CATS '93

CENTER FOR CHAOS AND TURBULENCE STUDIES

1993 ANNUAL REPORT

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Theoretical and experimental research in chaos, turbulence, and related problems in physics, chemistry and biology is pursued at the Center for Chaos and Turbulence studies, a cross-disciplinary center at the University of Copenhagen and the Technical University of Denmark. The Center is dedicated to promoting nonlinear science by conducting research, by organizing workshops and advanced courses, and by facilitating training and exchange of scientists and students.

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1 OVERVIEW

The Center for Chaos and Turbulence Studies (CATS) was established in 1993 by the Danish Natural Science Research Council (SNF) as a "center without walls". The purpose of CATS is to advance research and research training in nonlinear science, and to strengthen the role that nonlinear research in Denmark plays internationally. The Center consists of groups from the Niels Bohr Institute/Nordita (NBI), Chemistry Institute, H. C. Ørsted Institute (HCØ-KI), and Physics Department, The Technical University of Denmark (DTU-FI). The Center brings together the groups which are actively conducting research and education in this field, and enables them to share resources more effectively, ensure broader participation in research projects and offer better training for students and research fellows.

SNF has funded the Center with 15 million kroner for a 5-year period. In addition, the Center receives several million kroner annually from private foundations, Danish universities, and the European Union Mobility program, mostly through salaries for the non-tenured staff.

Even though we celebrated the establishment of the Center on 23 June 1993 by a reception at the Niels Bohr Institute, and the SNF funding first started per. 1 October 1993, the Center had commenced its scientific activities well before these dates, so this first annual report covers the entire 1993 calendar year.

Predrag Cvitanović
Niels Bohr Institute 19 May 94

1.1 Background

In the last decade, investigations of nonlinear phenomena in nature have developed into a field of great importance. The fundamental insight is that the simplest laws of nature can lead to bewilderingly complex dynamics, and that at the same time such dynamics exhibits universal features largely independent of the details of the particular underlying dynamical system. The new vistas, opened in no small part by the advent of modern computation, represent a natural and healthy shift in some research directions. Stressing the unity of the underlying concepts and benefitting from the accumulated experience of scientists of diverse backgrounds, they have transformed important problems that, until recently, have been considered intractable or even ill-posed, into promising new fields. Today a glance at the contents of leading professional publications reveals that nonlinear science has had a significant impact on a broad spectrum of natural sciences. Methods developed in the study of nonlinear systems are today routinely applied to such areas as turbulence, quantum theory, nuclear physics, atomic physics, biological control and self-organization, astronomy, cardiology, chemical reactions, nonlinear waves and diffusion, semiconductors, image processing, Josephson junctions and problems of classical mechanics (to name some that we have been interested in).

1.2 Participating groups: a brief profile

"Chaos" theory can be traced back a century to Poincaré, and to the fundamental contributions of Kolmogorov, Arnold and other great mathematicians, but the modern, physics inspired nonlinear dynamics is a young subject; the current blossoming has its roots in discoveries of 70's.

"Chaos" research on an international level started relatively early in Denmark. Even before "chaos" was recognized as a subject, the groups which today form the Center have made important contributions to nonlinear dynamics, such as the 1970 detection of subharmonics in acoustic wave amplification in piezoelectric semiconductors at DTU-FI, the 1974 discovery of multiperiodic oscillations ("burst oscillations") in chemical reactions at HCØ-KI, and the 1977 invention of the unbiased Josephson volt standard at HCØ-FL. Members of the NBI/Nordita group contributed in 1976 to one of the key advances of the modern theory, the *period doubling universality* at the onset of chaos; in 1981 Nordita published the "Chaos and Universality" reprint collection, and a year later the first Nordita "Chaos" summer workshop brought together many of the most distinguished contributors to the field, together with a large number of Nordic participants. In the early 80's, the research in solid state physics and phase transitions at HCØ-FL led to study of the dynamics of *mode-locking* (phenomena involving competing frequencies), and important contributions were made to this scenario of the onset of chaos at an early stage. Today the Danish nonlinear science is strong and has a high international profile. Besides the groups participating in CATS, there are other strong groups at DTU and Risø National laboratory, together with smaller but active groups at Århus University, Roskilde University Center, and Odense University. The CATS groups collaborate actively with many of these groups, and an important goal for the CATS center is to facilitate and catalyze further cooperation.

The reasons for the existing close CATS collaborations, and for our hopes of strengthening it further, lie in the complementary profiles of the groups involved. The current CATS collaborations formed gradually through the 80's, with some of the present members having worked both at HCØ, DTU and NBI/Nordita, and through jointly organized meetings and workshops. The **NBI/Nordita** group is the largest one and has established a strong theoretical, condensed matter and mathematical physics research effort, together with the newly started experimental effort. Connections to other theoretical groups (nuclear, high energy, astrophysics, solid state and neural networks) are essential to the group's activity, and the international connections are strong, in keeping with the best of the NBI/Nordita traditions. The main research topics at present are classical and quantum chaos, and turbulence. The **HCØ-KI** group has a long tradition for research on nonlinear chemical systems, and has contributed experimentally and theoretically to the present understanding of oscillatory chemical reactions. The method of characterizing complex chemical systems developed by the group forms a basis for the joint experimental and theoretical projects on chemical turbulence with the NBI/Nordita groups. The group has also a strong interest in models of biological self-organization and morphogenesis, a CATS theme that it shares with the DTU-FI group. The group at **DTU-FI** has specialized in applications of nonlinear dynamics to a wide variety of problems ranging from engineering to biology. They have good connections to industry and technology, and, most importantly, to the medical research. For many years the main interest of the group has been biological applications of nonlinear methods and the group has acquired

a unique position in this promising field.

One of the Center's key tasks will be to aid the continuity and stability of nonlinear dynamics research at the University of Copenhagen. Until now, a significant part of the research in these new directions has only been possible through temporary support from private foundations. Thus P. Alstrøm, T. Bohr, and P. Cvitanović are supported by the Novo and Carlsberg foundations, and M.H. Jensen is supported until the end of 1995 by a grant to the University of Copenhagen. Lack of permanent positions has hampered the long term research planning for both students and staff, and the level and quality of the future research activities will depend crucially on the Center's ability to contribute to the renewal of Danish nonlinear physics by providing support for young and dynamic researchers. To secure this goal, the Center provides support for several established scientists on lecturer level, and dedicates part of its funds to postdoctoral and visiting scientist positions.

1.3 Overview of activities

This section contains a brief overview over the planned CATS research activities. More details are given in section **??**.

Classical and Quantum Chaos:
The goal of this research is the development of a classical and quantum description of chaotic, non-integrable systems, of analytic and numerical quality comparable to those obtainable by the traditional techniques for nearly integrable systems. The NBI/Nordita group has established that strongly chaotic systems can be described surprisingly accurately by means of unstable periodic orbits. So far, the most exciting application of the theory has been to quantum chaos, where one can now experimentally measure the energy levels that the chaos theory predicts from the classical unstable orbits; the theory is under rapid development, with recent and highly impressive advances such as the first periodic-orbit calculation of the Helium spectrum. In the coming years we plan to apply this theory to a variety of solid state, mesoscopic, and other experimental systems. Some of the important open problems of this field that we intend to confront are: How can the quantum-mechanical unitarity be built into the periodic orbit theory? Can the periodic orbit quantization be made exact? How can the theory be extended to generic systems which exhibit a mixture of chaotic and integrable behavior?

Turbulence:
While today we know much about what deterministic chaos is and how to characterize it, our understanding of turbulence is still rudimentary. Progress in this field is difficult, but a serious theoretical advance is crucial in its implications for classical hydrodynamics, statistical mechanics, and strongly nonlinear field theories. The CATS groups study turbulence as chaotic phenomena in spatially extended systems; the main question is to what extent the methods developed in the context of low-dimensional chaotic dynamics can be applied to dynamical systems of high intrinsic dimensions. The groups plan to investigate transitions from coherent to incoherent motion, the turbulent motion of vortices in reaction-diffusion systems, the instability of boundary layers in convective systems, intermittency, and scaling exponents of turbulence. This involves extensive numerical investigations of **spatio-temporal chaos**: coupled-map lattices, cellular automata, neural networks, and a variety of discretized models of chemical and hydrodynamical turbulent

phenomena. Such models have enabled us to study the onset of turbulent vortex structures in models of inhomogeneous oscillatory chemical reactions, as well as to discover new types of phase transitions, subsequently observed experimentally in the Belousov-Zhabotinsky reaction and in charge-density waves. A common thread of all CATS projects is extensive **computer experimentation**; chaos researchers were among the first to utilize interactive graphical computation, and our future research will to a high degree rely on the advent of parallelism and other advances in computation.

Experimental Investigations:
Experimental investigations are crucial to the development of our understanding of complex systems. Carefully designed experiments help developing intuition and stimulate new theoretical advances. Some of the first experiments in chaotic systems were carried out by CATS groups, and it is of paramount importance for the Center to build a strong experimental program. Currently the CATS-groups have ongoing experiments in chemical reactions (quenching analysis, invariant manifolds, chemical waves), fluid dynamics (turbulence in pipe flows, soap film flows capillary waves, hydraulic jump), acoustics (spectral statistics of high Q resonators), biology (yeast colony growth), and solid state phenomena (relaxation oscillators, neural network electronics, dynamical friction, granular flows).

Chemical Reaction-Diffusion Systems:
Aspects of nonlinear dynamics, such as chaos and the theory of bifurcations, play an important role in chemical reaction-diffusion systems where chaotic behavior is by now an established experimental fact. The HCØ-KI group was one of the first to initiate quantitative experimental measurements of such dynamical systems. The group has shown that the Turing structures formed in these systems are relevant for biological pattern formation; the ability to describe complex chemical systems is essential for understanding key phenomena in biological control processes. The goal of the HCØ-KI/NBI collaboration, combining the experimental and theoretical strengths of the two groups, is to describe chemical turbulence in spatially extended systems.

Fractals:
One of the key notions in nonlinear science is the concept of fractals. Dendritic growth, dielectric breakdown, electrochemical deposition, viscous fingering of a fluid penetrating another fluid are some of the fractals generated in nature. The goal of the NBI experimental investigations of fractal structures and their theoretical descriptions is to discover, by techniques closely related to statistical mechanics, new observable consequences of chaoticity, such as universal scalings, phase transitions, fractal dimensions, and self-organizing critical phenomena.

Biology:
One of the most obvious areas in which to apply and further develop the tools of nonlinear dynamics is biology. Unstable phenomena are increasingly being recognized as significant in the regulation and function of living systems. Investigations performed during the last decade have revealed the existence of a large variety of biological rhythms with periods ranging from fractions of a second to hours or even days. Other investigations have demonstrated the occurrence of deterministic chaos in physiological systems. These observations pose a number of fundamental questions concerning possible advantages of complex behavior and the mechanisms of evolutionary processes. A particularly interesting topic is the evolution and development of biological forms. All of these closely related areas are experiencing dramatic breakthroughs, obtained through crossfertiliza-

tion of ideas from physics, mathematics, computer science and biology. The computer simulation of self-organization in realistic 3-dimensional systems based on bifurcations in reaction-diffusion systems was pioneered by the HCØ-KI group. Much of the DTU group research in the coming years will focus on collaborations with biologists, medical doctors and chemists in order to study oscillations, chaos and biological form formation in a number of concrete examples.

2 STAFF

2.1 Organization of the Center

Statens Naturvidenskabelige Forskningsråd (SNF) provides the funding for the Center for the initial 5 year period. The Center also seeks external funding from fundations, Forskerakademiet and international organisations for specific projects.

SNF appoints the director of the Center. SNF approves the scientific program, the budget of the Center and the yearly activity report as presented by the director.

The scientific program of the Center is supervised by a standing committee consisting of 4 staff members representing the permanent staff, 1 external expert, and is chaired by the director. For the period 1 oct. 1993 - 30 Nov. 1994 the committee consists of

Predrag Cvitanović (director)	Niels Bohr Institutet, KU
Hans C. Fogedby (external expert)	Physics Institute, Århus University
Preben Alstrøm (deputy director)	Niels Bohr Institutet, KU
Tomas Bohr	Niels Bohr Institutet, KU
Preben Graae Sørensen	Chemistry Institute, HCØ, KU
Erik Mosekilde	Physics Institute, DTU

2.2 Scientific staff

Preben Alstrøm	NBI	Novo Hallas-Møller stipendiat (until 1. Sep. 1997).
Tomas Bohr	NBI	Novo Hallas-Møller stipendiat (until 1. Sep. 1995).
Predrag Cvitanović	NBI	Carlsbergfonds professor (until 1. Sep. 1995).
Peter Dimon	NBI	CATS scientist.
Clive Ellegaard	NBI	lecturer.
Axel Hunding	KI HCØ	lecturer.
Finn Hynne	KI HCØ	lecturer.
Mogens Høgh Jensen	NBI	lecturer (KU funding until 1. Jan. 1995).
Mogens Colding Jørgensen	Novo	associated scientist.
Mogens T. Levinsen	NBI	lecturer.
Erik Mosekilde	FI DTU	docent.
Mogens Samuelsen	FI DTU	docent.
Preben Graae Sørensen	KI HCØ	lecturer.

2.3 Postdoctoral fellows

NBI:

S. Creagh (NBI-EU)
G. Huber (SNF)
R. Mainieri (NATO)
T. Sams (Carlsberg)
G. Vattay (Scezeny Fundation, NBI)
T. Guhr (DFG)
J.C. Larsen (DTU-Mathematics)
M. Oxborrow (NBI-EU)
K. Sneppen (SNF)
N. Whelan (NBI-EU)

DTU:
C. Kampmann

2.4 Students

In the course of 1993 calendar year, the following Ph.D and masters students were pursuing or have completed their graduate studies under the supervision of members of the CATS staff:

NBI:

C. Conrado (Ph.D.),
P. Rosenqvist (Ph.D.),
P. Bertelsen,
S. Hørlüch,
H. Lorenzen,
J. Nygård,
K. Schaadt,
J. Sparre Andersen
A. Vallespin Gomez
D. Zauner
K.T. Hansen (Ph.D.),
T. Schreiber (Ph.D.),
R. Hvass Hansen,
A. Johansen,
J. Nielsen,
V. Putkaradze (Ph.D.),
E. Schröder
M. Stoklund Johansen
C. Veje,
J. Zhang (Ph.D., experimental)

KI-HCØ:

R. Engelhardt
K. Geisshirt
Wang Jichang
T. Møller
V. Espensen
M. Ipsen(Ph. D.)
K. Kristiansen,

DTU:

C. Knudsen (Ph.D.)
J.S. Thomsen (Ph.D.)
C. Haxholdt (Ph.D.)
N. Kloster
O. Stauning
M. Christensen
P. Engelbrecht
V.O. Pannbacker
R. Feldberg (Ph.D.)
O. Lund (Ph.D.)
M. Olesen
O. Bruun
J. Rasmussen
G. Neergaard
O. Jensen

2.5 Administrative staff

Dorte Glass, CATS administrative secretary	tel:	+45: 353 25 328
Center for Chaos and Turbulence Studies		
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Center for Chaos and Turbulence Studies		
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The Technical University of Denmark	fax:	+45: 45 93 12 35
DK-2800 Lyngby		

3 TRAINING AND COLLABORATIONS

Nonlinear dynamics is a young field which attracts many young researchers, both Danish and foreign, and there is lively student participation, with relatively large enrollment in chaos courses. An important role is played by the weekly NBI/Nordita group meetings in chaos, quantum chaos and fluid dynamics in which some 10-20 young researchers discuss informally the latest advances in a broad spectrum of nonlinear problems. We believe that training young people in nonlinear science is a good investment, as the tools acquired should equip them to tackle a broad variety of problems. CATS acts as a *de facto* European center for graduate education, with Ph.D. students supported by and sent to us from Norway, Italy, Sweden, Germany, Brazil and other countries. The first generation of our Ph.D.'s is already professionally active, and the enthusiasm and the quality of the students and the research fellows that we see entering the field bring promise of a dynamic and vital coming generation.

Indicative of the level of activity and international standing are the number of researchers who have worked with the CATS-groups for more than 2 months during 1993: 13 post-doctoral fellows, 14 Ph.D. students, 9 master degree students, and 15 visiting foreign scientists. Many of the visitors, fellows, and students arrive with their own funding.

3.1 Master's and Ph.D. Projects

In 1993/94 the CATS staff is offering the following projects Master's and/or Ph.D. thesis topics to students that might chose to do their thesis work at CATS:

- Statistical mechanics as a maoscopic manifestation of the underlying chaotic dynamics (P. Cvitanović)
- Spatio-temporal chaos, instabilities and turbulence (T. Bohr)
- Spiral waves and chemical turbulence (P. Alstrøm and T. Bohr)
- Theoretical and experimental studies of bio-physical systems: Evolution and growing fungus/bacterium colonies (M. Høgh Jensen, K. Sneppen, T. Sams and C. Ellegaard)

- Dynamics and structures in turbulence and growing fronts: multifractals and multiscaling (M. Høgh Jensen and K. Sneppen)
- Acoustic resonances inspired by quantum chaos (C. Ellegaard and P. Cvitanović)
- The circular hydraulic jump (T. Bohr, P. Dimon and C. Ellegaard)
- Fluctuations in granular flows (P. Dimon)
- Self-organization in simple models of the brain (P. Alstrøm)
- Experimental and theoretical studies of pattern formation and turbulence in capillary waves (P. Alstrøm and M.T. Levinsen)
- Experimental and theoretical studies of turbulence in a soap film or a pipe (P. Alstrøm and M.T. Levinsen)
- Experimental front dynamics with proteins (G. Zocchi)
- Transition state metal catalyst variants of the Belousov-Zhabotinski reaction (P. G. Sørensen and F. Hynne)
- Biocybernetics. Modelling pattern formation in biochemical control systems (A. Hunding).
- Nonlinear dynamics in chemical kinetics. Quenching analysis (P. G. Sørensen and F. Hynne).
- Transient period doublings, Torus oscillations and chaos in a closed chemical system (P. G. Sørensen and F. Hynne).
- Turing Structures in Chemical Reaction Diffusion Systems (DTU)

3.2 Workshops

The members of the CATS staff have since 1981 been involved in a large variety of nonlinear physics activities. These include conferences, workshops, graduate education and courses, schools, contributions to television and radio programs, newspaper articles and colloquia to broader audiences. In 1993 members of the CATS staff have organized following schools, workshops and conferences:

Complex Systems: Turbulence, Chaos, Neural Networks EU-Nordic Summer School, Humlebæk, Denmark, August 21–28, 1993.

The aim of the school was to introduce graduate students and post-doctoral fellows into recent advances in physics of complex systems: chaos in low dimensional classical and quantum systems, turbulence in flows, neural networks, self-organized critical phenomena, fractals in oil research, interface growth. 75 students took part.

Lecturers: J. Feder (Oslo), P. Bak (Brookhaven), D. Rand (Warwick), A. Libchaber (Princeton), G. Grinstein (IBM), P. Cvitanović (CATS), S. Solla (AT&T and CONNECT), J. Hertz (Nordita), I. Procaccia (Weizmann), T. Bohr (CATS) and J. Krug (Jülich).

1st Copenhagen Symp. on Computer Simulation in Biology, Ecology and Medicine, Copenhagen, 23-25 Aug. 1993

This symposium was the first in a series of meetings in which biologists, ecologists, and

clinically orientated doctors can meet and exchange ideas with mathematicians, physicists and computer scientists in order to develop a more satisfactory description of the living world.

Invited lecturers: H. Meinhardt (Tübingen), N.-H. Holstein-Rathlow (Copenhagen), H. Herzel (Berlin), M. Marik (Prague), P. Borckmans (Brussels), and O.G. Mouritzen (Lyngby)

Pattern Formation, 15 Mar. 1993

DTU bld 206 room 250.

Center for Chaos and Turbulence Studies - 1/2 day meeting, 16 Sept. 1993.

Aud 38, bld. 306, DTU.
Seminars by E. Mosekilde, C. Knudsen, M. Samuelsen, K. Sneppen, and F. Hynne.

Center for Chaos and Turbulence Studies - 1/2 day meeting

Aud 6, H.C. Ørsted Institute.

P.G. Sørensen:	CATS at KI.
A. Hunding:	Pattern formation in biochemical control systems.
Laboratory tour:	Experimental equipment for investigations of temporal chaos and waves in chemical systems.
P. Dimon:	Dynamic phases in a spring block system.

3.3 Collaborations

CATS staff collaborates actively with various groups all over the world and the staff members frequently visit their collaborators abroad (for instance, P. Cvitanović is Adjunct Professor at the Rockefeller University two months a year, and P. Alstrøm is a Research Assistant Professor at the Boston University). It is no less important for the Danish research environment that we can bring some of the world's leading experts, post-doctoral fellows and students to Denmark. CATS Center facilitates such exchanges by giving us flexibility in arranging visits with short lead time, as well as helping us send the top Danish students to international schools and research centers.

Europe (in particular France, Russia, Germany, England, Belgium and Denmark) arguably leads the world in the research on nonlinear phenomena and complex systems, and the CATS groups are very active in European collaborative efforts. We already have many EU funded visitors and expect to attract still more EU post-doctoral fellows and Ph.D. students in coming years. Participation in the "European Network on Nonlinear Physics", coordinated by G. Nicolis, Brussels, with P. Cvitanovć acting as the Danish representative, strengthens the European role of the CATS Center. Some examples of other current European collaborations: E. Mosekilde and A. Hunding have visited the Brussels under auspices of the EU ERASMUS program. An EU twinning project "Dynamical and Geometrical Aspects of Correlations near Critical Points" between Copenhagen, Naples, Paris, Nice, and Cologne is coordinated by P. Alstrøm. G. Paladin (Rome) and M.H. Jensen coordinate an EU funded exchange of students and post-doctoral fellows between the nonlinear groups at the Physics Department, Rome University, and the

NBI/Nordita group. HCØ-KI participates in a Joint European TEMPUS Project "Nonlinear dynamics in Chemistry and Biology" together with M. Marek (Prague), G. Nicolis (Brussels), S. Müller (Dortmund), and F. W. Schneider (Würzburg). As a part of the program, P. G. Sørensen and F. Hynne have spent two weeks each in Prague, Ph.D. student J. Košek (Prague) has worked 3 months at HCØ-KI with quenching experiments, and master degree students T. Møller and M. Ipsen have been on visits for 3 months to Prague.

4 VISITORS

4.1 Short term visitors

D. Lohse, Marburg:	23 - 26 Jan. 1993
P. Sibani, Odense University:	26 - 27 Jan. 1993
P. Crehan, University College, Dublin:	23 - 25 Feb. 1993
O. Rössler, University of Tübingen:	14 - 17 Mar. 1993
H.R. Blank, Erlangen:	29 Mar. - 2 Apr. 1993
P. Dahlqvist, KTH, Stockholm:	3 - 7 Apr. 1993
S. Nagel, University of Chicago:	22 - 28 Apr. 1993
H.H. Rugh, ENS, Lyon:	4 - 11 May 1993
F. Christiansen, INO, Florence:	6 - 9 May 1993
H. Bruus, Yale:	25 - 28 May 1993
K. Wiesenfeld, Georgia Tech.:	12 - 15 Jun. 1993
V.A. Shneidman, Augsburg University:	22 - 26 Jun. 1993
R. Artuso, Italy:	1 - 6 Jul. 1993
M. Feigelman, Landau Institute, Moscow:	9 - 22 Aug. 1993
N. Obers, University of Bonn:	10 - 22 Aug. 1993
J. Krug, KFA, Jülich:	10 - 14 Aug. 1993
A. Fairhall, Australia:	16 - 20 Aug. 1993
J. Hietarinta, University of Turku:	12 - 26 Aug. 1993
S. Boatto, University of Chicago:	28 Aug. - 12 Sep. 1993
D. Stassinopoulos, Brookhaven:	5 - 9 Sep. 1993
J. Maher, Pittsburgh University:	9 - 13 Sep. 1993
A. Johansen, Nice:	25 - 30 Sep. 1993
A. Vulpiani, Italy:	30 Sep. - 7 Oct. 1993
J-L. Ottinger, Brussels:	4 - 8 Oct. 1993
J. Nycander, KTH, Stockholm:	9 - 12 Nov. 1993
M. Smrčinová, Dept. of Physics, STU, Bratislava:	8 - 24 Nov. 1993
C. Ghidaglia, ENS, Paris:	11 - 15 Dec. 1993
A. Richter, Darmstadt:	14 - 18 Dec. 1993
D. Rand, University of Warwick :	14 - 16 Dec. 1993

4.2 Long term visitors

M. Zaks, Bayreuth University:	3 Mar. - 30 Apr. 1993
C. Jayaprakash, Ohio State University:	15 May - 15 Jun. 1993
R. Mainieri, Los Alamos National Lab.:	31 May - 30 Jun. 1993
I. Halpern, USA:	13 Jun. - 12 Jul. 1993
M. Feigenbaum, USA:	1 -14 Jul. 1993
V. Baladi, ENS, Lyon:	2 - 16 Jul. 1993
G. Grinstein, Canada:	20 Aug. - 10 Sep. 1993
W. Goldburg, Pittsburgh University:	2 Sep. - 31 Oct. 1993
E.-J. Ding, Academia Sinica, Beijing:	25 Sep. - 25 Oct. 1993

5 CATS SEMINARS

- Per Bak, Brookhaven — NBI, 5 Jan. 1993.
  How to control economy
- Predrag Cvitanović, NBI — NBI, 20 Jan. 1993.
  The Fourier analysis of chaotic dynamics
- Detlef Lohse, Marburg — NBI, 25 Jan. 1993.
  Intermittency in turbulence
- Paolo Sibani, Odense University — Nordita, 28 Jan. 1993
  Slow dynamics from noise adaptation
- Gabor Vattay, NBI — NBI, 17 Feb. 1993.
  Level statistics in constant negative curvature billiards with nongeodetic walls
- Pat Crehan, Department of Math. Physics, University College, Dublin — NBI, 23 Feb. 1993.
  Wavelets, image compression and the analysis of brain waves
- Pat Crehan, Department of Math. Physics, University College, Dublin — NBI, 24 Feb. 1993.
  The geometry of degenerate poisson manifolds
- Kim Sneppen, NBI — HCØ, 24 Feb. 1993.
  Self organized pinning in growing interfaces
- Mikhail Zaks, Bayreuth University, and Inst. of Cont. Media Mechanics, Perm — NBI, 9 Mar. 1993.
  Double exponential scaling in unimodal mappings
- Aksel Walloe, Geophysical Institute, U. of Copenhagen — NBI, 9 Mar. 1993.
  The ozone hole developing above our heads
- Otto Rössler, University of Tübingen — NBI, 15 Mar. 1993.
  End of physics (complementarity derived form chaos)
- Otto Rössler, University of Tübingen — MIDIT, DTU, 16 Mar. 1993.
  Subjective history of chaos
- Otto Rössler, University of Tübingen — MIDIT, DTU, 17 Mar. 1993.
  Shilnikov and hyperchaos

- Lev Mikheev, Nordita — NBI, 17 Mar. 1993.
  Classical mechanics of phase transitions in layered magnets (brand new equations to play with)
- Hans-Richard Blank, Erlangen, Germany — NBI, 31 Mar. 1993.
  Measurement and evaluation of MEG spectra (MEG = magneto encephalogram)
- Preben Alstrøm, NBI — HCØ, 31 Mar. 1993.
  Vortex patterns in complex Ginzburg-Landau equations
- Per Dahlqvist, KTH, Stockholm — NBI, 5 Apr. 1993.
  Intermittency in zeta functions
- Thomas Schreiber, NBI — NBI, 14 Apr. 1993.
  Sandpiles and diffusion-limited reactions (on a paper by Joachim Krug)
- Carsten Knudsen, DTU — NBI, 21 Apr. 1993.
  Chaos on dense subsets of chaotic systems or Chaos without nonperiodicity (applies to Axiom A systems)
- Sid Nagel, University of Chicago — NBI, 26 Apr. 1993.
  Relaxation at the angle of repose: Flow and sound in sand
- Hans Henrik Rugh, ENS Lyon — NBI, 5 May, 1993.
  On the reality of the spectrum for Perron Frobenius operators
- Freddy Christiansen, INO Florence — NBI, 8 May, 1993.
  A symbolic dynamics for the Hamiltonian standard map
- Henrik Bruus, Yale — NBI, 19 May, 1993.
  Bifurcations in quantum chaotic dots
- Gabor Vattay, NBI — NBI, 27 May, 1993.
  Criticality in the Kohonen neural map
- Jens Chr. Larsen, Tomas Guhr, and Gabor Vattay, NBI — NBI, 25 May, 1993.
  Present situation in the nonlinear theory of everything
- Kurt Wiesenfeld, Georgia Tech. — NBI, 14 Jun. 1993.
  Noisy Josephson junction arrays
- Ronnie Mainieri, Los Alamos — NBI, 17 Jun. 1993.
  An exact solution to Kaufman's rugged landscape model
- V.A. Shneidman, Augsburg University — NBI, 23 Jun. 1993.
  Transient kinetics of nucleation
- Morten Colding Jørgensen, Panum Institute — NBI, 8 Jul. 1993.
  Is the brain a chemical computer?
- Viviane Baladi, Mathematics Department, ENS Lyon — NBI, 14 Jul. 1993.
  Generalized kneading matrices and zeta functions for 1-d dynamics
- M. Feigelman, Landau Institute, Moscow — NBI, 12 Aug. 1993.
  Vortices in high Tc-superconductors
- Joachim Krug, KFA, Jülich — NBI, 13 Aug. 1993.
  Multiscaling in growth processes
- Jarmo Hietarinta, Univ. of Turku — NBI, 18 Aug. 1993.
  Chaos in the one-dimensional gravitational three-body problem

- Stefanella Boatto, U. of Chicago — NBI, 1 Sep. 1993.
  Point vortices and particle advection
- Dimitris Stassinopoulos, Brookhaven — NBI, 7 Sep. 1993.
  Performing networks
- J. Maher, Pittsburgh University — HCØ, 10 Sep. 1993.
  Rough surfaces formed by sedimentation
- Kim Sneppen, NBI — NBI, 14 Sep. 1993.
  Olami-Procaccia-Zeitak-Leschhorn-Tang solution of the Sneppen model
- Mark Oxborrow, NBI — NBI, 14 Sep. 1993.
  Atoms, are they?
- Claudine Conrado, NBI — NBI, 15 Sep. 1993.
  Growth shapes and turbulent spots in unstable systems
- Anders Johansen, Nice — NBI, 28 Sep. 1993.
  Forest fire models
- Jean-Luc Ottinger, Brussels — NBI, 6 Oct. 1993.
  Scalar model for plasma turbulence
- Mark Oxborrow, NBI — NBI, 11 Oct. 1993.
  Why do quasi-crystals exist ?
- E.-J. Ding, Academia Sinica, Beijing — NBI, 13 Oct. 1993.
  Analytic treatment of a stick-slip model
- Hans Fogedby, Inst. of Physics and Astronomy, Univ. of Aarhus — NBI, 3 Nov. 1993.
  On the phase space approach to complexity
- Vakhtang Putkaradze, NBI — NBI, 3 Nov. 1993.
  Relaxing fast-linear oscillations
- Jonas Nycander, KTH Stockholm — NBI, 11 Nov. 1993.
  Existence and stability of stationary vortices in a uniform shear flow
- Miroslava Smrčinová, Dept. of Physics, STU, Bratislava — HCØ, 11 Nov. 1993.
  Different kinds of behavior of a photosynthetic system during illumination
- Kim Sneppen, Princeton Univ. — NBI, 16 Nov. 1993.
  Regeon theory for SOC
- Claude Ghidaglia, ENS Paris — NBI, 13 Dec. 1993.
  Hydrodynamics phenomena in deep bed filtration
- Achim Richter, Darmstad — NBIfAFG Fællescolloquium, 15 Dec. 1993.
  Microwave resonant cavities
- Achim Richter, Darmstad — NBI, 16 Dec. 1993.
  Ideas about studying resonances in crystals

6 RESEARCH PROJECTS

External collaborators are indicated by the parenthesis. The reference numbers indicate publications associated with a given project, listed in the publications list at the end of this report.

6.1 Classical and Quantum Chaos

A Fredholm Determinant for Semi-classical Quantization

We investigate a new type of approximation to quantum determinants, the "Quantum Fredholm Determinant", and test numerically the conjecture that for Axiom A hyperbolic flows such determinants have a larger domain of analyticity and better convergence than the quantum Selberg products derived from the Gutzwiller trace formula. The conjecture is supported by numerical investigations of the 3-disk repeller, a normal-form model of a flow, and a model 2-d map [**?**, **?**].

P. Cvitanović, P.E. Rosenqvist, G. Vattay, and H.H. Rugh

Entire Fredholm Determinants for Evaluation of Semi-classical and Thermodynamical Spectra

Proofs that Fredholm determinants of transfer operators for hyperbolic flows are entire can be extended to a large new class of multiplicative evolution operators. We construct such operators both for the Gutzwiller semi-classical quantum mechanics and for classical thermodynamic formalism, and introduce a new functional determinant which is expected to be entire for Axiom A flows, and whose zeros coincide with the zeros of the Gutzwiler-Voros zeta function. [**?**].

P. Cvitanović and G. Vattay

Creeping Orbits in Semi-classical Quantization

We try to incorporate the diffraction phenomena in the semi-classical description of classically chaotic systems. This leads us to introduction of periodic creeping orbits which are created by diffraction and thus have no classical particle trajectory analogue. We are studying their effects in simple 2-disk and 3-disk scattering systems, and are comparing our results to exact quantum mechanical results.

P. Cvitanović, P.E. Rosenqvist, G. Vattay, and (A. Wirzba)

Trace Formula for Dynamic Systems in the Presence of Noise

Evolution operator methods for evaluation of averages in chaotical dynamical systems require arbitrarily fine symbolic dynamics partition of the phase space; at the same time, the attainable accuracy in physical dynamical systems is always limited by external noise. We use external noise as a parameter which defines the smallest phase-space partitions that need to be taken into account in a given computation, and generate finite Markov diagram approximation to the dynamics.

P. Cvitanović and (R. Mainieri)

Advection of Vector Fields by Chaotic Flows

We have introduced a new transfer operator for chaotic flows whose leading eigenvalue yields the dynamo rate of the fast kinematic dynamo, and applied cycle expansion of the Fredholm determinant of the new operator to evaluation of its spectrum. The theory has been tested on a normal form model of the vector advecting dynamical flow. If the model is a simple map with constant time between two iterations, the dynamo rate is the same as the escape rate of scalar quantities. However, a spread in Poincaré section return times lifts the degeneracy of the vector and scalar advection rates, and leads to dynamo rates that dominate over the scalar advection rates. For sufficiently large time spreads we have even found repellers for which the magnetic field grows exponentially, in contrast to the scalar densities which decay exponentially [**?**].

(N.J. Balmforth), P. Cvitanović, (G.R. Ierley), (E.A. Spiegel), and G. Vattay

Periodic Orbit Expansions for Power Spectra of Chaotic Systems

We develop the periodic orbit theory of power spectra for chaotic dynamical systems. The theory is tested numerically on several 1-dimensional mappings.

P. Cvitanović, (M.J. Feigenbaum), and (A.S. Pikovsky)

Symbolic Dynamics Description of Chaotic Systems

We study a number of different chaotic systems using the theory of symbolic dynamics. We show how symbolic dynamics can be used to describe n-modal one-dimensional maps and two-dimensional maps of the Hénon type. Different billiard systems, such as the three disk system, hyperbola billiard, stadium billiard and the wedge billiard are described and we show how to construct a well ordered symbolic plane and find pruning fronts for these systems. These results will be applied to describe semiclassically quantum systems.

P. Cvitanović and K.T. Hansen

Sound Waves in Crystals

The experimental study of eigenmodes of microwave cavities has received considerable credit in recent years. For an essentially two dimensional cavity, the corresponding electrodynamical wave equation coincides mathematically with the quantum mechanical Schrödinger equation. The cavities have to be cooled to a temperature of about 2K to achieve a high resolution in the spectra. It is investigated whether a comparably high resolution can be achieved by exciting the eigenmodes of a crystal with sound waves. The study of three dimensional geometries should also be possible.

C. Ellegaard, T. Guhr, and M. Oxborrow

Correlation Holes in Microwave Spectra

In order to achieve a high resolution in microwave cavity spectra, it is necessary to cool the cavity down to a temperature of about 2K. The correlation hole technique, frequently used in molecular physics, decouples to some extent the statistical influences of the widths

and the positions of the resonances. We study if this method works for cavity spectra as well, to become less dependent on high resolution and cooling.

T. Guhr, (R. Hofferbert), and (A. Richter)

Fourier Bessel Analysis in Graded Matrix Spaces

The generating function of the correlation functions in theories involving random matrices can often be mapped onto theories, that show a so called hopping coupling and that are formulated in spaces of graded matrices. To evaluate the integrals an expansion is employed similar to the well known plane wave expansion. The angular degrees of freedom appear in generalized spherical harmonics and can thus be integrated by using results from representation theory.

T. Guhr

Mixing of a Fixed Matrix to a Gaussian Unitary Ensemble

A given matrix is, with a strength parameter, mixed to a Gaussian Unitary Ensemble. The correlation functions for arbitrary level number are evaluated by using the graded eigenvalue method. The results involve the graded symmetric functions which are related to the representations of supergroups.

T. Guhr

Thermoelectricity in Bimetallic Film

In 1821, Thomas Seebeck discovered that an D.C. electromotive force can be generated by heating a junction between two different conductors. This phenomenon, the so-called Seebeck effect, is reversible; it can be used to convert thermal energy into electrical energy with Carnot efficiency. Because irreversible phenomena (Joule heating and thermal conduction) inevitably accompany the Seebeck effect, however, no device based on the Seebeck effect, that can generate a *sizeable* amount of electricity at a *reasonable* efficiency, has ever been built. In 1993, I am attempting to generate an A.C.thermoelectric current, within a sputtered Al-Ni bilayer that lies on a piezoelectric PVDF substrate, by applying an ultrasonic (2MHz) wave normal to the substrate's surface. So far, I have generated only heat.

M. Oxborrow

Embedded Gaussian Unitary Ensemble

The Embedded Gaussian Ensembles model the forces in physical systems more realistically than the plain Gaussian Ensembles. The graded eigenvalue method and the graded Fourier Bessel Analysis are applied in order to study the Embedded Gaussian Unitary Ensemble.

T. Guhr

Four Spheres Scattering Problem

The scattering of a particle in a geometry of three discs in two dimensions was studied classically and quantum mechanically by several authors. Particularly, it was one of the first problems investigated with the periodic orbit theory. We study the analogous

problem in three dimensions, the tetrahedron of four spheres. We expect new information about the dependence of (semi)classical methods on the dimensionality. The model is also more realistic for applications in condensed matter physics.

(A. Wirzba) and T. Guhr

Periodic Orbits of Nonscaling Hamiltonian Systems from Quantum Mechanics.

The spectral formfactor is utilized to define a quantum equivalent of the classical (E, τ)-plot of the periodic orbits of a Hamiltonian system. For scaling systems where the classical dynamics is independent of the energy (except for a trivial scaling) the quantal (E, τ)-plot may be reduced to the Fourier transform of the energy spectrum. For nonscaling systems on the other hand, the quantal (E, τ)-plot reveals the richness of the classical dynamics with its bifurcations of periodic orbits. The analysis may be extended into the regime where the diagonal form of the semiclassical formfactor is not expected to hold and where the connection to Random Matrix Theories is largely unknown.

(M. Baranger), (M. Haggarty), B. Lauritzen, (D. Meredith), and (D. Provost)

Bound States in Twisting Tubes

It is known that an infinite tube with constant cross-section has propagating modes if the frequency is above a certain cutoff frequency. What is not so widely known is that if the tube is bent there can also be localized states around the bend. These states exist as solutions of both Schrodinger's equation and also of Maxwell's equations when the tube is very narrow in one dimension so as to be quasi two-dimensional. This contrasts with the corresponding classical motion which is not localized. I am interested in understanding the connection between the classical and quantum behaviour by using semiclassical approximations to the quantum Green function. Hopefully this project will result in predictions for microwave experiments.

N.D. Whelan

Classical Transport in Near-Integrable Chaotic Systems

The KAM theorem of classical mechanics asserts that if an integrable Hamiltonian is subject to a small perturbation then only a small fraction of the tori are destroyed. However, this theorem has certain smoothness conditions for the perturbation. These conditions are violated for a number of systems of common interest such as the Bunimovich stadium, the Sinai billiard and the wedge in a constant gravitational field. Because of this violation these three problems share the property of being integrable for one choice of parameter and completely chaotic for arbitrarily close parameter values. This non-KAM behaviour is not very well understood. I am interested in understanding the nature of the classical transport of these chaotic systems when their hamiltonians are close to the integrable limit. Results obtained so far indicate that a trajectory remains trapped for a long time in a small region of phase space before making a sudden jump to a different region. It would be good to understand the mechanism for this behaviour and the extent to which understanding the nearby integrable Hamiltonian helps in understanding the chaotic Hamiltonian. In the longer term, it would also be interesting to see how this is reflected in the corresponding quantum dynamics.

N.D. Whelan

Semiclassical Analysis of a Hyperbolic System

Much is now understood about the connection between classical periodic orbits and the quantum Green function. We want to use this understanding to study an apparently simple problem. We consider free motion between confocal hyperbolae. This problem is integrable both classically and quantum mechanically. The quantum eigenvalue problem can be solved in terms of Mathieu functions and the classical problem has two independent integrals of motion in involution. What makes this problem interesting is that there is only one classical periodic orbit, which is unstable. This should make the connection between the quantum and classical problems particularly clear. We hope to be able to describe scattering resonances in terms of the one unstable periodic orbit.

G. Vattay and N.D. Whelan

Determination of the Noise Level of Chaotic Time Series

We analyze analytically the influence of gaussian measurement noise on the shape of the estimated corelation integral of an otherwise low dimensional signal. If the data is embedded in a space of higher dimension than strictly required to reconstruct the dynamics, the extra dimensions are dominated by the noise which results in a certain shape of the correlation integral. For the case that only Gaussian noise is present, this shape can be calculated analytically as a function of the noise level. This leads to a method to determine the noise level present in the data. Furthermore, the analytical result shows that a noise level of more than 2% will obscure any possible scaling of the correlation integral and thus makes it impossible to estimate the correlation dimension.

T. Schreiber

Tunnelling in Multidimensional Integrable Systems

We examine tunnelling from a point of view that makes it possible to treat intrinsically multidimensional systems. The semiclassical theory of tunnelling in 1 dimension is very well understood, but much less so in more dimensions. The 1-dimensional theory is interpreted in terms of the construction of wavefunctions from complex Lagrangian manifolds in complexified phase space. With this interpretation it is possible to see how to generalize the calculation to more dimensions in cases where a quantum state can be identified with a Lagrangian manifold. The most obvious case is that of classically integrable systems where the quantum states are derived from invariant tori. We illustrate the theory by calculating the splittings in a 2-dimensional double well potential for which the Hamiltonian is integrable, but nonseparable. Because the system is nonseparable, it is not possible to reduce it to 1-dimensional problems — which has been the only way to solve tunnelling in integrable systems before.

S. Creagh

Anomalous Tunnelling Splittings in Near-integrable Systems

We compute the tunnelling rates in a near-integrable system under the assumption that the states can be associated with a complex Lagrangian manifold. The energy levels,

without tunnelling corrections, have been well predicted by applying EBK quantization to KAM tori. The tunnelling splittings are computed using the technique previously used for exactly integrable systems. The splittings so obtained are found to be significantly smaller that the actual splittings. This indicates that the mechanism for tunnelling in even near-integrable systems is significantly different from that in exactly integrable systems. This is in line with previous calculations that have shown that the introduction of nonintegrability significantly enhances tunnelling rates.

S. Creagh

Long Time Behaviour in Quantum Maps

I have been using quantum maps to investigate the long-time behaviour of semiclassical approximations. The recent advances of reorganization of the trace formula into Riemmann-zeta-function-like determinants has the potential problem that they require pushing the semiclassical approximations to times that are very large on classical scales, and it is not obvious that the approximations are always valid for such long times. I investigated the validity of the formalism by testing it for quantizations of a perturbed Arnol'd cat map. For sufficiently small perturbations, the map is hyperbolic and semiclassical approximations hold well for long times. It is found that the zeta-function formulation gives accurate results in this case, including an explicit and simple realization of the 'resurgence' predicted by Berry, (*Les Houches Lecture Series 52* (North-Holland, 1990)). As the perturbation is increased, nonhyperbolic structures appear and the twists and folds associated with these should make the semiclassical behaviour much worse. It is indeed found that the zeta function deteriorates after the transition to nonhyperbolic behaviour.

S. Creagh

Chaos in Periodically Driven Gunn Diodes

We have applied numerical simulation to explore the highly nonlinear dynamic phenomena that can arise in Gunn diodes by interaction between the internally generated domain mode and an external microwave signal. By adjusting the time of domain formation and the speed of propagation, the internal oscillation entrains with the external signal. This produces a devil's staircase of frequency-locked solutions. At higher microwave amplitudes, period-doubling and other forms of mode-converting bifurcations can be seen. In this interval the diode also exhibits spatio-temporal chaos. At still higher microwave amplitudes, transitions to delayed, quenched and limited space-charge accumulation modes take place.

R. Feldberg, C. Knudsen, E. Mosekilde, and J.S. Thomsen

Bifurcation Analyses for a Forced Chemical Oscillator

We have performed a detailed numerical bifurcation analysis of the forced Brusselator model, exposing local and global bifurcation curves that constitute the internal structure of the dominant Arnol'd tongues. The results are presented as phase diagrams and one-parameter bifurcation diagrams. Two theorems concerning the existence of global bifurcations near generic codimension-2 bifurcation points are stated and proved. It is argued that the results are generic to a class of periodically forced self-oscillating systems.

C. Knudsen, (J. Sturis), and J. S. Thomsen

Duffing's equation for a forced nonlinear oscillator

In collaboration with Y. Ueda, University of Kyoto, we have investigated the solutions to Duffing's equation for a forced nonlinear oscillator. This equation, which in many ways is prototypical to modern nonlinear dynamics, is known to exhibit a wealth of complex phenomena, including deterministic chaos, coexisting regular and chaotic solutions, fractal basin boundaries, cascades of period-doubling bifurcations, as well as blue sky catastrophes. We have determined the phase diagram for forcing amplitudes exceeding those hitherto studied. New regions of chaotic behavior have been found. The basins of attraction for two coexisting chaotic attractors have been determined, and the chaotic solutions have been characterized in terms of their Lyapunov exponents and fractal dimensions. We have also studied an extension to Duffing's equation to three variables with external forcing. Starting from a phase-space conserving chaos, three prototypes of chaotic attractors with a dimension larger than three have been obtained.

E. Mosekilde and J.S. Thomsen

Phase Locking in Long Josephson Diodes

Phase-locking of fluxon motion to an external signal has been extensively studied using a simple particle model that captures many of the characteristics observed experimentally in long Josephson junctions. When the influence of the external ac drive manifests itself only through the boundary conditions, and when there is exactly one fluxon present in the junction at all times, the simple model reduces the perturbed sine-Gordon equation into an implicit two-dimensional map. Hence, it is possible to obtain analytical expressions for the existence of fixed points and their stability. When the fluxon is in a phase-locked state, it has been observed that the stability of the fixed point changes under parameter variations. This can result in period-doubling bifurcations and chaotic oscillations in the soliton propagation.

We have investigated the effect of thermal noise on the phase-locked state and shown that the noise measurements can be used to detect internal bifurcations in the system. This is particularly relevant to the prediction of the line width of the emitted signal from a phase-locked Josephson junction, since the line width increases with the variance of the time of flight for the fluxon.

The project has also led to the development of a numerical scheme for solving a nonlinear partial integro-differential equation with nonlocal time dependence. In particular we have considered the dynamics of long Josephson diodes modeled by use of the microscopic theory for tunneling between superconductors. It is demonstrated that the detailed behavior of a solitonic mode is different from the results of the conventional sine-Gordon equation.

M. Samuelsen, (N. Grønbech-Jensen), and (Y.S. Kivskar)

Deterministic Approach to Die Tossing

Usually, the motion of a die is so rapid that the human eye is unable to follow the trajectory in detail. Combined with the actual complexity of the motion, which appears to have defied mathematical treatment for centuries, this may have contributed to the widespread notion that a die toss is random. However, by virtue of the dissipation of energy, a die must always come to rest within a finite time. Therefore, a die toss cannot

show the sensitive dependence on initial conditions found in chaotic systems or in systems with fractal basin boundaries. Nonetheless, die tossing has certain properties in common with these systems. In particular, the basins of attraction in the space of initial conditions show a certain self-similarity in structure.

In the present project we have investigated the structure of these basins of attraction by means of a recursive transformation involving the construction of flight and collision maps. We have proposed quantitative measures for the average size of the basins of attraction and for the mixing of these basins, and we have applied these measures to determine how the sensitivity to the initial conditions varies with the energy and orientation of the die.

R. Feldberg, C. Knudsen, and E. Mosekilde

Vehicle Dynamics

The dynamical behavior of railway vehicles is controlled by two nonlinear influences arising from the specific form of the wheel and rail profiles and from the dry friction in the contact between wheel and rail, respectively. Because of these nonlinearities, bifurcations, self-sustained oscillations and various forms of deterministic chaos may arise in the wheel/rail interaction. These phenomena can be observed in theoretical investigations and in experiments under laboratory conditions. In practical railway operation the deterministic response tends to be swamped by stochastic phenomena arising from various irregularities. However, the deterministic response may influence the wear of the wheels and the suspense system.

In the present project we have studied a dicone moving on a pair of cylindrical rails. Accounting for nonlinear friction, the equations of motion for this mechanical system consist of a set of nonlinear differential-algebraic equations. We have investigated the nonlinear dynamic behavior arising from these equations in detail with the aid of special in-house software and path-following algorithms.

C. Knudsen, R. Feldberg, (E. Slivsgaard), (H. True), and (M. Rose)

Chaos in Economic Systems

In collaboration with the Sloan School of Management, MIT, we have performed a series of experiments with human decision making behavior in simulated corporate environments. Participants were asked to operate a simplified production-distribution chain to minimize costs. Performance was systematically suboptimal, however, and in many cases the subjects were unable to secure the stable operation of the system. As a result, large-scale oscillations and various forms of highly nonlinear dynamic phenomena were observed.

A model of the applied ordering policy has been proposed. Econometric estimates show that the model is an excellent representation of the actual decisions. With different parameters, computer simulations of the estimated order policy produce a great variety of complex dynamic behaviors. Analyses of the parameter space reveal an extremely complex structure having a fractal boundary between the stable and unstable solutions, and with fingers of periodic solutions penetrating deeply intro regions representing quasiperiodic and chaotic solutions. In certain parts of the parameter space, any neighborhood of a given solution contains a qualitatively different solution. Thus, changes on the margin can produce a completely different system behavior.

Our results provide direct experimental evidence that chaos can be produced by the decision making behavior of real people in simple managerial systems. The consequent implications for the ability of human subjects to cope with complex dynamical systems are explored.

We have also studied how mode-locking and other nonlinear dynamic phenomena arise through the interaction of two capital-producing sectors in a disaggregated economic long-wave model. One sector might represent the construction of buildings and infrastructure capital with long lifetimes while the other represents production of machinery, transport means, etc., with much shorter lifetimes.

(E.R. Larsen), Chr. Haxholdt, Chr. Kampmann, E. Mosekilde, and J.S. Thomsen

Experimental Observation of the Lorentz Group in Nonlinear Circuits

We have measured Berry's geometrical phase of the Lorentz group in a pair of connected driven dissipative electronic oscillators in the vicinity of a period-doubling bifurcation. Surprisingly, under certain conditions, a small signal transforms just as the Lorentz transformation of a spinor, apart from some factors. By measuring along what corresponds to a closed loop on a hyperboloid (the invariant surface of the Lorentz group), we can determine Berry's phase of the Lorentz group.

H. Svensmark and P. Dimon

Recursive definition of global cellular-automata mappings

We have proposed a recursive definition of the global mappings for deterministic cellular automatas. The method is based on a graphical representation of global cellular-automata mappings. The recursive definition defines the change of the global mapping as the size of the automata is increased. A proof of lattice size invariance of the global mappings is derived from an approximation to the global mappings. The recursive definitions are applied to calculate the fractal dimension of the set of reachable states and of the set of fixed points of cellular automata on an infinite lattice.

R. Feldberg, C. Knudsen, and (S. Rasmussen)

6.2 Turbulence

Finite Time Singularities in Interface Dynamics

There is some evidence that certain models of dynamical interfaces have singularities, which develop at a finite time, after which they blow up. We are developing a new technique, based on projection from a space with one more spatial dimension, to check this.

V. Putkaradze, T. Bohr, and (J. Krug)

Advection of Particles and Passive Scalars in Nonlinear Field Theories

We have developed a new method for studying the statistical properties of the turbulent states of non-linear PDE's especially the so-called Kuramoto-Sivashinsky equation and its

generalizations. We seed the flow with fictitious particles (or a whole passively advected field). In this way one can study a wide class of transport phenomena in a setting which is much simpler than real 3d hydrodynamical turbulence, but has many of the same properties. We look at scaling properties (e.g. anomalous diffusion) and qualitative transitions, e.g. between pinned and flowing states in the presence of symmetry breaking terms.

T. Bohr and (A. Pikovsky)

Growth Shapes of Turbulent Spots in Unstable Systems

In a convectively unstable system, a localized disturbance will move "down-stream", while amplifying and spreading. We have studied the asymptotic form of such local disturbances in a very general setting of a PDE in one or 2 dimensions for a scalar field. In one dimension we have found analytically the form of the asymptotic field and predicted a novel type of instability for a generalized Kuramoto-Sivashinsky equation (NBI preprint 93-61). In two dimensions we show how to find the spatial shape of the spreading spots and note the transitions between different shapes and the appearance of non-convex growth forms (C. Conrado: *Localized Disturbances in Chaotic and Unstable Systems.* Ph.D. Thesis, University of Copenhagen 1993).

C. Conrado and T. Bohr

Space-Time Correlations in Turbulent Systems

Turbulent, or space-time chaotic, systems are characterized by a set of Lyapunov exponents of which the number of positive ones are believed to be an extensive quantity (i.e. proportional to the system volume). This implies that distant parts are decorrelated. Often, however, the systems can have long range correlations e.g. like the scale invariant structure functions of fully developed fluid turbulence or interface models. The chaotic fluctuations should show the lack of correlations and we are trying to figure out the best ways of showing this for various systems. Further we are studying the influence of conservation laws, e.g. on the Lyapunov spectra.

T. Bohr, (G. Grinstein), C. Jayaprakash, and (W. van de Water)

Predictability in Turbulence

We study the growth of small perturbations in fully developed turbulence. By using approximations to the Navier-Stokes equations, so-called shell models, we have shown that the predictability time in strongly turbulent flows depends crucially on the strength of the intermittency of the energy dissipation at the small scales. A shell model is local in k-space and therefore non-local in real space as is the Navier-Stokes equations due to the pressure term. To check the generality our results we must therefore formulate models in real space which include non-local interactions. One may for instance study simple systems of coupled maps which contain non-local couplings. Preliminary results indicate good agreement with our results obtained using the shell models and we plan to pursue this question in more details.

M.H. Jensen (A. Crisanti), (G. Paladin), and (A. Vulpiani)

Experimental Study of the Circular Hydraulic Jump

When a vertical jet of fluid is directed upon a horizontal surface, it spreads out radially and it is observed that at a certain radius the height of the fluid increases abruptly. This is known as the circular hydraulic jump. It is an interesting example of a nontrivial structure appearing on a fluid surface which, on average, remains fixed and shows a transition to turbulence. The height fluctuations of the surface have been studied using the absorption of a light beam. The resulting power spectra show interesting structures which may be understood in terms of classical orbits.

S. Hansen, S. Hørlück, D. Zauner, P. Dimon, C. Ellegaard, T. Sams, and S.C. Creagh

Shell Models for 2-d Turbulence

We investigate 2-d turbulence using shell models in k-space. A key point is to observe whether there are two scaling laws present for the energy spectrum, namely an inverse cascade with Kolmogorov scaling -5/3 and a forward cascade of scaling -3 below and above the point of energy input, respectively. All shell models, however, exhibit an inverse cascade with scaling -1, in contradiction to simple arguments based on dimensional analysis. We consider whether this discrepancy is due to fundamental reasons, and might thus be found in 2-d Navier-Stokes, or is due to the limitations of the shell models.

M.H. Jensen and (A. Brandenburg)

Experimental and Theoretical Studies of Turbulence in Soap Films and Pipe Flows

Soap films have attracted much interest because of their two-dimensional nature. Our laboratory soap film flows last up to 1 hour, with the flow velocity determined by the viscosity and the height difference between the two containers between which the soap film is spanned, and measured by He-Ne laser-Doppler anemometry. Turbulence is produced at sufficiently high velocities. The theoretical problem is to describe the flow and the boundary air layer.

Velocity fluctuations in a turbulent state We have recently developed equipment that with good precision measure velocity changes $\Delta v(\ell)$ over short space scales ℓ. Traditionally, the Taylor hypothesis is invoked to replace $\Delta v(\ell)$ by $\Delta v(\tau == \ell/U)$. An important question is to test the extent to which this assumption is justified in studying the scaling properties for turbulence. We have therefore initiated theoretical as well as experimental research on a pipe flow to resolve what role the Taylor hypothesis plays in extracting the scaling properties for the velocity fluctuations in a three-dimensional turbulent flow behind a grid. For this experiment, back-scatter Argon-Ion laser-Doppler anemometry is used to measure the velocity fluctuations.

P. Alstrøm, Mogens T. Levinsen and J. Zhang

Experimental and Theoretical Studies of Pattern Formation and Turbulence in Capillary Waves

Quasicrystals were observed in solid-state physics in 1984. Eight years later, we observed the first quasicrystal pattern in a fluid, a discovery that went into widely read journals like

Nature, Science, and Discovery. The quasicrystal pattern was made out of four standing capillary waves, formed on a water or alcohol surface undergoing vertical oscillations at high frequencies. At sufficient forcing capillary waves are formed with a frequency-dependent wavelength; we can attain about 50 wavelengths across the cell.

Besides the quasicrystal pattern, square and hexagonal patterns are observed. patterns are formed. We investigate experimentally how the patterns change when changing the viscosity of the fluid. Theoretically, the instability by which the surface waves are formed is an important issue in nonlinear hydrodynamics. A good theoretical project is to develop the nonlinear theory further.

At higher drive, the flow becomes turbulent. In this regime we consider diffusion and relative diffusion of particles placed on the surface. This is done using image processing and particle tracking programs. For comparison, simulations on particle diffusion in chaotic fields are carried out.

P. Alstrøm, M.T. Levinsen, J. Sparre Andersen and (W. Goldburg)

6.3 Fractals, critical phenomena

Dynamic Phases in a Spring-block System

When a block is pulled via a spring across a surface, there appear to be several different dynamic phases. These can be characterized by the pulling velocity v_0 and a dynamic velocity $v_D = g\sqrt{m/k}$ where g is the acceleration of gravity, m is the mass of the block, and k is the spring constant. For sufficiently small v_0, the block displays stick-slip (relaxation) motion. Then as a function of decreasing v_D, this stick-slip motion is first nearly periodic, then aperiodic with approximately an exponential slip size distribution, then aperiodic with possibly a power-law slip size distribution. When v_0 is increased, the block eventually ceases to stick and just slides across the surface. The motion can then be adequately described by a Langevin model.

A. Johansen, P. Dimon, C. Ellegaard, J.S. Larsen, and H.H. Rugh

Studies of Granular Flow

When small beads flow through a narrow channel, there will be fluctuations in the flow rate around the mean. We are studying the nature of the these fluctuations as a function of the various parameters which we can control, such as the ratio of the bead size to the channel width, and the tilt of the channel.

C. Veje and P. Dimon

Study of Fragmentation

The measured mass distributions of fragments from various objects show evidence of obeying a scaling law. The scaling exponent is found to depend on the dimensionality of the object but otherwise not on its morphology or composition. A cube, for example, has the same exponent as a sphere.

(L. Oddershede), (J. Bohr), and P. Dimon

Colored Activity in Self Organized Critical Interface Dynamics

We study roughening interfaces that become self organized critical by a rule similar to that of invasion percolation. We demonstrate that there is a fundamental difference between transient and critical dynamical exponents. The exponents break the Galilean invariance and temporal multiscaling is observed. We show that the activity along the interface exhibits non trivial power law correlations in both space and time even though only quenched Gaussian noise is applied. The results are compared with simulations where spatial power law correlated noise are used as input.

K. Sneppen and M.H. Jensen

Multidiffusion in Critical Dynamics of Strings and Membranes

We study dynamical roughening of strings which do not break the symmetry neither parallel nor vertical to the overall suspension. The suggested non local dynamics enforces a build up of long range correlations and the system becomes self organized critical with a new universality class which exhibits nontrivial temporal scalings and power law correlated activity along the string, even though only localized Gaussian noise is applied. We observe multiscaling of the temporal behaviour both perpendicular and parallel (multidiffusion) to the evolving string at criticality.

K. Sneppen and M.H. Jensen

Random Fractals, Phase Transitions, and Negative Dimension Spectra

We introduce an exactly soluble model of a random fractal where the corresponding dimension spectrum $f(\alpha)$, as a consequence of finite size effects in ensemble averaging, exhibits negative values of f corresponding to the strongest singularities of the probability measure, i.e. at the left part where $\alpha \approx 0$. The right part of the spectrum, which corresponds to the regular part of the measure, is not well defined as the scaling length $\ell \to 0$. These two effects are related to the fact that the generalized dimensions $\tau(q)$ exhibit: 1) a first order phase transition at $q = 1$ and 2) a non-existing thermodynamic limit for $q < 0$. We show that the scaling properties of the model can be understood as the thermodynamic limit of $f(\alpha) \ln \ell$ versus α. The connections to fractal aggregates is briefly discussed.

M.H. Jensen, (G. Paladin), (A. Vulpiani), and (W. v.d. Water)

Punctuated Equilibrium and the Critical Edge of an Evolving Ecosystem

We study a dynamic model for an evolving ecology of interacting species. The species are constrained by barriers in an underlying fitness landscape that parametrize the reproduction stability of each species and its potential mutants in the environment. A central element of the model is a variability in the couplings between species, which form a metric of the ecosystem that is self organized and subject to dynamic changes. This is obtained by a fine graining of the ecosystem into possible dependencies between species. In the limit of very rarely mutating species, the ecology expands its genetic space always keeping the system on the edge of generic scale invariance. Furthermore, an ecosystem governed by local punctuations within individual species morphology, will during its evolution show collective behaviour in terms of global punctuations, involving many simultaneous local

punctuations. The expanding ecology increases the diversity of life which in turn opens for an enhanced complexity of the individual species.

K. Sneppen, M.H. Jensen, (P. Bak), and (H. Flyvbjerg)

Inflation Rule for Canonical-cell Tilings

Despite the fact that quasicrystals were first discovered a decade ago, the atomic structures for all but a handful of them have yet to be determined. In other words, physicists have been measuring the properties of a class of solid-state material for ten years, but they still do not know what it is, exactly, they have been measuring the properties of. This must set some sort of record. I am presently trying to formulate the geometrical framework of an atomic model for icosahedral quasicrystals. This endeavour involves the search for an inflation rule, whereby quasiperiodic "canonical-cell" tilings can be generated. Such tilings, once decorated with atoms, specify the atomic structures of model quasicrystals; these structures should accurately approximate the structures of real quasicrystals. I have not yet found any such inflation rule.

M. Oxborrow

Self-Organized Critical Dynamics of Fronts: Intermittency and Multiscaling.

We present recent work on punctuated dynamics of fronts. This kind of dynamics drives the evolving front to a critical state with temporal multiscaling of the interface profile due intermittent activity characterized by avalanches which connect regions of recent local activity. This activity pattern exhibits non trivial power law correlations in both space and time. We present numerical results in both 1+1 and 2+1 dimensions and generalize a theory for the critical exponents to all dimensions.

(J. Falk), K. Sneppen, and M.H. Jensen

Punctuated Equilibrium and Criticality in a Simple Model of Evolution

A simple and robust model of biological evolution of an ecology of interacting species is introduced. The model self-organizes into a critical steady state with intermittent co-evolutionary avalanches of all sizes, i. e. it exhibits "punctuated equilibrium" behavior. This collaborative evolution is much faster than non-cooperative scenarios since no large and coordinated, and hence prohibitively unlikely, mutations are involved.

K. Sneppen and (P. Bak)

Mean Field Theory for a Simple Model of Evolution

A simple dynamical model for Darwinian evolution on its slowest time-scale is analyzed. Its mean-field theory is formulated and solved. A random neighbor version of the model is simulated, as is a one-dimensional version. In one dimension, the dynamics can be described in terms of a "repetitious random walker" and anomalous diffusion with exponent 0.4. In all cases the model self-organizes to a robust critical attractor.

K. Sneppen, (H. Flyvbjerg), and (P. Bak)

Self-Organization in Simple Models of the Brain

One of the most important goals in neuroscience is to understand the way the brain manage to undertake a diversity of tasks. The processing units are the neurons and their connections. On the neural level the underlying network dynamics is not task specific, and a realistic attempt to explain the brains ability to function must take this into account. In sharp contrast, the traditional artificial neural networks are heavily constrained, and the a priori knowledge of the task is crucial. Based on this, the system is trained to construct an appropriate input-output function, which is then used in different but similar situations. The brain is not hard-wired at birth, with all its connections coded into the DNA. Evolution is efficient, but not that efficient. The amount of information contained in DNA is vastly insufficient to specify all neural connections. The structure has to be self-organized rather than by design. Thus to understand the brain, we must understand the principles by which is organizes itself through interactions with the environment.

Recently, we have introduced a new class of adaptive networks that by construction are able to function in an changeable environment. Their performance is intimately connected with the adaptive nature, where information flows through a variety of ever changing paths. We denote this adaptive performance to emphasize that the dynamics cannot be divided into a learning mode and a retrieving mode as known from traditional artificial neural networks. Studies on the performing networks can follow one of two directions. Either a study on the relations to physiology and phycology, or a study on industrial applications. On the medical side, the point is to extract some important principles for brain functioning associated with the proposed network dynamics. This include a study of the chemistry and the 'electronics' involved when a neuron fires. On the network level, we would like to understand the processes of conditioning, and the behaviors like habituation and sensitization. Moreover, there is a growing interest in the variety of timescales present in recorded EEG signals. On the industrial side, there are a multitude of possible candidates in the literature, from control and robotics to analysis and diagnosis. Initial tests are carried out on computer, but electronic 'neurons' have been build for further testing.

P. Alstrøm, (P. Bak) and (D. Stassinopoulos)

Intermittent Dynamics and Self Organized Depinning in Propagating Fronts

We study the roughening dynamics of a 2 dimensional front where advances are made at minimal pinning sites, while slopes of the front are kept finite by additional lateral growth. The interface self organizes toward a critical state with long range correlations in space and time. The dynamics is governed by intermittent burst which give rise to a scale invariant avalanche distribution and multiscaling of the temporal roughening. Generalizing a recently proposed theory for the 1-d case, we demonstrate that the multiscaling can be explained by the static roughness exponent alone. The correlation between subsequent deposition activities exhibits the same power law as in the one dimensional case. The implication of such dimensional independence is discussed.

(J. Falk), M.H. Jensen, and K. Sneppen

Critical Punctuations and Metastability for Evolving Strings, Interfaces

We study the dynamics of interfaces that become self organized critical by a rule similar to that of invasion percolation. In the critical state we observe temporal multiscaling of the

interface profile and an activity pattern that exhibits non trivial power law correlations in both space and time even though only quenched Gaussian noise is applied. One way to characterize this intermittent activity is through avalanches consistent of connected regions of recent local activity. These avalanches display scale invariance, thus reflecting both small and large scale properties of the dynamics. An extension of the presented concepts to evolving strings leads to a new universality class with other scalings. Possible systems governed by the investigated algorithms are discussed.

K. Sneppen and M.H. Jensen

Scaling Behaviour in Daily Air Humidity Fluctuations

We show that the daily average air humidity fluctuations exhibit non-trivial $1/f^{\alpha}$ behaviour which is different from the spectral properties of other meteorological quantities. This feature and the fractal spatial structure found in clouds make it plausible to regard air humidity fluctuations as a manifestation of self-organized criticality. We give arguments why the dynamics of air humidity can be similar to dynamics of sandpile models of SOC.

G. Vattay and A. Harnos

6.4 Biology and Physiology

Chaos in Physiological Systems

Experimental investigations performed of The Department of Medicine, University of Chicago, have shown the existence of internally generated pulsations in human insulin production and release with a typical period of 100-120 min.

To understand the mechanisms underlying these pulses, we have developed a model of the glucose-insulin regulatory system in collaboration with K. Polonsky and J. Sturis, University of Chicago. By adding to the standard picture of this regulation mechanisms which allow for the distribution of insulin between the plasma and the intercellular spaces and for the delayed release of glucose from the liver, the model becomes unstable and generates self-sustained oscillations with a typical period of 2h.

By forcing the insulin release by means of a periodically varying intravenous glucose infusion, frequency-locking and other highly nonlinear dynamic phenomena have been experimentally observed.

R. Feldberg, E. Mosekilde, C. Knudsen, (J. Sturis), and J.S. Thomsen

Hyperchaos in Microbiological Population Dynamics

We have simulated a variety of different growth, competition and selection processes that may arise in interacting populations of bacteria and phages. Our model considers a culture containing several variants of the same bacterium, each sensitive to attacks from a specific phage. The culture grows in a chemostat with continuous supply of nutrients. Surplus bacteria and phages are removed through dilution. Depending of the rate of dilution, the model exhibits stable equilibrium, self-sustained oscillations, quasiperiodic behavior, deterministic chaos or hyperchaos. This last form of dynamics is characterized by two

or more positive Lyapunov exponents. It is likely that this type of behavior can arise in many biological systems when more than a few species are involved.

(G. Baier), (J. Engelbrecht), E. Mosekilde, and J.S. Thomsen

Chaos and hyperchaos in ecological models

We have studied the dynamics of a simple model of an aquatic ecosystem with four different species (bacteria, algae, zooplankton and fish) and three different dead pools (detritus and accessible nutrients in the water column and nutrients bound to the sediment). Depending on the characteristics of the species and on the inlet concentration of nutrients, various types of solutions from simple periodic oscillations to hyperchaos were found.

M. Barfred, M. Liberoth, and E. Mosekilde

Bifurcations in a Vocal Fold Model

An autonomous fourth-order model of vibrations of the vocal folds has been proposed. Each of the vocal folds is modeled by an upper and a lower mass. The aerodynamic forces are derived from a modified Bernoulli equation. In a wide parameter region the dynamics of the model exhibits many features of normal phonation. Near the borders of this parameter region coexistence of limit cycles, bifurcations, and chaos are observed. The implications for an understanding of pathological voices are addressed.

(H. Herzel) and C. Knudsen

Enhancement/Inhibition of HIV Infection

Antibodies specific for epitopes on HIV glycoprotein gp120 or gp41 can inhibit or enhance HIV infection of human cells in vito. These effects may have significant implications both for the pathogenesis of chronic HIV infection and for vaccine development. A particular puzzling finding in vito is antibody dependent enhancement at low concentrations of antibodies while high concentrations of the same antibody inhibit infection. Antibodies can inhibit infection by several mechanisms. However, by binding to receptors on target cells, virus bound antibodies can also enhance adhesion to these cells and thereby facilitate infection. We have developed a mathematical model which can account for these phenomena in detail.

(J. Hansen), (J.-E.S. Hansen), O. Lund, E. Mosekilde, and (J.O. Nielsen)

Glycosylation and Protein Conformation

Peptide linked carbohydrate affect such physical properties of glycoproteins as solubility, stability and conformation. Biologically, glycosylation modulates immunogenecity, tissue distribution and life time of glycoproteins. Increasing evidence indicates that surface carbohydrates play a major role in recognition, development, oncogenesis and viral infection. It is therefore of interest to investigate the relation between O- or N-linked oligosaccharides and glycoprotein conformation. We have applied artificial neural networks to stochastically analyse the local amino acid distribution and conformation at glycosylation sites in glycoproteins as inferred from the data bases.

(H. Clausen), (J.E. Hansen), (J.-E.S. Hansen), O. Lund, E. Mosekilde, (J.O. Nielsen), and (K. Rapacki)

Computerized Image Comparison

In the calculation of bone connectivity a comparison between two closely made cuts in bone tissue is performed. This comparison is normally carried out by copying the two cuts on two transparencies, and the alignment of the two sections is made by hand. However, inaccuracies in the copy process and distortion from the cutting of the bone samples often imply that the images will not match well over their whole area. We have developed a method of applying techniques from non-linear dynamics to computerize the processes of alignment and comparing two bone sections.

E. Mosekilde, (Li. Mosekilde), and J.S. Thomsen

Simulation of Multiple Remodeling Events in Human Cancellons Bone

Imbalances in the remodeling process have significant influence on the rate and severity of the microstructural alterations in the trabecular network and consequently on the strength of bone at the organ level. These alterations include changes in the remodeling frequency and changes in bone balance at the level of the remodeling unit due to altered resorption cell or osteoblastic cell activity. We have developed a model with published remodeling data from different treatment regimens and applied it to simulate multiple remodeling events and to predict the long time effects of various treatments on bone mass, trabecular thickness, and perforations.

E. Mosekilde, (Li. Mosekilde), and J.S. Thomsen

6.5 Chemical Reaction-Diffusion Systems

Quenching measurements of complex chemical dynamics

The research group has developed the theoretical and experimental basis of a method of analysing oscillatory chemical reactions (the quenching method). Quenching measurements for complex chemical systems in a CSTR operated close to a Hopf bifurcation is an effective tool for determining the essential species and elementary reactions from a comprehensive kinetic mechanism for the system. A reduced set of species and reactions obtained in this way may be a good starting point for a detailed understanding of bifurcations and chaotic behavior, as the operating conditions for a Hopf bifurcation is often not far from the operating conditions for chaotic oscillations. Quenching results on the Belousov-Zhabotinsky (BZ) reaction at a Hopf bifurcation have been compared with several established kinetic models. Qualitative or semiquantitative agreement has been obtained, but in a few cases considerable disagreements between the experiments and the predictions from existing models show the need for substantial changes of the models. We cooperate with R. J. Field from Montana on these problems.

A supercritical Hopf bifurcation has been discovered in the Briggs-Rauscher reaction, and quenching parameters have been measured. An interesting paradox unexplainable by the established models has turned up.

A model for the methylene blue-sulfide reaction has been investigated by numerical methods and an operating point on the thermodynamic branch corresponding to a supercritical Hopf bifurcation has been discovered. This behavior has not been confirmed experi-

mentally, but for very large flow rates with influx of hydrogenperoxide nearly harmonic oscillations have recently been observed suggesting the presence of a supercritical Hopf bifurcation.

F. Hynne, P. Graae Sørensen, V. Vukojević, K. Nielsen, M. Ipsen.

Analytic methods for systematic optimization of models of oscillatory reactions.

Analytical methods of assessing and optimizing networks of oscillatory chemical reactions are being developed. They exploit the close relationship that exists between quenching data of a system and the kinetics of its reactions. Two methods are currently being implemented and applied. Both can be used directly without integrating the kinetic equations.

A graphical method is based on an eigenvalue property of quenching data. For a given reaction network, it provides necessary conditions for the consistency of models with the experimental data. The other method depends on a unique property of a Hopf bifurcation: it uses an explicit representation of stationary states in terms of 'extreme currents'. It provides a way to systematically search all possible Hopf bifurcations of a network in an optimization, regardless of rate constants and stationary concentrations.

F. Hynne, P. Graae Sørensen, T. Møller.

Chaos in oscillatory chemical reactions.

The geometry of fully developed Feigenbaum chaos in a chemical system is best studied and described by determination of the (infinite) set of unstable limit cycles and the associated manifolds. Chemical chaos is extremely difficult to handle experimentally, and much work on it has been made under conditions that are not sufficiently well controlled, either from a chemical or from a dynamical point of view. Even early stages of the period-doubling sequence present a considerable challenge. Several types of chaos, in particular Feigenbaum chaos and homoclinic chaos, are being investigated experimentally in systems driven by periodic perturbations.

P. Graae Sørensen, K. Nielsen, F. Hynne.

Turing structures: Self-organization through bifurcations in biochemical control systems with diffusion

Spontaneous creation of stable concentration patterns (morphogenetic fields) is possible in autocatalytic biochemical control systems. Such patterns may emerge from the homogeneous state if the activity of certain enzyme passes over a critical threshold or simply if the size of the embryo grows over a critical value. The underlying mathematics describing the system shows profound analogies to models of transition to turbulence in fluid dynamics. The patterns arise through bifurcation in the nonlinear partial differential equation describing the autocatalytic reaction-diffusion system. At a NATO meeting in Brussels Dec 2-5, 1991 G. Nicolis pointed out that such Turing systems may be easier to analyze than hydrodynamic systems and recommended close cooperation in the two fields. H. Swinney (Austin, Texas), who is an expert on the transition to turbulence, presented experiments on chemically realized Turing structures. In collaboration with P.

Graae Sørensen we have established how Turing systems may adapt to size in growing embryos and how the effective diffusion constants in biosystems may vary sufficiently. This work has been used by the Bordeaux group in its recent discovery of experimentally realized Turing structures in a gel reactor. In the last decade we have pioneered the simulation of Turing structures in 3 curvilinear coordinates which make direct comparisons to biological systems possible. A number of otherwise unexplainable patterns have been interpreted by us as Turing patterns, and a possible link between such patterns and cell division (mitosis and cytokinesis) and segmentation in *Drosophila* has been suggested. Currently a model with 16 interacting species is used to simulate a combined model of Turing morphogens, gap- and primary pair-rule genes in 3D elliptical cylinder coordinates with a fast supercomputer code ($>$ 1300 MFLOPS on CRAY C92). This code is used by the national computer center to benchmark current supercomputers.

A. Hunding, [T. Lacalli (Saskatchewan), J. Boissonade (Bordeaux)]

Hopf bifurcation in biochemical systems.

Enzymatic reactions with oscillatory properties has been investigated by traditional kinetic methods for many years. We expect that quenching measurements may contribute to new theoretical understanding of this very important class of systems. Experiments with such systems require great experience in biochemical technique, and we are planning a collaboration with H.G.Busse at the University of Kiel on investigations of glycolysis.

K. Nielsen, F. Hynne, P. Graae Sørensen, [H. G. Busse (Kiel)].

Reinforced dynamics.

This study concerns relaxation oscillators, coupled by a reinforced dynamics, where the coupling strength between two neighboring oscillators increases when firing at one leads to firing at the other. We consider both the case, where the system has a constant drive, and the case, where the drive is a function of the outcome. Our goal is (i) to explore the patterns formed, and their stability, when the system is driven; (ii) to study the systems ability to perform well-defined tasks.

D. Stassinopoulos, P. Alstrøm.

Turing Structures in the CIMA Reaction

Using the Chlorite-Iodide-Malonic Acid (CIMA) reaction, in 1990 de Kepper and his group in Bordeaux finally succeeded in attaining stationary spatial structures. We have studied the one- and two-dimensional bifurcation diagrams of a model of the CIMA reaction. Because of multistability and pinning effects a great variety of localized structures may be produced, such as for instance, islands of hexagons surrounded by homogeneous steady state. We have determined the speed at which fronts between such structures propagate, and we have shown how one- and two-dimensional spirals with Turing induced cores can arise.

(P. Borckmans), (G. Dewel), O. Jensen, E. Mosekilde, and V.O. Pannbacker

7 STAFF ACTIVITIES

Members of the CATS staff have in 1993 delivered following seminars and conference talks, as well as participated in professional activities or received professional recognition:

Predrag Cvitanović:
Danish Physical Society Research Prize in Physics 1993.
Holds appointment as Adjunct Professor, Rockefeller University.
Member of the Royal Danish Academy of Sciences and Letters.
Editor of Cambridge University Nonlinear Science Series, and Nonlinearity journal.
Danish coordinator of the E.U. Nonlinear Science Network.
Member of advisory committees of the Dynamics Days and a variety of other international schools and conferences.

- Rockefeller University Theoretical Physics Seminar, New York, 25 Jan 1993.
- Stochastic Processes in Astrophysics workshop, University of Florida, Gainsville, 7 Feb 1993.
- Kaos i Fysik og Matematik, Ungd. Natur. Foren., Copenhagen 4 Mar 1993.
- Kvantekaos, Kgl. Danske Vid. Selsk., Copenhagen, 29 Apr 1993.
- Periodic Orbit Theory, Danish Physical Society Spring Meeting, Rødby, Denmark, 17 May 1993.
- Danish Physical Society Research Prize in Physics 1993, lecture by the recipient, Rødby, Denmark, 18 May 1993.
- Dynamics Days '93, Poznań, 11 June 1992.
- FSU-USA Conference on Chaos, Woods Hole, Mass., USA, 19–23 July 1993.
- Kinematic Fast Dynamo, WHOI seminar, Woods Hole, Mass., USA, 1 Aug 1993.
- Nordic/EC Complex school, HumlebæHumlebæk, Denmark, 21–28 Aug 1993.
- Rapporteur plenary talk, NATO "Chaos, Order and Patterns" conference, Villa Olmo, Como, Italy, 5-10 Sep 1993.
- 1st Int. Conference on "Complex Systems in Computational Physics", Buenos Aires, 18-22 Oct 1993.
- Buenos Aires University Physics Colloquium, 28 Oct 1993.
- Periodic Orbit Theory workshop, Bad Honeff, 2–4 Nov 1993.
- Georgetown University Physics Seminar, 9 Nov 1993.
- University of Maryland Dynamics Seminar, College Park, 10 Nov 1993.
- Univ. of Texas Mathematical Physics Seminar, Austin, 17 Nov 1993.
- Univ. of Texas Nonlinear Physics Seminar, Austin, 18 Nov 1993.
- Univ. of Houston Physics Seminar, 22 Nov 1993.
- City University Graduate School, Sullivan Seminar, Mathematics Department, New York, 14 Nov 1993.
- Rutgers Meeting on Statistical Physics (invited talk), 15 Dec 1993.

E. Mosekilde

- Chaos and Hyperchaos in an Experimental Economic System, Vth Rencontre de Blois: Chaos and Complexity - From Nonlinear Physics to the Shapes of Living Systems, Blois, France, June 21-26, 1993.
- Chaotic Hierarchy in a Model of Competing Populations, 1993 European Conf. on Artificial Life, Brussels, May 24-26, 1993.
- Behavior of the Solution to Duffing's Equation for Large Forcing Amplitudes, 1st European Nonlinear Oscillations Conference, Hamburg, Germany, Aug. 18-20, 1993.
- Chaos in Economic Production-Distribution Systems, Int. Summerschool on Nonlinear Dynamics, Univ. of Ljubljana, Slovenia, Sept. 26 - Oct. 4, 1993.
- Chaos and Nonlinear Dynamic Phenomena in Physiological Control Systems, Int. Summerschool on Nonlinear Dynamics, Univ. of Ljubljana, Slovenia, Sept. 26 - Oct. 4, 1993.
- Iterative Map Approach to Die Tossing, Int. Summerschool on Nonlinear Dynamics, Univ. of Ljubljana, Slovenia, Sept. 26 - Oct. 4, 1993.
- Turing Structures in Reaction-Diffusion Systems, Int. Summerschool on Nonlinear Dynamics, Univ. of Ljubljana, Slovenia, Sept. 26 - Oct. 4, 1993.
- Chaotic Hierarchies in Living Systems, Dept. of Physics and Mathematics, Univ. of Chicago, April 20, 1993.
- Chaos and Hyperchaos in Economic Systems, Dept. of Economics, Univ. of Wisconsin-Madison, April 26, 1993.
- Chaotic Hierarchies in Living Systems, Dept. of Chemical Engineering, Univ. of Virginia, Charlottesville, April 29, 1993.
- Chaotic Hierarchies in Living Systems, IBM Thomas J. Watson Research Center, Yorktown Heights, New York, April 30, 1993.
- Chaos in Human Decision Making Behavior, Swedish Defence Research Establishment, Linköping, June 16, 1993.

M. Samuelsen

- Coupling between two long Josephson junctions, Dept. of Theoretical Physics, Univ. of Salerno, Oct. 25, 1993.

C. Knudsen

- Models of Hyperchaotic Flows Euromech 308: Chaos and noise in Dynamical Systems, Spala, Poland, Sept. 20-25, 1993.

J.S. Thomsen

- A Stochastic Model of Vertebral Trabecular Bone Remodeling, 1st Copenhagen Symp. on Computer Simulation in Biology, Ecology and Medicine, Copenhagen, Aug. 23-25, 1993.
- A Stochastic Model of Vertebral Trabecular Bone Remodeling, 15th Annual Meeting of the American Society for Bone and Mineral Research, Tampa, Florida, Sept. 16-20, 1993.

O. Lund

- Enhancement and Inhibition of HIV Infection of Monocytes by Antibodies against HIV, 3rd Int. Symp. on Retroviruses in Multiple Sclerosis and Related Diseases, Copenhagen, Denmark, May 26-28, 1993.
- Enhancement and Inhibition of HIV Infection of Monocytes by Antibodies against HIV, Workshop on Theoretical Immunology, Centre International de Rencontre Mathematique, Marseille, France, April 14-17, 1993.
- Dynamic Interaction between HIV and other Sexually Transmitted Diseases, 1st Copenhagen Symp. on Computer Simulation in Biology, Ecology and Medicine, Copenhagen, Aug. 23-25, 1993.
- Complement-Antibody Mediated Enhancement of HIV Infection, Int. Solway Institutes of Physics and Chemistry, Nov. 15, 1993.

Preben Alstrøm
Research Associate at the Center for Polymer Studies, Boston University.

- Dynamical Systems, Poland, Sept. 20-25, 1993.

Tomas Bohr

- Dynamical Systems, Poland, Sept. 20-25, 1993.

Clive Ellegaard

- Dynamical Systems, Poland, Sept. 20-25, 1993.

Mogens Høgh Jensen
1993 Ørsted prize of the Royal Danish Academy of Sciences and Letters.
Nordita faculty member.
Director, EU-Nordic Summer School, HumlebæHumlebæk, August 1993.
Visiting scientist, University of Rome "La Sapienza"

- Dynamical Systems, Poland, Sept. 20-25, 1993.

Mogens Tweddell Levinsen

- Dynamical Systems, Poland, Sept. 20-25, 1993.

Preben Graae Sørensen

- From Excitability to Oscillations - a case study in spatially extended systems, (invited lecture) Nice June 2-5, 1993.

A. Hunding

- Bifurcations in 3 dimensional curvilinear coordinates. Prepatterns for mitosis, cytokinesis and segmentation. (invited lectures), Bordeaux, 30 aug. - 5 sept, 1993

- Simulation of 3 Dimensional Turing Patterns Related to Early Biological Morphogenesis, ECAL '93: Self organization and life: from simple rules to global complexity, Centre for Nonlinear Phenomena and Complex Systems, Bruxelles May 24-26, 1993.

8 Publications

Listed here are the CATS staff articles published in 1993. In addition (and not listed here), associated with most of the projects listed in section **??** are preprints and articles submitted but not published in 1993.

8.1 Refereed publications

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