The University of Michigan
Nuclear Engineering
Videotape Series

THERMAL ANALYSIS OF NUCLEAR REACTORS

COURSE MATERIALS

DEPARTMENT OF NUCLEAR ENGINEERING THE UNIVERSITY OF MICHIGAN ANN ARBOR

PREPARED EXCLUSIVELY FOR INDUSTRY, THIS SERIES COMBINES AUTHORITATIVE INSTRUCTION WITH OUTSTANDING TECHNICAL QUALITY AND FLEXIBILITY OF USAGE.

ELEMENTS OF NUCLEAR ENGINEERING — PART I

ELEMENTS OF NUCLEAR ENGINEERING — PART II

RADIATION DETECTION AND MEASUREMENT

NUCLEAR ENGINEERING MATERIALS

REACTOR THEORY AND ANALYSIS — PART I

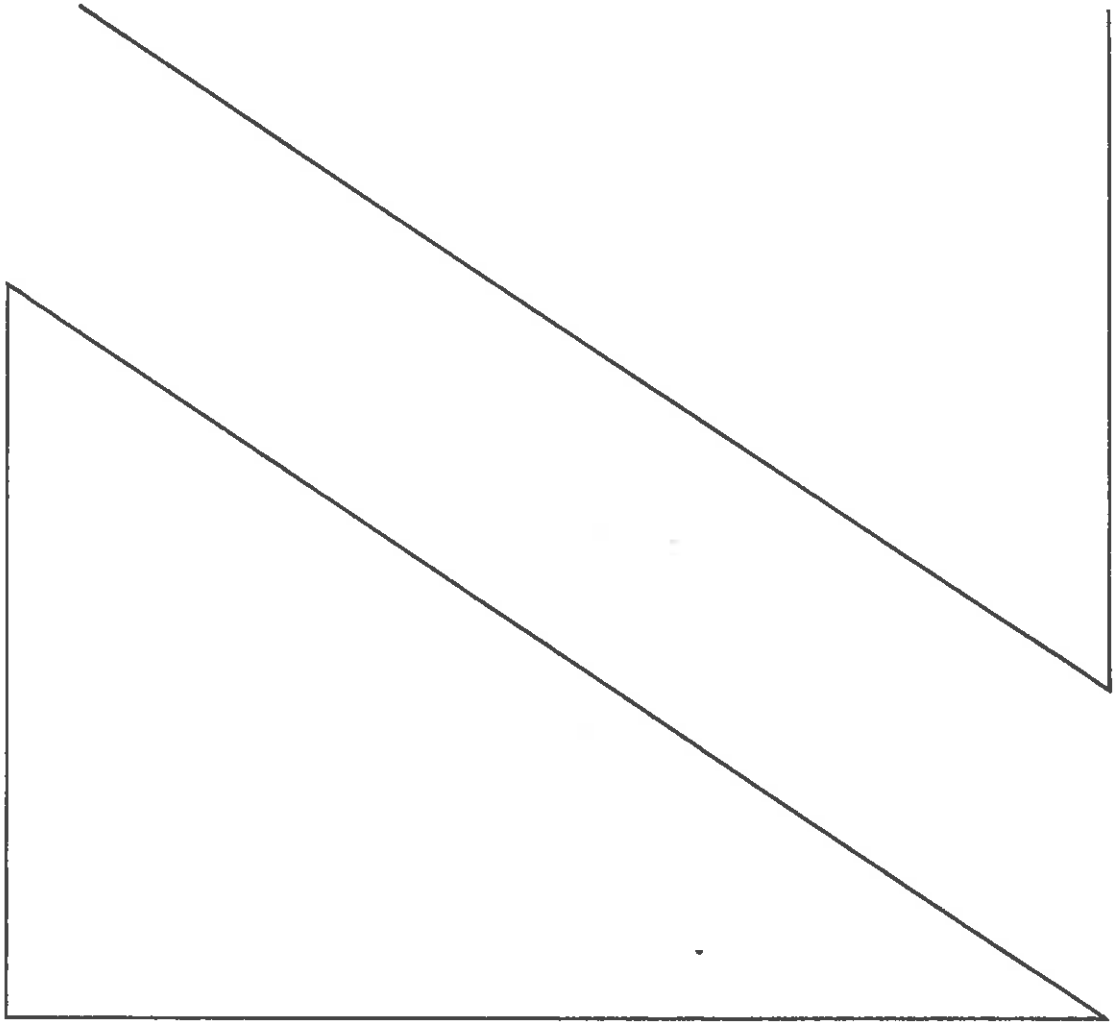
REACTOR THEORY AND ANALYSIS — PART II

THERMAL/HYDRAULIC ANALYSIS OF NUCLEAR PLANTS

FLUID MECHANICS

REACTOR SAFETY ANALYSIS

NUCLEAR POWER PLANT SYSTEMS



THERMAL ANALYSIS OF NUCLEAR REACTORS

James J. Duderstadt

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PREFACE

These materials are intended to accompany the University of Michigan videotape course in Nuclear Engineering entitled:

Thermal Analysis of Nuclear Reactors

A fundamental understanding of heat transfer and thermal effects is of critical importance in nuclear reactor design and operation. In this course our primary attention will be directed toward the study of these phenomena with particular emphasis on applications in nuclear power systems. However, because reactor thermal analysis is so closely related to other mechanical engineering topics such as fluid mechanics and stress analysis, we will include sufficient introductory material concerning these related subjects to make the course reasonably self-contained. Of particular interest will be the subjects of thermodynamic analysis of nuclear power systems, heat transfer (conduction, convection, and radiation), and thermal stress analysis.

REFERENCES: We have provided a list of representative references and reading assignments as supplements to the material covered in each lecture. Since topics such as heat transfer, thermodynamics, and thermal stress analysis are very important in many engineering disciplines, there is no shortage of textbooks. Unfortunately, however, no single textbook covers all of the topics of importance in nuclear reactor applications. Hence we have drawn material for the course from a wide variety of references. The complete list of references is provided in a subsequent section of these notes, and detailed reading assignments are drawn from this list. However we have relied particularly heavily on the following sources:

Heat Transfer:

E. R. G. Eckert and R. M. Drake, Jr., Heat and Mass Transfer (McGraw-Hill, New York, 1959)

Frank P. Incropera and David P. DeWitt, Fundamentals of Heat Transfer (John Wiley, New York, 1981)

Thermodynamics:

Gordon J. Van Wylen and Richard E. Sonntag, Fundamentals of Classical Thermodynamics (John Wiley, New York, 1981)

Thermal Stress Analysis:

Joseph Shigley, Mechanical Engineering Design (McGraw-Hill, New York, 1972)

Thermal Analysis of Nuclear Systems:

M. M. El-Wakil, Nuclear Heat Transport (American Nuclear Society, Lagrange Park, Illinois 1978)

J. Weisman, Elements of Nuclear Reactor Design (Elsevier, New York, 1977)

P. L. Pfenningworth and Bill Minkler, Heat Transfer and Fluid Flow with Applications to Pressurized Water Reactors (WAPD-TM-1000, Bettis Atomis Power Laboraory, 1971), Vols. I and II

Nuclear Reactor Analysis:

J. J. Duderstadt and L. J. Hamilton, Nuclear Reactor Analysis (Wiley, New York, 1976)

E. E. Lewis, Nuclear Power Reactor Safety (Wiley-Interscience, New York, 1977)

The reading assignments we have provided are intended to augment each of the lectures. While not essential to the course content, these references will provide the student with additional material that may assist them in mastering the material covered in the lectures.

NOTES AND VISUAL MATERIALS: We have provided copies of all visual materials used in the videotape lectures. These are presented in a form so that students can use them to follow the lectures and take notes in the margins. These notes coupled with the erading assignments should be sufficient for most purposes.

PROBLEMS AND EXERCISES: We have included a number of problems and exercises with each lecture. We are convinced that the only way to master the material covered by the lectures is by working such problems. Since the level and focus of the course will vary somewhat depending on the instructor and student background, we have provided a rather wide variety of problems, ranging from simple exercises to challenging problems requiring the use of computers. The instructor will assign those problems deemed appropriate for the course (either from this set or from other sources).

THERMAL ANALYSIS OF NUCLEAR REACTORS

COURSE OUTLINE

Introduction

- Introduction to the Course
- Importance of Thermal Analysis

Thermodynamic Analysis of Nuclear Power Systems

- Basic Concepts of Thermodynamics
- Basic Concepts of Thermodynamics -- Continued
- Thermodynamic Cycles
- The Rankine Cycle
- Practical Modifications to the Rankine Cycle
- Nuclear Power Systems as Thermodynamic Cycles

Heat Transfer in Nuclear Reactor Cores

- Fission Heat Sources
- Heat Generation in Reactor Cores
- Heat Transfer in Nuclear Reactors

Heat Conduction

- The Equation of Heat Conduction
- Solution of the Heat Conduction Equation
- Heat Conduction in Nuclear Fuel Elements
- Heat Transfer in Reactor Fuel Elements -- Continued
- Other Topics Concerning Heat Conduction in Reactors
- Multidimensional Heat Conduction
- Transient Heat Conduction

Forced Convection Heat Transfer

- Newton's Law of Cooling
- Convective Heat Transfer Coefficiencs
- Two-Phase Flow Phenomena
- The Slip-Flow Model of Boiling Flow
- Heat Addition to Boiling Flow

Radiative Heat Transfer

- Thermal Radiation
- Radiative Energy Transfer Between Black Bodies
- Radiative Heat Transfer Between Grey Bodies

Reactor Thermal Analysis

- Reactor Thermal Analysis
- Hot Channel Factors
- Applications of Reactor Thermal Analysis

Numerical Solution of the Heat Conduction Equation
Thermal-Hydraulic Computer Codes

Thermal Stress Analysis

Stresses in Nuclear Reactors
Stress and Strain
Introduction to Stress Analysis
Thermoelastic Stress Analysis
Thermal Stress Analysis in Simple Geometries - I
Thermal Stress Analysis in Simple Geometries - II
Additional Topics in Thermal Stress Analysis

Thermal Phenomena in Reactor Safety

Transient Models of Reactor Core Cooling
Modeling of Reactor Safety Transients

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Course Description
Prerequisites
Basic Approach
Course Outline

LECTURE 2: IMPORTANCE OF THERMAL ANALYSIS

Thermal Analysis of Nuclear Power Systems
Examples of Importance
Objectives and Limitations
Structure of the Course

LECTURE 3: BASIC CONCEPTS OF THERMODYNAMICS

The Plant as a Thermodynamic System
A Review of Thermodynamic Concepts
The Laws of Thermodynamics

LECTURE 4: BASIC CONCEPTS OF THERMODYNAMICS -- CONTINUED

The Laws of Thermodynamics - continued
Thermodynamic Properties
Equations of State
Thermodynamic Diagrams

LECTURE 5: THERMODYNAMIC CYCLES

Definition of a Thermodynamic Cycle
An Example: the PWR System
Heat Engines and Thermodynamic Efficiency
The Carnot Cycle
Idealized Models of Plant Components

LECTURE 6: THE RANKINE CYCLE

Limitations of the Carnot Cycle
The Rankine Cycle
Example: PWR
Limitations of the Rankine Cycle

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The Rankine Cycle -- Revisited
Reheating Cycles
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Other Ways to Improve Thermodynamic Efficiency

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Systems Based on the Rankine Cycle
(BWR, PWR, Candu, LMFBR, HTGR)
Advanced Cycles -- the Brayton Cycle

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An Overview of Core Thermal Analysis
Some Notation Concerning Power Densities
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Figures of Merit in Thermal Design
Thermal Constraints
Modes of Heat Transfer

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The Physical Process
Derivation of Energy Conservation Equation
Fourier's Law
Forms of the Heat Conduction Equation

LECTURE 13: SOLUTION OF THE HEAT CONDUCTION EQUATION

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Initial and Boundary Conditions
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Heat Transfer in Fuel
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Example

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Newton's Law of Cooling
Convective Heat Transfer Coefficient
Equivalent Hydraulic Diameter

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Heat Transport Equation for a Fluid
Laminar Flow
Turbulent Flow Correlations

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Pool Boiling Phenomena
Boiling Channel Phenomena
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Two-Phase Flow Models

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Static Quality and Void Fraction
Flow Quality and Slip
Heat Addition to Boiling Flow

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One-Dimensional Problems
Finite Difference Methods in 2-D and 3-D
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A Simple Single-Channel Code
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- Nuclear Reactor Stress Analysis
- Mechanical Loads
- Definition of Stress
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- Review of Stress
- Definition of Strain
- Elasticity and Hooke's Law
- Stress-Strain Curves

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- Lumped Parameter Models
- Power Transients
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LECTURE 40: MODELING OF REACTOR SAFETY TRANSIENTS

Two-Parameter Models
Overpower Transients
Cooling Failure Transients
Cooling Failure Accidents

REFERENCES

HEAT TRANSFER:

E. R. G. Eckert and R. M. Drake, Jr., Heat and Mass Transfer (McGraw-Hill, New York, 1959)

Frank P. Incropera and David P. DeWitt, Fundamentals of Heat Transfer (John Wiley, New York, 1981)

THERMODYNAMICS:

Gordon J. Van Wylen and Richard E. Sonntag, Fundamentals of Classical Thermodynamics, 2nd Edition (John Wiley, New York, 1976)

M. W. Zemansky, Heat and Thermodynamics, 6th Edition (McGraw Hill, New York, 1975)

STRESS ANALYSIS:

Joseph Shigley, Mechanical Engineering Design, 2nd Edition (McGraw-Hill, New York, 1972)

HEAT TRANSFER IN NUCLEAR SYSTEMS:

M. M. El-Wakil, Nuclear Heat Transport (American Nuclear Society, LaGrange Park, Illinois, 1978)

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L. S. Tong, Boiling Heat Transfer and Two-Phase Flow (John Wiley, New York, 1965)

L. S. Tong and J. Weisman, Thermal Analysis of Pressurized Water Reactors (American Nuclear Society, LaGrange Park, Illinois, 1970)

Joel Weisman and Robert W. Bowring, "Methods for Detailed Thermal and Hydraulic Analysis of Water Cooled Reactors", Nuc. Sci. Eng. 57, 255 (1975)

M. C. Edlund, et. al., Nuclear Engineering Computer Modules (Virginia Polytechnic Institute and State University,

Blacksburg, Virginia, 1973)

Owen C. Jones, Nuclear Reactor Safety Heat Transfer (Hemisphere, New York, 1981)

"Reactor Safety", Los Alamos Science 2 (Los Alamos Scientific Laboratory, Los Alamos, NM, 1981)

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James J. Duderstadt and Louis J. Hamilton, Nuclear Reactor Analysis (John Wiley, New York, 1976)

E. E. Lewis, Nuclear Power Reactor Safety (Wiley-Interscience, New York, 1977)

J. Weisman, Elements of Nuclear Reactor Design (Elsevier, New York, 1977)

John R. Lamarsh, Introduction to Nuclear Engineering, 2nd Edition (Addison-Wesley, Reading, Mass., 1983)

Alexander Sesonske, Nuclaer Power Plant Design Analysis, USAEC Document TID-26241 (National Technical Information Service, Springfiel+d, Virginia, 1973)

Harvey W. Graves, Jr., Nuclear Fuel Management (John Wiley, New York, 1979)

James H. Rust, Nuclear Power Plant Engineering (Haralson, Buchanan, Georgia, 1979)

LECTURE 1

INTRODUCTION TO THE COURSE

READING ASSIGNMENT:

None

EXERCISES:

None

Introduction to the Course

- Course Description
- Prerequisites
- Basic Approach
- Course Outline

Course: Thermal Analysis of Nuclear Reactors

Lectures: James J. Duderstadt
Professor of Nuclear Engineering
The University of Michigan

Thermal Analysis: Determine temperature distributions and heat transfer rates throughout nuclear plant

Why?

Integrity of system components

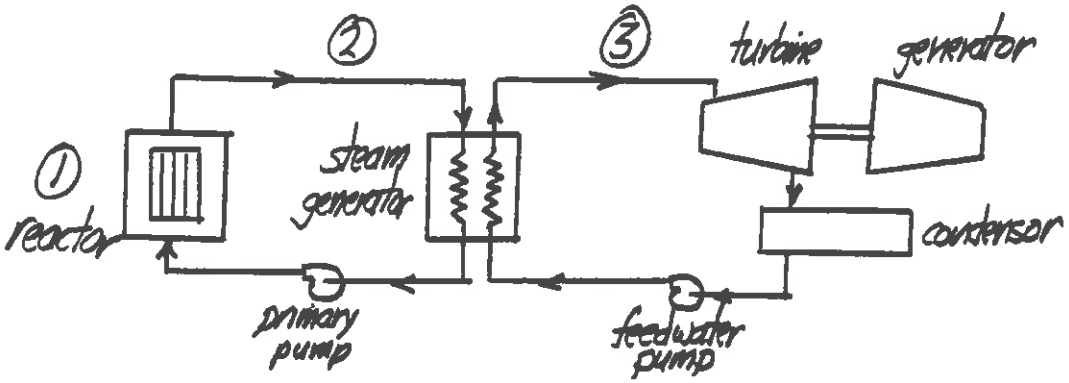
Economic operation

Levels of description:

① nuclear reactor core

② nuclear steam supply system

③ balance of plant



Steps in thermal analysis

① Determine heat source distributions (including time dependence) throughout reactor system

② Determine temperature distributions throughout solid components (fuel, structure, ...) using heat conduction

③ Analyze convective heat transfer in coolants (and determine surface temperatures of solid components)

④ Analyze heat transfer in out-of-core components

Elements of nuclear engineering - I & II
Radiation detection and measurement
Nuclear Engineering materials
Reactor theory and analysis - I & II
Thermal analysis of nuclear reactors
Fluid mechanics
Reactor safety analysis
Nuclear power plant systems

Course Prerequisites:

Engineering: Thermodynamics

Mathematics: Senior level standing

Nuclear engineering: Reactor analysis (concurrently)

Key subjects of concern:

Thermodynamics

Heat transfer:

- conduction
- convection
- radiation

Fluid mechanics

Stress analysis

Outline

Introduction to Course

- Importance of thermal analysis
- Path of thermal energy flow
- Objectives and limitations
- Structure of the course

Thermodynamic Analysis of Nuclear Power Systems

- Review of thermodynamic concepts
- The Laws of Thermodynamics
- Thermodynamic properties
- Equations of state
- Thermodynamic diagrams
- Thermodynamic cycle
- Example: PWR

Thermodynamics (continued)

- Heat engines & thermodynamic efficiency
- Carnot cycle
- Rankine cycle (including modifications)
- Brayton cycle
- Nuclear systems (PWR, BWR, Candu, HTGR, LMFBR)

Heat Transfer in Nuclear Reactor Cores

- Fission heat source distributions
- Heat generation within reactor components
- Reactor transients
- Fission product decay heating
- Path of thermal energy flow
- Figures of merit in thermal design
- Modes of heat transfer

Heat Conduction

Derivation of heat conduction equation

Mathematical properties, ic's bc

Solutions in media with & without sources

Heat conduction in nuclear fuel elements

Radiation heating

Multidimensional and transient problems

Convective Heat Transfer

Newton's Law of Cooling

Convective heat transfer coefficient

Laminar vs. turbulent flow

Two-phase flow phenomena

Slip-flow model

DNB -- departure from nucleate boiling

Axial temperature profiles

Radiative Heat Transfer

- General properties
- Kirchoff's Law, Lambert's Law, Stefan-Boltzmann Law
- Black bodies
- Radiative heat transfer between black bodies
- Radiative heat transfer between grey bodies

Reactor Thermal Analysis

- Goals, Constraints, Approaches, & Steps
- Hot channel analysis
- Some applications
- Numerical solution of heat conduction equation
- Single-channel thermal-hydraulic codes
- Subchannel analysis codes

Thermal Stress Analysis

Stresses in nuclear reactors

Definition of stress

Definition of strain

Elasticity and Hooke's law

Stress-strain curves

Strain tensor

Elastic stress theory

Thermal Stress Analysis -- Continued

Temperature-induced strains

Thermoelastic stress analysis

Plates, Spheres, cylinders

Hydrostatic stresses

Stress wave propagation

Creep

LECTURE 2

IMPORTANCE OF THERMAL ANALYSIS

READING ASSIGNMENT:

Duderstadt & Hamilton, pp. 447 - 465, pp. 467 - 475
J. Weisman, "Heat Generation and Transport", Chapter 6 in Weisman (Design)

EXERCISES:

2-1. List the various ways in reactor analysis that

i) nuclear (physics) analysis influences thermal analysis

ii) thermal analysis influences nuclear analysis

2-2. Sketch a diagram which indicates how the various computer code components involved in reactor analysis interact with one another. Take particular care to indicate the interaction between thermal-hydraulics calculations and nuclear (multigroup diffusion) calculations.

Importance of Thermal Analysis

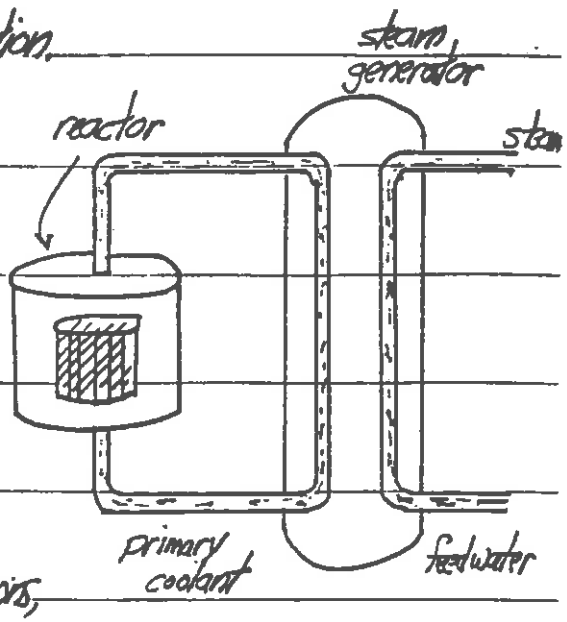
- Thermal analysis of nuclear power systems
- Examples of importance
- Objectives and limitations
- Structure of the course

Thermal analysis plays an important role in reactor design and operation.

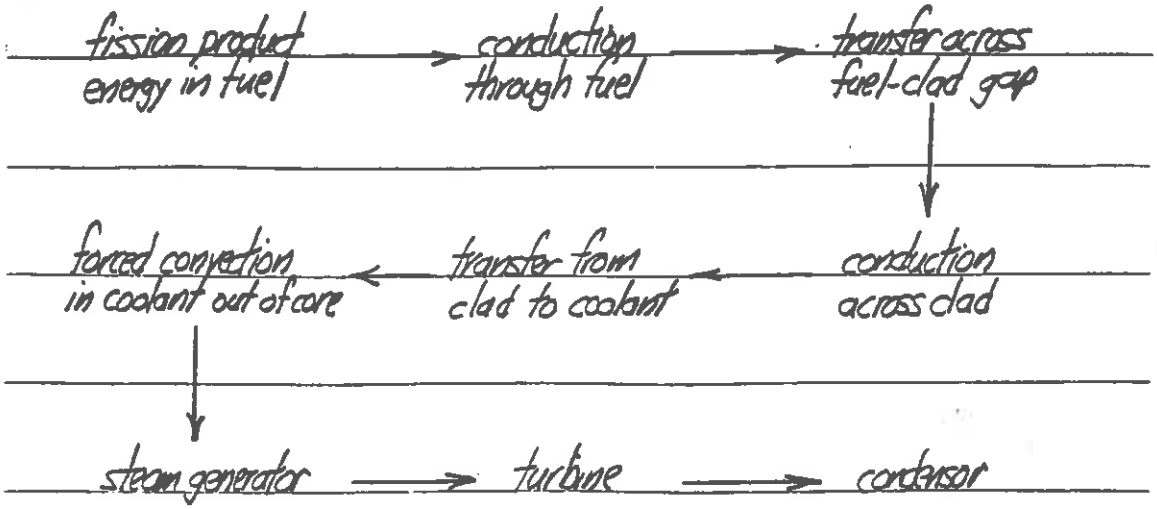
nuclear power reactor

⇒ exotic heat source for turning water into steam

Hence determination of heat transfer rates, temperature distributions, etc. are very important



Approach: Follow path of thermal energy flow



See over a complete diagram of

NSSS

and

BOP

Thermal phenomena play a critical role in determining reactor design and influencing reactor performance.

Example: Thermal limitations determine core size. Steps:

1) determine power density allowed by coolant

2) determine core size necessary to produce total MWt with this power density

3) determine if this is necessary for criticality

Example: thermal limitations determine fuel design

i) coolant requirements ⇒ coolant-to-fuel ratio

ii) hydrodynamic behavior of coolant flow in core

Example: strong interaction between thermal and nuclear behavior

$$k, \phi(\underline{r}) \Rightarrow P(\underline{r}) \Rightarrow T(\underline{r}), N_H(\underline{r}), N_F(\underline{r})$$

$$\Rightarrow \Sigma(\underline{r}) \Rightarrow k, \phi(\underline{r})$$

Example: Many systems in a nuclear plant play a critical role in safety analysis

i) LOCA (loss of coolant accident)

ii) ECCS (emergency core cooling system)

Some typical objectives of thermal design

i) high core power density (to reduce core volume and pressure vessel requirements)

ii) high fuel specific power (to reduce required fuel inventory)

iii) high coolant exit temperatures (higher thermodynamic efficiency, reduced turbine requirements)

Thermal limitations on plant design and operation

Fuel: $T_F < T_{MELT}$

Clad: fuel pellet cracking or densification
fission product gas pressure
avoid departure from nucleate boiling (DNB)

Coolant: PWR: $T_{cool} < T_{BOILING}$
ΔT_{cool} limited to avoid thermal shock

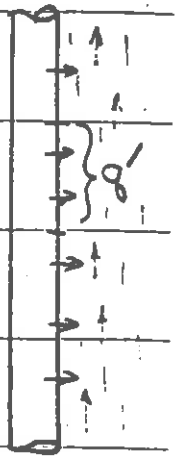
Nuclear: moderation
control margins

System: ECCS requirements
~~Thermodynamic~~ efficiency

Fuel Operating Limits

linear power density $q' \propto$

- peak fuel temperature T_c
- surface heat flux q''
- decay heat generation rate
- stored thermal energy



Example: Fuel centerline temperature

$$T_F|_{max} = T_\ell = f_{cn}(q')$$

We must keep

$$T_F < T_{melt} \cong 2800\,^\circ C \quad (UO_2)$$

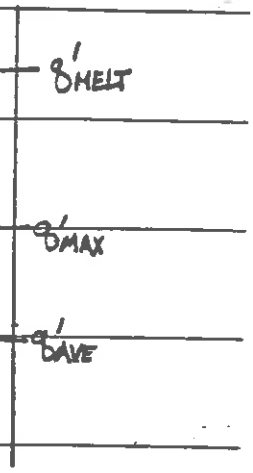
This implies a limit on linear power density

$$q' < 660 \text{ W/cm} \quad (UO_2)$$

[Reactors are designed to operate with

$$q'_{max} \sim 460 - 500 \text{ W/cm}$$]

Example: One frequently uses q' as a figure of merit



Note also

$$\frac{q'_{MAX}}{q'_{AVE}} = F$$

is determined by core design. Furthermore

$$\text{working capital charges} \sim \frac{1}{q'_{AVE}} \sim F$$

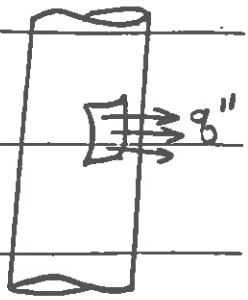
Yet another important quantity is the [illegible] q''

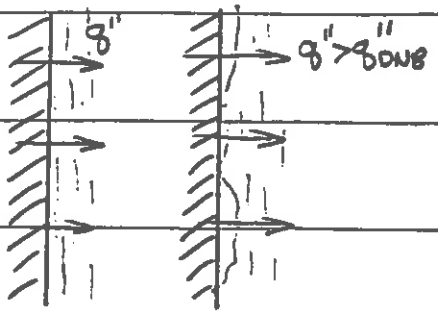
We must keep

$$q'' < q''_{DNB}$$

to avoid departure

from nucleate boiling





Objective of core thermal design:

Provision of adequate core cooling under

- normal, steady-state operation
- anticipated transients
- abnormal occurrences (LOCA)

Typical thermal design criteria for LWRs:

1) $T_{FUEL} < T_{melt}$ at 112% design overpower (at any time during core life)

2) $DNBR = \frac{q''_{DNB}}{q''} > 1.3$ at 112% overpower (so that cladding damage due to DNB is unlikely)
- (DNBR > 1.9 in BWRs)

3) no hydraulic instabilities occur during steady-state or anticipated transient conditions

4) a prescribed fraction of reactor vessel flow rate (95%) will be availdle for fuel element cooling (limits flow bypass)

Note: additional thermal design limitations arise when considering abnormal transients such as loss of coolant accidents.

These can be more restrictive than for steady-state operation (e.g., $q'_{LOCA} < q'_{\ell melt}$)

Steps in reactor thermal analysis

① Determine heat source distributions (including time dependence) through reactor system

② Determine temperature distributions through solid components (fuel, structure, ..) using heat conduction

③ Analyze convection heat transfer in coolant (and determine surface temperatures of solid components)

Two classes of analysis

Steady-state analysis: mapping of temperature distributions and heat transfer rates during full power operation

Transient analysis: mapping of temperature distributions during transients (both anticipated and abnormal)

Fundamental topics in course

thermodynamics

heat transfer

conduction

convection

radiation

thermal analysis

Structure of the Course

Thermodynamic Analysis of Nuclear Power Systems

Heat Transfer in Nuclear Reactors

Heat Conduction in Nuclear Fuel Elements

Forced Convection Heat Transfer

Thermal-Hydraulic Analysis

Thermal-Mechanical Interactions

Reactor Safety Analysis

LECTURE 3

BASIC CONCEPTS OF THERMODYNAMICS

READING ASSIGNMENT:

El-Wakil (Nuclear Energy Conversion), pp. 22 - 55
J. Anno, "Thermodynamics of Nuclear Power Systems", Chapter 3 in Weisman (Design)
Van Wylen and Sonntag, Chapters 2, 3, 5
Zemansky, Chapter 4

EXERCISES:

3-1. Verify that work in thermodynamic systems can be represented as:

$$dW = p\,dV$$

3-2. Does the First Law of Thermodynamics apply on a microscopic level of individual atoms and molecules? Why or why not?

3-3. An LMFBR operates at 3,400 MWt. Sodium enters the core at 400 C and leaves at 560 C. At what rate must sodium be pumped through the core to extract the thermal energy generated.

3-4. Derive the expressions for the specific heats at constant pressure and constant volume in terms of the internal energy and enthalpy.

Basic Concepts of Thermodynamics

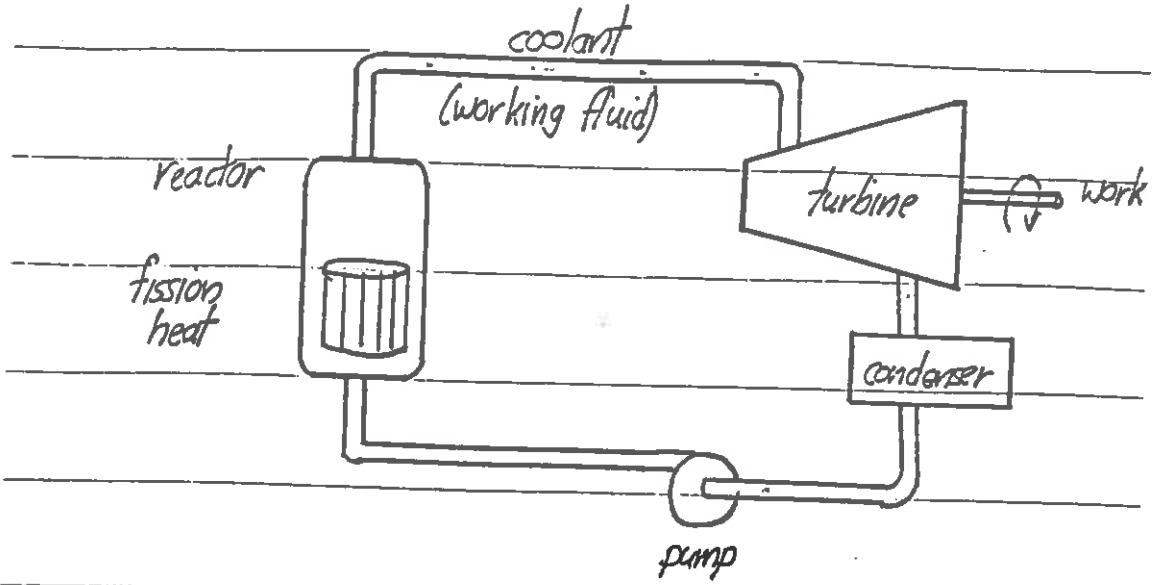
- The plant as a thermodynamic system
- A review of thermodynamic concepts
- The Laws of Thermodynamics

Nuclear power plant → thermodynamic system

Reactor → heat

working fluid

work → turbine generator



thermodynamic cycle

Thermodynamic efficiency = $\frac{\text{work produced}}{\text{heat input}}$

Why is this important?

i) fuel costs

ii) component size (core, heat exchangers)

iii) heat rejection to environment

(Note: not a driving factor in nuclear plant design)

A Review of Thermodynamic Concepts

Thermodynamics: a phenomenological theory of matter

Thermodynamic system: any macroscopic system

thermodynamic parameters: measurable quantities (p, V, T)

thermodynamic state: specified by a set of values of all thermodynamic parameters necessary to describe system

equation of state: relationship among thermodynamic parameters for system in equilibrium ($f(p, V, T) = 0$)

thermodynamic equilibrium: when thermodynamic state does not change with time

thermodynamic transformation: change of state

quasistatic: external conditions change so slowly that at any moment, system is approximately in equilibrium

reversible: if the transformation retraces its history in time if the external conditions retrace their history in time

work: concept taken from mechanics

$dW = \underset{\sim}{F} \cdot d\underset{\sim}{s} \rightarrow dW = PdV$

heat: energy which transfers from one system to another by virtue of a temperature difference

cycle: a process having identical end states

property: parameter determined by end states, not process itself

extensive: proportional to amount of substance

intensive: independent of amount of substance

heat capacity: $\Delta Q = C \Delta T$

heat reservoir: a system so large that ΔQ causes no ΔT

adiabatic: no heat transfer (thermally isolated)

The Laws of Thermodynamics

phenomenological statements (or "axioms")

Zeroth: $T_A = T_C$ & $T_B = T_C \Rightarrow T_A = T_B$

First: $\Delta U = \Delta Q - \Delta W$

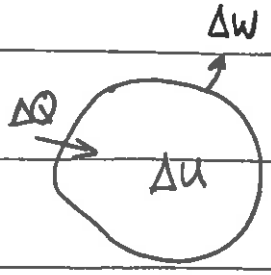
Second: $\Delta S = \frac{\Delta Q_{rev}}{T} \geq \frac{\Delta Q}{T}$

Third: $\lim_{T \to 0} S = 0$ for all substances

The First Law of Thermodynamics

= law of conservation of energy

Simple system:



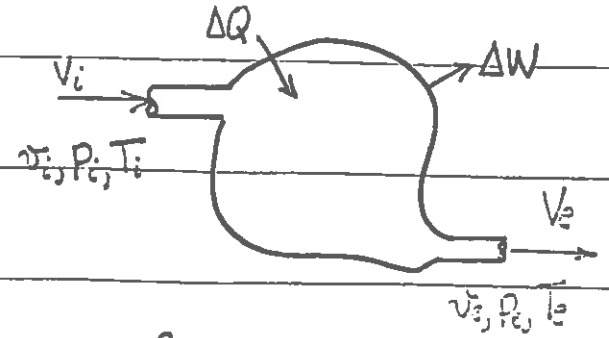
$\Delta U = \Delta Q - \Delta W$

ΔQ – heat absorbed by system

ΔW – work done by system

ΔU – change in internal energy of system

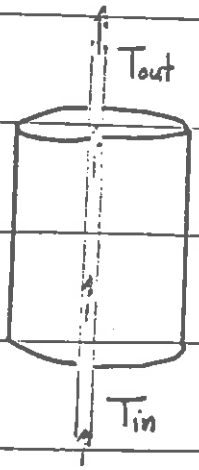
Steady Flow:



$$\Delta U = \frac{V_i^2}{2g} + P_i v_i + \Delta Q - \frac{V_e^2}{2g} - P_e v_e - \Delta W$$

Example:

$$\Delta U = \Delta Q - \cancel{\Delta W}^{\,0}$$



Note

$$\Delta U = C \Delta T$$

Hence

$$C(T_{out} - T_{in}) = \Delta Q$$

[Alternative:

$$q = w \int_{T_{in}}^{T_{out}} C_p(T)\, dT = w(h_{out} - h_{in})$$]

Additional Comments on First Law:

i) differential form

$$dU = dQ - dW$$

ii) $U = U(P,V) \Rightarrow dU = \left(\frac{\partial U}{\partial P}\right)_V dP + \left(\frac{\partial U}{\partial V}\right)_P dV$

Using this and $dW = PdV$ we can find

$$C_V \equiv \left(\frac{\Delta Q}{\Delta T}\right)_V = \left(\frac{\partial U}{\partial T}\right)_V$$

$$C_P \equiv \left(\frac{\Delta Q}{\Delta T}\right)_P = \left(\frac{\partial H}{\partial T}\right)_P$$

where $H = U + PV \rightarrow$ enthalpy of system

LECTURE 4

BASIC CONCEPTS OF THERMODYNAMICS -- CONTINUED

READING ASSIGNMENT:

Same as in Lecture 3
Van Wylen and Sonntag, Chapters 6 and 7

EXERCISES:

4-1. Derive the "T dS" equations.

4-2. By referring to any of the standard textbooks on thermodynamics, provide a physical interpretation of the concept of "entropy".

4-3. Derive an experssion for the steam production rate w (steam) in a BWR:

$$w(steam) = [q - w(water)\ (h(f) - h(in))]/h(fg)$$

in terms of the inlet flow rate w(water), the inlet specific enthalpy of water entering the core h(in) and saturated water leaving the core, h(f), the thermal power of the reactor q, and the heat of vaporization of the water h(fg) core

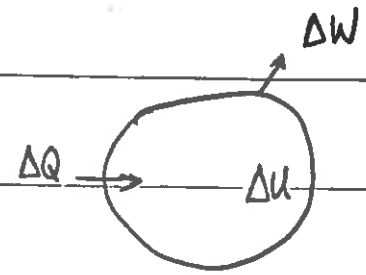
4-4. A BWR operates at a thermal power of 1593 MWt. Water enters the bottom of the core at 526 F and passes through the core at a rate of 48 x 10^6 lb/hr. The reactor pressure is 1035 psia. Using the result of Problem 4-1, compute the rate at which steam is produced in this reactor.

Basic Concepts of Thermodynamics – Continued

- The Laws of Thermodynamics – continued
- Thermodynamic properties
- Equations of state
- Thermodynamic diagrams

The First Law of Thermodynamics

(law of conservation of energy)



Simple system.

$$\Delta U = \Delta Q - \Delta W$$

ΔQ → heat absorbed by system

ΔW → work done by system

ΔU → change in internal energy of system

The Second Law of Thermodynamics

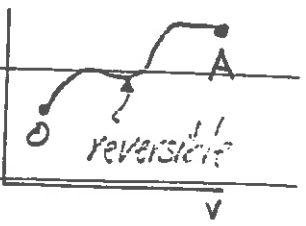
Governs the conversion of energy from one form to another.

Kelvin Statement: There exists no thermodynamic transformation whose sole effect is to convert a given amount of heat entirely into work.

Clausius Statement: There exists no thermodynamic transformation whose sole effect is to extract heat from a colder reservoir and transfer it to a hotter reservoir.

Define the entropy $S(A)$ of a thermodynamic state A as

$$S(A) \equiv \int_O^A \frac{dQ}{T}$$

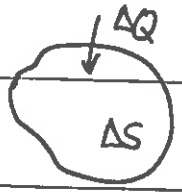


Hence

$$dS \equiv \frac{dQ}{T}$$

Mathematical Statement:

$$\Delta S \geq \frac{\Delta Q}{T}$$



$\left(\Delta S = \frac{\Delta Q}{T} \text{ for a reversible process}\right)$

(Note: $\Delta S \geq 0$ -- the entropy of an isolated system never decreases.)

Additional Comments on Second Law

i) note differential relationships

$$TdS = dU + PdV$$

$$TdS = dU - VdP$$

("TdS equations")

ii) The entropy of a state may be viewed as a measure of the unavailability of useful energy in that state

The Third Law of Thermodynamics

The entropy of a system at absolute zero is a universal constant, which may be taken to be zero.

(This implies the heat capacity of a system must vanish as $T \to 0$

Note: Third Law is a macroscopic manifestation of quantum effects

First Law: Energy can neither be created nor destroyed; it can only change form. ("You can't get something for nothing; you can only break even.")

Second Law: It is impossible to convert a given quantity of heat completely into work. ("You can't break even; you can only lose.")

Thermodynamic Properties

The state of a system can be characterized by its thermodynamic parameters or properties. Recall

extensive: proportional to amount of substance

V, S, U, C_v

intensive: independent of amount of substance

v, s, u, c_p, c_v

("specific volume", "specific entropy", ...)

Examples: p pressure

T temperature

v specific volume

ρ density

s specific entropy

h specific enthalpy $(h = u + pv)$

u specific internal energy

Note: The state of a substance (uniform, single phase) can be completely characterized by two independent thermodynamic properties.

[E.g., p, v or p, T or s, T or h, T . . .]

An equation of state is a functional relationship among the thermodynamic properties of a system in equilibrium:

$$f(p, v, T) = 0$$

[Need additional properties for multiple phases.]

Examples:

Ideal gas: $PV = NkT$ $(pv = RT)$

Incompressible substances: $h = c_p T$

Van der Waal: $p = \frac{RT}{(v-b)} - \frac{a}{v^2}$

Virial expansion: $\frac{pv}{RT} = 1 + \frac{B}{v} + \frac{C}{v^2} + \frac{D}{v^3} + \cdots$

Equations of state must either be measured or calculated from molecular considerations. Indeed, usually tabulate (and interpolate) equation of state data rather than using functional form:

Examples: ASME Steam Tables

Computer data bases

Charts of thermodynamic properties

What properties? p, T are common since they can be easily measured. Then equation of state data gives

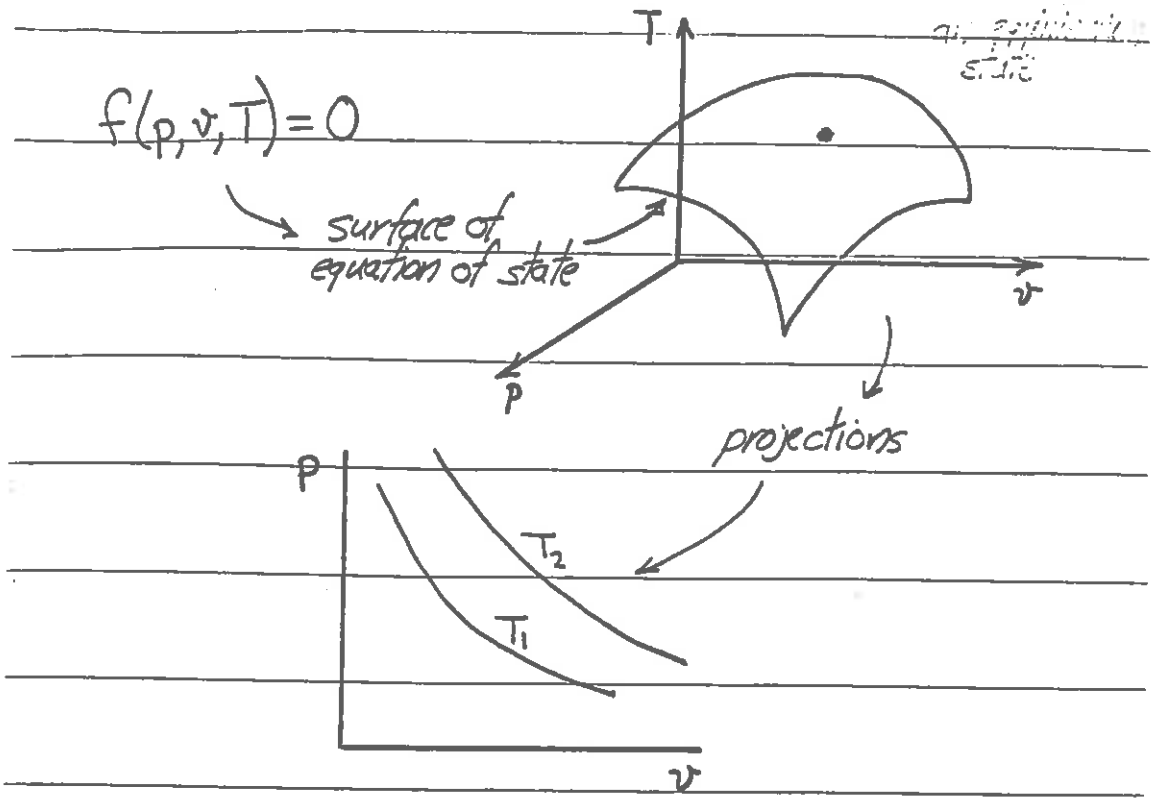
$$u = u(p, T)$$

$$h = h(p, T)$$

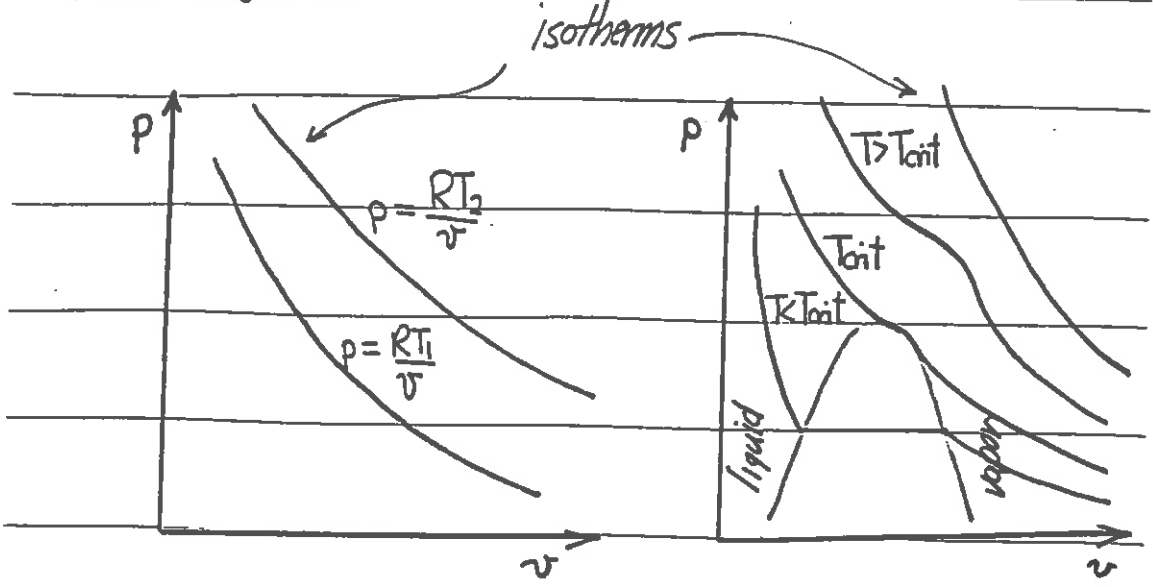
$$v = v(p, T)$$

$$s = s(p, T)$$

We can represent equation of state data using thermodynamic charts.

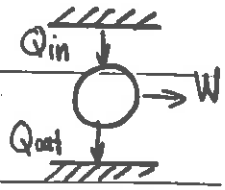


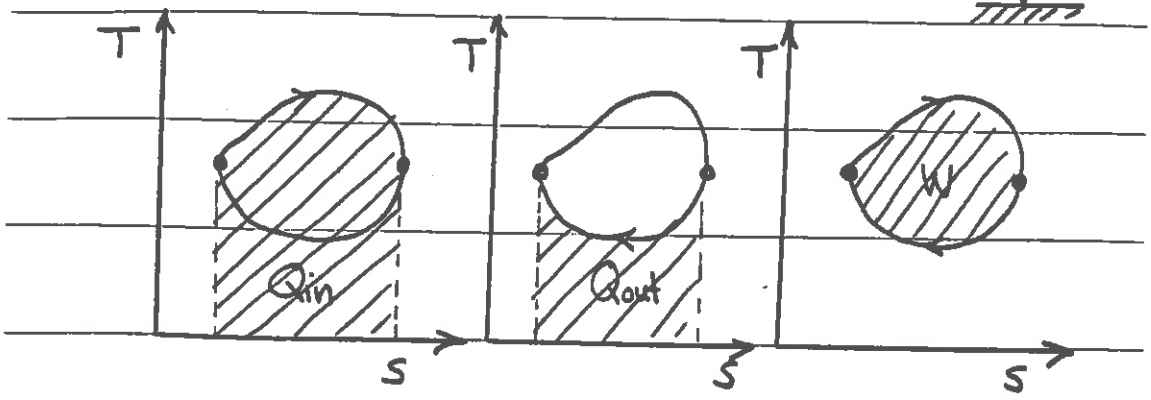
p-v diagrams



Perfect gas — Real gas

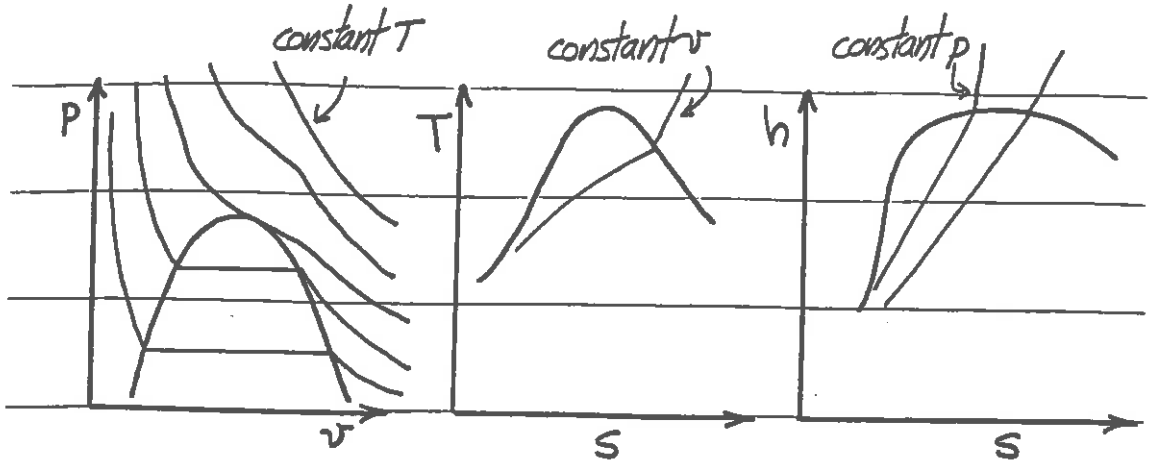
T-S diagrams are particularly useful for representing reversible thermodynamic cycles:





heat absorbed — heat rejected — work performed

Other types of thermodynamic diagrams



p-v diagrams | T-s diagrams | h-s diagrams (Mollier)

LECTURE 5

THERMODYNAMIC CYCLES

READING ASSIGNMENT:

Zemansky, Chapter 9
Van Wylen and Sonntag, Chapter 6

EXERCISES:

5-1. Using a simple diagram of a BWR NSSS, identify the path of states assumed by the working fluid on a T-S diagram and relate these to various points on the NSSS diagram.

5-2. Compare p-v, T-s, and h-s diagrams of a Carnot cycle.

5-3. Three Carnot cycles using mercury, sodium, and water as working fluids receive heat causing these fluids to vaporize at 10 atm. Determine for each cycle the pressure and temperature at which heat must be rejected if the efficiency in all cases is 35%.

5-4. Derive a relationship between the thermal pollution (heat rejected) from a power plant and the thermal efficiency of the plant. Show that the thermal pollution of a 30% efficient plant (e.g., PWR) is some 55% greater than from a 40% efficient plant (e.g., fossil-fuel fired), 133% greater than from a 50% efficient plant, and 250% greater than from a 60% efficient plant.

Thermodynamic Cycles

- Definition of thermodynamic cycle
- An example: PWR
- Heat engines and thermodynamic efficiency
- The Carnot Cycle
- Idealized models of plant components

Thermodynamic Cycles

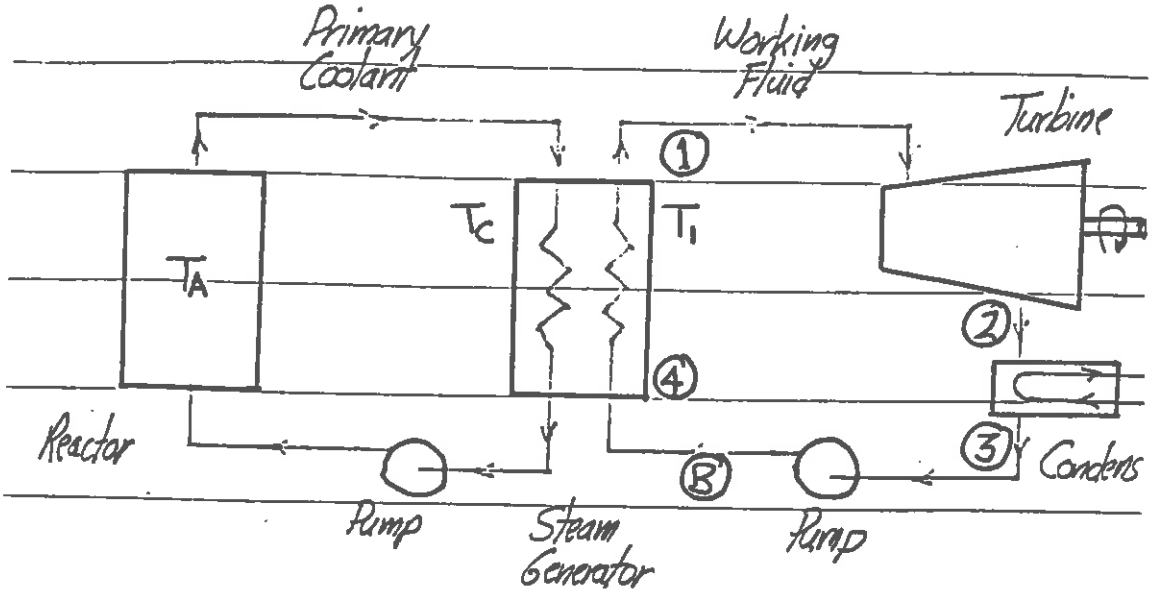
Thermodynamic cycle: closed path on a chart of thermodynamic properties of a substance.

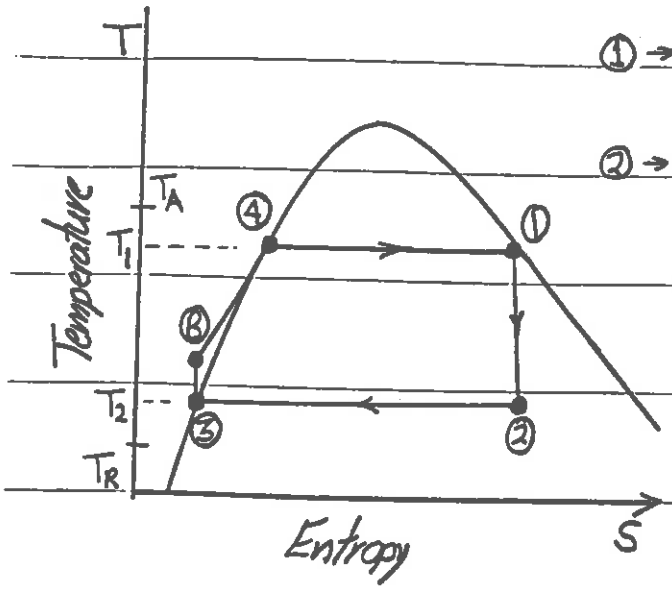
Each segment of path represents stages in the operation of the system.

Example: PWR Cycle (Rankine Cycle)

(a "heat engine" which uses cycle to convert heat into work)

PWR System





① → ②: adiabatic expansion in turbine

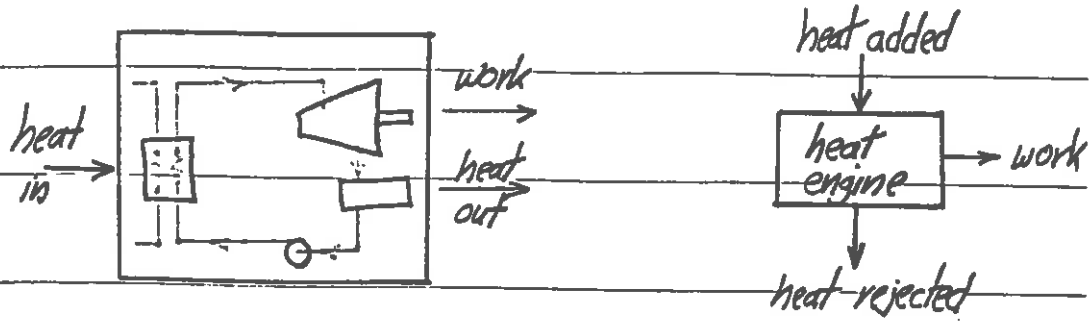
② → ③: vapor condensed at const. p

③ → Ⓑ: condensed fluid pumped to higher pressure

Ⓑ → ④: ~~sensible heat addition~~

④ → ①: latent heat addition (boiling)

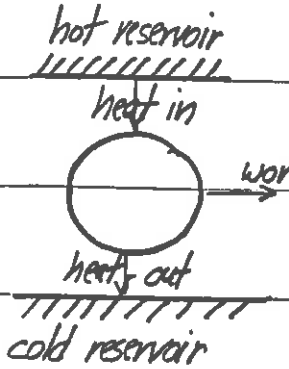
Note: This is an example of a heat engine which is a device for converting heat into work by cyclic operation



Note: Thermodynamic efficiency $= \frac{\text{work}}{\text{heat added}}$

The Carnot Cycle

The maximum efficiency is achieved with the Carnot cycle, an idealized cycle consisting of 4 reversible steps



① adiabatic compression (T↑)

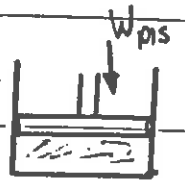
② isothermal expansion (heat absorbed)

③ adiabatic expansion (T↓, work)

④ isothermal compression (heat rejected)

More explicitly:

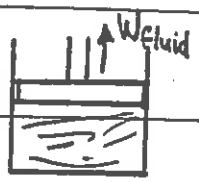
① adiabatic compression; piston does work on fluid to raise to temperature of hot reservoir



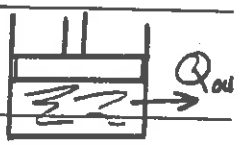
② isothermal expansion in which heat is absorbed by fluid from hot reservoir

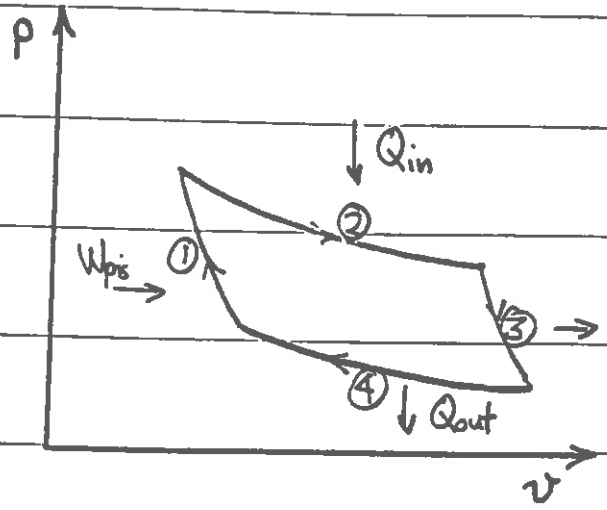


③ adiabatic expansion; fluid does work on piston; cools to temperature of cold reservoir

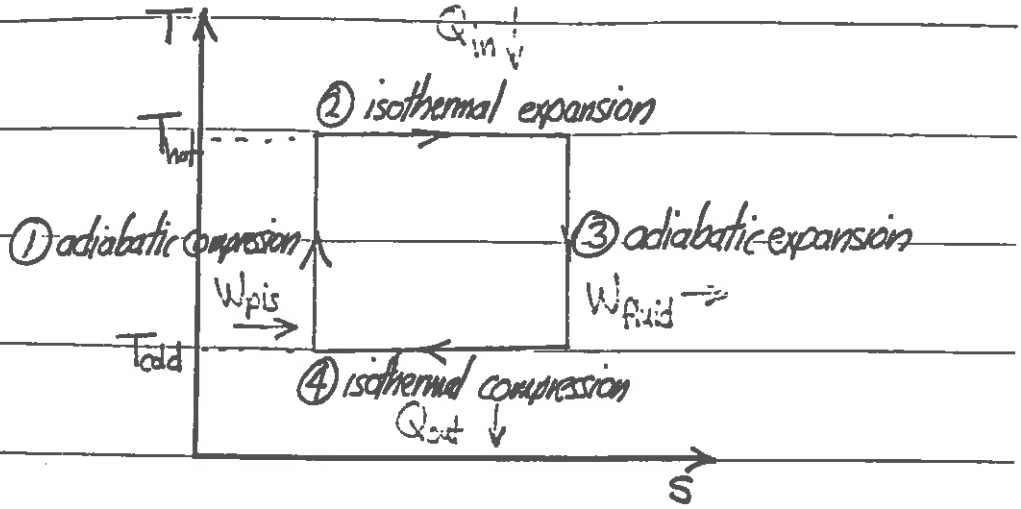


④ isothermal compression; fluid rejects heat to cold reservoir





It is more convenient to represent Carnot cycle on a T-s diagram.



Now note:

Along ② isothermal (reversible) expansion

$$\Delta S = \int_{②} \frac{dQ}{T} = \frac{\Delta Q_{in}}{T_{hot}} \Rightarrow \Delta Q_{in} = \Delta S\, T_{hot}$$

Similarly, along ④ isothermal (reversible) compression

$$\Delta S = \int_{④} \frac{dQ}{T} = \frac{\Delta Q_{out}}{T_{cold}} \Rightarrow \Delta Q_{out} = \Delta S\, T_{cold}$$

Now by First Law:

$$W_{fluid} - W_{pis} = \Delta W = \Delta Q_{in} - \Delta Q_{out}$$

Thus we can calculate the thermodynamic efficiency for the Carnot cycle as

$$\text{Efficiency} = \frac{\Delta W}{\Delta Q_{in}} = \frac{\Delta Q_{in} - \Delta Q_{out}}{\Delta Q_{in}}$$

$$= 1 - \frac{\Delta Q_{out}}{\Delta Q_{in}} = 1 - \frac{\Delta S\, T_{cold}}{\Delta S\, T_{hot}}$$

or

$$\text{Efficiency} = 1 - \frac{T_{cold}}{T_{hot}}$$

This is the maximum efficiency which can be achieved by a heat engine operating between reservoirs at T_{hot} and T_{cold}.

Example

T_{hot} ~ limiting temperature of practical materials

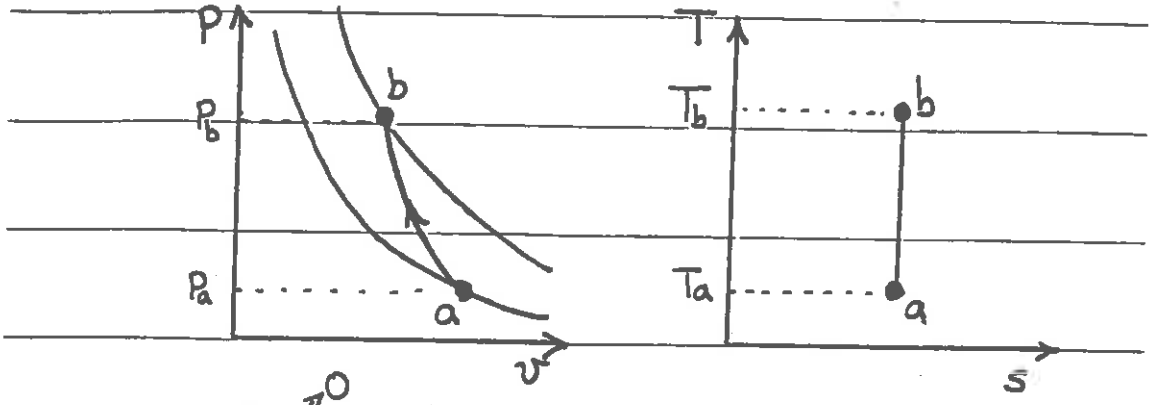
T_{cold} ~ temperature of atmospheric heat sink

$$\text{Carnot efficiency} = 1 - \frac{T_{cold}}{T_{hot}} = 2/3$$

Thermodynamic systems are composed of a series of devices, each of which causes a change of state corresponding to a section of the thermodynamic cycle path.

We can develop idealized models of such devices, just as the Carnot cycle was an idealized model of a heat engine.

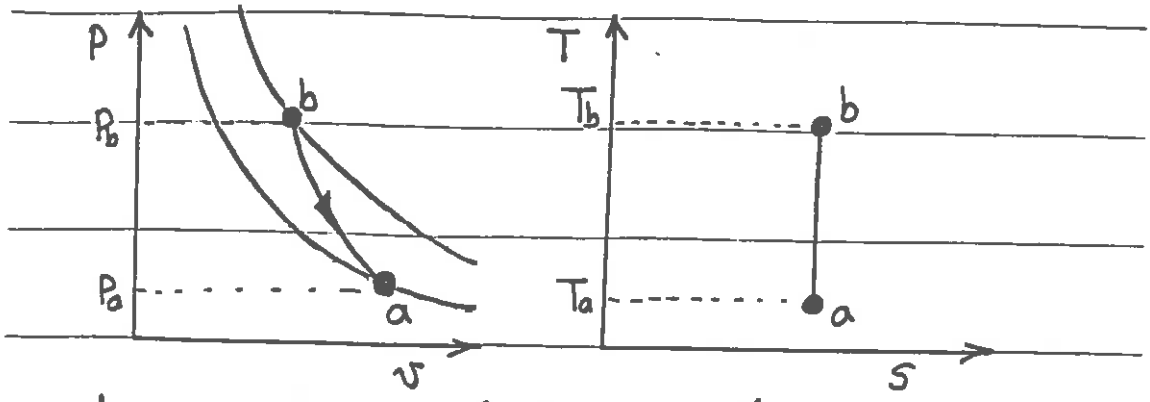
Pump (reversible, adiabatic compression)



Use $T\cancel{ds}^{0} = dh - vdp \Rightarrow dh = vdp$

$$\therefore \Delta W_{ab} = \int_a^b v\,dp = \int_a^b dh = h_b - h_a$$

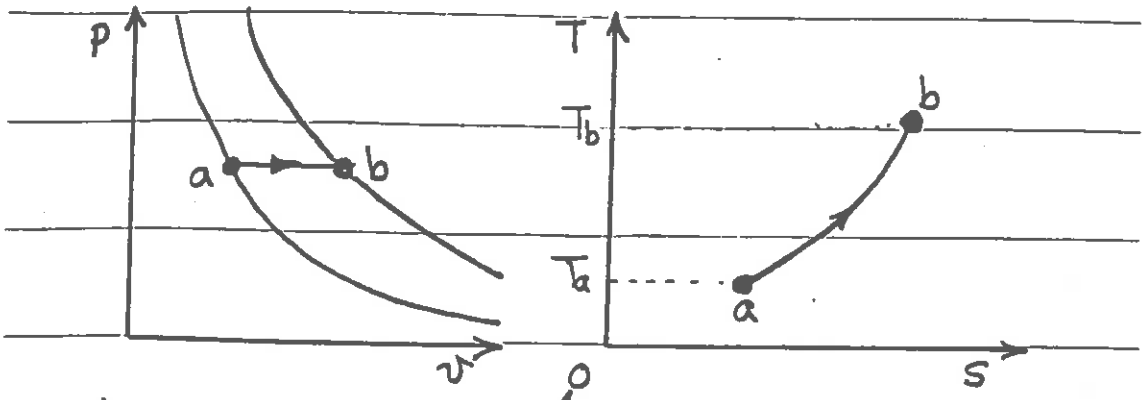
Turbine (reversible, adiabatic expansion)



We can use our results for the pump with a→b replaced by b→a

$$\Delta W_{ba} = \int_b^a v\,dp = \int_b^a dh = h_a - h_b$$

Heater or boiler (constant pressure heat addition)

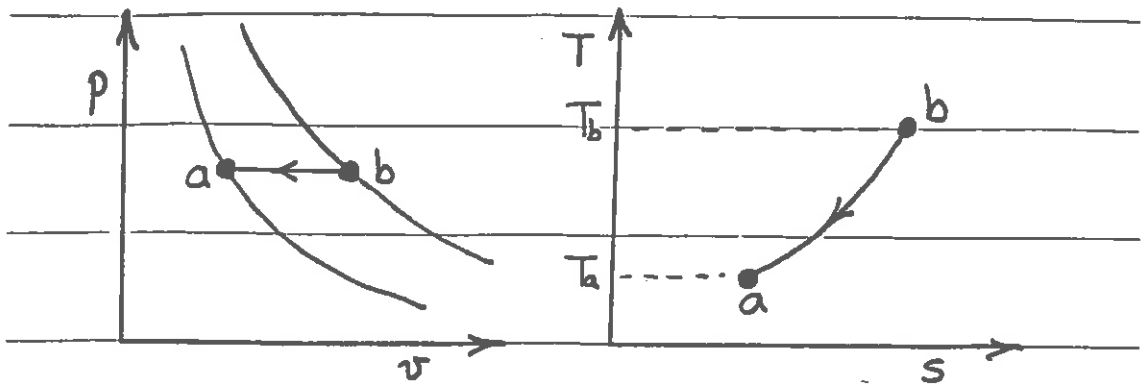


Use $Tds = dh - v\,dp = dh$

or $dQ = Tds = dh$

$$\Delta Q = \int_a^b dQ = \int_a^b dh = h_b - h_a$$

Cooler or condenser (constant pressure heat removal)

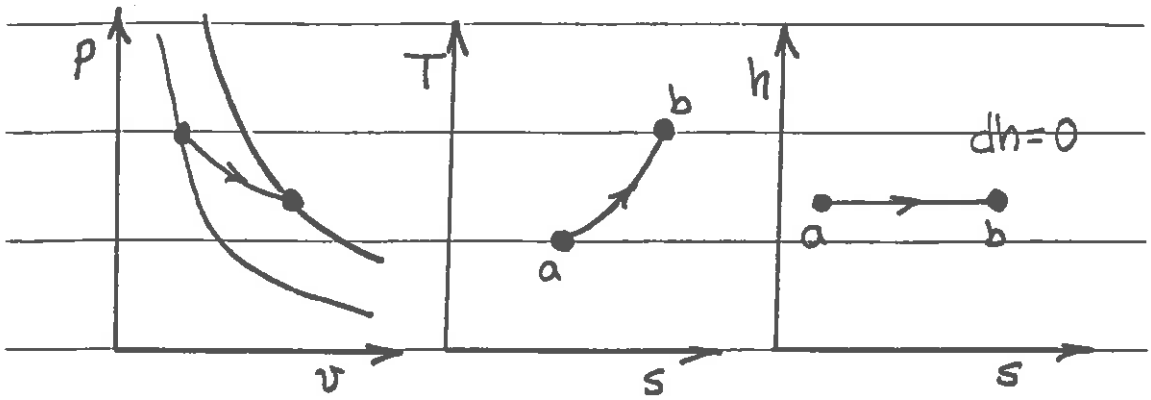


Can just reverse our results for a heater:

$$\Delta Q_{ba} = h_b - h_a$$

Note: Our results for a heater or cooler can be applied to describe a heat exchanger.

Pipe with frictional loss (irreversible adiabatic pressure drop)



LECTURE 6

THE RANKINE CYCLE

READING ASSIGNMENT:

El-Wakil (Nuclear Energy Conversion), pp. 22 - 55
J. Anno, "Thermodynamics of Nuclear Power Systems", Chapter 3 in Weisman (Design)
Rust, Chapter 2

EXERCISES:

6-1. Using a T-s diagram of water taken a source such as the Steam Tables, sketch the path taken by the working fluid in a typical PWR secondary loop.

6-2. Using the Steam Tables, verify the calculation of the efficiency of the steam cycle given in the Lecture.

The Rankine Cycle

- Limitations of the Carnot Cycle
- The Rankine Cycle
- Example: PWR
- Limitations of Rankine Cycle

The Carnot Cycle - Revisited

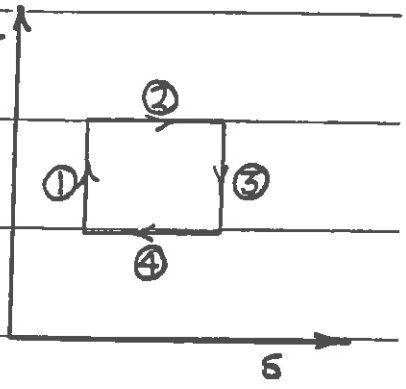
Recall the steps involved in the Carnot cycle

① adiabatic compression
(work done on system)

② isothermal expansion
(absorb heat from hot reservior)

③ adiabatic expansion
(work done by system)

④ isothermal compression
(rejects heat to cold reservior)



Limitations of the Carnot Cycle

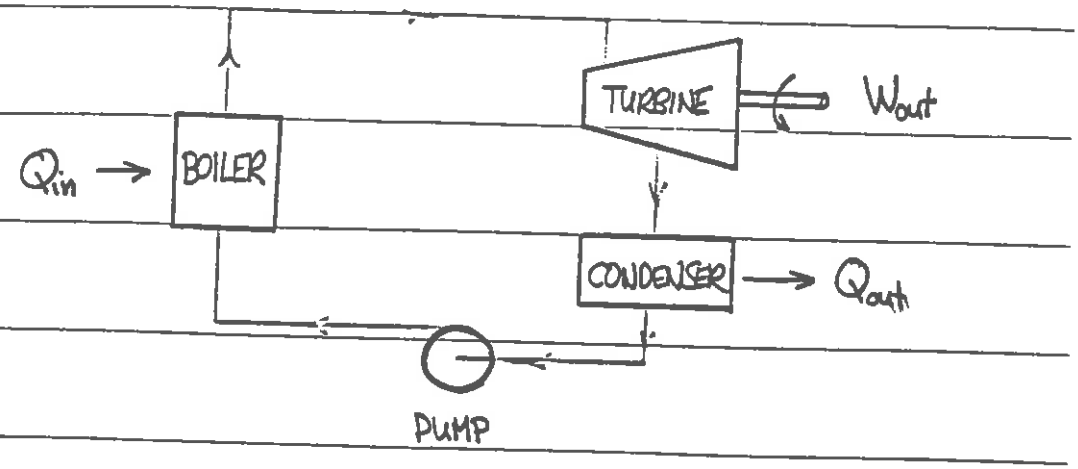
Isothermal heat transfer is impossible (by definition, heat transfer requires a temperature difference)

Irreversibility (fluid-pipe friction, pump & turbine losses, ...)

Let's turn our attention to a more realistic (although still idealized) thermodynamic cycle: the Rankine cycle based on liquid-vapor phase change.

The Rankine Cycle

Based on a condensable working fluid (such as steam)



Steps in the Rankine Cycle

a→b: isentropic compression in pump

b→c: constant pressure heating in boiler

b→b': heating subcooled liquid to saturation temperatur

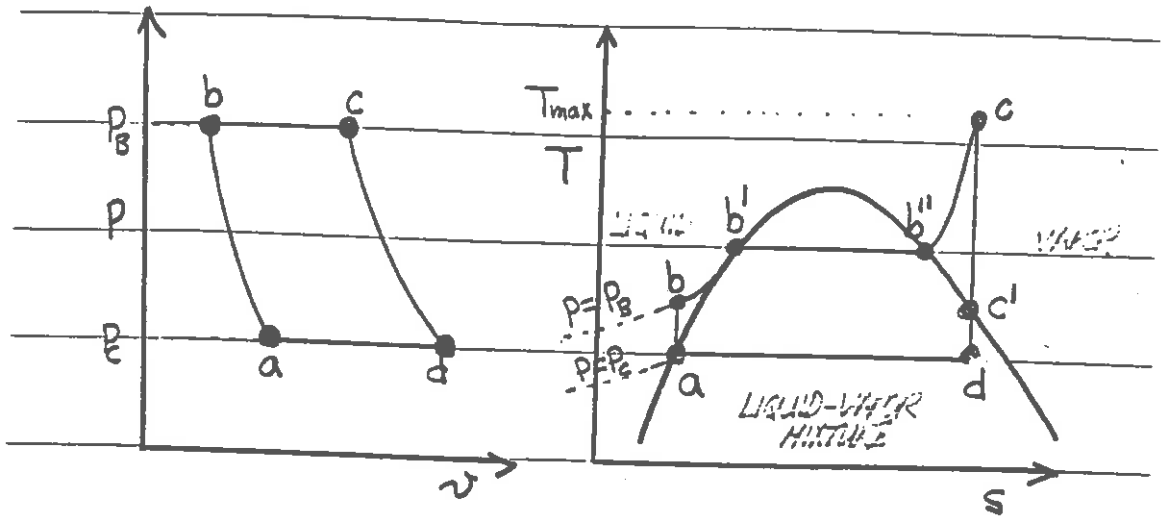
b'→b'': boiling liquid to vapor

b''→c: superheating vapor to T_{max}

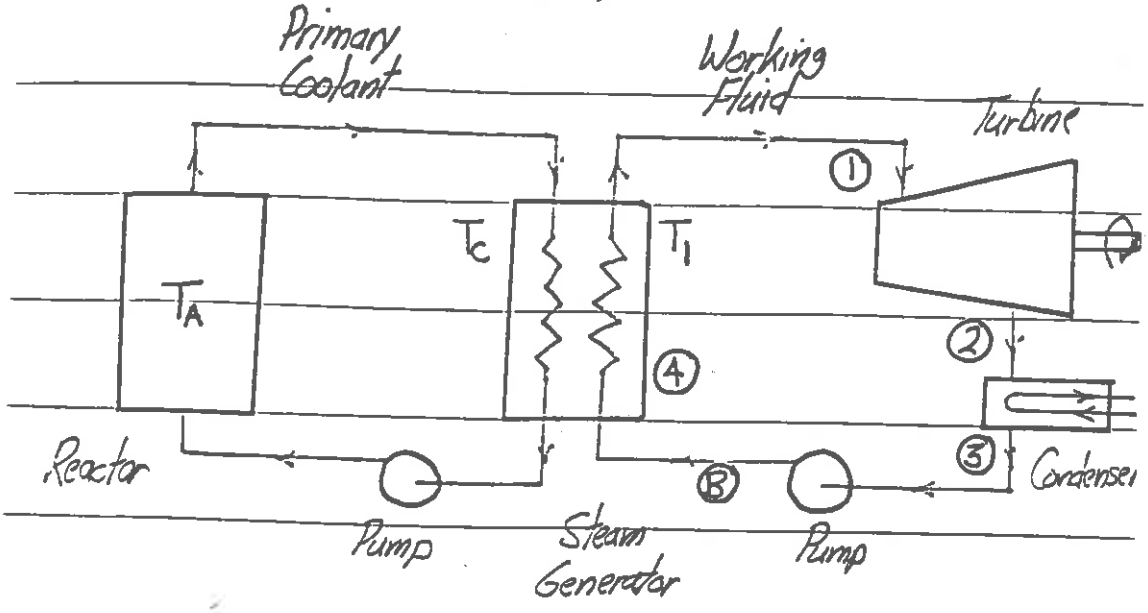
c→d: isentropic expansion in turbine (c→c', vapor; c'→d, condensation

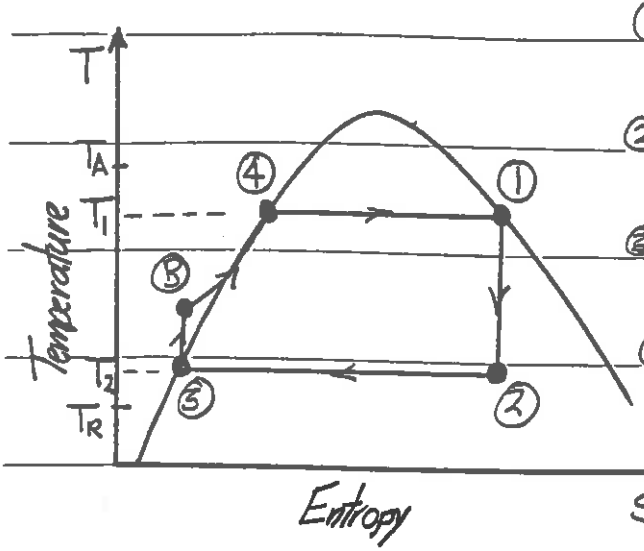
d→a: constant pressure cooling in condenser

Diagrams of Rankine Cycle



PWR System





① → ②: adiabatic expansion

② → ③: vapor condensation

② → Ⓑ: condensate pumped

Ⓑ → ④ → ①: heat addition

Example: Steam thermal cycle (typical parameters)

Conditions: Boiler pressure: $P_B = 2000$ psia

Max steam temp: $T_{max} = 1040°F = 1500°R$

Condenser temp: $T_c = 40°F = 500°R$

Condenser pressure: 0.1217 psia

a→b: From steam tables $h_a = 8.05$ Btu/lbm

$s_a = 0.0162$ Btu/lbm-F

From tables $s_b = s_a$

$h_b = h_a + 5.9 = 8.05 + 5.9 = 13.95$ Btu/lbm

Hence pump work is

$W_{pump} = W_{ab} = h_b - h_a = 5.9$ Btu/lbm

b→c: Heat addition in boiler: From tables

$$h_c = 1474.5 + \frac{1505.7 - 1474.5}{1050 - 1000}(1040 - 1000) = 1499.5 \text{ Btu/lb}$$

$$s_c = 1.5603 + \frac{1.5813 - 1.5603}{1050 - 1000}(1040 - 1000) = 1.5771 \text{ Btu/lbm}$$

Hence heat input to cycle is

$Q_{in} = h_c - h_b = 1499.5 - 13.95 = 1485.5$ Btu/lbm

c→d: Need to determine the fraction of vapor -- quality x --

$$s = x s_g + (1-x) s_f$$

$$1.5771 = x(2.1597) + (1-x)(0.0162) \Rightarrow x = 0.728$$

Hence enthalpy follows

$$h_d = x h_g + (1-x) h_f = h_f + x h_{fg}$$

$$= 8.05 + 1071.3x = 791 \text{ Btu/lbm}$$

Thus work output through turbine shaft is

$$W_{out} = h_c - h_d = 1499.5 - 791 = 708.5 \text{ Btu/lbm}$$

The net work output is the shaft work less the pumping power

$$W_{net} = W_{out} - W_{pump} = 708.5 - 5.9 = 702.6 \text{ Btu/lbm}$$

d→a: constant pressure cooling in condenser

Heat rejected is $Q_{out} = h_a - h_d = 791 - 8.05 = 783$ Btu/lbm

Summary: $Q_{in} = 1485.5$ Btu/lbm

$W_{net} = 702.6$ Btu/lbm

$Q_{out} = 783$ Btu/lbm

$W_{pump} = 5.9$ Btu/lbm

Thermal efficiency

$$\eta = \frac{W_{net}}{Q_{in}} = \frac{702.6}{1485.5} = 47.2\%$$

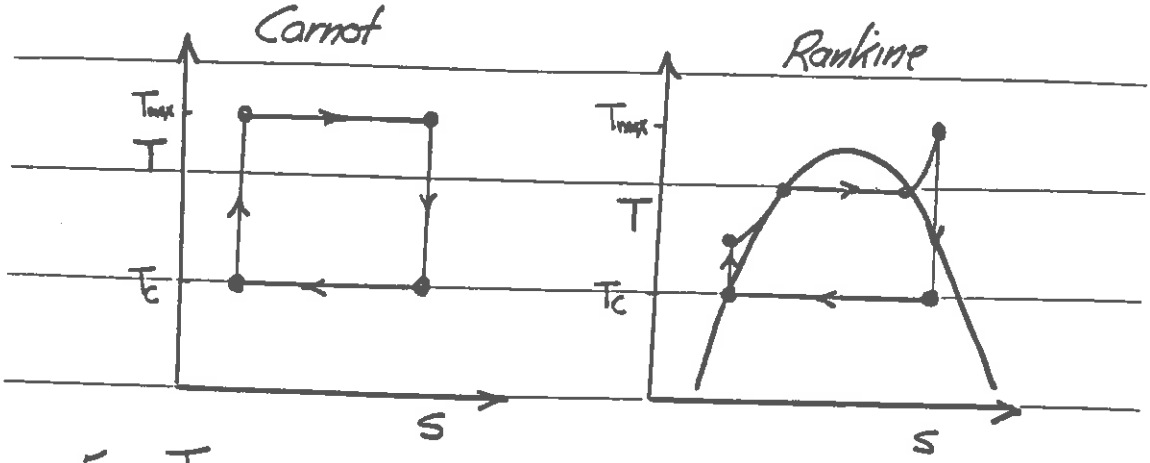
Notice something:

Rankine cycle: $\eta = \frac{W_{net}}{Q_{in}} = 47.2\%$

Carnot cycle: $\eta = 1 - \frac{T_c}{T_{max}} = 66.7\%$

Even though the Rankine cycle is idealized, there is still a significant loss in efficiency from the Carnot cycle operating at the same temperatures. (Primarily because in Carnot cycle, all heat is added isothermally at T_{max}.)

Note loss in efficiency



For $T_{max} = 1500\,R$, $T_c = 500\,R$

$\eta_C = 1 - \frac{500}{1500} = 66.7\%$, $\eta_R = 47.2\%$

Another problem:

Rankine cycle leads to a mixture of vapor and considerable liquid in turbine. Lowers turbine efficiency and causes erosion damage of blades due to liquid droplets.

LECTURE 7

PRACTICAL MODIFICATIONS TO THE RANKINE CYCLE

READING ASSIGNMENT:

El-Wakil (Nuclear Energy Conversion), pp. 22 - 55
J. Anno, "Thermodynamics of Nuclear Power Systems", Chapter 3 in Weisman (Design)
Rust, Chapter 2

EXERCISES:

7-1. Sketch the reheating configurations used in a typical PWR plant (working from FSARs, for example).

7-2. Demonstrate that the thermodynamic efficiency achieveable in an ideal regenerative cycle approaches that of the Carnot cycle.

7-3. Verify the effect of increased boiler pressure on the thermodynamic cycle path on a T-s diagram.

7-4. Why is superheat not utilized in PWR or BWR plants?

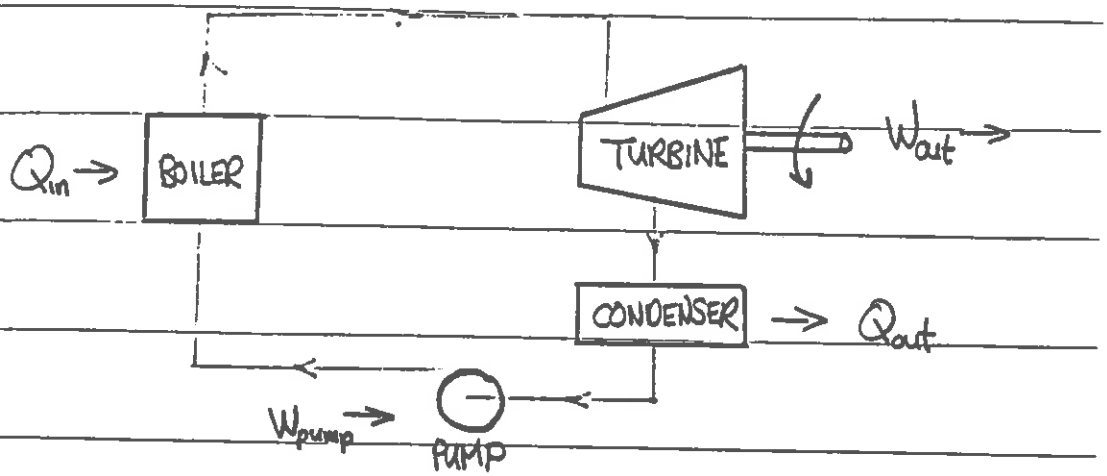
7-5. Provide a detailed explanation of the departure of the T-s path from ideal for a real turbine and pump.

Modifications to the Rankine Cycle

- The Rankine Cycle – Revisited
- Reheating Cycles
- Regenerative Cycles
- Other ways to improve efficiency

The Rankine Cycle – Revisited

Based on a condensable working fluid such as steam:



Limitations of the Rankine Cycle

a→b: isentropic compression in pump

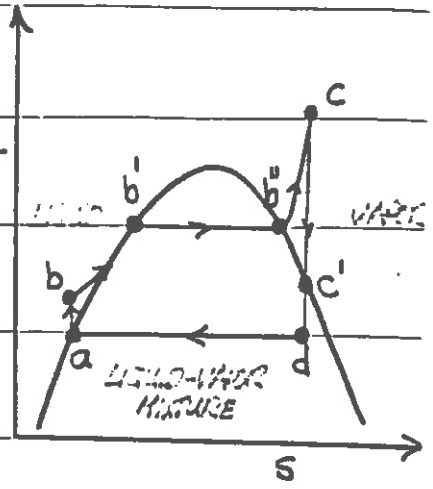
b→c: constant pressure heating in boiler

- b→b': subcooled
- b'→b'': boiling
- b''→c: superheat

c→d: isentropic expansion in turbine

- c→c': vapor
- c'→d: condensation

d→a: constant pressure cooling in condenser



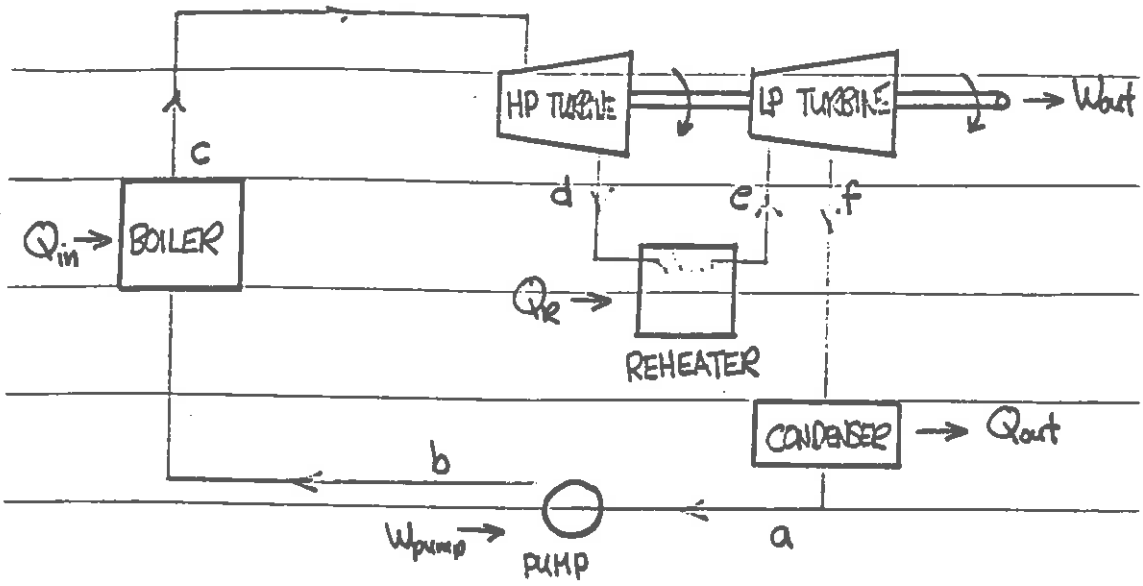
Limitations:

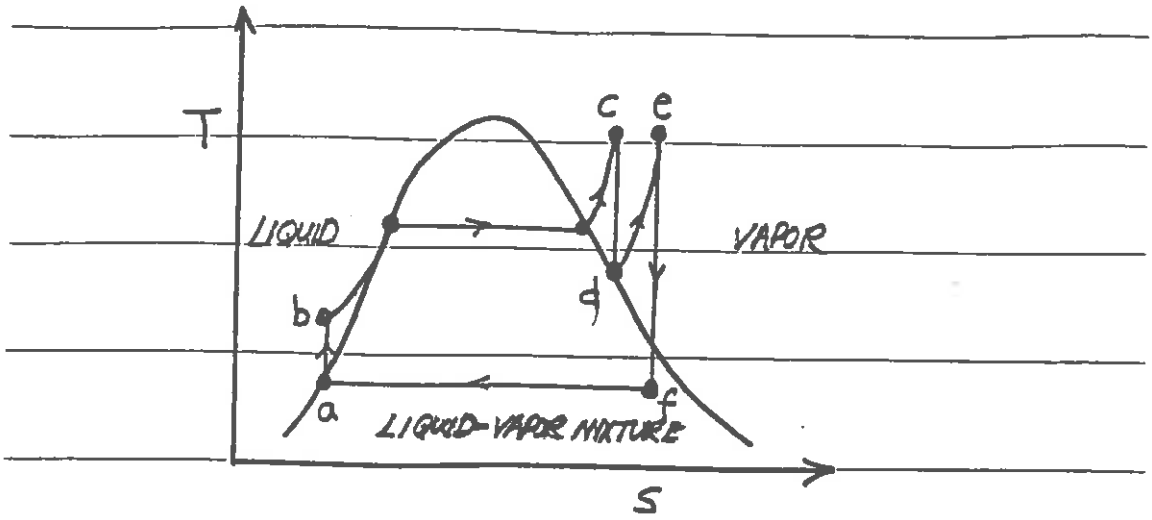
- low temperature heat addition
- condensation in turbine

Reheating Cycles:

Idea: Steam is removed from turbine before reaching saturation temperature and reheated in another device before passing through low pressure stages of turbine for further expansion

Reheating Cycle





Note: On the average, the reheating cycle adds heat at a higher temperature than the simple Rankine cycle

Hence reheating cycles yield two improvements

- lower moisture content in low pressure turbine stages
- improve thermal efficiency by allowing higher (average) temperature heat addition (thereby coming closer to ideal of Carnot cycle).

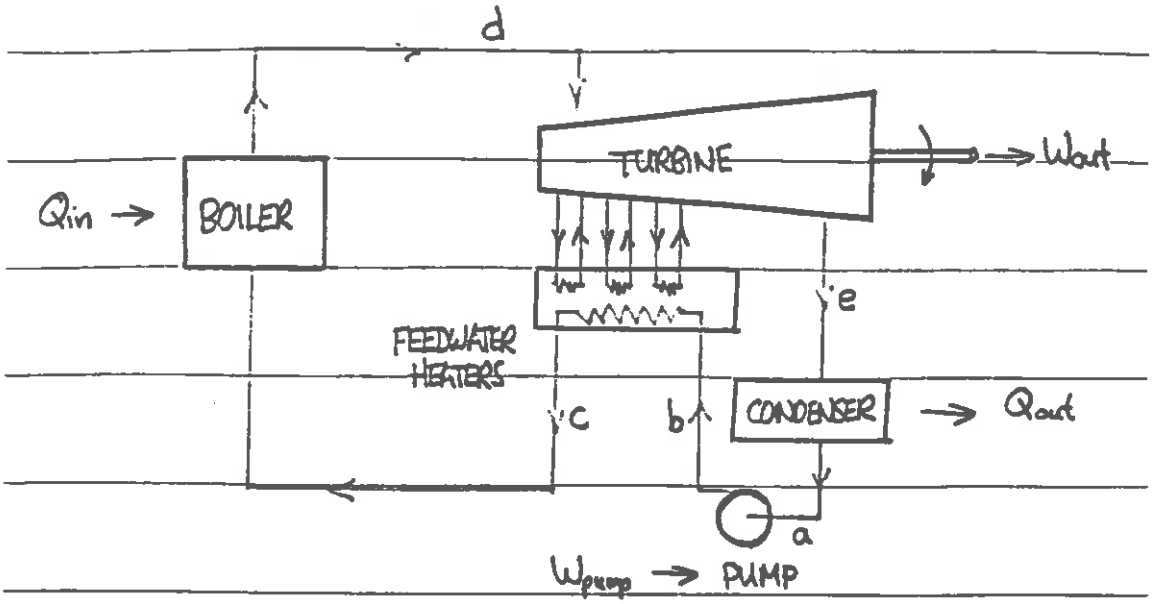
Obviously one could use several reheat stages.

Regenerative Cycle

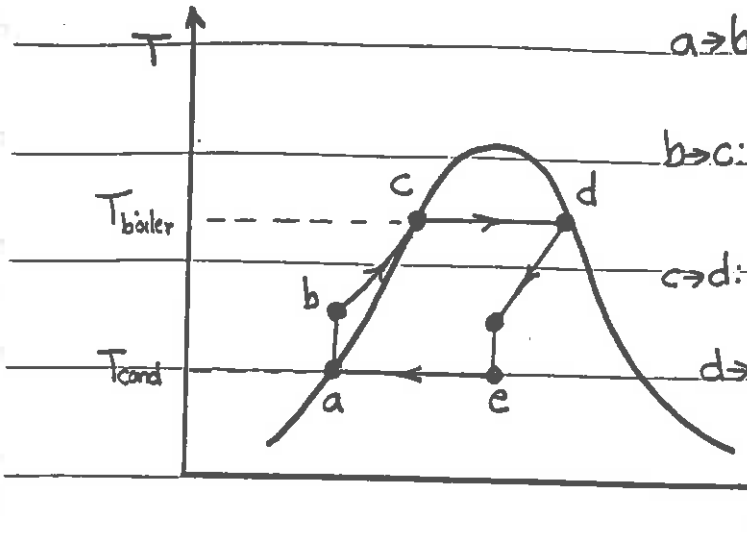
Idea: Limit heat addition to feedwater at low temperatures by transferring heat from turbine to feedwater via feedwater heaters.

(Recall Carnot efficiency $\eta_c = 1 - \frac{T_{REJECT}}{T_{ADD}}$)

Regenerative Cycle



Regenerative Cycle (idealized)



a→b: isentropic compression in pu

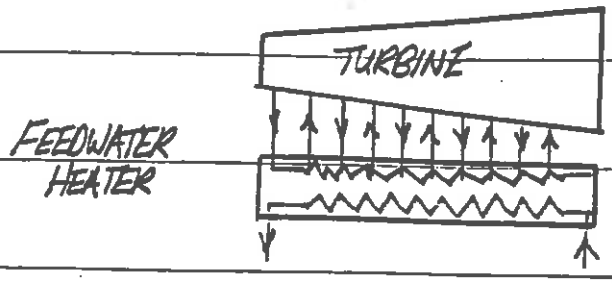
b→c: constant p heating in feedwat heater (heat from turbine)

c→d: constant p boiling

d→e: expansion in turbine with simultaneous heat transfer to feedwater

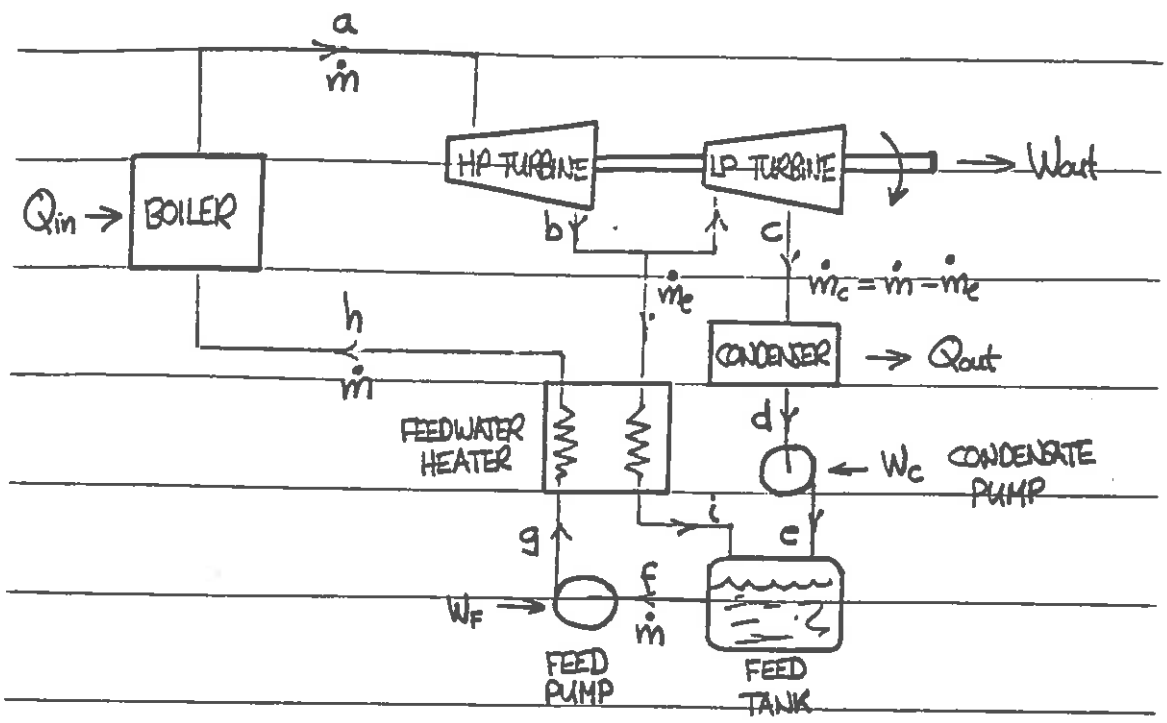
e→a: constant p condensation

In idealized regenerative cycle, all steps are reversible. Hence we can imagine local vapor temperature in turbine is kept only infinitesimally greater than the local temperature of the liquid in the feedwater heater:



Ideally, $\eta \to \eta_{CARNOT} = 1 - \frac{T_{CONDENSER}}{T_{CARNOT}}$

Practical (albeit simple) Regenerative Cycle

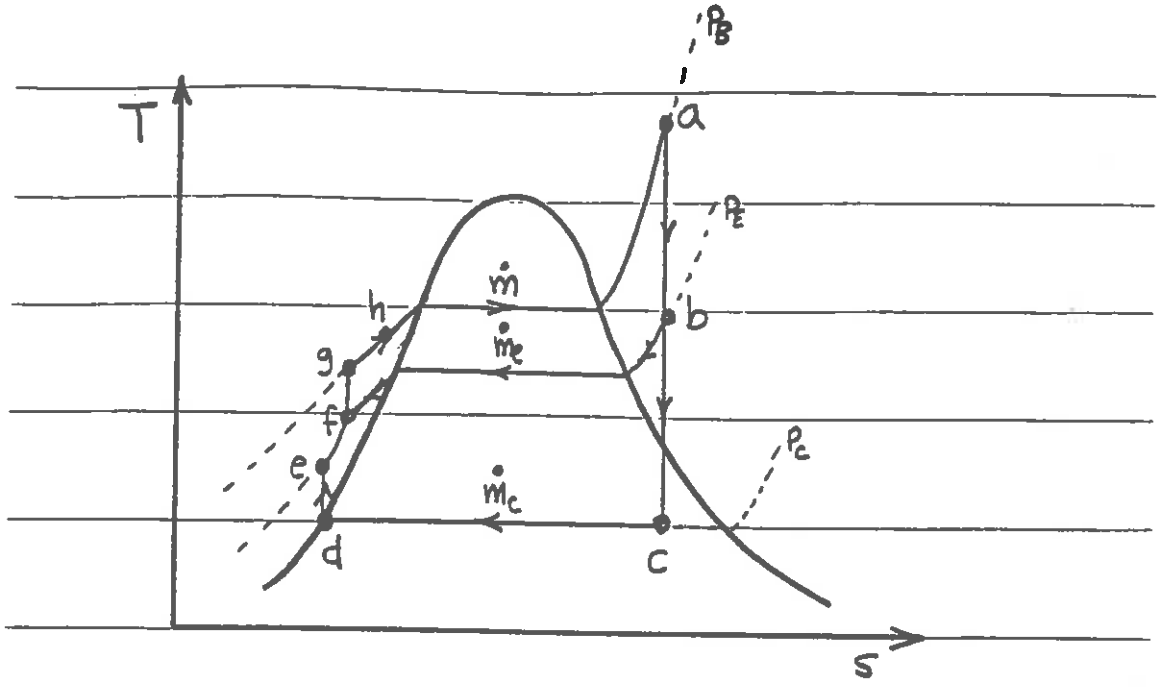


a→b: expansion to extraction pressure at b, stream splits

extraction stream:
- b→i: heat is given to feedwater stream
- i→f: stream is combined with condensate stream in tank

condensate stream:
- b→c: expansion to condenser pressure
- c→d: heat rejected in condenser
- d→e: pumped to pressure of extraction stream
- e→f: stream combined with extraction stream in feed tank

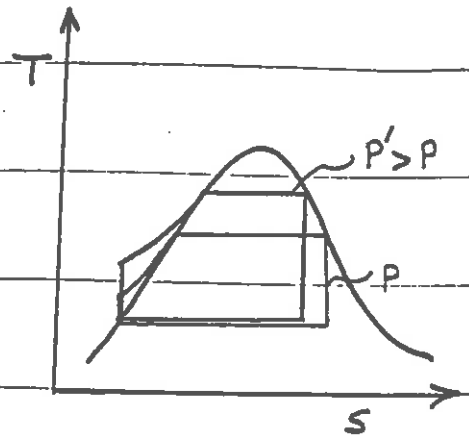
feedwater stream:
- f→g: pumped to boiler pressure
- g→h: heated by extraction stream in heat exchanger
- h→a: heated, vaporized, and superheated in boiler



In practice, use a number (~ 5) of feedwater heaters, approximately equally spaced between $T_{CONDENSER}$ and T_{BOILER}.

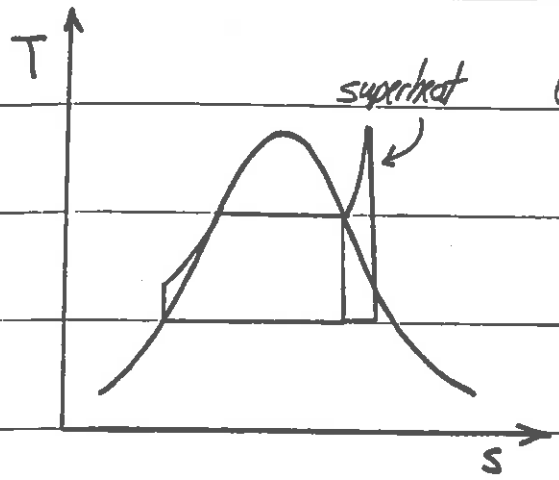
Other ways to improve the efficiency of the Rankine cycle:

Increasing boiler pressure: causes an increase in net work and decrease in heat rejected.



(But also increases moisture content of steam.)

Superheat: causes a net increase in temperature at which heat is added.



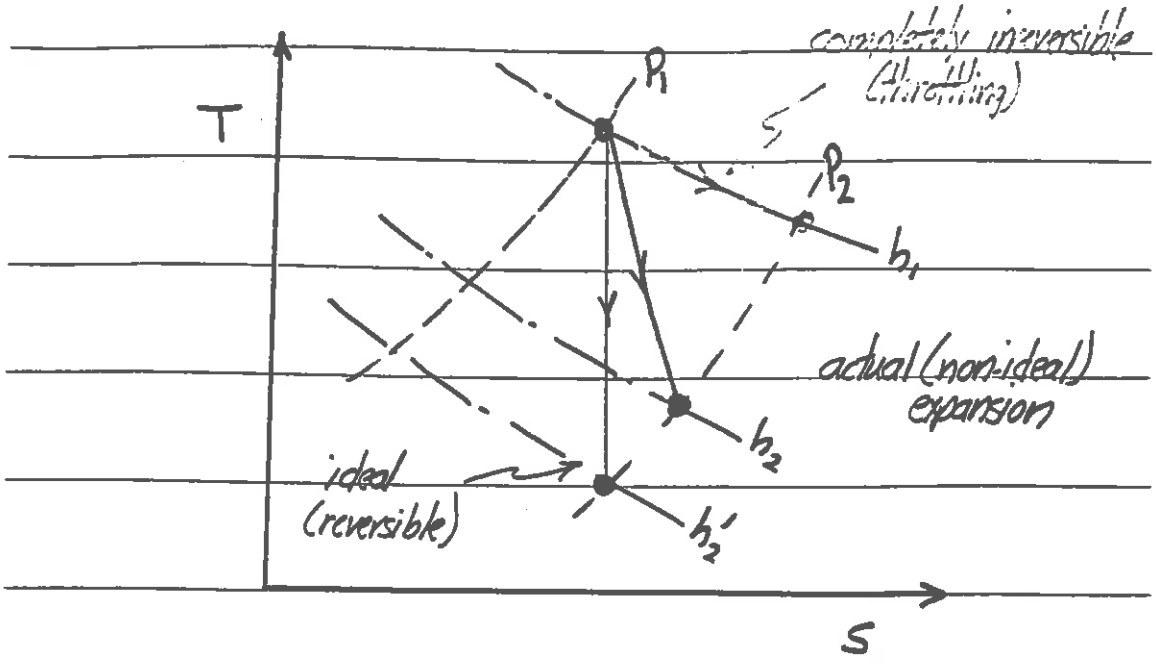
(But requires a high temperature heat source.)

Other Departures from Ideal Performance

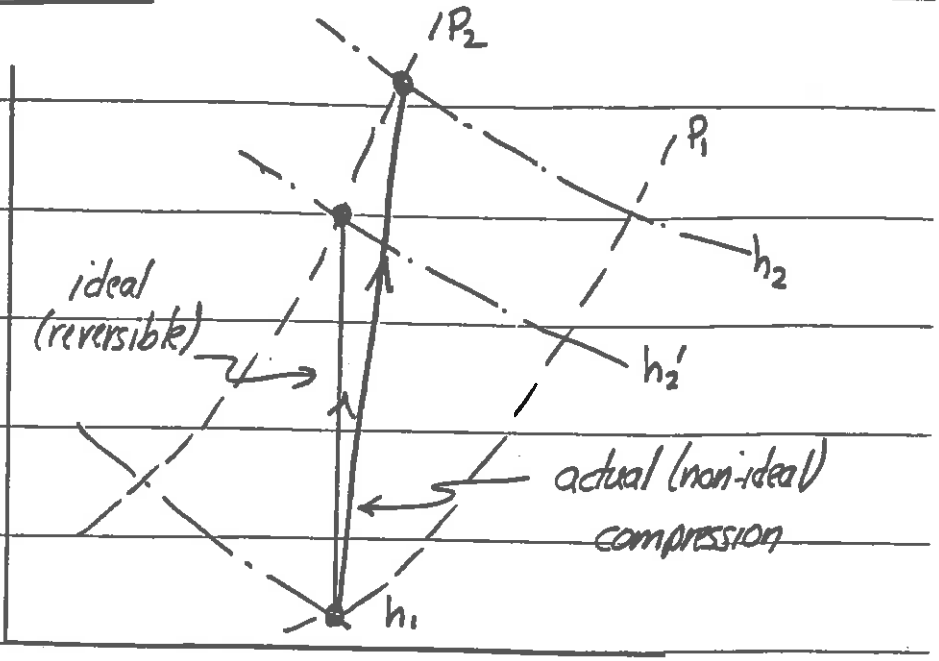
Irreversibilities in turbines & pumps:

- turbulence
- friction in fluid
- bearing friction

Turbine behavior



Pump behavior



Nuclear Power Systems as Thermodynamic Cycles

- Systems based on the Rankine Cycle
  (BWR, PWR, Candu, LMFBR, HTGR)
- Advanced cycles -- the Brayton cycle

LECTURE 8

NUCLEAR POWER SYSTEMS AS THERMODYNAMIC CYCLES

READING ASSIGNMENT:

Lamarsh, pp. 341 - 344
El-Wakil (Nuclear Energy Conversion), pp. 22 - 55
J. Anno, "Thermodynamics of Nuclear Power Systems", Chapter 3 in Weisman (Design)
Rust, Chapter 2
Sesonske, pp. 93 - 100

EXERCISES:

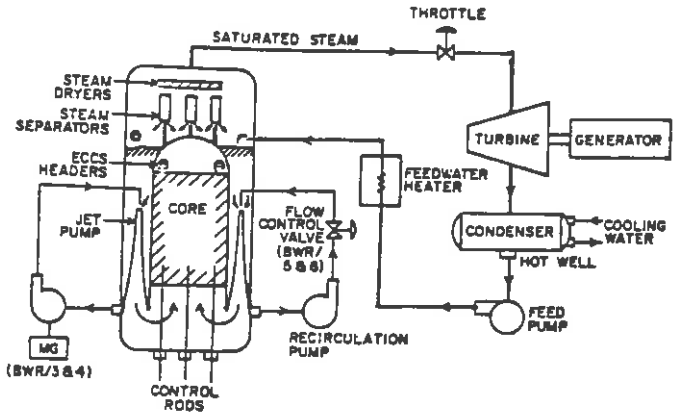
8-1. By referring to any standard text on thermodynamics, identify and explain several thermodynamic cycles other than the Rankine and Brayton. Explain why these are not serious candidates for nuclear power applications.

8-2. Provide the details in the calculation of the thermodynamic efficiency of the Brayton cycle.

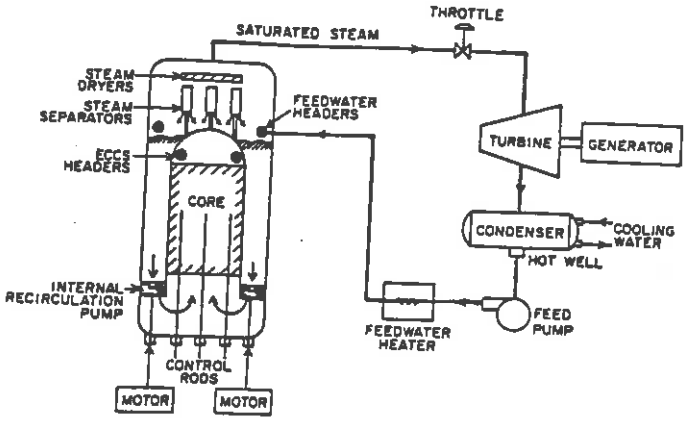
8-3. Estimate the maximum thermodynamic efficiency that would be possible (under realistic pressure conditions) for gas-cooled nuclear reactors based on He and CO-2.

Nuclear Systems based on the Rankine Cycle

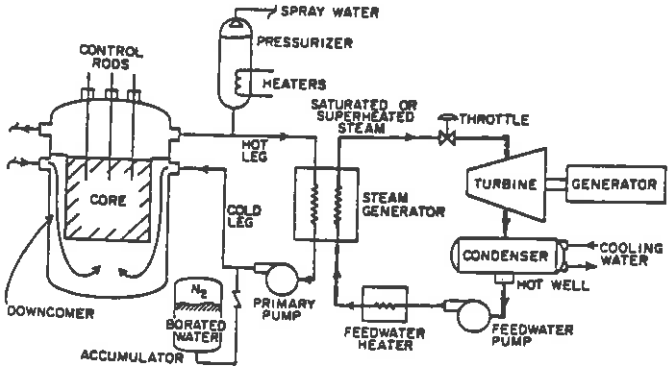
- BWR – direct cycle
- PWR – indirect (2) cycles
- Candu – indirect (2) cycles
- LMFBR – indirect (3) cycles
- HTGR – indirect (2) cycles



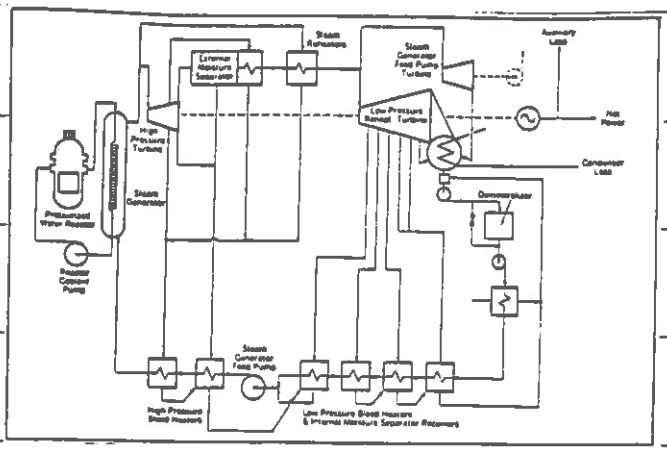
Modern BWR

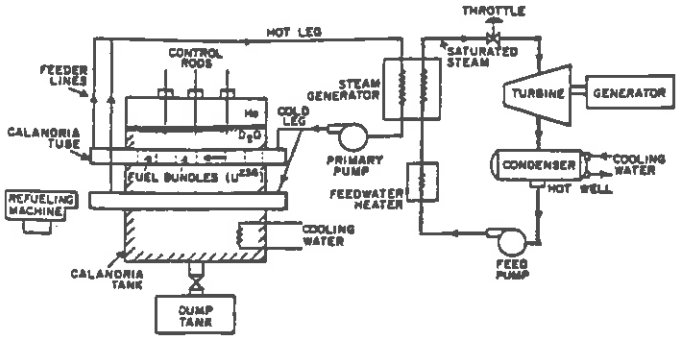


Modern BWR

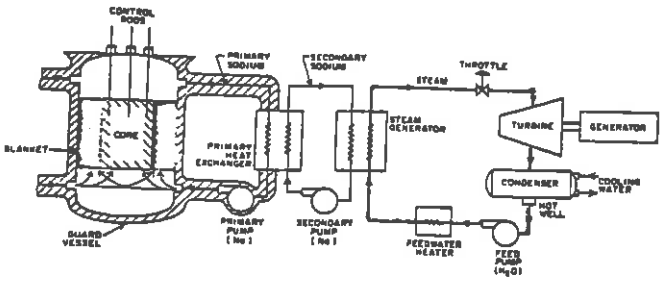


Modern PWR

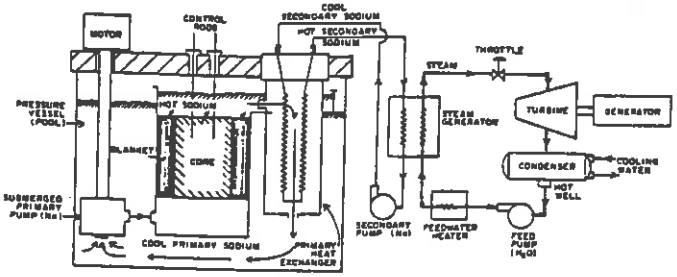



HOT LEG
THROTTLE
CONTROL RODS
FEEDER LINES
SATURATED STEAM
STEAM GENERATOR
TURBINE
GENERATOR
CALANDRIA TUBE
COLD LEG
PRIMARY PUMP
CONDENSER
COOLING WATER
FUEL BUNDLES
FEEDWATER HEATER
HOT WELL
REFUELING MACHINE
COOLING WATER
FEED PUMP
CALANDRIA TANK
DUMP TANK

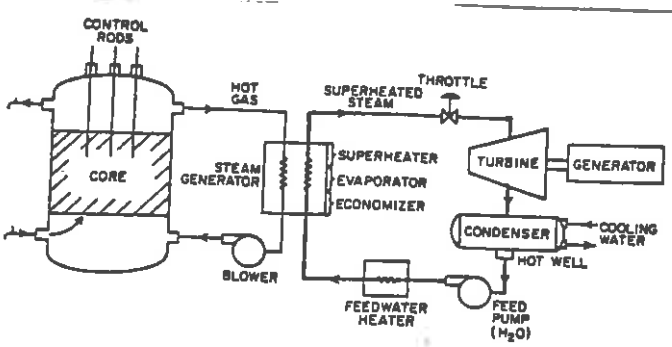
CANDU (HWR)



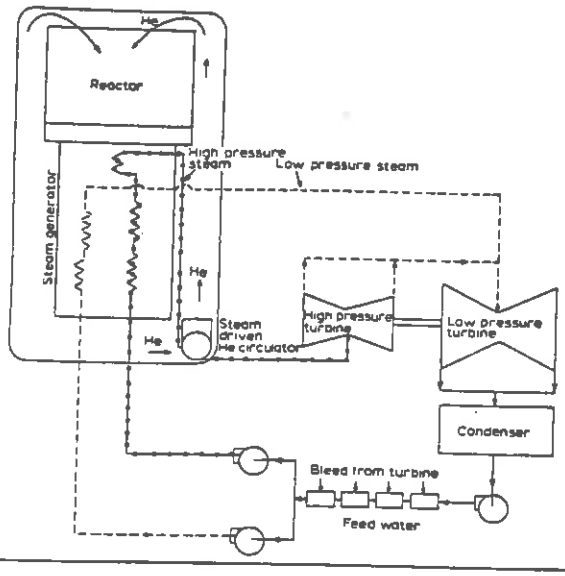
Loop Type LMFBR

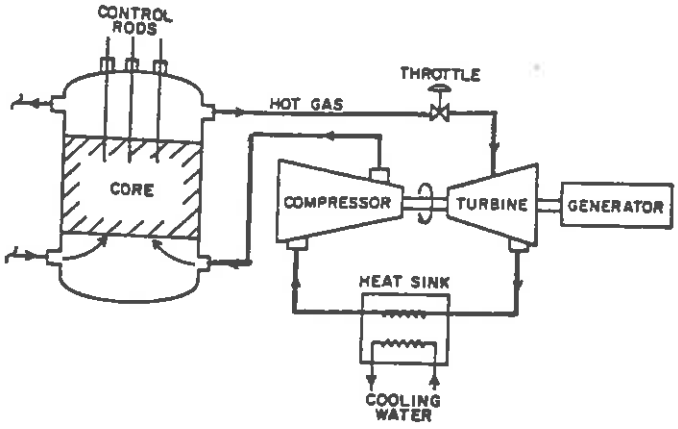


Pool Type LMFBR



Indirect, Closed Cycle High Temperature Gas-Cooled Reactor (HTGR)

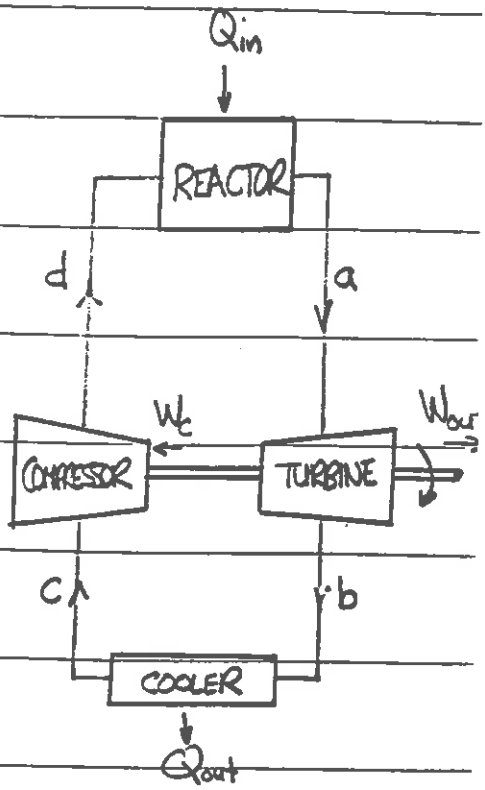




Direct, Closed Cycle High Temperature Gas-Cooled Reactor

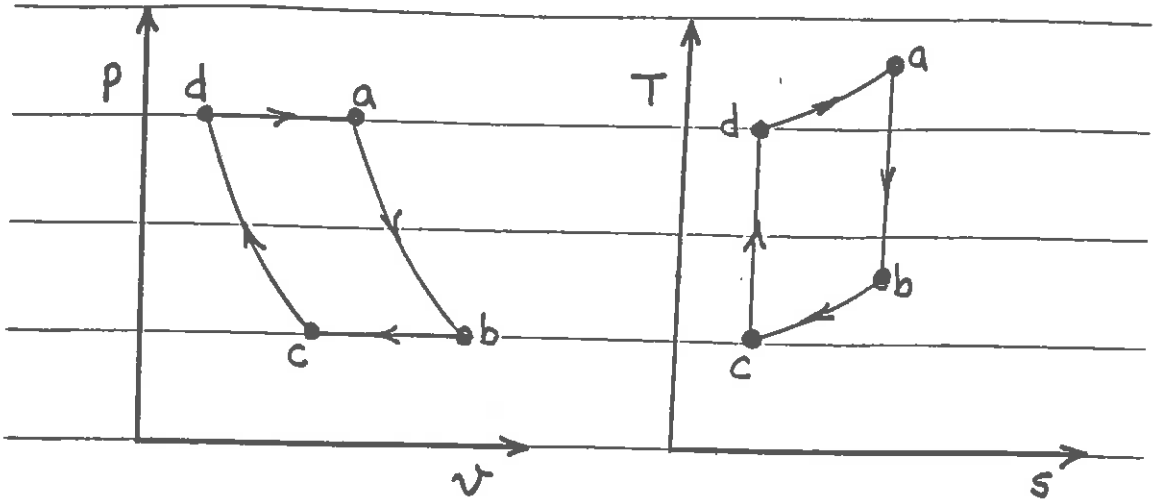
The Brayton Cycle

Advanced gas-cooled reactors may some day use the Brayton cycle (similar to that used in aircraft gas turbines)



a→b: expansion of high pressure gas to exhaust pressure

b→c: cooling at constant pressure



c→d: compression to working pressure

d→a: heating at constant pressure

Exercise: Calculate efficiency of Brayton cycle assuming perfect gas behavior: $p v^{\gamma} = \text{constant}$

a→b: isentropic expansion in turbine

$$P_a v_a^{\gamma} = P_b v_b^{\gamma} \; ; \quad T_a P_a^{\frac{\gamma-1}{\gamma}} = T_b P_b^{\frac{\gamma-1}{\gamma}}$$

$$W_{ab} = h_a - h_b = c_p (T_a - T_b)$$

b→c: isobaric cooling in cooler

$$Q_{bc} = c_p (T_c - T_b)$$

c→d: isentropic compression in compressor

$$T_d P_d^{\frac{\gamma-1}{\gamma}} = T_c P_c^{\frac{\gamma-1}{\gamma}} \; ; \quad W_{cd} = c_p (T_c - T_d)$$

d→a: isobaric heating in heater

$$Q_{da} = c_p (T_c - T_d)$$

Net work out is

$$W_{net} = W_{ab} + W_{cd} = c_p \left[(T_a - T_b) + (T_c - T_d) \right]$$

Heat input is

$$Q_{in} = Q_{da} = c_p (T_a - T_d)$$

It is most convenient to re-express all quantities in terms of high and low pressures, $P_1 = P_a$, $P_2 = P_b$,

$$\frac{T_b}{T_a} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \; ; \; \frac{T_c}{T_d} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

This yields

$$\eta_{BRAYTON} = \frac{W_{net}}{Q_{in}} = 1 - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

-- which depends only on the ratio of working pressures, $r_p = \frac{P_1}{P_2}$, and isentropic exponent, $\gamma = \frac{c_p}{c_v}$:

Gas	γ	$\frac{\gamma-1}{\gamma}$
He	1.659	.398
H_2	1.405	.288
CO_2	1.29	.225
air	1.4	.286
steam	1.335	.251

LECTURE 9

FISSION HEAT SOURCES

READING ASSIGNMENT:

J. Weisman, "Heat Generation and Transport", Chapter 6 in Weisman (Design)
Lamarsh, pp. 345 - 352
Rust, Chapter 5

EXERCISES:

9-1. Compare the average thermal power densities (q', q", q"') of each of the major reactor types (e.g., PWR, BWR, LMFBR, GCFR) along with that of the boiler of a fossil-fuel-fired plant. Use data from PSAR's for actual plants, if possible.

9-2. Discuss the importance of each of the following effects on fission heat source distributions: i) nonuniform fuel loading, ii) nonuniform fuel burnup, iii) control elements, iv) fuel assembly structure.

9-3. Calculate the mass of fuel fission per MWD (megawatt-day) burnup of U-235, U-238, and Pu-239.

9-4. Calculate the volumetric power density at the midpoint of a uniform, bare cylindrical reactor core generating 1,200 MWe. (Assume typical thermodynamic efficiencies.)

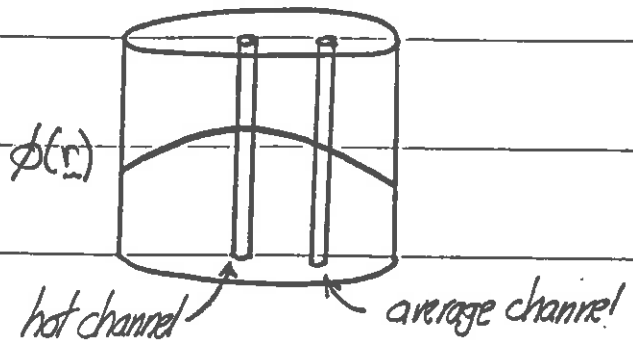
Fission Heat Sources

- An overview of core thermal analysis
- Some notation concerning power densities
- Fission heat source distributions
- Power generation in nuclear reactor cores

Thermal Analysis

Goal: Determine $T(\underline{r})$ for a given $\phi(\underline{r})$ to make sure temperature limits are not exceeded

(Use a "worst possible case" or "hot channel" approach)

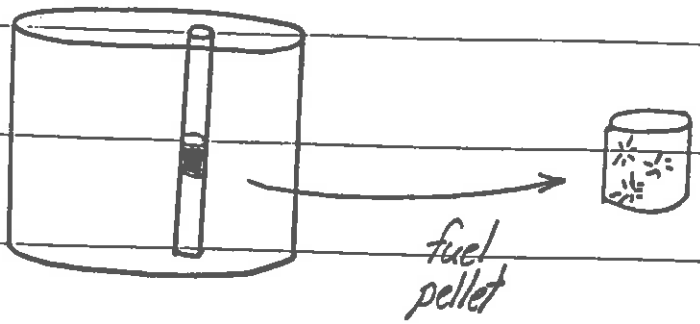


General approach:

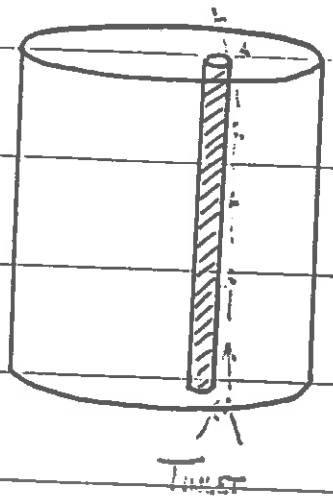
i) Determine volumetric fission heat source

$q'''(\underline{r}) \leftrightarrow P(\underline{r}) \leftrightarrow \phi(\underline{r})$

heat source — power density — neutron flux



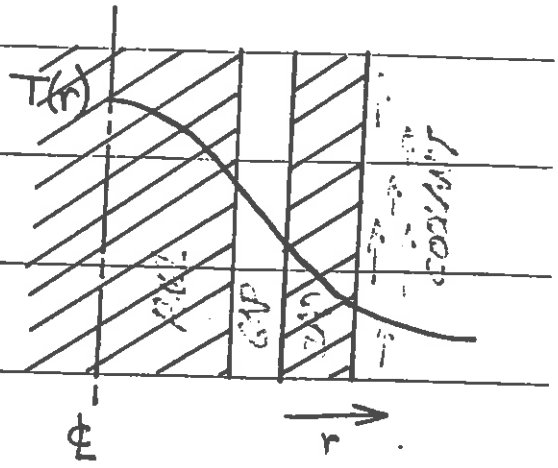
ii) In a steady-state, any heat produced in fuel must pass into coolant. Hence we can calculate temperature rise of coolant (given inlet conditions). (First law of thermodynamics)



iii) With this knowledge of $T_{cool}(z)$, we can work back into fuel to calculate temperature distribution in fuel $T(r)$ for each point z.

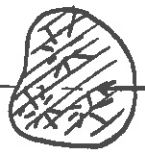
a) Newton's law of cooling
clad surface → coolant

b) Fourier's law of heat conduction
fuel-gap-clad



Some notation concerning power densities:

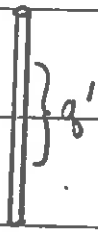
q''' → volumetric heat source (W/cm^3)



q'' → heat flux (W/cm^2)



q' → linear power density (W/cm)



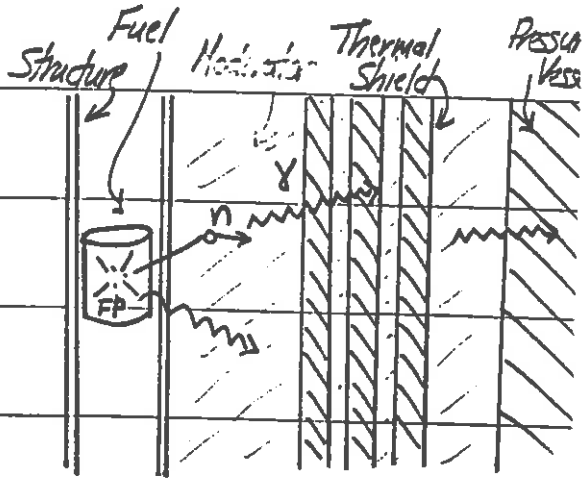
Fission Heat Source Distributions

coal or oil-fired boiler: constant temperature heat source

nuclear reactor: constant heat rate source

Very important to account for spatial and temporal variation.

Fission Heat Sources



Fission debri
- fission product KE
- prompt γs
- prompt neutrons

Decay products
- decay β
- decay γ
- neutrinos ν

Fission Energy Distribution	Energy	Range
Fission fragment KE	168 MeV	Short
Fission neutron KE	5	Medium
Prompt gammas	7	Long
FP decay β	8	Short
FP decay γ	7	Long
Neutrinos	12	Non-recoverable
Neutron capture decay	7-10	Short ; Long

Fission Energy Release for Various Isotopes

Isotope	Recoverable energy release
^{232}Th	196.2 ± 1.1
^{233}U	199.0 ± 1.1
^{235}U	201.7 ± 0.6
^{238}U	208.5 ± 1.1
^{239}Pu	210.7 ± 1.2
^{241}Pu	213.8 ± 1.0

Some rules of thumb:

Spatial:

~90% of recoverable energy deposited in fuel

~10% in structure, moderator, shields, vessel

Temporal:

~90% of recoverable energy released promptly

~10% from seconds to years later

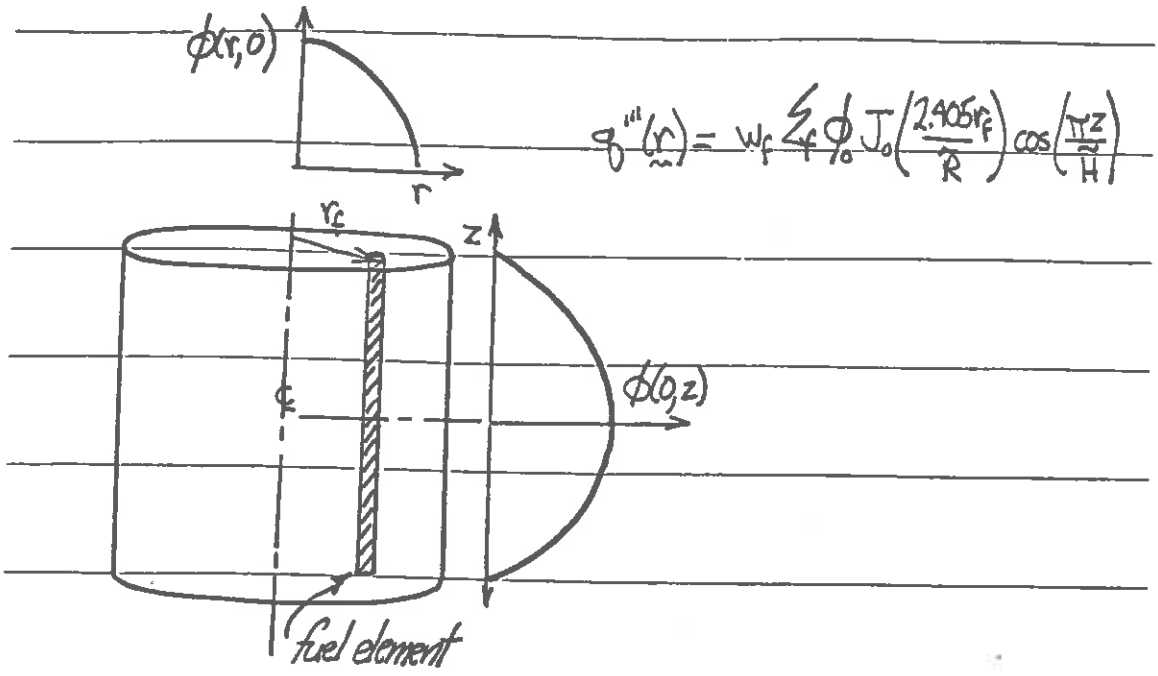
Power generation in nuclear reactor cores

Just use volumetric fission heat source

$$q'''(\underline{r}) = \sum_i w_f^{(i)} N_i(\underline{r}) \int_0^\infty dE\, \sigma_f^{(i)}(E)\, \phi(\underline{r}, E)$$

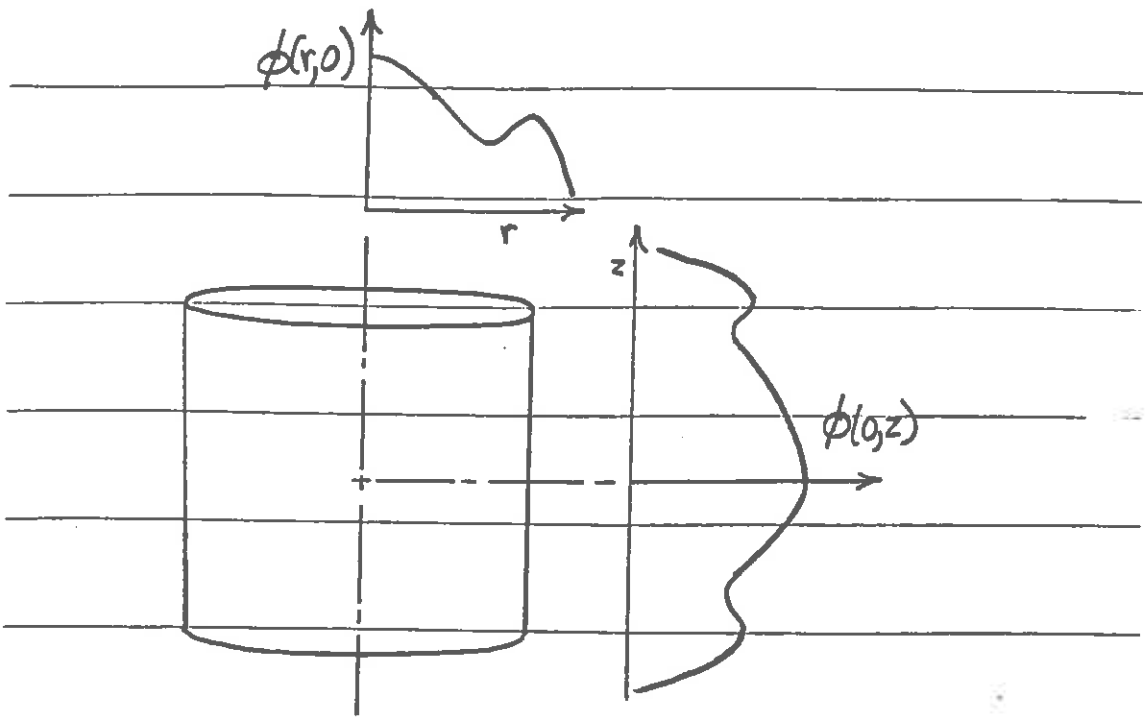
Note: Sometimes reassign energy deposition in coolant and structural materials to fuel to simplify analysis

Simplest model: bare, homogeneous core



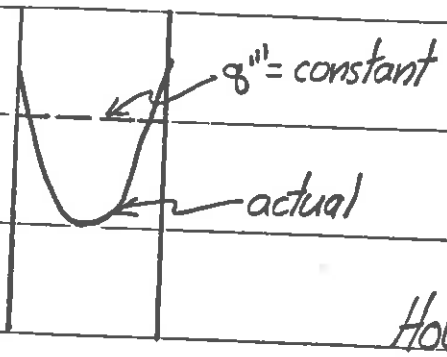
$$q'''(\underset{\sim}{r}) = w_f \Sigma_f \phi_0 J_0\left(\frac{2.405 r_f}{R}\right) \cos\left(\frac{\pi z}{H}\right)$$

Modification #1: Reflectors



Modification #2: Flux depression in fuel pin

Recall that self-shielding causes a flux depression in fuel pin



However it is common to assume a constant q''' and correct for the flux depression later.

Other modifications:

i) nonuniform fuel loading

ii) nonuniform fuel burnup

iii) control rods

iv) fuel assembly structure

We take these into account using "hot channel" or "power peaking" factors. (Thermal designer only needs to know q'(z) and hot channel factors.)

LECTURE 10

HEAT GENERATION IN REACTOR CORES

READING ASSIGNMENT:

J. Weisman, "Heat Generation and Transport", Chapter 6 in Weisman (Design)
Lamarsh, pp. 345 - 352
Rust, Chapter 5

EXERCISES:

10-1. A U-235 fueled reactor operates at a thermal power of 3,500 MWt for 1.5 years. Using the fission product decay curves, estimate the decay heat release 1 hour after shutdown and 1 year after shutdown.

10-2. A BWR plant operating at an efficiency of 34% has an electrical output of 1011 MW. (a) What is the maximum fission product decay heat in the reactor at shutdown? (b) What is the decay heat 6 months after shutdown?

10-3. A 1000 MWe reactor is operated at full power for one year, then at 10% power for one month, and then shut down. Calculate the fission product decay heat at shutdown and for one month after shutdown.

10-4. A reactor rated at 1,000 MWe has operated at full power for some time prior to shutdown. Following shutdown, determine the amount of decay heat that must be removed from the core during i) the first hour following shutdown, and ii) the second hour following shutdown.

Heat Generation in Reactor Cores

- Heat generation within reactor components
- Reactor transients
- Fission product decay heating
- Hot channel factors

Heat generation within reactor components

Fuel elements

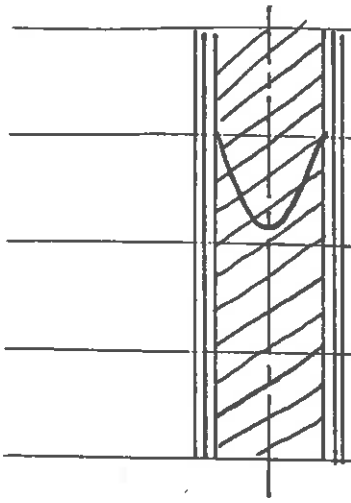
Structure and moderator

Thermal shields

Reactor fuel elements:

$$q'''(\underline{r}) = w_f N_F(\underline{r}) \int_0^\infty dE\, \sigma_f(E)\phi(\underline{r},E)$$

Must account for flux depression:



$$F = \frac{\phi_{SURFACE}}{\phi_{AVE}} \sim 1.0 - 1.1$$

$$= \frac{1}{P_F}$$

P_F = escape probability

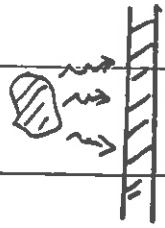
Structure and moderator:

metallic structure (fuel clad, fuel assembly cans, grids, ...)

heat sources due to gamma ray absorption

Since structural elements are thin, we can ignore attenuation:

$$q'''_\gamma \sim 0.105\, q''' \frac{\rho}{\rho_{ave}}$$



where q'''_γ heat source due to γs

q''' total heat source

ρ density of structural material

ρ_{ave} average density in core

moderator: heat generated by neutron thermalization (elastic scattering) and gamma absorption

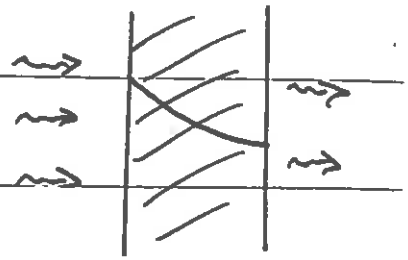
$$q'''_{gm} \approx 0.105\, q'''_{g} \frac{\rho_m}{\rho_{ave}} + w \Sigma_s \phi_{FAST} \Delta E$$

$$\Delta E = \frac{E_{FAST} - E_{TH}}{(\ln \frac{E_{FAST}}{E_{TH}})/\xi}$$

thick structural components (e.g., pressure vessel):

Now must account for gamma attenuation

$$\phi(x) = S_0 e^{-\mu x}$$



Hence

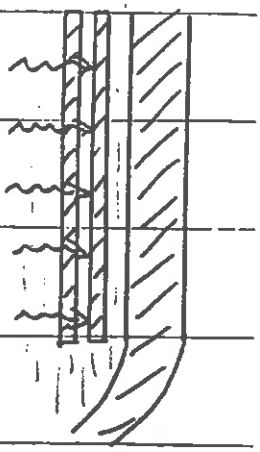
$$q'''_{\gamma} = \mu S_0 e^{-\mu x}$$

μ = energy absorption coefficient

[Also need buildup factors to account for Compton scattering.]

Thermal Shields

Thermal shields are used to protect vessel from high gamma flux.



Alternating water and steel also very efficient neutron shield.

Reactor transients

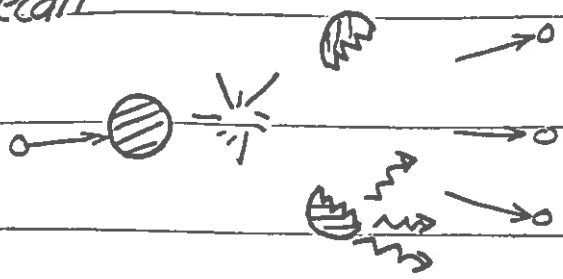
For reactor transients characterized by power levels greater than 50% of rated power:

$$q'''(t) \sim P(t) \sim \phi(t)$$

Below this level, however, fission product decay becomes an appreciable component of q''' and the time dependence may vary sharply from $\phi(t)$.

Fission Product Decay Heating

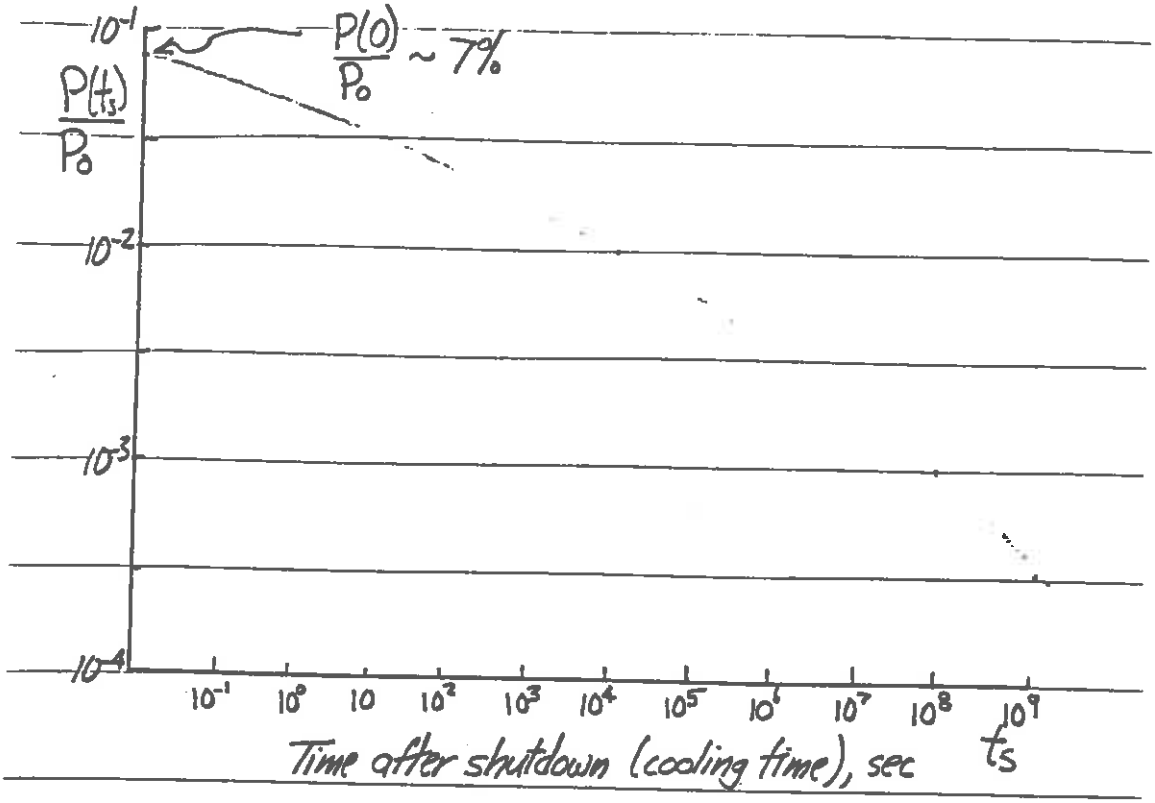
Recall



After a few days of reactor operation, FP γ and βs

$$q'''_{FP} \sim 7\% \; q'''_{TOTAL}$$

This is extremely important, since following reactor shutdown must still remove this decay heat.



One can also use simple formulas to estimate fission product decay rates:

$$R_\beta \sim 1.40\, t^{-1.2} \text{ MeV/fission-s}$$

$$R_\gamma \sim 1.26\, t^{-1.2} \text{ MeV/fission-s}$$

$$(10\text{s} < t < 100 \text{ days})$$

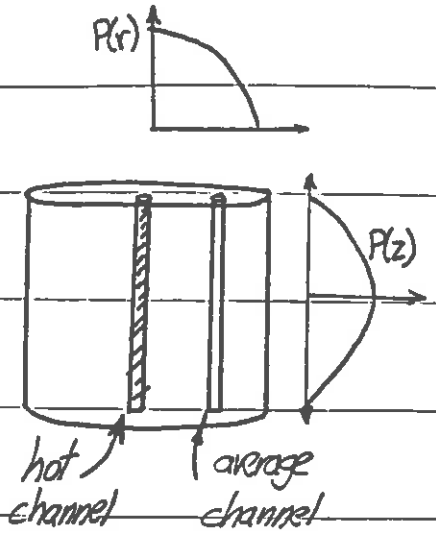
We can integrate over time T_o reactor operates at constant power level to determine fraction of power due to decay heat.

$$\frac{q''_\beta}{q'''_o} = 0.035\left[(t-T_o)^{-0.2} - t^{-0.2}\right]$$

$$\frac{q''_\gamma}{q'''_o} = 0.031\left[(t-T_o)^{-0.2} - t^{-0.2}\right]$$

Most commonly one uses empirical decay heat charts

A simple strategy: Can sometimes get by with only axial power distributions and hot channel factors



Radial hot channel factor

$$F_N^R = \frac{\text{ave heat flux in hot channel}}{\text{ave heat flux in ave channel}}$$

Axial hot channel factor

$$F_N^A = \frac{\text{max heat flux in hot channel}}{\text{ave heat flux in hot channel}}$$

Note: We can combine these to define a nuclear hot channel factor F_Q^N

$$F_Q^N = F_N^R F_N^A$$

$$= \frac{\text{max heat flux in core}}{\text{ave heat flux in core}}$$

It is common to solve only for

i) radial conduction in fuel element

ii) axial convection in coolant channel

Analyze both average channel and hot channel.

Immediate tasks:

- radial temperature distribution in fuel elements
  (heat conduction and forced convection)
- axial temperature distribution in coolant channels
  (energy balance – thermodynamics)
  (both single phase and two phase)

LECTURE 11

HEAT TRANSFER IN NUCLEAR REACTORS

READING ASSIGNMENT:

J. Weisman, "Heat Generation and Transport", Chapter 6 in Weisman (Design)
Duderstadt & Hamilton, Chapter 12
Rust, Chapter 6, 7

EXERCISES:

11-1. How might you expect the fuel temperature limitations to differ for:

i) a swimming pool reactor with MTR-type fuel elements

ii) a MAGNOX type (CO-2 cooled) reactor

iii) an HTGR

iv) a LMFBR

v) a GCFR

11-2. How might you expect clad thermal limitations to differ for each of the reactor types in 11-1.

11-3. How might you expect coolant temperature limitations to differ for each of the reactor types in 11-1.

11-4. In what situations might you expect thermal radiation to become an important heat transfer mechanism in reactors.

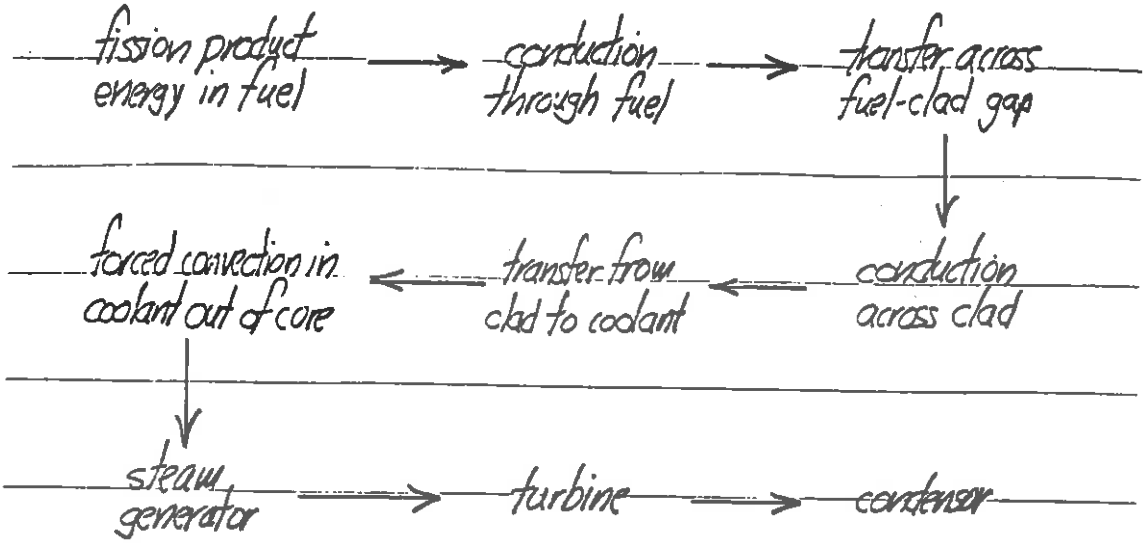
Heat Transfer in Nuclear Reactors

- Path of thermal energy flow
- Figures of merit in thermal design
- Thermal constraints
- Modes of heat transfer

Goals of thermal analysis

- determine temperature distribution in reactor core
- determine heat transfer rates throughout plant
- determine coolant characteristics (e.g., phase, density)
- determine plant thermodynamic efficiency

Approach: Follow path of thermal energy flow



(Note: steam is typically between plant thermal cycle and core.)

Some figures of merit:

- core power density [W/cm^3, kW/liter, ...]
- specific power [kW/kg U]
- coolant exit temperature
- linear power density [W/cm, kW/ft]
- fuel centerline temperature
- fuel or clad surface temperature
- fuel element surface heat flux [W/cm^2]

Objectives of Thermal Design

i) high core power density (to reduce required core volume for rated MWt hence pressure vessel size)

ii) high fuel specific power (to reduce required fuel inventory)

iii) high coolant exit temperatures (higher thermodynamic efficiency, higher temperature steam)

iv) linear power density q' ($T_\ell \propto q'$)

v) fuel centerline temperature ($T_\ell < T_{melt} \sim 2800\,^\circ C$)

vi) clad surface temperature ($T_{CLAD} < T_{failure}$)

viii) fuel element surface heat flux q''_s ($q''_s < q''_{DNB}$)

Fuel temperature limitations: prevent fuel temperature from exceeding melting point at any point in the core

$T_F < T_{melt}$: UO_2 – 2800 °C

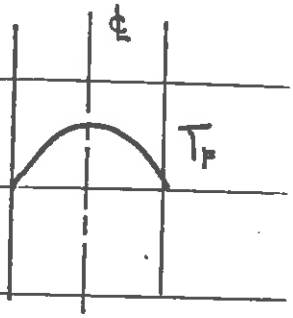
UC – 2500 °C

i) T_{melt} drops ~32 °C for each 10,000 MWD/MTU

ii) Maximum T_F occurs at fuel centerline

(can tolerate some ℄ fuel melting --

although fuel failure rate decreases)



iii) We will find

$T_F|_{max} \neq$ fcn (pin diameter)

$=$ fcn (q')

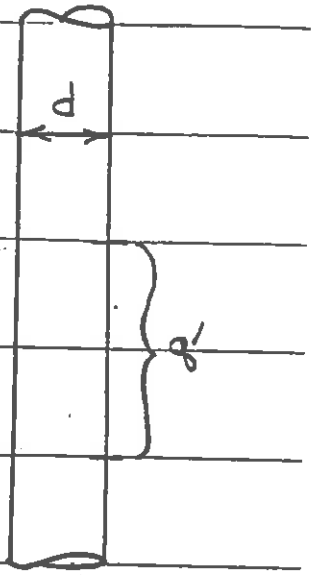
where q' = linear power density

Example: In UO_2-fueled reactors

limit: $q' < 660$ W/cm

(design reactors to operate with

$q'_{max} \sim 460-500$ W/cm)



Clad thermal limitations

T_{clad} rarely approaches T_{melt}

But large temperature gradients and other factors can cause clad failure

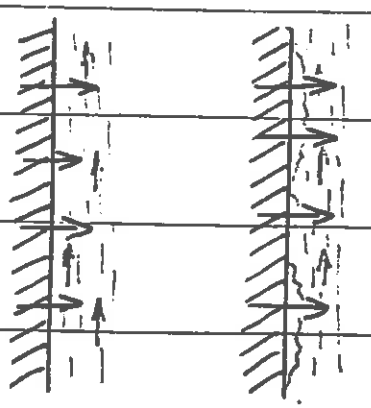
(Remember: clad serves as barrier to prevent fission products from escaping into coolant)

i) fuel pellet cracking or densification

ii) fission product gas pressure in fuel-clad gap

iii) limit heat flux across clad to avoid DNB (departure from nucleate boiling) or flow boiling crisis (surface boiling, film formation leads to rapid T_{clad} increase and clad failure for

$$q'' > q''_{DNB}$$



Coolant temperature limitations

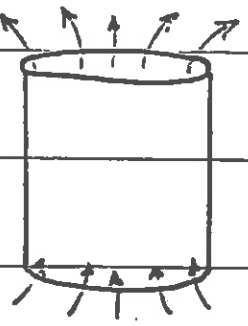
i) PWR: $T_{cool} < T_{boiling}$

LMFBR: T_{cool} sufficiently low to maximize heat transfer capability

ii) limits on ΔT_{cool} across core

(maximize for efficient heat transfer--
minimize to avoid thermal shock)

	PWR	LMFBR	HTGR
ΔT_{CORE}	20°	140°	500 °C



Other thermal limitations

i) nuclear limitations: moderation (reactivity)

ii) control limitations: control margins

Summary of limitations

i) avoidance of significant fuel centerline melting

ii) maintaining heat flux below maximum value allowed by coolant conditions (DNB)

iii) limiting stresses on clad due to fission gas release, fuel swelling, and thermal gradients

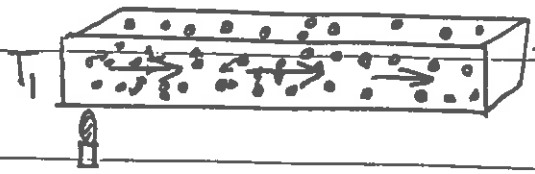
(Put limits on surface heat flux (q''), linear power density (q'), and volumetric power density (q'''))

Modes of heat transfer

- conduction
- convection
- radiation

Heat Conduction

transfer of thermal energy due to atomic or molecular motion



$T_1 > T_2$

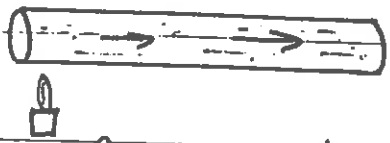
$$\underset{\sim}{q}'' = -k \underset{\sim}{\nabla} T$$

Fourier's Law of heat conduction

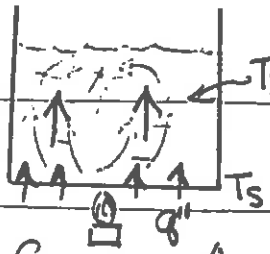
k = thermal conductivity

Heat Convection

transfer of thermal energy due to mass motion



forced convection



free convection

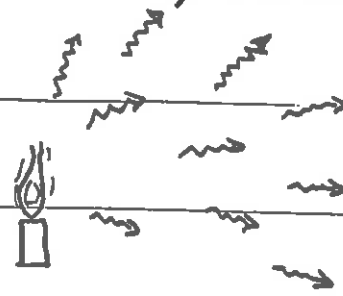
$$q'' = h_s (T_s - T_b)$$

Newton's Law of Cooling

h_s ≡ convective heat transfer coefficient

Radiation

heat transfer by electromagnetic radiation (photons)



$$q'' = \sigma (T_A^4 - T_B^4)$$ Stefan-Boltzmann Law

σ = Stefan-Boltzmann constant

In a reactor:

conduction: fuel elements

convection: coolant flow

radiation: UV/IR - safety transients

LECTURE 12

THE EQUATION OF HEAT CONDUCTION

READING ASSIGNMENT:

Eckert & Drake, pp. 25 - 32
Duderstadt & Hamilton, pp. 475 - 476
Lamarsh, pp. 353 - 354
Pfenningwerth & Minkler, pp. 99 - 109

EXERCISES:

12-1. Rederive the equation of heat conduction using the "pill-box" approach -- i.e., consider a differential volume element and balance energy exchange.

12-2. Refer back to the derivation of the neutron diffusion equation and the range of validity of Fick's Law to develop an argument estimating the range of validity of Fourier's Law.

12-3. Using a simple dimensional argument, provide a physical interpretation of the thermal diffusivity.

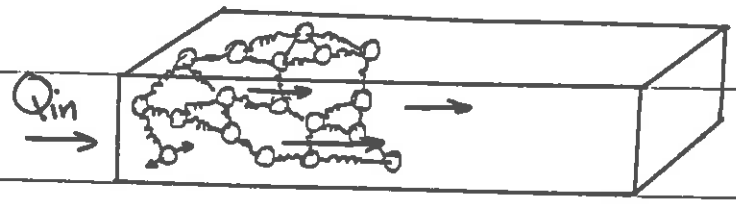
12-4. Compare the heat conduction equation with the neutron diffusion equation. How do the two equations differ in mathematical structure? Conjecture as to how the methods of solution might differ.

The Equation of Heat Conduction

- The Physical Process
- Derivation of Energy Conservation Equation
- Fourier's Law
- Forms of the Heat Conduction Equation

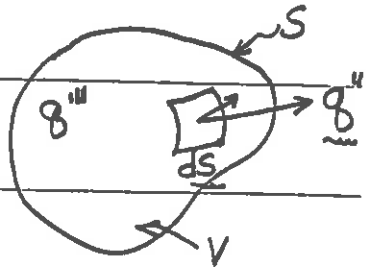
Heat Conduction

Heat transfer in solids involves molecular processes



To develop an equation to describe heat conduction, just set up an energy balance:

Consider an energy balance for an arbitrary control volume:



$$\begin{pmatrix}\text{time rate of change of energy in volume } V\end{pmatrix} = -\begin{pmatrix}\text{loss of heat through surface } S\end{pmatrix} + \begin{pmatrix}\text{heat sources in volume } V\end{pmatrix}$$

Mathematically,

$$\begin{pmatrix}\text{time rate of change of energy in volume } V\end{pmatrix} = \frac{d}{dt}\int_V d^3r\, U = \int_V d^3r \frac{\partial}{\partial t}(\rho c T)$$

ρ: mass density; c: specific heat; T: temperature

$$\begin{pmatrix}\text{heat sources in volume } V\end{pmatrix} = \int_V d^3r\, S(\underset{\sim}{r},t) = \int_V d^3r\, q'''(\underset{\sim}{r},t)$$

q''': volumetric heat source

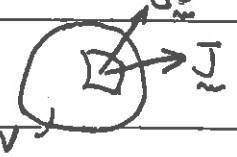
$$\begin{pmatrix}\text{loss of heat through surface } S\end{pmatrix} = -\int_S \underset{\sim}{J}\cdot d\underset{\sim}{S} = -\int_S \underset{\sim}{q}''\cdot d\underset{\sim}{S}$$

$\underset{\sim}{q}''$: heat flux

$$= -\int_V d^3r\, \underset{\sim}{\nabla}\cdot\underset{\sim}{q}''$$

where we have used Gauss's theorem

$$\int_S d\underset{\sim}{S}\cdot\underset{\sim}{J} = \int_V d^3r\, \underset{\sim}{\nabla}\cdot\underset{\sim}{J}$$



Rewrite as

$$\int_V d^3r \left[\frac{\partial}{\partial t}(\rho c T) + \underset{\sim}{\nabla} \cdot \underset{\sim}{q}'' - q''' \right] = 0$$

But since the control volume V is arbitrary, this implies

$$\frac{\partial}{\partial t}(\rho c T) = -\underset{\sim}{\nabla} \cdot \underset{\sim}{q}'' + q'''(\underset{\sim}{r}, t)$$

"Energy" conservation equation

In summary,

$$\frac{\partial}{\partial t}(\rho c T) = -\underset{\sim}{\nabla} \cdot \underset{\sim}{q}'' + q'''(\underset{\sim}{r}, t)$$

where

$\rho(\underset{\sim}{r}, t)$ mass density (g/cm^3)

c specific heat (J/kg)

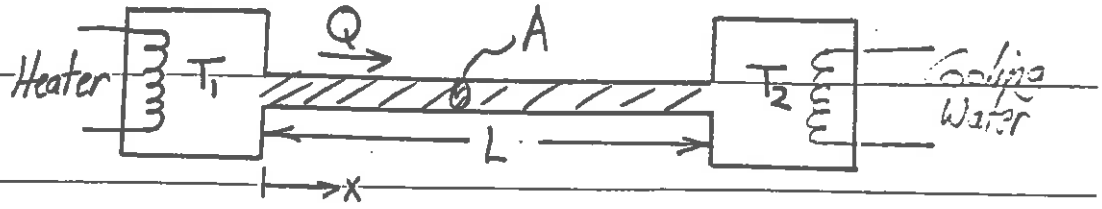
$T(\underset{\sim}{r}, t)$ temperature $(^\circ C)$

$\underset{\sim}{q}''(\underset{\sim}{r}, t)$ heat flux vector (W/cm^2) 

$q'''(\underset{\sim}{r}, t)$ volumetric heat source (W/cm^3)

Fourier's Law of Heat Conduction

Consider a simple experiment:

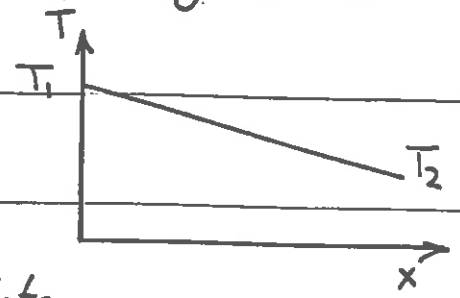


Measurements indicate

$$Q \sim [A][(T_1 - T_2)]\left[\frac{1}{L}\right]$$

$$= k A \frac{(T_1 - T_2)}{L}$$

If we also measure temperature along the rod:



$$\left(\frac{T_1 - T_2}{L}\right) \rightarrow -\frac{dT}{dx}$$

Hence we could also write

$$q'' = \frac{Q}{A} = -k\frac{dT}{dx}$$

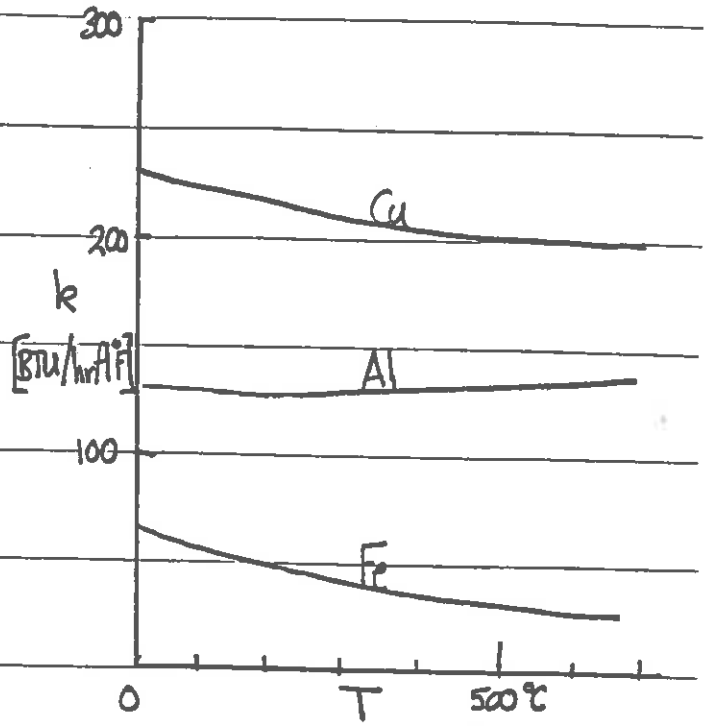
More generally one finds

$$\underset{\sim}{q''} = -k \underset{\sim}{\nabla} T \qquad \text{Fourier's Law}$$

k ← thermal conductivity

Note:

i) $k = k(T)$



ii) k may depend on direction in anisotropic materials (e.g., wood)

$$\underset{\sim}{q}'' = -\underset{\approx}{k} \cdot \nabla T$$

↖ dyadic (tensor) thermal conductivity

iii) Frequently one ignores $k = k(T)$ such that

$$\nabla \cdot \underset{\sim}{q}'' = \nabla \cdot (k \nabla T) = k \nabla^2 T$$

iv) Usually one ignores time dependence in ρ, c

$$\frac{\partial}{\partial t}(\rho c T) \rightarrow \rho c \frac{\partial T}{\partial t}$$

Forms of the Heat Conduction Equation

General Form:

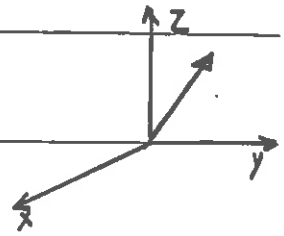
$$\frac{\partial}{\partial t}(\rho c T) = \underset{\sim}{\nabla} \cdot k \underset{\sim}{\nabla} T + q'''$$

More useful form:

$$\rho c \frac{\partial T}{\partial t} = k \nabla^2 T + q'''$$

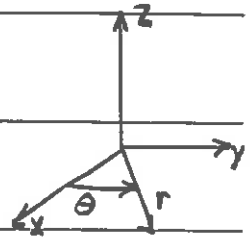
Rectangular:

$$\rho c \frac{\partial T}{\partial t} = k \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right] + q'''(x,y,z,t)$$



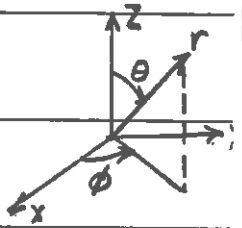
Cylindrical:

$$\rho c \frac{\partial T}{\partial t} = k \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} \right] + q'''(r,\theta,z)$$



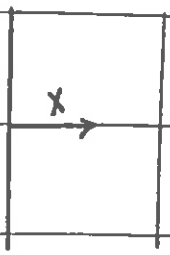
Spherical

$$\rho c \frac{\partial T}{\partial t} = k \left[\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2 T}{\partial \phi^2} \right] + q'''(r,\theta,\phi)$$



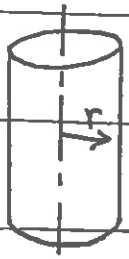
Plane:

$$\rho c \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} + \dot{q}'''(x,t)$$



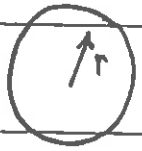
Cylindrical:

$$\rho c \frac{\partial T}{\partial t} = k \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) \right] + \dot{q}'''(r,t)$$



Spherical:

$$\rho c \frac{\partial T}{\partial t} = k \left[\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) \right] + \dot{q}'''(r,t)$$



Some Other Forms:

i) Divide by ρc

$$\frac{\partial T}{\partial t} = \underbrace{\left(\frac{k}{\rho c} \right)} \nabla^2 T + \frac{\dot{q}'''}{\rho c}$$

$$\alpha \equiv \frac{k}{\rho c} \quad \text{thermal diffusivity}$$

If no source term

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T$$

(Fourier's Equation)

ii) Time-independent heat conduction $\left(\frac{\partial T}{\partial t} = 0\right)$

$$-\underline{\nabla} \cdot k \nabla T(\underline{r}) = q'''(\underline{r})$$

$$-k \nabla^2 T(\underline{r}) = q'''(\underline{r})$$

$$-k \frac{d^2T}{dx^2} = q'''(x)$$

(Poisson's Equation)

iii) No internal heat generation $(q''' = 0)$

$$\nabla^2 T = 0$$

(Laplace's Equation)

We now have an equation

$$\rho c \frac{\partial T}{\partial t} = k \nabla^2 T + q'''$$

How do we solve it?

We need more information:

i) boundary conditions

ii) initial conditions

LECTURE 13

SOLUTION OF THE HEAT CONDUCTION EQUATION

READING ASSIGNMENT:

Eckert & Drake, pp. 36 - 75
Pfenningwerth & Minkler, pp. 111 - 161

EXERCISES:

13-1. Contrast the heat flux q" with the neutron flux, both in mathematical character and physical interpretation.

13-2. Under what circumstances can the temperature be discontinuous?

13-3. Demonstrate that the heat conduction equation is irreversible. (Assume t --> -t, $\underline{r}$ --> - $\underline{r}$ and compare the equations.)

13-4. Determine the values of a and b that will reduce the generalized form of boundary condition to each of the particular boundary conditions we have developed.

13-5. Verify each of the general forms of the solutions of the heat conduction equation provided for sourceless media.

13-6. Verify each of the general forms of the solutions of the heat conduction equation provided for media containing sources.

Solution of the Heat Conduction Equation

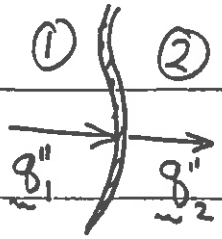
- Mathematical properties
- Initial and boundary conditions
- Heat transfer in sourceless media
- Heat transfer in media containing sources

Recall the heat conduction equation:

$$\rho c \frac{\partial T}{\partial t} = \underset{\sim}{\nabla} \cdot k \underset{\sim}{\nabla} T(\underset{\sim}{r}, t) + q'''(\underset{\sim}{r}, t)$$

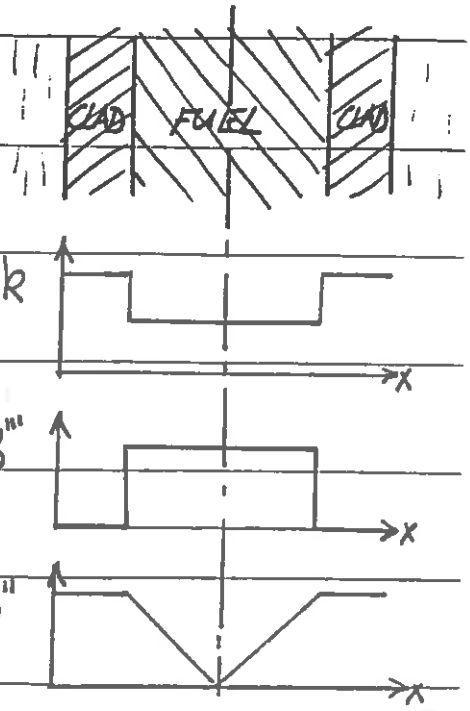
What are the mathematical properties of the solution $T(\underset{\sim}{r}, t)$? For example, is $T(\underset{\sim}{r}, t)$ continuous?

Recall $\underset{\sim}{q}''(\underset{\sim}{r}, t)$ is an energy flux? Hence since energy must be conserved, $\underset{\sim}{q}''$ must be continuous (cannot create or destroy energy across a boundary).



But while $\underset{\sim}{q}''$ is continuous, k and hence T can indeed be discontinuous

$$\underset{\sim}{q}'' = -k \underset{\sim}{\nabla} T(\underset{\sim}{r}, t)$$



Initial Conditions:

$$\rho c \frac{\partial T}{\partial t} = k \nabla^2 T + \dot{q}'''$$

Note Second Law implies "time arrow". (Equation is manifestly irreversible.

We need to specify

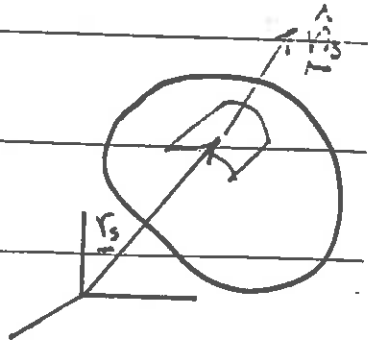
initial condition: $T(\underset{\sim}{r}, 0)$

--all $\underset{\sim}{r}$ in domain of interest

Boundary Conditions

i) Prescribed surface temperature

$T(\underset{\sim}{r}_s, t) = f(\underset{\sim}{r}_s, t)$ on S

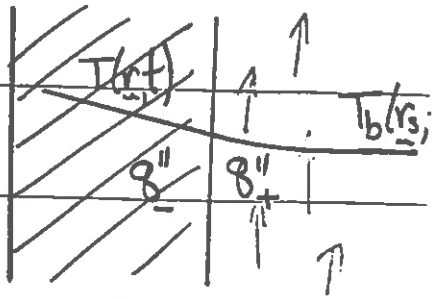


(i) Prescribed surface heat flux

$$\hat{n}_s \cdot \underset{\sim}{q}''(\underset{\sim}{r}_s, t) = -k \hat{n}_s \cdot \nabla T = -k \frac{\partial T}{\partial n_s} = f(\underset{\sim}{r}_s, t) \text{ on } S$$

(special case: $\frac{\partial T}{\partial n_s} = 0$ adiabatic)

iii.) Convection boundary condition



Recall

$$q''_+ = h_s(T - T_b)$$

Newton's Law of Cooling

Hence we require

$$\hat{\underline{n}}_s \cdot \underline{q}''_-(\underline{r}_s) = \hat{\underline{n}}_s \cdot \underline{q}''_+$$

or

$$-k\frac{\partial T}{\partial n_s} = h_s(T(r_s) - T_b)$$

Here the convective heat transfer coefficient h_s is assumed known:

	Typical h_s [BTU/hr-ft^2-°F]
1.) Natural convection, air (STP)	1
2.) Natural convection, steam (STP)	10
3.) Forced convection, PWR water	3000
4.) Nucleate boiling, PWR water	200,000

Aside:

$$k\frac{\partial T}{\partial n_s} = h_s(T - T_b)$$

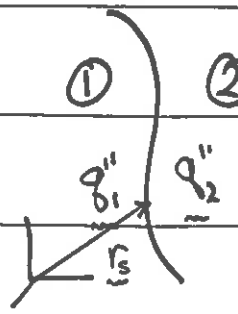
Define $Bo \equiv \frac{h_s L}{k} \equiv$ Biot number ← for solid

(Recall: $Nu \equiv \frac{h_s L}{k} \equiv$ Nusselt number ← for fluid)

iv) Two surfaces in contact

Use continuity:

$$\hat{n}_s \cdot \underset{\sim}{q}''_1(\underset{\sim}{r}_s) = \hat{n}_s \cdot \underset{\sim}{q}''_2(\underset{\sim}{r}_s) \quad \text{on } S$$



or

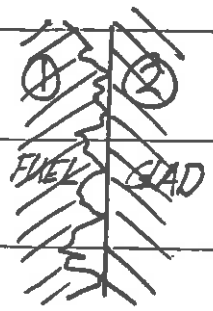
$$k_1\frac{\partial T_1}{\partial n_s} = k_2\frac{\partial T_2}{\partial n_s}$$

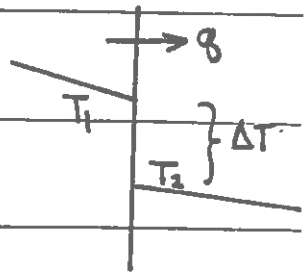
Approach: Solve heat conduction separately in each region (① and ②) and use continuity condition)

Imperfect contact ("gap conductance"):

Sometimes introduce a contact heat transfer coefficient h_s such that

$$\hat{n}_s \cdot \underset{\sim}{q}''(\underset{\sim}{r}_s) = h_s \left(T_1(\underset{\sim}{r}_s) - T_2(\underset{\sim}{r}_s) \right)$$





Summary of boundary conditions

i) $T(\underset{\sim}{r}_s)$ given on S

ii) $\hat{n}_s \cdot \underset{\sim}{q}''(\underset{\sim}{r}_s) = -\dfrac{\partial T}{\partial n_s}$ given on S

iii) $\hat{n}_s \cdot \underset{\sim}{q}_1''(\underset{\sim}{r}_s) = \hat{n}_s \cdot \underset{\sim}{q}_2''(\underset{\sim}{r}_s)$ on S

$$k_1 \frac{\partial T_1}{\partial n_s} = k_2 \frac{\partial T_2}{\partial n_s}$$

iv) $\hat{n}_s \cdot \underset{\sim}{q}''(\underset{\sim}{r}_s) = h_s (T - T_b)$

$$-k \frac{\partial T}{\partial n_s} = h_s (T - T_b)$$

We can write all of these in a generalized form

$$a\frac{\partial T}{\partial n_s} + bT(\underset{\sim}{r}_s) = d(\underset{\sim}{r}_s) \quad \text{on } S$$

Note: If $d \equiv 0$, bc is "homogeneous"

(Homogeneous bc are nice to work with.)

Idea: Use superposition:

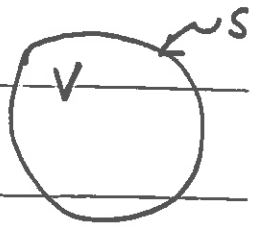
i) source terms + homogeneous bc

ii) no source terms + inhomogeneous bc

Heat transfer in sourceless media

$$\nabla^2 T = 0 \quad \text{in } V$$

$$a\frac{\partial T}{\partial n} + bT = d \quad \text{on } S$$



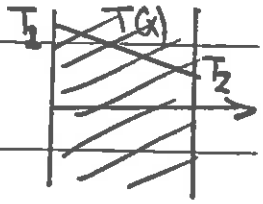
(Laplace's equation)

General solutions:

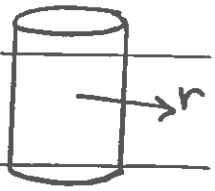
Rectangular: $\frac{d^2T}{dx^2} = 0$

$$T(x) = C_1 x + C_2$$

$$\left[T(x) = \left(\frac{T_2 - T_1}{L}\right)x + T_1 \right]$$

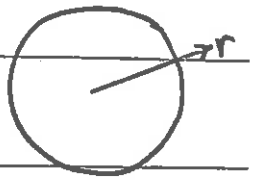


Cylindrical: $\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)=0$



$T(r)=C_1 \ln r + C_2$

Spherical: $\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)=0$

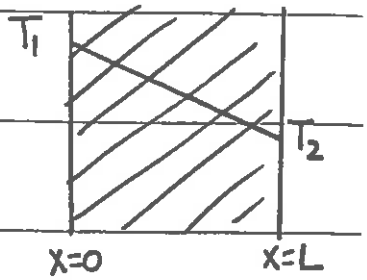


$T(r)=\frac{C_1}{r}+C_2$

Example: Slab

$$\frac{d^2T}{dx^2}=0$$

bc: $T(0)=T_1$; $T(L)=T_2$



Recall

$$T(x)=C_1x+C_2$$

bc: $T(0)=C_2=T_1$

$T(L)=C_1L+C_2=T_2 \Rightarrow C_1=\frac{T_2-T_1}{L}$

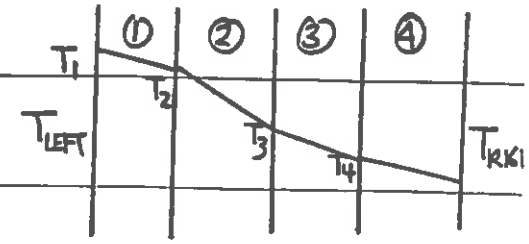
Hence

$$T(x)=\left(\frac{T_2-T_1}{L}\right)x+T_1$$

Aside: Heat conduction in composite slab

$$T_2 - T_1 = -q\frac{L_1}{k_1}$$

$$T_3 - T_2 = -q\frac{L_2}{k_2}$$



Hence $$T_{LEFT} - T_{RIGHT} = -q\left[\frac{L_1}{k_1} + \frac{L_2}{k_2} + \cdots \frac{L_N}{k_N}\right]$$

Sometimes define

$$R = \frac{1}{A}\left[\frac{1}{h_L} + \frac{L_1}{k_1} + \cdots + \frac{L_N}{k_N} + \frac{1}{h_R}\right]$$

thermal resistance

Also

$$r = AR = \left[\frac{1}{h_L} + \frac{L_1}{k_1} + \cdots + \frac{L_N}{k_N} + \frac{1}{h_R}\right]$$

= thermal resistivity

Can also define an overall heat transfer coefficient U

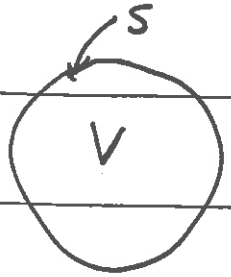
$$Q = qA = UA(T_{LEFT} - T_{RIGHT})$$

This $$\frac{1}{U} = r = \left[\frac{1}{h_L} + \frac{L_1}{k_1} + \cdots + \frac{L_N}{k_N} + \frac{1}{h_R}\right]$$

Heat transfer in media containing sources

$$-k\nabla^2 T = q''' \quad \text{in } V$$

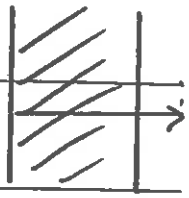
$$a\frac{\partial T}{\partial n} + bT = 0 \quad \text{on } S$$



General solutions:

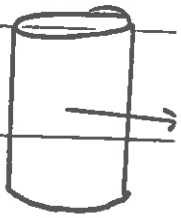
Rectangular: $-k\frac{d^2T}{dx^2} = q'''$

$$T(x) = -\frac{q'''}{2k}x^2 + C_1 x + C_2$$



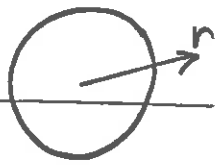
Cylindrical: $-\frac{k}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) = q'''$

$$T(r) = -\frac{q'''}{4k}r^2 + C_1 \ln r + C_2$$



Spherical: $-\frac{k}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = q'''$

$$T(r) = -\frac{q'''}{6k}r^2 + \frac{C_1}{r} + C_2$$



Other Comments:

i) Can use all standard solution methods for PDEs (separation of variables, orthogonal function expansions, Green's functions, ...)

ii) Can apply all of the machinery of neutron diffusion theory

iii) Usual numerical methods for 2-D and 3-D (Gaussian elimination, LU decomposition, SOR, ADI, ...)

LECTURE 14

HEAT CONDUCTION IN NUCLEAR FUEL ELEMENTS

READING ASSIGNMENT:

Duderstadt, pp. 475 - 482
Lamarsh, pp. 354 - 360
Sesonske, pp. 101 - 115

EXERCISES:

14-1. Develop an expression for the radial temperature distribution in a long, thin, hollow cylindrical fuel rod with inner radius Ri, outer radius Ro, and volumetric heat source density q"'. Assume that the inner surface is insulated and at a temperature Ti.

14-2. A long, thin hollow cylindrical fuel rod with volumetric thermal source strength q"' is cooled on both the inner and outer surfaces such that the surface temperatures are maintained at Ti and To, respectively. Derive an expression for the location of the maximum temperature in the fuel.

14-3. Show that for a cylindrical shell without internal heat generation, the use of an average constant thermal conductivity evaluated at the arithmetic mean of the inner and outer surface temperatures is equivalent to a linear variation of thermal conductivity with temperature.

14-4. Dimensional changes with temperature may be approximated by a relationship of the form

$$x(T) = x(To)\,[1 + a\,(T - To)]$$

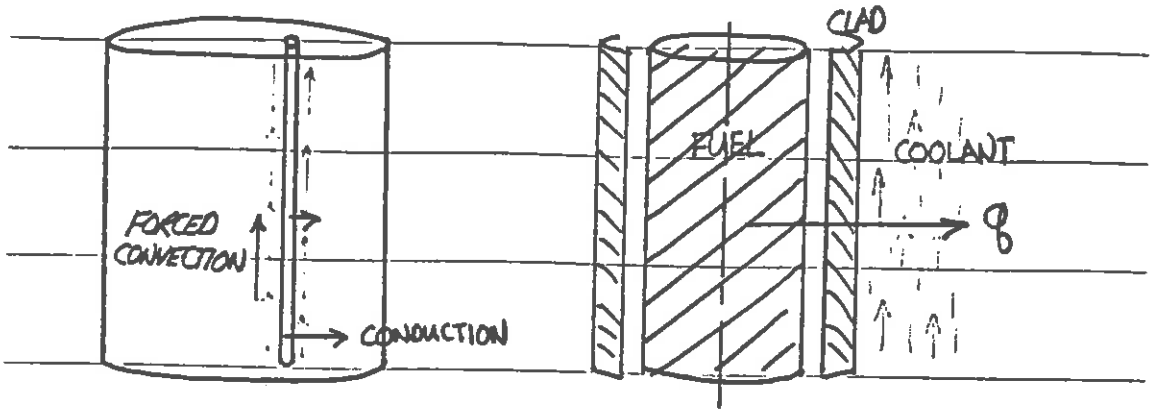
where $x(To)$ = linear dimension at temperature To
$x(T)$ = linear dimension at temperature T
a = linear expansion coefficient

Using this expression, determine the effect of expansion effects on fuel centerline temperature.

Heat Conduction in Nuclear Fuel Elements

- Assumptions
- Heat transfer in fuel
- Linear power density

Heat conduction in nuclear fuel elements

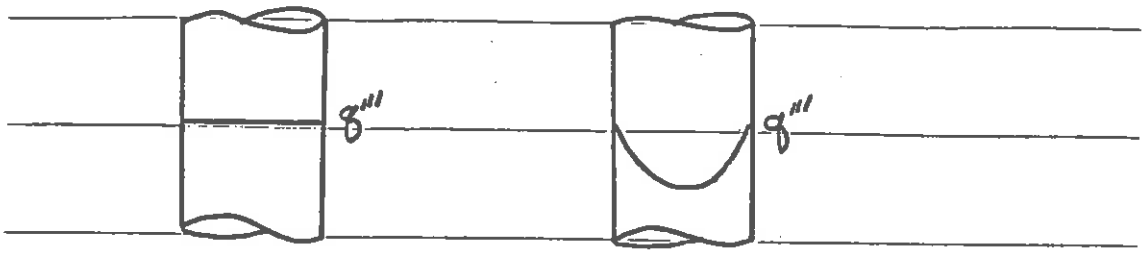


Assumptions:

i) Geometry → cylindrical

ii) Neglect axial heat conduction (axial transfer by convection)

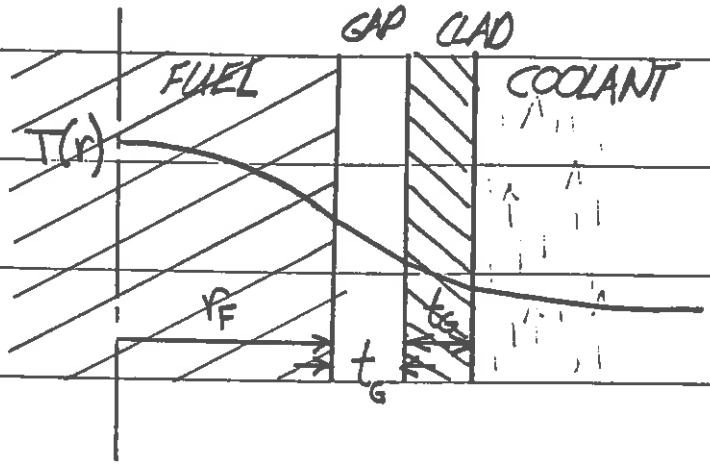
iii) For the moment we will assume fission heat sources are uniform across fuel pin:



assumed

actual
(due to self-shielding and flux depression)

Strategy: Solve equation of heat conduction for each region: fuel, gap, clad, clad surface to coolant



Simplifications of heat conduction equation

$$\rho c \frac{\partial T}{\partial t} = +\underset{\sim}{\nabla} \cdot k \underset{\sim}{\nabla} T + q'''(\underset{\sim}{r}, t)$$

i) Ignore time dependence: $\frac{\partial T}{\partial t} = 0$

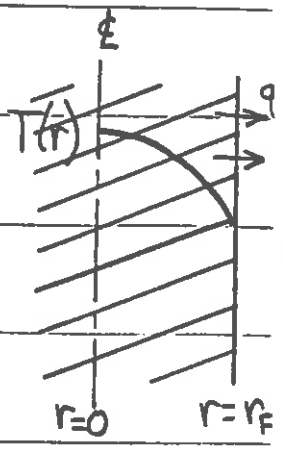
ii) Radial heat conduction: $\frac{1}{r}\frac{d}{dr} k r \frac{dT}{dr} = -q'''$

(Note: $k = k(T)$)

Heat transfer in fuel:

We will assume $q'''(r)$ = constant but allow $k_F = k_F(T)$. Then

$$\frac{1}{r}\frac{d}{dr}\left(k_F r \frac{dT}{dr}\right) = -q'''$$



bc: $T(0) = T_\ell$

$T(r_F) = T_F$

To handle the nonlinearity, multiply by r and integrate

$$\int_0^r dr\, r \frac{1}{r}\frac{d}{dr}\left(k_F r \frac{dT}{dr}\right) = -q''' \int_0^r dr\, r$$

to find

$$k_F r \frac{dT}{dr} = -q''' \frac{r^2}{2}$$

Next rearrange as

$$k_F(T)\, dT = -q''' \frac{r}{2} dr$$

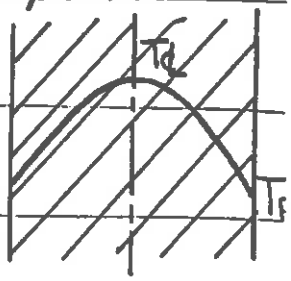
Then integrate

$$\int_{T_\ell}^{T_F} k_F(T) dT = -\frac{q'''}{2}\int_0^{r_F} r dr = -q''' \frac{r_F^2}{4}$$

To proceed further, define an average thermal conductivity:

$$\overline{k_F} = \frac{1}{T_\ell - T_F}\int_{T_F}^{T_\ell} k_F(T)\, dT$$

Aside: If k_F was independent of temperature (which it isn't), we could solve



$$-k_F \frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) = q'''$$

for $$T(r) = -\frac{q'''}{4k} r^2 + C_1 \ln r + C_2$$

Now apply bc:

$$T(0) = T_\ell \Rightarrow C_1 = 0,\ C_2 = T_\ell$$

Hence

$$T(r) = T_\ell - \frac{q'''}{4k_F} r^2$$

— parabolic profile

More generally, we cannot obtain an explicit solution.

But we can rewrite:

$$\int_{T_\ell}^{T_F} k_F(T)\,dT = -q''' \frac{r_F^2}{4}$$

as

$$(T_\ell - T_F)\,\bar{k}_F = -q''' \frac{r_F^2}{4}$$

Thus we can write the temperature drop across the fuel as

$$\Delta T\Big|_{FUEL} \equiv T_\ell - T_F = \frac{q''' r_F^2}{4\bar{k}_F}$$

Let's manipulate this a bit: Define the linear power density q' in the fuel pin by multiplying q''' by the cross sectional area πr_F^2 of the fuel pin:

$$q' = \pi r_F^2 q'''$$



Then the temperature drop across the fuel can be written as:

$$\Delta T|_{FUEL} = \frac{q'}{4\pi \bar{k}_F}$$

The thermal conductivity $\bar{k}_F$ is fixed (and quite low for ceramics -- $\bar{k}_F \sim 0.020\text{-}0.030$ W/cm K)

Hence

$$\Delta T|_{FUEL} \sim q'$$

This explains why the linear power density is such an important parameter in reactor analysis.

Example: LWR

$q' = 500$ W/cm $\Rightarrow$ $\Delta T|_{FUEL} \sim 1400$ °C

($q' = 15$ kW/ft $\Rightarrow$ $\Delta T|_{FUEL} \sim 2500$ °C)

Limiting Case:

$q' = 660$ W/cm $\Rightarrow$ $\Delta T|_{FUEL} \sim 1850$ °C

$\Rightarrow$ $T_{\ell} \sim 2800$ °C

General Operation:

$q' = 420 - 460$ W/cm

Comments:

i) Since $\Delta T|_{FUEL}$ depends only weakly on r_F, one can achieve the same q' without exceeding $T_{\ell}|_{max}$ by using smaller rods (thereby decreasing fuel inventory).

ii) Flux depression in pin due to self-shielding lowers estimate of $\Delta T|_{FUEL}$ (i.e., assuming q''' is constant is conservative).

LECTURE 15

HEAT TRANSFER IN REACTOR FUEL ELEMENTS -- CONTINUED

READING ASSIGNMENT:

Same as Lecture 14

EXERCISES:

15-1. Determine the temperature profile in plate-type fuel elements sandwiched between a clad. Assume the existence of a uniform gap between the fuel and the clad.

15-2. Determine the temperature distribution in a cylindrical fuel pin, assuming a source distribution given by

$$q'''(r) = q\ Io(r/L)$$

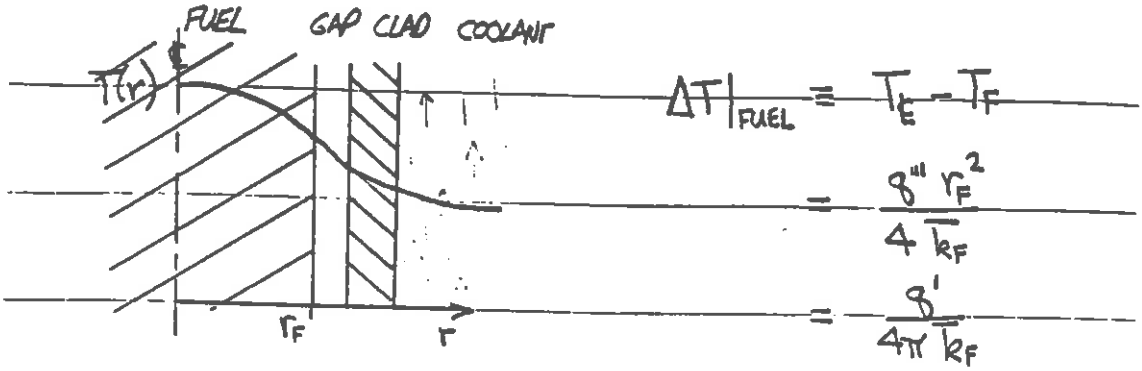
where the surface of the cylinder is at a temperature Ts. Assume that the thermal conductivity depends on temperature through the relationship $k(T) = ko\ [1 + a(T - Ts)]$. Sketch the temperature profile for the constant a positive, zero, and negative.

15-3. Calculate the equivalent heat transfer coefficient for a radial gas gap between pellet and clad of Ø.Ø1Ø and Ø.ØØ5 cm. Assume the fuel radius is much larger than the gap thickness and constant gas properties with temperature for both helium and fission product gas.

Heat Transfer in Reactor Fuel Elements - Continued

- Heat transfer in fuel pin
- Fuel-clad gap
- Heat conduction in clad
- Surface heat transfer

Recall our task:



$$\Delta T|_{FUEL} = T_E - T_F$$

$$= \frac{q''' r_F^2}{4 \overline{k}_F}$$

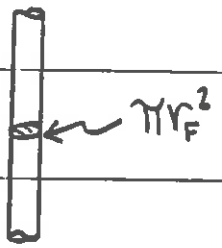
$$= \frac{q'}{4\pi \overline{k}_F}$$

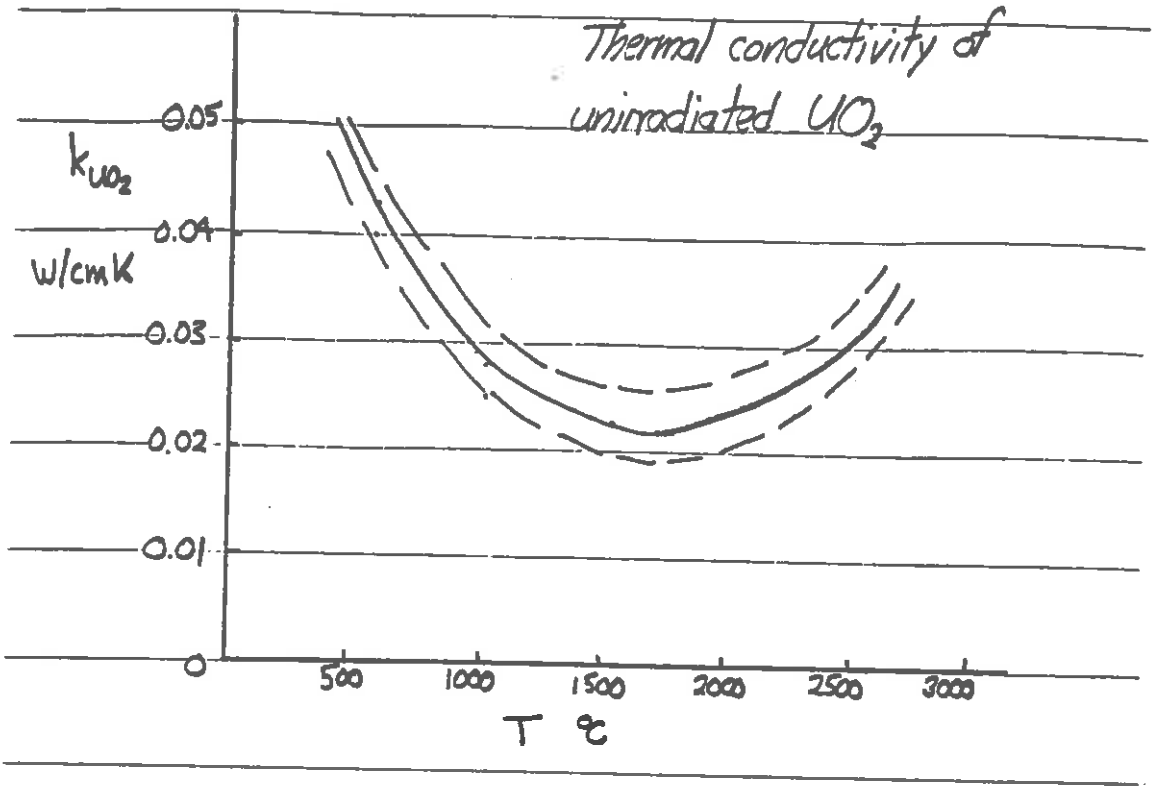
Here we have defined the linear power density

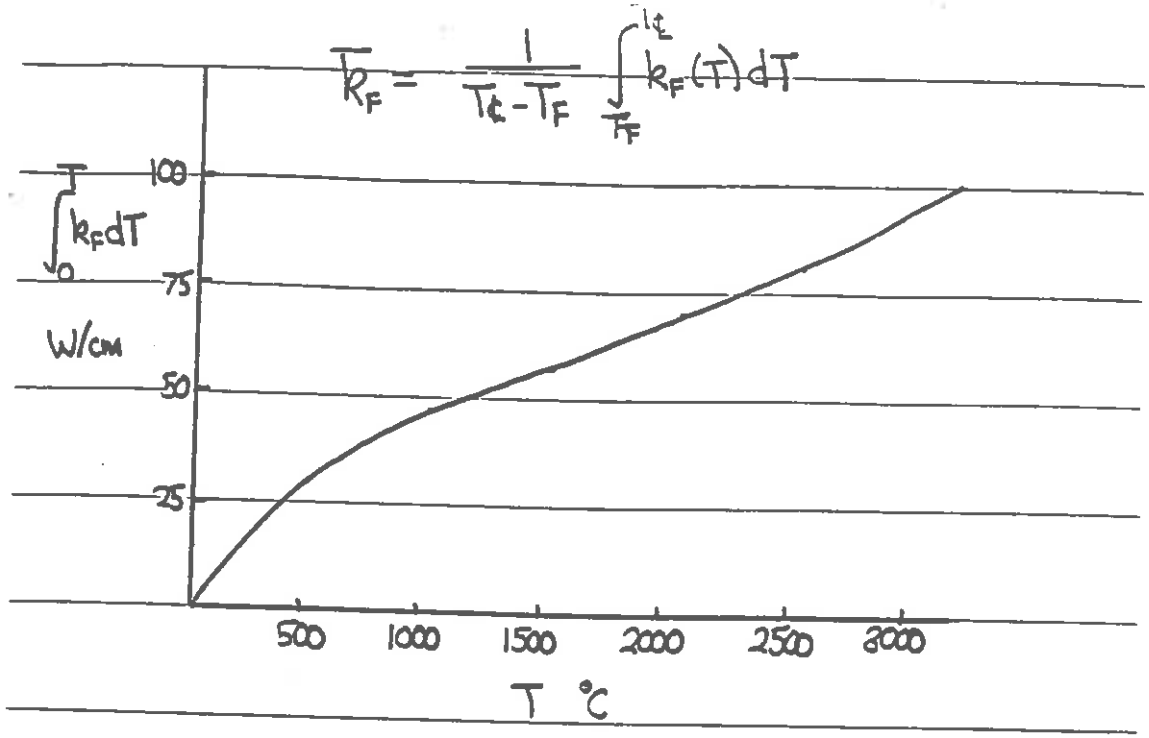
$$q' = \pi r_F^2 q'''$$

q': linear power density

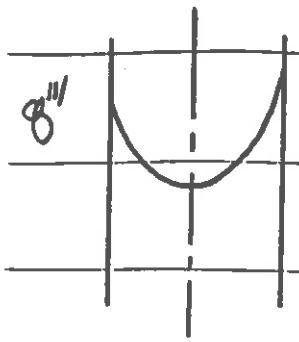
q''': volumetric heat source




Thermal conductivity of unirradiated UO_2
k_{UO_2}
W/cmK
0.05
0.04
0.03
0.02
0.01
0
500
1000
1500
2000
2500
3000
T °C


$\overline{k_F} = \frac{1}{T_\ell - T_F} \int_{T_F}^{T_\ell} k_F(T)\,dT$
$\int_0^T k_F dT$
W/cm
100
75
50
25
500
1000
1500
2000
2500
3000
T °C

Effect of nonuniform heat source distribution



$$\int_{T_s}^{T} k_F(T')dT' = \frac{q'}{4\pi}\left[\frac{I_0(\kappa r_F) - I_0(\kappa r)}{\frac{1}{2}\kappa r_F I_1(\kappa r_F)}\right]$$

$$\int_{T_s}^{T_c} k_F(T')dT' = \frac{q'}{4\pi}\left[\frac{I_0(\kappa r_F) - 1}{\frac{1}{2}\kappa r_F I_1(\kappa r_F)}\right] = \frac{q'}{4\pi} f$$

$\kappa = \frac{X}{L}$

Thus we can modify our earlier results

$$\Delta T|_{FUEL} = \frac{q'}{4\pi \bar{k}_F} f$$

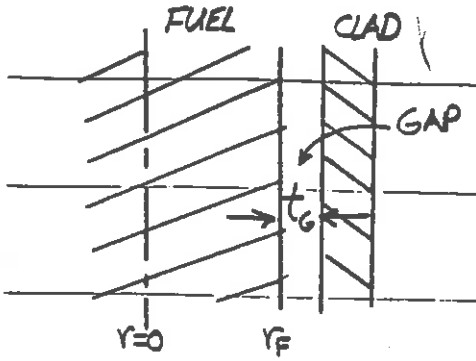
$$\Delta T|_{FUEL} = \frac{q'}{4\pi \bar{k}_F} f$$

Enrichment %	f
0	1.0
5	0.94
10	0.87
15	0.79
20	0.72

Other effects: Due to fuel irradiation

- changes in contact heat-transfer coefficients
- fuel densification
- formation of central void in cylindrical pellets
- fuel swelling due to heating and burnup
- decreases in $\bar{k}_F$ with burnup

Heat transfer across the fuel-clad gap



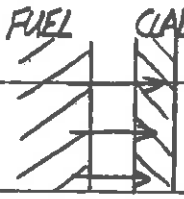
Typically the gap between the fuel pin and clad will contain a mixture of original fill gas (e.g., helium) and fission product gases (e.g., Xe, Kr)

A simple minded treatment: heat conduction in gap gases

$$\frac{1}{r}\frac{d}{dr}\left(k_G\, r \frac{dT}{dr}\right) = 0$$

b.c: $q''(r_F)$ specified

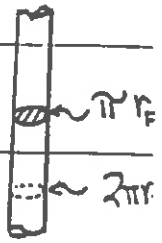
Hence we are assuming that the heat flux q'' through the fuel surface is given:



$$q'' = \text{given} = -k_G \left.\frac{dT}{dr}\right|_{r=r_F}$$

Note we can relate

$$q'' = \frac{q'}{2\pi r_F} = \frac{q''' r_F}{2}$$



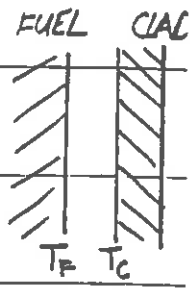
To proceed, we integrate

$$\frac{1}{r}\frac{d}{dr}\left(k_G r \frac{dT}{dr}\right) = 0$$

subject to bc: $T(r_F) = T_F$, $\left. -k_G \frac{dT}{dr}\right|_{r=r_F} = q'' = \frac{q''' r_F}{2}$

This leads to

$$\Delta T|_{GAP} \equiv T_F - T_C = \frac{q'''}{k_G}\frac{r_F^2}{2}\ln\left(\frac{r_F + t_G}{r_F}\right)$$



The gap thickness t_G is usually small ($t_G << r_F$)

so that

$$\ln\left(\frac{r_F + t_G}{r_F}\right) = \ln\left(1 + \frac{t_G}{r_F}\right) \sim \frac{t_G}{r_F}$$

Hence

$$\Delta T|_{GAP} \simeq \frac{q''' r_F t_G}{2k_G} = \frac{q'}{2\pi r_F}\left(\frac{t_G}{k_G}\right)$$

Typically k_G is small:

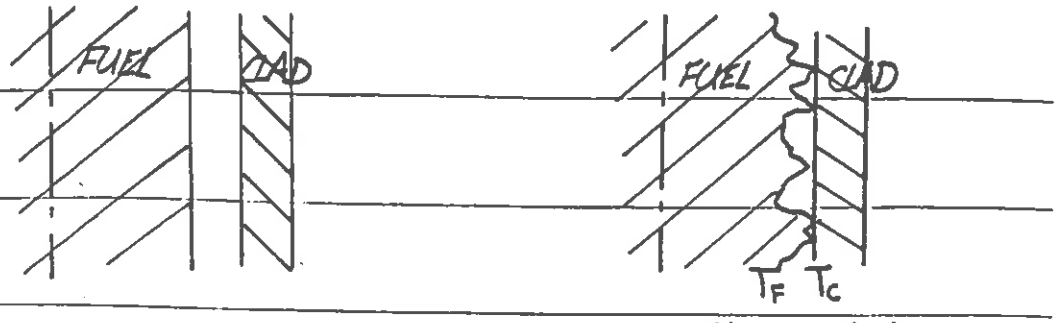
$$k_{He} = 0.002 \text{ W/cm K}$$

$$k_{Xe+Kr} = 0.0001 \text{ W/cm K}$$

Hence

$$\left.\begin{array}{l} q' = 500 \text{ W/cm} \\ t_G = 0.005 \text{ cm} \end{array}\right\} \Rightarrow \Delta T|_{GAP} \sim 300\,^{\circ}C$$

An important effect: fuel-clad contact



Prior to irradiation

After irradiation

To handle the complexities of fuel-clad contact, we use an empirical contact heat transfer coefficient h_S

$$q'' = h_G (T_F - T_C)$$

Here $h_G \sim 0.5 - 1.1 \text{ W/cm}^2\text{K}$.

Hence

$$\Delta T|_{GAP} = \frac{q''}{h_G} = \frac{q'}{2\pi r_F h_G}$$

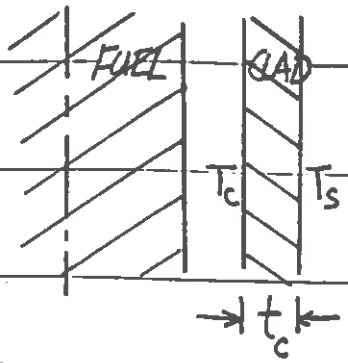
Example:

$q' = 500$ W/cm

$h_G = 1$ W/cm^{2}K

$\Delta T|_{GAP} \sim 150\,^\circ C$

Heat transfer across clad



Assume k_c is constant and that no heat is produced in clad:

$$\frac{1}{r}\frac{d}{dr}\left(r k_c \frac{dT}{dr}\right) = 0$$

We can solve for

$$\Delta T|_{CLAD} \equiv T_c - T_s = \frac{q''' r_F^2}{2k_c} \ln\left(\frac{r_F + t_c}{r_F}\right)$$

Again we assume $t_c << r_F$

$$\ln\left(\frac{r_F + t_c}{r_F}\right) = \ln\left(1 + \frac{t_c}{r_F}\right) \sim \frac{t_c}{r_F}$$

Hence

$$\Delta T|_{CLAD} \equiv \frac{q''' r_F}{2} \frac{t_c}{k_c} = \frac{q'}{2\pi r_F} \frac{t_c}{k_c}$$

Example:

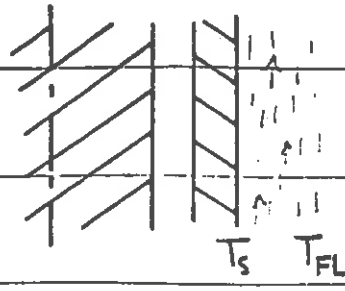
$q' = 500$ W/cm

$k_{Zr} = 0.11$ W/cm K

$t_c = 0.053$ cm

$\Rightarrow \Delta T|_{CLAD} \sim 80°C$

Heat transfer from clad surface to coolant



We'll get ahead of ourselves. Use Newton's law of cooling

$$q'' = h_s (T_S - T_{FL})$$

where h_s is a convective heat-transfer coefficient as a convective heat transfer boundary condition

$$\Delta T|_{COOL} = T_S - T_{FL} = \frac{q''}{h_s}$$

LECTURE 16

OTHER TOPICS CONCERNING HEAT CONDUCTION IN REACTORS

READING ASSIGNMENT:

El-Wakil (Nuclear Heat Transport), pp. 131 - 144

EXERCISES:

16-1. The flux depression in a cylindrical fuel pellet can be modeled by assuming that the radial dependence of q"'(r) is of the form q"'(r) = q"'o(1 + a r2). Derive an expression for the temperature rise in the fuel in terms of the linear power density q'o. In particular, compare this temperature drop with that resulting from a uniform heat flux for a 12% flux depression for the case in which q'o = 500 W/cm and k = 0.0245 W/cm K.

16-2. It is frequently of interest to determine the temperature distribution in shelding materials being heated by incident radiation. Such a calculation can easily be performed in analogy to our study of thermal conduction in fuel elements, provided one uses a non-uniform distributed heat source. Calculate the temperature distribution in a slab shield of thickness L with specified temperatures on each boundary if the shield is heated by incident gamma radiation.

16-3. Repeat the solution for the temperature distribution in the shield of problem 16-2 if:

i) One surface is thermal insulated so that no heat flow occurs across it while the other is kept at a constant temperature.

ii) Both surfaces are cooled by convective heat transfer.

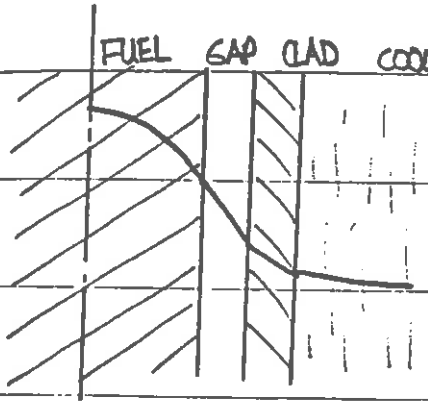
16-4. A plate-type fuel element is placed in a test reactor in a region of high thermal flux. Calculate (a) the heat production rate in the fuel, (b) the heat flux at the cladding surface, and (c) the difference in temperature between the center of the fuel and the cladding surface.

16-5. A 8 cm-thick flat-plate iron shield is subjected on one face to 3.0 MeV/photon gamma radiation of uniform flux of 10^13. The absorption cross section for 3 MeV gammas in iron is 0.282 1/cm. If the temperatures of both sides of the plate are to remain equal, find: (a) the temperature distribution in the plate, (b) the difference between maximum and surface temperatures, and (c) the surface cooling on each side of the plate necessary to maintain the above conditions.

Other Topics Concerning Heat Conduction in Reactors

- Total temperature drops across fuel elements
- Radiation heating

Summary of earlier results



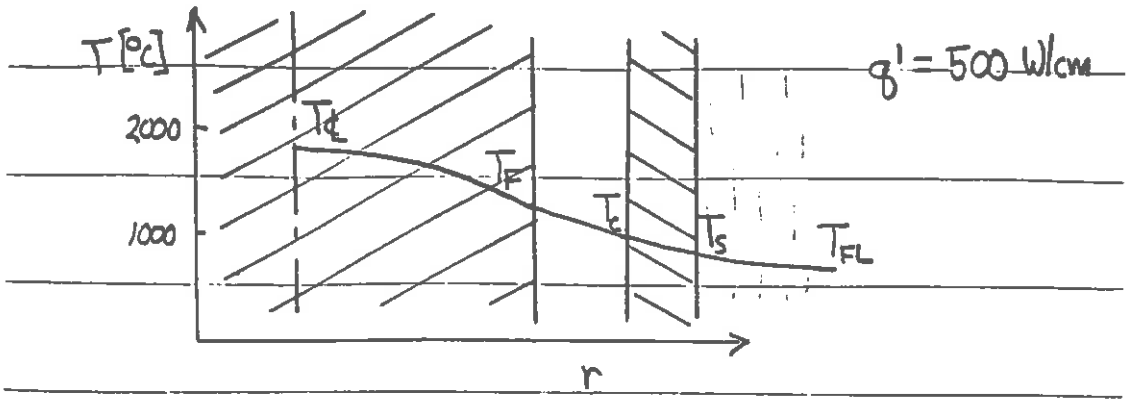
$$\Delta T|_{FUEL} = \frac{q'}{4\pi k_F}$$

$$\Delta T|_{GAP} = \frac{q'}{2\pi r_F h_G}$$

$$\Delta T|_{CLAD} = \frac{q'}{2\pi r_F} \frac{t_c}{k_c}$$

$$\Delta T|_{COOL} = \frac{q'}{2\pi h_S} \frac{1}{(r_F + t_c)}$$

Total temperature drop



$$T_{\ell} - T_{FL} = \frac{q'}{2\pi r_F}\left[\frac{r_F}{2\bar{k}_F} + \frac{1}{h_G} + \frac{t_c}{k_c} + \frac{r_F}{h_s(r_F + t_c)}\right]$$

Some typical values

$q' = 560$ W/cm

$r_F = 0.57$ cm

$t_c = 0.06$ cm

T_{ℓ}	fuel centerline	2358 °C
T_F	fuel pin surface	668
T_c	clad inner surface	431
T_s	clad outer surface	354
T_{FL}	coolant temperature	316

Note:

$$T_{\ell} - T_{FL} = \frac{q'}{2\pi r_F}\left[\frac{r_F}{2\bar{k}_F} + \frac{1}{h_G} + \frac{t_c}{k_c} + \frac{r_F}{h_s(r_F + t_c)}\right]$$

Here:

T_{FL} fixed by plant design

T_{ℓ} constrained by fuel melting

q' determines core size for a given power level

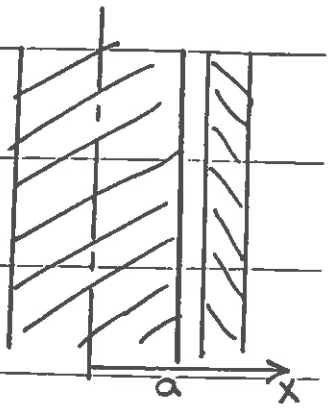
Also note:

$$q' = \frac{2\pi (T_{\ell} - T_{FL})}{\left[\frac{1}{2\bar{k}_F} + \frac{1}{r_F h_G} + \frac{t_c}{k_c r_F} + \frac{1}{h_s(r_F + t_c)}\right]}$$

$\Rightarrow$ strong motivation to maximize heat transfer parameters

$h_G, h_s, k_c, \bar{k}_F$

Aside: A very similar analysis applies for plate-type fuel elements.



If we take $k_F \cong$ constant

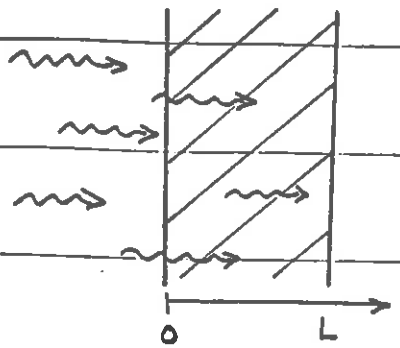
$$T(x) = T_\ell - \frac{q'''}{2k_F} x^2$$

(again parabolic). Alternative:

$$T(x) - T_F = \frac{q''' a^2}{2k_F}\left[1 - \left(\frac{x}{a}\right)^2\right] \quad \text{plate-type}$$

$$T(r) - T_F = \frac{q''' r_F^2}{4k_F}\left[1 - \left(\frac{r}{r_F}\right)^2\right] \quad \text{rod-type}$$

Radiation heating



We can assume

$$q'''(x) = q'''_0 e^{-\mu x}$$

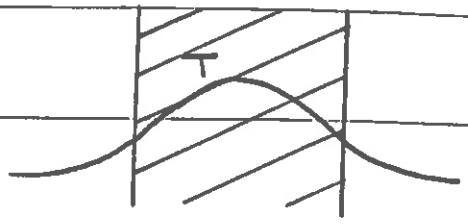
where μ is the absorption energy.

Hence we want to solve

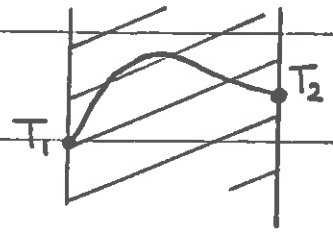
$$-k\frac{d^2T}{dx^2} = q'''_0 e^{-\mu x}$$

But what bc?

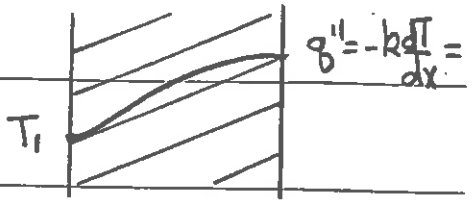
1) Both surfaces cooled by fluids (convective bc)



2) Temperature specified on both walls



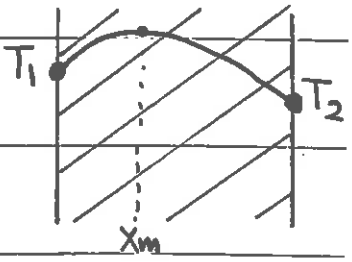
3) One wall insulated; T specified on other wall



Example: $-k\frac{d^2T}{dx^2} = q_0'''e^{-\mu x}$

$$T(x) = -\frac{q_0'''}{k\mu^2}e^{-\mu x} + C_1 x + C_2$$

bc: $T(0) = T_1$, $T(L) = T_2$



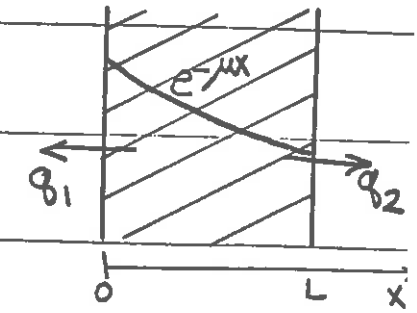
We find

$$T(x) = T_1 + (T_2 - T_1)\frac{x}{L} + \frac{q_0'''}{k\mu^2}\left[\frac{x}{L}\left(e^{-\mu L} - 1\right) - \left(e^{-\mu x} - 1\right)\right]$$

Can also solve for

$$x_m = -\frac{1}{\mu}\ln\left[\frac{\mu k}{q_0''' L}(T_2 - T_1) + \frac{1}{\mu L}\left(1 - e^{-\mu L}\right)\right]$$

We can also calculate the heat flux from each side of the slab.



$$q_1 = -k\left.\frac{dT}{dx}\right|_{x=0}$$

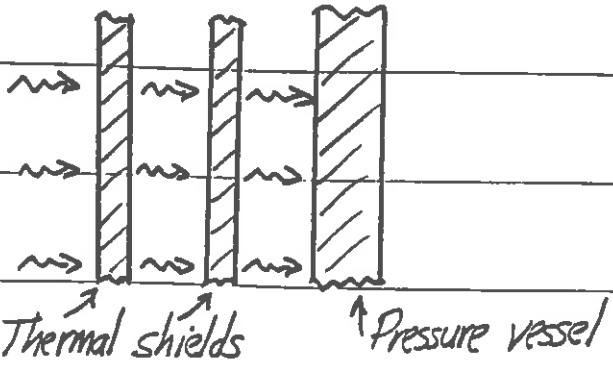
$$= \frac{k}{L}(T_2 - T_1) - \frac{q'''_{b0}}{\mu}\left(1 + \frac{e^{-\mu L} - 1}{\mu L}\right)$$

$$q_2 = -k\left.\frac{dT}{dx}\right|_{x=L}$$

$$= \frac{k}{L}(T_2 - T_1) - \frac{q'''_{b0}}{\mu}\left(e^{-\mu L} + \frac{e^{-\mu L} - 1}{\mu L}\right)$$

Note: Typically 80-90% absorbed in first 10-15% of thickness

Example: Thermal shields



Can apply same type of analysis.

LECTURE 17

MULTIDIMENSIONAL HEAT CONDUCTION

READING ASSIGNMENT:

Eckert & Drake, pp. 64 - 75
Pfenningwerth & Minkler, pp. 127 - 161

EXERCISES:

17-1. Frequently one encounters homogeneous boundary conditions in heat transfer problems. But the Third Law of Thermodynamics says that a system cannot achieve absolute zero. So what gives?

17-2. Provide the details for the solution of Laplace's equation for the "pie-shaped" segment in the example in the Lecture.

17-3. Using separation of variables, develop the eigenfunctions and eigenfunctions for the rectangular region considered in the example solution of Poisson's equation.

17-4. Solve for the temperature distribution in a cube subject to a uniform volumetric heat source and zero temperature boundary conditions.

17-5. Solve for the temperature distribution in a right circular cylinder containing a uniform volumetric heat source and subject to zero temperature boundary conditions.

17-6. There is something fishy about Poisson's equation. What is it?

Multidimensional Heat Conduction

- Laplace's equation
- Example
- Poisson's equation
- Example

We will assume that the thermal conductivity k can be taken as constant so that our two classes of problems become:

$$\nabla^2 T = 0$$

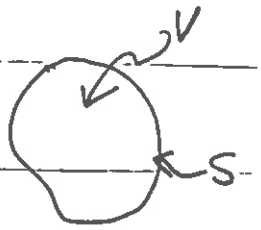
Laplace's Equation

$$\nabla^2 T = -\frac{\dot{q}'''(\underline{r})}{k}$$

Poisson's Equation

Solution of Laplace's Equation

We wish to solve Laplace's equation subject to some suitable boundary conditions:

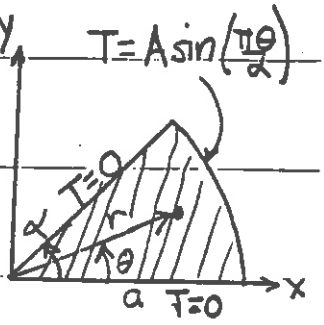


$$\nabla^2 T(r) = 0 \quad \text{in } V$$

b.c: $a\frac{\partial T}{\partial n} + bT = d$ on S

Example:

$$\nabla^2 T = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\left(\frac{\partial^2 T}{\partial \theta^2}\right) = 0$$



b.c: i) $T(r,0) = 0, \quad 0 \le r \le a$

ii) $T(r,\alpha) = 0, \quad 0 \le r \le a$

iii) $T(a,\theta) = A\sin\left(\frac{\pi\theta}{\alpha}\right), \quad 0 \le \theta \le a$

iv) $T < \infty$

Solution: Use separation of variables

$$T(r,\theta) = R(r)\Theta(\theta)$$

Substitute in and rearrange:

$$\frac{r}{R}\frac{d}{dr}\left(r\frac{dR}{dr}\right) = -\frac{1}{\Theta}\frac{d^2\Theta}{d\theta^2} = \gamma^2$$

Hence we find:

$$r\frac{d}{dr}\left(r\frac{dR}{dr}\right) - \gamma^2 R(r) = 0$$

$$\frac{d^2\Theta}{d\theta^2} + \gamma^2\Theta(\theta) = 0$$

We can solve these to find

$\gamma \neq 0$: $R = r^{\pm\gamma}$, $\Theta = e^{\pm i\gamma\theta}$

$\gamma = 0$: $R = A_0 \ln r + B_0$, $\Theta = C_0\theta + D_0$

Hence our general solution is of the form

$$T(r,\theta) = (A_0 \ln r + B_0)(C_0\theta + D_0) + \sum_{\gamma\neq 0}(A_\gamma r^\gamma + B_\gamma r^{-\gamma})(C_\gamma e^{i\gamma\theta} + D_\gamma e^{-i\gamma\theta})$$

Now apply boundary conditions:

iv) $T < \infty \Rightarrow A_0 = 0,\ B_\gamma = 0$ (from $r \to 0$)

i) $T(r,0) = 0 \Rightarrow D_0 = 0,\ C_\gamma = 0$

ii) $T(r,\alpha) = 0 \Rightarrow C_0 = 0,\ B_0 = 0,\ \gamma = \frac{n\pi}{\alpha},\ n = 1,2,3,\ldots$

iii) $T(a,\theta) = A\sin\left(\frac{\pi\theta}{\alpha}\right) \Rightarrow$? — wait a moment...

The solution thus far is:

$$T(r,\theta) = \sum_{n=1}^{\infty} (A_n C_n)\, r^{n\pi/\alpha} \sin\left(\frac{n\pi\theta}{\alpha}\right)$$

Now apply b.c. iii):

$$T(a,\theta) = A\sin\left(\frac{\pi\theta}{\alpha}\right) = \sum_{n=1}^{\infty} (A_n C_n)\, a^{n\pi/\alpha} \sin\left(\frac{n\pi\theta}{\alpha}\right)$$

$$\Rightarrow A_1 C_1 = A\, a^{-\pi/\alpha}$$

$$A_n C_n = 0, \quad n > 1$$

Thus our solution is

$$T(r,\theta) = A\left(\frac{r}{a}\right)^{\pi/\alpha} \sin\left(\frac{\pi\theta}{\alpha}\right)$$

Other common techniques:

Orthogonal function expansions (Fourier Series)

Integral transforms (Laplace or Fourier transform)

Conformal mapping

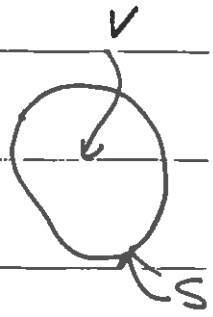
Numerical methods

Solution of Poisson's Equation

We now wish to solve Poisson's equation subject to some suitable boundary condition:

$$\nabla^2 T(\underset{\sim}{r}) = -\frac{q'''(\underset{\sim}{r})}{k} \quad \text{in } V$$

b.c: $a\frac{\partial T}{\partial n} + bT = d$ on S



Example:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = -\frac{q'''_0}{k}$$

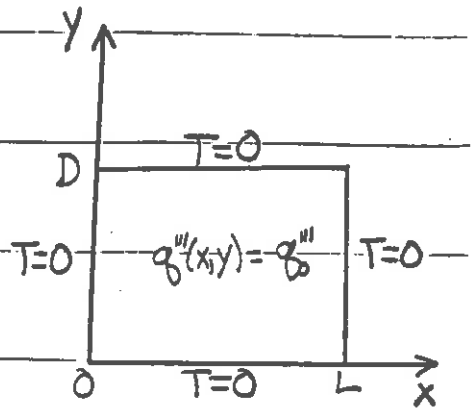
b.c: $T = 0$ on S

$[T(0,y) = 0$

$T(L,y) = 0$

$T(x,0) = 0$

$T(x,D) = 0]$



Solution: Lets try an orthogonal function expansion. The idea is to first use separation of variables to find an eigenvalue problem, and then use these eigenfunctions as the orthogonal expansion functions.

In this case the eigenvalue problem is

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}=-\lambda^2\psi(x,y)$$

b.c: $\psi=0$ on S

This yields

eigenfunctions: $\psi_{mn}(x,y)=\sin\frac{m\pi x}{L}\sin\frac{n\pi}{D}y$

eigenvalues: $\lambda_{mn}^2=\pi^2\left[\left(\frac{m}{L}\right)^2+\left(\frac{n}{D}\right)^2\right]$ $m=1,2,3\ldots$ $n=1,2,3,\ldots$

Thus we seek the solution as an expansion

$$T(x,y)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}A_{mn}\psi_{mn}(x,y)$$

$$=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}A_{mn}\sin\left(\frac{m\pi x}{L}\right)\sin\left(\frac{n\pi y}{L}\right)$$

Since this solution satisfies the bc, (the ψ_{mn} were constructed that way), we can plug into the Poisson equation

$$\frac{\partial^2T}{\partial x^2}+\frac{\partial^2T}{\partial y^2}=-\frac{q_0'''}{k}$$

to determine the expansion coefficients A_{mn}.

Substituting in, we find:

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn}\left[\left(\frac{m\pi}{L}\right)^2+\left(\frac{n\pi}{D}\right)^2\right]\sin\left(\frac{m\pi x}{L}\right)\sin\left(\frac{n\pi x}{D}\right) = -\frac{q'''_{00}}{k}$$

One equation -- an infinite number of unknowns! But that's OK. Just use orthogonality property:

$$\int_0^L dx \int_0^D dy \sin\left(\frac{m'\pi x}{L}\right)\sin\left(\frac{m\pi y}{L}\right)\sin\left(\frac{n'\pi x}{D}\right)\sin\left(\frac{n\pi y}{D}\right) = C_{mn}\,\delta_{m'm}\,\delta$$

$$\int dx \int dy\ \psi_{m'n'}(x,y)\,\psi_{mn}(x,y) = C_{mn}\,\delta_{m'm}\,\delta_{n'n}$$

Using orthogonality yields

$$A_{mn} = \frac{\frac{16\,q'''_{00}}{\pi^4 k}}{mn\left[\left(\frac{m}{L}\right)^2+\left(\frac{n}{D}\right)^2\right]} \qquad m,n = 1,3,5,\ldots$$

Hence our final solution is

$$T(x,y) = \frac{16\,q'''_{00}}{\pi^4 k}\sum_{\substack{m=1\\ m,n\ \text{odd}}}^{\infty}\sum_{n=1}^{\infty} \frac{\left(\sin\frac{m\pi x}{L}\right)\left(\sin\frac{n\pi x}{D}\right)}{mn\left[\left(\frac{m}{L}\right)^2+\left(\frac{n}{D}\right)^2\right]}$$

Other common techniques:

Integral transforms

Greens' functions

Numerical methods

LECTURE 18

TRANSIENT HEAT CONDUCTION

READING ASSIGNMENT:

Eckert & Drake, pp. 76 - 109
Pfenningwerth & Minkler, pp. 173 - 184

EXERCISES:

18-1. Provide the details for the solution of the space-time dependent temperature distribution given for the rectangular region in the Lecture.

18-2. What is the temperature distribution in the rectangular region for long times? (Examine the structure of the solution for long times.)

18-3. A temperature measuring device (a thermocouple) is mounted near the exit of a coolant channel in a nuclear reactor in order to measure the coolant temperature. The coolant suddenly changes from a steady value of 500 F to 600 F and remains there. How long will it take the thermocouple to reach 90% of its new final temperature?

(For the purposes of this calculation, the thermocouple can be approximated by a long cylinder, 1/8 inch in diameter, with the temperature sensitive element loacted at the centerline of the cylinder. The heat transfer coefficient between the thermocoupel and water is 50 BTU/hr-ft2-F. State any assumptions you may feel are necessary.)

18-4. Consider the temperature distribution in an infinite medium subject to a pulsed plane source at the origin. Use Laplace transforms to determine the temperature distribution throughout the medium at any time $t > 0$, assuming a zero temperature initial condition.

Transient Heat Conduction

- Newtonian temperature fields
- Space-time dependent problems
- Eigenfunction expansions
- Laplace transforms

Let's now consider the time-dependent form of the heat conduction equation

$$\rho c \frac{\partial T}{\partial t} = k \nabla^2 T + q'''$$

Here we have assumed constant properties ρ, c, k.

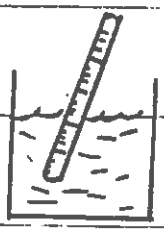
We will study several different classes of problems.

Newtonian Temperature Fields

In Newtonian problems, we assume we can neglect spatial dependence (e.g., the body is very small or the transient is very rapid):

$$\rho c \frac{\partial T}{\partial t} = q'''$$

Example: A thermometer in a fluid bath



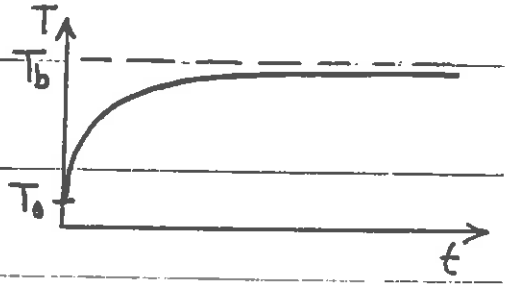
$$\rho c V \frac{dT}{dt} = UA(T_b - T)$$

where

$$U = \frac{1}{\frac{1}{h_s} + \frac{p}{k}}$$ overall heat transfer coefficient

Case 1: Thermometer, initially reading T_0, is immersed.

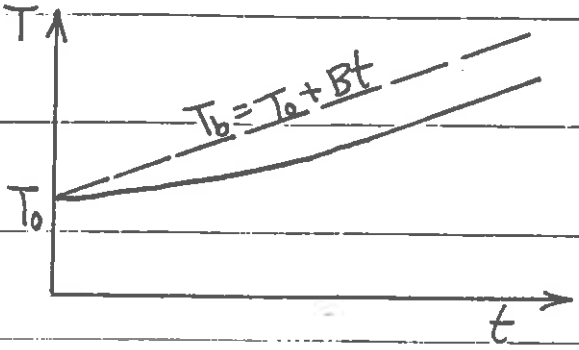
$$T(t) = T_b - (T_b - T_0)e^{-\left(\frac{UA}{\rho c V}\right)t}$$



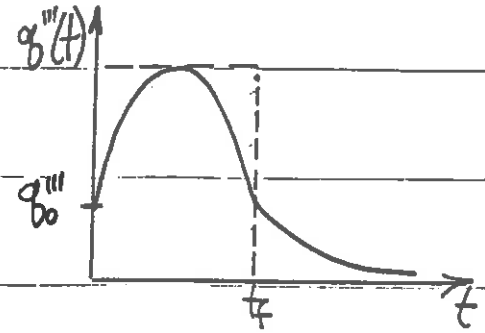
Case 2: Thermometer is at $T_b = T_0$, but bath temperature starts to rise as $T_b = T_0 + Bt$

$$\rho c V \frac{dT}{dt} = UA(T_0 + Bt - T)$$

$$T = T_0 + Bt - \frac{\rho c V}{UA} B\left(1 - e^{-\left(\frac{UA}{\rho c V}\right)t}\right)$$



Case 3: A fuel element at T_0 before a power transient $q'''(t)$.



Approximate transient as

$q'''(t) = q'''_0$, $t < 0$

$= q'''_p$, $0 \le t \le t_f$

$= 0$, $t > t_f$

We solve

$$\rho c \frac{dT}{dt} = q'''_p \qquad 0 \le t \le t_f$$

Solution is

$$T(t) = T_0 + \frac{q'''_p}{\rho c} t$$

At peak temperature

$$T(t_f) = T_0 + \frac{q'''_p}{\rho c} t_f$$

(Can be used as an upper bound on maximum fuel temperature.)

Space-time dependent problems

Consider:

$$\rho c \frac{\partial T}{\partial t} = k \nabla^2 T + q'''$$

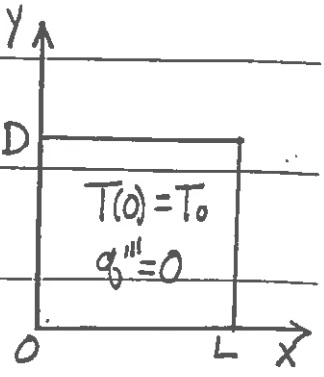
or

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T + \frac{q'''}{\rho c}$$

where $\alpha \equiv \frac{k}{\rho c}$ thermal diffusivity

Can apply all of the standard methods

Example: Consider the time dependance of a plate, initially at a temperature T_0 at $t=0$, with no sources



$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \quad \text{in } V$$

i.c: $T(x,y,0) = T_0$ in V

b.c: $T = 0$ on S, $t > 0$

If we substitute this into the equation

$$\sum_{m,n} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) A_{mn}(t)\psi_{mn}(x,y) = \frac{1}{\alpha}\sum_{m,n} \frac{dA_{mn}}{dt}(\psi_{mn}(x,y))$$

Using orthogonality yields

$$-\lambda_{mn}^2 A_{mn}(t) = \frac{1}{\alpha}\frac{dA_{mn}}{dt}$$

Hence

$$A_{mn}(t) = A_{mn}^0 e^{-\alpha\lambda_{mn}^2 t}$$

Let's try an eigenfunction expansion, just as we did for the steady-state problem.

$$\psi_{mn}(x,y) = \sin\left(\frac{m\pi x}{L}\right)\sin\left(\frac{n\pi y}{D}\right)$$

$$\lambda_{mn}^2 = \pi^2\left[\left(\frac{m}{L}\right)^2 + \left(\frac{n}{D}\right)^2\right], \qquad \begin{matrix} m = 1, 2, 3, \cdots \\ n = 1, 2, 3, \cdots \end{matrix}$$

Hence we seek:

$$T(x,y,t) = \sum_{m,n} A_{mn}(t)\,\psi_{mn}(x,y)$$

Once again this obviously satisfies the b.c.

Our solution is then:

$$T(x,y,t) = \sum_{m,n} A_{mn}^{o}\, e^{-\alpha\lambda_{mn}^2 t} \sin\left(\frac{m\pi x}{L}\right)\sin\left(\frac{n\pi y}{L}\right)$$

To determine A_{mn}^{o}, apply initial condition

$$T_o = \sum_{m,n} A_{mn}^{o} \sin\left(\frac{m\pi x}{L}\right)\sin\left(\frac{n\pi y}{L}\right)$$

We can again use orthogonality to evaluate A_{mn}^{o}

$$A_{mn}^{o} = \frac{4\,T_o}{LD}$$

Thus our solution finally becomes

$$T(x,y,t) = \frac{4T_0}{LD} \sum_{m,n} e^{-\alpha \lambda_{mn}^2 t} \sin\left(\frac{m\pi x}{L}\right) \sin\left(\frac{n\pi y}{L}\right)$$

An alternative approach:

Use a Laplace transform (only in time)

$$\mathcal{L}\{f\} = \tilde{f}(s) = \int_0^\infty f(t) e^{-st}\, dt$$

Now note

$$\mathcal{L}\left\{\frac{df}{dt}\right\} = s\tilde{f}(s) - f(0)$$

Hence we can transform our equation

$$\frac{\partial^2 \tilde{T}}{\partial x^2} + \frac{\partial^2 \tilde{T}}{\partial y^2} = \frac{s}{\alpha}\tilde{T} - \frac{1}{\alpha} T_0$$

(b.c. $\tilde{T} = 0$ on S)

Now that we have eliminated the time variable, we can solve the remaining spatial problem in the usual way -- e.g., eigenfunction expansions

$$\tilde{T}(x,y,s) = \sum_{m,n} A_{mn}(s)\, \psi_{mn}(x,y)$$

Our last step (after using bc to evaluate A_{mn}) is to invert. [Can sometimes use tables -- but frequently must use complex integration.]

Summary of methods:

Separation of variables

Eigenfunction (orthogonal) expansions

Laplace transforms

Numerical methods

LECTURE 19

NEWTON'S LAW OF COOLING

READING ASSIGNMENT:

Duderstadt and Hamilton, pp. 481 - 489
Lamarsh, pp. 363 - 375
J. H. Rust, "Fluid System Design", Chapter 7 in Weisman (Design)

EXERCISES:

19-1. Compute the equivalent hydraulic diameter for a hexagonal lattice.

19-2. Calculate the thermal resistance for heat transfer across the gap between the fuel and the clad (using the concept of the gap heat transfer coefficient).

19-3. The velocity profile of water at 1000 psia flowing through an insulated 8 in. ID coolant pipe can be fit by the expression

$$u = 60 - 540\ r^2 \quad ft/s$$

The measured temperature distribution across the pipe is

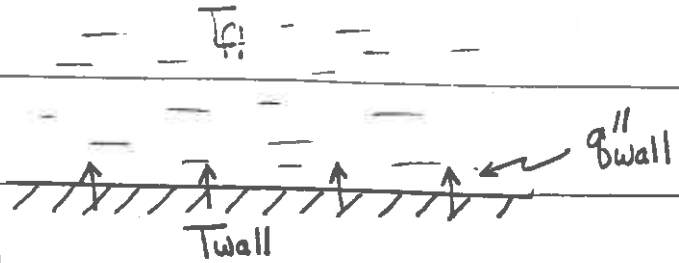
$$T = 155 - 720\ r^2\ F$$

(a) Compute the bulk temperature of the water in the pipe.
(b) Computer the average velocity of the water in the pipe.
(c) Compute the value of the Reynold's number for the flow.
(d) Compute the flow rate for the flow.
(Take the water density at 61.8 lb/ft-3 and the viscosity as 1.270 lb/hr-ft.

Newton's Law of Cooling

- Convection
- Newton's Law of Cooling
- Convective heat transfer coefficient
- Equivalent hydraulic diameter

We now turn our attention to the study of heat transfer in fluids. To gain some perspective, consider heat transfer from a heated surface into a fluid



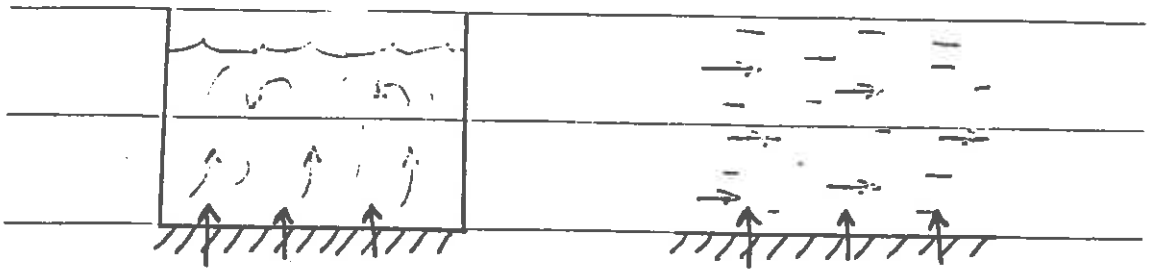
We have defined:

q''_{wall} wall heat flux

T_{wall} wall temperature

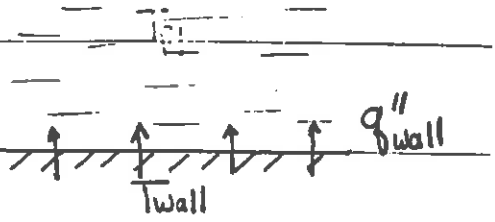
$T_{f,i}$ bulk fluid temperature

In a fluid, convection is usually the most important heat transfer mechanism. Two types:



free convection forced convection

Generally we are interested in determining a relationship among q''_{wall}, T_{wall}, and T_f. What physical phenomena are involved?



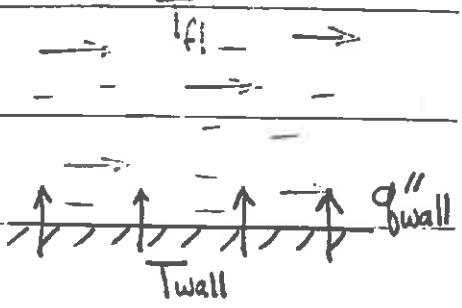
conduction: flow of heat due to molecular motion

convection: flow of heat due to mass motion (as internal energy)

radiation: transport of thermal energy by electromagnetic radiation

We can frequently describe heat transfer from a surface to a fluid by Newton's law of cooling

$$q''_{wall} = h_s (T_{wall} - T_{fl})$$

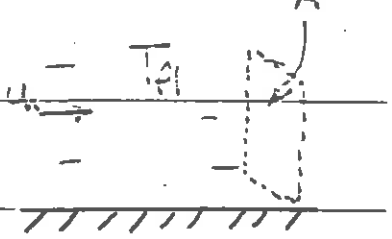


where h_s is known as the convective heat transfer coefficient.

Here, h_s depends on both fluid properties and flow characteristics.

Note: T_{fl} is the bulk fluid temperature (also called the mixing cup temperature)

$$T_{fl} = \frac{\int \rho c_p u_x T \, dA}{\int \rho c_p u_x \, dA}$$



A is the cross-sectional area of the flow. Also note

$$q_{wall} = h_s A (T_{wall} - T_{fl}) = \underbrace{\frac{1}{(1/h_s A)}}_{R_h \text{ thermal resistance}} (T_{wall} - T_{fl})$$

It is common to express h_s in terms of a dimensionless parameter, the Nusselt number Nu

$$h_s = \left(\frac{k}{D_h}\right) Nu$$

where

$k \equiv$ thermal conductivity of the fluid

$D_h \equiv$ equivalent hydraulic diameter of the flow channel

Our task is now to determine the Nusselt number Nu.

Dimensional analysis indicates that Nu depends at most on two other dimensionless parameters

$$Nu = \text{fcn}(Re, Pr)$$

where $Re \equiv \frac{\rho \bar{u} D_h}{\mu}$ Reynolds number

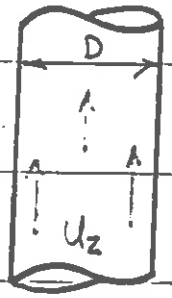
$Pr \equiv \frac{\mu c}{k}$ Prandtl number

Note:

Pr characterizes properties of the fluid

Re characterizes the flow

To characterize flow, we introduce dimensionless parameters:



$Re \equiv \frac{\rho \bar{u}_z D}{\mu} \equiv$ Reynold's number

ρ mass density

$Pr \equiv \frac{\mu c}{k} \equiv$ Prandtl number

μ shear viscosity

c specific heat

k thermal conductivity

For sufficiently low flow velocities ($Re \lesssim 2.00$), the flow is laminar, and we can calculate

$$Nu = 4.364 \quad \text{(laminar flow)}$$

However in reactors the flow is highly turbulent ($10^3 < Re < 10^5$), and we must use empirical correlations.

Fluids with $Pr \sim 1$ (H_2O or He)

$$Nu = 0.023\, Pr^{0.4} Re^{0.8}$$ Dittus-Boelter

Fluids with $Pr \gtrsim 0.1$ (liquid metals)

$$Nu = 6.3 + 0.03\,(Re\,Pr)^{0.8}$$ Lyon-Martinelli

Examples:

H_2O: $0.05 < h_s < 4.5$ W/cm²K

(single phase) (boiling)

Helium: $0.005 < h_s < 0.05$ W/cm²K

How do we calculate h_s?

Problem: Flow is highly turbulent

Must use "empirical correlations"

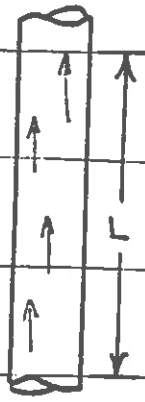
h_s = fcn (fluid properties ; flow characteristics)

Hydraulic Flow:

We can also use empirical correlations to calculate pressure drops

$$\Delta p = 2\left(\frac{L}{D}\right)\rho \bar{u}_z^2 f$$

where f is the Fanning friction factor



$$f = \begin{cases} \frac{16}{Re} & \text{laminar } (Re < 2100) \\ 0.0791\, Re^{-0.25} & (2100 < Re < 10^5) \end{cases}$$

Blasius formula

Comments:

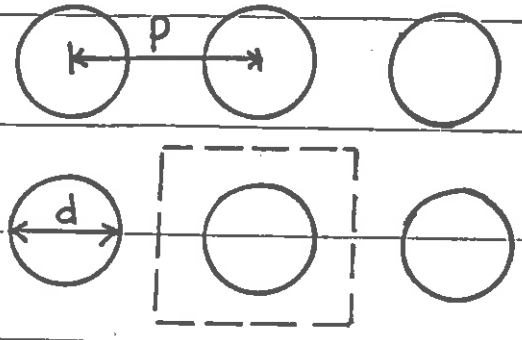
1. f will also depend on pipe surface smoothness -- more generally, use Moody diagram.
2. What about noncircular flow channels? Define an equivalent hydraulic diameter D_h:

$$D_h = \frac{4(\text{flow area})}{(\text{wetted perimeter})} = \frac{4S}{Z}$$

Example Flow through a bundle of circular fuel pins

$$S = p^2 - \frac{\pi d^2}{4}$$

$$Z = \pi d$$



Hence

$$D_h = \frac{4S}{Z} = \frac{4\left(p^2 - \frac{\pi d^2}{4}\right)}{\pi d} = d\left[\frac{4}{\pi}\left(\frac{p}{d}\right)^2 - 1\right]$$

Coolant	T_{inlet} [°C]	ΔT_{core} [°C]	Pressure [bar]	ΔP_{core} [bar]	u [cm/s]	ρ [g/cm³]	h_s [w/cm² °K]
H_2O	300	20	100	2	400	.70	2.8-4.5
He	500	400	40	1	16,000	.0024	.005-.05
Na	400	150	10	10	400	.85	2.2-5.5

The study of heat transfer from the clad surface is complex:

Flow complications: laminar vs. turbulent

compressible flow

Heat transfer complications: conduction

convection

radiation

Fluid properties complications: single-phase

two-phase

LECTURE 20

CONVECTIVE HEAT TRANSFER COEFFICIENTS

READING ASSIGNMENT:

Pfenningworth and Minkler, pp. 342 - 349
J. H. Rust, "Heat Transfer", Chapter 8 in Weisman (Design)

EXERCISES:

20-1. Provide the details for the derivation of the temperature profile for laminar flow between two stationary parallel plates (plane Pouiseuille flow).

20-2. Using Newton's law to define the convective heat transfer coefficient, determine this coefficient for plane Pouiseuille flow).

20-3. Consider laminar Couette flow between two infinite, parallel plates, one moving with a velocity V with respect to the other. Determine the temperature profile, if the upper plate experiences a constant wall heat flux. Determine the convective heat transfer coefficient for this plate.

20-4. Compare the convective heat-transfer coefficients for the following situations for 2 cm I.D. smooth-drawn tubing:

a) air at 10 bar, 100 m/s, and 200 C

b) CO2 at the same conditions as in a)

c) water at 1 bar, 10 m/s, and 200 C

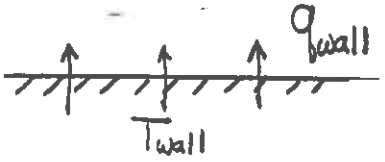
d) sodium at the same conditions as in c)

Convective Heat Transfer Coefficients

- Heat transport equation for a fluid
- Laminar flow
- Turbulent flow correlations

Recall Newton's Law of Cooling

$$q''_{wall} = h_s(T_{wall} - T_f)$$



How do we determine the convective heat transfer coefficient h_s?

Very complex: laminar vs. turbulent

incompressible vs. compressible

single-phase vs. two-phase

The transport of heat in a fluid is described by

$$\rho C_v \left(\frac{\partial}{\partial t} + \underset{\sim}{u}\cdot\underset{\sim}{\nabla}\right) T = -\underset{\sim}{\nabla}\cdot\underset{\sim}{q} - T\left(\frac{\partial p}{\partial T}\right)_v (\underset{\sim}{\nabla}\cdot\underset{\sim}{u}) - \underset{\approx}{\tau} : \underset{\sim}{\nabla}\underset{\sim}{u}$$

heat conduction; work done on (by) fluid pressure; viscous heating

Here we use

$$\underset{\sim}{q} = -k\underset{\sim}{\nabla}T \qquad \text{Fourier's Law}$$

$$\underset{\approx}{\tau}\cdot\underset{\sim}{\nabla}\underset{\sim}{u} = \mu\Phi_u \qquad \text{Newton's Law}$$

There are several simplifications of the energy equation:

1.) Ideal gas: $\left(\frac{\partial p}{\partial T}\right)_\rho = p/T$

$$\rho C_v \left(\frac{\partial}{\partial t} + \underset{\sim}{u}\cdot\underset{\sim}{\nabla}\right) T = k\nabla^2 T - p\,(\underset{\sim}{\nabla}\cdot\underset{\sim}{u})$$

2.) Fluid at constant pressure: $de = -p\,dv + c_p\,dT$

$$\rho c \left(\frac{\partial}{\partial t} + \underset{\sim}{u}\cdot\underset{\sim}{\nabla}\right) T = k\nabla^2 T \quad \text{(neglecting viscosity)}$$

3) Incompressible fluids: $c_p = c_v$, $\underset{\sim}{\nabla}\cdot\underset{\sim}{u} = 0$, ρ = constant

$$\rho C_p \left(\frac{\partial}{\partial t} + \underset{\sim}{u}\cdot\underset{\sim}{\nabla}\right) T = k\nabla^2 T$$

We can also write the energy equation for a fluid in terms of the specific enthalpy: $h \equiv e + p/\rho$

$$\rho\left(\frac{\partial h}{\partial t} + \underline{u}\cdot\underline{\nabla} h\right) = \underline{\nabla}\cdot(k\underline{\nabla}T) + \left(\frac{\partial p}{\partial t} + \underline{u}\cdot\nabla p\right) + \mu\Phi_u$$

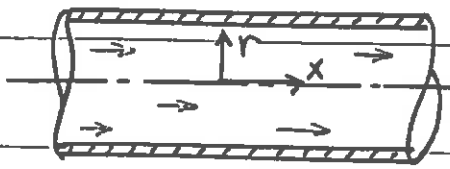
For many applications, $\frac{\partial p}{\partial t} + \underline{u}\cdot\nabla p$ and $\mu\Phi_u$ can be ignored

$$\rho\left(\frac{\partial h}{\partial t} + \underline{u}\cdot\underline{\nabla} h\right) = \underline{\nabla}\cdot(k\underline{\nabla}T)$$

Laminar Pipe Flow

$$\rho\left(\frac{\partial h}{\partial t} + \underline{u}\cdot\underline{\nabla} h\right) = \underline{\nabla}\cdot k\underline{\nabla}T + \left(\frac{\partial p}{\partial t} + \underline{u}\cdot\underline{\nabla} p\right) + \mu\Phi_u$$

Assume:



i) steady state

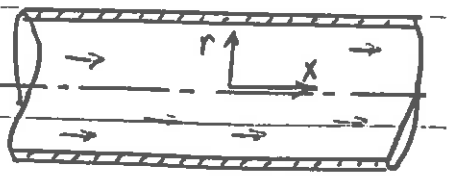
ii) constant physical properties (ρ, μ, k, c_p)

iii) $\underline{u}\cdot\underline{\nabla} p$, $\mu\Phi_u$ negligible

iv) fully-developed flow in x-direction

v) $dh = c_p\, dT$

Under these assumptions we find



$$\rho c_p u_x \frac{\partial T}{\partial x} = k \nabla^2 T$$

If the flow is fully developed laminar flow, we can use the velocity profile given by Hagen-Poiseulle flow

$$u_x = u_{max}\left(1 - \frac{r^2}{R^2}\right)$$

to solve for

$$h_s = \frac{k \frac{\partial T}{\partial n}\Big|_{wall}}{T_{wall} - T_{fl}} = 4.364\left(\frac{k}{d}\right)$$

Now recall our definition of the Nusselt number Nu

$$Nu = \frac{h_s D_h}{k}$$

Thus for fully-developed laminar flow we find

$$Nu = 4.364$$

Turbulent Flow

In a reactor, one generally finds $Re > 2100$ so that the flow is turbulent. Then one must resort to empirical correlations.

Most correlations take the form

$$Nu = c\, Re^{n} Pr^{m}$$

Correlations for LWR

Dittus-Boelter Correlation: $\frac{L}{D_h} > 60$, $Re > 10^4$

$$Nu = 0.023\, Re^{.8} Pr^{m}$$

$m = 0.4$ heating
$m = 0.3$ cooling

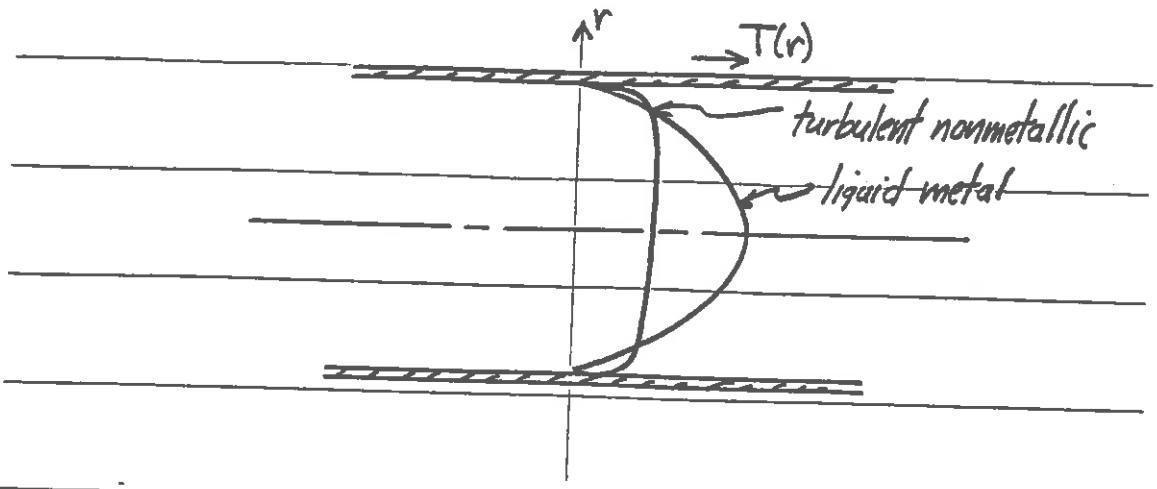
Thom Correlation: LWR conditions

$$Nu = 0.134\, Re^{.65} Pr^{.4}$$

Correlations for Liquid Metals

nonmetallic coolants: convection >> conduction

liquid metals: conduction > convection



$Pr < 0.1$ (liquid metals)

$$Nu = 6.30 + 0.03 (Re\,Pr)^{0.8}$$

(constant heat flux)

$$Nu = 4.80 + 0.03 (Re\,Pr)^{0.8}$$

(constant temperature)

Lyon-Martinelli

$0.5 \leq Pr \leq 1.0$

$$Nu = 0.022\ Pr^{0.6}\ Re^{0.8}$$

$Pr > 20$ (highly viscous fluids)

$$Nu = 0.0118\ Pr^{0.3}\ Re^{0.9}$$

Note:

$$Pr \sim \frac{\text{viscosity}}{\text{thermal conductivity}} \sim \frac{\nu}{\alpha}$$

$Pr < 0.1$ liquid metals

$1.0 < Pr < 20$ water, air, light liquids

$Pr > 20$ oils, highly viscous liquids

Transient Heat Transfer

Important for reactor transients (including LOCA)

Transient h_s are different than in steady-state

Increasing heat flux: $h_s^{tran} > h_s^{ss}$

Decreasing heat flux: $h_s^{tran} < h_s^{ss}$

LECTURE 21

TWO-PHASE FLOW PHENOMENA

READING ASSIGNMENT:

Duderstadt & Hamilton, pp. 490 - 493
Lamarsh, pp. 375 - 383
El-Wakil, pp. 291 - 326
Sesonske, pp. 116 - 126
J. H. Rust, "Fluid System Design", Chapter 7 in Weisman (Design)
J. H. Rust, "Heat Transfer", Chapter 8 in Weisman (Design)

EXERCISES:

21-1. Determine the saturation temperature for water in BWRs (at 70 bar) and PWRs (at 155 bar).

21-2. In an experiment on pool boiling of water, the heat flux and water temperature and pressure are simultaneously increased so that saturation boiling occurs at all times. Burnout (DNB) occurs when the pressures reaches 300 psia. Assuming for simplicity that burnout heat transfer occurs solely by radiation, and that the radiation heat transfer coefficient (similar in definition to convective heat transfer coefficient) is 200 BTU/hr-ft-2-F, estimate the temperature of the heating surface at burnout.

21-3. By referring to any of the standard references, contrast the slip flow and drif flux models of two phase flow.

Two-Phase Flow Phenomena

- Pool boiling phenomena
- Boiling channel phenomena
- Phase changes
- Two-phase flow models

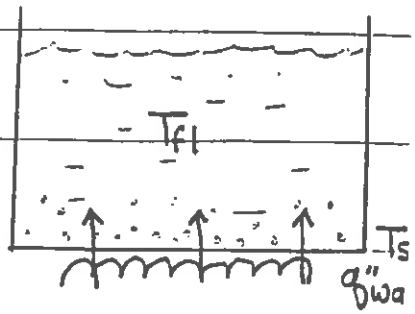
Thus far we have restricted our attention to single phase fluids: liquids or gases

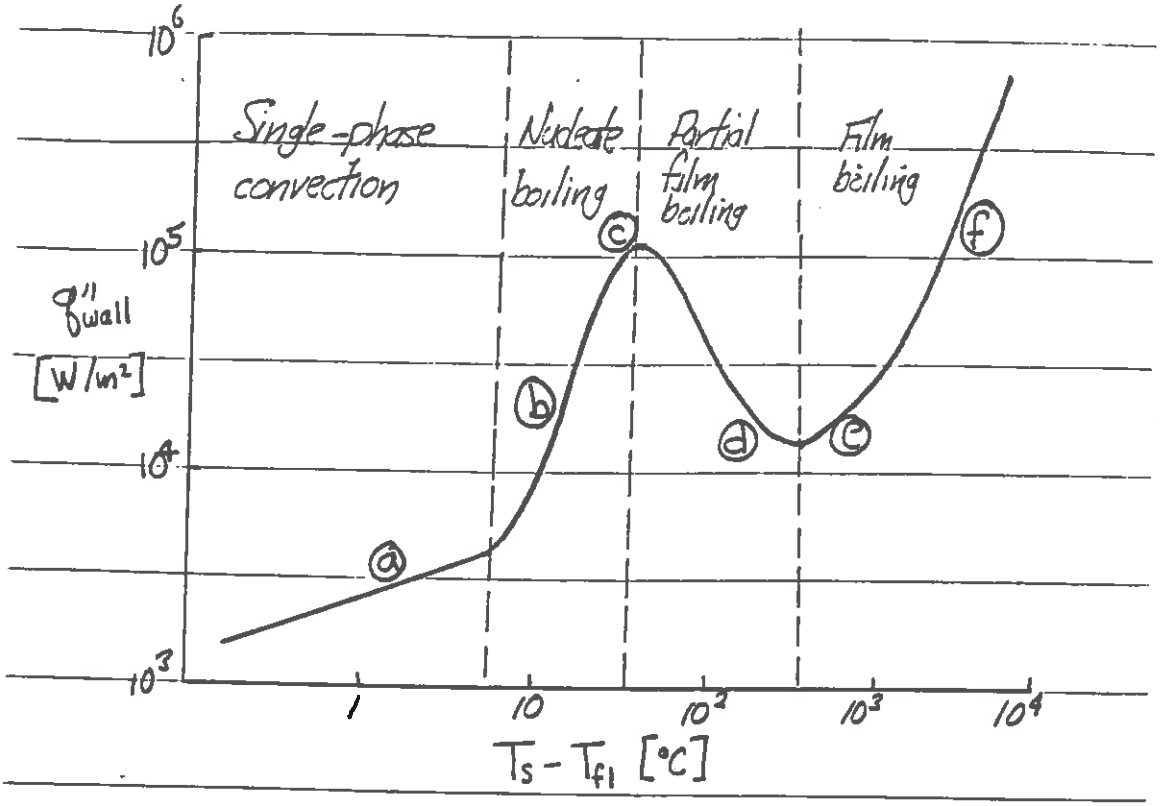
But of major importance in reactor applications are two-phase fluids -- mixtures of liquid and vapor

First, some background:

Pool Boiling Phenomena

Consider a pool of liquid containing a heating element. Suppose we measure the heat flux q_{wall} transported across the heated surface as a function of temperature difference $T_s - T_{f1}$. We will plot this for water:





o-a: little liquid superheat; natural convection

a-b: a few bubbles are formed, but collapse after leaving surface

a-c: number of bubbles increases -- nucleate boiling

c: departure from nucleate boiling (DNB or "burnout")

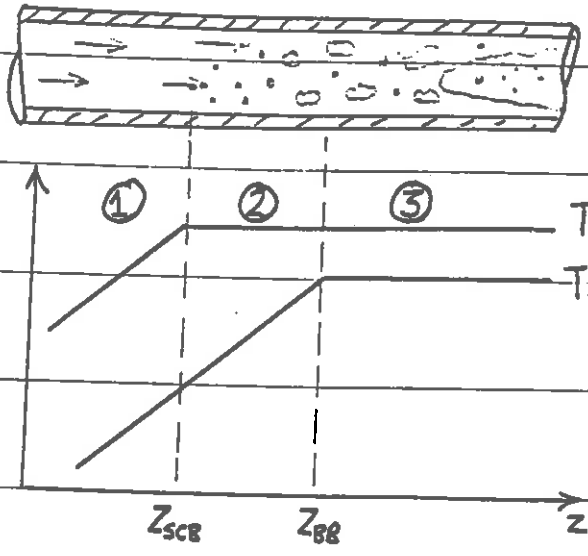
c-d: bubbles coalesce and clump near heating surface

d-e: continuous blanket of vapor forms over surface (film boiling)

e-f: thermal radiation becomes important (film and radiation)

Boiling Channel Phenomena

Consider a channel in which boiling occurs:



Region ①: Single-phase convection

We can use all of our earlier analysis -- e.g.,

$$q''_{wall} = h_s (T_{wall} - T_{fl})$$

and

$$h_s = \frac{k}{D_h} Nu = \frac{k}{D_h} \left[0.023 \, Pr^{0.4} \, Re^{0.8} \right]$$

Region ②: Subcooled Boiling [$T_s \geq T_{sat}$ but $T_{fl} < T_{sat}$]

Now bubbles form on the surface but collapse as they move into the bulk fluid. Heat transfer is very efficient. But the correlation is no longer linear

$$\Delta T|_{cool} = T_{wall} - T_{fl} = 45 \, e^{-p/62} \left(q''_{wall} \right)^{0.25}$$

Jens-Lottes correlation

Example: PWR, $p = 155$ bar

$q''_{wall} = 1.5 \times 10^6 \, W/m^2$ $\Rightarrow \Delta T|_{cool} = 3°C$

Region ③: Saturated or Bulk Boiling [$T_{f,1} = T_{sat}$]

Once $T_{f,1} \rightarrow T_{sat}$, bulk boiling occurs. Subsequent heat addition will cause further boiling, while $T_{f,1}$ remains at T_{sat} (isothermal flow)

Phase Changes:

Evaporation: process of conversion of liquid into vapor

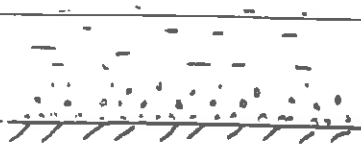
Boiling: when vapor forms within a continuous liquid

Pool boiling: boiling due to fluid contact with hot surface

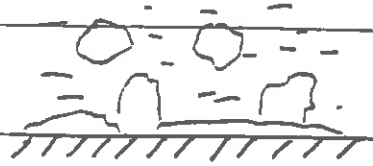
Bulk boiling: boiling due to bulk temperature of fluid reaching T_{sat} (also called saturated boiling)

Subcooled boiling: bulk of fluid at $T_{f,1} < T_{sat}$ -- boiling occurs only near heated surface

Nucleate boiling: bubbles of vapor are formed within liquid about a small nucleus of vapor



Film boiling: formation of a continuous film of vapor that covers heating surface



Two-phase flow: both vapor and liquid move together in a channel

diabatic: boiling occurs

adiabatic: boiling does not occur

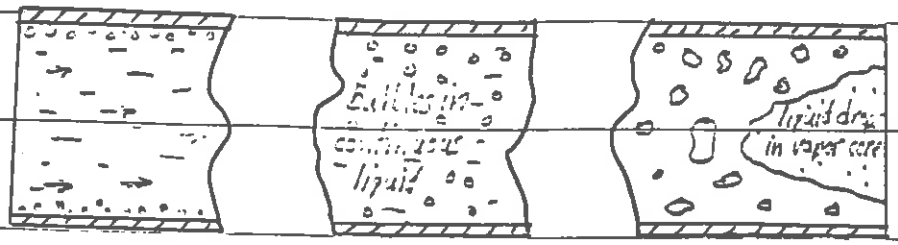
Condensation: process of conversion of vapor back to liquid

filmwise: continuous flow of liquid over cooling surface

dropwise: vapor condenses in drops; cooling surface is not completely covered by liquid

Qualitative Characteristics of Two-Phase Flow

We can identify a variety of two-phase flows



subcooled boiling flow | bubble flow | slug flow | annular flow

Models of Two-Phase Flow

Homogeneous model: Both liquid and vapor phases are assumed to be homogeneous and moving with the same velocity.

Slip flow model: Each phase is assumed to have a unique flow area and velocity as well as a unique density and specific energy.

Drift flux model: Multifield models which explicitly account for interactions between liquid and vapor phases.

Two-fluid model: Treat liquid and vapor as separate fluids, coupling hydrodynamics equations with mass, momentum, and energy exchange terms.

LECTURE 22

THE SLIP-FLOW MODEL OF BOILING FLOW

READING ASSIGNMENT:

Duderstadt & Hamilton, pp. 493 - 496
El-Wakil, pp. 326 - 334

EXERCISES:

22-1. Derive the S-alpha-chi relationship.

22-2. Consider three Carnot cycles using mercury, sodium, and water as working fluids which receive heat causing these fluids to vaporize at 10 atm with a thermodynamic efficienty of 35%. If the three fluids change from saturated liquid to saturated vapor during the heat-addition processes, determine in each case: a) the quality at the end of expansion, and b) the quality at the beginning of compression.

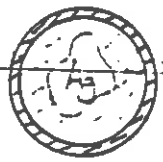
22-2. A BWR operates at a pressure of 1000 psia. In one of its fuel assemblies, the inlet velocity is 10 ft/s, inlet temperature is 520 F, and the coolant mass flow rate is 200,000 lbm/hr. If the heat generated in the afuel assembly is 2 x 10^7 BTU/hr, what is the fuel assembly exit void fraction?

The Slip Flow Model of Boiling Flow

- Static quality and void fraction
- Flow quality and slip
- Heat addition to boiling flow

Let's introduce some parameters to characterize two-phase flow.

First consider a static fluid.



Introduce

$$\text{Static quality} \equiv \chi_s \equiv \frac{\text{mass of vapor in mixture}}{\text{total mass of mixture}}$$

$$\text{Void fraction} = \alpha \equiv \frac{\text{volume of vapor in mixture}}{\text{total volume of liquid-vapor mixture}}$$

Let: ~~A_g = cross sectional area occupied by vapor~~

$A_g \equiv$ cross sectional area occupied by vapor

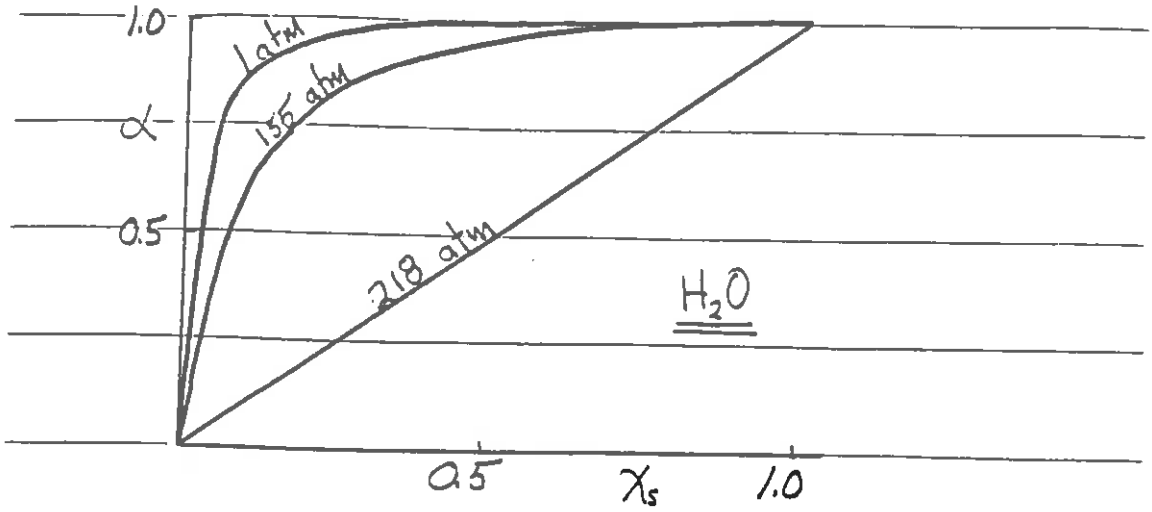
$A_\ell \equiv$ cross sectional area occupied by liquid

Then

$$\alpha = \frac{A_g}{A_g + A_\ell}$$

We can now calculate the static quality

$$\chi_s = \frac{\rho_g A_g}{\rho_g A_g + \rho_\ell A_\ell} = \frac{1}{1 + \frac{\rho_\ell}{\rho_g}\frac{A_\ell}{A_g}} = \frac{1}{1 + \frac{\rho_\ell}{\rho_g}\left(\frac{1-\alpha}{\alpha}\right)}$$



While we are at it, let's write the specific enthalpy h in terms of the quality χ_s. For a single-phase fluid

$$h = h_\ell(T) = \int_{T_{melt}}^{T} c_p(T')\,dT'$$

Let

$h_{\ell g} \equiv$ heat of vaporization of liquid

Then the enthalpy of a saturated vapor at T_{sat} is just

$$h_g(T_{sat}) = h_\ell(T_{sat}) + h_{\ell g}$$

The enthalpy of a two-phase mixture of liquid and vapor is

$$h_{mixture} = (1-x_s)h_\ell(T_{sat}) + x_s h_g(T_{sat})$$

$$= h_\ell(T_{sat}) + x_s h_{\ell g}$$

We can turn this around to calculate quality in terms of enthalpy

$$x_s = \frac{h_{mixture} - h_\ell(T_{sat})}{h_{\ell g}}$$

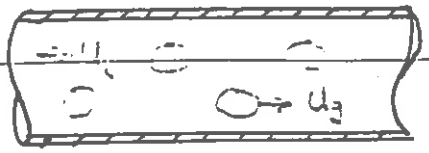
(Note this could be used to define a "negative quality.")

Example: Steam

P [atm]	T_{sat} [°C]	h_ℓ [MJ/kg]	h_g [MJ/kg]	c_p	x_s	$\frac{\rho_\ell}{\rho_g}$	α
1	100	0.42	2.26	4.2	0.1	1600	0.795
70	285	1.26	1.51	5.4	0.1	21	0.7
155	345	1.68	0.96	7.9	0.1	5.9	0.4
218	373	2.10	0	~62	0.1	1	0.1

Now consider a flowing two-phase mixture. We must now account for the fact that the vapor has a tendency to slip past the liquid because of its bouyancy. Define

$$\text{Slip ratio} = \frac{\text{vapor velocity}}{\text{liquid velocity}} = \frac{u_g}{u_\ell}$$



Now define a flow quality

$$\text{Flow quality} \equiv \chi \equiv \frac{\text{mass flow rate of vapor}}{\text{total mass flow rate}} = \frac{w_g}{w_g + w_\ell}$$

We can calculate

$$\chi = \frac{w_g}{w_g + w_\ell} = \frac{A_g \rho_g u_g}{A_g \rho_g u_g + A_\ell \rho_\ell u_\ell} = \frac{1}{1 + \frac{\rho_\ell}{\rho_g}\frac{A_\ell}{A_g}\frac{u_\ell}{u_g}}$$

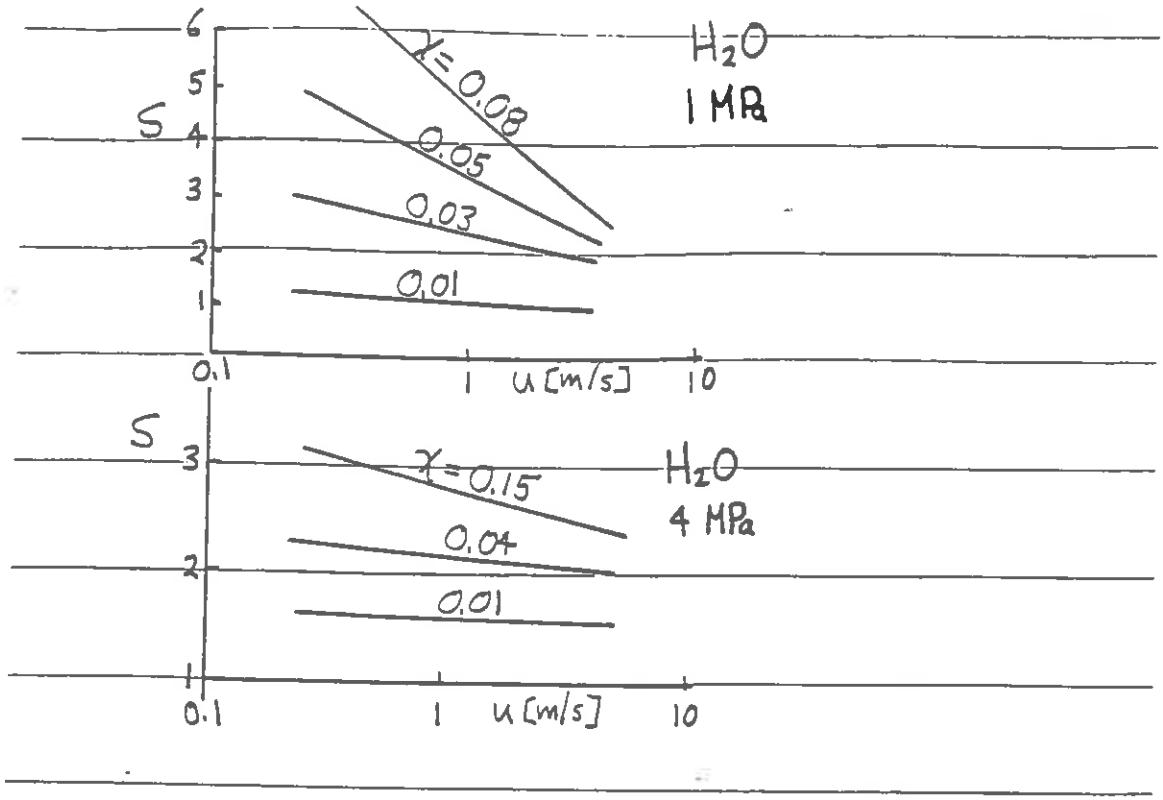
or

$$\chi = \frac{1}{1 + \frac{\rho_\ell}{\rho_g}\left(\frac{1-\alpha}{\alpha}\right)\frac{1}{S}}$$

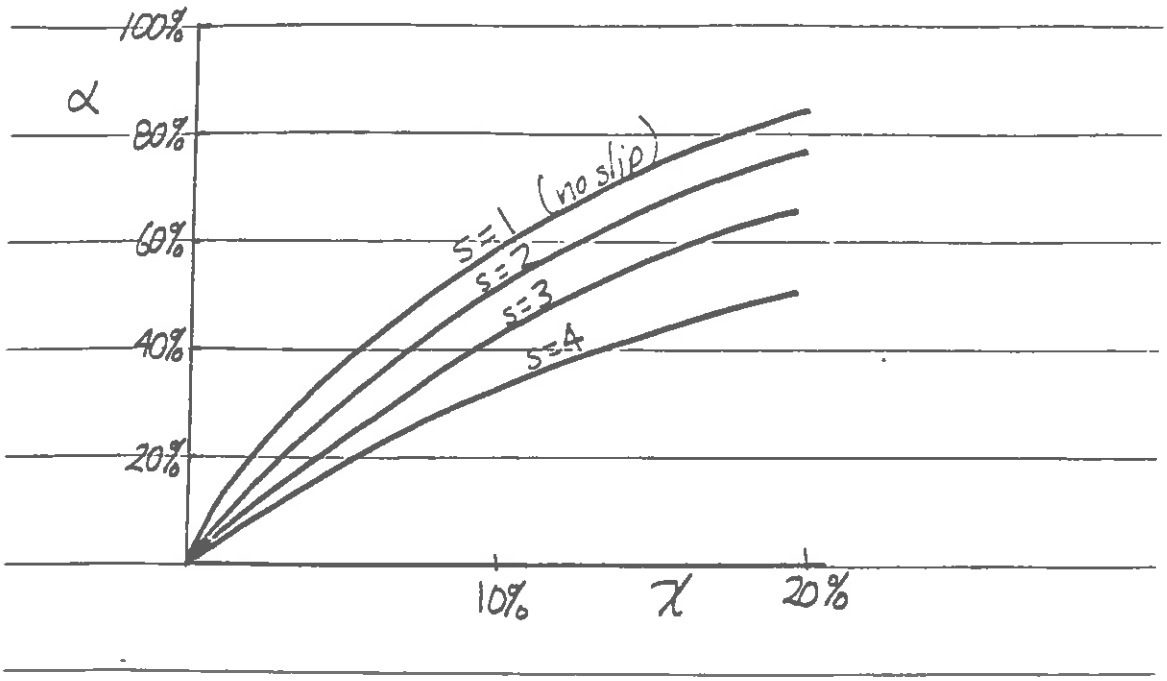
We can invert this to write the void fraction as

$$\alpha = \frac{1}{1 + \frac{\rho_g}{\rho_\ell}\left(\frac{1-\chi}{\chi}\right)S}$$

This is sometimes called the S-α-χ relationship.

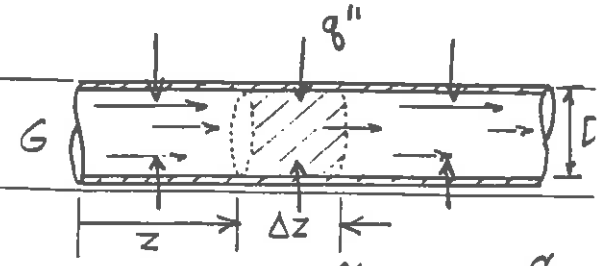


α vs. χ for H_2O (p = 6 MPa)



Heat addition to single-phase flow

We can perform an energy balance on a fluid control volume



G = fluid mass flux $= \rho u$

change in internal energy of fluid volume / sec $= G\left(\frac{\pi D^2}{4}\right) c_p \Delta T$

$= q''(\pi D)\Delta z =$ heat flux through surface

or as $\Delta z \to dz$

$$G\left(\frac{\pi D^2}{4}\right) c_p \, dT = q''(\pi D)\, dz$$

We can now integrate up the channel

$$\int_{T_{inlet}}^{T} G\left(\frac{\pi D^2}{4}\right) c_p \, dT = \int_0^z q''(\pi D)\, dz$$

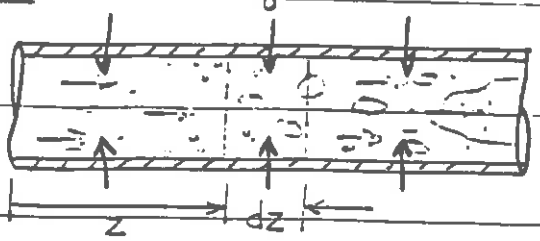
or

$$T(z) = T_{inlet} + \frac{4}{GDc_p}\int_0^z q''(z')\, dz'$$

Thus we can determine the bulk fluid temperature at any position in the channel (if we are given $q''(z)$)

Heat addition to boiling flow

We can again perform an energy balance



change in internal energy of fluid / volume/sec $= G\left(\frac{\pi D^2}{4}\right) dh$

$= q''(\pi D)\, dz =$ heat flux through surface

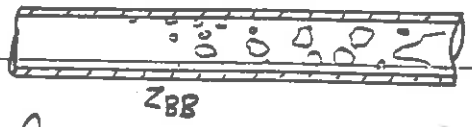
Rearranging

$$dh = \frac{4}{GD} q'' dz$$

Integrating

$$h(z) = h_{inlet} + \frac{4}{GD} \int_0^z q''(z')dz'$$

Suppose that bulk boiling begins at a point z_{BB}. We can determine z_{BB} from



$$h_f(T_{sat}) = h_{inlet} + \frac{4}{GD} \int_0^{z_{BB}} q''(z')dz'$$

for $z > z_{BB}$, remains at T_{sat} - our phase change occurs. Then

$$h(z) = h_{mixture} = h_f(T_{sat}) + \chi(z) h_{fg}, \quad z > z_{BB}$$

If we combine this with our energy balance equation, we find

$$\chi(z) = \frac{4}{GD} \frac{1}{h_{fg}} \cdot \int_{z_{BB}}^{z} q''(z')\, dz', \quad z > z_{BB}$$

Then if we know S (slip ratio), we can use S-α-χ relationship to determine $\alpha(z)$

LECTURE 23

HEAT ADDITION TO BOILING FLOW

READING ASSIGNMENT:

Duderstadt & Hamilton, pp. 496 - 501
El-Wakil, pp. 303 - 313, 336 - 364

EXERCISES:

23-1. Determine the axial clad surface and fuel centerline temperature distribution present in a bare, uniform cylindrical core with single-phase coolant flow. In particular, determine those positions at which the maximum fuel centerline and clad surface temperatures occur.

23-2. Water flows in a circular channel 3 m long by 1 cm I.D. The system pressure is 70 bar. The water enters the channel with average velocity 3 m/s and temperature 260 C. The wall heat flux has a cosine distribution (no extrapolation lengths) with peak value of 200 W/cm^2. Determine: a) axial coolant temperature distribution, b) axial quality distribution, c) axial void fraction distribution, assuming a constant slip ratio of 2, d) graph these qualitities versus z. Assume thermodynamic equilibrium exisits at each axial position (i.e., neglect subcooled boiling).

23-3. For the problem above (23-2), determine the axial position where subcooled boiling starts. What is the coolant temperature at this position? The wall temperature?

23-4. The power distribution in the axial direction of the 12 ft long fuel rods of a PWR is well represented by a cosine power distribution. The minimum DNBR is 2.1 and occurs 90 in. up the hottest channel, where the critical heat flux is computer to be 1.08 x 10^6 Btu/hr-ft-2. The hot channel factor for the reactor is 2.78 and the total heat transfer area is 48,000 ft-2. Calculate (a) the maximum heat flux, (b) the average heat flux, and (c) the operating power of the reactor.

Heat Addition to Boiling Flow

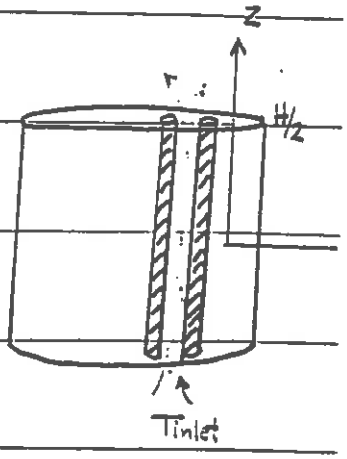
- Axial temperature profiles
- Two-phase heat transfer regimes
- DNB
- Critical heat flux correlations

Let's now see how we might calculate the axial temperature distribution in a coolant channel. Recall our energy balance

$$G\left(\frac{\pi D^2}{4}\right) c_p dT = q'' (\pi D) dz$$

or

$$W c_p dT = q' dz$$



(Here we are assuming for the moment that coolant remains in the liquid phase)

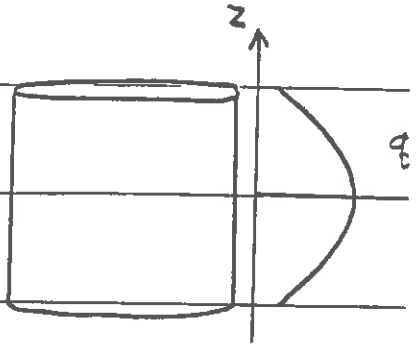
We can now integrate up the channel from the inlet conditions

$$WC_p \int_{T_{inlet}}^{T} dT = \int_{-H/2}^{z} q'(z)\,dz$$

All we need now is the axial power distribution $q'(x)$.

Example: Bare, uniform core

$$q'(z) = q'_0 \cos\left(\frac{\pi z}{\tilde{H}}\right)$$



Hence

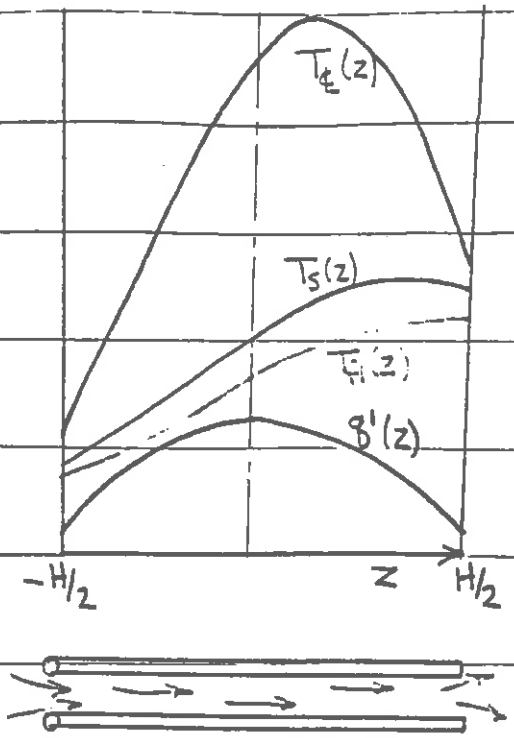
$$WC_p \int_{T_{inlet}}^{T_{f1}} dT = q'_0 \int_{-H/2}^{z} \cos\left(\frac{\pi z'}{\tilde{H}}\right) dz'$$

or

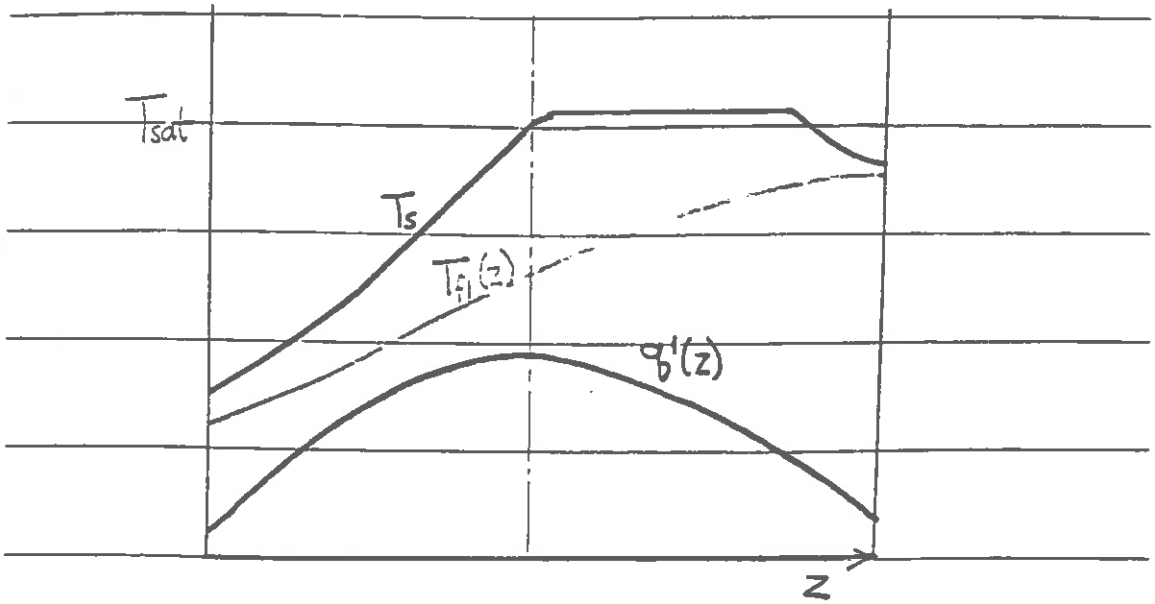
$$T_{f1}(z) - T_{inlet} = \frac{q'_0 \tilde{H}}{\pi C_p W}\left[\sin\left(\frac{\pi z}{\tilde{H}}\right) + \sin\left(\frac{\pi H}{2\tilde{H}}\right)\right]$$

We can determine the exit coolant temperature

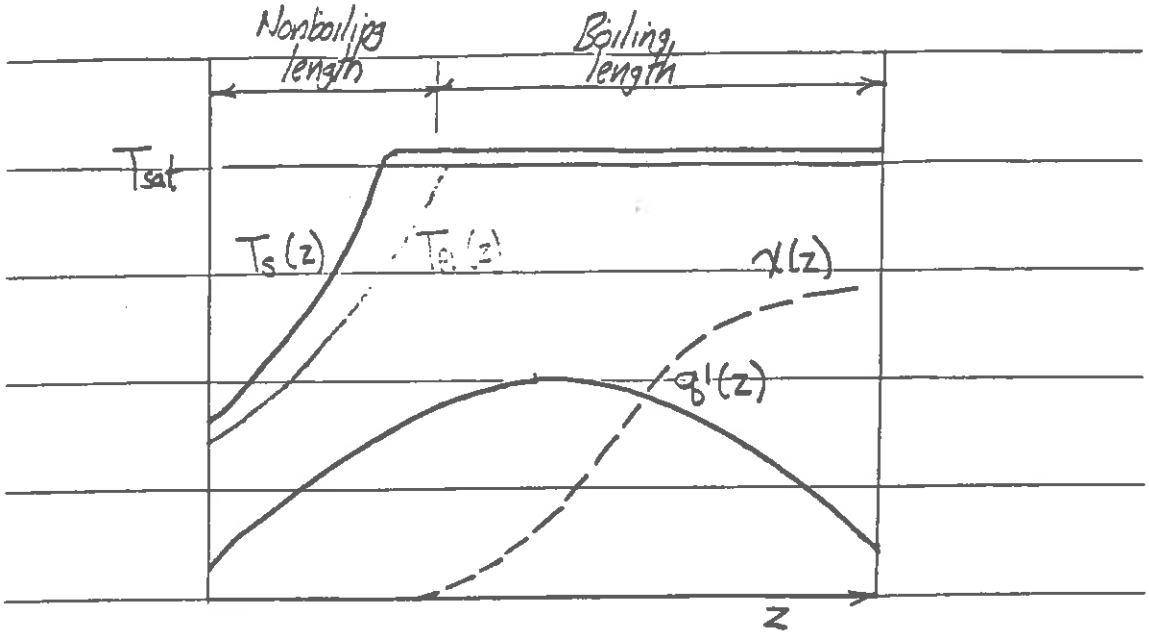
$$\Delta T|_{core} = T_{exit} - T_{inlet} = \frac{2q'_0 \tilde{H}}{WC_p \pi} \sin\left(\frac{\pi H}{2\tilde{H}}\right) \sim \frac{2q'_0 \tilde{H}}{\pi C_p W}, \quad \tilde{H} \sim H$$



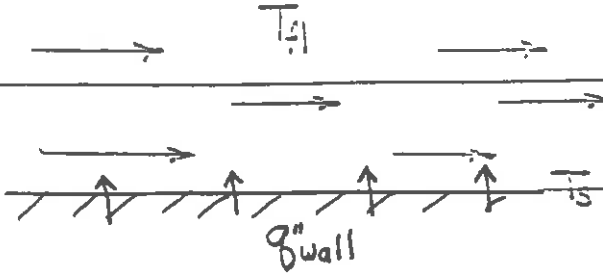
Temperature profiles in a PWR



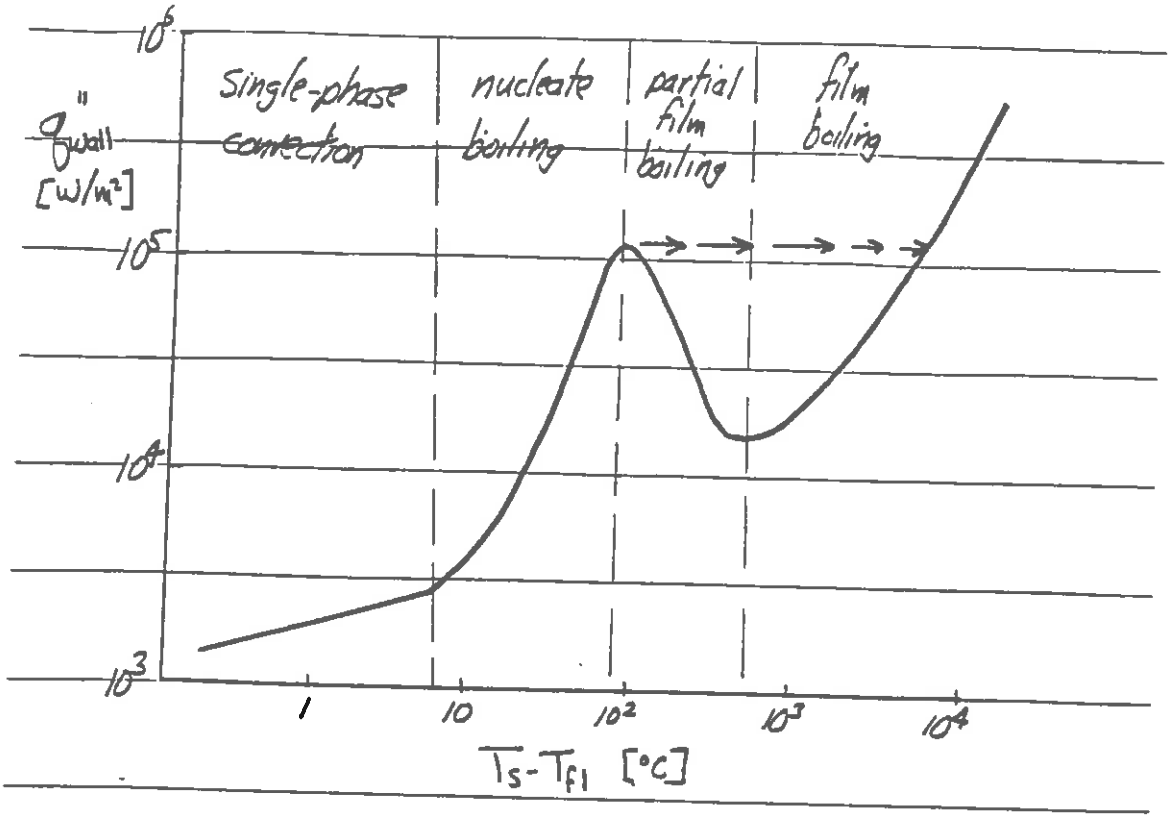
Temperature profiles in a BWR



Suppose our task is to extract heat from a surface with a flowing coolant:

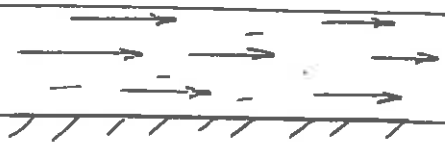


Let us assume q''_{wall} is increased from low to high fluxes, and then ask what happens to the resulting temperature difference $T_s - T_f$. Recall the heat transfer curve:



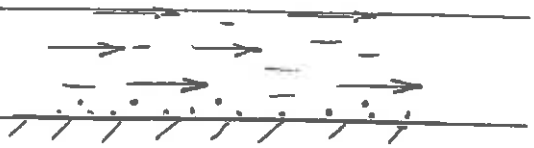
As q''_{wall} increases, the heat transfer mechanism changes:

① single-phase convection

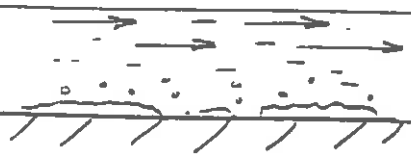


② nucleate boiling (subcooled)

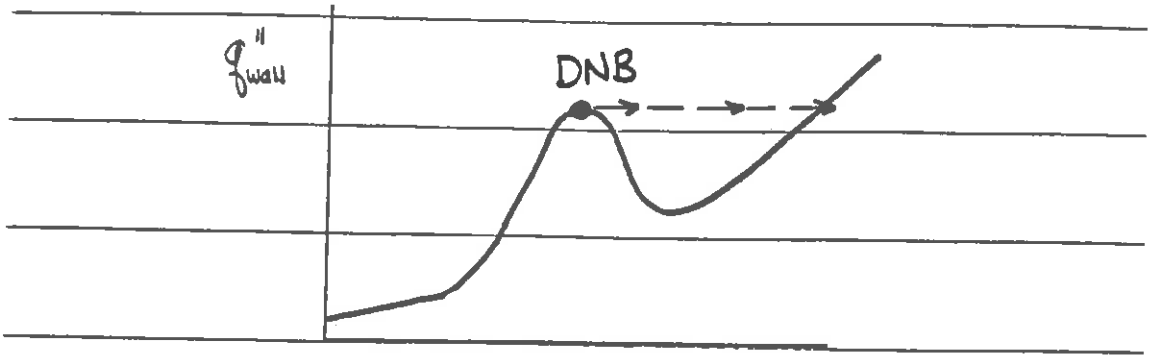
"departure from nucleate boiling" (DNB)

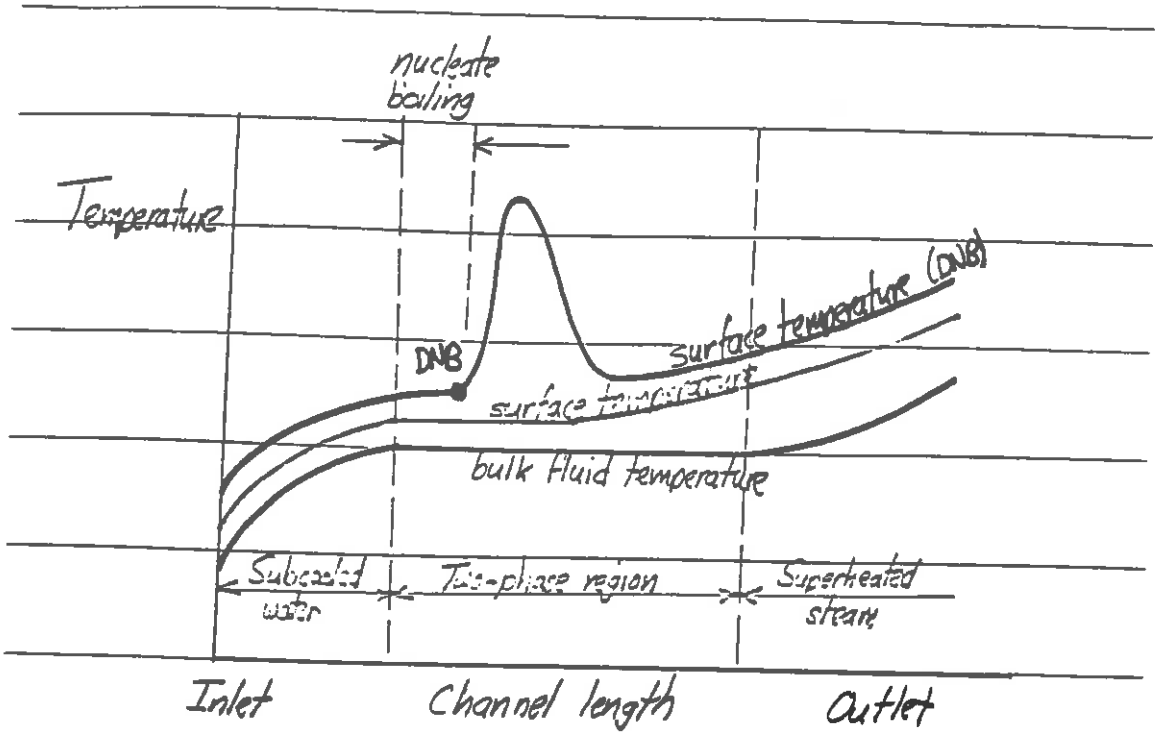


③ film boiling



Since film boiling is a far less effective heat transfer mechanism than nucleate boiling, the surface temperature rises rapidly at DNB:





It is extremely important to be able to predict the value of the critical heat flux q''_{crit} beyond which DNB occurs [E.g., nuclear reactors or rocket motors with liquid-fueled cooled walls.] These are given by critical heat flux empirical correlations.

Example: Jens-Lottes

$$q''_{crit} = C\left(\frac{G}{10^6}\right)^m (T_{sat} - T_{fl})^{0.22}$$

$C \sim 10^6$, $m \sim 0.2 - 0.7$ (depends on p)

Example: Macbeth

$$q'_{crit} = 0.00633 \times 10^6 \, h_{fg} \, D_h^{-0.1} \left(\frac{G}{10^6}\right)^{.5} (1 - \chi)$$

Example: Janssen-Levy (GE)

$$\frac{q''_{crit}}{10^6} = 0.705 + 0.237\left(\frac{G}{10^6}\right), \quad \chi < \chi_1$$

$$\frac{q''_{crit}}{10^6} = 1.634 - .270\left(\frac{G}{10^6}\right) - 4.710\,\chi_1 \quad \chi_1 < \chi < \chi_2$$

$$\frac{q''_{crit}}{10^6} = 0.605 - 0.164\left(\frac{G}{10^6}\right) - 0.653\,\chi_1 \quad \chi_2 < \chi$$

$$\chi_1 = 0.197 - 0.108\,(G/10^6)$$

$$\chi_2 = 0.254 - 0.026\,(G/10^6)$$

Example: W-3 (Tong)

$$\frac{q''_{crit}}{10^6} = \left\{(2.022 - 0.000430p) + (0.1722 - 0.0000984p)\right.$$
$$\left. \times \exp\left[(18.77 - 0.004129p)x\right]\right\}$$
$$\times \left[(0.1484 - 1.596x + 0.1729x|x|)\frac{G}{10^6} + 1.037\right]$$
$$\times \left[1.157 - 0.869x\right] \times \left[0.2664 + 0.8357\exp(-3.15D_h)\right]$$
$$\times \left[0.8258 + 0.000794(h_f - h_{in})\right]$$

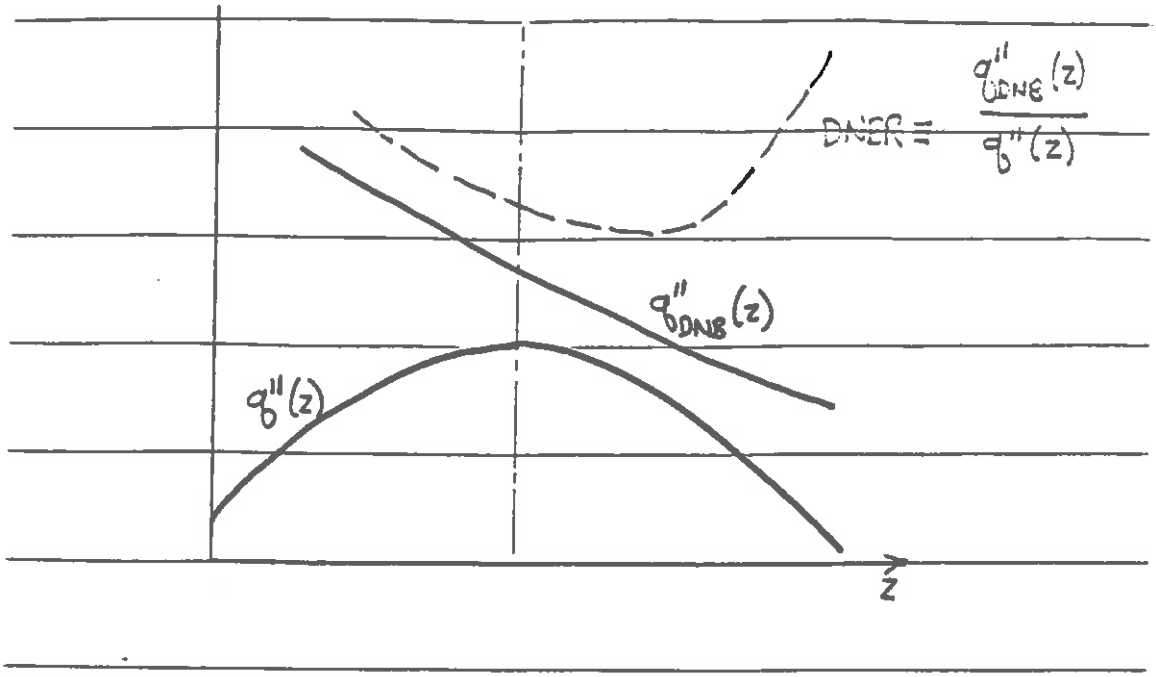
[$q''_{DNB} \sim 300$ W/cm^2 in LWRs]

Heat transfer correlation for nucleate boiling

Jens-Lottes:
$$q'' = \left(\frac{e}{60}\right)^{P/900} \Delta T_{sat}^4$$

Thom:
$$q'' = \left(\frac{e}{.072}\right)^{P/1260} \Delta T_{sat}^2$$

Levy:
$$q'' = \frac{p^{4/3}}{495} \Delta T_{sat}^3$$


DNBR = $\frac{q''_{DNB}(z)}{q''(z)}$
$q''_{DNB}(z)$
$q''(z)$
z

LECTURE 24

THERMAL RADIATION

READING ASSIGNMENT:

Eckert & Drake, pp. 353 - 370
Pfenningwerth & Minkler, pp. 404 - 408

EXERCISES:

24-1. Compare the wavelengths and photon energies of visible light, infrared radiation, radio waves, television transmissions, X-rays, and gamma rays.

24-2. What does Lambert's law imply about the photon source angular distribution on a surface.

24-3. A power plant is designed to generate power for a life support system on a space station. The plant must be nuclear because the length of the mission prohibits the lifting off of sufficient fossil-fuel supplies. The upper temperature of the power plant cycle is fixed by metallurgical considerations. Because the plant is in space, it can only reject heat by thermal radiation to space according to the Stefan-Boltzmann law. Noting that the radiator is by far the largest mass in the power plant, it is found that from a mass point of view it is preferable to operate the plant at relatively low thermodynamic efficiency. Derive a relationship between mass and cycle efficiency, assuming for simplicity that the plant operates on a Carnot cycle. Then find the optimum efficiency at which the mass is a minium. (Assume the mass is proportional to the radiator surface area.)

Thermal Radiation

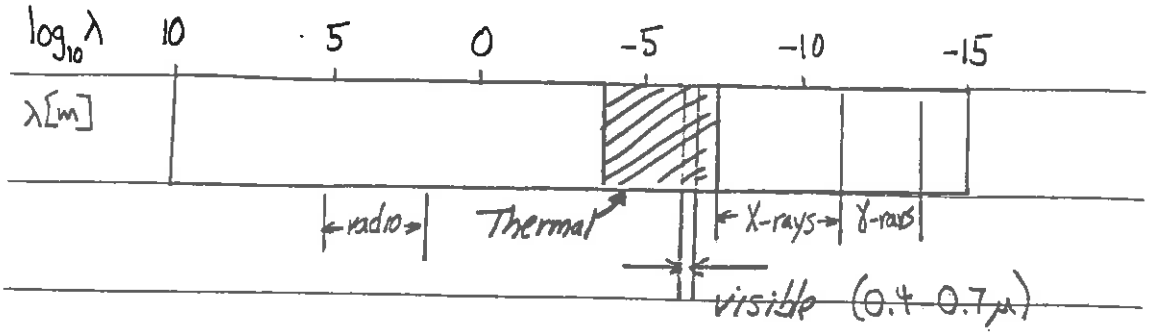
- General properties
- Conservation law
- Kirchoff's law
- Blackbody radiation
- Stefan-Boltzmann Law

Properties of thermal radiation

1) Thermal radiation → electromagnetic waves
(requires no propagation medium)



2) Electromagnetic waves characterized by wavelength

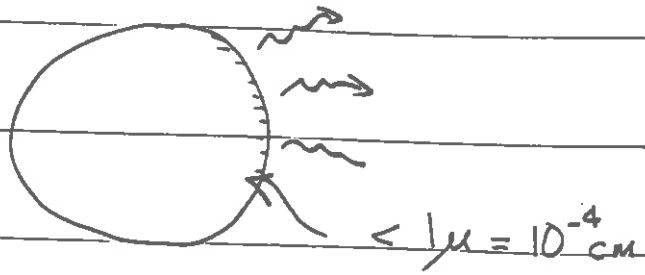


Some units: micron μ: 10^{-4} cm

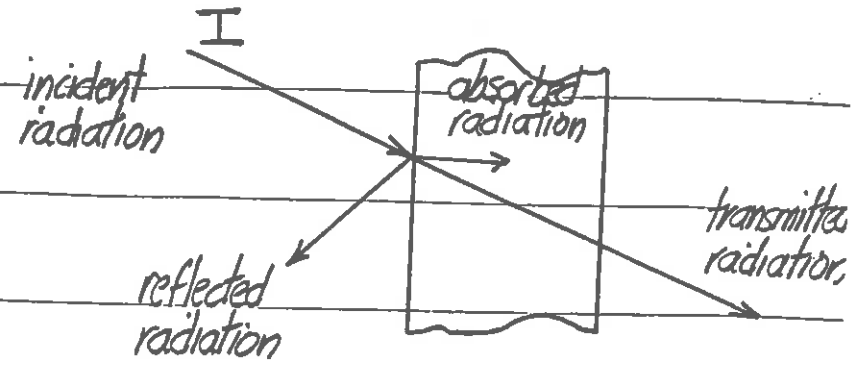
Angstrom Å: 10^{-8} cm

3) Thermal energy is radiated as a consequence of temperature, independent of surroundings

4) Radiation from solids is primarily a surface phenomenon



Conservation Law



Define

α absorptivity

ρ reflectivity

τ transmissivity

Our conservation law is

energy into surface = energy out

or

$$\underset{\text{energy}}{\text{incident}} = \underset{\text{energy}}{\text{absorbed}} + \underset{\text{energy}}{\text{reflected}} + \underset{\text{energy}}{\text{transmitted}}$$

$$I \qquad \alpha I \qquad \rho I \qquad \tau I$$

Thus

$$\alpha + \rho + \tau = 1$$

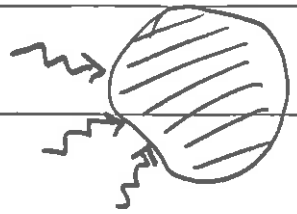
In a solid, the transmissivity τ is generally zero so

$$\rho + \alpha = 1$$

A black body is defined to be one which absorbs all radiation

$$\alpha_b = 1$$

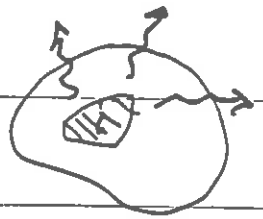
$$\rho_b = 0$$



Let's define several other concepts:

e_i emissive power (radiant flux density) of surface i

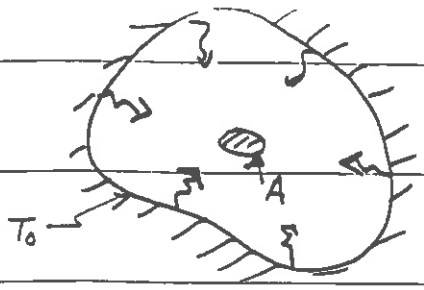
e_b emissive power of a black body



Now define:

$$\epsilon_i \equiv \frac{e_i}{e_b} \quad \text{emissivity}$$

Consider a cavity with a uniform wall temperature T_o: Radiation from walls will establish radiant flux density e_o in cavity. Insert a small body at this point with area A (so that e_o is undisturbed).



Total rate at which energy impinges on body $= e_o A$

Total rate at which body absorbs energy $= e_o \alpha_i A$

Now in thermal equilibrium, body is also at T_o so that

Total radiating energy rate from body $= e_i A$

From thermal equilibrium

$$\alpha_i e_o A = e_i A$$

or

$$\alpha_i e_o = e_i$$

Suppose we replace body by a black body ($\alpha = 1$)

$$e_o = e_b$$

Since e_b is the same in both cases

$$e_i = \alpha_i e_b$$

But recall the definition

$$e_i = \epsilon_i e_b$$

Thus we find

$$\epsilon_i = \alpha_i \qquad \text{Kirchoff's Law}$$

(emissivity = absorptivity)

Black body radiation

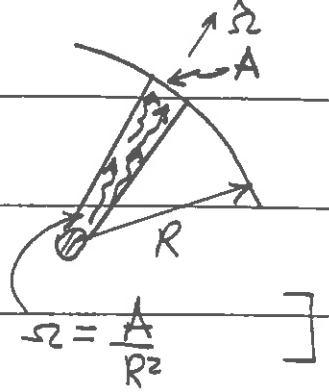
Recall

e_b = emissive power of black body

= total energy radiated per unit time per unit area

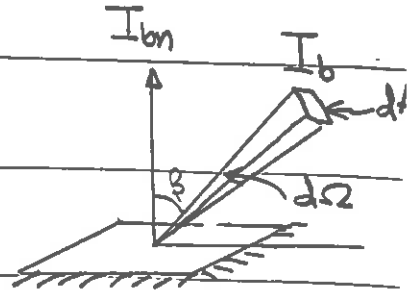
Define

$$I = \frac{de}{d\Omega} = \text{radiation intensity}$$



Ω solid angle $\left[\Omega = \frac{A}{R^2} \right]$

Lambert's Law for diffuse surfaces:



$$I_b = I_{bn} \cos\beta$$

↑ intensity normal to surface

Then total emissive power is

$$e_b = \int_{2\pi} I_b \, d\Omega$$

$$= I_{bn} \int_0^{\pi/2} \cos\beta \, (2\pi \sin\beta) \, d\beta = \pi I_{bn}$$

Planck's Law for black body radiation

$$I_{bn\lambda} = \frac{2hc^2}{\lambda^5 \left(e^{hc/\lambda kT} - 1 \right)}$$

Here

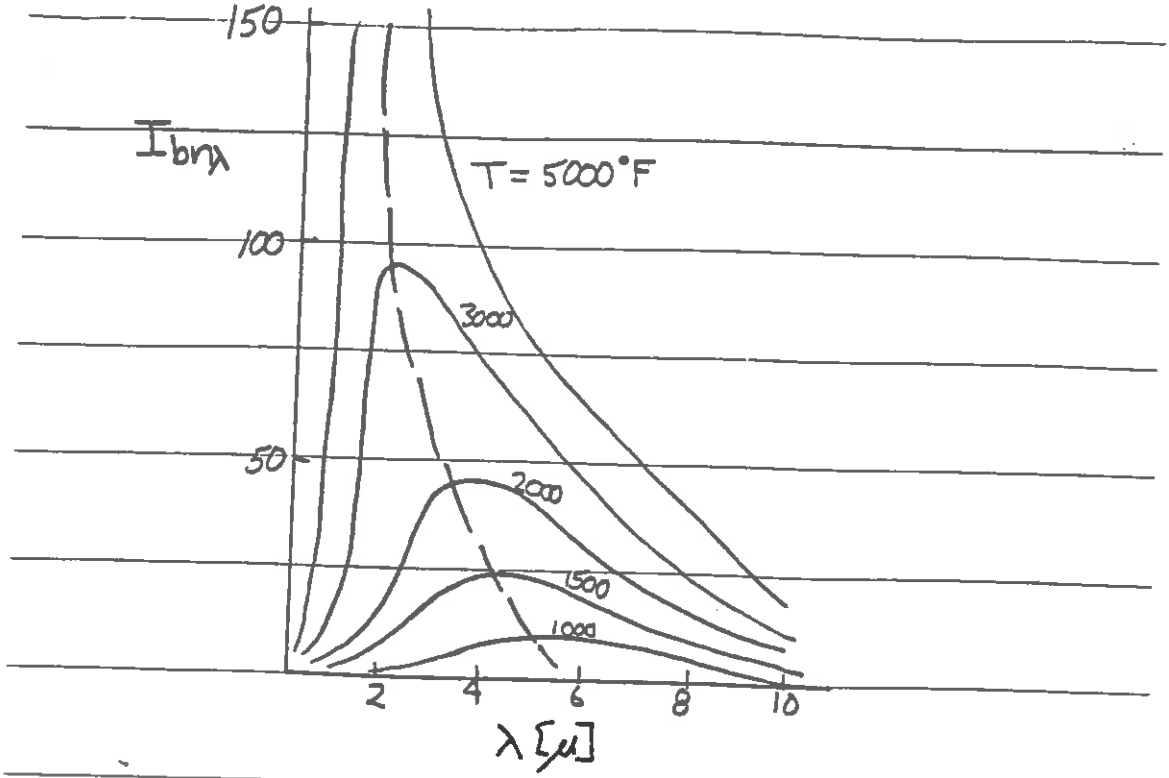
$I_{bn\lambda}$ = monochromatic black body intensity in normal direction

λ = radiation wavelength

h = Planck's constant (6.624×10^{-27} erg.sec)

c = speed of light (3×10^{10} cm/s)

k = Boltzmann's constant (1.38×10^{-16} erg/K)



The total intensity at temperature T is then

$$I_{bn} = \int_0^\infty I_{bn\lambda}\, d\lambda$$

$$= \int_0^\infty \frac{2hc^2\, d\lambda}{\lambda^5 \left(e^{hc/\lambda kT} - 1\right)}$$

$$= 2hc^2 \int_0^\infty x^3 \left(e^{\frac{hc}{kT}x} - 1\right)^{-1} dx$$

$$= C T^4$$

But

$$e_b = \pi I_{bn}$$

Hence

$$e_b = \sigma T^4$$

Stefan-Boltzmann Law

$$\sigma = 0.1713 \times 10^{-8} \text{ BTU/ft}^2\text{-hr-}^\circ R^4$$

Stefan-Boltzmann constant

LECTURE 25

RADIATIVE ENERGY TRANSFER BETWEEN BLACK BODIES

READING ASSIGNMENT:

Eckert & Drake, pp. 395 - 402
Pfenningwerth & Minkler, pp. 407 - 411

EXERCISES:

25-1. Calculate the view factor for the disk shown in the example given in the Lecture.

25-2. Calculate the view factor for a rectangular surface as shown in the example given in the Lecture.

25-3. Calculate the view factor for radiative heat exchange between two disks.

Radiative Energy Transfer Between Black Bodies

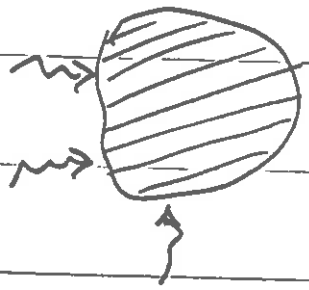
- Statement of problem
- Review of basic laws
- Calculation of energy exchange rates
- View factors

Consider a black body:

$\alpha = 1$

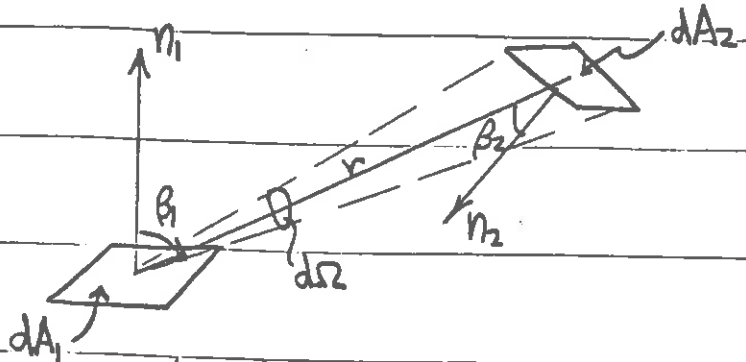
$\epsilon = 1$

$\rho = 0$



Since there is no reflection, energy exchange between black bodies reduces to a calculation in solid geometry

Consider heat exchange between two differential areas

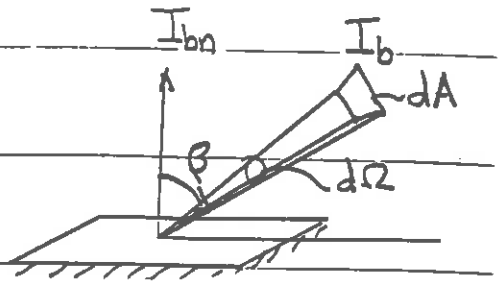


We will apply our two laws:

Lambert's Law

Stefan-Boltzmann Law

Lambert's Law



$$I_b = I_{bn} \cos\beta$$

$$e_b = \pi I_{bn}$$

Stefan-Boltzmann Law

$$e_b = \sigma T^4$$

Total rate of energy radiation from dA_1 to dA_2

$$d^2Q_{b1\to 2} = I_{bn_1} \cos\beta_1 \, d\Omega_1 \, dA_1$$

Further, note the solid angle subtended by dA_2 at surface 1 is

$$d\Omega_1 = \frac{dA_2 \cos\beta_2}{r^2}$$

Thus

$$d^2Q_{b_1\to 2} = I_{bn_1} \frac{\cos\beta_1 \cos\beta_2}{r^2} dA_1 dA_2$$

Similarly, the energy transfer rate from dA_2 to dA_1 is

$$d^2Q_{b2\to 1} = I_{bn_2} \frac{\cos\beta_1 \cos\beta_2}{r^2} dA_1 dA_2$$

But using $e_b = \pi I_{bn}$ and $e_b = \sigma T^4$ we can write

$$I_{bn_1} = \frac{\sigma}{\pi} T_1^4$$

$$I_{bn_2} = \frac{\sigma}{\pi} T_2^4$$

Hence the net rate of radiation exchange from dA_1 to dA_2 is

$$d^2Q_{b1} = \frac{\cos\beta_1 \cos\beta_2}{r^2} dA_1 dA_2 \frac{\sigma}{\pi} (T_1^4 - T_2^4)$$

For finite surface area, we just integrate

$$dQ_{b1} = \int_{A_2} d^2Q_{b1}$$

Assume A_2 is isothermal. Then

$$dQ_{b1} = \sigma F_{12} dA_1 (T_1^4 - T_2^4)$$

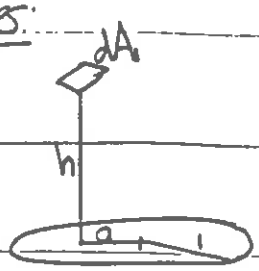
Here, F_{12} is a purely geometrical factor

$$F_{12} = \int_{A_2} \frac{\cos\beta_1 \cos\beta_2}{\pi r^2} dA_2$$

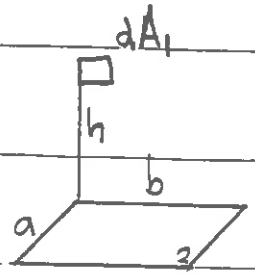
known as the view factor (or "shape" factor, "configuration" factor, or "geometrical" factor)

Fortunately the view factors F_{12} have been worked out for the standard geometries.

Examples:



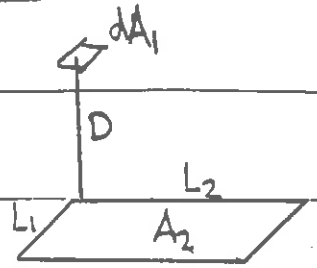
$$F_{1-2} = \frac{1}{2}\left[1 - \frac{h^2+a^2-1}{\sqrt{(h^2+a^2+1)^2-4a^2}}\right]$$

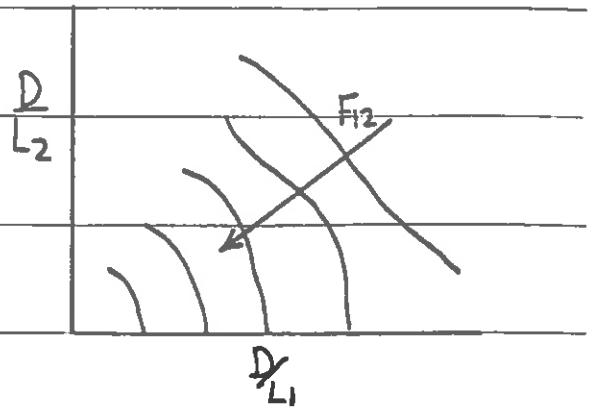


$$F_{n,1-2} = \frac{1}{2\pi}\left[\tan^{-1}\frac{b}{h} - \frac{h}{\sqrt{a^2+h^2}}\tan^{-1}\frac{b}{\sqrt{a^2+h^2}}\right]$$

(Note: One length dimension has been set equal to one.)

Example:





For heat exchange between two finite areas, we also integrate over dA_1:

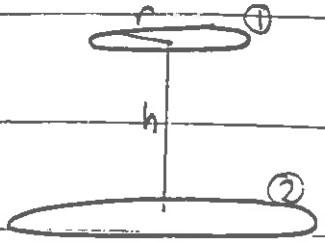
$$Q_{b1} = \bar{F}_{12} A_1 \sigma (T_1^4 - T_2^4)$$

where

$$\bar{F}_{12} = \frac{1}{A_1} \int_{A_1} F_{12} \, dA_1$$

$$= \frac{1}{A_1} \int_{A_1} \int_{A_2} \frac{\cos\beta_1 \cos\beta_2}{\pi r^2} \, dA_2 \, dA_1$$

Example:



$$\bar{F}_{1-2} = \frac{1}{2}\left[\frac{h^2+1}{r^2} + 1 - \sqrt{\left(\frac{h^2+1}{r^2}+1\right)^2 - \frac{4}{r^2}}\right]$$

Similarly, the net energy exchange rate from A_2 to A_1 is

$$Q_{b2} = \bar{F}_{21} A_2 \sigma (T_2^4 - T_1^4)$$

where

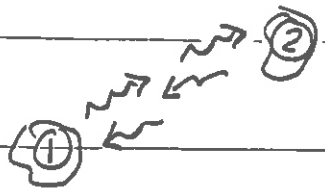
$$\bar{F}_{21} = \frac{1}{A_2} \int_{A_2} \int_{A_1} \frac{\cos\beta_1 \cos\beta_2}{\pi r^2} dA_1 dA_2$$

If we compare this with $A_1 \to A_2$ we find

$$\bar{F}_{12} A_1 = \bar{F}_{21} A_2$$

(can be useful for calculating view factors)

Remember: These results



$$Q_{b2} = \bar{F}_{21} A_2 \sigma (T_2^4 - T_1^4)$$

hold only for the ideal case of "black bodies."

LECTURE 26

RADIATIVE HEAT TRANSFER BETWEEN GRAY BODIES

READING ASSIGNMENT:

Eckert & Drake, Chapter 14
Pfenningwerth & Minkler, pp. 411 - 419

EXERCISES:

26-1. Provide the details in the derivation of the radiative exchange between two infinite, parallel, isothermal gray plates.

26-2. Verify the view factors and interchange factors for each of the cases given in the table in the Lecture.

26-3. Derive an expression for the thermal resistance characterizing radiative heat transfer from a surface.

26-4. Explain how the "gray body" model is related to the one-speed model in neutron transport. Fromt this analogy, how might one develop a "multigroup" model of radiative transfer.

Radiative Heat Transfer Between Gray Bodies

- Definition of a gray body
- Example: two infinite, isothermal, parallel gray planes
- Extension to more general geometries
- Heat transfer correlation for radiation

What about "gray" bodies? Recall that Kirchoff's Law implies that in thermal equilibrium

$$\alpha = \epsilon$$

(absorptivity = emissivity)

But even beyond this,

$$\alpha_\lambda = \epsilon_\lambda$$

Consider a radiating surface at temperature T_1.



The total emissivity of the surface depends on its temperature

$$\epsilon_1 = \epsilon(T_1)$$

The absorptivity is also a function of the wavelength of the incident radiation

$$\alpha_1 = \alpha_\lambda(T_1)$$

Define:

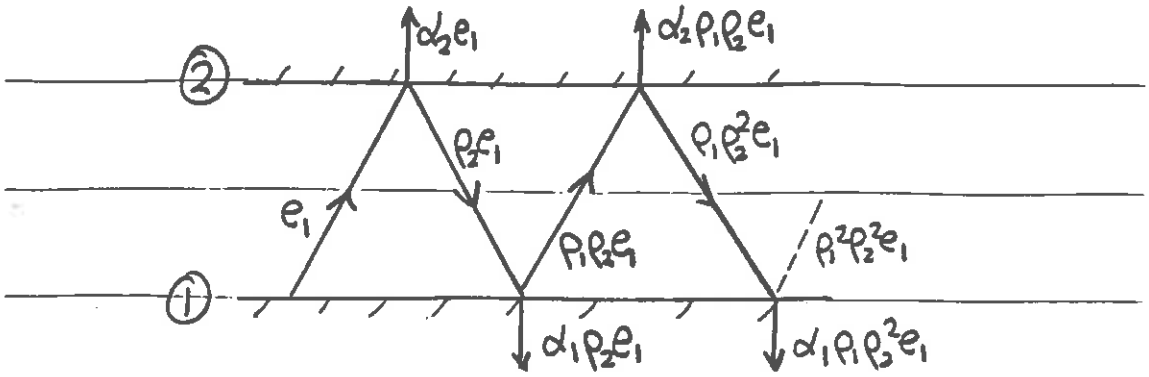
gray body ≡ a radiating surface which radiates and absorbs the same fraction as a black body, without regard to wavelength

That is

$$\alpha_\lambda = \alpha$$

$$\epsilon_\lambda = \epsilon$$

Example: Two infinite, parallel, isothermal gray planes



Net rate of loss of energy (per unit area) originating at ① is

$$q_{12} = (1 - \alpha_1 \rho_2 - \alpha_1 \rho_1 \rho_2^2 - \alpha_1 \rho_1^2 \rho_2^3 \cdots) e_1$$

$$= [1 - \alpha_1 \rho_2 (1 + \rho_1 \rho_2 + \rho_1^2 \rho_2^2 + \cdots)] e_1$$

Since the bodies are "non-black" with

$$\rho_1 < 1, \quad \rho_2 < 1$$

we can sum the series as

$$q_{12} = \left[1 - \frac{\alpha_1 \rho_2}{1 - \rho_1 \rho_2}\right] e_1$$

For opaque bodies, $\tau = 0$ and

$$\rho = 1 - \alpha$$

Then

$$q_{12} = \frac{\alpha_2 e_1}{\alpha_1 + \alpha_2 - \alpha_1 \alpha_2}$$

— or, using Kirchoff's Law, $\epsilon = \alpha$,

$$q_{b12} = \frac{\epsilon_2 e_1}{\epsilon_1 + \epsilon_2 - \epsilon_1 \epsilon_2}$$

But by the Stefan-Boltzmann Law, the emissive power of surface ① is

$$e_1 = \epsilon_1 e_{b_1} = \epsilon_1 \sigma T_1^4$$

Thus

$$q_{12} = \frac{1}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1} \sigma T_1^4$$

Similarly, the net emission from surface ② is

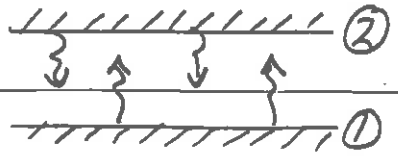
$$q_{21} = \frac{1}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1} \sigma T_2^4$$

Thus we can now calculate the net energy exchange from surface ① to surface ②:

$$q_1 = \frac{1}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1} \sigma (T_1^4 - T_2^4)$$

Lets rewrite this as

$$q_{01} = \bar{\epsilon}_{12}\,\sigma\,(T_1^4 - T_2^4)$$



where

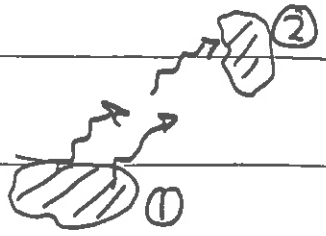
$$\bar{\epsilon}_{12} = \frac{1}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$$

interchange factor

Here, the interchange factor accounts for the "non-black" behavior of the surface.

More generally, we can combine this with our earlier result for black surfaces to write the total heat rate from surface A_1 as

$$Q_1 = \sigma\,\bar{\epsilon}_{12}\,\bar{F}_{12}\,A_1\,(T_1^4 - T_2^4)$$



Thus, the average heat flux from A_1 is

$$\bar{q}_{01}'' = \frac{Q_1}{A_1} = \sigma\,\bar{\epsilon}_{12}\,\bar{F}_{12}\,(T_1^4 - T_2^4)$$

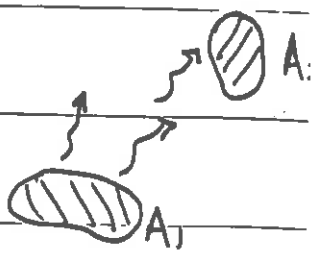
interchange factor — view factor

Case	Area	$\bar{F}_{12}$	$\bar{\epsilon}_{12}$
1) Completely enclosed body (A_1), small compared with enclosure	A_1	1	ϵ_1
2) Completely enclosed body (A_1), of same order as enclosure	A_1	1	$\frac{1}{\frac{1}{\epsilon_1}+\frac{1}{\epsilon_2}-1}$
3) Concentric spheres or infinite cylinders	A_1	1	$\frac{1}{\frac{1}{\epsilon_1}+\frac{A_1}{A_2}\left(\frac{1}{\epsilon_2}-1\right)}$
4) Element dA and finite area	dA	Calculate	$\epsilon_1\epsilon_2$

Heat transfer coefficient for radiation

Recall our result

$$q''_{b1} = \sigma \bar{\epsilon}_{12} \bar{F}_{12} (T_1^4 - T_2^4)$$



However, Newton's Law of Cooling is written as

$$q''_{b1} = h_s (T_1 - T_2)$$

Suppose we write Newton's law for radiating surfaces

$$q''_{b1} = h_r (T_1 - T_2)$$

where h_r – surface heat transfer coefficient for radiation,

If we compare

$$q_{01}'' = h_r(T_1 - T_2) \qquad q_{01}'' = \sigma \bar{\epsilon}_{12} \bar{F}_{12} (T_1^4 - T_2^4)$$

we identify

$$h_r = \sigma \bar{\epsilon}_{12} \bar{F}_{12} \frac{T_1^4 - T_2^4}{T_1 - T_2}$$

$$= \sigma \bar{\epsilon}_{12} \bar{F}_{12} f(T_1, T_2)$$

Here $f(T_1, T_2) = \frac{T_1^4 - T_2^4}{T_1 - T_2} = (T_1 + T_2)(T_1^2 + T_2^2)$

Rewrite the temperature factor using $\bar{T} = \frac{1}{2}(T_1 + T_2)$

$$f(T_1, T_2) = 4\bar{T}^3 \left[1 + \frac{(T_2 - T_1)^2}{4\bar{T}^2}\right]$$

For $|T_2 - T_1| << \bar{T}$ we can take

$$f(T_1, T_2) \sim 4\bar{T}^3$$

For the special case of black surfaces with unit view factors

$$\bar{\epsilon}_{12} \bar{F}_{12} = 1.0$$

we find

$$h_r = \sigma \frac{T_1^4 - T_2^4}{T_1 - T_2} \sim \sigma \bar{T}^3$$

Thus, as a crude approximation, we can use Newton's Law for the heat transfer from radiating surfaces:

$$q'' = h_r (T_s - T_2)$$

where

$$h_r \sim \sigma \bar{T}^3$$

(Remember: In radiative heat transfer, the temperature is always in absolute degrees.)

LECTURE 27

REACTOR THERMAL ANALYSIS

READING ASSIGNMENT:

Duderstadt & Hamilton, pp. 447 - 465, pp. 467 - 475
Lamarsh, pp. 383 - 389
J. Weisman, "Heat Generation and Transport", Chapter 6 in Weisman (Design)
Rust, Chapter 7
Sesonske, pp. 141 - 146
P. E. MacDonald and I-Chih Wang, "Fuel Element Design", Chapter 11 in Weisman (Design)

EXERCISES:

27-1. Consider two cylindrical fuel rods of radii a and 2a respectively which are tooperate under the limitation that the maximum center-to-outer surface temperature is the same for both rods. Assuming uniform heat generation in the rods, which rod would be able to supply more heat?

27-2. Using typical fuel element data for a BWR core, estimate the fuel centerline temperature that occurs at the point of maximum heat flux if it is assumed that the coolant temperature at this point is 290 C with a convective heat transfer coefficient of hs = 5.0 W/cm^2 K.

27-3. Repeat the calculation of 27-2 for an LMFBR core assuming a sodium temperature of 500 C and hs = 4.0 W.cm^2 K.

27-4. The outlet quality of an average coolant channel of a BWR is 14.6%. The system pressure is 70 bar and the inlet temperature is 220 C [h(220 C, 70 bar) = 1.23 MJ/kg]. Determine:

a) the exit void fraction for a slip ratio of 2.0.

b) the average heat generation (W/cm) of the fuel rods if the mass flow rate per fuel rod is 10,000 kg/hr and the core height is 375 cm.

c) the nonboiling length for a bottom-peaked power profile (given by Problem 29-2).

Reactor Thermal Analysis

- Goals
- Limitations
- Steps
- Approaches

Goals of reactor thermal analysis

fuel element integrity

economic plant operation

Specific concern: thermal limitations

1) fuel temperatures: avoid fuel melting or phase change

$T_c < T_{melt} \Rightarrow q' < q'_{max}$

[$q'_{max} \sim 600$ W/cm in UO_2 fuel elements]

2) clad surface heat flux: avoid excessive clad temperatures and clad failure

$q'' < q''_{DNB}$

[$q''_{DNB} \sim 300$ W/cm^2 in LWRs]

3) fuel burnup: limit fission product gas pressure, cladding creep, or embrittlement

[30,000 MWD/MTU – LWRs]

4) fuel specific power: optimize refueling times and fuel fabrication costs

5) abnormal transients (e.g., LOCA)

Objective of core thermal design:

Provision of adequate core cooling under

- normal, steady-state operation
- anticipated transients
- abnormal occurrences (LOCA)

Typical thermal design criteria for LWRs:

1) $T_{FUEL} < T_{melt}$ at 112% design overpower (at any time during core life)

2) $DNBR = \frac{q''_{DNB}}{q''} > 1.3$ at 112% overpower (so that cladding damage due to DNB is unlikely)

(DNBR > 1.9 in BWRs)

3) no hydraulic instabilities occur during steady-state or anticipated transient conditions

4) a prescribed fraction of reactor vessel flow rate (~95%) will be available for fuel element cooling (limits flow bypass)

Typical LOCA design criteria:

Additional thermal design limitations arise when considering abnormal transients such as loss of coolant accidents.

These can be more restrictive than for steady-state operation (e.g., $q'_{LOCA} < q'_{\ell\, melt}$)

Example: Criteria based upon calculated clad damage to judge the acceptability of design and operating limits in event of LOCA

Peak cladding temperature:

$T_{clad} < 2,200\ °F$

Maximum cladding oxidation:

Oxidation cannot exceed 17% of clad thickness

Maximum hydrogen generation:

Total hydrogen generation from clad-steam reaction less than 1%

Coolable geometry:

Changes in core geometry shall be such that core can continue to be cooled

Long-term cooling:

After LOCA, ECCS can keep core cooled

Steps in reactor thermal analysis

① Determine heat source distributions (including time dependence) throughout reactor system

② Determine temperature distributions through solid components (fuel, structure, ...) using heat conduction

③ Analyze convection heat transfer in coolants (and determine surface temperatures of solid components)

④ Analyze heat transfer in out-of-core components (steam generators, pressurizer, ...)

Two classes of analysis

Steady-state analysis: mapping of temperature distributions and heat transfer rates during full power operation

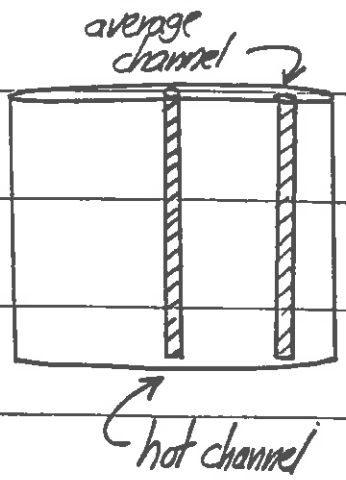
Transient analysis: mapping of temperature distributions during transients (both normal and abnormal)

Detailed approach:

Perform detailed 3-D calculation of core power distribution (including fuel burnup, fission product buildup, control distributions, moderator density distributions -- all over core life)

Use this information to determine coolant flow and temperature distribution throughout core

Alternative: hot channel analysis



Investigate how closely "hot channel" approaches operating limitations

Hot channel: coolant channel in which core heat flux and enthalpy rise are maximum.

Use "hot channel factors" to relate to average core behavior

LECTURE 28

HOT CHANNEL FACTORS

READING ASSIGNMENT:

Duderstadt & Hamilton, pp. 502 - 505
Weisman (Design) pp. 166 - 171
Sesonske, pp. 127 - 140
Lamarsh, pp. 385 - 387
Rust, pp. 326 - 338

EXERCISES:

28-1. How would one define the total nuclear hot channel factor for a spherical reactor core geometry? Calculate this assuming a uniform fuel distribution.

28-2. Calculate the axial nuclear hot channel factor for a bare cylindrical core in which the extrapolation is 10% of the core height. Compare this with that calculated for the case where the extrapolation length is zero.

28-3. Calculate the axial, radial, and total nuclear hot channel factors for a core design in which the power distribution is given by

$$P(r,z) = Po\ [1 - (r/R)^2]\ \cos\ (\pi\ z/L)$$

Hot Channel Factors

- Hot channel analysis
- Nuclear hot channel factors
- Engineering hot channel factors

Recall thermal limitations:

i) avoid fuel centerline melting ($T_\ell < T_{melt}$)

$q' < q'_{max}$ [~ 660 W/cm in UO_2]

ii) avoid DNB

$q'' < q''_{DNB}$ [~ 300 W/cm^2 in LWRs]

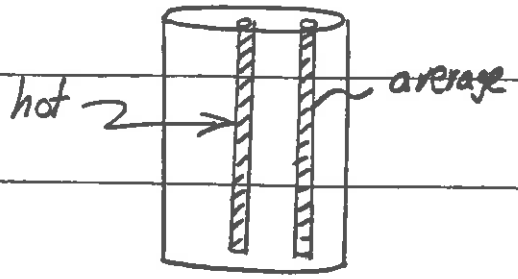
iii) other limits:

thermal and fission gas stresses on clad

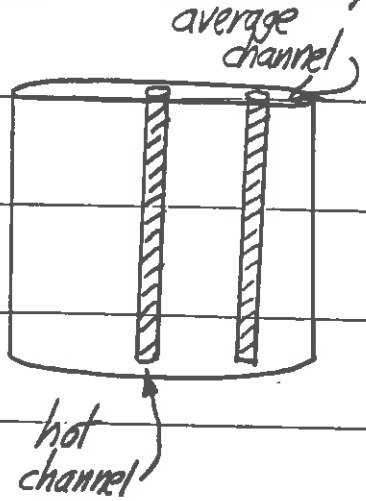
nuclear factors (control, feedback)...

Direct approach: Take $P(\underline{r})$ and calculate $T(\underline{r})$, $q'(\underline{r})$, $q''(\underline{r})$ ... through core

Simpler approach: Investigate how closely hot channel approaches operating limitations.



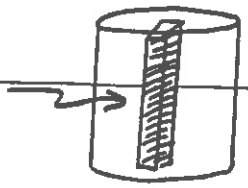
Hot channel analysis:



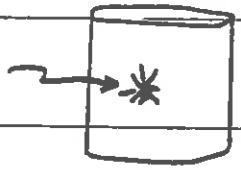
Goal: Investigate how closely hot channel approaches operating limitations; then relate to average core performance using hot channel factors or hot spot factors

Some definitions:

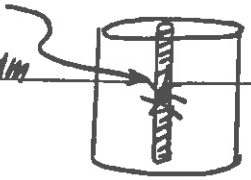
hot assembly: fuel assembly having maximum power output



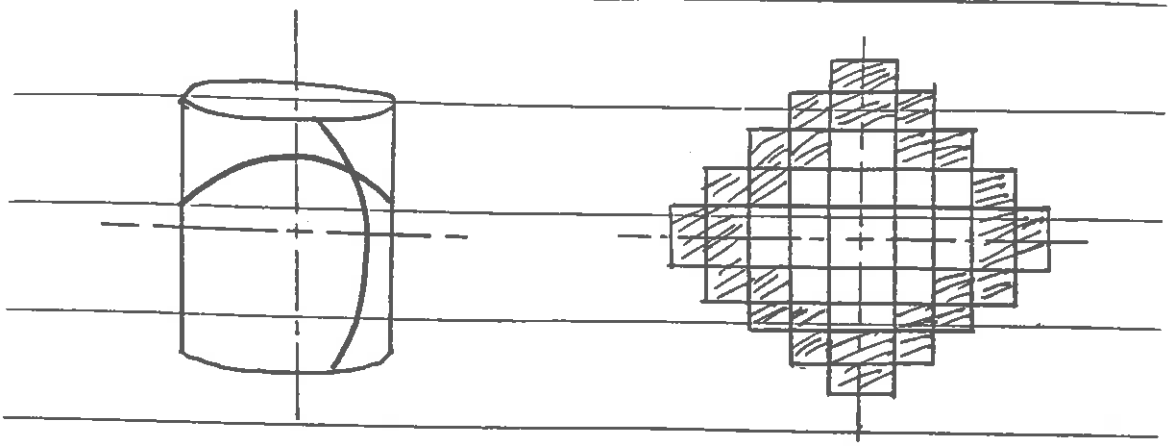
hot spot: point of maximum heat flux q'' or linear power density q'



hot channel: coolant channel in which hot spot occurs (or along which maximum coolant enthalpy rise occurs)



We introduce nuclear hot channel factors to take into account neutron flux and fuel enrichment variations in core:



Nuclear hot channel factors

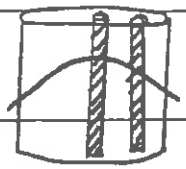
Take into account variation of neutron flux and fuel distribution in core

Radial nuclear hot channel factor F_R^N:

$$F_R^N = \frac{\text{average heat flux of hot channel}}{\text{average heat flux of all channels in core}}$$

or

$$F_R^N = \frac{\int_{-H/2}^{H/2} q''(\underline{r}_{HC})\,dz}{\frac{1}{I}\sum_{i=1}^{I}\int_{-H/2}^{H/2} q''(\underline{r}_i)\,dz}$$



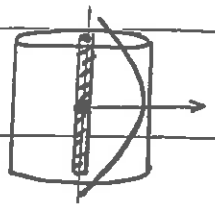
Example: bare, uniform core

$$F_R^N = \frac{J_0(0)\int_{-H/2}^{H/2}\cos\left(\frac{\pi z}{H}\right)dz}{\int_0^R J_0\left(\frac{2.405r}{R}\right)2\pi r\,dr\int_{-H/2}^{H/2}\cos\left(\frac{\pi z}{H}\right)dz} = 2.32$$

Axial nuclear hot channel factor F_z^N

$$F_z^N = \frac{\text{maximum heat flux of hot channel}}{\text{average heat flux of hot channel}}$$

$$= \frac{\max_z q''(\underline{r}_{HC})}{\frac{1}{H}\int_{-H/2}^{H/2} q''(\underline{r}_{HC})\,dz}$$



Example: bare, uniform core

$$F_z^N = \frac{\cos(0)J_0(0)}{\frac{1}{H}\int_{-H/2}^{H/2}\cos\left(\frac{\pi z}{H}\right)dz\,J_0(0)} = 1.57$$

Total nuclear hot channel factor F_q^N

$$F_q^N = \frac{\text{maximum heat flux in core}}{\text{average heat flux in core}}$$

$$= F_R^N F_z^N$$

Example: bare, uniform core

$$F_q^N = (2.32)(1.57) = 3.64$$

[Note: This is too conservative. A zone-loaded PWR will typically have

$$F_q^N \sim 2.6$$]

Enthalpy-rise hot channel factor $F_{\Delta H}$

$$F_{\Delta H} = \frac{\text{maximum coolant enthalpy rise}}{\text{average coolant enthalpy rise}}$$

$F_{\Delta H}$ depends on variations in power distribution and coolant flow

Example: bare, uniform core

$$\Delta h(r) = \frac{1}{W}\int_{-H/2}^{H/2} q'(r,z)\,dz = \frac{2HA}{\pi W} J_0\left(\frac{2.405r}{R}\right)$$

$$\overline{\Delta h} = \frac{1}{\pi R^2}\int_0^R \Delta h(r)\, 2\pi r\, dr = 0.862\,\frac{HA}{\pi W}$$

Hence

$$F_{\Delta H} = \frac{\Delta h|_{max}}{\overline{\Delta h}} = 2.32$$

Clad surface temperature hot channel factor F_{cs}

$$F_{cs} = \frac{(T_{clad} - T_{coolant})_{max}}{(T_{clad} - T_{coolant})_{ave}}$$

(Useful for HTGRs and LMFBRs where $T_{clad} - T_{coolant}$ is appreciable.)

Engineering hot channel factors

Engineering hot channel factors are introduced to deal with uncertainties such as:

- poorly known properties such as k_c
- manufacturing tolerances (pellet fuel loading, dimensions, ...)
- coolant flow distribution
- dimensional changes (burnup or thermal stress)

To take account of this, we introduce an engineering hot channel factor F^E so that the total hot channel factor is given by

$$F = F^N F^E$$

Here, F^E is further subdivided into subfactors:

- mechanical subfactors
- flow distribution subfactors

Mechanical subfactors

Due to factors such as

Variations in fuel density/enrichment

Dimension variations (tolerances)

Fuel and cladding thermal conductivities

Flow distribution subfactors

Due to factors such as

coolant flow bypassing fuel assemblies

gap in control rod slots

gap between core barrel and baffle

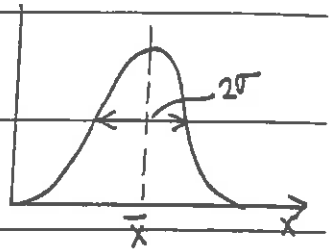
inlet flow from lower plenum

flow redistribution in fuel assemblies

subcooled or bulk boiling effects on flow

Generally must use a combination of experimental data and statistics to determine subfactors.

Example: Consider a subfactor F_x^E associated with the variation in a parameter x. Suppose x can be described by a normal distribution. We can define



$$F_x^E = \frac{\bar{x} + 3\sigma}{\bar{x}} = 1 + 3\frac{\sigma}{\bar{x}}$$

Question: How do we combine all the subfactors?

Deterministic: $F^E = F_1^E \times F_2^E \times \dots \times F_n^E$

-- but this is overconservative.

Statistical: Combine F_i^E using statistical methods

Comments:

1) Hot channel factors simplify thermal core analysis since they allow one to account for nonuniformities within a crude single-channel analysis of core.

2) However, more generally one must account for multiple-channel (cross flow) effects.

3) Also depend heavily on experimental measurement and empirical correlations.

LECTURE 29

APPLICATIONS OF REACTOR THERMAL ANALYSIS

READING ASSIGNMENT:

Duderstadt & Hamilton, pp. 505 - 507, 519 - 521

EXERCISES:

29-1. Consider a PWR core consisting of fuel elements 12 ft in length and 0.5 in in diameter. The reactor operates at a thermal power of 3000 MWt and has a maximum calculated heat flux of 539,000 Btu/hr-ft-2 and an overall hot channel factor of 2.80. Calculate the total heat transfer area of the core and the number of fuel rods in the core.

29-2. As we have seen, one of the primary limitations on thermal performance is the avoidance of fuel centerline melting. However it is obviously to our advantage to run the fuel at all points in the reactor as close to this limit as possible (consistent with safety requirements).

a) Determine the axial power profile that would yield a uniform axial fuel temperature in the core.

b) Determine the axial fuel loading distribution that would yield this power profile (use one-speed diffusion theory).

c) Compare the feasibility of obtaining such profiles for a HTGR and a PWR.

29-3. Consider a channel of length L in which there is a bottom-peaked power profile of the form

$$q'(z) = q'o\ (\ /H)\ (H - z)\ \sin[(\ /H)(H - z)]$$

Determine: a) the maximum/average heat flux in the channel, b) q'(z) in terms of the average channel heat flux, and c) the enthalpy rise at any position in the channel.

29-4. Consider a PWR operating in such a fashion that nucleate boiling occurs at the point of maximum power density. Howe might you expect the following changes to affect both the maximum fuel centerline temperature and DNBR, assuming a constant average linear power density:

a) 10% reduction of coolant flow in the hot channel.

b) modification of core analysis to account for nonuniform power generation in the fuel pellet.

c) radial fuel migration and densitication producing a 4% void volume along the fuel pellet axis.

d) the axial core power profile shifting from a centered to a bottom-peaked distribution (such that both the peak power and the integrated power remain the same).

e) the axial core power profile shifting to a top-peaked distribution.

Applications of Core Thermal Analysis

- Determination of reactor core size
- Thermal-hydraulic-neutronic coupling

Application: Determination of reactor core size

1) Determine hot channel factors (neutronics codes)

2) Next, determine maximum acceptable power level in hot channel -- expressed as a limit on linear power density

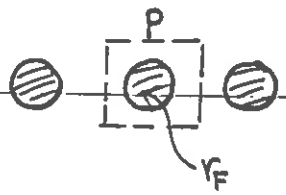
$$q'(z) < q'_{MAX}$$

Then

$$q'_{AVE} = \frac{q'_{MAX}}{F_q^N}$$

3) Fuel-to-moderator volume ratios and fuel pin radius are determined by nuclear considerations. Note

$$\frac{V_F}{V_M} = \frac{\pi r_F^2}{P^2 - \pi r_F^2}$$



(Usually specify pitch-to-diameter ratio. Typically

$$\frac{P}{2r_F} \sim 1.2 - 1.5 \;)$$

4) Thermal power output of core

$$MWt = \frac{q'_{AVE} \, L \, N_R}{C_1}$$

$N_R \equiv$ total number of fuel rods in core

$L \equiv$ average fuel rod length

$C_1 \equiv$ fraction of core power generated in fuel (~95-97%)

If we want N_A rods per assembly, then # of assemblies required is

$$\# \text{ assemblies} = \frac{MWt \, C_1 \, F_q^N}{q'_{MAX} \, L \, N_A}$$

5) Note that the average core power density q'''_{AVE} is

$$q'''_{AVE} = \frac{q'_{AVE}}{p^2}$$

Hence required core diameter D_{CORE} is determined by equating

$$MWt = q'''_{AVE}\left(\frac{\pi D_{CORE}^2}{4}\right)L = \frac{q'_{AVE}}{p^2}\left(\frac{\pi D_{CORE}^2}{4}\right)L$$

Thus required core diameter is just

$$D_{CORE} = \left[\frac{4p^2\, MWt\, F_q^N}{\pi\, q'_{MAX}}\right]^{1/2}$$

Comments:

$$D_{CORE} = \left[\frac{4p^2\, MWt\, F_q^N}{\pi\, q'_{AVE}}\right]^{1/2}$$

D_{CORE} can be reduced by

i) decreasing F_q^N

ii) decreasing p

iii) increasing q'_{MAX}

Application: Thermal-hydraulic-neutronic coupling

Neutronic-to-thermal-hydraulic

$\phi(\underline{r}) \Rightarrow P(\underline{r}) = q''' \Rightarrow q', T(\underline{r}), N_M(\underline{r})$

Thermal-hydraulic-to-neutronic

T_f will affect reactivity through Doppler effect

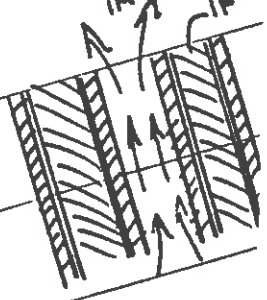
N_M will affect reactivity through spectrum effects

Reactivity ρ depends on the reactor power level

$\rho(P)$

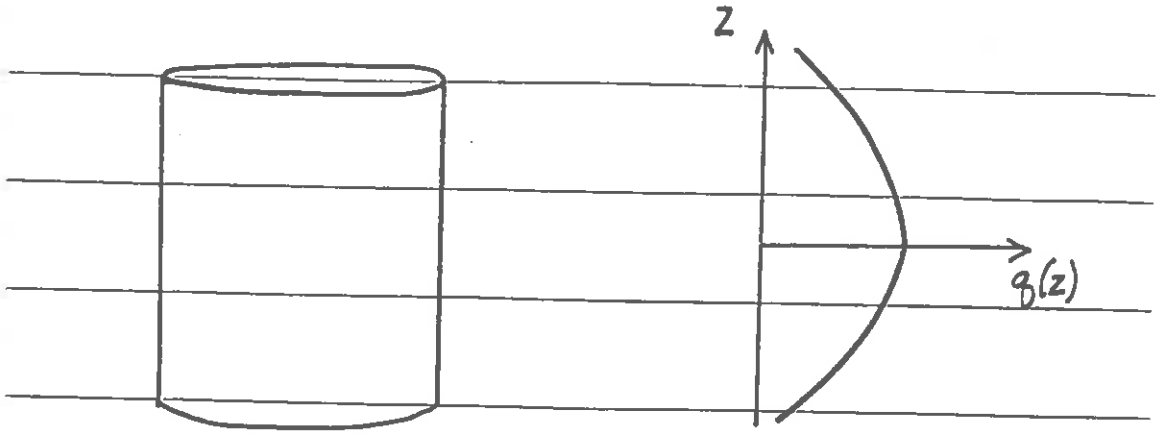
since both material densities N_i and cross sections σ depend on temperature. In a point model we might represent

$\Delta\rho = \Delta\rho(\Delta T_f, \Delta T_M, \Delta T_{cool} \ldots)$



Thermal-hydraulic interaction with core power distributions:

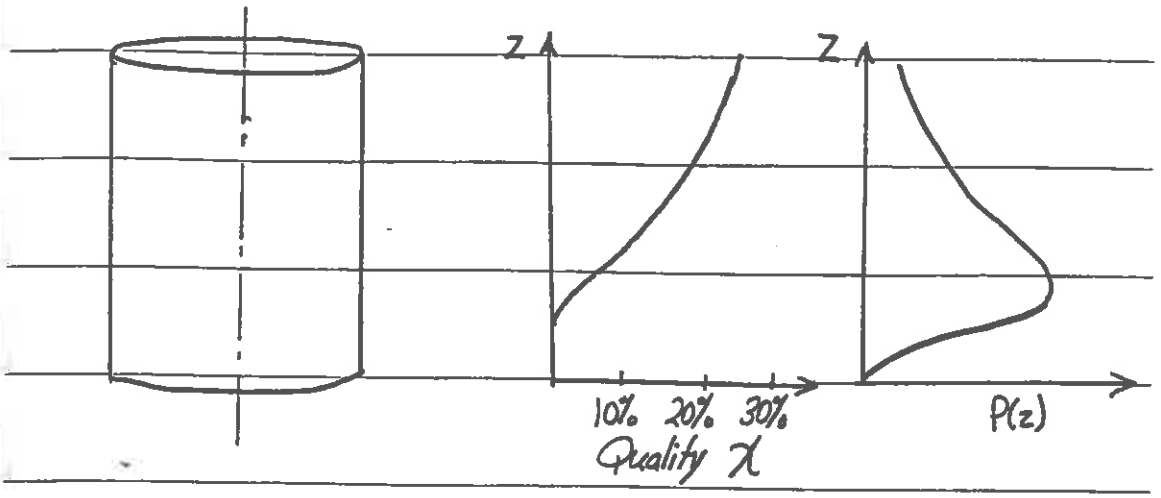
Recall our simple model of a bare, uniform core predicts



LWR: Cores are "undermoderated" so that

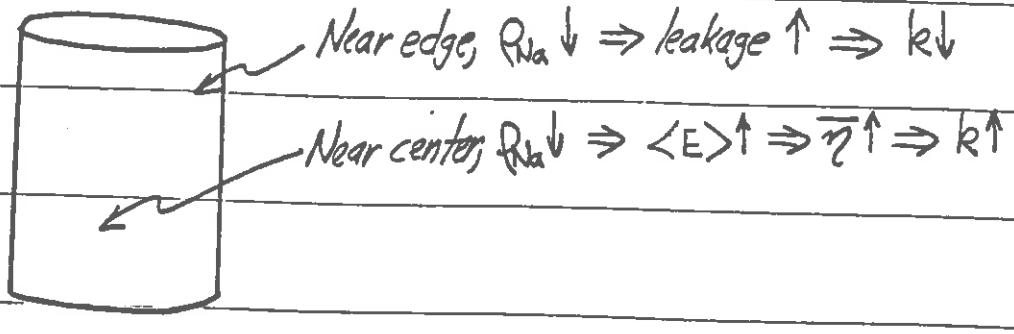
- $\rho_{H_2O} \downarrow \Rightarrow \phi \downarrow \Rightarrow P \downarrow$

Example: BWR -- strong bottom peaking

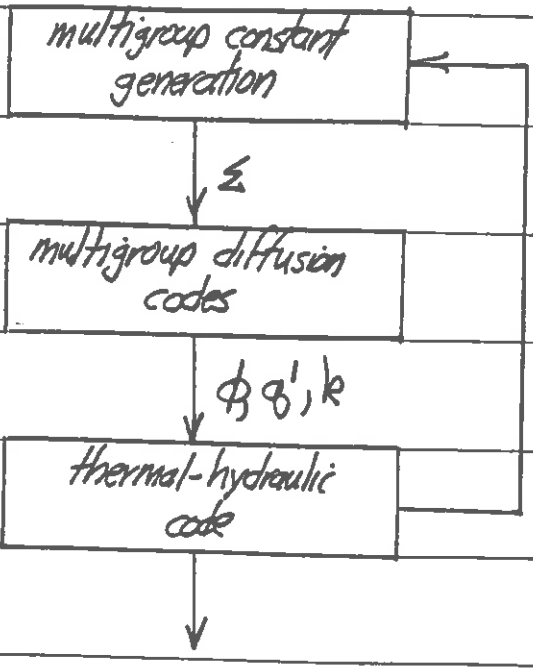


HTGR: Gas-cooled reactors have very little coolant-neutronic interaction

LMFBR:

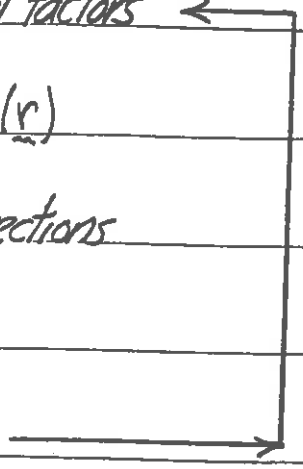


Interaction between thermal-hydraulic / neutronic calculations



Typical strategy

1) Guess $q'(z)$ and hot channel factors
2) Calculate $T_F(\underset{\sim}{r})$, $T_M(\underset{\sim}{r})$, $N_M(\underset{\sim}{r})$
3) Calculate macroscopic cross sections
4) Calculate k, ϕ, $q'(z)$, HCF
5) Iterate



LECTURE 30

NUMERICAL SOLUTION OF THE HEAT CONDUCTION EQUATION

READING ASSIGNMENT:

Duderstadt & Hamilton, pp. 176 - 193
VPI Computer Modules, TH-3, HTGR Thermal Hydraulics
Pfenningwerth & Minkler, pp. 186 - 208

EXERCISES:

30-1. Derive the three-point central difference formula for a second order derivative by using suitable difference formulas for the first derivative of a function.

30-2. Demonstrate that finite-differencing the 2-D form of the heat conduction equation yields a coefficient matrix with three central diagonal and two off-diagonals of nonzero terms.

30-3. Derive the nodal equations for a triangular-shaped region using the energy balance approach. (Use a small number of elements to keep things manageable.)

30-4. Does the absence of an "absorption" term in the heat conduction equation when compared to the neutron diffusion equation lead to any simplifications in numerical solutions?

Numerical Solution of the Heat Conduction Equation

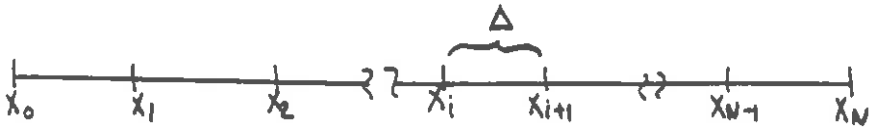
- 1-D Problems
- Finite Difference Methods in 2-D, 3-D
- Nodal (Finite Element) Methods

Review: 1-D Problems

Assume constant thermal conductivity so that

$$-k\frac{d^2T}{dx^2} = q(x)$$

Now introduce a spatial mesh



Now use a three-point central difference formula

$$\left.\frac{d^2T}{dx^2}\right|_i \cong \frac{T_{i+1} - 2T_i + T_{i-1}}{\Delta^2}$$

We can now write the heat conduction equation at a mesh point x_i as

$$-\frac{k}{\Delta^2} T_{i-1} + \frac{2k}{\Delta^2} T_i - \frac{k}{\Delta^2} T_{i+1} = q_i \qquad i = 1, N-1$$

or

$$a_{i,i-1} T_{i-1} + a_{i,i} T_i + a_{i,i+1} T_{i+1} = q_i \qquad i = 1, \ldots N-1$$

We can rewrite this in more convenient form

$$\begin{aligned} a_{11} T_1 + a_{12} T_2 &= q_1 \\ a_{21} T_1 + a_{22} T_2 + a_{23} T_3 &= q_2 \\ a_{32} T_2 + a_{33} T_3 + a_{34} T_4 &= q_3 \end{aligned}$$

or in matrix form

$$\begin{pmatrix} a_{11} & a_{12} & & \\ a_{21} & a_{22} & a_{23} & \\ & a_{32} & a_{33} & a_{34} \\ & \vdots & \vdots & \ddots \end{pmatrix} \begin{pmatrix} T_1 \\ T_2 \\ T_3 \\ \vdots \end{pmatrix} = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \\ \vdots \end{pmatrix}$$

or

$$\underline{\underline{A}}\ \underline{T} = \underline{q}$$

Hence we must solve

$$\underline{\underline{A}}\,\underline{T} = \underline{q}$$

to find

$$\underline{T} = \underline{\underline{A}}^{-1}\,\underline{q}$$

Since $\underline{\underline{A}}$ is tridiagonal

$$\underline{\underline{A}} = \begin{pmatrix} \diagdown\!\diagdown\!\diagdown & & 0 \\ & \ddots & \\ 0 & & \diagdown\!\diagdown\!\diagdown \end{pmatrix}$$

we can use a direct method such as Gaussian elimination.

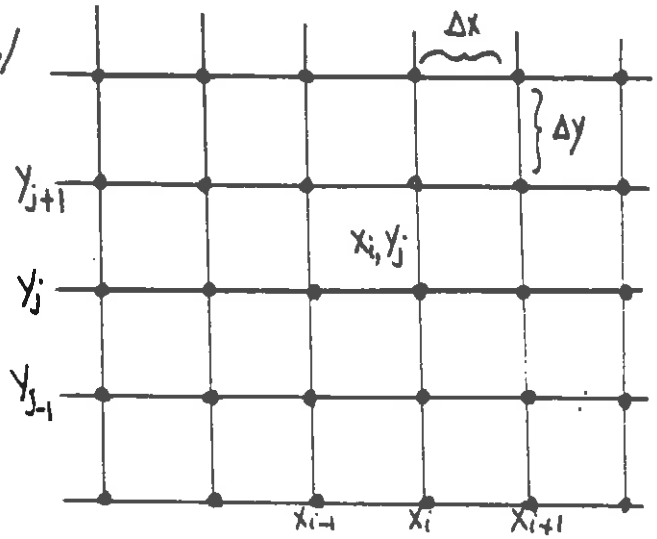
2-D Finite Difference Methods

Consider

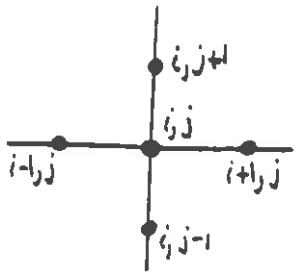
$$-k\frac{\partial^2 T}{\partial x^2} - k\frac{\partial^2 T}{\partial y^2} = q(x,y)$$

Again we define a spatial mesh:

$$T_{i,j} \equiv T(x_i, y_j)$$



We can again use central difference formulas



$$\left.\frac{\partial^2 T}{\partial x^2}\right|_{i,j} \cong \frac{T_{i-1,j} - 2T_{i,j} + T_{i+1,j}}{(\Delta x)^2}$$

$$\left.\frac{\partial^2 T}{\partial y^2}\right|_{i,j} \cong \frac{T_{i,j-1} - 2T_{i,j} + T_{i,j+1}}{(\Delta y)^2}$$

Hence our difference equation becomes

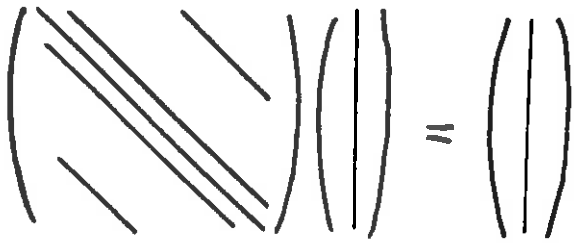
$$-\frac{k}{(\Delta x)^2}\left(T_{i+1,j} + T_{i-1,j}\right) - \frac{k}{(\Delta y)^2}\left(T_{i,j+1} - T_{i,j-1}\right)$$

$$+ 2k\left(\frac{1}{(\Delta x)^2} + \frac{1}{(\Delta y)^2}\right)T_{i,j} = q_{0(i,j}$$

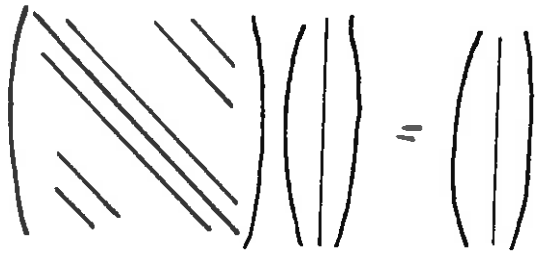
$i = 1, \cdots, N-1$

$j = 1, \cdots, M-1$

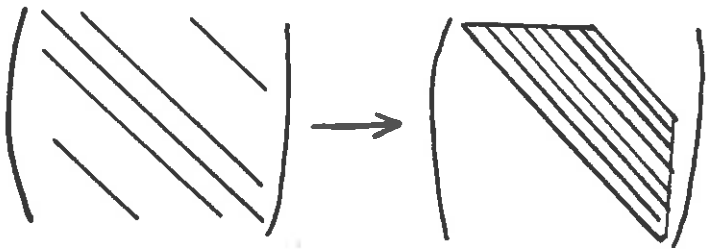
Now the matrix structure is



A similar structure is found for 3-D problems



If we try a direct method such as Gaussian elimination we run into trouble



Instead one uses iterative methods (Gauss-Seidel, SOR, ADI, ...)

General idea: LU decomposition



$$\underline{\underline{A}} = \underline{\underline{L}} - \underline{\underline{U}}$$

The rearrange $\underline{\underline{A}}\,\underline{T} = \underline{q}$ into

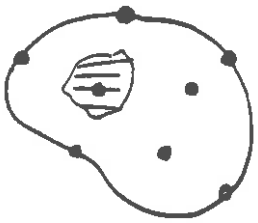
$$\underline{\underline{L}}\,\underline{T} = \underline{\underline{U}}\,\underline{T} + \underline{q}$$

or in an iterative scheme

$$\underline{T}^{(m+1)} = \underline{\underline{L}}^{-1}\,\underline{\underline{U}}\,\underline{T}^{(m)} + \underline{\underline{L}}^{-1}\,\underline{q}$$

Nodal Methods (Finite Element Methods)

For irregular shapes, represent region by a small number of nodes -- each of which can be thought of as having a well-defined volume, surface area, and temperature. How do we couple nodes?

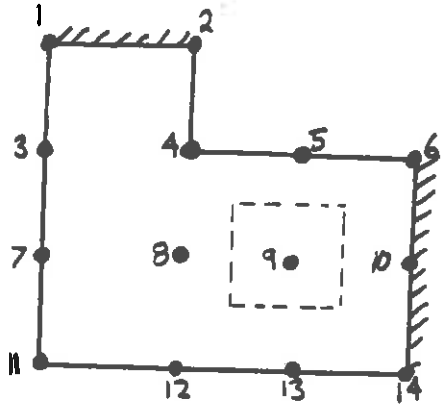


i) heat conduction equation

ii) energy balance

Example: Use:

$$-k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)=q'''$$



We can approximate

$$\left.\frac{\partial T}{\partial x}\right|_{8\text{-}9}=\frac{T_9-T_8}{\Delta x},\quad \left.\frac{\partial T}{\partial x}\right|_{9\text{-}10}=\frac{T_{10}-T_9}{\Delta x}$$

so that

$$\left.\frac{\partial^2 T}{\partial x^2}\right|_9=\frac{\left.\frac{\partial T}{\partial x}\right|_{9\text{-}10}-\left.\frac{\partial T}{\partial x}\right|_{8\text{-}9}}{\Delta x}=\frac{\frac{T_{10}-T_9}{\Delta x}-\frac{T_9-T_8}{\Delta x}}{\Delta x}=\frac{T_{10}+T_8-2T_9}{\Delta x^2}$$

Similarly

$$\left.\frac{\partial^2 T}{\partial y^2}\right|_9=\frac{\left.\frac{\partial T}{\partial y}\right|_{5\text{-}9}-\left.\frac{\partial T}{\partial y}\right|_{9\text{-}13}}{\Delta y}=\frac{T_5+T_{13}-2T_9}{\Delta y^2}$$

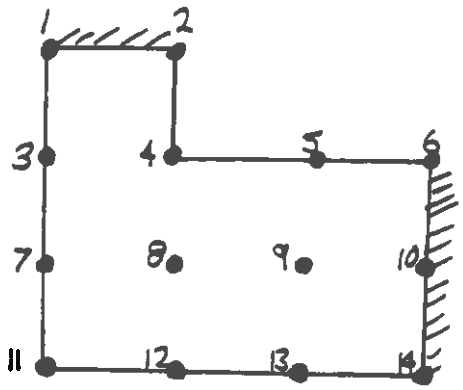
Thus the nodal equation for node 9 becomes

$$\frac{T_{10} + T_8 - 2T_9}{\Delta x^2} + \frac{T_5 + T_{13} - 2T_9}{\Delta y^2} = -\frac{\dot{q}'''}{k}$$

We would develop similar equations for each node.

Alternative: Energy balance

Again focus on node ⑨



In steady-state

$$q_{8\to9} + q_{10\to9} + q_{5\to9} + q_{13\to9} = -\dot{q}'''(\text{nodal volume})$$

Now we can use

$$q_n = -kA_n \frac{\partial T}{\partial x_n} = -kA_n \frac{\Delta T}{\Delta x_n}$$

Thus

$$q_{8\to 9} = k(\Delta y)\,\frac{T_8 - T_9}{\Delta x}$$

$$q_{10\to 9} = k(\Delta y)\,\frac{T_{10} - T_9}{\Delta x}$$

$$q_{5\to 9} = k(\Delta x)\,\frac{T_5 - T_9}{\Delta y}$$

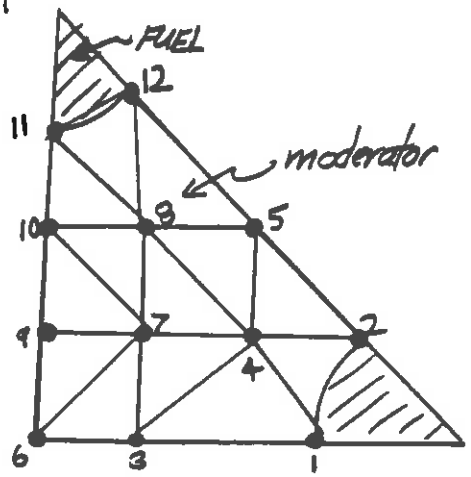
$$q_{13\to 9} = k(\Delta x)\,\frac{T_{13} - T_9}{\Delta y}$$

Thus our balance equation is

$$k\Delta y\,\frac{T_8 - T_9}{\Delta x} + k\Delta y\,\frac{T_{10} - T_9}{\Delta x} + k\Delta x\,\frac{T_5 - T_9}{\Delta y} + k\Delta x\,\frac{T_{13} - T_9}{\Delta y} = -q'''\,\Delta x \Delta y$$

(Same as before...)

Example: Hexagonal lattice (HTGR)



Can again just use energy balance

For node ④

$$q_{7\to 4} + q_{8\to 4} + q_{5\to 4} + q_{2\to 4} + q_{3\to 4} + q_{1\to 4} = -q'''(\text{volume})$$

LECTURE 31

THERMAL HYDRAULIC COMPUTER CODES

READING ASSIGNMENT:

Duderstadt & Hamilton, pp. 507 - 510
J. Weisman and R. Bowring, Nuc. Sci. Eng. 57, 253 (1975)

EXERCISES:

31-1. Using the description provided in the lecture, write a computer program capable to analyzing the hot channel of a LWR (either PWR or BWR).

31-2. Using a simple single-channel thermal-hydraulics code, evaluate the performance of a typical PWR core design. In particular,

a) Compare a bottom-peaked and centered profile from the standpoint of (i) peak linear power density, (ii) peak fuel centerline temperature, (iii) DNB ratio.

b) Roughly plot the following for the hot channel of the bottom-peaked profile: (i) coolant temperature, (ii) clad surface temperature, (iii) heat flux, and (iv) DNB heat flux.

c) Evaluate the effect of a simultaneous 10% reduction in flow and 10% increase in power level on DNB ratio and centerline fuel temperature.

31-3. Using a simple single-channel thermal-hydraulics code, evaluate the performance of a BWR core design. Use a bottom peaked power profile and determine:

a) Coolant temperature, fuel temperature, void fraction, and flow quality profiles.

b) The recirculation ratio (mass of recirculated coolant to coolant leaving the vessel as steam) and the reactor vessel inlet temperature (i.e., feedwater temperature).

Thermal-Hydraulic Computer Codes

- A Simple Single-Channel Code
- Subchannel Analysis
- Thermal-hydraulic Reactor Codes

Problems with thermal-hydraulic calculations:

highly nonlinear

empirical correlations

table lookups

Hence most suited to numerical solution

Example: Evaluation of temperature distributions and pressure drops in LWR channel

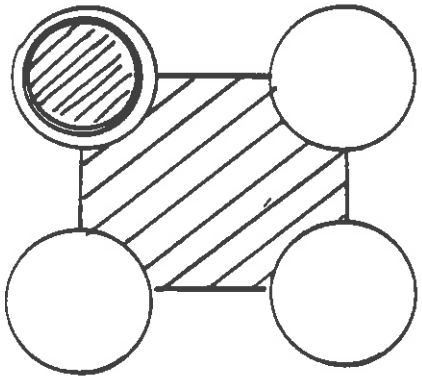
Geometry:

Metal clad oxide rods

Pellet diameter

Clad O.D.

Clad thickness



Pitch-to-diameter ratio

Assumptions:

Constant coolant flow in each channel

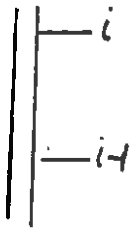
Same axial power shape in all channels

Hot channel analysis

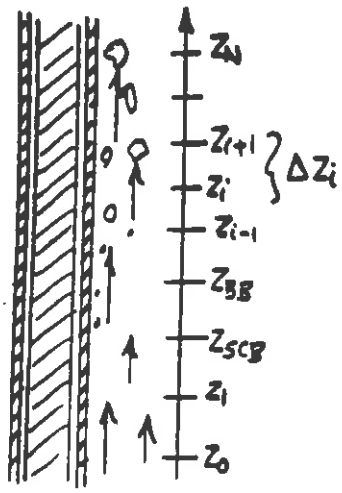
Approach: Divide channel into intervals and finite difference energy balance equation

$$h_i = h_{i-1} + \frac{q_i'}{w}$$

$$h_0 = h_{inlet}$$



Example: A simple thermal-hydraulic code



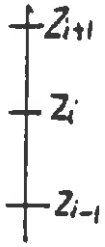
Divide channel into segments Δz_i,

then "discretize" equations.

Start with energy balance equation

$$w[h_{i+1} - h_i] = \int_{z_i}^{z_{i+1}} q'(z)\,dz$$

$$\cong \left(\frac{q_i + q_{i+1}}{2}\right)\Delta z_i$$



Hence we can solve for

$$h_{i+1} = h_i + \frac{\Delta z_i}{2w}(q_i + q_{i+1})$$

from neutronics

Next, calculate the pressure drop up the segment

$$P_{i+1} = P_i - \Delta p_i^f - \Delta p_i^h - \Delta p_i^{form}$$

friction elevation form

$$\Delta p_i^f = \frac{2\Delta z_i}{D_h} \rho_i \overline{u_{zi}}^2 f(Re_i)$$

$$\Delta p_i^h = \rho_i g \Delta z_i$$

$$\Delta p_i^{form} = \sum_i K_i \rho_i \frac{\overline{u_{zi}}^2}{2}$$

Since we now have h_{i+1} and P_{i+1}, we can go to thermodynamic tables (stored in memory) to find

$$\rho_{i+1} = \rho_{i+1}(h_{i+1}, P_{i+1})$$

$$T_{i+1} = T_{i+1}(h_{i+1}, P_{i+1})$$

Use these to compute Re_{i+1}, Nu_{i+1}, Pr_{i+1}

Now work inwards to compute $T_{S_{i+1}}$, $T_{F_{i+1}}$, $T_{\text{¢}_{i+1}}$ -- e.g.

$$T_{S_{i+1}} = T_{i+1} + \frac{q_{i+1}}{h_{S_{i+1}}}$$

$$T_{\ell_{i+1}} = T_{i+1} + \frac{q_{i+1}}{2\pi r_F}\left[\frac{r_F}{2\bar{k}_{F_{i+1}}} + \frac{1}{h_{G_{i+1}}} + \frac{t_c}{k_{c_{i+1}}} + \frac{r_F}{h_{s_{i+1}}(r_F+t_c)}\right]$$

Finally use mass flow conservation

$$W = \text{constant} = \rho_{i+1}\,\bar{u}_{z_{i+1}}\,A_{channel}$$

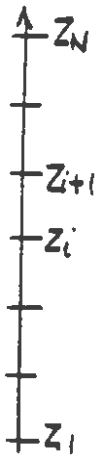
to determine fluid velocity

$$\bar{u}_{z_{i+1}} = \frac{W}{\rho_{i+1}\,A_{channel}}$$

Keep marching up channel until $T_i \rightarrow T_{sat}$, then switch to boiling channel correlations. (Remember S-α-χ relationship.)

Here single channel strategy is quite simple:

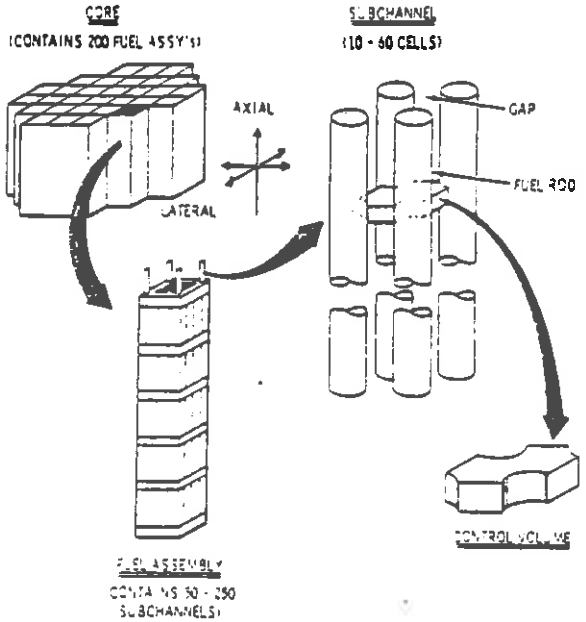
Just march up channel using $q'(z)$ and coolant inlet conditions as given. Determine temperatures T_{s_i}, T_{F_i}, T_{ℓ_i}, and other parameters such as DNBR.

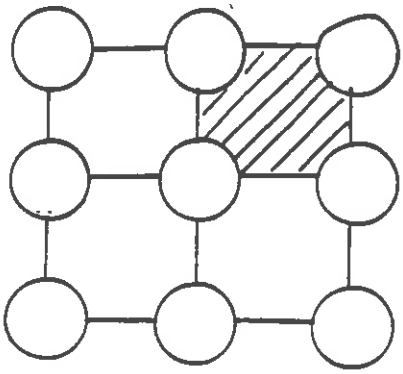


Subchannel Analysis

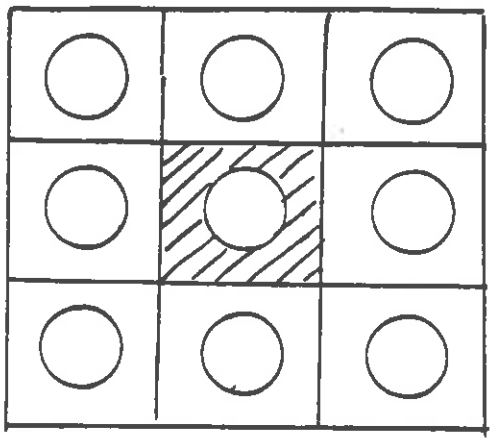
Fuel rod cluster is subdivided into a number of parallel, interacting flow subchannels between rods.

Equations of mass, momentum, and energy conservation are solved to give radial and axial variations in fluid enthalpy and mass velocity.



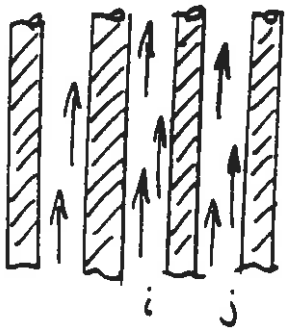


"Conventional"



Rod-centered

Conservation equations:



Mass conservation

$$\frac{\partial m_i}{\partial x} = -\sum_j w_{ij}$$

Energy conservation

$$m_i \frac{\partial h_i}{\partial x} = q_i - \sum_j (h^* - h_i) w_{ij} - \sum_j (h_i - h_j) w'_{ij}$$

Axial momentum conservation

$$\frac{\partial p}{\partial x} = -P_f - P_a - P_h - P_g - P_d$$

Solution methods

Marching procedures:

Knowing inlet conditions, proceed stepwise along channel length to exit -- iterative solution across channels

Simultaneous procedures:

Simultaneous solution at all axial intervals

Thermal Hydraulic Codes:

COBRA: slip flow

subchannel coupling

steady-state or transient

regular or reversed flow

THINC: assumes no pressure gradient exists within hot assembly

HAMBO (UK), FLICA (France)

LECTURE 32

STRESSES IN NUCLEAR REACTORS

READING ASSIGNMENT:

J. Weisman, "Elements of Stress Analysis", Chapter 9 in Weisman (Design)
Rust, Chapter 8
Shigley, pp. 24 - 42

EXERCISES:

32-1. Give examples of what you would regard as elastic and plastic behavior in nuclear reactor components.

32-2. Following closely the derivation given in the Lecture notes, derive the form of the stress tensor for a wo-dimensional plate.

32-3. By referring to any standard reference on stress analysis, provide the detailed derivation of the equations characterizing plane stress that lead to Mohr's circle.

Stresses in Nuclear Reactors

- Nuclear Reactor Stress Analysis
- Mechanical Loads
- Definition of Stress
- Mohr's Circle

Nuclear reactor components are subjected to

mechanical stresses

thermal stresses

Stress analysis in nuclear systems are complicated by:

unusual environment (radiation, pressure, thermal)

unusual materials

changes in reactor core

Steps in stress analysis

- Physics analysis to determine heat source distributions and material irradiations
- Thermal-hydraulics analysis to determine temperature distributions
- Stress analysis

Stress analysis requires a thorough knowledge of material behavior under stress

elastic materials: stress equations are linear -- mechanical and thermal stress can be analyzed seperately and added

plastic materials: stress equations are nonlinear -- mechanical and thermoplastic stresses must be treated simultaneously.

To minimize thermal stresses in reactor components, we desire

- small modulus of elasticity E
- small coefficient of thermal expansion α
- large thermal conductivity k
- small Poisson ratio ν

Mechanical loads

static loads: forces applied slowly and remaining constant over a relatively long period

repeated loads: forces applied a large number of times so that force on structure is repeated changed

impact loads: forces applied suddenly and maintained only a short time

Examples:

static loads: internal pressure

heating

repeated loads: startup / shutdown transients

flow vibrations

impact loads: pressure waves

accident conditions

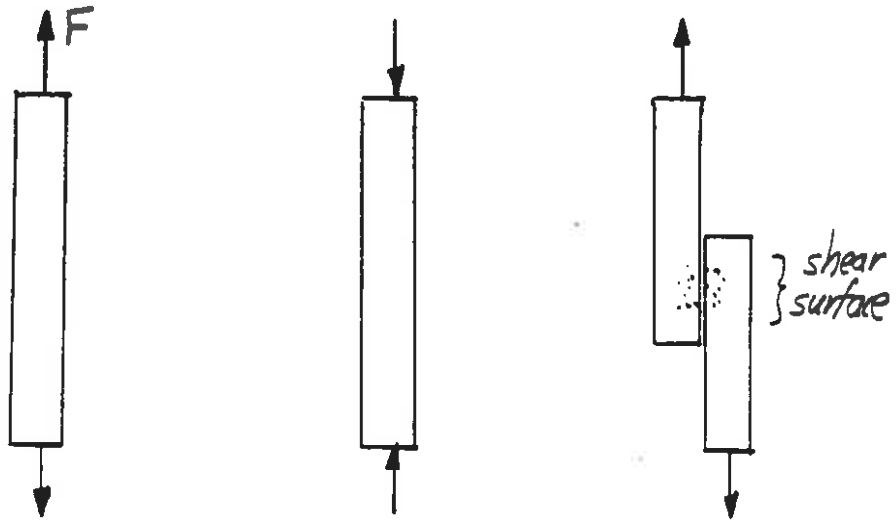
Definition of stress

Stress: internal forces per unit area induced by external applied force or load

Note:

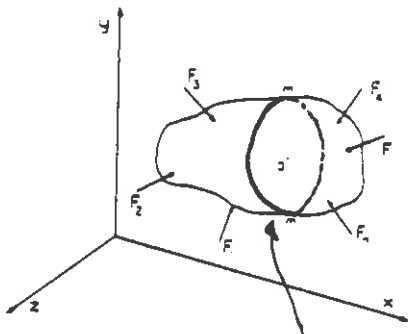
$$\text{Stress} = \frac{\text{force}}{\text{area}}$$

Types of stress

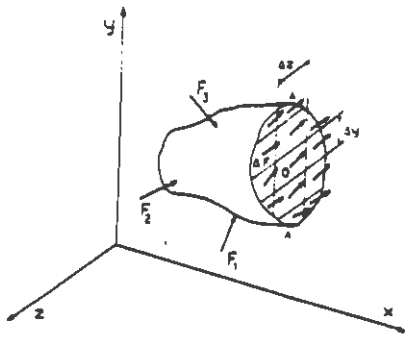


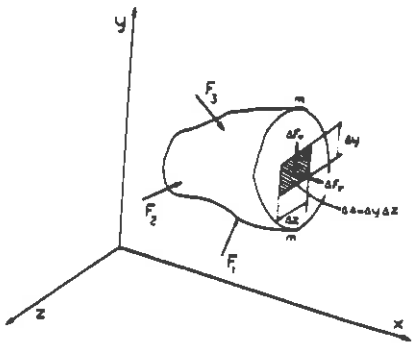
Tension Compression Shear

Let's be more precise: Consider a body in equilibrium under the action of a system of external forces

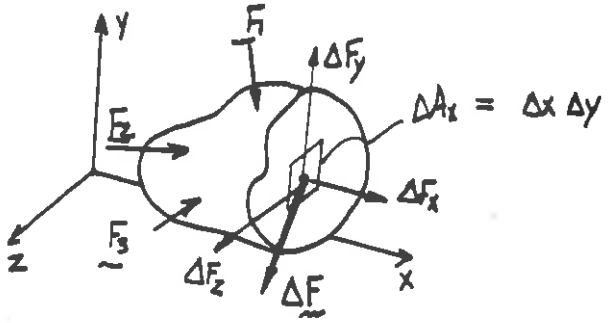


Now imagine body sliced in two





Examine magnitude and direction of internal forces over cut surface. By stress we mean the direction and intensity of internal force at a particular point on cut surface. (Note stress is an artificial concept.)

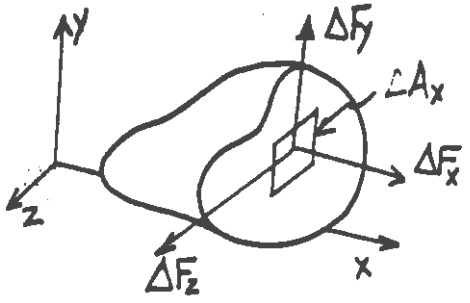


Let $\Delta \underset{\sim}{F}$ be the resultant of all internal forces at a point.

We define

$$\text{Stress} = \lim_{\Delta A_x \to 0} \frac{\Delta \underset{\sim}{F}}{\Delta A_x}$$

It is more convenient to describe stress in terms of its rectangular components

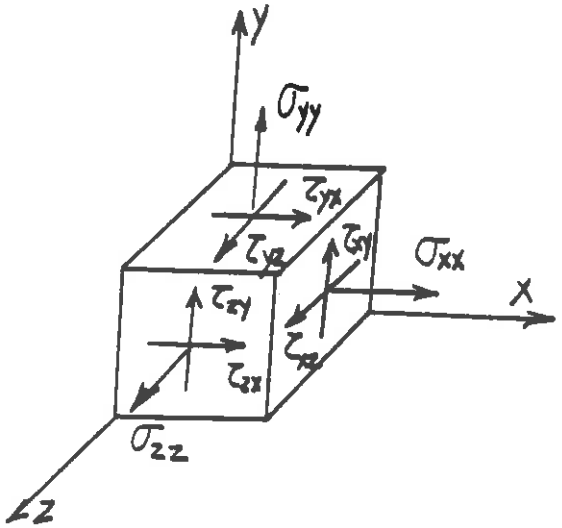


$$\sigma_{xx} \equiv \lim_{\Delta A_x \to 0} \frac{\Delta F_x}{\Delta A_x}$$

$$\tau_{xy} \equiv \lim_{\Delta A_x \to 0} \frac{\Delta F_y}{\Delta A_x}$$

$$\tau_{xz} \equiv \lim_{\Delta A_x \to 0} \frac{\Delta F_z}{\Delta A_x}$$

We can also define stress components along other axes:



$\tau_{xy}, \tau_{xz}, \tau_{yx}, \tau_{yz}, \tau_{zx}, \tau_{zy}$ → shear stress

$\sigma_{xx}, \sigma_{yy}, \sigma_{zz}$ → normal stress $(\sigma_x, \sigma_y, \sigma_z)$

Note: There are 9 components of stress.

Hence stress can be represented as a second-rank tensor quantity:

$$\underset{\approx}{\tau} = \begin{pmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{pmatrix}$$

Some symmetry considerations

<u>2-D (plane) stresses:</u>

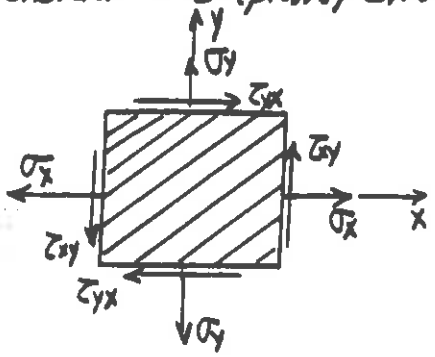
$$\tau_{xy} = \tau_{yx}$$

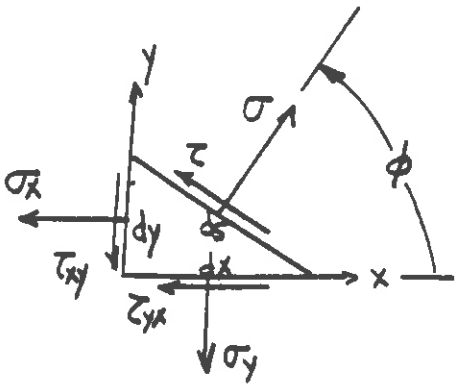
<u>3-D stresses:</u>

$$\tau_{xz} = \tau_{zx} \qquad \tau_{yz} = \tau_{zy} \qquad \tau_{xy} = \tau_{yx}$$

Mohr's Circle

Consider 2-D (plane) stress





If we sum all stress forces to zero, we can find σ and τ

$$\sum F_x = \sigma ds \cos\phi - \tau ds \sin\phi + \tau_{yx}(-dx) + \sigma_x(-dy) = 0$$

$$\sum F_y = \sigma ds \sin\phi + \tau ds \cos\phi + \tau_{xy}(-dy) + \sigma_y(-dx) = 0$$

Bypassing the algebra, we find

$$\sigma = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\phi + \tau_{xy}\sin 2\phi$$

$$\tau = -\frac{\sigma_x - \sigma_y}{2}\sin 2\phi + \tau_{xy}\cos 2\phi$$

If we take $\frac{d\sigma}{d\phi} = 0$ we find

$$\tan 2\phi = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

This defines the principal stresses and principal directions

maximum normal stress σ_1

minimum normal stress σ_2

We can also set $\frac{dr}{d\phi} = 0$ to find

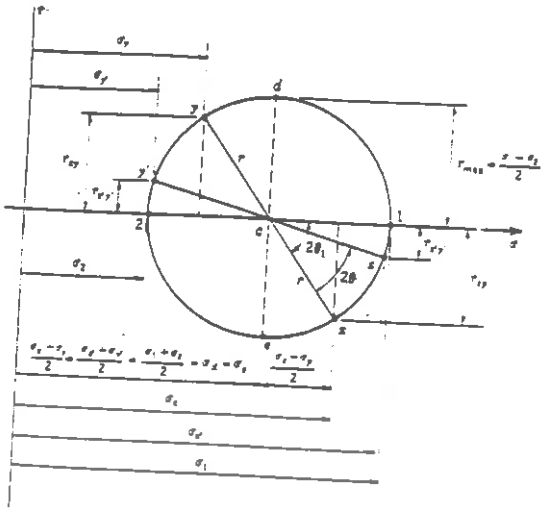
$$\tan 2\phi = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

We can solve these for

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \left[\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2\right]^{1/2}$$

$$\tau_{max} = \pm \left[\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2\right]^{1/2}$$

We can keep track of stress at a point using a graphical method, Mohr's circle diagram



Mohr's circle diagram for plane stress

LECTURE 33

STRESS AND STRAIN

READING ASSIGNMENT:

Shigley, pp. 24 - 42

EXERCISES:

33-1. A uniform bar of cross sectional area A is subjected to an axial tensile force P. What are the axial tensile stress, the axial tensile strain, and the reduction in cross sectional area, all assuming linear elastic behavior (Hooke's law). Define any symbols you may need.

33-2. A steel rod 0.5 in. in diameter and 40 in. long carries a tensile load of 3000 lbf. Find the stress and elongation of the rod.

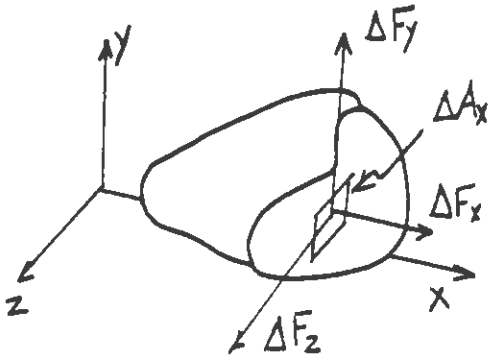
33-3. Derive the equation relating Young's modulus, the shear modulus, and Poisson's ratio.

33-4. Sketch the stress-strain curves for: i) steel, ii) glass, iii) rubber, iv) fiberglass.

Stress and Strain

- Review of Stress
- Definition of Strain
- Elasticity and Hooke's Law
- Stress-Strain Curves

Recall our definition of stress as the internal forces per unit area induced by external applied force or load



$$\sigma_{xx} \equiv \lim_{\Delta A_x \to 0} \frac{\Delta F_x}{\Delta A_x}$$

$$\tau_{xy} \equiv \lim_{\Delta A_x \to 0} \frac{\Delta F_y}{\Delta A_x}$$

$$\tau_{xz} \equiv \lim_{\Delta A_x \to 0} \frac{\Delta F_z}{\Delta A_x}$$

More generally, we can define a stress tensor

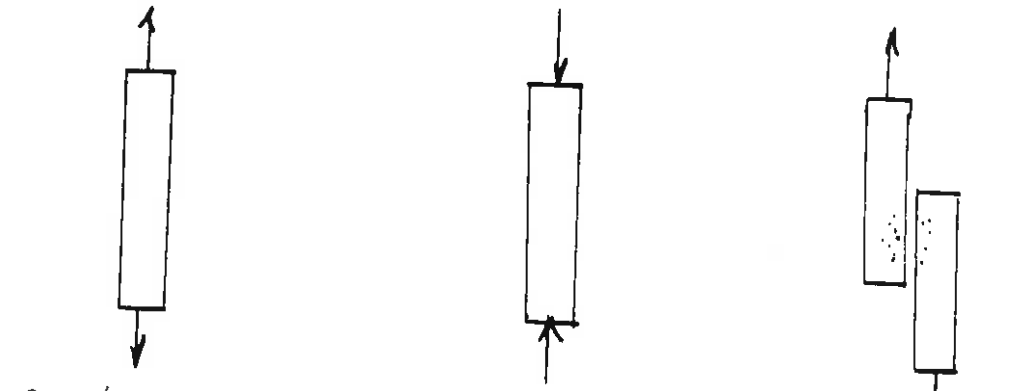
$$\underset{\approx}{\tau} = \begin{pmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{pmatrix}$$

Here

normal stresses: $\sigma_{xx} = \sigma_x$, $\sigma_{yy} = \sigma_y$, $\sigma_{zz} = \sigma_z$

shear stresses: $\tau_{xy}, \tau_{xz}, \tau_{yx}, \tau_{yz}, \tau_{zx}, \tau_{zy}$

Frequently one assumes a uniform distribution of stress -- pure tension, pure compression, or pure shear



Pure tension

$\sigma = \frac{F}{A}$

Pure compression

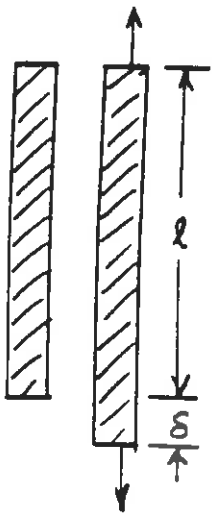
$\sigma = \frac{F}{A}$

Pure shear

$\tau = \frac{F}{A}$

Elastic Strain

Consider a straight bar subjected to a tensile load. The elongation of the bar is called strain.



unit strain = elongation per unit length = $\frac{\delta}{\ell} \equiv \epsilon$

[Note: "strain" → unit strain (ϵ)
"total strain" → total deformation (ℓ)]

Can similarly define

shear strain γ = change in right angle of a stress element subjected to pure shear

Elasticity -- that property of a material which enables it to regain its original shape and dimensions when load is removed

Hooke's Law:

stress is proportional to strain which produced it

(Elastic materials do not necessarily obey Hooke's Law.)

We can state Hooke's Law in mathematical form

$$\sigma = E\epsilon \qquad \text{(tension)}$$

$E \Rightarrow$ modulus of elasticity or Young's modulus

or for shear

$$\tau = G\gamma \qquad \text{(shear)}$$

$G \Rightarrow$ shear modulus of elasticity or modulus of rigidity

Note if we use

$$\epsilon = \frac{\delta}{\ell}, \quad \sigma = E\epsilon, \quad \sigma = \frac{F}{A}$$

We can determine the total deformation of a bar loaded in axial tension or compression

$$\delta = \frac{F\ell}{AE}$$



Note: When a material is placed in tension, there is both axial and lateral strain. Within the range of validity of Hooke's Law, these are proportional and given by



$$\text{Poisson's ratio} = \mu \equiv -\frac{\text{lateral strain}}{\text{axial strain}}$$

We can relate the three elastic constants

$$E = 2G(1+\mu)$$

where

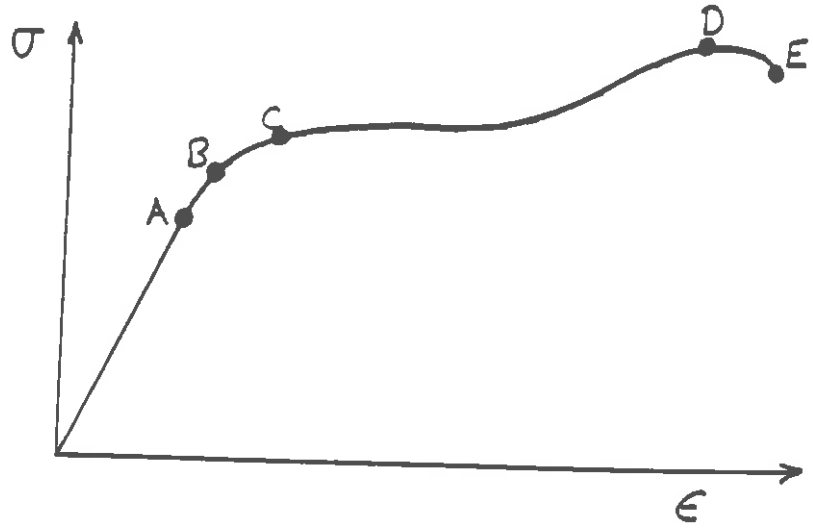
$E \rightarrow$ modulus of elasticity (Young's modulus)

$G \rightarrow$ shear modulus

$\mu \rightarrow$ Poisson's ratio

Stress-Strain Curves

Consider a typical ductile metal bar under pure tension



A: proportional limit (Hooke's Law)

O-A: elastic region (strain disappears when stress is removed)

B: elastic limit (strains beyond B will not disappear when stress is removed) -- plastic strains

[Note: For many materials

proportional limit = elastic limit]

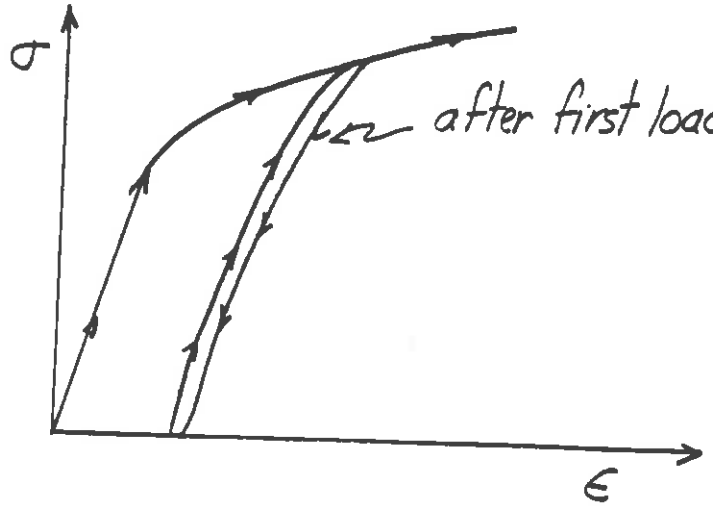
C: yield point (that stress where material continues to deform without an increase in load)

offset yield point (stress producing a given plastic strain -- 0.2%)

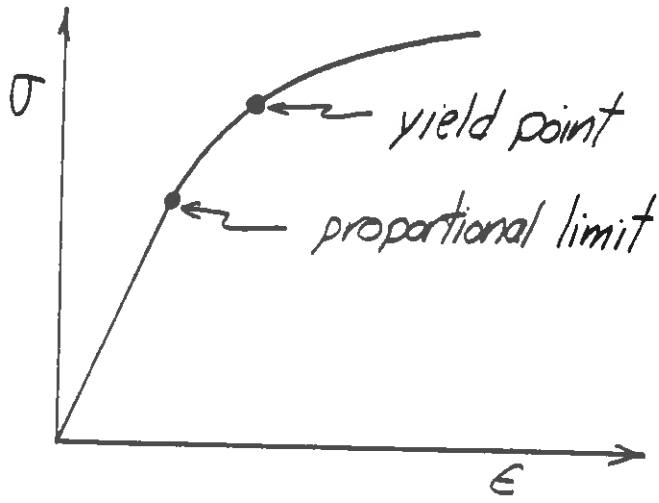
D: ultimate strength (maximum load material will bear before failure)

E: rupture strength (failure occurs)

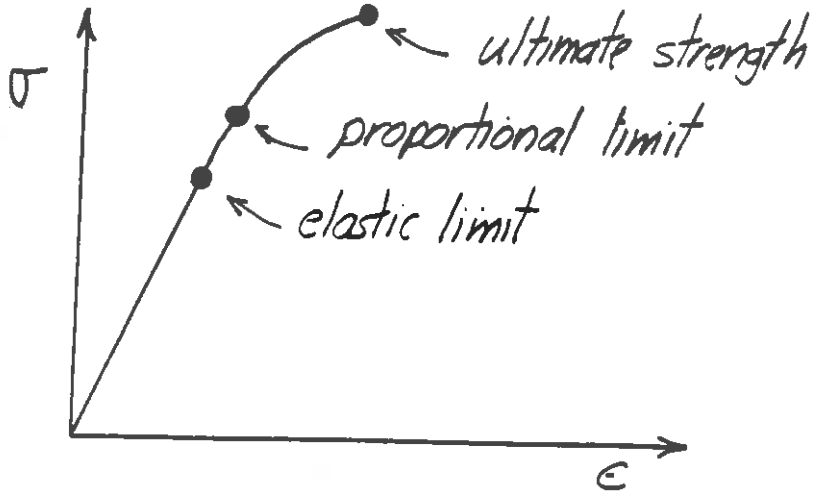
Consider a specimen stressed beyond yield point and then unloaded -- can extend elastic limit (strain hardening)



Stress-strain curve for ductile material in compession



Stress-strain curve for brittle material



LECTURE 34

INTRODUCTION TO STRESS ANALYSIS

READING ASSIGNMENT:

EXERCISES:

34-1. Provide an explicit demonstration that the off diagonal elements of the strain tensor do indeed represent the shear strains in terms of angular displacement.

34-2. By referring to any standard text on stress analysis, demonstrate the physical significance of the compatibility relations.

34-3. The equation describing the deflection of a beam is:

$$E\ I\ (d2y/dx2y) = M(x)$$

where M(x) is the moment of foces at x. Solve the differential equation, assuming constant E I, for the deflection at the end of the beam, for a load P placed at the end of the beam. Use as boundary conditions:

b.c: $y(\emptyset) = \emptyset$

$y'(\emptyset) = \emptyset$ (embedded end)

Introduction to Stress Analysis

- Temperature Effects
- State of Strain (Deformation)
- Strain Tensor
- Elastic Stress Theory

At moderate temperatures, most structural materials can be considered to behave elastically:

- strains proportional to stresses
- when stresses are removed, material returns to its original configuration

At higher temperatures, materials held in a stressed condition for prolonged periods will flow or creep to remove stresses.

Creep deformation is irreversible -- structure will not return to initial shape when loads are removed.

Time-dependent deformation is a type of viscous behavior -- "viscoelasticity." Viscoelastic and linear elastic behavior occurs simultaneously.

Problem:

viscoelastic stress theory is very complex

Instead:

use elastic thermal stress analysis

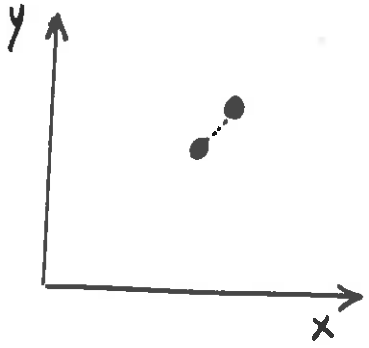
Note: Formal structure of theory is very similar to fluid mechanics.

State of strain (deformation) of a solid

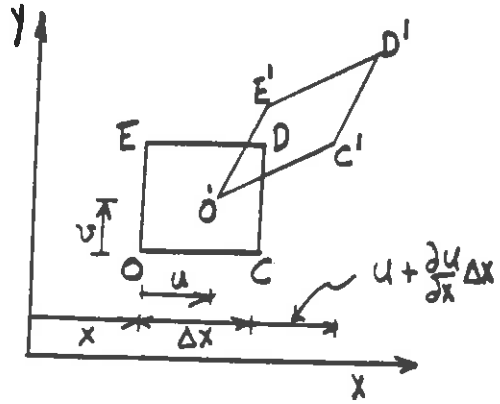
Consider the displacements that occur in plane strain by defining

$u = u(x,y)$ = displacement of a point in x-direction

$v = v(x,y)$ = displacement of a point in y-direction



Consider an element which has undergone a deformation



We can use our earlier definition of strain:

$$\epsilon_x = \lim_{\Delta x \to 0} \frac{O'C' - OC}{OC} = \lim_{\Delta x \to 0} \frac{\left(\Delta x + \frac{\partial u}{\partial x}\Delta x\right) - \Delta x}{\Delta x} = \frac{\partial u}{\partial x}$$

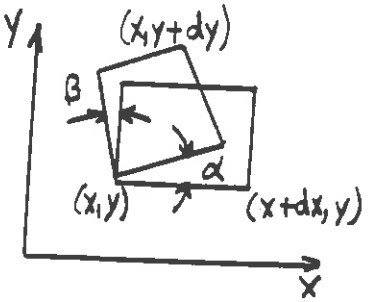
In a similar way

$$\epsilon_y = \lim_{\Delta y \to 0} \frac{O'E' - OE}{OE} = \lim_{\Delta y \to 0} \frac{\left(\Delta y + \frac{\partial v}{\partial y}\Delta y\right) - \Delta y}{\Delta y} = \frac{\partial v}{\partial y}$$

$$\gamma_{xy} = \lim_{\substack{\Delta x \to 0 \\ \Delta y \to 0}} \left(\frac{\pi}{2} - \angle C'O'E'\right) = \lim_{\substack{\Delta x \to 0 \\ \Delta y \to 0}} \left[\frac{\pi}{2} - \left(\frac{\pi}{2} - \frac{\frac{\partial v}{\partial x}\Delta x}{\Delta x} - \frac{\frac{\partial u}{\partial y}\Delta y}{\Delta y}\right)\right] = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

Here, the diagonal elements ϵ_{xx}, ϵ_{yy}, ϵ_{zz} represent the normal strain in each coordinate direction.

The off-diagonal elements $\gamma_{xy}, \gamma_{xz}, \gamma_{yx}, \gamma_{yz}, \gamma_{zx}, \gamma_{zy}$ represent the shear strains.

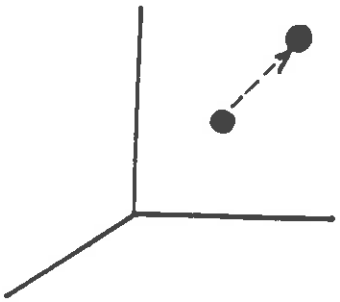


$$\alpha + \beta = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

= departure from 90°

More generally, one can define a displacement vector

$$\underline{V}(\underline{r}) \equiv u(\underline{r})\hat{\underline{i}} + v(\underline{r})\hat{\underline{j}} + w(\underline{r})\hat{\underline{k}}$$



Then the strain becomes a tensor

$$\underset{\approx}{\epsilon} = \begin{pmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{pmatrix}$$

The components of the strain tensor are

$$\epsilon_{xx} = \frac{\partial u}{\partial x} \equiv \epsilon_x$$

$$\epsilon_{yy} = \frac{\partial v}{\partial y} \equiv \epsilon_y$$

$$\epsilon_{zz} = \frac{\partial w}{\partial z} \equiv \epsilon_z$$

$$\epsilon_{xy} = \epsilon_{yz} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \equiv \gamma_{xy}$$

$$\epsilon_{xz} = \epsilon_{zx} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \equiv \gamma_{xz}$$

$$\epsilon_{yz} = \epsilon_{zy} = \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) \equiv \gamma_{yz}$$

There are a number of compatibility relations that imply that medium has not be "torn" or that no discontinuities in displacements occur:

$$\frac{\partial^2 \epsilon_{xy}}{\partial x \partial y} = \frac{1}{2}\left(\frac{\partial^2 \epsilon_{xy}}{\partial y^2} + \frac{\partial^2 \epsilon_{yy}}{\partial x^2}\right)$$

$$\frac{\partial^2 \epsilon_{xy}}{\partial y \partial z} = \frac{\partial}{\partial x}\left(-\frac{\partial \epsilon_{yz}}{\partial x} + \frac{\partial \epsilon_{xy}}{\partial y} + \frac{\partial \epsilon_{yy}}{\partial z}\right)$$

$$\frac{\partial^2 \epsilon_{xz}}{\partial x \partial z} = \frac{1}{2}\left(\frac{\partial^2 \epsilon_{xx}}{\partial z^2} + \frac{\partial^2 \epsilon_{zz}}{\partial x^2}\right)$$

$$\frac{\partial^2 \epsilon_{yy}}{\partial x \partial z} = \frac{\partial}{\partial y}\left(-\frac{\partial \epsilon_{xz}}{\partial y} + \frac{\partial \epsilon_{xy}}{\partial z} + \frac{\partial \epsilon_{yz}}{\partial x}\right)$$

$$\frac{\partial^2 \epsilon_{yz}}{\partial y \partial z} = \frac{1}{2}\left(\frac{\partial^2 \epsilon_{yy}}{\partial z^2} + \frac{\partial^2 \epsilon_{zz}}{\partial y^2}\right)$$

$$\frac{\partial^2 \epsilon_{xz}}{\partial x \partial y} = \frac{\partial}{\partial z}\left(-\frac{\partial \epsilon_{xy}}{\partial z} + \frac{\partial \epsilon_{yz}}{\partial x} + \frac{\partial \epsilon_{xz}}{\partial y}\right)$$

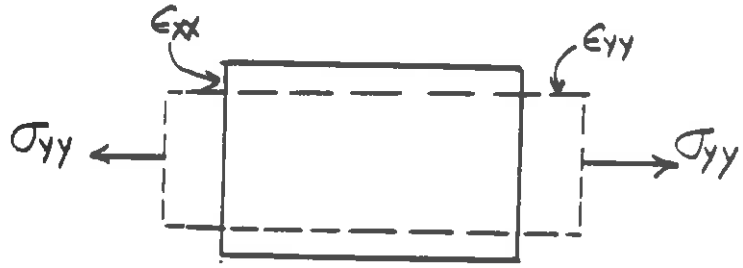
To complete the theory, we need a relation between stress and strain.

Example: For a pure tensile or compressive stress, we assume Hooke's Law to model elastic behavior

$$\epsilon_{xx} = \frac{1}{E}\sigma_{xx}$$

where E is Young's modulus.

But a solid cannot be compressed or elongated in one direction without altering the dimension perpendicular to the applied stress



We can relate axial and lateral strain by

$$\epsilon_{xx} = -\mu\,\epsilon_{yy} = -\mu\left(\frac{\sigma_{yy}}{E}\right)$$

where μ is Poisson's ratio.

More generally, if there are other normal stresses

$$\epsilon_{xx} = \frac{\sigma_{xx}}{E} - \mu\left(\frac{\sigma_{yy}}{E}\right) - \mu\left(\frac{\sigma_{zz}}{E}\right)$$

(Here we are using the principle of superposition.)
(-- allowed by our assumption of elastic behavior)

LECTURE 35

THERMOELASTIC STRESS ANALYSIS

READING ASSIGNMENT:

J. Weisman, "Elements of Stress Analysis", Chapter 9 in Weisman (Design)

EXERCISES:

35-1. Upon increasing the temperature of a piece of metal, how do you expect the following quantities to change?

a) Tensile strength

b) Young's modulus

c) stable creep rate

d) density

35-2. A rod of length L is placed between two immovable walls which touch each end. At temperature To, the rod is stress free. A heat source is not placed in the rod such that its temperature distribution is given by

$$T - To = 4\, T(max)\, (x/L)\, (1 - x/L)$$

What is the stress at the rod midpoint?

35-3. Using the expressions for the gradient operator in curvilinear coordinates, derive the form of the stress analysis equations in cylindrical and spherical coordinates.

35-4. Demonstrate that the equilibrium stress condition is really just a special case of the equations of fluid mechanics.

Thermoelastic Stress Analysis

- Review of Strain
- Temperature-Induced Strains
- Stress-Strain Relations
- Equilibrium Condition
- Curvilinear Coordinates

Recall we had defined the strain tensor

$$\underset{\approx}{\epsilon} \equiv \begin{pmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{pmatrix} = \begin{pmatrix} \epsilon_x & \gamma_{xy} & \gamma_{xz} \\ \gamma_{yx} & \epsilon_y & \gamma_{yz} \\ \gamma_{zx} & \gamma_{zy} & \epsilon_z \end{pmatrix}$$

where

$$\epsilon_{xx} = \frac{\partial u}{\partial x} \equiv \epsilon_x, \quad \epsilon_{yy} = \frac{\partial v}{\partial y} \equiv \epsilon_y, \quad \epsilon_{zz} = \frac{\partial w}{\partial z} \equiv \epsilon_z$$

$$\epsilon_{xy} = \epsilon_{yz} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \equiv \gamma_{xy} = \gamma_{yx}$$

$$\epsilon_{xz} = \epsilon_{zx} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \equiv \gamma_{xz} = \gamma_{zx}$$

$$\epsilon_{yz} = \epsilon_{zy} = \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) \equiv \gamma_{yz} = \gamma_{zy}$$

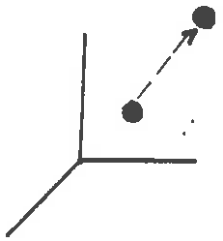
Note

$$\epsilon_{xx} = \frac{\partial u}{\partial x} \equiv \epsilon_x \rightarrow \text{normal strain}$$

$$\epsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \equiv \gamma_{xy} \rightarrow \text{shear strain}$$

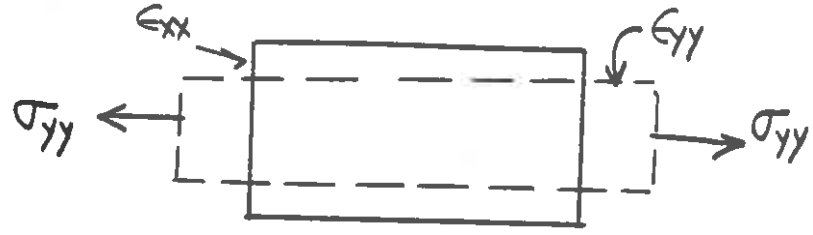
Here

$$\underline{V}(\underline{r}) = u(\underline{r})\,\hat{\underline{i}} + v(\underline{r})\,\hat{\underline{j}} + w(\underline{r})\,\hat{\underline{k}}$$



If we assume Hooke's Law is valid

$$\epsilon_{xx} = \frac{\sigma_{xx}}{E} - \mu\left(\frac{\sigma_{yy}}{E}\right) - \mu\left(\frac{\sigma_{zz}}{E}\right)$$



Temperature-induced strains



Consider the longitudinal expansion of a heated bar of original length l. If

α = coefficient of linear expansion

then bar will lengthen to

$$l = [1 + \alpha(T - T_0)]\, l_0$$

Hence the strain is just the elongation per unit length

$$\epsilon_{xx} = \frac{\Delta l}{l_0} = \alpha(T - T_0)$$

We can again use superposition to write

$$\epsilon_{xx} = \frac{\sigma_{xx}}{E} - \frac{\mu}{E}(\sigma_{yy} + \sigma_{zz}) + \alpha(T - T_0)$$

More generally

$$\epsilon_{xx} = \frac{\sigma_{xx}}{E} - \frac{\mu}{E}(\sigma_{yy} + \sigma_{zz}) + \alpha(T - T_o)$$

$$\epsilon_{yy} = \frac{\sigma_{yy}}{E} - \frac{\mu}{E}(\sigma_{xx} + \sigma_{zz}) + \alpha(T - T_o)$$

$$\epsilon_{zz} = \frac{\sigma_{zz}}{E} - \frac{\mu}{E}(\sigma_{xx} + \sigma_{yy}) + \alpha(T - T_o)$$

Now define a mean normal stress

$$\bar{\sigma} = \frac{1}{3}(\sigma_{xx} + \sigma_{yy} + \sigma_{zz})$$

and the volume dilitation

$$\nabla \cdot \underset{\sim}{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = \epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz}$$

Then we can combine our results to write

$$\nabla \cdot \underset{\sim}{V} = \left(\frac{1 - 2\mu}{E}\right)(3\bar{\sigma}) + 3\alpha(T - T_o)$$

If we subtract $\frac{\mu}{E}\sigma_{xx}$ from our expression for ϵ_{xx}

We can relate σ_{xx} to the normal strain components

$$\sigma_{xx} = 2\left[\frac{E}{2(1+\mu)}\right]\epsilon_{xx} + \left[\left(\frac{E}{1-2\mu}\right)\left(\frac{\mu}{1-\mu}\right)\right]\nabla\cdot\underset{\sim}{V} - \left[\left(\frac{E}{1+\mu}\right)\left(1+\frac{3\mu}{1-2\mu}\right)\right]\alpha(T-T_0)$$

Now define a shear modulus of elasticity (modulus of rigidity)

$$G \equiv \frac{E}{2(1+\mu)}$$

Then

$$\sigma_{xx} = 2G\left\{\epsilon_{xx} + \left(\frac{\mu}{1-2\mu}\right)\nabla\cdot\underset{\sim}{V} - \left(\frac{1+\mu}{1-2\mu}\right)\alpha(T-T_0)\right\}$$

We can generalize this stress-strain relation to account for shear as well as normal components

$$\sigma_{ij} = 2G\left\{\epsilon_{ij} + \left(\frac{\mu}{1-2\mu}\right)\nabla\cdot\underset{\sim}{V}\,\delta_{ij} - \left(\frac{1+\mu}{1-2\mu}\right)\alpha(T-T_0)\,\delta_{ij}\right\}$$

where $\delta_{ij} = \begin{cases} 1 & i=j \\ 0 & i\neq j \end{cases}$

Note that for shear, $i \neq j$ and

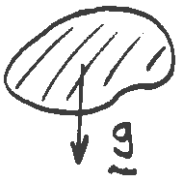
$$\sigma_{ij} = 2G\,\epsilon_{ij} \qquad i \neq j$$

-- hence shear stresses are unrelated to temperature changes.

Equilibrium Condition

In equilibrium, one can show

$$\underset{\sim}{\nabla}\cdot\underset{\approx}{\sigma} + \rho\underset{\sim}{g} = 0$$



where $\underset{\sim}{g}$ is the acceleration of gravity.

(This is a special case of the equations of fluid mechanics.)

If gravity is not of major concern

$$\underset{\sim}{\nabla}\cdot\underset{\approx}{\sigma} = 0$$

Let's write out the x-component

$$\frac{\partial\sigma_{xx}}{\partial x} + \frac{\partial\sigma_{xy}}{\partial y} + \frac{\partial\sigma_{xz}}{\partial z} + \rho g_x = 0$$

We can now use our stress-strain relation

$$\sigma_{ij} = 2G\left\{\epsilon_{ij} + \left(\frac{\mu}{1-2\mu}\right)\underset{\sim}{\nabla}\cdot\underset{\sim}{V}\,\delta_{ij} - \left(\frac{1+\mu}{1-2\mu}\right)\alpha(T-T_0)\delta_{ij}\right\}$$

to write the equilibrium condition as

$$G\left[\nabla^2 u + \left(\frac{1}{1-2\mu}\right)\frac{\partial}{\partial x}\left(\underset{\sim}{\nabla}\cdot\underset{\sim}{V}\right)\right] - 2G\left(\frac{1+\mu}{1-2\mu}\right)\alpha\frac{\partial T}{\partial x} + \rho g_x = 0$$

We can use this to analyze thermoelastic stress problems.

Curvilinear coordinate systems

To handle curved geometries (cylinders & spheres) we must transform:

- equilibrium condition
- strain tensor
- compatibility relation
- stress-strain relation

Cylindrical Coordinates:

Displacement vector: $\underset{\sim}{V}(u_r, u_\theta, u_z)$

Equilibrium conditions:

$$\frac{1}{r}\frac{\partial}{\partial r}(r\sigma_{rr}) + \frac{1}{r}\frac{\partial \sigma_{r\theta}}{\partial \theta} - \frac{1}{r}\sigma_{\theta\theta} + \frac{\partial \sigma_{rz}}{\partial z} + \rho g_r = 0$$

$$\frac{1}{r}\frac{\partial \sigma_{\theta\theta}}{\partial \theta} + \frac{\partial \sigma_{r\theta}}{\partial r} + \frac{2}{r}\sigma_{r\theta} + \frac{\partial \sigma_{\theta z}}{\partial z} + \rho g_\theta = 0$$

$$\frac{1}{r}\frac{\partial}{\partial r}(r\sigma_{rz}) + \frac{1}{r}\frac{\partial \sigma_{\theta z}}{\partial \theta} + \frac{\partial \sigma_{zz}}{\partial z} + \rho g_z = 0$$

Components of ϵ_{ij}:

$$\epsilon_{rr} = \frac{\partial u_r}{\partial r}, \quad \epsilon_{\theta\theta} = \frac{u_r}{r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta}, \quad \epsilon_{zz} = \frac{\partial u_z}{\partial z}$$

$$\epsilon_{r\theta} = \epsilon_{\theta r} = \frac{1}{r}\frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}$$

$$\epsilon_{rz} = \epsilon_{zr} = \frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}$$

$$\epsilon_{z\theta} = \epsilon_{\theta z} = \frac{\partial u_\theta}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial \theta}$$

Spherical coordinate systems

Displacement vector: $\underline{V}(u_r, u_\theta, u_\phi)$

Equilibrium conditions:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\sigma_{rr}\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left(\sigma_{r\theta}\sin\theta\right) + \frac{1}{r\sin\theta}\frac{\partial \sigma_{r\phi}}{\partial \phi} - \frac{\sigma_{\theta\theta} + \sigma_{\phi\phi}}{r} + \rho g_r = 0$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\sigma_{r\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left(\sigma_{\theta\theta}\sin\theta\right) + \frac{1}{r\sin\theta}\frac{\partial \sigma_{\theta\phi}}{\partial \phi} - \frac{\sigma_{r\theta}}{r} - \frac{\cot\theta}{r}\sigma_{\phi\phi} + \rho g_\theta = 0$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\sigma_{r\phi}\right) + \frac{1}{r}\frac{\partial \sigma_{\theta\phi}}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial \sigma_{\phi\phi}}{\partial \phi} + \frac{\sigma_{r\phi}}{r} + \frac{2\cot\theta}{r}\sigma_{\theta\phi} + \rho g_\phi = 0$$

Components of ϵ_{ij}:

$$\epsilon_{rr} = \frac{\partial u_r}{\partial r}, \quad \epsilon_{\theta\theta} = \frac{u_r}{r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta}, \quad \epsilon_{\phi\phi} = \frac{u_r}{r} + \frac{u_\theta}{r}\cot\theta + \frac{1}{r\sin\theta}\frac{\partial u_\phi}{\partial \phi}$$

$$\epsilon_{r\theta} = \epsilon_{\theta r} = \frac{1}{2}\left(\frac{1}{r}\frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}\right)$$

$$\epsilon_{r\phi} = \epsilon_{\phi r} = \frac{1}{2}\left(\frac{1}{r\sin\theta}\frac{\partial u_r}{\partial \phi} + \frac{\partial u_\phi}{\partial r} - \frac{u_\phi}{r}\right)$$

$$\epsilon_{\phi\theta} = \epsilon_{\theta\phi} = \frac{1}{2}\left(\frac{1}{r}\frac{\partial u_\phi}{\partial \theta} - \frac{\cot\theta}{r}u_\phi + \frac{1}{r\sin\theta}\frac{\partial u_\theta}{\partial \phi}\right)$$

LECTURE 36

THERMAL STRESS ANALYSIS IN SIMPLE GEOMETRIES -- I

READING ASSIGNMENT:

EXERCISES:

36-1. By referring back to your earlier notes, demonstrate that there are indeed 15 equations for 15 unknowns in thermal stress analysis. Write down these equations for rectangular coordinates.

36-2. Determine the stress, strain, and displacement in a flat place subject to a uniform heat source (e.g., a parabolic temperature distribution).

36-3. Provide the detail solution for the stress components in a spherical shell subject to a given radial temperature distribution.

36-4. Calculate the normal strain components and the radial displacement for a spherical shell subject to a given radial temperature distribution.

36-5. Calculate the normal stress, strain, and radial displacement for a solid sphere subject to a given radial temperature distribution.

Thermal Stress Analysis in Simple Geometries - I

- Outline of procedure
- Flat Plate with Plane Strain
- Spheres and Spherical Shells

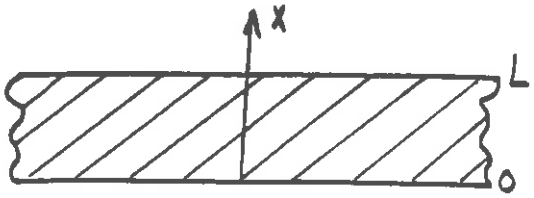
Basic Equations

Equilibrium conditions	3
Strain-displacement relations	6
Stress-strain relations	6
Boundary conditions	–
	15

Unknowns

Stresses $\underline{\underline{\sigma}}$ (symmetric)	6
Strains $\underline{\underline{\epsilon}}$ (symmetric)	6
Displacement $\underline{v}$	3
	15

Flat plate with plane strain



Assumptions: i) $T = T(x)$

ii) no displacement in y and z directions

$v = w = 0 \Rightarrow \epsilon_{yy} = \epsilon_{zz} = 0$ (plane strain)

iii) traction free surfaces

$\sigma_{xx}(0) = \sigma_{xx}(L) = 0$

Equilibrium conditions:

$$\frac{d\sigma_{xx}}{dx} = 0 \quad \Rightarrow \quad \sigma_{xx} = \text{constant} = 0$$

Strain-displacement condition:

$$\epsilon_{xx} = \frac{du}{dx}$$

Stress-strain relations:

$$\epsilon_{xx} = \cancel{\frac{\sigma_{xx}}{E}}^{0} - \frac{\mu}{E}\left(\sigma_{yy} + \sigma_{zz}\right) + \alpha\left(T - T_0\right)$$

$$\epsilon_{yy} = 0 = \frac{\sigma_{yy}}{E} - \frac{\mu}{E}\left(\cancel{\sigma_{xx}}^{0} + \sigma_{zz}\right) + \alpha\left(T - T_0\right)$$

$$\epsilon_{zz} = 0 = \frac{\sigma_{zz}}{E} - \frac{\mu}{E}\left(\cancel{\sigma_{xx}}^{0} + \sigma_{yy}\right) + \alpha\left(T - T_0\right)$$

We can solve the last two equations for

$$\sigma_{yy} = \sigma_{zz} = -\left(\frac{E\alpha}{1-\mu}\right)\left[T(x) - T_0\right]$$

We can combine the first two equations

$$\frac{du}{dx} = -\frac{2\mu}{E}\sigma_{yy} + \alpha(T - T_0)$$

$$= \left(\frac{1+\mu}{1-\mu}\right)\alpha(T - T_0)$$

Thus we can integrate to find

$$u(x) = \int_0^x \left(\frac{1+\mu}{1-\mu}\right)\alpha\left[T(x) - T_0\right] dx$$

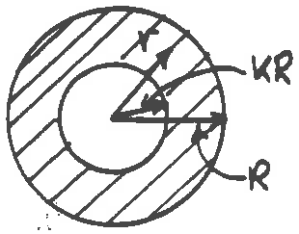
Hence we can write

$$\frac{\delta L}{L} = \frac{u(L)}{L} = \left(\frac{1+\mu}{1-\mu}\right)\alpha\left(\bar{T} - T_0\right)$$

where

$$\bar{T} = \frac{1}{L}\int_0^L T(x)\, dx$$

Sphere subject to radial temperature distribution



Assumptions:

i) $T = T(r)$

ii) $\sigma_{rr}(KR) = \sigma_{rr}(R) = 0$

(unrestrained inner and outer surfaces)

Note: symmetry in $\theta, \phi \Rightarrow$

$$\frac{\partial \sigma_{ij}}{\partial \theta} = \frac{\partial \sigma_{ij}}{\partial \phi} = 0$$

Also assume no applied shear stresses on surface. Hence since thermal stress cause no shear, $\sigma_{r\theta} = \sigma_{r\phi} = \sigma_{\theta\phi} = 0$

Equilibrium conditions:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \sigma_{rr}\right) - \frac{\sigma_{\theta\theta} + \sigma_{\phi\phi}}{r} = 0$$

$$\frac{1}{r \sin\theta}\frac{\partial}{\partial \theta}\left(\sigma_{\theta\theta} \sin\theta\right) - \frac{\cot\theta}{r}\sigma_{\phi\phi} = 0$$

Note the second equation reduces to

$$\frac{\cot\theta}{r}\sigma_{\theta\theta} - \frac{\cot\theta}{r}\sigma_{\phi\phi} = 0$$

$$\Rightarrow \qquad \sigma_{\theta\theta} = \sigma_{\phi\phi}$$

Strain-displacement:

Need only normal strain components

$$\epsilon_{rr} = \frac{du_r}{dr}$$

$$\epsilon_{\theta\theta} = \epsilon_{\phi\phi} = \frac{u_r}{r}$$

Only one compatibility relation is required

$$\frac{d}{dr}\epsilon_{\theta\theta} + \frac{\epsilon_{\theta\theta} - \epsilon_{rr}}{r} = 0$$

Stress-strain relations:

$$\epsilon_{rr} = \frac{\sigma_{rr}}{E} - \frac{\mu}{E}(\sigma_{\theta\theta} + \sigma_{\phi\phi}) + \alpha(T - T_0)$$

$$\epsilon_{\theta\theta} = \epsilon_{\phi\phi} = \frac{\sigma_{\theta\theta}}{E} - \frac{\mu}{E}(\sigma_{rr} + \sigma_{\phi\phi}) + \alpha(T - T_0)$$

We now have 6 equations and 6 unknowns:

$\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{\phi\phi}, \epsilon_{rr}, \epsilon_{\theta\theta}, u_r$

If we use $\sigma_{\theta\theta} = \sigma_{\phi\phi}$ we can simplify

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\sigma_{rr}\right) - \frac{2\sigma_{\theta\theta}}{r} = 0$$

$$\epsilon_{rr} = \frac{\sigma_{rr}}{E} - \frac{2\mu}{E}\sigma_{\theta\theta} + \alpha(T-T_0)$$

$$\epsilon_{\theta\theta} = \left(\frac{1-\mu}{E}\right)\sigma_{\theta\theta} - \frac{\mu}{E}\sigma_{rr} + \alpha(T-T_0)$$

We can eliminate to find

$$\epsilon_{rr} = \frac{\sigma_{rr}}{E} - \left(\frac{2\mu}{E}\right)\frac{1}{2r}\frac{d}{dr}\left(r^2\sigma_{rr}\right) + \alpha(T-T_0)$$

$$\epsilon_{\theta\theta} = \left(\frac{1-\mu}{E}\right)\frac{1}{2r}\frac{d}{dr}\left(r^2\sigma_{rr}\right) - \frac{\mu}{E}\sigma_{rr} + \alpha(T-T_0)$$

But from our strain-displacement equations

$$\frac{d}{dr}\epsilon_{\theta\theta} = \frac{1}{r}\frac{du_r}{dr} - \frac{u_r}{r^2} = \frac{1}{r}\epsilon_{rr} - \frac{1}{r}\epsilon_{\theta\theta}$$

or

$$r\frac{d\epsilon_{\theta\theta}}{dr} + \epsilon_{\theta\theta} = \epsilon_{rr}$$

or

$$\frac{d}{dr}\left(r\epsilon_{\theta\theta}\right) = \epsilon_{rr}$$

compatibility relation

If we now substitute our expressions for $\epsilon_{\theta\theta}$ and ϵ_{rr} into this, we find a differential equation for $\sigma_{rr}(r)$

$$\frac{1}{r^4}\frac{d}{dr}\left(r^4\frac{d\sigma_{rr}}{dr}\right)+\left(\frac{2\alpha E}{1-\mu}\right)\frac{1}{r}\frac{d}{dr}(T-T_0)=0$$

The solution to this equation subject to b.c:

$$\sigma_{rr}(kR)=\sigma_{rr}(R)=0$$

is given by

$$\sigma_{rr}(r)=\left(\frac{2\alpha E}{1-\mu}\right)\frac{1}{r^3}\left\{\frac{\left(\frac{r}{R}\right)^3-k^3}{1-k^3}\int_{kR}^{R}r^2(T-T_0)\,dr - \int_{kR}^{r}r^2(T-T_0)\,dr\right\}$$

We can use our earlier results to find

$$\sigma_{\phi\phi}(r)=\sigma_{\theta\theta}(r)=\left(\frac{2\alpha E}{1-\mu}\right)\frac{1}{r^3}\left\{\frac{\left(\frac{r}{R}\right)^3+\frac{1}{2}k^3}{1-k^3}\int_{kR}^{R}r^2(T-T_0)dr + \frac{1}{2}\int_{kR}^{r}r^2(T-T_0)dr-\frac{1}{2}r^3(T-T_0)\right\}$$

Finally, ϵ_{rr}, $\epsilon_{\theta\theta}=\epsilon_{\phi\phi}$, and u_r can be obtained from our earlier equations.

Note: We have also solved the problem for a solid sphere by letting $k\to 0$.

We now have the thermal stresses resulting from any type of radial temperature distribution in a sphere or spherical shell. Also have calculated the strain and displacement.

LECTURE 37

THERMAL STRESS ANALYSIS IN SIMPLE GEOMETRIES -- II

READING ASSIGNMENT:

EXERCISES:

37-1. Provide the details in the determination of the streses in an axially restrained cylindrical shell.

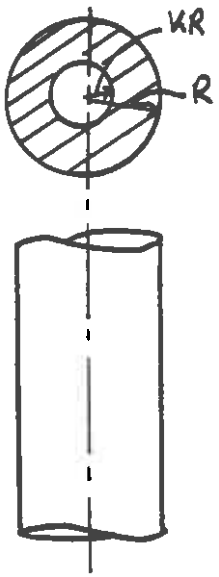
37-2. Provide the details in the determination of the stresses in an axially unrestrained cylindrical shell.

37-3. By applying St. Venant's principle, determine the strain and deformation on an unrestrained flat plate.

Thermal Stress Analysis in Simple Geometries - II

- Long Cylinders
- St. Venant's Principle
- Flat Plate -- Revisited

Thermal stress in a long cylinder



We will assume axial symmetry and zero applied shear stresses.

Equilibrium conditions:

$$\frac{\partial}{\partial r}(r\sigma_{rr}) - \sigma_{\theta\theta} = 0 \qquad \text{(radial)}$$

$$\frac{\partial}{\partial z}(\sigma_{zz}) = 0 \qquad \text{(axial)}$$

(Note this implies $\sigma_{zz} = \sigma_{zz}(r)$)

Strain-displacement:

$$\epsilon_{rr} = \frac{\partial u_r}{\partial r}$$

$$\epsilon_{\theta\theta} = \frac{u_r}{r}$$

$$\epsilon_{zz} = \frac{\partial u_z}{\partial z}$$

Just as in the case of spherical geometry, these imply a compatibility relation:

$$\frac{\partial}{\partial r}(r\epsilon_{\theta\theta}) = \epsilon_{rr}$$

Stress-strain relations:

$$\epsilon_{rr} = \frac{\sigma_{rr}}{E} - \frac{\mu}{E}(\sigma_{\theta\theta} + \sigma_{zz}) + \alpha(T - T_0)$$

$$\epsilon_{\theta\theta} = \frac{\sigma_{\theta\theta}}{E} - \frac{\mu}{E}(\sigma_{rr} + \sigma_{zz}) + \alpha(T - T_0)$$

$$\epsilon_{zz} = \frac{\sigma_{zz}}{E} - \frac{\mu}{E}(\sigma_{rr} + \sigma_{\theta\theta}) + \alpha(T - T_0)$$

Case I: Cylinder restrained axially

Then

$$\epsilon_{zz} = u_z = 0 \qquad \text{(plane strain)}$$

We can proceed as follows:

i) eliminate σ_{zz} in other two stress-strain relations

ii) use equilibrium condition to eliminate $\sigma_{\theta\theta}$ in equations for ϵ_{rr} and $\epsilon_{\theta\theta}$

iii) Use these equations to eliminate ϵ_{rr} and $\epsilon_{\theta\theta}$ in compatibility relation

This yields a differential equation

$$\frac{d^2\sigma_{rr}}{dr^2} + \frac{3}{r}\frac{d\sigma_{rr}}{dr} = \frac{1}{r^3}\frac{d}{dr}\left(r^3\frac{d\sigma_{rr}}{dr}\right) = -\left(\frac{\alpha E}{1-\mu}\right)\frac{1}{r}\frac{d}{dr}(T-T_0)$$

$$\text{b.c:} \quad \sigma_{rr}(kR) = \sigma_{rr}(R) = 0$$

We can integrate this subject to bc to find

$$\sigma_{rr} = \left(\frac{\alpha E}{1-\mu}\right)\frac{1}{r^2}\left\{\frac{\left(\frac{r}{R}\right)^2 - k^2}{1-k^2}\int_{kR}^{R} r(T-T_0)dr - \int_{kR}^{r} r(T-T_0)dr\right\}$$

Similarly

$$\sigma_{\theta\theta} = \left(\frac{\alpha E}{1-\mu}\right)\frac{1}{r^2}\left\{\frac{\left(\frac{r}{R}\right)^2 + k^2}{1-k^2}\int_{kR}^{R} r(T-T_0)dr + \int_{kR}^{r} r(T-T_0)dr - r^2(T-T_0)\right\}$$

Finally, we can use these relations to find

$$\sigma_{zz}(r) = \left(\frac{\alpha E}{1-\mu}\right)\left[\mu(\bar{T}-T_0) - (T-T_0)\right]$$

where $\bar{T}-T_0$ is the average temperature rise

$$\bar{T}-T_0 = \frac{2}{R^2(1-K^2)} \int_{KR}^{R} r(T-T_0)\,dr$$

Case II: Cylinder unrestrained axially

We can assume our solutions for $\sigma_{rr}(r)$ and $\sigma_{\theta\theta}(r)$ for Case I remain valid. Suppose we use our earlier result for $\sigma_{zz}(r)$ to calculate the average stress in the axial direction.

$$\bar{\sigma}_{zz} = \left(\frac{\alpha E}{1-\mu}\right)\left[\mu(\bar{T}-T_0) - (\bar{T}-T_0)\right]$$

$$= -\alpha E(\bar{T}-T_0)$$

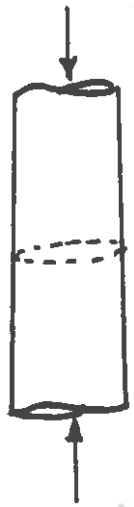
To proceed further, we use St. Venant's principle:

A system of forces acting over a small region of the boundary can be replaced by a statically equivalent system of forces without introducing appreciable changes in the distribution of stresses in regions well-removed from the area of load application.

Thus stress to be applied to ends of specimen to completely restrain it does not depend on radial variation of $\sigma_{zz}(r)$ near mid-plane.

Thus stress required for restraining specimen is

$$\bar{\sigma}_{zz} = -\alpha E (\bar{T} - T_0)$$



We can subtract $\sigma_{zz}(r)$ and $\bar{\sigma}_{zz}$ to get stress in unrestrained case

Thus

$$\sigma_{zz}(\text{unrestrained}) = \sigma_{zz}(\text{restrained}) - \bar{\sigma}_{zz}$$

$$= \frac{\alpha E}{1-\mu}(\bar{T}-T)$$

Note:

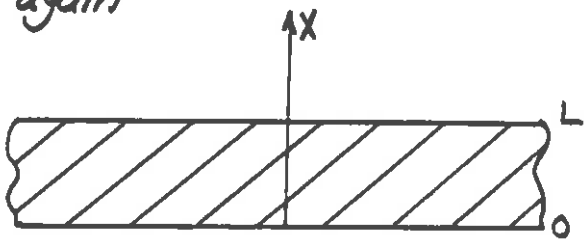
i) If an unrestrained specimen is subject to uniform heating ($T=\bar{T}=$ constant), the axial stresses are zero.

ii) This approximation is good for central section of long rods or pipes.

iii) Valid even for $T=T(z)$ -- only radial variation is important.

Flat plate — revisited

Consider again



Recall assumptions:

i) $T=T(x)$

ii) traction free surfaces

$$\sigma_{xx}(0) = \sigma_{xx}(L) = 0$$

Equilibrium conditions:

$$\frac{\partial \sigma_{xx}}{\partial x} = 0$$

Strain-displacement:

$$\epsilon_{xx} = \frac{du}{dx}$$

$$\epsilon_{yy} = \frac{dv}{dy}$$

$$\epsilon_{zz} = \frac{dw}{dz}$$

Stress-strain relations

$$\epsilon_{xx} = \frac{\sigma_{xx}}{E} - \frac{\mu}{E}(\sigma_{yy} + \sigma_{zz}) + \alpha(T - T_0)$$

$$\epsilon_{yy} = \frac{\sigma_{yy}}{E} - \frac{\mu}{E}(\sigma_{xx} + \sigma_{zz}) + \alpha(T - T_0)$$

$$\epsilon_{zz} = \frac{\sigma_{zz}}{E} - \frac{\mu}{E}(\sigma_{yy} + \sigma_{xx}) + \alpha(T - T_0)$$

Compatibility relation:

$$\frac{\partial^2}{\partial x^2}\epsilon_{yy} = 0 \quad , \quad \epsilon_{yy} = \epsilon_{zz}$$

Now

$$\frac{\partial \sigma_{xx}}{\partial x} = 0 \Rightarrow \sigma_{xx} = \text{const} = 0 \quad (\text{since } \sigma_{xx}(0) = \sigma_{xx}(L) = 0)$$

From stress-strain relations, since $\sigma_{xx}=0$, we get

$\sigma_{yy} = \sigma_{zz}$ and

$$\epsilon_{yy} = \epsilon_{zz} = \frac{\sigma_{yy}}{E}(1-\mu)+\alpha(T-T_0)$$

$$= \frac{\sigma_{zz}}{E}(1-\mu)+\alpha(T-T_0)$$

Hence

$$\frac{\partial^2}{\partial x^2}\epsilon_{yy} = \frac{d^2}{dx^2}\left[\frac{\sigma_{yy}}{E}(1-\mu)+\alpha(T-T_0)\right]=0$$

or

$$\sigma_{yy}(x) = -\left(\frac{\alpha E}{1-\mu}\right)(T-T_0)+Ax+B$$

We would like to set $\sigma_{yy}=0$ on edges of plate; however this is impossible unless $(T-T_0)$ is linear in x. However by St. Venant's principle, we can set average force and average moment on edge of plate equal to zero:

bc: $$\int_0^L \sigma_{yy}\,dx = \int_0^L x\,\sigma_{yy}\,dx = 0$$

This yields

$$\sigma_{yy}=\sigma_{zz}=\left(\frac{\alpha E}{1-\mu}\right)\left\{-(T-T_0)+(12\eta-6)\int_0^1 \eta(T-T_0)\,d\eta+(4-6\eta)\int_0^1 (T-T_0)\,d\eta\right\}$$

where $\eta = x/L$

LECTURE 38

ADDITIONAL TOPICS IN THERMAL STRESS ANALYSIS

READING ASSIGNMENT:

J. Weisman, "Elements of Stress Analysis", Chapter 9 in Weisman (Design)

EXERCISES:

38-1. Provide the detailed solution for the stresses in a spherical shell subject to a pressure differential.

38-2. Provide the detailed solution for the stresses in a cylindrical shell subject to a pressure differential.

38-3. Determine the maximum axial, radial, and tangential stresses in the cladding of a PWR fuel rod of 0.37 ID, 0.023 thickness, and 12 ft. length. The stresses will be due to fission product gas buildup and thermal expansion. The maximum linear heat rate is 15 kW/ft and the cladding is zircaloy with a thermal conductivity of 7.4 BTU/hr-ft-F. The fuel enrichment is 3%. Assume the reactor operating time is 10,000 hours and 0.5 gas atoms are produced per fission and one-half the gas atoms are released to a volume of 1.6 cu. in. Assume the gas temperature is 1000 F and $aE/k(1 - v)$ is 4.6. The fuel rod is initially filled with helium at 50 psi and room temperature. Assume a cosine power shape and an external pressure of 2300 psia.

Additional Topics in Thermal Stress Analysis

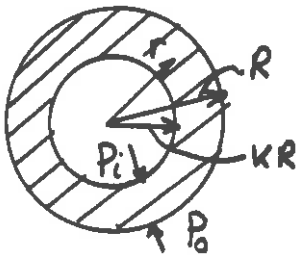
- Hydrostatic stresses
- Stress Wave Propagation
- Creep

In addition to the thermal stresses due to temperature non-uniformities, hydrostatic pressure differences across spherical or cylindrical shells may be present.

In elastic stress theory we can superimpose

total stresses = thermal stresses (no Δp)
+ hydrostatic stresses (no ΔT)

Example: Sphere



We can start with the differential equation we developed in thermoelastic stress theory -- taking $T - T_0 = 0$

$$\frac{1}{r^4}\frac{d}{dr}\left(r^4 \frac{d\sigma_{rr}}{dr}\right) = 0$$

bc: $\sigma_{rr}(KR) = -P_i$

$\sigma_{rr}(R) = -P_o$

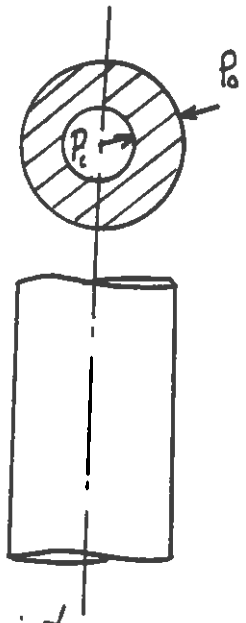
We can easily integrate this and apply the bc to find

$$\sigma_{rr}(r) = -P_o - \left[\frac{P_i - P_o}{\frac{1}{K^3} - 1}\right]\left[\left(\frac{R}{r}\right)^3 - 1\right]$$

Substituting σ_{rr} into the equilibrium condition yields

$$\sigma_{\theta\theta} = \sigma_{\phi\phi} = -P_o + \left[\frac{P_i - P_o}{\frac{1}{K^3} - 1}\right]\left[1 + \frac{1}{2}\left(\frac{R}{r}\right)^3\right]$$

Example: Cylinder



We will just give the solutions:

$$\sigma_{rr} = -P_o - \left[\frac{P_i - P_o}{\frac{1}{K^2} - 1}\right]\left[\left(\frac{R}{r}\right)^2 - 1\right]$$

$$\sigma_{\theta\theta} = -P_o + \left[\frac{P_i - P_o}{\frac{1}{K^2} - 1}\right]\left[\left(\frac{R}{r}\right)^2 + 1\right]$$

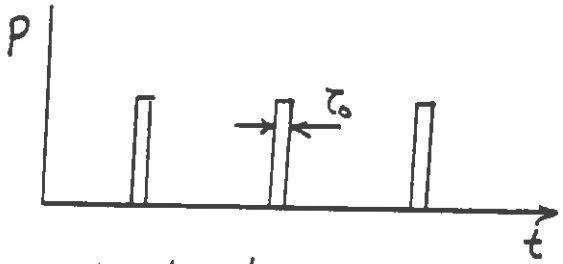
Also

$$\sigma_{zz} = 2\mu\left[-P_o + \frac{P_i - P_o}{\frac{1}{K^2} - 1}\right] \quad \text{complete restraint}$$

$$= 0 \quad \text{unrestrained}$$

Stress Wave Propagation

Consider a pulsed reactor where the power as a function of time is as follows:



Pulsing frequency 60–100 cps

Here, τ_o = heating time. If

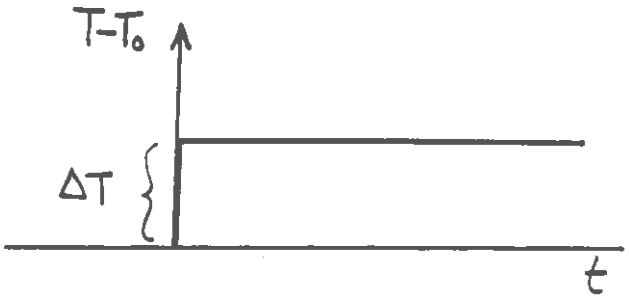
$$\tau_o \lesssim \tau_M \quad , \quad \tau_M = \frac{L}{c} \equiv \text{mechanical time}$$

stress wave propagation becomes important.

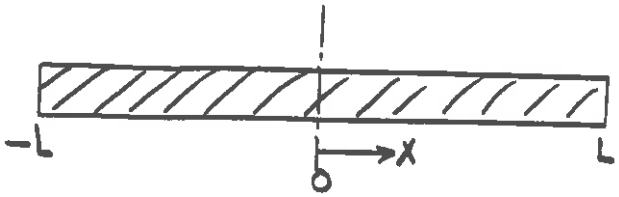
Consider a fuel rod which is heated very rapidly

$$\tau_0 \ll \tau_M$$

Then



Goal: Consider stress in rod due to a step change in temperature



Basic Equations:

i) $\underset{\sim}{F} = m\underset{\sim}{a}$: $\quad \frac{\partial \sigma_{xx}}{\partial x} = \rho \frac{\partial^2 u}{\partial t^2}$

ii) Strain-displacement: $\quad \epsilon_{xx} = \frac{\partial u}{\partial x}$

iii) Stress-strain: $\quad \epsilon_{xx} = \frac{\sigma_{xx}}{E} + \alpha (T - T_0)$

iv) Boundary and initial conditions:

$$\sigma_{xx}(-x,t) = \sigma_{xx}(x,t)$$

$$\sigma_{xx}(L,t) = 0$$

$$\sigma_{xx}(x,0) = \frac{\partial \sigma_{xx}}{\partial t}(x,0) = 0$$

$$T - T_0(0) = \frac{d}{dt}\, T - T_0(0) = 0$$

Now to manipulate:

$$\frac{\partial}{\partial x}\left[\frac{\partial \sigma_{xx}}{\partial x} = \rho \frac{\partial^2 u}{\partial t^2}\right] \Rightarrow \frac{\partial^2 \sigma_{xx}}{\partial x^2} = \rho \frac{\partial^3 u}{\partial x \partial t^2}$$

Combine

$$\frac{\partial u}{\partial x} = \epsilon_{xx} = \frac{\sigma_{xx}}{E} + \alpha(T - T_0)$$

Hence

$$\frac{\partial^3 u}{\partial x \partial t^2} = \frac{1}{E}\frac{\partial^2 \sigma_{xx}}{\partial t^2} + \alpha \frac{d^2}{dt^2}(T - T_0)$$

or

$$\underbrace{\frac{\partial^2 \sigma_{xx}}{\partial x^2} = \frac{\rho}{E}\frac{\partial^2 \sigma_{xx}}{\partial t^2}}_{\text{wave equation with } c^2 = E/\rho = (\text{sound speed})^2} + \alpha\rho \frac{d^2}{dt^2}(T - T_0)$$

Solution: Use Laplace transform

$$\tilde{\sigma}_{xx}(s) = \int_0^\infty e^{-st}\sigma_{xx}(t)\,dt$$

Then our Laplace transformed equation becomes

$$\frac{d^2\tilde{\sigma}_{xx}}{dx^2} = \frac{\rho}{E}s^2\tilde{\sigma} + \alpha\rho s^2\,(\widetilde{T-T_0})$$

But $$(\widetilde{T-T_0}) = \frac{\Delta T}{s}\,, \quad \frac{\rho}{E} = c^2$$

Thus

$$\frac{d^2\tilde{\sigma}_{xx}}{dx^2} - \frac{s^2}{c^2}\tilde{\sigma}_{xx} = \alpha\rho s\,\Delta T$$

The transformed bc's are:

$$\tilde{\sigma}_{xx}(-x,s) = \tilde{\sigma}_{xx}(x,s)$$

$$\tilde{\sigma}_{xx}(L,s) = 0$$

We find a general solution:

$$\tilde{\sigma}_{xx} = A\cosh\frac{s}{c}x + B\sinh\frac{s}{c}x - \alpha\rho c^2\frac{\Delta T}{s}$$

Bc's imply $B=0$, $A = \dfrac{\alpha\rho c^2\frac{\Delta T}{s}}{\cosh\frac{s}{c}L}$

Thus

$$\tilde{\sigma}_{xx}(x,s) = \frac{\alpha E \Delta T}{s}\left[\frac{\cosh \frac{s}{c}x}{\cosh \frac{s}{c}L} - 1\right]$$

Manipulate:

$$\frac{\tilde{\sigma}_{xx}(x,s)}{\sigma_0} = -\frac{1}{s} + \frac{1}{s}\frac{\left(e^{\frac{s}{c}x} + e^{-\frac{s}{c}x}\right)}{\left(e^{\frac{s}{c}L} + e^{-\frac{s}{c}L}\right)}$$

$$= -\frac{1}{s} + \frac{1}{s}\frac{\left(e^{\frac{s}{c}(x-L)} + e^{-\frac{s}{c}(x+L)}\right)}{\left(1 + e^{-2\frac{s}{c}L}\right)}$$

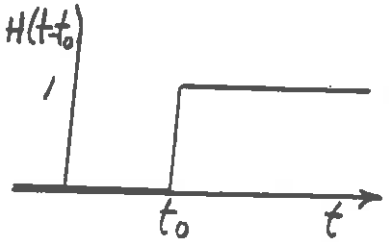
$$= -\frac{1}{s} + \sum_{n=0}^{\infty} \frac{(-1)^n}{s}\left\{e^{\frac{s}{c}(x-L-2nL)} + e^{\frac{s}{c}(-x-L-2nL)}\right\}$$

We can do a term-by-term inversion

$$\frac{\sigma_{xx}(x,t)}{\sigma_0} = -1 + \sum_{n=0}^{\infty}(-1)^n\left\{H\left[t - \left(\frac{-x+L+2nL}{c}\right)\right] + H\left[t - \left(\frac{x+L+2nL}{c}\right)\right]\right\}$$

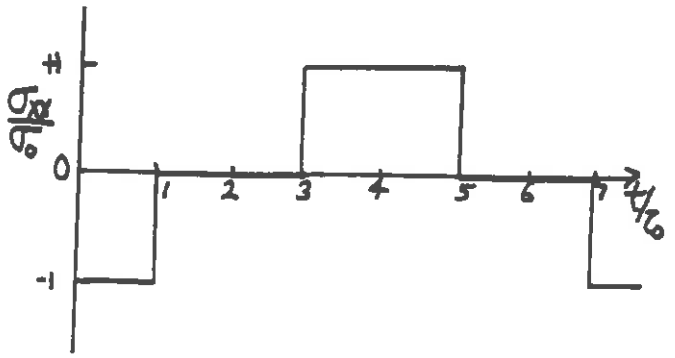
where $H(t)$ is the Heaviside step function

$$H(t-t_0) = \begin{cases} 0 & t < t_0 \\ 1 & t \geq t_0 \end{cases}$$



Consider $x = \frac{L}{2}$

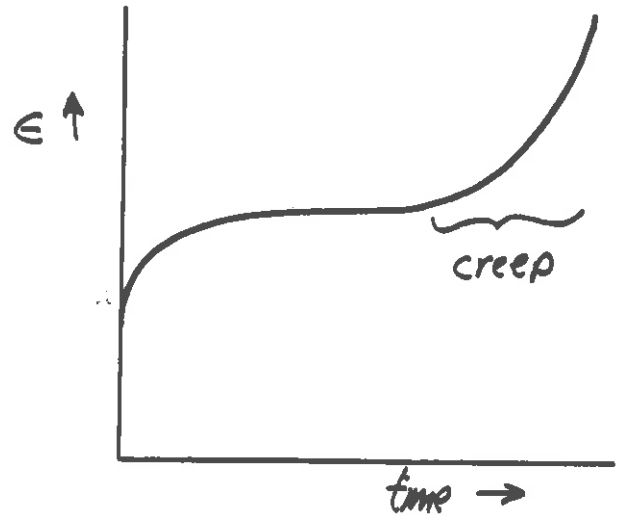
t	σ_{xx}/σ_0
$0 < t < \frac{L}{2c}$	-1
$\frac{L}{2c} < t < \frac{3L}{2c}$	0
$\frac{3L}{2c} < t < \frac{5L}{2c}$	$+1$
$\frac{5L}{2c} < t < \frac{7L}{2c}$	0
$\frac{7L}{2c} < t < \frac{9L}{2c}$	-1



Creep

A reminder: At room temperatures, structural materials develop the full strain they will exhibit as soon as such a load is applied.

At high temperatures, many materials will continue to deform at a slow rate as long as load is applied -- creep



Rate of creep is highly dependent on both stress and temperature:

$$\epsilon = A\sigma^n t$$

where A and n depend on material and temperature.

Note: analysis of creep requires viscoelastic theory (not just thermoelastic theory)

LECTURE 39

TRANSIENT MODELS OF REACTOR CORE COOLING

READING ASSIGNMENT:

Lewis, pp. 212 - 215, 265 - 288

EXERCISES:

39-1. Assuming that the mass flow rate and core inlet temperature remain constant, derive an equation for the time dependence of the coolant temperature rise through the core during a power transient. Also determine the initial equation for this equation under the assumption that the reactor has been operating at steady state prior to time t = Ø.

39-2. Solve the equation obtained in problem 39-1 to model the coolant outlet temperature behavior following a reactor trip at time t = Ø. Assume that the inlet temperature remains at its initial value.

39-3. Determine the outlet coolant temperature transient resulting from a power transient of the form:

$$P(t) = Po\ [1 + Ø.25\ t]$$

Transient Models of Reactor Core Cooling

- Lumped Parameter Model of Core
- Power Transients
- Thermal Time Constants
- Single Parameter Models

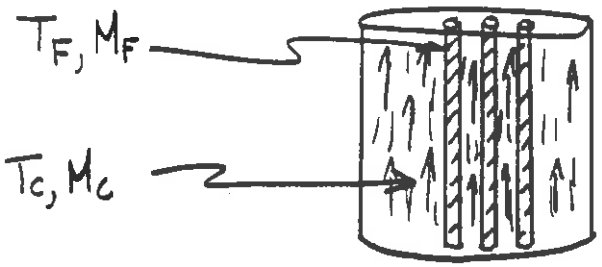
Core heat transport model:

Use two energy balances:

energy contained in fuel elements

energy contained in coolant (within core volume)

We will develop a lumped parameter model:



Fuel element energy balance:

$$M_F C_F \frac{dT_F}{dt} = P(t) - P_1(t)$$

- M_F: total fuel mass
- C_F: fuel specific heat
- $P(t)$: reactor power
- $P_1(t)$: energy transport rate across clad to coolant

Coolant energy balance:

$$M_c C_p \frac{dT_c}{dt} = P_1(t) - P_2(t)$$

- M_c: total coolant mass
- C_p: coolant specific heat
- $P_1(t)$: energy transport rate across clad to coolant
- $P_2(t)$: convective heat transport out of core

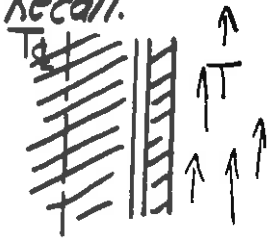
How do we model the heat transfer rates?

Fuel-to-coolant

$$P_1(t) = \frac{1}{R}\left[T_F(t) - T_c(t)\right]$$

$R \equiv$ core thermal resistance

Recall:



$$T_\ell - T_{FL} = \frac{q'}{2\pi r_F}\left[\frac{r_F}{2k_F} + \frac{1}{h_G} + \frac{t_c}{k_c} + \frac{r_F}{h_s(r_F + t_c)}\right]$$

$$\equiv q' R'$$

Hence the effective thermal resistance for a fuel element is

$$R' \equiv \frac{1}{2\pi r_F}\left[\frac{r_F}{2k_F} + \frac{1}{h_G} + \frac{t_c}{k_c} + \frac{r_F}{h_s(r_F+t_c)}\right]$$

The thermal resistance for a reactor core of height H with N fuel elements is then

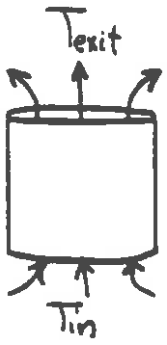
$$R = \frac{R'}{NH} = \frac{1}{2\pi r_F NH}\left[\frac{r_F}{2k_F} + \frac{1}{h_G} + \frac{t_c}{k_c} + \frac{r_F}{h_s(r_F+t_c)}\right]$$

Convective heat transfer out of core:

$$P_2(t) = W(t)\, c_p \left[T_{exit}(t) - T_{in}\right]$$

$W(t)$ = total core mass flow rate

T_{in}, T_{exit} = inlet, exit temperatures



Note: $T_c = \frac{T_{in} + T_{exit}}{2}$

$$T_{exit} - T_{in} = (2T_c - T_{in}) - T_{in} = 2(T_c - T_{in})$$

Thus

$$P_2(t) = 2W(t)\, c_p \left[T_c(t) - T_{in}\right]$$

Summary of model:

$$M_F C_F \frac{dT_F}{dt} = P(t) - P_1(t)$$

$$M_c C_p \frac{dT_c}{dt} = P_1(t) - P_2(t)$$

$$P_1(t) = \frac{1}{R}\left[T_F(t) - T_c(t)\right]$$

$$P_2(t) = 2W(t) C_p \left[T_c(t) - T_{in}(t)\right]$$

We can substitute in to find

$$M_F C_F \frac{dT_F}{dt} = P(t) - \frac{1}{R}\left[T_F(t) - T_c(t)\right]$$

$$M_c C_p \frac{dT_c}{dt} = \frac{1}{R}\left[T_F(t) - T_c(t)\right] - 2W(t) C_p \left[T_c(t) - T_{in}(t)\right]$$

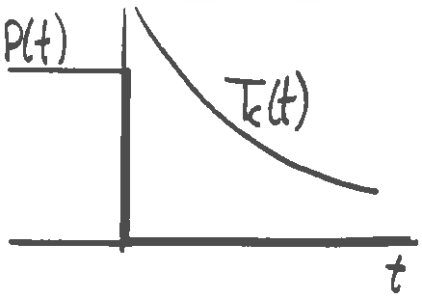
Note we can write

$$\frac{dT_F}{dt} = -\frac{1}{\tau}\left[T_F(t) - T_c(t)\right] + \frac{P(t)}{M_F C_F}$$

$$\tau \equiv R M_F C_F = \text{core time constant}$$

Example: Rough Simulation

$P(t) = 0, \quad t > 0$



Then we can solve

$$\frac{dT_F}{dt} = -\frac{1}{\tau}\left[T_F(t) - T_c\right]$$

to find

$$T_F(t) = T_c + \left[T_F(0) - T_c\right] e^{-t/\tau}$$

Typical Core Thermal Time Constants

	τ [sec]
PWR	4.1
BWR	7.1
HTGR	17.3
LMFBR	2.8
GCFR	4.0

Example: i) $M_F C_F >> M_c C_p$

ii) $T_F - T_c >> T_c - T_{in}$

Then $M_c C_p \frac{dT_c}{dt} = \frac{1}{R}[T_F(t) - T_c(t)] - 2W(t) C_p [T_c(t) - T_{in}(t)]$

$\Rightarrow T_F - T_c \sim 2W C_p R [T_c - T_{in}]$

$\Rightarrow 2W C_p R >> 1$

Hence we can uncouple equations

$$\frac{dT_F}{dt} = \frac{P(t)}{M_F C_F} - \frac{1}{\tau}[T_F(t) - T_{in}(t)]$$

$$\frac{d}{dt}[T_c(t) - T_{in}(t)] = \frac{P(t)}{2W(t) C_p \tau} - \left(\frac{1}{\tau} + \frac{1}{W}\frac{dW}{dt}\right)[T_c(t) - T_{in}(t)]$$

Note: core time constant τ dominates transient heat transport behavior <u>unless</u> there is a flow transient in progress that causes

$$\frac{1}{W}\frac{dW}{dt} \gtrsim \frac{1}{\tau}$$

Examples:

i) $\frac{1}{P}\frac{dP}{dt} << \frac{1}{\tau}$ (slow transients) $\Rightarrow$ steady state analysis

ii) $\frac{1}{P}\frac{dP}{dt} >> \frac{1}{\tau}$ (fast transients) $\Rightarrow$ adiabatic, prompt reactivity

iii) $\frac{1}{P}\frac{dP}{dt} \sim \frac{1}{\tau}$ $\Rightarrow$ more detailed analysis

must consider time dependence of

coolant inlet $T_{in}(t)$

flow transients $W(t)$

heat transfer $R(t)$

	PWR	BWR	HTGR	LMFBR	GCFR
q''' [kW/ℓ]	75	52	6.3	578	280
q' [kW/ft]	5.8	7.1	1.0	9.6	12.7
$M_F C_F$ [BTU/°F-ft]	0.044	0.079	0.196	0.019	0.029
h_s [BTU/hr-ft²-°F]	5300	5300	230	30,000	1880
τ core [sec]	4.1	7.1	17.3	2.8	4.0
τ' clad [sec]	0.63	1.10	2.94	0.50	0.59
$\frac{q'}{M_F C_F}$ adiabatic heat-up	122	84	5.0	472	409

Let's return to our equation for the core-averaged fuel element behavior

$$\frac{dT_{FE}}{dt} = \frac{P(t)}{M_{FE}C_{FE}} - \frac{1}{\tau}\left[T_{FE}(t) - T_c(t)\right]$$

We can convert this to a unit length basis

$$M_{FE} = NH\left(M'_F + M'_{CL}\right)$$

$$M'_F = \pi r_F^2 \rho_F$$

$$M'_{CL} = \pi\left(r_{CI}^2 - r_F^2\right)\rho_{CI}$$

$$C_{FE} = \frac{M'_F C_F + M'_{CL} C_{CL}}{M'_F + M'_{CL}}$$

Also recall

$$R'_{FE} = NHR$$

so that

$$\tau = R'_{FE}\left(M'_F C_F + M'_{CL} C_{CL}\right)$$

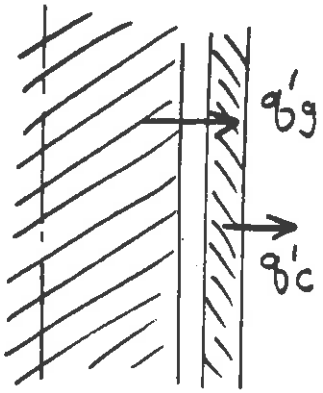
Thus we can write

$$\frac{dT_{FE}}{dt} = \frac{q'_0(t)}{M'_{FE}C_{FE}} - \frac{1}{\tau}\left[T_{FE}(t) - T_c(t)\right]$$

We can extend such models to treat the fuel and clad separately:

$$M_F' C_F \frac{dT_F}{dt} = q'(t) - q_g'(t)$$

$$M_{CL}' C_{CL} \frac{dT_{CL}}{dt} = q_g'(t) - q_c'(t)$$



Example: i) $M_F C_F >> M_c C_p$

ii) $T_F - T_c >> T_c - T_{in}$

Then $M_c C_p \cancel{\frac{dT_c}{dt}}^{0} = \frac{1}{R}\left[T_F(t) - T_c(t)\right] - 2W(t) C_p \left[T_c(t) - T_{in}(t)\right]$

$\Rightarrow \quad T_F - T_c \sim 2W C_p R \left[T_c - T_{in}\right]$

$\Rightarrow \quad 2W C_p R >> 1$

Hence we can uncouple equations

$$\frac{dT_F}{dt} = \frac{P(t)}{M_F C_F} - \frac{1}{\tau}\left[T_F(t) - T_{in}(t)\right]$$

$$\frac{d}{dt}\left[T_c(t) - T_{in}(t)\right] = \frac{P(t)}{2W(t) C_p \tau} - \left(\frac{1}{\tau} + \frac{1}{W}\frac{dW}{dt}\right)\left[T_c(t) - T_{in}(t)\right]$$

Note: core time constant τ dominates transient heat transport behavior <u>unless</u> there is a flow transient in progress that causes

$$\frac{1}{W}\frac{dW}{dt} \gtrsim \frac{1}{\tau}$$

Examples:

i) $\frac{1}{p}\frac{dp}{dt} << \frac{1}{\tau}$ (slow transients) $\Rightarrow$ steady state analysis

ii) $\frac{1}{p}\frac{dp}{dt} >> \frac{1}{\tau}$ (fast transients) $\Rightarrow$ adiabatic, prompt reactivity

iii) $\frac{1}{p}\frac{dp}{dt} \sim \frac{1}{\tau}$ $\Rightarrow$ more detailed analysis

must consider time dependence of

coolant inlet $T_{in}(t)$

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τ core [sec]	4.1	7.1	17.3	2.8	4.0
τ' clad [sec]	0.63	1.10	2.44	0.50	0.59
$\frac{q'}{M_F C_F}$ adiabatic heat-up	122	84	5.0	472	409

LECTURE 40

MODELING OF REACTOR SAFETY TRANSIENTS

READING ASSIGNMENT:

Lewis, pp. 288 - 292, Chapter 7
J. Weisman, "Loss-of-Cooling Accidents", Chapter 13 in Weisman (Design)
K. S. Ram, "Reactivity Insertion Accidents", Chapter 14 in Weisman (Design)
J. G. Collier, "Thermal Hydraulic Considerations in Safety Analysis", Chapter 3 in Jones

EXERCISES:

40-1. Using the fuel element heat transfer analysis of Lectures 14 and 15, derive expressions for the linear thermal resistivities R' characterizing heat transfer from the fuel to the clad and from the clad to the coolant.

40-2. Consider a reactor core subject to a complete coolant failure at $t = 0$ (i.e., $hc = 0$ for $t > 0$), but in which the power remains at its full value $q'(0)$. Using the two-lumped-parameter model, derive an analytical expression for the temperature dependence of the fuel and clad temperature. For typical PWR conditions, estimate how long after failure the cladding would reach the failure temperature of 1600 F.

40-3. Consider an incompressible coolant flowing through a channel of length L and area A with mass flow rate $w(0)$. A partial flow blockage at the channel entrance causes the pressure drop to fall instantaneously to one-half of its initial value. Assuming that the friction fraction, fluid density, and hydraulic diameter remain constant, determine the mass flow rate $w(t)$ as a function of time following the blockage.

Modeling of Reactor Safety Transients

- Two-Parameter Models
- Overpower Transients
- Cooling Failure transients
- Cooling Failure Accidents

Single Parameter Model:

$$\frac{dT_{FE}}{dt} = \frac{q'(t)}{M'_{FE} C_{FE}} - \frac{1}{\tau}\left[T_{FE}(t) - T_c(t)\right]$$

T_{FE} average fuel element temperature

M'_{FE} mass per unit length

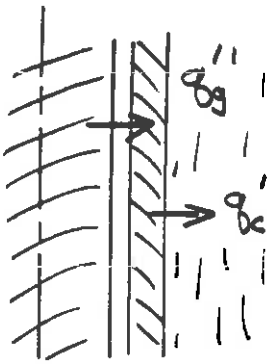
C_{FE} specific heat

$\tau \equiv R M_{FE} C_{FE}$ core time constant

R core thermal resistance

Two-Lumped Parameter Model:

We can treat fuel and clad seperately:



$$M'_F C_F \frac{dT_F}{dt} = q'(t) - q'_g(t)$$

$$M'_{CL} C_{CL} \frac{dT_{CL}}{dt} = q'_g(t) - q'_c(t)$$

We can take

$$q'_g(t) = \frac{1}{R'_g}\left[T_F(t) - T_{CL}(t)\right]$$

$$q'_c(t) = \frac{1}{R'_c}\left[T_{CL}(t) - T_c(t)\right]$$

Substituting in we find

$$M_F' C_F \frac{dT_F}{dt} = q'(t) - \frac{1}{R'_g}\left[T_F(t) - T_{CL}(t)\right]$$

$$M'_{CL} C_{CL} \frac{dT_{CL}}{dt} = \frac{1}{R'_g}\left[T_F(t) - T_{CL}(t)\right] - \frac{1}{R'_c}\left[T_{CL}(t) - T_c(t)\right]$$

or

$$\frac{dT_F}{dt} = \frac{q'(t)}{M_F' C_F} - \frac{1}{\tau_1}\left[T_F(t) - T_{CL}(t)\right]$$

$$\frac{dT_{CL}}{dt} = \frac{1}{\tau_2}\left[T_F(t) - T_{CL}(t)\right] - \frac{1}{\tau_3}\left[T_{CL}(t) - T_c(t)\right]$$

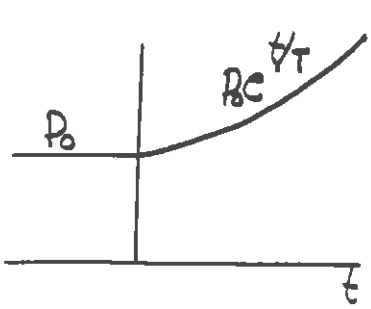
where the time constants are

$$\tau_1 = M_F' C_F R_g'$$

$$\tau_2 = M'_{CL} C_{CL} R_g'$$

$$\tau_3 = M'_{CL} C_{CL} R'_c$$

Example: Overpower transients



We will assume

$$q'(t) = q'_0 e^{t/T}$$

and the coolant temperature remains constant at T_c.

Seek

$$T_F(t) \sim T_c + A_F e^{t/T}$$

$$T_{CL}(t) \sim T_c + A_{CL} e^{t/T}$$

We can substitute these in and solve for A_F and A_C to find

$$T_F(t) = T_c + \frac{\left[1 + \left(\frac{1}{\tau_2} + \frac{1}{\tau_3}\right)T\right]}{\left[1 + \left(\frac{1}{\tau_1} + \frac{1}{\tau_2} + \frac{1}{\tau_3}\right)T + \frac{T^2}{\tau_2\tau_3}\right]} \frac{1}{M'_F C_F} q'_0 e^{t/T}$$

$$T_{CL}(t) = T_c + \frac{1}{\left[1 + \left(\frac{1}{\tau_1} + \frac{1}{\tau_2} + \frac{1}{\tau_3}\right)T + \frac{T^2}{\tau_1\tau_3}\right]} \frac{T^2}{\tau_2 M'_F C_F} q'_0 e^{t/T}$$

Case I: mild overpower transients $T >> \tau_1, \tau_2, \tau_3$

$$T_F(t) \sim T_c + \frac{(\tau_2 + \tau_3)}{M_F' C_F} q'_0 e^{t/T}$$

$$T_{CL}(t) \sim T_c + \frac{\tau_1\tau_3}{\tau_2 M'_F C_F} q'_0 e^{t/T}$$

Hence if we use our definitions of the time constants

$$T_F(t) \sim T_c + (R_g' + R_c') \frac{M_{CL}' C_{CL}}{M_F' C_F} q'(t)$$

$$T_{CL}(t) \sim T_c + R_c' q'(t)$$

Case II: super prompt critical $T \ll \tau_1, \tau_2, \tau_3$

$$T_F(t) \sim T_c + \frac{T}{M_F' C_F} q_0' e^{t/T}$$

$$T_{CL}(t) \sim T_c + \frac{T^2}{\tau_2 M_F' C_F} q_0' e^{t/T}$$

Recall

$$q'(t) = q_0' e^{t/\tau}$$

so that

$$\int_0^t q'(t)\,dt \sim T q_0' e^{t/T}$$

Thus when $t \gg T$

$$T_F(t) \sim T_c + \frac{1}{M_F' C_F} \int_0^t q'(t)\,dt$$

$$T_a(t) \sim T_c + \left(\frac{T}{\tau_2}\right) \frac{1}{M_F' C_F} \int_0^t q'(t)\,dt$$

Hence if $T \ll \tau_2$, fuel behaves adiabatically, and only a small fraction, T/τ_2, of heat is stored in clad.

Cooling Failure Accidents

loss of coolant accidents: primary cooling system is breached, and some or all of coolant escapes

coolant transients: coolant inventory is maintained, but heat transport from core is disrupted

Coolant transients: For slow changes, effects of core and coolant heat capacities can be ignored, and we can use static relationships:

$$T_{FE}(t) = T_{in}(t) + \left[R + \frac{1}{2W(t)C_p}\right] P(t)$$

$$T_{exit}(t) = T_{in}(t) + \frac{1}{W(t)C_p} P(t)$$

We can model flow failures by writing mass flow rate W as

$$W = A_T \left[\frac{2\rho_c \Delta p}{K} \right]^{1/2}$$

A_T flow area

Δp core pressure drop

K loss coefficient

We can substitute this into our earlier expressions:

$$T_{FE}(t) = T_{in}(t) + \left[R + \frac{1}{2A_T c_p} \sqrt{\frac{K}{2\rho_c \Delta p}} \right] P(t)$$

$$T_{exit}(t) = T_{in}(t) + \frac{1}{A_T c_p} \sqrt{\frac{K}{2\rho_c \Delta p}} \, P(t)$$

Examples: coolant pump failure $\rightarrow \Delta p \downarrow$

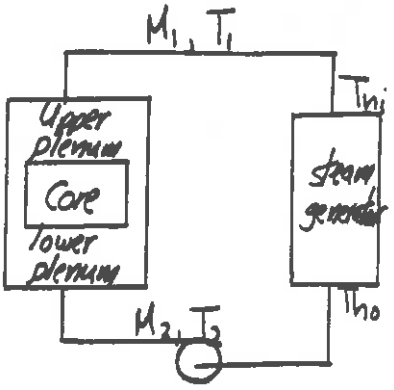
flow starvation $\rightarrow K \uparrow$ or $A_T \downarrow$

loss-of-heat-sink $\rightarrow T_{in} \uparrow$

Note: Most flow and heat-sink failures require a full time-dependent analysis

Such a lumped parameter model can be applied to a wide variety of accident situations

Example: loss-of-heat-sink



upper plenum and hot leg:

$$M_1 c_p \frac{dT_1}{dt} = W c_p \left[T_o(t) - T_{hi}(t)\right]$$

lower plenum and cold leg:

$$M_2 c_p \frac{dT_2}{dt} = W c_p \left[T_{ho}(t) - T_i(t)\right]$$

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