57.257	48.269	51.699	39.557	60.059	66.148
	M	45.768	51.352	53.577	57.714	58.118	54.482	59.231
	T	45.988	52.528	56.232	70.151	58.378	61.285	38.492
	J	45.984	59.324	47.646	54.165	62.003	63.405	77.359
	CC	46.053	55.101	59.692	53.761	37.966	2.787	46.813
	B	46.144	56.156	48.058	50.092	50.594	80.815	33.277
	H	46.416	52.371	33.747	43.906	63.035	57.541	25.711
	M	46.694	50.003	54.609	64.809	46.549	55.166	44.087
	Z	46.622	56.120	61.964	58.636	55.278	38.206	31.262
	D	46.761	58.901	55.478	49.293	54.840	17.634	28.460
	A	46.812	50.486	39.249	42.734	56.479	28.939	54.340
	N	47.000	52.829	63.307	46.771	33.286	57.419	63.893
	W	47.269	52.693	42.404	47.094	64.719	81.841	36.815
	BB	47.240	53.235	40.382	60.181	31.015	66.186	51.671
	F	47.326	49.911	57.016	41.806	54.999	59.693	84.598
	CC	47.594	46.963	53.768	55.462	17.409	71.897	56.993
	RR	47.724	49.946	47.998	39.727	55.320	46.522	68.982
	N	48.072	51.549	46.754	62.215	42.623	62.272	71.895
	AA	48.543	45.910	45.535	61.971	46.469	22.732	82.677
	Y	48.544	51.423	59.639	45.972	56.536	56.900	30.808
	H	48.749	54.446	43.883	44.375	50.514	67.869	13.192
	D	48.964	51.417	62.584	51.228	50.612	49.235	55.001
	P	48.922	49.498	53.874	41.879	43.401	72.493	60.206
	E	49.006	51.458	53.003	69.977	46.039	39.144	75.494
	X	49.141	51.146	54.588	54.206	58.836	49.972	55.175
	A	49.456	49.655	49.969	70.118	58.424	51.533	29.236
	F	49.435	54.229	50.717	28.820	64.592	45.123	59.626
	H	49.568	50.856	43.334	66.815	31.924	60.079	71.162
	M	49.681	52.754	57.116	39.834	56.576	30.181	66.441
	O	50.048	50.897	47.470	64.586	82.771	48.121	81.172
	AA	50.341	51.796	55.890	38.177	42.514	63.875	61.002
	V	50.579	50.593	43.917	59.797	73.104	43.930	43.686
12	EE	50.516	55.178	43.775	27.230	24.857	32.817	40.300
11	B	50.787	53.309	46.372	56.379	65.205	49.888	46.872
10	EE	50.977	50.082	46.405	32.601	42.123	31.827	24.567
9	FF	51.214	51.049	47.271	49.500	34.650	29.925	49.063
8	C	51.456	47.647	53.280	50.328	56.787	57.069	42.341
7	EE	51.401	48.347	59.290	48.129	53.258	52.215	54.799
6	G	52.107	49.303	47.970	44.050	46.254	54.597	46.100
5	H	52.323	45.213	59.339	45.640	27.636	37.949	51.987
4	I	52.310	46.384	54.614	42.551	31.323	37.579	41.706
3								

	P	52.598	45.407	60.569	39.881	11.665	55.459	30.181
	W	52.549	46.465	38.305	30.853	50.013	70.147	31.930
	FF	52.669	47.216	34.008	42.801	42.523	60.149	30.944
	P	52.764	47.294	51.901	66.961	60.250	63.027	54.276
	H	52.760	49.622	56.417	44.069	45.928	41.258	69.049
	R	53.132	44.576	48.526	38.200	25.525	32.280	60.312
	A	53.324	43.762	53.765	39.160	50.948	45.904	33.154
	A	53.488	42.661	38.527	44.537	74.085	55.803	59.440
	BB	53.567	42.331	49.479	54.104	45.072	30.975	43.655
	CC	53.567	49.108	49.027	53.858	41.539	61.122	59.963
	S	53.637	43.596	56.749	39.570	64.235	34.549	36.539
	M	53.667	44.926	44.746	50.866	50.691	65.414	86.732
	A	53.767	44.139	54.226	35.590	64.955	33.510	57.244
	H	53.841	45.555	44.867	48.160	44.085	63.660	91.348
	S	53.855	45.940	45.039	37.657	50.912	75.655	54.054
	C	54.274	49.480	59.222	44.397	44.906	65.498	43.541
	R	54.494	40.848	35.307	64.140	52.605	58.837	43.374
	W	54.497	42.654	66.764	42.476	56.999	53.346	32.855
	H	54.764	45.005	32.505	27.359	59.054	61.775	37.635
	G	54.882	43.063	53.147	48.463	39.708	66.033	39.207
	C	54.881	46.811	46.305	57.350	52.934	61.560	75.407
	A	55.068	42.518	51.211	47.141	52.204	39.333	61.460
	J	55.040	45.614	37.502	47.501	43.423	51.310	33.136
	DD	55.240	42.476	44.857	53.029	39.767	29.852	71.390
	Z	55.269	42.756	50.164	40.297	36.644	25.526	42.065
	E	55.213	44.053	59.277	46.272	29.721	29.977	58.610
	D	55.276	44.948	42.118	25.549	38.587	44.104	65.471
	V	55.304	43.906	39.214	38.871	60.837	57.741	41.249
	CC	55.605	37.753	54.550	43.210	45.936	42.935	64.111
	BB	55.981	47.283	42.805	55.269	41.322	77.090	49.137
	B	56.039	43.738	49.280	40.031	63.847	19.562	28.451
	N	56.068	44.176	43.679	25.287	54.522	27.100	25.222
	DD	56.184	42.163	53.163	32.157	45.104	58.575	54.613
	BB	56.233	41.699	36.952	44.345	36.176	46.964	50.442
	C	56.543	42.300	41.620	36.482	45.955	58.162	44.017
	AA	56.613	44.665	52.350	57.235	43.209	73.166	63.349
	H	56.804	44.717	44.332	27.898	48.611	42.214	18.995
	D	56.806	46.861	46.413	32.797	45.245	23.228	85.353
	D	57.106	44.794	33.214	54.346	46.520	72.234	77.006
	B	57.925	37.614	47.710	35.089	52.857	33.166	42.723
	N	57.959	40.289	46.059	45.100	30.806	35.047	34.506
	Z	58.106	42.328	40.220	45.671	32.543	20.196	53.187
	G	58.413	41.336	46.520	46.879	61.232	56.506	31.025
	E	58.928	40.821	37.937	37.824	38.444	25.577	50.280
	D	59.099	41.545	41.495	55.559	29.203	48.749	51.221
	E	59.108	37.712	41.489	42.645	58.421	55.413	65.082
	E	59.198	38.508	35.514	56.353	71.525	42.665	76.016
	FF	59.472	40.790	24.754	40.964	39.586	35.840	85.452
	V	59.675	44.964	43.689	46.325	59.262	53.349	63.628
	H	59.713	40.505	49.355	42.747	18.578	62.797	45.130
	C	59.741	40.642	45.478	45.930	37.940	67.427	37.547
12	P	59.819	40.999	42.319	46.563	50.636	21.507	34.503
11	I	59.943	39.381	47.955	42.124	41.032	27.502	19.200
10	A	59.980	43.395	32.997	35.466	52.189	56.302	48.903
9	FF	60.059	37.898	44.154	38.353	47.011	60.615	55.574
8	O	60.753	38.119	28.781	37.116	48.515	60.050	27.387
7	T	60.978	36.711	37.956	25.963	26.994	71.419	31.296
6	J	60.983	39.876	35.676	58.778	61.438	39.550	63.084
5	H	61.380	38.370	35.878	49.764	32.325	61.503	67.128
4	V	61.521	34.456	30.255	39.546	53.855	34.397	23.567
3								

AA	61.538	38.884	37.579	42.774	48.490	22.850	30.533
T	61.586	41.933	38.013	33.774	58.156	28.705	67.316
Y	62.101	37.322	33.549	50.453	44.925	69.724	27.651
EE	62.180	38.365	53.270	31.962	47.391	14.620	81.474
F	62.249	37.647	38.507	35.894	56.862	61.892	70.391
W	62.766	36.049	37.571	45.804	54.432	22.358	34.791
X	63.029	35.933	40.937	36.103	48.771	66.924	82.718
K	63.229	35.381	42.157	38.630	49.645	61.740	27.763
F	63.314	36.146	33.423	57.839	57.875	19.275	35.028
Y	63.427	37.180	41.995	38.034	30.292	56.652	35.685
T	63.726	40.368	42.631	37.614	45.030	31.415	48.326
A	63.911	39.726	37.041	56.365	26.740	13.532	17.051
C	64.466	31.777	39.057	31.456	40.346	17.298	46.107
H	64.554	36.017	43.222	53.206	33.970	26.815	57.363
T	64.730	37.987	45.726	24.983	51.936	30.023	74.809
W	65.022	34.287	32.479	51.638	19.373	58.967	57.826
K	65.350	37.809	39.589	54.670	62.152	47.465	71.625
Y	65.494	27.786	40.938	54.846	58.978	35.626	21.372
I	65.717	32.041	37.472	27.493	74.193	15.271	39.041
V	65.793	34.591	43.081	34.792	25.798	41.420	47.854
H	65.890	31.821	33.922	33.190	37.210	35.750	42.161
A	65.912	36.389	33.833	43.652	47.922	48.054	44.616
F	66.119	33.432	32.491	16.483	34.860	63.416	46.524
Y	66.360	32.705	22.726	46.053	35.875	51.857	14.214
D	66.671	38.332	36.681	46.521	15.631	44.890	42.214
K	66.721	37.602	35.013	29.254	15.846	17.052	72.395
C	66.844	36.172	30.625	24.595	21.061	85.668	37.871
O	67.051	33.702	45.473	19.237	34.104	61.909	58.174
P	67.602	36.125	40.040	52.525	35.229	39.611	58.035
K	67.849	29.279	25.734	22.722	22.060	16.201	61.402
I	68.040	32.230	35.138	40.703	18.509	31.461	79.162
G	68.288	34.193	18.608	41.610	55.990	31.406	32.925
T	68.503	28.180	20.819	28.272	47.426	31.791	55.181
H	69.720	32.853	31.264	32.318	19.421	49.712	41.859
P	68.810	29.109	31.213	38.180	34.965	30.661	34.063
M	69.233	28.999	49.482	20.091	44.892	33.359	40.929
AA	69.316	32.605	29.328	48.639	36.499	52.005	72.068
T	69.722	30.755	17.485	23.369	44.314	68.254	51.914
DD	69.911	32.787	41.990	44.708	37.112	45.977	83.793
G	70.267	31.902	26.771	38.959	42.792	20.147	51.756
K	70.501	29.980	37.620	42.348	50.917	40.332	64.351
EE	71.111	29.995	28.388	49.493	47.329	38.592	45.778
R	72.435	30.303	27.518	29.052	58.843	44.057	42.406
R	72.924	27.346	28.467	35.563	23.771	49.281	58.476
E	73.442	28.074	35.993	32.260	38.566	36.188	28.457
N	73.551	26.156	35.618	30.720	32.825	28.026	82.370
O	73.693	28.982	45.189	45.422	11.151	37.547	73.933
I	75.666	25.319	27.034	36.696	10.792	45.082	41.221
J	76.449	21.509	18.507	18.869	32.023	43.810	39.015
X	76.573	22.639	39.444	34.416	23.455	44.563	66.224
Z	76.552	27.233	21.203	31.828	48.723	36.556	42.311
CC	77.124	20.878	20.909	26.579	19.930	22.874	25.954
EE	77.291	24.052	20.457	20.251	39.144	43.915	34.542
AA	77.312	23.372	25.323	8.321	36.039	21.299	30.740
O	77.336	23.429	23.860	35.173	20.472	27.690	44.074
O	77.698	21.950	19.195	24.481	41.039	42.954	58.841
H	77.879	26.477	26.173	40.104	56.516	32.536	20.006
CC	79.951	21.253	36.618	34.436	27.408	23.897	58.405
P	80.186	19.184	18.446	34.012	42.212	10.068	89.462
T	80.450	20.555	34.766	1.971	55.424	29.600	52.981

R	80.987	20.433	23.793	-1.223	37.975	24.272	41.394
7	86.419	17.276	10.055	18.659	21.000	34.967	58.900
I	95.588	4.455	-.384	13.935	24.585	57.002	52.770
F	96.837	3.797	-7.050	9.911	1.320	34.462	43.789

1124 L

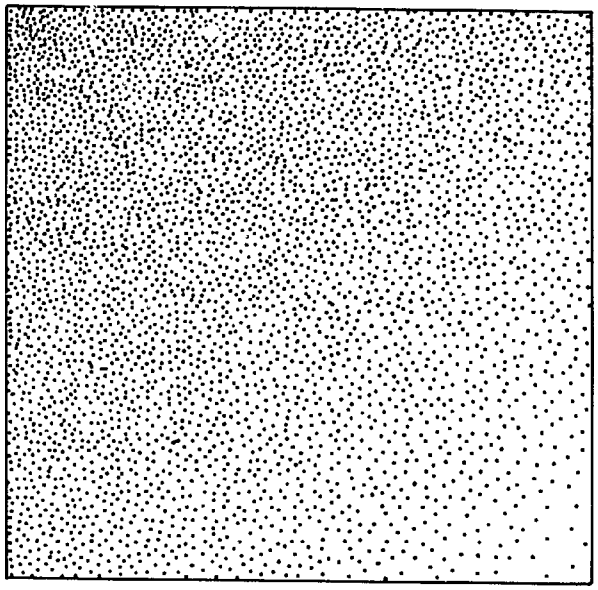
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APPENDIX C

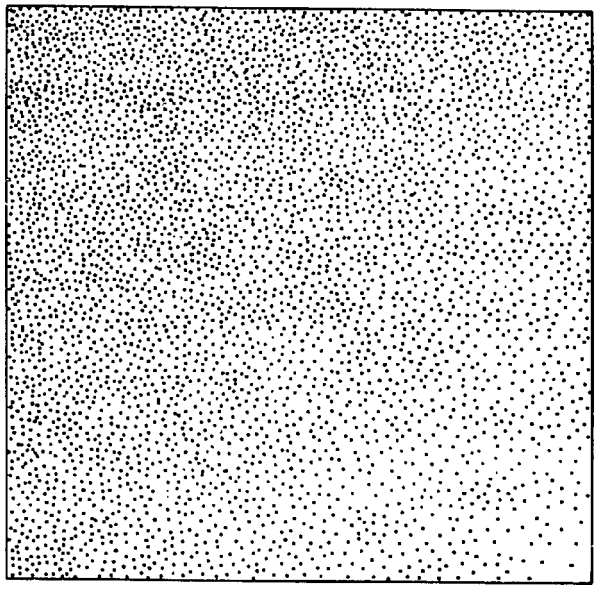
MAP TEST-SETS

CODE	r^2
26	.90 (+)
11	.80
29	.70
13	.60
28	.50
10	.40
19	.20
16/41	0
46	.20
44	.40
32	.50
45	.60
31	.70
43	.80
38	.90
40	1.00 (-)

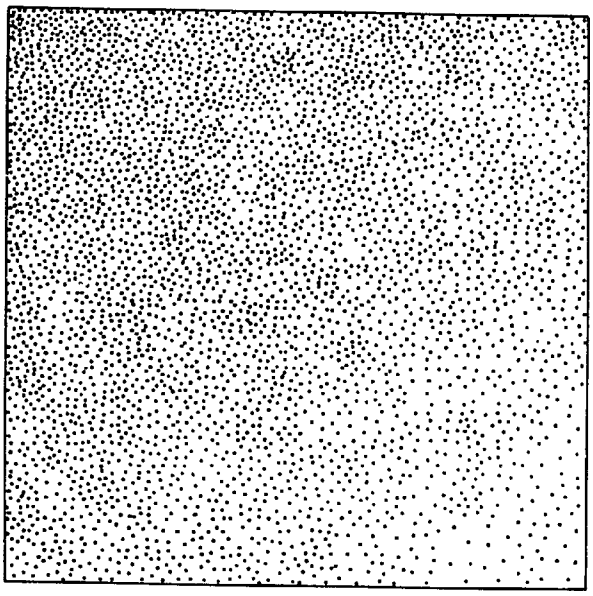
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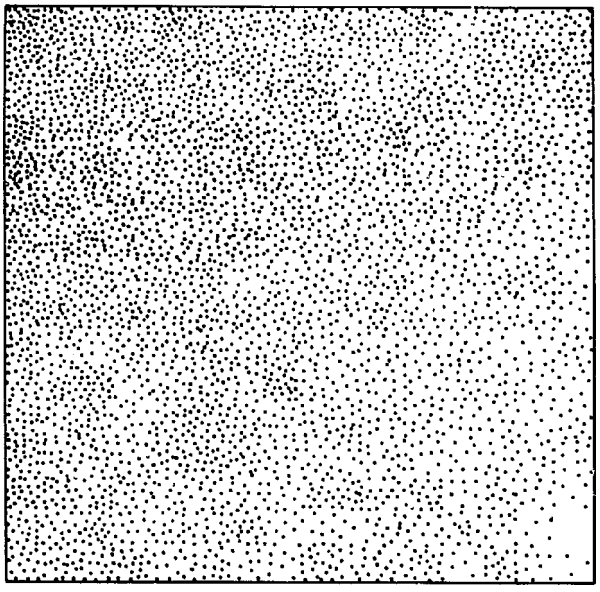
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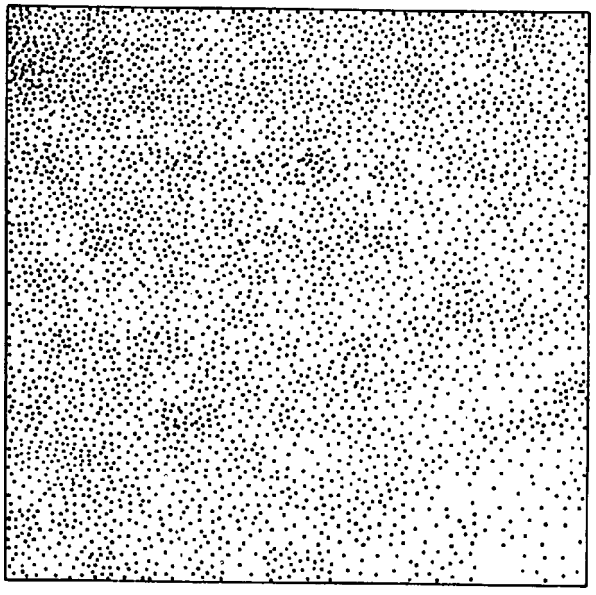
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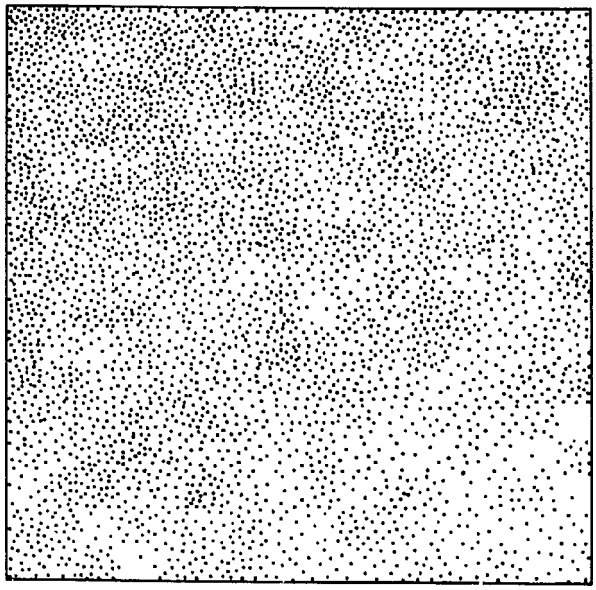
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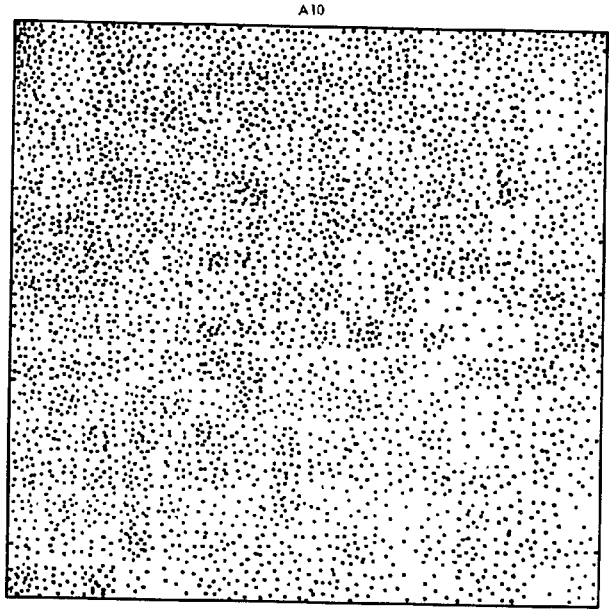
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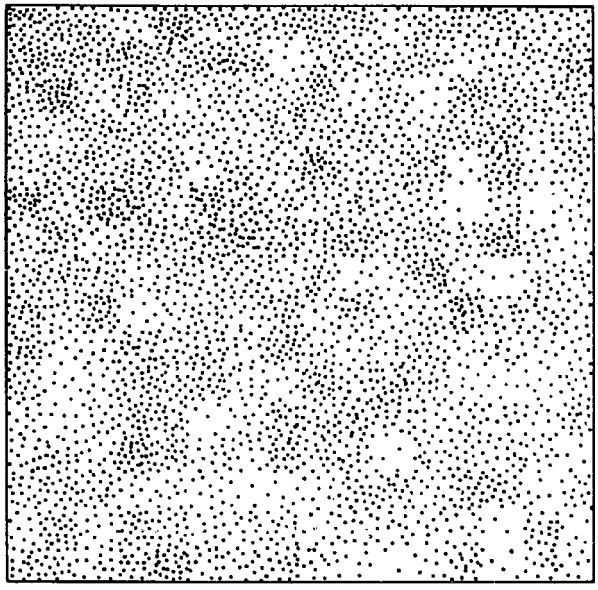
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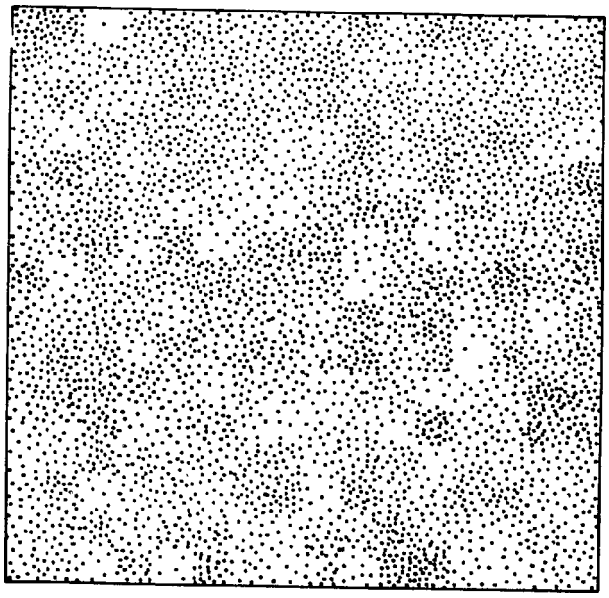
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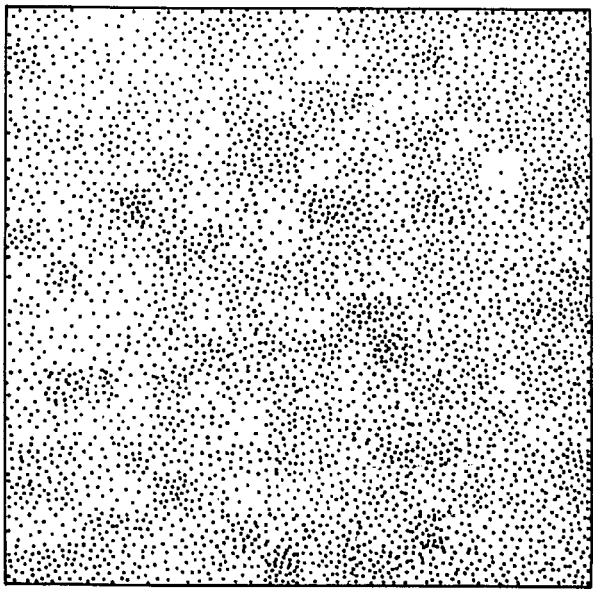
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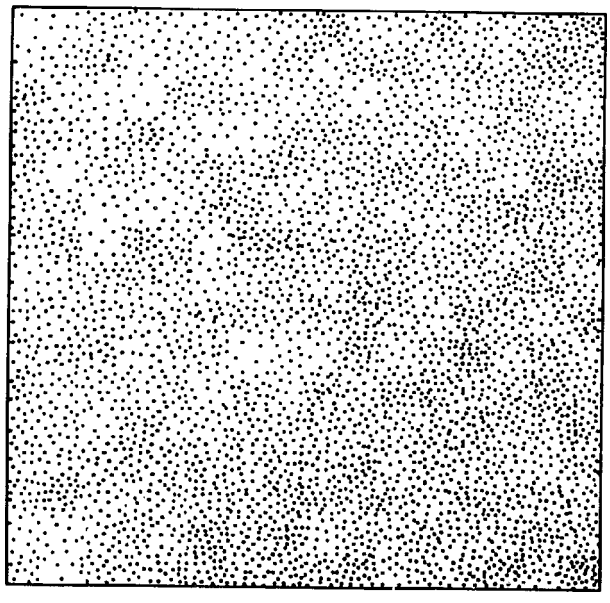
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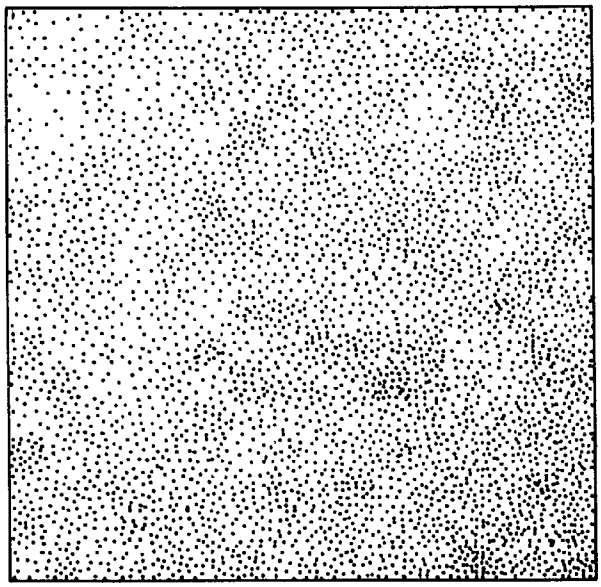
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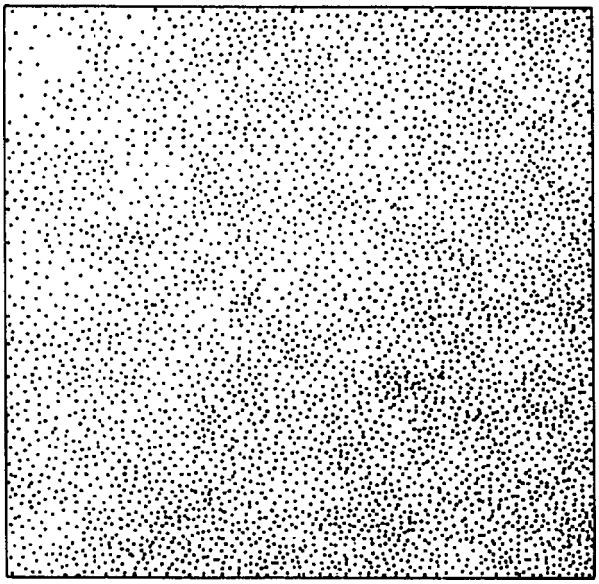
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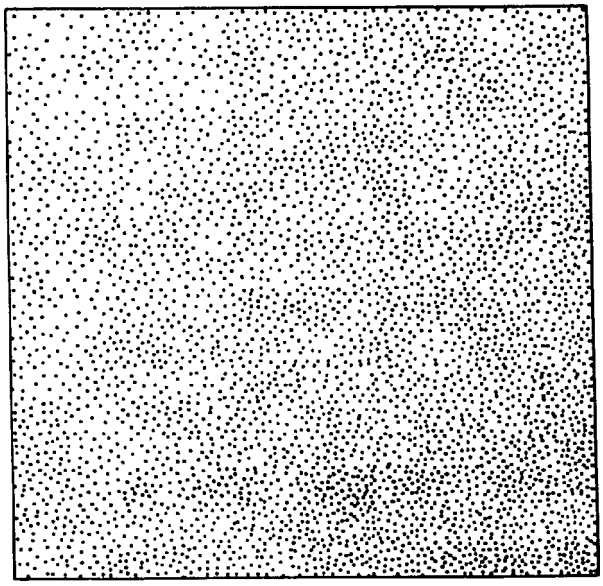
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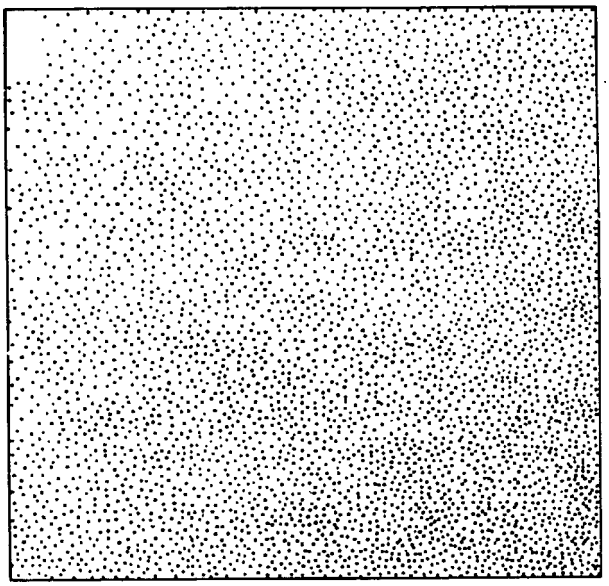
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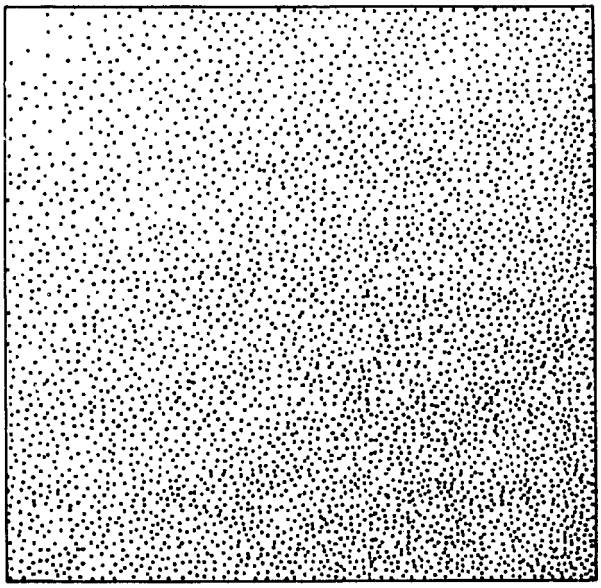
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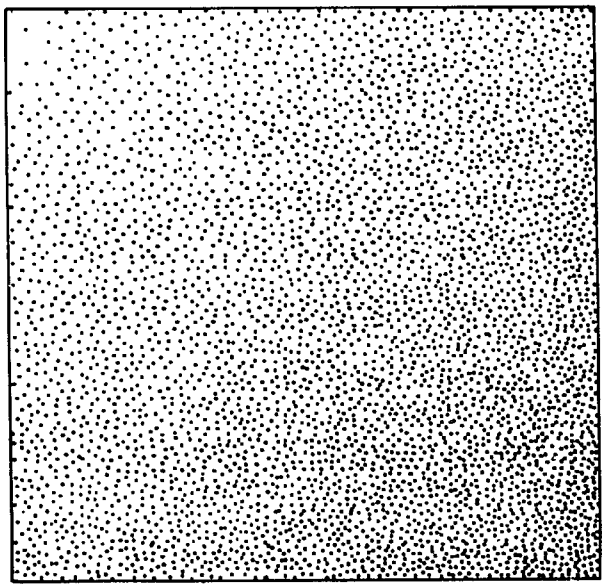
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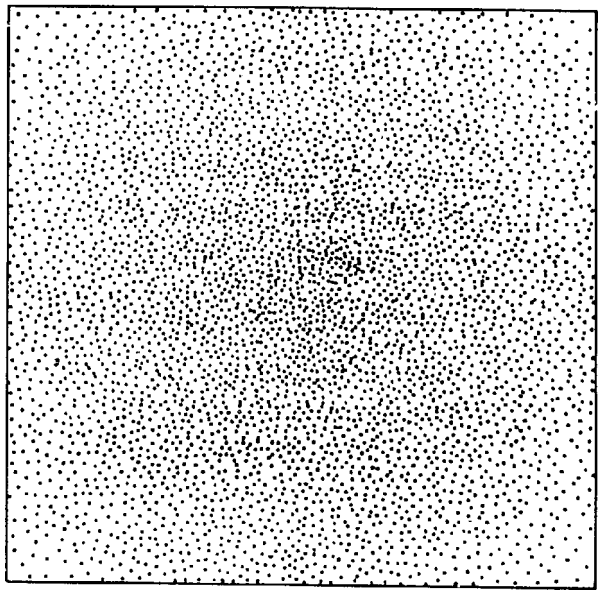
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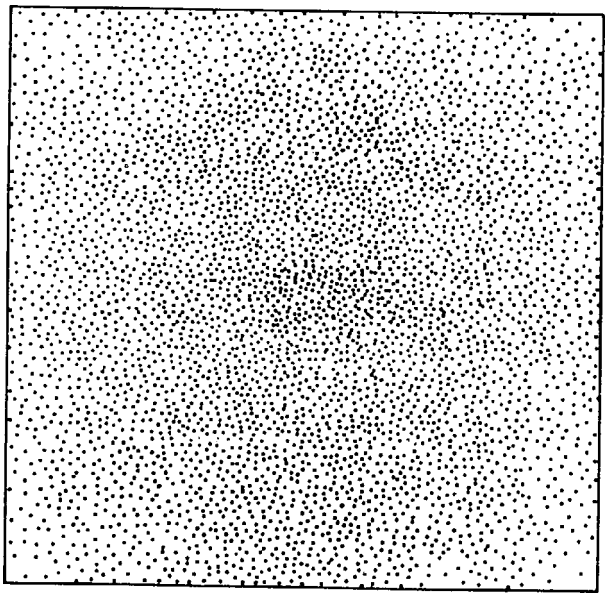
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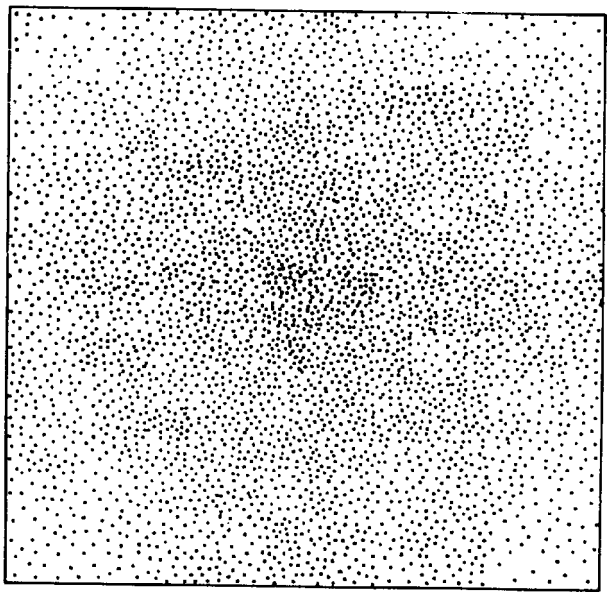
FORMAT (B4)



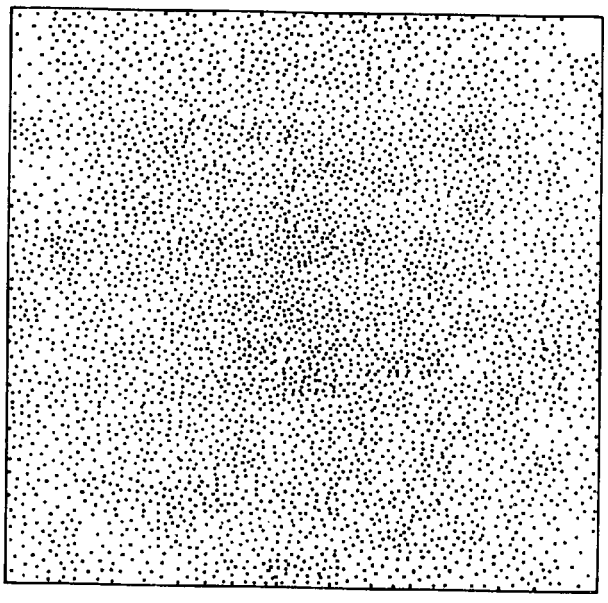
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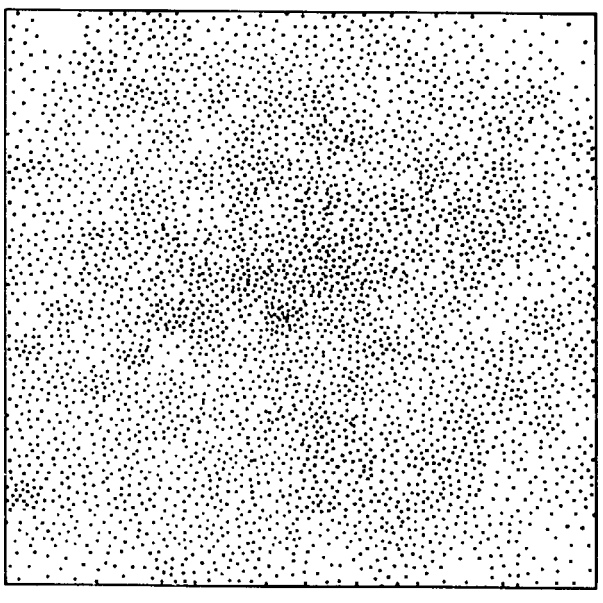
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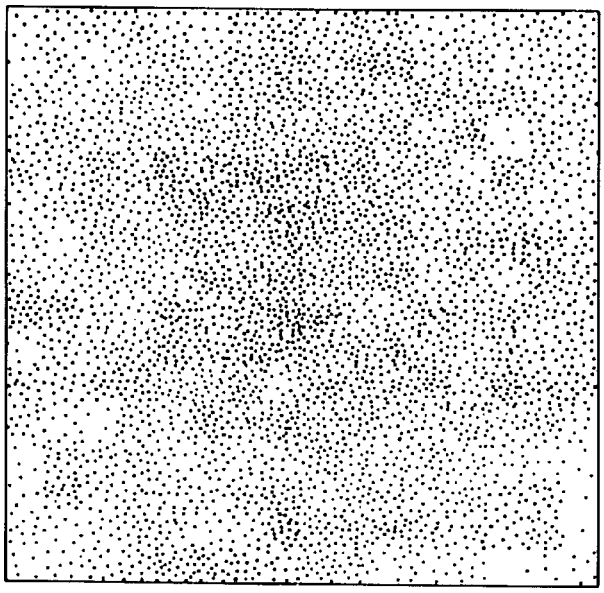
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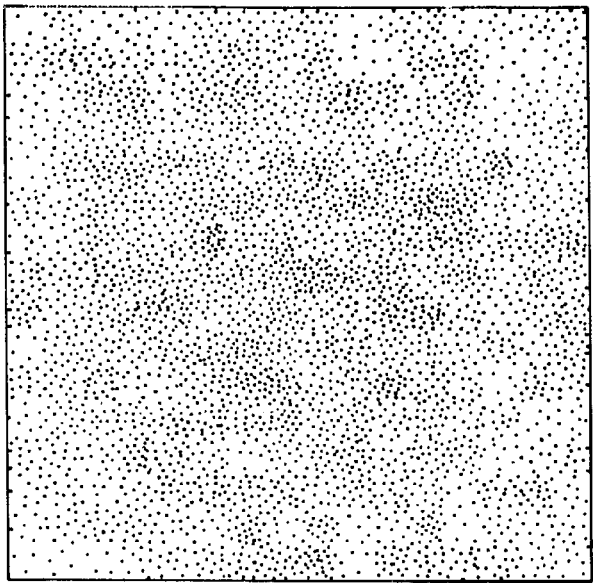
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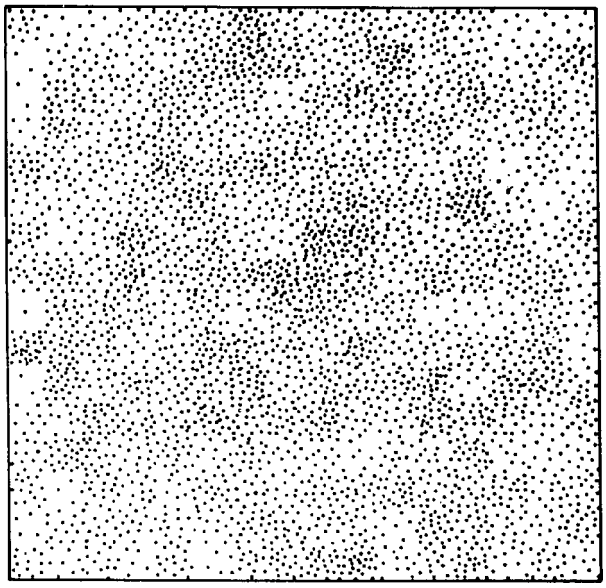
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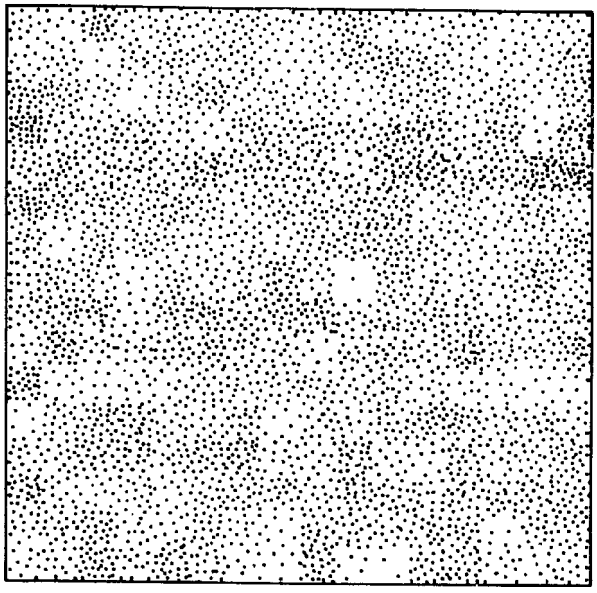
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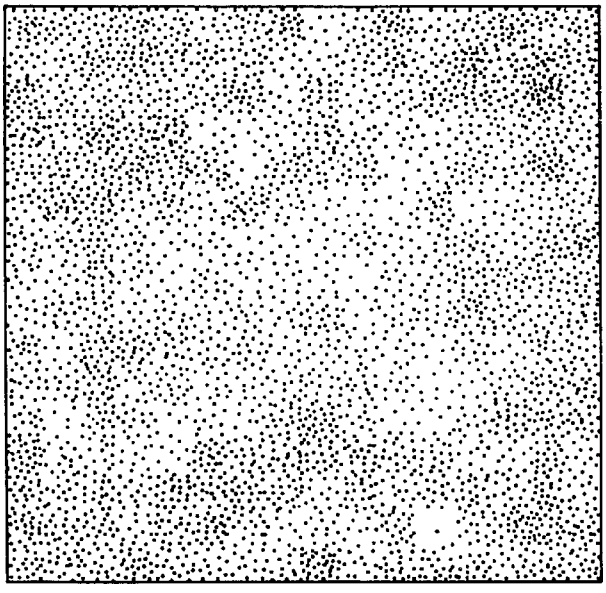
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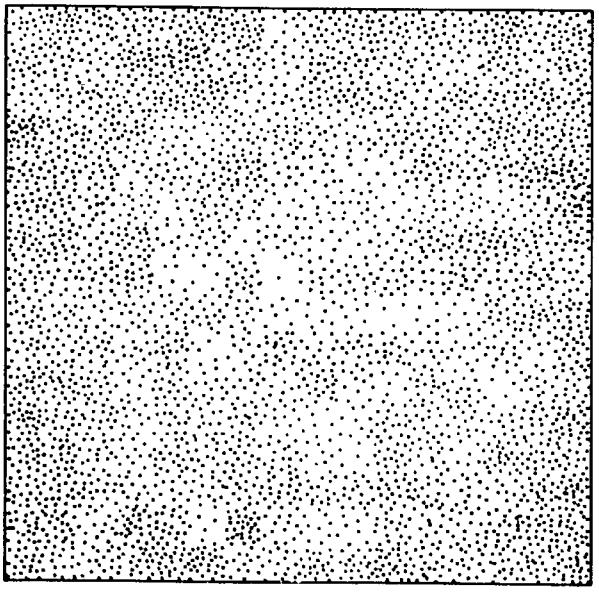
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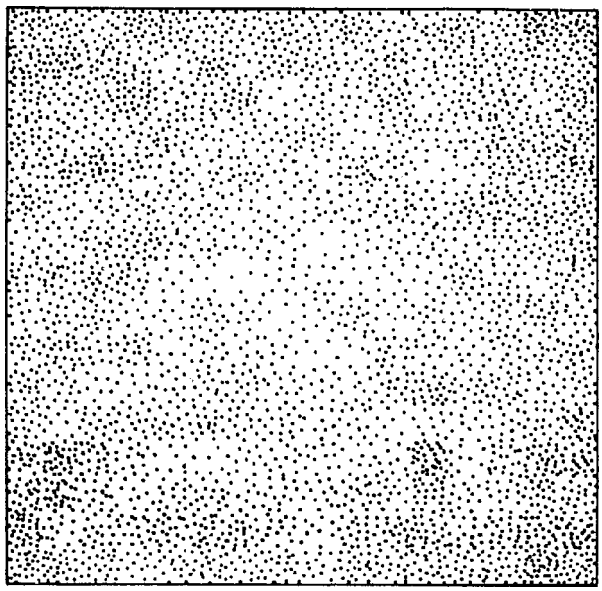
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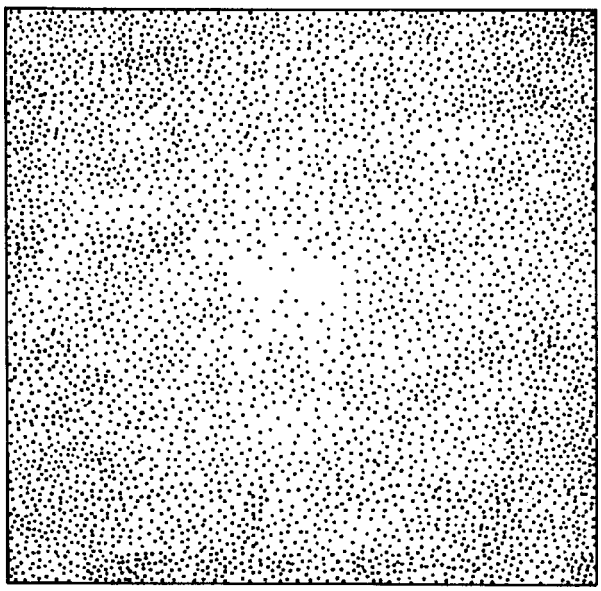
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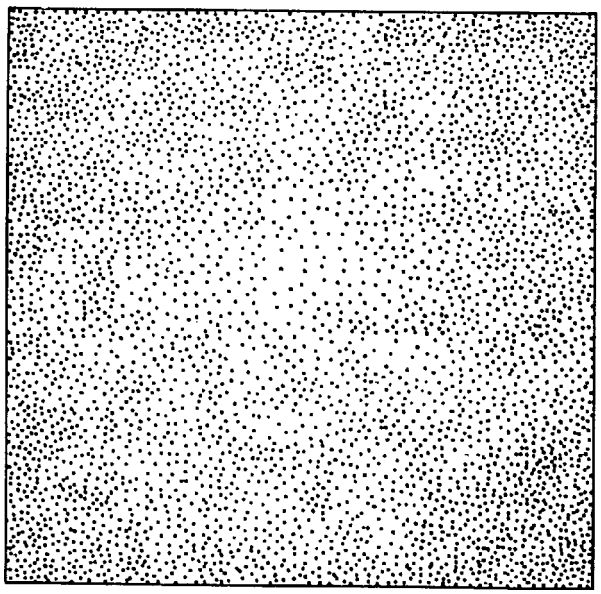
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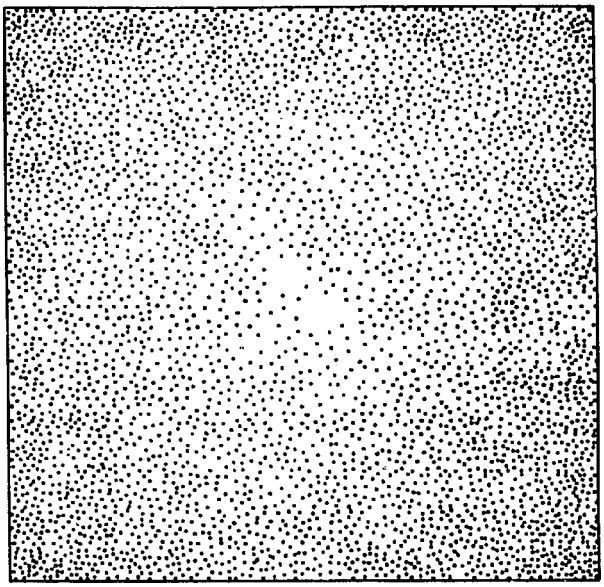
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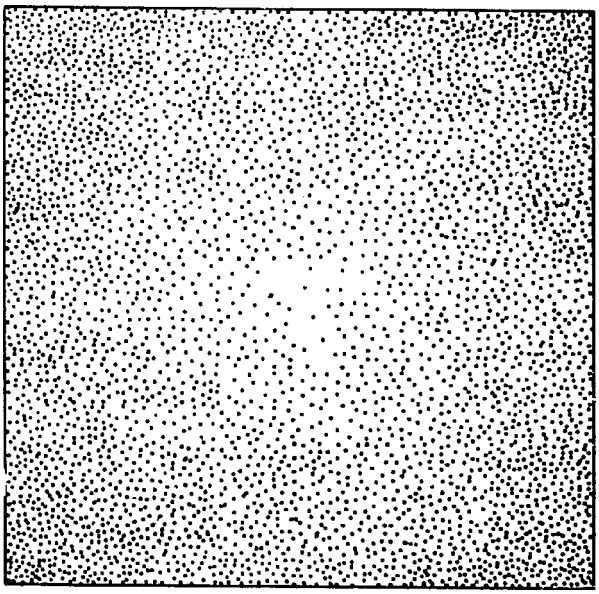
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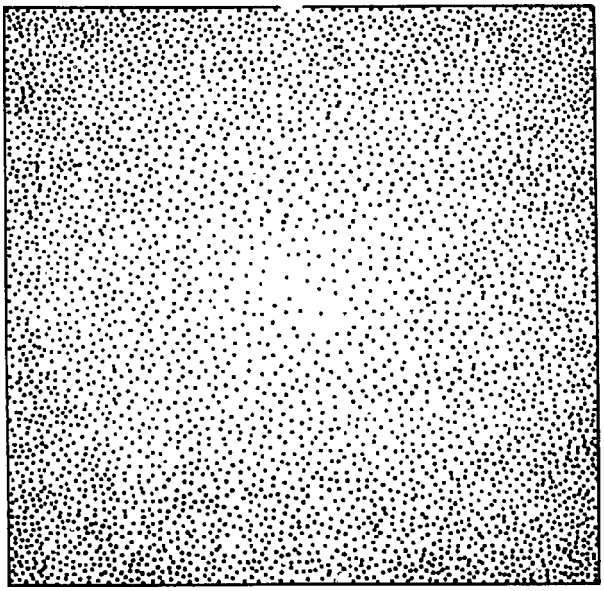
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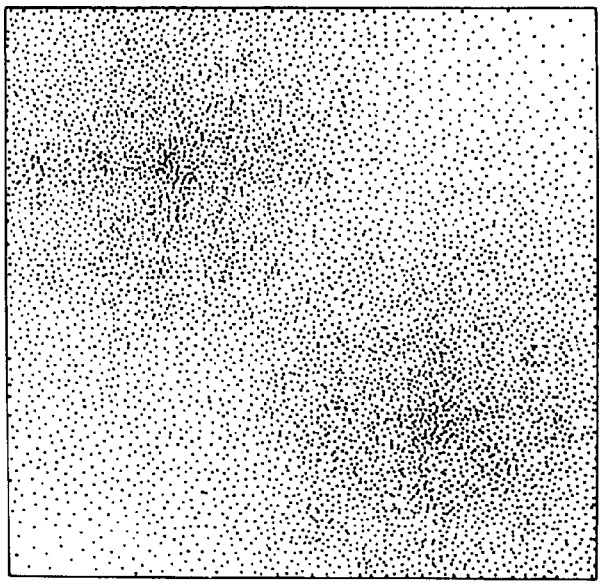
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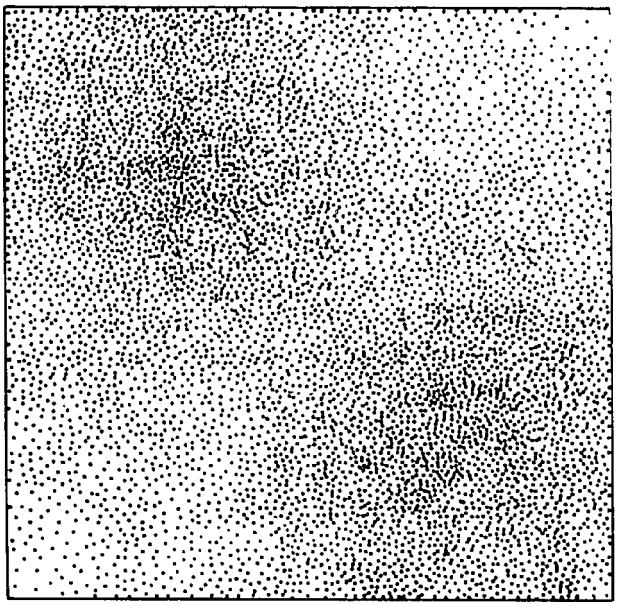
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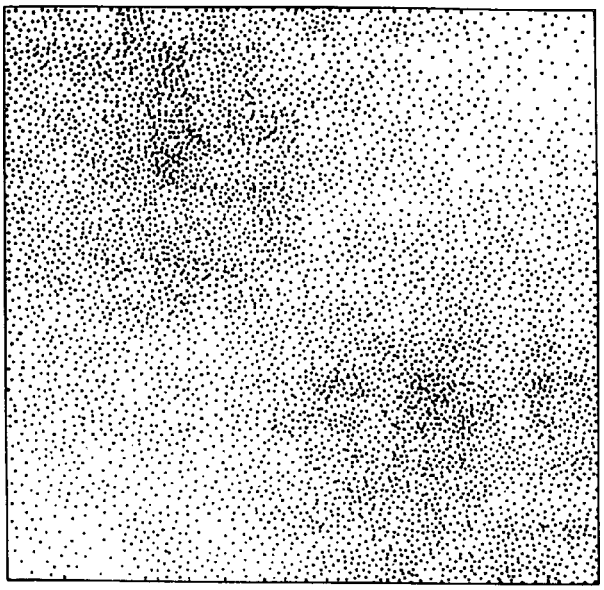
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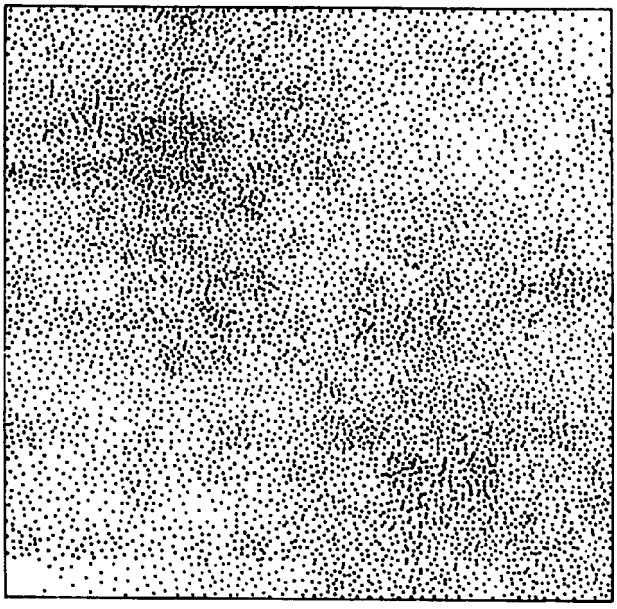
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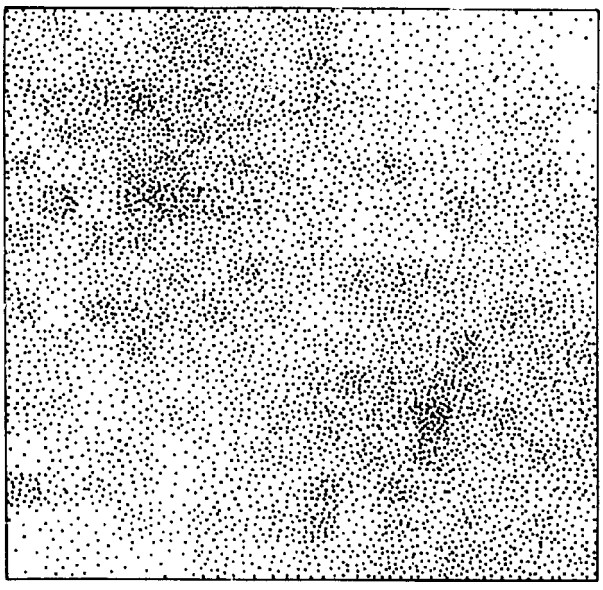
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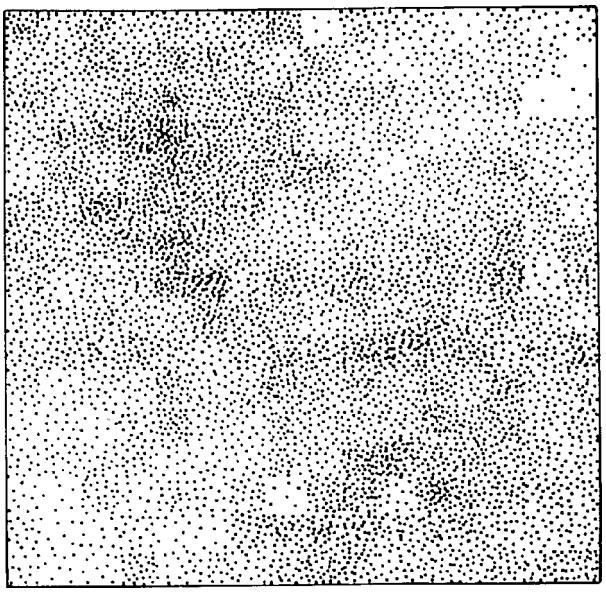
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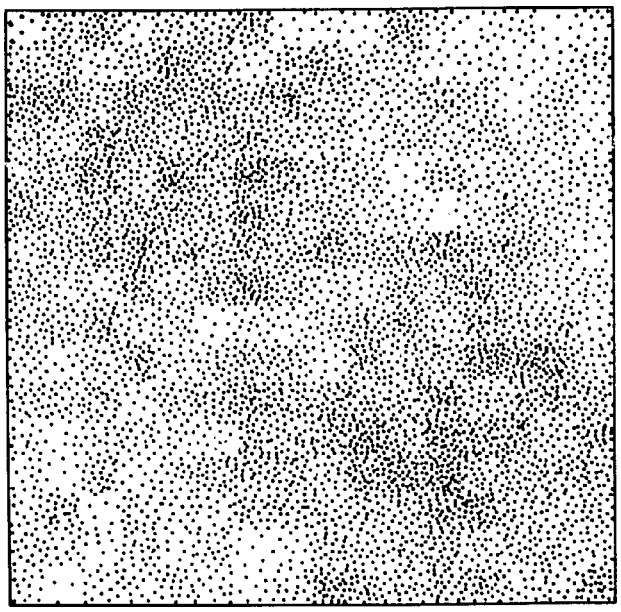
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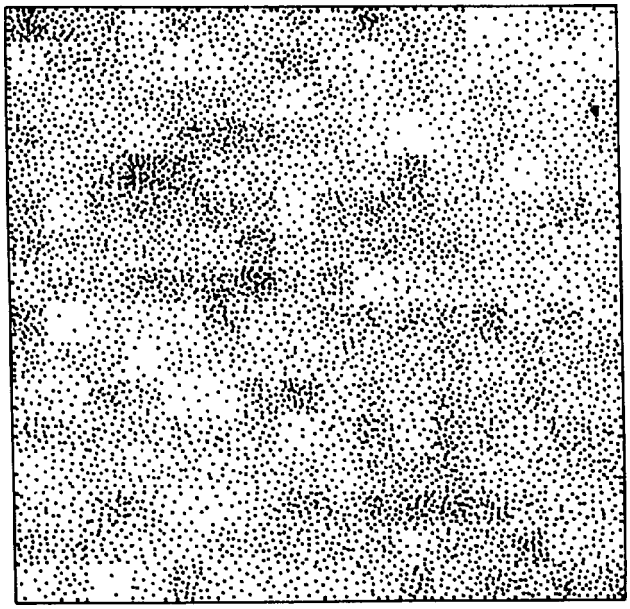
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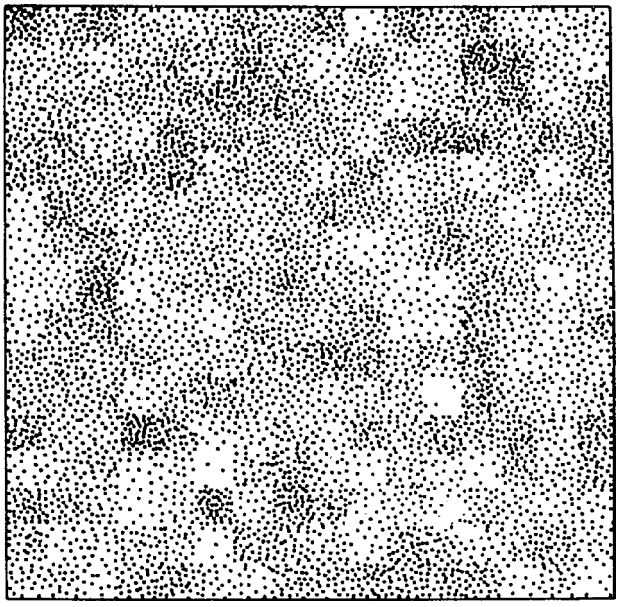
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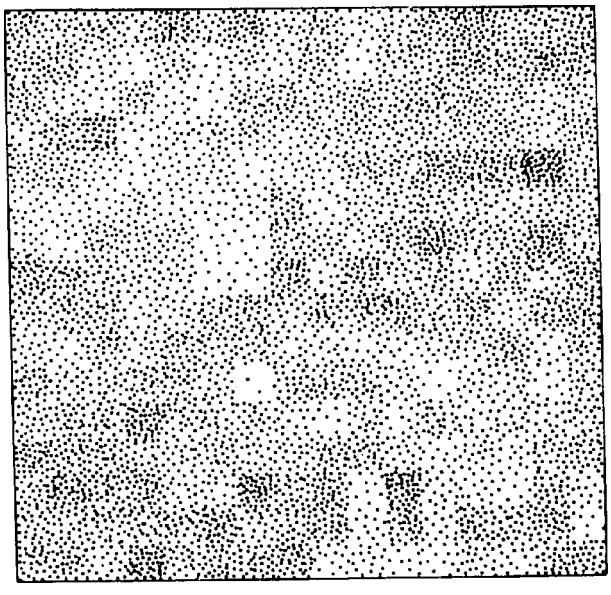
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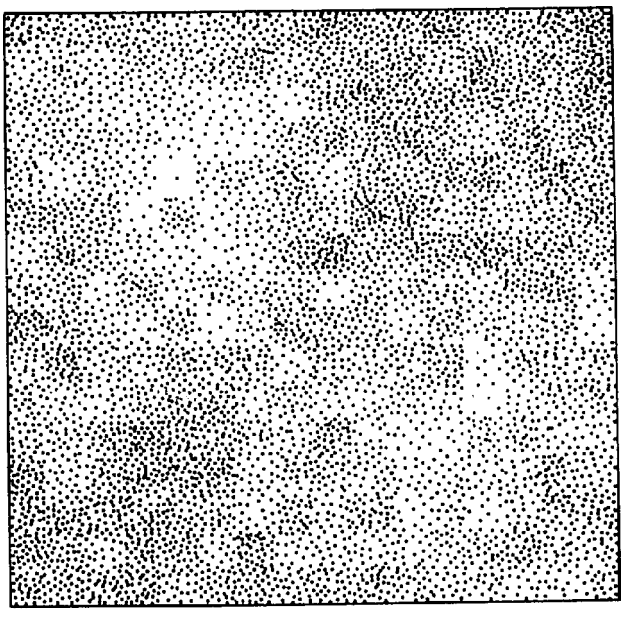
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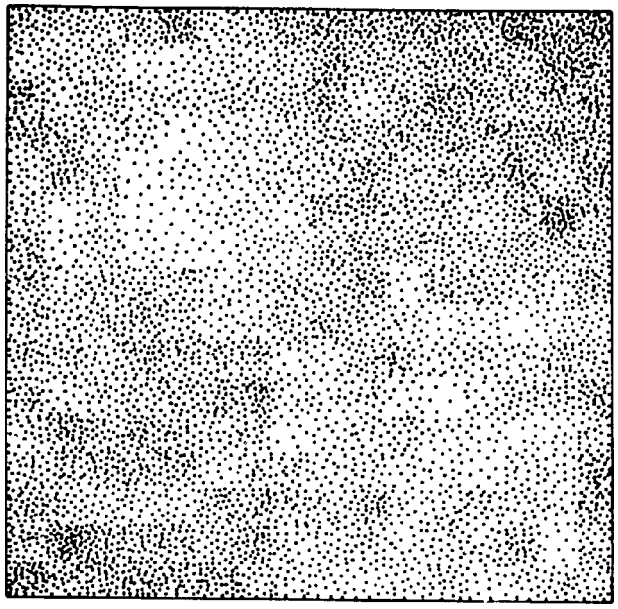
C46



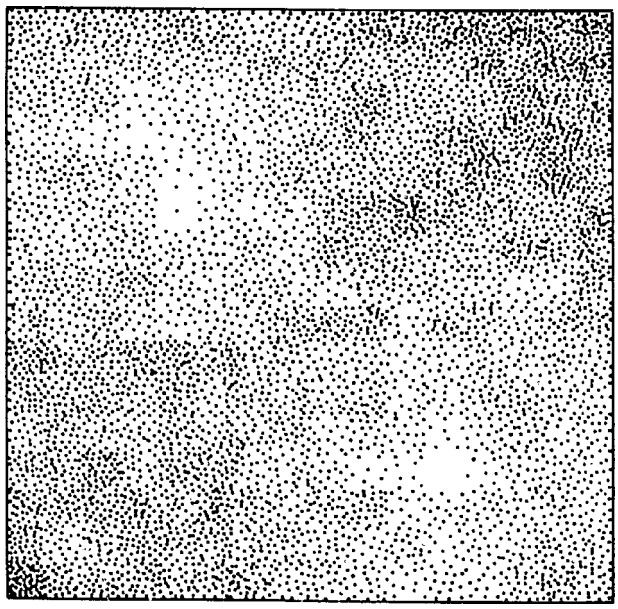
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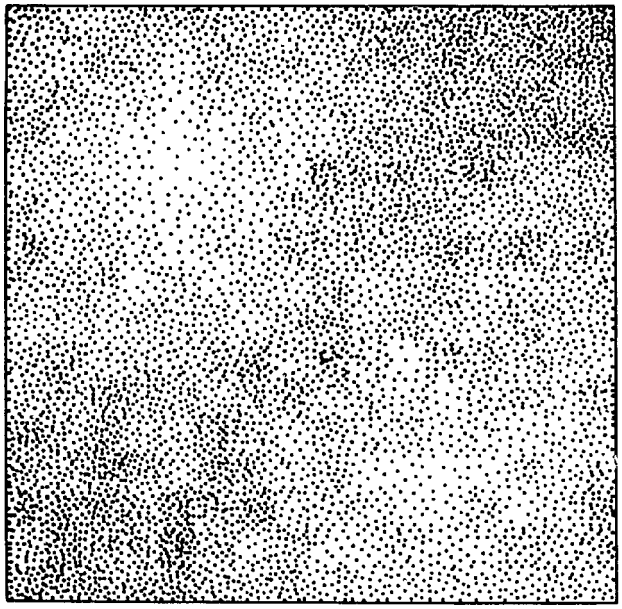
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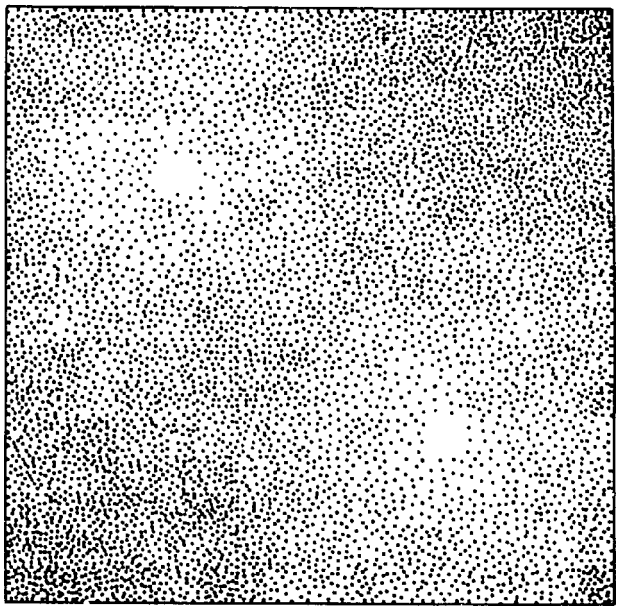
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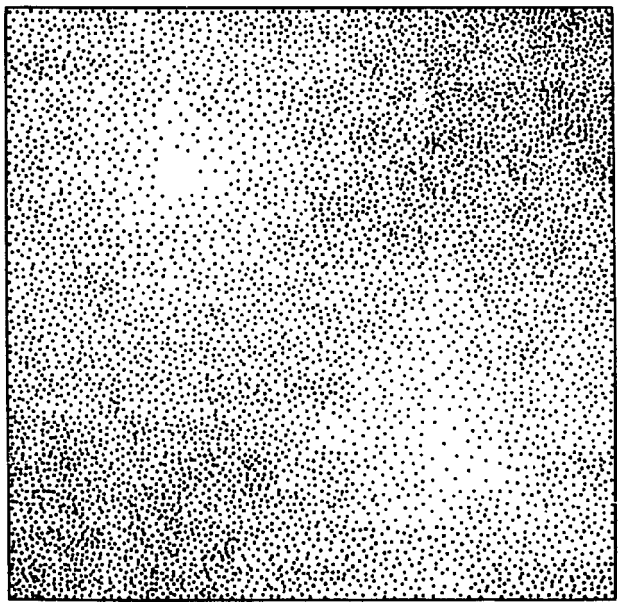
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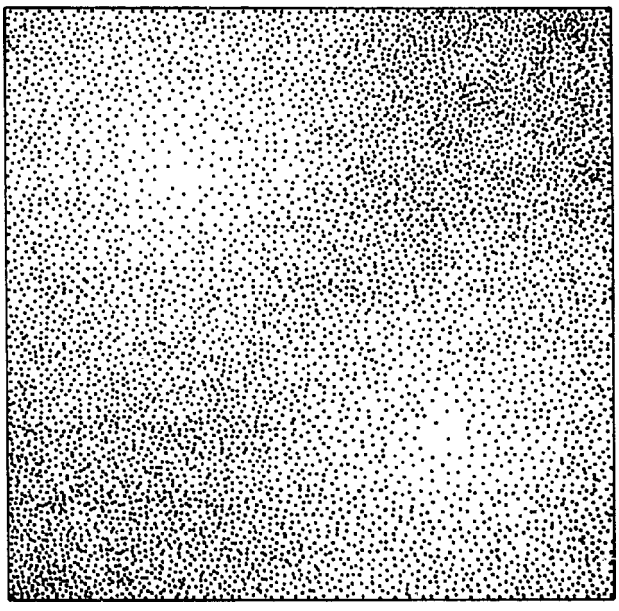
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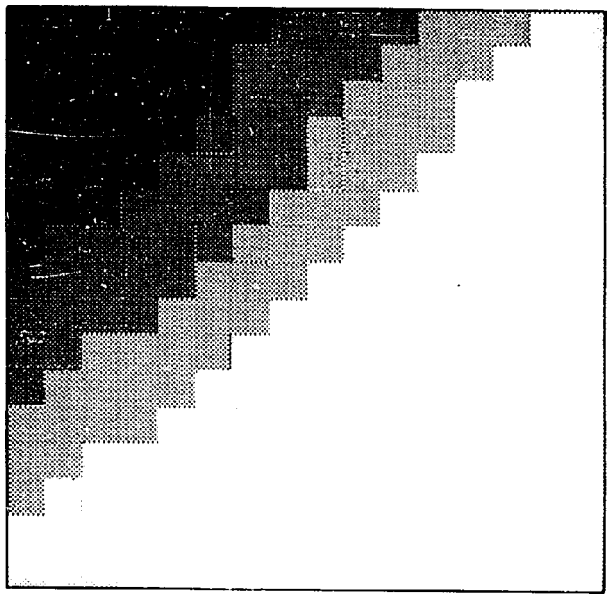
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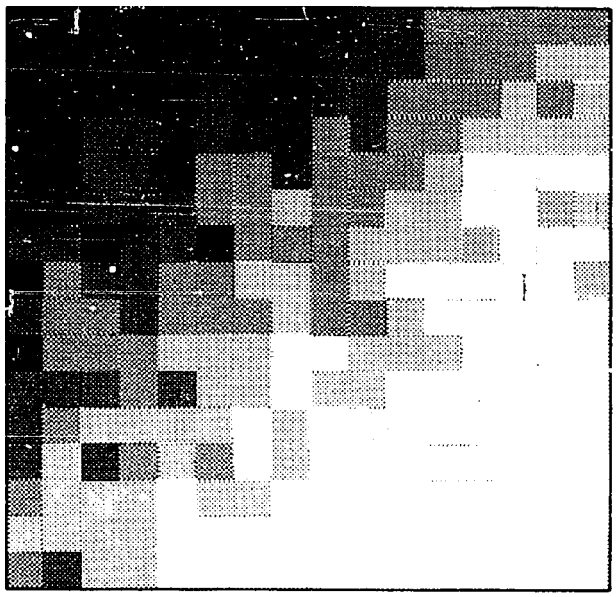
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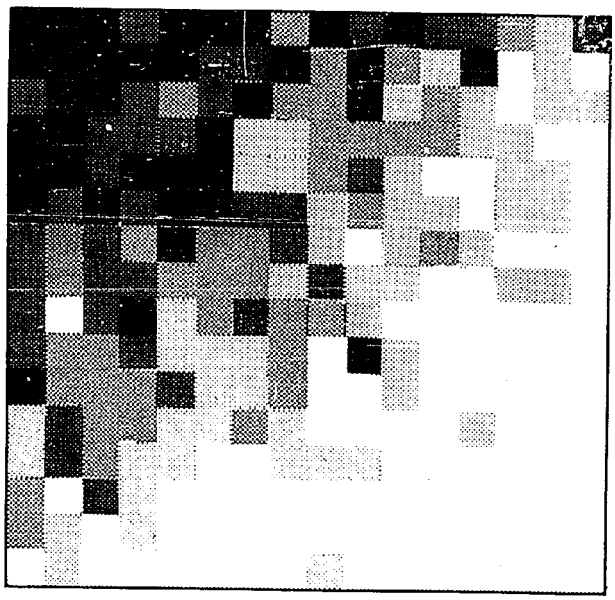
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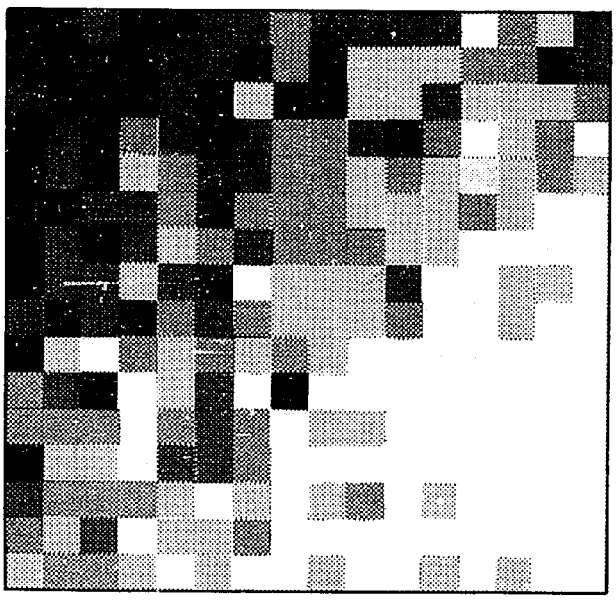
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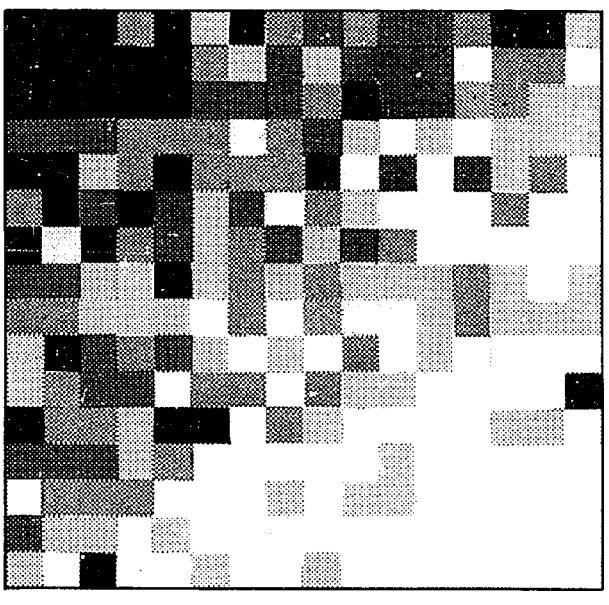
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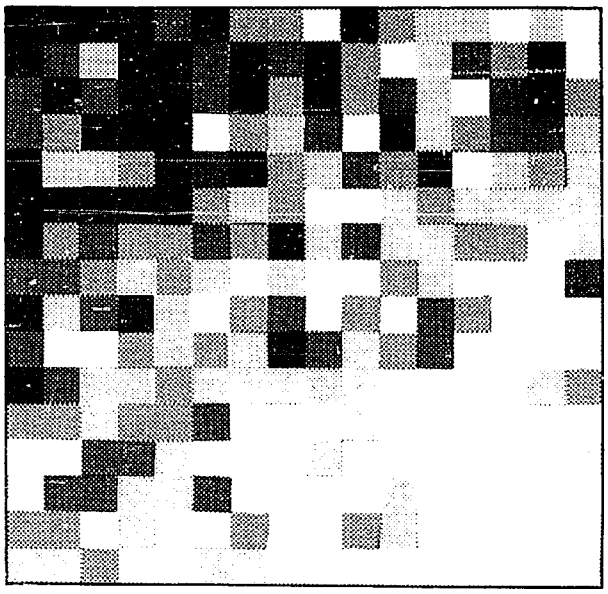
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A13



A28



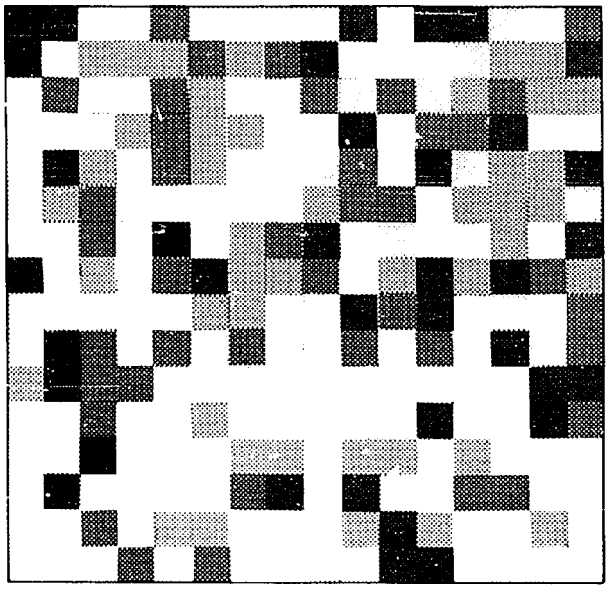
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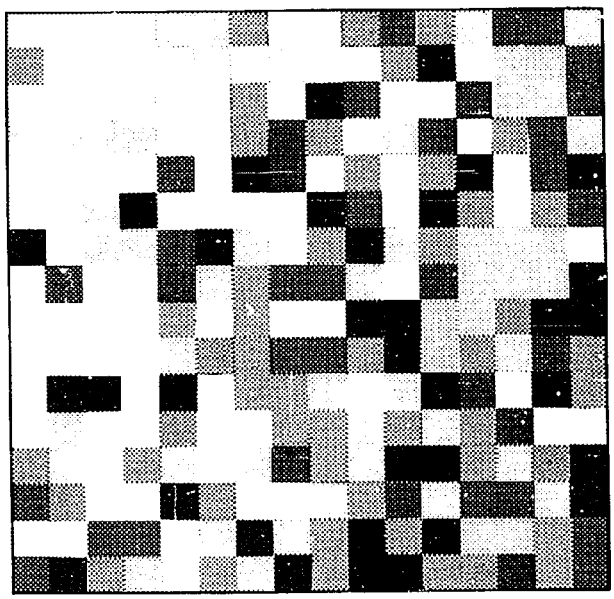
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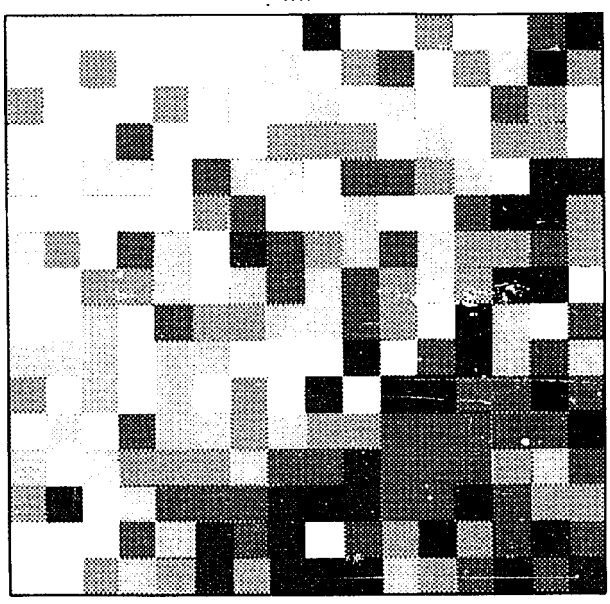
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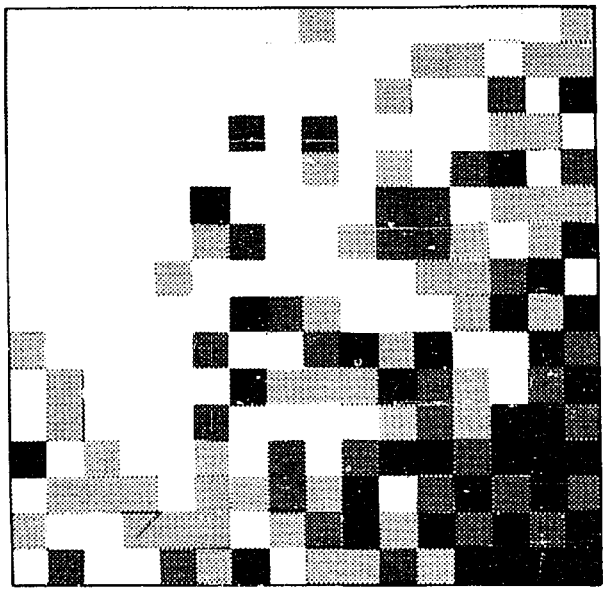


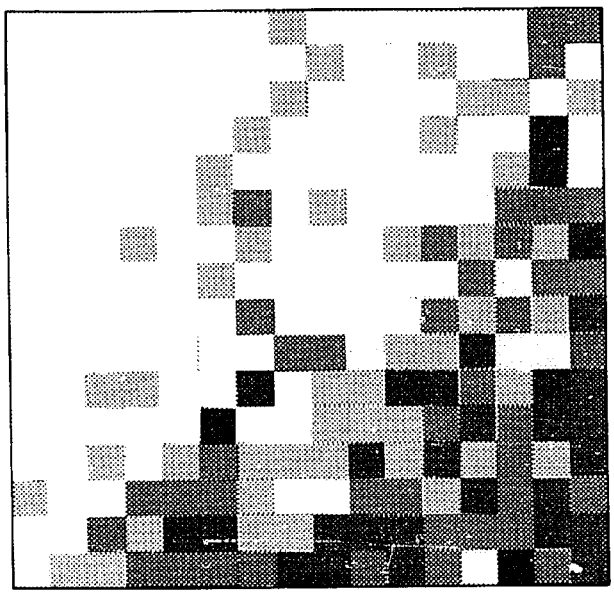
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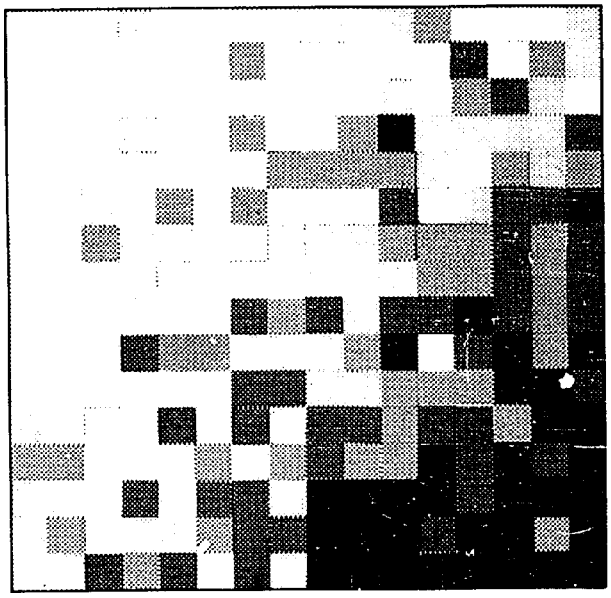


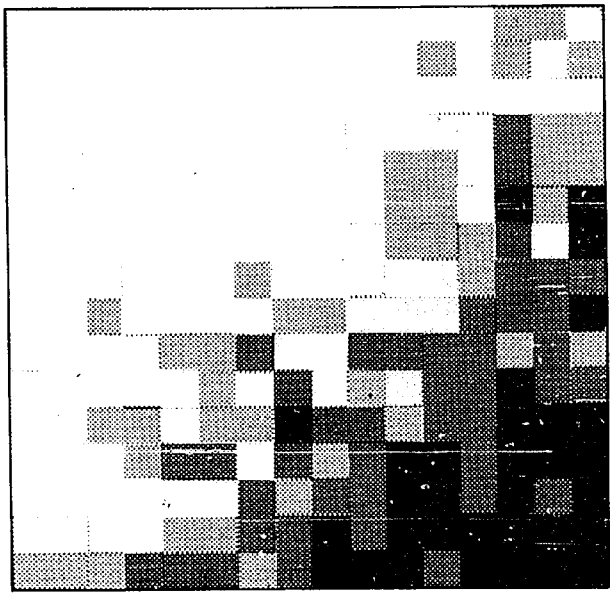
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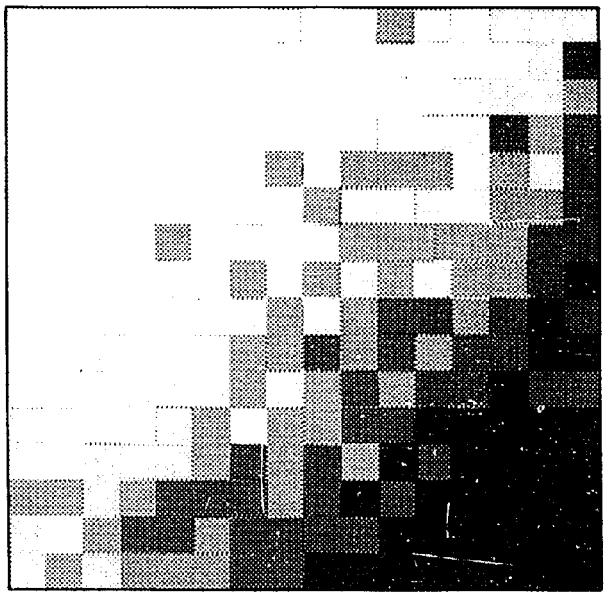


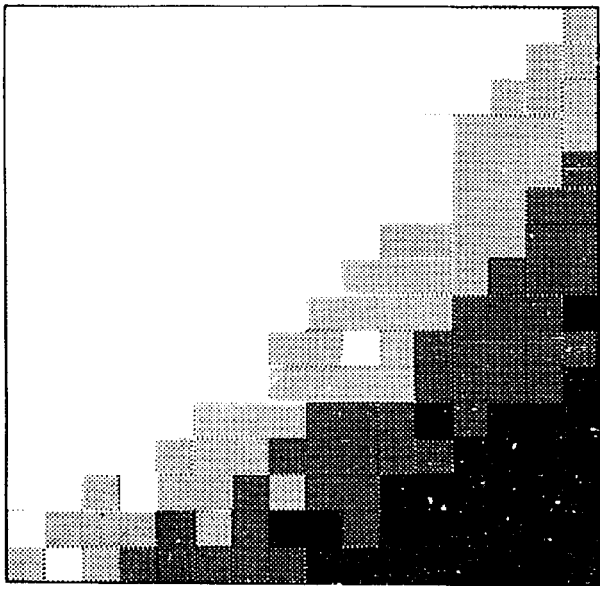


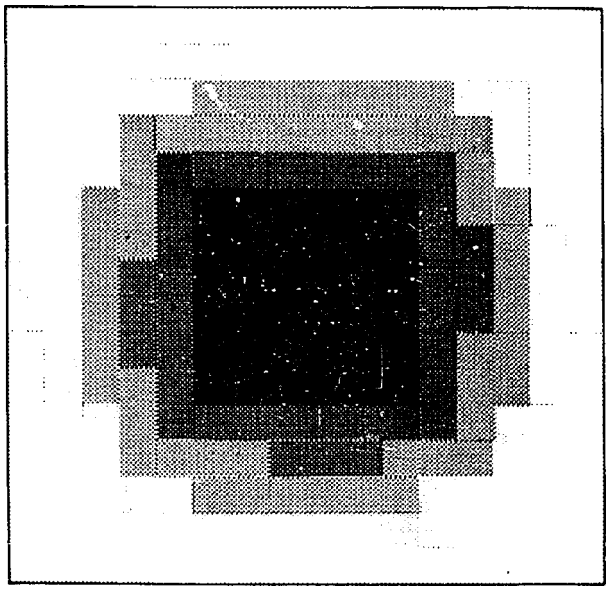




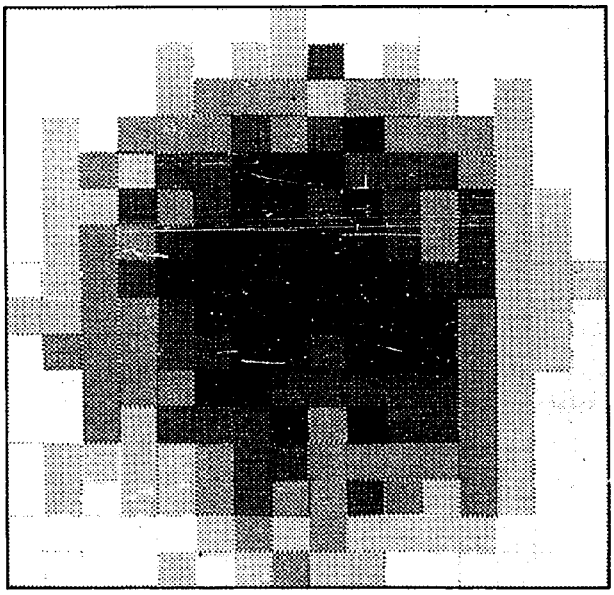




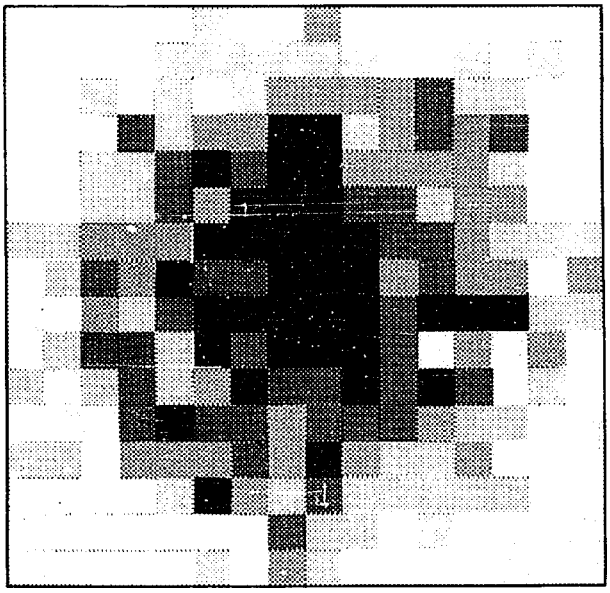




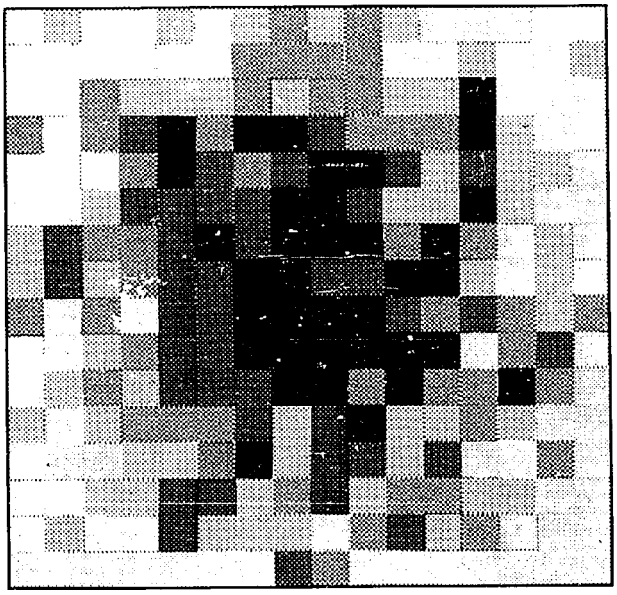
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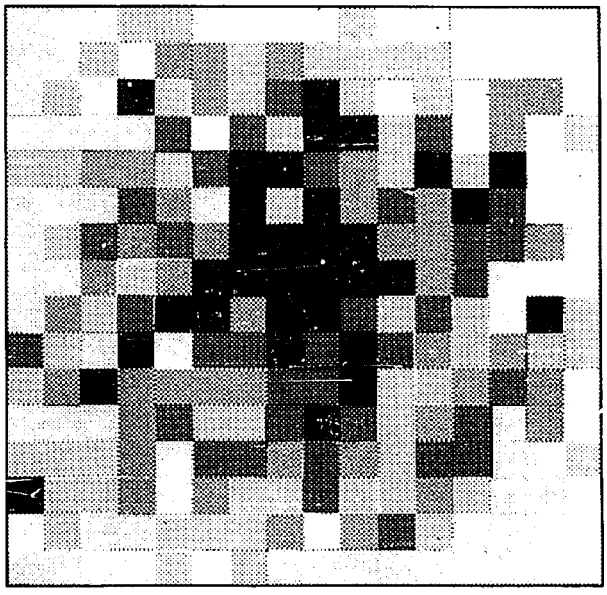
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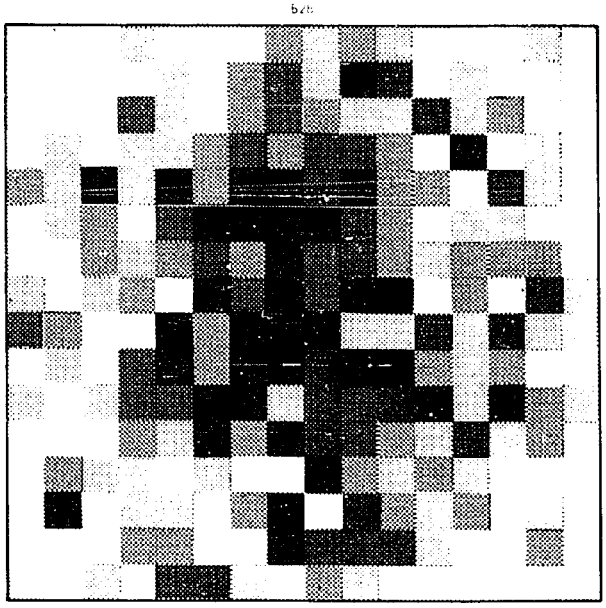


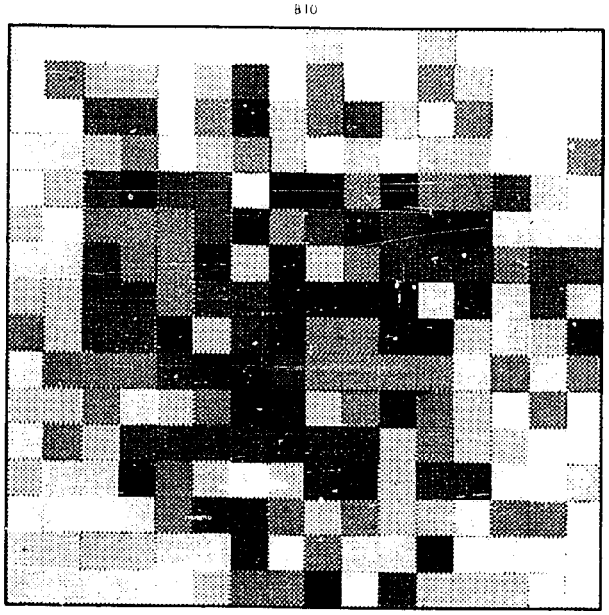
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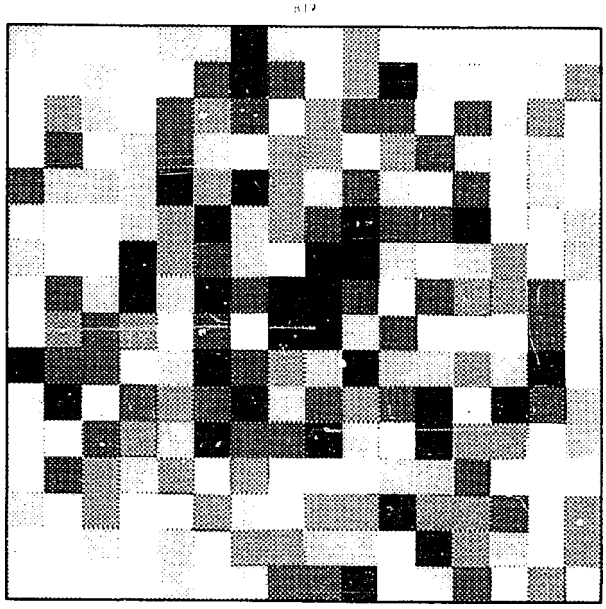


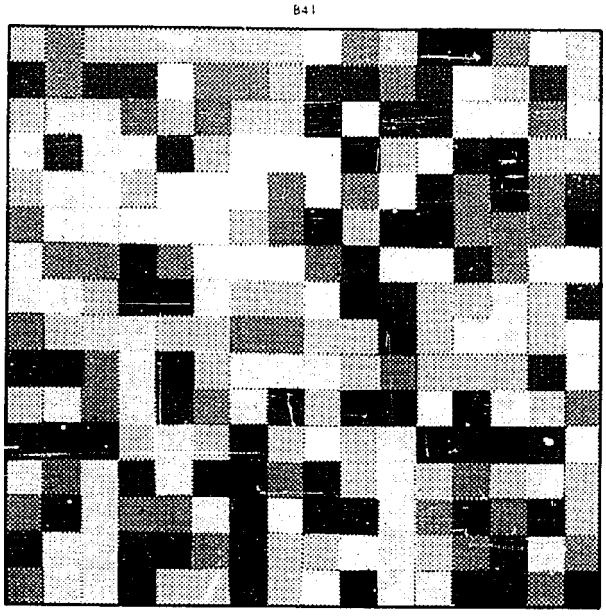
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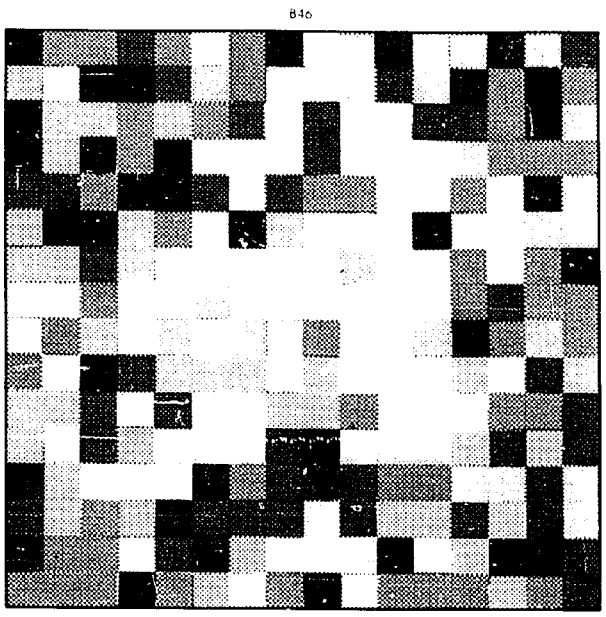


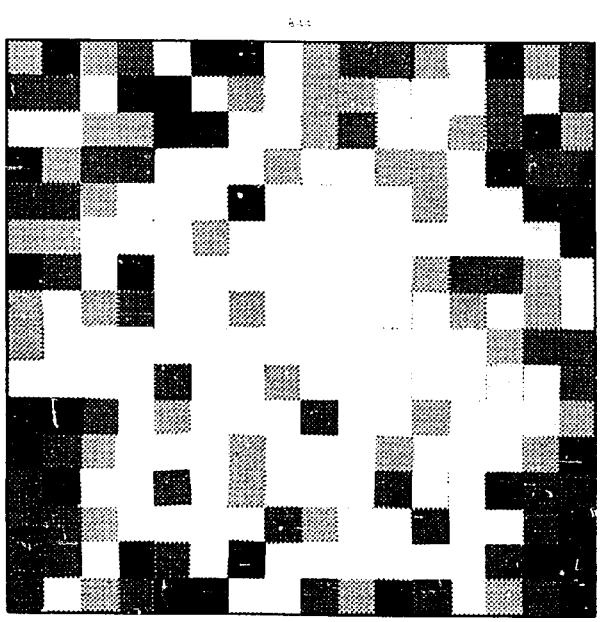


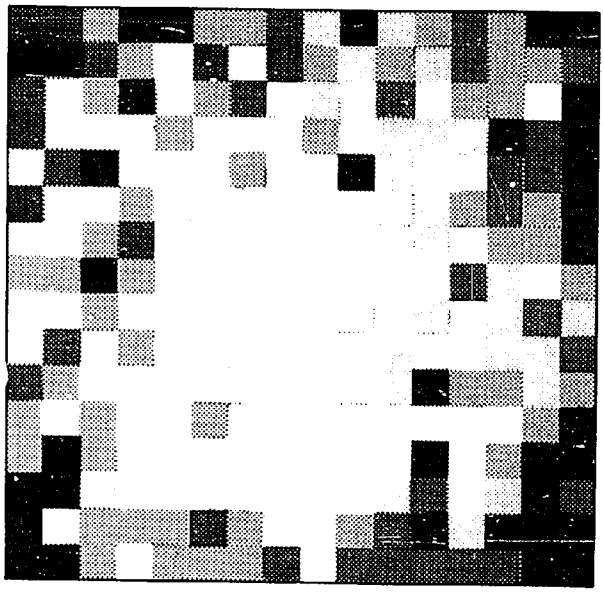


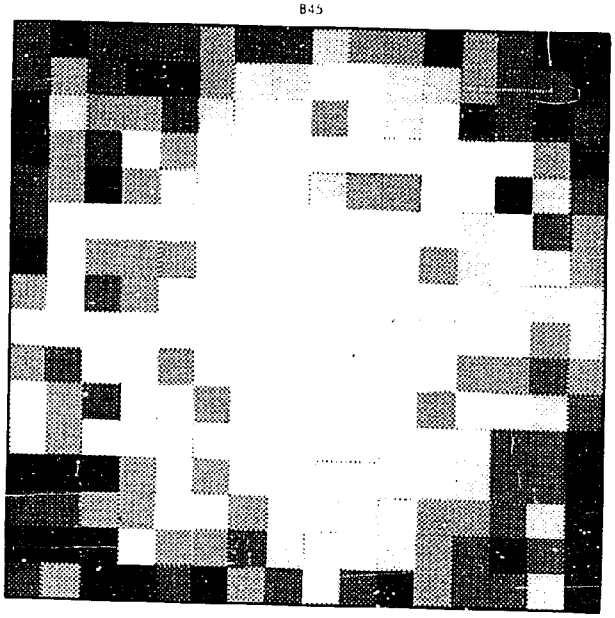


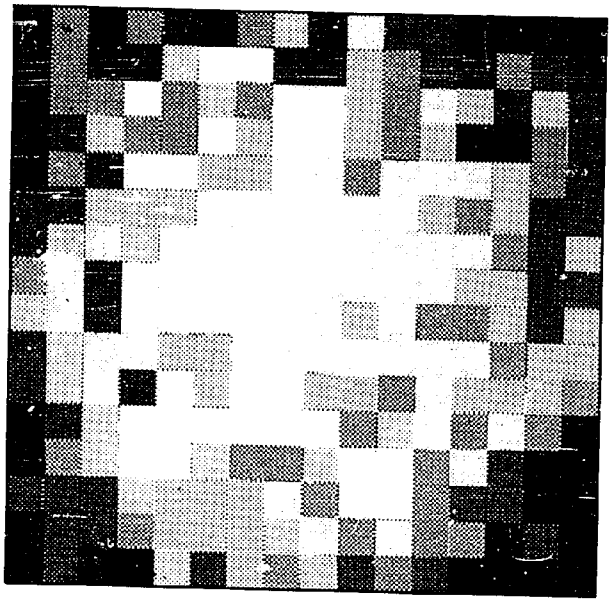


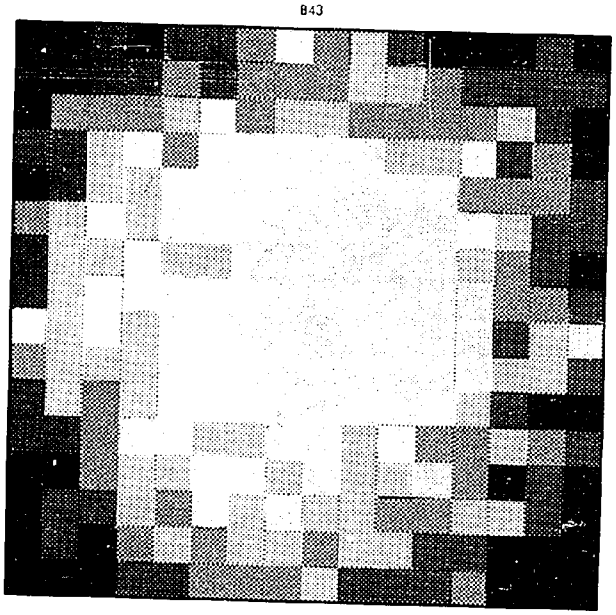


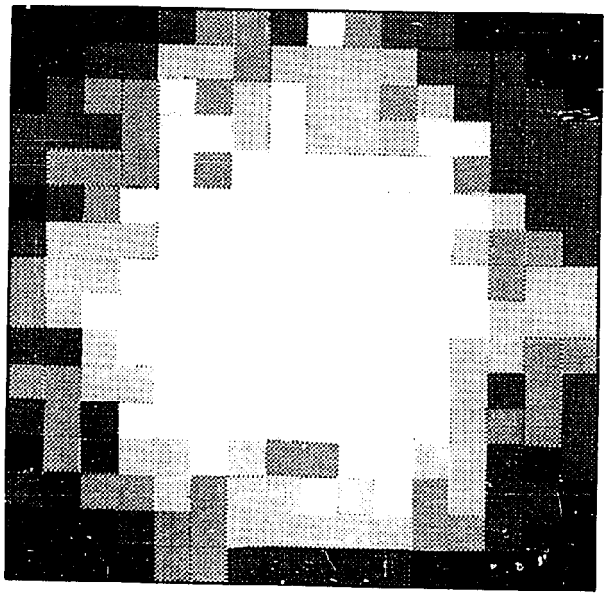


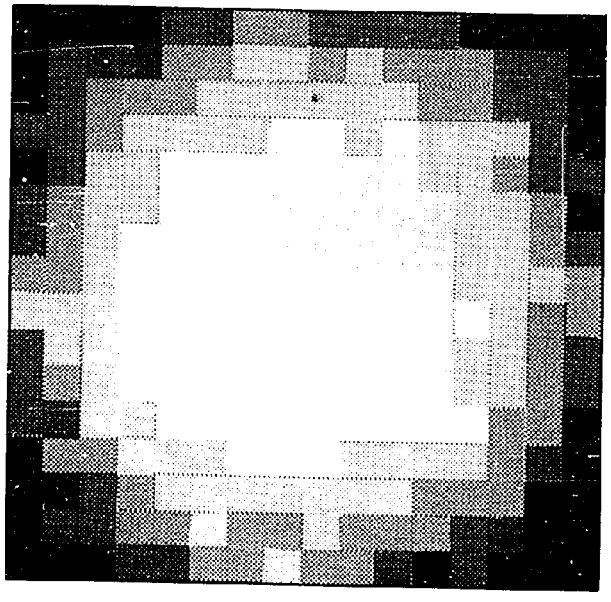




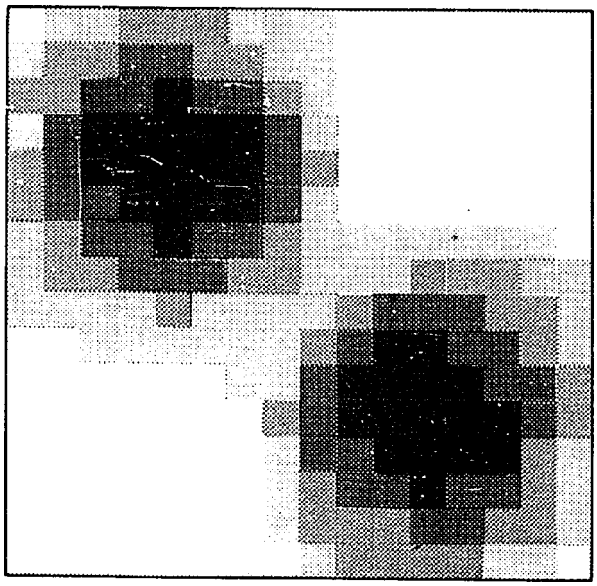




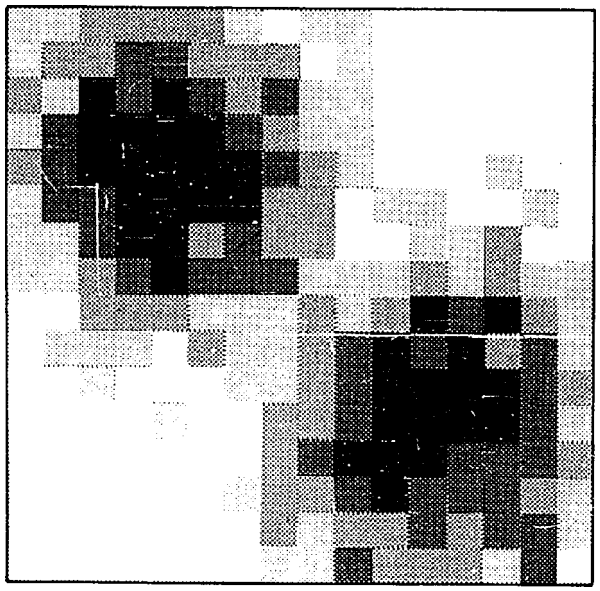




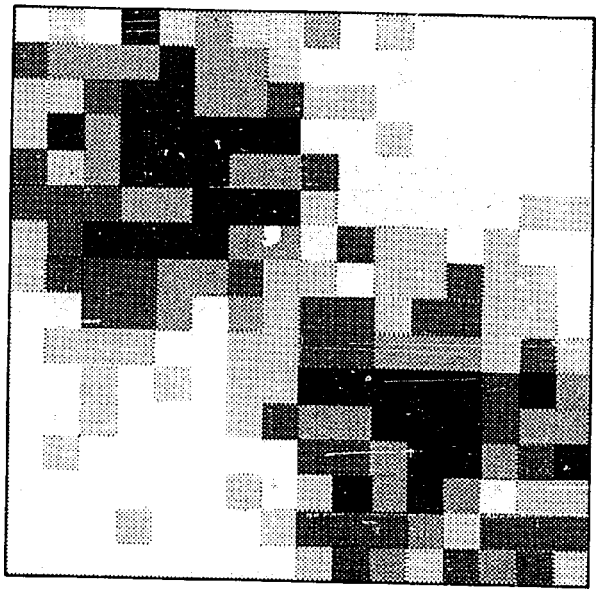
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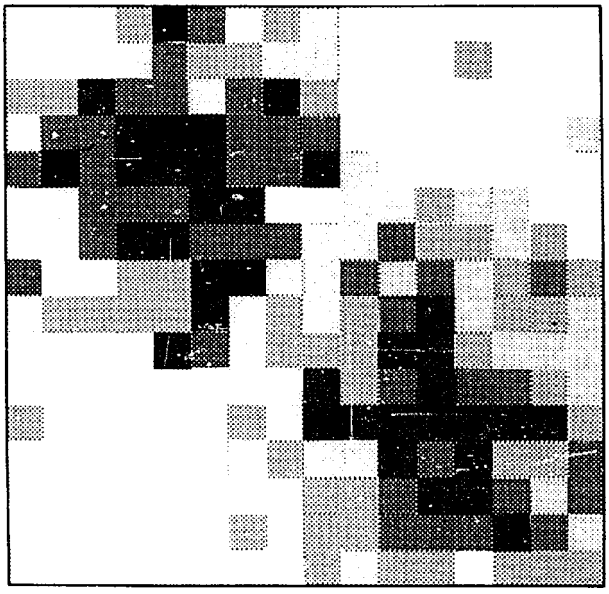
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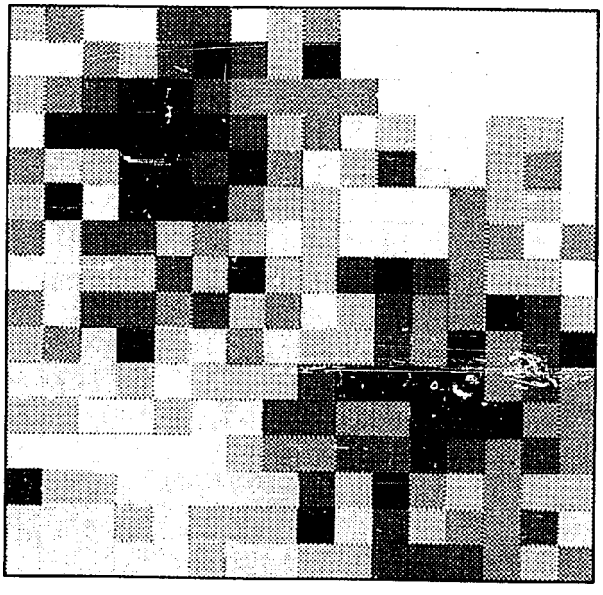
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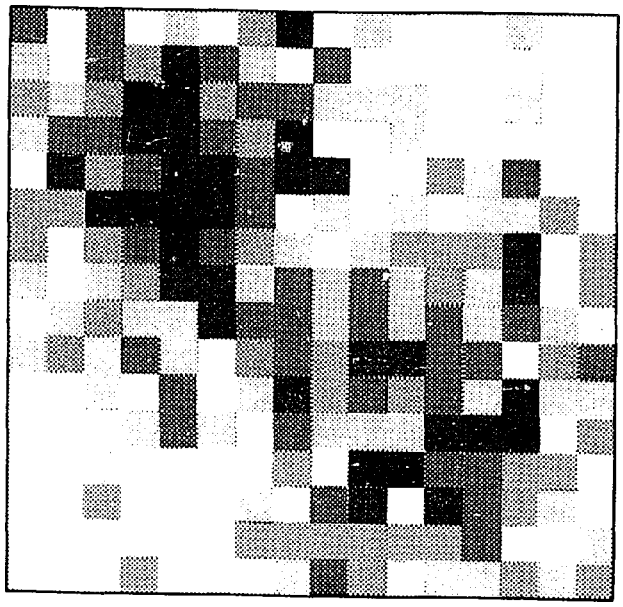
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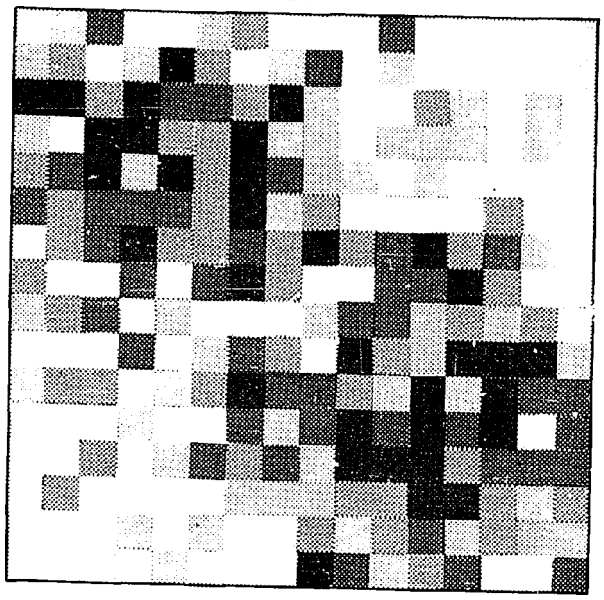
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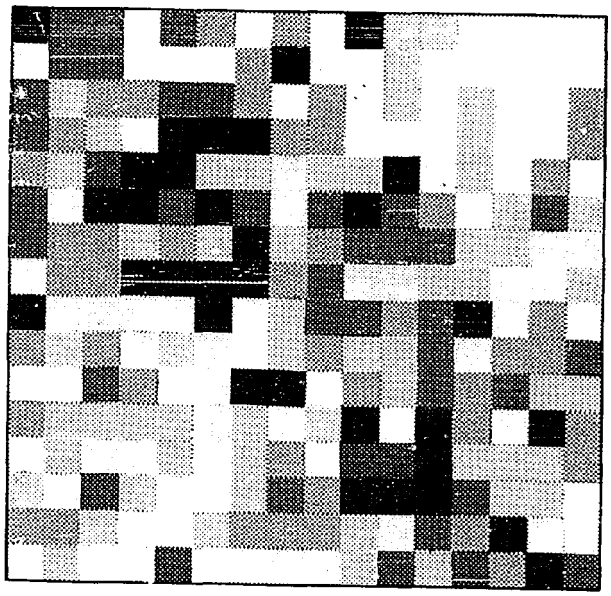
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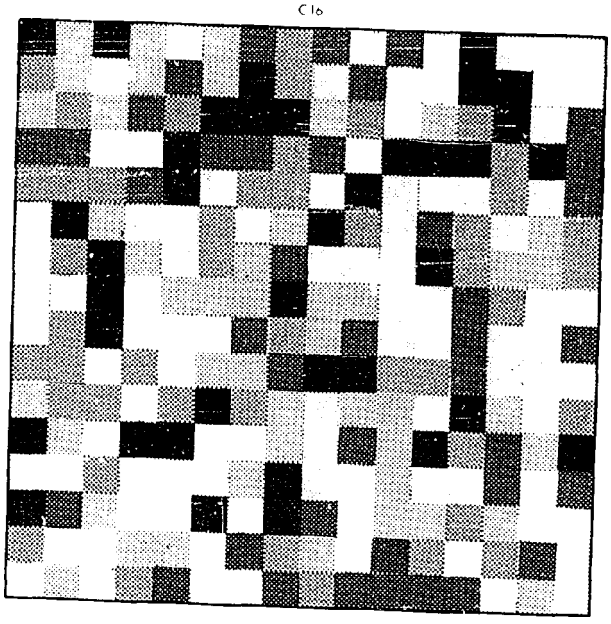
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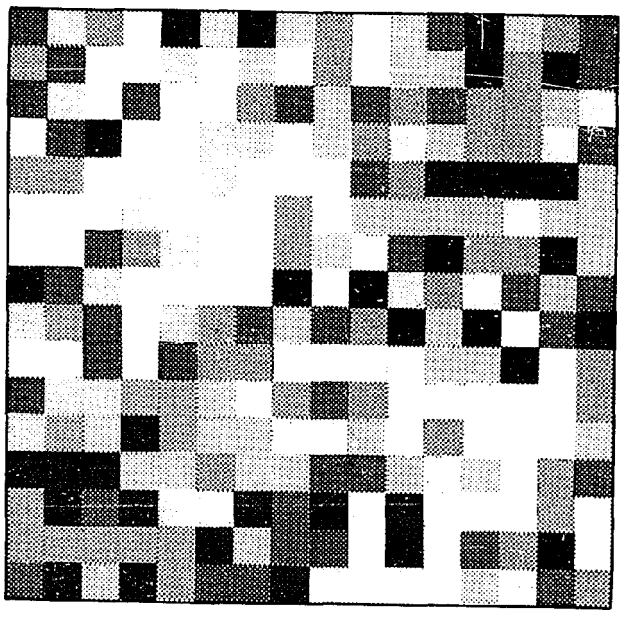
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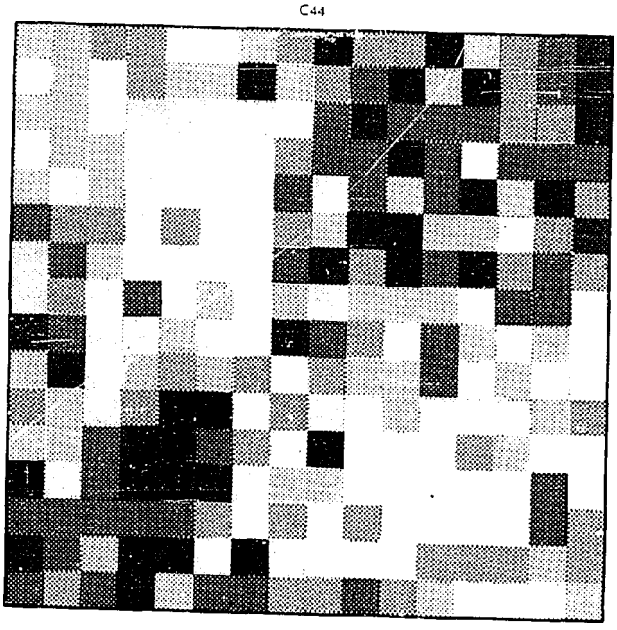
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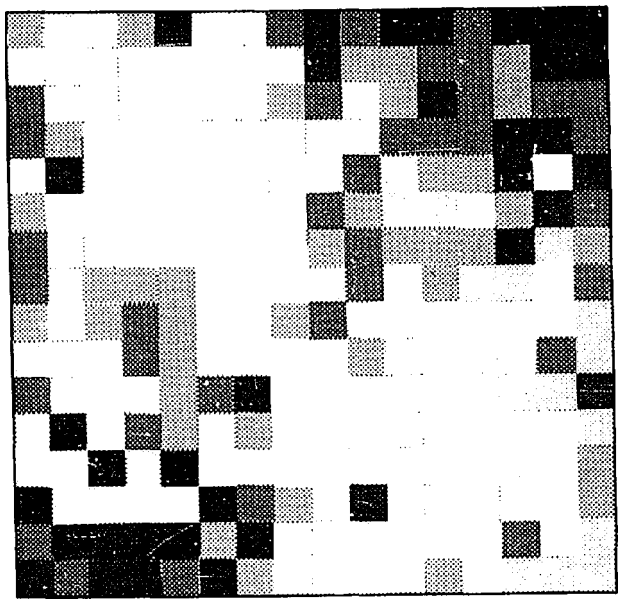
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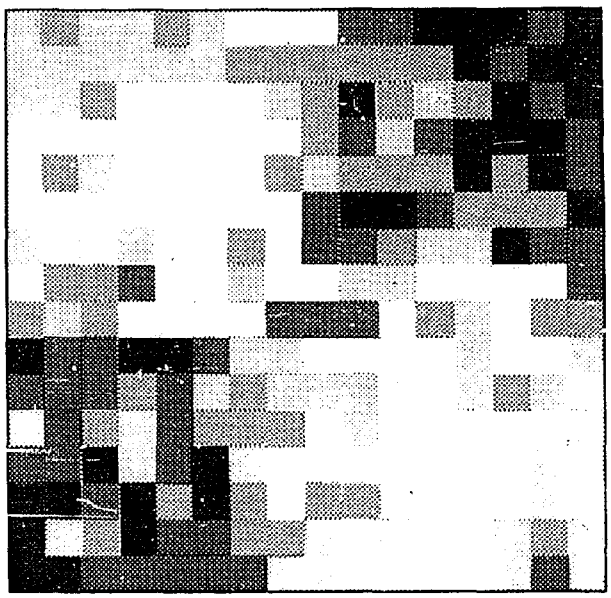
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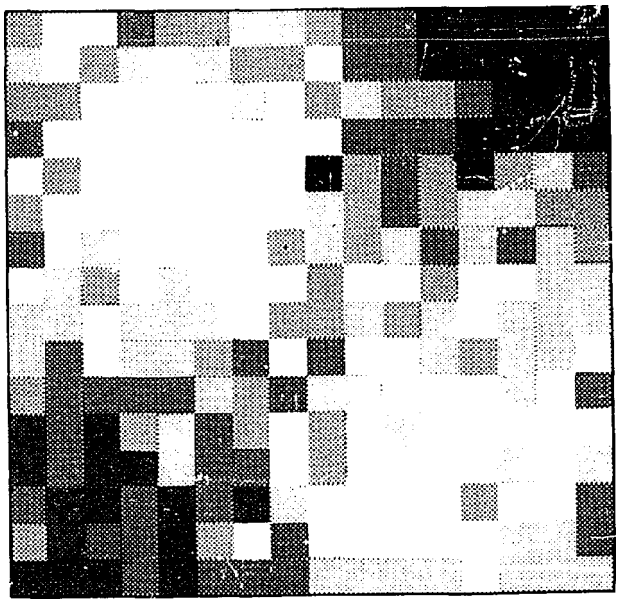
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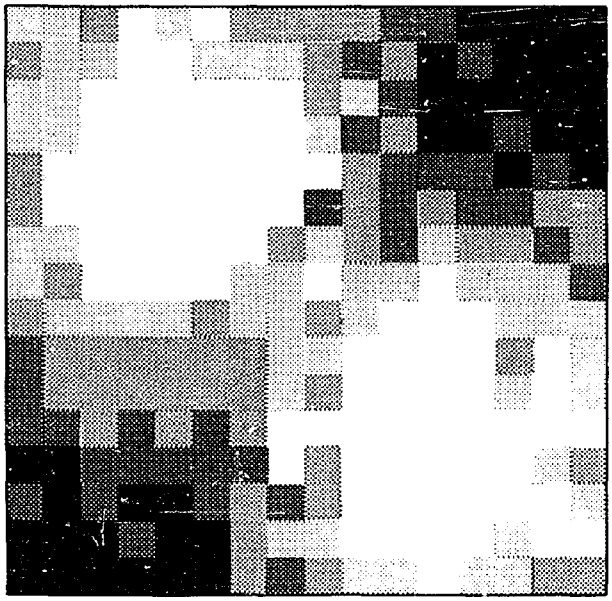
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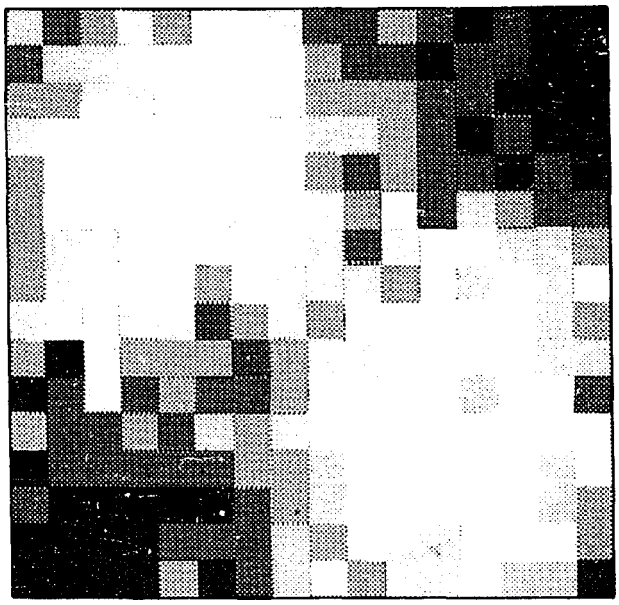
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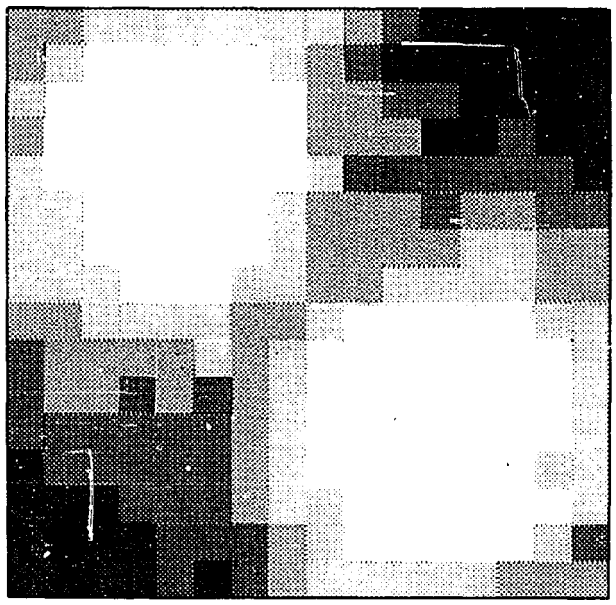
C38



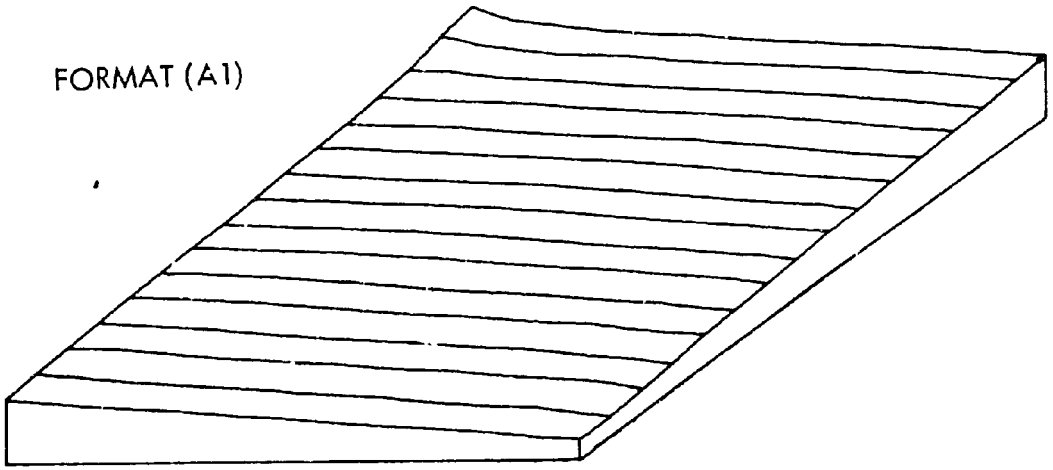
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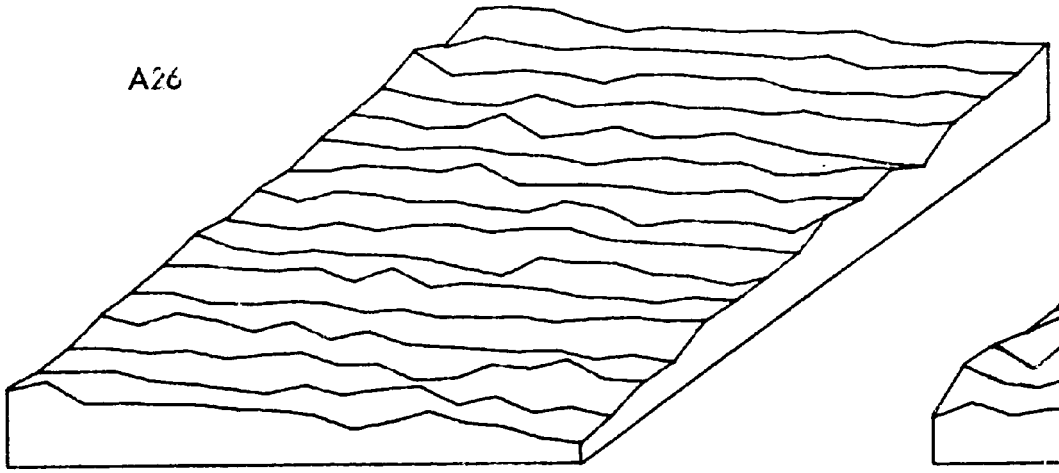
C40

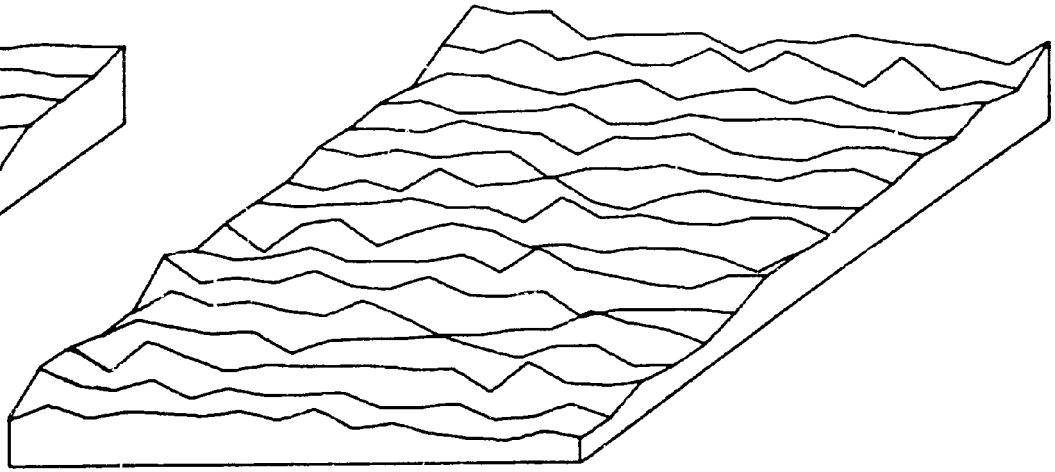


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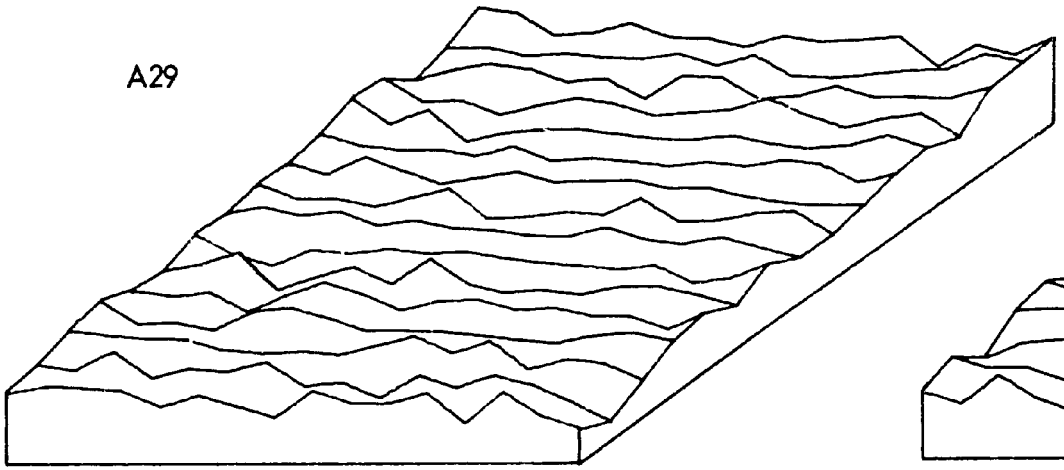


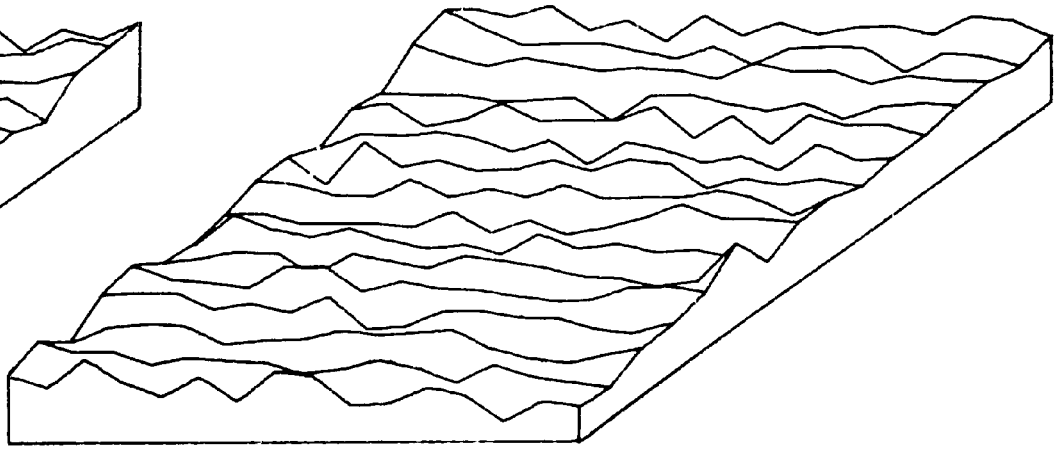
A26



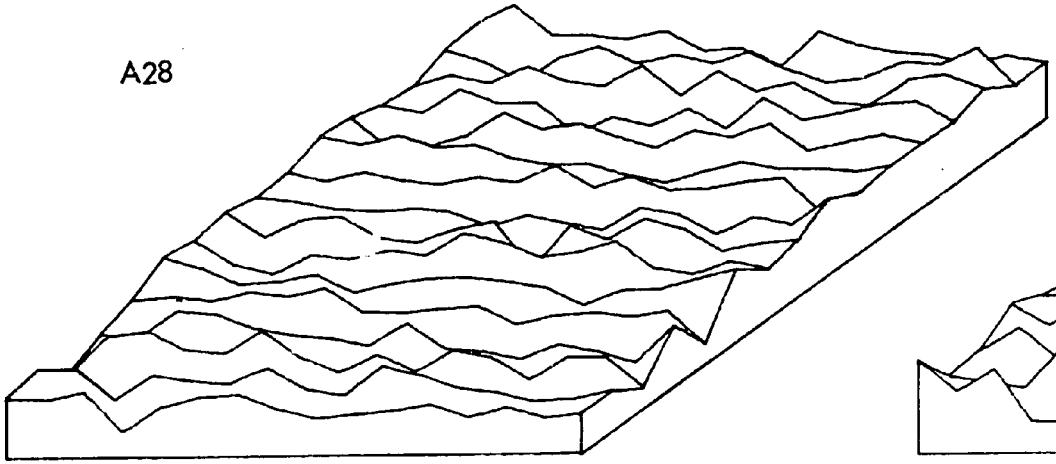


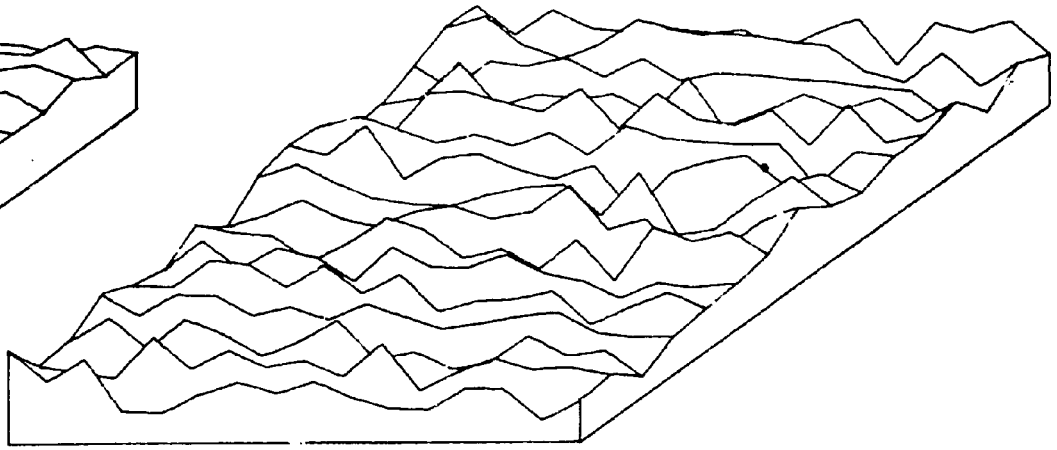
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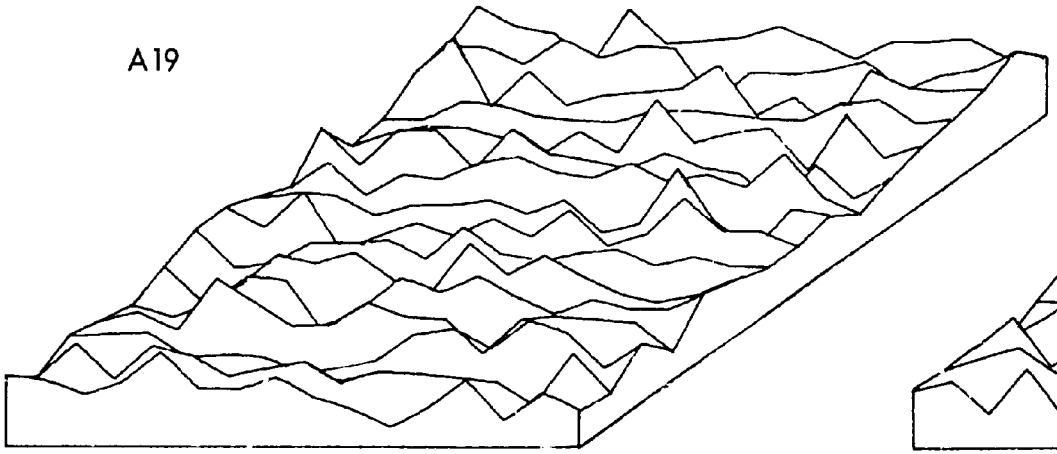


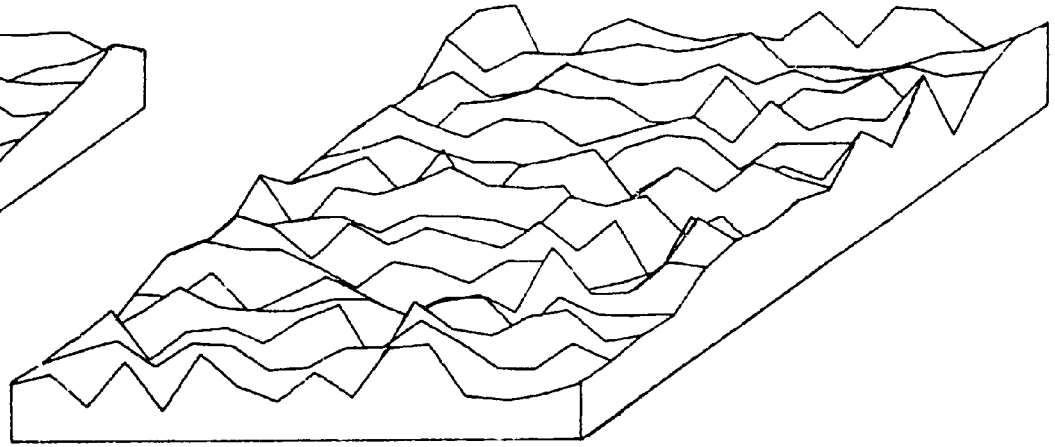
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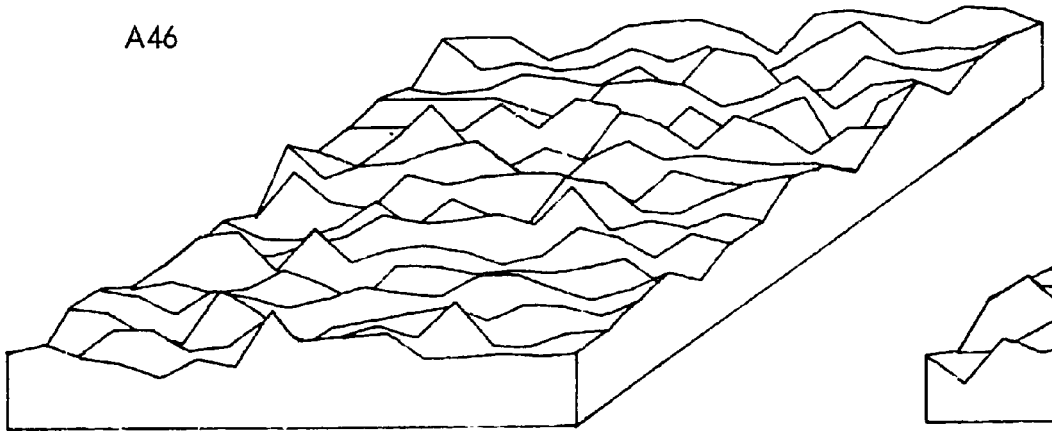


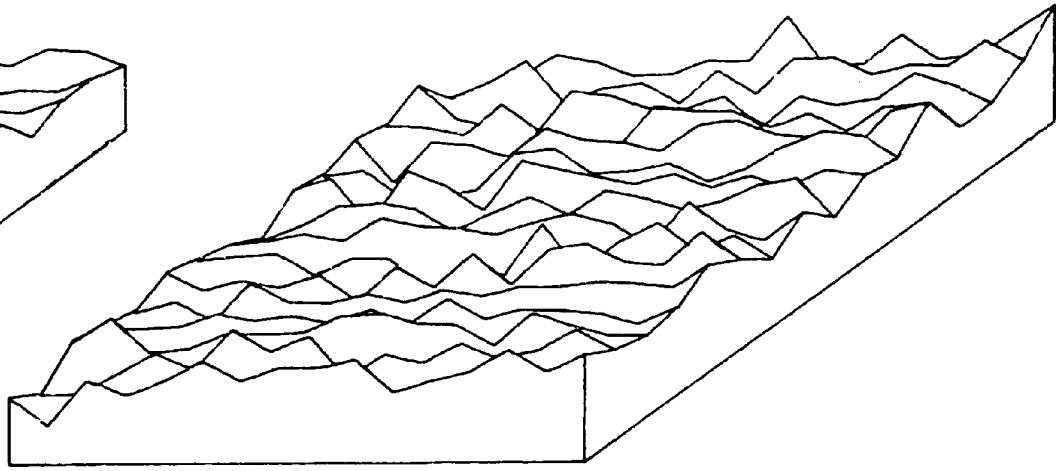
A19



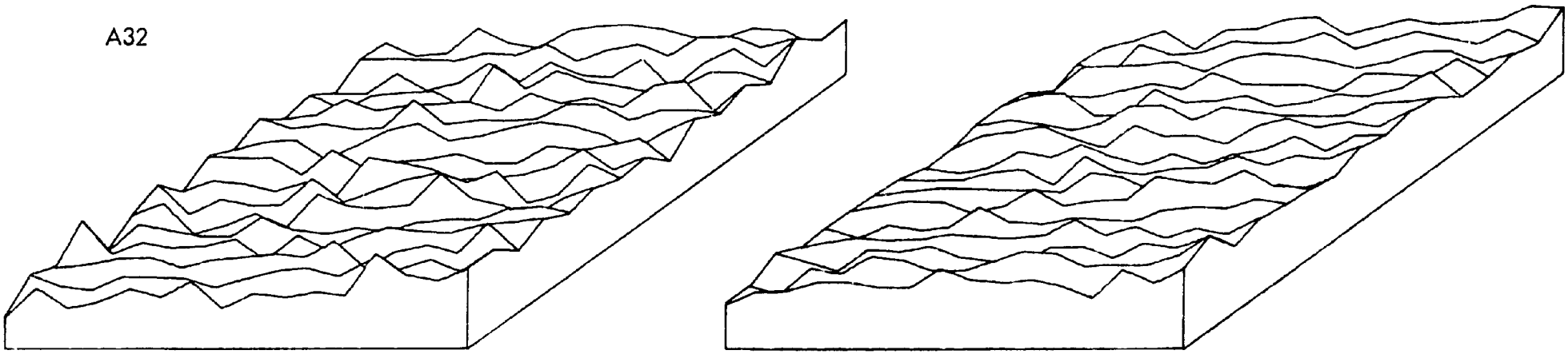


A46

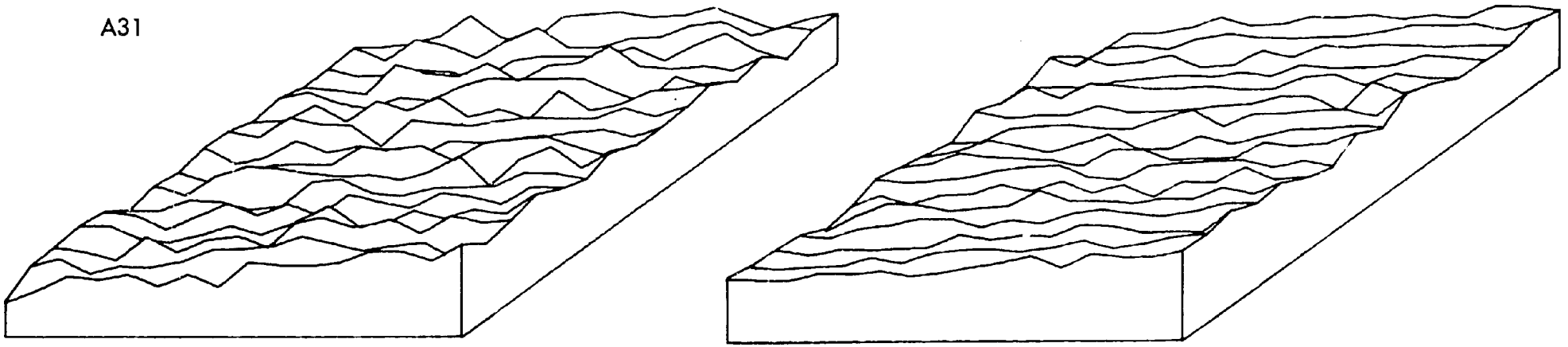




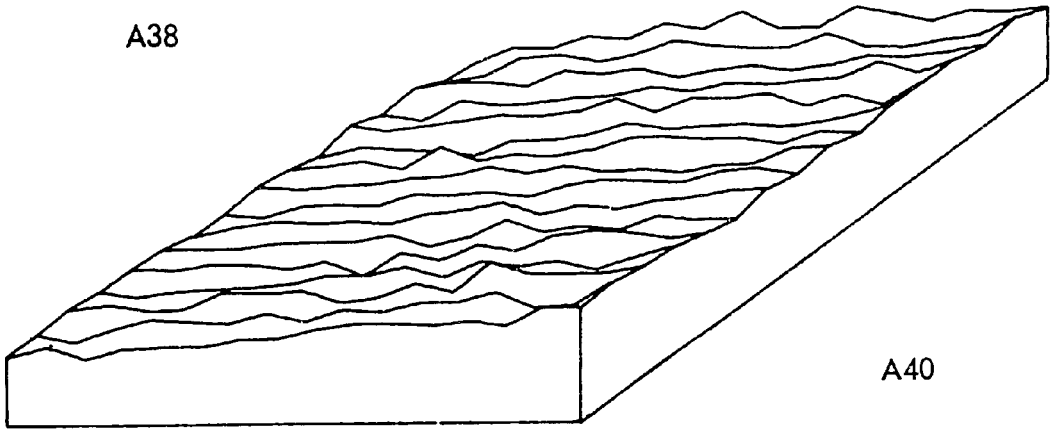
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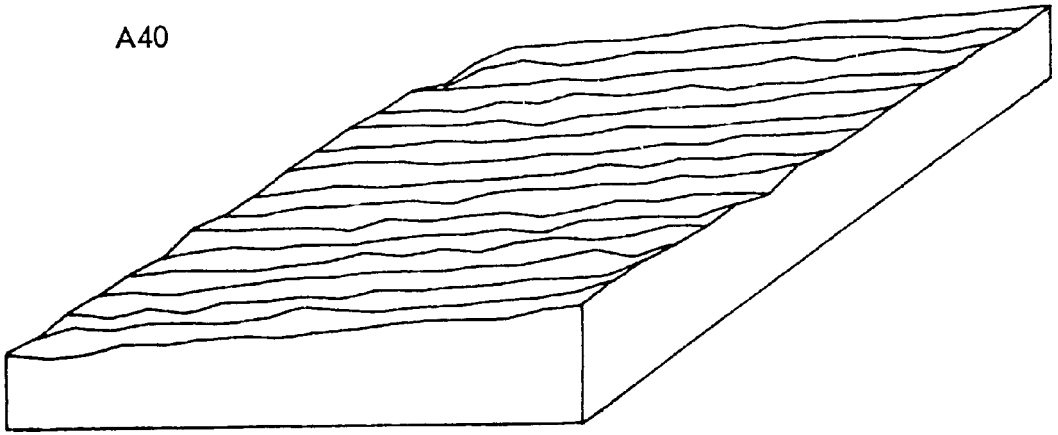
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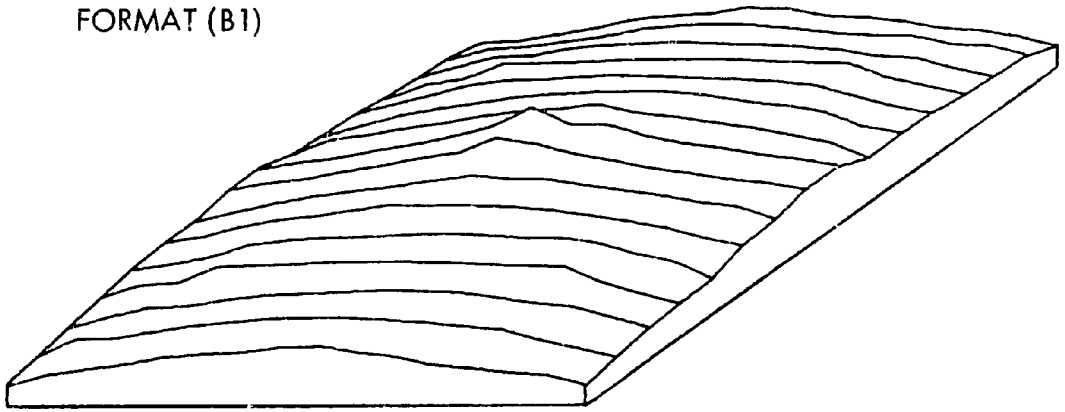
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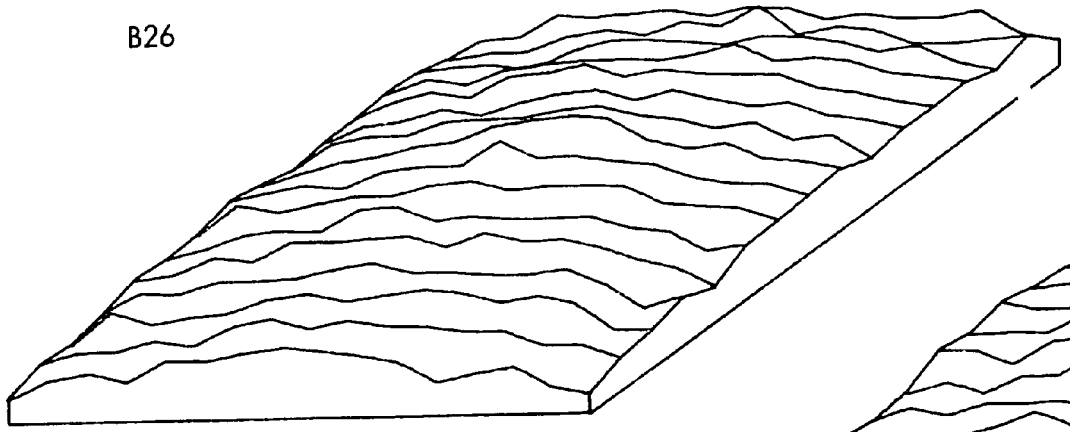
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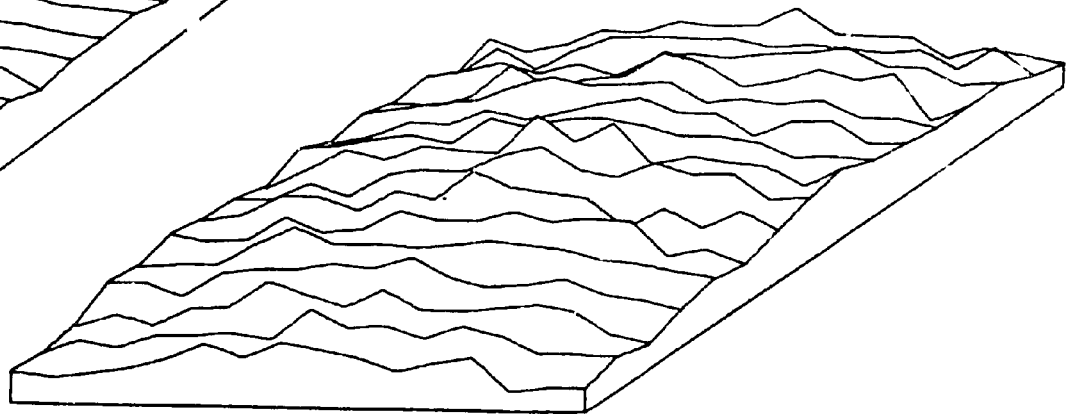


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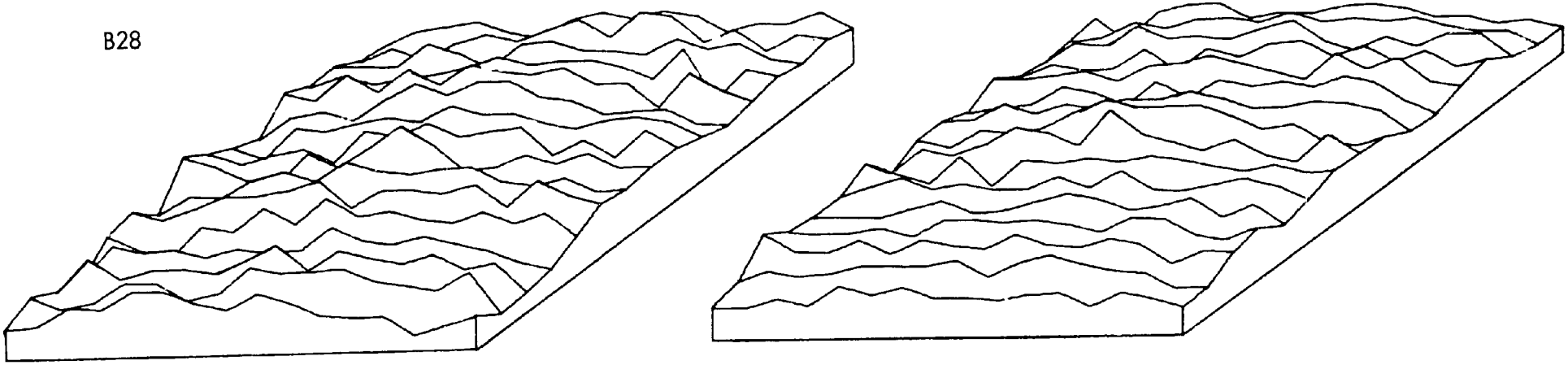


B26

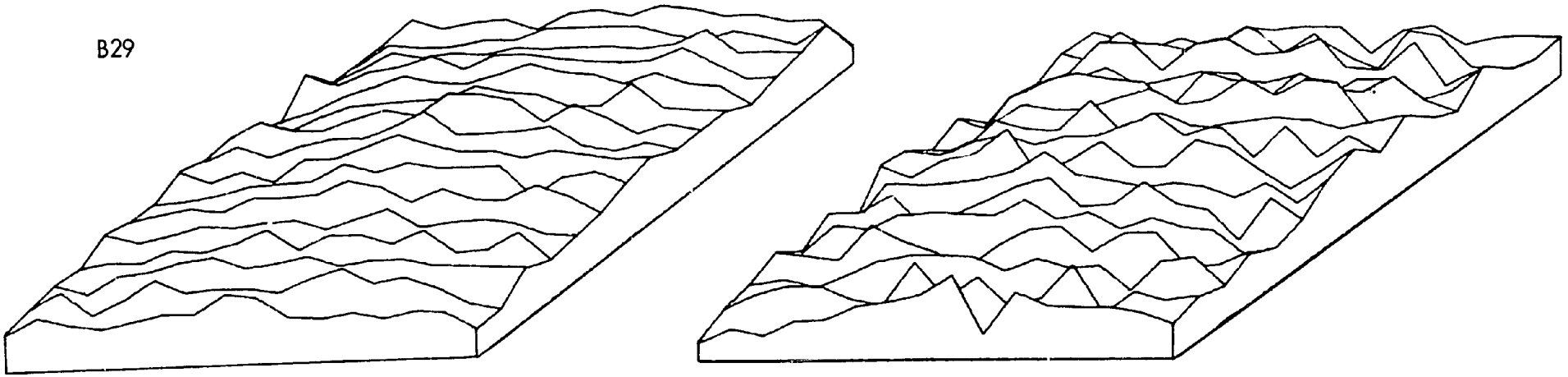


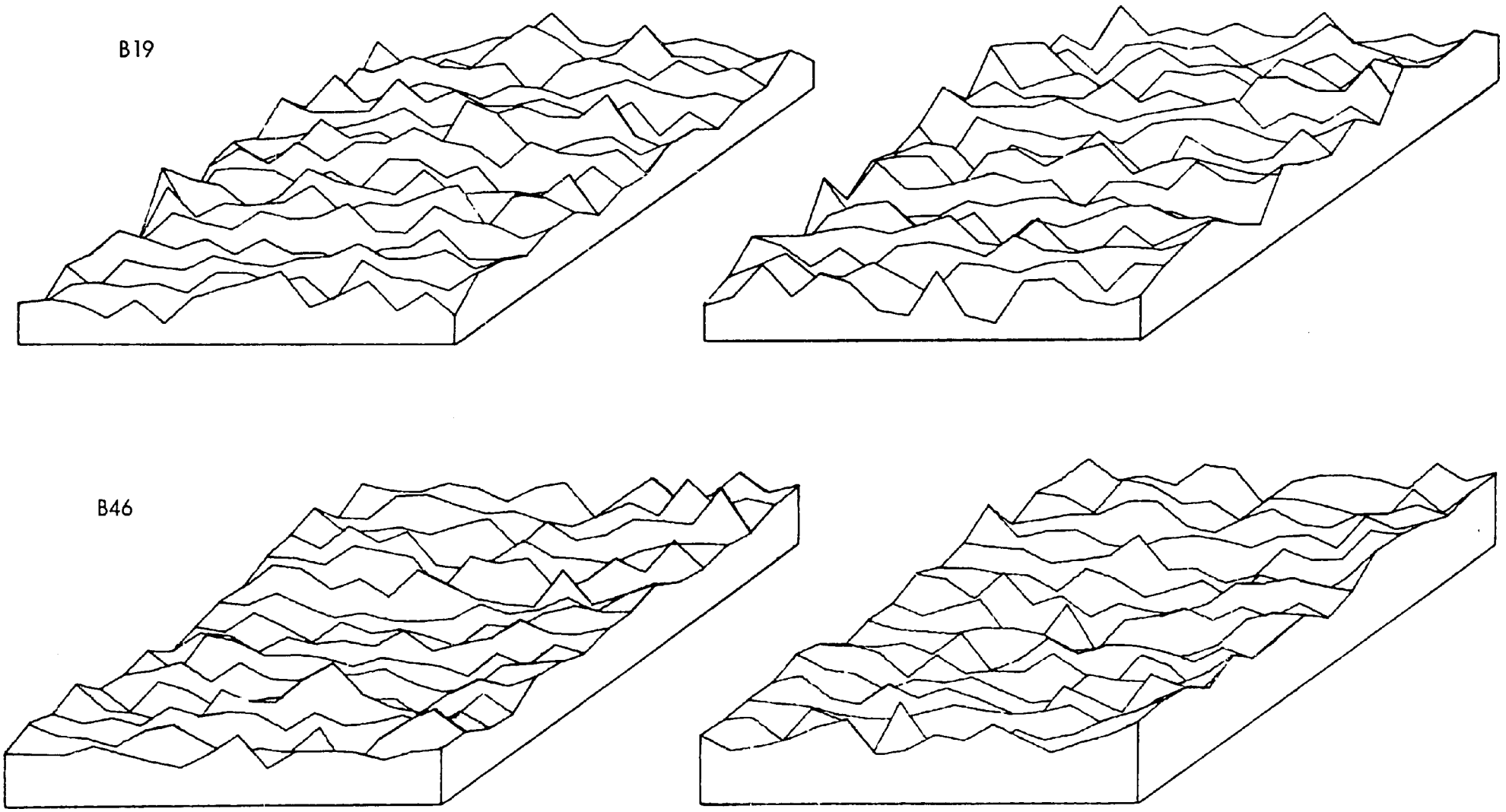


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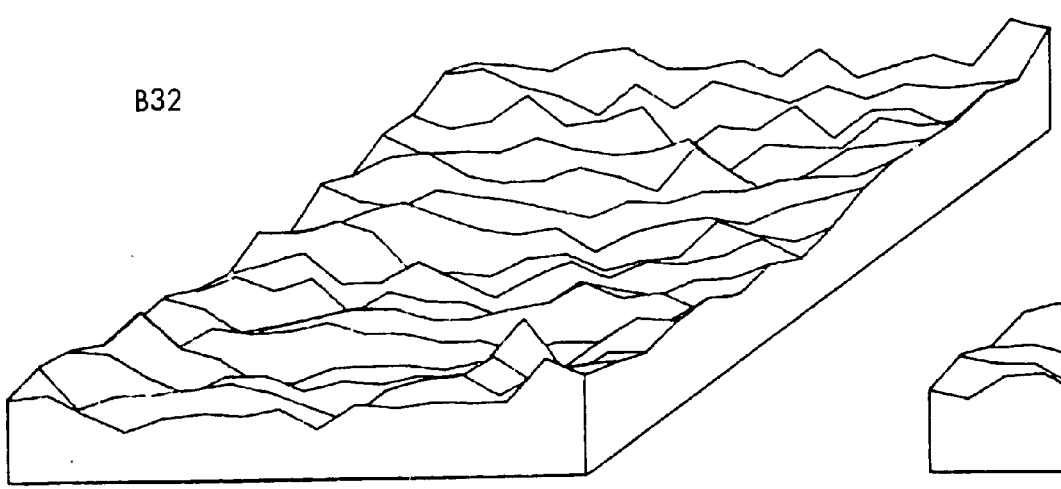


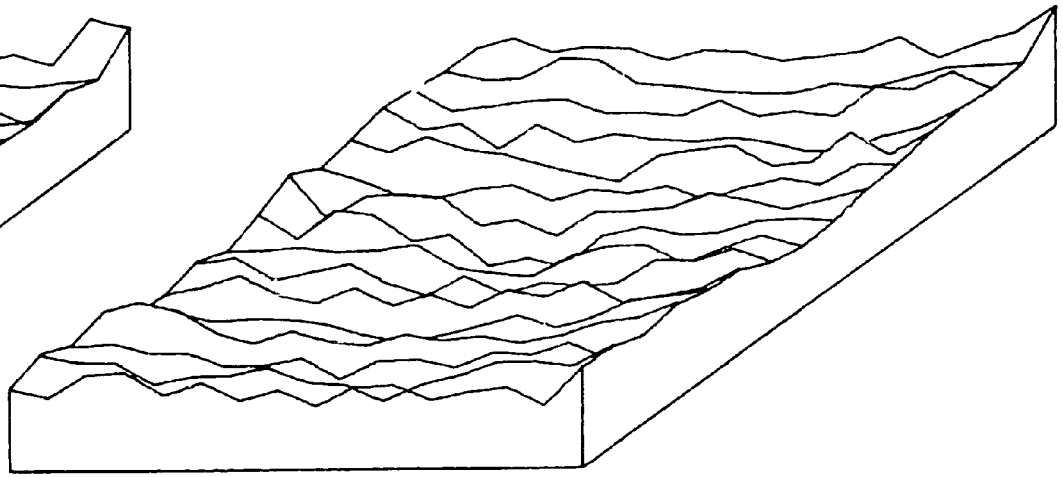
B29



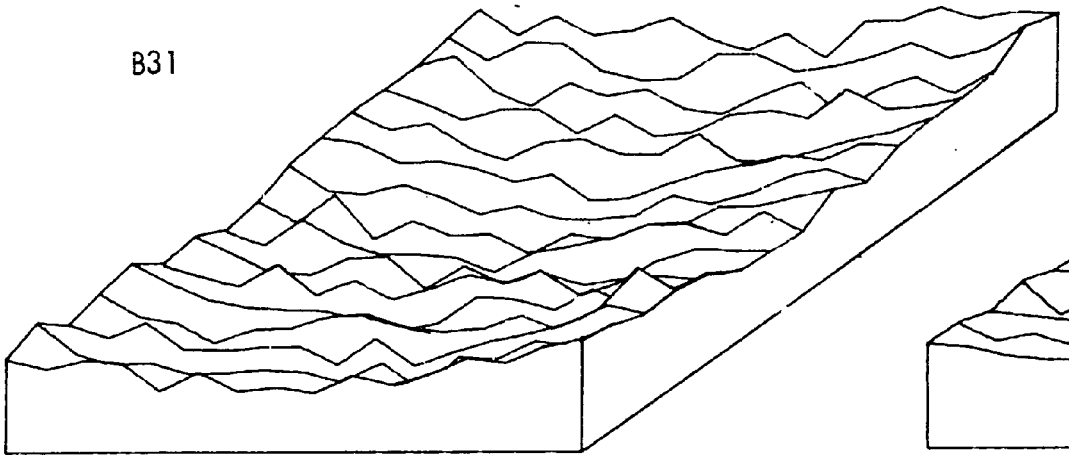

B19
B46

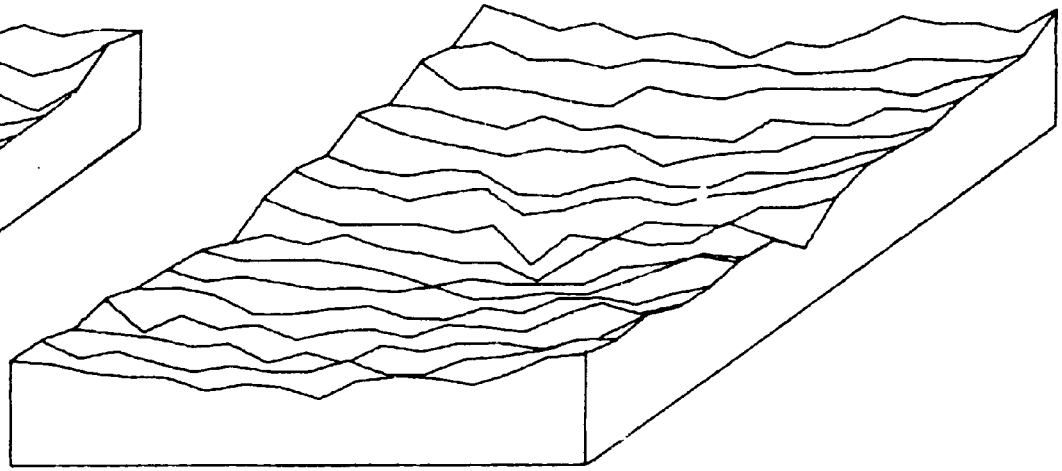
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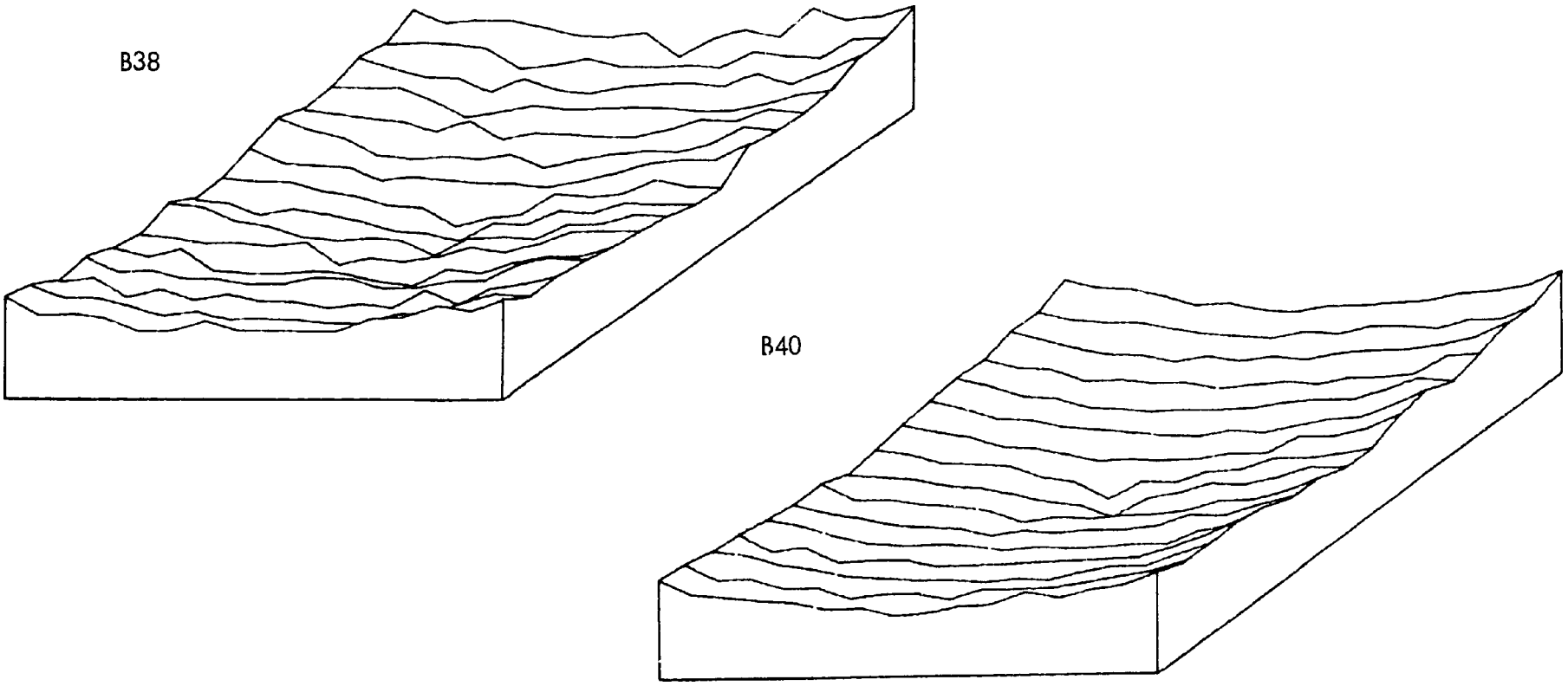




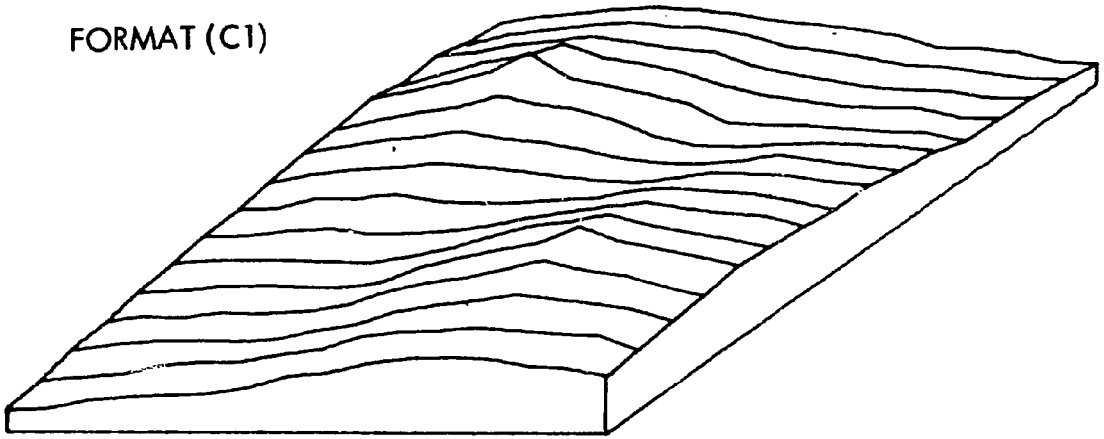
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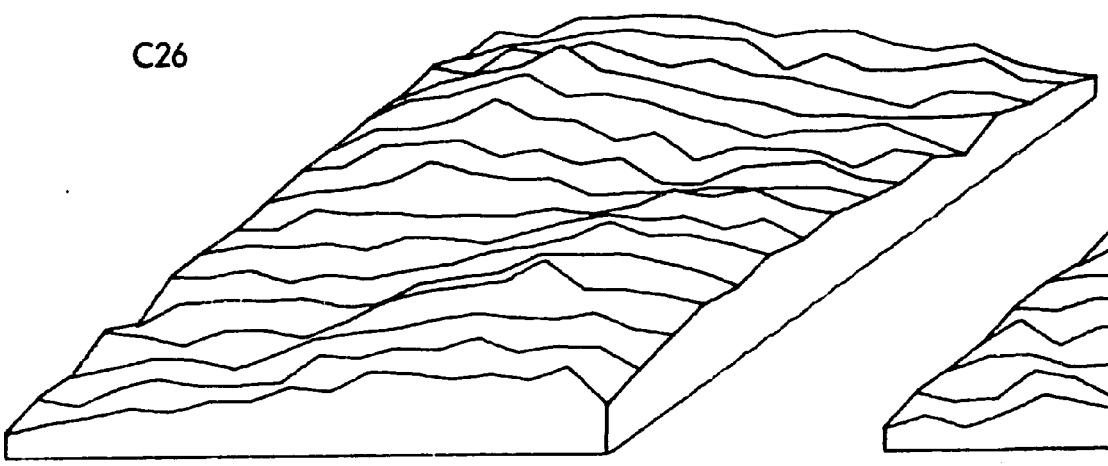


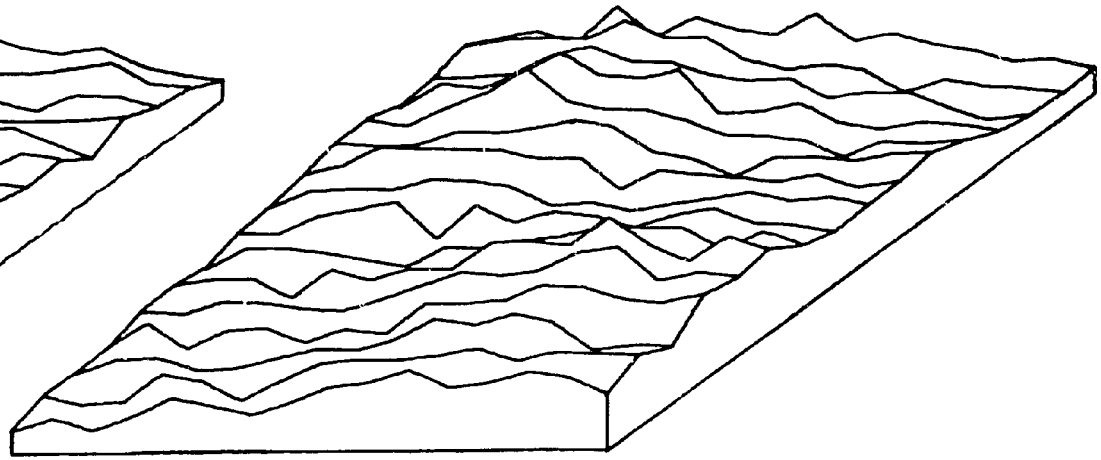

B38
B40

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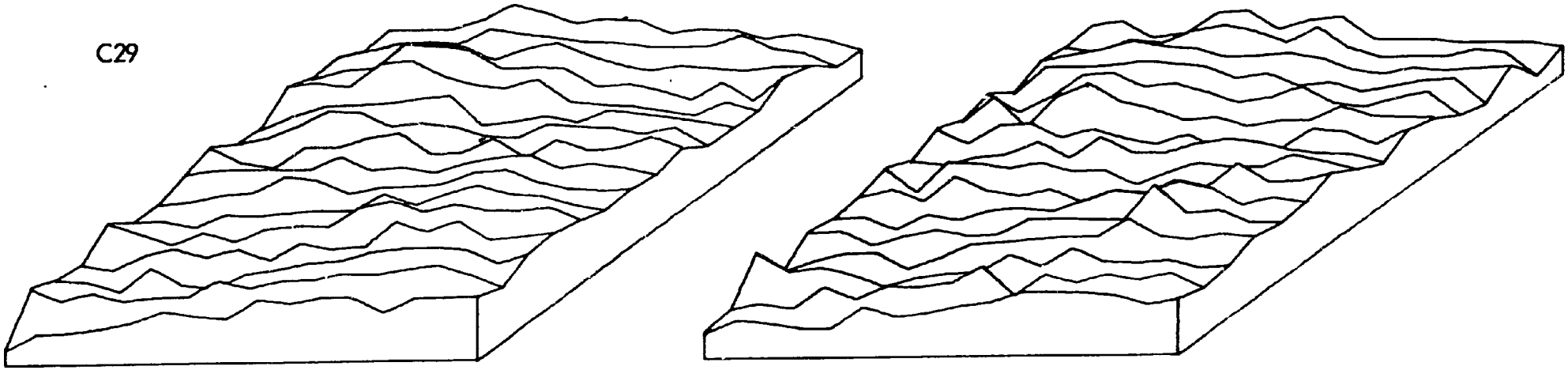


C26

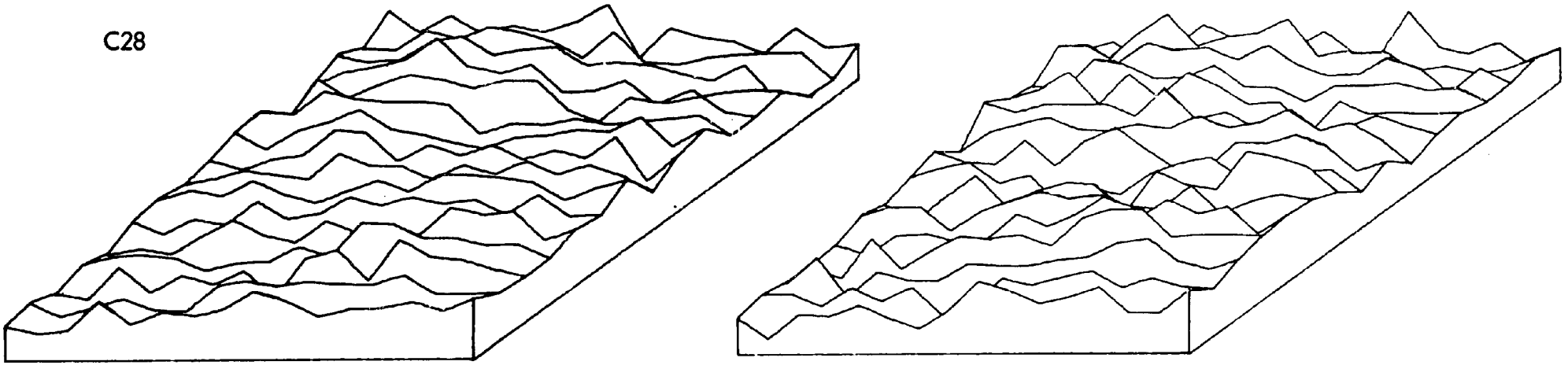


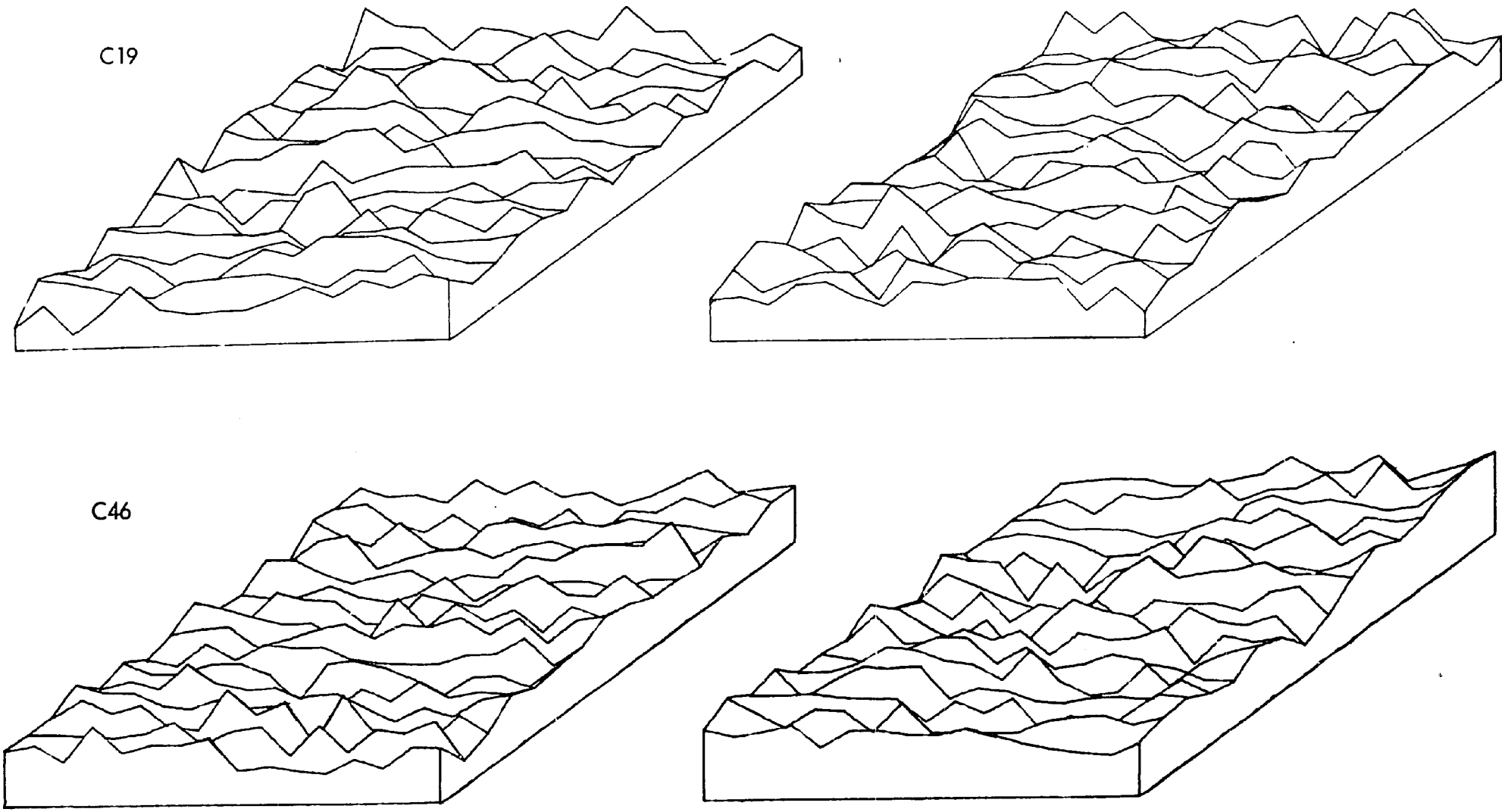


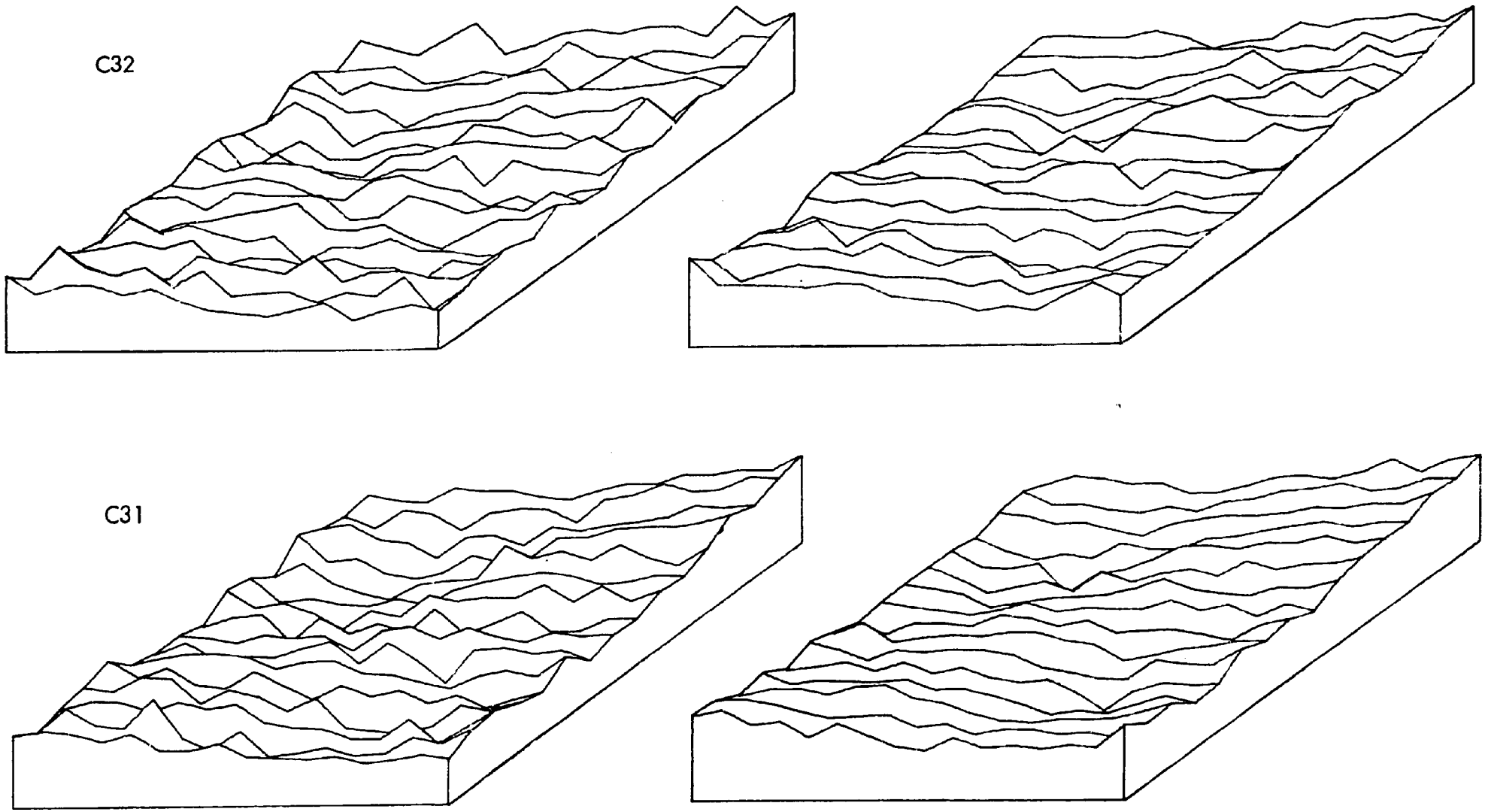
C29



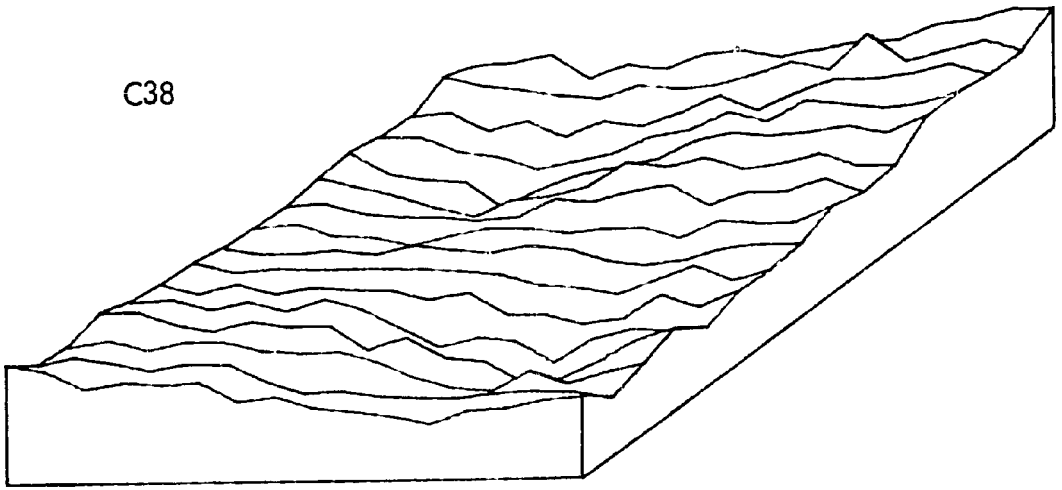
C28



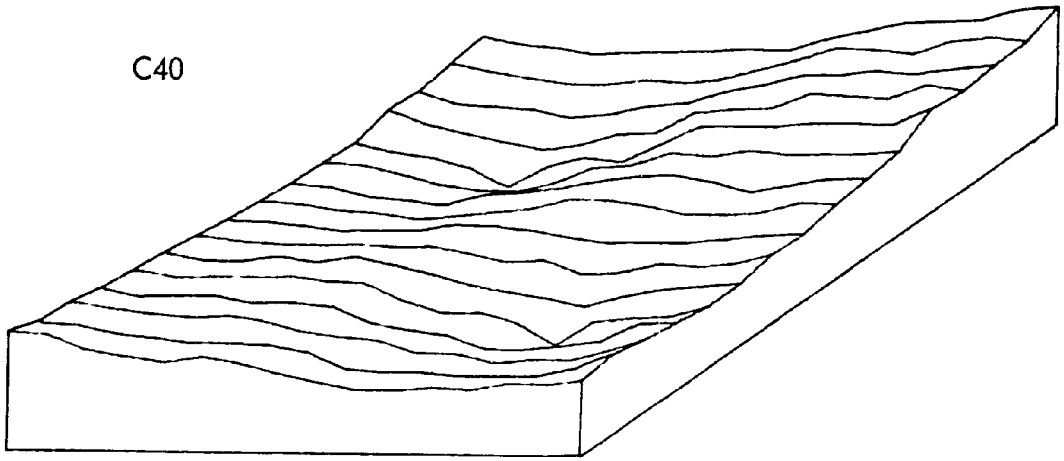

C19
C46


C32
C31

C38



C40



APPENDIX D

CONFUSION MATRICES

In Tables 1. . . 9, which may be called confusion matrices, rows represent stimuli (i = 1, 2, . . ., 16) and the columns stand for response categories (j = 1, 2, . . ., 16). The element f_{ij} at the intersection of the i^{th} row and j^{th} column gives the number of times the stimulus i was sorted into category j. If no ranking errors were made with stimulus i, then 100 would occur in the cell in row i falling on the main diagonal. Thus matrix rows depict the distribution of item (i = 1, 2, . . ., 16) rankings taken over the 100 subjects. Comparing successive rows or successive columns indicates differences in subjects' map comparison ability due to variations in map correlation. Comparison of successive matrices shows differences in subjects' map comparison ability due to variations in the map graphic and the complexity of the mapped distributions.

TABLE 1

LINEAR DOT MAPS[a]
(n = 100)

JUDGED RANK

ACTUAL RANK

R[b]	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	RE[c]	RD[d]
1	32	47	8	12		1											68	104
2	28	31	19	21	1												69	92
3	16	14	31	27	9	2		1									69	102
4	12	6	24	28	24	5	1										72	109
5	8	1	14	10	55	8	4										45	89
6	2		2	2	7	52	34	1									48	63
7	1		2		4	30	53	10									47	62
8	1				1	2	8	84	3				1				16	30
9							1	2	88	6	1		1			1	12	23
10								1	8	47	32		8	3	1		53	83
11									1	32	42	6	15	3	1		58	83
12										12	14	22	35	10	6	1	78	115
13									1	3	7	41	22	13	10	3	78	110
14											2	18	9	35	24	12	65	99
15										1		9	5	20	36	29	64	91
16												6	5	16	20	53	47	91

[a] In this and the following confusion matrices rows represent stimuli (i = 1, 2, . . ., 16) and the columns stand for response categories (j = 1, 2, . . ., 16). The element f_{ij} at the intersection of the i^{th} row and the j^{th} column gives the number of times the stimulus i was sorted into category j (n = 100).

[b] R = Rank Order (see explanation in text).

[c] RE = Total Ranking Errors (see explanation in text).

[d] RD = Total Ranking Displacement (see explanation in text).

TABLE 2

QUADRATIC DOT MAPS
(n = 100)

JUDGED RANK

ACTUAL RANK

R	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	RE	RD
1	29	67		2	2												71	81
2	54	25	8	7	6												75	94
3	4	6	30	31	26		2	1									70	110
4	7		18	37	23	10	5										63	97
5	2	2	18	30	19	10	1										70	110
6	2		10	5	11	43	28	1									57	91
7	2		13	1		22	44	8									46	97
8			2		1	2	5	80	9	1							20	33
9								4	64	21	8		2	1			36	54
10								3	20	52	15		9	1			48	72
11									3	17	49	2	22	5	1	1	51	93
12								1	2	3	22	5	55	6	1	5	95	128
13								1		3	3	27	10	33	10	13	90	139
14									1	1	5	28	3	28	19	15	72	132
15												21	2	19	37	21	63	107
16												17		5	32	46	54	110

TABLE 3

CUBIC DOT MAPS
(n = 100)

JUDGED RANK

ACTUAL RANK

R	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	RE	RD
1	45	51	2	1	1												55	62
2	45	32	11	10	2												68	82
3	8	14	44	29	2	3											56	72
4	2	1	27	35	16	17	2										65	91
5	1	1	8	16	46	22	5	1									54	74
6			8	7	25	43	16	1									57	81
7				1	6	12	63	14	3	1							37	40
8						3	13	74	7	2			1				26	31
9				2		1	2	7	76	7	5						24	41
10								2	9	51	32	2	4				49	61
11									6	28	44	9	10	4			56	71
12										7	13	32	42	1	2	3	68	89
13										2	6	40	27	5	5	15	73	118
14												10	11	29	23	27	71	108
15												5	4	21	46	24	54	68
16										1	2	2		40	24	33	69	128

TABLE 4

LINEAR CHOROPLETH MAPS
(n = 100)

JUDGED RANK

ACTUAL RANK

R	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	RE	RD
1	100																0	0
2		72	23	5													28	33
3		23	66	10	1												34	35
4		4	8	58	28	2											42	48
5			3	25	62	9	1										38	42
6		1		2	8	86	3										14	19
7					1	3	92	4									8	9
8							4	95	1								5	5
9									94	6							6	6
10								1	5	80	3	11					20	32
11										12	12	70	6				88	94
12											73	18	7	1	1		82	85
13											7	5	64	23	1		36	44
14											2		19	72	6	1	28	33
15											1		4	4	89	2	11	16
16															3	97	3	3

TABLE 5

QUADRATIC CHOROPLETH MAPS
(n = 100)

JUDGED RANK

ACTUAL RANK

R	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	RE	RD
1	97	3															3	3
2	3	78	17		2												22	26
3		16	73	9	1	1											27	30
4		3	8	74	14	1											26	30
5			2	13	75	10											25	27
6				5	7	79	9										21	26
7						9	90	1									10	10
8								99	1								1	1
9							1		97	2							3	4
10									2	95			3				5	11
11										1	81	13	5				19	24
12											16	66	18				34	34
13										1	3	21	70	2	2	1	30	39
14													5	44	50	1	56	57
15														50	46	4	54	54
16															2	98	2	2

TABLE 6

CUBIC CHOROPLETH MAPS
(n = 100)

JUDGED RANK

ACTUAL RANK

R	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	RE	RD
1	99	1															1	1
2	1	87	8	4													13	17
3		8	40	31	1												40	41
4		2	30	41	7	2											41	45
5		1	3	5	83	8											17	22
6					9	75	16										25	25
7						16	82	2									18	18
8							1	95	4								5	5
9							1	3	92	3	1						8	10
10									3	72	25						28	28
11									1	25	60	14					40	41
12											12	71	14	3			29	32
13												11	70	16	3		30	33
14												2	18	64	16		36	38
15													2	16	75	7	25	27
16														1	6	93	7	8

TABLE 7

LINEAR PROFILE MAPS
(n = 100)

JUDGED RANK

ACTUAL RANK

R	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	RE	RD
1	98	1		1													2	4
2	1	73	21	4													27	34
3		18	54	19	5	2	2										46	61
4		5	17	58	13	3	4										42	58
5	1	1	4	14	63	8	8		1								37	57
6			2	3	9	54	27	4	1								46	59
7		1	1		9	32	52	4	1								48	64
8		1	1		1		7	78	8	2	2						22	37
9				1				10	76	5	6	1			1		24	41
10								3	7	42	28	9	4	5		2	58	103
11								1	5	28	37	12	11	4	1	1	63	96
12										8	18	11	57	4	1	1	89	106
13										5	8	61	17	6	2	1	83	105
14									1	4		5	8	72	9	1	28	50
15										3	1			8	86	2	14	29
16										1	1	1	3	1	1	92	8	27

TABLE 8

QUADRATIC PROFILE MAPS
(n = 100)

JUDGED RANK

ACTUAL RANK

R	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	RE	RD
1	94	1	3		2												6	15
2	3	51	32	13	1												49	67
3	1	37	32	23	5					2							68	86
4		8	18	50	22		1		1								50	64
5	1	2	9	11	64	11	1				1						36	58
6			3	3	4	63	22	3	2								37	53
7					1	21	51	22	4	1							49	57
8		1			1	4	20	55	11	6	1	1					45	67
9				1			2	16	40	39	2						60	68
10	1					1	2	4	36	43	10	3					57	79
11				1			1		6	6	64	16	5	1			36	48
12										3	15	59	21	2			41	46
13											6	27	56	8	2	1	44	54
14											1	2	7	66	24		34	55
15											1	1	1	24	72	1	28	34
16													1	1		98	2	5

TABLE 9

CUBIC PROFILE MAPS
(n = 100)

JUDGED RANK

ACTUAL RANK R	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	RE	RD
1	97	2	1														3	4
2	3	51	28	17								1					49	75
3		28	35	33	2	1	1										65	114
4		11	28	47	9	1	1				1	2					53	87
5		4	5	4	55	17	10	1	1		2	1					45	89
6				1	18	37	26	7	3	1	4		3				63	114
7					6	23	38	17	7	2	3	1	2	1			62	108
8					6	9	13	43	20	3	2	1	3				57	100
9		2			3	6	4	20	36	14	10		5				64	126
10						5	3	7	18	35	21	4	5	1	1		65	114
11					1	1	1	1	12	18	21	8	37				79	142
12			1			1	2	3	2	19	32	10	30				90	143
13								1	2	6	6	66	10	5	2	2	90	124
14												4	2	73	21		27	31
15												2	1	21	75	1	25	30
16										1					2	97	3	8

APPENDIX E

One reason for inversions of particular map pairs could be attributed to imperfections in the cartography, photography, or printing. This is the case with maps of rank 11 and 12 in the linear choropleth set. The darker tones of map 11 appear to have been weighted more heavily than the degree of pattern randomness when choosing which map was more highly correlated with the format. This caused map 11 to be ranked next to the dark-tone map 13. Re-inversion of maps 11 and 13 produced a more realistic (i.e., smooth) response trend with both ranking errors and ranking displacement plots. Although the confusion matrix was left unaltered for data analysis, corrected (re-inverted) versions of the data are depicted on all graphic displays that follow. This is also the case with the corrections discussed below.

A second type of weighting error did not involve map imperfections. An example of this type of error can be seen in both the quadratic and cubic dot map sets. Each contained maps with empty cells in the low value zones in the medium-high to high r^2 value range.[4] (Footnote # 4 is listed with the references to Chapter V). These items, namely maps 12 and 13 in the quadratic dot set and maps 14, 15, and 16 in the cubic dot set, seem to have been mis-ranked by overweighting empty cells and underweighting the true degree of surface roughness. Since it is statistically insignificant if a cell contains 0 or 1 dot, this is a major source of deviation between the perceptual mechanism and the statistical model. A smoother response trend in ranking errors was produced by re-inversion of the quadratic maps (items 12 and 13). Since alteration of ranking displacement with maps 12 and 13 of the quadratic set and alteration of both ranking errors and ranking displacement with maps 14, 15, and 16 of the cubic set would have been ineffectual, these responses were left unchanged.

The quadratic choropleth map set also contains an obvious anomaly with maps 14 and 15. These maps appear to have been inverted because one two-cell dark block occurring in an otherwise light-toned area was weighted disproportionately high at the expense of pattern roughness. Strict inversion of this map pair would be ineffectual; thus, both ranking errors and ranking displacement values to be plotted were derived by averaging the values for items 14 and 15 over linear and cubic choropleth maps.

The profile map sets introduce a third example of perceptual overweighting. In particular, maps 12 and 13 in the linear set, and maps 12 and 13 in the cubic set, appear to have been interchanged because either the more extensive horizontal empty zones on the former (maps 12) -- often the result of hidden profiles -- or the rougher look of the latter (maps 13) were weighted more heavily when deciding which exhibited the higher degree of association with the format. This occurred in spite of the fact that there were fewer hidden lines with the more highly correlated items (13). These two pairs of maps were re-inverted on ranking errors plots but left intact on plots of ranking displacement.

Graphs 1 and 2 identify and display the results of map inversions. These graphs can be compared with the respective tables in Appendix E.

TABLE 1

TOTAL RANKING DISPLACEMENT [a]
(n = 100)

r^2	DOT SETS			CHOROPLETH SETS			PROFILE SETS		
	L [b]	Q	C	L	Q	C	L	Q	C
+.90	104	81	62	0	3	1	4	15	4
.80	92	94	82	33	26	17	34	67	75
.70	102	110	72	35	30	41	61	86	114
.60	109	97	91	48	30	45	58	64	87
.50	89	110	74	42	27	22	57	58	89
.40	63	91	81	19	26	25	59	53	114
.20	62	97	40	9	10	18	64	57	108
0	30	33	31	5	1	5	37	67	100
.20	23	54	41	6	4	10	41	68	126
.40	83	72	61	32	11	28	103	79	114
.50	83	93	71	39*	24	41	96	48	142
.60	115	128	89	42*	34	32	106	46	143
.70	110	139	118	44	39	33	105	54	124
.80	99	132	108	33	32*	38	50	55	31
.90	91	107	68	16	18*	27	29	34	30
-1.00	91	110	128	3	2	8	27	5	8
Total	1346	1548	1217	406	378	391	931	856	1409
	4111			1114			3196		

[a] This table consists of the row totals (RD) from the 9 confusion matrices found in Appendix E.

[b] L = linear; Q = quadratic; C = cubic

* These figures are not representative of the original subject response data. Refer to the text of this appendix (F) for further discussion.

TABLE 2

TOTAL RANKING ERRORS[a]
(n = 100)

r^2	DOT SETS			CHOROPLETH SETS			PROFILE SETS		
	L[b]	Q	C	L	Q	C	L	Q	C
+.90	68	71	55	0	3	1	2	6	3
.80	69	75	68	28	22	13	27	49	49
.70	69	70	56	34	27	40	46	68	65
.60	72	63	65	42	26	41	42	50	53
.50	45	70	54	38	25	17	37	36	45
.40	48	57	57	14	21	25	46	37	63
.20	47	46	37	8	10	18	48	49	62
. 0	16	20	26	5	1	5	22	45	57
.20	12	36	24	6	3	8	24	60	64
.40	53	48	49	20	5	28	58	57	65
.50	58	51	56	27*	19	40	63	36	79
.60	78	55*	68	30*	34	29	43*	41	34*
.70	78	73*	73	36	30	30	39*	44	69*
.80	65	72	71	28	35*	36	28	34	27
.90	64	63	54	11	22*	25	14	28	25
-1.00	47	54	69	3	2	7	8	2	3
Total	889	924	882	330	278	363	547	642	763
	2695			971			1952		

[a]This table consists of the row totals (RE) from the 9 confusion matrices presented in Appendix E.

[b]L = linear; Q = quadratic; C = cubic

*These figures are not representative of the original subject response data. Refer to the text of this appendix (F) for further discussion.

APPENDIX F

MISCELLANEOUS TABLES

TABLE 1

MEAN STANDARD DEVIATIONS IN
RANKING DISPLACEMENT
(n = 100)

r^2	DOT SETS			CHOROPLETH SETS			PROFILE SETS		
	L*	Q	C	L	Q	C	L	Q	C
+.90	1.03	.65	.59	.00	.17	.01	.32	.66	.22
.80	1.11	1.20	1.03	.57	.59	.49	.55	.74	.77
.70	1.33	1.25	1.04	.60	.62	.63	.98	1.34	.94
.60	1.39	1.29	1.14	.76	.63	.68	.98	1.03	1.43
.50	1.39	1.39	1.09	.68	.55	.58	.98	1.13	1.71
.40	1.10	1.33	1.12	.58	.56	.49	.92	.91	1.90
.20	1.04	1.57	.89	.33	.31	.39	.95	.85	1.56
0	1.02	.89	.81	.22	.01	.22	.87	1.17	1.54
.20	.86	.91	1.03	.24	.25	.38	.91	.97	1.91
.40	1.24	1.02	.91	.73	.54	.48	1.37	1.26	1.65
.50	1.22	1.36	1.18	.76	.54	.65	1.32	1.18	1.75
.60	1.38	1.47	1.24	.77	.59	.59	1.14	.77	1.74
.70	1.35	1.56	1.60	.78	.76	.64	1.01	.83	1.21
.80	1.31	1.52	1.27	.68	.59	.65	1.12	.69	.65
.90	1.33	1.51	.99	.60	.58	.54	.91	.66	1.41
-1.00	1.18	1.61	1.22	.17	.70	.31	.98	.36	.61

*L = linear; Q = quadratic; C = cubic

TABLE 2

MEAN RANKING DISPLACEMENT
(n = 100)

r^2	DOT SETS			CHOROPLETH SETS			PROFILE SETS		
	L*	Q	C	L	Q	C	L	Q	C
+.90	-1.01	-.76	-.57	00	-.03	-.01	-.04	-.15	-.03
.80	-.32	.11	.09	-.32	-.20	-.16	-.27	-.52	-.57
.70	-.09	-.76	-.13	.10	.02	-.25	-.23	-.10	-.14
.60	.29	-.18	-.23	-.15	-.03	.27	-.06	.05	.20
.50	.59	.28	.06	.19	.09	.06	.05	.23	-.48
.40	-.11	.28	.39	.13	.06	-.06	-.14	-.18	-.75
.20	.41	.84	.03	.01	.08	.13	.52	-.08	-.34
0	.17	.08	.03	.03	-.01	-.03	-.03	.09	-.04
.20	-.17	-.42	.05	-.06	00	00	-.16	-.19	.19
.40	-.66	-.14	-.35	-.18	-.07	-.22	-.69	.52	.08
.50	-.13	-.41	-.03	-.68	-.22	.13	-.11	00	-.24
.60	-.42	-.52	-.38	.65	-.02	-.03	-.32	-.16	.82
.70	.19	-.52	-.04	-.06	.21	-.10	.74	.24	.90
.80	.02	.23	-.42	.17	-.56	.06	.27	-.10	-.09
.90	.37	.74	.19	.14	.54	.13	.23	.30	.41
-1.00	.87	1.15	1.31	.03	.14	.08	.24	.05	.08

*L = linear; Q = quadratic; C = cubic

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