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NOISE CHARACTERISTICS OF CONVERGENT CROSSED-FIELD SPACE-CHARGE FLOWS

by
Shreedhar Govind Lele

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1966

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NOISE CHARACTERISTICS OF CONVERGENT CROSSED-FIELD SPACE-CHARGE FLOWS

by Shreedhar Govind Lele

The purpose of this study is to investigate the noise transport characteristics of space-charge flows in crossed d-c electric and magnetic fields. The theoretical work consists of a Monte Carlo analysis of the noise characteristics of Kino-type electron guns, a parallel plane diode gun and a crossed-field drift space. The experimental studies are concerned with the detection and control of signal (noise) transport along the beams in these guns.

In the Monte Carlo technique a random phenomenon described by a set of statistical quantities is simulated by postulating a stochastic model. The present analysis is nonrelativistic and the quantities are assumed to have no variation in the direction of the magnetic field.

An IBM-7090 high-speed digital computer is used to simulate random electron emission from the cathode and the subsequent space-charge flows in the gun and drift regions under both short- and open-circuit assumptions. Relevant data for noise parameter calculations are collected during these simulation runs and stored on magnetic tapes.

Kinetic noise-voltage and noise-current fluctuations are considered only in the direction of the d-c beam velocity at each point in the beam. Ψ , the Self-Power Density Spectrum (SPDS) of the noise-current fluctuations, and Φ , the SPDS of the kinetic noise-voltage fluctuations, are calculated on a cross-sectional and a rectangle basis to study their variation along the beam and over a given beam cross section respectively. Haus' other noise parameters, Π , S , etc., are calculated only along the beam.

A significant change in the values of the noise parameters at a given beam cross section is found to occur in the neighborhood of the hybrid frequency for that region of the beam. Φ increases by several orders between the cathode and the gun exit. Space-charge smoothing is not limited to the potential minimum region but continues for some distance along the beam. It is interesting to note that for Kino-type guns, Ψ has a peak in the region of the beam where the space-charge density changes from a value greater to one lower than the Brillouin space-charge density. Current density fluctuations over a beam cross section in the cathode region are correlated with the potential minimum profile. Π , S and $S-\Pi$ are not conserved along the beam. The noise parameter, $F_0(\text{db})$, increases rapidly near the cathode and the rate decreases along the beam. The kinetic temperature is minimum on the top layer of the electron beam in the gun region. There is some evidence of standing waves of Φ and Ψ along the beam in the drift space. In the present analysis the values of the noise parameters under the short- and open-circuit diode assumptions do not differ significantly.

An r-f probe technique was used to study signal (noise) transport in crossed-field guns. The cathode operating condition and beam velocity are found to influence the signal (noise) transport along the beam. Self-oscillations were detected in the frequency range of 90-125 MHz and the oscillation frequency was found to be a function of electric and magnetic field strengths and cathode temperature. These oscillations are suppressed by injecting a suitable r-f signal near the cathode.

In general, the theoretical and experimental studies provide a better understanding of noise transport in crossed-field guns. The results should be useful in designing electron guns for low-noise injected-beam crossed-field devices.

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CHAPTER I. INTRODUCTION

1.1 Characteristics of Crossed-Field Electron Devices

Microwave devices in which electrons move through mutually perpendicular static electric and magnetic fields are generally called crossed-field or M-type devices. These devices may be classified in many ways. One means of classification is the way in which electrons are emitted and caused to interact with an electromagnetic field. Three typical cases are shown in Fig. 1.1. Figure 1.1a indicates the familiar cylindrical magnetron in which the cathode is situated at the axis of the anode cylinder and emits electrons over its entire length. In Fig. 1.1b, a well-defined laminar electron beam is formed by means of an electron gun with a relatively narrow cathode and then is injected into the interaction (circuit-sole) region. The other possibility is that in which the sole electrode itself emits electrons along its entire length. Such a device, shown in Fig. 1.1c, is commonly called the emitting-sole crossed-field device. This is simply a linear version of the device shown in Fig. 1.1a. The second distinguishing characteristic is that the beam and the r-f circuit (wave-propagating circuit) may be both reentrant as in the case of the cylindrical magnetron or nonreentrant as is the planar crossed-field device. The third classification is based on the propagating characteristic of the slow-wave circuit. It may support an r-f standing wave, as in the case of a cylindrical magnetron, or a traveling wave; a forward wave for a crossed-field forward-wave amplifier and a backward wave for a crossed-field backward-wave amplifier or M-Carcinotron.

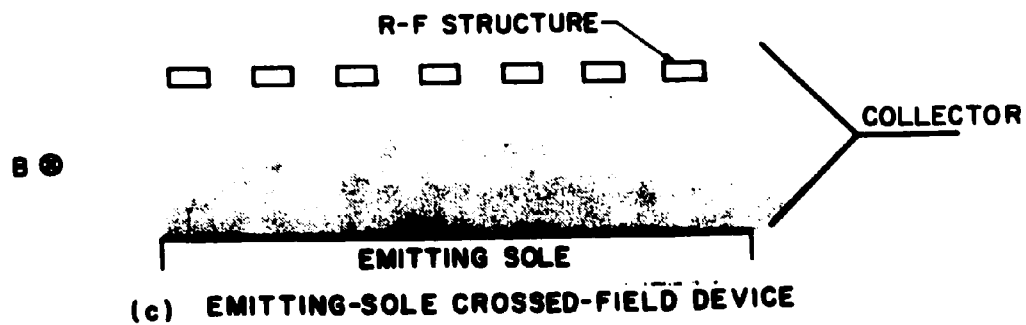
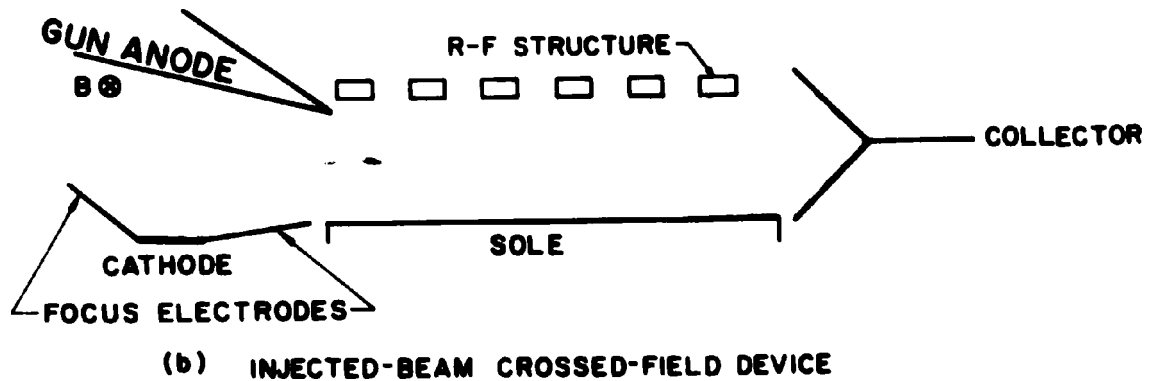
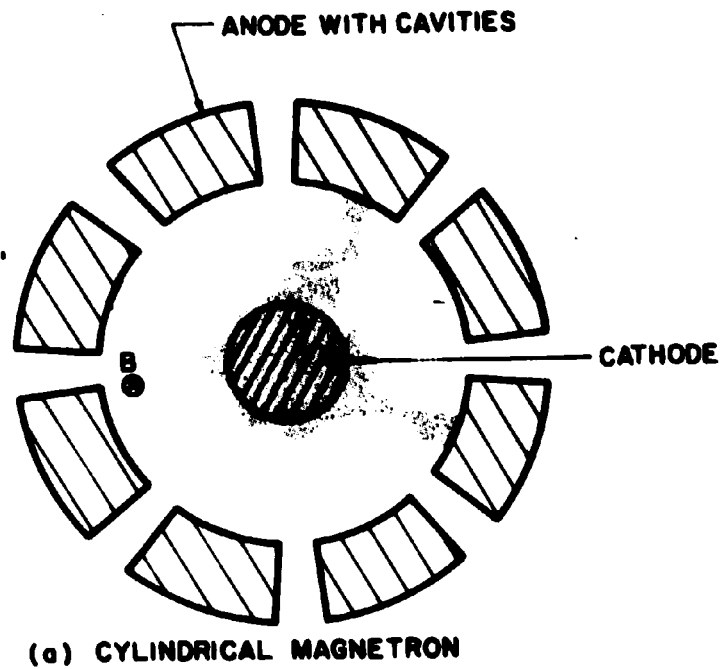


FIG. 1.1 TYPICAL CROSSED-FIELD ELECTRON DEVICES.

The energy exchange mechanism between the electrons and the slow wave on the r-f structure can easily be explained by using an injected-beam crossed-field device (see Fig. 1.1b) as an example. The electron beam is injected into the crossed-field interaction region with a drift velocity given by $v_d = E/B$, where E and B are, respectively, the static electric and magnetic fields. The phase velocity of the r-f wave on the slow-wave circuit is made very nearly equal to the drift velocity. The r-f field in the interaction region modulates the motion of the electrons. Those in the decelerating phase of the r-f field are slowed down and bunched about the maximum decelerating phase of the r-f cycle, $\phi = \pi/2$. These decelerated electrons move toward the anode and lose potential energy to the r-f field. On the other hand, electrons in the accelerating phase abstract energy from the r-f field and move toward the sole. It should be noted that the r-f fields vary hyperbolically away from the slow-wave circuit, causing a net transfer of potential energy of the electrons in the beam to the r-f wave on the circuit. The interaction is very efficient since the favorably phased electrons remain in the decelerating phase for a considerable time due to their constant drift velocity which is nearly in synchronism with the phase velocity of the r-f wave on the circuit. If the anode-sole voltage is large and the beam is injected close to the sole electrode in the interaction region, the favorably phased electrons, as they proceed toward the anode, will deliver a large amount of energy to the r-f wave. Thus relatively large power output can be obtained from these devices.

The high efficiency and the large power capacity of crossed-field devices have not been fully exploited because of the presence of the

large-amplitude, broad-band noise in their output. During the past few decades efforts have been directed toward devising ways and means of achieving substantial improvement in the noise figure of these tubes.

1.2 Review of Past Work on Noise in Crossed-Field Devices

Since the advent of the first crossed-field device, the cylindrical magnetron, numerous workers have occupied themselves with investigating various phenomena associated with the motion of electrons in crossed electric and magnetic fields. Excess noise associated with the cylindrical magnetron and the planar crossed-field device was first studied by Sproul¹, Mayper² and others during the period of 1947-50. Another puzzling phenomena is that current is drawn by the anode of a cylindrical magnetron when it is operated with a magnetic field beyond the cutoff value. To account for the rather gradual decrease in anode current with increasing magnetic field, cathode temperatures should have been 10^4 to 10^5 °K which is much higher than the actual cathode temperature. Lindsay and Sims³ maintain that the tangential emission velocities of the electrons are responsible for the slow cutoff in the anode current. However, in a recent time-dependent digital computer analysis of electron-wave interaction in crossed electric and magnetic fields Yu, Kooyers and Buneman⁴ have found that a fluctuating space-charge cloud rather than an absolutely steady space-charge configuration is present in such devices. Their computer results for a smooth-bore magnetron show that electrons do reach the anode under cutoff conditions. A maximum kinetic temperature exists in the interelectrode region and it is approximately equivalent to one tenth of the anode potential energy. This implies the existence of very high kinetic temperature in high-power magnetrons.

In an injected-beam crossed-field device it is found that electrons sometimes reach the sole even when it is held at a negative voltage with respect to the cathode. If the energy distribution of these electrons is assumed to be Maxwellian, then the equivalent temperature is estimated to be around 10^5 °K. Warnecke, Huber, Guenard and Doehler⁵ also noticed this phenomenon in their study of the M-Carcinotron. They attribute it to a growing-wave mechanism along the beam called the "diocotron" phenomenon. According to Miller⁶, the negative sole current is due to the exchange of energy between electrons trapped in a potential well in the gun region which causes the electrons to take a longer time to reach the anode-sole region. Midford⁷ attributes it to the instabilities of the crossed-field potential minimum. Guenard and Huber⁸ and Epsztein and Maillart⁹ observed that the sole current is considerably reduced when the cathode is operated under the temperature-limited condition, resulting in the absence of a potential minimum region near the cathode. These experiments thus support Midford's conjecture.

Little, Rupel and Smith¹⁰ and Epsztein and Maillart⁹ observed that for injected-beam crossed-field devices the noise increases appreciably when the cathode operation is changed from temperature-limited to space-charge-limited conditions. This behavior appears to contradict the smoothing action of the O-type potential minimum. Recently Monte Carlo analyses of a crossed-field diode by Wadhwa and Rowe¹¹ and Pollack and Whinnery¹² predicted space-charge smoothing and it has been experimentally verified by Mantena and Van Duzer¹³ and Sasaki et al.¹⁴. In a recent C.S.F. Laboratories report¹⁵ it is stated that for M-type guns (Charles-type), two cases should be considered,

depending on whether or not the B/B_c ratio (where B_c is the critical d-c magnetic field in the circuit-sole region) is sufficiently high to generate excess noise. In the first case excess noise disappears when the cathode is operated under the temperature-limited condition and it appears when the operation is changed to the space-charge-limited condition. In the second case the noise is only slightly affected by the cathode operating conditions. It is also found that at high beam currents and high magnetic fields the space-charge-limited guns are very noisy. This is probably due to the high diocotron gain under these operating conditions. Use of a screen grid at a small distance (a few millimeters) from the cathode is reported¹⁸ to have reduced the noise output by suppressing the feedback between the cathode emission and the instability of the space charge in the gun region. It is also suggested¹⁵ that the noise is strongly conditioned by a space correlation at the cathode. It is postulated that the noise is reduced because the grid subdivides the beam into a number of elementary beams. The validity of this conjecture has not been firmly established so far.

In addition to the Monte Carlo analyses mentioned above, Van Duzer¹⁷, Rando¹⁸ and Wadhwa¹⁹ have modified the Llewellyn-Peterson equations to study noise transport in a crossed-field diode. Wadhwa and Rowe²⁰ have generalized the density function method of Twiss²¹ by considering the initial tangential velocity of the electrons emitted from the cathode. He shows that the contribution of the tangential velocity is quite important in the potential minimum region. It should be noted that both the density function method and the modified Llewellyn-Peterson equations are based on a small-signal single-velocity assumption which is a poor approximation in the neighborhood of the potential minimum.

Rowe and Wadhwa²² have developed transmission-line analogs for crossed-field space-charge flows. They proposed velocity-jump noise transducers to reduce the beam noise. Sasaki et al.¹⁴ have discussed the boundary conditions and the transformation in the beam quantities through such a velocity jump. Recently Wadhwa and Van Duzer²³ have reported on a noise figure of 3.5 db for an S-band, medium-power, forward-wave, injected-beam crossed-field amplifier.

The investigations described above clearly indicate that several problems regarding noise in crossed-field devices have not yet been answered satisfactorily. Hence further analytical and experimental work is needed to provide a better understanding of crossed-field noise phenomena.

1.3 Outline of This Study

Chapter II deals with the Monte Carlo simulation of space-charge flows in crossed-field guns (an abbreviated Kino gun, a modified Kino gun and a parallel plane diode gun) and drift regions which follow the gun region. The geometry is divided into a number of small elementary rectangles. The digital computer simulation program gives the number and centroids of electrons in various rectangles under steady-state conditions. This knowledge is used to obtain a fairly accurate estimate of the beam configuration. In order to gain further information on the laminarity of the beam in the gun region, a digital computer program developed by Boers²⁴ is employed to give a few representative trajectories in each case. The d-c characteristics of the beams in the gun and drift regions are investigated.

Relevant noise parameters describing noise propagation in crossed-field electron beams are derived in Chapter III.

Chapter IV presents the results of the Monte Carlo noise analysis of space-charge flows in the abbreviated Kino gun, the modified Kino gun and the parallel plane crossed-field diode gun. A comparative study is attempted wherever possible.

In Chapter V analysis of noise transport along the abbreviated and the modified Kino gun beams is continued into the drift region following each gun. In the present work the first incremental drift space length analyzed is approximately one beam cyclotron wavelength.

Chapter VI summarizes the experimental investigation of the noise propagation on electron beams produced by the three crossed-field guns, i.e., the crossed-field diode gun, the crossed-field triode gun and the abbreviated Kino gun. The noise analyzer designed earlier in this laboratory has two parts--the electron gun and the drift region. In the electron gun region several coaxial r-f probes are mounted on the cathode block and used to modulate the beam. Two sets of coaxial r-f probes--one fixed at the entrance to the drift region and the other movable in the drift region--are used to pick up the signal (noise) from the beam. The former set of these probes was removed in the case of the abbreviated Kino gun for reasons explained in this chapter. The effects of cathode temperature, beam current, grid and gun anode voltages, and applied magnetic field on the signal (noise) propagation are studied and the data is presented and compared with theory where possible.

A summary of the present investigation and the conclusions drawn, together with suggestions for further study, are included in the last chapter.

CHAPTER II. MONTE CARLO SIMULATION OF ELECTRON FLOWS

2.1 Introduction

The density function^{20,21} and Llewellyn-Peterson^{17,18,19} analyses of noise transport in crossed-field space-charge flows are based on a small-signal single-velocity assumption. In the neighborhood of the potential minimum where the velocity spread of the electrons is comparable to their average velocity this becomes a poor approximation. In the Monte Carlo method (first used by Tien and Moshman²⁵ in their study of noise near the potential minimum of a high frequency diode) a random phenomenon described by a set of statistical quantities is simulated by postulating a stochastic model. In the case of thermionic emission the model consists of electrons randomly emitted from the cathode at discrete times according to a Poisson distribution²⁶ for the emission number and a Maxwellian distribution for the emission velocity of the electrons. At the beginning of each unit-time interval, Δt , i.e., the interval between successive, equally spaced, discrete times, electrons are emitted from the cathode. Poisson's equation is then solved to obtain the potential distribution and the electric field inside the gun geometry. In using Poisson's equation the space-charge density is assumed to have a smooth variation. The dynamic equations of motion are solved for each electron, and its new position and velocity coordinates are determined. Time is then incremented by Δt , and the program cycles back to the electron generation section.

The discrete nature of the electrons is retained and no small-signal assumption need be made in this analysis. Nevertheless this

method also has some disadvantages. In order to study the effect of a particular parameter on the process analyzed, e.g., noise propagation, the computer simulation program should be run for each selected value of that parameter. Also the accuracy of the various statistical estimates depends on the number of samples available for the analysis. Since samples are drawn for each unit-time interval, the total number of samples depends upon the sampling period (program execution time). Thus it becomes a very time-consuming and expensive method. Wadhwa and Rowe¹¹ have performed a Monte Carlo noise analysis of a two-dimensional crossed-field diode under the short-circuit assumption. Pollack and Whinnery¹² also used this method to study noise transport in a one-dimensional crossed-field diode under both short- and open-circuit assumptions. In the absence of a suitable analytical method based on a fairly satisfactory set of assumptions, the Monte Carlo method seems to be the best choice.

Monte Carlo simulations of two-dimensional space-charge flows for an abbreviated Kino gun, a modified Kino gun and a crossed-field diode are carried out first. Next, the simulation process is continued into the drift region for the abbreviated and modified Kino guns. The data obtained are used in Chapters IV and V for the noise-transport analysis in the gun regions and the drift spaces. The basic assumptions, the procedural details and the d-c characteristics of the beams in the various cases are described in the following sections.

2.2 Basic Assumptions and General Considerations in the Monte Carlo Simulation of Electron Flows

The Monte Carlo simulation of space-charge flows in electron guns can be conveniently divided into two parts. The first part consists of

the development of a stochastic model representing random electron emission from the cathode and the second part deals with the motion of electrons in the gun geometry. As described in the last section, this is a cyclic process and the cycle is repeated at the beginning of each Δt . The accuracy of the calculated noise spectra depends on the length of the unit-time interval Δt . The choice of a suitable Δt is discussed in Section 2.2.2. The so-called "big electron" is defined in Section 2.2.3 and the emission velocity components assigned to this hypothetical charged particle are discussed in Section 2.2.4.

2.2.1 Assumptions. Some other basic assumptions made in this analysis are:

1. Nonrelativistic mechanics.
2. Force due to r-f magnetic field on an electron is neglected.
3. Although the cathode has a finite length in the direction of the uniform applied magnetic field, no variation of any quantity is considered in this direction.
4. Direct collisions between electrons are not considered.
5. The emission process is ergodic.
6. The velocity distribution and the electron number distribution are uncorrelated.
7. A half-Maxwellian distribution for the normal component (y-component) and a full-Maxwellian distribution for the tangential component (z-component) of the initial velocity are assumed.
8. A Poisson distribution for electron emission number for each Δt is assumed.
9. Low-velocity electrons returning from the potential minimum and moving toward the cathode do not influence the emission process.

10. The electric field is constant over the unit-time interval Δt .

11. Space-charge-limited operation is assumed.

12. Two separate cases are considered: Case A involves the short-circuit diode assumption, whereas in Case B the open-circuit diode assumption is made.

13. Since the dimensions of the device are small compared to the shortest wavelength used in the noise analysis, Poisson's equation, instead of Maxwell's equations, is used to obtain the electric field configuration.

2.2.2 Choice of the Unit-Time Interval Δt . In the Monte Carlo noise analysis samples of electron velocities and current densities are drawn at times Δt apart. These samples represent the average values of these fluctuations over the time interval Δt . Dayem and Lambert²⁷ have shown that the effect of "averaging" over Δt and the effect of the "aliasing"²⁸ caused by the discrete nature of the data will be negligible if the highest frequency of interest in the noise analysis is less than $0.1/\Delta t$. In the simulation of space-charge flows in the abbreviated and modified Kino guns, Δt was chosen to be 10^{-11} sec.

2.2.3 Grouping of Electrons. It is impossible to follow the large number of electrons inside the geometry of a particular gun due to the limitation on computer memory and cost considerations. Thus a reduction factor D is used to reduce this number in such a way that one so-called "big electron" represents D electrons. The procedure for selecting the D factor is as follows.

Let J_s and (a) be the emission current density and the area of the cathode respectively. In Δt , an amount of charge equal to $(-J_s a \Delta t)$ is emitted from the cathode. In this model each "big electron" has a

charge equal to $-D|e|$, where $|e|$ is the absolute value of the charge of an electron. Assume arbitrarily that N "big electrons", if emitted from the cathode per Δt , will carry the same total charge, $(-J_s a \Delta t)$. Hence D is given by

$$D = \frac{J_s a \Delta t}{N|e|} . \quad (2.1)$$

Since the computer storage space available is limited, N is maximized by means of a few trial runs. This optimum value of N and the corresponding value of D are then used for obtaining the Monte Carlo simulation data.

2.2.4 Emission Velocity of the "Big Electron". Electron emission from the cathode is simulated in this model by emitting the so-called "big electrons". The "big electron" is assigned the same velocity components as those for an electron emitted from the cathode at a given operating temperature, T . Since both an ordinary electron and the "big electron" have the same charge-to-mass ratio it is clear from the Lorentz force equation that in a crossed electric and magnetic field configuration, both charged particles will experience the same acceleration if they possess the same velocity. Thus it can be easily seen that under identical field configurations their trajectories will also be identical. Thus the space-charge distribution in the present model dealing with "big electrons" will be essentially the same as that in the physical process involving electrons.

Since Maxwell's distribution is assumed for the emission-velocity components, the effective cathode temperature T_e for a "big electron" emitted with the same velocity as that of an electron emitted from a cathode at temperature T will be given by $T_e = DT$, where D could also be

considered as the ratio of the mass of the "big electron" and an electron. The effective cathode temperature will be discussed further in Chapter III.

2.3 Monte Carlo Simulation of the Space-Charge Flow in the Abbreviated Kino Gun

2.3.1 The Kino Short Gun and the Abbreviated Kino Gun. The analytical design of complete electrode systems capable of producing two-dimensional crossed-field space-charge flows under certain assumptions was developed by Kino²⁹ and has been used as the basis for "short" and "long" guns. In the Kino short gun the cathode defines the $y = 0$ plane and the uniform magnetic field is in the x-direction. The following assumptions are pertinent to the design of this gun:

1. Electrons are emitted from the cathode with zero initial velocity.
2. Electrons are confined to move in planes perpendicular to the x-axis, i.e., a two-dimensional space-charge flow is assumed.
3. No electric field exists in the z-direction.
4. All electrons flow in parallel paths with the current density, the space-charge density and the electron velocity independent of the z-coordinate. For this case J_y is constant.

It can be seen from the Kino short gun theory that at transit angle $\omega_c t = 2\pi$ along a trajectory there exists a theoretical infinite space-charge density. Although such a condition is not physically realizable, it suggests the possibility of beam instability around this point. This difficulty can be avoided if the gun is terminated at $\omega_c t < 2\pi$. Masnari³⁰ has studied the beam characteristics and the current-density variation at the cathode of such an abbreviated Kino

gun. Although the individual trajectories agreed well with the theoretical electron paths, the flow was quite nonlaminar. The cathode current density was found to be nonuniform over the length (z-direction) of the cathode. The center portion of the cathode had an essentially uniform current density which decreased at the front and rear ends of the cathode. He concluded that either laminarity of the beam or uniform cathode current density could be achieved by modifying the focus electrodes (Masnari's ramp electrodes). To obtain both uniform cathode current density and a laminar beam, additional modifications involving the gun anode (Masnari's focusing electrode) were also necessary.

In the abbreviated Kino gun studied by Masnari the upper and lower boundary electrons have transit angles of $\omega_c t = 280$ degrees and $\omega_c t = 250$ degrees, respectively, at the gun exit. The normalized cathode width, $W = 3$, corresponds to an actual value of 0.1 inch. The normalized potential Φ is chosen to be 25. The same gun is used in the present analysis but at a different operating gun anode potential and magnetic field. The pertinent data for this gun are given below.

1. Cathode width (along z-axis), $w = 0.1$ inch,
2. Cathode length (along x-axis), $d = 0.6$ inch,
3. Kino current density = 0.1115 amp/cm²,
4. Cathode emission current density = 0.4 amp/cm²,
5. Gun anode voltage = 900 volts,
6. Cathode and focus electrode voltages = 0 volt,
7. Magnetic field, $B = 1.688 \times 10^{-2}$ weber/m² and
8. Cathode temperature, $T = 1000^\circ\text{K}$.

2.3.2 Monte Carlo Simulation. The total simulation period is divided into unit-time intervals. The choice of the duration of Δt in

the present case is 10^{-11} sec. The cathode does not emit over its entire surface. Instead, the electrons are emitted from a prescribed set of six equally spaced points called the "cathode spots". On the average, these cathode spots share equally the cathode emission. During each Δt , 48 "big electrons" (eight from each spot) are emitted from the cathode. From Eq. 2.1 the D factor is found to be 2.0136×10^5 for this case. For convenience, the "big electrons" will be henceforth referred to as electrons unless otherwise specified.

The gun electrodes define the top and the bottom of the gun region shown in Fig. 2.1. The left-hand boundary (vertical plane No. 1) is chosen at some convenient distance from the left edge of the cathode. The right-hand boundary (vertical plane No. 13) is located slightly before the gun exit to avoid errors in the calculation of field components due to the presence of sharp edges in the geometry near the gun exit.

The computer simulation program for the short-circuit case (no a-c electric field) starts with the gun region devoid of any electrons. The sequence of steps shown in the flow diagram (Fig. 2.2) is followed for each Δt . Electrons are generated at each cathode spot according to a Poisson distribution with random velocity components in the y- and z-directions. Random emission time within a given Δt is taken care of by assigning advanced initial positions to the electrons when they are emitted. Details of these steps are given in Appendix A. Electrons having position coordinates outside the gun geometry are then eliminated. Poisson's equation is solved with the given boundary conditions to yield the potential distribution and the total electric field components (see Appendix B). The boundary conditions for the present case are:

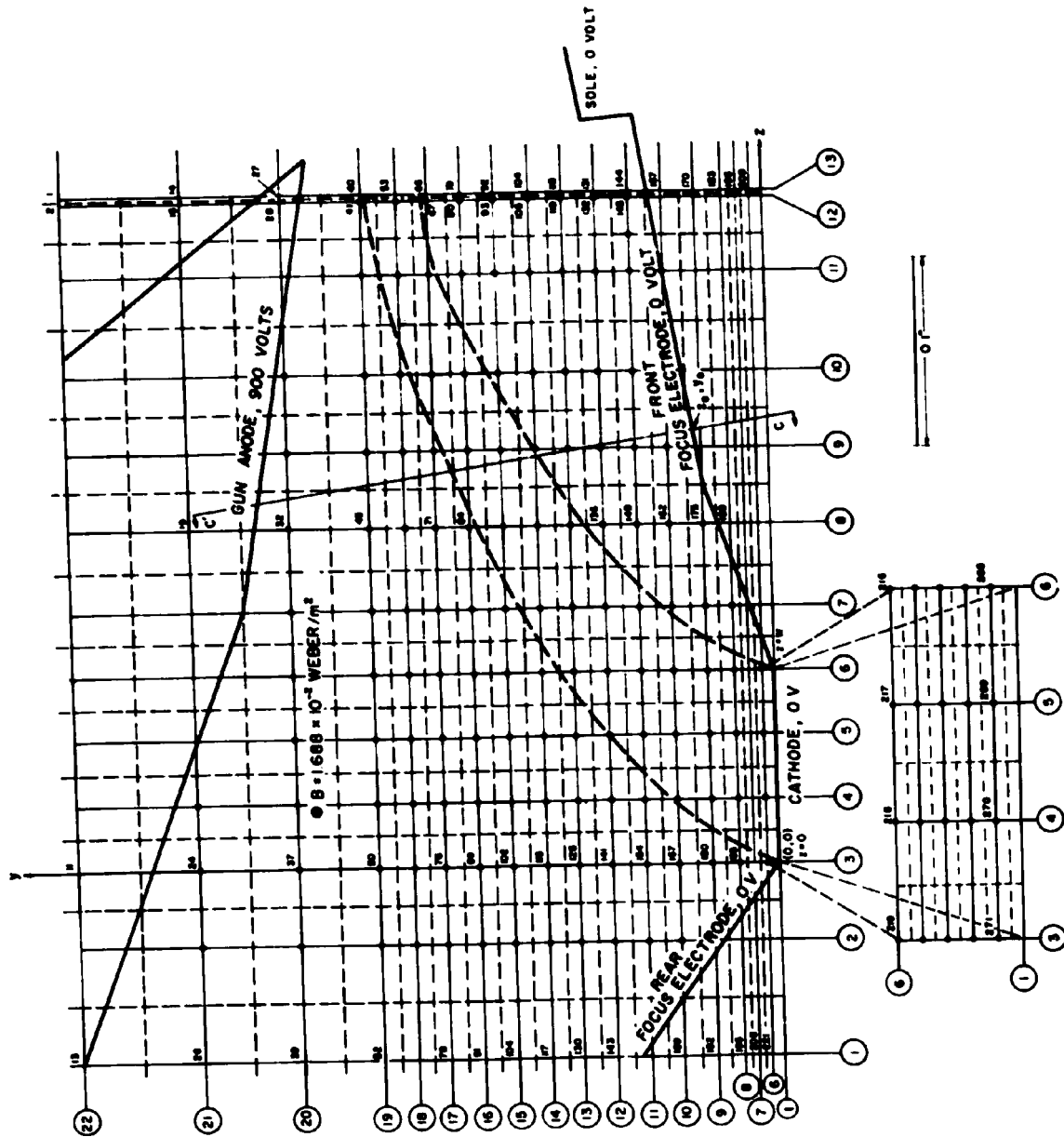


FIG. 2.1 ABBREVIATED KINO GUN WITH COMPLETE LAYOUT OF "SOLUTION POINTS".

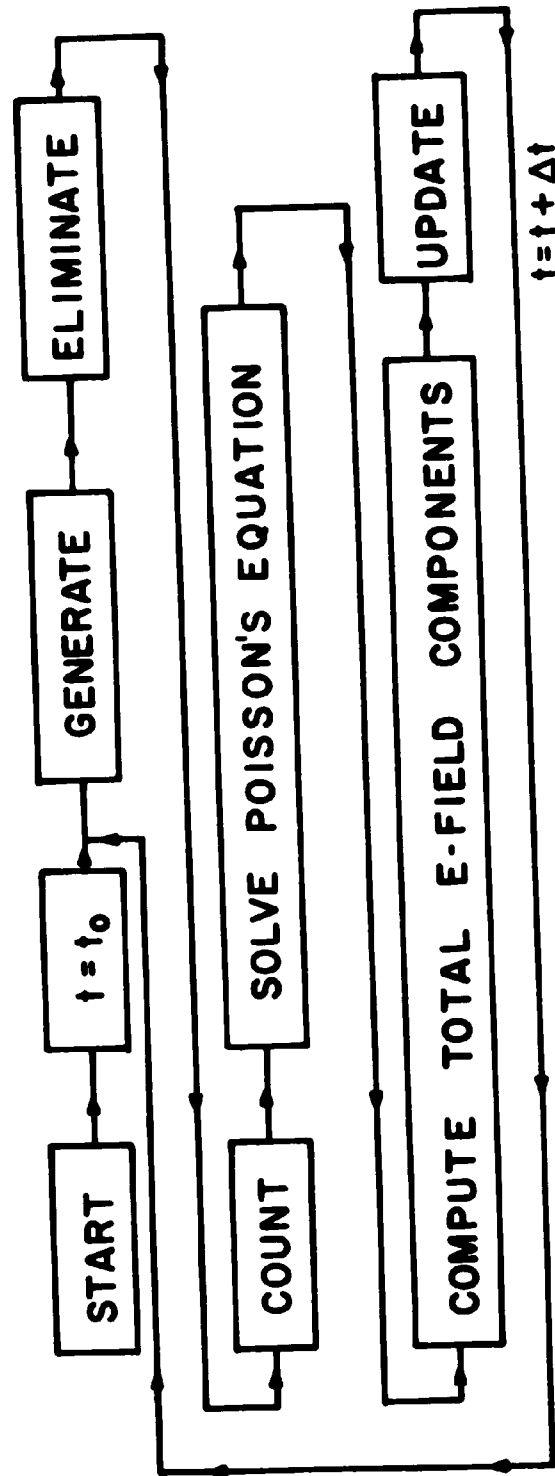


FIG. 2.2 FLOW DIAGRAM FOR MONTE CARLO SIMULATION.

1. Gun anode voltage = 900 volts.
2. Cathode and focus electrode voltage = 0 volt.
3. The left-hand boundary is given a fixed potential distribution obtained from the steady-state solution using a relaxation technique²³.
4. The potentials at the right-hand boundary fluctuate since the fluctuating beam passes through this plane.

Initially there are no electrons in the gun region and a linear potential variation is assumed on this boundary. As the electrons start filling up the gun region the potential at the boundary at a given point and for a given Δt is taken to be the potential at a point just to the left of this boundary point during the previous Δt . This process is continued and when a steady flow of electrons across the boundary is obtained, the boundary potentials settle down to steady-state values.

Next, the equations of motion (given in Appendix C) are solved and each electron is assigned the new position and velocity coordinates. This process is referred to as "updating" the electrons.

The above sequence of steps is continued until the steady state is reached. This condition is indicated by a small fluctuation of the potential values and the number of electrons in the gun region about their averages.

When the steady-state condition is reached, certain rectangles are chosen for the noise analysis. The simulation is continued for another 2000 unit-time intervals. For each Δt , the electron count and the average velocity components and position coordinates (averaged over all electrons in that rectangle) corresponding to each sampled

rectangle are recorded on a magnetic tape. These data are later used in the noise calculations. The number, position coordinates and velocity components of all electrons crossing an arbitrary plane cc' during each Δt are also recorded on another magnetic tape. These data are used in this chapter to simulate the electron beam which is injected into a crossed-field drift region. Time averages (taken over the sampling period T_n of L unit-time intervals) for the y - and z -components of velocity of all electrons in each rectangle inside the gun geometry are recorded on punched cards at the end of the sampling period. These averages are necessary for calculating the additional a-c electric field components in the space-charge simulation under the open-circuit diode assumption. The computer program also prints out the long-term averages of the following items:

1. The number, velocity components and position coordinates of electrons in each rectangle inside the gun geometry,
2. The potential at each solution point,
3. The number of electrons generated at the cathode per Δt and
4. The number of electrons eliminated at the various gun boundaries per Δt .

Next, the space-charge flow simulation under the open-circuit diode assumption (not in the strict sense) is undertaken. In this case the additional a-c electric field is taken into account in calculating the total electric field as described in Appendix B.2. Except for this, the simulation procedure is exactly the same as in the previous case with the short-circuit diode assumption. The computer program is restarted from where it was stopped in the previous case. A few hundred unit-time intervals are allowed to let the electron flow settle

down under the new conditions. The relevant data for the noise calculations and the input data for the simulation of the electron flow in the drift region are recorded on two magnetic tapes for the next 2000 unit-time intervals. No punched card output is necessary in this case. The various long-term averages mentioned in Nos. 1 through 4 of the previous paragraph are also printed out.

2.4 Steady-State Characteristics of the Space-Charge Flow in the Abbreviated Kino Gun

The long-term averages of "big electrons" generated per Δt at the cathode in the short-circuit and the open-circuit runs are 48.1 and 48.07 respectively. These numbers are in close agreement with the prescribed value of 48. Similarly, the beam current obtained in the Monte Carlo simulation is 44.8 ma in comparison to 43.2 ma according to the Kino theory.

Long-term averages of the number and centroids of electrons in each rectangle are used to define the beam with fair accuracy. The beam profile obtained by this method is shown, along with the theoretical beam profile determined from Kino theory, in Fig. 2.3. The beam convergence ratio is approximately 3:1.

Some representative electron trajectories obtained by a relaxation method²⁴ are plotted in Fig. 2.4. The trajectories are found to intersect in the gun region. The insert in this figure shows the current-density variation along the cathode obtained by the relaxation method. The theoretical J_c is uniform over the cathode and has a value of 0.1115 amp/cm².

A set of equipotentials for space-charge-limited flow in the abbreviated Kino gun is shown in Fig. 2.3. In the theoretical design

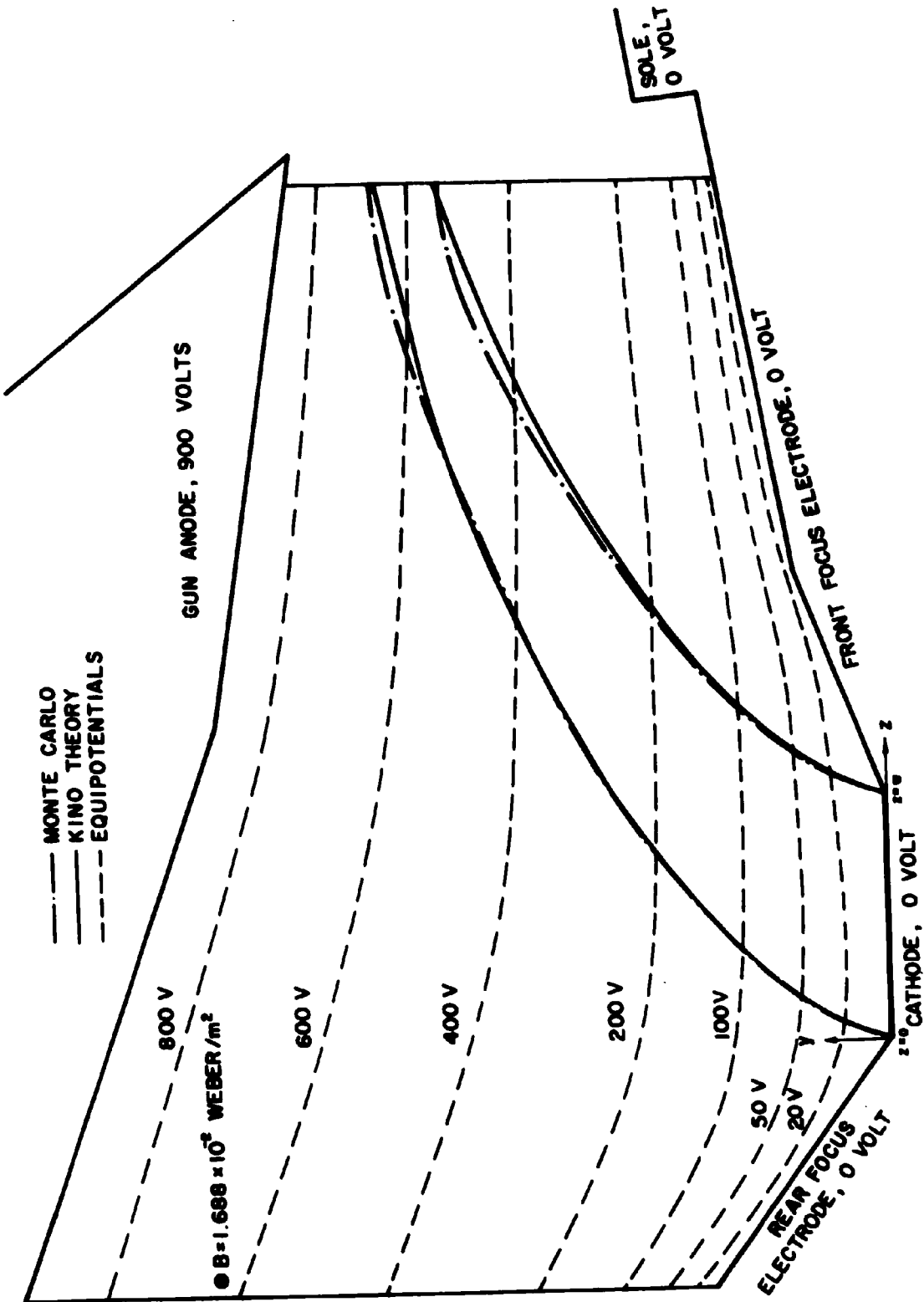
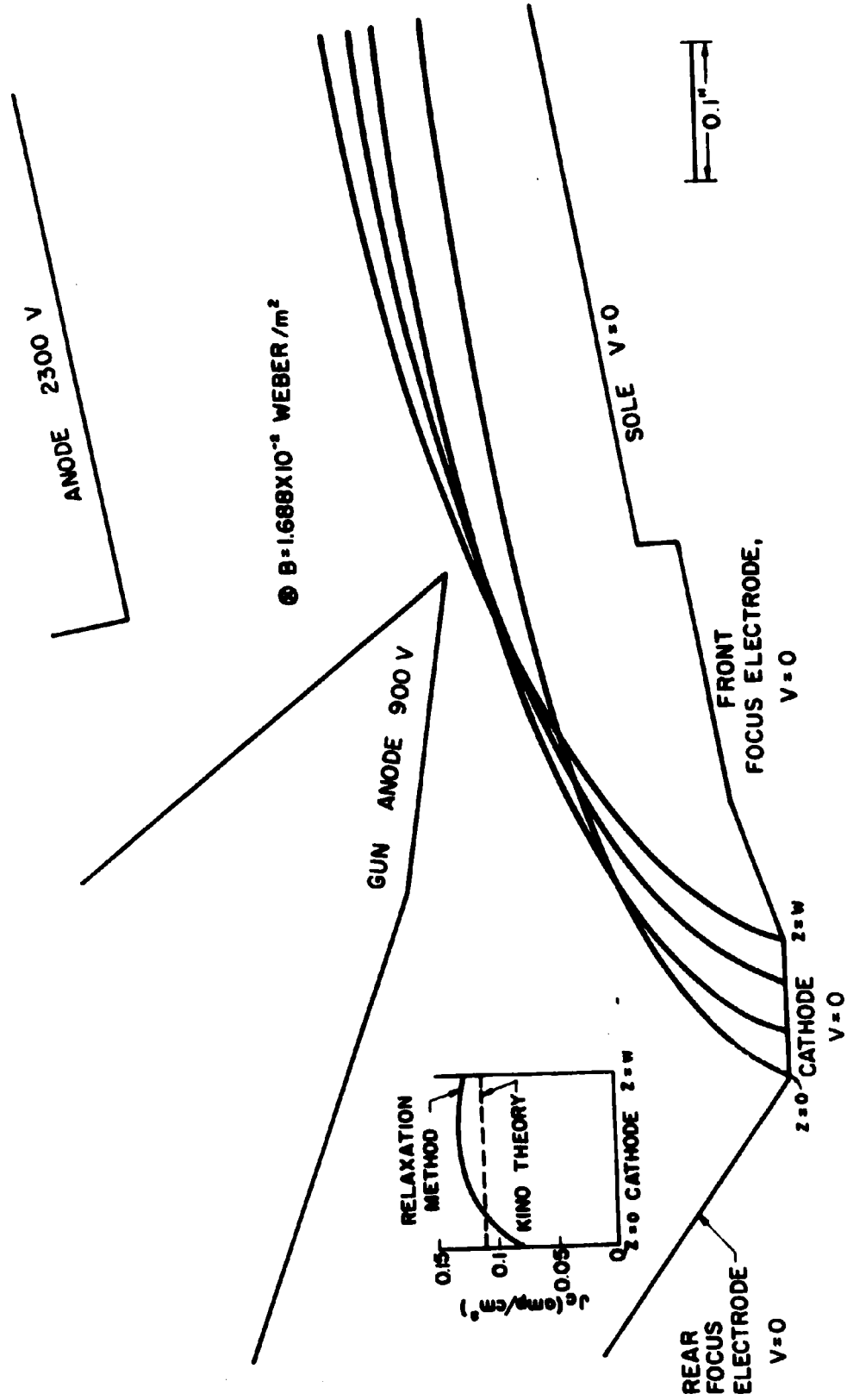


FIG. 2.3 THEORETICAL AND MONTE CARLO BEAM PROFILES AND EQUIPOTENTIALS FOR THE ABBREVIATED KINO GUN.



of the Kino gun no potential minimum exists. However, in the present case a potential minimum is present since the emission current density is nearly four times the Kino current density. As shown in Fig. 2.5a the magnitude of the potential minimum is approximately 0.1 volt over the length $z = 0$ to $z = 2w/3$, whereas at the right edge of the cathode it decreases to approximately 0.05 volt. The distance of the potential minimum from the cathode, y_m , behaves in a similar way. In Fig. 2.5b it can be seen that between $z = 0$ and $z = 2w/3$ the potential minimum is approximately 1×10^{-5} m from the cathode. At the right edge of the cathode the potential minimum appears at about 5×10^{-6} m from the cathode. The value of the potential at the minimum and the distance of the potential minimum from the right edge ($z = w$) of the cathode is observed to fluctuate significantly whereas these values are nearly constant over the rest of the cathode. This might be due to the presence of a corner in the geometry and the comparatively larger number of electrons returning to this area of the cathode due to the bending action of the magnetic field.

From the current-density curve (obtained from the relaxation method) shown in the insert of Fig. 2.4 it can be observed that the current density is least along the left edge of the cathode (around $z = 0$). It increases rapidly to a value which varies only slightly between $z = w/3$ and $z = w$. In the model, uniform emission current density is simulated by employing six emission spots on the cathode. Hence the potential minimum should be deepest around $z = 0$ and should not have much variation in magnitude and distance from the cathode plane. When the region around $z = w$ is neglected for reasons given above, the curves shown in Figs. 2.5a and 2.5b agree well with the

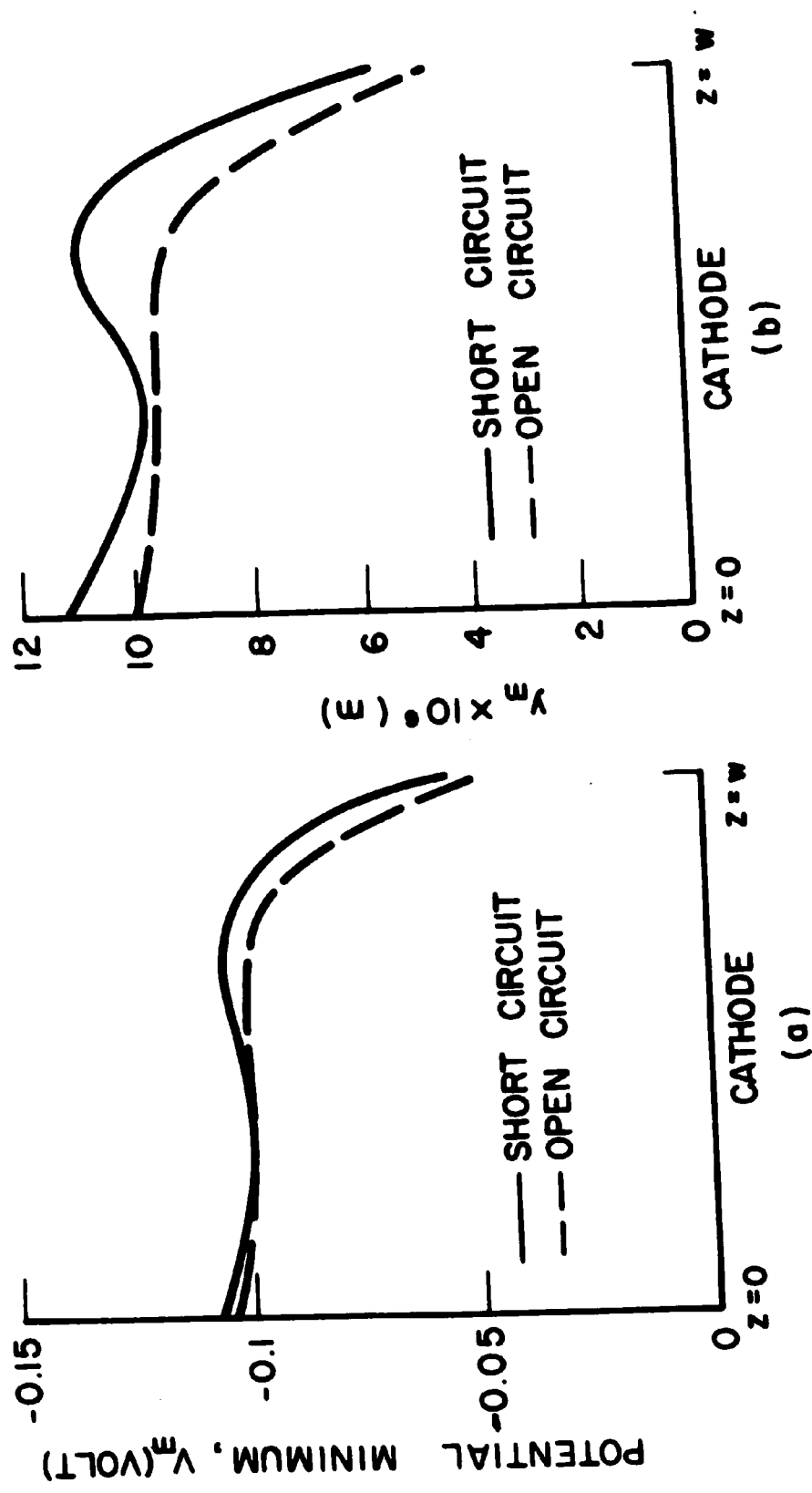


FIG. 2.5 (a) POTENTIAL MINIMUM VARIATION OVER THE CATHODE LENGTH OF THE ABBREVIATED KINO GUN;
 (b) VARIATION OF POTENTIAL MINIMUM DISTANCE FROM THE CATHODE OF THE ABBREVIATED KINO GUN.

expected behavior. No significant difference is observed between the short- and open-circuit cases.

2.5 Monte Carlo Simulation of the Space-Charge Flow in the Modified Kino Gun

2.5.1 The Modified Kino Gun. It can be seen from Fig. 2.4 that the abbreviated Kino gun does not produce a laminar flow because the electron trajectories intersect each other. Masnari³⁰ obtained a well-defined beam with nonintersecting trajectories by modifying the focus electrodes. He observed that in this so-called modified Kino gun the velocity gradient across the beam is approximately the same at different cross sections of the beam and equal to ω_c (the radian cyclotron frequency), the desired Brillouin value. However, the beam width at the gun exit is greater than that for Brillouin flow. The cathode current density is found to be slightly nonuniform over the length of the cathode but varies less than that for the abbreviated Kino gun. The electrode configuration of this gun along with the complete layout of solution points is shown in Fig. 2.6. The cathode emission current density and the electrode potentials are the same as for the abbreviated Kino gun described in the previous section.

2.5.2 Space-Charge-Flow Simulation. The space-charge-flow simulation under the short-circuit and open-circuit diode assumptions is carried out and the relevant data is stored on magnetic tapes according to the procedure outlined in Section 2.3.2.

2.5.3 Steady-State Characteristics of the Space-Charge Flow in the Modified Kino Gun. The average number of electrons generated per Δt is very close to the theoretical value. The beam current of 60.64 ma obtained from this simulation program checks fairly well with the beam

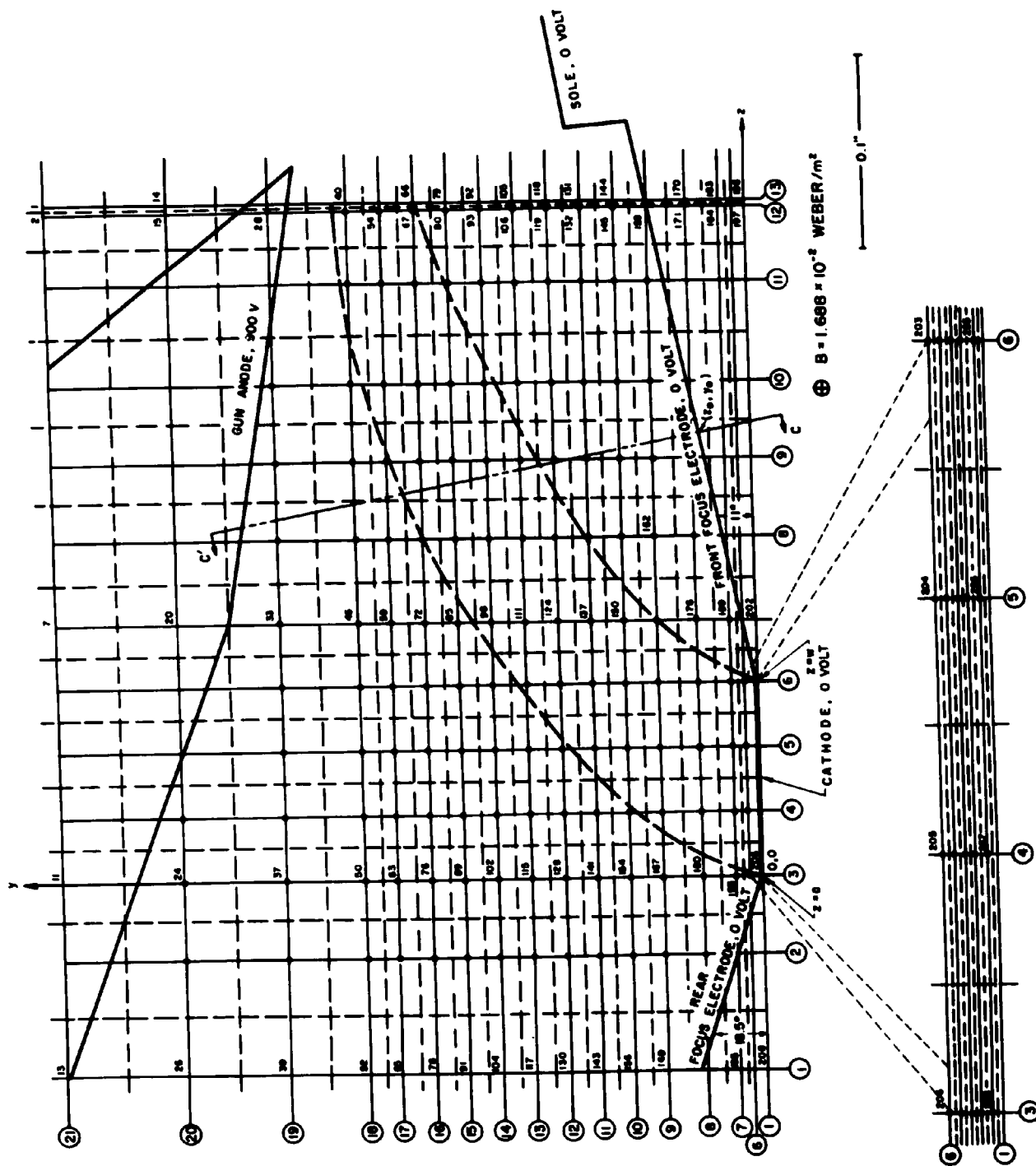


FIG. 2.6 MODIFIED KINO GUN WITH COMPLETE LAYOUT OF "SOLUTION POINTS".

current of 58.8 ma determined by the relaxation method. The beam current is about 1.35 times that obtained with the abbreviated Kino gun. The beam profiles obtained from the Monte Carlo and relaxation methods are shown in Fig. 2.7. The Monte Carlo beam profile is slightly wider than that obtained from the relaxation method. This is to be expected since the latter method does not take into account tangential components of emission velocities and does not allow for the formation of a potential minimum. The beam convergence ratio obtained from the Monte Carlo method is approximately 2.2:1. Figure 2.8 shows some representative trajectories obtained from the relaxation method. It should be noted that in contrast with the previous gun, the trajectories do not intersect in the gun region.

A set of equipotentials in the gun region corresponding to the space-charge-limited condition are shown in Fig. 2.7. The variation in the value and the distance of the potential minimum from the cathode, y_m , as a function of distance over the cathode length is shown in Fig. 2.9. The focus electrodes in the modified Kino gun make smaller angles with the plane $y = 0$ than the corresponding electrodes in the abbreviated Kino gun. Thus the electric field in the neighborhood of $z = 0$ and $z = w$ is higher in this case. This results in higher beam current emission from these cathode areas. The average depth of the potential minimum over the length of the cathode should be less. Also, the potential minimum should be shallower at the ends of the cathodes. Both of these inferences are borne out by the curves shown in Fig. 2.9. The behavior of the potential minimum in the neighborhood of the right edge ($z = w$) is similar to one observed in the previous case. The steady-state data obtained under short-circuit

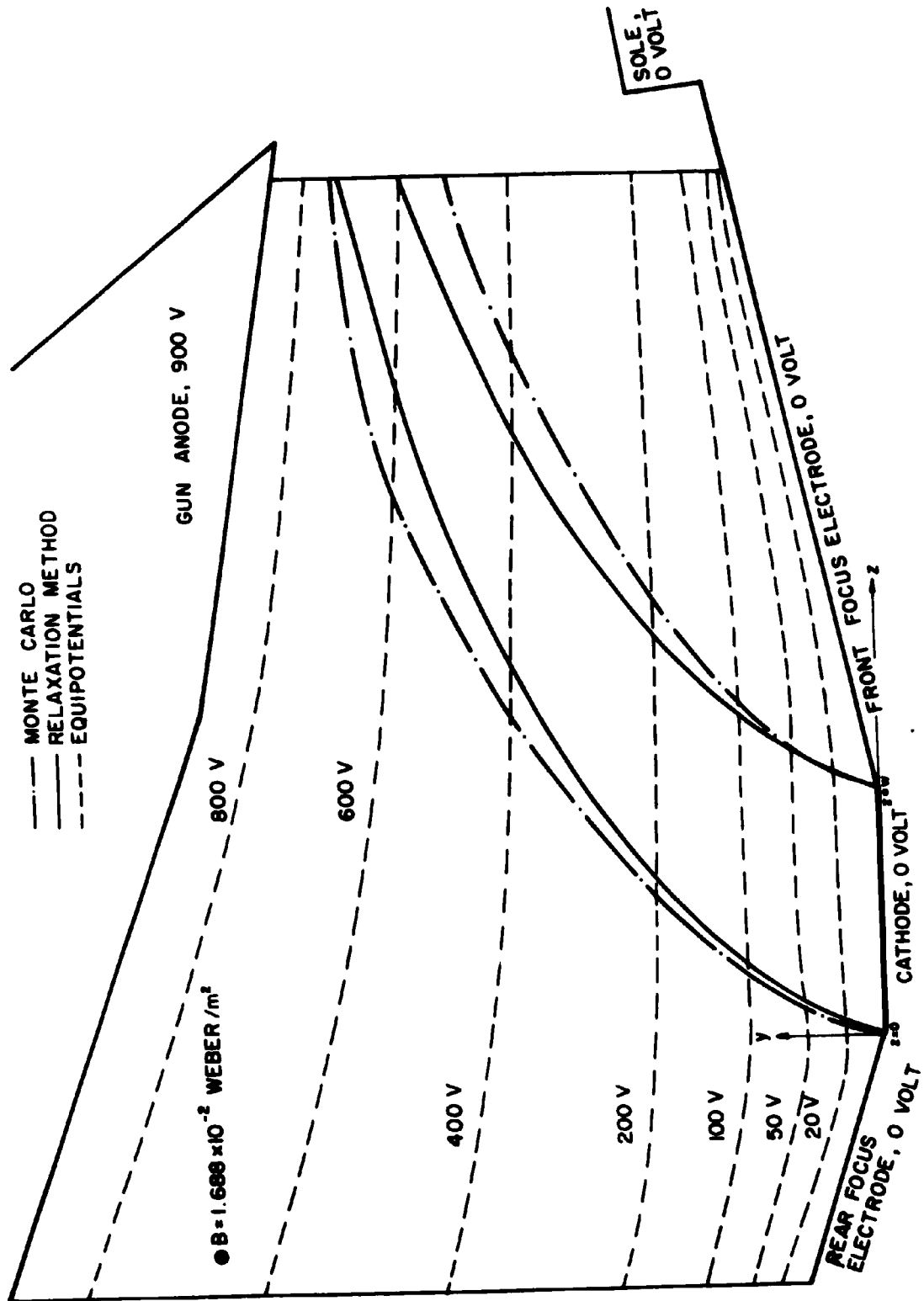


FIG. 2.7 EQUIPOTENTIALS ALONG WITH THE BEAM PROFILES OBTAINED FROM MONTE CARLO AND RELAXATION METHODS (MODIFIED KINO GUN).

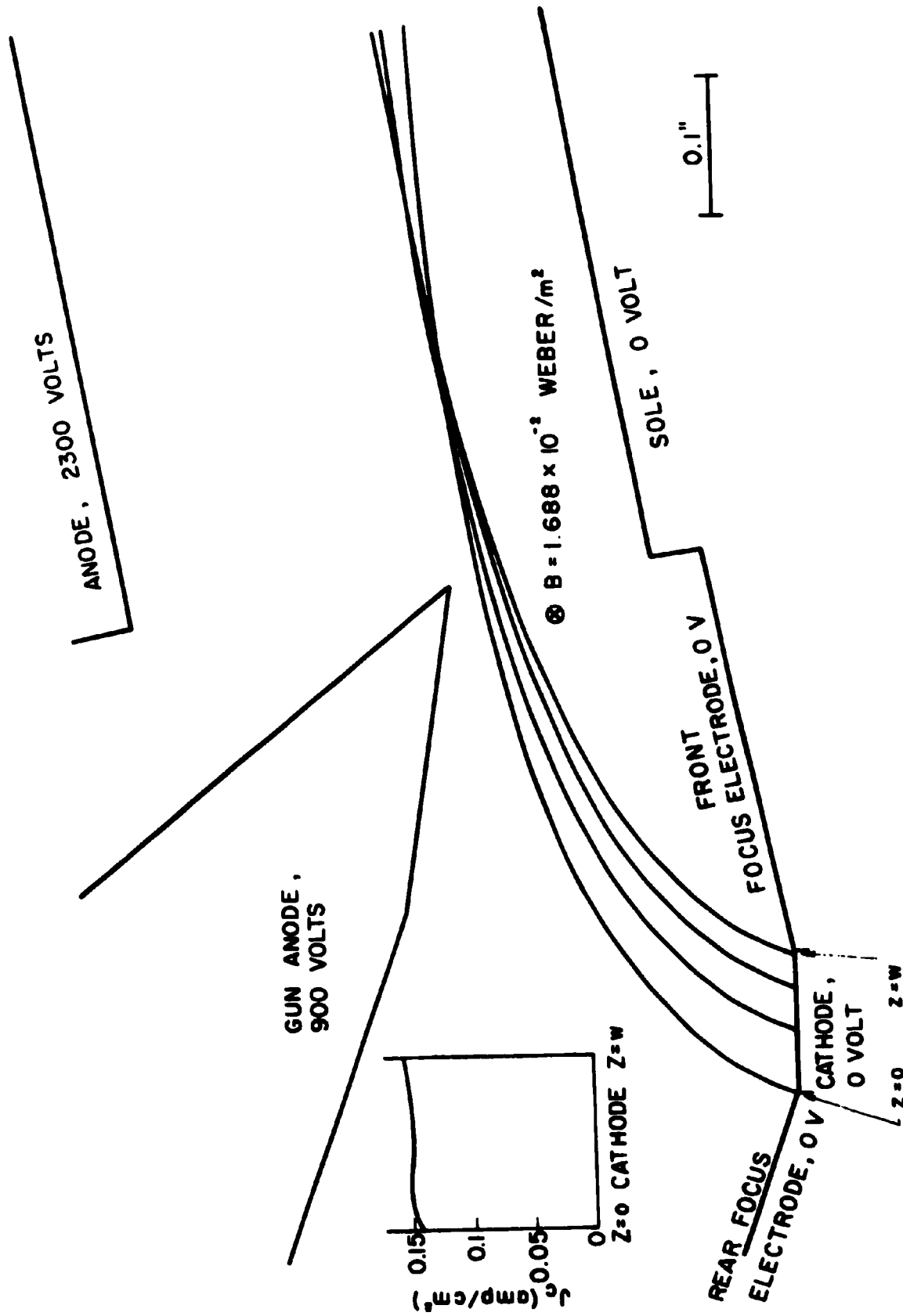


FIG. 2.8 SOME REPRESENTATIVE ELECTRON TRAJECTORIES IN THE MODIFIED KINO GUN.

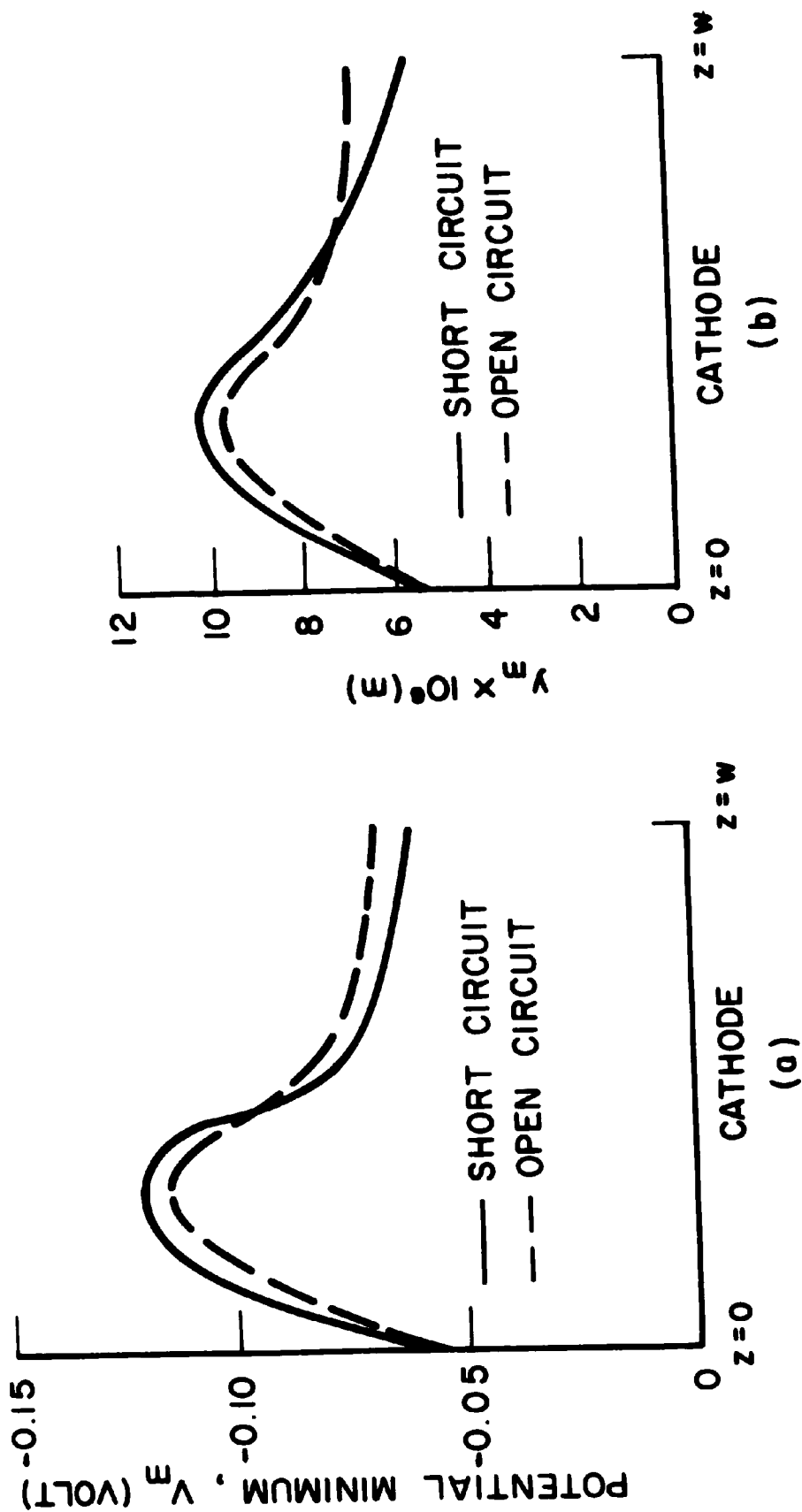


FIG. 2.9 (a) POTENTIAL MINIMUM VARIATION OVER THE CATHODE LENGTH OF THE MODIFIED KINO GUN; (b) VARIATION OF POTENTIAL MINIMUM DISTANCE, y_m , FROM THE CATHODE OF THE MODIFIED GUN.

and open-circuit diode assumptions does not indicate any significant difference.

2.6 Monte Carlo Simulation of the Space-Charge Flow in a Crossed-Field Diode

2.6.1 The Crossed-Field Diode. Wadhwa and Rowe¹¹ have carried out a Monte Carlo analysis of the noise transport in a two-dimensional crossed-field diode under the short-circuit assumption. A Monte Carlo simulation of the space-charge flow in a similar diode is carried out here for both the short-circuit and open-circuit diode assumptions. The purpose of using the same diode configuration is to enable a comparison between the results obtained here and those of Wadhwa and Rowe, at least for the short-circuit case. The pertinent data for the crossed-field diode is given below:

1. Width of the cathode (along z-axis), $w = 1 \times 10^{-3}$ m,
2. Length of the cathode (along x-axis), $d = 2 \times 10^{-2}$ m,
3. Cathode emission current density = 5×10^3 amp/m²,
4. Anode potential = 500 volts,
5. Magnetic field (along x-axis) = 2.7×10^{-2} weber/m² and
6. Cathode temperature, $T = 1000^\circ\text{K}$.

The electrode configuration and the complete layout of the solution points are shown in Fig. 2.10.

2.6.2 Space-Charge-Flow Simulation. The procedure for Monte Carlo simulation of the electron flow is the same as in the previous two cases. The D factor is 1.3×10^5 in this case. The relevant data for the noise analysis is recorded on one magnetic tape, while the number, velocity components and position coordinates of the electrons crossing the exit plane are stored on a second tape for future use.

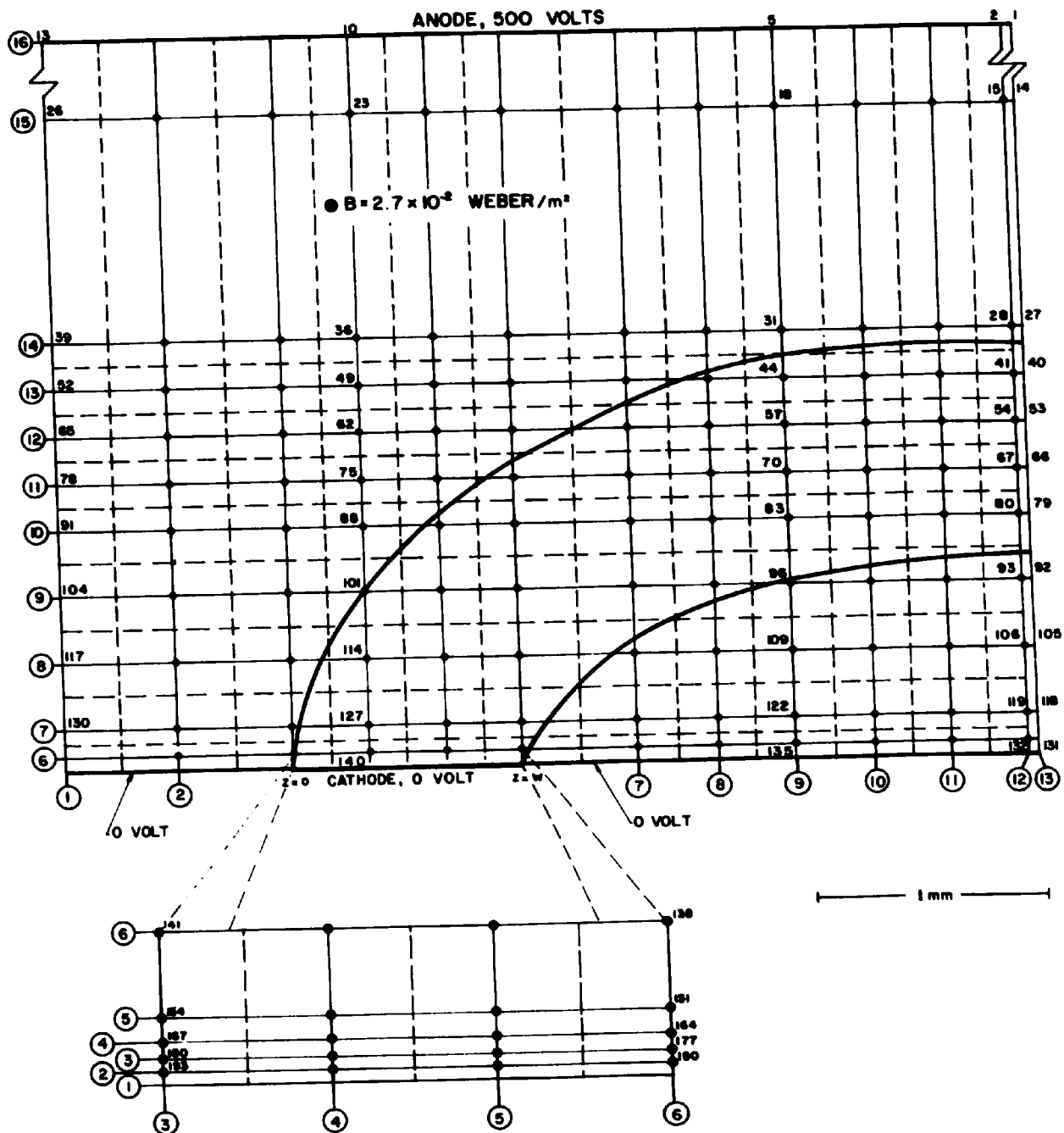


FIG. 2.10 CROSSED-FIELD DIODE GUN WITH COMPLETE LAYOUT OF "SOLUTION POINTS".

2.6.3 Steady-State Characteristics of the Space-Charge Flow in a Crossed-Field Diode. The beam obtained from the Monte Carlo simulation and the relaxation method is shown along with some equipotentials in Fig. 2.11. The insert shows the current-density variation (obtained from the relaxation method) over the cathode. Figure 2.12 shows the variation of the potential minimum over the length of the cathode. As mentioned earlier, the six cathode spots were provided to simulate as closely as possible the uniform emission from the cathode. The insert in Fig. 2.11 shows that the ratio of current going into the beam to the current emitted per unit area of the cathode is maximum around $z = 0$ and minimum over the region $z = 2w/3$ to $z = w$. This implies that the potential minimum should be deepest around $z = w$ and shallowest around $z = 0$. This is what is observed in Fig. 2.12.

In the Monte Carlo study of an identical diode, Wadhwa and Rowe¹¹ have found a similar variation of potential minimum over the length of the cathode. However, their $|V_m|$ was equal to 0.375, approximately twice the value obtained in this study. They attributed the large value of $|V_m|$ to a large value of the saturation current density. In both studies the several diode parameters including the saturation current density, herein referred to as emission current density, were identical. The beam current of 58.8 ma obtained in this study by Monte Carlo simulation agrees well with that of 60.6 ma obtained from the relaxation method. The ratio of the beam current to the total emission current is about 1:1.65.

The difference in the values of $|V_m|$ obtained from these two analyses could possibly be due to the following difference in the two simulation procedures. In the present analysis six cathode spots are

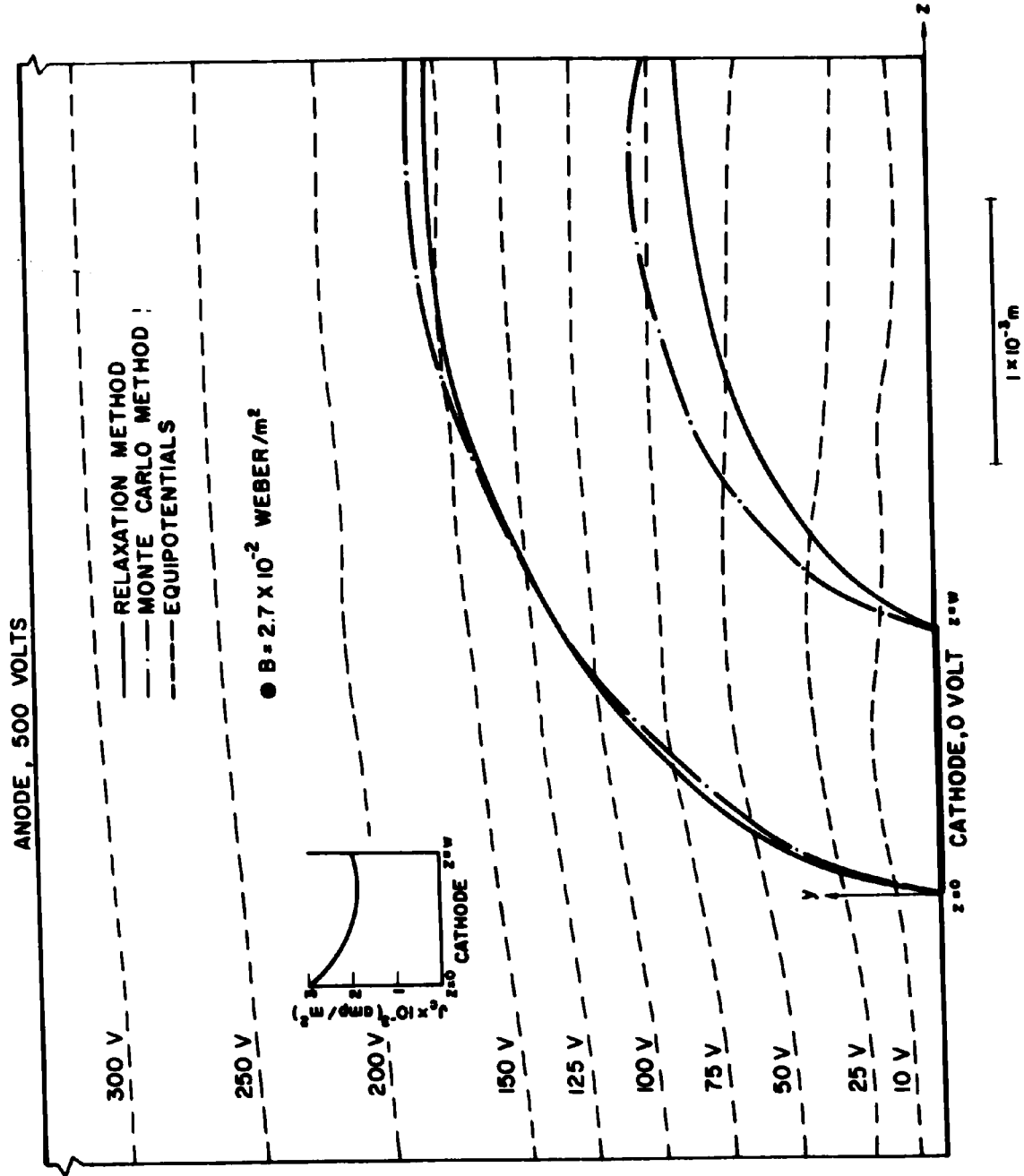


FIG. 2.11 EQUIPOTENTIALS ALONG WITH THE BEAM PROFILES OBTAINED FROM MONTE CARLO AND RELAXATION METHODS. (CROSSED-FIELD DIODE GUN)

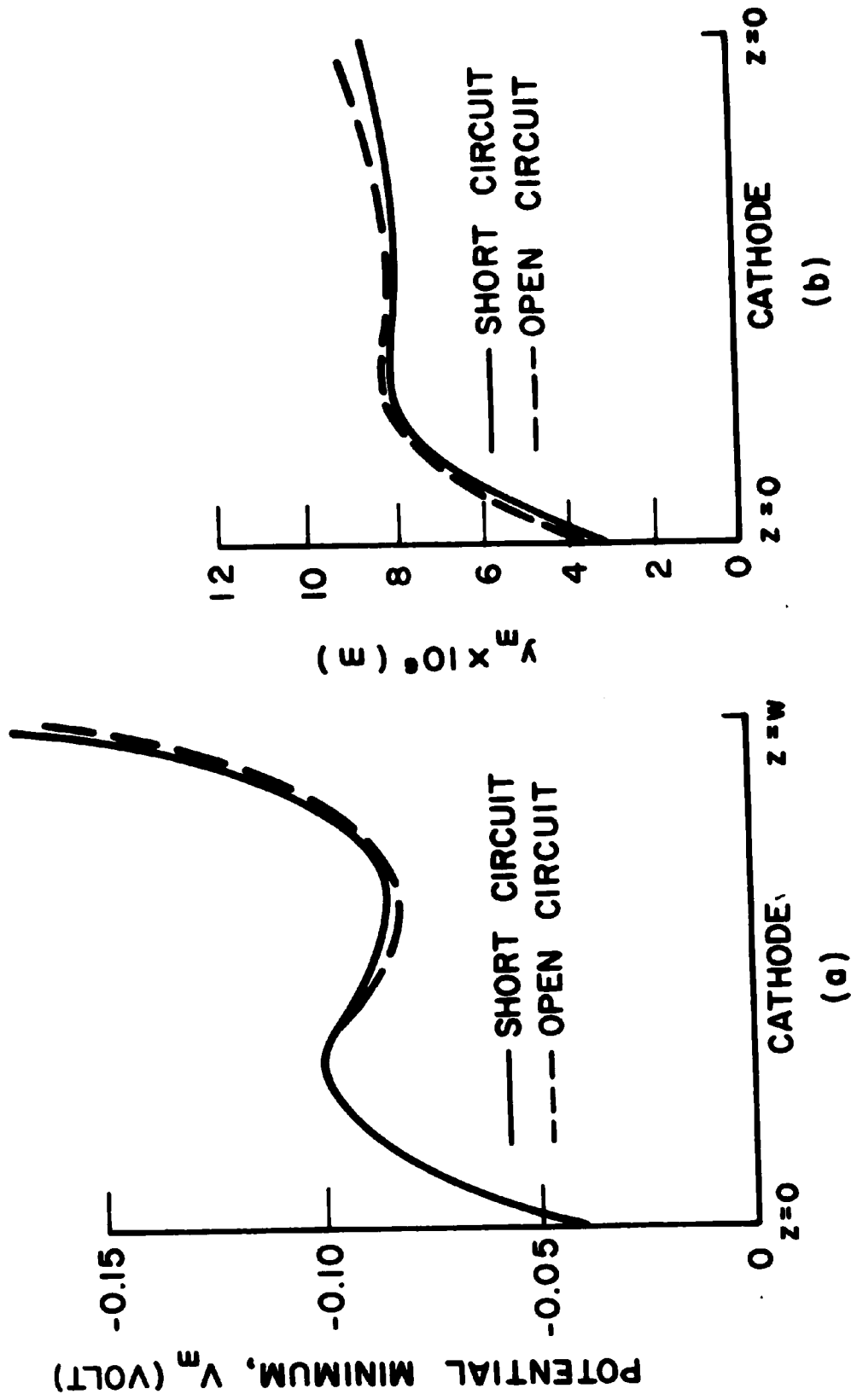


FIG. 2.12 (a) POTENTIAL MINIMUM VARIATION OVER THE CATHODE LENGTH OF THE CROSSED-FIELD DIODE GUN; (b) VARIATION OF THE POTENTIAL MINIMUM DISTANCE FROM THE CATHODE OF THE CROSSED-FIELD DIODE GUN.

used to simulate the emission from the cathode in contrast to the three used by Wadhwa and Rowe. Although both analyses use Poisson's distribution for the number of electrons emitted per Δt , the random number generator program and the cumulative distribution function inequality condition deciding the number of electrons emitted at the beginning of a given Δt are different. It is found that the procedure used in this analysis gives a long-term average of electrons emitted per Δt that is very close to the assumed mean value, λ , of Poisson's distribution. In the other analysis the above-mentioned long-term average was lower than the assumed value of λ . For the same amount of charge emitted per Δt , the number of "big electrons" emitted is 48 in this analysis as compared to 21 in the other analysis. It is therefore expected that the percentage error in space-charge density calculations will be less in this analysis than in the calculations of Wadhwa and Rowe.

It seems that the most important difference lies in the determination of the potential distribution in the diode region. Wadhwa and Rowe used a Green's function technique to derive an expression for the space-charge potential involving weighting constants. The numerical method used in this analysis for solving Poisson's equation is quite straightforward and involves no assumptions to reduce computation time. Furthermore, in the beam region the grid size in Wadhwa and Rowe's analysis was larger than that used here. Wadhwa and Rowe made some assumptions regarding the dimensions of the diode in the determination of the weighting constants to reduce the computation time. Since the two analyses followed procedures differing on several points, it is difficult to assess the superiority of one method over the other.

However, the results obtained in the present analysis agree fairly well with the results obtained using Boers' relaxation method²⁴ to find the trajectories and potential distribution.

2.7 Space-Charge-Flow Simulation in a Drift Region

Wadhwa and Van Duzer²³ have recently reported on a low noise, crossed-field amplifier which incorporates a crossed-field drift space that is one cyclotron wavelength long as a noise transformer. Since the excitation, propagation and coupling between the various waves on a crossed-field beam in a drift space have not been completely understood, a complete theoretical design procedure has not yet been developed. It is hoped that the study of noise transport in such a drift region based on a computer simulation of the space-charge flows could act as a guide in designing noise transformers and suggesting some analytical approaches to be followed in devising new noise reduction schemes.

It may be recalled that during the Monte Carlo simulation run for the gun region, the number, positions and velocity components of all electrons crossing a given plane cc' (making an angle α with the y -axis) are recorded on a magnetic tape for each unit-time interval for both short- and open-circuit cases. These data are used as input data for the space-charge-flow simulation in the drift region. The input plane cc' was chosen away from the gun exit in order to reduce the possibility of error in the field calculations in this region where the gun geometry has sharp corners. Furthermore, it is necessary to study the noise transport over incremental drift tube lengths due to the limitation on the number of electrons which can be handled by the available computer storage space. The first incremental distance

in this analysis is of the order of one mean beam cyclotron wavelength ($\lambda_c \approx 3.06$ cm).

2.8 Some Additional Remarks About Δt and D

The unit-time step Δt and the D factor have to be altered due to the limitation on the maximum number of electrons permitted at any time in the geometry. The need for changing Δt , and consequently D, can be explained as follows:

Assume that no electrons cross the upper and lower boundaries and that, on the average, the number of electrons entering the geometry at the left-hand boundary is equal to the number leaving the right-hand boundary. Furthermore, consider that an average electron takes a time, t , to go across the geometry and let the nearest integer number of unit-time intervals contained in this time be given by $T \approx t/\Delta t$. During each Δt , an average number of electrons, say n , enters through the left-hand boundary. Starting with no electrons inside the geometry there will be approximately $T \times n$ electrons present after the time, t . If this number exceeds the number of electrons which could be handled by the computer program, either T or n or both will have to be adjusted. In the present case, the input, i.e., the number of electrons entering the left-hand boundary in each Δt , is fixed since it is obtained from the previously recorded data during the Monte Carlo simulation of the gun region. Thus the only alternative is to adjust T . The time, t , depends on the length of the drift tube and the electron velocity. Since both are fixed for a given geometry and field configuration, Δt has to be changed to make the necessary adjustment in the value of T . The effect of this change in Δt on the D factor is explained below.

Assume two unit-time intervals: Δt used in the gun region simulation and $(\Delta t)'$ to be used in this simulation. In order that the charge crossing the input plane cc' in an arbitrary time τ [assuming that τ is exactly divisible by both Δt and $(\Delta t)'$] be the same in both cases, it is necessary that

$$-n \frac{\tau}{\Delta t} D|e| = -n \frac{\tau}{(\Delta t)'} D'|e| , \quad (2.2)$$

where $-D|e|$ and $D'|e|$ are the amounts of charge carried by each electron (strictly speaking, a "big electron") in the two cases. Thus

$$D' = \frac{(\Delta t)'}{\Delta t} D . \quad (2.3)$$

2.9 Transformation of Position and Velocity Coordinates of Electrons

The lower parallel plane of the drift space is parallel to the front focus electrode of the gun. The coordinates of the intersection of the line cc' with the front focus electrode are (z_0, y_0) , according to the coordinate system (for the gun region) with its origin at the left edge of the cathode. To facilitate computation, a primed coordinate system with its origin at (z_0, y_0) , rotated through an angle α with respect to the original system of coordinates, is chosen so that the z -axis lines up with the front focus electrode and the y -axis is along cc' . The positions and velocity coordinates of the electrons injected at cc' are transformed according to the following equations:

$$z' = (z - z_0) \cos \alpha + (y - y_0) \sin \alpha , \quad (2.4)$$

$$y' = -(z-z_0) \sin \alpha + (y-y_0) \cos \alpha , \quad (2.5)$$

$$\dot{z}' = \dot{z} \cos \alpha + \dot{y} \sin \alpha \quad (2.6)$$

and

$$\dot{y}' = -\dot{z} \sin \alpha + \dot{y} \cos \alpha . \quad (2.7)$$

In the next chapter velocity and current fluctuations which determine the various noise parameters at a given point in the beam are considered only in the direction of the beam. Hence these parameters will be independent of the system of coordinates used.

2.10 Drift-Space Configuration

The parallel plane drift space is designed such that the beam condition at the exit of the abbreviated Kino gun is matched as closely as possible to that of Brillouin flow in the drift region. The drift-space planes are parallel to the z' -axis. The operating conditions for the drift space attached to the modified Kino gun are taken to be the same as those for the drift space following the abbreviated Kino gun. They are as follows:

1. Upper parallel plane potential = 2300 volts,
2. Distance between the parallel planes = 0.365 inch,
3. Lower parallel plane and the front focus electrode = 0 volt,
4. Gun anode potential = 900 volts and
5. Magnetic field, $B = 1.688 \times 10^{-2}$ weber/m².

The geometry and the layout of the solution points for the two drift spaces used in this analysis are shown in Figs. 2.13 and 2.14.

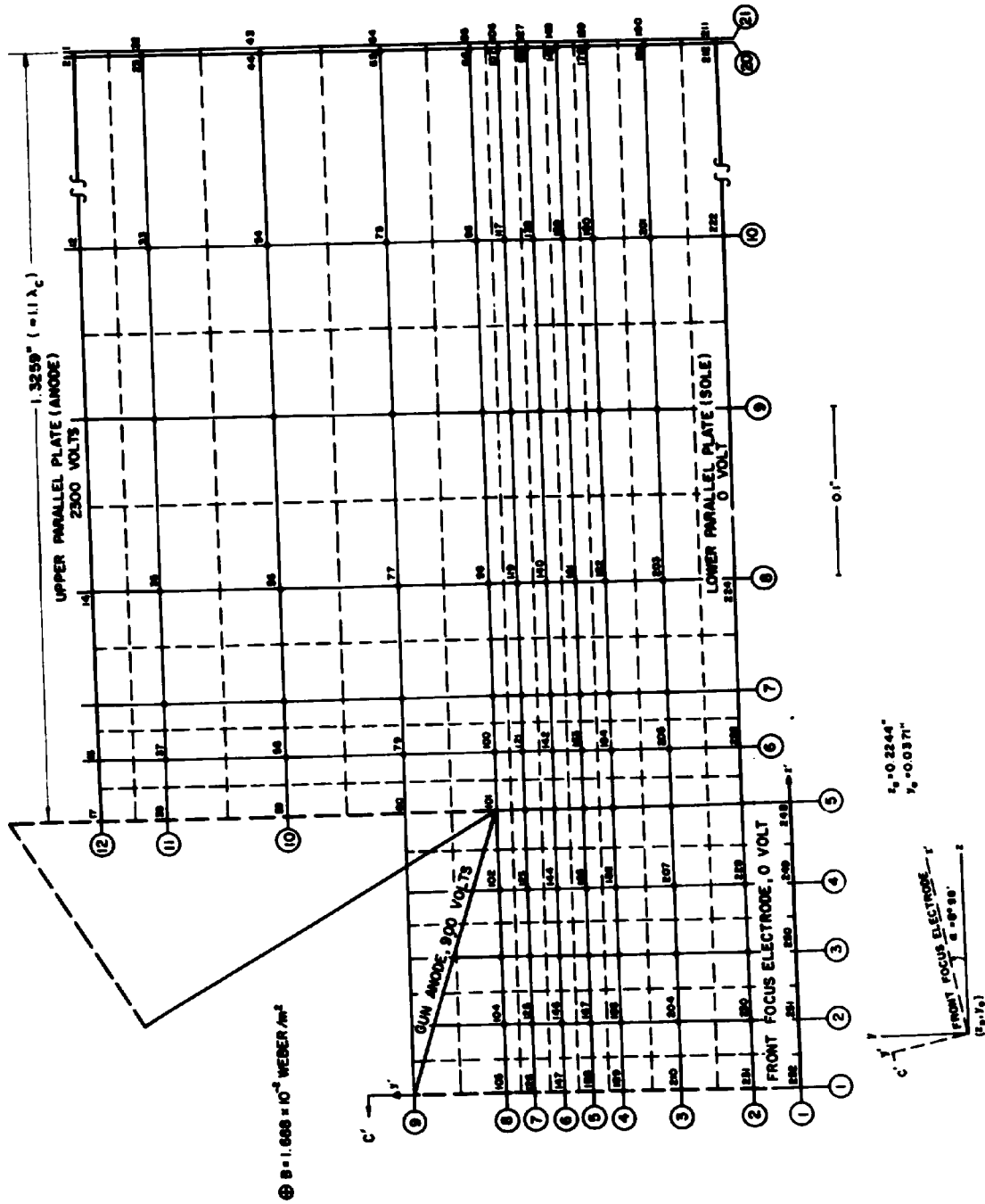


FIG. 2.13 THE GEOMETRY AND LAYOUT OF "SOLUTION POINTS" OF THE DRIFT SPACE FOLLOWING THE ABBREVIATED KINO GUN (DRIFT SPACE A).

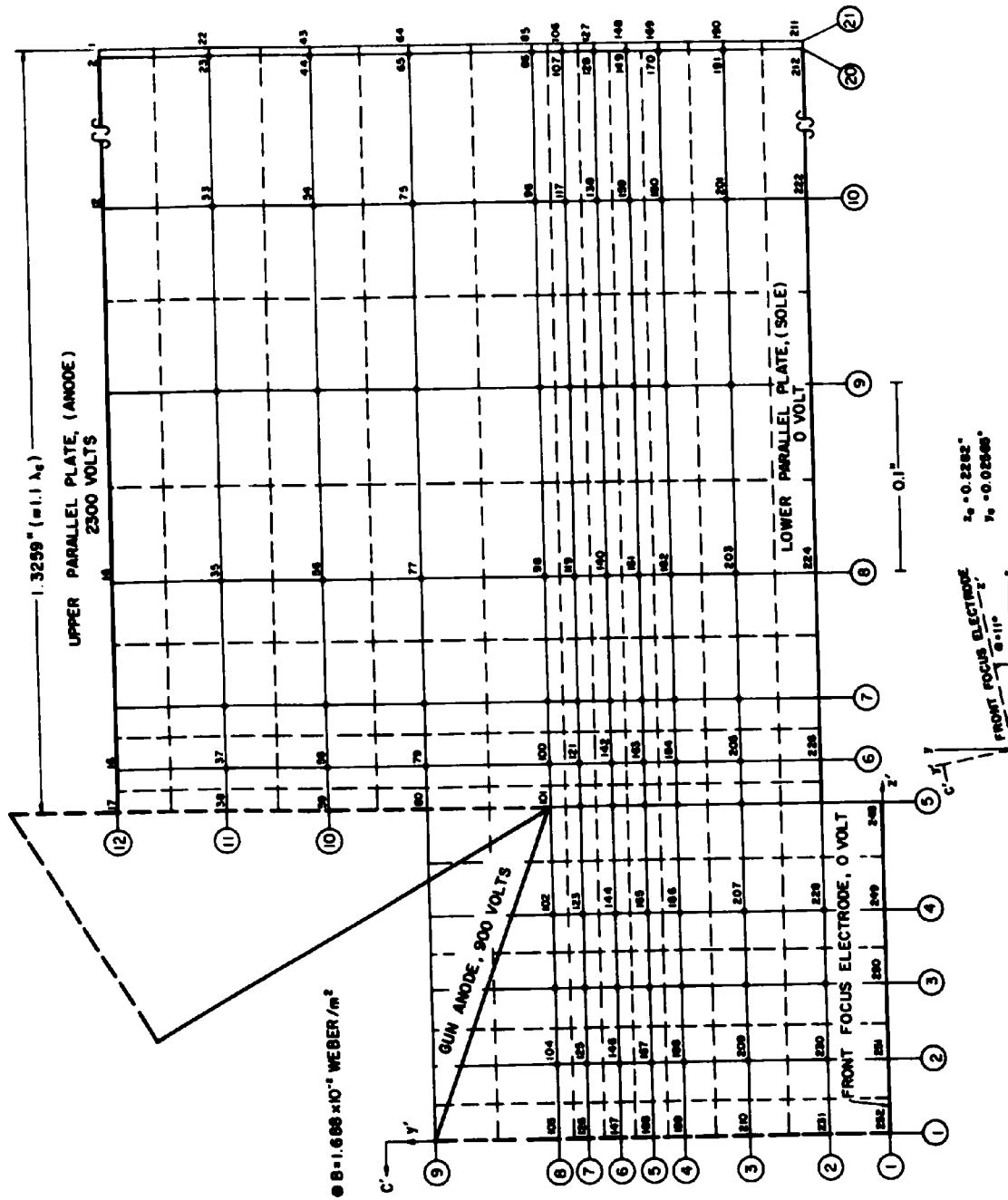


FIG. 2.14 THE GEOMETRY AND LAYOUT OF "SOLUTION POINTS" OF THE DRIFT SPACE FOLLOWING THE MODIFIED KINO GUN (DRIFT SPACE M).

2.11 Space-Charge-Flow Simulation

The simulation process is the same as described in Section 2.3.2 except that in this case the electrons are injected into the geometry at plane cc' instead of being emitted from the cathode. The number, positions and velocity components of electrons injected per unit-time interval are obtained from the data recorded earlier (during the electron gun Monte Carlo simulation run) and then are transformed into the primed coordinate system. The boundary conditions for solving Poisson's equation are the same for the two drift spaces. In addition to the electrode potentials forming the boundary, the following two boundary conditions are used at the input and exit planes of the geometry.

1. The input plane boundary cc' is given a fixed potential distribution obtained from the steady-state solution using a relaxation technique.

2. As the beam passes through the right-hand boundary, the potentials at the boundary fluctuate due to the fluctuating beam. The details of handling this boundary condition are the same as those for the right-hand boundary of the gun geometries discussed in Section 2.3.2.

Following the various steps in the simulation process described in Section 2.3.2, the data for the analysis of noise transport in both drift regions under short- and open-circuit diode assumptions is recorded on magnetic tape. At the same time the position and velocity coordinates of electrons leaving the right-hand boundary of the drift space in each Δt are recorded on another magnetic tape. These latter

data could be used for simulation of space-charge flow in the next incremental step in the drift region.

2.12 D-c Beam Characteristics

Masnari³⁰ has discussed the formation of undulating beams with varying thicknesses. He indicates that the theoretical variation of an electron trajectory in a space-charge-free field has a period of

$$L_{th} = \frac{2\pi\phi_a}{\eta dB^2} , \quad (2.8)$$

where ϕ_a = voltage between the two parallel planes,

d = spacing between the parallel planes,

B = magnetic field and

η = absolute value of electron charge-to-mass ratio.

The electron beam in both drift space A (following the abbreviated Kino gun) and drift space M (following the modified Kino gun) is undulating and has a varying thickness as shown in Figs. 2.15 and 2.16 respectively. The beam is wider in drift space M than in drift space A because of different beam widths at the entrance to the drift region. Since ϕ_a , B and d are the same for both drift spaces, the theoretical period calculated from Eq. 2.8 is 3.1 cm in both cases. The periods of approximately 3.2 cm and 3.1 cm obtained from the simulation process for the drift regions A and M, respectively, agree well with the theoretical value given above. As an aid in the study of the above beam shapes, the equipotentials for the two drift spaces are shown in Figs. 2.17 and 2.18 respectively.

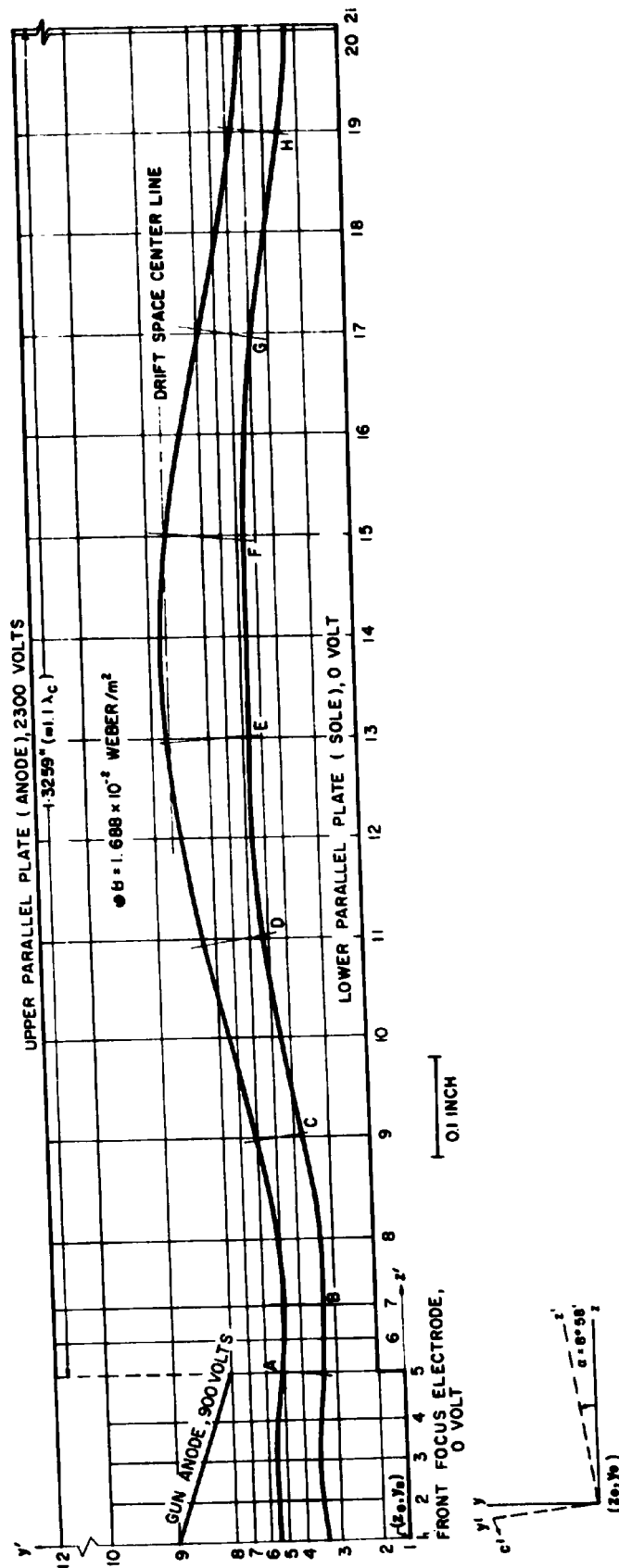


FIG. 2.15 UNDULATING BEAM WITH NOISE PLANES FOR DRIFT SPACE A.

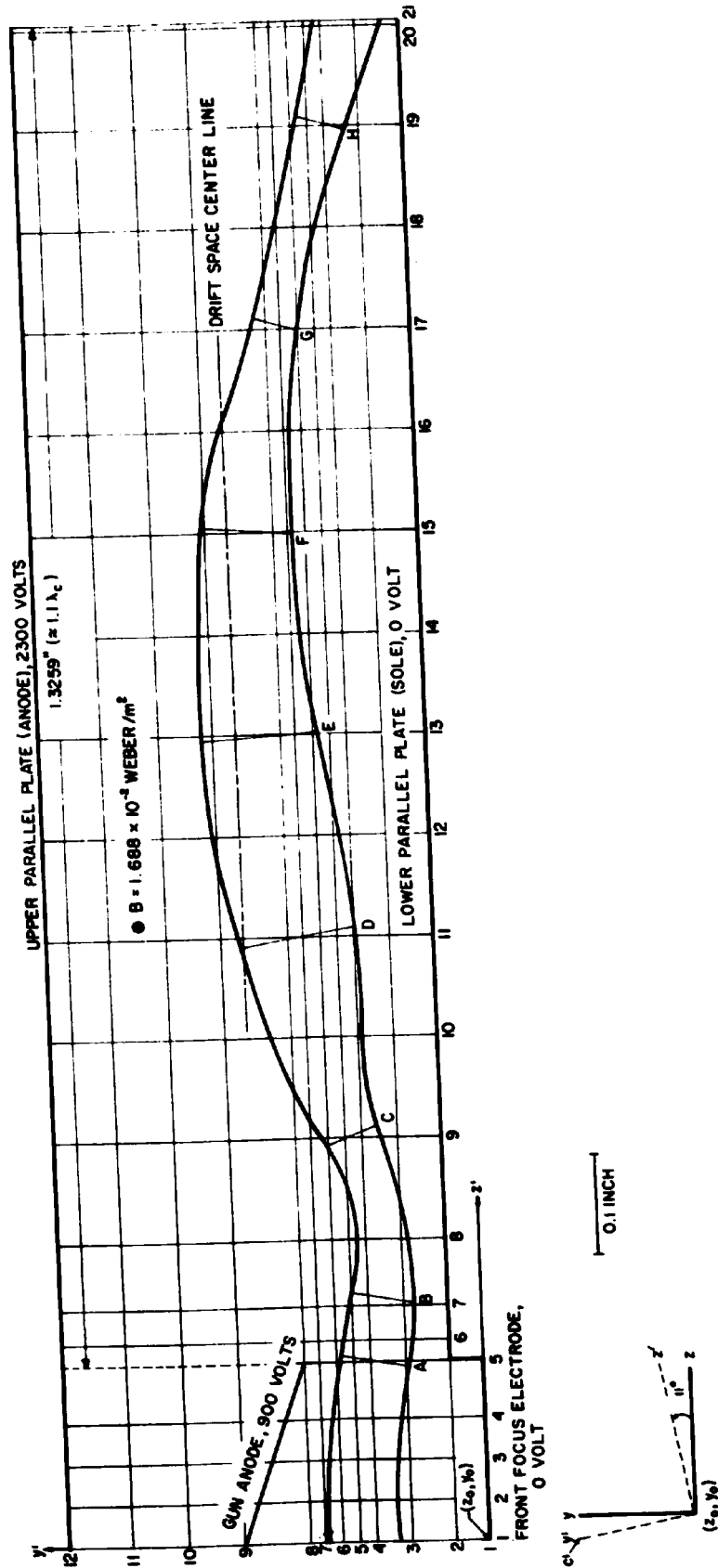


FIG. 2.16 UNDULATING BEAM WITH NOISE PLANES FOR DRIFT SPACE M.

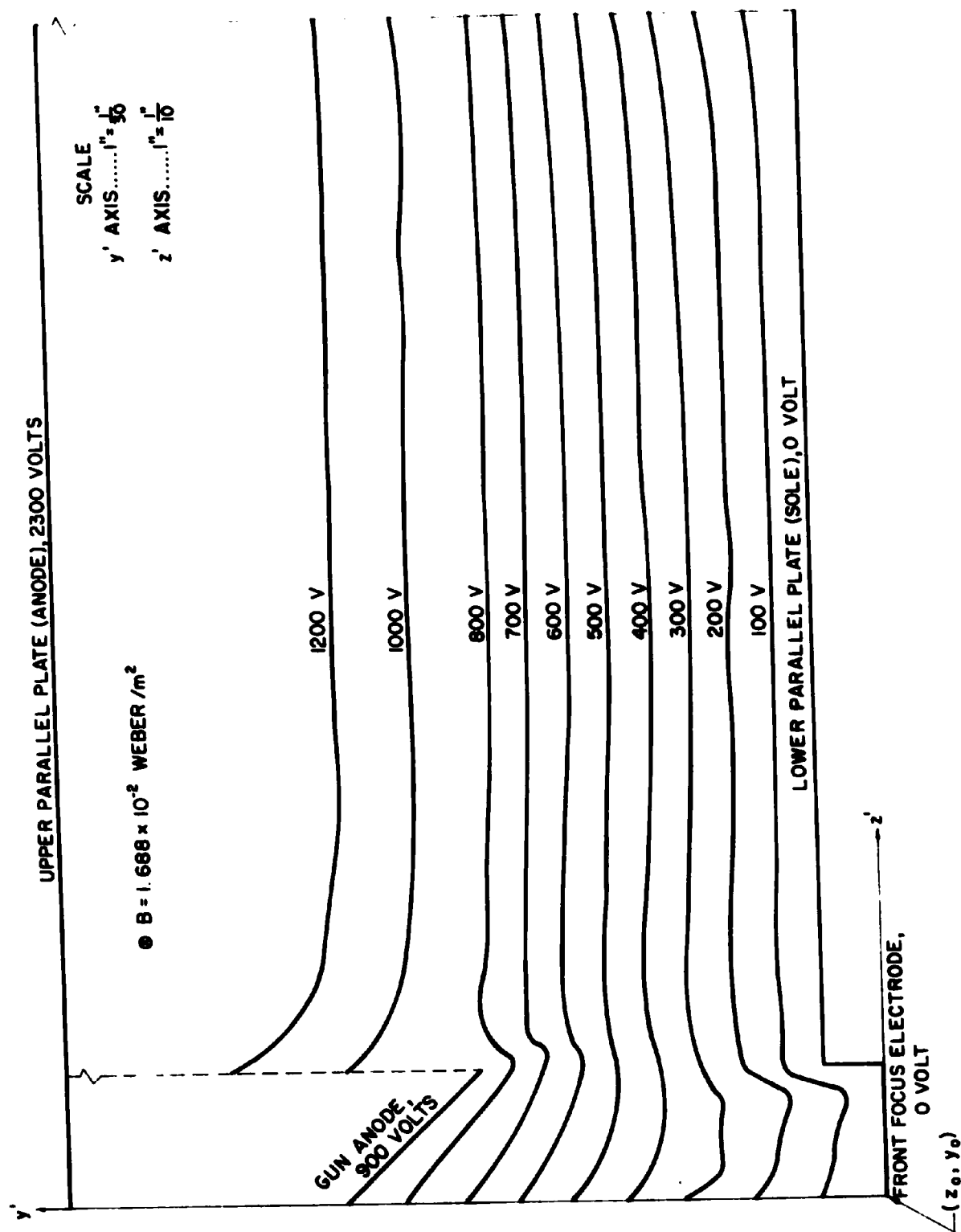


FIG. 2.17 EQUIPOTENTIALS FOR DRIFT SPACE A.

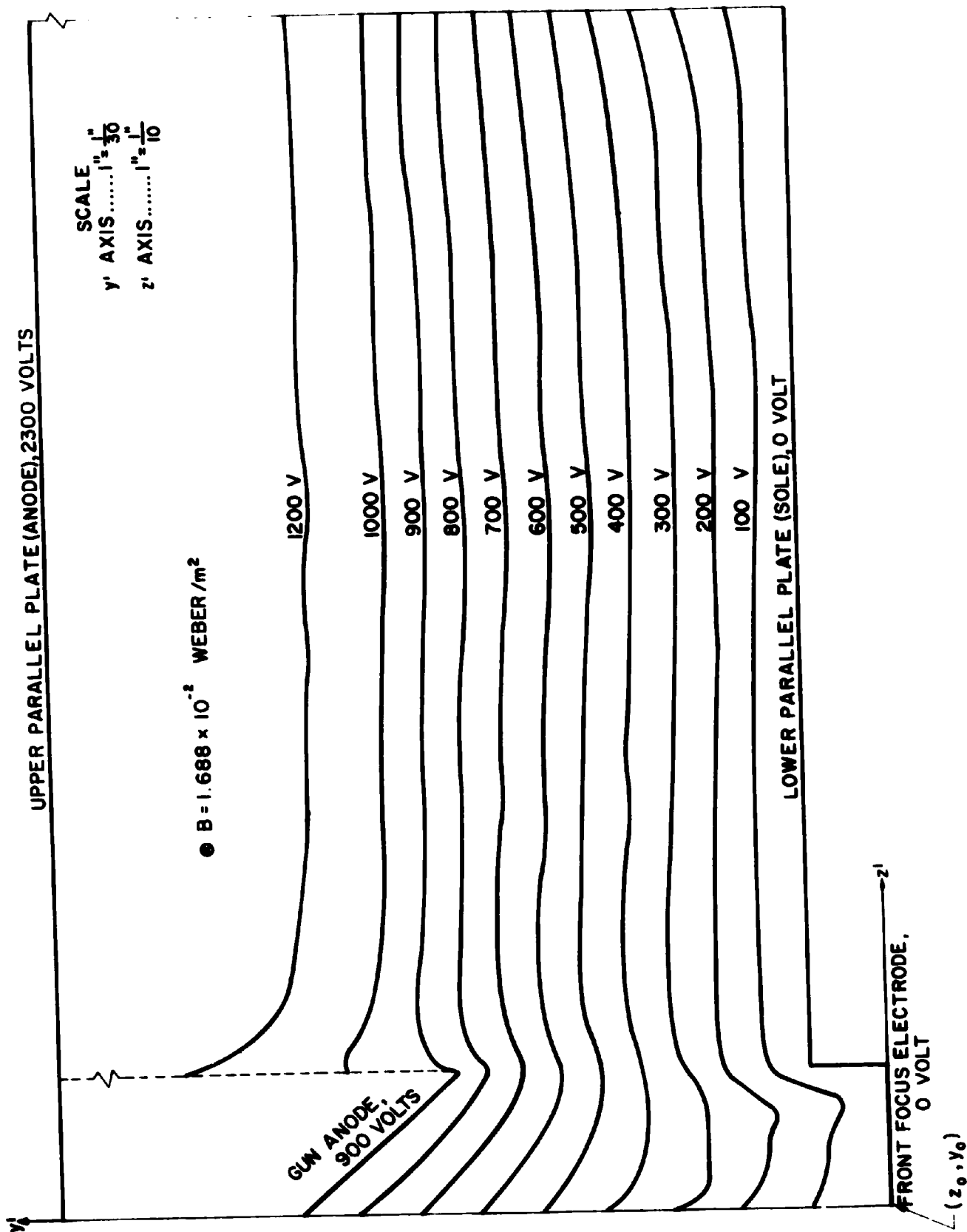


FIG. 2.18 EQUIPOTENTIALS FOR DRIFT SPACE M.

2.13 Discussion of Accuracy of Monte Carlo Simulation

The degree of accuracy achievable in a Monte Carlo simulation of space-charge flow in the gun region depends primarily on the accuracy of simulating electron emission at the cathode and on the accuracy in solving Poisson's equation. The long-time average of the number of "big electrons" emitted per Δt from the cathode was found to be within 0.2 percent of the prescribed value. A very satisfactory check of the accuracy and proper working of the matrix factorization program used in solving Poisson's equation was obtained by comparing the potential distribution in a parallel plane space-charge-free diode obtained from this computer program and the analytical solution. In the present work electrons are not considered individually; a "big electron" represents D electrons. The choice of the D factor has been discussed in Section 2.2.3. However, a check on the suitability of the D factor used in this work was made by running again the space-charge-flow simulation program for the parallel plane crossed-field diode gun using a different D factor (lower in value than that mentioned in Section 2.6.2). It was found that the potential distribution in the above gun for the two cases, i.e., with $D = 1.3 \times 10^5$ and 1.733×10^5 , differed by only 0.5 percent, except for a very small region near the cathode. A close agreement (the difference being approximately three percent) between the beam currents obtained from the Monte Carlo simulation of space-charge flow in the abbreviated Kino gun and the Kino theory gives further support to the satisfactory accuracy of the simulation program. A closer agreement between the beam currents obtained from the two methods is also not expected since Kino's theory does not consider the tangential component of emission

velocity and the formation of the potential minimum. Next, the space-charge-flow simulation in the drift region primarily depends upon the accuracy of solving Poisson's equation, which has been discussed earlier in this section.

CHAPTER III. DERIVATION OF NOISE PARAMETERS

3.1 Introduction

The noise content of a crossed-field electron beam will in general depend upon the current, velocity and position fluctuations along and normal to the principal direction of beam flow. The noise figure of a backward-wave amplifier, neglecting the contributions from the position fluctuations, was calculated by Mantena³¹ and found to be in close agreement with the experimental value. Wadhwa and Rowe¹¹ and Pollack and Whinnery¹² also considered only the current and velocity fluctuations in their Monte Carlo analyses of a crossed-field diode. In a similar manner, the following analysis will be concerned only with the current and velocity fluctuations in the beam.

The propagation of noise signals on curved electron beams in the gun and drift regions of the type illustrated in Chapter II can be studied in terms of Haus' noise parameters for a one-dimensional O-type electron beam, provided that these parameters are defined with some modifications.

In the present two-dimensional analysis the d-c and a-c components of the electron velocity and current density in the y- and z-directions are calculated first. These are then used to obtain velocity and current density fluctuations in the direction of the d-c beam velocity at each point in the beam. The expressions for a-c kinetic potential, a-c current fluctuations and other noise parameters defined along the beam are derived and discussed in the following sections.

3.2 Expressions for A-c Kinetic Potential and A-c Current Fluctuation

The a-c kinetic potential for a one-dimensional system under the small-signal single-velocity assumption is defined as

$$\tilde{V}(t) \triangleq - \frac{u_o v(t)}{\eta} , \quad (3.1)$$

where u_o = d-c velocity of the electron and

$v(t)$ = a-c component of the velocity of the electron.

The modified a-c kinetic potential under the same assumption may be defined as

$$\tilde{V}(t) = - \frac{\vec{u}_o \cdot \vec{v}(t)}{\eta} . \quad (3.2)$$

Here, in general, $\vec{u}_o$ and $\vec{v}(t)$ are not colinear. This definition is reduced to that given by Eq. 3.1 when $\vec{u}_o$ and $\vec{v}(t)$ are in the same direction (i.e., for a one-dimensional beam). Equation 3.2 is developed in detail below.

Consider a particular electron having an instantaneous velocity, $\vec{u}(t)$, and let $u_T(t)$ and $u_N(t)$, respectively, denote the components of $\vec{u}(t)$ along and normal to the direction of the d-c velocity, u_o , of the electron. $u_T(t)$ and $u_N(t)$ can also be written in terms of their d-c and a-c components as

$$u_T(t) = u_o + v_T \quad (3.3a)$$

and

$$u_N(t) = 0 + v_N , \quad (3.3b)$$

where v_T and v_N are a-c perturbations. Since the total velocity can be expressed as

$$|\vec{u}(t)|^2 = (u_0 + v_T)^2 + v_N^2, \quad (3.4)$$

the instantaneous kinetic energy, K_t , of the electron is given by

$$K_t = \frac{1}{2} m |\vec{u}(t)|^2 = \frac{1}{2} m [(u_0 + v_T)^2 + v_N^2], \quad (3.5)$$

whereas the steady-state kinetic energy, K_0 , of the electron is simply $(1/2)mu_0^2$. Hence the a-c kinetic energy of the electron is written as

$$K_{a-c} = K_t - K_0 = \frac{1}{2} m [2u_0 v_T + v_T^2 + v_N^2]. \quad (3.6)$$

If

$$2u_0 v_T \gg v_T^2 + v_N^2, \quad (3.7)$$

then Eq. 3.6 becomes

$$\begin{aligned} K_{a-c} &= mu_0 v_T \\ &= m \vec{u}_0 \cdot \vec{v}(t). \end{aligned} \quad (3.8)$$

Thus the a-c kinetic potential, $\tilde{V}(t)$, may be defined as

$$\tilde{V}(t) = - \frac{\vec{u}_0 \cdot \vec{v}(t)}{\eta} \quad (3.9)$$

$$= - \frac{[u_{oy}\tilde{y}(t) + u_{oz}\tilde{z}(t)]}{\eta} \quad (3.10)$$

when expressed in terms of the y- and z-direction velocity components (the symbol tilde denotes the a-c nature of the quantities).

As mentioned in Appendix B the gun and drift regions are divided into small rectangles and thus a given cross section of the beam is represented by a few rectangles. In the present analysis Eq. 3.10 is used to obtain the average value of $\tilde{V}(t)$ for the electrons in a given rectangle. The d-c and a-c velocity components in this equation referring to the average value of $\tilde{V}(t)$ are obtained from Appendix D. When $\tilde{V}(t)$ is not uniform across the beam it is desirable to take the weighted mean of $\tilde{V}(t)$ over all electrons in the rectangles located in that cross section as the a-c kinetic potential corresponding to that beam cross section as a whole. If $\tilde{V}_i(t)$ is the a-c kinetic potential for the i th rectangle falling in the given cross section and having n_i electrons, the weighted mean is given by

$$\tilde{V}(t) = \frac{\sum_i n_i \tilde{V}_i(t)}{\sum_i n_i} \quad (3.11)$$

Now consider a small portion of the beam where $\vec{J}(t)$ and $\vec{J}_0$ represent, respectively, the instantaneous and steady-state current densities. In general, $\vec{J}(t)$ and $\vec{J}_0$ will not be colinear. As in the case of the a-c kinetic potential defined above, consider the fluctuations in $\vec{J}(t)$ only in the direction of $\vec{J}_0$. Thus

$$J_T(t) = \frac{\vec{J}(t) \cdot \vec{J}_0}{|\vec{J}_0|} \quad (3.12)$$

$$= \frac{J_{0y} J_y(t) + J_{0z} J_z(t)}{|\vec{J}_0|} , \quad (3.13)$$

where $J_T(t)$ is the component of $\vec{J}(t)$ in the direction of $\vec{J}_0$.

Hence the a-c current-density fluctuation along $\vec{J}_0$ is given by

$$\tilde{J}_T(t) = J_T(t) - J_0 . \quad (3.14)$$

The current fluctuation across a given beam cross section can be expressed as

$$\tilde{I}(t) = \sum_{i=1}^n a_i \tilde{J}_T(t) , \quad (3.15)$$

where a_i is the area normal to $\vec{J}_{0i}$ for the i th rectangle falling in the cross section considered.

3.3 Correlation Functions and Power Spectra

In evaluating the noise characteristics of beam-type amplifiers the noise is assumed to be a stationary ergodic process. For an infinite, real and random process, $x(t)$, the autocorrelation function is defined by

$$C(\tau) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T x(t) x(t + \tau) dt \quad . \quad (3.16)$$

Similarly the cross-correlation function between two infinite and real random processes, $x(t)$ and $y(t)$, is given by

$$C_{xy}(\tau) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T x(t) y(t + \tau) dt \quad (3.17a)$$

and

$$C_{yx}(\tau) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T y(t) x(t + \tau) dt \quad . \quad (3.17b)$$

The self-power spectrum $W(f)$ is defined by

$$W(f) = \int_{-\infty}^{\infty} C(\tau) e^{-j2\pi f\tau} d\tau \quad . \quad (3.18)$$

Since $C(\tau)$ is an even function of τ ,

$$W(f) = 2 \int_0^{\infty} C(\tau) \cos(2\pi f\tau) d\tau , \quad (3.19)$$

where f denotes the frequency. Similarly the cross-power spectrum between the random processes $x(t)$ and $y(t)$ is defined by

$$\Theta(f) = \int_{-\infty}^{\infty} C_{xy}(\tau) e^{-j2\pi f\tau} d\tau \quad (3.20)$$

and

$$\Theta^*(f) = \int_{-\infty}^{\infty} C_{yx}(\tau) e^{-j2\pi f\tau} d\tau , \quad (3.21)$$

where the star denotes the complex conjugate. It is also true that

$$C_{xy}(\tau) = C_{yx}(-\tau) , \quad (3.22)$$

$C_{xy}(\tau) + C_{yx}(\tau)$ is an even function of τ , and $C_{yx}(\tau) - C_{xy}(\tau)$ is an odd function of τ . Hence

$$\text{Re } \Theta(f) = \frac{1}{2} [\Theta(f) + \Theta^*(f)] \quad (3.23)$$

$$= \frac{1}{2} \int_{-\infty}^{\infty} [C_{xy}(\tau) + C_{yx}(\tau)] e^{-j2\pi f\tau} d\tau \quad (3.24)$$

$$= \int_0^{\infty} [C_{xy}(\tau) + C_{yx}(\tau)] \cos(2\pi f\tau) d\tau , \quad (3.25)$$

and similarly

$$\text{Im } \Theta(f) = \int_0^{\infty} [C_{yx}(\tau) - C_{xy}(\tau)] \sin(2\pi f\tau) d\tau . \quad (3.26)$$

For a finite data sample it can be shown²⁰ that a smoothed estimate of the power spectrum in the neighborhood of the desired frequency is obtained by taking the Fourier transform of the product of the "apparent" correlation function (correlation function evaluated for $|\tau| \leq T_m < T_n$, where T_m is the maximum lag used) and $D(\tau)$, a suitable even function of τ . $D(\tau)$ can be considered as a window of variable transmission which modifies the values of the apparent correlation function differently for different lags. $D(\tau)$ is, therefore, called a "lag window". The lag window used in this analysis is the same as that used by Wadhwa and Rowe¹¹ and is given by

$$\begin{aligned} D(\tau) &= \frac{1}{2} \left(1 + \cos \frac{\pi\tau}{T_m} \right) , \quad \text{for } |\tau| < T_m \\ &= 0 , \quad \text{for } |\tau| \geq T_m . \end{aligned} \quad (3.27)$$

It is assumed in this analysis that various quantities, e.g., the velocity and position of electrons, the electric field, etc., do not change during each unit-time interval Δt . Thus the samples taken at the beginning or at the end of each Δt represent an average value of the sampled quantities during that interval. Since the samples are drawn at discrete times spaced Δt apart, the true power spectrum, $P(f)$, is imaged about nf_0 , where $n = 1, 2, \dots$ and $f_0 = 1/\Delta t$. This

process is called "aliasing" and produces an "aliased" power spectrum, $\tilde{P}(f)$, given by

$$\tilde{P}(f) = P(f) + \sum_{n=1}^{\infty} [P(f - nf_0) + P(f + nf_0)] \quad (3.28)$$

$P(f)$ can be recovered from $\tilde{P}(f)$ when the former vanishes for $|f| > f_0/2$. The "averaging" process fortunately produces a spectrum attenuated in this region and the errors due to the "averaging" and "aliasing" are reduced appreciably if $\Delta t < 0.1/|f|$, where $|f|$ is the highest frequency considered in the noise analysis.

On the basis of the above discussion, the cross-correlation and autocorrelation functions for the kinetic noise-voltage and noise-current modulations obtained from finite and discrete data samples can be written as

$$C_V^i(\mu\Delta t, 1) = \frac{1}{2L+1} \sum_{v=-L}^L \tilde{V}(v\Delta t) \tilde{V}[(\mu + v) \Delta t] \quad (3.29a)$$

$$C_I^i(\mu\Delta t, 1) = \frac{1}{2L+1} \sum_{v=-L}^L \tilde{I}(v\Delta t) \tilde{I}[(\mu + v) \Delta t] \quad (3.29b)$$

$$C_{V,I}^i(\mu\Delta t, 1) = \frac{1}{2L+1} \sum_{v=-L}^L \tilde{V}(v\Delta t) \tilde{I}[(\mu + v) \Delta t] \quad (3.29c)$$

and

$$C_{I,V}^i(\mu\Delta t, 1) = \frac{1}{2L+1} \sum_{v=-L}^L \tilde{I}(v\Delta t) \tilde{V}[(\mu + v) \Delta t] \quad (3.29d)$$

where primes denote the correlation functions for the discrete data,
i refers to the beam cross section and

$$T_n = L\Delta t ,$$

$$t = v\Delta t , \quad v = 1, 2, 3, \dots, L$$

and

$$\tau = \mu\Delta t , \quad |\mu| = 0, 1, 2, \dots, M . \quad (3.30)$$

Here M is equal to $T_m/\Delta t$ and L denotes the number of unit-time intervals contained in the total time-interval, T_n , over which the data is collected. Since the simulation process starts at $t = 0$ and the samples are drawn for positive values of t , the summation over v in Eqs. 3.29a through 3.29d will be carried out for positive values of v only. Furthermore, since the data are limited for L unit-time intervals,

$$\tilde{V}[(\mu + v)\Delta t] = \tilde{I}[(\mu + v)\Delta t] = 0 \quad (3.31)$$

whenever $(\mu+v) > L$. Thus the upper limit of v in the summation in Eqs. 3.29a through 3.29d will be $(L-\mu)$. These equations can then be rewritten as

$$C'_V(\mu\Delta t, i) = \frac{1}{L-\mu} \sum_{v=1}^{L-\mu} \tilde{V}(v\Delta t) \tilde{V}[(\mu + v)\Delta t] , \quad (3.32a)$$

$$C_I'(\mu\Delta t, i) = \frac{1}{L-\mu} \sum_{v=1}^{L-\mu} \tilde{I}(v\Delta t) \tilde{I}[(\mu + v)\Delta t] , \quad (3.32b)$$

$$C_{V,I}'(\mu\Delta t, i) = \frac{1}{L-\mu} \sum_{v=1}^{L-\mu} \tilde{V}(v\Delta t) \tilde{I}[(\mu + v)\Delta t] \quad (3.32c)$$

and

$$C_{I,V}'(\mu\Delta t, i) = \frac{1}{L-\mu} \sum_{v=1}^{L-\mu} \tilde{I}(v\Delta t) \tilde{V}[(\mu + v)\Delta t] . \quad (3.32d)$$

For the discrete data the expression (3.27) for the lag window can be rewritten as

$$\begin{aligned} D(\mu) &= \frac{1}{2} \left(1 + \cos \frac{\pi\mu}{M} \right) , \quad \text{for } \mu = 0, 1, \dots, M-1 \\ &= 0 , \quad \text{for } \mu \geq M . \end{aligned} \quad (3.33)$$

Multiplying Eqs. 3.32a through 3.32d by Eq. 3.33, taking the Fourier transform of the product and using Eqs. 3.19, 3.25 and 3.26 for the i th beam cross section gives

$$\Phi(f, i) = \Delta t \sum_{\mu=0}^{M-1} \left(1 + \cos \frac{\pi\mu}{M} \right) C_V'(\mu\Delta t, i) \cos(2\pi f\mu\Delta t) , \quad (3.34)$$

$$\Psi(f, i) = \Delta t \sum_{\mu=0}^{M-1} \left(1 + \cos \frac{\pi\mu}{M} \right) C_I'(\mu\Delta t, i) \cos(2\pi f\mu\Delta t) , \quad (3.35)$$

$$\Pi(f,i) = \Delta t \sum_{\mu=0}^{M-1} \frac{1}{2} \left(1 + \cos \frac{\pi \mu}{M} \right) [C'_{V,I}(\mu \Delta t, i) + C'_{I,V}(\mu \Delta t, i)] \cos(2\pi f \mu \Delta t) \quad (3.36)$$

and

$$\Lambda(f,i) = \Delta t \sum_{\mu=0}^{M-1} \frac{1}{2} \left(1 + \cos \frac{\pi \mu}{M} \right) [C'_{I,V}(\mu \Delta t, i) - C'_{V,I}(\mu \Delta t, i)] \sin(2\pi f \mu \Delta t) \quad , \quad (3.37)$$

where Φ is the Self-Power Density Spectrum (SPDS) of the kinetic noise-voltage modulation, Ψ is the SPDS of the noise-current modulation, Π is the real part of the Cross-Power Density Spectrum (CPDS) between the kinetic noise-voltage and noise-current modulations, Λ is the imaginary part of the CPDS between the kinetic noise-voltage and noise-current modulations. Haus' noise parameter S is given by

$$S = (\Phi \Psi - \Lambda^2)^{1/2} \quad (3.38)$$

and F , the noise figure in db for a one-dimensional single-velocity O-type beam is expressed as

$$F(f) = 10 \log_{10} \left[1 + \frac{2\pi}{kT_0} (S - \Pi) \right] \quad , \quad (3.39)$$

where k is the Boltzmann constant and $T_0 = 300^\circ\text{K}$.

To the author's knowledge no similar noise figure expression involving noise parameters derived from the kinetic noise-voltage fluctuations, noise-current fluctuations and possibly beam position

fluctuations (neglected in this study) is yet available. However, some indication of noise figure and its variation with frequency and distance along the beam could possibly be obtained by using Eq. 3.39. To avoid confusion between the quantity calculated here and the actual noise figure, F , the noise figure in db calculated from Eq. 3.39 will be denoted by the noise parameter F_0 (indicating that F is calculated on the basis of the O-type device noise formula) instead of F , the conventional symbol for noise figure.

In the simulation process the so-called "big electrons" (see Section 2.2.3) are emitted from the cathode and are followed through the gun region or the drift space. Calculations of noise parameters made with the data collected in this way will correspond to a model where the beam is made up of "big electrons". Noise parameters for the physical process, where real electrons constitute the beam, can be obtained from the noise parameters corresponding to the "big electron" model in the following manner.

From the theory of shot noise it is known that $\overline{i^2}$ is proportional to the "grain size" of the charge emitted and the d-c current I_0 . Thus the shot noise associated with two beams each carrying the same d-c current, I_0 , but with different "constituent" charged particles, will be different. Since a "big electron" is equivalent to D real electrons it is clear that Ψ obtained in this model, consisting of "big electrons", will be greater by a factor of D over the value of Ψ for the physical process involving real electrons.

It was shown in Section 2.2.4 that the "big electrons" are emitted from the cathode with an effective temperature $T_e = DT$ where T is the actual cathode temperature. The mean-square fluctuation in the

velocity of the electrons is proportional to the temperature corresponding to a Maxwellian velocity distribution for the electron emission. In the case of "big electrons" this temperature is T_e and is thus D times the actual temperature of the cathode from which the electrons would be emitted in the physical process. Hence the value of Φ , which is related to the velocity fluctuations, will be D times larger in this model than that in the real physical process.

Hence the noise parameters Φ and Ψ corresponding to the physical process involving real electrons are obtained by dividing the "big electron" values of Φ and Ψ (computed from Eqs. 3.34 and 3.35) by D . It can be easily shown that the same is true for Λ and Π . From Eq. 3.38 it is clear that S also follows the same rule. Equation 3.39, with F replaced by F_0 , is modified as

$$F_0(f) = 10 \log_{10} \left[1 + \frac{2\pi}{kT_0} \left(\frac{S - \Pi}{D} \right) \right], \quad (3.40)$$

where S and Π are calculated from the simulation data for the "big electron" model.

In order to study the variation of the noise content of the beam over a given cross section it is necessary to redefine the noise parameters Φ , Ψ , etc., so that they pertain to the values within a rectangle for a given cross section of the beam. Since, in general, the currents entering each rectangle of the beam are unequal, Ψ_J , the SPDS of the noise current-density fluctuations, should be considered as a basis of comparison between different rectangles. Equations 3.10 and 3.14, respectively, define the new a-c kinetic potential, $\tilde{V}(t)$, and a-c current-density fluctuation $\tilde{J}_T(t)$. C_V' and Φ are calculated as

before from Eqs. 3.32a and 3.34. When $\tilde{I}$ is replaced by $\tilde{J}_T$ in Eqs. 3.32b and 3.35, C_j' and Ψ_j are obtained. In the present analysis variations of Φ and Ψ are studied for a few selected cross sections of the beam in each case.

The data obtained in the previous chapter from the Monte Carlo simulation of space-charge flow in the gun and drift regions will be used in Chapters IV and V to calculate the various noise parameters on the basis of formulas developed in this chapter.

CHAPTER IV. NOISE CALCULATIONS FOR KINO-TYPE GUNS AND THE PARALLEL PLANE CROSSED-FIELD DIODE GUN

4.1 Introduction

The results of calculations pertaining to noise transport in the abbreviated and modified Kino guns and the parallel plane crossed-field diode gun are presented and discussed in this chapter. The calculations are made under both short-circuit and open-circuit assumptions for each case. It was mentioned in Chapter II that the geometries of the abbreviated and modified Kino guns differ only in the details of the focus electrode arrangement. This results in different beam currents and space-charge flow conditions. Various noise parameters for these two guns are discussed on an individual and comparative basis to determine if one gun is superior. Wadhwa and Rowe¹¹ have performed a Monte Carlo analysis of the noise transport in a crossed-field diode under a short-circuit assumption. A similar analysis is undertaken here for an identical diode under both short- and open-circuit assumptions to assess the effect of the open-circuit assumption on the various noise parameters and to serve as a basis of comparison for evaluation of the other electron gun configurations.

4.2 Variability of Spectrum Estimates

It was discussed in the last chapter how respectable estimates of the power spectra can be obtained from a finite data set consisting of samples taken at equally spaced discrete times, by proper selection of Δt and a suitable lag window, $D(\tau)$. Furthermore, Blackman and Tukey²⁸ have shown that, if a Gaussian random process is assumed, it is possible

to calculate a "confidence interval" about the true value of the spectrum such that the estimated value will fall inside this interval with a certain probability. The stability of the spectral estimates can be adequately described by an equivalent chi-square distribution with k degrees of freedom given by

$$k = 2 \left(\frac{T_n}{T_m} - \frac{1}{3} \right), \quad (4.1)$$

where T_n and T_m have been defined in Section 3.3. For an 80 percent probability the relation between k and the confidence interval is shown in Fig. 4.1. It can be seen from this figure that the confidence interval, i.e., the range in which the estimated value will fall with respect to the true value, decreases with increasing values of k . However, the rate of decrease in the confidence interval with an increase in k becomes very slow after $k = 100$. A suitable value of k is selected by properly choosing T_n and T_m . A large value of T_n is preferred since k is proportional to T_n , but cost considerations may prohibit the use of such a long T_n . Although k can be increased by lowering T_m , this cannot be done arbitrarily for several reasons. Blackman and Tukey²⁸ have shown that it is meaningless to evaluate the power spectra at frequencies differing by less than $\Delta f = 1/2T_m$ when a smoothed estimate of the power spectrum is obtained by using a suitable lag window. A small value of Δf is preferable and it can be obtained by raising the value of T_m . Thus the demands on the value of T_m for obtaining a large value of k and a small value of Δf are contradictory. Hence a suitable compromise is made in selecting T_m .

For the noise analyses of the guns described in this chapter, $\Delta t = 10^{-11}$ sec, $T_n = 2000 \Delta t$ and $\Delta f = 0.5$ GHz. This gives $T_m = 10^{-9}$ sec

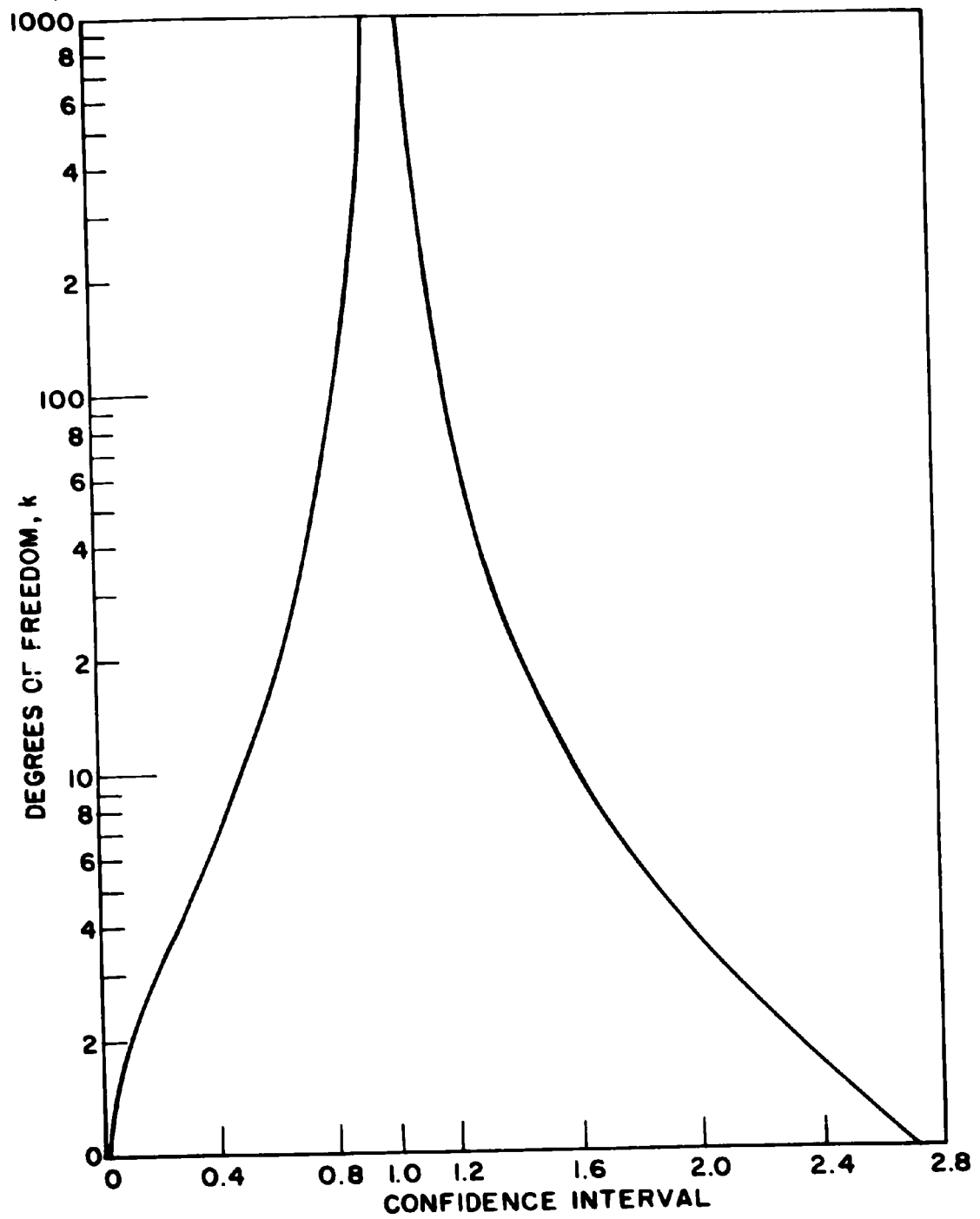


FIG. 4.1 DEGREES OF FREEDOM, k , VS. CONFIDENCE LIMITS
FOR CHI-SQUARE DISTRIBUTION (80 PERCENT
PROBABILITY).

and from Eq. 4.1 $k \approx 40$. If the true value of the power spectrum is x , then for $k \approx 40$, 80 percent of the estimates would fall between $0.72 x$ and $1.3 x$ while 10 percent would be below $0.72 x$ and the remaining 10 percent would be above $1.3 x$. It is believed that this should not mask any significant variations in the noise parameters.

4.3 Noise Parameters

The noise parameters describing the noise transport in the various guns under analysis are calculated in mks rationalized units from the formulas developed in Chapter III. A check run was made in each case to test the small-signal assumption (Eq. 3.7) made in deriving the a-c kinetic potential expression. This condition was completely satisfied everywhere in the beam except for a small region very close to the cathode. In the cathode region this condition is barely met or violated only during a few unit-time intervals. Although, for this region, the noise parameters, e.g., Φ , Θ , Π , etc., related to the a-c kinetic potential fluctuation are calculated from the a-c kinetic potential formula including second-order terms, the difference between these noise parameters with and without the second-order terms is very small except around $f = 0$ as shown for Φ in Fig. 4.2. One possible explanation is that since the sum of the second-order terms, $v_N^2 + v_T^2$, is always positive, the a-c kinetic potential defined with these terms included should be greater than when they are omitted. It follows then that C'_v defined by Eq. 3.32a should be similarly higher in the former case. It should be noted that in Eq. 3.34, $\cos(2\pi f \mu \Delta t)$ acts like a weighting factor which is identically equal to one (the maximum value for this factor) at $f = 0$ for all values of μ . Hence the difference

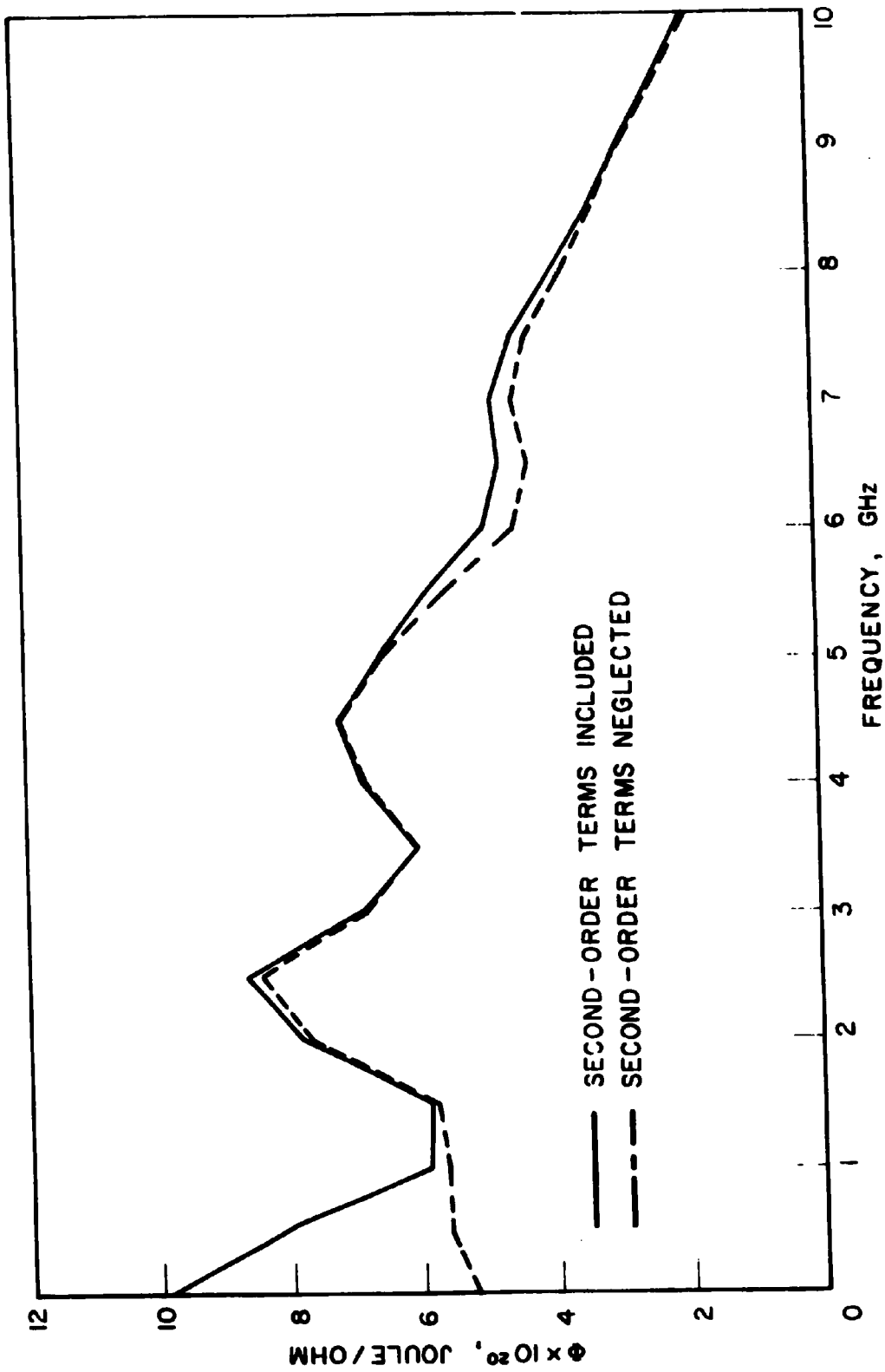


FIG. 4.2 ϕ VS. FREQUENCY, SHOWING THE EFFECT OF NEGLECTING SECOND-ORDER TERMS IN THE A-C KINETIC POTENTIAL FORMULA.

between the two values of Φ , corresponding to different values of C_v' , is highly accentuated around $f = 0$.

In order to study the variation of noise parameters along the beam two sets of planes are chosen and the noise parameters are calculated for each of these planes by averaging the kinetic noise-voltage and noise-current fluctuations over the particular plane. The first set consists of planes parallel to the cathode and occupies the portion of the beam before it bends appreciably. The location of each of these planes is expressed in terms of its y-distance from the cathode plane or in a normalized form $y/s_{\max}$ where $s_{\max}$ is the length of the central trajectory between the cathode and the gun exit. The second set of planes is normal to the beam and is located in the remaining part of the beam. The location of these planes can be expressed in the normalized form $s/s_{\max}$, where s is the distance measured along the central trajectory. The normalized distances for these planes generally extend over a few orders of magnitude; hence they are plotted on a logarithmic scale. The discontinuity in the curves results from the change from the parallel planes to the normal planes.

In order to discuss the variation of a particular noise parameter, e.g., Φ or Ψ , across the beam, the parameter is calculated for each rectangle in the given cross section and then plotted against the frequency. The parameter value calculated on the cross-sectional basis, i.e., using the weighted average of the kinetic-voltage and current-density fluctuations over the given cross section for each Δt , is plotted along with curves for the individual rectangles in that cross section. Henceforth short- and open-circuit conditions will be referred to as "SC" and "OC" respectively. For the sake of simplicity

the abbreviated Kino gun and the modified Kino gun will sometimes be referred to as KG-A and KG-M respectively.

4.4 Monte Carlo Analysis of Noise Transport in the Abbreviated Kino Gun

The geometry and the d-c characteristics of KG-A were described in Sections 2.3.1 and 2.4. Figure 4.3 shows the approximate beam profile and the positions of various beam cross sections (noise planes) used in the noise analysis. The rectangles falling in these cross sections are represented by the "solution point" they enclose. The boundaries of these rectangles are not shown here but can be seen in Fig. 2.1. Table 4.1 indicates the location of the noise planes. The ordinal number of each noise plane, its y- or s-distance and the normalized distance ($y/s_{\max}$ or $s/s_{\max}$) are given in Columns 1 through 3. It should be noted that according to Kino theory the transit angle is the same for all electrons located at a plane parallel to the cathode. For the second set of planes normal to the beam the transit angle denoting one of these planes is assumed to be that for an electron moving along the central trajectory. The transit angle ($\theta = \omega_c t$) corresponding to each of the noise planes is given in the fourth column. The last column identifies the planes as "Parallel Plane", meaning the noise plane parallel to the cathode, or "Normal Plane", meaning the noise plane normal to the direction of the beam. The various noise parameters describing the noise transport in the gun under the SC assumption are discussed in Sections 4.4.1 through 4.4.4.

4.4.1 Self-Power Density Spectrum (SPDS) of Kinetic Noise-Voltage Modulation, Φ (SC Case).

a. Variation of Φ with Frequency. Figures 4.4 and 4.5 show the variation of Φ with frequency for parallel planes 1 through 7. Φ assumes

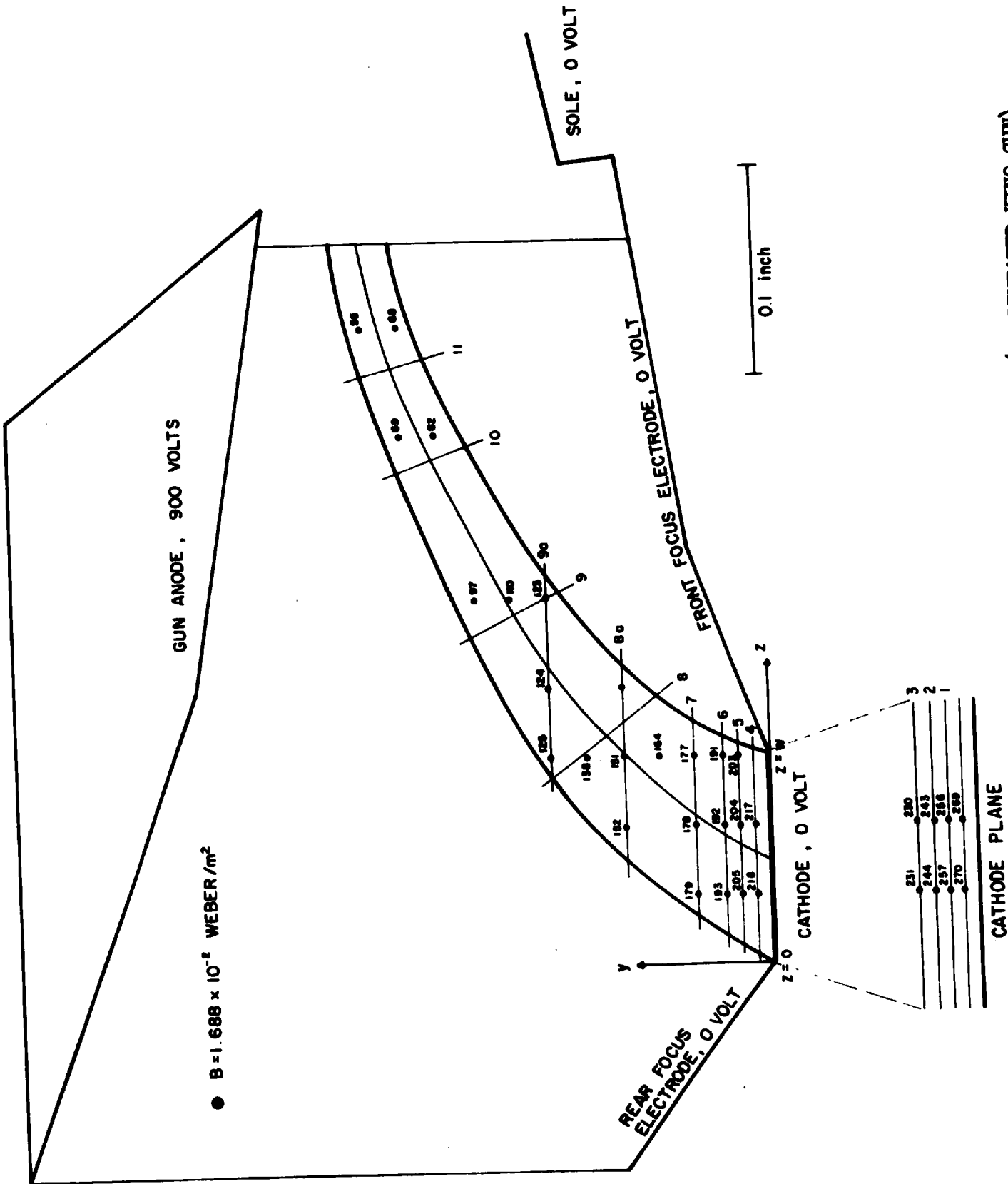


FIG. 4.3 APPROXIMATE BEAM PROFILE AND NOISE PLANE POSITIONS (ABBREVIATED KINO GUN).

Table 4.1
Noise Plane Positions for the Abbreviated Kino Gun

Noise Plane Number	Distance y or s (Inch)	Normalized Distance $y/s_{\max}$ or $s/s_{\max}$ ($s_{\max} = 0.38$ inch)	$\theta = \omega_c t$ (Degrees)	Remarks
1	y = 1.666×10^{-4}	$y/s_{\max} = 4.386 \times 10^{-4}$	18	Parallel Plane
2	= 3.333×10^{-4}	= 8.772×10^{-4}	22	Parallel Plane
3	= 2.000×10^{-3}	= 5.263×10^{-3}	41	Parallel Plane
4	= 6.666×10^{-3}	= 1.754×10^{-2}	62	Parallel Plane
5	= 1.333×10^{-2}	= 3.508×10^{-2}	79	Parallel Plane
6	= 2.000×10^{-2}	= 5.263×10^{-2}	92	Parallel Plane
7	= 3.333×10^{-2}	= 8.770×10^{-2}	111	Parallel Plane
8a	= 6.666×10^{-2}	= 0.1923	146	Parallel Plane
9a	= 0.1000	= 0.2631	176	Parallel Plane
8	s = 0.0960	$s/s_{\max} = 0.2526$	152	Normal Plane
9	= 0.1666	= 0.4386	189	Normal Plane
10	= 0.2516	= 0.6623	222	Normal Plane
11	= 0.2967	= 0.7807	241	Normal Plane

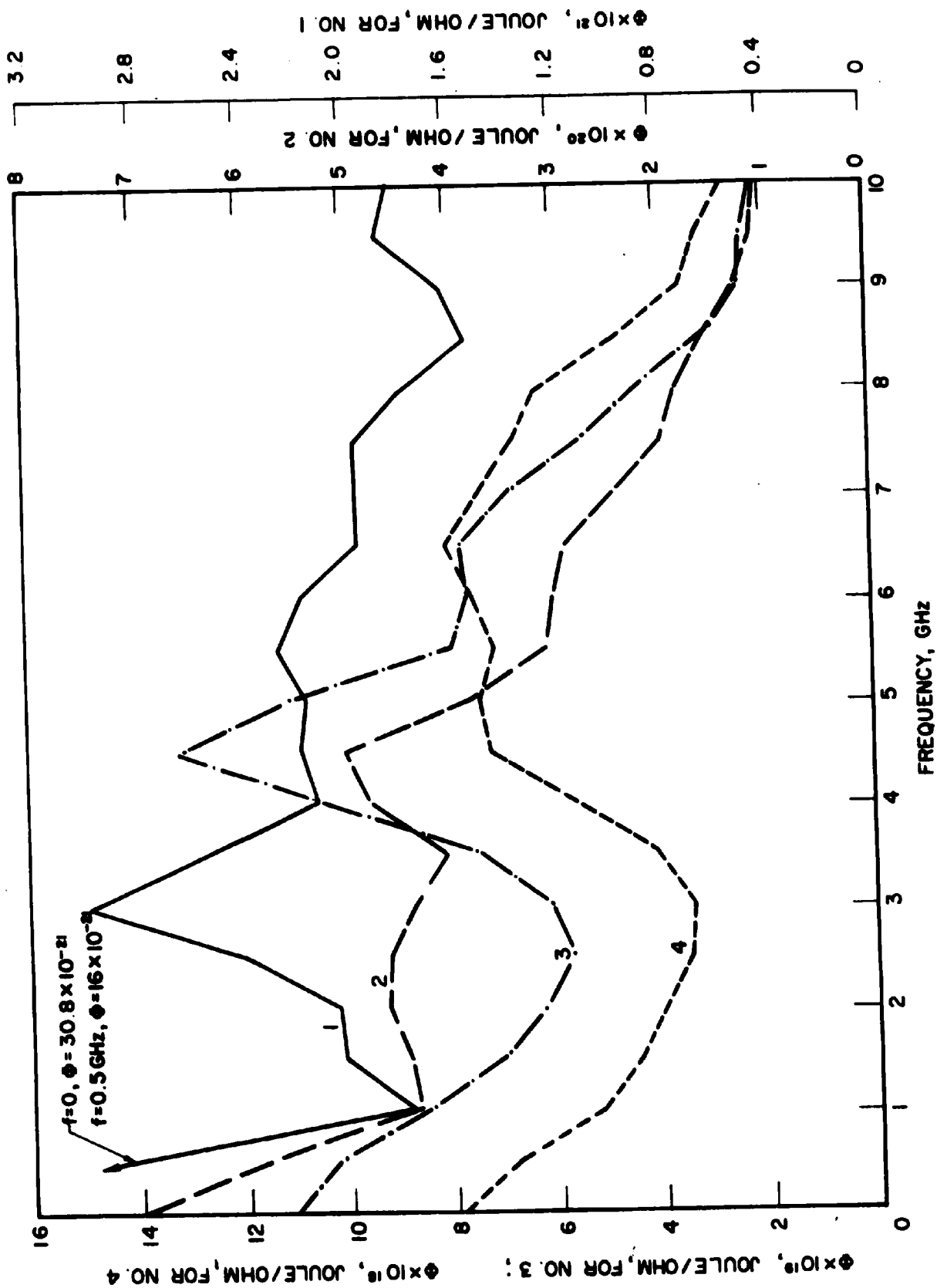


FIG. 4.4 ϕ VS. FREQUENCY FOR NOISE PLANES 1 THROUGH 4 (ABBREVIATED KINO GUN).

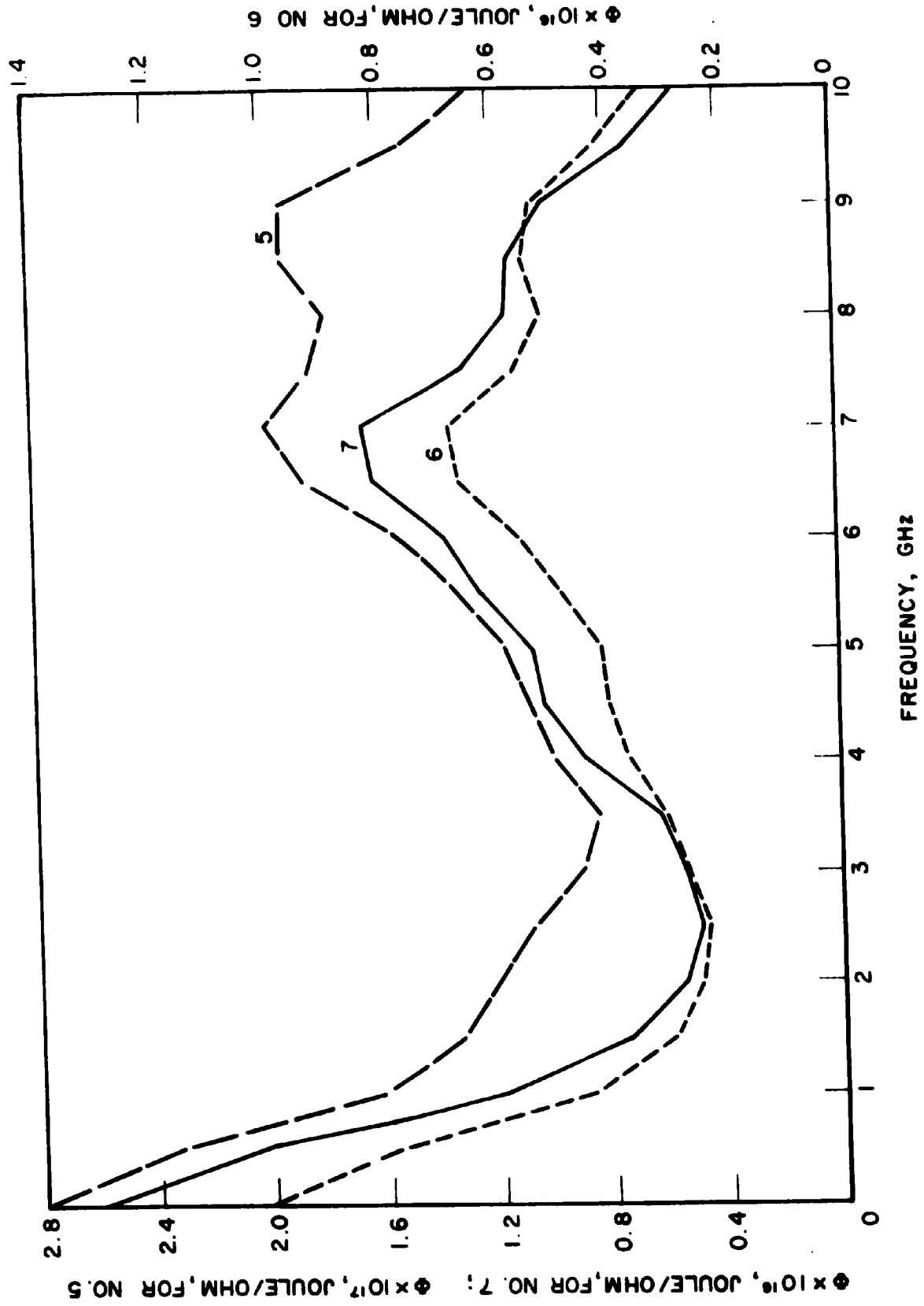


FIG. 4.5 ϕ VS. FREQUENCY FOR NOISE PLANES 5 THROUGH 7 (ABBREVIATED KINO GUN).

high values around $f = 0$ which agree well with the results of the density function analysis for the propagation of perturbations in a crossed-field diode as calculated by Wadhwa³². Except for planes 1 and 2, which are inside the potential minimum, all planes have relatively broad minimum values of Φ around 2.5 GHz which is close to the potential minimum plasma frequency, $f_{pm} (\approx 1.8 \text{ GHz})$, or the potential minimum hybrid frequency, f_{hm} , ($f_h = \sqrt{f_p^2 + f_c^2}$ where f_p and f_c are the plasma and the cyclotron frequencies respectively). It should be noted that f_p and f_h in this region are close to each other since $f_c = 0.47 \text{ GHz}$ is small compared to f_{pm} . Maximum values of Φ are reached in the mid-frequency band for planes 2 through 4, whereas planes 5 through 7 have peaks around 7 GHz. After passing through this maximum, Φ tends to decline toward the high frequency end. The reasons for the occurrence of a maximum in the mid-frequency region or around 7 GHz are not understood.

Figure 4.6 shows the variation of Φ for normal planes 8 through 11. The nature of these curves is quite different from the above set. Here Φ does not assume high values around $f = 0$ but has a peak around 1 GHz in all but one case. Φ decreases rapidly after passing through this peak and settles down to a low value for the rest of the frequency range. It appears that Φ for noise planes in this region is not related to the potential minimum hybrid or plasma frequency but is rather related to the local hybrid frequency, f_h (local plasma frequency lying between 0.3 GHz to 0.37 GHz). Later in Section 5.2.1 it will be seen that in the drift region following this gun Φ has peaks around 0.5 GHz or 1.0 GHz, close to the local hybrid frequency at the noise

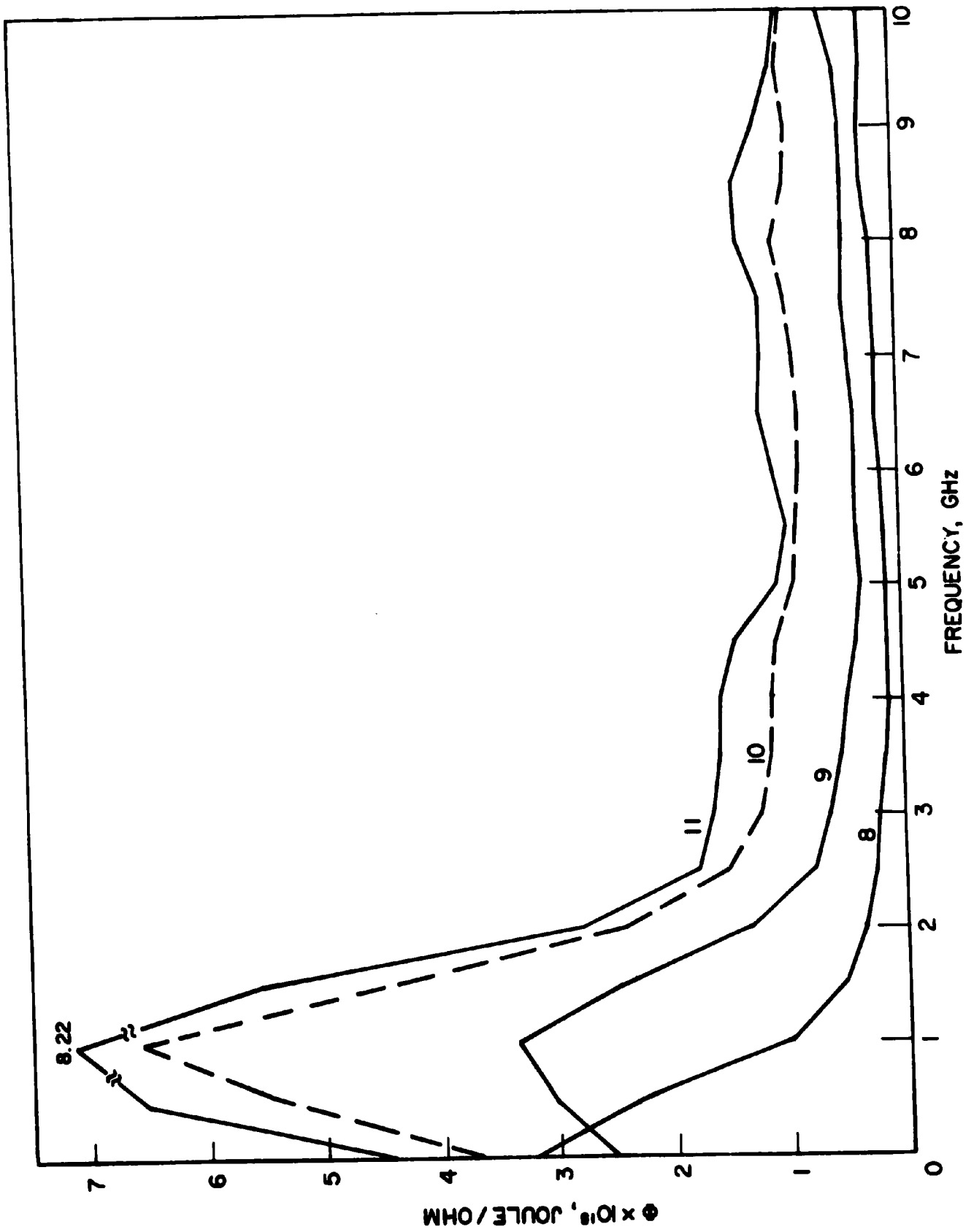


FIG. 4.6 ϕ VS. FREQUENCY FOR NOISE PLANES 8 THROUGH 11 (ABBREVIATED KINO GUN).

planes. Thus it appears that for beam cross sections not close to the cathode, Φ is affected by the local hybrid frequency.

b. Variation of Φ with Distance Along the Beam. Figure 4.7 shows the variation of Φ as a function of the normalized distance on a log-log scale for several selected frequencies. The identification number and the transit angle in degrees for each noise plane are given along with its normalized distance. For the mid-frequency range the variation of Φ with normalized distance in the "Parallel Plane" region may be approximated by two straight lines,

$$\Phi = K_1 \left(\frac{y}{s_{\max}} \right)^{4.4} \quad (4.2)$$

and

$$\Phi = K_2 \left(\frac{y}{s_{\max}} \right)^{1.6}, \quad (4.3)$$

where K_1 and K_2 are constants. Equation 4.2 represents the region up to the second noise plane which is just inside the potential minimum. The region lying beyond the second and up to the seventh noise plane is represented by Eq. 4.3. It can be seen from Eqs. 4.2 and 4.3 that Φ is an increasing function of position along the beam. This fact is borne out in Fig. 4.7 where Φ increases monotonically toward the gun exit and Φ at the eleventh noise plane (the last noise plane) is approximately six orders higher than that at the first noise plane. The rate of increase of Φ with distance along the beam is also found to increase between the first and the last noise plane.

c. Variation of Φ Across the Beam. The values of Φ corresponding to a typical normal plane (plane 9) and its constituent rectangles 97, 110 and 123 are plotted as a function of frequency in Fig. 4.8. It is

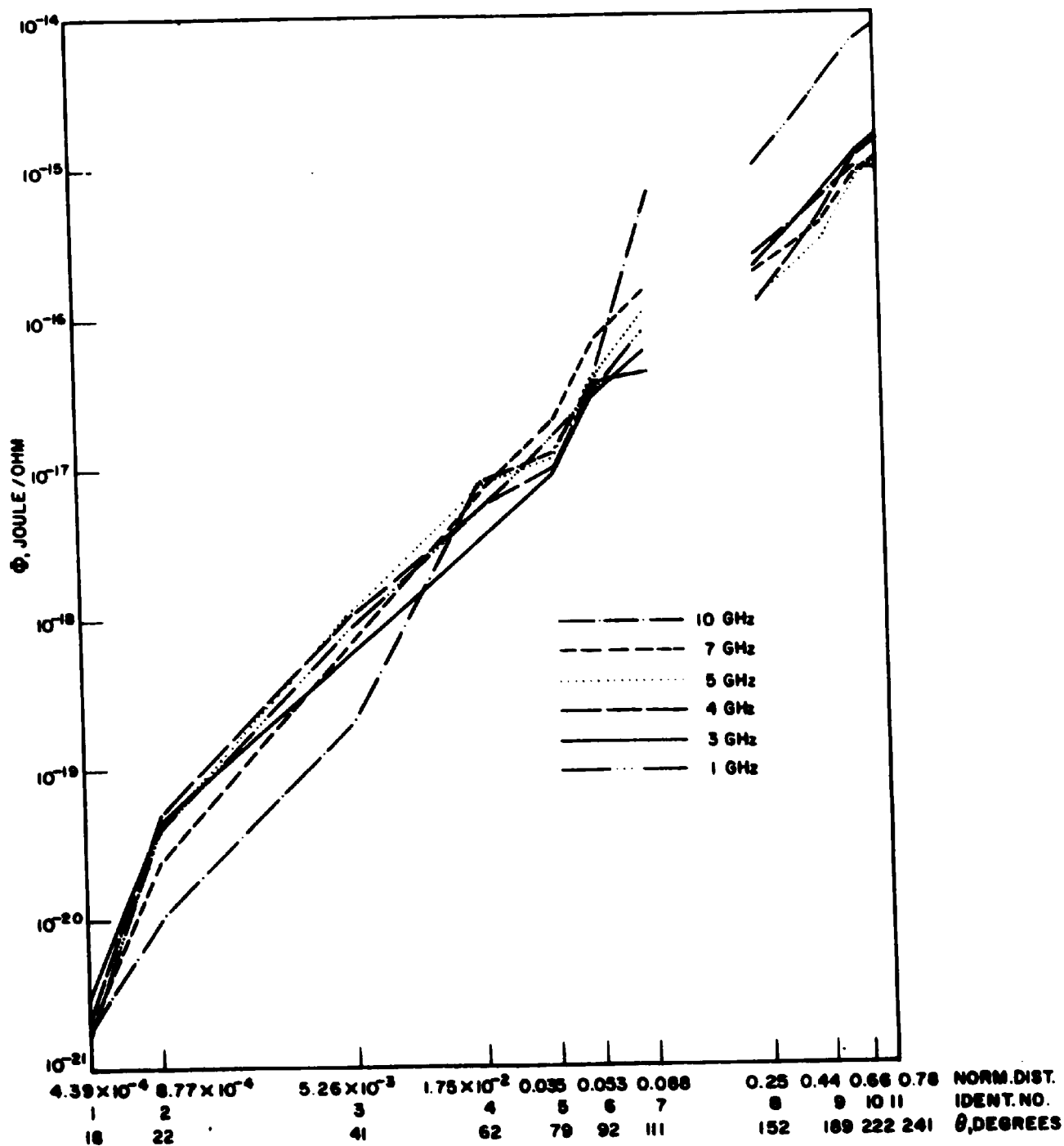


FIG. 4.7 Φ VS. NORMALIZED DISTANCE (ABBREVIATED KINO GUN).

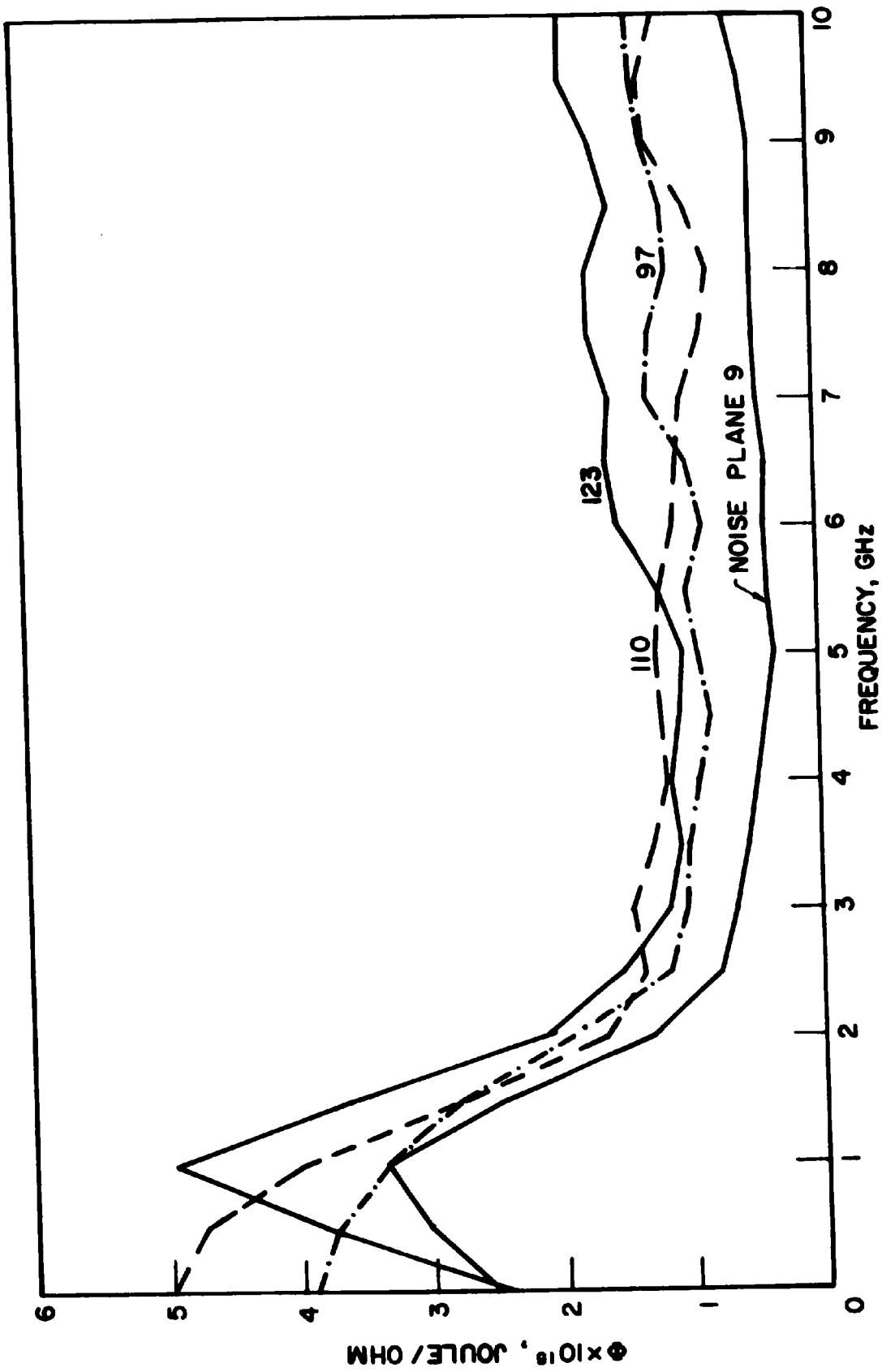


FIG. 4.8 ϕ VS. FREQUENCY FOR RECTANGLES 97, 110, 123 AND NOISE PLANE 9 (ABBREVIATED KINO GUN).

found that except around $f = 0$ there is no significant variation of Φ across the beam. Since the d-c velocity of electrons in the lower edge of the beam is smaller than that at the upper edge, it is clear from the definition of kinetic voltage, $\tilde{V}(t)$ given by Eq. 3.2, that the velocity fluctuations at the outer edge of the beam are smaller than those at the inner edge of the beam. For a parallel plane, Φ is generally found to be lowest for a rectangle at the outer edge of the beam. Here the d-c velocity of electrons is approximately the same in all rectangles belonging to the given parallel plane. Hence the velocity fluctuations should have the same variation as Φ over a given parallel plane. Thus it can be concluded that the velocity fluctuations are smallest at the outer beam edge.

In the above figure, Φ , calculated on the cross-sectional basis, is also plotted along with the curves for the constituent rectangles. It is found in this case as well as for other cross sections that Φ , considering the cross section as a whole, is generally lower than that for the constituent rectangles. An explanation for this observation is given later in Section 4.4.2c.

4.4.2 Self-Power Density Spectrum (SPDS) of the Noise-Current Modulation, Ψ (SC Case).

a. Variation of Ψ with Frequency. In Fig. 4.9 it can be seen that Ψ for every plane (Nos. 1 through 4) starts at a low value. For plane 1, Ψ reaches its maximum value around 3 GHz and does not vary significantly after this point. For other planes (Nos. 2, 3 and 4) Ψ has one or two peaks between 2.5 GHz and 4 GHz and it again assumes low values at the high frequency end. The peak around 2.5 GHz is close to the potential minimum plasma or hybrid frequency. The other small peak around 4 GHz

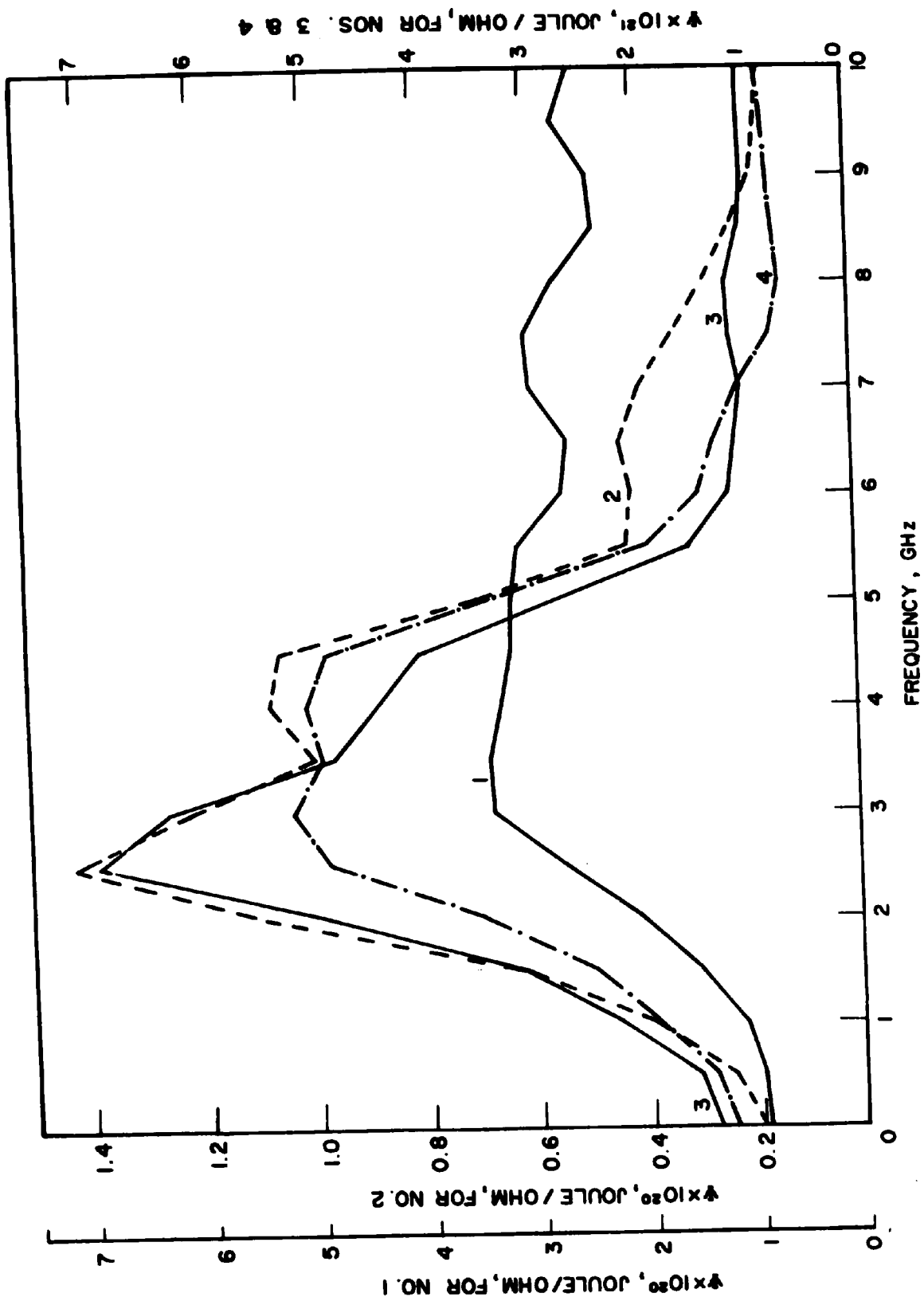


FIG. 4.9 ν VS. FREQUENCY FOR NOISE PLANES 1 THROUGH 4 (ABBREVIATED KINO GUN).

is approximately at $2f_{pm}$ or $2f_{hm}$. For a one-dimensional space-charge-limited diode, Siegman and Watkins³³ have shown that the reduction factor R^2 can have values of $R^2 \gg 1$ around f_{pm} under certain conditions. At least to the author's knowledge no such analysis exists for a crossed-field configuration. However, it is plausible that the hybrid frequency, f_{hm} , could possibly play the role of f_{pm} in such an analysis which could explain the above peaks of Ψ around f_{hm} . Ψ for the next set of planes (Nos. 5, 6 and 7) shown in Fig. 4.10 has a peak around 1 GHz in addition to one at 4 GHz. Except for plane 8 each normal plane (Nos. 9, 10 and 11) has the highest value of Ψ around $f = 0$. Ψ decreases rapidly until 2 GHz after which it does not change significantly. The variation of Ψ with frequency for these planes is shown in Fig. 4.11.

b. Variation of Ψ with Distance Along the Beam. Figure 4.12 shows the variation of Ψ with normalized distance (on a logarithmic scale) along the beam for several frequencies. It may be noted that between the first and the seventh noise planes Ψ decreases monotonically for a few frequencies, whereas for others Ψ has a minimum roughly between the third and the fourth plane and has a peak around the fifth plane. The rate of change of Ψ (negative or positive) is maximum between the first and second noise planes and then it decreases toward the gun exit such that between the last two noise planes it is about four orders below its maximum value. From the above description it is interesting to note that Ψ continues to decrease for some distance even beyond the potential minimum and that the rate of change of Ψ decreases toward the gun end. These facts cannot be explained completely by the commonly known "smoothing action" of the potential minimum on current

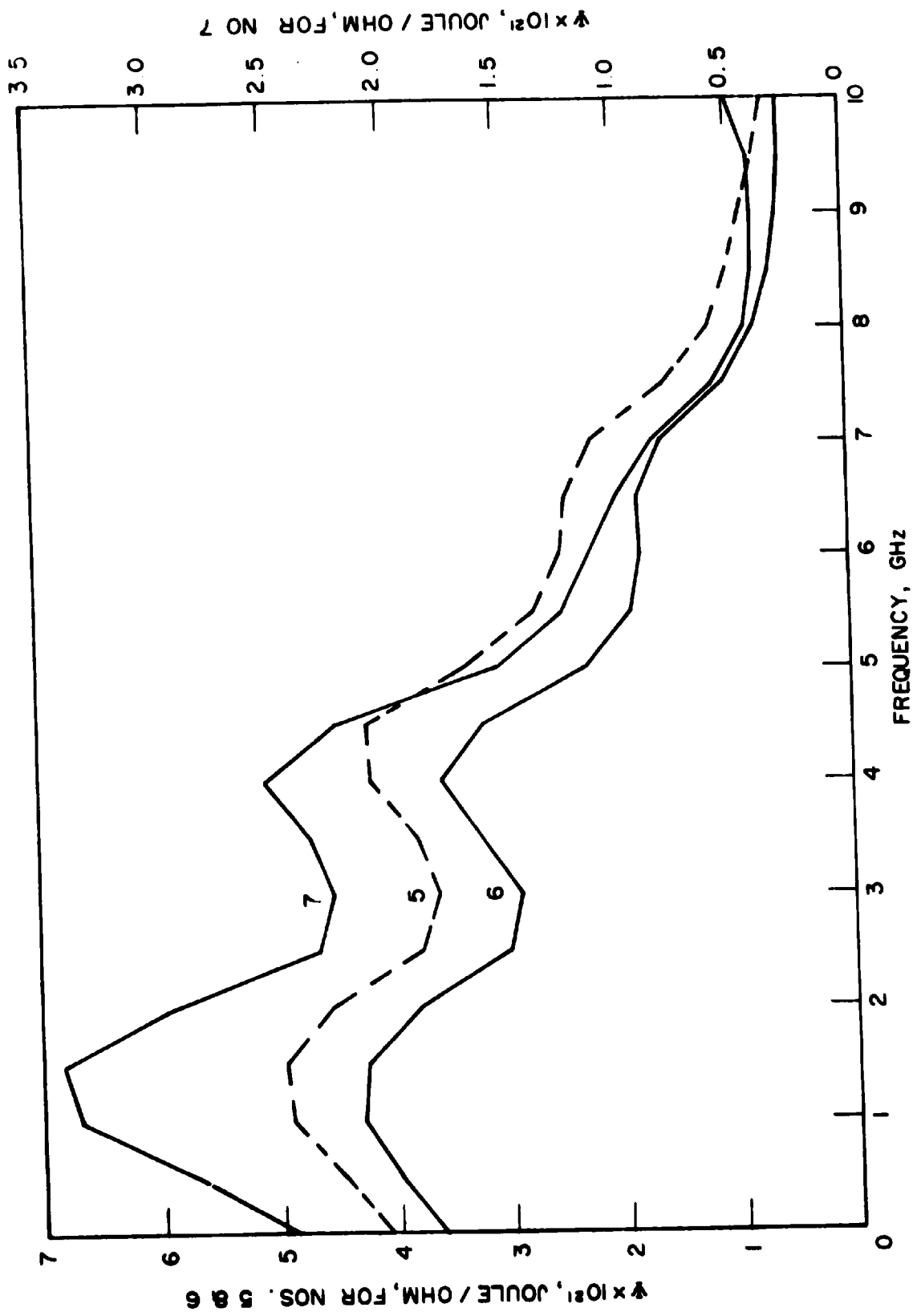


FIG. 4.10 Ψ VS. FREQUENCY FOR NOISE PLANES 5 THROUGH 7 (ABBREVIATED KINO GUN).

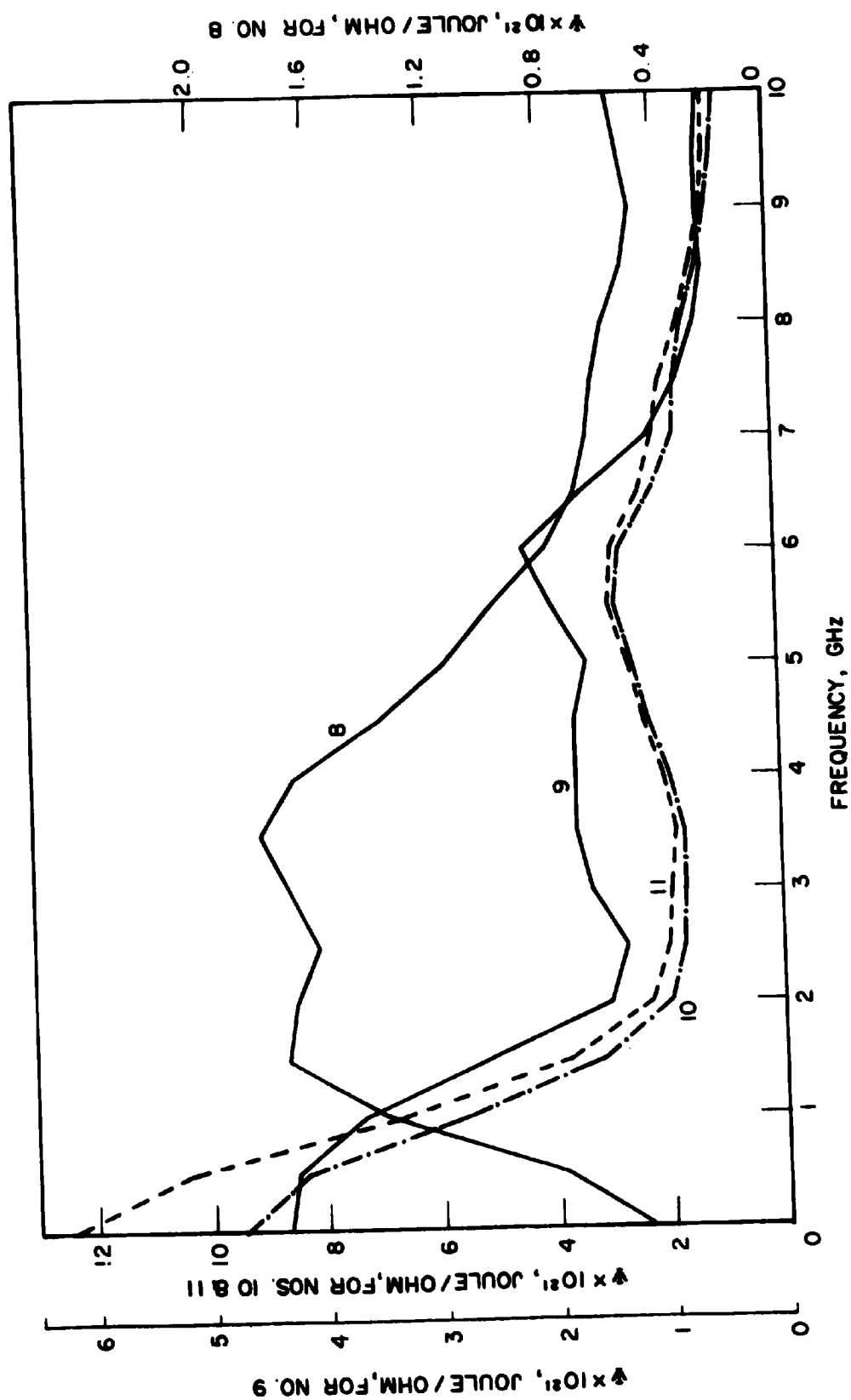


FIG. 4.11 Ψ VS. FREQUENCY FOR NOISE PLANES 8 THROUGH 11 (ABBREVIATED KINO GUN).

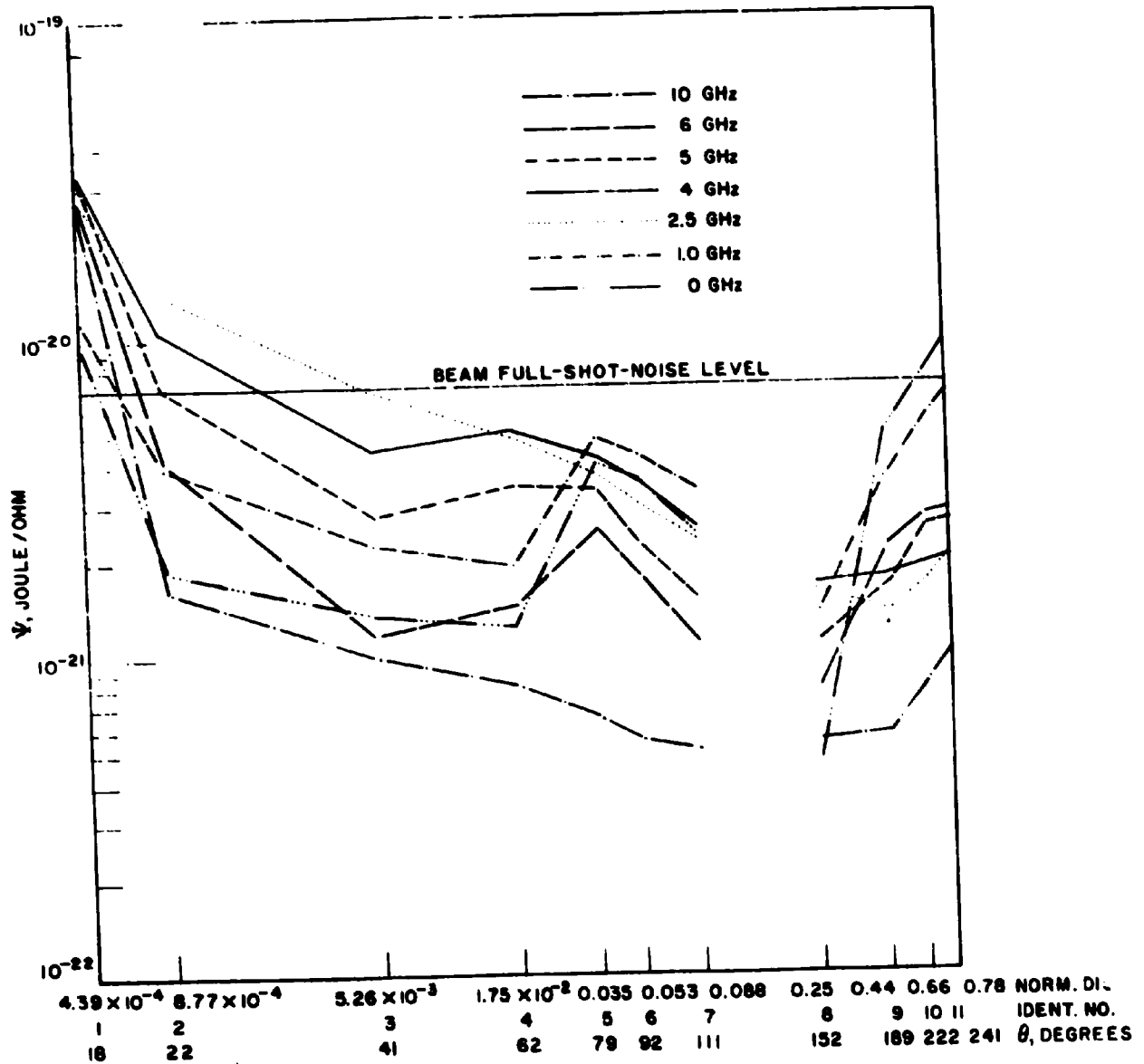


FIG. 4.12 Ψ VS. NORMALIZED DISTANCE (ABBREVIATED KINO GUN).

fluctuations. Another possible qualitative explanation may be found in the variation of space-charge density in the beam with transit angle (distance along the beam). The theoretical variation of ρ/ρ_0 (ρ_0 being the Brillouin density given by $\rho_0 = -\epsilon_0 \omega_c^2/\eta$) with transit angle for a Kino short gun beam is shown in Fig. 4.13. The rate of decrease in Ψ with transit angle, θ , is found to follow the theoretical variation of ρ/ρ_0 with θ , at least qualitatively. In Fig. 4.13 the cross-over point (where ρ changes from greater to less than the Brillouin value ρ_0) occurs at $\theta = \pi/2$. The space-charge-density variation obtained from the simulation run (Section 2.6) shows that this cross-over point lies in the neighborhood of the fifth noise plane where a peak in Ψ is observed for several frequencies. Beyond this point the rate of variation of Ψ follows the ρ/ρ_0 variation. Following the above observation, again a peak in Ψ might be expected around $\theta = 3\pi/2$ (another ρ/ρ_0 cross-over point) and Ψ should probably increase again as θ approaches 2π where ρ tends to infinity in the Kino theory. Since the last noise plane is at $\theta = 241$ degrees there is no possibility of checking the above tentative prediction in this case. It would be interesting therefore to study the variation of Ψ in a Kino short gun, where the gun exit is around $\theta \approx 2\pi$, to check the above hypothesis. Some analytical work is necessary in order to derive an expression for the reduction factor R^2 . This expression should probably have $(\rho - \rho_0)$ in the denominator so that when $\rho \rightarrow \rho_0$, R^2 goes to infinity.

From the above discussion it can be said that the contribution to beam noise from current fluctuations in crossed-field beams attains a relative maximum when the beam space-charge density attains the

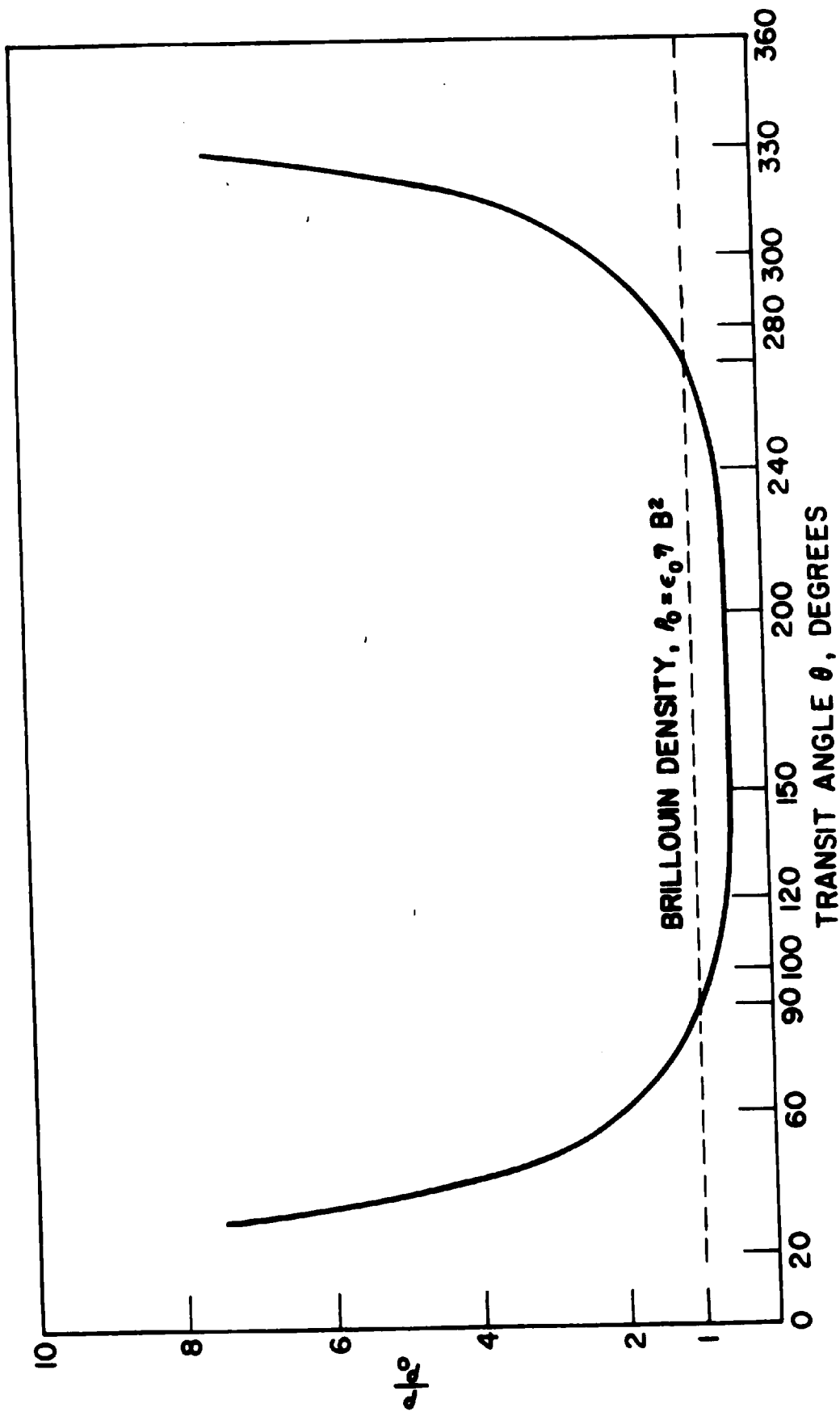


FIG. 4.13 ρ/ρ_0 VS. TRANSIT ANGLE θ FOR A KINO SHORT GUN (ρ_0 = BRILLOUIN DENSITY).

Brillouin value. This may also serve as one useful criterion for determining the optimum termination plane for a KG-A.

In this gun the first trajectory intersection occurs just before noise plane 9. The intersecting trajectories for this gun were shown earlier in Fig. 2.4. This might be another cause for the increasing Ψ in the set of normal planes (Nos. 8 through 11). It will be seen later in Section 4.5.2b that the variation of Ψ for the modified gun, where the trajectories do not intersect, has the same trend in a similar region of the gun. Thus the trajectory crossing does not appear to be the principal cause for the tendency of increasing Ψ in this region.

Finally it should be noted that space-charge smoothing of the current fluctuations exists in this gun since, after the third noise plane, Ψ for all frequencies is below the value of Ψ for the beam with full shot noise.

c. Variation of Ψ_J Across the Beam. In the region of the beam before it has appreciably converged, Ψ_J for both parallel and normal planes has the least value at the outer beam edge in the low frequency range up to 2.5 GHz. In this frequency range Ψ_J for the central part and the inner edge of the beam is not significantly different. After 2.5 GHz Ψ_J varies little over the beam cross section. A typical case of Ψ_J variation described above is shown in Fig. 4.14. Here rectangles 123 and 97 are, respectively, at the inner and outer beam edges. Ψ_J calculated on a cross-sectional basis is also shown in this figure. As in the case of Φ described in Section 4.4.1c, this is generally lower than that for the individual rectangles over most of the frequency range.

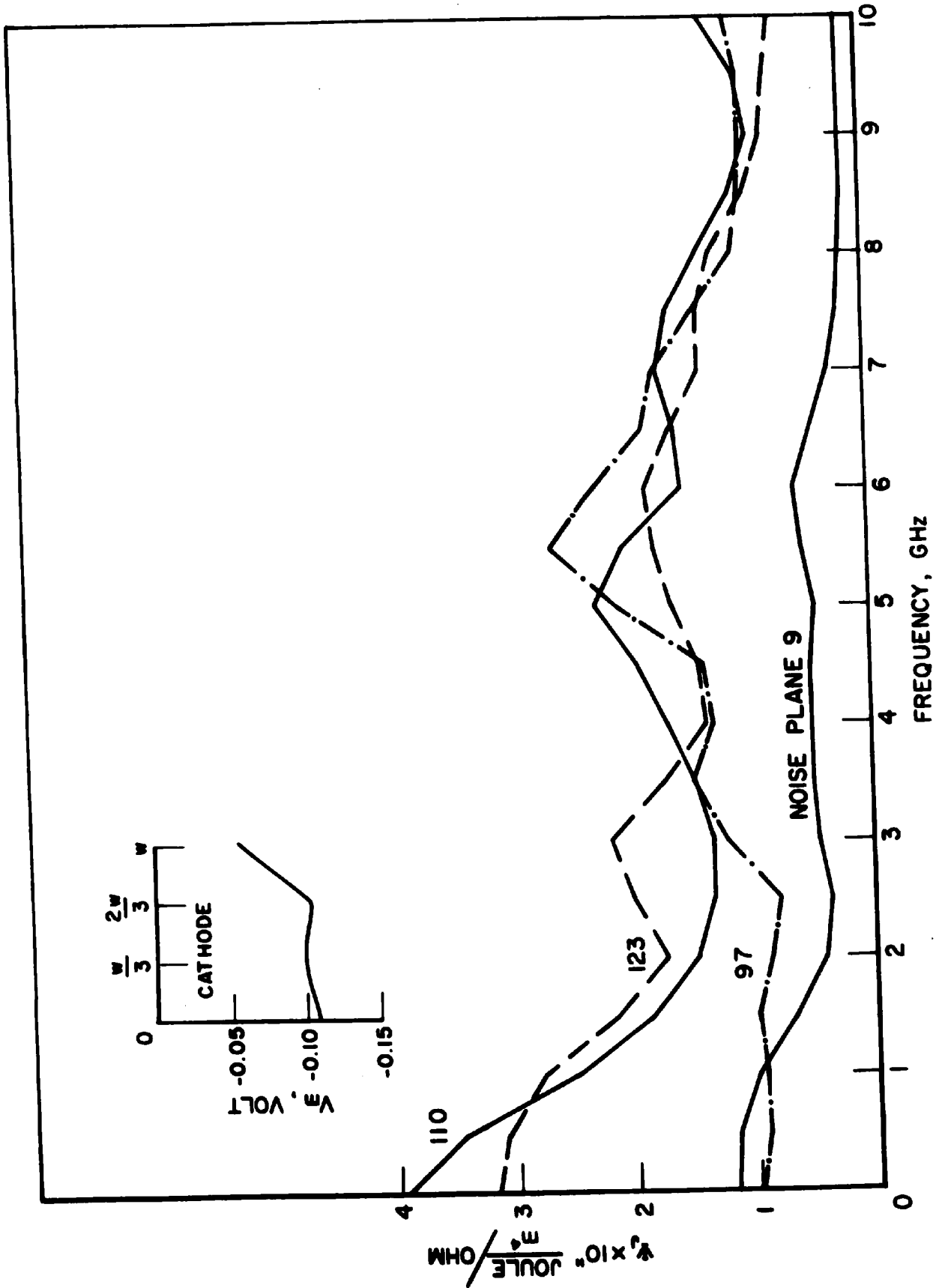


FIG. 4.14 Ψ_J VS. FREQUENCY FOR RECTANGLES 97, 110, 123 AND NOISE PLANE 9 (ABBREVIATED KINO GUN).

This is expected and can be explained on the basis of statistical theory in the following manner. Φ and Ψ_J are essentially variance parameters. These parameters defined on the cross-sectional basis are then variances for quantities (kinetic noise-voltage and noise current-density fluctuations) averaged over several beam layers (rectangles) in that beam cross section. It is well known that the variance of the average of several random variables is generally lower (unless the correlation coefficient between pairs of these variables is +1) than that for individual random variables. Thus Φ and Ψ_J defined on the cross-sectional basis should be generally lower than these parameters defined for the individual beam layers (rectangles) in that cross section.

Since the maximum frequency associated with the Ψ_J variation across the beam cross sections happens to be in the neighborhood of the hybrid frequency at the potential minimum, f_{hm} , it seems reasonable to conclude that the potential minimum strongly affects Ψ_J in this frequency range. Furthermore, some correlation between the Ψ_J variation over the beam cross section and the potential minimum profile over the length of the cathode seems to exist. The insert in Fig. 4.14 shows that the mean value of $|V_m|$ over the left one third of the cathode is highest, whereas it is lowest at the other end. Assuming that the smoothing action of the potential minimum is, to some extent, proportional to its magnitude, it is expected that Ψ_J should be minimum at the outer edge and maximum at the inner edge of the beam for a given cross section. This is borne out in Fig. 4.14 but the difference between Ψ_J for the central and inner layers is much smaller than expected.

4.4.3 Noise Parameters Π , S , and $S-\Pi$ (SC Case). The noise parameter Π represents the real kinetic power in the beam and also the correlation between the noise parameters Φ and Ψ . The problem of determining the number and nature of the waves propagating along the converging beam in a Kino gun is not understood at the present time. However, some guidance in the interpretation of the noise parameters Π , $S-\Pi$, etc., can be obtained from the present knowledge of wave propagation on a thin crossed-field beam drifting between two parallel conducting planes. Four waves are known to exist in this case, two appearing as a pair of fast and slow cyclotron waves and two as a pair of growing and declining space-charge waves. Rowe and Wadhwa²² have shown that the cyclotron modes are strongly excited when the beam is velocity modulated. The slow and fast cyclotron modes have negative and positive kinetic power respectively. The growing and declining diocotron waves cannot carry real kinetic power individually (otherwise the conservation of kinetic power would be violated) but they can do so together. Rowe and Wadhwa²² have also shown the existence of a pair of slow and fast hybrid waves under certain conditions. It should be noted that when the exponentially declining wave dies out, Π depends on the slow and fast modes only. The variation of Π with frequency and distance along the beam is discussed in the following paragraph.

Π is positive and increasing with distance for planes 1 through 4 throughout the frequency band. It shows a dip at 1 GHz in three out of four curves. Such a dip is absent in the OC case. The typical variation of Π with frequency in this group is shown in curve 3 in Fig. 4.15. The growing positive value of Π in this set of noise planes indicates that the fast wave having a positive kinetic power grows and

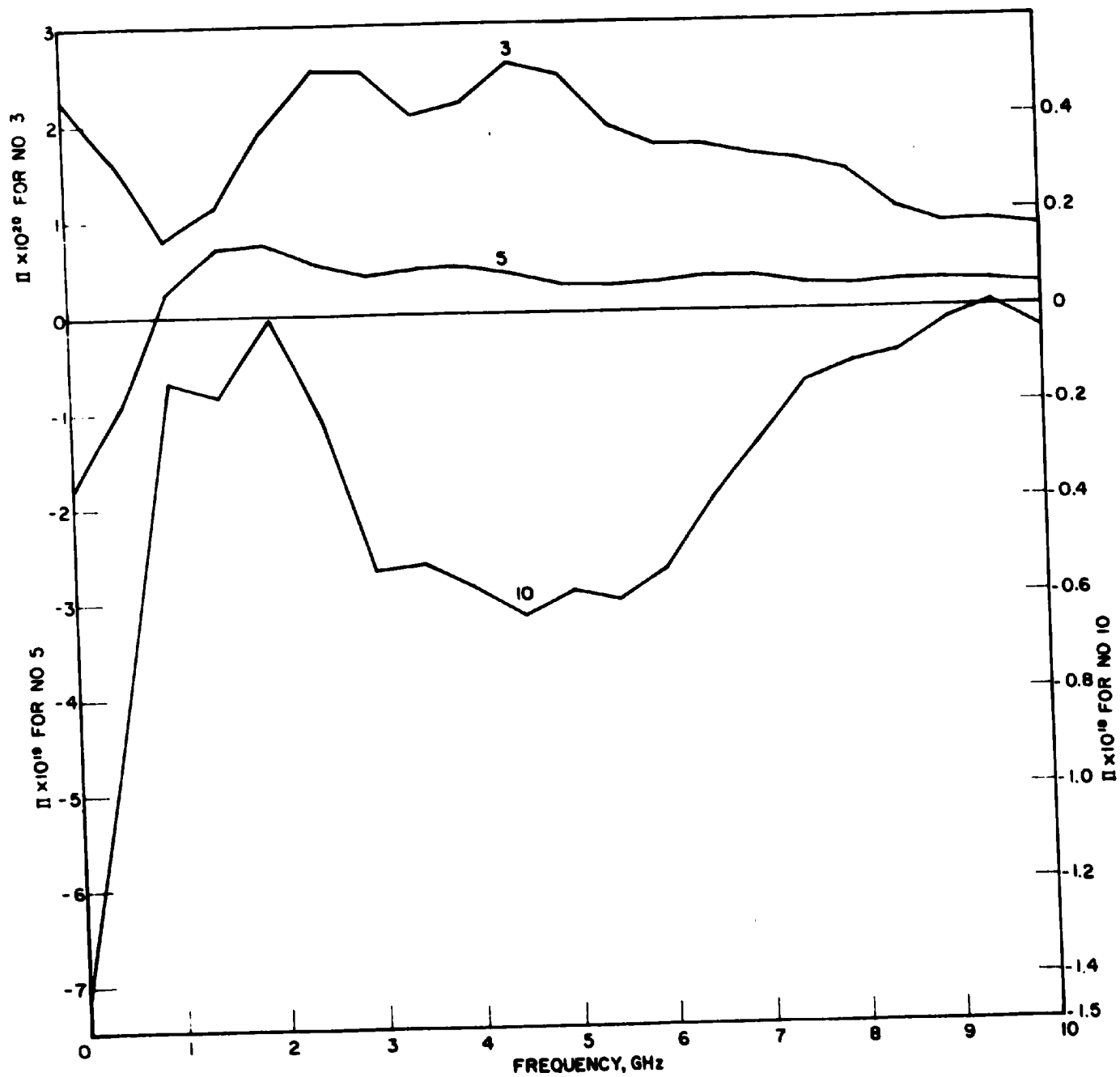


FIG. 4.15 Π VS. FREQUENCY FOR NOISE PLANES 3, 5 AND 10
(ABBREVIATED KINO GUN).

predominates in the region close to the cathode. In the next set of noise planes (Nos. 5, 6 and 7) Π has large negative values around $f = 0$ and the changeover from a negative to a positive value occurs at an increasing value of frequency for planes 5, 6 and 7 in order. The small positive value attained just after this point does not change significantly thereafter. This means that as the beam is accelerated, the slow wave grows and in this region its growth seems to be maximum for low frequencies and hence large negative values are observed around $f = 0$. As the beam is accelerated slow waves of progressively higher frequencies grow significantly. This explains the shift of the change-over frequency (from negative to positive values of Π) in the direction of increasing frequency for planes 5, 6 and 7 as described above. Curve 5 in the above figure represents the variation of Π in this set. As the beam is further accelerated, the negative power in the beam increases still further and Π for the set of normal planes is negative for almost all of the frequency band. For planes 8, 9 and 10 it has large negative values around $f = 0$. On the other hand for plane 11 it has high positive values in the same frequency range. Noise planes 10 and 11 have fairly large negative values in the mid-frequency band. A typical variation of Π in this set (normal plane set) is shown by curve 10 in Fig. 4.15. Thus it is apparent that Π is nonconservative along the beam.

The variation of S with frequency can be divided into two general groups. The first group consists of noise planes 1 through 4. For plane 1, S has large values in the low frequency range. For planes 2, 3 and 4 it has peaks around 2.5 and 4.5 GHz where the peaks of Ψ also appear for these noise planes (Fig. 4.9). The remaining planes 5, 6 and 7 of the parallel plane set and normal planes 9, 10 and 11 form the

other group. In this group S starts with a high value at $f = 0$ and drops sharply until $f = 2.5$ GHz after which there is little variation. For the normal plane set, variations of S and Ψ again are very similar. The reason for the close resemblance between the variation of S and Ψ with frequency however is not clear. The increase in S by two to three orders between the first and the last noise plane again shows that S is not conserved along the beam. A few representative curves of S vs. f are shown in Fig. 4.16.

The variation of $S-\Pi$ with frequency for noise planes 1 through 11 is similar to that described above. $S-\Pi$ is plotted against the normalized distance (on a logarithmic scale) for several frequencies in Fig. 4.17. A high value of $S-\Pi$ at $f = 0$ for the first noise plane can be correlated with a high value of Φ at this frequency. The corresponding values of Ψ are low; hence it might be thought that this very high value of Φ is probably due to this point falling out of the confidence interval. However the same thing happens in KG-M and this will be discussed later in Section 4.5.3. Hence this cannot be taken as a pure random event. The relatively large values of $S-\Pi$ in the low frequency range for noise planes after the fourth plane seem to be due to the combined effect of the relatively large values of Φ and Ψ in this frequency range. Since the noise parameter, F_0 , discussed in the next section, is directly related to $S-\Pi$, the same behavior is obtained for F_0 as illustrated in Fig. 4.18. It can be observed from Fig. 4.17 that $S-\Pi$ increases by about three orders of magnitude between the first and the last noise planes, indicating that it also is not constant along the beam.

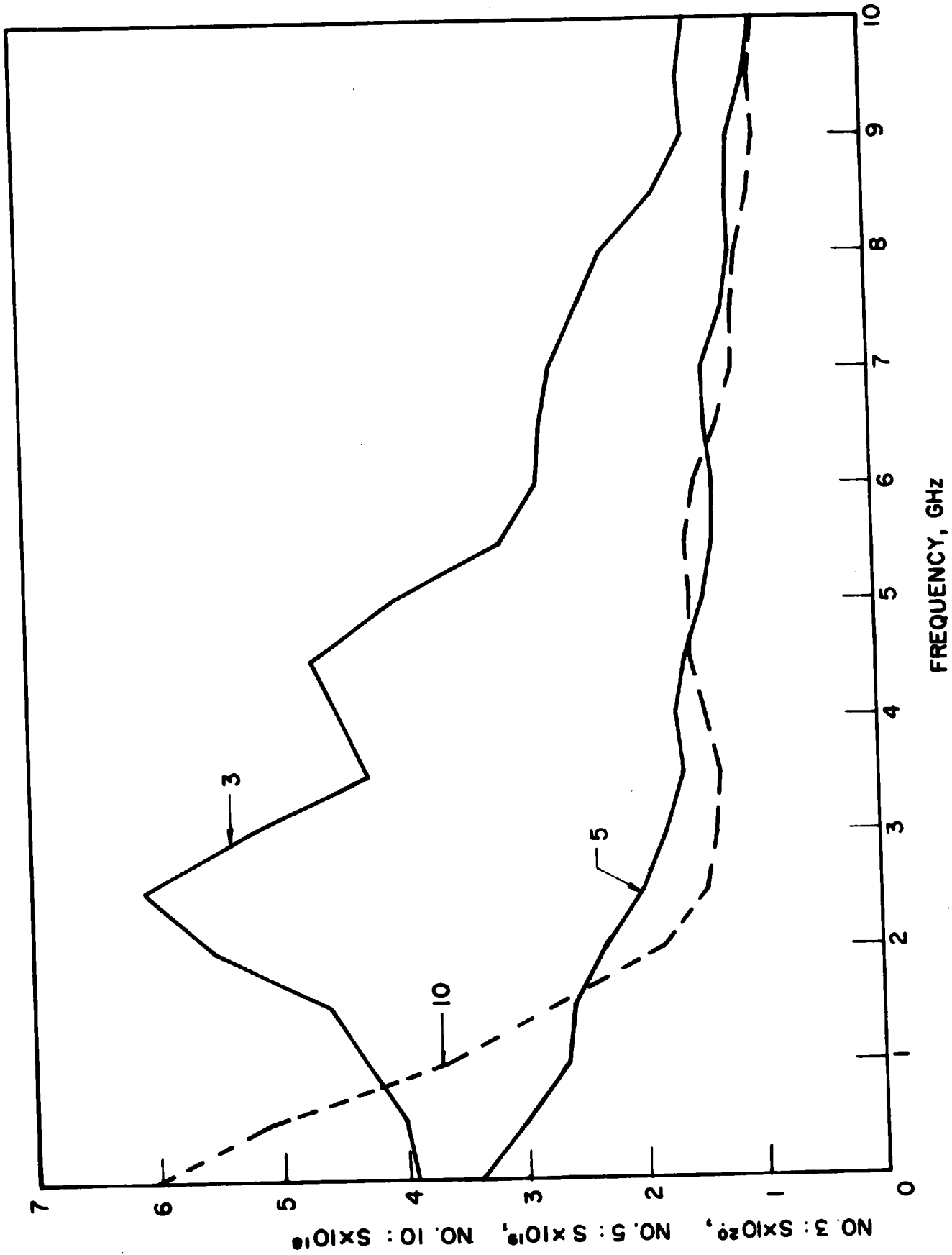


FIG. 4.16 S VS. FREQUENCY FOR NOISE PLANES 3, 5 AND 10 (ABBREVIATED KINO GUN).

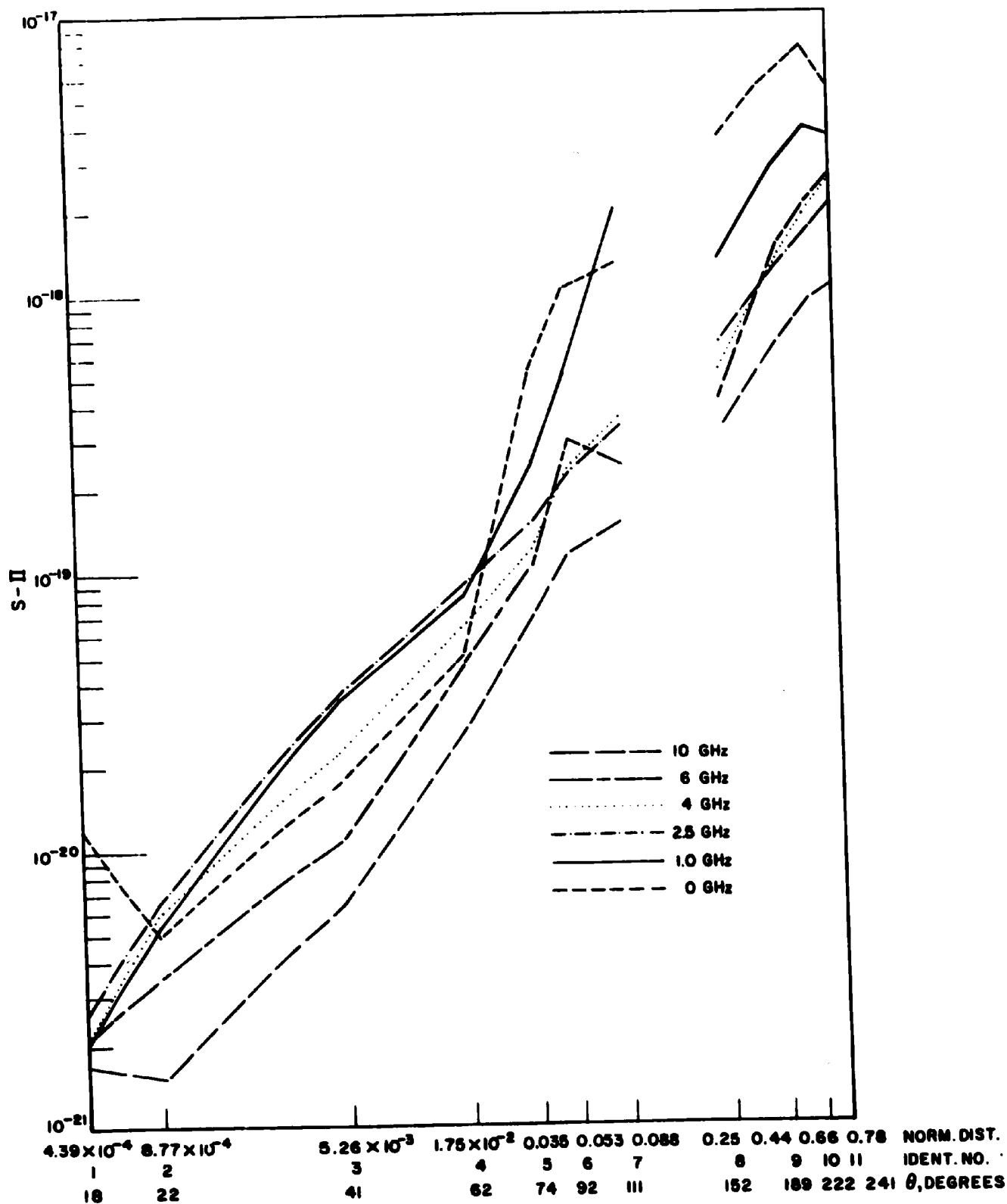


FIG. 4.17 $S-II$ VS. NORMALIZED DISTANCE (ABBREVIATED KINO GUN).

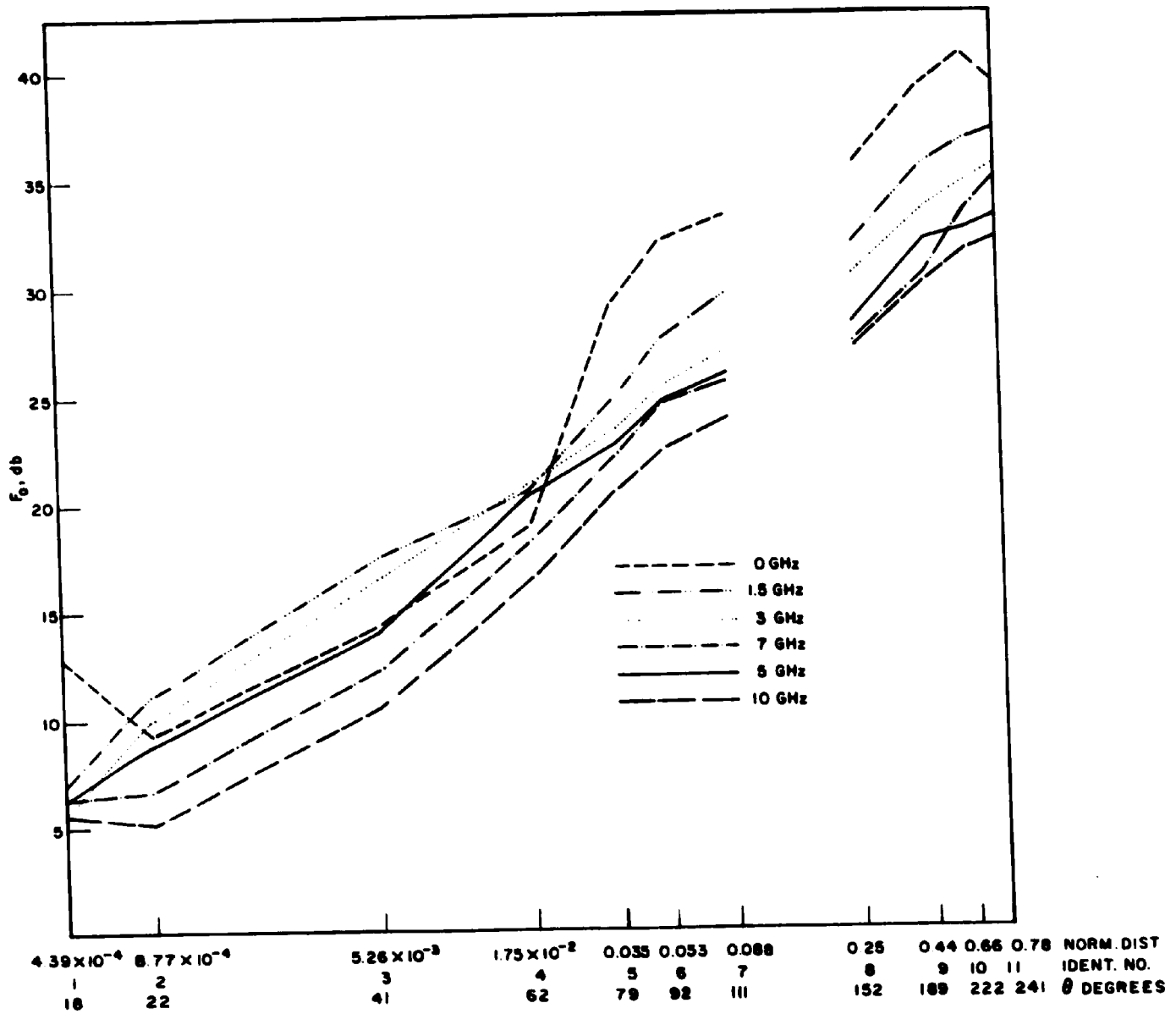


FIG. 4.18 F_0 VS. NORMALIZED DISTANCE (ABBREVIATED KINO GUN).

4.4.4 Noise Parameter F_o (SC Case). F_o is calculated from Haus' formula (Eq. 3.40) at the various noise planes. A discussion of the use of F_o has been given in Section 3.3. The relation between the noise figure $F_o(\text{db})$ and the logarithm of the normalized distance is approximately linear for several selected frequencies as shown in Fig. 4.18. This means that the rate of growth of noise in db is large near the cathode and the rate decreases with increasing distance from the cathode. A qualitative explanation for the above variation is as follows. One of the several mechanisms used to explain noise growth in M-type tubes is the diocotron phenomenon. The diocotron gain (db) per unit length of a thin beam with constant velocity u_o moving symmetrically between two parallel conducting planes is, according to Gould³⁴, inversely proportional to u_o^3 . Assuming the same relation to hold for an accelerating beam it is clear that a high diocotron gain per unit length (db/m) should be expected in the potential minimum region where u_o is small. Van Duzer and Whinnery³⁵ have proposed a similar explanation on the basis of growing edge waves on crossed-field beams. The density function analysis of a crossed-field diode by Wadhwa and Rowe²⁰ also predicts the above behavior.

Around $f = 0$, F_o assumes relatively large values at several noise planes. This is probably due to large values of Φ and Ψ in this frequency region for these noise planes. Otherwise F_o varies by 5 db over the mid-frequency band for most of the planes.

4.4.5 Noise Parameters Under the Open-Circuit (OC) Assumption.

It is found that in general the noise parameters under the OC assumption (not in the strict sense and discussed in detail in Appendix B) are slightly lower than but not significantly different from those under

the SC assumption except in the low frequency region. It may be recalled that in the OC case the total electric field is the sum of the electric field components (E_y and E_z) calculated from the solution of Poisson's equation and the additional a-c electric fields (ΔE_y and ΔE_z) due to the OC assumption. From the Monte Carlo simulation data it is found that ΔE_y and ΔE_z are two to three orders less than E_y and E_z . Since the total electric field configuration does not seem to be significantly different in the two cases it follows that in general the current and velocity fluctuations also should not significantly differ. However, the difference between the SC and OC values of the noise parameters in the low frequency range can be explained in the following manner. From Eq. 16 of Appendix B it is clear that the a-c electric field components ΔE_z and ΔE_y are proportional to $(\rho \dot{z})_{a-c}$ and $(\rho \dot{y})_{a-c}$ respectively. The latter are simply the a-c components of the current density fluctuation in the z- and y-directions. The power spectrum of the current density fluctuations, Ψ_J , has large values in the low frequency region (see Fig. 4.14 as an example) and does not vary much in the rest of the frequency band. Thus it can be concluded that the effect of the a-c electric field components will be maximum in the low frequency region.

4.4.6 Variances of the Total Velocity and the Velocity Components of Electrons. The kinetic temperature of electrons in a beam is proportional to the variance of the velocity of the electrons. The variances of the y- and z-velocity components and the total velocity can be used to study the variation of the kinetic temperature of electrons in the y- and z-directions and along the direction of the beam. The procedure for calculating these variances is described in Appendix E and the results are presented below.

On examining the variation of velocity variances among rectangles located at a beam cross section (both parallel and normal noise planes) it is found that the inner beam edge has the highest values of velocity variances (σ_u^2 , σ_y^2 and σ_z^2). The same conclusion was reached in Section 4.4.1c from the variation of Φ across the beam cross section. These values decrease toward the outer edge of the beam. This indicates that the kinetic temperature of electrons in the inner edge of the beam is higher than that at the outer edge, or in other words the inner edge of the beam is noisier. This is in accordance with the theoretical and experimental results of Wadhwa and Rowe¹¹ and Rao and Whinnery³⁶ respectively. A few rectangles are selected to represent only a part of the outer and inner edges of the beam. This is due to the fact that data on these variances for rectangles at the inner and outer edges of the beam in the neighborhood of the cathode are not available. The available data is presented in Tables 4.2 and 4.3. The values of the variances with and without parentheses refer to the OC and SC cases respectively.

From Table 4.2 it can be seen that σ_u^2 and σ_z^2 have a minimum at rectangle 125, the corresponding transit angle being approximately 180 degrees. It is interesting to note that at $\omega_c t = 180$ degrees, $\dot{y}$ attains its maximum value for the Kino short gun. Except for this beam region, σ_u^2 and σ_z^2 have only small variations along the length of the beam considered in Table 4.2. On the other hand σ_y^2 undergoes a significant increase in value for the same length of the beam.

Table 4.2

Variance Calculations for Rectangles Along the Upper Beam Edge
(KG-A)

Rectangle Number	$\sigma_u^2 \times 10^{-10}$	$\sigma_y^2 \times 10^{-11}$	$\sigma_z^2 \times 10^{-11}$
152	9.48 (9.45)	0.44 (0.38)	1.37 (1.32)
125	6.47 (6.60)	0.60 (0.50)	1.15 (1.05)
97	9.22 (8.10)	1.66 (1.46)	1.50 (1.49)
69	10.22 (8.73)	4.36 (3.04)	1.83 (1.48)
55	9.80 (7.80)	4.90 (3.64)	1.42 (1.10)

Table 4.3

Variance Calculations for Rectangles Along the Lower Beam Edge
(KG-A)

Rectangle Number	$\sigma_u^2 \times 10^{-10}$	$\sigma_y^2 \times 10^{-11}$	$\sigma_z^2 \times 10^{-11}$
150	15.46 (13.50)	2.56 (1.43)	3.37 (2.34)
123	15.33 (13.56)	4.57 (2.48)	3.40 (2.56)
82	15.20 (12.83)	7.64 (4.76)	3.12 (2.59)
68	15.77 (12.53)	8.32 (5.79)	2.50 (2.12)

Along the lower edge of the beam σ_u^2 remains almost constant whereas σ_y^2 increases toward the gun exit. Except for the drop in the value of σ_z^2 at rectangle 68 which might well be within the statistical error involved in these calculations, σ_z^2 does not show any significant

variation over the length of the beam considered in Table 4.3. All variance values are higher than the corresponding ones given in Table 4.2.

The noise planes used in Table 4.4 are planes parallel to the cathode. They extend between the cathode and the $\omega_c t \approx \pi$ plane. The dip in the value of σ_u^2 at plane 2 may not be significant since this plane is just inside the potential minimum and the amplitude of velocity fluctuations is of the order of the d-c velocity of an average electron in this region. σ_u^2 increases gradually up to the fifth plane and increases rather sharply between the fifth ($\omega_c t = 79$ degrees) and sixth ($\omega_c t = 92$ degrees) planes. The rise in σ_u^2 continues until the seventh plane. It shows no significant change for plane 8a and drops off at plane 9a ($\omega_c t \approx \pi$). This drop was also observed in Table 4.2 for rectangle 125 which lies in this plane. σ_y^2 increases steadily with increasing distance from the cathode. There is not much change in the value of σ_z^2 until the fourth plane. It shows a rather sharp rise between the fifth and seventh planes and then the rate of increase in σ_z^2 decreases. The region where σ_u^2 and σ_z^2 have a sharp rise coincides with the region where a peak of Ψ vs. normalized distance occurs as shown in Fig. 4.12. It should be recalled that in this region ρ changes from a value above to a value below ρ_0 , the Brillouin density.

On summarizing the observations made from the three preceding tables it can be said that the inner beam edge has a higher kinetic temperature than the outer beam edge and σ_y^2 increases with increasing distance from the cathode. A sharp increase in σ_u^2 and σ_z^2 is observed in the neighborhood of $\omega_c t = \pi/2$, where, according to Kino theory, ρ changes from a value above to one below ρ_0 , the Brillouin density.

σ_u^2 and σ_z^2 for the outer beam edge and σ_u^2 for the cross section as a whole exhibit a dip around $\omega_c t = \pi$ where $\dot{y}$ is maximum for the Kino short gun. After $\omega_c t \approx \pi$ there is not much variation in σ_u^2 and σ_z^2 with distance along the beam. Such a "geometric" variation of electron-beam temperature has been reported on by Pollack and Whinnery¹² in their Monte Carlo analysis of a crossed-field diode. The variance values under the OC assumption are slightly lower but not significantly different from those obtained under the SC case. This agrees with Pollack and Whinnery's conclusion¹² that the velocity spread introduced by the magnetic field is directly due to the average effects and r-f effects are masked to a considerable degree.

Table 4.4

Variance Calculations for the Parallel Set of Noise Planes (KG-A)

Noise Plane Number	$\sigma_u^2 \times 10^{-10}$	$\sigma_y^2 \times 10^{-11}$	$\sigma_z^2 \times 10^{-11}$
1	2.77 (2.40)	0.11 (0.10)	0.31 (0.26)
2	1.43 (1.26)	0.13 (0.12)	0.31 (0.27)
3	3.79 (3.30)	0.38 (0.34)	0.32 (0.32)
4	5.12 (4.82)	0.46 (0.44)	0.41 (0.42)
5	5.77 (5.23)	0.48 (0.44)	0.58 (0.49)
6	9.65 (8.80)	0.65 (0.61)	0.96 (0.73)
7	13.77 (11.40)	0.75 (0.60)	1.90 (1.30)
8a	13.53 (11.20)	1.14 (0.75)	2.79 (1.94)
9a	12.00 (10.26)	2.50 (1.43)	3.13 (2.13)

Lindsay and Sims³ have shown for a crossed-field diode (infinite cathode) that the magnetic field causes a redistribution of energy in the y- and z-directions. They showed that in this case electron temperature in the y-direction increases at the cost of the z-component of electron energy. Since the electric field configuration in KG-A is different from that for the crossed-field configuration analyzed by Lindsay and Sims³, their work cannot be used directly to check the above results. However, it does support the geometrical variation of temperature in the beam produced by KG-A. The same conclusion will be reached in Section 4.5.6 regarding KG-M.

4.5 Monte Carlo Analysis of Noise Transport in the Modified Kino Gun

It was mentioned in Section 2.5.1 that the modification in the focus electrodes of KG-A results in the so-called KG-M with nonintersecting trajectories in the gun region. It is noteworthy that this gun possesses the additional feature of having almost uniform cathode current density as opposed to the nonuniform variation found in the case of KG-A. Lastly, the potential minimum profiles (y_m and V_m across the cathode) shown in Figs. 2.5 and 2.9 for the two guns are quite different. The difference between the various d-c characteristics of these two guns is used, to some extent, as a basis for comparing the noise characteristics of the two guns.

The noise-parameter calculations are made on a rectangle as well as on a cross-sectional basis. The various noise planes (parallel and normal) are shown in Fig. 4.19 and described in Table 4.5 which is similar to Table 4.1 except for one additional column. To facilitate a comparison between the two guns, when a noise plane in this gun (KG-M)

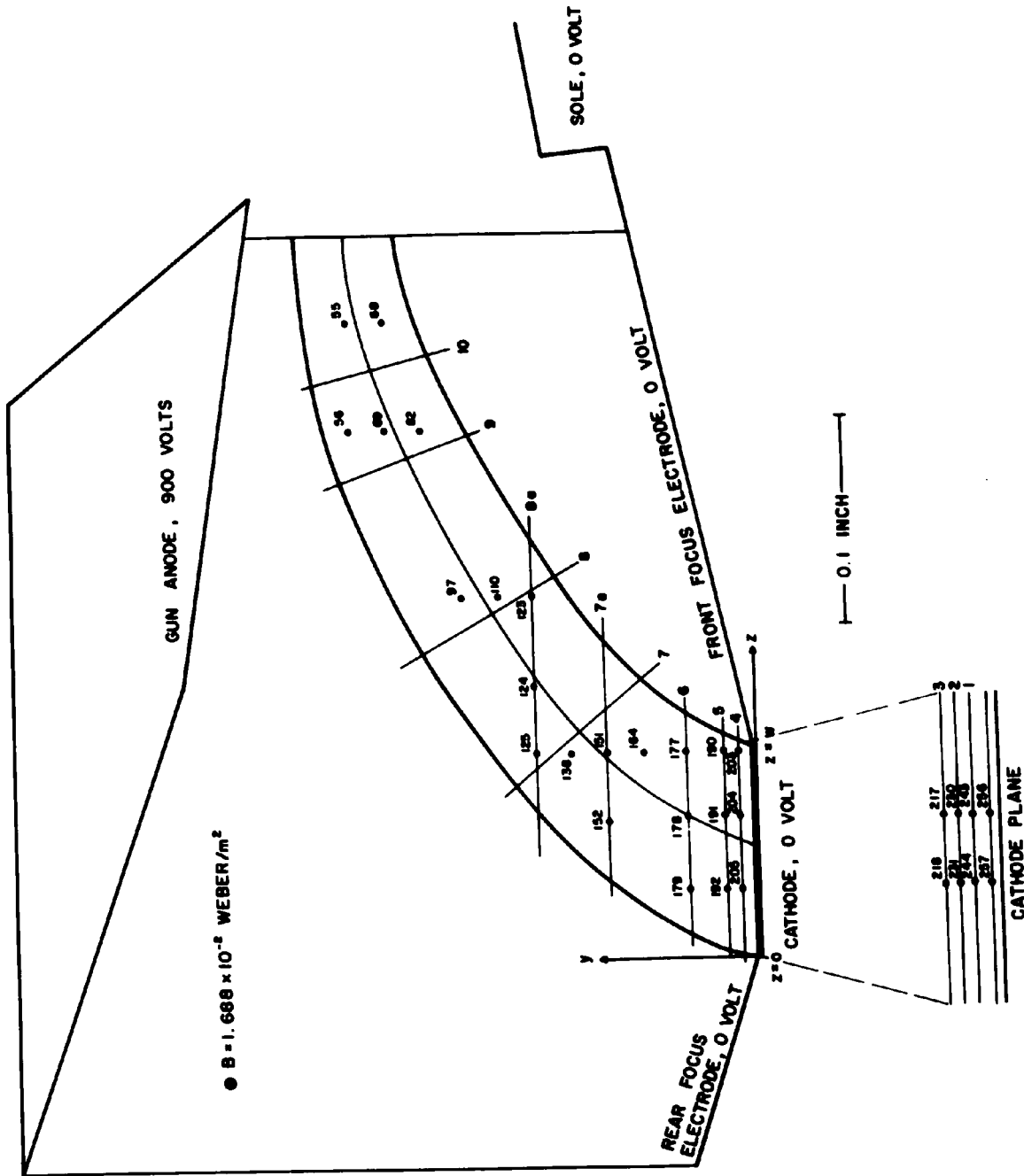


FIG. 4.19 APPROXIMATE BEAM PROFILE AND NOISE PLANE POSITIONS (MODIFIED KINO GUN).

Table 4.5
Noise Plane Positions for the Modified Kino Gun

Noise Plane Number KG-M	Corresponding Noise Plane Number KG-A	y or s (Inch)	Normalized Distance $y/s_{\max}$ or $s/s_{\max}$ ($s_{\max} = 0.3866$ Inch)	Transit Angle $\theta = \omega_c t$ (Degrees)	Remarks
1	1	$y = 1.666 \times 10^{-4}$	4.310×10^{-4}	18	Parallel Plane
2	2	$= 3.333 \times 10^{-4}$	8.621×10^{-4}	22	Parallel Plane
3	3	$= 2.000 \times 10^{-3}$	5.173×10^{-3}	41	Parallel Plane
4	4	$= 6.666 \times 10^{-3}$	1.724×10^{-2}	62	Parallel Plane
5	5	$= 1.333 \times 10^{-2}$	3.448×10^{-2}	79	Parallel Plane
6		$= 3.000 \times 10^{-2}$	7.760×10^{-2}	107	Parallel Plane
7a	8a	$= 6.666 \times 10^{-2}$	0.1724	146	Parallel Plane
8a	9a	$= 1.000 \times 10^{-1}$	0.2587	176	Parallel Plane
7	≈ 8	$s = 9.830 \times 10^{-2}$	0.2542	159	Normal Plane
8	≈ 9	$= 0.1683$	0.4353	192	Normal Plane
9	≈ 10	$= 0.2500$	0.6465	227	Normal Plane
10	≈ 11	$= 0.2966$	0.7672	243	Normal Plane

corresponds positionwise, exactly or approximately, to a noise plane in the previous gun (KG-A), the identification number of the latter is placed in this column. It should be noted that, in the absence of an exact analytical expression for the transit angle of an electron along a trajectory in this gun (KG-M), the transit angles shown in Column 5 of the preceding table are calculated by using the Kino short gun theory as was done for KG-A.

SC noise parameters for KG-M are discussed in the following sections. When the variation of a certain noise parameter in this gun is more or less similar to that for KG-A, the interpretation given previously is applicable in this case too and will not be repeated but reference will be made to the relevant section. On the other hand an attempt will be made to interpret, as far as possible, the difference in behavior of certain noise parameters in the two guns.

4.5.1 SPDS of Kinetic Noise-Voltage Modulation, Φ (SC Case).

a. Variation of Φ with Frequency. The variation of Φ with frequency for the set of parallel planes (Nos. 1 through 6) is shown in Fig. 4.20. Each of the curves has a high value of Φ at $f = 0$. After going through a minimum between 1.5 and 2.5 GHz ($f_{pm} \approx 1.8$ GHz), Φ for all planes rises and maintains a relatively large value between 3 and 8 GHz. In this range most of the curves have peaks around 4.5 and 7 GHz. Figure 4.21 shows the Φ variations for the set of normal planes. Φ for planes 8, 9 and 10 and their counterparts in KG-A has a peak around 1 GHz and has only small variations after about 3 GHz. On the other hand both plane 7 and the approximately corresponding plane 8 in KG-A do not exhibit any such peak.

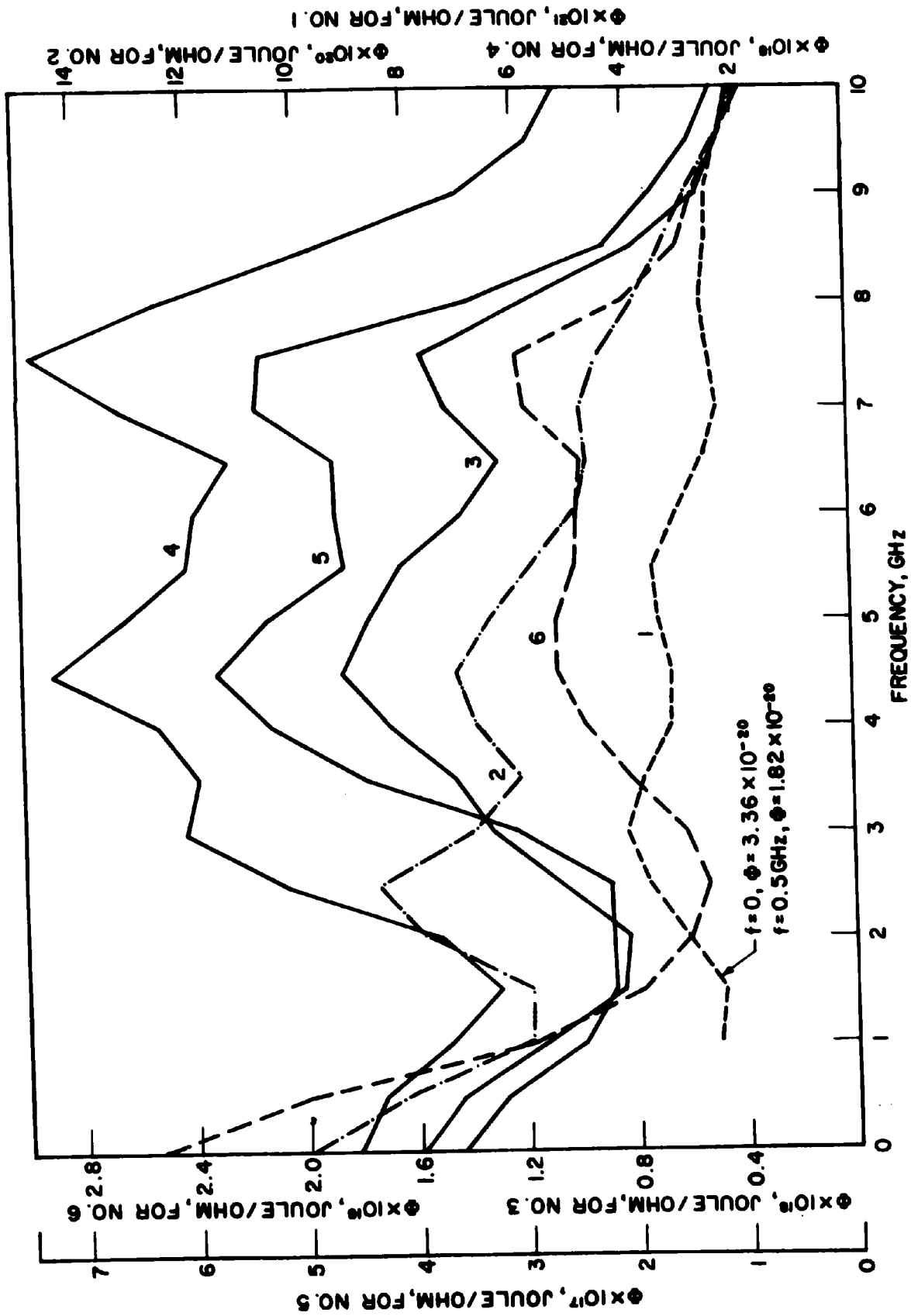


FIG. 4.20 ϕ VS. FREQUENCY FOR NOISE PLANES 1 THROUGH 6 (MODIFIED KINO GUN).

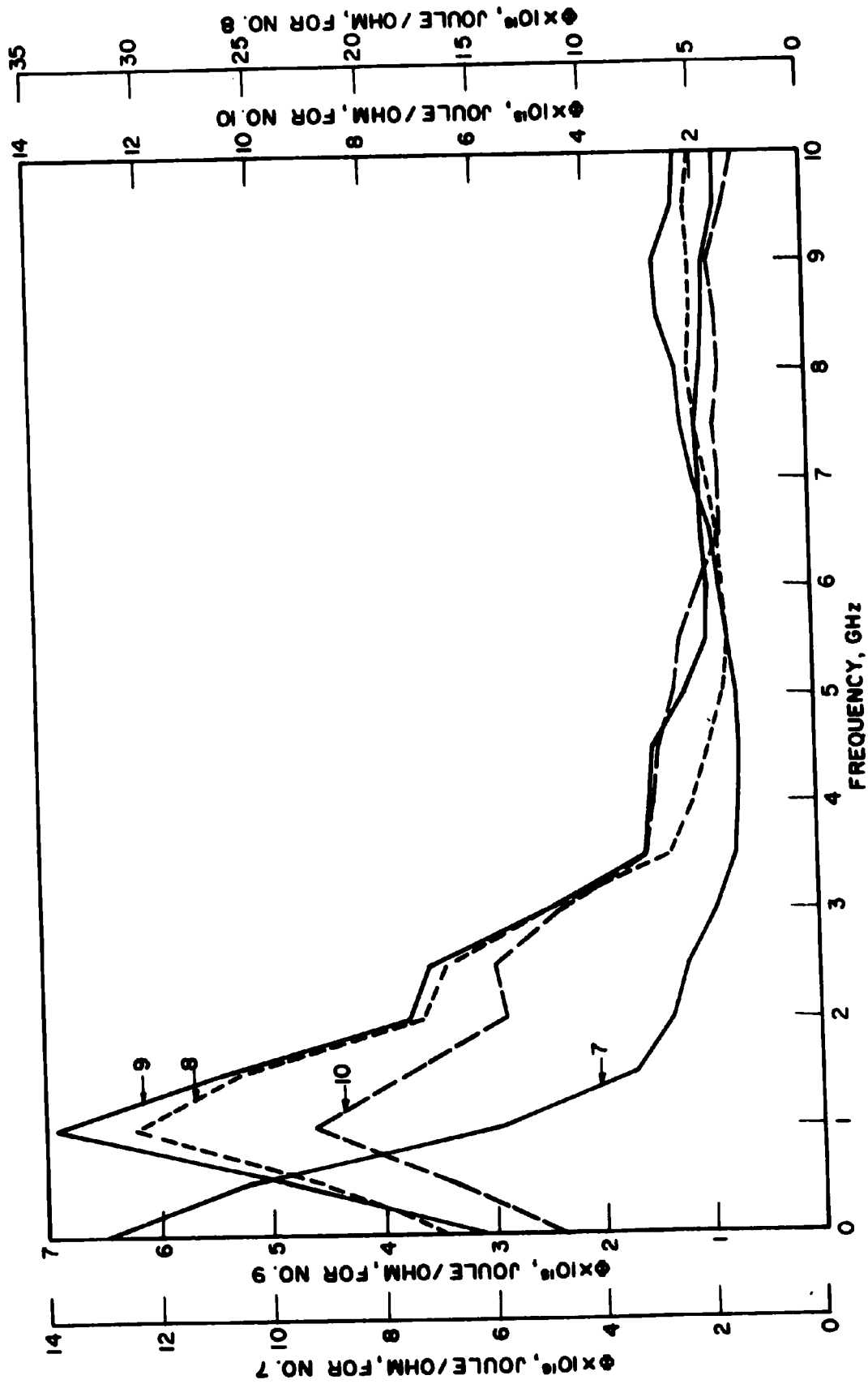


FIG. 4.21 ϕ VS. FREQUENCY FOR NOISE PLANES 7 THROUGH 10 (MODIFIED KINO GUN).

In general, it can be said that the variation in Φ with frequency in both guns is similar. The occurrence of a broad minimum near the potential minimum hybrid frequency for all planes in the parallel sets of both guns indicates some role of this hybrid frequency, f_{hm} , in decreasing Φ in the neighborhood of this frequency at least in the region not far from the potential minimum. Further along the beam it seems that the effect of the potential minimum is negligible and the hybrid frequency in this region of the beam affects Φ to produce a peak around 1 GHz. Since the steps in the calculation of Φ are 0.5 GHz apart it is difficult to locate the peak more accurately. It is not clear why Φ for planes in the neighborhood of the cathode, but not for planes in the normal set, should have high values around $f = 0$. It was mentioned in Section 4.4.1 that the density function analysis of Wadhwa³² predicts such high values of Φ around $f = 0$. In this gun (KG-M) the peaks of Φ around 4.5 GHz and 7.0 GHz are approximately at the second and third harmonics of the hybrid frequency at the potential minimum. Again the reason for the occurrence of these peaks is not known.

b. Variation of Φ with Distance Along the Beam. Figure 4.22 illustrates the variation of Φ along the beam. The increase in Φ with distance from the cathode follows the same pattern as in KG-A (Section 4.4.1b). In general for a given frequency the value of Φ for a noise plane in KG-M is slightly higher than that for the corresponding noise plane in KG-A. Thus with the rate of increase in Φ with distance being slightly higher in this case, Φ for the last noise plane (plane 10) becomes a few times larger than that for the corresponding noise plane in KG-A. This difference is expected since the two beams differ in

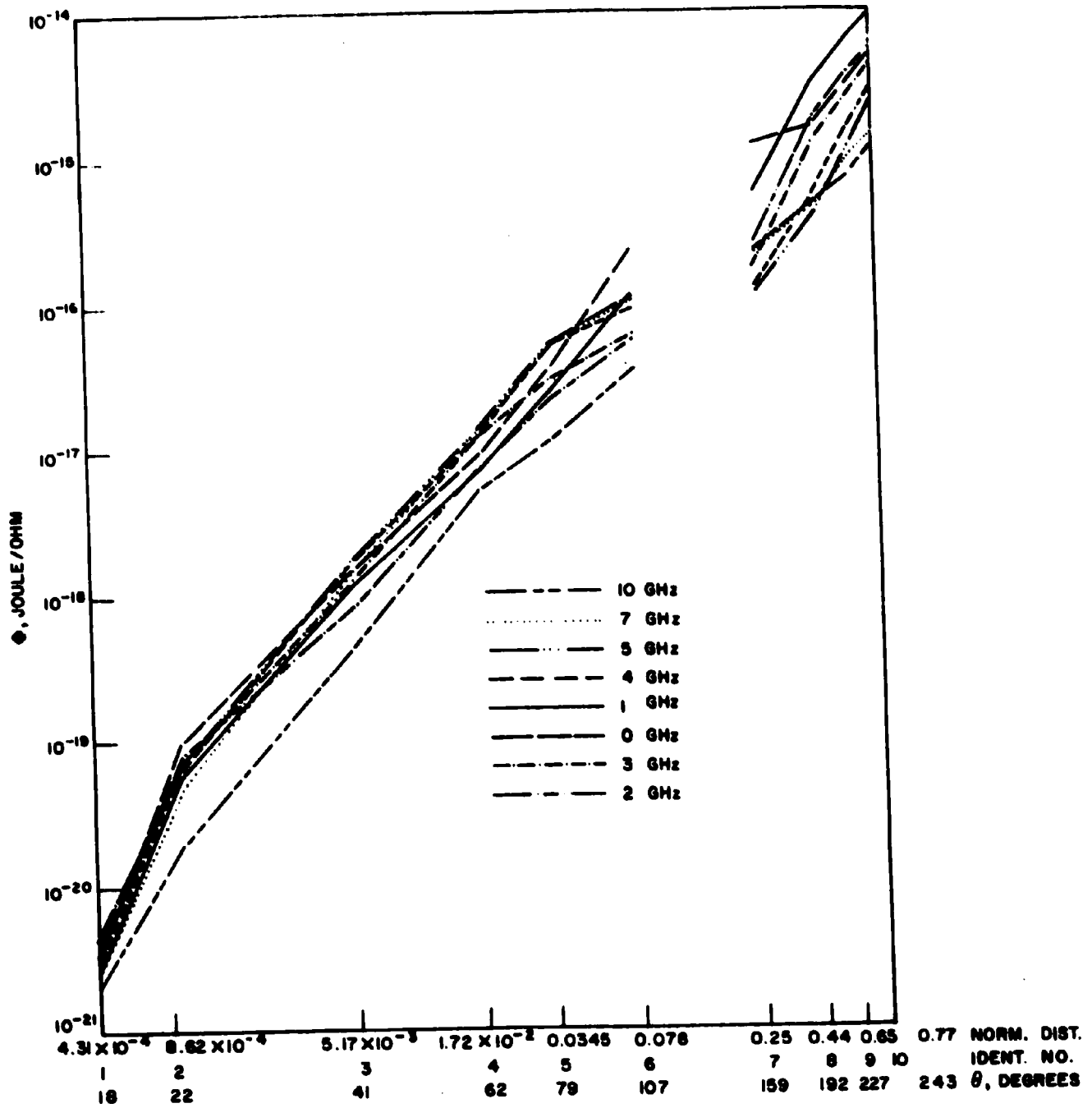


FIG. 4.22 Φ VS. NORMALIZED DISTANCE (MODIFIED KINO GUN).

thickness, average beam velocity and beam current, and the noise growth is a function of these parameters.

c. Variation of Φ Across the Beam. For both parallel and normal noise planes, Φ at the center of the beam is generally a maximum. At the inner edge it is slightly higher than at the outer edge. Consideration of the d-c velocity of each of the layers of the beam shows that the velocity fluctuations are minimum at the outer layer of the beam. The same conclusion was drawn in Section 4.4.1c in connection with KG-A. The occurrence of maximum Φ at the central layer of the beam in KG-M and its absence in KG-A suggests that probably the difference in the potential minimum profiles of these two guns (Figs. 2.5 and 2.9) is the suspected cause of this difference. Φ for a typical cross section (plane 7a) and its constituent rectangles is plotted against frequency in Fig. 4.23. Φ for the cross section as a whole is lower than the values for the individual layers (rectangles) of the beam, which is expected as explained in Section 4.4.2c.

4.5.2 SPDS of Noise-Current Modulation, Ψ (SC Case).

a. Variation of Ψ with Frequency. The general trend of the variation of Ψ with frequency (Figs. 4.24 and 4.25) is the same as that found in KG-A and described in Section 4.4.2a. The explanation in terms of the effect of the hybrid frequency near the potential minimum to explain relatively large values in the frequency range of 2.5 GHz to 4.5 GHz for planes in the parallel set is also applicable in this case. Very high values of Ψ around $f = 0$ and 0.5 GHz for noise plane 4 are possibly due to these points falling out of the confidence interval.

b. Variation of Ψ with Distance Along the Beam. In Fig. 4.26 Ψ is plotted against the normalized distance (on a logarithmic scale)

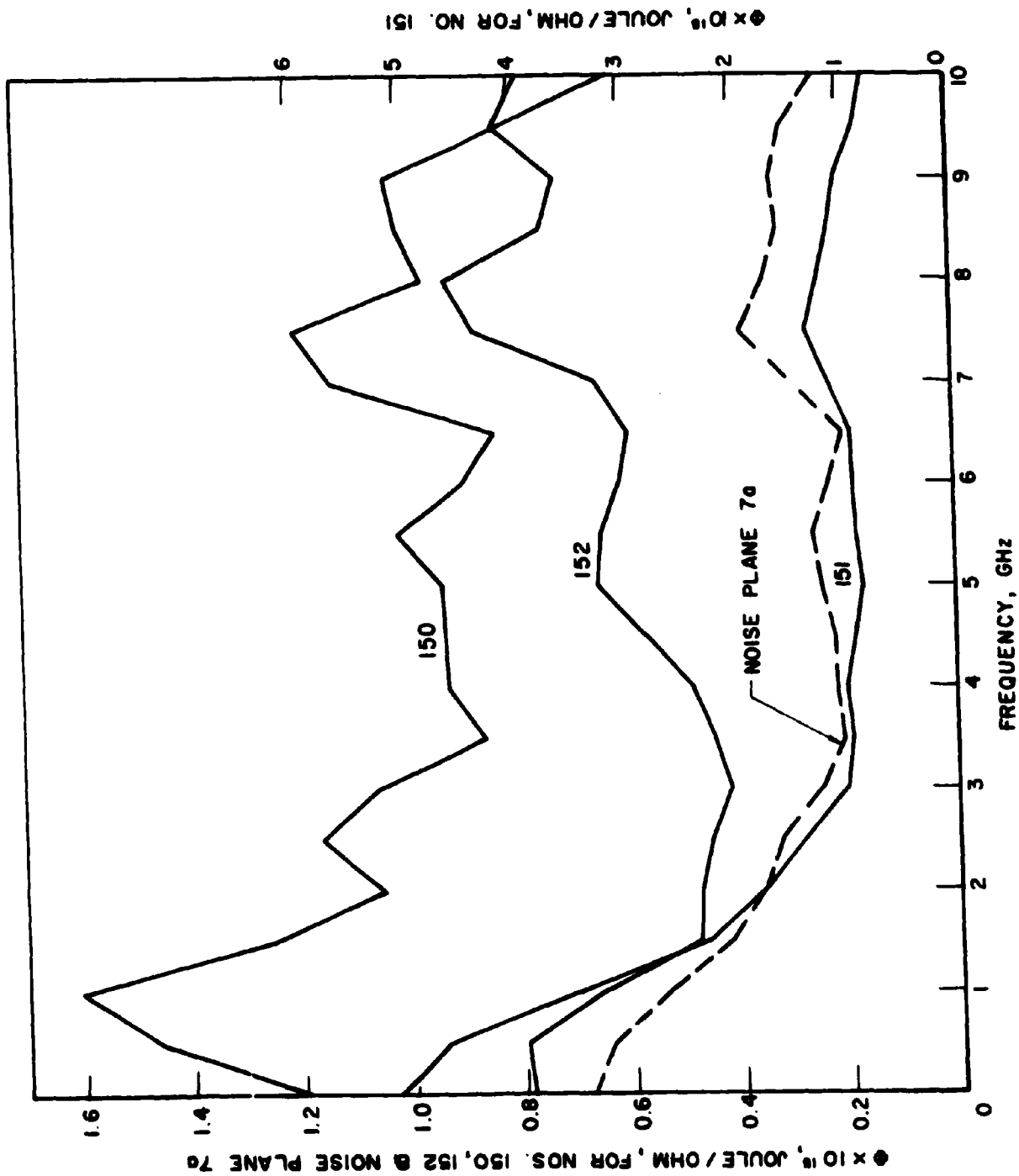


FIG. 4.23 ϕ VS. FREQUENCY FOR RECTANGLES 150, 151, 152 AND NOISE PLANE 7a (MODIFIED KINO GUN).

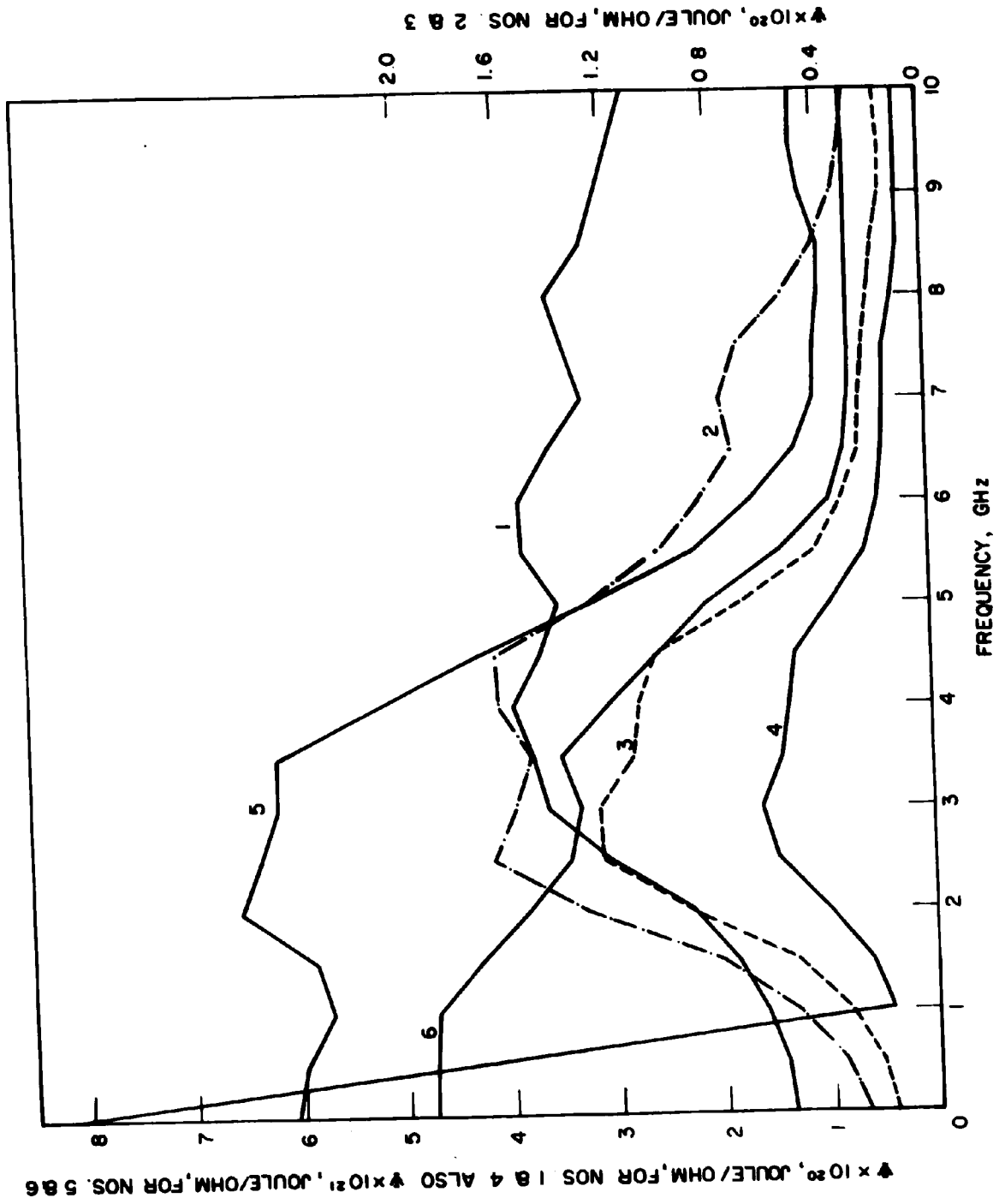


FIG. 4.24 ψ VS. FREQUENCY FOR NOISE PLANES 1 THROUGH 6 (MODIFIED KINO GUN).

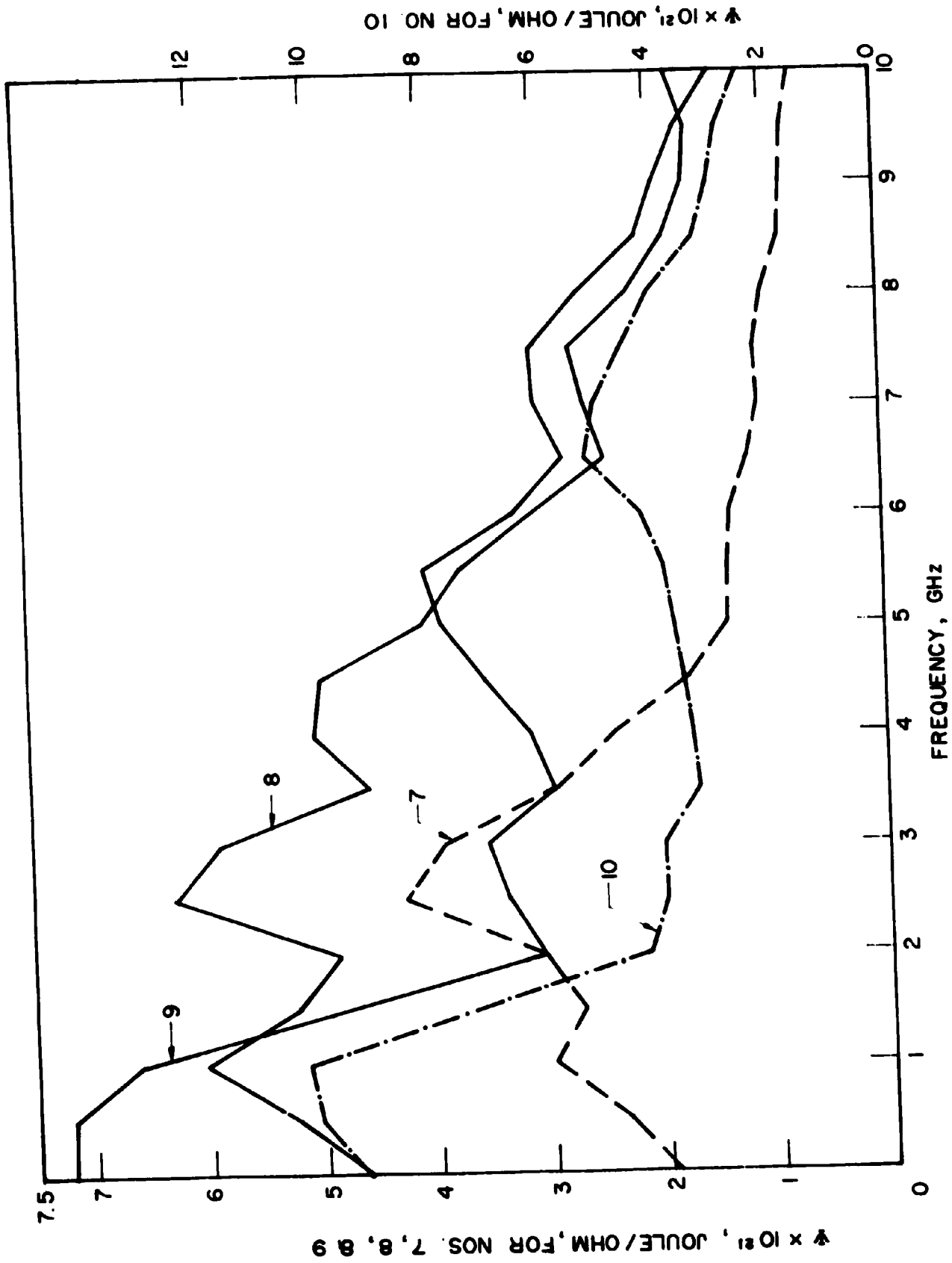


FIG. 4.25 Ψ VS. FREQUENCY FOR NOISE PLANES 7 THROUGH 10 (MODIFIED KINO GUN).

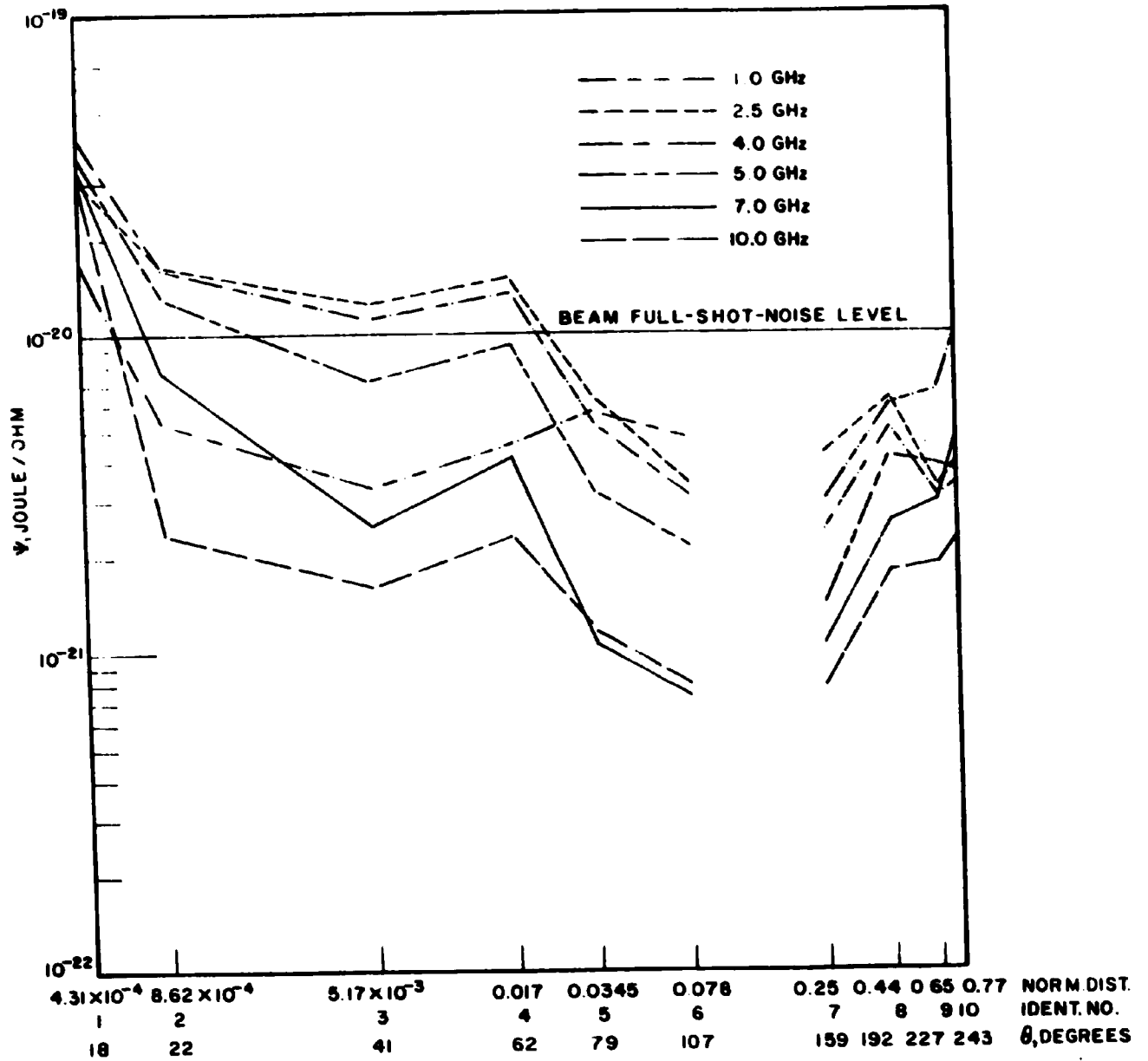


FIG. 4.26 Ψ VS. NORMALIZED DISTANCE (MODIFIED KINO GUN).

for several selected frequencies. In the set of parallel planes Ψ decreases up to the third plane, has a peak at the fourth plane, and then decreases again up to the sixth plane. The peak occurs approximately in the region where ρ changes from a value above to one below the Brillouin value ρ_0 . The rate of change of Ψ with distance is maximum between the first noise plane and the second which is in the neighborhood of the potential minimum. This rate decreases toward the gun end. The maximum rate of fall in Ψ for KG-A and KG-M is almost the same which is expected since the plasma frequencies (space-charge density), obtained from computer simulation data, at the potential minimum are almost identical for the two cases. Generally Ψ increases slowly for the set of normal planes. The above variation of Ψ with distance can be explained on the basis of the variation of ρ/ρ_0 with distance (transit angle) as was done in Section 4.4.2b to interpret a similar variation of Ψ in KG-A. In both guns current fluctuations are reduced below the full shot noise level in the beam within a very small distance from the cathode. In KG-M Ψ is somewhat higher than that in KG-A at the corresponding point. This is expected since current-fluctuation noise in a beam is proportional to the beam current which in KG-M is 1.35 times that in KG-A.

c. Variation of Ψ_J Across the Beam. In Fig. 4.27 the variation of Ψ_J across the beam cross section is only noticeable up to about 2.5 GHz which is in the neighborhood of the potential minimum hybrid frequency, f_{hm} . The same thing was observed with KG-A and the conclusion drawn in Section 4.4.2c that the potential minimum effects are strong up to the hybrid (or plasma) frequency at the potential minimum is thus true in this case also. In the same section some correlation

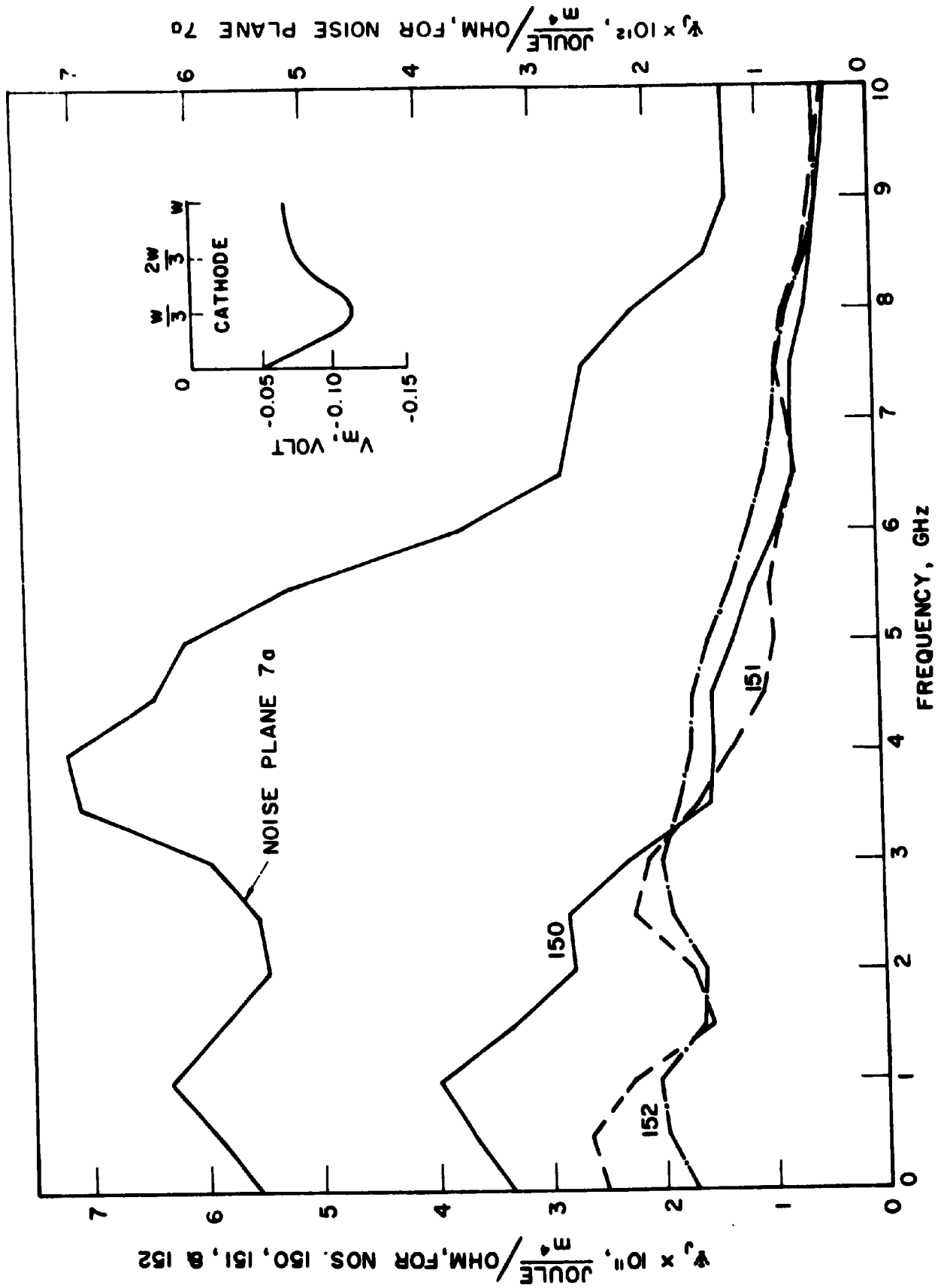


FIG. 4.27 j_j VS. FREQUENCY FOR RECTANGLES 150, 151, 152 AND NOISE PLANE 7a (MODIFIED KINO GUN).

between the Ψ_J variation across the beam and the potential minimum profile was suggested. The insert in Fig. 4.27 shows the potential minimum profile over the length of the cathode for this gun. Following Section 4.4.2c Ψ_J should be maximum at the inner beam layer and should not differ significantly over the central and outer parts of the beam since the mean $|V_m|$ over the left and central portions of the cathode is only slightly different. This is verified by the values of Ψ_J for rectangles 150 (inner), 151 (middle) and 152 (outer) for a typical cross section (plane 7a) of the beam. Ψ_J calculated on a cross-sectional basis is less than that for any layer of the beam for reasons explained in Section 4.4.2c.

4.5.3 Noise Parameters Π , S , and $S-\Pi$ (SC Case). Π , the real kinetic power in the beam, is found to vary and even change sign along the beam. On this basis the noise planes can be divided into three groups. The first group consists of noise planes 1 through 4. Π is always positive and increasing for these planes and a typical case (plane 3) is shown in Fig. 4.28. Noise planes 5 and 6 comprise the second group. In this group, represented by plane 5 in the above figure, Π is negative at the low frequency end and has a positive value for the major portion of the frequency band. The frequency of the changeover from negative to positive values shifts to a higher frequency from plane 5 to plane 6. Π is negative throughout the frequency band for all normal planes (Nos. 7, 8, 9 and 10). Ψ for plane 9 (shown in the above figure) typically represents the variation with frequency in this group.

The variation of Π described above is almost identical to that for KG-A (Section 4.4.3) except for the fact that the first group of noise planes (Nos. 1 through 4) with positive Π in this gun does not

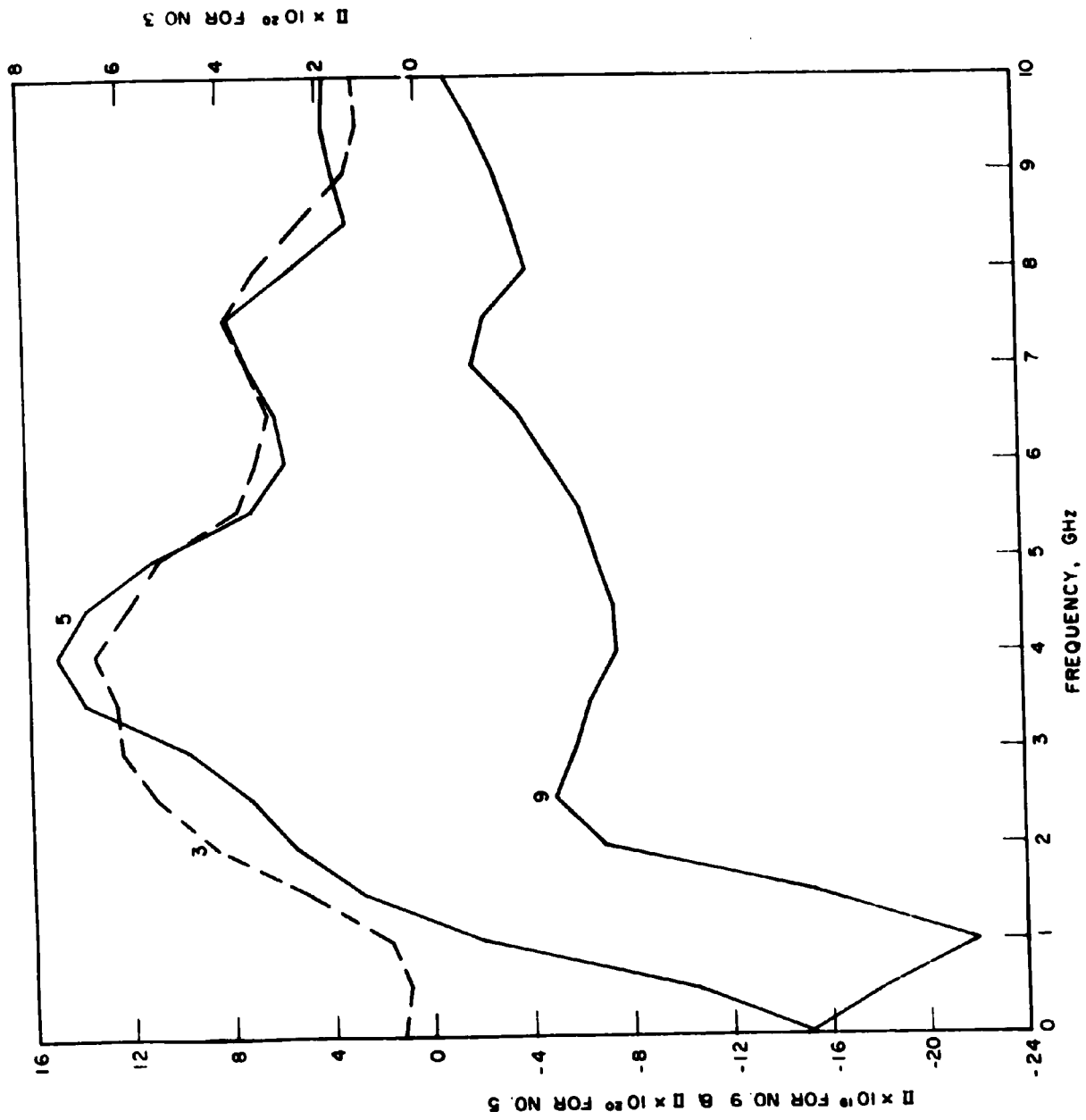


FIG. 4.28 Π VS. FREQUENCY FOR NOISE PLANES 3, 5 AND 9 (MODIFIED KINO GUN).

have a dip around 1 GHz which is present in the corresponding set for KG-A. The reason for this discrepancy is not clear. The variation of Π along the beam described above can be explained in terms of the amplitude of the slow and fast cyclotron or hybrid modes as a function of frequency and distance along the beam. The details are given in Section 4.4.3. It is found that Π is not conserved along the beam in either gun.

The variation of S can be discussed in terms of two principal groups. The first consists of noise planes 1 through 3. For planes 2 and 3, S has low values in the low and high frequency ranges, whereas it has large values in the mid-frequency range with peaks around 2.5 GHz and 4.5 GHz. Just as in the case of KG-A these peaks coincide with the peaks of Ψ for these noise planes. S for plane 1 has almost a constant value in the mid- and high frequency regions. This is also true for Ψ for this noise plane in KG-A. The rest of the noise planes in the parallel plane set and in the normal set start with high values of S around $f = 0$ and decrease rapidly until about 3 GHz after which the decrease is very slow. The variation of S with frequency for some representative noise planes is shown in Fig. 4.29. For these planes the S and Ψ variations with frequency are not very close but the general trend is the same. It is very interesting to note the striking similarity between the variations of S and Ψ with frequency and distance along the beam in both KG-A and KG-M although the cause is not apparent. The increase in S between the first and the last noise planes is of the same order as in the previous gun. Thus S is nonconservative along the crossed-field beams in both KG-A and KG-M.

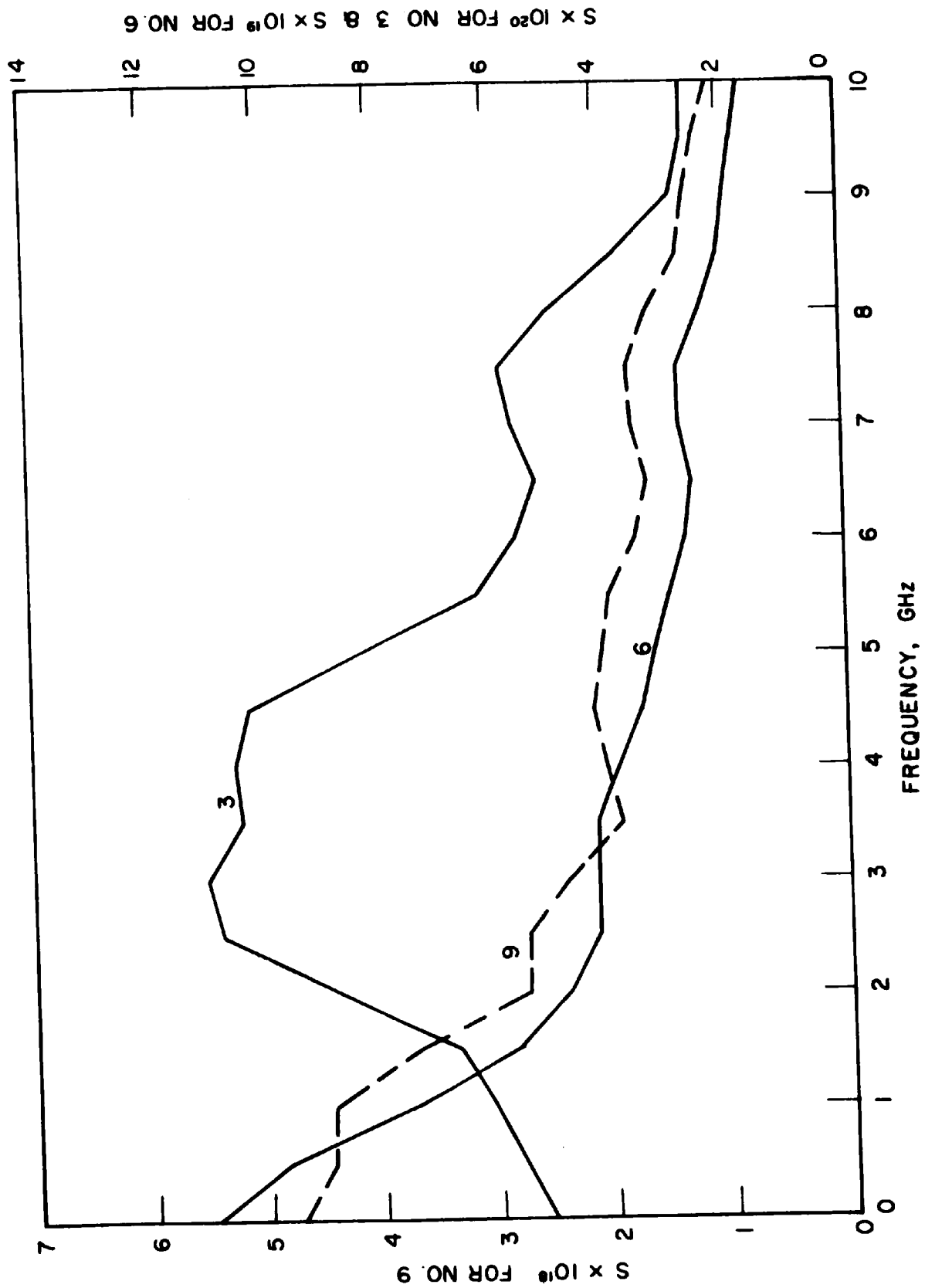


FIG. 4.29 S VS. FREQUENCY FOR NOISE PLANES 3, 6 AND 9 (MODIFIED KINO GUN).

The general variation of $S-\Pi$ with frequency is similar to that for S described above. As in KG-A, the large value of $S-\Pi$ at $f = 0$ for the first noise plane is due to the correspondingly high value of Φ . $S-\Pi$ has comparatively large values in the low frequency range for noise planes beyond the third plane. This can be correlated with high values of Ψ for these planes in this frequency range. Figure 4.30 shows a plot of $S-\Pi$ against normalized distance (logarithmic scale). Beyond the potential minimum the relation between $S-\Pi$ and the normalized distance, when both are plotted on a logarithmic scale, is approximately linear. $S-\Pi$ increases by about three orders between the first and the last noise plane. Similar behavior was observed for the $S-\Pi$ variation in the previous gun (KG-A).

4.5.4 Noise Parameter, F_o . The variation of F_o with frequency and distance follows the variations of $S-\Pi$ described in the last section. In this as well as in the previous gun, F_o is large at some frequencies in the low frequency region for certain noise planes. This is due to high values of Φ and/or Ψ at these beam locations. The diocotron gain, G , in a Kino gun is given by³⁷

$$G = \int_a^b \alpha(Z) dZ = \frac{\pi \eta \epsilon_o f}{(2)^{3/2}} \frac{w B^2}{J_y} \int_a^b \frac{K}{\Phi^{3/2}} dZ, \quad (4.4)$$

where a and b are two arbitrary planes along the beam, w is the width of the cathode, η is the absolute ratio of charge to mass of an electron, B is the magnetic flux density, f is the frequency, and J_y is the cathode current density. K is a geometric factor which takes into account the contribution due to nonparallel conducting planes in the gun region and is approximately equal to one between $\theta = \pi$ and $\theta = 2\pi$

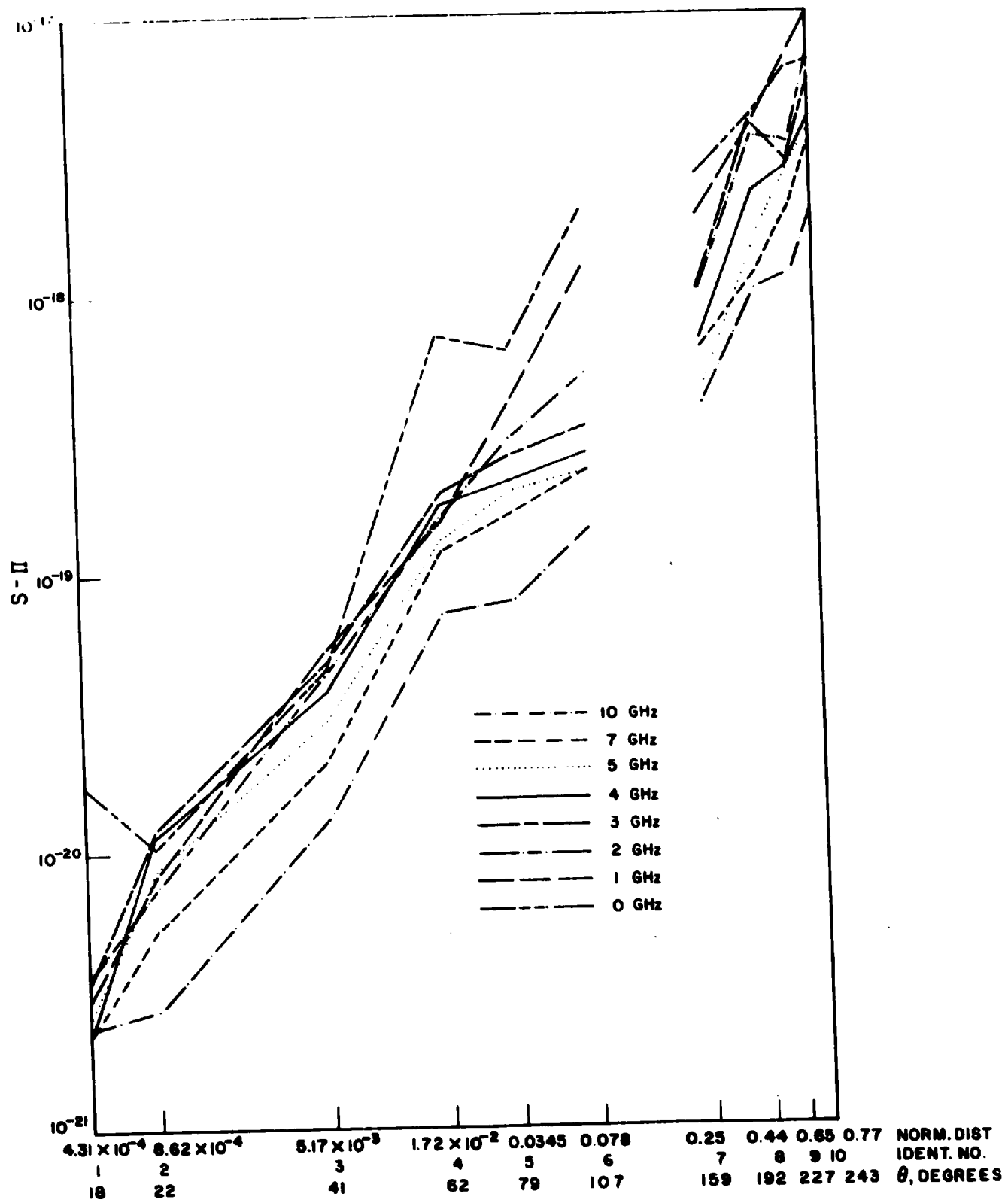


FIG. 4.30 $S-II$ VS. NORMALIZED DISTANCE (MODIFIED KINO GUN).

for a Kino short gun, and Φ and Z are the normalized voltage and distance respectively in Kino gun theory. The observation regarding high values of F_0 at some noise planes in the low frequency region does not agree with Eq. 4.4 where G is proportional to f . The diocotron gain between planes at $\theta = 180$ degrees and $\theta = 240$ degrees (approximately where the last noise plane is situated) is calculated from Eq. 4.4. For the mid-frequency range, say $f = 5$ GHz, it is about 2 db which is only slightly lower than 2.5 db obtained in this analysis for KG-A. It should be noted, however, that this is not true for the whole frequency range. The corresponding gain for KG-M is about 5.5 db which is expected since in Eq. 4.4 Φ and J_y for this case are different from the first gun.

F_0 (db) vs. the normalized distance (on a logarithmic scale) is plotted in Fig. 4.31. Though this variation is similar to that for the previous gun shown in Fig. 4.18, yet the noise parameter, F_0 , at the last noise plane in this gun is a few db higher than in the previous gun. This is also expected since Ψ is proportional to beam current which is higher in this case. The rate of change of F_0 (db) with distance is maximum in the low potential region which is true for the previous gun also. In both guns the rate of increase of F_0 with distance at the last noise plane is approximately three orders below that at the first noise plane. A qualitative explanation for the maximum rate of increase of F_0 in the low potential region, based on low beam velocity in this region, was given earlier in Section 4.4.4.

4.5.5 Noise Parameters for the OC Case. The noise parameters do not significantly differ under the OC and SC conditions. A similar observation was made in Section 4.4.6 regarding these parameters for KG-A and a possible explanation was given in the same section.

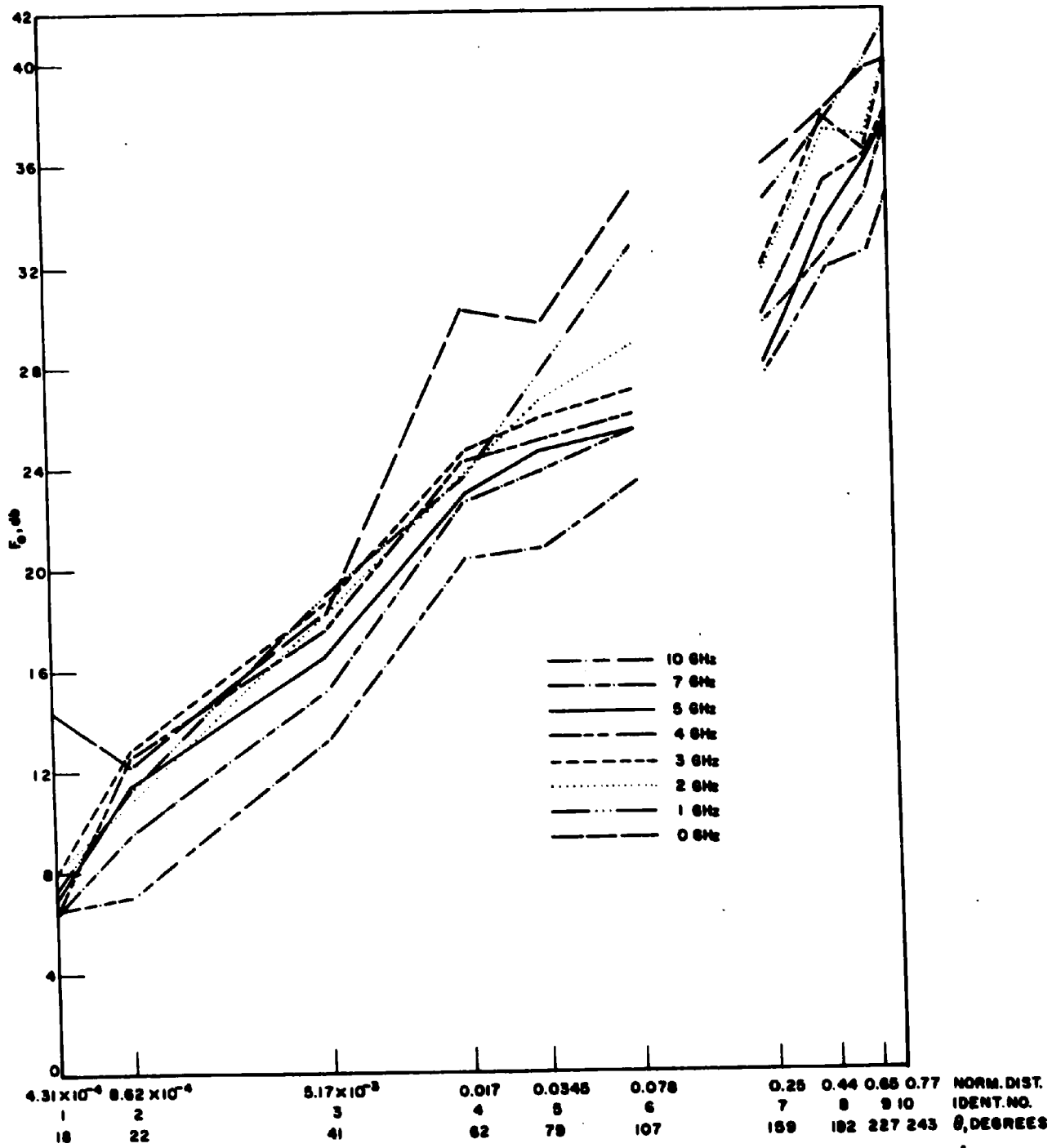


FIG. 4.31 F_0 VS. NORMALIZED DISTANCE (MODIFIED KINO GUN).

4.5.6 Variances of the Total Velocity and the Velocity Components of Electrons. The purpose of making these calculations has been explained in Section 4.4.6 in connection with similar calculations performed for KG-A. σ_u^2 , σ_y^2 and σ_z^2 have minimum values at the outer beam layer which was also true with the previous gun (KG-A). In Section 4.5.2c the same conclusion is drawn from the variation of ϕ across the beam. σ_u^2 and σ_z^2 are almost equal at the central and inner layers of the beam whereas σ_y^2 is generally maximum at the inner layer. This shows that the kinetic temperature at the central and inner parts of the beam is higher than that at the outer layer. This agrees well with the conclusions of Section 4.4.6.

The variation in the variances along the upper and lower edges of the beam can be studied from Tables 4.6 and 4.7 where the indicated rectangles are situated along a part of the upper and lower beam edges respectively. The parentheses in the tables have the same meanings as those in Tables 4.2 through 4.4.

In Table 4.6 σ_u^2 , σ_y^2 and σ_z^2 for the upper beam layer have a minimum at rectangle 125 ($\omega_c t \approx \pi$). Similar minima were observed for σ_u^2 and σ_z^2 in Table 4.2 for KG-A at the same transit angle. It was indicated in Section 4.4.6 that at this transit angle $\dot{y}$ is maximum according to the Kino theory. It is not clear how this correlation is brought about. After passing through this minimum σ_u^2 rises again, whereas it does not vary significantly in KG-A. Except for the minimum, σ_z^2 does not differ appreciably in the two guns studied. It should be noted that σ_y^2 did not have a minimum around $\theta = \pi$ in KG-A and the rate of increase in σ_y^2 along the upper beam edge after $\omega_c t = \pi$ in KG-M is lower than that found in KG-A. Similar calculations for the lower beam

edge are given in Table 4.7. The general variation of σ_u^2 , σ_y^2 and σ_z^2 for the lower beam edge is similar to that described above. In both guns the inner edge values are slightly higher than those at the outer edge, thereby confirming the existence of maximum velocity fluctuations at the lower beam edge.

Table 4.6

Variance Calculations for Rectangles Along the Upper Beam Edge (KG-M)

Rectangle Number	$\sigma_u^2 \times 10^{-10}$	$\sigma_y^2 \times 10^{-11}$	$\sigma_z^2 \times 10^{-11}$
192	20.22 (22.0)	1.49 (1.50)	1.14 (1.00)
179	16.44 (18.0)	0.91 (1.00)	1.74 (1.73)
152	12.33 (15.4)	0.90 (0.86)	1.68 (2.00)
125	9.50 (10.2)	0.87 (0.79)	1.14 (1.30)
97	11.90 (11.8)	1.48 (1.49)	1.70 (1.80)
55	17.30 (17.7)	1.86 (2.10)	2.10 (1.85)

It should be recalled that in this gun the focus electrodes are modified to yield a more or less laminar flow. The beam in this gun is wider than that in KG-A. This modification seems helpful in reducing the velocity fluctuations in the y- and z-directions since σ_y^2 and σ_z^2 are generally lower in KG-M than in KG-A. In the end it should be noted that in both guns σ_z^2 does not vary significantly along the upper and lower edges of the beam, whereas σ_y^2 increases toward the gun exit, indicating increasing kinetic temperature in the y-direction and almost constant temperature in the z-direction.

Table 4.7

Variance Calculations for Rectangles Along the Lower Beam Edge (KG-M)

Rectangle Number	$\sigma_u^2 \times 10^{-10}$	$\sigma_y^2 \times 10^{-11}$	$\sigma_z^2 \times 10^{-11}$
190	22.0 (19.5)	1.7 (1.4)	1.62 (1.37)
177	21.8 (21.5)	1.5 (1.2)	2.67 (2.50)
150	16.3 (17.2)	1.4 (1.1)	2.70 (2.78)
123	13.2 (13.2)	2.6 (2.1)	2.40 (2.49)
82	12.9 (11.9)	5.2 (4.6)	2.25 (2.10)
68	14.2 (12.9)	5.7 (5.0)	2.00 (1.83)

As in Table 4.4, the noise planes used in Table 4.8 are parallel to and in the neighborhood of the cathode plane. σ_u^2 for the second plane is lower than that for the first plane but this is probably due to the likelihood of appreciable statistical error in the variance calculations for planes in the very low velocity region. σ_u^2 , σ_y^2 and σ_z^2 all have sharp increases between the fourth and fifth noise planes where y has a peak as shown in Fig. 4.26. The same behavior was noted in Section 4.4.6 with KG-A. Thus it appears that a sharp rise in the velocity variances occurs when the space-charge density, which decreases with increasing transit angle, passes through the Brillouin value. After the sixth noise plane, σ_u^2 decreases toward noise plane 8a ($\omega_c t \approx \pi$) where it also has a low value in KG-A. σ_u^2 , σ_y^2 and σ_z^2 for the upper beam layer in this gun each attains a minimum at this noise plane. Llewellyn-Peterson coefficients for the Kino gun model were derived by Wadhwa³² and recalculated up to $\omega_c t = 2\pi$ by the author³⁸. Both the y- and z-velocity fluctuations for a given input velocity fluctuation have a

minimum around the transit angle $\omega_c t = \pi$. However, velocity fluctuations caused by the current fluctuations at the input plane have similar minima around 0.75π . Thus the minima observed in the velocity variances around $\theta = \pi$ in both Kino-type guns agree well with Wadhwa's results.

Table 4.8

Variance Calculations for the Set of Noise Planes Parallel to the Cathode (KG-M)

Noise Plane Number	$\sigma_u^2 \times 10^{-10}$	$\sigma_y^2 \times 10^{-11}$	$\sigma_z^2 \times 10^{-11}$
1	2.5 (1.8)	0.10 (0.10)	0.33 (0.26)
2	1.9 (1.7)	0.18 (0.16)	0.34 (0.28)
3	5.3 (4.7)	0.53 (0.46)	0.33 (0.30)
4	7.9 (6.7)	0.67 (0.63)	0.40 (0.42)
5	15.2 (17.2)	0.12 (0.13)	0.92 (1.00)
6	17.6 (18.7)	0.98 (0.90)	1.62 (1.78)
7a	14.6 (17.3)	1.03 (1.01)	1.99 (2.39)
8a	12.2 (13.1)	1.79 (1.79)	2.12 (2.41)

The OC values of these variances are not significantly different from their SC counterparts for both guns. Again the explanation given in Section 4.4.5 for a similar behavior of the noise parameters applies in this case also.

4.6 Monte Carlo Analysis of Noise Transport in a Crossed-Field Parallel Plane Diode Gun

The simulation and the d-c characteristics of the space-charge flow in a crossed-field parallel plane diode gun were described in Section 2.6. The noise characteristics of this diode gun are discussed in the following sections. As in the case of KG-A and KG-M one set each of the parallel planes (planes parallel to the cathode surface) and the normal planes (planes normal to the beam) are selected as shown in Fig. 4.32. Their identification numbers and positions, etc., are given in Table 4.9. The transit angle corresponding to each noise plane in terms of an electron moving along the central trajectory is given in Column 4. However, it is only approximate since the central trajectory is not the exact trajectory of an electron and, in addition, the formula used for calculation of transit angles is only exactly true for a space-charge-free case.

As mentioned earlier in Section 4.1 Wadhwa and Rowe¹¹ have performed a Monte Carlo analysis of a nearly identical diode under the SC assumption. Their analysis and the one presented here differ in the following important features. All noise parameters in their analysis are defined on a rectangle basis whereas in this analysis they are generally discussed on a cross-sectional basis unless the variation of a given noise parameter over a given beam cross section is to be studied. In the latter case these parameters are then defined on a rectangle basis. Wen and Rowe³⁹ have also defined the noise parameters on both the layer and cross-sectional bases in their Monte Carlo analysis of an O-type diode. The other important difference lies in the definitions of the kinetic potential and the autocorrelation and cross-correlation functions.

Table 4.9
Noise Plane Positions for the Parallel Plane Crossed-Field Diode Gun

Noise Plane Number	y or s Distance (Meter)	Normalized Distance $y/s_{\max}$ or $s/s_{\max}$ ($s_{\max} = 3.23 \times 10^{-3} \text{ m}$)	Transit Angle $\theta = \omega_c t$ (Degrees)	Remarks
1	$y = 3.00 \times 10^{-6}$	9.280×10^{-4}	5	Parallel Plane
2	$= 8.00 \times 10^{-6}$	2.470×10^{-3}	8	Parallel Plane
3	$= 18.00 \times 10^{-6}$	5.570×10^{-3}	12	Parallel Plane
4	$= 6.00 \times 10^{-5}$	1.855×10^{-2}	22	Parallel Plane
5	$= 2.00 \times 10^{-4}$	6.185×10^{-2}	42	Parallel Plane
6	$= 5.00 \times 10^{-4}$	0.155	69	Parallel Plane
7	$s = 1.05 \times 10^{-3}$	0.325	98	Normal Plane
8	$= 1.52 \times 10^{-3}$	0.469	120	Normal Plane
9	$= 2.18 \times 10^{-3}$	0.675	148	Normal Plane
10	$= 2.72 \times 10^{-3}$	0.840	170	Normal Plane

The various noise parameters, Φ , Ψ , etc., pertaining to noise transport in a crossed-field diode under the SC assumption are discussed in detail in the following sections.

4.6.1 SPDS of Kinetic Noise-Voltage Modulation, Φ (SC Case).

a. Variation of Φ with Frequency. Φ for the set of parallel planes 1 through 6 is plotted against frequency in Fig. 4.33. For the first two planes Φ assumes very large values around $f = 0$. Wadhwa and Rowe¹¹ have also reported the occurrence of maximum values of Φ at $f = 0$. For the rest of the planes in this set the variation of Φ with frequency is within the variability of the spectral estimates. Except for the seventh noise plane, Φ for all noise planes in the normal plane set has a peak around 1.5 GHz as shown in Fig. 4.34. Since calculations of Φ are made at steps of 0.5 GHz it is rather difficult to locate the peak precisely. Although this peak matches more closely twice the cyclotron frequency (≈ 0.9 GHz), it appears to be due to f_h since similar peaks were observed in the normal plane sets of KG-A and KG-M.

It is very interesting to note that the first plane in the normal set of noise planes has a similar variation of Φ for KG-A, KG-M and the crossed-field diode gun. For planes before and after this plane, this variation falls into two distinctly different types. These planes (Nos. 8, 7 and 7 respectively for KG-A, KG-M and the crossed-field diode) are approximately located such that the beam in each case has undergone a bend of about 45 degrees at this plane.

b. Variation of Φ with Distance Along the Beam. Φ is plotted against normalized distance (logarithmic scale) for several selected frequencies in Fig. 4.35. As in the case of KG-A and KG-M Φ increases with distance along the beam such that Φ at the last noise plane is

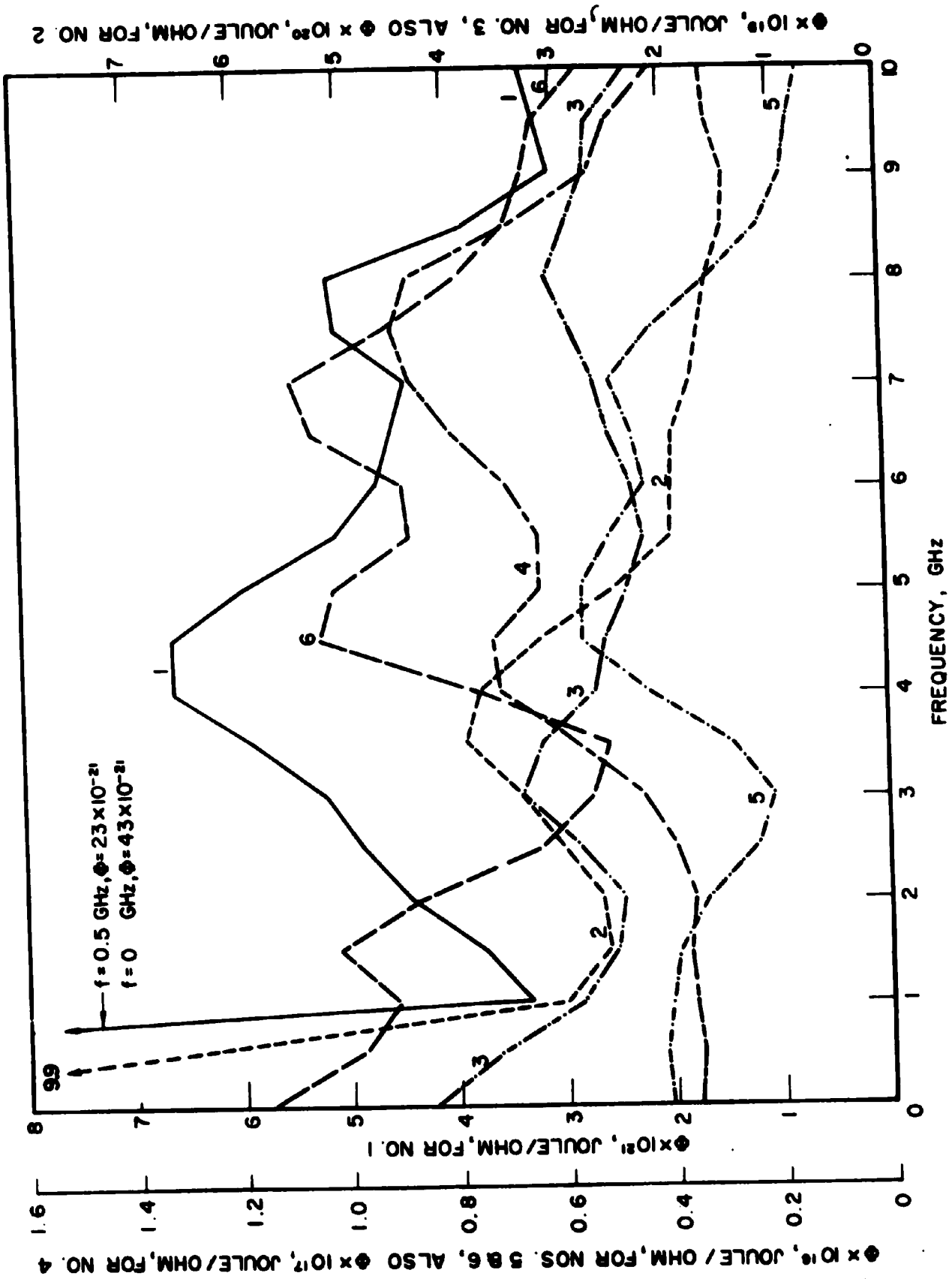


FIG. 4.33 ϕ VS. FREQUENCY FOR NOISE PLANES 1 THROUGH 6 (PARALLEL PLANE DIODE GUN).

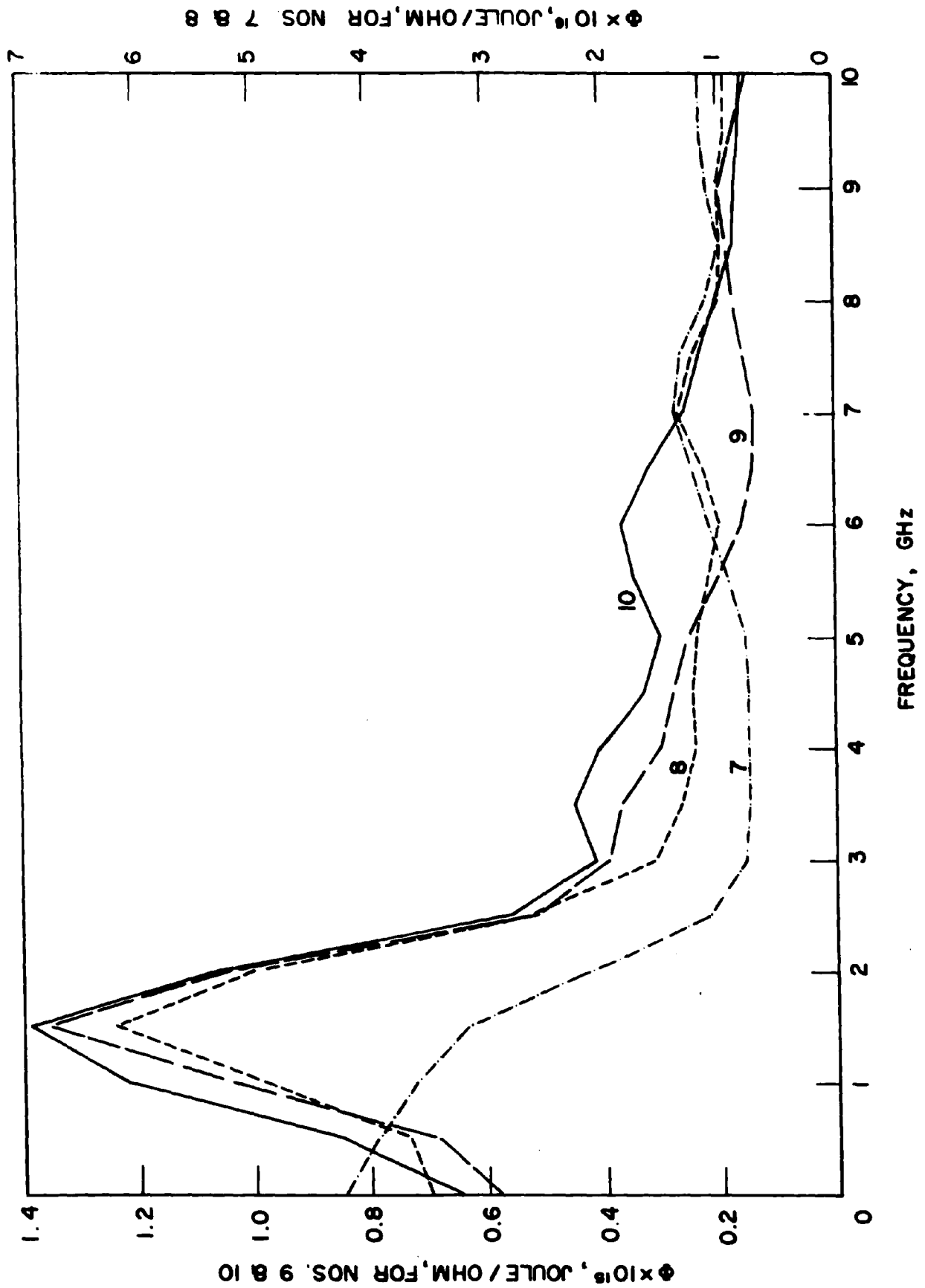


FIG. 4.34 ϕ VS. FREQUENCY FOR NOISE PLANES 7 THROUGH 10 (PARALLEL PLANE DIODE GUN).

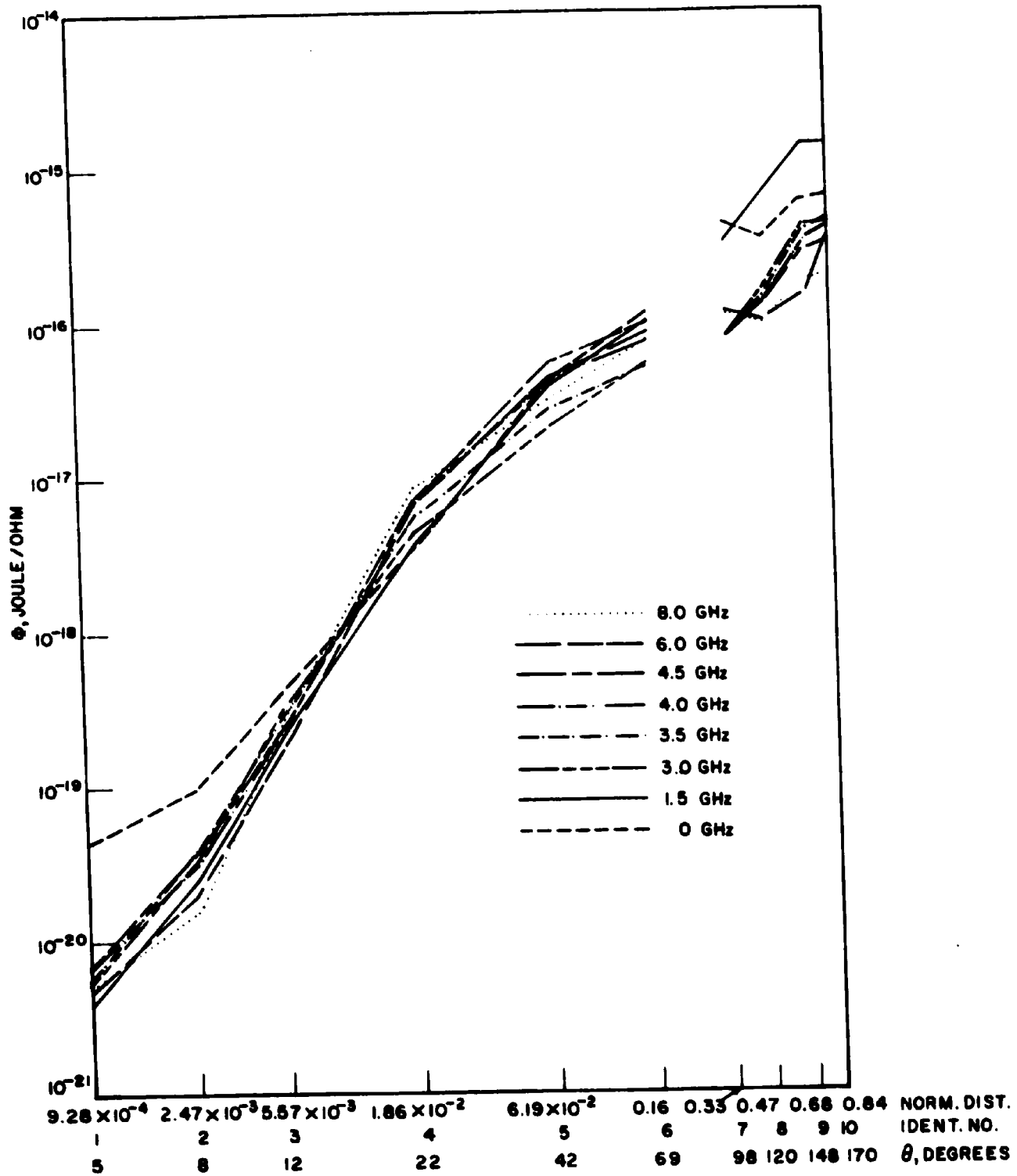


FIG. 4.35 Φ VS. NORMALIZED DISTANCE (PARALLEL PLANE DIODE GUN).

almost five orders higher than that at the first. This agrees well with Wadhwa's results³². However the actual values disagree by several orders and it appears that he did not consider the effect of the "grouping of electrons", i.e., the D factor (his D_1^2) in his calculations of Φ . This effect is not apparent in the case of other parameters since they are presented in normalized forms. F, the noise figure, is also not affected since he has used a normalized S- Π . In the present analysis it is also found that the rate of increase of Φ with distance along the beam also increases except in the region between the fifth and sixth noise planes. The reason for this discrepancy is not understood.

c. Variation of Φ Across the Beam. In this case as well as for the two previous guns (KG-A and KG-M) the variation of Φ across the beam cross section is significant in the low frequency region up to the hybrid or the plasma frequency at the potential minimum. A typical variation of Φ with frequency for the parallel plane set is shown by noise plane 5 in Fig. 4.36. In this frequency range Φ is minimum at the outer edge (rectangle 127) and its value at the center and inner edge (rectangle 125) is not very different. In the rest of the frequency band Φ for the center and the outer edge is almost the same. For the normal plane set Φ at the outer edge is again minimum. It is either maximum at the center of the beam or equal to that at the inner edge. A similar behavior was also found in KG-M. Consideration of the d-c beam velocity of the different layers of the beam again indicates, as in both KG-A and KG-M, that velocity fluctuations are minimum at the outer beam edge while the inner edge generally has large velocity fluctuations. As in the other two guns Φ calculated on the cross-sectional basis is lower than that for any layer of the beam.

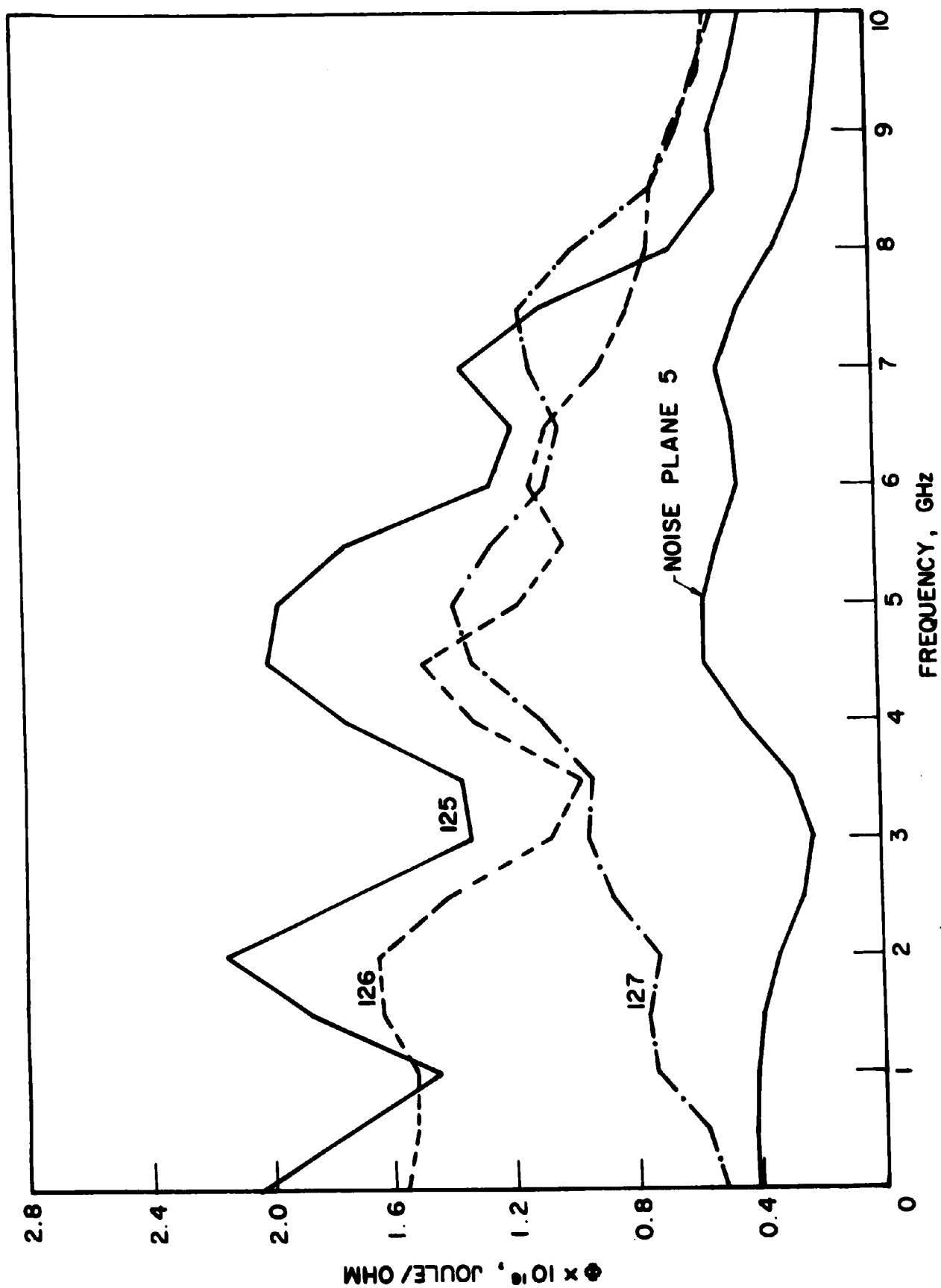


FIG. 4.36 Φ VS. FREQUENCY FOR RECTANGLES 125, 126, 127 AND NOISE PLANE 5 (PARALLEL PLANE DIODE GUN).

4.6.2 SPDS of the Noise-Current Modulation, Ψ (SC Case).

a. Variation of Ψ with Frequency. Ψ for the set of parallel planes (Fig. 4.37) is lowest around $f = 0$, goes through a peak somewhere in the range of 3.5 GHz to 4.5 GHz, and shows a moderate decreasing tendency toward the high frequency end. This range of frequencies where Ψ assumes rather large values is in the neighborhood of the potential minimum hybrid (or plasma) frequency. Similar behavior of Ψ was found in both KG-A and KG-M. The variation of Ψ with frequency for the set of normal planes shown in Fig. 4.38 is very different from that described above. Almost every curve shows a slight peak between 1.5 GHz and 2.0 GHz, and planes 9 and 10 have relatively high and approximately constant values of Ψ after 6 GHz. Ψ has relatively high values in the high frequency region, a condition which was not found in the other two guns. Wadhwa³² observed similar results but only for rectangles in the neighborhood of the cathode, whereas in the present analysis this happens for planes nearer the gun exit. The reason for this behavior is not clear.

b. Variation of Ψ with Distance Along the Beam. The variation of Ψ with normalized distance for a few selected frequencies is shown in Fig. 4.39. For all frequencies Ψ decreases between the first and sixth noise planes. However the rate of decrease is highest between the first noise plane and the second which is in the neighborhood of the potential minimum. This rate of decrease of Ψ decreases rapidly from one plane to the next such that between the fifth and sixth noise planes it is about three orders below that between the first and second noise planes. For the set of normal planes, Ψ increases slightly for most of the frequencies, yet the rate of change of Ψ with distance is very small. Similar variations of Ψ with distance were found in both KG-A and KG-M with the

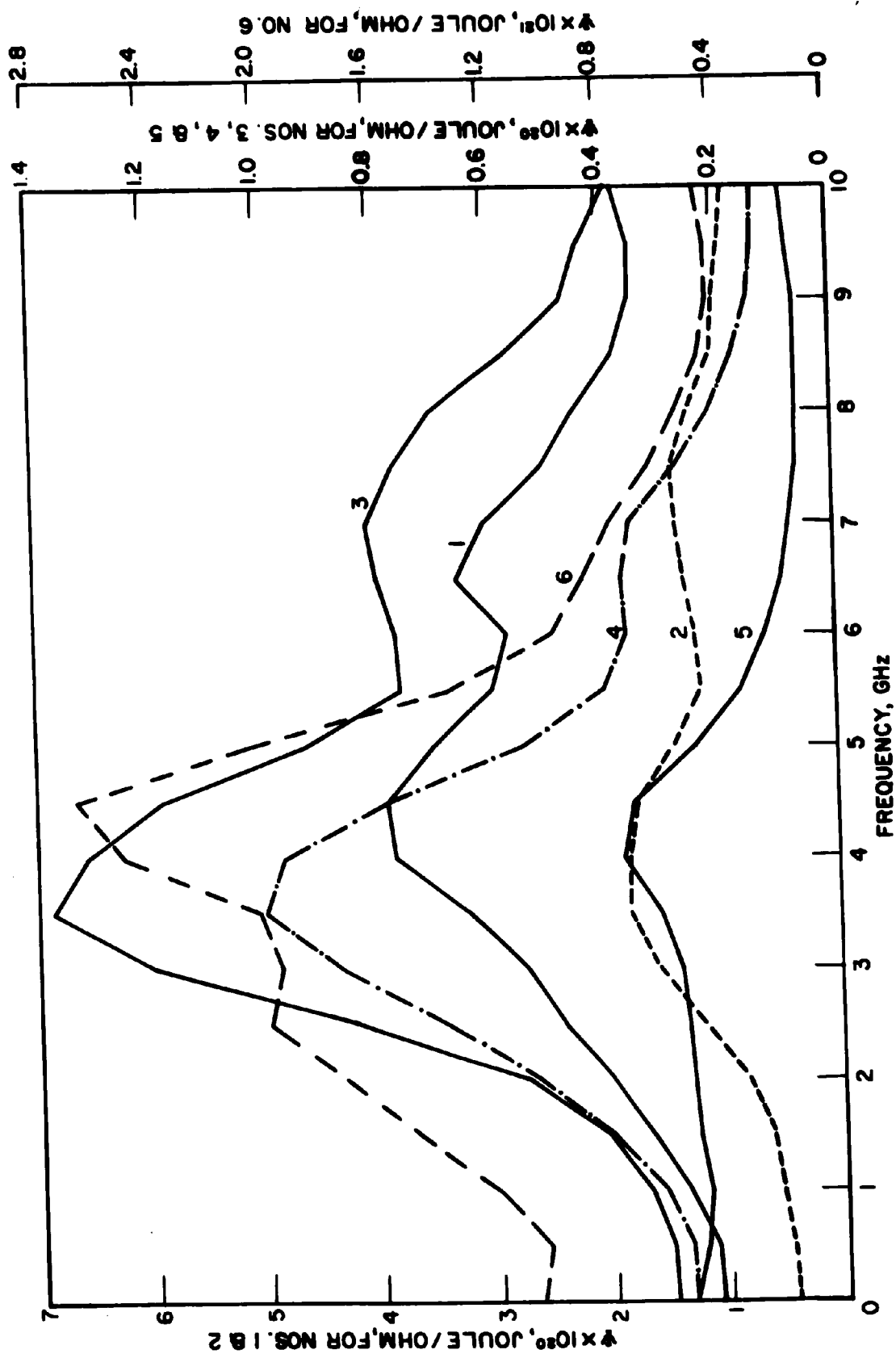


FIG. 4.37 Ψ VS. FREQUENCY FOR NOISE PLANES 1 THROUGH 6 (PARALLEL PLANE DIODE GUN).

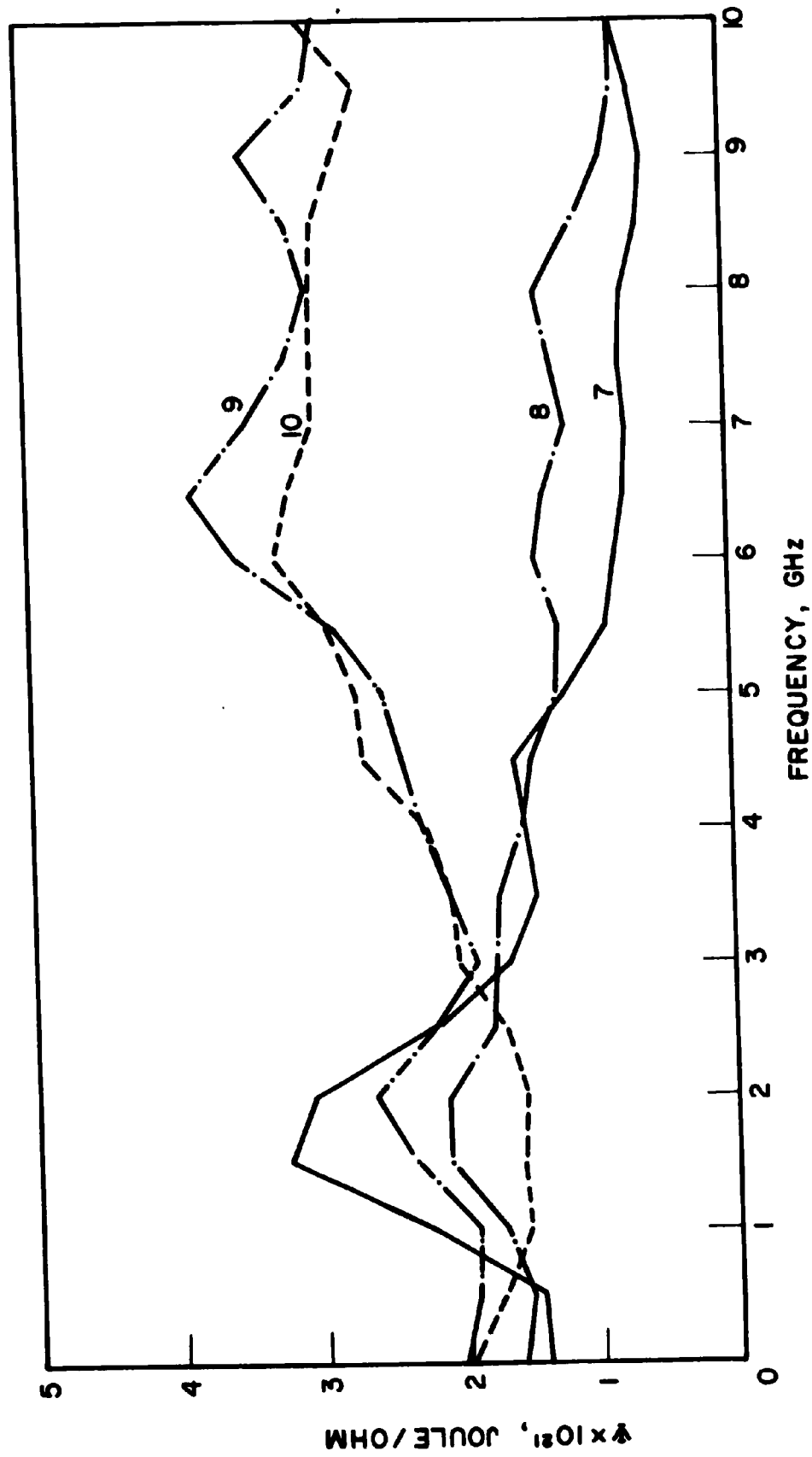


FIG. 4.38 Ψ VS. FREQUENCY FOR NOISE PLANES 7 THROUGH 10 (PARALLEL PLANE DIODE GUN).

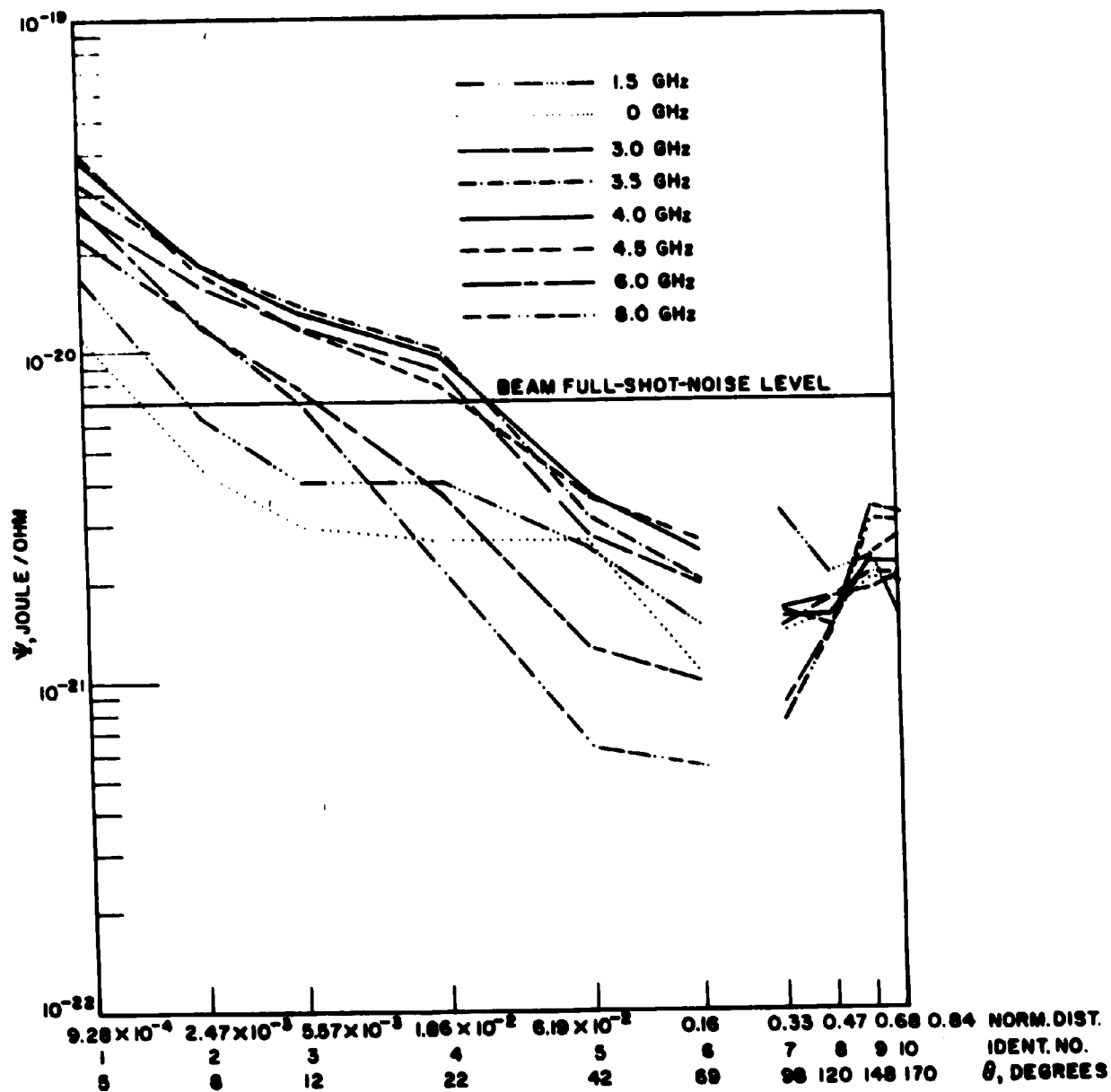


FIG. 4.39 Ψ VS. NORMALIZED DISTANCE (PARALLEL PLANE DIODE GUN).

exception that in this case no peak occurs in the region of the parallel plane set of noise planes. The variation of Ψ with distance was correlated in Sections 4.4.2b and 4.5.2b with the variation of ρ/ρ_0 along the beam. According to this, a peak is expected between the fifth and sixth planes. In this case ρ varies quite a bit over each cross section and hence the change of ρ from a value above ρ_0 to one below it is not very sharp. Hence it is probable that the peak does not show up. In the absence of any analytical expression for ρ/ρ_0 corresponding to a beam obtained in this case (crossed-field diode gun) it is difficult to check the above interpretation. As in the other two guns Ψ is reduced below the full-shot-noise level in the beam in a very small distance. Comparison between the values of Ψ at the gun end with the full-shot-noise level in the beam clearly shows that shot noise is partially smoothed out, which is in agreement with the results obtained by Wadhwa and Rowe¹¹.

c. Variation of Ψ_J Across the Beam. Ψ assumes its minimum value at the inner edge of the beam cross sections over the whole frequency band except for a few spot frequencies which could be due to the variability of estimates. The correlation between the potential minimum profile and the variation of Ψ_J across the beam cross section was put forward in Section 4.4.2c and used to explain the variation of Ψ across the beam cross section in both KG-A and KG-M. The potential minimum is shallower over the right edge of the cathode in both guns, whereas in this case (crossed-field diode gun) it is deepest around this region. Thus, this seems to be a good test case for the hypothesis of Section 4.4.2c where Ψ_J is a function of the potential minimum profile. Accordingly Ψ_J should have a minimum value at the inner edge of the

beam (rectangle 112) which is true as described above and shown in Fig. 4.40. The insert shows the potential minimum profile over the length of the cathode. Furthermore, since the mean value of $|v_m|$ is approximately the same in front of the center and the outer edge (left end) of the cathode, Ψ for these parts of the beam cross section should not differ appreciably from each other, which is found to be true for most of the cross sections analyzed. Figure 4.40 shows the variation of Ψ_J with frequency for a typical beam cross section (noise plane 6) and its constituent rectangles. Ψ_J for the cross section is lower than that for individual rectangles in this cross section. A similar situation was found earlier for Φ in Section 4.6.1c and for Φ and Ψ_J for KG-A and KG-M.

4.6.3 Noise Parameters Π , S and $S-\Pi$ (SC Case). Π varies along the beam and as in KG-A and KG-M experiences a change in sign also. In this case it remains positive up to the fourth noise plane. It has negative values at the low frequency end for noise planes 5 and 6. Planes 7 and 8 have large negative values of Π at the low frequency end and small positive values at the high frequency end. Π has moderate values over most of the frequency band for planes 9 and 10. Figure 4.41 illustrates the above behavior of Π by curves 3, 5, 7 and 10 which represent the groups described above. The interpretation for the above behavior of Π in terms of the growth of slow and fast cyclotron and hybrid modes as a function of distance along the beam and frequency goes along the same lines as detailed in Section 4.4.3. Thus Π is nonconservative in this case also.

Figure 4.42 shows S vs. the normalized distance (on a logarithmic scale) for a set of selected frequencies. For noise planes 1 through 4

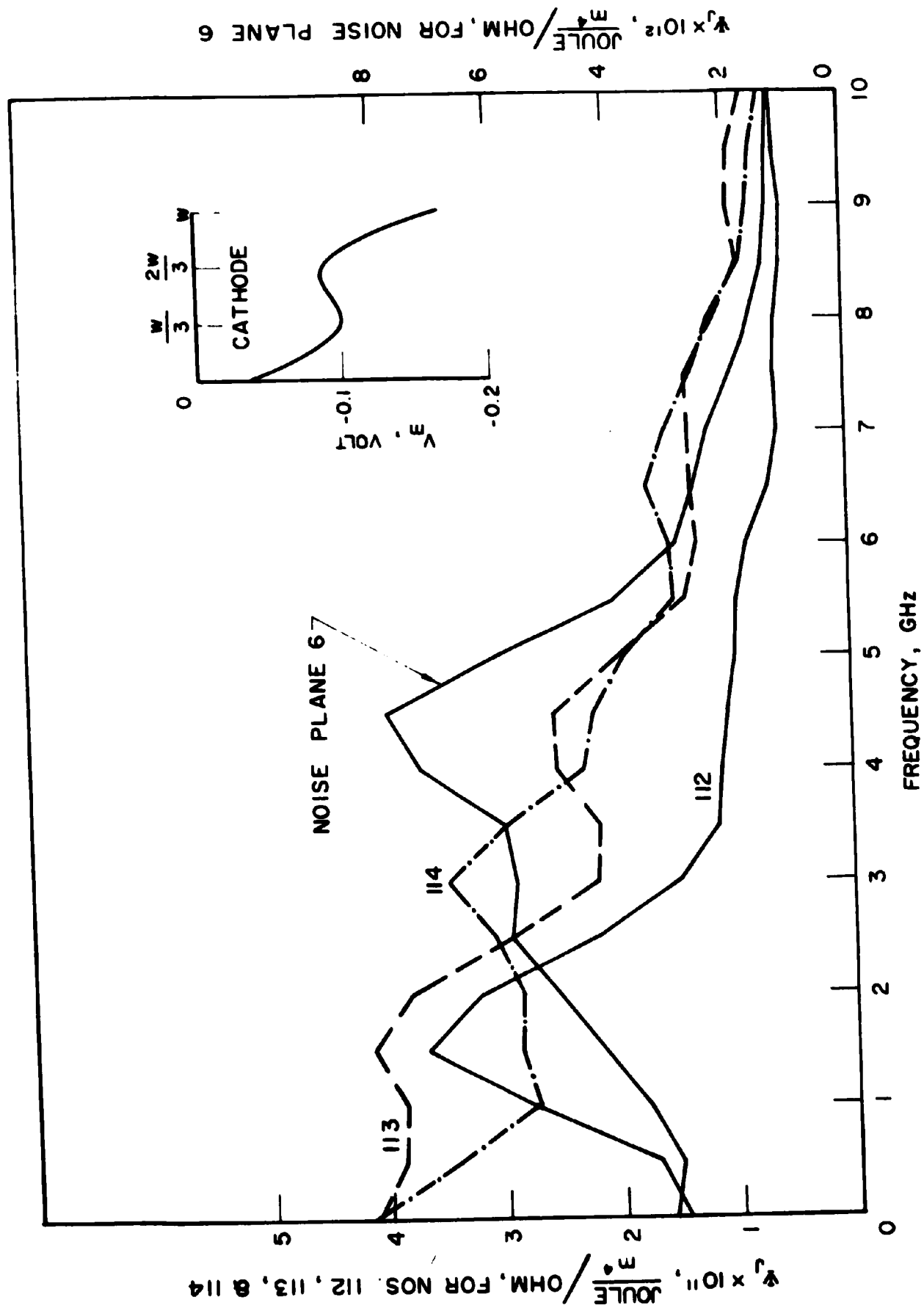


FIG. 4.40 Ψ_J VS. FREQUENCY FOR RECTANGLES 111, 112, 113 AND NOISE PLANE 6 (PARALLEL PLANE DIODE GUN).

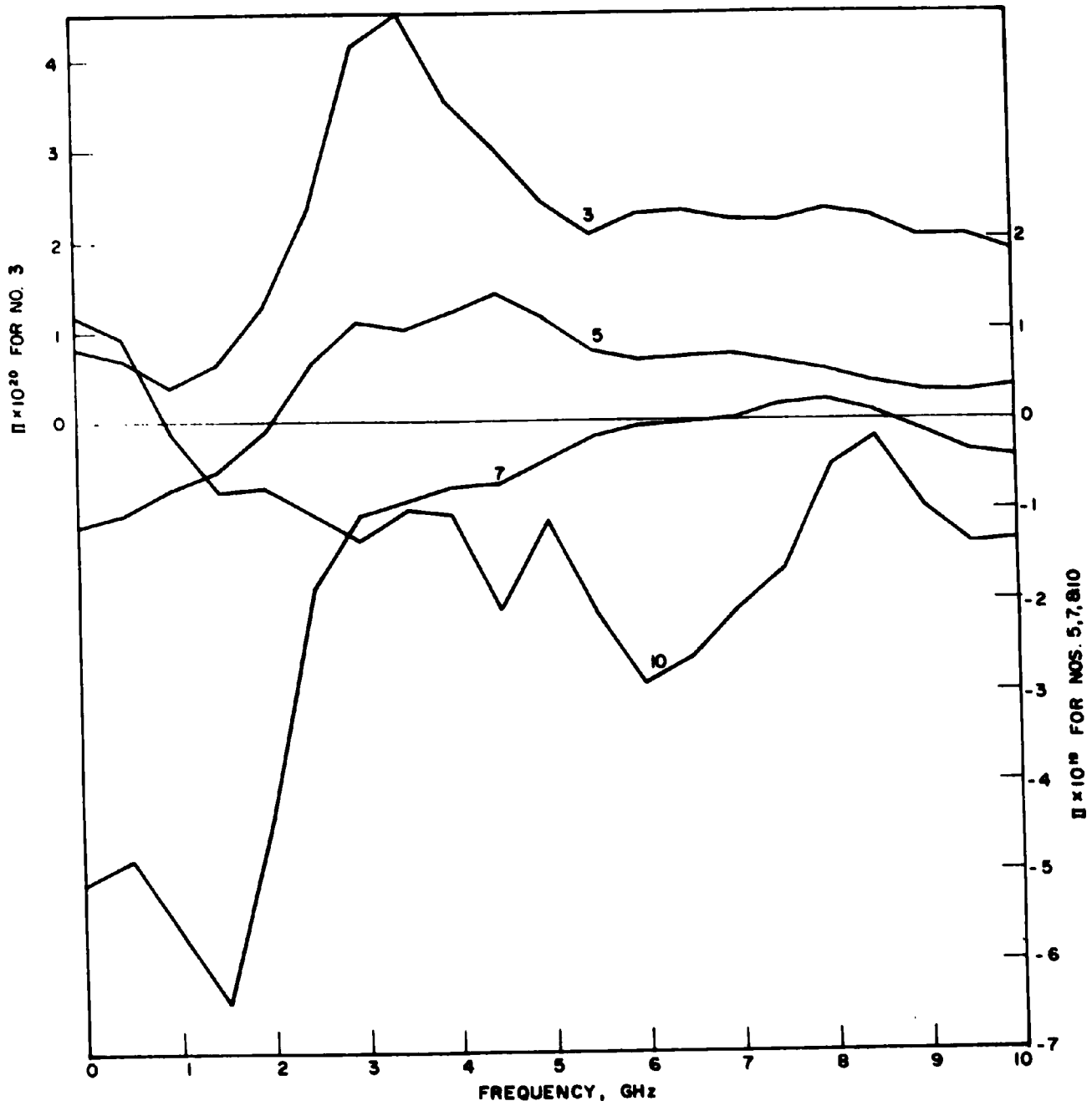


FIG. 4.41 Π VS. FREQUENCY FOR NOISE PLANES 3, 5, 7
AND 10 (PARALLEL PLANE DIODE GUN).

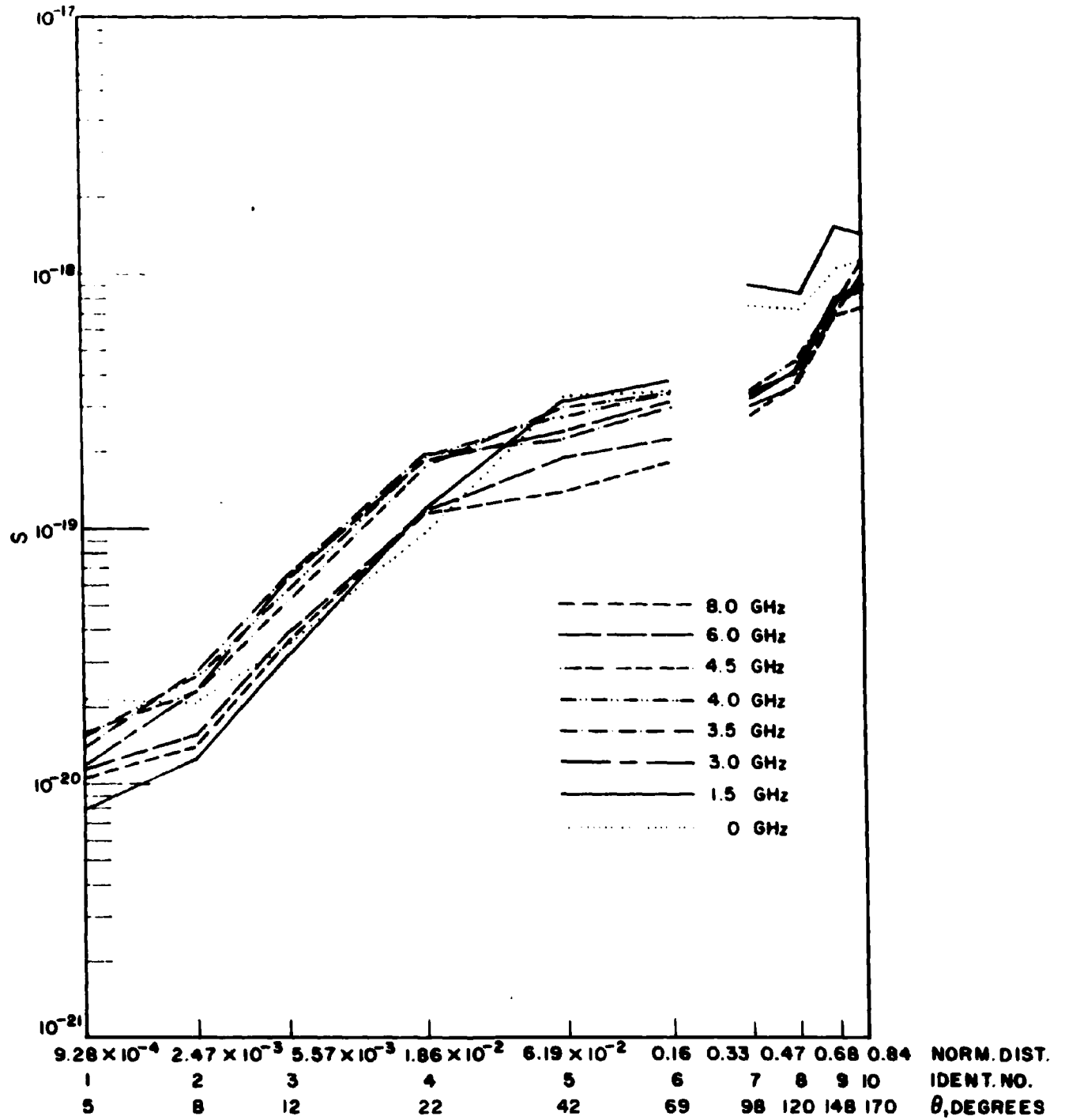


FIG. 4.42 S VS. NORMALIZED DISTANCE (PARALLEL PLANE DIODE GUN).

near the cathode, S has a peak around 3.5 GHz whereas for the set of normal planes the peak appears around 1.5 GHz. Similar peaks were found for Ψ for the corresponding noise planes in Section 4.6.2a. The increase in S between the first and tenth noise planes is slightly less than two orders of magnitude for most of the frequencies. Wadhwa and Rowe¹¹ also found S increasing over this portion of the beam.

$S-\Pi$ has large values around $f = 0$ for noise planes 1 and 2 which correspond to high values of Φ at these noise planes for the same frequencies. Similarly the peaks of $S-\Pi$ at the normal plane around $f = 1.5$ GHz coincide with those of Φ for the same set of noise planes. From Fig. 4.43 it can be seen that $S-\Pi$ increases by slightly less than three orders between the first and last noise planes. Thus $S-\Pi$ is also nonconservative along the beam.

4.6.4 Noise Parameter F_0 (SC Case). F_0 in db is plotted against the normalized distance of the noise planes (on a logarithmic scale) for several selected frequencies in Fig. 4.44. It can be seen that F_0 increases monotonically with distance along the beam for all frequencies up to the ninth noise plane. However, for some frequencies F_0 decreases slightly between the ninth and tenth noise planes. In the normal plane set relatively large values of F_0 at 1.5 GHz are probably due to a peak of Φ at this frequency for these planes. A separate calculation shows that the rate of change of F_0 (db) with distance (y or $y/y_{\max}$) is maximum between the first and second noise planes. This rate decreases rapidly with an increase in y such that between noise planes 5 and 6 it is only about one percent of its value between the first and second planes. In the set of normal planes the slow rate of increase of F does not change significantly between planes 7 and 9, whereas for the region between the

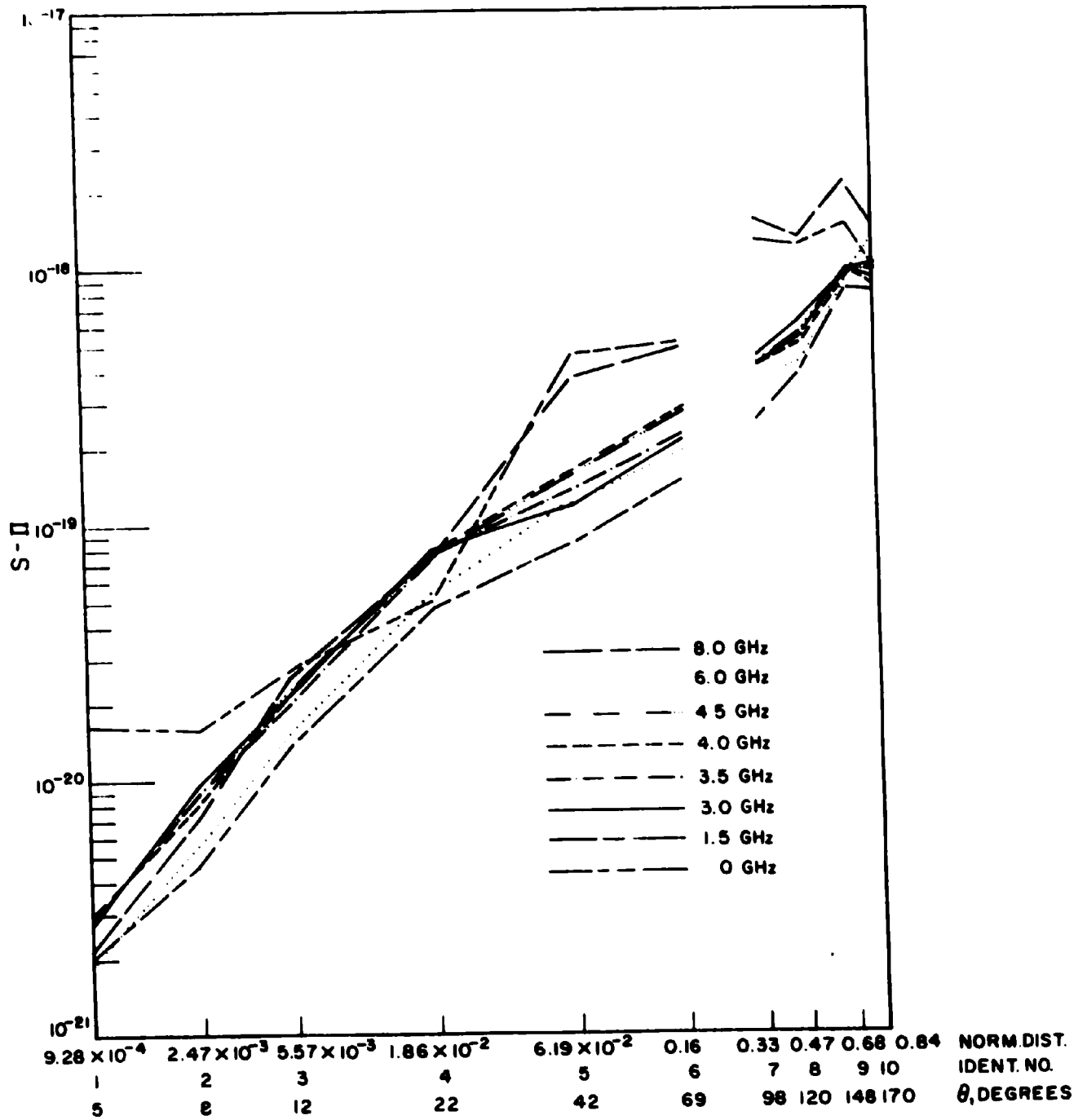


FIG. 4.43 $S-II$ VS. NORMALIZED DISTANCE (PARALLEL PLANE DIODE GUN).

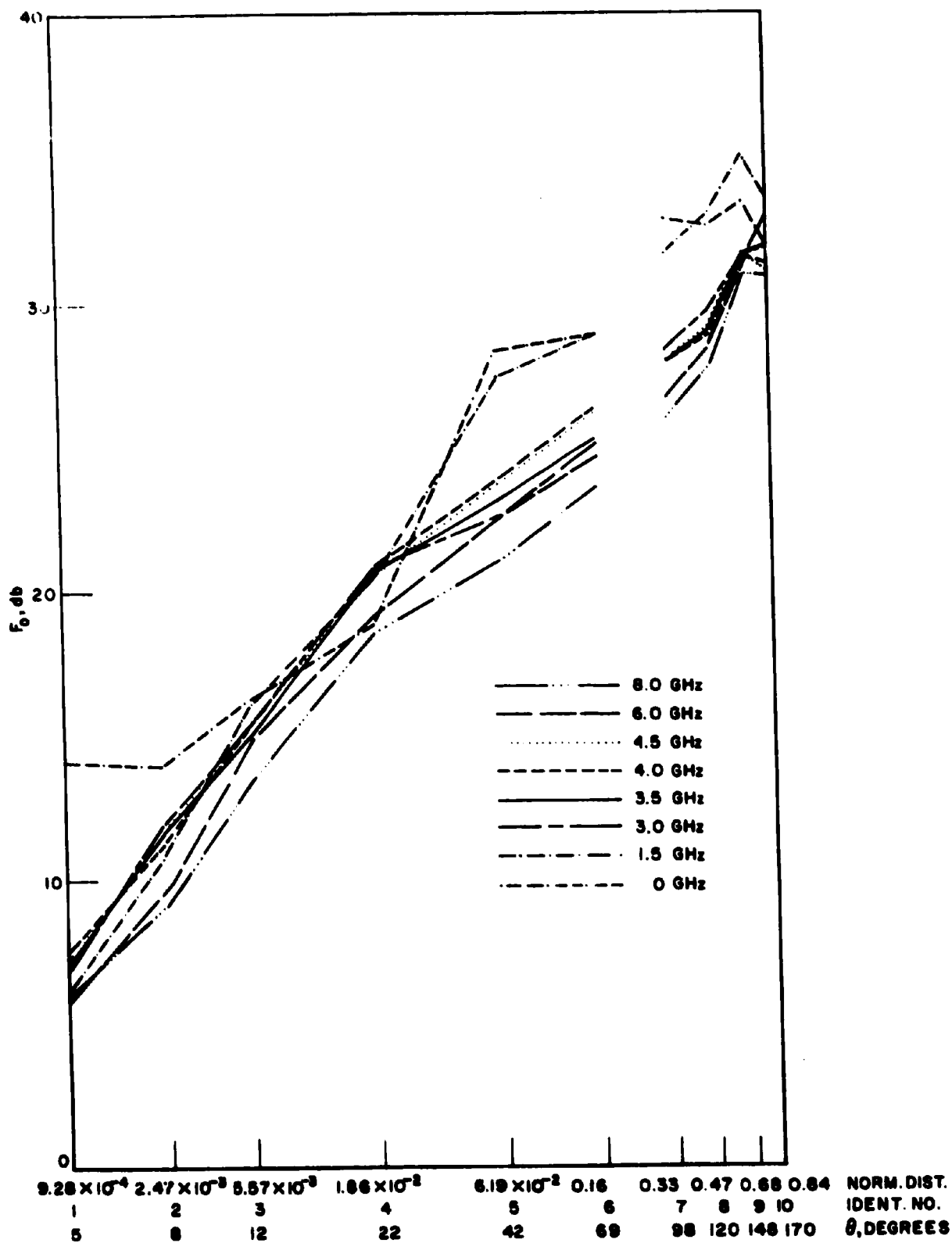


FIG. 4.44 F_0 VS. NORMALIZED DISTANCE (PARALLEL PLANE DIODE GUN).

last two planes (Nos. 9 and 10) it goes down and even changes sign for some frequencies. The maximum rate of increase of F_o with distance was also found near the cathode with the other two guns (KG-A and KG-M) and it is possibly due to large diocotron gain for low beam velocity in this region. The absolute noise figure curves of Wadhwa and Rowe¹¹ calculated on a rectangle basis (using Haus' formula for an O-type beam) qualitatively agree with the results obtained in this analysis. However their F (F_o in this analysis) values are generally a few db higher and in particular near the cathode the difference is significant. For most of the frequency band their F is around 18 db whereas in this analysis F_o is between 6 and 8 db. This discrepancy appears to be due to the difference in bases of calculation of F or F_o . Wadhwa and Rowe calculate F (or F_o) on the rectangle basis whereas in the present analysis F_o corresponds to a beam cross section as a whole. In studying the variation of Ψ_j and Φ across a beam cross section for KG-A, KG-M and crossed-field diode guns it is found that the noise parameter value calculated on the cross-sectional basis is less than that for any constituent rectangle in that cross section. Thus higher values of F (or F_o) could be expected when it is calculated on a rectangle basis. The other possible reason is the difference in the definition of the kinetic potential and autocorrelation and cross-correlation functions between Wadhwa and Rowe and the present analysis.

4.6.5 Noise Parameters for the OC Case. As in the case of the other two guns (KG-A and KG-M), the OC parameters are not significantly different, but are a bit smaller than the SC parameters. A probable explanation was elaborately discussed in Section 4.4.5.

4.6.6 Variances of Total Velocity and Velocity Components. σ_u^2 ,

σ_y^2 and σ_z^2 are minimum at the outer beam layer for both sets of parallel and normal planes. σ_u^2 for the central and inner layers is not very different but σ_z^2 and σ_y^2 increase from the outer to the inner edge. Thus the kinetic temperature is minimum at the outer layer of the beam. This is also found to be true in both KG-A and KG-M. It may then be concluded that the crossed-field beams have minimum kinetic temperature (across the beam cross section) at the upper layer.

The available data permits us to study only approximately the variations of σ_u^2 , σ_y^2 and σ_z^2 along the upper and the lower edges of the part of the beam lying between noise planes 6 and 10. As seen from Tables 4.10 and 4.11, σ_u^2 and σ_z^2 decrease monotonically whereas σ_y^2 varies somewhat between $\theta \approx 70$ degrees and the gun exit ($\theta \approx 180$ degrees). The variance values for the inner layer are higher than those along the upper layer of the beam. The same thing was noted for KG-A and KG-M.

Table 4.10

Variance Calculations for Rectangles Along the Upper Beam Edge
(Parallel Plane Diode Gun)

Rectangle Number	$\sigma_u^2 \times 10^{-10}$		$\sigma_y^2 \times 10^{-11}$		$\sigma_z^2 \times 10^{-11}$	
	SC	OC	SC	OC	SC	OC
114	8.69	9.37	0.47	0.49	0.87	0.91
86	5.93	6.00	0.75	0.66	0.78	0.80
72	4.10	4.20	0.85	0.72	0.48	0.48
42	2.75	2.79	0.53	0.63	0.26	0.28

Table 4.11

Variance Calculations for Rectangles Along the Lower Beam Edge
(Parallel Plane Diode Gun)

Rectangle Number	$\sigma_u^2 \times 10^{-10}$		$\sigma_y^2 \times 10^{-11}$		$\sigma_z^2 \times 10^{-11}$	
	SC	OC	SC	OC	SC	OC
112	19.10	20.90	0.83	0.83	4.57	5.33
98	9.33	12.52	2.10	2.31	2.12	2.26
81	6.44	5.83	1.10	1.03	0.66	0.58

The variances σ_u^2 , σ_y^2 and σ_z^2 calculated for noise planes 1 through 5 on a cross-sectional basis are given in Table 4.12. As before in KG-A and KG-M, σ_u^2 at the second noise plane is lower than at the first. This is, as explained before, probably due to appreciable statistical error involved in the calculation of σ_u^2 for the second noise plane in the neighborhood of the potential minimum. Otherwise, σ_u^2 , σ_y^2 and σ_z^2 all increase up to the fifth noise plane. The sudden rise in the variances between the fourth and fifth noise planes cannot be explained as before in Sections 4.4.6 and 4.5.6 in terms of a peak of Ψ occurring in the same region, since such a peak of Ψ is not observed in this gun. The possible reason for the absence of such a peak of Ψ was given in Section 4.6.2b. It should be noted that σ_u^2 and σ_z^2 grow up to a point along the beam in the cathode region and then decrease toward the gun exit. However, σ_y^2 increases in the cathode region as above but does not decrease along the beam at the gun exit. Similar behavior of σ_u^2 was noted for both KG-A and KG-M. Thus it can be concluded that the kinetic temperature of a crossed-field beam changes rapidly in the cathode region and

this variation becomes small for regions farther from the cathode. Furthermore, it is higher at the inner beam edge than at the outer edge.

Table 4.12

Variance Calculations for Some Noise Planes in the Parallel Plane Set
(Parallel Plane Diode Gun)

Noise Plane Number	$\sigma_u^2 \times 10^{-10}$		$\sigma_y^2 \times 10^{-11}$		$\sigma_z^2 \times 10^{-11}$	
	SC	OC	SC	OC	SC	OC
1	2.04	1.74	0.11	0.09	0.29	0.27
2	1.23	0.94	0.09	0.08	0.32	0.28
3	2.12	2.15	0.20	0.21	0.30	0.27
4	7.96	8.20	0.72	0.72	0.49	0.44
5	17.50	20.16	1.33	1.24	1.44	1.56

4.7 Conclusions

The noise parameters for the three crossed-field guns have been investigated and comparisons have been made wherever possible. A brief outline of the general conclusions of this chapter is given below. The noise parameter ϕ increases with an increasing rate toward the gun exit and has a minimum value at the outside edge of any beam cross section. In the cathode region ϕ assumes large values at $f = 0$ and has a broad minimum near the potential minimum hybrid or plasma frequency. For noise planes near the gun exit ϕ has a peak around the local hybrid or plasma frequency. Space-charge smoothing is observed in all of the guns. An interesting correlation is also found between the variations of Ψ and the space-charge density along the beam. Ψ_j is observed to vary over a

beam cross section mainly in the low frequency range up to the potential minimum hybrid or plasma frequency. This variation approximately follows the potential minimum profile over the length of the cathode in such a way that Ψ_J is minimum for the beam layer which passes through the deepest region in the potential minimum profile. Π has positive values in the cathode region which indicates that the fast hybrid or cyclotron wave predominates in this region. As the beam is accelerated the slow wave grows with maximum growth occurring at low frequencies. Π is generally negative over most of the frequency band at points remote from the cathode. S varies along the beam and has peaks where the peaks of Ψ also appear for these noise planes. $S-\Pi$ increases by several orders of magnitude over the length of the beam and has a relatively large value around $f = 0$. The rate of increase of the noise parameter, F_0 , is maximum near the cathode and decreases with increasing distance along the beam. Similar to $S-\Pi$, F_0 also has large values at $f = 0$ for several noise planes. Otherwise F_0 varies by a few db in the mid-frequency band for most of the noise planes in the guns. No significant difference, except for the low frequency region, is observed between the SC and OC noise parameters. The variance calculations made for the total velocity and velocity components of the electrons in various regions of the beams in the Kino-type guns show that σ_y^2 increases in general between the cathode and the gun exit. σ_u^2 and σ_z^2 increase in the cathode region but remain relatively constant throughout the remaining part of the beam. It is interesting to note that σ_u^2 , σ_y^2 and σ_z^2 increase sharply around $\theta = \pi/2$ where a peak in Ψ was observed and the space-charge density in the beam changes from greater to less than the Brillouin value, ρ_0 . Also σ_u^2 and σ_z^2 attain minimum values around $\theta = \pi$ where $\dot{y}$ attains the

maximum value in the Kino short gun. In the crossed-field parallel plane diode gun, σ_u^2 , σ_y^2 and σ_z^2 all increase, as before, in the cathode region, but σ_u^2 and σ_z^2 decrease in the remaining part of the beam whereas they do not vary much in KG-A and KG-M. The variance values at the inner layer of the beam in each of the above crossed-field guns are found to be higher than the corresponding values at the outer edge. This indicates higher kinetic temperature and more noise along the inner edge of the crossed-field beam.

CHAPTER V. NOISE TRANSPORT IN A PARALLEL PLANE

CROSSED-FIELD DRIFT SPACE

5.1 Introduction

The usefulness of the study of noise transport in a crossed-field drift space as a guide in designing noise transformers and in devising other means of noise reduction in crossed-field devices was indicated in Section 2.7. The space-charge-flow simulation and the d-c characteristics of the beam in an incremental distance (approximately equal to one mean beam cyclotron wavelength) of drift spaces A and M (A follows the abbreviated Kino gun and M follows the modified Kino gun) have been described in Chapter II. The beam shapes and the noise planes which are normal to the beam are shown in Figs. 2.15 and 2.16. It should be noted that noise planes identified by the same letter are approximately at the same position in the two drift spaces. The noise parameters Φ , Ψ , etc., corresponding to the noise planes in drift spaces A and M, are calculated for both short- and open-circuit assumptions. As can be seen from Figs. 2.15 and 2.16 the beams are undulating and of varying thickness. It was mentioned in Section 2.10 that drift space A is designed for use with the abbreviated Kino gun. The same drift space configuration is used to follow the electron gun denoted as KG-M. Since the position and width of the beam at the exit of KG-M are different than those for KG-A, a mismatch in electric field at the interface of the gun exit and the drift space is expected, resulting in a slightly asymmetrical beam (Fig. 2.16). It should be noted that because of the better match

at the entrance to drift space A the beam is symmetrical as shown in Fig. 2.15. Also the beam current in drift space M is approximately 1.35 times that in drift space A.

Due to the limitation on the maximum number of electrons permitted in the drift space geometry at any one time, it was indicated in Section 2.8 that both Δt and the D factor have to be altered. For drift spaces A and M, Δt is chosen as 1.5×10^{-11} sec and 2×10^{-11} sec respectively. This change in Δt affects the accuracy of the power spectra at the high frequency end of the investigated frequency band (0-10 GHz). Thus the noise power spectra calculated at the high frequency end for the drift space cases are a little less accurate than those for the electron guns described in Chapter IV. However, this should not significantly affect the conclusions drawn from the noise power spectra calculations made for the beams in the drift spaces considered in this chapter.

Various noise parameters Φ , Ψ , etc., for the two drift spaces under the SC assumption are described in the following sections. A comparison between the noise parameters for beams in drift spaces A and M is attempted, to some extent, on the basis of the difference in the d-c characteristics of the beams described above. In order to study the variation of the noise parameters along the beam, several frequencies of interest are selected and Φ , Ψ , etc., are calculated on the cross-sectional basis and plotted as a function of distance normalized with respect to the mean beam cyclotron wavelength ($\lambda_c \approx 3.06$ cm). For convenience, the beam with noise plane positions is shown as an insert in some of these figures.

5.2 SPDS of the Kinetic Noise-Voltage Modulation, Φ (SC Case)

5.2.1 Variation of Φ with Frequency. Figures 5.1 and 5.2 show the variation of Φ with frequency for beams in drift spaces A and M respectively. Φ for noise planes in both cases has a peak either at 0.5 GHz or 1.0 GHz which is in the neighborhood of the cyclotron frequency, 0.47 GHz, or the hybrid frequency at the noise plane considered. The plasma frequency is found to vary across a given beam cross section and also along the beam due to the varying beam thickness. Calculation of the average hybrid frequency in the neighborhood of each noise plane shows that this frequency corresponds more or less to the peak of Φ for that particular plane. This suggests the influence of the local hybrid frequency on the parameter Φ . Since f_c and f_h are close to each other, the above conclusion is only qualitative. After passing through the peak value described above, Φ drops sharply until $f = 2$ GHz, decreases slowly until $f = 6$ GHz, and then has little variation in the remaining frequency range for most of the noise planes in both drift spaces. A similar variation of Φ with frequency was observed in Figs. 4.6 and 4.20 for noise planes situated near the exit region of KG-A and KG-M respectively. The behavior of Φ described above indicates that, far from the potential minimum, the noise contribution from the kinetic noise-voltage fluctuation increases sharply at the local hybrid frequency or possibly the cyclotron frequency.

5.2.2 Variation of Φ with Distance Along the Beam. Φ for noise planes A through H in drift space A is plotted in Fig. 5.3 against the normalized distance of these planes from the drift space entrance. For some frequencies Φ shows a slight peak at plane B,

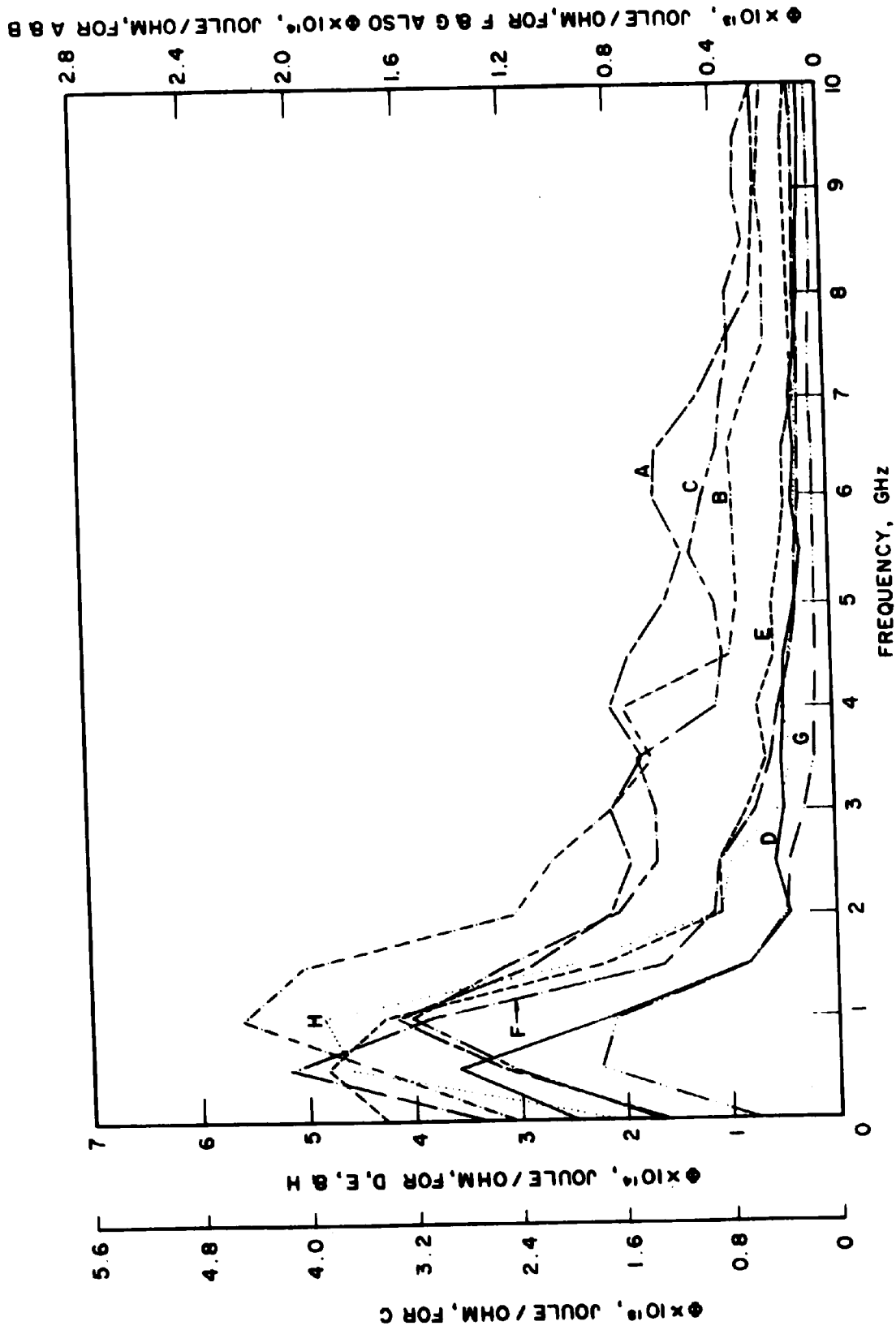


FIG. 5.1 ϕ VS. FREQUENCY FOR NOISE PLANES A THROUGH H (DRIFT SPACE A).

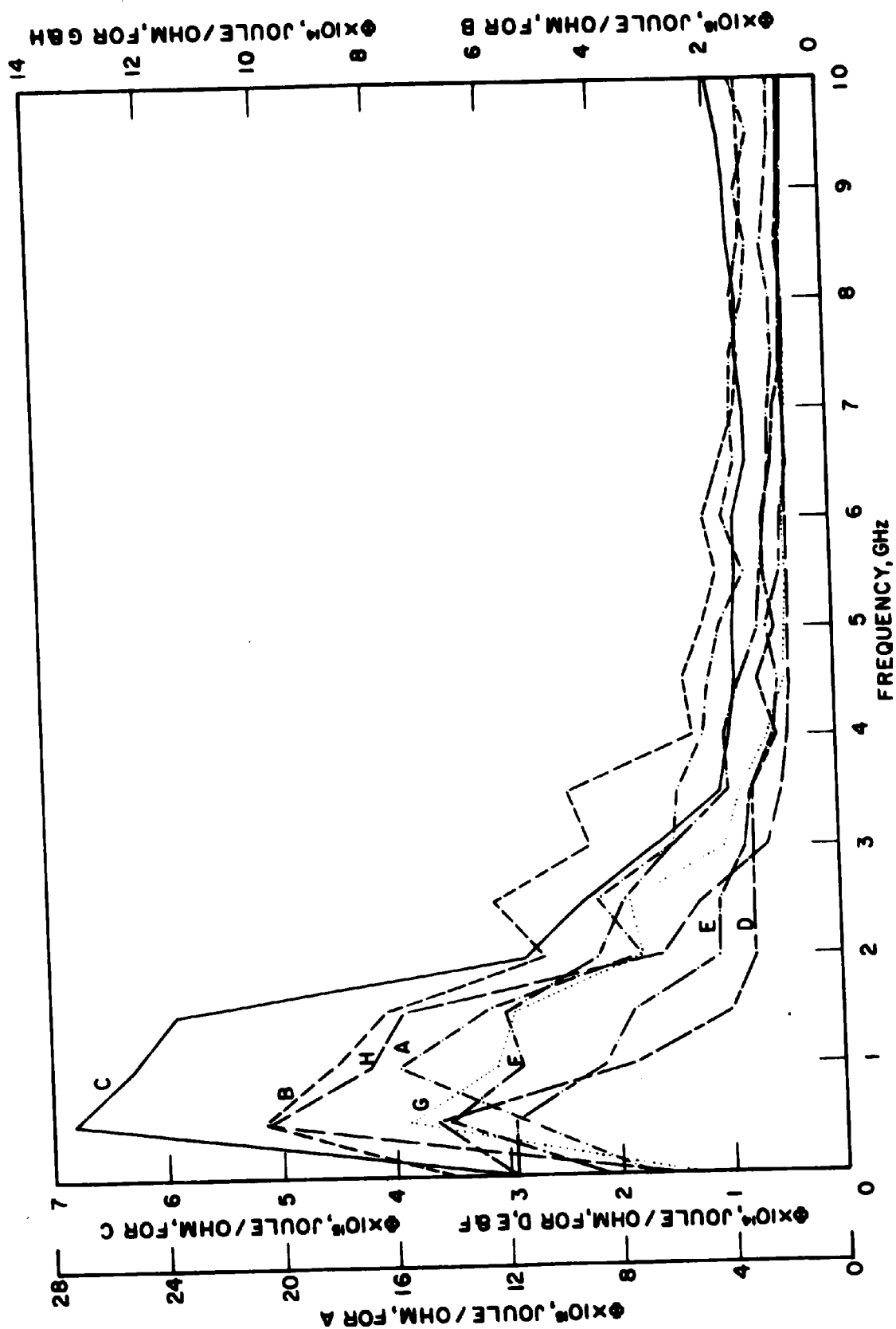


FIG. 5.2 Φ VS. FREQUENCY FOR NOISE PLANES A THROUGH H (DRIFT SPACE M).

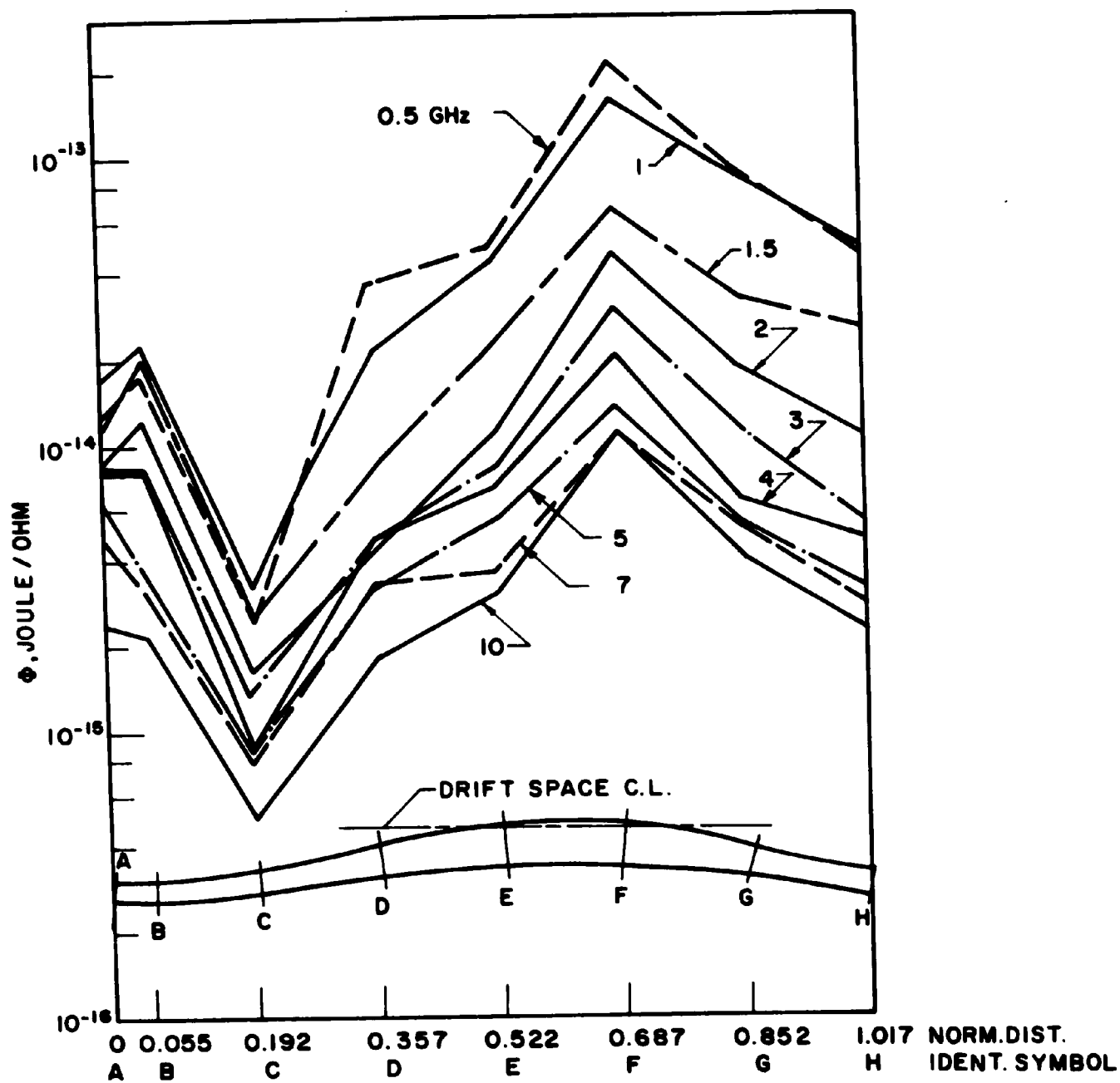


FIG. 5.3 Φ VS. NORMALIZED DISTANCE (DRIFT SPACE A).

otherwise $\phi_{\min}$ and $\phi_{\max}$ occur at planes C and F respectively for almost all frequencies. After passing through the maximum value, ϕ decreases toward the end of the drift tube geometry.

The variation of ϕ described above can be correlated, to some extent, with the beam location in the drift space. It is known that the diocotron gain per unit length, henceforth referred to as G_D , is maximum when the beam is symmetrical with respect to the two parallel conducting planes. The value of ϕ at a cross section of the beam thus depends on the distance along the beam (from the drift space entrance) and G_D which is a function of the beam position with respect to the two parallel conducting planes. Between planes C and E the beam is approaching the drift space center line and hence G_D should increase. Consequently between these two planes ϕ should also increase which agrees with Fig. 5.3. Between planes E and F, G_D is close to its maximum value and ϕ increases in this region also. After plane F the beam starts moving toward the lower parallel plate and hence G_D should decrease, but it is always positive. Therefore ϕ should increase at a slower rate in this region whereas it is actually found to decrease as shown in Fig. 5.3. Also $\phi_{\min}$ at plane C cannot be explained on the basis of the variation of diocotron gain per unit length as a function of the position of the beam with respect to the parallel conducting planes. Thus it appears that in addition to the diocotron gain along the beam there exists a standing wave pattern with maxima and minima of ϕ caused by the slow and fast cyclotron or hybrid waves as discussed in Section 4.4.3. Whether such a standing

wave pattern created by noise waves exists or not could be checked only if the noise analysis were continued over several beam cyclotron wavelengths. The presence of a growing space-charge mode responsible for the diocotron gain, however, corrupts the pattern to some extent.

A similar plot of ϕ vs. normalized distance for drift space M is shown in Fig. 5.4. $\phi_{\min}$ occurs somewhere between planes B and C or at plane C. After passing through this minimum, ϕ increases again up to plane G but the rate of change of ϕ is small between planes D and G. After plane G, ϕ decreases for some frequencies and increases for others. From Fig. 2.16 it can be seen that the beam position changes rapidly between planes C and D and the upper beam layer just crosses the center line of the drift space at plane D. As explained above, G_D should increase rapidly between planes C and D and attain a maximum value at the latter plane. This causes ϕ in Fig. 5.4 to grow rapidly between planes C and D and attain a fairly high value at plane D rather than at plane F as seen in Fig. 5.3. Since the upper layer of the beam moves away from the center line of the drift space between planes D and F, G_D in this region should decrease slightly below its maximum value around plane D. The upper beam layer again crosses the center line of the drift space somewhere between planes F and G. Hence G_D should increase in this region and help ϕ to increase likewise. In a manner similar to drift space A, the beam now starts moving toward the lower parallel plate resulting in a lower value of diocotron gain per unit length. As discussed earlier, ϕ should nevertheless increase but at a slower rate. In Fig. 5.4 ϕ increases up to plane F, and decreases for some frequencies and

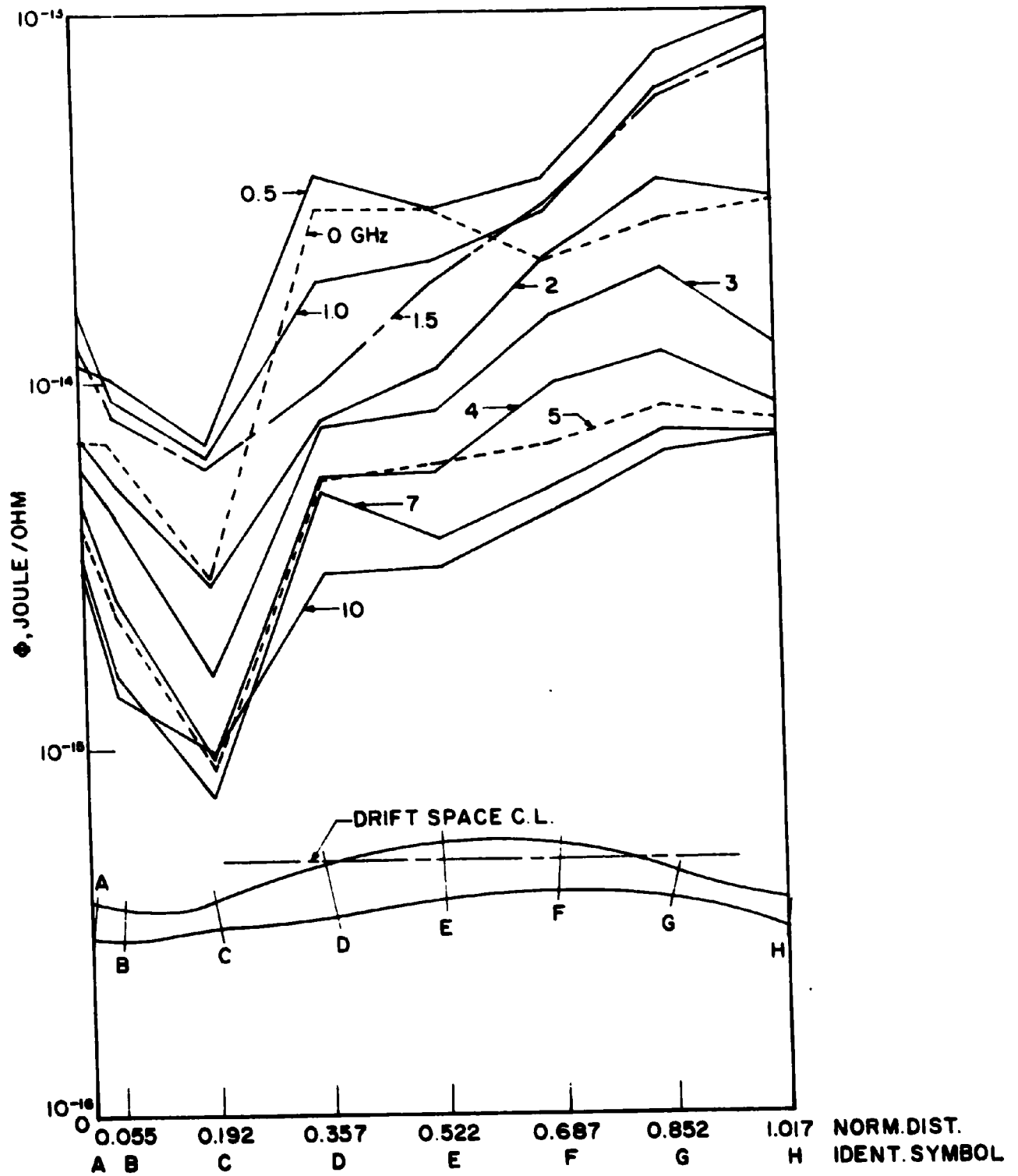


FIG. 5.4 Φ VS. NORMALIZED DISTANCE (DRIFT SPACE M).

increases for others between planes G and H. In Fig. 5.3 Φ decreases between planes F and G whereas this is not so in Fig. 5.4. One possible reason for this difference appears to be the peculiar beam shape in drift space B caused by the mismatch in electric field at the drift space entrance. This seems to have masked the declining tendency of Φ observed in Fig. 5.3. The variation of G_D with beam position relative to the parallel plates again cannot explain the occurrence of $\Phi_{\min}$ at plane C. Thus the possibility of the presence of a standing wave pattern of Φ , suggested earlier in this section, seems to exist. However, the effects of the undulating nature and varying thickness of the beam on the variation of Φ complicates the situation. Due to this, the standing wave pattern of Φ is not clearly discernible, particularly in Fig. 5.4.

5.3 SPDS of Noise-Current Modulation, Ψ (SC Case)

5.3.1 Variation of Ψ with Frequency. The variation of Ψ with frequency for all of the noise planes (A through H) of drift space A is shown in Fig. 5.5. Except for planes D and G, each plane has a large value of Ψ around 0.5 GHz which is very close to the cyclotron frequency of 0.47 GHz. Since the cyclotron and local hybrid frequencies are not far apart and the noise calculations are made in steps of 0.5 GHz, it is rather difficult to definitely assign these peaks of Ψ to either the cyclotron or local hybrid frequency. Ψ decreases rapidly until $f = 1.5$ GHz, varies slowly between 1.5 and 4.5 GHz and then remains constant at this value for the remaining high frequency region.

Ψ vs. frequency is plotted in Fig. 5.6 for all noise planes in drift space M. In contrast to Fig. 5.5, Ψ has several peaks and maintains relatively high values up to 3.5 GHz. Except for planes A and B

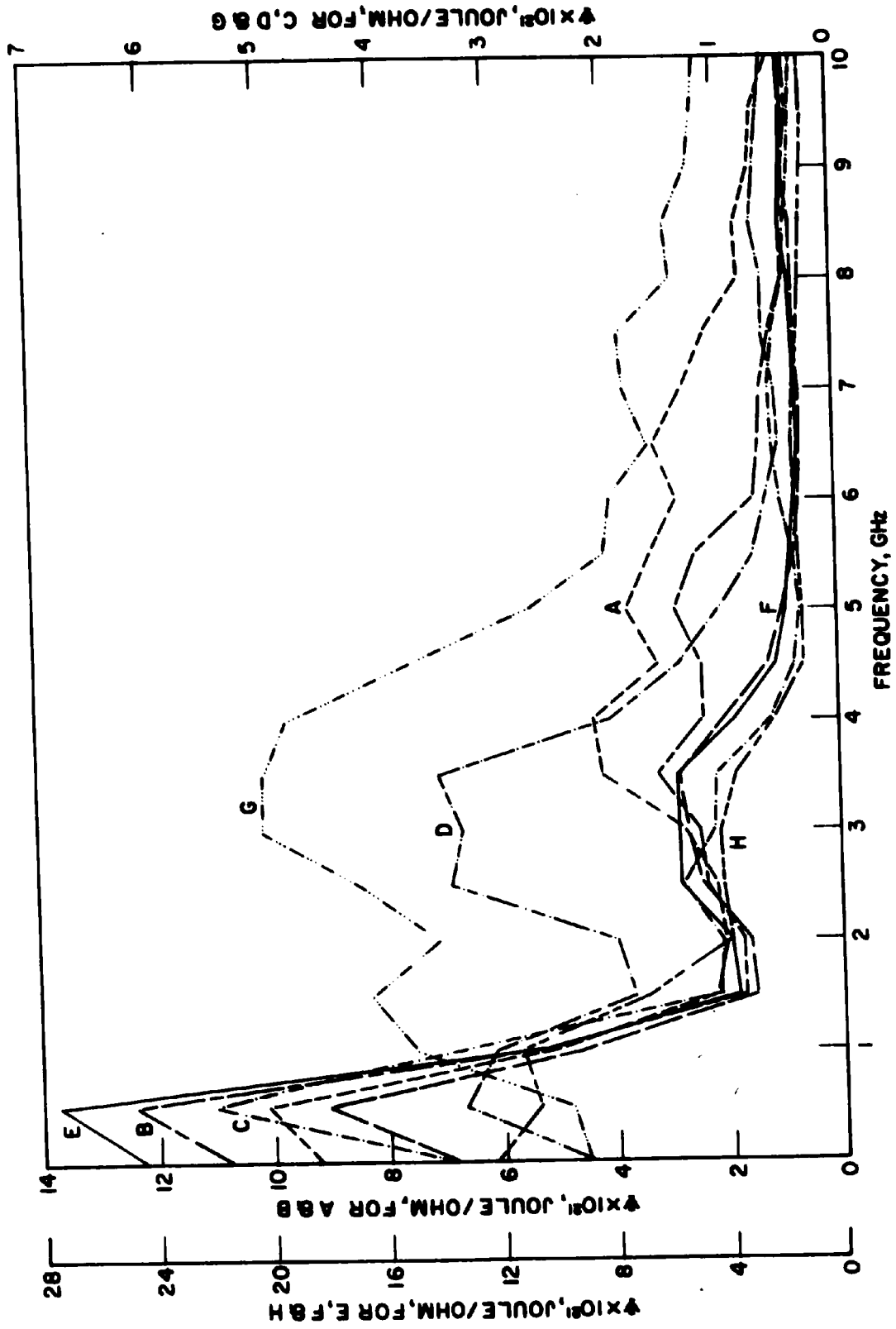


FIG. 5.5 γ VS. FREQUENCY FOR NOISE PLANES A THROUGH H (DRIFT SPACE A).

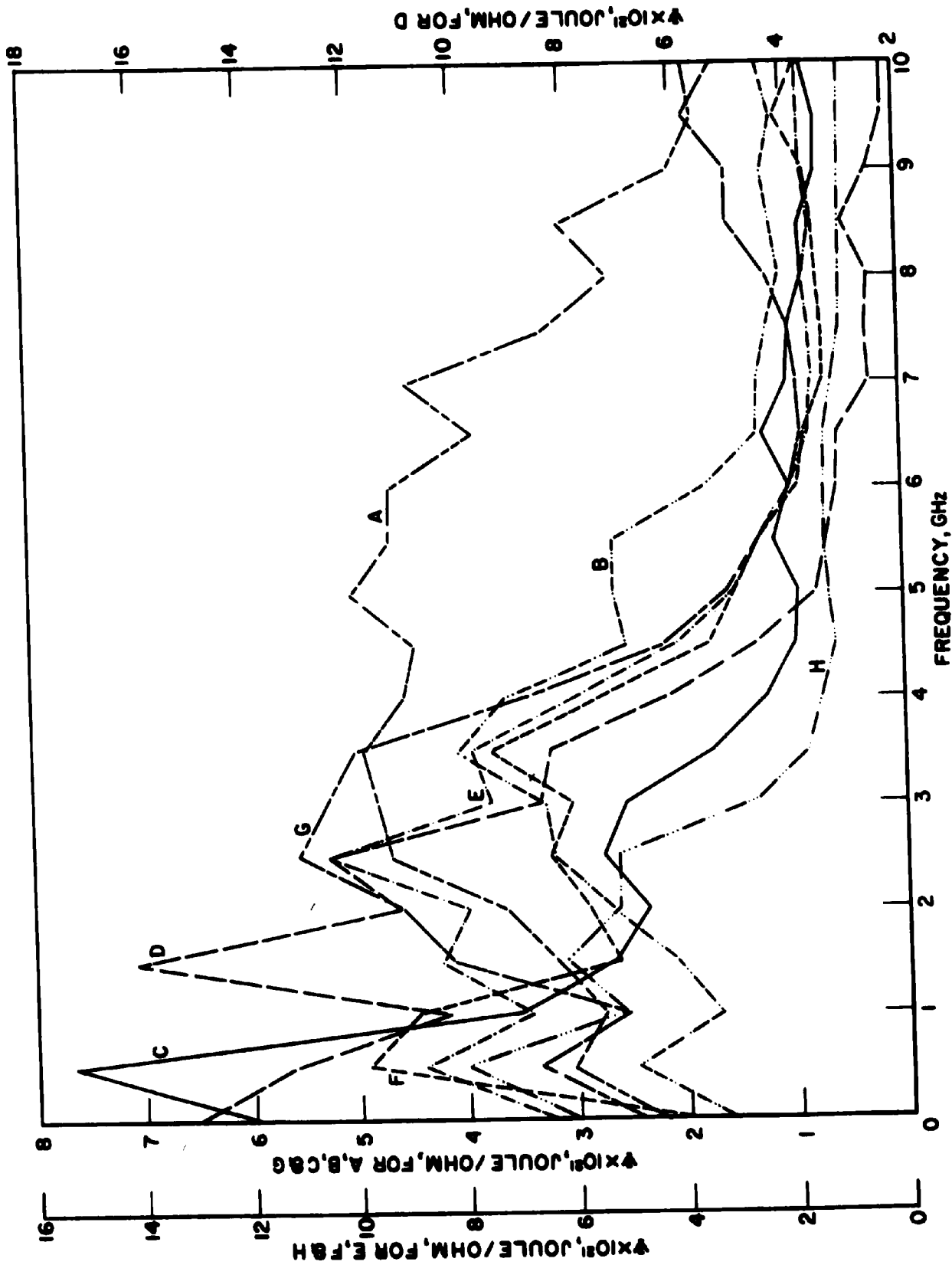


FIG. 5.6 γ VS. FREQUENCY FOR NOISE PLANES A THROUGH H (DRIFT SPACE M).

it then drops rapidly and varies little over the upper half of the frequency band. As mentioned at the end of the previous section the mismatch in electric field at the entrance to the drift space appears to affect Ψ mostly in the low frequency region.

5.3.2 Variation of Ψ with Distance. The variation of Ψ with normalized distance is shown in Fig. 5.7 for drift space A. For a majority of the noise planes, $\Psi_{\min}$ occurs at plane C, whereas for others it appears to lie between planes C and D. $\Psi_{\max}$ is generally found to lie between planes E and F where the beam upper surface is approximately at the midplane between the parallel plates of the drift space. At plane G, Ψ for some frequencies has a relatively small maximum. It then decreases toward plane H, whereas for some other frequencies just the opposite happens. It should be recalled that, in Fig. 5.3, $\phi_{\min}$ and $\phi_{\max}$ are at planes C and F respectively.

The variation of Ψ with normalized distance is plotted in Fig. 5.8 for drift space M. Generally the variation is similar to that indicated above, but, as in the case of ϕ , $\Psi_{\min}$ and $\Psi_{\max}$ occur at positions different from those for drift space A. In drift space M, $\Psi_{\min}$ for various noise planes is around plane B or between planes B and C, whereas Ψ attains large values at plane D. Thus $\Psi_{\max}$ and $\Psi_{\min}$ for drift space M are located closer to the drift space entrance than the same quantities for drift space A. A similar shift of the maximum and minimum values of ϕ for drift space M was noted in the last section. This could be due to the difference in the electric field configuration at the drift space entrance in the two cases.

The variation in Ψ with distance shown in Figs. 5.7 and 5.8 for drift spaces A and M respectively can be explained on the basis

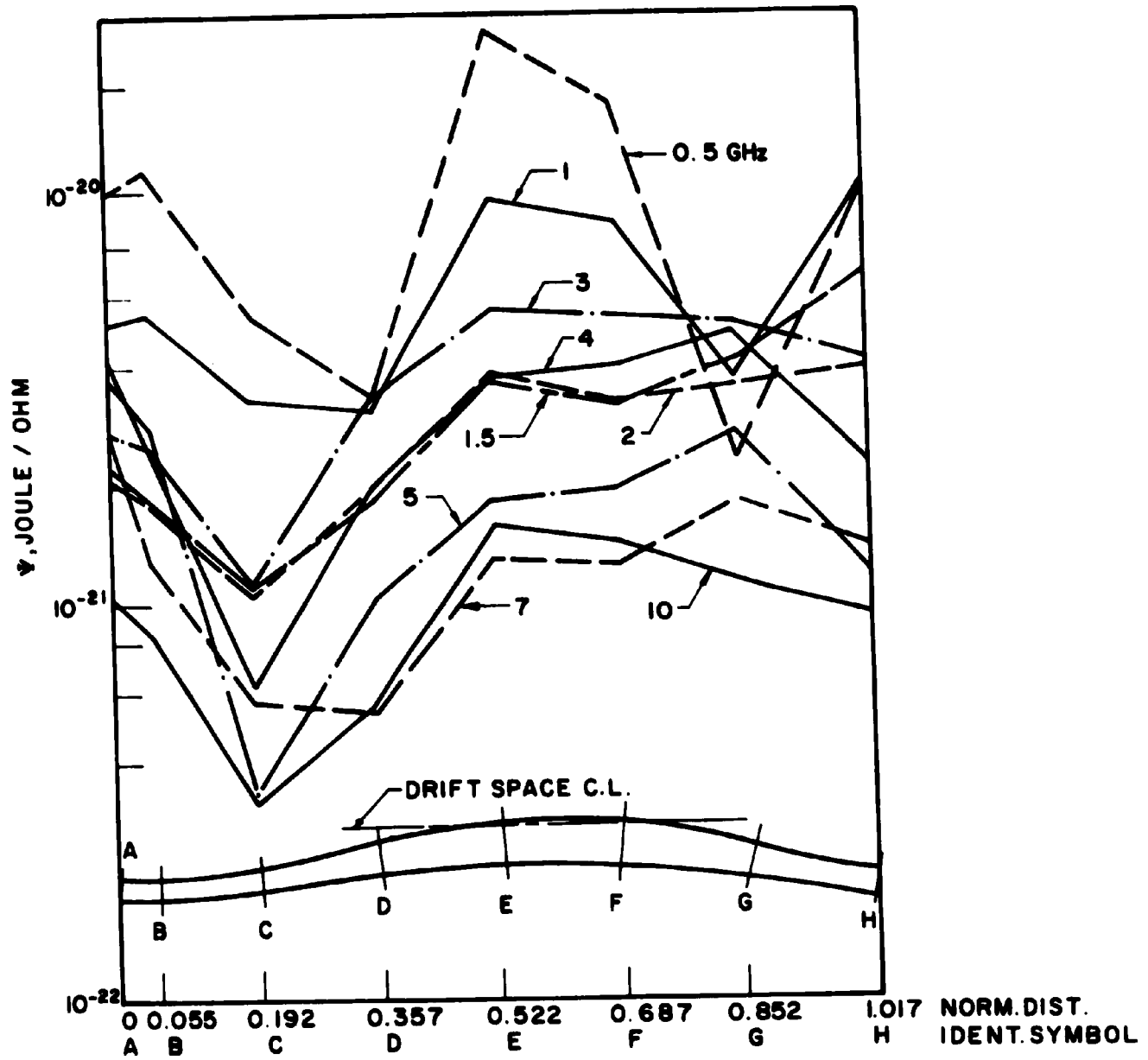


FIG. 5.7 Ψ VS. NORMALIZED DISTANCE (DRIFT SPACE A).

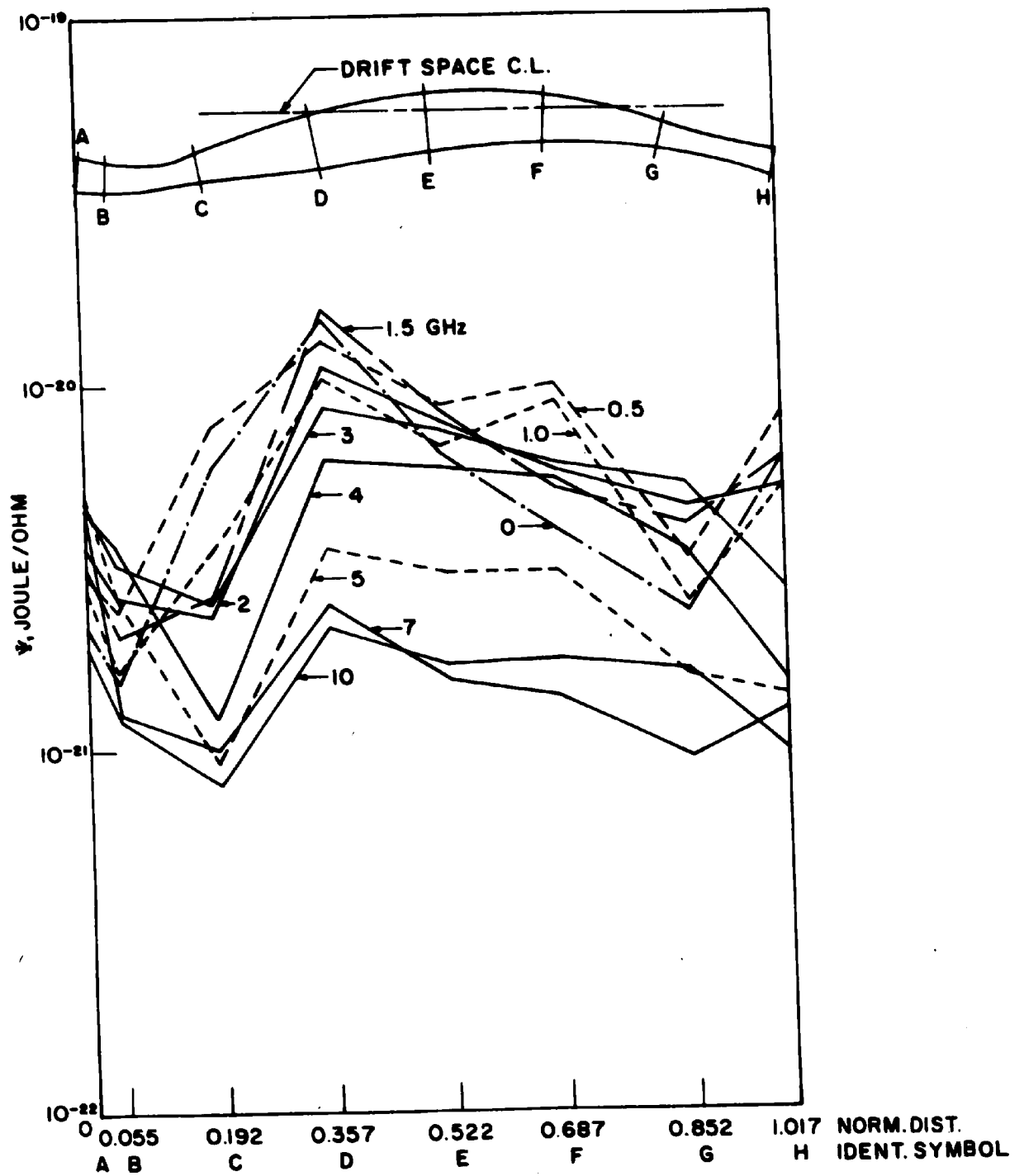


FIG. 5.8 Ψ VS. NORMALIZED DISTANCE (DRIFT SPACE M).

of the variation of G_D along the beam and the possible existence of a standing wave pattern of Ψ as in the case of Φ discussed at length in the previous section. Ψ values for drift space M are generally higher than those for drift space A. This is expected since the beam current in drift space M is approximately 1.35 times that in drift space A.

5.4 Noise Parameters Π , S , and $S-\Pi$ (SC Case)

The variation of Π with normalized distance in drift spaces A and M is shown in Figs. 5.9 and 5.10 for several selected frequencies. In both drift spaces Π generally has large negative values in the lower half of the frequency band and moderately negative values in most of the remaining frequency band for a majority of the noise planes. The same behavior of Π was found for noise planes near the exit of KG-A and KG-M. Thus it can be said that the slow cyclotron or hybrid modes are dominant for the low frequency region. From Fig. 5.9 it can be seen that except for planes A through C, all others have negative values of Π for almost all frequencies. $|\Pi|$ appears to assume minimum and maximum values at planes C and F respectively. At these planes Φ and Ψ also have their minimum and maximum values. In Fig. 5.10 for drift space M, $|\Pi|$ has a minimum somewhere between planes B and C and maxima at planes D and F. Both Ψ and Φ have a minimum between planes B and C and a maximum or a relatively high value at planes D and F.

The variation of S with normalized distance in drift space A is shown in Fig. 5.11 for several frequencies. S assumes large values at the low frequency end and its variation with frequency and normalized

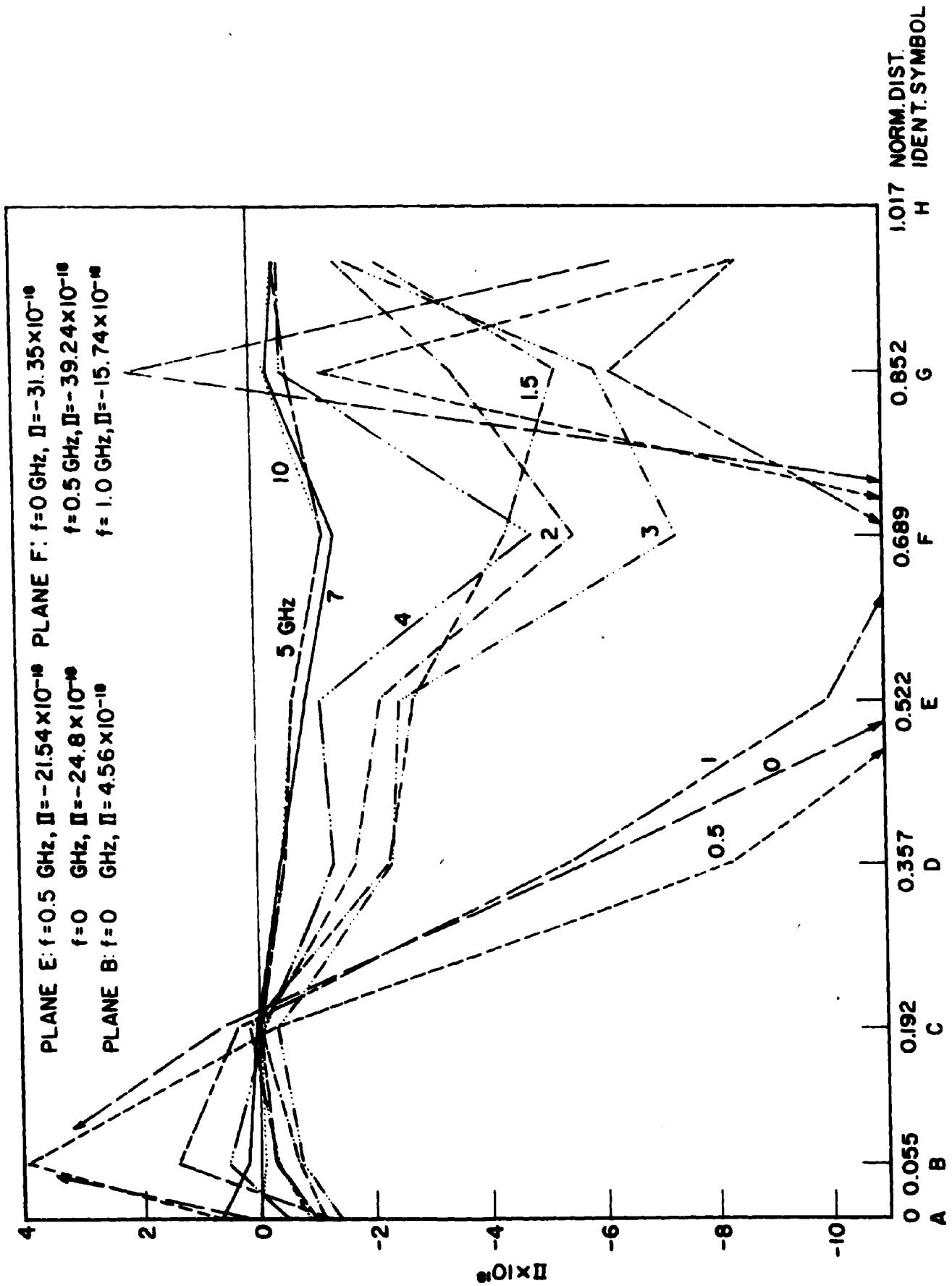


FIG. 5.9 Π VS. NORMALIZED DISTANCE (DRIFT SPACE A).

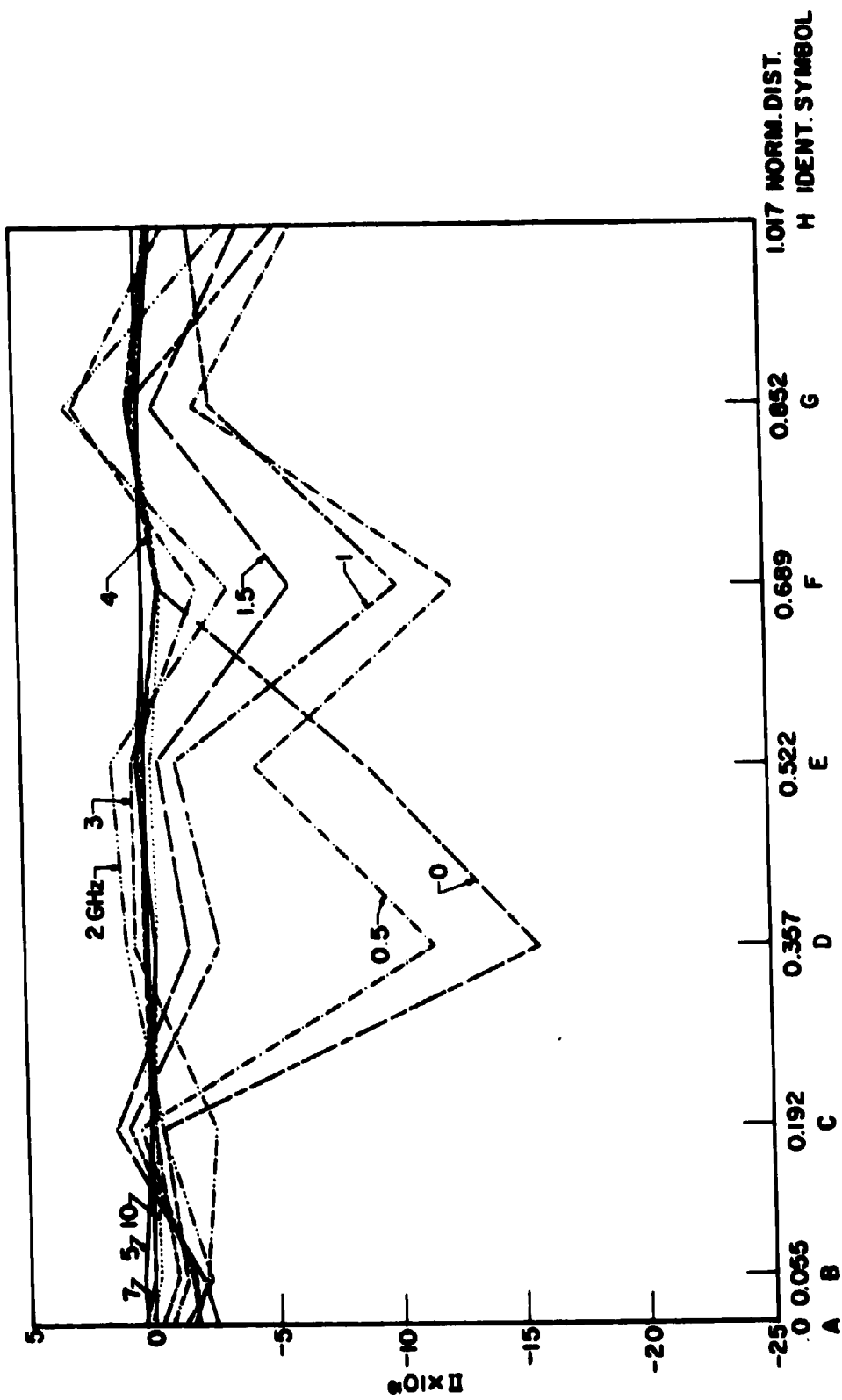


FIG. 5.10 II VS. NORMALIZED DISTANCE (DRIFT SPACE M).

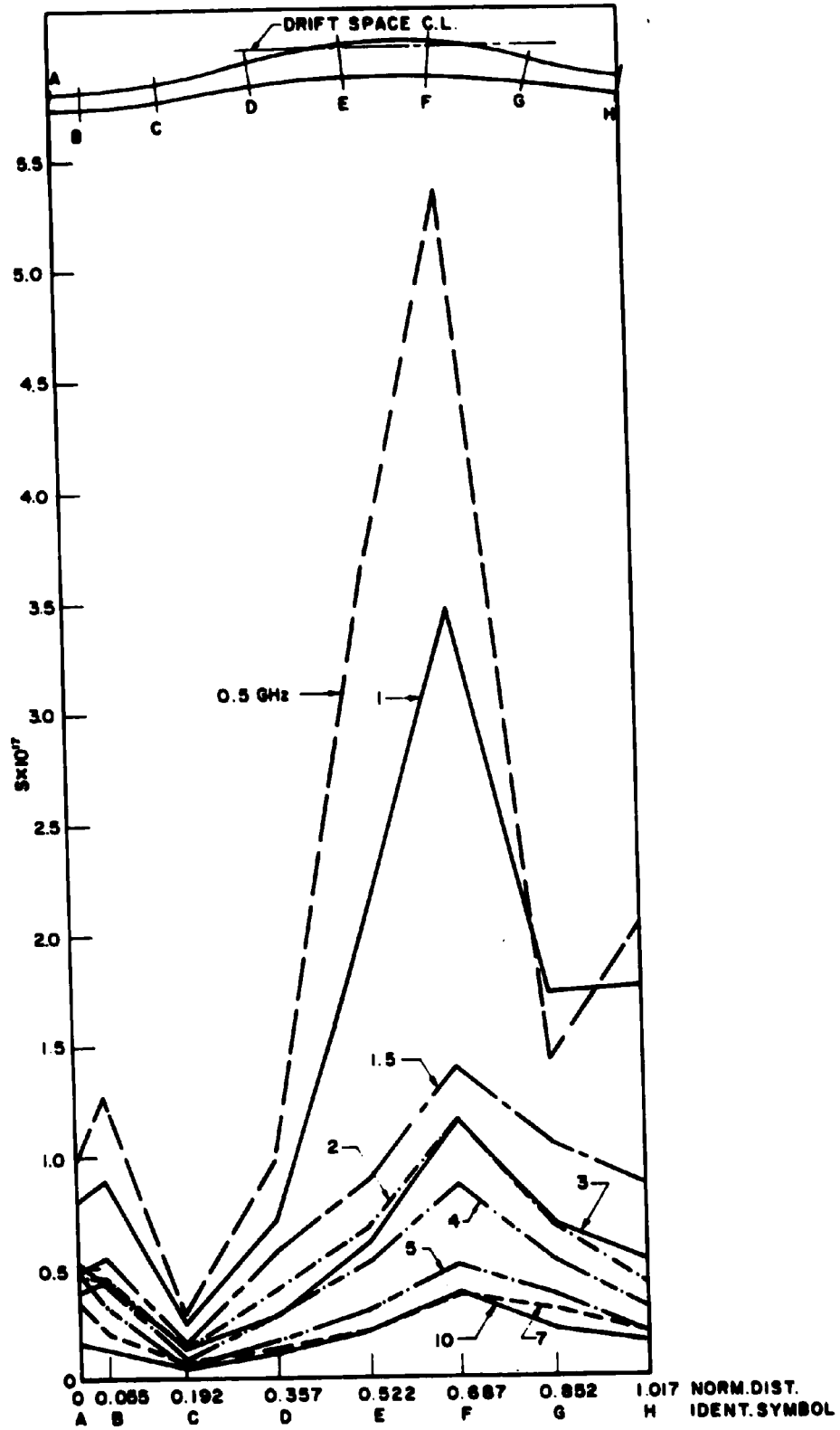


FIG. 5.11 S VS. NORMALIZED DISTANCE (DRIFT SPACE A).

distance is very close to that of ϕ shown in Fig. 5.3. In drift space M, S is also large in the low frequency region as shown in Fig. 5.12. The variation of S with distance along the beam follows somewhat the superposition of the ϕ and Ψ variations.

The variation of $S-\Pi$ with normalized distance, with frequency as a parameter, is shown in Figs. 5.13 and 5.14 for drift spaces A and M respectively. The $S-\Pi$ variation shown in Fig. 5.13 closely resembles that of S shown in Fig. 5.11. A similar resemblance exists between the variations of S and $S-\Pi$ shown in Figs. 5.12 and 5.14 for drift space M. This is expected since the variations of S and $S-\Pi$ with distance are almost similar. Like S, $S-\Pi$ for both drift spaces has large values at the low frequency end. This is, to some extent, due to large values of ϕ and Ψ in this frequency region.

The close similarity between the variations of S, Π and $S-\Pi$, and ϕ and Ψ with distance along the beam strongly suggests that variation in the position of the beam in the drift space has a great effect on the values of these parameters along the beam. Thus, for undulating beams, e.g., beams in drift spaces A and M, it is not possible to study the variation of these noise parameters along the beam without taking into account the influence of the beam position variation in the drift space.

5.5 Noise Parameter, F_0

The noise parameter F_0 , in db, computed from Eq. 3.40 for beams in drift spaces A and M is plotted in Figs. 5.15 and 5.16 as a function of normalized distance (measured from the drift space entrance) with

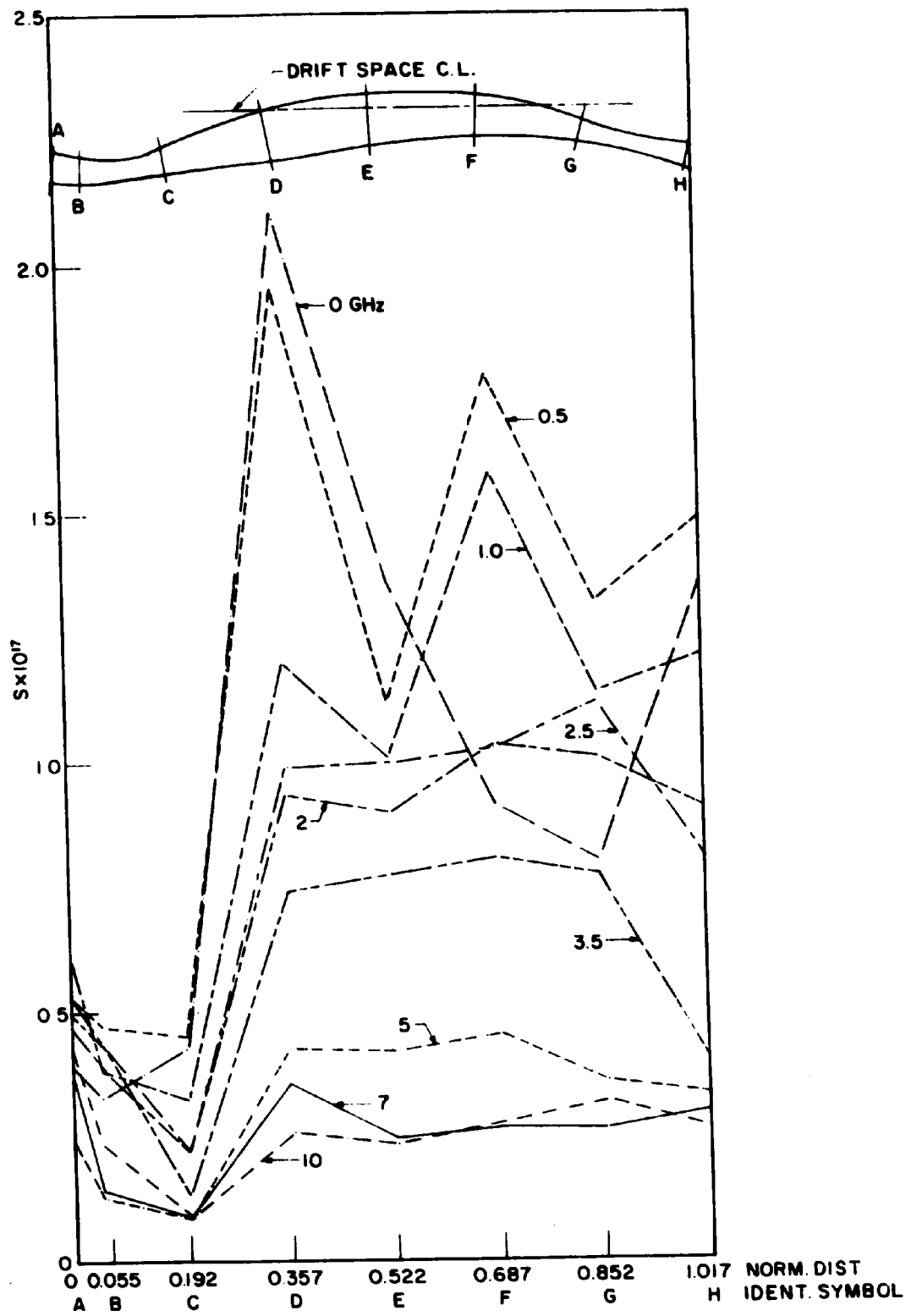


FIG. 5.12 S VS. NORMALIZED DISTANCE (DRIFT SPACE M).

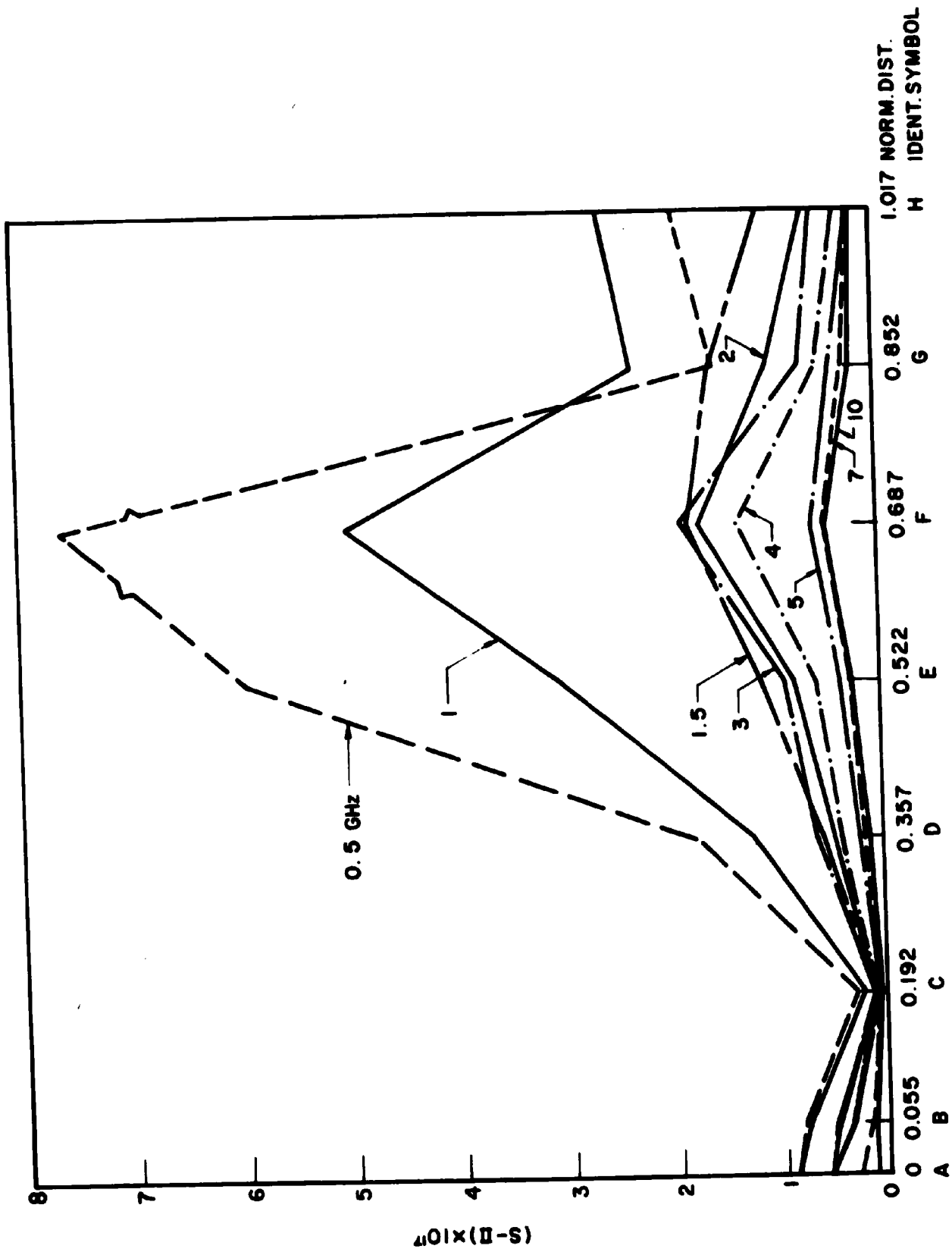


FIG. 5.13 S-II VS. NORMALIZED DISTANCE (DRIFT SPACE A).

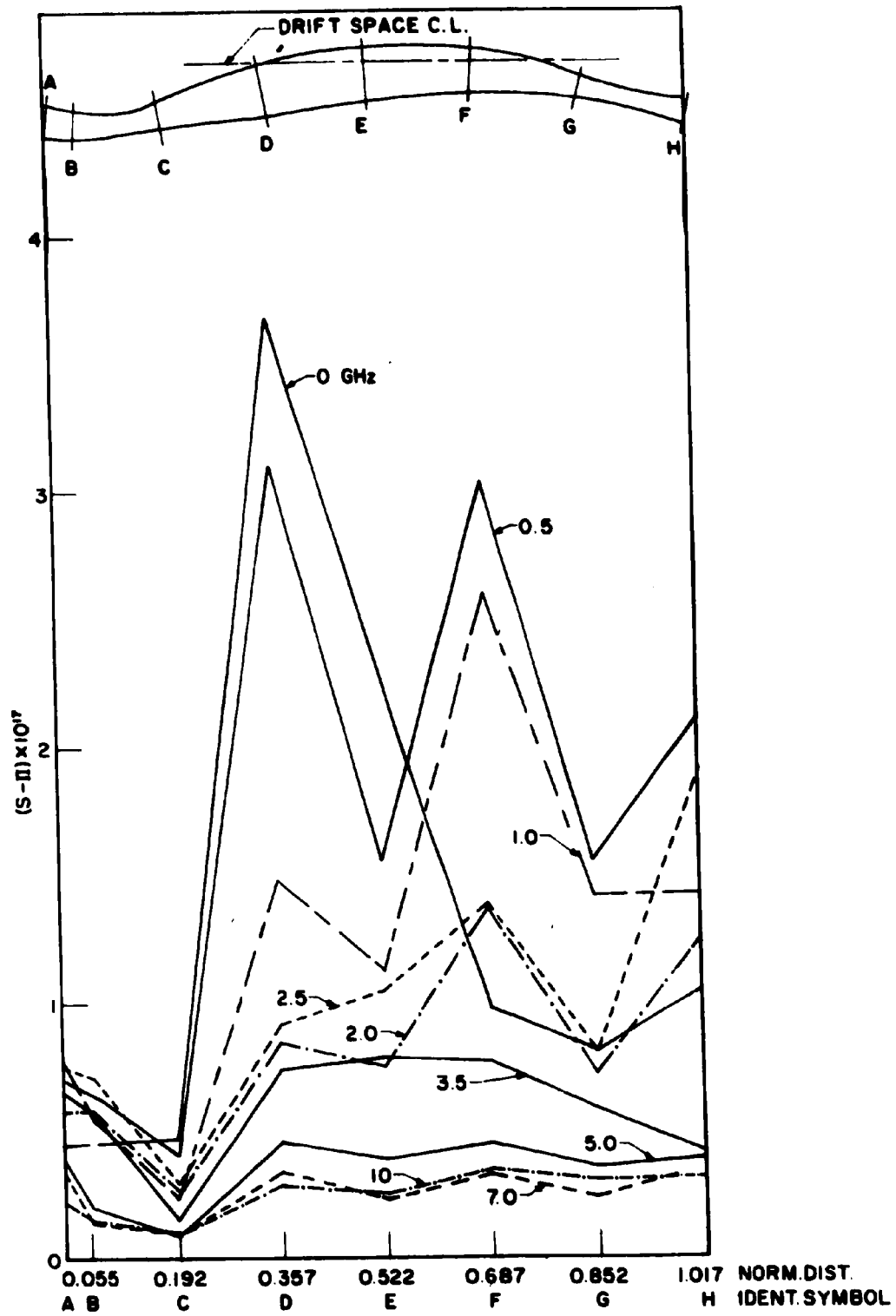


FIG. 5.14 S-II VS. NORMALIZED DISTANCE (DRIFT SPACE M).

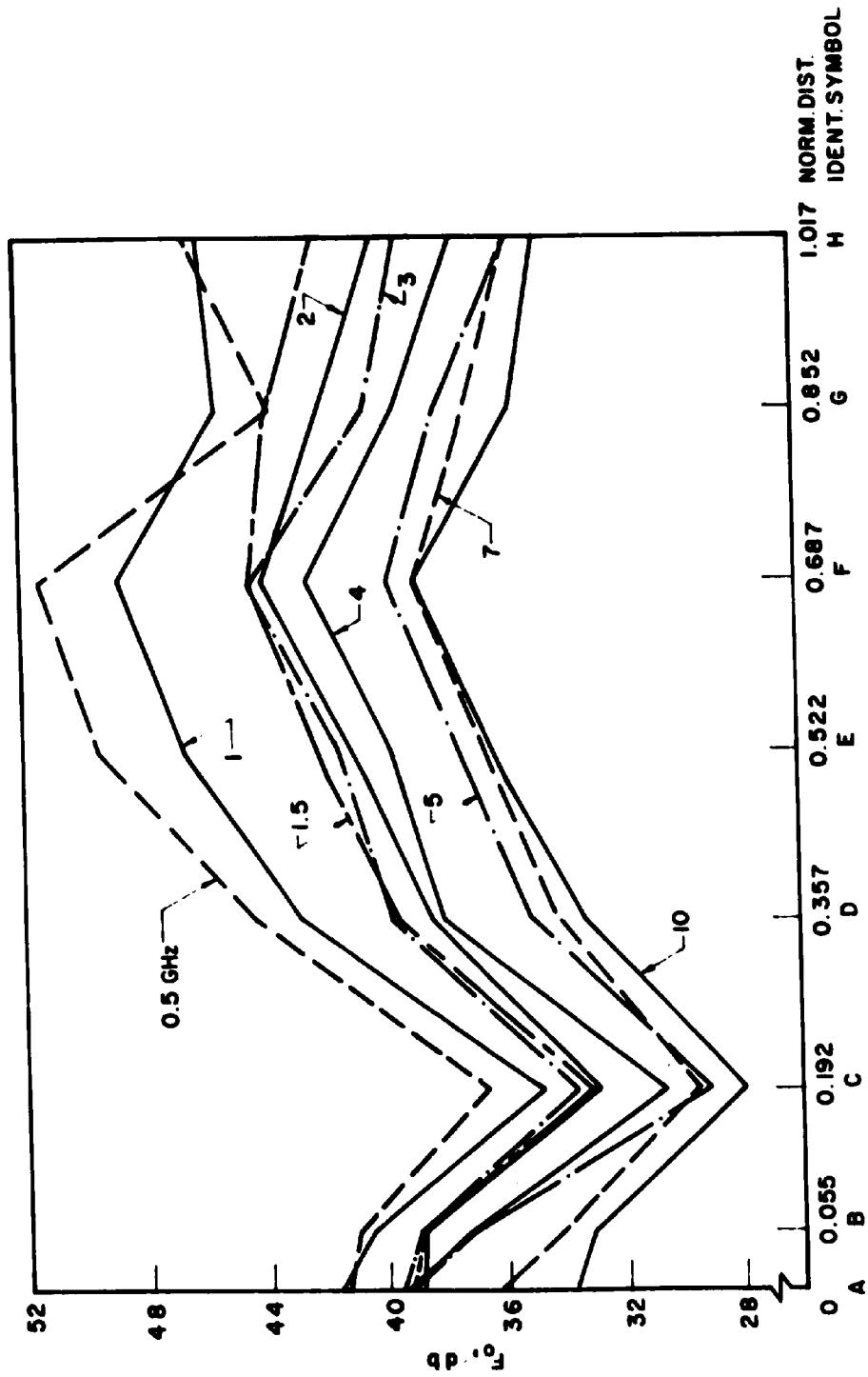


FIG. 5.15 F_0 VS. NORMALIZED DISTANCE (DRIFT SPACE A).

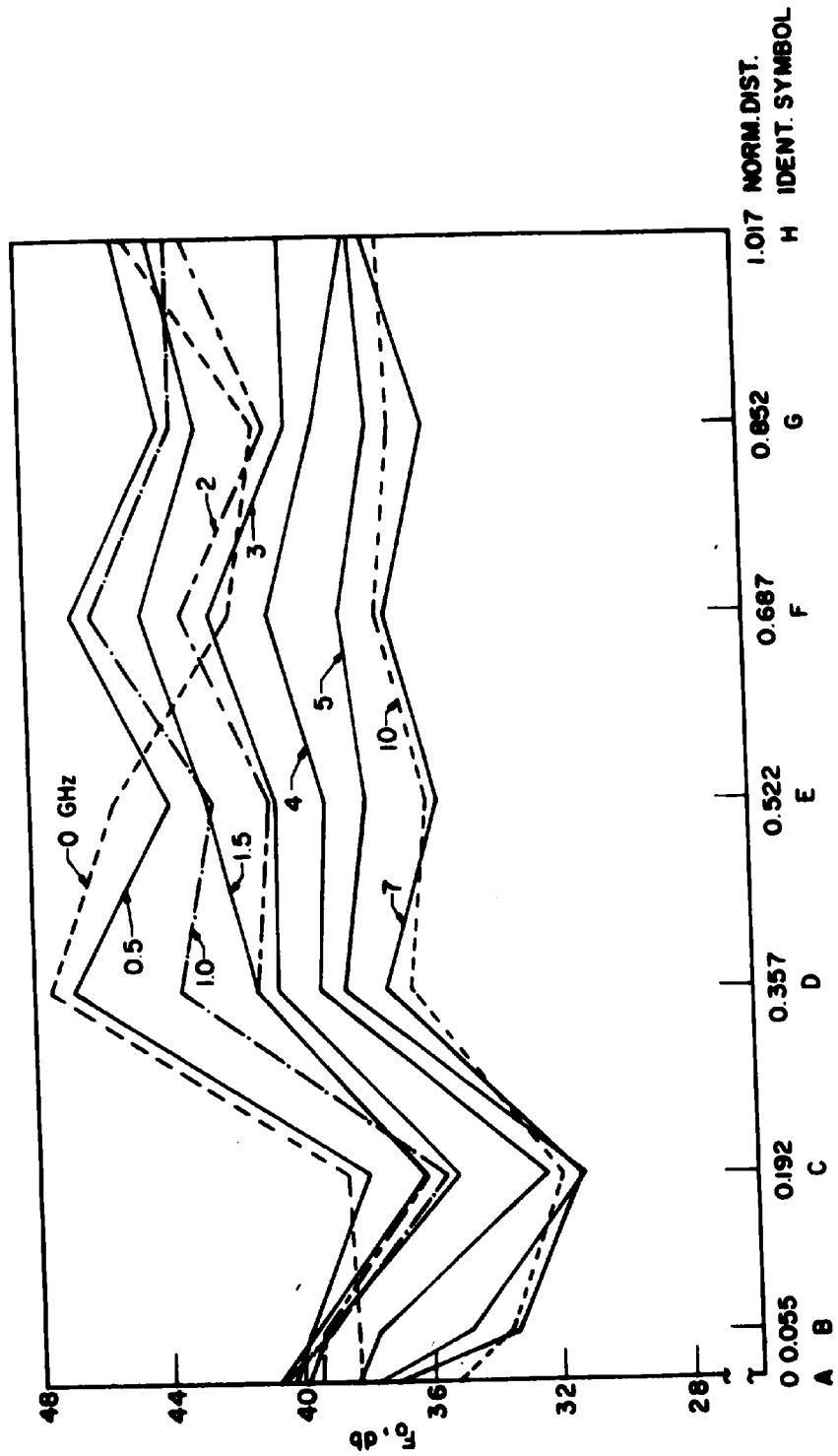


FIG. 5.16 P_0 VS. NORMALIZED DISTANCE (DRIFT SPACE M).

frequency as a parameter. From Fig. 5.15 it can be seen that, as in the case of other noise parameters, F_0 has a minimum at plane C and a maximum in the neighborhood of plane F. The rate of change of F_0 with distance (db/m) is large ($\approx 10^3$ db/m) between planes (B,C) and (C,D). It then decreases to about half of this value between planes D and F. The upper surface of the beam crosses the drift space center line slightly before plane E and then again at plane F as seen in Fig. 2.15. In this region the beam position does not change significantly and hence the diocotron gain per unit length, G_D , should remain almost constant which is found to be true in this case. Yet the variation of F_0 per unit length in this region is higher than would be expected on the basis of the diocotron gain formula for a thin drifting beam symmetrically situated between two conducting parallel planes. Furthermore, the gain per unit length is not found to increase monotonically with frequency as predicted by Gould's formula³⁴. Large values of F_0 at the low frequency end are probably due to the high values of Φ and Ψ found in this frequency range. After plane F the noise parameter F_0 starts generally declining toward the last noise plane. Though the beam is moving toward the lower parallel plane, resulting in lower G_D , it cannot explain the decrease in F_0 . It should be recalled that Φ and S-II in Figs. 5.3 and 5.13 also have a similar behavior in this region. The minimum value of F_0 occurs at plane C where the minima of Φ , Ψ , S, S-II, etc., are also found. As discussed earlier in detail in connection with the variation of Φ with distance in Section 5.2.2, the above variation of noise parameter F_0 again suggests the existence of a standing wave pattern of Φ and Ψ .

The variation of noise parameter F_0 with normalized distance shown in Fig. 5.16 for the beam in drift space M can be explained on the same lines as above. The minimum value of F_0 occurs at plane C as in the previous case. A sharp rise in F_0 is observed between planes C and D and no significant variation is seen after this noise plane. In Fig. 5.15 the maximum value of F_0 for drift space A occurs around plane F, and then falls toward the last noise plane. This difference in behavior between the F_0 variation in the two drift spaces is similar to one observed for Φ and discussed in Section 5.2.2 on the basis of beam shape, electric field match at the entrance to the drift region, and the beam location with respect to the parallel conducting planes. The variation of F_0 with frequency is similar to that for drift space A and the comments made there apply to this case also.

5.6 Noise Parameters for the OC Case

The noise parameters under SC and OC conditions do not differ significantly except for the low frequency region. In Chapter IV a similar observation was made in connection with the Kino-type guns and the crossed-field diode gun. The plausible explanation for this behavior is given in Section 4.4.5.

5.7 Conclusions

The noise parameters Φ and Ψ for both drift spaces have large values at the local hybrid frequency or the cyclotron frequency. As indicated earlier it is not possible to assign these Φ and Ψ peaks to either the local hybrid frequency or the cyclotron frequency since they are quite close. Most of these parameters, including Π , assume large values in the low frequency region, suggesting maximum noise

concentration in this region. Because of the undulating beam associated with varying thickness it was not possible to study the variation of these parameters along the beam without the effect of the changing diocotron gain per unit length. As mentioned earlier, the minimum value of various noise parameters in the neighborhood of plane C cannot be explained on the basis of the variation of diocotron gain per unit length with position of the beam in the drift space. There remains, however, a possibility of standing waves of ϕ and ψ with maxima and minima along the beam. This possibility could be checked if the noise calculations were evaluated over several beam cyclotron wavelengths, preferably with a beam having small amplitude undulations so that the effect of the variation of diocotron gain per unit length with beam position in the drift space could be minimized.

CHAPTER VI. EXPERIMENTAL INVESTIGATION OF NOISE TRANSPORT IN CROSSED-FIELD GUNS

6.1 Introduction

During the past two decades, several workers have studied theoretically and experimentally the noise phenomena in crossed-field devices. Monte Carlo analysis of noise transport in crossed-field guns and drift regions has been the topic of discussion in the previous two chapters. Due to the various assumptions made in the existing noise transport analysis, experimental investigation of noise transport in crossed-field devices is of great value in checking the analytical results and also in studying the problem from a different point of view. Several experimental observations, mentioned in Section 1.2, have not yet been satisfactorily explained. Little, Rupel and Smith¹⁰ and Epsztein and Maillart⁹ observed that noise increases for injected-beam crossed-field devices when the cathode operation is changed from temperature-limited to space-charge-limited operation. This result has been contradicted recently by the theoretical works of Wadhwa and Rowe¹¹ and Pollack and Whinnery¹², and the experimental works of Mantena and Van Duzer¹³ and Sasaki et al.¹⁴. Arnaud and Doehler¹⁶ have reported on experiments with a crossed-field short gun having a screen grid at a short distance (2-5 mm) from the cathode. They found that in this gun the noise appeared at a higher value of B/B_c (where B_c is the critical magnetic field in the circuit-sole region) than in the case of a similar short gun with no grid. In a recent C.S.F. report¹⁵ it is suggested that the noise is reduced because the grid subdivides the beam into a

number of separate streams, thus decreasing the space correlation at the cathode. Some intrinsic oscillations in the frequency range of 75 MHz to 100 MHz have been detected by Wadhwa³² in his experimental work on a crossed-field diode gun. To the author's knowledge no explanation has been given for the cause of these oscillations. Thus there seems to be ample reason for further experimental work in the study of noise in crossed-field devices.

The present investigation is more of a qualitative nature than a quantitative one. The noise characteristics of crossed-field diode and triode electron guns and an abbreviated Kino gun (KG-A) are studied experimentally using r-f probes rather than making actual noise figure measurements. A brief description of the crossed-field noise analyzer designed by Wadhwa³² and used in this investigation is given in Section 6.2. The effects of electric and magnetic fields, beam current, cathode temperature and the presence of a screen grid on signal (noise) transport in the electron guns mentioned above have been studied and the data are presented and discussed in Section 6.3. Some experiments performed to study the characteristics of self-oscillations are described and the results are discussed in Section 6.4.

6.2 Crossed-Field Noise Analyzer

The crossed-field noise analyzer shown in Figs. 6.1 and 6.2 was designed by Wadhwa³². The electron gun and the drift region are the principal parts of this device. The crossed-field diode gun is shown in Fig. 6.3a. A molybdenum screen grid (0.002-inch-diameter wires with 40 wires per inch) located approximately 2 mm above the cathode converts the above diode gun into the triode gun shown in Fig. 6.3b. The screen

- | | | | | |
|------------------------------------|------------------|---------------------------|--------------------------|---------------|
| 1.) PUSH PULL SEAL DRIVE-MECHANISM | 2.) PUMP OUTLET | 3.) PUSH PULL SEAL CASING | | |
| 4.) GLASS ENVELOPE | 5.) SOLE | 6.) BASE PLATE | 7.) ANODE | 8.) COLLECTOR |
| 9.) PROBE GUIDE RODS | 10.) PROBE GUIDE | 11.) TUBING | 12.) CATHODE BLOCK MOUNT | |
| 13.) PLATE FOR D-C PINS | | | | |

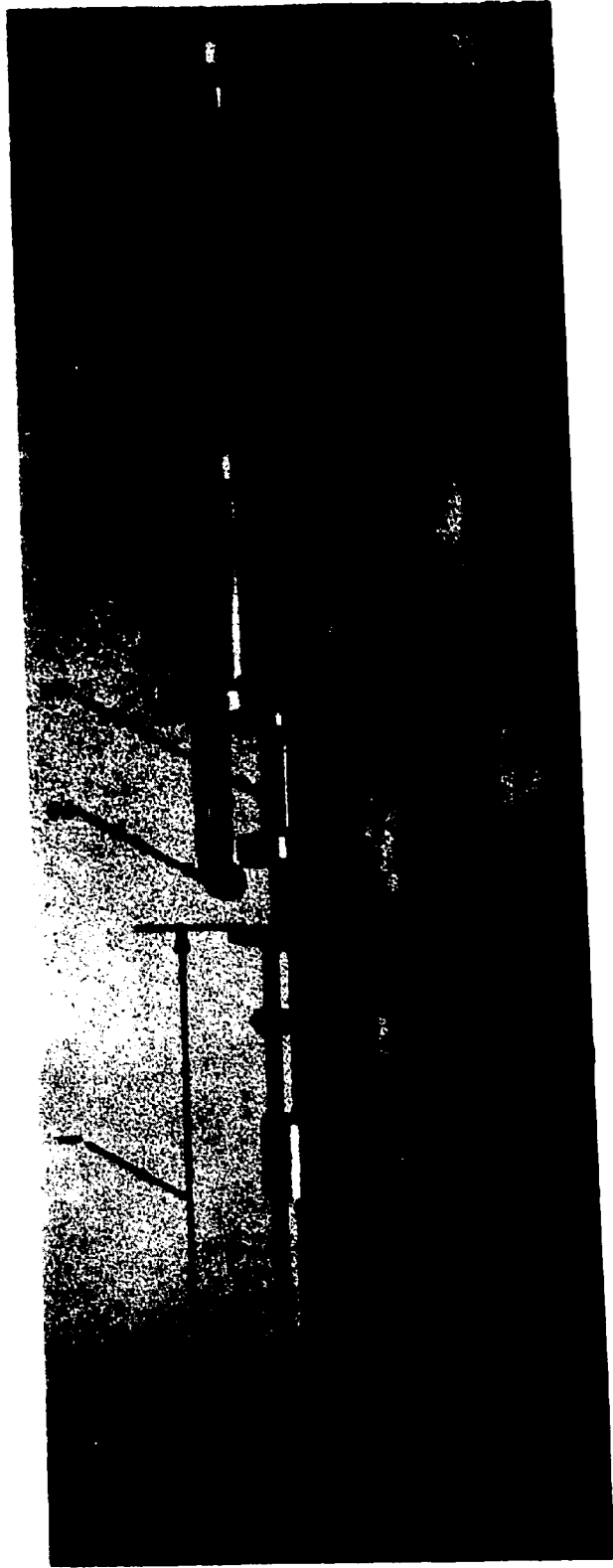


FIG. 6.1 PHOTOGRAPH OF THE CROSSED-FIELD NOISE ANALYZER (WADHWA³²).

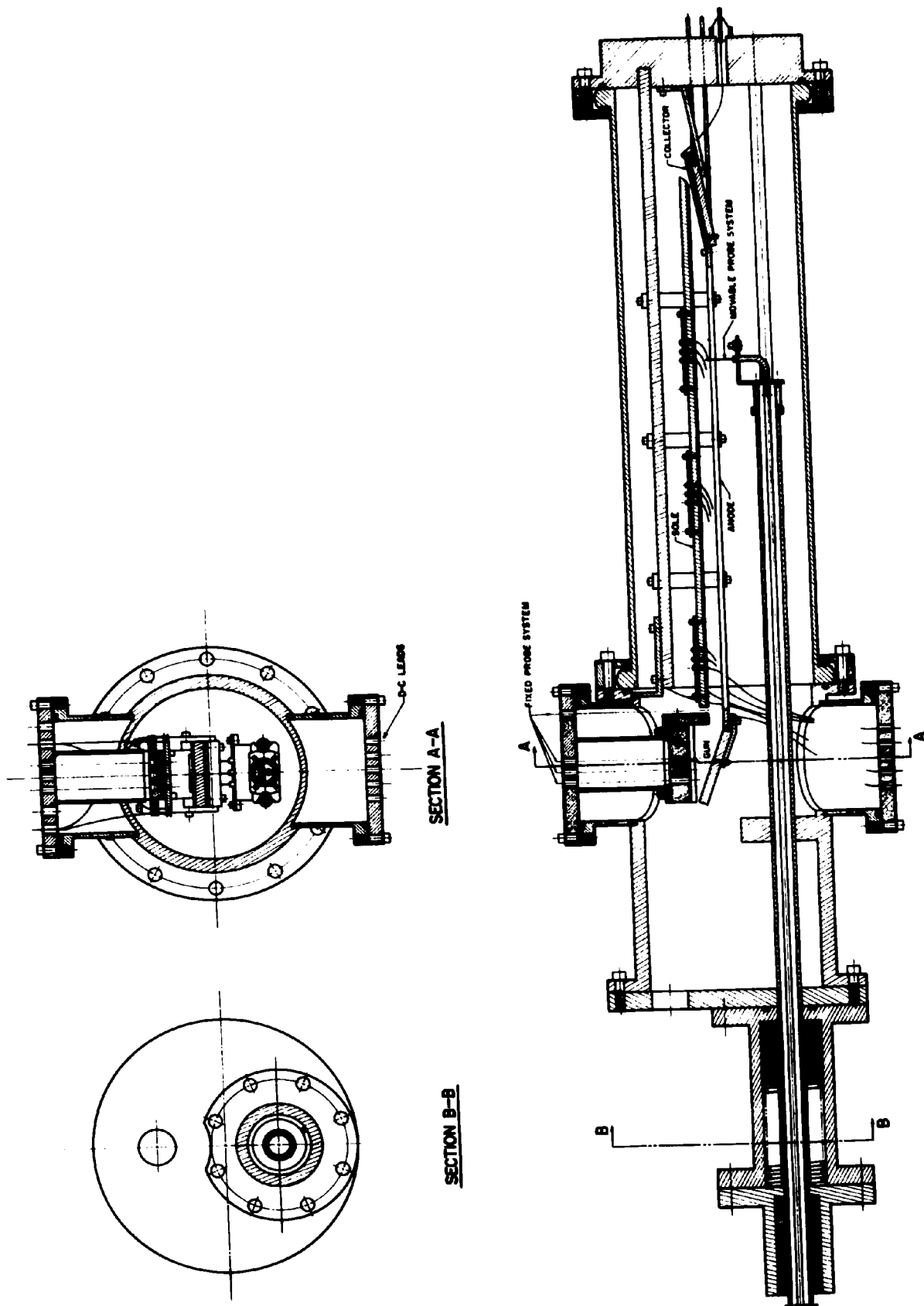


FIG. 6.2 ASSEMBLY OF THE CROSSED-FIELD NOISE ANALYZER (WADHWA³²).

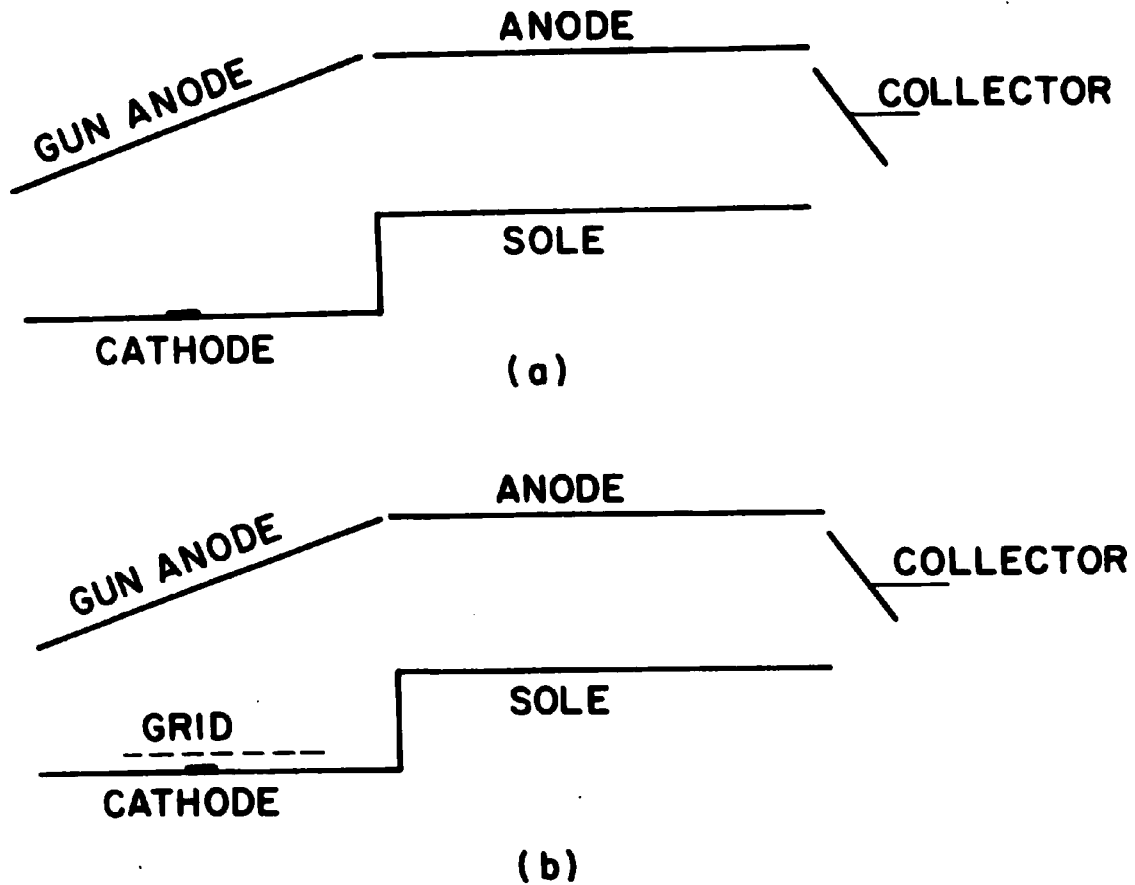


FIG. 6.3 (a) SCHEMATIC OF THE CROSSED-FIELD DIODE, DRIFT SPACE AND COLLECTOR ASSEMBLY; (b) SCHEMATIC OF THE CROSSED-FIELD TRIODE, DRIFT SPACE AND COLLECTOR ASSEMBLY.

grid was found to effectively shield the cathode so that the gun anode voltage had little influence on the cathode emission which was essentially controlled by the grid voltage. The gun anode for these guns is supported at the end of the anode plate (opposite to the collector end) with electrical insulation between the gun anode and the anode. The cathode block is mounted on a flange and is shown in Fig. 6.4. There is a provision for incorporating three cathodes to study multi-stream phenomena. However, only one cathode at a time was used in this study. The cathode block has a set of nine coaxial r-f probes (Nos. 4 through 12) in the neighborhood of the cathodes which are used to apply modulating signals to the beam. Three similar fixed probes (Nos. 1 through 3) at the entrance to the drift region are employed to detect the signal (noise) on the beam. Three movable probes (a, b, c) also used for signal (noise) detection pass through three slots in the anode plate and can be moved along the drift space by means of an external drive mechanism. The probe and the cathode configurations are shown in Fig. 6.5. These r-f probes were made according to a technique developed by Nunn and Paul⁴⁰.

In the experiments with the abbreviated Kino gun (KG-A) the complete gun was mounted on the removable flange. Side and top view of the gun indicating probe positions are shown in Figs. 6.6a and 6.6b respectively. R-f probes were situated at the entrance to the drift space for the crossed-field diode and triode guns. However, these were removed for the KG-A study since a gap between the gun front focus electrode and the sole would seriously affect the beam formation. Unfortunately some leaks through the teflon O-rings used in the vacuum seal eventually developed in the system and consequently in the later

- 1.) HOLES FOR R-F PROBES 2.) SLOTS FOR CATHODE
- 3.) CATHODE BLOCK 4.) CATHODE BLOCK MOUNT
- 5.) HOLES FOR COAXIAL CABLES 6.) HOLES FOR D-C LEADS

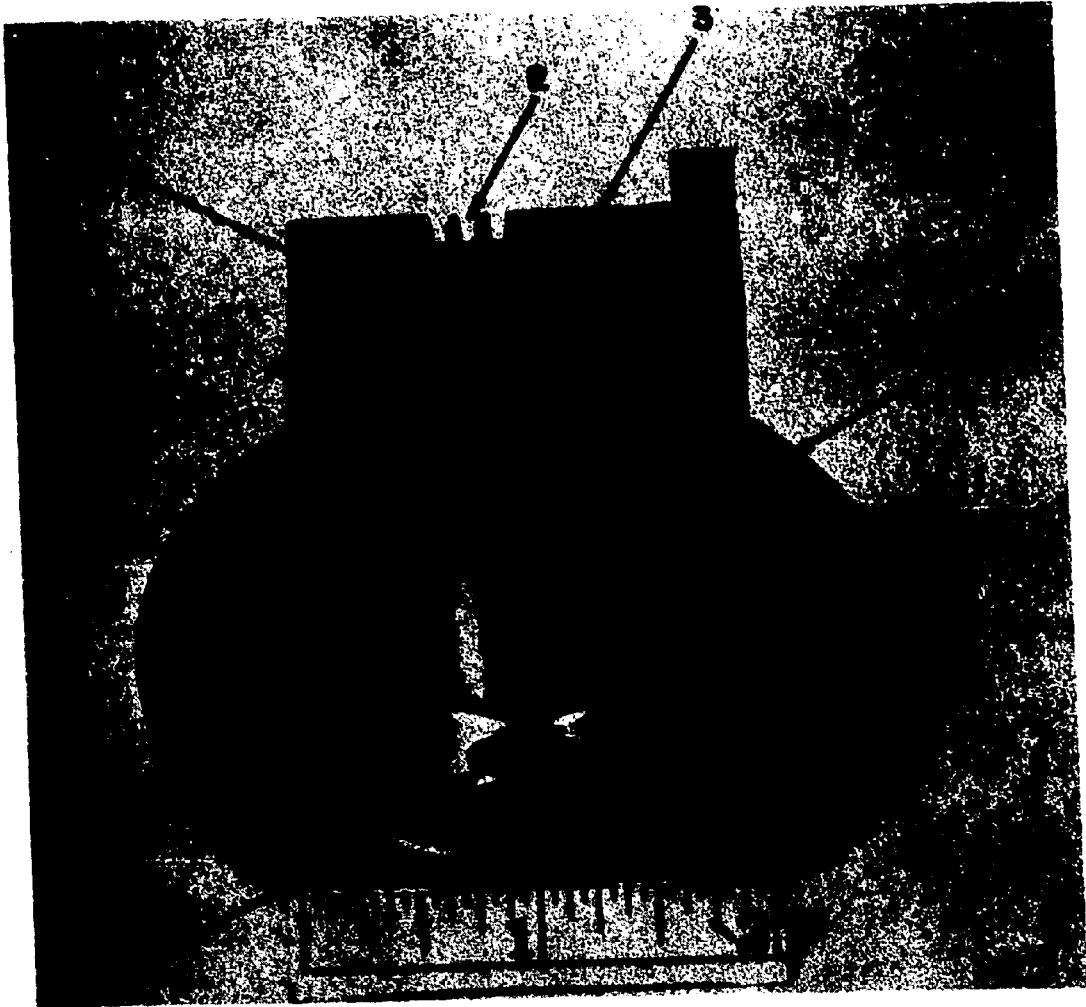


FIG. 6.4 CATHODE BLOCK WITH SLOTS FOR THREE CATHODES AND HOLES
FOR R-F PROBES MOUNTED ON THE CATHODE BLOCK MOUNT
(WADHWA³²).

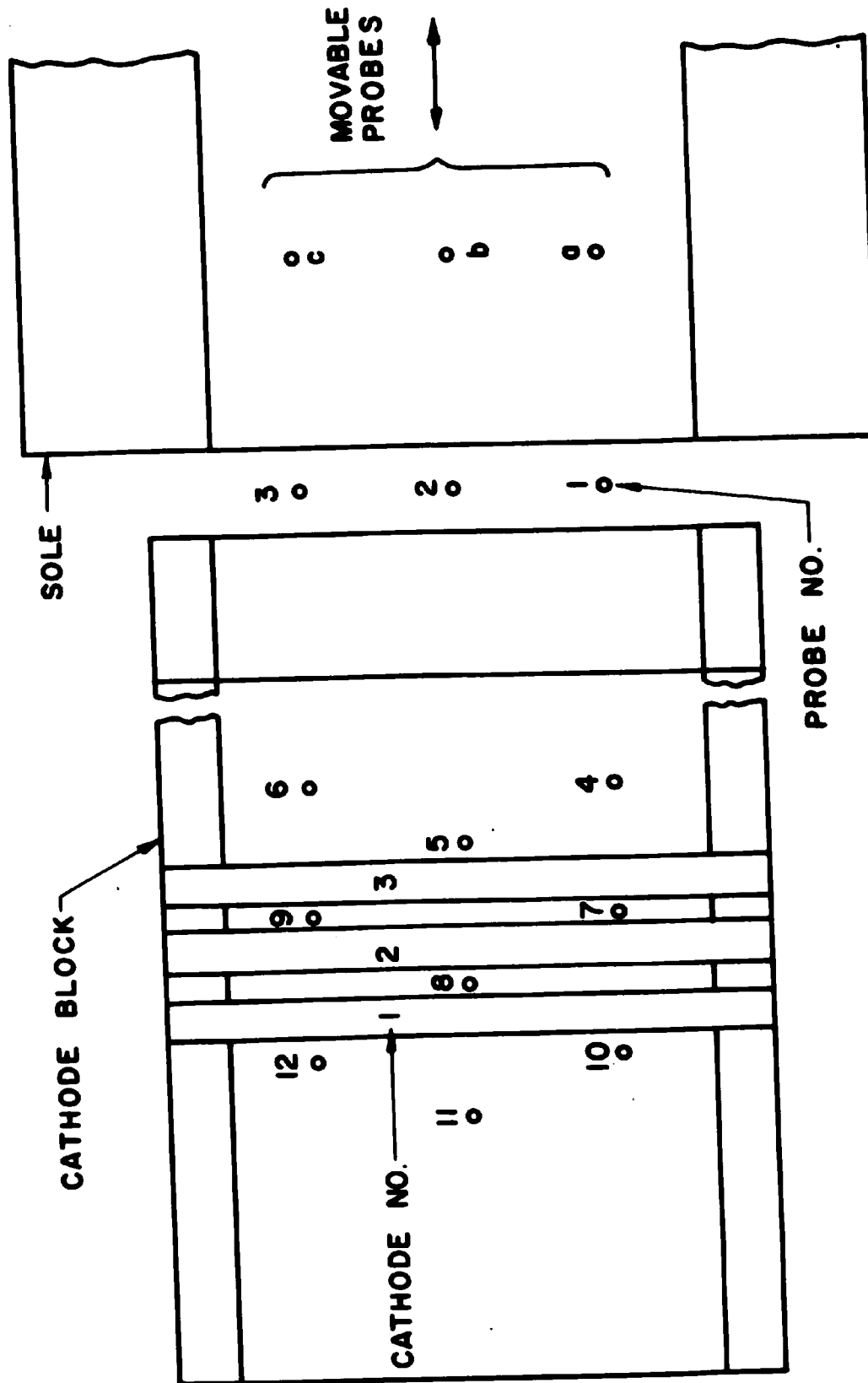


FIG. 6.5 PROBE AND CATHODE CONFIGURATIONS FOR THE CROSSED-FIELD
DIODE AND TRIODE GUNS (WADHWA³²).

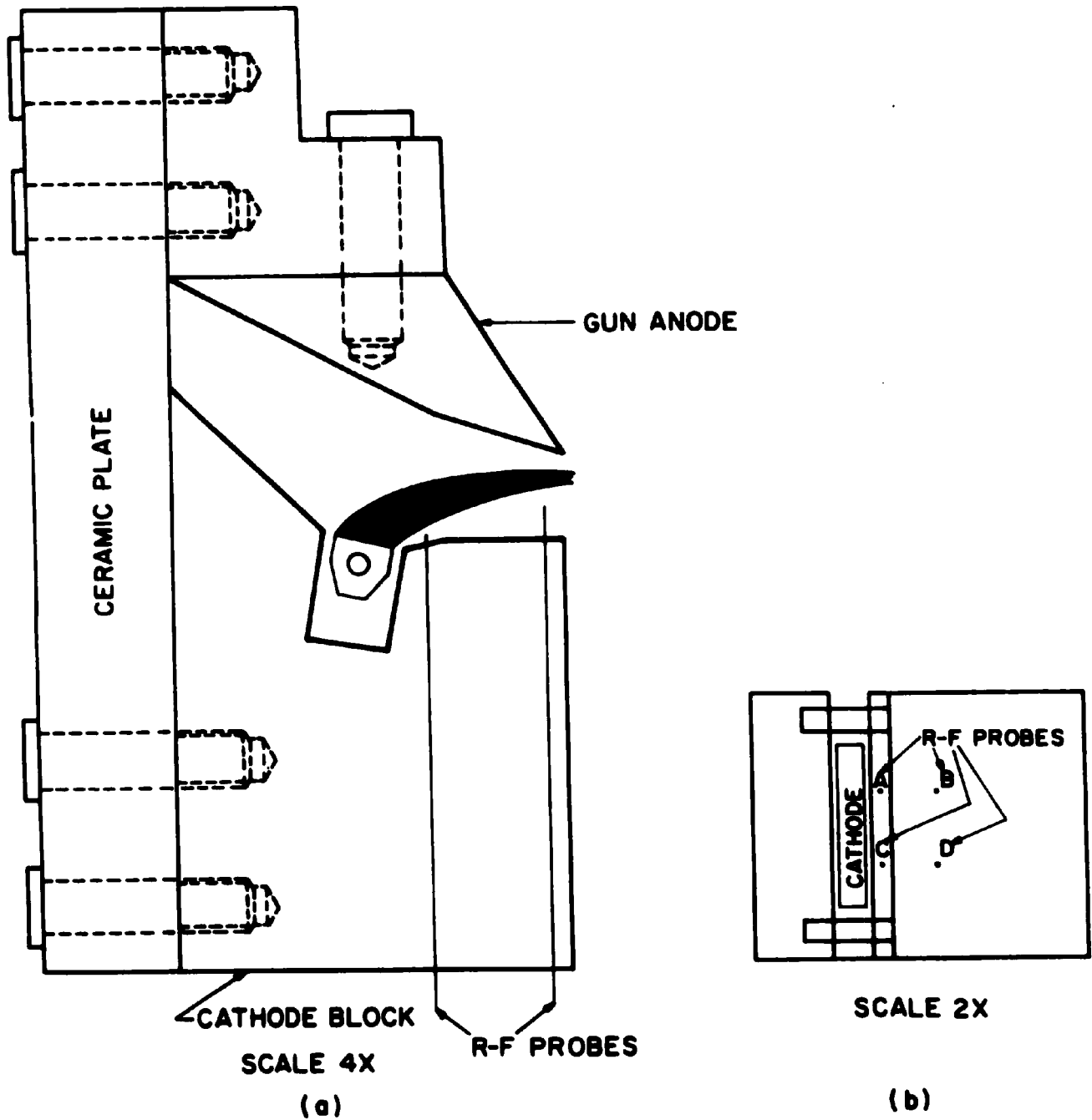


FIG. 6.6 SCHEMATIC OF THE ABBREVIATED KINO GUN:

(a) SIDE VIEW, (b) TOP VIEW.

experiments it was not possible to maintain the desired vacuum and the cathode became poisoned in a very short time. Hence several experiments which were performed on the crossed-field diode and triode guns could not be repeated with KG-A. However, data on noise transport as a function of frequency and input signal power was taken and is presented in Sections 6.3.1 and 6.3.2.

The drift space consists of two parallel conducting plates (anode and sole) separated by 0.365 inch. The collector consists of a sandwich of a beryllium oxide plate between two stainless steel plates, one of which is watercooled.

6.3 Noise Transport in Crossed-Field Guns

Since broadband noise is characteristic of crossed-field devices, the noise transport can be studied by modulating the beam with an r-f signal of a given frequency and power level (r-f signal injected into the beam by a coaxial r-f probe) and observing the corresponding signal power detected at the receiving r-f probe. The signal frequency is then varied over the frequency band of interest in the noise transport study. It was found that, in the absence of the beam, direct transmission of the signal between the injection and receiving probes was appreciable at high frequencies. This difficulty was avoided by operating below 250 MHz. Another reason for this choice of frequency limit was the occurrence of some self-oscillations in the beam in the frequency band of 90 MHz to 125 MHz under certain operating conditions.

Cathode No. 3 was used in experiments with the diode and triode guns, and r-f probe No. 5 was used to modulate the beam near the cathode. The set of fixed probes at the gun exit, viz., Nos. 1, 2 and 3, were used as receiving probes. The gun anode, anode and collector electrodes

were held at a common positive potential with respect to the cathode, whereas the sole electrode was at cathode potential. In the experiments with KG-A, probe A or probe C was used as the signal injection probe, and probe B or probe D respectively was employed for detecting the signal along the beam in the gun region.

The directional coupler shown in Fig. 6.7 was calibrated for the frequency range used in these experiments. The reflection coefficient of the r-f input probe was also determined for several frequencies of interest. This information was then used to determine the power radiated by the input probe. The part of this radiated power which modulates the beam cannot be evaluated without knowledge of the coupling coefficient between the r-f probe and the beam. However, the lack of such knowledge does not present problems in the present investigation.

Since signal power detected at the receiving probe is very small, an accurate method for power measurement becomes a necessity. Furthermore, to compare data taken at different times and possibly for different electron guns, the method adopted should be free from problems caused by variations in the operating equipment. The measurement setup shown in Fig. 6.7 achieves this purpose. The output power of the signal generator G_1 was monitored by means of a directional coupler and a power meter P_1 . With the coaxial switch S_1 in one position, the spectrum analyzer received the input from the receiving probe. The spectrum analyzer was calibrated each time by switching the coaxial switch to the other position, thereby connecting the analyzer to the calibration unit.

Instead of discussing experiments performed with these guns on an individual basis, the data is presented and analyzed to evaluate the

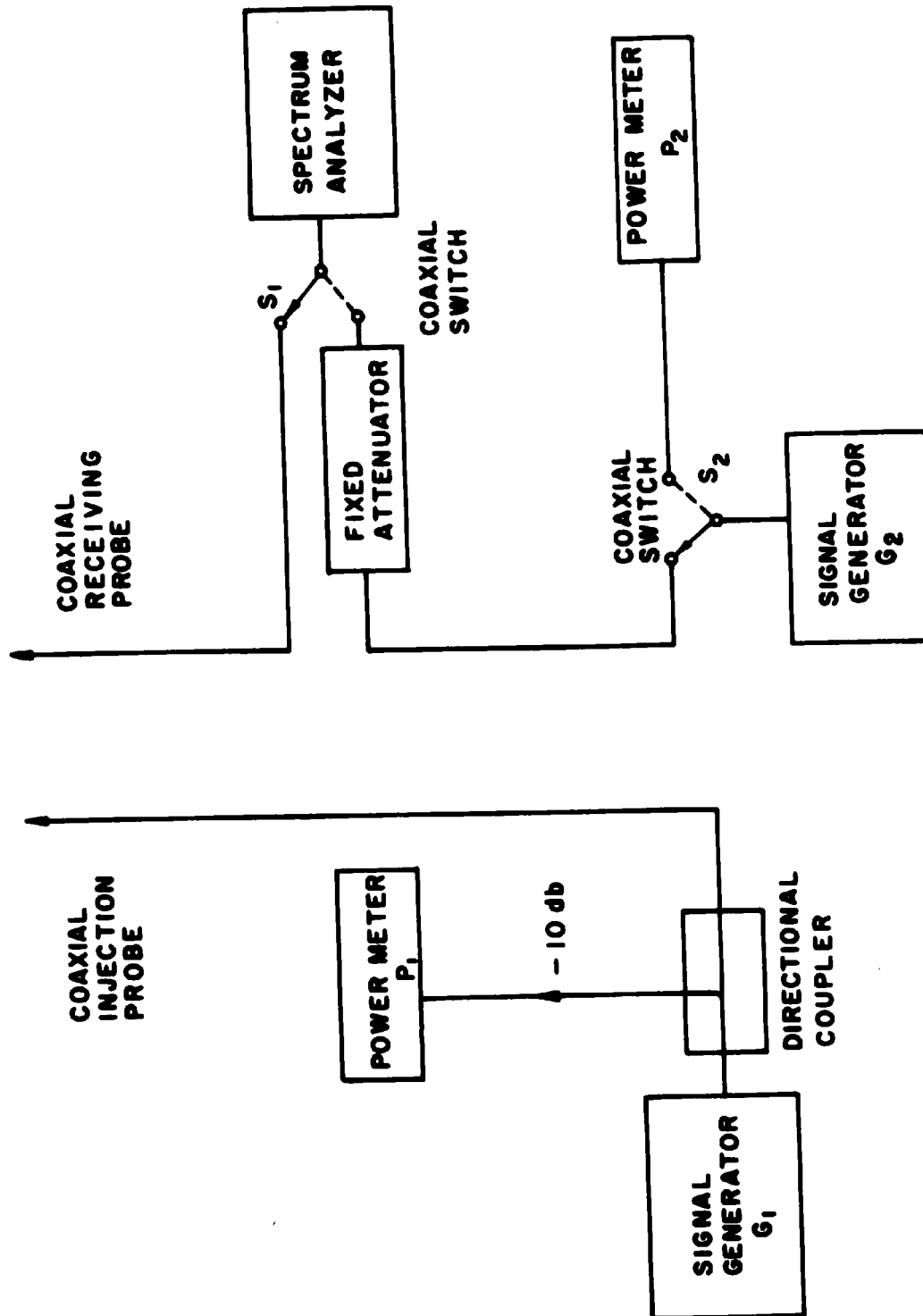


FIG. 6.7 BLOCK DIAGRAM OF THE SIGNAL-POWER MEASUREMENT SETUP.

role of various operating parameters on the noise transport along the beam in crossed-field guns.

6.3.1 R-f Signal Transport as a Function of Input Power.

Figure 6.8 shows the signal power at probe B (see Fig. 6.6b) as a function of input signal power at the injection probe A for two sample frequencies in the case of the KG-A beam. At low input modulation power it can be seen that, in both cases (100 MHz and 250 MHz), the relation between the signal powers at probes A and B is approximately linear. At higher input powers the output power at probe B becomes saturated for the 100 MHz case whereas for a 250 MHz signal this condition is not reached since the input power at probe A was limited by the maximum power output of the oscillator (signal generator) at this frequency. The variation of anode current, collector current and sole current as a function of input probe power for the 100 MHz input frequency case is shown in the same figure. It should be noted that these electrode currents do not change in the low-input signal power range. Then, as the input signal power increases, the collector current decreases and the anode and sole currents increase, whereas the total beam current remains approximately constant. This redistribution of beam current between these electrodes can be explained in the following way. In the presence of a large r-f signal, the beam spreads out and is more susceptible to interception by the anode and sole electrodes. Only a slight increase in sole current is expected since it is a negative electrode. Thus most of the current redistribution occurs between the collector and the anode.

6.3.2 Variation in Signal Power Detected at the Receiving Probe with Input Signal Frequency.

For a constant input signal power radiated by the input probe, the signal power detected at the receiving probe vs.

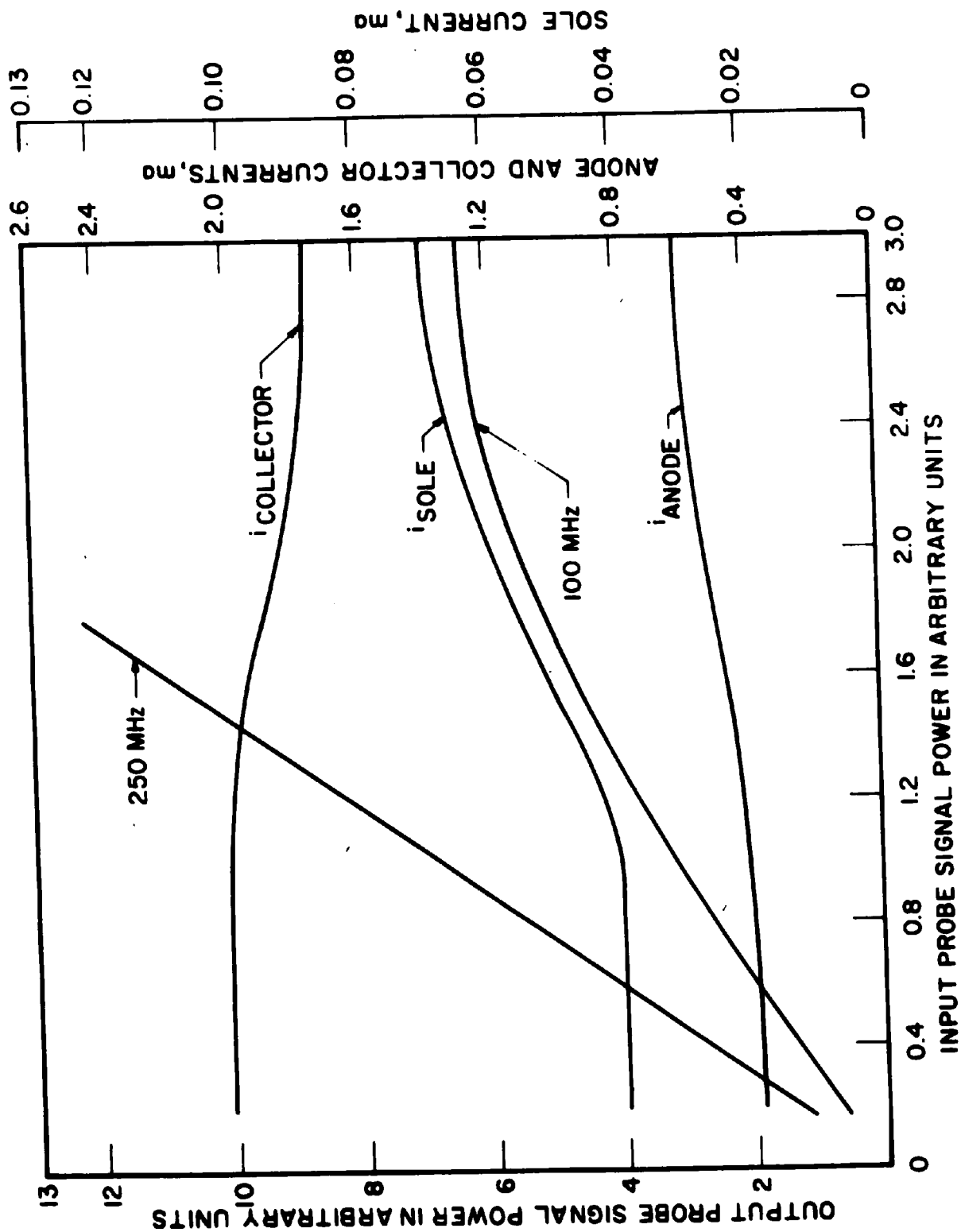


FIG. 6.8 OUTPUT PROBE SIGNAL POWER VS. INPUT PROBE SIGNAL POWER AT $f = 100$ MHz AND 250 MHz FOR KG-A. (GUN ANODE VOLTAGE = 100 VOLTS; ANODE AND COLLECTOR VOLTAGE = 256 VOLTS; $B = 73$ GAUSS; $f_c = 205$ MHz)

input signal frequency shows maxima and minima for beams in all three crossed-field guns. One such sample for the KG-A beam is shown in Fig. 6.9. Here the signal is again injected at probe A and received at probe B. The coupling between the beam and the probes can be reasonably assumed to be independent of frequency since the dimensions of the probes are very small compared to the range of wavelengths used in this experiment. The various maxima and minima in Fig. 6.9 could be possibly due to some resonances between the modulating signal and inherent perturbations on the beam.

It is also observed that the d-c currents drawn at the anode, collector and sole electrode are almost constant up to 150 MHz but for higher frequencies the collector current decreases, the anode current increases by almost the same amount, the sole current increases slightly and the total beam current remains almost constant. Exactly similar behavior of electrode currents was noted in the last section for a large input signal power. The change in the electrode currents at 150 MHz and a corresponding sharp rise in the signal received at probe B indicates some strong interaction between the input signal and the beam which enables the received signal to reach a large maximum around 175 MHz. The above-mentioned changes in the electrode currents suggest, as noted in the last section, some spreading of the beam when the input signal frequency is beyond 150 MHz. It should be noted, however, that the input signal power in this case was below the power range which resulted in beam spreading in the last section.

6.3.3 Variation of Signal Power Detected at the Receiving Probe as a Function of Cathode Temperature. In the case of the crossed-field diode gun, the signal detected at probe 2 (see Fig. 6.5) is plotted

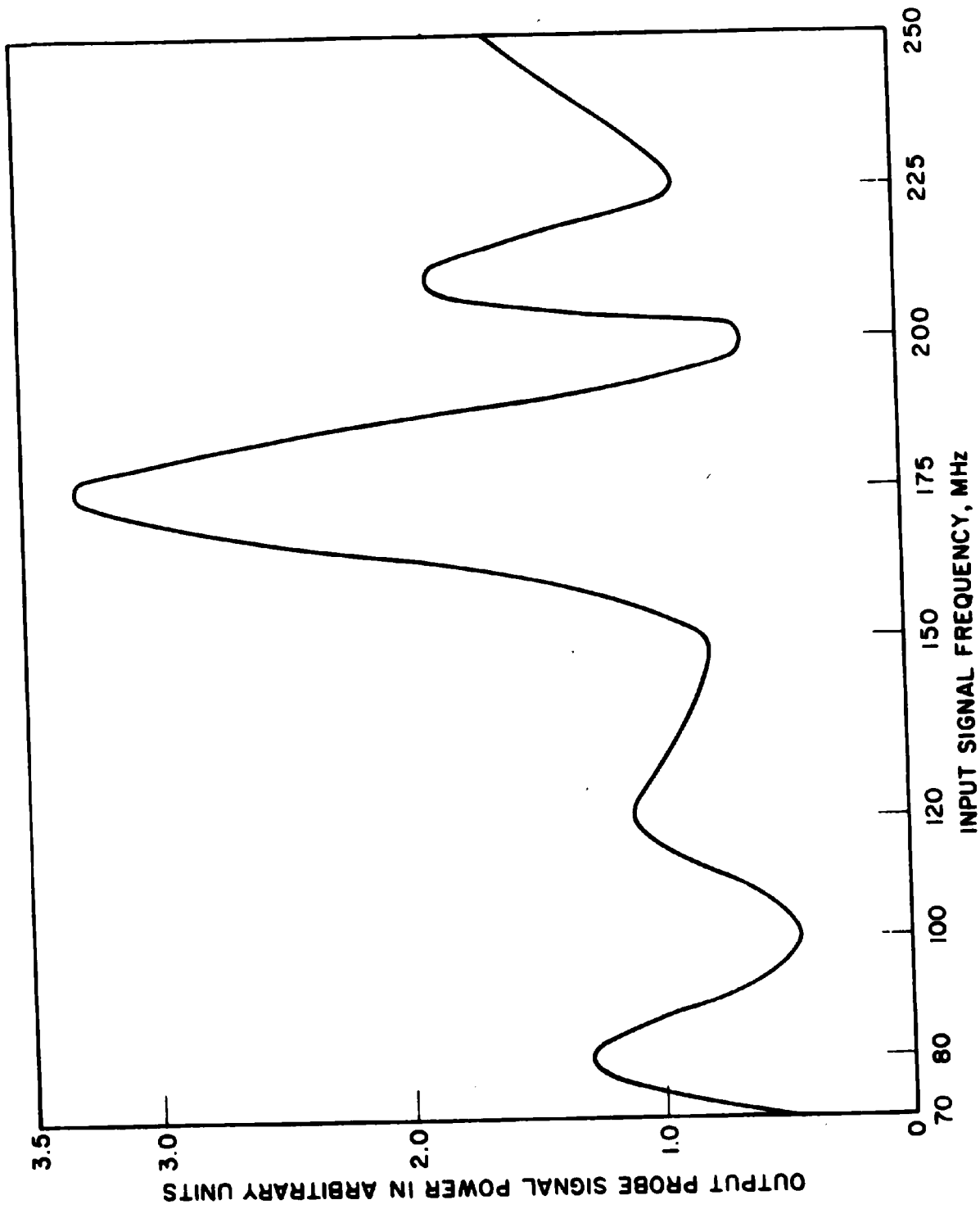


FIG. 6.9 OUTPUT PROBE SIGNAL POWER VS. INJECTED SIGNAL FREQUENCY AT CONSTANT INPUT POWER AT THE INJECTION PROBE FOR KG-A. (GUN ANODE VOLTAGE = 100 VOLTS; ANODE AND COLLECTOR VOLTAGE = 256 VOLTS; $B = 73$ GAUSS; $f_c = 205$ MHz.)

against heater power in Fig. 6.10 for injected signal power and frequency at probe 5. The electric and magnetic fields are held constant. Masnari³⁰ has shown that the beam location remains almost the same even when the cathode current is changed by 100 percent. Thus the coupling between the beam and the probe remains approximately constant in this experiment when the cathode current varies with cathode temperature. Since the frequency and power of the injected signal remain constant, these parameters do not affect the strength of the received signal. Hence the variation in the signal received at probe 2 is only due to changes in the cathode operating conditions. It is interesting to note that the detected signal power increases in the temperature-limited region and reaches a maximum in the neighborhood of the knee of the cathode current vs. heater power curve. When the heater power is further increased, cathode operation becomes space-charge limited and the detected signal decreases with an increasing degree of space-charge limitation. For large heater powers the change in the detected signal is very small. An exactly similar variation in the detected signal at probe 2 with a variation in the cathode operating condition is shown in Fig. 6.11 for the crossed-field triode gun. The grid and gun anode voltages and the magnetic field were held constant in this case.

In the experimental work of Wadhwa³², the detected r-f signal increases monotonically with heater power. The cathode current increases in steps which is probably due to not allowing sufficient time for attaining the equilibrium cathode temperature. When this is accounted for, the cathode operating condition is clearly temperature limited and thus Wadhwa's results agree with those obtained in this work.

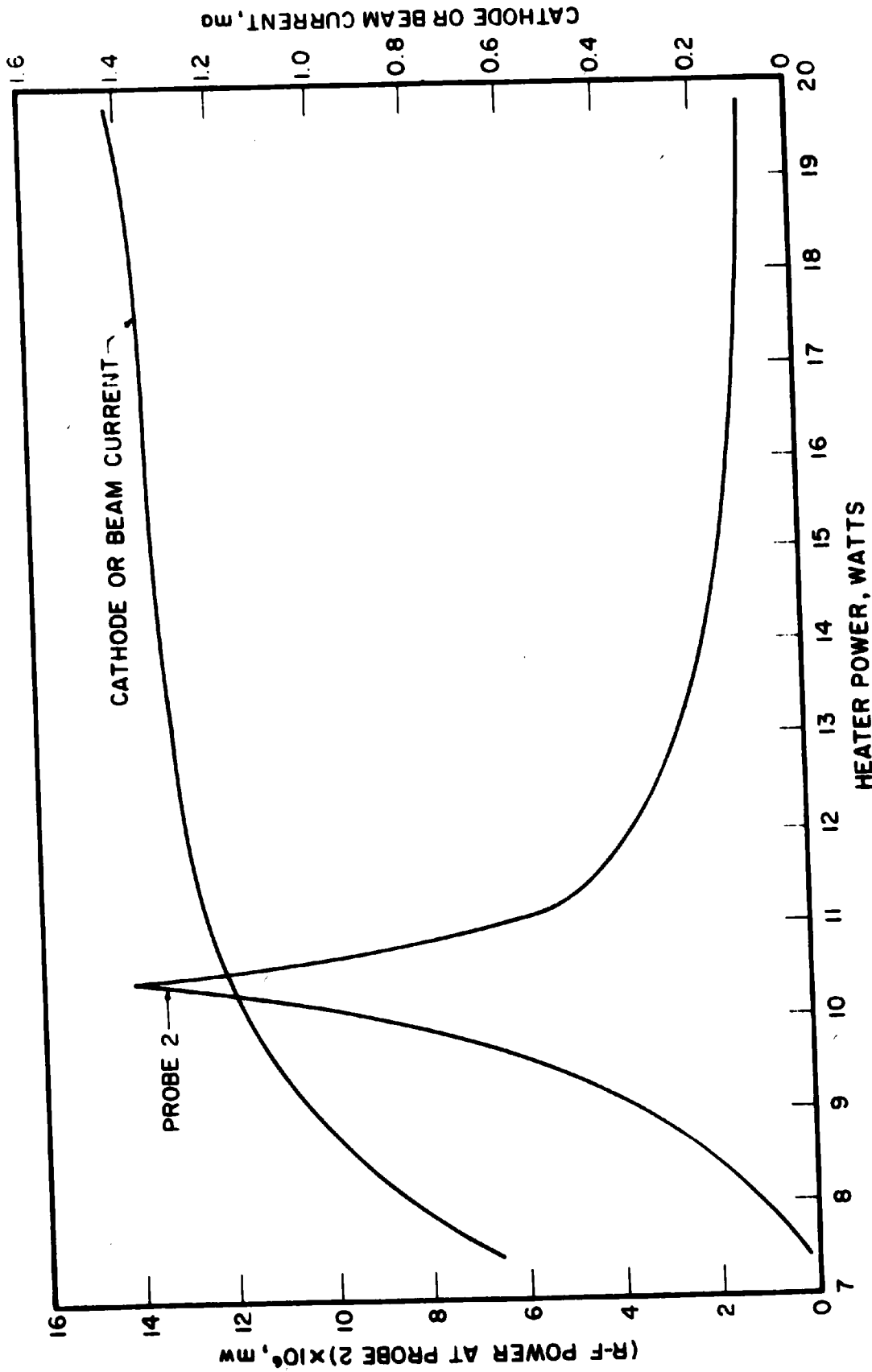


FIG. 6.10 VARIATION OF SIGNAL POWER ON RECEIVING PROBE 2 AS A FUNCTION OF HEATER POWER FOR AN INJECTED SIGNAL OF CONSTANT POWER AND FREQUENCY AT PROBE 5 FOR THE CROSSED-FIELD GUN. (GUN ANODE, ANODE AND COLLECTOR POTENTIAL = 190 VOLTS; CATHODE AND SOLE POTENTIAL = 0 VOLT; $B = 87$ GAUSS; $f = 100$ MHz AND $f_c = 244$ MHz)

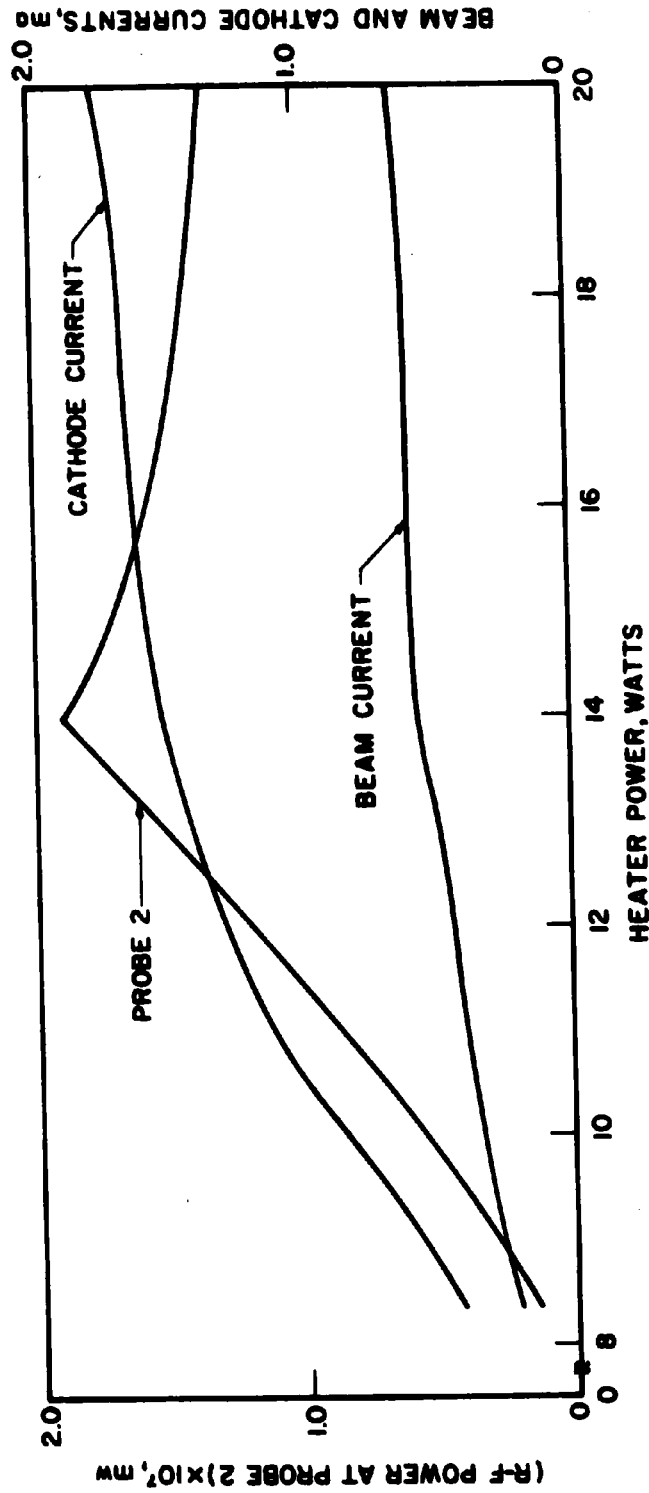


FIG. 6.11 VARIATION OF SIGNAL POWER ON RECEIVING PROBE 2 AS A FUNCTION OF HEATER POWER FOR AN INJECTED SIGNAL OF CONSTANT POWER AND FREQUENCY AT PROBE 5 FOR THE CROSSED-FIELD TRIODE GUN. (GUN ANODE, ANODE AND COLLECTOR POTENTIAL = 190 VOLTS; CATHODE AND SOLE POTENTIAL = 0 VOLT; GRID VOLTAGE = 12.8 VOLTS; $B = 87$ GAUSS; $f = 100$ MHz AND $f_c = 244$ MHz)

Though the r-f signal is injected beyond the minimum region, the results described above indicate that the signal (noise) transport along the crossed-field beam does depend on the cathode operating condition. This suggests that the characteristics of the beam beyond the potential minimum, which influence the signal (noise) transport, are modified appreciably by the cathode operating condition. Next, the decrease in the detected signal power in the space-charge-limited region agrees well with the results of Monte Carlo analyses of crossed-field guns described in Chapter IV where space-charge smoothing was observed even in the regions beyond the potential minimum.

6.3.4 Effect of Grid Voltage Variation on the Detected Signal Power. In Fig. 6.12 the signal powers received at detecting probes 1, 2 and 3 at the exit of the crossed-field triode gun (see Fig. 6.5) are shown to decrease with increasing cathode current produced by increasing the grid voltage. From the insert in Fig. 6.12 showing the variation of the cathode current, i_c , and the grid current, i_g , as a function of grid voltage, V_g , it can be seen that the cathode was operating under space-charge-limited conditions. Depending upon the degree of the space-charge limitation, according to Section 6.3.3, the signal should have increased when the cathode current was increased by increasing the grid voltage. In the present case the detected signal decreased. The explanation for this behavior is as follows. The variation in the grid voltage from 10.5 volts to 19.6 volts greatly affects the velocity of electrons in the cathode-grid region. It can be easily seen that, for a constant magnetic field, the electron paths between the cathode and the grid become less curved as the grid voltage is increased. The electric and magnetic field configurations in the gun region beyond

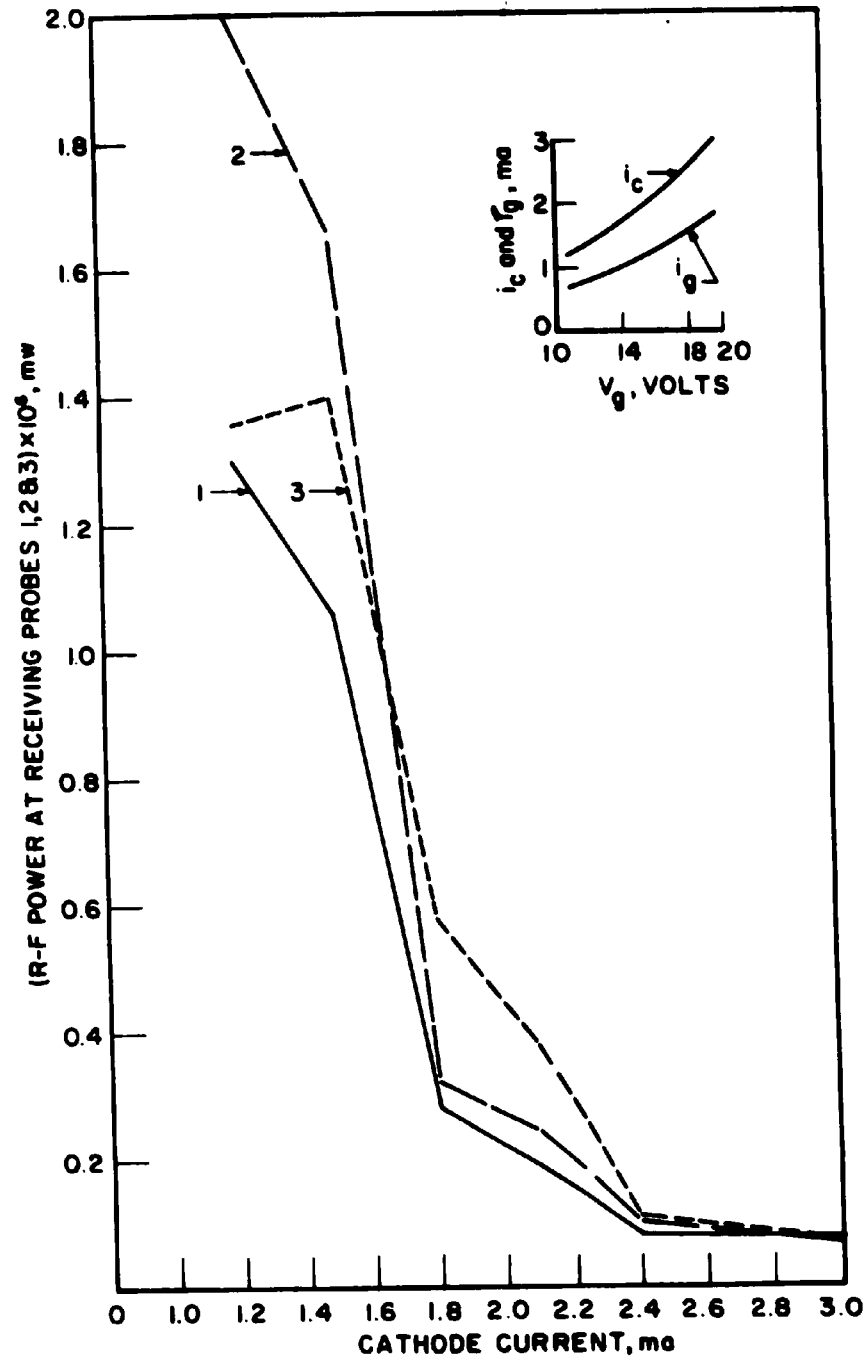


FIG. 6.12 VARIATION OF SIGNAL POWER AT RECEIVING PROBES 1, 2 AND 3 WITH CATHODE CURRENT DUE TO A VARIATION OF GRID VOLTAGE FOR THE CROSSED-FIELD TRIODE GUN. (GUN ANODE, ANODE AND COLLECTOR POTENTIAL = 380 VOLTS, CATHODE AND SOLE POTENTIAL = 0 VOLT; $B = 100$ GAUSS; $f = 100$ MHz AND $f_c = 280$ MHz)

the grid essentially remain the same since only a 2.5 percent change occurs in the gun anode-grid voltage. However, the electrons are injected into this region with increasing velocity as a result of increasing grid voltage. The electron trajectories in a crossed electric and magnetic field configuration depend a great deal on the injection velocity. Hence by changing the grid voltage the electron trajectories are essentially changed and thus the position of the beam with respect to the detecting probe changes, resulting in the coupling change between the beam and the probe. In Fig. 6.12 the beam presumably moves away from the detecting probe as the grid voltage is increased. Hence the signals on probes 1, 2 and 3 decrease. The signal power received on probe 2 is the largest of the three since this probe is located at the center of the beam and the signal is also injected at the center of the beam near the cathode. Probes 1 and 3 are equidistant on each side of the central probe and hence the signal power received on these probes should be approximately equal which is found to be true as shown in Fig. 6.12. Under other operating conditions the signal power at the detecting probe passes through several maxima and minima when the grid voltage is varied. A typical case is shown in Fig. 6.13. In this case it appears that the beam alternately moves toward the probe and then away from it as the grid voltage is varied. Thus it is clear that, for a signal of fixed power and frequency injected near the cathode, the variation in the signal detected at a point along the beam with varying grid voltage is essentially due to the coupling change between the probe and the beam.

6.3.5 Effect of Beam Velocity on the Signal Power Detected

Along the Beam. Table 6.1 shows the signal detected at receiving probes 1, 2 and 3 under two operating conditions of the crossed-field

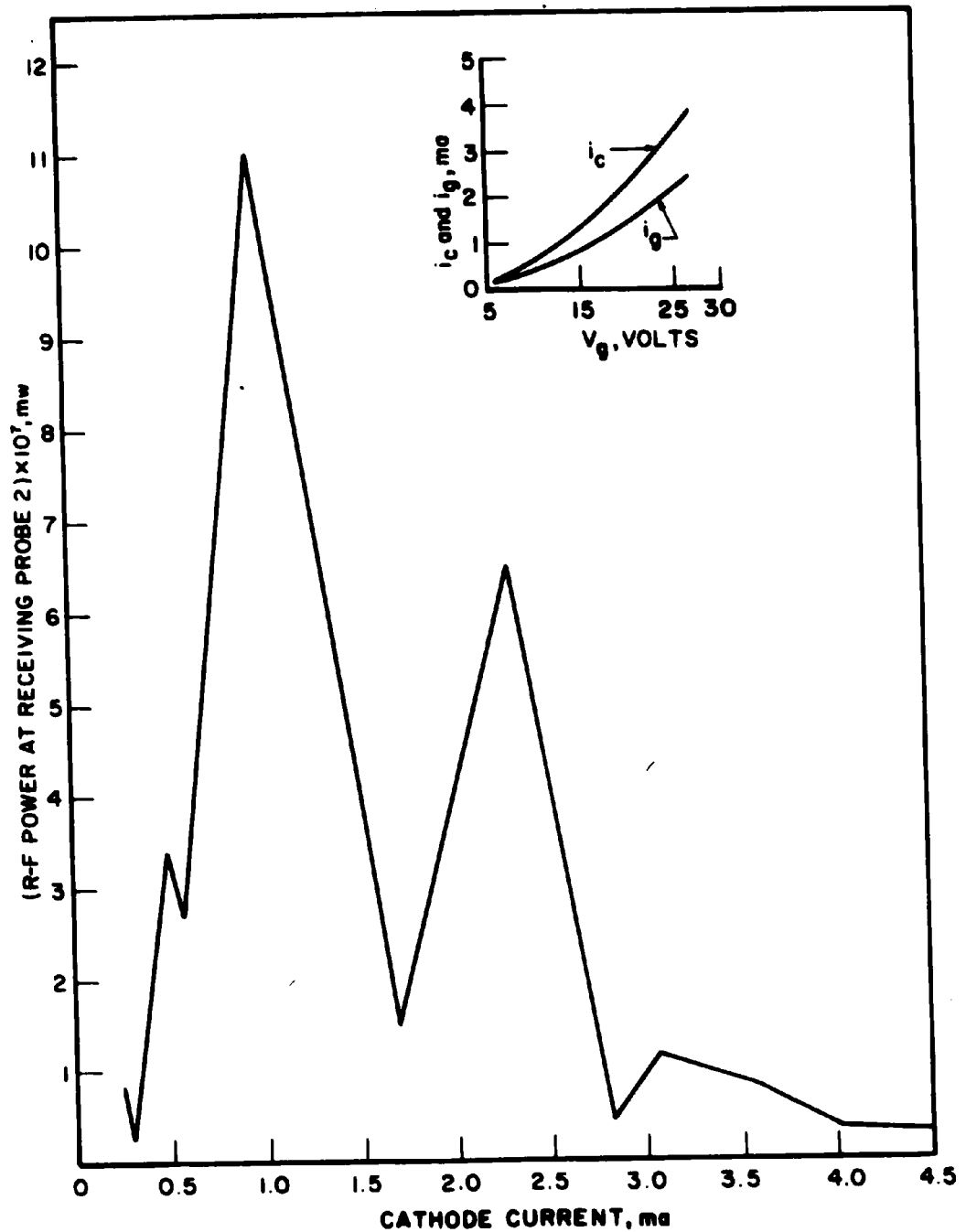


FIG. 6.13 VARIATION OF SIGNAL POWER AT RECEIVING PROBE 2 WITH CATHODE CURRENT DUE TO A VARIATION OF GRID VOLTAGE FOR THE CROSSED-FIELD TRIODE GUN. (GUN ANODE, ANODE AND COLLECTOR POTENTIAL = 300 VOLTS; CATHODE AND SOLE POTENTIAL = 0 VOLT; $B = 123$ GAUSS; $f = 100$ MHz AND $f_c = 344$ MHz)

Table 6.1
Variation in Signal Power Detected at Receiving Probes 1, 2 and 3
with Beam Velocity (Crossed-Field Triode Gun)

Gun Anode, Anode and Collector Potential (Volts)	Cathode and Sole Potential (Volts)	Cathode Current, i_c , (ma)	Grid Potential (Volts)	Grid Current, i_g , (ma)	Beam Current ($i_c - i_g$) (ma)	Magnetic Field B (Gauss)	Detected Signal Power		
							Probe 1 (mw)	Probe 2 (mw)	Probe 3 (mw)
190	0	2.0	16.0	1.36	0.64	85	2.0×10^{-8}	3.0×10^{-8}	2×10^{-8}
380	0	2.0	17.0	1.30	0.70	120	1.9×10^{-8}	2.4×10^{-8}	2×10^{-8}

triode gun when the frequency and power of the injected signal were kept constant. In the second operating condition the gun anode, anode-sole and collector voltages were doubled while the magnetic field was increased by a factor of $\sqrt{2}$. The grid voltage was adjusted to keep the cathode current constant (cathode temperature remaining constant) in the two cases. The ratio V/B^2 is thus essentially constant throughout the gun region in both cases since the necessary change in the grid voltage is small. Consequently the electron trajectories and the beam location are practically the same and no discernible change in coupling between the beam and the detecting probes occurs. The cathode operating conditions remain the same in the two cases since the cathode temperature and the cathode current, i_c , are identical. Due to the change in the grid current, i_g , the beam current ($i_c - i_g$) changes by about 9.4 percent. Although it was not possible to evaluate (as explained in Section 6.3.6) the effect, if any, of beam current on the signal power detected at the receiving probes, it is believed that the above-mentioned change in the beam current should not have any significant effect on the signal power received at probes 1, 2 and 3. The only important difference in the two cases appears to be the beam velocity which, in the second case, is $\sqrt{2}$ times that in the first case. Since all of the other parameters (discussed in Sections 6.3.1 through 6.3.4) which might affect the signal detected at the receiving probes are almost the same in the two cases, the large ratio between the signal powers received at probes 1, 2 and 3 in the two cases can only be attributed to the difference in the beam velocities and thereby possibly to an increase in the kinetic potential detected by the probe. In short, with other parameters remaining constant, an increase in beam velocity

facilitates signal (noise) transport along the beam. Finally it should be noted that the increase in the detected signal level with an increase in beam velocity was several times larger than that observed in previous sections in connection with the variation of other parameters.

6.3.6 Effect of Beam Current on the Signal Power Detected at the Receiving Probe. It was not possible to change the beam current without altering other parameters, viz., the electric and magnetic fields, the beam location and the cathode operating condition, which affect the transport of signal (noise) along the beam. Hence any change produced by the variation in beam current on the signal transport along the beam could not be evaluated in this work.

6.4 Experimental Study of Self-Oscillations

During the investigations on the r-f signal transport along the beams in the crossed-field diode and triode guns and KG-A, it was observed that, for certain electrode potentials and magnetic fields, some intrinsic oscillations were generated within the frequency band of 90 MHz to 125 MHz. The oscillation frequency changed by a small amount due to changes in the electric field, the magnetic field and the cathode temperature. It was also observed that these self-oscillations were associated with fluctuations in the d-c anode and collector currents. When an r-f signal below 300 MHz was injected in the neighborhood of the cathode, the self-oscillations as well as the d-c anode and collector current fluctuations were quenched.

The self-oscillations generated on the crossed-field diode and triode guns are described in the following sections. The self-oscillations were detected both at the fixed probes (Nos. 1, 2 and 3) at

the gun exit and at the movable probes (a, b and c) in the drift region. The signal power in the latter case was lower so it was thought desirable to study these oscillations by using one of the fixed probes. No significant difference was observed between the signal strength at these probes. The experimental results are discussed below.

6.4.1 Variation of the Oscillation Frequency with Magnetic Field. Figures 6.14 and 6.15 show the variation of the cathode current and the self-oscillation frequency as a function of the magnetic field for the crossed-field diode gun. In both cases the cathode temperature is the same but the gun anode voltage is different. As mentioned earlier, these oscillations exist only for a small range of the magnetic field. The cathode current decreases with the increase in the magnetic field. In Fig. 6.14 the oscillation frequency goes through a maximum, whereas in the other case (Fig. 6.15) it increases monotonically with increasing magnetic field. In the case of the crossed-field triode gun, these plots are made for one set of fixed gun anode and grid voltages. It is interesting to note that in this case (Fig. 6.16) the frequency variation curve has a negative slope.

To interpret the above behavior of the change in the oscillation frequency with magnetic field, first consider Fig. 6.14 which has both positive and negative slopes. Under space-charge-limited operation, it appears that the E/B (electric to magnetic field) ratio determines the slope of the frequency variation curve. Now, assume that the E/B ratio for the maximum of the frequency curve is the critical value of E/B which turns out to be 170 volts/meter/gauss. The values of $E/B > (E/B)_{cr}$ correspond to the positive slope of the curve and vice-versa. This hypothesis will now be applied to the other

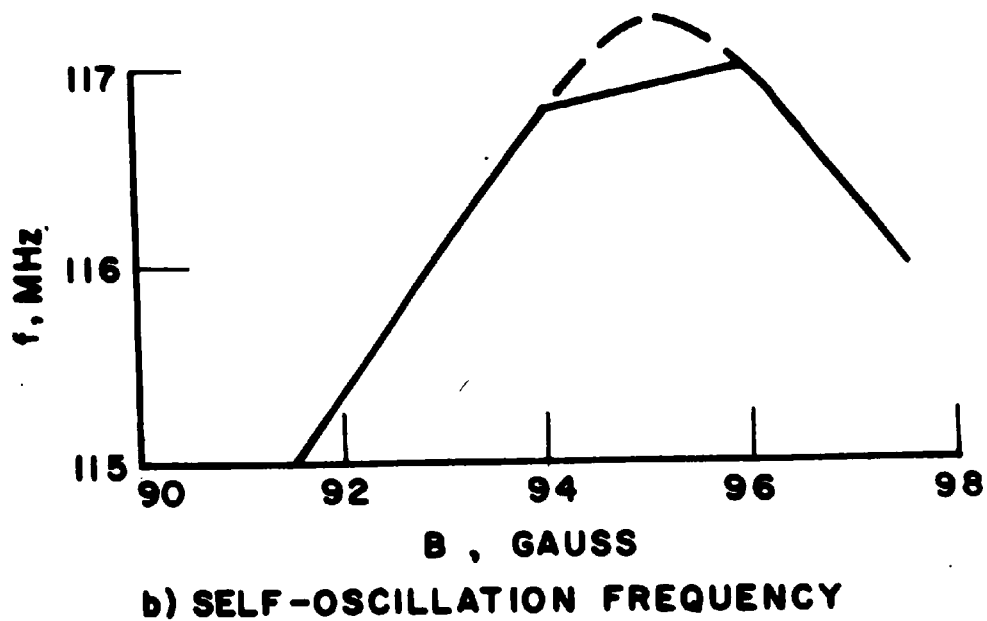
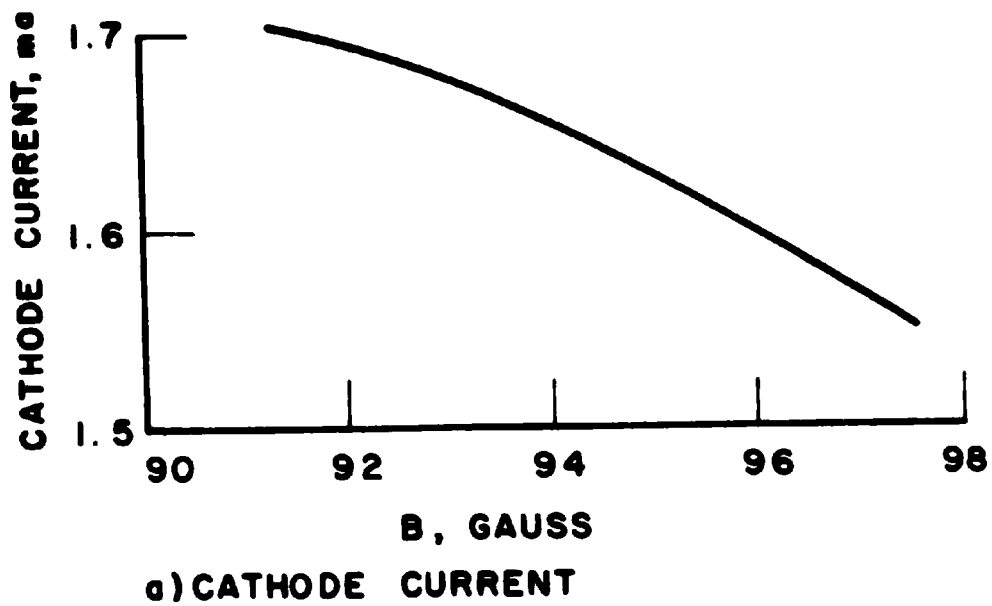


FIG. 6.14 VARIATION OF THE CATHODE CURRENT AND SELF-OSCILLATION FREQUENCY WITH MAGNETIC FIELD FOR THE CROSSED-FIELD DIODE GUN.
(FILAMENT CURRENT = 1.5A; GUN ANODE VOLTAGE = 190 VOLTS)

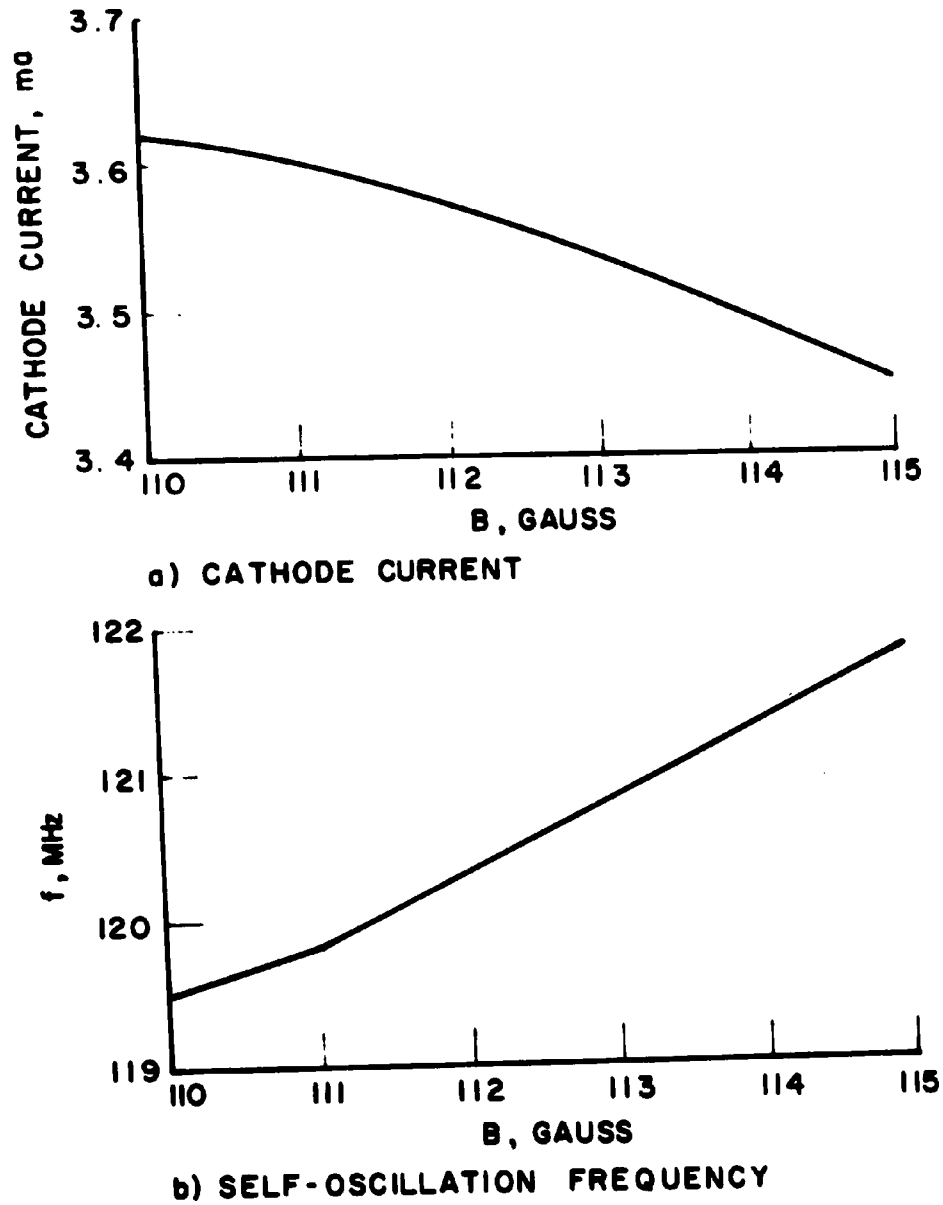
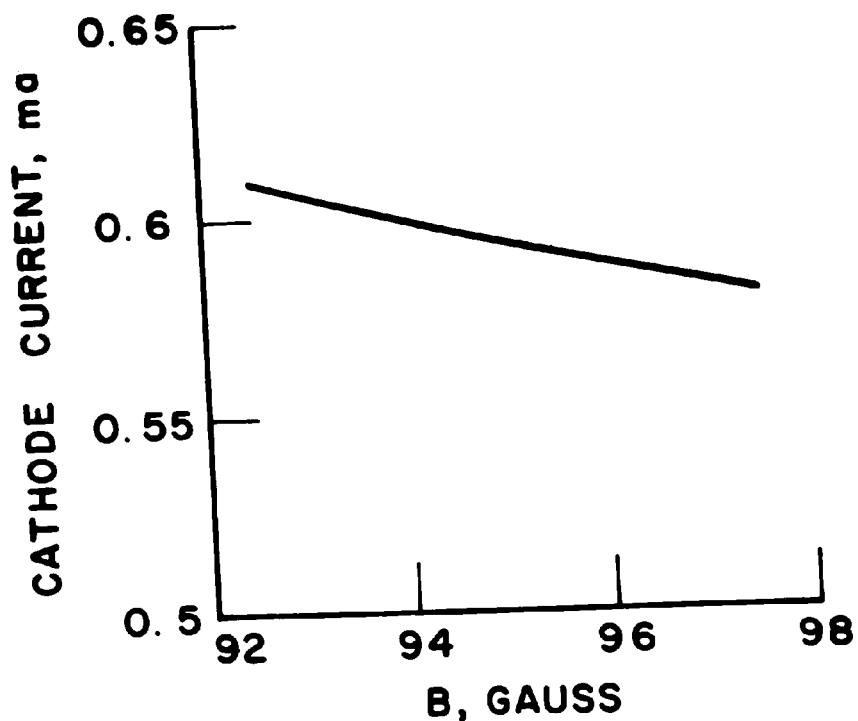
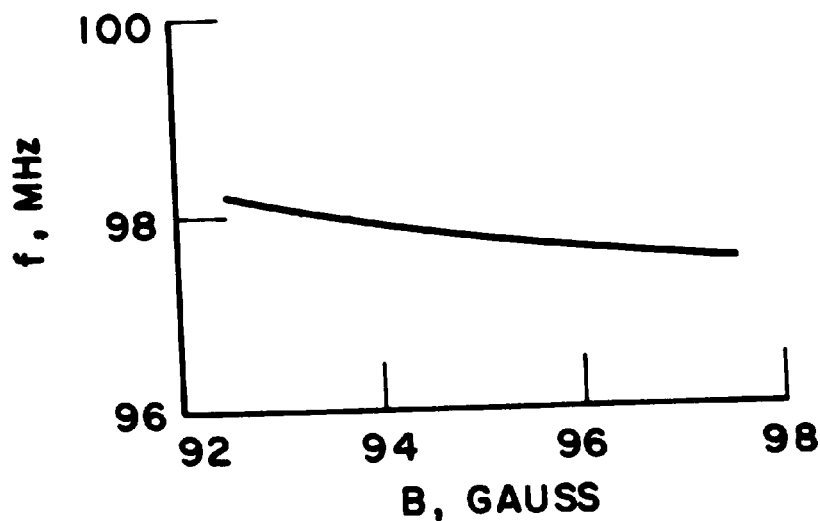


FIG. 6.15 VARIATION OF THE CATHODE CURRENT AND SELF-OSCILLATION FREQUENCY WITH MAGNETIC FIELD FOR THE CROSSED-FIELD DIODE GUN. (FILAMENT CURRENT = 1.5A; GUN ANODE VOLTAGE = 300 VOLTS)



a) CATHODE CURRENT



b) SELF-OSCILLATION FREQUENCY

FIG. 6.16 VARIATION OF THE CATHODE CURRENT AND SELF-OSCILLATION FREQUENCY WITH MAGNETIC FIELD FOR THE CROSSED-FIELD TRIODE GUN. (FILAMENT CURRENT = 1.5A; GUN ANODE VOLTAGE = 300 VOLTS; GRID VOLTAGE = 6.8)

two cases shown in Figs. 6.15 and 6.16. In the second case (Fig. 6.15), E/B lies between 233 and 244 over the self-oscillation frequency range. Hence in this case the slope of the frequency vs. magnetic field curve should be positive. This agrees well with Fig. 6.15. In the third case, viz., the crossed-field triode gun, the grid practically screens the cathode from the gun anode. Consequently, the electric field at the cathode is approximately the same as that produced by the grid alone. In this case E/B varies between 34.8 and 36.7 for the oscillation-frequency range which, according to the above hypothesis, should correspond to a negative slope of the frequency vs. magnetic field curve. This is borne out by Fig. 6.16.

6.4.2 Variation of the Oscillation Frequency with Grid Voltage.

In the last case mentioned above, the grid and gun anode voltages were constant and the magnetic field was varied. Here the magnetic field and the gun anode potential are held constant and the grid voltage (which essentially creates the field in the neighborhood of the cathode) is varied. It should be noted that these two cases are the counterparts of each other. In a crossed-field system, increasing the magnetic field has the same effect on the E/B ratio as decreasing the electric field. Hence, for a given range of E/B ratio, when the frequency vs. magnetic field curve has a negative slope, the frequency vs. grid voltage curve should have a positive slope. In Fig. 6.17, the ratio E/B varies between 34.6 and 41.3 which is much less than $(E/B)_{cr}$. The positive slope of the frequency curve in this figure is in agreement with the discussion above.

6.4.3 Variation of the Oscillation Frequency with the Cathode Temperature. The variation of the self-oscillation frequency as a

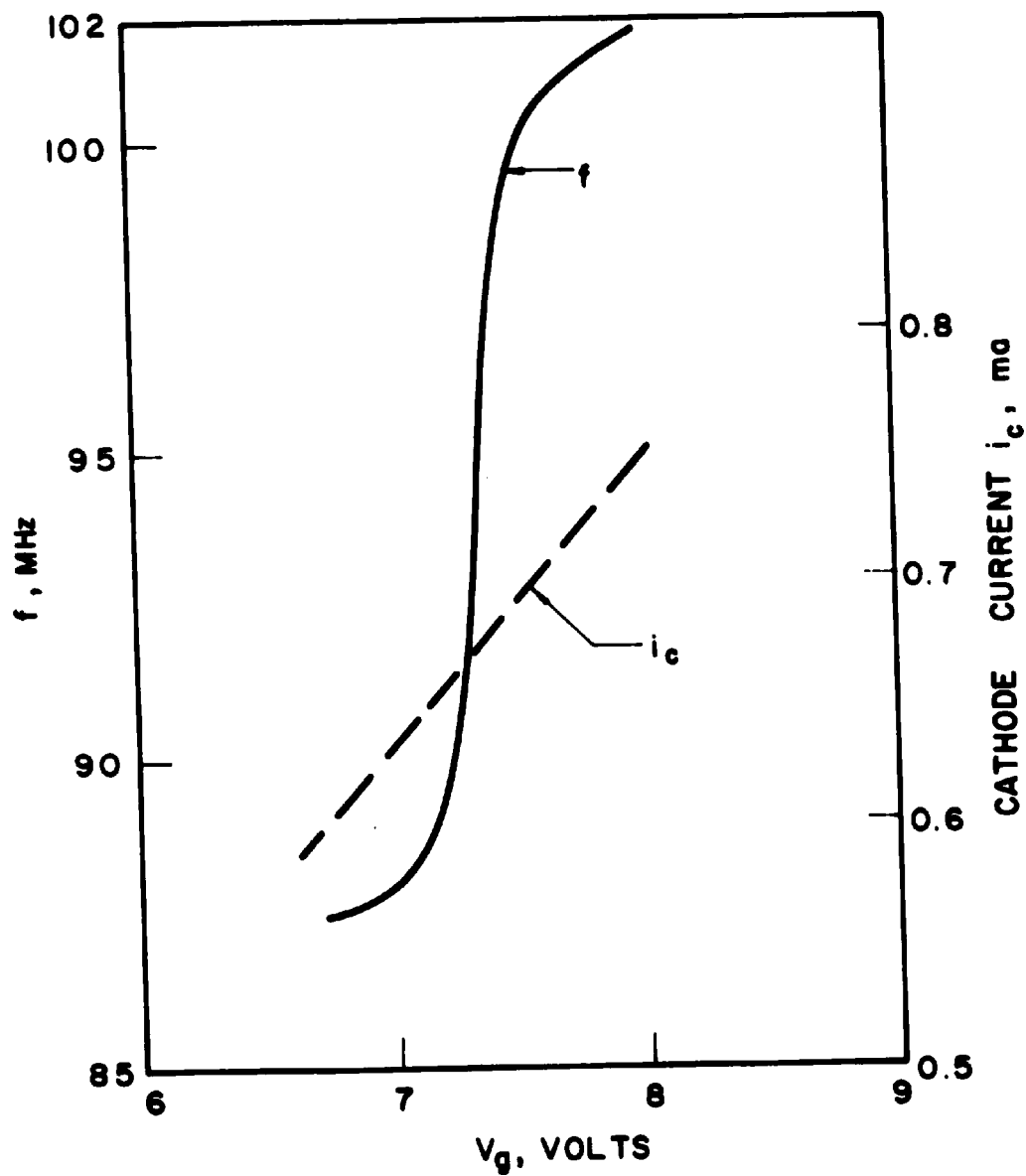


FIG. 6.17 VARIATION OF THE CATHODE CURRENT AND THE SELF-OSCILLATION FREQUENCY WITH GRID VOLTAGE FOR THE CROSSED-FIELD TRIODE GUN. (FILAMENT CURRENT = 1.5A; GUN ANODE VOLTAGE = 300 VOLTS; $B = 97$ GAUSS)

function of the filament current is shown in Figs. 6.18 and 6.19 for the crossed-field diode and crossed-field triode respectively. In each case the E/B ratio is held constant. If $(E/B)_{cr}$ is assumed to be some increasing function of the space-charge density in the neighborhood of the cathode, the frequency variation curves in Figs. 6.18 and 6.19 may be explained as follows. Under space-charge-limited operation when the electrode potentials are held constant, the space-charge density near the cathode increases with increasing cathode temperature. As a result, $(E/B)_{cr}$ is also some increasing function of the cathode temperature (or the filament current). In Fig. 6.18, $E/B \approx (E/B)_{cr}$ initially and then $E/B < (E/B)_{cr}$ because $(E/B)_{cr}$ increases with an increasing filament current. In Fig. 6.19, E/B initially is much smaller than $(E/B)_{cr}$ for this operating cathode temperature. As the filament current is increased, $(E/B)_{cr}$ increases and hence $E/B < (E/B)_{cr}$ throughout this range of cathode temperature. Thus the negative slope of the frequency curve in these two cases is the one expected since the slope should be negative whenever $E/B < (E/B)_{cr}$.

6.4.4 Suppression of the Self-Oscillations. The self-oscillations were quenched when an r-f signal below 300 MHz was injected in the neighborhood of the cathode. It was found that if the signal was injected at one of the fixed probes at the exit of the gun, the oscillations were unaffected. One set of observations on the quenching of the self-oscillations generated in the crossed-field diode is briefly described below. With the gun anode potential at 300 volts and a magnetic field of 115 gauss, the generated oscillation frequency f_{osc} was 121.8 MHz. The oscillations were quenched immediately by the application of injected signal frequencies f_{in} up to 180 MHz. When

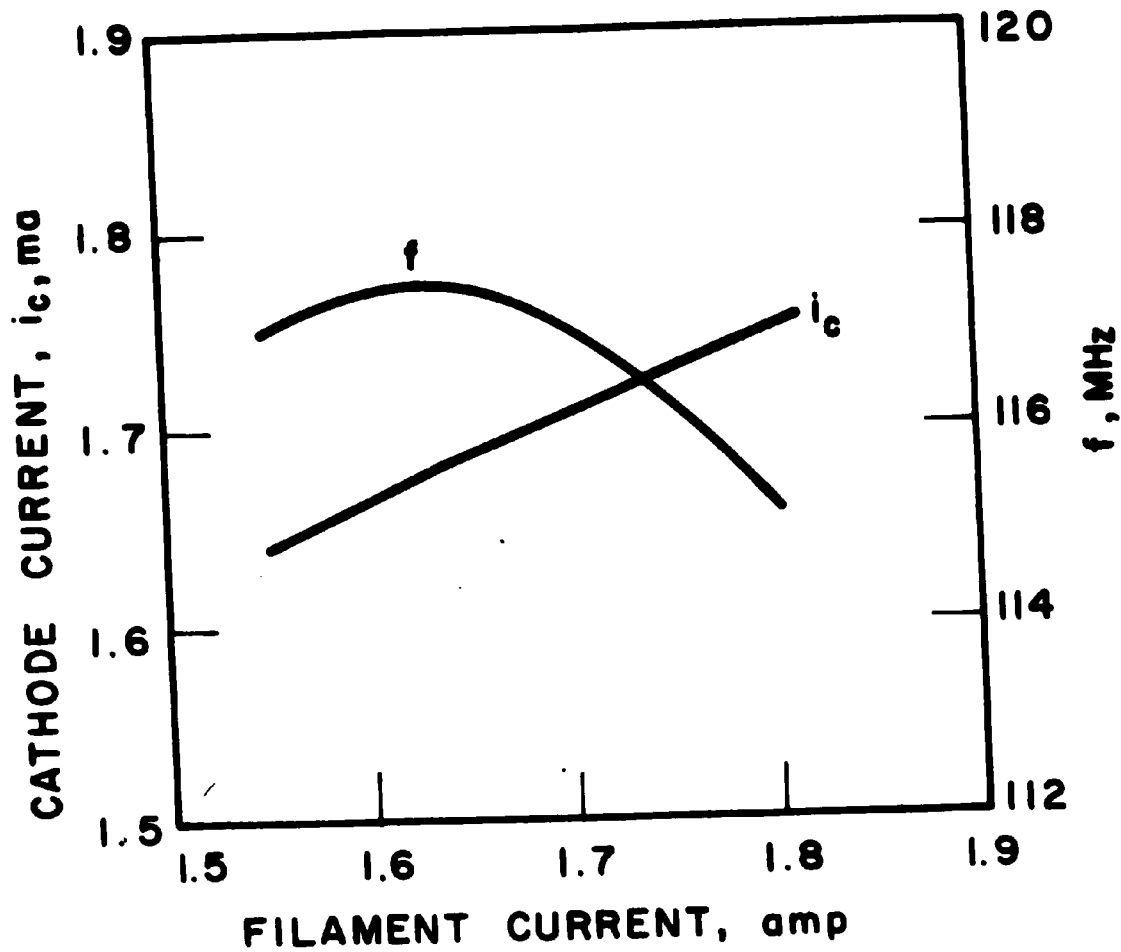


FIG. 6.18 VARIATION OF THE CATHODE CURRENT AND THE SELF-OSCILLATION FREQUENCY WITH FILAMENT CURRENT FOR THE CROSSED-FIELD DIODE GUN. (GUN ANODE VOLTAGE = 190 VOLTS; $B = 97$ GAUSS)

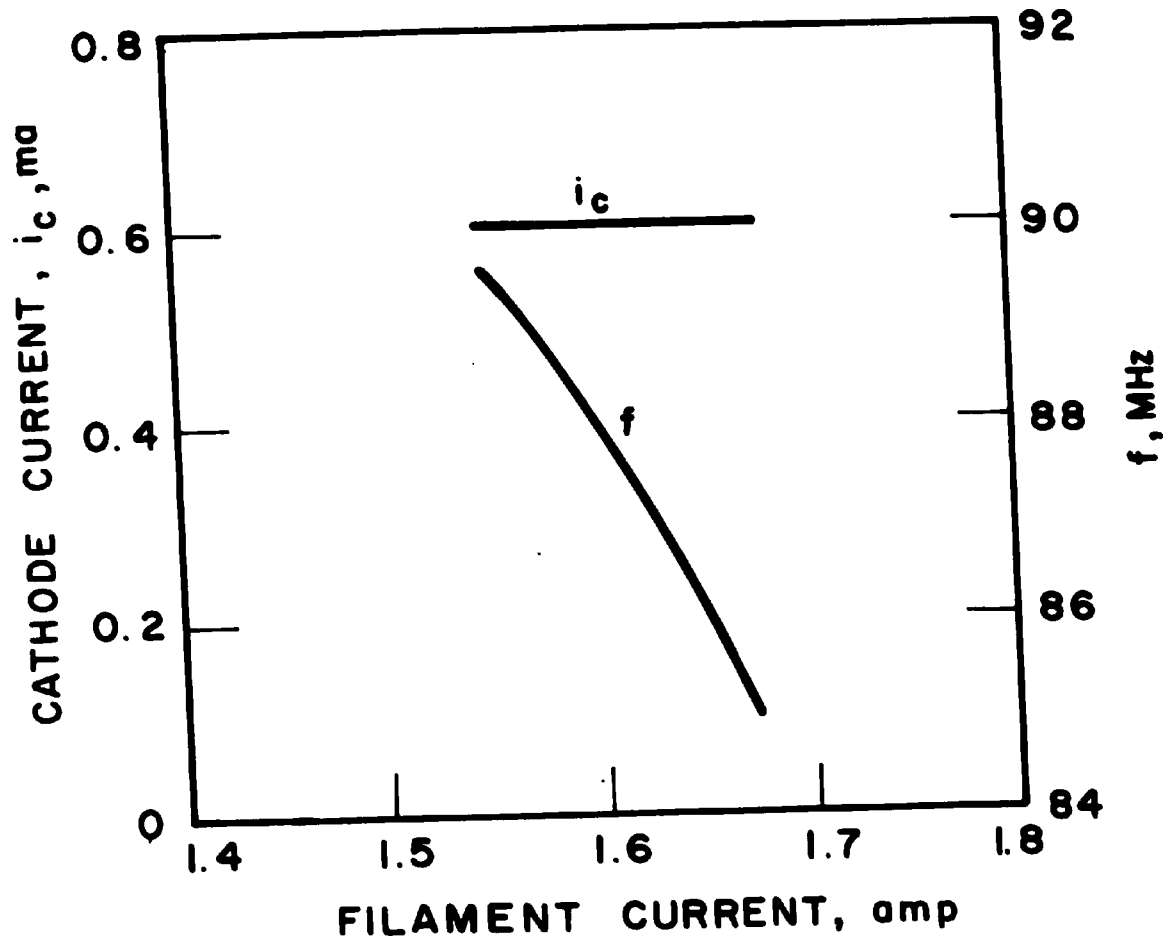


FIG. 6.19 VARIATION OF THE CATHODE CURRENT AND THE SELF-OSCILLATION FREQUENCY WITH FILAMENT CURRENT FOR THE CROSSED-FIELD TRIODE GUN. (GUN ANODE VOLTAGE = 300 VOLTS; GRID VOLTAGE = 6.8 VOLTS; $B = 97$ GAUSS)

f_{in} was between 180 and 300 MHz, f_{osc} decreased slightly before quenching. The maximum shift in f_{osc} was 1.2 MHz. Similar behavior was observed for the oscillations generated in the crossed-field triode and KG-A.

6.5 Conclusions

Experiments with crossed-field guns described above indicate that signal (noise) transport along a crossed-field beam depends on the cathode operating condition and the beam velocity. The signal power detected along the beam is proportional to the injected signal power up to a point beyond which saturation sets in. The signal transport is also a function of injected signal frequency.

The frequency range of the self-oscillations is not in the vicinity of the plasma frequency, the cyclotron frequency or the ion-oscillation frequency. The quenching of these oscillations when an r-f signal is injected in the neighborhood of the cathode suggests that the source of these oscillations lies in the vicinity of the cathode.

The strong r-f field of the injected signal seems to suppress the development of the weaker space-charge field, causing these oscillations. The same conclusion was drawn by Epsztein⁴¹ in his experimental study (an analogue method) of large-signal behavior in M-type tubes. He found that the presence of a large r-f field considerably reduces the amplitude of noise. He thought that, because of the nonlinearity of the motion, the stronger applied r-f field suppresses the development of the weaker space-charge field. Similar oscillations have been reported on by Wadhwa³² but no satisfactory explanation regarding the origin and the nature of these oscillations has been found. Some qualitative explanations given in the previous sections are based on $(E/B)_{cr}$ as

some increasing function of the cathode temperature (with the cathode operating under space-charge-limited conditions). A suitable theory incorporating the role of $(E/B)_{cr}$ could probably explain the above results quantitatively.

CHAPTER VII. SUMMARY, CONCLUSIONS AND SUGGESTIONS

FOR FURTHER STUDY

7.1 Summary and Conclusions

The main purpose of this study has been to investigate noise transport along the space-charge flow in crossed d-c electric and magnetic fields. The Monte Carlo technique is employed to calculate parameters describing noise transport along the beams in an abbreviated Kino gun (KG-A), a modified Kino gun (KG-M), a crossed-field parallel plane diode gun and drift regions which follow the gun region. Some experimental data on signal (noise) transport and self-oscillations generated under certain operating conditions in these electron guns has been presented and discussed.

The Monte Carlo analysis is superior to the density function and Llewellyn-Peterson methods of analysis since it does not require the small-signal single-velocity assumption. This fact is especially important near the potential minimum where the velocity spread of the electrons is comparable to their average velocity and the small-signal single-velocity assumption becomes a poor approximation. The present analysis is nonrelativistic and the quantities are assumed to have no variation in the direction of magnetic field. The thermionic emission is simulated by randomly emitting the so-called "big electrons" using random number transform techniques. The big electrons are then followed through the electron gun and drift space geometries by numerical integration of the dynamic equations of motion. A satisfactory accuracy (discussed in detail in Section 2.13) was obtained in the space-charge-flow simulation in the gun and drift regions.

In the present two-dimensional analysis the d-c and a-c components of electron velocity and the current density in the y- and z-directions are calculated first. These are then used to obtain velocity and current density fluctuations in the direction of the d-c beam velocity at each point in the beam. The Self-Power Density Spectra Φ for the kinetic noise-voltage fluctuations and Ψ for noise-current fluctuations are defined on both a cross-sectional and a rectangle basis to study the variation of these noise parameters along the beam and across a given beam cross section respectively. Other noise parameters, viz., Π , S , $S-\Pi$ and F_0 , are also calculated along the beam on the cross-sectional basis. A discussion of the confidence interval, i.e., the range in which the estimated value of a given noise parameter will fall with respect to the true value, is given in Section 4.2. The noise parameters are calculated for the frequency range 0 to 10 GHz at intervals $\Delta f = 0.5$ GHz apart. It is believed that the statistical error involved in the calculation of power spectra should not mask any significant variations of these parameters with frequency.

In the cathode region Φ for all three guns is large around $f = 0$, especially for the first one or two noise planes. Near the potential minimum hybrid or plasma frequency, Φ has a broad minimum and attains high values in the mid-frequency region. Away from the cathode Φ does not assume large values around $f = 0$ but has a peak around the local hybrid or plasma frequency. The rate of change of Φ with distance increases along the beam such that Φ increases by several orders between the cathode and the gun exit. The velocity fluctuations are minimum along the outer beam edge.

In each of the three guns space-charge smoothing is observed not only in the potential minimum region but also further along the beam although the rate of change of Ψ is maximum near the cathode. It is very interesting to note that Ψ has a peak in both KG-A and KG-M in the region of the beam where the space-charge density changes from a value greater to one lower than the Brillouin space-charge density. This observation could be an important consideration in deciding on the termination point of KG-A. It is also important to note that Ψ_J (due to current density fluctuations) across a given beam cross section in the gun region has a significant variation only up to the potential minimum hybrid or plasma frequency and that the Ψ_J variation is correlated with the potential minimum profile across the cathode. The portion of the beam which passes through the deepest potential minimum has minimum Ψ_J . Thus a deep and uniform potential minimum profile across the cathode is desirable in minimizing noise contributions from the current fluctuations. In the cathode region Ψ for all three guns has peaks around the potential minimum hybrid or plasma frequency and harmonics of the hybrid or plasma frequency, whereas in regions away from the cathode Ψ generally has a high value around $f = 0$.

In each of the three guns Π varies in magnitude and sign along the beam. The fast wave is seen to predominate in the low velocity region, whereas the slow wave grows as the beam is accelerated and the growth is maximum in the low frequency region. S and $S-\Pi$ both increase by several orders between the cathode and gun exit planes. An interesting similarity between the variation of S and Ψ with frequency is also observed. The noise parameter $F_0(\text{db})$ increases along the beam with the

maximum spatial rate of change occurring in the neighborhood of the cathode.

The variance calculations show that σ_u^2 , σ_y^2 and σ_z^2 increased in the cathode region for all of the guns analyzed. In the Kino-type guns these variances show a sharp rise around $\theta = \pi/2$ where a peak of Ψ appears and the space-charge density in the beam changes from a value higher to one lower than the Brillouin space-charge density. σ_u^2 and σ_z^2 attain minimum values around $\theta = \pi$ where $\dot{y}$ attains its maximum value according to the Kino theory. σ_y^2 increases, in general, between the cathode and gun exit in Kino-type guns. In regions of the beam away from the cathode, σ_u^2 and σ_z^2 remain relatively constant in the Kino-type guns whereas these quantities decrease in the parallel plane diode gun. Variance values are highest at the inner edge of the beam for each gun, indicating higher kinetic temperatures along the inner edge of crossed-field beams. The above observations also indicate a "geometric" variation of the kinetic temperature in crossed-field beams.

Except for the lower beam convergence several d-c characteristics of KG-M, e.g., uniformity of cathode current density across the cathode and beam laminarity, are superior to those of KG-A. The modified focus electrodes in KG-M help to yield a more or less laminar beam (nonintersecting trajectories) but the change in electric field and beam shape due to this modification results in a higher rate of change of Φ with distance and consequently a value of Φ about four to five times higher than that for KG-A over the same beam lengths. However, σ_y^2 and σ_z^2 for upper and lower edges of the beam near the exit plane of KG-M are smaller than those over the same portion of the beam in KG-A. This indicates that, with respect to laminarity, KG-M is better than

KG-A. The space-charge smoothing of noise-current fluctuations in KG-M is less than that found with KG-A. This is probably because of the nonuniform potential minimum profile and larger beam current than that in KG-A for the same cathode emission current density, J_s , assumed in the Monte Carlo simulation. Since an exact noise figure expression in terms of Φ , Ψ , etc., is not yet available for crossed-field devices it is not possible to compare these Kino-type guns (KG-A and KG-M) on the basis of noise figure. Thus, on the basis of data available, it is not possible to decide whether KG-A or KG-M is better on the whole. Since a crossed-field diode gun does not produce a convergent beam, it is not suitable for producing high density beams. The noise characteristics of such a gun obtained in this work cannot be compared meaningfully with those of KG-A and KG-M because of the difference in various operating parameters of the diode and Kino-type guns. The main object of analyzing noise transport in such a crossed-field diode gun was to compare the results obtained here with those of Wadhwa and Rowe¹¹ for a nearly identical diode under the short-circuit assumption and also to study the effect of the OC assumption on the various noise parameters. The analysis presented here and that of Wadhwa and Rowe differ on several counts discussed in Section 4.6. There is a qualitative agreement between the results obtained from the two analyses. The quantitative difference is most probably due to the difference in the definitions of various noise parameters.

The noise transport analysis in an incremental length of the drift space following KG-A or KG-M indicates large noise concentration in the low frequency region. Peaks of Φ and Ψ occur near the local hybrid or plasma frequency. The variation of noise parameters along

the undulating beam could not be explained completely on the basis of the variation of diocotron gain per unit length along the beam. The existence of standing waves of Φ and Ψ is also indicated and could be checked by continuing the calculations for a few more incremental steps along the beam, preferably with small amplitude undulations.

The short- and open-circuit noise parameters calculated for beams in the gun and drift regions are approximately the same, possibly because of the very small difference between the electric fields in the two cases.

The experimental study of signal (noise) transport along the crossed-field beams was made using an r-f probe technique. An r-f signal is injected at a probe located near the cathode and is detected at a later point along the beam by another r-f probe. The detected signal power is found to vary with input frequency at a constant input signal power and shows saturation at higher input signal powers. The signal power detected along the beam depends on the cathode operating condition (temperature-limited or space-charge-limited) and in particular, a higher degree of space charge reduces the signal strength. A higher beam velocity also favors signal propagation along the beam.

For some values of electric and magnetic fields, intrinsic oscillations in the frequency range of 90 to 125 MHz were detected in the crossed-field guns used in this experimental study. The oscillations were generally associated with fluctuations in the anode and collector currents and were quenched when a suitable r-f signal was injected near the cathode. The self-oscillation frequency is found to be a function of electric and magnetic field strengths and cathode temperature.

In short, the theoretical and experimental results summarized above have contributed toward a better understanding of the noise (signal) propagation along crossed-field electron beams. This knowledge is very useful in designing electron guns for low noise injected beam crossed-field devices.

7.2 Suggestions for Further Study

In the abbreviated and the modified Kino guns a peak in the value of Ψ occurred in the neighborhood of $\theta = \pi/2$ where the space-charge density changes from a value greater to one lower than the Brillouin space-charge density. In Section 4.4.2b it was stated that a similar peak might be expected around $\theta = 3\pi/2$ and Ψ might increase again as θ approaches 2π . A Monte Carlo analysis of a Kino short gun extending up to $\theta \approx 2\pi$ would be very useful in verifying this possibility. The results of this analysis could also be used to study the noise transport along a Kino gun beam between the abbreviated Kino gun exit and $\theta = 2\pi$.

Monte Carlo analysis of a crossed-field gun with the cathode operating under temperature-limited conditions would help in deciding the controversy over the role of the cathode operating condition on the crossed-field gun noise. The results would also be useful in verifying the idea of Mantena and Van Duzer¹³ that a partial smoothing of current fluctuations due to the sorting action of the magnetic field can occur even in the absence of a potential minimum.

Noise transport analysis for a drift space several beam cyclotron wavelengths long and having a beam with little undulation should be undertaken to check whether standing waves of Φ and Ψ exist. This

knowledge would be very useful in designing noise transformers for low noise crossed-field devices.

The experimental work on signal (noise) transport in the crossed-field triode gun should be extended for different mesh sizes of the grid used. The results would be useful in verifying the claim of workers in C.S.F. Laboratories of Paris, France, that the gun noise is lowered by the action of the grid which reduces the space correlation at the cathode by splitting the beam into many elementary beams. Some ingenious way to study the effect of beam current only on the signal (noise) transport along the crossed-field beams is needed.

The experimental study of signal (noise) transport in the abbreviated Kino gun or Kino-type guns, in general, using probe techniques and actual noise figure measurements would help in understanding the noise transport in injected beam crossed-field amplifiers.

APPENDIX A. RANDOM ELECTRON EMISSION FROM THE CATHODE

A.1 Random Electron Emission Number

It was mentioned in Section 2.3.2 that electrons are emitted from a prescribed set of six equally spaced points on the cathode called the "cathode spots". Since the number of electrons emitted from the cathode in each Δt varies randomly, the same will be true for each cathode spot. Under the assumption that the probability of emission of an electron during an interval is statistically independent of the number of electrons emitted in other similar intervals, it is shown²⁶ that the random process can be described by the Poisson distribution

$$f(S) = \frac{e^{-\lambda} \lambda^S}{S!} , \quad (A.1)$$

where $f(S)$ is the probability that exactly S electrons are emitted from a given cathode spot during the given time interval Δt and λ is the average number of electrons emitted in each Δt .

The cumulative distribution function is given by

$$F(S) = \sum_{S=0}^{S=S} f(S) \quad \text{and} \quad 0 < F(S) \leq 1 , \quad (A.2)$$

where $F(S)$ is the probability that at most S electrons are emitted during Δt . If a random number R uniformly distributed between 0 and 1 maps $F(S)$ having the same range such that

$$F(S-1) < R \leq F(S) ,$$

then S electrons are emitted in the interval Δt . No electrons are emitted when $R \leq F(0)$.

A.2 Random Emission-Velocity Components

In order to simulate the random emission process from a given cathode spot, each electron emitted according to Section A.1 must be assigned the appropriate random y -component (normal to the cathode) and z -component (along the cathode) of emission velocity. The procedure for assigning these random velocity components to the emitted electrons is described in this appendix.

Assuming a half-Maxwellian distribution for the y -component of initial velocity of the emitted electrons gives

$$f(\dot{y}) = \frac{m\dot{y}}{kT} e^{-m\dot{y}^2/2kT}, \quad (\text{A.3})$$

where $f(\dot{y})$ is the probability density of an emitted electron having its y -component of velocity lying between $\dot{y}$ and $\dot{y}+d\dot{y}$, k is the Boltzmann constant and T is the cathode temperature in degrees Kelvin.

The cumulative distribution function $F(\dot{y})$ is given by

$$F(\dot{y}) = \int_0^{\dot{y}} f(\dot{y}) d\dot{y} \quad (\text{A.4})$$

$$= 1 - e^{-m\dot{y}^2/2kT} \quad (\text{A.5})$$

and

$$0 \leq F(\dot{y}) \leq 1. \quad (\text{A.6})$$

Let R_y be a random number uniformly distributed over the range between 0 and 1. Then $(1-R_y)$ is another such random number having the same range. Hence

$$(1 - R_y) = 1 - e^{-m\dot{y}^2/2kT} \quad (A.7)$$

or

$$R_y \triangleq e^{-m\dot{y}^2/2kT} \quad (A.8)$$

and

$$\dot{y} = \left(\frac{2kT}{m} \right)^{1/2} \left(-\ln R_y \right)^{1/2} . \quad (A.9)$$

A full Maxwellian distribution is assumed for the z-component of emission velocity of the electrons. Hence

$$f(\dot{z}) = \sqrt{\frac{m}{2\pi kT}} e^{-m\dot{z}^2/2kT} , \quad (A.10)$$

where $f(\dot{z})$ is the probability density of an emitted electron having its z-component of velocity lying between $\dot{z}$ and $\dot{z}+d\dot{z}$.

The cumulative distribution function $F(\dot{z})$ is given by

$$F(\dot{z}) = \int_{-\infty}^{\dot{z}} \sqrt{\frac{m}{2\pi kT}} e^{-m\dot{z}^2/2kT} d\dot{z} . \quad (A.11)$$

On evaluating the integral one obtains

$$F(\dot{z}) = \frac{1}{2} \left\{ 1 + \operatorname{erf} \left[\left(\frac{m}{2kT} \right)^{1/2} \dot{z} \right] \right\} . \quad (\text{A.12})$$

Since $\operatorname{erf} \left[\left(\frac{m}{2kT} \right)^{1/2} \dot{z} \right]$ is in the range 0 to 1 for values of $\dot{z}$ between 0 and ∞ , a random number R_z uniformly distributed over the range 0 to 1 can be chosen such that

$$R_z = \operatorname{erf} \left[\left(\frac{m}{2kT} \right)^{1/2} \dot{z} \right] \quad (\text{A.13})$$

or

$$\dot{z} = \left(\frac{2kT}{m} \right)^{1/2} \operatorname{Inverse} \operatorname{erf} (R_z) . \quad (\text{A.14})$$

The proper sign (plus or minus) for $\dot{z}$ is decided randomly by testing the last bit of the generated random number R_z for even or odd character.

The procedure for assigning $\dot{z}$ described in the above paragraph requires that an error function table be stored in the computer memory. Consideration of the computation time and the storage space available in the computer memory leads to the following alternative procedure for assigning the z-component of velocity to the emitted electrons. In the case of thermionic emission

$$\overline{\dot{y}^2} = \frac{2kT}{m} \quad \text{and} \quad \overline{\dot{z}^2} = \frac{kT}{m} . \quad (\text{A.15})$$

From this point of view, the expression for $\dot{z}$ will be the expression for $\dot{y}$ multiplied by $1/\sqrt{2}$ with the sign selected in the random manner described above. It should be noted that this procedure does not, however, simulate accurately the z-component of emission velocity in the neighborhood of $\dot{z} = 0$.

As an alternative to the storage of an error function table in the computer memory one can use a polynomial representation of the error function.

A.3 Random Emission Times

Electrons emitted from the cathode during an interval Δt are not emitted simultaneously. Their emission times are distributed randomly over the time interval Δt . Thus the electrons are under the action of the electric and magnetic fields for different portions of the unit-time interval Δt in which they are emitted. This added complication is avoided if the electrons are emitted simultaneously but at different advanced positions in front of the cathode. The advanced position for a particular electron is given by

$$y = R_1 \dot{y} \Delta t \quad (\text{A.16})$$

and

$$z = z_0 \pm R_2 \dot{z} \Delta t , \quad (\text{A.17})$$

where R_1 and R_2 are random numbers in the range 0 and 1 and $(z_0, 0)$ are the coordinates of the given cathode spot. Again the sign of the second term in Eq. A.17 is decided by testing the last bit of the random number R_2 for odd or even character. The error involved in this procedure is negligible if Δt is small.

APPENDIX B. NUMERICAL SOLUTION OF POISSON'S EQUATION AND CALCULATION OF TOTAL ELECTRIC FIELD COMPONENTS

B.1 Numerical Solution of Poisson's Equation

Due to the irregular geometry of some electron guns, e.g., the Kino-type electron gun, it is impossible analytically to obtain the potential distribution throughout the region. Hence Poisson's equation must be solved numerically for a given space-charge distribution and set of boundary conditions. The choice of numerical method depends upon the computer storage space needed, the required accuracy of the solution and the execution time. Since a normal simulation run consists of approximately 1500 to 2000 unit-time intervals and Poisson's equation has to be solved for each Δt , a fast numerical method becomes a necessity.

If there is no variation along the x-axis (the direction of the applied uniform magnetic field), the three-dimensional Poisson equation is simplified so that the potential depends only on y- and z-coordinates. The gun region is divided into small rectangles by unequally spaced lines drawn parallel to the y- and z-axes. To obtain the precise potential distribution in the immediate neighborhood of the cathode (along $y = 0$ plane), some of the lines parallel to the z-axis are drawn very close to one another in this region. The points of intersection of these lines falling inside the gun region are called "solution points" of Poisson's equation. As shown in Fig. B.1, surrounding each solution point denoted by indices i, j , there is a rectangular region whose boundaries are centrally located between the solid mesh lines. These rectangles are used in solving Poisson's

equation and also for noise parameter calculations. They are numbered according to the solution points they enclose.

The two-dimensional Poisson equation is written as

$$V_{zz} + V_{yy} = -\rho/\epsilon_0, \quad (B.1)$$

where V is the potential and the subscripts denote partial derivatives with respect to direction coordinates. Integrating Eq. B.1 over the region $r_{i,j}$ shown in Fig. B.1 results in the equation

$$-\int_{r_{i,j}} \int (V_{zz} + V_{yy}) dzdy = \frac{1}{\epsilon_0} \int_{r_{i,j}} \int \rho dzdy, \quad (B.2)$$

which can be approximated by

$$-\int_{r_{i,j}} \int (V_{zz} + V_{yy}) dzdy \approx \frac{1}{\epsilon_0} \rho_{i,j} a_{i,j}, \quad (B.3)$$

where the space-charge density $\rho_{i,j}$ is assumed to be uniform over the region $r_{i,j}$ with the area $a_{i,j}$. If there are $N_{i,j}$ "big electrons" in $r_{i,j}$ and the length of the cathode in the direction of the magnetic field is d , then

$$-\int_{r_{i,j}} \int (V_{zz} + V_{yy}) dzdy \approx \frac{-|e|DN_{i,j}}{\epsilon_0 d}, \quad (B.4)$$

where $-|e|$ is the charge of an electron. When Green's theorem is applied,

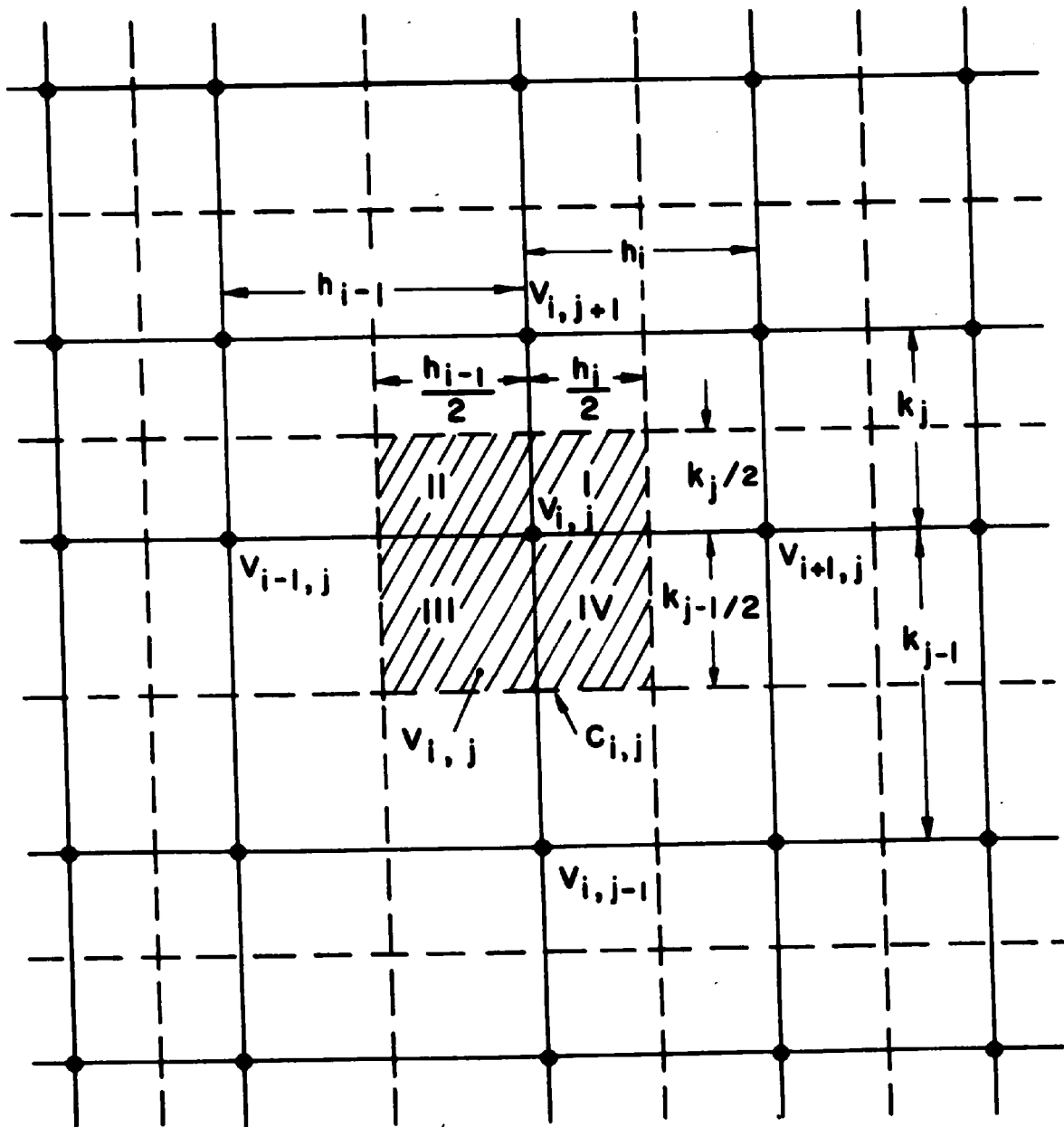


FIG. B.1 A TYPICAL "SOLUTION POINT" $v_{i,j}$ AND THE ASSOCIATED REGION (SHADED) OF INTEGRATION OF POISSON'S EQUATION.

$$-\oint_{C_{1,j}} (V_z dy - V_y dz) \approx \frac{-|e|DN_{1,j}}{\epsilon_o d}, \quad (B.5)$$

where $C_{1,j}$ is the boundary of the region $r_{1,j}$.

By use of mean values for the first partial derivatives of potentials in Eq. B.5, the equation can be written as

$$E_{1,j}V_{1,j} + A_{1,j}V_{1+1,j} + C_{1,j}V_{1,j+1} + B_{1,j}V_{1-1,j} + D_{1,j}V_{1,j-1} = \rho_{a_{1,j}}, \quad (B.6)$$

$$\begin{aligned} \text{where } A_{1,j} &\triangleq -(\epsilon_o/h_1)(k_j + k_{j-1})/2, \\ B_{1,j} &\triangleq -(\epsilon_o/h_{1-1})(k_j + k_{j-1})/2, \\ C_{1,j} &\triangleq -(\epsilon_o/k_j)(h_1 + h_{1-1})/2, \\ D_{1,j} &\triangleq -(\epsilon_o/k_{j-1})(h_1 + h_{1-1})/2, \\ E_{1,j} &\triangleq -(A_{1,j} + C_{1,j} + B_{1,j} + D_{1,j}) \text{ and} \\ \rho_{a_{1,j}} &\triangleq -|e|DN_{1,j}/d \end{aligned}$$

and where distances h_1 , h_{1-1} , k_j and k_{j-1} are shown in Fig. B.1.

There are n such equations for n solution points. In the event that some of the four points surrounding the solution point fall outside the boundary, extrapolated potentials determined from the steady-state relaxation solution²⁴ are used for potentials corresponding to these points to preserve the symmetry of the equations.

Finally the general set of equations can be put in the following matrix form:

$$[A][V] = [\rho_{BC}] + [\rho_A] \quad (B.7)$$

and

$$[A] [V] = [\rho_C] , \quad (B.8)$$

where $[V]$, the potential solution vector, is a column vector,

$[\rho_{BC}]$ is a column vector which takes account of boundary conditions,

$[\rho_A]$ is a column vector which refers to the known space-charge density and

$[A]$ is a $(n \times n)$ real, symmetric, positive definite band matrix involving only the mesh structure.

The matrix $[A]$ having such properties can be factored into a product of a lower triangular matrix $[L]$ and its transpose $[L]^T$. This is accomplished by the Square Root Matrix Factorization Method⁴¹. Since $[A]$ can be written as

$$[A] = [L] [L]^T , \quad (B.9)$$

Eq. B.8 becomes

$$[L] [L]^T [V] = [\rho_C] . \quad (B.10)$$

With the substitution

$$[L]^T [V] = [y] , \quad (B.11)$$

Eq. B.10 assumes the form

$$[L] [y] = [\rho_C] . \quad (B.12)$$

Equations B.11 and B.12 are solved for V using a variable back substitution method to reduce the computation time. Thus, when the space-charge distribution in a bounded region is given, the potential distribution consistent with the specified potentials on the boundaries can be obtained by the above method.

B.2 Calculation of Total Electric Field Components

In Fig. B.1 the shaded area is divided into four parts designated I through IV. The electric field components (obtained from the solution of Poisson's equation) acting on electrons in these subregions are given by

$$(E_y)_{I,II} = - \frac{V_{1,j+1} - V_{1,j}}{k_j} , \quad (B.13a)$$

$$(E_z)_{I,IV} = - \frac{V_{1+1,j} - V_{1,j}}{h_1} , \quad (B.13b)$$

$$(E_y)_{III,IV} = - \frac{V_{1,j} - V_{1,j-1}}{k_{j-1}} \quad (B.13c)$$

and

$$(E_z)_{II,III} = - \frac{V_{1,j} - V_{1-1,j}}{h_{1-1}} . \quad (B.13d)$$

In a one-dimensional diode where the beam extends to infinity in the transverse directions the total a-c current is zero when the diode is assumed to be open-circuited to a-c current. In the two-dimensional model where a fringing space-charge field exists at the beam boundaries the assumption of zero total a-c current is not strictly true. However, as discussed by Dayem and Lambert²⁷, if the beam velocity is

small compared to ωb , where ω is the radian frequency and b is the beam radius, the assumption that the total a-c current is zero for a two-dimensional diode open-circuited to a-c is reasonable. In the present work the above condition was satisfied. The additional a-c electric field to be added to that given by Eqs. B.13a through B.13d to obtain the total electric field for the open-circuit diode assumption is calculated in the following manner:

$$\text{The total a-c current} = \vec{J}_{a-c} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} = 0 \quad . \quad (\text{B.14})$$

The two components of Eq. B.14 can be written as

$$\epsilon_0 \frac{\partial E_z}{\partial t} + (\rho \dot{z})_{a-c} = 0 \quad (\text{B.15a})$$

and

$$\epsilon_0 \frac{\partial E_y}{\partial t} + (\rho \dot{y})_{a-c} = 0 \quad . \quad (\text{B.15b})$$

The following expressions are obtained from Eqs. B.15a and B.15b:

$$\Delta E_{z,p}(t_1) = -[(\rho \dot{z})_{p,t_1}]_{a-c} \frac{\Delta t}{2\epsilon_0} \quad (\text{B.16a})$$

and

$$\Delta E_{y,p}(t_1) = -[(\rho \dot{y})_{p,t_1}]_{a-c} \frac{\Delta t}{2\epsilon_0} \quad , \quad (\text{B.16b})$$

where $\Delta E_{z,p}(t_1)$ and $\Delta E_{y,p}(t_1)$ are the additional a-c electric field components (averaged over Δt) acting on all electrons present in the

pth rectangle during the t_1 th unit-time interval for the open-circuit diode assumption.

The expressions for $(\rho \dot{z})_{a-c}$ and $(\rho \dot{y})_{a-c}$ are derived in Appendix D (Eqs. D.8a and D.8b). When these are used in Eqs. B.16a and B.16b the following equations are obtained:

$$\Delta E_{y,p}(t_1) = \frac{\Delta t |e| D}{2\epsilon_0 d d y_p d z_p} \left[\sum_{k=1}^{N_p(t_1)} \dot{y}(t_1, k) - \frac{1}{L} \sum_{t_1} \sum_{k=1}^{N_p(t_1)} \dot{y}(t_1, k) \right] \quad (B.17a)$$

and

$$\Delta E_{z,p}(t_1) = \frac{\Delta t |e| D}{2\epsilon_0 d d y_p d z_p} \left[\sum_{k=1}^{N_p(t_1)} \dot{z}(t_1, k) - \frac{1}{L} \sum_{t_1} \sum_{k=1}^{N_p(t_1)} \dot{z}(t_1, k) \right] \quad (B.17b)$$

Under the short-circuit diode assumption no additional a-c electric field is present and the total electric field is the same as that given by Eqs. B.13a through B.13d.

APPENDIX C. TRAJECTORY EQUATIONS

The equations of motion for an electron moving in crossed electric and magnetic fields are given by

$$\ddot{x} = -\eta E_{x_T} , \quad (C.1a)$$

$$\ddot{y} = -\eta [E_{y_T} + B\dot{z}] \quad (C.1b)$$

and

$$\ddot{z} = -\eta [E_{z_T} - B\dot{y}] , \quad (C.1c)$$

where E_{x_T} , E_{y_T} and E_{z_T} are the components of the total electric field along the three axes respectively and B is the uniform magnetic field in the x -direction.

If there is no variation of quantities in the x -direction and the electrons are assumed to be emitted from the cathode as rods of charge extending in the x -direction, Eq. C.1a can be ignored and the three-dimensional problem is reduced to a two-dimensional one. When Eqs. C.1b and C.1c are integrated, the position and the velocity components of an electron at the end of a given unit-time interval can be expressed in terms of their values at the beginning of this interval (say at time t_1) in the following manner:

$$y(t_1 + \Delta t) = \frac{\eta E_{z_T}(t_1)}{\omega_c^2} (\omega_c \Delta t - \sin \omega_c \Delta t) - \left(\frac{\eta E_{y_T}(t_1)}{\omega_c^2} + \frac{\dot{z}(t_1)}{\omega_c} \right) \cdot (1 - \cos \omega_c \Delta t) + \frac{\dot{y}(t_1)}{\omega_c} \sin \omega_c \Delta t + y(t_1) \quad , \quad (C.2)$$

$$\dot{y}(t_1 + \Delta t) = \frac{\eta E_{z_T}(t_1)}{\omega_c} (1 - \cos \omega_c \Delta t) - \left(\frac{\eta E_{y_T}(t_1)}{\omega_c} + \dot{z}(t_1) \right) \sin \omega_c \Delta t + \dot{y}(t_1) \cos \omega_c \Delta t \quad , \quad (C.3)$$

$$z(t_1 + \Delta t) = - \frac{\eta E_{z_T}(t_1)}{\omega_c^2} (1 - \cos \omega_c \Delta t) - \left(\frac{\eta E_{y_T}(t_1)}{\omega_c^2} + \frac{\dot{z}(t_1)}{\omega_c} \right) \cdot (\omega_c \Delta t - \sin \omega_c \Delta t) + \frac{\dot{z}(t_1)}{\omega_c} (1 - \cos \omega_c \Delta t) + \dot{z}(t_1) \Delta t + z(t_1) \quad (C.4)$$

and

$$\dot{z}(t_1 + \Delta t) = - \frac{\eta E_{z_T}(t_1)}{\omega_c} \sin \omega_c \Delta t - \left(\frac{\eta E_{y_T}(t_1)}{\omega_c} + \dot{z}(t_1) \right) (1 - \cos \omega_c \Delta t) + \dot{y}(t_1) \sin \omega_c \Delta t + \dot{z}(t_1) \quad . \quad (C.5)$$

The above set of trajectory equations (Eqs. C.2 through C.5) hold both for electrons and "big electrons" since they have the same charge-to-mass ratio and the "big electrons" are emitted from the cathode with the same initial velocity components as those of an electron. The same set of equations was used by Wadhwa and Rowe¹¹.

APPENDIX D. DERIVATION OF VELOCITY AND CURRENT-DENSITY COMPONENTS

The relevant equations used in obtaining the a-c velocity and current-density components of the "big electrons" located in an arbitrary elementary rectangle p at a time t_1 are given below.

Let $\dot{z}_p(t_1)$ and $\dot{y}_p(t_1)$ be the average values of the velocity components taken over all "big electrons", $N_p(t_1)$, occupying the rectangle p at time t_1 . Thus the average values can be expressed as

$$\dot{z}_p(t_1) = \frac{1}{N_p(t_1)} \sum_{k=1}^{N_p(t_1)} \dot{z}_p(t_1, k) \quad (D.1a)$$

and

$$\dot{y}_p(t_1) = \frac{1}{N_p(t_1)} \sum_{k=1}^{N_p(t_1)} \dot{y}_p(t_1, k) \quad , \quad (D.1b)$$

where $\dot{z}_p(t_1, k)$ and $\dot{y}_p(t_1, k)$ refer to the velocity components of the k th electron in the rectangle p at time t_1 .

The long-term average values of the number and the velocity components of "big electrons" in the rectangle p can be written as

$$N_{op} = \frac{1}{L} \sum_{t_1=1}^L N_p(t_1) \quad , \quad (D.2)$$

$$\dot{z}_{op} = \frac{1}{N_{op}L} \sum_{t_1=1}^L \sum_{k=1}^{N_p(t_1)} \dot{z}_p(t_1, k) \quad (D.3a)$$

and

$$\dot{y}_{op} = \frac{1}{N_{op}L} \sum_{t_1=1}^L \sum_{k=1}^{N_p(t_1)} \dot{y}_p(t_1, k) \quad (D.3b)$$

Hence the a-c velocity components for the pth rectangle at time t_1 are given by

$$\begin{aligned} \tilde{\dot{z}}_p &= \frac{N_p(t_1)}{N_{op}} [\dot{z}_p(t_1) - \dot{z}_{op}] \\ &= \frac{N_p(t_1)}{N_{op}} \left[\frac{1}{N_p(t_1)} \sum_{k=1}^{N_p(t_1)} \dot{z}_p(t_1, k) - \frac{1}{N_{op}L} \sum_{t_1=1}^L \sum_{k=1}^{N_p(t_1)} \dot{z}_p(t_1, k) \right] \end{aligned} \quad (D.4a)$$

and

$$\begin{aligned} \tilde{\dot{y}}_p &= \frac{N_p(t_1)}{N_{op}} [\dot{y}_p(t_1) - \dot{y}_{op}] \\ &= \frac{N_p(t_1)}{N_{op}} \left[\frac{1}{N_p(t_1)} \sum_{k=1}^{N_p(t_1)} \dot{y}_p(t_1, k) - \frac{1}{N_{op}L} \sum_{t_1=1}^L \sum_{k=1}^{N_p(t_1)} \dot{y}_p(t_1, k) \right], \end{aligned} \quad (D.4b)$$

where a weighting factor $N_p(t_1)/N_{op}$ is introduced such that the average values of $\tilde{y}_p$ and $\tilde{z}_p$ taken over L unit-time intervals approach zero.

The d-c space-charge density in the p th rectangle, ρ_{op} , can be expressed as

$$\begin{aligned}\rho_{op} &= \frac{-|e|DN_{op}}{ddy_p dz_p}, \\ &= \frac{-|e|D}{Lddy_p dz_p} \sum_{t_1=1}^L N_p(t_1),\end{aligned}\quad (D.5a)$$

where dy_p and dz_p are the sides of the small rectangle p and L is the length of the cathode in the direction of the magnetic field.

Similarly,

$$\rho_p(t_1) = \frac{-|e|D}{ddy_p dz_p} N_p(t_1), \quad (D.5b)$$

where $\rho_p(t_1)$ is the space-charge density in the p th rectangle at time t_1 .

The current-density components corresponding to the p th rectangle at time t_1 can be written as

$$\begin{aligned}J_{z,p}(t_1) &= \rho_p(t_1)\dot{z}_p(t_1) = \frac{-|e|DN_p(t_1)}{ddy_p dz_p} \dot{z}_p(t_1) \\ &= \frac{-|e|D}{ddy_p dz_p} \sum_{k=1}^{N_p(t_1)} \dot{z}_p(t_1, k)\end{aligned}\quad (D.6a)$$

and

$$J_{y,p}(t_1) = \rho_p(t_1)\dot{y}_p(t_1) = \frac{-|e|D}{dd\mathbf{y}_p d\mathbf{z}_p} \sum_{k=1}^{N_p(t_1)} \dot{y}_p(t_1, k) \quad (D.6b)$$

The long-term average current-density components corresponding to the pth rectangle are as follows:

$$J_{oz,p} = \rho_{op}\dot{z}_{op} = \frac{-|e|D}{Ldd\mathbf{y}_p d\mathbf{z}_p} \sum_{t_1=1}^L \sum_{k=1}^{N_p(t_1)} \dot{z}_p(t_1, k) \quad (D.7a)$$

and

$$J_{oy,p} = \rho_{op}\dot{y}_{op} = \frac{-|e|D}{Ldd\mathbf{y}_p d\mathbf{z}_p} \sum_{t_1=1}^L \sum_{k=1}^{N_p(t_1)} \dot{y}_p(t_1, k) \quad (D.7b)$$

From Eqs. D.6a and D.7a,

$$\begin{aligned} [\rho_p(t_1)\dot{z}_p(t_1)]_{a-c} &= \rho_p(t_1)\dot{z}_p(t_1) - \rho_{op}\dot{z}_{op} \\ &= \frac{-|e|D}{dd\mathbf{y}_p d\mathbf{z}_p} \left[\sum_{k=1}^{N_p(t_1)} \dot{z}_p(t_1, k) - \frac{1}{L} \sum_{t_1=1}^L \sum_{k=1}^{N_p(t_1)} \dot{z}_p(t_1, k) \right] \quad (D.8a) \end{aligned}$$

Similarly, from Eqs. D.6b and D.7b,

$$[\rho_p(t_1)\dot{y}_p(t_1)]_{a-c} = \frac{-|e|D}{dd\mathbf{y}_p d\mathbf{z}_p} \left[\sum_{k=1}^{N_p(t_1)} \dot{y}_p(t_1, k) - \frac{1}{L} \sum_{t_1=1}^L \sum_{k=1}^{N_p(t_1)} \dot{y}_p(t_1, k) \right] \quad . \quad (D.8b)$$

APPENDIX E: CALCULATION OF THE VARIANCES OF THE TOTAL VELOCITY AND THE VELOCITY COMPONENTS OF ELECTRONS

The variances of the total velocity and the y- and z-velocity components of electrons occupying a given rectangle in the beam are calculated in the following manner. It is known from the theory of statistics that if samples of size n are taken from a population and the variance of the samples is, say, σ_s^2 , then the variance of the population, σ_p^2 , is given by

$$\sigma_p^2 = n\sigma_s^2, \quad (E.1)$$

and the population mean and the mean of the samples are the same.

During the simulation of the beam on the high-speed digital computer, as described in Chapter II, average velocity components and the number of electrons in each rectangle for each unit-time interval are recorded on a magnetic tape. Equations D.1a and D.1b give the average velocity components $\dot{z}_p(t_1)$ and $\dot{y}_p(t_1)$ of electrons $N_p(t_1)$ occupying the rectangle p at the time t_1 . The long-term averages of the number and velocity components are given by Eqs. D.2, D.3a and D.3b. Thus for L unit-time intervals, L samples of size $N_p(t_1)$ for $\dot{z}_p(t_1)$ and $\dot{y}_p(t_1)$ are available. It is found that the variance of $N_p(t_1)$ is small, and since $N_p(t_1)$ is not small, negligible error results from assuming that all samples are of equal size N_{op} . Thus the sample variances for the velocity components are given by

$$\sigma_{\dot{z}_p(t_i)}^2 = \frac{\sum_{t_i=1}^L [\dot{z}_p(t_i) - \dot{z}_{op}]^2}{L} \quad (E.2)$$

and

$$\sigma_{\dot{y}_p(t_i)}^2 = \frac{\sum_{t_i=1}^L [\dot{y}_p(t_i) - \dot{y}_{op}]^2}{L} \quad (E.3)$$

Hence the population variances for the velocity components of electrons in the pth rectangle are given by

$$\begin{aligned} \sigma_{\dot{z}_p(t_i,k)}^2 &= N_{op} \sigma_{\dot{z}_p(t_i)}^2 \\ &= N_{op} \frac{\sum_{t_i=1}^L [\dot{z}_p(t_i) - \dot{z}_{op}]^2}{L} \end{aligned} \quad (E.4)$$

and

$$\sigma_{\dot{y}_p(t_i,k)}^2 = N_{op} \frac{\sum_{t_i=1}^L [\dot{y}_p(t_i) - \dot{y}_{op}]^2}{L} \quad (E.5)$$

For the rectangle p, the variance of the total velocity of electrons, u_T , (in the direction of the beam) is obtained in a different manner. The a-c kinetic potential was defined by Eq. 3.9 as

$$\tilde{V}(t) = - \frac{\vec{u}_0 \cdot \vec{v}(t)}{\eta} , \quad (E.6)$$

which becomes

$$\tilde{V}(t_1) = - \frac{u_0 v_T(t_1)}{\eta} , \quad (E.7)$$

where u_0 is the d-c total velocity and $v_T(t_1)$ is the a-c component of the velocity fluctuation in the direction of $\vec{u}_0$. As stated previously, in the calculation of the autocorrelation function, $C_V(\tau)$, the quantities $\tilde{V}(t_1)$, u_0 and $v_T(t_1)$ represent an average electron in the rectangle p.

From Eq. E.7 it is clear that

$$C_V(\tau) = \frac{u_0^2}{\eta^2} C_{v_T}(\tau) , \quad (E.8)$$

so that

$$C_{v_T}(0) = \frac{\eta^2}{u_0^2} C_V(0) . \quad (E.9)$$

It is well known that

$$C_{v_T}(0) = \sigma_{v_T}^2(t_1) . \quad (E.10)$$

From Eq. 3.3a,

$$u_T = u_0 + v_T , \quad (E.11)$$

and, since the long-term average values of u_T and v_T are u_0 and zero respectively, it follows that

$$\sigma_{u_T}^2(t_i) = \sigma_{v_T}^2(t_i) \quad . \quad (E.12)$$

Equations E.9, E.10 and E.12 yield

$$\sigma_{u_T}^2(t_i) = \frac{1}{u_0^2} C_V(0) \quad . \quad (E.13)$$

It should be noted that $\sigma_{u_T}^2(t_i)$ is again a sample variance. Hence, on the basis of the discussion for the variances of the velocity components, the variance of the total velocity of the electrons in the rectangle p is given by

$$\begin{aligned} \sigma_{u_T}^2(t_i, k) &= N_{op} \sigma_{u_T}^2(t_i) \\ &= N_{op} \frac{1}{u_0^2} C_V(0) \quad , \end{aligned} \quad (E.14)$$

where $C_V(0)$ is readily available from the noise parameter calculations.

Now, consider planes parallel to and in the neighborhood of the cathode. The average d-c velocity is approximately the same for all electrons in each of the rectangles belonging to such a plane. The variance of the velocity components and the total velocity can be calculated for the cross section as a whole if the various quantities

$\dot{z}_p(t_1)$, $\dot{y}_p(t_1)$, $\dot{z}_{op}$ and $\dot{y}_{op}$ are now taken to be the weighted averages over all rectangles in the given cross section p . N_{op} is then the sum of the long-term averages of the number of electrons in each rectangle in this cross section and $C_v(0)$ refers to the cross section as a whole since the kinetic voltage $\tilde{V}(t)$ is then the weighted average as defined in Eq. 3.11. The error involved in these calculations is of the same order as that for the calculation on a rectangle basis. However, the error will be quite appreciable if the plane considered is one which is normal to the beam, since it has an appreciable variation in velocity. For this reason, no attempt is made to make similar calculations for normal planes.

It should be noted in the end that the calculated velocity variances are not exact since an approximation (see page 237) is made in generating the z -component of the emission velocity of the electrons. Since the variation of these variances along and across the beam are studied here in a qualitative manner, this error does not seem to materially affect the conclusions.

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LIST OF SYMBOLS

a	Area of the cathode.
B	D-c magnetic field in the x-direction.
B_c	D-c critical magnetic field in the circuit-sole region.
$C(\tau)$	Auto-correlation function.
$C_{xy}(\tau), C_{yx}(\tau)$	Cross-correlation function, $C_{xy}(\tau) = C_{yx}(-\tau)$.
d	Cathode length along the x-axis.
d	Anode-sole spacing (Eq. 2.8).
D	Reduction factor used in grouping of electrons.
$D(\tau)$ or $D(\mu)$	Lag window.
Drift Space A	Drift space following the abbreviated Kino gun.
Drift Space M	Drift space following the modified Kino gun.
$ e $	Absolute value of electron charge.
E_y, E_z	Electric field components obtained from solution of Poisson's equation.
$\Delta E_y, \Delta E_z$	Additional a-c electric field components for the open-circuit assumption.
f	Frequency.
f_p	Plasma frequency, $f_p = (1/2\pi)\sqrt{\eta\rho/\epsilon_0}$.
f_c	Cyclotron frequency, $f_c = \eta B/2\pi$.
f_h	Hybrid frequency, $f_h = \sqrt{f_p^2 + f_c^2}$.
f_{pm}	Potential minimum plasma frequency.
f_{hm}	Potential minimum hybrid frequency.
f_o	$1/\Delta t$.
F_o	Noise parameter in db calculated by using Haus' formula for an O-type one-dimensional beam.

G	Diocotron gain between two arbitrary planes a and b along a Kino gun beam.
G_D	Diocotron gain per unit length (db/m) along a beam in a parallel plane crossed-field drift space.
i_c	Cathode current.
i_g	Grid current.
$\tilde{I}(t)$	A-c current fluctuation across a given beam cross section.
$\text{Im } \Theta$	Imaginary part of Θ .
$\vec{J}_0$	D-c current density vector at a given point in the beam.
$\vec{J}(t)$	Instantaneous current density vector at a given point in the beam.
J_c	Cathode current density.
J_s	D-c cathode emission current density.
J_y	D-c cathode current density in the y -direction.
$J_T(t)$	Component of $\vec{J}(t)$ in the direction of $\vec{J}_0$.
$\tilde{J}_T(t)$	A-c current-density fluctuations along $\vec{J}_0$.
k	Boltzmann's constant.
k	Number of degrees of freedom (Eq. 4.1).
K_t	Instantaneous kinetic energy of an electron.
K_o	Steady-state kinetic energy of an electron.
K_{a-c}	A-c kinetic energy of an electron.
KG-A	Abbreviated Kino gun.
KG-M	Modified Kino gun.
L	Number of unit-time intervals for the entire sampling period.
L_{th}	Theoretical period of electron motion in constant crossed electric and magnetic fields (space-charge-free case).
m	Mass of an electron.

M	$T_m/\Delta t$.
N	Number of "big electrons" emitted from the cathode per Δt .
OC	Open-circuit diode assumption.
$P(f)$	True power spectrum (Eq. 3.28).
$\tilde{P}(f)$	Aliased power spectrum (Eq. 3.28).
R, R_y, R_z, R_1 and R_2	Random numbers.
Re Θ	Real part of Θ .
R^2	Reduction factor for current fluctuations.
s	Distance measured along the central trajectory.
$s_{\max}$	Length of the central trajectory between the cathode and the gun exit plane.
S	Haus' noise parameter, $S = (\phi V - A^2)^{1/2}$.
SC	Short-circuit diode assumption.
t	An instant of time.
Δt	Unit-time interval.
T	Cathode temperature.
T	Time interval (Eqs. 3.16 and 3.17).
T_e	Effective cathode temperature.
T_m	Maximum lag used.
T_n	Total time interval over which the data is collected in the simulation run.
T_o	Ambient temperature.
$\vec{u}(t)$	Instantaneous velocity vector of the electron.
u_o	D-c velocity of the electron.
$u_T(t)$	Component of the instantaneous velocity of the electron along the d-c velocity, u_o .
$u_N(t)$	Component of the instantaneous velocity of the electron normal to the d-c velocity, u_o .

$\vec{v}(t)$	A-c velocity vector of the electron.
v_T	A-c component of the velocity of the electron along u_0 .
v_N	A-c component of the velocity of the electron normal to u_0 .
V	D-c voltage of an electrode.
$\tilde{V}(t)$	A-c kinetic potential.
$ V_m $	Magnitude of the potential minimum.
w	Cathode width (along the z-axis).
W	Normalized cathode width (Kino theory).
$W(f)$	Self-power spectrum.
$x(t)$	An infinite, real random process.
y	y-coordinate of the electron in the unprimed coordinate system.
$\dot{y}$	y-component of velocity of the electron in the unprimed coordinate system.
$\tilde{y}$	y-component of the a-c velocity of an electron.
y'	y-coordinate of the electron in the primed coordinate system.
$\dot{y}'$	y-component of the velocity of the electron in the primed coordinate system.
y_m	Distance of the potential minimum from the cathode.
y_0	y-coordinate of the point of intersection of plane CC' and the front focus electrode in Kino-type guns.
z	z-coordinate of the electron in the unprimed coordinate system.
$\dot{z}$	z-component of the velocity of the electron in the unprimed system.
$\tilde{z}$	z-component of the a-c velocity of an electron.
z'	z-coordinate of the electron in the primed coordinate system.

$\dot{z}'$	z-component of the velocity of the electron in the primed coordinate system.
z_0	z-coordinate of the point of intersection of plane CC' and the front focus electrode in Kino-type guns.
α	Angle between the plane CC' and the y-axis.
ϵ_0	Free-space dielectric constant.
η	Absolute value of the electron charge-to-mass ratio.
θ	Electron transit angle, $\theta = \omega_c t$.
Θ	Cross-Power Density Spectrum between the kinetic noise-voltage and noise-current fluctuations.
Θ^*	Complex conjugate of Θ .
λ	Mean value of Poisson's distribution.
λ_c	Mean beam cyclotron wavelength.
Λ	Imaginary part of the Cross-Power Density Spectrum Θ between the kinetic noise-voltage and noise-current modulations.
μ	$\tau/\Delta t$, $ \mu = 0, 1, \dots, M$.
ν	$t/\Delta t$, $\nu = 1, 2, \dots, L$.
Π	Real part of the Cross-Power Density Spectrum between the kinetic noise-voltage and noise-current modulations.
ρ	Space-charge density.
ρ_0	Brillouin space-charge density, $\rho_0 = -\epsilon_0 \omega_c^2 / \eta$.
σ_u^2	Variance of total velocity, u , of an electron.
$\sigma_{\dot{y}}^2$	Variance of the y-component of velocity of an electron.
$\sigma_{\dot{z}}^2$	Variance of the z-component of velocity of an electron.
τ	Time delay used in defining correlation functions.
φ_a	Anode-sole voltage.
ϕ	Normalized potential (Kino theory).

- Φ Self-Power Density Spectrum of the kinetic noise-voltage fluctuation.
- Ψ Self-Power Density Spectrum of the noise-current fluctuation.
- Ψ_J Self-Power Density Spectrum of the noise-current density fluctuation.
- ω_c Radian cyclotron frequency.