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A COMPARISON OF THE MATHEMATICAL UNDERSTANDINGS OF PROSPECTIVE ELEMENTARY TEACHERS IN COLLEGES HAVING DIFFERENT MATHEMATICS REQUIREMENTS

by
Melvin Cleo Withnell

A dissertation submitted in partial fulfillment
of the requirements for the degree of
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Doctoral Committee:

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ABSTRACT

A COMPARISON OF THE MATHEMATICAL UNDERSTANDINGS OF PROSPECTIVE ELEMENTARY TEACHERS IN COLLEGES HAVING DIFFERENT MATHEMATICS REQUIREMENTS

By Melvin C. Withnell

The general problem of this study was to determine whether colleges with mathematics requirements of three, six, or nine semester hours are satisfactorily preparing prospective elementary teachers for today's elementary curriculum. The related question, what should be a minimum number of required hours of mathematics for the satisfactory preparation of prospective elementary teachers, was considered. Relationships among individual differences in ability, attitude, high school and college mathematics backgrounds, teaching preference, and mathematical achievement were also studied.

Procedure

Prospective elementary teachers at thirteen colleges were tested during the last three weeks of their last required mathematics courses. The testing was completed during May and June, 1967. The schools were classified as large universities, large colleges, and small colleges. For each classification, schools were included requiring three, six, or nine semester hours of mathematics for elementary education students. The sample consisted of 2513 students.

The test instrument consisted of three parts. Part I was a short questionnaire used to gather information about individual differences; Part II was a short ability test which provided for control of ability differences among students and colleges and served as a basis for classification by abilities in the statistical analyses; and Part III was a test of mathematics concepts classified as arithmetic, algebraic, or geometric. The mathematics concepts were also classified as basic (concepts taught in K-6) and related (not basic).

The main statistical comparisons were based on analyses of covariance of individual scores on the five subtests and the total test. The treatment groups consisted of students who had completed three, six, or nine semester hours of mathematics requirements. Analyses of covariance were completed for students classified by type of college, by ability level, and for the entire sample. Relationships among individual differences and achievement were analyzed by stepwise regression analyses.

Findings

1. Prospective elementary teachers' understanding of mathematical concepts was considered low. The highest mean was attained by high ability students completing their third required mathematics course; however, this mean represented only a 58% correct response to the twenty-four basic questions.

2. There were no significant differences among group means for students from large universities for arithmetic, geometric, and basic subtests and the total test. Likewise, there were no significant differences in arithmetic and total test means for high ability groups. For all other classifications significant differences occurred among the group means for all subtests and the total test.

3. Students completing a second required course attained the highest algebra mean; students completing a third required course attained the highest geometry mean.

4. The best predictors of achievement were ability, attitude, and high school and college mathematics backgrounds. High school mathematics background was an especially important factor in algebra and geometry achievements.

Conclusions

1. Prospective elementary teachers are not being satisfactorily prepared by a three, six, or nine semester hour mathematics requirement.

2. The number of courses needed for satisfactory preparation of prospec-elementary teachers may be far beyond current recommendations.

3. Most students make gains in arithmetic understanding corresponding to increased mathematics requirements; algebraic and geometric concepts are best understood while they are being studied.

4. Mathematics courses for elementary teachers are aimed at low and middle ability students.

5. Widespread differences in abilities and mathematical backgrounds exist among students in mathematics courses for elementary teachers.

6. Different problems occur in the preparation of prospective elementary teachers at different types of colleges due to ability differences.

Recommendations

1. Mathematics courses for elementary teachers should include a more intuitive approach to basic concepts, especially in first courses.

2. The content of mathematics courses for elementary teachers should be related to the elementary school.

3. The areas of arithmetic, algebra, and geometry should be integrated and the spiral curriculum idea applied.

ACKNOWLEDGMENTS

I wish to express my sincere appreciation to the chairman of my doctoral committee, Mr. Arthur F. Coxford, Jr., for his advice, encouragement, and time with respect to this study. I also wish to thank the other committee members for their constructive criticism during the design of this project and especially to Mr. M. Clemens Johnson for his aid in the statistical analysis of the data.

I express my deepest gratitude to my wife, LeeAnn, for her patience, encouragement, and moral support during my graduate study.

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CHAPTER I

INTRODUCTION AND RESEARCH DESIGN

The revolution in mathematics education was initially concerned with the content of secondary school mathematics. As the revolution continued, it grew not only in the number of persons involved but also in its scope. Experimentation and change in the elementary curriculum soon became an important and extensive part of the research.

The sweeping changes in the content of the elementary and secondary mathematics curriculums produced the need for corresponding changes in the mathematics education of prospective elementary and secondary teachers as well as the need for inservice training of those teachers already in the field. Much inservice work has been accomplished by National Science Foundation institutes, by institutes supported by industry, by mathematics coordinators working with local teachers, by workshops, and by teachers returning to college to study "modern" mathematics.

The problem of preparing prospective elementary and secondary mathematics teachers has been left mostly to the colleges. It can be argued that the problem of preparing prospective secondary mathematics teachers for the new curriculums has had a simpler solution than the problem

of preparing prospective elementary teachers. This follows from the fact that most secondary mathematics teachers acquire a college major or minor in mathematics; therefore, the problem with them is not one of requiring a certain number of mathematics courses but one of including the proper types of background courses in their majors and minors. Since the elementary teacher is usually expected to teach a self-contained classroom and thus must have a knowledge of many academic areas, she usually is only required to have a limited background in mathematics. In fact, according to a study by Fisher[1] the average number of required hours of mathematics for elementary education students in 1960 was 1.97 semester hours and in 1965 was 4.15 semester hours. Even though there has been an increase in the average number of required hours in mathematics, the number is less than that recommended by various authorities.

The most authoritative current recommendation concerning mathematics requirements is that of the Committee on the Undergraduate Program in Mathematics (CUPM) of the Mathematical Association of America.[2] The CUPM recommendation

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[1]John J. Fisher, "Extent of Implementation of CUPM Level I Recommendations," The Arithmetic Teacher, XIV (March, 1967), pp. 194-97

[2]The Committee on the Undergraduate Program in Mathematics, "Recommendations of the Mathematical Association of American for the Training of Teachers of Mathematics," American Mathematical Monthly, LXVII (December, 1960), pp. 982-991.

specifies that twelve semester hours of mathematics be taken by all prospective elementary teachers. During the twelve semester hours, the students are to study concepts of number systems, algebra, and geometry. Few colleges have adopted a twelve semester hour mathematics requirement. In fact the colleges have various requirements for the prospective teachers' preparation ranging from three to twelve semester hours.

The general problem of this study is to determine whether the colleges with different mathematics requirements for prospective elementary teachers are satisfactorily preparing teachers for today's elementary curriculum. Since the elementary curriculum involves concepts not only from arithmetic but also from algebra and geometry, it is necessary for the elementary teachers to understand concepts from these areas. Thus, the prospective elementary teachers' understandings of arithmetic, algebra, and geometry are investigated in this study to determine whether or not their preparation is satisfactory. In this study, satisfactory preparation is arbitrarily defined as the attainment of a mathematical understanding which is measured by correct responses to 75% or more of the basic questions in the test instrument. (See Appendix A for identification of basic questions.) A related question is also investigated, namely, what should a minimum number of required hours of mathematics be for the satisfactory preparation of prospective elementary teachers for

today's elementary curriculum. Finally, other concerns related to the major questions are investigated. These include a study of the individual differences due to different high school mathematics backgrounds, the number of college mathematics courses completed, the completion of a methods course, ability, attitude, and preferred level of teaching.

VALUE OF THE STUDY

The results of this study should be useful to educators in at least the following three ways. First the study includes an overall measure of the effectiveness of the various present day college mathematics requirements. In particular, the results indicate whether or not arithmetic, algebraic, and geometric concepts, found in the elementary school curriculum are being satisfactorily understood by the prospective teachers. Certainly, algebraic and geometric concepts have become an important part of the elementary school mathematics curriculum during the past few years, and the teachers need to know more than just arithmetic.

Secondly, the study should be helpful to colleges considering a change in their mathematics requirements in elementary education programs because it will provide them with research evidence concerning the minimum credit hour needs in order to provide an adequate background for their students. Since there has been only a limited

movement of the colleges toward the adoption of the CUPM recommendation of a twelve semester hour requirement it seems that many college education and/or mathematics departments do not believe in the need for a twelve semester hour requirement. Research evidence is essential to either endorse or refute these beliefs.

Thirdly, relationships among individual differences due to ability, attitude, high school and college mathematics backgrounds, completion of an elementary mathematics methods course, and the student's teaching level preference, were analyzed. The problem of providing for individual differences is present in every classroom. Thus, research considering individual differences of prospective elementary teachers should have value for the instructors of the college elementary mathematics courses. Since the consideration of individual differences was not the main concern of this study, a solution to the general problem was not expected nor found. However, the individual differences, mentioned above, were considered in the analysis of results and recommendations based on this analysis are given.

Research Design

General Design

The measurement of the understanding of mathematics concepts and the collection of other pertinent data was accomplished by the use of a test and questionnaire. The

instrument was given to the students enrolled in their last required mathematics course during the last three weeks of the quarter or semester.

Instrumentation

The instrument used (see Appendix A) consists of three parts. Part I, a short questionnaire, was employed for the collection of personal data concerning: high school mathematics background, college mathematics background, completion of an elementary mathematics methods course, attitude, teaching level preference, and class standing. Part II consists of a Short Ability Test copyrighted by Educational Testing Service from whom permission was obtained for its use in this study. The Short Ability Test was originally designed for college bound high school seniors and was modified by the author for college students. All questions, other than three, five, seven, eight, and twelve, are from the original test. Part III is the Mathematics Concepts Test which includes forty multiple choice questions. These questions are classified in two different ways (see Appendix A). Twenty questions are concerned with arithmetic concepts, ten questions are concerned with algebraic concepts, and ten questions are concerned with geometric concepts. The number of questions in each classification was chosen to conform to the 2:1:1 ratio which is recommended by CUPM as the number of courses in each of these areas, respectively.

The forty questions of the Mathematics Concepts Test are also classified as basic or related. The twenty-four questions classified as basic involve concepts actually taught in the elementary grades, K-6. The sixteen related questions involve concepts not actually taught in the elementary school but considered as background or extensions of basic concepts. The basic questions were determined after reviewing the concepts included in five contemporary series of K-6 mathematics text books.[1] The related questions involve concepts recommended in the CUPM study guide[2] of courses for elementary teachers.

The test instrument is designed to be completed in a fifty minute class period. Most students can complete it in this time; however, a maximum of fifty-five minutes was allowed.

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[1]Edwina Deans et al., The Modern Mathematics Series Grades 1 through 6 (New York: American Book Company, 1963).

Greater Cleveland Mathematics Program Grades 4 through 6 (Chicago: Science Research Associates, 1964-1965).

Robert L. Morton et al., Modern Mathematics Through Discovery Grades 3 through 6 (Morristown, New Jersey: Silver Burdett Company, 1964-66).

Joseph N. Payne et al., Elementary Mathematics Grades 3 through 6 (New York: Harcourt, Brace, and World, 1966).

Henry Van Engen et al., Seeing Through Arithmetic Grades 1 through 6 (Chicago: Scott, Forsman and Company, 1964).

[2]Committee on the Undergraduate Program in Mathematics, Course Guides for the Training of Teachers of Elementary School Mathematics (Berkeley: Committee on the Undergraduate Program in Mathematics, Fourth Draft, 1964).

Sample

The sample consisted of students from colleges categorized as follows:

A. large university

B. large college (predominantly involved in teacher education)

C. small teacher college.

For each of the above three categories, at least one school was included which required the following number of semester hours in mathematics for the elementary education program:

A. 3 semster hours

B. 6 semester hours

C. 8 or 9 semester hours (12 quarter hours was considered equivalent to 8 semester hours).

Because it was necessary to obtain schools for each of these classifications, it was not possible to use random sampling procedures. However, schools were included from six states - - Indiana, Michigan, New Mexico, New York, South Dakota, and Wisconsin. Their enrollment ranged from 1000 to 35,000 students. An alphabetical listing of the schools follows:

1. Ball State University, Muncie, Indiana.
2. Central Michigan University, Mt. Pleasant Michigan
3. General Beadle State College, Madison, South Dakota.

4. Michigan State University, East Lansing, Michigan.
5. Northern Michigan University, Marquette, Michigan.
6. Northern State College, Aberdeen, South Dakota.
7. State University College at Brockport, Brockport, New York.
8. State University College at Buffalo, Buffalo, New York.
9. University of Michigan, Ann Arbor, Michigan.
10. University of New Mexico, Albuquerque, New Mexico.
11. Wayne State University, Detroit, Michigan.
12. Western New Mexico University, Silver City, New Mexico.
13. Wisconsin State University at Plattville, Plattville, Wisconsin.

Even though the sample was not random, it did consist of schools having enrollments over a wide range and located in different geographic areas. Therefore, the sample was representative of schools other than liberal arts colleges which are preparing elementary teachers.

The testing was completed during the last three weeks of the semester or quarter which ended in May or June, 1967. The tests were administered by the instructor of the class. The testing was completed at every school which had been selected for the sample. The cooperation of the individuals and institutions involved was outstanding.

Statistical Analysis and Hypotheses

The main statistical comparisons were based on analysis

of covariance, using the ability scores determined by the Short Ability Test and the number of semesters of high school mathematics completed as the initial variables. The major question asked whether or not there was a significant difference in understanding of mathematical concepts by prospective elementary teachers as the college mathematics requirements varied from three to nine semester hours.

The numerous null hypotheses related to the above question may best be stated by use of variables as follows:

> There is no significant difference among groups 1, 2, and 3 for students of type x as to mathematical understanding of type y mathematical concepts.
>
> Groups 1, 2, and 3 represent students having completed 3, 6, and 8 or 9 semester hours of mathematical requirements respectively.
>
> x = large universities, large colleges, small teacher colleges, the total sample, high ability, middle ability, or low ability.
>
> y = arithmetic, algebraic, geometric, basic (taught in elementary school), related (not basic), or the total test.

Other statistical analyses included were a stepwise regression analysis and an intercorrelation analysis. The stepwise regression analysis was used on the entire sample in order to find the best predictors of achievement on the Mathematics Concepts Test from among the variables

of ability, attitude, high school and college backgrounds in mathematics, completion of a methods course, and teaching level preference. The intercorrelation analysis was used to find relationships among the various qualitative variables as well as the quantitatively measured variables of ability and achievement. The regression and intercorrelation analyses were useful in the study of individual differences of prospective elementary teachers.

Also, an item analysis was completed on the Short Ability Test for one-tenth of the sample (250 tests). Likewise, item analyses were completed on the Mathematics Concepts Test for certain representative subsets of the sample. In any study involving testing, the test instrument is always open to criticism. The item analyses were made to provide statistical information as a basis for criticism of the test. This information includes the mean, standard deviation, range, frequency distribution, reliability, achievement on each question, and discriminatory power of each question.

The presentation and analysis of results are found in chapter three.

CHAPTER II

RELATED LITERATURE

One of the main factors involved in the attempts to improve elementary mathematics education is the change in the content of the mathematics curriculum. The contemporary textbooks contain much new material, and the subject matter of various research projects and proposals for grades K-6 indicates that further changes can be expected. Closely related to the change in content is the increased emphasis on understanding mathematical concepts rather than the older emphasis on memorization of facts, algorithms, and types of problems. The change in content and emphasis made necessary a stronger mathematical background for elementary teachers.

The mathematical background of the elementary school teacher is, therefore, another major factor involved in the improvement of elementary mathematics education and is the concern of this review of the literature. This review is divided into four sections. The first section includes statements and recommendations about college mathematics requirements for elementary teachers. The second section is a survey of the studies related to the prospective teachers' understandings of mathematical concepts and their

attitudes toward mathematics. The third part is a review of the studies concerned with inservice teachers of elementary school mathematics. In some cases prospective elementary teachers were also included. The fourth part of this chapter includes statements and recommendations concerning miscellaneous problems related to the mathematics education of the elementary teacher.

MATHEMATICS REQUIREMENTS FOR ELEMENTARY TEACHERS

The lack of mathematical background of elementary teachers has often been given as the reason for the ineffective teaching of elementary arithmetic. During World War II there was little doubt as to this lack of background since, according to Ruddell, Dutton, and Reckzeh,[1] many people were teaching on emergency certificates which required little or no college training.

Supporting evidence concerning the lack of mathematical background is supplied by Layton,[2] who reported that the mathematics requirements for elementary certification in 1947 ranged from zero to five semester hours with a mean of .52 semester hours; the average required

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[1] A. K. Ruddell, Wilbur Dutton, and John Reckzeh, "Background Mathematics for Elementary Teachers," Instruction in Arithmetic, Twenty-fifth Yearbook of the National Council of Teachers of Mathematics, (Washington D.C.: National Council of Teachers of Mathematics 1960), pp. 296-319.

[2] W. I. Layton, "The Certification of Teachers of Mathematics," XLII The Mathematics Teacher, (December, 1949) pp. 377-380.

number of semester hours of mathematics methods was .16. The very low means indicate that most prospective elementary teachers had no required mathematics background. Of course, certification requirements are minimums determined by the states. These minimums may be exceeded by the college requirements for graduation. However, in 1951, Layton[1] reported the mean mathematics requirement for eighty-five colleges as only 1.63 semester hours for prospective elementary teachers. This low mean requirement also implies that many teachers were graduating with no required mathematics background.

A recent survey by Fisher[2] of a random sample of 117 colleges and universities out of 822 selected schools indicated a 1960 mean requirement in mathematics of 1.97 semester hours for elementary education students. The 1965 mean requirement in the same schools showed a significant increase to 4.15 semester hours. The change in median was from zero semester hours in 1960 to 4.30 semester hours in 1965.

A comparison of the preceding data indicates only a slight change in mathematics requirements for prospective elementary teachers up to 1960 but a significant increase

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[1]W. I. Layton, "Mathematical Training Prescribed by Teachers Colleges in the Preparation of Elementary Teachers," The Mathematics Teacher, XLIV (December, 1951) pp. 551-556.

[2]John J. Fisher, "Extent of Implementation of CUPM Level I Recommendations," The Arithmetic Teacher, XIV (March, 1967) pp. 194-197.

in recent years. The recent change is further substantiated by a CUPM survey[1] of all colleges in the United States involved in teacher education. The results show that approximately 23% of the colleges required no mathematics for elementary education students in 1962, but only about 8% had no mathematics requirement in 1966. Also, five or more semester hours were required by 32% of the colleges in 1962 and by 50% of the colleges in 1966. However, fifty-eight schools decreased their mathematics requirements for prospective elementary teachers during this time of overall increase in college mathematics requirements. Forty-one of the fifty-eight schools had decreases of three semester hours or more. The reasons for these reversals were not reported; however, such reductions may be an indication that some mathematics educators are not convinced of either the need or the value of twelve or more semester hours of mathematics background for prospective elementary teachers.

The mathematics requirements discussed above do not fulfill the current recommendations of the Committee on the Undergraduate Program in Mathematics (CUPM)[2], a committee

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[1]The Committee on the Undergraduate Program in Mathematics, Eleven Conferences on the Training of Teachers of Elementary School Mathematics, (Berkeley: The Committee on the Undergraduate Program in Mathematics, April, 1966) Appendix p. 143.

[2]The Committee on the Undergraduate Program in Mathematics, "Recommendations of the Mathematical Association of America for the Training of Teachers of Mathematics," LXVII American Mathematical Monthly, (December, 1960) pp. 982-991.

of the Mathematical Association of America. The CUPM recommendation for teachers at the K-6 grade level is twelve semester hours of required college mathematics for those with two years of high school mathematics background. This recommendation has been widely discussed in CUPM sponsored conferences of mathematics educators and certification directors of state departments of public instruction. There has been approval of the CUPM recommendation by these conferences and also by the National Association for the Advancement of Science[1].

Progress toward a twelve hour mathematics requirement seems slow however, and actually today's college requirements in mathematics for prospective elementary teachers approximately fulfill recommendations made fifteen to thirty years ago. For example, in 1939 R. L. Morton[2] proposed six to ten hours of college mathematics for prospective teachers of arithmetic. In the years immediately after World War II recommendations for at least one course in mathematics content for K-6 teachers were made by the

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[1]"Report of the NASDTEC-AAAS Studies," CUPM Report: Ten Conferences on the Training of Teachers of Elementary School Mathematics, No. 9, April, 1964, pp. 103-110.

[2]R. L. Morton, "Mathematics in the Training of Arithmetic Teachers," The Mathematics Teacher, XXXII (March, 1939), pp. 106-110.

Commission on Post-War Plans[1], Grossnickle[2], Newsom[3], Schaaf[4], Wren[5], and others. Actually Grossnickle and Newsom strongly preferred a six semester hour requirement. Also, many suggested that the courses should be designed for the elementary teachers. That is, the recommended coursework was not the usual college algebra, trigonometry, and analytic geometry sequence. The topics suggested for the courses were closely related to the mathematics taught at the elementary level. Many of the recommended topics for such background courses are now standard topics in the "modern" mathematics courses for elementary teachers.

The above summary shows that recommendations are not accepted immediately. One reason for the time lag in accepting recommendations may have been the lack of pertinent

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[1]Commission on Post-War Plans, "Guidance Report of the Commission on Post-War Plans," The Mathematics Teacher, XL (November, 1947), pp. 315-339.

[2]F. E. Grossnickle, "The Training of Teachers of Arithmetic," The Teaching of Arithmetic, Fiftieth Yearbook of the National Society for the Study of Education, Part II (Chicago: University of Chicago Press, 1951), pp. 203-231.

[3]C. V. Newsom, "Mathematical Background Needed by Teachers," The Teaching of Arithmetic, Fiftieth Yearbook of the National Society for the Study of Education, Part II (Chicago: University of Chicago Press, 1951), pp. 232-250.

[4]William L. Schaaf, "Arithmetic for Arithmetic Teachers," School Science and Mathematics, LIII (October, 1953), pp. 537-543.

[5]F. L. Wren, "The Professional Preparation of Teachers of Arithmetic," Arithmetic 1948, Supplementary Educational Monographs, No. 66 (Chicago: University of Chicago Press, October, 1948), pp. 80-90.

research evidence. Since the author's study compares mathematical achievement by prospective elementary teachers from colleges with different mathematics requirements, this study supplies research evidence concerning current recommendations. Therefore, the results of this study should be useful to mathematics educators contemplating changes in mathematics requirements. The study should be helpful in the implementation of current recommendations or in the modification of these recommendations.

Most statements concerning the question of whether or not current recommendations of mathematics requirements for prospective elementary teachers should be implemented seem to be on the affirmative side. Garsten[1] notes that elementary classroom teachers spend 16% of their time teaching mathematics; however, only 2% of their undergraduate course work is in mathematics. Jones[2] says "The roots of most mathematical ideas as well as the development of attitudes toward mathematics are to be found in the elementary schools. Elementary school teachers must be trained both to understand and to enjoy mathematics." He also points out the need for special mathematics courses for prospective elementary teachers, which should be taught by sympathetic

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[1]Helen L. Garstens, "Mathematics and Elementary Education Majors," The Arithmetic Teacher, XI (December, 1964), pp. 540-542.

[2]Phillip S. Jones, "Recent Research in Mathematics: Implications for Teacher Education," American Mathematical Monthly, LXVII (June-July, 1960), pp. 589-590.

but mathematically sound and informed teachers. Stone[1] in his critical review of Goals for School Mathematics, The Report of the Cambridge Conference on School Mathematics says that he believes a better answer than the implementation of the Cambridge Report can be given before 1990. He says, "The real obstacle lies not in our knowledge of mathematics or our knowledge of pedagogy, but in the outmoded and inadequate preparation we are still giving to our future mathematics teachers. It is high time we stop prating about this obstacle and take counsel as to how we can remove it."[2]

These statements and others by leading mathematics educators emphasize the need for more understanding of mathematical concepts by elementary teachers in order to teach the contemporary and future mathematics curricula. It also is assumed by many that requiring more courses in mathematics for elementary teachers will provide this needed understanding. The need for understanding of mathematical concepts by elementary teachers is accepted by "everyone". It is a commonly accepted fact that "you cannot teach what you don't know". However, the question as to whether or not prospective elementary teachers attain a better understanding of mathematical concepts due to increased college

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[1]Marshall H. Stone, "Reviews and Evaluations," The Mathematics Teacher, LVIII (April, 1965) pp. 353-360.

[2]Ibid p. 360.

requirements in mathematics is an unanswered question and is an important aspect of this study. The related studies of the past, which are reviewed in the next two sections of this chapter, do not answer this question specifically but provide support for both possibilities.

STUDIES OF PROSPECTIVE ELEMENTARY TEACHERS' UNDERSTANDING AND ATTITUDES

One of the early studies involving prospective elementary teachers was conducted at the University of Illinois by Phillips[1]. He collected data about the students' high school mathematics background as well as achievement data based on the Hundred-Problem Arithmetic Test and mimeographed tests covering algebra and geometry. The 268 students of the study were enrolled in a Teaching of Arithmetic course. Phillips concluded from his research that the amount of elementary and high school mathematics completed gave little indication of the achievement in meaning, understanding, and mechanical mastery of arithmetic. Competence in algebra was higher than in geometry. He decided that the achievement in meaning and understanding of arithmetic as measured by the tests to be extremely low.

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[1]Clarence Phillips, "Background and Mathematical Achievement of Elementary Education Students in Arithmetic for Teachers," School Science and Mathematics, LIII (January, 1953), pp. 48-52.

Likewise, Shryock[1] administered a mathematics test to prospective elmentary teachers. His test was based on topics from eight college elementary mathematics texts and returns from a questionnaire. The questionnaire was used to obtain reactions about elementary mathematics from experienced elementary teachers. The results of his test showed no significant difference between the sixty-nine prospective teachers who had taken no college mathematics and the one hundred sixteen who studied at least one mathematics course.

Dutton[2], However, found a significant change in fifty-five students' understanding of arithmetic concepts during a one semester mathematics course. He used the forty-six item California Arithmetic Comprehension Test for 6th grade as his measuring instrument. Nevertheless, he concluded that the students clung to traditional methods in explaining many of these topics.

Similarly using a pretest and a posttest on 287 students in sixteen sections of a CUPM type number systems course, Todd[3] found significant changes in mathematical

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[1]Alfred Jerry Shryock, "A Study of Mathematics Background Courses for Elementary School Teachers," Dissertation Abstracts, XXIII (November, 1962), p. 1562.

[2]Wilbur Dutton, "University Students Comprehension of Mathematical Concepts," The Arithmetic Teacher, VIII (February, 1961) pp. 60-64.

[3]Robert M. Todd, "A Mathematics Course for Elementary Teachers: Does it Improve Understanding and Attitude?" The Arithmetic Teacher, XIII (March, 1966), pp. 198-202.

understanding and attitude. He used Glennon's Test of Basic Mathematical Understanding. Todd felt that one course was not sufficient to insure competence in basic mathematical understanding.

Grossnickle[1] studied student achievement in mathematics as measured by standard tests for six groups of elementary education students at Jersey City State College from 1954 to 1959. He compared their scores at the end of their high school work with their scores at completion of their senior year in college. He found significant gains for all groups. All students in the study completed six semester hours of college mathematics and four hours of methods for the teacher of arithmetic. These gains, he says, justify the accepted assumption that elementary teachers benefit in mathematical competency from taking courses in background mathematics and methods courses.

In a comparison of the mathematical facility and understanding of selected prospective elementary teachers from Boston University, the University of Illinois, and the University of Alberta, Nelson and Worth[2] used Phillips' Achievement Test in Elementary Arithmetic and Glennon's

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[1] Foster E. Grossnickle, "Growth in Mathematical Ability Among Prospective Teachers of Arithmetic," The Arithmetic Teacher, IX (May, 1962), pp. 278-279.

[2] L. Doyal Nelson and Walter H. Worth, "Mathematical Competence of Prospective Elementary Teachers in Canada and the United States," The Arithmetic Teacher, VIII (April, 1961), pp. 147-151.

Test of Basic Mathematical Understanding. The comparison showed the Canadian students to be significantly higher in achievement and understanding as measured by the tests; however, most of the Canadian students had completed three years of high school mathematics by grade twelve. Most of the United States students had not completed three years of high school mathematics. The researchers conclusion was that competence in mathematics varies directly with the amount of experience in mathematics.

Another comparison of elementary education students from different colleges was conducted in Minnesota by Curtis[1] in 1954-55. His fifty multiple choice test was designed to measure understandings of mathematical concepts found in elementary textbooks. He also administered the Stanford Advanced Arithmetic Test and the test of Quantitative relationships of the Analytical Scales of Attainment. He concluded that students were more successful in problem solving than in computation. The students preferring to teach in intermediate grades had more high school and college mathematics than those choosing the primary level, and they also had significantly higher scores on all the tests. Also students electing college mathematics had higher mean scores than those with no college mathematics;

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[1]E. L. Curtis." A Study of the Mathematical Backgrounds of Students Who are Preparing to be Elementary Teachers and Who are enrolled in Certain Colleges in Minnesota," Dissertation Abstracts, XVI (November, 1956), p. 2391.

likewise, students with more than two years of high school mathematics did significantly better than those with less than two years.

A related area of study concerning elementary education students involves their attitude toward mathematics. Dutton[1] asked 211 students to bring to class a list of their most favorable and unfavorable attitudes toward arithmetic. He classified 76% of the responses as unfavorable. He found 65% of these unfavorable responses centered around lack of understanding, teaching disassociated from life, pages of word problems, boring drill, poor teaching, lack of interest and poor motivation, and fear of making mistakes.

A study by O'Donnell[2] indicated that students who expressed a liking for mathematics were not necessarily the ones who had taken the most mathematics in high school. He also found that measured attitude showed a low correlation with arithmetic achievement and problem solving. However, only one student scored below the norm on the Remmer Attitude Scale used in the study. Perhaps results would be different for other groups.

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[1]Wilbur H. Dutton, "Attitudes of Prospective Teachers Toward Arithmetic," Elementary School Journal, LII (October, 1951), pp. 84-90.

[2]J. R. O'Donnell, "Levels of Arithmetic Achievement; Attitudes toward Arithmetic, and Problem-Solving Behavior Shown by Prospective Elementary Teachers," Dissertation Abstracts, XIX (December, 1958), p. 1300.

In a survey of 645 elementary student teachers from six California colleges, Groff[1] obtained a surprising result. He had the student teachers rank eleven subject matter areas of the elementary curriculum. The order was their self-judgment of their competence for teaching these subjects. Reading ranked first and arithmetic ranked second. Arithmetic was not expected to rank this high.

All of the above studies have been concerned with the prospective elementary teachers' understandings of mathematics or their attitudes toward mathematics. In most cases the studies involve students from only one college. In the two projects using more than one college there was no attempt to evaluate different mathematical requirements. The conclusions of these two studies did imply that understanding of mathematical concepts increased with the amount of mathematics studies. However, in one case this conclusion was based on the amount of high school mathematics background and in the other case on the amount of elected college mathematics.

The research study being reported in this paper is similar to the studies reported here in that it attempts to measure the mathematical understanding of prospective elementary teachers and relate this understanding to factors such as attitude, high school mathematics background,

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[1]Patrick J. Groff, "Self-Estimate of Ability to Teach Arithmetic," The Arithmetic Teacher, X (December, 1963), pp. 479-480.

etc. However, this study differs significantly from the others in that it compares the mathematical understanding of students who have been required to take different numbers of mathematics courses at various colleges. Hence, this study is a central theme of the measurement of the relative effectiveness of different present day mathematics requirements. The results of this study provide research evidence which should be helpful to educators considering changes in mathematics requirements for elementary education.

The next section includes studies of the inservice teachers' understanding and attitudes related to mathematics. In some of these studies prospective teachers were also included.

STUDIES OF INSERVICE TEACHERS' UNDERSTANDING AND ATTITUDES

The results of studies involving inservice teachers are inconclusive as to the value of teaching experience and mathematics background as related to understanding of mathematical concepts. Studies by Brown[1], Fulkerson[2], and

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[1]Edward Dietz Brown, "Arithmetical Understandings and Attitudes toward Arithmetic of Experienced and Inexperienced Teachers," Dissertation Abstracts, (July-September, 1961) p. 775.

[2]E. Fulkerson, "How well do 158 Prospective Elementary Teachers Know Arithmetic," The Arithmetic Teacher, VII (March, 1960), pp. 141-146, 158.

Glennon[1] showed experienced elementary teachers to have a better understanding of mathematics than prospective elementary teachers. The measure in Brown's and Glennon's studies was Glennon's Test of Mathematical Understanding, whereas Fulkerson used his own forty item test. Fulkerson reported a mean of twenty or 50% on his test. Glennon had a 55% mean for inservice teachers and a 43% mean for prospective teachers. With respect to mathematics background, Brown found that inservice teachers who had completed college mathematics courses demonstrated a better understanding of concepts than those who had taken no college mathematics. Fulkerson's results showed an increase in test performance corresponding to an increase in the amount of high school mathematics background. However, Glennon found no difference in achievement between inservice teachers with a methods course and inservice teachers without such a course.

Another study which showed college mathematics to be of value was the one conducted by Harper[2]. He tested 396 classroom teachers in seventy-four schools in Colorado. He made comparisons by grouping teachers according to the

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[1]Vincent J. Glennon, "A Study in Needed Redirection in the Preparation of Teachers of Arithmetic," The Mathematics Teacher, XLII (December, 1949), pp. 389-396.

[2]E. Harold Harper, "Elementary Teachers' Knowledge of Basic Arithmetic Concepts and Symbols," The Arithmetic Teacher, XI (December, 1964), pp. 543-546.

amount of college mathematics completed, years of teaching experience, teaching grade level, and recency of study. In every comparison involving a group of teachers with six or more hours of college mathematics, the results were significantly in favor of the group with the mathematics background. The test he designed for his study consisted of sixty-one items and the scores ranged from zero to fifty-five. He also found that teachers profited from a "modern" mathematics course.

In contrast to studies showing results favorable to those with college mathematics background are others such as the one by Barnes, Cruickshank, and Foster[1]. They found no significant relation between the number of college mathematics courses completed and the quality of the teachers's instruction. It should be noted, however, that the quality of the teachers' instruction was a subjective ranking by the teachers' principals. Kenney[2], however, administered a fifty item test to 365 elementary teachers and found that those having had recent courses in arithmetic did no better than those without such courses. He also concluded that understanding did not increase with experience.

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[1]Kenneth Barnes, Raymond Cruickshank, and James Foster, "Selected Educational and Experience Factors and Arithmetic Teaching," The Arithmetic Teacher, VII (December, 1960), pp. 418-420, 430.

[2]Russell A. Kenney, "Mathematical Understandings of Elementary School Teachers," The Arithmetic Teacher, XII (October, 1965) pp. 431-442.

Likewise, Carroll's[1] study of 317 students at Wayne State University demonstrated no significant relationships between the mathematical understanding of the students and the number of years of high school mathematics, the number of college mathematics courses studied, or the number of years of teaching experience. The criterion for measuring understanding was a test of seventy-eight items. The scores ranged from thirteen to sixty-three and the mean was forty. Using her own thirty-three item "modern" test, Melsom[2] tested forty-one new teachers in six leading school districts around Philadelphia. Twenty-seven out of the forty-one got more than half of the problems wrong even though most had taken college courses in modern mathematics.

The studies of inservice teachers' understanding of mathematics reviewed in this section are not directly related to the author's study involving prospective elementary teachers. Still the inconclusive results concerning the value of college mathematics training for elementary teachers leads to speculation as to the values of the different mathematics requirements. This inconclusiveness may actually be a difference of opinions among college

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[1]Emma C Carroll, "A Study of the Mathematical Understandings Possessed by Undergraduate Students Majoring in Elementary Education," Dissertation Abstracts, (July-September, 1961), p 494.

[2]Ruth Melson, "How Well are Colleges Preparing Teachers for Modern Mathematics," The Arithmetic Teacher, XII (February, 1965), pp. 51-53.

mathematics instructors and some researchers as to what the elementary teacher should know about mathematics. In considering the concepts for the test used in this study, the author reviewed elementary (K-6) mathematics texts and recommendations of the Committee on the Undergraduate Program in Mathematics[1]. There is no assurance that college mathematics instructors are emphasizing these concepts, however.

Although the studies related to the elementary teacher's attitudes toward mathematics tend to favor more college mathematics training, there is still some unfavorable evidence. Using Glennon's Test of Basic Mathematical Understandings and a questionnaire in his study of 450 classroom teachers, Bean[2] found a positive relation between the teacher's perception of his arithmetic competence and his test score. There was a significant increase in the number who considered themselves below average after taking the test. The mean for the eighty item test was 65.58% which compares favorably with Glennon's results.

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[1]Committee on the Undergraduate Program in Mathematics, Course Guides for the Training of Teachers of Elementary School Mathematics, (Berkeley: Committee on the Undergraduate Program in Mathematics, Fourth Draft 1964).

[2]John E. Bean, "Arithmetical Understandings of Elementary School Teachers," The Elementary School Journal, (May, 1959), pp. 447-450.

In a survey by Reys[1] of 250 recent elementary education graduates of the University of Missouri, 78% of those responding expressed a need for additional mathematics preparation in order to assume the responsibilities of their teaching positions.

Huettig and Newell[2] surveyed 115 inservice elementary teachers concerning attitudes toward modern mathematics. They found that those with more experience had a more negative attitude than those with less experience. They found a significant increase in positive attitude for those with college mathematics training compared to those with no college mathematics. They showed that as the number of courses increased so did the measure of positive attitude. There was no significant difference between the attitudes of primary (1-3) and intermediate (4-6) teachers, however.

Even though the studies reviewed above do not indicate significant differences in all cases for variations in the high school mathematics background, the number of college mathematics and methods courses studied, or the length of teaching experience, still one result does seem to be common

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[1]Robert E. Reys, "Are Elementary School Teachers Satisfied With Their Mathematics Preparation," The Arithmetic Teacher, XIV (March, 1967), pp. 190-193.

[2]Alice Huettig and John M. Newell, "Attitudes toward Introduction of Modern Mathematics Program by Teachers with Large and Small Number of Years Experience," The Arithmetic Teacher, XIII (February, 1966), pp. 125-130.

to all. The commonality is the lack of understanding of mathematical concepts by both prospective and inservice elementary teachers. The mean scores on the various tests represented in most cases an understanding of only about half of the concepts which the researcher expected the student or teacher to know. Even though many of the older tests would be criticized today with respect to contemporary content, stress, and vocabulary, still when one considers that these tests were designed to measure what the teacher was expected to know at that time, one concludes that many teachers have been and are "teaching" without understanding the concepts involved. Such results are obviously the basis for many of the recommendations calling for a better mathematical background for prospective elementary teachers. The research study reported in this paper was designed to compare the effectiveness of some of these recommendations. In a sense this study is an updating of various aspects of some of the studies reviewed here; however, the uniqueness of the study is the comparison of the various requirements in mathematics for elementary education students. Most of the previous studies have involved students who were not required to take a certain amount of mathematics. The few studies involving students who were required to complete a specified amount of mathematics were conducted at a single school with a single mathematics requirement.

RELATED MISCELLANEOUS PROBLEMS

There have been various suggestions for improving the mathematical background of prospective elementary teachers besides the common one of requiring more credit hours. As at other levels, a change in content has been advised by many for the mathematics courses involving elementary education students.

Stipanowich[1] surveyed seventy mathematics educators to obtain their opinions about the part of the elementary education program concerned with mathematics. Besides finding that most of these educators desired more mathematics in the teacher training program, he also found a strong opinion for separate courses in mathematics for elementary teachers. More than 75% of the panel suggested topics for these courses which involved arithmetic. Many also mentioned the need for qualified instructors to teach these courses and the need for better textbooks. This survey was conducted in 1952.

In a 1959 article Mueller[2] requested people in teacher education to recognize that arithmetic is not mumbo-jumbo to be memorized. He called for courses which provide elementary teachers with an awareness and an appreciation of the

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[1]Joseph Stipanowich, "The Mathematical Training of Prospective Elementary-School Teachers," The Arithmetic Teacher, IV (December, 1957), pp. 240-248.

[2]Francis J. Mueller, "Arithmetic Preparation for Elementary Teachers," Education (January, 1959), pp. 299-302.

structure of arithmetic.

The geometry content of the course work for elementary teachers was the concern of John[1] in 1960. She recommended topics concerning intuitive or nonmetric geometry, the use of set language, the geometric and algebraic descriptions of certain geometric figures, and the underlying assumptions for measurement. The topics suggested are now commonly found in elementary (K-6) texts and in texts for elementary education students. This is an indication of the relative ease of changing content involved in curriculum development as compared to the difficulty of changing the teachers' mathematics background by increasing requirements. The test designed by the author for this study reflects the contemporary content and language found in the elementary curriculum with respect to all areas, including geometry.

Of course the most influential suggestions concerning course content for mathematics courses for elementary teachers has come from the Committee on the Undergraduate Program in Mathematics, (CUPM)[2]. This committee has outlined course content for two courses involving number

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[1]Lenore John, "Geometry for Elementary School Teachers," The American Mathematical Monthly, (April, 1960), pp. 374-375.

[2]The Committee on the Undergraduate Program in Mathematics, Course Guides for the Training of Teachers of Elementary School Mathematics, (Berkeley: Committee on The Undergraduate Program in Mathematics, Fourth Draft, 1964).

systems, for an algebra course, and for a geometry course. These course outlines stress the structure, logic, and proof involved in the topics from arithmetic, algebra, and geometry. It certainly seems that if elementary teachers understood the many and varied concepts suggested in this guide then the means for unprecendented progress in elmentary mathematics teaching would be available.

Related to increasing requirements in mathematics and to changing the content of the courses involved are some other problems at the college level. Both Marks[1] and Young[2] discuss the lack of manpower at the college level for implementing a twelve hour mathematics requirement for elementary education students. They also discuss the need for obtaining effective teachers for such courses. Marks suggests an inservice training of college instructors for such teaching. Young discusses the difficulty of getting colleges to require twelve or more hours of mathematics and introduces the question of allowing graduate credit for mathematics courses following a twelve hour prerequisite.

These questions are of course important and have a bearing on the changing elementary mathematics program.

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[1]John L. Marks, "The Uneven Progress of the Revolution in Elementary School Mathematics," The Arithmetic Teacher, X (December, 1963), pp. 474-478.

[2]Gail S. Young,"Problems in the Training of Elementary School Teachers," The Arithmetic Teacher, XIII (May, 1966), pp. 380-384.

As suggested, there may be a lack of manpower for the teaching of a twelve hour mathematics requirement, it is doubtful that this is the main reason for the twelve hour recommendation not being implemented. The reason for the inaction is more likely due to educators not knowing whether there is value in increased mathematics requirements. Research evidence, such as supplied by this study, is needed to aid in making such a decision.

CHAPTER III

ANALYSIS OF DATA

This chapter is divided into six sections. The first section provides statistical and other pertinent information about the short ability test. The next three sections contain the analyses of variance and covariance for the total sample, for the types of colleges, and for three ability levels. The fifth section includes the analysis of the relationships among the qualitative and quantitative variables involved in the study. The method of stepwise regression was used for the entire sample. The last section in this chapter provides statistical and other pertinent information about the Mathematics Concepts Test. Item analyses of the test, along with comparisons based on these analyses, provide statistical data and information concerning the test and test items.

Conclusions and recommendations based on interpretations of the analyses are the major emphasis of the next chapter.

SHORT ABILITY TEST

This study compares the mathematical understanding of elementary education students who have completed three,

six, or nine semester hours of college mathematics. Since the students were enrolled at thirteen different colleges, some of which limit enrollment to high ability students and others which accept all high school graduates, differences in student ability were expected between colleges as well as within colleges. In order to control for these ability differences a common measure was desired.

The common measure of ability used in this study was a short ability test copyrighted in 1955 by Educational Testing Service of Princeton, New Jersey. The ten minute test, consisting of twelve vocabulary items and eight arithmetic items, was originally designed for a National Science Foundation sponsored study[1] involving 33,000 high school seniors in 478 schools. Permission for the use of the test in this research study was granted gratuitously by Educational Testing Service.

To determine the suitability of the short ability test for college students an item analysis of the test was completed using the responses of students enrolled in elementary mathematics methods courses at Eastern Michigan University and the University of Michigan. On the basis of this analysis five vocabulary items, which nearly everyone marked correct, were replaced by five other items

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[1] Background Factors Relating to College Plans and College Enrollment among Public High School Students, (Princeton, N.J.: Educational Testing Service, 1957), pp. 7-8.

drafted by the author. The replacement items are numbered 3, 5, 7, 8, and 12 in the short ability test. (See Appendix A.)

In order to obtain statistical information concerning the modified short ability test, an item analysis of the test responses of a randomly chosen subset of one-tenth of the total sample in the study was completed. The information obtained from this analysis of the modified short ability test follows.

Range	3-20
Mean	12.5
Median	12.7
Standard Deviation	3.04
Reliability Coefficient	.62

The frequency distribution was nearly normal and the discrimination power for each test item was positive. (See Appendix B for the frequency distribution and the discrimination power of each test item.) Correlations were computed between the short ability test scores and available college entrance test scores. These correlation coefficients are listed below.

Correlation of short ability test with:

American College Test (ACT) Math	.593
American College Test (ACT) Composite	.680
Scholastic Aptitude Test (SAT) Verbal	.537
Scholastic Aptitude Test (SAT) Math	.576

The reliability and validity coefficients obtained were

considered adequate for a test of only twenty items.

The students' scores on the short ability test were used as one of the covariates in the analyses of covariance. Also the short ability test scores were used for the grouping of students into high, middle, and low ability groups.

ANALYSIS OF COVARIANCE-TOTAL SAMPLE

A one way analysis of covariance was completed for the total sample using treatment groups 1, 2, and 3 representing 3, 6, and 8 or 9 semester hours of required mathematics, respectively. The covariates used were:

A = number of semesters of high school mathematics background.

B = short ability test scores.

The means and standard deviations for the covariates are given in Table 1. The number of semesters of high school mathematics background among the Groups 1, 2, and 3 do not differ significantly as determined by an anlysis of variance. However, the mean ability scores among the three groups do differ significantly in an analysis of variance. The standard deviations show Group 2 to have the greatest variance and Groups 3 to have the least. Thus, Group 3 would be more homogeneous. There is no known reason as to why Group 2 has the greatest variance. The high mean ability score attained by Group 1 may be due to the large number of students in Group 1 who are enrolled at two large universities. The students at these two universities have

above average ability as well as above average high school mathematics backgrounds as indicated in the next section.

TABLE 1.--Means and standard deviations of covariates A and B for Groups 1, 2, and 3 of total sample

COVARIATE		GROUPS 1	2	3	
A	Mean	5.30	5.28	5.43	Total sample
	SD	1.95	2.11	1.73	
B	Mean	12.66	12.28	12.43	Total sample
	SD	3.09	3.26	2.85	

Group means, standard deviations, and adjusted group means for the six achievement categories of the Mathematics Concepts Test are given in Table 2. The achievement categories represent subsets of the forty test items. (See Appendix A for the classification.) The possible scores for each category are as follows: Arithmetic 20, Algebra 10, Geometry 10, Basic 24, Related 16, and Total Test 40.

The information for each of the Groups 1, 2, and 3 for each of the six achievement categories may be read opposite the group number with the caption at the top of the table representing the statistic. For example, the mean for Group 2 on the arithmetic items is 9.21; the standard deviation is 3.03; the effect -.04; the

TABLE 2.--Group means, standard deviations, effects, and adjusted means of subtest and total test scores of the entire population included in this study

ACHIEVEMENT	GROUP	MEAN	SD	EFFECT	ADJUSTED MEANS A	ADJUSTED MEANS B
Arithmetic	1	9.10	3.00	-.16	9.10	9.02
	2	9.21	3.03	-.04	9.23	9.29
	3	9.72	2.80	.47	9.66	9.74
Algebra	1	3.55	1.68	-.09	3.55	3.51
	2	3.89	1.67	.25	3.90	3.92
	3	3.29	1.44	-.34	3.26	3.30
Geometry	1	2.77	1.71	-.17	2.77	2.74
	2	2.77	1.62	-.17	2.78	2.79
	3	3.75	2.08	.81	3.72	3.76
Basic	1	10.95	3.76	-.24	10.96	10.85
	2	10.94	3.56	-.25	10.96	11.04
	3	12.33	3.70	1.14	12.25	12.35
Related	1	4.47	2.08	-.17	4.47	4.43
	2	4.93	2.09	.29	4.94	4.97
	3	4.44	1.77	-.20	4.40	4.45
Total test	1	15.41	5.18	-.41	15.43	15.27
	2	15.86	4.94	.04	15.90	16.01
	3	16.76	4.77	.94	16.64	16.80

adjusted mean using covariate A is 9.23; and the adjusted mean using covariate B is 9.29.

The effect represents the difference between the mean of the three groups combined and the mean of the particular group. "If mathematical understanding increases with the amount of mathematics required" is a true assumption, then the effect should be negative for Group 1 and positive for Group 3 with the effect for Group 2 somewhere between. This pattern is found only twice among the six achievement categories. The positive effect for Group 2 in algebra and for Group 3 in geometry represent the usual emphasis of algebra in the second course and geometry in the third course. This pattern is repeated in every partition of the sample except one.

The adjusted means may be interpreted as the expected achievement means of the groups, assuming all subjects of the groups to have the same "score" on the covariate or initial variable. The adjustment is the product of the correlation coefficient between the initial and final variables within the group multiplied by the group effect. This adjustment is subtracted from the group mean of the final variable in order to obtain the adjusted group mean.

For the basic category, which was used in the definition of satisfactory preparation, the mean for Group 3 is the only mean representing correct responses for more than 50% of the basic items; however, a correct response for at least 75% of the basic items was defined as representing

satisfactory preparation. The arithmetic means represent correct responses for nearly half of the items, the algebra means represent about a one-third correct response, and the geometry means represent from a one-fourth to a three-eighths correct response. The basic items were answered correctly about one-half of the time and the related items were answered correctly only about one-fourth of the time.

The analysis of variance and the analyses of covariance for the total sample are found in Table 3. These analyses were completed for the scores on each of the achievement categories. For any category the analysis of variance and the analyses of covariance using covariates A and B can be read horizontally in the table and its continuance by interpreting the numbers as representing the captions at the top of the columns. The abreviations S. S., M. S., and adj. S. S. represent sums of squares, mean square, and adjusted sum of squares, respectively. The between groups sum of squares represents the variation of the group means around the grand mean. The within group sum of squares represents the variation of the individual scores of the groups about the group means. The total sum of squares is the sum of the between and within groups sums of squares. The total sum of squares is the sum of the deviations of all scores from the grand mean. The mean square is the quotient of the sum of squares divided by

TABLE 3.--Analyses of variance and covariance of individual scores on arithmetic, algebra, geometry, basic, and related subtests and total test for total sample groups 1, 2, and 3

CRITERION	SOURCE OF VARIATION	S. S.	M. S.	F-RATIO
Arithmetic	Between groups	126.84	63.42	7.14**
	Within groups	22269.03	8.88	
	Total	22395.87		
Algebra	Between groups	122.43	61.22	22.75**
	Within groups	6744.62	2.69	
	Total	6867.06		
Geometry	Between groups	349.76	174.88	57.20**
	Within groups	7664.63	3.06	
	Total	8014.39		
Basic	Between groups	700.20	350.10	25.93**
	Within groups	33845.29	13.50	
	Total	34545.48		
Related	Between groups	132.90	66.45	16.07**
	Within groups	10369.51	4.14	
	Total	10502.42		
Total test	Between groups	578.37	289.18	11.47**
	Within groups	63195.20	25.21	
	Total	63775.57		

*$F_{.95}(2,2507) = 3.00$ **$F_{.99}(2,2507) = 4.61$

TABLE 3.--(continued)

(Analyses of covariance)

COVARIATE A			COVARIATE B		
adj. S.S.	M. S.	F-RATIO	adj. S.S.	M. S.	F-RATIO
98.95	49.48	6.44**	169.90	84.95	12.09**
19257.29	7.68		17614.48	7.03	
19356.24			17784.38		
r = .368			r = .454		
136.51	68.25	28.46**	144.23	72.11	30.11**
6010.10	2.40		6001.73	2.39	
6146.60			6145.96		
r = .324			r = .324		
324.05	162.02	58.27**	357.25	178.63	63.06**
6967.76	2.78		7098.70	2.83	
7291.81			7455.95		
r = .300			r = .264		
601.98	300.99	26.44**	750.89	375.45	35.64**
28532.36	11.39		26400.03	10.53	
29134.34			27150.93		
r = .396			r = .463		
146.74	73.37	20.18**	171.01	85.50	23.09**
9109.97	3.64		9281.59	3.70	
9256.71			9452.59		
r = .344			r = .316		
475.13	237.56	11.57**	786.79	393.40	20.13**
51451.02	20.53		48971.97	19.54	
51926.15			49758.77		
r = .431			r = .469		

*$F_{.95}(2,2506) = 3.00$ **$F_{.99}(2,2506) = 4.61$

the degrees of freedom. The degrees of freedom between groups is two in this case since it is one less than the number of groups. The degrees of freedom within groups is 2507 for the analysis of variance and 2506 for the analysis of covariance. The F-ratio is the quotient of the between group mean square divided by the within group mean square. In the analysis of covariance the adjusted means replace the unadjusted means in the analysis of variance. The r represents the correlation coefficient between the covariate and the scores on the achievement subtest.

The significance of the F-ratio is determined by reference to a distribution table for the F variable. If the F-ratio is smaller than the table value for a certain confidence level, then the null hypothesis is accepted. The .95 and .99 confidence levels for the degrees of freedom involved in this analysis are given at the bottom of the table.

The analysis of variance and the analyses of covariance of the scores of the total sample indicate significant differences beyond the .99 confidence level among the group means for every achievement category. This does not necessarily indicate significant differences between any two particular groups, however. Actually, the use of a post hoc test of differences between most pairs of group means for all categories revealed significant differences.

Of course, the total sample includes students from different types of colleges and of different abilities. Thus, the analyses of the total sample are useful for only very general comparisons. Specific comparisons are made in the next two sections through analysis of student achievement at various types of colleges and for various ability levels.

ANALYSIS OF COVARIANCE-CLASSIFICATION: TYPE OF COLLEGE

The colleges which participated in this study had enrollments which ranged from approximately 1000 to 35,000 students. Their functions varied from single purpose institutions (teacher education) to the multiple purpose large universities. In order to make the comparisons most useful, the colleges were classified into three types as follows:

Large University--These are large multiple purpose institutions which are not, predominantly, involved in teacher education.

Large College--These are schools with enrollments over 5,000 which are predominantly involved in teacher education. These schools have changed their names from teachers college to university. They are rapidly becoming multiple purpose institutions.

Small College--These are basically teachers colleges with enrollments less than 5,000. Their names likewise have been changed to state colleges or universities in order

to reflect their increasing functions.

The sample for this study included elementary education students who were required to take three, six, or nine semester hours at each of the three types of institutions. One-way analyses of covariance were completed for test responses from each type of college using the different requirements in mathematics as the basis for the treatment groups. The covariates were:

A = number of semesters of high school mathematics background.

B = short ability test score.

The means and standard deviations of the covariates for each type of college are presented in Table 4. Groups 1, 2, and 3 represent students having had one course (3 semester hours), two courses (6 semester hours), or three courses (8 or 9 semester hours). The mean number of semesters of high school mathematics background for groups 1, 2, and 3 differ significantly for each type of college by an analysis of variance of the backgrounds of the individuals from each type of college. The small college groups have means representing less than two and one-half years of mathematics in each case. Small college Group 1 has an exceptionally low high school mathematics background of 2.63 semesters. Large college Group 1 has a lower mean number of semester hours of high school mathematics than Groups 2 and 3 from large colleges; however, the large college Group 1 mean exceeds all the group means for the

TABLE 4.--Means and standard deviations of covariates A and B for Groups 1, 2, and 3 from small colleges, large colleges, and large universities

COVARIATE		GROUPS 1	GROUPS 2	GROUPS 3	TYPE OF SCHOOLS
A	Mean	2.63	4.38	4.65	Small College
	SD	1.46	1.91	1.66	
B	Mean	9.86	11.55	11.42	" "
	SD	3.19	3.13	3.46	
A	Mean	5.04	5.18	5.56	Large College
	SD	1.75	2.26	1.64	
B	Mean	12.50	11.77	12.32	" "
	SD	2.88	3.29	2.76	
A	Mean	5.61	4.91	5.15	Large University
	SD	1.80	1.92	1.98	
B	Mean	13.38	13.92	13.09	" "
	SD	2.85	2.81	2.84	

small colleges. The small and large college group means for covariate A increase from Group 1 to Group 2 to Group 3; this may be an indication that some students with less high school background fail the prerequisite courses at colleges requiring more than one mathematics course for prospective elementary teachers. The large university Group 1 has the highest mean number of semesters of high school mathematics for all groups from all types of colleges with 5.61 semesters. The increase in covariate A for Groups 1 to 2 to 3, which was observed for small and large colleges, is not found among the large university groups; the reason may be the large universities selection procedures which limit

enrollment to well-prepared students. The students at large universities with the lesser amounts of high school mathematics may have the ability to compensate for their lack of high school mathematics.

The means on the short ability test are also significantly different for Groups 1, 2, and 3 from small and large colleges but not from large universities as determined by analyses of variance for each type of college. The small college means range from 9.86 to 11.55 (possible score 20); the large college range is 11.77 to 12.50; and the large university range is 13.09 to 13.92. The apparent positive relationship in Table 4 between high school mathematics background and ability is supported by the correlation between these two variables of .32 for the entire sample. The data presented in Table 4 indicate that students with high ability and more high school mathematics attend the large colleges and universities.

Tables 5, 6, and 7 contain the means, standard deviations, effects, and adjusted means of the achievement scores on arithmetic, algebraic, geometric, basic, and related items and the total test for students from large universities, large colleges, and small colleges, respectively. Achievement scores represent correct responses for subsets of the mathematics concepts test. (See Appendix A for classification of test items into subsets.)

A comparison of the achievement means for students from large universities and large colleges favors the large

TABLE 5.--Group means, standard deviations, effects, and adjusted means of achievement scores for students from large universities

ACHIEVEMENT	GROUP	MEAN	SD	EFFECT	ADJUSTED MEANS A	ADJUSTED MEANS B
Arithmetic	1	9.68	2.89	.00	9.58	9.70
	2	9.96	2.55	.28	10.30	9.72
	3	9.36	3.47	-.32	9.55	9.52
Algebra	1	3.97	1.73	.08	3.92	3.97
	2	4.09	1.68	.20	4.24	4.00
	3	3.26	1.46	-.63	3.35	3.32
Geometry	1	2.82	1.70	-.04	2.76	2.82
	2	3.10	1.72	.24	3.27	3.00
	3	2.78	1.89	-.07	2.88	2.85
Basic	1	11.41	3.61	-.11	11.28	11.44
	2	12.00	3.36	.48	12.43	11.69
	3	11.55	4.35	.04	11.79	11.76
Related	1	5.05	2.11	.14	4.98	5.06
	2	5.15	1.83	.24	5.39	5.02
	3	3.85	1.99	-1.06	3.98	3.93
Total test	1	16.47	5.12	.04	16.26	16.50
	2	17.15	4.34	.72	17.82	16.72
	3	15.40	5.65	-1.02	15.77	15.69

TABLE 6.--Group means, standard deviations, effects and adjusted means of achievement means for students from large colleges

ACHIEVEMENT	GROUP	MEAN	SD	EFFECT	ADJUSTED MEANS A	ADJUSTED MEANS B
Arithmetic	1	9.24	2.65	.05	9.35	9.10
	2	8.43	3.18	-.75	8.48	8.57
	3	9.99	2.51	.81	9.86	9.93
Algebra	1	3.25	1.50	-.13	3.31	3.20
	2	3.52	1.58	.14	3.54	3.57
	3	3.31	1.42	-.07	3.25	3.28
Geometry	1	3.48	1.69	-.02	3.54	3.42
	2	3.01	1.76	-.49	3.04	3.07
	3	4.07	2.08	.57	3.99	4.07
Basic	1	11.97	3.46	.39	12.13	11.79
	2	10.43	3.78	-1.24	10.41	10.53
	3	12.72	3.45	1.13	12.52	12.63
Related	1	4.00	1.90	-.48	4.07	3.94
	2	4.62	2.11	.14	4.65	4.68
	3	4.65	1.65	.18	4.57	4.62
Total test	1	15.97	4.67	-.09	16.20	15.73
	2	14.96	5.23	-1.10	15.06	15.21
	3	17.37	4.39	1.31	17.10	17.25

TABLE 7.--Group means, standard deviations, effects and adjusted means of achievement scores for students from small colleges

ACHIEVEMENT	GROUP	MEAN	SD	EFFECT	ADJUSTED MEANS A	ADJUSTED MEANS B
	1	5.63	2.61	-3.19	6.50	6.36
Arithmetic	2	9.51	3.23	.69	9.39	9.39
	3	7.65	2.59	-1.17	7.39	7.60
	1	2.49	1.61	-.99	2.91	2.81
Algebra	2	3.68	1.68	.20	3.63	3.63
	3	3.19	1.52	-.29	3.06	3.17
	1	1.54	1.20	-.89	1.96	1.73
Geometry	2	2.49	1.35	.06	2.04	2.46
	3	3.19	1.52	.76	3.07	3.18
	1	6.57	3.32	-3.67	7.76	7.47
Basic	2	10.86	3.72	.62	10.71	10.72
	3	10.23	3.02	-.01	9.87	10.17
	1	3.09	1.61	-1.41	3.61	3.42
Related	2	4.82	2.12	.32	4.75	4.76
	3	3.81	1.82	-.68	3.65	3.78
	1	9.66	4.43	-5.08	11.37	10.89
Total test	2	15.68	5.15	.95	15.46	15.48
	3	14.04	3.95	-.70	13.51	13.95

universities in a majority of the cases. Likewise, a comparison of large and small college means for the various achievement categories favors the large colleges. There are some exceptions however, since Group 2 of the small colleges has higher means in four out of the six achievement categories as compared to Group 2 of the large colleges. Likewise, Group 3 of the large colleges has a higher mean than Group 3 of the large universities in each category. The only explanation is that there are differences among the small colleges, large colleges, and large universities just as there are differences between any two of the types of schools.

The analysis of variance and the analysis of covariance for the achievement scores of students from large universities are given in Table 8. The analysis of variance for groups 1, 2, and 3 from large universities shows significant differences in means only for algebra and related items. It should be noted that seven of the sixteen related items are algebraic and thus the significant differences may be due to the same test items. When the means are adjusted for the number of semesters of high school mathematics in the analysis of covariance, then there are significant differences among the group means for every achievement subscore. The analysis of covariance using ability scores as the covariate, however, gives only the same significant differences as the analysis of variance.

Examination of the means and adjusted means for the

TABLE 8.--Analyses of variance and covariance of individual scores on arithmetic, algebra, geometry, basic, and related subtests and total test for large university groups 1, 2, 3

CRITERION	SOURCE OF VARIATION	S. S.	M. S.	F-RATIO
Arithmetic	Between groups	17.65	8.82	1.03
	Within groups	5963.75	8.58	
	Total	5981.39		
Algebra	Between groups	43.35	21.67	7.54**
	Within groups	1998.38	2.88	
	Total	2041.72		
Geometry	Between groups	7.31	3.66	1.22
	Within groups	2086.49	3.00	
	Total	2093.80		
Basic	Between groups	29.54	14.77	1.09
	Within groups	9454.72	13.60	
	Total	9484.26		
Related	Between groups	119.61	59.80	14.07**
	Within groups	2953.34	4.25	
	Total	3072.95		
Total Test	Between groups	150.55	75.27	2.90
	Within groups	18054.08	25.98	
	Total	18204.63		

*$F_{.95}(2,695) = 3.01$ **$F_{.99}(2,695) = 4.64$

TABLE 8.--(continued)

(Analyses of covariance)

COVARIATE A			COVARIATE B		
adj. S.S.	M. S.	F-RATIO	adj. S.S.	M. S.	F-RATIO
45.93	22.96	3.19*	2.83	1.42	.21
4995.73	7.20		4658.62	6.71	
5041.66			4661.45		
r = .396			r = .470		
40.42	20.21	7.80**	34.66	17.33	6.59**
1796.87	2.59		1826.19	2.63	
1837.28			1860.85		
r = .316			r = .298		
22.01	11.01	4.17*	2.56	1.28	.48
1829.67	2.14		1862.42	2.68	
1851.68			1864.98		
r = .340			r = .331		
118.37	59.18	5.19**	11.54	5.77	.55
7906.80	11.39		7335.09	10.57	
8025.17			7346.63		
r = .392			r = .475		
105.20	52.60	14.78**	101.38	50.69	13.41**
2469.77	3.56		2622.90	3.78	
2574.97			2724.28		
r = .403			r = .337		
250.52	125.26	6.08**	61.52	30.76	1.53
14292.24	20.59		13930.18	20.07	
14542.76			13991.70		
r = .448			r = .481		

*$F_{.95}(2,694) = 3.01$ **$F_{.99}(2,694) = 4.64$

achievement categories which have significant differences in the analyses shows that group 2 has the highest means in every case except one; the group 3 means are the lowest in eight out of the ten achievement categories for which significant differences exist. Since Group 2 had the highest ability means (covariate B), the adjusted means for ability should be examined. These adjusted means show that much of the higher achievement is due to the higher ability of Group 2; however, the Group 2 adjusted means remain the highest in most of the categories. Group 3 had the lowest mean ability among the large university groups. The low means and adjusted means on the achievement categories for Group 3 are indicative of the dependence of achievement on ability.

The analysis of variance and the analyses of covariance for large colleges are given in Table 9. The analyses for groups 1, 2, and 3 indicate significant differences among the means in every category except for the comparison of the algebra means in the analysis of variance. Examination of the achievement means for categories in which significant differences exist reveal Group 3 as having the highest mean in thirteen of the seventeen cases. Group 2 has the highest mean in the other four cases. These other cases are in the algebra and related categories. Since the related category has seven algebra items out of the sixteen items of the category, the related mean is quite dependent upon algebraic achievement. Thus the high Group 2 means in

TABLE 9.--Analyses of variance and covariance of individual scores on arithmetic, algebra, geometry, basic, and related subtests and total test for large college groups 1,2, and 3

CRITERION	SOURCE OF VARIATION	S. S.	M. S.	F-RATIO
Arithmetic	Between groups	421.75	210.88	26.23**
	Within groups	7324.10	8.04	
	Total	7745.85		
Algebra	Between groups	12.26	6.13	2.69
	Within groups	2079.00	2.28	
	Total	2091.26		
Geometry	Between groups	192.15	96.07	27.54**
	Within groups	3178.35	3.49	
	Total	3370.50		
Basic	Between groups	1013.38	506.69	39.21**
	Within groups	11772.97	12.92	
	Total	12786.35		
Related	Between groups	68.37	34.19	9.39**
	Within groups	3315.73	3.64	
	Total	3384.11		
Total test	Between groups	1001.85	500.92	21.60**
	Within groups	21118.72	23.18	
	Total	22120.57		

*$F_{.95}(2,911) = 3.00$ **$F_{.99}(2,911) = 4.63$

TABLE 9.--(continued)

(Analyses of covariance)

COVARIATE A			COVARIATE B		
adj. S.S.	M.S.	F-RATIO	adj. S.S.	M.S.	F-RATIO
335.02	167.51	23.32**	314.58	157.29	23.46**
6537.05	7.18		6102.19	6.71	
6872.07			6416.77		
r = .336			r = .414		
16.57	8.29	3.96*	23.21	11.61	5.52**
1903.80	2.09		1913.34	2.10	
1920.37			1936.55		
r = .286			r = .272		
155.62	77.81	24.22**	162.81	81.40	24.89**
2923.98	3.21		2975.64	3.27	
3079.60			3138.45		
r = .294			r = .262		
854.91	427.45	38.24**	760.81	380.40	35.87**
10171.92	11.18		9651.55	10.61	
11026.82			10412.36		
r = .371			r = .431		
50.37	25.18	7.59**	87.25	43.62	12.97**
3018.98	3.32		3059.56	3.36	
3069.35			3146.80		
r = .305			r = .265		
715.55	357.78	18.25**	741.41	370.70	19.54**
17842.33	19.61		17266.75	18.97	
18557.89			18008.15		
r = .401			r = .431		

$^{*}F_{.95}(2,910) = 3.00$ $^{**}F_{.99}(2,910) = 4.63$

algebra support the pattern of course two having an emphasis on algebra. Of course, Group 3 having the highest means in the other cases supports the pattern of course three having an emphasis on geometry. However, Group 3 also has the highest mean for arithmetic, indicating that some students from large colleges are making gains in this area with increased mathematics requirements.

For small colleges, the analysis of variance and the analyses of covariance in Table 10 indicate significant differences among the means of groups 1, 2, and 3 for every achievement category. Group 2 has the highest means and adjusted means in fifteen cases, and Group 3 has the highest means and adjusted means in the other three cases. The group 3 means and adjusted means which are the highest, all represent geometric items. Thus the group means for scores from small colleges follow the pattern of Group 2 being the highest in algebra and Group 3 being the highest in geometry.

ANALYSIS OF VARIANCE-CLASSIFICATION: ABILITY GROUPS

As part of this study, it was desired to determine the effect of different mathematics requirements on students of different abilities. Therefore, the 2513 data sets of the study were separated into three ability levels. The data were separated into three groups according to scores on the short ability test. The ability levels were defined

TABLE 10.--Analyses of variance and covariance of individual scores on arithmetic, algebra, geometry, basic, and related subtests and total test for small college groups 1,2, and 3

CRITERION	SOURCE OF VARIATION	S. S.	M. S.	F-RATIO
Arithmetic	Between groups	489.61	244.80	25.19**
	Within groups	2575.80	9.72	
	Total	3065.40		
Algebra	Between groups	45.13	22.56	8.15**
	Within groups	733.74	2.77	
	Total	778.87		
Geometry	Between groups	43.46	21.73	11.79**
	Within groups	488.46	1.84	
	Total	531.92		
Basic	Between groups	551.84	275.92	20.92**
	Within groups	3495.40	13.19	
	Total	4047.24		
Related	Between groups	103.18	51.59	12.36**
	Within groups	1105.81	4.17	
	Total	1208.99		
Total test	Between groups	1100.39	550.19	22.18**
	Within groups	6573.80	24.81	
	Total	7674.19		

*$F_{.95}(2,265) = 3.06$ **$F_{.99}(2,265) = 4.75$

TABLE 10.--(continued)

(Analyses of covariance)

COVARIATE A			COVARIATE B		
adj. S.S.	M. S.	F-RATIO	adj. S.S.	M. S.	F-RATIO
292.23	146.11	16.84**	309.41	154.71	21.54**
2290.00	8.67		1869.12	7.18	
2582.23			2205.54		
r = .397			r = .530		
19.34	9.67	3.83*	22.47	11.24	4.91**
666.59	2.52		603.98	2.29	
685.93			626.25		
r = .345			r = .442		
17.14	8.57	5.34**	31.56	15.78	9.35**
423.68	1.60		445.76	1.69	
440.82			477.32		
r = .414			r = .320		
242.72	121.36	10.80**	306.34	153.17	16.38**
2967.81	11.24		2468.15	9.35	
3210.53			2774.49		
r = .455			r = .561		
56.97	28.49	7.50**	66.78	33.39	9.15**
1002.18	3.80		963.10	3.65	
1059.15			1029.88		
r = .352			r = .385		
504.56	252.28	12.16	629.34	314.67	17.91**
5474.94	20.74		4638.12	17.57	
5979.50			5267.46		
r = .470			r = .560		

*$F_{.95}(2,264) = 3.06$ **$F_{.99}(2,264) = 4.75$

by the ranges of the test scores indicated in Table 11.

TABLE 11.--Ability level classification

	Range	N	%	Mean Ability Group 1	Group 2	Group 3
High Ability	15-20	670	27%	16.17	16.12	16.04
Middle Ability	12-14	957	38%	13.00	12.95	12.89
Low Ability	3-11	886	35%	9.15	8.99	9.40
Total Sample	3-20	2513	100%	12.66	12.28	12.43

Again the treatment Groups 1, 2, and 3 represent the number of required mathematics courses, 3, 6 and 8 or 9 semester hours, respectively. A one-way analysis of variance was completed for each ability level.

The means and standard deviations of the achievement scores on the arithmetic, algebra, geometry, basic, and related subtests and the total test for the three ability groups are given in Tables 12, 13, and 14. A comparison of these three tables indicates clearly the dependence of achievement on ability. For example, on the basic items the low ability Group 3 mean is 10.82, the middle ability Group 3 mean is 12.64, and the high ability Group 3 mean is 14.03. The established patterns of Group 2 having the highest mean in algebra and Group 3 having the highest mean in geometry holds for each of the ability levels.

The analysis of variance results are given for high

TABLE 12.--Group means, standard deviations, and effects of achievement scores for students of high ability

ACHIEVEMENT	GROUP	MEAN	SD	EFFECT
Arithmetic	1	11.04	2.53	.18
	2	10.59	2.67	-.27
	3	10.90	2.58	.05
Algebra	1	4.21	1.74	-.06
	2	4.61	1.60	.34
	3	3.67	1.37	-.60
Geometry	1	3.48	1.89	-.11
	2	3.40	1.77	-.19
	3	4.37	2.09	.78
Basic	1	13.36	3.34	.06
	2	12.89	3.19	-.41
	3	14.03	3.34	.73
Related	1	5.37	2.24	-.05
	2	5.70	2.07	.29
	3	4.91	1.75	-.50
Total test	1	18.73	4.91	.01
	2	18.60	4.56	-.12
	3	18.94	4.49	.23

TABLE 13.--Group means, standard deviations, and effects of achievement scores for students of middle ability

ACHIEVEMENT	GROUP	MEAN	SD	EFFECT
Arithmetic	1	9.15	2.57	-.40
	2	9.77	2.72	.22
	3	9.99	2.53	.44
Algebra	1	3.63	1.55	-.10
	2	4.05	1.58	.33
	3	3.27	1.45	-.45
Geometry	1	2.79	1.62	-.16
	2	2.75	1.57	-.20
	3	3.70	2.15	.75
Basic	1	11.01	3.26	-.47
	2	11.43	3.17	-.06
	3	12.64	3.40	1.15
Related	1	4.56	1.92	-.18
	2	5.15	1.94	.40
	3	4.33	1.85	-.41
Total test	1	15.57	4.41	-.66
	2	16.57	4.33	.35
	3	16.97	4.54	.74

TABLE 14.--Group means, standard deviations, effects, and adjusted means of achievement scores for students of low ability

ACHIEVEMENT	GROUP	MEAN	SD	EFFECT	ADJUSTED MEAN A
Arithmetic	1	7.30	2.71	-.40	7.37
	2	7.72	2.95	.02	7.75
	3	8.60	2.85	.90	8.37
Algebra	1	2.86	1.50	-.19	2.89
	2	3.24	1.58	.19	3.25
	3	3.05	1.42	.00	2.94
Geometry	1	2.11	1.32	-.32	2.14
	2	2.35	1.42	-.08	2.36
	3	3.40	1.90	.96	3.29
Basic	1	8.72	3.25	-.53	8.82
	2	9.12	3.32	-.13	9.16
	3	10.82	3.68	1.57	10.50
Related	1	3.55	1.67	-.38	3.58
	2	4.18	2.02	.26	4.20
	3	4.23	1.63	.30	4.10
Total test	1	12.27	4.21	-.91	12.40
	2	13.30	4.55	.12	13.36
	3	15.05	4.55	1.87	14.60

ability students in Table 15 and for middle ability students in Table 16. For the low ability students an analysis of variance and an analysis of covariance with the number of semesters of high school mathematics as the covariate are reported in Table 17. Analyses of covariance for high and middle ability groups were completed but are not reported since the number of semesters of high school mathematics background did not differ significantly and caused only negligible changes in F-ratios. The mean number of semesters of high school mathematics are shown below for the ability groups.

	Group 1	Group 2	Group 3
High Ability	6.13	6.19	5.89
Middle Ability	5.40	5.43	5.40
Low Ability	4.44	4.52	5.16

The analysis of variance, reported in Table 15 for Groups 1, 2, and 3 of high ability students shows significant differences in the mean scores of the groups for the algebra, geometry, basic, and related items but not for the arithmetic items and the total test. The lack of significant differences for the arithmetic category and the attainment of the highest mean by Group 1 may indicate that high ability students understand arithmetic concepts upon their first contact with them. At least they do not make gains in arithmetic with increased requirements as the courses are now taught. As mentioned before, the high

TABLE 15.--Analysis of variance of individual scores on arithmetic, algebra, geometry, basic, and related subtests and total test for high ability groups 1, 2, and 3

CRITERION	SOURCE OF VARIATION	S. S.	M. S.	F-RATIO
Arithmetic	Between groups	28.55	14.28	2.12
	Within groups	4485.69	6.73	
	Total	4514.25		
Algebra	Between groups	66.89	33.44	12.42**
	Within groups	1796.29	2.69	
	Total	1863.18		
Geometry	Between groups	76.94	38.47	10.83**
	Within groups	2369.19	3.55	
	Total	2446.13		
Basic	Between groups	96.92	48.46	4.46*
	Within groups	7240.98	10.86	
	Total	7337.89		
Related	Between groups	47.17	23.58	5.29**
	Within groups	2975.65	4.46	
	Total	3022.82		
Total test	Between groups	8.90	4.45	.20
	Within groups	14964.08	22.43	
	Total	14972.98		

*$F_{.95}(2,667) = 3.01$ **$F_{.99}(2,667) = 4.64$

TABLE 16.--Analysis of variance of individual scores on arithmetic, algebra, geometry, basic, and related subtests and total test for middle ability groups 1, 2, and 3

CRITERION	SOURCE OF VARIATION	S. S.	M. S.	F-RATIO
Arithmetic	Between groups	119.83	59.91	8.69**
	Within groups	6574.86	6.89	
	Total	6694.69		
Algebra	Between groups	80.50	40.25	16.90**
	Within groups	2271.57	2.38	
	Total	2352.06		
Geometry	Between groups	127.78	63.89	21.59**
	Within groups	2823.10	2.96	
	Total	2950.88		
Basic	Between groups	335.61	167.80	15.80**
	Within groups	10131.42	10.62	
	Total	10467.03		
Related	Between groups	105.15	52.57	14.33**
	Within groups	3500.10	3.67	
	Total	3605.25		
Total test	Between groups	320.59	160.30	8.24**
	Within groups	18565.21	19.46	
	Total	18885.80		

*$F_{.95}(2,954) = 3.00$ **$F_{.99}(2,954) = 4.62$

TABLE 17.--Analyses of variance and covariance of individual scores on arithmetic, algebra, geometry, basic, and related subtests and total test for low ability groups 1, 2, and 3

CRITERION	SOURCE OF VARIATION	S. S.	M. S.	F-RATIO
Arithmetic	Between groups	183.93	91.96	11.40**
	Within groups	7118.12	8.07	
	Total	7302.05		
Algebra	Between groups	26.11	13.05	5.62**
	Within groups	2049.00	2.32	
	Total	2075.10		
Geometry	Between groups	183.30	91.65	41.76**
	Within groups	1935.95	2.19	
	Total	2119.25		
Basic	Between groups	486.28	243.14	21.54**
	Within groups	9958.03	11.29	
	Total	10444.31		
Related	Between groups	90.74	45.37	13.74**
	Within groups	2912.63	3.30	
	Total	3003.37		
Total test	Between groups	843.60	421.80	21.62**
	Within groups	17210.19	19.51	
	Total	18053.79		

*$F_{.95}(2,882) = 3.00$ **$F_{.99}(2,882) = 4.63$

TABLE 17.--(continued)

(Analysis of covariance)

COVARIATE A		
adj. S. S.	M. S.	F-RATIO
109.18	54.59	7.41**
6490.04	7.37	
6599.22		
r = .310		
26.23	13.11	6.05**
1908.88	2.17	
1935.11		
r = .260		
145.10	72.55	35.20**
1815.53	2.06	
1960.63		
r = .274		
307.45	153.72	15.44**
8772.00	9.96	
9079.45		
r = .361		
75.39	37.69	12.15**
2732.17	3.10	
2807.56		
r = .255		
540.74	270.37	15.97**
14918.45	16.93	
15459.19		
r = .379		
*$F_{.95}(2,881) = 3.00$	**$F_{.99}(2,881) = 4.63$	

ability students in Group 2 achieve the highest in algebra and those in Group 3 achieve the highest in geometry.

Even though the pattern indicates that high ability students do best on what they have just studied, there is cause for concern when one examines the achievement means with respect to the total possible scores on the concepts test. The means in most cases represent achievement of less than 50% of the possible scores and in no case do the means exceed 60% of the total possible. On the basic items which reflect arithmetic, algebraic, and geometric concepts acutally taught in the elementary school the means represent 56%, 54%, and 58% of the total score possible for Groups 1, 2, and 3 respectively. This represents a low understanding by prospective teachers of concepts to be taught and certainly connot be considered satisfactory preparation. Examination of the middle and low ability groups' achievement reveals an even more pessimistic situation.

Significant differences among the means of the treatment groups are found in every achievement category for both middle ability and low ability students (Tables 16 and 17). The highest means for the algebra category are attained by Group 2 and the highest means for geometry are achieved by Group 3 for both ability levels. However there are increases in the means from Group 1 to Group 2 and from Group 2 to Group 3 for the categories of arithmetic, basic, and total test items. These gains by low and middle

ability students which did not occur for high ability students may be an indication that the mathematics for elementary teachers courses are often aimed at the middle and low ability students. It may be that these students need to see some of the concepts a number of times and from different viewpoints in order to understand them. Nevertheless, there should be a means of providing the high ability students with an opportunity to also make significant gains in understanding of mathematical concepts.

The analyses of variance and covariance presented in this and the preceding sections show significant differences among the means for a majority of the cases. Since it is usual to expect increased understanding with increased study, concern actually arises when significant differences do not occur or when the differences are due to decreases in means. The lack of significant differences occurred among the high ability groups and the large university groups. Since the students at the large universities demonstrated higher ability scores, it is likely that the lack of significant differences is due to the same factor i.e. aiming courses at the lower ability levels. Significant differences which are decreases may be explained in some cases by the fact that students may forget what they once knew. It is likely that the decreases from Group 2 to Group 3 in the algebra category for the various classifications is due to forgetting. Of course, some of the differences in the analyses by types of college may actually

represent differences between schools. The analyses by ability levels avoid differences between colleges and may be the most valid for generalizations.

STEPWISE REGRESSION ANALYSIS-TOTAL SAMPLE

Stepwise regression analyses were run for the entire sample of the study using the achievement scores for the categories of arithmetic, algebra, geometry, basic, related, and total test as the dependent variables. The independent variables used in the analyses were the type of college, the number of required courses in mathematics, the number of semesters of high school mathematics background, the number of college courses in mathematics completed, completion of a methods course, teaching preference as to grade level, attitude, and ability scores. For each dependent variable a prediction equation was formulated using the eight independent variables.

The independent variables were coded as shown in Table 18. Using these coded values for the variables, intercorrelations were computed among the independent variables and the total test scores. These correlations are found in Table 19. The two variables having the highest correlation with the total test scores are the ability scores and the number of semesters of high school mathematics; these are the two variables used as covariates in the analyses of covariance reported earlier.

In Table 19 there are some negative correlation

TABLE 18.--Coding of variables for purpose of computing intercorrelations

VARIABLE	BREAKDOWN	CODE
Type of college	large university	1
	large college	2
	small college	3
Number of required courses	one course	1
	two courses	2
	three courses	3
Number of semesters of high school mathematics	zero through eight	0-8
	more than eight	9
Number of college mathematics courses completed	zero through eight	0-8
	more than eight	9
Completion of a methods course	no	0
	yes	1
Teaching preference	K-3	1
	4-6	2
	7-8	3
Attitude	strong dislike	1
	dislike	2
	neutral	3
	like	4
	strong like	5
Ability score	raw score	0-20

TABLE 19.--Intercorrelation of independent variables and total test scores for the total sample

VARIABLE[1]	B	C	D	E	F	G	H	I
A	.320	-.133	.018	.189	-.082	.101	.365	.431
B		-.185	-.041	.063	-.063	.065	.176	.473
C			.296	.065	-.095	-.056	-.020	-.074
D				.601	.057	-.026	.078	.093
E					.131	.112	.240	.256
F						.008	.041	-.058
G							.118	.077
H								.375

[1]The letters represent the following variables:

A = number of semesters of high school mathematics background

B = Short Ability Test score

C = type of college

D = required number of college mathematics courses

E = number of college mathematics courses completed

F = methods course completed

G = teaching preference

H = attitude

I = Mathematics Concepts Test total score

coefficients displayed. For those involving variable C, the type of college, it should be noted that the coding assigned 1 to large universities, 2 to large colleges, and 3 to small colleges. Therefore, for example, the -.185 correlation between short ability test scores and type of college (variables B and C) indicates that students from large universities attained higher ability scores than students from small colleges. Some of the other negative correlation coefficients are near to zero and represent no significant relationship.

The correlation between ability scores and the number of semesters of high school mathematics completed is .320. The correlation between ability scores and the number of college mathematics courses completed is .063. This drop is explained by the fact that the number of college mathematics courses completed includes the required courses in mathematics for prospective elementary teachers, and in most cases this was all the college mathematics which had been taken. It is possible that many of the prospective teachers will elect more than the required amount of college mathematics since many of them are freshmen, sophomores, and juniors. Those who elected high school mathematics have, of course, already completed it.

Attitude toward mathematics correlates the highest with ability and mathematics background. However, attitude also correlates positively with teaching preference. This means that the prospective teachers with the greatest

dislike for mathematics expect to teach at the lowest grade level.

The correlation between the short ability test scores and the Mathematics Concepts Test scores was .473. This correlation compares favorably with the correlation of .369 between the available Scholastic Apptitude Test verbal scores and the Mathematics Concepts Test scores. Between the Scholastic Aptitude Test Mathematics scores and the Mathematics Concepts Test scores the correlation was .587. The Scholastic Aptitude Test scores were available for two schools.

Many of the correlations are near zero and may be considered as representing no significant relation between the variables. Some of these variables would not be expected to show any relationship to each other. However, one might expect the completion of a methods course to be positively correlated with the total test scores, since a methods course provides one more contact with basic concepts. The correlation between these two variables was -.058, indicating no positive relationship.

The purpose of the stepwise regression analysis is to formulate a linear regression equation of the form $Y=B_c + B_1X_1 + B_2X_2 + \ldots + B_kX_k$ where Y is the dependent variable and X_1, $X_2 \ldots X_k$ are the independent variables. The coefficients, B_c, B_1, $\ldots B_k$, are determined so as to minimize the sum of the squares of the deviations between the observed values of Y and the values of Y predicted by

the regression equation.

The stepwise regression procedure introduces the independent variables into the regression equation in their order of relative importance. For each dependent variable, the independent variables are listed in Table 20 in their order of entry. Also the multiple correlation coefficients and the regression equation coefficients are given in Table 20. The multiple correlation coefficient represents the correlation between the dependent variable and a linearly additive combination of the independent variables. In Table 20 the multiple correlation coefficients represent a linear combination of the particular variable with all variables listed above it. The regression equation coefficients listed are the coefficients obtained after the last variable has entered. The prediction equation in standard form is obtained by multiplying the regression equation coefficients times the corresponding independent variables, adding these products to the constant term, and setting the sum equal to the dependent variable.

The prediction equations for the dependent variables (i.e. achievement scores) show that the ability, attitude, high school mathematics background, and college mathematics background of the student are the best predictors of achievement. A comparison of the prediction equations for the areas of arithmetic, algebra, and geometry as represented in the Mathematics Concepts Test shows the value of the high school mathematics background in the areas of algebra and

TABLE 20.--Coefficients for linear regression equations and multiple correlation coefficients

DEPENDENT VARIABLE (Y)	INDEPENDENT VARIABLE	MULTIPLE CORRELATION COEFFICIENT	REGRESSION EQUATION COEFFICIENTS
Arithmetic	Ability	.458	.351
	Attitude	.534	.595
	H. S. Math	.556	.233
	College Math-completed	.562	.206
	Methods Course	.563	-.313
	College Math-required	.563	.101
	Type of College	.564	-.071
	Teaching Preference	.564	.030
	Constant Term		1.440
Algebra	Ability	.324	.128
	H. S. Math	.398	.160
	Attitude	.413	.166
	College Math-completed	.417	.182
	College Math-required	.422	-.232
	Teaching Preference	.423	-.073
	Type of College	.424	.072
	Methods Course	.425	-.065
	Constant Term		.849

TABLE 20.--(continued)

DEPENDENT VARIABLE (Y)	INDEPENDENT VARIABLE	MULTIPLE CORRELATION COEFFICIENT	REGRESSION EQUATION COEFFICIENTS
Geometry	H. S. Math	.300	.147
	College Math-completed	.379	.315
	Ability	.419	.110
	Attitude	.431	.186
	Type of College	.433	.091
	Methods Course	.435	-.164
	College Math-required	.436	.083
	Teaching Preference	.436	-.019
	Constant Term		-.378
Basic	Ability	.468	.444
	Attitude	.550	.719
	H. S. Math	.577	.332
	College Math-completed	.592	.394
	Methods Course	.593	-.305
	College Math-required	.594	.189
	Teaching Preference	.594	.027
	Type of College	.594	.014
	Constant Term		.802

TABLE 20.--(continued)

DEPENDENT VARIABLE (Y)	INDEPENDENT VARIABLE	MULTIPLE CORRELATION COEFFICIENT	REGRESSION EQUATION COEFFICIENTS
Related	H. S. Math	.344	.209
	Ability	.406	.146
	Attitude	.426	.229
	College Math-completed	.439	.311
	College Math-required	.442	-.237
	Methods Course	.445	-.236
	Teaching Preference	.446	-.089
	Type of College	.447	.078
	Constant Term		.111
Total Test	Ability	.473	.590
	Attitude	.558	.948
	H. S. Math	.596	.540
	College Math-completed	.612	.705
	Methods Course	.613	-.541
	Type of College	.613	.093
	Teaching Preference	.614	-.062
	College Math-required	.614	-.047
	Constant Term		1.910

geometry. The number of semesters of high school mathematics was the second variable to enter the algebra prediction equation. The first variable entering the prediction equation is the best single predictor of the dependent variable. The second variable entering the prediction equation is not necessarily the second best single predictor of the dependent variable, but in the algebra equation the amount of high school mathematics was the second best predictor and had a simple correlation of .322 with the algebra scores. Since the correlation of algebra and ability scores was .324, the difference in determining the best predictor of algebra was negligible. Since many of the prospective elementary teachers complete only one course in college mathematics, it is probable that they do not study much algebra and geometry in college. That they are dependent upon their high school background in these areas is shown by the prediction equations. The same pattern is repeated in the prediction equation for the related subtest scores. This is support for the conclusion that the prospective teachers are dependent upon their high school mathematics for their achievement in algebra and geometry since eleven of the sixteen related items are concerned with algebraic and geometric concepts.

The concepts included in the arithmetic subtest were probably studied by students in their first required course and thus studied by everyone. With somewhat equivalent backgrounds, ability became the best predictor of arithmetic

scores. Fifteen out of twenty-four of the basic items and one-half of the total test are arithmetic items. Therefore, the prediction equations for the basic and total test scores follow the pattern for arithmetic.

MATHEMATICS CONCEPTS TEST-ITEM ANALYSIS

The Mathematics Concepts Test is the very foundation of this study. It is the instrument used to measure the understanding of mathematical concepts of the students who have studied various amounts of required mathematics. The test is, therefore, the basis of all the comparisons and decisions made in the study. Design and statistical information concerning the test and test questions is presented in this section.

In designing the Mathematics Concepts Test there were two possible alternatives. One way would have been to assume that students who had studied the most mathematics were satisfactorily prepared to become elementary teachers. Using a trial sample of these students one could have then designed a test to obtain the expected results. With such a test, comparisons could then have been made among groups having various mathematics requirements to determine the suitability of preparation of those students having the lesser amounts of mathematics. However, the initial assumption would enter into all decisions. The second method for designing such a test would have been to decide what concepts should be understood by prospective elementary

teachers and to write questions testing this understanding. This method allows for the possibility that no amount of mathematics now specified is satisfactory preparation for elementary teaching. The second method was used in this study.

The design of the test and selection of concepts to be tested was based on certain criteria. The first criterion was a partition of test emphasis into areas of arithmetic or number systems, algebra, and geometry. The main emphasis of each test question refers to a concept from one of these three areas. However, some questions are naturally related to more than one area. The second criterion was to place the major emphasis of the test on basic concepts actually taught at the K-6 level. Twenty-four of the forty test questions are classified as basic. (See Appendix A for the classification of questions and for a listing of concepts being tested.) The remaining sixteen questions were designed to test some of the concepts recommended by CUPM for study in mathematics courses for elementary teachers. The concepts involved in these questions are related to concepts taught in the elementary school but are not actually taught at the K-6 level.

The procedure used in preparing the test was as follows. First, a list of basic concepts found in elementary school mathematics books and related concepts recommended by CUPM was prepared. Then approximately eighty questions were designed to test understanding of these

concepts. The number of questions was reduced to forty on the basis of consultations with mathematics educators. Rewording of some questions also took place at this time.

The questions were separated into sets of eight composed of four arithmetic, two algebraic, and two geometric questions. In each of these sets the eight questions were randomly distributed and then the five sets were randomly distributed to make up the test.

The forty item test was then given to students in an elective mathematics course for elementary teachers at the Dearborn Campus of The University of Michigan. This class was asked to comment on the test and test items in order to determine the presence of any difficulties in directions or wording. There were no conspicuous difficulties.

Most people reviewing the test rated it as difficult. Much of this difficulty was attributed to the related questions, however, since there was agreement that the basic questions involved concepts that every elementary teacher should know.

The final form of the test was printed commercially in order to enhance its appearance. It was felt that the appearance of the test would be a motivating factor in obtaining the student's best effort. Another motivating factor was the appeal as part of the general directions to the student's professional responsibility toward efforts for improvements in mathematics education. The teachers administering the test were requested to instruct the

students to read the directions.

Statistical information about the Mathematics Concepts Test was obtained from item analyses of 250 randomly selected students' responses from each of Groups 1, 2, and 3. (3, 6, and 8 or 9 semester hours of required mathematics, respectively). Means, median, standard deviations, ranges, and reliability coefficients are reported for each group in Table 21.

TABLE 21.--Means, medians, standard deviations, ranges, and reliability coefficients for Groups 1, 2, and 3 on the Mathematics Concepts Test.

STATISTIC	GROUP		
	1	2	3
Mean	15.15	15.85	16.37
Median	15.06	15.46	16.32
Standard Deviation	4.84	5.13	4.68
Range	4-32	3-35	3-30
Reliability Coefficient	.72	.74	.69

Frequency distributions of test scores for each group of 250 students are reported in Appendix C. In each case the distribution is nearly normal.

The discrimination power of each test item as determined for each group is given in Tables 22, 23, and 24. The discrimination power of any test item represents the difference between the percent of the upper and lower

TABLE 22.--Discrimination power of Mathematics Concepts Test items for Group 1

ITEM	PERCENT UPPER	PERCENT LOWER	DIFFERENCE
1	54	16	38
2	12	13	- 1
3	96	67	29
4	38	9	29
5	39	25	14
6	85	63	22
7	24	16	8
8	45	24	21
9	31	18	13
10	16	3	13
11	68	38	30
12	19	8	11
13	92	67	25
14	33	8	25
15	85	60	25
16	73	18	55
17	91	51	40
18	9	4	5
19	28	12	16
20	80	28	52

TABLE 22.--(continued

ITEM	PERCENT UPPER	PERCENT LOWER	DIFFERENCE
21	46	12	34
22	56	14	42
23	33	12	21
24	48	19	29
25	86	44	42
26	59	38	21
27	19	7	12
28	16	6	10
29	30	7	23
30	86	37	49
31	85	51	34
32	71	36	35
33	25	3	22
34	91	56	35
35	65	25	40
36	4	0	4
37	59	9	50
38	26	10	16
39	22	2	20
40	78	44	34

TABLE 23.--Discrimination power of Mathematics Concepts Test items for Group 2

ITEM	PERCENT UPPER	PERCENT LOWER	DIFFERENCE
1	63	24	39
2	30	19	11
3	98	78	20
4	51	8	43
5	51	26	25
6	95	60	35
7	31	22	9
8	43	25	18
9	28	15	13
10	20	7	13
11	60	42	18
12	53	20	33
13	86	54	32
14	28	4	24
15	83	45	38
16	72	26	46
17	95	63	32
18	10	1	9
19	33	15	18
20	79	40	39

TABLE 23.--(continued)

ITEM	PERCENT UPPER	PERCENT LOWER	DIFFERENCE
21	49	16	33
22	62	13	49
23	31	8	23
24	39	15	24
25	81	45	36
26	60	26	34
27	14	2	12
28	18	6	12
29	31	13	18
30	84	46	38
31	85	50	35
32	75	44	31
33	28	8	20
34	91	57	34
35	65	12	53
36	12	3	9
37	62	10	52
38	34	6	28
39	22	12	10
40	83	31	52

TABLE 24.--Discrimination power of Mathematics Concepts Test items for Group 3

ITEM	PERCENT UPPER	PERCENT LOWER	DIFFERENCE
1	59	27	32
2	25	21	4
3	91	71	20
4	68	19	49
5	49	33	16
6	96	62	34
7	24	18	6
8	56	37	19
9	53	16	37
10	22	4	18
11	68	44	24
12	7	7	0
13	92	63	29
14	20	8	12
15	81	34	47
16	65	27	38
17	95	67	28
18	13	1	12
19	57	14	43
20	72	38	34

TABLE 24.--(continued)

ITEM	PERCENT UPPER	PERCENT LOWER	DIFFERENCE
21	39	12	27
22	69	25	44
23	25	13	12
24	51	15	36
25	90	59	31
26	43	22	21
27	13	12	1
28	9	6	3
29	13	6	7
30	87	57	30
31	89	57	32
32	78	46	32
33	45	3	42
34	89	66	23
35	60	13	47
36	3	0	3
37	46	9	37
38	68	27	41
39	9	3	6
40	83	53	30

one-thirds of the group responding correctly to the item. Every question except question 2 was a positive discriminator for each group. Question 2 showed a -1 difference for Group 1 but was a positive discriminator for Groups 2 and 3. Questions are often rated as good discriminators if the difference lies between 30% and 70%. Twenty-six of the forty questions would be rated as good discriminators using this criterion. This does not mean that the fourteen other questions were necessarily poor discriminators, since in many of these cases the total number responding correctly was so low that the difference could not be 30%.

Two patterns are evident in Tables 22, 23, and 24 of discrimination powers. One pattern shows certain algebraic and geometric questions to be the most discriminating in Groups 2 and 3, respectively. Since algebra and geometry are usually emphasized in second and third courses, this result was somewhat expected. The other trend shows certain questions to be high discriminators in Group 1 but not as high in Groups 2 and 3. Examination of the percent correct for upper and lower one-thirds of the groups shows that nearly the same percent of the upper groups is responding correctly and a higher percent of the lower groups is responding correctly. This may be evidence that courses for elementary teachers are aimed at the lower achieving students.

The discrimination power between all possible pairs of Groups 1, 2, and 3 for the items of the Mathematics Concepts Test is presented in Table 25. The entries represent the differences in the percent of correct responses for each item between the two groups as designated in the headings. For example, using item number 4 the 18 in the $G_3 - G_1$ column represents an 18% greater correct response by Group 3 than by Group 1. In the next column the 15 likewise represents a 15% greater correct response by Group 3 than by Group 2. Since item four is a geometry question, this is evidence that students are learning some geometry in a third required course. Other geometry items showing this same pattern are numbers 9, 19, 32, 33, and 38. Item 22 has the same discrimination pattern but is an arithmetic item.

The negative discriminators in Table 25 represent a greater correct response rate by the second mentioned group rather than the first one. For example, item 12 has a -23 under the $G_3 - G_2$ column. This means that Group 2 had a 23% greater correct response than Group 3. For the same item Group 2 has a 16% greater correct response than Group 1. Item 12 is an algebraic item involving modular arithmetics. This topic is probably studied in a second required course and students evidently do best on the topics they have just studied. Items 7 and 20 are the other algebra items showing the same pattern.

TABLE 25.--Discrimination power between groups of items in the Mathematics Concepts Test

ITEM NUMBER	$G_3 - G_1$	$G_3 - G_2$	$G_2 - G_1$
1	8	10	-2
2	3	- 6	9
3	2	- 2	4
4	18	15	3
5	6	8	-2
6	1	1	0
7	0	- 9	9
8	6	8	-2
9	15	15	0
10	3	0	3
11	-2	1	-3
12	-7	-23	16
13	3	2	1
14	-4	0	-4
15	-3	4	-7
16	1	2	-1
17	0	2	-2
18	1	0	1
19	15	15	0
20	0	- 6	6

TABLE 25.--(continued)

ITEM NUMBER	$G_3 - G_1$	$G_3 - G_2$	$G_2 - G_1$
21	1	-8	9
22	14	14	0
23	-2	-2	0
24	-3	0	-3
25	6	8	-2
26	-7	-7	0
27	-1	0	-1
28	-4	-4	0
29	-3	-3	0
30	11	8	3
31	8	-1	9
32	8	8	0
33	9	12	-3
34	4	5	-1
35	0	3	-3
36	-2	-4	2
37	-3	-3	0
38	31	30	1
39	-4	-4	0
40	8	9	-1

The rank orders of difficulty for the items in the Mathematics Concepts Test for Goups 1, 2, and 3 are presented in Table 26 . The easiest item has rank 1; the most difficult has rank 40. The basic items were the easiest in the test as evinced by eight of the first ten ranks being basic questions. Seven out of the ten most difficult items were related items. Nine of the ten easiest items involved arithmetic or number systems. The arithmetic, algebraic, and geometric items after the ten easiest appear randomly distributed.

The percent of students responding to each choice for each item in the Mathematics Concepts Test are found in Appendix C. High rates of incorrect responses are sometimes indicative of the students' misunderstandings. For example, on test item 8, it is evident that many students associated multiplication only with Cartesian products of sets and missed the idea of multiplication as repeated addition shown by the union of disjoint sets. Approximately one-half of the test questions showed a high percent of the incorrect responses as a particular choice.

TABLE 26.--Rank order of item in the mathematics concepts test for Groups 1, 2, and 3

	GROUP 1		GROUP 2		GROUP 3	
RANK	MEAN PCT. CORRECT	ITEM NUMBER	MEAN PCT. CORRECT	ITEM NUMBER	MEAN PCT. CORRECT	ITEM NUMBER
1	82	17	86	3	84	3
2	82	3	80	17	82	17
3	80	6	80	6	82	34
4	78	34	79	13	81	6
5	78	13	77	34	81	13
6	72	15	75	31	78	30
7	67	25	70	30	74	31
8	67	30	65	15	73	25
9	67	31	65	25	71	40
10	63	40	62	40	69	15
11	57	32	60	20	65	32
12	55	11	57	32	54	20
13	54	20	52	11	53	11
14	48	16	47	16	52	38
15	43	26	43	26	49	16
16	40	35	37	8	49	22
17	39	8	37	1	47	1
18	39	1	37	35	45	8
19	35	22	35	22	42	4
20	35	5	34	21	41	5

TABLE 26.--(Continued)

	GROUP 1		GROUP 2		GROUP 3	
RANK	MEAN PCT. CORRECT	ITEM NUMBER	MEAN PCT. CORRECT	ITEM NUMBER	MEAN PCT. CORRECT	ITEM NUMBER
21	34	24	33	5	40	35
22	33	37	33	37	38	9
23	25	21	31	12	37	19
24	24	4	31	24	36	26
25	23	9	29	7	31	24
26	22	19	27	2	30	37
27	21	38	27	4	26	33
28	20	23	23	9	26	21
29	20	7	22	38	21	2
30	20	14	22	19	20	7
31	18	2	20	23	18	23
32	17	33	16	14	16	14
33	15	12	15	29	13	10
34	15	29	14	33	12	27
35	13	27	13	10	12	29
36	12	39	12	39	8	39
37	11	28	12	27	8	12
38	10	10	11	28	7	28
39	5	18	7	36	6	18
40	5	36	6	18	3	36

CHAPTER IV

SUMMARY AND CONCLUSIONS

This chapter contains a brief restatement of the problem, the findings and conclusions relevant to the stated hypotheses and questions, and suggestions for further research.

RESTATEMENT OF THE PROBLEM

This study was designed to obtain evidence concerning the relative effectiveness of three different mathematics requirements of elementary education as preparation for teaching today's elementary curriculum. The measure of understanding of mathematical concepts from the areas of arithmetic, algebra, and geometry was obtained by testing students in their last required mathematics courses at thirteen colleges. These colleges required from three to nine semester hours of mathematics for prospective elementary teachers.

The study was concerned with conceptual understanding in the areas of arithmetic, algebra, and geometry, since concepts from these areas are commonly found in the elementary curriculum. Hence, the items are classified as arith-

metic, algebraic, and geometric. With respect to preparation for the teaching of today's curriculum, some of the items measured understanding of basic concepts actually taught in the elementary school today. The remaining items tested concepts related to the basic concepts. Therefore, the test items are also classified as basic and related. Satisfactory preparation of prospective teachers of elementary school mathematics was defined to be a correct response to at least 75% of the basic items on the test. A concern related to the measuring of the effectiveness of college mathematics requirements is the question, what is the minimum number of required semester hours in mathematics for propective elementary teachers which will produce satisfactory understanding.

Other aspects of the study concerned the relationships of individual differences due to various variables and student achievements.

FINDINGS AND CONCLUSIONS

To measure the relative effectiveness of the various mathematics requirements in the areas of arithmetic, algebra, and geometryas well as for the concept classifications of basic, related, and total test, comparisons of the achievements by Group 1 (3 semester hours), Group 2 (6 semester hours), and Group 3 (8 or 9 semester hours) were made for each achievement category. The comparisons were based

on analyses of variance and covariance. In order to make these comparisons most meaningful the analyses were completed not only for the entire sample but also for Groups 1, 2, and 3 from each of the following classifications: large university, large college, small teachers college, high ability, middle ability, and low ability.

For the statistical analyses the questions to be investigated were stated in the form of null hypotheses to be rejected or accepted. The forty-two null hypotheses are considered in the following subsection for the total sample, by types of college, and by ability levels.

Hypotheses 1 - 6: Total sample

There are no significant differences among Groups 1, 2, and 3 for students of the total sample as to mathematical understanding of (1) arithmetic concepts, (2) algebraic concepts (3) geometric concepts, (4) basic concepts, (5) related concepts, (6) the total combination of mathematical concepts involved in this study.

Findings. There are significant differences beyond the .99 confidence level among Groups 1,2, and 3 for each of the six sets of mathematical concepts as evidenced by the analyses of variance and the analyses of covariance reported in Chapter Three. There are significant differences also for pairs of groups in each achievement category according to a post hoc test of differences used after analysis of variance or covariance.

Conclusions. Hypotheses 1-6 are rejected on the basis of the findings. The means for the arithmetic concepts of Groups 1, 2, and 3 show an increase corresponding to increased mathematics requirements. Group 2 has the highest mean for algebra and Group 3 has the highest mean in geometry which indicates that students do best on concepts from these areas while they are currently studying them; however, the gains in arithmetic means corresponding to increased requirements are important because they indicate that prospective elementary teachers can attain increased understanding of arithmetic concepts through repeated exposure to these concepts in different ways. Perhaps even greater gains in arithmetic and similar gains in algebra and geometry would be made if related concepts from these three areas were unified, sequenced, and taught throughout all the required mathematics courses for elementary teachers. The algebra and geometry means indicate that the emphasis is now on algebra course two and on geometry in course three.

The basic and total test categories contain more arithmetic concepts than algebraic or geometric concepts and the results for these two categories are the same as for the arithmetic means. The related category has seven algebraic items out of sixteen and shows the same pattern as mentioned above for algebra.

Even though there are significant differences among the means for all acheivement categories, these differences are still quite small. The mean achievements indicate that there is much room for improvement in the understanding

of mathematical concepts by prospective elementary teachers.

Hypotheses 7-24: Type of College

There are no significant differences among Groups 1, 2, and 3 for students from (Hypotheses 7-12) large universities, (Hypotheses 13-18) large colleges, and (Hypotheses 19-24) small teachers colleges as to mathematical understanding of arithmetic concepts, algebraic concepts, geometric concepts, related concepts, and the total combination of mathematical concepts involved in this study.

Findings. The analyses of variance and the analyses of covariance indicate significant differences beyond the .99 confidence level among the large university group means for the algebra and related categories. The analysis of variance and the analysis of covariance using ability as the covariate do not indicate significant differences among the large university group means for the four other achievement categories; however, the analysis of covariance using high school mathematics background as the covariate does indicate significant differences among the large university group adjusted means for the four categories of arithmetic, geometry, basic, and total test.

There are significant differences among the large college group means for every achievement category in the analyses of covariance. In the analysis of variance significant differences are indicated for each category other

than algebra.

Among the small college group means there are significant differences for all categories of achievement in all the analyses of variance and covariance.

Conclusions. Hypotheses 8 (algebraic concepts) and 11 (related concepts) are not accepted on the basis of the findings. Hypotheses 7 (arithmetic concepts), 9 (geometric concepts), 10 (basic concepts), and 12 (total test concepts) are accepted because significant differences do not occur in two out of three analyses. Accepting the null hypotheses does not mean that there are no differences but only that the probability of the differences being caused by the treatment is less than a specified confidence level. For the algebra and related categories, which have significant differences among the large university group means, the highest mean is attained by Group 2 in each case; the lowest mean is attained by Group 3 in each case. The Group 2 high algebra mean continues the pattern for the category of algebra.

The lack of significant differences among large university Groups 1, 2, and 3 in four of the six achievement categories indicates inefficient use of increased mathematics requirements. Normally increased understanding of mathematical concepts is expected with increased study of mathematics. Even though the differences were not significant, the attainment by Group 3 of the lowest mean in

every achievement category is a reversal of what is usually expected. Group 2 attained the highest mean in each case. Since Group 2 had the highest ability level and Group 3 had the lowest ability level among the large university groups, the analysis of covariance using ability scores as the covariate is an important analysis to examine. The results of the analysis of covariance, however, indicate no significant differences for the categories of arithmetic, geometry, basic, and total test. The adjustments in the group means do cause changes so that Group 3 has the lowest adjusted mean in only four out of the six categories. Since students from large universities are of high ability and are not making significant gains in mathematical understanding corresponding to increased mathematics requirements, it is probable that the courses are aimed at the low and middle ability students. Study of ways of providing for the high ability prospective elementary teachers' special needs and possibilities is highly recommended. This should be a special concern of large universities.

Hypotheses 13 - 24 concerning significant differences among large college and small college groups for the achievement categories are rejected on the basis of the findings.

For the large colleges the high means attained by Group 3 in every category except algebra support the assumption of increased understanding of mathematical concepts with increased requirements in mathematics. In the case of

algebra the high mean was attained by Group 2 which shows again that students do best in what they are currently studying. A comparison of the large college Group 3 means with the large university or the small college group means shows the large college Group 3 to have the highest mean in every category except algebra and related Even though the large college groups compare favorably with groups from other types of colleges, the total achievement is quite low and leaves much room for improvement.

The small college groups follow the pattern of Group 2 having the highest mean in algebra and Group 3 having the highest mean in geometry. However, Group 2 also has the highest mean for arithmetic, basic, related, and total test items. The achievement by the small college Group 2 may be due to the higher ability of this group as compared to the other small college groups. Also Group 2 consists of students from two colleges; the students from the larger of these two schools had higher ability scores and more high school mathematics background. The corresponding higher achievement of the students from the larger school is the cause of the higher means attained by Group 2 and therefore, the differences among the small college groups are partially attributed to differences among colleges. The means for students from small colleges for Groups 1 and 3 and for the smaller of the colleges in Group 2 are much lower than the means for students from large colleges and universities. Certainly part of the lower achievement may be attributed

to the lower abilities and less adequate mathematics backgrounds of these students.

The results of the analyses by types of colleges indicate the dependence of achievement upon ability. The truth of this is ascertained by the analyses using ability levels (Hypotheses 25-42). The large universities and large colleges need to examine their mathematics offerings for elementary teachers to make sure that their high ability students are profiting from their courses. The small teachers colleges with more of the lower ability students need to find ways of increasing the mathematical understanding of their students. The small gains corresponding to increased mathematics requirements implies the need for a more creative prescription than just more mathematics.

Hypotheses 25-42: Ability levels

There are no significant differences among Groups 1, 2, and 3 for students of (25-30) high ability, (31-36) middle ability, and (37-42) low ability as to mathematical understanding of arithmetic concepts, algebraic concepts, geometric concepts, basic concepts, related concepts and the total combination of mathematical concepts involved in this study.

Findings. In the analyses of variance among Groups 1, 2, and 3 of high ability students there are significant differences in the categories of algebra, geometry, basic, and related concepts but there are no significant differences

in the categories of arithmetic and for the total test. The analyses of variance for middle ability students indicate significant differences among the groups for every achievement category with a confidence level beyond .99. The differences among the low ability group's achievements in each category are also significant beyound the .99 confidence level in the analyses of variance and covariance.

Conclusions. Hypotheses 25 (arithmetic concepts) and 30 (total test concepts) for high ability students are accepted on the basis of the various analyses. All other hypotheses, 26-29 and 31-42, are rejected on the basis of the findings.

The lack of significant differences among high ability Groups 1, 2, and 3 on the arithmetic concepts shows that high ability students learn such concepts upon their first introduction to them. The algebra and geometry means for high ability students follow the pattern of other classifications in that Group 2 has the highest algebra mean and Group 3 has the highest geometry mean. On the basic items, Group 3 has the highest mean as expected. However, it is disturbing to note that the high ability students with three courses in college mathematics could only answer 58% of the basic questions. The means for the related category show Group 2 with the highest mean. This follows the pattern for algebraic concepts, as expected, since the area of algebra makes up a large part of

the related category. The absence of significant differences among the total test means for Groups 1, 2, and 3 is partly due to the small differences between any two groups for the categories in which significant differences exist. The lack of increase on the total test score for the high ability students indicates that the mathematics courses for elementary teachers are not aimed at the high ability students. Certainly some provision should be made, however, to provide high ability students with the opportunity and stimulation to make significant gains in mathematical understanding corresponding to increased requirements.

The significant differences among middle ability and low ability Groups 1, 2, and 3 for every achievement category and the increases in means corresponding to increased mathematics requirements for every category except algebraic and related concepts indicate that the mathematics courses for elementary teachers are aimed at the middle and low ability students. The gains, even though small, do indicate that low and middle ability students profit from taking more mathematics. The low ability students make the greatest overall gains between Group 1 and Group 3; however, the low ability Group 3 means are less than the middle ability Group 1 means in every case except geometry.

The different achievements of the various ability groups creates a very real problem for the instructors of mathematics courses for elementary teachers. There is a need for a study of the ways of providing for the differences in

ability, achievement, and background of prospective elementary teachers.

GENERAL PROBLEM

The general problem of this study questioned whether or not the colleges with their different mathematics requirements were satisfactorily preparing teachers for today's curriculum. For the purpose of this study, satisfactory preparation was defined as at least 75% achievement on the basic questions of the Mathematics Concepts Test.

In every breakdown of students into Groups 1 (3 semester hours), 2 (6 semester hours), or 3 (9 semester hours) and partitioning by types of college or ability levels, the average achievement on the set of twenty-four basic questions was less than 60%. Thus, using the above definition for satisfactory preparation, the answer to the general problem is that the colleges are not satisfactorily preparing elementary teachers. Of course, the selection of 75% was an entirely arbitrary choice. However, if the level of expected achievement had been set as low as 50% the majority of students would not have been satisfactorily prepared as evidenced by the means on the basic concepts for the various classifications shown in Table 27.

Of the 2513 students tested in this study, 108 correctly answered 18 or more of th 24 basic questions for a 75% or higher rate of achievement. This group of 108 students had approximately seven semesters of high school mathematics, three college courses in mathematics, and an

average score on the short ability test of 15 (possible 20).

TABLE 27.--Achievement means on basic concepts (possible score 24) for Groups 1, 2, and 3 from each classification

CLASSIFICATION	GROUP 1	GROUP 2	GROUP 3
Large University	11.41	12.00	11.55
Large College	11.97	10.34	12.72
Small College	6.57	10.86	10.23
High Ability	13.36	12.89	14.03
Middle Ability	11.01	11.43	12.64
Low Ability	8.72	9.12	10.82
Total Sample	10.95	10.94	12.33

This group's averages indicate a high ability and an extensive mathematics background. There are exceptions, however, as indicated by the ranges of these variables. The high school mathematics background ranged from three to nine semesters; the college mathematics completed varied from a first required course to nine or more courses; and the ability scores range from eight to nineteen. The achievement on the basic questions by the 108 students indicates that the basic concepts are being presented in courses for elementary mathematics teacher; however, only 4% of the students are obtaining a satisfactory understanding of these concepts. Perhaps the presentations are too rigorous for

the elementary education students.

The question of whether or not prospective elementary teachers are receiving training in algebra and geometry as well as arithmetic can only be partially answered. That algebraic and geometric concepts are being presented in second and third courses, respectively, is evidenced by the higher means by Group 2 in algebra and by Group 3 in geometry. That the prospective teachers are not understanding these concepts is evidenced by the low achievement in these areas. The algebra means for the various classifications represent approximately 25% to 46% achievements and the geometry means represent 15% to 44% achievement. The high means do not indicate satisfactory preparation. The low means were achieved by students in a first course and indicate the need for further study of algebra and geometry.

RELATED PROBLEM

The related question, what is the minimum number of required hours of mathematics for the satisfactory preparation of prospective elementary teachers for today's elementary curriculum, cannot be answered specifically. None of the various requirements now in use is sufficient to supply the desired understanding of mathematical concepts. A possible conclusion is that the required number of hours of mathematics must be greater than nine semester hours. However, upon careful examination of the differences

in means (many of them significant beyond the .99 confidence level) one finds the averages to increase very slowly or in some cases decrease with the increase of mathematics requirements. Extrapolation of increased requirements to obtain the desired amount of understanding indicates the need for as many as eight courses or twenty-four semester hours of mathematics.

Examination of the evidence shows students of low ability who completed three required courses to have a lower average achievement than middle ability students with one required course, and, likewise, middle ability students with three courses attain lower means than high ability students with one course. These results give an indication of the widespread differences in achievement that would result if as many as eight courses were required. Actually, the differences in abilities are a problem in the present day requirements, and the high ability students are the ones being slighted. The solution may not be in the direction of increased requirements for everyone. The evidence indicates that many students planning to become elementary teachers have little understanding of mathematical concepts. There is reason to believe that many students are unable to grasp the desired concepts. Perhaps the solution is to extend to the elementary level the idea of having special teachers of mathematics such as is done at the secondary level.

That some progress has been made toward the training of special mathematics teachers for the elementary level is evidenced by communications from some of the schools participating in this study. These schools reported that they have elective mathematics courses for elementary teachers, and in some of these schools the elementrary teachers have to complete a concentration in some academic area. Mathematics is one of the areas which may be elected for a concentration.

The remaining aspect of this study was the consideration of individual differences and their relationships, especially to achievement. As expected the best predictor of overall achievement was ability; however, in the case of geometric and related concepts the best predictor was the number of semesters of high school mathematics completed. The other variables which were of most importance in the prediction of achievement were attitude and the amount of college mathematics completed. The intercorrelations among these four variables were all positive and were the highest meaningful correlations found in this study. These variables should be condidered in any attempt to find a solution to the problem of providing for the individual differences among the students in the mathematics courses for elementary teachers.

SUMMARY

The various conclusions obtained from this study are

summarized here for clarity and emphasis. The conclusions are restricted to the schools of the sample, since the selection of the sample was not random.

1. Propective elementary teachers of mathematics are not being satisfactorily prepared by a three, six, or nine semester hour mathematics requirement.
2. The overall understanding of mathematical concepts by propective elementary teachers is very low.
3. The number of courses needed for satisfa ct ory preparation of prospective elementary teachers cannot be determined from this study; however, the number needed may be far beyond current recommendations.
4. Algebraic concepts are being presented in a second course for elementary teacher and geometric concepts in a third course. Students do best on these concepts while they are studying them.
5. Students continue to make gains in arithmetic understanding by studying more mathematics.
6. The mathematics courses for elementary teachers are aimed at low and middle ability students.
7. There are widespread differences in abilities and mathematical backgrounds of the students in the mathematics courses for elementary teachers.
8. The high school mathematics background of a prospective elementary teacher is an important factor in his college mathematics achievement, especially in algebra and geometry.

9. Ability, attitude, and high school and college mathematics background are the most important variables in predicting prospective elementary teachers' achievement in mathematics.
10. The preparation of prospective elementary teachers at different types of colleges poses different problems caused by the differences in ability of their students.
11. There is a need for a more creative solution to the problem of preparing prospective elementary teachers.

RECOMMENDATIONS FOR FURTHER RESEARCH

The low mathematical achievement by prospective elementary teachers shows that many students are not understanding the concepts they will be teaching in the elementary school. This suggests the possibility of the need for special teachers of mathematics at the elementary level. Research is needed to determine the effect of special mathematics teachers (and probably for other academic areas also) upon the elementary students as compared to the teacher who teaches the self-contained classroom. Such research needs to consider not only the psychological aspects of the student-teacher relationship but also the academic value of the subject being studied.

A second consideration for research with respect to the mathematics training of prospective elementary teachers concerns the apparent separation of the areas of arithmetic,

algebra, and geometry. The integration of the three areas and the application of the spiral curriculum idea is suggested as possibly being a much more efficient use of mathematics requirements.

If all students continue to be required to take the same mathematics courses, then research concerning methods of providing for widespread individual differences of prospective elementary teachers is needed. The present study indicated ability, attitude, amount of high school mathematics, and amount of college mathematics to be the best predictors of achievement by prospective elementary teachers. There may also be other important variables to be analyzed.

The basis for these suggestions is the concern for the lack of mathematical understanding by prospective elementary teachers. They are not prepared to teach today's elementary mathematics curriculum. Many questions need answers before progress can be made toward curriculums suggested for tomorrow.

APPENDIX A

NAME _____
(FIRST) (INITIAL) (LAST

GENERAL DIRECTIONS

This test is part of a research project involving 2,500 prospective elementary teachers in fourteen colleges. The test is designed for students from colleges requiring one, two, three, or four mathematics courses. Therefore, some questions may involve concepts that you have not studied. Your best score will be obtained by first answering the questions that you know and then returning to those about which you have doubts. You should consider every question in the test but should not spend an excessive amount of time on any one. There is no penalty for guessing.

The results of the test will be used to suggest ways of providing the most appropriate mathematical background for all prospective elementary teachers. It is expected that you will answer all questions to the best of your ability in order to make the results most meaningful and beneficial to the teaching profession.

- -

PART I.

1. Circle the number of semesters of high school (9-12) mathematics that you completed.

   0 1 2 3 4 5 6 7 8 more than 8

2. Circle the number of college courses in mathematics completed other than this one.

   0 1 2 3 4 5 6 7 8 more than 8

3. Have you completed or are you now taking a methods course for the teaching of elementary mathematics?

_____NO _____YES

4. At which of the following grade levels do you desire to teach? (Check one.)

_____K-3 _____4-6 _____7-8

5. Circle the word or words which best describes your feeling toward mathematics.

Strong dislike Dislike Neutral Like Strong like

6. Check your present college class standing.

____Freshman ____Sophomore ____Junior ____Senior

____Other (graduate, special, etc.)

PART II. SHORT ABILITY TEST*

DIRECTIONS: In the blank place the letter representing the word or phrase which is nearest in meaning to the given word.

_____1. IMPAIR:

(A) make uneven (B) put in order

(C) make worse (D) double (E) insinuate

_____2. ASSENT:

(A) protest (B) agree (C) deliver

(D) rest (E) retard

*This is a modified version of an experimental test copyright © 1955 by Educational Testing Service. All rights reserved. Reprinted by special permission of Educational Testing Service.

_____3. CREDENCE:

(A) philosophy of life (B) view

(C) background (D) reputation (E) belief

_____4. DIPLOMACY:

(A) guile (B) graduation

(C) secrecy (D) tact

(E) arbitration

_____5. ASSUAGE:

(A) to cover up for (B) give pleasure to

(C) convince (D) make less harsh

(E) to depress

_____6. ACKNOWLEDGE:

(A) assure (B) favor (C) yield

(D) learn (E) admit

_____7. INCONGRUOUS:

(A) out of place (B) paradoxical

(C) not logical (D) foolish (E) silly

_____8. LIMPID:

(A) soft (B) tranquil (C) dull

(D) clear (E) flexible

_____9. FOSTER:

(A) nourish (B) boil over (C) indorse

(D) surmount (E) solicit

_____10. DIRE:

(A) far-off (B) contorted

(C) death-like (D) prophetic

(E) extremely bad

_____11. APPREHENSION:

(A) attachment (B) regret (C) dread

(D) suffering (E) gratitude

_____12. OSTENSIBLE:

(A) practical (B) apparent (C) obvious

(D) showy (E) actual

Indicate the correct answer to the following problems.

_____13. $4\frac{1}{4} - 3\frac{1}{3} =$

(A) $\frac{3}{4}$ (B) $\frac{5}{6}$ (C) $\frac{11}{12}$

(D) $1\frac{1}{12}$ (E) $1\frac{11}{12}$

_____14. $.02\overline{)632}$

(A) 3.16 (B) 31.6 (C) 316

(D) 3160 (E) 31,600

_____15. $1\frac{1}{2}\%$ of 500

(A) .75 (B) 75 (C) 750

(D) 7500 (E) none of these

_____16. 8 pounds 4 ounces $\div$ 3 =

(A) 2 pounds $1\frac{1}{3}$ ounces

(B) 2 pounds 2 ounces

(C) 2 pounds 8 ounces

(D) 2 pounds 12 ounces (E) none of these

_____17. Change $1\frac{1}{4}\%$ to a decimal.

(A) .00125 (B) .0125 (C) .125

(D) 1.25 (E) 125

_____18. 0.1% of $2500.00

(A) $2.50 (B) $25.00 (C) $250.00

(D) $25,000.00 (E) none of these

_____19. 6/ 9 feet 6 inches

(A) 1 foot $1\frac{1}{2}$ inches (B) 1 foot 6 inches

(C) 1 foot 7 inches (D) 1 foot 9 inches

(D) none of these

_____20. Write $\frac{1}{500}$ as a decimal.

(A) .0002 (B) .0005 (C) .002

(D) .005 (E) .02

PART III. MATHEMATICS CONCEPTS TEST

DIRECTIONS: In the blank place the letter indicating the correct answer to the problem, question, or statement. (There is only one correct response.)

_____1. In 19,534 the number named by 9 is how many times the number named by 3?

(A) three (B) one hundred (C) three hundred
(D) one thousand (E) three thousand

_____2. Any two infinite sets can be:

(A) placed in a 1-1 correspondence with each other.

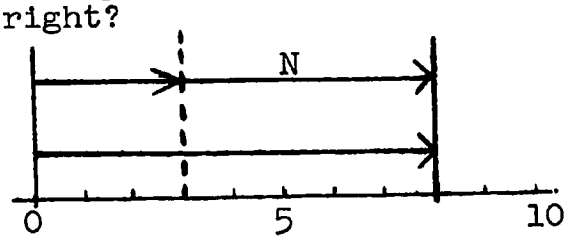
(B) placed in a 1-1 correspondence with every subset of themselves.

(C) placed in a 1-1 correspondence with the whole numbers.

(D) shown to have subsets which are infinite.

(E) shown to have an intersection set which is infinite.

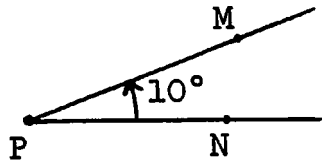
_____3. Which number sentence is represented by the diagram at the right?



(A) 5 + N = 8
(B) 3 + N = 8
(C) 3 - N = 5
(D) 5 - N = 3
(E) N = 8

_____4. If $\angle MPN$ is called a unit angle, then the sum of the measures of the angles of any triangle is:

(A) 3 units
(B) 9 units
(C) 18 units
(D) 36 units
(E) 180 units



_____5. If the following polygons had the same perimeter, then the one enclosing the maximum area would be the:

(A) triangle (B) square (C) trapezoid
(D) regular pentagon (E) regular hexagon

_____6. A team won 75% of its games for the year. If the team won 12 of its games, how many did it play?

(A) 9 (B) 15 (C) 16 (D) 18 (E) 21

_____7. In a modular (clock) arithmetic the field properties hold:

(A) if the modulus is any positive integer other than 1.
(B) if the modulus is a composite integer.
(C) if the modulus is a prime integer.
(D) if the modular arithmetic contains zero divisors.
(E) if the modulus is relatively prime to 2.

_____8. The multiplication of 3 times 5 may be represented by sets using:

(A) the union of three mutually disjoint sets each containing five elements.

(B) the intersection of three sets each containing five elements.

(C) the Cartesian product of three sets each containing five elements.

(D) the Cartesian product of two sets each containing fifteen elements.

(E) the union of any five sets each containing three elements.

_____9. In spherical geometry, lines are defined as great circles on a sphere. Using this definition, determine which statement is <u>true</u> in spherical geometry.

(A) All lines intersect.

(B) No lines intersect.

(C) For any given line, there is a unique parallel line passing through any given point.

(D) All lines are parallel.

(E) There is a unique line passing through any two points.

_____10. In the partial product, 7542 of the partially completed multiplication problem at the right, the encircled 7 represents:

$$\begin{array}{r} 2514 \\ \times\quad 31 \\ \hline 2514 \\ \textcircled{7}542 \\ \hline \end{array}$$

(A) multiplying 3 x 2 and adding 1

(B) 7,000

(C) $(3 \times 10^1) \times (2 \times 10^3) + 1$.

(D) $(3 \times 10^1) \times (2 \times 10^3) + (1 \times 10^4)$.

(E) $(3 \times 10^1) \times (2 \times 10^3) + (1 \times 10^3)$.

_____11. Which equation represents the following situation? A turkey must be baked 35 minutes per pound at a temperature of 350 degrees. The turkey weighs 12 pounds and has been in the oven for an hour. How many hours longer should it be baked?

(A) $\frac{(12 \times 35)}{350} - 1 = X$

(B) $X + 1 = \frac{12 \times 35}{60}$

(C) $(12 \times 35) + 60 = X$

(D) $(12 \times 35) - X = 1$

(E) $(12 \times 35) - (1 \times 350) = X$

_____12. The solution set of $2X = 2$ (modulo 6) is:

(A) $\emptyset$ (B) $\{1\}$ (C) $\{1,3\}$ (D) $\{1,4\}$

(E) $\{1,2,4\}$

_____13. If a, b, and c are real numbers such that $a \div b = c$ then:

(A) $a.b = c$ (B) $a \cdot c = b$ (C) $b \cdot c = a$

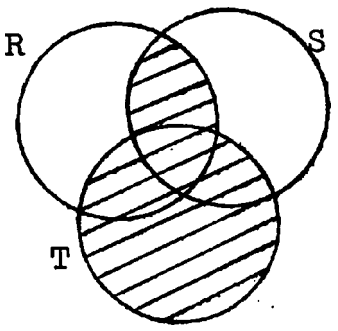
(D) $c \div b = a$ (E) $c \div a = b$

_____14. In the base seven numeral, 16234_{seven}, the number named by 6 is how many times the number named by 3?

(A) twice (B) fourteen (C) forty-nine

(D) ninety-eight (E) two hundred

_____15. The shaded portion of the Venn diagram to the right represents:



(A) $R \cap (S \cup T)$

(B) $(R \cap S) \cup T$

(C) $(R \cup S) \cap T$

(D) $(R \cap T) \cup (S \cap T)$

(E) $(R \cup T) \cap S$

_____16. The area of triangle ABC is found by:

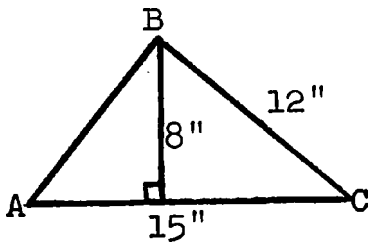
(A) adding 8, 12, and 15.

(B) multiplying 8 by 15.

(C) multiplying 8 by 12 and dividing by 2.

(D) multiplying 8 by 15 and dividing by 2.

(E) multiplying 8 by 15 and dividing by $\frac{1}{2}$.



_____17. Which numeral names the number represented in the chart?

THOUSANDS	HUNDREDS	TENS	ONES
2	12	8	23

(A) 212,823 (B) 21,231 (C) 32,103

(D) 3,303 (E) 3,231

_____18. If p, q, and r represent distinct positive prime integers, then the integer formed from their product, namely p x q x r, has how many divisors greater than 1?

(A) 1 (B) 3 (C) 6 (D) 7 (E) 8

_____19. Which of the following is <u>false</u>?

(A) A point separates a line into two half-lines.

(B) A line separates a plane into two half-planes.

(C) A plane separates space into two half-spaces.

(D) A ray separates a plane into two half-planes.

(E) An angle separates a plane into two planar regions.

_____20. If $|x - 2| < 3$ and x is an integer then the solution set is:

(A) $\{1,2,3\}$ (B) $\{1,2\}$ (C) $\{-2, -1, 0, 1, 2\}$

(D) $\{-1, 0, 1, 2, 3, 4, 5\}$ (E) $\{0, 1, 2, 3, 4\}$

_____21. Which of the following subsets of the integers is closed with respect to addition?

(A) the set of odd integers

(B) the set of primes

(C) the set of composites

(D) $\{-1, 0, 1\}$

(E) the set of even integers

_____22. Consider the partially completed division problem at the right. The 1 indicates:

$$23\overline{)475}\quad 2,\ 46,\ 1$$

(A) a remainder of 1 after two 23's have been subtracted from 47.

(B) a remainder of 1 after twenty 23's have been subtracted from 47.

(C) a remainder of 1 after 46 has been subtracted from 47.

(D) a remainder of 10 and some more after 460 has been subtracted from 475.

(E) that 10 is a possible replacement for $\underline{r}$ in $475 = (23 \times q) + r$.

_____23. The graph of $2x - 2y = 5$ has a slope of :

(A) 1 (B) -1 (C) $\frac{2}{5}$ (D) $\frac{-2}{5}$

(E) $\frac{5}{2}$

_____24. If $N - 3 < 7$ and N represents a positive integer, then the solution set is:

(A) $\{1,2,3,4,5,6\}$

(B) $\{0,1,2,3,4,5,6\}$

(C) $\{1,2,3,4,5,6,7,8,9,10\}$

(D) $\{0,1,2,3,4,5,6,7,8,9\}$

(E) $\{1,2,3,4,5,6,7,8,9\}$

_____25. Consider the subtraction number sentences:

$(8 - 6) - 1 = 1$ and $8 - (6 - 1) = 3$

These sentences illustrate that:

(A) subtraction is commutative

(B) subtraction is associative

(C) subtraction is not commutative

(D) subtraction is not associative

(E) subtraction is both commutative and associative

_____26. Which of the following statements about real numbers if <u>false</u>?

(A) If $x < y$ then $2x < 2y$.

(B) If $a < 0$ then $a^{-1} < 0$.

(C) If $a^2 + b^2 = 0$ then $a = 0$ and $b = 0$.

(D) If $a < 1$ then $a^2 < 1$.

(E) $-3 \leq 1$.

_____27. Four of the following numerals name the same number. Which one names a different number?

(A) .333 (B) $\frac{1}{3}$ (C) $33\frac{1}{3}\%$ (D) $\frac{7}{21}$

(E) $\frac{3}{10} + \frac{3}{100} + \frac{1}{300}$

_____28. If the radius of a circular cylinder were doubled and its height halved, then the volume would:

(A) be doubled. (B) remain the same.

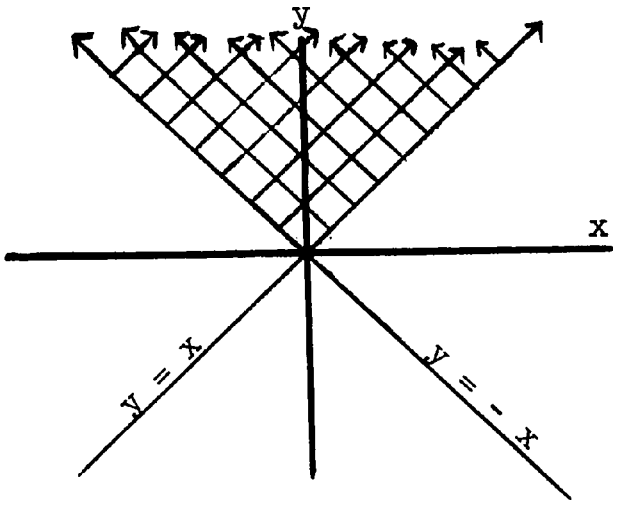
(C) be four times the original.

(D) be $\frac{1}{2}$ the original.

(E) be 2π times the original.

_____29. The shaded area of the graph represents:

(A) $y \geq x$
(B) $y \geq |x|$
(C) $x \geq y$
(D) $|x| \geq y$
(E) $x + y \geq 0$



_____30. Which of the following number sentences is <u>true</u>?

(A) $\frac{7}{2} \div \frac{2}{3} = \frac{2}{3} \div \frac{7}{2}$

(B) $\frac{7}{2} \div \frac{2}{3} = \frac{3 \times 7}{2 \times 2}$

(C) $\frac{7}{2} \div 0 = 0 \div \frac{7}{2}$

(D) $(\frac{7}{2} \div \frac{2}{3}) \div \frac{3}{4} = \frac{7}{2} \div (\frac{2}{3} \div \frac{3}{4})$

(E) If $r = \frac{7}{2} \div \frac{2}{3}$ then $r = \frac{4}{21}$.

_____31. A counterexample to the statement "If an integer is prime then it is odd" is:

(A) 0 (B) 1 (C) 2 (D) 3 (E) 9

_____32. A parallelogram has:

(A) congruent diagonals.
(B) congruent adjacent sides.
(C) complementary consecutive angles.
(D) opposite sides congruent and opposite angles congruent.
(E) congruent opposite sides but not congruent opposite angles.

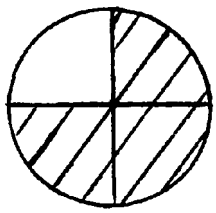
_____33. The intersection set for a straight line and a rectangle <u>cannot</u> be:

(A) the empty set.
(B) a set of exactly one point.
(C) a set of exactly two points.
(D) a set of exactly three points.
(E) a set of infinitely many points.

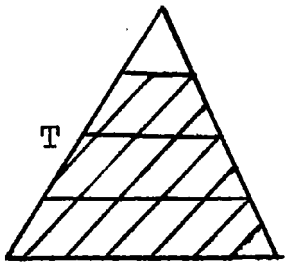
_____34. The number sentence, $7 \times 3\frac{3}{5} = (7 \times 3) + (7 \times \frac{3}{5})$, illustrates the:

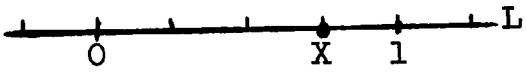
(A) associative law for addition.
(B) associative law for multiplication.
(C) commutative law for addition.
(D) commutative law for multiplication.
(E) distributive law for multiplication over addition.

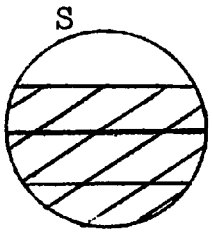
_____35. the shaded portion of the figure at the right represents a rational number. Which figures below represent the same rational number?

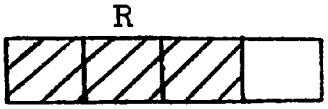


(A) All of them.
(B) R and L.
(C) R, T, and S.
(D) Only L.
(E) R, L, and S.









_____36. The polynomial, $ax^2 + bx + c$, with a, b, and c representing rational numbers and $a \neq 0$, always can be factored into two polynomials with coefficients that are

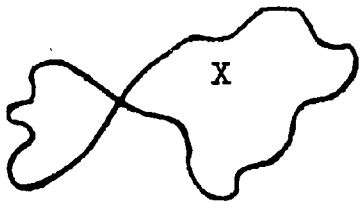
(A) whole numbers
(B) integers
(C) rational numbers
(D) real numbers
(E) complex numbers

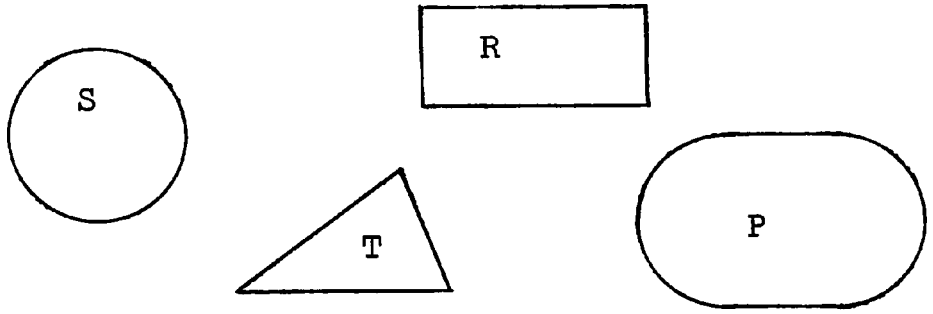
_____37. Which of the following proves "The product of two odd integers is an odd integer"?

(A) $3 \times 5 = 15$
(B) $(2k + 1)(2m + 1) = 4km + 2k + 2m + 1$
(C) $(2k + 1) + (2m + 1) = 2(k + m + 1)$
(D) $(2k + 1)(2m + 1) = 2k + 2m + 1$
(E) $(2k + 1)(2m + 1) = 4km + 1$

_____38. Which of the following are simple closed curves?

(A) All of them
(B) Only S and P
(C) Only R and T
(D) Only S, P, and X
(E) Only P, R, S, and T





_____39. If the cardinal number of set A is 11, the cardinal number of $A \cap B$ is 3, and the cardinal number of $A \cup B$ is 18, then the cardinal number of B is:

(A) 7 (B) 8 (C) 10 (D) 14 (E) 15

_____40. Which numeral represent $2 \times 1000 + \frac{4}{10} + \frac{3}{1000}$?

(A) 2.43
(B) 2000.43
(C) 2000.043
(D) 2000.0403
(E) 2000.403

- -

Some states require the student's permission for use of his college entrance test scores. Please grant such permission by signing below.

I hereby permit the college or university in which I am enrolled to release my college entrance test scores (ACT, SAT, or others) to Mr. Melvin C. Withnell for use in this research project. These scores will be used in the statistical analysis but individual's scores will not be published in any report of the project. Records of such scores will be destroyed after the statistical analysis is completed.

SIGNATURE_____

TABLE 28.--Classification of items in the Mathematics Concepts Test

QUESTION NUMBER	CLASSIFICATION	QUESTION NUMBER	CLASSIFICATION
1	Arithmetic--Basic	21	Arithmetic--Basic
2	Arithmetic--Related	22	Arithmetic--Basic
3	Algebraic---Basic	23	Geometric---Related
4	Geometric---Basic	24	Algebraic---Basic
5	Geometric---Related	25	Arithmetic--Basic
6	Arithmetic--Basic	26	Algebraic---Related
7	Algebraic---Related	27	Arithmetic--Basic
8	Arithmetic--Basic	28	Geometric---Related
9	Geometric---Related	29	Algebraic---Related
10	Arithmetic--Basic	30	Arithmetic--Basic
11	Algebraic---Basic	31	Arithmetic--Related
12	Algebraic---Related	32	Geometric---Basic
13	Arithmetic--Basic	33	Geometric---Basic
14	Arithmetic--Basic	34	Arithmetic--Basic
15	Arithmetic--Related	35	Arithmetic--Basic
16	Geometric---Basic	36	Algebraic---Related
17	Arithmetic--Basic	37	Algebraic---Related
18	Arithmetic--Related	38	Geometric---Basic
19	Geometric---Basic	39	Arithmetic--Related
20	Algebraic---Related	40	Arithmetic--Basic

TABLE 29.--Concept classification of items in the Mathematics Concepts Test

QUESTION NUMBER	GENERAL CLASSIFICATION	MAIN CONCEPT	RELATED CONCEPT
1	Numeration	Place value in base ten	Number versus numeral
2	Sets	Properties of infinite sets a) different cardinalities b) infinite subsets	1 - 1 correspondence
3	Number sentence	Representation by number line	180° in triangle
4	Measurement	Unit is arbitrary	Regular polygons
5	Measurement	Relation of area to perimeter	
6	Rational numbers	Application of percent	Modular arithmetic
7	Field properties	Property of prime modulus	Disjoint sets, union intersection, Cartesian product
8	Whole numbers-sets	Relationship of multiplication to sets	
9	Spherical-nonmetric geometry	Use of definition out of usual context	Uniqueness, parallel
10	Integers	Multiplication algorithm	Exponents
11	Number sentence	Symbolism to represent physical situation	Too much information
12	Number sentence	Cancellation law dependent on number system involved	Solution set, modular arithmetic
13	Real numbers	Division is related to multiplication	
14	Numeration	Place value in base seven	Number versus numeral
15	Sets	Representation of operations by diagrams	Use of parentheses
16	Measurement	Use of formula for area of triangle	Dividing by 2 versus dividing by $\frac{1}{2}$
17	Numeration	Regrouping in base ten	Expanded notation
18	Integers	Factors	Combinations

TABLE 29.--(continued)

QUESTION NUMBER	GENERAL CLASSIFICATION	MAIN CONCEPT	RELATED CONCEPT
19	Nonmetric	Separation in one, two, and three dimensions	Ray, line, angle
20	Number sentence	Solution set for inequalities	Absolute value
21	Integers	Closure	Odd-even, prime-composite
22	Integers	Meaning of steps in division	Division algorithm, place value
23	Graph-number sentence	Slope of graph of linear equation	
24	Number sentence	Solution set for inequality	0 is not a positive integer
25	Field properties	Identifying property (or lack of it) from given sentence	Commutativity, associativity
26	Real numbers	Theorems and examples about ordered fields	Order
27	Numeration	Rational number representation as repeating decimal, quotient of two integers, percent	Number versus numeral
28	Measurement	Ratio and proportion	Volume of cylinder
29	Graph	Intersection set of two inequalities	Absolute value
30	Rational numbers	Properties (or lack of them) of division	Division by zero is not defined
31	Integers	Counterexample	2 is an even prime
32	Nonmetric geometry	Congruence	Consecutive, opposite

TABLE 29.--(continued)

QUESTION NUMBER	GENERAL CLASSIFICATION	MAIN CONCEPT	RELATED CONCEPT
33	Nonmetric geometry	Intersection of line and polygon	
34	Field properties	Distributive law applied to multiplication of mixed numbers	
35	Rational numbers	Representation of part of a whole by use of rational numbers	Areas of circles, triangles, rectangles and their parts
36	Polynomials	Factorization dependent upon set of numbers allowed	Various number systems
37	Integers	Proof dependent upon generalization	Multiplication of factors
38	Nonmetric geometry	Properties of simple closed curve	
39	Whole numbers-sets	Dependence of addition upon disjoint sets	Cardinal number, set operations
40	Numeration	Expanded notation in rationals	Decimal notation

APPENDIX B

TABLE 30.--Frequency distribution of scores on the short ability test

SCORE	FREQUENCY	PERCENT OF SAMPLE	CUMULATIVE PERCENT
20	1	.4	.4
19	4	1.6	2.0
18	5	2.0	4.0
17	13	5.2	9.2
16	13	5.2	14.4
15	29	11.6	26.0
14	31	12.4	38.4
13	37	14.8	53.2
12	33	13.2	66.4
11	27	10.8	77.2
10	16	6.4	83.6
9	16	6.4	90.0
8	11	4.4	94.4
7	6	2.4	96.8
6	2	.8	97.6
5	4	1.6	99.2
4	1	.4	99.6
3	1	.4	100.0

The above frequency distribution represents the scores of 250 students randomly selected from the 2513 students tested in the study.

TABLE 31.--Correct responses and discrimination power of each item in the short ability test

ITEM	CORRECT RESPONSES	PERCENT CORRECT	PERCENT UPPER 1/3	PERCENT LOWER 1/3	DIFFERENCE
1	222	88	97	75	22
2	170	67	83	45	38
3	150	59	79	38	41
4	215	85	92	78	14
5	73	29	49	15	34
6	199	79	93	55	38
7	130	51	66	39	27
8	22	8	14	4	10
9	185	73	84	60	24
10	182	72	87	54	33
11	193	77	95	53	42
12	55	21	37	14	23
13	200	79	95	67	28
14	173	69	83	53	30
15	145	57	77	37	40
16	185	73	92	54	38
17	128	51	84	27	57
18	127	50	73	28	45
19	202	80	93	61	32
20	175	69	86	48	38

APPENDIX C

TABLE 32.--Frequency distributions of Mathematics Concepts Test scores for random samples of 250 students from each of Groups 1, 2, and 3

SCORE	GROUP 1	GROUP 2	GROUP 3
35		1	
34		0	
33		0	
32	1	0	
31	0	0	
30	1	0	2
29	0	0	0
28	1	2	3
27	0	2	1
26	3	2	0
25	2	8	7
24	5	6	3
23	6	7	7
22	7	7	9
21	6	12	9
20	10	10	17
19	15	13	21
18	13	22	26
17	24	12	17
16	20	20	17
15	25	27	26

TABLE 32.--(Continued)

SCORE	GROUP 1	GROUP 2	GROUP 3
14	18	15	13
13	14	16	22
12	22	19	17
11	15	10	9
10	10	13	7
9	10	10	7
8	14	8	4
7	3	1	3
6	1	3	0
5	2	3	1
4	2	0	1
3		1	1

TABLE 33.- Percentages of Groups 1, 2, and 3 responding to each of the choices for each of the items in the Mathematics Concepts Test

ITEM	CHOICE	TOTAL	GROUPS 1	2	3
1	0	.02	.02	.02	.01
	1	.06	.06	.08	.05
	2	.32	.34	.30	.32
	3*	.40	.39	.37	.47
	4	.10	.10	.12	.06
	5	.10	.10	.10	.09
2	0	.02	.01	.02	.02
	1	.56	.61	.52	.54
	2	.12	.11	.11	.15
	3	.04	.04	.05	.04
	4*	.22	.18	.27	.21
	5	.04	.05	.03	.04
3	0	.01	.01	.01	.01
	1	.05	.06	.04	.07
	2*	.84	.82	.86	.84
	3	.01	.01	.01	.00
	4	.03	.02	.03	.02
	5	.06	.07	.05	.06
4	0	.05	.06	.06	.03
	1	.29	.31	.27	.28
	2	.04	.03	.05	.04
	3*	.28	.24	.27	.42
	4	.04	.04	.06	.02
	5	.30	.33	.31	.22
5	0	.06	.06	.05	.06
	1	.18	.17	.18	.18
	2	.26	.25	.27	.24
	3	.09	.10	.09	.08
	4	.07	.08	.07	.03
	5*	.35	.35	.33	.41
6	0	.01	.01	.00	.01
	1	.04	.05	.03	.05
	2	.07	.07	.08	.06
	3*	.80	.80	.80	.81
	4	.04	.04	.03	.02
	5	.04	.03	.07	.04

*Correct response

TABLE 33.--(Continued)

ITEM	CHOICE	TOTAL	GROUPS	1	2	3
7	0	.19		.22	.11	.31
	1	.28		.30	.27	.25
	2	.13		.11	.16	.11
	3*	.23		.20	.29	.20
	4	.11		.12	.10	.07
	5	.06		.05	.06	.06
8	0	.02		.01	.02	.02
	1*	.40		.39	.37	.45
	2	.05		.05	.06	.04
	3	.32		.32	.29	.35
	4	.03		.03	.03	.02
	5	.19		.19	.23	.12
9	0	.06		.06	.05	.06
	1*	.26		.23	.23	.38
	2	.11		.10	.13	.07
	3	.18		.19	.18	.15
	4	.06		.06	.07	.04
	5	.34		.36	.34	.29
10	0	.02		.01	.02	.04
	1	.28		.29	.29	.24
	2	.32		.33	.32	.30
	3	.10		.10	.11	.10
	4*	.12		.10	.13	.13
	5	.16		.17	.14	.19
11	0	.04		.03	.04	.06
	1	.15		.15	.15	.13
	2*	.53		.55	.52	.53
	3	.08		.09	.08	.06
	4	.15		.13	.15	.18
	5	.05		.05	.06	.04
12	0	.12		.12	.06	.23
	1	.10		.09	.11	.08
	2	.43		.50	.35	.46
	3	.10		.10	.09	.12
	4*	.20		.15	.31	.08
	5	.05		.05	.07	.04
13	0	.01		.00	.01	.02
	1	.02		.02	.02	.01
	2	.14		.15	.14	.11
	3*	.79		.78	.79	.81
	4	.03		.03	.02	.04
	5	.02		.02	.01	.02

TABLE 33.--(Continued)

ITEM	CHOICE	TOTAL	GROUPS 1	2	3
14	0	.07	.06	.08	.11
	1	.14	.12	.17	.13
	2	.11	.09	.13	.11
	3	.39	.45	.33	.36
	4*	.18	.20	.16	.16
	5	.11	.08	.13	.14
15	0	.02	.01	.02	.05
	1	.04	.04	.03	.03
	2*	.69	.72	.65	.69
	3	.09	.08	.10	.07
	4	.15	.13	.17	.15
	5	.02	.01	.02	.01
16	0	.04	.05	.03	.04
	1	.05	.05	.04	.05
	2	.09	.10	.08	.09
	3	.08	.08	.09	.05
	4*	.48	.48	.47	.49
	5	.26	.23	.28	.28
17	0	.01	.01	.01	.01
	1	.11	.11	.12	.10
	2	.01	.01	.01	.01
	3	.03	.03	.03	.04
	4*	.81	.82	.80	.82
	5	.02	.02	.02	.01
18	0	.06	.06	.06	.08
	1	.13	.12	.14	.11
	2	.56	.55	.58	.55
	3	.15	.16	.13	.17
	4*	.06	.05	.06	.06
	5	.04	.06	.04	.02
19	0	.07	.08	.07	.06
	1	.18	.17	.20	.14
	2	.08	.08	.09	.09
	3	.15	.15	.14	.14
	4*	.24	.22	.22	.37
	5	.27	.29	.29	.19
20	0	.04	.04	.03	.09
	1	.08	.10	.07	.09
	2	.07	.08	.05	.10
	3	.17	.18	.18	.15
	4	.06	.07	.07	.04
	5*	.56	.54	.60	.54

TABLE 33.--(Continued)

ITEM	CHOICE	TOTAL	GROUPS	1	2	3
21	0	.08		.07	.06	.14
	1	.05		.05	.06	.06
	2	.12		.14	.11	.10
	3	.15		.16	.14	.13
	4	.32		.33	.30	.31
	5*	.29		.25	.34	.26
22	0	.02		.01	.02	.01
	1	.39		.42	.39	.32
	2	.04		.04	.05	.05
	3	.10		.11	.09	.10
	4*	.38		.35	.35	.49
	5	.07		.07	.09	.04
23	0	.15		.15	.13	.19
	1*	.19		.20	.20	.18
	2	.12		.12	.13	.09
	3	.14		.13	.13	.17
	4	.15		.15	.17	.13
	5	.25		.26	.25	.25
24	0	.04		.04	.04	.08
	1	.06		.07	.05	.07
	2	.09		.10	.18	.09
	3	.05		.06	.04	.05
	4	.43		.39	.49	.40
	5*	.32		.34	.31	.31
25	0	.02		.01	.02	.02
	1	.02		.03	.02	.01
	2	.11		.12	.11	.09
	3	.12		.11	.13	.12
	4*	.67		.67	.65	.73
	5	.06		.06	.07	.03
26	0	.05		.04	.05	.09
	1	.04		.05	.04	.03
	2	.15		.15	.15	.13
	3	.10		.09	.11	.08
	4*	.42		.43	.43	.36
	5	.24		.23.	.22	.31
27	0	.03		.02	.03	.03
	1*	.12		.13	.12	.12
	2	.01		.01	.01	
	3	.11		.11	.11	.10
	4	.08		.09	.09	.06
	5	.65		.65	.64	.68

TABLE 33.--(Continued)

ITEM	CHOICE	TOTAL	GROUPS	1	2	3
28	0	.11		.11	.11	.14
	1*	.10		.11	.11	.07
	2	.57		.57	.56	.62
	3	.05		.05	.05	.03
	4	.07		.07	.08	.06
	5	.09		.10	.09	.07
29	0	.20		.19	.18	.29
	1	.20		.22	.20	.17
	2*	.14		.15	.15	.12
	3	.15		.16	.15	.13
	4	.08		.08	.09	.06
	5	.22		.20	.23	.23
30	0	.05		.04	.07	.03
	1	.04		.05	.04	.01
	2*	.70		.67	.70	.78
	3	.11		.14	.09	.07
	4	.06		.06	.06	.06
	5	.05		.05	.04	.04
31	0	.05		.04	.06	.06
	1	.09		.12	.07	.07
	2	.04		.06	.03	.05
	3*	.71		.67	.75	.74
	4	.04		.06	.03	.03
	5	.06		.06	.06	.05
32	0	.09		.08	.09	.09
	1	.05		.04	.05	.08
	2	.05		.06	.06	.02
	3	.13		.16	.12	.09
	4*	.58		.57	.57	.65
	5	.09		.09	.11	.07
33	0	.10		.08	.11	.12
	1	.38		.40	.39	.31
	2	.14		.13	.15	.13
	3	.06		.06	.06	.06
	4*	.17		.17	.14	.26
	5	.15		.17	.16	.11
34	0	.05		.04	.07	.05
	1	.01		.01	.01	.00
	2	.08		.09	.08	.07
	3	.01		.01	.01	.01
	4	.06		.07	.06	.04
	5*	.78		.78	.77	.82

TABLE 33.--(Continued)

ITEM	CHOICE	TOTAL	GROUPS	1	2	3
35	0	.07		.06	.08	.09
	1	.22		.23	.22	.21
	2*	.39		.40	.37	.40
	3	.14		.15	.14	.15
	4	.02		.03	.01	.02
	5	.15		.13	.17	.14
36	0	.19		.17	.18	.25
	1	.13		.14	.12	.15
	2	.12		.15	.11	.08
	3	.28		.29	.26	.27
	4	.22		.20	.25	.22
	5*	.06		.05	.07	.03
37	0	.10		.08	.12	.14
	1	.36		.39	.32	.36
	2*	.33		.33	.33	.30
	3	.06		.05	.07	.05
	4	.06		.05	.07	.05
	5	.10		.10	.09	.10
38	0	.06		.05	.07	.07
	1	.08		.08	.10	.05
	2	.38		.43	.38	.23
	3	.01		.01	.01	.00
	4	.20		.23	.21	.12
	5*	.27		.21	.22	.52
39	0	.14		.11	.16	.19
	1	.46		.49	.42	.51
	2	.11		.12	.11	.10
	3*	.11		.12	.12	.08
	4	.06		.05	.07	.05
	5	.11		.12	.13	.07
40	0	.07		.05	.10	.09
	1	.03		.02	.03	.01
	2	.07		.06	.07	.06
	3	.10		.12	.10	.05
	4	.09		.11	.08	.08
	5*	.64		.63	.62	.71

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