

Modeling the Dynamic Response Of Multilayer Plates with Shear Thickening Fluids

by

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Abstract

Multilayered plates have become a field of interest for vehicle lightweighting and improving survivability due to their variety of material and structural configurations. A new material to consider in multilayer plates is shear thickening fluids (STFs). STFs have been shown to improve the performance of body armor and to have structural damping applications. A new multilayer plate design is proposed in this dissertation that uses layers of STFs sandwiched between solid plate layers to act as a damping mechanism. The objective of this work is to investigate how STFs affect the dynamic response of a multilayer plate using computer modeling tools.

A reduced order model in LS-DYNA was used to validate the significance of implementing a damping mechanism similar to an STF within a multilayer plate. Then a microscale model was developed in the computational fluid dynamics (CFD) program STAR-CCM+. This model used the discrete element method (DEM) to simulate the particles in the STF that are linked to its non-Newtonian behavior. The results of this microscale model were validated against published STF rheometry data. A macroscale model of the multilayer plate with layers of a STF was developed using the reverberation matrix method (RMM). The relationship between the shear rate and measured viscosity from the microscale results were input into the macroscale model. This multiscale modeling method was used to investigate how changing the material composition of a STF, such as particle size, affects the dynamic response of a multilayer plate.

The results of these models indicate that incorporating the STF into a multilayer plate reduces its dynamic response.

Chapter One: Multilayer Plates and Shear Thickening Fluids

1.1 Introduction

Providing occupant protection without impeding mobility through overweighting is a critical design goal for vehicles, especially vehicles in combat theatres [TARDEC, 2014]. These vehicles must withstand a variety of impacts from shrapnel, missiles, improvised explosive devices (IEDs), and other hazards. While standard safety mechanisms such as seatbelts and airbags are critical for protecting passengers, safety design in military vehicles mainly focuses on armor [Hogg, 2006]. This thesis uses the Merriam-Webster's dictionary definition of armor, "*a protective outer layer (as of a ship, a planet or animal, or a cable)*".

Traditionally, military armor has consisted of placing dense structural components, such as steel plates, between the projectile or blast source and the occupant. This strategy utilizes conservation of momentum to slow down projectiles and Newton's second law to reduce the acceleration passengers are subjected to in blast loading conditions [Hogg, 2006]. However, adding weight to a vehicle reduces its mobility, which impedes the vehicle's ability to perform its fundamental objective of being a form of transport.

Multilayer plates were recognized early on as a solution to this conflict in design goals. Multilayer plates, as the name implies, are plates that consist of clearly stratified layers of differing materials. These layers can consist of ceramics, metals, or composite materials and can be selected to utilize the properties of multiple materials within the plate's design. The plate acts as a single unit, meaning that the plate deforms as a single unit and stresses are transferred

between layers. Layers are bonded along their interface by glue, lamination, or some other binding mechanism. Figure 1 shows an example of what a multilayer plate may look like.

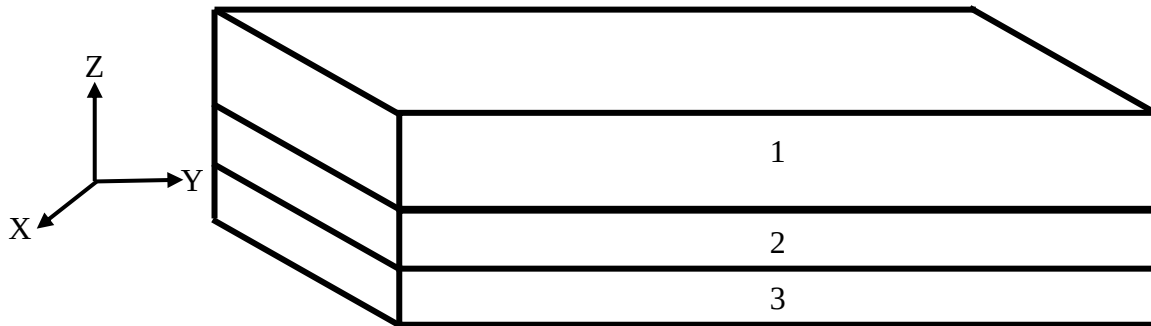


Figure 1. Simple drawing of a multilayer plate with three layers (denoted by the numbers one through three)

Incorporating a damping mechanism into a multilayer plate is another method of improving armor performance without necessarily using dense materials. An early example of utilizing multilayer plates with a damping mechanism in armor is Chobham armor. Developed in the 1970s at the British Army research facility in Chobham, England, Chobham armor consists of a ceramic composite layer sandwiched between two steel plates [Clancy and Gresham, 1994]. It is currently used on the M1 Abrams, Challenger 1, and Challenger 2 tanks as a protectant against high-explosive anti-tank (HEAT) warheads [Hogg, 2006]. The ceramic layer within Chobham armor is less dense than steel and has a honeycomb structure that allows it to act as an energy absorption mechanism. The steel plates on either side of the ceramic center layers provide a hardened, dense surface that prevents shrapnel penetration.

Another way to implement a damping mechanism into multilayer plates may be to sandwich layers of shear thickening fluid (STF) between solid layers. STFs are non-Newtonian fluids, meaning that their viscosity is dependent upon shear rate. Unlike Newtonian fluids, whose viscosity is independent of shear rate, the viscosity of STFs increases at higher shear rates

[Wagner et al, 2002. Lee et al, 2003]. Figure 2 illustrates the difference in the relationship between viscosity and shear rate for a Newtonian fluid and for a STF.

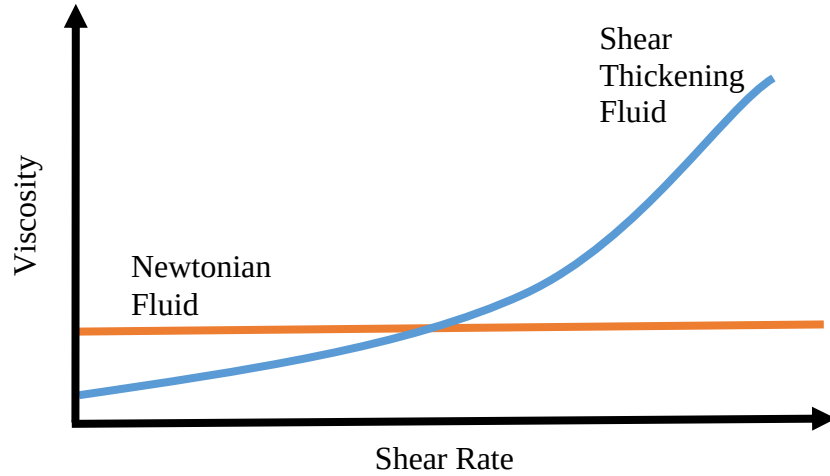


Figure 2. Illustration of a generalized relationship between viscosity and shear rate for a shear thickening fluid (in blue) and for a Newtonian fluid (in orange)

Due to their viscous behavior, the amount of work required to shear a STF increases with shear rate. The energy dissipated through shearing a STF can be also be orders larger than traditional Newtonian fluids such as water due to their large increase in viscosity. Despite being called ‘fluids’, many STFs are mixtures of densely packed, micro-scale particles suspended within a fluid phase [Decker et al, 2007]. A famous example of such a STF is ‘oobleck’, a mixture of water and cornstarch [Raborn, Daniel, 1999. Fall et al, 2008]. The non-Newtonian behavior of these STFs is caused by the particle interactions [Barnes, 1989]. The STF used for this research consists of silica particles ranging from 500 to 750 nanometers in diameter suspended within a polyethylene glycol (PEG 200).

The proposed design investigated in this research is a multilayer plate with a layer of a STF sandwiched between solid layers as shown in Figure 3. It is hypothesized that adding a STF into the multilayer plate design will decrease the dynamic response of the plate due to energy loss caused by shearing the STF. The dynamic response is defined as the amplitude of the plate’s

displacement caused by a dynamic load, such as vibration. The work required to shear the STF is directly related to the fluid's viscosity. Due to its non-Newtonian properties, the viscosity of the STF increases significantly at high shear rates such as those experienced under large dynamic loads.

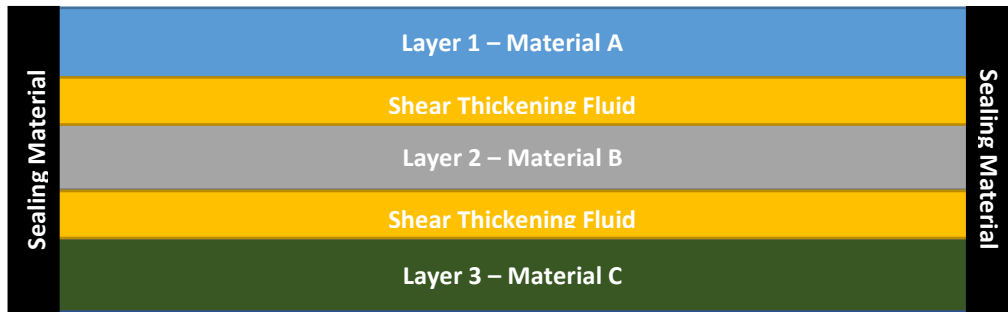


Figure 3. Proposed multilayer plate design; layers of STF are sandwiched between solid layers

The field of integrated computational materials engineering (ICME) provides an alternative means of investigating the effects of STFs in a multilayer plate. ICME is a holistic engineering approach that uses multiple computational tools at various time and length scales to study a material's composition affects its behavior and determine how the material performs within an application [Horstemeyer, 2012]. This approach is useful for determining how to optimize a material for a new application.

This research utilizes an engineering method commonly used in ICME called multiscale modeling. This modeling method uses smaller scale models to determine the material properties that are then input into larger scale models. There are two forms of multiscale modeling: sequential and concurrent. In the concurrent method, both simulations run at the same time with information passing between the models throughout the simulation. This research uses sequential multiscale modeling, meaning that a smaller scale model is first developed to solve for a set of material behaviors that are then input into a larger scale model [Weinan and Jianfeng, 2011].

The models used within this study are a small scale computational fluid dynamics (CFD) model that utilizes the discrete element method (DEM) and a structural-scale model that uses the reverberation matrix method (RMM). The discrete element method is a meshless, Lagrangian method of modeling particles. The location, velocity, and acceleration of each particle is calculated at each time step by summing the forces acting on the particle. The effects of a fluid on the particles are taken into account by calculating drag and lift forces on the particles. The DEM allows for the effects of small scale particle motion and interaction to be accurately captured. The RMM uses material stiffness matrices and the plane wave theory to calculate the stresses within a structure and its dynamic response. The versatility of this method broadens its application to a variety of material types.

The tools used to develop these models are the finite element analysis (FEA) code LS-DYNA, the math language software MATLAB, and the computational fluid dynamics (CFD) software STAR-CCM+.

1.3 Literature Review

Multilayer plates are comprised of clearly stratified layers of materials which makes them a useful structural component due the ability to optimize their material properties. For armor, these layers traditionally have comprised of metal alloys or ceramics and are bonded together through lamination [Rettenbacher et al, 2006. Van der Loo et al, 1994. Vives et al, 1992]. However, engineers have begun to incorporate more innovative materials such as metal foams, sandwich or truss structures, and even memory alloys [Qiao et al, 2008. Davalos, et al. 2001., Wahed et al, 2002]. The goal of many of these new structural components is to provide shock absorption to improve the dynamic response of the plate [Qiao et al, 2008].

One of the earliest example of such a design is Chobham armor. As previously discussed, this armor is a multilayer plate that consists of a honeycomb structure made of a composite material sandwiched between steel plates [Clancy and Gresham, 1994]. The goal of this design is to combine the stiffness of the denser steel layers with the energy absorbing capabilities of the lighter composite layer to provide an armor that can withstand high impact rounds without overweighting a vehicle [Sparks, 1998].

Outside of military applications, researchers have recognized that incorporating innovative materials into the design of multilayer plates can improve their desired performance [Khatua, Cheung, 1973. Mawenya, Davies, 1974, Leissa, 1986]. Many of these research efforts are in the field of aerospace engineering, where lightweight and thin structural components are critical [Kant and Swaminathan, 2000. Conlon, Hambric, 2009]. Other efforts have focused on improving the impact survivability of the plate by using material layers as a damping mechanism [Kay, 1987. Scherzer, 1985.] An example of this is the research published in 2008 by engineers from the Institut Supérieur de Mécanique de Paris regarding analyzing the effects of using visco-hyperelastic polymer layers on the plate's response [Gacem et al, 2009]. However, it is only recently that the application of shear thickening fluids as a structural dampening mechanism has been examined [Decker et al, 2007, Asigja et al, 2017.].

The number of publications pertaining to the application of STFs as a damping mechanism has greatly increased since Lee, Wetzel, and Wagner's 2003 publication on the improved performance of Kevlar woven fabrics impregnated with STFs to ballistic loads [Lee et al, 2003]. In the design used in their work, layers of composite fabric were impregnated with a STF and then joined together to form a multilayer sheet. When the fabric is dynamically loaded, the STF is sheared as the fabric fibers move past each other, resulting in energy dissipation

similar to energy dissipation through friction. Further research showed that impregnating STF into a body armor design consisting layers of Kevlar fabric significantly improved its stab resistance [Decker et al, 2007].

Other applications for STF besides impregnated body armor fabrics have been investigated [Lee et al, 2003. Asiaga et al, 2016. Wagner, Wetzel, 2019. Decker et al, 2007, Wetzel et al, 2004]. One such application is to directly use STF within rheological dampers. Dr. Yeh of the National Center for Research on Earthquake Engineering in Taipei and his colleagues studied using various STF in damper systems as an alternative to power-based structural dampers such as electro rheological dampers [Yeh, Chang, and Chen, 2012]. Dr Zhang et al published their work on designing and testing a piston-like STF damper in 2008 [Zhang et al, 2008]. Another proposed application is to use STF within lithium ion batteries to improve resistance to mechanical abuse [Ding et al, 2013]. Other researchers have proposed incorporating STF into structural components such as STF filled sandwich structures [Fischer et al, 2006].

The main approach researchers use to analyze these applications is to build a prototype and test it. Computer simulations of the design are developed after experiments on the prototype are completed [Zhang, Li, 2008. Dekcer et al, 2007. Sigja et al, 2017. Wetzel et al, 2004. Wu et al, 2017. Caglayan et al, 2018]. For example, Dr. Wetzel and his colleagues, as well as Dr. Lee at the Korea Advanced Institute of Science and Technology, modified the Coulomb's friction between Kevlar fibers in LS-DYNA simulations using prototype results to simulate the effects of impregnating the fabric with a STF [Wetzel et al, 2004. Lee, Kim, 2012]. Dr. Mossakovsky and his colleagues at the Research Institute of Mechanics in Russia use the Alternative Lagrangian-Eulerian (ALE) method to simulate the effects of having a fluid within the Kevlar fabric, but the viscous effects of the STF on the fabric is still simulated using Coulomb friction along the fluid-

fabric interface [Mossakovsky et al, 2011]. Sometimes prototypes are presented without a corresponding method of computer simulation [Lee et al, 2003. Yeh et al, 2012]. However, some researchers have developed equations to describe the relationship between the STF viscosity and shear rate using experimental data [Mossakovsky, 2011. Galindo-Rosales et al, 2011].

While the previously discussed modeling methods are able to capture the effects of an STF, they are dependent on developing a prototype first to ‘train’ the simulation with [Mirrahimi et al, 2017. Egres et al, 2006]. This is not a cost effective method to explore a design space.

Integrated computational materials engineering (ICME) would provide such a method. ICME is a design approach that analyzes a product by linking the results from multiple models of its materials at various time or length scales [National Research Council, 2008. Allison et al, 2006. Joost, 2012]. This holistic approach uses both material science and traditional engineering analysis methods such as finite elements. The first papers published on this subject, such as the work published by researchers at Kobe University, focused on developing multiscale models to study the effects of steel fabrication on its strength [Olson, 2000]. This method has been successfully applied to defense and light weighting applications [Allison et al, 2006. Joost, 2012]. An example of using ICME to study the effects of STFs on a design would be to develop a small scale of model capable of simulating multiple STFs to study their behavior and then use these results within a full-scale simulation of a design.

Multiscale modeling is a key component of ICME [Allison et al, 2006]. Multiscale modeling is a simulation method that links the results from various models at different time and length scales together to generate accurate results of how a system or material will perform. Although it originated before ICME, researchers recognize it as an integral tool of the field [Olson, 2000]. Multiscale modeling has been used to study the effects of molecular dynamics

and Brownian motion [Fabritiis et al, 2006, Zheng et al, 2008. Feig et al, 2004]. This work shows that multiscale modeling can be used to study a material such as a STF.

Its applications range from determining how molecular dynamics affect gas dynamics to studying how crystal lattice structure affect crack propagation [Weinan and Jianfeng, 2011]. This method, which has been used for both solid and fluid continuum, could be a useful tool for determining the effects of STFs on multilayer plates.

A potential simulation method to develop the small scale model is to couple the discrete element method (DEM) into a CFD program to simulate the two phases of the STF. Implementing DEM into a CFD program has been a topic of interest for over the last twenty years. The first DEM-CFD model was reported in literature in 1992 for plug flow [Tsuji et al, 1992]. After this publication, coupled DEM-CFD simulations have been utilized for a variety of applications. One the main application of DEM-CFD simulations is to model fluidized beds [Di Renzo, Di Maio, 2007. Deen et al, 2007. Rhodes et al, 2001]. Fluidized beds consist of a pressurized fluid phase, such as water or air passes through a packed bed of solid particulates, causing the fluid-particle mixture to behave as a single fluid. Hydroclusters and dilatation do not occur in fluidized beds, but their simulations have shown that DEM can be successfully implemented into a CFD simulation to study fluid like behavior. DEM-CFD has also been used to simulate the behavior of wet granular materials [Zhao, Shan, 2013. Apostolou, Hrymak, 2008]. Despite the variety of applications of the DEM-CFD model, so far it has not been used to study the particle behavior observed in STFs.

The reverberation matrix method (RMM) could be used to develop the macro-scale model of a plate design that uses an STF. RMM has been applied to studying the dynamic response of multilayer plates, beams, and fluid columns [Jiang et al, 2016. Yan, Wang, Chen,

2011. Pao, Su, Tian, 2000. Su, Tian, Pao, 2002]. RMM was successfully incorporated the effects of dampers within a truss-like structure [Cai et al, 2009]. It has not been used to study the application of non-linear materials such as STFs [Jiang et al, 2016. Cai et al, 2009. Pao, Su, Tian, 2000. Su, Tian, Pao, 2002]. However, the versatility of this simulation allows the algorithm to be applied to a variety of applications and material types.

This literature review demonstrates that the material behavior of STFs makes them a potential damping mechanism within multilayered plates. It also supports the idea of using DEM-CFD and RMM within the ICME approach of multiscale modeling to study a shear thickening fluid and its application within a multilayered plate.

1.4 Objectives

The main objective of this research is to use modeling tools to study how STFs affect the dynamic response of multilayer plate when they are placed between solid plate layers. This main objective is met by completing the following objectives:

1. To examine the effects of implementing a reduced order damping mechanism along layer interfaces on the plate's dynamic response and energy losses.
2. To use the results of aforementioned study to evaluate the validity of investigating the multilayer plate design that incorporates a STF.
3. To develop an accurate microscale model of a STF using the CFD program STAR-CCM+ that can be used to study the relationship between the STF's viscosity and shear rate.
4. To formulate a macroscale model of a multilayer plate with STF layers using the reverberation matrix method (RMM) that uses the viscous behavior from the microscale model.

5. To examine how changing material properties in the microscale model of the STF affects the dynamic response of a multilayer plate.

1.5 Methodology

To achieve the stated objectives, the following steps were taken. First, a reduced order model of a multilayer plate acting as a vehicle floor was developed in LS-DYNA. Friction was used to simulate the effects of having a material similar to a STF between the plate layers. The measured dynamic response and energy dissipation of this model was used to determine the effectiveness of implementing a damping mechanism similar to a STF between plate layers.

Next, a small scale model of the STF subjected to a cone-in-plate rheometry test was developed in the CFD program STAR-CCM+. Because the STF consists of both a fluid phase and suspended particles, a multiphase model was developed. The particles were modeled using the discrete element method (DEM). This model was used to develop the relationship between shear rate and the STF's viscosity.

Then, the reverberation matrix method was used to model a multilayer plate that contains both layers of solid materials and a STF. Viscous effects had to be considered, which necessitated a new formulation of the RMM stiffness matrix. The relationship between shear rate and the STF's viscosity was taken into account through an iterative method. This model was used to analyze the effects of adding a STF to a multilayer plate on its dynamic response.

Finally, to gain a better understanding of the effects of an STF on the dynamic response, a brief parametric study was conducted. This study varied the particle packing fraction and size separately to first see the effect of changing these parameters on the viscosity measured in the STAR-CCM+ simulation. The relationships between shear rate and viscosity were developed for

these modified parameters were then input into the RMM model. The dynamic response of these simulations were used to determine how material properties of the STF affect the dynamic response of a multilayer plate.

Chapter Two: The Reduced Order Model

The proposed multilayer plate design outlined in the previous chapter is an intriguing design concept, but the feasibility and effectiveness of such a design should be investigated. In this chapter, a reduced order model of a multilayer plate representing a vehicle floor is developed in LS-DYNA. Coulomb friction is used as a simplified model of the viscous effects of a STF. The results from subjecting the plate to a dynamic load is used to evaluate whether or not a damping mechanism such as a STF would be effective at reducing the dynamic response of a multilayer plate.

2.1 LS-DYNA Modeling

LS-DYNA is finite element analysis (FEA) code developed by Livermore Software Technology Corporation (LSTC) that is also capable of multiphysics analysis. It originates from the 3D FEA program DYNA3D, which was developed by the Lawrence Livermore National Laboratory for the purpose of simulating the impact of the Full Fusing Option (FUFO) nuclear bomb from low altitude release [Benson, 2007]. Due to its history and the further development of the program for the automotive industry, LS-DYNA's applications are strongly focused on the dynamic response of structures consisting of a variety of material types to impact loading conditions [LSTC, 2018]. This capability as well as ease of use and availability of LS-DYNA contributed to its selection as the software to use for modeling the dynamic response of the proposed plate design.

LS-DYNA uses the pre- and post-processor LS-PrePost to generate its simulation files and view its output files. The simulation files consist of information cards called keywords.

Keywords contain information ranging from material properties to mesh information to loading conditions.

The plate loading conditions are based on blast loading conditions examined in the Federal Emergency Management Agency (FEMA) Building Design for Homeland Security report [FEMA 426]. Table 1 gives the expected structural damage based upon the blast's incident pressure as reported by FEMA. A point of interest is that FEMA used TNT blast data to develop this table and its values.

Table 1. Damage to structures based upon incident pressure from blast load [FEMA 426]

Damage	Incident Pressure (psi)	Incident Pressure (kPa)
Panels of sheet metal buckled	1.1-1.8	7.6-12.4
Failure of unreinforced concrete block walls	1.8-2.9	12.4-20
Serious damage to steel framed buildings	4-7	27.6-48.3
Serious damage to reinforced concrete structures	6-9	41.4-62.1

The Friedlander waveform was used to construct the loading curve for the simulation.

Figure 4 shows an example of the pressure loading curve generated by the Friedlander waveform, which is described in equation 1 [Goel et al, 2012]. The main variables of interest are the peak pressure, P_{SO} , the time of arrival, t_A , and time of duration, t_o . The Friedlander waveform is used to represent blast loads that originate from a single point, such as an improvised explosive device (IED), on a flat plane with no obstructions between the point of origin and the point the load is measured [Souichi, 2006]. The impulsive loads from these blast loads are calculated by integrating the Friedlander waveform function with respect to time.

$$P(t) = P_{SO} e^{\frac{-(t-t_a)}{t_o}} \left(1 - \frac{t-t_a}{t_o}\right) \quad (1)$$

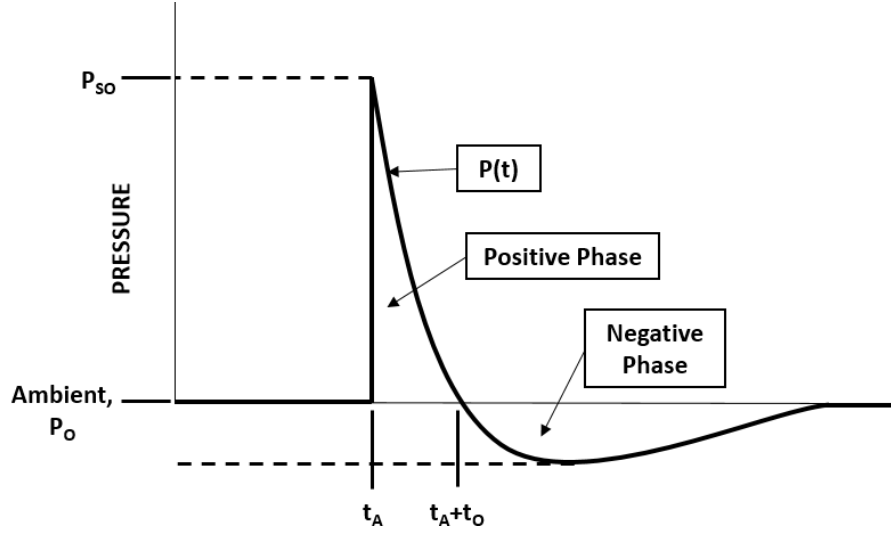


Figure 4. Friedlander waveform for blast load

The generated pressure-time curve from the Friedlander wave equation was input into LS-DYNA in table form. This was done by inputting the values given in table 2 below into the keyword *DEFINE_CURVE. These values correspond to a time of arrival at zero seconds, meaning the plate was instantaneously loaded; t_o was set to 0.01 seconds. The peak pressure was 50 kPa to represent a serious blast load based upon the information given in Table 1. The underside face of the plate is loaded by inputting the defined curve into the keyword *LOAD_SEGMENT.

Table 2. Time and pressure values used to define Friedlander waveform in LS-DYNA

Time (s)	Pressure (kPa)
0.00	50.00
0.001	40.72
0.0025	29.21
0.005	15.16
0.01	0.00

To measure the effectiveness of implementing friction in the multilayer plate, the total energy of the plate was measured at each time step in LS-DYNA. Equation 2a is the equation

used by LS-DYNA to measure total energy. This equation states that the total energy, E_{total} , of the part at each time step is equal to the current kinetic energy E_{kin} , current internal energy (due to deformation or bending), E_{int} , the current interface energy, E_{si} , the current rigid wall energy, E_{rw} , the current damping energy, E_{damp} , and the current hourglass energy, E_{hg} . The total energy is also defined by equation 2b. In this equation, the total energy is equal to the initial kinetic energy of the part, its initial internal energy, and the external work done during the simulation. It should be noted that since the plate is not given an initial velocity and is not initially deformed. Therefore, the total energy is equal to the work done by external forces acting on the model. Therefore, the results will show that the total energy in the simulation increases with time.

$$E_{total} = E_{kin} + E_{int} + E_{si} + E_{rw} + E_{damp} + E_{hg} \quad (2a)$$

$$E_{total} = E_{kin}^0 + E_{int}^0 + W_{ext} \quad (2b)$$

The sliding energy, E_{si} , is equal to the sum of the work done by friction along the selected parts' interfaces and the energy stored in the contact model, as shown in equations 3. The first term is the energy stored in the contact model, which has a stiffness of k_s . The second term is the work done by friction along the interface; χ represents the coefficient of friction.

$$E_{si} = \frac{1}{2} k_s \Delta_z^2 + \chi \Delta_y F_z \quad (3a)$$

LS-DYNA uses the keyword *CONTACT to define the interaction of two parts with separate meshes. One mesh is defined as the master mesh, while the other mesh is defined as the slave mesh. Nodes of the slave mesh are prevented from penetrating the element surfaces of the master mesh by imposing a contact stiffness force on the slave mesh that acts normal to the contact interface of the two parts and only appears during penetration. This contact stiffness can be enacted to prevent the master mesh from penetrating the slave mesh in newer keywords. For

the simulations used in this thesis, the keyword `*CONTACT AUTOMATIC` `_SURFACE_TO_SURFACE` was used to define the contact between the plate layers. This keyword enacted contact stiffness to act on both the master and slave meshes. Figure 5 illustrates two elements from two different parts in contact. The different parts are denoted using differing element colors, blue and black. The two elements on the left hand side are in contact but not penetrating; the area of contact, A , is highlighted in yellow. The elements in the center of the figure are penetrating a distance, Δ_z , which will engage the contact stiffness. The work done by friction is a result of the elements coming into contact with each other and then slide a distance, Δ_y .

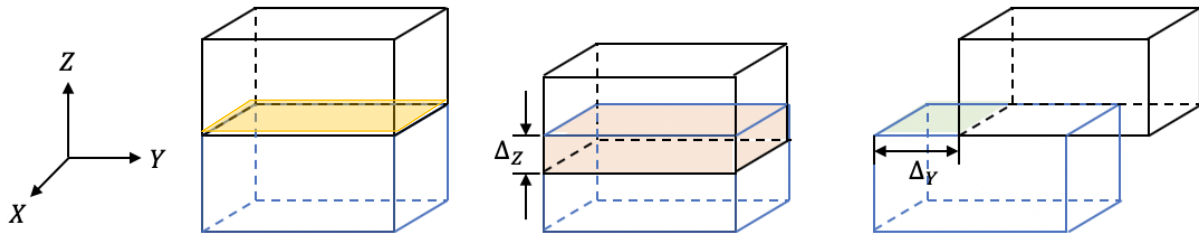


Figure 5. Various contact states: on the left elements are in contact but not penetrating, the elements in the center are penetrating each other, and the elements on the right are in contact but sliding past each other.

Equation 4 defines the contact stiffness, k_s , in terms of the volume of the element being penetrated, V , the area of the element surface being penetrated, A , the bulk modulus of the element material, K , and the penalty factor, α .

$$k_s = \frac{\alpha K A^2}{V} \quad (4)$$

Coulomb friction is used along the interface, given by equation 5a. Coulomb friction uses the static friction coefficient, χ_s , the dynamic friction coefficient, χ_D , and the decay coefficient, DC . The values published by Dr Wetzel and his colleagues were used to represent the effects of STFs on fabric fibers were used to define the frictional coefficients, given in equations 5b, 5c, and 5d below [Wetzel et al, 2008].

$$\chi = \chi_D + (\chi_S - \chi_D)e^{-DC|v_{rel}|} \quad (5a)$$

$$\chi_D = 1.42 \quad (5b)$$

$$\chi_S = 1.53 \quad (5c)$$

$$DC = 1.23 \quad (5d)$$

The sliding energy should be small if friction is not present. If it becomes negative, then the slave mesh has penetrated the master mesh to a degree that the simulation should be stopped and α should be increased. When friction is present, the sliding energy is a metric used to determine the energy dissipated by friction.

Other terms defined in equation 2a are not of interest. Hourglass energy should not appear in the simulation if the elements are properly formed. Damping energy only appears if linear damper elements are used in the model. Rigid wall energy is only used in simulations where parts impact a rigid boundary, such as a wall. Hence for these simulations, the main variables of interest are the total, kinetic, internal, and sliding energies.

The basic plate model configuration is shown in Figure 6. The multilayer plate represents a vehicle floor and consists of two layers that do not share nodes between their meshes. These separate meshes allow sliding to occur along the layer interfaces. The layers are held together using stiff springs around the edge of the plate. A point mass is attached to the top layer using

rigid nodal connections to simulate the vehicle weight. The contact between the two layers has friction; however, the frictional coefficients can be set equal to zero.

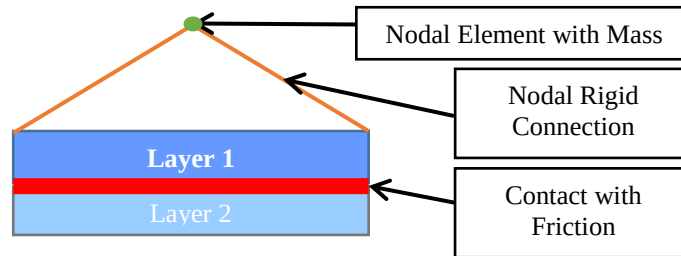


Figure 6. Schematic of simplified model of plate used in simulations; the plate represents a vehicle floor while the nodal mass element represents the vehicle weight

2.2.2. Small Plate Simulations

The initial simulation consisted of a concave multilayer plate that represented the floor of a vehicle, shown in figure 7. The plate has two layers, shown in red and blue. The plate layers are made of steel, with a mass density of 7820 kg/m^3 , a Poisson's ratio of 0.3, and a Young's modulus of elasticity of 200 GPa. The simulation of the plate was an explicit time simulation, with a time step of $1\text{E-}4$ seconds. The length time simulated was 0.1 seconds.

The plate dimensions for this simulation are 0.1 by 0.1 meters wide with each layer one centimeter thick. The point mass is equal to 10,000 kg. This mass is far larger than what would be expected in a civilian vehicle, but is meant to represent an armored vehicle. The spring stiffness of the springs binding the two plate layers together is 1,000 MN/m. The curvature of the plate is based upon the curvature used in truck leaf springs.

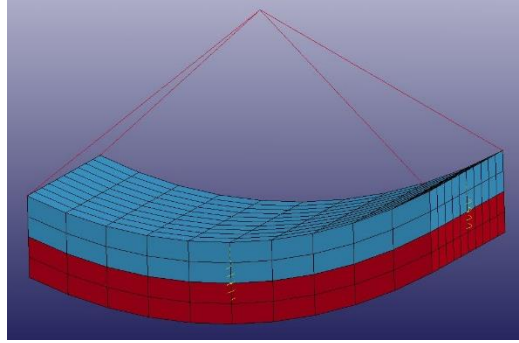


Figure 7. Initial reduced order model of curved plate

Figures 8 and 9 show the total energy and sliding energy results of the model for simulations with and without friction plotted against the simulation time. Looking at these, it is apparent that the addition of friction has a large impact upon the results of the plate.

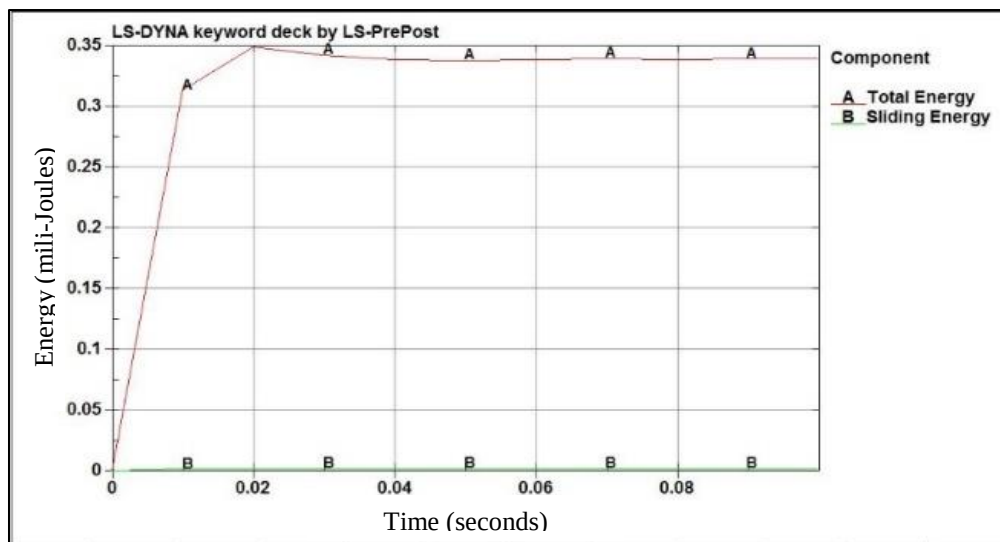


Figure 8. Total energy and sliding energy from the curved plate without friction plotted against time

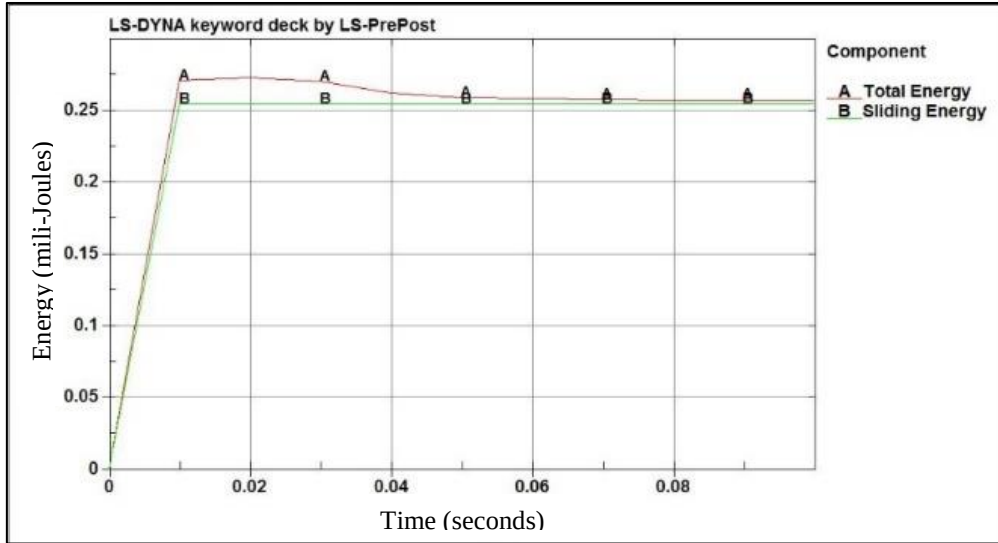


Figure 9. Total energy and sliding energy from the curved plate with friction plotted against time

The first observation made about these two data sets is that the total energy in the plate significantly decreases for the simulations with friction. Because the plate does not have an initial velocity or displacement at the beginning of the simulation, the total energy plotted at each time step is equal to the work done by the external forces at each time step. By introducing friction into the model, almost all of the work done by the external forces is converted into work done by friction, represented by the sliding energy.

The second main observation about this data is that the sliding energy is almost equal to the total energy in simulations with friction, meaning that most of the work done by the external loading force is accounted for by frictional dissipation. There is a significant reduction in the internal energy of the system when friction is present since the total energy is the sum of the kinetic, internal, and sliding energies. Therefore, the dynamic response of the plate is significantly reduced. The fact that at the end of the simulation almost ninety nine percent of the total energy in the system reflects that implementing a damping mechanism along the plate layer interfaces can significantly impact the plate's dynamic response.

Another plate configuration tested was a v-hull shape, shown in Figure 10. A ten centimeter by ten centimeter plate was modeled. The blast load used in this simulation is the same as the previously discussed simulation. This simulation also used the same point mass value to represent the vehicle's mass with rigid connections to the edges of the plate. The plate layers were bound with the same type of spring element as the previously discussed simulation.

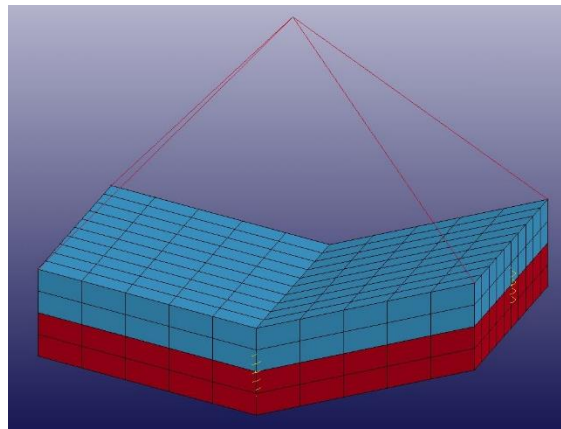


Figure 10. Initial reduced order model V-Shaped plate

Figures 11 and 12 show the results for the total energy and sliding energy for this model with and without friction plotted against the simulation time. Again, the total energy of the plate decreases when friction is present, indicating that the total sum of the kinetic, potential, and sliding energies decreases with friction. Similarly to the previously discussed model's results, almost the entirety of the total energy of the system is accounted for by sliding energy when friction is implemented into the simulation. By the end of the simulation, almost ninety nine percent of the total energy was attributed to the sliding energy, meaning that less than two percent of the total energy in the plate could be accounted for by kinetic or internal energy. Therefore, most of the energy input to the plate by the blast load was dissipated through the contact friction. This results further supports the conclusion that the addition of a shear based

damping mechanism along the contact interface could have significant effects on the response of the plates.

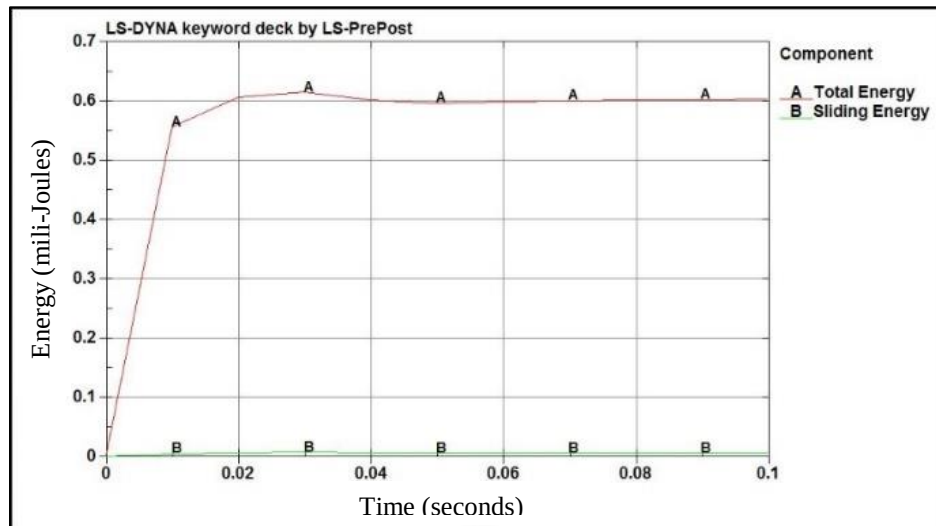


Figure 11. Total energy and sliding energy for the v-shaped plate without friction plotted against time

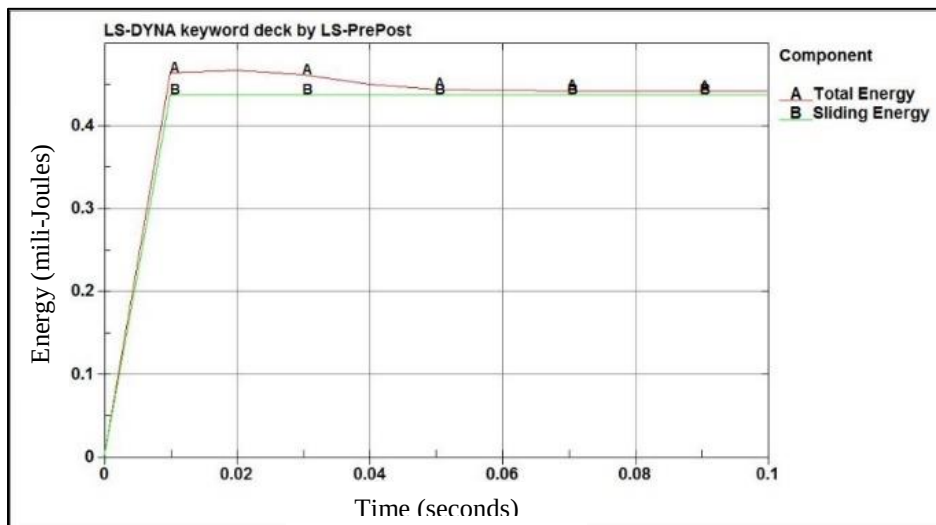


Figure 12. Total energy and sliding energy for the v-shaped plate with friction plotted against time

Table 3 shows the resulting energy values for both plate models at the end of their simulations. The difference between the total energy and the sliding energy is the sum of the kinetic and potential energy of the plate since hourglassing did not occur. For both the curved plate and v-shaped plate, implementing friction between the plate layers significantly reduced the internal and kinetic energy of the plate both in magnitude and the percent of the total energy they

represent. Therefore, it can be assumed the dynamic response of the plate is reduced. Based upon these conclusions a larger scale model was produced in LS-DYNA to better understand how a friction-like damping mechanism between plate layers was generated.

Table 3. Energy Results at end of simulation for both curved and v-shaped plates

	Total Energy (mili-Joules)	Sliding Energy (mili- Joules)	Difference (mili- Joules)	Percent Total Energy Accounted for by Internal and Kinetic Energy	Percent Total Energy Accounted for by Sliding energy
Curved Plate – No Friction	0.34	0.001	0.339	~100	>0.001
Curved Plate – With Friction	0.258	0.255	0.003	1.16	98.84
V-Shaped Plate – No Friction	0.6	0.01	0.599	99.83	0.17
V-Shaped Plate – With Friction	0.441	0.438	0.003	0.68	99.32

2.2.3 Large Scale LS-DYNA Simulations

Finally, larger simulations were conducted in LS-DYNA to investigate how implementing friction between the plate layers affects the energy absorption of the plate. Figure 13 shows a screen capture of the model. For this model, a concave one meter by one meter plate was used to represent the vehicle floor. The point element used to represent the rest of the vehicle structure had a mass of 1E7 kg. The top of the plate was rigidly attached to the point element every ten nodes, shown by the red lines in figure 13. Two plate layers were modeled with separate meshes that did not share nodes. The layers were bound together using the spring elements previously described. The same loading condition was applied to the plate as the

previous model. Because the surface area of the plate increased, the overall force from the pressure load applied increases as well.

An explicit solver was used to simulate the response of the plate over 0.1 seconds. The blast load started at the beginning of the simulation with a time of duration of 0.01 seconds. The characteristic element length of the mesh is one centimeter. An eight point node element was used in the simulation to prevent hourglassing.

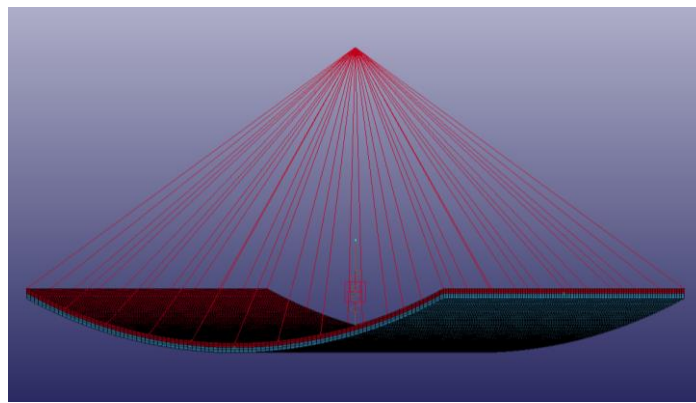


Figure 13. Larger plate simulation for LS-DYNA

Figures 14 and 15 shows the results of this simulation for the plate. The light blue line is the total energy in the simulation. The internal energy, which is equal to the plate's bending energy, is shown in green. The kinetic energy of the model is in red, and the sliding energy is shown in dark blue. For both the simulations with and without friction, the total energy at the end of the simulation is about the same.

Interestingly, the peak in total energy peak that beings around 0.02 seconds into the simulation significantly decreases when friction is present. As expected, the sliding energy of the simulation becomes significant when friction is present. The results also show that introducing friction into the plate simulation significantly reduces the kinetic energy of the plate after the time of duration of the blast and the internal energy in the plate.

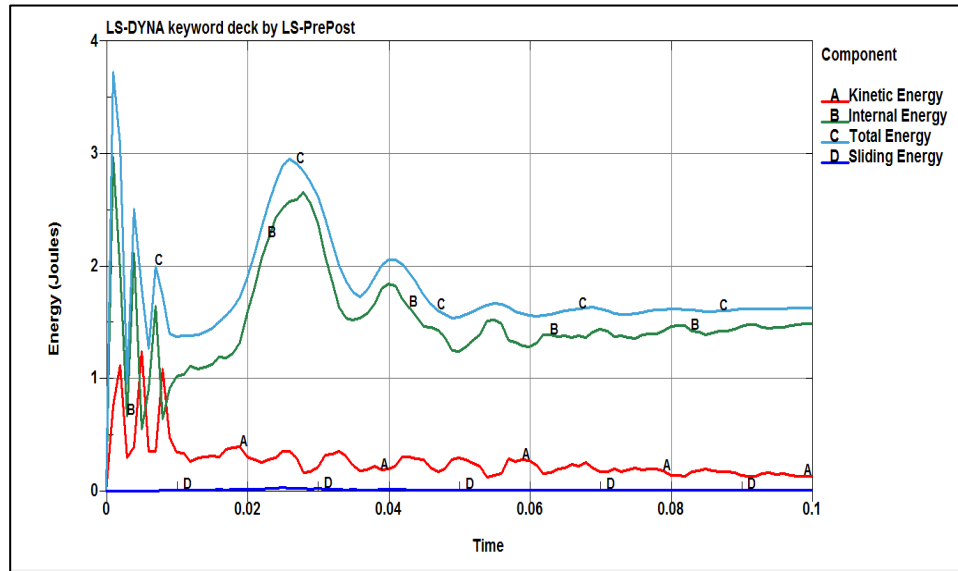


Figure 14. LS-DYNA Energy values for plate without friction plotted against time

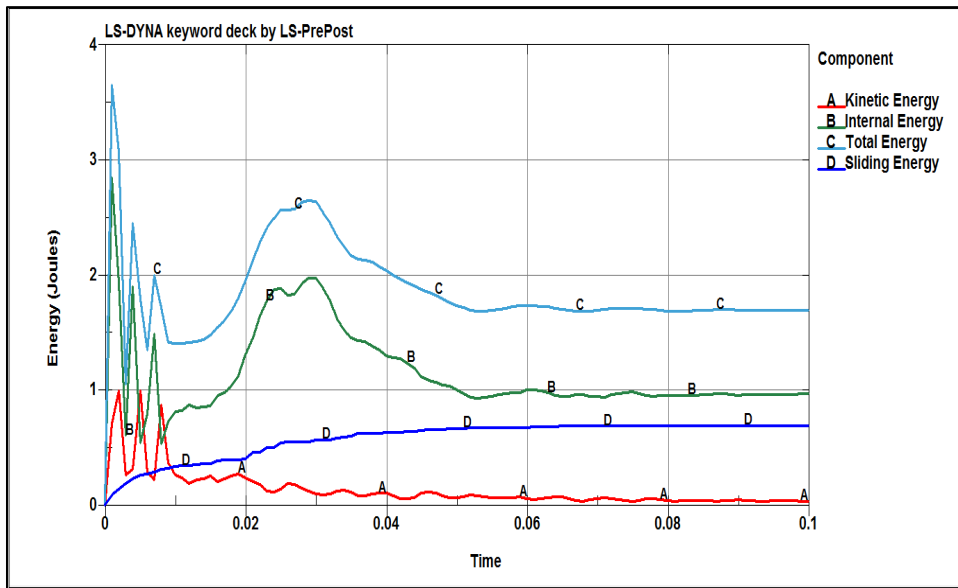


Figure 15. LS-DYNA Energy values for plate with friction plotted against time

It should be noted though that with the larger plate, the sliding energy accounts for only 41 percent of the total energy at the end of the simulation. These values are reflected in table 4. While this is not as dramatic of a change in energy distribution in the system as observed in the previous simulation, it still shows friction significantly affects the deformation of the plate.

Table 4. Total energy and sliding energy results for larger scale curved plate model at the end of the simulation

	Total Energy (mili-Joules)	Sliding Energy (mili- Joules)	Difference (mili- Joules)	Percent Total Energy Accounted for by Internal and Kinetic Energy	Percent Total Energy Accounted for by Sliding energy
No Friction	1.68	0.001	1.68	~100	>0.001
With Friction	1.73	0.71	1.01	69	41

Figure 16 shows the measured vertical displacement measured at the node located at the center of plate on the side opposite to the blast load plotted against time. The results for the simulation without friction are plotted in red. The vertical response of the plate with friction is plotted in green. These results show that when friction is present the vertical response is significantly dampened as time progresses in comparison to the vertical response when no friction is used. Therefore, adding in a damping mechanism similar to friction between the plate layers will yield a significant decrease in the dynamic response of a multilayer plate.

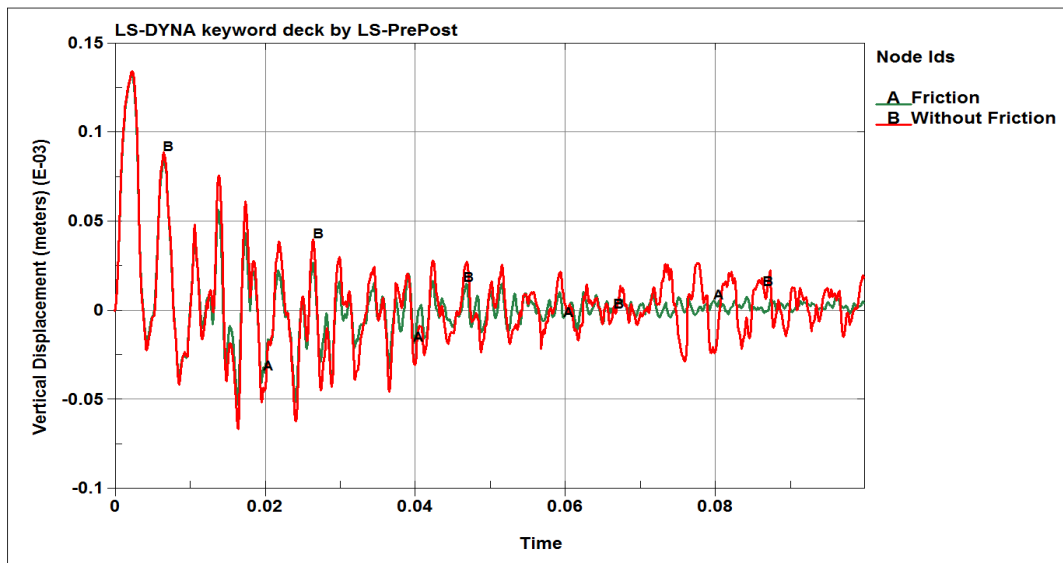


Figure 16. Vertical response of plate with and without friction at the top, center of the plate

Multilayer plates often usually comprised of layers of different materials, rather than layers of a single material. To see the effect of implementing friction along the interface of two varying materials, the material in the second layer was changed to aluminum with a Young's modulus of 73.7 GPa, a mass density of 2700 kg per cubic meter, and a Poisson's ratio of 0.334.

Figures 17 and 18 show the energy values for the plate with a bottom layer of aluminum and top layer of steel with and without friction plotted against time. Unexpectedly, the total energy, shown in light blue, exhibits a large peak around 0.03 seconds in the simulation without friction. Observing the plate deformations in LS-DYNA's postprocessing showed that at this time in the simulation the lower layer of the plate, the aluminum layer, forcefully clashed with the top layer. The large internal energy results from large deformations in the bottom layer, which is aluminum, as it strikes the top layer. It should also be noted that there is a small peak in the sliding energy at this point too for the simulation without friction. This indicates that the deformations were so large that the layers began to penetrate each other slightly.

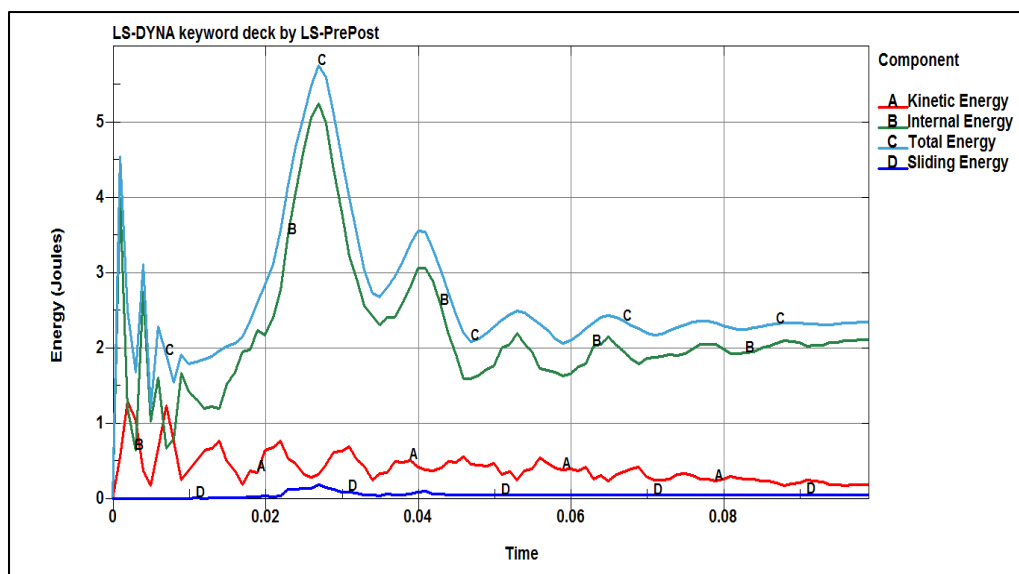


Figure 17. The energy results for the model with steel and aluminum plate layers without friction plotted against time

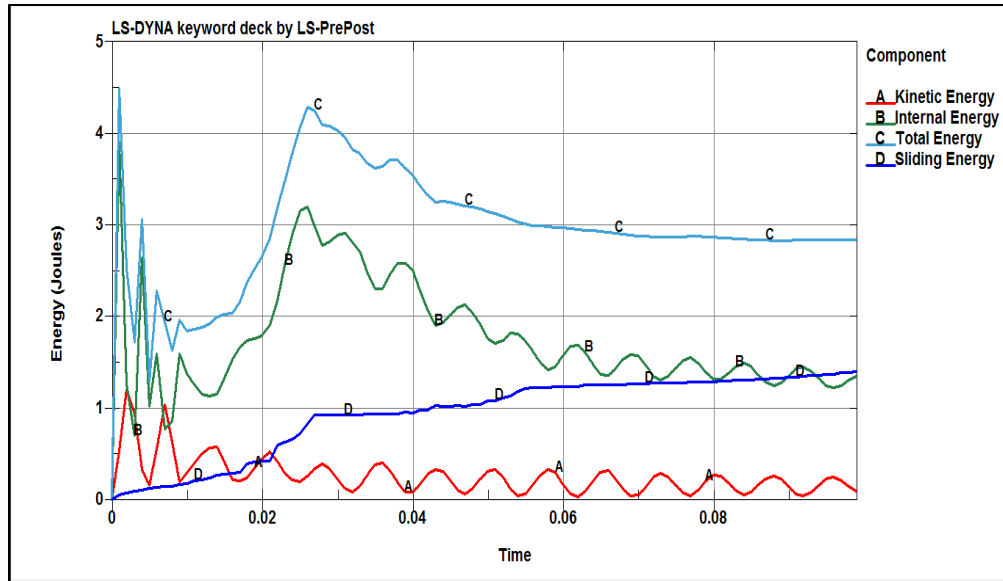


Figure 18. The energy results for the model with steel and aluminum plate layers with friction plotted against time

Implementing friction into the simulation significantly decreased the internal energy, which had a significant effect on the peak in total energy observed at about 0.03 seconds in the simulation. At the end of the simulation sliding energy, which takes into account the energy dissipated by friction along the plate layer interface, accounts for about 50 percent of the total energy of the simulation with friction. This is larger than the percentage of the total energy than the percent total energy sliding energy accounted for at the end of the steel only simulation with friction. These results are expected since the work done by friction is directly proportional to the relative motion of the two layers. The aluminum-steel plate exhibits larger deformations than the steel only plate when neither have friction along their layer interfaces; therefore it makes sense that more energy would be lost through the work done by friction in the aluminum-steel plate.

The total energy at the end of the simulations is higher for the simulation with friction than the simulation without friction. This is because the sliding energy has significantly increased at this time step due to the work done by friction. Table 5 shows the results for this simulation at the end of the simulation time.

Table 5. Total energy and friction energy for the aluminum-steel plate at the end of simulation

	Total Energy (mili-Joules)	Sliding Energy (mili- Joules)	Difference (mili- Joules)	Percent Total Energy Accounted for by Internal and Kinetic Energy	Percent Total Energy Accounted for by Sliding energy
No Friction	2.43	0.001	2.429	~100	>0.001
With Friction	2.8	1.41	1.39	50	50

To determine that the method of attaching the point mass to the plate was not affecting the simulation's results by over stiffening the top layer further analysis was performed. In these simulations the same plate was used with both layers made of steel, but the point mass was distributed to point masses located every ten centimeters along the top edge of the plate, highlighted in yellow in Figure 19.

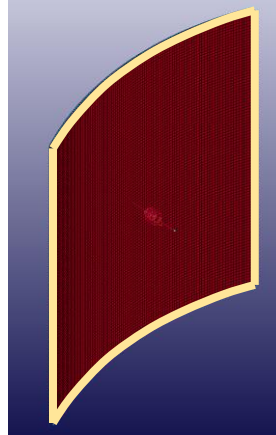


Figure 19. New plate simulation with new point attachments

Figures 20 and 21 shows the results from these simulations. The same colors are used in these figures as the previous figures to represent the different energy values. It should be noted that changing the method of attaching the point mass, which represents the weight of the vehicle, to the plate, which represented the vehicle floor, does not have a significant effect on how much of the total energy is accounted for by the sliding energy at the end of the simulation. Overall, the

values from these simulations were in similar to those shown in figure 14 and 15, which is to be expected.

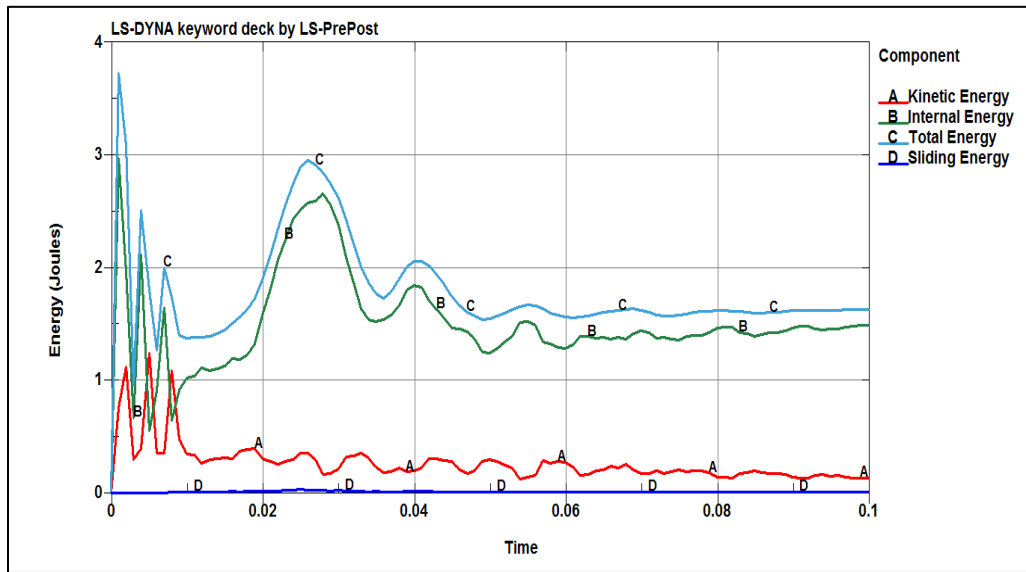


Figure 20. The energy results for the steel plate with modified boundary conditions and no friction plotted against time

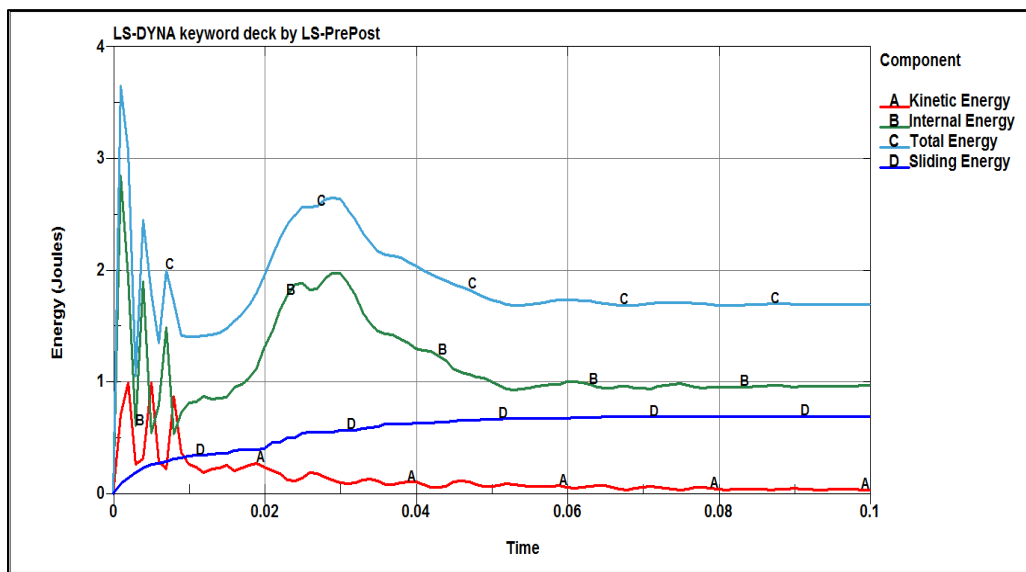


Figure 21. The energy results for the steel plate with modified boundary conditions and friction plotted against time

Table 6 shows the comparison for the energy terms in the steel plate. The values in this table agree as the values given in table 4. This further reflects that the boundary conditions are not affecting the results.

Table 6. Total energy and friction energy for steel plate with new boundary conditions at the end of the simulation

	Total Energy (mili-Joules)	Sliding Energy (mili- Joules)	Difference (mili- Joules)	Percent Total Energy Accounted for by Internal and Kinetic Energy	Percent Total Energy Accounted for by Sliding energy
No Friction	1.71	0.001	1.71	~100	>0.001
With Friction	1.73	0.74	0.99	71	39

As expected by the energy results, the dynamic response of the plate is significantly affected by the presence of friction. To illustrate this, the vertical response of two nodes were recorded during the simulation. Figure 22 illustrates where the nodes of interest are located on the plate. The first node is the node in the center of the plate face that is on the opposite side of the face being loaded, represented as a red dot. The second node is located at the center of the face of the plate that is loaded with the pressure wave, represented as a green dot .

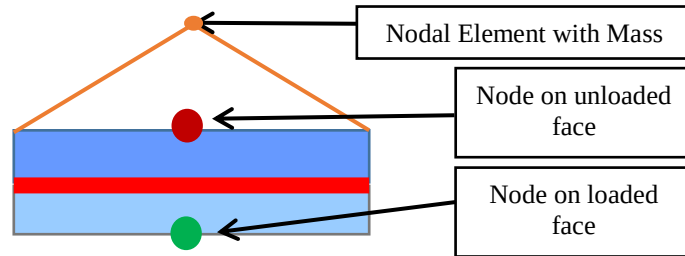


Figure 22. Illustration of node sampling

Figure 23 shows the vertical response recorded at the node located at the center, top side of the upper layer of the plate in red and the vertical response recorded at the node located on the center of the bottom side of the bottom layer of the plate in green. On the left are the results for the simulation without friction while on the right are the results for the plate with friction. Both simulations show that during the blast event the plate layers have very similar responses. After the time of duration of 0.01 seconds the top layer's response is far smaller in magnitude than the

bottom layer for both simulations. The main difference in the response is that the bottom layer has a peak displacement on the order of one tenth of a millimeter. Once friction is introduced this peak decreases almost 25 percent.

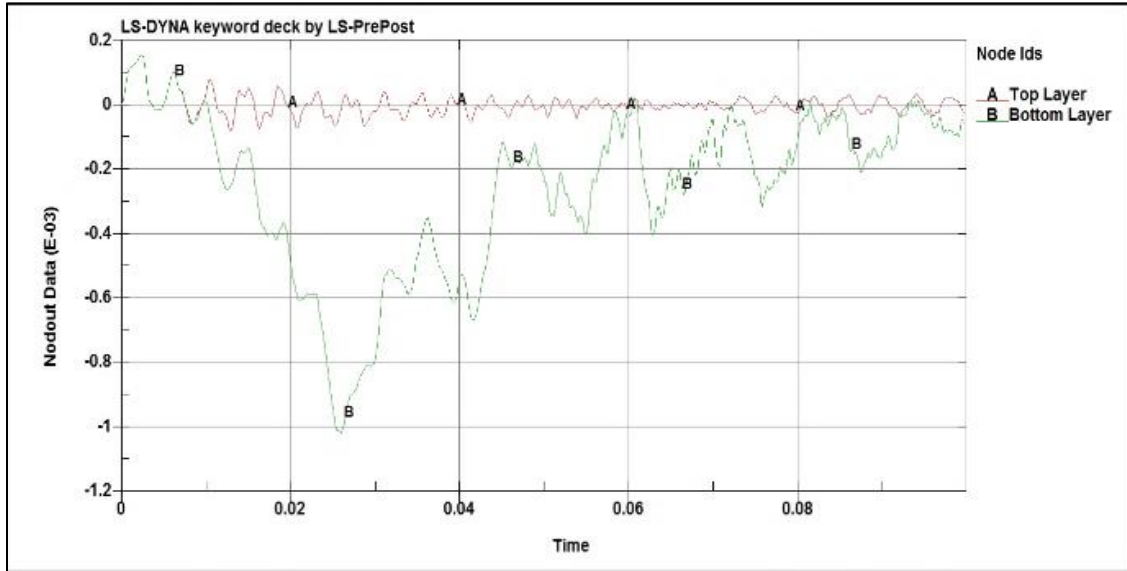


Figure 23. Vertical response of reduced order plate without friction

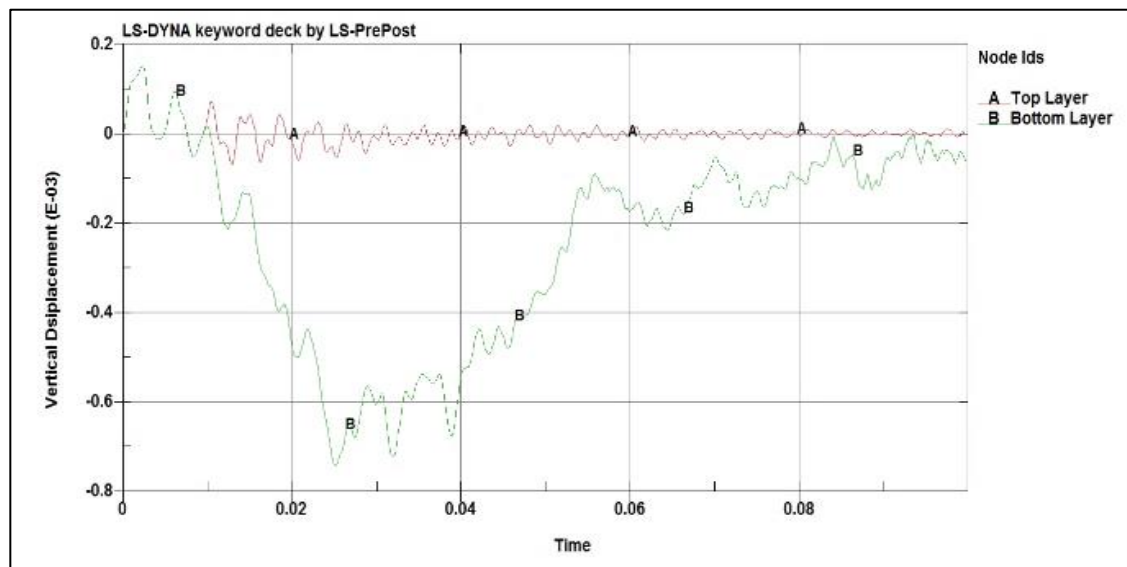


Figure 24 Vertical response of the multilayer plate with friction

2.3 Further Analysis

A reduced order model of a multilayer plate acting as a vehicle floor was developed in LS-DYNA. The plate layers consisted of combinations of steel and aluminum, two metals widely

used in vehicle armor and structure. The model was subjected to a blast loading condition congruent to loading conditions observed in combat.

The LS-DYNA simulations show that implementing a simple damping mechanism along the interface of the two plate layers had a significant effect on the amount of energy dissipated during the simulation. This result was observed regardless of the material configuration used in the plate layers and the boundary conditions applied to the plate edges. More importantly, this energy dissipation resulted in a decrease in the plate's vertical dynamic response regardless of the previous parameters mentioned. This decrease in the dynamic response observed by measuring nodal displacements and by the significant decrease in the bending energy of the plate when friction is present.

These results show that implementing a friction-like damping mechanism, like a layer of a STF, into multilayer plates could improve the dynamic response of these plates. Based upon this study it was determined that the research should move forward with developing the multiscale modeling method.

Chapter Three: Modeling Shear Thickening Fluids in STAR-CCM+

STFs consist of two phases: a fluid phase and solid particles. The simulation method used to develop the small scale model of both phases is to couple the discrete element method (DEM) into a CFD program. Implementing DEM into a CFD program has been a topic of interest for over the last twenty years. The first DEM-CFD model was reported in literature in 1992 for plug flow [Tsuji et al, 2012]. After this publication, coupled DEM-CFD simulations have been utilized for a variety of applications. One of the main applications of DEM-CFD simulations is to model fluidized beds that show that DEM can be successfully implemented into a CFD simulation to study multiphase behavior [Deen et al, 2007. Rhodes et al, 2001. Zhao, Shan, 2013]. DEM-CFD has also been used to simulate the behavior of wet granular materials [Apostolou, Hrymak, 2008]. Despite the variety of applications of the DEM-CFD model, so far it has not been used to study the particle behavior observed in STFs.

In this chapter, the causes of shear thickening in STFs are described, then the theory used to simulate the STF is presented along with the modeling method, and finally the results of this small scale model are presented.

3.1 Shear Thickening Behavior

Despite their name, shear thickening fluids often consist of a fluid phase with densely packed, small particles. These fluid-particle mixtures often fall into the category of colloidal suspensions. Colloidal suspensions are suspensions with particles ranging from two to 500

nanometers in diameter. The particles do not settle out of the fluid phase when the suspension is at rest [Wagner, Brady, 2009].

The two main causes of shear thickening in colloidal suspensions are hydroclusters and dilatancy[Mewis, Wagner, 2012. Gurgun et al, 2016. Brown, Jaeger, 2014. Barnes, 1989]. Both of these mechanisms relate to the particles' behavior within the fluid phase. This dependence on particle behavior is expected since STFs have high packing fractions. The volume packing fractions in STFs can range from 0.4 up to 0.62 [Brown, Jaeger, 2014].

Hydroclusters are clusters of particles that form when the applied forces on the particles overcome the lubrication forces between the individual particles that arise due to squeeze flow [Mewis, Wagner, 2012]. These clusters do not easily flow past each other due to their size and the larger lubrication forces that arise between them. This flow resistance causes the observed viscosity to increase as the particle clusters are unable to flow past each other. Figure 25 illustrates the formation of hydroclusters.

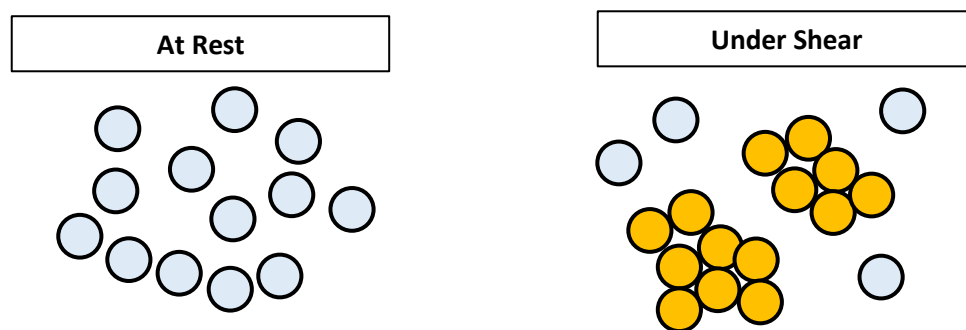


Figure 25. Formation of hydroclusters: particle are randomly distributed throughout the fluid phase until subjected to large shear rates. At these shear rates lubrication forces between particle clusters (shown in yellow) become significant and push the formed clusters away from each other, generating hydroclusters

Dilatancy is a granular material property where the total volume occupied by the particles increases at high shear rates due to the particles being unable to slide past one another due to frictional particle-to-particle forces [Gurgun et al, 2016. Barnes, 1989]. Dilatancy is dependent on the shear strain exerted on the material, but not the shear rate exerted on the material [Bolton,

1986]. It is linked to the shear rate dependent behavior of STF's by the fact that when a material is subjected to a shear rate for a constant time, larger shear strains will be observed for higher shear rates. The increase in volume increases the normal forces acting on the particles, causing the frictional forces between the particles and the STF's wall boundary conditions to increase.

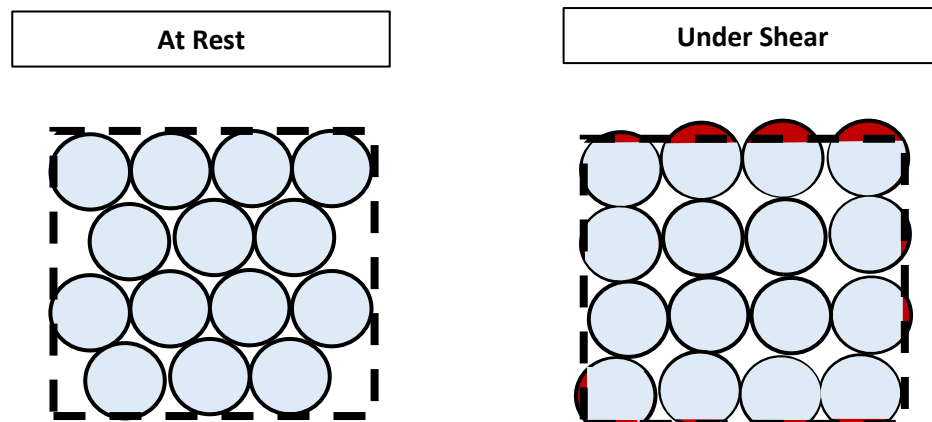


Figure 26. Example of dilatancy. The particles on the left are at rest and fit into the fixed volume outlined in black dashes. When they are subjected to shear, as shown on the right, they move and occupy space outside their original volume.

While both of these mechanisms are recognized as significant causes of the non-Newtonian behavior of STF's, there is not an agreed upon, explicit relationship between the observed viscosity and these behaviors [Brown, Jaeger, 2014]. This research will focus on modeling both of these phenomena.

3.2 Discrete Element Method

The discrete element method (DEM) is a versatile numerical method that describes the mechanical behavior of a large number of independent and separate particles. It is used a variety of fields, such as geophysics and the agricultural and food industry, to model the behavior of granular materials and solid continua [Luding, 2008]. DEM differs from molecular dynamics in its inclusion of rotational degrees of freedom and use of contact models [Rojeck et al, 2012].

The basic scheme of DEM is as follows. First, the domain in which the particles reside and its boundary conditions are defined. Then the number of particles, their size, shape, location, and initial velocity are defined. The forces acting on each particle is solved for at each time step. These forces include, but are not limited to, body forces, contact forces, adhesion forces, and drag forces. Unlike other particle based methods, such as smoothed-particle hydrodynamics (SPH), the effect of particles on each other are not weighted using a kernel function [Luding, 2008]. Equation 5 lists the forces solved for at each time step in STAR-CCM+. There are body forces, F_{body} , contact forces from particle collision, $F_{contact}$, drag forces, F_{drag} , and any user-defined forces such as lubrication forces, F_{user} . The lubrication forces will be discussed further on in this chapter.

$$m_{particle} \frac{dv}{dt} = F_{drag} + F_{body} + F_{contact} + F_{user} \quad (5)$$

Soft particle DEM assumes that particles can slightly overlap each other and the wall boundary conditions and uses the distance of overlap to calculate the restorative force [Luding, 2008. Rojeck et al, 2012]. The contact models used to calculate the restorative force range from linear springs and dampers to more complicated models such as the Hertz-Mindlin contact model. Figure 27 below shows an example of the particle-to-particle interactions with simplified spring and dampers relating the particle motions to each other.

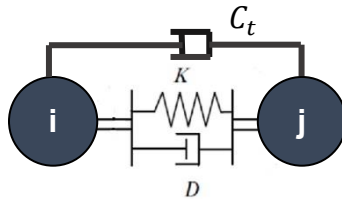


Figure 27. The contact model between particle i and particle j . There are the normal spring stiffness, k , the normal damping constant, D , and the tangential damping coefficient, C_t .

3.3 Initial Modeling Attempt Using LS-DYNA

Due to the ease of working with LS-DYNA and its versatility and effort was made to use LS-DYNA's DEM capabilities. LS-DYNA's DEM solver was developed mainly to model dry continuum, such as powders and soil particles [Karajan, 2013]. Moisture can be modeled, however the particles are not assumed to be fully surrounded by the liquid. Instead, LS-DYNA simulates a 'fluid bridge' between the particles by modifying the particle contact model to include capillary forces that hold the particles together. These parameters then are input into the LS-DYNA control card *CONTROL_DISCRETE_ELEMENT.

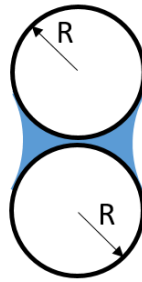


Figure 28. Particles with fluid bridge between them; the fluid bridge does not completely surround the particles and can be broken

However, this model is not appropriate for a multiphase material where particles are mixed into the fluid such as a STF. Due to the contact model limitations and other simulation issues, efforts to model a STF with LS-DYNA were not continued.

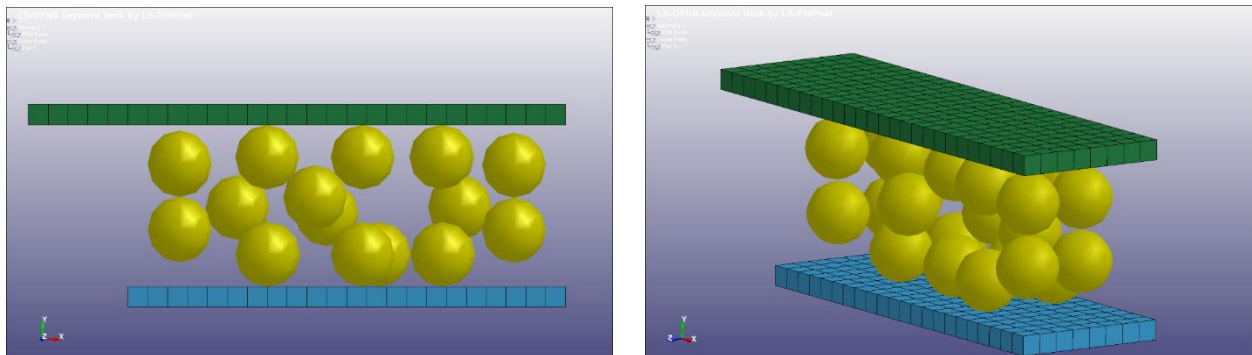


Figure 29. Example of LS-DYNA DEM model used to simulate a Couette flow; particles are constrained to stay between the two plates as the top green plate moves laterally

3.4 STAR-CCM+ Methodology

STAR-CCM+ is a computational fluid dynamic (CFD) program with multi-phase modeling capabilities. It was developed by CD-adapco (Computational Dynamics-Analysis & Design Application Company Ltd) in 2004. It was selected for modeling due to its multiphase capabilities and license availability.

For the normal contact stiffness model, the Hertz-Midlin contact model is used. This non-linear model incorporates the particles' Young's modulus, E_p , Poisson's ratio, ν_p , radius, R , the distance overlap, d , the wall's Young's modulus, E_w , and Poisson's ratio, ν_w . Equation 6a describes the normal contact force between the particle and wall [STAR-CCM+, 2017]. Equation 6b describes the normal contact force between the particles [STAR-CCM+, 2017]. Tangential contact model forces are calculated by multiplying a frictional coefficient, χ , by the normal force, as shown in equation 7.

$$\mathbf{F}_{Contact}^N = \frac{4}{3} \left(\frac{E_p E_w}{\frac{(1-\nu_w^2)}{E_p} + \frac{(1-\nu_p^2)}{E_w}} \right) \left(\frac{R}{2} \right)^{\frac{1}{2}} d^{\frac{3}{2}} \quad (6a)$$

$$\mathbf{F}_{Contact}^N = \frac{\frac{4}{3} E_p}{1-\nu_p^2} \left(\frac{R}{2} \right)^{\frac{1}{2}} d^{\frac{3}{2}} \quad (6b)$$

$$\mathbf{F}_{Contact}^T = \chi \mathbf{F}_{Contact}^N \quad (7)$$

3.4.1 STAR-CCM+ Multi-phase Modeling

STAR-CCM+ has a variety of multi-phase modeling capabilities. These capabilities can be used to model multiple Eulerian phases, such as two fluids mixing, or Lagrangian and Eulerian phases, such as DEM particles in a fluid. This research uses the later capability.

To model the particle and fluid phases, the volume of interest is first meshed with an Eulerian mesh. After the Eulerian mesh is completed, an injector is defined to identify where the particles will be placed. STAR-CCM+ has three main types of injectors: point injectors, surface injectors, and part injectors. Point injectors are single point locations at where particles are generated. Surface injectors are surfaces where particles are generated; they can be on regional faces or on a user defined plane. Part injectors, unsurprisingly, are pre-defined parts in which particles are generated in. For this research, a part of injector was used. The injector populates the volume with the number of desired particles based upon user defined particle appearance rates and injection points. Figure 31 shows an example of a volume populated by particles using a part injector. The particles are allowed to move freely between the Eulerian mesh's cells, as shown in Figure 32. Each particle is a discrete element in the DEM.

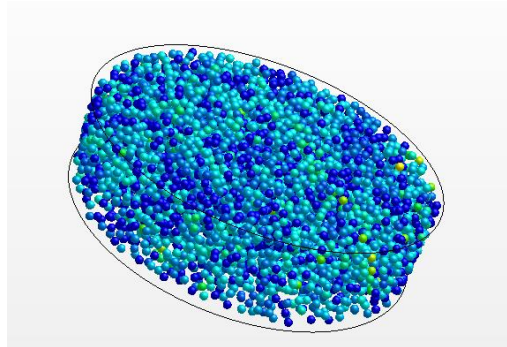


Figure 30. Example of a volume, outlined in black, populated with DEM particles

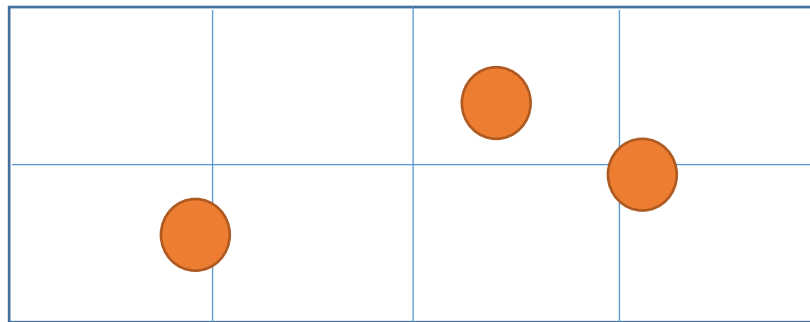


Figure 31. The Lagrangian DEM particles, shown in orange, moving through the Eulerian mesh elements, shown in blue.

As discussed in the previous section, the formation of hydroclusters is a large factor in shear thickening behavior. Lubrication forces are an important part of how these clusters interact with one another. Because LS-DYNA does not explicitly model the flow of the fluid around the surface of the particles, the lubrication forces must be input into the particle forces used in the DEM. The lubrication forces acting between the particles and the particles and wall boundary conditions are given in equations 8a and 8b respectively. In equation 8a, the distance between the particles, H_p , is set to a specified distance to indicate when squeeze flow begins between the particles. The fluid phase's viscosity, η_f , is used along with the particles' radii, R . H_w in equation 8b is the distance between the particle and the wall at which the lubrication force arises. The user specifies it. The lubrication force acts along the normal vector that passes through the particle's centroid, shown in Figure 32.

$$\mathbf{F}_{Lubrication_i}^n = \frac{-6\eta_f\pi R^2}{H_p} \left((\mathbf{v}_i - \mathbf{v}_j) \cdot \mathbf{n} \right) \mathbf{n} \quad (8a)$$

$$\mathbf{F}_{Lubrication_k}^n = \frac{-6\eta_f\pi R^2}{H_w} \left((\mathbf{v}_k) \cdot \mathbf{n}_w \right) \mathbf{n}_w \quad (8b)$$

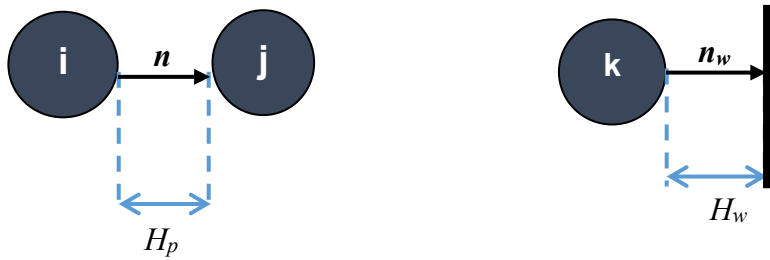


Figure 32. Two particles, i and j , are shown on the left at H_p distance apart, with the normal vector, $\mathbf{n}$, showing the direction the particle to particle force acts in. The lubrication force acting between particle k , on the right, and the wall acts in the direction of the normal vector, $\mathbf{n}_w$.

The effects of the particle motion and particle drag forces on the fluid phase appear as a source term in the Navier-Stokes' momentum equations for each Eulerian cell, given by equation 9. This source term for each cell, given in equation 10, is calculated by first taking the sum of the

drag force acting on the particle and the product of the particle's mass flow rate into the cell and the particle's velocity. This sum is then integrated with respect to time. This is repeated for each particle that resides in the cell and the sum of these integrals is divided by the time step, Δt [STAR-CCM+, 2017].

$$\frac{\delta \rho \mathbf{v}_f}{\delta t} + \nabla \cdot (\rho \mathbf{v}_f \mathbf{v}_f) = -\frac{1}{\rho} \nabla P - \nabla \boldsymbol{\tau} + \rho \mathbf{g} + \mathbf{S} \quad (9)$$

$$\mathbf{S} = -\left(\frac{1}{\Delta t}\right) \sum_i \int_0^{\delta t} (\mathbf{F}_{drag} + \dot{m}_i \mathbf{v}) \delta t \quad (10)$$

The drag force on a particle is given in equation 11. C_d is the drag coefficient. A is the cross sectional area normal to the fluid fluid. The relative velocity of fluid to the particle is given by $\mathbf{v}_f$, and ρ is the fluid's mass density. The Di Felice drag coefficient is used because it takes into account the presence of other particles in densely packed particle regions [STAR-CCM+, 2017].

$$\mathbf{F}_{drag} = \frac{1}{2} C_D A \rho \mathbf{v}_f |\mathbf{v}_f| \quad (11)$$

Figure 33 is a diagram of how STAR-CCM+ solves for the particle displacement and motion using the DEM and CFD. The CFD solver is mainly unaffected, except that particle information and locations are input into solver at the beginning of the time step to calculate source terms. An unsteady implicit CFD solver was used for this work. After the CFD time step is completed the DEM solver is engaged to update the position, velocities, and forces acting on the DEM particles.

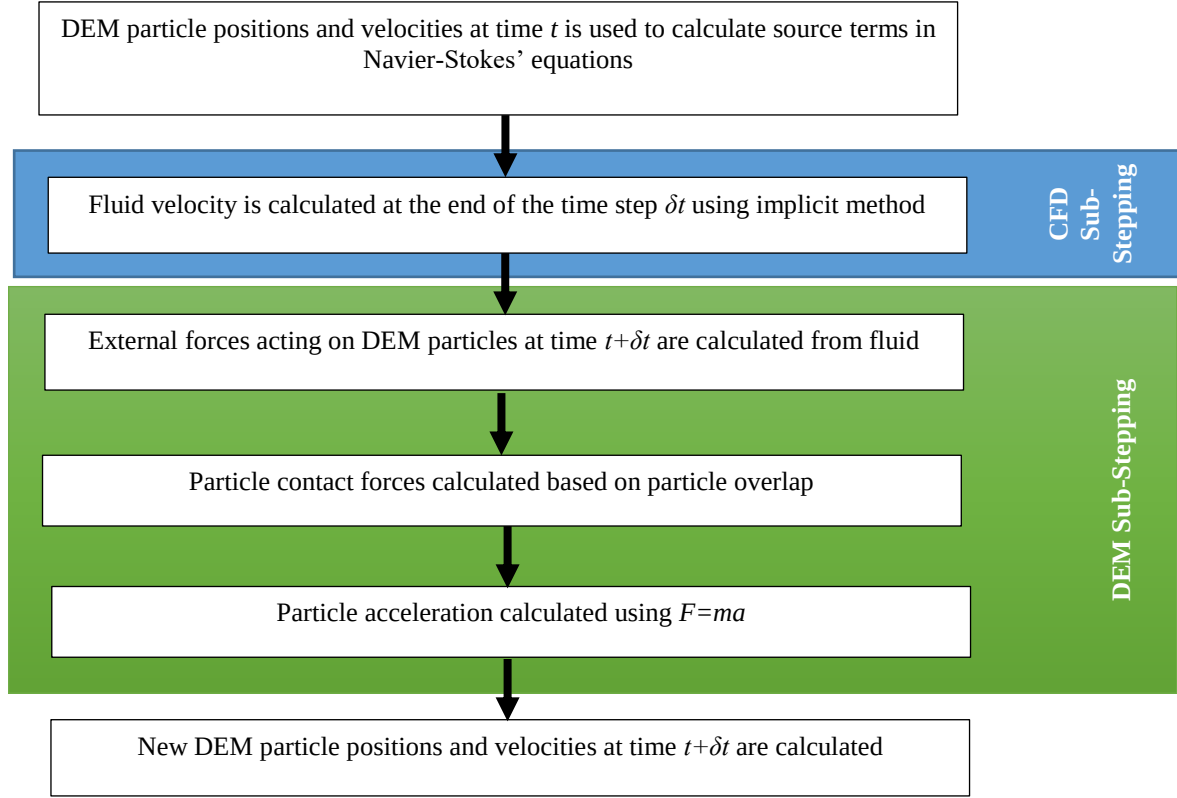


Figure 33. Method of calculating fluid and DEM forces in STAR-CCM+

3.5 Rheological Modeling

The STF used for these simulations consists of silica particles 500 nanometers in diameter suspended in polyethylene glycol (PEG) 200. The packing fraction of the particles is 0.57. The goal of these simulations is to replicate the rheometer data from Lee et al, 2003. This data was collected using a strain controlled SR-500 Rheometrics cone-and-plate rheometer. Figure 34 shows the general geometry of a cone and plate rheometer and its sample. The diameter of the rheometer is 25 mm, with a gap height of 1.9 mm, and a cone angle of 0.1 radians. The rheometer measures the shear rate and uses the relationship between shear stress, τ , and shear rate, $\dot{\gamma}$, given in equation 12, to calculate the viscosity, μ , of the fluid.

$$\tau = \mu \dot{\gamma} \quad (12)$$

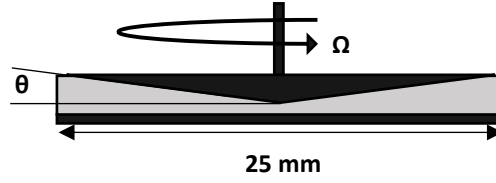


Figure 34. Example of a rheometer test. The fluid is shown in grey and is sandwiched between a flat plate of the bottom, and a small angled cone on the top. The cone is rotated at a fixed angular velocity.

The shear rate for a cone and plate rheometer with cone angle, α , less than or equal to 0.1 radians is given in equation 13 [Mewis, 2012]. Ω is the rotational speed in rad s^{-1} prescribed to the top wall of the fluid domain. The cone-in-plate geometry of the rheometer allows for uniform shear across the sample.

$$\dot{\gamma}(r) \cong \frac{\Omega}{\theta} \quad (13)$$

The shear stress is calculated using the measured torque on the cone, T . Equation 14 gives the relationship between the shear stress, τ , and T [Mewis, 2012]. R_c is the radius of the cone itself. It should be noted that these equations assume that there is laminar flow in the sample.

$$\tau = \frac{3T}{2\pi R_c^3} \quad (14)$$

The volume is meshed using a polyhedral mesher with a base element size of 1E-6 meters. The prism layer, which is used to resolve the boundary layer flow, is set to a width of 1E-6 meters along each wall boundary condition, with three layers of elements. This yielded a mesh with over 200,000 elements.

The boundary conditions are given in Figure 35. The boundaries with the atmospheric pressure are defined as pressure outlets, meaning that the fluid phase can have a velocity normal to the surface of these elements. However, a phase impermeable boundary condition was

implemented on this boundary for the particle phase, meaning that the particles were not allowed to pass through these boundaries.

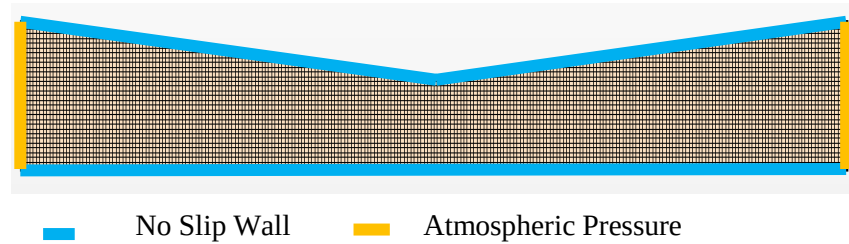


Figure 35. Example of the mesh used in STAR-CCM+ with the boundary conditions

Along the top wall, which represents the cone of the rheometer, a constant rotational velocity, Ω , is defined. This rotational velocity is the value used in equation 14 to define the shear rate on the STF. Figure 36 shows an example of the resulting velocity profile on the top wall boundary condition. The total torque exerted on this wall boundary is then calculated at each time step. The final torque measured at the end of the simulation is then used in equation 8 to calculate the shear stress exerted on the STF.

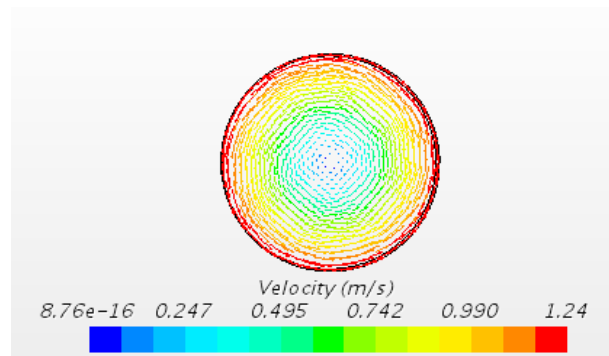


Figure 36. Velocity profile of top wall boundary condition. The top wall is given a constant rotation rate, resulting in a top wall velocity profile that varies with radius

3.6 STAR-CCM+ Results

The initial friction coefficients of friction for the particle contact models are given in table 7 below. The results for these initial simulation are given in figure 37. Initial simulations of

the STF yielded surprisingly low viscosities. In these simulations, the friction coefficient between particles and particles and the wall was set to 0.4. Figure 37 shows the results from these simulations plotted against the values presented in Lee's work [Lee et al, 2003]. The STAR-CCM+ results are shown in orange while Lee's values are shown in blue. It is apparent that the STAR-CCM+ simulation are in agreement with the results at lower shear rates, but do not accurately capture the fluid's behavior at higher shear rates.

Table 7. The coefficient of friction for the particle contact forces

Coefficient of friction between wall and particle	0.2
Coefficient of friction between particle and particle	0.2

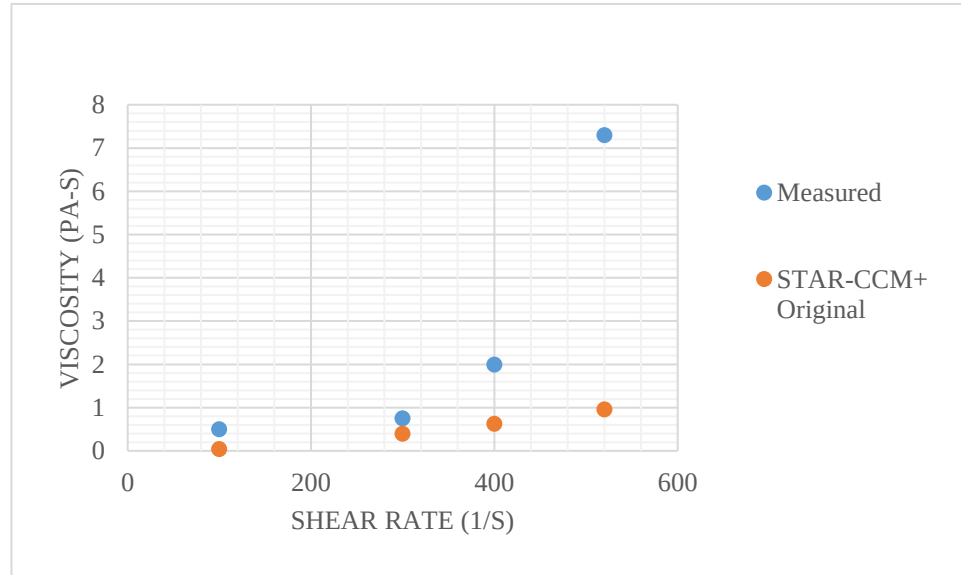


Figure 37. Viscosity vs shear rate for the initial STAR-CCM+ simulations plotted alongside published values

A brief investigation into the STAR-CCM+ simulation showed that even though the effects of the particles on the fluid flow was taken into account through source terms in the Navier-Stokes' equations, this did not mean that lubrication forces between the particles were

being modeled. Lubrication forces are based upon squeeze film theory, which requires the fluid to be squeezed between two surfaces. Because the DEM particles travel through the Eulerian mesh and do not affect the geometry of the cells themselves, this phenomenon cannot occur. To implement the effects of the lubrication forces that arise due to squeeze flow between the particles, a user-defined force was added to the normal force contact model. The lubrication forces are given by equations 8a and 8b. For these initial lubrication forces the variables H_p and H_w were set to 0.2 times the particle radius.

There also was a question about whether the friction coefficient of the particle surface could be modified. Increasing friction between the particles will not only increase the torque they exert on the cone surface when they are in contact with it, but will also help with modeling dilatancy.

Figure 38 shows the resulting measured viscosities for a series of STAR-CCM+ simulations conducted at the shear rates of 100 (1/s), 300 (1/s), and 520 (1/s) plotted against the published values. The results in orange are for simulations where the friction coefficient was increased from a value of 0.4 to 2.0 and no lubrication forces were added. The results in black were for the same friction value, except with lubrication forces. Seeing that these two simulations yielded viscosities greater than the measured values, it was determined that the friction coefficient of the particles should be decreased to 1.75. The results for these simulations are shown in green.

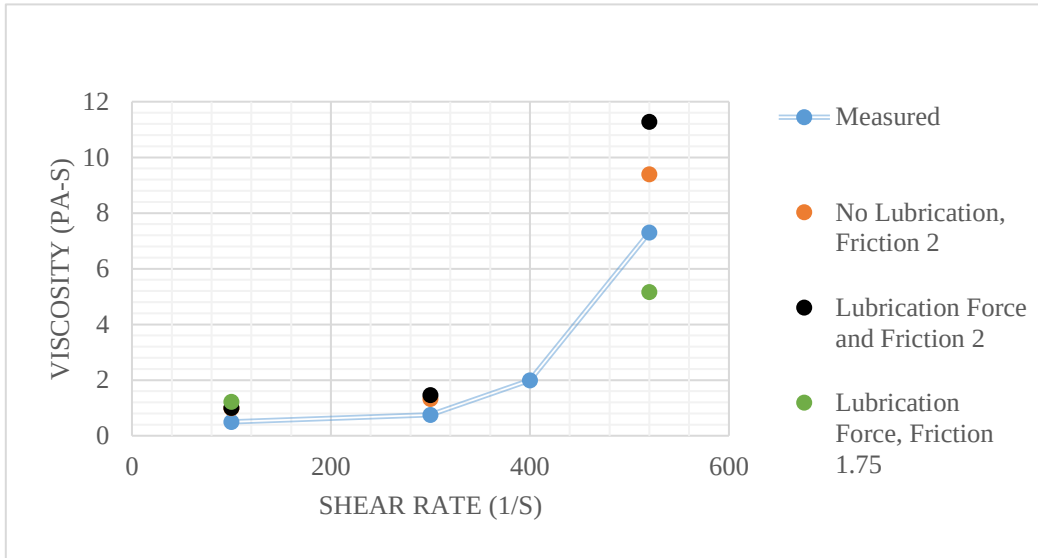


Figure 38. STAR-CCM+ results with modified friction and the addition of lubrication forces

To further investigate the proper friction coefficient, multiple simulations were run at the shear rate 520 (1/s) with varying coefficients of friction. Figure 39 shows the resulting viscosity of these simulations plotted against the coefficient of friction used in the analysis. The solid blue line at 7.3 Pa s marks the measured viscosity value. From these results it is apparent that increasing the frictional coefficient is critical to properly capturing the behavior of the STF at high shear rates. However, it is unlikely that the silica particles have a coefficient of friction greater than one. This indicated that further investigation into the lubrication force terms needed to be conducted.

Figure 39 shows the results for STAR-CCM+ simulations run with varying coefficients of friction at a constant shear rate of 520 (1/s). The blue line at 7.3 (Pa s) represents the viscosity shown in Lee's work. There is a jump in the measured viscosity between the coefficients of friction 0.75 and 1.0. It is unlikely that the silica particles have a coefficient of friction greater than one. There also is not much of a change between the derived viscosity values for the simulations with coefficients of friction 0.5 and 0.75. Based on these results, it was determined

to keep the coefficient of friction at 0.75 and continue the model development by focusing on the lubrication force modeling. The results shown in the graph are also given in table 8.

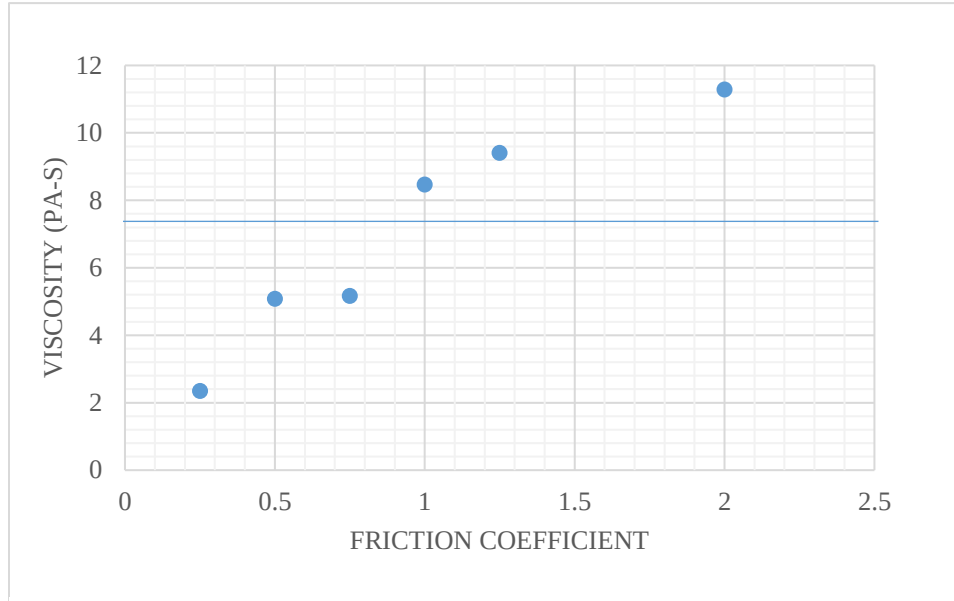


Figure 39. Resulting viscosity for STAR-CCM+ simulations plotted against the coefficient of friction of the particles

Table 8. The coefficient of friction used in the particle contact models and the resulting viscosity at 520 (1/s)

Coefficient of Friction	Measured Viscosity (Pa s)
0.25	2.35
0.5	5.08
0.75	5.17
1.0	8.46
1.25	9.4
2.0	11.23
Measured Value	7.3

To modify the lubrication force model two new variables were introduced. The first variable was R_w , which is the value the particle radius is multiplied by to determine H_w in the lubrication force term in the particle-to-wall contact model. When R_w is 0.5, H_p in equation 8a is

set to half the particle radius for the particle to wall contact model. The other variable of interest is R_p , which is the value the particle radius is multiplied by to determine H_p in the lubrication force term in the particle-to-particle contact model. Like R_w , when R_p is set to 0.5, H_p and H_w are set to half the particle's radius. The lubrication forces have an inverse relationship with these two variables.

Figure 40 shows the results of the STAR-CCM+ simulations where R_w is set to 0.1 and R_p is set to 0.1 in blue. The particles' coefficient of friction is kept at 0.75. The data measured by Lee is presented in orange, with a dotted trend line showing the overall behavior of the STF's viscosity with respect to shear rate. Increasing the lubrication forces greatly improved the model's accuracy at shear rates greater than 400 (1/s). The STAR-CCM+ simulation still results in a slightly lower viscosity at these shear rates, but the results closely follow the pattern shown by the measured data.

These results may be somewhat surprising considering that lubrication force act to push the particles away from each other. However, the particles in the STF have a high packing fraction at 0.57. This means that it is more difficult for particles to stay out of contact with each other. These high lubrication forces in the contact model increase the forces exerted by the particles on the wall boundary conditions, which increases the measured viscosity. This is not unlike what happens with hydroclusters, where large lubrication forces arise between clusters of particles and make it difficult for them to slide past each other in a confined volume.

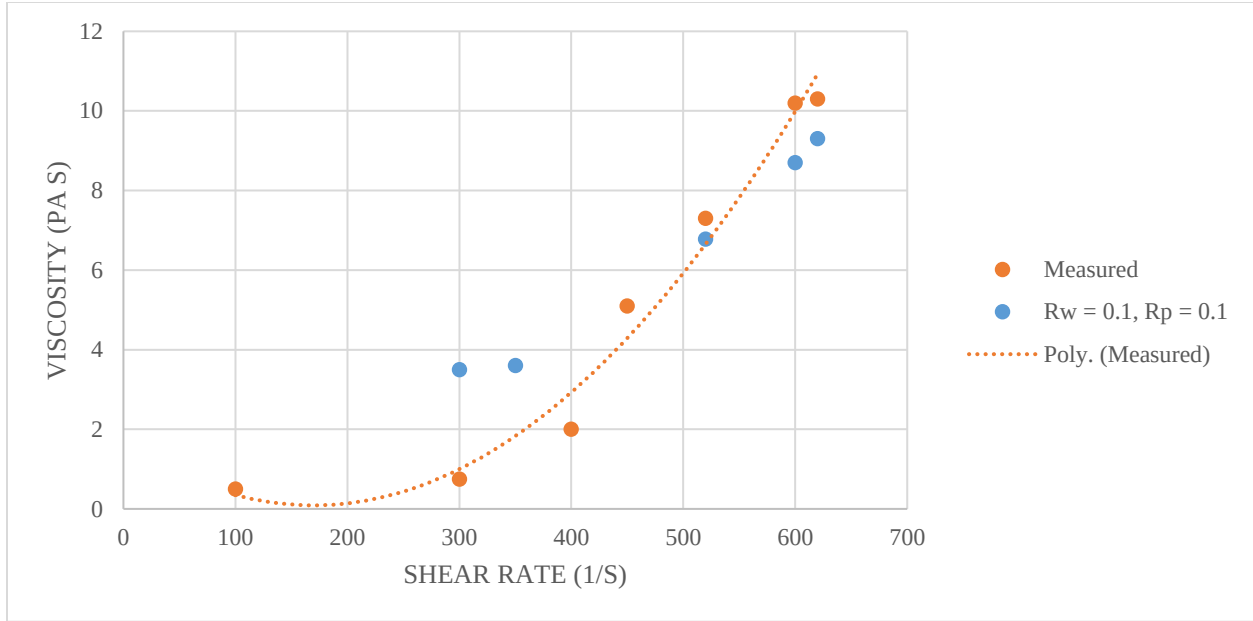


Figure 40. Measured viscosity and STAR-CCM+ resultant viscosity for a coefficient of friction of 0.75, $R_w = 0.1$, and $R_p = 0.1$ plotted against shear rate

While modifying the model to have larger lubrication forces between the particles improved results at higher shear rates, it did not improve the model's accuracy at lower shear rates. Figure 38 shows that the STAR-CCM+ models with high particle coefficients of friction more accurately matched the measured data results at shear rates less than 400 (1/s). Figure 41 shows an example of such a data set for the STAR-CCM+ model that had a coefficient of friction equal to 2.0.

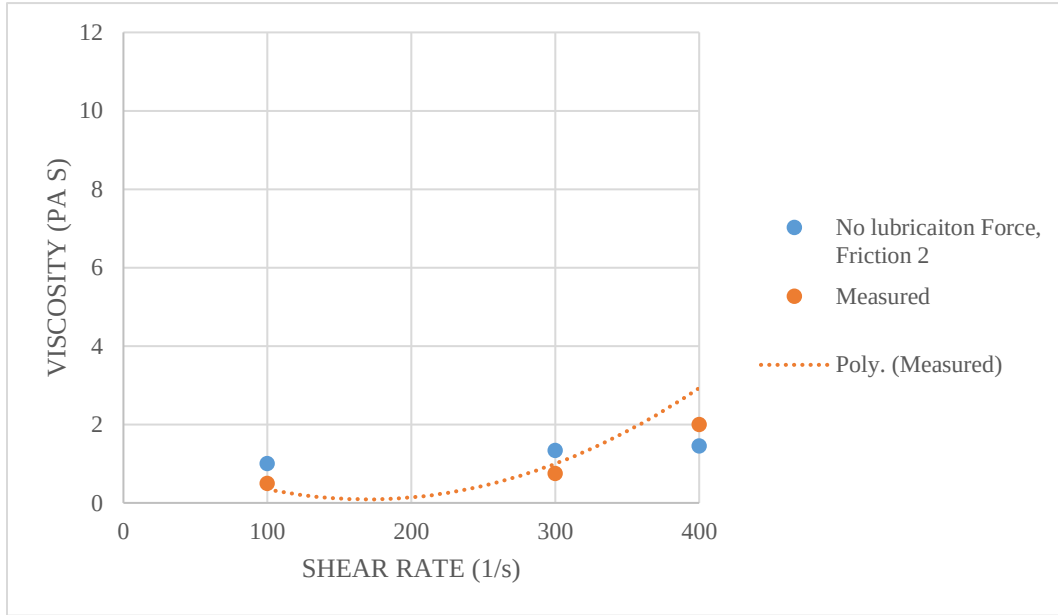


Figure 41. STAR-CCM+ results for simulations with no added lubrication forces and a particle coefficient of friction equal to 2.0

The results indicate that the particles' coefficient of friction is important to the overall modeling of the STF in STAR-CCM+. However, lubrication forces play a critical role in capturing the shear thickening behavior at higher shear rates. Therefore, the STAR-CCM+ model proposed is to use a high coefficient of friction of 1.75 at shear rates less than 400 (1/s), akin to the fact that static coefficients of friction are greater than dynamic coefficient of friction. At shear rates greater than this value the coefficient of friction will be reduced to 0.75 and lubrication forces that use the variable R_w and R_p set to 0.1 will implemented into the particle contact models.

This shift in the particle contact model may reflect the non-linear relationship between shear rate and viscosity. When Lee's data is plotted, it is apparent there is a quadratic relationship between these two variables. Around the shear rate 400 (1/s) the slope of this relationship begins to increase significantly. This shift in behavior may be caused by an increase in lubrication forces, hence the need to explicitly model lubrication forces.

The trend line shown to represent the published rheological data is a second order polynomial. Based on this trend line alone, it would be expected that the STF would exhibit a viscosity greater than one Pascal-second at very low shear rates. However, at low shear rates the STF has a viscosity close to its fluid phase, which is 0.06 Pascal-seconds. This indicates that there are some limits to using a second order polynomial to represent the relationship between the STF viscosity and shear rate.

From the results of these simulations an accurate model of the STF was developed that can be used to derive the relationship between the STF's viscosity and shear rate. This model is the first part of the multiscale modeling effort made to determine the effects of implementing a STF into a multilayer plate.

Chapter Four: Reverberation Matrix Method Model

4.1 Introduction

The reverberation matrix method (RMM) is a linear algebra algorithm that uses planar wave theory and elastodynamic definitions to solve for the dynamic response of a medium at one location due to a harmonic excitation at another location [Howard, and Pao, 1998. Pao et al, 2013. Shao et al, 2016]. This is done by solving for the response in terms of the arriving and departing wave amplitudes at the locations of interest [Pao et al, 2000]. First a scattering matrix is computed based upon the continuum properties of the media, then a phase propagation matrix is computed. Finally, if multiple locations are being evaluated, a permutation matrix is computed to take into account local coordinates. The reverberation matrix is the product of the scattering, phase, and permutation matrices [Howard, and Pao, 1998]. The desired responses are calculated using the reverberation matrix. The excitation is usually a harmonic force, although base excitations are also acceptable [Pao et al, 2000]. The RMM solves for the responses in the frequency domain, however time domain solutions can be calculated using an inverse Fourier transform [Shao et al, 2016].

After its initial inception to solve for the response of a planar truss, the RMM has been used extensively with multilayered media. Its ability to solve for the response at multiple points of interest at once makes it an appealing method when trying to conserve time and computational resources [Maio et al, 2015]. The fact that it uses the frequency domain makes the RMM a useful tool for spectral analysis and allows it to be easily applied to a variety of load conditions.

The flexibility of the RMM for a variety of applications for vibration and acoustic analysis makes it an attractive method for solving a structural problem involving both fluid and solid domains. Although this method has been used extensively for structural applications, it has not been applied to the propagation of waves through layers of fluids and solids [Howard and Pao, 1998. Shao et al, 2016. Miao et al, 2016. Pao et al, 2000]. This new method provides a new method for analyzing wave propagation along a fluid-solid interface and the effectiveness of the proposed STF damping mechanism in the frequency domain. The RMM is an attractive method due to the ease of using various materials within it, and the fact it can be used within widely distributed software such as MATLAB.

In this chapter, the reverberation matrix method (RMM) is defined for both solid and fluid materials. Four different methods of modeling the fluid-solid multilayer plate are investigated. Then the results for the fluid-solid RMM models with a Newtonian fluid are presented with validation from LS-DYNA simulations. Then the results for the fluid-solid RMM plate model with the shear thickening fluid are presented.

4.2 Reverberation Matrix Method Definition

The reverberation matrix method (RMM) is broken down into the following steps: defining the local coordinate systems, transforming the fundamental wave equations into the spectral domain, generating local scattering matrices from the interface boundary conditions, assembling the reverberation matrix from the scattering, phase, and permutation matrices, and then calculating the dynamic response of the continua.

4.2.1 Local Coordinate Definition

Both the solid and fluid layers of the plate are modeled explicitly as separate layers. The N-layer multilayer plate is modeled as infinitely wide, meaning that edge boundary conditions

are not considered. The global two-dimensional coordinate system (x,z) is defined at the 0th interface in the same plane as the shear and pressure wave normals. Two dimensional local coordinates are defined at each layer interface, as shown in Figure 42.

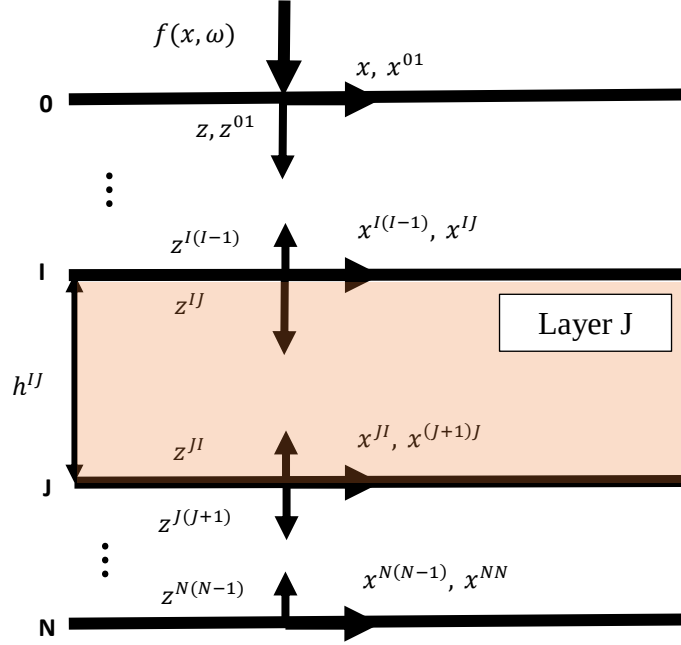


Figure 42. Local coordinates for an infinitely wide plate with $N-1$ layers

4.2.2 Wave Propagation in the Frequency Domain in Solid Materials

Within each solid layer the general solution for the p- and s-wave potentials, $\tilde{\phi}_{IJ}$ and $\tilde{\psi}_{IJ}$, are assumed to be harmonic solutions to the wave equation. Only the p-wave potential is considered for fluid layers since shear waves do not propagate in fluids [White, 1983]. A double Fourier transform is performed in the space and time domains on the fundamental wave equations to derive their forms in the radial frequency, ω , and wave number, k , domains. Equations 16 and 17 are the general results of this transform [Jiang et al, 2016].

$$\frac{d^2 \tilde{\phi}}{dz^2} + \alpha^2 \tilde{\phi} = 0 \quad (16)$$

$$\frac{d^2 \tilde{\psi}}{dz^2} + \beta^2 \tilde{\psi} = 0 \quad (17)$$

The wave number in the z direction for the pressure and shear waves are given by Equations 18 and 19, respectively. C_p is the pressure wave speed; C_s is the shear wave speed [Jiang et al, 2016].

$$\alpha = \sqrt{\frac{\omega^2}{C_p^2} - k^2} \quad (18)$$

$$\beta = \sqrt{\frac{\omega^2}{C_s^2} - k^2} \quad (19)$$

The wave potentials are defined in terms of the spectral amplitude of the arriving, $\tilde{a}$, and departing, $\tilde{d}$, waves at the interface. The resulting solutions for the wave potentials are:

$$\tilde{\varphi}(k, z, \omega) = \tilde{a}_p(k, \omega)e^{-i\alpha z} + \tilde{d}_p(k, \omega)e^{i\alpha z} \quad (20)$$

$$\tilde{\psi}(k, z, \omega) = \tilde{a}_s(k, \omega)e^{-i\alpha z} + \tilde{d}_s(k, \omega)e^{i\alpha z} \quad (21)$$

The wave potentials are then used to calculate the displacements and stresses within the plate. These are crucial for determining shock absorption capabilities. Equations 22 and 23 define the vertical and horizontal displacements respectively in terms of the wave potentials [Jiang et al, 2016]. It should be noted that for fluid layers the s-wave potential terms are dropped for reasons previously discussed.

$$u_x = \frac{\partial \varphi}{\partial x} - \frac{\partial \psi}{\partial z} \quad (22)$$

$$u_z = \frac{\partial \varphi}{\partial z} + \frac{\partial \psi}{\partial x} \quad (23)$$

The Cauchy stress tensor is used for both the solid and fluid layers of the plate. Equations 24 and 25 define the stresses in the solid layers using Lamé's constants [Barber, 2010].

$$\sigma_{zz} = \lambda \left(\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial z^2} \right) + 2\mu \left(\frac{\partial^2 \varphi}{\partial z^2} + \frac{\partial^2 \psi}{\partial x \partial z} \right) \quad (24)$$

$$\sigma_{xz} = 2\mu \left(\frac{\partial^2 \varphi}{\partial x \partial z} + \frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi}{\partial z^2} \right) \quad (25)$$

Lame's constants for solid materials are defined by equations 26 and 27, where E is the Young's modulus and ν is Poisson's ratio [Barber, 2010].

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)} \quad (26)$$

$$\mu = \frac{E}{2(1+\nu)} \quad (27)$$

4.2.2 Wave Propagation in the Frequency Domain in Fluid Materials

Shear waves do not propagate through fluids. Therefore, in fluid materials only the p-wave potential is considered. The lateral and vertical displacements within the fluid are determined using equations 22 and 23, except the shear wave terms are ignored.

If the fluid is inviscid, then σ_{xz} is equal to zero in the fluid domain. σ_{zz} is defined by equations 28a and 28b.

$$\sigma_{zz} = -p \quad (28a)$$

$$p = \rho \frac{\partial^2 \varphi}{\partial t^2} \quad (28b)$$

For viscous fluids, the solution for the p-wave number is altered to include the damping effects of viscosity. Equation 29 gives the new term for α in terms of the fluid's mass density, ρ , and dynamic viscosity, η [White (1983)]. Previous research has not used a complex wave number within the RMM. For the STF the dynamic viscosity used is that of the fluid phase of the STF. For the STF the material properties of the fluid phase are used to calculate α .

$$\alpha = \sqrt{\frac{\omega^2}{C_P^2 \left(1 + \frac{i4\omega\eta}{3\rho C_P^2}\right)}} \quad (29)$$

Navier-Stokes' equations are used to calculate the stress terms in the fluid layers.

Viscosity is considered when calculating the stresses in the fluid layers. Equations 30 and 31 define the stresses in the fluid layer based off of Navier-Stokes' equations [Pao et al, 2013]. The dynamic viscosity of the fluid is represented by η . Previous work has not used the viscous terms in the stresses. It should be noted that by setting η equal to zero, the stresses become equal to the equations used to define the inviscid fluid properties in the previous section.

$$\sigma_{zz} = -\left(p + \frac{2}{3}\eta \left(\frac{\partial^3 \varphi}{\partial t \partial x^2} + \frac{\partial^3 \varphi}{\partial t \partial z^2}\right)\right) + 2\eta \left(\frac{\partial^3 \varphi}{\partial t \partial z^2}\right) \quad (30)$$

$$\sigma_{xz} = 2\eta \left(\frac{\partial^3 \varphi}{\partial t \partial x \partial z}\right) \quad (31)$$

4.2.3 Boundary Conditions Along Layer Interfaces

Four assumptions are used to determine the boundary conditions at the layer interfaces: all layers are made of elastic materials, layers do not penetrate each other, if the fluid is explicitly modeled between solid layers the no-slip boundary condition applies, and equilibrium of forces is maintained along the interfaces. Due to these assumptions, the layer interface boundary conditions are based on displacements and stresses.



Figure 43. Material configurations at interface J, solid-solid layers (left) or solid-fluid layers (right)

There are three types of interfaces: solid-solid, solid-fluid(viscous), solid-fluid(inviscid).

Figure 2 illustrates these possible material arrangements. Equations 32a through 32d describe

the boundary conditions for the solid-solid and the solid-fluid(viscous) interfaces. The superscripts ($J-1$) and ($J+1$) indicate if the displacement or stress is measured on the upper or lower side of the interface respectively.

$$u_x^{J(J-1)} - u_x^{J(J+1)} = 0 \quad (32a)$$

$$u_z^{J(J-1)} + u_z^{J(J+1)} = 0 \quad (32b)$$

$$\sigma_{xz}^{J(J-1)} + \sigma_{xz}^{J(J+1)} = 0 \quad (32c)$$

$$\sigma_{zz}^{J(J-1)} - \sigma_{zz}^{J(J+1)} = 0 \quad (32d)$$

For interfaces between solid and fluid layers, such as the one shown on the right hand side in Figure 43, a no slip boundary condition is imposed along the interface. This means that the displacement for each layer on either side of the boundary interface is the same.

Free surface boundary conditions are assumed along the top and bottom faces of the plate. Therefore, the shear stress is interpreted to go to zero and lateral displacements are negligible. Due to these assumptions, the shear stress and lateral motion in the x-direction are not calculated at the 0th and Nth interfaces [Jiang et al 2016].

There are two methods of taking into account viscosity: to model the fluid as inviscid and use a damper to take into account viscous effects or to model the fluid as viscous. The first method is attractive because it is similar to other implementations of the RMM in fluids [Pao et al, 2000]. The second method is a more accurate material model. For both methods, the fluid is defined as incompressible.

For the first method, where the fluid is inviscid, a lateral damper spans the fluid layer, like shown in Figure 44. Pao uses the RMM to model the propagation of acoustic waves through

inviscid fluid layers, but he and his colleagues did not take into account viscous effects [Pao et al, 2000]. The lateral damper differs the work presented in this dissertation from previous work.

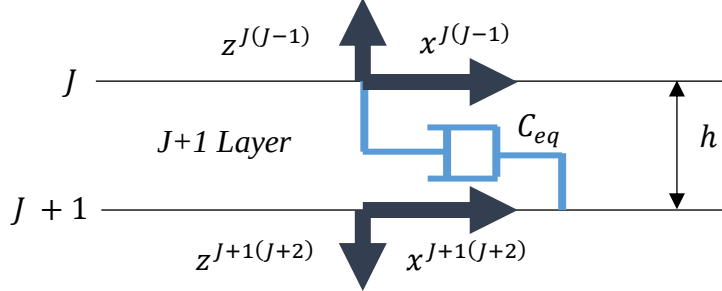


Figure 44. Damper in fluid layer between layer interfaces J and J+1

The equivalent damping coefficient of the fluid damper is calculated by setting the work absorbed by a damper during a single period of oscillation, given in equation 33a, to the work required to shear a layer of fluid during a single period of oscillation, given in equation 33b. By setting these two equations equal to each other the equivalent damping coefficient of the fluid is calculated, given in equation 33c.

$$W = \oint C_{eq} \dot{u}_x dx = \int_0^{2\pi/\omega} C_{eq} \dot{u}_x^2 dt \quad (33a)$$

$$W = \oint \tau dx = \oint \eta \dot{\gamma} dx = \int_0^{2\pi/\omega} \eta \frac{\partial \dot{u}_x}{\partial z} \dot{u}_x dt \approx \int_0^{2\pi/\omega} \frac{\eta}{h} \dot{u}_x^2 dt \quad (33b)$$

$$C_{eq} = \frac{\eta}{h} \quad (33c)$$

The effects of the damper are then implemented into the relationship between the shear stresses acting on the solid layers the fluid lies between, as shown in equation below. This equation is then incorporated into equation 34.

$$\sigma_{xz}^{J-1(J)} - \sigma_{xz}^{J+1(J+2)} - C_{eq}(\dot{u}_x^{J+1(J+2)} - \dot{u}_x^{J-1(J)}) = 0 \quad (34)$$

For the second method, where the fluid is considered viscous, the p-wave number is calculated using equation 29. The first three boundary conditions given in equations 32a through 32c are used at the layer interfaces. The Cauchy stresses in the fluid are calculated using equation 30 and 31. The shear stress in the solid layers on either side of the $J+1$ layer fluid, like shown in figure 44, are set equal and opposite, as defined in equation 35, to take into account the wall shear stress.

$$\sigma_{xz}^{J(J-1)} + \sigma_{xz}^{J+1(J+2)} = 0 \quad (35)$$

4.2.4 Reverberation Matrix Definition

The boundary conditions at each interface are rewritten in terms of the wave potentials. A series of linear equations are then written in terms of the arriving and departing wave amplitudes, given in equation 36. The matrix $\mathbf{A}$ is the coefficients for the arriving wave amplitudes. The matrix $\mathbf{D}$ consists of the coefficients for the departing wave amplitudes. The vector $\tilde{\mathbf{g}}^J$ consists of external stresses and excitations applied at the interface J.

$$\tilde{\mathbf{g}}^J = \mathbf{A}^J \tilde{\mathbf{a}}^J + \mathbf{D}^J \tilde{\mathbf{d}}^J \quad (36)$$

Equation 36 is then rewritten into equation 37, where $\mathbf{S}^J$ is the local scattering matrix at interface J and $\tilde{\mathbf{s}}^J$ is the local source vector. At the 0th and Nth interfaces of the plate, the scattering matrix collapses into a 2x2 matrix due to the boundary conditions for reasons previously discussed.

$$\tilde{\mathbf{d}}^J = \mathbf{S}^J \tilde{\mathbf{a}}^J + \tilde{\mathbf{s}}^J \quad (37)$$

The local scattering equations at each interface are combined form a single, global system of equations, given in equation 38. This results in the local scattering matrices are combined to become a global scattering matrix, $\mathbf{S}$.

$$\begin{bmatrix} \tilde{\mathbf{a}}^0 \\ \tilde{\mathbf{a}}^1 \\ \vdots \\ \tilde{\mathbf{a}}^{N-1} \\ \tilde{\mathbf{a}}^N \end{bmatrix} = \begin{bmatrix} \mathbf{S}^0 & 0 & \cdots & 0 & 0 \\ 0 & \mathbf{S}^1 & & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \\ 0 & 0 & \cdots & \mathbf{S}^{N-1} & 0 \\ 0 & 0 & & 0 & \mathbf{S}^N \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{a}}^0 \\ \tilde{\mathbf{a}}^1 \\ \vdots \\ \tilde{\mathbf{a}}^{N-1} \\ \tilde{\mathbf{a}}^N \end{bmatrix} + \begin{bmatrix} \tilde{\mathbf{s}}^0 \\ \tilde{\mathbf{s}}^1 \\ \vdots \\ \tilde{\mathbf{s}}^{N-1} \\ \tilde{\mathbf{s}}^N \end{bmatrix} \quad (38)$$

The waves that depart from an interface on one side of the plate layer become the arriving waves at the other side of the plate layer with a phase lag. Therefore, all arrival and departure waves within a plate layer can be linked using a phase matrix, $\mathbf{P}_J(h_J)$, using equation 38. The vector $\tilde{\mathbf{a}}^*$ has the same values as the vector $\tilde{\mathbf{a}}$, except the values are reordered. If the layer is a solid material, then $\mathbf{P}_J(h_J)$ is defined by equation 40a. If the layer is made of a fluid material, then $\mathbf{P}_J(h_J)$ is defined equation 40b. These local phase matrices are used to construct the global phase matrix, $\mathbf{P}$, given in equation 40c. This research differs from other implementations of the RMM in that two different phase matrices must be used to take into account the two different types of materials used in the multilayer plates.

$$\tilde{\mathbf{a}}_J = \mathbf{P}_J(h_J) \tilde{\mathbf{a}}_J^* \quad (39)$$

$$\mathbf{P}_{Js}(h_J) = \begin{bmatrix} e^{i\alpha_J h_J} & 0 \\ 0 & e^{i\beta_J h_J} \end{bmatrix} \quad (40a)$$

$$\mathbf{P}_{Jf}(h_J) = e^{i\alpha_J h_J} \quad (40b)$$

$$\mathbf{P} = \begin{bmatrix} \mathbf{P}_1(h_1) & 0 & \cdots & 0 \\ 0 & \mathbf{P}_1(h_1) & & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \mathbf{P}_N(h_N) & 0 \\ & & 0 & \mathbf{P}_N(h_N) \end{bmatrix} \quad (40c)$$

The elements in $\tilde{\mathbf{d}}^*$ and $\tilde{\mathbf{d}}$ for each layer are the same and are linked through Equation 42, where $\mathbf{u}$ is the local permutation matrix. Equation 42a describes the local solid permutation matrix, $\mathbf{u}_s$. Equation 42b gives the local fluid permutation matrix, $\mathbf{u}_f$ [Pao *et al.* (2003)]. Equation 42c describes the global permutation matrix, $\mathbf{U}$. Equation 41c has the fluid and solid permutations alternating because the proposed multilayer plate design has alternating layers of fluid and solid materials. The global permutation matrix follows the layer material arrangements. This variation in the permutation matrix is yet another way that this implementation of the RMM differs from other research.

$$\tilde{\mathbf{d}}^* = \mathbf{u} \tilde{\mathbf{d}} \quad (41)$$

$$\mathbf{u}_s = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad (42a)$$

$$\mathbf{u}_f = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad (42b)$$

$$\mathbf{U} = \begin{bmatrix} \mathbf{u}_s & 0 & \dots & 0 & 0 & 0 \\ 0 & \mathbf{u}_f & & 0 & 0 & 0 \\ \vdots & & \ddots & & \vdots & \\ 0 & 0 & & \mathbf{u}_s & 0 & 0 \\ 0 & 0 & \dots & 0 & \mathbf{u}_f & 0 \\ 0 & 0 & & 0 & 0 & \mathbf{u}_s \end{bmatrix} \quad (42c)$$

The reverberation matrix, $\mathbf{R}$, is defined as the product of the scattering, phase, and permutation matrices, as shown in Equation 43. Equations 44a-b state the relationship between $\mathbf{R}$ and the arriving and departing wave amplitudes. The displacements and stresses of the plate can be determined using the fully solved-for wave potentials.

$$\mathbf{R}(k, \omega) = \mathbf{S} \mathbf{P} \mathbf{U} \quad (43)$$

$$\tilde{\mathbf{a}} = \mathbf{P}\mathbf{U}[\mathbf{I} - \mathbf{R}]^{-1}\tilde{\mathbf{s}} \quad (44a)$$

$$\tilde{\mathbf{d}} = [\mathbf{I} - \mathbf{R}]^{-1}\tilde{\mathbf{s}} \quad (44b)$$

Neumann's series is used to calculate the inverse of $[\mathbf{I} - \mathbf{R}]$, given in equation 44. By inserting this approximation into equation 44a, the values of $\tilde{\mathbf{a}}$ can easily be calculated.

$$[\mathbf{I} - \mathbf{R}]^{-1} = \mathbf{I} + \mathbf{R} + \mathbf{R}^2 + \dots + \mathbf{R}^n \quad (45)$$

Equation 46 is used to calculate the response of the plate at certain points within the plate in terms of the global coordinates. The matrices $\mathbf{A}_u$ and $\mathbf{D}_u$ are the global receiver matrices, which comprise of the coefficients in the elastodynamic solutions that contain the global coordinates for the point of interest.

$$\mathbf{W}(k, z, \omega) = \mathbf{A}_u\tilde{\mathbf{a}} + \mathbf{D}_u\tilde{\mathbf{d}} = (\mathbf{A}_u\mathbf{P}\mathbf{U} + \mathbf{D}_u)[\mathbf{I} + \mathbf{R} + \mathbf{R}^2 + \dots + \mathbf{R}^n]\tilde{\mathbf{s}} \quad (46)$$

An inverse Fourier transform is then calculated in the wave number domain to transform the response into the space-frequency domain, given in equation 47 below.

$$\mathbf{W}(x, z, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathbf{W}(k, z, \omega) e^{ikx} dx \quad (47)$$

4.3 Comparison of RMM Fluid Models

Both RMM fluid models offer a method of taking into account the fluid's viscosity. The results of both of these models for a plate with two solid layers and with a single layer of fluid were compared to one another. The solid layers were five centimeters thick, and the fluid layers were five millimeters thick. Five different materials were used in the solid layers: steel, aluminum, bronze, and titanium. The material properties of these metals are given in table 9.

Table 9. Material properties used in LS-DYNA analysis

	Modulus of Elasticity (GPa)	Mass Density (kg/m ³)	Poisson's Ratio	C _p (m/s)	Bulk Modulus (GPa)
Steel	200	7820	0.3	--	--
Aluminum	73.7	2704	0.34	--	--
Bronze	129	8940	0.348	--	--
Titanium	114	4430	0.342	--	--
PEG 200	--	1125	--	1800	3.645

The RMM models were subjected to a unit load in the z-direction at the top of the plate. The load frequencies ranged from 1000 Hz to 5000 Hz, with a 10 Hz interval. The amplitude of the response in the z-direction at the bottom of the plate for each frequency was measured. The sum of these results are plotted in figure 45 below. The sum of the dynamic response is taken to simulate the dynamic response of an impulsive load, which is represented by a delta function in the time spectrum.

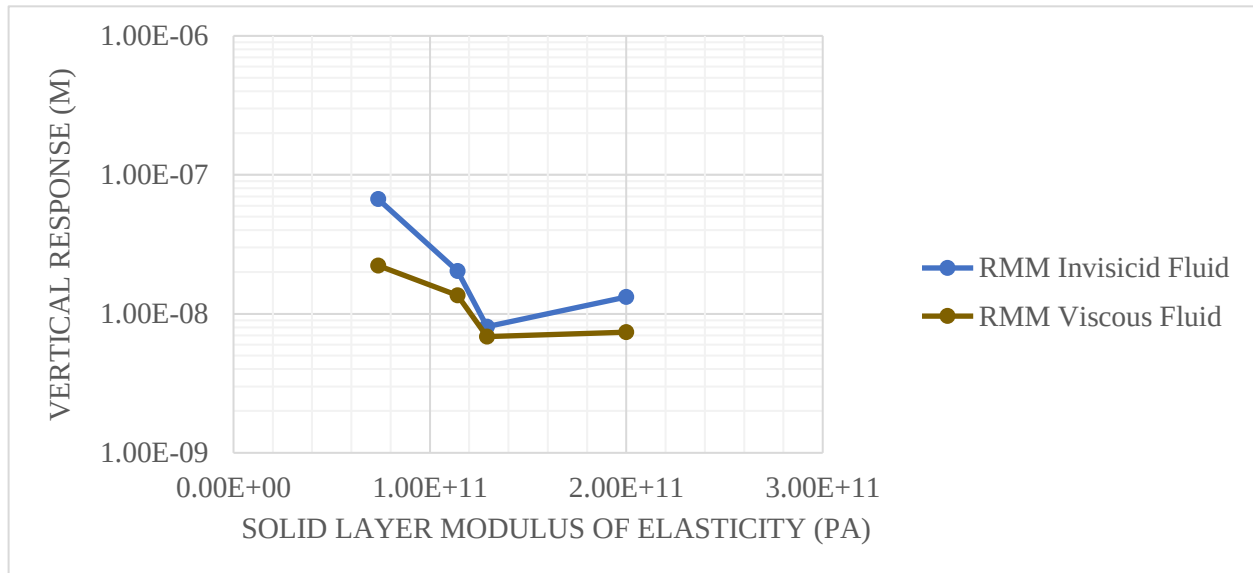


Figure 45. The vertical dynamic response of the RMM models with viscous and inviscid fluid layers

The dynamic responses of the plate for both methods are similar to each other, as expected. Overall, both methods have low values for the dynamic response. It is logical that

these models would have small responses with the plate being over ten centimeters thick. The inviscid RMM model exhibits a greater response. This indicates that the complex p-wave number and taking viscosity into account when calculating stress are important. Based upon this comparison it was decided that further work would use the viscous fluid model in the RMM.

4.3 Verification of Reverberation Matrix Method with LS-DYNA

To verify the new method of incorporating a viscous fluid into the RMM, the dynamic response to an impulsive load calculated by the RMM was compared to the dynamic response calculated by the finite element code LS-DYNA. The objective of this assessment is to determine if the changes in the dynamic response of the RMM model due to implementing a fluid layer are similar to the changes observed in the dynamic response of a FEA model when a fluid is added into it.

A one meter by one-meter flat plate was modeled in LS-DYNA. The plate comprised of three layers: a five centimeter thick layer, a five millimeter thick layer, and another five centimeter thick layer. The layers are defined as separate parts; elements of bordering layers share the nodes along the plate layer interface. Figure 46 shows a side view of the plate with each layer having its own color. The characteristic element length used in the simulation is one centimeter. The default element type, constant stress, was used for the mesh.

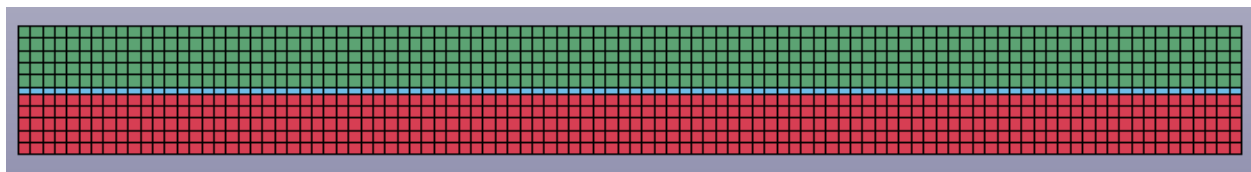


Figure 46. Side view of multilayer plate; a thin, blue fluid layers lies between the red and green layers

A fixed location boundary condition is applied to all the nodes on the edge of the plate, which are highlighted in white in Figure 47. If the fixed boundary condition is not applied to nodes along the plate edge, rigid body motion arises when fluid materials are used.

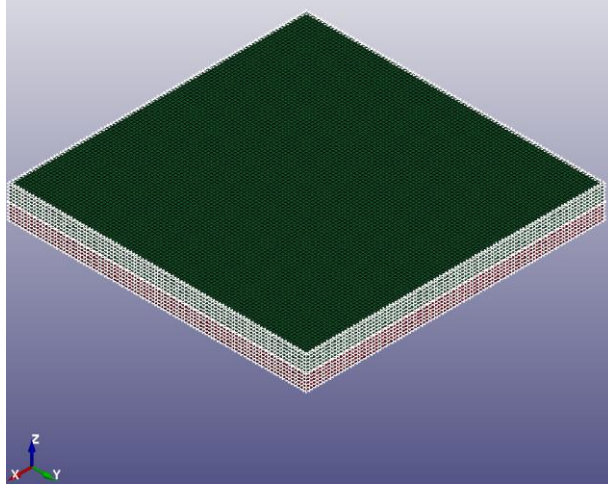


Figure 47. The nodes at which the pinned boundary condition is applied are shown highlighted in white

The fluid was modeled in LS-DYNA using the material card *MAT_ELASTIC_FLUID. For this material card the shear modulus is set equal to zero. The bulk modulus, K , is used to calculate the pressure rate in the fluid, shown in equation 48. Equation 49 is used to calculate the shear stress, σ_{ij} . ΔL is the characteristic length of the fluid's element length, C_p is the fluid's speed of sound, and ρ is the fluid's density. The deviatoric strain rate is given by $\dot{\epsilon}'_{ij}$, and ζ is the user defined constant. The product of ζ , ΔL , ρ , and C_p is the fluid's dynamic viscosity [LS-DYNA Manual, 2018].

$$\dot{p} = -K \dot{\epsilon}_{ii} \quad (48)$$

$$\sigma_{ij} = \zeta \cdot \rho \cdot \Delta L \cdot C_p \cdot \dot{\epsilon}'_{ij} \quad (49)$$

The material card *MAT_ELASTIC is used for the solid materials. Four different materials were used: steel, aluminum, bronze, and titanium. These materials are commonly used

in armor and other structural components. Table 9 gives the material properties used for the solid and fluid materials.

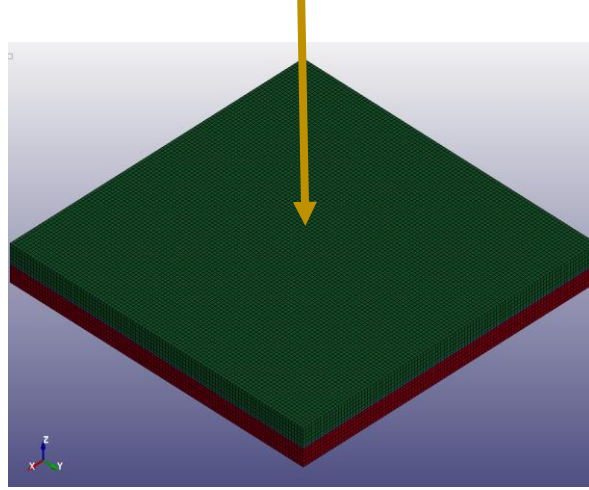


Figure 48. The LS-DYNA model of the plate. The yellow arrow represents the unit load applied to the center of the plate.

The frequency response function (FRF) at the center, bottom of the plate was calculated for a unit load applied to the top of the plate over the frequency range of 1000 Hz to 5000 Hz at every 10 Hz. The reasoning behind this is that a single unit load represents an impulsive load from zero to infinity in the spectral domain. Figure 48 shows where the unit load is applied to the plate. The FRF is a transfer function that outputs the amplitude displacement per unit force applied, as shown in equation 50. It was calculated using the first 500 eigenmodes of the plate.

$$U(\omega) = FRF(\omega) * F(\omega) \quad (50)$$

For the RMM, a unit load was applied to one side of the plate and the vertical displacement at the other side was recorded for a frequencies ranging from 1000 Hz to 5000 Hz at every 10 Hz, as done previously. The sum of the responses was then compared to the sum of the LS-DYNA responses.

Figure 49 shows the vertical responses of the completely solid plate and the plate with a fluid layer calculated by the RMM model and LS-DYNA. The magnitude of the dynamic responses are small, which is expected considering the plate's total thickness. The results for the solid plate's results calculated by the RMM, shown in orange, are similar to those calculated by LS-DYNA, shown in blue, in that there is not a dramatic change in the response of the plate as the modulus of elasticity increases. Both methods show that increasing the plate's modulus of elasticity decreases its dynamic response.

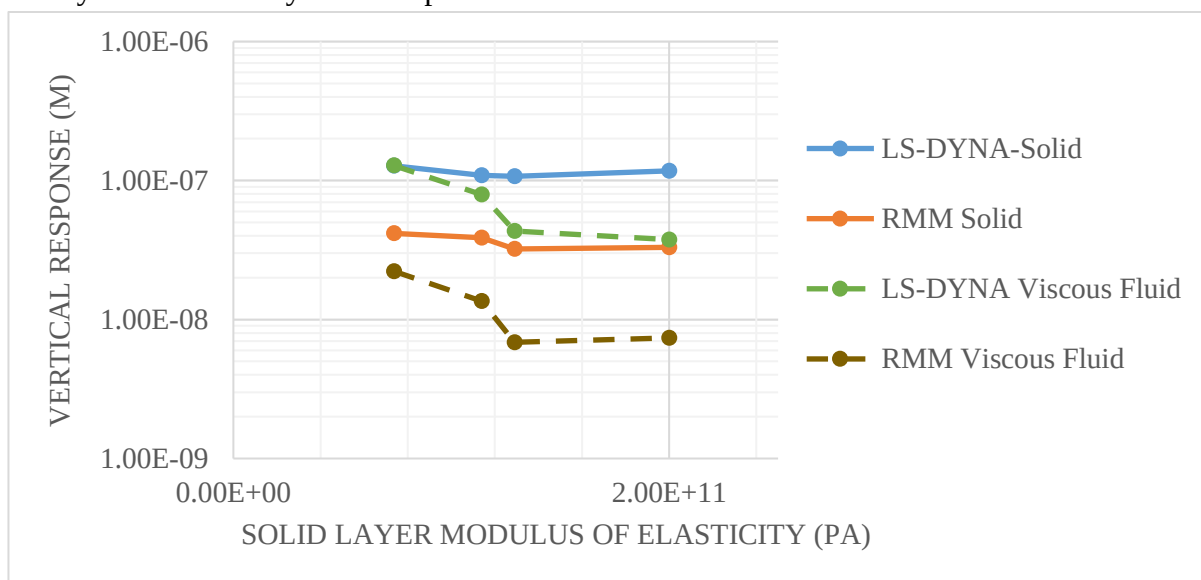


Figure 49. The dynamic response of a solid plate and a plate with a fluid layer calculated using LS-DYNA and the RMM

The dynamic response of the plate with a fluid layer calculated by the RMM, shown in brown, have a strikingly similar trend as the dynamic response calculated by LS-DYNA, shown in green. Both LS-DYNA and the RMM show the implementing a fluid layer causes a decrease in the dynamic response of the plate. This difference is the smallest for the plates with aluminum, which has the lowest modulus of elasticity, and increases as the modulus of elasticity increases. The greatest difference in the dynamic response for the solid plates and the plates with a fluid layer is observed for steel, which has the largest modulus of elasticity.

These results show that the RMM can successfully capture how implementing a fluid layer into a multilayer plate can affect its dynamic response. This is a novel application of the RMM and provides not only a method of analyzing the STF multilayer plate, but also raises the possibility of the RMM being applied to other fluid-structure interactions.

4.4 RMM Method for STF

The two previous RMM algorithms assume that the fluid material used is a Newtonian fluid, meaning that viscosity is constant and independent of the shear rate. However, the observed viscosity for a STF is shear rate dependent, hence its non-Newtonian behavior. This kind of non-linear behavior is difficult to model in FEM and similar programs due to its interdependence.

In the previous methods, the viscosity input into the RMM did not need to be validated since the fluids were assumed to be Newtonian. Errors may arise in the analysis of an STF due to selecting a viscosity that corresponds to a shear rate lower or higher than the actual shear rate the fluid experiences during the dynamic response of the plate. Therefore an iterative method must be introduced to take into account that the viscosity input into the RMM at the beginning of the simulation may not be the correct viscosity the fluid exhibits. The proposed method discussed in this section provides an innovative method for analyzing the dynamic response of a multilayer plate. Figure 50 shows the iterative method used to modify the viscosity of the STF in the RMM.

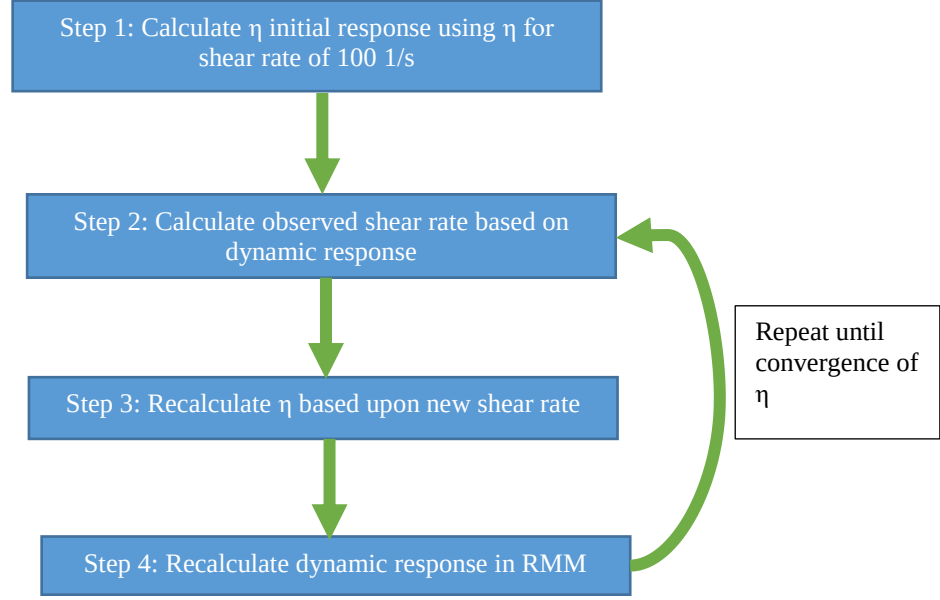


Figure 50. Method of RMM for STF; this takes into account the dependence between viscosity and shear rate

The relationship between the viscosity and shear rate was calculated using data published by Lee et al in 2003 for an STF consisting of 500 nanometer silica particles in PEG200. The packing fraction of the STF is 0.57. The shear rate the STF experiences in the RMM is calculated using equation 51. In this equation, h_{fluid} is the thickness of the fluid in the z direction, $u_{x_{top}}$ is the velocity in the x-direction along the top interface of the fluid layer, and $u_{x_{bottom}}$ is the velocity in the x-direction at the bottom interface of the fluid layer.

$$\dot{\gamma} = \frac{\omega |u_{x_{top}} - u_{x_{bottom}}|}{h_{fluid}} \quad (51)$$

This method provides a new method of implementing non-linear material behaviors into the RMM. It also provides a method for linking the material model of the STF discussed in the next chapter to an analysis of the multilayer plate. A comparison of this method with the original, viscous fluid layer method, referred to as the Newtonian fluid, are shown in figure 51.



Figure 51. Steel plate layouts. Green bands represent STF layers that are 0.005 meters thick

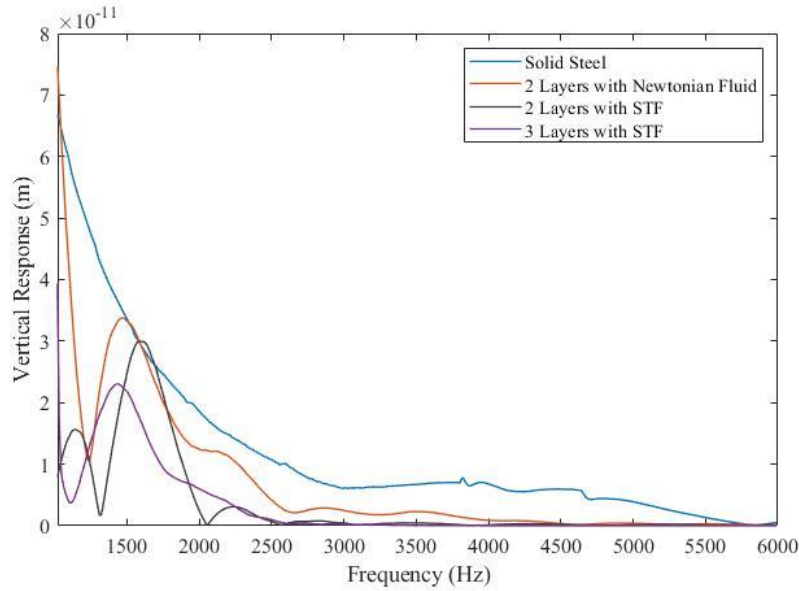


Figure 52. The vertical response of a solid plate, a plate with a Newtonian fluid, and a plate with STF

Figure 52 shows a comparison between the dynamic response for a solid steel plate, a plate with two steel layers and a Newtonian fluid between them, and finally plates with two and three layers of steel and the STF method implemented. The blue line is the vertical response of a solid steel plate, while the orange line is the response of a steel plate with a 0.005 meter layer of a Newtonian fluid. The Newtonian fluid has the viscosity of 0.75 Pa/s, the measured viscosity of the STF at a shear rate of 1/100 seconds. The black line represents the results of the simulation when using the new iterative method. It shows that implementing the iterative method has a major effect upon the dynamic response of the plate. This response, while different, is still reasonably in agreement with the Newtonian fluid method.

These results show that the presence of a fluid layer greatly affects the dynamic response of the plate at higher frequencies, as shown in the difference between the blue and orange lines. As expected, the results calculated by the iterative method, shown in purple, are similar to the results for the plate with a Newtonian fluid layer. It was expected that there would be a change in response; however, a drastic change would indicate possible issues with the implementation of the STF behavior. Using the iterative method did decrease the response of the plate, showing that the non-linear behavior of the STF needs to be taken into account.

The black line in figure 52 is the dynamic response of the plate with three solid layers as shown in figure 51. This response was calculated using the iterative method. The results for this plate configuration are significantly lower than the results for the two solid layers plate configuration. This decrease in the dynamic response can be attributed to the fact that the STF is moved away from the neutral axis of the plate in the three solid layers plate. The shear stresses and shear rates within the plate increase with distance from the neutral axis. Therefore the effects of the STF would be expected to be greater when it is further away from the neutral axis. The results of the three solid layers plate reflect this.

4.5 STAR-CCM+ Implementation

Now that the RMM has successfully been developed to model the dynamic response of a plate with non-Newtonian fluid layers, the next step is to implement material properties derived from high fidelity simulations of STFs into the RMM. High fidelity simulations, like those done by the CFD program STAR-CCM+ are computationally expensive and are not well suited towards the rapid exploration of a design space of a multilayer plate. Instead, the simulations of an STF in STAR-CCM+ will be used to develop a viscosity and shear rate relationship. Then, this relationship will be implemented into the previously discussed RMM iterative method. The

RMM will be used to determine how the STF affects the dynamic response of the plate. This methodology will allow for further analysis into how material properties of the STF, such as particle packing fractions and the fluid phase's density, affect the dynamic response of the plate.

Linking the RMM model to the STAR-CCM+ results will not only allow for a better understanding of how STFs affect the dynamic response of a plate, but it will achieve the objective of creating a multiscale modeling method.

Chapter Five: Implementation of Integrated Computational Materials Engineering Into Plate Design

As discussed in the first chapter, one of the goals of this work is to develop a multiscale modeling method to analyze the effects of modifying a STF has on the dynamic response of multilayer plates that use STF in their design. Accomplishing this objective will allow for the analysis of multiple plate designs using available software without having to invest resources and time into fabricating prototypes and testing them. It also would allow researchers to explore how changing parameters within a STF affects a multilayer plate design that incorporates layers of STFs. This type of analysis could allow for engineers to optimize STFs to better suit different applications.

In the previous two chapters, two different models were developed that are optimal for multiscale modeling. The STAR-CCM+ model is a small scale model of the STF that captures the relationship between viscosity and shear rate. The RMM model is a computationally inexpensive, large-scale model that can determine the dynamic response of a structure, in this case a multilayer plate, using material properties such as viscosity. Both of these models were externally validated using published data. From these two models, a multiscale modeling method was proposed at the end of chapter four.

In this chapter, a parametric study is conducted to determine how changing material properties within the STF affects the dynamic response of a multilayer plate. This study also serves as a means to evaluate the effectiveness of the proposed multiscale modeling method.

To reiterate, the modeling method proposed is to use the relationship between viscosity and shear rate generated by the small scale STAR-CCM+ model within the large scale RMM model to determine how using a STF in a multilayer plate will affect its dynamic response. The end goal of such a study is to see how changing parameters within the STF will affect its material properties and how these changes will affect the dynamic response of the multilayer plate.

5.1 Parameter Study Model in STAR-CCM+

Because the overall material behavior of the STF, its viscosity, is derived from this model, the parametric must begin with the STAR-CCM+ model. Changing parameters within the STAR-CCM+ model is relatively simple since many of these parameters are explicitly modeled within the simulation. Understanding how changing these parameters affects the model is critical to evaluating its use as a design tool. After this investigation, the resulting relationship between the modified STF's viscosity and shear rate will be used in the RMM model.

5.1.1 Parameters of Interest

Multiple materials parameters can affect the behavior material of a STF, but the majority of these parameters can be broken down into two categories: the fluid phase properties and the particle properties. Fluid phase properties mainly focus upon the fluid phase's material properties such as density and viscosity [Barnes, 1989]. To change one of these properties requires changing the fluid phase to a new material entirely, which is not ideal when conducting a parametric study where ideally only one variable is changed at a time. The particle phase properties include, but are not limited to the particle shape, size, and packing fraction [Brown and Jaeger, 2014]. While changing the material used for the particle can also be changed, this is not a major source of interest within studies into STFs [Wetzel et al, 2004. Wagner and Brady, 2009]. For example, Barnes in his 1989 literature review article on STFs heavily discusses the

effects of the packing density of particles on the behavior of STFs and particle size, but does not discuss the material of the particles beyond denoting the material type and the fact the particles are solids [Barnes, 1989]. This is because the particles are assumed to not deform, therefore material properties such as Young's modulus are not as important.

For this research, the two parameters of interest are the particles' diameter and the packing fraction of the particles. These two parameters were selected due to the widespread interest in these parameters in works done by Barnes and Wetzal and the ease of changing these parameters in the STAR-CCM+ model.

The packing fraction was increased from 0.57 to 0.62 in the first study. This increase of 0.05 is a common stepping increment for changing the STF packing fraction, as shown in multiple studies [Brown and Jaeger, 2009, Jones et al, 1991]. All other parameters were held constant, including the particle size and the contact model parameters. This was to make sure that the results only reflect the effects of changing the packing fraction.

The particle diameter was increased from 500 nm to 750 nm. This 50 percent increase in particle diameter is based upon the sizing silica particles available for study through companies such as Sigma Aldrich. If this research is to reflect the ICME design process, it should take into account the materials available from vendors.

5.1.2 STAR-CCM+ Small Scale Modeling Method

The main changes to the STAR-CCM+ model were implemented in the particle phase injector. For each new model all particles were removed from the meshed volume. To change the packing fraction, the particle limit for the injector was increased to represent the increased packing fraction. To change the particle size, the particle size used in the injector is changed

from 500 nm to 750 nm. After the injector parameters are changed, the volume was repopulated with randomly placed particles until the number of desired particles was reached.

The new models are subjected to constant shear rates at 100 (1/s), 300 (1/s), 400 (1/s), and 520 (1/s). These shear rates are selected based upon the data available for the measured results and to capture the behavior of the STF over a range of shear rates. For the shear rates less than 400 (1/s) the coefficient of friction of the particles was set to 2.0 and no lubrication forces were added to the contact models. For the shear rates 400 (1/s) and above the coefficient of friction was reduced to 0.75, but lubrication forces were added to the model with the variables H_w and H_p set to 0.1 of the particle radius.

5.2. STAR-CCM+ Results

Figure 53 shows the results from the first parametric study where the packing fraction of the STF was changed from 0.57 to 0.62. Lee's measured data for the STF with 500 nm diameter particles and a packing fraction of 0.57 is shown in orange with an orange dotted trend line. The STAR-CCM+ results with the same parameters is shown in grey. The new STAR-CCM+ results are shown in blue with a blue dotted trend line representing the derived relationship between viscosity and shear rate. It is apparent that increasing the packing fraction of the shear thickening fluid increases the magnitude of the observed viscosity. The data shows that packing fraction appears to have a large effect at higher shear rates as well. It should be noted that the data point at the shear rate 400 (1/s) is lower than expected. Shear rates within this range have proven difficult to model, as shown in Chapter Three. The results from that chapter showed that there is some uncertainty about this shear rate regarding which contact model (high friction or large lubrication forces) should be around this shear rate.

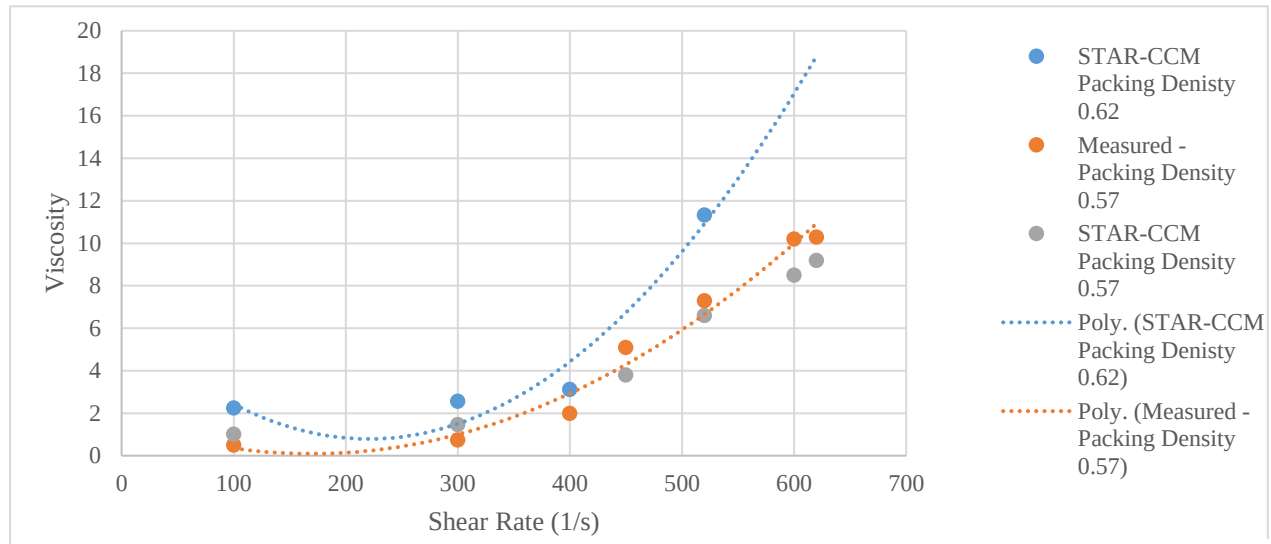


Figure 53. Viscosity vs shear rate for the packing fraction parametric study

This increase in viscosity at the higher shear rates is expected. Lee's rheological data includes the viscosity of a STF with 500 nanometer diameter particles, a 0.62 packing fraction, and a PEG200 fluid phase. His results showed a marked increase the in slope of the relationship between shear rate and viscosity when compared to his data for a STF with a packing fraction of 0.57. This rapid increase may indicate that lubrication forces contact model should be implemented at lower shear rate values when the packing fraction is increased.

Figure 54 shows the results from the second parametric study in which the particle diameter was changed from 500 nm to 750 nm. Lee's measured data is shown in orange with an orange trend line representing the relationship between shear rate and viscosity. The results from the STAR-CCM+ simulations that use the original STF parameters are shown in grey. The results of the new simulations with the larger particle sizes are shown in navy. The results of this parametric study do not show as dramatic of an increase at the higher shear rates at the packing fraction parametric study did. The values of the STAR-CCM+ for the 750 nanometer particles at

the shear rates 100 (1/s) and 300 (1/s) are strikingly similar to those for the original STAR-CCM+ simulation with 500 nanometer particles and to Lee's results at those shear rates as well.

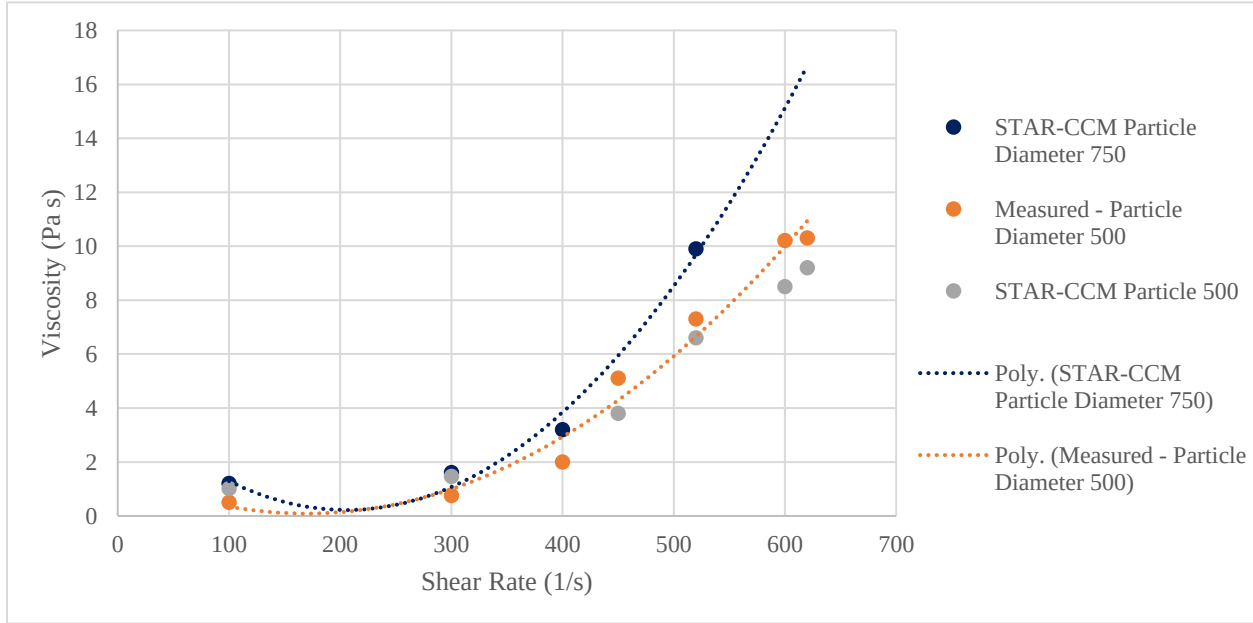


Figure 54. Viscosity vs shear rate for the particle size parametric study

From the results least squares trend lines were developed to describe the relationship between the viscosity, η , and the shear rate, $\dot{\gamma}$. These trend lines are used within the MATLAB simulation to show to effects of changing material parameters within the STF have on the dynamic response of a multilayer plate. Quadratic trend lines were selected to match the trend exhibited by Lee's data. The limits to using a second order polynomial are discussed at the end of Chapter 3.

Equation 52 describes the relationship between viscosity and shear rate for Lee's measured data. The relationship between viscosity and shear rate for the STAR-CCM+ simulation with the same material parameters is given in equation 53. Equation 54 gives the relationship between viscosity and shear rate for the simulations with a packing fraction

increased to 0.62. Equation 55 gives the relationship between viscosity and shear rate for the simulations with the 750 nm diameter particles.

$$\eta = 5 * (10^{-5}) * \dot{\gamma}^2 - 0.0181\dot{\gamma} + 1.6283 \quad (52)$$

$$\eta = 2 * (10^{-5}) * \dot{\gamma}^2 - 0.0023\dot{\gamma} + 0.8516 \quad (53)$$

$$\eta = 1 * (10^{-3}) * \dot{\gamma}^2 - 0.0498\dot{\gamma} + 6.280 \quad (54)$$

$$\eta = 1 * (10^{-4}) * \dot{\gamma}^2 - 0.0394\dot{\gamma} + 4.275 \quad (55)$$

Comparing these equations it is apparent increasing the number of particles to the STF has the greater effect on the STF's viscous behavior. By increasing the particle fraction by a mere five percent, the slope of the viscosity function increases by two orders of magnitude. Increasing the size of the particles only increased the slope by a single order. Besides Lee's data, it is logical that adding more particles to a STF would have a greater effect on its viscosity since particles that causes the shear-thickening behavior.

Based off these initial findings, it is hypothesized that the STF with the highest packing fraction will have the greatest effect on the dynamic response of the plate. However, the large scale model needs to be run before any conclusions can be drawn.

5.3 Implementing STAR-CCM+ Results into RMM Model

The iterative method described in Chapter 4 is used to model the dynamic response of the multilayer plate using the RMM. In the first step the calculated viscosity of the STF for the shear rate of 100 (1/s) is input into the RMM as the viscosity of STF layer. After the dynamic response is calculated, the actual shear rate the fluid undergoes is calculated. This new shear rate is used to recalculate the viscosity of the STF. The dynamic response is then recalculated using the RMM

with the new viscosity value. This iterative method will continue to loop until the viscosity does not change by more than 10%.

The plate configuration used in this study is the two layer plate discussed in Chapter Four. A 0.005-meter thick layer of a STF is sandwiched between two layers of solid materials that are 0.05 meters thick each. Simulations were completed with both solid layers being made of steel and both layers being made of aluminum. The material properties of the steel and aluminum are those listed in Table 6. The mass density listed in this table for PEG200 is also used in the model.

The viscosity used in the RMM is calculated using the results from the STAR-CCM+ small scale models. In this way, material properties are passed from a small scale model to a structure-scale model.

5.3.1 RMM Results

The dynamic response of the multilayer plate was calculated using the iterative method four times using equations 52 through 55. For brevity and clarity's sake, the results obtained using Dr. Lee's data, which is for a STF with particles 500 nanometers in diameter and a packing fraction of 0.57, will be referred to as results obtained using the 'measured' data.

As expected, the difference in the dynamic response calculated from the RMM using the measured data's shear-rate and viscosity relationship and the dynamic response calculated using the initial STAR-CCM+ model were insignificant. However, differences in the dynamic response became apparent when comparing the results from the measured data to the results from the parametrically altered STF data.

Figure 55 shows the dynamic response of the STF with the 0.62 packing fraction divided by the dynamic response of the measured data for the plate with steel layers. This normalized dynamic response provides a measure of how changing the STF parameter affects the dynamic response of the multilayer plate. The results of this graph show that increasing the packing fraction by 0.05 resulted in up to a seven percent decrease in the dynamic response of the plate at peak frequencies.

There are also frequencies where the normalized response that are greater than one. This means that the dynamic response at those frequencies for the plate with the more viscous STF is greater than the dynamic response for the plate that has the measured STF. This is because there

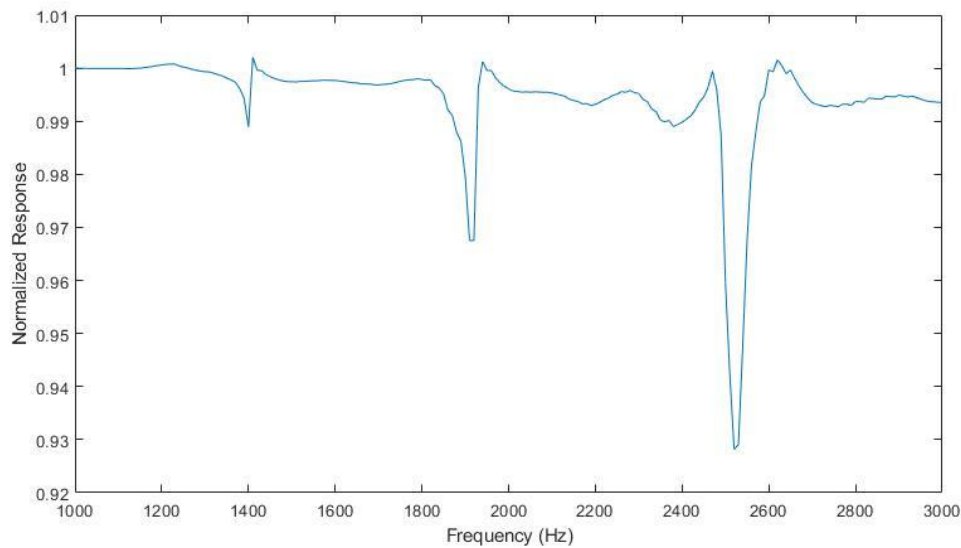


Figure 55. The dynamic response of the steel multilayer plate with a STF packing fraction of 0.62 divided by the dynamic response of the steel multilayer plate with the measured STF data

is a slight shift in the dynamic response of the multilayer plate when more viscous STFs are used, which results in normalized values greater than one even though the dynamic response over the entire frequency domain has decreased.

Figure 56 shows the normalized results for the aluminum multilayer plate with the 0.62 packing fraction STF. Using the more viscous STF appears to have had a greater effect on the

dynamic response of the aluminum multilayer plate than it did on the dynamic response of the steel multilayer plate. The dynamic response of the plate is reduced up to 11 percent at the higher frequencies by using the 0.62 packing fraction STF in plate rather than the 0.57 packing fraction STF. Some normalized values are greater than one for the same reason previously discussed.

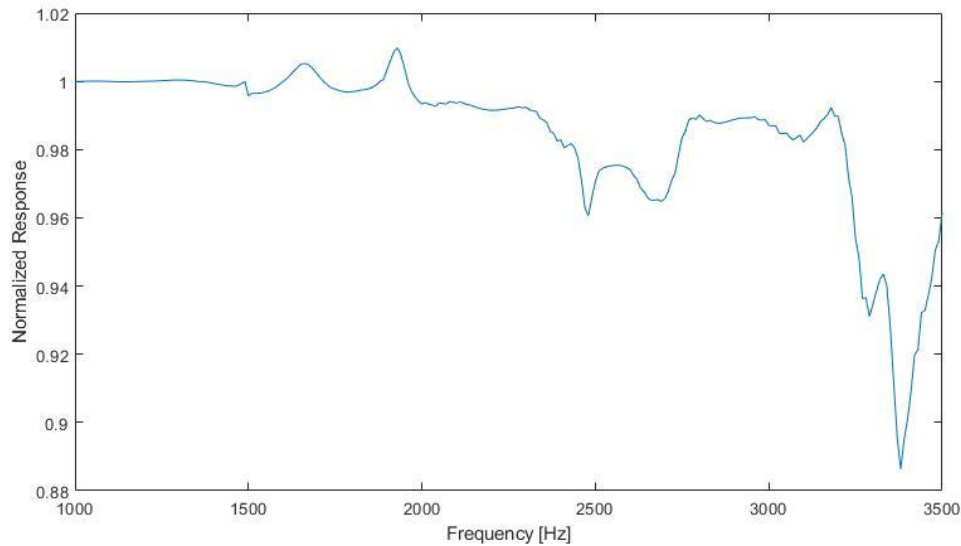


Figure 56. The dynamic response of the aluminum multilayer plate with a STF packing fraction of 0.62 divided by the dynamic response of the steel multilayer plate with the measured STF data

The RMM model then was used to study the effects of using the STF with the 750-nanometer diameter particles on the dynamic response of the multilayer plates. Figure 57 shows the dynamic response of the steel multilayer plate with the 750-nanometer diameter particle STF normalized by the dynamic response of the steel multilayer plate with the 500-nanometer diameter particle STF. There are fewer values above one, representing that the phase shift between the original dynamic response and the STF with larger particles is not as pronounced as the phase shift observed in the dynamic response of the more densely packed STF. Using the larger particle STF in the multilayer plate only reduced the dynamic response of the plate by up

to about four percent. This is in agreement with the fact the larger particle STF is still more viscous than the measured STF.

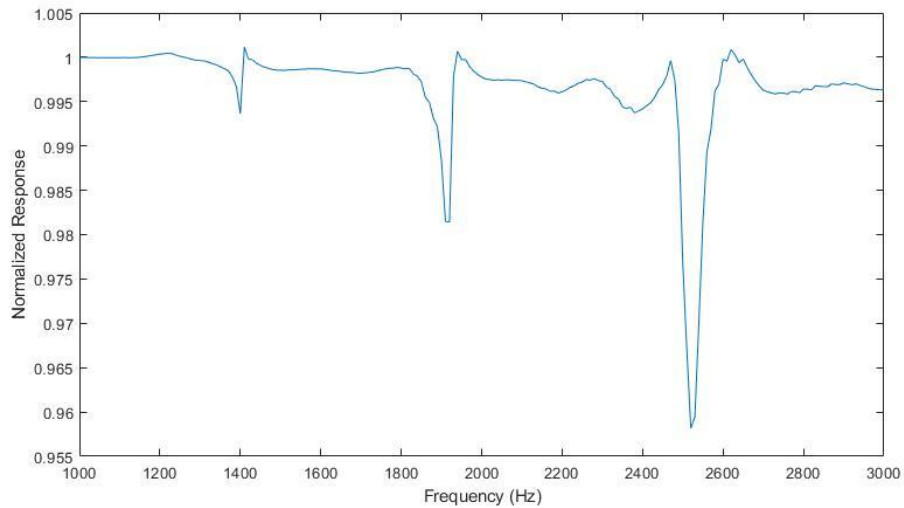


Figure 57 The dynamic response of the steel multilayer plate with a STF packing fraction of 0.62 divided by the dynamic response of the steel multilayer plate with the measured STF data

Figure 58 shows the normalized dynamic response of the aluminum multilayer plate with the larger particle STF. The overall trend of this normalized response is similar to the one shown in Figure 56. However, the response of the plate is reduced by about 6.5 percent when using this STF. This is to be expected since the more densely packed STF is more viscous.

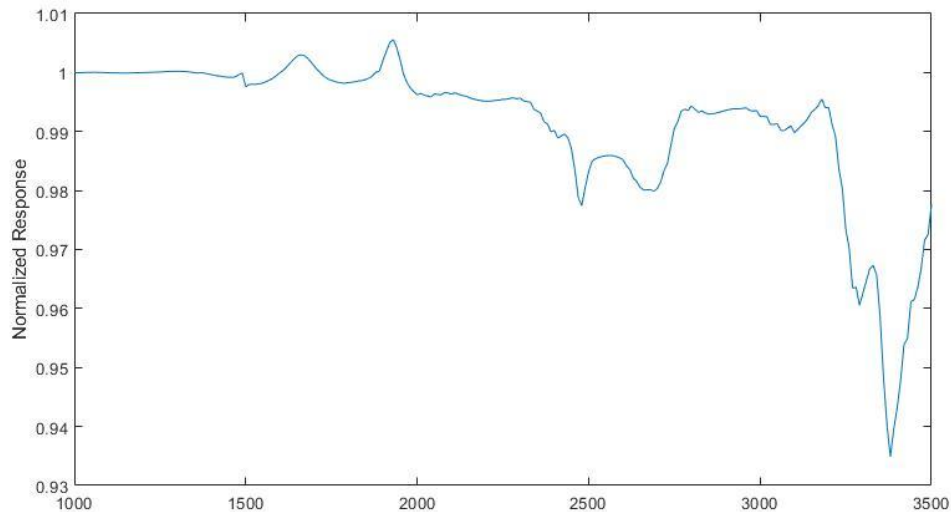


Figure 58 The dynamic response of the aluminum multilayer plate with a STF packing fraction of 0.62 divided by the dynamic response of the steel multilayer plate with the measured STF data

5.4 Analysis

The STAR-CCM+ small scale model successfully modeled how changes in the composition of a STF would affect its viscosity. As discussed earlier, literature has shown that increasing the packing fraction of particles in a STF increases the STF's viscosity. Increasing the particle size can also increase the viscosity of the STF. The results from the STAR-CCM+ simulation reflected both of these trends.

Furthermore, the large-scale RMM model successfully incorporated the new viscosity and shear rate relationships. The iterative method succeeded in reaching convergence. More importantly, the dynamic response of the multilayer plates was reduced when the new STF properties were used. This was to be expected; incorporating a more viscous material into a design should result in a greater damping effect. Using STFs that are more viscous also had a greater effect on the dynamic response of aluminum multilayer plates than the dynamic response of steel multilayer plates. Aluminum is a far weaker material than steel, so it is logical that its response would be more effected.

The proposed multiscale modeling method was successfully executed. The results from the microscale model in STAR-CCM+ were used within the macroscale RMM model to study the effects of changing STF material parameters on the dynamic response of a plate.

Chapter Six: Conclusions, Contributions, and Future Work

6.1 Conclusions

The overarching goal of this work is to model how shear thickening fluids (STFs) can affect the dynamic response of a multilayer plate. The motivation behind this objective is to resolve the ongoing design conflict between mobility and survivability in military vehicles [TARDEC, 2014]. To achieve this objective, five smaller goals that focused on modeling STFs and their effects were outlined and fulfilled.

The first aim was to study the effects of using a reduced order damping mechanism similar to a STF along the layer interfaces of a multilayer plate. The effects examined for this study were the plate's dynamic response to a blast load and plate's energy losses under this loading condition. This was successfully achieved by developing a multilayer plate model in LS-DYNA that represented a vehicle floor and using Coulomb friction along the plate layer interfaces to represent the effects of the STF. Coulomb friction was used to due to similarity how energy is lost shearing a STF and how energy is loss through friction [Decker et al, 2007]. The results of these simulations showed that using a damping mechanism along the plate interfaces causes a significant energy dissipation and a reduction of the plate's bending energy. Furthermore, the plates with inter-layer friction had a lower dynamic response than those without any inter-layer friction.

Based upon these simulations the second objective was also achieved. This was to use the results of the previously mentioned study to evaluate whether or not to continue investigating the

use STFs in multilayer plates. Implementing friction in the LS-DYNA simulations resulted in a 40 percent reduction in the bending energy of the plate. This, coupled with the reduced response of the plate, led to the decision that further work should be taken to investigate how using STFs in a multilayer plate would affect the plate's dynamic response.

In order to do this, research efforts focused on meeting the third goal to develop a small scale model of the STF needed to be created in order to extract material properties for a large-scale structural model of the plate. The discrete element method (DEM) was used to model the particle phase of a STF in a computational fluid dynamics (CFD) simulation developed in the software STAR-CCM+. Lubrication force terms had to be added to the particle contact model at higher shear rates. This reflected the increase in the slope of the function that describes the viscosity of the STF in terms of the shear rate.

Once this small scale model was established, a macroscale model of a multilayer plate with layers of a STF was developed using the reverberation matrix method (RMM). The RMM is a versatile tool that uses a plane wave theory and material stiffness properties to solve for the dynamic response of a plate. To achieve this goal of developing a macroscale model of the multilayer plate, the RMM method had to be modified so that it could solve for the dynamic response and stresses in a structure consisting of alternating layers of fluids and solids. Then, the viscous effects of the fluids had to be incorporated into the model. Finally, an iterative method was developed to compensate for the non-linear relationship between the STF's viscosity and shear rate. These efforts yielded a RMM based plate model that could simulate a multilayer plate with a viscous fluid with non-linear material properties. Analysis of this model showed that implementing a STF in a multilayer plate did reduce its dynamic response. The RMM model also

showed that a STF is more effective at reducing the dynamic response than using a viscous, Newtonian fluid.

After the two models at different scales were established, the fifth objective was fulfilled. This objective was to use multiscale modeling to examine how changes in the material compositions of a STF affect the dynamic response of a multilayer plate. This was accomplished by performing a parametric study with the small scale STAR-CCM+ model and applying the results of this study to the large scale RMM model. The parametric study focused on the effects of increasing the particle packing fraction of the STF and increasing the particle size. Both of these changes caused the STF's viscosity to increase, but the packing fraction had a greater effect on the STF's viscosity. These new relationships between viscosity and shear rate were input into the RMM model. As expected, increasing the viscosity of the fluid reduced the dynamic response of the multilayer plate. Interestingly, it was found that modifying the material properties of the STF had a larger effect on a multilayer plate made of softer materials such as aluminum than stronger materials such as steel.

By completing these goals, the overall objective of determining how STFs affect the dynamic response of a multilayer plate was achieved. Implementing layers of a STF between the plate layers reduces the dynamic response of the plate. Relatively small changes within the STF can result in significant effects on the plate's dynamic response. Furthermore, the effects of the STF on the dynamic response of the plate are more pronounced when the solid layers of the plate are made of softer materials.

6.2 Contributions

This research successfully expanded on the applications of the RMM. The RMM was successfully utilized to analyze a multilayer plate containing both solid and fluid media. To do so

required novel methods of constructing the scattering matrices at the layer interface boundaries. New approaches of incorporating the viscous effects of the fluid within the RMM, which had not been done before, were developed. Result comparisons to FEA simulations from the software LS-DYNA showed the importance of including viscous material properties in the RMM. Finally, the RMM was employed to develop a new method of modeling non-linear materials.

The STAR-CCM+ model of the STF is also represents a new innovation. The discrete element method (DEM) has been used to model densely packed particle flows, such as fluidized beds, but these simulations were not trying to simulate non-Newtonian behavior. The STAR-CCM+ model was used to further explore the relationship between particle friction, lubrication forces, and shear rate when trying to accurately capture the behavior of the STF.

Furthermore, the proposed multiscale modeling method provides a new method for analyzing the incorporation of new materials into multilayer plate design. Using the previously discussed models developed for this method allow for the modeling and analysis of a design that incorporates STFs without having to use expensive and time consuming prototypes.

6.2.1 Publications

This research has yielded the following publications:

- Jiang, W., Bennett, A., Vlahopoulos, N., Castanier, M., & Thyagarajan, R. (2016). A Reduced-Order Model for Evaluating the Dynamic Response of Multilayer Plates to Impulsive Loads. *SAE International Journal of Passenger Cars - Mechanical Systems*, 9(1), 83-89. doi:10.4271/2016-01-0307
- A. Bennett, N. Vlahopoulos, W. Jiang, M. Castanier, R. Thyagarajan, S. Shurin, (2018). Modeling of Shear Thickening Fluids for Analysis of Energy Absorption Under Impulse Loading. *Ground Vehicle System Engineering & Technology Symposium*, Michigan
- W. Jiang, A. Bennett, N. Vlahopoulos, G. Zhan (2019). A Reduced-Order Modeling Based Design and Optimization for a Lightweight Multilayer Armor Plate against Blast and Impact. *Advance Materials for Defense: Development, Analysis and Applications*, Springer, Minho

6.3 Future Work

Another publication detailing the RMM model that includes viscous effects and the new STAR-CCM+ model of the STF will be published.

There are three major areas of future research for this work: continuing to improve the STAR-CCM+ model, developing a more accurate material model of the STF in the reverberation matrix model, and expanding the ICME analysis.

While changing particle size and packing fraction within the STAR-CCM+ model yield the expected change in the viscosity, the accuracy of the model results could be improved to better match published results. In Lee's work, increasing the packing fraction to 0.62 caused the STF to exhibit higher viscosities than those given at STAR-CCM+ [Lee et al, 2003]. This could be done by developing relationships between the particle frictional coefficient, packing fraction, and the particle size. Further studies on the development of the lubrication force term and particle packing fraction could also be conducted.

The current STAR-CCM+ model applies a constant shear rate to the fluid sample. This does not reflect the harmonic loading conditions that the STF is subjected to in the RMM model. To improve the microscale model, a harmonic displacement could be applied to the STF model to get the relationship between harmonic shear rates and the fluid's viscosity.

To improve the accuracy of the reverberation matrix method model, the STF could be modeled as a fluid filled porous material rather than only as a viscous fluid. Currently the STF is modeled as a viscous fluid. This would account for the effects the particles would have on how vibrations penetrate the STF.

The ICME analysis can be expanded by incorporating an optimization code that would evaluate which solid materials to use within the multilayer plates to better utilize the STF's behavior. Another area of optimization is to further investigate what particle parameters the STF should have to have the greatest impact on the dynamic response of the plate. Such optimization would provide a better link between the manufacturing of the STF and its effects on the plate's dynamic response.

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