

A Novel Sandwich Design for Improved Out-of-Plane Stiffness in Double Parallelogram Flexure Mechanism

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ABSTRACT

This paper proposes a novel sandwich design for improved out-of-plane stiffness in the Double Parallelogram Flexure Mechanism (DPFM). In DPFMs and flexure mechanisms comprising DPFMs, low in-plane stiffness along the motion direction helps improve the range of motion and reduces the actuation effort. On the other hand, high out-of-plane stiffness is essential to support large payloads and minimize out-of-plane parasitic motions. Achieving both of these design objectives simultaneously is challenging in DPFMs due to the inherent tradeoff between the in-plane and out-of-plane mechanics. The novel sandwich DPFM design proposed in this paper achieves a significant improvement in the out-of-plane bearing stiffness without affecting the in-plane motion-direction stiffness, thereby overcoming the tradeoff between the out-of-plane and in-plane stiffness values. Analytical models are presented for the out-of-plane and in-plane stiffness of the sandwich DPFM, which match very closely with Finite Element Analysis (FEA) predictions. The superior performance of the sandwich DPFM is demonstrated and several design insights are presented using the analytical stiffness models.

1) INTRODUCTION

Flexure mechanisms are designed to allow relative motion between two bodies along certain directions, referred to as Degrees of Freedom (DoFs) or motion directions, and restrict motion along other directions, referred to as Degrees of Constraint (DoCs) or bearing directions. The design objective in flexure mechanisms is to maximize the displacement and minimize the stiffness along DoFs, and minimize the displacement and maximize the stiffness along the DoCs.

Complex flexure mechanisms typically consist of simpler building blocks, referred to as *flexure modules*, arranged in different configurations throughout the mechanism to achieve the design objectives along the DoFs and DoCs. The Double Parallelogram Flexure Mechanism (DPFM) is a common flexure module that is used by itself and also as a building block in various multi-axis flexure mechanisms [1-3]. A DPFM (e.g.

layer 'A' or 'B' in Figure 1) is typically designed to provide guided in-plane motion along a Y-translation DoF while restricting motions in all other displacements and rotations DoC. Low in-plane stiffness along the Y-translation DoF direction helps improve the range of motion and reduce the actuation effort. On the other hand, high stiffness along the DoC directions provides load-bearing capability, which is important in many applications. However, there remains an inherent tradeoff between the low stiffness requirement in the DoF directions and the high stiffness requirement in the DoC directions, in any flexure mechanism. This tradeoff between the Y-translation DoF and planar DoCs (X-translation and Θ_z -rotation) for the DPFM has been overcome by various designs that help constrain the secondary stage [3, 4].

Nonetheless, there are not many reported design solutions to help overcome the tradeoff between the low stiffness desired in the Y-translation DoF and the high stiffness desired in the out-of-plane DoCs (Z-translation, Θ_x -rotation, and Θ_y -rotation). Common strategies include the use of reinforcement sections '4' in the flexure blades, increasing their height H , or reducing their length L (referring to DPFM layer 'A' in Figure 1). But all these geometric optimization strategies inevitably come at the expense of increasing the in-plane stiffness K_Y , which is undesirable. This paper presents a novel sandwich architecture that fundamentally overcomes the tradeoff between the out-of-plane and in-plane stiffness by employing two identical DPFMs (layers 'A' and 'B' in Figure 1) that are spaced apart in the Z direction and employ out-of-plane interconnects '9', '10', '11', and '12'. This design offers a significantly higher out-of-plane stiffness while preserving the in-plane K_Y stiffness compared to the conventional single-layer DPFM. Analytical results, corroborated by FEA, are presented to show this superior design performance.

2) SANDWICH DPFM DESIGN

A schematic view of the proposed sandwich DPFM is shown in Figure 1. The sandwich DPFM comprises two identical DPFMs spaced apart by S

along the Z direction. Each DPFM consists of four reinforced flexure blades, each with an overall length L , two flexible segments of length B , height H , and thickness T . The respective ground stages, rigid reinforcements, secondary stages, and motion stages between the two DPFM layers 'A' and 'B' are rigidly interconnected via out-of-plane interconnects '9', '10', '11', and '12'.

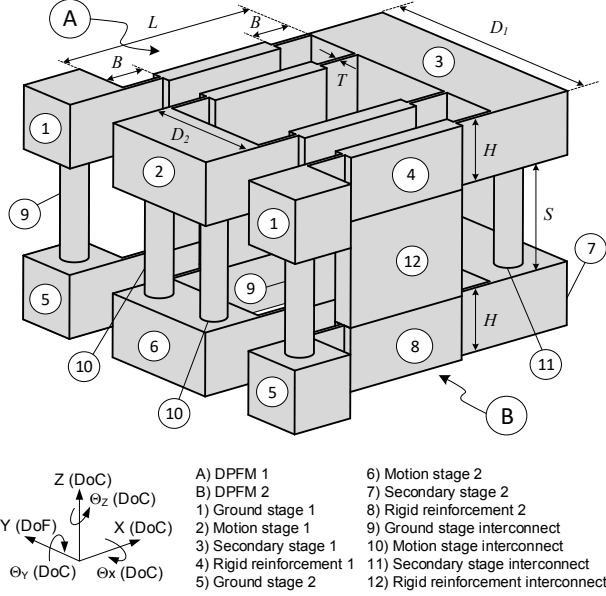


Figure 1. Schematic view of a sandwich DPFM

The out-of-plane deformation of the sandwich DPFM is derived from the out-of-plane deformation of the constitutive flexure blades (W) in the Z direction, and comprises bending (W_b) and shear deformations (W_s), i.e. $W = W_b + W_s$. At zero plate spacing, the out-of-plane bending deformations W_b of the constitutive flexure blades in the single-layer and sandwich designs are equal [5, 6]. However, the out-of-plane shear deformation in the sandwich DPFM is slightly larger than that of the single-layer DPFM. This is because in the sandwich design, the flexible segments of DPFM layer 'A' are disconnected from the flexible segments of DPFM layer 'B', allowing them to undergo a small amount of in-plane shear deformation w.r.t. each other along X direction. This in-plane shear deformation gives rise to a corresponding shear deformation along the Z direction [5, 6]. Therefore, at zero plate spacing, the K_z stiffness of the sandwich DPFM is slightly smaller than the single-layer design. By increasing the plate spacing S , the bending deformation of the flexure blade (i.e. W_b) decreases. Meanwhile, the in-plane shear deformation of DPFM layer 'A' w.r.t. DPFM layer 'B' is suppressed by the out-of-plane interconnects between the rigid reinforcements of DPFM layers 'A' and 'B'. As a result, by increasing the plate spacing in the sandwich DPFM,

the out-of-plane stiffness K_z increases and quickly overtakes that of the single-layer DPFM. On the other hand, the in-plane DoF stiffness of the sandwich DPFM (i.e. K_y) is derived from the bending stiffness of the constitutive flexure blades in the XY plane and is not affected by the plate spacing S . Therefore, by increasing the plate spacing S in the sandwich DPFM, one can improve the out-of-plane stiffness without affecting the in-plane DoF stiffness K_y . This is in contrast to any geometric optimization strategies where increasing the out-of-plane stiffness inevitably leads to an increase in the in-plane DoF stiffness.

To demonstrate the superior performance of the sandwich DPFM compared to a conventional DPFM, analytical models for the out-of-plane and in-plane stiffness for both designs are provided next. In all of the subsequent discussions, the constitutive flexure blades of the single-layer and sandwich DPFMs are assumed to have the same physical parameters B , T , L . Furthermore, the overall height of the flexible segments in the constitutive flexure blades is $2H$.

3) OUT-OF-PLANE STIFFNESS MODELS

3-1) Out-of-plane stiffness K_z and $K_{\theta y}$

The out-of-plane stiffness of single-layer and sandwich Parallelogram Flexure Mechanisms (PFMs) were previously derived in [6]. A DPFM (either single-layer or sandwich), can be considered as two PFMs connected in series: an outer PFM with the width D_1 (referred to as PFM 1) connected to the ground stage and an inner PFM with the width D_2 (referred to as PFM 2) connected to the PFM 1. Therefore, the out-of-plane stiffness K_z and $K_{\theta y}$ of a DPFM (either single-layer or sandwich) is obtained as half of that of the constitutive PFMs. Accordingly, one can say

$$K_{Z-\text{single}}^{\text{DPFM}} = \frac{48AEI_y Gk}{B(2AkG(3L^2 - 3LB + 2B^2) + 48EI_y)} \quad (1)$$

$$K_{\theta y-\text{single}}^{\text{DPFM}} = \frac{4EI_y}{B} \quad (2)$$

$$K_{Z-\text{sandwich}}^{\text{DPFM}} = \frac{12AEGI_y k(A(S+H)^2 + 4I_y)}{\Delta_1(S)} \quad (3)$$

$$\text{where: } \Delta_1(S) = B(A(S+H)^2(AGB^2k + 12EI_y) + 8AGI_y k(3L^2 - 3LB + 2B^2) + 48EI_y^2)$$

$$K_{\theta y-\text{sandwich}}^{\text{DPFM}} = \frac{E(A(S+H)^2 + 4I_y)}{4B} \quad (4)$$

where E and G are Young's modulus of elasticity and shear modulus, $I_y = TH^3/12$ is the second moment of area of the flexure blade cross-section w.r.t. the Y-direction, $A = TH$ is the cross-section area and k is the shear correction factor.

Investigating the above analytical stiffness models (Eqs. (1)-(4)), the following conclusions are drawn:

- 1) The K_Z stiffness of the sandwich DPFM at zero plate spacing is always less than that of the single-layer DPFM, regardless of the geometric parameters of the constitutive flexure blades. By increasing the plate spacing S , the K_Z of the sandwich DPFM increases and quickly overtakes that of the single-layer DPFM. The plate spacing value beyond which the K_Z of the sandwich DPFM exceeds that of the single-layer DPFM is called S_{min-Z}^{DPFM} and is given by the following expression

$$S_{min-Z}^{DPFM} = -H + 2\sqrt{\frac{I_Y}{AL}\left(3L + \frac{2B^2}{(L-B)}\right)} \quad (5)$$

- 2) The K_Z stiffness of the sandwich DPFM increases by increasing the plate spacing S and asymptotically plateaus out at large values of S . The theoretical maximum value of the K_Z for the sandwich DPFM is given by

$$K_{Z-sandwich}^{DPFM-theo-max} = \lim_{S \rightarrow \infty} K_{Z-sandwich}^{DPFM} = \frac{12AEGI_Y k}{B(AGB^2 k + 12EI_Y)} \quad (6)$$

The theoretical maximum value given by Eq. (6) can only be achieved for infinite S which is practically infeasible. Therefore, taking advantage of Eq. (6), we provide the S_{max-Z}^{DPFM} which achieves 90% of the theoretical maximum K_Z and is given by the following expression

$$S_{max-Z}^{DPFM} = -H + 2\sqrt{\frac{I_Y(AGk(54L(L-B) + 26B^2) - 12EI_Y)}{A(AGB^2 k + 12EI_Y)}} \quad (7)$$

It is recommended to select the plate spacing as $S < S_{max-Z}^{DPFM}$ to avoid diminishing return in K_Z with increasing S .

- 3) The theoretical maximum value of $K_{Z-sandwich}^{DPFM}$ is always greater than $K_{Z-single}^{DPFM}$, regardless of the geometric parameters of the constitutive flexure blades.
- 4) The $K_{\theta Y}$ stiffness of the sandwich DPFM is equal to that of the single-layer DPFM at zero plate spacing. This is because, at zero plate spacing, the resisting out-of-plane moment acting on the motion stages of the single-layer and sandwich DPFMs are equal, leading to the $K_{\theta Y}$ stiffness of these designs being equal [5, 6].
- 5) The $K_{\theta Y}$ stiffness of the sandwich DPFM is always greater than that of the single-layer DPFM at non-zero plate spacing, regardless of the physical parameters of the flexure blades.

According to the above-mentioned observations, the K_Z and $K_{\theta Y}$ of the single-layer and sandwich DPFMs can be sorted as follows

$$\begin{cases} K_{Z-sandwich}^{DPFM} < K_{Z-single}^{DPFM} & S < S_{min-Z}^{DPFM} \\ K_{Z-sandwich}^{DPFM} = K_{Z-single}^{DPFM} & S = S_{min-Z}^{DPFM} \\ K_{Z-single}^{DPFM} < K_{Z-sandwich}^{DPFM} & S > S_{min-Z}^{DPFM} \end{cases} \quad (8)$$

$$\begin{cases} K_{\theta Y-sandwich}^{DPFM} = K_{\theta Y-single}^{DPFM} & S = 0 \\ K_{\theta Y-sandwich}^{DPFM} > K_{\theta Y-single}^{DPFM} & S > 0 \end{cases} \quad (9)$$

3-2) Out-of-plane stiffness $K_{\theta X}$

A DPFM (either single-layer or sandwich) subjected to an out-of-plane moment M_X can be considered as the serial connection of the constitutive PFMs. The $K_{\theta X}$ of single-layer and sandwich PFMs were previously derived in [6]. Therefore, the $K_{\theta X}$ stiffness of the single-layer and sandwich DPFMs are obtained as follows

$$K_{\theta X-single}^{DPFM} = \frac{48AEI_Y Gk}{B(2kAG(3L^2 - 6LB + 4B^2) + 96EI_Y)} \frac{D_1^2 D_2^2}{D_1^2 + D_2^2} \quad (10)$$

$$K_{\theta X-sandwich}^{DPFM} = \frac{6AEGI_Y k (A(S+H)^2 + 4I_Y)}{\Delta_2(S)} \frac{D_1^2 D_2^2}{D_1^2 + D_2^2} \quad (11)$$

$$\Delta_2(S) = B(A(S+H)^2 (AGB^2 k + 12EI_Y) + 4AGI_Y k (3L^2 - 6LB + 4B^2) + 48EI_Y^2)$$

Inspecting the analytical $K_{\theta X}$ expressions given by Eqs. (10) and (11), the following observations are concluded:

- 1) The $K_{\theta X}$ stiffness of the sandwich DPFM is smaller than that of the single-layer DPFM at zero plate spacing, regardless of the geometric parameters of the constitutive flexure blades. This is due to the fact that the $K_{\theta X}$ stiffness of a DPFM (either single-layer or sandwich) is derived from the K_Z stiffness of the constitutive flexure blades. Furthermore, the K_Z stiffness of a sandwich flexure blade is smaller than that of the single-layer flexure blades at zero plate spacing.
- 2) The $K_{\theta X}$ stiffness of the sandwich DPFM increases by increasing S and overtakes that of the single-layer DPFM beyond a specific plate spacing value called $S_{min-\theta X}^{DPFM}$ which is given by

$$S_{min-\theta X}^{DPFM} = -H + 2\sqrt{\frac{I_Y}{AL}\left(3L + \frac{4B^2}{(L-2B)}\right)} \quad (12)$$

It has to be noted that $S_{min-\theta X}^{DPFM}$ given in Eq.(12) is larger than S_{min-Z}^{DPFM} given in Eq. (5) (the second term under the square root in Eq. (12) is larger than the corresponding term in Eq. (5) while all the other terms are the same). Hence, selecting $S > S_{min-\theta X}^{DPFM}$ in a sandwich DPFM ensures that both K_Z and $K_{\theta X}$

are higher than those of a single-layer DPFM with similar physical parameters.

- 3) The $K_{\theta X}$ of the sandwich DPFM increases with increasing the plate spacing and plateaus at large values of S . The theoretical maximum value of the $K_{\theta X}$ for the sandwich DPFM when $S \rightarrow \infty$ is given by the following expression

$$K_{\theta X-sandwich}^{DPFM-asy} = \frac{12EI_y A^2 Gk}{B(A^2 GB^2 k + 12AEI_y)} \frac{D_1^2 D_2^2}{D_1^2 + D_2^2} \quad (13)$$

The theoretical maximum value given by Eq. (13) can only be achieved for infinite plate spacing which is practically infeasible. Using Eq.(13), we can determine $S_{max-\theta X}^{DPFM}$ which provides 90% of the theoretical maximum of $K_{\theta X-sandwich}^{DPFM}$ and is given by the following expression

$$S_{max-\theta X}^{DPFM} = -H + 2\sqrt{\frac{I_y (AGk(27L(L-2B)+26B^2)-12EI_y)}{A(AGB^2k+12EI_y)}} \quad (14)$$

The $S_{max-\theta X}^{DPFM}$ presented in Eq. (14) is smaller than the S_{max-Z}^{DPFM} given in Eq. (7), (the first term in the numerator under the square root in Eq. (14) is smaller than the corresponding term in Eq. (7) while all the other terms are the same). Thus, choosing $S > S_{max-Z}^{DPFM}$ ensures that both K_Z and $K_{\theta X}$ reach their maximum values.

Based on observations 1-3, the $K_{\theta X}$ of the single-layer and sandwich DPFMs can be sorted as follows

$$\begin{cases} K_{\theta X-sandwich}^{DPFM} < K_{\theta X-single}^{DPFM} & S < S_{min-\theta X}^{DPFM} \\ K_{\theta X-sandwich}^{DPFM} = K_{\theta X-single}^{DPFM} & S = S_{min-\theta X}^{DPFM} \\ K_{\theta X-single}^{DPFM} < K_{\theta X-sandwich}^{DPFM} & S > S_{min-\theta X}^{DPFM} \end{cases} \quad (15)$$

3-3) Validation of the out-of-plane stiffness models

To corroborate the accuracy of the presented analytical stiffness models, nonlinear FEA was conducted using ANSYS. A single-layer and a sandwich DPFM with the following physical parameters were considered: $L = 100mm$, $B = 40mm$, $H = 15mm$, $T = 0.5mm$, $E = 70GPa$, $G = 26.92GPa$, $D_1 = 120mm$, $D_2 = 80mm$. The constitutive flexure blades were discretized using 100 beam elements and the rigid reinforcements, motion stage, and secondary stage were modeled as rigid bodies. The out-of-plane stiffness K_Z , $K_{\theta Y}$ and $K_{\theta X}$ of the single-layer and sandwich DPFMs are illustrated in Figs. 2-4. All the figures show excellent agreement between the analytical stiffness models and the FEA results ($< 3\%$ error). As expected, the K_Z , $K_{\theta Y}$, and $K_{\theta X}$ stiffness of the sandwich DPFM increases significantly with increasing the spacing S . The percentage improvements in the out-of-plane stiffness of the

sandwich DPFM compared to the single-layer DPFM at various values of S are indicated in the figures.

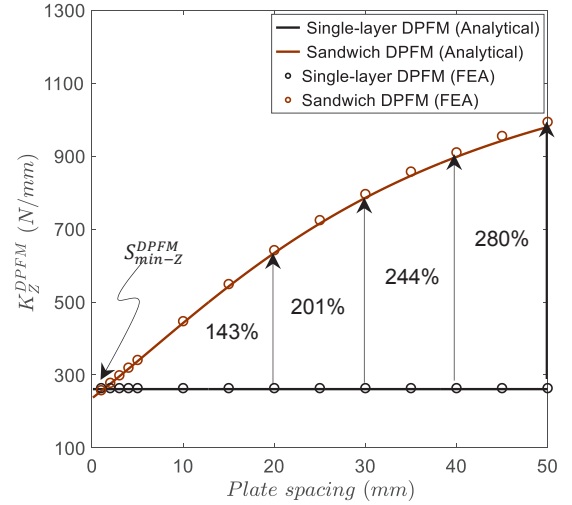


Figure 2. K_Z stiffness of the single-layer and sandwich DPFMs versus the plate spacing

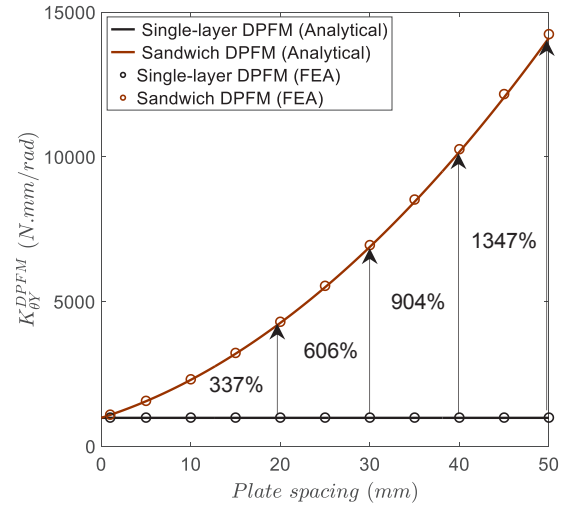


Figure 3. $K_{\theta Y}$ stiffness of the single-layer and sandwich DPFMs versus the plate spacing

4) NON-NOMINAL OUT-OF-PLANE STIFFNESS

The stiffness of a DPFM (either in-plane or out-of-plane) when the motion stage displacement along the DoF direction is zero, is referred to as the *nominal stiffness*. The stiffness of a DPFM when the motion stage has been displaced by a finite amount along the DoF direction, is referred to as the *non-nominal stiffness*. The non-nominal out-of-plane stiffness of a DPFM (either single-layer or sandwich), drops by increasing the DoF (Y-direction) displacement due to the elastokinematic effect [7].

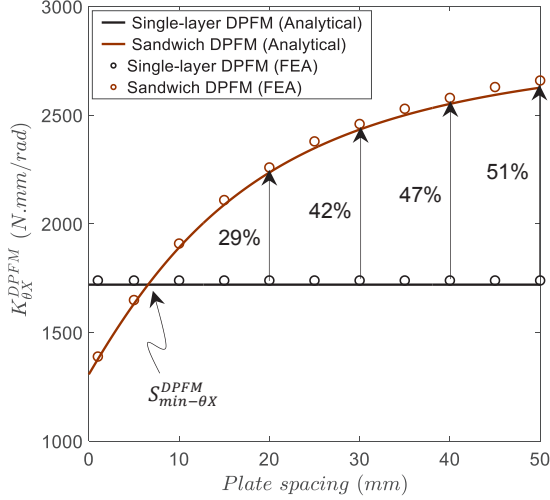


Figure 4. $K_{\theta X}$ stiffness of the single-layer and sandwich DPFMs versus the plate spacing

We used FEA with a similar procedure outlined in Section 3 to obtain the non-nominal K_Z stiffness of the single-layer and sandwich DPFMs. To this end, a Y-direction displacement varying between -10mm to 10mm was applied on motion stages of the single-layer and sandwich DPFMs across 21 steps, over which the out-of-plane stiffness K_Z was calculated. Figure 5 shows the non-nominal K_Z stiffness of the single-layer and sandwich DPFMs obtained from the FEA versus the Y-direction displacement of the motion stage for $S = 20\text{mm}$. The sandwich DPFM exhibits a smaller non-nominal stiffness drop by increasing the Y-direction displacement compared to the single-layer DPFM. The out-of-plane K_Z stiffness of a DPFM (either single-layer or sandwich) is derived from the K_Z stiffness of the constitutive flexure blades. In the non-nominal case (when the motion stage has been displaced by a finite amount along the Y-direction), the out-of-plane displacement W of a flexure blade is given by [7]:

$$W = C^E F_Z + C^{E,K} (Y, \theta_X, \theta_Z) F_Z + C^K (Y, \theta_X, \theta_Z) \quad (16)$$

where C^E , C^K and $C^{E,K}$ are the elastic, kinematic and elastokinematic compliance terms. The term $C^{E,K}$ is a quadratic function of the Y-direction (DoF) displacement of the motion stage that captures the decrease in the out-of-plane stiffness with increasing the DoF displacement [7]. The constitutive flexure blades in the sandwich DPFM exhibit higher out-of-plane translational and rotational stiffness compared to those of the single-layer DPFM [5]. Therefore, according to Eq. (16), the elastokinematic term of the non-nominal out-of-plane displacement of the constitutive flexure blades in the sandwich design is smaller than the corresponding term in the single-layer design. As a result, the elastokinematic drop in the

non-nominal out-of-plane stiffness of the sandwich DPFM is smaller than that of the single-layer DPFM as shown in Figure 5. Similar trends also exist for the out-of-plane rotational stiffness $K_{\theta Y}$ and $K_{\theta X}$.

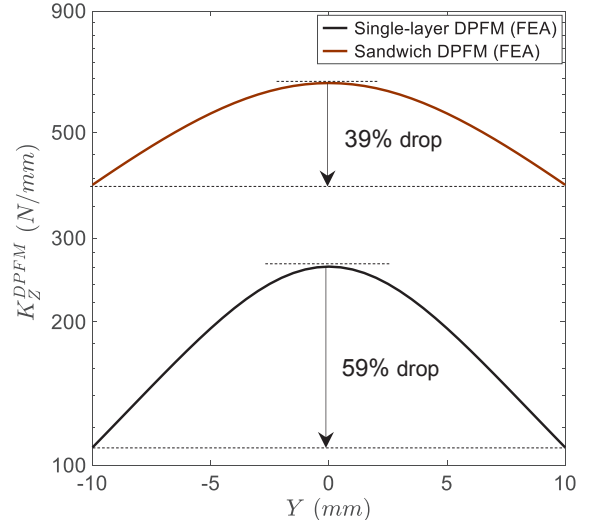


Figure 5. K_Z stiffness of the single-layer and sandwich DPFMs versus the Y-displacement of the motion stage

5) IN-PLANE DOF STIFFNESS

The in-plane Y-direction stiffness of the sandwich DPFM is derived from the bending stiffness of the constitutive flexure blades in the XY plane. The second moment of area of the constitutive flexure blades w.r.t. the Z axis in sandwich DPFM is equal to that of the single-layer DPFM and does not depend on the plate spacing S ($I_{Z-single} = I_{Z-sandwich} = HT^3/6$). As a result, the in-plane Y-translation of the sandwich DPFM is equal to that of the single-layer DPFM at all plate spacing values, implying that the in-plane K_Y stiffness of the sandwich DPFM is theoretically equal to that of the single-layer DPFM at all plate spacing values. Accordingly, one can say [3]

$$K_{Y-sandwich}^{DPM} = K_{Y-single}^{DPM} = \frac{6EI_Z}{B(4B^2 - 6BL + 3L^2)}, \quad I_Z = \frac{2HT^3}{12} \quad (17)$$

The K_Y stiffness of the single-layer and sandwich DPFMs are shown in Figure 6 versus the plate spacing S , which shows excellent agreement between the analytical stiffness models and the FEA results (< 2% error). As the figure shows, the in-plane DoF stiffness of the sandwich DPFM does not vary with the plate spacing S . This highlights the unique attribute of the sandwich DPFM that allows one to achieve significantly higher out-of-plane stiffness (K_Z , $K_{\theta Y}$, and $K_{\theta X}$) without affecting the K_Y stiffness. Therefore, the sandwich DPFM design proposed in this paper overcomes the inherent tradeoff between the out-of-

plane and in-plane DoF stiffness in DPFMs and flexure mechanisms comprising DPFMs.

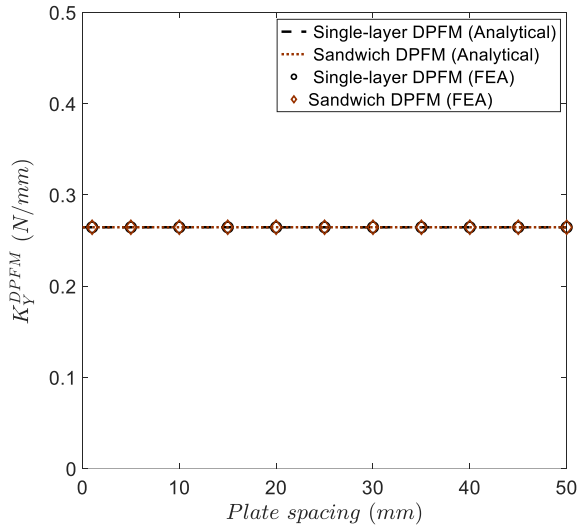


Figure 6. K_Y stiffness of the single-layer and sandwich DPFMs versus the plate spacing

6) CONCLUSION

We have presented a novel sandwich architecture for the Double Parallelogram Flexure Mechanism (DPFM) that offers significantly higher out-of-plane stiffness compared to conventional single-layer DPFMs, without impacting the in-plane DoF stiffness. Analytical models were derived for the out-of-plane and in-plane DoF stiffness of the single-layer and sandwich DPFMs at the nominal condition (where the motion stage displacement along the DoF direction is zero) that closely match the FEA predictions. At zero plate spacing ($S = 0$), the sandwich DPFM exhibits slightly lower K_Z and $K_{\theta X}$ compared to the corresponding single-layer DPFM with similar physical properties. However, by increasing the plate spacing in the sandwich DPFM, the K_Z and $K_{\theta X}$ stiffness quickly increase and overtake those of the single-layer DPFM. In addition, the $K_{\theta Y}$ stiffness of the sandwich DPFM is always larger than that of the single-layer DPFM at non-zero plate spacing values. For the physical parameters given in Section 3-3, at the plate spacing $S = 50\text{mm}$, the sandwich DPFM exhibits 280% higher K_Z , 1347% higher $K_{\theta Y}$ and 51% higher $K_{\theta X}$ stiffness compared to the corresponding single-layer DPFM, while the K_Y stiffness for both designs remains the same. Furthermore, in the non-nominal case (where the motion stage has been displaced by a finite amount along the DoF direction), the sandwich DPFM exhibits a smaller out-of-plane stiffness drop by increasing the DoF displacement. The proposed sandwich architecture is an effective design solution for improving the out-of-plane DoC stiffness of any

flexure mechanism with in-plane DoF. Future work includes experimental testing and validation of the superior performance of the sandwich DPFM design.

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