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WEISBERG, Herbert Frank, 1941-

DIMENSIONAL ANALYSIS OF LEGISLATIVE ROLL CALLS.

The University of Michigan, Ph.D., 1968
Political Science, general

University Microfilms, Inc., Ann Arbor, Michigan

DIMENSIONAL ANALYSIS OF LEGISLATIVE ROLL CALLS

by
Herbert Frank Weisberg

A dissertation submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy in
The University of Michigan
1968

Doctoral Committee:

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DIMENSIONAL ANALYSIS OF LEGISLATIVE ROLL CALLS

by Herbert Frank Weisberg

Chairman: Donald E. Stokes

This study investigates the methodology of dimensional analysis of legislative roll calls. Scaling techniques have been applied to roll calls with considerable frequency in recent years, but without systematic attention to the basis of such analysis. Alternative scaling models are presented in this study along with a consideration of the techniques for analysis under these models.

Dimensional analysis of roll calls has meant the application of the Guttman scaling model. An alternative proximity model is presented which assumes that individuals approve adjacent items on a scale rather than a cumulative set of items on that scale. Roll call scales may be found under this proximity model which do not satisfy the usual Guttman scaling requirements, suggesting that the extent of dimensional voting in legislatures may be more than is usually believed.

Theoretical reasons have led to the conjecture that proximity scales will be found mainly under conditions of extreme conflict when compromise is rejected. The proximity model may be useful in explaining legislative voting in multi-party legislatures and in such historical periods as that preceding the American Civil War. Non-metric multidimensional scaling techniques may be used for scaling under the proximity model and for differentiating between the proximity model and the cumulative Guttman model.

The consideration of alternative scaling models also leads to a better understanding of the basis of the Guttman scale model in legislative

voting. An important explanation of the frequency of Guttman scales in roll call materials has to do with the paired comparison nature of the roll call--the legislator expressing his preference between the state of affairs in which the motion passes and the state of affairs in which it fails. Guttman scales arise when legislators with unidimensional preference functions respond to roll calls in this paired comparison sense. This study of response models focusses attention on how little is known as to how legislators structure their attitude space and as to how they react to the voting situation.

The past success in the application of the Guttman scale model dictates that attention be given to the procedures for such scaling. It is necessary to have objective procedures for Guttman scaling which can handle such problems as multidimensionality, error, item reflections, and missing data. Correlational analysis procedures are generally more robust in the face of such problems. Therefore cluster analysis, factor analysis, and similarity analysis are valuable techniques for the dimensional analysis of roll calls.

While the classic measure of correlation is the Pearson product-moment r, analysis of this coefficient does not lend itself to a valid portrayal of the dimensional properties of the data. The dimensionality is overestimated and the dimensional specifications of the items are misstated. This is due to the fact that Pearson's r is not a pure measure of the unidimensionality of the items.

Measures of one-way association such as Yule's Q, tetrachoric r, and Yule's Y, do measure the extent of unidimensionality between the items. These coefficients range in value from zero under statistical

independence to unity under unidimensionality and their sign indicates the need for item reflections. Analysis of these coefficients leads to a valid portrayal of the dimensional properties of the data. Factor analysis of these coefficients indicates the dimensionality of the data and indicates the unidimensional scales. Cluster analysis does not handle complex multidimensional situations adequately. Similarity analysis may provide a more accurate solution, but it is considerably more time-consuming and complicated. The factor analysis of measures of one-way association is therefore recommended as a means for scaling under the general cumulative model. Roll calls of the 85th Congress are analyzed as a demonstration of the utility of this form of dimensional analysis.

ACKNOWLEDGEMENTS

This dissertation represents the culmination of many years of study. Thanks must first be given to my parents who encouraged my academic interests and to the faculty of the Political Science Department at the University of Minnesota who guided me toward graduate study in Ann Arbor. The Political Behavior Program of the Survey Research Center has provided a congenial intellectual atmosphere at Michigan. The encouragement of all members of that program has been of incalculable assistance. Particular appreciation is expressed for the facilities, data, and computer support which were made available. Also I should like to express gratitude to Mr. Lutz Erbring of the Political Behavior Program for the many discussions we have had on the topics treated in this work. I am also grateful to the members of my committee who have given me guidance in the writing of this thesis.

The very frequent references still do not fully indicate the extent of the intellectual debt which this work has to the teachings of Professor Clyde H. Coombs. His courses have provided an organization for my own interests in legislative roll call analysis. His numerous insights have provided the basis for most of the good ideas in this thesis.

Most of all I must express my gratitide to Professor Donald E. Stokes for his persistent guidance in this project. His penetrating judgment has brought a focus to this work, suggesting valuable ideas and

helping me avoid numerous errors in judgment. His encouragement and his interest in this project also have done much to sustain me throughout the period of research and writing.

In completing this dissertation and in looking forward to the future, I should wish to give a final word of thanks to everyone at Michigan who has made the past years so enjoyable.

Herbert F. Weisberg

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CHAPTER I

INTRODUCTION

Legislative roll call analysis is one of the oldest branches of empirical political science. Historians and political scientists recognized the value of roll call materials for the study of decision-making in the legislative setting long before such research tools as the sample survey became popular. While additional means of studying legislative decision-making are now available, roll call analysis remains as a basic research technique.

In 1896 Orin Libby issued his "Plea for the Study of Votes in Congress." A. Lawrence Lowell (1901) became the first person to utilize roll call materials in political research with his comparative study of the role of the political party in the American and British legislatures. Lowell's pioneering work was then followed by Stuart Rice's (1928) analysis of consensus and cleavage within and between the parties of legislatures. Herman Beyle (1931) extended Rice's work on the structural properties of legislative parties.

Roll call analysis did not become an important research tool in political science until 1950. In fact the major published roll call studies of the 1940's were all methodological inquiries by psychologists (Carlson and Harrell, 1942; Brimhall and Otis, 1948; Harris, 1948; Gage and Shimberg, 1949). Since 1950, sparked by the controversial

American Political Science Association Committee on Political Parties (1950) report "Toward a More Responsible Two-Party System" and by the methodological innovation of the psychologists, roll call analysis has emerged as a major device for the analysis of legislative behavior.

The importance of roll call analysis may be seen in part by noting the wide variety of recent applications. Roll call analysis has been used to examine the bases of division during the ministry of Peel (Aydelotte, 1963), the changes in Congressional attitude on foreign aid (Rieselbach, 1964), the actions on European unification by the French National Assembly (Wood, 1964), the behavior of state delegations in the American House of Representatives (Truman, 1956), the pattern of voting blocs in the United Nations General Assembly (Lijphart, 1963), and the behavior of boarding-house groups in the early Congresses (Young, 1966). Lowell's original concerns with the nature of representation, the structure of the legislative party, and the bases of legislative conflict remain as the central foci of the recent major roll call studies of the American Congress by Julius Turner (1951), Duncan MacRae (1958), David Truman (1959), and Warren Miller and Donald Stokes (1963). Thus roll call analysis has consistently been utilized for the study of the most basic questions of legislative behavior.

Roll call analysis is an important tool for the analysis of legislative behavior in that roll calls are part of the full process of legislative decision. The roll calls represent actions taken by the legislature as a part of its decision-making process. The roll calls constitute behavior of the legislators in their official positions.

Nevertheless roll call analysis has always been controversial. In the following paragraphs we shall point out several justifications of roll call analysis. In doing so we shall indicate some of the research interests which might lead the investigator to inspect roll call behavior. The intention of this discussion is not to restrict the class of research interests which can lead to roll call analysis to only those which are mentioned, but rather to emphasize the variety of different research interests which are leading investigators to employ roll call techniques.

Data on legislative decision-making can be collected by means of observation and interviewing as well as through the public records. While observation and interviewing have obvious utility, there are severe problems of access to be faced. There are notable instances of Congressmen giving enormous amounts of their time to persistent investigators, as in the communications audit of a week in the life of each of seven Congressmen. Such occasions only serve to emphasize the general problem of getting good empirical data on Congress. Even if access were not a problem in the present tense, the techniques of observation and interviewing can be of no help in the historical analysis of legislative decision-making. The public record, and especially the recorded divisions, must remain a prime source of data for the legislative analyst.

Roll call analysis employs data from only a single stage of the legislative process, emphasizing floor action at the expense of actions behind the scenes and in committee. It has been argued that the roll call vote is not significant behavior since the "real" decisions are made at these earlier stages.

The significance of the roll call vote should be considered in terms of the intended purpose of the roll call analysis. Roll call analysis might seek to describe the legislative process or to describe the behavior of individual legislators at a particular point in the process. The latter might be intended as an end in its own right or it might be a means to enable the investigator to analyze the process leading to legislative outputs. The significance question may be a serious problem when the goal is to describe the legislative process. It is much less of a problem when the goal is to understand the behavior of legislators. The roll call vote has a special importance in this latter case. When the legislator who is accountable to his constituents takes a position with which he can be publicly identified, the roll call becomes important to the legislator. The vote thereby becomes endowed with properties which make it of interest for the study of the decision-making behavior of legislators.

This interpretation makes the significance of the roll call vote to a large degree contextual. In a legislature in which the roll call votes are subject to the scrutiny of interested voters and pressure groups, the legislators consider roll calls important. Conversely, roll calls may have a much different meaning where there are no pressure groups and where the constituents are unlikely to obtain communications on the positions of their legislators. This may limit the possibility of roll call analysis historically. In an earlier period the voters may have had much less knowledge of the behavior of their particular Congressmen and there may have been much less pressure group activity. Thus roll call votes may have had very

different meanings one hundred or more years ago than they have today. While this would not fully negate the analysis of the roll calls of the early Congresses, it suggests the need for great caution in the interpretation of such studies.

It should be added that roll call data is little different from any other voting data. We can identify the roll call votes with the voter and we have a large amount of public information about the voter. As with any voting data, a large number of causes are compressed into a single event. This multiplicity of causes might constitute a problem for analysis if there was but a single roll call for each class of phenomena under consideration. The multiplicity of roll calls in the domain, however, compensates for the multiplicity of causes. We can take advantage of the multiplicity of roll calls to determine the relative importance of the various causes. The multiplicity of events makes roll call analysis easier than the analysis of some other types of single event voting, such as referendum voting.

Actually one would be justified in considering roll call data to be of very high quality. Whereas observational and interviewing techniques are dependent on the quality of the observer and the interviewer as well as on the quality of subsequent coding, roll call materials are generally transcribed with virtually complete accuracy. Roll call data are among the most accurate types of data available in social science generally.

The one question regarding the quality of roll call materials which can be raised is the extent to which the roll call indicates the real position of the legislator. There are a large number of roll calls in a session of a legislature, some on issues of monumental

importance and others on issues of true insignificance. On the latter type the legislator may decide how to vote on the basis of a brief explanation of the question involved together with a suggestion on how to vote from whichever fellow legislator he happens to ask for information. This element of response uncertainty does exist, but the legislator is probably as concerned with it as is the roll call analyst. The legislator does not want accidentally to vote the wrong way on a measure which unexpectedly later turns out to be important to him. So even when the legislator must decide how to vote on the basis of minimal information, he will generally seek to obtain that information from a source he considers reliable. Roll call data compare favorably in quality with other types of data since they are obtained from elites who themselves wish to minimize the amount of response uncertainty.

METHODOLOGICAL ASPECTS

As noticeable as the breadth of subject matter which has been considered through roll call analysis has been the variety of methodological application. Lowell (1901) himself began the tradition of methodological innovation in roll call research with his development of an index of party voting. Rice (1928) next contributed his indices of party cohesion and likeness and, with modifications by Beyle (1931), the cluster-bloc technique. The methodological sophistication of some of these early studies might be questioned by today's standards, but their importance in the development of empirical political science cannot be denied. Each innovated methodologically in order to provide answers to substantive questions.

The methodological emphasis has remained apparent in later roll call studies. The few roll call analyses of the 1940's, for example, included applications of two forms of factor analysis (Carlson and Harrell, 1942; Harris, 1948) and also Guttman scaling (Gage and Shimberg, 1949). The importance of methodology to roll call analysis was acknowledged by two recent works: a handbook on Legislative Roll-Call Analysis by Lee Anderson and his colleagues (1966) and a text on "methods of statistical analysis" of legislative voting by Duncan MacRae (forthcoming). In the midst of this methodological innovation, a systematic framework is now necessary.

A new analysis technique is employed because of its promise for a more valid portrayal of substantive reality. However the innovator is too rarely concerned with stating the relation of his technique to earlier techniques, especially since his posture is one of advocacy. The result is methodological disarray. Cluster-bloc analysis, factor analysis, Guttman scaling, and similarity analysis have been applied to legislative roll calls. Anderson (1966) provides good summaries of the first three of these methods as applied to roll calls and MacRae (forthcoming) examines all of these procedures, but there remains a need to provide systematic attention to their interrelations.

The substantive implications of this methodological disarray are the vital concerns. What interpretation can be given when two different techniques yield two different sets of substantive conclusions? Indeed how are the results of a single study to be interpreted when they are so dependent on methodological considerations? Methodological clarity must be attained if substantive conclusions are to have generality.

DIMENSIONALTIY

There are two basic traditions of roll call analysis -- one which makes explicit allowance for the dimensionality of the data and one which does not. This is not to insist that dimensionality need always be present in the data. It might be that the dimensionality of a set of items would not involve a meaningful reduction of their original complexity, but an allowance for that possibility should still be made in the analysis. The dimensional tradition of roll call analysis yields the most complete portrayal of the structural properties of the data. A true understanding of the voting space cannot be obtained if the dimensional phenomenon is disregarded.

The vital connection between the methodological and the substantive and the importance of the dimensional construct are both well illustrated by an exchange regarding the measurement of power. Robert Dahl (1957) provided a very careful definition of the amount of power of one individual over another, the respondent, using a given means with respect to a specified response. He expressed this entity as the probability that the respondent would perform the response if the means were employed minus the probability that the respondent would perform that response without the employment of the means. Dahl further obtained a measure of power comparability in roll call behavior by subtracting the probability that a bill passes if the legislator works against it from the probability that it passes if he works for it. This measure is operationalized as the proportion of motions which the legislator votes for which pass minus the proportion of motions which the legislator votes against which pass. This percentage difference would seem to be an obvious measure of the legislator's legislative success or power.

The Dahl indicator of power does turn out to have some very basic methodological faults. Dahl himself noted the problem of high power being ascribed to the chameleon who guesses perfectly how the chamber will vote and proceeds invariably to vote along with the majority. He also recognized the problem of the satellite who is ascribed considerable power only because he votes along with some more powerful legislator. The chameleon and the satellite necessarily confound any roll call analysis since they cast their votes on the basis of non-content specific concerns. The more critical difficulty with Dahl's operational definition was pointed out by Duncan MacRae and Hugh Price (1959). They showed that Dahl's measurement of power is hopelessly confounded with the effects of dimensionaltiy.

Reconsider Dahl's definition from the perspective of dimensionality. Say that power is being measured with reference to only one issue area, which is in fact what Dahl does. Say further that this issue area is entirely unidimensional. Let us suppose that the roll calls which are employed all satisfy a Guttman scale perfectly. MacRae and Price show that the legislators nearest the median position on the scale will necessarily be given the highest indices of power.

Table 1.1 demonstrates this problem. A perfect Guttman scale is shown which fully accounts for the voting on six roll calls in a seven-man legislature. The legislator with the median score on the scale is legislator four. All of the items which he votes for pass while none of the items which he votes against pass.[1] Therefore his power according to the Dahl index is the maximum possible value: plus one. Legislators

---

[1]This is always true of the median individual on a perfect Guttman scale, when the number of individuals is odd.

TABLE 1.1

VOTING PATTERNS ON A HYPOTHETICAL DIMENSION

Legislator	Item A	B	C	D	E	F	Scale Score	p_{1+}*	p_{2+}*	Power
1	Yes	No	No	No	No	No	1	1.00	0.40	0.60
2	Yes	No	No	No	No	No	1	1.00	0.40	0.60
3	Yes	Yes	No	No	No	No	2	1.00	0.25	0.75
4	Yes	Yes	Yes	No	No	No	3	1.00	0.00	1.00
5	Yes	Yes	Yes	Yes	No	No	4	0.75	0.00	0.75
6	Yes	Yes	Yes	Yes	No	No	4	0.75	0.00	0.75
7	Yes	Yes	Yes	Yes	Yes	No	5	0.60	0.00	0.60
Outcome:	Pass	Pass	Pass	Fail	Fail	Fail				

*In MacRae and Price's (1959) notation, p_{1+} is the proportion of items favored by the legislator which pass and p_{2+} is the proportion of items opposed by the legislator which pass. Power equals $p_{1+} - p_{2+}$.

---

having higher scale scores than this median legislator less often see items they vote for pass, so that the power index declines monotonically as one moves away from the median individual toward the end of the scale at which the legislators vote for even the hardest items. Similarly legislators having lower scale scores than the median legislator more often see items they vote against pass, so that the power index also declines monotonically as one moves away from the median individual toward the end of the scale at which the legislators vote against even the easiest items. Thus the power ascribed to the legislator depends completely on his distance from the median scale position. Centrality on an ideological dimension might be equivalent to high power in some situations, but one would hope to be able to obtain independent measures of each. Instead the substantive notion of power cannot be disentangled from the methodological feature of dimensionality in the Dahl index.

The concept of dimensionaltiy can similarly confound other forms of analysis. Such an effect can be seen in the indiscriminate use of cluster-bloc analysis. Again assume that perfect unidimensionality holds. In general, cluster-bloc analysis yields more than a single cluster-bloc although the data perfectly fit the Guttman scale model. When there is a long scale with some consistent response patterns missing (which yields separation of blocs), one would obtain several blocs, though all that need be present is a common cultural framework with independent individual decisions within that framework. A cohesive bloc does not imply the influence of bloc leadership. A cohesive bloc can result instead from similar positions along an ideological continuum. Table 1.2 shows the inter-legislator agreement matrix corresponding to the unidimensional votes of Table 1.1. The cell entry shows on how many of the six votes the row legislator and the column legislator vote together, either for or against the motion. As indicated in the agreement matrix, cluster-bloc analysis of this matrix would yield two fairly distinct cluster-blocs. Thus there is no necessary correspondence between the number of dimensions (here one) and the number of cluster-blocs (here two). Nor are these results unique. The cluster-bloc analysis of two- or three-dimensional data structures might also have yielded cluster-bloc results identical to those of Table 1.2. Some forms of cluster-bloc analysis can be misleading when dimensionality is ignored.

A useful comparison along this very line can be made between two important works utilizing roll call techniques in analyses of the 81st Congress (1949-50). Duncan MacRae's Dimensions of Congressional

TABLE 1.2

INTER-LEGISLATOR AGREEMENT MATRIX FOR HYPOTHETICAL VOTES OF TABLE 1.1

	Legislator						
Legislator	1	2	3	4	5	6	7
1	-	6	5	4	3	3	2
2	6	-	5	4	3	3	2
3	5	5	-	5	4	4	3
4	4	4	5	-	5	5	4
5	3	3	4	5	-	6	5
6	3	3	4	5	6	-	5
7	2	2	3	4	5	5	-

Voting (1958) studies the bases of divisions within the parties through Guttman scaling of roll calls. Within each party, scales are obtained by finding sets of items, each of which is scalable in a pairwise sense with the five adjacent items on the scale. MacRae's method yields voting dimensions, basic unidimensional continua along which the legislators can be arrayed. Ideological reasons for voting are the obvious explanations of such dimensions, though it is also possible that interactional patterns account for the structure. At least the scaling approach keeps the content of the roll calls within the investigator's view, enabling him to judge the substantive unity of the dimensions.

David Truman's The Congressional Party (1959) studies the structure of the legislative parties within the same Congress through a cluster-bloc analysis of the legislators. Within each legislative party, a matrix of inter-legislator agreements is inspected to reveal the structural elements of the party. Intra-party blocs are identified by finding sets of legislators, each of whom vote with each other member of the set on at least a specified number of votes. Truman's method yields groups

of legislators, but the method itself cannot explain why the members of a group vote together and why the several groups are separate. Truman infers that this structure is due to interactional patterns, but it could as well result from ideological similarity.

These two works both employ a clustering technique with the two parties being analyzed separately. The similarity between them ends at this point. MacRae obtains a direct representation of the data in a multidimensional setting. Truman obtains a dimensionless set of blocs which must be some complex function of the distributions along the several scales. Truman's blocs in no way dispute the presence of a space of low dimensionality. However a knowledge of the dimensionality might make us desire to employ a different approach in describing the bloc structure. It should also be noted that cluster-bloc analysis need not always ignore dimensionality. Some forms of cluter-bloc analysis are in the dimensional tradition while others are not. Both MacRae and Truman employed clustering procedures in their analysis, for example,but the former did so in the dimensional context while the latter totally ignored the dimensionality question.

The dimensional tradition of roll call analysis has yielded one basic theoretical conclusion--the importance of the dimensional construct in the explanation of legislative behavior. Numerous studies confirm the necessity to study legislative decision-making separately within different issue areas. Legislative conflict involves multiple arenas of dispute (MacRae, 1958). Representation may take different forms within different issue areas (Miller and Stokes, 1963). The legislative party may exhibit different structural properties within different issue areas (MacRae, 1965).

RESPONSE MODELS

Dimensional analysis has come to mean Guttman scaling and factor analysis where the latter, according to two of its prime advocates, "may loosely be considered a multidimensional generalization of Guttman scale analysis" (Alker and Russett, 1965, p. 29). The Guttman cumulative model and its extensions exemplify what can be termed a "dominance response model." The legislator's vote is determined, according to this model, by whether the legislator's preference dominates the roll call--whether the legislator desires at least as much of the qualities represented by the dimensions as is contained in the roll call.

The dominance model may be the most familiar form of dimensional construction, but it is not the only form. The other main response model is the "proximity model." The vote of the legislator in this model is determined by his distance in the space from the roll call. He will vote for items which are close to him and against items which are far from him, regardless of which side of him they are on. That is, the legislator votes for only those items which contain approximately as much of the qualitites represented by the dimensions as he desires. Each legislator has a region of acceptance and will vote yes only on the motions falling within this region of acceptance.

The basic form of a Guttman scale is illustrated by the perfect scale of Table 1.1. An ordering of the items is possible such that each legislator votes yes on only those items beneath his own location on that scale. Of course most Guttman scales are not perfect. The responses satisfy the Guttman scale pattern only within a certain degree of fit. Table 1.3 shows such a scale with "error". While

TABLE 1.3

A HYPOTHETICAL VOTING DIMENSION

Scale Type	Items A	B	C	D	Frequency
1	No	No	No	No	2
2	Yes	No	No	No	4
3	Yes	Yes	No	No	4
4	Yes	Yes	Yes	No	4
5	Yes	Yes	Yes	Yes	2
6	No	Yes	Yes	No	1
7	No	Yes	No	No	1
8	No	No	Yes	No	1
9	No	No	No	Yes	1
					20

there is error in this scale, the proportion of reponses fitting the scale, Guttman's coefficient of reproducibility, is a high 0.95.

At least four explanations of such deviation from a pure Guttman scale are possible. The individuals might be behaving probabilistically, as in the model behind Lazarsfeld's latent structure analysis. Second, there might not be a common frame of reference for all individuals. Perhaps, for example, the members of one party have one frame of reference and the members of the other party have a different frame of reference. Then scaling each party separately might yield perfect Guttman scales for each party, though the order of the items in the two party scales would be different. A third possible explanation is that the data may be better explained in terms of a multidimensional model. A final possibility is that the data were generated in accordance with a response model other than the dominance model. Indeed the data of Table 1.3 can be explained perfectly in terms of a single dimension according to the proximity model.

The proximity scale has a different interpretation than the usual Guttman scale. Each legislator is seen as having a position on the single dimension and he votes in favor of those items within a given distance of his position on that scale. This means that each person votes yes only on adjacent items on the scale, as is the case in Table 1.3. Different legislators have not only different positions on the dimension but they also have acceptance regions of different widths. Thus some of the legislators in the middle of the scale endorse only a single item while others approve several of the middle items. Some legislators vote only for the items at one end of the scale while others vote only for those at the other end of the scale. Note also that some legislators have acceptance regions so wide that they vote for all the items under consideration while others have acceptance regions so narrow that they vote against all of the items. There is insufficient information to locate the positions of either of these latter two groups on the scale.

As an illustration of such a scale, imagine a country with many parties arrayed along a single dimension. Each member of the legislature of this country is asked to indicate which parties he would accept as members of the government coalition. Some legislators are only willing to vote for a government composed of their own party. They are not at all willing to join in a compromise with members of other parties because their acceptance regions are very narrow. Other legislators are willing to accept coalitions of their party together with certain adjacent parties on the dimension. Some legislators may be willing to support a grand coalition of all of the parties. A few

legislators may withhold their support from any possible party or coalition because they feel that none of the main parties represent their views adequately. Some possible coalitions would elicit no support since, for example, a coalition of the extreme left-wing parties with the extreme right-wing parties which excludes the parties of the center would violate the common dimension underlying the party structure.

The proximity model on its face would seem to be a plausible model for the explanation of legislative voting, perhaps more plausible than the dominance model. Guttman scaling was developed in the area of mental testing. In that area there is a natural cumulation in the items, as in the amount of ability and knowledge necessary to perform various mathematical operations. In attitude work in general and legislative behavior in particular, it is more natural to describe actors in terms of preference functions over the array of possible items, basically a proximity scheme. The mechanisms behind both models require further exploration if one is to determine the legislative situations in which each model would be expected to hold.

If there are two very different response models which could underlie legislative voting, one would desire empirical tests to discriminate between the models. Procedures should be developed by which it can be determined which model best explains the data. The simplest of such empirical procedures is to compare the proportion of responses which fit the scale according to the two models. Additionally some attention to the face content of the scale orderings is necessary. Thus in the example of Table 1.3 it is evident that a more parsimonious solution is

achieved with a proximity interpretation, but this is no guarantee that the proximity scale can be interpreted more successfully than the dominance scale of these items. Discrimination between these two response models requires the utilization of substantive information in addition to whatever statistical procedures can be devised.

The proximity scale merits much more attention than it has received in legislative roll call analysis. It might help explain voting in some legislative settings. Still the importance of the dominance model must be acknowledged. The large number of successful applications of the dominance model to roll calls demonstrates the vital importance of the dominance model in explaining legislative voting. Legislative roll call analysis requires an understanding of both models.

DOMINANCE SCALING

While the dominance model is the most popular model in the explanation of legislative voting, the techniques for the analysis of this model are still inadequate. The basic dominance scaling technique is Guttman scaling which was developed for the unidimensional scaling of small numbers of stimulus items. Legislative analysis can involve hundreds of roll call votes in a multidimensional space. Computer algorithms have been developed to cope with the problem of numbers, but generally their view of multidimensionality is overly restricted. Overall much too little attention has been given to multidimensional dominance scaling--its meaning and its algorithms. Multidimensionality is generally understood in the sense of there being more than one set of items satisfying a unidimensional scale. While there may be several

dimensions, each item is seen as belonging to only a single dimension. Actually multidimensionality must permit each item to be partially related to several dimensions.

One technique which can fully cope with the problems of large numbers of items and multidimensionality is factor analysis. However factor analysis has not been used often for dominance scaling because it has been felt that factor analysis is inconsistent with Guttman scales. In fact the most vigorous argument against the use of factor analysis for dominance scaling was given by Louis Guttman (1950) in his basic presentation of Guttman scaling.

The argument against using factor analysis of dominance scaling bears re-examination. Factor analysis of a matrix of Pearson's product-moment correlation coefficients seriously misstates the dimensional structure as Guttman noted. There are, however, other measures of association which could be employed instead of the product-moment coefficient on which so much of classic factor analysis has rested. These other coefficients measure different properties of the data; some measure properties which are closely related to dominance scaling. Therefore the factor analysis of such measures of association as Yule's Q and the tetrachoric coefficient yields results quite analogous to dominance scaling.

While the factor analysis of dominance data structures requires elaborate justification which will be given later, its utility can be seen by reviewing the results of the first roll call study to employ R factor analysis. In 1948 the League of Women Voters published the votes of the Senate on ten roll calls taken that year. Chester Harris

(1948) employed nine of these roll calls in a factor analysis, deleting one item with an excessive amount of missing data and then adding a dummy item indicating party affiliation. He then computed an inter-item tetrachoric correlation matrix and factored this matrix. Three factors were obtained. The first was interpreted as a big business factor, the second as a foreign relations factor, and the third could not be interpreted.

Table 1.4 shows Harris's factors along with those obtained by a reanalysis of the same matrix of tetrachoric correlations.[2] The values obtained in these two analyses do differ, apparently due to the use of a somewhat different factoring schemes. Yet both analyses lead to the same basic set of conclusions. Five items have high loadings on the first factor. Their high loadings are evident in Harris's original results and are even more striking in the reanalysis. Ordering these items in terms of their marginals, they are: national defense investigation, party affiliation, Taft-Hartley Bill, income tax reduction, and rent increase. The coefficient of reproducibility of these five items is 0.97, confirming the existence of a Guttman scale.[3] While there is evidence of a foreign relations factor, it is much weaker according to its factor loadings. There seems to be little gain in treating two or

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[2]Harris employed the centroid method of factor analysis followed by oblique rotation. In the reanalysis the principal components method has been used followed by oblique (biquartimin) rotation. The rotated results are followed in the discussion.

[3]The inclusion of party affiliation in this scale means that items on either side of this item differentiate only members of one party. In other words, nearly all Republicans voted for Taft-Hartley, for income tax reductions, and for rent increases. The Democrat response to these three items does still show the Guttman scale pattern, the reproducibility being 0.96.

TABLE 1.4

CHESTER HARRIS'S FACTOR ANALYSIS OF 1948 SENATE VOTES

	Harris's Factors			Reanalyzed Factors		
	I	II	III	I	II	III
Taft-Hartley labor bill--passage	.95	.06	-.01	.94	.25	-.14
Allow 15% rent increase	.78	-.02	.42	.98	.31	.26
Income tax reduction--passage	.78	.00	.45	.97	.31	.36
Party affiliation--Republican	.62	-.13	.61	.94	.24	.51
National defense investigation	.70	.02	.58	.96	.39	.50
Housing agency consolidation	.33	.22	.74	.76	.58	.64
Lilienthal nomination to AEC	.40	.57	.39	.77	.77	.35
Wool supports and tariffs	.00	.01	.85	.42	.31	.93
Greek-Turkish aid bill--passage	-.11	.57	.36	.22	.89	.38
Cut foreign relief funds	-.02	.98	.02	.31	.95	.02

three item scales, so one should conclude that the only Guttman scale in the data is the five item scale. Harris was the first person to find a Guttman scale of legislative roll calls through factor analysis, although he did not give such an interpretation to his findings.

The importance of this result is to be emphasized. Dominance scaling of roll calls has been difficult since there are so many roll calls in a session of Congress and since they have a complicated multidimensional form. If factor analysis gives equivalent results, dominance scaling of legislative roll calls can become a simple routine task.

CHAPTER PREVIEWS

A roll call analysis always involves a series of decisions and assumptions, both explicit and implicit. The set of roll calls to be included in the analysis must be defined. The set of legislators to be included in the analysis must be delimited. The meaning of each of the many possible forms of non-voting must be specified. The analysis

technique must be chosen, with appropriate attention to the underlying response theory being employed. Anderson's Legislative Roll-Call Analysis (1966) is a useful compendium of information on the various techniques which have been used in the analysis of roll call materials. Yet more consideration of the stakes involved in the several necessary decisions is required. We shall emphasize a desire to maintain all reliable features in the observations for the actual analysis, along with a desire to determine the uncertainties which will affect the results by the analysis rather than by prior assumption. The analytic framework of Clyde Coombs and his associates (Coombs, 1964; Coombs, et.al., in press) on measurement and data analysis will be employed in the next chapter, "A Theory of Data Framework," in considering these problems of models and methods. One important aspect of these materials will be a clear presentation of the alternative dominance and proximity response models which will be considered at length in the later chapters.

The third chapter, "Models of Legislative Voting," will further explore the dominance and proximity response models. The a priori assumptions which lead to each model will be clarified. Legislative settings which are likely to lead to each of these models will be presented. The possibilities of empirical testing procedures on actual data to discriminate between the two response models will be considered.

The following chapter, "Correlation in a Dominance Space," will detail the use of various correlation measures in connection with correlational analysis under the dominance model. We shall find that there are several different basic properties of a cross-tabulation of items and that correlation coefficients can be classified in terms of

which of these properties they measure. In turn each class of correlation coefficients yields a different patterning for the matrix of inter-item correlation coefficients so that different analysis techniques are required for each.

The various techniques for the analysis of these correlation coefficients will be presented in the chapter on "Dominance Scaling," concentrating on the measures and algorithms which are appropriate for determining the dimensionality of a set of items. This will include a detailed consideration of conventional applications of factor analysis and a detailed justification of the unconventional usage of factor analysis illustrated in the previous section. The use of similarity analysis as a non-metric alternative to factor analysis for the same purpose will also be presented. Throughout this chapter avowedly multidimensional data analysis techniques will be applied to known unidimensional and multidimensional structures in order to determine their sensitivities as applied to a variety of measures of correlation.

The results of this study should lead to a much better understanding of procedures for the analysis of legislative roll calls. This will be tested in chapter six, "Dimensions of Voting in the Eighty-Fifth Congress." This chapter will begin with a reanalysis of the roll call materials used in the Miller and Stokes study of representation in the American Congress. The multidimensional structure of their roll calls will be considered, with special attention being given to the structure of Congressional voting on foreign policy. Also the full set of roll calls of that Congress will be analyzed by the techniques suggested in the previous chapter in order to obtain their dimensional structure.

The results are a drastic reduction in the number of factors and dimensions underlying the roll calls of the 85th Congress, but a reduction which calls attention to the complexities of multidimensional analysis.

The final chapter will summarize "The State of the Art." The various steps of a dimensional analysis of roll call materials will be listed. Several areas which required further attention will be suggested and the principal conclusions of this work will be reviewed.

A METHODOLOGICAL NOTE

The preparation of this study has involved some procedures which are unusual in political science. As this final document does not do justice to the process by which the results were generated, the methods of this methodological inquiry should be noted.

A basic technique employed is the analysis of "known" data structures. Psychologists have long employed the analysis of known data structures as a means of discerning the sensitivities of the analysis techniques, but such model testing is virtually unknown in political science. Its use in this study was originally for the purpose of checking out the new iterative similarity analysis procedures before applying them to large sets of unknown data. The results of this model testing were continually so unexpected that they have become the focus of this study.

The process being described begins with the construction of a simple set of observations which are known to fit a given model perfectly, such as the votes consistent with a Guttman scale which are given in Table 1.1. The analysis technique is then applied to the

observations to check whether the relevant properties of the observations are captured correctly by the technique. In the case of correlational analysis, this latter step becomes a two-stage process--first the correlation coefficient is applied and it is determined whether the coefficient maintains the relevant properties of the observations and then the correlational analysis technique is applied and it is determined whether the technique still indicates these properties in its results.

This process is actually much less trivial than it may seem. Sometimes the results come out as expected--the technique does correctly capture the properties of the data. This does serve to confirm the theory motivating the application, but obviously no single example can prove a theory. A success therefore just serves to open up an area for more extended analysis. A rigorous theoretic proof becomes necessary along with careful experimentation concerning the limits within which the theory does hold.

Often the results do not come out as expected--the theory does not correctly capture the properties which were directly built into the data. Such exciting failures force upon one a direct confrontation with one's theory. If the data do contain the properties which were intended, then the analysis technique is in some way inappropriate in the circumstances. This might mean that the technique is appropriate only in some limited sets of circumstances. When correlational analysis is involved, it might mean that the coefficient does not tap the properties of the data which the analysis technique is designed to utilize. The situation practically becomes indeterminate when the technique is expected to represent only

some of the properties of the data so that several different data structures could yield effectively identical representations. In this case the analysis yields a solution different from the initial data structure but one similar to that structure in certain essential characteristics. Here one must carefully determine whether the two structures are really similar in these essential characteristics or if the solution is indeed incorrect. The iterative procedures of similarity analysis further compound these difficulties. A good solution would be one with a very low degree of error since the data are known to be perfect. A solution with a moderate degree of error might indicate an error in the theory or it might simply indicate that the best solution was not obtained by the iteration which was performed.

This preoccupation with studying known perfect sets of data may seem strange. I would contend that it is not possible to interpret the results of an application of a technique to unknown sets of data unless its use on known data sets is well understood. Data which fit a model perfectly are used since the application of the technique to data with error cannot be understood without a prior knowledge of its application to errorless data. The robustness of the technique with respect to error can be studied after its behavior on perfect data has been explained. But one cannot seriously attempt to use a technique on real imperfect data if it does not perform as desired on known perfect data.

This philosophy should be followed more frequently in political science, particularly in methodological work. Model testing and the employment of simulated data are valuable, indeed essential, techniques

in the development of systematic methodology. Many, perhaps most, data analyses would profit from a preliminary run through with hypothetical data. Experimentation with known data can considerably clarify the working of both old and new data analysis techniques.

CHAPTER II

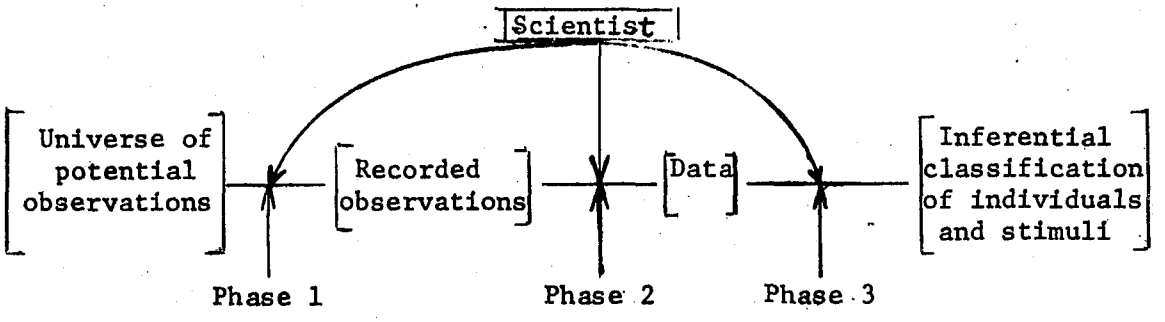
A THEORY OF DATA FRAMEWORK

Dimensional analysis of legislative voting seeks a classification of the legislators and/or the roll calls. This classification may itself be the basic goal of the analysis or it may be a preliminary to a further analysis that utilizes the classification of individuals and stimuli which the dimensional analysis has made possible. The classification obtained is the end-product of a process of inference based upon the empirical observation of behavior. This process of inference is the topic of the present chapter.

Clyde Coombs (1964) has abstracted three phases of such an inference process. First, the features of the behavior which are considered relevant are selected. Second, the behavior is classified in terms of response models. Third, the structure of the data is obtained through the employment of the appropriate analysis techniques. This process has been illustrated by Coombs with the flow diagram of Figure 2.1.

The first phase of the inference process involves the observation of the behavior and the selection of the features of the behavior which are relevant. The legislature records its own roll calls, but relevance must still be determined by the analyst. Which roll calls and which legislators should be included in the analysis? What is missing data

Fig. 2.1 Flow Diagram from the Real World to Inferences*



*Coombs (1964, p. 4)

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in this context and how should it be handled in the analysis? These decisions, usually made very early in the analysis, may mold to a very large extent the final results.

The second phase involves the conversion of the observations into that which will be analyzed, the "data." One does not simply analyze the observations. The observations are interpreted prior to the use of the actual analysis technique when the analyst assumes that one response model underlies the behavior rather than an alternate response model. This interpretative stage may be critical in that the investigator often employs only the methods appropriate to the particular interpretation chosen without actually comparing alternative response models. Data theory treats this crucial stage of interpretation.

The third phase of the inference process involves the use of scaling techniques to obtain the desired classifications. Several different types of analysis techniques can generally be applied to a given type of data. The analyst may, for example, choose that analysis technique which best embodies his posture toward error. Additional assumptions regarding the level of measurement, the dimensionality,

and the nature of the multidimensional space may also be necessary. Scaling theory treats the issues involved in the choice among alternative scaling techniques on the basis of such considerations.

Data analysis is thus not a simple objective operation. A very real interaction with the observations is required. Decisions are made throughout the analysis which will have implications for the final results. Theory is inherent in these decisions, so there certainly cannot be an analysis without theory. These decisions must be tested if the theory itself is to be proved.

This perspective on data analysis differs from the classical tradition of hypothesis testing and from the more recent spurts of raw empiricism. The acknowledgment of the subjectivity of all analysis and of the need to make arbitrary decisions throughout the analysis distinguishes this view from one which the traditionalist would accept. Simultaneously the recognition of the importance of one's assumptions and of the necessity of model testing elevates this position above brute empiricism.

We shall now turn to the stages of a roll call analysis, following the outline suggested by Coombs. Special attention will be given to the choices faced within a roll call analysis, noting the degree of importance of each choice as well as the determinants of each necessary decision.

PHASE 1: THE STRUCTURE OF LEGISLATIVE ROLL CALLS

As the first stage of a roll call analysis, a series of assumptions must be made concerning which observations should be treated. The roll calls and legislators to be included in the analysis must be chosen.

Also which of the possible responses to a roll call should be considered valid responses as opposed to missing data must be decided as well as which of these should be treated as positive responses and which should be treated as negative responses.

The basic rule to follow in making these decisions is to make as few assumptions as possible which might affect the results. The analyst should not, wittingly or unwittingly, miss the opportunity to allow his analysis to answer a question which is relevant to his argument. The worst cases of the violation of this rule are those in which the investigator, usually without realizing it, forces a certain outcome to his analysis by one or more of his preliminary decisions. As a corollary, those decisions which are made should be as neutral as possible relative to the final results. This usually means using all information which is available, provided it is deemed reliable. While this general philosophic perspective is important, the specific decisions will always depend on the exact circumstances of the analysis. Indeed the investigator may often be justified in feeling that he can make many of these preliminary decisions immediately without in any way biassing his basic results.

Selection of Votes

Selection of Roll Calls.--Roll call analysis always requires a selection of roll calls. The item selection may involve the choice of a time period, the choice of content areas, the sampling of roll calls, and/or the exclusion of various types of roll calls. This selection process is clearly vital as votes dropped at this stage have no chance of affecting the conclusions of a study.

Roll call analysis always involves the selection of a time period. This includes a decision as to the length of the time period to be studied. Scaling techniques implicitly assume relative stability of the preferences of the individuals. Some legislators, however, may change their issue positions during the time period. Actually it is possible that such change would not adversely affect the scaling. A general movement of legislators in one direction over time, for example, might be reflected in the placement of items on the dimension without any "scale error." This "temper of the times effect" is probably very common but it in no way detracts from the actual scaling. One might also desire to verify the degree of stability over time instead of simply assuming stability. Studies of the American Congress, for example, usually assume that the positions of legislators are constant throughout the two year time span between elections. On the one hand this assumption is open to challenge (Riker and Niemi, 1962) while on the other hand it would be interesting to try to scale within an issue area across Congresses to test the degree of constancy.

Within a time period, the selection of roll calls by content is frequently employed. The investigator himself defines the issue area or areas of interest and then classifies the roll calls to obtain those falling within the relevant policy areas. However it is not a simple matter to delimit issue areas. Should civil rights votes, for example, be placed into a separate issue area or should they be put into a broader civil liberties category? Whether a special roll call actually belongs to a specified issue area is frequently debatable. These difficulties suggest that one should avoid an initial specification of

issue areas as much as possible and that an analysis technique should be chosen which has sufficient power to describe the actual issue areas without such prior specification.

The number of roll calls to be analyzed must still be kept manageable. Computerized analysis techniques are expensive when there are hundreds of roll calls in the analysis as there may be in the analysis of a typical Congress. The only solution which avoids biassing the results of the study is to employ representative sampling of roll calls.

Another solution to the problem of numbers would be to choose only the most competitive roll calls, those with the narrowest winning margins. The rationale behind such an approach would be that the high conflict votes best demonstrate the bases of legislative conflict. One could choose to analyze the most competitive and the least competitive roll calls separately in order to contrast legislative behavior in the high and low conflict situations. David Truman (1959) follows such an approach for each party separately, using Rice's index of cohesion to measure the competitiveness of the roll calls within parties. One could instead analyze only the highly competitive roll calls. William Riker (1959) has suggested a measure of competitiveness which combines the size of the winning margin with the number of absences. The most competitive vote is the one with the tightest result and the fewest absences. Riker labels his measure a "significance coefficient," though the term significance has connotations which can be challenged in this context. Lee Anderson (1966) has used Riker's index to find the most competitive roll calls for subsequent analysis. Such an analysis of high conflict votes yields quite different

information from that of an analysis of a random sample of votes. Random sampling of roll calls is simply a method of cutting down the number of votes to be analyzed whereas the Rice and Riker indices have substantive implications.

Finally most roll call studies exclude certain types of roll call votes. Unanimous votes are generally excluded, which cannot affect the results, and near-unanimous votes are often excluded. Items with a large number of absences are also frequently excluded. Alternative strategies for dealing with absences will be considered below. Party line divisions are often excluded as they only indicate the effect of the partisan factor. However the effect of the partisan factor may in itself be very interesting (MacRae, 1968) as will be seen in the analysis of Chapter VI below.

Selection of Legislators.--Roll call analysis also requires a selection of the legislators to be included. Legislators are sometimes excluded from an analysis if they have too much missing data on the roll calls under consideration. Thus the votes of a legislator who died midway through the time period being studied are often dropped. This strategy inevitably results in the loss of some available information. It would be preferable to keep such legislators in the analysis if the analysis technique makes suitable allowance for missing data. This possibility will be discussed in detail in the next section.

The selection of individuals also involves the decision of whether or not to analyze the several political parties together. Dimensional analysis assumes that all individuals have the same perception of the dimensional locations of the items, though the individuals differ from

one another in their preferences. It would be a clear violation of the model to analyze two parties together if their members have different perceptions of the dimensional attributes of the roll calls.

Several different circumstances involving different frames of reference can be mentioned. One possibility is that each party behaves on roll calls from a single issue area in a manner consistent with a single dimension, but with a very different ordering of the items on that dimension for each party. Another possibility is that a scale can be found such that each half of the scale differentiates only the members of a single party. Each party is unanimous on the roll calls on one half of the scale and shows a scalar pattern on the roll calls on the other half of the scale. Such a scale can sometimes represent only random fluctuations from party positions, a situation which should be avoided (Clausen, 1967a). In multi-party legislatures, most parties might share a common frame of reference while a single-issue or single-social group party (such as an agrarian or Christian party) has a unique frame of reference.

Parties with different frames of reference should be analyzed separately. This would permit inferences on the internal division of the party and its factional characteristics, but would yield limited comparisons between the parties. Parties with the same frame of reference can be analyzed separately or together. Analysis of the parties together permits one to judge the extent of partisan conflict as well as permitting the discovery of bipartisan factions. The behavior of a single party can be observed even when the parties are analyzed together, so that analysis of the parties together yields more information than separate analyses of the parties.

Analysis of the parties together is justified only if they share the same frame of reference, but this can be ascertained only through the analysis. The commonality of perceptions can be tested through comparing separate dimensional analyses of the several parties. However such a test is difficult since inter-party differences would have to be differentiated from error. The assumption of a common frame of reference would be rejected only if the separate analyses yield totally incompatible results. The commonality of perceptions can also be tested through a single analysis of the several parties together. The combined analysis would yield a poor fit to the observations if the parties have different conceptual frameworks. There would be considerable disarray in the results with a sizable overestimate of the dimensionality.

Classification of Votes

Missing Data.--The rules of a legislature determine the various possible forms of response to a roll call. In the American Congress, for example, a legislator may vote on a motion, join in a pair, announce a position, abstain, or be absent on any particular roll call. Other legislatures provide different combinations of alternatives, but this list includes most possible alternatives. Which of these should be considered as missing data? Once missing data is defined, how should it be handled in the analysis? These decisions "define" the observations to be subsequently analyzed.

Pairings and announcements are not as formal in character as recorded votes. On occasion they represent parliamentary maneuvering, as when a legislator announces in favor of a motion which requires a

fixed number of votes to pass so that by not voting he actually prevents its passage. Usually in the American Congress pairings and announcements result simply from the absence of the legislator, though at times they do result from bargains with party leaders who do not want the legislators to actually vote against them. Converting these responses to missing data would lose the information provided by the legislators on the direction of their preferences. One might choose to consider these as responses of intermediate strength, but such a distinction is not reliable if pairings and announcements just indicate absence from the chamber. Pairings and announcements should not be differentiated from actual votes unless such a differentiation can be contextually justified.

Abstentions frequently seem to carry more substantive meaning than absences. The legislator who is present but decides against voting and the legislator who is absent but decides against announcing his position are consciously choosing not to indicate their preferences. Such behavior is clearly different from that of the Speaker of the House who votes only to break the occasional tie and the replacement who fills a vacancy in the second half of a session, both of whom are virtually precluded from indicating their preferences on a large number of motions. While a distinction can be made, it is often difficult to assess whether the member purposefully chose not to indicate his preference. One knows nothing about the preferences of those who abstain and those who are absent. Except for legislative settings such as the United Nations where there are traditional distinctions between abstentions and absences, there is little gain in differentiating between such forms of behavior.

Once the definition of missing data has been chosen, one must decide how to handle the missing data. The decision depends considerably on the analysis technique to be employed. Some techniques require that there be no missing data, making the problem acute. In such situations one must either assign all missing data or delete all cases having missing data. Other techniques, particularly correlational procedures, have a high tolerance for missing data, so long as it is distributed non-systematically. These techniques permit the deletion of missing observations without deleting the entire case.

Several methods of assigning missing data have been used. Missing data can be randomly assigned, either in proportion to the actual vote totals on the roll call or on an equally likely basis. Random assignment tends to mask the patterning inherent in the actual data. Most dimensional analysis techniques are not sufficiently robust to tolerate the addition of much extra random noise to the data. At the opposite extreme is the assignment of missing data on the basis of the known predispositions of the legislator. This strategy would result in the strengthening of the patterns thought to exist in the data. It is an unnecessary intrusion of the subjective into the data analysis. A closely related approach is to "recover" the missing observations through prediction from the available data, as through multiple regression using highly correlated items as predictors. This would seem likely to strengthen systematically the dominant patterns in the available data. Such a prediction scheme would be particularly inappropriate if it were not taken into account in subsequent tests of the fit of the results to the data. A final assignment procedure is to

assign the missing data into an intermediate category. This device is often used in correlational analysis where one computes correlation statistics on each pair of items and then analyzes the matrix of coefficients. The assignment of missing data to an intermediate category for correlational analysis can also affect the results. Even if only an intermediate ordinal value is assigned, the magnitude of the correlation coefficient would be affected. Since the missing data is not generally related to the available observations, the magnitude of the correlation coefficient is usually lowered. This would in turn affect the analysis of the correlation matrix and the interpretation of that analysis. Thus all procedures for the assignment of missing data are likely to affect the final results.

It is generally preferable to delete the missing data rather than use an assignment procedure. Deletion of missing data can mean the dropping of legislators and/or roll calls with any or much missing data. This deletion strategy would, however, result in the loss of considerable available information. Correlational analysis techniques instead permit one to delete missing data "pairwise." The absence of a legislator on a roll call need only affect the correlation coefficients calculated between that roll call and the other roll calls, not the coefficients which measure the correlation between two roll calls on which he actually voted. Pairwise deletion of missing data means that all individuals who voted on both items involved in the computation of a particular correlation coefficient will be included while only those individuals having missing values on either of that specific pair of items will be deleted. This strategy results in the maximum utilization

of available data without making additional assumptions about the missing data. Reliable estimates of the values of the correlation coefficients can be obtained by deleting missing data pairwise so long as the missing data is neither excessive nor systematic. Some techniques for the analysis of the correlation matrix are sensitive to patternings of coefficients which could not have occurred with complete data, but such problems are best left to a discussion of the techniques themselves. In general handling missing data by pairwise deletion in correlational analysis is least likely to affect the results of the analysis.

By deleting missing data (abstentions and absences) pairwise while not differentiating between actual votes, pairs, and announcements, legislative votes are converted into binary data. All that is analyzed is whether the legislator responds affirmatively or negatively to the roll call.

Item Direction.--It is generally necessary to specify the direction of the roll call vote. The direction of an item depends on its wording, whether the motion is stated affirmatively or negatively. There is a very real difference between voting yes on a passage motion and a tabling motion on the same bill. Item direction can be a problem in that it is defined only in a relative sense--relative to the polarity of other items or to the polarity of the dimensions. An item which is positive relative to a pro-civil rights dimension, for example, would be negative relative to an anti-civil rights dimension.

The direction to be ascribed to a roll call is not always evident. It is especially difficult to determine item reversals when there are multiple dimensions, since the direction of the item relative to all

the dimensions must be taken into account. An item given the wrong direction might lead to an overestimate of the dimensionality when certain analysis techniques are utilized. In such cases one would want to reverse the items to obtain the proper dimensionality, but it is not always possible to specify how an item relates to several dimensions simultaneously.

A priori assignment of item directions should not be permitted to result in erroneous conclusions. Some analysis models virtually ignore the construct of item direction. When item direction is relevant, analysis techniques might be used which determine the proper item polarity without having to specify the polarity in advance. Correlational techniques can be particularly useful in solving for item polarities. The correlation coefficient can be defined such that item reversals do not affect the magnitude of the coefficient but only reverse its sign. A correlational technique which is capable of distinguishing sign patterns is therefore suitable for the determination of item polarities.

Item reflections cause a somewhat different problem when inter-individual relations are analyzed. We would not regard some legislators as having been reversed relative to other legislators, while we would regard some roll calls as having been reversed relative to other roll calls. This introduces a degree of asymmetry between inter-item and inter-individual analysis (MacRae, 1966b). This can be removed by performing all appropriate item reversals prior to the inter-individual analysis. Alternatively, analysis techniques can be employed which are not susceptible to the reversal problem in the inter-individual case.

Table 2.1 summarizes the many decision which must be made during this initial phase of a roll call analysis. The general posture advocated has been to select roll calls by random selection within time periods, include members of all parties within the analysis, treat legislative data as binary data, and resolve item polarity analytically. However it is impossible to reach authoritative conclusions on these issues. The decisions faced during the first phase of a roll call analysis can be answered only within the specific research context which will frequently modify the solutions suggested here.

PHASE 2: DATA THEORY

As the second stage of a roll call analysis, the underlying response model and the corresponding analysis procedure must be selected. Two basic dimensions are involved in both of these selections, though the choice of analysis procedure is to a degree independent of the choice of response model. Data theory treats these two basic dimensions.

Response Models.--An important aspect of a roll call analysis which is generally treated only implicitly is the choice of the response model. It is necessary to interpret the observations as to the underlying response model. Questions of the response model also are involved in the topic of scaling theory which will be treated later in this chapter, but the most basic categorizations are those to be discussed here. Two basic dimensions of a categorization of response models have been outlined by Coombs (Coombs, et. al., in press).

1) The relations being analyzed can be either the relations among one set or the relations between two sets, where the legislators

TABLE 2.1

PHASE 1 OF A ROLL CALL ANALYSIS

I. Selection of votes

  A. Selection of roll calls

    1. Time period

    2. Content selection

    3. Random selection

    4. Competitiveness criterion

    5. Exclusion of roll calls

      a. Unanimous and near-unanimous votes

      b. Votes with excessive absences

      c. Party votes

  B. Selection of legislators

    1. Exclusion of legislators with considerable missing data

    2. Inclusion of multiple parties

II. Classification of votes

  A. Missing data

    1. Definition of missing data

      a. Pairings and announcements as valid data or missing data

      b. Abstentions as distinct from missing data

    2. Handling of missing data

      a. Assignment of missing data
        i. Random assignment
        ii. Subjective assignment
        iii. Predictive assignment
        iv. Intermediate value

Table 2.1 (Continued)

b. Deletion of missing data
   i. Deletion of full case
   ii. Pairwise deletion by correlational procedures

B. Item direction

1. A priori assignment

2. Analytical determination

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can be considered as constituting one set and the roll calls the other. In other words, the votes may depend on relations among the roll calls, relations among the legislators, or relations between the legislators and the roll calls.

To illustrate this distinction we will compare two possible theories of the legislative process. One theory is an interactional theory which suggests that legislators consciously agree to vote together over a wide range of roll calls as part of a highly developed system of mutual logrolling. According to this theory legislative votes are determined by the relations among the legislators, a single set hypothesis. The other theory is an ideological theory which suggests that legislators respond to roll calls by comparing their preference on the dimensions with the location of the roll call with respect to the dimensions. This would mean that the votes are determined by the relations between the legislators and the roll calls, a two set view. Both of these theories have been suggested as the appropriate legislative response model, David Truman (1959) being a prime adherent of the interactional theory while Duncan MacRae (1958) is one of the main adherents of the ideological theory.

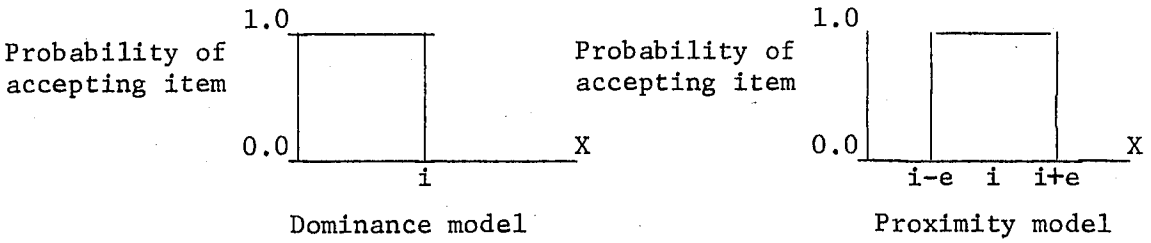
2) The relations being analyzed can be either dominance or proximity relations. If we were comparing tones, for example, the

statement that one tone is louder than another involves a dominance relation while the statement that two tones are not alike involves a proximity relation. Both dominance and proximity relations have their counterparts in the legislative voting context.

To illustrate this distinction we will consider two forms of ideological theories of legislative voting. A dominance theory would specify that the legislator votes for a motion if and only if he favors at least as much of the qualities represented by that motion as is contained in its provisions. A proximity theory, on the other hand, would specify that the legislator votes yes only on motions representing approximately as much of the qualities involved in the measure as he desires. The dominance model utilizes the form of cumulation along the dimensions which is inherent in Guttman scaling. The proximity model involves a special form of non-cumulation, the rejection of both extremes by the moderate. It is only reasonable to admit that both models probably underlie legislative voting in some situations, though legislative analysts have concentrated their attention exclusively on the dominance theory.

A convenient means of representing a response model is through its individual trace line. The individual trace line plots the probability of a given individual's acceptance of items as a function of their location on the dimension. Figure 2.2 shows the forms of the individual trace lines for the unidimensional dominance and proximity models. In the dominance case, the individual whose trace line is given is certain to accept all items below point i on dimension X and is certain to reject all items above that point. By way of contrast the individual

Fig. 2.2 Two Cases of Individual Trace Lines in a Unidimensional Space



whose trace line is shown for the proximity model is certain to accept all items within an acceptance region of radius e around his "ideal point" i, but he is certain to reject all items to either the left or right of this region.

The two classifications of response models given here are independent. That is, both one set and two set relations of both the dominance and proximity types can theoretically occur. These four possible situations have been represented schematically by the four quadrants of Figure 2.3 by Coombs (Coombs, et. al., in press). Legislative roll call votes could be generated by any of the four underlying processes of this representation.

Studies of legislative voting too frequently ignore the possibility of alternative response models. Data theory's single most important contribution is to sensitize one to this possibility. One interprets the observations when determining the nature of the relations which constitute the data. It is necessary to test the interpretation which is made, rather than simply following through the interpretation to its logical consequences which then become the results of the analysis. Model testing can be seen as the core of dimensional analysis.

Fig. 2.3 The Four Quadrants of the Theory of Data*

	Dominance Relations	Proximity Relations
Two sets	Quadrant II	Quadrant I
One set	Quadrant III	Quadrant IV

*Adapted from Coombs (Coombs, et. al., in press)

Analysis Procedures.--Coombs's typology actually classifies several different concepts simultaneously. It is generally seen as a classification of data collection techniques, though we are using it as a means of classifying interpretations of the underlying response model for data collected by a single method. It can also be seen as a categorization of data matrices and, therefore, as a categorization of procedures by which to analyze data matrices.

There are two fundamental types of data analysis. One may directly analyze the raw matrix of votes. This matrix would be a rectangular matrix with the rows representing legislators and the columns representing the roll call divisions. The cell entry then indicates the vote of the individual on the motion. As both the roll calls and the legislators are involved in the analysis simultaneously, this type of analysis is called joint space analysis.

Instead of analyzing the raw matrix, one can instead intercorrelate the items or the legislators and analyze the correlation matrix. Stimulus space analysis deals with the analysis of the square inter-item correlation matrix while population space analysis refers to the

analysis of the square inter-individual correlation matrix. We will refer to these two forms of analysis together as correlational analysis in contrast to joint space analysis.

The entries of both the raw matrix of observations and the derived matrix of correlations can be interpreted as either dominance or proximity measures. For the joint space matrix, a yes vote would indicate that the legislator dominates the roll call for the dominance interpretation and that the legislator is near the roll call for the proximity interpretation, or the opposite in both cases if the item is reflected. For the correlation matrix, the coefficient indicates the extent to which cumulation exists between the column item and the row item for the dominance interpretation while it indicates the inter-item distance for the proximity interpretation. Thus the typology of Figure 2.3 can be interpreted as indicating four types of data matrices.

The decision as to the form of the analysis procedure is fairly independent from the choice of response model. Regardless of whether joint space analysis or correlational analysis is adopted, one must decide which form of response model underlies the observations. In both cases one must decide whether the relations among one set or between two sets are involved as well as whether dominance or proximity relations are involved.

The independence between the form of the analysis procedure and the response model may be illustrated by returning to the alternative hypotheses of Truman and MacRae. Truman (1959) argued in favor of a single set log-rolling hypothesis while MacRae (1958) favored a two

set ideological hypothesis. Truman analyzed his data by means of correlational analysis, but his data could have been derived from a pattern of behavior which conformed to MacRae's two set hypothesis. The use of correlational analysis does not prove the existence of single set behavior. Truman obtained an interesting set of results concerning the internal structures of the parties, but without testing his basic logrolling and generalized influence theory.

Correlational analysis has been defined broadly here at the expense of joint space analysis. Keeping the class of joint space analysis techniques small forces one to make explicit all of the assumptions employed when a correlational technique is used under a two set response theory. This emphasizes further the importance of testing alternative response models.

The translation process involved in shifting from a two set response model to a correlational analysis calls attention to Coombs's distinction between observations and data. The observations are that which are observed while the data are that which are analyzed. The two are different when the observations are translated into correlations. They are also different, though in a more subtle sense, even when such translations into correlations are unnecessary. The real difference is that which is analyzed is a set of relations, abstractions from the observations. We analyze relations between the individuals and the stimuli, relations among the individuals, or relations among the stimuli.

Assumptions.--In discussing response models and analysis procedures, the geometric metaphor of "data points" is frequently employed. The roll call items can be seen as located in a space of relevant dimensions

and the individuals are located in that same space according to their preferences. The points representing the individuals in this space will be termed their "individual points." These individual points may be thought of as their "ideal points" in the context of the proximity model or as their "tolerance values" in the context of the dominance model. The stimuli will be represented as "stimulus points." The data then represent relations between stimulus points and individual points, relations among the individual points, or relations among the stimulus points. In general the data can be viewed as relations on points.[1] The analysis of these relations is intended to map out the space of individual points and/or stimulus points.

The formality of the definition of data as "relations on points" also extracts some of the implicit assumptions of scaling. If the observations result from the relations between the locations of the individual legislators and the roll call items, then three assumptions are inherent. Each individual can be represented in that space. The individual preferences remain fixed in their locations in the space over the time period of the observations. The individuals agree on the locations they ascribe to the stimuli in the space, or at least their responses are consistent with some single set of locations for those stimuli.

Each of these assumptions is critical. The third assumption amounts to a common frame of reference for all individuals. The

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[1] For some models the items or individuals are instead represented as lines (not necessarily straight lines) in two-dimensional space, planes in three-dimensional space, and, in general, (n-1)-dimensional hyperplanes in n-dimensional space.

analysis must be changed if this assumption is violated. If, for example, Republicans have one frame of reference and Democrats have another, then simultaneous analysis of the two parties would violate the assumption. The analysis should then proceed within the two parties separately.

The second assumption simply states that scaling is a static technique. Some kinds of change, such as a uniform shift in individual attitudes over time, can be incorporated in the model with little difficulty. Slight random fluctuation over time can also be incorporated in some models. In general, however, the static assumption is violated if individuals change their positions within the time period being studies. The time period of the study should be shortened if there seem to be substantial dynamic elements.

The first assumption, that each individual can be represented in the space, is the most basic assumption. However, the individuals may not have sufficient interest to ever consider seriously what they really desire so that their responses are essentially random. Such response uncertainty is a more serious problem in the survey setting, but it may also be evidenced in roll calls of minor importance.

PHASE 3: SCALING THEORY

As the third stage of a roll call analysis, a scaling technique must be chosen and then applied. The decisions regarding the type of data involved, which were discussed in the previous section, yield a set of analysis techniques which could be employed. Questions of theory and pragmatic efficiency enter into the choice of the specific analysis technique. The matter of efficiency is especially important

as the analyst must balance off the appropriateness of the assumptions of each technique against the resultant pay-off in terms of results. Would the type of results which the technique can yield be sufficient to justify the use of a technique which may not, strictly speaking, be appropriate? This question must often be faced directly if the analysis is to proceed.

Dimensional analysis has much in common with the task facing the geographer. The geographer seeks to construct a geometric model of the world, creating a representation of certain features of the world using a space of relevant dimensions which indicates the relations between the data points. A mapping procedure is necessary to establish the desired correspondence between the world and this geometric model. The mapping procedure is inevitably inexact. This state of affairs is acceptable so long as certain basic properties are mapped accurately and so long as the nature of the inexactitude is known. Dimensional analysis likewise involves the construction of a geometric model to represent certain of the relations between the data points. Measurement theory treats the characteristics of the mapping procedure whereas error theory describes the resultant inexactitude.

Measurement Theory.--Measurement theory deals with the mapping from the relations of the data, the "empirical relational system," to the relations of geometry, a "numerical relational system" (Coombs, et. al., in press). Measurement theory involves the assignment of numbers, which raises the question of what type of numbers are to be assigned. Numerical relational systems can be

characterized in terms of their measurement level, their dimensionality, and the nature of their multidimensionality.

The level of measurement of the numbers assigned usually ranges from nominal to interval. The lowest level is the nominal scale which involves only categorization. The ordinal scale introduces the notion of an ordering of the numbers. The ordered metric scale further adds the notion of an ordering on the differences between the numbers. The interval scale also permits a comparison of the intervals between scale values, such as stating that the length of one interval is a certain multiple of the length of another interval. The interval and the ordered metric scales are generally considered high levels of measurement in contrast to the nominal and ordinal scales.

The choice of level of measurement raises what Coombs (1953) has termed "the dilemma of the social scientist." A high level of measurement is required for many sophisticated analysis techniques which one might wish to utilize after the scaling. Therefore one would want to obtain as high a level as possible from the scaling. Generally scaling yields a high level of measurement only when a scaling technique is employed which assumes a similar high level of measurement for the data. However social science data do not usually merit such a high level. Should the assumption of a high level of measurement of the data be made when it cannot be strictly justified or is the scaling technique which can yield the type of results required for later analysis to be discarded?

A conservative approach would permit the analyst to make only those assumptions which his data can justify, even if he therefore cannot complete the desired analysis. An alternative risk-taking

approach would instead permit the analyst to balance the likelihood of obtaining a wrong conclusion in his analysis due to making questionable assumptions against the loss incurred when the desired analysis cannot be performed. This risk-taking approach acknowledges the possibility that the subsequent analysis will yield correct conclusions even when some of the preliminary assumptions are debatable. The degree of risk depends upon the robustness of the analysis techniques and the structure of the data. In many situations the risk-taking perspective would justify a form of analysis which could not otherwise be performed.

The ability to obtain identical results from techniques assuming different measurement levels suggests the use of the most convenient technique or the technique which most fully describes the dimensional structure even if that technique could not be justified formally. Also scaling techniques can be chosen which yield scales of a high measurement level while still making only low level assumptions on the data. Similarity analysis in particular assumes only ordinal data while it yields ordered metric results, the latter being very close to interval level under many circumstances (Shepard, 1966). Because one can obtain scales at the high levels without making the strong assumptions which have always seemed necessary, the level of measurement problem may not be as important as most of the literature suggests.

In addition to the question of measurement level, one must consider the dimensional properties of the numerical relational system. The numbers which are to be assigned can either be single numerals or they can be the n-tuples of an n-dimensional space. The analysis

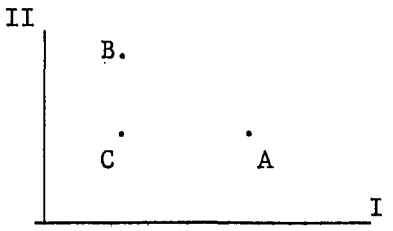
technique may determine the appropriate dimensionality, but some analysis techniques require one to specify in advance the dimensionality as well as the dimensional composition.

The composition of a multidimensional space is a topic too rarely discussed explicitly. Multidimensionality frequently takes on the meaning of the existence of multiple separate dimensions. If ten items were found to constitute two perfect five item unidimensional scales, such a multiple dimensional space would be involved. The more general case is when an item falls on more than one scale or when more than a single dimension is required to account for the variance of the item. Dimensional analysis techniques must be able to cope with such truly multidimensional situations.

Multidimensionality requires further specification of the nature of dominance and proximity relations. Figure 2.4 shows a two-dimensional space with three points. Point C has been located at the intersection of the projections of point A onto the vertical axis (dimension II) and of point B onto the horizontal axis (dimension I). If we consider this space as a dominance space, we would find that point A is above point B on dimension I but is lower than point B on dimension II. Does this mean that A dominates B or that B dominates A? In order to determine which point dominates which other point in a multidimensional dominance space, it is necessary to specify a composition axiom which defines how the several dimensions combine.

If we consider the diagram on the following page to represent a proximity space, we could measure the distance between points A and B by finding the length of the straight line connecting them. This

Fig. 2.4 Points in a Two-Dimensional Space



Euclidean measure of distance would imply that multiple dimensions combine as the square root of the sum of squares of differences on the dimensions, a most unusual psychological process in the legislative context. Instead one might just take the simple sum of the differences on the dimensions, the sum of the distance from A to C and the distance from C to B, to measure the distance between points A and B in Figure 2.4. Other non-Euclidean definitions of distance could also be used to describe the way in which the several dimensions of a proximity space combine. The appropriate measure of distance is an empirical question.

In summary, choices do exist relative to both the measurement level and the dimensional composition. Level of measurement is not the central concern that it may at times appear to be. Dimensional composition, on the other hand, is a basic aspect of dimensional analysis which is generally disregarded. Once these properties of the numerical relational system are chosen and the measurement rule is selected, it remains only to determine how to treat error.

Error Theory.--Empirical observations which violate the model of behavior underlying the measurement rule are usually termed errors. The use of this term is not intended in any value-laden sense. A

legislator may have perfectly good reasons to behave in a fashion different from that suggested by the model. There is no intent to judge his behavior. Error in this context signifies an inadequacy in the analyst's model rather than a mistake on the part of the legislator. Since the model intends to reduce the full richness of the roll call voting to a summary of dominant patterns, it is inevitable that the model will not fit the observations perfectly, causing error. Error theory deals with the choices which must be made concerning the definition and measurement of error.

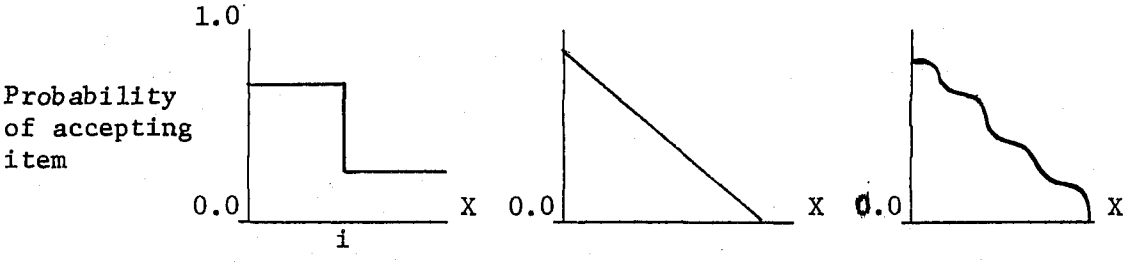
What is an error and how is it to be identified? Error exists only in a relative sense--error relative to the measurement model. What one model would classify as error might fit another model perfectly. Deterministic models predict exactly which response patterns will occur so that any other pattern would constitute error. Probabilistic models build in a degree of violation of the expected response patterns as a parameter of the model. This leads to a continuum of tolerance for error.

When describing the dominance and proximity response models earlier in this chapter, the forms of the individual trace lines for the two models were given. The trace lines of Figure 2.2 showed the deterministic versions of these models. Knowledge of the location of the item relative to the person (and possibly the width of his acceptance region) is sufficient in these deterministic models to indicate with certainty whether the individual will accept or reject the item. The likelihood of the individual's acceptance of an item took on only the values of zero and one. Probabilistic models permit

this probability to assume an intermediate value between zero and one as well.

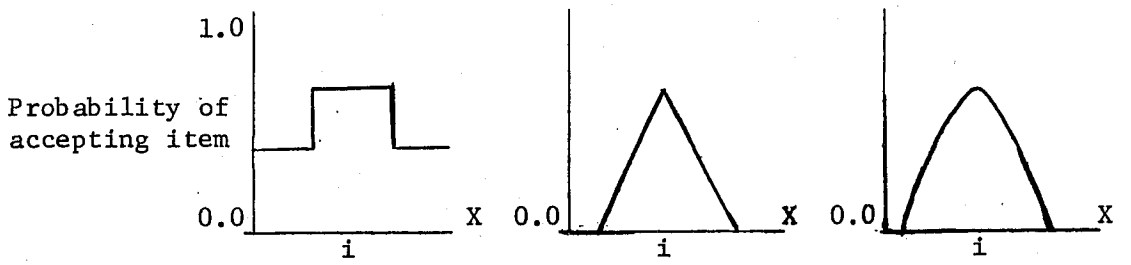
In the probabilistic version of the dominance model, the individual is more likely to accept an item if it is below rather than above his position. The only restriction on the shape of the individual trace line in this generalized unidimensional dominance model is that it be monotonically non-increasing. Figure 2.5 indicates some of the possible trace lines. It should be noted that the entire notion of the location of the individual on the dimension is supplanted by the individual trace line in some of these models.

Fig. 2.5 Some Probabilistic Unidimensional Dominance Models



In the probabilistic version of the proximity model, the individual is more likely to accept an item the closer it is to his ideal point. We shall assume symmetry on either side of the individual's ideal point, though asymmetric models might also be imagined. The only actual restriction on the shape of the individual trace line is that it be single-peaked with that peak occurring at the individual's ideal point. Thus the general shape of the probabilistic unidimensional proximity trace line corresponds to Duncan Black's notion of a single-peaked

Fig. 2.6 Some Probabilistic Unidimensional Proximity Models



preference function (Black, 1948, 1958). Three possible unidimensional probabilistic proximity trace lines are shown in Figure 2.6.

Scaling techniques range from those intolerant of error to those which are totally invulnerable. A deterministic theory considers any violation of the small number of compatible response patterns to constitute an error. This strict intolerance of error causes such a theory to be, strictly speaking, disconfirmed whenever a single error occurs. With a multidimensional deterministic situation, it may be very unclear as to which of several competing models is correct since each would be disconfirmed to some extent by the data. A probabilistic theory introduces the possibility of error as a parameter of the model. Such theories are generally invulnerable as in the case of some scaling techniques which will find the best unidimensional representation of any data structure. A measure of goodness of fit is frequently used with probabilistic models in order for there to be some possibility of rejecting the model.

There are several different mechanisms which can lead to a probabilistic model of behavior. The probabilistic character of a model can be an aggregate or an individual phenomenon. An aggregate probabilistic model is one in which each individual behaves in a

deterministic fashion but there are several individuals with the same location in the space who behave deterministically according to different parameters. In this case the probabilistic feature derives from a process of averaging over individuals with the same location in the space. It is also possible for the individuals themselves to behave probabilistically.

Why would rational individuals behave probabilistically? Coombs and his colleagues have suggested two processes yielding individual probabilistic behavior (Coombs, et. al., in press). The first is one in which the probabilistic behavior derives from difficulties in discrimination. In a dominance model, for example, the individual would vote for all items up to a given point. However he might have considerable difficulty in determining whether items near that tolerance point are on one side or the other of that point. This difficulty in discrimination leads to probabilistic behavior. The second process leading to probabilistic behavior is one in which there is some random variability in the location of the individual in the space. As the individual's point varies and as the stimulus points vary independently, the votes of the individual also vary. The variability of the points thus yields probabilistic behavior. These two processes give essentially similar predictions as to individual behavior so that we need not be concerned with distinguishing between them.

The continuum of error tolerance described in this section also relates to the choice between joint space analysis and correlational analysis. Joint space techniques tend to be deterministic whereas correlational techniques tend to be probabilistic. That means that correlational techniques are especially useful for multidimensional

analysis as they are less vulnerable. Also, however, there remains the possibility that the correlational model cannot be disconfirmed which makes model testing difficult.

Scaling Techniques.--The interpretation of the recorded observations into data is not sufficient to imply a single appropriate scaling technique by which the data should be analyzed. There are several different scaling techniques which are appropriate for the analysis of each variety of data matrix implied by Coombs's theory of data. A measurement rule and a set of error procedures must therefore be specified for the actual scaling. When the measurement rule and the error procedures are combined, one obtains the "algorithm for arriving at the numerical assignments" which constitutes a scaling theory (Coombs, et. al., in press).

With the application of the scaling technique, one arrives at the desired classification of the legislators and/or the roll calls. The classification is desired for one of two reasons--to discover the dimensional properties of the observations or to obtain measures of individual behavior for subsequent analysis. The choice of scaling technique depends in part on which of these reasons is dominant.

The dimensional properties of a set of roll call votes can be the focus of the research. At one level this can amount to seeking an objective delineation of issue areas. Alternatively the goal might be to describe the nature of legislative conflict in terms of the number of basic dimensions, their relative importance, and their interrelations. Such concentration on the dimensional properties of the observations per se has been referred to as dimensioning.

Dimensioning represents a search for the model which best explains the observations in order to be able to describe the underlying process.

The purpose of roll call analysis may instead be to obtain measures of the behavior of legislators which can be used in later portions of the analysis. The analyst seeking to describe the nature of representation, for example, might want to correlate a measure of constituency attitude on an issue area with a measure of the legislator's roll call behavior in that issue area. The roll call votes to be used would have to be selected with some attention to their functional unity and then the legislators' positions on the combined measure would have to be obtained. Such manipulation of the observations to score the individuals on scales to be further analyzed has been termed positioning by Donald Stokes to distinguish the motivation of the analysis from that which underlies an analysis that seeks only a dimensional characterization of the relations of items of individuals. Positioning emphasizes the need to obtain valid measurement of individual behavior.

The distinction between dimensioning and positioning is important. One might, for example, choose to employ very different criteria in the two modes of analysis because of their different ultimate objectives. Thus dimensioning should not be seen as simply a preliminary phase in a positioning analysis. It is true that the problem of validation must be confronted in dimensioning as well as in positioning and that the problem of model testing must be confronted in positioning as well as in dimensioning. Still it is too easy to identify dimensional analysis completely with the search for valid measures without acknowledging the importance of the descriptive function served by dimensioning. This

distinction then is useful in suggesting the existence of alternative reasons to undertake dimensional analysis as well as in pointing out the basic aspects of a dimensional analysis.

Validation.--The emphasis on validation in dimensional analysis involves problems which are far from trivial. How can one determine whether a scale measures the desired concept? How might one detect that some items in the scale are not valid indicators of that concept?

There are several procedures for the validation of a measure. The simplest involves face validity, checking subjectively whether the content of the items corresponds to the concept which the scale seeks to measure. A second procedure is to check the consistency of the scale, determining statistically the level of error in the scale and the amount which each item contributes to that overall level. A third procedure is to consider the scale in its relation to other variables, prediction to or from criteria. If the criteria can be assumed to be valid, the correlations of the scale with the criteria can be calculated. For the scale to be valid, the correlations with the criteria should be high and the removal of some of its constituent items should not drastically increase the correlations. The use of the criteria procedure is generally complicated by the fact that the validity of the criteria may be equally uncertain.

It should be admitted that these general guides for validation may require modification in a given research context. Validity depends on the context because it has little meaning in the abstract. In some situations the most valid measure of a concept might violate the guides suggested above. Indeed there may be no "best" measure, but instead several measures of a concept each of which is best for a

specific purpose. Different uses would then require different measures of the same concept. Even more important is the possibility that the concept itself is inappropriate in the given situation. That criteria scales exist in non-roll call data does not guarantee that the same dimension is at work in roll call behavior. The possibility exists that a valid measure of a concept cannot be constructed from any set of the available roll calls.

It is worth considering explicitly the relation of an invalid scale to the various criteria. If the scale has no relation to the concept, its behavior might be random. Low correlations would be expected if the scale totally lacks validity. A more subtle possibility is that a scale measures a concept which is similar but not identical to that intended. This might be due to the two concepts being highly related or to one of the concepts being a subset of the other. Consider, for example, the measurement of the foreign policy area in Congressional roll call votes. Many, if not most, foreign policy roll calls concern mutual security legislation. On the one hand the mutual security votes by themselves might prove as effective a single measure of foreign policy behavior as any other possible scale. On the other hand the mutual security votes might not fully reflect the concept of foreign policy so that the results would be similar to those which are expected but would differ systematically from the correct results. It can be very difficult to assure oneself that a scale measures what one desires to measure.

The problem of validation is even more complex in the multidimensional situation. Say that one is trying to obtain valid scales in two issue areas. The various validation procedures suggested for the unidimensional

case can be adapted to the multidimensional case. Face validity would suggest that each item in a given scale should have more in common in content terms with the meaning attributed to that scale than with the meaning attributed to the other scale. Consistency implies that each item should have a closer relation statistically to its own scale than to the other scale. This can be tested at the level of the correlations among the total set of items--checking whether all the intra-scale correlations are larger than any of the inter-scale correlations. This test might be too severe so that one might instead check whether in the multidimensional space of the entire set of items each item loads higher on its own dimension than on any other dimension. Validation by use of criteria can also be used in the multidimensional case if multiple criteria are available, at least one for each roll call scale. Validity suggests that each roll call scale should have a higher correlation with the criteria with which it is supposed to correspond than with the other criteria.

The analysis of validation in the multidimensional context raises the question of the nature of the underlying multidimensionality. Generally one would consider the degree of correlation of issue areas to be an empirical question so that independence should not be assumed. If the scales are not independent, the correlations between items from different scales could be reasonably high. Mere visual inspection of the correlation matrix is therefore a questionable means of determining validity. Another problem which arises with multidimensionality is that some items could be related to more than one dimension. Is this to be permitted? Should an item with such loadings be excluded from consideration all together? Or should it be acknowledged that individuals view

an item as belonging to more than one dimension so that their votes should be apportioned statistically to these dimensions by differential weightings. The use of an item which loads on more than one dimension in a scale without differential weights may be dubious. Most other solutions are reasonable, simple exclusion of such items probably being the easiest procedure.

Roll call analysis permits one additional and unusual procedure for validation--the locations of the items from the scales can be found in the overall roll call space. The locations of these items relative to each other and to the other roll calls can serve as useful clues as to the meaning of the scales. For example, mutual security roll calls may scale very well, but an analysis of the universe of roll calls may indicate that other economic questions fall onto this same dimension. The mutual security items might then be interpreted as being more related to the question of the level of federal expenditures than to the question of American involvement abroad. The multitude of roll calls is a resource which is generally ignored when attempting to interpret or validate one or two roll call scales. An investigation of the overall space could lead to some rethinking of conventional notions as to the dimensions of roll call behavior.

Table 2.2 summarizes the theoretical decisions which are made in the second and third phases of a data analysis. While each of these decisions is faced in any scaling analysis of legislative roll calls, most of these decisions are usually determined implicitly. One generally chooses a convenient analysis technique without noting its theoretical implications instead of choosing that technique which

TABLE 2.2 RESPONSE AND SCALING MODELS

1. One set or two set response model
2. Dominance or proximity response model
3. Joint space or correlational analysis procedure
4. Dominance or proximity measure
5. High or low measurement level
6. Unidimensional or multidimensional space
7. Multiple dimensions or multidimensional composition
8. Deterministic or probabilistic error theory
9. Aggregate or individual probabilistic theory
10. Dimensioning or positioning analysis
11. Face validity, consistency, and/or criteria validity

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embodies the most satisfying theoretical properties. Increased attention to the theory behind the analysis technique is imperative.

SCALING LEGISLATIVE DATA

Dimensional analysis of legislative votes seeks a classification of legislators and/or roll calls in order to either describe the dimensional characteristics of legislative voting or to measure the behavior of individual legislators for subsequent analysis. There are two aspects to such an analysis--valid measurement and model testing.

Dimensional analysis confronts the problem of valid measurement. Consider first the several initial decisions which must be made. The roll calls must be selected with the hope that the selection does not bias the eventual results. The legislators to be included in the analysis must be selected, with special attention as to whether the

parties should be analyzed separately or together. The several forms of voting and non-voting must be collapsed into a positive response on the roll call, a negative response on the roll call, and missing data with the added possibility of an intermediate category for abstentions. It is also necessary to decide how to handle the missing data in the subsequent scaling so that it does not affect the results. In all of these matters one desires the scaling techniques to throw light on the proper decisions, rather than having to settle each question on prior grounds. Dimensional analysis then is, in large part, the search for robust objective scaling techniques for the dimensional characterization of observations.

Dimensional analysis also emphasizes the futility of the search for valid measurement without sensitivity to model testing. There are a very large number of dimensional theories which could reasonably explain legislative voting. The distinctions between dominance and proximity models, between one set and two set models, between unidimensional and multidimensional models, between deterministic and probabilistic models, and between different multidimensional compositions make model testing critical.

Dimensional analysis requires intelligent utilization of scaling techniques. In general the joint space analysis techniques are vulnerable with respect to missing data, item reflections, error, and unknown dimensionality whereas correlational techniques are more robust. Correlational analysis is therefore frequently employed even when a two set response model is assumed. In the following chapters the dominance and proximity response models will be presented in more detail, along with a consideration of the most efficient scaling techniques for each model.

CHAPTER III

MODELS OF LEGISLATIVE VOTING

The two basic response models which might pertain to legislative voting are the dominance and proximity models. What is the correct model? Which response mechanism best explains legislative voting? Legislative analysts have been very successful in employing the dominance scaling technique of Guttman scaling. This could be interpreted as indicating that nearly all legislative voting behavior satisfies the dominance model. On the other hand there has been a noticeable amount of error in Guttman scales of legislative votes. It is therefore possible that the proximity model would yield a better overall degree of fit. Still it would be too simple to believe that any single model can explain all of legislative voting behavior. Realistically one would expect both models to hold, though in different legislatures and/or different issue areas.

In this chapter we shall pursue the choice of the response model using two distinct though complementary approaches. It is of interest to determine which model prevails in legislative voting in general and to obtain specific procedures to find which model holds in a given situation. First, we shall consider conceptually the properties of the two models in legislative settings so as to determine the conditions under which each model is likely to occur. Then we shall examine the extent to which the actual voting data can be used to determine empirically the response process.

RESPONSE PROCESSES

We shall assume that each individual behaves in a deterministic fashion according to his "response function." In attitude situations this response function would be a preference function, indicating the individual's degree of preference for all possible alternative states of nature. In economic situations this response function might be a utility function, indicating the amount of utility for the individual of all possible alternative states of nature. In the area of arithmetic testing this response function would indicate the degree of difficulty of all possible arithmetic items. In other situations the response function might indicate the degree of an attribute the individuals considers various objects to possess, such as how green all possible objects are.

To state that the individual behaves in a deterministic fashion according to his response function is to state that his response function strictly determines his response. Thus an individual will favor items which fall above his base preference level and he will oppose items which fall below this level. In the area of mental testing this corresponds to his passing items which are less difficult than his level of ability and his failing items which are beyond his level of ability. When an individual must choose between a pair of items instead of simply reacting to a single item, he will choose that item with the higher value on his response function. In choosing between two goods, for example, he will choose that good which gives him the greater utility.

Response processes can be classified by the shape of the individual's response function. The general family of response functions with which

we will be concerned is that of single-peaked response functions, the response function having a single maximum point and decreasing as one moves from that maximum point in any direction. A special case of particular interest is when everyone's maximum point is at the same end of the space--at plus or minus infinity. The response functions are then "monotonic" with the space, having their maxima at one end and decreasing in value as one moves away from that end in any direction. Monotonic shape response functions in a single dimension may be exemplified by graphing the degree of difficulty of a set of arithmetic items which have been ordered from simple addition to the calculus.

Response processes can also be classified by the nature of the response task. In some response tasks the individual responds to a single alternative, indicating acceptance or rejection. In other response tasks the individual chooses between a pair of alternatives, indicating which he prefers.

Four varieties of response processes are obtained when combining the two shapes of response functions--single-peaked and monotonic--with the two types of response tasks--responding to a single alternative or choosing between a pair of alternatives. Figure 3.1 indicates these four processes. We shall now consider these four processes in detail and then discuss the conditions of legislative voting which lead to each.

The Pure Dominance Model.--We begin with the classic model from the area of testing, the pure dominance model. Each individual has a response function which is monotonic with the space and responds to a single alternative. The individual is conceived of possessing a "tolerance value" such that he approves those alternatives for which

Fig. 3.1 Four Varieties of Response Processes

Response task \ Response Function	Single-peaked	Monotonic
Single alternative	Pure proximity	Pure dominance
Paired alternatives	Derived dominance	Unanimous dominance

the value of his response function is above that tolerance value and he opposes those alternatives for which the value of his response function is below that tolerance value. Since his response function is monotonic with the space, this means that he responds affirmatively to alternatives on one side of that tolerance value and responds negatively to alternatives on the other side. His response depends on which side of this value is the alternative. Since we are requiring that all the response functions are monotonic in the same direction, this amounts to the dominance response model.

Arithmetic items are generally felt to be characterized by this model. Each item has a difficulty level and the individual has an ability level. The individual responds to a single arithmetic item at a time. The level of difficulty of the arithmetic items is monotonic with the order of the items on the dimension, which we assume to be unidimensional. The individual passes those arithmetic items for which his level of ability surpasses the difficulty level of the items. That is, whether he passes the item depends on which side of his ability level the item's difficulty is located.

The pure dominance model generalizes to the multidimensional situation. In the multidimensional case, the item is represented in the space by a line or plane (not necessarily straight) which indicates the combination of the dimensions required to pass the item. There are many possible composition rules for the dimensions. One composition rule is the compensation rule--the more an individual possesses of one dimension, the less he need possess of the other dimensions in order to pass the item. Other composition rules are not compensatory. An example of a non-compensatory composition rule is the conjunctive rule.

An illustration of the conjunctive model is the French history test example which Coombs (1964, p. 246) attributes to J.F. Bennett. In order to answer questions on a history test written in French, one must both be able to read French well enough to understand the question and know enough history to know the answer. It does not matter how much history one knows if one does not know French and vice versa. An answer to a question requires a minimal knowledge of both French and history. An excess of one does not compensate for a deficiency in the other. The individual passes the French history question only if he passes it on both dimensions, the conjunctive composition rule. This conjuctive composition rule is entirely consistent with the pure dominance model--the individual has a response function which is monotonic with the space (the easiest items being those which require the least knowledge of both French and history and the items being more difficult as one moves away from that point in any direction) and he responds to a single alternative.

In requiring monotonic response functions, the pure dominance model assumes that the individual's maximum value point is at infinity. In the area of attitude measurement, this would mean that the individual's ideal point is at infinity.[1] All individuals desire an infinite amount of the dimensions.[2] This type of issue can be illustrated by considering attitudes toward corruption in government--everyone presumably would

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[1]This identity between the dominance model and individuals with infinite ideals is due to the discussion of Tucker's model of preference behavior by Coombs (1964, p. 282).

[2]Stokes (1963) has termed such issues on which everyone has the same position "valence issues" in contrast to "position issues" on which individuals have different positions.

advocate the complete absence of corruption if we ignore the possibility of their gaining personally from the corruption. The pure dominance model also requires individuals to respond to a single alternative at a time. In the area of attitude measurement, this suggests that each individual not only has an infinite ideal but also has a point of minimum tolerance. Even if everyone agrees that there should not be corruption in government, people with the same amount of knowledge about the government might disagree as to whether or not it is corrupt because they have different tolerance values for corruption. The amount of corruption which one individual would consider inevitable in any government might be beyond the pale according to another individual. Therefore individuals with the same ideal points (at infinity) would respond differently to attitude items under the pure dominance model, permitting the analyst to order the attitude items in the space.

The Pure Proximity Model.--In the pure proximity model the alternatives are still voted upon separately but the individual response functions have the more general single-peaked shape. The individual's response function has a maximum and also he has a tolerance region around that maximum such that he approves those alternatives within that region and opposes those alternatives outside of that region. Given the single-peaked shape of the response function, in the unidimensional case this means that the individual may approve a set of items in the middle of the dimension which possibly rejecting the items at either end of the dimension.

An example of such a model is the tuning of volume on a radio or television. A person has an ideal for loudness and usually would not want his set to be either too loud or too soft. He has a tolerance

range corresponding to the volumes which are acceptable while he would find corrective action to be necessary when the volume of the set is outside of this region. His response depends on the proximity of the actual volume of the set to his ideal volume.

This model is also relevant in the area of attitude measurement. In contrast with the pure dominance model the individual ideal point, in general, is not at infinity and different individuals have different ideals. The internationalist-isolationist dimension may be considered to be of this type. Every American has an ideal degree of intervention which he desires his government to display in its international dealings. Some prefer virtual isolation, others complete intervention, and still others intermediate positions. Every American also has a range of intervention which he will tolerate. Both too little intervention and too much intervention would be labelled unsatisfactory. Requesting individuals to either approve or disapprove of specified acts of intervention would yield information which the analyst could use to order the acts in terms of their degree of intervention.

An additional perspective on these response processes is gained by introducing the notion of a negative ideal. So far we have considered only positive ideals, the maximum points of individual response functions. One can also consider negative ideals, the minimum points of individual response functions. In a preference situation, a negative ideal represents the state of affairs which the individual least prefers.

There is a single positive ideal in both the pure dominance and the pure proximity models. In the unidimensional pure dominance case there is a single negative ideal while in the unidimensional pure proximity case there are, in general, two negative ideals. The

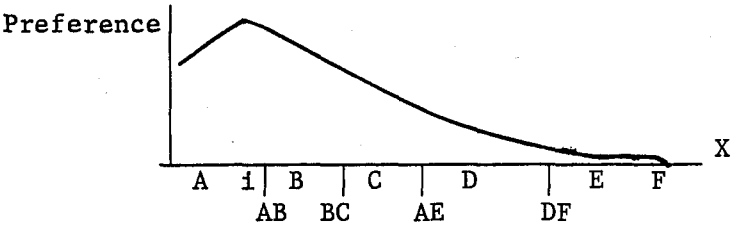
individual in the latter model would have negative ideals at both ends of the dimension, plus and minus infinity, and a positive ideal some place in between. Thus the distinction between the pure dominance model and the pure proximity model has much in common with the distinction between single and multiple negative ideals.

The range of alternatives considered often determines whether or not an area is seen to possess a single negative ideal. Many areas really involve two negative ideals, but appear to possess only a single negative ideal when only the limited range of alternatives which might command majority support are considered. Restrictions on the range of alternatives which are considered thus can shift the model from the pure proximity model to the pure dominance model.

The Derived Dominance Model.--Most work on attitudes and preferences assumes a single-peaked response function as opposed to the special case of a monotonic response function. We have seen that, given a sufficient range of alternatives, the single-peaked response function leads to the pure proximity model. However it is critical to realize that the single-peaked response function may lead to dominance behavior, depending on the nature of the response task. The pure proximity model results if the individual responds to a single alternative under the single-peaked response function. If, instead, he chooses between a pair of alternatives under the single-peaked response function, dominance behavior results. We shall label this model the "derived dominance model" to distinguish it from the pure dominance model described above.

Figure 3.2 shows the response function of individual i and the locations of six alternative states--A, B, C, D, E, and F. Several

Fig. 3.2 Paired Comparison Voting on a Single-Peaked Response Function



midpoints between alternatives have also been indicated in this figure. Individual i will vote for alternative A against alternative B since A is closer to i on dimension X than is B. This can be stated equivalently in terms of the relative locations of the individual and the AB midpoint. Since individual i is closer to A than to B, he is on the left (A) side of the AB midpoint. The individual's vote in paired alternative voting is determined by the side of the midpoint between the alternatives on which he is located, a dominance situation.

If the individual is asked to pick the three alternatives which he likes most, he will pick those nearest him. Individual i in Figure 3.2 will select alternatives A, B, and C. A total of four response patterns are consistent with the scale order ABCDEF--the selection of A, B, and C, the selection of B, C, and D, the selection of C, D, and E, and the selection of D, E, and F. Conversely, given the occurrence of only these four response patterns, the scale order ABCDEF can be deduced. When people pick the three alternatives they like most from a set of six alternatives, the scale order of the alternatives can be deduced from their responses using a technique known as parallelogram analysis (Coombs, 1964).

Under the conditions of the derived dominance model, the individual does not simply choose the alternatives he likes the most. He is given several pairs of alternatives and must choose the alternative he prefers from each pair. Say, for example, individual i in Figure 3.2 had to choose first between alternatives A and E, then between B and C, and finally between D and F. Being on the left side of the AE midpoint, he is closer to A than to E and so will choose A over E. A similar line of reasoning shows that he will also choose B over C and D over F. Individual i thus selects alternatives A, B, and D when he must choose within these specified pairs of alternatives, rather than choosing alternatives A, B, and C which are the three alternatives he likes most.

Choosing within specified pairs of alternatives in this fashion yields less information for subsequent scaling. In particular the scale order of the alternatives cannot be deduced from the response patterns. We have seen that individual i in Figure 3.2 would choose alternatives A, B, and D when choosing between alternatives A and E, B and C, and D and F. Four other response patterns are consistent with the scale order ABCDEF given this same response task--the selection of A, C, and D, the selection of B, D, and E, the selection of C, D, and E, and the selection of C, E, and F. It can be verified that the same five response patterns are also consistent with the scale order ABDCEF. Thus more than one scale order of alternatives is consistent with a specified set of response patterns when individuals choose within pairs of alternatives.

What information is obtained if the scale order of the alternatives is not found? The scale order of the midpoints of the paired alternatives

is obtained instead. In the present example, alternatives A and E were paired as were alternatives B and C and alternatives D and F. The occurrence of the four response patterns ABD, ACD, CDE, and CEF indicates that the order of the paired alternatives midpoints is BC, AE, DF. The midpoint order is obtained without indicating which of the orderings of the alternatives which are consistent with the obtained midpoint order is correct.

Guttman scaling amounts to the scaling of midpoints (Coombs, 1964), p. 217). Let the paired alternatives be represented by the midpoint between them so that the midpoint represents the item. The individual votes for the item if he is on one side of it and against the item if he is on the other side. Scaling the midpoints is identical to scaling the items, though not to scaling the constituent alternatives. The individual's response depends on which side of the item he is located, so a dominance pattern is found in the scaling. The pairing of alternatives under the single-peaked response function leads to the derived dominance response model.

This derived dominance model assumes that all individuals have single-peaked response functions over the set of alternatives with a common specification of the alternatives. Individuals agree not only on the ordering of the alternatives, but also on the exact placement of these alternatives up to an ordered metric transformation.[3] Coombs (1964, p. 86) has termed this condition a "quantitative J scale" in contrast to the "qualitative J scale" in which the individuals agree

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[3]That is, the order of the inter-point distances must be maintained by a transformation if it is to be valid as well as maintaining the order of the points themselves on the dimension.

only on the ordering of the alternatives. Coombs (1964, pp. 193-195) has shown that the distinction between quantitative and qualitative J scales can also be stated in terms of the shapes of the preference functions. If all the preference functions are symmetric and strictly monotonic, then there is a quantitative J scale. If either of these conditions is violated while all the preference functions are still single-peaked, there is a qualitative J scale. Thus the derived dominance model requires symmetric and strictly monotonic response functions.

To illustrate the derived dominance model, we shall consider pairwise voting over the scale ABCDEF where the individuals must choose between alternatives A and E, B and C, and D and F. We define a yes vote as a vote for the alternative closer to the left (A) end of the dimension ABCDEF, so that votes for A over E, B over C and D over F are yes votes. Table 3.1 lists the three paired comparison votes across the top of the table. The rows of this table correspond to the rank orders consistent with one of the quantitative J scales over the dimension ABCDEF--the quantitative J scale in which the distance between alternatives E and F is greater than that between A and D, the distance between alternatives D and E is greater than that between A and C, and the distance between alternatives C and D is greater than that between A and B. Every rank order consistent with this quantitative J scale leads to a response pattern on the three paired alternative votes which is consistent with a dominance scale. The dominance scale indicates the order of the alternative midpoints on the dimension--the B/C item being the hardest to pass, the D/F item being the easiest, and the A/E item being in the middle of the scale.

TABLE 3.1 PAIRWISE VOTES OVER A QUANTITATIVE J SCALE

	Pairwise vote		
Rank Order	B/C	A/E	D/F
ABCDEF	Yes	Yes	Yes
BACDEF	Yes	Yes	Yes
BCADEF	Yes	Yes	Yes
CBADEF	No	Yes	Yes
CBDAEF	No	Yes	Yes
CDBAEF	No	Yes	Yes
DCBAEF	No	Yes	Yes
DCBEAF	No	No	Yes
DCEBAF	No	No	Yes
DECBAF	No	No	Yes
EDCBAF	No	No	Yes
EDCBFA	No	No	Yes
EDCFBA	No	No	Yes
EDFCBA	No	No	Yes
EFDCBA	No	No	No
FEDCBA	No	No	No

With actual data we would never know the quantitative J scale nor even the order of the alternatives on the dimension. We would be given only the response patterns which occur and, from these, we would derive the item (read midpoint) order on the scale. What is significant is that we would obtain a dominance scale because the individuals have single-peaked response functions over a common quantitative J scale.

If the individuals all have single-peaked response functions but these response functions are over a qualitative rather than a quantitative J scale, scaling the items would yield a dominance scale with error. The amount of error may often be small, so that a derived dominance interpretation could be given even with the qualitative J scale. That is, the derived dominance model may hold if the individuals agree only on the order of the alternatives on the dimension.

This derived dominance model also generalizes to the multidimensional case. Again a vote must be considered a paired comparison between two alternatives and both of these alternatives should be represented as points in the multidimensional space. In two-dimensional Euclidean space, for example, the locus of points equidistant from these two points is a straight line in the space. Individuals on one side of the line will all vote in one direction on the paired comparison vote and individuals on the other side of this line will all vote in the opposite direction. The votes of the individuals depend on which side of the line they are on, a dominance form of response. Scaling the items actually involves scaling the lines of equidistance. This multidimensional model corresponds to a linear compensation rule between the dimensions. One may recall that the linear compensation rule was not the only multidimensional composition rule compatible with the pure dominance model, but it is the only composition rule compatible with the derived dominance model in Euclidean space.

It does not really matter for a unidimensional analysis whether the pure or derived dominance model holds. It also does not matter for multidimensional analysis when a linear compensation rule mediates the dimensions. Whenever dominance behavior is found, dimensioning and positioning can proceed with little difficulty, without bothering to determine the basis of that dominance behavior.

The derived dominance model is of particular importance in the area of attitude measurement. Many attitude items involve a paired comparison of alternatives, either explicitly or implicitly. A vote between the administration bill and the opposition substitute involves such a paired comparison. Likewise asking whether an individual favors

a proposed action can be interpreted as requesting him to state a preference between that action and the status quo. Thus the derived dominance model explains to a great degree the frequent occurrence of dominance behavior.

The Unanimous Dominance Model.--We have seen that pairing of alternatives leads to dominance behavior under a single-peaked response function. Under the special case of a monotonic response function with everyone's response function beingmonotonic in the same direction, the pairing of alternatives leads to unanimous behavior. Say that individuals with infinite ideal points and monotonic response functions choose between alternatives in a pairwise fashion. Individuals vote for the alternative in each pair which ranks higher on their response functions. But if everyone has a monotonic response function, at least in the unidimensional instance, then everyone agrees as to which alternative ranks higher on their response functions and so all votes are unanimous.

An example of this model would be to have individuals indicate which of a pair of objects was more green. Presumably the attribute of being green is unidimensional and everyone has the same conception of this attribute. They would respond that the object which is higher on the dimension of greenness, whichever object that was, was the more green. Having individuals respond to all possible pairs of objects would then permit one to rank the set of objects along this dimension. Having individuals respond to specified pairs of alternatives indicates which of each pair has more of the dimension but cannot order the several alternatives on the dimension.

Attitude questions can be of this type, though such attitude items are generally uninteresting. If everyone had the same conception of corruption in government and if, as suggested above, corruption in government is an area with monotonic response functions, then asking individuals to vote on a bill to decrease corruption in government would lead to a unanimous vote. Everyone would prefer less corruption to the status quo, regardless of how good or bad the status quo is, and so everyone would favor the bill. There is generally little interest in analyzing variables lacking variance, so the unanimous dominance model merits little attention.

LEGISLATIVE VOTING

Four response processes have now been considered and we have seen how each can arise in attitude measurement. Next we shall consider the circumstances in which each of these response processes will arise in legislative voting.

Legislative voting is generally characterized by a pairing of alternatives. The legislator votes between a motion and a substitute motion, between an unamended bill and a proposed amendment, or between the status quo and a bill. Under certain circumstances the paired comparison interpretation of the votes might be ignored in the legislature, but such circumstances require further specification.

Monotonic Response Functions.--Monotonic response functions do characterize certain issue areas. The proverbial God, motherhood, and country are of this type. Morality, corruption, patriotism, and institutional loyalty are, under most circumstances, issue areas with monotonic response functions. Voting on such issues would typically

be given the paired comparison interpretation. Therefore the unanimous dominance model explains the unanimous voting which usually occurs on this type of issue.

The paired comparison interpretation may on occasion be ignored in legislative voting. It may be that the legislators are reluctant to pass legislation of the given type regardless of their stand on the overall issue and furthermore they have different tolerance values on the issue. Consider a motion to censure a fellow legislator for corrupt behavior. While everyone might agree that corruption is bad, legislators are generally reluctant to censure fellow legislators. Some might be willing to vote for censure only on the most grave offenses while others might vote for censure whenever any sign of corruption is evident. One might expect the pure dominance model to hold in such a situation.

Often it is unclear whether or not the conditions for the pure dominance model are met in legislative voting. Consider the area of federal intervention in civil rights. We shall assume that everyone opposes federal intervention when that intervention is unnecessary. Assuming that this area is unidimensional, this corresponds to response functions having their maxima at no federal intervention and descending monotonically from that point. The legislator evaluating a motion for a certain amount of federal intervention compares that amount to the maximum amount which he can tolerate under the present state of affairs. Each person would vote for up to a certain amount of intervention, a dominance form of behavior. His vote does not depend on a comparison of the status quo with the motion and so does not

involve a pairing of alternatives. Thus one could argue that the pure dominance model underlies voting on federal intervention in civil rights. On the other hand, the assumptions made in this argument can be challenged. Some legislators might have a degree of intervention as their maximum preference points. Even if zero intervention were everyone's maximum preference point, the pure proximity model would hold rather than the pure dominance model if motions for federal intervention against civil rights were considered as well as motions for federal intervention for civil rights. A decision as to whether the pure dominance model holds in the area of federal intervention in civil rights requires more knowledge of the shape of preference functions than is presently available.

Single-Peaked Response Functions.--Most issue areas are probably characterized by single-peaked response functions. Different individuals favor different amounts of federal intervention in the social welfare area, for example, as well as different degrees of American intervention in international affairs.

Voting on such issues would typically be given the paired comparison interpretation. The question might be whether or not to add to an appropriation for a given purpose. Each legislator has an ideal value for the appropriation. If the proposed additional appropriation is closer to his ideal than the present value, he will favor the additional expenditure. His vote depends on a paired comparison between the present amount and the proposed amount. This leads directly to the derived dominance model.[4]

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[4]Appropriations may instead involve monotonic response functions with the legislators being willing to support up to a certain amount of appropriation. Once again there is insufficient knowledge of the shape of preference functions of legislators to determine which model is appropriate.

If the derived dominance model is not to hold with single-peaked response functions, the legislators must ignore the paired comparison interpretation of the motions. They must vote on the motion on an absolute basis rather than voting for the alternative which they prefer. The legislator must be willing to vote only for the motions falling within his acceptance region. Such behavior would be expected mainly in situations in which compromise is spurned. The legislator might vote against a motion instituting a program he favors if that motion does not go far enough in the desired direction.

Legislative voting in the United States today generally involves a willingness to compromise. Occasional mavericks in Congress do vote against bills because they feel the bills are too weak, but such behavior is very rare. Such analysis in the United States today would amount to deviant case analysis.

The proximity model might be useful in explaining roll call behavior in other countries, particularly those with multi-party systems. In a multi-party system, there is not always a premium on being on the winning side. A party might consider it so important to maintain its separate identity that it is unwilling to compromise its position. It would vote against bills yielding desired changes in the status quo which do not follow the precise party position. Imagine, for example, a party system with five parties arrayed along a conventional left-right dimension. Say that each party will vote only for its own motions plus possibly those submitted by the parties in immediately adjacent scale positions. Thus the left-wing parties would vote against motions of the most conservative party while the right-wing

parties would vote against motions of the most liberal party. The middle party would vote for motions of the other center parties, but would vote against the motions of the extreme conservatives and the extreme liberals.

The proximity model might also be useful in explaining legislative voting in more volatile periods of American history. Consider, for example, the era of great cleavage preceeding the Civil War. Some favored abolition of slavery and rejected all compromise while others favored the unlimited expansion of slavery and rejected all compromise. Motions sponsored by either of these extreme groups were opposed by the middle and the opposite extreme. Motions sponsored by the middle, such as a positive affirmation of the status quo, might be viewed as threatening by both extremes. Similarly a compromise bill giving neither side an advantage might be opposed by both extremes. Such situations would lead to the proximity response model.

The possible relevance of the proximity model in the pre-Civil War period is particularly interesting because of the increased attention to this period by empirically-oriented historians. Thomas Alexander (1967) and Joel Silbey (1967) have both utilized Guttman scaling on roll calls of this period. Our theory suggests that the proximity model might be better suited to an investigation of this voting.

We will consider a single example, House voting on the Compromise of 1850. The parliamentary circumstances of voting on the Compromise were quite complex. There were seven components of the Compromise--statehood for California, creation of the New Mexico territory,

resolution of the Texas boundary with New Mexico,the Texas debt, creation of the Utah territory, the fugitive slave bill, and a prohibition on slave trade in the District of Columbia. The first five of these components were combined in the Omnibus bill which was defeated in the Senate, only the Utah territory provision surviving. The Senate later passed several of the components in separate bills, including a "little Omnibus" combining the three measures relating to Texas and New Mexico. The House then passed each of these separate bills.

Alexander (1967) used some of the House votes in his sectional issues scale for the 31st Congress. This scale differentiates well between Northerners and Southerners. Additionally the Northern Whigs tended to be at one end of the scale and the Southern Democrats at the opposite end.

Silbey (1963, 1967) included these issues in two scales. The first is a scale of sectional division. Like Alexander's sectional issues scale, this scale differentiates between Northerners and Southerners with the Northern Whigs and Southern Democrats occupying the bulk of the extreme positions. Silbey constructed a second scale on the Compromise issue. Both parties and both sections were "badly fragmented" on this Compromise scale (Silbey, 1963, p. 291). The Southern Whigs all were on the pro-Compromise end of this scale. The Southern Democrats were found equally divided on the Compromise. Most Northern Whigs favored the Compromise while most Northern Democrats opposed it. The reproducibility of the Compromise scale is 0.947. This value, however, considerably overstates the quality of this scale. Of the 118 Representatives who voted on all thirteen roll calls included

in this scale, 54% had perfect scale patterns on this scale. Only 32% of the 72 Representatives who did not consistently vote for the Compromise had perfect scale patterns. Eight of the fourteen scale positions have no cases with perfect patterns. Twelve of the fourteen scale positions have fewer people with perfect patterns than people with error patterns. In summary, Silbey's Compromise scale is very weak and provides little evidence that voting on the Compromise followed a dominance model.

Table 3.2 shows a proximity scale of the five votes on final House passage of the bills of the Compromise. The source of the data is Holman Hamilton's Prologue to Conflict (1964). The reproducibility of this scale is 0.996. Of the 131 Representatives who voted on all five of these roll calls, 98.5% had perfect scale patterns on this proximity scale. Including the Representatives who did not vote on all of the items, only four of 223 Representatives had response patterns inconsistent with this scale. The Guttman scale which fits these votes best consists of only the first six response patterns of Table 3.2. The reproducibility of this Guttman scale is 0.960 with 84.7% of the 131 Representatives who voted on all five items having a perfect response pattern. While the Guttman scale of these roll calls is good, the proximity scale is noticeably better.

The proximity scale of Table 3.2 differentiates the Northerners from the Southerners. At one extreme are the Northerners who accepted only the portions of the Compromise restricting slavery in the District of Columbia and granting California statehood. At the opposite extreme are the Southerners who accepted only the part of the Compromise enacting fugitive slave legislation. In the middle of the scale are those who

TABLE 3.2 HOUSE VOTING ON THE COMPROMISE OF 1850 BY REGION AND PARTY*

Fugitive slaves	Utah territory	Texas/ N. Mex.	California statehood	D.C. slavery	Total	Northern Free Soil	Northern Whig	Northern Dem.	Southern Whig	Southern Dem.
No	No	No	Yes	Yes	48	7	32	9	0	0
No	No	Yes	Yes	Yes	6	0	5	1	0	0
No	Yes	Yes	Yes	Yes	5	0	3	2	0	0
Yes	Yes	Yes	Yes	Yes	28	0	0	25	2	1
Yes	Yes	Yes	Yes	No	9	0	0	0	6	3
Yes	Yes	Yes	No	No	15	0	0	0	4	11
Yes	Yes	No	No	No	5	0	0	0	0	5
Yes	No	No	No	No	13	0	0	0	1	12
Yes	No	Yes	Yes	Yes	1	0	1	0	0	0
Yes	Yes	Yes	No	Yes	1	0	0	0	0	1
					131	7	41	37	13	33

*Only those Representatives who voted on all five items are included.

were willing to support all five Compromise bills. The scale thus does not range from pro-Compromise to anti-Compromise but instead ranges from pro-Northern to pro-Southern with the pro-Compromise position in the middle. The proximity scale also points out that the Utah territory provision and the bill dealing with Texas and New Mexico were opposed by both the extreme Northerners and the extreme Southerners. These bills permitted the territorial legislatures of Utah and New Mexico to determine whether slavery would be permitted in their territories, rather than explicitly permitting or prohibiting slavery as a condition of becoming a territory. Thus these two bills did not settle matters fully to the satisfaction of either side and so were opposed by extremists from both sections.

Strong partisan differences are seen in the Compromise voting. The Free Soilers followed the Northern position without any inclination to compromise. The Northern Whigs tended to follow the extreme Northern

position, though some accepted parts of the Compromise. The Northern Democrats are seen to provide the critical support for the passage of the full Compromise. The Southern Whigs tended to accept some of the Compromise provisions. The Southern Democrats were split into two main blocs--one which supported compromise to the extent of voting for the Utah and New Mexico bills and another which rejected all attempts for compromise. This division between the Northern Whigs at one end of the scale and the Southern Democrats at the other end is consistent with the sectional scales of Alexander and Silbey, though it bears little resemblance to Silbey's Compromise scale. While the Compromise of 1850 merits a much more careful analysis than that given here, it seems clear that such an analysis must make allowance for the existence of the proximity response model.

While we have here associated the pure proximity model with an unwillingness to compromise, it must be admitted that there are empirical questions involved which require further investigation. We know too little about the shape of individual response functions and how legislators conceive of motions. Further investigation might suggest that factors beside willingness to compromise are also relevant.

Response Models.--We have now specified the legislative circumstances leading to each response model. The proximity model will occur only when legislators have single-peaked preference functions but vote on motions in an absolute sense. We have identified this situation with an unwillingness to compromise on the part of the legislators, though additional consideration of other possible factors leading to this model is warranted.

The dominance model will occur when legislators have monotonic preference functions and vote on motions in an absolute sense and when

legislators have single-peaked preference functions and vote on motions in the paired alternative fashion. While further investigation may indicate the relative frequencies of these two forms of dominance models, it is not necessary to differentiate between these two forms in order to conduct a dimensional analysis of the votes. For this reason we shall not differentiate in later sections and chapters between the pure and derived dominance models.

This discussion of response models suggests that it would be of some interest to obtain from legislators an indication of how they structure their attitude space and how they consider legislative votes. When are their points of maximum preference at infinity? In what types of issue areas do the legislators have monotonic preference functions? Do legislators generally conceive of motions as involving a choice between a pair of alternatives? Under what circumstances do legislators respond to roll calls in an absolute sense without comparing their preference for the passage and defeat of the motion? These are questions which can be answered only by interviewing legislators directly, probing in detail the structure of their attitude spaces. Raising these questions indicates how little we do know about the nature of legislative attitudes governing roll call behavior.

MODEL TESTING

While the results of the previous section suggest that the proximity model is unlikely to hold in contemporary American legislative behavior, it is still important to be able to determine exactly when this model does hold. Given a set of observations, how can one decide whether a dominance or a proximity response model is at work?

TABLE 3.3 UNIDIMENSIONAL PROXIMITY SCALE

	Item		
Individual	A	B	C
1	Yes	No	No
2	Yes	Yes	No
3	No	Yes	No
4	No	Yes	Yes
5	No	No	Yes
6	Yes	Yes	Yes
7	No	No	No

Empirical testing between the two models is difficult. The logical relationship between the two models is such that the response patterns consistent with a dominance model are a subset of those consistent with a proximity model. This is illustrated by the listing of the patterns consistent with the three item proximity scale ABC as given in Table 3.3. Each of these seven patterns fits the proximity rule since each individual accepts only an adjacent set of items on the scale. The only one of the eight logically possible response patterns for binary responses to three items which is inconsistent with the proximity scale is the pattern in which the individual votes yes on A and C but not on the middle item B. It can be seen that the response patterns of individuals 1, 2, 6, and 7 in Table 3.3 are the response patterns which fit the Guttman scale ABC. Indeed a perfect Guttman scale is typified by all individuals approving only adjacent items on the scale, which is completely compatible with a proximity scale. Thus the response patterns consistent with a dominance model are a subset of those consistent with a proximity model. In general with n items there are 2^n logically possible response patterns. Of these, $1 + n$ are consistent with the unidimensional dominance model

compared to $1 + (1 + 2 + \ldots + n-1 + n) = 1 + 1/2\ n + 1/2\ n^2$ which are consistent with the unidimensional proximity model. On the one hand this means that the dominance model represents a much more severe restriction on the number of consistent response patterns. On the other hand the occurrence of only the dominance response patterns might signify nothing more than a proximity response process in which some of the possible patterns are missing. A perfect Guttman scale is also a perfect proximity scale.

Model testing is therefore in large part theoretical in nature. Given the successful application of a dominance scaling technique, one must examine the resultant scale or space and decide whether a dominance or proximity interpretation is more natural. Indeed if the response space has been determined, it may not be necessary to determine the underlying response model.

What, however, if dominance scaling does not succeed? To state that the responses consistent with a dominance model are a subset of those consistent with a proximity model does not preclude the possible occurrence of patterns consistent with only a proximity model in the given dimensionality. The occurrence of extra response patterns can easily be noticed when performing unidimensional joint space dominance analysis, but may be more difficult to detect if other dominance scaling techniques are employed. It is therefore of interest to determine the results of dominance correlational analysis when a proximity model is operating. To do so we shall consider the properties of the cross-tabulation of a pair of proximity items on which a correlation coefficient would be based.

Proximity Cross-Tabulation Properties.--Individuals may vote no on unidimensional proximity items for two different reasons--because the item is too far to the left of the person's ideal point or because the item is too far to the right of his ideal (Coombs, 1964, pp. 298-299). We will label the former a "no+" response and the latter a "no-" response. Figure 3.3 shows the full notation to be employed for the proportion of votes in cells of the cross-tabulation of proximity items where the two forms of no votes are explicitly distinguished. Here item A is assumed to be to the right of item B on the continuum.

Fig. 3.3 Cross-Tabulation of Proximity Items

	Item B			
Item A	No-	Yes	No+	Total
No-	p_{--}	p_{-0}	p_{-+}	$p_{-.}$
Yes	p_{0-}	p_{00}	p_{0+}	$p_{0.}$
No+	p_{+-}	p_{+0}	p_{++}	$p_{+.}$
Total	$p_{.-}$	$p_{.0}$	$p_{.+}$	1.0

Unidimensionality is evidenced in the three-by-three cross-tabulation by the occurrence of three zero cells. The p_{0-}, p_{+-}, and p_{+0} cells would be empty under unidimensionality. If the p_{+-} cell were not empty, for example, then some individual would be to the right of the right-most item (A) while being simultaneously to the left of the left-most item (B), a direct contradiction. Similar contradictions would result if the p_{0-} and p_{+0} cells were not also empty.

Although the three-by-three cross-tabulation of Figure 3.3 conceptually underlies the inter-item cross-tabulation of proximity items, it

cannot be generated from empirical data. To obtain this full table would require a knowledge of whether each no vote was of the no+ or the no- variety. This cannot be known unless the proximity scale itself is known in advance. In practice the two opposite types of no votes cannot be differentiated. Only a fourfold table results from the cross-tabulation of the yes and no responses on two items.

Figure 3.4 shows the fourfold table resulting from the cross-tabulation of two proximity items. The cell entries indicate which cells of the three-by-three cross-tabulation of Figure 3.3 map into each of the cells of the fourfold table. Note, for example, that there are four different reasons for voting against both of the items so that the no-no cell of the fourfold table is the sum of four cells of the three-by-three table. There is no way to apportion the values in the fourfold table back to the cells of the underlying three-by-three cross-tabulation.

Fig. 3.4 Fourfold Table of Proximity Items

		Item B: Yes	Item B: No
Item A	Yes	p_{00}	$p_{0-} + p_{0+}$
	No	$p_{-0} + p_{+0}$	$p_{--} + p_{-+} + p_{+-} + p_{++}$

Correlational analysis of two-by-two tables of proximity items is therefore most difficult. Only a single cell in the full three-by-three table is mapped one-to-one into the fourfold representation--the cell indicating the proportion of individuals favoring both items. Were one to admit the possibility of item reflections, it would be impossible to tell even which cell in the fourfold table is the invariant cell. Of

particular importance is that no cell of the fourfold table would be empty under unidimensionality. The three cells of the full table which would be zero under unidimensionality map into three different cells of this fourfold table and even under unidimensionality three cells of the full table map into a single cell of the fourfold table. There is no zero cell to indicate unidimensionality and a complete association between the two items. In fact it is impossible to judge the direction of the association between the pair of items on the basis of the fourfold table since it could differ from the direction in the underlying tabulation.

Measures of association based on fourfold tables of proximity items are thus meaningless. Correlational analysis of such values will lead to nonsense results. The dimensionality is likely to be overestimated and the fit is likely to be poor. Dominance correlational analysis will not yield a good solution if response patterns occur which are consistent with only a proximity model.

Proximity Scaling.--A final question deals with the feasibility of scaling if response patterns occur which are consistent with only a proximity model. There are two possibilities which merit brief mention. Both involve similarity analysis, a procedure to be described in detail in a later chapter. Similarity analysis seeks a representation in a space of minimum dimensionality of a set of points such that the inter-point distances are monotonic with the available measures of inter-point similarities. Joint space analysis is possible through similarity analysis. In general terms this amounts to seeking a configuration of points such that each person is closer to the items he votes for than

those he votes against. In practice this joint space analysis is frequently indeterminate as there are relatively few constraints on the locations of the points in the space with only binary legislative voting data. Correlational space analysis of the votes may also be possible through similarity analysis. We have seen that measures of association based on proximity items are meaningless. However it may be possible to obtain one measure of inter-item distance from the fourfold table of proximity items--the proportion of individuals approving both items. This joint approval score is the only value of the three-by-three table which can be estimated validly from the fourfold table. The joint approval score is monotonic with inter-point distance under some circumstances, though its general validity requires further exploration. Similarity analysis of this score may often be an effective means of scaling proximity items.[5]

It should be added that the results of a similarity analysis of joint approval scores depend considerably on whether response patterns occur which are not consistent with a dominance scale. We will first consider the eight item scale ABCDEFGH where each response pattern consistent with that proximity scale occurs exactly once. The closer together a pair of items are on the scale, the greater the proportion of individuals approving both. This yields the patterning in the matrix of joint approval scores which is evident in Table 3.4. Within each row the highest entry is along the diagonal and the entries fall off as one moves to either end of the row. Because of this patterning, similarity analysis will yield a unidimensional result. The solution

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[5]Both procedures would ignore the possibility of item reflections, a concept which might not be necessary for proximity items.

TABLE 3.4 PROPORTION OF INDIVIDUALS APPROVING PAIRS OF PROXIMITY ITEMS

	Item							
Item	A	B	C	D	E	F	G	H
A	-	.19	.16	.14	.11	.08	.05	.03
B	.19	-	.32	.27	.22	.16	.11	.05
C	.16	.32	-	.41	.32	.24	.16	.08
D	.14	.27	.41	-	.43	.32	.22	.11
E	.11	.22	.32	.43	-	.41	.27	.14
F	.08	.16	.24	.32	.41	-	.32	.16
G	.05	.11	.16	.22	.27	.32	-	.19
H	.03	.05	.08	.11	.14	.16	.19	-

to Table 3.4 given by similarity analysis would be a set of locations such as A at 0.00, B at 0.28, C at 0.40, D at 0.48, E at 0.52, F at 0.60, G at 0.72, and H at 1.00. Adjacent items in the middle of the scale would be found to be closer to each other than would adjacent items at the ends of the scale.

The same matrix calculated on a dominance model would have a quite different patterning. According to a Guttman scale, each person who approves a given item should approve all easier items, except for the intrusion of error. In the perfect scale, the proportion of individuals approving a fixed item and any other easier item would be a constant, equal to the proportion of individuals approving that fixed item itself. Table 3.5 shows the matrix of joint approval scores calculated on the scale ABCDEFGH where each response pattern consistent with that dominance scale occurs exactly once. Values decrease monotonically with distance in each row on the right of the diagonal, showing the relation of the fixed row item with harder items. Values are constant in each row on the left of the diagonal, showing the relation of the fixed row item with easier items. Similarity analysis of this matrix by rows should

TABLE 3.5 PROPORTION OF INDIVIDUALS APPROVING PAIRS OF DOMINANCE ITEMS

Item	Item A	B	C	D	E	F	G	H
A	-	.78	.67	.56	.44	.33	.22	.11
B	.78	-	.67	.56	.44	.33	.22	.11
C	.67	.67	-	.56	.44	.33	.22	.11
D	.56	.56	.56	-	.44	.33	.22	.11
E	.44	.44	.44	.44	-	.33	.22	.11
F	.33	.33	.33	.33	.33	-	.22	.11
G	.22	.22	.22	.22	.22	.22	-	.11
H	.11	.11	.11	.11	.11	.11	.11	-

still yield a unidimensional result, though the numerical results would be artifacts due to the cumulation of items. The solution of Table 3.5 would yield values such as A at 0.000, B at 0.008, C at 0.024, D at 0.055, E at 0.118, F at 0.244, G at 0.496, and H at 1.000. Adjacent items at the easier end of the scale are much closer to each other than those at the harder end of the scale. Each item would be found closer to all items easier than it than to any of the items harder than it. Thus a monotonic increase in the distances between adjacent items on a scale is more consistent with a dominance scale.

The patterning of the joint approval matrix and the results of the similarity analysis of this matrix depend in the unidimensional case on whether response patterns occur which are not consistent with a dominance scale.[6] The multidimensional extensions of these ideas are less clear because of the variety of possible multidimensional models and because of the difficulty of portraying the inter-item relations in a multidimensional space. Similarity analysis of the approval scores

[6] See Leik and Matthews (1968) for a related treatment of this subject.

should still be an effective means of scaling the proximity items.

LEGISLATIVE VOTING MODELS

The proximity model may account directly for legislative voting in some legislative situations, particularly those involving an unwillingness to compromise. In a formal sense the dominance scale is a special case of a proximity scale which makes model testing difficult. More critical than a determination of the proper model is the correct scaling of the data. The scaling of proximity data by dominance correlational techniques may yield an overestimate of the dimensionality because correlation coefficients cannot be validly estimated on proximity items. When a proximity space is suspected, either on a priori grounds or on the basis of an unexpectedly high dimensionality obtained by correlational analysis, similarity analysis may be attempted as a means of scaling the data. Similarity analysis of joint approval scores is particularly useful as it is sensitive to the occurrence of response patterns which are not consistent with the dominance model.

We would expect, however, that the dominance model would be the most common model of legislative voting. The reasons for this have been clarified in this chapter. The implicit paired comparison nature of the roll calls in most legislative situations leads directly to a dominance scale structure. For that reason a thorough understanding of dominance scaling techniques is necessary for legislative voting analysis. We turn next to a consideration of dominance analysis, beginning with the correlation of items in a dominance space.

CHAPTER IV

CORRELATION IN A DOMINANCE SPACE

The dominance response model has been the model used most in legislative scaling. In the previous chapter we found that the dominance model should hold in most legislative applications. For these reasons it is important to consider the techniques for analysis under the dominance model.

Correlational analysis has several advantages over joint space analysis. In general, joint space analysis techniques are difficult to apply in the presence of missing data, error, item reflections, and unknown dimensionality. Correlational techniques have notable advantages in terms of robustness in the face of such difficulties. We shall therefore emphasize correlational analysis procedures for the dominance model.

There are various properties in cross-tabulations which correlation coefficients can measure. Different analysis procedures make different assumptions as to the properties which the correlation coefficient measures. Thus correlational analysis requires the concomitant choice of a correlation coefficient and an analysis technique. The varieties of correlation coefficients will be considered in this chapter from the perspective of dominance scaling and then the corresponding analysis techniques will be considered in the next chapter.

CROSS-TABULATION PROPERTIES

Correlation coefficients measure various properties of cross-tabulations. It is therefore necessary to begin with a discussion of the properties of the cross-tabulation itself. Figure 4.1 shows the notation we shall use for the inter-item cross-tabulation, where missing observations have been excluded. The frequencies in this table have been converted to percentages so that the sum of all the entries is one.

Fig. 4.1 Cross-Tabulation of Dominance Items

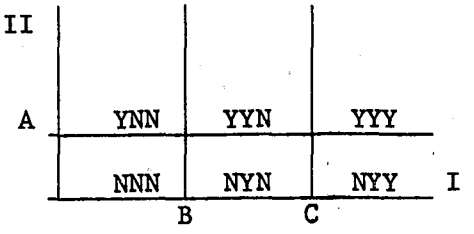
		Item B: Yes	Item B: No	Total
Item A	Yes	p_{11}	p_{12}	$p_{1.}$
	No	p_{21}	p_{22}	$p_{2.}$
	Total	$p_{.1}$	$p_{.2}$	1.0

Unidimensionality directly implies a specific pattern of entries for this cross-tabulation. Under perfect unidimensionality, no one would pass the harder item while failing to pass the easier item. Say that the items are ordered such that item B is harder to pass than item A ($p_{.1} \leq p_{1.}$). Perfect unidimensionality would then mean that no one who says yes to item B would say no to item A. The p_{21} cell of the cross-tabulation should be empty. Since observations in this p_{21} cell violate unidimensionality, this cell is spoken of as the error cell. The size of this error cell may be taken as a rough indicator of the degree of unidimensionality between the items.

An equivalent statement of unidimensionality is that everyone who passes the harder item also passes the easier item. The set of individuals who pass the harder item is a subset of the set who pass the easier item. Thus much of the notion of cumulation involves the nesting of the passing sets of the items. Perfect cumulation means that p_{21} would be zero while p_{12} would be non-zero. The size of p_{12} relative to the size of p_{21} might therefore be taken as a rough measure of the degree of nesting between the passing sets of a pair of items.

An additional property which one would desire to be able to measure from the inter-item cross-tabulation is the inter-item distance. It is difficult to measure distance, however, in a dominance space. Figure 4.2 shows the responses to the three items A, B, and C in a two-dimensional space where individuals respond affirmatively to an item if and only if they are both above and to the right of the item. The proportion of individuals responding yes to the pair of items is monotonic with inter-item distance here though the proportion of individuals giving the same response to both items is not monotonic with inter-item distance. The monotonicity between the proportion responding yes to the pair of items and the inter-item distance is seen in the fact that more people respond yes to both items A and B then respond yes to both items A and C. Yet more people respond no to both items A and C than respond no to both items A and B. Thus, depending on the distribution of individuals in the space, the number of cases in the major diagonal cells of the cross-tabulation of items A and C may be greater than the number of the major diagonal cells of the cross-tabulation of A and B even though B is closer to A than is C. The simple agreement score obtained by computing the proportion of individuals giving the same response to a pair of items is therefore not always monotonic with inter-item distance

Fig. 4.2 Responses in a Two-Dimensional Space



in a dominance space. When all items have the same direction, the proportion of individuals who respond yes to both of a pair of items is monotonic with inter-item distance from a fixed item, a result which was used in the previous chapter. However it should also be remembered that the proportion of people approving a fixed item and easier items on the same dimension is constant so that only a weak form of monotonicity holds between the proportion approving a pair of items and their inter-item distance. The possibility of analyzing this measure by similarity analysis was discussed in the previous chapter.

We shall concentrate on obtaining measures of the unidimensionality and nesting properties so as to be able to determine the dimensionality of the set of items. The proportion of cases in the error cell indicates the extent to which the items share a common dimension while the relative sizes of the two minor diagonal cells indicate the extent to which the passing sets of the two items nest. In this chapter we shall be concerned with the correlation coefficients which measure these two properties and in the next chapter we shall consider the algorithms for the analysis of these correlation coefficients.

Before treating the correlation coefficients for measuring these properties, we shall note one significant difference between inter-item

and inter-individual cross-tabulations. As pointed out earlier, there is an asymmetry between inter-item and inter-individual analysis due to our willingness to reflect items but not individuals (MacRae, 1966b). In an inter-individual cross-tabulation, only the sum of cases in the main diagonal cells and the sum of cases in the minor diagonal cells have any real meaning. The exact distribution of cases within either diagonal depends on the directions of the several items. If the pair of individuals both voted for an item, for example, they would have both voted against it had its wording been reversed in direction. Thus unless the direction of the items can be solved for and standardized, the properties of interest in the inter-individual cross-tabulation cannot be measured. In particular one would not seek a zero cell in the cross-tabulation of individuals under a dominance model so measures of unidimensionality and nesting are inappropriate for inter-individual analysis.

MEASUREMENT OF CORRELATION

There are a wide variety of measures of correlation which can be analyzed by correlational procedures. We shall now consider the properties of these various measures so as to better judge their suitability for the analysis techniques. It should be noted that the term "correlation coefficient" is being used here in its most general sense as denoting any of the symmetric measures which might be analyzed by correlational procedures.

We have interpreted legislative roll calls as binary data so we shall concentrate upon the correlation coefficients which are appropriate

for fourfold tables.[1] Although the correlation of dichotomous variables has been studied for a very long time, there is no single authoritative treatment of this subject. Therefore we shall explore the measurement of correlation in two-by-two cross-tabulations in considerable detail.

Goodman and Kruskal's (1954) insistence on operational interpretations for measures of association has strongly influenced subsequent discussions of the choice between correlation coefficients. It is helpful to be able to interpret the values of a correlation coefficient so that meaningful comparisons of values can be made. The contextual meaning of the statistic is what is relevant. There are, however, several different types of operational interpretations possible. Kruskal (Kruskal, W.H., 1958) has emphasized the importance of probabilistic interpretations whereas Costner (1965) sees the proportional-reduction-in-error interpretation as all important. A somewhat different emphasis is more relevant for dimensional analysis.

In the context of dimensional analysis, it is essential that the matrix of correlation coefficients be susceptible to analysis. The patterning of the matrix must be a function of relevant aspects of the dimensional structure and there must exist analysis techniques to discover the dimensional structure from that patterning. The operational interpretability of a measure of correlation will do nothing to save a measure that is without theoretical utility in the analysis. Thus the interpretation of measures of correlation in this setting requires attention to the behavior of the coefficient in the full matrix of values and demands simultaneous attention to analysis techniques.

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[1] The choice of measures of correlation often depends on the level of measurement of the data, but this consideration is irrelevant here so long as the roll call votes are viewed as dichotomous.

Types of Correlation Coefficients.--For present purposes we shall be concerned with two types of correlation coefficients. The first of these is the measure of "one-way association." Such a coefficient measures the degree of unidimensionality between a pair of items. If two items partake of the same dimension without error, the error cell of the inter-item cross-tabulation would be zero. Correspondingly the measure of one-way association attains its maximum value when there is a single zero cell in the fourfold table. Statistical independence indicates the absence of a common dimension. Measures of one-way association have statistical independence as their null condition. We shall find several correlation coefficients with these properties.

Measures of "two-way association" have frequently been employed in dimensional analysis. A pair of items covary perfectly when everyone gives the same response to both items. Correspondingly the maximum value condition for a measure of two-way association is that the items are completely identical--there are two zero cells in the fourfold table. Statistical independence indicates the absence of covariation. The null value condition for a measure of two-way association is therefore statistical independence. The Pearson product-moment r is the classic measure of two-way association.

Measures of one-way association indicate the degree of unidimensionality between a pair of items while measures of two-way association indicate the extent of covariation. Measures of two-way association are not pure measures of unidimensionality because their maximum value condition is two, rather than one, zero cells. Neither is a pure measure of distance because both have statistical independence as their

null condition. Statistical independence is not a useful null condition for a measure of distance. That item A is statistically independent from items B and C does not imply that item A is equidistant from items B and C. A distance measure should be able to differentiate the distance between items A and B from the distance between items A and C, even when A is independent of both B and C. For distance to always be measured, the value of the statistic should not be affected by statistical independence. However statistical independence is the null condition for measures of both one-way and two-way association so neither is a pure measure of distance.

This distinction between measures of one-way association and measures of two-way association should not be confused with the more common distinction between asymmetric measures and symmetric measures. An asymmetric measure distinguishes explicitly between the independent variable and the dependent variable so that, in general, its value as calculated on a single bivariate distribution depends on which variable is chosen as independent. A symmetric measure has the same value regardless of which variable is seen as independent. The regression coefficient (or slope) is a typical asymmetric measure while the correlation coefficient is a typical symmetric measure. Measures of both one-way and two-way association are symmetric measures in that their values do not depend on which variable is taken to be the independent variable. The asymmetric regression coefficient is not equal to its maximum value when calculated on a bivariate frequency distribution under unidimensionality while the measure of one-way association has unidimensionality as its maximum value condition. Thus measures of one-way association are different than asymmetric measures.

Analysis Implications.--Because of their different value conditions, these two types of coefficients yield correlation matrices with different patternings. We shall indicate here the different patternings of coefficients which might result on a single dominance data structure. This brings out what each type of coefficient measures and clearly shows that the same analysis procedure cannot be used on both types of coefficient.

Table 4.1 shows the voting on two independent Guttman scales. Items A, B, C, and D form one perfect scale while items E, F, and F form a second perfect scale. These scales have been constructed so that each item on one scale is statistically independent of each item on the other scale.

TABLE 4.1 VOTING ON INDEPENDENT GUTTMAN SCALES

	Item						
Individual	A	B	C	D	E	F	G
1	Yes	Yes	Yes	Yes	Yes	Yes	Yes
2	Yes	Yes	Yes	Yes	No	Yes	Yes
3	Yes	Yes	Yes	Yes	No	No	Yes
4	Yes	Yes	Yes	Yes	No	No	No
5	No	Yes	Yes	Yes	Yes	Yes	Yes
6	No	Yes	Yes	Yes	No	Yes	Yes
7	No	Yes	Yes	Yes	No	No	Yes
8	No	Yes	Yes	Yes	No	No	No
9	No	No	Yes	Yes	Yes	Yes	Yes
10	No	No	Yes	Yes	No	Yes	Yes
11	No	No	Yes	Yes	No	No	Yes
12	No	No	Yes	Yes	No	No	No
13	No	No	No	Yes	Yes	Yes	Yes
14	No	No	No	Yes	No	Yes	Yes
15	No	No	No	Yes	No	No	Yes
16	No	No	No	Yes	No	No	No
17	No	No	No	No	Yes	Yes	Yes
18	No	No	No	No	No	Yes	Yes
19	No	No	No	No	No	No	Yes
20	No	No	No	No	No	No	No

Table 4.2 shows the matrix of one-way association coefficients for this set of data, where it makes no difference which particular coefficient is used. A one-way association coefficient has its maximum value of unity when there is a zero cell in the inter-item cross-tabulation. Items on the same perfect scale therefore have correlations of unity. Statistical independence is the null condition for a one-way association coefficient, so the correlations between items from different scales are zero.

TABLE 4.2 MATRIX OF ONE-WAY ASSOCIATION MEASURES

	Item						
Item	A	B	C	D	E	F	G
A	-	1.00	1.00	1.00	0.00	0.00	0.00
B	1.00	-	1.00	1.00	0.00	0.00	0.00
C	1.00	1.00	-	1.00	0.00	0.00	0.00
D	1.00	1.00	1.00	-	0.00	0.00	0.00
E	0.00	0.00	0.00	0.00	-	1.00	1.00
F	0.00	0.00	0.00	0.00	1.00	-	1.00
G	0.00	0.00	0.00	0.00	1.00	1.00	-

Measures of one-way association thus indicate the extent to which a pair of items belong to the same dimension. A matrix of such coefficients should be analyzed by a technique which ascribes high values to the same dimension and low values to different dimensions. Table 4.2 also indicates clearly that in the case of perfect data the measures of one-way association yield no clues as to inter-item distance. The values for the items in the same scale are all equal so that inter-item distance is not reflected in the values. If there were error in the data, the correlations between items in the same scale would be

unequal. This would indicate the effect of error more than inter-item distances. Measures of one-way association indicate only the dimensional properties of the items and not their relative locations on the dimensions.

Table 4.3 shows the matrix of values of a measure of two-way association for the data of Table 4.1, where the Pearson product-moment correlation coefficient is the particular coefficient employed. The order of the items within each of the scales is evident in the matrix. Among the items E, F, and G, for example, item E has its larger correlation with F and its smaller correlation with G, indicating that F is closer to E than is G. It can also be seen that F is closer to G than is E. Together these conditions imply the scale order EFG. The independence of the two scales is also visible in the matrix, as shown by the zero correlations between items in the two scales. However the fact that the correlations between item A and items E, F, and G are all equal means that one cannot judge the relative distances of items E, F, and G from item A by means of their correlations. Indeed there is no indication of the inter-point distance for items from different scales. Measures of two-way association are affected by statistical independence and so do not always indicate inter-point distance.

Different types of correlation coefficients measure different properties of the inter-item cross-tabulation and therefore matrices of these coefficients have different structural patternings. Measures of one-way association show whether the items belong to the same dimension. Measures of two-way association indicate the extent of covariation of the items. They show the order of inter-item distances for items from the same dimension and show the correlation between

TABLE 4.3 MATRIX OF TWO-WAY ASSOCIATION MEASURES

Item	Item A	B	C	D	E	F	G
A	-	.67	.44	.25	.00	.00	.00
B	.67	-	.67	.44	.00	.00	.00
C	.44	.67	-	.67	.00	.00	.00
D	.25	.44	.67	-	.00	.00	.00
E	.00	.00	.00	.00	-	.58	.33
F	.00	.00	.00	.00	.58	-	.58
G	.00	.00	.00	.00	.33	.58	-

different dimensions. These two types of coefficients are not interchangeable because they measure different properties.

The other aspect of correlational analysis is the analysis technique. Different analysis techniques make different assumptions as to the meaning of various structural patternings. One would not expect a single analysis technique to give the same interpretation to matrices as different as those of Tables 4.2 and 4.3. The assumptions of the analysis technique must be matched with the properties measured by the correlation coefficient for correlational analysis. This shall be the concern of the next chapter.

CORRELATION COEFFICIENTS

We shall now determine which correlation coefficients belong to the types described above. The usual norming conventions for correlation coefficients will be followed here so that the maximum values of the coefficients will be unity and the null values will be zero. It is also often desirable that only the sign of a coefficient does change when one of the pair of items is reflected.

Covariation.--Measures of association are built around a measure of the covariation of the pair of items. Let p_{21} denote the size of the error cell and $E(p_{21})$ its expected value under statistical independence. Following Yule and Kendall (1950) and Alker (1965), we denote δ (delta) as the difference between $E(p_{21})$ and p_{21} when calculated on a percentage table. Delta is formally equivalent to the covariation of the pair of items.

If two items belong to the same dimension, the error cell of the fourfold table would be empty. If two items belong to different independent dimensions, the value of the error cell would be that which would occur under statistical independence (the product of the marginals which contain the cell). Thus delta will equal $E(p_{21})$ if the items belong to the same dimension and will equal zero if the items belong to different dimensions. The value of delta ranges from zero under statistical independence to a maximum of 0.25, attainable only when both item marginals are 50-50. Delta must be normed if the conventional range of zero to one is to be used. Delta is the basis of several correlation coefficients because it indicates the extent to which the items share a common dimension by contrasting the actual size of the error cell with that expected by chance.

Delta is equivalent to the "cross-products difference" when both are calculated on a percentage table. Using the notation of Figure 4.1, $\delta = E(p_{21}) - p_{21} = p_{2.}p_{.1} - p_{21} = (p_{21} + p_{22})\ (p_{11} - p_{21}) - p_{21} = p_{11}p_{21} + p_{11}p_{22} + p_{21}^2 + p_{21}p_{22} - p_{21} = p_{11}p_{22} + p_{21}(1 - p_{12}) - p_{21} = p_{11}p_{22} - p_{21}p_{12}$.

It should be noted that delta in no way indicates whether p_{12} or p_{21} is smaller. The interpretation given here depends on the error cell being the smaller negative diagonal cell, but this is not necessarily the case in the cross-tabulation of all pairs of items which are being considered for scaling.

Measures of Two-Way Association.--Since delta ranges in value from zero to 0.25, the simplest means of norming it so that it ranges in value from zero to 1.00 is simply to multiply delta by four. This, in effect, is what Kendall's tau-c does when it is calculated on a two-by-two table. Tau-c has a value of zero under statistical independence since it is a constant multiple of delta which is zero under statistical independence. The norming gives tau-c a maximum value of unity when both item marginals are 50-50 with the two negative diagonal cells of the cross-tabulation both being empty. Thus tau-c has the properties specified for a measure of two-way association.

Table 4.4b shows an inter-item cross-tabulation which we will use to demonstrate the calculation of the various coefficients. Table 4.4c shows the cell values which correspond to statistical independence given the marginals of Table 4.4b. Table 4.4a shows the cell values which yield the maximum association between the items given the marginals of Table 4.4b. Each of these tables is given in percentage terms, corresponding to the usage in Figure 4.1.

The value of delta calculated on Table 4.4b is .10. This can be obtained either by subtracting the actual value of the error cell (.10) from its expected value (.20) or by subtracting the product of the minor diagonal cells (.10 x .20) from the product of the major

TABLE 4.4 INTER-ITEM CROSS-TABULATIONS

	Yes	No	Total
Yes	.40	.10	.50
No	.00	.50	.50
Total	.40	.60	1.00

Table 4.4a

	Yes	No	Total
Yes	.30	.20	.50
No	.10	.40	.50
Total	.40	.60	1.00

Table 4.4b

	Yes	No	Total
Yes	.20	.30	.50
No	.20	.30	.50
Total	.40	.60	1.00

Table 4.4c

diagonal cells (.30 x .40). Tau-c is four times delta so tau-c equals .40.

The classic measure of correlation is Pearson's product-moment correlation coefficient. For the special case of a two-by-two table, the Pearson coefficient is usually called the phi coefficient. The phi coefficient is also the equivalent of Kendall's tau-b calculated on dichotomous variables. Phi can be computed as the cross-products difference (the product of the major diagonal cells less the product of the minor diagonal cells) divided by the square root of the product of the four marginal terms. In other words, delta is normed for the calculation of phi by dividing it by the square root of the product of the four marginals. Phi in Table 4.4b, for example, is .41 (= .10/ $\sqrt{.40 \times .50 \times .50 \times .60}$).

Phi has the value of zero under statistical independence since its numerator is delta which is zero under statistical independence. Delta is normed to have its maximum value of unity only with equal item marginals and therefore two zero cells in the fourfold table. Thus phi has the properties specified for a measure of two-way association.

Measures Of One-Way Association.--We have required that a measure of one-way association have a maximum of unity with a single zero cell, have a null value of zero under statistical independence, and have a negative value in the case of an item reversal. It should measure the degree to which the items share a common dimension--the degree of departure from statistical independence. The measure of one-way association will therefore depend directly upon the quantity delta.

Both tau-c and phi have maximum values below unity when the item marginals are unequal. Thus with the marginals of Table 4.4, the maximum value of tau-c is .80 (=4 x .20) and the maximum value of phi is .83 (= .20/ $\sqrt{.40 \times .50 \times .50 \times .60}$) which correspond to the values for Table 4.4a. It is possible to adjust the values of tau-c and phi with unequal marginals by dividing them by their respective maximum values attainable with the given marginals. The ratio of tau-c to the maximum tau-c is identical to the ratio of phi to the maximum phi. This ratio is termed phi/phi max. Phi/phi max on Table 4.4b is .50 (=.40/.80 = .41/.83).

An equivalent derivation of phi/phi max is to divide delta, the difference between the expected and actual sizes of the error cell, by the expected value of the error cell when the error cell is the smaller of the minor diagonal cells. According to this latter formulation, phi/phi max measures the proportion by which the size of the error cell deviates from its expected value. Thus phi/phi max on Table 4.4b can also be calculated as delta (.10) divided by the expected value of the error cell (.20) which again yields a result of .50.

Phi attains its maximum value with a given set of marginals when there is a single zero cell in the tabulation. Therefore phi/phi max

has a value of one with a single zero cell. This coefficient also satisfies the other requirements of a measure of one-way association.

Phi/phi max can be extremely sensitive to small shifts in frequencies. Since phi/phi max is the ratio of delta to the expected value of the error cell, its value can change considerably with slight changes in frequencies when the expected value of the error cell is small. As an extreme example, phi/phi max is unity if $p_{11} = .98$ and $p_{22} = .02$ but phi/phi max falls to less than one-half if $p_{11} = .97$ and $p_{22} = p_{12} = p_{21} = .01$.

A different method of norming delta would be to norm the equivalent cross-products difference. The difference between the main diagonal cross-product and the minor diagonal cross-product could be divided by the main diagonal cross-product. This solution would lack certain desirable norming and symmetry properties, so instead we divide the cross-products difference by the cross-products sum. The result is the index of association, Yule's Q, which is identical to Goodman and Kruskal's gamma for the special case of a fourfold table. The value of Yule's Q for Table 4.4b is .71 (= .10/(.30 x .40 + .10 x .20).

Yule's Q satisfies the stated requirements for a measure of one-way association, but it has a higher value than phi even when the items have equal marginals. If both item marginals were 50-50, we could utilize the difference between two conditional probabilities--the conditional probability of favoring item B given that item A is favored minus the conditional probability of favoring item B given that item A is not favored, the percentage difference for the special case of equal 50-50 item marginals. The index of colligation, Yule's Y

(Yule, 1912), determines what the value of this conditional probability would be if both items did indeed have 50-50 marginals. Y is calculated by dividing the difference between the square roots of the cross-products by the sum of the square roots of the cross-products. Y is a monotonic function of Q, but differs in never having a higher absolute value. The value of Y for Table 4.4b is .42 (= $(\sqrt{.30 \times .40} - \sqrt{.10 \times .20})/(\sqrt{.30 \times .40} + \sqrt{.10 \times .20})$). Y is an appropriate measure of dimensional properties if we agree that dimensional specifications are conceptually independent of the artificial inequality of the item marginals. Such does not seem to be the case, however, and particularly not when the inequality of the item marginals is very real and meaningful as with legislative roll call votes.

Another procedure would be to assume that both variables are continuous with a bivariate normal distribution. One might then want to obtain the same coefficient value from a fourfold table as would have been obtained by the use of the Pearson product-moment coefficient on the full bivariate cross-tabulation. The tetrachoric correlation coefficient, r_t, adopts this approach. The tetrachoric coefficient solves for the value of the Pearson r correlation coefficient which, under a bivariate normal distribution, would yield the obtained fourfold table when the two variables are dichotomized so as to result in the given marginals.

As an example of the theoretic formulation of the tetrachoric r, consider its behavior when there is a single zero cell in the cross-tabulation. If a pair of variables has a bivariate normal distribution with a correlation of unity, then the variables fall along the same

dimension. If this dimension is dichotomized at one point by one variable and at another point by the other variable, then the variables will have unequal marginals and there will be a zero cell in the cross-tabulation of these dichotomized variables. According to this logic, the zero cell in the fourfold table results from a pair of variables with a correlation of unity. For this reason the tetrachoric r has the value of one with a single zero cell.

The formula for r_t is very complex, but it can generally be approximated by the sine of $\pi/2$ times Yule's Y.[2] The tetrachoric r is always at least as large in absolute value as Yule's Y which is always at least as large as phi. The tetrachoric r value on Table 4.4b is approximately .61 (= sine ($\pi/2$ x .42)).

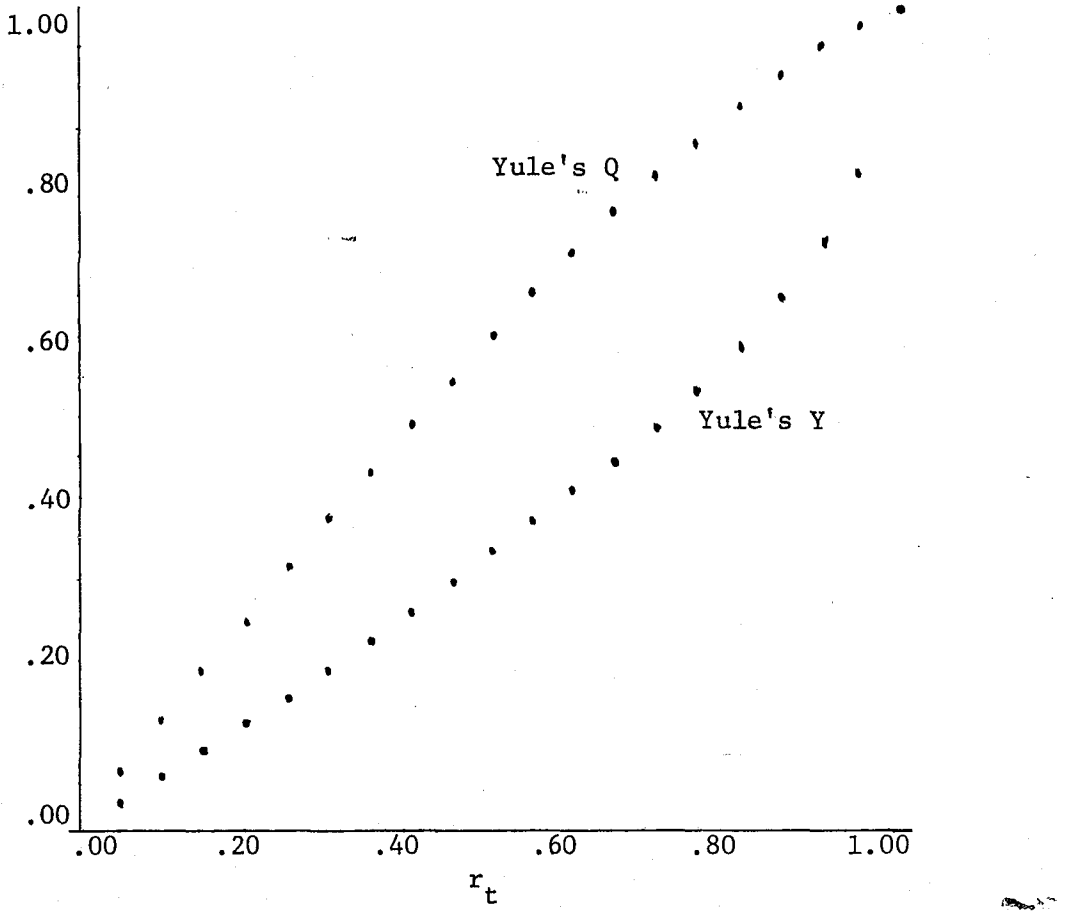
Three of these measures of one-way association, Yule's Q, Yule's Y and r_t, have now been seen to be monotonically related to one another. Figure 4.3 shows the corresponding values for these three coefficients. Observe that the values of Yule's Y decrease very rapidly as r_t is decreased while the values of Yule's Q decrease very slowly as r_t is decreased. The tetrachoric r has values between those of the other two coefficients, but its rate of decrease is closer to that of Yule's Q than to that of Yule's Y.

Yule's Y can be interpreted as the percentage difference when both items have 50-50 marginals. The Yule's Y value of 0.590, for example,

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[2] This approximation is identical to the common "cosine-pi" approximations for r_t, $\cos\left(\pi\sqrt{p_{12}p_{21}}/(\sqrt{p_{11}p_{22}} + \sqrt{p_{12}p_{21}})\right)$ and $\cos\left(\pi/(1 + \sqrt{(p_{11}p_{22}/p_{12}p_{21})})\right)$. Davidoff and Goheen (1953) discuss these approximations in more detail, proving their equivalence and discussing their precision.

Fig. 4.3 Corresponding Values for Yule's Q, r_t, and Yule's Y



would be obtained when, with 50-50 marginals for both items, $p_{11} = p_{22} = .3975$ and $p_{12} = p_{21} = .1025$ so that the percentage difference is 59%. Thus when both items have 50-50 marginals and Yule's Y is 0.590, the size of the error cell, p_{21}, is .1025. It can be shown that for any given size of the error cell, the product of the two percentage differences is at its maximum when the items have 50-50 marginals. However when the items have 50-50 marginals, each of these percentage differences equals Yule's Y. Thus for any given size of the error cell, Yule's Y has its maximum when the items have 50-50 marginals. Conversely, for any given value of Yule's Y, the maximum compatible

TABLE 4.5 CORRESPONDING VALUES FOR YULE'S Q, r_t, YULE'S Y, AND THE SIZE OF THE ERROR CELL

Yule's Q	r_t	Yule's Y	Maximum Error Cell
1.000	1.000	1.000	.000
.975	.950	.798	.051
.945	.900	.713	.072
.912	.850	.647	.088
.876	.800	.590	.102
.836	.750	.540	.115
.794	.700	.494	.127
.749	.650	.450	.137
.702	.600	.410	.148
.652	.550	.371	.157
.600	.500	.333	.167
.546	.450	.297	.176
.490	.400	.262	.185
.433	.350	.228	.193
.374	.300	.194	.202
.314	.250	.161	.210
.252	.200	.128	.218
.190	.150	.096	.226
.127	.100	.064	.234
.064	.050	.032	.242
.000	.000	.000	.250

size of the error cell is that which occurs when the items have 50-50 marginals. As an example, when the size of the error cell is .1025, the maximum value of Yule's Y is .590 which is the value corresponding to 50-50 marginals. If the size of the error cell is above .1025, the maximum value of Yule's Y would have to be below .590. Thus if Yule's Y has a value of .590, the size of the error cell cannot be above .1025. Since Yule's Q and r_t are monotonically related to Yule's Y, it is also possible to specify the maximum size of the error cell corresponding to given values of these coefficients. The last column of Table 4.5 indicates the maximum value of the error cell which is possible with the given values of Yule's Q, r_t, and Yule's Y.

The tetrachoric r is an unbiassed estimator of r only if its stringent assumptions are met. This coefficient might still be used for descriptive purposes if these conditions are not met. Its descriptive use depends on its being a measure of one-way association that is related to Yule's Q and Y. The tetrachoric r is a monotonic, though non-linear, function of both Yule's Q and Y. In general it takes on lower values than Yule's Q but higher values than Yule's Y. As seen in Table 4.5, Yule's Q has a tendency to take on very large values even when the degree of unidimensionality is only moderate. The high values of Yule's Q can lead to problems when certain analysis techniques are employed. Yule's Y has a tendency to take on small values even when the evidence of unidimensionality is substantial, which can again lead to problems when certain analysis techniques are employed. The more moderate values of the tetrachoric coefficient avoid some of the problems encountered with the use of the high-valued Yule's Q and the low-valued Yule's Y. The relative advantages of different measures of one-way association will be considered more fully in the next chapter, but it should be realized here that pragmatic considerations may motivate the use of tetrachoric r as a descriptive measure of one-way association even though its continuity and normality assumptions may not be satisfied.

One last measure of one-way association emphasizes the notion of nesting which was described at the beginning of this chapter.[3] Much of the essence of the nesting of a pair of items is captured in the

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[3]The nesting concept and this measure are both due to Donald Stokes (personal communication).

inequality of the minor diagonal cells of the cross-tabulation. With perfect nesting, the passing set of one item is wholly contained within the passing set of the other item. When the nesting is no greater than would be expected on the basis of statistical independence alone, then the two items do not partake of the same dimension or the inclusion of both of the items in the same scale yields no increased power in differentiating individuals when positioning people on the scale.

The eta statistic measures this nesting relationship.[4] The raw value of eta is the difference between the minor diagonal cells divided by their sum. This measures the relative distribution of mass in the minor diagonal cells of the fourfold table. The nesting coefficient eta measures how large this raw value of eta is as compared to its expected value under statistical independence with the given marginals. The raw value is corrected for statistical independence by subtracting off the value expected under independence and this difference is then normed by dividing it by the complement of the expected value. Thus eta indicates how much better the nesting is than would be expected by chance. As an example of the calculation of eta, the raw value of eta for Table 4.4b is .33 (= (.20 - .10)/(.20 + .10)) and its expected value as calculated from Table 4.4c is .20 (= (.30 - .20)/(.30 + .20)). The value of eta is then .17 (= (.33 - .20)/(1.00 - .20)).

The correction of raw eta for statistical independence is actually equivalent to multiplying it by phi/phi max.[5] Since eta is the product

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[4]It should be emphasized that the eta coefficient described here has no relationship to the eta coefficient employed in the analysis of variance context.

[5]Thus the value of eta in Table 4.4b is the raw value of eta (1/3) times the value of phi/phi max found above (1/2), which again yields a final value of 1/6 for eta.

of a measure of one-way association with another factor, it represents an extremely severe test of nesting and is a very low-valued coefficient. Eta always has the value of zero when the two items have equal marginals since the negative diagonal cells are then equal. The logic of the situation considers it redundant to include both items in a scale for positioning of individuals when the items have equal marginals. The inclusion of the second item puts as much error into the scale as it puts in power to differentiate individuals. Items with equal marginals may belong to the same dimension, as would be revealed by an inspection of their behavior with respect to other items on that dimension, but they should not be included in the same scale when that scale is to be used for further analysis of the individuals. The eta coefficient can be used to measure this nesting relationship for purposes of positioning the individuals.

All five of the coefficients described in this section are measures of one-way association. Each has a maximum value of unity with a single zero cell and a null value of zero under statistical independence. In addition we have seen that Yule's Q, Yule's Y, and the tetrachoric r are functionally interrelated. Phi/phi max and eta are also related to each other. All of these coefficients depend directly on delta. When both items have 50-50 marginals without perfect two-way association between them, Q has the highest value, r_t a medium value, phi/phi max and Y equal lower values, and eta equals zero. If $p_{11} = p_{22} = 3/8$ and $p_{12} = p_{21} = 1/8$, for example, $Q = 0.80$, $r_t \simeq 0.71$, phi/phi max = Y = 0.50, and eta = 0.00.

Each of these measures indicates the extent to which the items share a common dimension. It is essential to employ one of these

measures of one-way association when the analysis technique assumes a coefficient with its stated properties. The choice among these coefficients is generally less important than is the use of any one of these coefficients when a measure of one-way association is necessary. The exact choice of a coefficient will be discussed in the next chapter in relation to its effects on the results of an analysis.

Correlation Coefficients.--We have now considered a large number of measures of correlation for dichotomous variables. Table 4.6 lists these coefficients and gives their computational formulas. These coefficients have been classified into two types of coefficients, each of which is defined in terms of its formal properties.

TABLE 4.6 CORRELATION COEFFICIENTS

Coefficient	Formula*
Kendall's tau-c	$= 4(p_{11}p_{22} - p_{12}p_{21})$
Phi[1]	$= (p_{11}p_{22} - p_{12}p_{21})/\sqrt{p_{1.}p_{2.}p_{.1}p_{.2}}$
Phi/phi max	$= (p_{11}p_{22} - p_{12}p_{21})/(p_{2.}p_{.1})$
Yule's Q[2]	$= (p_{11}p_{22} - p_{12}p_{21})/(p_{11}p_{22} + p_{12}p_{21})$
Yule's Y	$= (\sqrt{p_{11}p_{22}} - \sqrt{p_{12}p_{21}})/(\sqrt{p_{11}p_{22}} + \sqrt{p_{12}p_{21}})$
Tetrachoric r	$\simeq$ Sine $(1/2\ \pi\ Y)$
Eta	$=$ (phi/phi max)$(p_{12} - p_{21})/(p_{12} + p_{21})$

*The notation is as follows:

		Item B: Yes	Item B: No	Total
Item A	Yes	p_{11}	p_{12}	$p_{1.}$
	No	p_{21}	p_{22}	$p_{2.}$
	Total	$p_{.1}$	$p_{.2}$	1.0

Without loss of generality, we require $p_{1.} \geq p_{.1}$ and $p_{11}p_{22} \geq p_{12}p_{21}$.

[1]This is equivalent to Pearson's r and Kendall's tau-b.

[2]This is equivalent to Goodman and Kruskal's gamma.

The symmetric measures of two-way association are not generally appropriate for dimensional analysis. They do not attain their maximum value condition under unidimensionality. They are influenced by statistical independence, so the use of these measures as distance analogues is questionable. Thus the meaning of such coefficients as phi, r, tau-b, and tau-c in dimensional analysis is unclear.

The symmetric measures of one-way association should instead be used for the measurement of the dimensional properties of the data. These indicate the degree to which the variables share a common dimension. We have seen that there are several such measures--Yule's Q, Yule's Y, tetrachoric r, phi/phi max, and eta--each of which can be used with dichotomous data. We turn in the next chapter to the use of these coefficients in dominance scaling through correlational analysis procedures.

CHAPTER V

DOMINANCE SCALING

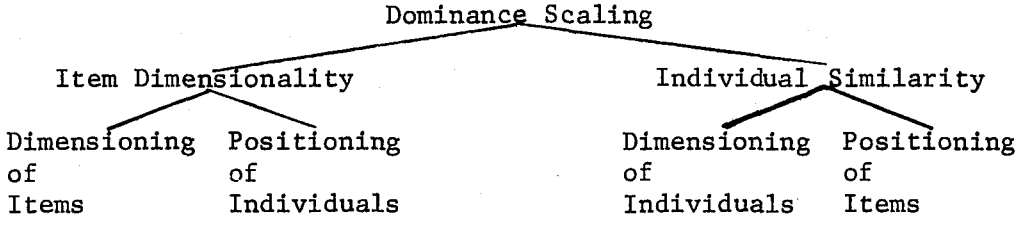
We have emphasized in previous chapters the advantages of correlational techniques as means of dominance scaling. We shall now consider the techniques themselves in some detail. The technique must be chosen so as to combine the robustness which motivates the use of correlational analysis with an accurate analysis of the chosen correlation coefficient over a wide variety of data structures. A consideration of correlational analysis thus involves an interaction of properties, coefficients, techniques, and data.

Dimensional analysis encompasses several different properties of the data. One wishes to explain the dimensionality of the items--to obtain the dimensional properties of the items and then to locate the individuals in the resultant space. One also wishes to explain the similarity of the individuals--to obtain the dimensional properties of the individuals and then to locate the items in the resultant space. This distinction between the dimensionality of the items and the similarity of the individuals corresponds very closely to the common distinction between the inter-item R analysis of the stimulus space and the inter-individual Q analysis of the population space. We shall restrict our attention to the former, the item dimensionality.

There are two distinct aspects of explaining the dimensionality of the items. The first is the dimensioning concern--obtaining the

dimensional specifications of the items. The second is the positioning concern--locating the individuals in the space and scoring them on the dimensions. Positioning is often the second stage of an anlysis, the stage following dimensioning. Figure 5.1 diagrams these several aspects of dominance scaling. We shall concentrate on the dimensioning concerns, occasionally noting the possibilities for positioning.

Fig. 5.1 Dominance Analysis



ITEM DIMENSIONALITY

We shall now focus on one precise property which we want to extract from the data--the dimensional specifications of the items. Correlational analysis of this property requires the selection of an analysis coefficient which measures this property and the concomitant selection of an analysis technique which operates upon matrices of that coefficient to yield the desired dimensional specifications of the items.

Classic statistics provides one technique for the analysis of the dimensional specifications of a set of data--factor analysis. However factor analysis has been found to have a number of undesirable properties having to do with overestimating the dimensionality of data. In the present context of dominance scaling, factor analysis may not accurately reproduce known Guttman scales. These difficulties have caused factor

analysis to fall into disrepute as a means of obtaining the dimensional specifications of data under the dominance model.

Our main purpose in this chapter is to reinterpret the use of factor analysis for dimensioning dominance items. We shall begin by demonstrating how unsuited factor analysis is for this task when the usual product-moment correlation coefficient is utilized. We shall then suggest that the problem is not so much in the technique as in the coefficient. Although the theory of factor analysis is based on the product-moment coefficient, we shall suggest that factor analysis of dominance items should be employed only with measures of one-way association. Factor analysis captures very well the dimensional specifications of items under the dominance model when the correlation coefficient chosen is one of the one-way association measures described in the previous chapter.

That factor analysis may be inappropriate for the analysis of the product-moment coefficient on which its theory is based may seem paradoxical. A preliminary explanation of this paradox can be obtained by dissecting a different paradox--the paradox of index construction. Say that one wished to build an additive index from a battery of items. For example, one might count the number of liberal votes cast by a legislator as an indicator of his position on a left-right ideological continuum. First, however, the items to be included in this index must be determined. Let the items be chosen for the index through a visual inspection of the matrix of inter-item correlations. Items with very high correlations need not be included in the same index due to their very considerable overlap. Items with very

low correlations should not be included in the same index since they measure different properties. What then is an index if items having high correlations and items having low correlations are excluded?

In describing the paradox of index construction, we have remained vague as to which correlation coefficient was being employed. The paradox clearly does occur when a measure of two-way association such as the classic product-moment phi is used. Items with a large phi value do basically overlap whereas items with low phi values do belong to different dimensions. The paradox can be avoided if a measure of one-way association is instead used.

Say that the matrix of correlations being inspected were a matrix of measures of one-way association. A high correlation indicates that the pair of items belongs to the same dimension. The high value does not indicate excessive overlap since inter-item distance is not being measured. Two items having low correlations according to a measure of one-way association do tap different latent dimensions. There is no contradiction here. An index should consist only of items having high inter-correlations as measured by one-way association coefficients.

Thus there is no paradox when measures of one-way association are employed, but the paradox of index construction arises when the classic measures of two-way association are used. The paradox results from the fact that the product-moment phi is not a pure measure of the dimensioning properties which are of greatest significance in index construction.

To return to the original problem, how can factor analysis be inappropriate for the analysis of the product-moment coefficient on

which its theory is based? The product-moment statistic is not a pure measure of the dimensional properties of the data. The fact that it is affected by the inter-item distances decreases its utility for the analysis of dominance items. The factor analysis of measures of one-way association is a more reliable procedure for the determination of item dimensionality.

FACTOR ANALYSIS OF MEASURES OF TWO-WAY ASSOCIATION

It is well known that there are difficulties in the factor analysis of the product-moment coefficient. A classic study of the problem is provided by Coombs and Kao (Coombs, 1964, chap. 8). The authors wished to determine whether the robust factor analysis procedure could give a solution essentially similar to that provided by the laborious and vulnerable technique of multidimensional unfolding. Beginning with known data structures, they obtained the preference orderings of individuals, correlated these preference orderings, and factor analyzed these correlations. The obtained solutions were found extremely congruent with the original known data structures, but with an extra dimension appearing in the factor space. This extra dimension was found to indicate the distance of the individual from the median individual in the space, corresponding to their theoretical conjecture of a "social utility" dimension. Thus Coombs and Kao found that factor analysis overestimates dimensionality with the added dimension having to do with distance from the centroid of the space.

Ross and Cliff (1964) have followed upon the findings of Coombs and Kao. They have produced an elegant proof showing that the factor analysis of the product-moment coefficient overestimates the

dimensionality because the wrong matrix is being factor analyzed. If the matrix of squared inter-point distances were available, exact statements about the dimensionality would be possible. The correct dimensionality would be obtained if the squared distance matrix were double-centered. The use of the product-moment coefficient on the data without a knowledge of the inter-point distances yields an overestimate of the dimensionality, at times by more than the one extra dimension which Coombs and Kao found. Ross and Cliff indicate which matrix should be factor analyzed, but it is not generally available with real data. When the usual product-moment matrix is factor analyzed instead, one cannot know by how much the dimensionality of the space is being overestimated.

Dimensionality.--Ross and Cliff provide devastating results regarding the general use of factor analysis to obtain the dimensionality of data. We are more concerned with one special case--the factor analysis of dominance structures. Factor analysis of dominance items using the product-moment phi coefficient does indeed misrepresent the dimensionality of the space.

Factor analysis of the phi coefficient overestimates the dimensionality of dominance items. That the factor analysis of items differing only in their level of difficulty results in more than a single factor was observed by Ferguson as early as 1941. Wilkins (1962) found that the factor analysis of several perfect scales yields two dimensions. This was further explored by Clausen (1964) who found that the factoring of items with passing sets of size .80, .75, .55, .50, .25, and .20, for example, would lead to three factors.

To illustrate this overestimation, a ten item Guttman scale was constructed with an equal number of individuals being given each of the

eleven consistent response patterns. Table 5.1 shows the resultant product-moment phi matrix.

TABLE 5.1 PHI MATRIX FOR A GUTTMAN SCALE

					Item					
Item	A	B	C	D	E	F	G	H	I	J
A	-	.67	.52	.42	.35	.29	.24	.19	.15	.10
B		-	.77	.62	.52	.43	.36	.29	.22	.15
C			-	.81	.67	.56	.46	.38	.29	.19
D				-	.83	.69	.57	.46	.36	.24
E					-	.83	.69	.56	.43	.29
F						-	.83	.67	.52	.35
G							-	.81	.62	.42
H								-	.77	.52
I									-	.67
J										-

We have used the principal components form of factor analysis (Harman, 1967). This amounts to the mathematical extraction of the characteristic roots and vectors from the correlation matrix where unities have been placed in the diagonal cells of the correlation matrix. Each factor then accounts for the maximum amount of the variance remaining in the correlation matrix after the previous factors are extracted, subject to the constraint of orthogonal (independent) axes. We shall employ Kaiser's (1960) criterion to determine the number of factors--stopping the factoring so that only the factors explaining at least one unit of variance are used, where the total number of units of variance is the number of items being factored.

The factor analysis of the matrix of Table 5.1 yields two factors which explain more than one unit of the variance each. The first factor

accounts for 55% of the variance while the first two together account for 73.33% of the variance.[1]

This does not mean that the principal components solution always overstates the dimensionality by a value of one when the data are generated according to a perfect unidimensional Guttman scale. Experiments were made with varying numbers of items equally spaced along the difficulty continuum. The more items in the scale, the more factors were found. For example, the factor analysis of a four item scale yields one factor while that of a twenty item scale yields four.[2]

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[1]One might argue that Kaiser's criterion should not be used to determine the number of factors so that a unidimensional solution might be obtained. This argument will be considered after the two-dimensional solution is shown in the next section.

[2]The experimentation with the factor analysis of Guttman scales with items being equally spaced along the difficulty continuum has led to certain definite patterns of results. While a mathematical proof is not available, the patterns found empirically are exact down to a sufficient number of decimal places to guarantee their validity. These results will be given here to further emphasize the mathematical formalism involved in the application of factor analysis to phi matrices.

The table below shows the fraction of the total variance explained by the k-th factor where there are n items, for selected values of k and n. The entries in the upper-left three-by-three square in this table were obtained empirically. The entries in the k and n columns and the n and infinity rows were obtained by extrapolation from the values obtained empirically.

Fraction of total variance explained by factor k

Number of Items	1	2	3	...	$k(k \leq n)$	...	n
4	5/8	5/24	5/48		5/4k(k+1)		1/16
10	11/20	11/60	11/120		11/10k(k+1)		1/100
20	21/40	21/120	21/240		21/20k(k+1)		1/400
...							
n	(n+1)/2n	(n+1)/6n	(n+1)/12n		(n+1)/nk(k+1)		$1/n^2$
...							
∞	1/2	1/6	1/12		1/k(k+1)		0

The above results invalidate the practice of estimating the dimensionality of dominance items from the factor analysis of the product-moment phi. Dimensionality should be independent of the number of items, but factor analysis introduces an element of dependence. The analyst who works in the area of short scales may feel that these results do not apply to him. It is true that the factor analysis of a four item Guttman scale yields only one factor. However this one factor explains only 62.5% of the total variance. Few would interpret a single factor which explains only 62.5% of the variance as resulting from a perfect Guttman scale. Furthermore these results are based on the simple case of equal spacing of items. Clausen's example showed three factors arising from one set of unequal spacing of six items on a scale. This certainly means that even with a small number of items one would obtain more than one factor with many instances of real unidimensional data.

Do these results with hypothetical data structures have implications for the real world of dimensional analysis? Indeed, for how can one interpret the results from an unknown data structure if the technique yields incorrect results when applied to simple known data structures? If the factor analysis of phi matrices cannot solve correctly for the dimensionality of a perfect Guttman scale, how can it be used in the

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[2](Continued) Finally we can also obtain the number of factors which Kaiser's criterion would yield. The Kaiser criterion states that the number of factors to be extracted from a factor analysis of n items is equivalent to the number which account for more than 1/n of the variance. The k-th factor passes this test if the number of items, n, is greater than the value which causes $(n+1)/k(k+1)$ to equal unity. Therefore two factors will be obtained if there are more than five items, three if more than eleven items, four if more than nineteen items, five if more than 29 items, and so on. The number of factors depends directly on the number of items!

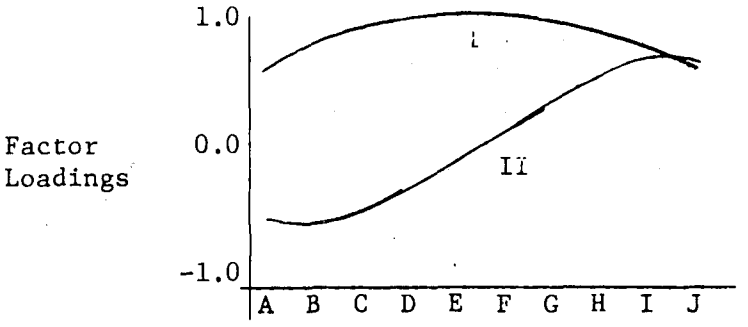
more complex situations with error and possible multidimensionality? Factor analysis of phi coefficients is not to be recommended as a means of determining the dimensionality of a dominance data structure.

The generalization of these results to multiple category items has never been explored. Alker and Russett (1965) and Anderson (1966) employ factor analysis using product-moment coefficients based on trichotomous roll call votes. The product-moment coefficient is still constrained by the item marginals in this case. The results of Coombs and Kao and those of Ross and Cliff are sufficient to suggest that the dimensionality may not be accurately determined by such analysis. The determination of dimensionality would be particularly misleading if a dominance structure were involved.

Item and Individual Locations.--The determination of dimensionality is but one of the tasks for which factor analysis is designed. It also locates the items in the space and permits one to position the individuals in the space. How well are these functions performed when one factor analyzes phi coefficients on dominance items?

We return to the ten item Guttman scale and its phi matrix shown in Table 5.1. This matrix yields two factors. Their factor loadings are plotted in Figure 5.2. Neither factor reveals the scale ordering. The scale ordering would be indicated by a factor if the order of the item loadings on the factor were monotonic with the order of the items on the original dimension. Neither factor in Figure 5.2 has the monotonic shape which this would imply. The factor analysis of phi coefficients on a Guttman scale does not permit one to subsequently locate the items along that scale.

Fig. 5.2 Factor Loadings of Correlation Matrix of Table 5.1



The first factor in Figure 5.2 has a special interpretation which has already been mentioned. The middle items on the dimension have the highest loadings on this factor while the extreme end items have the lowest loadings. This factor is the dominance scale "folded" at its midpoint. The first principal component is not always the folded scale, but some linear combination (rotation) of the principal components will yield the folded scale. Coombs (1964) terms this factor a "social utility" factor because it corresponds to the preference ordering which would win in a majority election. Note that all items have a positive loading on this factor. A negative loading of an item on this factor would indicate that the item is reflected. Thus the sign patterns on the social utility factor can be used to obtain item polarities.[3]

If the first factor measures only the extremeness of the item, then does the extraction of this factor mean that the remaining variance of the correlation matrix consists of exactly what we would want to analyze? Unfortunately this is not the case. The inspection of the plot of the second factor in Figure 5.2 shows that even after the social utility

[3]The sign of the item on a later factor might be used to determine its polarity in a multidimensional situation if its loading on the first factor were near zero.

factor is removed the next factor does not indicate the order of the items on the scale.

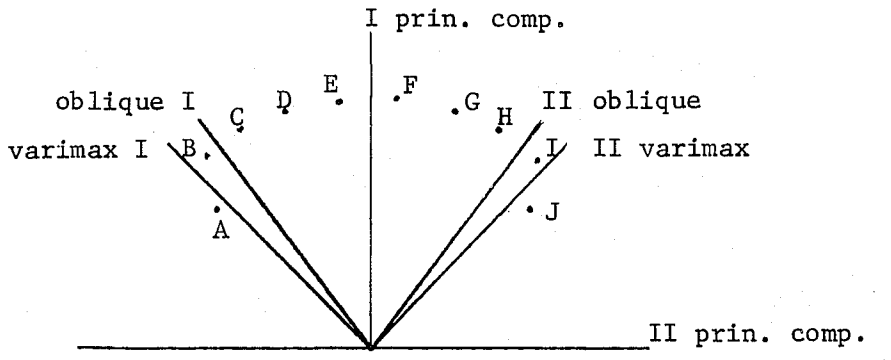
In a factor analysis of product-moment coefficients based upon a perfect Guttman scale, the scores of the individuals on the first factor are monotonic with their positions on the scale. Since each item has a positive loading on the first factor, the sum of the loadings on the first factor for the items which an individual approves (his factor score) varies with the number of items he approves (his position on the perfect scale). The possibility of positioning the individuals on the scale by scoring them on the first factor is still of limited utility. One would not realize from the factor analysis that unidimensionality was present and therefore factor scores would not be computed on the first factor. Note also that if one were to compute factor scores on subsequent factors, the results would be mathematical artifacts--Guttman's (1954) principal components of intensity, closure, and involution. An analysis of the factor scores on these factors would be very confused if one attempted to base substantive conclusions on such mathematical artifacts.

Factor analysis does not yield a unique positioning of the axes in the space. The principal components solution positions the axes to satisfy a set of variance maximization requirements, but these axes are known to be difficult to interpret. Thurstone (1935) suggested that the axes should be rotated to satisfy a set of criteria which he termed "simple structure." Simple structure requires that as many variables as possible load on fewer than the full set of dimensions. This makes it possible to differentiate the factors readily and should

simplify the interpretation of the factors. These criteria do not lend themselves directly to an analytic solution. For actual rotations, we shall employ Kaiser's orthogonal varimax rotation and Carroll's oblique biquartimin rotation (Cooley and Lohnes, 1962).

Such rotation does not resolve our earlier difficulties in interpreting the order of item loadings. Figure 5.3 plots the two factors from Figure 5.2 against each other, with the principal components, orthogonal varimax, and oblique biquartimin axes shown. In the rotated solutions, the items on the left half of the scale have their highest loadings on one factor while the items on the right half of the scale have their highest loadings on the other factor. Thus rotation separates out clusters of items. Factor analysis with rotation may be considered analogous to a metric cluster analysis. Still the rotation does not give the desired item order--it does not reproduce the order of the items on the Guttman scale.

Fig. 5.3 Factor Analysis of Phi Matrix on Guttman Scale



The importance of these results is to be emphasized. When we perform a factor analysis we naturally interpret the order of the items on the factors as meaningful. However when a dominance structure underlies

the data, the item order is completely artificial. Regardless of which factor is observed and regardless of which rotation is employed, the order of the item loadings on a factor does not show the order of the items on the scale. Extreme caution must be employed in the interpretation of the factor matrix. A similar caution would be appropriate for the factor analysis of product-moment correlations on multiple category items when a dominance structure is at work.

We should explore some possible simple explanations of these results. One such explanation would be that the apparent doubling-back of items along the scale order in Figure 5.3 is due simply to a misstatement of the dimensionality. Indeed there is nothing sacred about Kaiser's criterion which we have used to establish the dimensionality. However changing the dimensionality does not allow the recovery of the item order. While one might claim that only a single dimension is necessary in our example, the first factor does not indicate the correct item order. One could also argue that the items are linear in a space of larger dimensionality but that their projections in two dimensions are not. This would argue in favor of extracting more than two factors. Yet the third factor extracted from the correlation matrix of Table 5.1 has a U-shape when it is plotted against the item order as in Figure 5.2. That is, adding an extra dimension would not straighten out the item order, but would actually make the non-linearity even more severe than in the two-dimensional solution of Figure 5.3.

Another possibility is that the items would be found unidimensional if the origin of the space were simply centered. Say that the origin was moved from its location in Figure 5.3 to the center of gravity of the

space, the mean on each of the dimensions. The origin is already centered relative to the second dimension so it would only have to be moved up to about .73 on the first dimension in order to center it in the two-dimensional space. Centering, however, cannot straighten up the curve of Figure 5.3. The variation on the first dimension would be so slight as to permit one to conclude that a single dimension suffices--dimension two. Still the item order on this dimension does not correspond to the item order of the original scale because the items at the ends of the scale are misplaced. Dropping the first dimension would be even more disastrous from the point of view of factor scoring in that the factor scores on the second dimension are far from monotonic with scale position. Thus the factor analysis of phi coefficients cannot be saved by changing the dimensionality criterion or by centering the results.

The problems which have been seen here with factor analysis of the phi coefficient also occur when conducting such an analysis of the inter-individual matrix with the Q form of factor analysis. An additional complication is introduced by the asymmetry between inter-item and inter-individual analysis. Items can be reflected but individuals cannot be. The inter-item cross-tabulation contains a zero cell under the perfect Guttman scale. The inter-individual cross-tabulation need contain no zero cell under the perfect Guttman scale unless all the items have the same direction. Unless all items do have the same direction, the only meaningful bits of information in the inter-individual cross-tabulation are the proportion of items on which the pair of individuals agree and the proportion on which they disagree. The exact cell in which the agreement or disagreement on any particular item is located depends on

the direction of that item. Reversing an item would change the phi value. If, on the basis of face content or an analytic solution, the directions of the items are standardized, the phi coefficient can be meaningfully calculated on the inter-individual cross-tabulation. With the item directions standardized, the inter-individual cross-tabulations for the perfect Guttman scale will contain zero cells and the patterning of the correlation matrix will be identical to that described for inter-item analysis. There would then be a duality between the R and Q forms of analysis, with the roles of the items and individuals reversed. This duality means that the factor analysis of product-moment coefficients on inter-individual relations will overstate the dimensionality in the same way as in inter-item analysis. The loadings of the individuals on the Q factors will not correspond to their order on the scale, as the dual to the fact that with inter-item analysis the loadings of the items on the R factors do not correspond to their order on the scale.

We have now seen that the factor analysis of the phi coefficients under a dominance structure misstates the dimensionality of the data and distorts the ordering of the items. We can gain some small amount of information from such an analysis--an upper estimate on the dimensionality and an indication of reflected items. Nevertheless there are other ways of gaining these small bits of information. Overall, factor analysis of the phi coefficient is an inappropriate means of obtaining knowledge of the dimensional specifications of the items.

FACTOR ANALYSIS OF MEASURES OF ONE-WAY ASSOCIATION

While factor analysis in general has difficulties, it proves useful for exploring the dimensional specifications of dominance items when

measures of one-way association are used rather than the product-moment phi. Factor analysis of such coefficients is not unknown. Harris (1948) factor analyzed a matrix of tetrachoric r's which were calculated on a set of Senate roll calls. The tetrachoric r is a one-way association statistic and, as we saw in the first chapter, Harris in effect discovered a Guttman scale through his factor analysis of this coefficient. MacRae (MacRae and Schwarz, 1968; MacRae, 1968, forthcoming) has employed factor analysis on measures of one-way association to analyze Guttman scale structures.

Inconsistency.--The factor analysis of coefficients other than the product-moment has been called into severe doubt. Clausen (1964, pp. 70-71), for example, observed that the use of the tetrachoric r or the phi/phi max coefficient would resolve some of the difficulties encountered with the phi coefficient, but he continues:

> Unfortunately, the use of either of these coefficients tends to create problems in the factor analysis of the resulting correlation matrix. The problems arise from the nonsensical results which may be obtained in the intercorrelation of items. Using either a tetrachoric r or phi/phi max coefficient, Guttman (Stouffer et. al., 1950, pp. 196-7) shows that it is possible to have the situation in which two items (A and B) are perfectly and positively correlated with a third (C), yet the correlation between A and B may be perfectly negative. The inconsistency occurs according to Guttman because the assumptions required for the use of these correlations--bivariate normal distributions--are not met. As a result, the definition of the factor space may be marked by irregularities, such as factor loadings in excess of 1.0, which cast doubt upon the definition of the factor space provided by the factor analysis.

The internal inconsistency problem mentioned by Clausen is, in mathematical terms, that the correlation matrix of one-way association statistics is not necessarily positive semi-definite and therefore may be singular. As a simple example of what is meant by such inconsistency,

two items which correlate perfectly with a third item need not correlate perfectly with each other. With the product-moment coefficient, of course, two items which correlate perfectly with a third item would necessarily correlate perfectly with each other. Why this is not true with measures of one-way association can be seen by examining the multidimensional space shown in Figure 2.4. Items A and B are both unidimensional with respect to item C but not with respect to each other. Measures of one-way association between items A and C and between items B and C would be unity while measures of one-way association between items A and B would be less than unity. Guttman's example referred to by Clausen above is shown in Table 5.2. In this most extreme case, two items have a perfect negative correlation with one another while having perfect positive correlations with a third item, assuming that correlation is being measured by a one-way association statistic. Problems do occur in the factor analysis as a result of such violation of the internal consistency of the correlation matrix. Factor loadings can be greater than unity, factors can have negative characteristic roots, and items can have more than 100% of their variance explained by the factors. Yet these difficulties are not as serious as they may seem.

TABLE 5.2 VOTES VIOLATING THE CONSISTENCY REQUIREMENTS*

Type of Person	Item A	Item B	Item C	Frequency
I	Yes	Yes	Yes	10
II	Yes	Yes	No	30
III	Yes	No	No	40
IV	No	Yes	No	20
				100

*Adapted from Guttman (1950, p. 196).

We should first point out that even if difficulties do exist they are not for the reasons to which they are usually attributed. In particular they are not due to the violation of the "assumptions required for the use of these correlations--bivariate normal distributions." These problems occur with any measure of one-way association--phi/phi max, Yule's Q, and Yule's Y which do not assume a bivariate normal distribution as well as the tetrachoric r which does. It is true that the tetrachoric r is an unbiassed estimate of the value of an underlying r only when a bivariate normal distribution is assumed. However the descriptive purposes of the factor analysis of a one-way association statistic need not require the use of an unbiassed estimate of the product-moment r. Actually a similar problem sometimes exists when the product-moment r itself is used. The correlation matrix of the product-moment coefficient may be internally inconsistent when missing observations are deleted in a pairwise fashion from the computation of the correlation coefficients. Thus the mere existence of difficulties does not mean that critical assumptions are being violated or that factor analysis should automatically be discarded.

It is also important to realize that whatever difficulty that does exist is in the interpretation of the results rather than in the computational algorithm. Several algorithms exist for principal components analysis. The Hotelling procedure (Horst, 1965, pp. 160-167) extracts a single factor at a time in the order of decreasing proportion of the variance explained. This algorithm can extract only the most important factors when the correlation matrix is internally inconsistent. However the Jacobi algorithm which extracts all of the factors simultaneously can succeed regardless of such properties of the matrix.

The interpretation of the factor analysis of measures of one-way association must differ from the conventional interpretation of factor analysis particularly in regard to the meaning ascribed to the factor loadings. The factor loading is usually taken to represent the square root of the proportion of the variance of the item which is accounted for by the factor. This permits the calculation of the proportion of the variance of the item which is explained by all the factors and the calculation of the proportion of the total variance which is explained by each factor. However the conventional interpretation is based on the direct interpretation of the product-moment coefficient in terms of the explanation of variance. Measures of one-way association have no ready interpretation in terms of the explanation of variance. Therefore the variance interpretation is appropriate for the factor analysis of the product-moment coefficient but not for the factor analysis of measures of one-way association. We shall suggest an appropriate alternative interpretation of factor loadings when measures of one-way association are employed as we explore the interpretation to be given to the results of the factor analysis of measures of one-way association.

The real question is whether a reasonable interpretation can be given to the results of the factor analysis in the face of these difficulties. Clausen states that "the definition of the factor space may be marked by irregularities...which cast doubt upon the definition of the factor space provided by the factor analysis." But is there no possibility of recovering useful information? When we discussed the paradox of index construction, we noted that items which have a high correlation as measured by a measure of one-way association belong to

the same dimension while items which have a low correlation as measured by such a statistic belong to different dimensions. This is the type of information which we wish to recover from the factor analysis of measures of one-way association--the extent to which the items belong to the same dimensions. Indeed the factor analysis of these coefficients provides exactly this information. We might note that even Guttman's data of Table 5.2 do not lead to wrong conclusions when factor analyzed using measures of one-way association, such as the tetrachoric r. Factor analysis of these data yields two factors, but it is always possible to locate three points in two dimensions. Thus the factor analysis indicates the lack of a dominance structure in these data. In fact this procedure gives a better portrayal of the data than Guttman scaling, which would conclude that there is a single Guttman scale here with a coefficient of reproducibility of 0.97.

We shall employ a geometric argument to indicate the interpretation to be given to the results of the factor analysis of measures of one-way association. Factor analysis can be conceived of as mapping the correlation structure into a dimensional configuration. Whenever two items have a correlation of unity they are placed together on the same dimension. A correlation of minus one is represented by placing the items at opposite ends of a single dimension--that is, at the positive and negative ends of the dimension. A correlation of zero results in the placement of the items on different orthogonal dimensions. This interpretation of factor analysis can be stated formally in terms of an identity between the product-moment correlation coefficient and the cosine of the angle between the two items, with the lengths of the item vectors

being taken into account if they are not both of unit length. This geometric interpretation conforms roughly to the placement of axes under a simple structure rotation. We shall now employ this geometric machinery, but without the product-moment coefficient.

Unidimensional Case.--Consider first a perfect Guttman scale. Each cross-tabulation of a pair of items would contain a zero cell and therefore the matrix of measures of one-way association would consist entirely of unities. According to the geometric interpretation of factor analysis, items with correlations of unity are placed together on the same dimension. Therefore factor analysis of a matrix of measures of one-way association calculated upon a perfect Guttman scale would yield only a single factor with each item having a loading of unity on that factor. Factor analysis finds the correct dimensionality for this simple case and puts each item on the correct dimension. That factor analysis can obtain the correct dimensionality for items differing only in their difficulty by the use of a measure of one-way association was first found by Wherry and Gaylord (1944). They employed the tetrachoric coefficient so as to avoid the previously described problems with factor analysis of phi coefficients calculated upon dominance scales.

Note, however, that the factor analysis of one-way association measures on a perfect Guttman scale locates all items at the same point. It does not determine the item order on the dimension.[4] The factor loading is showing the extent to which the item partakes of the dimension, rather than the position of the item on the dimension. One common

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[4]The item order for this unidimensional case may be determined by the order of the item marginals though the possibility of inversions must be taken into account for items with similar marginals when there is missing data.

interpretation of a factor loading is that it indicates the correlation between the item and the factor. Here the factor loading shows the extent to which the item shares the same dimension as the factor, as if one were calculating the amount of one-way association between the item and the factor. When a perfect Guttman scale is factor analyzed with one-way association coefficients, each item has a loading of unity on the factor because each item falls exactly on that dominance dimension.

If some items of a perfect scale were reversed, then items having opposite polarities would have inter-correlations of minus one. The minus sign indicates negative association between the items while the magnitude of unity denotes the perfect scalar relationship. The geometric interpretation of factor analysis places items with correlations of minus one at opposite ends of the same dimension. Therefore only a single factor will be needed to account for the measures of one-way association on a perfect Guttman scale even when some of the items are reflected. The reflected items have loadings of minus one on the factor while the other items have loadings of plus one. This indicates immediately which items were reversed. In this case the items do appear at different locations on the factor, but this still has nothing to do with their locations on the dimension. The negative loadings indicate reversals of items, not item order.

How would error affect these results? If the amount of error in the scale is moderate, one would expect the values of the one-way association coefficients to remain reasonably high. Factor analysis should still extract a single factor which accounts for an overwhelming proportion of the total "variance." The item loadings on the factor

would be below unity, showing that the factor does not fully account for their dimensional properties. The loadings do not show the order of the items on the scale but instead indicate the extent to which the item is explained by the common dimension. These results may not hold under every possible distribution of error, but one would generally be expected to obtain a reliable estimate of the dimensionality of the space. Further model sampling would be necessary if the exact effects of scale error are to be established.

Factor scores can also be calculated on the factors obtained from an analysis of one-way association coefficients. In the perfect unidimensional case, the ordering of the individuals will be monotonic with their ordering on the scale. Since each item has a positive loading on the first factor (after reversal, if necessary), the sum of the loadings on the first factor for the items which an individual approves (his factor score) varies with the number of items he approves (his position on the perfect scale). In the presence of error, each item is, in effect, weighted by the degree to which it measures the factor. Factor scores are likely to be reliable measures of individual positions on the dimension so long as the amount of error is not excessive.

Factor analysis of measures of one-way association calculated on unidimensional Guttman scales is a useful technique. It allows for a test of dimensionality. It uncovers item reflections. It finds which partake of the common dimension. It yields an ordering of the individuals on the dimension. The item ordering, however, cannot be obtained. The usual variance interpretation of factor analysis must also be modified. The factor loading is not related to the proportion of the

item's variance which is explained by the factor. Instead it seems to indicate the extent of association between the item and the factor, the extent to which the item belongs to the dimension. The difficulties described earlier in the factor analysis of measures of one-way association do not preclude one from gaining useful information from such an analysis, provided necessary adjustments are made in the interpretation of the results. We shall return to this point when we consider the factor analysis of measures of one-way association in the multidimensional setting.

Obviously there are easier ways to perform Guttman scaling. Why not, for example, simply inspect the matrix of one-way association coefficients? Items with high correlations would be placed on the same dimension while items with low correlations would be placed on different dimensions. Visual inspection of the correlation matrix would be sufficient in the unidimensional case. However it would be quite difficult to obtain an accurate understanding of a real multidimensional structure by simply eyeballing a correlation matrix. Hopefully factor analysis, in that it is an avowedly multidimensional technique, can aid in the analysis of multidimensional dominance structures.

Multiple Scales.--We turn next to the simplest multidimensional situation--the case of multiple scales. Each item belongs to exactly one dimension, but not all items belong to the same dimension. These multiple scales may or may not be correlated. Factor analysis of measures of one-way association works very well for this case.

Table 5.3 shows the voting on two independent Guttman scales. Items A, B, C, and D form one scale, with the first two items reflected

TABLE 5.3 VOTING ON INDEPENDENT GUTTMAN SCALES

Individual	Item A	B	C	D	E	F	G
1	Yes	Yes	No	No	No	Yes	Yes
2	Yes	Yes	No	No	Yes	Yes	Yes
3	Yes	Yes	No	No	Yes	No	Yes
4	Yes	Yes	No	No	Yes	No	No
5	No	Yes	No	No	No	Yes	Yes
6	No	Yes	No	No	Yes	Yes	Yes
7	No	Yes	No	No	Yes	No	Yes
8	No	Yes	No	No	Yes	No	No
9	No	No	No	No	No	Yes	Yes
10	No	No	No	No	Yes	Yes	Yes
11	No	No	No	No	Yes	No	Yes
12	No	No	No	No	Yes	No	No
13	No	No	Yes	No	No	Yes	Yes
14	No	No	Yes	No	Yes	Yes	Yes
15	No	No	Yes	No	Yes	No	Yes
16	No	No	Yes	No	Yes	No	No
17	No	No	Yes	Yes	No	Yes	Yes
18	No	No	Yes	Yes	Yes	Yes	Yes
19	No	No	Yes	Yes	Yes	No	Yes
20	No	No	Yes	Yes	Yes	No	No

relative to the second two items. Items E, F, and G form a second scale, with item E reflected relative to the other two items. The corresponding matrix of one-way association coefficients is shown in Table 5.4. This matrix has a very clear structure. There are correlations of unity for items in the same scale with the same direction, correlations of minus one

TABLE 5.4 ONE-WAY ASSOCIATION COEFFICIENTS ON VOTES OF TABLE 5.3

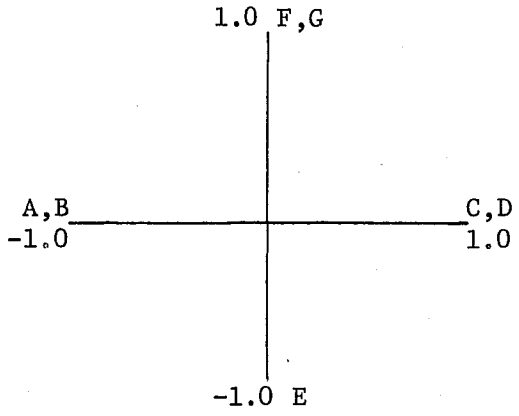
Item	Item A	B	C	D	E	F	G
A	-	1.00	-1.00	-1.00	0.00	0.00	0.00
B	1.00	-	-1.00	-1.00	0.00	0.00	0.00
C	-1.00	-1.00	-	1.00	0.00	0.00	0.00
D	-1.00	-1.00	1.00	-	0.00	0.00	0.00
E	0.00	0.00	0.00	0.00	-	-1.00	-1.00
F	0.00	0.00	0.00	0.00	-1.00	-	1.00
G	0.00	0.00	0.00	0.00	-1.00	1.00	-

for items in the same scale with opposite directions, and values of zero for items in different scales.

Recall the geometric interpretation of factor analysis. Items with correlations whose absolute value is unity are placed on the same factor while items with correlations of zero are placed on orthogonal dimensions. Items with correlations of minus one are placed on opposite ends of the same dimension while items with correlations of plus one are placed together on the dimension. Therefore factoring the above matrix would result in the same number of factors as scales--two. Each item would load on only a single factor, the sign of the loading indicating the polarity of the item. The factors would be orthogonal since the original scales were uncorrelated. Figure 5.4 plots the two factors against each other. It is easy to verify that items with correlations of unity are placed together on the factor, items with correlations of minus one are placed at opposite ends of the factor, and items with correlations of zero are on independent dimensions. It can also be seen in this figure that the loadings of the items indicate the scales to which they belong rather than their locations on the scales.

In general, multiple scales would be correlated. Therefore the one-way association coefficients between items in different scales would be non-zero. To state that two scales are not independent is equivalent to stating that not all items on one scale are independent of all the items on the other scale. The independence of a pair of items depends on the distribution of individual responses to those two items compared to the distribution expected on the basis of the item marginals. That is, the independence of items and therefore the independence of perfect

Fig. 5.4 Factor Analysis of Table 5.4



scales depends on the distribution of the individuals in the space. The degree of correlation between the scales of Table 5.3 depends on the distribution of individuals in the space or, equivalently, the relative frequencies of each of the listed response patterns. Changing the relative frequencies of these response patterns does not affect the unidimensional properties of the two scales but does affect the correlation of these scales. We can therefore explore the case of separate perfect scales which are correlated by just changing the relative frequencies of the response patterns of Table 5.3.

Table 5.5 shows the matrix of Yule's Q coefficients for the votes of Table 5.3 with the first response pattern occurring twice. Changing the frequency of this response pattern means that the two scales are correlated, as evidenced by the non-zero values of Yule's Q for items from different scales. However by the nature of measures of one-way association, all the correlations between items in different scales do not have the same magnitude even when the scales themselves are perfect. As seen in Table 5.5 the magnitudes of the correlations between items

TABLE 5.5 MATRIX OF YULE'S Q COEFFICIENTS FOR VOTES OF TABLE 5.3 WITH FIRST RESPONSE PATTERN REPEATED TWICE

	Item						
Item	A	B	C	D	E	F	G
A	-	1.00	-1.00	-1.00	-0.33	.20	.14
B	1.00	-	-1.00	-1.00	-0.20	.11	.08
C	-1.00	-1.00	-	1.00	.14	-0.08	-0.05
D	-1.00	-1.00	1.00	-	.11	-0.06	-0.04
E	-0.33	-0.20	.14	.11	-	-1.00	-1.00
F	.20	.11	-0.08	-0.06	-1.00	-	1.00
G	.14	.08	-0.05	-0.04	-1.00	1.00	-

in the two scales range from .04 to .33 as measured by Yule's Q when the first response pattern in Table 5.3 occurs twice.

The geometric interpretation of factor analysis relates the correlation between a pair of items to their angular separation, with allowance for their distances from the origin if not equal to unity. Since the correlations between items from the different scales are unequal in the correlation matrix of Table 5.5, the factor analysis of this matrix cannot yield a perfect two-dimensional solution with all the items the same distance from the origin on oblique axes. Logically this can be resolved in one or more of three ways--introducing one or more extra factors, adjusting the distances of the items from the origin, and not locating the items exactly on the oblique axes. While there may be some tendency to resolve the correlations by introducing extra factors, it would seem most unlikely that these factors would satisfy any reasonable criterion concerning the determination of the number of factors. Instead the items will be moved about in the two-dimensional space so as not to have loadings of one on one factor and zero on the other and so as not to have distances of one from the origin.

We shall not explore here the exact placement of the items further, except to indicate the implications of these results for the interpretation of the factor analysis. The factor analysis of the correlation matrix of Table 5.5 would yield the correct number of dimensions--two. Each item will have a high loading on one factor and a low (near-zero) loading on the other factor, with item reversals being appropriately indicated. Even with perfect scales, the loading of an item on its own factor will not be exactly equal to one and its loading on the other factor will not be exactly equal to zero. The factor loadings indicate the factor to which the item belongs only in a rough sense. Yet the correspondence is sufficient for correct conclusions to be reached. Each item will be associated with the correct factor. A comparison of the sum of squares of the factor loadings for each factor indicates the number of items on each scale as a rough gauge of its relative importance. The sum of squares of the factor loadings for a given item will be much less than unity only if it does not belong to any of the extracted scales. Thus the conventional interpretation of factor analysis will hold in an approximate sense but with item locations not being indicated by their loadings and without the variance interpretation.[5]

An additional problem which may occur with correlated multiple scales is the underestimation of the dimensionality. Say that there are three perfect scales which are correlated with each other. It may

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[5]The results described in the text correspond to the oblique simple structure rotation. The principal axis solution would be a rotation of this oblique simple structure so as to maximize the "proportion of the variance" for which each successive orthogonal factor accounts. The actual locations of the principal axes relative to the oblique axes depend on the relative number of items on each scale. Substantive interpretation of the principal axes should be extremely cautious.

happen that the correlations of the three scales are such as to be satisfied in just two dimensions. If the angular separation between two of the scales is approximately equal to the sum of the angles separating each of these scales from the third scale, two dimensions may suffice. Thus the number of correlated scales may be greater than the number of factors. Three clusters of points would emerge from the factor analysis. No rotation of the two axes could pass a separate axis through each of the three clusters. More axes are required than there are factorial dimensions. The points are located by the factor analysis to indicate the scales correctly, but even a rotation does not yield an identity between the scales and the factorial axes. One solution to this problem involves a clustering of the inter-point distances in the factor space, as suggested by MacRae and Schwarz (1968). In this three scale example, the distances in the two-dimensional factor space between items in the same scale would be small while the distances between items in different scales would be large, where the calculation of the distances is independent of the placement of the orthogonal axes. Clustering the matrix of inter-point distances would thus yield the three scales. Another solution might be to simply extract an extra factor. The oblique rotation of the three axes might then allow the axes to be uniquely identified with the scales. Whenever the conventional stopping criterion seems to find fewer factors than expected scales, one need just extract additional factors before rotating.

In the case of multiple scales the dimensionality and the dimensional attributes of each item can be obtained from factor analysis of the matrix of measures of one-way association, but the item ordering along

the dimensions can still not be obtained. There are techniques which can do all of these tasks for the cases considered so far. The Multiple Scalogram Analysis (MSA) procedure developed by James Lingoes (1963) is one such technique. MSA starts a scale with the most extreme item, then adds the closest item which has little error with respect to the other items in the scale. This process continues until the first scale is exhausted--no other items fit with the items already in the scale. Then the most extreme item not in the first scale is chosen to start a second scale and the process is repeated for the items not in the previous scale. This technique has been used for roll call analysis by both Lingoes (1962) and Clausen (1964).

MSA is a useful technique for verifying the minimum acceptability of a set of items which are thought to be unidimensional. It also allows for the exploration of multiple dimensions. However it restricts each item to a single dimension. Thus it is useful for the case of multiple scales, particularly when they are independent. It cannot cope with a complex multidimensional composition in which some items load on more than one dimension. This is an overly restrictive view of possible multidimensional compositions. Factor analysis permits items to load on more than one dimension and permits factors to be oblique without restriction, so that factor analysis should be a more powerful analysis procedure than techniques such as MSA.

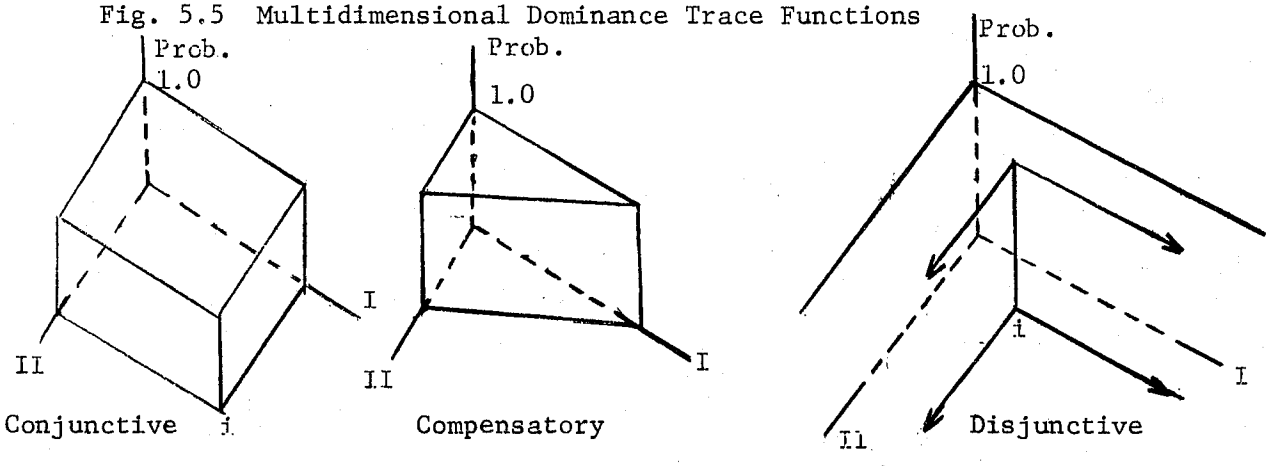
Other Multidimensional Models.--In the case of multiple scales considered above each item belonged to only a single scale though the distribution of people in the space might have caused the multiple scales to be correlated. We will now turn to more complicated multidimensional

compositions in which items belong to more than a single scale. We shall discuss scaling under these compositions, but first it is necessary to describe the composition rules in more detail.

The multidimensional composition rules of interest are the conjunctive, disjunctive, and compensatory rules. The differences between these various composition rules can be seen by examining the individual trace functions for each model. Figure 5.5 shows three possible trace functions in a two-dimensional space, with the height of the function at any given point representing the probability of the person accepting an item located at that point. The composition rule by which the dimensions combine is indicated on the bottom plane. The individual, according to the deterministic dominance interpretation, is certain to accept those items located between this line and the axes of this bottom plane. He is equally certain to reject those items outside this region.

The left-hand chart of Figure 5.5 represents the conjunctive model in which the individual accepts only those items which he passes on <u>all</u> dimensions. If the item is above point i on either dimension, individual i will vote against it. The right-hand figure shows the trace line for

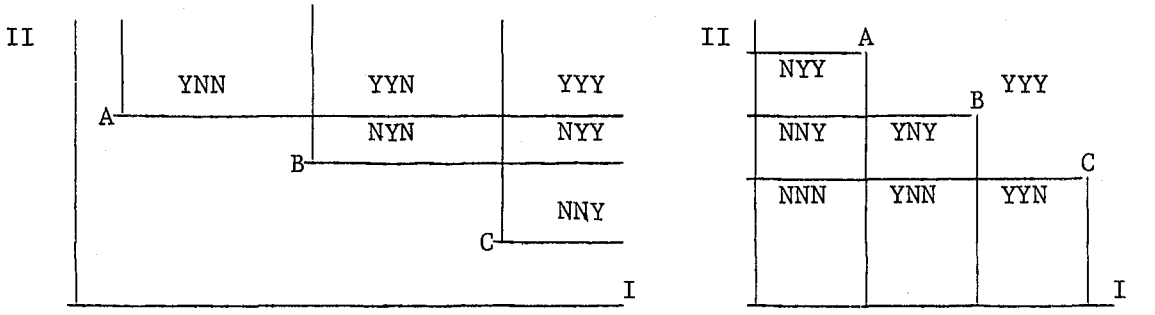
Fig. 5.5 Multidimensional Dominance Trace Functions



a disjunctive model. The person accepts the item if he passes it on any dimension. Individual i votes against only those items which are above point i on both dimensions. The center chart shows an intermediate trace function, the compensatory model. The individual will accept an item which he fails on one dimension if he passes it by a wide enough margin on the other dimension. A higher value on one dimension compensates for a lower value on the other dimension. The compensatory model requires the specification of the exact shape of the function by which the dimensions combine. One might recall from Chapter III that the linear compensation rule is the only composition rule compatible with the "derived dominance model," though all the composition rules are compatible with the "pure dominance model."

An important question is how the measures of one-way association reflect the existence of multidimensional compositions. To answer this question we will consider the case of three items in a two-dimensional space. In order to obtain the most general case, we require that the order of the projections of the points on the two axes be opposite. Figure 5.6 shows the conjunctive and disjunctive cases. The response pattern of the individuals within each region is signified by listing their votes on items A, B, and C in that order. The people in the upper right-hand corners of both diagrams, for example, vote yes on all three items because they dominate the item points according to the appropriate composition rule. Seven regions are defined in both cases. The YNY response pattern is missing from the conjunctive model and the opposite NYN response pattern is missing from the disjunctive model. In either case one would expect only one of the eight logically possible response patterns to be missing. The same

Fig. 5.6 Trivariate Multidimensional Compositions

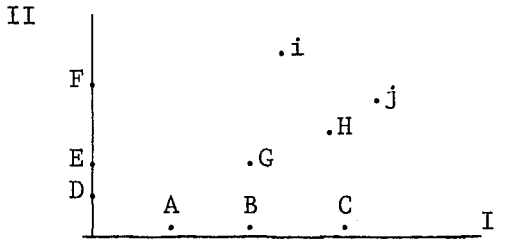


is true for the compensatory model (Coombs, 1964, pp. 264-265). Therefore bivariate cross-tabulations of pairs of the three items would not be expected to have any zero cells, even when the dominance model holds. Measures of one-way association would not be particularly sensitive to such compositions.

The analysis of the general multidimensional models is difficult, but a particularly interesting special case can be treated more easily. One can imagine a multidimensional space containing some clearly unidimensional scales with some items falling on more than one scale or only partially satisfying any single scale. The most common notion of multidimensionality probably does involve several unidimensional scales with a few items falling between them. The use of measures of one-way association leads to clusters of unities in the correlation matrix, corresponding to the unidimensional scales, with the items which do not belong completely to any single scale having intermediate correlations.

Figure 5.7 shows such a set of items in a two-dimensional space. Consider a conjunctive composition rule for this space. Each person will approve only those items which are both to the left of and below his position in the space. Person i will approve items A, B, D, E,

Fig. 5.7 Dominance in a Multidimensional Space



F, and G while person j will approve items A, B, C, D, E, G, and H. One of these individuals accepts item H while rejecting item F while the other has the exact opposite pattern of response to this pair of items. Thus items F and H will not have an empty cell in their cross-tabulation. Similar reasoning leads to the conclusion that if all consistent response patterns occur, measures of one-way association between items C and G, C and H, F and G, and F and H will be below unity. In other words, items which load on more than one dimension will have one-way association coefficients below unity with some of the items from the separate scales. An item can fit perfectly with two scales only if it falls exactly at their intersection. Strictly speaking, an item cannot be said to fall on more than one Guttman scale.

How exactly would the factor analysis of measures of one-way association work on the structure of Figure 5.7? A matrix of Yule's Q coefficients was calculated on Figure 5.7 with a uniform distribution of cases over the space. The resultant matrix is shown in Table 5.6. The two perfect scales are signified by clusters of unities in the matrix whereas the remaining two items have intermediate correlations with some of the items in the scales. Indeed factor analysis of this matrix yields two factors. The principal axis solution puts the first axis

TABLE 5.6 YULE'S Q COEFFICIENTS ON FIGURE 5.7

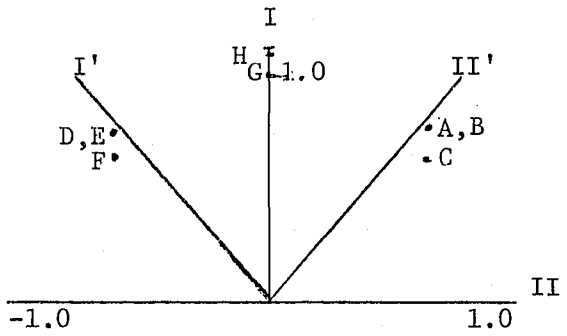
Item	Item A	B	C	D	E	F	G	H
A	-	1.00	1.00	0.00	0.00	0.00	1.00	1.00
B		-	1.00	0.00	0.00	0.00	1.00	1.00
C			-	0.00	0.00	0.00	0.56	0.71
D				-	1.00	1.00	1.00	1.00
E					-	1.00	1.00	1.00
F						-	0.56	0.71
G							-	1.00
H								-

through points G and H.[6] Figure 5.8 shows the more useful oblique axes along with the principal components.[7] Items A and B have the highest loadings on one of the oblique factors with item C having a slightly lower loading and H and G having much lower loadings. Items D and E have the highest loadings on the other oblique factor with item F having a slightly lower loading and H and G having much lower loadings. In each case the set of items having loadings near unity on the oblique factors form a perfect Guttman scale. Items G and H do not fit into either scale perfectly, but they are related to both scales. This is indicated by their intermediate loadings on the two factors. Note that the plot of Figure 5.8 does not preserve the relative locations of the items that are given in Figure 5.7. The factor analysis of measures

[6] Loadings on this first principal axis indicate item polarities. Items with loadings near zero on this first principal axis may have their polarities indicated by a later principal axis. Cf. MacRae and Schwarz (1968, p. 187).

[7] Because of the symmetry of the data and the independence of the two scales, an orthogonal varimax rotation gives approximately the same results as the oblique axes shown. The oblique rotation yielded non-independent axes due to the internal inconsistency of the correlation matrix. In general, oblique rotation seems more appropriate since the scales may not be independent.

Fig. 5.8 Factor Analysis of Multidimensional Space



of one-way association indicates the dimensionality and provides the dimensional specifications of the items, but does not show the locations of the items in the dimensional space.

The solution of Figure 5.8 carries with it all the difficulties which can occur when the correlation matrix is inconsistent due to the use of a correlation coefficient other than the product-moment. Some of the factors have negative characteristic roots. Items G and H have loadings greater than unity on one of the factors. Items A, B, D, E, G, and H have more than 100% of their variance explained by the two factors. Notwithstanding these violations of factor analytic theory, the representation of Figure 5.8 reveals very well the dimensional specifications of the items. The results do not show the locations of the items in the space, but they do yield the dimensionality and do indicate to which scale each item belongs. The classic cautions against factor analyzing coefficients other than the product-moment overlook the possibility of gaining such basic dimensional information.

As in the case of correlated multiple scales, one may obtain fewer factors than scales with these more complicated multidimensional compositions. Rotation of the axes is of little use in such a situation unless

one had intentionally over-factored. Inter-point distances can be calculated in the factor space and the matrix of inter-point distances can be cluster analyzed in order to obtain the scales (MacRae and Schwarz, 1968). There may be some advantage in then placing an axis through each of the clusters in order to judge the relationships among the clusters and between the clusters and the items not in any of the clusters. Items which are related to more than a single scale would have high loadings with respect to multiple axes. Items which are unrelated to all the scales will have near-zero loadings on all axes. Thus, as MacRae and Schwarz (1968) point out, clusters of items near the origin of the factor space should not be sought as they would not represent common scales. It might be noted that in the example in this section, a cluster analysis of the inter-point distances in Figure 5.8 would suggest the existence of three correlated Guttman scales. While this interpretation of the data is different from that intended, it is entirely reasonable given the actual response patterns underlying Figure 5.7.

Although further multidimensional compositions have not been explored formally, speculation as to the form of the general results is in order. I would expect this procedure of applying factor analysis to measures of one-way association to succeed with a conjunctive, disjunctive, or compensatory model when some of the items satisfy a Guttman scale. There will be clusters of values near unity in the correlation matrix which will be translated into clusters in the factor space. Other items which fall between scales will be given intermediate locations between the clusters. Unique items unrelated

to all the scales will be located near the origin of the space. In the most general case of a dominance model without subsets of items which scale unidimensionally, there would not be clusters of unities in the matrix so such unidimensional scales would not be separated out by the factor analysis. Hopefully the rough dimensional properties of the items would be recovered by a factor analysis of the type being proposed, but we lack the apparatus for a strict proof.

Relation to Cluster Analysis.--Throughout this discussion of the factor analysis of one-way association measures, we have relied on an intuitive form of cluster analysis together with an implicit claim of similarity between cluster analysis and factor analysis. If cluster analysis has so much in common with factor analysis, why not simply employ cluster analysis without the extra assumptions and the inconsistency problem associated with factor analysis?

Duncan MacRae (1965) has noted the ability to pull out separate Guttman scales through cluster analysis of such coefficients as Yule's Q. His basic clustering procedure, however, takes too restricted a view of multidimensionality in allowing an item to fall on only a single scale. The factor analysis procedure can instead show the extent to which each item belongs to each dimension. An ordinal or even interval level of measurement is assumed by most clustering procedures, as, for example, when a cluster is defined as a set of items having inter-correlations which are all above a value such as 0.8. The assumptions of factor analysis are not much more severe, but they yield much more information.

MacRae (1966a) has also applied Robert Tryon's clustering techniques (Tryon and Bailey, 1966) to legislative votes, again using Yule's Q coefficient. Let us quote MacRae's description of the technique employed:[8]

> ....The input data are a matrix of correlations or other coefficients of association. The program first searches for a suitable "pivot" variable for the initial cluster. It computes the variance of r^2 for each variable (row or column) in the matrix, and considers the variable with the highest variance as a possible pivot variable. If the program can find at least one other variable sufficiently similar to the first, it allows this group of two or more to define a cluster.
>
> Similarity between two variables is judged in terms of their patterns of correlation with all other variables in the matrix. It is measured by an index analogous to the product-moment correlation coefficient squared, differing only in that moments between two rows in the matrix are computed around zero. This index of "collinearity" is:
>
> $$p_{ab}^2 = \frac{(\Sigma r_{ai} r_{bi})^2}{\Sigma r_{ai}^2 \Sigma r_{bi}^2}$$
>
> For a pivot variable to be chosen, it must have one or more other variables related to it by values of P^2 of at least 0.4. If the trial pivot variable fails to pick up any partners, the variable with next highest variance of r^2 is tried; after four unsuccessful tries, factoring is terminated.
>
> After the identification of the first cluster, a factorial axis is passed through the composite location of the two or more items that define the cluster. Residual correlations are then calculated in the usual manner, as done in factor analysis, and cluster search is repeated for the residuals. Factoring may be terminated by a criterion dependent on exhaustion on the residual communalities, or exhaustion of the correlations...(page 633)

Here MacRae allows a fuller multidimensional view of the stimulus space, one which is more satisfactory than that permitted by the usual forms

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[8] MacRae uses the symbol "r" to represent the correlation coefficient, regardless of which coefficient is employed. Here "r" may be read as "Q".

of cluster analysis. The assumptions are, however, about as strong as those involved in the more conventional factor analysis. The variance of r^2 and the index of collinearity are both interval level measures. The use of residual correlations also employs the interval level properties of the coefficients of association and makes this form of analysis very similar to factor analysis. Thus it is unclear what advantages the Tryon procedure has as compared to the better-known factor analysis technique.

Duncan MacRae and Susan Borker Schwarz (1968) have provided an empirical comparison between the various factor analysis and cluster analysis procedures which have been described. Their data consist of roll calls cast by House Republicans in the 84th Congress. MacRae (1965) originally employed cluster analysis on a matrix of Yule's Q coefficients. Six clusters were obtained when it was required that all pairs of items in a cluster must have Q values above 0.8. MacRae (1966a) performed a cluster analysis of the same matrix using Tryon's BC TRY system. This analysis "confirmed [the] description of the issues [of the original cluster analysis] and added detail to it. In the results of [the BC TRY] analysis, the roll calls in Clusters 1-5 could be assigned positions as points in a three-dimensional space....In general the previous clusters found in the Q-matrix corresponded to groupings of points near one another..." (MacRae and Schwarz, 1968).

MacRae and Schwarz then factor analyzed the same Q matrix. Six factors were extracted in an attempt to match the six clusters found originally. An orthogonal varimax rotation was employed. The original six clusters were found related to three factors--three clusters to

one factor, two clusters to a second factor, and one cluster to a third factor. The foreign trade and foreign aid clusters, for example, load on the same factor. MacRae and Schwarz note, however, that the foreign aid items also have moderate loadings on a second factor on which the foreign trade items have low loadings which suggests that a separation of these two sets of items is in order. Two oblique rotation procedures did not solve this problem.

The matrix of distances between points in the six-dimensional factor space was then cluster analyzed, excluding the roll calls near the origin. Five clusters were obtained with the chosen cutting point (corresponding to a minimum Q value of 0.9 in the original clustering). These five clusters closely resembled the original clusters. MacRae and Schwarz suggest the original clusters were more homogeneous in content, but they do not indicate which set best approximates Guttman scales. They do mention at one point that an item relating to the debt limit was in the original foreign aid cluster, but was found to have a distinct pattern of factor loadings. This at least does support the present argument that factor analysis can explain the dimensional structure more fully than simple cluster analysis. MacRae and Schwarz do not report their results fully enough to permit a conclusion as to which technique is the superior method for dominance scaling, though it seems clear that one would be unlikely to obtain drastically different conclusions from the application of different of these techniques.

The difficulty that MacRae and Schwarz encountered in the identification of factor axes with scales might not indicate a serious flaw in the technique of factor analysis of measures of one-way association.

They extracted only six factors in an attempt to obtain six factors comparable to the original six clusters. The factors might have corresponded better to the clusters had the rotation included more factors. The need for clustering within the factor space may be due to under-factoring the correlation matrix.

Inter-Individual Analysis.--One can also factor analyze measures of one-way association calculated on inter-individual cross-tabulations. Examples of such work are the early Q analysis of roll calls with the tetrachoric coefficient by Carlson and Harrell (1942) and the recent Q analysis of roll calls with phi/phi max coefficients by Grumm (1963).

As stated earlier, the inter-individual cross-tabulation does not contain a zero cell even under a perfect Guttman scale unless all the items have the same direction. If one were to standardize the directions of all of the items, then the factor analysis of measures of one-way association on inter-individual relations would show the correct dimensionality on a perfect Guttman scale. Indeed in the unidimensional case when the items have standardized directions there will be a complete duality between R and Q analysis with the roles of the individuals and the items reversed. Thus the loadings of the individuals on the Q factors will not correspond to their order on the scale. With inter-item analysis, the scores of the individuals on the first factor indicate properly their order on an underlying Guttman scale. Correspondingly, for inter-individual analysis the scores of items on the first factor indicate properly their order on an underlying Guttman scale. This is one type of information which could not be obtained from R factor analysis of measures of one-way association.

However Q factor analysis yields the item ordering only in the unidimensional case. There is no duality between R and Q factor analysis for the multidimensional case. For example, it is very different for the votes to be decomposable into two independent unidimensional scales of three and four items each than for the votes to be decomposable into two independent unidimensional scales of say ten individuals each. That the votes can be decomposed into two independent Guttman scales does not imply that measures of one-way association calculated on the inter-individual cross-tabulations all have magnitudes of either zero or unity. Thus Q analysis of measures of one-way association need not in general yield the correct dimensionality and should probably be avoided.

Factor Analysis of Measures of One-Way Association.--Factor analysis of measures of one-way association based on inter-item cross-tabulations can provide the dimensional specifications of the items. The dimensionality can be obtained. Item reversals can be detected. Item orderings cannot be obtained, but individual orderings can be found by scoring individuals on the factors of an R analysis. In some cases the factoring may have to be supplemented by a cluster analysis of distances within the factor space, but in any case the Guttman scales can be extracted from the data.

The analyst who applies factor analytic methods to one-way measures of association in the presence of a dominance response model will use the sign and magnitude of his factor loadings to draw certain inferences as to the dimensionality of the space and the relations of the items to these dimensions. The analyst's inferences as to the dimensional properties will not be wrong, even though he will not be able to avail himself of several interpretations of his resulting statistics to which they would

lend themselves if he had analyzed a matrix of product-moment correlations that were reflective of inter-item distances. The inconsistency of the correlation matrix does not preclude the possibility of obtaining useful information from the factor analysis.

Once the dimensional specifications of the items are obtained, one may proceed to a positioning analysis. An efficient scale for positioning can be obtained through refinement of the dimensioning scale, as by removing items which do not nest well with the other items as indicated by their eta values.

SIMILARITY ANALYSIS OF MEASURES OF ONE-WAY ASSOCIATION

An important new technique for dimensional analysis is similarity analysis. Its first utilization in a roll call context was an analysis of United Nations roll calls by Hayward Alker (1967). MacRae and Schwarz (1968) also used this technique in their methodological exploration of the use of alternative analysis techniques on roll call votes. Because of the novelty of these ideas, at least within the political science discipline, we shall describe the basic procedures at some length before indicating how similarity analysis can be employed in dominance scaling.

Similarity Analysis.--Consider a correlation matrix. Say that the higher the correlation is between a pair of points in this matrix, the closer together the points should be in the underlying space. A pair of points with a very high correlation should be very near one another while a pair with a very low correlation should be far apart from each other. This is the monotonicity axiom on which similarity analysis is based. Similarity analysis is an appropriate analysis

technique whenever this monotonicity axiom is reasonable. If the size of the correlation coefficient is thought to depend on the inter-point distance, then similarity analysis can be used to analyze the correlation matrix.

Another fundamental concept in similarity analysis is the stress coefficient. Say that one has both a correlation matrix and some configuration for the points in a space. One can compute the inter-point distances in the space and can compare the order of these distances with the order of the inter-point correlations in the matrix. According to the monotonicity axiom, there will be a perfect inverse relationship between these two orders for the correct configuration. The stress coefficient is used to express how close any given configuration comes to satisfying this inverse order. It is a measure of goodness of fit to the monotonicity axiom.

The similarity analysis procedures are iterative. One starts with some configuration and computes the stress as compared to the correlation matrix. If the stress is too high, one seeks another configuration with lower stress. When minimum stress is reached with a configuration in a given dimensionality, a similar procedure is followed in another dimensionality. One continues until the lowest stress solution is found in a space of minimum dimensionality. Actually the goals of minimum stress and minimum dimensionality are conflicting--stress in general varies inversely with dimensionality. The determination of the proper dimensionality is therefore somewhat subjective. Typically this problem is not too severe as decreasing the number of dimensions by one usually has little effect on the stress until one goes below

the correct number of dimensions. Thus inspection of the stress values as a function of the dimensionality should lead to a proper determination of the dimensionality.

Similarity analysis thus employs only the ordinal properties of the correlation values. The monotonicity axiom itself is ordinal. This monotonicity axiom imposes a series of constraints on the locations to be assigned to the points. The simultaneous satisfaction of all of these constraints has the effect of virtually pin-pointing the location of each of the points. The use of ordinal assumptions thus leads to an order metric level or even stronger solution. Here lies much of the importance of this technique--it achieves a solution involving a high level of measurement while only making the assumptions of a low level of measurement.

We have been speaking of the monotonicity axiom as if it were to be employed throughout the entire correlation matrix. Although one could do so, a weaker assumption would sometimes be preferable. Monotonicity may be maintained only within rows of the matrix rather than throughout the matrix. This conditional form of the monotonicity axiom amounts to stating that the order of points in the space from a fixed point should be inverse to the size of their correlation with that fixed point. We shall refer to this latter form of the monotonicity axiom as conditional in contrast to the symmetric form originally described.

Similarity analysis developed mainly from the theoretic work of Roger Shepard (1962) and Joseph Kruskal (1964). Their computer program for the symmetric analysis of the square correlation matrix is the classic routine. MacRae and Schwarz used this program, MDSCAL, in their

study. Louis Guttman and James Lingoes have collaborated on the development of a similar set of procedures. Their programs, called smallest space analysis, provide for virtually every form of similarity analysis. Terry Gleason (1967) has performed the task of theoretical integration of the work on similarity analysis. His own program provides the maximum flexibility for conditional analysis. It should be added that the theory of similarity analysis specifies that any solution which satisfies the monotonicity axiom is correct, regardless of how it was obtained. This makes the exact choice of a program from the set of appropriate programs relatively unimportant from a theoretical standpoint.

The Type of Coefficient.--The logic of similarity analysis requires a correlation coefficient to be employed which can be construed as a distance analogue. However the simplicity of similarity analysis has caused analysts to apply it to a variety of correlation coefficients. Since the technique assumes only an ordinal level of measurement, it is thought that any of the wide class of ordinal correlation coefficients can be subjected to analysis in this way. It is also thought that similarity analysis represents a means of analyzing only the ordinal properties of the product-moment coefficient. Actually much care is required in the application of similarity analysis to coefficients other than pure distance measures.

One difficulty encountered regardless of the coefficient used has to do with item polarities. Similarity analysis interprets a minus sign as showing a greater inter-point distance than a plus sign. Reversals therefore are not handled as would usually be desired. One

must either give the proper direction to each item prior to the similarity analysis, subjectively or through utilizing the signs of the items on the first principal components of a factor analysis, or else one must be willing to argue substantively that the polarity concept is irrelevant in the given substantive application.

Measures of one-way and two-way association are sensitive to considerations of independence. Consider a two-dimensional space with point A falling upon one dimension and points B and C falling along the orthogonal dimension. Any coefficient sensitive to independence would find an association of zero between A and B and between A and C although A need not be equidistant from B and C. The inter-point distances from A to B and A to C would not be distinguished, though they need not be identical. Independence can thus be a confounding factor when such coefficients are analyzed by an algorithm which considers them pure distance analogues.

This independence argument is based upon a multidimensional space. If unidimensionality is present, a coefficient such as the product-moment can be considered a pure measure of inter-point distance so it can be used in similarity analysis. Of course one does not know the dimensionality of the space in advance, so this exception is of little practical advantage.

The similarity analysis of measures of one-way association raises several interesting questions. Consider a unidimensional scale with error. The value of the one-way association coefficient is inversely related to the amount of error between the pair of points. The similarity procedure would then interpret the coefficient as a distance

analogue--the higher the correlation, the less error and the smaller the inter-point distance. In point of fact the opposite interpretation might be more reasonable--the smaller the inter-point distance, the more error. One might actually prefer conducting a similarity analysis of measures of one-way association by construing the measures of one-way association as dissimilarity coefficients. From this perspective the use of measures of one-way association as distance analogues depends too much on the distribution of errors and therefore should be avoided.

MacRae and Schwarz (1968) provide an alternative justification of the similarity analysis of measures of one-way association. Similarity analysis can be conceived of as a non-metric form of factor analysis. We have seen that measures of one-way association can be factor analyzed, where the results indicate the dimensional attributes of the items rather than their locations in the space. By analogy, similarity analysis can be used to provide the same information with weaker assumptions as to the level of measurement of the coefficients.

This alternative justification of similarity analysis involves a reinterpretation of the monotonicity axiom--the higher the correlation, the more the items belong to the same dimension. This is an entirely reasonable interpretation of measures of one-way association.

The axes in such a similarity analysis have even less meaning than they had in the factor analysis of measures of one-way association. Again it is possible that there will be more clusters of points than dimensions, so that a rotation of the axes may not suffice for determining the scales. Additionally the origin of the space is arbitrary and a negative loading on a dimension does not indicate item reflections. As

a result there will be frequently one cluster of items with high positive loadings on an axis and another cluster of items with high negative loadings on the same axis. The usual factor analysis interpretation that this common axis can be interpreted by contrasting the content of the two clusters would be in error here. All that would be evident is that the clusters are far apart in the space so that very different dimensional considerations underlie each one. The inability to identify axes with scales even under rotation again suggests that a cluster analysis of the inter-point distances in the multidimensional space may be necessary (MacRae and Schwarz, 1968).

MacRae and Schwarz (1968) provide an empirical comparison between similarity analysis and the cluster and factor analysis procedures described previously. They applied Kruskal's MDSCAL routine to the same matrix of Yule's Q coefficients based on House of Representatives roll calls. They obtained a solution in six dimensions to permit comparison with their factor analysis and cluster analysis. Four of the original clusters can be clearly identified in the space by their loadings on the axes. MacRae and Schwarz then calculated inter-point distances between the points in the six-dimensional space and cluster analyzed the matrix of inter-point distances in a manner designed to correspond to the original cluster analysis with a minimum value of 0.9 required between all items in a cluster. Five of the same clusters were found as in the original clustering, with only minor additions and deletions of items. Once again MacRae and Schwarz do not report the gains or losses in terms of the scalability of the clusters. They do note that one of the original clusters was so heterogeneous as regards

the location of its items on the dimensions that it should not be regarded as an "identifiable issue." They seem to suggest that clustering the Q matrix with a minimum of 0.9 would be an equally acceptable approach to scaling this set of roll calls, though the similarity analysis approach avoids the arbitrary element in setting a minimum value for clustering.

It seems clear from the example of MacRae and Schwarz that similarity analysis of measures of one-way association is an appropriate means of obtaining information as to the dimensional structure of a set of items. The principal disadvantage of this method in contrast to the others previously discussed is the need to obtain the item polarities in advance, often making it necessary to perform both a factor analysis and a similarity analysis. As with the factor analysis of measures of one-way association, it may sometimes be necessary to perform a cluster analysis on the results of the similarity analysis in order to locate the main scales.

THE CHOICE OF ANALYSIS TECHNIQUE

Cluster analysis, factor analysis, and similarity analysis all can be used on measures of one-way association for the purpose of dimensioning the items. Which of these techniques is best has not yet been fully determined. The choice among these three techniques involves at least three questions--which best identifies the unidimensional scales among the set of items, which yields the least amount of distortion in its multidimensional representation of the data, and how much difference the choice of technique actually makes.

We do not know enough at this time to be able to state which technique best identifies the unidimensional scales among a set of items. Resolution of this question would require a determination of the scalability of the clusters of items which each technique yields and a comparison of the scalability values obtained. It may well be that one technique is biassed in favor of obtaining scales with little error while another technique is biassed in favor of obtaining slightly more broad scales. It may be postulated that cluster analysis will frequently insert items into the scale with large amounts of error because cluster analysis is insensitive to the multidimensional properties of the items. Factor analysis and similarity analysis, on the other hand, may be expected to yield somewhat more narrow and issue-specific scales.

Even less is known about the amount of distortion in the multidimensional representation of the data which each technique yields. There may be an inevitable degree of distortion when measures of one-way association are analyzed. The distortion may involve a slight misstatement of the degree of correlation between scales or it may involve the placing of certain items in locations somewhat distant from their correct locations. It may be possible to analyze such distortion effects mathematically. More likely, experimentation with known data structures may be necessary to determine the parameters governing distortions. Cluster analysis may be expected to yield the most severe distortions, particularly when the clustering criterion is that each item can belong to only a single cluster and when no distinction is made as to how much each item belongs to a given cluster. Similarity

analysis, on the other hand, uses the minimization of distortion as part of its scaling criterion and so may be expected to yield the most valid portrayal of the multidmensional space.

These techniques vary considerably in their ease and expense. Cluster analysis can actually be done by hand, resulting in the saving of considerable computer time if not human time. Similarity analysis is generally the most costly of these techniques, partly because it must generate separate solutions for each number of dimensions considered and partly because of the need to obtain item polarities by a procedure such as factor analysis prior to the use of similarity analysis. Given these relative cost considerations, the question of how much difference the choice of analysis technique actually makes is very important.

Further exploration is needed of the differences in results which these techniques give. It would be particularly useful to score the individuals on the scales obtained from each of these techniques and then to correlate the scale scores as a means of determining how much difference the choice of technique makes. We need to find out whether the correlations are high enough that wrong conclusions would not be obtained from using a technique other than that technique which is most satisfying on theoretic grounds. There might be considerable savings if such were the case. I would conjecture that such an analysis would find that cluster analysis does result in real differences from the other techniques, making it useful for a quick, cheap overview of data but not for a careful, systematic analysis. On the other hand factor analysis is likely to prove a less expensive substitute for similarity analysis, even if it yields a solution which is slightly less exact mathematically.

THE CHOICE OF THE CORRELATION COEFFICIENT

We have seen that analysis of measures of one-way association is an effective means of determining the dimensionality of items. While several analysts have utilized these techniques on the Pearson product-moment r, a measure of two-way association, such applications generally will not portray the dimensional properties correctly. Measures of one-way association measure the unidimensionality of an pair of dominance items in a less ambiguous manner than do measures of two-way association. Factor analysis and similarity analysis of these coefficients can, with a proper interpretation, indicate which sets of items are unidimensional.

We have not discussed which measure of one-way association should be used because the choice of statistic should be made together with the choice of analysis technique. Now that the various analysis techniques have been considered, we turn to the question of which measure of one-way association should be used with each technique. Five measures of one-way association were presented in Chapter IV--Yule's Q, Yule's Y, the tetrachoric r, phi/phi max, and eta. We shall now consider when each of these coefficients should be used.

When one is interested in developing or refining a scale for purposes of positioning the individuals, analysis of the eta statistic is appropriate. That statistic is particularly sensitive to the nesting of the passing sets of the items. When one wants to employ the scale to locate the individuals on the scale as a prelude to correlating the scale with other variables, one would desire each item to differentiate the individuals clearly instead of contributing considerable error. Analysis of the matrix of eta values should lead to a scale with the

desired properties. When cluster analysis procedures are employed, the minimum values should be set quite low because of the low values of this statistic. Further refinement of the scale can be achieved by examining the matrix of eta values for the scale and excluding items which cause exceptionally low entries in that matrix.

When one is more interested in determining the dimensional properties of the data, dimensioning, one would generally choose one of the other measures of one-way association. An important consideration in the choice between these remaining coefficients is that three of them--Yule's Q, Yule's Y, and r_t--are monotonic transformations of one another. If the correlation between one pair of variables is higher than the correlation between another pair according to one of these coefficients, the same will be true when either of the other two coefficients is employed. This means that for the purposes of dimensioning, the main choice is between a coefficient of this set and phi/phi max.

The stakes involved in the choice between phi/phi max and the other coefficients are not altogether clear. MacRae (1965) argues that Yule's Q involves a desirable weighting of errors by the distance between the pair of items. Wood (1968) suggests that phi/phi max is a better choice than Yule's Q since phi/phi max provides an indicator of reproducibility as well as correlation. It was pointed out in the previous chapter that phi/phi max can be extremely sensitive to error, which suggests that the use of the other measures of one-way association may be preferable. We shall concentrate on these other measures, though further analysis of the differences between these measures and phi/phi max is desirable.

Several criteria can be suggested for choosing between Yule's Q, Yule's Y, and r_t. One could choose between them on the basis of their assumptions. The assumptions behind Y and r_t are quite elegant, but they may not be appropriate with roll call data. This does not necessarily invalidate their use for descriptive purposes. These coefficients are not unbiassed estimates of certain other statistics if their assumptions are not met, but their possible use for descriptive purposes is a separate question. Indeed the monotonic relationship between them tends to suggest that their assumptions are not critical for descriptive purposes.

One could instead choose between these three coefficients on the basis of convenience. While Yule's Q is the simplest of these coefficients to compute, computational convenience is not a critical factor in the computer era. There are advantages in employing well-known coefficients. While the tetrachoric coefficient has been used in some legislative behavior research (Harris, 1948; MacRae, 1968), it is not as well-known as Yule's Q. Duncan MacRae has popularized Yule's Q and it has been used in several recent studies (MacRae, 1965, 1966a; Anderson, 1966; Clausen, 1967a). An additional advantage of Yule's Q is its ease in generalization to multiple category items. Goodman and Kruskal's gamma is the generalization of Yule's Q. Like other measures of one-way association, it has the value of one only under unidimensionality though a proof of this assertion would take us too far from the matters at hand. While convenience might thus suggest Yule's Q as the measure of one-way association, it is still best to consider the choice of coefficient within the context of the analysis technique.

Because of the monotonic relationship between Yule's Q, Yule's Y, and r_t, identical results are obtained regardless of which coefficient

is chosen for cluster analysis and similarity analysis. These techniques analyze only the order of the correlation values and this order is the same for each of these coefficients. The only adjustment which must be made in the analysis is that the minimum value used to define a cluster should be set according to the coefficient chosen. To require all inter-item Q values in a cluster to be above 0.8, for example, is identical to requiring all Y values to be above 0.5. When using cluster analysis or similarity analysis, the most convenient coefficient may as well be used.

Because factor analysis is a metric procedure, the choice of coefficient will directly affect the results of the factor analysis. Experimentation with different coefficients in the next chapter will suggest that oblique rotation yields higher correlation of the items with the factors but less clear separation of the factors when a high-valued coefficient such as Q is used. The use of a coefficient with intermediate values would yield results closer to the classical simple structure criteria of each item having a high loading on one factor and low loadings on all other factors. The need to discriminate scales may sometimes suggest the use of a low-valued coefficient such as Y, but the analysis of the low-valued statistics may often raise unnecessary doubt as to the integrity of the scales themselves. Above all when the analyst interprets the results of his factor analysis, he should realize the effect of his choice of coefficient on those results.

We must settle for a lack of closure regarding the choice of a measure of one-way association. No single coefficient is always best. Sometimes there are definite advantages in the use of one statistic,

such as eta for positioning. Other times the choice of coefficient has no effect, such as the identical results obtained from cluster analysis or similarity analysis of Yule's Q, Yule's Y, and r_t. The choice of coefficient does affect the interpretation of the factor analysis to some extent. Still it is most critical to realize the fundamental identity between the different measures of one-way association. Each measures the extent to which the items share a common dimension. The choice between these coefficients is much less important than the use of one of them rather than any other type of coefficient when the analysis technique assumes such a coefficient. Item dimensionality should be investigated through the use of measures of one-way association.

DOMINANCE ANALYSIS

In this chapter we have achieved several results concerning scaling under a dominance model. A brief review of the conclusions is now appropriate.

First, the product-moment coefficient does not lead to a valid analysis of the item dimensionality when a dominance model is at work. Factor analysis of that coefficient misstates the dimensionality while also giving a misleading view of the locations of the items. Similarity analysis of that coefficient is improper under multidimensionality since it is not a distance analogue. The product-moment coefficient may be the classic measure of correlation, but it is not particularly useful in the analysis of the item dimensionality under a dominance data structure.

Second, measures of one-way association capture the dimensional properties of a dominance structure quite well. R factor analysis of these coefficients indicates the dimensionality and the dimensional specifications of the items, though without indicating the locations of the items in the space. Scoring the individuals on the factors orders them in the space.

One particular measure of one-way association which measures the nesting relationship well is the eta coefficient. For positioning of individuals on a Guttman scale, this coefficient is a particularly powerful measure in indicating which items should be retained in a scale.

In general, the strategy for scaling under a dominance model should involve a combination of these procedures--factor analysis of measures of one-way association to ascertain the dimensionality and the scales and positioning by means of the nesting coefficient eta to treat the unidimensional scales.

CHAPTER VI

DIMENSIONS OF VOTING IN THE EIGHTY-FIFTH CONGRESS

In the previous chapters we have considered the methods for the analysis of legislative roll calls. We will now demonstrate the utility of these methods in an actual analysis situation.

Warren Miller and Donald Stokes have conducted a study of representation in the American Congress, employing interview material in a sample of Congressional districts with Congressmen and constituents as well as roll call votes. Their study provides an unusually rich setting for a consideration of roll call behavior, so the substantive data set chosen for the analysis of this chapter is that employed in the Miller and Stokes representation study--the roll calls taken in the United States House of Representatives during the 85th Congress (1957-58).

The distinctive feature of the Miller and Stokes study of representation is that they viewed representation within the dimensional context. They considered it likely that different modes of representation would hold in different issue areas and therefore they were concerned with obtaining measures in each issue area. They obtained attitude information from the constitutents and Congressmen on a number of items and then constructed Guttman scales for each of three issue areas for both sets of respondents. They also asked the Congressmen for their perceptions of the attitude of their constituency on

each of the issue areas. All together there were four variables--elite behavior as measured by roll call scales, elite attitude as measured by attitude scales, elite perceptions as measured by direct questioning, and mass attitude as measured by attitude scales. The dependent variable of elite behavior could be caused by elite attitude and/or by elite perception, with the latter two variables possibly being caused by mass attitudes.

Three issue areas were chosen in this study--social welfare, civil rights, and foreign policy. Their preliminary publication (Miller and Stokes, 1963) indicates that a different mode of representation was found in each of these issue areas. In the social welfare area the dominant mode is through the attitude of the Congressman. The constituency attitude is highly correlated with the attitude of the Congressman which is, in turn, highly correlated with his roll call behavior. In the civil rights area the dominant mode is through the perception of the constituency attitude. Constituency attitude is highly correlated with the Congressman's perception of that attitude which is, in turn, highly correlated with his roll call behavior. The foreign policy area conforms to neither of these patterns. Additional analysis indicates that voting in foreign policy depends on elite cues, such as willingness to follow the lead of the administration. The employment of the dimensional context permits the authors to find such important differences in behavior between different issue domains.

Such a study faces several problems in the construction of its roll call scales. First one must determine whether roll call behavior can indeed be divided into these three issue areas. In Stokes's terms,

does a dimensioning analysis of the roll calls indicate the existence of three such dimensions? Second, the individuals must be positioned along the three roll call dimensions. Furthermore this scaling must be performed in an explicitly multidimensional context.

This discussion will inevitably bring us to a consideration of the problems of item selection in scaling. This is not meant to call into question the item selection in the Miller and Stokes study, which is not of central interest here.[1] Rather, our interest is in determining the consequences of various means of item selection in such a multidimensional context. Their study is an ideal setting for an analysis of this matter as the consequences can be seen very directly in the various networks of intercorrelations. It should additionally be stated that the topic of item selection is not a trivial question. Scaling would present no difficulty if the item selection problem did not exist. It is because the item selection problem is magnified in multidimensional analysis that makes this particular set of data so interesting.

In accordance with the discussion of validation in Chapter II, we shall employ three related approaches in considering these roll call scales. We shall first examine each roll call scale individually, considering its properties as a scale. Second, we shall examine the multidimensional space of the three roll call scales together, considering the properties of this multidimensional space. Third, we

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[1]The roll call scales from their preliminary analysis are being used here. A thorough re-examination of the scales has been undertaken by the authors of that study as a part of their wider analysis.

shall examine these roll call scales within the broader context of the universe of votes taken during the 85th Congress.[2]

THE ROLL CALL SCALES

Civil Rights.--The civil rights area is meant to include items relating to the rights of Negroes. While this area is closely related to the more general area of civil liberties, it has been hypothesized that attitudes toward Negro rights can be measured separately.

One means of determining the meaning of an issue area, as a prelude to testing the validity of the corresponding roll call scale, is to observe the questions used by the investigators to determine Congressional and constituent attitudes. There is a degree of circularity involved here, determining the validity of one measure by reference to other measures whose validity is unknown. Nevertheless the investigators could choose attitude questions representing their understanding of the issue domain while having to settle for the roll call votes which were actually taken. This could make the attitude questions more representative of the issue area than the roll calls, but thereby makes the domain of the attitude questions somewhat different from that of the roll calls. The overall fit between the attitude questions in this study and the available roll call votes in the issue areas seems quite good, though the reader should make his own judgment on this matter.

Two questions were used to tap the civil rights issue area in the interviews with the constituents. One of these concerned the rights

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[2]We shall restrict our attention to scales including both parties in order to maintain compatibility with the Miller and Stokes analysis.

against discrimination in employment and housing and the other concerned the federal role in school desegregation. Three questions were asked of Congressmen relating to this same issue area. These involved their attitudes toward federal action to protect Negro voting rights, to prevent job discrimination, and to promote school desegregation.

The universe of civil rights votes taken during the 85th Congress is small. Five votes were taken relating to the Civil Rights Act of 1957 and one vote was taken to finance the Civil Rights Commission instituted under that act. The Civil Rights Act of 1957 did not directly legislate as regards discrimination in school, jobs, or housing. It did act to protect Negro voting rights. Perhaps its greatest importance was symbolic--it was the first civil rights legislation passed by Congress since Reconstruction. Thus the votes on this bill must certainly be considered to involve actions on civil rights, although their content is not identical with the questions used to tap constituent and Congressional attitudes on civil rights.

The six roll call votes were employed in a single scale in the original publication (Miller and Stokes, 1963). These items are shown in Table 6.1. The range of the passing sets of these roll calls is small. No other votes were taken during this Congress which directly involved civil rights, so this range cannot be improved.

The unidimensional properties of this scale are most striking. Its reproducibility is 0.99. The inter-item correlations, as measured by one-way association coefficients, are all very high. The lowest value of Yule's Q, for example, is 0.98. Factor analysis of measures of one-way association calculated on this scale yields a single factor

TABLE 6.1 85TH CONGRESS CIVIL RIGHTS SCALE

Bill	Year	CQ#	Content	Proportion "liberal"
HR 6127	1957	96	Adopt jury trial provision.	73%
HR 6127	1957	95	End debate on amendment involving jury trial.	72%
HR 10589	1958	22	Civil Rights Commission appropriation.	72%
HR 259	1957	40	Open rule for debate of Civil Rights Bill.	71%
HR 6127	1957	42	Passage of Civil Rights Act.	69%
HR 6127	1957	41	Recommit to modify jury trial provision.	61%*

*Reflected

accounting for an overwhelming proportion of the "variance."[3] All of the items have very high loadings on this factor. To give specific values, the factor analysis of Yule's Q coefficients yields a single factor accounting for over 99% of the variance and every item has a loading above 0.99 on this factor. The unidimensionality of these items cannot be doubted.

The civil rights scale is something of a paradox. On the one hand it is very clear that all of these items belong to the same dimension. On the other hand one is tempted to discount the scale because of the nearly equal passing sets. One could discard some of the items with passing sets of virtually identical size through the use of the eta statistic if the scale is to be used for positioning, but the overall level of error in the scale is so slight that this would have little effect.

[3]While the term "variance" is not appropriate when the coefficient analyzed is not the product-moment r, we shall quote variance figures in order to provide summary statistics on the relative importance of the factors.

The correlations between roll call behavior and the explanatory variables, Congressman's attitude and perception of constituency attitude, are higher for civil rights than for any other issue area in this study. Perception of constituency attitude has the highest correlation with roll call behavior--0.85. The correlation between the Congressman's own attitude and his score on the civil rights roll call scale is 0.74. Employing both explanatory variables in a multiple regression equation yields a multiple correlation coefficient of 0.89. These high values again attest to the validity of this scale as a measure of civil rights position.

Social Welfare.--The second issue area of the Miller and Stokes study is domestic social and economic welfare. The questions used to tap this area in the constituent interviews included questions on the role of the federal government with respect to electric power and housing, employment, and building public schools. Congressmen were asked their views on public housing, public works and full employment, public atomic power, and federal aid to education.

A very large number of roll calls could be coded into the social welfare area. Ten items were employed in a roll call scale in this area in their original publication (Miller and Stokes, 1963) as shown in Table 6.2 These roll calls relate to the debt limit, aid to education, housing, depressed areas aid, school construction, and appropriations to the Departments of Labor and Health, Education, and Welfare (HEW). There is a wide range of item marginals on these questions, in sharp contrast to the constricted range for the civil rights items.

TABLE 6.2 85TH CONGRESS SOCIAL WELFARE SCALE

Bill	Year	CQ#	Content	Proportion "liberal"
HR 9955	1958	3	Passage of $5 billion debt limit increase.	82%
HR 675	1958	73	Open rule for National Defense Education Act.	71%
S 4035	1958	80	Passage of Housing Act under rules suspension.	65%
HR 13247	1958	74	Recommit Defense Education Bill.	61%*
HR 682	1958	78	Open rule for depressed areas aid.	57%
S 3683	1958	79	Recommit depressed areas resolution.	52%*
HR 1	1957	56	Kill School Construction Bill.	49%*
HR 6287	1957	17	Cut Labor Department appropriation.	49%*
HR 6287	1957	19	Cut unemployment funds for federal employees.	40%*
HR 6287	1957	20	Cut Mexican farm labor program funds.	19%*

*Reflected

This ten item social welfare scale has a reproducibility of 0.94. The factor analysis of measures of one-way association on this scale yields a single factor. This scale is noticeably weaker than the civil rights scale. All of the social welfare items, for example, have a lower loading on the common factor than did the civil rights items on their factor. Still there is little doubt that most of the social welfare items belong in this scale. In the factor analysis of Yule's Q coefficients, the single factor accounts for nearly 88% of the "variance" with the lowest item loadings on the factor being 0.88 for the debt limit vote and 0.92 for the Mexican farm labor program item. Since Yule's Q is a high-valued statistic, these two loadings may be considered somewhat marginal.

Inspection of the inter-item eta values indicates values below 0.10 among the fifth, sixth, and seventh items in the scale. These

low eta values are at least partially due to the nearly equal passing sets for these items. The Labor Department appropriation item and, even more so, the school construction item have low average eta values with the other items in the scale. It would seem that these items might profitably be dropped for purposes of positioning the legislators.

The social welfare roll call scale has sizable correlations with the predictor variables. Its correlation with the Congressman's own attitude is 0.69 and its correlation with his perception of his district is 0.49. The multiple correlation coefficient of these two predictor variables with the roll call scale is 0.72. This value indicates a good level of correspondence between the content of the social welfare domain as measured in the interviews and by the selected roll calls.

This review of the internal validity of the social welfare scale suggests that some modifications may be in order. The debt limit item has a relatively low loading on the common factor. In content terms, the size of the debt limit would seem to have a greater relation to economic policy than social welfare policy. The school construction roll call has a weak relationship with some of the other items as measured by its nesting properties. In content terms, the issue of school construction in the 1950's had as much to do with the area of civil rights as with the area of social welfare.

One means of comparing the validity of alternative scales within an issue area is to compare their multiple correlations with the two predictor variables. The roll call scale with the highest multiple correlation is most likely to tap the same content dimension. The use of this test does not bias the determination of the path of greatest influence on the Congressman's roll call behavior.

In an attempt to determine the effects of modifications of the social welfare scale, Congressmen were scored on roll call scales which deleted the debt limit and school construction items. The deletion of either of these items increased the multiple correlation with the predictor variables. The highest multiple correlation was obtained when both of these roll calls were removed. In this last case, the multiple correlation coefficient is 0.75, compared to an original value of 0.72.[4] This gain in the multiple correlation obtained by dropping the two items is nearly as large as the gain in using both predictor variables instead of only the single most powerful predictor.[5]

Foreign Policy.--The social welfare and civil rights scales are both adequate, though not perfect. Modifications and refinements in these scales may be advisable, but there is no doubt that these scales measure their respective issue areas. The acceptability of the foreign policy scale is less clear. Indeed the nature of the foreign policy issue area is open to question.

The questions relating to American foreign policy which were asked in the constituency sample involved attitudes on American involvement abroad, economic assistance to poor countries, and the maintenance of American troops overseas. Congressmen were asked about foreign economic aid, foreign military aid, aid to underdeveloped countries which are neutral, and the overall level of American foreign commitments.

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[4]Correlation is being measured here in terms of the Pearson product-moment coefficient. As a result the differences from the original results may be due to the reduction in the number of categories in the dependent variable.

[5]This change in the scale does not alter the original conclusions as to the relative importance of the various paths of influence.

Congressional responsibility in the area of foreign policy is less than its responsibility in the areas of civil rights and social welfare policy and this is particularly true of the role of the House of Representatives. The legislative output in the area of foreign policy may consist as much of committee hearings as of roll call votes. The foreign policy roll call votes which do occur in the House tend to center on the issues of mutual security (foreign aid) and trade policy. Rarely is the House able to vote directly on the nature and extent of American involvement abroad. This makes it difficult to obtain roll call scales in the foreign policy area which match with the foreign policy attitude scales.

There were 26 roll call votes taken in the House during this Congress which had some direct bearing on foreign policy. Five of these were included in the original scale, as shown in Table 6.3.

The reproducibility of this five item scale is 0.95. The factor analysis of measures of one-way association calculated on the items of this scale yields a single factor. With Yule's Q as the coefficient, this factor accounts for just over 87% of the "variance." The items with the lowest loadings on this factor are the cut for international organizations (0.88) and the item dealing with atomic energy (0.90). An equivalent statement is that the three mutual security items in this scale have the highest loadings on the common factor.

An inspection of the nesting properties of the passing sets indicates a lack of cumulation between the third and fourth items in the scale. Their eta value is a very low 0.08. In fact the fourth item, funding for international organizations, has a lower average

TABLE 6.3 85TH CONGRESS FOREIGN POLICY SCALE

Bill	Year	CQ#	Content	Proportion "liberal"
HR 8922	1957	70	Congress not required to approve International Atomic Energy Agency transfers of fissionable materials.	75%
HR 13192	1958	56	Passage of Mutual Security Act.	65%
S 2130	1957	79	Adoption of conference report on mutual security.	58%
HR 6871	1957	30	Cut funds for international organizations.	55%*
HR 9302	1957	81	Recommit Mutual Security Act to restore cut.	34%

*Reflected

value of eta with the other items in the scale than was the case for any other item in the three roll call scales. There is serious doubt as to the utility of this item for purposes of positioning individuals on a foreign policy scale.

The correlations with predictor variables are somewhat lower in the foreign policy area than in the other issue areas. The correlation of roll call behavior with the Congressman's attitude is 0.56 and the correlation with his perception of the district attitude is 0.55. Using both variables as predictors leads to a multiple correlation coefficient of 0.63. This values indicates a degree of fit with the foreign policy domain as defined in the interview questions, but the degree of fit is limited.

There are a variety of means of constructing alternative foreign policy scales, three of which will now be considered. The simplest is to drop the international organization item from the original scale and use a four item scale. A second possibility is to scale the mutual security items and see whether mutual security alone is an adequate

measure of behavior on foreign policy. The third approach is to analyze the full set of 26 foreign policy roll calls taken during this Congress to determine their dimensional properties.

We begin by examining the effects of removing the international organization roll call from the original foreign policy scale. Removing this item results in a substantial increase in the reproducibility of the scale--from 0.95 to 0.98 for the sample of representatives in the study. Also there is a marked increase in the correlation of the roll call scale with perception of the district (from 0.55 to 0.62) and a corresponding increase in the multiple correlation coefficient with the two predictors (from 0.63 to 0.68).

Two considerations motivate against the use of this reduced scale even though it is a clear improvement over the original version. First, given the existence of 26 foreign policy roll call in the 85th Congress, the use of a scale with only four items must be questioned. That the four item scale is better than the original five item scale does not necessarily imply that the four item scale is an adequate measure of the full foreign policy area. Second, three of the items in this reduced scale concern mutual security so this scale may be measuring the specific area of mutual security voting more than the general area of foreign policy. A scale which measures mutual security voting is not necessarily a valid measure of behavior on the full foreign policy area.

It is evident that mutual security legislation was one of the main foreign policy concerns of the 85th Congress. Ten of the 26 foreign policy roll calls concerned mutual security. A unidimensional scale can be obtained from these ten items, as shown in Table 6.4.

TABLE 6.4 85TH CONGRESS MUTUAL SECURITY VOTES

Bill	Year	CQ#	Content	Proportion "liberal"
HR 9302	1957	82	Passage of Mutual Security Appropriations of 1958.	64%
HR 13192	1958	56	Passage of Mutual Security Appropriations of 1959.	65%
HR 12181	1958	31	Passage of Mutual Security Act of 1958.	65%
HR 12181	1958	51	Recommit Mutual Security Act of 1958 to conference.	63%*
S 2130	1957	54	Passage of Mutual Security Act of 1957.	62%
HR 9302	1957	100	Conference report on 1958 appropriations.	59%
S 2130	1957	79	Conference report on 1957 Mutual Security Act.	58%
S 2130	1957	53	Recommit 1957 Mutual Security Act to delete Development Loan Fund.	55%*
HR 13192	1958	55	Recommit 1959 Appropriations to increase defense funds.	44%
HR 9302	1957	81	Recommit and cut 1958 appropriations.	34%

*Reflected

The reproducibility of this scale is high--0.975. Factor analyzing measures of one-way association on this scale yields a single factor accounting for a large proportion of the "variance." Using tetrachoric r as the coefficient, this factor accounts for 93% of the "variance." The only item with a loading below 0.95 on this factor is the one dealing with defense funds which has a value of 0.86. There are several sets of items in the scale with nearly identical marginals. Correspondingly there are some low inter-item eta values, suggesting that such items as the 1959 appropriations bill passage might be deleted for positioning.

The correlations between the ten item mutual security scale and the predictors are larger than those of the original five item foreign

policy scale. The correlation with perception of district is 0.60, the correlation with own attitude is 0.57, and the multiple correlation is 0.67. The latter may be compared to the 0.63 multiple correlation found for the five item foreign policy scale.

The ten item mutual security scale might not be the best measure of foreign policy behavior, but it is important to realize that it is a reasonably good measure. The measure may be improved by dropping some items and there may be some advantage in broadening the scope from a narrow concern with mutual security. The multiple correlation with the predictors is not quite as high for the mutual security scale as for the reduced four item foreign policy scale considered previously. Yet the restriction of the scale to mutual security items does not seriously damage its relation to the predictors. Mutual security voting is so much the core of House voting on foreign policy that it may be as good an indicator of foreign policy position as any measure available.

This directly raises the question of the structure of foreign policy voting in the House. Using a broad definition of foreign policy, the House voted on 26 roll calls concerning foreign policy during the 85th Congress. Table 6.5 shows the oblique factor structures for the 23 of these items on which there was at least 10% dissent from the majority position, the tetrachoric r being used as the measure of association.

Four factors were obtained. It can be seen from Table 6.5 that the ten mutual security roll calls have their highest values on one factor, the two trade votes have their highest on the second factor, and the three votes on the Mid-East Doctrine have their highest on a

TABLE 6.5 OBLIQUE FACTOR STRUCTURE OF FOREIGN POLICY ROLL CALLS

Year	CQ#	Content	I	II	III	IV
1957	82	Mutual Security Appropriations of 1958	.86	.60	.64	.53
1958	56	Mutual Security Appropriations of 1959	.84	.67	.65	.55
1958	31	Mutual Security Act of 1958.	.85	.66	.66	.53
1958	51	Recommit Mutual Security Act of 1958.*	.83	.65	.63	.52
1957	54	Mutual Security Act of 1957.	.90	.56	.65	.48
1957	100	Conference report on 1958 appropriations.	.88	.58	.62	.53
1957	79	Conference report on 1957 Mutual Security Act.	.89	.55	.62	.51
1957	53	Development Loan Fund.*	.90	.48	.63	.48
1958	55	Increase defense funds.	.92	.29	.36	.21
1957	81	Cut 1958 mutual security appropriations.	.99	.24	.56	.25
1958	40	Recommit reciprocal trade legislation*	.38	.94	.36	.34
1958	41	Pass reciprocal trade legislation.	.37	.95	.37	.35
1957	2	Rule to consider Mid-East Doctrine.	.47	.23	.94	.12
1957	3	Pass Mid-East Doctrine.	.50	.36	.98	.32
1957	6	Accept Senate version of Mid-East Doctrine.	.55	.38	.98	.34
1957	28	Postpone payment on British loans.	.45	.64	.56	.65
1957	30	Cut funds for international organizations.*	.69	.56	.41	.52
1957	65	Suspend rules on annuity to Panama.	-.06	.49	.29	.74
1957	70	International Atomic Energy Agency transfers.	.71	.29	.49	.58
1957	98	Revise immigration laws.	.90	.27	.30	.04
1958	5	Suspend import duties on coarse wools.	.23	.19	.13	.76
1958	49	Authorize additional Assistant Secretary of State.	.77	.48	.44	.40
1958	86	Cut U.S. Information Agency funds.	.77	-.11	.22	-.12

Factor				
I	1.00	.27	.41	.22
II		1.00	.37	.45
III			1.00	.32
IV				1.00

*Reflected

third factor. In each of these cases there is some variation in the location in the space ascribed to the items, but the variation is minimal. The remaining eight items have less of their "variance"

explained by the four factors than do any of the mutual security, reciprocal trade, or Mid-East Doctrine roll calls. Three of these eight items have their highest values on a fourth factor. The other five, including two items from the original foreign policy scale, have their highest correlations with the mutual security factor.

The foreign policy area is thus seen to be composed of more than one dimension. Reciprocal trade, for example, involves a quite different dimension than does mutual security. The voting on the Mid-East Doctrine in 1957 involved a third dimension. This analysis does not prove that mutual security voting represents a totally distinct dimension. Indeed careful analysis may suggest that certain of the remaining foreign policy roll calls should be placed together with the mutual security items on the same scale. Yet it is clear from Table 6.5 that mutual security legislation represents the core of one dimension of Congressional voting on foreign policy. All five items of the original foreign policy scale are related to this dimension.

THE MULTIDIMENSIONAL SPACE

We have now considered the unidimensional properties of the three roll call scales of the Miller and Stokes study. We will next examine their multidimensional properties. The procedure to be employed is a factor analysis of the items in all three scales together using measures of one-way association.

Table 6.6 shows the factor structure obtained under oblique rotation, with three alternative measures of one-way association. The item ordering corresponds to that of Tables 6.1, 6.2, and 6.3 above.

TABLE 6.6 OBLIQUE FACTOR STRUCTURE OF REPRESENTATION ROLL CALLS

Scale & Item	Yule's Q			Tetrachoric r			Yule's Y		
	I	II	III	I	II	III	I	II	III
CR 1	.98	.37	.55	.98	.37	.47	.94	.30	.35
CR 2	.99	.37	.53	.98	.37	.44	.95	.30	.32
CR 3	1.00	.37	.48	.99	.37	.40	.92	.31	.28
CR 4	1.03	.35	.33	1.01	.35	.26	.97	.29	.17
CR 5	1.01	.36	.41	1.01	.36	.33	.99	.29	.23
CR 6*	.98	.39	.50	.98	.38	.41	.97	.31	.29
FP 1	.54	.71	.65	.45	.71	.55	.30	.62	.41
FP 2	.58	.82	.77	.47	.89	.66	.31	.88	.49
FP 3	.62	.85	.71	.52	.92	.59	.35	.92	.42
FP 4*	.47	.65	.80	.37	.68	.72	.23	.62	.56
FP 5	.81	.96	.26	.74	.94	.16	.57	.86	.06
SW 1	.53	.68	.83	.43	.72	.75	.28	.67	.60
SW 2	.55	.46	.91	.46	.48	.87	.31	.42	.76
SW 3	.01	.39	.98	-.02	.37	.94	-.04	.28	.84
SW 4*	.48	.47	.94	.39	.49	.91	.26	.42	.81
SW 5	.35	.27	.98	.28	.29	.96	.17	.28	.88
SW 6*	.43	.28	.97	.35	.30	.95	.22	.24	.87
SW 7*	.77	.38	.90	.71	.40	.85	.55	.36	.74
SW 8*	.51	.22	.95	.43	.25	.92	.30	.22	.82
SW 9*	.46	.26	.95	.37	.29	.92	.25	.24	.81
SW 10*	.49	.15	.95	.40	.22	.90	.28	.22	.78
Factor									
I	1.00	.31	.37	1.00	.30	.30	1.00	.24	.20
II		1.00	.31		1.00	.31		1.00	.26
III			1.00			1.00			1.00

*Reflected

Regardless of the coefficient chosen, each factor can be identified with a single scale. Factor I is a civil rights factor, factor II a foreign policy factor, and factor III a social welfare factor. All civil rights items have their highest correlations with the civil rights factor. While the social welfare factor also stands out strongly, some of its items have substantial relationships to other factors. The debt

limit increase actually has a higher correlation with the foreign policy factor than the social welfare scale when Yule's Y is used. The school construction item has a rather high correlation with the civil rights factor. The unidimensional analysis of the social welfare scale earlier in this chapter similarly suggested that these two items are weak links in the social welfare scale. The association of the debt limit increase with the foreign policy scale not only calls into question its placement in the social welfare domain but also calls into question the meaning of the foreign policy scale. The foreign policy roll calls tend to have high correlations with more than one factor. When Yule's Q or tetrachoric r is used, the vote on cutting the funds for international organizations has a higher correlation with social welfare than with the foreign policy factor. It might also be noted that the fifth foreign policy item, a motion to recommit a mutual security bill to restore a cut, has a different pattern of values than the other foreign policy items. Thus the severe problems in the foreign policy scale which were seen earlier are reflected in this multidimensional analysis.[6]

Three different correlation coefficients have been employed for Table 6.6 in order to give a better feel for the stakes involved in the choice of a coefficient. Yule's Q has the highest value of these coefficients and Yule's Y has the lowest. Correspondingly the correlations of the items with the factors are generally greatest for Q and lowest for Y. Simultaneously Y provides the clearest factor separation

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[6] An alternative means of gauging the fit of the factors with the original scales is to rotate the factor matrix into a binary target matrix which associates each item uniquely with its scale. Two means of target rotation were employed and both confirm the results described here.

whereas the item correlations with each factor are most nearly equal with Q. Yule's Y provides the best discrimination between the scales, but it raises questions as to the integrity of the scales themselves. The correlations of the foreign policy items with factor II and even the correlations of the social welfare items with factor III are sufficiently below unity when Y is analyzed to call the actual existence of these scales into doubt. Tetrachoric r yields results closer to Thurstone's simple structure criteria--each item having a high loading on one factor and low loadings on the others. The separation of the scales is somewhat less distinct but their integrity is confirmed. Still since these coefficients are monotonic with one another, one would obtain similar basic conclusions from the use of any of these measures of one-way association. While there are occasional differences as to the factor with which an item has the greatest correlation, it remains clear as to which items are marginal in the extent to which they belong to a given scale. Different investigators may disagree as to which measure of one-way association they prefer, but they should obtain essentially similar conclusions.

In the discussion of the individual scales, it was suggested that the mutual security items could serve as an effective measure of foreign policy position. This at least raises the question of whether the mutual security items adhere more to a single factor in the multidimensional space that do the foreign policy scale items. Table 6.7 shows the results of a factor analysis of tetrachoric r's for all the items from the civil rights, social welfare, and mutual security scales.

TABLE 6.7 OBLIQUE FACTOR STRUCTURE OF MUTUAL SECURITY AND OTHER SCALES

Scale & Item	I	II	III
CR 1	.97	.50	.43
CR 2	.97	.50	.41
CR 3	.98	.52	.36
CR 4	1.01	.49	.22
CR 5	1.00	.51	.29
CR 6*	.96	.55	.38
MS 1	.46	.95	.57
MS 2	.41	.96	.59
MS 3	.40	.97	.60
MS 4*	.38	.96	.58
MS 5	.50	.97	.53
MS 6	.43	.98	.53
MS 7	.46	.98	.52
MS 8*	.61	.93	.43
MS 9	.77	.83	.12
MS 10	.69	.96	.09
SW 1	.38	.81	.69
SW 2	.41	.58	.85
SW 3	-.07	.46	.94
SW 4*	.34	.56	.89
SW 5	.23	.41	.96
SW 6*	.30	.42	.96
SW 7*	.67	.56	.84
SW 8*	.39	.40	.92
SW 9*	.33	.42	.92
SW 10*	.34	.45	.89
Factor			
I	1.00	.38	.21
II		1.00	.37
III			1.00

*Reflected

The factor analysis of the civil rights, mutual security, and social welfare scales also yields three factors, each corresponding to one of the scales. All of the civil rights items have their highest values on a single factor. The social welfare items cohere to a second scale,

though the item relating to the debt limit has its highest value on the mutual security factor and the school construction item has a high value on the civil rights factor. These findings confirm the previous results concerning the social welfare scale. All of the mutual security items have their highest correlations with the second factor, though some, particularly the item concerning defense assistance, also relate to the civil rights scale. Thus the mutual security items do have greater coherence in the multidimensional space that the original foreign policy scale items.

The utility of factor analysis of measures of one-way association should now be reviewed. Four factor analyses of single Guttman scales have been reported along with two factor analyses of three Guttman scales and one analysis of a set of items of unknown dimensionality. In each instance the analysis yielded a reasonable number of factors. The correct dimensionality was found where it was known; the nature of the four factors obtained in the case of unknown dimensionality suggests that three Guttman scales exist in that instance along with a few unrelated items. We found in the last chapter that this form of factor analysis can underestimate the dimensionality, but that danger has been avoided in the analyses reported so far.

In addition the factor analysis of measures of one-way association reveals many of the properties of the items. The signs of the loadings on the first principal component indicate the item polarities correctly. One finds which items belong to which scales and which items have ambiguous dimensional properties.

Factor analysis suggests the basic dimensions, but the problem of interpretation is still that of the analyst. We find in Table 6.7, for example, that the debt limit increase roll call has a closer relationship to the mutual security factor than to the social welfare factor. This suggests that the debt limit item should be removed from the social welfare scale. It was indicated earlier that doing so increases the relation of the social welfare scale to the predictor variables. This also suggests that the debt limit increase has something in common with the mutual security issue area. Both explicitly involve economic matters. Thus it is possible that the mutual security roll call scale (and the foreign policy roll call scale as well) really measure economic policy rather than foreign policy concerns. This logic would also explain the relatively low correlations of the mutual security (and foreign policy) scales with the predictor variables--the predictor variables concern foreign policy more than economic policy. The factor analysis of the scales cannot determine the true meaning of the mutual security scale, but it does serve the important function of calling into question that very meaning.

VOTING IN THE EIGHTY-FIFTH CONGRESS

One final means remains by which to examine the validity of the three roll call scales--to examine the universe of roll call votes taken during the 85th Congress. How many dimensions were there in the voting? How do these dimensions relate to those chosen in the representation study? Do the representation study scales accurately reflect the dimensions of conflict in the 85th Congress? Can the larger universe of items indicate more clearly the meaning of the mutual security scale?

The number of roll call votes taken in the House of Representatives during the 85th Congress was 193. A factor analysis of 193 variables would be very expensive, if it could even fit within the capacity of available computer programs. Therefore it is necessary to cut down the number of items.

Some of these roll call votes are virtually unanimous while others have large amounts of missing data. The correlation matrix of even the Pearson product-moment coefficient may be internally inconsistent when there is missing data. The correlation matrix with measures of one-way association is usually internally inconsistent; the inclusion of items which are virtually unanimous and which have large amounts of missing data further exacerbate the problem. While we have argued that the internal inconsistency problem does not invalidate the factor analysis of measures of one-way association, it seems prudent to keep the extent of this problem in bounds by excluding items which are virtually unanimous or which have large amounts of missing data.

For the analysis to be reported here, votes were excluded if 15% of more of the 444 Congressmen who served in this Congress have missing data. This amounts to an exclusion of roll calls in which 13 1/4% or more of the 435 members who served at any moment of time did not indicate their position. Additionally roll calls supported by 91% or more of those voting and roll calls supported by 9% or less of those voting were dropped from this analysis. These tests may seem stringent, but I believe that even more stringent requirements might well have been established without affecting the results. Of the total 193 roll calls, 140 survived these tests.

The treatment of party in this analysis bears special mention. Members of both parties were included in the analysis to maintain comparability with the Miller and Stokes study. Additionally it was decided that party line roll calls would not be excluded. Roll calls were kept in the analysis even if most members voted by party. In fact the vote for the Speaker of the House, a roll call which is completely determined by party, was included in the analysis. It was felt that the inclusion of these party roll calls would allow a better evaluation of the role of party in the overall voting. Other studies have deleted items which follow party lines. The fact that they must set arbitrary limits on what constitutes a party roll call makes it difficult to evaluate the role of party in their final results. By contrast we shall be able to determine immediately when a scale is partisan. We shall return later to this topic to evaluate the wisdom of this decision.

The correlation coefficient chosen for this analysis is the tetrachoric r. This choice was based on its tendency to yield factor analysis structures between the extreme forms of Yule's Q and Y, as was seen in the previous section. Once again it is expected that essentially the same conclusions would have been obtained if any other measure of one-way association had been used. The decision to employ a measure of one-way association for the factor analysis is of far greater importance than the choice among these coefficients.

A cluster analysis has been conducted on the matrix of tetrachoric r's in order to better evaluate the performance of the factor analysis. A minimum tetrachoric r value of 0.9 was chosen for the clustering. This corresponds to a requirement of a minimum Yule's Q of nearly 0.95.

The amount of error in an inter-item cross-tabulation cannot exceed 7.2% if the tetrachoric r is to have this minimum value.

The clustering process is particularly useful in calling attention to the varying specificity of scales. Setting a high clustering criterion tends to yield quite specific scales while setting a low clustering criterion tends to yield very general scales. This effect can be seen in MacRae's (1965) cluster analysis where he first required a minimum Q of 0.8 and obtained several scales. He then scored the legislators on these scales and correlated these scores. These scale correlations were clustered as an additional stage of the analysis, yielding a small number of general issue areas.

The corollary of the above principle is that specific issues satisfy a higher clustering criterion than more general issue areas. Roll calls on a single bill thus tend to have a very high minimum value. Thus the lowest tetrachoric r among the roll calls on the National Defense Education Act is 0.96 while much lower values exist between these roll calls and other social welfare items. Something like two dozen clusters of roll calls concerning specific bills or issues exist among the votes of this Congress.

Table 6.8 shows the minimum correlations between fourteen scales of specific bills or issues, with the number of items in each scale shown in parenthesis. The smallest correlation between the three budget cuts items and the two military construction items is, for example, shown to be 0.95. The lowest correlations within each scale are shown in the diagonal cells. Also the minimum correlations with party are shown in the top row of this table. In analyzing these

TABLE 6.8 MINIMUM CORRELATIONS BETWEEN SPECIFIC SCALES*

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
A Party (1)	-	.97	.98	.88	.89	.84	.64	.40	.32	.11	.48	.23	.01	.04	.40
B Budget cuts (3)	1.00		.95	.85	.88	.70	.62	.42	.31	.13	.46	.09	.05	.01	.37
C Military construction (2)				.98	.75	.82	.68	.62	.43	.39	.20	.47	.23	.01	.01	.30
D Legislative appropriations (2)					.98	.66	.71	.57	.41	.24	.05	.40	.35	.00	.09	.07
E Atomic Energy Commission (3)						.95	.57	.60	.35	.30	.16	.46	.17	.02	.03	.03
F Illinois Waterway (3)							.98	.46	.22	.17	.01	.30	.18	.03	.01	.33
G Area redevelopment (2)								1.00	.84	.79	.64	.82	.59	.13	.42	.22
H Federal preemption doctrine (4)									.98	.77	.78	.78	.66	.04	.66	.88
I National Defense Education Act (3)										.96	.67	.71	.64	.28	.56	.21
J Alaskan statehood (4)											.97	.55	.43	.10	.44	.60
K Labor and HEW Appropriations (9)												.90	.56	.01	.38	.09
L Debt limit increase (2)													.94	.53	.67	.22
M Mid-East Doctrine (3)														.91	.53	.29
N Mutual security (8)															.92	.47
O Civil rights (6)																.97

*Absolute values are shown.

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correlations it should be realized that three of the scales, those dealing with the appropriations for the Departments of Labor and Health, Education, and Welfare, mutual security, and civil rights, are considerably longer than the other scales. The greater length tends to increase the chances for low correlations with other items as the clustering criterion by which these scales were formed is not very severe.

The specificity effect described above is apparent in this matrix. The lowest correlation between any single scale and any other scale is below the lowest correlation within that first scale. This is true of the single bill scales and also the scales, such as the mutual security and civil rights scales, which involve several bills on the same issue.

Two separate large clusters emerge when a clustering criterion of 0.50 is used. All the correlations between budget cuts, military construction, legislative appropriations, Atomic Energy Commission, and the Illinois waterway items are above 0.50 as are all the correlations between the federal preemption doctrine, National Defense Education Act, Alaskan statehood, and Labor and Health, Education, and Welfare Department appropriations items. These two groups of items bear a strong resemblance to the two broad issue domains identified in the longitudinal analysis of House voting by Aage Clausen (1964, 1967b). His "economic policy" scales have content similar to the first cluster described here while his "welfare policy" scales have content similar to the second cluster.

The correlations between the items in these two domestic issues clusters and party are of particular interest. Since the measure of association being used is a measure of one-way association, a correlation of unity with party would indicate that the members of one party are unanimous on the item so the item differentiates only members of the other party. The lowest correlation of any item from the first cluster with party is 0.84 while the highest correlation for any item from the second cluster is 0.67. Thus the first cluster has a great deal to do with partisan factors while the second does not. Clausen (1967b) finds a similar effect with his two longitudinal dimensions. He emphasizes that

the first dimension is not simply party line voting since the measure of correlation being employed takes on high values when only a single party is cohesive. The responsible party model defines party voting as when members of both parties are cohesive while the parties vote in opposition to one another. Here instead only the members of one party are cohesive. Yet this latter situation is probably the typical partisan situation in the American Congress. An issue may polarize the situation sufficiently to unite one party without uniting the other. While it may not seem appropriate to label such a dimension a "partisan" dimension, it is clear that the first cluster found has very much to do with the phenomenon of party.

The relation of these two clusters with the other scales of Table 6.8 should also be observed. The area redevelopment issues bear a substantial relation to both clusters. Otherwise the issues of the first cluster have little relation to the remaining scales, those concerning the debt limit, Mid-East Doctrine, mutual security, and civil rights. The Mid-East Doctrine and mutual security scales are related to the debt limit as well as to each other. The civil rights scale is related strongly to the federal preemption doctrine, is related moderately to Alaskan statehood, and also shows a relation to mutual security voting. The mutual security and debt limit items also have moderate correlations with the roll calls from the second cluster, particularly the federal preemption doctrine roll calls. The scales of the second cluster are thus related to each other and to other issues. The voting space of the 85th Congress has a complex multidimensional structure.

The federal preemption doctrine items are of particular interest. These roll calls concerned a bill which would bar courts from ruling that a federal law nullifies a state law in the same field unless specified by Congress, including a vote on an amendment stating specifically that federal subversive activities laws should not be construed as invalidating state anti-subversive laws. Issues of states rights and civil liberties are involved in this matter. The items from this bill have a strong relation to the civil rights items, yielding a states rights or civil liberties scale. Yet the federal preemption doctrine votes have a strong relationship to the social welfare items which the civil rights votes do not possess. A cluster analysis which requires each item to be associated with only a single cluster would miss these strongly multidimensional characteristics of the data. The inspection of Table 6.8 begins to suggest the multidimensional structure, but a multidimensional analysis technique would be required in order to specify that structure in more detail.

There are several algorithms available for conducting a principal components factor analysis. All should provide the same results, though some cannot extract all the factors when the matrix is internally inconsistent. The Jacobi algorithm which extracts all of the factors simultaneously can succeed regardless of such properties of the matrix. However other algorithms, such as the Hotelling procedure which extracts a single factor at a time in the order of decreasing proportion of the variance explained (Horst, 1965, pp. 160-167), can extract only the most important factors under such conditions. While the Jacobi procedure is preferable when analyzing measures of one-way association, the available computer capacity made it necessary to employ the Hotelling procedure for the analysis of the 140 roll calls of the 85th Congress.

Five factors, accounting for approximately 85% of the "variance", were successfully extracted. These first five factors are correct, but it is evident that additional factors exist which could not be extracted. Since more dimensions exist than were extracted, oblique rotation cannot possibly identify each dimension with a separate axis. A clustering procedure would have to be used if the various scales are to be identified.

Since it is difficult to comprehend the complete structure of a 140 variable factor analysis, we will show only some of the values. Table 6.9 shows the highest correlation with each factor for each of the clusters previously described. It is important to realize that no single variable has the set of values listed for the cluster but rather that these represent the highest of any of the items on the particular factor. In particular, the signs of correlations near zero should not be interpreted as being meaningful. Correlations above one appear in Table 6.9 as a result of the inconsistency of the correlation matrix.

The factor structure strongly reinforces the conclusions from the cluster analysis. The two main clusters described above correspond to the first and second factors, respectively. Party is seen to be highly related to the first factor and weakly related to the second. The scales from the first factor do not have high loadings on the other factors. The scales from the second factor do have moderately high loadings on the third and fourth factors. The third factor involves mutual security, the debt limit increase, and the Mid-East Doctrine. The fourth factor is seen to relate to civil rights. All of the scales shown here have low loadings on the fifth factor, but some roll calls on farm legislation and minerals legislation do have high loadings on this

TABLE 6.9 HIGHEST CORRELATIONS WITH OBLIQUE FACTORS

	I	II	III	IV	V
A Party (1)	1.07	.35	.22	-.28	.11
B Budget cuts (3)	1.06	.39	.19	-.26	.11
C Military construction (2)	1.00	.42	.26	-.23	.09
D Legislative appropriations (2)	.86	.56	.47	-.23	-.08
E Atomic Energy Commission (3)	.98	.50	.30	-.32	-.12
F Illinois Waterway (3)	.96	.28	.28	-.34	-.09
G Area redevelopment (2)	.55	.90	.49	.25	-.18
H Federal preemption doctrine (4)	.30	1.16	.52	.41	.05
I National Defense Education Act (3)	.46	.81	.64	.48	-.26
J Alaskan statehood (4)	.16	.81	.35	.58	-.33
K Labor & HEW appropriations (9)	.59	.97	.55	.38	-.17
L Debt limit increase (2)	.34	.66	.87	.47	-.08
M Mid-East Doctrine (3)	-.24	.25	.85	.16	-.25
N Mutual security (8)	.14	.65	.91	.55	.15
O Civil rights (6)	-.63	.62	.37	1.27	.13
Factor					
I	1.00	.28	.19	-.32	-.02
II		1.00	.42	.39	.01
III			1.00	.15	-.06
IV				1.00	-.02
V					1.00

factor. It can be seen that this fifth factor is independent of the other factors and therefore we are safe in disregarding it in the subsequent analysis. The effect of under-factoring the matrix is evident mainly on the third factor. Inspection of the correlations with the other factors indicates that the Mid-East Doctrine items have quite different locations in the space than the mutual security and public debt items.

The under-factoring remains bothersome as it does not permit a full test of the factor analysis procedure. One would expect the under-factoring to result in a loss of detail in two respects. First, some scales will not be shown at all since they have little in common with the first few principal components. This effect does occur in this instance. The roll calls on some bills, such as one to permit the Navy to use margerine, do scale among themselves but have no relation to the first five principal components. As a result they have low loadings on the factors of Table 6.9 whereas further factoring would be likely to demonstrate the existence of such scales. Second, the under-factoring can cause more than one scale to load on the same factor since there are fewer factors than scales. We shall be primarily concerned with such inability to uniquely identify the major scalar dimensions.

In order to avoid the under-factoring problem, the number of roll calls was next reduced from 140 to 97, so as to fit within a computer program which would extract all of the factors regardless of the inconsistency of the matrix. The items chosen were those with correlations above 0.75 with any of the oblique factors. This criterion

has the effect of eliminating the possibility of discovering scales of items with little relation to the first few principal components. Instead it will permit a complete determination of the structure of the scales related to these principal components. It happens that no item had a correlation above 0.75 with the fifth factor so that factor is in effect eliminated. There is little harm in its elimination since it is independent of the other factors. The specific scales which have been used for the presentation of Table 6.8 and 6.9 contribute 55 of the 97 items which were used in this next phase of the analysis.

The factor analysis of the 97 roll calls yielded ten factors. There might actually be a degree of over-factoring as, because of the inconsistency of the correlation matrix, the first eight factors account for 100% of the "variance" and all ten factors account for 103%. Oblique rotation of the ten factors did not converge, so we will have to employ the orthogonal varimax rotation results. Since the various scales are not mutually independent, there may not be a unique identification of scales with axes.

Table 6.10 indicates the highest loadings of the items of the various scales with the ten rotated factors. As in the case of Table 6.9 no single item necessarily has the row of loadings shown for the scale. The items with the highest loadings on two of the ten factors do not belong to the scales which we have been considering, so these items have been listed additionally in the table.

The first factor corresponds to the first cluster found earlier. Again it should be observed that party has the highest loading on this factor. Party and the other issues in this cluster, budget cuts, military

TABLE 6.10 HIGHEST LOADINGS ON ROTATED FACTORS

	Factor									
	I	II	III	IV	V	VI	VII	VIII	IX	X
A Party (1)	1.03	.15	.08	-.04	-.01	.03	.00	-.04	.06	.01
B Budget cuts (3)	1.03	.21	.06	-.04	-.04	.05	-.01	.07	.03	-.05
C Military construction (2)	.96	.22	.14	.10	.03	.05	.12	-.17	.05	-.10
D Legislative appropriations (2)	.85	.29	.33	-.17	.07	.26	-.14	-.09	.06	.36
E Atomic Energy Commission (3)	.97	.32	.18	.10	-.09	-.04	.03	.17	-.24	-.07
F Illinois Waterway (3)	.91	.14	.10	-.07	-.06	-.04	-.03	.13	.56	.10
G Area redevelopment (2)	.48	.77	.27	.12	.15	.10	.33	.03	.04	.06
H Federal preemption doctrine (4)	.30	.92	.39	.14	-.15	.35	.08	-.03	.09	.10
I National Defense Education Act (3)	.38	.70	.46	.15	.12	-.05	.49	-.08	-.09	.07
J Alaskan statehood (4)	.15	.74	.22	.63	-.06	.06	.08	.08	-.06	.06
K Labor & HEW appropriations (9)	.50	.90	.34	-.08	-.13	.22	-.08	-.16	.06	.14
L Debt limit increase(2)	.25	.54	.72	.02	-.04	-.03	.12	-.12	.14	.44
M Mid-East Doctrine (2)	-.14	.14	.72	-.11	.75	-.04	.07	-.03	-.03	.06
N Mutual security (8)	-.22	.45	.90	.09	.05	.11	.09	-.07	-.07	-.18
O Civil rights (6)	-.80	.79	.36	.18	.08	.34	.13	.02	-.13	.15
Overturn Mallory decision (1)	.46	.81	.46	.20	-.17	.44	-.04	.19	-.17	.09
Postal pay (1)	.40	.74	.02	-.04	.12	.15	-.04	-.61	-.01	-.06
Factor contributions	41.9	29.7	14.7	2.6	2.1	2.0	1.6	1.5	1.5	1.4

construction, legislative appropriations, Atomic Energy Commission, and the Illinois Waterway, tend to have low loadings on the other factors. The only exception is that the Illinois Waterway items have the highest loadings on the ninth factor, though they still have much higher loadings on the first factor.

The second factor corresponds to the second cluster found originally. The federal preemption doctrine and Labor and HEW appropriations have the highest loadings on this factor. The civil rights items have very high loadings on both of the first two factors. This is one place where an oblique rotation might have been helpful. The civil rights items fell on a separate factor in the earlier analysis, and here they still have a distinctive pattern of loadings.

The mutual security items have the highest loadings on the third factor. The debt limit and Mid-East Doctrine items also have high loadings on this factor, though they also have high loadings on later factors. The Alaskan statehood roll calls have the highest loadings on the fourth factor. The Mid-East Doctrine items have the highest loadings on the fifth factor. The bill to overturn the Mallory decision has the highest loading on the sixth factor. Its pattern of loadings is actually very close to that of the federal preemption doctrine items, both having to do with Supreme Court decisions in the area of civil liberties. The National Defense Education Act items have the highest loadings on the seventh factor. The postal pay measure has the highest loading on the eighth factor. Other miscellaneous appropriations measures are also located on this factor. The Illinois Waterway items have the highest

loadings on the ninth factor and the debt limit increase items have the highest loadings on the tenth factor.[7]

There were a large number of dimensions of voting during the 85th Congress. Ten factors were found when analyzing a subset of the items. Perhaps fifteen to twenty factors would have been found had it been feasible to analyze the full set of roll calls.

While the number of factors is large, it is clear that a small number of these factors account for most of the items. Factors IV through X in Table 6.10 are specific factors which involve only small numbers of items. Even most of these items have their highest loadings on the first three factors, though this might be due to the inability to obtain an oblique solution. These first few factors represent the general scalar dimensions.

The principal dimension of conflict is partisan. It corresponds to a dimension which Clausen has identified in his longitudinal analysis of House voting. He has interpreted one variant of this dimension as an economic policy dimension (Clausen, 1964, 1967b) and another variant as a Public-Private Economy dimension (Clausen, 1967a). By explicitly including party as a variable in this analysis and by including both parties in the analysis, we have been able to indicate the close connection of this dimension with party.

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[7] Some items not shown here do have high loadings on several factors. The most severe case is a motion to cut appropriations for the U.S. Information Agency. Its correlations with the ten factors are in order, -.55, .68, .43, .02, .10, -.04, -.47, .19, .34, and .37. This pattern of values is different enough from that of other items that one would conclude that it does not enter any unidimensional scales, but its multidimensional properties are most complex.

A few of the other general dimensions which were obtained are of the type expected to have a continuing existence across several Congresses. One of these has to do with social welfare, the proper role of the federal government. A civil rights dimension is also evident. There is an indication of a civil liberties dimension, related to but distinguishable from the civil rights dimension. Mutual security legislation constitutes the core of another dimension. Some other items relating to foreign policy and to the national debt limit are on this dimension. The essence of this dimension remains elusive. It seems to combine elements of American intervention abroad with elements of fiscal conservatism. The remaining minor dimensions are related to single bills or short-range issues; the number of general dimensions is indeed small.

To focus more directly on the representation study scales, Table 6.11 shows the loadings of the items from the civil rights, foreign policy, mutual security, and social welfare scales on the ten rotated factors. Not all of the items from these scales are shown as some were dropped when the subset of 97 roll calls was selected.

The civil rights scale is seen to be rather cohesive in its factor loadings. Its fourth item, relating to the rule for considering the Civil Rights Act, does have a greater relation to the party factor than do the other items. Otherwise there is little variation in the loadings of its items.

Only two of the foreign policy scale items were included in this analysis and both deal with mutual security. Eight of the ten mutual security items were included in the analysis. The mutual security items are very cohesive in their loadings. The eighth mutual security item,

TABLE 6.11 LOADINGS OF REPRESENTATION STUDY SCALES ON ROTATED FACTORS

Scale & Item	Factor I	II	III	IV	V	VI	VII	VIII	IX	X
CR 1	-.59	.79	.25	.18	.08	.28	.13	-.02	-.05	.15
CR 2	-.62	.78	.26	.16	.07	.24	.09	.00	-.06	.16
CR 3	-.65	.79	.25	.15	-.05	.25	.04	.00	-.08	.04
CR 4	-.80	.74	.23	.15	-.09	.30	-.09	.03	-.13	-.03
CR 5	-.68	.76	.26	.16	.00	.32	-.12	.02	-.01	-.07
CR 6*	-.49	.73	.36	.07	-.05	.34	-.11	.00	.00	-.06
FP 2	.05	.44	.89	.07	.03	.02	.08	-.04	-.07	-.05
FP 3	-.05	.43	.88	.01	.04	.08	.02	.04	.00	-.03
MS 1	.00	.44	.87	.08	.05	.11	.03	.01	-.06	-.06
MS 2	.05	.44	.89	.07	.03	.02	.08	-.04	-.07	-.05
MS 3	.05	.42	.90	.08	.04	.01	.09	.02	-.04	-.02
MS 4*	.06	.43	.89	.07	.00	.00	-.01	.03	-.01	.00
MS 5	-.08	.44	.88	.09	.02	.11	.03	-.06	.00	-.04
MS 6	-.04	.42	.89	-.01	.05	.05	.02	-.07	-.06	-.02
MS 7	-.05	.43	.88	.01	.04	.08	.02	.04	.00	-.03
MS 8*	-.22	.45	.85	-.03	.00	.07	.02	.05	.04	-.18
SW 1	.08	.54	.72	.02	.09	-.03	.12	-.12	.10	.44
SW 2	.17	.70	.45	.09	.03	-.02	.49	-.08	-.01	.02
SW 4*	.30	.67	.46	.11	.05	-.01	.47	-.02	.06	.07
SW 5	.48	.72	.27	.12	.14	.04	.33	-.03	.04	.06
SW 6*	.46	.77	.27	.02	.15	.10	.26	.03	.02	.06
SW 7*	.08	.84	.41	.16	.02	.04	.18	.08	.12	-.18
SW 8*	.39	.84	.26	.01	.09	-.08	-.04	.05	.01	.03
SW 9*	.44	.80	.30	.05	.02	-.14	-.08	-.10	-.05	.02
SW 10*	.46	.77	.35	-.06	-.14	-.09	-.22	-.16	-.27	.03

*Reflected

relating to the development loan fund, has a somewhat greater relation to the party factor than do the other items. Still the basic unity of these items is clear.

Nine of the ten social welfare scale items were included in this analysis. These items do not evidence as much cohesion as the other two scales. The first item, involving the size of the debt limit, has substantial loadings on the mutual security factor and its own factor. The

second and fourth items, both dealing with the National Defense Education Act, also have noticeable loadings on a separate factor. All but the public debt item have their highest loading on the same factor. Yet there is considerable variation in their relation to the party factor.

The variation in the civil rights, mutual security, and social welfare scale item loadings shows the amount of variation which is compatible with a Guttman scale. Minor modifications in these three scales aside, the variation present in these loadings is small relative to the variation between the scales.

Comparing Tables 6.10 and 6.11 suggests that the representation study scales do reflect three of the dimensions of conflict in the 85th Congress--civil rights, mutual security, and social welfare. The near-vanishing values of the loadings of most of the scale items in Table 6.11 on factor IV through X indicate the centrality of these scales. The further analysis has not clarified the meaning of the mutual security scale, though it reinforces the previous finding that there is a relation between this scale and voting on the size of the national debt.

Some final comments on the factor analysis of measures of one-way association are in order. The fact that the correlation matrix is internally inconsistent does create some difficulties in the analysis, though these problems have more to do with specific algorithms than with general principles. Some caution is necessary in the interpretation of the factor loadings, but the basic dimensions can be found from this procedure. The factor analysis is superior to mechanical cluster analysis in that the multidimensional properties of the items can be

explored without arbitrarily restricting each item to a single scale. The interpretation of the factor analysis may be difficult in such multidimensional situations, but it would be less valid to ignore the multidimensional complications. The factor analysis of measures of one-way association is not a simple procedure, but no simple procedure suffices for the analysis of multidimensional dominance structures.

CHAPTER VII

THE STATE OF THE ART

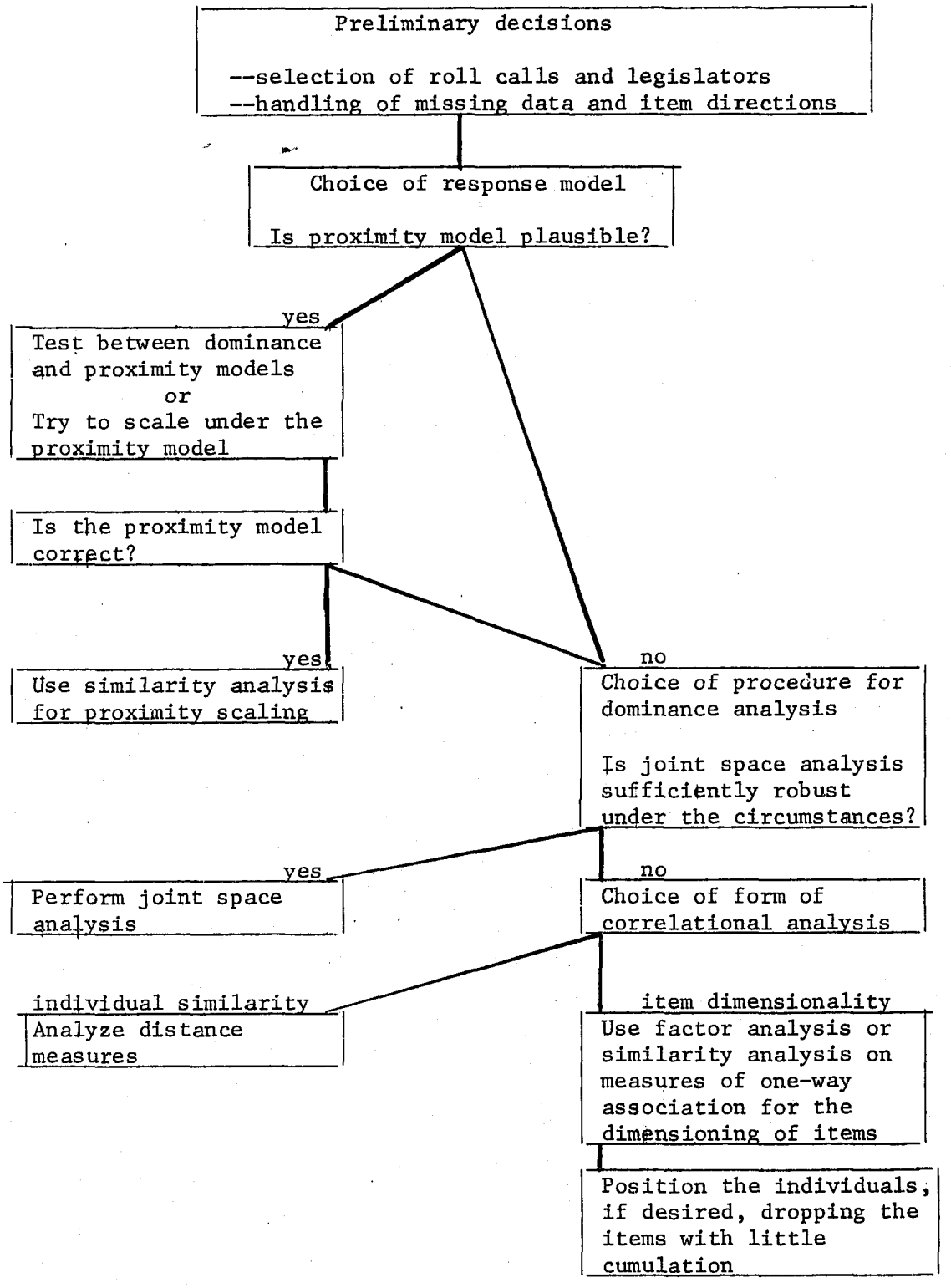
Having now discussed the dimensional analysis of legislative roll call votes at some length, we shall conclude with a consideration of the "state of the art." We shall concentrate on two aspects--summarizing the procedures for dimensional analysis of roll calls which are presently available and suggesting the lines for future developments.

AN ALGORITHM FOR ROLL CALL ANALYSIS

One cannot give a simple set of rules for dimensional analysis, but Figure 7.1 provides a rough flow chart of the various steps in such an analysis.

As we saw in Chapter II, a roll call analysis begins with the making of several assumptions as to which observations should be treated. This includes selecting the roll calls and the legislators as well as determining the missing data and the item directions. The decisions made at this preliminary stage do affect the final conclusions. For example, the decision as to whether to scale members of different parties separately or together determines whether a partisan dimension will be found, a dimension which does explain a large amount of the voting in contemporary Congresses. While this initial phase is quite important, we shall not restate further the decisions which are made at this stage.

Fig. 7.1 Dimensional Analysis of Legislative Roll Calls



After the preliminary phase, the analyst must select the underlying response model and the corresponding analysis procedure. Response models can be classified in several ways, but the most important distinction we have considered is that between dominance response models and proximity response models. According to the dominance model, the individual responds to an item on the basis of whether it provides more or less of the various dimensions than he desires. According to the proximity model, the individual responds to an item on the basis of whether or not it provides about as much of the various dimensions as he desires. We have shown that the proximity model assumes single-peaked individual response functions with the individual responding to a single alternative at a time. If all the individual response functions are monotonic with the space and/or if the individuals vote by implicitly comparing their preference for the state of affairs resulting from the passage of the motion with their preference for the state of affairs resulting from the defeat of the motion, then the votes will fit the dominance model. Pending further knowledge concerning the nature of the response functions of legislators and their interpretation of the voting situation, we have suggested that the proximity model occurs mainly under conditions of unwillingness to compromise. Empirical procedures for discriminating between these two response models have also been suggested here. The principal device considered was the inspection of the matrix of inter-item "joint approval scores," the proportion of individuals who vote in favor of both items. This matrix has a different structure in the general proximity model than it has in the dominance case. Similarity analysis of this matrix is a means of proximity scaling. Similarity

analysis of the matrix of raw votes is another possible means of proximity scaling, though it may often be indeterminate because of the small number of constraints.

The decision as to whether to assume a dominance model or a proximity model is quite important. Roll call analysis has generally assumed a dominance model as a matter of course without even permitting the discovery of possible proximity behavior. We have shown that the proximity model is a plausible model of legislative voting in certain circumstances and in those circumstances it provides a much better fit to the data. Thus an early step in a dimensional analysis of roll calls should be to ponder the possible relevance of a proximity response model and, if that model seems at all plausible substantively, to test between the dominance and proximity model or to attempt to scale under the proximity model.

If one decides, either theoretically or empirically, that the proximity model is not appropriate, one must choose between the available analysis procedures for the dominance model. A basic decision must be made concerning whether the analysis is to be of the raw vote matrix or of a derived correlation matrix. Joint space analysis of the raw vote matrix generally presents problems having to do with the robustness of the analysis technique in the presence of missing data, error, unknown item directions, and multidimensionality. The best solution at present is to switch from joint space analysis to correlational analysis.

In shifting from the raw vote matrix to the derived correlation matrix for the purpose of correlational analysis of dominance data,

one must choose which correlation coefficient is to be employed. We have seen in Chapter IV that some measures of association measure the dominance relation between items while other measures of association measure neither the dominance relation nor the proximity relation in a pure sense. Because they measure different properties, these two types of coefficients lead to different patternings of the correlation matrix even when they are computed on the basis of the same data structure. Correspondingly, an analysis technique gives different interpretations to these different patternings. Therefore it is well to consider the choice of correlation coefficient together with the choice of analysis technique so as to insure that the two when used together yield the analysis desired.

There are two basic forms of dominance analysis--the analysis of the dimensionality of the set of items and the analysis of the similarity of the set of individuals. We have concentrated on the former, obtaining the dimensional properties of the items. In this form of analysis, measures of one-way association provide the purest measure of the dominance relations between the items which are of interest. We have shown in particular that analysis of the measures of two-way association, including Pearson's r, leads to a misstatement of the dimensionality and erroneous placement of the items in the space. While analysis of measures of one-way association is not a conventional technique, we have shown very carefully that under most circumstances it yields correct results. Cluster analysis of these measures does not handle multidimensional situations as well as would be desired. Instead factor analysis or similarity analysis should be used in this connection. The interpretation

to be given to the results of the factor analysis and the similarity analysis of these measures is different from the classic interpretation of their results. Instead of indicating the placement of the items in the space, the present usage results in clusters of items in the space which can be said to fall along the same dimension. Under factor analysis, it is likely that these clusters will be located on the rotated axes, simplifying the analysis. When the dimensionality of the set of items is analyzed only as a prelude to positioning the individuals on the resultant scales, a final stage of the dimensional analysis may be to purify the scales by removing items which are near one another but do not cumulate well with one another.

PERSPECTIVES ON THE FUTURE

Roll call analysis has long been a popular means of analyzing legislative behavior. After the psychologists developed the concept of dimensionality, it was immediately seen as relevant to roll call analysis. Roll calls within single issue areas were scaled and analysts attempted to discover the main attitudinal dimensions of roll call voting. The importance of this pioneering work has not been questioned here, but instead we have emphasized the need for more complex analysis.

Dimensional analysis was originally conceived as consisting solely of the cumulative Guttman scale model. This model does indeed yield a good fit for much roll call analysis in contemporary American legislatures. However an alternative response model, the proximity model, merits greater consideration in roll call analysis. For

theoretic reasons one would anticipate it to be useful in some historical situations and in multi-party legislatures. Its applicability in contemporary American legislatures requires further investigation.

The consideration of alternative response models directly raises questions about how legislators structure the voting dimensions. What frame of reference is used by legislators when deciding how to vote on a motion? What differentiates legislative issues on which everyone has the same ideal from those on which they have different positions? When do legislators view voting as a paired choice task and when is there a bias in favor of a yea or nay vote? It may be difficult to devise interview questions which elicit this information from Congressmen, but such information would considerably increase our knowledge about Congressional voting behavior.

Dimensional analysis was originally concerned exclusively with unidimensional phenomena. The Guttman scale itself is unidimensional, of course, but even when legislative analysts have conceived of multi-dimensionality they have generally meant simply the existence of several unidimensional Guttman scales. The exclusive concern with unidimensionality permitted the use of rather simply analysis techniques such as cluster analysis in order to obtain the voting dimensions. So long as the data consist of separate unidimensional scales, cluster analysis generally yields the dimensions correctly.

Legislative roll call voting, however, represents the results of a complicated process with complex relationships between issue areas. Some separate unidimensional areas exist, but the basic areas of contemporary legislative conflict are sufficiently intertwined as to

require analysis which is explicitly multidimensional. Cluster analysis is not satisfactory in such situations as it takes too simplistic a view of the structure of the data. Factor analysis and similarity analysis provide more adequate means of analyzing such a multidimensional structure. They can be viewed as generalizations of cluster analysis which are more appropriate with a complex multidimensional data structure.

As we are still accustomed to thinking in terms of single dimensions, the handling of the results of a multidimensional analysis remains awkward. Generally one quickly shifts to a consideration of some set of axes within the multidimensional space in order to return to the familiar grounds of unidimensional analysis. Yet this results in the loss of the true advantages of multidimensional analysis. As experience with multidimensional analysis accumulates, it should become easier to deal with the multidimensional properties of the data structure and their relations to the predictor variables. The difficulties at present may have more to do with our state of mind than with real methodological obstacles.

The means of multidimensional analysis also require further refinement. The relative advantages of factor analysis and similarity analysis require further consideration. It is necessary to find the extent to which each technique distorts data structures. It is of particular importance to judge the extent to which either technique would yield misleading conclusions. We have demonstrated that factor analysis can be useful, but we have not proceeded into a determination of which technique would be preferable. This would require extensive experimentation on known data structures, focussing upon how well each technique reproduces known properties of the data.

A by-product of our analysis has to do with the choice of correlation coefficient for use in these techniques. Measures of one-way association, such as Yule's Q, tetrachoric r, and Yule's Y, are particularly sensitive to the dimensional properties of the data. These measures are well suited for cluster analysis, factor analysis, and similarity analysis. However measures of two-way association, such as the classic Pearson product-moment r, tap neither the dimensional properties of the data nor the inter-item distances in a pure form.

What then does it mean to perform a factor analysis on Pearson r's? It is too easy to pass this procedure off as dimensional analysis without coming to grips with exactly what it measures. Its classic basis notwithstanding, a careful justification of this procedure is necessary. We have refrained from considering possible justifications, feeling that this task is best left to the advocates of this procedure. Yet the real meaning, if any, of factor analysis of Pearson r's is one of the most important unanswered questions in the area of dimensional analysis.

Dimensional analysis remains as an important device for the analysis of legislative voting behavior. The analysis to date has not given sufficient attention to alternative response models and to the multidimensionality of the voting. Yet these possibilities only serve to increase estimates of the extent of dimensional voting in legislatures. Studies of the contemporary American Congress already converge in the finding that a small number of basic dimensions can explain a large amount of Congressional voting. The longitudinal analysis which has been done suggests that some of these dimensions have a degree of

permanence. Studies combining roll call data with interviews with Congressmen and their constitutents suggest that some of the same dimensions have a basis in elite and mass attitudes as well, though this does not imply that roll call voting simply mirrors these attitudes. Further analysis of roll calls can clarify the nature of conflict in previous periods of our history, indicating when and how different response models become important and when and how the structure of the multidimensional voting space changes. A better understanding of the basis of dimensional analysis of roll calls serves to increase the number and the complexity of the questions on which dimensional analysis may be brought to bear.

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