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The University of Michigan, Ph.D., 1976
Physics, elementary particles and
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TOPICS IN HIGH ENERGY PHYSICS

by

Ilmun Ju

A dissertation submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
(Physics)
in The University of Michigan
1976

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ACKNOWLEDGMENTS

I would like to thank Professor Yukio Tomozawa for suggesting the topics and his unlimited kindness, encouragement, and support. My sincere gratitude and respect is due to Professor David N. Williams for his kindness and valuable guidance. It is my pleasure to thank Professors Eugene Feenberg and Edwin T. Jaynes for their inspiring lectures and guidance. Chapters 1 and 2 were supported in part by the United States Atomic Energy Commission, and Chapter 3 was supported in part by the United States Energy Research and Development Administration. I would also like to thank the Horace H. Rackham School of Graduate Studies for the University Fellowship in the 1971-72 academic year and the Department of Physics for the financial support through teaching fellowships. Finally, it is a deep personal pleasure to thank my mother for her love and sacrifice and my wife for her understanding and encouragement.

PREFACE

This thesis is composed of 3 independent topics. Chapter 1 is published in Physical Review D 8, 3103 (1973). Chapter 2 is published in Physical Review D 7, 3529 (1972) in collaboration with Professor Yukio Tomozawa.

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CHAPTER 1

MUON POLARIZATION IN THE NEUTRINO-INDUCED INCLUSIVE REACTION--A TEST OF V-A WEAK INTERACTION AT HIGH ENERGY

Introduction

The conventional V-A theory of the weak interaction [1] has been able to describe low-energy leptonic interactions remarkably well, in spite of the fact that higher-order contributions in perturbation theory badly diverge. But this seemingly satisfactory consistency of the V-A theory with experiment cannot be regarded as proof of the validity of the theory; rather it implies that present data are not sufficient fo fix uniquely all the general nonderivative coupling constants. In fact, Jarskog [2] claims that up to 30 percent scalar and tensor couplings would not be contradictory to the experimental muon decay data. Moreover, even at the phenomological level, there is no fundamental reason why the current-current theory should prevail at higher momentum transfers. The idea of a mediating particle for the weak interaction analogous to the photon in quantum electrodynamics or the Yukawa-type pion field in the strong

interactions has long been attractive, because this non-local theory reduces to the local current-current interaction at low energies. Although the intermediate-vector-boson theory is no less renormalizable,, theoretical consequences were examined extensively in the literature. But this line of effort has always been embarrassed by the fact that intermediate vector bosons are not observed experimentally. This is also the case for the renormalizable theories of spontaneous symmetry breakdown, which employ unobserved gauge bosons and/or heavy leptons.

On the other hand, it would be very interesting to know what kind of theory, even with a phenomenological Hamiltonian, would explain the experimental data far above the unitary limit [3]. Recently a model [4] of weak interactions at high energy by Applequist, Bjorken, and Chanowitz extending the conventional current-current theory to incorporate possible higher-order effects has been proposed. In this paper we investigate some theoretical consequences of a derivatively coupled intermediate vector boson (IVB), or, equivalently, a phenomenological momentum-dependent vector current, in the Bjorken-Johnson-Low scaling limit. Previously, IVB theorists excluded the possibility of a derivative coupling on the grounds that: (a) in the observed leptonic weak interactions, momentum transfer q is so small as to be neglected, and (b) introduction of a derivative coupling provides a momentum q

at each vertex in higher orders, making already divergent Feynman diagrams more so. Nevertheless, high-energy accelerators available at present or in the future may be able to detect the momentum dependence of the leptonic weak vector current directly, if it exists at all. Also the argument of divergent graphs may be bypassed in the meanwhile because we are dealing with the phenomenological Lagrangian anyway. In fact, it turns out that a certain derivative coupling is no worse than the conventional V-A intermediate-vector-boson theory of weak interactions as we will see later. In the following section we present a model Lagrangian and vector current. This model is used to calculate the polarization of the muon in the deep-inelastic neutrino-nucleon scattering in the final section of this chapter.

Phenomenological Lagrangian and Vector Current

Consider a general coupling of lepton and charged vector boson fields [5],

$$\begin{aligned}\mathscr{L} = {} & g_W^S \bar{\psi}_\ell(x)(a_1 - b_1\gamma_5)\psi_\nu(x)\partial_\lambda \bar{W}_\lambda(x) \\ & - ig_W^V \bar{\psi}_\ell(x)\gamma_\lambda(1 - \gamma_5)\psi_\nu(x)\bar{W}_\lambda(x) \\ & + \tfrac{1}{2}ig_W^T \bar{\psi}_\ell(x)\sigma_{\lambda\mu}(a_2 - b_2\gamma_5)\psi_\nu(x) \\ & \cdot [\partial_\lambda \bar{W}_\mu(x) - \partial_\lambda \bar{W}_\mu(x)] + g_W^V J_\lambda(x)\bar{W}_\lambda(x), \quad (1\text{-}1)\end{aligned}$$

where J (x) is the hadronic current. We can write this in another form, viz.,

$$\mathscr{L} = g_W^V [j_\lambda(x) + J_\lambda(x)] \bar{W}_\lambda(x) + \text{H.c.}, \tag{1-2}$$

where

$$j_\lambda(x) = \bar{\psi}_\ell(x) \left[i \frac{\eta}{m} (a_1 - b_1\gamma_5) q_\lambda - i\gamma_\lambda(1-\gamma_5) \right.$$

$$\left. - i \frac{\xi}{m} \sigma_{\lambda\mu} q_\mu (a_2 - b_2\gamma_5) \right] \psi_\nu(x),$$

$$\bar{\psi}_\ell(x) q_\lambda \psi_\nu(x) \equiv -i \frac{\partial}{\partial x_\lambda} [\bar{\psi}_\ell(x) \psi_\nu(x)], \tag{1-3}$$

and

$$\frac{\eta}{m} = \frac{g_W^S}{g_W^V}, \quad \frac{\xi}{m} = \frac{g_W^T}{g_W^V}, \tag{1-4}$$

where m is the muon mass. We introduce the muon mass in (1-3) to make ξ and η dimensionless. In general, a_i and b_i are invariant functions of the momentum transfer q. We may regard (1-3) as a natural modification of the conventional V-A leptonic weak current. Notice that these derivative-coupling terms when applied to hadronic physics with strongly interacting intermediate fields yield Weinberg's form factors [6] with a specific choice of a_i and b_i. Derivative couplings similar to (1-1) in principle have been used by Igarashi et al in their strong-meson-dominance model [7] to get the V - αA theory of β decay even though they used the V - A leptonic

current. Also it is well known that the decay $\pi^- \to \mu^- \bar{\nu}_\mu$ can be described phenomenologically by the derivative coupling of the pion directly to the lepton current [8], i.e.,

$$\mathscr{L} = g_\pi [\bar{\mu}\gamma_\lambda (1 - \gamma_5)\nu_\mu + \bar{e}\gamma_\lambda (1 - \gamma_5)\nu_\ell] \partial_\lambda \Phi_{\pi^-} + \text{H.c.} \tag{1-5}$$

Therefore, it may be interesting to see the consequences of a current-current theory with $j_\lambda(x)$ defined as in (1-3). This would not change the well established low-energy predictions of the V - A theory for sufficiently small η and ξ. Then this result can readily be translated to that of the intermediate-vector-boson theory, Lagrangian (1).

Cheng and Tung [9] have discussed tests of the V - A theory in neutrino-scattering processes assuming the most general form of local (nonderivative V - A and S - T) interaction in the helicity formalism. Here we are interested in the nonlocal deviation from the V - A theory, which does not originate from the second-order radiative corrections. As for the scalar and tensor hadronic currents, their commutators, hence those hadronic structure functions, are relatively unknown to us simply because only those of the vector and axial-vector hadronic currents have been the focus of

attention up to now. Therefore we will restrict ourselves to the vector and axial-vector hadronic currents.

Polarization of the Muon in $\nu N \to \mu X$

The phenomenological current (1-3) discussed in the previous section will be used to compute the polarization of the muon in the neutrino-induced inclusive reaction. To simplify the calculation we invoke the following assumptions:

(1) Only the left handed neutrino takes part in the leptonic current, i.e., $a_1 = b_1$, $a_2 = b_2$. Choose $a_1 = a_2 = 1$ without loss of generality.

(2) Time-reversal invariance, i.e., η and ξ are real. Then if we put

$$\frac{(g_W^V)^2}{m_W^2} = \frac{G}{\sqrt{2}} ,$$

we have [10]

$$\frac{d^2\sigma}{dQ^2 d\nu} = \frac{G^2}{4\pi M^2 E^2} \mathcal{M}_{\mu\lambda} W_{\mu\lambda} ,$$

where

$$\mathcal{M}_{\mu\lambda} = \sum_{\substack{\nu \\ \text{spin}}} m m_\nu \langle \mu(k') | j_\mu(0) | \nu(k) \rangle \langle \nu(k) | j_\lambda^\dagger(0) | \mu(k') \rangle \tag{1-6}$$

and

$$W_{\mu\lambda} = \int \frac{d^4x}{4\pi} e^{iq\cdot x} \langle N(p) | [J_\mu(x), J_\nu(0)] | X(p') \rangle$$

$$= -(g_{\mu\lambda} - \frac{q_\mu q_\lambda}{q^2}) W_1$$

$$+ \frac{1}{M^2} (p_\mu - \frac{\nu}{q^2} q_\mu)(p_\lambda - \frac{\nu}{q^2} q_\lambda) W_2$$

$$- i \frac{\epsilon_{\mu\lambda\alpha\beta} p^\alpha q^\beta}{2M^2} W_3 + \frac{q_\mu q_\lambda}{M^2} W_4 + \frac{(p_\mu q_\lambda + p_\lambda q_\mu)}{2M^2} W_5 \qquad (1\text{-}7)$$

The average over the proton spin is taken in $W_{\mu\lambda}$ and $q = k - k' = p' - p$, $\nu = q \cdot p = M(E - E')$, $q^2 = -Q^2 < 0$, where E and E' are the laboratory energies of the neutrino and the muon, respectively. For the sake of illustration, we use the quark model version of the structure functions, which exhibits a scaling behavior. A more general case can be worked out, leading to similar conclusions. Thus we assume that [11, 12]:

$$\lim_{Bj} W_1(q^2, \nu) = F_1(x),$$

$$\lim_{Bj} (\frac{\nu}{M^2}) W_{2,3}(q^2, \nu) = F_{2,3}(x)$$

$$\lim_{Bj} (\frac{\nu}{M^2})^2 W_{4,5}(q^2, \nu) = F_{4,5}(x), \qquad (1\text{-}8)$$

where $x = -(q^2/2\nu)$. On the other hand, from (1-3) to (1-6), it follows that [10]:

$$\mathcal{M}_{\mu\nu} = \mathcal{M}^{(0)}_{\mu\nu} + \mathcal{M}^{(s)}_{\mu\nu},$$

where

$$\mathcal{M}^{(0)}_{\mu} = (k'_{\mu}k_{\nu} + k'_{\mu}k_{\nu} - \delta_{\mu\nu}k \cdot k')(1 + \xi)^2$$

$$\mp i\epsilon_{\mu\nu\sigma\rho}k'^{\sigma}k^{\rho}(1 + \xi)^2$$

$$+ \frac{1}{m^2}(k\cdot k')q_{\mu}q_{\nu}\eta^2 \mp \frac{1}{m^2}(k\cdot k')$$

$$[q_{\mu}(k + k')_{\nu} + q_{\nu}(k + k')_{\mu}]\eta\xi$$

$$+ \frac{1}{m^2}(k\cdot k')(k + k')_{\mu}(k + k')_{\nu}\xi^2$$

$$\pm(k_{\nu}q_{\mu} + k_{\mu}q_{\nu})(\eta + \eta\xi)$$

$$- [k_{\mu}(k + k')_{\nu} + k_{\nu}(k + k')_{\mu}[(\xi + \xi^2)$$

and

$$\mp\mathcal{M}^{(s)}_{\mu\nu} = -m[s_{\mu}k_{\nu} + s_{\nu}k_{\mu} - \delta_{\mu\nu}s\cdot k](1 + \xi)^2$$

$$\pm im\epsilon_{\mu\nu\sigma\rho}s^{\sigma}k^{\rho}(1 + \xi)^2 + \frac{\eta^2}{m}(s\cdot k)q_{\mu}q_{\nu}$$

$$+ \frac{i}{m}\epsilon_{\sigma\rho\lambda\nu}k'^{\sigma}s^{\rho}k^{\lambda}[-q_{\mu}(\eta + \eta\xi)$$

$$\pm(k + k')_{\mu}(\xi + \xi^2)] -$$

$$- \frac{i}{m}\epsilon_{\sigma\rho\lambda\mu}k'^{\sigma}s^{\rho}k^{\lambda}[-q_{\nu}(\eta + \eta\xi)$$

$$\pm (k + k')_\nu(\xi + \xi^2)]$$

$$\pm \frac{1}{m}[(s\cdot k)k'_\nu q_\mu - (k\cdot k')s_\nu q_\mu](\eta + \eta\xi)$$

$$\pm \frac{1}{m}[(s\cdot k)k'_\mu q_\nu - (k\cdot k')s_\mu q_\nu](\eta + \eta\xi)$$

$$\mp \frac{1}{m}(s\cdot k)[q_\mu(k + k')_\nu + q_\nu(k + k')_\mu]$$

$$+ \frac{1}{m}(s\cdot k)(k + k')_\mu(k + k')_\nu\xi^2$$

$$+ \frac{1}{m}[(k\cdot k')s_\mu(k + k')_\nu - k'_\mu(k + k'_\nu)(s\cdot k)](\xi + \xi^2)$$

$$+ \frac{1}{m}[(k\cdot k')s_\nu(k + k')_\mu - k'_\nu(k + k')_\mu(s\cdot k)](\xi + \xi^2), \qquad (1\text{-}9)$$

where the upper and lower signs refer to the neutrino and antineutrino, respectively. Furthermore, from now on, the following expressions will be used:

$$E = \frac{q^2}{2Mxy}, \quad E' = -\frac{(1 - y)}{2Mxy} q^2,$$

$$s\cdot k \cong \frac{1}{2m}(-q^2 - m^2 \frac{1+y}{1-y} - m^2 \frac{2M^2x^2y^2}{(1-y)^2q^2},$$

$$s\cdot p \cong -\frac{(1-y)}{2mxy} q^2 + m \frac{M^2xy}{(1-y)q^2}, \qquad (1\text{-}10)$$

where $y = \nu/ME = 1 - E'/E$ and s is the longitudinal polarization vector of the muon. Now averaging over the muon spin, we get the Callan-Gross relation

$$F_1 = F_2/2x,$$

$$\frac{d^2\sigma}{dQ^2 d\nu} = \frac{G^2 x^2 y^2}{\pi q^4} \overline{\sum} \sum \mathcal{M}_{\mu\nu} W_{\mu\nu}$$

$$= \frac{G^2 x}{\pi} \left\{ (1-y)\frac{\xi^2}{m^2} F_2(x) + \frac{1}{2Q^2} \left[F_2(x) \right.\right.$$

$$\left[y^2 + 2(1-y) + 2y^2\xi - y(2-y)\xi^2 \right]$$

$$\mp x F_3(x) \left(\frac{y(2-y)}{2} \right) (1+\xi)^2 +$$

$$\left.\left. + \frac{M^2 \gamma \eta^2 y^2}{m^2} F_2(x) \pm \frac{2M^2 x^2 y(2-y)}{m^2} F_5 \eta\xi \right] \right\}. \tag{1-11}$$

Thus we see that

$$\lim_{Bj} \frac{d^2\sigma}{dQ^2 d\nu} = (1-y) \frac{G^2 \xi^2}{\pi m^2} x F_2(x) + O(\frac{1}{Q^2}), \tag{1-12}$$

while the V - A theory yields the term of the order of $1/Q^2$.

Next we turn to the computation of the muon polarization. We define the polarization P and the longitudinal muon polarization vector S such that

$$P = \frac{d\sigma(\leftarrow) - d\sigma(\rightarrow)}{d\sigma(\leftarrow) + d\sigma(\rightarrow)}, \quad s = (\frac{E'}{m}\hat{k}', \frac{|k'|}{m}). \tag{1-13}$$

Then a straightforward calculation yields the longitudinal polarization P^L:

$$P^L = \frac{c_1 + q^2\left[F_1 a_1 + \frac{F_2}{2x} a_2 \mp F_3 a_3 + c_2 a_4 - c_3 \mp c_4\right]}{c_1 - q^2\left[F_1 b_1 + \frac{F_2}{2x} b_2 \mp F_3 b_3 + c_2 b_4 + c_3 \pm c_4\right]} \tag{1-14}$$

where

$$a_1 = 1 + \xi + \xi^2,$$

$$a_2 = \frac{2(1-y)}{y^2} - \frac{(4-y)}{y}\xi + \frac{2}{y}\xi^2,$$

$$a_3 = \frac{1}{2}\left[\frac{(2-y)}{y} + 2\frac{(4-y)}{y}\xi + \frac{(6-y)}{y}\xi^2\right],$$

$$a_4 = -\frac{(1+y)}{1-y}\xi^2,$$

$$b_1 = 1 + 2\xi,$$

$$b_2 = \frac{2(1-y)}{y^2} - \frac{(2-y)}{y}\xi^2,$$

$$b_3 = \frac{(2-y)}{2y}(1+\xi)^2,$$

$$b_4 = \mp\eta \mp \eta\xi + \xi + \frac{1}{2}(\eta^2 + \xi^2),$$

$$\gamma = \frac{4x^3 F_4 - 2x^2 F_5}{F_2}$$

$$c_1 = \frac{q^4}{2m^2}\,\xi^2\left[-F_1 + \frac{F_2}{2x}\frac{(2-y)^2}{y^2}\right]$$

$$c_2 = F_1 - \frac{F_2}{2x}$$

$$c_3 = \frac{F_5 M^2 \gamma \eta^2}{2xm^2}$$

$$c_4 = \frac{F_2 2M^2 x^2}{2xm^2}\,\frac{(2-y)}{y}\,\eta\xi \qquad (1\text{-}15)$$

Furthermore, if we adopt the spin-1/2 parton model, we have the Callan-Gross relation $F_1 = F_2/2x$. Then:

$$P^L = \pm\,\frac{d_1 - \frac{m^2}{2Q^2}\left[d_2 - \frac{4}{y}\xi + \frac{2}{y}\xi^2 \mp \frac{xF_3}{F_2}\left(d_3 + \frac{4}{y}\xi + \frac{4}{y}\xi^2\right) - d_4 \mp c_4\right]}{d_1 + \frac{m^2}{2Q^2}\xi\left[d_2 - \frac{2}{y}\xi^2 \mp \frac{xF_3}{F_3}\,d_3 + d_4 \pm c_4\right]} \qquad (1\text{-}16)$$

where

$$c_4 = \frac{2M^2x^2}{m^2}\,\frac{F_5}{F_2}$$

$$d_1 = \frac{(1-y)}{y^2}\,\xi^2$$

$$d_2 = (1+\xi)^2 + \frac{2(1-y)}{y^2}$$

$$d_3 = \frac{(2-y)}{y}(1+\xi)^2$$

$$d_4 = \frac{M^2\gamma\eta^2}{m^2}$$

We note that in the vector-gluon model [12] the parameter γ in Eq. (1-15) is a constant given by $\gamma = (m_p/M)^2$, where m_p is the bare proton quark mass. The constants $\xi/m < 0.1\ \text{GeV}^{-1}$, $\eta/m < 0.1\ \text{GeV}^{-1}$, i.e., $\xi < 10^{-2}$, $\eta < 10^{-2}$ seem to be reasonable from the low-energy experiments which agree with the V - A theory. Moreover as is well known, the positivity condition gives $0 \leq -xF_3 \leq F_2$. Therefore, for practical purposes we may write, if $y \neq 1$,

$$P^L = \mp 1 \pm \frac{m^2 x y^2 [2F_1 \mp F_3 - (1-y)(F_1 - \frac{F_2}{2x})]}{Q^2(1-y)\{xy^2F_1 + (1-y)F_2 \mp xy(\frac{2-y}{2})F_3}, \quad \text{(1-17a)}$$

for pure V - A,

$$P^L = \pm 1 \mp \frac{m^2}{Q^2\xi^2}\left[\frac{y^2}{1-y} + 2 \mp \frac{xF_3}{F_2}\frac{y(2-y)}{(1-y)}\right], \quad \text{(1-17b)}$$

for η, ξ, and V - A,

$$P^L = \mp \frac{1 + \frac{2(1-y)}{y^2} \mp \frac{xF_3}{F_2}(\frac{2-y}{y}) - \frac{M^2\gamma\eta^2}{m^2}}{1 + \frac{2(1-y)}{y^2} \mp \frac{xF_3}{F_2}(\frac{2-y}{y}) + \frac{M^2\gamma\eta^2}{m^2}}, \quad \text{(1-17c)}$$

for $\xi = 0$, and V - A.

These results indicate that P^L approaches 1 instead of -1 for large Q^2 and $0 < y < 1$ if $\xi \neq 0$. We illustrate

P^L for $y = 1/2$ in Figure 1. But P^L will not change sign if $\xi = 0$ and $(M^2\gamma\eta^2/m^2) << (2/y^2)$, as long as the assumption $F_2 = -xF_3$ is used. This corresponds to $\gamma << 8 \times 10^2$, which means that $m_p << 28M$ in the vector-gluon model, when $\eta = 10^{-2}$ and $y = 1/2$. An estimate [12] is that $\gamma = 1.6 \times 10^{-2}$, i.e., $m_p = 120$ MeV, which leads to $P^L = -1$.

An important feature in the estimate described above is that the momentum transfer at which the change of the muon polarization occurs is very sensitive to the value of ξ and η. Because complete right-handed polarization needs extremely large Q^2, it would be interesting

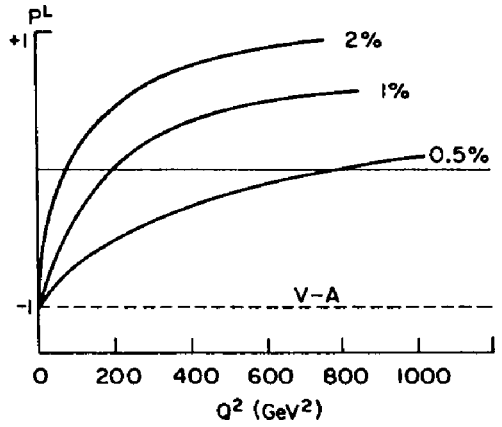


Figure 1. Longitudinal polarization of the muon in $\nu N \to \mu X$ with the assumptions $F_4 = F_5 = 0$, $-xF_3 = F_2$, $F_1 = F_2/2x$, only for illustration. Percentage on the curve refers to that of mg_W^T with respect to the vector coupling constant g_W^V. We put $y = 1/2$, which is the average value of y calculated in [12].

to see the value of Q^2 at which P goes to zero before it changes sign. In Figure 1, we see that, for $\xi/m = 0.1\ \text{GeV}^{-1}$, P^L vanishes at $Q^2 = 200\ \text{GeV}^2$, while for $\xi/m = 0.05\ \text{GeV}^{-1}$ that occurs at $Q^2 = 800\ \text{GeV}^2$. The perpendicular polarization would not give any insight about the ξ or η term, because it is proportional to $m\sin\theta$, which is very small at high energy.

If we do the above calculation in the intermediate-vector-boson theory, i.e., Lagrangian (1), we can easily obtain the result by simply replacing ξ, η, and G in Eqs. (1-11) - (1-17) by

$$\eta \to \frac{m^2}{m_W^2} + \eta\left(\frac{q^2}{m_W^2} - 1\right),\quad \xi \to \xi,$$

$$G^2 = \frac{G^2}{(1 + 2MxyE/m_W^2)^2} \tag{1-18}$$

Therefore we conclude that unless $Q^2/m_W^2 >> 1$, the previous results would not be modified. Notice that as far as the ξ term and polarization are concerned we cannot distinguish between the intermediate-vector-boson theory, or a momentum-dependent weak current. In fact the merit of the polarization measurement rather than that of the cross section in view of testing the V - A theory as formulated here, is that the change of the polarization is not as sensitive to the validity of the

Callan-Gross relation or the breakdown of scaling as the cross section is. Nevertheless, one might expect that the general IVB theory given by the Lagrangian(1) would be appropriate to describe high-energy phenomena rather than the current-current theory with $j_\alpha(x)$ in (1-3). In that case, if we assume $F_2 = 2xF_1 = -xF_3$ and $F_4 = F_5 = 0$, the violation of scaling in the expression $d^2\sigma/dxdy$ may be parameterized by

$$F_2^{\nu}(x) \approx F_2^{\nu}(x) \left[2 + \frac{\xi^2 Q^2}{m^2}(1-y)\right]/\left(1 + \frac{Q^2}{m_W^2}\right)^2 \qquad (1\text{-}19a)$$

and

$$F_2^{\bar{\nu}}(x) \approx F_2^{\bar{\nu}}(x) \left[2(1-y)^2 + \frac{\xi^2 Q^2}{m^2}(1-y)\right]/\left(1 + \frac{Q^2}{m_W^2}\right)^2. \qquad (1\text{-}19b)$$

As far as power counting in Feynman diagrams is concerned, the ξ term is not worse than the V - A theory with an IVB, because $\gamma_\mu q_\mu q_\alpha \approx m q_\alpha$, and $\sigma_{\mu\nu} q_\mu q_\nu q_\alpha = 0$. We summarize all the results in Table I. Obviously the realization of the limits in Table I depends upon the value of $y = 1 - E'/E$. A larger y requires higher Q^2.

In the recent analysis of momentum-independent S-T-P weak currents, Cheng and Tung [9] state that:

> to the extent that the lepton mass can be neglected, a purely left-handed outgoing lepton indicates V interaction, a purely right-handed one indicates S-T interaction, and the coexistence of both indicates a mixture of the two.

TABLE I

Longitudinal Polarization of the Muon in the $\nu N \to \mu X$ at the Scaling Limit

V - A	$P = \mp 1$	IVB V - A	$P = \mp 1$
MDV ξ, V - A	± 1	ξ, V - A	± 1
ξ, η, V - A	± 1	ξ, η, V - A	± 1
η, V - A	depend upon γ	η, V - A	$\pm$ if $\frac{Q^2}{m_W^2} \gg 1$
			$\mp$ if $\frac{Q^2}{m_W^2} \ll 1$

MDV refers to the momentum-dependent weak vector current The values of P^L given are the limit of P^L as $Q^2 \to \infty$. These values seem to be realized because of the smallness of ξ and η, at much larger Q^2 than the one at which structure functions begin to scale in the Bjorken-Johnson-Low limit.

Accepting the premise that no matter what the additional term is, it must be small compared with the V - A term, Cheng and Tung predict $P^L = -1 \pm \alpha$ when there exists a small contamination of S - T interaction. Here α is a small positive constant. However, our result is that, as can be seen in Table I, a purely right-handed outgoing muon at high energy indicates the existence of the ξ term whether the interaction is via current-current theory or IVB theory. It is interesting to note that even though the ξ term contains momentum transfer q which

makes us anticipate that it might dominate over the V - A term at large momentum transfer, it turns out that all the higher momentum-dependent parts involving η cancel out so that it does not dominate over the V - A term. But a small contamination of the η term would mean $P^L = -1 \pm \alpha$ as in Cheng's and Tung's case with the momentum-independent S - T interaction. Therefore, if a future experiment indicates that $P^L = -1 \pm \alpha$, a very elaborate analysis would be necessary to decide whether that is due to Cheng's and Tung's momentum-independents S - T interaction or our momentum-dependent η-type coupling. However, we notice that the result described above concerning the η term is a consequence of the assumption (1-8). If we assume a different scaling law, such as

$$\lim_{Bj} \left(\frac{\nu}{M^2}\right) W_{4,5} = F_{4,5}(x),$$

the role of the η terms would be similar to that of the ξ term.

Another interesting prediction of the momentum-dependent vector current (ξ type) is that the ratio $\sigma(\bar{\nu})/\sigma(\nu)$ increases as the neutrino energy gets larger, i.e.,

$$R \equiv \frac{\sigma(\bar{\nu})}{\sigma(\nu)}$$

$$R = \frac{\frac{ME\xi^2}{6m^2} + \left(\frac{4}{3} - \frac{2}{3}\frac{|\langle xF_3\rangle|}{\langle F_2\rangle} + \frac{M^2\gamma\eta^2}{2m^2} - \frac{2M^2\eta\xi\langle x^2F_5\rangle}{3m^2\langle F_2\rangle}\right)}{\frac{ME\xi^2}{6m^2} + \left(\frac{4}{3} + \frac{2}{3}\frac{|\langle xF_3\rangle|}{\langle F_2\rangle} + \frac{M^2\gamma\eta^2}{2m^2} + \frac{2M^2\eta\xi\langle x^2F_5\rangle}{3m^3\langle F_2\rangle}\right)}$$

and

$$\lim_{Bj} R = 1 \qquad (1\text{-}20)$$

where

$$\langle x^n F_m\rangle \equiv \frac{\int_0^1 dx \int_0^1 dy\ x^n F_m \frac{d^2\sigma}{dxdy}}{\int_0^1 dx \int_0^1 dy \frac{d^2\sigma}{dxdy}} \quad .$$

The above conclusion will not be modified as long as the ratios F_3/F_2, F_4/F_2, and F_5/F_2 scale, as in the IVB theory. The CERN result [13,14] $R = 0.377 \pm 0.023$ may indicate the deviation from $R = 1/3$, although it is obviously too early to draw any definite conclusion. However, various models predict different values of R. The spin-1/2 parton model yields $R = 1/3$, while other constituents would give $1/3 < R < 3$. In general, from the positive semidefinite property of $W_{\mu\nu}$ and the scaling behavior of structure functions given in (1-8), it follows that $F_2 \geq 2xF_1 \geq -xF_3$, which leads to $R \geq 1/3$. However, the above ratio is constant unless scaling breaks down to the extent $F_2(x) = xF_3(x)G(E) = xF_3(x)G = xF_3(x)G(Q^2/2Mxy)$, where G(E) is some function of E.

Therefore in order to detect the ξ or η term, we have to look at the energy dependence of the ratio R.

One might expect that the ξ term (or η term) could be obtained as a result of the radiative corrections in the framework of the V - A theory as in the eeA vertex correction in quantum electrodynamics. However, that this is not the case can be seen by the following argument. First, the radiative corrections [15] to the $\mu\nu W$ coupling (which do not have vertex corrections in the second order) fail to induce the ξ-type effective coupling. Similarly in the Weinberg theory [16], the $\mu\nu W$ vertex correction [17] due to a neutral vector boson Z_μ rules out this possibility because of the $1 \pm \gamma_5$ structure of the interactions. Thus we cannot as of now tell where the extra derivative coupling may come from. Nevertheless, the other higher-order corrections might well lead to the same conclusions about the polarization of the muon deduced from our formalism [4]. On the phenomenological level, however, we conclude that the polarization measurement of the muon in the inclusive reaction $\nu N \to \mu X$ at high energy will provide a good criterion for the existence or nonexistence of a phenomenological momentum-dependent weak vector current or of the intermediate-vector-boson theory with Lagrangian(1).

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$$\mathcal{M}^{(0)}_{\mu\nu} \quad \text{and} \quad \mathcal{M}^{(s)}_{\mu\nu}$$

in the expressions for $W_{\mu\nu}$ and $\mathcal{M}_{\mu\nu}$, remain as given in the text.

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CHAPTER 2

ANTIPROTON-NEUTRON BOUND STATE

A recent experiment [1] of antiproton-neutron annihilation at rest has shown a narrow peak in positive G-parity states, and $\bar{p}$-n bound-state formation was suggested for its explanation. Although the possibility of a nonrelativistic bound state of the $\bar{p}$-n system has already been studied by several authors [2,3], none of these predictions could account for the reported narrow width ($\leq$8 MeV) at the binding energy 83.3 $\pm$ 1.4 MeV. In this study, we point out that the observed state should correspond to a higher angular momentum state, possibly a p state. Let us simplify the problem by assuming a central square-well potential for the antinucleon-nucleon force. (Let the potential depth be -V, and the range R.) Certainly we are aware of the fact that the nuclear force depends on the spin, isospin, and velocity of the nucleons through a complicated exchange mechanism. However, the essential part of the discussion will not depend on the detailed form of the potential, as can be seen by comparing our result with some of the more elaborate calculations [3]. Since we have the narrow peak only in the positive G-parity state, we are dealing

with $^3S_1$, $^1P_1$, $^3D_{1,2,3}\cdots$ states for the I = 1 NN system and have to assume its potential for negative G-parity states to be very different from that of positive G-parity states. (The latter should be more attractive than the former.) Such a difference in the potentials for both G-parity states may be partly due to an L-S coupling force and partly due to a short-range force which results from more complicated t-channel exchange and s-channel annihilation diagrams. Whether this is true or not is to be seen by a further investigation.

The formula for computing the width Γ_ℓ of annihilation from a $\bar{p}$-n bound state with orbital angular momentum ℓ may be given by

$$\Gamma_\ell = \sigma_\ell \, v \, \rho_\ell \tag{2-1}$$

where σ_ℓ designates the ℓth partial cross section for $\bar{p}$-n annihilation and the densities of states are expressed as

$$\rho_s = \frac{1}{\frac{4}{3}\pi R'^3} \int_0^{R'} \int |\psi_0|^2 \, r^2 \, dr \, d\Omega \tag{2-2}$$

for s state and

$$\rho_p = \frac{1}{\frac{4}{3}\pi R'^3} \int_0^{R'} \int \frac{|\nabla\psi_1|^2}{k_1^2} \, r^2 \, dr \, d\Omega \tag{2-3}$$

for p state.

Here R' stands for the annihilation radius, which is assumed to be smaller than R, and

$$k_1 = [2\mu(V - |E|)]^{1/2}, \tag{2-4}$$

μ and E being the reduced mass and the binding energy, respectively. The formulas (2-1) and (2-3a) for the p wave reduce to those given by Jackson et al [4], when $R' \to 0$, i.e.,

$$\rho_p \to \frac{1}{k_1^2} |\nabla\psi_1(0)|^2.$$

We might as well use the density for the p state,

$$\rho_p = \frac{1}{\frac{4}{3}\pi R'^3} \int_0^{R'} \int |\psi_1|^2 \, r^2 \, dr \, d\Omega, \tag{2-3b}$$

which vanishes in the limit $R' \to 0$. As will be seen later (Table II), the formulas (2-3a) and (2-3b) lead to similar results as long as R' = 0.5 - 1.2 F. (For definiteness, we have fixed the value of R to be 1.2 F.)

Using the boundary condition at r = R

$$\frac{k_1 j_\ell'(k_1 R)}{j_\ell(k_1 R)} = \frac{ik_2 h_\ell^{(1)'}(ik_2 R)}{h_\ell^{(1)}(ik_2 R)} \tag{2-5}$$

with

$$k_2 = (2\mu|E|)^{1/2}, \tag{2-6}$$

we obtain the depth of the potential:

TABLE II

Densities and Decay Widths of the $\bar{p}n$ Bound States with Positive G Parity

State	Annihilation Radius (R')	1.2F a	1.2F b	0.5F a	0.5F b	0.1F a	0.1F b	Zero a	Zero b
$L = 1(^1P_1)$	4ρ (MeV/mb)	9.5	8.0	5.6	5.9	18.96	0.31	19.45	0
	Γ (MeV)	6.4	5.3	3.8	4.0	12.7	0.21	13.0	0
$L = 0(^3S_1)$	$\frac{4}{3}\rho$ (MeV/mb)	2.8		6.4		7.5		7.55	
	Γ (MeV)	28.4		64.8		76.5		77.0	

Note: a and b for the p-wave width refer to Eqs. (9a) and (9b) in the text.

$$V = 226 \text{ MeV for s-wave bound state,} \qquad (2\text{-}7a)$$

$$V = 420 \text{ MeV for p-wave bound state} \qquad (2\text{-}7b)$$

for the observed binding energy $E = -83.3$ MeV.

From Equations (1)-(6), it follows that [5]:

$$\Gamma_s = (\sigma v)_s \frac{|A_0|^2}{3\pi} \frac{3}{2}[j_0^2(k_1R') + n_0(k_1R')\, j_1(k_1R')] \qquad (2\text{-}8)$$

and

$$\Gamma_p = (\sigma v)_p \frac{|A_1|^2}{3\pi} 9 \left\{ \frac{1}{2}[j_1^2(k_1R') - j_0(k_1R')j_2(k_1R')] + \frac{j_1(k_1R')}{k_1R'}[j_0(k_1R') - \frac{2}{k_1R'} j_1(k_1R')] \right\} \qquad (2\text{-}9a)$$

or

$$\Gamma_p = (\sigma v)_p \frac{3}{2\pi} |A_1|^2 [j_1^2(k_1R') - j_0(k_1R')j_2(k_1R')], \qquad (2\text{-}9b)$$

where A_0 and A_1 are the normalization constants for the s wave and the p wave, respectively, given by

$$|A_0|^2 = \frac{1}{\frac{1}{2}R^3[j_0^2(k_1R)(1 + 1/k_2R) + n_0(k_1R)j_1(k_1R)]},$$

$$|A_1|^2 = \frac{-1}{\frac{1}{2}R^3(1 + k_1^2/k_2^2)j_0(k_1R)(j_2(k_1R))}.$$

As for the estimate of the partial annihilation cross section, we make an assumption that

$$(\sigma v)_s \approx a,$$
$$(\sigma v)_p \approx bk^2, \tag{2-10}$$

where a, b are constants.

Although no direct beam experiments are available up to now, an early indirect determination of $\sigma(\bar{p}n)$ by subtracting $\bar{p}p$ cross sections from those of $\bar{p}d$, coupled with the Glauber shadow-effect correction, seems to indicate that $\sigma(\bar{p}p) \approx \sigma(\bar{p}n)$ within the experimental error [6-8]. Therefore, assuming the same cross sections for both antiproton-nucleon annihilations, and from the data of $\bar{p}p \rightarrow$ even number of pions, we get

$$(\sigma v)_s \approx 10.2 \text{ mb},$$
$$(\sigma v)_p \approx 0.67 \text{ mb} \tag{2-11}$$

for a positive G-parity state, the latter being the value at the momentum 0.66 GeV/c, which is the c.m. momentum of the antiproton-neutron bound system in the p state. Since we have neglected the k-dependent term of the s-wave cross section which would give an ambiguity in the determination of the p-wave cross section, Eq. (2-11), the value quoted above for the p-wave cross section must be taken as an indication of the order of magnitude.

The result of numerical computation of the decay widths, using Eqs. (2-1)-(2-11), is given in Table II.

Notations a and b for the p wave refer to the adopted formulas, (2-9a) and (2-9b). From the table, one can immediately notice that the s-wave annihilation gives too large a decay width, while the p-wave annihilation is consistent with the observed width ≤ 8 MeV. (We note, however, that in a recent work [9], some questions are raised about the estimate of the width of the peak in the experiment.) As we mentioned earlier, formulas (2-9a) and (2-9b) give approximately equal widths except for a small annihilation radius.

Accepting the premise that the observed narrow peak is due to a p-wave bound state of the p-n system $(^1P_1)$, one encounters a question regarding why the ground s-wave bound state has not been observed in the experiment. In order to answer this question, we estimate the binding energy of the s-state using the potential (7b), obtaining

$$E_{\text{ground state}} = -248 \text{ MeV}.$$

Since both states are positive G-parity states, and

$$E_{\text{p state}} - E_{\text{ground state}} = 165 \text{ MeV} < 2\, m_\pi,$$

the transition $(^1P_1) \rightarrow (^3S_1)$ by strong interaction is forbidden. The transition by electromagnetic interaction $(^1P_1) \rightarrow (^3S_1) + \gamma$, has a small width ($\approx 0.28$ MeV).

Thus the $^3S_1$ ground state is difficult to observe, unless it is formed directly from $\bar{p}$ in an outer orbit. Finally, it should be pointed out that our argument is merely of a qualitative nature, since the obtained potential (2-7b) is so deep that a nonrelativistic treatment may not be warranted.

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CHAPTER 3

EFFECT OF NEUTRAL CURRENTS ON THE NEUTRINO BREMSSTRAHLUNG

Introduction

Whenever an electron is accelerated, a pair of neutrinos can be emitted as photons are so via quantum electrodynamics. This is because of the (νe) coupling either in the form of universal four fermion weak interaction proposed by Feynman and Gell-Mann and others [1] or intermediate vector boson theory of weak interaction. In the presently available experimental conditions, we cannot detect it due to the extremely small cross section and the small density of the matter on earth. However, an appropriate condition in terms of the density, temperature, and chemical potential of the matter are thought to be realized in the course of the evolution of the stars. Moreover, due to the large mean free path of the neutrino beam in the dense star than that of the light, the neutrino emitting reactions are more important mechanisms in cooling a star in certain regions of density and temperature. Consequently, as soon as the universal weak interaction theory appeared in the literature, many authors applied that theory to calculate

neutrino opacity of the star to get the cooling time or the other features of the stellar evolution such as the supernovae explosion and the formation of the neutron star. The neutrino emission mechanisms which are important in this context are photoneutrino emission [2], which is the photon electron inelastic scattering resulting in a neutrino pair and annihilation of a pair of electrons into a neutrino pair, plasma neutrino [4] which comes from the longitudinal or transverse plasmon, i.e.,

$$\gamma + e^{\pm} \rightarrow e^{\pm} + \bar{\nu}e,\mu + \nu e,\mu$$

$$e^{+} + e^{-} \rightarrow \bar{\nu}e,\mu + \nu e,\mu$$

$$\text{"}\gamma\text{"}_{\text{plasma}} \rightarrow \bar{\nu}e,\mu + \nu e,\mu$$

and URCA process of Gamow and Schonberg [5] and neutrino bremsstrahlung from the nuclear coulomb field, i.e.,

$$e^{-} + (z,A) \rightarrow (z - 1,\ A) + \nu e$$
$$\hookrightarrow (z.A) + e^{-} + \bar{\nu}e$$

$$e^{-} + (z.A) \rightarrow e^{-} + (z.A) + \bar{\nu}e,\mu + \nu e,\mu$$

Neutrino bremsstrahlung process has been investigated by Grandel'man and Pinaev [6] in the non-relativistic non-degenerate region and Festa and Ruderman [7] in the

extremely relativistic degenerate electron energy region. P. Cazzola, G. De Zotti, and A. Saggion [8] computed numerically in a relativistic context at various densities and temperatures. However, all the above calculations were based on the conventional V-A theory of weak interaction, or in terms of the language of the current-current theory, on the charged weak currents. Now that some of the unified gauge theory of weak and electromagnetic interactions predict the existence of the neutral currents and this current has been confirmed by experiments, any future astrophysical application of the neutrino emission mechanism has to incorporate the neutral current in the calculations. The aim of this work is to recalculate the energy loss rate due to the neutrino bremsstrahlung in terms of the charged and neutral currents even though the exact space time and isospin structure of the neutral currents are yet to be determined by experiments.

In the following section, we calculate energy loss rate with Weinberg's SU(2) × U(1) theory of weak and electromagnetic interactions up to the second order in the neutrino energy (Q_o) expansion, and discussion and comparison with other processes appear in the third section.

As is seen in the text, the neutrino energy expansion method greatly simplifies the whole calculation

and allows the analytic expression of the integrals. In Appendix I we discuss the integrals of the changed variables and in Appendix II we present the details of the $\vec{Q}$ integrations. Finally, the method of distribution integral is given in Appendix III.

Energy Loss Rate

The relevant part of Weinberg's Hamiltonian [10] is:

$$\mathcal{H} = - e\bar{\psi}_e\gamma^\alpha(1 - \gamma_5)\psi_e A_\alpha - \frac{g}{2\sqrt{2}}\bar{\psi}_{\nu e}\gamma^\alpha(1 - \gamma_5)\psi_e W_\alpha^\dagger$$

$$- \frac{g}{2\sqrt{2}}\bar{\psi}\gamma^\alpha(1 - \gamma_5)\psi_{\nu e}W_\alpha - \frac{1}{4}\frac{1}{\sqrt{g^2 + g'^2}}\bar{\psi}_e\gamma^\alpha$$

$$(a + b\gamma_5)\psi_e z_\alpha - \frac{1}{4}\sqrt{g^2 + g'^2}\,\bar{\psi}_{\nu_e}\gamma^\alpha(1 - \gamma_5)\psi_{\nu_e} z_\alpha$$

$$- \frac{1}{4}\sqrt{g^2 + g'^2}\,\bar{\psi}_{\nu_\mu}\gamma^\alpha(1 - \gamma_5)\psi_{\nu_\mu} z_\alpha \qquad (3\text{-}1)$$

where $a = 3g'^2 - g^2$, $b = g^2 + g'^2$, $e = \frac{gg'}{\sqrt{g^2 + g'^2}}$,

$\tan\theta\omega = g'/g$.

The lowest order Feynman diagrams for the neutrino bremsstrahlung are shown in Figure 2. The matrix elements corresponding to the W exchange diagrams (e) and (f) are:

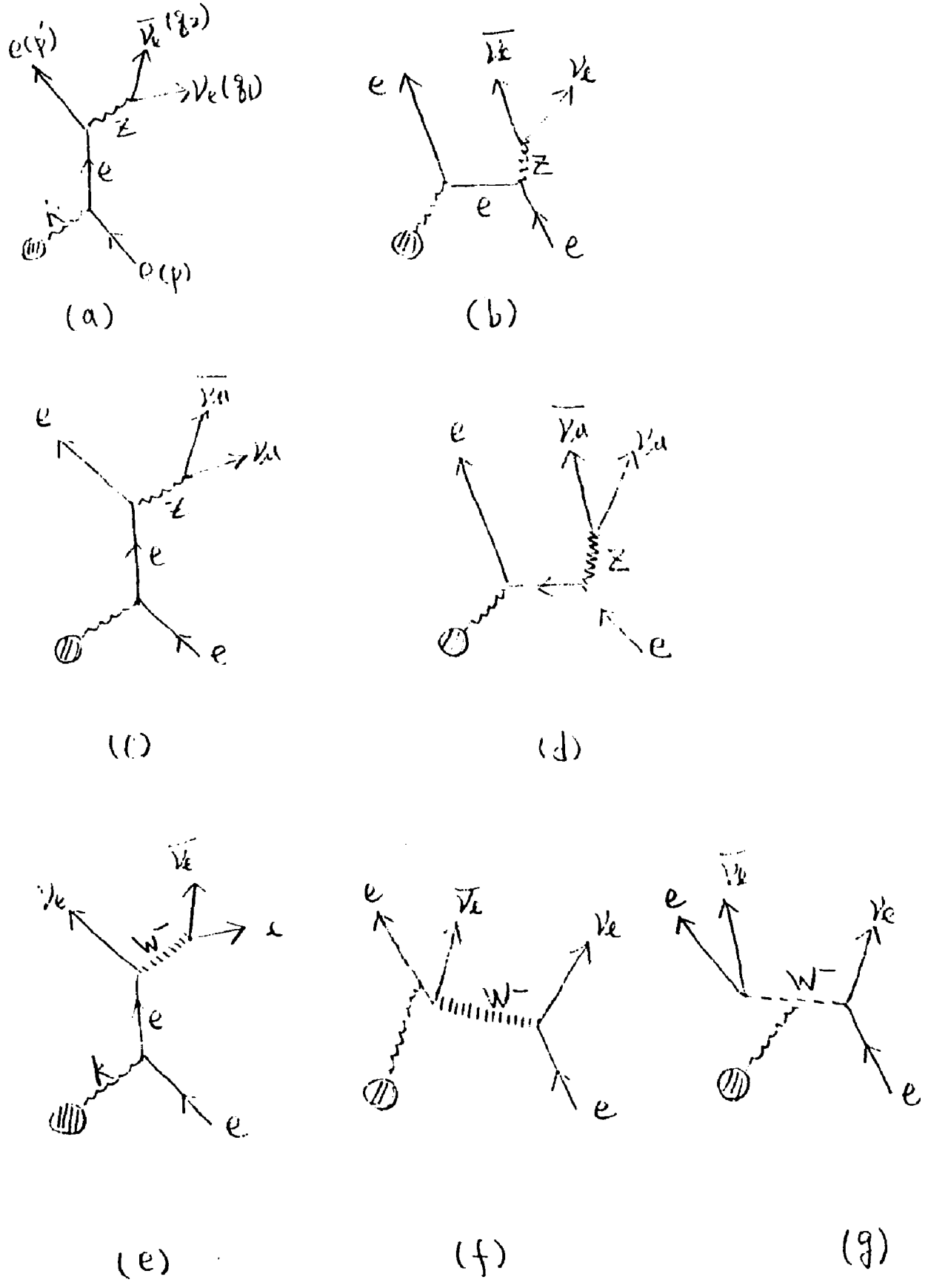


Figure 2. Feynman diagrams for neutrino bremsstrahlung.

$$M_{ef} = -\left(\frac{4\pi^2 e^2 Z}{k^2 + a^2}\right)\left(\frac{ig^2}{8Mw^2}\right)\bar{U}_e(p_f)\gamma^\circ \frac{\not{p}_f - \not{k} + m}{(p_f-k)^2-m^2}\gamma^\mu$$

$$(1 - \gamma_5)V_{\bar{\nu}e}(q_2)\bar{U}_\nu(q_1)\gamma_\mu(1 - \gamma_5)U_e(p_i)$$

$$- \left(\frac{4\pi^2 e^2 Z}{k^2 + a^2}\right)\left(\frac{ig^2}{8M_w^2}\right)\bar{U}_{\nu e}(q_1)\gamma^\mu(1 - \gamma_5)\frac{\not{p}_i+\not{k}+m}{(p_i+k)^2-m^2}$$

$$\gamma^\circ U_e(p_i)\bar{U}_e(p_f)\gamma_\mu(1 - \gamma_5)V_{\bar{\nu}e}(q_2) \qquad (3\text{-}2)$$

and Z exchange diagram's contribution to the matrix element is:

$$M_{ab} = \left(\frac{4\pi^2 e^2 Z}{k^2+a^2}\right)\left(\frac{ig^2}{16Mz^2}\right)\bar{U}_e(p_f)\gamma^\alpha(a + b\gamma_5)\frac{\not{p}_i + \not{k} + m}{(p_i+k)^2 - m^2}$$

$$\gamma^\circ U_e(p_i)\bar{U}_{\nu_e}(q_1)\gamma_\alpha(1 - \gamma_5)V_{\bar{\nu}_e}(q_2)$$

$$+ \left(\frac{4\pi^2 e^2 Z}{k^2+a^2}\right)\left(\frac{ig^2}{16Mz^2}\right)\bar{U}_e(p_f)\gamma^\circ \frac{\not{p}_f - \not{k} + m}{(p_f-k)^2 - m^2}\gamma^\alpha$$

$$(a + b\gamma_5)U_e(p_i)\bar{U}_{\nu_e}(q_1)\gamma_\alpha(1 - \gamma_5)V_{\bar{\nu}_e}(q_2) \qquad (3\text{-}3)$$

$$M_{cd} = \left(\frac{4\pi^2 e^2 Z}{k^2+a^2}\right)\left(\frac{ig^2}{16Mz^2}\right)\bar{U}_e(p_f)\gamma^\alpha(a + b\gamma_5)\frac{\not{p}_i + \not{k} + m}{(p_i+k)^2-m^2}$$

$$\gamma^\circ U_e(p_i)\bar{U}_{\nu_\mu}(q_1)\gamma_\alpha(1 - \gamma_5)V_{\bar{\nu}_\mu}(q_2)$$

$$+ \left(\frac{4\pi^2 e^2 Z}{k^2+a^2}\right)\left(\frac{ig^2}{16Mz^2}\right)\bar{U}_e(p_f)\gamma^\circ \frac{\not{p}_f - \not{k} + m}{(p_f-k)^2-m^2}\gamma^\alpha$$

$$(a + b\gamma_5)U_e(p_i)\bar{U}_{\nu_\mu}(q_1)\gamma_\alpha(1 - \gamma_5)V_{\nu_\mu}(q_2) \qquad (3\text{-}4)$$

After the Fierz transformation $M_{ef} + M_{ab} = M_{abef}$ and M_{cd} can be rewritten as follows:

$$M_{abef} = -\left(\frac{4\pi^2 e^2 Z}{k^2+a^2}\right) i \frac{G}{\sqrt{2}} \bar{U}_e(p_f)\gamma^\alpha(C_V - C_A\gamma_5)\frac{\not p_i + \not k + m}{B}$$

$$\gamma^\circ U_e(p_i)\bar{U}_{\nu_e}(q_1)\gamma_\alpha(1-\gamma_5)V_{\bar{\nu}_e}(q_2)$$

$$-\left(\frac{4\pi^2 e^2 Z}{k^2+a^2}\right) i \frac{G}{\sqrt{2}} \bar{U}_e(p_f)\gamma^\circ \frac{\not p_f - \not k + m}{B'} \gamma^\alpha$$

$$(C_V - C_A\gamma_5)U_e(p_i)\bar{U}_{\nu_e}(q_1)\gamma_\alpha(1-\gamma_5)(V_{\bar{\nu}_e}(q_2) \qquad (3\text{-}5)$$

$$M_{cd} = -\left(\frac{4\pi^2 e^2 Z}{k^2+a^2}\right) i \frac{G}{\sqrt{2}} \bar{U}_e(p_f)\gamma^\alpha(C_V - 1 + C_A\gamma_5)$$

$$\frac{\not p_i + \not k + m}{B} \gamma^\circ U_e(p_i)\bar{U}_{\nu_\mu}(q_1)\gamma_\alpha(1-\gamma_5)$$

$$V_{\bar{\nu}_\mu}(q_2) - \left(\frac{4\pi^2 e^2 Z}{k^2+a^2}\right) i \frac{G}{\sqrt{2}} \bar{U}_e(p_f)\gamma^\circ \frac{\not p_f - \not k + m}{B'}$$

$$\gamma^\alpha(C_V - 1 + C_A\gamma_5)U_e(p_i)\bar{U}_{\nu_\mu}(q_1)\gamma_\alpha(1-\gamma_5)V_{\bar{\nu}_\mu}(q_2) \qquad (3\text{-}6)$$

where

$$C_A = 1 - \frac{b}{2Mz^2}\frac{Mw^2}{g^2} = 1/2$$

$$C_V = 1 + \frac{q}{2Mz^2}\frac{Mw^2}{g^2} = 1/2 + \frac{e^2}{4Mw^2}\sqrt{\frac{2}{G}} = \frac{1}{2} + 2\sin^2\theta w$$

$$B = k^2 + 2p_i \cdot k$$

$$B' = k^2 - 2p_f \cdot k \qquad (3-7)$$

and a corresponds to the shielding of the coulomb potential. The diagram (g) contains two W boson propagaters, i.e.,

$$\frac{-g_{\mu\alpha} + \frac{(p-q_1)_\mu (p-q_1)_\alpha}{Mw^2}}{(p-q_1)^2 - M_w^2} \gamma_o \frac{-q_{\alpha\nu} + \frac{(p+k-q_1)_\alpha (p+k-q_1)_\nu}{Mw^2}}{(p+k-q_1)^2 - Mw^2} \qquad (3-8)$$

But, the astrophysical electron energy is always less than 100 MeV and $M_w \geq 37.3$ GeV/c^2. The contribution of diagram (g) is less than the rest of the diagrams by the factor of p^2/M_w^2 which is very small. Therefore, we will neglect the contribution of the diagram (g) altogether.

Now the energy loss rate is

$$\mathscr{E} = \int (E_i - E_f) f(E_i) (1 - f(E_f)) \frac{1}{(2\pi)^{15}} \frac{1}{2} |\mathscr{M}|^2 d^3k \frac{d^3p_i d^3p_f}{2E_i 2E_f} (2\pi)^4 \delta^4 (p_i + k - p_f - q_1 - q_2) \frac{d^3q_1}{2q_{10}} \frac{d^3q_2}{2q_{20}} \qquad (3-9)$$

where $f(E_i)$ is the occupation number density of the electron at the energy E_i and $1 - f(E_f)$ is the empty number density at E_f, and

$$|\mathcal{M}|^2 = \frac{(\frac{G}{\sqrt{2}})^2(4\pi e^2)z^2}{(k^2+a^2)^2} T^{\mu\nu}t_{\mu\nu} \equiv \frac{(\frac{G}{\sqrt{2}})^2(4\pi e^2)z^2}{(k^2+a^2)^2}|M|^2 \tag{3-10}$$

where $T^{\mu\nu}$ is electron part trace tensor and $t_{\mu\nu}$ is the neutrino part trace tensor that appear when we square the matrix elements and sum over the final polarization of the electrons and neutrinos and average over the initial electron polarizations. To evaluate Eq (3-9), first we make a change of variables such that $Q = q_1 + q_2$, $q_1 = q_1$, and rewrite the last three integrals in equation (3-9) as follows.

$$I = \int d^3k \frac{d^3q_1}{2q_{10}} \frac{d^3q_2}{2q_{20}} \delta^4(p_i+k-p_f-q_1-q_2)$$

$$= \int d^4k\delta(k_0)d^4q_1d^4q_2\delta(q_1^2)\theta(q_{10})\delta(q_2^2)\theta(q_{20})$$

$$\delta^4(p_i+k-q_1-q_2).$$

After d^4k integration we have

$$I = \int \delta(E_i-E_f-Q_0)\, d^4Q \frac{d^3q_1}{2q_{10}} \delta((Q-q_1)^2)\theta(Q_0-q_{10})\theta(q_{10}).$$

It is easy to see that, as in Appendix I, with $t_{\mu\nu}$ in page 60,

$$\int \frac{d^3q_1}{2q_{10}}\ \delta(Q-q_1)^2)\,\theta(q_{10})\,\theta(Q_0-q_{10})\,t_{\mu\nu} = \frac{4\pi}{3}(Q_\mu Q_\nu - g_{\mu\nu}Q^2) \tag{3-11}$$

The $T^{\mu\nu}$ is

$$\begin{aligned}
T^{\mu\nu} = 2x\Big[& \{p_f{}^\mu p_f{}^\nu + p_f{}^\nu p_f{}^\mu - g^{\mu\nu} p_f\cdot p_f\} \\
& \{\frac{4E_i{}^2+2(m^2-p_i\cdot p_f)-2p_i\cdot Q}{B^2} + \frac{2m^2-2p_i\cdot Q}{BB'}\} \\
& + \{p_i{}^\mu p_i{}^\nu + p_i{}^\nu p_i{}^\mu - g^{\mu\nu} p_i\cdot p_i\} \\
& \{\frac{4E_f{}^2+2(m^2-p_i\cdot p_f)+2p_f\cdot Q}{B'^2} + \frac{2m^2+2p_f\cdot Q}{BB'}\} \\
& + \{p_f{}^\mu p_i{}^\nu + p_f{}^\nu p_i{}^\mu - g^{\mu\nu} p_i\cdot p_f\}\{\frac{2}{B} + \frac{2}{B'} + \frac{8E_iE_f-4p_i\cdot p_f}{BB'}\} \\
& + \{p_f{}^\mu Q^\nu + p_f{}^\nu Q^\mu - g^{\mu\nu} p_f\cdot Q\} \\
& \{\frac{4E_i{}^2+2(m^2-p_i\cdot p_f)-2p_i\cdot Q}{B^2} + \frac{2m^2-2p_i\cdot Q+2p_f\cdot p_i-4E_iE_f}{BB'}\} \\
& + \{p_i{}^\mu Q^\nu + p_i{}^\nu Q^\mu - g^{\mu\nu} p_i\cdot Q\} \\
& \{-\frac{4E_f{}^2+2(m^2-p_i\cdot p_f)+2p_f\cdot Q}{B^2} \\
& -\frac{2m^2+2p_f\cdot Q+2p_i\cdot p_f-4E_iE_f}{BB'}\} \\
& + \{p_f{}^\mu g^{\nu o} + p_f{}^\nu g^{\mu o} - g^{\mu\nu} E_f\}\{-\frac{4E_i}{B}\} \\
& + \{p_i{}^\mu g^{\nu o} + p_i{}^\nu g^{\mu o} - g^{\mu\nu} E_i\}\{-\frac{4E_f}{B'}\} \\
& + \{Q^\mu Q^\nu + Q^\nu Q^\mu - g^{\mu\nu} Q^2\}\{\frac{2p_i\cdot p_f-4E_iE_f}{BB'}\}
\end{aligned}$$

$$+ \{Q^{\mu}g^{\nu o}+Q^{\nu}g^{\mu o}-g^{\mu\nu}Q_o\}$$

$$\{\frac{4(E_i-E_f)(m^2-p_i\cdot p_f)+4E_f p_i\cdot Q+4E_i p_f\cdot Q}{BB'}\}$$

$$+g^{\mu o}g^{\nu o}\frac{4p_i\cdot p_f\{2(m^2-p_i\cdot p_f)+Q^2+2Q\cdot(p_f-p_i)}{BB'}$$

$$\{\frac{+2\{2(m^2-p_i\cdot p_f)-2p_i\cdot Q\}\{2(m^2-p_i\cdot p_f)+2p_f\cdot Q\}}{BB'}\}$$

$$+ g^{\mu\nu}\frac{(2p_i\cdot p_f-4E_iE_f)\{2(m^2-p_i\cdot p_f)+Q^2+2Q\cdot(p_f-p_i)\}}{BB'}\Big]$$

$$+ 2m^2Z\Big[g^{\mu\nu}\frac{8E_iE_f+4(m^2-p_i\cdot p_f)+2Q^2+4Q\cdot(p_f-p_i)}{BB'}$$

$$+ g^{\mu\nu}\frac{4E_i^2+2(m^2-p_i\cdot p_f)+Q^2+2Q\cdot(p_f-p_i)}{B^2}$$

$$+ g^{\mu\nu}\frac{4E_f^2+2(m^2-p_i\cdot p_f)+Q^2+2Q\cdot(p_f-p_i)}{B'^2}$$

$$-g^{\mu o}g^{\nu o}\frac{8(m^2-p_i\cdot p_f)+4Q^2+8Q\cdot(p_f-p_i)}{BB'}$$

$$-\frac{4(p_f-p_i+Q)^{\mu}(p_f-p_i+Q)^{\nu}}{BB'}\Big]$$

where

$$x = |C_V|^2 + |C_A|^2,\quad z = |C_V|^2 - |C_A|^2. \tag{3-13}$$

The exact integral with respect to Q and the subsequent integrals are impossible to carry out analytically, but we observe that the distribution function integral can be expanded in the power series of the chemical potential ν such that, as in Appendix III,

$$\iint dE_i dE_f \; f(E_i)(1-f(E_f))E_i{}^r(E_i-E_f)^n$$

$$= \sum_{m=0}^{\infty} a_m \nu^{r-n-m} + \text{smaller terms} \qquad (3\text{-}14)$$

Therefore, if $\nu >> 1$, the higher order terms in the $Q_o = E_i - E_f$ expansion are negligible compared to the lower order terms. Thus we keep the terms in $|M|^2$ which has the power of Q less than 2. Then the Q integral would be of the form

$$\int (E_i-E_f)\, d^3Q(\alpha + \beta Q) = \alpha' Q_o{}^4 + \beta' Q_o{}^5 \qquad (3\text{-}15)$$

which would result in the final integral of the form $a + \frac{b}{\nu}$ as we will see later. After dropping higher order terms in Q, we finally have

$$|M|^2 = \frac{8\pi}{3} \times \Bigg[\frac{Q^2}{BB'} \{-2p_i\cdot p_f(-4p_i\cdot p_f+8E_iE_f)\}$$

$$+ \frac{m^2Q^2}{BB'} \{4(m^2-p_i\cdot p_f)+24E_iE_f-8m^2\}$$

$$+ \frac{m^2Q_o{}^2}{BB'} 8(m^2-p_i\cdot p_f) + \frac{m^2Q^2}{B^2}\{4E_i{}^2+2(m^2-p_i\cdot p_f)\}$$

$$+ \frac{m^2Q^2}{B'^2} \{4E_f{}^2 + 2(m^2-p_i\cdot p_f)\} + m^2\left(2+\frac{B'}{B}+\frac{B}{B'}\right)\Bigg]$$

$$+$$

$$+ \frac{8\pi}{3} Z \left[\frac{m^2Q^2}{BB'} \{-24E_iE_f + 4(m^2-p_i\cdot p_f)\} \right.$$

$$- \frac{m^2Q_o^2}{BB'} \{8(m^2-p_i\cdot p_f)\} - \frac{3m^2Q^2}{B^2} \{4E_i^2+2(m^2-p_i\cdot p_f)\}$$

$$\left. - \frac{3m^2Q^2}{B'^2} \{4E_f^2+2(m^2-p_i\cdot p_f)\} - m^2\left(2+\frac{B'}{B}+\frac{B}{B'}\right)\right] \tag{3-16}$$

where

$$B' = Q^2 - 2p_i\cdot Q, \quad B = Q^2 + 2p_f\cdot Q$$

Let I be

$$I \equiv \int |M|^2 d^3Q = \frac{8\pi^2}{3p_i^2} Q_o^3 [x(A_1+\frac{Q_o}{p_i} B_1)+z(A_2+\frac{Q_o}{p_i} B_2)],$$

then,

$$\mathscr{E} = \iint \frac{1}{2} \frac{G^2}{2} \frac{(4\pi e^2)Z^2(2\pi)^4}{(2\pi)^{15}} (E_i-E_f) I \frac{d^3p_f}{2E_i} \frac{d^3p_f}{2E_f}$$

$$f(E_i)(1-f(E_f)) \tag{3-17}$$

Now define

$$E_i' = \frac{E_i}{kT}, \; E_f' = \frac{E_f}{kT}, \; \nu = \frac{\epsilon_F}{kT} = \frac{\mu}{kT}, \; \lambda = \frac{kT}{mc^2}$$

and drop the prime from now on. Here m is the electron mass and μ is the chemical potential. In terms of these new variables the energy loss rate can be expressed as

$$\mathscr{E} = \frac{G^2Z^2e^4}{2} \frac{(kT)^6}{12\pi^5} \int_{\lambda^{-1}}^{\infty} dE_i \int_{\lambda^{-1}}^{dE_i} dE_f \, Q_o^4 [x(f_1+\frac{Q_o}{p_i} g_1)$$

$$+ Z(f_2+\frac{Q_o}{p_i} g_2)] \cdot f(E_i)(1-f(E_f)) \tag{3-18}$$

where

$$f_1 = A_1, \qquad f_2 = A_2, \qquad \beta \equiv \frac{E_i}{p_i},$$
$$g_1 = B_1 - \beta A_1, \qquad g_2 = B_2 - \beta A_2,$$

Here use is made of

$$\frac{p_f}{p_i} = (1 - \frac{Q_o}{p_i}\beta);$$

Explicit forms of the functions f_1, g_1, f_2, and g_2 are given by:

$$f_1 = -2h(R_{22} - 2R_{21}) - (\beta^2 - 1)hR_{21} - 2(h - \tfrac{1}{3})R_{21}$$

$$\begin{aligned} g_1 = {} & 6\beta h(R_{22} - 2R_{21}) - 4\beta h(R_{33} - 2R_{32}) \\ & + \beta(\beta^2 - 1)hR_{21} + 4\beta(h - \tfrac{1}{3})R_{21} \\ & - \{(\frac{5\beta^2 - 1}{\beta})h - \frac{2}{3\beta}\}\{R_{22} - 2R_{21}\} \\ & - 4\beta(h - \tfrac{1}{3})R_{32} - 2\beta(\beta^2 - 1)hR_{32} \\ & - \frac{R_{21}}{2}(\beta^2 + 1)\{(\frac{3\beta^2 - 1}{\beta})(h - \tfrac{1}{3}) + \frac{\beta^2 - 1}{\beta}\} \end{aligned}$$

$$f_2 = 5(\beta^2 - 1)hR_{21} + 2(h - \tfrac{1}{3})R_{21}$$

$$\begin{aligned} g_2 = {} & -9\beta(\beta^2 - 1)hR_{21} - 4\beta(h - \tfrac{1}{3})R_{21} \\ & + 4(\beta^2 - 1)\{(\frac{5\beta^2 - 1}{2\beta})h - \frac{1}{3\beta}\}R_{21} \\ & + \frac{(\beta^2 + 1)(3\beta^2 - 1)}{2\beta}(h - \tfrac{1}{3})R_{21} + \beta(\beta^2 - 1)R_{21} \\ & + 10\beta(\beta^2 - 1)hR_{32} + 4\beta(h - \tfrac{1}{3})R_{32} \end{aligned}$$

where

$$h = -(\beta^2 - 1)\left\{\frac{\beta}{2}\ln\frac{\beta+1}{\beta-1} - 1\right\} + \frac{1}{3},$$

$$\alpha \equiv \frac{a^2}{2p_i^2} = \frac{\beta}{215},$$

$$R_{20} = \frac{2}{\alpha(2+\alpha)},$$

$$R_{21} = \ln\frac{2+\alpha}{\alpha} - \frac{2}{\alpha+2},$$

$$R_{22} = 2 - 2\alpha\ln\frac{2+\alpha}{\alpha} + \frac{2\alpha}{2+\alpha},$$

$$R_{30} = \frac{1}{2\alpha^2} - \frac{1}{2(2+\alpha)^2},$$

$$R_{31} = \frac{1}{2\alpha} - \frac{1}{2+\alpha} + \frac{\alpha}{2(2+\alpha)},$$

$$R_{32} = \ln\frac{2+\alpha}{\alpha} - \frac{3}{2} + \frac{2\alpha}{2+\alpha} - \frac{\alpha^2}{2(2+\alpha)^2},$$

$$R_{33} = 2 - \frac{5}{2}\alpha - 3\alpha\ln\frac{2+\alpha}{\alpha} - \frac{3\alpha^2}{(2+\alpha)} + \frac{\alpha^3}{2(2+\alpha)^2}.$$

Finally, if $\nu - \lambda^{-1} >> 1$, then the distribution integral is given by

$$\int_{\lambda^{-1}}^{\infty} dE_i \int_{\lambda^{-1}}^{E_i} dE_f \left[f_1 Q_o^4 + \frac{Q_o^5}{p_i} g_1\right] \frac{1}{\left(1 + e^{\frac{E_i - \mu}{kT}}\right)} \left(1 - \frac{1}{1 + e^{\frac{E_i - \mu}{kT}}}\right) \equiv J, \qquad (3\text{-}21)$$

$$J = 4![\{2\eta(6) + 4\eta(2)\eta(4)\}f_1(\nu) + \{5\eta(6) + 8\eta(2)\eta(5) + 4\eta(3)\eta(4) + \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}(-1)^{m+n}\frac{2}{\eta^5(m+n)}\}f_1'(\nu)]$$

$$+ 5![21n2\eta(6) + 2\eta(2)\eta(5) + 2\eta(3)\eta(4)]g_1(\nu)$$

$$\approx 122.07\{f_1(\nu) + 2.97\, f'_1(\nu) + \frac{4.59}{p_i}\, g_1(\nu)\} \qquad (3\text{-}22)$$

where $\eta(x)$ is Rieman's eta function,

$$\eta(x) = \sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n^x}$$

and

$$f'_1 \equiv \left.\frac{df_1}{dE_i}\right|_{E_i=\nu} = \frac{1}{p_i}\Big[\{(\frac{3\beta^2-1}{\beta})(h-\tfrac{1}{3}) + \frac{\beta^2-1}{\beta}\}2(R_{22}-2R_{21})$$

$$+ 2h(\frac{\beta^2-1}{\beta})\{R_{22} + \frac{12}{2+\alpha} - 4\} + 2\beta(\beta^2-1)hR_{21}$$

$$+ (\beta^2+1)\{\frac{(3\beta^2-1)}{\beta}(h-\tfrac{1}{3}) + (\frac{\beta^2-1}{\beta})\}R_{21}$$

$$- 4\frac{(\beta^2-1)^2}{\beta}\frac{1}{(2+\alpha)^2}h$$

$$- 8\frac{(\beta^2-1)}{\beta}(h-\tfrac{1}{3})\frac{1}{(2+\alpha)^2}\Big]$$

$$f'_2 \equiv \left.\frac{df_2}{dE_i}\right|_{E_i=\nu} = \frac{1}{p_i}\Big[\frac{20(\beta^2-1)h}{(2+\alpha)^2\beta} + 8(h-\tfrac{1}{3})(\frac{\beta^2-1}{\beta})\frac{1}{(2+\alpha)^2}$$

$$- 5(\beta^2-1)R_{21}\{\frac{(3\beta^2-1)}{\beta}(h-\tfrac{1}{3}) + \frac{\beta^2-1}{\beta}\}$$

$$-2\{(\frac{3\beta^2-1}{\beta})(h-\tfrac{1}{3}) + \frac{\beta^2-1}{\beta}R_{21}\}\Big].$$

In fact integral (3-21) contains many other terms which we have neglected here. For example, the term

$$\frac{(\nu - \lambda^{-1})^5}{5} \ln 2 \ln(1 + e^{-(\nu-\lambda^{-1})}) \qquad (3\text{-}23')$$

has its maximum at $\nu - \lambda^{-1} = 5$ with the numerical value 2.9 which is much smaller than 122. But, when $\nu - \lambda^{-1} \approx 0$, we have to add the following terms to equation (3-18).

$$4! f_1(\nu)\{-4\eta(2)\eta(4) - 2\ln 2\eta(5) + 2\eta(2)\eta(4) - (\eta(3))^2$$

$$+ \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} (-1)^{m+n} \frac{1}{n^5(m+n)} - \eta(6)\}$$

$$+ 4! f_1'(\nu)\{- 5\ln 2\eta(6) + 3\eta(5)\eta(2) - \eta(3)\eta(4)$$

$$- 4\eta(2)\eta(5) - \ln 2\eta(5) - 2\eta(2)\eta(4) - \ln 2\eta(5)$$

$$- \eta(7) + 5\ln 2\eta(6) - 3\eta(2)\eta(5)$$

$$- \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} (-1)^{m+n} \frac{1}{n^6(n+m)^2} \}$$

$$+ 5! g_1(\nu)\Big\{- 2\ln 2\eta(6) - 2\eta(3)\eta(4) - 2\eta(2)\eta(5)$$

$$- \eta(7) + \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} (-1)^{m+n} \frac{1}{n^6(m+n)} \Big\} .$$

Similarly we can obtain the distribution integral for f_2 and g_2, simply by replacing $f_1(g_1)$ in Equations (3-22) and (3-24) by $f_2(g_2)$.

Discussion and Conclusions

Consider the extreme relativistic degenerate region, first neglecting the second order contributions in Q_o expansion, i.e., put $g_1 = g_2 = f_1' = f_2' = 0$. Then, the energy loss rate is

$$\mathcal{E} = \frac{G^2Z^2e^4}{2}\frac{(kT)^6}{12\pi^5}\,4!\{2\eta(6) + 4\eta(2)\eta(4)\}(xf_1 + zf_2)$$

$$= G^2Z^2e^4\,\frac{2(kT)^6}{\pi^5}\{2\eta(6) + 4\eta(2)\eta(4)\}$$

$$\left[\frac{(C_V^2 + C_A^2)}{2}\{2h(2R_{21} - R_{22}) - (\beta^2-1)hR_{21} - 2(h - \tfrac{1}{3})R_{21}\} + \frac{(C_V^2 - C_A^2)}{2}\{5(\beta^2-1)hR_{21} + 2(h - \tfrac{1}{3})R_{21}\}\right] \qquad (3\text{-}25)$$

This result is the same as that of Ruderman and Festa if $C_V = C_A = 1$ and

$$4!\{2\,\eta(6) + 4\eta(2)\eta(4)\} = 122.07$$

is replaced by 120. Recall that f_1, f_2, g_1, g_2 come from the electron neutrino emission diagrams which correspond

to the matrix element M_{abef} in Equation (3-5). If we include the muon neutrino emission, i.e., matrix element M_{cd} in Equation (3-6), we need the following replacement:

$$\frac{(C_V^2 + C_A^2)}{2} \rightarrow \frac{(2C_V^2 + 2C_A^2 - 2C_V + 1)}{2} \xrightarrow[C_V=C_A=1]{} \frac{3}{2}$$

$$\frac{(C_V^2 - C_A^2)}{2} \rightarrow \frac{(2C_V^2 - 2C_A^2 - 2C_V + 1)}{2} \xrightarrow[C_V=C_A=1]{} -\frac{1}{2} \qquad (3\text{-}26)$$

Notice that the coefficients in Equation (3-26) do not return to the V-A theory value even if we put $C_V = C_A = 1$, because the Weinberg theory opens a new Z vector meson annihilation channel which is absent in the conventional V-A theory of weak interactions. Now to have the emission rate per gram rather than per atom with atomic number Z and mass number A, we divide Equation (3-25) by AM_p to obtain $\mathscr{E}' = \frac{\mathscr{E}}{AM_p}$. If ν goes to infinity, $\beta^2 - 1 = 0$, which is the infinite density limit, $h - \frac{1}{3} = 0, 2R_{21} - R_{22} = 8$. Therefore,

$$\mathscr{E}' = \frac{(2C_V^2 + 2C_A^2 - 2C_V + 1)}{2} \cdot 0.285 \frac{Z^2}{A} \left(\frac{T}{10^{8\circ}K}\right)^6 \frac{8}{3} \text{ erg/gm-sec.}$$

$$= \frac{(2C_V^2 + 2C_A^2 - 2C_V + 1)}{2} 0.76 \frac{Z^2}{A} \left(\frac{T}{10^{8\circ}K}\right)^6 \text{ erg/gm-sec} \qquad (3\text{-}27)$$

Thus the ratio R of the energy loss rate per gram

predicted by the Weinberg theory to the conventional one is:

$$R = \frac{(2C_V^2 + 2C_A^2 - 2C_V + 1)}{2} = \frac{1}{2} + 4\sin^4\theta w \quad (3\text{-}28)$$

It has been shown that the ratio R is, respectively

$$\frac{1}{2} + 4\sin^4\theta w$$

for pair annihilation neutrino emission at the relativistic region and photoneutrino emission at the relativistic region,

$$\frac{1}{2} + 8\sin^4\theta w$$

for pair annihilation at non-relativistic non-degenerate region and plasma neutrino emission, and

$$\frac{1}{6}\ (3 + 8\sin^4\theta w)$$

for the photoneutrino emission at non-relativistic region. We quote in Figure 3 the plot of R as a function of the intermediate meson mass. As we can see from the plot, Weinberg theory elevates the importance of the plasma process and diminishes that of the non-relativistic photoneutrino process against the bremsstrahlung neutrino emission. As for the non-degenerate non-relativistic neutrino bremsstrahlung reaction, we can easily see from the matrix element (16) and Reference [7] that the ratio is also

$$\frac{1}{2} + 4\ \sin^4\theta w\ .$$

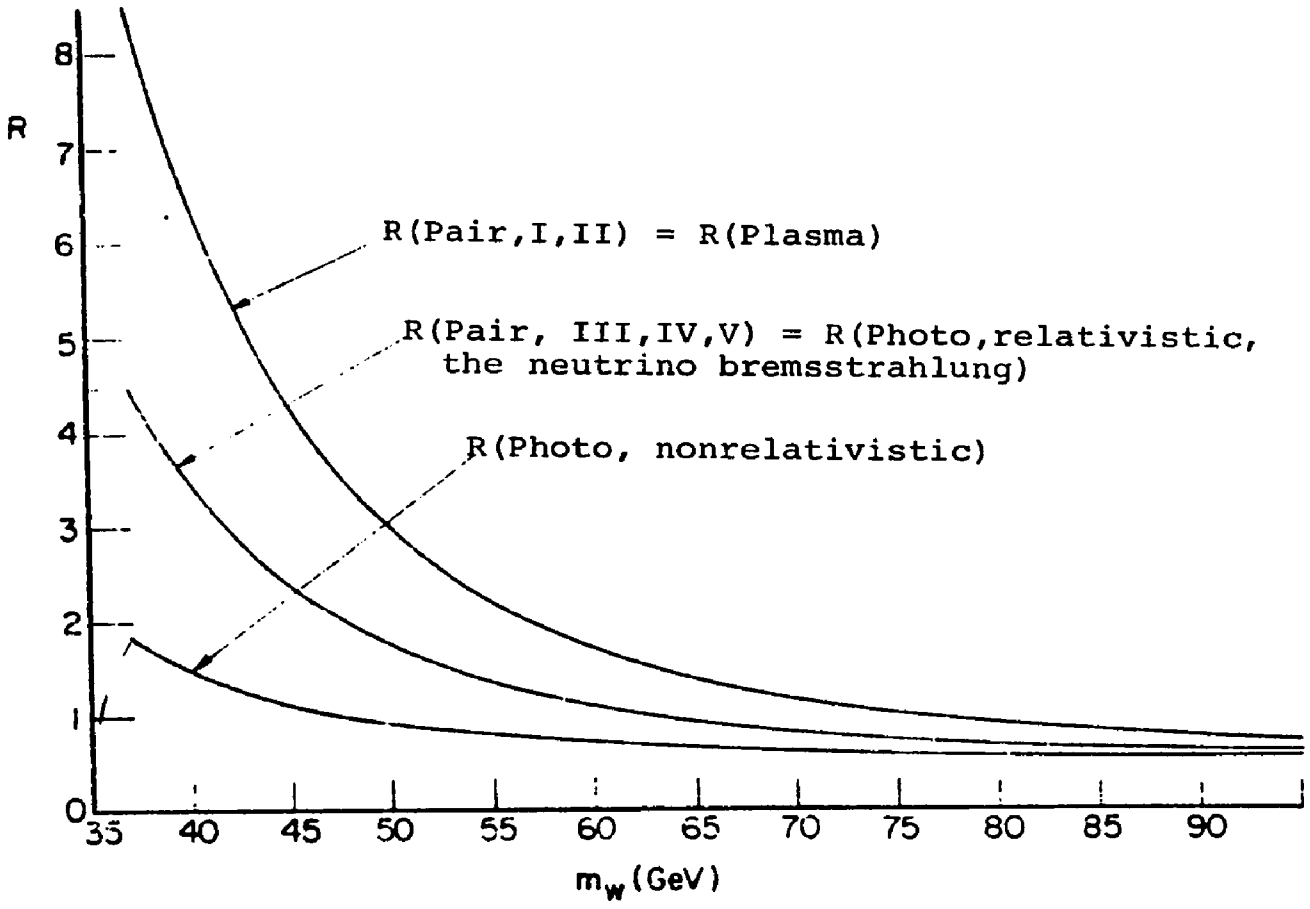


Figure 3. The ratios of the energy-loss rates as calculated here to the energy-loss rates calculated in the ordinary theory vs the mass of the W meson. R (Pair; I,II) is the ratio for the pair annihilation in the regions I and II discussed in the text. All three lines approach R = 1/2 as m_W approaches ∞. Adapted from Duane Dicus, Phys. Rev. D 6, 941 (1972).

In general when one applies the energy loss rate to the astrophysical problems, one expresses this rate as a function of $\frac{\rho}{\mu e}$, ν, and T, unlike the rates given here as a function of β, ν, and T. Customarily, the conversion is done as the following: As the star we have in mind is in the thermodynamic equilibrium with temperature T and density ρ and it is fully ionized neutral gas which consists of n_i atoms with atomic number Z_i and mass number A_i, we define the abundance in weight X_i of the atomic number Z_i. Then the proton number density in weight $1/\mu e$ is

$$X_i = \frac{n_i A_i}{\sum_j n_j A_j}$$

$$\frac{1}{\mu_e} \frac{\sum_i n_i Z_i}{\sum_j n_j A_j} = \sum_i \frac{Z_i X_i}{A_i} \tag{3-29}$$

Thus we have

$$\frac{N_p}{V} = \sum_i \frac{n_i Z_i}{V} = \frac{\sum_j n_j A_j}{V \mu_e} = N \frac{\rho}{\mu_e}$$

where

$$\frac{\rho}{\mu_e} = C(G_o^- - G_o^+)$$

$$C = \frac{1}{\pi^2} \left(\frac{mc^2}{\hbar}\right)^2 \frac{1}{N} = 2.922 \times 10^6 \text{ gm/cm}^3 \tag{3-30}$$

and

$$G_n^{\pm} = \lambda^{3+2n} \int_{\lambda^{-1}}^{\infty} \frac{\lambda^{2n+1} (x^2 - \lambda^{-2})^{1/2}}{1 + e^{x-\nu}} dx \tag{3-31}$$

Unfortunately the integral in Equation (3-31) cannot be done analytically, except for the certain extreme T and ν regions. A plot obtained from the numerical evaluation and some of the analytic form are given in Ref. [9], such as

$$\lambda\nu = \frac{E_F}{mc^2} = \left(\frac{\frac{\rho}{\mu_e}}{0.974 \times 10^6 \text{ gm/cm}^3} \right)^{1/3}$$

for extreme relativistic degeneracy, i.e., $\nu >> \lambda^{-1} >> 1$ and

$$\lambda\nu = \frac{E_F}{mc^2} = \left(\frac{\frac{\rho}{\mu_e}}{3.425 \times 10^6 \text{ gm/cm}^3} \right)^{2/3}$$

for non-relativistic degeneracy, i.e., $\lambda << \lambda\nu - 1$. These formulae applied in the border region in ν and λ would introduce large error in the correct energy loss rate in the process of conversion. Therefore we would not attempt to convert the bremsstrahlung rate analytically from β, ν, T to ρ/μ_e, ν, T. Now the higher order corrections come from two sources, one from the higher order terms in Q_o and the other from the distribution integral for a given Q_o. A result of the former correction is shown in Table III. The corrections are within 5 percent for all the densities considered. This result is, in fact, expected, because as we can see from equations (3-19) and (3-23), all the higher order terms are proportional to either

TABLE III

Contribution of the Higher Order Terms at $T=10^8 K$

ρ/μ_e	ν	p	f_1	C*	C/f_1
10^5	64.49	23.66	2.363	-0.095	-4.0%
$5x10^5$	70.41	36.44	2.657	-0.087	-3.3%
10^6	84.83	59.97	3.223	-0.120	-3.7%
10^7	124.52	109.11	4.040	-0.171	-4.2%
10^8	268.27	261.47	4.952	-0.151	-3.0%

*$C = (2.97\ f'_1 + 4.59/pi)$

$f_.$, f_1', and g_1 are defined in the text.

$$\frac{(\beta^2 - 1)}{p_i} \quad \text{or} \quad \frac{1}{p_i}(h - \frac{1}{3})$$

and $p_i = \sqrt{\nu^2 - \lambda^{-2}}$ are from 23.6 to 261.5. The corrections due to the distribution integral have many terms. However, if $\nu - \lambda^{-1} >> 1$, practically all the other terms are negligible compared to the leading constant term. However, for the small values of $\nu - \lambda^{-1}$, especially when $\nu - \lambda^{-1} \simeq 0$, the change in the constant term is large and we need the correction of the form $\frac{}{e}(\nu-\lambda^{-1})$ as we can see from equations (3-23), (3-24) and Appendix III.

In conclusion, the neutrino bremsstrahlung energy loss rate in a unified theory of weak and electromagnetic interactions with neutral current is proportional to

$$\frac{1}{2} + 4\sin^4\theta w$$

in both extreme relativistic degenerate and non-relativistic non-degenerate regions. The energy loss rate is proportional to T^6 and independent of the density at the extreme degeneracy, which makes it dominate other processes. The asymptotic expression for the energy loss rate is of the form

$$A(\beta) + (\frac{\rho_e}{\mu})^{-1/3} kTB(\beta).$$

If the Weinberg angle were determined experimentally and the structure of the neutral current were known, we would have a definite ratio R numerically. However, even

if the Weinberg theory were proved incorrect, still the neutral current would play an indispensable role in the future astrophysical investigations.

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APPENDIX I

CHANGE OF VARIABLES IN EQUATION (3-11)

When we change the variables such that $q_1 + q_2 = Q$, $q_1 = q_1$, we have to evaluate the following form of integral:

$$I = \int \frac{d^3q_1}{2q_{10}} \delta(Q - q_1)^2)\theta(Q_o - q_{10})\theta(q_o) F(A\cdot q_1\ldots).$$

Taking $\vec{Q}$ as z axis, we have

$$I = \int_{\frac{Q_o-|Q|}{2}}^{\frac{Q_o+|Q|}{2}} \frac{q_{10}{}^2 dq_{10}}{2q_{10}} \int_0^{2\pi} \frac{d\phi}{2|Q|q_{10}}$$

$$F(A\cdot q_1\ldots)\Big|_{\cos\theta_{q_1} = \frac{2Q_o q_{10} - Q^2}{2|Q|q_{10}}}$$

Then,

$$\int \frac{d^3q_1}{2q_{10}} \delta((Q-q_1)^2)\,\theta(q_{10})\,\theta(Q_o-q_{10})q_{1\mu} = \frac{\pi}{4} Q\mu$$

$$\int \frac{d^3q_1}{2q_{10}} \delta((Q-q_1)^2\theta(q_{10})\,\theta(Q_o-q_{10})q_{1\mu}q_{1\nu}$$

$$= \frac{\pi}{24}[2Q_\mu Q_\nu + g_{\mu\nu}Q^2]$$

Because $t_{\mu\nu}$ is

$$t_{\mu\nu} = 4\{(q_{1\mu}q_{2\nu} + q_{1\nu}q_{2\mu} - g_{\mu\nu}\, q_1\cdot q_2 - i\varepsilon_{\mu\nu\alpha\beta}q_{1\alpha}q_{2\beta}\}$$

$$= 4\{q_{1\mu}Q_\nu + q_{1\nu}Q_\mu - g_{\mu\nu}Q\cdot q_1 - i\varepsilon_{\mu\nu\alpha\beta}q_{1\alpha}Q_\beta$$

$$-2q_{1\mu}q_{1\nu} + i\varepsilon_{\mu\nu\alpha\beta}q_{1\alpha}q_{1\beta}\}$$

$$\int\frac{d^3q_1}{2q_{10}}\ \delta((Q-q_1)^2)\,\theta(q_{10})\theta(Q_o-q_{10})\,t_{\mu\nu} = \frac{4}{3}\pi(Q_\mu Q_\nu - g_{\mu\nu}Q^2).$$

APPENDIX II

THE $\vec{Q}$ INTEGRATION

A. We expand $Q^2/B^2(\quad)^2$ and $Q^2/B'^2(\quad)^2$ in the following way:

$$\frac{Q^2}{B^2(\ )^2} = \frac{Q^2}{(2p_f\cdot Q)^2p^2} - \frac{2Q^4}{p^2(2p_f\cdot Q)^3} + \frac{2Q^2}{p^3(2p_f\cdot Q)} - \frac{4(p_i\cdot Q)Q^2}{p^3(2p_f\cdot Q)^2} \qquad \text{II-1}$$

$$\frac{Q^2}{B'^2(\ )^2} = \frac{Q^2}{(2p_i\cdot Q)^2p^2} + \frac{2Q^4}{p^2(2p_i\cdot Q)^3} - \frac{2Q^2}{p^3(2p_i\cdot Q)} + \frac{4(p_f\cdot Q)\ Q^2}{p^3(2p_i\cdot Q)^2} \qquad \text{II-2}$$

where

$$\frac{1}{(\ )^2} = \frac{1}{\{|\overrightarrow{p_f-p_i}|^2 + a^2 + 2(\overrightarrow{p_f-p_i})\cdot\vec{Q} + |\vec{Q}|^2\}^2}$$

$$\frac{1}{p^2} = \frac{1}{(|\overrightarrow{p_f-p_i}|^2 + a^2)^2}$$

In our approximation we can put $p_i = p_f$ whenever it is possible in the last three terms in II-1 and II-2. In this approximation scheme, after angle integration, we find that, in effect,

$$\frac{Q^2}{B^2(\ )^2} = \frac{Q^2}{(2p_i\cdot Q)^2 p^2} + \frac{2Q^4}{p^2(2p_i\cdot Q)^3} + \frac{2Q^2}{p^3(2p_i\cdot Q)}$$

$$- \frac{4(p_f\cdot Q)Q^2}{p^3(2p_i\cdot Q)^2} \qquad \text{II-3}$$

because

$$\int \frac{Q^2 d\Omega}{(p_f\cdot Q)^2} = \frac{4\pi Q^2}{(E_f{}^2 Q^2 - p_f{}^2|Q|^2)} \simeq \frac{4\pi Q^2}{(E_i{}^2 Q_o{}^2 - p_i{}^2|Q|^2)}$$

$$+ \frac{4\pi 2E_i Q_o Q^4}{(E_i{}^2 Q_o{}^2 - p_i{}^2|Q|^2)^2}$$

This implies that

$$\frac{Q^2}{(p_f\cdot Q)^2} \approx \frac{Q^2}{(p_i\cdot Q)^2} + \frac{2Q^4}{(p_i\cdot Q)^3}$$

From the following formula

$$\int \frac{Q^2 d^3Q}{4(p_i\cdot Q)^2} = \frac{\pi}{p_i{}^2} Q_o{}^3 h$$

$$\int \frac{Q^4 d^3Q}{4(p_i\cdot Q)^3} = \frac{\pi}{p_i{}^2}\frac{Q_o{}^4}{p_i}\left\{\left(\frac{5\beta^2-1}{2\beta}\right)h - \frac{1}{3\beta}\right\}$$

$$\int \frac{Q^2 d^3Q}{(p_i\cdot Q)} = \pi\frac{Q_o{}^4}{p_i}\left\{-\left(\frac{\beta^2-1}{\beta}\right)h + \frac{2}{3\beta}\right\}$$

$$\int \frac{(p_f \cdot Q) Q^2 d^3Q}{(p_i \cdot Q)^2} = \pi \frac{Q_o^4}{p_i} \left[(1-z) \left\{ \left(\frac{5\beta^2-1}{\beta} \right) h - \frac{2}{3\beta} \right\} \right.$$

$$\left. + \left\{ - \left(\frac{\beta^2-1}{\beta} \right) h + \frac{2}{3\beta} \right\} \right] \qquad \text{II-4}$$

and with

$$\frac{1}{p^2} \sim \frac{1}{4p_i^4} \left[\frac{1}{(y+\alpha)^2} + \frac{Q_o}{p_i} \frac{2\beta y}{(y+\alpha)^3} \right]$$

$$\frac{1}{p^3} \sim \frac{1}{8p_i^6} \frac{1}{(y+\alpha)^3}$$

where

$$y \equiv 1 - z \equiv 1 - \cos\theta_{p_f},$$

$$\beta \equiv \frac{E_i}{p_i}$$

$$\alpha \equiv \frac{a^2}{2p_i^2}$$

$$m^2 = p_i^2 (\beta^2 - 1)$$

$$h \equiv -(\beta^2-1) \left\{ \frac{\beta}{2} \ln \frac{\beta + 1}{\beta - 1} - 1 \right\} + \frac{1}{3} .$$

We have from Equations II-2, II-3, and II-4,

$$\int \frac{Q^2 d^3Q}{B^2 (\ )^2} = \frac{\pi}{p_i^2} \frac{Q_o^3}{4p_i^4} \left[\frac{h}{(y+\alpha)^2} + \frac{Q_o}{p_i} \left\{ \frac{1}{(y+\alpha)^2} \right. \right.$$

$$\left. \left. \left\{ \left(\frac{5\beta^2-1}{2\beta} \right) h - \frac{1}{3\beta} \right\} - \frac{y}{(y+\alpha)^3} \left\{ \left(\frac{\beta^2-1}{2\beta} \right) - \frac{1}{3\beta} \right\} \right\} \right]$$

and

$$\int \frac{Q^2 d^3Q}{B'^2(\ )^2} = \frac{\pi}{p_i^2} \frac{Q_o^3}{4p_i^4} \left[\frac{h}{(y+\alpha)^2} + \frac{Q_o}{p_i} \left\{ \frac{1}{(y+\alpha)^2} \right.\right.$$

$$\left.\left. \{ (\frac{5\beta^2-1}{2\beta})h - \frac{1}{3}\beta \} + \frac{y}{(y+\alpha)^3} \{ (\frac{9\beta^2-1}{2\beta})h - \frac{1}{3\beta} \} \right\} \right] \qquad \text{II-5}$$

B. Using Feynman's formula and after an appropriate expansion we find that

$$\frac{1}{ab} = \int_0^1 \frac{dx}{[a + x(b-a)]^2}$$

$$\frac{1}{BB'(\ )^2} = - \frac{1}{(2p_i \cdot Q)(2p_f \cdot Q)p^2}$$

$$\int \frac{d\Omega}{(p_i \cdot Q)(p_f \cdot Q)} = 4\pi \int_0^1 dx\ N^{-1}$$

where

$$N \equiv (E_i^2 Q_o^2 - p_i^2 |Q|^2) + 2x \{ -E_i Q_o^3 + |Q|^2 p_i^2 (1-z)$$

$$+ E_i Q_o z |Q|^2 \} + x^2 \{ Q_o^4 - |Q|^2 2p_i^2 (1-z)$$

$$+ |Q|^2 2E_i Q_o (1-z) \} .$$

If we expand N^{-1} around $(E_i^2 Q_o^2 - p_i^2 |Q|^2)$, we will have infinite number of terms with the same power of Q and the

worst thing is that the $\vec{Q}$ integration of the terms proportional to (1-z) diverges when $E_i = p_i$ limit is taken. However, we know that in the relativistic region only the forward scattering is important. Therefore, putting z = 1 would not incur great error. In fact, the authors in Reference [8] performed numerical integration to find that the error is less than 5 percent. So we put z = 1 and expand to find that

$$\frac{1}{(p_i\cdot Q)(p_f\cdot Q)} \approx \frac{1}{(p_i\cdot Q)^2} + \frac{Q^2}{(p_i\cdot Q)^3} \qquad \text{II-6}$$

Thus we have

$$\int \frac{Q^2 d^3Q}{BB'(\ )^2} \approx -\frac{\pi}{p_i^2} Q_o^3 \left[h + \frac{Q_o}{p_i}\left\{\left(\frac{5\beta^2-1}{2\beta}\right)h - \frac{1}{3\beta}\right\}\right]$$

$$\cdot \left[\frac{1}{4p_i^4}\left\{\frac{1}{(y+\alpha)^2} + \frac{Q_o}{p_i}\ \frac{2\beta y}{(y+\alpha)^3}\ \right\}\right]$$

and

$$\int \frac{Q_o^2 d^3Q}{BB'} \approx \frac{\pi}{p_i^2}\frac{Q_o^3}{(\beta^2-1)}\left[(h-\tfrac{1}{3}) + \frac{Q_o}{p_i}\left\{\left(\frac{3\beta^2-1}{2\beta}\right)(h-\tfrac{1}{3})\right.\right.$$

$$\left.\left. + \frac{\beta^2-1}{2\beta}\right\}\right]\cdot\left[\frac{1}{4p_i^4}\left\{\frac{1}{(y+\alpha)^2} + \frac{Q_o 2\beta y}{p_i(y+\alpha)^3}\ \right\}\right]$$

We find that the second term in II-6 is necessary to make this approximation scheme reasonably good, i.e., R_{20} and R_{31} terms are absent in f_1, f_2, g_1, g_2.

C.

$$\frac{B'}{B} + \frac{B}{B'} \simeq - \left(\frac{p_f \cdot Q}{p_i \cdot Q} + \frac{p_i \cdot Q}{p_f \cdot Q}\right) \frac{1}{p2}$$

$$\int \frac{d^3Q}{p^2} \left(\frac{B'}{B} + \frac{B}{B'}\right) = - \frac{\pi}{p_i^4} \left[\{hy + \frac{2}{3}\} \right.$$

$$+ \frac{Q_o}{p_i} \left\{ \frac{y}{(y+\alpha)^2} \left\{ \left(\frac{3\beta^2-1}{2\beta}\right) \left(h - \frac{1}{3}\right) + \frac{\beta^2-1}{\beta} \right\} \right.$$

$$\left. \left. + \frac{1}{(y+\alpha)^3} \{2\beta h y^2 + \frac{4}{3}\beta y\} \right\} \right] \qquad \text{II-8}$$

where use has been made of the integration

$$\int \frac{p_f \cdot Q}{p_i \cdot Q} d^3Q = Q_o^3 \left[\frac{m^2 \left(E_f - \frac{E_i}{p_i} p_f z\right)}{p_i^2 E_i} \right.$$

$$\left. \left(\frac{E_i}{2p_i} \ln \frac{E_i + p_i}{E_i - p_i} - 1\right) + \left(\frac{E_f}{2E_i} - \frac{p_f z}{6p_i}\right) \right]$$

D. The R_{ij} defined in the text in Equation (3-20) comes from

$$\int_0^2 dy \frac{y^j}{(y+\alpha)^i} \equiv R_{ij}$$

where $j = 0, 1, 2, 3$ and $i = 2, 3$.

APPENDIX III

THE DISTRIBUTION INTEGRAL

Use has been made of the following series expansion.

$$\frac{1}{1 + e^x} = \sum_{n=1}^{\infty} (-1)^n e^{nx}, \quad x < 0$$

$$\frac{1}{1 + e^x} = \sum_{n=1}^{\infty} (-1)^{n+1} e^{-nx}, \quad x > 0$$

Then

$$I_n \equiv \int_{\lambda^{-1}}^{\infty} dx \int_{\lambda^{-1}}^{x} dy \left(\frac{1}{1 + e^{x-\nu}}\right)\left(1 - \frac{1}{1 + e^{y-\nu}}\right)(x-y)^n f(x) \qquad \text{III-1}$$

would have the form

$$I_n = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} (-1)^{m+n} \left[- \int_{\lambda^{-1}}^{\nu} dx e^{mx} e^{-m\nu} \int_{\lambda^{-1}}^{x} dy e^{ny} e^{-n\nu} \right.$$

$$- \int_{\lambda^{-1}}^{\nu} dx (-1)^m \int_{\lambda^{-1}}^{x} e^{ny-n\nu} dy$$

$$+ \int_{\nu}^{\infty} dx e^{-mx} e^{m\nu} \int_{\lambda^{-1}}^{\nu} dy e^{ny-n\nu} dy$$

$$- \int_{\nu}^{\infty} e^{-mx+m\nu} \int_{\nu}^{x} e^{-ny} e^{n\nu}$$

$$\left. - \int_{\nu}^{\infty} dx \ -mx-m\nu \ (-1)^n \int_{\nu}^{\infty} dy \right] (x-y)^n f(x) \qquad \text{III-2}$$

$= A + B + C + D + E$. The order of A, B, C, D, E is the same as that of the integral in Equation III-2. We take one step further to rewrite the integral I. With a change of variables,

$$x - y = z,\ y = w + \lambda^{-1} - z,\ x - \lambda^{-1} = w,$$
$$\text{and } x = w + \lambda^{-1}.$$

$$A = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} (-1)^{m+n} e^{(m+n)(\lambda^{-1}-\nu)} \int_0^{\nu-\lambda^{-1}} dw e^{(m+n)w} \int_0^{w} dz e^{-nz} z^n f(w+\lambda^{-1})$$

$$B = -\sum_{n=1}^{\infty} (-1)^n e^{n(\lambda^{-1}-\nu)} \int_0^{\nu-\lambda^{-1}} dw e^{nw} \int_0^{w} e^{-nz} z^n dz\, f(w+\lambda^{-1}).$$

Next, with a new change of variables

$$x - y = w + z,\ y = x - w - z = \nu - z,$$
$$z = \nu - y$$

$$C = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} (-1)^{m+n} \int_0^{\infty} dw e^{-mw} \int_0^{\nu-\lambda^{-1}} dz e^{-nz} (w+z)^n f(w+\nu)$$

And with $x - y = z$, $y = x - z = w + \nu - z$, $x - \nu = w$, $z = w + \nu - y$

$$D = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} (-1)^{(m+n)} \int_0^{\infty} e^{-(m+n)w} dw \int_0^{w} dz\, e^{nz}\, z^4\, f(w+\nu)$$

$$E = -\sum_{m=1}^{\infty} (-1)^m \int_0^{\infty} dw \int_0^{w} e^{-mw}\, z^n\, dz\, f(w+\nu)$$

Then the successive partial integration yields a series solution of the integral I. It is easy to show that when $\nu - \lambda^{-1} >> 1$, the leading term in I is, if $f(E_i) = E_i^{\,r}\, \phi(E_i)$

$$I_n \approx \sum_{m=0}^{\infty} a_m \nu^{r-n-m} \phi(\nu) + \sum_{m=1}^{\infty} b_m \nu^{r-n-m+1} \phi^m(\nu), \text{ if } r > n.$$

$$I_n \approx \sum_{m=n}^{\infty} a'_m \nu^{n-m} \phi(\nu) + \sum_{m=1}^{\infty} b'_m \nu^{1-m} \phi^m(\nu), \text{ if } r \leq n$$

where $\phi^m(x) \equiv \dfrac{d^m \phi(x)}{dx^m}$

As a simple example, we write down I when $f(x) = 1$ and $r = 4$.

$$I = 4!\{2\eta(6) + 4\eta(2)\eta(4)\} - \frac{(\nu-\lambda^{-1})^5}{5} \ln 2 \ln(1+e^{-(\nu-\lambda^{-1})}) + \cdots$$

$$+ (\nu-\lambda^{-1})^4 \sum_{n=1}^{\infty} (-1)^n \frac{e^{-n(\nu-\lambda^{-1})}}{n^2}$$

$$+ 4(\nu-\lambda^{-1})^3 \sum_{n=1}^{\infty} (-1)^n e^{-n(\nu-\lambda^{-1})} \{\frac{2\eta(2)}{n} + \frac{1}{n^3}\}$$

$$+ 12(\mu-\lambda^{-1})^2 \sum_{n=1}^{\infty} (-1)^n e^{-n(\nu-\lambda^{-1})} \{\frac{2\eta(2)}{n^2} + \frac{1}{n^4}\}$$

$$+ 24(\nu-\lambda^{-1}) \sum_{n=1}^{\infty} (-1)^n e^{-n(\nu-\lambda^{-1})}$$

$$\{\frac{\eta(4)+\eta(5)}{n} + \frac{\eta(4)}{n^2} + \frac{\eta(2)+\eta(3)}{n^3} + \frac{\eta(2)}{n^4} + \frac{1+\ln 2}{n^5}\}$$

$$+ 24 \sum_{n=1}^{\infty} (-1)^n e^{-n(\nu-\lambda^{-1})} \{\frac{2\eta(2)}{n^4} + \frac{1}{n^6} + \frac{2\eta(4)}{n^2}\}$$

$$+ 24 \ln(1 + e^{-(\nu-\lambda^{-1})}) \sum_{m=1}^{\infty} (-1)^m e^{-m(\nu-\lambda^{-1})} \frac{1}{m^6}$$

$$+ 24 \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} e^{-(m+n)(\nu-\lambda^{-1})} \frac{(-1)^{m+n}}{n^5(m+n)} .$$

Finally, when we write down Equation (3-22), we neglect all the terms proportional to the $f(\lambda^{-1})$.